Estimating Discrete Games with Many Firms and Many Decisions: An Application to Merger and Product Variety*

Ying Fan [1] Chenyu Yang [2]

[1] University of Michigan, NBER, CEPR

[2] University of Maryland

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* Researchers' own analyses calculated (or derived) based in part on (i) retail measurement/consumer data from Nielsen Consumer LLC ("NielsenIQ"); (ii) media data from The Nielsen Company (US), LLC ("Nielsen"); and (iii) marketing databases provided through the respective NielsenIQ and the Nielsen Datasets at the Kilts Center for Marketing Data Center at The University of Chicago Booth School of Business. The conclusions drawn from the NielsenIQ and Nielsen data are those of the researchers and do not reflect the views of Nielsen. Nielsen is not responsible for, had no role in, and was not involved in analyzing and preparing the results reported herein

What We Do In This Paper

- Develop a new method to estimate large, multiple-discrete choice models
  - many players; many actions
- Examine the entry defense of merger
  - merger→higher prices→entry→competition compensated→merger effects (partially) mitigated
  - merger→internalized business stealing incentives→variety reduced→merger effects worsened

Empirical Setting

- Retail craft beer market in California.
  - craft breweries: popular targets of acquisition by large breweries like ABI; antitrust concerns
  - California: the state with the highest craft volume, most breweries and most products
- Retail market (off-premise): Nielsen Scanner and Panel data
  - rich demographic variations across markets aiding estimation

Related Work

- Entry defense
  - theory: Spector (2003), Anderson et al. (2020), Caradonna et al. (2020), ...
  - empirics/simulation: Werden and Froeb (1998), Cabral (2003), Gandhi et al. (2008), Ciliberto et al. (2021), ...
- Empirical work on product variety
  - Seim (2006), Draganska et al. (2009), Eizenberg (2014)
- Empirical work on merger and product variety
  - Fan (2013), Wollmann (2018), Li et al. (2019), Fan and Yang (2020), Garrido (2020), ...
- Estimation
  - Ciliberto and Tamer (2009), Ciliberto, Murry and Tamer (2021)
  - Grieco (2014), Magnolfi and Roncoroni (2021)...
  - Ho (2009), Eizenberg (2014), PPHI (2015), ...

Plan of the Talk

- An entry-game example: how our bounds work
- Empirics
  - model
  - empirical context
  - estimation
  - counterfactual

An Illustrative Model

- Two firms – later, multiple firms
- A single binary decision – later, a vector of binary decisions
- Complete information pure strategy Nash equilibrium – later, more flexible information environment

Model Setup

$$Y_{1m} = 1\left[\pi_{1m}\left(Y_{2m}\right) - C_1 - \zeta_{1m} \geqslant 0\right]$$
$$Y_{2m} = 1\left[\pi_{2m}\left(Y_{1m}\right) - C_2 - \zeta_{2m} \geqslant 0\right]$$

- Y_{nm}: binary entry decision of firm $n = 1, 2$ in market m
- $\pi_{nm}(Y_{-nm})$: variable profit function, known to the researchers
- C_1, C_2: fixed costs of entry
- $\zeta_{nm} \sim F_{\zeta}(\cdot, \boldsymbol{\sigma})$, fixed-cost shocks unknown to the researchers, $\boldsymbol{\sigma}$: parameters to be estimated

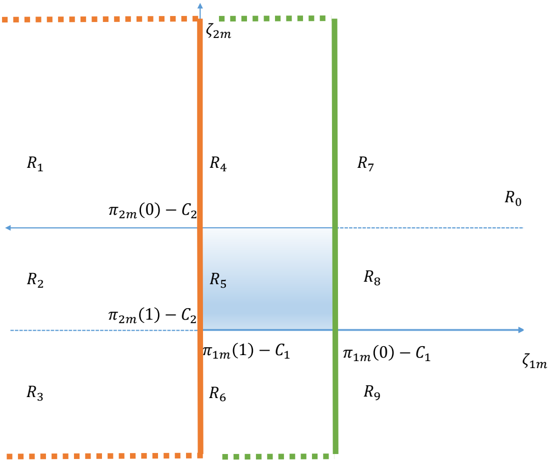
Estimation Challenges

- Multiple equilibria
  - "Incomplete" model (Tamer 2003)
- Selection
  - $E(\zeta_{nm}|Y_{nm}=1) \neq 0$, $E(\zeta_{nm}|Y_{nm}=0) \neq 0$
  - e.g. cannot average $\pi_{1m}(Y_{2m}) - C_1 - \zeta_{1m} \geqslant 0$ conditional on entry

Our Approach

- We construct bounds for $\Pr(Y_{nm} = 1)$
- Main assumption:
  observed equilibrium strategies are not dominated
  - level-1 rationality

Level-1 Rationality: $\Pr(Y_{1m} = 1)$



Probability Bounds

$$\underbrace{\Pr\left(\zeta_{1m} \in \cup_{\ell=1}^{3} R_\ell\right)}_{\text{small } \zeta_{1m}} \leqslant \Pr\left(Y_{1m} = 1\right) \leqslant \underbrace{\Pr\left(\zeta_{1m} \in \cup_{\ell=1}^{6} R_\ell\right)}_{\text{large } \zeta_{1m}}$$

$$\Longrightarrow$$

$$F_{\zeta_{1m}}\left(\pi_{1m}(1) - C_1\right) \leqslant \Pr\left(Y_{1m} = 1\right) \leqslant F_{\zeta_{1m}}\left(\pi_{1m}(0) - C_1\right)$$

Can construct similar bounds for $\Pr\left(Y_{2m} = 1\right)$

Our Approach (Cont.)

