Life Cycle Modeling is (Almost) Ready for Prime Time

Christopher Carroll[1]

Johns Hopkins University

Econ-ARK

Alan Lujan

Johns Hopkins University

Econ-ARK

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Abstract

The 'life cycle model' of optimal saving for retirement is familiar to anyone who has taken an introductory economics class. When hiring a financial advisor, such people probably think the advisor's job is just to tailor optimal life-cycle-model choices to their particular circumstances. But academics and advisors know that the advice about both saving and portfolio choice provided by standard academic life-cycle models is deeply problematic – for example, most such models imply that retirees should plan to run their wealth down to zero or some small amount and then (optimally!) live pension-check to pension-check (at least approximately). This paper makes the case that recent developments in the economics literature have finally given us the tools and insights required to construct rigorous life cycle models whose advice is sensible. We provide one example of a simple model that can solve a number of problems by putting wealth in the utility function.

1 Synopsis {-}

The retirement planning landscape is evolving rapidly due to shifting demographics, longer life expectancy, and greater market volatility. As individuals live longer, the need for pension plans

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[1]Correspondence to: ccarroll@llorracc.org

and retirement funds to provide flexible, rigorous, and sustainable investment advice has become increasingly urgent.

Traditional economic models, however, have struggled to keep pace with the demands of the financial advice industry, particularly in accurately capturing how individuals manage their portfolios and decumulate wealth post-retirement. For example, the "life cycle model" of optimal saving for retirement is familiar to anyone who has taken an introductory economics class. When hiring a financial advisor, clients likely expect the advisor's job is to tailor these life-cycle model choices to their particular circumstances. However, academics and advisors know that most life-cycle models provide unrealistic guidance—suggesting retirees should run their wealth down to zero and live pension-check to pension-check. This example highlights that the significant shortcomings in economic modeling have so far limited fund managers' ability to translate sound economic principles into actionable insights for their clients.

Recent advancements in theory and computation, however, have given economists new tools—structural models—that bridge the gap between theoretical modeling and real-world data. In this report, we argue that these mathematical and computational innovations enable the creation of portfolio models that can offer advice that is not only data-driven but also sensible, transparent, and customizable for individual clients.

This report presents a model that leverages advances in structural economic modeling, developed using the Econ-ARK platform. Rather than relying on ad hoc theoretical assumptions, we utilize **XYZ data** to guide the theoretical structure and parameter values, ensuring greater accuracy and relevance to how people actually behave.

1.1 Key Insights from the Model {-}

- **Dynamic Asset Allocation**: Our model provides an improved estimate of consumer risk preferences. This allows the glidepath to incorporate more realistic growth asset allocations that balance the need for portfolio growth with the requirement of securing income in retirement.

- **Wealth Preservation Post-Retirement**: Using **XYZ** (e.g., the TRP wealth in the utility function?), we precisely estimate the value retirees place on wealth. Unlike traditional models, this approach reflects the reality that retirees tend to preserve wealth longer, resulting in more sustainable retirement plans that align with observed behaviors.

1.2 Practical Applications {-}

This model serves as a proof of concept for applying cutting-edge structural economic modeling to financial advice. The open-source tools presented with this report can generate customized scenarios based on participant preferences and be adapted to new fund and survey data.

The model tools developed here also allow for ongoing refinement. The modelling can be enriched with more granular investment strategies and client preferences, and ultimately integrated into existing financial workflows to accommodate factors like risk tolerance, investment goals, and demographics.

2 Introduction

Franco Modigliani and Richard Brumberg (1954)[1] were the first to propose trying to understand consumer financial choices as optimal responses to the realities of the paths of income and of spending needs over the lifetime. An enormous academic literature has followed their pioneering work (see the literature appendix for an expansive summary), but it has proven difficult to build rational optimizing models that give sensible advice about both life-cycle saving choices and about investment decisions. That is, the models yield implausible answers to questions about how much wealth should be retained later in life, and how much of one's retirement savings should be invested in the stock market. Indeed, the subtitle of a recent paper by Daga et al. [2023] captures the current state of affairs nicely: "Why Practitioners Have Not Adopted the Lifecyle Model – Yet"; see also De Nardi et al. [2016].

In this paper, we argue that the elements are already available to construct a model that 'practitioners can adopt,' and all that is needed is to combine the relevant academic contributions with some wisdom from practitioners' own experience of advising clients. Our paper's central contribution is to provide a small open-source computational model that incorporates some of the features that make it possible to build rigorous optimizing life cycle models whose advice is not obviously wrong.

We begin with a (very) brief literature synopsis, then present a set of models that illustrate the difficulty of matching empirical facts with the life-cycle approach. We end with our proposed solution, which involves a small twist to the old idea (Carroll [1998]) of incorporating wealth directly in the utility function.

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[1]200 [2005]

2.1 Recent Developments in the Literature

2.1.1 Computation/Uncertainty/Complexity

Consumers face many uncertainties: about their own income, stock returns, interest rates, health expenditures, mortality, and much more. Further complications arise because of liquidity constraints and other financial frictions.

The incorporation of realistic descriptions of these complexities makes computation of the mathematically optimal solution to the consumer's astonishingly difficult. It is more difficult than, say, the calculation of optimal trajectories for spacecraft, and perhaps comparable to the complexity of figuring out how to drive a car roughly as well as a human (another problem where adequate computational solutions have only recently become available). The remarkable advance of computational power has now finally made it possible to compute a credible answer to the question "what saving and portfolio choices are truly mathematically optimal?" in a context where the key complexities are properly represented.

2.1.2 Survey Data on Expectations and Preferences

Another academic development has been a new openness to the idea that people's beliefs and preferences can be probed by *asking them* about their beliefs and preferences. In the context of motivations for saving, this leads us to take seriously the answers to a survey question about their 'most important' reason for saving that respondents to the Federal Reserve's Survey of Consumer Finances (SCF) have been asked for many years. Among retirees, one answer dominates the rest: 'Liquidity/The Future.' (See the discussion below for details). A natural interpretation of the importance consumers put on 'liquidity' is that precautionary saving motives matter for many households – highlighting the need for the aforementioned computational advancements.

