Optimising Formula 1 Race Strategies

How does the application of integration, Monte Carle simulations, and game theory aid in the optimisation of Formula 1 race strategies?

Mathematics

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Table of Contents

Introduction

How does the application of integration, Monte Carlo simulations, and game theory aid in the optimization of Formula 1 race strategies?

Strategy is an essential part of F1 races as the difference between cars and drivers is extremely small. Drivers and teams must decide when to pit and what tyres to use. They must also take into account the decisions of other drivers to gain an advantage. In this essay, regression will initially be utilised to calculate the rate of tyre wear to set up a race time model of the Bahrain Grand Prix. Integration will then be applied to the model to calculate the different possible total race times. The various race times will then be applied to a Monte Carlo simulation which includes random variables such as safety cars. The results of the Monte Carlo simulation will be placed into several different race situations where the game theory will be applied. The game theory will be used to optimise the strategy meaning that the best possible decisions will be taken while considering other drivers and their possible strategies. This includes the use of both pure and mixed strategies, and finding the Nash equilibrium for these games.

Race Time Model

To predict the total length for any possible race, a model utilising various equations is generated to find the value of T (total race time). There are two components of time in a race: the pit stop and the stint time. In each race, Formula 1 drivers have various tyres with different speeds and lifespans. Therefore, depending on the compound utilised, the total time driving on the track is composed of varying stints combined into the total stint time (S_T). For example, $S_T = S_4 + S_3 + S_2$. S_4 is the total stint time for C4 (soft) compound tyres. S_3 and S_2 for the C3 (medium) and C2 (hard) tyres, respectively. Total race time (T) then is defined as $T = nP_t + S_T$ in which n is the number of pit stops, and (P_t) represents the time it takes to complete each pit stop.

As the tyre gets used it wears down, meaning that the stint time is defined as a linear relationship between L (the number of laps driven with that compound) and that lap's time. The slope of this linear equation would then be W which is the time added per lap as a result of tyre wear. The final linear equation is $S_c = W_c L_c + (I_c - W_c)$ with I representing the ideal lap times on new tyres without any wear. The constant was concluded as being $(I_c - W_c)$ because the value of S_c should be equal to I when $L_c = 0$.

Applying The Model

To test and apply the model to a real-life situation, the Bahrain GP was chosen as it is a race in which strategy is vital due to the increased tyre wear from the intense desert heat. Firstly, the values of W had to be calculated. The first step was to find the time increase per lap due to tyre wear. Three separate regressions were calculated for each tyre compound. The C4 (soft), C3 (medium), and C2 (hard).

For the C4 compound, all lap times with this compound were placed in a table with the tyre life as the independent variable. Only tyre lives with more than five laps were used to achieve consistent results. Out laps (lap immediately after pit) and in laps (lap immediately before pit) were ignored as the lap times were inflated due to the driver slowing down in the pit lane.

Figure 1

Tyre Life (Lap)	Norris	Leclerc	Ricciardo	Sainz	Stroll	Ocon	Russell	Latifi	Alonso	Average
1	97.893	98.294	97.571	97.112	97.046	97.855	100.223	98.390	97.652	98.004
2	96.906	97.148	97.205	97.139	97.209	97.991	99.318	98.518	97.366	97.644
3	97.724	97.886	97.639	97.829	97.700	98.234	98.317	98.784	98.006	98.013
4	97.360	98.667	97.858	98.396	97.797	97.866	98.415	98.027	99.062	98.161
5	97.523	97.518	97.782	98.012	97.614	98.818	98.386	98.280	97.758	97.966
6	97.162	97.729	97.314	97.562	97.536	97.945	98.471	98.781		97.813
7			98.454	97.716		98.201	98.719	98.526		98.323

The average lap times were then taken as the Y values while the tyre age was the X values. After placing them in a table, I used the least squares method to find the slope of the regression line. The following equation was used with n being the number of data points, which was 7 in this table. I then calculated the XY and X^2 columns. I then found the sum of the X, Y, XY, and X^2 numbers, and used those values for the regression line formula. The goal of the regression line formula is to find the gradient of the curve by setting up squares between various sets of X and Y points. This method can be compiled into the following equation.

$$\beta_1 = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2}$$

C4 Table for Regression Formula

X Tyre Age (Laps)	Y Lap Time (s)	XY	X^2
1	98.004	98.004	1
2	97.644	195.289	4
3	98.013	294.040	9
4	98.161	392.644	16
5	97.966	489.828	25
6	97.813	586.875	36
7	98.323	688.262	49
28	685.924	2,744.942	140

$$\beta_1 = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2}$$

$$\beta_1 = \frac{(7)(2,744.942)-(28)(685.924)}{(7)(140)-(28)^2}$$

$$\beta_1 = \frac{19,214.594-19,205.872}{980-784}$$

$$\beta_1 = \frac{8.722}{196}$$

$$\beta_1 = 0.0445$$

This formula gave me the value 0.0445 which in this situation is the increase of time per lap in seconds. This can be modelled as W_4 which is the wear per lap on the C4 compound.

I then repeated this process two more times with the C3 Compound and the C2 Compound respectfully.

$$\beta_1 = \frac{n(\Sigma xy)-(\Sigma x)(\Sigma y)}{n(\Sigma x^2)-(\Sigma x)^2}$$

$$\beta_1 = \frac{(17)(14,876.879)-(153)(1,651.777)}{(17)(1,785)-(153)^2}$$

$$\beta_1 = \frac{252,906.943-252,721.881}{30,345-23,409}$$

$$\beta_1 = \frac{185.062}{6,936}$$

$$\beta_1 = 0.0267\ (C3)$$

$$\beta_1 = \frac{n(\Sigma xy)-(\Sigma x)(\Sigma y)}{n(\Sigma x^2)-(\Sigma x)^2}$$

$$\beta_1 = \frac{(23)(26{,}470.792)-(276)(2{,}203.449)}{(23)(4{,}324)-(276)^2}$$

$$\beta_1 = \frac{608{,}828.216-608{,}151.924}{99{,}452-76{,}176}$$

$$\beta_1 = \frac{676.292}{23{,}276}$$

$$\beta_1 = 0.0291\ (C2)$$

The two calculations produced the values $W_3 = 0.0267$ and $W_2 = 0.0291$.

In addition to finding the values of W for each compound, the pit stop time for the Bahrain GP had to be calculated as every track has a different pit lane. P_t was determined as 24.647 as all complete pit stop times from the 2021 Bahrain GP were taken and the average was found. Next, the value of I was concluded to be 93.096s for the C4 (soft) compound, this was determined by finding the average of the initial stint laps set in a practice session (see appendix). Using data from Pirelli the tyre manufacturer, at Bahrain the C3 medium compound is 0.9s slower per lap than the C4 and the C2 (hard) compound is 1.3s slower than the C4. Making the value of T 93.996 for C3 and 94.396 for C2. Additional information for the formulas is the domain of the various variables. $L_4 \leq 15$, this is because the soft tyres were determined to not last more than 15 tyres while keeping optimal performance. Extending the stint past this many laps runs the risk of the tyres not only popping, but also drastically reduces times as a result of wear. Additionally, $L_3 \leq 20$ and $L_2 \leq 30$.

