TOPIC 4: BONDS, BOND PRICES, AND THE DETERMINATION OF INTEREST RATES*

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Any financial arrangement involving the current transfer of resources from a lender to a borrower, with a transfer back at some time in the future, is a form of a bond. This lecture covers the relationship between bond prices and interest rates, the determination of bond prices in the market (by supply and demand), and why bonds are risky.

1 BOND PRICES

A standard bond specifies the fixed amounts to be paid and the exact dates of the payments. We will consider the pricing of four different types of bonds. For simplicity, this section assumes away the risk of holding bonds.

Zero-coupon bonds. **U.S. Treasury bill** or **T-bill** is an example of **zero-coupon bond** because each T-bill represents a promise by the U.S. government to pay \$100 on a fixed future date but without coupon payments. Two remarks:

- Let i be the interest rate and n the time until the payment is made. Then the price of a T-bill (TB) is

$$\$P_{TB} = \frac{\$100}{(1+i)^n} \tag{1.1}$$

- Given the price of a zero-coupon bond, we can also compute the interest rate as

$$i = \sqrt[n]{\$100/\$P_{TB}} - 1 = \left(\frac{\$100}{\$P_{TB}}\right)^{1/n} - 1 \tag{1.2}$$

Fixed-payment loans. Conventional home mortgages and car loans are examples of **fixed-payment loans** because they promise a fixed number of equal payments at regular intervals. Let i be the interest rate, $\$C$ the fixed payment, and n the number of payments. Then the

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Disclaimer: these are notes that I used by myself to lecture from and for educational purposes only. The material presented here is largely based upon the undergraduate textbook by Stephen Cecchetti and Kermit Schoenholtz (2014), *Money, Banking and Financial Markets*, 4th Edition, McGraw-Hill/Irwin. Please do NOT circulate.

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price of a fixed-payment loan (FL) is

$$\$P_{FL} = \frac{\$C}{1+i} + \frac{\$C}{(1+i)^2} + \cdots + \frac{\$C}{(1+i)^n} \tag{1.3}$$

Coupon bonds. See Topic 2 for details.

Consols. **Consols** or **perpetuities** are bonds whose interest payments last forever and the borrower never repays the principal. Let i be the interest rate and $\$C$ the interest payment. Then the price of a consol (C) is

$$\$P_C = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{\$C}{(1+i)^k} = \lim_{n\to\infty} \frac{\$C}{i}\left[1 - \frac{1}{(1+i)^n}\right] = \frac{\$C}{i} \tag{1.4}$$

2 Bond Yields

Bonds of different maturities each have a price and an associated interest rate called the **yield to maturity**, or simply the **yield**.[1] The relation between maturity and yield is called the **yield curve**, or the **term structure of interest rates**. With the information about the promised payments and the bond price, we can obtain the bond yield.

Yield to maturity. **Yield to maturity** measures the return on holding a bond to its maturity when the principal payment is made. For example, the price of one-year 5% coupon bond (CB) is

$$\$P_{CB} = \frac{\$5}{1+i} + \frac{\$100}{1+i} \tag{2.1}$$

where the value of i that solves this equation is the yield. Three remarks:

- If $\$P_{CB}$ equals the \$100 face value, then the yield equals the 5% coupon rate.
- If $\$P_{CB}$ is below the face value, say \$99.06, then the yield, which is 6%, is above the coupon rate. This is because except for the \$5 coupon payment, the bondholder also receives the rise in bond value from \$99.06 to \$100, referred to as a **capital gain**.
- If $\$P_{CB}$ is above the face value, say \$100.96, then the yield, which is 4%, is below the coupon rate. This is because though the \$5 coupon payment, the bondholder now incurs a **capital loss**, which is the fall in bond value from \$100.96 to \$100.

[1] Yields on bonds with a short maturity, typically a year or less, are called short-term interest rates. Yields on bonds with a longer maturity are called long-term interest rates.

Note that the above remarks also apply to coupon bonds with longer maturities (Check it!).

