Managing Expectations in the New Keynesian Model

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Question

- Time inconsistency is often an issue for policymaking
  – monetary policy, macroprudential policy, taxation, etc.
- Ability for commitment $\neq$ Reputation for commitment
- What is the optimal policy when policymaker **can** commit but is not fully trusted to?
  - Two types of policymaker: one can commit and the other cannot
  - Both types are optimizing
  - Private sector does not observe policymaker's type but is learning
  - Private sector has forward-looking expectations
- Application: monetary policy in a New Keynesian model.

A pedagogical model

- Forward-looking expectations
- Random regime switch

$$\pi_t{}^C \qquad \pi_{t+1}{}^C \qquad \pi_{t+2}{}^C$$

$$\pi_t{}^D \qquad \pi_{t+1}{}^D \qquad \pi_{t+2}{}^D$$

A pedagogical model

- Forward-looking expectations
- Random regime switch

$$
\begin{tikzcd}
{\pi_t}^C \arrow[r, "\rho"] \arrow[dr, "1-\rho"] & {\pi_{t+1}}^C & {\pi_{t+2}}^C \\
{\pi_t}^D & {\pi_{t+1}}^D & {\pi_{t+2}}^D
\end{tikzcd}
$$

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\begin{tikzcd}
{\pi_t}^C & {\pi_{t+1}}^C \arrow[l] & {\pi_{t+2}}^C \\
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\end{tikzcd}
$$

Our model

- Forward-looking expectations
- Incomplete information and learning

$$\pi_t{}^C \qquad \pi_{t+1}{}^C \quad \pi_{t+2}{}^C \ldots$$

$$\pi_t{}^D \qquad \pi_{t+1}{}^D \quad \pi_{t+2}{}^D \ldots$$

Our model

- Forward-looking expectations
- Incomplete information and learning

$\rho_t \quad {\pi_t}^C \qquad\qquad {\pi_{t+1}}^C \quad {\pi_{t+2}}^C \ldots$

π_t

$1-\rho_t \quad {\pi_t}^D \qquad\qquad {\pi_{t+1}}^D \quad {\pi_{t+2}}^D \ldots$

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Our model

- Forward-looking expectations
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$$\rho_t \quad {\pi_t}^C \qquad\qquad \rho_{t+1} \quad {\pi_{t+1}}^C \quad {\pi_{t+2}}^C \ldots$$

$$\pi_t \longrightarrow E_t \pi_{t+1}$$

$$1-\rho_t \quad {\pi_t}^D \qquad\qquad 1-\rho_{t+1} \quad {\pi_{t+1}}^D \quad {\pi_{t+2}}^D \ldots$$

What we do in this paper

- Show how to obtain equilibrium as a solution to a recursive optimization
  - formulate the dynamic game as a principal-agents problem
  - principal: committed policymaker
  - agents: discretionary policymaker, private sector
  - recursive formulation of the problem ala Marcet and Marimon (2019):
- Develop an efficient algorithm to compute the solution.
  - rational expectation fcn is "parameterized" by a Lagrangian multiplier.
  - IC set: discretionary policies motivated by rational expectation fcns
  - committed policymaker directly optimizes over IC set

What we do in this paper

- Equilibrium dynamic under discretionary CB is consistent with U.S. inflation experience in 60s and 70s
  - lengthy real stimulations with gradually rising actual and expected inflation
  - reputation gradually erodes
  - ends with stagflation
- Optimal committed policy depends nonlinearly on the CB's reputation for commitment
  - good reputation: close to standard solution under full commitment
  - poor reputation: aggressive disinflation policies with real output costs

Central banker and private sector

- Committed type chooses and follows through on a policy plan $\{a_t\}_{t=0}^{\infty}$

$$\max \sum_{t=0}^{\infty} \beta^t \widetilde{u}\left(\pi_t, x_t\right)$$

- Discretionary type chooses policy α_t period-by-period:

$$\max \widetilde{v}\left(\pi_t, x_t\right)$$

- π_t is random outcome of policy

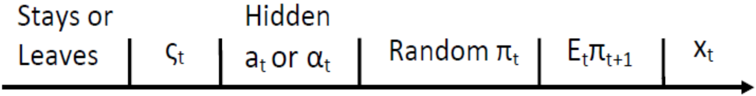
$$\pi_t = \begin{cases} a_t + \varepsilon_t & \text{if committed type in place} \\ \alpha_t + \varepsilon_t & \text{if discretionary type in place} \end{cases}$$

