LECTURE 12: EXPECTATIONS: THE BASIC TOOLS*

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Economic agents have foresight, so beliefs about the future can affect the present decisions. This lecture provides the basic tools by which consumers and firms can price future economic opportunities. We lay the groundwork for a look at consumption and investment decisions when agents are forward-looking, a discussion of asset markets, and an integration of expectations into the IS-LM analysis. Future economic opportunities can be expressed in terms of the present by using a discount factor, which acts like a price. The sum of a sequence of payments, each priced at an appropriate discount factor, is called the present discounted value of the sequence. In practice, future variables are unknown, so we calculate the present discounted value of the expected sequence of payments.

1 NOMINAL VERSUS REAL INTEREST RATES

When we borrow (lend), we care about how many goods—rather than dollars—we give up (get) in the future in exchange for the goods we get (give up) today. The presence of inflation makes the distinction between nominal and real interest rates important:

- Interest rates expressed in terms of units of the national currency are called **nominal interest rates**. If the nominal interest rate for year t is i_t, borrowing one dollar this year requires you to pay $1 + i_t$ dollars next year. So i_t can be thought as this year's price of one dollar relative to next year.

- Interest rates expressed in terms of baskets of goods are called **real interest rates**. If the real interest rate for year t is r_t, borrowing the equivalent of one basket of goods this year requires you to pay the equivalent of $1 + r_t$ baskets of goods next year. So r_t can be thought as this year's price of one basket of goods relative to next year.

Now we relate nominal interest rate, which we do observe, and real interest rate, which we typically do not observe, by taking into account expected inflation. See Figure 1 below.

- Suppose we want to consume one more basket of goods with price P_t this year. Then we must borrow P_t dollars and repay $(1 + i_t)P_t$ dollars next year.

- Let P^e_{t+1} be the expected price of one basket of goods next year. Then we would expect to repay $(1 + i_t)P_t/P^e_{t+1}$ baskets of goods next year.

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These are notes that I used by myself to lecture from and for educational purposes only. The material presented here is largely based upon the undergraduate textbook by Blanchard and Johnson (2012), *Macroeconomics*, 6th Edition, Prentice Hall. Please do NOT circulate.

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- Putting together the definition and the above derivation of real interest rate, it follows that

$$1 + r_t = (1 + i_t)\frac{P_t}{P^e_{t+1}} \tag{1.1}$$

Denote the expected inflation between t and $t+1$ by $\pi^e_{t+1} \equiv (P^e_{t+1} - P_t)/P_t$. Then (1.1) can be rewritten as

$$1 + r_t = \frac{1 + i_t}{1 + \pi^e_{t+1}} \tag{1.2}$$

which gives us the exact relation among real interest rate, nominal interest rate, and expected inflation.

- When nominal interest rate and expected inflation are not too large (less than 20%), a close approximation to equation (1.2) is given by the simpler relation

$$r_t \approx i_t - \pi^e_{t+1} \tag{1.3}$$

In what follows, we treat the above relation as if it were an equality.

