TOPIC 5: THE RISK AND TERM STRUCTURE OF INTEREST RATES[*]

Fei Tan[†]

This lecture introduces three factors that affect bond yields: default risk, taxes, and maturity. We will focus on the maturity by studying the relationship among bonds with the same risk characteristics but different maturities, called the term structure of interest rates, and its two explanations. The lecture concludes with an analysis of the information contained in the risk structure and term structure of interest rates.

1 RATINGS AND THE RISK STRUCTURE OF INTEREST RATES

Assigned as reading. You should know the impact of bond ratings on bond yields: everything else held equal, a **ratings downgrade** signifies a higher risk of default, shifting the bond demand curve to the left [Core Principle 2]; as a result, bond price falls and yield rises. We can think of any bond yield as follows:

$$\text{bond yield} = \underbrace{\text{U.S. Treasury yield}}_{\text{benchmark}} + \underbrace{\text{default risk premium}}_{\text{risk spread}} \tag{1.1}$$

which implies that when Treasury yields move, all other yields move with them. See Figure 7.2 below.

2 DIFFERENCES IN TAX STATUS AND MUNICIPAL BONDS

Bondholders must pay income tax on the interest income they receive from owning privately issued bonds, called **taxable bonds**. In contrast, the coupon payments on bonds issued by state and local governments, called **municipal** or **tax-exempt bonds**, are exempt from taxation. The relation between the yields on taxable and tax-exempt bonds is given by

$$\text{tax-exempt bond yield} = \text{taxable bond yield} \times (1 - \text{tax rate}) \tag{2.1}$$

For example, with a 30% tax rate, the tax-exempt yield on a 10% bond is 7%. The higher the tax rate, the wider the gap between the two yields. Note that investors base their decisions

[*] *Date:* February 7, 2015.

Disclaimer: these are notes that I used by myself to lecture from and for educational purposes only. The material presented here is largely based upon the undergraduate textbook by Stephen Cecchetti and Kermit Schoenholtz (2014), *Money, Banking and Financial Markets*, 4th Edition, McGraw-Hill/Irwin. Please do NOT circulate.

[†]Department of Economics, Indiana University Bloomington. E-mail: tanf@indiana.edu

on the tax-exempt yield.

3 The Term Structure of Interest Rates

The relationship among bonds with the same risk characteristics but different maturities is called the **term structure of interest rates**. See Figure 7.3 below. Three observations:

- Interest rates of different maturities tend to move together.
- Yields on short-term bonds are more volatile than yields on long-term bonds.
- Long-term yields tend to be higher than short-term yields.

Figure 7.2 The Risk Structure of Interest Rates

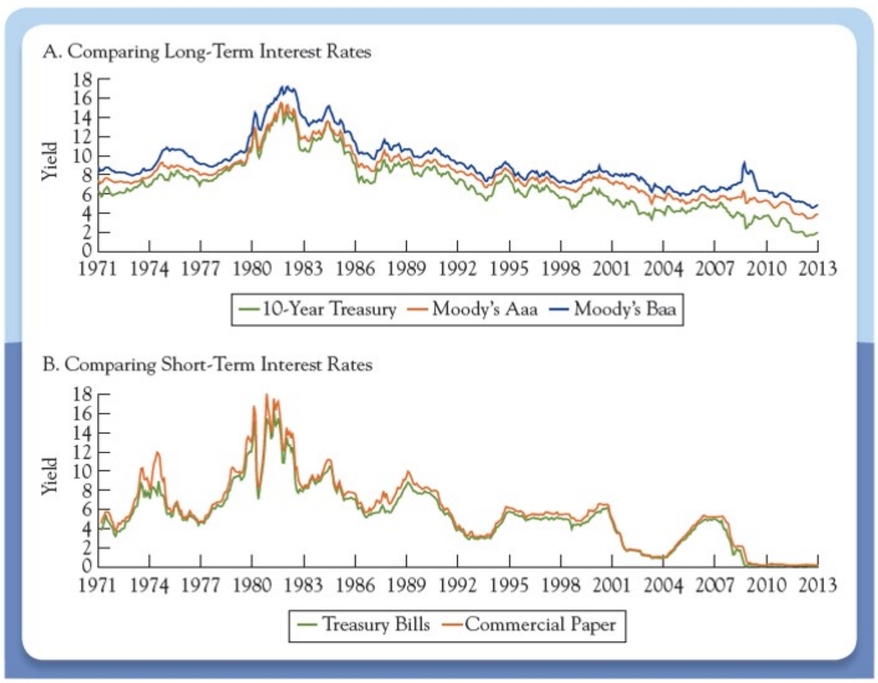
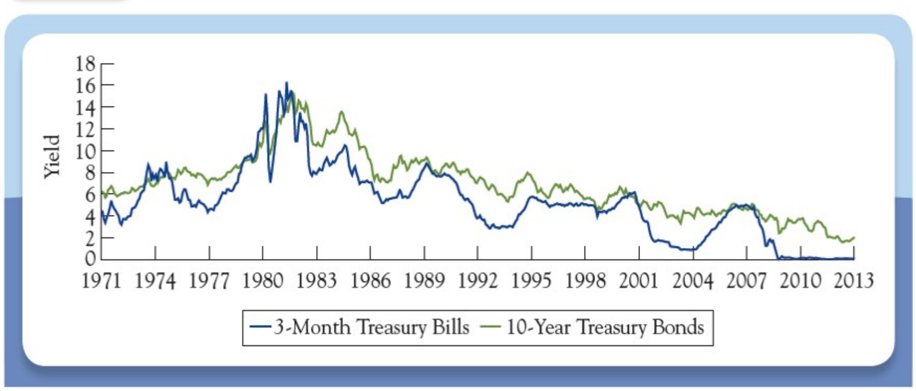


