Benefits of FDI subsidies: The role of funding sources

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Abstract

Using a two-country model with heterogeneous firms, we show that the optimal level and welfare gains of FDI subsidies critically depend on how they are funded. In a setting that resembles tax distortions in emerging markets, we compare the effects of distortionary taxes that are imposed to fund FDI subsidies and examine their cross-country spillovers. We find that the optimal level of FDI subsidies and the associated welfare gains are much lower than those for non-distortionary taxes. FDI subsidies funded by distortionary taxes are also found to be beggar-thy-neighbor, although they generate positive cross-country spillovers if funded by non-distortionary taxes.

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1 Introduction

Many emerging markets provide tax and other incentives to attract foreign direct investment (FDI), and there has been a dramatic increase in FDI flows into these economies in the past three decades. Although subsidizing FDI has become an increasingly common practice, it is not immediately apparent that such policies will raise the welfare of host countries. In this paper, we examine in a two-country model with heterogeneous firms the welfare effect of FDI subsidies and their cross-country spillovers under different tax schemes to fund these subsidies.

Host countries treat FDI favorably due to its various benefits. One benefit is the consumption gain discussed in Chor (2009): FDI can reduce the price of consumption goods by shifting the production of previously imported goods to host countries, which saves international trade costs. In a model with heterogeneous firms, Chor (2009) shows that it is optimal to subsidize FDI through a tax on labor income.[1] FDI is also believed to boost economic growth in host countries by directly introducing new technologies and/or managerial skills. In addition, empirical studies document convincing evidence of technology spillovers from FDI to local firms through technology diffusion, labor turnover, and many other channels such as promoting competition among local firms.[2] FDI could also play an additional role, especially in emerging markets, by providing liquidity to credit-constrained firms, which would promote investment and trade in host countries (e.g., Alquist, Berman, Mukherjee, and Tesar (2019) and Manova, Wei, and Zhang (2015)). If such externalities are not fully captured in investment returns of multinational companies (MNCs), it could be socially optimal for host countries to subsidize FDI firms.

[1]For examples of other studies on the welfare effect of FDI subsidies, see Haaland and Wooton (1999), Pennings (2005), and Fumagalli (2003), among others.

[2]See Grossman and Helpman (1995) for a survey on articles about FDI and technology spillovers. Recent studies on this topic include Javorcik (2004), Yasar and Paul (2007), Keller and Yeaple (2009), Alfaro and Chen (2018), and Bao and Chen (2018), among others.

In this paper, we focus on FDI's benefit of consumption gain that is discussed in Chor (2009), as the over welfare effect of other benefits of FDI on host countries remains a highly debatable issue. For instance, Aitken and Harrison (1999) document that FDI has a negative spillover effect on the productivity of domestic firms, although it has a positive effect on FDI firms in the firm-level data of Venezuela. The two effects are almost canceled out, leaving a very small overall effect. Alfaro, Chanda, Kalemli-Ozcan, and Sayek (2004) document that the role of FDI in contributing to economic growth depends on the development of financial markets in host countries: FDI can significantly benefit countries with well-developed financial markets, although the results are ambiguous in other countries.[3] Using macro-level data, Borensztein, De Gregorio, and Lee (1998) and Carkovic and Levine (2005) find little evidence that FDI has a positive effect on host country's economic growth.

Our paper complements the literature by examining the welfare costs of funding FDI subsidies. Our main point is remarkably simple but has been overlooked in the literature so far. The distortionary effect of funding FDI subsidies is surprisingly underinvestigated, although both empirical and theoretical studies have been done on other costs and factors that impede positive spillovers of FDI to the productivity and economic growth of host countries.[4] Governments must finance FDI subsidies from certain tax income, which may be distortionary. This issue is particularly pronounced in emerging markets as these countries in general have less-efficient tax systems than advanced economies.[5] Obviously, distortionary taxes will reduce the welfare gain of attracting FDI. However, it is critical to examine how much welfare gain declines in the host country and the cross-country spillover effect of FDI subsidies when they are funded by distortionary taxes. We fill this gap in the literature by

[3] Wang and Wang (2015) and Alquist, Berman, Mukherjee, and Tesar (2019) find evidence that many foreign mergers and acquisitions in emerging markets are driven by multinational companies' financial advantages rather than advantaged technologies, which is consistent with Alfaro, Chanda, Kalemli-Ozcan, and Sayek (2004).

[4] In addition, FDI subsidies face implementation challenges as studied in Janeba (2002) and Janeba (2004).

[5] For instance, see Tanzi and Zee (2000).

studying optimal FDI subsidies in a model with heterogeneous firms and different types of taxes to fund these subsidies. We find in our model that the welfare gains under distortionary taxes are usually less than 15% of that under the non-distortionary tax. In addition, the cross-country spillover can turn negative when the funding of FDI subsidies is distortionary, which may cause retaliations from other countries if a country provides generous subsidies to attract FDI.

Chor (2009) abstracts from tax distortions by assuming that FDI subsidies are funded by a lump sum labor income tax. As shown in Tanzi and Zee (2000), income taxes, which are generally less distortionary than other taxes, only account for a small fraction of total government revenues in developing countries. Income taxes are difficult to collect in these countries as most workers are employed in agriculture or small, informal enterprises. Emerging markets usually have to rely heavily on consumption and corporate taxes, or even tariffs in some countries, for government revenues. For instance, taxes on corporate revenues and profits in Chile and Columbia accounted for 22.1% and 25.5% of total government revenues in 2017. Consumption taxes are also more distortionary in emerging markets than advanced economies. The effective consumption tax rates in emerging markets usually change substantially across different goods and services as transactions of many goods and services are "off the books," while advanced economies display greater similarity of tax rates across goods and services, which allows consumers' buying decisions to be based on less distorted market prices.

We examine welfare effects of the above tax distortions on FDI subsidies in a two-country (Home and Foreign) model with heterogeneous firms. Following Chor (2009), FDI in our model increases host-country consumption by saving international trade costs through production relocation. The Home government provides FDI subsidies and fund the subsidies through multiple sources of distortionary taxes: the corporate revenue tax (equivalent to the value added tax (VAT) in our model), consumption tax, and labor income tax.[6] To capture tax distortions in emerging markets, we assume that the consumption and corporate revenue taxes are imposed on differentiated goods, not homogeneous goods, which induces price distortions between homogeneous and differentiated goods in the model. Firms in each country are heterogeneous in productivity and can decide whether they want to export to the foreign country or open overseas subsidiaries to serve the foreign market. Firms have to pay transportation costs if they export or alternatively, pay additional fixed and variable production costs if they decide to open foreign subsidiaries.

We show that the optimal FDI subsidy rate and the associated welfare gains are much lower after taking into account tax distortions. Consumption and corporate revenue taxes negatively affect Home welfare through their distortionary effects on the variety of goods, monopolistic prices, and average firm productivity. The distortionary taxes reduce available product varieties, raise product prices, and decrease the average productivity of firms. Among the three taxes, we find the consumption tax is the most distortionary. The optimal subsidy rate decreases from 26% to only 8% if the funding source of FDI subsidies changes from non-distortionary labor income taxes to consumption taxes. Accordingly, the welfare gains for the Home country decrease from over 0.7% to less than 0.05%. We find similar results for the corporate revenue tax, with the optimal FDI subsidy rate being about 10% and the welfare gains being slightly above 0.05%.

We also find that Home FDI subsidies funded by distortionary consumption and corporate revenue taxes reduce Foreign welfare, although the cross-country spillovers are positive for FDI subsidies funded by non-distortionary labor income taxes. When Home FDI subsidies are funded by non-distortionary labor income tax, it increases the *ex ante* profits of Foreign

[6] In the data, these three taxes together account for over 90% of government revenues in many emerging markets. Following Chor (2009), we assume a passive Foreign government. This simplification allows us to derive analytical results, but it also abstracts from important issues such as the strategic competition between governments. We leave those issues for future study.

firms and encourages more firms to enter. As a result, Foreign households become better off as the variety of Foreign products and the average productivity of Foreign firms increase. However, under distortionary taxes, the varieties of both Home and Foreign goods that are available to Foreign households decrease, inducing negative welfare spillovers to the Foreign country.

In one exercise, we calibrate the model to match the composition of government revenues in emerging markets assuming that FDI subsidies are jointly funded by labor income taxes, consumption taxes, and corporate revenue taxes. The optimal FDI subsidy rate does not deviate substantially from the one under non-distortionary labor income tax. However, the welfare gains of FDI subsidies are much lower for both the Home country and the world due to the distortionary effects of the taxes that are collected to fund such subsidies. It highlights the importance of recognizing the sources of funding in evaluating the effects of FDI subsidies. We also conduct several robustness checks on the format of utility functions and the level of home bias in consumption. Our results hold well qualitatively. The welfare gains of FDI subsidies become even smaller under more general utility functions such as CES or under higher home bias in consumption.

Our paper is not trying to deny the benefits of FDI and the policies designed to attract FDI. We acknowledge that our model does not include many other benefits of FDI that are studied in the literature. The purpose of our paper is to remind emerging markets of the potential distortionary effects of funding FDI subsidies. After taking into account such costs, the benefits of subsidizing FDI may not be as large as people expected. Specifically, if FDI subsidies are beggar-thy-neighbor, the retaliation from foreign countries could make such policies counterproductive.

The remainder of the paper is arranged as follows. Section 2 reviews the related literature and discusses our contributions. Section 3 presents a two-country model with heterogeneous firms and MNCs, from which we derive our analytical results. Section 4 shows our model's numerical results and robustness checks. Then, concluding remarks follow in section 5.

