Forecasting the Forecasts of Others on Social Networks†

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Abstract

While social networks often exhibit strong connectivity, persistent divergence in opinions remains a widespread phenomenon. We address this "failure of social consensus" puzzle by developing a dynamic game model where rational agents can observe *noisy* opinions of their social network neighbors. The communication frictions motivate agents to outguess one another, thereby creating an infinite regress in beliefs that poses significant analytical challenges. We introduce a simple approach to characterizing equilibrium strategies for any network structure. Applying our framework to media networks, we find that mainstream media's incentive to misreport strengthens under strategic substitution, perpetuating opinion disparities. However, the inclusion of citizen journalism significantly improves information transmission and accelerates opinion convergence. These findings highlight the importance of decentralized information channels in mitigating misinformation and shaping public discourse.

Keywords: Social learning; Network game; Incomplete information; Frequency-domain methods.

JEL Classification: C6, C7, D8, L8

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1 Introduction

Empirical networks are well-documented to exhibit small-world properties. For example, the maximal distance between any two nodes scales logarithmically rather than linearly with the network size. This observation is often illustrated by the "six degrees of separation" hypothesis—any two individuals worldwide are connected through no more than six acquaintances. The advent of social media platforms, such as Facebook and TikTok, which facilitate global connections, has made these properties increasingly evident. Grounded in the imitation principle, theoretical models of social learning suggest that rational agents in a strongly connected network should eventually reach a consensus in their opinions. Yet, this implication is starkly contradicted by empirical observations, such as the persistent divergence in forecasts of election results or stock prices.

The contributions of this article are twofold, one substantive and the other methodological. First, we address the "failure of social consensus" puzzle by developing a dynamic game model with a finite number of rational agents (Section 2.1). Each agent is disparately informed and acquires information through three channels: a public endogenous signal (e.g., stock price), a private exogenous signal, and the opinions of its social network neighbors. While such private information is largely transmitted via network linkages, a key novelty of the model is that we allow opinions to be misreported so that agents have to "forecast the forecasts of others" on the network. These communication frictions impede efficient information aggregation, leading to sustained divergence in opinions even within strongly connected networks.

Second, we introduce a simple approach to solving and analyzing dynamic network games with continuous action space and general information frictions (Section 2.2). In such games, the need for agents to outguess one another creates an infinite regress in expectations that poses significant analytical challenges. We show how the frequency-domain method of Tan and Wu (2022) can be adapted to characterize equilibrium strategies for *any* network structure. Our approach enables the study of how network structure influences opinion formation among rational agents with disparate information. This advancement is particularly relevant to economics and finance, where market participants formulate opinions about future economic states or asset prices through various informational linkages such as social networks. It also contributes to game theory and network science by providing a general framework to study network formation games under incomplete information.

Examples of imperfect communication abound in reality. Media channels, particularly mainstream media and citizen journalism, play a critical role in information dissemination. These channels differ fundamentally in their approaches: mainstream media adheres to structured editorial standards, whereas citizen journalism offers real-time updates and diverse perspectives. Recently, public confidence in mainstream media has declined dramatically, with Gallup (a leading pollster) reporting that 64% of U.S. adults express little to no confidence in newspapers.

Meanwhile, the influence of social media has grown significantly, as evidenced by the public backlash against the U.S. TikTok ban. This dual shift in trust raises two key questions: Does centralized mainstream media have an economic incentive to disseminate information truthfully? If not, can decentralizing information channels through citizen journalism improve the public's ability to withstand misinformation?

To examine these questions, we first consider a three-agent Bayesian game on a strongly connected network (Section 3.1). The informed mainstream media serves as a central node, broadcasting to and receiving feedback from two uninformed audiences that do not directly communicate with each other. Our analysis reveals that the mainstream media's incentive to share misinformation strengthens when strategic substitution dominates agent interactions. Under such misreporting, despite the social exchange of information, the public signal remains non-revealing and opinion disparities persist among audiences. We then extend the game to include a fourth agent representing citizen journalism (Section 3.2). This expanded network is no longer strongly connected because the informed citizen journalist communicates unilaterally to uninformed audiences. We find that even a minimal presence of citizen journalism significantly improves information transmission and leads to a quick convergence in opinions. These findings hold particular relevance in the context of social media, where the spread of misinformation has become a pressing issue.

Related Literature. First, we draw connections to the literature on social learning in network games. Among others, Gale and Kariv (2003) show that, even when rational agents possess different private information, the uniformity of social behavior remains a robust feature of strongly connected networks. Huang, Strack and Tamuz (2024) further establish that all agents will learn at the same bounded rate. Our model extends these studies by introducing communication frictions among network neighbors so that agents can rationally hold diverse opinions in equilibrium. Second, this article is closely related to the literature on rational expectations models with endogenous and heterogeneous information [see, e.g., Tan and Wu (2022) and Huo and Takayama (2024)]. While such models often impose ad hoc assumptions on the information structure, social networks provide a natural justification for what economic agents can or cannot observe. We demonstrate how Bayesian network games can be accommodated within this framework and solved numerically using frequency-domain techniques. Lastly, we add to the literature on macroeconomic effects of small shocks [see Acemoglu, Akcigit and Kerr (2016) for a comprehensive overview]. Unlike existing studies that rely on input-output linkages, our model emphasizes the role of informational linkages as an amplification mechanism for idiosyncratic shocks. We show that misinformation from even a small number of "rumor-mongers" can propagate throughout the entire community and persistently distort social learning.

2 Model

We set up a dynamic game of incomplete information among a network of rational agents. There is an underlying unknown state of asset supply that changes over time. Agents simultaneously choose continuous asset demand after learning their types, which consist of differential information about the state and the noisy opinions of their network neighbors. They also understand the entire market structure and this fact is common knowledge among all agents. While we adopt this asset pricing example to ground the discussion, our framework is applicable to a broad range of economic and social settings involving informational frictions, strategic interactions, and network effects.

