Lecture 6: Repeated Games

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Date: October 30, 2025

The Road Ahead

Finite Repeated Games

If we play the Prisoner's Dilemma repeatedly, surely the shadow of the future will encourage cooperation?

	Cooperate	Defect
Cooperate	3, 3	1, 4
Defect	4, 1	2, 2

In any **finitely repeated** game where the end is known, backward induction destroys all hope of cooperation

Why? Players can build trust, signal intentions, and seek rewards/punishments—but a cold, backward-flowing logic prevents cooperation when the end is certain

Backward Induction

Final Round N: All previous payoffs are locked in. Players must play the one-shot Nash Equilibrium (Defect, Defect) with payoffs (2, 2)

Round N-1: Players know the final round outcome is predetermined as mutual defection

- Known future: Final round will be (Defect, Defect) regardless
- Empty promise: Cooperating now cannot influence future behavior
- Inevitable choice: This round becomes strategically identical to a one-shot game

Unstoppable collapse: This logic continues backward through every round:

- Stage N → Stage N-1 → Stage N-2 → ... → Stage 1
- The only subgame perfect equilibrium is (Defect, Defect) in every single round

A Glimpse of Hope

The entire unraveling depends on certainty about the end. Two ways to eliminate the end → prevent backward induction:

1. **Infinitely repeated games**: No final round exists
2. **Unknown termination**: Players don't know when the game ends

Result: The incentive to build reputation and encourage future cooperation remains powerful because there's always a "next round" to consider

Folk Theorem: This opens a complex world where almost anything can be justified as rational

Discount Factors

How do we compare infinite payoff streams?

- Player 1 always defects: $4 + 4 + 4 + ... = \infty$
- Player 1 always cooperates: $1 + 1 + 1 + ... = \infty$
- Paradox: Best and worst outcomes both equal infinity!

Solution: The discount factor δ where $0 < \delta < 1$. Two interpretations:

1. Time value: $3 today > $3 tomorrow (can invest, consume sooner)
2. Continuation probability: $\delta =$ probability the game continues to next period

Infinite cooperation payoff: $3 + 3\delta + 3\delta^2 + 3\delta^3 + ...$

- High δ (close to 1) = Patient player
- Low δ (close to 0) = Impatient player

Geometric Series

How to calculate $X + X\delta + X\delta^2 + X\delta^3 + ...$

Magic trick:

- Finite series: $S = X + X\delta + X\delta^2 + ... + X\delta^{n-1}$
- Multiply by δ: $\delta S = X\delta + X\delta^2 + ... + X\delta^{n-1} + X\delta^n$
- Subtract: $S - \delta S = X - X\delta^n \rightarrow S = \frac{X(1-\delta^n)}{1-\delta}$
- As $n \to \infty$, $\delta^n \to 0$

Infinite series formula: $X + X\delta + X\delta^2 + ... = \frac{X}{1-\delta}$

One-Shot Deviation Principle

In a 16-decision game, there are 65,536 possible strategies! Traditional approach: check against all 65,535 alternatives

One-shot deviation principle: A strategy is a subgame perfect equilibrium if and only if no player can profitably deviate at a single stage while maintaining their strategy everywhere else. Why It Works?

- If a strategy is truly optimal, it must beat any single-change alternative
- If you can't improve by changing one decision, you can't improve by changing multiple decisions

Benefit: Without principle → Check 65,535 alternatives; With principle → Check only 16 one-shot deviations. Transform an impossible task into a manageable one!

Grim Trigger Strategy

Unforgiving rule: If anyone has defected at any point previously, defect forever. Otherwise, cooperate. Two simple rules:

1. Start by cooperating (offer initial trust)
2. Permanent punishment for any defection (no forgiveness, ever)

Two possible paths: Cooperation path (steady payoff of 3) vs. Punishment path (mutual defection: 4, 2, 2, 2, ...)

