

Optimal fiscal policy with heterogeneous agents

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The aim of this paper is to study the relationship between the intertemporal behavior of taxes and wealth distribution. The optimal-taxation literature has often concentrated on representative-agent models, in which it is optimal to smooth distortionary taxes. When tax liabilities are unevenly spread in the population, deviations from tax smoothing lead to interest rate changes that redistribute wealth. When a “bad shock” hits the economy, the optimal policy will then call for smaller or larger deficits, depending on the political power of different groups. This effect is particularly relevant in the case of large shocks to government finances, such as wars.

KEYWORDS. Optimal taxation, heterogeneous agents, asset prices, distortion, net trade.

JEL CLASSIFICATION. E62, H21.

1. INTRODUCTION

The tax structure in an economy is in part the result of a struggle over the distribution of resources. The aim of this work is to study one aspect of this struggle, the choice of an optimal intertemporal tax plan. The intertemporal aspect of fiscal policy is important because any government constantly faces fiscal shocks. These may come from a wide variety of sources: business cycles, financial crises, the transition from a centralized to a decentralized economic system, and wars. In any of these cases, the government must choose among various policies for accommodating the shock. For example, a negative shock can be absorbed by an increase in taxes, a low return on previously issued state-contingent debt, or new issues of debt to be repaid with future taxes.

Different groups of agents in the economy have different preferences over these policies, and the goal of this paper is to study how the government can manipulate intertemporal prices to favor some groups over others.

On the normative side, I am interested in studying the characteristics of second-best tax policies, which will trace a (second-best) Pareto frontier. A benevolent government would choose one of these policies; which one depends on its relative preferences for

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I am indebted to Robert Lucas, Casey Mulligan, José Scheinkman, Nancy Stokey, a co-editor, two anonymous referees, and especially Thomas Sargent for helpful comments. I also benefited from conversations with Fernando Alvarez, Gary Becker, Mariacristina De Nardi, Larry Jones, Kenneth Judd, Adriano Rampini, Andrew Sellgren, and François Velde. Any remaining errors are my own. Financial support from the Alfred P. Sloan Foundation and the ESRC grant to the Centre for Macroeconomics is gratefully acknowledged.

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DOI: 10.3982/QE362

the different classes of agents in our economy. Along the Pareto frontier, we can also inquire whether there is a trade-off between “equity and efficiency.” For extreme values for the Pareto weights, the incentive to redistribute wealth may lead the government to impose significant distortions in the economy.

In addition, although in reality the political process is more complicated than a benevolent planner, a good political system will select policies close to the Pareto frontier. For this reason, my analysis is also likely to have some positive implications. Moreover, these are more evident the sharper the welfare differences among the possible policies, that is, in cases of large fiscal shocks, such as wars.

A striking example of different experiences in war financing comes from England and France in the 17th and 18th centuries. As discussed in [Sargent and Velde \(1995\)](#), England relied heavily on debt to finance its wars, while France made heavy use of temporary tax increases. These differences cannot be easily explained by representative-agent models, but I show through an example that they can be accounted for by the theory I am proposing.

The main conflict I study in this paper opposes the “taxpayers,” who bear the burden of taxes, and the “rentiers,” simply identified as all the other agents in the economy. In the model I present, I concentrate on labor-income taxation by considering an economy without capital, similar to the setup of [Lucas and Stokey \(1983\)](#): output is produced by a constant-returns-to-scale (CRS) technology requiring only labor; the taxpayers’ labor income is taxed by the government to finance an exogenous stream of public spending. Government spending is the source of fundamental uncertainty. The government and the two classes of agents trade in complete markets.

The environment I analyze is closest to [Persson and Svensson \(1986\)](#), who study optimal taxation in an open economy, where heterogeneity is given by the presence of domestic and foreign consumers. However, their emphasis is mainly on the conditions required for time consistency of the optimal policy; my research is instead aimed at characterizing the optimal policy and studying the economic forces that drive it. My work is also related to [Ben-Gad \(1996\)](#), who studies the effect of the timing of taxes in two-period models. In his environment, the major impact of government debt is due to the presence of incomplete markets. The two-class division that I assume in my numerical examples is the same as in [Conklin \(1994\)](#), who studies debt sustainability in the absence of commitment.¹

I study the pattern of net trades that emerges among the government, the rentiers, and the taxpayers in a competitive equilibrium, and I show how optimal policy is connected to it. Specifically, consider what happens when government spending varies over time (or across states). Government spending is ultimately paid for by the taxpayers; when it is temporarily high, they need to borrow, either directly or through government debt backed by future tax revenues. A lower tax rate in periods of high spending (i.e., a larger deficit) encourages production and, consequently, it allows the taxpayers to borrow at more favorable terms; for this reason, a government that wishes to favor taxpayers

¹In contrast to my work, Conklin assumes that taxpayers are denied access to any financial market in his model. As a consequence, the government acts mainly as a financial intermediary between the taxpayers and the rentiers.

will run bigger deficits than one that does not have redistribution motives (or that wishes to favor rentiers).

The relationship between net trades and the incentive to manipulate prices that appears quite transparently here is a general feature that applies across a wide variety of models. Net trades between the government and households are important for the results in the representative-agent economy of [Lucas and Stokey \(1983\)](#), as well as [Lustig, Sleet, and Yeltekin \(2008\)](#); they play an important role even in incomplete-market models such as [Yared \(2013\)](#) and [Golosov and Sargent \(2012\)](#).² The pattern of net trades also explains the nature of the deviation from the uniform commodity taxation results of [Atkinson and Stiglitz \(1976\)](#): in their environment, the households' initial endowment is given by a single good (leisure), whereas here households start with leisure and claims to goods across many periods.

I illustrate the key results of the paper through a calibration to 18th century wars. This example makes two points:

- The preferred policy by different groups is quantitatively very different.
- Most papers on optimal Ramsey policy assume that the first-order conditions of the Ramsey problem are necessary and sufficient. I show here that, for the application at hand, this is not always the case. When the government favors sufficiently the rentiers, an incentive arises to engage in random taxation. I thus provide a strategy to compute the optimal path when this possibility is present.³

The plan of the paper is the following: Section 2 presents the results for the general model; Section 3 specializes it to the two-class economy I study in greater detail and presents a calibrated experiment. Finally, Section 4 concludes. Appendices A–C follow. Additional appendices and data are available in supplementary files on the journal website, <http://qeconomics.org/supp/362/supplement.pdf> and http://qeconomics.org/supp/362/code_and_data.zip.

2. THE MODEL

In this section we present the general setup of the economy: we introduce the preferences, the technology, and the government; we define the equilibrium concept; and we provide a few general results that will be useful for characterizing the solution.

We consider an economy populated by N agents, which may differ by their preferences, their initial wealth, and their productivity while at work.

2.1 *Endowment and technology*

There is an exogenous stream of public spending: public spending does not affect the utility of the agents.⁴ We define g_t to be public spending in period t ; we also use the

²This paper is also related to [Niepelt \(2004\)](#), where taxes redistribute intertemporally in a direct way, rather than acting through intertemporal prices.

³The role for randomization in taxes is explored in [Stiglitz \(1982\)](#) and [Brito, Hamilton, Slutsky, and Stiglitz \(1985\)](#). We will discuss the relationship with their work in Section 4.

⁴As usual, the results do not change if public spending does enter in the utility function of the individuals, but only in a separable way.

convenient notation $g_t^s \equiv \{g_u\}_{u=t}^s$ and $g^s \equiv \{g_u\}_{u=0}^s$. We assume that $\{g_t\}_{t=0}^\infty$ is a stochastic process with a finite range G .⁵

Since the optimal taxation plan will sometimes feature randomization, we also introduce “public randomization” or a “sunspot” variable h_0 that is revealed right after consumption and tax plans are chosen at time 0, and that takes a finite number of values. As for all other papers on Ramsey taxation, we will maintain the assumption that taxes must be set in each period before agents make their consumption/leisure decision for that period;⁶ conditional on this assumption, the [Appendix](#) shows that using a single, time-0 random variable is sufficient to achieve an optimum.⁷ We will denote as s^t the history of random shocks up to time t , including government spending and the public randomization variable: $s^t := (g^t, h_0)$. The set of possible time- t histories is denoted S^t . Expectations as of time t (denoted by E_t) will be conditional on the information revealed by one such history.

There is no storage and only one consumption good. Output is produced through a CRS technology: in each period and each state, 1 unit of time spent working by agent i produces w^i units of output. Each agent is endowed with 1 unit of time. Each agent must choose a plan for consumption and leisure $\{(c^i(s^t), x^i(s^t))\}_{t=0}^\infty$, where $c^i(s^t)$ is consumption of the i th agent and x^i_t is leisure of the i th agent in period t conditional on history s^t . In addition, households may have an endowment of goods that cannot be taxed by the government, such as the land of the nobility and the clergy in France under the ancien régime, or black-market income that is present to varying degrees in all countries.

The government can levy proportional taxes on (or provide subsidies to) the labor income of each agent in the economy. Following [Ramsey \(1927\)](#), we assume that the tax rate is constrained to be equal across agents and that the marginal tax rate is constant on all labor income (i.e., there is proportional taxation). The incentive to manipulate prices would generalize to environments with nonlinear taxation, but it would require a more substantial description of the limits to government policy and their relationship to the existence of private credit markets. The tax rate depends on s^t and (as previously mentioned) is thus known by the consumers when they implement their consumption/leisure decision for the period.

There are complete markets, both for privately issued and publicly issued securities; the government is not allowed to default on previously issued debt instruments, so privately and publicly issued claims are perfect substitutes.

⁵The working paper version ([Bassetto \(1999\)](#)) treats the more general case, to which results extend straightforwardly.

⁶This assumption means that we do not allow the government to “toss a coin” to determine the amount of taxes due *after* production has taken place.

⁷In the main text, we assume that h_0 takes a finite number of values; this permits us to adopt a notation that is similar to most other papers on Ramsey taxation. With appropriate notation, it is straightforward to consider the case in which the range of h_0 has the power of the continuum, as we do in the [Appendix](#). The [Appendix](#) shows that h_0 can be restricted to be uniform in $[0, 1]$ without loss of generality. In the special case discussed in Section 3, Appendix D in the supplementary file shows that optimal allocations, prices, and policy will take at most two values as a function of h_0 , so a random variable with two values would be sufficient if the government is allowed the choice of the probability of each realization. For more general cases, it would be straightforward to prove that optimal allocations, prices, and policy would take a finite set of values.

