TOPIC 12: A SIMPLE MODEL OF MONEY[*]

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This lecture introduces a very simple version of Samuelson (1958)'s overlapping generations model that has been applied to study a large number of topics in monetary theory. This basic model is highly tractable and has two essential features to monetary economics: [i.] some form of trade "friction" keeping people away from directly acquiring the desired goods in the absence of money is required, or otherwise there would be no role for money; [ii.] some people are willing to hold money from one period to the next since money is an asset held over some period of time before it is spent, i.e. the model must be dynamic.

1 THE ENVIRONMENT

We assume that each individual lives for two periods, "young" and "old". The economy begins with N_0 members of the initial old in period 1. In each period $t \geq 1$, N_t individuals are born. Thus, there are N_t young individuals and N_{t-1} old individuals alive in period t. Individuals are endowed with y units of "perishable" goods when young and nothing when old. See Figure 1 below.

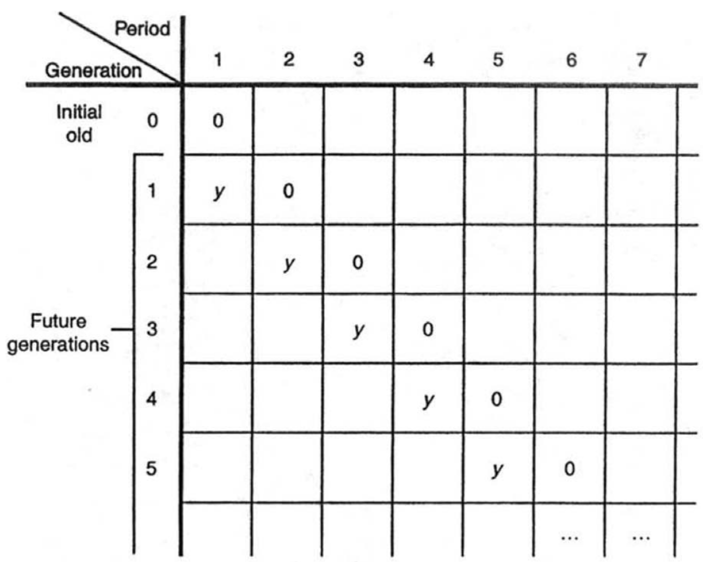

Figure 1. Endowment Pattern.

[*]*Date:* April 14, 2015.

Disclaimer: these are notes that I used by myself to lecture from and for educational purposes only. The material presented here is based upon chapter 1 of the undergraduate textbook by Bruce Champ, Scott Freeman, and Joseph Haslag (2011), *Modeling Monetary Economies*, 3rd Edition, Cambridge University Press. (Also see other chapters for various extensions of the baseline model.) Please do NOT circulate.

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Preferences. Individuals born in period t derive **utility** from consuming the goods when young, $c_{1,t}$, and when old $c_{2,t+1}$. The initial old simply derives utility from $c_{2,1}$. **Indifference curve**, which connects all consumption bundles yielding equal utility, is a graphical representation of an individual's preference. See Figure 2 below. We make three assumptions about preference:

- Assumption 1: to receive another unit of consumption tomorrow, an individual is willing to give up more consumption today if the good is currently abundant than if it is scare. See Figure 2 below. Note that the curve becomes flatter from left to right.

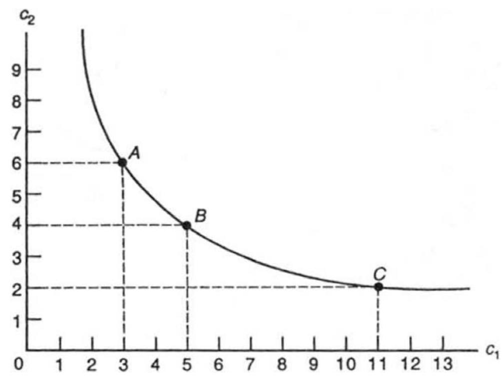

Figure 2. Indifference Curve.

- Assumption 2: an individual prefers the consumption of positive amounts of goods in both periods of life over the consumption of any amount of goods in only one period of life. Note that the curve never crosses either axis.

- Assumption 3: for a given amount of consumption in one period, an individual's utility increases with the consumption in the other period. See Figure 3 below. Note that utility increases in the direction of the arrow.