2.1 Bayesian Network Game Let time be indexed by the set of integers $\mathbb{Z}$. There is a finite set of speculative agent types indexed by $i \in N$, each consisting of a non-atomic continuum of identical agents. Henceforth, the continuum of type i agents is replaced by a single, representative agent i. In each period t, agent i may finance her asset purchases by borrowing at the constant margin rate $r > 0$. She then chooses the asset demand $d_{i,t}$ according to

$$d_{i,t} = \mathbb{E}_{i,t} p_{t+1} - (1+r) p_t, \tag{2.1}$$

taking the asset price p_t as given. Here $\mathbb{E}_{i,t}(\cdot) = \mathbb{E}[\cdot|\mathcal{I}_{i,t}]$ is a mathematical expectation operator conditional on agent i's information set at time t, $\mathcal{I}_{i,t}$, as will be specified below. Intuitively, agent i will take a long action if her expected capital gain, i.e., $\mathbb{E}_{i,t} p_{t+1} - (1+r)p_t$, is positive, or a short action otherwise. More precisely, the linear decision rule (2.1) can be rationalized as the best response function (up to some normalizing constant) of a myopic agent with exponential utility and facing normally distributed sources of uncertainty [see, e.g., Singleton (1987)].

There is also an exogenous asset supply s_t driven by both a persistent component and a transitory component

$$s_t = \frac{1}{1-\rho \mathrm{L}} \theta_t + \eta_t, \qquad \theta_t \sim \mathcal{N}(0, \sigma_\theta^2), \qquad \eta_t \sim \mathcal{N}(0, \sigma_\eta^2), \tag{2.2}$$

where L is the lag operator, $\mathrm{L}^k \theta_t = \theta_{t-k}$, and $0 < \rho < 1$. For example, the asset may be supplied by some non-speculative agents, such as financial intermediaries, who do not seek to optimize through speculative trading. The innovations $\{\theta_t, \eta_t\}$ are unobservable to agents, and have normal distributions with zero mean and variances $\{\sigma_\theta^2, \sigma_\eta^2\}$, respectively.

Information. During each period, every agent i acquires three sources of information about the market. First, she observes the full history of past and current prices $\{p_s\}_{s \leqslant t}$, which serves as a globally public endogenous signal to all agents. Second, she receives a private exogenous signal $q_{i,t}$ on the persistent innovation θ_t

$$q_{i,t} = \theta_t + v_{i,t}, \qquad v_{i,t} \sim \mathcal{N}(0, \sigma_{v_i}^2), \tag{2.3}$$

where the noise term $v_{i,t}$ has a normal distribution with zero mean and variance $\sigma_{v_i}^2$. Contrary to what is typically assumed in the literature, these agent-specific noises are not dispersed so that they are not washed out after aggregation. Third, she learns some noisy forecasts of the future price from a subset $N_i \subseteq N$ of agents who are her *social network neighbors*

$$\mathbb{E}^*_{j,t} p_{t+1} = \mathbb{E}_{j,t} p_{t+1} + u_{j,t}, \qquad u_{j,t} \sim \mathcal{N}(0, \sigma_{u_j}^2), \qquad j \in N_i, \tag{2.4}$$

where the noise term $u_{j,t}$ follows a normal distribution with zero mean and variance $\sigma_{u_j}^2$. Thus, a complete collection of neighborhood sets $\{N_i\}_{i\in N}$ pins down the entire network structure. We will refer to $\mathbb{E}_{j,t} p_{t+1}$ as agent j's *opinion* and $\mathbb{E}^*_{j,t} p_{t+1}$ as her *noisy opinion*. The coarse communication (2.4) represents a locally public endogenous signal that arises from the common situation in which individuals may not express their opinions to an arbitrarily accurate degree.[1] Because neighbors' noisy opinions reflect the signals they receive, social learning might help agents make better inferences about the fundamental shock s_t. In what follows, both the agent-specific innovations $\{(u_{i,t}, v_{i,t})\}_{i\in N}$ and the aggregate innovations $\{\theta_t, \eta_t\}$ are assumed to be independent with each other at all leads and lags.

In sum, the information set available to agent i at time t is given by

$$\mathcal{I}_{i,t} = \{p_s\}_{s\leqslant t} \vee \{q_{i,s}\}_{s\leqslant t} \vee \{\mathbb{E}^*_{j,s} p_{s+1}\}_{j\in N_i, s\leqslant t}, \tag{2.5}$$

where $\vee$ denotes the smallest closed subspace generated by the history of all signals. As such, agents are disparately informed about one another's opinions as well as the asset supply. Accordingly, they will form heterogeneous opinions about the future price and hence hold different asset positions as in (2.1).

Equilibrium. The asset demand and supply from all market participants must be equal, yielding

$$p_t = \frac{1}{1+r}\left(\overline{\mathbb{E}}_t p_{t+1} - s_t\right), \qquad \overline{\mathbb{E}}_t(\cdot) = \sum_{i\in N} \lambda_i \mathbb{E}_{i,t}(\cdot), \tag{2.6}$$

where $\overline{\mathbb{E}}_t(\cdot)$ defines the average expecation operator and $\lambda_i > 0$ captures the mass of agent i such that $\sum_{i\in N} \lambda_i = 1$. The market-clearing condition (2.6) makes clear that today's asset price depends concurrently on the average opinion of tomorrow's price $\overline{\mathbb{E}}_t p_{t+1}$ and the underlying state s_t.

[1] More generally, the communication error in (2.4) can be easily made bilateral in the sense that it depends on not only the "listener" i but also the "speaker" j.