We will define $b^i(s^t)$ to be the amount of government-issued contingent claims payable at time t that the i th agent holds at the beginning of period 0; if this is a negative number, then it will mean that the i th agent owes to the government. We will also define $\eta^i(s^t)$ to be an entitlement to state-contingent goods owned by agent i as of time 0. This entitlement is the sum of pure financial claims against other private agents and of the endowment of goods beyond the reach of tax authorities. When only financial claims are present, $\sum_{i=1}^N \eta^i(s^t) \equiv 0$, whereas the sum is strictly positive if it represents a physical endowment. We assume that both b and η only depend on g^t and not on the sunspot process h_0 .⁸

2.2 Preferences

The preferences of the i th consumer are described by

$$U^i \equiv E_0 \sum_{t=0}^{\infty} \beta^t u^i(c^i(s^t), x^i(s^t)). \quad (1)$$

We assume u^i is strictly concave, is continuously differentiable, and satisfies Inada conditions.

The preferences of the government are described by the social welfare function

$$W \equiv \sum_{i=1}^N \alpha^i U^i, \quad (2)$$

where α^i is the Pareto weight of the i th agent (a single individual or the representative agent of the i th group).

2.3 Competitive equilibrium and Ramsey outcome

We will consider Ramsey outcomes, where the government is free to choose its preferred allocation among all of those that represent a competitive equilibrium.⁹ As is well known, this choice is, in general, time inconsistent: the preferred outcome as of time 0 is no longer the best outcome among all possible competitive equilibria from time 1 onward. The working paper version (Bassetto (1999)) discusses conditions under which the debt maturity can be used to ensure time consistency, generalizing results of Lucas and Stokey (1983). The private agents take the policy parameters as given. Since markets are complete, we can assume that they choose their optimal contingent plans at time 0 based on a single Arrow–Debreu budget constraint. The economy starts at time 0 with some given level of public spending g_0 .

⁸If b and η depended on h_0 , this random variable would be part of the fundamentals of the economy and a new public randomization device would be needed.

⁹We sidestep the question of implementation, that is, exactly what government strategy is needed to ensure that such outcome is the unique equilibrium of the economy. On this issue, see Bassetto (2005).

The i th agent has the Arrow–Debreu budget constraint¹⁰

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} p(s^t) [c^i(s^t) - \eta^i(s^t) - b^i(s^t) - (1 - \tau(s^t))w^i(1 - x^i(s^t))] = 0, \quad (3)$$

where $p(s^t)$ is the time-0 price of an Arrow–Debreu security that pays 1 at date t in state s^t .

The government budget constraint is

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} p(s^t) \left[g_t + \sum_{i=1}^N (b^i(s^t) - \tau(s^t)w^i(1 - x^i(s^t))) \right] = 0. \quad (4)$$

This constraint automatically holds whenever (3) holds for all agents.

DEFINITION 1 (Competitive Equilibrium). A competitive equilibrium is given by a policy $\{\{\tau(s^t)\}_{s^t \in S^t}\}_{t=0}^{\infty}$, an allocation $\{\{(c^i(s^t), x^i(s^t))\}_{i=1}^N\}_{s^t \in S^t}\}_{t=0}^{\infty}$, a price system $\{p(s^t)\}_{s^t \in S^t}\}_{t=0}^{\infty}$, and initial conditions $\{\{(b^i(s^t), \eta^i(s^t))\}_{i=1}^N\}_{s^t \in S^t}\}_{t=0}^{\infty}$ such that the following situations exist:

(i) Given the price system, the government policy, and the initial conditions, the allocation maximizes the utility of each consumer subject to her budget constraint described by (3).

(ii) The markets clear, that is,

$$\sum_{i=1}^N c^i(s^t) + g_t = \sum_{i=1}^N w^i(1 - x^i(s^t)) + \sum_{i=1}^N \eta^i(s^t) \quad \forall t \geq 0 \quad \forall s^t \in S^t. \quad (5)$$

DEFINITION 2 (Ramsey Outcome). Within the set of competitive equilibria, a Ramsey outcome is any element that attains the maximal utility for the government according to its preferences (2).

We first look at the conditions for a competitive equilibrium. The first-order conditions for the private agents of our economy are

$$\frac{u_x^i(c^i(s^t), x^i(s^t))}{u_c^i(c^i(s^t), x^i(s^t))} \geq w^i(1 - \tau(s^t)) \quad \forall t \geq 0 \quad \forall s^t \in S^t, i = 1, \dots, N, \quad (6)$$

and

$$\frac{\beta^t \Pr(s^t) u_c^i(c^i(s^t), x^i(s^t))}{u_c^i(c^i(\bar{s}^0), x^i(\bar{s}^0))} = \frac{p(s^t)}{p(\bar{s}^0)} \Pr(\bar{s}^0) \quad \forall t \geq 0 \quad \forall s^t \in S^t, i = 1, \dots, N, \quad (7)$$

¹⁰More precisely, for each individual agent, the constraint (3) has to hold as a weak inequality, with the left-hand side being less than or equal to 0. We assume nonsatiation, so households will always exhaust their budget constraint. Writing the constraint as an equality is convenient way to write the Ramsey problem, where either side of the inequality may be binding, depending on the government preferences for redistribution.

where \bar{s}^0 is an arbitrary initial history (realization of h_0) that we use as the numeraire, $\Pr(s^t)$ is the probability of observing history s^t , and (6) must hold with equality if $x_t^i < 1$, that is, when the agent is supplying a positive amount of labor. The nonnegativity constraints on consumption and leisure are never binding because of the Inada conditions.

A competitive equilibrium is characterized by equations (3), (5), (6), and (7). The Ramsey outcome is thus the solution to maximizing (2) by choice of an allocation, a price system, and a tax policy, subject to (3), (5), (6), and (7).

In solving this problem, it is common to simplify the constraints by substituting out either prices and the tax policy (the “primal” approach) or the allocation (the “dual” approach). Before we undertake either, it is useful to gain intuition directly from the problem as it is. Equations (3), (6), and (7) correspond to the missing tax instruments that prevent the Ramsey outcome from achieving first best. Specifically, equation (3) for individual i rules out lump-sum transfers or taxes on individual i (or a tax on the initial wealth of individual i). The Lagrange multiplier on this constraint represents the marginal value of transferring 1 unit of numeraire from household i to the government. In the case of a representative agent, this corresponds to the deadweight cost of distortionary taxation and it is always positive. With heterogeneous agents, the multiplier can be negative if the government would be willing to give lump-sum transfers to household i if allowed. Equation (6) imposes equality of labor-income tax rates across households. Finally, equation (7) states that households are allowed to trade in intertemporal asset markets beyond the reach of the tax man (this may be because trades are anonymous and unobserved by the government).

Of these constraints, the one that is essential to our results is (3): the inability of the government to attain the desired redistribution of wealth and raise the appropriate amount of resources for public spending without imposing any distortions. If we dropped (6) and (7), our results would be qualitatively similar. Conversely, if we dropped (3), (6) and (7) would not bind: with type-specific lump-sum taxes and transfers, the government could attain the first-best allocation, which satisfies (6) and (7) without imposing them.

In comparing an agent’s welfare across competitive equilibria, the relevant measures are the intertemporal price of consumption, $p(s^t)$, and (after-tax) labor, $(1 - \tau(s^t))p(s^t)$. From standard comparative statics, we see that a change in $p(s^t)$ (holding $(1 - \tau(s^t))p(s^t)$ constant) will increase the welfare of an agent i when $c^i(s^t) - \eta^i(s^t) - b^i(s^t) < 0$ and decrease it when the reverse inequality holds. When $c^i(s^t) - \eta^i(s^t) - b^i(s^t) < 0$, the i th agent is a net seller of the considered good: her endowment in the Arrow–Debreu market, whether from untaxed income or from initial financial claims, is larger than her consumption.¹¹ In the absence of nondistorting methods of redistribution, the government will often face a temptation to distort intertemporal prices to favor some agents at the expense of others. To do so, there needs to be a clear pattern of net trade that the government can exploit. In the applications that we will present, this pattern arises from the assumption that government spending varies and that only some

¹¹A similar intuition is valid for the intertemporal price of labor; however, this price will not play as important a role in our applications.

agents are taxed to pay for it. There are many political and informational reasons why the policy of redistributing indirectly by distorting intertemporal prices may be easier to implement than direct redistribution toward the classes most favored by the government; it is in these cases that manipulating interest rates through the deficit policy will be a desirable policy instrument.

2.4 Some analytic results

In this section, we follow [Lucas and Stokey \(1983\)](#) and consider the first-order conditions of the Ramsey problem. To simplify notation and avoid lengthy discussions of complementary slackness conditions, we assume that the nonnegativity constraint on labor supply is always binding for agents 1 through M (i.e., we assume that $w^i = 0$ for $i = 1, \dots, M$) and that it is never binding for agents $M + 1$ through N (i.e., $u_x^i(c, 1) = 0$ for $i = M + 1, \dots, N$). We also assume that preferences are strongly separable in consumption and leisure, so that $u_{cx}(c, x) \equiv 0$.

We adopt the primal approach, and use the competitive equilibrium necessary conditions (6) and (7) to substitute prices and tax rates in the government problem. We thus transform the household budget constraints (3) into the implementability constraints¹²

$$E_0 \sum_{t=0}^{\infty} \beta^t u_c^i(c^i(s^t))(c^i(s^t) - \eta^i(s^t) - b^i(s^t)) = 0, \quad i = 1, \dots, M, \quad (8)$$

and

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta^t [u_c^i(c^i(s^t))(c^i(s^t) - \eta^i(s^t) - b^i(s^t)) - u_x^i(x^i(s^t))(1 - x^i(s^t))] \\ = 0, \quad i = M + 1, \dots, N. \end{aligned} \quad (9)$$

Further constraints on the Ramsey problem impose equality of the marginal rates of substitution across households:

$$\frac{u_x^i(x^i(s^t))}{u_c^i(c^i(s^t))} = \frac{w^i u_x^N(x^N(s^t))}{w^N u_c^N(c^N(s^t))} \quad \forall t \geq 0 \quad \forall s^t \in S^t, i = M + 1, \dots, N - 1, \quad (10)$$

and

$$\frac{u_c^i(c^i(s^t))}{u_c^i(c^i(\bar{s}^0))} = \frac{u_c^N(c^N(s^t))}{u_c^N(c^N(\bar{s}^0))} \quad \forall t \geq 0 \quad \forall s^t \in S^t, i = 1, \dots, N - 1. \quad (11)$$

To take first-order conditions, rather than using person N as a benchmark, it is more convenient to treat all households symmetrically. We can do so by introducing two new

