

## QUICK CONTENT REVIEW

### • Lists

- list elements can be accessed or changed by indexing into the list using '[]'  
↳ remember we have 0-indexing!!

i.e.) >> list = ['hi', 2, 'bye']

>> list[0]

'hi'

>> list[1]

2

>> len(list)

3

remember for list we can use  
len(listname) to figure out  
the length of any list

- list slicing → kind of like indexing but getting more than one element at a time

listname[startIndex (inclusive): stopIndex (exclusive): stepSize]

i.e.) >> list = [1, 3, 4, 2, 7]

>> list[1:4:2] ← start at index 1,

increment by 2  
until you reach index 4 (remember we don't include index 4)

[3, 2]

lst = []

for i in range(5):  
 lst.append(i+1)

- list comprehension → quick/compact way to create a list

[ <what you want in list> for <variable> in <some iteration> if <cond> ]  
 i=0,1,2,3,4  
 this part is optional

i.e.) [ (i+1) for i in range(5) if (i%2) == 0 ] → [ 1 3 5 ]

↳ will create a list of all #s from 0 → 5 that are even, incremented by 1

### • List Mutation

↳ can change the contents of an existing list (since lists are mutable)

→ all mutation operations except 'pop' return 'None' ('pop' returns the popped off element)

#### ④ methods④

Let lst = [] be the list we want to modify / mutate.

① append(el) : adds an element 'el' to the end of the list whom 'append' is applied to

ex.) lst.append(1)

lst

>> [1]

lst.append([1, 2, 3])

lst

>> [1, [1, 2, 3]]

the entire thing passed into  
'append' is added  
as a single item  
to the end of  
the list

must be some  
sort of  
sequence

② extend(lst2) : extends the list by concatenating it with 'lst2' ('lst2' must be an iterable)

ex.) lst.extend([1])  
 ↳ will not work/will cause an error

lst.append(1)

lst.extend([1, 2, 3])

lst

>> [1, 1, 2, 3]

all items in sequence passed  
into 'extend' are added  
as their own items to the  
list one by one

③ `insert(i, el)`: insert element 'el' at index i (doesn't replace previous value)

