Lecture 9: Bayesian DSGE Models

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Course: Introduction to Bayesian Statistics

Date: January 31, 2026

Introduction

- DYNARE has played a large role in fitting DSGE models
- However, it cannot handle *high-dimensional* DSGE models or models with
  - Student-t shocks, e.g., Chib & Ramamurthy (2014)
  - stochastic volatility, e.g., Justiniano & Primiceri (2008)
- Chib, Shin, & Tan (2021) provide a user-friendly MATLAB toolbox for such models that contains
  - training sample priors
  - efficient sampling of parameters by the TaRB-MH algorithm of Chib & Ramamurthy (2010)
  - fast computation of the marginal likelihood by the method of Chib (1995) and Chib & Jeliazkov (2001)
  - post-estimation tools, e.g., point and density forecasts

The Road Ahead

1. DSGE Model
2. Prior Distribution
3. Posterior Sampling
4. Marginal Likelihood
5. Prediction

Illustration

- Dynamic IS relation

$$\hat{y}_t = \mathbb{E}_t\hat{y}_{t+1} + \hat{g}_t - \mathbb{E}_t\hat{g}_{t+1} - \frac{1}{\tau}(\hat{R}_t - \mathbb{E}_t\hat{\pi}_{t+1} - \mathbb{E}_t\hat{z}_{t+1})$$

- New Keynesian Phillips curve

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \kappa(\hat{y}_t - \hat{g}_t)$$

- Monetary policy rule

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\psi_1\hat{\pi}_t + (1-\rho_R)\psi_2(\hat{y}_t - \hat{g}_t) + \epsilon_{R,t}$$

- Exogenous shocks

$$\hat{g}_t = \rho_G\hat{g}_{t-1} + \epsilon_{G,t}, \quad \hat{z}_t = \rho_Z\hat{z}_{t-1} + \epsilon_{Z,t}$$

- t-innovation with SV: for $s \in \{R, G, Z\}$

$$\epsilon_{s,t} \sim t_\nu(0, e^{h_{s,t}}), \quad h_{s,t} = (1-\phi_s)\mu_s + \phi_s h_{s,t-1} + \eta_{s,t}, \quad \eta_{s,t} \sim N(0, \omega_s^2)$$

State Space Form

- We rely on Sims' (2001) method to solve for the solution of the structural model, for *each* value of θ
- After (log) linearizing around the steady state, the structural model is expressed in canonical form as

$$\Gamma_0(\theta)x_t = \Gamma_1(\theta)x_{t-1} + \Psi\epsilon_t + \Pi\eta_t$$

- Applying Sims' method, the unique, bounded solution takes the form

$$x_t = G(\theta)x_{t-1} + M(\theta)\epsilon_t$$

  where $G(\theta)$ and $M(\theta)$ are non-linear unspecified functions of θ determined by the solve step

State Space Form (Cont'd)

- Model completed by the measurement equations

$$\begin{pmatrix} \text{YGR}_t \\ \text{INF}_t \\ \text{INT}_t \end{pmatrix} = \begin{pmatrix} \gamma^{(Q)} \\ \pi^{(A)} \\ \pi^{(A)} + r^{(A)} + 4\gamma^{(Q)} \end{pmatrix} + \begin{pmatrix} \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t \\ 4\hat{\pi}_t \\ 4\hat{R}_t \end{pmatrix}$$

- We show results for an extended version of this model that has 51 parameters, 21 variables, 8 shocks, 8 observables, and 1,494 non-Gaussian and nonlinear latent variables

Example

- Set MATLAB directory to the 'DSGE-SVt' folder
- Specify the model, data, and save directories

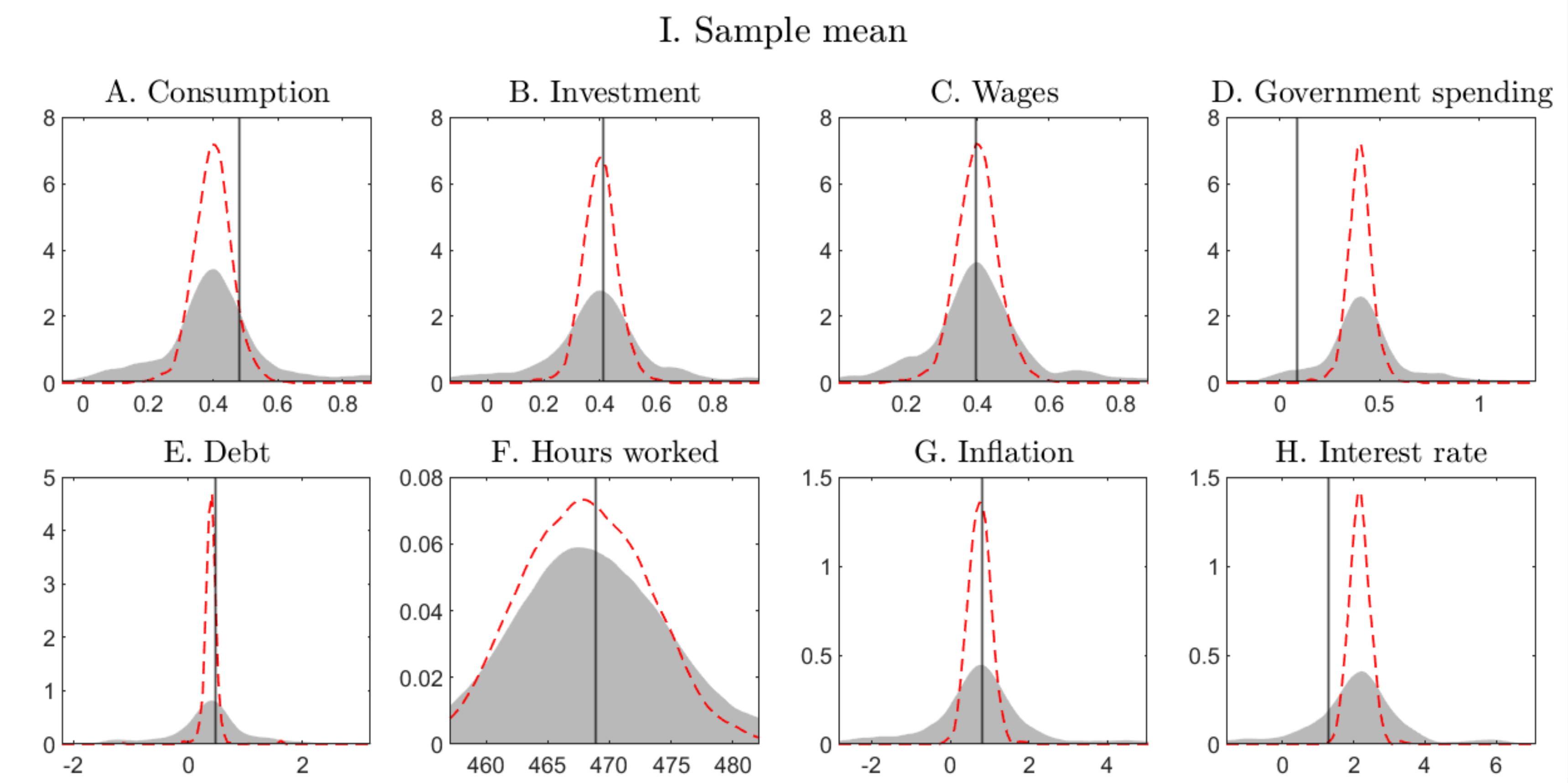
```
%% Housekeeping
clear
close all
clc

%% User search path & mex files
modpath = ['user' filesep 'ltw17'];
datpath = ['user' filesep 'ltw17' filesep 'data.txt'];
savepath = ['user' filesep 'ltw17'];
spec = tarb_spec([],'modpath',modpath,'datpath',datpath,'savepath',savepath);

OneFileToMexThemAll
```