¹²With some abuse of notation, we use only one argument for the partial derivatives of u , since they are independent of the other argument by strong separability. Also, we assume that consumption and leisure are such that the appropriate sums are well defined, and that it is permissible to exchange the order of expectations and sums. By changing the way the sums are computed, the [Appendix](#) provides a formal proof that this is the case.

variables $\{\{\tilde{\kappa}(s^t)\}_{s^t \in S^t}\}_{t=0}^\infty$ and $\{\{\kappa(s^t)\}_{s^t \in S^t}\}_{t=0}^\infty$, and imposing that the marginal rates of substitution of all households (including household N) should be equal to the new variables¹³

$$u_x^i(x^i(s^t)) = \tilde{\kappa}(s^t) w^i u_c^i(c(s^t)) \quad \forall t \geq 0 \quad \forall s^t \in S^t, i = M+1, \dots, N, \quad (12)$$

and

$$\kappa(s^t) u_c^i(c(s^t)) = u_c^i(c(\bar{s}^0)) \quad \forall t \geq 0 \quad \forall s^t \in S^t \setminus \{\bar{s}^0\}, i = 1, \dots, N. \quad (13)$$

The Ramsey problem thus maximizes (2) subject to the feasibility constraint (5), and subject to (8), (9), (12), and (13). Let λ^i be the Lagrange multiplier associated with (3), $\beta^t \Pr(s^t) \psi^i(s^t)$ be associated with (12), $\beta^t \Pr(s^t) \phi^i(s^t)$ be associated with constraint (13),¹⁴ and $\beta^t \Pr(s^t) \mu(s^t)$ be associated with (5). The first-order conditions are then

$$\begin{aligned} & u_c^i(c^i(s^t))(\alpha^i + \lambda^i) + \lambda^i u_{cc}^i(c^i(s^t)) [c^i(s^t) - \eta^i(s^t) - b^i(s^t)] \\ & \quad - \psi^i(s^t) \tilde{\kappa}(s^t) w^i u_{cc}^i(c^i(s^t)) + \phi^i(s^t) \kappa(s^t) u_{cc}^i(c^i(s^t)) - \mu(s^t) \\ & = 0 \quad \forall t \geq 0 \quad \forall s^t \in S^t \setminus \{\bar{s}^0\}, i = 1, \dots, N, \end{aligned} \quad (14)$$

$$\begin{aligned} & u_c^i(c^i(\bar{s}^0))(\alpha^i + \lambda^i) + \lambda^i u_{cc}^i(c^i(\bar{s}^0)) [c^i(\bar{s}^0) - \eta^i(\bar{s}^0) - b^i(\bar{s}^0)] \\ & \quad - \psi^i(\bar{s}^0) \tilde{\kappa}(\bar{s}^0) w^i u_{cc}^i(c^i(\bar{s}^0)) + u_{cc}^i(c^i(\bar{s}^0)) E_0 \sum_{t=0}^\infty \beta^t \phi^i(s^t) - \mu(\bar{s}^0) \\ & = 0, \quad i = 1, \dots, N, \end{aligned} \quad (15)$$

$$\begin{aligned} & u_x^i(x^i(s^t))(\alpha^i + \lambda^i) - \lambda^i u_{xx}^i(x^i(s^t)) [1 - x^i(s^t)] - \psi^i(s^t) u_{xx}^i(x^i(s^t)) - \mu(s^t) w^i \\ & = 0 \quad \forall t \geq 0 \quad \forall s^t \in S^t, i = M+1, \dots, N, \end{aligned} \quad (16)$$

$$\sum_{i=M+1}^N \psi^i(s^t) w^i u_c^i(c(s^t)) = 0 \quad \forall t \geq 0 \quad \forall s^t \in S^t, \quad (17)$$

and

$$\sum_{i=1}^N \phi^i(s^t) u_c^i(c(s^t)) = 0 \quad \forall t \geq 0 \quad \forall s^t \in S^t \setminus \{\bar{s}^0\}. \quad (18)$$

To gain intuition, assume first that $\psi^i(s^t) \equiv \phi^i(s^t) \equiv 0$, so that the government can manipulate household-specific intertemporal prices. Consider the case of an agent for

¹³The variables with respect to which the optimization takes place are thus $\{\{c^i(s^t)\}_{i=1}^N, \{x^i(s^t)\}_{i=M+1}^N, \tilde{\kappa}(s^t), \kappa(s^t)\}_{s^t \in S^t}\}_{t=0}^\infty$.

¹⁴Notice there is no equation (13) for the reference history \bar{s}^0 . To simplify writing the expressions, we define the missing Lagrange multipliers as $\phi^i(\bar{s}^0) := 0, i = 1, \dots, N, \psi^i(s^t) := 0, i = 1, \dots, M$.

whom $\lambda^i > 0$: if allowed, the government would like to take wealth away from agent i by levying a lump-sum tax on her. From equation (14), if we compare two histories s^t and $s^{t'}$ such that $c^i(s^t) - \eta^i(s^t) - b^i(s^t) > 0$ and $c^i(s^{t'}) - \eta^i(s^{t'}) - b^i(s^{t'}) < 0$, we obtain

$$\frac{u_c^i(s^t)}{u_c^i(s^{t'})} > \frac{\mu(s^t)}{\mu(s^{t'})}. \quad (19)$$

Since $\mu(s^t)$ represents the shadow cost to the planner of goods after history s^t , equation (19) shows that the Ramsey allocation distorts the marginal utility (and hence the price) upward for histories for which agent i is a net buyer relative to histories after which it is a net seller of the good: the planner wants the agent to sell cheap and buy expensive. The inequality reverses in the case of an agent for whom $\lambda^i < 0$: in this case, if allowed, the planner would like to offer a lump-sum transfer to the agent.

The constraints (12) and (13) imply that the planner cannot freely distort individual prices, but it must distort prices for all agents in the same way. In this case, not all the multipliers $\phi^i(s^t)$ and $\psi^i(s^t)$ can be zero, but it is a weighted average (history by history) that will be zero instead, as shown by equations (17) and (18). Distorting prices will then be particularly helpful for the planner if the most favored and least favored agents tend to be on opposite sides of the trades.

To gain further intuition, it is useful to compare our results with Werning (2007), whose benchmark involves no price distortions by virtue of the uniform commodity taxation results in Atkinson and Stiglitz (1976).¹⁵ The key difference between our setup and his is the presence of income from financial assets or other factors that cannot be taxed (either because the income is hidden to the planner or because of other political constraints).¹⁶ To see this, assume that the planner can tax initial contingent claims $\eta^i(s^t) + b^i(s^t)$ at some rate $\theta(s^t)$.¹⁷ It is straightforward to prove that the first-order condition for $\theta(s^t)$ would add the additional condition

$$\sum_{i=1}^N \lambda^i u_c^i(c^i(s^t)) (\eta^i(s^t) + b^i(s^t)) = 0 \quad \forall t \geq 0 \quad \forall s^t \in S^t. \quad (20)$$

either the government fully expropriates the initial contingent claims or an appropriately weighted sum of the households' marginal utilities is 0, so that further redistribution through the tax on initial wealth does not pay off. In Werning's benchmark, preferences are isoelastic in consumption, so that $u^i(c, x) = \frac{c^{1-\gamma}}{1-\gamma} + v(x)$. The first-order condi-

¹⁵A more formal comparison with Atkinson and Stiglitz appears in the supplementary file.

¹⁶As discussed by Werning, our assumption also implies that consumption must also be partly hidden from the planner.

¹⁷In Werning (2007), initial wealth is composed of claims to capital, which can be converted into time-0 consumption goods at a fixed price, and bonds maturing in period 0. For this reason, a single initial tax rate is sufficient. Whenever the value of initial wealth may depend on intertemporal prices, his results only apply if there are as many tax rates as types of initial wealth. In our environment, in general, there is a full set of initial contingent claims, which requires a full set of contingent taxes.

tion (14) yields then

$$\begin{aligned}
 & (\alpha^i + \lambda^i - \lambda^i \gamma)(c^i(s^t))^{-\gamma} \\
 & + \gamma(c^i(s^t))^{-\gamma-1}[-\lambda^i(1 - \theta(s^t))(\eta^i(s^t) + b^i(s^t)) \\
 & + \psi^i(s^t)\tilde{\kappa}(s^t)w^i - \phi^i(s^t)\kappa(s^t)] - \mu(s^t) \\
 & = 0 \quad \forall t \geq 0 \quad \forall s^t \in S^t \setminus \{\bar{s}^0\}, i = 1, \dots, N.
 \end{aligned} \tag{21}$$

Next multiply equation (21) by $c^i(s^t)$ and sum across households, using equations (17) and (18). If (and, generically, only if!) $\theta(s^t)$ is a choice variable and thus (20) holds, then we obtain¹⁸

$$\sum_{i=1}^N (\alpha^i + \lambda^i - \lambda^i \gamma)(c^i(s^t))^{1-\gamma} = \mu(s^t) \sum_{i=1}^N c^i(s^t) \quad \forall t \geq 0 \quad \forall s^t \in S^t. \tag{22}$$

Using equation (13), we then obtain

$$\frac{c^i(s^t)}{c^i(\bar{s}^0)} = \frac{\mu(s^t)}{\mu(\bar{s}^0)} \quad \forall t \geq 0 \quad \forall s^t \in S^t, i = 1, \dots, N: \tag{23}$$

the planner equates the marginal rate of substitution to the ratio of the shadow cost of resources across histories. The reason for this result is intuitive: if the planner is allowed to directly tax away the initial state-contingent wealth, there is no reason to achieve the same outcome through the indirect means of distorting intertemporal prices.¹⁹

In our paper, the tax $\theta(s^t)$ is not available. As an example, the governments of many countries find it difficult to eradicate a substantial black-market sector that evades taxation. More generally, political constraints might imply that opposition to setting up institutions for direct redistribution might be stronger than the resistance to intertemporal price manipulation. We do not model the institutional and/or information frictions that prevent the direct tax from taking place. We study the implications of this differential tax across types of income on optimal fiscal policy.

While the first-order conditions of the Ramsey problem are useful in developing intuition, unfortunately in our environment with heterogeneous agents, they are sometimes not sufficient and random policy may occur.²⁰ While this issue only arises for somewhat extreme values of the government preferences, the potential for this to occur requires developing a method to probe this possibility. The [Appendix](#) develops useful aggregation results. Two conclusions are important for computational purposes:

¹⁸The expression corresponding to (21) for history \bar{s}^0 is slightly different, but nonetheless (22) is the same.

¹⁹If, in addition, preferences are isoelastic in the labor supply as well, as assumed in [Werning's \(2007\)](#) benchmark, then perfect tax smoothing follows.