While formulating their own opinions, agents are motivated to outguess one another's opinions at all levels, giving rise to two types of higher-order expecations (HOE) that are essential for characterizing the equilibrium. The first type, i.e., *intertemporal HOE*, has been emphasized by Allen, Morris and Shin (2006) and stems from the failure of the law of iterated expectations for average opinions. That is, by iterating (2.6) forward, it is generally true that $\overline{\mathbb{E}}_t\overline{\mathbb{E}}_{t+1}\cdots\overline{\mathbb{E}}_{t+k}(\cdot) \neq \overline{\mathbb{E}}_t(\cdot)$ for all $k > 0$. We highlight a second type, i.e., *intratemporal HOE*, that is new to the literature and unique to our network model of imperfect communication. Specifically, the contemporaneous noisy opinions that agent i learns from her network neighbors in (2.4) contain noises to filter so that she needs to evaluate $\mathbb{E}_{i,t}\mathbb{E}_{j,t}p_{t+1}$ for all $j \in N_i$. For the subsequent analysis, we adopt the following strategy and equilibrium concepts.

Definition 2.1 (Pure-Strategy Nash Equilibrium)**.** A pure strategy of agent i consists of a stochastic process for her asset demand $\{d_{i,t}\}_{t\in\mathbb{Z}}$ and an information flow $\{\mathcal{I}_{i,t}\}_{t\in\mathbb{Z}}$. A profile of all agents' pure strategies $\{(d_{i,t},\mathcal{I}_{i,t})\}_{i\in N,t\in\mathbb{Z}}$ is a Nash equilibrium if for all $t \in \mathbb{Z}$,

1. given $\{\mathcal{I}_{i,t}\}_{i\in N}$, $\{d_{i,t}\}_{i\in N}$ are chosen optimally as in (2.1);

2. given $\{d_{i,t}\}_{i\in N}$, p_t clears the market as in (2.6) and is consistent with $\{\mathcal{I}_{i,t}\}_{i\in N}$.

Several remarks about Definition 2.1 are in order. As usual, the first condition ensures that no agent can achieve a strictly higher expected utility by unilaterally deviating from the equilibrium strategy profile. The second condition, on the other hand, is standard in rational expectations models where agents are heterogeneously informed and endogenous variables convey information; it represents a dual fixed point in both the market-clearing price and the information flows of all agents. Lastly, because the equilibrium price and strategy profile are determined by each agent's opinion, it suffices to characterize the equilibrium by solving for the profile of stochastic processes $\{\mathbb{E}_{i,t}p_{t+1}\}_{i\in N,t\in\mathbb{Z}}$, as will be explained below.

2.2 Frequency-Domain Solution

For notational convenience, let $\boldsymbol{\epsilon}_t$ be a column vector containing all innovations that drive the uncertainty at time t. Throughout this section, we work in the space spanned by $\{\boldsymbol{\epsilon}_s\}_{s\leqslant t}$. The linear and Gaussian model structure implies that conditional expectations reduce to optimal linear projections. Therefore, we seek a linear covariance-stationary solution to each agent i's opinion formation problem

$$\mathbb{E}_{i,t}p_{t+1} = \sum_{k=0}^{\infty} P_{i,k}\,\mathrm{L}^k\,\boldsymbol{\epsilon}_t, \tag{2.7}$$

where $\{P_{i,k}\}_{k=0}^{\infty}$ is a square-summable sequence of coefficient row vectors. The moving average representation (2.7) has a straightforward economic interpretation—$P_{i,k}$ measures exactly the k-period-ahead impulse response of her opinion to one unit increase in $\boldsymbol{\epsilon}_t$.

Rather than solving the infinite sequence $\{P_{i,k}\}_{k=0}^{\infty}$ in the time domain, which can be a daunting task, we adopt a frequency-domain approach that solves an equivalent yet simpler problem of finding its analytic policy function

$$P_i(z) = \sum_{k=0}^{\infty} P_{i,k} z^k, \qquad z \in \mathbb{D}, \tag{2.8}$$

where $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ denotes the open unit disk on the complex plane $\mathbb{C}$. The transformation (2.8) replaces the lag operator L in (2.7) with the complex variable z and is called the z-transform of $\mathbb{E}_{i,t} p_{t+1}$. Analogously, let $\{P(z), Q_i(z), P_i^*(z), S(z)\}$ denote the z-transforms of $\{p_t, q_{i,t}, \mathbb{E}_{i,t}^* p_{t+1}, s_t\}$, respectively. Substituting $P_i(z)$, $P(z)$, and $S(z)$ into the equilibrium condition (2.6) gives

$$P(z) = \frac{1}{1+r}\left(\sum_{i \in N} \lambda_i P_i(z) - S(z)\right). \tag{2.9}$$

The functional equation (2.9) forms the basis for the following iterative procedure to solve for the profile of analytic policy functions $\{P_i(z)\}_{i \in N}$.

Algorithm 2.2 (Analytic Policy Function Iteration)**.** The solution algorithm can be outlined by a sequence of generic steps as follows:

1. **Initialization.** Choose all agents' initial opinions $\{P_i(z)\}_{i \in N}$ as in (2.8). For example, set $P_i(z) = 0$ for all $z \in \mathbb{D}$ and $i \in N$.

2. **Aggregation.** Substitute $\{P_i(z)\}_{i \in N}$ into (2.9) to obtain the implied price $P(z)$.