The traditional approach to constructing such models has been for economists to try to measure the necessary inputs (income uncertainty, e.g.), and to assume that agent beliefs incorporate whatever it is that the economist has measured. However, the newly collected survey data on consumer expectations show that the beliefs that many people \textit{actually} hold differ substantially from what economists postulate they 'should' believe based on their empirical measurements. Moreover, there is now considerable evidence that the decisions people make reflect their actual beliefs and preferences rather than whatever it is that economists think they *should* believe and prefer.[2]

Some recent work suggests that taking beliefs data into account could resolve many of the problems that have long beset the life cycle modeling literature. For example, Velásquez-Giraldo [2023] shows

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[2]A particularly troubling possibility is raised by the work of GABAIX et al. [2010], who point out that at least some elderly decision-makers (say, those with dementia) may be beyond the 'age of reason.'

that even college-educated people systematically have held beliefs about stock market returns that are pessimistic compared to what the market has historically delivered. He argues that this explains why people have been less eager to invest in stocks than would be predicted by models calibrated with economists' more optimistic expectations, and that the portfolio investment behavior of college-educated people over most of their lives is reasonably consistent with *subjectively* rational decision-making (i.e. given their beliefs). Concretely, many people think that investment in stocks is a lousy deal, yielding a low return with high risk, so it's no mystery why such people do not invest.

It is not astonishing to discover that many people hold beliefs that differ from those of experts, especially on subjects whose mastery requires considerable domain-specific education, like the returns and risk of stock investments. Indeed, the existence of a large industry offering financial advice is *prima facie* evidence that many people are not confident that they understand everything necessary to make good financial financial choices on their own.

Financial advice, however, is fraught with potential conflicts of interest. That is one reason that justifying such advice with an explicit and transparent modeling framework is so attractive. If the advice is consistent with the model, and the model can be checked both for mathematical correctness and conceptual soundness (by outside experts), then it is reasonable for a client hiring an advisor to trust the advice.

2.1.3   Model Specification and Estimation

In the Models section below, we provide a formal description of the mathematical and computational structure of our optimizing models, beginning with the Life Cycle Portfolio model, which calculates optimal saving and optimal portfolio shares over the life cycle. In the Estimation, we report that the model implies a rapid drawdown of wealth after retirement that is simply not observed in empirical observations, confirming a longstanding problem with life cycle models (see, e.g., Hurd [1987], HEIMER et al. [2019]).[3] We call this the 'drawdown failure,' which has been the subject of a large literature with both U.S. evidence (see, e.g., Hurd [1989], De Nardi et al. [2016], Kopecky and Koreshkova [2014], Mortenson et al. [2019], Poterba et al. [2018]) and international evidence (see, e.g., Christensen et al. [2022], Ventura and Horioka [2020], Niimi and Horioka [2019]).

It would be interesting to see how our results might change if we were to include such shocks, but the inexorable logic behind the model's prediction of a rapid drawdown of wealth should still apply as the threat of mortality grows with age. Furthermore, the drawdown failure is also present in

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[3]Some impressive recent work by Ameriks, Caplin, and coauthors (Ameriks et al. [2011], Ameriks et al. [2020]) has argued that concerns about the possibility of extremely large medical costs (e.g. nursing home or other long term care) may be behind the drawdown failure for some people (see De Nardi et al. [2016]) for a survey).

other countries whose social insurance systems are much more generous than the U.S., so it seems unlikely that uninsurable medical risk is a sufficient explanation.

We next modify the model by adding a bequest motive, because the literature has extensively explored whether such a motive could explain the drawdown failure (],De Nardi et al. [2016]). But in the Estimation section we confirm the consensus in the literature that the bequest motive does not seem to have much force for most households. (Note that Hendricks [2002] finds that for the great majority of households, bequests amount to less than 2 percent of lifetime resources).

This leads us into more speculative territory. If what consumers care most about is to hold wealth for 'Liquidity/The Future' but that wealth is not explainable by precautionary saving against measurable shocks, a potential interpretation is that consumers value the ownership of wealth *in and of itself*. After fleshing out this idea a bit, we propose a final model that makes wealth a direct input to the utility function in a slightly different way than in the existing life cycle literature.

The main quantitative result of this paper is to show that this final model does a better job jointly matching the data on wealth profiles and portfolio choice than either the Life Cycle Portfolio or the Warm Glow Bequest models. More broadly, our modeling and conceptual tools have recently advanced to the point where it is finally possible to construct rational optimizing models of life-cycle financial choice that can serve as a credible justification for normative advice. In particular, these conceptual advances include the idea that "softer" data like surveys on household expectations and beliefs should be taken seriously.

3 Models

The academic literature on life-cycle modeling is vast, and we cannot hope to do it justice (even in the broader sampling in the literature appendix). But the intrinsic nature of papers in any academic literature is to focus narrowly on one specific question at a time. Here, our goal is to examine the "big picture" question of what elements are needed to craft a model that can provide credible advice to retirees about spending and portfolio choices, while remaining reasonably consistent with the relevant well-established facts from the academic literature, as well as one new kind of further evidence that we view as vital: the experience of financial advisors themselves in interactions with their clients. We have been told,[4] for example, that a good way to get fired as a financial advisor is to recommend the LCP model's conclusion that it is optimal for retirees to plan to run their wealth down to zero then live pension-check to pension-check.

For purposes like 401(k) or other pension plan design, the optimization problem should be constrained to be one that satisfies the legal obligations employers have to their employees. For

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[4]Personal communication, James Tzitzouris with Christopher Carroll, 2024-05-15.

example, the employer's contract is with the employee, not with the employee's heirs. The employer's duty is to craft a plan that is expected to permit the employee to have adequate resources for their own expenditures during retirement. These legal considerations effectively prohibit the advisor from including a bequest motive in its optimization objective.[5]

3.1 The Baseline Academic Model

We begin by describing the optimal consumption-saving problem over the life cycle for a consumer, focusing on the dynamics of their income while ignoring how returns to saving work. After we have finished describing the basic life cycle model, we will augment it to add optimal portfolio choice between a safe asset and a risky asset (like the stock market) with a higher expected rate of return.