2 Literature Review

In the 1990s, emerging markets began to offer various financial incentives to attract FDI, and this practice has become increasingly common as countries compete for the investment of MNCs. According to the *Global Investment Competitive Report (2017—2018)* of the World Bank, "46 percent of developing countries introduced new incentives or made existing incentives more generous for at least one sector between 2009 and 2015."

The theoretical motive for FDI subsidies by host countries is to internalize the externalities of foreign investment such as spillovers of foreign technology and skills to local firms. Local firms may be able to improve their productivity as a result of forward or backward linkages with FDI firms. They may also learn from their FDI competitors either by imitating their technologies or hiring workers trained by FDI firms. For instance, using micro data from the Indonesian Census of Manufacturing, Arnold and Javorcik (2009) find that foreign acquisitions lead to significant productivity improvements in the acquired plants. Javorcik (2004) documents that FDI has a consistently positive spillover effect on productivity of the domestic firms that supply inputs to FDI firms (backward linkages). Newman, Rand, Talbot, and Tarp (2015) document evidence of productivity gains for the domestic firms that receive inputs from foreign-owned firms (forward linkages). Girma and Görg (2007) and Balsvik (2011) provide evidence for spillovers through labor mobility. The increase in competition due to foreign entry may also be considered as a benefit if it forces local firms to introduce new technology and work harder. Bao and Chen (2018) show that competition pressures from foreign multinationals prompt domestic firms to upgrade productivity, raise innovation, and alter product composition.[7] Such externalities are usually not fully captured

[7]Other positive findings include Gorodnichenko, Svejnar, and Terrell (2014), Yasar and Paul (2007),

in MNCs' investment returns, and it is socially optimal for local governments to subsidize FDI firms in this case.

However, the spillover of FDI to host countries remains a highly debatable issue. Many empirical studies using firm-level data from developing countries have often failed to detect positive spillovers, especially for horizontal FDI. These negative findings include Haddad and Harrison (1993) for Morocco; Aitken and Harrison (1999) for Venezuela; Djankov and Hoekman (2000) for the Czech Republic; Konings (2001) for Bulgaria, Romania, and Poland; and Hu and Jefferson (2002) and Hale and Long (2011) for China. Foreign multinationals can negatively affect local firms through a market-stealing effect as discussed in Aitken and Harrison (1999), while they also have a positive agglomeration effect on local firms through channels such as knowledge spillovers, input sharing, and labor pooling. Lu, Tao, and Zhu (2017) present evidence for both of these effects and find a negative overall spillover effect of horizontal FDI on the performance of domestic firms in China. The effects of FDI on local firms and the local economy are also found to depend on various factors such as the absorptive capacity of local firms (Girma (2005) and Blalock and Gertler (2009)), the development of financial markets in host countries (Alfaro, Chanda, Kalemli-Ozcan, and Sayek (2004)) and the sources of FDI (Javorcik and Spatareanu (2011)).

Despite mixed empirical findings about FDI spillover, developing countries continue to adopt more policies to attract FDI. The discrepancy between government policy and academic research calls for more welfare analysis on FDI subsidies based on the FDI benefits and costs that researchers agree upon. Relative to technology spillovers, a less controversial benefit of FDI to host countries is the saving of international trade costs if exporters switch production to its market countries. Through production reallocation, FDI firms can lower the prices of previously traded goods and increase host countries' consumption.[8] Under this

Guadalupe, Kuzmina, and Thomas (2012), and Keller and Yeaple (2009), among others.

[8]Job creation is another often-cited benefit of FDI. See Javorcik (2015) for a review on related studies.

framework, Chor (2009) analyzes the welfare effects of FDI subsidies when firms are heterogeneous in their productivity levels. He finds that FDI subsidies can successfully attract the most-productive exporters to switch production to host countries, resulting in a net welfare gain if the subsidies are funded by a non-distortionary tax on labor incomes. Chor (2009) also shows that a subsidy to variable production costs of FDI yields higher welfare gains than a subsidy to fixed costs of FDI, as the former can partially correct the inefficiency caused by FDI firms' pricing power.

We contribute to this line of research by analyzing the effects of distortionary taxes that fund FDI subsidies. Many emerging markets face some formidable challenges in establishing effective and efficient tax systems. As discussed in the review article by Tanzi and Zee (2000), developing countries usually have to rely heavily on distortionary taxes such as corporation taxes and even tariffs to raise government revenues, while personal income taxes are the most important government revenue source for developed countries. For instance, OECD data show that less than 15% of government revenues were from taxes on individual incomes and profits in developing countries in 2017, compared to over 30% in advanced economies. Consumption taxes are also more distortionary in emerging markets than advanced economies because of the large role played by informal activities and limited reporting requirements. As a result, effective consumption tax rates can vary substantially across different products and services, creating price distortions in the economy. Given the tax systems in emerging markets, it could be very welfare costly for these countries to fund FDI subsidies. It is critical to take into account the costs of funding FDI subsidies through distortionary taxes when we examine the welfare effects of such subsidies.

3 Model and Analytical Results

This section presents our benchmark two-country model with heterogeneous firms and the welfare analysis based on this model.

3.1 Model Description

Figure 1 displays the structure of our two-country model. The two countries (Home and Foreign) are mostly symmetric, and we focus on the Home country in the model description. Variables of Foreign counterparts are indicated by an asterisk. We only describe the model briefly and leave the details to the online appendix. Following Chor (2009), our model makes several simplifications to derive analytical results. A more general setup with numerical solutions is also considered later.

3.1.1 Household

The representative Home household maximizes its utility subject to the budget constraint:

$$\max \quad V = C_0 + \tfrac{1}{\theta}\left(C^H\right)^\theta + \tfrac{1}{\theta}\left(C^F\right)^\theta \quad \text{subject to}$$

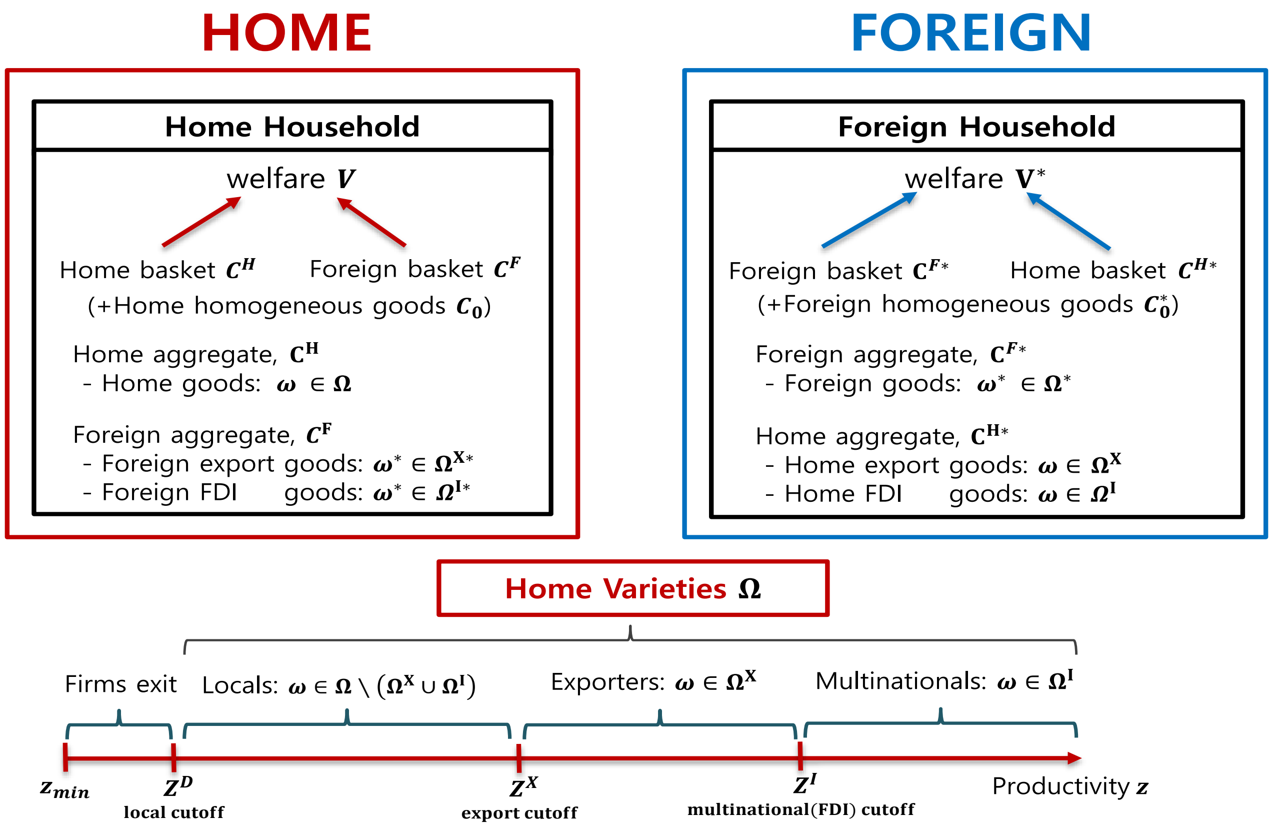
$$\begin{bmatrix} & C_0 + \int_{\omega\in\Omega}(1+\tau_C)p^D(\omega)y^D(\omega)d\omega \\ + & \int_{\omega^*\in\Omega^{X*}}(1+\tau_C)p^X(\omega^*)y^X(\omega^*)d\omega^* + \int_{\omega^*\in\Omega^{I*}}(1+\tau_C)p^I(\omega^*)y^I(\omega^*)d\omega^* \end{bmatrix} = WL(1-\tau_L), \tag{1}$$

where C_0 is non-tradable homogeneous goods that serve as the numeraire.[9] The homogeneous good is produced from labor in a perfectly competitive industry with constant returns to scale.[10]

[9] In equations (1) and (2), Ω denotes the set of products produced by all Home firms; Ω^{X*} represents the set of products imported from Foreign exporters; Ω^{I*} stands for the set of products produced by FDI affiliates of Foreign multinationals.

