“Singing” Tube ---- Excitation and resonance of airflow in corrugated cavity

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Abstract: The sound tube is a kind of tube toy with corrugated surface. Under certain conditions, the airflow will make a sound after flowing inside the sound tube. This article applies the resonance principle and the Bernoulli principle to explain the sounding conditions of the sound tube and the reason for the air flow, and explain the mechanism of whistling by combining the sounding of wooden frog. Quantitatively, this article finds that the natural frequency of the sound tube is inversely proportional to the length of the sound tube, and proportional to the ratio of the corrugation volume to the total volume; the excitation frequency of the sound tube is directly proportional to the flow velocity; the relationship of discrete natural frequency and corresponding discrete excitation frequency is given. This article also validates the theoretical results from experiments quantitatively and effectively. The phenomenon that some low-order natural frequency modes of the sound tube cannot be excited is observed. We analyzed the reason for this phenomenon in combination with the turbulent flow state of the fluid: there is a minimum excitation frequency that makes the natural frequency less than this frequency never sound. The experimental results support our theoretical analysis.

Keywords: Natural frequency; excitation frequency; resonation; fluid flow state; onset frequency; centrifugal effect; Doppler effect

Contents

Introduction

Vibration and sound are crucial parts of physics and play a pivotal role in the cutting-edge field of research and teaching. Sound tube whistling is an interesting phenomenon in this field. The sound tube is a simple and common toy, usually composed of plastic bellows with open ends. When the user holds one end of it and rotates the sound tube, the sound tube will make a strange sound. When the rotation speed increases, the frequency of the sound emitted by the sound tube also increases. Studies have shown that the sound of the sound tube is caused by the vibration of the air flowing through the pipe and colliding with the sound tube, and it has been found that the frequency excited by the sound tube is positively related to its speed [4]. Other studies have given expressions for the natural frequency of corrugated sound tubes [1][3], indicating that its natural frequency is discretely distributed, and the relationship between the fluid velocity and the excitation frequency that excites the vibration of the sound tube. Other studies have shown that under certain conditions, air flow through the sound tube will produce vortexes in the pitch of the bellows [3].

Through these theories and experiments, it is possible to quantitatively calculate the frequency value of the sound tube for a certain fluid velocity under static conditions, but there are still many details to be studied and explained in this question: 1) Although the natural frequency of the sound tube and the excitation frequency generated by the external force are known, the relationship between them and the correlation with the received frequency has not yet been clearly figured out. For example, the natural frequency of the sound tube changes discretely, while the excitation frequency changes continuously; but the received frequency signal changes discretely; 2) Even if it can theoretically predict the value of the frequency excited by a sound tube at a certain fluid velocity, there is no clear theory that can explain the physical meaning behind the equation; 3) When the sound tube rotates to produce sound, no matter where the sound source is, the Doppler effect is a variable that cannot be ignored. However, in existing studies, the effect on the frequency of the signal collected by the pickup has not been accurately verified by quantitative experiments; 4) When the air velocity is small, no obvious sound can be heard, but when the fluid velocity reaches a certain threshold, the sound tube excites the frequency. Existing theories cannot explain why there is a critical velocity here, nor can it be quantitatively explored.

This article believes that this phenomenon may become the root cause of the sound tube sounding phenomenon, and the research on this critical fluid velocity and critical frequency is of considerable significance. In summary, this article starts with the excitation frequency caused by the natural frequency and wind velocity of the sound tube, and then goes to the study of the dispersion of the frequency emitted by the sound tube, exploration of air flow caused by Bernoulli effect, the explanation of the sound tube sounding because of air flow state and the discussion of the sound tube's initial frequency and initial air velocity, and research on the influence of Doppler effect on the received audio signal. We have verified the above research by means of physics simulation to the sound tube mechanism theory simulation and quantitative experiment.

1.Theoretical analysis

1.1 Geometric Model: The Longitudinal Section of Sound Tube

By modeling the theoretical structure of sound tube, we quantify the mechanism of sound tube and the characteristic of produced sound. Fig. 1 shows the longitudinal section of a sound tube, which is basically the periodical corrugation structure. The length of the tube is L; the corrugation is isosceles triangle of height h and base d. The outer diameter is 2R, inner diameter is 2r. The flow velocity in the tube is U.

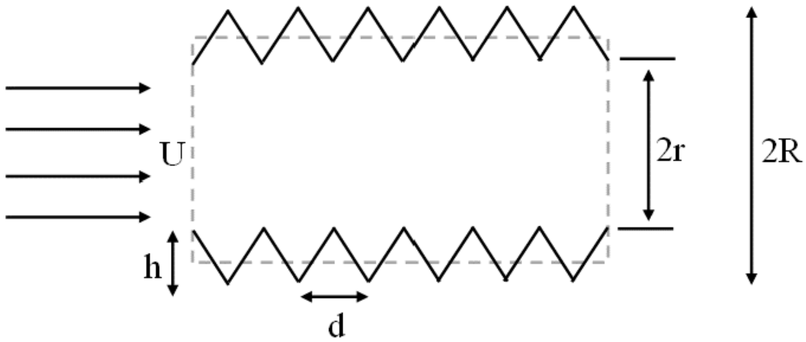

Fig.1. The Longitudinal Section of Sound Tube

Natural Frequency, which is mainly determined by the geometric structure, is closely related to the sound frequency. Thus, we firstly calculate the natural frequency of the structure in Fig. 1.

1.2 The Natural Frequency of Sound Tube

Natural frequency is the frequency at which a system tends to oscillate in the absence of any driving force.

