

# Lecture 6: Repeated Games

**Instructor:** Fei Tan

 @econdojo    @BusinessSchool101    Saint Louis University

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## The Road Ahead

1. Finite Repeated Games
2. Infinite Repeated Games
3. Grim Trigger Strategy
4. Tit-for-Tat Strategy
5. Folk Theorem

## Finite Repeated Games

If we play the Prisoner's Dilemma repeatedly, surely the shadow of the future will encourage cooperation?

	Cooperate	Defect
Cooperate	3, 3	1, 4
Defect	4, 1	2, 2

In any **finitely repeated** game where the end is known, backward induction destroys all hope of cooperation

**Why?** Players can build trust, signal intentions, and seek rewards/punishments—but a cold, backward-flowing logic prevents cooperation when the end is certain

## Backward Induction

**Final Round N:** All previous payoffs are locked in. Players must play the one-shot Nash Equilibrium (Defect, Defect) with payoffs (2, 2)

**Round N-1:** Players know the final round outcome is predetermined as mutual defection

- Known future: Final round will be (Defect, Defect) regardless
- Empty promise: Cooperating now cannot influence future behavior
- Inevitable choice: This round becomes strategically identical to a one-shot game

**Unstoppable collapse:** This logic continues backward through every round:

- Stage N  $\rightarrow$  Stage N-1  $\rightarrow$  Stage N-2  $\rightarrow$  ...  $\rightarrow$  Stage 1
- The only subgame perfect equilibrium is (Defect, Defect) in every single round

## A Glimpse of Hope

The entire unraveling depends on certainty about the end. Two ways to eliminate the end → prevent backward induction:

1. **Infinitely repeated games:** No final round exists
2. **Unknown termination:** Players don't know when the game ends

**Result:** The incentive to build reputation and encourage future cooperation remains powerful because there's always a "next round" to consider

**Folk Theorem:** This opens a complex world where almost anything can be justified as rational

## Discount Factors

How do we compare infinite payoff streams?

- Player 1 always defects:  $4 + 4 + 4 + \dots = \infty$
- Player 1 always cooperates:  $1 + 1 + 1 + \dots = \infty$
- Paradox: Best and worst outcomes both equal infinity!

**Solution:** The discount factor  $\delta$  where  $0 < \delta < 1$ . Two interpretations:

1. Time value: \$3 today > \$3 tomorrow (can invest, consume sooner)
2. Continuation probability:  $\delta$  = probability the game continues to next period

**Infinite cooperation payoff:**  $3 + 3\delta + 3\delta^2 + 3\delta^3 + \dots$

- High  $\delta$  (close to 1) = Patient player
- Low  $\delta$  (close to 0) = Impatient player

## Geometric Series

How to calculate  $X + X\delta + X\delta^2 + X\delta^3 + \dots$

**Magic trick:**

- Finite series:  $S = X + X\delta + X\delta^2 + \dots + X\delta^{n-1}$
- Multiply by  $\delta$ :  $\delta S = X\delta + X\delta^2 + \dots + X\delta^{n-1} + X\delta^n$
- Subtract:  $S - \delta S = X - X\delta^n \rightarrow S = \frac{X(1-\delta^n)}{1-\delta}$
- As  $n \rightarrow \infty$ ,  $\delta^n \rightarrow 0$

**Infinite series formula:**  $X + X\delta + X\delta^2 + \dots = \frac{X}{1-\delta}$

## One-Shot Deviation Principle

In a 16-decision game, there are 65,536 possible strategies! Traditional approach: check against all 65,535 alternatives

**One-shot deviation principle:** A strategy is a subgame perfect equilibrium if and only if no player can profitably deviate at a single stage while maintaining their strategy everywhere else. Why It Works?

- If a strategy is truly optimal, it must beat any single-change alternative
- If you can't improve by changing one decision, you can't improve by changing multiple decisions

**Benefit:** Without principle → Check 65,535 alternatives; With principle → Check only 16 one-shot deviations. Transform an impossible task into a manageable one!



