

Lecture 1 Linear Classification

Fei Tan

Department of Economics
Chaifetz School of Business
Saint Louis University

E6930 Introduction to Neural Networks

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Image Classification



08	02	22	97	38	15	00	40	00	75	04	05	07	78	52	12	50	77	97	29
49	49	99	40	17	81	18	57	60	87	17	40	98	43	69	45	74	56	62	00
81	49	31	73	55	79	14	29	93	71	40	67	58	85	30	03	49	13	36	65
52	70	95	23	04	60	11	42	68	44	68	56	01	32	56	71	37	02	36	91
22	31	16	71	51	60	05	89	41	92	36	54	22	40	40	28	66	33	13	80
24	47	33	60	99	03	45	02	44	75	33	53	78	36	84	20	35	17	12	50
02	95	81	28	64	23	67	10	26	38	40	67	59	54	70	66	18	38	64	70
67	26	20	68	02	62	12	20	95	63	94	39	63	08	40	91	66	49	94	21
24	35	38	05	66	73	99	26	97	17	78	78	96	83	14	88	34	89	63	72
21	36	23	09	75	00	76	44	20	45	35	14	00	61	33	97	34	31	33	95
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16	39	05	42	96	35	31	47	55	58	88	24	00	17	54	24	36	29	85	57
86	56	00	48	35	71	89	07	05	44	44	37	44	60	21	58	51	54	17	58
19	80	81	68	05	94	47	69	28	73	92	13	86	52	17	77	04	89	55	40
04	32	08	83	97	35	99	16	07	97	57	32	16	26	26	79	33	27	98	66
65	36	68	87	57	62	20	72	03	46	33	67	46	55	12	32	63	93	53	69
04	42	16	73	35	26	39	11	24	94	72	18	08	46	29	32	40	62	76	36
20	69	36	41	72	30	23	88	34	52	82	69	82	67	59	85	74	04	36	16
20	73	35	29	78	31	90	01	74	31	49	71	38	24	61	16	23	57	05	54
01	70	54	71	83	51	54	69	16	92	33	48	61	43	52	01	89	29	62	48

What the computer sees

image classification

82% cat
15% dog
2% hat
1% mug

The Road Ahead...

① Linear Classifiers

② Gradient Descent

③ Backpropagation

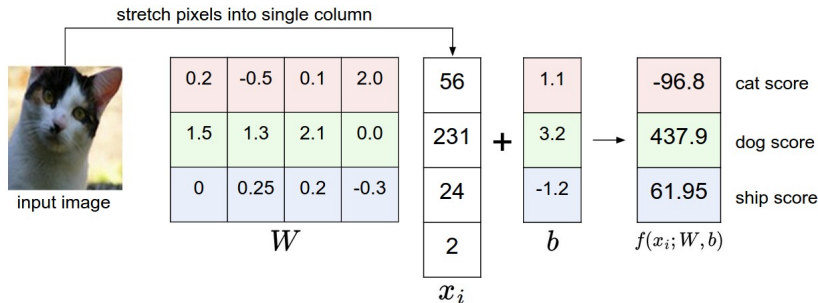
Score Function

Linear score

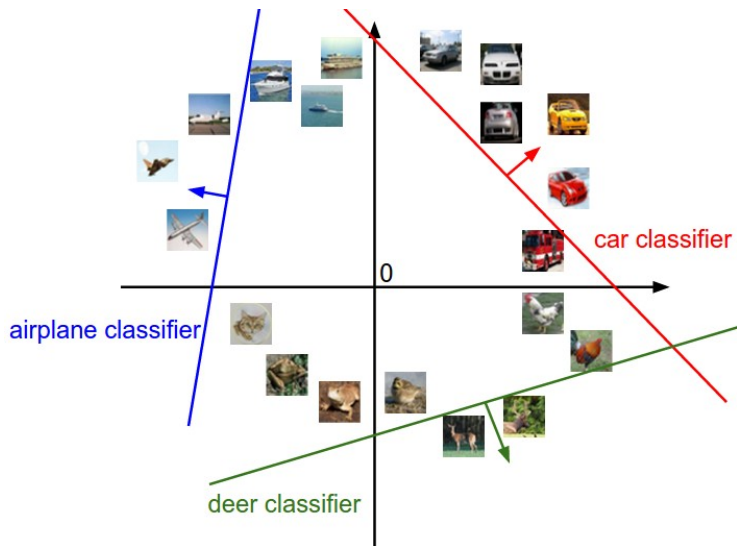
$$f(x_i, W, b) = Wx_i + b$$

- ▶ Notation
 - ▶ dataset of N examples $\{(x_i, y_i)\}_{i=1}^N$, D (normalized) features $x_i \in \mathbb{R}^D$, K categories of *single* attribute $y_i \in 1, \dots, K$
 - ▶ $K \times D$ weights W , $K \times 1$ bias b
- ▶ Pipeline
 - ▶ split data into training, validation, and test sets
 - ▶ train parameters and (cross) validate hyperparameters
 - ▶ evaluate on test set only once at the end