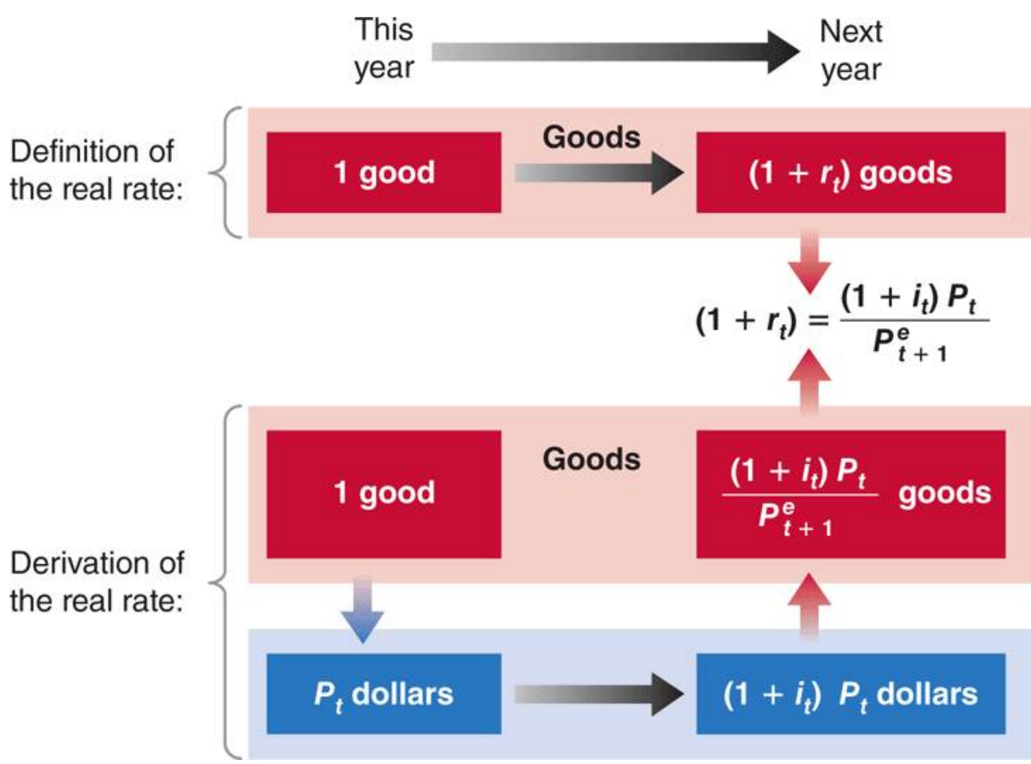

Figure 1. Definition and derivation of the real interest rate

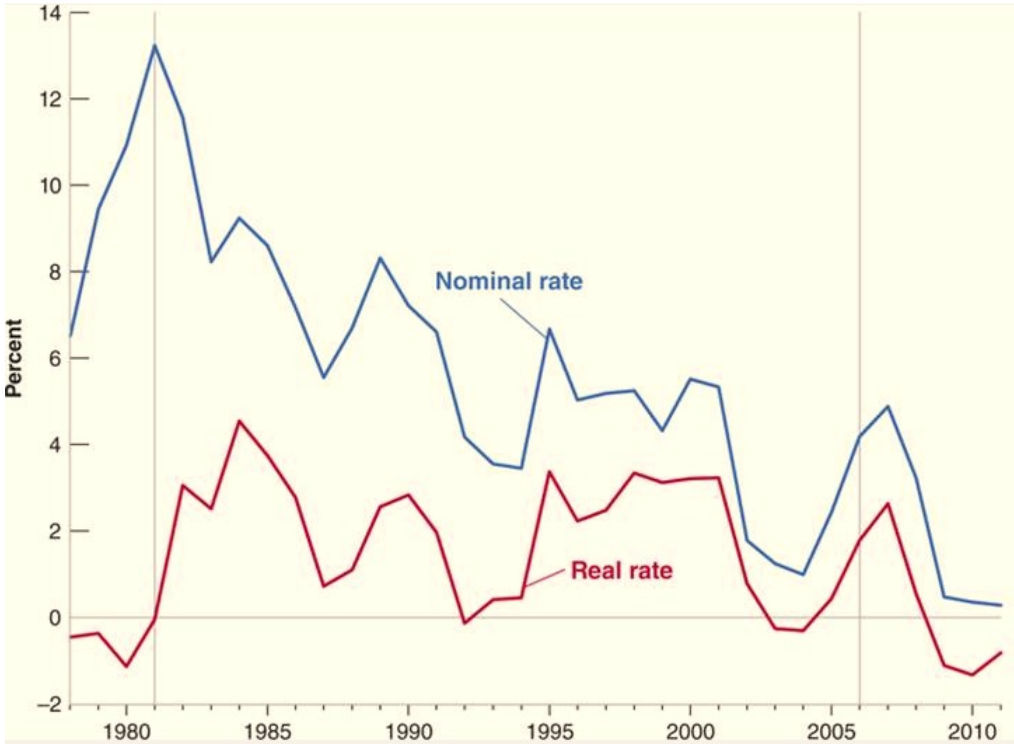

Figure 2. Nominal and real one-year T-bill rates in the U.S. since 1978

Because expected inflation is typically positive, the real interest rate is typically lower than the nominal interest rate. Moreover, for a given nominal interest rate, the higher the expected rate of inflation, the lower the real interest rate.[1] See Figure 2 above.

2 Nominal and Real Interest Rates, and the IS-LM Model

In the IS-LM model developed earlier, "the" interest rate not only affected investment in the IS relation, but also the choice between money and bonds in the LM relation.

- Firms, in deciding how much to invest, want to know how much they have to repay in terms of goods. So the IS relation should read as

$$Y = C(Y - T) + I(Y, r) + G$$

- When people decide whether to hold money or bonds, they take into account the opportunity cost, i.e. the nominal interest rate, of holding money. Thus, the LM relation

[1]Note that the real interest rate is based on expected inflation. If actual inflation turns out to be different from expected inflation, the realized real interest rate will be different from the real interest rate. For this reason, the real interest rate is sometimes called the *ex-ante* real interest rate, whereas the realized real interest rate is called the *ex-post* real interest rate.

