Problem Set 7

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1 Lecture note

($\implies$) Suppose not, so $Z = g(X_i, \theta_0) - g(X_i, \theta) = 0$. Then $Z^2 = 0$ and $EZ^2 = 0$, which is a contradiction.

($\impliedby$) Suppose that $P(Z \neq 0) = P(Z^2 > 0)$. Then $\exists n \in \mathbb{N}$ s.t. $P(Z^2 \geq n^{-1}) > 0$ so that

$$EZ^2 \geq EZ^2 \mathbb{1}\{Z^2 \geq n^{-1}\} \geq n^{-1} P(Z^2 \geq n^{-1}) > 0$$

2 Lemma 11.1

Lemma 11.1 is that Θ compact, $Q(\theta)$ continuous, and θ_0 uniquely minimizes $Q(\theta)$ over $\theta \in \Theta$ implies $\forall \epsilon < 0, \inf_{\theta \notin B(\theta_0, \epsilon)} Q(\theta) > Q(\theta_0)$. Suppose not, so $\exists \epsilon > 0$ s.t. $\inf_{\theta \notin B(\theta_0, \epsilon)} Q(\theta) \leq Q(\theta_0)$. Since Θ is compact, $\Theta \backslash B(\theta_0, \epsilon)$ is also compact. Since $Q(\theta)$ is continuous, by the extreme value theorem $\exists \theta^* \in \Theta \backslash B(\theta_0, \epsilon)$ s.t. $Q(\theta^*) = \inf_{\theta \notin B(\theta_0, \epsilon)} Q(\theta)$. Then $Q(\theta^*) \leq Q(\theta_0)$ which violates the final assumption that θ_0 be the unique minimizer of $Q(\theta)$ on Θ.