ex.) `lst.append(1)`

`lst.extend([1, 2, 3])`

`lst`

`>> [1, 1, 2, 3]`

`lst.insert(2, 0)`

`lst`

`>> [2, 1, 1, 2, 3]` just added a new

element at index 0 and moved the rest of the elements down

④ `remove(el)`: removes the first occurrence of 'el' in the list

ex.) `lst.append(1)`

`lst.extend([1, 2, 3])`

`lst`

`>> [1, 1, 2, 3]`

`lst.remove(1)`

`lst`

`>> [1, 2, 3]` just removed the first occurrence

of '1' from the list

⑤ `pop(i)`: removes & returns the element at index i

ex.) `lst.append(1)`

`lst.extend([1, 2, 3])`

`lst`

`>> [1, 1, 2, 3]`

`a = lst.pop(0)`

`a`

`>> 1`

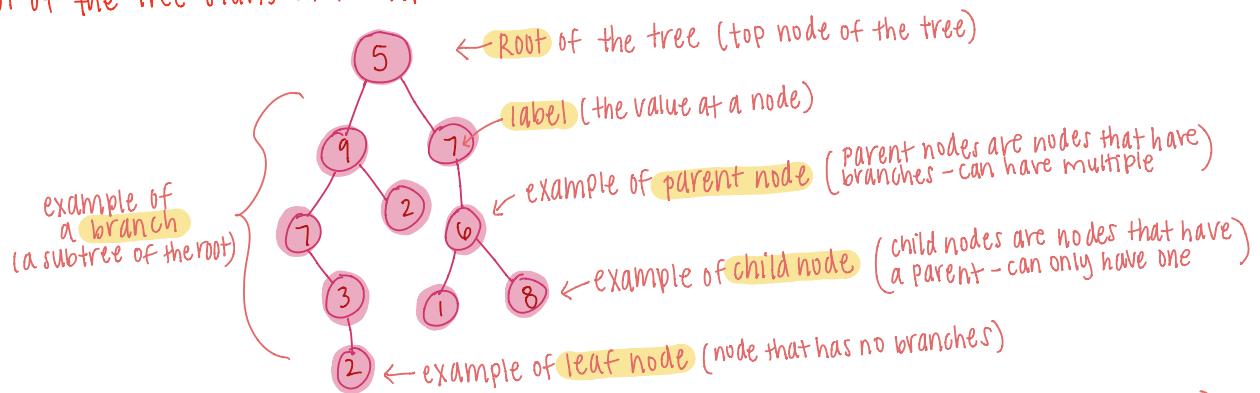
`lst`

`>> [1, 2, 3]`

• Trees (conceptually)

- recursive data structure

- root of the tree starts at the top and the leaves are at the bottom



⑥ `depth of a node` (how far away a node is from the root - # of edges between root & node)

⑦ `height of a tree` (depth of the lowest leaf)

⑧ `height of a tree` (depth of the lowest leaf)

• Trees (implementation)

- tree has value for root node and a sequence of branches

• represent branches as lists of trees

- constructor for 'tree' data type

• if no 2nd parameter is given, the default value [] is used

• `t = tree(root_label, list_of_branches)`

• to access label & branches

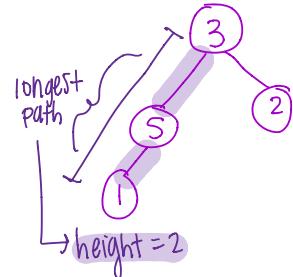
`label(t)` → will return root\_label  
`branches(t)` → will return list\_of\_branches (list of trees)

## Warmup: Height (Q6)

Write a function that returns the height of a tree.

↳ length of longest path from root to leaf

```
def height(t):  
    // base case? simplest case  
  
    // recursive call(s)
```



Solution:

```
def height(t):  
    if is_leaf(t):  
        return 0  
    else:  
        all_branch_heights = [height(branch) for branch in branches(t)]  
        longest_branch_height = max(all_branch_heights)  
        return 1 + longest_branch_height
```

## MAX Product (Q2)

### Q2: (Tutorial) Max Product

Write a function that takes in a list and returns the maximum product that can be formed using nonconsecutive elements of the list. The input list will contain only numbers greater than or equal to 1.

```
1 def max_product(s):
2     """Return the maximum product that can be formed using non-consecutive
3     elements of s.
4
5     >>> max_product([10, 3, 1, 9, 2]) # 10 * 9
6     90
7     >>> max_product([5, 10, 5, 10, 5]) # 5 * 5 * 5
8     125
9     >>> max_product([])
10    1
11
12    *** YOUR CODE HERE ***
13
14
15
```

if  $s == []$ :  
 return 1

$S = [x | 2 | 3 | S | S]$

a  $\rightarrow S[0] * \max\_p(S[2:])$   
b  $\rightarrow \max\_p(S[1:])$

return  $\max(a, b)$

→ Base case?

→ what choice do we have to make at every element of list?

• limitation, if we use  $S[i]$  we cannot use  $S[i+1]$  or  $S[i-1]$

Solution:

```
def max_product(s):
    if  $s == []$ :
        return 1
    elif len(s) == 1:
        return s[0]
    else:
        dont_include_first = max_product(s[1:])
        include_first = s[0] * max_product(s[2:])
        largest_product = max(dont_include_first, include_first)
        return largest_product
```

## Add this many (Q5)

Two mutation we must make to list s

2.4 Tutorial: Write a function that takes in a value  $x$ , a value  $e1$ , and a list  $s$  and adds as many  $e1$ 's to the end of the list as there are  $x$ 's. Make sure to modify the original list using list mutation techniques.

```
def add_this_many(x, e1, s):\n    """ Adds e1 to the end of s the number of times x occurs\n    in s.\n    >>> s = [1, 2, 4, 2, 1]\n    >>> add_this_many(1, 5, s)\n    >>> s\n    [1, 2, 4, 2, 1, 5, 5]\n    >>> add_this_many(2, 2, s)\n    >>> s\n    [1, 2, 4, 2, 1, 5, 5, 2, 2]\n    """
```

