Lecture 3 A Minimal Case Study

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Introduction to Neural Networks

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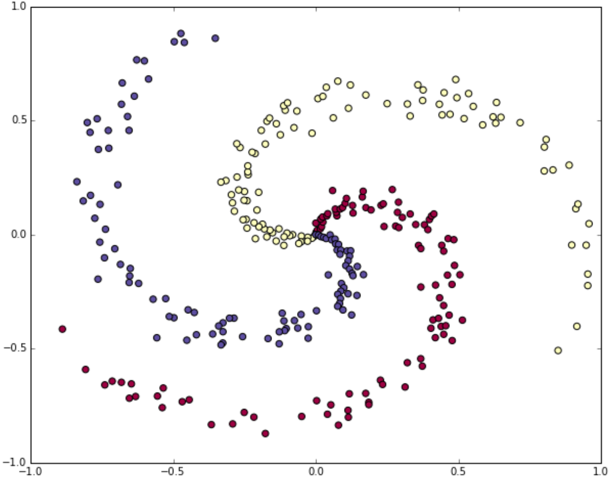
Nonlinear Classification



The Road Ahead...

1. Training Softmax Classifier
2. Training Neural Network
3. Becoming Backprop Ninja

Throwback

L^2 regularized loss

$$L = \frac{1}{N}\sum_i L_i + \frac{1}{2}\lambda \sum_{i,j} W_{ij}^2$$

- Softmax classifier
  - linear score: $f(x_i, W, b) = Wx_i + b$
  - cross-entropy loss: $L_i = -\log(p_{y_i})$, $p_k = \frac{e^{f_k}}{\sum_j e^{f_j}}$
- Backpropagation

$$\frac{\partial L}{\partial W_{k\cdot}} = \frac{1}{N}\sum_i \underbrace{\frac{\partial L_i}{\partial f_k}}_{p_k - \mathbb{1}(y_i = k)} \underbrace{\frac{\partial f_k}{\partial W_{k\cdot}}}_{x_i'} + \lambda W_{k\cdot}$$

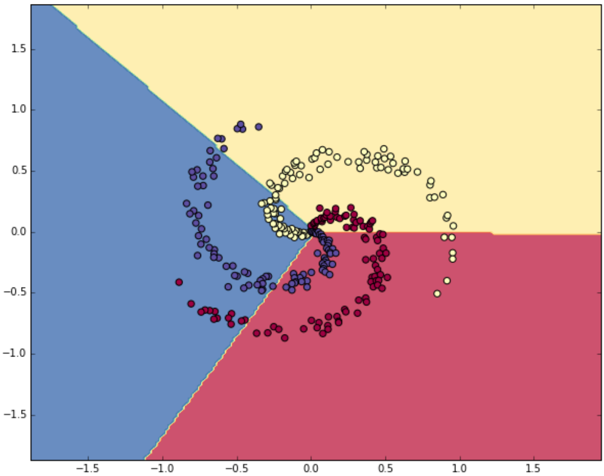
Forward-Backward Pass

```
import numpy as np

# Forward pass
exp_scores = np.exp(np.dot(X, W) + b)
probs = exp_scores / np.sum(exp_scores, axis=1,
    keepdims=True)
correct_logprobs = -np.log(probs[range(
    num_examples), y])
loss = np.sum(correct_logprobs) / num_examples +
    0.5 * reg * np.sum(W * W)

# Backward pass
probs[range(num_examples),y] -= 1
dscores = probs / num_examples
dW = np.dot(X.T, dscores)
db = np.sum(dscores, axis=0, keepdims=True)
dW += reg*W
```

Softmax Classifier



The Road Ahead...

