Lecture 4   Language Modeling

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Introduction to Neural Networks

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Makemore

- Autoregressive character-level language model

  - input: a text file of words

  - output: similar words as input

- Application to name generation

```
# Input: 32K common names from ssa.gov in 2018
$ python makemore.py -i names.txt -o names

# Output: name-like words
dontell
khylum
camatena
aeriline
najlah
sherrith
```

The Road Ahead...

Bigram Counts

Preprocessing

```python
import torch

# Training dataset
xs, ys = [], []
for w in words:          # list of names
    chs = ['.'] + list(w) + ['.']
    for ch1, ch2 in zip(chs, chs[1:]):
        ix1 = stoi[ch1]  # char to index
        ix2 = stoi[ch2]
        xs.append(ix1)
        ys.append(ix2)
xs = torch.tensor(xs)
ys = torch.tensor(ys)

# Weight initialization
g = torch.Generator().manual_seed(2147483647)
W = torch.randn((27, 27), generator=g,
    requires_grad=True)
```

Training

```
import torch.nn.functional as F

# Gradient descent (maximum likelihood estimation)
for k in range(10000):
    # Forward pass
    xenc = F.one_hot(xs, num_classes=27).float() #
        one-hot encoding to avoid ordering
    logits = xenc @ W  # softmax classifier
    counts = logits.exp()
    probs = counts / counts.sum(1, keepdims=True)
    loss = -probs[torch.arange(num), ys].log().
        mean() + 0.01*(W**2).mean()
    print(loss.item())

    # Backward pass
    W.grad = None       # zero gradients
    loss.backward()
    W.data += -0.1 * W.grad
```
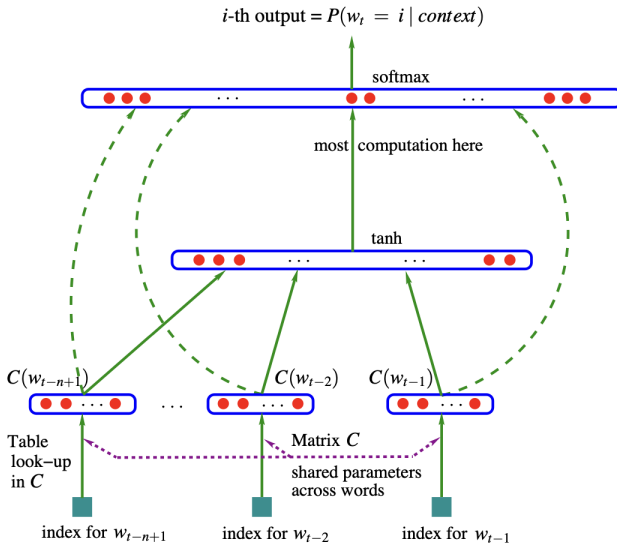
Prediction

```python
# Sampling
for i in range(5):
    out = []
    ix = 0
    while True:
        xenc = F.one_hot(torch.tensor([ix]),
            num_classes=27).float()
        logits = xenc @ W
        counts = logits.exp()
        p = counts / counts.sum(1, keepdims=True)
        ix = torch.multinomial(p, num_samples=1,
            replacement=True, generator=g).item()
        out.append(itos[ix])
        if ix == 0:
            break
    print(''.join(out))  # mor. axx. minaymoryles.
        kondlaisah. anchthizarie.
```

The Road Ahead...

Model Architecture

i-th output = $P(w_t = i \mid context)$

softmax

most computation here

tanh

$C(w_{t-n+1})$

$C(w_{t-2})$

$C(w_{t-1})$

Table
look−up
in C

Matrix C
shared parameters
across words

index for w_{t-n+1}

index for w_{t-2}

index for w_{t-1}

PyTorch-Like API

```python
class Linear:   # torch.nn.Linear
    def __init__(self, fan_in, fan_out, bias=True):
        self.weight = torch.randn((fan_in, fan_out
            ), generator=g)   # or Kaiming init
        self.bias = torch.zeros(fan_out) if bias
            else None
    def __call__(self, x):
        self.out = x @ self.weight
        if self.bias is not None:
            self.out += self.bias
        return self.out
    def parameters(self):
        return [self.weight] + ([] if self.bias is
            None else [self.bias])

class Tanh:     # torch.nn.Tanh
    def __call__(self, x):
        self.out = torch.tanh(x)
        return self.out
```

PyTorch-Like API (Cont'd)

```python
class BatchNorm1d:   # torch.nn.BatchNorm1d
    def __init__(self, dim, eps=1e-5, momentum
        =0.1):
        self.eps = eps
        self.momentum = momentum
        self.training = True
        # Trained with backprop
        self.gamma = torch.ones(dim)
        self.beta = torch.zeros(dim)
        # Trained with running momentum update
        self.running_mean = torch.zeros(dim)
        self.running_var = torch.ones(dim)

    def parameters(self):
        return [self.gamma, self.beta]
    ...
```