- The inequalities hold
  - when there are multiple Nash equilibria
  - when the equilibrium selection mechanism is not the same across markets
  - when there does not exist a pure strategy equilibrium for some values of ζ_{nm} (when there are more than two firms)
  - with a complete or incomplete information game.
- One-dimensional CDF, easy to calculate
  - can be used for settings with a large number of firms or firms with a long vector of actions additional results-identification

Elephant in the room: are they too loose?

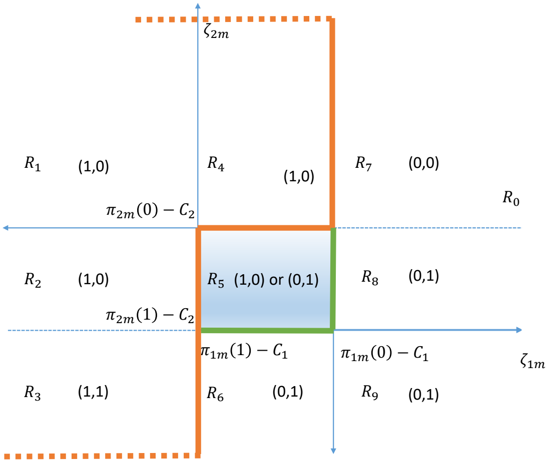
Comparison to Ciliberto and Tamer (2009)

- Ciliberto and Tamer (2009): bounds for prob of eqm $\Pr(Y_{1m}, Y_{2m})$. They are

$$
\begin{aligned}
& \Pr\left((Y_{1m}, Y_{2m}) \text{ is a unique pure-strategy Nash eqm}\right) \\
\leqslant & \Pr\left(Y_{1m}, Y_{2m}\right) \\
\leqslant & \Pr\left((Y_{1m}, Y_{2m}) \text{ is a pure-strategy Nash eqm}\right)
\end{aligned}
$$

  - need to find all equilibria for given draws of (ζ_{1m}, ζ_{2m})
  - good for settings with few firms and a short vector of decisions
- How are our bounds related to the CT bounds?

$\Pr(Y_{1m} = 1)$; CT 2009



Compare the Bounds; CT 2009

CT bounds:

$$\Pr\left(\zeta_{1m} \in \cup_{\ell=1}^{4} R_\ell\right) \leqslant \Pr\left(Y_{1m} = 1\right) \leqslant \Pr\left(\zeta_{1m} \in \cup_{\ell=1}^{5} R_\ell\right)$$

Our bounds:

$$\Pr\left(\zeta_{1m} \in \cup_{\ell=1}^{3} R_\ell\right) \leqslant \Pr\left(Y_{1m} = 1\right) \leqslant \Pr\left(\zeta_{1m} \in \cup_{\ell=1}^{6} R_\ell\right)$$

- lose some information from the Nash assumption
- gain tractability
- how does our method compare with CT bounds in finite samples?

Compare the Bounds; PPHI 2015

- Our approach:
  - assumptions on "structural error" ζ_{nm}: unobservable fixed-cost shocks
  - estimate the parameters of the distribution
  - account for this distribution in counterfactual simulations
- PPHI (2015):
  - assumptions on the equilibrium object $E\left(\zeta_{nm} \middle| Y_{nm}\right) = 0$

Implementation and Monte Carlo

- Estimator based on Chernozhukov, Chetverikov and Kato (2019)

$$E\left(F_{\zeta_{nm}}\left(\pi_{nm}(1) - C_n\right) - \Pr\left(Y_{nm} = 1\right)\right) \leqslant 0$$

$$E\left(\Pr\left(Y_{nm} = 1\right) - F_{\zeta_{nm}}\left(\pi_{nm}(0) - C_n\right)\right) \leqslant 0$$

- Faster, looser but still informative confidence intervals compared to CT 2009
  - For 15 firms, 10^5 faster

monte details

Model: A Two-Stage Game

- Stage 1: in each market m and year τ, firms choose sets of products to sell in a market
  - empty set: no entry
  - product decisions fixed for a year
- Stage 2: in each market m and month t, firms observe demand and marginal cost shocks and set prices
  - mixed logit demand model with rich random coefficients

Demand

- Utility of household i in market m from product j in month t

$$
\begin{aligned}
u_{ijmt} =& (\sigma_0 \nu_i + \kappa_0 y_i) + (\alpha + \kappa_\alpha y_i) p_{jmt} \\
& + \sigma^{\text{ale}} \nu_i^{\text{ale}} x_j^{\text{ale}} + \sigma^{\text{lager}} \nu_i^{\text{lager}} x_j^{\text{lager}} + \sigma^{\text{light}} \nu_i^{\text{light}} x_j^{\text{light}} \\
& + \sigma^{\text{import}} \nu_i^{\text{import}} x_j^{\text{import}} + (\sigma^{\text{craft}} \nu_i^{\text{craft}} + \kappa^{\text{craft}} y_i) x_j^{\text{craft}} \\
& + \beta X_{jm} + FE_j^{\text{demand}} + FE_m^{\text{demand}} + FE_t^{\text{demand}} + \xi_{jmt},
\end{aligned}
$$

- $\nu_i^{(\cdot)}$: household-specific unobserved taste shocks
- y_i: log household income
- X_{jm}: j-m specific characteristics (flexible distance controls)
- ξ_t: month fixed effects
- ξ_j, ξ_m: product and market fixed effects
- ξ_{jmt}: month-to-month variations of demand shocks

