Problem Set 2: Stochastic environment

Sciences Po - Macroeconomics 3 - Fall 2024
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The deadline submission is 6th, October 23:59. Send a unique pdf file on Moodle and a Julia file by email.

Literature: Rare disaster and asset pricing

Present and comment on an **empirical** result or a stylized fact about "rare disaster and its consequences on asset prices" of an article of your choice, typically a table or a graph. Please confirm the article with me by email (paloma.peligry@sciencespo.fr). Once it has been approved, register the paper on the Google Sheet.

Exercise 1: Asset pricing in complete and incomplete markets

Environment

It is an endowment economy, where agents face some risks at period 2, but not at period 1. At period 1 agents consume $y_0 = \mu$ (for sure). The endowment in $t = 1$ depends on the state of the economy, either "good" and "bad", as follows:

- probability of 1/2 that we are in a **good state** where: $y_1 = \mu$
- probability of 1/2 that we are in a **bad state** where:
  - probability of $(1 - \lambda)$ not to be affected $y_1 = \mu$
  - probability of λ to be affected $y_1 = (1 - \frac{\phi}{\lambda})\mu$

Assets

The economy is composed of three assets:

- one **safe asset**, denoted b at price q, which returns 1 whatever the state of the economy,
- a **risky asset 1**, denoted a_1 at price p_1, which delivers $1 + \pi_1$ in good states and -1 in bad states,
- a **risky asset 2**, denoted a_2 at price p_2, which delivers -1 in good states and $1 + \pi_2$ in bad states.

Moreover, we assume that the assets are not yet introduced in the economy (e.g. zero net supply). In other words, we price this asset considering exogenous consumption profile and equal to the income in both period 0 and 1.

Preferences

We assume that the consumer favors his inter temporal period utility. π_1 and π_2 are exogenous and we want to determine the prices of the assets p_1, p_2 and q.

$$\max_{b,a_1,a_2} ln(c_0) + \mathbb{E}ln(c_1)$$

Part A: Preliminaries

1. What is the coefficient of relative risk aversion of the agent?
2. Compute the aggregate income/consumption in a bad state of the economy.
3. Write the budget constraints in $t = 0$ and $t = 1$

Part B: Asset pricing in complete market

1. Write down the program.
2. Derive the pricing equation of the three assets: risk free, risky 1 and risky 2.
3. How do the price ratios $\frac{p_1}{q}$ and $\frac{p_2}{q}$ vary with λ? With ϕ? Compare and interpret the results.

Part C: Asset pricing in incomplete market

1. Write down the program.
2. Derive the pricing equation of the three assets: risk free, risky 1 and risky 2.
3. How do the price ratios $\frac{p_1}{q}$ and $\frac{p_2}{q}$ vary with λ? With ϕ? Compare and interpret the results.

Exercise 2: Asset pricing with internal habits

Assume that agents have the period utility function:

$$u(c_t - hc_{t-1}, l_t), \text{ with } h < 1$$

where l_t is labor and c_t denotes consumption. They are assumed to have a discount factor β and they understand that consumption affects their habit. The budget constraint is (w is constant) :

$$c_t + a_{t+1} = (1 + r_t)a_t + wl_t$$

1. Write the Bellman equation of this problem.
2. Derive the pricing kernel.
3. What is the steady state interest rate?
4. Assume that $a_t = 0$ and that

$$u(c, l) = \frac{c^{1-\sigma} - 1}{1 - \sigma} - \frac{l^{1+\varepsilon}}{1 + \varepsilon}$$

   How does the steady-state labor supply change when agents care more about habits?

Exercise 3 (Julia): Markov Chain and stable distribution

For this exercise, iterate on the initial code `PS2_init.jl` (in the `PS2_init.zip` file).

Part A: Tauchen's method on process discretization

In numerical work it is sometimes convenient to replace a continuous model with a discrete one. In particular, Markov chains are routinely generated as discrete approximations to AR(1) processes of the form

$$log(y') = \rho log(y) + u$$

Here u is assumed to be i.i.d. and with variance σ^2.

Tauchen (1986) method is the most common method for approximating this continuous state process with a finite state Markov chain.

Krueger et al. (2015) assume that, conditional on being employed, log-labor earnings of households follow a simple AR(1) process. They estimate this process for household labor earnings after taxes (after first removing age, education and time effects) from annual PSID data and find estimates of $(\rho, \sigma^2) = (0.8, 0.1225)$.

1. As Krueger et al. (2015), discretize the process into a five state Markov chain using the Tauchen function from QuantEcon.
2. What is the probability matrix? What are the possible values of income y? *Hint: careful with the log of income!*
3. Create a function to compute the stable distribution. It musts take as input an initial guess of the distribution (vector of $N \times 1$) and the probability matrix (matrix of $N \times N$). The goal is to find the fixed point. The output should be the stable distribution (vector of $N \times 1$).
4. What is the mean income? What is the share of the households with the highest income?

Part B: Hamilton recessions probability

Using US unemployment data, Hamilton (2005) estimated the stochastic matrix

$$P = \begin{pmatrix} 0.971 & 0.029 & 0.000 \\ 0.145 & 0.778 & 0.077 \\ 0.000 & 0.508 & 0.492 \end{pmatrix}$$

where the frequency is monthly, the first state represents "normal growth", the second state represents "mild recession", the third state represents "severe recession".

For example, the matrix tells us that when the state is normal growth, the state will again be normal growth next month with probability 0.971.

1. Assume we are at normal growth, what is the probability of going in recession (mild **or** severe) next month? In 6 months?
2. Compute the stable distribution. What is the probability of being in a severe recession in the long run?

References

Hamilton, J. D. (2005): "What's real about the business cycle?" .

Krueger, D., K. Mitman, and F. Perri (2015): "Macroeconomics and heterogeneity, including inequality," *Handbook of Macroeconomics (forthcoming)*.

Tauchen, G. (1986): "Finite state markov-chain approximations to univariate and vector autoregressions," *Economics letters*, 20, 177–181.