²⁰For some examples, these first-order conditions may not even be necessary, as the solution becomes degenerate, with the supremum of the Ramsey problem being attained with tax rates on labor converging to 1 with probability converging to 0.

- If randomization is optimal, it is sufficient to consider a single sunspot variable, whose outcome is realized at time 0, before τ_0 is set.
- Without loss, consumption and leisure of all agents and the tax rate can be set to the same values after two histories s^t and $s^{t'}$ if they share the same realization of the sunspot h_0 , the same level of government spending, and the same initial contingent claims: $\eta^i(s^t) + b^i(s^t) = \eta^i(s^{t'}) + b^i(s^{t'})$, $i = 1, \dots, N$.

These results, along with Theorem 1 below, reduce an infinite-dimensional optimization problem to a finite-dimensional one.

3. THE TWO-CLASS ECONOMY

In this section, we specialize the general framework presented above. We consider an economy populated by two types of agents: M agents of type 1 and (by normalization) one agent of type 2.²¹ Type-1 agents are “rentiers.” Their productivity is 0, so they always choose $x_t^1 = 1$. They have no labor income and live only out of their assets. As the discussion of the previous section has emphasized, the important assumption for our analysis is not that the rentiers do not work, but that they are not subject to taxes.²² Type-2 agents are identified as the “taxpayers,” as they are the only ones who have labor income and therefore pay taxes. We normalize their productivity to be $w^2 = 1$.

We assume agents to be completely homogeneous within groups. When $\alpha^1 = 0$, that is, when the government maximizes the welfare of the taxpayers only, we can interpret this as an open economy where the government does not have the authority to tax foreigners; in this case, our setup is the same as that in Persson and Svensson (1986).²³

We assume the agents have the utility functions

$$u^1(c_t^1, x_t^1) = \frac{(c_t^1)^{1-\gamma} - 1}{1-\gamma} \quad (24)$$

and

$$u^2(c_t^2, x_t^2) = \frac{(c_t^2)^{1-\gamma} - 1}{1-\gamma} + \xi \frac{(x_t^2)^{1-\sigma} - 1}{1-\sigma}. \quad (25)$$

The form of the utility function for the leisure component for type-1 agents is irrelevant, since they will always choose $x_t^1 \equiv 1$.²⁴

To prove the main theorem that allows us to compute the solution, we also assume the following condition.

²¹Formally, this economy coincides with the general case if $N = M + 1$ and the first M agents share the same endowment and preferences. Nonetheless, for simplicity of notation, we use the superscript 1 for all of the first M agents and use the superscript 2 for agent N .

²²We could easily adjust the analysis to allow for labor income to be earned by the rentiers, with little difference in the results, as long as their income was not subject to taxes.

²³Our economy is closest to the second setup in Persson and Svensson (1986), in which they allow for perfect capital mobility.

²⁴Although the functional form we chose is different from that in Chari, Christiano, and Kehoe (1994), it is consistent with their baseline preferences if we set $\gamma = \sigma = 1$ and $\xi = 3$.

CONDITION 1. Either of the following properties is met.

- $\gamma \geq 1$ and there exists a policy that satisfies the government budget constraint (4) and $Mc^1(s^t) + c^2(s^t) > 0$ almost surely (a.s.).
- $(g_t, \eta^1(s^t) + b^1(s^t), \eta^2(s^t) + b^2(s^t)) \neq \mathbf{0}$ a.s.

Under this condition, we can establish the following theorem.

THEOREM 1. *Assume that there exists a solution to the Ramsey problem. Then the allocation and the tax rate, as functions of the sunspot h_0 , assumes almost surely at most two values for each value of the vector $(g_t, b^1(s^t), b^2(s^t), \eta^1(s^t), \eta^2(s^t))$.*

For the proof, see Appendix D in the supplementary file.

Theorem 1 is reminiscent of [Stiglitz \(1982\)](#). However, in Stiglitz (as well as in Brito et al. 1985) agents are not allowed to trade based on the realization of the tax rate (alternatively, their trades are observable and can be stopped by suitable taxes); this greatly simplifies the analysis, because the problem becomes linear in the probabilities. In our case, redistribution occurs precisely because households trade based on the realization of the tax rate: randomization affects the asset pricing kernel.

3.1 Example: No government spending

In this example, we show how heterogeneity may lead to situations in which the first-order conditions are not sufficient even in the simplest case.

Let $g_t \equiv 0 \forall t \geq 0$, $b^i(s^t) \equiv 0 \forall t \geq 0$, $i = 1, 2$, and $\eta^i(s^t) \equiv \hat{\eta}_0^i$, $i = 1, 2$. The government has no public spending to finance and no debt to repay (or credit to distribute). Furthermore, the only outstanding private claims are annuities that pay a fixed amount every period.

In this setup, the government has no need to raise taxes ever; in a representative-agent model, the government would achieve a first best by setting²⁵ $\tau(s^t) \equiv 0 \forall t \geq 0 \forall s^t \in S^t$. With this tax policy, we would have

$$c^1(s^t) = \hat{\eta}_0^1 \quad \forall t \geq 0 \forall s^t \in S^t, \quad (26)$$

$$u_c^2(c^2(s^t)) = u_x^2(x(s^t)) \quad \forall t \geq 0 \forall s^t \in S^t, \quad (27)$$

and

$$c^2(s^t) + x^2(s^t) = 1 + \hat{\eta}_0^2 \quad \forall t \geq 0 \forall s^t \in S^t. \quad (28)$$

Equation (27) comes directly from (6) and describes the allocation of resources between leisure and consumption given that labor income is not taxed.

Equation (28) states that in each period, the sum of each agent's consumption and leisure is equal to her time endowment and the income (which may be negative) from the annuities she holds. The price system in this competitive equilibrium is $p(s^t) =$

²⁵Note that here s^t includes only the realization of the sunspot h_0 , since everything else is deterministic.

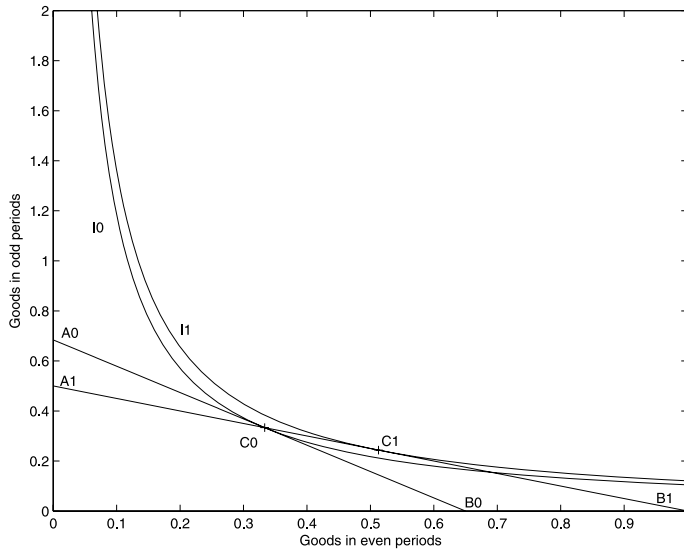


FIGURE 1. The benefits of randomization for the rentiers.

$\beta^t \Pr(s^t) \forall t \geq 0 \forall s^t \in S^t$. Given this price system, the choice of a constant profile of consumption and leisure implied by (26)–(28) is optimal, as it is implied by (7); furthermore, the budget constraints of each agent (3) are satisfied and so is the market clearing condition (5).

It is easy to check that this equilibrium satisfies the first-order conditions of the Ramsey problem, equations (14)–(18). While the no-tax solution always satisfies the first-order conditions in this example, it is not the optimal solution when the rentiers have a sufficiently large weight in the government. To understand why this is the case, we concentrate on the welfare of the rentiers.

Let us consider deviations from the no-tax policy that involve one tax rate in all even periods and another one in all odd periods.²⁶ Figure 1 shows what happens in this case. The no-tax solution is represented by the point C_0 : the rentiers consume in each period exactly the amount of resources they are owed by the taxpayers, that is, $c_t^1 = \hat{\eta}_0^1 \forall t \geq 0$. The line from A_0 to B_0 represents the Arrow–Debreu budget constraint of the rentiers in the no-tax policy: its slope is $-\frac{1}{\beta}$, as we assume the first period to be even (period 0). The indifference curve through C_0 is tangent to the budget constraint, reflecting the optimality of C_0 when the pricing kernel is constant. Suppose now the government varies the tax rates in odd and even periods. The rentiers are not affected directly by the change in the tax rate; they are only affected indirectly, as different tax rates lead to different relative prices between odd and even periods. As a consequence the budget constraint of the rentiers rotates (e.g., to A_1, B_1); however, it still goes through C_0 , as $c_t^1 = \hat{\eta}_0^1 \forall t \geq 0$ is always feasible. Since the utility function is assumed to be strictly concave, the indifference curves are strictly convex and the rentiers are strictly better off when the relative

²⁶As we argued previously, this policy is equivalent to a policy that sets two different constant tax rates depending on whether h is larger or smaller than $\frac{\beta}{1+\beta}$.

price of goods varies in either direction: the new choice is C_1 , which lies on a higher indifference curve. This welfare improvement is locally of second-order magnitude and this is why the no-tax policy satisfies the first-order conditions.

By taxing labor in odd periods and subsidizing it in even periods (or vice versa), the government generates an artificial scarcity of some goods with respect to others, and this is beneficial to the rentiers. Of course, this policy is very costly to the taxpayers. In the economy we consider, if the government were allowed to transfer resources directly between the two agents, it would never choose to distort prices, as a constant consumption stream for both agents would be Pareto efficient. The taxpayers, therefore, pay both for the gains of the rentiers and for the distortions introduced by taxes and subsidies. These losses are also of second order in a neighborhood of the no-tax policy.

To compute when the government would resort to randomization, I solved numerically for the optimal policy. Based on my computations, choosing different tax rates when all exogenous variables (government spending and maturing coupons) are the same is a very costly way of redistributing wealth among the agents. For instance, consider a case in which $\gamma = 2$, $\sigma = 1.1$, $\beta = 0.95$, $M = 1$, $\eta_t^1 = -\eta_t^2 = 1/3 \forall t \geq 0$, and $\xi = 3^{\gamma-\sigma}$. In this case, taxpayers and rentiers reach the same consumption when the government implements a no-tax policy. Deviating from this policy becomes desirable for the government only when the Pareto weight of the rentiers exceeds 0.58.

3.2 A calibration: France versus Britain

In this section, we illustrate the characteristics of the Ramsey outcomes of the two-class economy by looking at a quantitative experiment. The insights that we will obtain are robust across a variety of parameter choices. Only the magnitude of changes in the interest rate will be greatly amplified by choosing higher values for the risk aversion.

