# Double/Debiased Machine Learning for Static Games with Incomplete Information: An Application to Pharmacy Deserts

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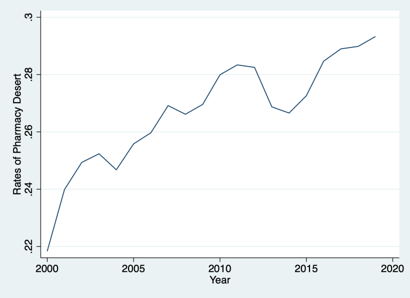
# Motivation I

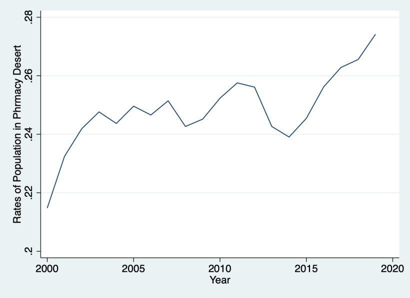
- Access to pharmacies is crucial for medication availability/adherence.
  - "Pharmacy deserts": areas with limited access to pharmacies and prescription drugs within walking distance (5-10 miles) (Amstislavski et al. (2012), Qato et al. (2014)).
- Health disparities: Minorities in rural areas are particularly vulnerable to pharmacy deserts.
  - Lower patient adherence rates, including chronic disease prescriptions and multiple pharmacological therapies, have an effect on increased patients' health (ED visits, hospitalization) Di Novi et al. (2020).
- **Stylized Facts:** In the rural town, there has been an increased trend in pharmacy deserts from 2000-2019.

# Stylized Facts

Figure 1: Trends in Pharmacy Deserts in Rural Areas Township in Midwest 2000-2019



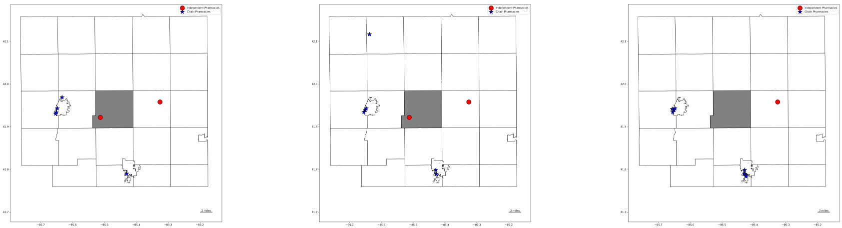

(a) Proportions of Pharmacy Desert in the U.S.




(b) Proportions of Population Pharmacy Desert in the U.S

**Note**: Samples are from township-level in the U.S. Midwest region 2000-2019. Based on the census RUCA (Rural-Urban Commuting Area Codes), I construct township-level rural areas if township areas include rural regions from RUCA. I used three years moving average for this calculation.

# Figure 2: An Example: Spatial Distribution in Independent/Chain Pharmacy




(a) Year: 2000
\# of Chain: 4

(b) Year: 2009
\# of Chain: 6

(c) Year: 2019
\# of Chain: 9

**Note**: Samples are from Nottawa township in MI, 2000-2019.

- Key Take Away:
  1. Independent pharmacies have disappeared which caused prevalence in pharmacy deserts in the U.S.
  2. Chain pharmacies are more common and dense within big towns.

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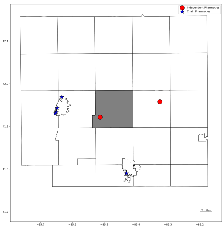

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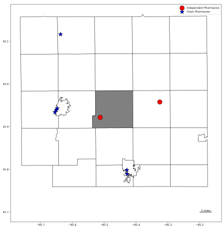
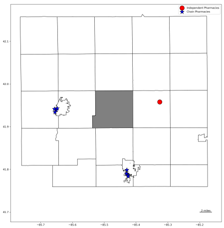


(b) Year: 2009
\# of Chain: 6



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**Note**: Samples are from Nottawa township in MI, 2000-2019.