PyTorch-Like API (Cont'd)

```python
class BatchNorm1d:  # torch.nn.BatchNorm1d
    def __call__(self, x):
        if self.training:
            xmean = x.mean(0, keepdim=True)
            xvar = x.var(0, keepdim=True)
        else:
            xmean = self.running_mean
            xvar = self.running_var
        xhat = (x - xmean) / torch.sqrt(xvar +
            self.eps)  # normalization
        self.out = self.gamma * xhat + self.beta
        if self.training:
            with torch.no_grad():
                self.running_mean = (1 - self.
                    momentum) * self.running_mean
                    + self.momentum * xmean
                self.running_var = (1 - self.
                    momentum) * self.running_var +
                     self.momentum * xvar
        return self.out
```

Preprocessing

```
for w in words:
    context = [0] * block_size
    for ch in w + '.':
        ix = stoi[ch]
        X.append(context)
        Y.append(ix)
        context = context[1:] + [ix]    # shift
X = torch.tensor(X)
Y = torch.tensor(Y)

C = torch.randn((vocab_size, n_embd), generator=g)
layers = [
    Linear(n_embd * block_size, n_hidden, bias=
        False), BatchNorm1d(n_hidden), Tanh(),
    Linear(n_hidden, n_hidden, bias=False),
        BatchNorm1d(n_hidden), Tanh(),
    ...
    Linear(n_hidden, vocab_size, bias=False),
        BatchNorm1d(vocab_size)]
```

Training

```
for i in range(200000):
    # Forward pass
    ix = torch.randint(0, X.shape[0], (batch_size
        ,)) # minibatch
    emb = C[X[ix]]
    x = emb.view(emb.shape[0], -1) # concatenate
    for layer in layers:
        x = layer(x)
    loss = F.cross_entropy(x, Y[ix])

    # Backward pass
    for layer in layers:
        layer.out.retain_grad()    # debug
    for p in parameters:
        p.grad = None
    loss.backward()
    lr = 0.1 if i < 100000 else 0.01
    for p in parameters:
        p.data += -lr * p.grad
```
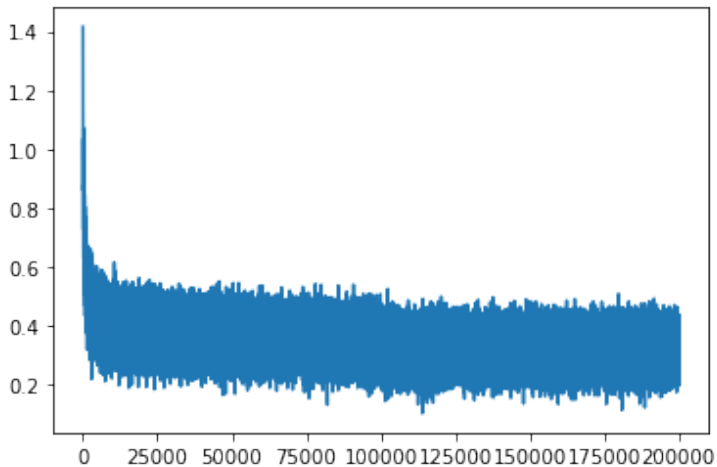
Prediction

```python
for _ in range(5):
    out = []
    context = [0] * block_size
    while True:
        emb = C[torch.tensor([context])]
        x = emb.view(emb.shape[0], -1)
        for layer in layers:
            x = layer(x)
        probs = F.softmax(x, dim=1)
        ix = torch.multinomial(probs, num_samples
            =1, generator=g).item()
        context = context[1:] + [ix]
        out.append(ix)
        if ix == 0:
            break
    print(''.join(itos[i] for i in out))  # carpah
        . qarlileif. jmrix. thty. sacansa.
```
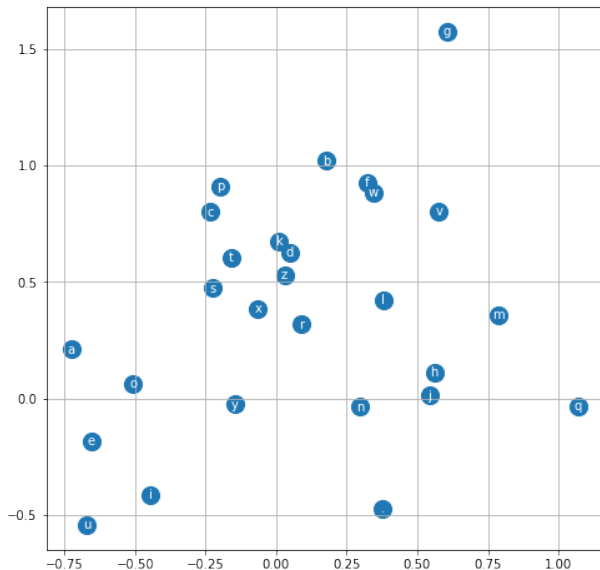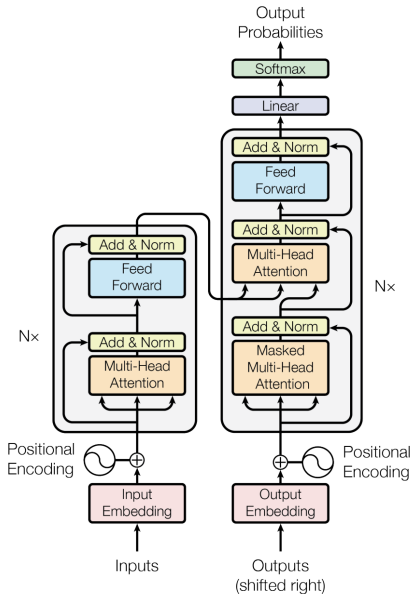
Diagnoses
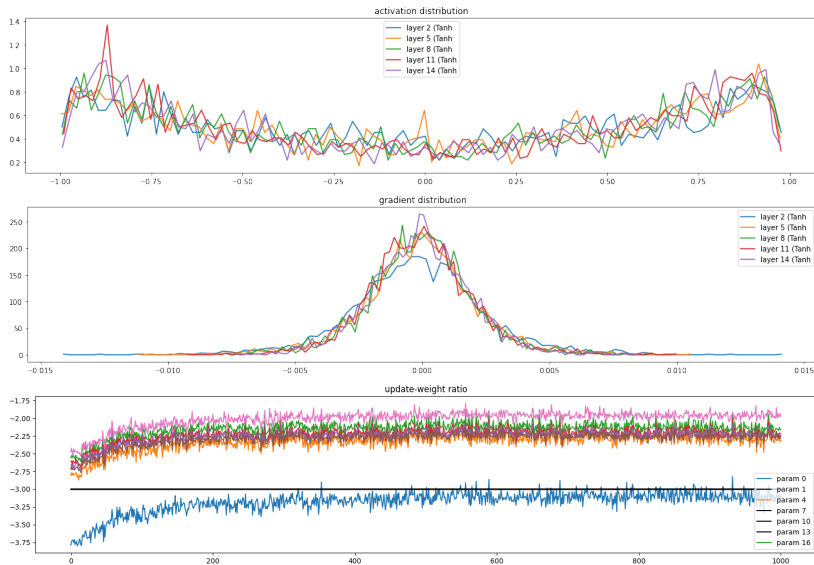
Loss Trace (log10)

