Dynamic Information Acquisition and Entry into New Markets

Snehal Banerjee and Bradyn Breon-Drish*

September 2018

Abstract

We model dynamic information acquisition and entry by a strategic trader into a new trading opportunity. Instead of restricting the trader to make her choices before the market opens, we allow her to optimally choose when to enter in response to public news. We show that there exists a unique equilibrium in which optimal entry exhibits delay. The model provides novel implications for how the likelihood and timing of entry, and choice of precision, depend on news volatility and the trading horizon. Our results shed light on the entry behavior of institutional investors into new asset classes like cryptocurrencies.

JEL: D82, D84, G12, G14

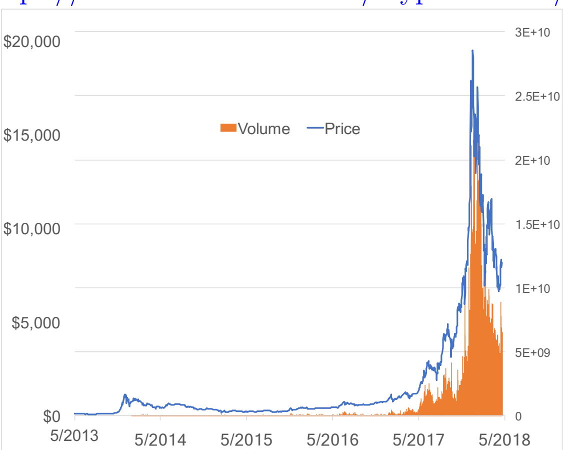
Keywords: dynamic information acquisition, entry, cryptocurrencies

*Banerjee (snehalb@ucsd.edu) and Breon-Drish (bbreondrish@ucsd.edu) are at the University of California, San Diego. All errors are our own. We thank Brett Green for numerous, invaluable discussions during an early stage of this project. We also thank Samanvaya Agarwal, Stathi Avdis, Kerry Back, Jesse Davis, Darrell Duffie, Joey Engelberg, Itay Goldstein, Naveen Gondhi, Jungsuk Han, Mariana Khapko, Igor Makarov, Seymon Malamud, Sophie Moinas, Dmitry Orlov, Christine Parlour, Chris Parsons, Uday Rajan, Michael Sockin, Allan Timmermann, Dimitri Vayanos, and participants at Collegio Carlo Alberto, UC Davis, LSE, UC Riverside, University of Colorado Boulder, the University of Minnesota Junior Finance Faculty Conference, the FIRS 2017 Meeting, Barcelona GSE Summer Forum, the 2017 Western Finance Association Meeting, the 2017 TAU Finance Conference, the 2018 American Finance Association Meeting, and the 2018 Finance Symposium at INSEAD for helpful suggestions. This paper subsumes part of an earlier paper, titled "Dynamic Information Acquisition and Strategic Trading."

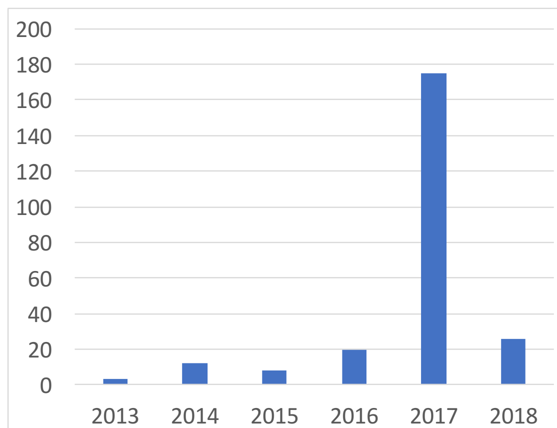
1 Introduction

Investors often choose to delay entry into new opportunities. For instance, the bitcoin network came into existence in January 2009 and the first exchange, BitcoinMarket.com, started operating in March 2010. However, as Figure 1 suggests, trading volume and participation by institutional investors remained relatively low until 2017. And in spite of the trading opportunities generated by the recent volatility, many traditional institutional investors remain out of the market.[1]

Figure 1: Bitcoin Price, Volatility and Number of Crypto Funds by Inception Date
Panel (a) plots the price of bitcoin in USD and the daily volume, using data from CoinMarketCap (https://coinmarketcap.com/currencies/bitcoin/historical-data/). Panel (b) plots the number of new crypto funds, as of April 2018, as collected by Autonomous Next (https://next.autonomous.com/cryptofundlist/).



(a) Bitcoin Price and Volume



(b) Crypto Hedge Funds by Inception Date

Understanding the nature of these delays is important. As highlighted by Duffie (2010) and others, slow arrival of investment capital to trading opportunities can lead to substantial and persistent dislocations between prices and fundamentals. Delayed entry by informed investors can also lead to less informative prices, which, in turn, can decrease allocative efficiency (see Bond et al. (2012) for a recent survey). Following Grossman and Stiglitz (1980), a large literature has studied how investors choose to acquire information and participate in investment opportunities. Despite the inherently dynamic nature of these choices, however, the existing literature has largely focused on the "static acquisition / entry" decision by requiring that investors make their information choices / entry decisions before trading commences.[2] As such, these models do not provide insight into when investors acquire

[1]For example, see "Big investors yet to invest in bitcoin" by Alice Ross and Aliya Ram in the Financial Times, Oct 19, 2017 (https://www.ft.com/content/4c700f9a-b267-11e7-aa26-bb002965bce8).

[2]As we discuss below, some of these models allow for dynamic trading. However, the acquisition decision

information.

We study the dynamic information acquisition and entry decision of a strategic investor. When uncertainty about an investment opportunity varies over time, so does the value of acquiring information and trading in the market. As a result, the optimal acquisition and entry decision exhibits delay: the investor optimally waits until uncertainty is sufficiently high before entering. We derive predictions that relate the likelihood and timing of entry to characteristics of the investment opportunity (e.g., uncertainty and trading horizon) and market conditions. Moreover, we show that allowing for dynamic information acquisition has qualitatively distinct implications for the likelihood of information acquisition and the choice of information precision from those that arise in a static acquisition setting.

As described in Section 2, we begin with a continuous-time Kyle (1985) framework that builds on Back and Baruch (2004) and Caldentey and Stacchetti (2010). There is a single risky investment opportunity, traded by a risk-neutral, strategic investor and a mass of noise traders. We introduce a publicly observable news process, which affects the market's uncertainty about the risky opportunity and evolves stochastically over time. A risk-neutral market maker competitively sets the asset's price, conditional on the public signal and aggregate order flow. The trading opportunity disappears at a random time when the risky payoff is publicly revealed.[3] In contrast to earlier work, we do not constrain the investor to make her information and entry choices before trading begins. Instead, we allow her to pay a cost at any point in time to privately acquire (noisy) information about the investment opportunity and enter the market.[4]

For concreteness, consider the decision of an institutional investor deciding whether or not to begin trading bitcoin.[5] Information acquisition and entry are costly and irreversible:[6] the institution must invest in research, information technology and infrastructure, and relevant expertise (analysts and traders) before it can begin trading. Importantly, the expected

is still "static" because investors choose to acquire information / enter before trading begins.

[3]The assumption of a random horizon is largely for tractability and is not qualitatively important for our primary results. What is key is that a random horizon induces the trader to discount future profits. We expect our results to carry over to settings with fixed horizon that feature discounting for other reasons (e.g., if the trader has a subjective discount factor or the risk-free rate is nonzero).

[4]We treat acquisition and entry as a joint decision. As we discuss in Section 2.1 this is an economically reasonable assumption.

[5]For instance, see "Goldman Sachs to open a bitcoin trading operation", Nathaniel Popper, New York Times, May 2, 2018 (https://www.nytimes.com/2018/05/02/technology/bitcoin-goldman-sachs.html), "George Soros set to trade cryptocurrencies", Alastair Marsh, Saijel Kishan, and Katherine Burton, Bloomberg, April 6, 2018 (https://www.bloomberg.com/news/articles/2018-04-06/george-soros-prepares-to-trade-cryptocurrencies-as-prices-plunge), "Andreessen Horowitz lends credence to crypto with new fund", Klint Finley, Wired, June 25, 2018 (https://www.wired.com/story/andreessen-horowitz-lends-credence-to-crypto-with-new-fund/).

[6]Irreversible in the sense that the associated costs of entry/acquisition cannot be fully recovered if one exits the market.

value from trading bitcoin varies over time and with economic conditions. For instance, when the price is stable and trading activity is limited, the value from trading bitcoin is low. In contrast, high volatility and increased trading by less sophisticated traders (retail or noise traders), increase the value from participation. As a result, acquiring information and entering the market immediately need not be optimal; instead, the investor might prefer to wait until uncertainty about the investment opportunity is sufficiently high.

Appealing to standard results on optimal stopping, we characterize the investor's optimal strategy in Section 3. We show that it follows a cutoff rule: she chooses to acquire information only when public uncertainty reaches a threshold. Furthermore, the optimal decision exhibits delay relative to a static "NPV" rule that prescribes entry as soon as the benefit from entry exceeds the cost. Intuitively, the investor's entry / acquisition decision resembles exercising a call option. By immediately acquiring information and entering the market (i.e., exercising the option), the investor can begin to exploit her informational advantage by trading against uninformed, noise traders. By not entering immediately (i.e., by delaying exercise), she preserves her potential informational advantage and can wait for uncertainty to increase, which makes information and entry more valuable. The investor optimally waits until uncertainty is sufficiently high (i.e., her option is deep enough "in the money") before acquiring information and entering the market. Consistent with the intuition from option exercise problems, the optimal boundary (i) increases in the cost of acquisition / entry and the volatility of public news (both of which make waiting more attractive), and (ii) decreases in the volatility of noise trading and in the precision of the private signal (which make trading more valuable).

We then characterize the economic implications of dynamic acquisition and entry in Sections 4 and 5. As a baseline, when the investor is constrained to make her entry / acquisition decision before trading begins, we show that: (i) when costs are sufficiently low, there is always information acquisition and entry, (ii) the likelihood of acquisition increases with the expected trading horizon, since the investor expects to exploit her informational advantage for longer, and (iii) from a given set of signals with varying precisions and correspondingly varying costs, the investor optimally chooses the signal with the higher "bang for buck" i.e., with the lowest cost-benefit ratio.

The implications are qualitatively different when the investor can choose when to acquire information and enter. First, for any given cost, the probability that acquisition / entry occurs is less than one when the volatility of the public news process is sufficiently high. This is because higher news volatility increases the likelihood that public uncertainty will be higher in the future, and consequently, increases the value from waiting. Since the investment opportunity can disappear before the investor enters, this decreases the likelihood of entry.

In fact, when news volatility is sufficiently high, the likelihood of entry decreases with further increases in volatility.

Second, we show that the likelihood of acquisition / entry is hump-shaped in the trading horizon. When the payoff is expected to be revealed quickly (i.e., the horizon is short), the value from being informed is very low since there is little time over which to profit at the expense of noise traders. However, as the expected trading horizon increases, there are two offsetting effects. On the one hand, the value from being informed increases with the horizon since the trader expects her information advantage to last longer. On the other hand, the cost of waiting decreases with the horizon, since the likelihood that the payoff is revealed before acquisition is low. We find that initially the first effect dominates, while eventually the second one does. As a result, the trader is less likely to acquire information when the trading horizon is very long or very short.

Third, we show that the investor's choice of precisions does not depend only on the relative relative costs across signals, since this ignores option to wait. Consider a given pair of signal precisions and costs, where the lower precision signal is also cheaper (otherwise, the investor should pick the high precision, low cost signal). When considered separately, the low precision-low cost signal has a lower optimal threshold i.e., the relatively cheaper signal would be acquired earlier. Given both signals, the optimal choice of precision then depends on whether the strategic investor finds it worthwhile to wait for the higher precision signal.

For a fixed pair of signals, we show that the investor always prefers the high precision signal, irrespective of its relative cost, if either the news volatility is sufficiently high or the trading horizon is sufficiently long. In either case, the cost of waiting is relatively low, so it is not optimal to forgo the opportunity to acquire the more precise signal, even when it is relative more expensive. As such, the optimal choice of signal in a dynamic setting is not pinned down by just the relative precisions and costs of the signals, but also depends on the dynamics of the investment opportunity.

Our analysis also permits a characterization of the expected delay, conditional on entry.[7] We show that the expected time of acquisition / entry is increasing in the cost of entry / acquisition, decreasing in the volatility of noise trading and in the precision of signals. We show that the effect of news volatility and trading horizon are possibly non-monotonic, and depend on the cost of information acquisition. However, the conditional expected time to entry is decreasing in news volatility and increasing in trading horizon when either the cost is sufficiently high, news volatility is sufficiently high, or the trading horizon is sufficiently high.

[7]Note that since the likelihood of acquisition / entry is less than one, the unconditional expected time of entry is infinite. Instead, we focus on the expected time of entry, conditional on there being entry.

Although stylized, our model provides some guidance for understanding the delay of entry into new investment opportunities like bitcoin. Specifically, bitcoin is characterized by (i) an evidently long trading horizon[8], (ii) a sharp increase in prices and uncertainty in 2017, and (iii) increased demand from uninformed, retail investors (noise traders), especially in the second half of 2017. Our model predicts that the likelihood of acquisition / entry is low when the cost of acquisition is high and when the trading horizon is long, which suggests that we expect to see relatively low entry and participation by institutional investors. The sharp increase in new crypto funds in 2017 appears consistent with the model's prediction that the likelihood of entry increases with public uncertainty about the value of the opportunity and with noise trading. Finally, our model predicts that when the trading horizon is sufficiently long, investors choose the highest precision signals available to them, and consequently, wait longer. This suggests that more sophisticated investors remain out of the market for longer, which appears consistent with the decisions of many large asset management firms.[9]

Our predictions also appear broadly consistent with the entry behavior of institutional investors into internet stocks in the 1990s, and help understand the difference in entry dynamics across these two markets. Most dot-com firms came into existence in the mid-1990's.[10] And yet, by late 1999, ownership by institutional investors was already substantial, as illustrated by Figure 1(b) in Griffin et al. (2011). Relative to cryptocurrencies, entry into tech stocks featured less delay and was more widespread. This is consistent with our model's predictions since (i) market uncertainty was arguably lower for tech stocks, (ii) the trading horizon for tech stocks was neither extremely short nor extremely long.

Our paper relates to the large literature on asymmetric information models with endogenous information acquisition that was initiated by Grossman and Stiglitz (1980). While a number of papers extend this basic setting to allow for dynamic trading (e.g., Mendelson and Tunca (2004), Avdis (2016)), to allow traders to condition their information acquisition decision on a public signal (e.g., Foster and Viswanathan (1993)), to allow traders to pre-commit to receiving signals at particular dates (e.g., Back and Pedersen (1998), Holden and Subrahmanyam (2002)), to incorporate a time-cost of information (e.g., Kendall (2017), Dugast and Foucault (2017), and Huang and Yueshen (2018)), or to incorporate a sequence of one-period information acquisition decisions (e.g., Veldkamp (2006)), the information ac-

[8]Despite the fact that bitcoin has been traded for almost a decade, the true value of the currency remains uncertain and hotly debated. For a recent survey, see "What 12 major analysts from banks like Goldman, JPMorgan, and Morgan Stanley think of bitcoin," by Will Martin in Business Insider, on January 18, 2018 (http://www.businessinsider.com/bitcoin-round-up-wall-street-cryptocurrencies-bull-bear-market-2018-1).

[9]See "Bitcoin extends gains as BlackRock looks into crypto and blockchain" by Ryan Browne and Fred Imbert for CNBC on July 16, 2018 (https://www.cnbc.com/2018/07/16/bitcoin-jumps-after-report-says-blackrock-exploring-cryptocurrencies.html).

[10]For instance, Amazon was founded in July 1994, Yahoo in March 1995, and eBay in September 1995.

quisition decision remains essentially static — investors make their information acquisition decision before the start of trade.[11] To the best of our knowledge, however, our model is the first to allow for dynamic information acquisition in that the strategic trader can choose to become privately informed at any point in time. Our analysis implies that allowing for dynamic information acquisition and entry has economically important consequences.

