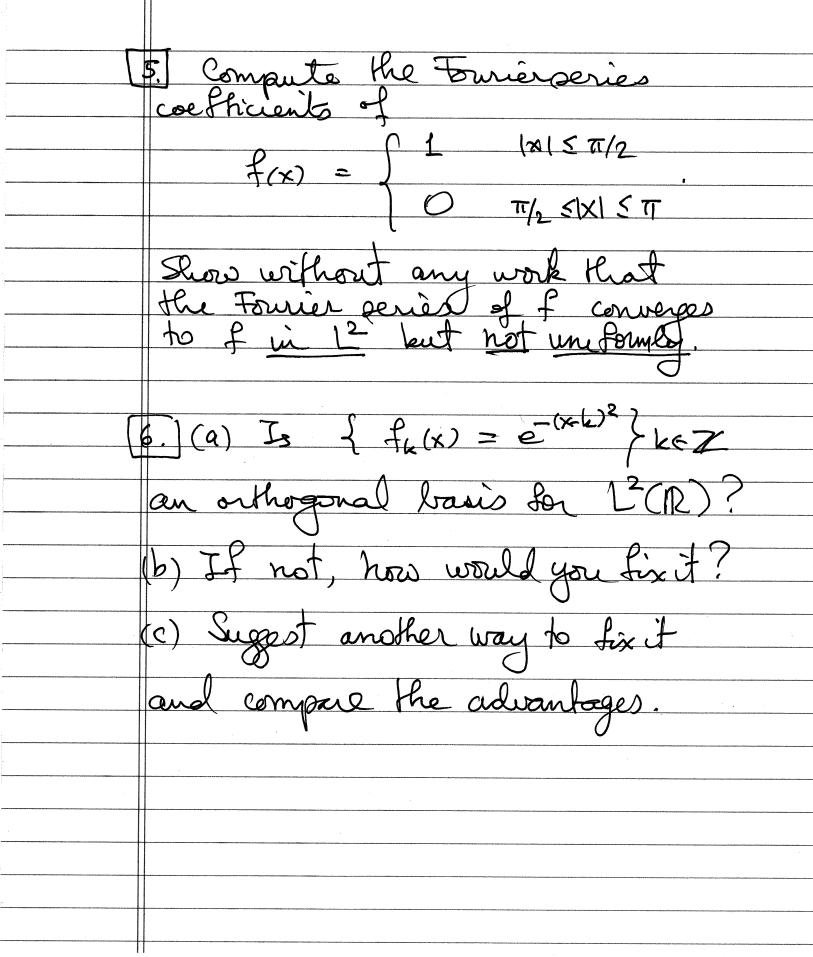


for $f(x) = -2xe^{x^2}$.

3. Compute the Fourier transform of $f(x) = \frac{-2x}{(1+x^2)^2}$

and use it to evaluate $\int_{-\frac{\pi}{2}}^{\infty} \frac{dx}{(1+x^2)^4} dx.$

4. I compute the Fourier series coefficients $\hat{f}(k) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-ikx} dx \quad k \in \mathbb{Z}$ $f(x) = \pi^2 x^2.$