should read as

$$\frac{M}{P} = YL(i)$$

- Putting together the modified IS relation, the LM relation, and the relation between real and nominal interest rate, we have the extended IS-LM model

$$\text{IS relation: } Y = C(Y - T) + I(Y, r) + G \tag{2.1}$$

$$\text{LM relation: } \frac{M}{P} = YL(i) \tag{2.2}$$

$$\text{real interest rate: } r = i - \pi^e \tag{2.3}$$

An immediate implication of the above relations to policymaking: since nominal interest rate is directly affected by monetary policy while real interest rate affects spending and output, the effects of monetary policy on output depend on how movements in the nominal interest rate translate into movements in the real interest rate.

3 MONEY GROWTH, INFLATION, NOMINAL AND REAL INTEREST RATES

We look at how an increase in money growth affects the nominal interest rate and the real interest rate, both in the short run and in the medium run.

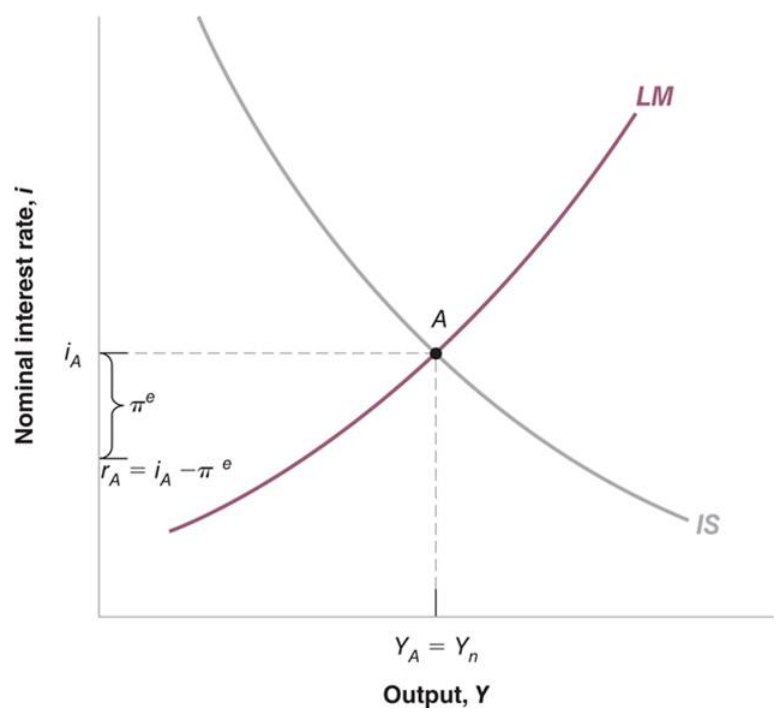

Figure 3. Equilibrium output and interest rates

Revisiting the IS-LM model. For given values of P, M, G, T, and π^e, the associated IS and LM curves are drawn in Figure 3 above.