3.1.1 Basic Life Cycle Consumption-Saving Model

In each discrete time period (indexed by t), a consumer's flow of utility depends on how much they consume from their available resources: We assume that the utility function is of the standard Constant Relative Risk Aversion form:

$$u(\mathbf{c}) = \frac{\mathbf{c}^{1-\rho}}{1-\rho} \tag{1}$$

At the time they choose how much to consume, the consumer has total market resources of $\mathbf{m}_t$, representing their previously owned resources (bank balances) and current income flow $\mathbf{y}_t$. Whatever the agent does not consume is retained as assets $\mathbf{a}_t$, which accrue interest by factor $\mathcal{R}_{t+1}$ between period t and period $t+1$. We follow most of the literature and assume that the consumer faces a hard liquidity (or borrowing) constraint: they cannot end any period with negative assets. The intertemporal budget constraint can thus be expressed as:

$$\mathbf{a}_t = \mathbf{m}_t - \mathbf{c}_t, \quad \text{remaining assets are market resources less consumption} \tag{2}$$

$$\mathbf{a}_t \geq 0, \quad \text{consumer cannot borrow} \tag{3}$$

$$\mathbf{b}_t = \mathcal{R}_{t+1}\mathbf{a}_t, \quad \text{bank balances are assets after yielding interest} \tag{4}$$

$$\mathbf{m}_{t+1} = \mathbf{b}_{t+1} + \mathbf{y}_{t+1}. \quad \text{future market resources are bank balances plus income} \tag{5}$$

$$\tag{6}$$

One of the fundamental discoveries of the past 40 years or so is the extent to which optimal choice is profoundly altered by the presence of uncertainty. Friedman [1957] proposed a simple formulation of the labor income process that remains an excellent description of annual income shocks even today, proffering that there are two components to income. The 'permanent' component is roughly

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[5]One way to accommodate this requirement would be to limit the empirical sample used to estimate the model to childless people. This might not be feasible with public datasets like the SCF because the sample sizes might be too small; but with large administrative data of the kind available to 401(k) providers it should be possible.

what they would expect to earn in a 'normal' year (say, their annual salary), and the 'transitory' component reflects events like unemployment spells or lottery winnings, which make a given year's realized income deviate from its expected value. From a modeling perspective, the upshot is that a consumer's financial circumstances can be fully captured with two variables. First, the consumer's permanent income level $\mathbf{p}_t$ is the non-capital income they would normally expect to receive. Second, total market resources $\mathbf{m}_t$ represent the sum of financial assets and current income: the pool of resources that can be immediately spent, or 'money' in the colloquial sense of 'how much money does grandma have?'. The transitory component of income need not be tracked at all: as soon as this uncertainty is resolved, its information is fully incorporated by market resources $\mathbf{m}_t$.

A consumer's 'value' of having a given amount of market resources $\mathbf{m}_t$ right now, and of knowing their current permanent income level to be $\mathbf{p}_t$, is the sum of consumption utility they will experience from today onward into the indefinite future, assuming that they make optimal choices in all periods. Potential future utility flows matter only to the extent that the agent expects to survive to that period, and might be further discounted due to placing more weight on the present than the future.

In formal mathematical terms, the consumer's objective is to maximize expected present discounted utility from consumption over a life cycle that ends no later than date T:

$$\mathbf{v}_t(\mathbf{m}_t, \mathbf{p}_t) = \max_{\{c\}_t^T} \; u(\mathbf{c}_t) + \mathbb{E}_t \left[\sum_{n=1}^{T-t} \mathcal{L}_t^{t+n} \beta^n u(\mathbf{c}_{t+n}) \right]. \tag{7}$$

$\mathcal{L}_t^{t+n}$: probability to $\mathcal{L}$ive until age $t + n$ given you are alive at age t

- $\mathcal{L}_{120}^{121} = 0.0$ says that a 120 year old has zero probability of living to 121
- $\mathcal{L}_{80}^{90} \; = 0.3$ says that an 80 year old has a 30 percent chance of reaching 90

$\beta = 1$: time discount factor (captures degree of present bias)

To capture the predictable patterns that non-capital income follows over the life cycle (i.e. rising with age and experience, and falling at retirement to the level of any regular pension payments), we define a sequence to characterize the Modiglianian life cycle:

$$\Gamma_{t+1} : \text{typical life cycle permanent income growth factor by age} \tag{8}$$

For any particular consumer, deviations from the typical life cycle permanent income growth pattern arise from "permanent income shocks" that we represent with the variable ψ. At any given age, such shocks could be positive ($\psi > 1$), corresponding to an unexpected promotion or a switch to a higher-paying job, or negative ($\psi < 1$), possibly representing a failure to be promoted or a change to a lower paying job. An individual is also subject to purely transitory shocks that affect only this period's income(e.g.\ an unexpected absence or bonus), represented by θ.

This gives us the following description of the dynamics of income:

$$\mathbf{p}_{t+1} = \mathbf{p}_t \Gamma_{t+1} \psi_{t+1}, \tag{9}$$

$$\mathbf{y}_{t+1} = \mathbf{p}_{t+1} \theta_{t+1}, \tag{10}$$

$$\mathbb{E}_t[\mathbf{p}_{t+1}] = \mathbf{p}_t \Gamma_{t+1}, \tag{11}$$

where the third line follows because the expected value of the permanent shock is $\mathbb{E}_t[\psi] = 1$.

The transitory component θ of income has two modes. In unemployment spells, the consumer earns no income; we assume that such spells occur with probability $\wp$ each period. Otherwise, the consumer receives a transitory income shock $\xi > 0$ from some (mean one) distribution, rescaled to preserve the unit mean of θ:[6]

$$\xi_s = \begin{cases} 0 & \text{with probability } \wp > 0 \\ \xi_s/(1-\wp) & \text{with probability } (1-\wp) \end{cases} \tag{12}$$

It is conventional to assume that shocks to permanent income and to the transitory income of the employed are (mean one) lognormally distributed:

$$\log \psi_s \sim \mathcal{N}(-\sigma^2_{[\psi,t]}/2, \sigma^2_{[\psi,t]}) \tag{13}$$

$$\log \xi_s \sim \mathcal{N}(-\sigma^2_{[\xi,t]}/2, \sigma^2_{[\xi,t]}) \tag{14}$$

Under the assumptions we have made about the structure of the utility function (homotheticity), budget constraint (linearity and geometric returns), and income process (permanent and transitory shocks) it is possible to recast the problem entirely in terms of *ratios* of the model variables to permanent income $\mathbf{p}$. So, for example, italic $c = \mathbf{c}/\mathbf{p}$ is the ratio of the (boldface) level of consumption to the level of permanent income $\mathbf{p}$ (see Carroll [2023] for the math).