Training Sample Prior

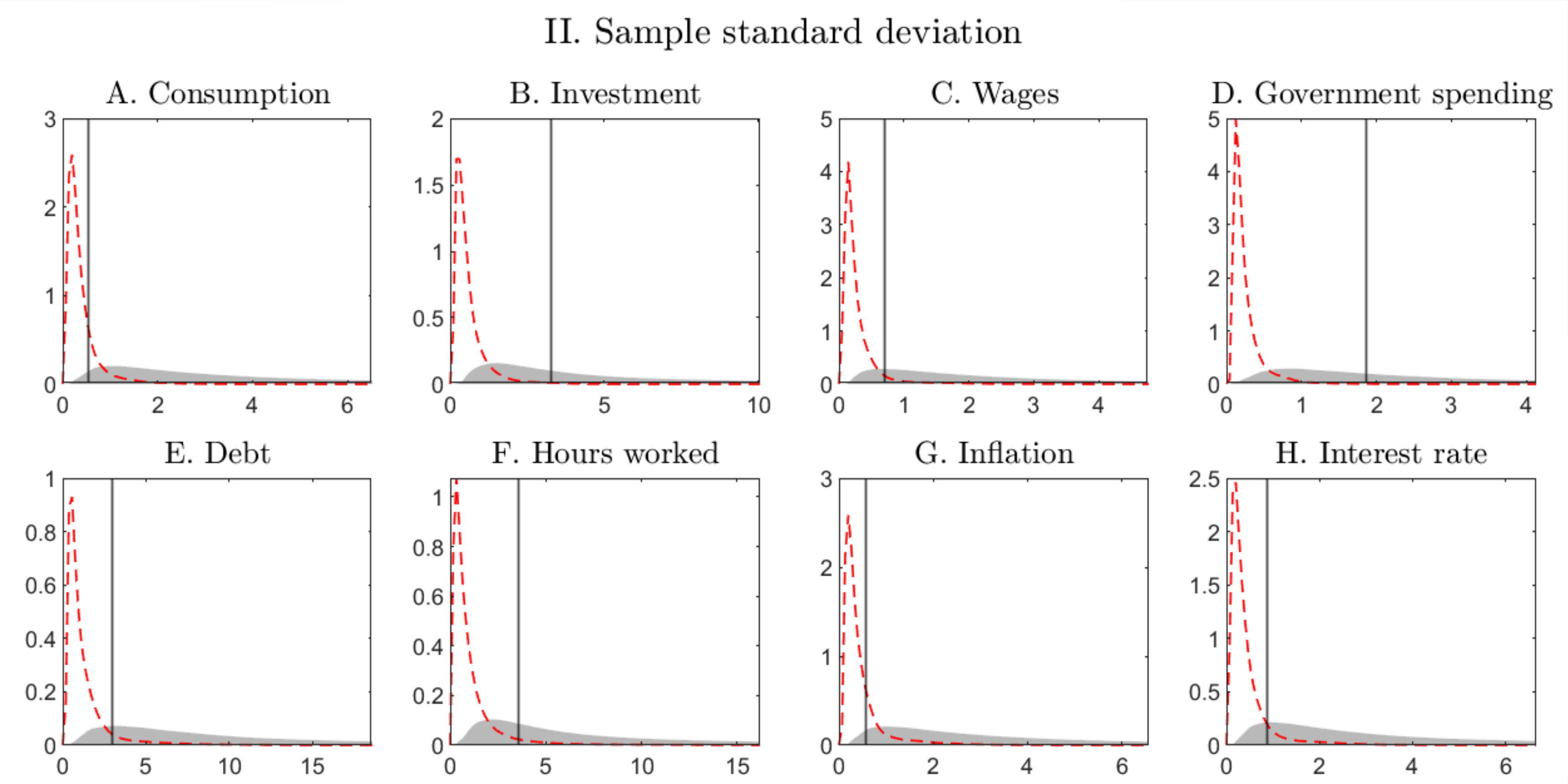
- In high dimensions, formulating an appropriate prior is difficult due to the complex mapping from θ to $G(\theta)$ and $M(\theta)$
- Standard choices often produce prior-sample conflict
- We supply two ways of dealing with this: training sample priors and Student-t family of distributions for location-type parameters
- A sampling the prior function is available to calculate the implied distribution of the outcomes

```
%% Sampling the prior
npd = 10000;          % number of prior draws
T = 200;              % number of periods
sdof = 5;             % shock degrees of freedom
SamplePrior(npd,T,sdof,savepath)
```