Figure 7.3 The Term Structure of Treasury Interest Rates



Explanation 1: The expectations hypothesis. The **expectations hypothesis of the term structure** assumes that there is no uncertainty about the future. As a result, bonds of different maturities are perfect substitutes for each other. For example, an investor with two-year horizon is indifferent between two strategies:

- Invest in a two-year bond and hold it to maturity. We denote the associated interest rate (or yield) by i_{2t}, where 2 stands for two years and t for the current period. Then investing one dollar in this bond yields $(1+i_{2t})^2$ dollars in two years.

- Invest in two one-year bonds, one today and a second when the first matures. We denote the interest rate (or yield) associated with the second bond by i^e_{1t+1}, where $t+1$ stands for next period and e for *expected*.[1] Then investing one dollar using this strategy yields $(1+i_{1t})(1+i^e_{1t+1})$ dollars in two years.

- Indifference between the two strategies means they must have the same return

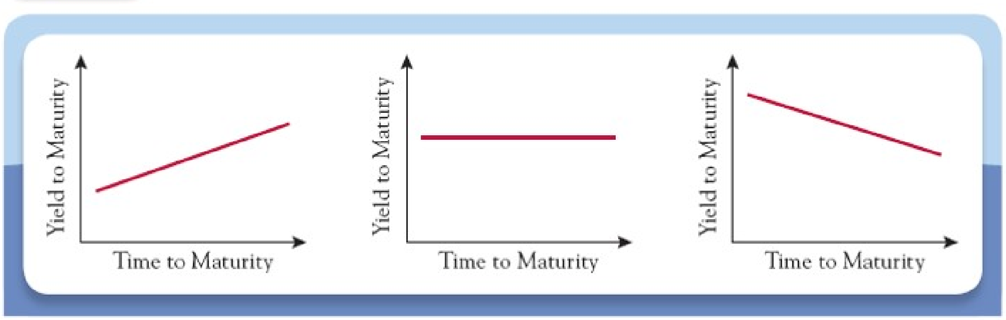
$$(1+i_{2t})^2 = (1+i_{1t})(1+i^e_{1t+1}) \tag{3.1}$$

  A useful approximation to the above relation is given by

$$i_{2t} \approx \frac{i_{1t}+i^e_{1t+1}}{2} \quad \Rightarrow \quad i^e_{1t+1} \approx 2i_{2t} - i_{1t} \tag{3.2}$$

  which suggests that when the long-term interest rate (i_{2t}) is higher (lower) than the short-term interest rate (i_{1t})—financial markets expect the short-term rate (i_{1t+1}) to be higher (lower) in the future—the yield curve slopes upward (downward). See Figure 7.5 below.