- Private sector has forward-looking expectations:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \varsigma_t$$

Bayesian learning and rational expectation

- Within each period:



- Private sector observes π_t to update belief
  ρ : likelihood that the current CB is the committed type

$$\rho_{t+1} = b\left(\pi_t | \rho_t, a_t, \alpha_t\right)$$

- Private sector forms rational expectation about π_{t+1}

$$E_t \pi_{t+1} = \rho_{t+1} E_t a_{t+1} + \left(1 - \rho_{t+1}\right) E_t \alpha_{t+1}$$

Public Perfect Bayesian Equilibrium

- Public history: $h_t = (h_{t-1}, \pi_{t-1}, \varsigma_t)$ with $h_0 = \{\varsigma_0\}$.
- Public Equilibrium: $a(h_t), \alpha(h_t), e(h_t, \pi_t)$.
- Belief Consistency – private sector

$$\begin{aligned} e(h_t, \pi_t) &= \beta\left\{\rho_{t+1} E_t[a(h_{t+1})] + (1-\rho_{t+1}) E_t[\alpha(h_{t+1})]\right\} && (1) \\ \text{with } \rho_{t+1} &= b(\pi_t | \rho_t, a(h_t), \alpha(h_t)) && (2) \end{aligned}$$

- Sequential Rationality – both types of central banker

Sequential rationality of discretionary type

$$
\begin{aligned}
\alpha_t &= \arg\max_{\alpha} \int \widetilde{v}\left(\pi_t, x_t\right) f\left(\pi_t | \alpha\right) d\pi_t \\
\text{s.t. } x_t &= \frac{1}{\kappa}\left(\pi_t - e_t - \varsigma_t\right).
\end{aligned}
$$

- *Discretionary type takes $e\left(h_t, \pi_t\right)$ as given*:

$$
\alpha_t = \arg\max_{\alpha} \int v\left(\pi_t, e\left(h_t, \pi_t\right), \varsigma_t\right) f\left(\pi_t | \alpha\right) d\pi_t, \tag{3}
$$

with the FOC:

$$
\int v\left(\pi_t, e\left(h_t, \pi_t\right), \varsigma_t\right) f_{\alpha}\left(\pi_t | \alpha_t\right) d\pi_t = 0. \tag{4}
$$

Sequential rationality of committed type

$$\max_{\{a_t\}_{t=0}^{\infty}} E_0 \{ \sum_{t=0}^{\infty} \beta^t \int u(\pi_t, e(h_t, \pi_t), \varsigma_t) f(\pi_t | a_t) d\pi_t \}$$

- s.t.
  - Belief Consistency, γ

$$e(h_t, \pi_t) = \beta \left\{ \rho_{t+1} E_t [a(h_{t+1})] + (1 - \rho_{t+1}) E_t [\alpha(h_{t+1})] \right\}$$

  - ICC: FOC of discretionary policy problem, ϕ

$$\int v(\pi_t, e(h_t, \pi_t), \varsigma_t) f_\alpha(\pi_t | \alpha_t) d\pi_t = 0$$

- *Committed type chooses* $e(h_t, \pi_t)$.

Recursive formulation

$$W(\rho,\eta,\varsigma) = \min_{\phi,\gamma(\pi)} \max_{a,\alpha,e(\pi)} E^a \left\{ w + \beta E^{\varsigma} W\left(\rho',\eta',\varsigma'\right) \right\}, \tag{5}$$

where $E^a(\cdot)=\int(\cdot)f(\pi|a)d\pi,\ E^{\varsigma}(\cdot)=\sum_{\varsigma'}\delta(\varsigma',\varsigma)(\cdot),$

$$\begin{aligned} w &= u(\pi, e(\pi), \varsigma) && (6)\\ &\quad +\gamma(\pi)\, e(\pi) - \eta\left[\rho a + (1-\rho)\alpha\right] && (7)\\ &\quad +\phi \frac{f_\alpha(\pi|\alpha)}{f(\pi|a)} \upsilon(\pi, e(\pi), \varsigma), && (8) \end{aligned}$$

