Double/Debiased Machine Learning for Static Games with Incomplete Information: An Application to Pharmacy Deserts

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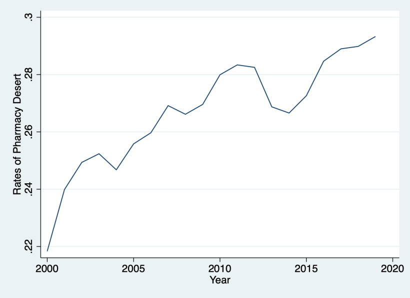
Motivation I

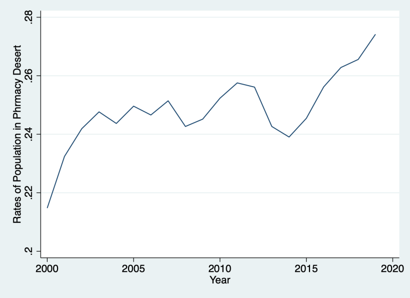
- Access to pharmacies is crucial for medication availability/adherence.
  - "Pharmacy deserts": areas with limited access to pharmacies and prescription drugs within walking distance (5-10 miles) (Amstislavski et al. (2012), Qato et al. (2014)).
- Health disparities: Minorities in rural areas are particularly vulnerable to pharmacy deserts.
  - Lower patient adherence rates, including chronic disease prescriptions and multiple pharmacological therapies, have an effect on increased patients' health (ED visits, hospitalization) Di Novi et al. (2020).
- **Stylized Facts:** In the rural town, there has been an increased trend in pharmacy deserts from 2000-2019.

Stylized Facts

Figure 1: Trends in Pharmacy Deserts in Rural Areas Township in Midwest 2000-2019



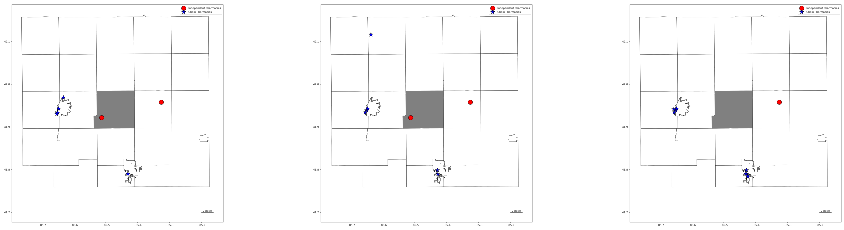
(a) Proportions of Pharmacy Desert in the U.S.



(b) Proportions of Population Pharmacy Desert in the U.S

Note: Samples are from township-level in the U.S. Midwest region 2000-2019. Based on the census RUCA (Rural-Urban Commuting Area Codes), I construct township-level rural areas if township areas include rural regions from RUCA. I used three years moving average for this calculation.

Figure 2: An Example: Spatial Distribution in Independent/Chain Pharmacy



(a) Year: 2000
# of Chain: 4

(b) Year: 2009
# of Chain: 6

(c) Year: 2019
# of Chain: 9

Note: Samples are from Nottawa township in MI, 2000-2019.

- Key Take Away:
    1. Independent pharmacies have disappeared which caused prevalence in pharmacy deserts in the U.S.
    2. Chain pharmacies are more common and dense within big towns.

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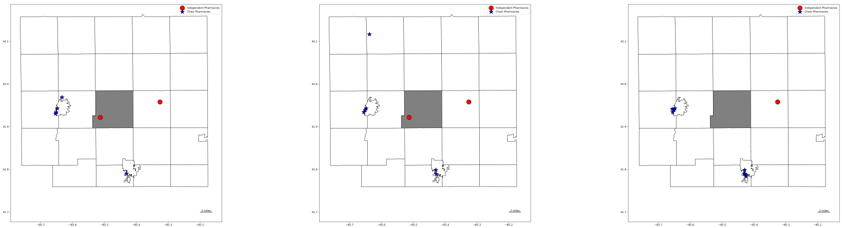

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- Key Take Away:
  1. Independent pharmacies have disappeared which caused prevalence in pharmacy deserts in the U.S.
  2. Chain pharmacies are more common and dense within big towns.
- **Research Question**: The mechanism of pharmacy desert is under-studied.