Stage 2: Pricing

- Each firm n chooses prices for its products $\mathcal{J}_{nm\tau}$
- Firm n's problem in Stage 2:

$$\max_{p_{jmt}, j \in \mathcal{J}_{nm\tau}} \sum_{j \in \mathcal{J}_{nm\tau}} \left(p_{jmt} - mc_{jmt}\right) D_{jmt}\left(p_{jmt}, p_{-jmt}\right).$$

- Marginal cost

$$mc_{jmt} = \omega_t + \omega_j + \omega_m + \gamma X_{jm} + \omega_{jmt}.$$

Stage 1: Product Decisions

- Each firm n chooses $\mathcal{J}_{nm\tau}$ from the set of potential products $\mathcal{J}_{n\tau}$
- Firm n's problem in Stage 1:

$$\max_{\mathcal{J}_{nm\tau} \subseteq \mathcal{J}_{n\tau}} \underbrace{\pi_{nm}\left(\mathcal{J}_{nm\tau}, \mathcal{J}_{-nm\tau}\right)}_{\text{expected variable profit}} - \underbrace{C_{nm}\left(\mathcal{J}_{nm\tau}\right)}_{\text{fixed cost}}.$$

- $\pi_{nm}\left(\mathcal{J}_{nm\tau}, \mathcal{J}_{-nm\tau}\right)$: sum of expected profits across 12 months in a year

Stage 1 Fixed Cost

- Additively separable across products
$$C_{nm}\left(\mathcal{J}_{nm\tau}\right) = \sum_{j \in \mathcal{J}_{nm\tau}} \left(\theta W_{jm} + \sigma_m \zeta_{jm\tau}\right).$$
- W_{jm}: covariates such as market size fixed effects and craft fixed effects
- Unobserved shock $\zeta_{jm\tau} \sim N(0,1)$
- Extension: (dis-)economies of scope; market-level unobservables

Data

- Nielsen Retail Scanner Data and Nielsen Consumer Panel: 2010-2016
- Craft designation by the Brewers Association
- Hand-collected data on identities of breweries and corporate owners (firms), locations of breweries
- County demographics from the Census

other details

Summary Statistics (2010-2016)

	Total Quantity (12 pk equiv)	Avg. Price (2016 $)	# Firms	# Products
Craft	4,914,209	17	36	135
All	53,465,658	11	54	269

▸ flavors

Estimation of Demand and Marginal Costs

- Micro-moments identify σ's (dispersion in unobs. heterogeneity) and κ's (income effect on taste)
  - persistence of tastes

$$E\left(\sum_{t=1}^{12} q_{it}^{\tilde{f}} \,\middle|\, \sum_{t=1}^{12} q_{it}^{f} \geqslant 1\right)$$

  - correlations between income and purchases
- Macro-moments identify the mean price coefficient
  - global barley prices interacted with beer types (light, lager, ale, ...)
- Marginal cost: inverted from pricing first-order conditions and projected onto fixed effects

Demand Estimates

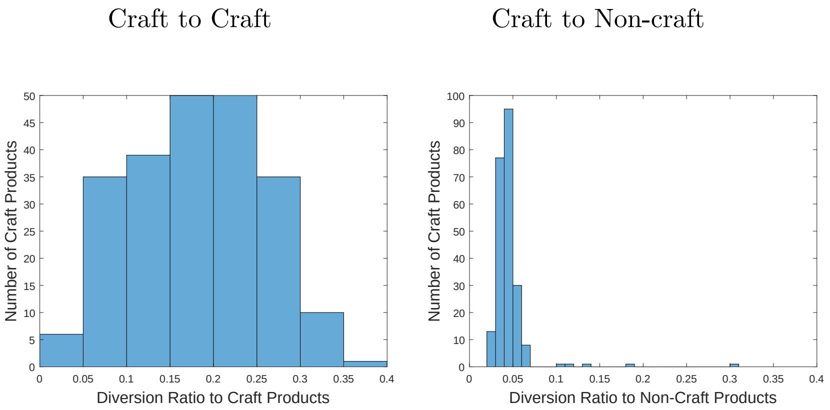
Unobs. Heter.	σ_0	0.00	Income Effect (log)	κ_0	-2.15
		(0.02)			(0.02)
	σ^{ale}	1.98		κ^{craft}	1.08
		(<0.01)			(0.02)
	σ^{lager}	0.89		κ_α	0.15
		(<0.01)			(<0.01)
	σ^{light}	2.67			
		(<0.01)			
	σ^{import}	2.14	Price Coefficient	α	-2.26
		(<0.01)			(0.03)
	σ^{craft}	2.44			
		(<0.01)			
	$\rho^{\text{craft-light}}$	-0.28			
		(<0.01)			

σ^{craft}: 3.81 dollars of discount for a household of $50,000 annual income

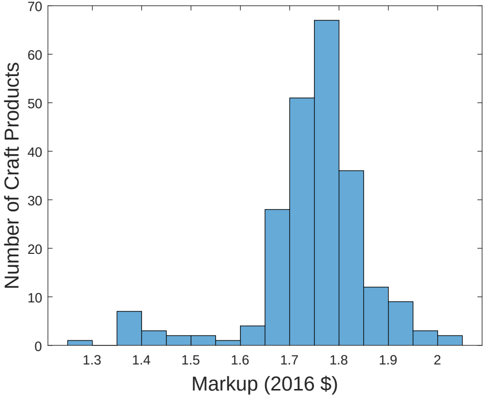
Elasticities

	Craft			Main		
Craft	-10.09	0.14	0.02	0.01	0.01	0.01
	0.22	-9.52	0.02	0.01	0.01	0.01
	0.04	0.03	-9.16	0.01	0.03	0.01
Main	0.00	0.00	0.00	-5.87	0.04	0.67
	0.00	0.00	0.00	0.08	-6.81	0.08
	0.00	0.00	0.00	0.68	0.04	-5.88

Diversion Ratio



Markup Distribution



A sources in 2014 suggest an 8% margin on the retail price, which is $1.4, a little lower than our estimates.