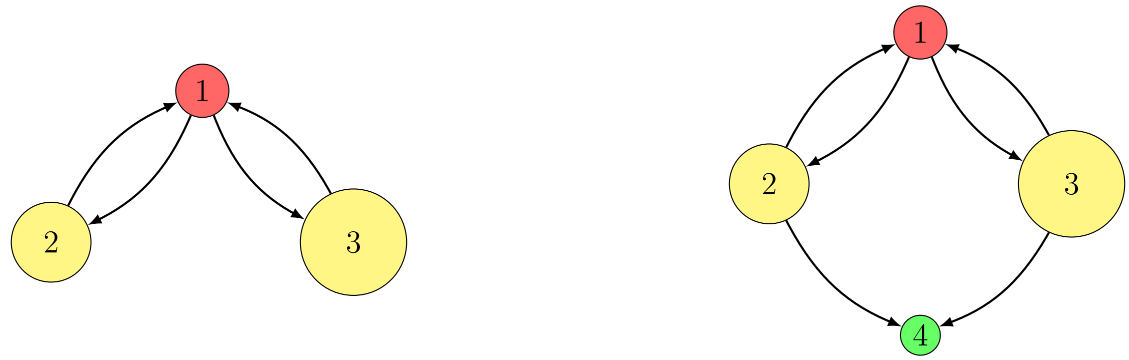
3. **Updating.** Update each agent i's information set $\{P(z), Q_i(z), \{P_j^*(z)\}_{j \in N_i}\}$ and compute the associated opinion $P_i'(z)$ using the Wiener-Hopf optimal prediction formula.

4. **Recursion.** If the relative distance between the guess and updated opinions is smaller than a pre-specified criterion ϵ for all agents, i.e.,

$$\max_{i \in N} \frac{\|P_i(z) - P_i'(z)\|}{\|P_i'(z)\|} < \epsilon,$$

   where $\|\cdot\|$ denotes some functional norm, then stop and treat $\{P_i'(z)\}_{i \in N}$ as the true policy functions. Otherwise, set $P_i(z) = P_i'(z)$ for all $i \in N$ and go back to step 2.

Practically, we adapt the MATLAB toolbox developed by Tan and Wu (2022) to implement a numerical version of Algorithm 2.2. Appendix A demonstrates how to cast our Bayesian game into their canonical representation of linear rational expecations models with general information



(a) Three-agent game

(b) Four-agent game

Figure 1: Directed and unweighted graphs. Notes: Nodes correspond to agents and directed links signify observational relationships. Panel (a) highlights agent 1 (red node) as the mainstream media disseminating information. Panel (b) introduces a fourth agent (green node) as a citizen journalist relaying information.

structures. We conclude this section by noting that the presence of endogenous information (i.e., asset price and neighbors' noisy opinions) makes it difficult to provide a general characterization for the equilibrium existence and uniqueness. Instead, we examine the equilibrium sensitivity—if the algorithm converges—to different initial conjectures and find that they all lead to the same solution.

3 Analysis

We now examine the effects of imperfect communication on opinion formation within two different networks. For ease of understanding, Figure 1 represents both networks as directed, unweighted graphs, where nodes of different sizes correspond to agents with varying mass and a directed link from node i to j signifies that agent i can observe j's noisy opinion. Our equilibrium analysis addresses the following questions: [i.] Can well-connected agents, despite having disparate information, rationally hold diverse opinions? [ii.] How do strategic interactions affect a central agent's incentive to share her opinion truthfully? [iii.] Does decentralizing information channels through peripheral agents enhance the network's resilience to misinformation?

3.1 Mainstream Media We first consider a three-agent Bayesian game on a *strongly connected* network—there is an observational path between every pair of agents. This assumption precludes the possibility that divergence in opinions arises solely from the lack of communication channels between certain agents. As evinced by Figure 1-a, the network structure resembles a stylized social setting in which the mainstream media (agent 1) acts as a conduit of information, disseminating news to distinct audience groups (agents 2 and 3) while also receiving feedback from them. These audience groups, however, remain disconnected from one another in their direct interactions. The neighborhood sets of this three-agent network are given by $N_1 = \{2, 3\}$,

Table 1: Agent-specific parameter values

	(a) Three-agent game			(b) Four-agent game			
Agent i	1	2	3	1	2	3	4
λ_i: mass	0.1	0.3	0.6	0.09	0.3	0.6	0.01
σ_{u_i}: local opinion uncertainty	2	0.1	0.1	2	0.1	0.1	0.1
σ_{v_i}: private signal uncertainty	0.1	0.3	5	0.1	0.3	5	0.1

Notes: Global parameters are $r = 0.05$, $\rho = 0.7$, $\sigma_\theta = 0.5$, and $\sigma_\eta = 0.5$.

$N_2 = \{1\}$, and $N_3 = \{1\}$.

Table 1-a sumarizes the parameter values for each agent. Our choice of the mass λ_i and the private signal uncertainty σ_{v_i} imposes a hierarchy of information precision $1/\sigma^2_{v_i}$ among agents. Specifically, more informed agents possess smaller masses, i.e., $\lambda_1 < \lambda_2 < \lambda_3$ and $1/\sigma^2_{v_1} > 1/\sigma^2_{v_2} > 1/\sigma^2_{v_3}$, reflecting their concentrated yet influential role in the network. On the other hand, we choose the local opinion uncertainty σ_{u_i} to allow agent 1, the most informed participant, to unilaterally misreport her opinion to agents 2 and 3, i.e., $\sigma^2_{u_1} \gg \sigma^2_{u_2} = \sigma^2_{u_3} \approx 0$. Overall, this asymmetric information structure mirrors the enduring media practice where the mainstream media acts as a potentially biased provider of information to its broader audience. Lastly, we set the global parameters to $r = 0.05$, $\rho = 0.7$, $\sigma_\theta = 0.5$, and $\sigma_\eta = 0.5$.