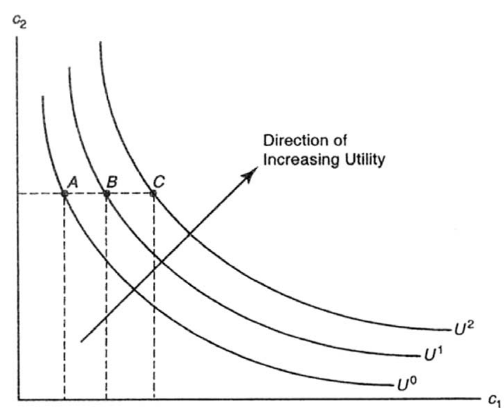

Figure 3. Indifference Map.

One last concept is that an individual's rankings of preference is **transitive**—if she prefers bundle B to bundle A and bundle C to bundle B, then she must prefer bundle C to bundle A. Graphically, this means indifference curves cannot cross. Figure 4 below violates this transitivity property and assumption 3.

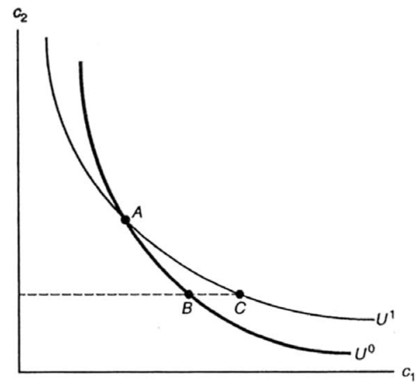

Figure 4. Indifference Curves Cannot Cross.

The problem facing each generation is that, since each individual is endowed with perishable goods only when young but wants to consume in both periods of life, she must find a way to acquire consumption when old. In what follows, we will examine two solutions to this economic problem.

2 Centralized Solution

The first, centralized solution proposes that a benevolent central planner with complete knowledge and total control over the economy will allocate the economy's resources between the young and the old at each point in time.

Feasible allocations. We first look at the resource constraint facing the central planner.

- The total amount of goods available in period t is $N_t y$. For the sake of equity, suppose each individual of generation t is assigned the same lifetime allocation $(c_{1,t}, c_{2,t+1})$. The resource constraint requires that consumption by the young and the old do not exceed total resources available

$$N_t c_{1,t} + N_{t-1} c_{2,t} \leq N_t y, \quad t \geq 1 \tag{2.1}$$

- For simplicity, we assume that the population is constant, $N_t = N$ for all t, and focus on the **stationary allocation** in which members of each generation are assigned the same consumption pattern, $c_{1,t} = c_1$ and $c_{2,t} = c_2$ for all t.[1] Then we can derive a simple per capita resource constraint

$$c_1 + c_2 \leq y \tag{2.2}$$

See Figure 5 below. The shaded area represents the set of all feasible per capita allocations.

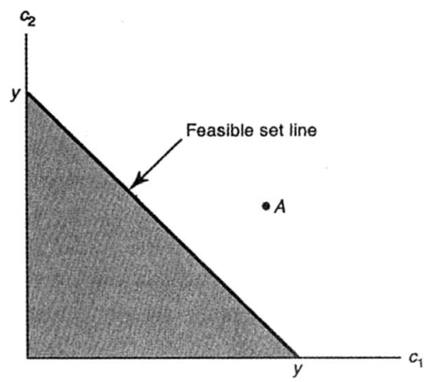

Figure 5. Feasible Set.

The golden rule allocation. We are now ready to find the **golden rule** that maximizes the utility of future generations.

- The golden rule occurs at the unique tangency point between the boundary of the feasible set and an indifference curve. Any other point either yields a lower level of utility or is infeasible. See Figure 6 below.

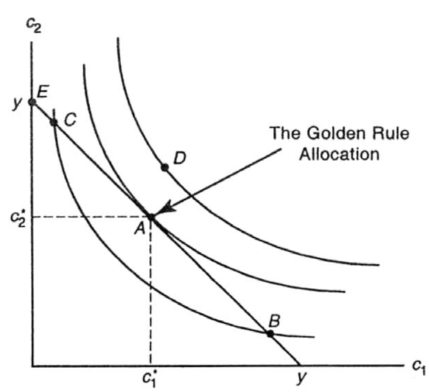

Figure 6. The Golden Rule Allocation.

[1]This is because all generations have the same endowments and preferences, and expect the same future pattern of endowments and preferences. Note that a stationary allocation does not necessarily imply $c_1 = c_2$.