Fixed Cost Estimates, 2016 Data

Projection of the 95% confidence set

Craft (θ_1)	[229.14, 1093.24]
In State× Craft (θ_2)	[-387.82, 208.18]
Market-size Specific Fixed Cost	
Small Market	[308.95, 938.33]
Medium Market	[1027.77, 1468.10]
Large Market	[3325.71, 4177.69]
Market-size Specific Std Dev. (σ_ζ)	
Small Market	[0.00, 522.79]
Medium Market	[679.41, 863.25]
Large Market	[2511.65, 3424.06]

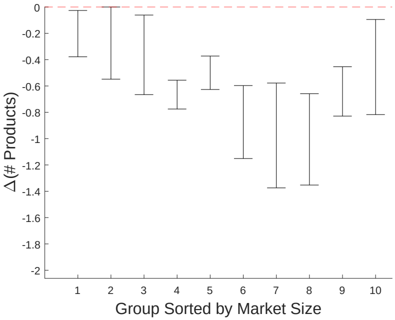
Counterfactual

- Counterfactual merger: a large brewery acquires top 3 craft breweries in 2016
  - exclude Boston Beer Company and Sierra Nevada Brewing
  - 149 markets where at least one craft product is observed
  - 44% of craft sales; 4.6% of all sales
  - what happens when the craft segment is as concentrated as the overall market?
- Complete information pure strategy Nash equilibria
- Report 95% CI
  - merger effects across markets sorted into 10 groups

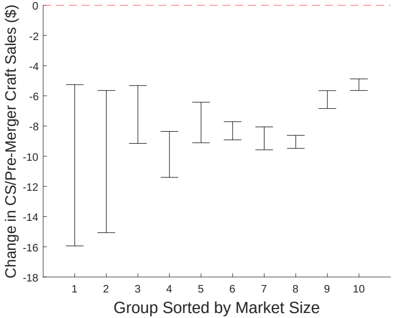
details

Main Results

Change in the Number of Products



Change in (CS/pre-merger craft quantity)



Average Results

Average Change Per Market		
(1)	# of firms	[-2.93, -2.82]
(2)	# new entrants	[0.02, 0.14]
(3)	# of products	[-0.86, -0.33]
(4)	merging firms	[-0.90, -0.49]
(5)	non-merging incumbents	[0.02, 0.08]
(6)	new entrants	[0.02, 0.14]
(7)	average price ($)	[0.00, 0.00]
(8)	craft products ($)	[0.04, 0.07]
(9)	craft, merging firms ($)	[0.13, 0.15]

Aggregate Results

Aggregate Change Across Markets		
(10)	quantity (1000)	[-266.94, -251.57]
(11)	craft	[-249.76, -230.46]
(12)	craft, merging firms	[-301.64, -283.55]
(13)	consumer surplus ($1000)	[-639.00, -602.81]
(14)	craft beer profits ($1000)	[97.95, 111.07]
(15)	merging firms	[24.78, 27.68]
(16)	total surplus ($1000)	[-533.03, -504.86]
ΔCS decomposition ($1000)		
(17)	due to variety change	[-155.43, -106.54]
(18)	due to entry	[6.85, 26.09]
(19)	due to incumbent product adj.	[-164.55, -123.48]

Merger Efficiency

- Acquisition of craft mergers unlikely to reduce short-run marginal costs
  - craft products brewed at same locations
  - no savings from the transportation costs
- Potential reduction in fixed costs
  - could benefit from the macro brewery's marketing network
  - set the craft parameter $\theta_1 = 0$: the parameter measured the difference in fixed costs between independent craft and craft owned by macro breweries
  - average change in the number of products: [0.00, 1.43]
  - no new entrants
  - loss in consumer surplus

Conclusion

- A new estimation method for multiple-discrete choice games
    - weak conduct and information assumptions
    - computationally attractive
    - reasonable statistical performance
- Retail craft beer market in California
    - net effects tend to reduce product variety and worsen consumer welfare loss
    - larger per-capita loss in medium-sized markets than the largest markets

Monte Carlo Simulation

- There are M markets. Each market has N firms. Firm n's profit upon entry in market m is

$$\pi_{nm}\left(Y_{-nm}\right) = O_m \cdot \prod_{k \neq n} x_{knm}^{Y_{km}} - C - \sigma\zeta_{nm}$$

  - $O_m \sim \text{Uniform}\,[0,2]$, $x_{knm} \sim \text{Uniform}\,[a,1]\,, C = \sigma = 1$
  - $\zeta_{nm} \sim \text{N}\,(0,1)$
- Complete information pure strategy Nash
- Identification: variation in O_m, x_{knm} across firms and markets
- Monte Carlo exercises
  - vary a to change the tightness of our bounds
  - estimator: Chernozhukov, Chetverikov and Kato (2019) (CCK)
  - compute the finite sample coverage probabilities of 95% confidence set for candidate parameters $\hat{C} \in [0,3]\,, \hat{\sigma} \in [0,3]$