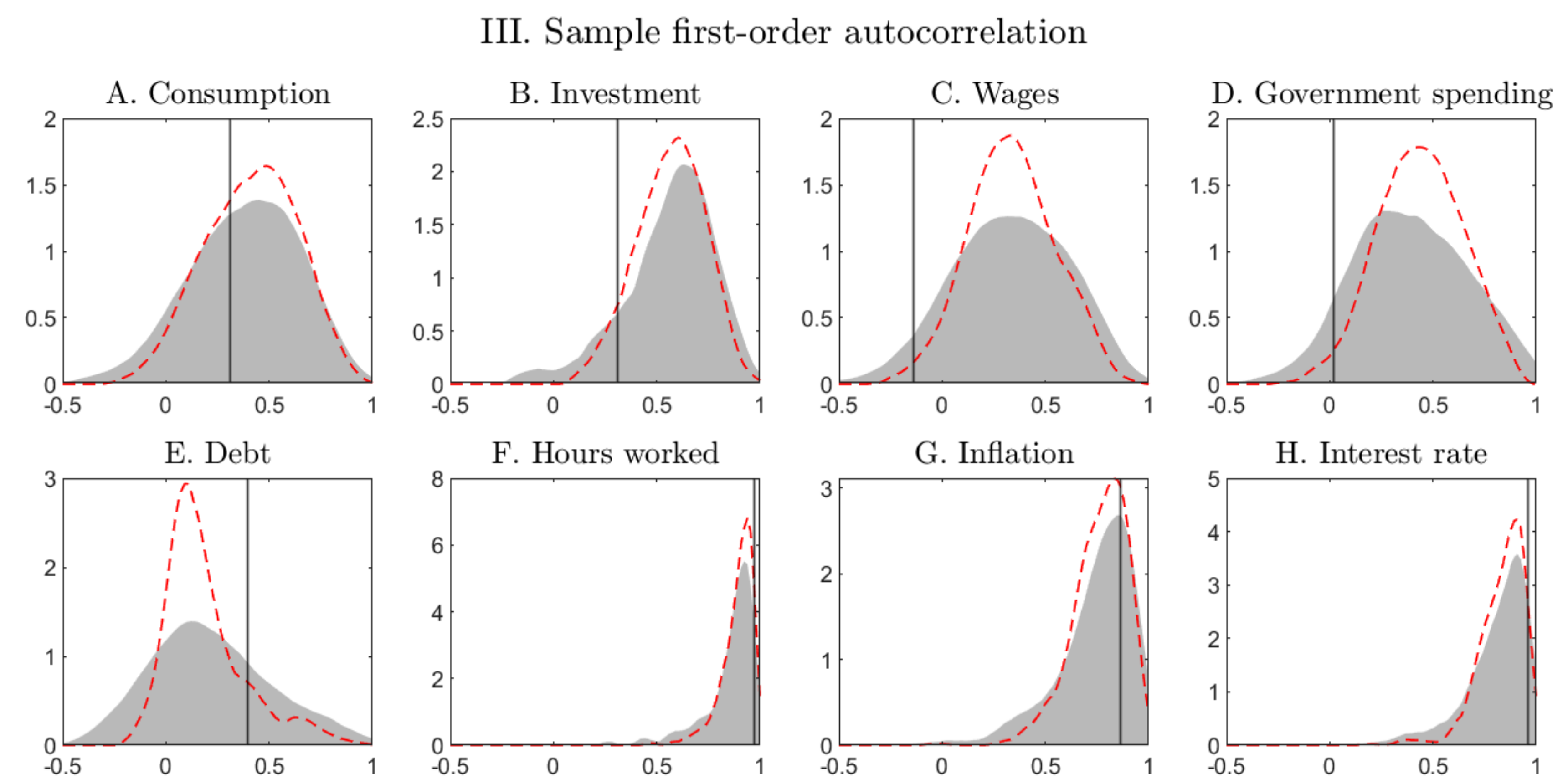
Prior Predictive Distribution



Prior Predictive Distribution (Cont'd)



Prior Predictive Distribution (Cont'd)



Estimation

- The centerpiece of the estimation procedure is the TaRB-MH algorithm of Chib & Ramamurthy (2010)
- Used to draw samples of $(\theta, z_{1:T})$ from
$$\pi(\theta, z_{1:T} | y_{1:T}) \;\propto\; f(y_{1:T}, z_{1:T} | \theta) \cdot \pi(\theta) \cdot \mathbf{1}\{\theta \in \Theta_D\}$$
- TaRB-MH is coded up in DYNARE, but the implementation there is somehow not efficient