[10] We normalize the price of non-tradable homogeneous goods to be one, which implies $W = 1$ under constant returns to scale in production.

Figure 1: Model Structure



C^H is a CES aggregate of differentiated Home goods, $y^D(\omega)$; C^F is a CES aggregate of differentiated goods imported from Foreign country, $y^X(\omega^*)$, and differentiated goods produced by Foreign FDI firms in the Home country, $y^I(\omega^*)$. They are given by

$$\begin{aligned} C^H &= \left(\int_{\omega\in\Omega} \left[y^D(\omega)\right]^\rho d\omega\right)^{\frac{1}{\rho}}, \\ C^F &= \left(\int_{\omega^*\in\Omega^{X*}} \left[y^X(\omega^*)\right]^\rho d\omega^* + \int_{\omega^*\in\Omega^{I*}} \left[y^I(\omega^*)\right]^\rho d\omega^*\right)^{\frac{1}{\rho}}, \end{aligned} \tag{2}$$

where $0 < \rho < 1$ and $\frac{1}{1-\rho} \equiv \sigma$ is the elasticity of substitution among differentiated goods. A larger ρ implies higher substitutability among differentiated goods and less monopoly power for firms.

τ_C is the consumption tax rate for differentiated goods. It captures the consumption tax distortions that are prevalent in emerging markets. τ_L is the tax rate on labor incomes, and W and L are wage and labor endowments, respectively. Following Chor (2009), we assume that household preference is separable among Home homogeneous goods, Home differentiated goods, and Foreign differentiated goods. In addition, the utility is linear in homogeneous goods. Under this preference and exogenous wage and labor endowments in the model, the labor income tax is equivalent to a non-distortionary lump-sum tax. This simplification allows us to derive analytical solutions that illustrate underlying mechanisms of our results. More general model setups are considered later.

3.1.2 Heterogeneous Firms

Firms are heterogeneous in their productivity and have to incur sunk entry costs F^D (F^{D*}) to enter the Home (Foreign) market. After paying entry costs, each firm takes an independent random draw of productivity, z, from a Pareto distribution $G(z) = 1 - (z_{min})^{\eta} z^{-\eta}$. Given its productivity, the firm can choose to exit or run its business. Provided that the productivity level exceeds three different thresholds, the Home firm serves the Home country only (as a domestic firm), or serves both the Home and Foreign countries by exporting (as an exporter) or by establishing FDI affiliates (as a multinational). It is similar for Foreign firms in the Foreign country.

Local firms in Home: If a firm of productivity z chooses to serve the domestic market only, its profit maximization problem is:

$$\begin{aligned} &\pi^D(z) = \max \quad \left\{(1-\tau_R)p^D(z)y^D(z) - Wl^D(z) - Wf^D\right\} \\ s.t. \quad &\pi^D(z) \geq 0, \qquad l^D(z) = \tfrac{y^D(z)}{z}, \qquad y^D(z) = \left((1+\tau_C)p^D(z)\right)^{-\sigma}\left(C^H\right)^{\theta\sigma+1-\sigma}, \end{aligned} \tag{3}$$

where $0 \leq \tau_R < 1$ captures corporate revenue tax, which is equivalent to VAT in our model. $p^D(z)$ and $y^D(z)$ are price and output for firm z, and $l^D(z)$ is the labor hired by the firm. f^D is the fixed cost (in terms of labor) each firm has to pay for production. The first constraint is the participation constraint of the firm, and the other two constraints are the production function and the demand function of $y^D(z)$.

Exporters in Home: An exporter of productivity z earns profits by both serving the domestic market and exporting to the foreign market:

$$\pi^X(z) = \max \begin{bmatrix} & \left\{(1-\tau_R)p^{D,X}(z)y^{D,X}(z) - Wl^{D,X}(z) - Wf^D\right\} \\ + & \left\{(1-\tau_R)p^{X*}(z)y^{X*}(z) - Wl^X(z) - Wf^X\right\} \end{bmatrix} \tag{4}$$

$$\begin{aligned} s.t. \quad \pi^X(z) &\geq \pi^D(z), \\ l^{D,X}(z) &= \tfrac{y^{D,X}(z)}{z}, \qquad l^X(z) = \tau \tfrac{y^{X*}(z)}{z}, \\ y^{D,X}(z) &= \left((1+\tau_C)p^{D,X}(z)\right)^{-\sigma}\left(C^H\right)^{\theta\sigma+1-\sigma}, \\ y^{X*}(z) &= \left((1+\tau_C^*)p^{X*}(z)\right)^{-\sigma}\left(C^{H*}\right)^{\theta\sigma+1-\sigma}, \end{aligned}$$

where the Home exporter's total profits comprise of two parts: those from serving the domestic market and those from exporting to the Foreign country.[11] The first constraint is the participation condition of the exporter, and others are the production functions and demand functions. The transport cost $\tau > 1$ captures that additional labor is required for exporting. Home firms pay the fixed exporting costs f^X to establish export platforms only when exporters' profits exceed domestic firms' profits.

Multinationals in Home and Foreign: Multinational firms produce in both Home and Foreign countries. For a Home MNC of productivity z, which has affiliates in the Foreign

[11] Superscript D, X denotes the variables for the products that are made by exporters and sold in the domestic market. For instance, $p^{D,X}(z)$ is the price of the products that are made by Home exporters and sold in the Home country. Superscript X represents the variables for the exported products. For instance, $p^{X*}(z)$ is the price of Home exports to the Foreign country.

country, the profit is:

$$\pi^I(z) = \max \left[\begin{array}{ll} & \left\{(1-\tau_R)p^{D,I}(z)y^{D,I}(z) - Wl^{D,I}(z) - Wf^D\right\} \\ + & \left\{(1-\tau_R^*)p^{I*}(z)y^{I*}(z) - (1-s_V^*)W^*l^{I*}(z) - W^*f^{I*}\right\} \end{array} \right] \tag{5}$$

$$\begin{aligned} s.t.\quad \pi^I(z) &\geq \pi^X(z), \\ l^{D,I}(z) &= \tfrac{y^{D,I}(z)}{z}, \qquad l^{I*}(z) = \tfrac{y^{I*}(z)}{z}, \\ y^{D,I}(z) &= \left((1+\tau_C)p^{D,I}(z)\right)^{-\sigma}\left(C^H\right)^{\theta\sigma+1-\sigma}, \\ y^{I*}(z) &= \left((1+\tau_C^*)p^{I*}(z)\right)^{-\sigma}\left(C^{H*}\right)^{\theta\sigma+1-\sigma}, \end{aligned}$$

where the Home MNC' profits comprise of two parts: those from producing in the domestic market and those from operating in the Foreign country by FDI.[12] Note that Foreign affiliates of Home MNCs pay revenue taxes in the Foreign country at a rate of τ_R^*. To produce in the foreign country, FDI firms have to pay a fixed production cost (f^{I*} for Home MNCs in Foreign). Suppose the host country provides subsidies for FDI firms to cover part of their variable production costs.[13] For instance, many developing countries provide job creation subsidies and/or tax cuts to FDI firms. s_V^* is the subsidy rate for variable production costs provided by the Foreign country to Home MNCs. The first constraint is the participation condition for Home MNCs to conduct FDI in the Foreign country. It implies firms will run their FDI affiliates if FDI profits exceed exporting profits.

Foreign MNCs that operate their FDI affiliates in the Home country have a symmetric profit function:

$$\pi^{I*}(z^*) = \max \left[\begin{array}{ll} & \left\{(1-\tau_R^*)p^{D,I*}(z^*)y^{D,I*}(z^*) - W^*l^{D,I*}(z^*) - W^*f^{D*}\right\} \\ + & \left\{(1-\tau_R)p^I(z^*)y^I(z^*) - (1-s_V)Wl^I(z^*) - Wf^I\right\} \end{array} \right] \tag{6}$$

[12] Superscript I denotes corresponding variables for multinationals.

[13] Chor (2009) also considers a subsidy for FDI firms' fixed production costs. But he finds that the subsidies for variable production costs are quantitatively much more important than the fixed production subsidy. We find similar results and only focus on the variable production subsidy in the paper.

$$
\begin{aligned}
s.t.\quad \pi^{I*}(z^*) &\geq \pi^{X*}(z^*), \\
l^{D,I*}(z^*) &= \tfrac{y^{D,I*}(z^*)}{z^*}, \qquad l^{I}(z^*) = \tfrac{y^{I}(z^*)}{z^*}, \\
y^{D,I*}(z^*) &= \left((1+\tau_C^*)p^{D,I*}(z^*)\right)^{-\sigma}\left(C^{F*}\right)^{\theta\sigma+1-\sigma}, \\
y^{I}(z^*) &= \left((1+\tau_C)p^{I}(z^*)\right)^{-\sigma}\left(C^{F}\right)^{\theta\sigma+1-\sigma}.
\end{aligned}
$$

3.1.3 Government Budget Balance

In our welfare analysis, we assume a passive government in the Foreign country with all taxes and subsidies being set at zero. The Home government taxes household incomes, consumption, and corporate revenues. The tax revenues are used for FDI subsidies. We focus on subsidies for variable production costs (s_V) as Chor (2009) shows that such subsidies are more welfare-improving than subsidies for fixed production costs in this type of model setup.