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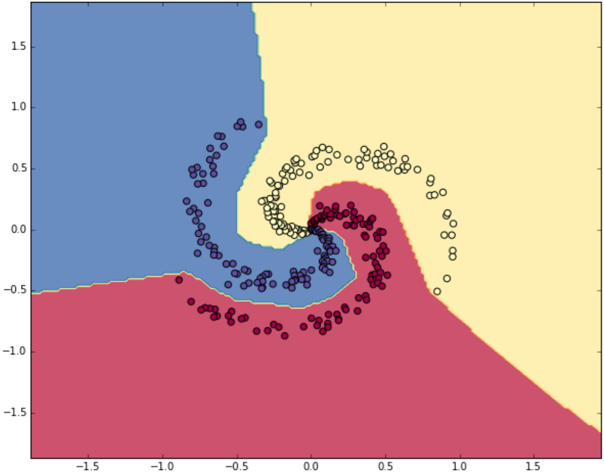
Forward-Backward Pass

```
# Forward pass
hidden_layer = np.maximum(0, np.dot(X, W) + b)
scores = np.dot(hidden_layer, W2) + b2

# Backward pass
dW2 = np.dot(hidden_layer.T, dscores)
db2 = np.sum(dscores, axis=0, keepdims=True)
dhidden = np.dot(dscores, W2.T)
dhidden[hidden_layer <= 0] = 0  # ReLU
dW = np.dot(X.T, dhidden)
db = np.sum(dhidden, axis=0, keepdims=True)
```

Neural Network



PyTorch Implementation

```
import torch
import torch.nn as nn
import torch.optim as optim

X_tensor = torch.tensor(X, dtype=torch.float32)
y_tensor = torch.tensor(y, dtype=torch.long)

class SimpleNN(nn.Module):
    def __init__(self, D, h, K):
        super(SimpleNN, self).__init__()
        self.fc1 = nn.Linear(D, h)
        self.relu = nn.ReLU()
        self.fc2 = nn.Linear(h, K)

    def forward(self, x):
        x = self.fc1(x)
        x = self.relu(x)
        x = self.fc2(x)
        return x
```

PyTorch Implementation (Cont'd)

```
# Initialization
model = SimpleNN(D, h, K)
criterion = nn.CrossEntropyLoss()
optimizer = optim.SGD(model.parameters(), lr=1e-0,
    weight_decay=1e-3)

# Training
for epoch in range(num_epochs):
    # Forward pass
    scores = model(X_tensor)
    loss = criterion(scores, y_tensor)
    optimizer.zero_grad()  # zero grad!

    # Backward pass
    loss.backward()
    optimizer.step()
```

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Deep Learning Then vs. Now

Bonus Question

- Consider vanilla neural network

$$
\begin{aligned}
z_i^{(l)} &= B^{(l)} a_i^{(l-1)} + b^{(l)}, \quad l = 1, \ldots, L+1 \\
a_i^{(l)} &= \max(0, z_i^{(l)}), \quad l = 1, \ldots, L
\end{aligned}
$$

  with parameters

$$
\beta = [\beta^{(1)'}, b^{(1)'}, \ldots, \beta^{(L+1)'}, b^{(L+1)'}]', \quad \beta^{(l)} = \mathrm{vec}(B^{(l)'})
$$

- Prove backprop recursion

$$
\begin{aligned}
V_i^{(l)} &= \frac{\partial z_i^{(L+1)}}{\partial b^{(l)'}} = V_i^{(l+1)} B^{(l+1)} D_i^{(l)}, \quad D_i^{(l)} = \frac{\partial a_i^{(l)}}{\partial z_i^{(l)'}} \\
U_i^{(l)} &= \frac{\partial z_i^{(L+1)}}{\partial \beta^{(l)'}} = V_i^{(l)} (I \otimes a_i^{(l-1)'}), \quad l = 1, \ldots, L
\end{aligned}
$$

References

- cs231n.stanford.edu – CS231n: Deep Learning for Computer Vision
- A visual proof that neural nets can compute any function [link], by Michael Nielsen
- PyTorch internals [link], by Edward Yang