Policy Rule Regressions with Survey Data

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November 2, 2024

Motivation

1. Policy rule regressions are crucial (understanding current policies; forecast of future policies)
   Example: Taylor rule (1993)

$$i_t = i^* + \psi_1(\pi_t - \pi^*) + \psi_2 y_t + \varepsilon_t \tag{1}$$

2. Reduced form OLS estimations could be problematic when feedback variables are endogenous to policy shocks.
3. This paper: survey data as an IV under various information settings.

Model

A linear regression model with endogeneity (policy rule regression)

$$R_t = \alpha_0 + \beta_0 \pi_t + \gamma_0 y_t + \varepsilon_t \tag{2}$$

where

- π_t and y_t can be correlated with ε_t
- Goal: identify and estimate $\theta_0 = (\alpha_0, \beta_0, \gamma_0)$

Idea: use survey data as an instrument for y_t and π_t

Model

We replace variables with two separate terms: forecast and forecast error terms.

$$
\begin{aligned}
R_t &= \alpha_0 + \beta_0 \pi_t + \gamma_0 y_t + \varepsilon_t \\
&= \alpha_0 + \beta_0 \mathbb{E}_{t-1}\pi_t + \gamma_0 \mathbb{E}_{t-1} y_t + \underbrace{\beta_0[\pi_t - \mathbb{E}_{t-1}\pi_t] + \gamma_0[y_t - \mathbb{E}_{t-1}y_t] + \varepsilon_t}_{\varepsilon_t^{new}} \qquad (3)
\end{aligned}
$$

Full-information rational expectations

$$
v_{t,t-1} = x_t - \mathbb{E}_{t-1}x_t \qquad (4)
$$

where $v_{t,t-1}$ is the full-information rational expectation error and is thus uncorrelated with information dated $t-1$ or earlier.
θ_0 is identified iff π_t and y_t are not (perfectly) multicollinear.

Model: Potential problem

Example: Sticky information (analogous to sticky price)

Definition: Rational Inattention

Economic decision-makers cannot absorb all available information but can choose which pieces of information to process [Sims, 2003]

Definition: Sticky information

In each period, a fraction of inattentive agents obtain new information about the state of the economy and computes a new path of optimal prices. Other agents continue to set prices based on old plans and outdated information [Mankiw and Reis, 2002].

Model: Potential problem

Full-information rational expectations

$$\mathbb{E}_{t-1}x_t = x_t - v_{t,t-1} \tag{5}$$

where $v_{t,t-1}$ is the full-information rational expectation error and is thus uncorrelated with information dated $t-1$ or earlier.

However, with sticky-information, the average forecast at time $t-1$ can be written as

$$\mathbb{F}_{t-1}x_t = (1-\lambda)\mathbb{E}_{t-1}x_t + \lambda\mathbb{F}_{t-2}x_t \tag{6}$$

Combining (5) and (6) yields the predicted relationship between the ex post mean forecast error across agents and the ex ante mean forecast revision

$$\underbrace{x_t - \mathbb{F}_{t-1}x_t}_{\text{ex post mean forecast error}} = \frac{\lambda}{1-\lambda} \underbrace{(\mathbb{F}_{t-1}x_t - \mathbb{F}_{t-2}x_t)}_{\text{ex ante mean forecast revision}} + v_{t,t-1} \tag{7}$$

Model: Potential problem

Our example:

$$\pi_t \equiv \mathbb{F}_{t-1}\pi_t + \underbrace{\pi_t - \mathbb{F}_{t-1}\pi_t}_{\text{the ex post mean forecast error}} \tag{8}$$

Intuitively, we want to split the ex post mean forecast error into two parts:

1. part that is correlated with the error term.
2. part that is uncorrelated with the error term.

Estimate the following empirical specification [Coibion and Gorodnichenko, 2015]

$$x_{t+1} - \mathbb{F}_t x_{t+1} = \gamma_0 + \gamma_1(\mathbb{F}_t x_{t+1} - \mathbb{F}_{t-1} x_{t+1}) + \epsilon_t \equiv \Delta x_{t+1,t} \tag{9}$$

We get estimator $\Delta\widehat{x}_{t,t-1}$ from Eq. (9).

