Lecture 1: Basic Strategic Form Games

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The Road Ahead

- 1. What is Game Theory?
- 2. Dominant Strategies
- 3. Pure Strategy Nash Equilibrium
- 4. Best Response
- 5. Mixed Strategy Nash Equilibrium

What is Game Theory?

Game Theory is the study of **strategic interdependence** - situations where your actions affect both your welfare and others' welfare, and vice versa. Key elements include:

- Players: Decision makers (individuals, firms, countries)
- Strategies: Available choices for each player
- Payoffs: Outcomes that result from the combination of all players' choices
- Information: What each player knows about the game

Why Study Game Theory?

- Understanding competition and cooperation in business and markets
- Analyzing political and economic policies and their outcomes
- Making better strategic decisions in interactive situations

Example: Prisoner's Dilemma

Two thieves are arrested for trespassing. Police suspect they planned to rob a store but lack evidence. Each prisoner is offered a deal:

- If you confess and your partner doesn't: You go free, partner gets 12 months
- If both confess: You each get 8 months
- If neither confesses: You each get 1 month (trespassing only)
- If your partner confesses and you don't: You get 12 months, partner goes free

Payoff Matrix

	Player 2: Quiet	Player 2: Confess
Player 1: Quiet	-1, -1	-12, 0
Player 1: Confess	0, -12	-8, -8

Note: Payoffs represent negative months in jail (higher numbers are better)

Reading the Matrix

- Player 1 chooses rows, Player 2 chooses columns
- First number in each cell = Player 1's payoff
- Second number in each cell = Player 2's payoff

Solving the Prisoner's Dilemma

Player 1's Analysis

If Player 2 stays quiet:

• Quiet: -1 months

Confess: 0 months ✓ (Better!)

If Player 2 confesses:

• Quiet: -12 months

Confess: -8 months ✓ (Better!)

Conclusion: Player 1 should **always confess** regardless of what Player 2 does!

Player 2's Analysis (by symmetry)

Dominant Strategies

A **dominant strategy** is a strategy that gives a player the highest payoff regardless of what other players do. There are two types of dominance:

- Strictly Dominant: Always gives strictly higher payoffs
- Weakly Dominant: Always gives higher or equal payoffs (at least as good)

The Prisoner's Dilemma

- "Confess" is a strictly dominant strategy for both players, leading to the dominant strategy equilibrium: (Confess, Confess) or (-8, -8)
- Both staying quiet would give (-1, -1) better for everyone!
- But (-1, -1) is **not stable** each player has incentive to deviate

Iterated Elimination of Strictly Dominated Strategies

	Left	Center	Right
Up	13, 3	1, 4	7, 3
Middle	4, 1	3, 3	6, 2
Down	-1, 9	2, 8	8, -1

Step 1: Center dominates Right for Player 2 → eliminate Right

Step 2: Middle dominates Down for Player 1 → eliminate Down

Step 3: Center dominates Left for Player 2 → eliminate Left

Result: (Middle, Center) with payoffs (3, 3) (order does not matter!)

Caution: Does this work for weakly dominated strategies?

Pure Strategy Nash Equilibrium

A **Pure Strategy Nash Equilibrium** is a set of strategies where each player's strategy is a best response to the other players' strategies.

Key Property: No player wants to unilaterally change their strategy.

The Prisoner's Dilemma

(Confess, Confess) is the unique pure strategy Nash equilibrium

- If Player 2 confesses, Player 1's best response is confess
- If Player 1 confesses, Player 2's best response is confess

Example: Stag Hunt Game

	Player 2: Stag	Player 2: Hare
Player 1: Stag	3, 3	0, 2
Player 1: Hare	2, 0	1, 1

Multiple Equilibria

- (Stag, Stag): High payoff but risky requires coordination
- (Hare, Hare): Lower payoff but safe
- Coordination and trust matter for achieving better outcomes

Example: Stoplight Game

	Player 2: Go	Player 2: Stop
Player 1: Go	-5, -5	1, 0
Player 1: Stop	0, 1	-1, -1

Multiple Equilibria

- Real-world solution: Traffic lights coordinate behavior by telling players which equilibrium, (Go, Stop) or (Stop, Go), to play
- **Self-enforcing**: No police needed players naturally want to follow the signal because it makes their strategy optimal

Best Response

A **best response** is the optimal strategy for a player given what all other players are doing

Method for Finding Nash Equilibria

- 1. For each possible combination of strategies by all other players, mark each player's best response in the payoff matrix
- 2. Nash equilibria occur where **all players** are simultaneously playing best responses

Why This Works

- Nash Equilibrium Property: No player wants to unilaterally deviate, and best responses ensure no player can improve by changing strategies
- Mutual Best Response: When all players are simultaneously best responding, the outcome is stable and self-reinforcing

Example: 4×4 Safety in Numbers

Two generals each have 3 units. They can send 0, 1, 2, or 3 units to battle. The side with more troops wins (+1), fewer troops loses (-1), equal troops draw (0), no battle means (0,0)

	0 units	1 unit	2 units	3 units
0 units	0*, 0*	0, 0	0, 0	0*, 0*
1 unit	0*, 0	0, 0	-1, 1*	-1, 1*
2 units	0*, 0	1*, -1	0, 0	-1, 1*
3 units	0*, 0*	1*, -1	1*, -1	0*, 0*

Note: Nash equilibria are marked with * for both players

Mixed Strategy Nash Equilibrium

A **mixed strategy** is a probability distribution over a player's pure strategies. Consider matching pennies:

	Player 2: Heads	Player 2: Tails
Player 1: Heads	1, -1	-1, 1
Player 1: Tails	-1, 1	1, -1

Mixed Strategy Solution

- Each player randomizes: 50% Heads, 50% Tails
- This makes the opponent indifferent between their strategies
- Mixed strategy equilibrium exists when pure strategy equilibrium doesn't

Example: Zero-Sum Game

	Player 2: Left	Player 2: Right
Player 1: Up	3, -3	-2, 2
Player 1: Down	-1, 1	0, 0

Step 1: Let Player 1 play Up with probability p, Down with probability 1-p

Step 2: Player 2's expected payoffs:

ullet Left: p(-3) + (1-p)(1), Right: p(2) + (1-p)(0)

Step 3: Player 2 must be indifferent between Left and Right $ightarrow p = rac{1}{6}$

Note: By symmetry, Player 2 plays Left with probability $\frac{1}{3}$, Right with probability $\frac{2}{3}$

Example: Battle of the Sexes

A couple wants to go on a date. She prefers the ballet, he prefers the fight. Both prefer being together to being alone

	She: Ballet	She: Fight
He: Ballet	1, 2	0, 0
He: Fight	0, 0	2, 1

All Nash Equilibria

- Pure strategy: (Ballet, Ballet) and (Fight, Fight)
- Inefficiency of mixed strategy: He plays Ballet $\frac{1}{3}$, Fight $\frac{2}{3}$; She plays Ballet $\frac{2}{3}$, Fight $\frac{1}{3}$. Both earn only $\frac{2}{3}$ and fail to coordinate $\frac{5}{9}$ of the time