Sticky Expectations and Consumption Dynamics

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Consumption Dynamics: Macro vs Micro

Macro

- *Aggregate* consumption exhibits **'excess smoothness'**

Micro

- *Idiosyncratic* consumption does not

Consumption Dynamics: Macro vs Micro

Modeling response: 'habits'

Macro

- *Aggregate* consumption exhibits **'excess smoothness'**

Micro

- *Idiosyncratic* consumption does not

Consumption Dynamics: Macro vs Micro

Modeling response: 'habits'

Macro

- *Aggregate* consumption exhibits **'excess smoothness'**

Micro

- *Idiosyncratic* consumption does not $\longrightarrow$ Habits strongly rejected

Consumption Dynamics: Macro vs Micro

Modeling response: 'habits'

$\uparrow$

Macro

- *Aggregate* consumption exhibits **'excess smoothness'**

Micro

- *Idiosyncratic* consumption does not $\longrightarrow$ Habits strongly rejected

This paper

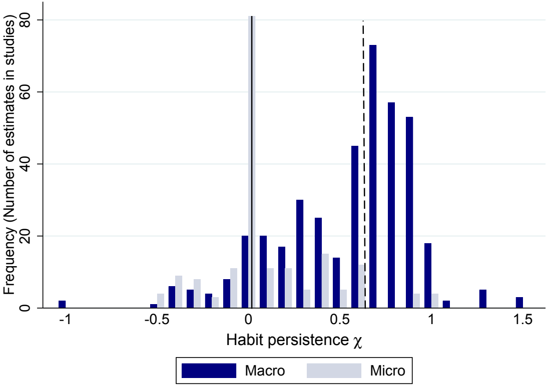
- Builds a model that reconciles these empirical facts
  - Sticky Expectations replace habits
  - Tractable
  - Quantitatively plausible (both micro and macro)

Excess Smoothness: Macro vs Micro

Estimate χ in

$$\Delta \log \mathbf{C}_{t+1} = \varsigma + \chi \Delta \log \mathbf{C}_t + \epsilon$$

597 estimates of χ in Havranek, Rusnak, and Sokolova (2017)



$\chi^{\text{Macro}} \approx 0.6$

$\chi^{\text{Micro}} \approx 0.1$

Claim: It's Not Habits, It's (Macro) Inattention!

Our Income Process

- Idiosyncratic Component: Perfectly Observed
- **Aggregate Component: Stochastically Observed**
  - Updating à la Calvo (1983)

Advantages

- Resolves macro/micro habits dissonance
- Simple to apply in heterogenous agent settings
  - e.g. Auclert, Rognlie, and Straub (2019)

Why Macro Inattention Is Plausible

Idiosyncratic Variability Is $\sim 100\times$ Bigger

- If Same Specification Estimated on Micro vs Macro Data
- Pervasive Lesson of All Micro Data

Utility Cost of Inattention Small

- Micro: Critical (and Easy) To Notice You're Unemployed
- Macro: *Not* Critical To Instantly Notice If U $\uparrow$

Quadratic Utility 'Toy Model'

Hall (1978) Random Walk

- Total Wealth (Human + Nonhuman):

$$\mathbf{o}_{t+1} = (\mathbf{o}_t - \mathbf{c}_t)\mathsf{R} + \zeta_{t+1}$$

- Euler Equation:

$$\mathrm{u}'(\mathbf{c}_t) = \mathsf{R}\beta\mathbb{E}_t[\mathrm{u}'(\mathbf{c}_{t+1})]$$

- $\Rightarrow$ Random Walk (for $\mathsf{R}\beta = 1$):

$$\Delta\mathbf{c}_{t+1} = \epsilon_{t+1}$$

- Expected Wealth:

$$\mathbf{o}_t = \mathbb{E}_t[\mathbf{o}_{t+1}] = \mathbb{E}_t[\mathbf{o}_{t+2}] = \ldots$$

Sticky Expectations—Individual $\mathbf{c}$

- Consumer who happens to update at t and $t+n$

$$
\begin{aligned}
\mathbf{c}_t &= (\mathrm{r}/\mathrm{R})\mathbf{o}_t \\
\mathbf{c}_{t+1} &= (\mathrm{r}/\mathrm{R})\widetilde{\mathbf{o}}_{t+1} = (\mathrm{r}/\mathrm{R})\mathbf{o}_t = \mathbf{c}_t \\
\vdots &\quad \vdots \\
\mathbf{c}_{t+n-1} &= \mathbf{c}_t
\end{aligned}
$$