Model: Two-step estimation

We can estimate the reduced-form policy rule in two steps in the model with sticky information

1. **First stage:** Regress $\Delta x_{t+1,t} \equiv x_{t+1} - \mathbb{F}_t x_{t+1}$ on $\mathbb{F}_t x_{t+1} - \mathbb{F}_{t-1} x_{t+1}$ & obtain predicted values of $\Delta \widehat{x}_{t+1,t}$ which contains variation in $\Delta x_{t+1,t}$ that is uncorrelated with ε_t

$$\Delta \widehat{x}_{t+1,t} = \hat{\gamma}_0 + \hat{\gamma}_1 (\mathbb{F}_t x_{t+1} - \mathbb{F}_{t-1} x_{t+1}) \tag{10}$$

2. **Second stage:** Regress R_t on $\mathbb{F}_{t-1} x_t + \Delta \widehat{x}_{t,t-1}$

$$\hat{R}_t = \hat{\alpha}_0 + \hat{\beta}_0 \underbrace{[\mathbb{F}_{t-1} \pi_t + \Delta \widehat{\pi}_{t,t-1}]}_{\text{new estimator}} + \hat{\gamma}_0 [\mathbb{F}_{t-1} y_t + \Delta \widehat{y}_{t,t-1}] \tag{11}$$

Simulation (FIRE)

We consider a textbook version of the new Keynesian model presented in Woodford (2003) and Gali(2008) but augmented with a fiscal policy rule

$$
\begin{aligned}
\text{Dynamic IS equation:} \quad & \hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \tau^{-1}(\hat{R}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \mathbb{E}_t \hat{z}_{t+1}) && (12)\\
\text{New Keynesian Phillips curve:} \quad & \hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa \hat{y}_t + \kappa \tau^{-1} \hat{u}_t && (13)\\
\text{Monetary policy:} \quad & \hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R)(\psi_\pi \hat{\pi}_t + \psi_y \hat{y}_t) + \epsilon_{R,t} && (14)\\
\text{Fiscal policy:} \quad & \hat{s}_t = \rho_s \hat{s}_{t-1} + (1 - \rho_s)\psi_s \hat{b}_{t-1} + \epsilon_{s,t} && (15)\\
\text{Government budget constraint:} \quad & \hat{b}_t = \beta^{-1}(\hat{b}_{t-1} + \hat{R}_{t-1} - \hat{\pi}_t - \hat{z}_t) - (\beta^{-1} - 1)\hat{s}_t && (16)\\
\text{Technology shock:} \quad & \hat{z}_t = \rho_z \hat{z}_{t-1} + \epsilon_{z,t} && (17)\\
\text{Markup shock:} \quad & \hat{u}_t = \rho_u \hat{u}_{t-1} + \epsilon_{u,t} && (18)
\end{aligned}
$$

We rely on Sim's (2001) method to solve for the solution of the structural model.

Simulation (FIRE)

Table: Parameters

Parameter	Description	value
τ	elasticity of intertemporal substitution	2.00
κ	the slope of the new Keynesian Philips curve	0.20
β	discount factor	0.99
ψ_{π}	coefficients of Taylor rule on inflation	1.5
ψ_{y}	coefficients of Taylor rule on output gap	1
ψ_{s}	coefficients of fiscal policy on output gap	0.5
ρ_{R}	monetary policy smoothing	0.5
ρ_{s}	fiscal policy smoothing	0.9
ρ_{z}	AR(1) coefficient on technology shock	0.6
ρ_{u}	AR(1) coefficient on Markup shock	0.7

NOTES:

Simulation

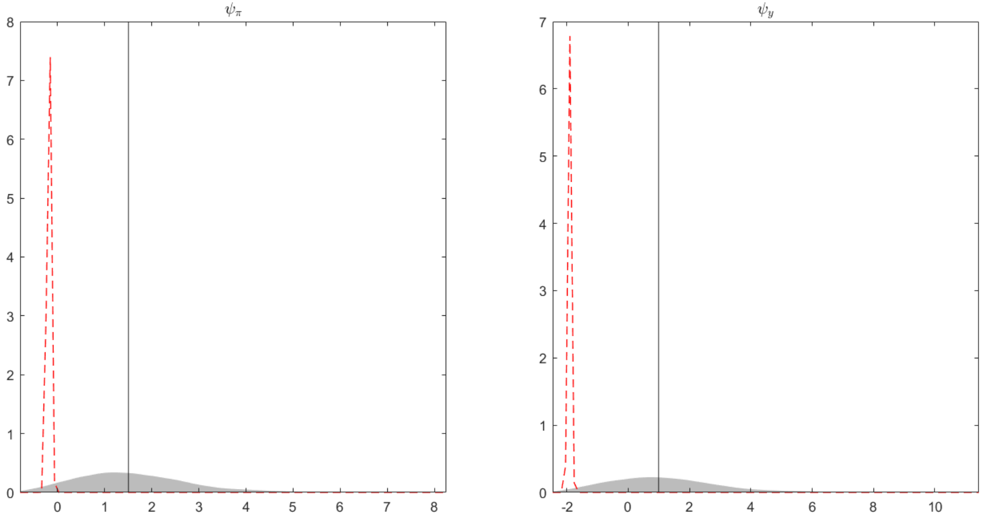

Figure: Distribution of 500 simulations

Estimation: Data

- Forecast of Inflation: Survey of Professional Forecasters (SPF), Federal Reserve Bank of Philadelphia
- Output gap: Tealbook (formerly Greenbook) Data Set, Federal Reserve Bank of Philadelphia
- Sample: 1987.Q3 to 2006.Q4 prior to the Great Recession

Estimation: model specification

Empirical specifications

$$\mathcal{M}_1: \qquad R_t = \psi_1 \pi_t + \psi_2 y_t + \varepsilon_t \tag{19}$$

$$\mathcal{M}_2: \qquad R_t = \psi_1 \mathbb{E}_{t-1} \pi_t + \psi_2 \mathbb{E}_{t-1} y_t + \varepsilon_t \tag{20}$$

$$\mathcal{M}_3: \qquad R_t = \psi_1 (\mathbb{E}_{t-1} \pi_t + \widehat{\Delta\pi}_{t,t-1}) + \psi_2 (\mathbb{E}_{t-1} y_t + \widehat{\Delta y}_{t,t-1}) + \varepsilon_t \tag{21}$$

Estimation: model specification

Empirical specifications: one-period lag

$$\mathcal{M}_4: \qquad R_t = \rho_1 R_{t-1} + \psi_1 \pi_t + \psi_2 y_t + \varepsilon_t \tag{22}$$

$$\mathcal{M}_5: \qquad R_t = \rho_1 R_{t-1} + \psi_1 \mathbb{E}_{t-1}\pi_t + \psi_2 \mathbb{E}_{t-1} y_t + \varepsilon_t \tag{23}$$

$$\mathcal{M}_6: \qquad R_t = \rho_1 R_{t-1} + \psi_1(\mathbb{E}_{t-1}\pi_t + \widehat{\Delta\pi}_{t,t-1}) + \psi_2(\mathbb{E}_{t-1} y_t + \widehat{\Delta y}_{t,t-1}) + \varepsilon_t \tag{24}$$

where $\psi_1 \equiv (1-\rho_1)\psi_1^*$ and $\psi_2 \equiv (1-\rho_1)\psi_2^*$.

Estimation: model specification

Empirical specifications: two-period lag

$$\mathcal{M}_7\colon \quad R_t = \rho_1 R_{t-1} + \rho_2 R_{t-2} + \psi_1 \pi_t + \psi_2 y_t + \varepsilon_t \tag{25}$$

$$\mathcal{M}_8\colon \quad R_t = \rho_1 R_{t-1} + \rho_2 R_{t-2} + \psi_1 \mathbb{E}_{t-1}\pi_t + \psi_2 \mathbb{E}_{t-1} y_t + \varepsilon_t \tag{26}$$

$$\mathcal{M}_9\colon \quad R_t = \rho_1 R_{t-1} + \rho_2 R_{t-2} + \psi_1(\mathbb{E}_{t-1}\pi_t + \widehat{\Delta\pi}_{t,t-1}) + \psi_2(\mathbb{E}_{t-1} y_t + \widehat{\Delta y}_{t,t-1}) + \varepsilon_t \tag{27}$$

where $\psi_1 \equiv (1-\rho_1-\rho_2)\psi_1^*$ and $\phi_2 \equiv (1-\rho_1-\rho_2)\psi_2^*$.

Estimation

MCMC

- Our model-specific priors are denoted by $\boldsymbol{\theta}$. For example, in $\mathcal{M}_7$, $\boldsymbol{\theta} = \{\rho_1, \rho_2, \psi_1, \psi_2\}$.
- We use an initial 15% of the total sample over the time period 1987:IV to 2006:IV to form the training sample prior. We sample posterior distribution by 10,000 MCMC draws and discard the burn-in draws, saying the first 1,000.
- The following tables report posterior mean, sd, the .025, and the .975 quantiles of $\boldsymbol{\theta}$. The posterior distribution of asterisk parameters is calculated based on parameters in $\boldsymbol{\theta}$. For example, $\psi_1^* = \psi_1/(1 - \rho_1 - \rho_2)$ in $\mathcal{M}_6$, we calculate ψ_1^* for each draw of ψ_1, ρ_1, ρ_2, and get 10,000 samples.
- The log marginal likelihood is calculated by the method in [Chib, 1995]