2 Model

Our framework is based on the continuous-time Kyle (1985) model with random horizon in Back and Baruch (2004) and Caldentey and Stacchetti (2010). Fix a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ on which is defined a 2-dimensional standard Brownian motion $\bar{W} = (W_\Delta, W_Z)$ with filtration $\mathcal{F}_t^W$, independent random variables ξ and T, and random variable S independent of all random variables except ξ. Let $\mathcal{F}_t$ denote the augmentation of the filtration $\sigma(\{\bar{W}_s\}_{\{0\le s\le t\}})$. Suppose that the random variable T is exponentially distributed with rate r and that $\xi \in \{0,1\}$ is binomial with probability $\alpha = \Pr(\xi = 1)$.

There are two assets: a risky investment opportunity and a risk-free asset with interest rate normalized to zero. The risky investment opportunity pays off a terminal value v at random time T. The risky opportunity can have either a high or low payoff, denoted by $v \in \{H_T, L_T\}$ where $\xi \in \{0,1\}$ is the indicator for whether the payoff is high. We normalize the low payoff to $L_t \equiv 0$ and let $H_t = L_t + \Delta_t = \Delta_t$. Hence Δ_t represents the time-t expected difference between the high and low payoffs. The difference Δ_t is publicly observable and follows a geometric Brownian motion

$$\frac{d\Delta_t}{\Delta_t} = \sigma_\Delta \, dW_{\Delta t}, \tag{1}$$

where $\sigma_\Delta > 0$ and the initial value Δ_0 is a constant, which we normalize to $\Delta_0 \equiv 1$. Compactly, the risky payoff at time T is

$$v = L_T + \xi\Delta_T = \xi\Delta_T, \tag{2}$$

and the time-t conditional expected asset value, given knowledge of ξ and the history of Δ_t is $v_t = \xi\Delta_t$.

[11] Kendall (2017) studies whether or not investors wait for better quality information when there is no explicit monetary cost. In Dugast and Foucault (2017), investors can acquire a raw (less precise) signal which arrives early or a processed (more precise) signal which arrives later. In Huang and Yueshen (2018), investors can acquire both information and speed, which allows them to exploit their informational advantage in an earlier period. However, in all these papers, information acquisition decision is implicitly (i) publicly observable, and (ii) made prior to the start of trading.

There is a single, risk-neutral strategic trader ("institutional investor") who can pay a fixed cost $c > 0$ at any time τ to investigate the investment opportunity and enter the market, and can optimally choose when to do so. If the investor acquires information and enters, she observes a noisy signal $S \in \{l, h\}$ about ξ that has a precision $q > 1/2$ i.e., $q = \Pr(S = h|v = H_T) = \Pr(S = l|v = L_T)$. This signal structure implies that the trader to has a conditional expectation given by:

$$\mathbb{E}[\xi|S] = \begin{cases} \frac{\alpha q}{\alpha q + (1-\alpha)(1-q)} \equiv \xi_h & \text{when } S = h \\ \frac{\alpha(1-q)}{\alpha(1-q)+(1-\alpha)q} \equiv \xi_l & \text{when } S = l \end{cases} \tag{3}$$

Let X_t denote the cumulative holdings of the trader, and suppose the initial position $X_0 = 0$. Further, suppose X_t is absolutely continuous and let $\theta(\cdot)$ be the trading rate (so $dX_t = \theta(\cdot)dt$).[12] There are noise traders who hold Z_t shares of the asset at time t, where

$$dZ_t = \sigma_Z dW_{Zt}, \tag{4}$$

with $\sigma_Z > 0$ a constant.

There is a competitive, risk neutral market maker who sets the price of the risky asset equal to the conditional expected payoff given the public information set. Let $\mathcal{F}_t^P$ denote the public information filtration, which we describe formally below. The price at time $t < T$ is given by

$$P_t = \mathbb{E}\left[v \middle| \mathcal{F}_t^P\right]. \tag{5}$$

Let $I_t = \mathbf{1}_{\{\tau \le t\}}$ denote an indicator for whether the institutional investor has entered the market at time-t or before. Because the market maker observes the public signal Δ_t and order flow processes $Y_t = X_t + Z_t$, and the entry status of the investor, the public information filtration $\mathcal{F}_t^P$ is the augmentation of the filtration $\sigma(\{\Delta_t, Y_t, I_t\})$.[13] Let $\mathcal{T}$ denote the set of $\mathcal{F}_t^P$ stopping times. We require that the trader's information acquisition time $\tau \in \mathcal{T}$. That is, we require acquisition to depend only on public information up to that point. Let $\mathcal{F}_t^I$ denote the augmentation of the filtration $\sigma(\mathcal{F}_t^P \cup \sigma(S))$. Thus, $\mathcal{F}_t^I$ represents the institution's information set, post-entry. We require the trader's pre-entry trading strategy to be adapted to $\mathcal{F}_t^P$ and her post-acquisition strategy to be adapted to $\mathcal{F}_t^I$.

Finally, let π_t denote the market maker's conditional probability that the trader has

[12] Back (1992) shows that it is optimal for the trader to follow strategies of this form in a model in which she is exogenously informed.

[13] To reduce clutter, we abuse notation somewhat by using $\mathcal{F}_t^P$ to denote both the market maker's information set, which includes the acquisition indicator I_t in this case, as well as the institution's pre-acquisition (public) information set, which includes only the news process and order flow variables, and defines the admissible class of stopping times for acquisition.

observed a high signal, $S = h$. Note that zero and one are absorbing states for π_t. As such, following Back and Baruch (2004), we must rule out trading strategies that first drive the risky asset price to $\xi_l \Delta_t$ or $\xi_h \Delta_t$, incurring infinite losses, and then yield infinite profits by trading against a pricing rule that is unresponsive to order flows. To do so, we add to the existing smoothness and measurability restrictions on trading strategies a further condition which requires that the trading strategy for a trader informed of $S = h$ satisfies

$$\mathbb{E}\left[\int_0^T \Delta_u(1-\pi_u)\theta_u^- \, du\right] < \infty, \tag{6}$$

and analogously for a trader informed of $S = l$,

$$\mathbb{E}\left[\int_0^T \Delta_u \pi_u \theta_u^+ \, du\right] < \infty. \tag{7}$$

A trading strategy that is smooth, satisfies the measurability restrictions, and satisfies (6) and (7) is *admissible*.

Our definition of equilibrium is standard, but modified to account for endogenous entry.

Definition 1. An equilibrium with pure strategy information acquisition is an entry time $\tau \in \mathcal{T}$ and *admissible* trading strategy θ for the trader, and a price process P_t such that, given the trader's strategy the price process satisfies (5) and, given the price process, the trading strategy and acquisition time maximize the ex-ante expected profit

$$\mathbb{E}\left[\int_0^T \theta(v_u - P_u)\, du\right]. \tag{8}$$

We focus on pure entry strategies. As we discuss below, we are in fact able to rule out the existence of equilibria with mixed-strategy entry.

2.1 Discussion of Assumptions

The assumption that information acquisition and entry occur simultaneously is made for simplicity, but it is economically reasonable. It is unlikely that a trader who has incurred the cost of acquiring private information about the risky opportunity will wait to begin trading, given that the opportunity may quickly disappear. Similarly, large investors are likely to be reluctant to enter and trade in a new market without some private informational advantage, especially since it is difficult to credibly convey to other market participants that one is uninformed.

We also assume that the acquisition / entry decision is detected by the market maker. As the news articles cited in the introduction suggest, entry by large investors into new markets is publicly scrutinized by the financial media. For instance, speculation about whether larger investment firms are setting up cryptocurrency trading desks has been a recurring theme in recent news.[14] The addition of star traders, portfolio managers, and executives also garners significant media attention. Even if not covered by the popular press, participation by large traders is often known to other market participants. For instance, prime brokers observe the cash and securities positions of their clients, and counter-parties in OTC derivative transactions disclose their interests to each other through ISDA agreements. Finally, many institutional investors are subject to regulatory reporting requirements, and disclosures about trading positions and capital adequacy can provide noisy information about an investor's trading strategies and private information.

Note that the value of entering the market varies over time with the publicly observable news about Δ_t. Since acquiring information provides information about whether the value is H_t or L_t, this information is more valuable when the difference Δ_t is larger. More generally, the specification of the public news process allows us to introduce stochastic volatility in a parsimonious and tractable manner, since the conditional variance of the payoff under the public information set prior to acquisition is

$$\mathrm{var}[v|\Delta_t] = \alpha\,(1-\alpha)\,\Delta_t^2. \tag{9}$$

Without variation in public news ($\Delta_t \equiv 1$), the above setting reduces to the one analyzed by Back and Baruch (2004) but with endogenous, noisy information acquisition. In this case, however, the trader's acquisition decision is effectively static since the value of information is constant over time.[15] With a stochastic news process, the value of information evolves over time, which introduces dynamic considerations to the acquisition decision. We expect alternative specifications that generate time-variation in uncertainty about fundamentals would generate similar predictions, although at the expense of tractability or a less natural economic interpretation.[16]

[14]For example, see "Goldman Is Setting Up a Cryptocurrency Trading Desk" by Hugh Son, Dakin Campbell and Sonali Basak for Bloomberg.com on December 21, 2017 (https://www.bloomberg.com/news/articles/2017-12-21/goldman-is-said-to-be-building-a-cryptocurrency-trading-desk), and "BlackRock is evaluating cryptocurrencies, CEO Fink says" by Trevor Hunnicutt for Reuters.com on July 16, 2018 (https://www.reuters.com/article/us-blackrock-cryptocurrency/blackrock-is-evaluating-cryptocurrencies-ceo-fink-says-idUSKBN1K61MC)

[15]The acquisition decision would also be effectively static if uncertainty always decreased over time (e.g., if v was normally distributed, and the publicly observable signals were conditionally normal). In this case, uncertainty about v is highest at the beginning, and consequently, so is the value of acquisition and entry.

[16]Arguably, a more standard specification of the model would be one in which the value v is normally distributed with stochastic volatility (e.g., variance Σ_t). In order for this volatility to impact the acquisition decision, it must be publicly observable. However, this poses a difficulty: how does one interpret a setting

Finally, note that the assumptions that the public signal is perfectly informative about Δ_t and that Δ_t has zero drift are solely for tractability and are, economically, without loss of generality. More generally, one could introduce a public signal about Δ_t and replace Δ_t with $\mathbb{E}[\Delta_T|\mathcal{F}_t^P]$ in the pricing rule and trading strategy without qualitatively affecting the rest of the analysis. It is also straightforward to generalize to a general continuous, positive martingale for Δ, but at the expense of closed-form solutions to the optimal acquisition problem in most cases.

3 Equilibrium

In this section we construct an overall equilibrium of the model by working backwards. First we characterize the equilibrium in the financial market given an entry time τ, and then we solve for optimal entry. We show that, generally, optimal entry exhibits delay. The entry decision by the investor resembles the exercise of a real option, and as such, the standard assumption that the investor makes a one-shot entry / information acquisition decision when the financial market opens is restrictive. Moreover, as we show in the next section, allowing for dynamic, endogenous entry has qualitatively novel implications for the likelihood of and timing of information acquisition and entry.

3.1 Financial market equilibrium

In the following result, we characterize the financial market equilibrium, conditional on an arbitrary acquisition time.

Proposition 1. *Fix an information acquisition time $\tau \in \mathcal{T}$. There exists an equilibrium in the trading game in which the price of the risky asset is given by $P_t = \Delta_t(\xi_h \pi_t + \xi_l(1-\pi_t))$, where*

$$\pi_t \equiv \mathbb{P}\left[S = h \mid \mathcal{F}_t^P\right] = \begin{cases} \hat{\alpha} & 0 \leq t < \tau \\ \Phi\left(\Phi^{-1}\left(\hat{\alpha}\right)e^{r(t-\tau)} + \sqrt{\frac{2r}{\sigma_Z^2}}\int_\tau^t e^{r(t-s)}dY_s\right) & \tau \leq t < T \end{cases} \tag{10}$$

where $\hat{\alpha} = \alpha q + (1-\alpha)(1-q)$ is the prior probability that the trader will observe a high signal. Prior to information acquisition, the investor does not trade (i.e., $\theta^U \equiv 0$), and conditional

in which the value of an asset is unobservable, but exhibits observable stochastic volatility? An alternative specification, in which there is a public signal with an error that exhibits stochastic volatility (e.g., $\Delta_t = v + \epsilon_t$, where ϵ_t exhibits stochastic volatility σ_t), necessitates the introduction of two state variables (i.e., the signal N_t and the conditional variance of v under the public information set, $\Sigma_{P,t}$), which limits tractability.

on entry / information acquisition, her strategy depends only on π and her signal, and is given by

$$\theta^h(\pi) = \frac{\sigma_Z^2 \lambda(\pi)}{\pi}, \text{ and } \quad \theta^l(\pi) = -\frac{\sigma_Z^2 \lambda(\pi)}{1-\pi}, \tag{11}$$

where θ^i, $i \in \{U, h, l\}$, denotes the trading strategy corresponding to prior to entry, informed of $S = h$, and informed of $S = l$. In this equilibrium, conditional on entry, the investor's value function is given by

$$J^h(\pi_t, \Delta_t) = \Delta_t(\xi_h - \xi_l) \int_{\pi_t}^1 \frac{1-a}{\lambda(a)} da, \text{ and } \quad J^l(\pi_t, \Delta_t) = \Delta_t(\xi_h - \xi_l) \int_0^{\pi_t} \frac{a}{\lambda(a)} da, \tag{12}$$

where $\lambda(\pi) = \sqrt{\frac{2r}{\sigma_Z^2}} \phi\left(\Phi^{-1}(1-\pi)\right)$.

Our equilibrium characterization naturally extends the equilibrium in Back and Baruch (2004) to (i) accommodate the public news process Δ_t, (ii) account for the possibility that the investor is uninformed before the acquisition / entry time τ, and (iii) account for the noisy signal about ξ. Before entry, the investor does not trade,[17] and consequently, the order flow is uninformative and the market-maker does not update his beliefs from order flow. As a result, before τ the price is $P_t = (\hat{\alpha}\xi_h + (1-\hat{\alpha})\xi_l)\Delta_t = \alpha\Delta_t$ which is a geometric Brownian motion that evolves linearly with Δ_t. Conditional on information acquisition, the trader optimally trades according to θ^S characterized in the proposition. Since $\theta^h \neq \theta^l$, the order flow provides a noisy signal about S (and therefore ξ) to the market maker. The market maker's conditional beliefs about S, given by π_t, depend on the cumulative (weighted) order flow since the acquisition date (i.e., $\int_\tau^t e^{r(t-s)} dY_s$), and consequently, so does the price P_t.

3.2 Optimal entry

Given the value function in Proposition 1, we characterize the optimal entry decision in the following result.

Proposition 2. *Given the financial market equilibrium in Proposition 1, there is a unique optimal entry strategy: the investor optimally enters the first time Δ_t hits the optimal entry*

[17] Under the posited price function, the pre-acquisition trading strategy is indeterminate. Any strategy that uses only public information earns zero expected profit under the public information set. Given such a trading strategy, it also remains optimal for the market maker to set $P_t = N_t\alpha$. Without loss of generality, we focus on the case in which the trader does not trade before time τ. In the presence of transaction costs, this would be the uniquely optimal strategy.

boundary $\Delta^* = \frac{\beta}{\beta-1}\frac{c}{K}$ *from below, where*

$$K = (\xi_h - \xi_l)\phi\left(\Phi^{-1}\left(1-\hat{\alpha}\right)\right)\sqrt{\frac{\sigma_Z^2}{2r}}, \text{ and } \beta = \tfrac{1+\sqrt{1+8r/\sigma_\Delta^2}}{2}. \tag{13}$$

Moreover, the optimal acquisition boundary Δ^* *increases in* c *and* σ_Δ*, decreases in* σ_Z *and* q*, is U-shaped in* α *(minimized at* $\alpha = 0.5$*), and is U-shaped in the expected trading horizon* $1/r$*.*

In contrast, the standard approach in the literature restricts the strategic trader to make her information choices before trading begins. In this case, she follows a naive "NPV" rule — she only acquires information if the value from becoming informed is higher than the cost i.e., $\bar{J}\left(\Delta_0\right) \geq c$. As the following corollary highlights, the resulting information acquisition decision is effectively a static one.