Motivation II

- Through the lenses of empirical IO, independent pharmacies face two different competitions:
  - Endogenous competition from **rival independent pharmacies** in the same market.
  - Exogenous competition from **chain pharmacies** in more distant regions.
- **Motivation for structural model**: To decompose the two sources of competition, I estimate discrete games of incomplete information following Bajari et al. (2010).
- **Challenges**: Existing models often rely on strong assumptions about payoff-relevant market characteristics:
  1. Relevant market characteristics include low-dimensional.
  2. Functional forms are correctly specified.
- To relax these assumptions, I **develop econometric models** that incorporate **high-dimensional covariates**.

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1. **Reduced Form Analysis** (Not today)
   - Approximately, a 100% increase in chain pharmacies ($2 \rightarrow 4$ units) is associated with a decrease of 0.06 units (8.4% decrease) in independently-owned pharmacies.

Preview of Results

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2. **Methodology Development**
   - Neyman Orthogonal Moment/Asymptotic Theory
     - I propose a new orthogonal moment based on Bajari et al. (2010).
     - To achieve this, I combine the double machine learning (DML) literature Chernozhukov et al. (2018) and cross-sample splitting methods.

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     - To achieve this, I combine the double machine learning (DML) literature Chernozhukov et al. (2018) and cross-sample splitting methods.
     - The developed model is applicable to **a wide range of discrete games with interactions**, including entry/exit games, quality/location choices, and pricing strategies such as promotions.

Preview of Results (Cont'd)

3. **Structural Analysis**: Applications in local pharmacy markets
    - ML performs better in the estimation of first-stage nuisance parameters, in terms of ROC-AUC (accuracy under the curve).
    - The developed methodology successfully eliminates biases induced by machine learning (ML).
    - A 100% increase (2$\rightarrow$4 units) in chain pharmacies will lead to a decrease in the probabilities of being active by:
        - 11.47% in the conventional method.
        - 8.58% in the Naive-plug-in method.
        - 21.89% in the Orthogonal moments.

Literature Review

- Pharmacy Desert
  - Qato et al. (2014), Olson et al. (2018), Di Novi et al. (2020).
- Static Games Literature
  - Bresnahan and Reiss (1991), Berry (1992), Mazzeo (2002), Seim (2006), Jia (2008), Ellickson and Misra (2008), Holmes (2011), Nishida (2015), Ellickson and Misra (2011), Bajari et al. (2010), Bajari et al. (2013), Grieco (2014).
- Neyman/Double Machine Learning Literature (DML)
  - Neyman (1959), Chernozhukov et al. (2018), Chernozhukov et al. (2021), Chernozhukov et al. (2022b), Chernozhukov et al. (2022a).
- Influence function with Game Settings
  - Bajari et al. (2009), Semenova (2018), Adusumilli and Eckardt (2019), Nekipelov et al. (2022).

	Data Dimensions	
	Low	High
Conventional Method	Bajari et al. (2010)	Naive-plug-in
Developed Neyman Orthogonal		Orthogonal

Model Framework

- I closely follow standard notations in a static game setting as described in Bajari et al. (2010) and Bajari et al. (2013).
- In a static game with incomplete information settings, I consider a finite number of players $i = (1, ..., N)$ with binary choices, $J = 2$.

$$a_i = \begin{cases} 1 & \text{if Player } i\text{'s being active.} \\ 0 & \text{if Player } i\text{'s being inactive.} \end{cases} \tag{1}$$

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- For simplicity, I illustrate with two players. The payoff function for being active is given by:

$$\pi_i(a_i = 1 | d, x) = \theta_r \sigma_{-i}(d_{-i}, x) + \theta_d d_i + \underbrace{x\beta}_{\text{high dimensions}} + \epsilon_i(1). \tag{2}$$

where

- a_{-i} denote the rival's action.
- $\sigma_{-i}(d_{-i}, x)$ denotes the probability of rival's being active.
- d_{-i} denote rival specific productivity shock and d_i denote player i specific productivity shock.
- x denotes market characteristics (common demands) to every player in the market.
- $\epsilon_i(1)$ denote i.i.d from a Type 1 Extreme Value Distribution.