Estimation

Table: Tests of inflation expectations and output gap process

Inflation					output gap				
Name	postmean	postsd	[2.5%	97.5%]	Name	postmean	postsd	[2.5%	97.5%]
γ_0^π	-0.048	0.082	[-0.209	0.112]	γ_0^y	0.026	0.066	[-0.104	0.154]
γ_1^π	0.249	0.207	[-0.157	0.664]	γ_1^y	-0.099	0.097	[-0.290	0.095]
σ^2	0.580	0.100	[0.416	0.805]	σ^2	0.406	0.071	[0.290	0.564]

$$\Delta\widehat{x}_{t+1,t} = \hat{\gamma}_0 + \hat{\gamma}_1(\mathbb{F}_t x_{t+1} - \mathbb{F}_{t-1}x_{t+1}) \tag{28}$$

Estimation

Table: Estimates of monetary policy without lag

$\mathcal{M}_1$	postmean	postsd	[2.5%	97.5%]
$\psi_1 : \pi_t$	1.893	0.103	[1.692	2.092]
$\psi_2 : y_t$	0.661	0.130	[0.406	0.922]
σ^2	6.809	1.059	[5.042	9.141]
log marginal likelihood: -143.9046				
$\mathcal{M}_2$	postmean	postsd	[2.5%	97.5%]
$\psi_1 : \mathbb{E}_{t-1}\pi_t$	1.998	0.048	[1.903	2.092]
$\psi_2 : \mathbb{E}_{t-1}y_t$	0.702	0.060	[0.584	0.821]
σ^2	1.261	0.215	[0.908	1.740]
log marginal likelihood: -94.90397				

Estimation

Table: Estimates of monetary policy without lag

$\mathcal{M}_3$	postmean	postsd	[2.5%	97.5%]
$\psi_1 : \mathbb{E}_{t-1}\pi_t + \widehat{\Delta\pi}_{t,t-1}$	2.026	0.059	[1.910	2.140]
$\psi_2 : \mathbb{E}_{t-1}y_t + \widehat{\Delta y}_{t,t-1}$	0.691	0.072	[0.550	0.836]
σ^2	1.832	0.308	[1.325	2.518]
log marginal likelihood: -105.7976				

a $\mathcal{M}_1$: OLS estimates with contemporaneous data

b $\mathcal{M}_2$: our proposed one-step method estimates with survey data

c $\mathcal{M}_3$: our proposed two-step method estimates with survey data

Estimation

Table: Estimates of monetary policy without one-period lag

$\mathcal{M}_4$	postmean	postsd	[2.5%	97.5%]
$\rho_1 : R_{t-1}$	0.870	0.024	[0.823	0.917]
$\psi_1 : \pi_t$	0.258	0.049	[0.162	0.353]
$\psi_2 : y_t$	0.167	0.027	[0.113	0.221]
$\psi_1^* : (1-\rho_1)\pi_t$	1.983	0.156	[1.685	2.303]
$\psi_2^* : (1-\rho_1)y_t$	1.310	0.245	[0.908	1.873]
σ^2	0.174	0.030	[0.124	0.241]

log marginal likelihood: -33.52841

Estimation

Table: Estimates of monetary policy without one-period lag

$\mathcal{M}_5$	postmean	postsd	[2.5%	97.5%]
$\rho_1 : R_{t-1}$	0.724	0.033	0.659	0.789
$\psi_1 : \mathbb{E}_{t-1}\pi_t$	0.554	0.068	0.421	0.687
$\psi_2 : \mathbb{E}_{t-1}y_t$	0.252	0.029	0.195	0.309
$\psi_1^* : (1-\rho_1)\mathbb{E}_{t-1}y_t$	2.005	0.060	1.886	2.125
$\psi_2^* : (1-\rho_1)\mathbb{E}_{t-1}y_t$	0.916	0.083	0.765	1.091
σ^2	0.126	0.022	0.090	0.175