Figure 7.5 The Expectations Hypothesis and Expectations of Future Short-term Interest Rates



- The general statement of the expectations hypothesis is that the interest rate on an n-

[1] Because we have assumed away future uncertainty, this expectation is certain to be correct.

year bond is (approximately) the average of n expected future one-year interest rates

$$i_{nt} \approx \frac{i_{1t} + i^e_{1t+1} + i^e_{2t+1} + \cdots + i^e_{1t+n-1}}{n} \tag{3.3}$$

- We look at whether the expectations hypothesis can explain the three observations of the term structure:
  - (3.3) makes it clear that interest rates of different maturities will move together. For example, if i_{1t} changes, then all the yields at higher maturities will change in the same direction.
  - Since long-term yields are averages of a sequence of expected future short-term yields, they will be less volatile than the short-term yields.
  - It implies that the yield curve slopes upward only when interest rates are expected to rise. But the data shows that interest rates have been trending downward. So the expectations hypothesis cannot explain why long-term yields are normally higher than short-term yields.

To account for the third observation, we need to extend the expectations hypothesis to include risk.

Explanation 2: The liquidity premium theory. If we think about a bond yield as having two parts, risk free rate and risk premium, then the expectations hypothesis explains the risk free rate, and inflation and interest-rate risk explain the risk premium. Together they form the **liquidity premium theory of the term structure**. Mathematically, this theory says

$$i_{nt} = rp_n + \frac{i_{1t} + i^e_{1t+1} + i^e_{1t+2} + \cdots + i^e_{1t+n-1}}{n} \tag{3.4}$$

where rp_n denotes the risk premium associated with an n-year bond and is an increasing function of n—the longer the maturity, the higher the inflation and interest rate risk and hence risk premium. Two remarks:

- Because long-term bonds are riskier than short-term bonds, the liquidity theory predicts that long-term yields are higher than short-term yields.
- Under the liquidity premium theory, the yield curve normally slopes upward; a flat curve means interest rates are expected to fall; a downward-sloping curve means interest rates are expected to decline significantly.

4 The Information Content of Interest Rates

Assigned as reading. You should know what information is contained in the risk and term structure of interest rates and how it can be useful in predicting the overall economic conditions. In particular, know why increasing risk spread or **inverted yield curve** predicts a general economic slowdown. See Figures 7.8 and 7.9 below.

Figure 7.8 The Risk Spread and GDP Growth

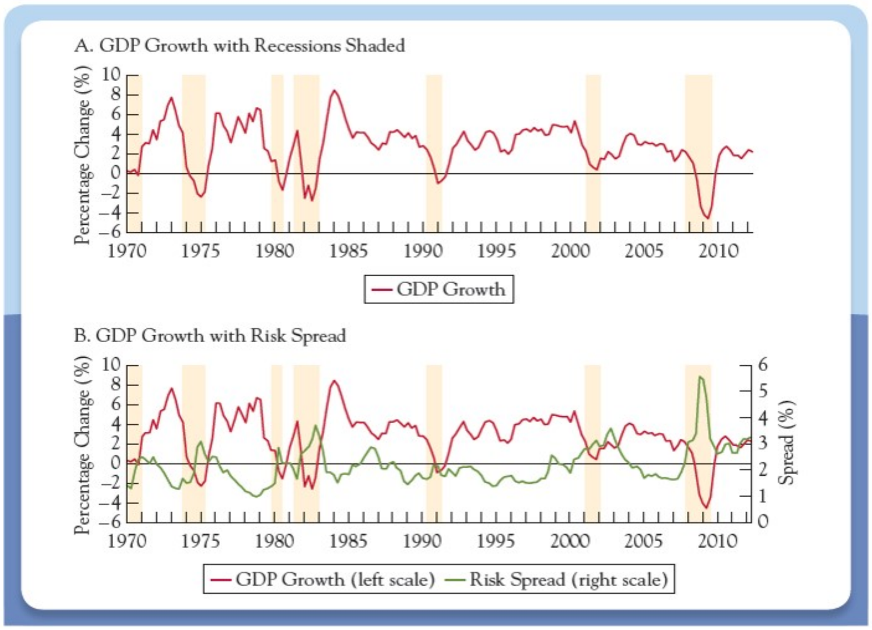


Figure 7.9 The Term Spread and GDP Growth