- The golden rule for stationary allocation does not maximize the utility of the initial old, which solely depends on c_2; if the central planner were to maximize the welfare of the initial old, she would set $c_2 = y$, implying that people consume nothing when young, which is inferior to the more balanced allocation (c_1^*, c_2^*).

So far we have made stringent assumptions about the central planner; she has not only the power to reallocate endowments costlessly between generations, but also the wisdom about individuals' exact preferences so as to determine (c_1^*, c_2^*).

3 Decentralized Solution

The second, decentralized solution proposes that individuals use money to trade for what they want. In other words, we let the market do the job of a central planner with the help of money.

Competitive equilibrium. We first define the notion of **competitive equilibrium** that has three properties: [i.] each individual tries to achieve the highest level of utility that she can afford through mutually beneficial trades with other individuals; [ii.] all individuals are **price-takers**—their actions have no effect on prices (rates of exchange); [iii.] markets clear—supply equals demand in all markets. We consider two competitive equilibria:

- Without money, there is a lack of trades among individuals that captures the "lack of double coincidence of wants"—each generation wants what the next generation has but does not have what the next generation has. The resulting equilibrium is autarky.

- A **monetary equilibrium** is an equilibrium in which there is a valued supply of **fiat money**—an intrinsically useless object that is widely accepted as a means of payment. We assume that the government can costlessly produce fiat money that cannot be counterfeited. By valued, it means that the fiat money can be traded for goods.

Monetary equilibrium with constant economy. By backward induction, we can infer that fiat money will have no value if it is *believed* to be valueless at any future date. Instead, we consider a monetary equilibrium in which money has a positive value in all future periods.

- Suppose the economy begins with a fixed stock of fiat money, M, and each of the initial old shares an equal amount, M/N. Define v as the value of one unit of fiat money (call the unit a dollar) in terms of goods; v_t is the inverse of the dollar price, p, of one unit of goods.[2] For example, if an apple costs 50 cents, then $p = .5$ and $v = 2$.

[2]Since the economy has only one good, p can be viewed as the aggregate price level in this economy.

- Let the number of dollars acquired by an individual be denoted by m. The budget constraint facing the individual in the first period of life is

$$c_{1,t} + v_t m_t \leq y \tag{3.1}$$

The budget constraint facing the individual in the second period of life is

$$c_{2,t+1} \leq v_{t+1} m_t \tag{3.2}$$

Combining (3.1) and (3.2) gives the lifetime budget constraint

$$c_{1,t} + \frac{v_t}{v_{t+1}} c_{2,t+1} \leq y \tag{3.3}$$

where $v_t > 0$ for all t by the definition of a monetary equilibrium. Again, the $(c^*_{1,t}, c^*_{2,t+1})$ combination that maximizes the utility of future generations occurs at the unique point where the budget line is tangent to an indifference curve. See Figure 7 below.

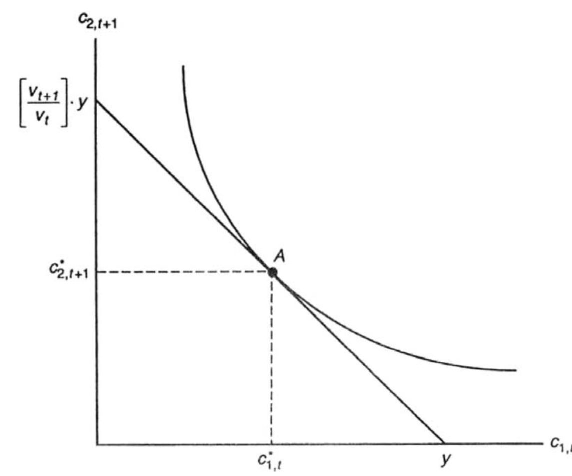

Figure 7. The Choice of Consumption with Fiat Money.

With valued fiat money, individuals can trade for goods despite the lack of double coincidence of wants. Therefore, it is welfare-improving.