- Implies that $\Delta^n \mathbf{o}_{t+n} \equiv \mathbf{o}_{t+n} - \mathbf{o}_t$ is white noise
- So **individual** $\mathbf{c}$ is RW across updating periods:

$$
\mathbf{c}_{t+n} - \mathbf{c}_t = (\mathrm{r}/\mathrm{R}) \underbrace{(\mathbf{o}_{t+n} - \mathbf{o}_t)}_{\Delta^n \mathbf{o}_{t+n}}
$$

Sticky Expectations—Aggregate $\mathbf{C}$

- Aggregate: $\mathbf{C}_t = \int_0^1 \mathbf{c}_{t,i}\, \mathrm{d}i$
- **Calvo (1983)-Type Updating of Expectations:**
  - Probability $\Pi = 0.25$ (per quarter)
- Economy composed of many sticky-$\mathbb{E}$ consumers:

$$
\begin{aligned}
\mathbf{C}_{t+1} &= (1-\Pi)\underbrace{\mathbf{C}^{\not\pi}_{t+1}}_{=\mathbf{C}_t} + \Pi\mathbf{C}^{\pi}_{t+1} \\
\Delta\mathbf{C}_{t+1} &\approx \underbrace{(1-\Pi)}_{\equiv\chi=0.75}\Delta\mathbf{C}_t + \epsilon_{t+1}
\end{aligned}
$$

- **Substantial persistence ($\chi = 0.75$) in aggregate C growth**

One More Ingredient . . .

Idiosyncratic shocks: Frictionless observation

- I notice if I am fired, promoted, somebody steals my wallet

Aggregate shocks: Sticky observation

- May not instantly notice changes in aggregate productivity

So...

Idiosyncratic $\Delta \mathbf{c}$: dominated by frictionless dynamics

But law of large numbers $\Rightarrow$ idiosyncratic part vanishes

Aggregate $\Delta \mathbf{C}$: highly serially correlated

Full Model

Partial Equilibrium/Small Open Economy

- CRRA Utility
- Idiosyncratic Shocks Calibrated From Micro Data
- Aggregate Shocks Calibrated From Macro Data
- Markov Process (Discrete RW) for Agg. Income Growth
  - Handles changing growth 'eras'
- Liquidity Constraint
- Mildly Impatient Consumers
- Blanchard (1985) Mortality and Insurance

Solving the Model

All results are generated using the open-source Econ-ARK toolkit:

- `http://econ-ark.org`

Income Process

- Individual's labor productivity is

$$\boldsymbol{\ell}_{t,i} = \overbrace{\theta_{t,i}\Theta_t}^{\equiv \boldsymbol{\theta}_{t,i}} \overbrace{p_{t,i}P_t}^{\equiv \boldsymbol{p}_{t,i}}$$

- Idiosyncratic and aggregate p evolve according to

$$\begin{aligned} p_{t+1,i} &= p_{t,i}\psi_{t+1,i} \\ P_{t+1} &= \Phi_{t+1} P_t \; \Psi_{t+1} \end{aligned}$$

- **Φ is Markov 'underlying' aggregate pty growth**
  - Discrete (bounded) random walk
  - Calibrated to match postwar US pty growth variation
  - Generates predictability in income growth (for IV regressions)

Sticky Expectations about Aggregate Income

Calvo Updating of Perceptions of Aggregate Shocks

- *True* Permanent income: $P_{t+1} = \Phi_{t+1} P_t \Psi_{t+1}$
- Tilde ($\widetilde{P}$) denotes perceived variables
- Perception for consumer who has not updated for n periods:

$$\widetilde{P}_{t,i} = \mathbb{E}_{t-n}\left[P_t \middle| \Omega_{t-n}\right] = \Phi^n_{t-n} P_{t-n}$$

because Φ is random walk

Key Assumption:

- People act as if their perceptions about aggregate state $\{\widetilde{P}_{t,i}, \widetilde{\Phi}_{t,i}\}$ are the true aggregate state $\{P_t, \Phi_t\}$