The Home government budget balance is:

$$M^{I*}\int_{Z^{I*}}^{\infty} s_V W l^{I}(z)\tfrac{dG^*(z)}{1-G^*(Z^{I*})} = \tau_L W L + \tau_C G_C + \tau_R G_R, \tag{7}$$

where M^{I*} is the mass of Foreign MNCs operating in the Home country. The left side of the equation is the total amount of subsidies provided by the Home government to Foreign FDI firms. The right side of the equation includes all tax revenues to fund the FDI subsidies. G_C denotes the consumption tax base which is the total consumption expenditure on differentiated goods in the Home country, comprising consumption of domestic goods, imported goods, and FDI-produced goods:

$$G_C = \left(\begin{array}{ll} & M\int_{Z^D}^{\infty} p^D(z)y^D(z)\tfrac{dG(z)}{1-G(Z^D)} \\ + & M^{X*}\int_{Z^{X*}}^{Z^{I*}} p^X(z^*)y^X(z^*)\tfrac{dG^*(z^*)}{G^*(Z^{I*})-G^*(Z^{X*})} + M^{I*}\int_{Z^{I*}}^{\infty} p^I(z^*)y^I(z^*)\tfrac{dG^*(z^*)}{1-G^*(Z^{I*})} \end{array}\right), \tag{8}$$

where M and M^{X*} are the variety of Home local goods and the variety of imported goods from the Foreign country, respectively. G_R represents the corporate revenue tax base which is the total revenue of all firms operating in the Home country, including domestic local firms, domestic exporters, and FDI firms:

$$G_R = \begin{pmatrix} M \int_{Z^D}^{\infty} p^D(z) y^D(z) \frac{dG(z)}{1-G(Z^D)} \\ + \quad M^X \int_{Z^X}^{Z^I} p^{X*}(z) y^{X*}(z) \frac{dG(z)}{G(Z^I)-G(Z^X)} + M^{I*} \int_{Z^{I*}}^{\infty} p^I(z^*) y^I(z^*) \frac{dG^*(z^*)}{1-G^*(Z^{I*})} \end{pmatrix}, \quad (9)$$

where M^X is the variety of Home exports to the Foreign country.

3.2 Analytical Results

We derive demand functions and substitute them to Home's household budget constraints and utility function:

$$V = WL(1-\tau_L) + \left(\tfrac{1}{\theta} - 1\right) \left(C^H\right)^{\theta} + \left(\tfrac{1}{\theta} - 1\right) \left(C^F\right)^{\theta}, \quad (10)$$

where

$$\begin{aligned} \left(C^H\right)^{\theta} &= \int_{\omega \in \Omega} (1+\tau_C) p^D(\omega) y^D(\omega) d\omega, \\ \left(C^F\right)^{\theta} &= \int_{\omega^* \in \Omega^{X*}} (1+\tau_C) p^X(\omega^*) y^X(\omega^*) d\omega^* + \int_{\omega^* \in \Omega^{I*}} (1+\tau_C) p^I(\omega^*) y^I(\omega^*) d\omega^*. \end{aligned} \quad (11)$$

Equation (10) shows that the labor income tax decreases the consumption of Home homogeneous goods, and it has no other distortionary effect. We can further derive in equilibrium that

$$\left(C^H\right)^{\theta} = \left(\underbrace{M}_{\text{Variety}} \underbrace{\left(\frac{\sigma}{\sigma - 1} \frac{(1+\tau_C)}{(1-\tau_R)} \right)^{-(\sigma-1)}}_{\text{Effective Price}} \underbrace{\left(\widetilde{Z}^D\right)^{\sigma-1}}_{\text{Average Productivity}} \right)^{\frac{\theta}{(\sigma-1)(1-\theta)}} \quad (12)$$

where $\sigma > 1$ and $\theta \in (0, 1)$. Equation (12) shows that the consumption basket of Home goods are affected by three factors: the variety of Home goods, the effective price of the consumption basket, and the average productivity of Home firms that produce the consumption basket.[14] The utility from Home goods consumption is higher with more product varieties, lower effective prices, and higher average productivity. The effective price effect in equation (12) suggests that taxes on consumption and firm revenue amplify the price markup in Home firms' monopolistic pricing ($\frac{\sigma}{\sigma-1}$). These taxes will also affect C^H through product varieties (M) and average productivity ($\widetilde{Z}^D$). All in all, consumption and corporate revenue taxes turn out to cause more distortions than the labor income tax studied in Chor (2009).

The benefits and additional costs of providing FDI subsidies can also be seen from the consumption of Foreign goods in the Home country:

$$\left(C^F\right)^\theta = \left(\begin{array}{l} M^{X*}\left(\frac{\tau\sigma}{\sigma-1}(1+\tau_C)\right)^{-(\sigma-1)}\left(\widetilde{Z}^{X*}\right)^{\sigma-1} \\ + \underbrace{M^{I*}}_{\text{Variety}} \underbrace{\left(\frac{(1-s_V)\sigma}{\sigma-1}\frac{(1+\tau_C)}{(1-\tau_R)}\right)^{-(\sigma-1)}}_{\text{Effective Price}} \underbrace{\left(\widetilde{Z}^{I*}\right)^{\sigma-1}}_{\text{Average Productivity}} \end{array} \right)^{\frac{\theta}{(\sigma-1)(1-\theta)}} \tag{13}$$

where $\widetilde{Z}^{X*}$ and $\widetilde{Z}^{I*}$ are the average productivity levels of Foreign exporters and MNCs, respectively.[15] If τ_C and τ_R are zero as in Chor (2009), the price markup of imported goods is $\frac{\tau\sigma}{\sigma-1}$, which is higher than that of FDI goods $\frac{(1-s_V)\sigma}{\sigma-1}$ due to the trade cost $\tau > 1$ and the FDI subsidy $s_V \in(0, 1)$. Since $\frac{(1-s_V)\sigma}{\sigma-1}$ decreases with s_V, FDI subsidies will reduce the price distortions of FDI goods and also alleviate price distortions of imported goods by converting Foreign exporters to FDI firms. However, if FDI subsidies are funded by consumption and/or corporate revenue taxes, there are offsetting effects: the consumption and corporate revenue

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[14] The average productivity of Home firms is defined as $\widetilde{Z}^D \equiv \left[\int_{Z^D}^{\infty} z^{\sigma-1}\frac{dG(z)}{1-G(Z^D)}\right]^{\frac{1}{\sigma-1}} = \left[\frac{\eta}{\eta-\sigma+1}\right]^{\frac{1}{\sigma-1}} Z^D$, where Z^D is the cutoff productivity for Home local firms.

[15] The average productivity of Foreign exporters is defined as $\widetilde{Z}^{X*} \equiv \left[\int_{Z^{X*}}^{Z^{I*}} z^{\sigma-1}\frac{dG^*(z)}{G^*(Z^{I*})-G^*(Z^{X*})}\right]^{\frac{1}{\sigma-1}} = \left[\frac{\eta^*}{\eta^*-\sigma+1}\right]^{\frac{1}{\sigma-1}}\left[\frac{\left(Z^{X*}\right)^{-(\eta^*-\sigma+1)}-\left(Z^{I*}\right)^{-(\eta^*-\sigma+1)}}{(Z^{X*})^{-\eta^*}-(Z^{I*})^{-\eta^*}}\right]^{\frac{1}{\sigma-1}}$. Likewise the average productivity of Foreign MNCs is defined as $\widetilde{Z}^{I*} \equiv \left[\int_{Z^{I*}}^{\infty} z^{\sigma-1}\frac{dG^*(z)}{1-G^*(Z^{I*})}\right]^{\frac{1}{\sigma-1}} = \left[\frac{\eta^*}{\eta^*-\sigma+1}\right]^{\frac{1}{\sigma-1}} Z^{I*}$. Here Z^{X*} and Z^{I*} denote their cutoff productivity levels.

taxes will amplify the monopolistic pricing for imported and FDI goods, lowering the welfare in the Home country. In addition to their distortionary effects on C^H that we show earlier, these taxes will also negatively affect the varieties and the average productivity of Foreign goods that are consumed in the Home country.

Our numerical analysis in section 4 reveals that even when FDI subsidies are funded with distortionary consumption and corporate revenue taxes, FDI subsidies can still raise welfare in the host country, although the welfare gain is lower than for non-distortionary labor income tax. In the following propositions, we assume consumption and corporate revenue taxes are imposed only on domestic sales[16], whereas the numerical analysis takes into account the full tax bases of (8) and (9). The proposition 1 establishes that the slope of the Home welfare function when the subsidy level is zero is smaller under the distortionary taxes than under the labor income tax.

Proposition 1. ***[Home welfare under distortionary financing for FDI subsidies]*** *Suppose the FDI subsidy is financed through the balanced government budget (7) where the tax bases, G_C and G_R, are simplified as $G_C = G_R = M \int_{Z^D}^{\infty} p^D(z) y^D(z) \frac{dG(z)}{1-G(Z^D)}$. Suppose the ranking of cutoff productivity of firms is preserved: $z_{min} < Z^D < Z^X < Z^I$. The optimal FDI subsidies funded by taxes on consumption or firm revenue bring lower consumption gains to the Home country than that funded by the labor income tax.*

Chor (2009) shows that there exists a strictly positive subsidy level which maximizes Home welfare when it is financed by the labor income tax.[17] Under tax on consumption or firm revenue, the positivity of optimal subsidy is not analytically guaranteed.[18] The

[16] That is, in propositions 1 and 2, the government tax bases, (8) and (9), are reduced to $G_C = G_R = M \int_{Z^D}^{\infty} p^D(z) y^D(z) \frac{dG(z)}{1-G(Z^D)}$. We take this simplification because the full tax bases, (8) and (9), render analytical derivations infeasible under endogenous mass of firms. If we take exogenous mass of firms for the analytical analysis as in the proposition 2 of Chor (2009), the distortionary effects of consumption and corporate revenue taxes disappear and the optimal FDI subsidy trivially becomes 100%.