- Key Take Away:
  1. Independent pharmacies have disappeared which caused prevalence in pharmacy deserts in the U.S.
  2. Chain pharmacies are more common and dense within big towns.
- **Research Question**: The mechanism of pharmacy desert is under-studied.

# Motivation II

- Through the lenses of empirical IO, independent pharmacies face two different competitions:
  - Endogenous competition from **rival independent pharmacies** in the same market.
  - Exogenous competition from **chain pharmacies** in more distant regions.
- **Motivation for structural model**: To decompose the two sources of competition, I estimate discrete games of incomplete information following Bajari et al. (2010).
- **Challenges**: Existing models often rely on strong assumptions about payoff-relevant market characteristics:
  1. Relevant market characteristics include low-dimensional.
  2. Functional forms are correctly specified.
- To relax these assumptions, I **develop econometric models** that incorporate **high-dimensional covariates**.

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# Preview of Results

1. **Reduced Form Analysis** (Not today)
    - Approximately, a 100% increase in chain pharmacies ($2 \rightarrow 4$ units) is associated with a decrease of 0.06 units (8.4% decrease) in independently-owned pharmacies.

# Preview of Results

1. **Reduced Form Analysis** (Not today)
   - Approximately, a 100% increase in chain pharmacies ($2 \rightarrow 4$ units) is associated with a decrease of 0.06 units (8.4% decrease) in independently-owned pharmacies.
2. **Methodology Development**
   - Neyman Orthogonal Moment/Asymptotic Theory
     - I propose a new orthogonal moment based on Bajari et al. (2010).
     - To achieve this, I combine the double machine learning (DML) literature Chernozhukov et al. (2018) and cross-sample splitting methods.

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     - To achieve this, I combine the double machine learning (DML) literature Chernozhukov et al. (2018) and cross-sample splitting methods.
     - The developed model is applicable to **a wide range of discrete games with interactions**, including entry/exit games, quality/location choices, and pricing strategies such as promotions.

# Preview of Results (Cont'd)

3. **Structural Analysis**: Applications in local pharmacy markets
   - ML performs better in the estimation of first-stage nuisance parameters, in terms of ROC-AUC (accuracy under the curve).
   - The developed methodology successfully eliminates biases induced by machine learning (ML).
   - A 100% increase (2$\rightarrow$4 units) in chain pharmacies will lead to a decrease in the probabilities of being active by:
     - 11.47% in the conventional method.
     - 8.58% in the Naive-plug-in method.
     - 21.89% in the Orthogonal moments.

# Literature Review

- Pharmacy Desert
  - Qato et al. (2014), Olson et al. (2018), Di Novi et al. (2020).
- Static Games Literature
  - Bresnahan and Reiss (1991), Berry (1992), Mazzeo (2002), Seim (2006), Jia (2008), Ellickson and Misra (2008), Holmes (2011), Nishida (2015), Ellickson and Misra (2011), Bajari et al. (2010), Bajari et al. (2013), Grieco (2014).
- Neyman/Double Machine Learning Literature (DML)
  - Neyman (1959), Chernozhukov et al. (2018), Chernozhukov et al. (2021), Chernozhukov et al. (2022b), Chernozhukov et al. (2022a).
- Influence function with Game Settings
  - Bajari et al. (2009), Semenova (2018), Adusumilli and Eckardt (2019), Nekipelov et al. (2022).

| | Data Dimensions | |
|---|---|---|
| | Low | High |
| Conventional Method | Bajari et al. (2010) | Naive-plug-in |
| Developed Neyman Orthogonal | | Orthogonal |

# Model Framework

- I closely follow standard notations in a static game setting as described in Bajari et al. (2010) and Bajari et al. (2013).
- In a static game with incomplete information settings, I consider a finite number of players $i = (1, ..., N)$ with binary choices, $J = 2$.

$$a_i = \begin{cases} 1 & \text{if Player } i\text{'s being active.} \\ 0 & \text{if Player } i\text{'s being inactive.} \end{cases} \tag{1}$$

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$$a_i = \begin{cases} 1 & \text{if Player } i\text{'s being active.} \\ 0 & \text{if Player } i\text{'s being inactive.} \end{cases} \tag{1}$$