TaRB-MH MCMC

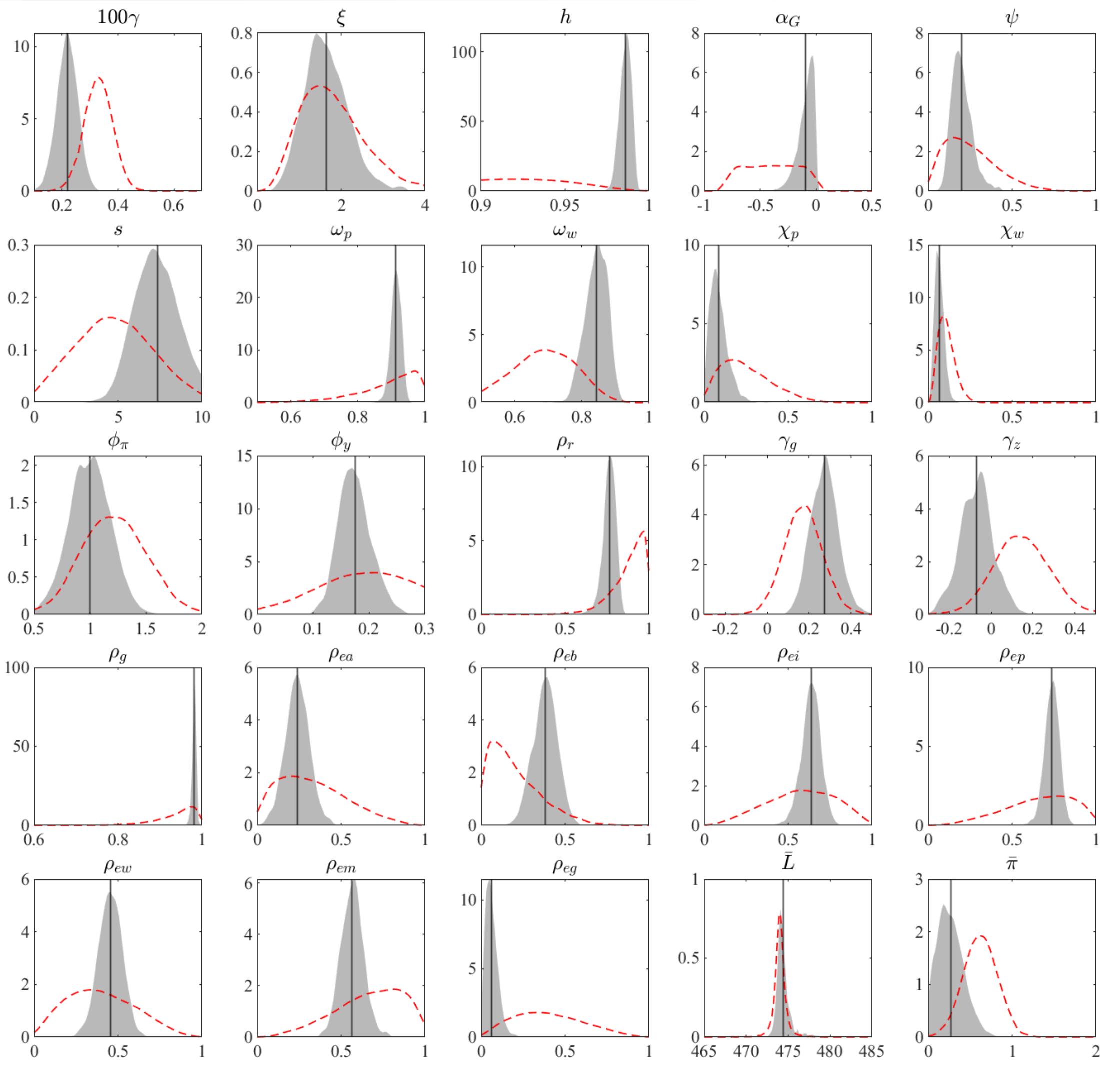
- Two hallmarks of TaRB-MH
  - randomize the number and components of blocks
  - tailor the proposal density to the posterior location and curvature, i.e., $q_b(\theta_b|y_{1:T}, z_{1:T}, \theta_{-b}) = t_v(\theta_b|\hat{\theta}_b, \hat{V}_b)$, $b = 1, \ldots, B$, using simulated annealing and Chris Sims' `csminwel`
- The toolbox provides a fast implementation of these steps by
  - randomly tailoring proposal every few iterations
  - MEX loop-intensive functions from C/C++ code
  - `csminwel` to get $\hat{V}_b$ as output
- Although intensive it all happens seamlessly

Example

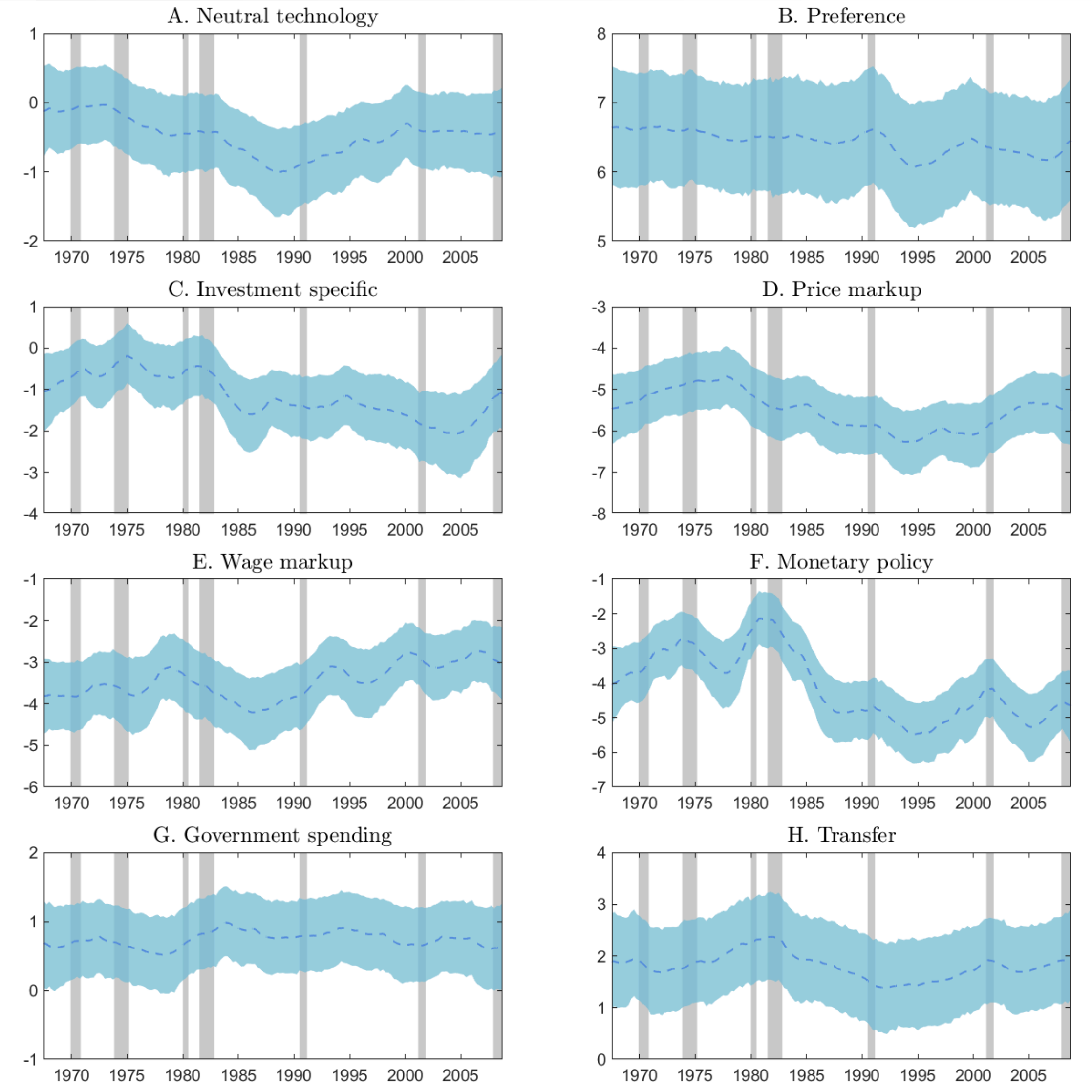
- Set up the TaRB-MH algorithm
- The estimation results are stored in the MATLAB data file `tarb_full.mat`

```
%% TaRB-MH (full run)
p = 0.7;                    % blocking probability
w = 0.5;                    % tailoring frequency
spec = tarb_spec(spec,'prob',p,'freq',w);

M = 11000;                  % number of draws
burn = 1000;                % number of burn-in
tarb_full(M,burn,spec)
```