[17] See the proposition 3 in Chor (2009).

[18] We can show that the lower bound for the FDI subsidy under distortionary taxes is negative.

proposition 1 implies that if the government tax schemes distort market demand or pricing of firms through consumption or corporate revenue taxes, the welfare gains from FDI subsidies are lower than under the labor income tax.

We will also show from the numerical analysis in the following section that the cross-country spillover of FDI subsidies turns negative if the subsidies are funded by consumption or corporate revenue taxes, even if the spillover is positive when the subsidy is funded by the non-distortionary labor income tax. The proposition 2 states that the slope of the Foreign welfare function for all positive subsidy levels is smaller under the distortionary taxes than under the labor income tax.

Proposition 2. ***[Cross-country spillovers of FDI subsidies]*** *Suppose the FDI subsidy is financed through the balanced government budget (7) where the tax bases, G_C and G_R, are simplified as $G_C = G_R = M \int_{Z^D}^{\infty} p^D(z) y^D(z) \frac{dG(z)}{1-G(Z^D)}$. Suppose the ranking of cutoff productivity of firms is preserved: $z_{min} < Z^D < Z^X < Z^I$. Then the welfare in the Foreign country increases when Home FDI subsidies are financed by the non-distortionary labor income tax. If Home FDI subsidies are funded by taxes on consumption or firm revenues, the positive spillovers of FDI subsidies are reduced due to the adverse effect of distortionary taxes on Home exports to Foreign.*

In proposition 2, the positive spillovers of Home FDI subsidies onto Foreign under the labor income tax is the reminiscence of lemma 1 in Chor (2009). The Home FDI subsidies lead to higher ex-ante profits for Foreign firms. More Foreign firms enter in Foreign markets and more varieties of Foreign goods are available. Thus Foreign consumers are better off by the increase of Foreign-good consumption. On the other hand, if Home FDI subsidies are financed by Home distortionary taxes, it leads to not only higher prices for Home-produced goods, but also lower ex-ante profits for Home entrants and lower Home-good varieties. These adversely affect Foreign households' consumption on Home exports, reducing their welfare.

The theoretical proof of the above qualitative results are provided in the appendix A.2. In the next section, we show the quantitative significance of these findings in our numerical results and conduct robustness checks under more general settings.

4 Numerical Results and Robustness Checks

4.1 Benchmark Results

This section presents our numerical results of the optimal FDI subsidy with four different tax schemes to fund the subsidies in the Home country. In the first three scenarios, the subsidies are fully funded by the labor income tax, the consumption tax, or the corporate revenue tax (or VAT). In addition, we also consider a hybrid tax regime, in which FDI subsidies are funded by a combination of these three taxes. In this case, the model is calibrated such that the shares of these three taxes in total government revenues match the data in emerging markets. In all cases, we find that Home welfare gains become much lower and the cross-country spillover turns negative if FDI subsidies are funded by distortionary taxes relative to the non-distortionary labor income tax.

Table 1 displays the baseline parametrization. The relative weight on homogeneous goods in utility (θ) is set to 0.32 to assign 47% labor to the homogeneous goods sector as in Chor (2009). The elasticity of substitution between differentiated goods $\left(\sigma = \frac{1}{1-\rho}\right)$ is given by 3.80, implying a 36% price markup for differentiated goods as in Bernard, Eaton, Jensen, and Kortum (2003).

Labor endowment is normalized to unity. We follow the standard parameter values for the Pareto distribution: the lowest support for the productivity is 0.10 and the shape parameter is set to 3.30 following Bernard, Eaton, Jensen, and Kortum (2003). All fixed costs are parameterized to have a 50% mass of local firms, a 30% mass of exporters, and a 20% mass of multinationals in equilibrium. We take transport costs of international trade as 20% of output, which is consistent with Anderson and van Wincoop (2004). Following Chor (2009), we assume there is no government sector in the Foreign country and only the Home government grants subsidies for FDI through different tax schemes to maximize Home welfare.

Figure 2 summarizes our main findings. The first two charts in the upper row show the optimal FDI subsidy rates under different tax schemes (the chart on the left) and the corresponding tax rates (the chart on the right). In the lower row, we compare the welfare levels in Home, Foreign, and the world under different tax schemes. All welfare gains are measured as the percentage changes relative to their levels when there is no FDI subsidy. Two interesting findings emerge immediately. First, the welfare gains of FDI subsidies for the Home country decline substantially under the consumption and corporate revenue taxes relative to that under the labor income tax. The Home welfare gains under these distortionary taxes are less than 15% of that under the non-distortionary labor income tax. This is consistent with Proposition 1 in the last section. Second, the cross-country spillover becomes negative under distortionary consumption and corporate revenue taxes, resulting a net world welfare loss, although Home FDI subsidies funded by the non-distortionary labor income tax is welfare-improving for both Home and Foreign countries. This finding confirms Proposition 2 in the last section.

As shown in Chor (2009), Home FDI subsidies funded by a non-distortionary labor income tax can improve the welfare in both countries. The *laissez-faire* world economy in our model is inefficient due to the presence of transportation costs and monopolistic competition. Home FDI subsidies to variable production costs can attract the most-productive exporters to become FDI firms, which saves trade costs and alleviates monopolistic price distortions of Foreign goods. These benefits of FDI subsidies initially exceed their costs—the decrease in the consumption of homogeneous Home goods—if the subsidies are funded by non-distortionary labor income taxes. Home FDI subsidies also benefit the Foreign country because they increase the *ex ante* profits of Foreign firms, and thus more firms enter in the Foreign country, which increases the product varieties enjoyed by Foreign households.

However, the distortionary effects of consumption and corporate revenue taxes substantially reduce the welfare gains from the FDI subsidies, and the Foreign country even becomes worse off. For instance, the Foreign country loses 0.1% of utility if the FDI subsidies in the Home country are financed by a consumption tax, compared to an increase of 0.25% when the subsidies are funded by the labor income tax. When FDI subsidies are funded by a combination of income, consumption and corporate revenue taxes (hybrid case), the Foreign country loses about 0.3% of utility, while the Home country's utility gain is slightly below 0.3%, leaving the world total welfare barely changed. It is apparent that in all cases in which FDI subsides are funded by distortionary taxes in the Home country, the policy is beggar-thy-neighbor.

In the following subsections, we discuss the underlying mechanisms that drive our results by delving deeper into equilibrium allocations under these taxes.

4.1.1 Tax on Labor Income

Figure 3 displays the equilibrium allocations of some variables in our model with respect to different FDI subsidy rates when the subsidy bill is financed by non-distortionary taxes on labor income. The labor income tax rate increases with the FDI subsidy rate to cover the rising bill for FDI subsidies, as shown in the top left chart. In the top right chart, the optimal subsidy rate that maximizes Home welfare is 26%, and the corresponding labor-income tax rate is 8% in the chart on the left. These findings are similar to those in Chor (2009). The Foreign welfare also increases with the subsidy rate, indicating a positive cross-country spillover. The welfare gains from FDI subsidies relative to the *laissez-faire* economy for Home, Foreign, and the world are 0.74%, 0.25%, and 0.50% respectively.

FDI subsidies funded by the labor income tax shift Home consumption toward Foreign heterogeneous goods and improve the Home welfare through several channels that will be explained shortly. In equation (10), the labor income tax reduces consumption of homogeneous non-tradable goods and the subsidies funded by the tax help FDI firms lower their prices. As a result, Home consumption shifts toward Foreign heterogeneous goods as shown in the middle left chart in Figure 3. The subsidy improves Home welfare through several channels. First, it directly reduces the price markup of FDI firms. In addition, FDI subsidies switch the most-productive Foreign exporters to FDI firms, allowing Home households to enjoy Foreign goods at lower prices and smaller price markups than before. At the same time, the least-productive exporters are squeezed out of the market to become local firms in the Foreign country. Intuitively, the Home FDI subsidies are subsidizing the most-productive Foreign firms such that the more productive Foreign firms (FDI firms) expand their output, and less productive Foreign firms (exporters) shrink and even exit from exporting. This production reallocation increases the overall productivity of Foreign goods consumed in the Home country. Of course, the product varieties of Foreign goods consumed in the Home country decrease as some Foreign firms stop exporting. But the decrease in Foreign product prices and the increase in productivity of Foreign goods that are consumed by Home households dominate the decrease in Foreign product varieties, and thus Home welfare increases. When the subsidy rate continues to rise, the costs of FDI subsidies eventually exceed their benefits.

The positive welfare spillover into the Foreign country is due to an increase in the consumption basket of Foreign goods in the Foreign country (the middle right chart in Figure 3), while the consumption of homogeneous goods and Home differentiated goods in the Foreign country remain constant. The subsidy from the Home country increases the *ex ante* expected profits of Foreign firms. As a result, more Foreign firms enter the market and the variety of

Foreign goods consumed in the Foreign country (the mass of Foreign firms) rises (the bottom left chart in Figure 3). A larger mass of Foreign firms leads to lower market demand for each firm and stronger competition among firms. As a result, the cutoff productivity (as well as the average productivity) of Foreign local firms increases as shown in the bottom right chart in Figure 3.[19]

4.1.2 Tax on Consumption and Corporate Revenue

Figure 4 presents the results for consumption and corporate revenue taxes. It is clear from the top row of the figure that Home welfare gains are much smaller in these two cases relative to the labor income tax. In addition, Foreign welfare decreases with the Home FDI subsidy rate, indicating negative cross-country spillovers of the policy.