Model Framework

Commonly Made Assumptions in IO Literature

- Assumption 1. Incomplete Information Appendix
- Assumption 2. Normalization of Outside Choice Appendix
- Assumption 3. Correct Beliefs (Rational Expectation over Rival's action) Appendix
- Assumption 4. Equilibrium Selection Appendix
- Assumption 5. Exclusion Restriction Appendix
    - Remark 1. Identification condition holds with high-dimensional controls. Appendix

Two Step Estimation

Step 1: Estimation of choice probabilities:

$$\gamma_{-i} = E[a_{-i}|d_{-i}, x] \text{ for all } -i = 1, ..., N. \tag{3}$$

where

- a_i denote the rival's action.
- d_{-i} denote player $-i$ specific productivity shock.
- x denote market characteristics (common demands) to every player in the market.

Step 2: Recovering the Structural Parameters With variables z, GMM can be constructed as:

$$\arg\min_{\theta,\beta} g(w;\theta,\beta) = E[z(a_i - \Lambda(\theta\hat{\gamma}_{-i}, X\hat{\beta}))]. \tag{4}$$

where Λ denotes the logistic link function

Two-step methods with high dimensional data

- In the two-step estimation, the **regularization/model selection bias** in $\hat{\eta} = (\hat{\gamma}_{-i}, \hat{\beta})$ could be transmitted into the second stage estimation of structural parameters θ.
- To correct local mis-specification, I use the orthogonality condition:

$$||\partial_{\eta} E_P[\psi(w_i;\theta,\eta)][\eta-\eta_0]|| = 0 \text{ for all } \eta \in \mathcal{T}_N,$$

  where $\psi(w_i, \theta, \eta)$ denote the Neyman orthognal moment function.
- Conventional methods do not satisfy Neyman orthogonality condition:

$$\partial_{\gamma_{-i}} E[g(w_i;\theta,\eta)][\gamma_{-i}-\gamma_{-i0}] = E\left[z_i \cdot \Lambda' \cdot \left(-\theta_{\gamma} \sum_{a_{-i}} \prod_{s\neq i,i} (1-\gamma_s)\right) \cdot (\gamma_{-i}-\gamma_{-i0})\right] \neq 0$$

$$\text{for } -i = 1, \ldots i-1, i+1, \ldots n.$$

Asymptotic Analysis: Neyman Orthogonal Moment

- I give the new orthogonal moment function:

$$\psi(w_i,\theta,\eta) = (z_i - x'\mu_x)[a_i - \Lambda(\theta_\gamma\gamma_{-i}, X\beta)] - (z_i - x'\mu_x)\Lambda(\cdot)(1-\Lambda(\cdot))\theta_\gamma[a_{-i} - \gamma_{-i}] \quad (5)$$

$$\text{where } \mu_x = \text{argmin}_{\mu_x} E[f_i(z_i - x'\mu_x)]^2$$

Lemma 1

$E\left[\psi(W;\theta_0,\eta_0)\right] = 0$

Theorem 2

The moment (5) obeys Neyman orthogonality moment conditions at (θ_0,η_0) with respective to the nuisance realization set $\mathcal{T}_N \subset \mathcal{T}$ if Lemma 1 holds and

$$||\partial_\eta E_P[\psi(W;\theta_0,\eta_0)][\eta-\eta_0]|| = 0 \text{ for all } \eta \in \mathcal{T}_N,$$

Estimation Procedures

Algorithm

1. Let size $n = N/K$. Use a K-fold random partition $(I_k)_{k=1}^{K}$ Also, for each $k \in K = \{1, ..., K\}$, define I_k^c as the complement of I_k, i.e., the set of indices 1,...,N excluding those in I_k.
2. For each $k \in K$, estimate nuisance parameters $\hat{\eta}_k$ using I_k^c

$$\hat{\eta}_k = \hat{\eta}((W_i)_{i \in I_k^c}), \hat{\eta}_k = (\hat{\gamma}_{-ik}, \hat{\beta}_k, \hat{\mu}_{zk}).$$

   1. Obtain $\hat{\gamma}_{-ik}$ from Logit Lasso
   2. Obtain $\hat{\beta}_k$ from Logit Lasso
   3. Estimate $\hat{\mu}_{zk}$ from the Lasso estimator
3. Construct the orthogonal moment estimator, then obtain $\hat{\theta}$ as

$$\frac{1}{K} \sum_{k=1}^{K} \frac{1}{n} \sum_{i \in I_k} \psi(w_i, \theta, \eta_k)) = 0$$