Another way to make the problem easier to understand is to combine several of the multiplicative terms into portmanteau variables. Particularly, define $\boldsymbol{\beta}_{t+1}$ as

$$\boldsymbol{\beta}_{t+1} = \beta(\psi_{t+1}\Gamma_{t+1})^{1-\rho}. \tag{15}$$

The consumer's problem can be expressed more simply by realizing that it boils down to a "now versus later" problem. All the consumer needs to know about the future is summarized by the value they will expect as a consequence of ending the current period with a certain ratio of assets to permanent income, $a = \mathbf{a}/\mathbf{p}$. We can represent the value of ending the period with assets of a_t using the Gothic variant of the letter v:

$$\mathfrak{v}_t(a_t) = \mathbb{E}_t[\boldsymbol{\beta}_{t+1}\mathrm{v}_{t+1}(m_{t+1})]. \tag{16}$$

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[6]It is straightforward to extend the model to allow for a more realistic treatment of unemployment, for example by taking account of the existence of an unemployment insurance system; such an adjustment does not change the substantive conclusions we are interested in.

With this definition, the period t choice problem can be summarized in Bellman form as simply:

$$v_t(m_t) = \max_{\mathbf{c}_t} \left\{ u(\mathbf{c}_t) + \mathcal{L}_{t+1} \mathfrak{v}_t(a_t) \right\} = \max_{\mathbf{c}_t} \left\{ u(\mathbf{c}_t) + \mathcal{L}_{t+1} \mathfrak{v}_t(m_t - c_t) \right\}. \tag{17}$$

Because a_t measures available market resources that are unspent, this formulation makes it crystal clear that the consumer faces a tradeoff between the utility of consumption today and the expected value of preserving assets $a = m - c$ for the future.[7]

3.1.2 The Life Cycle Portfolio ('LCP') Model

We are now ready to add portfolio choice to the problem and discuss how the interest factor $\mathcal{R}_{t+1}$ is determined. Suppose the consumer can invest their assets in a risk-free asset with return factor R, and in a risky asset with returns $\log \mathbf{R}_{t+1} \sim \mathcal{N}(\mathsf{r} + \varphi - \sigma^2/2, \sigma^2)$, where $\mathsf{r} = \log(\mathsf{R})$. That is, we make the conventional assumption that risky returns are lognormally distributed with an expected equity premium of φ. The portfolio return the consumer earns will depend on the *share* ς_t of their assets that they invest in the risky asset:

$$\mathcal{R}_{t+1} = \mathsf{R} + (\mathbf{R}_{t+1} - \mathsf{R})\varsigma \tag{18}$$

Now the consumer makes *two* choices in each period t: how much to consume c_t and the share ς_t of his assets to put into the risky asset. These choices are made simultaneously, but they can be thought of as being made sequentially, one immediately after the other: first consumption (conditioned on market resources m_t) and then the risky asset share (conditioned on assets a_t after consumption). The consumer makes the optimal choice of portfolio share, so we redefine the "gothic" value function as:

$$\mathfrak{v}_t(a_t) = \max_{\varsigma_t} \ \mathbb{E}_t \left[\boldsymbol{\beta}_{t+1} v_{t+1}(\mathcal{R}_{t+1} a_t + \theta_{t+1}) \right]. \tag{19}$$

A split second before choosing the risky share, the consumer's objective in the consumption stage of the problem is exactly the same as the Bellman form above, but with the redefined continuation value that accounts for optimal portfolio choice.

object	meaning
m, c, a	market resources, consumption, and end-of-period assets, normalized by permanent income
v	the normalized value function
$\mathcal{L}_{t+1} \equiv \mathcal{L}_t^{t+1}$	probability that a person alive at age t survives to age $t+1$

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[7]The normalization for value function involves more than just division by $\mathbf{p}$; see Carroll [2023] for details.

3.1.3 Calibration

Many of the parameters of the basic life-cycle consumption-saving model can be calibrated from well measured empirical data. For example, we use standard calibrations of both of the income shock processes during the working life, based on [INSERT REFERENCE], and survival probabilities by age are taken directly from actuarial mortality tables published by the Social Security Administration. We set the 'pure' rate of time preference to $\beta = 1$, meaning that the optimal choice is to care exactly as much your future self as much as your present self (conditional on surviving into the future).

Beyond those basic assumptions, we calibrate the model to include two kinds of uncertainty after retirement. First, to address the possibility that the drawdown failure reflects a fear of medical expenses, we incorporate estimates from Velásquez-Giraldo [2023] of the size of shocks to medical expenditures for retirees. Our estimation results show that even when we include this calibration of medical shocks, the model still predicts much more drawdown of wealth than the data show. Second, we assume that there are 'ordinary' expenditure shocks in retirement that are of similar magnitude to income shocks during working life (following recent estimates from Fulford and Low [2024]). Again, in principle, the presence of such shocks provides a precautionary motive to draw down wealth more slowly.

3.2 Alternative Preferences

The specification of preferences in the LCP model is the standard assumption of time-separable Constant Relative Risk Aversion This is the workhorse tool for intertemporal choice models because it has a number of convenient mathematical properties and its implications satisfy some plausible economic desiderata (cf. Kimball [1993]). However, mathematical convenience provides no guarantee that the utility specification is *right* in the sense of giving a proper representation of what people's preferences really are. Economists have explored a number of modifications to these standard preferences in an attempt to make their models' predictions match various facts.

3.2.1 Habit Formation and Epstein-Zin Preferences

One intuitive idea is that people care not only about the current level of their consumption but also about how it compares to their past levels of consumption (they have "habit formation" in their preferences). For example, Carroll et al. [2000] show that a model with multiplicative habits can provide an explanation for otherwise puzzling patterns in the relationship between saving and growth across countries, and Michaelides [2002] examine the implications of such a model for life-cycle choices.

A second common modification to preferences considered by a substantial literature is the use of Epstein and Zin [1991] preferences, which allow agents to be extremely risk averse with respect to financial risk at a point of time, while simultaneously allowing the agents to be willing to be quite willing to tolerate changes in consumption over time. Such preferences have been proposed as a way to solve various puzzles related to the relatively high rate of return that equity investments have exhibited over time.

Both of these formulations are motivated mostly by the goal of matching macroeconomic data. In our view, however, they are both difficult to defend given some facts we can robustly observe in microeconomic data. In particular, both models would imply that microeconomic households would be extremely eager to buy insurance to smooth away almost any risk to their microeconomic circumstances. While people do typically have insurance against large risks (fire insurance for the home, auto insurance for the car), the parameter values in these models required to match the macroeconomic facts would justify consumers in spending a large fraction of their income on insurance of all kinds. One particular example stands out: Households with either strong habits or a high Epstein-Zin instantaneous coefficient of relative risk aversion would be extremely eager to buy private unemployment insurance to supplement the default UI system provided by the state. Indeed, such private UI is available– and at prices such that a large fraction of households would be eager to buy it if they had such preferences– yet the fraction of households who participate in the private UI market is vanishingly small.