- For simplicity, I illustrate with two players. The payoff function for being active is given by:

$$\pi_i(a_i = 1|d, x) = \theta_r \sigma_{-i}(d_{-i}, x) + \theta_d d_i + \underbrace{x\beta}_{\text{high dimensions}} + \epsilon_i(1). \tag{2}$$

where

- $a_{-i}$ denote the rival's action.
- $\sigma_{-i}(d_{-i}, x)$ denotes the probability of rival's being active.
- $d_{-i}$ denote rival specific productivity shock and $d_i$ denote player $i$ specific productivity shock.
- $x$ denotes market characteristics (common demands) to every player in the market.
- $\epsilon_i(1)$ denote i.i.d from a Type 1 Extreme Value Distribution.

# Model Framework

## Commonly Made Assumptions in IO Literature

- Assumption 1. Incomplete Information Appendix
- Assumption 2. Normalization of Outside Choice Appendix
- Assumption 3. Correct Beliefs (Rational Expectation over Rival's action) Appendix
- Assumption 4. Equilibrium Selection Appendix
- Assumption 5. Exclusion Restriction Appendix
    - Remark 1. Identification condition holds with high-dimensional controls. Appendix

# Two Step Estimation

**Step 1: Estimation of choice probabilities**:

$$\gamma_{-i} = E[a_{-i}|d_{-i}, x] \text{ for all } -i = 1, ..., N. \tag{3}$$

where

- $a_i$ denote the rival's action.
- $d_{-i}$ denote player $-i$ specific productivity shock.
- $x$ denote market characteristics (common demands) to every player in the market.

**Step 2: Recovering the Structural Parameters** With variables $z$, GMM can be constructed as:

$$\arg\min_{\theta,\beta}\ g(w;\theta,\beta) = E[z(a_i - \Lambda(\theta\hat{\gamma}_{-i}, X\hat{\beta}))]. \tag{4}$$

where $\Lambda$ denotes the logistic link function

# Two-step methods with high dimensional data

- In the two-step estimation, the **regularization/model selection bias** in $\hat{\eta} = (\hat{\gamma}_{-i}, \hat{\beta})$ could be transmitted into the second stage estimation of structural parameters $\theta$.
- To correct local mis-specification, I use the orthogonality condition:

$$||\partial_\eta E_P[\psi(w_i;\theta,\eta)][\eta - \eta_0]|| = 0 \text{ for all } \eta \in \mathcal{T}_N,$$

  where $\psi(w_i, \theta, \eta)$ denote the Neyman orthognal moment function.
- Conventional methods do not satisfy Neyman orthogonality condition:

$$\partial_{\gamma_{-i}} E[g(w_i;\theta,\eta)][\gamma_{-i} - \gamma_{-i0}] = E\left[z_i \cdot \Lambda' \cdot \left(-\theta_\gamma \sum_{a_{-i}} \prod_{s \neq i, i} (1-\gamma_s)\right) \cdot (\gamma_{-i} - \gamma_{-i0})\right] \neq 0$$

$$\text{for } -i = 1, \ldots i-1, i+1, \ldots n.$$

# Asymptotic Analysis: Neyman Orthogonal Moment

- I give the new orthogonal moment function:

$$\psi(w_i, \theta, \eta) = (z_i - x'\mu_x)[a_i - \Lambda(\theta_\gamma \gamma_{-i}, X\beta)] - (z_i - x'\mu_x)\Lambda(\cdot)(1 - \Lambda(\cdot))\theta_\gamma[a_{-i} - \gamma_{-i}] \tag{5}$$

$$\text{where } \mu_x = \text{argmin}_{\mu_x} E[f_i(z_i - x'\mu_x)]^2$$

## Lemma 1

$$E\left[\psi(W; \theta_0, \eta_0)\right] = 0$$

## Theorem 2

*The moment (5) obeys Neyman orthogonality moment conditions at $(\theta_0, \eta_0)$ with respective to the nuisance realization set $\mathcal{T}_N \subset \mathcal{T}$ if Lemma 1 holds and*

$$||\partial_\eta E_P[\psi(W; \theta_0, \eta_0)][\eta - \eta_0]|| = 0 \text{ for all } \eta \in \mathcal{T}_N,$$