subject to the state evolution equations for $\varsigma', \rho' = b(\pi|\rho, a, \alpha)$, and

$$\eta' = \gamma, \text{ with } \eta_0 = 0. \tag{9}$$

Rational expectation function parameterized

Given the state and (a, α), $e(\pi)$ is **uniquely** pinned down by ϕ

- The FOC w.r.t. e :

$$u_e + \eta' + \phi v_e \frac{f_\alpha(\pi|\alpha)}{f(\pi|a)} = 0.$$

  - e is to smooth the path of output: $u_e \propto (\pi - e - \varsigma - \kappa x^*)$.
  - e has implications for future committed policy: η'.
  - e affects optimal discretionary policy α: $\phi v_e f_\alpha(\pi|\alpha)$

- Rational expectation:

$$e = \widehat{M}(\rho', \eta', \varsigma)$$

where $\widehat{M}(\rho', \eta', \varsigma) = E^{\varsigma}[\rho' a(\rho', \eta', \varsigma') + (1 - \rho')\alpha(\rho', \eta', \varsigma')]$.

Incentive compatible set (IC set)

For each a, $\hat{\alpha}$ is incentive compatible if there exists $e\left(\pi|\phi, a, \hat{\alpha}\right)$ such that

$$\begin{aligned} \int \upsilon\left(\pi, e\left(\pi|\phi; a, \hat{\alpha}\right), \varsigma\right) f_{\alpha}\left(\pi|\hat{\alpha}\right) d\pi_t &= 0 \\ \int \upsilon\left(\pi, e\left(\pi|\phi; a, \hat{\alpha}\right), \varsigma\right) f_{\alpha\alpha}\left(\pi|\hat{\alpha}\right) d\pi_t &< 0 \end{aligned}$$

- IC set: given a, a set of α that are incentive compatible.
- Each α in the set is associated with a unique $e\left(\pi|\phi, a, \alpha\right)$.

Direct optimization

Optimal (a, α) is the point in IC set that maximizes the committed type's payoff.

Trade-offs:

- flow utility: (a, e) to smooth π and x
- continuation value:
  - η' measures the cost of delivering e,
  - ρ' measures reputation gain from the gap between a and α.
- to deliver the promise made in the last period: $-\eta\left[\rho a + (1-\rho)\alpha\right]$

Numerical algorithm

Start with a set of guessed functions $a(\rho, \eta, \varsigma), \alpha(\rho, \eta, \varsigma), W(\rho, \eta, \varsigma)$

1. Given a state (ρ, ς), identify the incentive compatible set of α for each a.
2. Find (a^*, α^*) for each η that maximizes the committed type's payoff.

Update the guessed functions.

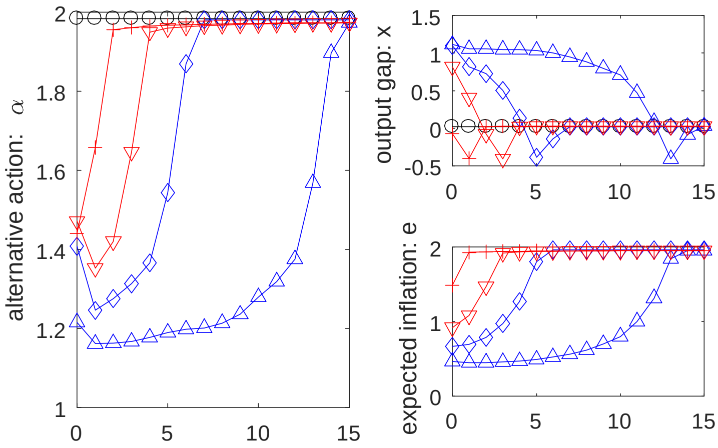
Iterate until policy functions converge.