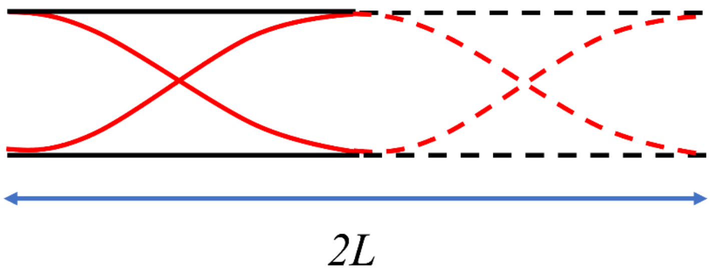

Fig. 2. The Fundamental Frequency of an Ideal Open-ended Tube

In Fig. 2, for an ideal open-ended tube, the wave length of fundamental frequency is[1]:

$$\lambda = 2L \tag{1}$$

The fundamental frequency is:

$$f = \frac{v}{\lambda} = \frac{v}{2L} \tag{2}$$

Here, v is the speed at which sound travels in the tube. As the increase of tube length, the fundamental frequency decreases. In addition to the fundamental frequency, the tube can sustain harmonic frequency with modes n. Fig 3 shows the second harmonic frequency with wave length:

$$\lambda = L \tag{3}$$

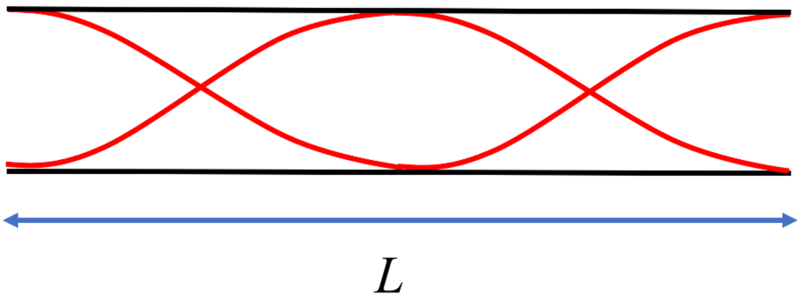

Fig. 3. The Second Harmonic Frequency of an Ideal Open-ended Tube

In conclusion, the wave length of harmonic frequency of the tube changes discretely:

$$\lambda = \frac{2L}{n} \tag{4}$$

Besides, sound tube is placed in air for research. So, we assume the speed in air is:

$$v = c \tag{5}$$

Thus, the Natural Frequency of an Ideal Open-ended Tube is:

$$f_n = \frac{v}{\lambda} = \frac{nc}{2L} \tag{6}$$

Here, f_n is the natural frequency with modes n. However, due to the effect of sound radiation, we need to use effective length L_e to account for it.[2]

$$f_n = \frac{nc}{2L_e} \tag{7}$$

The effective length equals to the tube lenght adding the correction length ΔL on both openings:

$$L_e = L + 2\Delta L \tag{8}$$

Normally, the correction length $\Delta L = 0.61r$[3]. Thus, the natural freuqency after correction is:

$$f_n = \frac{nc}{2(L + 1.22\mathrm{r})} \tag{9}$$

Now we consider the corrugated sound tube in Fig. 1. The corrugated sound tube could be regarded as ‘compress’ tube, which leading to complex flow behavior, for example, the vortex, inside sound tube. Due to the influence of corrugation, the sound travels along the corrugation at a slower speed, compared to a smooth one. To account for the effect of corrugation, we introduce effective sound velocity c_e[3]:

$$f_n = \frac{nc_e}{2L_e} \tag{10}$$

$$c_e = c_0 \frac{(1-M^2)}{\left[1+\frac{h}{r}\left(\frac{\frac{d}{2}}{d}\right)\left(1+\frac{h}{2r}\right)\right]} \tag{11}$$

M is Mach Number:

$$M = \frac{U}{c} \tag{12}$$

It can be seen that the effective sound velocity given in Formula (11) is closely related to the corrugation height and base, but has no obvious relationship with the corrugation shape. In experiment environment, $U \approx 20m/s$, far below the sound velocity. Thus, $M = \frac{U}{c} < 0.2$, meaning the air flow inside tube is incompressible. We can simply neglect the Mach number and simplify the Formula (11):

$$c_e = c_0 \frac{1}{\sqrt{1 + \frac{V_c}{SL}}} \tag{13}$$

Here, V_c is the volume of corrugations; S is the inner cross-sectional area; L is the tube lenght. In total, $S \times L$ is the volume of sound tube without corrugations. Because the corrugations volume is far less than the total volume (about one tenth of the total one), we can simplify the Formula (13):

$$c_e \approx c_0 \left(1 - \frac{V_c}{2SL}\right) \tag{14}$$

Formula (14) states that when $V_c \ll SL$, effective sound velocity will decrease linearly with corrugation volume, which means the natural frequency will increase with the decrease of corrugation volume according to Formula (10).

For simplicity, we use $V_{in} = SL$ to represent the tube volume without corrugations, $V_{tot} = V_{in} + V_c$ to represent total tube volume:

$$c_e = c_0 \sqrt{\frac{V_{in}}{V_{tot}}} \tag{15}$$

According to Formula (15), the effective sound velocity is related to the ratio between V_{in} and V_{tot}. The corrugations behave like an extra volume of air, decreasing the sound speed. When corrugations volume decrease gradually, the sound tube tends to become a smooth tube and effective sound speed tends to approach normal sound speed.

By combining Formula (9) and 15, we get the natural frequency of sound tube:

$$f_n = \frac{nc_0}{2(L + 1.22r)} \sqrt{\frac{V_{in}}{V_{tot}}} \tag{16}$$

Corrugations volume, sound tube length, and sound tube diameter will all influence the natural frequency. The longer the length of the sound tube, the smaller the ratio of the corrugation volume to the total volume, and the lower the natural frequency of the sound tube. However, although natural frequency is an inherent property of a sound tube, it still need external driving force to produce sound. According to Mechanical Vibration Theory, when the frequency of a periodically applied force is closed to a natural frequency of a system, the system will increase oscillation amplitude, which finally leading to producing sound. This is the phenomenon of resonance, the basic mechanism for sound tube. Thus, we need to investigate the frequency of the periodically driving force, which is called Excited Frequency.

1.3 The Excited Frequency of Sound Tube

The result of preliminary experiment shows that the sound tube needs both corrugations and air flow to produce sound. In sound tube system, both corrugations and air flow play a part in exciting sound. Due to the collision between air and corrugations, sound will be excited, as the wooden frog showing in Fig. 4. By using the stick to rub the corrugation on frog's back, we can produce stable sound. This is because the collision between the stick and the corrugation. Obviously, the speed of rubbing and corrugation pitch are closely related to the produced sound. In sound tube system, the air flow acts like the stick in the wooden frog system.