- The IS curve is downward sloping: given π^e, a decrease in nominal interest rate leads to an equal decrease in real interest rate, leading to an increase in spending and output.
- The LM curve is upward sloping: given M, an increase in output, which leads to an increase in the demand for money, requires an increase in nominal interest rate.
- The equilibrium is given by the intersection of the IS and LM curves.

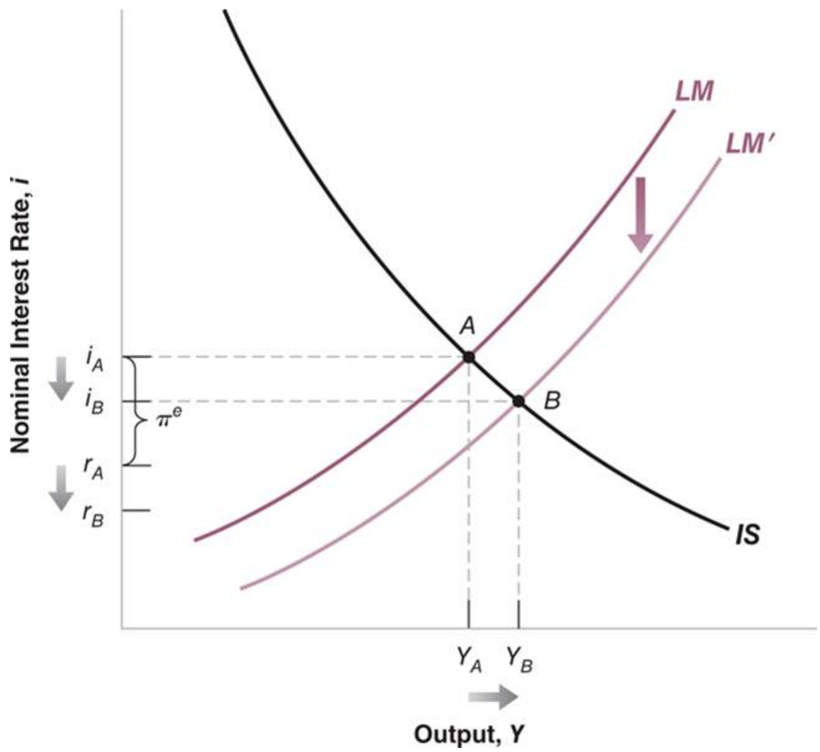

Figure 4. The short-run effects of an increase in money growth

Nominal and real interest rates in the short run. Assume the economy is initially at the natural level of output. We consider the effects of increases in the growth rate of money, i.e. faster increases in nominal money, on output, nominal interest rate, and real interest rate in the short run. See Figure 4 above.

- The LM curve shifts down. This is because the faster increase in nominal money will not be matched by an equal increase in the price level in the short run, leading to an increase in the real money stock (M/P). At any given level of output, higher real money stock leads to lower nominal interest rate.

- The IS curve does not shift. This is because people and firms do not revise expectations of inflation immediately. Given expected inflation, a given nominal interest rate corresponds to the same real interest rate and the same level of spending and output.

- In equilibrium, output is higher, nominal interest rate is lower, and given expected inflation, so is the real interest rate.

Nominal and real interest rates in the medium run. We consider the effects of a permanent increase in the growth rate of money g_M on output, nominal interest rate, and real interest rate in the medium run.