Our attention in this paper is therefore directed toward other modifications in preferences that seem to be plausible in both microeconomic and macroeconomic data.

3.2.2 The "Warm Glow" Bequest Motive

The LCP model sketched above assumes that the only reason to hold wealth is to spend it later, which means that eventually an age must come at which the wealth starts being spent down. As the literature has demonstrated, and as we will confirm below using data from SCF's from 1995 to 2022, the path of the median wealth ratio after retirement does not look anything like what that model predicts.

Of course, the model can make no sense at all of the behavior of the very rich. Bill Gates, for example, has chosen to allocate a large portion of his lifetime wealth to the Bill and Melinda Gates foundation rather than spending it on himself; and even with the relative pittance that remains to

him ($153 billion, per Business Insider) he would need to spend about $22 million per day to *avoid getting richer*.[8] In fact, he has pledged to give away nearly all of his wealth before he dies.

But the model also fails for people of much more modest means - the 'drawdown failure' articulated above. From a mathematical point of view, it is clear that some other motive for holding onto wealth must be added to the framework if it is to explain the broad facts, never mind Bill Gates. A natural candidate is a bequest motive: The idea that people take pleasure in the thought of leaving something to their heirs.

This idea can be incorporated by adding another term to the sources of utility: the value the consumer places on the bequest, which we will denote as $e(a)$, the utility they experience from the thought of leaving an *e*state. The flow of utility that the consumer receives includes both their utility from consumption *and* the pleasure they take from the thought that, if they pass away before next period (which happens with probability $1 - \mathcal{L}$), their assets will pass to their heirs.

The consumer's new value function is therefore:

$$v_t(m_t) = \max_{c_t} \overbrace{u(c_t)}^{\text{present}} + \overbrace{\underbrace{\mathcal{L}_{t+1}\mathfrak{v}(a_t)}_{\text{live}} + \underbrace{(1 - \mathcal{L}_{t+1})e(a_t)}_{\text{die}}}^{\text{future}}. \tag{20}$$

The literature has commonly used a 'warm glow utility from bequests' motive of the form:

$$e(a) = \alpha \frac{(a + \underline{a})^{1-\rho}}{1 - \rho} \tag{21}$$

where the ρ coefficient is the same as in the utility function for consumption (see, e.g., De Nardi [2004]), and the α coefficient controls the importance of the bequest motive relative to the utility from consumption.

Our estimation finds little evidence that a bequest motive of this kind is important for the median college-educated retiree. It also seems unlikely to be important for much richer people, at least according to the evidence from the historian Fredrick Cople Jaher [1980]'s chronicle of the behavior of the richest Americans since the Revolution. Jaher presents a feast of quotations articulating a host of motivations for extreme wealth accumulation; but among their many explanations of their behavior, almost none of the tycoons under study mention anything resembling the bequest motive as formulated in the standard academic life cycle literature. Andrew Carnegie was most explicit: 'I would rather leave my son a curse than the almighty dollar.'[9]

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[8]As of 2024-05-15, the Fed Funds rate is 5.3 percent at an annual rate. $153b $\times$ 0.053/365 days $\approx$ $22 million. At the current inflation rate of 3.4 percent, he would only have to spend a little over $8 million a day to run down his real wealth – assuming the Fed Funds rate is the highest rate of return he can earn.

[9]He made good on this: He gave away more than 90 percent of his wealth before he died, to the astonishment of many skeptics.

As mentioned above, the 202 [2023] has for many years asked respondents a question about their motivations for saving.[10] While respondents' answers are fairly heterogeneous, the SCF has a suggested aggregation of the many different answers into categories that correspond approximately to some of the motivations that the academic literature has considered. The category that best matches the 'bequest' motivation is 'Family' (which includes 'to help the kids out' and 'to leave an estate' but also includes saving for 'weddings and other ceremonies' and 'to have children/a family.')

An ambitious agenda would be to tabulate the answers to this survey question for people at different ages and then to construct a model that would imply the same age pattern of motivations as the data. For example, one might find that for people who have just entered the labor market (say, the 26-30 age group) the survey responses showed that saving for 'retirement' was not a priority, while saving for 'purchases' and 'liquidity' were important. In order for a model to be credible, its implications would need to comport with the survey data. Our aim here is to take a first step in that direction, by constructing a model that is at least consistent with the responses of retirees.

The table below presents the responses to this question for college-educated households older than age 70 from the 1995 to the 2022 waves of the SCF. If bequests were a primary motivation for saving for most (college-educated) people, it would be surprising for them to mention this motivation so rarely. Given these (and other) objections to the bequest motive, as well as the problems of the model without a bequest motive, it is natural to consider alternative modifications to the framework.

3.2.3 Table: Most Important Reason for Saving

Reason	Proportion	Explanation
'Family'	0.06	bequests; weddings, etc
'Retirement'	0.27	
'Liquidity/The Future'	0.40	
'Purchases'	0.13	cars, vacation homes, etc
'Cannot save'	0.06	
Other	0.08	

3.3 Wealth in the Utility Function

Explaining the motivation to save is one of those places where economists' new openness to the idea of taking seriously what people say about their motivations has bite. While it is reasonable to be skeptical about taking quotations from Jaher [1980] at face value, Carroll [2000] shows that

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[10]See the material starting at line 848 in the documentation.

essentially all of the motivations articulated (wealth brings power; wealth allows philanthropy; wealth is a way of 'keeping score'; and more) can be captured in a mathematical formulation in which wealth enters the utility function directly.

The most general way that economists have for incorporating people's motivations into models of behavior is simply to assume that the decision-maker directly values something – in this case, wealth. The question is how best to incorporate the item in the utility function to study any particular question. Carroll [2000], for example, proposed a utility function specifically designed to capture saving behavior as wealth approached infinity, and accomplishing that goal required some mathematical structure that delivered the desired results but was unwieldy (and not obviously necessary for explaining the behavior of the bottom 99 percent, whose wealth does not approach infinity).[11]

3.3.1   Money in the Utility Function

There is a literature in macroeconomics, pioneered by Miguel Sidrauski [1967], that has long included 'money' (in the monetary sense) in the utility function of the representative agent in one form or another. A well-known paper by Rotemberg [1984] proposed a specific utility function designed to capture the stability of the ratio of money to GDP, and Rotemberg along with James Poterba estimated this model on U.S. data in Poterba and Rotemberg [1986].