Fig. 4. Wooden Frog

In Fig. 5, when corrugation pitch is d and air flow speed is U, the time between air collide with two corrugations is:

$$T = \frac{d}{U} \tag{17}$$

Thus, the excited frequency is:[5]

$$f = \frac{1}{T} = \frac{U}{d} \tag{18}$$

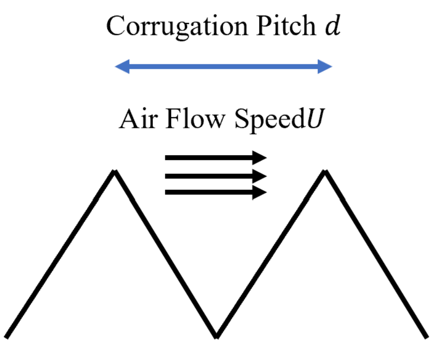

Fig. 5. The Diagram of Excited Frequency

Formula (18) states the influence of corrugation pitch and air flow speed on excited frequency. For a smooth tube, the corrugation pitch d approaches infinite, so the excited frequency for smooth tube approaches 0. That's why a smooth tube cannot produce sound. In forced vibration, only general external force to generate vibration is not enough to produce sound. The vibration driven by external force must be close to the natural frequency of the system to generate resonance. In the following part, we will discuss the resonance phenomenon of sound tubes. The phenomenon is that the air flow acts like a stick to rub the corrugations and generate vibration. The condition is that the air flow can flow into the corrugation pitches and interact with them. It's obvious that the excited frequency is continuous. However, in our experiment, the produced sound is discrete. This is due to resonance, which will be discussed further in next part.

1.4 The Resonance of Sound Tube

Resonance is the basic mechanism for sound tube. To produce sound, the excited frequency must approach natural frequency of sound tube to increase vibration ampplitude.

Here, the sound frequency produced by sound tube is:

$$f_n = \frac{nc_e}{2L_e} \approx \frac{U}{d} \tag{19}$$

The excited frequency is close to the natural frequency of the sound tube. Although the excited frequency is continuous, the natural frequency of sound tube is discrete. That's why the sound we hear is discrete. The sound tube will produce sound frequency of harmonic frequency with mode n when:

$$f_n \leq \frac{U}{d} < f_{n+1} \tag{20}$$

In other word, when the air flow speed is:

$$f_n d \leq U < f_{n+1} d \tag{21}$$

The sound tube produce sound of $f = f_n$. Fig. 6 shows the theoretically curve of excited frequency and natural frequency.

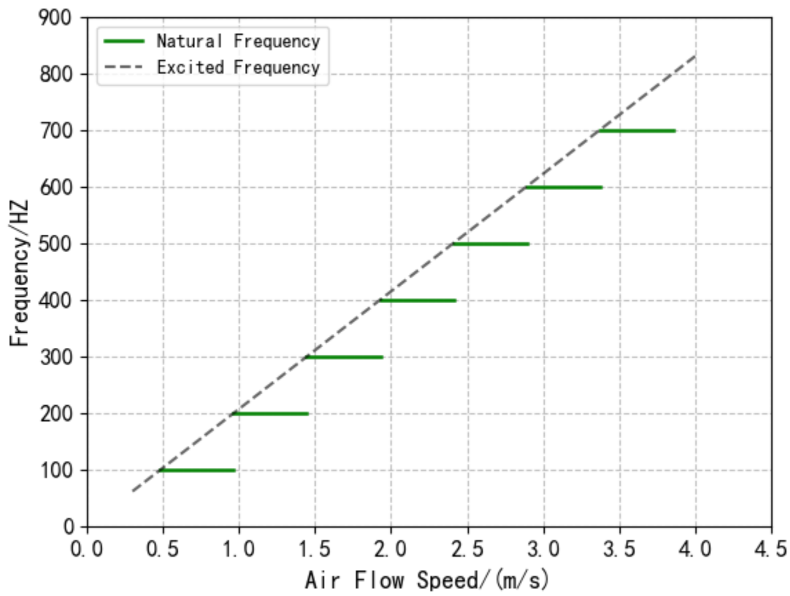

Fig. 6. The Relationship between Air Flow Speed and Sound Frequency

1.5 "Hidden" Fundamental Frequency – The Onset Frequency.

In our experiment, we discover the phenomenon of 'hidden' fundamental frequency: the fudamental frequency of sound tube wouldn't be produced; instead, it is the second harmonic frequency that being produced firstly. If we choose a longer sound tube to decrease the fundamental frequency of the sound, we discover that even the second and third harmonic frequency might be hidden. This is due to the air flow pattern inside the tube.

When air flows, eddies form in corrugations because of the airflow's pattern. Eddyies inside corrugations would strike the corrugations and vibrate the whole system. In the theoretical analysis before, airflow strikes the corrugation like a mallet, and thus cause the tube to whistle. It means that when the tube can produce sound, the airflow hits the corrugation and excites the whole tube. Therefore, the airflow will flow into grooves and cause turbulence, as shown in Fig. 7.

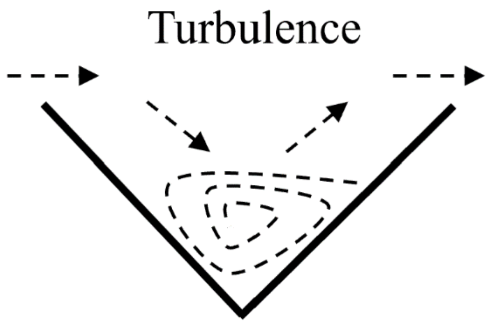

Fig. 7. The Vortex inside Corrugation

We introduce Reynolds Number to calculate the air flow pattern inside the tube:

$$Re = \frac{\rho_{air} U L_c}{\mu} \tag{22}$$

Re is Reynolds Number, U is air flow speed, L_c is characteristic length, and μ is dynamic viscosity of the fluid.