log marginal likelihood: -23.53923

Estimation

Table: Estimates of monetary policy without one-period lag

$\mathcal{M}_6$	postmean	postsd	[2.5%	97.5%]
$\rho_1 : R_{t-1}$	0.758	0.031	[0.698	0.819]
$\psi_1 : \mathbb{E}_{t-1}\pi_t + \widehat{\Delta\pi}_{t,t-1}$	0.494	0.064	[0.366	0.620]
$\psi_2 : \mathbb{E}_{t-1}y_t + \widehat{\Delta y}_{t,t-1}$	0.228	0.028	[0.172	0.283]
$\psi_1^* : (1-\rho_1)\mathbb{E}_{t-1}\pi_t + \widehat{\Delta\pi}_{t,t-1}$	2.041	0.073	[1.898	2.187]
$\psi_2^* : (1-\rho_1)\mathbb{E}_{t-1}\pi_t + \widehat{\Delta\pi}_{t,t-1}$	0.946	0.099	[0.768	1.158]
σ^2	0.135	0.024	[0.097	0.188]
log marginal likelihood: -25.64392				

a $\mathcal{M}_4$: OLS estimates with contemporaneous data

b $\mathcal{M}_5$: our proposed one-step method estimates with survey data

c $\mathcal{M}_6$: our proposed two-step method estimates with survey data

Estimation

Table: Estimates of monetary policy without two-period lag

$\mathcal{M}_7$	postmean	postsd	[2.5%	97.5%]
$\rho_1 : R_{t-1}$	1.465	0.095	[1.273	1.651]
$\rho_2 : R_{t-2}$	-0.550	0.086	$[-0.716$	$-0.380]$
$\psi_1 : \pi_t$	0.161	0.041	[0.081	0.242]
$\psi_2 : y_t$	0.086	0.025	[0.038	0.135]
$\psi_1^* : (1-\rho_1-\rho_2)\pi_t$	1.908	0.203	[1.515	2.316]
$\psi_2^* : (1-\rho_1-\rho_2)y_t$	1.032	0.282	[0.560	1.638]
σ^2	0.107	0.019	[0.076	0.150]

log marginal likelihood: -18.4747

a $\mathcal{M}_7$: OLS estimates with real-time data

Estimation

Table: Estimates of monetary policy without two-period lag

$\mathcal{M}_8$	postmean	postsd	[2.5%	97.5%]
$\rho_1 : R_{t-1}$	1.249	0.114	[1.021	1.467]
$\rho_2 : R_{t-2}$	-0.437	0.091	[-0.614	$-$ 0.255]
$\psi_1 : \mathbb{E}_{t-1}\pi_t$	0.370	0.071	[0.235	0.509]
$\psi_2 : \mathbb{E}_{t-1}y_t$	0.150	0.033	[0.086	0.215]
$\psi_1^* : (1-\rho_1-\rho_2)\mathbb{E}_{t-1}\pi_t$	1.970	0.082	[1.800	2.131]
$\psi_2^* : (1-\rho_1-\rho_2)\mathbb{E}_{t-1}y_t$	0.799	0.107	[0.597	1.013]
σ^2	0.095	0.017	[0.067	0.133]
log marginal likelihood: -14.64363				

b $\mathcal{M}_8$: our proposed one-step method estimates with survey data

Estimation

Table: Estimates of monetary policy without two-period lag

$\mathcal{M}_9$	postmean	postsd	[2.5%	97.5%]
$\rho_1 : R_{t-1}$	1.297	0.118	[1.061	1.522]
$\rho_2 : R_{t-2}$	-0.453	0.096	$[-0.639$	$-0.260]$
$\psi_1 : \mathbb{E}_{t-1}\pi_t + \widehat{\Delta\pi}_{t,t-1}$	0.312	0.068	[0.181	0.447]
$\psi_2 : \mathbb{E}_{t-1}y_t + \widehat{\Delta y}_{t,t-1}$	0.126	0.033	[0.063	0.191]
$\psi_1^* : (1-\rho_1-\rho_2)\mathbb{E}_{t-1}\pi_t + \widehat{\Delta\pi}_{t,t-1}$	1.993	0.107	[1.772	2.200]
$\psi_2^* : (1-\rho_1-\rho_2)\mathbb{E}_{t-1}\pi_t + \widehat{\Delta\pi}_{t,t-1}$	0.804	0.137	0.542	1.076]
σ^2	0.102	0.018	[0.072	0.143]
log marginal likelihood: -16.95793				

$\mathcal{M}_9$: our proposed two-step method estimates with survey data

Conclusion

1. Policy rule regressions with Survey Data reduce endogenous bias under different information settings.
2. No evidence shows that there are information rigidities in forecasts of inflation and output gap over the time period 1987 IV -2006 IV
3. One-step method has the best performance under different model settings
4. Empirical results prefer the two-period lag in monetary policy rule and one-period lag in government spending rule.

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