A Lagrangian-Informed Long-Term Dispatch Policy for Coupled Hydropower and Photovoltaic Systems

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***Abstract*—Large-scale deployment of floating photovoltaic (FPV) cells on hydropower reservoirs offers a promising approach to integrating more renewable energy into the power grid. However, due to the limiting capacity of the transmission line at the reservoir, the scheduling of water releases must be coordinated with solar power generation to maximize the revenue from electricity generation. This paper presents a multi-period optimization framework to manage the energy dispatch of a coupled hydropower-FPV system. We introduce a temporal decomposition algorithm based on partial Lagrangian relaxation to address long-term water contract constraints. We derive a real-time, non-anticipatory control policy based on water contract pricing, suitable for practical implementation. Our framework is evaluated with a case study using real-world hydrology and power system data from Lake Mead and Lake Powell, on the Colorado River, demonstrating competitive performance against commercial solvers for both linearized and nonlinear reservoir models. We conduct a sensitivity analysis on transmission constraints and water contract pricing, showing that contract prices are significantly influenced by the transmission capacity and electricity prices. Our findings offer insights to inform future reservoir operation strategies.**

***Index Terms*—Hydropower, floating photovoltaics, long-term dispatch, water contract, Lagrangian relaxation**

I. Introduction

Decarbonizing energy systems through a cleaner power grid is essential for combating climate change. Reservoir-based hydropower has the potential to play a key role in balancing renewable solar and wind energy sources due to its flexibility and unique capability to rapidly deploy energy given prior water storage, while incurring no additional fuel costs and negligible emissions [1]. Reservoirs serve a range of purposes such as hydropower generation, water supply, and flood control. However, leveraging multiple uses creates conflicts in planning operations. While reservoirs are primarily managed based on monthly water release targets, power system operates on hourly schedules [2]. Existing optimization approaches for reservoir management require both short-term and long-term forecasting information, while long-term forecasts are often unreliable or unavailable [3].

Co-locating photovoltaic (PV) systems with hydropower reservoirs enhances the utilization of local power infrastructure while supporting reliable clean energy generation. Floating solar photovoltaics (FPVs), which consist of PV modules mounted on floating platforms, are gaining traction due to their ability to conserve land, reduce water evaporation, and achieve higher efficiency than land-based PV systems [4]. Large-scale FPV installations on hydropower reservoirs leverage existing transmission and distribution networks. Additionally, coupling hydropower with PV improves generation reliability by balancing peak electricity demand through the complementary nature of these two generation methods [5]. This synergy provides an opportunity for innovative water release policies that promotes resiliency and integrates water management, solar generation, and grid requirements effectively across their relevant time scales [6].

New, lightweight algorithms are essential for managing coupled PV-hydro systems and integrating them into electricity markets, which dictate power generation schedules across much of North America. Optimization of large-scale hydropower systems face challenges of nonlinearity and long-term inter-temporal water release constraints [7]. Typically, these complexities are divided into separate optimization tasks, including long-term capacity planning, medium-term water allocation, and short-term unit commitment [8]. Integrating medium- and short-term planning is especially beneficial, as it enables reservoir operators to control monthly water releases to both maximize energy generation and meet water demands without conflict. This approach supports grid stability under decarbonization pathways, motivating more sophisticated hydropower models [9].

This paper proposes a multi-period optimization framework for controlling the energy dispatch of a hydropower unit with FPV. The framework includes a hydraulics-based power production model and constraints to account for mandatory water release requirements. Since handling long-term constraints directly in energy dispatch is intractable, we employ a sequential non-anticipatory decision-making approach that decomposes the multi-time-period optimization problem into a series of single-horizon optimizations. This decomposition is achieved via partial Lagrangian relaxation on the water contract constraint, enabling an efficient and user-friendly decision-making algorithm suitable for reservoir operators. The primary contributions of this paper are as follows:

- We propose a formulation for coordinated hydropower reservoir and FPV management and demonstrate that the multi-period optimization framework can be effectively decomposed, achieving near-optimal revenue results.
- We show the relaxed algorithm offers a practical approach for online control strategies by deriving the shadow price

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of the long-term water release constraint, providing a valuable tool for real-time reservoir management.