Parameter values

β	Discount factor	0.995
q	Replacement probability	0.03
h	Output weight	0.017
x^*	Output target	0.05
κ	PC output slope	0.17
σ_ε	Std of implementation error	0.5%
σ_ξ	Std of cost-push shock	0.5%
δ	Persistence of cost-push shock	0.9

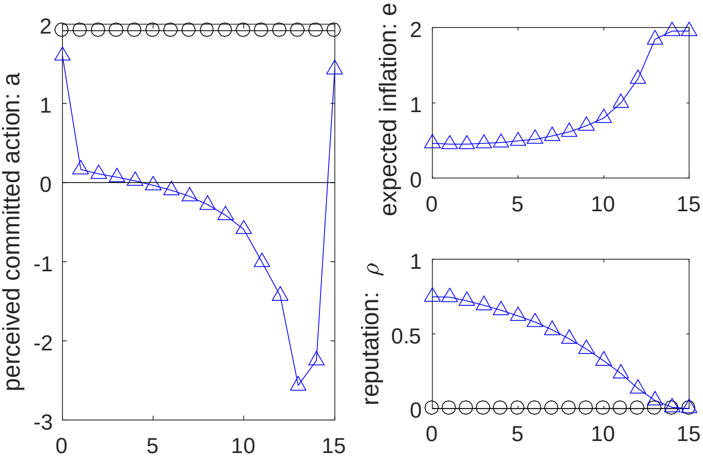
- Initial reputation after replacement: $1\% + 0.5\rho_{-1}$.

D1: Transitional Dynamics



	$*$	$\triangle$	$\Diamond$	∇	$+$	o
$\rho_0 =$	-	.75	.4	.2	.1	0

D1: Transitional Dynamics



	$*$	$\triangle$	$\lozenge$	$\triangledown$	$+$	o
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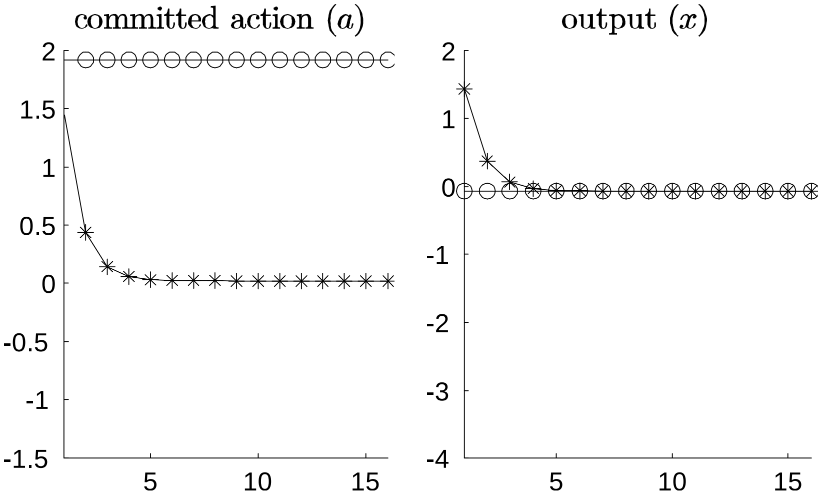
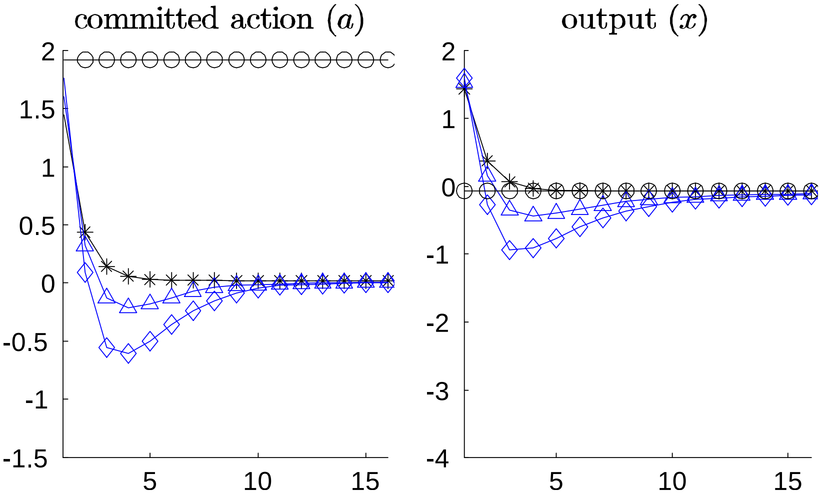
C1: Transitional Dynamics