Calculating Optimal Race Strategy

Now that the formulas and variables are all defined, they can be used to find the optimal strategy based on the number of laps each stint runs. For the Bahrain race strategy, a two-stop is the best because no combination of two tyres can add up to 56 without extending a compound beyond its tyre life.

First I took the following formulas and substituted them into the $S_T = S_4 + S_3 + S_2$ equation.

$S_4 = W_4 L_4 + (I_4 - W_4)$

$S_3 = W_3 L_3 + (I_3 - W_3)$

$S_2 = W_2 L_2 + (I_2 - W_2)$

This gave me $S_T = W_4 L_4 + (I_4 - W_4) + W_3 L_3 + (I_3 - W_3) + W_2 L_2 + (I_2 - W_2)$. This formula was then substituted into the initial equation $T = nP_t + S_T$ for the final equation:

$$T = nP_t + W_4 L_4 + (I_4 - W_4) + W_3 L_3 + (I_3 - W_3) + W_2 L_2 + (I_2 - W_2)$$

However for the value of T to be calculated, the integral of S_T has to be found to find the times of all the laps from each stint added up.

$$T = nP_t + \int L_4 + (I_4 - W_4) + W_3 L_3 + (I_3 - W_3) + W_2 L_2 + (I_2 - W_2) dL$$

$$T = n24.647 + \int 0.0445L_4 + (93.0515) + 0.0267L_3 + (93.9693) + 0.0291L_2 + (94.3669) dL$$

$$= 49.294 + \int 0.0445L_4 + (93.0515) + 0.0267L_3 + (93.9693) + 0.0291L_2 + (94.3669)\, dL$$

$$= 49.294 + (0.0445L_4^2 + 93.0515L_4) + (0.0267L_3^2 + 93.9693L_3) + (0.0291L_2^2 + 94.3669L_2)$$

Once the indefinite integral of S_T was found, Lagrange's multiplier was utilised to optimise the integral containing three variables. This would reveal the combination of laps from each compound resulting in the lowest total stint time.

$$f(L_4, L_3, L_2) = (0.0445L_4^2 + 93.0515L_4) + (0.0267L_3^2 + 93.9693L_3) + (0.0291L_2^2 + 94.3669L_2) + 49.294$$

The following constraint was added as the number of laps on each compound has to add up to 56.

$$g(L_4, L_3, L_2) = 56$$

$$g(L_4, L_3, L_2) = L_4 + L_3 + L_2$$

A system of equations was then set up using λ as a constant.

$$f(L_4) = \lambda g(L_4)$$

$$0.0445L_4 + 93.0515 = \lambda(1)$$

$$0.0445L_4 = \lambda - 93.0515$$

$$L_4 = 22.472\lambda - 2091.045$$

$$f(L_3) = \lambda g(L_3)$$

$$0.0267L_3 + 93.9693 = \lambda(1)$$

$$0.0267L_3 = \lambda - 93.9693$$

$$L_3 = 37.453\lambda - 3519.449$$

$$f(L_2) = \lambda g(L_2)$$

$$0.0291L_2 + 94.3669 = \lambda(1)$$

$$0.0291L_2 = \lambda - 94.3669$$

$$L_2 = 34.364\lambda - 3242.849$$

The three equations were then substituted into the constraint to find the value of λ.

$$56 = L_4 + L_3 + L_2$$

$$56 = (22.472\lambda - 2091.045) + (37.453\lambda - 3519.449) + (34.364\lambda - 3242.849)$$

$$56 = 94.289\lambda - 8853.343$$

$$94.289\lambda = 8909.343$$

$$\lambda = 94.490$$

$$L_4 = 22.472\lambda - 2091.045$$

$$L_4 = 22.472(94.490) - 2091.045$$

$$L_4 = 32.328$$

$$L_3 = 37.453\lambda - 3519.449$$

$$L_3 = 37.453(94.490) - 3519.449$$

$$L_3 = 19.478$$

$$L_2 = 34.364\lambda - 3242.849$$

$$L_2 = 34.364(94.490) - 3242.849$$

$$L_2 = 4.195$$

Rounding the values to integers as there can only be a whole number of laps. (19.478 was rounded to 20 because the three integers had to add up to 56). These can be substituted into the original race time equation.

$$f(L_4, L_3, L_2)$$

$$= (0.0445(32)^2 + 93.0515(32)) + (0.0267(20)^2 + 93.9693(20)) + (0.0291(4)^2 + 94.3669(4)) + 49.294$$

$= (3000.432) + (1884.726) + (377.700) + 49.294$

$= 5312.152\ seconds$

$1:28:32.984\ race\ time$

$f(L_4, L_3, L_2)$

$= (0.0445(15)^2 + 93.0515(15)) + (0.0267(20)^2 + 93.9693(20)) + (0.0291(21)^2 + 94.3669(21)) + 49.294$

$= (1400.779) + (1884.726) + (1988.121) + 49.294$

$= 5322.920\ seconds$

$1:28:42.551\ race\ time$

Monte Carlo Simulations

Although the race time model can be used to determine the optimal strategy, it does not account for random events that can occur during the race such as a crash. In this investigation, I used a Monte Carlo Simulation to account for these random variables. Monte Carlo simulations are primarily used for betting or business uses however they are an actual technique used by Formula 1 teams to account for the vast possible circumstances. The main component of the simulation is the safety car which includes the virtual safety car. By using a computer simulation, thousands of different scenarios can be run to provide an extensive amount of possible race scenarios.

I. **Safety Cars**

Safety Cars (SCs) are extremely common in Formula 1 races as they are deployed whenever there is an incident. Under safety car conditions, all cars are required to slow down above a set time and queue up behind the safety car.

Safety cars can impact strategy as they can occur at any time during the race. In order to simulate the probabilities of a safety car coming out and the race time it could add, data from the past eight F1 Bahrain races was compiled (the last eight were chosen because 2014 marks the beginning of the current car era). Using this data it was concluded that there is a probability of 37.5%% for a safety car coming out before the first pit stop and 37.5% of coming out after the second pit stop. The standard deviation was then found using the sample standard deviation formula in which each sample was compared to the mean in order to set up a bell curve distribution. The standard deviation was concluded to be 51.75%. The number of laps the safety car lasts is also another random variable as it depends on the severity of the incident. The mean number of laps during a safety car was 4.167 with a standard deviation of 1.472.

	Safety Car Frequency	
	Before Lap 15	After Lap 36
2021	1	0
2020	1	1
2019	0	1
2018	0	0
2017	1	0
2016	0	0
2015	0	0
2014	0	1
Mean	37.50%	37.50%

	Safety Car Length (laps)	Mean	Difference from Mean	Difference Squared
1	3	4.167	-1.167	1.361
2	6		1.833	3.361
3	3		-1.167	1.361
4	3		-1.167	1.361
5	4		-0.167	0.028
6	6		1.833	3.361
			Sum of Squares	10.833
			Divided by Sample Number Minus 1 (5)	2.167
			Standard Deviation ($\sqrt{2.167}$)	1.472

A. Virtual Safety Cars

Virtual safety cars (VSCs) are another form of safety car which can be deployed during races although they are less common. Virtual safety cars consist of a “virtual” car that essentially requires all the drivers to drop their speed. The probability of a VSC was calculated the same way as with the safety car with the mean being 12.5% and the standard deviation being 35.4%.