Corollary 1. *If the investor is restricted to choosing acquisition and entry only at* $t = 0$*, she optimally acquires information if and only if* $\Delta_0 \equiv 1 \geq \Delta_{NPV}$*, where* $\Delta_{NPV} = \frac{c}{K}$*. Moreover, the optimal acquisition boundary* Δ_{NPV} *increases in* c*, decreases in* σ_Z*, is U-shaped in* α *(minimized at* $\alpha = 0.5$*), and decreases in the expected trading horizon (i.e., increases in* r*).*

As we show in the proof of Proposition 2, the expected profit immediately prior to entry at any date t (i.e., the value function the instant before ξ is observed) is given by

$$\bar{J}\left(\Delta_t\right) \equiv \mathbb{E}_t\left[\hat{\alpha}J^h\left(\hat{\alpha},\Delta_t\right) + \left(1-\hat{\alpha}\right)J^l\left(\hat{\alpha},\Delta_t\right)\right] = K\Delta_t. \tag{14}$$

Intuitively, the value function given information acquisition at date t (i.e., $K\Delta_t$) increases in the uncertainty about v. Specifically, note that $\bar{J}\left(\Delta_t\right)$ increases linearly in $\Delta_t = H_t - L_t$, the difference between the high and low payoff values. For a fixed prior uncertainty α about whether v is high or low, an increase in Δ_t leads to an increase in uncertainty about v. Similarly, the expected value from acquiring information also increases in the prior uncertainty about v (i.e., when α is closer to 0.5). The payoff from acquisition and entry is also higher when there is more noise trading (i.e., higher σ_Z), when the signal S is more precise (i.e., q is higher), and when the information advantage is expected to be longer lived (i.e., when r is smaller).

Given this expected payoff from information acquisition and entry, the optimal time to enter is characterized by the following optimal stopping problem:

$$J^U\left(\delta\right) \equiv \sup_{\tau\in\mathcal{T}}\mathbb{E}\left[1_{\{\tau<T\}}(\bar{J}(\Delta_\tau) - c)\middle|\Delta_t = \delta\right] = \sup_{\tau\in\mathcal{T}}\mathbb{E}\left[e^{-r\tau}\left(K\Delta_\tau - c\right)^+\left|\Delta_t = \delta\right.\right]. \tag{15}$$

This problem is analogous to characterizing the optimal exercise time for a perpetual American call option.[18] Notably, the optimal entry decision exhibits delay: the investor does not enter the first instant that $K\Delta_t = c$, as would be implied by a naive, static NPV rule. The intuition for this effect is analogous to that for investment delay in a real options problem. At any point, the investor faces the following trade-off: she can enter now to begin exploiting her informational advantage against noise traders, or she can wait until uncertainty (i.e., Δ_t) is higher and her expected payoff from entry is larger. Since entry irreversibly sacrifices the ability to wait, it is optimal to enter only when doing so is sufficiently profitable to overcome this opportunity cost. Consistent with the intuition from real options problems, the option to wait is more valuable (and hence Δ^* is higher) when the volatility of the news process (i.e., σ_Δ) is higher.

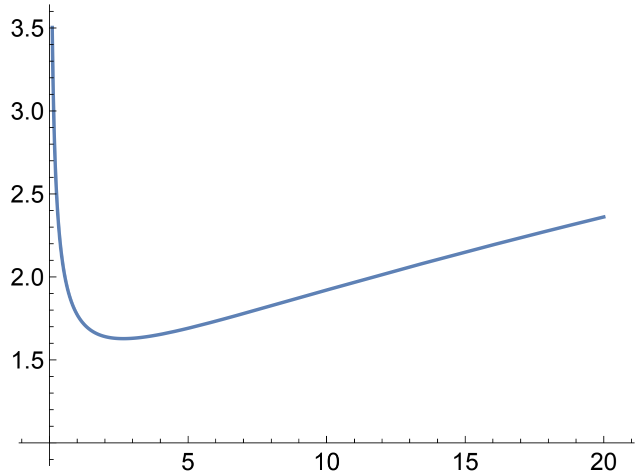
3.2.1 The effect of trading horizon on the optimal boundary

A key difference between the static entry boundary of Corollary 1 and the dynamic entry boundary of Proposition 2 is how they respond to the expected trading horizon (i.e., $1/r$). In the static case, an increase in the expected trading horizon (i.e., an decrease in r) leads to an decrease in the boundary Δ_{NPV}. This is intuitive: a longer trading horizon makes acquisition and entry more valuable since the trader can exploit her informational advantage over a longer window.

With dynamic entry, the trader also accounts for the cost of waiting to enter. Specifically, an increase in the trading horizon (i.e., an decrease in r) has two offsetting effects. First, as in the static case, it increases the value of acquisition, which pushes the boundary Δ^* downwards. Second, it decreases the cost of waiting since the likelihood that the value will be revealed before she can trade on the opportunity is lower. This pushes the boundary Δ^* upwards. As Figure 2 illustrates, this implies that the exercise boundary Δ^* is non-monotonic in the trading horizon ($1/r$). When the expected trading horizon is extremely short, the boundary is high because entry is not very valuable. Initially, the first effect dominates: an increase in the trading horizon leads to a decrease in the boundary Δ^*. However, eventually, the second effect over-comes the first — when the trading horizon is sufficiently high, further increases make waiting more attractive and so increase the boundary Δ^*.

[18] Hence, appealing to standard results, we establish that the optimal stopping time is a first hitting time for the Δ_t process and show that the given Δ^* is a solution to this problem.

Figure 2: Exercise Boundary Δ^* versus trading horizon
Unless otherwise specified, parameters are set to $\sigma_Z = \sigma_\Delta = 1$, $c = 0.25$ and $\alpha = 0.5$.



Δ^* versus trading horizon (i.e., $1/r$)

4 The likelihood and timing of entry

In this section, we characterize the likelihood that the investor optimally acquires information and enters the market before the trading opportunity disappears. We show that allowing for dynamic entry / acquisition yield novel economic predictions that are not captured by standard models with static entry/acquisition. We then characterize how the expected time of entry, conditional on entry, depends on the underlying parameters of the model. Finally, we interpret the recent evidence of limited participation and delay in new investment opportunities, like bitcoin, through the lens of our model.

4.1 Likelihood of information acquisition and entry

The likelihood of entry depends on two forces. First, the cost of doing so may be too high relative to the value of acquiring it: given c, the trader might never find it optimal to exploit the investment opportunity if uncertainty does not become sufficiently high. Second, even if the cost of entry is not too high, the asset payoff may be revealed before the investor chooses to enter the market. The following result characterizes how these effects interact to determine the likelihood of entry.

Proposition 3. *Suppose acquisition does not occur immediately (i.e., $\Delta_0 = 1 < \Delta^*$). The probability that information is acquired is $\Pr(\tau < \infty) = \left(\frac{1}{\Delta^*}\right)^\beta$. The probability is decreasing in c, increasing in σ_Z and q, symmetric and hump-shaped in α (around $\frac{1}{2}$), and hump-shaped* *in the expected trading horizon $1/r$. If the cost c is sufficiently small (i.e., $c \leq K$), the probability is decreasing in σ_Δ; otherwise, it is hump-shaped in σ_Δ.*

Accounting for the possibility that the payoff is revealed before Δ_t hits Δ^* implies that information is not always acquired. More interestingly, it reveals novel comparative statics relative to those suggested in a static entry / acquisition setting.

First, the effect of changes in expected trading horizon (changes in $1/r$) is inherited from the effect of such changes on the optimal boundary Δ^*. When the trading horizon is short, the probability of acquisition and entry is low because, conditional on acquiring, the trader has little time to profit from her informational advantage. On the other hand, when the trading horizon is long, the probability of acquisition and entry is also low because in this case the cost of waiting is sufficiently low to offset the longer trading horizon conditional on acquiring. As such, the likelihood of entry is highest for intermediate trading horizons. In contrast, when the trader is restricted to choosing entry at $t = 0$, an increase in the expected trading horizon leads to an increase in the likelihood of entry.

Second, incorporating the possibility that the payoff is revealed before the trader acquires information also changes the effect of the volatility σ_Δ on the likelihood of acquisition. Increasing the volatility σ_Δ of Δ_t has two effects on the probability of acquisition: (i) it increases the acquisition boundary (i.e., Δ^* increases in σ_Δ), which tends to reduce the probability of acquisition, and (ii) fixing the boundary, it increases the likelihood that Δ_t will hit the boundary by any given time (i.e., Δ_t is more volatile), which tends to increase the probability of acquisition. The overall effect of σ_Δ therefore depends on the relative strength of these two forces.

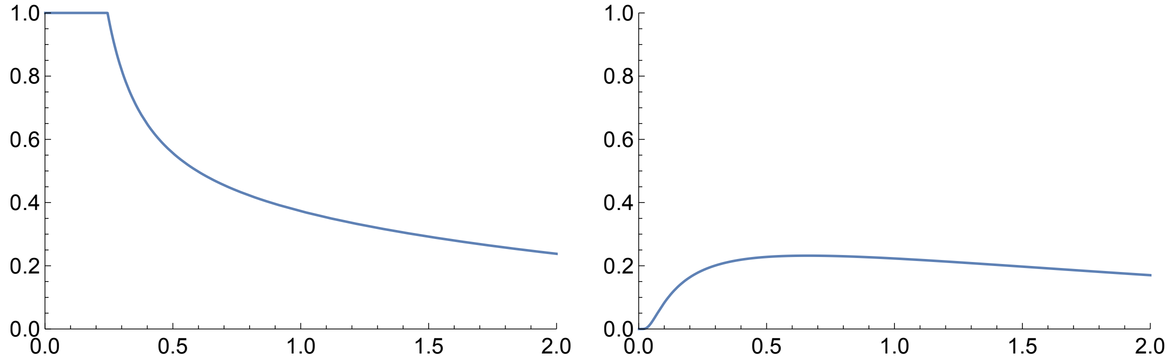
To gain some intuition for how the effect of σ_Δ depends on the acquisition cost c, note that the risky payoff is either $v = L_T = 0$ or

$$v = H_T = \Delta_T = e^{-\frac{1}{2}\sigma_\Delta^2 T + \sigma_\Delta W_T}.$$

As a result, the uncertainty about v, and consequently, the benefit of acquiring information depends on σ_Δ. When c is sufficiently low (i.e., $c \leq K$), uncertainty about v is already high enough for information acquisition and entry to be relatively valuable. Appealing to the analogy with an American call option, the option to acquire information starts in the money. In this case, an increase in volatility makes waiting more attractive (consequently, increasing the boundary Δ^*) and as a result, the likelihood of entry decreases.

In contrast, when the cost of entry c is relatively high (i.e., $c > K$), uncertainty about v is too low for entry to be valuable, i.e., the option starts out of the money. When volatility σ_Δ is low, the second effect initially dominates: an increase in σ_Δ increases the likelihood

Figure 3: Probability that information is acquired $\Pr(\tau < \infty)$ versus σ_Δ.
Unless otherwise specified, parameters are set to $\sigma_Z = 1$, $c = 0.25$ $r = 1.5$, $\alpha = 0.5$.



(a) $c = 0.2$ (i.e., $c < K$)   (b) $c = 0.25$ (i.e., $c > K$)

that Δ_t will hit the boundary, and so increases the likelihood of entry. However, once σ_Δ is sufficiently high, the effect on the boundary overwhelms this effect: further increases in σ_Δ make waiting more attractive and decrease the likelihood of entry.

Figure 3 presents an example of this non-monotonic effect of σ_Δ on the probability of information acquisition. In panel (a), the cost of acquisition and entry is sufficiently low (i.e., $c \leq K$) so that the probability of information acquisition is decreasing in σ_Δ. In panel (b), the cost is relatively high (i.e., $c > K$) so that the probability of information acquisition initially increases and then decreases in σ_Δ.

4.2 Expected time of entry

Using the distribution of τ derived in the proof of Proposition 3, we characterize the expected time of information acquisition in the following result.

Proposition 4. *Suppose acquisition does not occur immediately (i.e., $\Delta_0 < \Delta^*$). All unconditional moments of τ are infinite. The expected time of entry, conditional on entry occurring, is*

$$\mathbb{E}[\tau|\tau < \infty] = \frac{2\log(\Delta^*)}{\sigma_\Delta^2\sqrt{1 + \frac{8r}{\sigma_\Delta^2}}}. \tag{16}$$

Moreover, $\mathbb{E}[\tau|\tau < \infty]$ is increasing in c, decreasing in σ_Z and q, U-shaped in α. When the cost c is sufficiently large, the conditional expected time is decreasing in σ_Δ and increasing in expected trading horizon ($1/r$); otherwise, it may be non-monotonic in either parameter.

Since information acquisition / entry does not always occur, the *unconditional* moments of τ are infinite. However, conditional on entry, the expected time of entry is characterized by expression (16). To gain some intuition, consider the numerator and denominator of the expression in (16) separately. Intuitively, an increase in the acquisition boundary Δ^* implies there is more delay in entry / acquisition since it takes longer for Δ_t to cross Δ^* — this is reflected by the numerator. Moreover, for a fixed Δ^* and conditional on entry, a higher volatility (i.e., higher σ_Δ) implies entry must have occurred faster (on average) — a more volatile Δ_t process would take less time to cross the threshold. Similarly, conditional on entry having happened, a shorter trading horizon (i.e., higher r) implies entry must have occurred earlier since it occurred before the value is publicly revealed. These conditional effects are reflected in the denominator of (16).

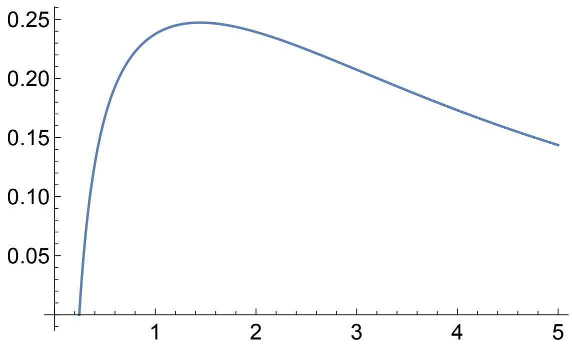
The comparative statics with respect to c, σ_Z, q and α are intuitive and inherited from the dependence on Δ^*. Recall that delaying acquisition / entry becomes more attractive (i.e., Δ^* increases) with a decrease in noise trading (σ_Z), signal precision (q), or prior uncertainty about v (i.e., $\alpha(1-\alpha)$), and with an increase in the cost of acquisition / entry (c). As a result, the conditional expected time to acquisition increases with each of these changes.

As Figure 4 illustrates, the effects of volatility (σ_Δ) and trading horizon ($1/r$) are more nuanced since they impact both the numerator and the denominator of (16). When the cost c is sufficiently large, the denominator channel dominates: the conditional expected time of entry decreases with volatility σ_Δ and increases with the trading horizon $1/r$ — see panels (b) and (d), respectively. However, when the cost c is relatively small, the effects through the Δ^* term in the numerator interact with those from through the denominator to generate non-monotonic effects, as illustrated in panels (a) and (c). Notably, when both the cost of entry (c) and volatility (σ_Δ) are sufficiently small, the option to enter is deep in the money and immediate entry is optimal — in this case, the conditional expected time is zero. For low levels of σ_Δ, an increase in volatility can lead to a higher acquisition boundary, and consequently, a higher conditional expected time (e.g., see panel (a)). Similarly, when the trading horizon is short (i.e., r is large), the conditional expected time initially increases, then decreases and then increases in $1/r$.

