**Your Name: JOSEPH AGOR** **Problem Set 2, Due: Mar. 18, 2025**

(1) Use the probability integral transformation method to simulate from the distribution

$$f(x) = \begin{cases} \frac{2}{a^2}x, & \text{if } 0 \le x \le a \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

where $a > 0$. Set a value for $a$, simulate various sample sizes, and compare results to the true distribution.

## Question 1: Probability Integral Transformation

Given the probability density function:

$$f(x) = \begin{cases} \frac{2}{a^2}x, & 0 \le x \le a \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

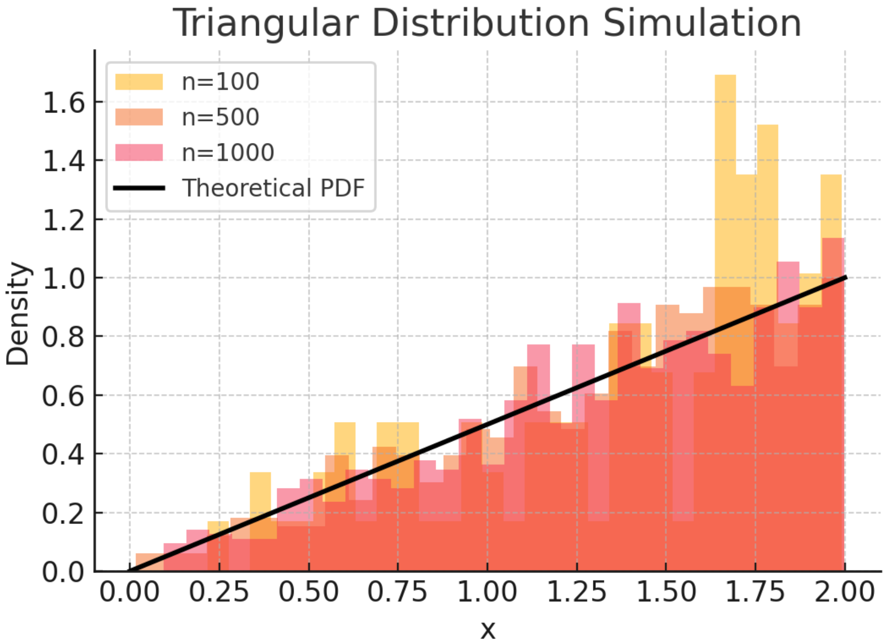
We use the inverse transform method:

$$F(x) = \int_0^x \frac{2}{a^2} t dt = \frac{x^2}{a^2}. \quad (3)$$

Solving for $x$ in terms of $U \sim U(0,1)$:

$$x = a\sqrt{U}. \quad (4)$$

We simulate samples for different sizes and compare them with the theoretical distribution.


Triangular Distribution Simulation
n=100
n=500
n=1000
Theoretical PDF
Density
x
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

(2) Generate samples from the distribution

$$f(x) = \frac{2}{3}e^{-2x} + 2e^{-3x} \tag{5}$$

using the finite mixture approach.