Estimation Bounds

- For each (n, m), define

$$\boldsymbol{X}_{nm} = \left(\max_{Y_{-nm}} \pi_{nm}\left(Y_{-nm}\right), \min_{Y_{-nm}} \pi_{nm}\left(Y_{-nm}\right) \right)$$
$$= \left(\pi_{nm}\left(1, 1, \ldots, 1\right), \pi_{nm}\left(0, 0, \ldots, 0\right)\right)$$
$$= \left(O_m, O_m \cdot \prod_{k \neq n} x_{knm} \right)$$

- Lower bound (sufficient condition for entry)

$$L_{nm} = \Pr\left(\zeta_{nm} < O_m \cdot \prod_{k \neq n} x_{knm} - C \left| \boldsymbol{X}_{nm} \right. \right) - \Pr\left(Y_{nm} = 1 \left| \boldsymbol{X}_{nm} \right.\right)$$

- Upper bound (necessary condition for entry)

$$H_{nm} = \Pr\left(Y_{nm} = 1 \left| \boldsymbol{X}_{nm} \right.\right) - \Pr\left(\zeta_{nm} < O_m - C \left| \boldsymbol{X}_{nm} \right.\right)$$

Fixed Cost Estimation: Moments

- Conditional moment conditions

$$E\left[L_{nm}\left|\boldsymbol{X}_{nm}\right.\right] \leqslant 0, E\left[H_{nm}\left|\boldsymbol{X}_{nm}\right.\right] \leqslant 0$$

- Unconditional moment conditions

$$E\left[\sum_{n} L_{nm} \cdot g_{nm}^{(k)}\right] \leqslant 0, E\left[\sum_{n} H_{nm} \cdot g_{nm}^{(k)}\right] \leqslant 0$$

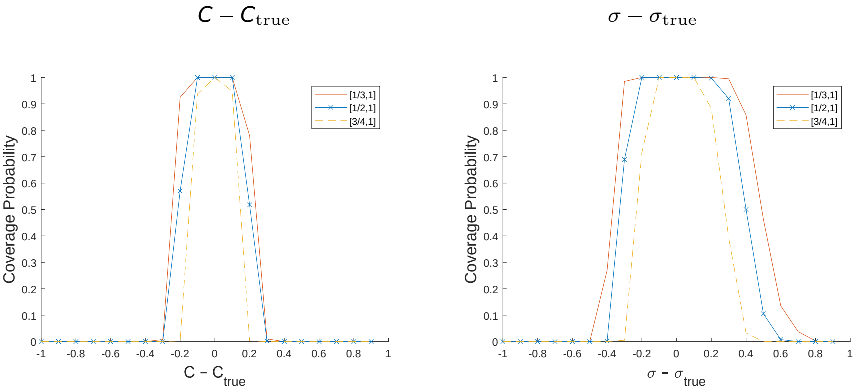
  - nonnegative functions $g_{nm}^{(k)}, k = 1, ..., K$ of $\boldsymbol{X}_{nm}$
  - $g_{nm}^{(k)}$: discretize the space of $\boldsymbol{X}_{nm}$ into hypercubes $\mathcal{C}_k, k = 1 \ldots K$; define

$$g_{nm}^{(k)} = 1\left[\boldsymbol{X}_{nm} \in \mathcal{C}_k\right]$$

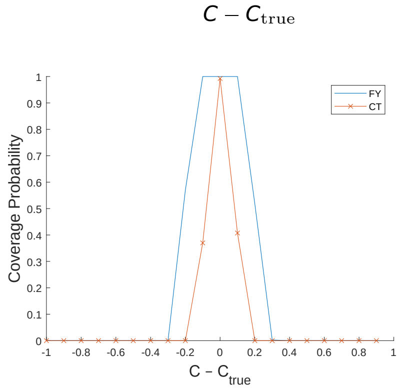
  - expectation with respect to m: entry decisions correlated across products within a market but conditionally independent across markets

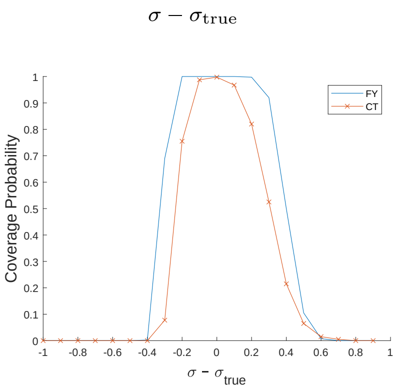
▸ CCK test statistic

$N = 2, M = 2000$


$C - C_{\mathrm{true}}$
$\sigma - \sigma_{\mathrm{true}}$
Coverage Probability
C - C true
σ - σ true
[1/3,1]
[1/2,1]
[3/4,1]

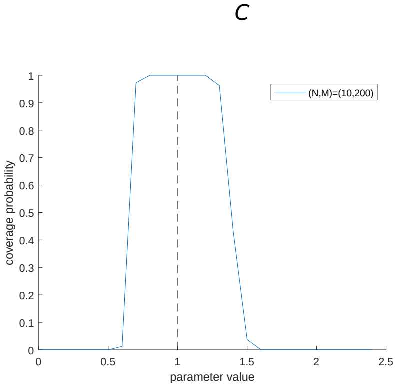
Comparison with CT Bounds: $N = 2, M = 2000, d \sim \left[\frac{1}{2}, 1\right]$