II. Simulating Safety Cars

Now that the means and standard deviations have been calculated, the various race scenarios were simulated in Microsoft Excel to generate random probabilities of a safety car occurring and the number of laps it lasts.

Projected Race Time Change		Max	Min	Mean	StdDev
SC Deployment		1	0	0.375	0.5175
VSC Deployement		1	0	0.125	0.353553
Laps Under Safety Car		6	3	4.166667	1.47196

Once everything was set up, I condued 5 simulations of 5000 races to get the most data points possible.

Simulation	Number of Races without any SC	%
1	3920	78.4
2	3901	78.02
3	3946	78.92
4	3963	79.26
5	3875	77.5

Utilising these 5 simulations, the percentage of races out of the 25,000 that had a safety car was 21.58%. Meaning that about 1 out of every 5 races had a safety car. Now I had to find the average amount of laps each safety car lasted for using five simulations again.

Laps Under Safety Car	Simulation 1 Frequency	Simulation 2 Frequency	Simulation 3 Frequency	Simulation 4 Frequency	Simulation 5 Frequency	%
1	72	62	63	98	66	1.44
2	376	291	273	345	310	6.38
3	898	699	686	780	696	15.04
4	1417	1187	1250	1132	1196	24.73
5	717	1317	1285	1616	1315	25.00
6	977	900	864	601	904	16.98
7	427	403	435	289	364	7.67
8	98	111	112	87	121	2.12
9	7	17	19	26	15	0.34
10	1	3	2	0	1	0.03
11	1	0	0	0	0	0.00

The probability of laps under a safety car is dependent on the probability of a safety car being deployed. This can be used to set up another probability equation in which the probability of a safety car being deployed is multiplied by all the probabilities of safety car laps.

Probability of laps under safety car given that a safety car is deployed

1 lap	2 laps	3 laps	4 laps	5 laps	6 laps	7 laps	8 laps	9 laps	10 laps	11 laps
0.311%	1.377%	3.246%	5.337%	5.395%	3.664%	1.655%	0.457%	0.073%	0.006%	0.000%

A. Virtual Safety Cars

The probability of a VSC is extremely low as I conducted five simulations to see the frequency of VSCs and found that less than 1 percent of all the races simulated had a VSC deployed.

Simulation	Number of Races with a VSC	%
1	41	0.82%
2	34	0.68%
3	44	0.88%
4	27	0.54%
5	32	0.64%

Applying Game Theory

These previous models can be successfully utilised to create the fastest race strategies, however, they fail to account for other drivers. This is where game theory can be applied. Game theory is defined as “the study of mathematical models of action between intelligent rational decision-makers” (Myerson 1). This highlights two key components of game theory, firstly there must be more than one “player” and secondly all the “players” are rational meaning that they will always attempt to maximise their “pay-off” or rewards. This applies perfectly to Formula 1 in which 20 different drivers all attempt to run the most optimal strategy in order to win the race.

Game theory allows for a new element to be introduced. How can altering the optimal strategy pit stops allow a driver to gain the upper hand against a rival? This can be addressed by both the planning of the tyre strategy and the decision making when safety cars come out.

For the various different games, the race situation will involve two players, drivers A and B. We will assume that both their cars are the same speed and driver A will start 1st on the grid with driver B starting behind A in second.

I. Number Of Pit Stops (Two Player Zero Sum Game)

Driver B is starting behind driver A, and they will both run the optimal two-stop strategy as calculated before. However, driver B could opt for the three-stop strategy. The initial optimal strategy calculated involved 32 laps on the soft (C4), 20 on the medium (C3), and 4 on the hard (C2). This was not possible with the two-stop but if the laps on the C4 were decreased to 30 (two stints of 15) and the laps on hards increased to 6 then a three-stop strategy could work. This was substituted into the equation from section 1 with an extra 24.647 added as a result of the third pitstop.

$$= 2(0.0445(15)^2 + 93.0515(15)) + (0.0267(20)^2 + 93.9693(20)) + (0.0291(6)^2 + 94.3669(6)) + 3(24.647)$$

$$= 2(1405.785) + (1890.066) + (567.249) + 73.941$$

$$= 5342.826\ seconds$$

$$1:29:02.493\ race\ time$$

The resulting time was 19.906 seconds slower than the optimal two-stop strategy. This is too much for it to be a feasible strategy for contesting against driver A. The three-stop shown above has all three compounds yet the F1 rules state that only two compounds are needed to be used in a race. This could allow for another three-stop strategy with 36 laps on softs (three stints of 12), 20 on mediums, and 0 on the hards.

$$= 3(0.0445(12)^2 + 93.0515(12)) + (0.0267(20)^2 + 93.9693(20)) + (0.0291(0)^2 + 94.3669(0)) + 3(24.647)$$

$$= 3(1123.026) + (1890.066) + (0) + 73.941$$

$$= 5333.085\ seconds$$

$$1:28:53.560\ race\ time$$

This strategy is 10.165 seconds slower than the optimal two-stop strategy. Although it is slower, it can represent a feasible strategy during the race if the time can be made up through safety cars. The following two-player zero-sum game can be set up with the payoffs as (time change driver B, time change driver A) with time in seconds.

Driver A

	2 stop strategy	3 stop strategy	Maximin
2 stop strategy	(0,0)	(-10.165,10.165)	(-10.165, 0)
3 stop strategy	(10.165, -10.165)	(0,0)	(0,-10.165)
Minimax	(10.165, 0)	(0, 10.165)	

Driver B

The saddle point for the row player is 0 meaning that the optimum strategy for driver B would be to run the 3 stop strategy. The saddle point for driver A is also 0 meaning that their optimum strategy is the 2 stop. However, driver B could risk running the 2 stops in hopes that driver A runs the 3 stops as that is the only situation where driver B would win. This is however very unlikely meaning that driver B can not gain an advantage by changing the number of pit stops.

II. Pitting During a Safety Car

Safety cars can have a huge impact on strategy as drivers lose 4 seconds less per stop than under regular conditions. The race can be split into three phases of the optimal two-stop strategy before the 1st pit stop, after the 1st pit stop, and after the 2nd pit stop. Assuming a uniform distribution of safety car probability, there is a 0.385% chance of a safety car coming out each lap. $21.58\%/56 = 0.385\%$. The probability of a safety car during each phase can be calculated by multiplying 0.358% by the number of laps. These are the phases for the strategy with softs as the starting tyre.