# Estimation Procedures

## Algorithm

1. Let size $n = N/K$. Use a K-fold random partition $(I_k)_{k=1}^{K}$ Also, for each $k \in K = \{1, ..., K\}$, define $I_k^c$ as the complement of $I_k$, i.e., the set of indices 1,...,N excluding those in $I_k$.
2. For each $k \in K$, estimate nuisance parameters $\hat{\eta}_k$ using $I_k^c$

$$\hat{\eta}_k = \hat{\eta}((W_i)_{i \in I_k^c}), \hat{\eta}_k = (\hat{\gamma}_{-ik}, \hat{\beta}_k, \hat{\mu}_{zk}).$$

   1. Obtain $\hat{\gamma}_{-ik}$ from Logit Lasso
   2. Obtain $\hat{\beta}_k$ from Logit Lasso
   3. Estimate $\hat{\mu}_{zk}$ from the Lasso estimator

3. Construct the orthogonal moment estimator, then obtain $\hat{\theta}$ as

$$\frac{1}{K} \sum_{k=1}^{K} \frac{1}{n} \sum_{i \in I_k} \psi(w_i, \theta, \eta_k)) = 0$$

# Asymptotic Analysis

## Theorem 3

*Suppose that assumption 1-5, and assumption 6 holds. For $V = M^{-1}E[\psi_0(W)\psi_0(W)']M^{-1}$, the DML static game estimators (5) obeys Neyman Orthogonal Moments and it follows:*

$$\sqrt{N}(\hat{\theta} - \theta_0) \to N(0, V), \ \hat{V} \xrightarrow{p} V.$$

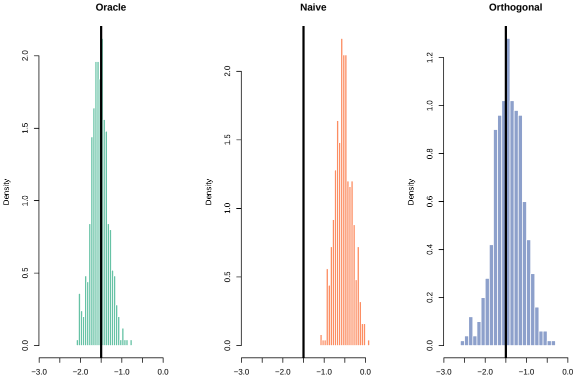
$$\text{where } \hat{V} = \Big(\frac{1}{K}\sum_k^K E[M]\Big)^{-1}\frac{1}{K}\sum_k^K E[\psi^2(w, \hat{\theta}, \hat{\eta}_k)]\Big(\frac{1}{K}\sum_k^K E[M]\Big)^{-1}$$

Appendix: Assumption 6 Appendix: Multiplayers

# Monte Carlo Simulation

Figure 3: The distribution of the estimated structural parameters from simulation (Lasso/Logit Lasso)

Sample Size: 2,000, Dimension of $X$: 203




*Notes*: The estimated effect of rival coefficients are based on 500 simulations. The true value of the rival effects is $\theta_0 = -1.5$. The Oracle method uses low-dimensional relevant covariates, while the naive plug-in method uses high-dimensional covariates without correcting for biases. The orthogonal method also uses high-dimensional covariates like the naive plug-in method, but it corrects for biases using the proposed Neyman orthogonal method.

# Structural Analysis

- Scope of Analysis
  - Rural townships in the U.S. Midwest region in 1997-2021.
  - I focus on 2000-2019 time periods.
  - I limit the township with at least one independent pharmacy during 1997-2021.
  - Two-player game between independent pharmacies with external competition from chain pharmacies.[1]

[1] In the data, 99.2% of markets have at the most two independent pharmacies.