Character Embeddings

# The Road Ahead...

Model Architecture

Attention

Scaled dot-product attention

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V$$

- ▶ Key concepts

    - ▶ *query* (Q) represents current word, *key* (K) captures other words, *value* (V) carries information

    - ▶ encoder vs. decoder, self-attention vs. cross-attention

- ▶ Why attention mechanism?

    - ▶ eliminate need for recurrence or convolution

    - ▶ superior performance while maintaining parallelization

Scaled Dot-Product Attention
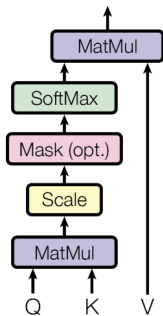
Multi-Head Attention
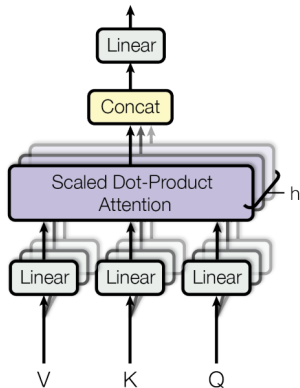
Single-Head Attention

```python
class Head(nn.Module):
    def __init__(self, head_size):
        super().__init__()
        self.key = nn.Linear(n_embd, head_size,
            bias=False)
        self.query = nn.Linear(n_embd, head_size,
            bias=False)
        self.value = nn.Linear(n_embd, head_size,
            bias=False)
        self.register_buffer('tril', torch.tril(
            torch.ones(block_size, block_size)))
            # not updated during backprop
        self.dropout = nn.Dropout(dropout)
    ...
```