- We show that incorporating a nonlinear and nonconvex hydraulic head to storage function is feasible and computationally efficient within the relaxed formulation, improving the model's practicality for real-world applications.
- We validate the framework through a case study on the Hoover and Glen Canyon Dams, modeled in aggregate with a hypothetical FPV installation. These hydropower units, located on the Colorado River, are operated to conform to an agreed long-term water release policy [10].

The remainder of the paper is organized as follows. Section II presents the problem formulation. Section III introduced the partial Lagrangian relaxation method. Section IV describes the numerical studies to demonstrate the effectiveness of the proposed method. Finally, we conclude this paper in Section V.

II. Problem Formulation

We consider a coupled generation system combining hydropower and FPV. The coupled generation system operates under the same operator and transmission line. Hydropower manages reservoir water release for energy generation and other purposes, while FPV controls solar power based on available solar radiation. While the formulation proposed here is deterministic, we will show in section III that it serves as a baseline for developing a non-anticipatory dispatch policy.

A. Formulation

We formulate a multi-period optimization problem in (1) to maximize total generation revenue over the time period set $\mathcal{T} = \{1,2,\dots,T\}$ based on price of electricity λ_t (\$/MWh). All units are normalized by the time step duration which is one hour in our case study.

$$\max_{h_t,s_t,u_t,V_t} \sum_{t\in\mathcal{T}} \lambda_t(h_t+s_t) \tag{1a}$$

$$\text{s.t.} \sum_{t\in\mathcal{T}} u_t = U : \theta \tag{1b}$$

$$V_t - V_{t-1} = I_t - u_t,\ t\in\mathcal{T} \tag{1c}$$

$$0 \le h_t \le \eta g \rho \phi(V_{t-1}) u_t / 3.6e^9,\ t\in\mathcal{T} \tag{1d}$$

$$\underline{u} \le u_t \le \overline{u},\ t\in\mathcal{T} \tag{1e}$$

$$-\underline{R} \le u_t - u_{t-1} \le \overline{R},\ t\in\mathcal{T} \tag{1f}$$

$$0 \le s_t \le \alpha_t S,\ t\in\mathcal{T} \tag{1g}$$

$$0 \le s_t + h_t \le P,\ t\in\mathcal{T} \tag{1h}$$

where decision variables include s_t (MWh) denotes the dispatched energy from FPV at time period t; h_t (MWh) denotes the dispatched energy from hydropower at time period t; u_t (m^3) denotes the hourly water release at time period t; V_t (m^3) denotes the reservoir volume at time period t.

(1b) models the monthly water contract U (m^3) and θ is the associated shadow price variable. (1c) tracks the water balance in the reservoir system based on the inflow I_t (m^3). The hydropower generation from water release is limited by (1d), in which η denotes the hydropower generation efficiency; $\phi(\cdot)$ (m) denotes the hydraulic head function; g denotes the gravitational constant; ρ ($\mathrm{kg/m^3}$) denotes the density of water. A conversion from joule to MWh (1MWh=$3.6e^9$Joule) ensures unit consistency on both sides of (1d). (1e) models the upper and lower water release bounds $\underline{u}$ and $\overline{u}$, and (1f) models the water flow ramp rates $\underline{R}$ and $\overline{R}$, all in m^3. (1g) limits the solar power based on the available solar radiation with the capacity factor α_t and the installed solar capacity S (MWh). (1h) limits the total generation from hydro and FPV with the transmission capacity P (MWh).

B. Fitting the Nonlinear Hydraulic Head Function

We incorporate the nonlinear hydraulic head function for hydropower, assuming a concave relationship between reservoir volume and height.

$$\phi(V_t) = aV_t^b \tag{2}$$

where parameters a and b are obtained by fitting real-world data[1], as illustrated in Fig. 1. Preserving nonlinearity in the hydraulic head mirrors the real-world hydrology dynamics as opposed to simplifications introduced by traditional approximations such as a piece-wise linear functions or convex counterparts [11].