Asymptotic Analysis

Theorem 3

Suppose that assumption 1-5, and assumption 6 holds. For $V = M^{-1}E[\psi_0(W)\psi_0(W)']M^{-1}$, the DML static game estimators (5) obeys Neyman Orthogonal Moments and it follows:

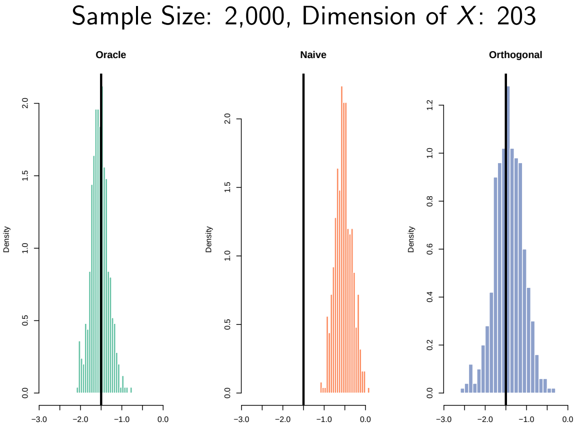
$$\sqrt{N}(\hat{\theta} - \theta_0) \to N(0, V), \ \hat{V} \xrightarrow{p} V.$$

$$\text{where } \hat{V} = \Big(\frac{1}{K}\sum_k^K E[M]\Big)^{-1}\frac{1}{K}\sum_k^K E[\psi^2(w, \hat{\theta}, \hat{\eta}_k)]\Big(\frac{1}{K}\sum_k^K E[M]\Big)^{-1}$$

Appendix: Assumption 6 Appendix: Multiplayers

Monte Carlo Simulation

Figure 3: The distribution of the estimated structural parameters from simulation (Lasso/Logit Lasso)



Notes: The estimated effect of rival coefficients are based on 500 simulations. The true value of the rival effects is $\theta_0 = -1.5$. The Oracle method uses low-dimensional relevant covariates, while the naive plug-in method uses high-dimensional covariates without correcting for biases. The orthogonal method also uses high-dimensional covariates like the naive plug-in method, but it corrects for biases using the proposed Neyman orthogonal method.

Structural Analysis

- Scope of Analysis
  - Rural townships in the U.S. Midwest region in 1997-2021.
  - I focus on 2000-2019 time periods.
  - I limit the township with at least one independent pharmacy during 1997-2021.
  - Two-player game between independent pharmacies with external competition from chain pharmacies.[1]

[1] In the data, 99.2% of markets have at the most two independent pharmacies.

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- Data
  - Panel data of Pharmacies Entry/Exit: Data Axle (1997-2020)
  - Socio-Economic Characteristics: Population, Per capita Income, Race composition, Age distributions (over 65), Household composition, Education, Vehicle Availability. (Source: Census 2000, 2010)
  - Number of Physician Offices (county) (Source: ZBP)
  - Summary Statistics in Appendix
-

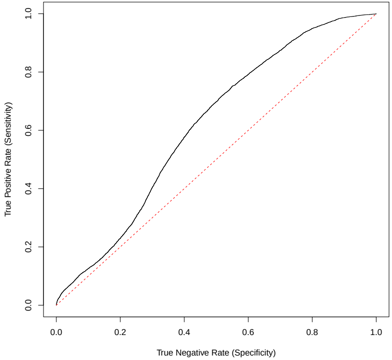
Table 1: Comparison of Methods

Method	Covariates	Dimensions for covariates
Bajari et al. (2010)	Soci-economic variables	33
Naive-plug in	Soci-economic variables + their interactions	497
Orthogonal moment with cross-fitting	Soci-economic variables + their interactions	497

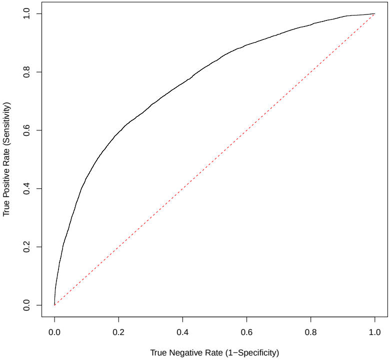
[1] In the data, 99.2% of markets have at the most two independent pharmacies.

