Lecture 7: Incomplete Information

Instructor: Fei Tan

@econdojo @BusinessSchool101 Saint Louis University

Date: November 6, 2025

The Road Ahead

Why Information Matters

In strategic interactions, what you know—and what you don't know—is everything

Two types of uncertainty:

- **Imperfect information**: Uncertainty about past actions (simultaneous moves)
- **Incomplete information**: Uncertainty about payoffs, preferences, or "types"

Key question: What's the difference between not knowing what someone just did versus not knowing what they truly want?

Imperfect Information

Definition: A player does not know the previous actions of other players

Classic example: Prisoner's Dilemma

- Both prisoners choose simultaneously
- Player 2 doesn't observe Player 1's choice
- But both know all possible payoffs

	Cooperate	Defect
Cooperate	3, 3	1, 4
Defect	4, 1	2, 2

Key insight: Imperfect information but complete information—payoffs are known to all

Incomplete Information

Definition: Uncertainty about another player's payoffs, preferences, or "type"

Examples of hidden characteristics:

Scenario	Unknown Information
Soccer Penalty Kick	Striker's accuracy to left vs. right
Volunteer's Dilemma	Cost of calling for help (busy vs. leisurely)
Battle of the Sexes	Intensity of preference for ballet

Key insight: You don't know what things are worth to your opponent

The Critical Distinction

Feature	Imperfect Information	Incomplete Information
Nature of Uncertainty	Uncertainty about prior action	Uncertainty about payoffs/preferences
Guiding Question	"What did they just do?"	"What do they truly want?"
Classic Example	Prisoner's Dilemma	Soccer Penalty Kick
State of Payoffs	All payoffs known	At least one payoff unknown

Why it matters: Different information structures require completely different solution concepts

Building Blocks

From Complete to Incomplete Information:

Complete Information	Incomplete Information
Players	Players
Actions	Actions
Payoffs	Payoffs
	Player Types
	Beliefs

The new elements:

- **Types**: Different versions of a player (e.g., left-accurate vs. right-accurate striker)
- **Beliefs**: Probabilities assigned to opponent types (e.g., 50% chance of left-accurate)

Player Types

Definition: Different variations of a player with private information about payoffs or capabilities

Key properties:

- A player always knows their own type
- Opponents do not know the player's type
- Types capture informational asymmetry

Example: Soccer striker

- Type A: More accurate kicking left (private information)
- Type B: More accurate kicking right (private information)
- Goalie must form beliefs about which type they face

Beliefs

Definition: Probabilities a player assigns to possible types of opponents

Purpose: Provide a framework for rational decision-making under uncertainty

Example: Goalie's beliefs about striker

- 50% belief: Striker is more accurate to left
- 50% belief: Striker is more accurate to right

Common Prior Assumption: All players know each other's beliefs, and know that they know, infinitely

Bayesian Nash Equilibrium

Definition: A set of strategies—one for each type of player—such that no type has incentive to deviate, given beliefs about types and what other types are doing

Three key upgrades from Nash Equilibrium:

1. **Focus on types**: Check for deviations by each type, not just each player
2. **Strategies per type**: One strategy for each possible type
3. **Role of beliefs**: Use beliefs to calculate expected payoffs

Solution concept for: Simultaneous-move games with incomplete information

BNE Example: One-Sided Incomplete Information

Setup: Player 1 has two types with probabilities p and $1 - p$. Player 2 is uncertain which type they face.

Two possible games:

Type A (prob = p)	Left	Right
Up	3, 3	1, 0
Down	4, 3	2, 0

Type B (prob = 1-p)	Left	Right
Up	6, 2	0, 4
Down	5, 2	-1, 4

Finding Player 1's Strategy

Type A analysis: Compare Up vs. Down

- If Player 2 plays Left: Down (4) > Up (3)
- If Player 2 plays Right: Down (2) > Up (1)
- **Result**: Down strictly dominates Up for Type A

Type B analysis: Compare Up vs. Down

- If Player 2 plays Left: Up (6) > Down (5)
- If Player 2 plays Right: Up (0) > Down (-1)
- **Result**: Up strictly dominates Down for Type B

Player 1's equilibrium strategy: Type A plays Down; Type B plays Up

Finding Player 2's Strategy

Player 2's dilemma: Cannot use simple dominance because optimal action depends on Player 1's type

Expected utility approach:

- Expected utility of Left: $3p + 2(1 - p) = p + 2$
- Expected utility of Right: $0p + 4(1 - p) = 4 - 4p$

Player 2's best response:

- Play Left if $p + 2 > 4 - 4p$ → $p > \frac{2}{5}$
- Play Right if $p < \frac{2}{5}$
- Indifferent if $p = \frac{2}{5}$

Complete BNE Solution

The equilibrium strategy profile:

- Player 1 (Type A): Down
- Player 1 (Type B): Up
- Player 2: Left if $p > \frac{2}{5}$; Right if $p < \frac{2}{5}$

Two key insights:

1. **BNE is a contingency plan**: Specifies action for every possible type
2. **Beliefs drive rationality**: Player 2's optimal choice pivots on probability p

Note: From Player 2's perspective, Player 1 appears to be "mixing" even though each type plays a pure strategy

Solving for BNE: The Matrix Method

Challenge: When games have multiple matrices (one per opponent type), how do we find the BNE?

