Lecture 1: Basic Strategic Form Games

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The Road Ahead

What is Game Theory?

Game Theory is the study of **strategic interdependence** - situations where your actions affect both your welfare and others' welfare, and vice versa. Key elements include:

- **Players**: Decision makers (individuals, firms, countries)
- **Strategies**: Available choices for each player
- **Payoffs**: Outcomes that result from the combination of all players' choices
- **Information**: What each player knows about the game

Why Study Game Theory?

- Understanding competition and cooperation in business and markets
- Analyzing political and economic policies and their outcomes
- Making better strategic decisions in interactive situations

Example: Prisoner's Dilemma

Two thieves are arrested for trespassing. Police suspect they planned to rob a store but lack evidence. Each prisoner is offered a deal:

- **If you confess and your partner doesn't**: You go free, partner gets 12 months
- **If both confess**: You each get 8 months
- **If neither confesses**: You each get 1 month (trespassing only)
- **If your partner confesses and you don't**: You get 12 months, partner goes free

Payoff Matrix

	Player 2: Quiet	Player 2: Confess
Player 1: Quiet	-1, -1	-12, 0
Player 1: Confess	0, -12	-8, -8

Note: Payoffs represent negative months in jail (higher numbers are better)

Reading the Matrix

- Player 1 chooses rows, Player 2 chooses columns
- First number in each cell = Player 1's payoff
- Second number in each cell = Player 2's payoff

Solving the Prisoner's Dilemma

Player 1's Analysis

If Player 2 stays quiet:

- Quiet: -1 months
- Confess: 0 months ✓ (Better!)

If Player 2 confesses:

- Quiet: -12 months
- Confess: -8 months ✓ (Better!)

Conclusion: Player 1 should **always confess** regardless of what Player 2 does!

Player 2's Analysis (by symmetry)

Dominant Strategies

A **dominant strategy** is a strategy that gives a player the highest payoff regardless of what other players do. There are two types of dominance:

- **Strictly Dominant**: Always gives strictly higher payoffs
- **Weakly Dominant**: Always gives higher or equal payoffs (at least as good)

The Prisoner's Dilemma

- "Confess" is a **strictly dominant strategy** for both players, leading to the **dominant strategy equilibrium**: (Confess, Confess) or (-8, -8)
- Both staying quiet would give (-1, -1) - better for everyone!
- But (-1, -1) is **not stable** - each player has incentive to deviate

Iterated Elimination of Strictly Dominated Strategies

	Left	Center	Right
Up	13, 3	1, 4	7, 3
Middle	4, 1	3, 3	6, 2
Down	-1, 9	2, 8	8, -1

Step 1: Center dominates Right for Player 2 → eliminate Right

Step 2: Middle dominates Down for Player 1 → eliminate Down

Step 3: Center dominates Left for Player 2 → eliminate Left

Result: (Middle, Center) with payoffs (3, 3) (order does not matter!)

Caution: Does this work for weakly dominated strategies?

Pure Strategy Nash Equilibrium

A **Pure Strategy Nash Equilibrium** is a set of strategies where each player's strategy is a best response to the other players' strategies.

Key Property: No player wants to unilaterally change their strategy.

The Prisoner's Dilemma

(Confess, Confess) is the **unique pure strategy Nash equilibrium**

- If Player 2 confesses, Player 1's best response is confess
- If Player 1 confesses, Player 2's best response is confess

Example: Stag Hunt Game

	Player 2: Stag	Player 2: Hare
Player 1: Stag	3, 3	0, 2
Player 1: Hare	2, 0	1, 1

Multiple Equilibria

- **(Stag, Stag)**: High payoff but risky - requires coordination
- **(Hare, Hare)**: Lower payoff but safe
- Coordination and trust matter for achieving better outcomes

Example: Stoplight Game

	Player 2: Go	Player 2: Stop
Player 1: Go	-5, -5	1, 0
Player 1: Stop	0, 1	-1, -1

Multiple Equilibria

- **Real-world solution**: Traffic lights coordinate behavior by telling players which equilibrium, (Go, Stop) or (Stop, Go), to play
- **Self-enforcing**: No police needed - players naturally want to follow the signal because it makes their strategy optimal

Best Response

A **best response** is the optimal strategy for a player given what all other players are doing

Method for Finding Nash Equilibria

1. For each possible combination of strategies by all other players, mark each player's best response in the payoff matrix
2. Nash equilibria occur where **all players** are simultaneously playing best responses

Why This Works

- **Nash Equilibrium Property**: No player wants to unilaterally deviate, and best responses ensure no player can improve by changing strategies
- **Mutual Best Response**: When all players are simultaneously best responding, the outcome is stable and self-reinforcing

Example: 4×4 Safety in Numbers

Two generals each have 3 units. They can send 0, 1, 2, or 3 units to battle. The side with more troops wins (+1), fewer troops loses (-1), equal troops draw (0), no battle means (0,0)

	0 units	1 unit	2 units	3 units
0 units	0*, 0*	0, 0	0, 0	0*, 0*
1 unit	0*, 0	0, 0	-1, 1*	-1, 1*
2 units	0*, 0	1*, -1	0, 0	-1, 1*
3 units	0*, 0*	1*, -1	1*, -1	0*, 0*

Note: Nash equilibria are marked with * for both players

Mixed Strategy Nash Equilibrium

A **mixed strategy** is a probability distribution over a player's pure strategies.

Consider matching pennies:

	Player 2: Heads	Player 2: Tails
Player 1: Heads	1, -1	-1, 1
Player 1: Tails	-1, 1	1, -1

Mixed Strategy Solution

- Each player randomizes: 50% Heads, 50% Tails
- This makes the opponent indifferent between their strategies
- Mixed strategy equilibrium exists when pure strategy equilibrium doesn't

Example: Zero-Sum Game

	Player 2: Left	Player 2: Right
Player 1: Up	3, -3	-2, 2
Player 1: Down	-1, 1	0, 0

Step 1: Let Player 1 play Up with probability p, Down with probability $1 - p$

Step 2: Player 2's expected payoffs:

- Left: $p(-3) + (1 - p)(1)$, Right: $p(2) + (1 - p)(0)$

Step 3: Player 2 must be indifferent between Left and Right → $p = \frac{1}{6}$

Note: By symmetry, Player 2 plays Left with probability $\frac{1}{3}$, Right with probability $\frac{2}{3}$

Example: Battle of the Sexes

A couple wants to go on a date. She prefers the ballet, he prefers the fight. Both prefer being together to being alone

	She: Ballet	She: Fight
He: Ballet	1, 2	0, 0
He: Fight	0, 0	2, 1

All Nash Equilibria

- **Pure strategy**: (Ballet, Ballet) and (Fight, Fight)
- **Inefficiency of mixed strategy**: He plays Ballet $\frac{1}{3}$, Fight $\frac{2}{3}$; She plays Ballet $\frac{2}{3}$, Fight $\frac{1}{3}$. Both earn only $\frac{2}{3}$ and fail to coordinate $\frac{5}{9}$ of the time