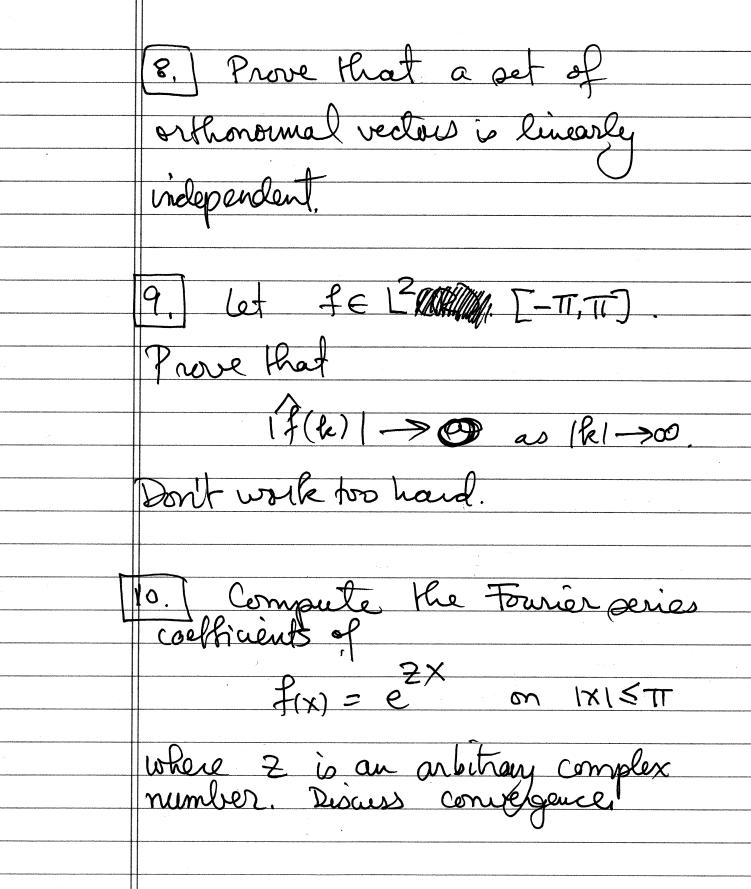


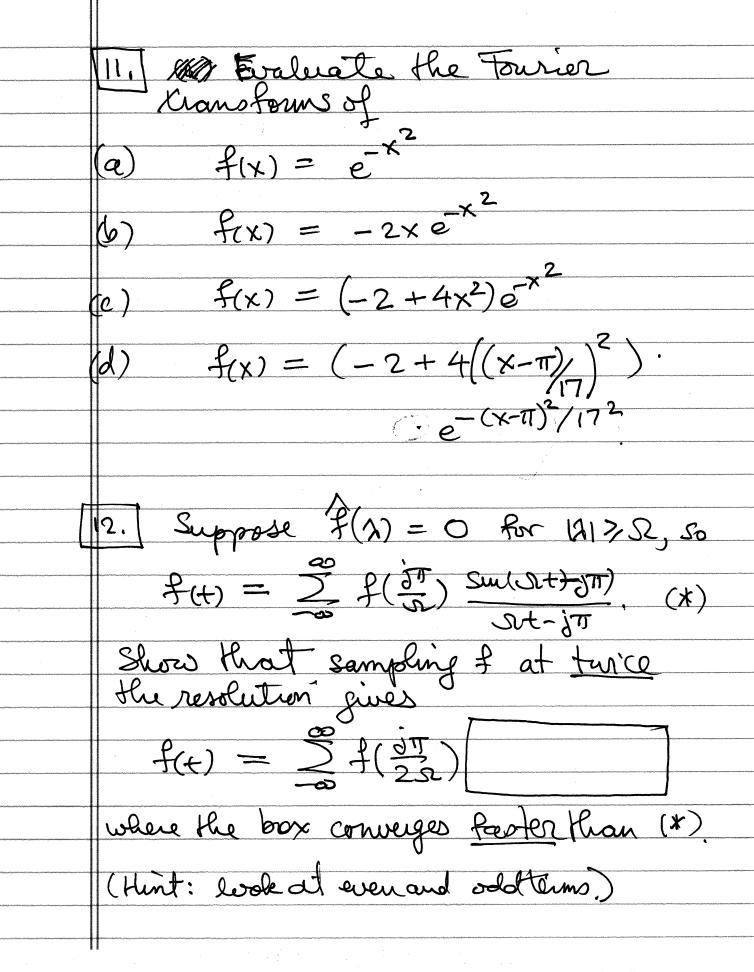
$$f_{k}(x) = x^{k-1} e^{-x^{2}/2}$$

$$S_4 \longrightarrow S_2 \oplus T_2$$
.