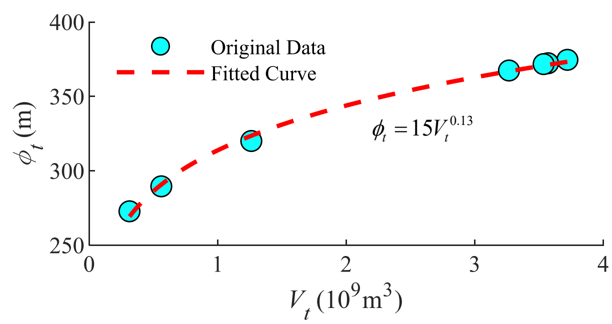

Fig. 1. Visualization of fitting the nonlinear hydraulic head function.

III. Methodology

We propose a partial Lagrangian relaxation approach to decompose the inter-temporal constraint of the water contract delivery. This allows the optimization problem to be solved independently at each time step. The proposed optimization framework can be further solved by online decision-making approaches [12], [13] with uncertainty realizations from forecasts or observed data.

A. Partial Lagrangian Relaxation

The formulation in (1) has only one long-term time coupling constraint (1b). All other constraints only consider a single or two neighboring time steps, while the objective is also separable. Hence, we utilize the Lagrangian relaxation of the long-term water contract to generate a two-level formulation:

$$\min_{\theta} \Big(U - \sum_{t\in\mathcal{T}} u_t^*\Big)^2 \text{ where} \tag{3a}$$

$$u_t^* \in \arg\max_{h_t,s_t,u_t,V_t} \Big\{\lambda_t(h_t+s_t) - \theta u_t \text{ s.t. (1c) - (1h)}\Big\} \tag{3b}$$

[1]https://www.usbr.gov/lc/region/g4000/LM AreaCapacityTables2009.pdf

where u_t^* represents the optimized water release from the time-decoupled subproblem. The subproblem (3b) is a non-anticipatory which only optimizes the generation and water release during time step t based on the input water contract price θ, while the master objective (3a) ensures the water contract is fulfilled by the lower-level non-anticipatory reservoir operations by find the optimal θ. Hence, the master problem is a water contract pricing problem, while the lower-level subproblems simulate realistic reservoir operations. Note that the minimized master problem will be zero representing the original water release contract (1b).

This decomposed formulation provides two advantages compared to (1). First, it simulates non-anticipatory reservoir operation, allowing the water contract to be priced in a realistic manner. Each subproblem only includes decision variables at the current step and considers past and current information without looking into the future, as (1c)-(1h) only includes the current time step and all previous time steps. Second, it significantly reduces the computation complexity in solving the nonlinear hydraulic head function in (1d). Note that solving (1) over a typical water contract duration, such as one month, at hourly resolution can be extremely computation-challenging with an arbitrary hydraulic head function ϕ, but in the decomposed structure we only need to solve one step at a time and the optimization can be efficiently solved.

B. Bi-section Solution Method of the Master Problem

We leverage the fact that the master problem (3a) has only a single unknown θ. Based on this, we propose a bisection algorithm to find the optimal solution. The bisection algorithm relies on the following proposition:

***Proposition** 1:* The sum of the water releases, $\sum_{t\in\mathcal{T}} u_t^*$, decreases monotonically with respect to θ if the hydraulic head function ϕ is constant.

The proof of this proposition is as follows. First, we observe from (3b) that θ represents the price associated with water release. The upper bound of hydropower generation h_t is proportional to the water release u_t, as modeled by the hydraulic head function in (1d). Therefore, as θ increases, the effective price of water release also increases, leading to a reduction in u_t over each time step t.

There are two complicating factors in this proposition, the ramp constraint (1f) and the hydraulic head function (1d). Since the subproblem (3b) simulates non-anticipatory reservoir operation, we perform the analysis in sequential order. For the ramp constraint, we start from the first period $t = 1$, and it is trivial to see what the proposition holds. Then, in the next time period, a higher value of θ not only further penalizes the water release but also reduces the water release upper bound due to the ramp constraint (1f) because the earlier water release must also be lower. Hence, the proposition holds for the (1f). While for the hydraulic head function, we have to assume it to be a constant, hence (1d) be a linear constraint for the proposition to hold rigorously, hence we finished the proof.

