# Forecasting the Forecasts of Others on Social Networks


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# Introduction

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  - social networks exhibit strong connectivity and diverse opinions
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  - develop a dynamic game of imperfect communication to explain "failure of social consensus"
  - introduce a simple method to solve Bayesian games for *any* network structure
- We study opinion formation in media networks
  - does mainstream media have incentive to report truthfully?
  - can citizen journalism improve information transmission?

# The Road Ahead...

1. Model: Bayesian Network Game, Frequency-Domain Solution

2. Analysis: Mainstream Media, Citizen Journalist

# Bayesian Network Game

- Finite network of rational agents with asset demand

$$d_{i,t} = \mathbb{E}_{i,t} p_{t+1} - (1+r) p_t, \quad i \in N$$

Exogenous asset supply

$$s_t = \frac{1}{1-\rho\,\mathrm{L}} \theta_t + \eta_t, \quad \theta_t \sim \mathcal{N}(0, \sigma_\theta^2), \quad \eta_t \sim \mathcal{N}(0, \sigma_\eta^2)$$

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- Agent $i$'s information set (i.e. type)

$$\mathcal{I}_{i,t} = \{p_s\}_{s \le t}$$

  - public signal: $p_t$

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$$\mathcal{I}_{i,t} = \{p_s\}_{s \le t} \vee \{q_{i,s}\}_{s \le t}$$

  - public signal: $p_t$
  - private signal: $q_{i,t} = \theta_t + v_{i,t}$, $v_{i,t} \sim \mathcal{N}(0, \sigma_{v_i}^2)$

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$$\mathcal{I}_{i,t} = \{p_s\}_{s \le t} \vee \{q_{i,s}\}_{s \le t} \vee \{\mathbb{E}^*_{j,s} p_{s+1}\}_{j \in N_i, s \le t}$$

  - public signal: $p_t$
  - private signal: $q_{i,t} = \theta_t + v_{i,t}$, $v_{i,t} \sim \mathcal{N}(0, \sigma_{v_i}^2)$
  - neighbors' *noisy* opinions: $\mathbb{E}^*_{j,t} p_{t+1} = \mathbb{E}_{j,t} p_{t+1} + u_{j,t}$, $j \in N_i$, $u_{j,t} \sim \mathcal{N}(0, \sigma_{u_j}^2)$

# Nash Equilibrium

- Market clearing introduces higher-order expectations (HOE)

$$p_t = \frac{1}{1+r}\left(\sum_{i \in N} \lambda_i \mathbb{E}_{i,t} p_{t+1} - s_t\right), \quad \sum_{i \in N} \lambda_i = 1$$

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  - informational network linkages $\Rightarrow$ intratemporal HOE

# Nash Equilibrium

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- Pure-strategy Nash equilibrium: profile of asset positions and information flows $\{(d_{i,t}, \mathcal{I}_{i,t})\}_{i\in N, t\in\mathbb{Z}}$ satisfying $\forall t \in \mathbb{Z}$
  - given $\{\mathcal{I}_{i,t}\}_{i\in N}$, $\{d_{i,t}\}_{i\in N}$ are chosen optimally
  - given $\{d_{i,t}\}_{i\in N}$, $p_t$ clears market and is consistent with $\{\mathcal{I}_{i,t}\}_{i\in N}$

# Frequency-Domain Solution

1. **Initialization.** Initialize agents' $z$-transform opinions

$$\mathbb{E}_{i,t} p_{t+1} = \sum_{k=0}^{\infty} P_{i,k} \, \mathrm{L}^k \, \boldsymbol{\epsilon}_t \quad \Leftrightarrow \quad P_i(z) = \sum_{k=0}^{\infty} P_{i,k} z^k$$

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2. **Aggregation.** Aggregate to obtain implied price

$$P(z) = \frac{1}{1+r} \left( \sum_{i \in N} \lambda_i P_i(z) - S(z) \right)$$

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4. **Recursion.** If $\|P_i(z) - P_i'(z)\| < \epsilon$, stop; otherwise, set $P_i(z) = P_i'(z)$ and go to step 2

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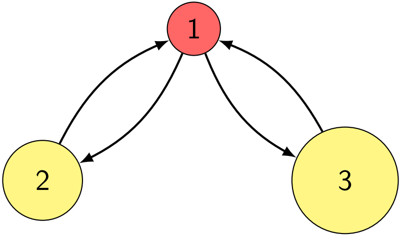
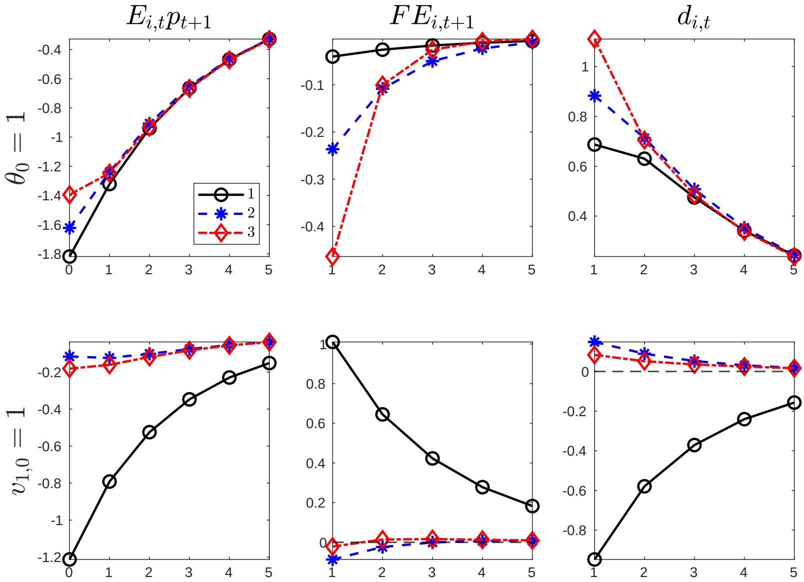
# Mainstream Media