For a smooth tube, L_c is the diameter of the tube. Thus, we can deduce the L_c of corrugated sound tube as $L_c = r + R$.[5]

In that case:

$$Re = \frac{\rho_{air} f d(r+R)}{\mu} \tag{23}$$

In Formula (23), Reynolds Number is proportional to excited frequency. Because air would enter corrugations, the turbulence is the air flow pattern inside the tube. Thus, $Re > 2300$:

$$Re = \frac{\rho_{air} f d(r+R)}{\mu} > 2300 \tag{24}$$

In that case, for a corrugated sound tube, there must be an onset frequency f_{min}, leading the form of vortex inside tube:

$$f_{min} = \frac{Re_{min}\mu}{\rho_{air} d(r+R)} \tag{25}$$

Here, $Re_{min} = 2300$. In Fig. 8, when $f < f_{min}$, $Re < Re_{min} = 2300$, the flow pattern inside tube is laminar, which means the air flow wouldn't collide with corrugations. That's why when the fundamental frequency of the tube f_{min} is less than the onset frequency, the tube wouldn't excite the fundamental frequency.

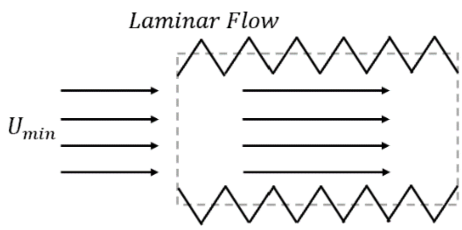

Fig. 8. Laminar Flow inside Sound Tube

However, due to the limits of the experiment instrument and complexity of sound tube structure, we can hardly measure the Flow Field precisely by experiment. In that case, we employ simulation to further our research in the flow pattern in next part.

1.6 Flow Field inside Sound Tube

By using COMSOL Multiphysics, we can simulate the flow field inside sound tube. In the physical scenarios covered in this article, the CFD (Computational Fluid Dynamics) module is the most appropriate. Because of the special mechanism of sound tube whistles (the vortex in the corrugated pitch space collides with the pipe wall), we use Largy Eddy Simulation (LES) in the module Turbulent Flow. Large eddy simulation is a spatial average of turbulent vortices. That is to separate the vortices of different scales through a certain filter function, directly simulate the large-scale vortices, and use the model to close(converge) the small-scale vortices. By accurately solving all turbulent motions above a certain scale, the LES can capture many unsteady fluid flows that the RANS method cannot do. Of course, the selected inertial sub-scale must be smaller than the corrugation scale, because the larger vortex inside the pitch is our focus.

Parameters in the simulations are velocity v, height of triangles h, and the base line l. Let $\lambda = \frac{h}{l}$. Larger λ means deeper and narrower pitch. In the experiments, $h \approx 0.5cm$, $l \approx 0.5cm$, $v < 20m/s$.

Shown in Fig. 9, initial velocity is 10m/s, geometrical parameters of corrugation pitch are the same as that of sound tubes used in our experiments. It can be clearly seen from the figure that when all conditions are the same as the experimental conditions, the airflow will indeed enter the corrugated pitch space and form a vortex, which verifies the theory above.

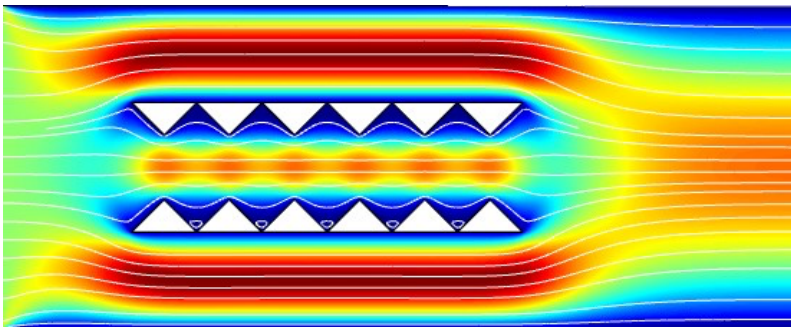

Fig. 9. Simulation of fluid flow inside the tube

The partial enlarged view of the flow in pitches is shown in Fig. 10. It it clear that the airflow streamlines in it have complex behaviors, such as vortexes, which form turbulence. The result is consistent with our previous analysis and prediction in Section 1.5. Thus, through simulation, this article demonstrates the turbulence mentioned in the theoretical analysis, which shows that the gair does enter the pitch and form turbulence under certain conditions.

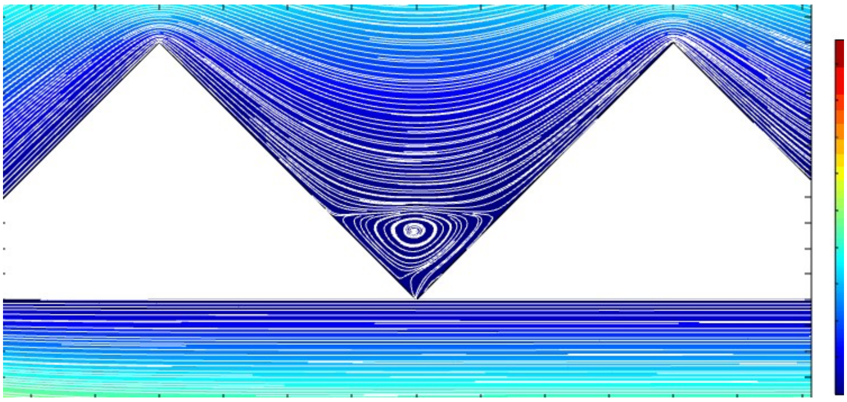

Fig. 10. The partial enlarged view of the flow in pitches

1.7 Prerequisite for Sound Tube

In conclusion, resonance and turbulance are the prerequisites for sound tube to produce sound. Not only the flow pattern inside the sound tube need to be turbulence, but also the excited frequency need to exceed the fudamental frequency.

The least air flow speed U_{min}^{a} needed to create turbulence:

$$U_{min}^{a} = \frac{Re_{min}\mu}{\rho_{air}L_c} \tag{26}$$

The least air flow speed U_{min}^{b} needed to create resonance:

$$U_{min}^{b} = \frac{c_e d}{2L_e} \tag{27}$$

By combining (26) and (27):

$$U \geq \frac{U_{min}^{a} + U_{min}^{b}}{2} + \frac{\theta\left(U_{min}^{a} - U_{min}^{b}\right)}{2}\left(U_{min}^{a} - U_{min}^{b}\right) \tag{28}$$

In which:

$$\theta(x) = \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases} \tag{29}$$

is jump function.