The welfare costs of FDI subsidies are higher if they are funded by distortionary consumption and corporate revenue taxes relative to being funded by non-distortionary labor income tax, inducing lower welfare gains for the Home country. First, the consumption substitution in the Home country is mainly between Home heterogeneous goods (C^H) and Foreign heterogeneous goods (C^F) under consumption and corporate revenue taxes (the middle left chart in Figure 4), while it is mainly between Home homogeneous goods and Foreign heterogeneous goods under the labor income tax (the middle left chart in Figure 3). Under the model setup, the substitution between homogeneous goods and heterogeneous goods are not distortionary, but the substitution between Home and Foreign heterogeneous goods are.[20] Therefore, FDI subsidies funded by these distortionary taxes are more welfare costly

[19] This numerical finding is from the analytical result that $Z^{D*} = \left(\frac{f^{D*}}{(\frac{1}{\sigma})(\rho)^{\sigma-1}A^{F*}}\right)^{\frac{1}{\sigma-1}} = \Xi_{Z^{D*}}(M^*)^{\frac{(1-\theta)\sigma-1}{\theta(\sigma-1)}}$, where Z^{D*} is the average productivity of Foreign local firms and $\Xi_{Z^{D*}}$ is a positive constant. Z^{D*} increases with the mass of Foreign firms (M^*) since $\frac{(1-\theta)\sigma-1}{\theta(\sigma-1)} > 0$ under our parametrization.

[20] As we mentioned, the non-distortionary substitution between homogeneous and heterogeneous goods is due to the separable quasi-linear utility function in the benchmark model. A more general CES utility function will be considered in the robustness checks.

than the subsidies funded by non-distortionary taxes. Second, consumption and corporate revenue taxes create price distortions that reduce Home welfare. As shown in equations (12) and (13), taxes on consumption and corporate revenues induce price distortions for domestic local goods, imported goods, and FDI goods that are consumed in the Home country. Specifically, equation (13) shows that consumption and corporate revenue taxes partially offset the benefit of FDI subsidies in alleviating price distortions for FDI goods. For instance, the price markup $\frac{(1-s_V)\sigma}{\sigma-1}\frac{1+\tau_C}{1-\tau_R}$ for FDI goods increases from 1.02 in the case of the labor income tax to 1.27 in the case of the consumption tax under the optimal FDI subsidy.[21]

Due to the distortionary effects of consumption and corporate revenue taxes, Home welfare peaks at a lower rate of FDI subsidies and the welfare gains are also much smaller than that in the case of the labor income tax (the charts in the upper row of Figure 4). Comparing the two distortionary taxes, the optimal welfare gain for the Home country is slightly lower under a consumption tax than that under a corporate revenue tax. Consumption and corporate revenue taxes have similar effects on the Home-good consumption basket (C^H) but affect the Foreign-good consumption basket (C^F) differently. From equation (13), we can see that a consumption tax is equivalent to the international trade cost (τ) in rasing import prices. As a result, for a given FDI subsidy, there is more FDI and less exporting by Foreign firms under a consumption tax than under a corporate revenue tax. The middle right chart in Figure 4 shows that Home households enjoy more varieties of Foreign goods under a consumption tax than under a corporate revenue tax for a given subsidy rate. However, the increase in trade costs reduces the number of Foreign exporting firms and the total mass of Foreign goods consumed in the Home country. This explains the lower welfare gains for the Home country under a consumption tax.

Home FDI subsidies financed by distortionary consumption and corporate revenue taxes turn out to be beggar-thy-neighbor: Foreign households are worse off and the net world

[21] The price markup is 36% without tax and subsidies.

welfare decreases. Consumption and corporate revenue taxes reduce the *ex ante* profits of Home firms, causing fewer firms to enter the market. As a result, the variety of Home goods and the average productivity of producing those goods decrease and their prices increase (the bottom left chart in Figure 4). These effects reduce the welfare in the Foreign country. The firm mass in the Foreign country also decreases slightly as the profits of exporting to and conducting FDI decrease for Foreign firms when they face less demand (due to the consumption tax) or higher costs (due to the revenue tax), which is shown in the bottom right chart.

4.1.3 Hybrid Tax

Now we consider a case in which FDI subsidies are jointly funded by a labor income tax, a consumption tax and a corporate revenue tax. We calibrate the tax rates in our model to match the empirical composition of government revenues in emerging market countries. In 2017 OECD data, the median of ten emerging market countries finances 34% of government revenue by tax on labor income, 41% by tax on consumption, and 25% by corporate tax.[22]

"Hybrid τ" in Figure 2 reports the result. Tax rates that match the empirical composition of emerging-market government revenue are 2.1% for labor income tax, 4.8% for consumption tax, and 3% for firm revenue tax. The collected taxes are used to fund FDI subsidies in the Home country. Under this hybrid tax scheme, the optimal subsidy rate is 24%, which is very close to the optimal subsidy rate under a labor income tax (26%). However, the Home welfare gains are much smaller under the hybrid tax scheme: they are less than half of that under the labor income tax. This example clearly illustrates the importance of recognizing different funding sources for FDI subsidies. Although the subsidy rates are very similar, the welfare gains can differ substantially if the subsides are funded by different taxes. In

[22]Data source is OECD Global Revenue Statistics Database. The ten emerging markets include Chile, Colombia, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovenia, and Turkey, which are based upon data availability. See the appendix for more details.

addition, the Foreign country becomes worse off: its welfare decreases by -0.31%, while the home country's welfare increases by 0.27%, leaving the world welfare barely changed.

4.2 Robustness Checks

4.2.1 Robustness to CES Utility Function

The quasi-linear utility function in equation (1) features two layers of separable preference: homogeneous and differentiated goods are separable and Home and Foreign differentiated goods are separable. This enables us to pin down allocations and welfare in closed form and allows for non-distortionary tax on labor income, τ_L. With this quasi-linear utility, the benefit of FDI subsidies comes at a minimized cost under τ_L since tax on labor income only reduces consumption on homogeneous goods, which corresponds to a lump-sum payment. Following Chor (2009), we relax this restriction to consider a general income effect by taking the CES utility:

$$V = \left[(\varkappa)^{\frac{1}{\epsilon}} \left[C_0\right]^{\frac{\epsilon-1}{\epsilon}} + (1-\varkappa)^{\frac{1}{\epsilon}} \left[\tfrac{1}{\theta} \left(C^H\right)^{\theta} + \tfrac{1}{\theta} \left(C^F\right)^{\theta} \right]^{\frac{\epsilon-1}{\theta\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{14}$$

where $0 < \frac{\epsilon-1}{\epsilon} < \theta < \frac{\sigma-1}{\sigma} < 1$ and $\varkappa \in (0,1)$. As in Chor (2009), Home and Foreign differentiated goods are assigned the same weight in consumption.[23]

Figure 5 presents the results of the CES utility function.[24] Note that with the CES utility in equation (14), there is no non-distortionary tax scheme available to fund FDI

[23] $\frac{\epsilon-1}{\epsilon} < \theta$ is required so that differentiated goods are better substitutes for each other compared to homogeneous goods. Likewise, $\theta < \rho = \frac{\sigma-1}{\sigma}$ implies differentiated goods from the same country are better substitutes for each other compared to differentiated goods from different countries. If we use the Cobb-Douglas preference between Home and Foreign differentiated goods (C^H, C^F), then tax on labor income is isomorphic to tax on consumption. If we apply the Armington preference with non-unitary substitutability between Home and Foreign differentiated goods (C^H, C^F), then tax on consumption is more distortionary than tax on labor income since the consumption tax distorts firm demand further than the labor income tax.

[24] We assign equal weight to homogeneous and differentiated goods by setting $\varkappa = \frac{1}{2}$. The substitutability between homogeneous and differentiated goods is set to $\epsilon = 1.20$. All other parameters follow the same calibration as in our benchmark model.

subsidies anymore. The income effect influences both homogeneous-goods consumption and differentiated-goods consumption under the CES utility. This implies that the labor income tax will depress demand for Home and Foreign differentiated goods and distort allocations of heterogeneous firms. As a result, the optimal subsidy rate and Home welfare gains are lower than those in the benchmark model.

In addition, Home FDI subsidies are beggar-thy-neighbor and the world welfare declines for all tax schemes including the labor income tax. The beggar-thy-neighbor effect is much stronger under the CES utility than in the benchmark model. The welfare loss of the Foreign country ranges from about 0.3% to over 0.6%, compared to about 0.2% or less in the benchmark model. The labor income tax actually records the largest welfare loss for the Foreign country, although its distortionary effect for the world welfare is the least among four tax schemes. This is because the Home country can offer a higher subsidy rate under labor income tax as the tax scheme is less distortionary for the Home country than other tax schemes.

4.2.2 Robustness to Home Bias in Consumption

So far we have assigned the same weight in consumption for the goods produced by Home and Foreign firms. However, the presence of home bias in consumption is a well-known characteristic of international trade data. Obstfeld and Rogoff (2000) point out that a strong preference for home goods in consumption is one of the major puzzles in international economics. In this subsection, we analyze the effect of home bias in consumption on FDI subsidies and the associated welfare under different tax schemes. Following the literature, we incorporate a home bias parameter, ν, into the CES preference:

$$V = \left[(\varkappa)^{\frac{1}{\epsilon}} \left[C_0\right]^{\frac{\epsilon-1}{\epsilon}} + (1-\varkappa)^{\frac{1}{\epsilon}} \left[\chi \left((\nu)^{1-\theta} \left[C^H\right]^{\theta} + (1-\nu)^{1-\theta} \left[C^F\right]^{\theta} \right)^{\frac{1}{\theta}} \right]^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \quad (15)$$

where $0 < \frac{\epsilon-1}{\epsilon} < \theta < \rho = \frac{\sigma-1}{\sigma} < 1$, $\varkappa \in (0,1)$, and $\nu \in (0,1)$.[25] $\nu > \frac{1}{2}$ implies that Home households value goods produced by home firms more than products supplied by foreign firms.