$\Longrightarrow$ Model solution is *exactly* the same as the frictionless model

Key Parameter Values

		Preference Parameters
ρ	2	Coefficient of Relative Risk Aversion
β	0.970	Discount Factor (SOE Model)
Π	0.25	Probability of Updating Expectations (if Sticky)
K/K^{γ}	12.0	SS Capital to Output Ratio
		Shock Parameters
σ_{θ}^{2}	0.120	Variance Idiosyncratic Tran Shocks (=4× Annual)
σ_{ψ}^{2}	0.003	Variance Idiosyncratic Perm Shocks ($=\frac{1}{4}\times$ Annual)
σ_{Θ}^{2}	0.00001	Variance Aggregate Transitory Shocks
σ_{Ψ}^{2}	0.00004	Variance Aggregate Permanent Shocks
$\wp$	0.050	Probability of Unemployment Spell
D	0.005	Probability of Mortality

Full Calibration

Regressions on Simulated and Actual Data

Dynan (2000)/Sommer (2007) Specification:

$$\Delta \log \mathbf{C}_{t+1} \approx \varsigma + \chi \mathbb{E}[\Delta \log \mathbf{C}_t] + \eta \mathbb{E}[\Delta \log \mathbf{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

- χ: **Extent of habits**
  Data: Micro: $\chi^{\text{Micro}} = 0.1$ (EER 2017 paper)
  Macro: $\chi^{\text{Macro}} = 0.6$
- η: **Fraction of Y going to 'rule-of-thumb' C = Y types**
  Data: Micro: $0 < \eta^{\text{Micro}} < 1$ (Depends ...)
  Macro: $\eta^{\text{Macro}} \approx 0.5$ (Campbell and Mankiw (1989))
- α: **Precautionary saving (micro) or IES (Macro)**
  Data: Micro: $\alpha^{\text{Micro}} < 0$ (Zeldes (1989))
  Macro: $\alpha^{\text{Macro}} < 0$ (but small)
  [In GE r depends roughly linearly on A]

Micro Regressions: Frictionless

$$\Delta \log \mathbf{c}_{t+1,i} = \varsigma + \chi \Delta \log \mathbf{c}_{t,i} + \eta \mathbb{E}_{t,i}[\Delta \log \mathbf{y}_{t+1,i}] + \alpha \bar{\mathbf{a}}_{t,i} + \epsilon_{t+1,i}$$

Model of Expectations	χ	η	α	$\bar{R}^2$
Frictionless				
	0.019			0.000
	(–)			
		0.011		0.004
		(–)		
			-0.190	0.010
			(–)	
	0.061	0.016	-0.183	0.017
	(–)	(–)	(–)	

Micro Regressions: Sticky

$$\Delta \log \mathbf{c}_{t+1,i} \quad = \quad \varsigma + \chi \Delta \log \mathbf{c}_{t,i} + \eta \mathbb{E}_{t,i}[\Delta \log \mathbf{y}_{t+1,i}] + \alpha \bar{a}_{t,i} + \epsilon_{t+1,i}$$

Model of Expectations	χ	η	α	$\bar{R}^2$
Sticky				
	0.012			0.000
	(–)			
		0.011		0.004
		(–)		
			-0.191	0.010
			(–)	
	0.051	0.015	-0.185	0.016
	(–)	(–)	(–)	

Empirical Results for U.S.

$$\Delta \log \mathbf{C}_{t+1} = \varsigma + \chi \Delta \log \mathbf{C}_t + \eta \mathbb{E}_t[\Delta \log \mathbf{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

Expectations : Dep Var Independent Variables			OLS or IV	2nd Stage $\bar{R}^2$	Hansen J p-val
Nondurables and Services					
$\Delta \log \mathbf{C}_t^*$	$\Delta \log \mathbf{Y}_{t+1}$	A_t			
0.468 (0.076)			OLS	0.216	
0.830 (0.098)			IV	0.278	0.439
	0.587 (0.110)		IV	0.203	0.319
		−0.17e−4 (5.71e−4)	IV	−0.005	0.181
0.618 (0.159)	0.305 (0.161)	−4.96e−4 (2.94e−4)	IV	0.304	0.825
Memo: For instruments $\mathbf{Z}_t, \Delta \log \mathbf{C}_t = \mathbf{Z}_t \zeta$, $\bar{R}^2 = 0.358$					

Notes: Data source is NIPA, 1960Q1–2016Q. Robust standard errors are in parentheses. Instruments $\mathbf{Z}_t = \{\Delta \log \mathbf{C}_{t-2}, \Delta \log \mathbf{C}_{t-3}, \Delta \log \mathbf{Y}_{t-2}, \Delta \log \mathbf{Y}_{t-3}, A_{t-2}, A_{t-3}, \Delta_8 \log \mathbf{C}_{t-2}, \Delta_8 \log \mathbf{Y}_{t-2}$, lags 2 and 3 of differenced Fed funds rate, lags 2 and 3 of the Michigan Index of Consumer Sentiment Expectations$\}$.