4:34

(+) I will share a link in the chat. Please open it & we can collectively write our solution together. If you think you have a solution or partial solution, feel free to type it up & we will collectively go through all the solutions (more than one right answer! :))

## SOLUTION:

```
def add_this_many(x, e1, s):\n    num_appearances = 0\n    for elem in s:\n        if elem == x:\n            num_appearances += 1\n    while num_appearances > 0:\n        s.append(e1)\n        num_appearances -= 1
```

First iterate through all elements of  $s$  & count # of times  $x$  appears

add ' $e1$ ' to end of  $s$  the # of times  $x$  occurs (num\_appearances)

(+) Key to remember (+)  
you can't append to a list while iterating through it!

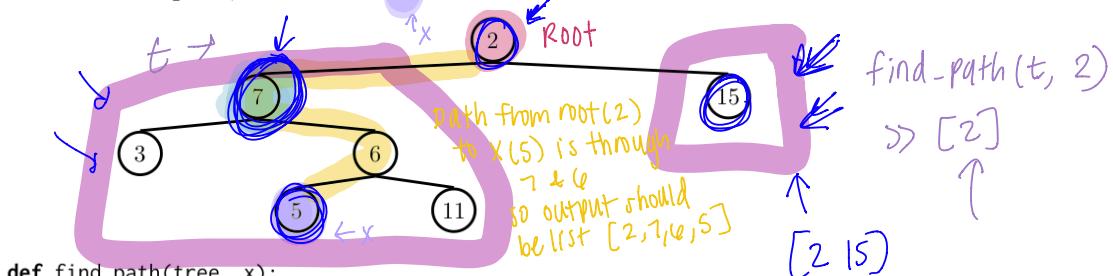
(+) can avoid having to do this 2-part sol if we iterate through indices instead of elements (or using 'list')  
↳ can just append directly in first 'for' loop

## Find Path (Q8)

- 1.4 **Tutorial:** Write a function that takes in a tree and a value  $x$  and returns a list containing the nodes along the path required to get from the root of the tree to a node containing  $x$ .  
 If  $x$  is not present in the tree, return None. Assume that the entries of the tree are unique.

don't  
have to  
worry  
about ' $x$ '  
appearing  
more than  
once

For the following tree,  $\text{find\_path}(t, 5)$  should return  $[2, 7, 6, 5]$



```
def find_path(tree, x):
    """
    >>> t = tree(2, [tree(7, [tree(3), tree(6, [tree(5), tree(11)])]), tree(15)])
    >>> find_path(t, 5)
    [2, 7, 6, 5]
    >>> find_path(t, 10) # returns None
    """

```

think about simplest case

think about how to iterate through all branches

implicit 'return None'

Base Case  
 if label(tree) == x:  
 return [x]

Recursive Case  
 for branch in branches(tree):  
 path = find-path(branch, x)  
 if path is not None:  
 return [label(tree)] + path  
 → return None

[label(tree)] + path  
 $\rightarrow [7, 6, 5]$

def find-path(tree, x):  
 if label(tree) == x:  
 → again think simplest possible case! just a tree with a single node (which is  $x$ ) so path from root to  $x$  includes just  $x \Rightarrow$  we return a list including only  $x$ !

return [label(tree)]

for b in branches(t):  
 → very common line for tree problems! (need to iterate through all branches)

path = find-path(b, x)  
 → recursive call on the branches (common in tree problems)  
 (x doesn't change cuz destination is still the same)

remember if  
 we ever exit  
 what's return?  
 there is implicit  
 return None

if path:  
 return [label(tree)] + path  
 → think about all the possible return values of this function: it's either a list or None.  
 (if it's not None, we have smthg to add to our list cuz it means we're on the right path)

If we're here, we have found the path from a branch to  $x$ , so the final list we return should be that path w/ the root's label added to the front

## COLOR CODED EXAMPLE /EXPLANATION:

