Lecture 10   Bayesian DSGE Models

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Introduction to Bayesian Statistics

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Introduction

- ▶ DYNARE has played a large role in fitting DSGE models

- ▶ However, it cannot handle *high-dimensional* DSGE models or models with

  - ▶ Student-t shocks, e.g., Chib & Ramamurthy (2014)

  - ▶ stochastic volatility, e.g., Justiniano & Primiceri (2008)

- ▶ Chib, Shin, & Tan (2021) provide a user-friendly MATLAB toolbox for such models that contains

  - ▶ training sample priors

  - ▶ efficient sampling of parameters by the TaRB-MH algorithm of Chib & Ramamurthy (2010)

  - ▶ fast computation of the marginal likelihood by the method of Chib (1995) and Chib & Jeliazkov (2001)

  - ▶ post-estimation tools, e.g., point and density forecasts

The Road Ahead...

Illustration

▶ Dynamic IS relation

$$\hat{y}_t = \mathbb{E}_t\hat{y}_{t+1} + \hat{g}_t - \mathbb{E}_t\hat{g}_{t+1} - \frac{1}{\tau}(\hat{R}_t - \mathbb{E}_t\hat{\pi}_{t+1} - \mathbb{E}_t\hat{z}_{t+1})$$

▶ New Keynesian Phillips curve

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \kappa(\hat{y}_t - \hat{g}_t)$$

▶ Monetary policy rule

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1 - \rho_R)\psi_1\hat{\pi}_t + (1 - \rho_R)\psi_2(\hat{y}_t - \hat{g}_t) + \epsilon_{R,t}$$

▶ Exogenous shocks

$$\hat{g}_t = \rho_G\hat{g}_{t-1} + \epsilon_{G,t}, \quad \hat{z}_t = \rho_Z\hat{z}_{t-1} + \epsilon_{Z,t}$$

▶ t-innovation with SV: for $s \in \{R, G, Z\}$

$$\epsilon_{s,t} \sim t_\nu(0, e^{h_{s,t}}), \quad h_{s,t} = (1 - \phi_s)\mu_s + \phi_s h_{s,t-1} + \eta_{s,t}, \quad \eta_{s,t} \sim N(0, \omega_s^2)$$

State Space Form

- ▶ We rely on Sims' (2001) method to solve for the solution of the structural model, for *each* value of θ

- ▶ After (log) linearizing around the steady state, the structural model is expressed in canonical form as

$$\Gamma_0(\theta)x_t = \Gamma_1(\theta)x_{t-1} + \Psi\epsilon_t + \Pi\eta_t$$

- ▶ Applying Sims' method, the unique, bounded solution takes the form

$$x_t = G(\theta)x_{t-1} + M(\theta)\epsilon_t$$

where $G(\theta)$ and $M(\theta)$ are non-linear unspecified functions of θ determined by the solve step

State Space Form

- ▶ Model completed by the measurement equations

$$\begin{pmatrix} \text{YGR}_t \\ \text{INF}_t \\ \text{INT}_t \end{pmatrix} = \begin{pmatrix} \gamma^{(Q)} \\ \pi^{(A)} \\ \pi^{(A)} + r^{(A)} + 4\gamma^{(Q)} \end{pmatrix} + \begin{pmatrix} \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t \\ 4\hat{\pi}_t \\ 4\hat{R}_t \end{pmatrix}$$

- ▶ We show results for an extended version of this model that has 51 parameters, 21 variables, 8 shocks, 8 observables, and 1,494 non-Gaussian and nonlinear latent variables

Example

- Set MATLAB directory to the 'DSGE-SVt' folder

- Specify the model, data, and save directories

```
%% Housekeeping
clear
close all
clc

%% User search path & mex files
modpath = ['user' filesep 'ltw17'];
datpath = ['user' filesep 'ltw17' filesep '
    data.txt'];
savepath = ['user' filesep 'ltw17'];
spec = tarb_spec([],'modpath',modpath,'datpath
    ',datpath,'savepath',savepath);

OneFileToMexThemAll
```

The Road Ahead...

Training Sample Prior

- In high dimensions, formulating an appropriate prior is difficult due to the complex mapping from θ to $G(\theta)$ and $M(\theta)$

- Standard choices often produce prior-sample conflict

- We supply two ways of dealing with this: training sample priors and Student-t family of distributions for location-type parameters

- A sampling the prior function is available to calculate the implied distribution of the outcomes

```matlab
%% Sampling the prior
npd = 10000;          % number of prior draws
T = 200;              % number of periods
sdof = 5;             % shock degrees of freedom
SamplePrior(npd,T,sdof,savepath)
```

Prior Predictive Distribution

I. Sample mean

A. Consumption · B. Investment · C. Wages · D. Government spending · E. Debt · F. Hours worked · G. Inflation · H. Interest rate

II. Sample standard deviation
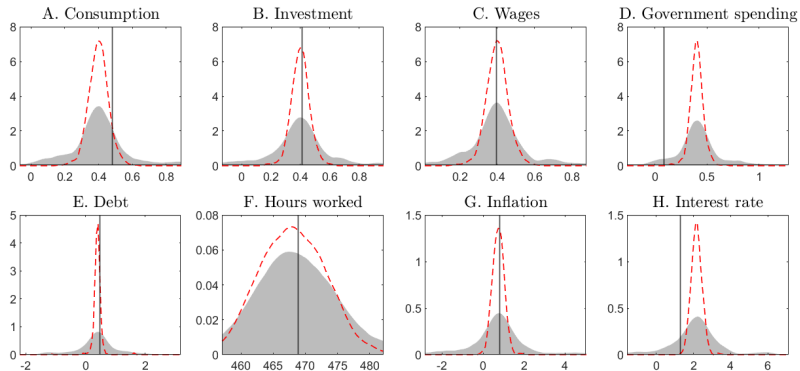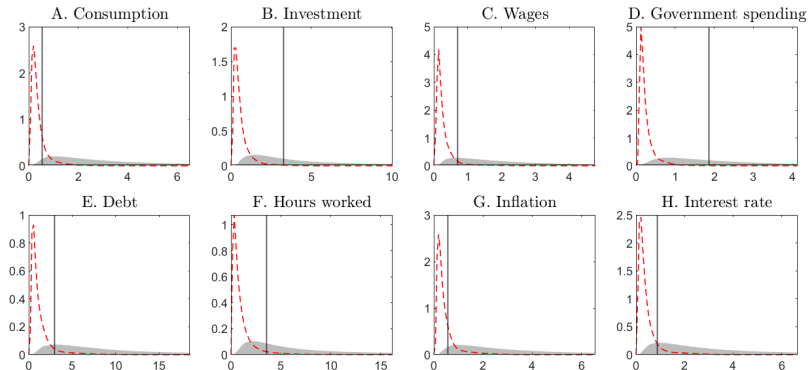
Prior Predictive Distribution (Cont'd)

III. Sample first-order autocorrelation

The Road Ahead...

Estimation

- The centerpiece of the estimation procedure is the TaRB-MH algorithm of Chib & Ramamurthy (2010)

- Used to draw samples of $(\theta, z_{1:T})$ from

$$\pi(\theta, z_{1:T} | y_{1:T}) \propto f(y_{1:T}, z_{1:T} | \theta) \cdot \pi(\theta) \cdot \mathbf{1}\{\theta \in \Theta_D\}$$

- TaRB-MH is coded up in DYNARE, but the implementation there is somehow not efficient

TaRB-MH MCMC

- ▶ Two hallmarks of TaRB-MH

  - ▶ randomize the number and components of blocks

  - ▶ tailor the proposal density to the posterior location and curvature, i.e., $q_b(\theta_b|y_{1:T}, z_{1:T}, \theta_{-b}) = t_v(\theta_b|\hat{\theta}_b, \hat{V}_b)$, $b = 1, \dots, B$, using simulated annealing and Chris Sims' `csminwel`