For a sound tube, if the fundamental frequency is always less than the excited frequency, then the sound tube will never produce sound:

$$\frac{Re_{min}\mu}{\rho_{air}dL_c} > \frac{nc_0}{2(L + 1.22r)}\sqrt{\frac{V_{in}}{V_{tot}}} \tag{29}$$

Simply put:

$$\frac{2Re_{min}\mu}{\rho_{air}nc_0L_c} > \frac{d}{L_e}\sqrt{\frac{V_{in}}{V_{tot}}} \tag{30}$$

If we ignore the correction of effective sound speed and length:

$$\frac{2Re_{min}\mu}{\rho_{air}nc_0L_c} > \frac{d}{L} \tag{31}$$

Formula (31) states that when the corrugation pitch $d \to \infty$, the air flow speed needed to reach fundamental frequency is infinite. Thus, the sound tube will never produce sound. This is why a smooth tube wouldn't produce sound.

1.8 Swinging Sound Tube

As a type of toy, sound tube could simply produce sound by rotating with hands. When holding one side of sound tube and rotate it, the air will be pumped into the tube according to Bernoullis' Principle.

In the non-inertial reference frame, we consider infinitesimal tube length dx. The Pressure-induced force $F_p = -\pi D^2 dp$ is balance to centrifugal force $F_c = \rho_{air}(\pi D^2 dx)\omega^2 x$ to make the air flow uniformly in the tube.

According to equilibrium of force:

$$\rho_{air}\omega^2 x dx = dp \tag{32}$$

After intergration:

$$P(L) - P(0) = \frac{1}{2}\rho_{air}(\omega L)^2 \tag{33}$$

According to Bernoullis' Principle:

$$P(L) - P(0) = \frac{1}{2}\rho_{air}U^2 \tag{34}$$

By combining (29) and (30):

$$U = \omega L \tag{35}$$

Formula (35) states the relationship between air flow speed and angular velocity, tube length. Without friction, the air flow velocity inside sound tube equals to the tangential speed of tube tail. As angular velocity increases, the air flow velocity increases and the excited frequency increases as well.

However, in reality, the friction is non-negligible. Due to the friction, the pressure difference between both ends of tube will be reduced. The change of pressure difference is:

$$\Delta P = 4c_f\left(\frac{1}{2}\rho_{air}U^2\right)\frac{L}{2r} \tag{36}$$

c_f is the frictional coefficient of the tube.

Taking the frictional loss into account:

$$P(L) - P(0) = \frac{1}{2}\rho_{air}U^2\left(1 + 2c_f\frac{L}{r}\right) \tag{37}$$

Combing (29) and (30):

$$U = \frac{\omega L}{\sqrt{1 + 2c_f\frac{L}{r}}} \tag{38}$$

According to Formula (38), we can deduce the air flow speed from angular velocity . The friction is only related to the inner diameter and tube length.

1.9 Doppler Effect

When the sound tube is rotating, the position of sound source is constantly changing relative to the receiver. Thus, we need to take Doppler Effect into consideration.

Doppler Effect:

$$f' = f\frac{c_0 \pm v_l}{c_0 \mp v_s} \tag{39}$$

v_l is the velocity of listener(receiver); v_s is the velocity of sound source; c_0 is sound speed.
The position of sound source:

$$\underline{x}_s(t) = \begin{bmatrix} r_s\cos(\omega t) \\ r_s\sin(\omega t) \end{bmatrix} \tag{40}$$

Assuming that the position of receiver is static:

$$x_l(r_l, 0, h) \tag{41}$$

Thus, the velocity of sound source is:

$$\underline{v}_s = \frac{d}{dt}\underline{x}_s(t) = \begin{bmatrix} -r_s\omega\sin(\omega t) \\ r_s\omega\cos(\omega t) \end{bmatrix} \tag{42}$$

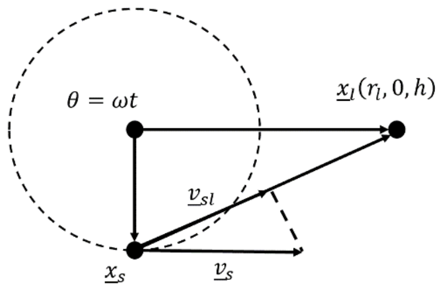

Fig. 11. Top View of Swinging Sound Tube

As shown in Fig 11, the relative velocity $\underline{v}_{sl}$ is:

$$\underline{v}_{sl} = \frac{r_l r_s \omega \sin(\omega t)}{\sqrt{{r_l}^2 + h^2 + {r_s}^2 + 2r_s r_l \cos(\omega t)}} \tag{43}$$

Thus, the doppler effect is:

$$f' = f\left(\frac{c_0}{c_0 + \underline{v}_{sl}}\right) \tag{44}$$

By employing Python, we could visualize the curve of doppler effect:

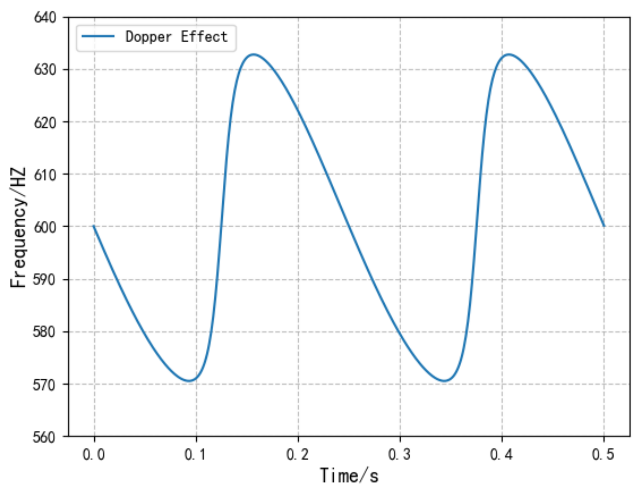

Fig. 12. Doppler Effect

When the extension line of tangential velocity of sound source passes the projection of listener, the relative velocity is max.

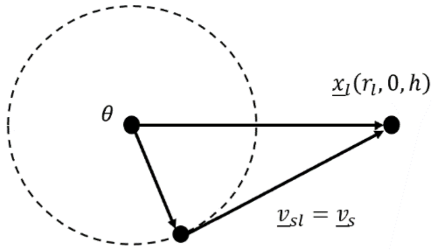

Fig. 13. The maximum Doppler Effect

$$v_{max} = \frac{\omega r\sqrt{r_l{}^2 - r_s{}^2}}{\sqrt{r_l{}^2 + h^2 - r_s{}^2}} \tag{45}$$

At the moment, the doppler effect is the most obvious one.