This example is suggested by an observation in [Sargent and Velde \(1995\)](#) about war financing in France and Britain in the 17th and 18th centuries. The clearest description of the difference between the two regimes is their quote of Montyon, a senior civil servant in the French finance ministry in 1770s. He made the following point:

Great Britain finances by taxation neither all nor part of the costs of war, it finances them by loans (...). In wartime it is our habit to increase taxes (...). Indeed in wartime the country suffers enough from the labor withdrawn from agriculture and manufactures to be sent into the army, the navy, and into the production activities necessitated by war.

Montyon wrote to express his dissatisfaction with the French policies. As Sargent and Velde argue convincingly, that dissatisfaction was one of the factors that led eventually to the French revolution.

France and Britain had very different political regimes at that time; the noble class had much more clout in France than in Britain. In terms of our model, we interpret this as meaning that the rentiers had a higher Pareto weight in France than in Britain.

In this numerical example, the government has thus to finance wars. I assume a two-state Markov process for wars, with the peace state lasting 11 years on average and war lasting 10 years; this roughly matches the average duration of peace and war between

France and Britain in 1688–1792. The economy starts in peacetime in period 0. Following Chari, Christiano, and Kehoe (1994), I set $\beta = 0.96$, $\gamma = \sigma = 1$, and ξ such that the amount of work in market activities would be approximately 1/3 of the available time for a nonstochastic economy with government spending set at its peacetime level. To calibrate the level of government spending, I use data from Mitchell (1988);²⁷ based on these data, I set wartime spending at 13.5% of gross domestic product (GDP) and peacetime spending at 6%. In addition, the government started with a 6% debt/GDP ratio, which I assume to be made entirely of consols. This flat structure across maturities ensures that government debt is not a reason to manipulate interest rates, thereby isolating the role of redistribution across classes of economic agents. The relative number and wealth of rentiers and taxpayers derives from data on income inequality from Morrisson and Snyder (2000). I set the fraction of rentiers in the population at 2%, and calibrate their tax-free income endowment so that the fraction of their income in the nonstochastic version of the economy would be 9%. Morrisson and Snyder also provide a guesstimate that about 50% of the income of what we call taxpayers was also unreported for tax purposes; we set η^2 to match this fact as well. This large fraction of income that escaped taxation explains why a level of spending of 13.5% represented a significant strain on government finances, while in contemporary times, government easily finances much larger burdens.

Figures 2–4 show the Ramsey allocation depending on the Pareto weight of the rentiers in the range where the government does not wish to randomize its policy.²⁸ These figures trace the constrained Pareto frontier by plotting the allocation, the price system, and government policy as a function of the relative weight of the rentiers in the government’s objective function. The vertical line represents the relative Pareto weight that

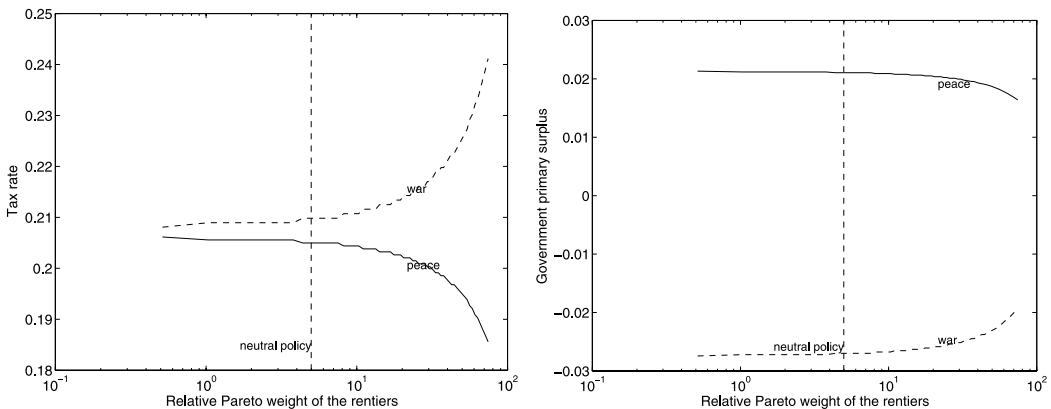


FIGURE 2. Fiscal policy in the France versus Britain example.

²⁷ See the Appendix for further details on the calibration.

²⁸ The government will randomize its policy when the Pareto weight of the rentiers is really large, but this range does not produce empirically plausible predictions. When the government has such a high desire to redistribute wealth from the taxpayers to the rentiers, it is likely that it will try to adopt alternative, less costly schemes.

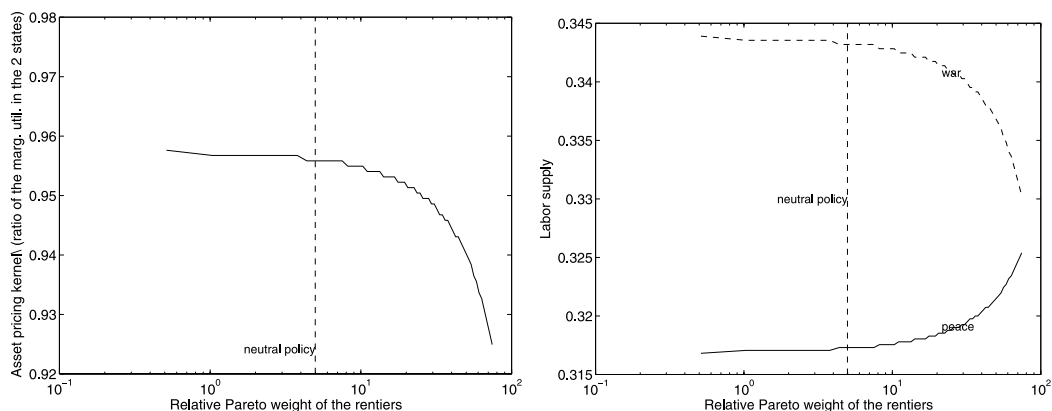


FIGURE 3. Asset prices and labor supply in the France versus Britain example.

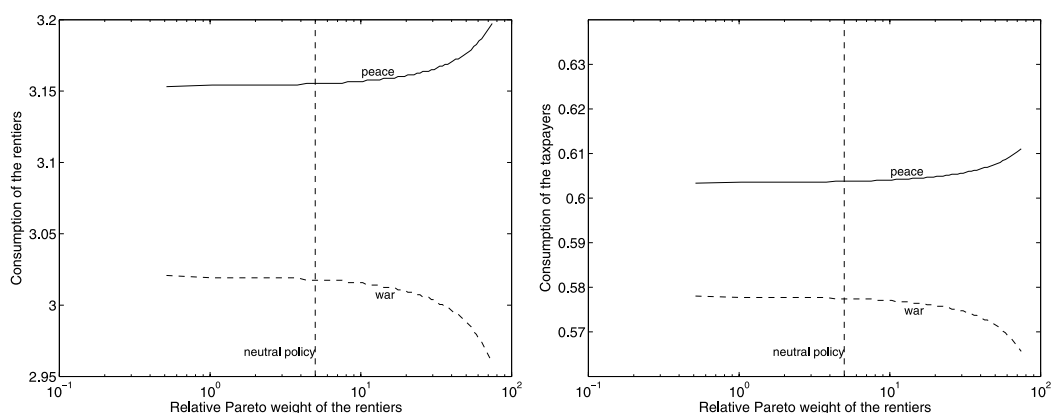


FIGURE 4. Consumption in the France versus Britain example.

coincides with the ratio of the marginal utilities of the two groups in the resulting equilibrium: at this point, the government does not wish to redistribute either way. To the right of this policy, which we label “neutral,” the government distorts prices to favor the rentiers, while the opposite is true on the left. The neutral policy corresponds to a relative Pareto weight of about 5: since the nobles and the clergy started with much more wealth, a desire for redistribution would arise unless the government regarded these classes with favor.

The main conclusion that these pictures suggest is that the optimal way of financing a war is influenced by the attitude of the government with respect to redistribution. Furthermore, the effect is consistent with the mentioned pattern of war finance in Britain and France: a government run by the taxpayers will run larger deficits in wartime, whereas the preferred policy for the rentiers involves a large increase in taxes during the war.

The intuition of this result is the following. Sooner or later, a war will have to be financed in this model by raising funds through taxes. This means that, implicitly (i.e.,

through the government) or explicitly (i.e., through direct transactions) the taxpayers will have to borrow from the rentiers to smooth their consumption stream. When the government “sides with the taxpayers,” it will try to get the best possible deal to raise the funds it needs. This can be achieved by distorting prices so that the price of the consumption good is kept low during wartime relative to peacetime. To achieve this, the best strategy for the government is to run a huge deficit by cutting taxes during the war. The deficit is then covered by increasing taxes during peacetime and gradually repaying the large debt accumulated during the war. When the government is influenced more by the rentiers, it will respond to a war by running a much smaller deficit and having a higher tax rate during wartime. The scarcity of goods will then be more acute. The rentiers will pay a lower interest rate on the war chest being built before the war and will demand a higher interest rate on the debt they subscribe during wartime.

At the neutral policy, taxes are just dictated by efficiency considerations. For the preferences we specified, this policy implies slightly higher taxes in wartime.²⁹

In the no-spending example, the optimal tax policy is unaffected by the different Pareto weights the government can attach to the agents over a large range. That happens because in the Ramsey policy, neither agent is “borrowing” in some state and lending in some other state; the rentiers are consuming exactly their “endowment” stemming from the maturing coupons on the annuities, and the taxpayers are consuming their proceeds from labor supply net of taxes and the coupon payment on the annuities. Because of this, deviating from the Ramsey policy has only second-order distributional effects. When government spending varies, this is no longer the case. The rentiers are now lending to the taxpayers in wartime and being repaid in peacetime. Figure 3 shows that their consumption is below the level implied by their coupons in wartime, and it is above in peacetime. A change in the relative tax rates the government applies during the war and in peace brings thus first-order distributional effects. Because of this, the optimal tax rates change when the Pareto weights change.

We have so far argued that the volume of public borrowing in France and Britain was consistent with the predictions of the model. It would be interesting to examine whether the other predictions are consistent with the data. In particular, we would predict that the real interest rate during the war was higher in France than in England, both for private and for public loans; we would also predict that, compared with Britain, France had a larger flow of financial resources from the rentiers to the taxpayers in wartime and a much larger flow from the taxpayers to the rentiers in peacetime.

While in principle these hypotheses are empirically testable, in practice the available data do not allow sharp conclusions.

