Literature Appendix

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Keywords

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Citation	Abstract
Samuelson [1975]	Publisher Summary This chapter reviews the optimal consumption-investment problem for an investor whose utility for consumption over time is a discounted sum of single-period utilities, with the latter being constant over time and exhibiting constant relative risk aversion (power-law functions or logarithmic functions). It presents a generalization of Phelps' model to include portfolio choice and consumption. The explicit form of the optimal solution is derived for the special case of utility functions having constant relative risk aversion. The optimal portfolio decision is independent of time, wealth, and the consumption decision at each stage. Most analyses of portfolio selection, whether they are of the Markowitzâ"Tobin mean-variance or of more general type, maximize over one period. The chapter only discusses special and easy cases that suffice to illustrate the general principles involved and presents the lifetime model that reveals that investing for many periods does not itself introduce extra tolerance for riskiness at early or any stages of life.
Merton [1969]	OST models of portfolio selection have M been one-period models. I examine the combined problem of optimal portfolio selection and consumption rules for an individual in a continuous-time model whzere his income is generated by returns on assets and these returns or instantaneous "growth rates" are stochastic. P. A. Samuelson has developed a similar model in discrete-time for more general probability distributions in a companion paper [8]. I derive the optimality equations for a multiasset problem when the rate of returns are generated by a Wiener Brownian-motion process. A particular case examined in detail is the two-asset model with constant relative riskaversion or iso-elastic marginal utility. An explicit solution is also found for the case of constant absolute risk-aversion. The general technique employed can be used to examine a wide class of intertemporal economic problems under uncertainty. In addition to the Samuelson paper [8], there is the multi-period analysis of Tobin [9]. Phelps [6] has a model used to determine the optimal consumption rule for a multi-period example where income is partly generated by an asset with an uncertain return. Mirrless [5] has developed a continuous-time optimal consumption model of the neoclassical type with technical progress a random variable.
Merton [1971]	
Samuelson [1989]	Maximizing expected utility over a lifetime leads one who has constant relative risk aversion and faces random-walk securities returns to be "myopic" and hold the same fraction of portfolio in equities early and late in life–a defiance of folk wisdom and casual introspection. By assuming one needs to assure at retirement a minimum ("subsistence") level of wealth, the present analysis deduces a pattern of greater risk-taking when young than when old. When a subsistence minimum is needed at every period of life, the rentier paradoxically is least risk tolerant in youth–the Robert C. Merton paradox that traces to the decline with age of the present discounted value of the subsistence-consumption requirements. Conversely, the decline with age of capitalized human capital reverses the Merton effect.

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