To conserve space, we focus on the impacts of a unit shock in agent 1's private signal at time 0, i.e., $q_{1,0} = \theta_0 + v_{1,0} = 1$, on the equilibrium opinion formation. Figure 2 depicts the dynamic responses of each agent i's opinion $\mathbb{E}_{i,t}p_{t+1}$, forecast error $FE_{i,t+1} = p_{t+1} - \mathbb{E}_{i,t}p_{t+1}$, and asset demand $d_{i,t}$ following either $\theta_0 = 1$ or $v_{1,0} = 1$. We begin with an increase in the aggregate persistent innovation ($\theta_0 = 1$) that signals higher asset supplies now and in the future to *all* agents. In response, more informed agents will revise their opinions and hence expected capital gains not only further downward but also with greater accuracy. This revision leads to initial divergence in opinions, learning rates, and asset positions.[2] For instance, agent 1 has the most precise private information and, therefore, learns faster than agents 2 and 3. She purchases fewer assets, however, as her superior information signals much lower future prices and capital gains. Subsequently, agents will communicate with one another over a strongly connected network. While agents 2 and 3 may leverage agent 1's disclosed opinion to refine their own, they are also aware of the fact that her disclosure is highly noisy. Consequently, despite the social exchange of information, the publicly observed price remains non-revealing and disparate opinions persist in equilibrium.

[2]Formally, agent i's learning rate can be quantified using the reduction in her mean squared forecast errors over a given period, i.e., $\sum_{t=1}^{T} FE^2_{i,t}$. A faster reduction indicates a higher learning rate, reflecting agent i's ability to more efficiently incorporate new information into her opinion.

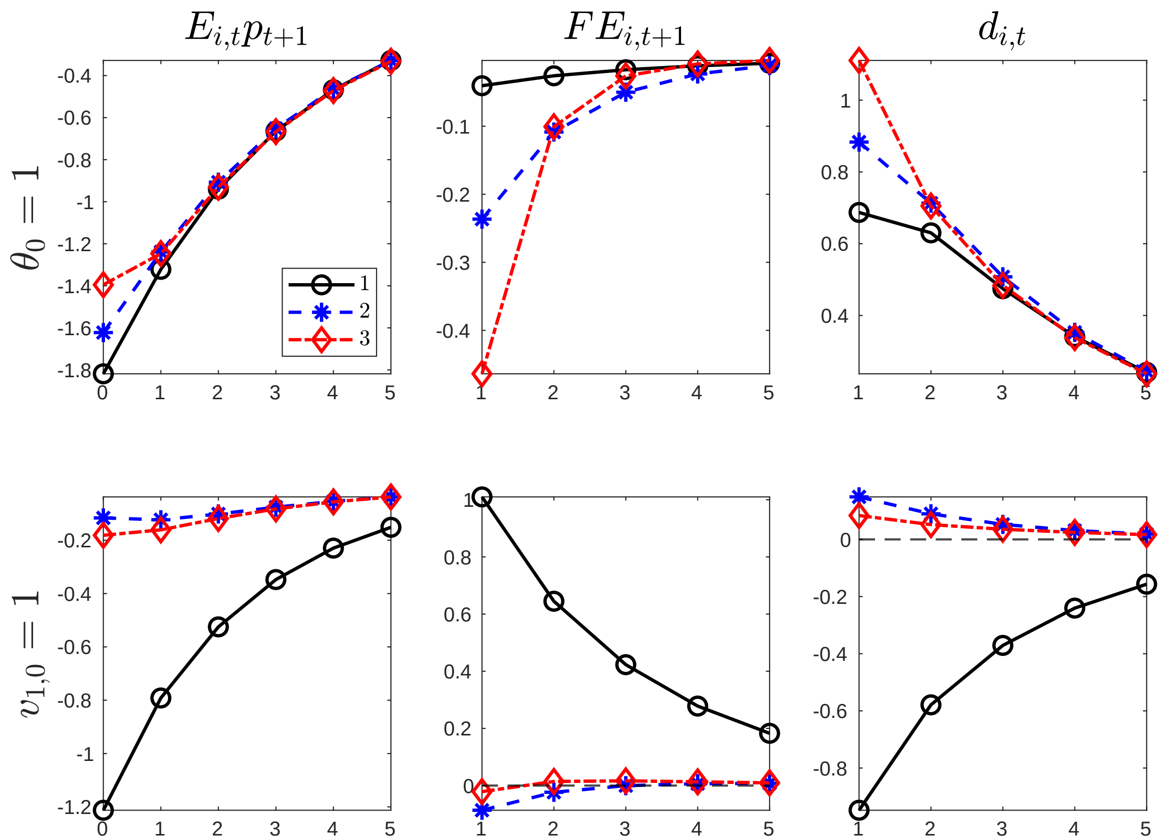


Figure 2: Impulse response functions under three-agent game. Notes: Dynamic responses of each agent i's opinion $\mathbb{E}_{i,t}p_{t+1}$ (1st column), forecast error $FE_{i,t+1} = p_{t+1} - \mathbb{E}_{i,t}p_{t+1}$ (2nd column), and asset demand $d_{i,t}$ (3rd column) following either $\theta_0 = 1$ (1st row) or $v_{1,0} = 1$ (2nd row).

On the other hand, consider a higher noise in agent 1's private signal ($v_{1,0} = 1$) that induces her to dramatically adjust her opinion downward, even though the aggregate fundamental θ_0 remains unchanged. As a result, agent 1 initiates a short position, falsely anticipating future price drops. Meanwhile, agents 2 and 3, aware of the strong bias in agent 1's disclosure, only mildly adjust their opinions and asset positions. Over time, interactions on the strongly connected network enable agents to mitigate the effects of communication errors. Nevertheless, persistent asymmetries in opinions remain in equilibrium, as the market price continues to be non-revealing and fails to fully align agents' opinions. Notably, although agent 1 constitutes only 10% of market participants, her actions are magnified throughout the network, demonstrating how a small shock can propagate and cause pronounced market deviations. This amplification, driven by the informational linkages within the network, sets our model apart from other studies on the aggregate effects of idiosyncratic shocks.[3]

[3]While beyond the scope of this article, our model also exhibits bubble-like dynamics—small non-fundamental disturbances, amplified by informational frictions and network effects, can drive significant mispricing and sustained deviations from fundamentals.