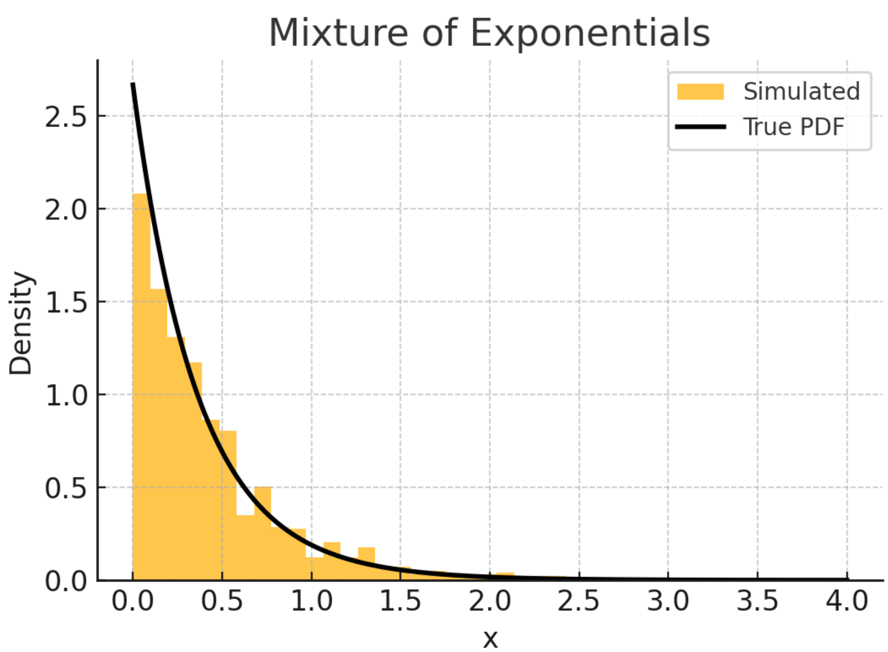
## Question 2: Mixture of Exponentials

The given density function is:

$$f(x) = \frac{2}{3}e^{-2x} + 2e^{-3x}. \tag{6}$$

We use the finite mixture approach where each component is selected with equal probability. The sampling method follows:

- Generate $U \sim U(0,1)$.
- If $U < 0.5$, sample $X \sim \text{Exp}(2)$, else sample $X \sim \text{Exp}(3)$.



(3) Draw 500 observations from Beta$(3,3)$ using the accept-reject algorithm. Compute the mean and variance of the sample and compare them to the true values.

## Question 3: Beta(3,3) using Accept-Reject

Using $g(x) = U(0,1)$ as the proposal distribution, we have:

$$f(x) = \frac{\Gamma(6)}{\Gamma(3)\Gamma(3)} x^2(1-x)^2. \tag{7}$$

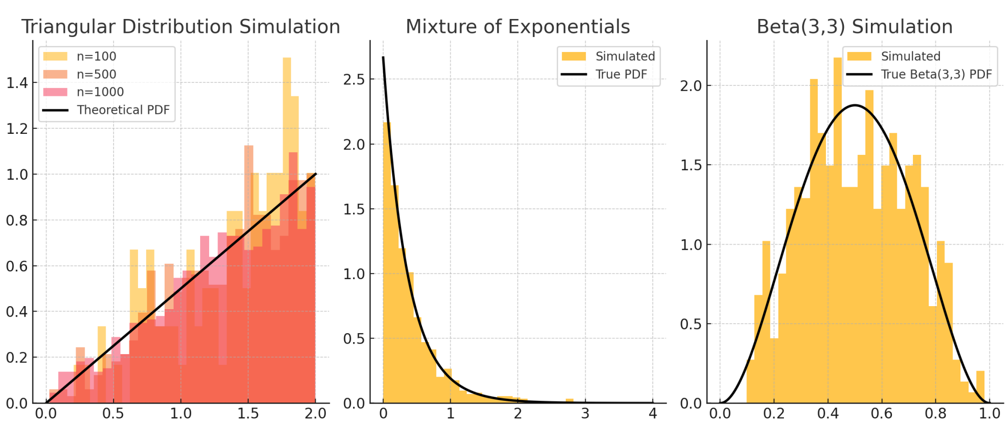
The constant $c$ is chosen such that:

$$c \geq \sup_x \frac{f(x)}{g(x)} = 1.5. \tag{8}$$

The accept-reject algorithm follows:

- Generate $Y \sim U(0,1)$.
- Generate $U \sim U(0,1)$.
- Accept $Y$ if $U \leq \frac{f(Y)}{cg(Y)}$.

The empirical mean and variance are compared with theoretical values.




Empirical Mean: `0.508045063498666`, Theoretical Mean: `0.5`
Empirical Variance: `0.03895205175094688`, Theoretical Variance: `0.0278`