- ▶ The toolbox provides a fast implementation of these steps by

  - ▶ randomly tailoring proposal every few iterations

  - ▶ MEX loop-intensive functions from C/C++ code

  - ▶ `csminwel` to get $\hat{V}_b$ as output

- ▶ Although intensive it all happens seamlessly

Example

- Set up the TaRB-MH algorithm

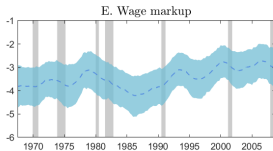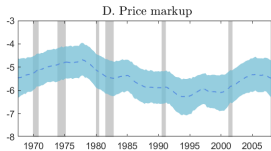- The estimation results are stored in the MATLAB data file `tarb_full.mat`
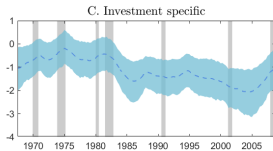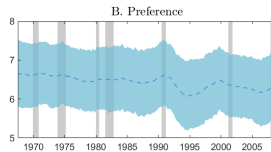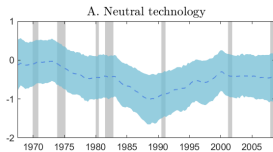
```
%% TaRB-MH (full run)
p = 0.7;                % blocking probability
w = 0.5;                % tailoring frequency
spec = tarb_spec(spec,'prob',p,'freq',w);

M = 11000;              % number of draws
burn = 1000;            % number of burn-in
tarb_full(M,burn,spec)
```
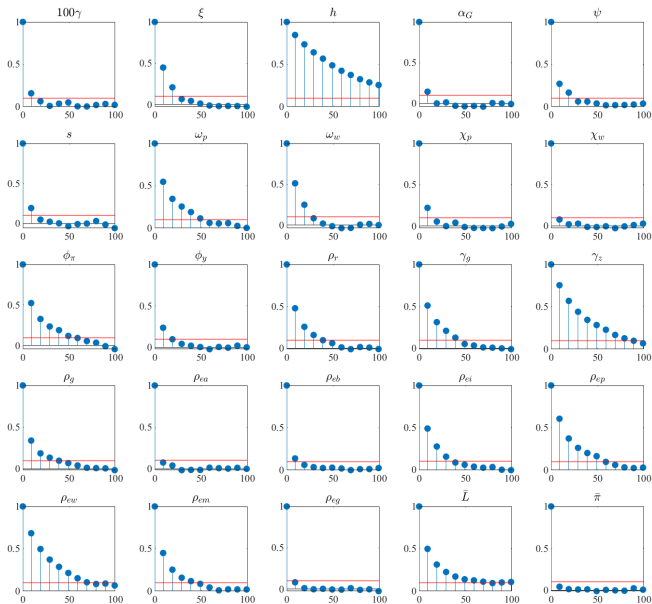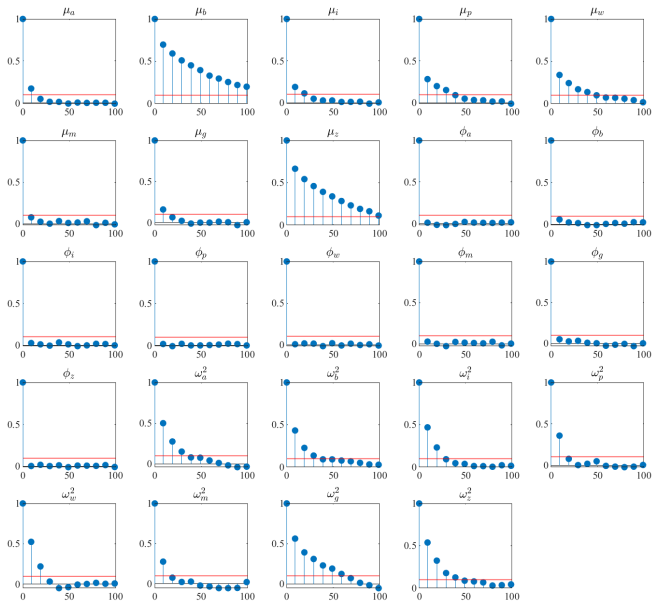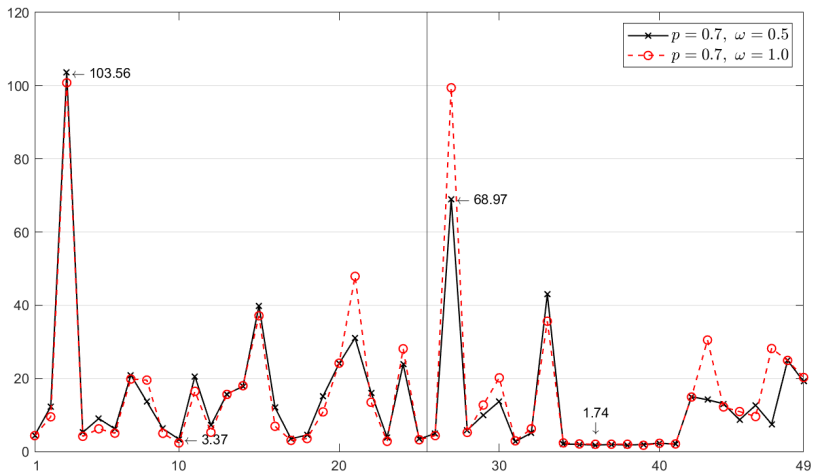
Prior-Posterior
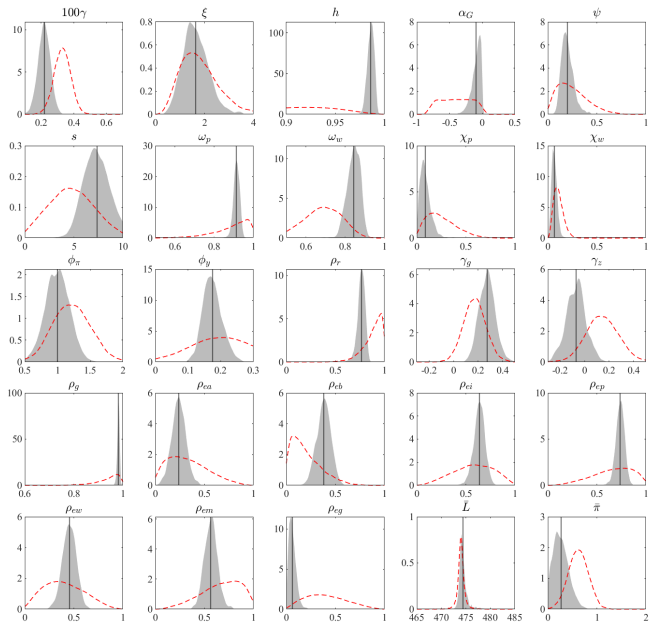
Stochastic Volatility
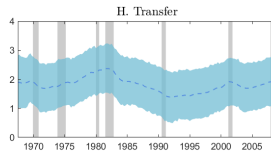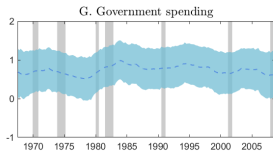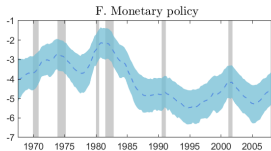
Autocorrelation