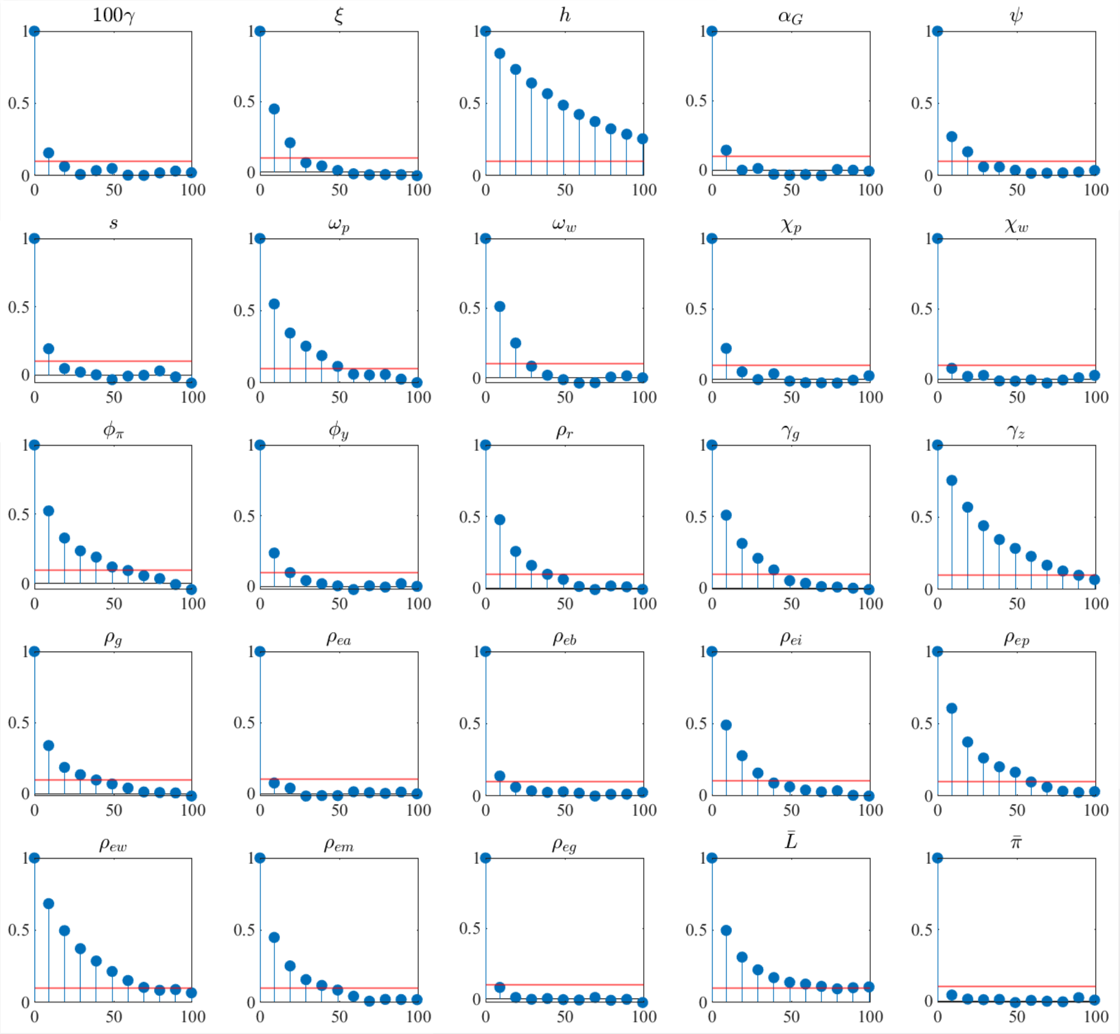
Prior-Posterior



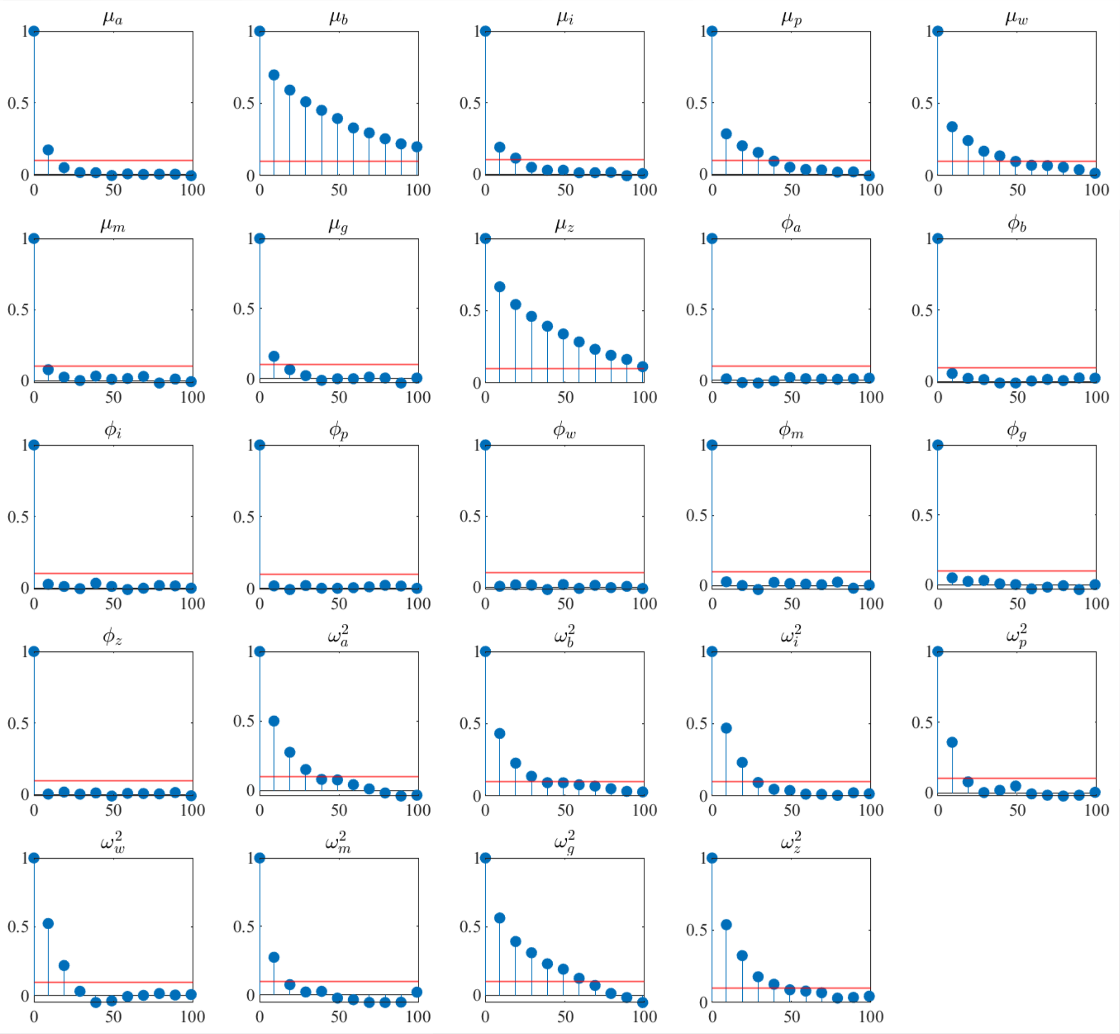