Structural Analysis (Cont'd)

Figure 4: ROC Curve in First Stage Estimation



(a) Conventional methods (Logit)
AUC: 0.6151



(b) Logit Lasso (CV)
AUC: 0.7660

Note: I use 10 K-fold cross-fitting to learn penalty terms λ_1 of logit lasso. AUC denotes the area under the accuracy

Structural Analysis (Cont'd)

Table 2: Results from the Structural Model

Parameters	Variables	Bajari et al. (2010)	Naive plug-in	Neyman Orthogonal Moments
θ_i	Pr. of rival independent pharmacies	-3.311	-2.674	-3.826
		(0.040)	(0.040)	(0.074)
θ_c	log(No. of chain pharmacies	-0.457	-0.403	-1.027
	within 20 miles)	(0.016)	(0.016)	(0.101)
Observations		31,280	31,280	31,280
No. of Socio-Economic Variables		33	497	497
Interaction		No	Yes	Yes
Counties FE		Yes	Yes	Yes
Year FE		Yes	Yes	Yes

Notes: Standard errors are in parenthesis.

- Interpretation of Maginal Effects: A 100% increase (from 2 to 4 units) in chain pharmacies will lead to a decrease in the probability of being active
  - by 11.47 percent: Bajari et al. (2010)
  - by 8.58 percent: Naive-plug-in method.
  - by 21.89 percent: Orthogonal moments.
- Ridge estimators produce similar results.

Conclusion

- Summary
  - Reduced form evidence suggests that entry of chain pharmacies is negatively correlated with the number of local pharmacies.
  - I established asymptotic theory/MCMC results.
  - Empirical application illustrated that Neyman Orthogonal moment corrected ML biases from the first stage.
- Future Plan
  - Report (more) goodness of Fit
  - Conduct Robustness check
  - Implement Counterfactuals

Thank you so much for attending my presentation!

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I greatly appreciate your valuable comments and feedback.

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Preliminary Analysis (Cont'd)

Table 3: The number of local pharmacies and chain pharmacies

	(1) # of Independent Pharmacies	(2) # of Independent Pharmacies	(3) # of Independent Pharmacies
log(# of Chain Pharmacies within 20 miles)	-0.080**	-0.124**	-0.120**
	(0.0210)	(0.026)	(0.027)
County FE	No	YES	Yes
Year FE	YES	No	Yes
Demographic controls	YES	YES	Yes
Observations	15,480	15,480	15,480
# of Counties	459	459	459
Adj. R^2	0.078	0.3528	0.3541
Outcome mean	0.710	0.710	0.710

Note Estimates are from fixed effects regression of the number of independent pharmacies in the township on the log(number of chain/supermarket pharmacies) in the county c and year t. Column (1)-(3) includes the total observations. Column (2) includes county-level fixed effects. Column (3) includes demographic controls from Census, County FE, and year FE. Standard errors are clustered at the county level. Significance levels are denoted by $+$ p$<$0.10, * p$<$0.05, ** p$<$0.05

- A 100 % change in the number of chain pharmacies is associated with a decrease in the number of independent pharmacies of -0.06 units.

Preliminary Analysis (Contd)

Table 4: Distribution (of the Number) of Active Local Pharmacies in 2000-2019 by Market Size (%)

	Active Local Pharmacies		
	0	1	2+
Total Pop. $< 1{,}500$			
No Chain	28.03	67.77	4.20
Chain ≥ 1	48.52	49.50	1.98
Total Pop. $\geq 1{,}500$			
No Chain	17.96	72.21	9.84
Chain ≥ 1	34.20	60.26	5.53

Note: Samples are from RUCA (Rural-Urban Commuting Area Codes). Total Population of 1,500 is the median of the rural area population in the mid-west region.

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Preliminary Analysis (Cont'd)

Table 5: Descriptive Statistics

		2000 Census						2010 Census				
Variables	Obs.	Mean	Std. dev.	Median	Min.	Max	Obs.	Mean	Std. dev.	Median	Min.	Max
Market characteristics												
Population	774	1703.68	1191.67	1447.5	33	14388	774	1685.87	1231.71	1408.5	39	14738
Population density (sq, miles)	774	548.40	660.79	102.63	1.07	3166.62	774	536.07	650.60	106.13	1.26	3434.14
Per capita income	774	16716.43	2595.45	16631	8360	35705	774	21244.63	3906.68	21100.5	10063	42282
Proportion of No vehicles in HH	774	0.073	0.048	0.068	0	0.54	774	0.063	0.054	0.054	0	0.72
Proportion of Age over 65	774	0.22	0.065	0.22	0.05	0.48	774	0.22	0.06	0.21	0.05	0.49
Store characteristics												
Independent pharmacy	774	0.81	0.51	1	0	2	774	0.70	0.53	1	0	2
Employee size	313	5.52	3.78	5	0	35	258	6.41	4.03	5	1	25