Figure 6 shows the results from varying the degree of home bias under different tax schemes. The optimal subsidy rates and welfare gains in the Home country decrease with the level of home bias in consumption for all tax schemes. Intuitively, higher home bias reduces the benefits of FDI subsidies to the Home country since households value foreign goods C^F less than home goods C^H. In addition, under the CES preference, all tax schemes adversely affect the production of home goods C^H, and the costs of Home FDI subsidies increase with the level of home bias. Therefore, the optimal FDI subsidy rate and Home welfare gains decrease with the level of home bias in consumption. Indeed, the Home welfare gain drops substantially even for a moderate increase in home bias. For instance, the welfare gain decreases more than 60% (from over 0.5% to less than 0.2%) if the home bias parameter simply changes from 0.5 to 0.65.

5 Conclusion

Many emerging markets provide financial subsidies to MNCs, assuming that such policies would attract more international investment and raise the host country's welfare. We argue that the benefits of FDI subsidies may critically depend on how they are funded. For countries with significant tax distortions, which is generally true in many emerging markets, the net welfare gains of FDI subsidies could be much smaller than that in countries with less distortionary tax systems. In addition, the cross-country spillover could turn negative when FDI subsidies are funded by distortionary taxes. In this case, it risks policy retaliation from

[25] The parameter χ is set to $\left(\frac{(2)^{1-\theta}}{\theta}\right)^{\frac{1}{\theta}}$ obtaining the same marginal rate of substitution between homogeneous and differentiated goods under $\nu = \frac{1}{2}$ as in the utility (14). We also assign a half to $\varkappa$, set ϵ to 1.20, and follow the same parametrization in Table 1 for the numerical analysis.

the foreign country, which could eliminate any welfare gains that FDI subsidies bring to the host countries.

Previous studies have also warned about some implementation challenges faced by FDI subsidy policies. For instance, Janeba (2002) argues that upfront subsidies are not sufficient to attract FDI if the host-country government is unable to make a long-term commitment to its FDI policies. Janeba (2004) shows that FDI subsidies redistribute wealth from workers to firms, and the scope for large FDI subsidies could be limited if workers have sufficient political power. However, in the emerging markets with weak governance and limited political power for workers, FDI subsidies may create serious corruption and income inequality issues. These potential drawbacks of FDI subsidies have to be taken into account when considering policies for the developing countries.

Our paper abstracts from important issues such as strategic interactions among countries in setting FDI and trade policies. For instance, Wang (2020) finds that the 2017 U.S. corporate tax cut reduces real incomes in other countries but raises the U.S. real income due to the production reallocation of MNCs. He shows that international tax cooperation can increase real incomes for all participating countries. In addition, foreign investment itself may also influence the policy making in host countries through its government connections in either the host countries or in the home country. For instance, Antràs and Padró i Miquel (2011) show that foreign interests can have important influence on tariff policies and such influences depend on whether countries can engage in international agreements. The same argument may also apply to the making of FDI policies. These are important relevant issues for future work.

References

Aitken, B. J. and A. E. Harrison (1999). Do domestic firms benefit from direct foreign investment? Evidence from Venezuela. *American Economic Review 89*(3), 605–618.

Alfaro, L., A. Chanda, S. Kalemli-Ozcan, and S. Sayek (2004). FDI and economic growth: The role of local financial markets. *Journal of International Economics 64*(1), 89–112.

Alfaro, L. and M. X. Chen (2018). Selection and market reallocation: Productivity gains from multinational production. *American Economic Journal: Economic Policy 10*(2), 1–38.

Alquist, R., N. Berman, R. Mukherjee, and L. Tesar (2019). Financial constraints, institutions, and foreign ownership. *Journal of International Economics 118*, 63–83.

Anderson, J. E. and E. van Wincoop (2004). Trade costs. *Journal of Economic Literature 42*(3), 691–751.

Antràs, P. and G. Padró i Miquel (2011). Foreign influence and welfare. *Journal of International Economics 84*(2), 135 – 148.

Arnold, J. M. and B. S. Javorcik (2009). Gifted kids or pushy parents? Foreign direct investment and plant productivity in Indonesia. *Journal of International Economics 79*(1), 42–53.

Balsvik, R. (2011). Is labor mobility a channel for spillovers from multinationals? Evidence from Norwegian manufacturing. *The Review of Economics and Statistics 93*(1), 285–297.

Bao, C. G. and M. X. Chen (2018). Foreign rivals are coming to town: Responding to the threat of foreign multinational entry. *American Economic Journal: Applied Economics 10*(4), 120–57.

Bernard, A. B., J. Eaton, J. B. Jensen, and S. Kortum (2003). Plants and productivity in international trade. *American Economic Review 93*(4), 1268–1290.

Blalock, G. and P. J. Gertler (2009). How firm capabilities affect who benefits from foreign technology. *Journal of Development Economics 90*(2), 192–199.

Borensztein, E., J. De Gregorio, and J.-W. Lee (1998). How does foreign direct investment affect economic growth? *Journal of International Economics 45*(1), 115–135.

Carkovic, M. and R. Levine (2005). Does foreign direct investment accelerate economic growth? In T. Moran, E. M. Graham, and M. Blomstrom (Eds.), *Does FDI Promote Development*, Chapter 8, pp. 195–220. Institute for International Economics.

Chor, D. (2009). Subsidies for FDI: Implications from a model with heterogeneous firms. *Journal of International Economics 78*(1), 113–125.

Djankov, S. and B. Hoekman (2000). Foreign investment and productivity growth in Czech enterprises. *The World Bank Economic Review 14*(1), 49–64.

Fumagalli, C. (2003). On the welfare effects of competition for foreign direct investments. *European Economic Review 47*(6), 963–983.

Girma, S. (2005). Absorptive capacity and productivity spillovers from FDI: A threshold regression analysis. *Oxford Bulletin of Economics and Statistics 67*(3), 281–306.

Girma, S. and H. Görg (2007). Evaluating the foreign ownership wage premium using a difference-in-differences matching approach. *Journal of International Economics 72*(1), 97–112.

Gorodnichenko, Y., J. Svejnar, and K. Terrell (2014). When does FDI have positive spillovers? Evidence from 17 transition market economies. *Journal of Comparative Economics 42*(4), 954–969.

Grossman, G. M. and E. Helpman (1995). Technology and trade. *Handbook of International Economics 3*, 1279–1337.

Guadalupe, M., O. Kuzmina, and C. Thomas (2012). Innovation and foreign ownership. *American Economic Review 102*(7), 3594–3627.

Haaland, J. I. and I. Wooton (1999). International competition for multinational investment. *Scandinavian Journal of Economics 101*(4), 631–649.

Haddad, M. and A. Harrison (1993). Are there positive spillovers from direct foreign investment? Evidence from panel data for Morocco. *Journal of Development Economics 42*(1), 51–74.

Hale, G. and C. Long (2011). Are there productivity spillovers from foreign direct investment in China? *Pacific Economic Review 16*(2), 135–153.

Hu, A. G. and G. H. Jefferson (2002). FDI impact and spillover: Evidence from China's electronic and textile industries. *The World Economy 25*(8), 1063–1076.

Janeba, E. (2002). Attracting FDI in a politically risky world. *International Economic Review 43*(4), 1127–1155.

Janeba, E. (2004). Global corporations and local politics: Income redistribution vs. FDI subsidies. *Journal of Development Economics 74*, 367–391.

Javorcik, B. S. (2004). Does foreign direct investment increase the productivity of domestic firms? In search of spillovers through backward linkages. *American Economic Review 94*(3), 605–627.

Javorcik, B. S. (2015). Does FDI bring good jobs to host countries? *The World Bank Research Observer 30*(1), 74–94.

Javorcik, B. S. and M. Spatareanu (2011). Does it matter where you come from? Vertical spillovers from foreign direct investment and the origin of investors. *Journal of*

Development Economics 96(1), 126–138.

Keller, W. and S. R. Yeaple (2009). Multinational enterprises, international trade, and productivity growth: firm-level evidence from the United States. *The Review of Economics and Statistics 91*(4), 821–831.

Konings, J. (2001). The effects of foreign direct investment on domestic firms: Evidence from firm-level panel data in emerging economies. *Economics of Transition 9*(3), 619–633.

Lu, Y., Z. Tao, and L. Zhu (2017). Identifying FDI spillovers. *Journal of International Economics 107*, 75–90.

Manova, K., S.-J. Wei, and Z. Zhang (2015). Firm exports and multinational activity under credit constraints. *Review of Economics and Statistics 97*(3), 574–588.

Newman, C., J. Rand, T. Talbot, and F. Tarp (2015). Technology transfers, foreign investment and productivity spillovers. *European Economic Review 76*, 168–187.

Obstfeld, M. and K. Rogoff (2000). The six major puzzles in international macroeconomics: Is there a common cause? *NBER Macroeconomics Annual 15*, 339–390.

Pennings, E. (2005). How to maximize domestic benefits from foreign investments: The effect of irreversibility and uncertainty. *Journal of Economic Dynamics and Control 29*(5), 873–889.

Tanzi, V. and H. Zee (2000). Tax policy for emerging markets: Developing countries. *National Tax Journal 53*, 2.

Wang, J. and X. Wang (2015). Benefits of foreign ownership: Evidence from foreign direct investment in China. *Journal of International Economics 97*(2), 325–338.

Wang, Z. (2020). Multinational production and corporate taxes: A quantitative assessment. *Journal of International Economics*, 103353.

World Bank (2018). Global investment competitiveness report 2017/2018: Foreign investor perspectives and policy implications.