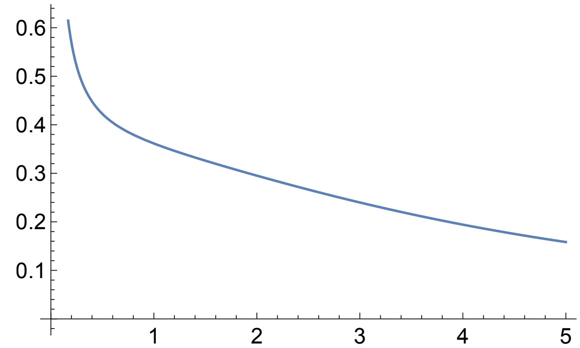
4.3 Limited participation and delayed entry

The results of this section help interpret the evidence of limited participation and delayed entry into new opportunities like bitcoin. Even though bitcoin has been available for trade since 2010, there is wide and persistent disagreement about its true value. Together with the observation that the volatility of bitcoin is high and time-varying, this suggests that bitcoin is characterized by (i) a long expected trading horizon, and (ii) high volatility of

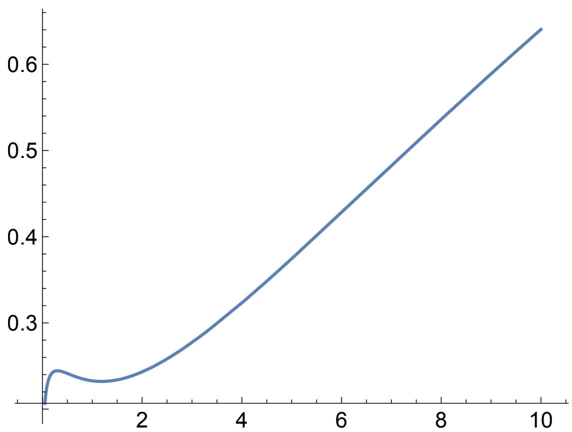
Figure 4: Conditional expected time of entry $\mathbb{E}[\tau|\tau < \infty]$ vs. σ_Δ and $1/r$. Unless otherwise specified, parameters are set to $\sigma_Z = 1$, $\sigma_\Delta = 1$, $r = 1.5$, $\alpha = 0.5$.



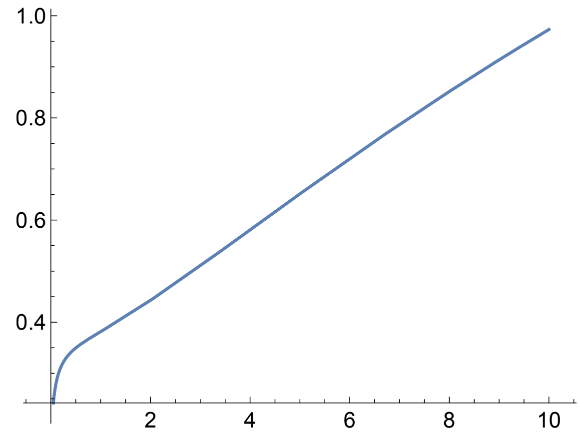
(a) $\mathbb{E}[\tau|\tau < \infty]$ vs. σ_Δ for $c = 0.2$



(b) $\mathbb{E}[\tau|\tau < \infty]$ vs. σ_Δ for $c = 0.25$



(c) $\mathbb{E}[\tau|\tau < \infty]$ vs. $1/r$ for $c = 0.2$



(d) $\mathbb{E}[\tau|\tau < \infty]$ vs. $1/r$ for $c = 0.25$

uncertainty/news. As such, Proposition 3 implies a low likelihood of entry, which is consistent with the observation that many large institutional investors have chosen to remain out of the market. Proposition 4 suggests that a long trading horizon is also consistent with more delay, conditional on entry, which appears consistent with the empirical evidence. Finally, to the extent that smaller, specialized firms have lower cost of entry / acquisition, the above predictions are consistent with the empirical observation that there has been staggered entry by different types of investors, led by smaller, more specialized firms in the earlier part of the decade.

Our model also provides insight on the entry behavior of institutional investors into technology stocks in the 1990s, and how it differs from that into bitcoin. Griffin et al. (2011) document that institutions such as hedge funds largely delayed entry until late 1999. Furthermore, they show that individual investors (noise traders) also entered the market heavily in the later years of the decade. It is plausible that the trading horizon of technology stocks in the 90s was neither extremely short, nor extremely long.[19] Also, uncertainty about the potential value of tech stocks while substantial, was arguably less volatile than the uncertainty investors face about bitcoin. Relative to cryptocurrencies, our model would suggest that tech stocks should exhibit less delay and greater entry by sophisticated investors, especially during the period with increased noise trading.

5 Precision choice and entry dynamics

In this section, we allow the institutional investor to choose not only the timing of information acquisition and entry, but also the precision of her signal. Specifically, suppose at the time of entry, the investor can either acquire a signal with high precision q_h at cost c_h or acquire one with a with low precision q_l at cost c_l, where $\frac{1}{2} < q_l < q_h \leq 1$ and $0 < c_l \leq c_h$. As such, her decision depends not only on the tradeoff between cost and precision, but also on the timing of entry: she can acquire the cheaper, less precise signal but begin trading earlier or wait to acquire the more informative, more expensive signal.

As before, we assume that the market maker can detect the timing of entry. To ensure tractability, we further assume that the market maker can detect the precision choice of the strategic trader. As discussed earlier, entry into a new market involves hiring of analysts and traders. The reputations and track records of such individuals are likely to convey information (albeit noisy) about their quality.[20]The assumption ensures that the financial market equilibrium in the current setting is given by the characterization in Proposition 1, given the optimal choice of precision $q^* \in \{q_l, q_h\}$. In what follows, let K_l and K_h denote the the value function coefficient characterized by (13) that correspond to the precision choice of q_l and q_h, respectively. We begin with an observation about the static acquisition / entry setting.

Lemma 1. *If the investor is restricted to choosing acquisition and entry only at $t = 0$, the trader acquires the high precision signal iff $\frac{c_h}{K_h} \leq \min\left\{1, \frac{c_l}{K_l}\right\}$ and acquires the low precision signal iff $\frac{c_l}{K_l} \leq \min\left\{1, \frac{c_h}{K_h}\right\}$.*

When the acquisition / entry is a static decision, the investor optimally chooses the signal

[19]The first graphical web browser, Mosaic (the precursor to Netscape) was released in 1993 ("April 22, 1993: Mosaic Browser Lights Up Web With Color, Creativity", Michael Calore, Wired, April 22, 2010, https://www.wired.com/2010/04/0422mosaic-web-browser/), while the abrupt decline of technology stocks began in March 2000 (Griffin et al., 2011).

[20]For instance see "Goldman Sachs just made its first crypto hire to explore a potential bitcoin trading desk" by Frank Chaparro in Business Insider on April 23, 2018 (http://www.businessinsider.com/goldman-sachs-bitcoin-trading-desk-new-hire-2018-4), which discusses the importance of Goldman Sachs' hire of Justin Schmidt to explore creating a bitcoin trading desk.

with the highest "bang for buck," or equivalently, the lowest cost-benefit ratio. In contrast, the following result characterizes the optimal choice of the investor with dynamic acquisition and entry.

Proposition 5. *Let $\bar{\Delta} \equiv \frac{c_h - c_l}{K_h - K_l}$, $\Delta_l^* = \frac{\beta}{\beta-1}\frac{c_l}{K_l}$ and $\Delta_h^* = \frac{\beta}{\beta-1}\frac{c_h}{K_h}$, where β, K_h and K_l are characterized as in (13). Given the financial market equilibrium in Proposition 1 , there is a unique optimal entry strategy: the investor optimally enters the first time Δ_t hits the optimal entry boundary Δ^* from below, where*

$$\Delta^* = \begin{cases} \Delta_l^* & \text{if } \Delta_l^* < \bar{\Delta} \text{ and } K_h^\beta c_h^{1-\beta} \leq K_l^\beta c_l^{1-\beta} \\ \Delta_h^* & \text{otherwise} \end{cases}. \tag{17}$$

The equilibrium characterized by Proposition 5 is intuitive. As in the equilibrium of the benchmark model, the optimal acquisition time is given by the first time Δ_t hits the optimal acquisition boundary Δ^*. Moreover, the optimal boundary corresponds to either the boundary when only the high precision signal is available (i.e., $\Delta_h = \frac{\beta}{\beta-1}\frac{c_h}{K_h}$), or when only the low precision signal is available (i.e., $\Delta_l = \frac{\beta}{\beta-1}\frac{c_l}{K_l}$). Finally, note that $\bar{\Delta}$ is the threshold at which the investor is indifferent between acquiring the high precision and low precision signals i.e.,

$$K_h\bar{\Delta} - c_h = K_l\bar{\Delta} - c_l. \tag{18}$$

It is straightforward to show that if $\frac{c_h}{K_h} < \frac{c_l}{K_l}$ then the conditions for a low-precision signal are never met. Hence, the investor prefers the low precision signal only when (i) the cost-benefit ratio for the low precision signal is better (i.e., $\frac{c_l}{K_l} < \frac{c_h}{K_h}$), and (ii) the low precision leads to a sufficiently high value function that waiting and later acquiring a high-precision signal is not optimal. In contrast, the investor prefers the high precision signal if either the cost-benefit ratio for the high precision signal is better i.e., $\frac{c_l}{K_l} \geq \frac{c_h}{K_h}$, or if the the effective cost of waiting to hit Δ_h^* is not too high.

Importantly, unlike the case where acquisition / entry is a static decision, the optimal choice of precision does not only depend on the relative costs (i.e., c_l vs c_h) and precisions of the two signals (i.e., q_l and q_h, via K_l and K_h). Instead, it also depends on the volatility of the news process (i.e., σ_Δ) and the expected trading horizon (i.e., $1/r$) though their effect on β and therefore Δ_l^*. In fact, this leads to the following striking result.

Corollary 2. *Fixing any pair of precisions and costs (i.e., for fixed $\{q_l, c_l\}$ and $\{q_h, c_h\}$), the investor always prefers the high precision signal if either the volatility σ_Δ of the news process is sufficiently high, or if the expected trading horizon $1/r$ is sufficiently long (i.e., r is sufficiently small).*

The result implies that when news volatility is sufficiently high or the trading horizon is sufficiently long, the investor optimally chooses the high precision signal, irrespective of its relative cost-benefit ratio. If news volatility is sufficiently high, or the trading horizon is sufficiently long, the cost of waiting is relatively low. Since acquiring a low-precision signal irreversibly forgoes the ability to acquire a high-precision signal, it is never optimal to acquire a low-precision signal, regardless of its direct cost-benefit ratio.

The result suggests that, under such circumstances, larger and more sophisticated institutions that have access to better expertise (and hence more "precise signals") are likely to wait longer to enter new investment opportunities. This appears consistent with the observed pattern of entry into bitcoin: while smaller and (arguably) less sophisticated firms have entered the market, larger and more sophisticated firms appear to be taking a wait and see approach.

6 Conclusions

When do traders choose to enter into new markets and exploit new investment opportunities? To study this question, we develop a strategic trading model in which a trader can endogenously choose *when* to acquire information in response to the evolution of a public signal. While a number of papers have studied entry/information acquisition in financial markets, this work typically assumes that investors make a one-shot decision at the time that the market opens. As such, these models are not well-suited for studying the optimal timing of information acquisition and entry.

We show that when a trader can optimally choose when to enter, there is generally delay beyond what is prescribed by a naive "NPV" rule. Furthermore, allowing for dynamic entry/acquisition provides qualitatively novel economic implications relative to a model of "static entry". In particular, we derive new predictions for how the likelihood and timing of entry, as well as optimal precision choice, depend on news volatility and the expected trading horizon. While our model is stylized, its predictions are broadly consistent with the entry behavior of large asset managers into cryptocurrencies in recent years, which is characterized by minimal initial participation, followed by an abrupt spike starting in 2017. As discussed above, our results also shed light on the entry behavior of institutions into technology stocks in the late 1990s.

More broadly, our analysis suggests that key features of the standard strategic trading framework may be difficult to reconcile with the dynamic entry/acquisition decisions of large traders. Exploring the robustness of our results to various assumptions is a natural next step. While entry by large institutions into new opportunities is likely to be observed by other market participants, information acquisition by existing investors may be more difficult to detect. In related work (Banerjee and Breon-Drish, 2018), we study an alternative setting in which an investor's information acquisition is not detected by the market maker, but must instead be filtered from order flow. We show that unobservable acquisition can lead to market breakdown.

Another important extension would be to consider competition among multiple strategic traders. It is natural to expect that competition will tend to reduce delay, though we conjecture that as long as traders' private information is not perfectly correlated, the key qualitative results on delay would remain. It would also be interesting to study the robustness of our results to different information acquisition technologies (e.g., a continuously optimized flow of private information, or a sequence of "lumpy" signals), and to understand the effect of endogenous public news (e.g., in the form of strategic disclosure by firms or regulators). We hope to explore these extensions in future work.

References

Avdis, E., 2016. Information tradeoffs in dynamic financial markets. *forthcoming in* Journal of Financial Economics. 1

Back, K., 1992. Insider trading in continuous time. Review of financial Studies 5 (3), 387–409. 12

Back, K., Baruch, S., 2004. Information in securities markets: Kyle meets glosten and milgrom. Econometrica 72 (2), 433–465. 1, 2, 2, 2.1, 3.1, A, A, A, A

Back, K., Pedersen, H., 1998. Long-lived information and intraday patterns. Journal of Financial Markets 1 (3), 385–402. 1

Banerjee, S., Breon-Drish, B., 2018. Strategic trading, unobservable information acquisition and market breakdown. Working paper. 6

Bond, P., Edmans, A., Goldstein, I., 2012. The real effects of financial markets. Annu. Rev. Financ. Econ. 4 (1), 339–360. 1

Caldentey, R., Stacchetti, E., January 2010. Insider trading with a random deadline. Econometrica 78 (1), 245–283. 1, 2

Duffie, D., 2010. Presidential address: Asset price dynamics with slow-moving capital. The Journal of finance 65 (4), 1237–1267. 1

Dugast, J., Foucault, T., 2017. Data abundance and asset price informativeness. 1, 11

Foster, F. D., Viswanathan, S., 1993. The effect of public information and competition on trading volume and price volatility. Review of Financial Studies 6 (1), 23–56. 1

Griffin, J. M., Harris, J. H., Shu, T., Topaloglu, S., August 2011. Who drove and burst the tech bubble? Journal of Finance 66 (4), 1251–1290. 1, 4.3, 19

Grossman, S. J., Stiglitz, J. E., 1980. On the impossibility of informationally efficient markets. American Economic Review 70 (3), 393–408. 1

Holden, C. W., Subrahmanyam, A., January 2002. News events, information acquisition, and serial correlation. Journal of Business 75 (1), 1–32. 1

Huang, S., Yueshen, B. Z., 2018. Speed acquisition. 1, 11

Karatzas, I., Shreve, S. E., 1998. Brownian Motion and Stochastic Calculus, 2nd Edition. Vol. 113 of Graduate Texts in Mathematics. Springer, New York. A

Kendall, C., 2017. The time cost of information in financial markets. working paper. 1, 11

Kyle, A. S., 1985. Continuous auctions and insider trading. Econometrica, 1315–1335. 1, 2

Mendelson, H., Tunca, T. I., 2004. Strategic trading, liquidity, and information acquisition. Review of Financial Studies 17 (2), 295–337. 1

Peskir, G., Shiryaev, A., 2006. Optimal Stopping and Free-Boundary Problems. Lectures in Mathematics, ETH Zürich. Birkhäuser, Boston. A

Protter, P. E., 2003. Stochastic Integration and Differential Equations, 2nd Edition. Vol. 21 of Applications of Mathematics. Springer, New York. A

Veldkamp, L. L., June 2006. Media frenzies in markets for financial information. American Economic Review 96 (3), 577–601. 1

A Proofs

Proof of Proposition 1. To establish the equilibrium in the Proposition, we need to show: (i) the proposed price function is rational, and (ii) the informed trader's strategy is optimal. Fix any $\tau \in \mathcal{T}$.

Rationality of pricing function

Consider the set $\{t : t < \tau\}$ on which the trader has not acquired information. Then, because $\{\Delta_t\}$, $\{Z_t\}$ and ξ are independent, and under the proposed trading strategy $Y_t = Z_t$ for $t < \tau$, it is immediate that

$$\mathbb{E}[\xi\Delta_T|\mathcal{F}_t^P] = \mathbb{E}[\xi|\mathcal{F}_t^P]\mathbb{E}[\Delta_T|\mathcal{F}_t^P] = \alpha\mathbb{E}[\Delta_T|\mathcal{F}_t^P].$$

Since T is almost surely finite and is independent of the process Δ_t we have $\mathbb{E}[\Delta_T|\mathcal{F}_t^P] = \Delta_t$, and so $\mathbb{E}[\xi\Delta_T|\mathcal{F}_t^P] = \alpha\Delta_t$.