- In the medium run, output returns to its natural level Y_n.[2] Thus, for given values of G and T, the real interest rate must be such that

$$Y_n = C(Y_n - T) + I(Y_n, r_n) + G \tag{3.1}$$

  where r_n is called the **natural real interest rate**. That is, *in the medium run, the real interest rate returns to its natural rate, which is independent of the growth rate of money.*

- In the medium run, since $r = r_n$, $\pi^e = \pi$, and $\pi = g_M$, it follows that

$$i = r + \pi^e = r_n + \pi = r_n + g_M \tag{3.2}$$

  That is, *in the medium run, an increase in money growth leads to an equal increase in both inflation and nominal interest rate.*

- The result that, in the medium run, the nominal interest rate increases one-for-one with inflation is known as the **Fisher hypothesis** after Irving Fisher.

From the short to the medium run. We look at the adjustment process of nominal and real interest rates over time. Figure 5 below.

- So long as $r < r_n$, $Y > Y_n$ and $u < u_n$. From the Phillips curve relation, inflation increases.

- As inflation increases, it eventually becomes higher than nominal money growth, leading to negative real money growth, i.e. monetary contraction.[3] Thus, given expected inflation, both nominal and real interest rates start increasing.

[2]For simplicity, we ignore output growth and assume that the natural level of output is constant over time. If output is growing at rate g_Y, then $i = r_n + g_M - g_Y$, where g_Y is the trend growth rate of output.

[3]Note that $g_{M/P} = g_M - \pi$, where $g_{M/P}$ is real money growth.

- In medium run, the real interest rate increases back to r_n, output decreases back to Y_n, unemployment increases back to u_n, and inflation is no longer changing. As a result, the nominal interest rate converges to a new higher value, equal to the natural real interest rate plus the new higher growth rate of nominal money.

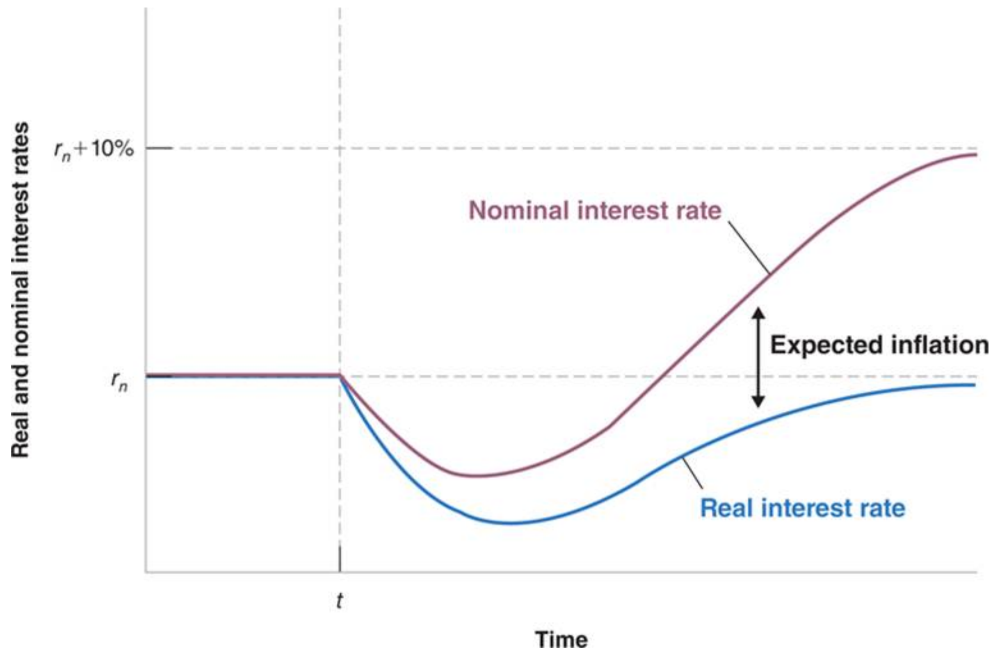

Figure 5. The adjustment of real and nominal interest rates to an increase in money growth from 0% to 10%

4 EXPECTED PRESENT DISCOUNTED VALUES

Suppose that there is a manager considering whether or not to buy a new machine. Her question is whether the **expected present discounted value** of the sequence of future profits generated by the machine exceeds the cost of buying and installing the machine today.