Small Open Economy: Sticky

$$\Delta \log \mathbf{C}_{t+1} = \varsigma + \chi \Delta \log \mathbf{C}_t + \eta \mathbb{E}_t[\Delta \log \mathbf{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

Expectations : Dep Var Independent Variables			OLS or IV	2nd Stage $\bar{R}^2$	Hansen J p-val
Sticky : $\Delta \log \mathbf{C}^*_{t+1}$ (with measurement error $\mathbf{C}^*_t = \mathbf{C}_t \times \xi_t$);					
$\Delta \log \mathbf{C}^*_t$	$\Delta \log \mathbf{Y}_{t+1}$	A_t			
0.508 (0.058)			OLS	0.263	
0.802 (0.104)			IV	0.260	0.554
	0.859 (0.182)		IV	0.198	0.233
		−8.26e–4 (3.99e–4)	IV	0.066	0.002
0.660 (0.187)	0.192 (0.277)	0.60e–4 (5.03e–4)	IV	0.261	0.546
Memo: For instruments $\mathbf{Z}_t$, $\Delta \log \mathbf{C}^*_t = \mathbf{Z}_t \zeta$, $\bar{R}^2 = 0.260$; $\text{var}(\log(\xi_t)) = 5.99\text{e–}6$					

Notes: Reported statistics are the average values for 100 samples of 200 simulated quarters each. Instruments

$\mathbf{Z}_t = \{\Delta \log \mathbf{C}_{t-2}, \Delta \log \mathbf{C}_{t-3}, \Delta \log \mathbf{Y}_{t-2}, \Delta \log \mathbf{Y}_{t-3}, A_{t-2}, A_{t-3}, \Delta_8 \log \mathbf{C}_{t-2}, \Delta_8 \log \mathbf{Y}_{t-2}\}$.

Small Open Economy: Frictionless

$$\Delta \log \mathbf{C}_{t+1} = \varsigma + \chi \Delta \log \mathbf{C}_t + \eta \mathbb{E}_t[\Delta \log \mathbf{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

Expectations : Dep Var Independent Variables			OLS or IV	2nd Stage $\bar{R}^2$	Hansen J p-val
Frictionless : $\Delta \log \mathbf{C}^*_{t+1}$ (with measurement error $\mathbf{C}^*_t = \mathbf{C}_t \times \xi_t$);					
$\Delta \log \mathbf{C}^*_t$	$\Delta \log \mathbf{Y}_{t+1}$	A_t			
0.295			OLS	0.087	
(0.066)					
0.660			IV	0.040	0.600
(0.309)					
	0.457		IV	0.035	0.421
	(0.209)				
		−6.92e–4	IV	0.026	0.365
		(5.87e–4)			
0.420	0.258	0.45e–4	IV	0.041	0.529
(0.428)	(0.365)	(9.51e–4)			
Memo: For instruments $\mathbf{Z}_t$, $\Delta \log \mathbf{C}^*_t = \mathbf{Z}_t \zeta$, $\bar{R}^2 = 0.039$; var$(\log(\xi_t)) = 5.99\text{e–}6$					

Notes: Reported statistics are the average values for 100 samples of 200 simulated quarters each. Instruments

$\mathbf{Z}_t = \{\Delta \log \mathbf{C}_{t-2}, \Delta \log \mathbf{C}_{t-3}, \Delta \log \mathbf{Y}_{t-2}, \Delta \log \mathbf{Y}_{t-3}, A_{t-2}, A_{t-3}, \Delta_8 \log \mathbf{C}_{t-2}, \Delta_8 \log \mathbf{Y}_{t-2}\}$.