Inefficiency Factor

The Road Ahead...

Model Comparison

▶ Marginal likelihood is computed by a fast implementation of the Chib & Jeliazkov (2001) estimator

▶ This estimator is based on the identity of Chib (1995)

$$m(y_{1:T}) = \frac{f(y_{1:T}|\theta^*)\pi(\theta^*)}{\pi(\theta^*|y_{1:T})}, \quad \forall \theta^*$$

Model Comparison (Cont'd)

▶ With multiple block sampling, the posterior ordinate is estimated from the decomposition

$$\pi(\theta^*|y_{1:T}) = \pi(\theta_1^*|y_{1:T})\pi(\theta_2^*|y_{1:T},\theta_1^*)\cdots\pi(\theta_B^*|y_{1:T},\theta_1^*,\ldots,\theta_{B-1}^*)$$

using several reduced TaRB-MH runs

▶ Key point: all reduced runs can be parallelized for the cost of only one reduced run, regardless of the number of blocks

▶ This speeds up the calculation enormously

Example

- Set up the TaRB-MH reduced run

- The estimation results are stored in the MATLAB data file
  `tarb_reduce.mat`

```
%% TaRB-MH (reduced run)
M = 11000;            % number of draws
burn = 1000;          % number of burn-in
B = 7;                % number of blocks
tarb_reduce(M,burn,B,spec)
```