Current yield. **Current yield** measures that part of the return from buying the bond that arises solely from the coupon payments. For example, the current yield of a coupon bond with yearly coupon payment $\$C$ and price $\$P_{CB}$ is

$$\text{current yield} = \frac{\$C}{\$P_{CB}} \tag{2.2}$$

which ignores the capital gain or loss. Back to the one-year 5% coupon bond. Three remarks:

- If $\$P_{CB}$ equals the \$100 face value, then the current yield, yield to maturity, and coupon rate all equal because there is neither capital gain nor loss.
- If $\$P_{CB}$ is below the face value, say \$99, then the current yield, $\$5/\$99 = 5.05\%$, is above the coupon rate but below the yield to maturity, 6.06%, which can be solved from

  $$\$99 = \frac{\$5}{1+i} + \frac{\$100}{1+i} \tag{2.3}$$

  because of the $\$100 - \$99 = \$1$ capital gain.
- If $\$P_{CB}$ is above the face value, say \$101, then the current yield, $\$5/\$101 = 4.95\%$, is below the coupon rate but above the yield to maturity, 3.96%, which can be solved from

  $$\$101 = \frac{\$5}{1+i} + \frac{\$100}{1+i} \tag{2.4}$$

  because of the $\$101 - \$100 = \$1$ capital loss.

Similar to yield to maturity, current yield also moves in the opposite direction from the bond price. See Table 6.1 below for a summarization.

Bond price < Face value:	Coupon rate < Current yield < Yield to maturity
Bond price = Face value:	Coupon rate = Current yield = Yield to maturity
Bond price > Face value:	Coupon rate > Current yield > Yield to maturity

Table 6.1. Relationship among bond price, coupon rate, current yield, and yield to maturity

Holding period returns. Since the bond price can change between the purchase time and the sale time, the **holding period return**—return to buying a bond and selling it before it matures—can differ from the yield to maturity. Suppose that we pay \$100 for a 10-year 6% coupon bond and sell it as a 9-year bond one year later. Three cases:

- If the interest rate stays at 6%, then the one-year holding period return equals the yield to maturity, which is $\$6/\$100 = 6\%$.

- If the interest rate falls from 6% to 5% over the year we hold the bond, then the 9-year bond price is

$$\$107.11 = \sum_{k=1}^{9} \frac{\$6}{(1+5\%)^k} + \frac{\$100}{(1+5\%)^9} \tag{2.5}$$

  and so the one-year holding period return is

$$13.11\% = \frac{\$6}{\$100} + \frac{\$107.11 - \$100}{\$100} \tag{2.6}$$

  which is above the yield to maturity because of the "surprise" \$7.11 capital gain.

- If the interest rate rises from 6% to 7% over the year we hold the bond, then the 9-year bond price is

$$\$93.48 = \sum_{k=1}^{9} \frac{\$6}{(1+7\%)^k} + \frac{\$100}{(1+7\%)^9} \tag{2.7}$$

  and so the one-year holding period return is

$$-0.52\% = \frac{\$6}{\$100} + \frac{\$93.48 - \$100}{\$100} \tag{2.8}$$

  which is below the yield to maturity because of the "surprise" \$6.52 capital loss.

In general, the one-year holding period return can be written as

$$\begin{aligned} \text{1-year holding period return} &= \frac{\text{yearly coupon payment}}{\text{price paid}} + \frac{\text{price sold} - \text{price paid}}{\text{price paid}} \\ &= \text{current yield} + \text{capital gain rate} \end{aligned} \tag{2.9}$$

Therefore, the potential for interest rate movements and hence changes in bond prices create risk. The longer the bond term, the greater those price movements and the associated risk.

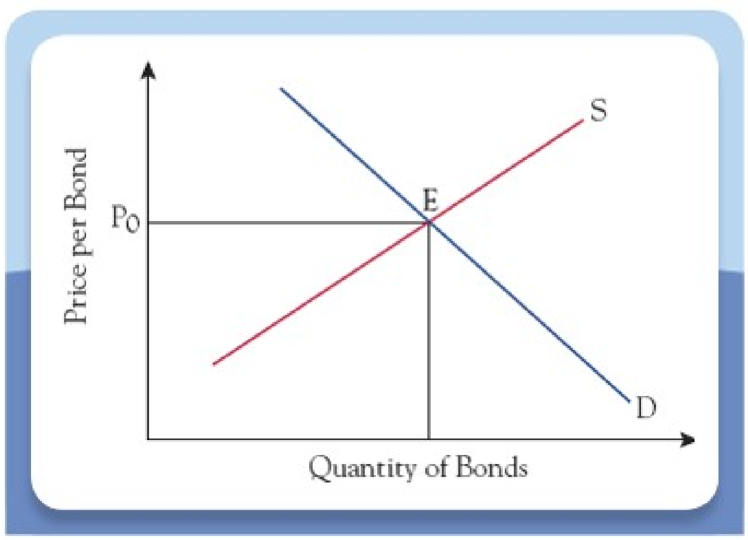
3 The Bond Market and the Determination of Interest Rates

To see how bond prices are determined, we will look at bond supply, bond demand, and equilibrium prices in the bond market. Note that once we know the bond price, we know the bond yield.