The structure of their utility function is

$$\mathrm{u}(c,\ell) = \frac{\left(c^{1-\delta}\ell^{\delta}\right)^{1-\rho}}{1-\rho} \tag{22}$$

where ℓ captures the the ℓiquidity services provided by money-holding.

To be clear, the aim of that literature was to explain the holding of ℓ defined as dollar cash holdings, to study questions like the 'velocity' of money and the role of money supply and money demand in determining interest rates – not to explain saving behavior.

3.3.2   Wealth In the Utility Function: Cobb-Douglas Form

But for the question of how to incorporate wealth in the utility function, Tzitzouris [2024] proposed a mathematically identical formulation in which assets a takes the place of ℓ in the Rotemberg-Poterba utility function.[12] The Cobb-Douglas functional form is commonly used in other contexts, but does not seem to have been explored as a formulation for how to put a direct wealth-holding motive in the utility function.

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[11]Specifically, a separable utility-from-wealth function was added to the maximizer's objective and with a coefficient of relative risk aversion smaller than that for the utility from consumption.

[12]The question of whether a or m should be in the utility function is of little importance; here we prefer a because assets after consumption are immune to considerations of whether the time period is a year, a quarter, or a month.

The upshot is that if we credit the proposition that the ownership of wealth yields utility, then there is good precedent for the functional form of Tzitzouris [2024]. Henceforth we will call this the Tzitzouris-Rotemberg-Poterba or 'TRP' utility function. It is a relatively simple matter to solve the revised problem with wealth in the utility function using the TRP utility specification. The revised utility and value functions of the problem are:

$$u_t(c_t, a_t) = \frac{\left(c_t^{1-\delta} a_t^{\delta}\right)^{1-\rho}}{1-\rho}, \tag{23}$$

$$v_t(m_t) = \max_{c_t} \ u(c_t, a_t) + \mathcal{L}_{t+1} \mathfrak{v}_t(a_t). \tag{24}$$

We are open to the possibility that wealth in the utility function is a reduced form for other motivations – indeed, that was the thesis of Carroll [2000]. In particular, the fact that in our SCF table above, 'Liquidity/The Future' is the most popular answer among retirees for the most important reason to save might signal that the forms of uncertainty that we can measure – like the Ameriks et al. [2020] calculations about nursing home expense risks – constitute only a fraction of the matters retirees might worry about. Maintaining a buffer stock of wealth to protect oneself against 'unknown unknowns' is possibly perfectly rational, and also nearly impossible to calibrate in a quantitative model in which we would need to have an accurate representation of people's beliefs about the magnitude, frequency, and persistence of 'unknown unknowns.' Even if you knew those answers, they would be, at best, 'known unknowns.'

3.4 Comparisons to Other Models Familiar to Practitioners

Gordon and Joseph [2014] asserted that financial planning practitioners mostly used rules of thumb and heuristics to provide their advice. That paper aimed to introduce the key concepts of formal life-cycle modeling to the audience of practitioners. Since its publication, there appears to have been considerable movement in the direction advocated by its authors. A number of leading financial institutions have made available partial descriptions of proprietary life cycle models that they are developing. For this report, we decided to describe only models that have been published in peer-reviewed journals and whose detailed characteristics are knowable.

Daga et al. [2023] looks at the post-retirement period and normalizes all variables by retirement income, but it does not incorporate risk to permanent income (as we do), nor does it attempt to reconcile post-retirement with pre-retirement behavior. The model uses an additive utility of bequest with no shifter term, which has the implication that even income-poor households have a powerful bequest motive. The multigoal framework additionally disaggregates consumption expenses into different categories, each with the same CRRA coefficient. This reduces the effect of the diminishing marginal utility of consumption, such that the sum of utilities of consumption

is greater than the utility of the sums of consumption. Moreover, each goal has an allocation coefficient (relative importance) that is set subjectively by the researchers.

O'Hara and Daverman [2015] incorporate an additively separable utilty of bequest like the bequest motive we explored above, and like our models it incorporates both permanent and transitory shocks to income. However, the model uses Epstein-Zin preferences (see our above objections to this feature). Their model also assumes that there is no uncertainty (except for mortality) in the post-retirement period, because that assumption has the convenient implication that the post-retirement period is extremely simple; the portfolio share, in particular, should remain constant at the infinite-horizon Merton-Samuelson solution.

Lanski et al. [2022] uses a life-cycle consumption and portfolio choice model without any consideration of bequests like the one discussed above (although other aspects of the model are simpler than our specification).

There have also been publications that partake of the mathematical flavor of life cycle modeling but do not use the intertemporal choice framework. Idzorek [2023] uses a static model of portfolio optimization, where the objective is to maximize the mean-variance trade off of a portfolio position with additional features such as non-pecuniary benefits. Because this model is not dynamic, it can not consider and/or anticipate risks such as future market volatility, changes in preferences over the life cycle, mortality, and retirement horizon.

4   Estimation

4.1   Indirect Inference Described

Even if one knew all the parameters of the model (the consumer's coefficient of relative risk aversion, etc), solving an optimization problem that includes the many real-world complications described above (especially those due to uncertainty) is such a formidable problem that it only became possible about 25 years ago (and solving the models took days).

But of course we do not know the best values to choose for unobservable parameters like relative risk aversion and time preference rates. The increasingly standard approach to this problem is the method of 'indirect inference.' Essentially, this means specifying the structure of your model except for the values of parameters that you cannot measure well (like time preference and risk aversion), and asking a numerical search algorithm to seek the values of those parameters that lets the model fit the data as well as it is capable of doing. This requires the computer to solve the problem perhaps thousands of times, which is why indirect inference has only begun to come into its own recently, as computer speeds have become fast enough to tackle the problem.

4.2 Indirect Inference Implemented: the Method of Simulated Moments

We are particularly interested in finding the optimal post-retirement choices, both for the rate of spending and for portfolio allocation between safe and risky assets. The method of simulated moments consists of finding the parameters that make the model's simulated moments (statistics), like the median wealth and the median portfolio share, match the corresponding empirical facts as closely as possible.