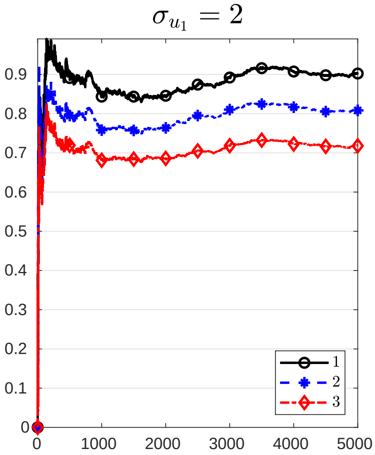

| Agent $i$ | 1 | 2 | 3 |
|---|---|---|---|
| $\lambda_i$: mass | $0.1$ | $0.3$ | $0.6$ |
| $\sigma_{u_i}$: local opinion uncertainty | $2$ | $0.1$ | $0.1$ |
| $\sigma_{v_i}$: private signal uncertainty | $0.1$ | $0.3$ | $5$ |

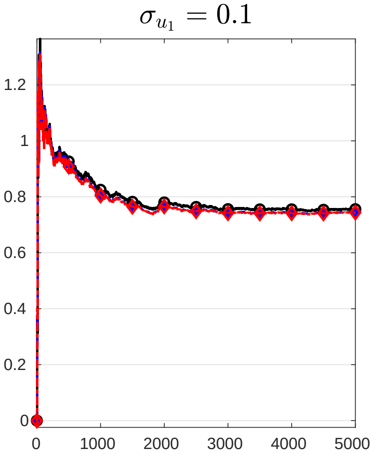
Notes: Global parameters are $r = 0.05$, $\rho = 0.7$, $\sigma_\theta = 0.5$, and $\sigma_\eta = 0.5$.

# Disparate Opinions



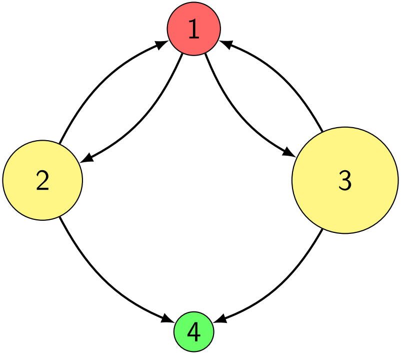
# Wealth Distribution




- $w_{i,t+1} = (1+r)w_{i,t} + [p_{t+1} - (1+r)p_t]d_{i,t},\ i \in N$

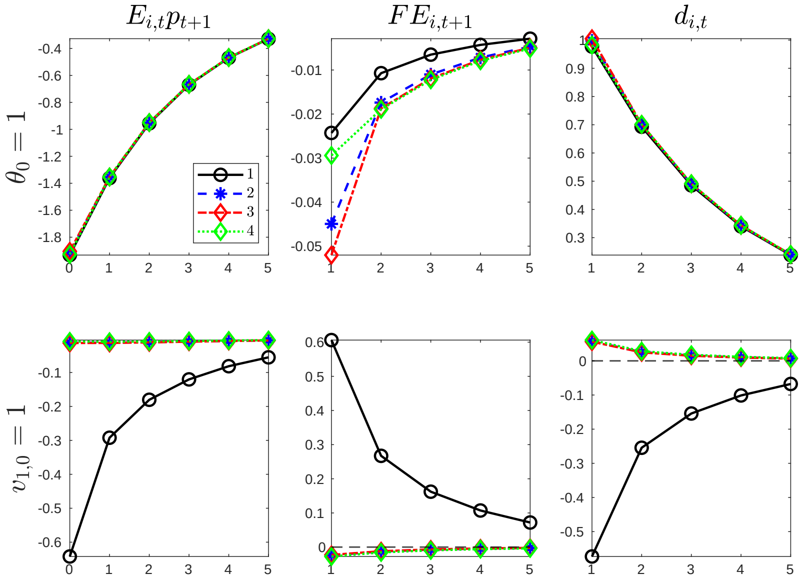
# Citizen Journalist




| Agent $i$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $\lambda_i$: mass | $0.09$ | $0.3$ | $0.6$ | $0.01$ |
| $\sigma_{u_i}$: local opinion uncertainty | $2$ | $0.1$ | $0.1$ | $0.1$ |
| $\sigma_{v_i}$: private signal uncertainty | $0.1$ | $0.3$ | $5$ | $0.1$ |

Notes: Global parameters are $r = 0.05$, $\rho = 0.7$, $\sigma_\theta = 0.5$, and $\sigma_\eta = 0.5$.

# Consensus Opinion



# Conclusion

- We provide a new framework to study social learning in network games
  - continuous action space and general information-network structure
  - frequency-domain method to characterize equilibrium

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- We provide a new framework to study social learning in network games
  - continuous action space and general information-network structure
  - frequency-domain method to characterize equilibrium
- Two promising extensions
  - DeGroot-type learning with weighted networks
  - network formation games