Now, consider the set $\{t : \tau \le t < T\}$ on which the trader has entered and the asset payoff has not yet occurred. Up to the addition of the news process and the noisy signal, the problem now resembles that considered in Back and Baruch (2004), and we can adapt the proof offered there. Specifically, consider the updating rule from Back and Baruch (2004), adapted for the fact that the signal is acquired at time τ,

$$d\pi_t = \lambda(\pi)dY_t, \quad \pi_\tau = \hat{\alpha},$$

where $\lambda(\pi)$ is given in the statement of the Proposition. (Later we will show that this pricing rule can be written in the explicit form in eq. (10).) Note that the proposed trading strategy depends only on S and π, the process π depends only on the order flow, and $\{\Delta_t\}$ is independent of S and $\{Z_t\}$, so $(S, \{\pi_t\})$ is conditionally independent of $\{\Delta_t\}$, and therefore

$$\mathbb{E}[\xi\Delta_T|\mathcal{F}_t^P] = \mathbb{E}[\xi|\mathcal{F}_t^P]\mathbb{E}[\Delta_T|\mathcal{F}_t^P] = \mathbb{P}[\xi = 1|\mathcal{F}_t^P]\mathbb{E}[\Delta_T|\mathcal{F}_t^P] = \mathbb{P}[\xi = 1|\{Y_s\}_{s\le t}]\Delta_t = (\xi_h\pi_t + \xi_l(1-\pi_t))\Delta_t,$$

where the next-to-last equality follows since $\mathbb{E}[\Delta_T|\mathcal{F}_t^P] = \Delta_t$. Furthermore, since $Y_t = Z_t$ for $t < \tau$ under the proposed trading strategy and S is independent of $\{Z_t\}$ it follows that $\mathbb{P}[\xi = 1|\{Y_s\}_{s\le t}] = \mathbb{P}[\xi = 1|\{Y_s\}_{\tau\le s\le t}]$.

Recall that as of time τ, the informed trader begins trading according to the strategy $\theta^S(\pi)$ and the order flow becomes informative. The market maker's conditional expectation is simply equal to her prior $\hat{\alpha}$ since before this time only noise traders have been active. It follows that starting at time τ the market maker's filtering problem becomes identical to that of the market maker in Back and Baruch (2004), modified to account for the fact that she is filtering one of two signal realizations rather than ξ itself. Hence, their Theorem 1 implies that for $t \geq \tau$ the pricing rule

$$d\pi_t = \lambda(\pi) dY_t, \qquad \pi_\tau = \hat{\alpha},$$

satisfies $\pi_t = \mathbb{P}[\xi = 1 | \{Y_s\}_{s \geq \tau}]$.

To complete the proof of the rationality of the proposed price, it suffices to show that the explicit form of $\pi(\cdot)$ for $\tau \leq t < T$ in eq. (10) satisfies $d\pi_t = \lambda(\pi) dY_t$. Applying Ito's Lemma to the function $f(\pi) = \sqrt{\frac{\sigma_Z^2}{2r}}\Phi^{-1}(\pi)$ to the above process for π_t gives

$$\begin{aligned} df(\pi_t) &= \frac{1}{2}\sigma_Z^2 \lambda^2(\pi_t) \frac{\frac{2r}{\sigma_Z^2} f(\pi_t)}{\lambda^2(\pi_t)}\, dt + \frac{1}{\lambda(\pi_t)} \lambda(\pi_t) dY_t \\ &= r f(\pi_t)\, dt + dY_t. \end{aligned}$$

Now applying Ito's lemma to the function $e^{-rt} f(\pi_t)$ and integrating allows one to express

$$f(\pi_t) = f(\pi_\tau) e^{r(t-\tau)} + \int_\tau^t e^{r(t-s)} dY_s.$$

Note that $f(\pi_\tau) = \sqrt{\frac{\sigma_Z^2}{2r}}\Phi^{-1}(\hat{\alpha})$, so returning to the explicit form of the function $f(\pi)$ and inverting it follows that

$$\pi_t = \Phi\left(\Phi^{-1}\left(\hat{\alpha}\right) e^{r(t-\tau)} + \sqrt{\frac{2r}{\sigma_Z^2}} \int_\tau^t e^{r(t-s)} dY_s\right).$$

Optimality of trading strategy

Next, we demonstrate the optimality of the proposed trading strategy, taking as given the acquisition time τ. This analysis closely follows the proof in Back and Baruch (2004). Define $V(\pi) \equiv (\xi_h - \xi_l) \int_\pi^1 \frac{1-a}{\lambda(a)} da$ and consider the proposed post-acquisition value function for the case $S = h$ (the case for $S = l$ is analogous)

$$J^h\left(\pi_t, \Delta_t\right) = \Delta_t V(\pi_t).$$

We begin by showing that the given J characterizes the value function for $t \geq \tau$. Consider $\{t : \tau \leq t < T\}$ and suppose $S = h$. Direct calculation on the function V yields

$$V' = (\xi_h - \xi_l) \frac{\pi - 1}{\lambda} \tag{19}$$

$$rV = \frac{1}{2}\sigma_Z^2 \lambda^2 V'', \tag{20}$$

which coincides with eq. (1.15) and (1.16) in Back and Baruch (2004).

Let θ_t denote an arbitrary admissible trading strategy. Following Back and Baruch (2004), let $\hat{\pi}_t$ denote the process defined by $\hat{\pi}_s = \hat{\alpha}$ for $s \le \tau$ and $d\hat{\pi}_t = \lambda(\hat{\pi})dY_t$ for $t > \tau$ and $0 < \hat{\pi}_t < 1$, with Y_t generated when the trader follows the given arbitrary trading strategy. In order to condense notation, in this section, we denote $\mathbb{E}[\cdot|\mathcal{F}_t^P] = \mathbb{E}_t[\cdot]$. Since θ is admissible, we know that

$$\mathbb{E}_\tau \left[\int_\tau^T \Delta_u (1-\pi_u)\theta_u^- du \right] = \mathbb{E}_\tau \left[\int_\tau^\infty e^{-r(u-\tau)} \Delta_u (1-\hat{\pi}_u)\theta_u^- du \right] < \infty,$$

from which it follows that

$$\int_\tau^\infty e^{-r(u-\tau)} \Delta_u (1-\hat{\pi}_u)\theta_u^- du < \infty$$

almost surely, and therefore that the integral

$$\int_\tau^\infty e^{-r(u-\tau)} \Delta_u (1-\hat{\pi}_u)\theta_u du$$

is well-defined, though is possibly infinite.

Let $\hat{T} = \inf\{t \ge \tau : \hat{\pi} \in \{0,1\}\}$. Applying Ito's lemma to $e^{-r(t-\tau)}J$ yields

$$\begin{aligned}
&e^{-r(t\wedge\hat{T}-\tau)} J^h(\hat{\pi}_{t\wedge\hat{T}}, \Delta_{t\wedge\hat{T}}) - J^h(\hat{\pi}_\tau, \Delta_\tau) \\
&\quad = \int_\tau^{t\wedge\hat{T}} e^{-r(u-\tau)} \Delta \left(-rV(\hat{\pi}_u) + \lambda\theta V'(\hat{\pi}_u) + \frac{1}{2}\sigma_Z^2\lambda^2 V''(\hat{\pi}_u) \right) du \\
&\qquad\qquad + \sigma_Z \int_\tau^{t\wedge\hat{T}} e^{-r(u-\tau)} \Delta\lambda V'(\hat{\pi}_u) dW_{Zu} + \sigma_N \int_\tau^{t\wedge\hat{T}} e^{-r(u-\tau)} \Delta V(\hat{\pi}_u) dW_{\Delta u} \\
&\quad = -(\xi_h - \xi_l) \int_\tau^{t\wedge\hat{T}} e^{-r(u-\tau)} \Delta_u \theta_u (1-\hat{\pi}_u) du - \sigma_Z(\xi_h - \xi_l) \int_\tau^{t\wedge\hat{T}} e^{-r(u-\tau)} \Delta_u (1-\hat{\pi}_u) dW_{Zu} \\
&\qquad\qquad + \sigma_\Delta \int_\tau^{t\wedge\hat{T}} e^{-r(u-\tau)} \Delta_u V(\hat{\pi}_u) dW_{\Delta u}
\end{aligned} \tag{21}$$

where the last equality uses eq. (19) and (20). Since $V \geq 0$, the above implies

$$(\xi_h - \xi_l) \int_\tau^{t \wedge \hat{T}} e^{-r(u-\tau)} \Delta_u \theta_u (1 - \hat{\pi}_u) du \leq \Delta_\tau V(\alpha) + x(t), \tag{22}$$

where we define $x(t) = \sigma_\Delta \int_\tau^{t \wedge \hat{T}} e^{-r(u-\tau)} \Delta_u V(\hat{\pi}_u) dW_{\Delta u} - \sigma_Z (\xi_h - \xi_l) \int_\tau^{t \wedge \hat{T}} e^{-r(u-\tau)} \Delta_u (1 - \hat{\pi}_u) dW_{Zu}$. The integrands in the stochastic integrals are locally bounded and hence the integrals are local martingales (Thm. 29, Ch. 4, Protter (2003)). It follows that $x(t)$ is itself a local martingale (Thm. 48, Ch. 1, Protter (2003)).

Let $\hat{\tau}_n$ be a localizing sequence of stopping times for $x(t)$. That is, $\hat{\tau}_{n+1} \geq \hat{\tau}_n$, $\hat{\tau}_n \to \infty$, and $x(t \wedge \hat{\tau}_n)$ is a martingale for each n. Because $x(t)$ is a local martingale such a sequence exists (e.g., because $x(t)$ is continuous we can take $\hat{\tau}_n = \inf\{t : |x(t)| \geq n\}$). Further considering the sequence $n \wedge \hat{\tau}_n$, eq. (22) implies

$$(\xi_h - \xi_l) \int_\tau^{n \wedge \hat{\tau}_n \wedge \hat{T}} e^{-r(u-\tau)} \Delta_u \theta_u (1 - \hat{\pi}_u) du \leq \Delta_\tau V(\alpha) + x(n \wedge \hat{\tau}_n).$$

Applying Fatou's lemma,[21] along with this inequality, yields

$$\begin{aligned}
\mathbb{E}_\tau \left[(\xi_h - \xi_l) \int_\tau^{\hat{T}} e^{-r(u-\tau)} \Delta_u \theta_u (1 - \hat{\pi}_u) du \right] &\leq \liminf_{n \to \infty} \mathbb{E}_\tau \left[(\xi_h - \xi_l) \int_\tau^{n \wedge \hat{\tau}_n \wedge \hat{T}} e^{-r(u-\tau)} \Delta_u \theta_u (1 - \hat{\pi}_u) du \right] \\
&\leq \Delta_\tau V(\alpha) + \liminf_{n \to \infty} \mathbb{E}_\tau \left[x(n \wedge \hat{\tau}_n) \right] \\
&\leq \Delta_\tau V(\alpha).
\end{aligned}$$

Note that for $\hat{T} < \infty$ we have $\hat{\pi}_{\hat{T}} = 1$ since $\hat{\pi}_{\hat{T}} = 0$ would imply a violation of the admissibility condition. To establish this, note that eq. (21) implies

$$-\mathbb{E}_\tau \left[(\xi_h - \xi_l) \int_\tau^{n \wedge \hat{\tau}_n \wedge \hat{T}} e^{-r(u-\tau)} \Delta_u \theta_u (1 - \hat{\pi}_u) du \right] = \mathbb{E}_\tau \left[e^{-r(t \wedge \hat{T} - \tau)} \Delta_{t \wedge \hat{T}} V(\hat{\pi}_{t \wedge \hat{T}}) - \Delta_\tau V(\alpha) \right] - J^h(\hat{\pi}_\tau, \Delta_\tau),$$

and therefore

$$-\mathbb{E}_\tau \left[(\xi_h - \xi_l) \int_\tau^{\hat{T}} e^{-r(u-\tau)} \Delta_u \theta_u (1 - \hat{\pi}_u) du \right]$$

[21] The typical formulation of Fatou's Lemma requires that the integrands f_n be weakly positive. However, if f_n^- is bounded above by an integrable function g, considering $f_n + g$ in Fatou's lemma delivers the result. Here, due to the admissibility condition we can take $g = N_u(1 - p_u)\theta_u^-$.

$$
\begin{aligned}
&\geq \limsup_{n\to\infty} \mathbb{E}_\tau \left[-(\xi_h - \xi_l) \int_\tau^{n\wedge\hat{\tau}_n\wedge\hat{T}} e^{-r(u-\tau)} \Delta_u \theta_u (1-\hat{\pi}_u) du \right] \\
&= \limsup_{n\to\infty} \mathbb{E}_\tau \left[e^{-r(n\wedge\hat{\tau}_n\wedge\hat{T}-\tau)} \Delta_{n\wedge\hat{\tau}_n\wedge\hat{T}} V(\hat{\pi}_{n\wedge\hat{\tau}_n\wedge\hat{T}}) - \Delta_\tau V(\alpha) \right] - J^h(\hat{\pi}_\tau, \Delta_\tau) \\
&\geq \mathbb{E}_\tau \left[e^{-r(\hat{T}-\tau)} \Delta_{\hat{T}} V(\hat{\pi}_{\hat{T}}) \right] - J^h(\hat{\pi}_\tau, \Delta_\tau) \\
&= \infty,
\end{aligned}
$$

where the first line applies the 'reverse' Fatou's Lemma, the second line uses the equality in the previous displayed equation, the third line applies Fatou's Lemma and the final line follows because $V(0) = \infty$. Furthermore, $\hat{\pi}_u = \hat{\pi}_{\hat{T}} = 1$ for all $u \geq \hat{T}$ since 1 is an absorbing state. It follows that

$$
\mathbb{E}_\tau \left[(\xi_h - \xi_l) \int_\tau^\infty e^{-r(u-\tau)} \Delta_u \theta_u (1-\hat{\pi}_u) du \right] = \mathbb{E}_\tau \left[(\xi_h - \xi_l) \int_\tau^{\hat{T}} e^{-r(u-\tau)} \Delta_u \theta_u (1-\hat{\pi}_u) du \right] \leq \Delta_\tau V(\alpha). \tag{23}
$$

Furthermore, this inequality is trivially true for $\hat{T} = \infty$, so it holds regardless of the behavior of $\hat{T}$. It follows that

$$
\Delta_\tau V(\alpha) \geq \mathbb{E}_\tau \left[(\xi_h - \xi_l) \int_\tau^\infty e^{-r(u-\tau)} \Delta_u \theta_u (1-\hat{\pi}_u) du \right] = \mathbb{E}_\tau \left[(\xi_h - \xi_l) \int_\tau^T \Delta_u \theta_u (1-\pi_u) du \right],
$$

since $\hat{\pi} = \pi$ for $t \leq T$. Hence $\Delta_\tau V(\alpha)$ is an upper bound on the post-acquisition value function.