- The term v_{t+1}/v_t equals the (gross real) rate of return of fiat money. (Why?) Note that v_t depends on an infinite chain of expectations about the future values of fiat money. We assume that individuals have **perfect foresight**—their expectations of future variables equal the actual values.[3] In competitive equilibrium, v (price of money)

[3] In a more general context where the economy is hit by random shocks, we may reasonably assume **rational expectations**—individuals' expectations of future variables are not systematically wrong. Mathematically, this requires that their expectations equal the true statistical expected values.

is determined as the price at which money demand equals money supply:

$$\underbrace{v_t M}_{\text{total supply of money}} = \underbrace{N(y - c_{1,t})}_{\text{total demand of money}} \tag{3.4}$$

We again consider a stationary equilibrium in which $c_{1,t} = c_1$ and $c_{2,t} = c_2$ for all t. This implies a constant value of money and hence a constant price level

$$v_{t+1} = v_t \quad \Rightarrow \quad p_{t+1} = p_t \tag{3.5}$$

- In a stationary equilibrium, (3.4) implies that

  $$p_t = \frac{1}{v_t} = \frac{M}{N(y - c_1)} \tag{3.6}$$

  Thus, the price level p is proportional to the stock of money M and hence the "quantity theory of money" holds in our basic model.

- Figure 7 suggests that an individual's choices of consumption and real money balances do not depend on the nominal stock of money M but do depend on v_{t+1}/v_t, which is unaffected by M. This property is referred to as the **neutrality of money**.

- Since the budget line of Figure 7 and the feasible set line of Figure 5 are identical, the stationary monetary equilibrium obeys the golden rule. The initial old also become better off because each of them will receive $v_1 m_0 = v_1(M/N)$ units of goods.

Monetary equilibrium with growing economy. We now allow the economy to grow over time by assuming that the population grows at a constant gross rate n, i.e. $N_t = nN_{t-1}$ for all t.

- We first consider the stationary allocation in the centralized solution. The resource constraint facing the central planner is the same as (2.1), which can be simplified to

  $$c_1 + \frac{1}{n} c_2 \leq y \tag{3.7}$$

  The golden rule occurs at the unique tangency point between the feasible allocation line and an indifference curve. See Figure 8 below. Note that the feasible allocation line becomes steeper than before.

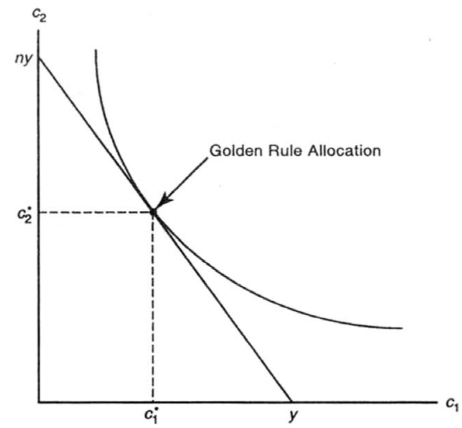

Figure 8. The Golden Rule Allocation with Growing Economy.

- Next we consider the stationary monetary equilibrium. Since money demand must equal money supply, we have

$$v_t M = N_t(y - c_1) \tag{3.8}$$

implying that the real rate of return on money is given by

$$\frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1}(y-c_1)}{M}}{\frac{N_t(y-c_1)}{M}} = \frac{N_{t+1}}{N_t} = n \tag{3.9}$$

Since $n > 1$, the value of money is increasing over time, or the price level is falling over time. Thus, the individual's lifetime budget constraint becomes

$$c_1 + \frac{v_t}{v_{t+1}} c_2 \leq y \quad \Rightarrow \quad c_1 + \frac{1}{n} c_2 \leq y \tag{3.10}$$

- Since (3.7) and (3.10) are identical, the stationary monetary equilibrium obeys the golden rule. Note that the above analysis also applies to a shrinking economy in which $n < 1$. (Try!)

4 Math Appendix

We will derive the real demand of fiat money, denoted by $q_t = v_t m_t$, from a specific utility function given by $(c_{1,t})^{1/2} + (c_{2,t+1})^{1/2}$. Plugging the budget constraints in both periods into the utility function gives

$$(y - q_t)^{1/2} + \left(\frac{v_{t+1}}{v_t} q_t\right)^{1/2}$$

Differentiating with respect to q_t and setting the derivative to zero gives

$$-\frac{1}{2}(y - q_t^*)^{-1/2} + \frac{1}{2}\left(\frac{v_{t+1}}{v_t}\right)^{1/2}(q_t^*)^{1/2} = 0$$

Solving for q_t^* yields the real money demand

$$q_t^* = \frac{y}{1 + \frac{v_t}{v_{t+1}}}$$

References

SAMUELSON, P. A. (1958): "An Exact Consumption-Loan Model of Interest With or Without the Social Contrivance of Money," *Journal of Political Economy*, 66(6), 467–482.