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- Data
  - Panel data of Pharmacies Entry/Exit: Data Axle (1997-2020)
  - Socio-Economic Characteristics: Population, Per capita Income, Race composition, Age distributions (over 65), Household composition, Education, Vehicle Availability. (Source: Census 2000, 2010 )
  - Number of Physician Offices (county) (Source: ZBP)
  - Summary Statistics in Appendix
-

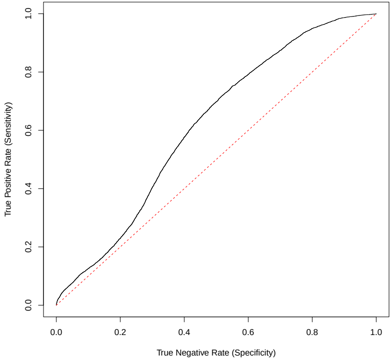
Table 1: Comparison of Methods

| Method | Covariates | Dimensions for covariates |
|---|---|---|
| Bajari et al. (2010) | Soci-economic variables | 33 |
| Naive-plug in | Soci-economic variables + their interactions | 497 |
| Orthogonal moment with cross-fitting | Soci-economic variables + their interactions | 497 |

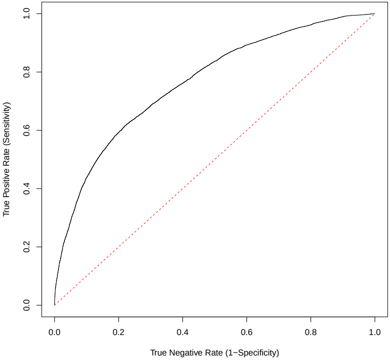
[1] In the data, 99.2% of markets have at the most two independent pharmacies.

# Structural Analysis (Cont'd)

Figure 4: ROC Curve in First Stage Estimation




(a) Conventional methods (Logit)
AUC: 0.6151




(b) Logit Lasso (CV)
AUC: 0.7660

**Note**: I use 10 K-fold cross-fitting to learn penalty terms $\lambda_1$ of logit lasso. AUC denotes the area under the accuracy

# Structural Analysis (Cont'd)

Table 2: Results from the Structural Model

| Parameters | Variables | Bajari et al. (2010) | Naive plug-in | Neyman Orthogonal Moments |
|---|---|---|---|---|
| $\theta_i$ | Pr. of rival independent pharmacies | -3.311 | -2.674 | -3.826 |
| | | (0.040) | (0.040) | (0.074) |
| $\theta_c$ | log(No. of chain pharmacies | -0.457 | -0.403 | -1.027 |
| | within 20 miles) | (0.016) | (0.016) | (0.101) |
| Observations | | 31,280 | 31,280 | 31,280 |
| No. of Socio-Economic Variables | | 33 | 497 | 497 |
| Interaction | | No | Yes | Yes |
| Counties FE | | Yes | Yes | Yes |
| Year FE | | Yes | Yes | Yes |

*Notes*: Standard errors are in parenthesis.

- Interpretation of Maginal Effects: A 100% increase (from 2 to 4 units) in chain pharmacies will lead to a decrease in the probability of being active
  - by 11.47 percent: Bajari et al. (2010)
  - by 8.58 percent: Naive-plug-in method.
  - by 21.89 percent: Orthogonal moments.
- Ridge estimators produce similar results.

# Conclusion

- Summary
  - Reduced form evidence suggests that entry of chain pharmacies is negatively correlated with the number of local pharmacies.
  - I established asymptotic theory/MCMC results.
  - Empirical application illustrated that Neyman Orthogonal moment corrected ML biases from the first stage.
- Future Plan
  - Report (more) goodness of Fit
  - Conduct Robustness check
  - Implement Counterfactuals

Thank you so much for attending my presentation!

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I greatly appreciate your valuable comments and feedback.