Velde and Weir (1992) show that France paid substantially higher interest rates than Britain did; however, France also defaulted frequently on its debt. They also show that observed interest rates on government debt oscillated mainly in anticipation of government defaults. The appropriate comparison for our purposes should thus adjust the interest rates for the expected defaults. There seems to be evidence that the premium paid

²⁹The tax choice that minimizes distortions requires equality of the marginal tax distortions across states. Depending on the preferences, this may imply higher or lower taxes during wartime. See Lucas and Stokey (1983).

by France was more than enough to offset the default risk, with the exception of the Law Affair.³⁰ Further research is required though to study whether the interest rates, net of the default premium, were higher in France than in Britain in wartime and lower in periods preceding a war.

As for the private credit markets, there are very few studies on microeconomic data that would allow us to distinguish flows between social classes. Rosenthal (1994) studies credit in a rural area, where the shocks to local agriculture seem to be much more important than wars or other government intervention. Wars seem much more important for Paris,³¹ but yearly data have been estimated only for aggregate series. The aggregate volume of credit is not a good measure of the series we are interested in because the net position of the different social classes on the market was about even.³²

4. CONCLUSION

I have used my model to explore some of the ways in which distributional motives affect the choice of an intertemporal tax plan, interacting with efficiency considerations.

In the presence of real shocks, the possibility of distorting intertemporal prices gives the government an important redistribution tool. This seems especially relevant for large shocks, such as wars. We have shown that the size of the deficit a government chooses to run during a war will be heavily influenced by its constituency. A government that draws its main support from the people who pay taxes should optimally run larger deficits and wait for the end of the war to levy the taxes necessary to repay the defense expenses. On the other hand, a king supported by a privileged class of rentiers, largely exempt from taxes, should run a much smaller deficit and force the taxpayers to borrow at bad terms from the rentiers.

The model implies that the introduction of a balanced-budget requirement on the government has undesirable redistribution implications for the taxpayers of an economy.

I have shown that the pattern of net trades across heterogeneous agents over time is an essential element to understand the government's incentives to manipulate real interest rates. This insight can be applied to much more general environments, where heterogeneity could arise from a number of alternative sources, such as age differences, or a differential access to credit.

APPENDIX A: PROOFS

A.1 *General properties*

As mentioned in the main text, it may be beneficial under some circumstances for the government to deliberately introduce randomness in its tax policy. This presents a compu-

³⁰See Hoffman, Postel-Vinay, and Rosenthal (1995).

³¹See Hoffman, Postel-Vinay, and Rosenthal (1994).

³²Had this not been the case, we could have estimated the flows from measures of aggregate volume of credit. For example, suppose that aristocrats were mainly lending, whereas the bourgeois were mainly borrowing. In this case, we would have expected the bourgeois to borrow even more in wartime, thereby increasing the size of the aggregate volume of private credit. The reverse effect would have arisen if the bourgeois had been the lenders.

tational challenge. In this appendix, we establish useful aggregation results to overcome the challenge.

In the main text, we assume that government spending and the sunspot process h_0 take a finite number of values. In this appendix, we will prove that restricting h_0 in this way is without loss. To do so, we start from a generic real-valued process $\{h_t\}_{t=0}^\infty$ with values in a measurable space (H, \mathcal{H}) . Equation (3) becomes

$$E_0 \sum_{t=0}^{\infty} \beta^t p_t [c_t^i - \eta_t^i - b_t^i - (1 - \tau_t)w^i(1 - x_t^i)] = 0, \quad (29)$$

where p_t is the asset pricing kernel (which is $\beta^{-t} \Pr(s^t) p(s^t)$ in the discrete case).

THEOREM 2. *For any competitive equilibrium of the economy described in Section 2, there exist functions $\{C^i\}_{i=1}^N, \{X^i\}_{i=1}^N, P: G \times \mathbb{R} \rightarrow \mathbb{R}$ such that*

$$\begin{aligned} c_t^i &= C^i(g_t, \tau_t) \quad \forall t \geq 0 \quad \forall s^t \in S^t, i = 1, \dots, N, \\ x_t^i &= X^i(g_t, \tau_t) \quad \forall t \geq 0 \quad \forall s^t \in S^t, i = 1, \dots, N, \\ p_t &= P(g_t, \tau_t) \quad \forall t \geq 0 \quad \forall s^t \in S^t. \end{aligned} \quad (30)$$

PROOF. From the first-order conditions of the consumers, we have

$$\begin{aligned} u_c^i(c_t^i, x_t^i) &= \nu^i p_t \quad \forall t \geq 0 \quad \forall s^t \in S^t, i = 1, \dots, N, \\ u_x^i(c_t^i, x_t^i) &\geq \nu^i p_t (1 - \tau_t) w^i, \quad = \quad \text{if } x_t^i < 1, \end{aligned} \quad (31)$$

where ν^i are the Lagrange multipliers associated with the budget constraints of the agents. Because of the strict concavity of u , (31) can be inverted to get

$$\begin{aligned} c_t^i &= \hat{C}^i(\nu^i, p_t, (1 - \tau_t)w^i) \quad \forall t \geq 0 \quad \forall s^t \in S^t, i = 1, \dots, N, \\ x_t^i &= \hat{X}^i(\nu^i, p_t, (1 - \tau_t)w^i) \quad \forall t \geq 0 \quad \forall s^t \in S^t, i = 1, \dots, N, \end{aligned} \quad (32)$$

where both \hat{C}^i and \hat{X}^i are strictly decreasing in p_t .³³ We now use (32) in the feasibility constraints, which gives us

$$\begin{aligned} &\sum_{i=1}^N \hat{C}^i(\nu^i, p_t, (1 - \tau_t)w^i) + g_t \\ &= \sum_{i=1}^N w^i [1 - \hat{X}^i(\nu^i, p_t, (1 - \tau_t)w^i)] + \sum_{i=1}^N \eta_t^i \quad \forall t \geq 0 \quad \forall s^t \in S^t. \end{aligned} \quad (33)$$

Given the monotonicity properties of \hat{C}^i and \hat{X}^i , (33) is an implicit equation that can at most have one solution for p_t as a function of $(\{\nu^i, w^i\}_{i=1}^N, g_t, \tau_t)$. Since we are considering an allocation and a price system that form a competitive equilibrium, the asset

³³ \hat{X}^i is strictly decreasing when strictly less than 1.

pricing kernel must be a solution of (33) given the Lagrange multipliers.³⁴ We can thus define the function P as the unique solution to (33). By substituting this function into (32) we get consumption and leisure as a function of (g_t, τ_t) given the value of ν^i : this defines the functions C^i and X^i . \square

Note that the functions C^i , X^i , and P we just derived depend on which competitive equilibrium we are in, since the Lagrange multipliers do.

As a technical remark, the functions C^i , X^i , and P are measurable and identified up to sets of measure 0. This is because the equations (32) and (33) that define them involve only measurable functions and are valid almost surely.

Theorem 2 states that in any given competitive equilibrium, the consumption and leisure choices of all agents in the economy will be the same in all periods and/or states in which government spending and the tax rate are the same.

THEOREM 3. *For given initial conditions $\{(b_t^i, \eta_t^i)\}_{i=1}^N\}_{t=0}^\infty$ and a given process $\{g_t\}_{t=0}^\infty$ for government spending, let $\{\tau_t\}_{t=0}^\infty$ and $\{\tilde{\tau}_t\}_{t=0}^\infty$ be two policies satisfying the requirements*

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t \Pr((g_t, \{(b_t^i, \eta_t^i)\}_{i=1}^N, \tau_t) \in A) \\ = \sum_{t=0}^{\infty} \beta^t \Pr((g_t, \{(b_t^i, \eta_t^i)\}_{i=1}^N, \tilde{\tau}_t) \in A) \quad \forall A \in \mathcal{G} \times \mathcal{B}^{2N+1}, \end{aligned} \quad (34)$$

where \mathcal{G} is the σ -algebra of the measurable space within which the process $\{g_t\}_{t=0}^\infty$ lies, and \mathcal{B} is the Borel σ -algebra.

Let

$$\begin{aligned} c_t^i &= C^i(g_t, \tau_t) \quad \forall t \geq 0 \quad \forall s^t \in S^t, i = 1, \dots, N, \\ x_t^i &= X^i(g_t, \tau_t) \quad \forall t \geq 0 \quad \forall s^t \in S^t, i = 1, \dots, N, \\ p_t &= P(g_t, \tau_t) \quad \forall t \geq 0 \quad \forall s^t \in S^t \end{aligned} \quad (35)$$

describe an allocation and a price system that form a competitive equilibrium given the initial conditions, the spending process, and the policy $\{\tau_t\}_{t=0}^\infty$. Then the same functions

$$\begin{aligned} c_t^i &= C^i(g_t, \tilde{\tau}_t) \quad \forall t \geq 0 \quad \forall s^t \in S^t, i = 1, \dots, N, \\ x_t^i &= X^i(g_t, \tilde{\tau}_t) \quad \forall t \geq 0 \quad \forall s^t \in S^t, i = 1, \dots, N, \\ p_t &= P(g_t, \tilde{\tau}_t) \quad \forall t \geq 0 \quad \forall s^t \in S^t \end{aligned} \quad (36)$$

describe an allocation and a price system that form a competitive equilibrium given the initial conditions, the spending process, and the policy $\{\tilde{\tau}_t\}_{t=0}^\infty$. Furthermore, the utility of

³⁴If we had to actually compute the competitive equilibrium, we should take into account the fact that the Lagrange multipliers depend on the price system. Our problem here is, however, different: given that we are in a competitive equilibrium with some multipliers ν^i , we want to show that in this equilibrium, there can be only one level of p_t associated with any level of (g_t, τ_t) .

each agent is the same in either equilibrium and, hence, the same is true for government welfare.

PROOF. We need to prove that the allocation and the price system described by (36), together with the initial conditions, the spending process, and the policy $\{\tilde{\tau}_t\}_{t=0}^\infty$, satisfy equations (5), (29), and (31).