Bond supply, bond demand, and equilibrium in the bond market. Bond prices (and hence bond yields) are determined by the interaction between bond supply and bond demand. Three remarks:

- The bond supply curve is the relationship between the price and the quantity of bonds people are willing to sell. The higher the price, the larger the quantity supplied—it slopes upward.
- The bond demand curve is the relationship between the price and the quantity of bonds that investors demand. The higher the price, the smaller the quantity demanded—it slopes downward.
- Equilibrium in the bond market is the point at which supply equals demand. See Figure 6.1 below.

Figure 6.1 Supply, Demand, and Equilibrium in the Bond Market



Factors that shift bond supply. We will look at three factors that shift the bond supply curve:

- Changes in government borrowing. Any increase in government's borrowing needs increases the quantity of bonds outstanding, shifting the supply curve to the right. As a result, bond price falls and interest rate rises.

- Changes in general business conditions. As business condtions improve, firms have increasing borrowing needs, shifting the supply curve to the right. As a result, bond price falls and interest rate rises.

- Changes in expected inflation. At a given nominal interest rate, higher expected inflation lowers real interest rate and hence borrowing cost, shifting the supply curve to the right. As a result, bond price falls and interest rate rises.

Factors that shift bond demand. We will look at five factors that shift the bond demand curve:

- Wealth. Increases in wealth increase the investment in bonds, shifting the demand curve to the right. As a result, bond price rises and interest rate falls.

- Expected inflation. At a given nominal interest rate, lower expected inflation increases the real return on bonds, shifting the demand curve to the right. As a result, bond price rises and interest rate falls.

- Expected returns and expected interest rates. If the expected return on bonds rises (e.g. due to lower expected interest rate) relative to the return on alternative investments, the quantity of bonds demanded at each price will rise, shifting the demand curve to the right. As a result, bond price rises and interest rate falls.

- Risk relative to alternatives. If a bond becomes less risky relative to alternative investments, investors are willing to pay a higher price for it, shifting the demand curve to the right. As a result, bond price rises and interest rate falls.

- Liquidity relative to alternatives. If a bond becomes more liquid relative to alternative investments, investors are willing to pay a higher price for it, shifting the demand curve to the right. As a result, bond price rises and interest rate falls.

Understanding changes in equilibrium bond prices and interest rates. We will look at two factors that shift both the bond supply and demand curves:

- Changes in expected inflation. Higher expected inflation shifts the supply curve to the right and demand curve to the left. As a result, bond price falls and interest rate rises.

- Changes in general business conditions. A business-cycle downturn shifts the supply and demand curves both to the left. As a result, bond price can either rise or fall.

4 Why Bonds Are Risky

Bondholders face three major risks. We will look at how each risk affects the premium investors require over the risk-free return [Core Principle 2].

- **Default risk** is the chance that the bond's issuer may fail to make the promised payment. For example, suppose that the one-year risk-free interest rate is 5%. Without default risk the price of a one-year 5% coupon bond is

$$\$P_1 = \frac{\$100 + \$100 \times \text{coupon rate}}{1 + \text{risk-free rate}} = \frac{\$105}{1.05} = \$100 \tag{4.1}$$

  Suppose further that with probability of 0.1 the bondholder receives \$0. With default risk the bond price, or the expected present value (EPV) of bond payment, becomes

$$\underbrace{\$P_2}_{\text{EPV}} = \frac{\text{EV of bond payment}}{1 + \text{risk-free rate}} = \frac{0.9 \times \$105 + 0.1 \times \$0}{1.05} = \$90 \tag{4.2}$$

  Note that only risk-neutral investors are willing to pay $\$P_2$ for this bond. Now the promised yield to maturity is

$$16.67\% = \frac{\$105}{\$90} - 1 \tag{4.3}$$

  which implies a default-risk premium of $16.67\% - 5\% = 11.67\%$. Also note that risk-averse investors are only willing to pay a price below $\$P_2$ so as to get a yield to maturity above 16.67% [Core Principle 2].

- **Inflation risk** is the chance that inflation may turn out to be higher than expected, thereby reducing the real return on holding the bond. To account for such risk, consider the following modified version of the Fisher relation:

$$i_t \approx r_t + \pi^e_{t+1} + \text{compensation for inflation risk} \tag{4.4}$$

  Thus, the greater the inflation risk (measured by inflation s.d.), the larger the compensation for it, the higher the nominal interest rate.

- **Interest-rate risk** is the chance that the bond price may fall between the time a bond is purchased and the time it is sold. See the example of holding period return above.