Consider an empirical moment q_i where $i \in \{1, ..., N\}$ and the corresponding simulated moment $\hat{q}_i(\theta)$, where θ is the vector of parameters that we are interested in estimating. By solving and simulating our structural model with different θ parameters, we can calculate the simulated moments $\hat{q}_i(\theta)$ for each parameter set. The method of simulated moments then consists of searching for the parameter set θ that minimizes the distance between the simulated versus empirical moments. This is done by minimizing the following objective function:

$$\min_{\theta} \sum_{i=1}^{N} \left(\omega_i [q_i - \hat{q}_i(\theta)] \right)^2 \tag{25}$$

Here, ω_i is the weight of each moment in the objective function, representing the relative importance of each moment in the estimation process. For example, we might be more interested in matching the median wealth than the median portfolio share, and thus assign a higher weight to the former.

For our exercise, we are interested in matching the median wealth to income ratios throughout the life cycle, and the median portfolio share of risky assets after retirement. Because aggregate age data can be noisy and subject to selection bias and measurement error, we will aggregate the data into 5-year age bins to smooth out the noise and reduce the impact of selection bias. Starting at age 25, we calculate the median wealth-to-income ratio as follows: Wealth is defined as the sum of all assets and liabilities, including financial assets, housing, vehicles, and debt. For income, we use the sum of all wages, salaries, Social Security, and retirement income, excluding capital gains and other non-recurring income. We then calculate the wealth to income ratio of every household in the age bin and remove households with an income of zero. The median wealth-to-income ratio is calculated from the remaining households.

In our structural model, we hard-code retirement to occur at age 65, whereas in the data we observe retirement at different ages, but predominantly between ages 60 and 70. Therefore, we avoid the data for ages 60 to 69 to prevent any bias in the estimation process, but keep the data for ages 70 and above to capture the behavior of retirees. Similarly, we calculate the median portfolio share of risky assets after retirement for ages 70 and above given by Aboagye et al. [2024].

Considering the selection of moments we have chosen, it is clear that there is an imbalance between the wealth-to-income moments and the portfolio share moments. There are more wealth to income moments than portfolio share moments (12 vs 5), and the portfolio share moments lie between 0

and 1, whereas the wealth to income ratios can be much larger. To account for this, we set the weights to normalize the wealth to income ratios by the highest ratio in the data, making them all lie between 0 and 1, and adjust the weights for the portfolio share moments by a factor of 12/5, so that the two sets of moments are about equally weighted in the estimation process. This ensures that our estimation process puts even weight on the two sets of moments, despite the difference in scale and number of moments.

Having chosen the moments we are interested in matching and their respective weights, we can now proceed to a discussion of estimating the parameters of our various models. We use the `Econ-ARK` project's `HARK` package to solve and estimate the models, and `optimagic` (Gabler [2022], former `estimagic`) to perform the estimation process. Our exercise consists of estimating one parameter (the coefficient of relative risk aversion) for the Life Cycle Portfolio Choice Model and up to three parameters (CRRA, the weight of the bequest motive, and the wealth shifter of the bequest motive) for the `LCP+WarmGlow` model, so we develop a robust and efficient estimation process that can handle a varying number of parameters.

Our estimation process is computationally expensive, requiring the solving and simulation of the model given a parameter set many times. Because our simulated moments indeed require simulation, our moment generating functions $\hat{y}_i(\theta)$ have no analytical derivatives with respect to the parameters, so we must rely on numeric differentiation and clever optimization algorithms to find the optimal parameter set. We use the `tranquilo` algorithm (Gabler et al. [2024]), which stands for TrustRegion Adaptive Noise robust QuadratIc or Linear approximation Optimizer, to find the optimal parameter set. The `tranquilo` optimizer has many attractive features, such as being able to evaluate the function in parallel and estimate even noisy objective functions with many parameters, as well as being especially designed for least squares problems, such as the MSM.

4.3 Indirect Inference Results

Table 1 shows the results of our estimation exercise. For the standard Life-Cycle-Portfolio (LCP) model, we estimate a CRRA ρ coefficient of about 8.8, which lines up with the literature finding that portfolio choice models require a very high ρ in order to prevent agents from putting all their assets in the risky form. The Warm-Glow Portfolio Choice model also estimates a ρ coefficient that also rounds to 8.8. as well as a bequest factor α of about 44, and a bequest shifter $\underline{a}$ of 26. The fact that the ρ parameter is nearly unchanged is a clue to something that we will see in our figures: Although the bequest factor α is large, the estimated bequest shifter $\underline{a}$ is also large enough that it renders the bequest motive almost irrelevant for the median college-educated household we are examining. This is consistent with previous findings in the literature such as those by De Nardi [2004], who finds that the bequest motive comes into play only for the richest households. Finally,

Table 1: Estimated parameter values for the life-cycle, warm-glow, and TRP portfolios.

Name	Criterion	CRRA (ρ)	Wealth Share (δ)	Bequest Factor (α)	Bequest Shift ($\underline{a}$)
Life-Cycle Portfolio	0.594	8.810			
Warm-Glow LC Portfolio	0.594	8.765		43.773	26.256
TRP LC Portfolio	0.397	3.472	0.531		

the TRP Portfolio Choice model estimates a CRRA ρ coefficient of 3.472 and a wealth share of utility δ coefficient of 0.531. This result is significant because the CRRA ρ coefficient required to match the wealth accumulation patterns is significantly lower than that of the Standard Life-Cycle Portfolio choice model, whose high CRRA ρ has long been a puzzle in the literature. The addition of a non-separable utility of wealth factor δ allows the TRP Portfolio model to match the wealth accumulation patterns with a lower CRRA ρ coefficient, which is more in line with the literature on risk aversion. Finally, we can observe the column labeled "Criterion", which represents the minimum value of the objective function attained for each model during the estimation process. The TRP Portfolio model has the lowest criterion value, indicating that it is the best-fitting model among the three.

Figure 1 shows the median portfolio share for the agents in our simulation, along with the target share moments that come from Aboagye et al. [2024]. That paper presents the typical glide-path of target-date funds (TDFs) which provide a basis for much of commercial financial advice. While the red line shows the whole life-cycle glidepath of optimal advice, here we only target those moments starting at age 70. As we can see, the TRP Portfolio model does a good job of matching the targeted portfolio shares post-retirement, while the Life-Cycle Portfolio and the Warm-Glow Portfolio models come close but persistently under-estimate the share of risky assets.