Define a measure Q on $(G \times \mathbb{R}^{2N+1}, \mathcal{G} \times \mathcal{B}^{2N+1})$ by

$$\begin{aligned} Q(A) &\equiv \sum_{t=0}^{\infty} \beta^t \Pr((g_t, \{(b_t^i, \eta_t^i)\}_{i=1}^N, \tau_t) \in A) \\ &= \sum_{t=0}^{\infty} \beta^t \Pr((g_t, \{(b_t^i, \eta_t^i)\}_{i=1}^N, \tilde{\tau}_t) \in A). \end{aligned} \quad (37)$$

Given any measurable function $f: G \times \mathbb{R}^{2N+1} \rightarrow \mathbb{R}$, (37) implies that

$$\begin{aligned} E \sum_{t=0}^{\infty} \beta^t f(g_t, \{(b_t^i, \eta_t^i)\}_{i=1}^N, \tau_t) \\ = \int_{G \times \mathbb{R}^{2N+1}} f dQ = E \sum_{t=0}^{\infty} \beta^t f(g_t, \{(b_t^i, \eta_t^i)\}_{i=1}^N, \tilde{\tau}_t). \end{aligned} \quad (38)$$

It then follows immediately that (29) is satisfied for the policy $\{\tilde{\tau}_t\}_{t=0}^\infty$ whenever it is satisfied for $\{\tau_t\}_{t=0}^\infty$. In the same way, we can prove that the expected utility of each agent is the same in both equilibria.

Let us now consider equation (5). Assume by contradiction that it does not hold almost surely for the policy $\{\tilde{\tau}_t\}_{t=0}^\infty$. Then there is some time \hat{t} such that

$$\Pr\left(\sum_{i=1}^N C^i(g_{\hat{t}}, \tilde{\tau}_{\hat{t}}) + g_{\hat{t}} \neq \sum_{i=1}^N w^i(1 - X^i(g_{\hat{t}}, \tilde{\tau}_{\hat{t}})) + \sum_{i=1}^N \eta_{\hat{t}}^i\right) > 0.$$

It then follows that

$$\sum_{t=0}^{\infty} \beta^t \Pr\left(\sum_{i=1}^N C^i(g_t, \tilde{\tau}_t) + g_t \neq \sum_{i=1}^N w^i(1 - X^i(g_t, \tilde{\tau}_t)) + \sum_{i=1}^N \eta_t^i\right) > 0,$$

which implies

$$\sum_{t=0}^{\infty} \beta^t \Pr\left(\sum_{i=1}^N C^i(g_t, \tau_t) + g_t \neq \sum_{i=1}^N w^i(1 - X^i(g_t, \tau_t)) + \sum_{i=1}^N \eta_t^i\right) > 0. \quad (39)$$

Equation (39) contradicts equation (5), which must hold almost surely for all periods t given the policy $\{\tau_t\}_{t=0}^\infty$ because of the assumptions of the theorem.

In the same way we can prove that (31) holds for the policy $\{\tilde{\tau}_t\}_{t=0}^\infty$, holding the Lagrange multiplier fixed at its value associated with the equilibrium (35). \square

DEFINITION 3 (Policy Equivalence). We call two policies equivalent whenever (34) holds.

Theorem 3 justifies the definition of equivalence. Intuitively, it does not matter what kind of randomization over taxes the government chooses or its distribution over time: in an Arrow–Debreu economy, it only matters how often it takes a given value and how it co-moves with the “fundamentals,” that is, government spending and the coupon payments. This result arises from the presence of Arrow–Debreu markets, and from the fact that our preferences are additive both with respect to time (strong time separability) and with respect to different events (a property of von Neumann–Morgenstern preferences).

As an example of equivalent policies, consider a world where government spending is constant and all outstanding claims at time 0 are annuities, so $(g_t, \{(b_t^i, \eta_t^i)\}_{i=1}^N)$ are constant and deterministic. The first policy sets the tax rate to some level τ_1 in even periods and to some other level τ_2 in odd periods. The second policy sets the tax rate permanently to either τ_1 or τ_2 , depending on the outcome of h_0 ; the policy is designed in such a way that the probability of the tax rate being τ_1 is $\frac{1}{1+\beta}$. It is easy to see that (34) holds for these policies.

The government would be indifferent between two equivalent policies, and so would each of the private agents. Furthermore, the allocation and the price system are described by the same functions in the competitive equilibria associated with the two different policies. Therefore, if we solve for the competitive equilibrium associated with a given policy, we can infer immediately the allocation and the price system that form a competitive equilibrium with any policy that is equivalent to it.

Guided by Theorem 3, we will now restrict our attention to a simpler set of policies.

COROLLARY 1. *Let $\{h_t\}_{t=0}^\infty$ be a sunspot process described as $(H, \mathcal{H}) = ([0, 1], \mathcal{B}([0, 1]))$; h_0 is distributed according to a uniform distribution and is independent of $\{g_t\}_{t=0}^\infty$; $h_t = h_0 \forall t \geq 0 \forall s^t \in S^t$. Let $\{\tau_t\}_{t=0}^\infty$ be the best policy among those adapted to the information generated by $\{g_t\}_{t=0}^\infty$ and the sunspot process $\{h_t\}_{t=0}^\infty$, that is, the one that leads to the competitive equilibrium with the highest value W for the government. Then $\{\tau_t\}_{t=0}^\infty$ achieves a payoff that is greater than or equal to the payoff that the government can achieve using the best policy adapted to the information generated by g_t and any sunspot process $\{\tilde{h}_t\}_{t=0}^\infty$.*

PROOF. We proceed in two steps.

- Consider an arbitrary sunspot process $\{\tilde{h}_t\}_{t=0}^\infty$, and an arbitrary policy $\{\tilde{\tau}_t\}_{t=0}^\infty$ adapted to the information generated by $\{g_t\}_{t=0}^\infty$ and $\{\tilde{h}_t\}_{t=0}^\infty$. In the first step, we show that we can find a measurable function $f: G \times \mathbb{R}^{2N+1}$ such that whenever we choose $\tau_t = f(g_t, \{(b_t^i, \eta_t^i)\}_{i=1}^N, h_0)$,

$$\begin{aligned} Q(A) &\equiv \frac{1}{1-\beta} \sum_{t=0}^{\infty} \beta^t \Pr((g_t, \{(b_t^i, \eta_t^i)\}_{i=1}^N, \tau_t) \in A) \\ &= \frac{1}{1-\beta} \sum_{t=0}^{\infty} \beta^t \Pr((g_t, \{(b_t^i, \eta_t^i)\}_{i=1}^N, \tilde{\tau}_t) \in A) \\ &\equiv \tilde{Q}(A) \quad \forall A \in \mathcal{G} \times \mathcal{B}^{2N+1}; \end{aligned} \tag{40}$$

in other words, $\{\tau_t\}_{t=0}^\infty$ is equivalent to $\{\tilde{\tau}_t\}_{t=0}^\infty$. We scaled the measures in (40) so that they are probability measures; this is just for convenience, as we can now call conditional expectation the projection operator.

Notice that $Q(A)$ and $\tilde{Q}(A)$ coincide by their definition on all events that do not depend on τ and $\tilde{\tau}$, that is, on all sets of the form $A = A_1 \times \mathbb{R}$, $A_1 \in \mathcal{G} \times \mathcal{B}^{2N}$: this is because we are keeping the same spending process and the same initial conditions under both policies.

It is natural to call $(g, \{(b^i, \eta^i)\}_{i=1}^N, \tilde{\tau})$ the random vector whose probability distribution is $\tilde{Q}(A)$.

We can then decompose $\tilde{\tau} = E^{\tilde{Q}}(\tilde{\tau} | (g, \{(b^i, \eta^i)\}_{i=1}^N)) + \tilde{\tau}^\perp$. Let $F_{\tilde{\tau}^\perp}$ be the cumulative distribution function (c.d.f.) of $\tilde{\tau}^\perp$. Let us then choose the function f as

$$\tau_t = f(g_t, \{(b_t^i, \eta_t^i)\}_{i=1}^N, h_0) = E^{\tilde{Q}}(\tilde{\tau} | (g, \{(b^i, \eta^i)\}_{i=1}^N)) + \tilde{f}(h_0) \quad (41)$$

with

$$\tilde{f}(x) \equiv \min\{y : x \leq F_{\tilde{\tau}^\perp}(y)\}. \quad (42)$$

Note that this choice implies

$$\Pr(\tilde{f}(h_0) \leq x) = F_{\tilde{\tau}^\perp}(x) \quad \forall x \in \mathbb{R}. \quad (43)$$

To prove that \tilde{Q} coincides with Q , it is enough to show that they coincide on all sets in the π -system

$$\begin{aligned} \mathcal{A} \equiv \{A : \{z \in G \times \mathbb{R}^{2N+1} : z_i \leq \bar{z}_i, i = 1, \dots, 2N+1, \\ z_{N+2} \leq E^Q(\tau | z_1 \leq \bar{z}_1, \dots, z_{2N+1} \leq \bar{z}_{2N+1}) + \bar{z}_{2N+2} \\ \text{for some } \bar{z} \in \mathbb{R}^{2N+2}\}\}. \end{aligned} \quad (44)$$

By construction, given a set $A \in \mathcal{A}$ characterized by a vector \bar{z} , we have $\tilde{Q}(A) = \tilde{Q}((-\infty, \bar{z}_1] \times \dots \times (-\infty, \bar{z}_{2N+1}] \times \mathbb{R})F_{\tilde{\tau}^\perp}(\bar{z}_{2N+2})$.

Furthermore, for such sets

$$\begin{aligned} Q(A) &\equiv \frac{1}{1-\beta} \sum_{t=0}^{\infty} \beta^t \Pr((g_t, \{(b_t^i, \eta_t^i)\}_{i=1}^N, \tau_t) \in A) \\ &= \frac{1}{1-\beta} \sum_{t=0}^{\infty} \beta^t \Pr(g_t \leq \bar{z}_1, b_t^1 \leq \bar{z}_2, \eta_t^1 \leq \bar{z}_3, \dots, \\ &\quad b_t^N \leq \bar{z}_{2N}, \eta_t^N \leq \bar{z}_{2N+1}, \tilde{f}(h_0) \leq \bar{z}_{2N+2}) \\ &= \frac{1}{1-\beta} \sum_{t=0}^{\infty} \beta^t \Pr(g_t \leq \bar{z}_1, b_t^1 \leq \bar{z}_2, \eta_t^1 \leq \bar{z}_3, \dots, \\ &\quad b_t^N \leq \bar{z}_{2N}, \eta_t^N \leq \bar{z}_{2N+1}) F_{\tilde{\tau}^\perp}(\bar{z}_{2N+2}) \\ &= M((-\infty, \bar{z}_1] \times \dots \times (-\infty, \bar{z}_{2N+1}] \times \mathbb{R}) F_{\tilde{\tau}^\perp}(\bar{z}_{2N+2}) \\ &= \tilde{Q}(A). \end{aligned} \quad (45)$$

• We now know that the best policy that is adapted to the information generated by $\{g_t\}_{t=0}^\infty$ and an arbitrary sunspot process $\{\tilde{h}_t\}_{t=0}^\infty$ is equivalent to some policy adapted to the information generated by $\{g_t\}_{t=0}^\infty$ and $\{h_t\}_{t=0}^\infty$. The implication then follows trivially. \square

Corollary 1 greatly simplifies our problem. Rather than having to deal with an infinite sunspot process, it establishes that it is enough to allow tax policy to respond to a single, time-0 public randomization device that is uniformly distributed. We will study the Ramsey outcome where the allocation and the asset-pricing kernel only depend on $(g_t, \{(b_t^i, \eta_t^i)\}_{i=1}^N, h_0)$. This is done simply for convenience; using Theorem 3, we can characterize Ramsey outcome in which the government follows different (but equivalent) policies, such as deterministic variations of the tax rate over time even when the fundamentals are constant.