Computing expected present discounted values. Since the expected present discounted values are not directly observable, they must be constructed from information on the sequence of expected payments and expected interest rates. See Figure 6 below.

- One dollar next year is worth $1/(1+i_t)$ dollars this year. We call $1/(1+i_t)$ the present discounted value of one dollar next year. We also call $1/(1+i_t)$ the **discount factor** and i_t the **discount rate**. Because the nominal interest rate is always positive, the discount factor is always less than one: a dollar next year is worth less than a dollar today.

- More generally, consider a sequence of known payments in dollars with known interest rates, starting today and continuing into the future. Denote today's payment by

$\$z_t$, the payment next year by $\$z_{t+1}$, and so on. Then the present discounted value of this sequence of payments is given by

$$\$V_t = \$z_t + \frac{1}{1+i_t}\$z_{t+1} + \frac{1}{(1+i_t)(1+i_{t+1})}\$z_{t+2} + \cdots \tag{4.1}$$

Note that future payments are discounted more heavily and so their present discounted value is lower.

- Actual decisions are based on expectations of future payments and interest rates. Replacing the known future payments and interest rates in the above formula with their expectations, the expected present discounted value is given by[4]

$$\$V_t = \$z_t + \frac{1}{1+i_t}\$z^e_{t+1} + \frac{1}{(1+i_t)(1+i^e_{t+1})}\$z^e_{t+2} + \cdots \tag{4.2}$$

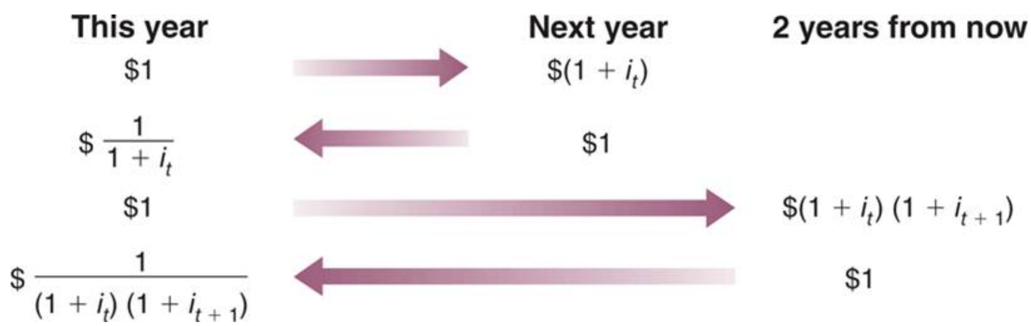

Figure 6. Computing present discounted values

Using present values: examples. Equation (4.2) has two important implications:

- The present value depends positively on current and expected future payments.
- The present value depends negatively on current and expected future interest rates.

As an exercise, you should work through the examples of computing expected present discounted values in section 14-4, all of which are special cases of the general formula (4.2).

Nominal versus real interest rates, and present values. Now we look at the expected present discounted value of a sequence of real payments. Replacing the expected future nominal payments and nominal interest rates in equation (4.2) with their real terms, we can otain

$$V_t = z_t + \frac{1}{1+r_t}z^e_{t+1} + \frac{1}{(1+r_t)(1+r^e_{t+1})}z^e_{t+2} + \cdots \tag{4.3}$$

[4]This argument ignores riskiness. If people dislike risk, the value of an uncertain payment will be lower than the value of a riskless payment, even if both have the same expected value.

These two ways of writing the present discounted value turn out to be equivalent and can be linked up by the following relation

$$\$V_t/P_t = V_t \tag{4.4}$$

See the appendix of chapter 14 for its proof. Which one of the two formulas is more helpful depends on the context.