To establish the optimality of the trader's post-acquisition strategy and the expression for the value function, it remains to show that the expected profits generated by the strategy attain the bound $\Delta_\tau V(\alpha)$. (We show below that the trader's overall trading strategy is admissible.) Compute the trader's expected profit at time τ. We have

$$
\begin{aligned}
\mathbb{E}_\tau \left[(\xi_h - \xi_l) \int_\tau^T \theta^h(\pi_u) \Delta_u (1-\pi_u)\, du \right] &= \int_\tau^\infty (\xi_h - \xi_l) \mathbb{E}_\tau \left[\mathbf{1}_{\{t\leq T\}} \theta^h(\pi_u) \Delta_u (1-\pi_u) \right] du \\
&= \int_\tau^\infty (\xi_h - \xi_l) \mathbb{E}_\tau[\Delta_u] \mathbb{E}_\tau \left[\mathbf{1}_{\{t\leq T\}} \theta^h(\pi_u)(1-\pi_u) \right] du \\
&= (\xi_h - \xi_l) \Delta_\tau \int_\tau^\infty \mathbb{E}_\tau \left[\mathbf{1}_{\{t\leq T\}} \theta^h(\pi_u)(1-\pi_u) \right] du \\
&= (\xi_h - \xi_l) \Delta_\tau \mathbb{E}_\tau \left[\int_\tau^T \theta^h(\pi_u)(1-\pi_u)\, du \right],
\end{aligned}
$$

where the first equality applies Fubini's theorem which is permissible because the integrand is positive, the second equality uses the fact that N is independent of T and $\{p_u\}$, the next-to-last equality follows because N is a martingale, and the final equality applies Fubini's theorem again. The proof in Back and Baruch (2004) establishes that under the given trading strategy and pricing rule, $V(\alpha) = \mathbb{E}_\tau\left[(\xi_h - \xi_l)\int_\tau^T \theta^h(\pi_u)(1-\pi_u)\,du\right]$. Hence,

$$\Delta_\tau V(\alpha) = \mathbb{E}_\tau\left[(\xi_h - \xi_l)\int_\tau^T \theta^h(\pi_u)\Delta_u(1-\pi_u)\,du\right],$$

which establishes the optimality of the post-acquisition trading strategy.

Let $J^U(\Delta)$ denote the pre-acquisition value function (i.e., the value function for a trader prior to information acquisition and entry). Note that because $\pi \equiv \hat{\alpha}$ for $t < \tau$, J^U effectively depends only on the news process in this case. We need to characterize this function and establish that the overall posited trading strategy, involving no trade prior to acquisition, is optimal. Under the given trading strategy, we have

$$\begin{aligned} J^U(\Delta) &= \mathbb{E}\left[\mathbf{1}_{\{\tau<T\}}(\xi_h - \xi_l)\int_\tau^T \theta^S(\pi_u)\Delta_u(\mathbf{1}_{S=h} - \pi_u)\,du\right] \\ &= \mathbb{E}\left[\mathbf{1}_{\{\tau<T\}}J^S(\pi_\tau, \Delta_\tau)\right] \end{aligned}$$

Let $\check{\theta}$ be any admissible trading strategy that is adapted to $\mathcal{F}_t^P$ and $\hat{\theta}$ any admissible strategy that is adapted to $\mathcal{F}_t^I$. Then $\theta = \mathbf{1}_{\{t<\tau\}}\check{\theta} + \mathbf{1}_{\{t\geq\tau\}}\hat{\theta}$ is an arbitrary admissible strategy that obeys the restriction that the investor does not observe ξ until time τ. The expected profits from following this strategy are

$$\begin{aligned} &\mathbb{E}_0\left[\mathbf{1}_{\{\tau<T\}}\int_0^\tau \check{\theta}_u\Delta_u(\xi-\alpha)\,du + \mathbf{1}_{\{\tau<T\}}(\xi_h - \xi_l)\int_\tau^T \hat{\theta}_u\Delta_u(\mathbf{1}_{S=h} - \pi_u)\,du + \mathbf{1}_{\{\tau\geq T\}}\int_0^T \check{\theta}_u\Delta_u(\xi-\alpha)\,du\right] \\ &= \mathbb{E}_0\left[\mathbf{1}_{\{\tau<T\}}(\xi_h - \xi_l)\int_\tau^T \hat{\theta}_u\Delta_u(\xi - \pi_u)\,du\right] \\ &= \mathbb{E}_0\left[\mathbf{1}_{\{\tau<T\}}\mathbb{E}\left[(\xi_h - \xi_l)\int_\tau^T \hat{\theta}_u\Delta_u(\xi - \pi_u)\,du|\mathcal{F}_\tau^I\right]\right] \\ &\leq \mathbb{E}_0\left[\mathbf{1}_{\{\tau<T\}}J^S(\pi_\tau, \Delta_\tau)\right] \\ &= J^U(\Delta), \end{aligned}$$

where the first equality takes expectations over ξ, the second equality uses the law of iterated expectations, and the inequality follows since it was shown above that as of time τ, our posited trading strategy achieves higher expected profit than any other admissible strategy. □

Proof of Proposition 2. Let $\bar{J}(\Delta_t)$ denote the value of entry at instant $t$when the news process is equal to Δ_t. Using the expression for the post-acquisition value function in Proposition 1, we have

$$\bar{J}\left(\Delta_t\right) = \Delta_t(\xi_h - \xi_l)\left(\hat{\alpha}\int_{\alpha}^{1}\frac{1-a}{\lambda(a)}\,da + (1-\hat{\alpha})\int_{0}^{\alpha}\frac{a}{\lambda(a)}\,da\right) \equiv \Delta_t K.$$

Make the change of variables $x = \Phi^{-1}(1-a)$ in the integrals in the expression for $J^U(N_t)$

$$\begin{aligned}
K &= (\xi_h - \xi_l)\left(\hat{\alpha}\sqrt{\tfrac{\sigma_Z^2}{2r}}\int_{\hat{\alpha}}^{1}(1-a)\frac{1}{\phi(\Phi^{-1}(1-a))}\,da + (1-\alpha)\sqrt{\tfrac{\sigma_Z^2}{2r}}\int_{0}^{\hat{\alpha}} a\frac{1}{\phi(\Phi^{-1}(1-a))}\,da\right)\\
&= (\xi_h - \xi_l)\left(-\hat{\alpha}\sqrt{\tfrac{\sigma_Z^2}{2r}}\int_{\Phi^{-1}(1-\alpha)}^{-\infty}\Phi(x)dx - (1-\hat{\alpha})\sqrt{\tfrac{\sigma_Z^2}{2r}}\int_{\infty}^{\Phi^{-1}(1-\alpha)}(1-\Phi(x))\,dx\right)\\
&= (\xi_h - \xi_l)\left(\hat{\alpha}\sqrt{\tfrac{\sigma_Z^2}{2r}}\int_{-\infty}^{\Phi^{-1}(1-\alpha)}\Phi(x)dx + (1-\hat{\alpha})\sqrt{\tfrac{\sigma_Z^2}{2r}}\int_{\Phi^{-1}(1-\alpha)}^{\infty}(1-\Phi(x))\,dx\right).
\end{aligned}$$

Now integrate by parts

$$\begin{aligned}
K &= (\xi_h - \xi_l)\left(\hat{\alpha}\sqrt{\tfrac{\sigma_Z^2}{2r}}\int_{-\infty}^{\Phi^{-1}(1-\hat{\alpha})}\Phi(x)dx + (1-\hat{\alpha})\sqrt{\tfrac{\sigma_Z^2}{2r}}\int_{\Phi^{-1}(1-\hat{\alpha})}^{\infty}(1-\Phi(x))\,dx\right)\\
&= (\xi_h - \xi_l)\hat{\alpha}\sqrt{\tfrac{\sigma_Z^2}{2r}}\left(-\int_{-\infty}^{\Phi^{-1}(1-\hat{\alpha})}x\phi(x)\,dx + x\Phi(x)\Big|_{-\infty}^{\Phi^{-1}(1-\hat{\alpha})}\right)\\
&\qquad + (\xi_h - \xi_l)(1-\hat{\alpha})\sqrt{\tfrac{\sigma_Z^2}{2r}}\left(\int_{\Phi^{-1}(1-\hat{\alpha})}^{\infty}x\phi(x)\,dx + x(1-\Phi(x))\Big|_{\Phi^{-1}(1-\hat{\alpha})}^{\infty}\right)\\
&= (\xi_h - \xi_l)\hat{\alpha}\sqrt{\tfrac{\sigma_Z^2}{2r}}\left(-\int_{-\infty}^{\Phi^{-1}(1-\hat{\alpha})}x\phi(x)\,dx + (1-\hat{\alpha})\Phi^{-1}(1-\hat{\alpha})\right)\\
&\qquad + (\xi_h - \xi_l)(1-\hat{\alpha})\sqrt{\tfrac{\sigma_Z^2}{2r}}\left(\int_{\Phi^{-1}(1-\hat{\alpha})}^{\infty}x\phi(x)\,dx - \hat{\alpha}\Phi(1-\hat{\alpha})\right)\\
&= (\xi_h - \xi_l)\sqrt{\tfrac{\sigma_Z^2}{2r}}\int_{-\infty}^{\Phi^{-1}(1-\hat{\alpha})}-x\phi(x)\,dx = (\xi_h - \xi_l)\sqrt{\tfrac{\sigma_Z^2}{2r}}\phi(\Phi^{-1}(1-\hat{\alpha})),
\end{aligned}$$

since $\int -x\phi(x)dx = \int \phi'(x)dx = \phi\left(x\right)$.

The pre-entry value function under optimal stopping is

$$J^U(\delta) \equiv \sup_{\tau\in\mathcal{T}}\mathbb{E}\left[\mathbf{1}_{\{\tau<T\}}(K\Delta_\tau - c)\mid \Delta_t = \delta\right] = \sup_{\tau\in\mathcal{T}}\mathbb{E}\left[e^{-r\tau}(K\Delta_\tau - c)^+\mid \Delta_t = \delta\right],$$

where the second equality follows because T is independently exponentially distributed and it suffices to consider only the positive part of $K\Delta_\tau - c$ since the trader can always guarantee herself zero profit by not acquiring. Note that this problem is similar to pricing a perpetual American call option on an asset with price process KN_t that follows a geometric Brownian motion and with strike price c. Hence, standard results (Peskir and Shiryaev (2006), Chapter 4) imply that there is a uniquely optimal stopping time and this time is is a first hitting time of the N_t process,

$$T_\Delta = \inf\{t > 0 : \Delta_t \geq \Delta^*\},$$

where $\Delta^* > 0$ is a constant to be determined.

The value function and optimal N^* solve the following free boundary problem

$$\begin{aligned}
rJ^U &= \tfrac{1}{2}\sigma_\Delta^2 \Delta_t^2 J^U_{\Delta\Delta} && \text{for } \delta < \Delta^* \\
J^U(\Delta^*) &= K\Delta^* - c && \text{`value matching'} \\
J^U_\Delta(\Delta^*) &= K && \text{`smooth pasting'} \\
J^U(\delta) &> (\delta - c)^+ && \text{for } \delta < \Delta^* \\
J^U(\delta) &= (\delta - c)^+ && \text{for } \delta > \Delta^* \\
J^U(\delta) &= 0.
\end{aligned}$$

To determine the solution in the continuation region $\delta < \Delta^*$, consider a trial solution of the form $J^U(\delta) = A\delta^\beta$. Substituting and matching terms in the differential equation yields

$$r = \tfrac{1}{2}\sigma_\Delta^2 \beta(\beta - 1), \quad \beta = \tfrac{1}{2} \pm \tfrac{1}{2}\sqrt{1 + \tfrac{8r}{\sigma_\Delta^2}}$$

and the boundary condition at $\Delta = 0$ requires that one take the positive root

$$\beta = \tfrac{1}{2} + \tfrac{1}{2}\sqrt{1 + \tfrac{8r}{\sigma_\Delta^2}}.$$

Applying the above conjecture to the value-matching and smooth pasting conditions implies:

$$\Delta^* = \frac{\beta}{\beta - 1}\frac{c}{K}, \qquad A = \frac{K}{\beta}\left(\frac{\beta}{\beta - 1}\frac{c}{K}\right)^{1-\beta} = \frac{c}{\beta - 1}\frac{1}{(\Delta^*)^\beta},$$

and the resulting function satisfies $J^U(\delta) > \delta - c$ in the continuation region, which establishes the result. The comparative statics with respect to c, σ_Δ, and σ_Z are immediate from the explicit expression for Δ^*.

To establish the remaining results, note that q and α appear only in K, so their effects on the boundary will follow from establishing their effects on K. Straightforward algebra shows that

$$\xi_h - \xi_l = \frac{\alpha(1-\alpha)(2q-1)}{q(1-q)+\alpha(1-\alpha)(2q-1)^2} = \frac{2q-1}{\frac{q(1-q)}{\alpha(1-\alpha)}+(2q-1)^2}$$

Therefore, we have

$$K = \frac{2q-1}{\frac{q(1-q)}{\alpha(1-\alpha)}+(2q-1)^2}\sqrt{\frac{\sigma_Z^2}{2r}}\phi\left(\Phi^{-1}\left(1-\hat{\alpha}\right)\right).$$

It is now immediate that K is hump-shaped and symmetric around $1/2$ in α.[22] Therefore the optimal acquisition boundary is U-shaped in α and symmetric around $\alpha = 1/2$.

To establish the result for q, define

$$f \equiv \frac{\alpha(1-\alpha)(2q-1)}{q(1-q)+\alpha(1-\alpha)(2q-1)^2} \tag{24}$$

$$g \equiv \phi\left(\Phi^{-1}\left(1-\hat{\alpha}\right)\right) = \phi\left(\Phi^{-1}\left(\alpha(1-q)+(1-\alpha)q\right)\right) \tag{25}$$

and note that

$$K = \sqrt{\frac{\sigma_Z^2}{2r}}fg. \tag{26}$$

Taking the log of K and differentiating with respect to q, it is equivalent to sign

$$\frac{\partial \log K}{\partial q} = \frac{f_q}{f} + \frac{g_q}{g}. \tag{27}$$

It is straightforward to show that $f_q \geq 0$ and $g_q \leq 0$, so the sign depends on the relative sizes of these two terms.

We have

$$\begin{aligned}
\frac{f_q}{f} &= \frac{1-2q+2q^2-2\alpha(1-\alpha)(2q-1)^2}{(2q-1)\left(q(1-q)+\alpha(1-\alpha)(2q-1)^2\right)} && (28)\\
&\geq \frac{1-2q+2q^2-\frac{1}{2}(2q-1)^2}{(2q-1)\left(q(1-q)+\frac{1}{4}(2q-1)^2\right)} && (29)\\
&= \frac{2}{2q-1}, && (30)
\end{aligned}$$

[22] Recall that $\phi\left(\Phi^{-1}\left(\cdot\right)\right)$ is hump-shaped around $1/2$ and also that $\phi\left(\Phi^{-1}\left(1-\hat{\alpha}\right)\right) = \phi\left(\Phi^{-1}\left(\hat{\alpha}\right)\right)$), which establishes that the $\phi(\Phi^{-1}(\cdot))$ term is hump-shaped in α since replacing $\hat{\alpha}$ with $1-\hat{\alpha}$ simply exchanges which of α and $(1-\alpha)$ multiplies q and $1-q$ in this term.

where the second line follows from taking $\alpha = 1/2$ and the final line does some tedious algebra.

Turning to g, we can bound the magnitude of the derivative term. Without loss of generality, due to the symmetry of K in α, suppose $\alpha \leq 1/2$, which implies $1 - \hat{\alpha} \geq 1/2$. We have

$$\begin{aligned}
\frac{|g_q|}{g} &\leq \frac{|(1-2\alpha)|\,|\Phi^{-1}(1-\hat{\alpha})|}{\phi\left(\Phi^{-1}\left(1-\hat{\alpha}\right)\right)} && (31)\\
&\leq \frac{|\Phi^{-1}(1-\hat{\alpha})|}{\phi\left(\Phi^{-1}\left(1-\hat{\alpha}\right)\right)} && (32)\\
&= \frac{\Phi^{-1}(1-\hat{\alpha})}{\phi\left(\Phi^{-1}\left(1-\hat{\alpha}\right)\right)}. && (33)
\end{aligned}$$

The upper-tail inequality for the standard normal pdf implies that for $x \geq 0$

$$1 - \Phi(x) \leq \frac{\phi(x)}{x} \Rightarrow \frac{x}{\phi(x)} \leq \frac{1}{1-\Phi(x)}. \tag{34}$$

Setting $x = \Phi^{-1}(1-\hat{\alpha}) \geq \Phi^{-1}(1/2) = 0$ in this inequality yields

$$\begin{aligned}
\frac{\Phi^{-1}(1-\hat{\alpha})}{\phi(\Phi^{-1}(1-\hat{\alpha}))} &\leq \frac{1}{1-\Phi(\Phi^{-1}(1-\hat{\alpha}))} && (35)\\
&= \frac{1}{\hat{\alpha}} && (36)\\
&\leq 2, && (37)
\end{aligned}$$

where the final line uses $\hat{\alpha} \leq 1/2$.