Algebraic Interpretation



Geometric Interpretation



Loss Function

Regularized loss

$$L = \frac{1}{N} \sum_i L_i + \lambda R(W), \quad \lambda > 0$$

- ▶ Examples of data loss
 - ▶ multiclass support vector machine (SVM) classifier uses hinge loss: $L_i = \sum_{j \neq y_i} \max(0, f_j - f_{y_i} + \Delta)$, $\Delta > 0$
 - ▶ Softmax classifier uses cross-entropy loss: $L_i = -\log \left(\frac{e^{f_{y_i}}}{\sum_j e^{f_j}} \right)$
- ▶ Examples of regularization/prior
 - ▶ L^1 regularization: $R(W) = \sum_{i,j} |W_{ij}|$
 - ▶ L^2 regularization: $R(W) = \sum_{i,j} W_{ij}^2$

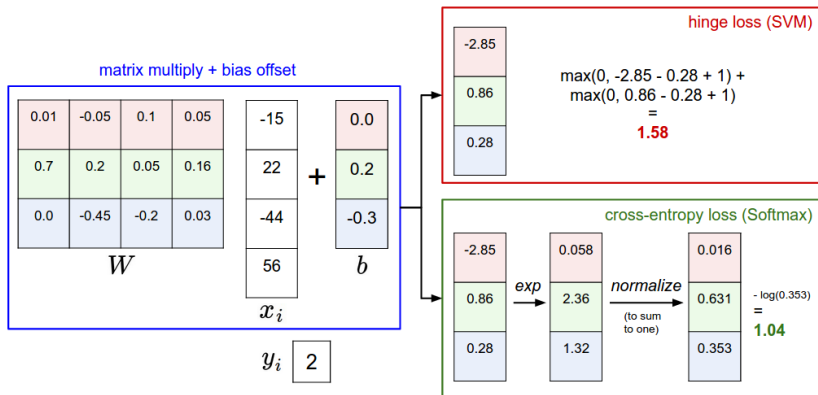
Loss Function (Cont'd)

```
import numpy as np

# SVM classifier
def L1_i(x_i, y_i, W, b):
    delta = 1.0
    f = W.dot(x_i) + b
    margins = np.maximum(0, f - f[y_i] + delta)
    margins[y_i] = 0 # ignore true class
    return np.sum(margins)

# Softmax classifier
def L2_i(x_i, y_i, W, b):
    f = W.dot(x_i) + b
    f -= np.max(f) # avoid potential blowup
    p = np.exp(f) / np.sum(np.exp(f))
    return -np.log(p[y_i])
```


SVM vs. Softmax



The Road Ahead...

- ① Linear Classifiers
- ② Gradient Descent
- ③ Backpropagation

Derivative of 1-D function

$$\frac{df(x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- ▶ Gradient of multi-D function
 - ▶ vector of partial derivatives in each dimension
 - ▶ examples of 2-D function

$$f(x, y) = xy \quad \rightarrow \quad \nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right] = [y, x]$$

$$f(x, y) = x + y \quad \rightarrow \quad \nabla f = [1, 1]$$

$$f(x, y) = \max(x, y) \quad \rightarrow \quad \nabla f = [\mathbb{1}(x \geq y), \mathbb{1}(x \leq y)]$$