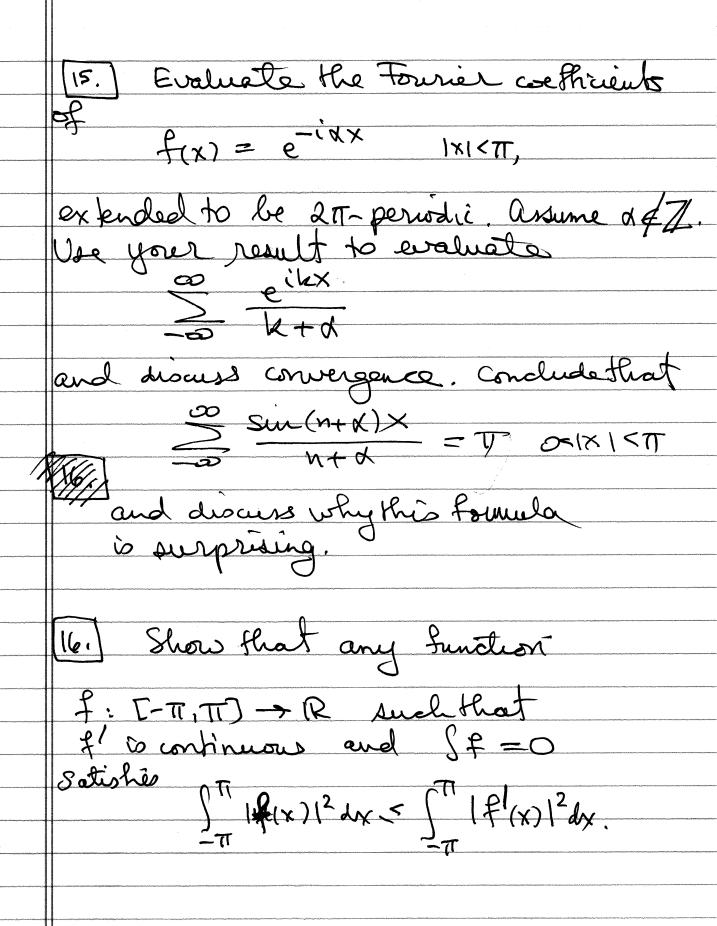
$$\begin{cases} 2 \\ 3 = 1 \end{cases} \Rightarrow \begin{cases} 3 = 1 \end{cases} \Rightarrow \begin{cases} 3 \\ 3 = 1 \end{cases} \Rightarrow \begin{cases} 3 = 1$$

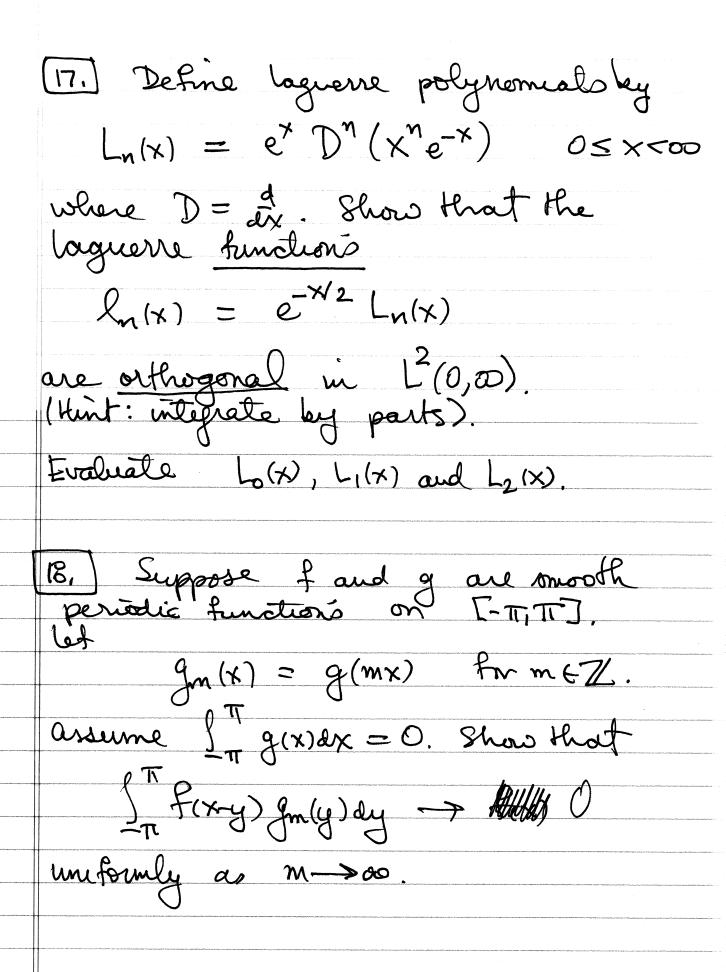
$$M: \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \longrightarrow \begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \end{bmatrix}$$