1.10 Theory Summary

- The natural frequency of sound tube is discrete and stable, being inversly proportional to the effective sound length and direct proportional to the volume of corrugations as shown in Formula (16).
- Because of Bernouli's Principle, when one end of the sound tube is fixed and the other end is rotating, the pressure difference will suck the air in and create air flow inside the sound tube.
- As air flow speed increases, the flow patter inside the tube will gradually become turbulence, leading to the collision between air and corrugation. This is excited frequency.
- Because of turbulence, an onset frequency is the key to sound production. If the natural frequency is less than the onset frequency, the natural frequency wouldn't be excited.
- When the position of listener is fixed, dopper effect will shift the received frequency.
- To sum up, the pressure difference, air entering the sound tube, and the turbulence and vortex under certain conditions are all key factors. When the frequency reaches the natural frequency of the sound tube, resonance occurs and a clear sound is emitted.

2.Experimental Verification

2.1 Exploration of The Relation between Flow Velocity and Whistling Frequency

According to Formula (18) in the theory section, as the flow velocity in the corrugated pipe increases, the excitation frequency by forced vibration to the pipe system increases. Through Formula (19), Formula (20), and Formula (21), we can derive the velocity-frequency figure like the Fig. 6.

2.1.1 Experimental Setup and Audio Analysis

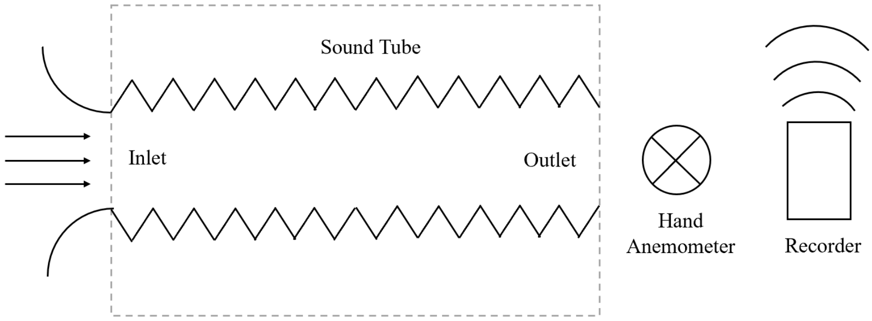

Fig. 14. Experimental Setup

We designed a set of experimental arrangement as shown in the Fig. 14 in order to research the relationship between flow velocity and whistling frequency.

As shown in the Fig. 14, in the experiment, air is pumped into the pipe from one end by a fan. After air flow becomes stable, the flow velocity is determined by handheld anemoscope at another end. After hearing the whistling sound, sound pick-up collects the whistling sound at the end of the pipe with length $L = 0.7m$ and corrugated interval $d = 0.5cm$. We imported the sound signal collected over a period of time into Python and applied the Fast Fourier Transform (FFT) to the audio signals, and we can get the spectrum diagram of the sound signal. In Fig. 15, we exhibit the spectrum diagram of sound signals after being collected and applied with FFT when the flow velocity $v = 2.3m/s$ in the Fig.1.

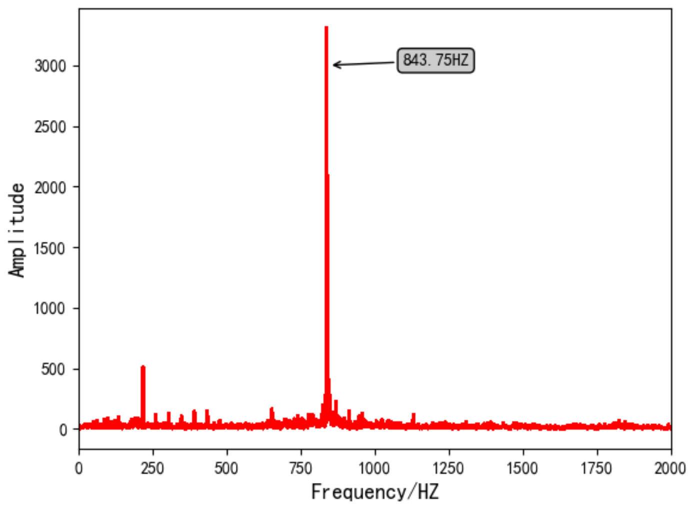

Fig. 15. the spectrum diagram corresponding to 843.75 audio was analyzed by FFT

From Fig. 15, there is a peak frequency of 843.75Hz, corresponding the most intense amplitude. There is also a relatively lower peak frequency near 240Hz. By analysis and comparison, the lower peak frequency is noise caused by the fan. Other part of the frequency spectrum diagram is environmental noise. By using this method, the relation between the whistling frequency f and the flow velocity v is obtained at different flow velocity.