Race Phase	Lap 1-15 (before 1st pit stop)	Lap 16-36 (after 1st pit stop)	Lap 37-56 (after 2nd pit stop)
Probability of SC	5.78%	8.09%	7.70%

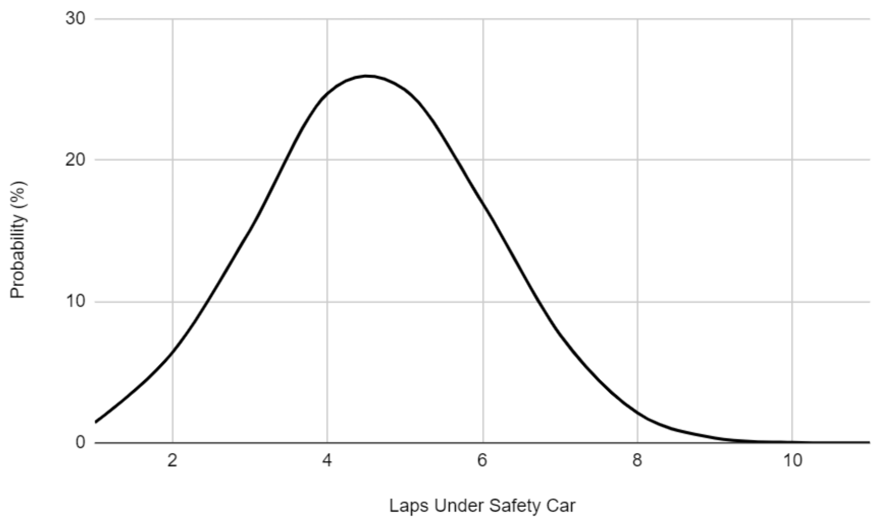
Conversely, for the strategy with mediums as the starting tyre, these are the probabilities of safety cars during each phase.

Race Phase	Lap 1-20 (before 1st pit stop)	Lap 21-41 (after 1st pit stop)	Lap 42-56 (after 2nd pit stop)
Probability of SC	7.70%	8.09%	5.78%

Hard tyres are not used at the beginning because they do not have enough grip for the race start. They are the second stint as drivers do not like finishing on them.

Situation 1: Safety Car Before 1st Stop

If there is a safety car before the first pit stop then the decision for what tyres to put on depends on the length of the safety car, the lap of the safety car, and starting tyre of the driver. Using the data from the previous section, the majority of safety cars last 3-6 laps with 5 laps being the most probable. The change in race time will be calculated assuming that the safety cars lasted 5 laps with a lap time of 139.14 seconds per lap (see appendix for table). Additionally, four seconds will be taken off from the total pitstop time.



The possible scenarios were decided by either pitting the driver or leaving them out when a safety car came out. This determined the number of laps on each compound because the number of laps per compound had to fit within the constraints ($L_4 \leq 15,\ L_3 \leq 20,\ L_2 \leq 25$).

Scenario 1 is the "control variable" that will be used to see how much the race time changes compared to the optimal strategy without a safety car.

Scenarios for Drivers that Started on Soft Tyres

Scenario	Probability	SC Deployment Lap	Laps on Softs (C4)	Laps on Mediums (C3)	Laps on Hards (C2)	Pit Stops	Pitted Under Safety Car?
1	78.42%	No SC	15	20	21	2	No
2	1.93%	Laps 11-15	10	20	21	2	No
3	3.85%	Laps 1-10	10	20	21	2	No
4	3.85%	Laps 1-10	5	0	46 (2 stints of 23)	2	Yes

Race Time:

Scenario 2

$= (0.0445(10)^2 + 93.0515(10)) + (0.0267(20)^2 + 93.9693(20)) + (0.0291(21)^2 + 94.3669(21)) + (2(24.647) - 4) + 5(139.14)$

$= (934.965) + (1890.066) + (1994.538) + (45.295) + (695.7)$

$= 5560.564\ seconds$

$1:32:40.335\ race\ time$

Scenario 3

$= (0.0445(10)^2 + 93.0515(10)) + (0.0267(20)^2 + 93.9693(20)) + (0.0291(21)^2 + 94.3669(21)) + (2(24.647)) + 5(139.14)$

$= (934.965) + (1890.066) + (1994.538) + (49.295) + (695.7)$

$= 5564.564\ seconds$

$1:32:44.335\ race\ time$

Scenario 4

$= (0.0445(5)^2 + 93.0515(5)) + (0.0267(0)^2 + 93.9693(0)) + 2(0.0291(23)^2 + 94.3669(23)) + (2(24.647) - 4) + 5(139.14)$

$= (466.37) + (0) + 2(2185.833) + (45.295) + (695.7)$

$= 5579.030\ seconds$

$1:32:59.149\ race\ time$

Medium Tyres

Scenario	Probability	SC Deployment Lap	Laps on Softs (C4)	Laps on Mediums (C3)	Laps on Hards (C2)	Pit Stops	Pitted Under Safety Car?
5	3.85%	Laps 1-10	15	15	21	2	No
6	3.85%	Laps 1-10	0	5	46 (2 stints of 23)	2	Yes
7	1.93%	Laps 11-15	15	15	21	2	No
8	1.93%	Laps 11-15	0	30 (a stint of 10 and another of 20)	21	2	Yes
9	1.93%	Laps 16-20	15	15	21	2	Yes

The same formula can be applied to calculate the race times for these scenarios.

Scenario 5 and 7

$$= (0.0445(15)^2 + 93.0515(15)) + (0.0267(15)^2 + 93.9693(15)) + 2(0.0291(21)^2 + 94.3669(21)) + (2(24.647)) + 5(139.14)$$

$$= (1405.785) + (1415.547) + (1994.538) + (49.295) + (695.7)$$

$$= 5558.166\ seconds$$

$$1{:}\,32{:}\,38.958\ race\ time$$

Scenario 6

$$= (0.0445(0)^2 + 93.0515(0)) + (0.0267(5)^2 + 93.9693(5)) + 2(0.0291(23)^2 + 94.3669(23)) + (2(24.647) - 4) + 5(139.14)$$

$$= (0) + (470.514) + 2(2185.833) + (45.295) + (695.7)$$

$$= 5583.175\ seconds$$

$$1{:}\,33{:}\,03.103\ race\ time$$

Scenario 8

$$= (0.0445(0)^2 + 93.0515(0)) + (0.0267(20)^2 + 93.9693(20)) + (0.0267(10)^2 + 93.9693(10)) + (0.0291(21)^2 + 94.3669(21)) + (2(24.647) - 4) + 5(139.14)$$

$$= (0) + (1890.066) + (942.363) + (1994.538) + (45.295) + (695.7)$$

$$= 5567.962\ seconds$$

$$1{:}\,32{:}\,47.574\ race\ time$$

Scenario 9

$$= (0.0445(15)^2 + 93.0515(15)) + (0.0267(15)^2 + 93.9693(15)) + (0.0291(21)^2 + 94.3669(21)) + (2(24.647) - 4) + 5(139.14)$$

$$= (1405.785) + (1415.547) + (1994.538) + (45.295) + (695.7)$$

$$= 5556.865\ seconds$$

$$1{:}\,32{:}\,36.515\ race\ time$$

Situation 2: Safety Car After 1st Stop

A safety car after the first pit stop means that pitting is not always viable as it would require the driver to extend to three stops. All drivers will have hard tyres for their second stint regardless of starting tyre..