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# Preliminary Analysis (Cont'd)

Table 3: The number of local pharmacies and chain pharmacies

| | (1)<br># of Independent Pharmacies | (2)<br># of Independent Pharmacies | (3)<br># of Independent Pharmacies |
|---|---|---|---|
| log(# of Chain Pharmacies within 20 miles) | -0.080** | -0.124** | -0.120** |
| | (0.0210) | (0.026) | (0.027) |
| County FE | No | YES | Yes |
| Year FE | YES | No | Yes |
| Demographic controls | YES | YES | Yes |
| Observations | 15,480 | 15,480 | 15,480 |
| # of Counties | 459 | 459 | 459 |
| Adj. $R^2$ | 0.078 | 0.3528 | 0.3541 |
| Outcome mean | 0.710 | 0.710 | 0.710 |

**Note** Estimates are from fixed effects regression of the number of independent pharmacies in the township on the log( number of chain/supermarket pharmacies) in the county $c$ and year $t$. Column (1)-(3) includes the total observations. Column (2) includes county-level fixed effects. Column (3) includes demographic controls from Census, County FE, and year FE. Standard errors are clustered at the county level. Significance levels are denoted by + p$<$0.10, * p$<$0.05, ** p$<$0.05

- A 100 % change in the number of chain pharmacies is associated with a decrease in the number of independent pharmacies of -0.06 units.

# Preliminary Analysis (Contd)

Table 4: Distribution (of the Number) of Active Local Pharmacies in 2000-2019 by Market Size (%)

| | Active Local Pharmacies | | |
|---|---|---|---|
| | 0 | 1 | 2+ |
| Total Pop. $< 1{,}500$ | | | |
| No Chain | 28.03 | 67.77 | 4.20 |
| Chain $\geq 1$ | 48.52 | 49.50 | 1.98 |
| Total Pop. $\geq 1{,}500$ | | | |
| No Chain | 17.96 | 72.21 | 9.84 |
| Chain $\geq 1$ | 34.20 | 60.26 | 5.53 |

**Note**: Samples are from RUCA (Rural-Urban Commuting Area Codes). Total Population of 1,500 is the median of the rural area population in the mid-west region.

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# Preliminary Analysis (Cont'd)

Table 5: Descriptive Statistics

| Variables | 2000 Census Obs. | Mean | Std. dev. | Median | Min. | Max | 2010 Census Obs. | Mean | Std. dev. | Median | Min. | Max |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Market characteristics** | | | | | | | | | | | | |
| Population | 774 | 1703.68 | 1191.67 | 1447.5 | 33 | 14388 | 774 | 1685.87 | 1231.71 | 1408.5 | 39 | 14738 |
| Population density (sq, miles) | 774 | 548.40 | 660.79 | 102.63 | 1.07 | 3166.62 | 774 | 536.07 | 650.60 | 106.13 | 1.26 | 3434.14 |
| Per capita income | 774 | 16716.43 | 2595.45 | 16631 | 8360 | 35705 | 774 | 21244.63 | 3906.68 | 21100.5 | 10063 | 42282 |
| Proportion of No vehicles in HH | 774 | 0.073 | 0.048 | 0.068 | 0 | 0.54 | 774 | 0.063 | 0.054 | 0.054 | 0 | 0.72 |
| Proportion of Age over 65 | 774 | 0.22 | 0.065 | 0.22 | 0.05 | 0.48 | 774 | 0.22 | 0.06 | 0.21 | 0.05 | 0.49 |
| **Store characteristics** | | | | | | | | | | | | |
| Independent pharmacy | 774 | 0.81 | 0.51 | 1 | 0 | 2 | 774 | 0.70 | 0.53 | 1 | 0 | 2 |
| Employee size | 313 | 5.52 | 3.78 | 5 | 0 | 35 | 258 | 6.41 | 4.03 | 5 | 1 | 25 |

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- Shifters: Number of employees three years ago. (Source: Data Axle)

# Model Framework

## Assumption 1 (Incomplete information)

1. *The error terms are independently and identically distributed across actions and players, and are drawn from a Type 1 Extreme Value Distribution.*
2. *The structural shock affecting the error term is privately observed by each player.*
3. *The additive components of the error term are not observed by analysts.*
4. *The state variable $d, x$ is observable to both players in the same market and analysts.*

# Model Framework

- Having private information $\epsilon$ and observable state variable $d$, the payoff of player $i$ can be expressed as

$$\pi_i(a, d, \epsilon_i; \theta) = \pi_i(a_i, a_{-i}, d; \theta) + \epsilon_i(a_i). \tag{6}$$

where flow utility $\pi_i(a_i, a_{-i}, d; \theta)$ depends on agent's discrete choice actions $a_i$, other agents' action $a_i$, and the state variable $d$.