Stochastic Volatility



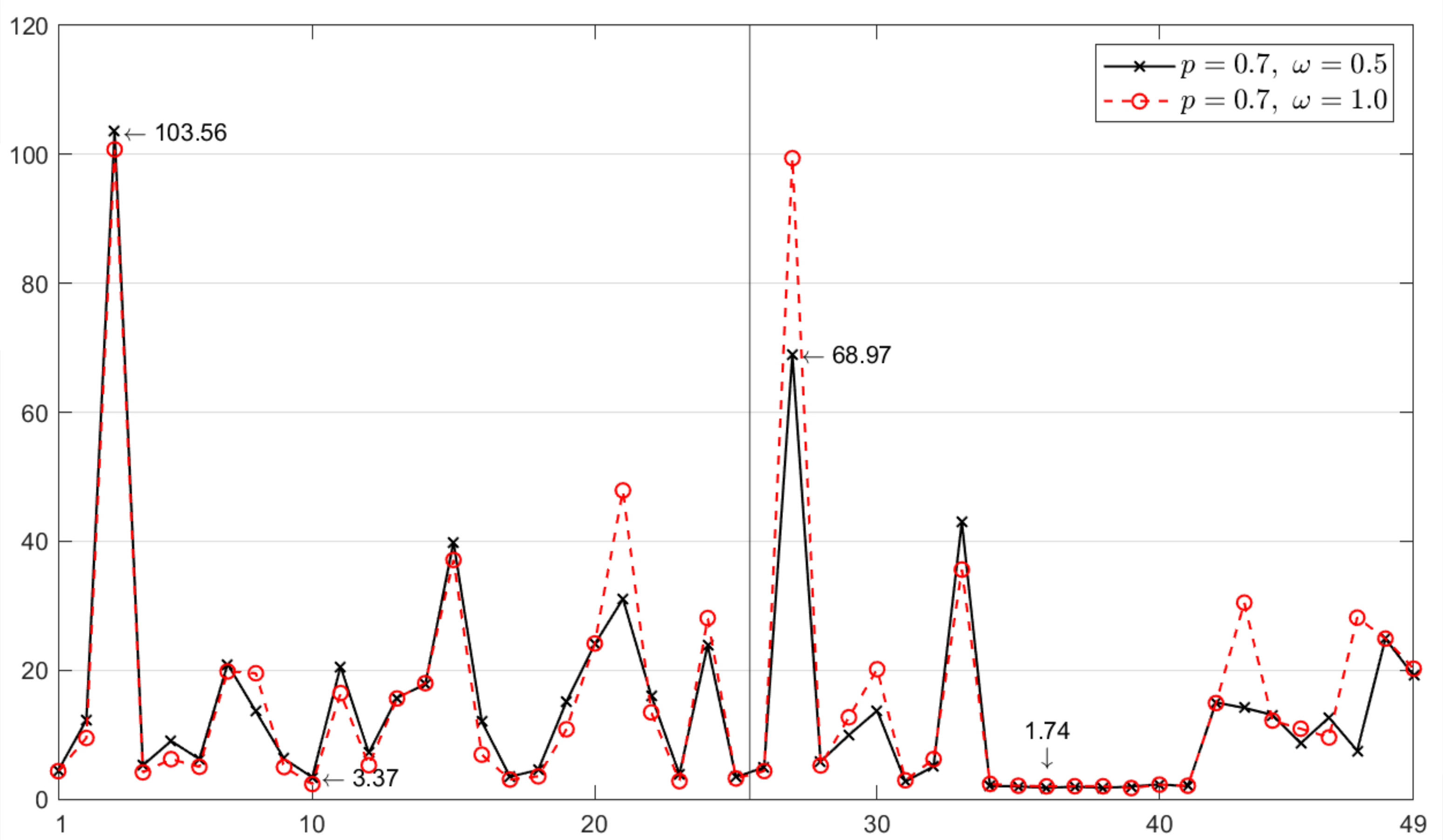
Autocorrelation



Autocorrelation (Cont'd)