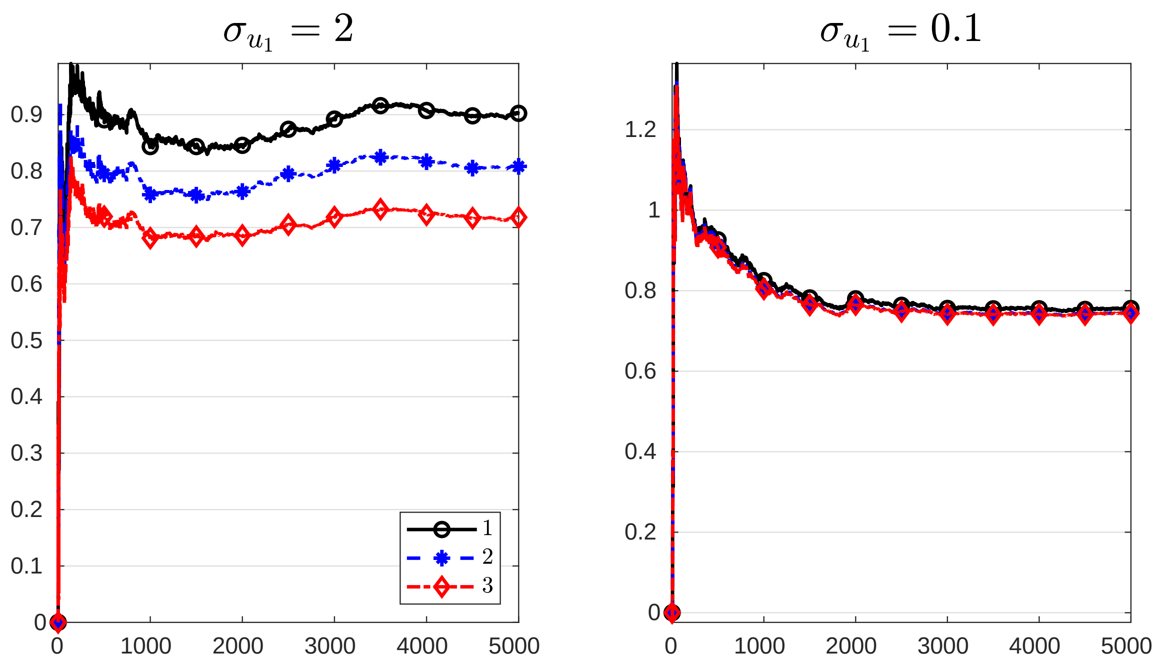


Figure 3: Wealth distribution under three-agent game. Notes: Comparison of each agent i's average wealth, i.e., $\frac{1}{T}\sum_{t=1}^{T} w_{i,t}$, between agent 1's opinion distortion (left panel) and her truthful reporting (right panel).

Another key distinction between these two cases lies in the nature of strategic interactions among agents. When $\theta_0 = 1$, the higher asset supply prompts all agents to buy more assets. This behavior reflects strategic complementarity, where agents' actions reinforce one another in equilibrium. Conversely, when $v_{1,0} = 1$, the asset supply remains unchanged. As a result, agent 1's overreaction leads her to sell assets, which are purchased by agents 2 and 3. This behavior exhibits strategic substitution, where agents' actions offset each other in equilibrium. Overall, these contrasting behaviors highlight the interplay between the structure of shocks and agents' strategic incentives in determining equilibrium outcomes.

The preceding discussion raises the natural question of how the type of strategic interactions shapes agent 1's economic incentive to share her opinion truthfully. Intuitively, strategic complementarity aligns her interest with accurate reporting, whereas strategic substitution creates incentives for opinion distortion. To formalize this intuition, we conduct a comparative analysis of each agent's wealth accumulation by simulating the game under alternative disclosure schemes for agent 1.[4] Given agent i's optimal position $d_{i,t}$, her wealth evolves according to

$$w_{i,t+1} = (1+r)w_{i,t} + [p_{t+1} - (1+r)p_t]d_{i,t}, \tag{3.1}$$

[4]A more formal analysis would require internalizing agent 1's strategic choice of opinion disclosure into her payoff function, which we do not pursue in this article.

which directly reflects her realized payoffs over time. Figure 3 displays each agent's wealth trajectory, time-averaged across 5,000 simulation periods to ensure stationarity. When agent 1 distorts her opinion ($\sigma_{u_1} = 2$), she accumulates significantly more wealth than others. In contrast, when she reports truthfully ($\sigma_{u_1} = 0.1$), she not only accumulates less wealth but also performs similarly to others. This is because strategic substitution dominates in the current parameter setting, where the relative volatilities of aggregate innovations to agent-specific innovations are low. Under such conditions, agent 1 has a stronger incentive to misrepresent her opinion, as she can profit at others' expense. These findings underscore the challenges of ensuring information reliability in short-term markets characterized by zero-sum trading and thus strong strategic substitution.

3.2 Citizen Journalist We then extend the game to include a fourth agent who acts as an alternative intermediary to relay information within the network (see Figure 1-b). Agent 4 emerges as a diminutive variant of agent 1: she shares the same level of informativeness but discloses her opinion truthfully (see Table 1-b). This extension is motivated by the observation that public access to citizen journalism has expanded substantially, allowing individuals to engage with a broader range of perspectives beyond traditional mainstream sources. Unlike the mainstream media (agent 1), the citizen journalist (agent 4) disseminates news unilaterally to the public audiences (agents 2 and 3), rendering the network no longer strongly connected. The neighborhood sets of this four-agent network are given by $N_1 = \{2, 3\}$, $N_2 = \{1, 4\}$, $N_3 = \{1, 4\}$, and $N_4 = \varnothing$.