- Present value of cooperation: $\frac{3}{1-\delta}$
- Present value of defection: $4+\frac{2\delta}{1-\delta}$
- Cooperation condition: $\frac{3}{1-\delta} \geq 4+\frac{2\delta}{1-\delta} \rightarrow \delta \geq \frac{1}{2}$

Remark: Cooperation requires sufficient patience ($\delta \geq 1/2$). If players are too impatient (low δ), the immediate temptation payoff outweighs future punishment, and defection becomes rational

Grim Trigger Is Self-Enforcing

Grim Trigger is not an imposed rule—it's a **voluntary strategy** that rational players choose because it's in their self-interest:

- No external enforcement needed—players follow it because deviation makes them worse off
- It's a Subgame Perfect Equilibrium with **credible** threats (when $\delta \geq 1/2$): Once in punishment phase, continuing to defect is optimal

Bottom line: Grim Trigger works precisely because rational players commit to it knowing that both cooperation and punishment are self-enforcing

US Dollar As Reserve Currency

Players: United States and other countries holding dollar reserves

- Cooperation path: US maintains sound monetary policy; other countries hold dollar reserves → Global financial stability, low transaction costs, US seigniorage benefits (payoff: 3, 3)

- Temptation to defect: US prints money excessively, inflates away debt; other countries diversify reserves early to avoid dollar devaluation risk (payoff: 4 today)

Grim Trigger punishment: Permanent loss of dollar dominance → US loses seigniorage benefits, higher borrowing costs forever; global financial instability (payoff: 2, 2, 2, ...)

Self-enforcing strategy: No treaty needed—Both sides value long-term cooperation over short-term gains when patient ($\delta \geq 1/2$)

Tit-for-Tat Strategy

A forgiving strategy that doesn't hold grudges forever. Two simple rules:

1. Start by cooperating (be nice first)
2. Copy opponent's last move (forgive quickly, but retaliate immediately)

Trade-off: Cooperation $(3, 3, 3, \ldots)$ vs. Alternating $(4, 1, 4, 1, \ldots)$ after defection

Cooperation condition: $\delta \geq \frac{1}{2}$ (same as Grim Trigger)

Credibility problem: Tit-for-Tat's threat isn't credible! After opponent defects:

- Punish: Defect → $(4, 1, 4, 1, \ldots)$ vs. Forgive: Cooperate → $(3, 3, 3, \ldots)$
- To cooperate initially: Need $\delta \geq 1/2$; To punish credibly: Need $\delta \leq 1/2$
- Nash equilibrium but not subgame perfect equilibrium (except at $\delta = 1/2$)

Folk Theorem

In infinitely repeated games, any outcome that gives all players payoffs strictly better than their punishment payoff can be supported as a subgame perfect equilibrium. The mechanism includes:

1. Agreement: Players commit to a specific strategy profile
2. Punishment: Any deviation triggers permanent reversion to Nash equilibrium

Generalized Grim Trigger:

- Player 1: Cooperate 100% of the time → Expected payoff: 2.85
- Player 2: Cooperate 95% of the time, defect 5% → Expected payoff: 3.15
- Both payoffs > 2 (punishment level), so both accept this asymmetric deal

Infinitely many equilibria: As long as players are patient enough (high δ), they won't risk losing any arrangement that beats permanent punishment

Prediction Problem

Challenge: When everything is possible, nothing is predictable

Example:

- Imagine observing this 4-period sequence: 1. Mutual Cooperation → 2. Mutual Defection → 3. Mutual Defection → 4. (Defect, Cooperate)
- Strategy: Follow the prescribed periods 1-4, then cooperate forever; Any deviation triggers permanent defection
- Seemingly random? Folk Theorem shows this can be a rational equilibrium!

Insight: The value of infinite future cooperation vastly outweighs any finite sequence of strange payoffs. For patient players, any 4-period "cost" becomes irrelevant compared to infinite future benefits