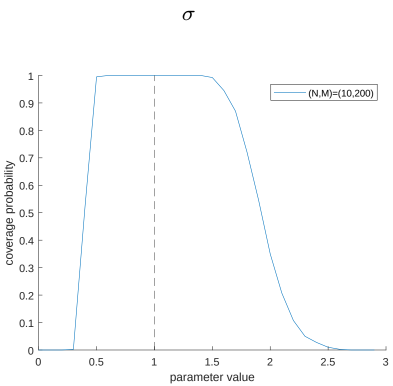




additional results-many firms

$N = 2, M = 2000$ vs $N = 10, M = 200$





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monte summary

Definitions and Sample

- Market: county-retailer pair
  - about 80% of the households purchased all of their beers in 2016 from one retailer-county combination
- Firm: corporate owner (e.g., Boston Beer Company)
  - a firm can own multiple breweries and products
- Product: a brand in Nielsen data (e.g., Samuel Adams Boston Lager)
  - homogenize size to be 12-ounce-12-pack equivalents
  - aggregate to product/month level

Additional Data Details

- A product is in a market in a year: the product was sold more than 20 units in a month for more than 6 months in the market/year
- Market size: average monthly alcohol $\times$ 8 (average number of trips to grocery stores)
- For craft products, we keep those by the top 60 craft breweries (by national volume in 2015) in the Brewers Association data
  - our sample covers 83% of California craft beer quantity in the Nielsen Scanner Data

return

Flavors

	light	lager	ale
quantity			
craft	0.44%	27.13%	71.53%
all	40.02%	46.42%	12.50%
number of products			
craft	0.41%	26.87%	66.33%
all	7.23%	39.76%	44.19%

return

Identification of Variance of FC Shock

$$\Pr\left(\zeta_{jm} < \pi_{jm}(1) - C_j\right) \leqslant \Pr(Y_{jm} = 1) \leqslant \Pr\left(\zeta_{jm} < \pi_{jm}(0) - C_2\right)$$

- When σ_ζ is very large, LHS and RHS do not vary with $W_m \longrightarrow$ more likely to be violated
- e.g, symmetric distribution, when $\sigma_\zeta = \infty$, LHS = RHS = 0.5
- When σ_ζ decreases to 0, (Chebyshev's inequality)
  - both bounds approach to 1 if $\underline{\pi}_{im} - C(W_{im}, \theta) > 0$
  - both bounds approach to 0 if $\overline{\pi}_{im} - C(W_{im}, \theta) < 0$

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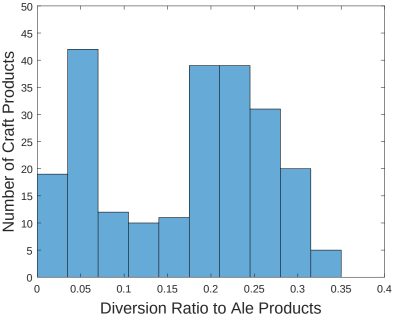
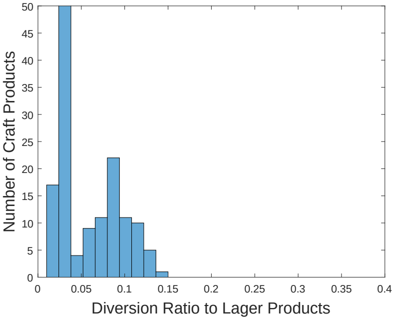
Diversion Ratio

Craft to Ale



Craft to Lager



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CCK Estimator

- In market m, define

$$Z_m = \left(\left(\sum_n L_{nm} \cdot g_{nm}^{(k)}\right)_{k=1}^{K}, \left(\sum_n H_{nm} \cdot g_{nm}^{(k)}\right)_{k=1}^{K}\right)$$

and use $Z_{\tilde{k}m}$ to denote an element; $\tilde{k} = 1, \ldots, 2K$.

$$\hat{\mu}_{\tilde{k}} = \frac{1}{M}\sum_{m=1}^{M} Z_{\tilde{k}m}, \hat{\sigma}_{\tilde{k}} = \sqrt{\frac{1}{M}\sum_{m=1}^{M}\left(Z_{\tilde{k}m} - \hat{\mu}_{\tilde{k}}\right)^2}$$

- Test statistic

$$\max_{1 \leqslant \tilde{k} \leqslant 2K} \frac{\sqrt{M}\hat{\mu}_{\tilde{k}}}{\hat{\sigma}_{\tilde{k}}}$$

- Critical value with size α

$$\frac{\Phi^{-1}\left(1 - \alpha/2K\right)}{\sqrt{1 - \Phi^{-1}\left(1 - \alpha/2K\right)^2/M}}$$

Fixed Cost Estimation: Notation

- Firm n's decision on project j: $a_{jm} \in \{0, 1\}$
- Firm n's decision on its products other than j and the product choices of firm n's rivals

$$a_{-jm} \in \bigtimes \{0, 1\}$$

- Change in variable profit by adding product j of firm n in market m:

$$\Delta_j \left(a_{-jm}, X_m\right) = \Pi_n(a_{jm} = 1, a_{-jm}, X_m) - \Pi_n(a_{jm} = 0, a_{-jm}, X_m)$$

- Min and max of the change in variable profit:

$$\underline{\Delta}_j \left(X_m\right) = \min_{a_{-jm}} \Delta_j \left(a_{-jm}, X_m\right), \ \overline{\Delta}_j \left(X_m\right) = \max_{a_{-jm}} \Delta_j \left(a_{-jm}, X_m\right)$$

- Covariates and parameters: $X_{jm} = (X_{jm}, W_{jm})$ and $\theta = (\theta, \sigma_\zeta)$

Fixed Cost Estimation: Notation (Cont.)