2.1.2 The Relation of Flow Velocity and The Whistling Frequency

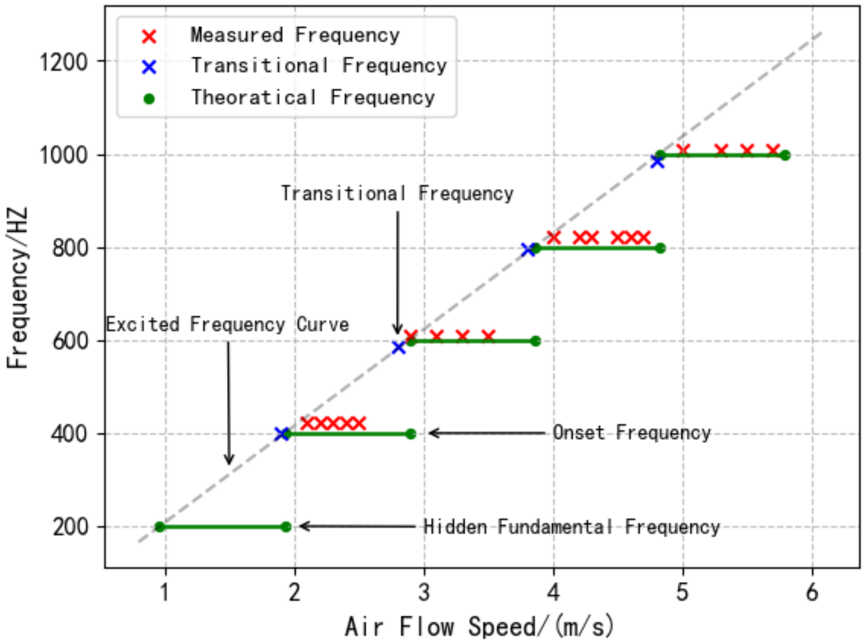

Fig. 16. the relation of flow velocity and the whistling frequency

The relation of flow velocity and the whistling frequency is shown in Fig. 16. Green line segments represent theoretical velocity-whistling frequency diagram; the gray dotted line represents theoretical velocity-excitation frequency diagram. The excitation frequency is in direct proportion to flow velocity. Through the two kinds of lines, the whistling frequency exists in a "platform" regulation within certain velocity condition, like the discussion in the theory part. Red crunodes represent whistling frequencies detected in experiment; blue crunodes represent the alternating transition frequencies. Because of their indeterminacy, we cannot record a massive amount of data to analyze them. According to the Fig. 16, the base frequency of the sound tube, the first mode of frequency (the lowest green platform) is not able to be excited, which is the phenomenon of the "hidden" fundamental frequency in the theory.

As the flow velocity increases, the first detectable frequency is the sound tube's second mode. Within a certain velocity boundary, the whistling frequency is constant; they are discrete in the diagram. It is consistent with the theoretical analysis before and also verifies out explanation of the whistling frequency: if and only if the excitation frequency caused by air flow is corresponding to a mode of the natural frequency of a sound tube, the tube could resonate and the frequency could be amplified.

2.2 The Relation of Tube's Length and "Hidden" Frequency

From Formula (9), as the length of the sound tube increases, the natural frequency decreases; however, the onset frequency is consistent. It means longer tube's more frequencies would be hidden, while shorter tube has more excitable natural frequencies.

In the experiment of 2.1, we use the tube with length of 0.7m, and its lowest excitable mode is the second mode. The natural frequency of a tube with length of 0.35m is twice higher than that of 70-centimeter tube. The onset frequency is irrelevant to tube's length, given by Formula (24); thus, "hidden" frequencies for 70-centimeter tube are excitable for 35-centimeter tube. To verify the hypothesis, the relation between different sound tube lengths and excitable frequencies is researched. Based on Formula (25), we limit the onset frequency for tubes:

$$f_{min} = \frac{Re_{min}\mu}{\rho_{air}d(r+R)} = \frac{2300 \times 1.83 \times 10^{-4}}{1.293 \times 10^{-3} \times 0.5 \times 2.3} = 283.06\text{Hz} \tag{46}$$

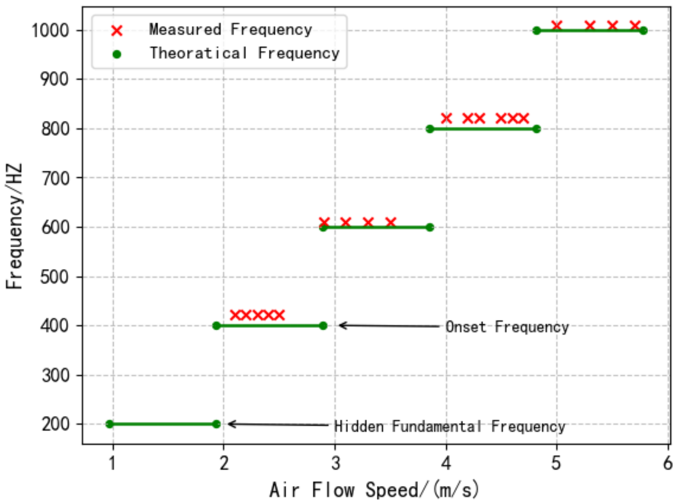

Fig.17(a). The frequencies with length 70cm

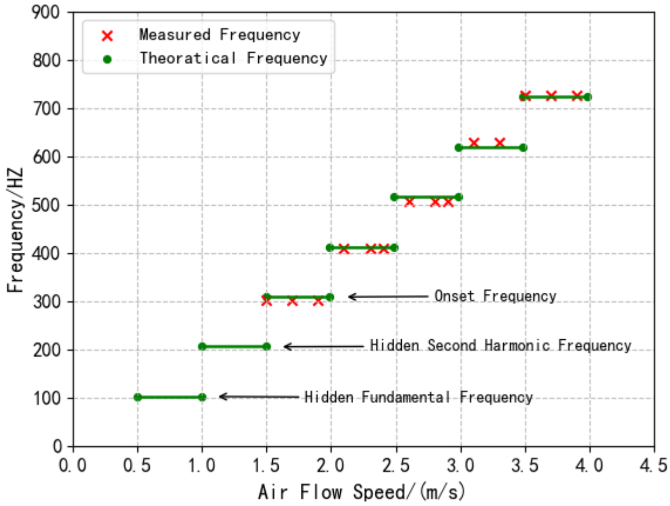

Fig.17(b). The frequencies with length 135.6cm

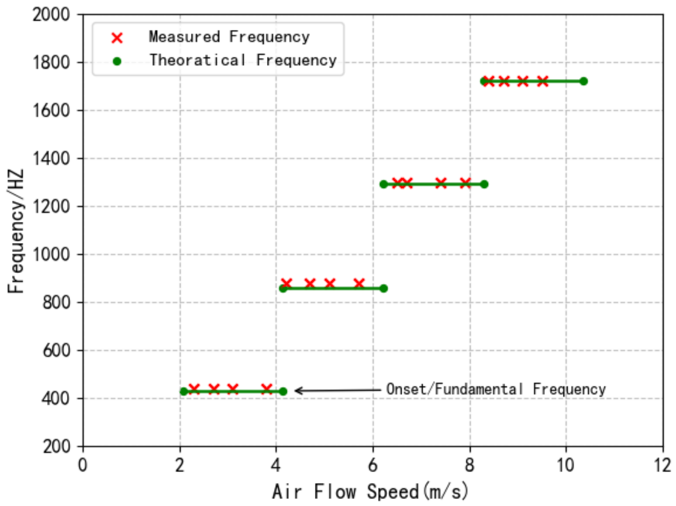

Fig.17(c). The frequencies with length 32.6cm

Table 1 the relation between the number of "hidden" frequencies and length and fundamental frequencies

Length(cm)	1st.mode(Hz)	Number	Onset frequency(Hz)	Onset whistling(Hz)
32.6	430.03	0	283.06	430.03
70.0	200.00	1	283.06	400.00
135.6	103.24	2	283.06	309.72

Table 1 represents organized data in three figures of Fig. 17. The number of "hidden" frequencies changes as the fundamental frequency changes. When the length is low (32.6cm), the first mode is excitable. When the length increases to 70cm, the first mode cannot be excited; when the length is 135.6cm, the second mode is also not excitable. It corresponds to the theory. In conclusion, longer tube as lower natural frequencies; because the lowest excitable frequency is irrelevant to its length, longer tube has more “hidden” frequencies.

2.3 Exploration of the Influence of the Doppler Effect on Swinging Tube

At the end of the theory part before, the influence of the Doppler effect on swinging tube is discussed, and the result is shown by Formula (44). In the experiment above, we all utilize the methodology of pumping air into the motionless pipe by the fan to conduct the experiment in order to eliminate the influence of Doppler effect. In the experiment below, we are going to research the change of the frequency when the tube is rotating.

2.3.1 Experimental Setup

The experimental arrangement is shown in Fig. 18. The sound signals are collected by the microphone, and the tube is open at both ends and rotates at a constant angular velocity carried by the electric motor. We can continuously change the rotational speed of the tube by altering the volts between the electric motor. The microphone is on the rotating plain of the tube.

Fig. 18. Setup’s front view (left) and top view (right)

2.3.2 Tube’s Whistling and Doppler Effect

We can get different signals of one tube by altering the rotational speed. In Fig. 19, red crunodes are experimental data points, which fit well with blue theoretical curve. As the relative translational speed changes, the frequency fluctuates following the blue curve. As the rotational speed increases, the amplitude of fluctuation of frequency increases. When the rotational speed increases to a certain value, the frequency also “jumps” to next mode. It means the Doppler effect only leads to frequency fluctuation and does not change the fact of excitation frequency and natural frequency. The conclusion also corresponds to the theoretical analysis.

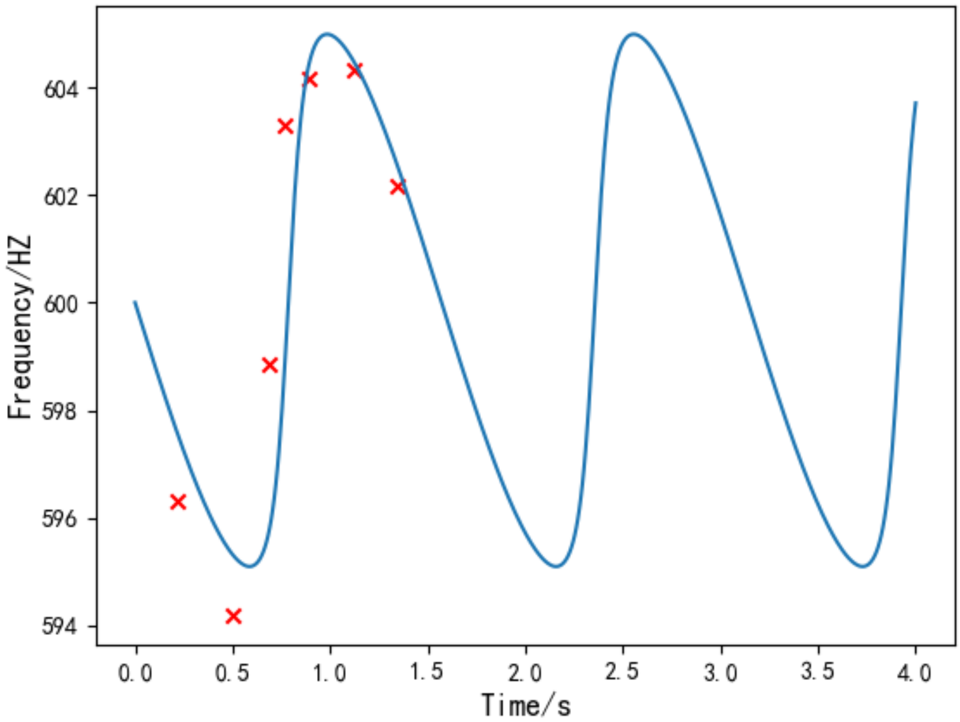

Fig. 19. influence of Doppler effect on received signals

3.Conclusion

In the paper, the natural frequency of the sound tube and the reason of air flowing through the tube are studied. When the tube rotates, the air is pumped into the tube based on the Bernoulli's Principle. The velocity of the air inside the tube is consistent and directly proportional to rotational speed and tube's length. The excitation frequency caused by the air flow is direct proportional to air's velocity and inversely proportional to corrugated intervals. We verified that the smooth tube is silent and not able to be excited to sing. For a sound tube, the natural frequency is inversely proportional to its length, directly proportional to ratio of corrugated volume and total volume. Combined with resonation theory, the paper derives that whistling frequency is discrete, in positive correlation to air velocity, and the magnitude corresponds to tube's natural frequency. Quantitative experiment verifies the theory.

In relation to the state of fluid flow, the paper explains the sound mechanism of the sound tube: if and only if the air flows turbulently inside the tube and vortexes form, the air flow can strike the tube system and oscillate the tube, like wooden frog; meanwhile, based on the conditions for turbulence and the value of Reynolds number, the paper explains the reason why some of natural frequencies are "hidden": turbulence must exist and accordingly, the excitation frequency can be generated. The condition requires the air velocity achieves a threshold. Then, the onset frequency is derived. Experiment successfully verifies the theory.

Finally, the paper points out the influence of Doppler effect to signals collected: when the pick-up is stationary on the rotational plain of the tube, signals collected would fluctuate in the vicinity of a certain frequency, and the theory is also verified by the experiment. Based on the research, we propose that a new kind of instrument could be designed based on the sound tube. Several tubes with different length are controlled by SCM to rotate in different speed. They sing together, producing different frequencies, and create a “physical song” by using the “singing” sound tubes.

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