Soft Tyre

Scenario	Probability	SC Deployment Lap	Laps on Softs (C4)	Laps on Mediums (C3)	Laps on Hards (C2)	Pit Stops
10	1.93%	Laps 16-20	15	20	16	2
11	1.93%	Laps 16-20	30 (2 stints of 15)	20	1	3
12	1.93%	Laps 21-25	15	20	16	2
13	1.93%	Laps 21-25	30 (2 stints of 15)	16	5	3
14	1.93%	Laps 26-30	15	20	16	2
15	1.93%	Laps 26-30	41 (a stint of 15 and 2 of 13)	0	10	3
16	1.93%	Laps 31-35	15	20	16	2
17	1.93%	Laps 31-35	15	0	36 (a stint of 15 and another of 21)	2
18	0.39%	Lap 36	15	16	20	2

Medium Tyre

Scenario	Probability	SC Deployment Lap	Laps on Softs (C4)	Laps on Mediums (C3)	Laps on Hards (C2)	Pit Stops
19	1.93%	Laps 21-25	15	20	16	2
20	1.93%	Laps 21-25	30 (2 stints of 15)	20	1	3

21	1.93%	Laps 26-30	15	20	16	2
22	1.93%	Laps 26-30	26 (2 stints of 13)	20	5	3
23	1.93%	Laps 31-35	15	20	16	2
24	1.93%	Laps 31-35	21 (a stint of 10 and another of 11	20	10	3
25	1.93%	Laps 36-40	15	20	16	2
26	1.93%	Laps 36-40	0	36 (a stint of 20 and another of 16)	15	2
27	0.39%	Lap 41	11	20	20	2

Scenario 10, 12, 14, 16, 19, 21, 23, and 25

$= (0.0445(15)^2 + 93.0515(15)) + (0.0267(20)^2 + 93.9693(20)) + 2(0.0291(16)^2 + 94.3669(16)) + (2(24.647)) + 5(139.14)$

$= (1405.785) + (1890.066) + (1517.32) + (49.295) + (695.7)$

$= 5558.166\ seconds$

$1:32:38.958\ race\ time$

Scenario 11 and 20

$= 2(0.0445(15)^2 + 93.0515(15)) + (0.0267(20)^2 + 93.9693(20)) + (0.0291(1)^2 + 94.3669(1)) + (3(24.647) - 4) + 5(139.14)$

$= 2(1405.785) + (1890.066) + (94.396) + (69.941) + (695.7)$

$= 5561.673\ seconds$

$1:32:41.402\ race\ time$

Scenario 13

$= 2(0.0445(15)^2 + 93.0515(15)) + (0.0267(16)^2 + 93.9693(16)) + (0.0291(5)^2 + 94.3669(5)) + (3(24.647) - 4) + 5(139.14)$

$= 2(1405.785) + (1510.344) + (472.562) + (69.941) + (695.7)$

$= 5560.117\ seconds$

$1:32:40.071\ race\ time$

Scenario 15

$= (0.0445(15)^2 + 93.0515(15)) + 2(0.0445(13)^2 + 93.0515(13)) + (0.0267(0)^2 + 93.9693(0)) + (0.0291(10)^2 + 94.3669(10)) + (3(24.647) - 4) + 5(139.14)$

$= (1405.785) + 2(1217.19) + (0) + (946.579) + (69.941) + (695.7)$

$= 5552.385\ seconds$

$1{:}\,32{:}\,32.236\ race\ time$

Scenario 17

$= (0.0445(15)^2 + 93.0515(15)) + (0.0267(0)^2 + 93.9693(0)) + (0.0291(15)^2 + 94.3669(15)) + (0.0291(21)^2 + 94.3669(21)) + (2(24.647) - 4) + 5(139.14)$

$= (1405.785) + (0) + (1422.051) + (1994.538) + (45.294) + (695.7)$

$= 5563.368\ seconds$

$1{:}\,32{:}\,43.225\ race\ time$

Scenario 18

$= (0.0445(15)^2 + 93.0515(15)) + (0.0267(16)^2 + 93.9693(16)) + (0.0291(20)^2 + 94.3669(20)) + (2(24.647) - 4) + 5(139.14)$

$= (1405.785) + (1510.344) + (1898.970) + (45.294) + (695.7)$

$= 5556.093\ seconds$

$1{:}\,32{:}\,36.534\ race\ time$

Scenario 22

$= 2(0.0445(13)^2 + 93.0515(13)) + (0.0267(20)^2 + 93.9693(20)) + (0.0291(5)^2 + 94.3669(5)) + (3(24.647) - 4) + 5(139.14)$

$= 2(1217.190) + (1890.066) + (472.562) + (69.941) + (695.7)$

$= 5562.649\ seconds$

$1{:}\,32{:}\,42.386\ race\ time$

Scenario 24

$= (0.0445(10)^2 + 93.0515(10)) + (0.0445(11)^2 + 93.0515(11)) + (0.0267(20)^2 + 93.9693(20)) + (0.0291(10)^2 + 94.3669(10)) + (3(24.647) - 4) + 5(139.14)$

$= (934.965) + (1028.951) + (1890.066) + (946.579) + (69.941) + (695.7)$

$= 5566.202\ seconds$

$1{:}\,32{:}\,46.127\ race\ time$

Scenario 26

$= (0.0445(0)^2 + 93.0515(0)) + (0.0267(20)^2 + 93.9693(20)) + (0.0267(16)^2 + 93.9693(16)) + (0.0291(15)^2 + 94.3669(15)) + (2(24.647) - 4) + 5(139.14)$

$= (0) + (1890.066) + (1510.344) + (1422.051) + (45.294) + (695.7)$

$= 5563.455\ seconds$

$1:32:36.534\ race\ time$

Scenario 27

$= (0.0445(11)^2 + 93.0515(11)) + (0.0267(20)^2 + 93.9693(20)) + (0.0291(20)^2 + 94.3669(20)) + (2(24.647) - 4) + 5(139.14)$

$= (1028.951) + (1890.066) + (1898.970) + (45.294) + (695.7)$

$= 5558.981\ seconds$

$1:32:38.585\ race\ time$

Situation 3: Safety Car After 2nd Stop

A safety car after the second pit stop means that the driver does not need to pit anymore and can finish the race on their current tyres. However, pitting onto tyres can still be a viable option as they can go onto newer faster tyres that will give them an advantage.

Soft Tyres

Scenario	Probability	SC Deployment Lap	Laps on Softs (C4)	Laps on Mediums (C3)	Laps on Hards (C2)	Pit Stops
28	1.93%	Laps 37-41	15	15	21	2
29	1.93%	Laps 37-41	29 (a stint of 15 and another of 14)	1	21	3
30	1.93%	Laps 42-46	15	15	21	2
31	1.93%	Laps 42-46	25 (a stint of 15 and another of 10)	5	21	3
32	1.93%	Laps 47-51	15	15	21	2
33	1.93%	Laps 47-51	20 (a stint of 15 and another of 5)	10	21	3
34	1.93%	Laps 52-56	15	15	21	2

Scenario 28, 30, 32, and 34

$$= (0.0445(15)^2 + 93.0515(15)) + (0.0267(15)^2 + 93.9693(15)) + 2(0.0291(21)^2 + 94.3669(21)) + (2(24.647)) + 5(139.14)$$

$$= (1405.785) + (1415.547) + (1994.538) + (49.295) + (695.7)$$

$$= 5558.166\ seconds$$

$$1:32:38.958\ race\ time$$

Scenario 29

$$= (0.0445(15)^2 + 93.0515(15)) + (0.0445(14)^2 + 93.0515(14)) + (0.0267(1)^2 + 93.9693(1)) + 2(0.0291(21)^2 + 94.3669(21)) + (3(24.647) - 4) + 5(139.14)$$

$$= (1405.785) + (1311.443) + (93.996) + (1994.538) + (69.941) + (695.7)$$

$= \ 5571.403\ seconds$

$1:32:51.241\ race\ time$

Scenario 31

$= (0.0445(15)^2 + 93.0515(15)) + (0.0445(10)^2 + 93.0515(10)) + (0.0267(5)^2 + 93.9693(5)) + 2(0.0291(21)^2 + 94.3669(21)) + (3(24.647) - 4) + 5(139.14)$

$= (1405.785) + (934.965) + (470.514) + (1994.538) + (69.941) + (695.7)$

$= \ 5571.443\ seconds$

$1:32:51.263\ race\ time$

Scenario 33

$= (0.0445(15)^2 + 93.0515(15)) + (0.0445(5)^2 + 93.0515(5)) + (0.0267(10)^2 + 93.9693(10)) + 2(0.0291(21)^2 + 94.3669(21)) + (3(24.647) - 4) + 5(139.14)$

$= (1405.785) + (466.370) + (942.363) + (1994.538) + (69.941) + (695.7)$

$= \ 5574.796\ seconds$

$1:32:54.415\ race\ time$

Medium Tyres

Scenario	Probability	SC Deployment Lap	Laps on Softs (C4)	Laps on Mediums (C3)	Laps on Hards (C2)	Pit Stops
35	1.93%	Laps 42-46	10	20	21	2
36	1.93%	Laps 47-51	10	20	21	2
37	1.93%	Laps 52-56	10	20	21	2

Scenarios 35. 36, and 37

$= (0.0445(10)^2 + 93.0515(10)) + (0.0267(20)^2 + 93.9693(20)) + (0.0291(21)^2 + 94.3669(21)) + (2(24.647)) + 5(139.14)$

$= (934.965) + (1890.066) + (1994.538) + (49.295) + (695.7)$

$= 5564.564\ seconds$

$1:32:44.335\ race\ time$

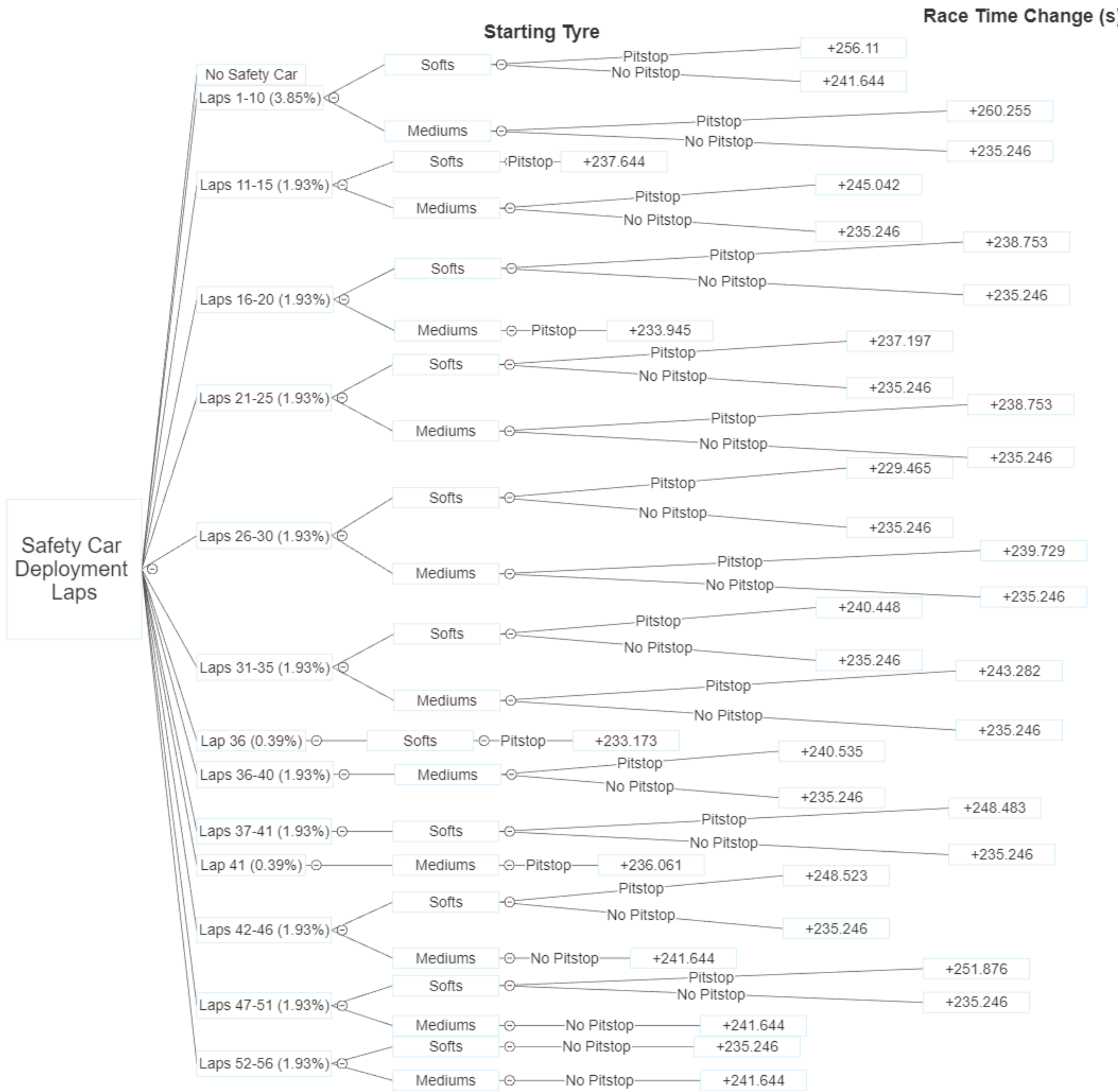
Scenario Table

Scenario	Starting Tyre	Safety Car Deployment Laps	Probability	Pit Stops	Race Time (s)	Gap to Control (s)
Control	S/M (Optimal Strategy)	No SC	78.42%	2	5322.920	-
2	Softs (C4)	Laps 11-15	1.93%	2	5560.564	237.644
3	Softs (C4)	Laps 1-10	3.85%	2	5564.564	241.644
4	Softs (C4)	Laps 1-10	3.85%	2	5579.030	256.11
5	Mediums (C3)	Laps 1-10	3.85%	2	5558.166	235.246
6	Mediums (C3)	Laps 1-10	3.85%	2	5583.175	260.255
7	Mediums (C3)	Laps 11-15	1.93%	2	5558.166	235.246
8	Mediums (C3)	Laps 11-15	1.93%	2	5567.962	245.042
9	Mediums (C3)	Laps 16-20	1.93%	2	5556.865	233.945
10	Softs (C4)	Laps 16-20	1.93%	2	5558.166	235.246
11	Softs (C4)	Laps 16-20	1.93%	3	5561.673	238.753
12	Softs (C4)	Laps 21-25	1.93%	2	5558.166	235.246
13	Softs (C4)	Laps 21-25	1.93%	3	5560.117	237.197
14	Softs (C4)	Laps 26-30	1.93%	2	5558.166	235.246
15	Softs (C4)	Laps 26-30	1.93%	3	5552.385	229.465
16	Softs (C4)	Laps 31-35	1.93%	2	5558.166	235.246
17	Softs (C4)	Laps 31-35	1.93%	2	5563.368	240.448
18	Softs (C4)	Lap 36	0.39%	2	5556.093	233.173
19	Mediums (C3)	Laps 21-25	1.93%	2	5558.166	235.246
20	Mediums (C3)	Laps 21-25	1.93%	3	5561.673	238.753
21	Mediums (C3)	Laps 26-30	1.93%	2	5558.166	235.246
22	Mediums (C3)	Laps 26-30	1.93%	3	5562.649	239.729
23	Mediums (C3)	Laps 31-35	1.93%	2	5558.166	235.246

24	Mediums (C3)	Laps 31-35	1.93%	3	5566.202	243.282
25	Mediums (C3)	Laps 36-40	1.93%	2	5558.166	235.246
26	Mediums (C3)	Laps 36-40	1.93%	2	5563.455	240.535
27	Mediums (C3)	Lap 41	0.39%	2	5558.981	236.061
28	Softs (C4)	Laps 37-41	1.93%	2	5558.166	235.246
29	Softs (C4)	Laps 37-41	1.93%	3	5571.403	248.483
30	Softs (C4)	Laps 42-46	1.93%	2	5558.166	235.246
31	Softs (C4)	Laps 42-46	1.93%	3	5571.443	248.523
32	Softs (C4)	Laps 47-51	1.93%	2	5558.166	235.246
33	Softs (C4)	Laps 47-51	1.93%	3	5574.796	251.876
34	Softs (C4)	Laps 52-56	1.93%	2	5558.166	235.246
35	Mediums (C3)	Laps 42-46	1.93%	2	5564.564	241.644
36	Mediums (C3)	Laps 47-51	1.93%	2	5564.564	241.644
37	Mediums (C3)	Laps 52-56	1.93%	2	5564.564	241.644

Game Tree

The game tree was made from all the possible scenarios based on the lap that the safety car was deployed with the change in race time depending on the starting tyre and whether the driver pitted under the safety car.



The following table shows the time gained by each compound based on when the safety car is deployed. The highlighted number is the lowest increase in time for that lap range from the four data values. The mediums are the better tyres for 35.71% of the race (20/56), the softs are better for 46.42% of the race (26/52), and both tyres are equal for 17.86% of the race (10/56).

(Time gained if driver pits under SC, time gained if driver stays out under SC) in seconds.

SC Deployment Lap	Softs	Mediums
Laps 1-10	(256.110, 241.644)	(260.255, 235.246)
Laps 11-15	(237.644, N/A)	(245.042, 235.246)
Laps 16-20	(238.753, 235.246)	(233.945, N/A)
Laps 21-25	(237.197, 235.246)	(238.753, 235.246)
Laps 26-30	(229.465, 235.246)	(239.729, 235.246)
Laps 31-35	(240.448, 235.246)	(243.282, 235.246)
Laps 36-40	(233.173, N/A)	(240.535, 235.246)
Laps 37-41	(248.483, 235.246)	(236.061, N/A)
Laps 42-46	(248.523, 235.246)	(N/A, 241.644)
Laps 47-51	(251.876, 235.246)	(N/A, 241.644)
Laps 52-56	(N/A, 235.246)	(N/A, 241.644)

Mixed Strategy Nash Equilibrium

As demonstrated earlier, we can set up a payoff matrix between the two players to determine the optimum strategy each can take. The two players will be the same as before. So drivers A and B will have the same speed car and driver A will start ahead of B on the grid.

With the safety car scenarios, it was determined which starting tyre is faster than the other for what portion of the race. However as shown earlier from the Monte Carlo simulation, there is only a 21.58% chance of a safety car being deployed. Multiplying this by the probability of the faster starting tyre, as it is a dependent event, gives you the probability of one tyre being better than the other given that a safety car is deployed.

$0.2158 * 0.3571 = 0.0771 = 7.71\%\ chance\ of\ mediums\ being\ faster\ starting\ tyre$

$0.2158 * 0.4642 = 0.1002 = 10.02\%\ chance\ of\ softs\ being\ faster\ starting\ tyre$

$0.2158 * 0.1786 = 0.0385 = 3.85\%$

$1 - 0.2158 = 78.42\%\ probability\ of\ no\ safety\ car$

$78.42\% + 3.85\% = 82.27\%\ of\ there\ being\ no\ faster\ starting\ tyre$

Because driver A and B are the same speed, if their starting tyres have equal race times then driver A will always win the race as they start ahead of driver B.

(Probability of driver B winning, probability of driver A winning)

Driver A

Driver B

	Softs (q)	Mediums $(1 - q)$
Softs (p)	(0%, 100%)	(10.02%, 89.98%)
Mediums $(1 - p)$	(7.71%, 92.29%)	(0%, 100%)

Now the optimal strategy can not be calculated using the saddle point as this situation involves mixed strategies as a result of a driver's outcome depending on the other driver's decision which they do not know. The drivers would rather randomise their strategies as there is no pure strategy Nash equilibrium.

To find the probability of each driver choosing a strategy, we will need to find the mixed strategy Nash equilibrium. This can be calculated by defining that driver B chooses softs with probability p and mediums with probability $(1 - p)$ as driver B chooses mediums the rest of the time they do not choose softs. This can also be applied to driver A with softs as probability q and mediums with probability $(1 - q)$.

We will calculate the utility of softs for driver A by multiplying probability p by the payoff of driver A for p and adding this to the payoff of driver A for $(1 - p)$ multiplied by $(1 - p)$. We then do the same to find the utility of mediums for driver A and set it equal to the utility of softs.

$$U_q = 1(p) + 0.9229(1 - p)$$

$$U_{1-q} = 0.8998(p) + 1(1 - p)$$

$$1(p) + 0.9229(1 - p) = 0.8998(p) + 1(1 - p)$$

$$p + 0.9229 - 0.9229p = 0.8998p + 1 - p$$

$$0.1773p = 0.771$$

$$p = 0.4349$$

The same method can be applied to find the value of q using the payoffs for driver B instead.

$$U_p = 0(q) + 0.1002(1 - q)$$

$$U_{1-p} = 0.0771(q) + 0(1 - q)$$

$$(q) + 0.1002(1 - q) = 0.0771(q) + 0(1 - q)$$

$$0.1002 - 0.1002q = 0.0771q$$

$$0.1002 = 0.1773q$$

$$q = 0.5651$$

These results show that driver B would use the softs 43.49% of the time and the mediums for the other 56.51%. Driver A, would use softs for 56.51% of the time and the mediums for 43.49% of the time. The table below shows the probabilities of each situation occurring, by multiplying the probability of driver B choosing that tyre by driver A choosing the tyre that corresponds to it.

Sample Calculation

$0.4349 * 0.4349 = 18.91\%$

Driver A

Driver B

	Softs	Medium
Softs	18.91%	24.58%
Medium	24.58%	31.93%

These percentages can then be used to calculate the probability of driver A winning the race and driver B winning the race taking into account the probability of a safety car coming out and changing the optimal starting tyre, and the probability of each driver choosing each starting tyre.

The following probability of each driver winning the race was calculated

$18.91\% + 31.93\% = 50.84\%$

There is a 50.84% chance of both drivers choosing the same tyre. The other two situations in which driver A would win is 89.98% of the times that they are on mediums and driver B is on softs, and 92.29% of the times that they are on softs and driver B is on mediums. Multiplying these probabilities with the mixed strategy probabilities gives us 22.12% and 22.68% chances of driver A winning. Adding these to 50.84% gives us a 95.64% probability of driver A winning the race.

$89.98\% \times 24.58\% = 22.12\%$

$92.29\% \times 24.58\% = 22.68\%$

$50.84\% + 22.12\% + 22.68\% = 95.64\%$

The probability of driver B winning the race can be calculated by subtracting 95.64% from 100% or by repeating the same steps used for driver A, to give us a 4.26% probability of driver B winning the race with all factors taken into account.

$7.71\% \times 24.58\% = 1.90\%$

$10.02\% \times 24.58\% = 2.46\%$

$1.90\% + 2.46\% = 4.36\%$

Conclusion

In conclusion, the use of mathematics can greatly aid in the optimisation of Formula 1 strategies. The race time model produced using regression and integration is greatly effective as it can provide a better view of the various tyre strategies available. Additionally, the simulation required standard deviation and other probabilistic maths to set up. Finally, the game theory allowed me to predict various race scenarios using the data produced from the first two sections of the essay. In the end, the use of the maths allowed for great decision-making when comparing two drivers (drivers A and B) and what their strategies may look like.

A possible extension in the future is taking into consideration more factors for the race time model as the data that is available online is limited. With access to more on-track data the F1 teams can create more complex models that account for every detail so getting in touch with people closer to the sport for data analysis could aid in creating better models. Additionally, the game theory was only done taking into account two drivers when in reality there are 20 drivers that race every weekend. Applying the same game theory concepts but with many more players would allow for even further probabilistic analysis and more complete decision-making.

There were no ethical issues that arose in the essay. In addition, bridging the gap between mathematics and sports is more important than ever now as teams are always looking to gain an edge and concepts such as game theory can greatly help them. Also, it is important to increase the applicability of maths and always look for new ways to implement the concepts that are taught in the classroom.

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Appendix

C3 Table for Regression Formula

X Tyre Age (Laps)	Y Lap Time (s)	XY	X^2
1	96.790	96.790	1
2	96.964	193.928	4
3	97.034	291.103	9
4	97.319	389.276	16
5	97.014	485.068	25
6	97.218	583.308	36
7	97.223	680.562	49
8	97.177	777.415	64
9	97.188	874.688	81
10	96.875	968.750	100
11	97.058	1,067.633	121
12	97.168	1,166.012	144
13	96.998	1,260.970	169
14	97.336	1,362.708	196

15	97.456	1,461.846	225
16	97.495	1,559.921	256
17	97.465	1,656.901	289
153.000	1,651.777	14,876.879	1,785.000

C2 Table for Regression Formula

X Tyre Age (Laps)	Y Lap Time (s)	XY	X^2
1	95.402	95.402	1
2	95.472	190.945	4
3	95.545	286.636	9
4	95.589	382.354	16
5	95.615	478.074	25
6	95.656	573.934	36
7	95.801	670.606	49
8	95.739	765.914	64
9	95.634	860.705	81
10	95.702	957.017	100

11	95.875	1,054.621	121
12	95.923	1,151.079	144
13	96.069	1,248.896	169
14	95.778	1,340.895	196
15	95.778	1,436.664	225
16	95.802	1,532.825	256
17	95.781	1,628.273	289
18	95.755	1,723.595	324
19	95.900	1,822.098	361
20	95.917	1,918.346	400
21	96.242	2,021.074	441
22	96.106	2,114.327	484
23	96.370	2,216.514	529
276	2,203.449	26,470.792	4,324

Pit Stop Times from 2021 Bahrain GP

24.373
24.899
24.925
24.884
24.839
24.688
24.107
25.226
24.621
26.046
25.798
24.353
25.046
24.262
24.767
24.105
24.317
24.626
24.076
25.525

24.046
24.775
24.223
24.471
24.176
24.655
23.983
25.640
24.328
25.343
24.248
24.341
24.191
23.848
24.983
24.566
24.647 (average)

Free Practice 2 Times on New C4 compound Tyres from the 2021 Bahrain GP

93.215
93.587
92.823
94.849
92.78
96.485
92.088
92.094
93.12
92.327
92.787
92.397
94.049
91.842
91.647
95.136
92.05
93.05
92.495
93.096 (average)

Average Lap Times During Safety Car from 2021 Bahrain GP

	1 SC Lap	2 SC Laps	3 SC Laps
	2:22.712	2:36.656	1:44.932
	2:22.406	2:38.001	1:44.343
	2:21.478	2:34.199	1:48.046
	2:20.999	2:32.067	1:49.988
	2:20.666	2:15.246	1:58.724
	2:21.183	2:35.188	1:46.103
	2:20.642	2:31.045	1:51.438
	2:19.070	2:28.127	1:53.477
	2:19.603	2:29.035	1:52.919
	2:18.520	2:22.096	2:00.403
	2:19.993	2:25.649	1:56.558
	2:18.879	2:25.126	1:53.761
	2:18.821	2:23.946	1:57.137
	2:18.994	2:19.901	1:57.443
	2:18.294	2:22.630	1:57.475
	2:19.552	2:17.719	2:09.259
	2:22.263	2:32.610	2:06.772
	2:19.416	2:18.832	2:01.239
	2:19.776	2:30.630	1:52.313
Average	2:10.224	2:20.172	2:27.300

$Average\text{: } 2{:}19.232 = 139.14\ seconds$