- Lower bound

$$L_{jm} = \Pr\left(\zeta_{jm} < \underline{\Delta}_j(X_m) - c(X_{jm}, \theta) \,|\, X_{jm}, X_{-jm}\right) \\ - \Pr\left(A_{jm} = 1 \,|\, X_{jm}, X_{-jm}\right)$$

- Upper bound

$$H_{jm} = \Pr\left(A_{jm} = 1 \,|\, X_{jm}, X_{-jm}\right) \\ - \Pr\left(\zeta_{jm} < \overline{\Delta}_j(X_m) - c(X_{jm}, \theta) \,|\, X_{jm}, X_{-jm}\right)$$

Fixed Cost Estimation: Moments and Objective Function

- Conditional moment conditions

$$E\left[L_{jm} \mid X_{jm}, X_{-jm}\right] \leqslant 0, E\left[H_{jm} \mid X_{jm}, X_{-jm}\right] \leqslant 0$$

- Unconditional moment conditions

$$E\left[\sum_j L_{jm} \cdot g_{jm}^{(k)}\right] \leqslant 0, E\left[\sum_j H_{jm} \cdot g_{jm}^{(k)}\right] \leqslant 0$$

  - nonnegative functions $g_{jm}^{(k)}, k = 1, ..., K$ of X_{jm}, X_{-jm} and θ
  - conditioning variables: the bounds and the covariates in the fixed cost function
  - $g_{jm}^{(k)}$: given pairs of (c_1, c_2), define the corresponding hypercube as $\mathcal{C}(c_1, c_2) = [c_1, \infty] \times [c_2, \infty]$, and construct the g functions as $(x, \tilde{x}) \in \mathcal{C}(c_1, c_2)$ for all pairs of conditioning variables $(x, \tilde{x}) \subset (X_{jm}, X_{-jm})$ and all pairs of (c_1, c_2).
  - expectation with respect to m: entry decisions correlated across products within a market but conditionally independent across markets

Fixed Cost Estimation: Implementation Details

- Potential products $\mathcal{J}_{n\tau}$: observed in year τ in any market in CA
- $\overline{\Delta}_j, \underline{\Delta}_j$

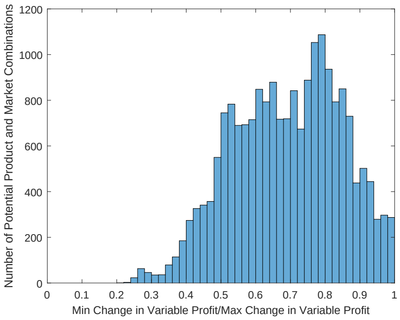
$$\underline{\Delta}_j(X_m) \approx \Delta_j((1, ..., 1), X_m)$$

$$\overline{\Delta}_j(X_m) \approx \Delta_j((0, ..., 0), X_m).$$

return

Identification

distribution of lower bound/upper bound



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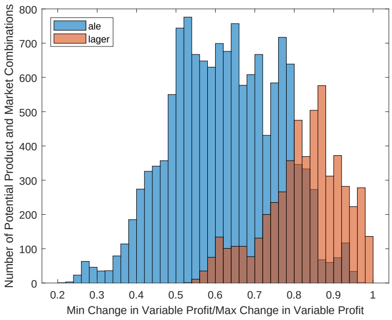

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Counterfactual Design

- CF1: price adjustment, product repositioning, firm entry
- CF2: price adjustment, product repositioning
  - remove products added by new entrants in CF1, recompute pricing eqm
- CF3: price adjustment
  - restore products to pre-merger, recompute pricing eqm
- Main results: CF1
- Decomposition
  - overall product variety effect: CF1 - CF3
  - product variety effect due to new entry: CF1 - CF2
  - product variety effect due to incumbent repositioning: CF2 - CF3

return

FC Draws

- Draw parameter values from the 95% confidence set
- Draw fixed cost shocks from the estimated distribution while taking into account selection, i.e., being consistent with the observed outcome
- Calculate the post-merger equilibrium using the algorithm in Fan and Yang (2020)

For each firm n, we simulate the fixed cost shocks to its potential products $\zeta_{nm} = (\zeta_{jm} : j \in \mathcal{J}_n)$ as follows

1. Simulate ζ_{jm} from a truncated normal distribution with the underlying normal distribution parameterized by mean 0 and variance $\hat{\sigma}^2_\zeta$ and the truncation being $\tilde{L}_{jm} < \zeta_{jm} < \tilde{H}_{jm}$, where

$$\tilde{L}_{jm} = -\infty, \tilde{H}_{jm} = \Delta_{jn}(X_m), \text{ if } j \text{ is in the market before the merger}$$
$$\tilde{L}_{jm} = \Delta_{jn}(X_m), \tilde{H}_{jm} = \infty, \text{ otherwise}$$

2. Verify ζ_{nm} indeed support the equilibrium by checking whether $\mathcal{J}_{nm}$ is the best response to $\mathcal{J}_{-nm}$
3. If not, go back to Step 1 and re-draw the fixed cost shocks

return

CF Results

- Draw θ from CS
- Draw fixed cost shock ζ
  - ζ rationalizes the observed market outcomes
- Compute

$$\operatorname*{Mean}_{\zeta} \mathit{Outcome}_m(\zeta, \theta)$$

- Sort markets by size O_m; 10 groups G

$$\text{Avg Outcome}_G(\theta) = \sum_{m \in G} O_m \operatorname*{Mean}_{\zeta} \mathit{Outcome}_m(\zeta, \theta) / \sum_m O_m$$