Numerical Gradient

```
import numpy as np

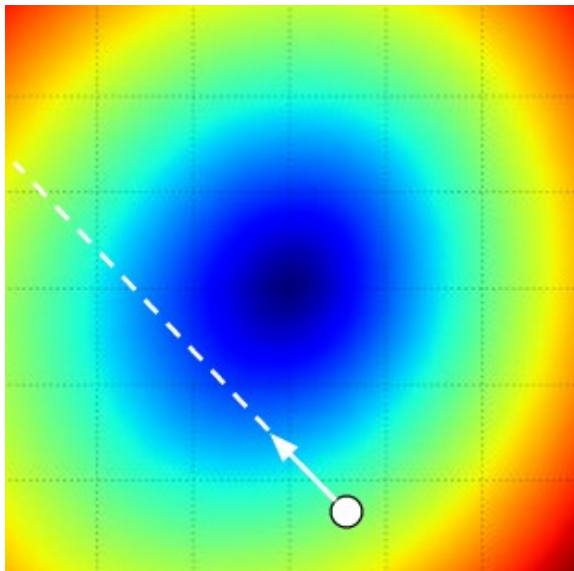
def num_grad(f, x): # finite difference method
    fx = f(x)
    grad = np.zeros(x.shape)
    h = 0.00001
    it = np.nditer(x, flags=['multi_index'],
        op_flags=['readwrite'])
    while not it.finished:
        ix = it.multi_index
        old_value = x[ix]
        x[ix] = old_value + h
        fxh = f(x)
        x[ix] = old_value
        grad[ix] = (fxh - fx) / h # alternatively
            [f(x+h)-f(x-h)]/2h
        it.iternext()
    return grad
```

Gradient Descent

- ▶ Repeated local search to minimize loss function
 - ▶ update in *negative* gradient direction
 - ▶ validate learning rate (step size)
- ▶ Mini-batch/stochastic gradient descent

```
while True:
    data_batch = sample_data(training_data,
                              256)
    weights_grad = eval_grad(loss_fun,
                              data_batch, weights)
    weights += - step_size * weights_grad
```

Gradient Descent (Cont'd)



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- ① Linear Classifiers
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Backpropagation

Chain rule

$$\frac{dz}{dx} = \frac{dz}{dy} \frac{dy}{dx}$$

► Example of composite function

$$f(v(p(x,y), q(z,w))) = 2 \left[\underbrace{xy}_{p \text{ (* gate)}} + \underbrace{\max(z,w)}_{q \text{ (max gate)}} \right]$$

$\underbrace{\hspace{15em}}_{v \text{ (+ gate)}}$

► forward pass: $[x, y, z, w] = [3, -4, 2, -1]$, $f = -20$

► backward pass: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}, \frac{\partial f}{\partial w} \right] = [-8, 6, 2, 0]$

Backpropagation (Cont'd)

```
# Forward pass
```

```
x = 3; y = -4; z = 2; w = -1
```

```
p = x * y          # -12
```

```
q = max(z, w)      # 2
```

```
v = p + q          # -10
```

```
f = 2 * v           # -20
```

```
# Backward pass
```

```
dfdvdv = 2
```

```
dvdvp = 1
```

```
dpdx = y
```

```
dpdy = x
```

```
dvdq = 1
```

```
dqdz = (z > w)
```

```
dqdw = (w > z)
```

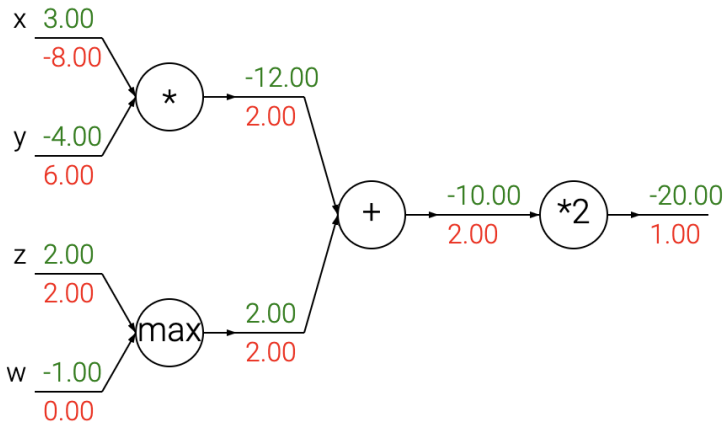
```
dfdx = dfdv * dvdvp * dpdx    # -8
```

```
dfdy = dfdv * dvdvp * dpdy    # 6
```

```
dfdvdz = dfdv * dvdq * dqdz   # 2
```

```
dfdvw = dfdv * dvdq * dqdw    # 0
```

PyTorch's Computational Graph



References

- ▶ cs231n.stanford.edu – CS231n: Deep Learning for Computer Vision
- ▶ Tang (2013), “Deep Learning using Linear Support Vector Machines”, [arXiv:1306.0239](https://arxiv.org/abs/1306.0239)
- ▶ pytorch.org – PyTorch package for deep learning
- ▶ github.com/karpathy/micrograd – tiny autograd engine by Andrej Karpathy