Inefficiency Factor



Model Comparison

- Marginal likelihood is computed by a fast implementation of the Chib & Jeliazkov (2001) estimator
- This estimator is based on the identity of Chib (1995)

$$m(y_{1:T}) = \frac{f(y_{1:T}|\theta^*)\pi(\theta^*)}{\pi(\theta^*|y_{1:T})}, \quad \forall \theta^*$$

Model Comparison (Cont'd)

- With multiple block sampling, the posterior ordinate is estimated from the decomposition

$$\pi(\theta^*|y_{1:T}) = \pi(\theta_1^*|y_{1:T})\pi(\theta_2^*|y_{1:T},\theta_1^*)\cdots\pi(\theta_B^*|y_{1:T},\theta_1^*,\ldots,\theta_{B-1}^*)$$

  using several reduced TaRB-MH runs

- Key point: all reduced runs can be parallelized for the cost of only one reduced run, regardless of the number of blocks

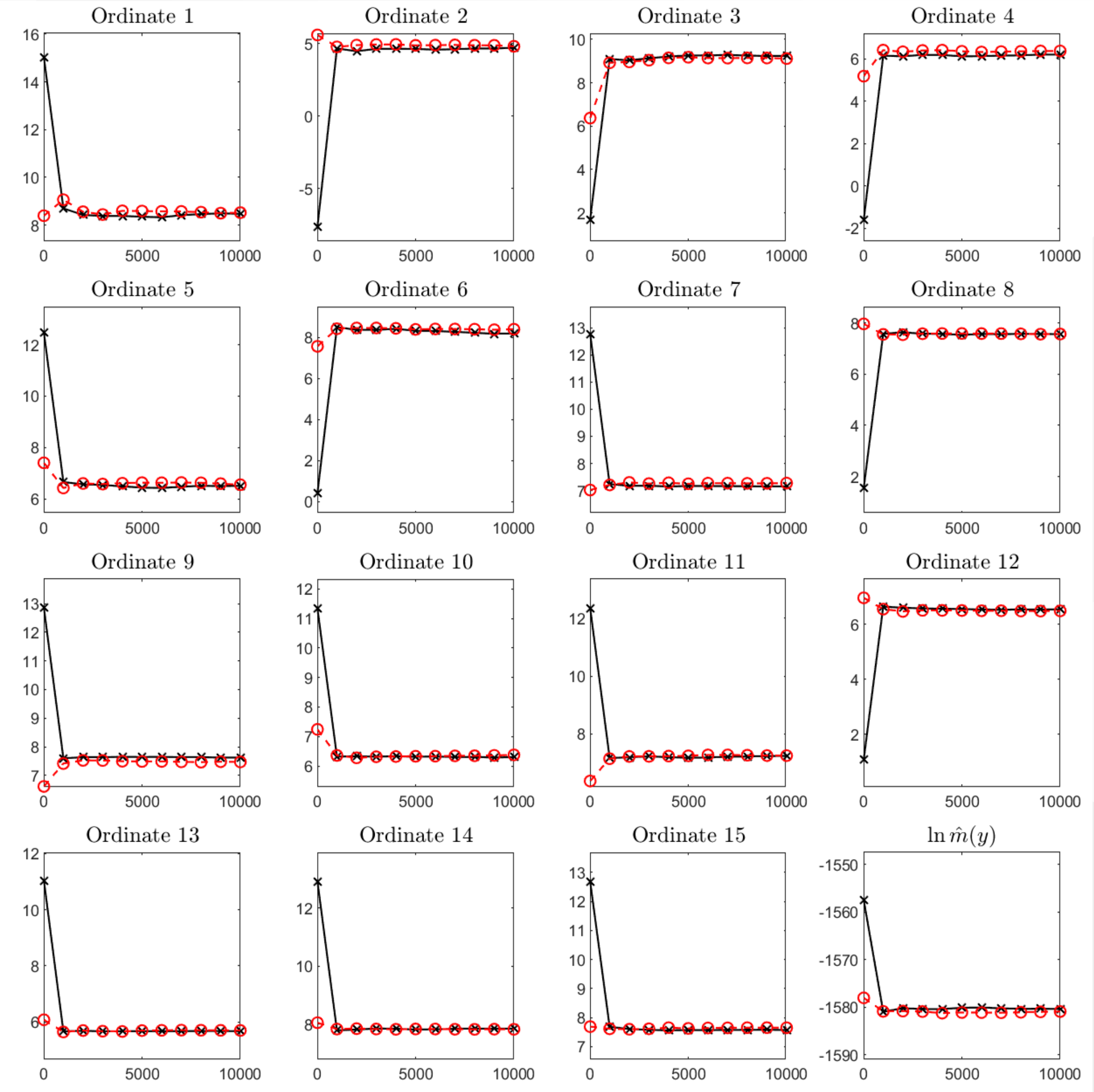
- This speeds up the calculation enormously

Example

- Set up the TaRB-MH reduced run
- The estimation results are stored in the MATLAB data file `tarb_reduce.mat`

```
%% TaRB-MH (reduced run)
M = 11000;              % number of draws
burn = 1000;            % number of burn-in
B = 7;                  % number of blocks
tarb_reduce(M,burn,B,spec)
```

Marginal Likelihood



Simulation Evidence

- Chib-Jeliazkov estimator

DGP 1: regime-M with ν=15			**DGP 2: regime-F with ϕ=0.5**		
ν	M	F	ϕ	M	F
30 (light)	4	0	0.1 (weak)	0	9
15 (fat)	15	0	0.5 (moderate)	0	10
5 (heavy)	1	0	0.9 (strong)	0	1

- Modified harmonic mean estimator

DGP 1: regime-M with ν=15			**DGP 2: regime-F with ϕ=0.5**		
ν	M	F	ϕ	M	F
30 (light)	0	0	0.1 (weak)	0	0
15 (fat)	0	0	0.5 (moderate)	0	0
5 (heavy)	20	0	0.9 (strong)	0	20

Notes: Number of picks for each model specification.