13. (a) Evaluate Sn(x) = _ sm(j'x) (b) Show that the Cesaro sum $\frac{1}{N+1}\sum_{n=0}^{N} S_n(x) \rightarrow \frac{1}{2} \cot(\frac{N}{2})$ as N->0. 14. lef f(x) = x2 for 1x15TT and F(x) = x4 for 1x15T. Is it true that $F''(k) = 12\hat{f}(k)$? Why or why not?





[19.] (a) Use the Possson sumformula
to show that
$$\frac{2}{1-e^{-\frac{1}{4}}} = 1 + \sum_{n \in \mathbb{Z}} \frac{2t}{t^2 + 4\pi^2 n^2}.$$

(b) Define the Beinoulli numbers by key
$$\frac{1}{1-e^{-\frac{1}{4}}} = \frac{1}{e^{-\frac{1}{4}}} + \frac{1}{2} + \sum_{N=1}^{\infty} \frac{k_N}{N!} (-+)^N + \infty$$

Compute b, and bz.

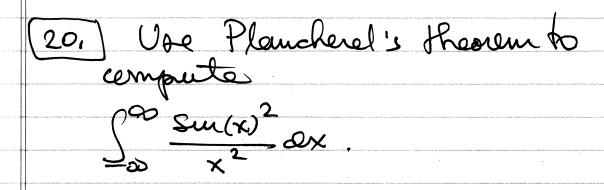
c) The Riemann zeta function is defined by

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$$

Use (a) and (b) to show that $\frac{5(2m) = \frac{(-1)^{m-1} 2^{2m-1}}{(2m-1)!} b_{2m-1} \pi^{2m}$

and evaluate

$$\sum_{1}^{\infty} \frac{1}{n^2} \text{ and } \sum_{1}^{\infty} \frac{1}{n^4}$$



21.] One the following filters

(A) unear (B) time-invariant (c) causal?

1. Let (+) = $f(t) - \int_{t}^{2} f(s) ds$ 2. Let (+) = $\int_{t}^{\infty} f(x) e^{-(x+t)^{2}} dx$

3. L3 f(*) = (F(x-t) et dt

(22.) Suppose P: H-> H to a brounded operator on an uner product operator of P = P* = P.

Show that P(I-P) = O. Give a nontrivial example. Give an example on $H = L^2(IR)$ where P is an integral operator.

23. Show that $\hat{c}(\hat{s}) = 2\sqrt{\frac{2}{11}} \frac{\sin^2(\hat{s}/2)}{\hat{s}^2\sqrt{1-\frac{2}{3}}\sin^2(\hat{s}/2)}$ has orthonormal shefts. < 4(·-k), 4(·-l)> = 501.

24. Show that suc(x) = Sui(TX)

 $\varphi(x) = \varphi(2x) + \sum_{i} \frac{2(-i)^{k}}{(2k+i)!} \varphi(2x-2k-1)$

25.