Marginal Likelihood
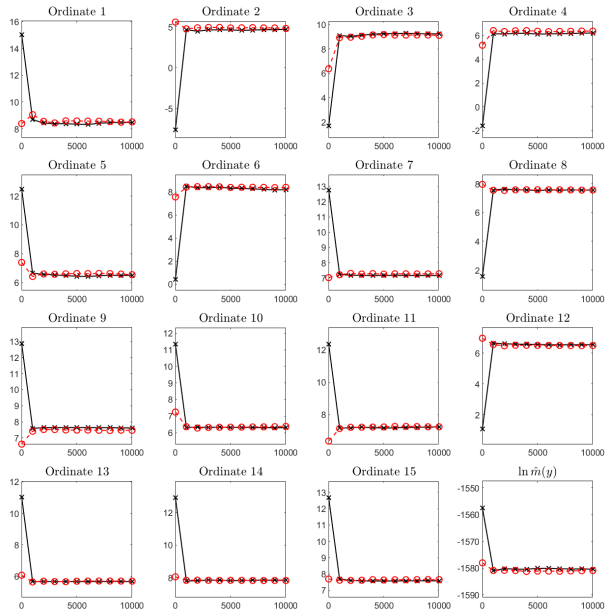
Simulation Evidence

Table 1: Chib-Jeliazkov estimator

DGP 1: regime-M with $\nu = 15$			DGP 2: regime-F with $\phi = 0.5$		
ν	M	F	ϕ	M	F
30 (light)	4	0	0.1 (weak)	0	9
15 (fat)	15	0	0.5 (moderate)	0	10
5 (heavy)	1	0	0.9 (strong)	0	1

NOTES: Number of picks for each model specification.

Table 2: Modified harmonic mean estimator

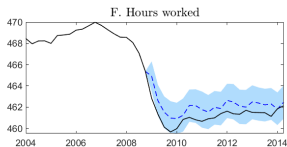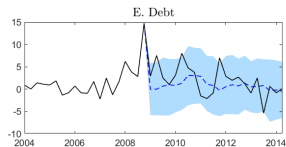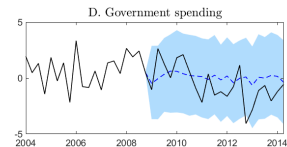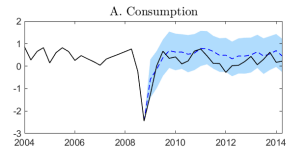| DGP 1: regime-M with $\nu = 15$			DGP 2: regime-F with $\phi = 0.5$		
ν	M	F	ϕ	M	F
30 (light)	0	0	0.1 (weak)	0	0
15 (fat)	0	0	0.5 (moderate)	0	0
5 (heavy)	20	0	0.9 (strong)	0	20

The Road Ahead...

Post-Estimation Tools

- Toolbox includes post-estimation tools, e.g., functions for conducting impulse response, variance decomposition, and prediction

- For instance, suppose we want the one-quarter-ahead prediction density

$$p(y_{T+1}|y_{1:T}) = \int p(y_{T+1}|y_{1:T}, \theta, z_{T+1}) \cdot \pi(\theta, z_{T+1}|y_{1:T}) d(\theta, z_{1:T})$$

- This is available through the TaRB-MH specification

```
%% Out-of-sample forecast
head = 50;          % training sample size
tail = 22;          % forecasting sample size
h = 1;              % forecasting horizon
spec = tarb_spec([],'datrange',[head tail],'
    pred',h);
```

Out-of-Sample Forecast

A. Consumption   B. Investment   C. Wages   D. Government spending   E. Debt   F. Hours worked   G. Inflation   H. Interest rate
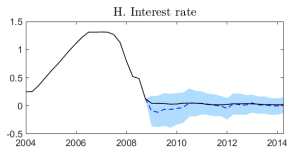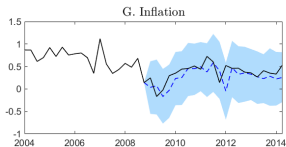
Outroduction

- ▶ Going beyond DYNARE for Next-Gen DSGE models

- ▶ Efficient and fast estimation via TaRB-MH and parallel computing

- ▶ MATLAB toolbox: publicly available at sites.google.com/a/slu.edu/tanf/research

- ▶ Readily applied. Current application in progress, an open economy DSGE model that is about twice as big as the largest DSGE model estimated to date

References

▶ Chib & Ramamurthy (2010), "Tailored Randomized Block MCMC Methods with Application to DSGE Models," *Journal of Econometrics*

▶ Chib, Shin, & Tan (2021), "DSGE-SVt: An Econometric Toolkit for High-Dimensional DSGE Models with SV and t Errors," *Computational Economics*

▶ Sims (2001), "Solving Linear Rational Expectations Models," *Computational Economics*