***Remark** 2:* For nonlinear hydraulic head function, we show that Proposition 1 may not necessarily hold by substituting (1d) into the objective of (3b) assuming it is binding. The objective then becomes

$$\max_{h_t,s_t,u_t,V_{t-1}} \lambda_t s_t + (\lambda_t \eta g \rho \phi(V_t)/3.6e^9 - \theta)u_t \tag{4}$$

Note that a lower water release in prior periods due to higher θ will lead to a higher V_t hence $\phi(V_t)$ will have a higher value. Hence, $\lambda_t \eta g \rho \phi(V_t)/3.6e^9 - \theta$ may not change monotonically with θ. Hence, we cannot conclude that Proposition 1 will definitely hold. However, we will show later in the simulation that using realistic data, the proposed algorithm provides similar results to benchmark IPOPT solvers.

Building on this proposition, we introduce an auxiliary variable, $\sigma(\theta)$, to represent the total water release as a function of the θ. We formally define this as:

$$\sigma(\theta) = \sum_{t\in\mathcal{T}} u_t^*, \tag{5}$$

where u_t^* is the solution obtained from (3b).

We can now develop the algorithm by picking an arbitrary θ and simulate the hourly reservoir operation as stated in (3b). Assume θ^* is the minimizer to (3a) we have:

$$\begin{aligned} &\text{If } \sigma(\theta) > U \rightarrow \theta < \theta^*, \text{ If } \sigma(\theta) < U \rightarrow \theta > \theta^*, \\ &\text{If } \sigma(\theta) = U \rightarrow \theta = \theta^* \end{aligned} \tag{6}$$

C. Analytical Solution to the Subproblem

We now propose an analytical algorithm for solving the decomposed subproblem (3b) for fast and efficient computation without the need for commercial solvers especially regarding the nonlinear hydraulic head function.

***Proposition** 3:* The solution to (3b) with a given θ is as follows if $\lambda_t \geq 0$:

$$s_t^* = \min\{\alpha_t S, P\} \tag{7a}$$

$$u_t^* = \max\{u_{t-1} - \underline{R}, \underline{u}, \min\{\overline{u}, u_{t-1} + \overline{R}, \hat{u}_t\}\} \tag{7b}$$

$$V_t^* = V_{t-1} + I_t - u_t^* \tag{7c}$$

$$h_t^* = \min\{P - s_t^*, \eta g \rho \phi(V_t^*) u_t^*\} \tag{7d}$$

where

$$\hat{u_t} = \begin{cases} \dfrac{(P - s_t^*) \cdot 3.6e^9}{\eta g \rho \phi(V_{t-1})} & \text{if } \hat{\theta} > \theta \\ 0 & \text{else} \end{cases} \tag{7e}$$

$$\hat{\theta} = \lambda_t \eta g \rho \phi(V_{t-1})/3.6e^9 \tag{7f}$$

The proof of this proposition is as follows. We start by assuming $\lambda_t \geq 0$ then it is trivial to see the shadow price of (1b) θ should also be a non-negative value and the right-hand side of (1d) must always be binding due to the positive value of releasing water and (1d) is the only constraint coupling h_t and u_t. Since s_t has no marginal cost, then s_t should always be upper bounded by the lesser of (1g) and (1h) before considering h_t as in (7a). (1e) and (1f) are both box constraints in the decomposed subproblem - note u_{t-1} is treated as a parameter. Then (7e) calculates u_t without these two box constraints based on the coefficient of u_t by subsisting (1d) into the objective to replace h_t as in (5), in which u_t would reach the transmission capacity minus s_t if the weight is positive. Finally, (7b) enforces the box constraints back over u_t and we update V_t and h_t accordingly.

D. Full Algorithm

We implement a bi-section search tree algorithm to iteratively determine the optimal value of θ and the corresponding dispatch solutions. A confident search range is to choose $L = 0$, $R = 1$ and set $\epsilon = 10^{-6}$.