**Assumption 2 (Normalization of Outside Choice)**

$$\textit{For all } a_{-i} \in A_{-i} \textit{ and all } d,\ \pi_i(a_i = 0, a_{-i}, d) = 0. \tag{7}$$

- By setting the flow utility of being inactive to zero, the utility of being active becomes relative to being inactive.

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## Assumption 3 (Correct Beliefs)

*Coupled with incomplete information structure, players form correct beliefs over rival's actions.*

- With private information assumptions 1 and correct beliefs assumption 3, I can express a choice-specific value function:

$$\pi_i(a_i = 1, d) = \sum_{a_{-i} \in A_{-i}} \sigma_{-i}(a_{-i}|d)\pi_i(a_i = 1, a_{-i}, d) \text{ for all } i = 1, ...N. \tag{8}$$

$$\text{where } \sigma_{-i}(a_{-i}|d) = \prod_{s \neq i} \sigma_s(a_s|d)$$

- A decision rule choice rule can be expressed as:

$a_i^* = 1$ if and only if

$$\underbrace{\pi(a_i = 1, d)}_{=\sum_{a_{-i} \in A_{-i}} \sigma_{-i}(a_{-i}|d)\pi_i(a_i=1,a_{-i},d)} + \epsilon_i(1) > \underbrace{\pi(a_i = 0, d)}_{=0} + \epsilon_i(0).$$

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# Model Framework

Fixing state variable $d$, $\sigma_i(a_i|d)$ will be the solution to the system of $N$ equations:

$$\sigma_1(a_1 = 1|d) = \frac{\exp\Big(\sum_{a_{-1}\in A_{-1}} \sigma_{-1}(a_{-1}|d)\pi_i(a_1 = 1, a_{-1}, d)\Big)}{1 + \exp\Big(\sum_{a_{-1}\in A_{-1}} \sigma_{-1}(a_{-1}|d)\pi_i(a_1 = 1, a_{-1}, d)\Big)} \tag{9}$$

$$\sigma_2(a_2 = 1|d) = \frac{\exp\Big(\sum_{a_{-2}\in A_{-2}} \sigma_{-2}(a_{-2}|d)\pi_i(a_2 = 1, a_{-2}, d)\Big)}{1 + \exp\Big(\sum_{a_{-2}\in A_{-2}} \sigma_{-2}(a_{-2}|d)\pi_i(a_2 = 1, a_{-2}, d)\Big)}$$

$$\vdots$$

$$\sigma_N(a_N = 1|d) = \frac{\exp\Big(\sum_{a_{-N}\in A_{-N}} \sigma_{-N}(a_{-N}|d)\pi_i(a_N 1, a_{-N}, d)\Big)}{1 + \exp\Big(\sum_{a_{-N}\in A_{-N}} \sigma_{-N}(a_{-N}|d)\pi_i(a_N = 1, a_{-N}, d)\Big)}$$

# Model Framework

**Assumption 4 (Equilibrium Selection)**

*The data are generated by a single equilibrium from the set of possible equilibria and this observed equilibrium may vary across different markets.*

**Assumption 5 (Exclusion Restriction)**

$$\pi_i(a_i, a_{-i}, d) = \pi(a_i, a_{-i}, d_i, x).$$

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# Model Framework: Identification

## Remark 1 (Identification)

1. *Suppose that Assumption 2 and Assumption 5 are satisfied. As long as there are* $2^{N-1}$ *points in the support of conditional distribution* $d_{-i}|d_i$*, then the necessary condition holds.*
2. *Allowing high dimensional market characteristics* $d$ *does not change necessary conditions for identification.*