- Report

$$\left[\min_{\theta \in \mathrm{CS}} \text{Avg Outcome}_G(\theta), \max_{\theta \in \mathrm{CS}} \text{Avg Outcome}_G(\theta)\right]$$

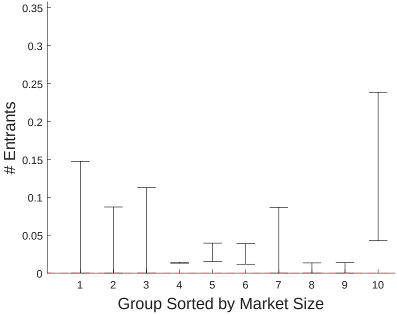
Market Size Cutoffs for Market Groups

Group ID	Mkt Size Cutoffs (10^6)
1	0.051
2	0.076
3	0.099
4	0.149
5	0.197
6	0.258
7	0.362
8	0.591
9	1.202
10	5.844

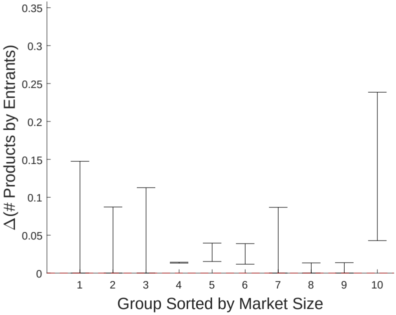
return

CF Results: Entry

Number of New Entrants

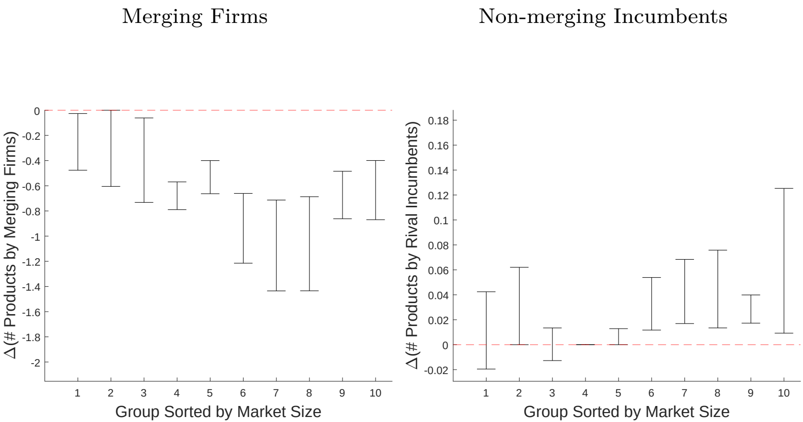


Number of Products Added by Entrants



CF Results: Incumbent Product Adjustment

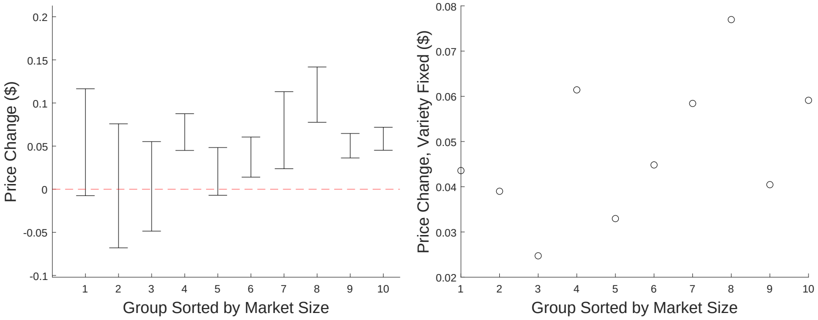
Change in the Number of Products by



CF Results: Prices

Change in Sales-Weighted Average Price

with Product Adjustment and Entry   without Product Adjustment and Entry



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Regression Results: Correlation with Market Structure

	(a) Merging Firms	(b) Other Firms	(c) Other Firms	(d) Market	(e) Market
Δp	[-6.98, -3.91]*	[0.73, 1.61]*	[0.86, 1.52]*	[-6.03, -3.08]*	[-6.00, -2.88]*
$\overline{FC}_{\text{merge}}$	[-0.10, 0.05]			[-0.05, 0.00]	[-0.05, 0.00]
$\overline{FC}_{\text{other}}$		[-0.05, -0.02]*	[-0.05, -0.02]*	[-0.13, 0.01]	[-0.13, 0.01]
$\overline{y_i}$			[-0.01, 0.01]		[-0.01, 0.01]
O	[0.04, 0.08]	[0.10, 0.13]*	[0.10, 0.13]*	[0.14, 0.21]*	[0.14, 0.21]*
R^2	[0.14, 0.34]	[0.20, 0.34]	[0.20, 0.35]	[0.13, 0.32]	[0.13, 0.32]
obs	149	149	149	149	149

Linear regressions where the dependent variables are the changes in the numbers of products by merging firms and other firms and the net change in a market. Each observation is a market. We report the range of estimates from the parameters in the confidence set. * indicates significance above 95% confidence level for all parameters in the sampled confidence set.

Δp: average price increase, merging firms ($), variety fixed

$\overline{FC}_{\text{merge}}$: average fixed cost ($1000), merging firms

$\overline{FC}_{\text{other}}$: average fixed cost ($1000), other firms

$\overline{y_i}$: average household income ($10,000)

O: market size (10^6)

return