Solution: Transform the game into a single, unified matrix

Key insight: The uninformed player needs a complete contingency plan specifying what to do for each possible type they might be

Method:

1. Create compound strategies for the uncertain player
2. Calculate expected payoffs weighted by type probabilities
3. Solve for Nash equilibria in the combined matrix
4. These Nash equilibria are the Bayesian Nash equilibria of the original game

Example Setup: PD/SH Mixed Game

Player 1: Has one type (Stag Hunt preferences)

Player 2: Two possible types

- Prisoner's Dilemma type (probability 0.2)
- Stag Hunt type (probability 0.8)

P2: PD Type (0.2)	Left	Right
Up	3, 3	0, 4
Down	2, 1	1, 2

P2: SH Type (0.8)	Left	Right
Up	3, 3	0, 2
Down	2, 0	1, 1

Building the Combined Matrix

Player 2's compound strategies: (Action if PD, Action if SH)

- (Left, Left): PD type plays Left AND SH type plays Left
- (Left, Right): PD type plays Left AND SH type plays Right
- (Right, Left): PD type plays Right AND SH type plays Left
- (Right, Right): PD type plays Right AND SH type plays Right

Calculating expected payoffs: Weight outcomes by type probabilities

Example: Up, (Left, Right)

- Player 1: $(0.2)(3) + (0.8)(0) = 0.6$
- Player 2: $(0.2)(3) + (0.8)(2) = 2.2$

The Combined Matrix

	(L, L)	(L, R)	(R, L)	(R, R)
Up	3, 3	0.6, 2.2	2.4, 3.2	0, 2.4
Down	2, 0.2	1.2, 1	1.8, 0.4	1, 1.2

Critical step: Ensure arithmetic is correct! Wrong calculations → Wrong equilibria

Next step: Eliminate dominated strategies and find Nash equilibria

Eliminating Dominated Strategies

Key insight: PD type has dominant strategy to play Right (4 > 3, 2 > 1)

Eliminate (Left, Left): Dominated by (Right, Left)

- Compare Player 2's payoffs: 3.2 > 3 and 0.4 > 0.2
- Confirms PD type prefers Right when SH type plays Left

Eliminate (Left, Right): Dominated by (Right, Right)

- Compare Player 2's payoffs: 2.4 > 2.2 and 1.2 > 1
- Confirms PD type prefers Right when SH type plays Right

Simplified 2×2 game:

	(R, L)	(R, R)
Up	2.4, 3.2	0, 2.4
Down	1.8, 0.4	1, 1.2

Finding the Equilibria

Pure strategy equilibria (mutual best responses):

1. **(Up, Right-Left)**: Player 1 plays Up; P2's PD type plays Right, SH type plays Left
2. **(Down, Right-Right)**: Player 1 plays Down; P2 plays Right regardless of type

Mixed strategy equilibrium:

- Player 1 mixes between Up and Down
- P2's PD type plays pure strategy Right (still dominant)
- P2's SH type mixes between Left and Right

Interpretation: What looks like Player 2 "mixing" might actually be different types playing different pure strategies

Dominance Concepts

Two mindsets for evaluating strategies:

Mindset	Timing	Focus
Ex Ante	Before knowing your type	Overall player's grand strategy
Interim	After knowing your type	Specific type's optimal action

Poker analogy:

- **Ex Ante**: Planning what to do with Aces vs. Sevens before cards are dealt
- **Interim**: Choosing best move after seeing you have Aces

Interim Dominance

Definition: A strategy for a type is interim dominated if an alternative strategy for that type provides higher payoff, regardless of other players' strategies

Focus: Single type after knowing their identity

Example: Player 2 as PD type

- If Player 1 plays Up: Right (4) > Left (3)
- If Player 1 plays Down: Right (2) > Left (1)
- **Conclusion**: Left is interim dominated for PD type

Key question: "Given that I'm this type, is there a move that's always better for me?"

Ex Ante Dominance

Definition: A strategy for a player is ex ante dominated if an alternative strategy for that player provides higher payoff, regardless of other players' strategies

Focus: Player before knowing their type

Example: Player 2's complete strategies

- Any plan where "PD type plays Left" is flawed
- Better plan: Switch PD type to Right, keep everything else same
- Eliminates: (Left, Left) and (Left, Right)

Key question: "Is there an overall plan that's always better for me, considering all types I could be?"

The Relationship Between Dominance Types

Key result: If a strategy is interim dominated for a type, then any complete strategy including that action is ex ante dominated for the player

Logic:

- Interim dominance: "Playing Left is always bad for PD type"
- Ex ante implication: "Any plan that says 'if I'm PD type, play Left' must be suboptimal"

Practical application: Use interim dominance to identify and eliminate ex ante dominated strategies in the combined matrix

Summary: Key Takeaways

Incomplete vs. Imperfect Information:

- Imperfect: Don't know opponent's past action
- Incomplete: Don't know opponent's payoffs/preferences

Bayesian Nash Equilibrium:

- Solution concept for incomplete information games
- Assigns one strategy to each type of player
- Uses beliefs to calculate expected payoffs

Solving for BNE:

- Matrix method: Combine multiple games into one
- Create compound strategies for uncertain player
- Find Nash equilibria in combined matrix

Dominance: Interim (type-specific) vs. Ex Ante (player-level)