Algorithm 1: Bi-section Search Tree Algorithm

Input: Initial search range $[L,R]$ and accuracy ϵ.
Output: Optimal price of water θ^*.
Search Algorithm:

```
while R − L > ε do
    Set θ = (L + R)/2
    Update subproblem solutions as in (7)
      sequentially from t = 1 to T
    Calculate σ(θ) as in (5)
    if σ(θ) > U then
        R ← θ (increase the penalty)
    else
        L ← θ (decrease the penalty)
    end
Return θ as the optimal price of water.
```

IV. Numerical Studies

A. Set-up

We demonstrate the effectiveness of the proposed method through real-world case studies on the coupled hydropower and FPV systems at Lake Powell and Lake Mead, USA. Draining and overtopping of the reservoir are not considered in the model, as the large reservoir capacity sufficiently governs the release dynamics. The hourly electricity price data is from CAISO, the hourly available solar radiation is from NREL, and the daily water inflow data from the US Bureau of Reclamation[2] over 2022-2023 is collected and available online[3]. The system configurations and parameters are shown in Table I, which are derived from [14] and [15]. The optimization is coded in Julia and the programming environment is an Apple M3 Core @ 4.06 GHz with RAM 16 GB.

TABLE I
System Parameters

Parameter	Value	Parameter	Value
$\overline{u}$	707.9 m^3	$\underline{u}$	141.6 m^3
$\overline{R}$	113.3 m^3	$\underline{R}$	70.4 m^3
P	1300 MW	S	1000 MW
η	0.775	g	9.8 m/s^2
ρ	1000 kg/m^3	T	720 h

B. Benchmarking with Gurobi and IPOPT

We compare the results with different methodologies:

1) M1: Multi-period optimization as (1).
2) M2: Decomposed optimization as (3).

[2]CAISO: http://oasis.caiso.com; NREL: https://nsrdb.nrel.gov; USBR: https://www.usbr.gov/uc/rm/crsp/gc/
[3][Code Available]: https://github.com/ecohn44/fpv-hydro-dispatch

The original multi-period formulation (M1) is not tractable with a nonlinear hydraulic head for the full duration of the water contract. Gurobi solves only the linearized version and IPOPT has a sharp degradation in performance for any horizon larger than two weeks. The advantages of our proposed algorithm (M2) is that the nonlinear hydraulic is feasible over any finite time periods. Because the full month cannot be solved for M1 and M2, they are benchmarked via IPOPT over a one week period in January of 2022. The cumulative revenue under M1 and M2 is 10.013 M$ and 10.012 M$, the total water release is 1.696 x10^8 m^3 and 1.695 x10^8 m^3, and the cpu time is 5039.1 ms and 17.6 ms respectively. We can conclude that M2 is nearly optimal because (i) the revenue of M1 is only 0.1% larger and (ii) M1 only releases 0.06% more water than M2. Furthermore, M2 sees significant advantages in its reduced runtime.

The methodologies are next bench-marked with a linearized hydraulic head via Gurobi over the same one week period. The cumulative revenue under M1 and M2 is 10.29 M$ and 10.10 M$, the total water release is 1.696 x10^8 m^3 and 1.694 x10^8 m^3, and the cpu time is 19.8 ms and 17.4 ms respectively. The conclusions of near-optimality are the same as above.

C. Sensitivity Analysis

In Figure 2, the shadow price of water solved for in 1 is overlaid with the average price of electricity for each month. The correlation coefficient between the two prices is 0.88, demonstrating how the price of electricity drives the dispatch of both FPV generation and water release. Under the deterministic formulation, a knowledge of the next period average price of electricity means it is possible to predict the shadow price and control release under near-optimal operating guidelines.

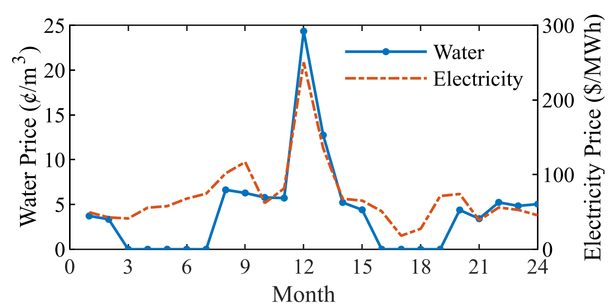

Fig. 2. Shadow Price of Water and Marginal Price of Electricity for a transmission capacity of 1300 MW over 2022-2023.