Putting things together,

$$\begin{aligned}
\frac{\partial \log K}{\partial q} &= \frac{f_q}{f} + \frac{g_q}{g} && (38)\\
&\geq \frac{2}{2q-1} - 2 && (39)\\
&\geq 0, && (40)
\end{aligned}$$

where the final line uses $1/2 \leq q \leq 1$. Hence, K is increasing in q and therefore Δ^* is decreasing in q.

Moreover, since

$$\frac{\partial}{\partial r}\Delta^* = \frac{c}{\sigma_Z^2 \phi\left(\Phi^{-1}\left(1-\hat{\alpha}\right)\right)} \frac{4\sqrt{2}\left(\sqrt{r} - 2\sqrt{\frac{r}{\frac{8r}{\sigma_\Delta^2}+1}}\right)}{\left(\sigma_\Delta - \sqrt{\sigma_\Delta^2 + 8r}\right)^2} \tag{41}$$

we know that Δ^* is decreasing in r when $r < \frac{3}{8}\sigma_\Delta^2$, but increasing otherwise. Because the trading horizon $h = 1/r$ is strictly decreasing and maps $(0, \infty)$ onto itself, it follows that the derivative of the acquisition boundary with respect to the horizon is also first decreasing and then increasing. This is easily established by appealing to the result for r and writing $r = 1/h$. $\square$

Proof of Proposition 3. In what follows, it is useful to define T_Δ as the first time $\Delta_t \geq \Delta^*$. Then, the time at which information is acquired can be expressed as

$$\tau = T_\Delta 1_{\{T_\Delta \leq T\}} + \infty \times 1_{\{T_\Delta > T\}}, \tag{42}$$

where, as before, $\tau = \infty$ corresponds to no entry. To avoid the trivial case, assume $\Delta_0 \equiv 1 < \Delta^*$. We begin with the following observation.

Lemma 2.

Suppose $1 < \Delta^*$. *For* $0 \leq t < \infty$, *the probability that* $T_\Delta \in [t, t+dt]$ *is given by*

$$\Pr\left(T_\Delta \in [t, t+dt]\right) = \frac{\log\left(\Delta^*\right)}{\sigma_\Delta\sqrt{2\pi t^3}} \exp\left\{-\frac{\left(\frac{1}{\sigma_\Delta}\log\left(\Delta^*\right) + \frac{1}{2}\sigma_\Delta^2 t\right)^2}{2t}\right\} dt. \tag{43}$$

The probability that T_Δ *is not finite is given by* $\Pr\left(T_\Delta = \infty\right) = 1 - \frac{1}{\Delta^*}$.

Proof. Note that

$$\begin{aligned} \Delta_t \geq \Delta^* &\iff \log(\Delta_t) \geq \log(\Delta^*) \\ &\iff -\frac{1}{2}\sigma_\Delta t + W_{\Delta t} \geq \frac{1}{\sigma_\Delta}\log(\Delta^*), \end{aligned}$$

so that the first time that Δ_t hits Δ^* is the first time that a Brownian motion with drift $-\frac{1}{2}\sigma_\Delta$ hits $\frac{1}{\sigma_\Delta}(\log(\frac{\Delta^*}{\Delta_0}))$. It follows from Karatzas and Shreve (1998) (Chapter 3.5, Part C, p.196-197) that for $\Delta_0 < \Delta^*$ the density of T_Δ is

$$\Pr\left(T_\Delta \in [t, t+dt]\right) = \frac{(\log(\Delta^*))}{\sigma_\Delta\sqrt{2\pi t^3}} \exp\left\{-\frac{\left(\frac{1}{\sigma_\Delta}\log(\Delta^*) + \frac{1}{2}\sigma_\Delta t\right)^2}{2t}\right\} dt.$$

Moreover, since $\frac{1}{\sigma_\Delta}\log(\Delta^*) > 0$ but the drift is $-\frac{1}{2}\sigma_\Delta < 0$, it follows from Karatzas and Shreve (1998) (p.197) that $\Pr(T_\Delta = \infty) > 0$. Specifically, note that

$$\Pr\left(T_\Delta < \infty\right) = \int_0^\infty \frac{\log(\Delta^*)}{\sigma_\Delta\sqrt{2\pi t^3}} \exp\left\{-\frac{\left(\frac{1}{\sigma_\Delta}\log(\Delta^*) + \frac{1}{2}\sigma_\Delta t\right)^2}{2t}\right\} dt = \frac{1}{\Delta^*}, \tag{44}$$

which implies $\Pr\left(T_\Delta = \infty\right) = 1 - \frac{1}{\Delta^*}$. $\square$

Given the definition of τ, we have that for $0 \le t < \infty$,

$$\begin{aligned}
\Pr\left(\tau \in [t, t+dt]\right) = & \quad \Pr\left(\tau \in [t, t+dt] \,\middle|\, T_\Delta \le T\right)\Pr\left(T_\Delta \le T\right) \\
& + \Pr\left(\tau \in [t, t+dt] \,\middle|\, T_\Delta > T\right)\Pr\left(T_\Delta > T\right) && (45)\\
= & \Pr\left(T_\Delta \in [t, t+dt] \,\middle|\, T_\Delta \le T\right)\Pr\left(T_\Delta \le T\right) && (46)\\
= & \Pr\left(T_\Delta \in [t, t+dt]\right)\Pr\left(T \ge t\right) && (47)\\
= & e^{-rt}\Pr\left(T_\Delta \in [t, t+dt]\right). && (48)
\end{aligned}$$

Integrating gives us

$$\begin{aligned}
\Pr\left(\tau < \infty\right) &= \int_0^\infty e^{-rt}\frac{\log(\Delta^*)}{\sigma_\Delta\sqrt{2\pi t^3}} \exp\left\{-\frac{\left(\frac{1}{\sigma_N}\log(\Delta^*) + \frac{1}{2}\sigma_\Delta t\right)^2}{2t}\right\} dt && (49)\\
&= \frac{e^{-\frac{\log(\Delta^*)\sqrt{8r+\sigma_\Delta^2}}{2\sigma_\Delta}}}{\sqrt{\Delta^*}} = \left(\frac{1}{\Delta^*}\right)^\beta && (50)
\end{aligned}$$

The comparative statics for c, σ_Z, q, and α follow from plugging in the expressions for Δ^* and β, and using the results from Proposition 2 for the effect of q and α on Δ^*. To establish the comparative statics for σ_Δ, first note that since $\lim_{\sigma_\Delta \to 0}\beta = \infty$, $\lim_{\sigma_\Delta \to \infty}\beta = 1$, and $\Delta^* = \frac{\beta}{\beta-1}\frac{c}{K}$,

$$\lim_{\sigma_\Delta \to \infty}\Pr\left(\tau < \infty\right) = 0 \tag{51}$$

$$\lim_{\sigma_\Delta \to 0} \Pr(\tau < \infty) = \begin{cases} 0 & \text{if } c > K \\ 1 & \text{if } c \leq K \end{cases}. \tag{52}$$

Let

$$\zeta \equiv \frac{\partial}{\partial \beta} \left(\log\left(\Pr\left(\tau < \infty\right)\right)\right) = \log\left(\frac{1}{\Delta^*}\right) + \beta \frac{\partial}{\partial \beta} \log\left(\frac{1}{\Delta^*}\right) = \log\left(\frac{1}{\Delta^*}\right) + \frac{1}{\beta - 1} \tag{53}$$

which implies $\lim_{\sigma_\Delta \to 0} \zeta = \lim_{\beta \to \infty} \zeta = \log\left(\frac{K}{c}\right)$, $\lim_{\sigma_\Delta \to \infty} \zeta = \lim_{\beta \to 1} \zeta = \infty$, and

$$\frac{\partial}{\partial \sigma_\Delta} \zeta = \frac{\partial \zeta}{\partial \beta} \frac{\partial \beta}{\partial \sigma_\Delta} = -\frac{1}{\beta (1-\beta)^2} \frac{\partial \beta}{\partial \sigma_\Delta} > 0. \tag{54}$$

Since $\frac{\partial}{\partial \sigma_\Delta} \log\left(\Pr\left(\tau < \infty\right)\right) = \zeta \frac{\partial \beta}{\partial \sigma_\Delta}$, we have the following results:

- When $c \leq K$, since $\zeta \geq 0$ for $\sigma_\Delta \to 0$ and $\frac{\partial}{\partial \sigma_\Delta} \zeta > 0$ we have $\zeta > 0$ for all σ_Δ, which in turn implies $\frac{\partial}{\partial \sigma_\Delta} \log\left(\Pr\left(\tau < \infty\right)\right) < 0$ for all σ_Δ.
- When $c > K$, ζ crosses zero once, from below, as σ_Δ increases, which implies $\frac{\partial}{\partial \sigma_\Delta} \log\left(\Pr\left(\tau < \infty\right)\right) = 0$ at exactly this one point. In this case, $\Pr\left(\tau < \infty\right)$ is hump-shaped.

Similarly, for r, $\frac{\partial}{\partial r} \log(\Pr(\tau < \infty)) = \zeta \frac{\partial}{\partial r} \beta - \frac{\beta}{2r}$. We have $\frac{\partial}{\partial r} \zeta = -\frac{1}{\beta(\beta-1)^2} \frac{\partial}{\partial r} \beta - \frac{1}{2r} < 0$. Since $\frac{\partial}{\partial r} \beta = \frac{1}{\sigma_\Delta^2 \left(\beta - \frac{1}{2}\right)} > 0$ this implies $\frac{\partial}{\partial r} \log(\Pr(\tau < \infty))$ crosses zero as most once as r increases and from above if it does so. Consider the limit as r tends to zero,

$$\begin{aligned} \lim_{r \to 0} \frac{\partial}{\partial r} \log(\Pr(\tau < \infty)) &= \lim_{r \to 0} \left(\zeta \frac{\partial}{\partial r} \beta - \frac{\beta}{2r}\right) \\ &= \lim_{r \to 0} \frac{2r\zeta - \sigma_\Delta^2 \beta \left(\beta - \frac{1}{2}\right)}{2\sigma_\Delta^2 r \left(\beta - \frac{1}{2}\right)}. \end{aligned} \tag{55}$$

If it can be shown that the numerator in eq. (55) has a finite, positive limit it will follow that the overall limit is ∞. Considering the numerator, we have

$$\begin{aligned} \lim_{r \to 0} \left(2r\zeta - \sigma_\Delta^2 \beta \left(\beta - \tfrac{1}{2}\right)\right) &= 2 \lim_{r \to 0} r \left(\frac{1}{\beta - 1} - \log \frac{\beta}{\beta - 1} - \log \sqrt{2r}\right) - \tfrac{1}{2} \sigma_\Delta^2 \\ &= \sigma_\Delta^2 - 2 \lim_{r \to 0} \frac{\frac{1}{\beta(\beta-1)}}{\frac{1}{r^2}} - \tfrac{1}{2} \sigma_\Delta^2 \\ &= \tfrac{1}{2} \sigma_\Delta^2 - 2 \lim_{r \to 0} \frac{2r}{(2\beta - 1) \frac{\partial}{\partial r} \beta} = \tfrac{1}{2} \sigma_\Delta^2 \end{aligned}$$

where the second equality applies l'Hôspital's rule to the three different terms and uses the fact $\frac{\partial}{\partial r} \beta \to \frac{2}{\sigma_\Delta^2}$ as $\beta \to 1$. The third equality rearranges the expression in the remaining limit to place r^2 in the numerator and uses l'Hôspital's rule again. Returning to eq. (55), this implies $\lim_{r \to 0} \frac{\partial}{\partial r} \log(\mathbb{P}(\tau < \infty)) = \infty$.

Now, consider $\lim_{r\to\infty} \frac{\partial}{\partial r} \log(\Pr(\tau < \infty))$. We have

$$\lim_{r\to\infty} \zeta = \lim_{r\to\infty} \left(\tfrac{1}{\beta-1} - \log \tfrac{\beta}{\beta-1} - \log \sqrt{2r}\right) = \lim_{\beta\to\infty} \left(\tfrac{1}{\beta-1} - \log \tfrac{\beta}{\beta-1}\right) - \lim_{r\to\infty} \log \sqrt{2r} = -\infty.$$

Because $\frac{\partial}{\partial r}\beta > 0$, it follows that $\lim_{r\to\infty} \frac{\partial}{\partial r} \log(\Pr(\tau < \infty)) = -\infty$. Because the trading horizon $h = 1/r$ is strictly decreasing and maps $(0,\infty)$ onto itself, it follows that the derivative of the acquisition probability with respect to the horizon is also first decreasing and then increasing. This is easily established by appealing to the result for r and writing $r = 1/h$. This completes the proof. $\square$

Proof of Proposition 4. It was shown in Proposition 3 that τ is infinite with strictly positive probability. Hence, all unconditional moments (of positive order) are infinite.

Using the density from eq. (49) to compute the conditional density of τ gives

$$\mathbb{P}(\tau \in [t, t+dt] | \tau < \infty) = \frac{e^{-rt} \frac{\log(\Delta^*)}{\sigma_\Delta \sqrt{2\pi t^3}} \exp\left\{ -\frac{\left(\frac{1}{\sigma_N} \log(\Delta^*) + \frac{1}{2}\sigma_\Delta t\right)^2}{2t} \right\} \mathbf{1}_{\{t<\infty\}}}{\mathbb{P}(\tau < \infty)} dt$$

Integrating and doing some simplifying algebra gives

$$\begin{aligned}\mathbb{E}[\tau | \tau < \infty] &= \int_0^\infty t \frac{e^{-rt} \frac{\log(\Delta^*)}{\sigma_\Delta \sqrt{2\pi t^3}} \exp\left\{ -\frac{\left(\frac{1}{\sigma_N} \log(\Delta^*) + \frac{1}{2}\sigma_\Delta t\right)^2}{2t} \right\}}{\mathbb{P}(\tau < \infty)} dt \\ &= \frac{2\log(\Delta^*)}{\sigma_\Delta^2 \sqrt{1 + \frac{8r}{\sigma_\Delta^2}}}\end{aligned}$$

The conditional expectation inherits the comparative statics of Δ^* with respect to c, σ_Z^2, q, and α since these parameters appear only in Δ^* and log is an increasing transformation.

To determine the dependence on σ_Δ^2, note that by solving $\beta = \frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{8r}{\sigma_\Delta^2}}$ for σ_Δ^2 and substituting, we can write

$$\mathbb{E}[\tau | \tau < \infty] = 2 \frac{1}{\left(\frac{2r}{\beta(\beta-1)}\left(\frac{2r}{\beta(\beta-1)} + 8r\right)\right)^{1/2}} \log(\Delta^*) \tag{56}$$

$$= 2\sqrt{\frac{\beta(\beta-1)}{2r\left(\frac{2r}{\beta(\beta-1)} + 8r\right)}} \log(\Delta^*) \tag{57}$$

$$= 2\sqrt{\frac{\beta^2(\beta-1)^2}{4r^2 + 16r^2\beta(\beta-1)}} \log(\Delta^*) \tag{58}$$

$$= \frac{1}{r}\sqrt{\frac{\beta^2(\beta-1)^2}{1+4\beta(\beta-1)}}\log(\Delta^*) \tag{59}$$

$$= \frac{\beta(\beta-1)}{r(2\beta-1)}\log\left(\frac{\beta}{\beta-1}\frac{c}{K}\right), \tag{60}$$

We have

$$\frac{\partial}{\partial\beta}\mathbb{E}[\tau|\tau<\infty] = \frac{1-2\beta+2\beta^2}{r(2\beta-1)^2}\log\left(\frac{\beta}{\beta-1}\frac{c}{K}\right) - \frac{\beta(\beta-1)}{r(2\beta-1)}\frac{1}{\beta(\beta-1)} \tag{61}$$

$$= \frac{(1-2\beta+2\beta^2)\log\left(\frac{\beta}{\beta-1}\frac{c}{K}\right)-(2\beta-1)}{r(2\beta-1)^2} \tag{62}$$

$$= \frac{(2\beta(\beta-1)+1)\log\left(\frac{\beta}{\beta-1}\frac{c}{K}\right)-(2\beta-1)}{r(2\beta-1)^2} \tag{63}$$

$$\tag{64}$$

The denominator of this expression is strictly positive, so the sign of the derivative depends on the sign of the numerator

$$\frac{\partial}{\partial\beta}\mathbb{E}[\tau|\tau<\infty] \gtrless 0 \iff (2\beta(\beta-1)+1)\log\left(\frac{\beta}{\beta-1}\frac{c}{K}\right)-(2\beta-1) \gtrless 0 \tag{65}$$

$$\iff \log\left(\frac{\beta}{\beta-1}\frac{c}{K}\right) \gtrless \frac{(2\beta-1)}{(2\beta(\beta-1)+1)} \tag{66}$$

$$\iff \log\left(\frac{c}{K}\right) \gtrless \underbrace{\frac{(2\beta-1)}{(2\beta(\beta-1)+1)} - \log\left(\frac{\beta}{\beta-1}\right)}_{\equiv g(\beta)} \tag{67}$$

Calculating the derivative of g, we have

$$g'(\beta) = \frac{(2\beta-1)^2}{\beta(\beta-1)(2\beta(\beta-1)+1)^2} > 0. \tag{68}$$

Furthermore,

$$\lim_{\beta\downarrow 1} g(\beta) = -\infty \tag{69}$$

$$\lim_{\beta\to\infty} g(\beta) = 0. \tag{70}$$

Hence, $g(\beta) < 0$ for any $\beta > 1$ and strictly increases towards 0 as β increases.