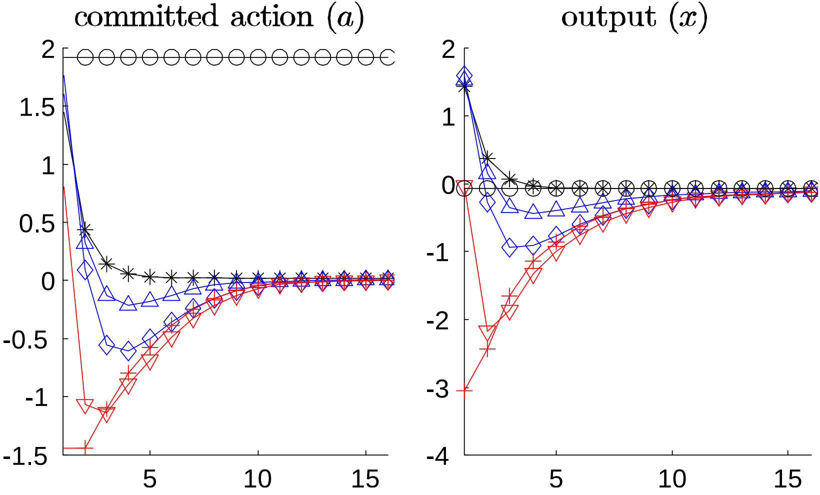
	$*$	$\triangle$	$\lozenge$	$\triangledown$	$+$	o
$\rho_0 =$	1					0

C1: Transitional Dynamics



	$*$	$\triangle$	$\Diamond$	∇	$+$	o
$\rho_0 =$	1	.8	.5			0

C1: Transitional Dynamics



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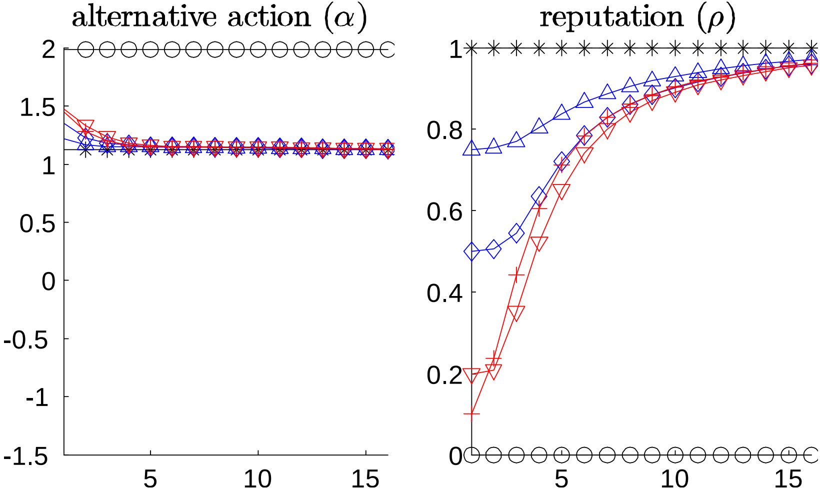

	$*$	$\triangle$	$\Diamond$	∇	$+$	o
$\rho_0 =$	1	.8	.5	.2	.1	0

Conclusions

- We provide a method to compute optimal committed and discretionary policy
  - when policymaker has imperfect credibility
  - private sector is learning about the policymaker's type
  - private sector has forward-looking expectations
- Equilibrium dynamics under discretionary type is consistent with U.S. inflation experience in 60s and 70s.
  - lengthy real stimulation with gradually rising actual and expected inflation
  - reputation gradually erodes
  - stagflation in the end
- Optimal committed policy depends nonlinearly on CB's initial reputation.
  - good initial reputation: close to standard solution under full commitment
  - poor initial reputation: anti-inflation policies with real output costs

Calibration details

- $\{h, x^*, \kappa, \beta\}$ consistent with
  - the elasticity of marginal cost with respect to the output ($A = 2$);
  - the demand elasticity ($\epsilon = 10$); implying a gross markup 1.11.
  - the probability of reoptimizing price each period ($1 - \theta = 0.25$).
  - a steady-state interest rate of about 2% annually.
- Std of ε: 1% annually, matching Mishkin and Schmidt-Hebbel (2007).
- $\varsigma_t = \varsigma_{t-1}$ with probability δ and $\varsigma_t = \xi_t$ with probability $1 - \delta$
  - $\delta = 0.9$;
  - ξ_t is uniformly over $[-\tilde{\varsigma}, \tilde{\varsigma}]$ with the std $\sigma_\xi = 0.5\%$ quarterly.