The system behavior under a range of transmission capacity ratings is examined in in Figures 3 and 4. Figure 3 shows that the price of water increases proportionally with transmission capacity. This indicates that increased capacity for generation increases the value of water released. However, there is a limit in this potential value through observing that price of water for 2 GW and 3 GW is nearly the same. For a small transmission capacity rating such as 0.5 GW, the system can rely solely on FPV generation to drive profits and the water used for generation becomes less valuable. Figure 4 shows that revenue also increases proportionally with transmission capacity. Again, there is a limit in total earnings as the transmission capacity nears 3 GW.

These results provide insights into predicting the shadow price of water associated with the water release for hydropower generation. The average monthly price of electricity highly correlates with the water contract shadow price and can be used to control the water release. This is advantageous because the average monthly price is far easier to predict than the higher resolution forecasts of inflow, solar radiation, or hourly price of electricity used in standard hydropower operations.

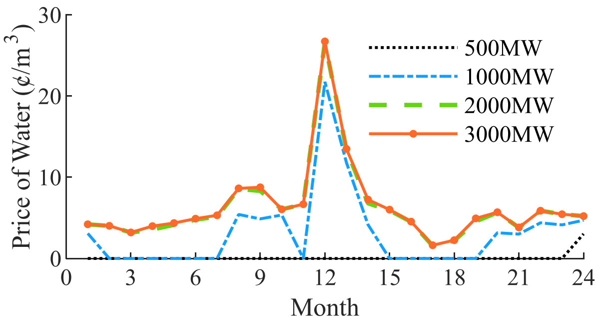

Fig. 3. Shadow Price of Water over Transmission Capacity from 2022 to 2023.

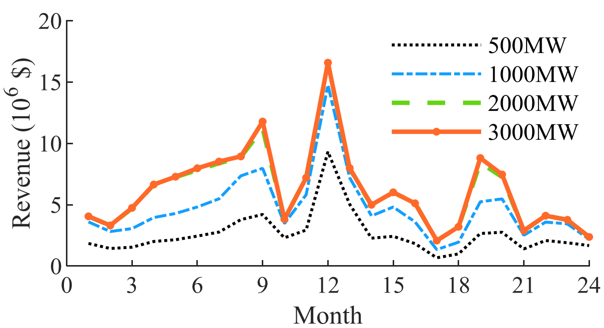

Fig. 4. Revenue for Varying Transmission Capacity from 2022 to 2023.

V. Discussion and Conclusion

This paper proposes a coordinated framework for the coupled operation of a FPV-hydropower system. Due to the incorporation of long-term water release constraint, an algorithm is proposed that decomposes the water release constraint via a partial Lagrangian relaxation. Our findings are validated in a real-world case study and offer a practical control policy for reservoir operators that incorporate a long-term water contract often left out of traditional electricity dispatch models.

An added benefit of installing FPV cells onto existing reservoirs is water conservation. To the water sensitive Western US, the reduction in evaporation achieved from covering the reservoirs is critical. To calculate the quantity of water saved from the hypothetical FPV installation proposed in this paper, we use 10,000 m^2/MW as the average capacity density of floating solar in the US [16]. A proposed capacity of 1 GW over a total surface area of 500 sq. miles translates into 3.25 sq. mile (0.75%) coverage of the reservoir surface area. Given that approximately 267.5 cm/year of water is lost at the case study site location [15] and that flexible FPV modules can reduce evaporation by 42% [17], this coverage would save 10.9 million cubic meters of water per year.

Future work will include the incorporation of uncertainty and stochastic scenario generation. The proposed approach demonstrates that with a carefully tuned shadow price of water and knowledge of the monthly average price of electricity, near-optimality can be achieved. We will investigate how sensitive this shadow price is under varying scenarios of hourly solar availability, inflow, and price of electricity in order to explore if the average monthly price of electricity can still be a reliable factor in determining operations.

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