(1) Use the probability integral transformation method to simulate from the distribution

$$f(x) = \begin{cases} \frac{2}{a^2}x, & \text{if } 0 \leq x \leq a \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where $a > 0$. Set a value for a, simulate various sample sizes, and compare results to the true distribution.

Problem 1: Probability Integral Transformation

Using the probability integral transformation method, we generate samples from the given distribution:

$$f(x) = \begin{cases} \frac{2}{a^2}x, & \text{if } 0 \leq x \leq a \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

The inverse CDF method is used to transform uniform samples into the target distribution:

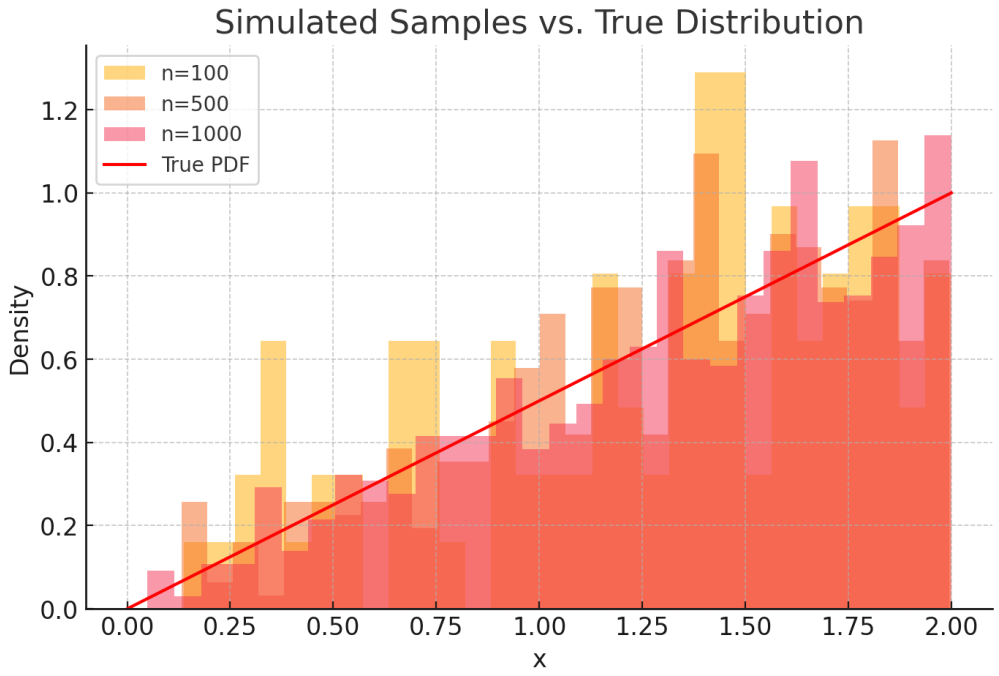

Figure 1: Simulated Samples vs. True Distribution

(2) Generate samples from the distribution

$$f(x) = \frac{2}{3}e^{-2x} + 2e^{-3x} \tag{3}$$

using the finite mixture approach.

Problem 2: Finite Mixture Simulation

The given density function:

$$f(x) = \frac{2}{3}e^{-2x} + \frac{1}{3}3e^{-3x} \tag{4}$$

is a mixture of two exponential distributions. We simulate this using a component selection approach:

(3) Draw 500 observations from Beta$(3,3)$ using the accept-reject algorithm. Compute the mean and variance of the sample and compare them to the true values.

Problem 3: Accept-Reject Algorithm for Beta(3,3)

We use the accept-reject method to generate samples from Beta$(3,3)$:

$$\begin{aligned} \text{Sample Mean: } 0.5119, \quad & \text{True Mean: } 0.5000 \\ \text{Sample Variance: } 0.0322, \quad & \text{True Variance: } 0.0357 \end{aligned}$$