APPENDIX B: THE COMPUTATIONAL ALGORITHM

We now refer to the two-class economy of Section 3. We will use aggregate private consumption as the variable in the optimization problem. Our first step is to show that specifying aggregate private consumption allows us to recover all of the other elements of a competitive equilibrium, given the initial conditions.

Define the measure m as

$$m(A) \equiv \sum_{t=0}^{\infty} \beta^t \Pr((g_t, \{(b_t^i, \eta_t^i)\}_{i=1}^N, h_0) \in A) \quad \forall A \in \mathcal{G} \times \mathcal{B}^{2N+1}. \quad (46)$$

We will use this measure in evaluating (1), (2), (3), and (4).

We define $v \equiv (h_0, g_t, b_t^1, b_t^2, \eta_t^1, \eta_t^2)$. By Corollary 1, the allocation, the policy, and the price system in the Ramsey outcome will be functions of v . We will also denote $g_t = g(v)$ and so on, where these are just selector functions that take the appropriate component of the vector v . We define $e(v)$ to be aggregate private consumption in the Ramsey outcome.

THEOREM 4. *Given a function $e(v)$, there exists at most one competitive equilibrium whose aggregate consumption is given by $e(v)$.*

PROOF. Using (31) we find that in a competitive equilibrium,

$$c^i(v) = k^i e(v) \quad \forall t \geq 0 \quad \forall s^t \in S^t, i = 1, 2, \quad (47)$$

and the asset-pricing kernel is given by

$$p(v) = e(v)^{-\gamma} \quad \forall t \geq 0 \quad \forall s^t \in S^t. \quad (48)$$

We can compute k^1 from the budget constraint of the agents of type 1 after substituting (47), (48), and the definition of the measure m :

$$k^1 = \left[\int (b^1(v) + \eta^1(v)) e(v)^{-\gamma} dm(v) \right] \left[\int e(v)^{1-\gamma} dm(v) \right]^{-1}. \quad (49)$$

Since aggregate private consumption is $e(v) \equiv Mc^1(v) + c^2(v)$, we can compute k^2 from the requirement $Mk^1 + k^2 = 1$:

$$k^2 = 1 - Mk^1. \quad (50)$$

We use the market-clearing condition (5) to determine leisure:

$$\begin{aligned} x^2(v) &= 1 - c^2(v) - Mc^1(v) - g(v) + M\eta^1(v) + \eta^2(v) \\ &= 1 - e(v) - g(v) + M\eta^1(v) + \eta^2(v). \end{aligned} \quad (51)$$

We finally determine the tax policy using (6):

$$\begin{aligned} \xi x^2(v)^{-\sigma} &= (1 - \tau(v))c^2(v)^{-\gamma}, \\ \tau(v) &= 1 - \xi(1 - e(v) - g(v) + M\eta^1(v) + \eta^2(v))^{-\sigma} (1 - Mk^1)^\gamma e(v)^\gamma. \end{aligned} \quad (52) \quad \square$$

We now wish to establish which functions $e(v)$ are compatible with a competitive equilibrium. For this to happen, the following situations must hold:

- (i) Equations (47)–(52) must have a well defined solution in the admissible range; if this happens, (5) and (31) will be satisfied, and so will (3) for type-1 agents. In (47), we need $c^i(v) \geq 0 \forall t \geq 0 \forall s^t \in S^t$, $i = 1, 2$; this requires $e(v) \geq 0 \forall t \geq 0 \forall s^t \in S^t$, $i = 1, 2$, and $k^i \geq 0$, $i = 1, 2$, which can be rewritten as requiring $k^1 \in [0, \frac{1}{M}]$. We also need $x^2(v) \geq 0 \forall t \geq 0 \forall s^t \in S^t$, which requires $e(v) + g(v) \leq 1 + M\eta^1(v) + \eta^2(v) \forall t \geq 0 \forall s^t \in S^t$.
- (ii) Either (3) for type-2 agents or (4) must hold (the other one will hold by Walras' law).

We will use the budget constraint of the government (4). Using equations (47)–(52), this constraint can be rewritten, after some algebra, as

$$\begin{aligned} &\log \xi + \gamma \log(1 - Mk^1) + \log \left[\int (1 - e(v) - g(v) + M\eta^1(v) + \eta^2(v))^{-\sigma} dm(v) \right. \\ &\quad \left. - \int (1 - e(v) - g(v) + M\eta^1(v) + \eta^2(v))^{1-\sigma} dm(v) \right] \\ &\quad - \log \left[\int e(v)^{1-\gamma} dm(v) \right. \\ &\quad \left. - \int e(v)^{-\gamma} (M\eta^1(v) + \eta^2(v) + Mb^1(v) + b^2(v)) dm(v) \right] \\ &= 0. \end{aligned} \quad (53)$$

We assume that, given $\{(\eta_t^i, b_t^i)\}_{i=1,2}\}_{t=0}^\infty$, there exist functions $e(v)$ that satisfy (i) and (ii). Intuitively, this requires the rentiers not to be so poor that their wealth is negative under any government policy or so rich that their wealth exceeds the value of the highest attainable output net of government spending; it also requires government

spending not to be too large and the government not to be too heavily indebted against private agents. These requirements are necessary for existence of a competitive equilibrium given the initial conditions and the spending process.

Assuming thus that we have at least one competitive equilibrium, we wish to find now the one that maximizes the objective function of the government, which is the Ramsey outcome.

Using (47)–(51), the expected utility of type-1 agents is

$$U^1 = (k^1)^{1-\gamma} \int \frac{e(v)^{1-\gamma}}{1-\gamma} dm(v), \quad (54)$$

the expected utility of type-2 agents is

$$\begin{aligned} U^2 = & (1 - Mk^1)^{1-\gamma} \int \frac{e(v)^{1-\gamma}}{1-\gamma} dm(v) \\ & + \xi \int \frac{(1 - e(v) - g(v) + M\eta^1(v) + \eta^2(v))^{1-\sigma}}{1-\sigma} dm(v), \end{aligned} \quad (55)$$

and the objective function of the government is thus

$$\begin{aligned} W = & [\alpha^1 M(k^1)^{1-\gamma} + \alpha^2 (1 - Mk^1)^{1-\gamma}] \int \frac{e(v)^{1-\gamma}}{1-\gamma} dm(v) \\ & + \alpha^2 \xi \int \frac{(1 - e(v) - g(v) + M\eta^1(v) + \eta^2(v))^{1-\sigma}}{1-\sigma} dm(v). \end{aligned} \quad (56)$$

We proceed as follows. In each case, we first specify what possible values

$$\tilde{v} \equiv (g_t, b_t^1, b_t^2, \eta_t^1, \eta_t^2) \quad (57)$$

can take at any date and in any state. We will restrict this to be a finite number of possibilities. Let (v^1, \dots, v^J) be the possible values for \tilde{v} .

Consider first the case in which $J = 1$. The deterministic solution can be found by solving (53) for a constant e . We can then check whether second-order conditions are satisfied locally. Finally, to rule out a local maximum that is not a global maximum, using Theorem 1, we search for solutions of (53) that involve two points, each of which has positive probability; we impose a grid for one realization of e and its probability, and solve (53) for the other value of e . We then evaluate the objective function.

In the case of $J = 2$, we again first consider the case in which the tax policy does not depend on public randomization. In this case, we grid aggregate consumption in the first state and solve for aggregate consumption in the second state using (53). This step is independent of the Pareto weights used by the planner. We then find the optimum in this menu of choices by evaluating the objective function; at this step, the optimal solution will depend on the Pareto weights. To check for randomness, we again check whether the second-order conditions are satisfied locally. To rule out local maxima, we rely on Theorem 1 again. We need to consider random deviations that are described by five parameters: aggregate consumption in the four states (two fundamental states

times two realizations of the sunspot) and the probability of the sunspot trigger. This is still doable on a grid, albeit a somewhat coarser one.

To analyze cases beyond $J = 3$ (which are not presented in this paper), the procedure described above becomes too cumbersome. The first step (finding a nonrandom policy) relies on solving the first-order conditions for the problem of maximizing (56) subject to (53); the vector of aggregate consumption to be found is of dimension J . Once a solution is found, checking second-order conditions can be done exactly as in the cases in which $J = 1$ or $J = 2$. The final step looks for alternative solutions to maximizing (56) subject to (53), where aggregate consumption may depend on the sunspot, but may only take at most two values. This requires finding a vector of dimension $2J + 1$: aggregate consumption in each of the J states times the two sunspot states, plus the probability of the sunspot (notice that we need a single sunspot because of Corollary 1). This final step requires starting from alternative points on a coarse grid and using an optimization routine that does not rely on concavity, such as simulated annealing.

APPENDIX C: FURTHER DETAILS ON THE CALIBRATION

To obtain an estimate of wartime and peacetime spending, I use data from Mitchell (1988), proceeding as follows. First, I obtain nominal spending from pages 578–580. I subtract debt charges—to consider only primary spending—and deflate primary spending by a price index obtained by splicing the tables on pages 719 and 720. For GDP, Mitchell provides estimates at three points in time on page 821. I deflate these estimates, and interpolate assuming that *real* GDP grows at a constant rate. I compute the average of the government primary spending/GDP ratio during peacetime and wartime. I use 1692 as the initial level of debt.

The fraction of rentiers in the population is set to match the fraction of nobles and clergy in Table 1 of Morrisson and Snyder (2000). Morrisson and Snyder do not attempt to separately identify the income of nobles and clergy versus the bourgeois. I assume that the two groups share the same per capita income.

The full set of parameter values that I choose is the following: $\xi = 0.9$, $g = 0.044$ in peacetime and $0.057 * 0.135/0.06$ in wartime, $M = 540/(28,000 - 540)$, $b = 0.00175$, $\eta^1 = 3.1 - b/M$, and $\eta^2 = 1/3$. By normalization, all of the government debt is assumed to be held by the rentiers; nothing would change if debt were split across the two groups, provided η^1 and η^2 are adjusted correspondingly.

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Submitted April, 2013. Final version accepted October, 2013.