## Grim Trigger Strategy

**Unforgiving rule:** If anyone has defected at any point previously, defect forever. Otherwise, cooperate. Two simple rules:

1. Start by cooperating (offer initial trust)
2. Permanent punishment for any defection (no forgiveness, ever)

**Two possible paths:** Cooperation path (steady payoff of 3) vs. Punishment path (mutual defection: 4, 2, 2, 2, ...)

- Present value of cooperation:  $\frac{3}{1-\delta}$
- Present value of defection:  $4 + \frac{2\delta}{1-\delta}$
- Cooperation condition:  $\frac{3}{1-\delta} \geq 4 + \frac{2\delta}{1-\delta} \rightarrow \delta \geq \frac{1}{2}$

**Remark:** Cooperation requires sufficient patience ( $\delta \geq 1/2$ ). If players are too impatient (low  $\delta$ ), the immediate temptation payoff outweighs future punishment, and defection becomes rational

## Grim Trigger Is Self-Enforcing

Grim Trigger is not an imposed rule—it's a **voluntary strategy** that rational players choose because it's in their self-interest:

- No external enforcement needed—players follow it because deviation makes them worse off
- It's a Subgame Perfect Equilibrium with **credible** threats (when  $\delta \geq 1/2$ ): Once in punishment phase, continuing to defect is optimal

**Bottom line:** Grim Trigger works precisely because rational players commit to it knowing that both cooperation and punishment are self-enforcing

## US Dollar As Reserve Currency

**Players:** United States and other countries holding dollar reserves

- Cooperation path: US maintains sound monetary policy; other countries hold dollar reserves → Global financial stability, low transaction costs, US seigniorage benefits (payoff: 3, 3)
- Temptation to defect: US prints money excessively, inflates away debt; other countries diversify reserves early to avoid dollar devaluation risk (payoff: 4 today)

**Grim Trigger punishment:** Permanent loss of dollar dominance → US loses seigniorage benefits, higher borrowing costs forever; global financial instability (payoff: 2, 2, 2, ...)

**Self-enforcing strategy:** No treaty needed—Both sides value long-term cooperation over short-term gains when patient ( $\delta \geq 1/2$ )

## Tit-for-Tat Strategy

A forgiving strategy that doesn't hold grudges forever. Two simple rules:

1. Start by cooperating (be nice first)
2. Copy opponent's last move (forgive quickly, but retaliate immediately)

**Trade-off:** Cooperation  $(3, 3, 3, \dots)$  vs. Alternating  $(4, 1, 4, 1, \dots)$  after defection

**Cooperation condition:**  $\delta \geq \frac{1}{2}$  (same as Grim Trigger)

**Credibility problem:** Tit-for-Tat's threat isn't credible! After opponent defects:

- Punish: Defect  $\rightarrow (4, 1, 4, 1, \dots)$  vs. Forgive: Cooperate  $\rightarrow (3, 3, 3, \dots)$
- To cooperate initially: Need  $\delta \geq 1/2$ ; To punish credibly: Need  $\delta \leq 1/2$
- Nash equilibrium but not subgame perfect equilibrium (except at  $\delta = 1/2$ )

## Folk Theorem

In infinitely repeated games, any outcome that gives all players payoffs strictly better than their punishment payoff can be supported as a subgame perfect equilibrium. The mechanism includes:

1. Agreement: Players commit to a specific strategy profile
2. Punishment: Any deviation triggers permanent reversion to Nash equilibrium

### Generalized Grim Trigger:

- Player 1: Cooperate 100% of the time → Expected payoff: 2.85
- Player 2: Cooperate 95% of the time, defect 5% → Expected payoff: 3.15
- Both payoffs  $> 2$  (punishment level), so both accept this asymmetric deal

**Infinitely many equilibria:** As long as players are patient enough (high  $\delta$ ), they won't risk losing any arrangement that beats permanent punishment

## Prediction Problem

**Challenge:** When everything is possible, nothing is predictable

**Example:**

- Imagine observing this 4-period sequence: 1. Mutual Cooperation → 2. Mutual Defection → 3. Mutual Defection → 4. (Defect, Cooperate)
- Strategy: Follow the prescribed periods 1-4, then cooperate forever; Any deviation triggers permanent defection
- Seemingly random? Folk Theorem shows this can be a rational equilibrium!

**Insight:** The value of infinite future cooperation vastly outweighs any finite sequence of strange payoffs. For patient players, any 4-period "cost" becomes irrelevant compared to infinite future benefits