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# Characterization of Neyman Orthogonal Moment

$$\frac{\partial\psi(w_i, \theta_0; \gamma_{-i0}, \beta, \mu_{z0})}{\partial\beta} = E\left[-\mu_z\Lambda' x \cdot (\beta - \beta_0)\right] = -E[u_i f_i x_i \cdot (\beta - \beta_0)] = 0 \ (\because \text{ equation ??})$$

$$\begin{aligned}\frac{\partial\psi(w_i, \theta_0; \gamma_{-i}, \beta_0, \mu_{z0})}{\partial\gamma_{-i}} &= E[(-\mu_z\Lambda'\theta_\gamma - \alpha) \cdot (\gamma_{-i} - \gamma_{-i0})] \\ &= E[(-\mu_z\Lambda'\theta_\gamma + \mu_z\Lambda'\theta_\gamma, \mu_{z0}) \cdot (\gamma_{-i} - \gamma_{-i0})] = 0. \\ \frac{\partial\psi(w_i, \theta_0; \gamma_{-i}, \beta_0, \mu_z)}{\partial\mu_z} &= E[a_i - \Lambda(\theta_\gamma\gamma_{-i0}x\beta_0) \cdot (\mu_z - \mu_{z0}] = 0.\end{aligned}$$

$Q.E.D.$ Back

# Asymptotic Analysis

## Assumption 6 (Regularity Condition)

*i*) $W_i = (A_i, D_{-i}, D_i, X_i)$ *are bounded.*

*ii*) $M$ *is twice differentiable with uniformly bounded derivatives bounded from zero.*

*iii*) $E[\{Y - \hat{\gamma}(X)\}^2 | X]$ *and* $\hat{\alpha}$ *are bounded.*

*iv*) $E[m(W, \gamma_0, \theta_0)^2] < \infty$ *and* $\int ||m(w, \hat{\gamma}_\ell, \theta_0) - m(w, \gamma_0, \theta_0)||^2 F_0(dw) \xrightarrow{p} 0$

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# Extension: Multi Players Game

$$\psi(w_i;\theta,\eta) = m(W;\theta_0,\eta_0) + \phi(W;\theta_0,\alpha_0,\eta_0) \tag{10}$$

$$\text{where } m(W;\theta_0,\eta_0) = \underbrace{\mu_z}_{=z_i - x_i'\mu_x} \left[a_i - \Lambda(\gamma_{-i}\theta_\gamma, X\beta)\right],$$

$$\phi(W,\theta_0,\alpha_0,\eta_0) = -\sum_{a_{-i}} \prod_{s\neq i,-i} \underbrace{\alpha_{-i}}_{=\mu_z\Lambda(\cdot)(1-\Lambda(\cdot))(1-\gamma_s)\theta_\gamma} [a_{-i} - \gamma_{-i}]$$

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# Monte Carlo Simulation Result

Table 6: Simulation Results
(Sample Size: 2,000, Dimension of $X$: 203)

| | | Oracle | | | | Naive | | | | Orthogonal | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DGP (1) | Estimates (2) | Bias(%) (3) | CI (4) | RMSE (5) | Estimates (6) | Bias(%) (7) | CI (8) | RMSE (9) | Estimates (10) | Bias(%) (11) | CI (12) | RMSE (13) |
| $\theta_r$ | -1.5 | -1.535 | -2.325 | 0.934 | 0.211 | -0.531 | 64.593 | 0.000 | 0.990 | -1.427 | 4.857 | 0.940 | 0.356 |
| | | (0.209) | | | | (0.204) | | | | (0.349) | | | |

*Notes:* Mean and Standard Deviation for 500 simulations. Column (1) represents the true value for the rival effect under the model. Columns (2)-(5) used Oracle and columns (6)-(9) used Naive plug-in estimators. Columns (10)-(13) used developed Neyman orthogonal moments. Column (2), (6), and (10) shows the mean and standard deviations for the estimated parameters. CI denotes the probability of a 95 percent confidence interval based on standard deviation from 500 simulations. RMSE denotes the root mean square error between estimated parameters and true parameters.