To explore the equilibrium opinion formation in the expanded network, we consider again the effects of a unit shock in agent 1's private signal on each agent's opinion, forecast error, and asset demand (see Figure 4). Compared to the three-agent game, the inclusion of agent 4 introduces several notable changes. First, despite agent 4's minimal mass, her informational pathway enables less informed agents to adjust their opinions more accurately in response to the aggregate persistent innovation. This adjustment leads to a rapid convergence in opinions, learning rates, and asset positions even in a network lacking strong connectivity. Second, by leveraging agent 4's information source, less informed agents exhibit smaller deviations in their opinions and asset positions in response to the noise in agent 1's private signal. Consequently, agent 1's exaggerated adjustments to her opinion and asset demand are counterbalanced by the more tempered responses of other agents.

In sum, the expanded network fosters greater resilience to misinformation. While agent 1 can still exert some influence due to her central position, agent 4's emergence at the periphery helps prevent information distortions from dominating equilibrium opinions. These results highlight the importance of decentralizing information channels to promote more accurate and balanced opinion formation in social and economic networks.

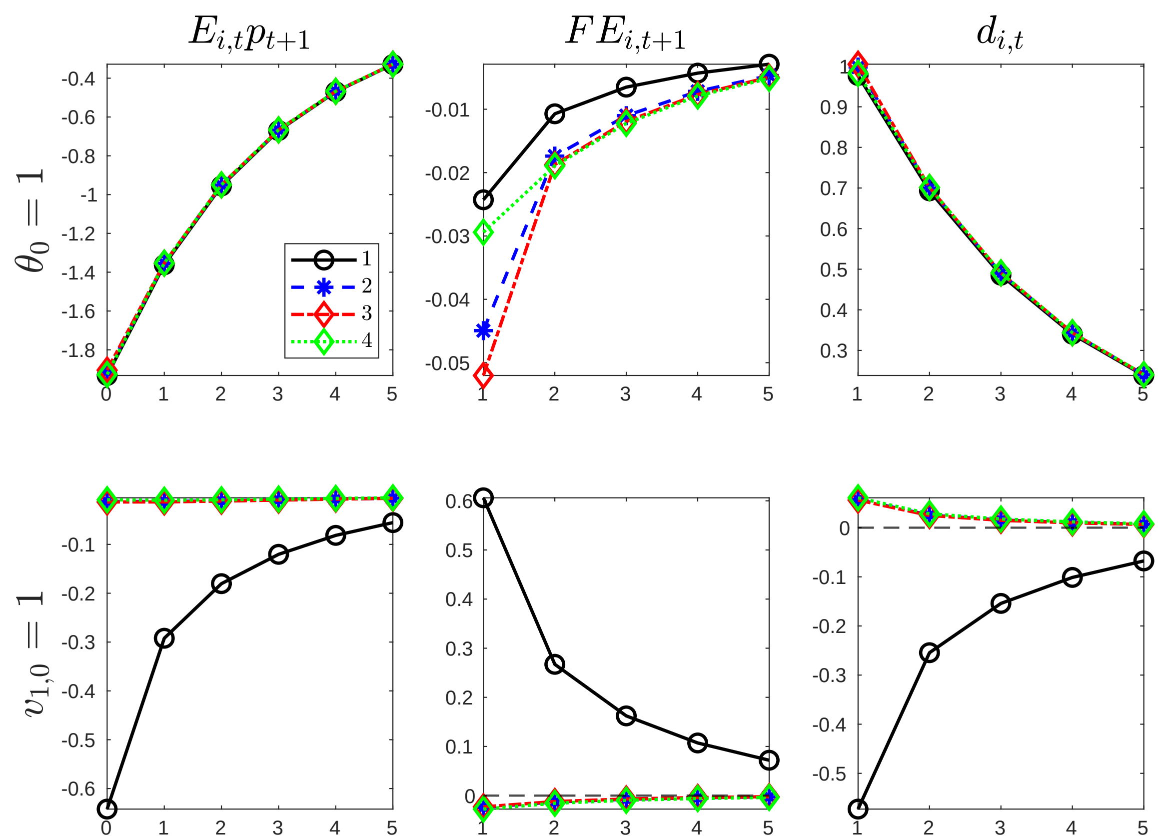


Figure 4: Impulse response functions under four-agent game. Notes: See Figure 2.

4 Conclusion

This article advances the understanding of opinion formation in social networks by addressing the puzzle of persistent belief heterogeneity in strongly connected networks. Using a dynamic game model with communication frictions, we demonstrate how the need to "forecast the forecasts of others" creates equilibrium opinion divergence. Our frequency-domain method of solving dynamic network games provides a general toolkit for studying information transmission in various network structures. Several promising extensions warrant future investigation, including the DeGroot (1974)-type social learning with weighted networks (see Appendix B) and the endogenous network structure through network formation games. These extensions would further illuminate how network topology and informational frictions shape public opinions, particularly in an era where information integrity is increasingly vital for social and economic outcomes.

Appendix

A Canonical Representation Tan and Wu (2022) study a class of linear rational expectations models with general information structures

$$\sum_{k=0}^{l} A_k \boldsymbol{y}_{t-k} + \sum_{k=0}^{h} B_k \mathbb{E}_t \boldsymbol{y}_{t+k} = \mathbf{0}, \tag{A.1}$$

where the model variables $\boldsymbol{y}_t$ and their coefficient matrices $\{A_k, B_k\}$ are partitioned as

$$\boldsymbol{y}_t = \begin{bmatrix} \boldsymbol{x}_t \\ \boldsymbol{a}_t \\ \boldsymbol{s}_t \end{bmatrix}, \qquad A_k = \begin{bmatrix} A_k^x & A_k^a & A_k^s \end{bmatrix}, \qquad B_k = \begin{bmatrix} B_k^x & B_k^a & B_k^s \end{bmatrix}.$$

The generalized expectation operator $\mathbb{E}_t(\cdot)$ allows for heterogeneous conditional expectations. That is, each expecational equation can be associated with a different information set.