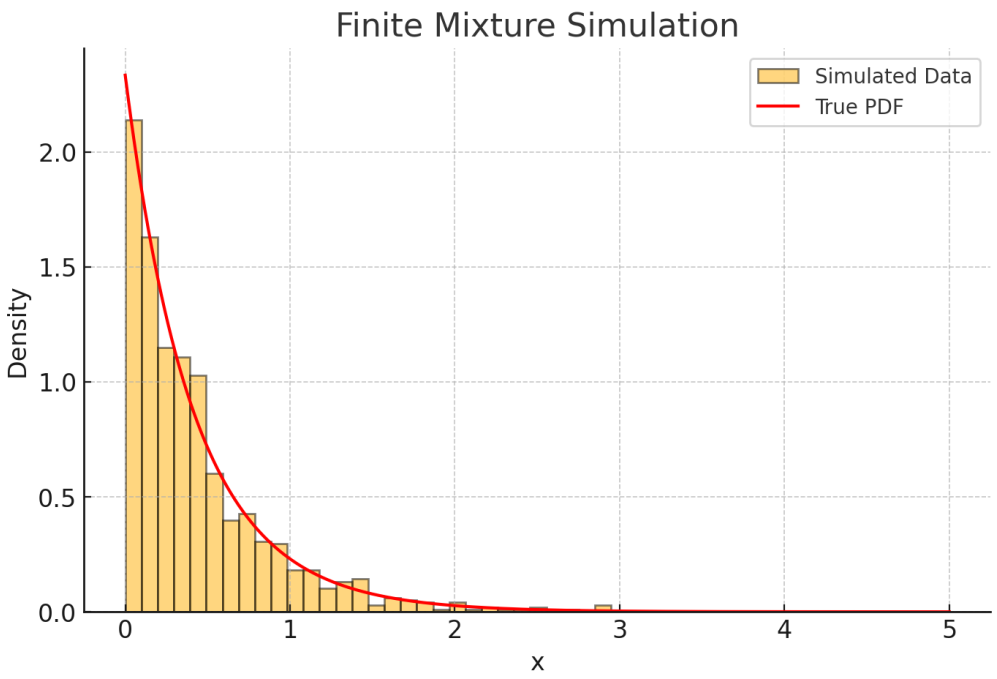

Figure 2: Finite Mixture Simulation

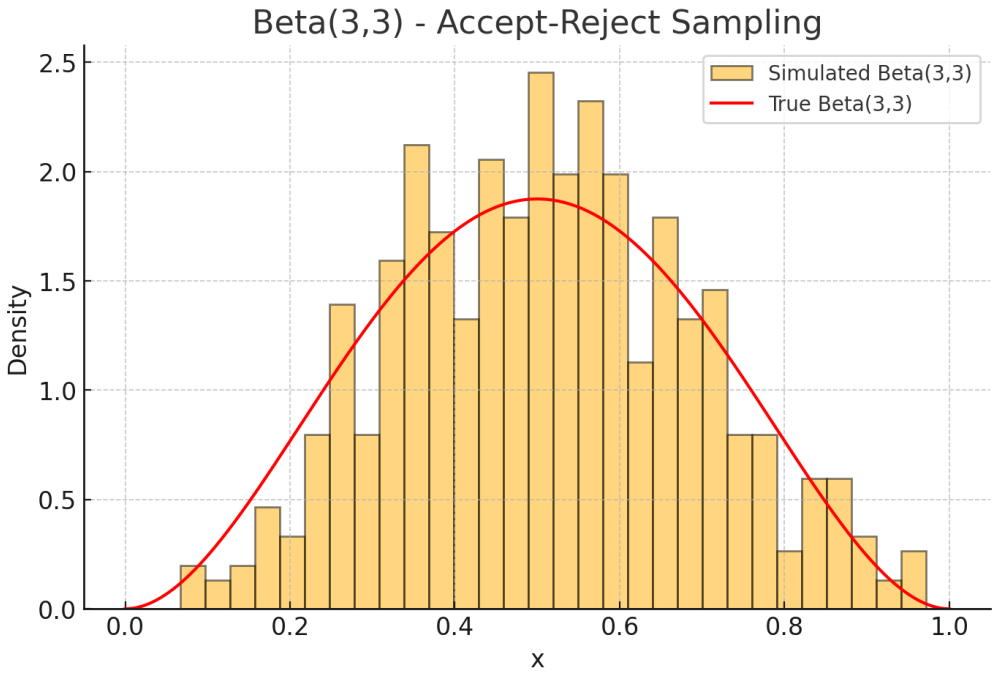

Figure 3: Beta(3,3) - Accept-Reject Sampling