(1a) Use Gram-Schmidt orthonormalization to find an orthonormal basis e_1, e_2 for the range of the matrix

$$A = \begin{bmatrix} 1 & 5 & 9 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \\ 4 & 8 & 12 \end{bmatrix} = [a_1|a_2|a_3]$$

(1b) Find the 4×4 matrix P which projects orthonormally onto the range of A.

(2a) Compute the complex Fourier coefficients of

$$f(x) = |x|$$

on the interval $-\pi < x < \pi$.

(2b) Evaluate the sum

$$S = \sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} = \frac{\pi^2}{8}.$$

(2c) State a theorem justifying (2b) and verify its hypotheses on $f(x) = |x|$.

(3a) Let $u(x,t)$ be the solution of the Schrodinger equation

$$u_t = iu_{xx},$$

where $i = \sqrt{-1}$, which is 2π-periodic in x and satisfies the initial condition $u(x,0) = g(x)$ where $g \in L^2(-\pi,\pi)$. Find the complex Fourier coefficients $\hat{u}(k,t)$ in terms of $\hat{g}$.

(3b) Show that
$$\int_{-\pi}^{\pi} |u(x,t)|^2 dx = \int_{-\pi}^{\pi} |g(x)|^2 dx$$
for all $t \geq 0$.

(3c) Show that u is 2π-periodic in t:

$$u(x, t + 2\pi) = u(x, t)$$

for all $t \geq 0$.