There are three types of model variables: [i.] $\boldsymbol{x}_t$ is a vector of endogenous variables; [ii.] $\boldsymbol{a}_t = \int_0^1 \boldsymbol{x}_t di$ is the aggregation of $\boldsymbol{x}_t$ across a continuum of agents indexed by $i \in [0,1]$; [iii.] $\boldsymbol{s}_t$ is a vector of exogenous shocks that follows a covariance-stationary process

$$\boldsymbol{s}_t = \sum_{k=1}^{p_s} C_k^s \boldsymbol{s}_{t-k} + \sum_{k=0}^{q_s} D_k^s \boldsymbol{\epsilon}_{t-k}, \qquad \boldsymbol{\epsilon}_t \sim \mathcal{N}(0, \Sigma_{\boldsymbol{\epsilon}}), \tag{A.2}$$

where $\boldsymbol{\epsilon}_t$ is a vector of independent Gaussian innovations with covariance matrix $\Sigma_{\boldsymbol{\epsilon}}$.

Tan and Wu (2022) provide a user-friendly MATLAB toolbox called z-Tran that implements Algorithm 2.2 (Analytic Policy Function Iteration) of the main text in just seconds. The toolbox is publicly available at github.com/econdojo/ztran. As an illustrative example, we show how to cast the three-agent game of Section 3.1 into the canonical form (A.1)–(A.2).

First, define the 7×1 vector of endogenous variables $\boldsymbol{x}_t = [a_{1,t}, a_{2,t}, a_{3,t}, b_{1,t}, b_{2,t}, b_{3,t}, p_t]'$. The following linear equations constitute the equilibrium system (A.1)

$$\begin{aligned} \text{Agent } i\text{'s true forecast:} &\qquad a_{i,t} = \mathbb{E}_{i,t} p_{t+1}, \qquad i = 1,2,3, \\ \text{Agent } i\text{'s noisy forecast:} &\qquad b_{i,t} = a_{i,t} + c_{i,t}, \qquad i = 1,2,3, \\ \text{Market-clearing:} &\qquad p_t = \frac{1}{1+r}(\lambda_1 a_{1,t} + \lambda_2 a_{2,t} + \lambda_3 a_{3,t} - s_t). \end{aligned}$$

Next, define the 7×1 vector of exogenous shocks $\boldsymbol{s}_t = [c_{1,t}, c_{2,t}, c_{3,t}, q_{1,t}, q_{2,t}, q_{3,t}, s_t]'$ and the 8×1 vector of innovations $\boldsymbol{\epsilon}_t = [u_{1,t}, u_{2,t}, u_{3,t}, v_{1,t}, v_{2,t}, v_{3,t}, \theta_t, \eta_t]'$. The following equations provide the law of motion for shock processes (A.2)

$$\begin{aligned}
\text{Agent } i\text{'s forecast noise:} \quad & c_{i,t} = u_{i,t}, \qquad i = 1,2,3, \\
\text{Agent } i\text{'s private signal:} \quad & q_{i,t} = \theta_t + v_{i,t}, \qquad i = 1,2,3, \\
\text{Asset supply:} \quad & s_t = \rho s_{t-1} + \theta_t + \eta_t - \rho\theta_{t-1}.
\end{aligned}$$

Finally, we map the above system into the canonical form (A.1)–(A.2) as

$$A_0^x = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ -\frac{\lambda_1}{1+r} & -\frac{\lambda_2}{1+r} & -\frac{\lambda_3}{1+r} & 0 & 0 & 0 & 1 \end{bmatrix}, \; A_0^s = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{1+r} \end{bmatrix},$$

$$B_1^x = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \; C_1^s = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho \end{bmatrix}, \; D_0^s = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix},$$

$$D_1^s = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho \end{bmatrix}, \ \Sigma_{\boldsymbol{\epsilon}} = \begin{bmatrix} \sigma_{u_1}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{u_2}^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{u_3}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{v_1}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{v_2}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{v_3}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\theta}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\eta}^2 \end{bmatrix}.$$

To complete the model input, we specify the endogenous and exogenous signals that define the information set of each expecational equation

$$\begin{aligned} &\text{Agent 1's signal set:} \qquad \{b_{2,t}, b_{3,t}, p_t\} \cup \{q_{1,t}\}, \\ &\text{Agent 2's signal set:} \qquad \{b_{1,t}, p_t\} \cup \{q_{2,t}\}, \\ &\text{Agent 3's signal set:} \qquad \{b_{1,t}, p_t\} \cup \{q_{3,t}\}. \end{aligned}$$

B WEIGHTED NETWORK Following DeGroot (1974), we use a (row) stochastic matrix G to represent the directed and weighted network. Specifically, its element $G_{i,j}$ captures the weight or trust that agent i places on j's noisy opinion in forming her own opinion. In the three-agent game, this amounts to redefining $\boldsymbol{x}_t = [a_{1,t}, a_{2,t}, a_{3,t}, b_{1,t}, b_{2,t}, b_{3,t}, d_{1,t}, d_{2,t}, d_{3,t}, p_t]'$ and augmenting the equilibrium system by

$$\text{Agent } i\text{'s weighted opinion:} \qquad d_{i,t} = G_{i,1}b_{1,t} + G_{i,2}b_{2,t} + G_{i,3}b_{3,t}, \qquad i = 1, 2, 3.$$

Accordingly, the signal sets can be specified as

$$\text{Agent } i\text{'s signal set:} \qquad \{d_{i,t}, p_t\} \cup \{q_{i,t}\}, \qquad i = 1, 2, 3.$$

Mapping the above augmented system into the canonical form is straightforward and omitted for brevity.

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