Utility Costs of Stickiness

- Simulate expected lifetime utility when market resources nonstochastically equal to W_t at birth under **frictionless**

$$\overline{\mathrm{v}}_0 \equiv \mathbb{E}[\mathrm{v}(\mathsf{W}_t, \cdot)]$$

  and **sticky expectations**: $\overline{\widetilde{\mathrm{v}}}_0 \equiv \mathbb{E}[\widetilde{\mathrm{v}}(\mathsf{W}_t, \cdot)]$
- Expectations taken over state variables other than $m_{t,i}$
- Newborn's willingness to pay (as fraction of permanent income) to avoid having sticky expectations:

$$\omega = 1 - \left(\frac{\overline{\widetilde{\mathrm{v}}}_0}{\overline{\mathrm{v}}_0} \right)^{\frac{1}{1-\rho}}$$

- $\omega \approx 0.05\%$ **of permanent income**

Excess Sensitivity to Fiscal Stimulus

Replicate Parker, Souleles, Johnson, and McClelland (2013) results:

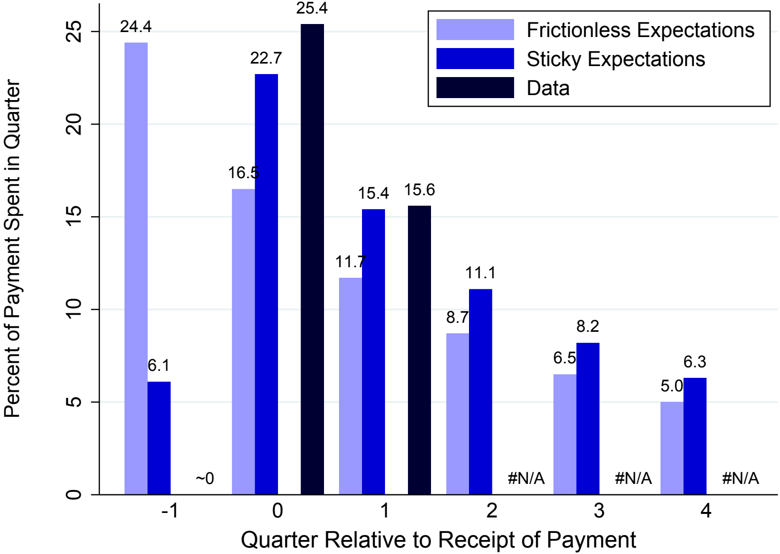
- Little response when stimulus announced
- MPC 0.12-0.3 on arrival of check

We model stimulus as *macro* news - only some households notice the announcement

Calibrate to *distribution* of liquid wealth to achieve high MPCs

Announcement occurs one quarter before check arrives

Excess Sensitivity to Fiscal Stimulus



Conclusion

Model with 'Sticky Expectations' of aggregate variables can match both micro and macro consumption dynamics

$$\Delta \log \mathbf{C}_{t+1} \quad \approx \quad \varsigma + \chi \Delta \log \mathbf{C}_t + \eta \mathbb{E}_t[\Delta \log \mathbf{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

	χ	η	α
Micro			
Data	≈ 0	$0 < \eta < 1$	< 0
Theory: Habits	≈ 0.75	$0 < \eta < 1$	< 0
Theory: Sticky Expectations	≈ 0	$0 < \eta < 1$	< 0
Macro			
Data	≈ 0.75	≈ 0	< 0
Theory: Habits	≈ 0.75	≈ 0	< 0
Theory: Sticky Expectations	≈ 0.75	≈ 0	< 0

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Calibration I

	Macroeconomic Parameters	
γ	0.36	Capital's Share of Income
δ	$1 - 0.94^{1/4}$	Depreciation Rate
σ^2_{Θ}	0.00001	Variance Aggregate Transitory Shocks
σ^2_{Ψ}	0.00004	Variance Aggregate Permanent Shocks
	Steady State of Perfect Foresight DSGE Model	
	$(\sigma_{\Psi} = \sigma_{\Theta} = \sigma_{\psi} = \sigma_{\theta} = \wp = \mathrm{D} = 0,\ \Phi_t = 1)$	
K/K^{γ}	12.0	SS Capital to Output Ratio
K	48.55	SS Capital to Labor Productivity Ratio ($= 12^{1/(1-\gamma)}$)
W	2.59	SS Wage Rate ($= (1-\gamma)K^{\gamma}$)
r	0.03	SS Interest Rate ($= \gamma K^{\gamma-1}$)
$\mathscr{R}$	1.015	SS Between-Period Return Factor ($= 1 - \delta + \mathrm{r}$)