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- Shifters: Number of employees three years ago. (Source: Data Axle)

Model Framework

Assumption 1 (Incomplete information)

1. *The error terms are independently and identically distributed across actions and players, and are drawn from a Type 1 Extreme Value Distribution.*
2. *The structural shock affecting the error term is privately observed by each player.*
3. *The additive components of the error term are not observed by analysts.*
4. *The state variable d, x is observable to both players in the same market and analysts.*

Model Framework

- Having private information ϵ and observable state variable d, the payoff of player i can be expressed as

$$\pi_i(a, d, \epsilon_i; \theta) = \pi_i(a_i, a_{-i}, d; \theta) + \epsilon_i(a_i). \tag{6}$$

where flow utility $\pi_i(a_i, a_{-i}, d; \theta)$ depends on agent's discrete choice actions a_i, other agents' action a_i, and the state variable d.

Assumption 2 (Normalization of Outside Choice)

$$\textit{For all } a_{-i} \in A_{-i} \textit{ and all } d, \ \pi_i(a_i = 0, a_{-i}, d) = 0. \tag{7}$$

- By setting the flow utility of being inactive to zero, the utility of being active becomes relative to being inactive.

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Assumption 3 (Correct Beliefs)

Coupled with incomplete information structure, players form correct beliefs over rival's actions.

- With private information assumptions 1 and correct beliefs assumption 3, I can express a choice-specific value function:

$$\pi_i(a_i = 1, d) = \sum_{a_{-i} \in A_{-i}} \sigma_{-i}(a_{-i}|d)\pi_i(a_i = 1, a_{-i}, d) \text{ for all } i = 1, ...N. \tag{8}$$

$$\text{where } \sigma_{-i}(a_{-i}|d) = \prod_{s \neq i} \sigma_s(a_s|d)$$

- A decision rule choice rule can be expressed as:

$a_i^* = 1$ if and only if

$$\underbrace{\pi(a_i = 1, d)}_{=\sum_{a_{-i} \in A_{-i}} \sigma_{-i}(a_{-i}|d)\pi_i(a_i=1,a_{-i},d)} + \epsilon_i(1) > \underbrace{\pi(a_i = 0, d)}_{=0} + \epsilon_i(0).$$

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Model Framework

Fixing state variable d, $\sigma_i(a_i|d)$ will be the solution to the system of N equations:

$$\sigma_1(a_1 = 1|d) = \frac{\exp\Big(\sum_{a_{-1}\in A_{-1}} \sigma_{-1}(a_{-1}|d)\pi_i(a_1 = 1, a_{-1}, d)\Big)}{1 + \exp\Big(\sum_{a_{-1}\in A_{-1}} \sigma_{-1}(a_{-1}|d)\pi_i(a_1 = 1, a_{-1}, d)\Big)} \tag{9}$$

$$\sigma_2(a_2 = 1|d) = \frac{\exp\Big(\sum_{a_{-2}\in A_{-2}} \sigma_{-2}(a_{-2}|d)\pi_i(a_2 = 1, a_{-2}, d)\Big)}{1 + \exp\Big(\sum_{a_{-2}\in A_{-2}} \sigma_{-2}(a_{-2}|d)\pi_i(a_2 = 1, a_{-2}, d)\Big)}$$

$$\vdots$$

$$\sigma_N(a_N = 1|d) = \frac{\exp\Big(\sum_{a_{-N}\in A_{-N}} \sigma_{-N}(a_{-N}|d)\pi_i(a_N 1, a_{-N}, d)\Big)}{1 + \exp\Big(\sum_{a_{-N}\in A_{-N}} \sigma_{-N}(a_{-N}|d)\pi_i(a_N = 1, a_{-N}, d)\Big)}$$

Model Framework

Assumption 4 (Equilibrium Selection)

The data are generated by a single equilibrium from the set of possible equilibria and this observed equilibrium may vary across different markets.