Post-Estimation Tools

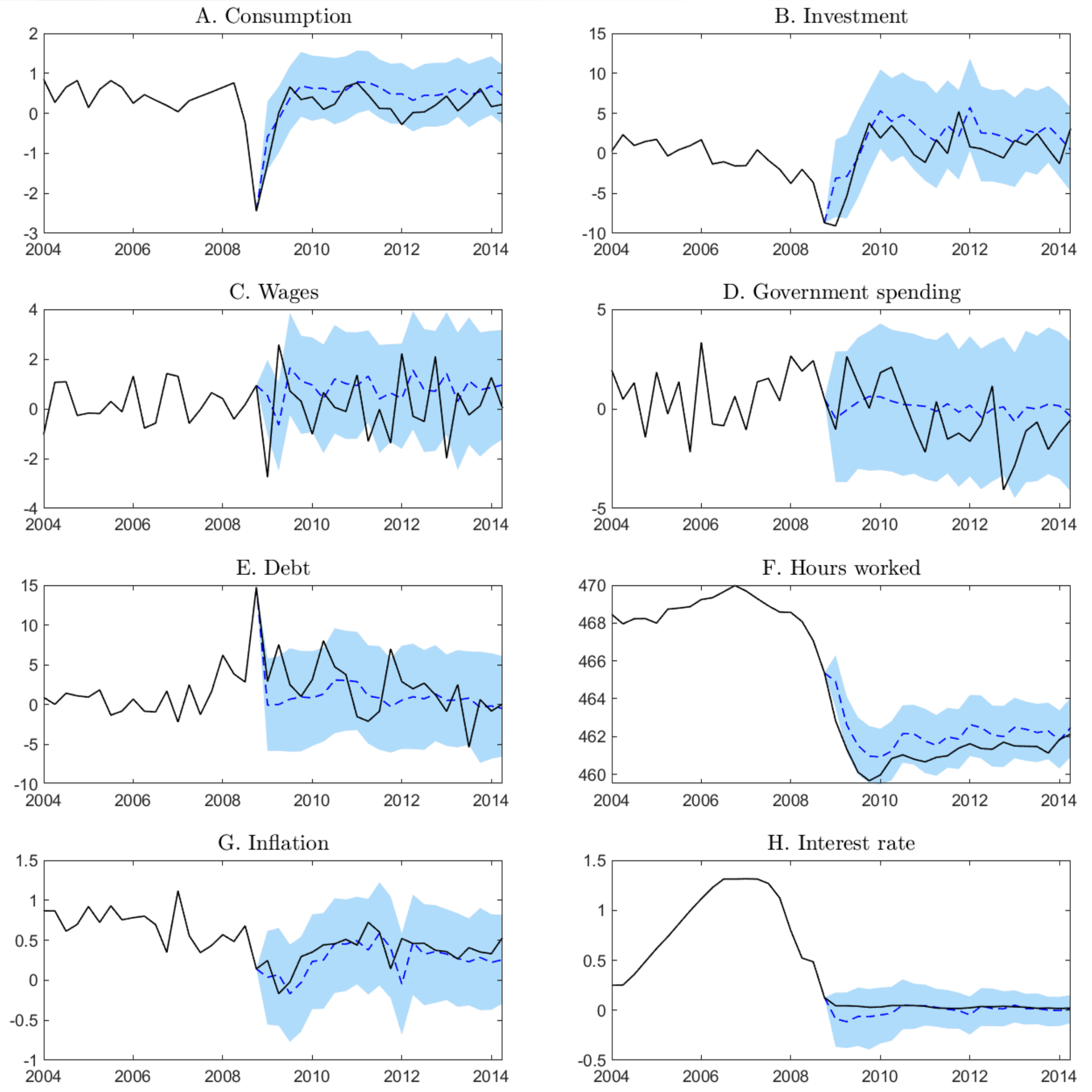
- Toolbox includes post-estimation tools, e.g., functions for conducting impulse response, variance decomposition, and prediction
- For instance, suppose we want the one-quarter-ahead prediction density

$$p(y_{T+1}|y_{1:T}) = \int p(y_{T+1}|y_{1:T}, \theta, z_{T+1}) \cdot \pi(\theta, z_{T+1}|y_{1:T}) d(\theta, z_{1:T})$$

- This is available through the TaRB-MH specification

```
%% Out-of-sample forecast
head = 50;          % training sample size
tail = 22;          % forecasting sample size
h = 1;              % forecasting horizon
spec = tarb_spec([],'datrange',[head tail],'pred',h);
```

Out-of-Sample Forecast



Outroduction

- Going beyond DYNARE for Next-Gen DSGE models
- Efficient and fast estimation via TaRB-MH and parallel computing
- MATLAB toolbox: publicly available at github.com/econdojo/dsge-svt
- Readily applied. Current application in progress, an open economy DSGE model that is about twice as big as the largest DSGE model estimated to date

References

- Chib & Ramamurthy (2010), "Tailored Randomized Block MCMC Methods with Application to DSGE Models," *Journal of Econometrics*
- Chib, Shin, & Tan (2021), "DSGE-SVt: An Econometric Toolkit for High-Dimensional DSGE Models with SV and t Errors," *Computational Economics*
- Sims (2001), "Solving Linear Rational Expectations Models," *Computational Economics*