Back

Calibration II

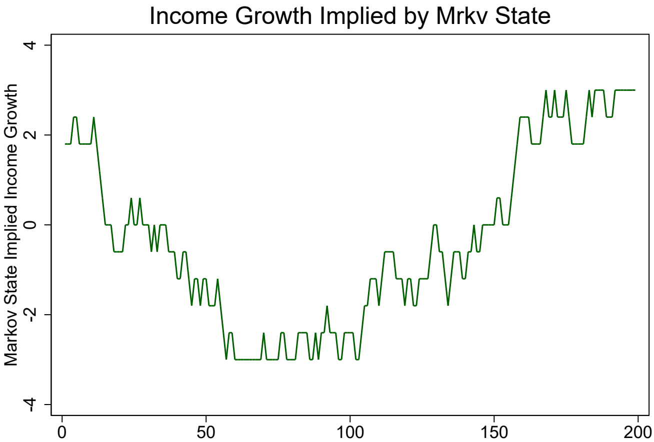
		Preference Parameters
ρ	2.	Coefficient of Relative Risk Aversion
β	0.970	Discount Factor (SOE Model)
Π	0.25	Probability of Updating Expectations (if Sticky)
		Idiosyncratic Shock Parameters
σ_θ^2	0.120	Variance Idiosyncratic Tran Shocks ($=4\times$ Annual)
σ_ψ^2	0.003	Variance Idiosyncratic Perm Shocks ($=\frac{1}{4}\times$ Annual)
$\wp$	0.050	Probability of Unemployment Spell
D	0.005	Probability of Mortality

Back

Markov Process for Aggregate Productivity Growth Φ

$$\boldsymbol{\ell}_{t,i} = \theta_{t,i}\Theta p_{t,i}P_t, \quad p_{t+1,i} = p_{t,i}\psi_{t+1,i}, \quad P_{t+1} = \Phi_{t+1}P_t\Psi_{t+1}$$

- Φ_t follows bounded (discrete) RW
- 11 states; average persistence 2 quarters
- Flexible way to match actual pty growth data



Equilibrium

	SOE Model		HA-DSGE Model	
	Frictionless	Sticky	Frictionless	Sticky
Means				
A	7.49	7.43	56.85	56.72
C	2.71	2.71	3.44	3.44
Standard Deviations				
Aggregate Time Series (‘Macro’)				
$\log A$	0.332	0.321	0.276	0.272
$\Delta \log \mathbf{C}$	0.010	0.007	0.010	0.005
$\Delta \log \mathbf{Y}$	0.010	0.010	0.007	0.007
Individual Cross Sectional (‘Micro’)				
$\log \mathbf{a}$	0.926	0.927	1.015	1.014
$\log \mathbf{c}$	0.790	0.791	0.598	0.599
$\log p$	0.796	0.796	0.796	0.796
$\log \mathbf{y} \mid \mathbf{y} > 0$	0.863	0.863	0.863	0.863
$\Delta \log \mathbf{c}$	0.098	0.098	0.054	0.055
Cost of Stickiness	4.82e–4		4.51e–4	

Is Muth–Lucas–Pischke Kalman Filter Equivalent?

No.

Muth (1960)–Lucas (1973)–Pischke (1995) Kalman filter

- All you can see is Y
  - Lucas: Can't distinguish agg. from idio.
  - Muth–Pischke: Can't distinguish tran from perm
- Here: Can see own circumstances perfectly
- Only the (tiny) aggregate part is hard to see
- **Signal extraction for aggregate $\mathbf{Y}_t$ gives too little persistence in $\Delta\mathbf{C}_t$: $\chi \approx 0.17$**

Muth–Pischke Perception Dynamics

- Optimal signal extraction problem (Kalman filter):
  Observe $\mathbf{Y}$ (aggregate income), estimate P, Θ
- Optimal estimate of P:

$$\hat{P}_{t+1} \quad = \quad \Pi \mathbf{Y}_{t+1} + (1 - \Pi)\hat{P}_t,$$

  where for signal-to-noise ratio $\varphi = \sigma_\Psi / \sigma_\Theta$:

$$\Pi \quad = \quad \varphi\sqrt{1 + \varphi^2/4} - \varphi^2/2, \tag{1}$$

- But if we calibrate φ using observed macro data
  - $\Rightarrow \Delta \log \mathbf{C}_{t+1} \approx \mathbf{0.17}\ \Delta \log \mathbf{C}_t$
  - **Too little persistence!**