Since β is strictly decreasing in σ_Δ^2, this implies

$$\frac{d}{d\sigma_\Delta^2} g(\beta) = \frac{\partial g}{\partial \beta} \underbrace{\frac{\partial \beta}{\partial \sigma_\Delta^2}}_{<0} < 0 \tag{71}$$

$$g(\beta) \mid_{\sigma_\Delta^2 \to \infty} = -\infty \tag{72}$$

$$g(\beta) \mid_{\sigma_\Delta^2 \to 0} = 0. \tag{73}$$

Putting things together, if $c \geq K$ then it is immediate that

$$\log\left(\frac{c}{K}\right) \geq 0 > g(\beta), \tag{74}$$

so that $\mathbb{E}[\tau|\tau < \infty]$ is decreasing in σ_Δ^2.

On the other hand, if $c < K$, then $\log\left(\frac{c}{K}\right) < 0$ and therefore for sufficiently small σ_Δ^2 we have

$$0 \gtrapprox g(\beta) > \log\left(\frac{c}{K}\right), \tag{75}$$

so that $\mathbb{E}[\tau|\tau < \infty]$ is increasing in σ_Δ^2 for small . For sufficiently large σ_Δ^2 we have

$$g(\beta) < \log\left(\frac{c}{K}\right) \tag{76}$$

so that $\mathbb{E}[\tau|\tau < \infty]$ is decreasing in σ_Δ^2. Given the monotonicity of $g(\beta)$ there is a unique β at which the sign of the dependence flips and therefore we conclude that if $c \geq K$ then $\mathbb{E}[\tau|\tau < \infty]$ is decreasing in σ_Δ^2, and if $c < K$ then $\mathbb{E}[\tau|\tau < \infty]$ is first increasing in σ_Δ^2 and then decreasing.

To determine the dependence on r, let $\kappa = (\xi_h - \xi_l)\sqrt{\sigma_Z^2}\phi\left(\Phi^{-1}\left(1 - \hat{\alpha}\right)\right)$, which does not depend on r, and recall that

$$\mathbb{E}[\tau|\tau < \infty] = \frac{\beta(\beta-1)}{r(2\beta-1)} \log\left(\frac{\beta}{\beta-1}\frac{c}{K}\right) = \frac{1}{r}\frac{\beta(\beta-1)}{(2\beta-1)} \log\left(\frac{\beta}{\beta-1}\frac{c\sqrt{2r}}{\kappa}\right) \tag{77}$$

Taking the total derivative with respect to r gives

$$\frac{d}{dr}\mathbb{E}[\tau|\tau < \infty] = \frac{\partial \mathbb{E}[\tau|\tau < \infty]}{\partial r} + \frac{\partial \mathbb{E}[\tau|\tau < \infty]}{\partial \beta}\frac{\partial \beta}{\partial r} \tag{78}$$

$$= -\frac{1}{r^2}\frac{\beta(\beta-1)}{(2\beta-1)} \log\left(\frac{\beta}{\beta-1}\frac{c\sqrt{2r}}{\kappa}\right) + \frac{1}{r}\frac{\beta(\beta-1)}{(2\beta-1)}\frac{1}{2r} \tag{79}$$

$$+\frac{\partial}{\partial\beta}\left(\frac{\beta(\beta-1)}{r(2\beta-1)}\log\left(\frac{\beta}{\beta-1}\frac{c}{K}\right)\right)\frac{\partial\beta}{\partial r} \tag{80}$$

$$=-\frac{1}{r^2}\frac{\beta(\beta-1)}{(2\beta-1)}\log\left(\frac{\beta}{\beta-1}\frac{c}{K}\right)+\frac{1}{r}\frac{\beta(\beta-1)}{(2\beta-1)}\frac{1}{2r} \tag{81}$$

$$+\frac{\frac{2\beta(\beta-1)+1}{2\beta-1}\log\left(\frac{\beta}{\beta-1}\frac{c}{K}\right)-1}{r(2\beta-1)}\frac{\partial\beta}{\partial r} \tag{82}$$

We have

$$\frac{\partial\beta}{\partial r}=\frac{2}{\sigma_\Delta^2\sqrt{1+\frac{8r}{\sigma_\Delta^2}}} \tag{83}$$

$$=\frac{2}{\sigma_\Delta^2}\frac{1}{2\beta-1} \tag{84}$$

$$=\frac{\beta(\beta-1)}{r(2\beta-1)}, \tag{85}$$

where the second equality substitutes for σ_Δ^2 in terms of β.

Plugging in to the previous expression gives

$$\frac{d}{dr}\mathbb{E}[\tau|\tau<\infty]=-\frac{1}{r^2}\frac{\beta(\beta-1)}{(2\beta-1)}\log\left(\frac{\beta}{\beta-1}\frac{c}{K}\right)+\frac{1}{r}\frac{\beta(\beta-1)}{(2\beta-1)}\frac{1}{2r} \tag{86}$$

$$+\frac{\frac{2\beta(\beta-1)+1}{2\beta-1}\log\left(\frac{\beta}{\beta-1}\frac{c}{K}\right)-1}{r(2\beta-1)}\frac{\beta(\beta-1)}{r(2\beta-1)} \tag{87}$$

$$=\frac{\beta(\beta-1)}{r^2(2\beta-1)^2}\left(\frac{-2\beta(\beta-1)}{(2\beta-1)}\log\left(\frac{\beta}{\beta-1}\frac{c}{K}\right)+\frac{2\beta-3}{2}\right) \tag{88}$$

Hence,

$$\frac{d}{dr}\mathbb{E}[\tau|\tau<\infty]\gtrless 0 \iff \frac{-2\beta(\beta-1)}{(2\beta-1)}\log\left(\frac{\beta}{\beta-1}\frac{c}{K}\right)+\frac{2\beta-3}{2}\gtrless 0 \tag{89}$$

$$\iff \frac{(2\beta-3)(2\beta-1)}{4\beta(\beta-1)}-\log\left(\frac{\beta}{\beta-1}\right)-\log\left(\frac{c}{\kappa}\sqrt{2r}\right)\gtrless 0 \tag{90}$$

Rearranging the expression for β yields

$$\sqrt{2r}=\frac{1}{2}\sqrt{\sigma_\Delta^2}\sqrt{(2\beta-1)^2-1} \tag{91}$$

Hence,

$$\frac{d}{dr}\mathbb{E}[\tau|\tau<\infty] \gtrless 0 \iff \underbrace{\frac{(2\beta-3)(2\beta-1)}{4\beta(\beta-1)} - \log\left(\frac{\beta}{\beta-1}\right) - \log\left(\frac{c}{2\kappa}\sqrt{\sigma_\Delta^2}\sqrt{(2\beta-1)^2-1}\right)}_{\equiv h(\beta)} \gtrless 0. \tag{92}$$

Hence, the desired sign depends on the sign of $h(\beta)$ for $\beta \in (1,\infty)$. To begin, we will find its maximum. Differentiate h and set equal to zero

$$h'(\beta) = \frac{3-6\beta+4\beta^2}{4\beta^2(\beta-1)^2} + \frac{1}{\beta(\beta-1)} - \frac{2\beta-1}{2\beta(\beta-1)} = 0 \tag{93}$$

$$\iff 3-6\beta+4\beta^2+4\beta(\beta-1)-2\beta(\beta-1)(2\beta-1) = 0 \tag{94}$$

$$\iff -(2\beta-1)(2\beta^2-6\beta+3) = 0 \tag{95}$$

This is a cubic with one (real) root at $\beta = 1/2$, and two others (both real), which the quadratic formula gives as

$$\beta = \frac{1}{2}(3\pm\sqrt{3}). \tag{96}$$

The root $\beta = \frac{1}{2}(3+\sqrt{3})$ is the only one in the interval $\beta \in (1,\infty)$, so it is the only relevant critical point. The second-order condition is

$$\frac{3-12\beta+18\beta^2-12\beta^3+2\beta^4}{2\beta^3(\beta-1)^3} \leq 0, \tag{97}$$

and at $\beta = \frac{1}{2}(3+\sqrt{3})$ we have

$$\left.\frac{3-12\beta+18\beta^2-12\beta^3+2\beta^4}{2\beta^3(\beta-1)^3}\right|_{\beta=\frac{1}{2}(3+\sqrt{3})} = 2-\frac{4}{\sqrt{3}} < 2-\frac{4}{\sqrt{4}} = 0. \tag{98}$$

Hence, h has a local maximum at $\beta = \frac{1}{2}(3+\sqrt{3})$. At the boundaries, we have

$$\lim_{\beta\downarrow 1} h(\beta) = -\infty \tag{99}$$

$$\lim_{\beta\to\infty} h(\beta) = -\infty. \tag{100}$$

Because h is negatively infinite at the boundaries and has a single critical point in $(1,\infty)$, it follows that it achieves a *global* maximum at $\beta = \frac{1}{2}(3+\sqrt{3})$. The value of h at this point is

$$h\left(\frac{1}{2}(3+\sqrt{3})\right) = \frac{1}{2}\underbrace{\left(1+\log\left(\frac{1}{18}(2\sqrt{3}-3)\right)\right)}_{<0} - \log\left(\frac{c}{2\kappa\sqrt{\sigma_\Delta^2}}\right). \tag{101}$$

If $\frac{c}{2\kappa\sqrt{\sigma_\Delta^2}}$ is sufficiently large, this expression is negative, $h < 0$ for all $\beta \in (1, \infty)$, and therefore for all $r > 0$. Hence, in this case $\frac{d}{dr}\mathbb{E}[\tau|\tau < \infty] < 0$. On the other hand, if $\frac{c}{2\kappa\sqrt{\sigma_\Delta^2}}$ is not sufficiently large then h is first negative, then positive, then negative, as β increases, which implies that $\frac{d}{dr}\mathbb{E}[\tau|\tau < \infty]$ changes sign from negative to positive and back to negative as r increases. This completes the proof. $\square$

Proof of Proposition 5 When evaluating entry when $\Delta_t = \Delta$, the payoff can be written concisely as

$$\max\{K_h\Delta - c_h, K_l\Delta - c_l, 0\}$$

Since the payoff function is monotonic in Δ, standard results imply that the optimal entry time is a first hitting time, from below, for the Δ process. The value function J^U and entry boundary Δ^* satisfy

$$\begin{aligned}
rJ^U &= \tfrac{1}{2}\sigma_\Delta^2\Delta^2 J_{\Delta\Delta} && \text{continuation region}\\
J^U(\Delta^*) &= \max\{K_h\Delta^* - c_h, K_l\Delta^* - c_l, 0\} &&\\
J^U_\Delta(\Delta^*) &= \left.\frac{d}{d\delta}\right|_{\delta=\Delta^*} \max\{K_h\delta - c_h, K_l\delta - c_l, 0\} &&\\
J^U(\delta) &> \max\{K_h\delta - c_h, K_l\delta - c_l, 0\} && \text{continuation region}\\
J^U(\delta) &= \max\{K_h\delta - c_h, K_l\delta - c_l, 0\} && \text{outside continuation region}\\
J^U(0) &= 0
\end{aligned}$$

Consider a trial solution of the form $J^U(\delta) = A\delta^\beta$. Note that for there to be a single optimal acquisition point Δ^* it must either acquire a low precision signal in the region $K_h\Delta - c_H \leq K_l\Delta - c_l \iff \Delta \leq \frac{c_h - c_l}{K_h - K_l} \equiv \bar{\Delta}$ or a high precision signal in the region $\Delta \geq \bar{\Delta}$.

Let's first search for a low-precision solution. Plugging the conjectured solution into the differential equation yields

$$\beta = \frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{8r}{\sigma_\Delta^2}}.$$

The value-matching and smooth-pasting conditions require

$$\begin{aligned} A_l &= \frac{K_l}{\beta}\left(\frac{c_l}{K_l}\frac{\beta}{\beta-1}\right)^{1-\beta} \\ \Delta_l^* &= \frac{c_l}{K_l}\frac{\beta}{\beta-1}. \end{aligned}$$

Now, let's search for a high-precision solution. The differential equation still implies

$$\beta = \frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{8r}{\sigma_\Delta^2}}.$$

The value-matching and smooth-pasting conditions imply

$$\begin{aligned} A_h &= \frac{K_h}{\beta}\left(\frac{c_h}{K_h}\frac{\beta}{\beta-1}\right)^{1-\beta} \\ \Delta_h^* &= \frac{c_h}{K_h}\frac{\beta}{\beta-1}. \end{aligned}$$

The choice between a low- and high-precision signal therefore depends on which of the value functions from the two candidate solutions is larger. Comparing the value functions from implies that the choice of precision depends on

$$K_h^\beta c_h^{1-\beta} \gtreqless K_l^\beta c_l^{1-\beta},$$

as well as whether the candidate Δ_j^* is greater or less than $\bar{\Delta} = \frac{c_h - c_l}{K_h - K_l}$ as discussed above.

Note that a high-precision signal is always optimal if $c_h/K_h < c_l/K_l$ since in that case we have

$$\begin{aligned} c_h/K_h < c_l/K_l &\Rightarrow K_l^\beta c_l^{-\beta} < K_h^\beta c_h^{-\beta} \\ &\Rightarrow K_l^\beta c_l^{1-\beta} < K_h^\beta c_l c_h^{-\beta} \\ &\Rightarrow K_l^\beta c_l^{1-\beta} < K_h^\beta c_h^{1-\beta}, \end{aligned}$$

where the second line multiplies through by $c_l > 0$ and the final line uses $c_l \leq c_h$. Furthermore, in this case

$$\bar{\Delta} = \frac{c_h - c_l}{K_h - K_l} = \frac{\frac{1}{K_l}\frac{c_h}{K_h} - \frac{1}{K_h}\frac{c_l}{K_l}}{\frac{1}{K_l} - \frac{1}{K_h}}$$

$$
\begin{aligned}
&\leq \frac{\frac{1}{K_l}\frac{c_h}{K_h} - \frac{1}{K_h}\frac{c_h}{K_h}}{\frac{1}{K_l} - \frac{1}{K_h}} \\
&= \frac{c_h}{K_h} \\
&\leq \frac{\beta}{\beta - 1}\frac{c_h}{K_h} = \Delta_h^*.
\end{aligned}
$$

Now, consider the case $c_h/K_h \geq c_l/K_l$. Note that in this case, the only situation in which the optimal signal is not immediate is when $\Delta_l^* \leq \bar{\Delta} \leq \Delta_h^*$. In that case, to determine the optimal signal, one must directly compare the two value functions, which reduces to comparing A_h and A_l, as well as $\Delta_l^* \leq \bar{\Delta}$, as in the Proposition.