Assumption 5 (Exclusion Restriction)

$$\pi_i(a_i, a_{-i}, d) = \pi(a_i, a_{-i}, d_i, x).$$

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Model Framework: Identification

Remark 1 (Identification)

1. *Suppose that Assumption 2 and Assumption 5 are satisfied. As long as there are 2^{N-1} points in the support of conditional distribution $d_{-i}|d_i$, then the necessary condition holds.*
2. *Allowing high dimensional market characteristics d does not change necessary conditions for identification.*

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Characterization of Neyman Orthogonal Moment

$$\frac{\partial \psi(w_i, \theta_0; \gamma_{-i0}, \beta, \mu_{z0})}{\partial \beta} = E\left[-\mu_z \Lambda' x \cdot (\beta - \beta_0)\right] = -E[u_i f_i x_i \cdot (\beta - \beta_0)] = 0 \text{ } (\because \text{ equation } \textbf{??})$$

$$\begin{aligned}\frac{\partial \psi(w_i, \theta_0; \gamma_{-i}, \beta_0, \mu_{z0})}{\partial \gamma_{-i}} &= E[(-\mu_z \Lambda' \theta_\gamma - \alpha) \cdot (\gamma_{-i} - \gamma_{-i0})] \\ &= E[(-\mu_z \Lambda' \theta_\gamma + \mu_z \Lambda' \theta_\gamma, \mu_{z0}) \cdot (\gamma_{-i} - \gamma_{-i0})] = 0.\end{aligned}$$

$$\frac{\partial \psi(w_i, \theta_0; \gamma_{-i}, \beta_0, \mu_z)}{\partial \mu_z} = E[a_i - \Lambda(\theta_\gamma \gamma_{-i0} x \beta_0) \cdot (\mu_z - \mu_{z0}] = 0.$$

$Q.E.D.$ Back

Asymptotic Analysis

Assumption 6 (Regularity Condition)

i) $W_i = (A_i, D_{-i}, D_i, X_i)$ *are bounded.*

ii) M *is twice differentiable with uniformly bounded derivatives bounded from zero.*

iii) $E[\{Y - \hat{\gamma}(X)\}^2|X]$ *and* $\hat{\alpha}$ *are bounded.*

iv) $E[m(W, \gamma_0, \theta_0)^2] < \infty$ *and* $\int ||m(w, \hat{\gamma}_\ell, \theta_0) - m(w, \gamma_0, \theta_0)||^2 F_0(dw) \xrightarrow{p} 0$

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Extension: Multi Players Game

$$\psi(w_i;\theta,\eta) = m(W;\theta_0,\eta_0) + \phi(W;\theta_0,\alpha_0,\eta_0) \tag{10}$$

$$\text{where } m(W;\theta_0,\eta_0) = \underbrace{\mu_z}_{=z_i - x_i'\mu_x} \left[a_i - \Lambda(\gamma_{-i}\theta_\gamma, X\beta)\right],$$

$$\phi(W,\theta_0,\alpha_0,\eta_0) = -\sum_{a_{-i}} \prod_{s\neq i,-i} \underbrace{\alpha_{-i}}_{=\mu_z\Lambda(\cdot)(1-\Lambda(\cdot))(1-\gamma_s)\theta_\gamma} \left[a_{-i} - \gamma_{-i}\right]$$

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Monte Carlo Simulation Result

Table 6: Simulation Results
(Sample Size: 2,000, Dimension of X: 203)

		Oracle				Naive				Orthogonal			
	DGP (1)	Estimates (2)	Bias(%) (3)	CI (4)	RMSE (5)	Estimates (6)	Bias(%) (7)	CI (8)	RMSE (9)	Estimates (10)	Bias(%) (11)	CI (12)	RMSE (13)
θ_r	-1.5	-1.535	-2.325	0.934	0.211	-0.531	64.593	0.000	0.990	-1.427	4.857	0.940	0.356
		(0.209)				(0.204)				(0.349)			

Notes: Mean and Standard Deviation for 500 simulations. Column (1) represents the true value for the rival effect under the model. Columns (2)-(5) used Oracle and columns (6)-(9) used Naive plug-in estimators. Columns (10)-(13) used developed Neyman orthogonal moments. Column (2), (6), and (10) shows the mean and standard deviations for the estimated parameters. CI denotes the probability of a 95 percent confidence interval based on standard deviation from 500 simulations. RMSE denotes the root mean square error between estimated parameters and true parameters.