Single-Head Attention (Cont'd)

```python
class Head(nn.Module):
    def forward(self, x):
        B,T,C = x.shape # (batch, time, channels)
        k = self.key(x)
        q = self.query(x)
        wei = q @ k.transpose(-2,-1) * k.shape
            [-1]**-0.5   # attention scores
        wei = wei.masked_fill(self.tril[:T, :T] ==
             0, float('-inf'))   # decoder
        wei = F.softmax(wei, dim=-1)
        wei = self.dropout(wei)
        v = self.value(x)
        out = wei @ v    # (batch, time, head size)
        return out
```

Multi-Head Attention

```python
class MultiHead(nn.Module):
    def __init__(self, num_heads, head_size):
        super().__init__()
        self.heads = nn.ModuleList([Head(head_size
            ) for _ in range(num_heads)])
        self.proj = nn.Linear(head_size *
            num_heads, n_embd)
        self.dropout = nn.Dropout(dropout)

    def forward(self, x):
        out = torch.cat([h(x) for h in self.heads
            ], dim=-1)
        out = self.dropout(self.proj(out))
        return out
```

Feed Forward

```python
class FeedFoward(nn.Module):
    def __init__(self, n_embd):
        super().__init__()
        self.net = nn.Sequential(
            nn.Linear(n_embd, 4 * n_embd),
            nn.ReLU(),
            nn.Linear(4 * n_embd, n_embd),
            nn.Dropout(dropout))

    def forward(self, x):
        return self.net(x)
```

Transformer Block

```python
class Block(nn.Module):
    def __init__(self, n_embd, n_head):
        super().__init__()
        head_size = n_embd // n_head
        self.sa = MultiHeadAttention(n_head,
            head_size)
        self.ffwd = FeedFoward(n_embd)
        self.ln1 = nn.LayerNorm(n_embd)
        self.ln2 = nn.LayerNorm(n_embd)

    def forward(self, x):
        x = x + self.sa(self.ln1(x))
        x = x + self.ffwd(self.ln2(x))
        return x
```

GPT

```python
class GPT(nn.Module):
    def __init__(self):
        super().__init__()
        self.token_embedding_table = nn.Embedding(
            vocab_size, n_embd)
        self.position_embedding_table = nn.
            Embedding(block_size, n_embd)
        self.blocks = nn.Sequential(*[Block(n_embd
            , n_head=n_head) for _ in range(
            n_layer)])
        self.ln_f = nn.LayerNorm(n_embd)
        self.lm_head = nn.Linear(n_embd,
            vocab_size)
    ...
```

GPT (Cont'd)

```
class GPT(nn.Module):
    def forward(self, idx, targets):
        B, T = idx.shape
        tok_emb = self.token_embedding_table(idx)
        pos_emb = self.position_embedding_table(
            torch.arange(T))
        x = tok_emb + pos_emb
        x = self.blocks(x)
        x = self.ln_f(x)
        logits = self.lm_head(x)
        B, T, C = logits.shape
        logits = logits.view(B*T, C)
        targets = targets.view(B*T)
        loss = F.cross_entropy(logits, targets)
        return logits, loss

device = torch.device("mps" if torch.backends.mps.
    is_available() else "cpu")
model = GPT().to(device)
```

Prediction

```
# wget https://raw.githubusercontent.com/karpathy/
    char-rnn/master/data/tinyshakespeare/input.txt
First Citizen:
Before we proceed any further, hear me speak.

All:
Speak, speak.

# Output: Shakespeare-like text

VALHASINA:
Nobleman; go, then both groans to us.

AUFIDIUS:
O those prepation!
```

Tokenizer

- Byte-pair encoding for tokenizing text
  - iteratively replace most frequent pair of consecutive UTF-8 bytes (characters) with single byte
  - reduce vocabulary while being able to encode *any* word

- Example: "aaabdaaabac" → "XdXac", X=ZY, Y=ab, Z=aa

```
from minbpe import BasicTokenizer
tokenizer = BasicTokenizer()
text = "aaabdaaabac"
tokenizer.train(text, 256 + 3)  # 256 byte
    tokens + 3 merges
print(tokenizer.encode(text))   # [258, 100,
    258, 97, 99]
print(tokenizer.decode([258, 100, 258, 97,
    99]))  # aaabdaaabac
```

References

- ▶ github.com/karpathy/makemore – An autoregressive character-level language model for making more things

- ▶ github.com/karpathy/nanoGPT – The simplest, fastest repository for training/finetuning medium-sized GPTs

- ▶ github.com/karpathy/minbpe – Minimal, clean code for the Byte Pair Encoding (BPE) algorithm commonly used in LLM tokenization

- ▶ Bengio et al. (2003), "A Neural Probabilistic Language Model", *Journal of Machine Learning Research*

- ▶ Vaswani et al. (2017), "Attention is All You Need", arXiv:1706.03762