Figure 2 shows the median wealth to income ratio[13] for achieved during the simulation. Again, the red line represents the target wealth to income ratios, or the empirical moments of our simulation, that come from the Survey of Consumer Finances. One might be concerned about the presence of a "zig-zag" in the aggregate empirical data, but this is likely due to selection/sampling bias and the reduction of participating households at higher ages due to death. The Life-Cycle and Warm-Glow Portfolio models overshoot the wealth accumulation over the life-cycle, but as expected, the agents

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[13]Wealth is `networth` and income consists of wages, social security, and retirement income as defined by the Survey of Consumer Finances.
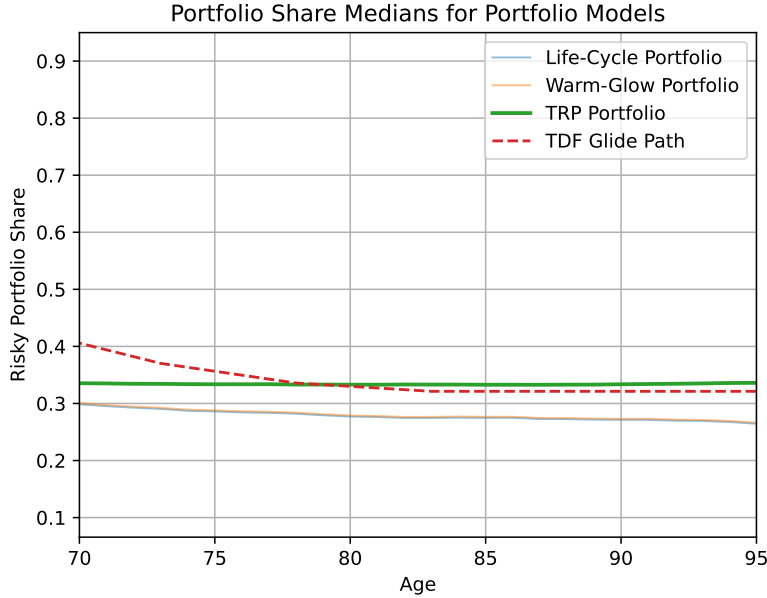
Figure 1: Median Portfolio Share for different portfolio models. The red line shows the target moments from Aboagye et al. [2024].

in these simulations start to run down their wealth right after retirement. The TRP Portfolio model similarly overshoots wealth accumulation for working-age agents, but the agents in this simulation do not run down their wealth post-retirement. Overall, the TRP Portfolio model does a good job of matching the observed pattern post-retirement for the median household: peak of wealth to income accumulation post-retirement and a slow decline in the wealth to income ratio after that.

5 Conclusion

To thoughtful academics, it has long been disturbing that the financial advice industry has paid so little attention to our hard work in constructing and solving impressively sophisticated dynamic stochastic optimization models of financial behavior. Those of us with a bit of humility have always suspected that the failure has been on our side: If all we could offer was models that produced risible advice like "everyone should spend down their wealth to zero and live pension-check to pension-check," while financial analysts' real world experience told them that such advice would get them fired, then it was reasonable to disregard the academic literature.

The thesis of this paper, however, is that a confluence of factors has now finally brought us to a point where state-of-the-art mathematical/computational life-cycle optimization models can provide advice that makes sense– so long as the model assumptions are also disciplined by survey data and the practical knowledge of financial advisors.[ĥousing]
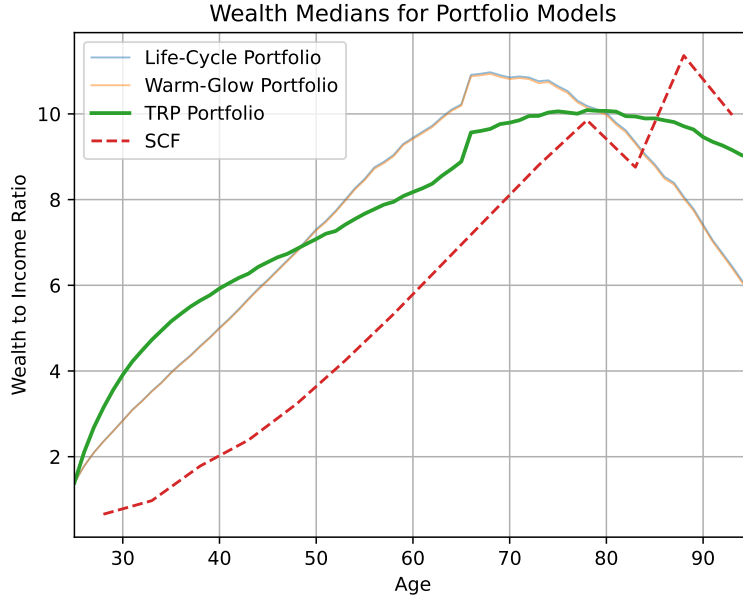
Figure 2: Median Wealth to Income Ratio for different portfolio models. The red line indicates median wealth-to-income ratios for College educated households in the Survey of Consumer Finances. Wealth is `networth` and income consists of wages, social security, and retirement income.

Much more remains to be done to improve the models further; for example, a question of great practical importance that is now just at the edge of possibility of being computationally solved is to calculate the implications of nonfinancial (principally, housing) wealth for optimal financial choice. Because homeownership is such a complex phenomenon, the academic literature is only now reaching the point at which it may be possible to answer questions like "if I own a house, how should I modify my spending and portfolio plans to take that into account?" We do know the *direction* of the effect. Kimball [1993] shows that the addition of a new uncontrollable risk reduces the optimal choice of exposure to controllable risks like the stock market. But *by how much* one's stock exposure should be reduced because of house-price risk can only be answered by solving a quantitatively plausible model.

It would be a better world if financial advice could be justified as reflecting the mathematically optimal solution to a well-defined problem. Not only would academics have the satisfaction of knowing that they had finally come close to fulfilling the vision of Modligliani and Brumberg 70 years ago. Financial analysts could also sleep more soundly in the knowledge that the advice they were giving were what many people probably think it already is: The adaptation to the client's particular circumstances of the advice that is the best that can be delivered by the latest high-tech computational optimization tools.

The time seems ripe for a much closer collaboration between academia and the financial industry in building this better world. This paper's open-source code, built with the associated open-source Econ-ARK project's tools, would be a good place to start.[14]

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