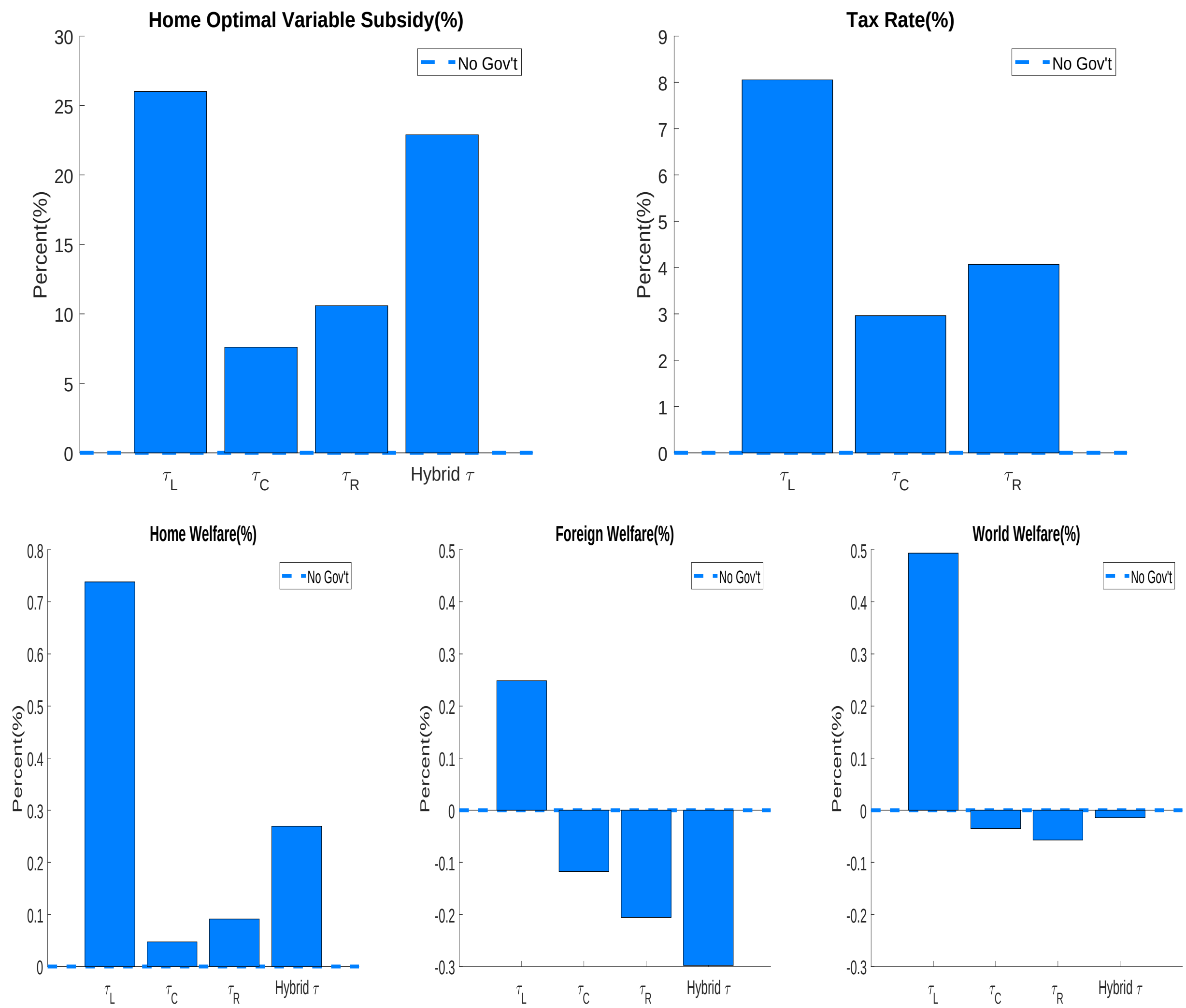
Yasar, M. and C. J. M. Paul (2007). International linkages and productivity at the plant level: Foreign direct investment, exports, imports and licensing. *Journal of International Economics 71*(2), 373–388.

Table 1: Benchmark Model Calibration

Parameter	Description	Value	Source
θ	The relative weight on homogeneous goods	0.32	47% employment in homogeneous sector as in Chor (2009)
$\sigma = \frac{1}{1-\rho}$	Substitutability between differentiated goods	3.80	Bernard, Eaton, Jensen, and Kortum (2003): Monopoly markup $\frac{\sigma}{\sigma-1} = 1.36$, i.e. 36.0%
L	Labor endowment of the Home country	1.00	Normalization
L^*	Labor endowment of the Foreign country	1.00	Symmetry
z_{min}, z^*_{min}	Lower bound in Pareto distribution for Home and Foreign firms	0.10	Normalization
η, η^*	Dispersion in Pareto distribution for Home and Foreign firms	3.30	Bernard, Eaton, Jensen, and Kortum (2003)
F^D	Home fixed sunk entry costs	1.00	1 unit of labor
f^D	Home fixed production costs	0.30	30% of sunk entry costs
f^X	Home fixed export costs	0.40	40% of sunk entry costs
f^I	Home fixed FDI costs	1.00	100% of sunk entry costs
τ	Home transport costs	1.20	20% trade costs
F^{D*}	Foreign fixed sunk entry costs	1.00	1 unit of labor
f^{D*}	Foreign fixed production costs	0.30	30% of sunk entry costs
f^{X*}	Foreign fixed export costs	0.40	40% of sunk entry costs
f^{I*}	Foreign fixed FDI costs	1.00	100% of sunk entry costs
τ	Foreign transport costs	1.20	20% trade costs
s_V	Home Government's variable subsidy for Foreign Multinationals	$[-1, 1]$	Maximize Home Utility
s^*_V	Foreign Government's variable subsidy for Home Multinationals	0	No Subsidy in Foreign

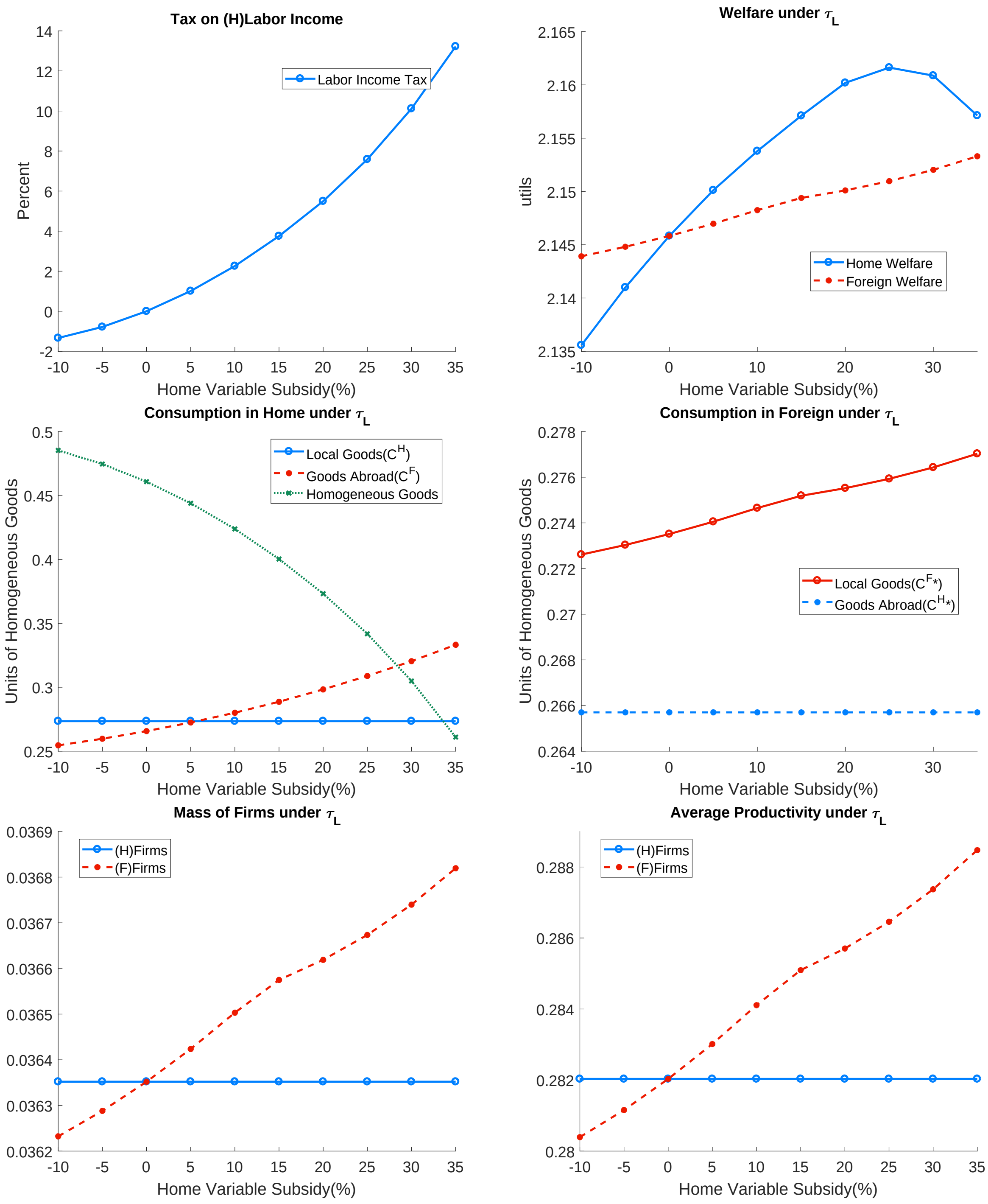
Note – The Home and Foreign fixed costs are symmetric. These costs are set such that the masses of local firms, exporters and MNCs are 50%, 30%, and 20%, respectively.

Figure 2: Optimal Subsidy, Tax Rates, and Welfare



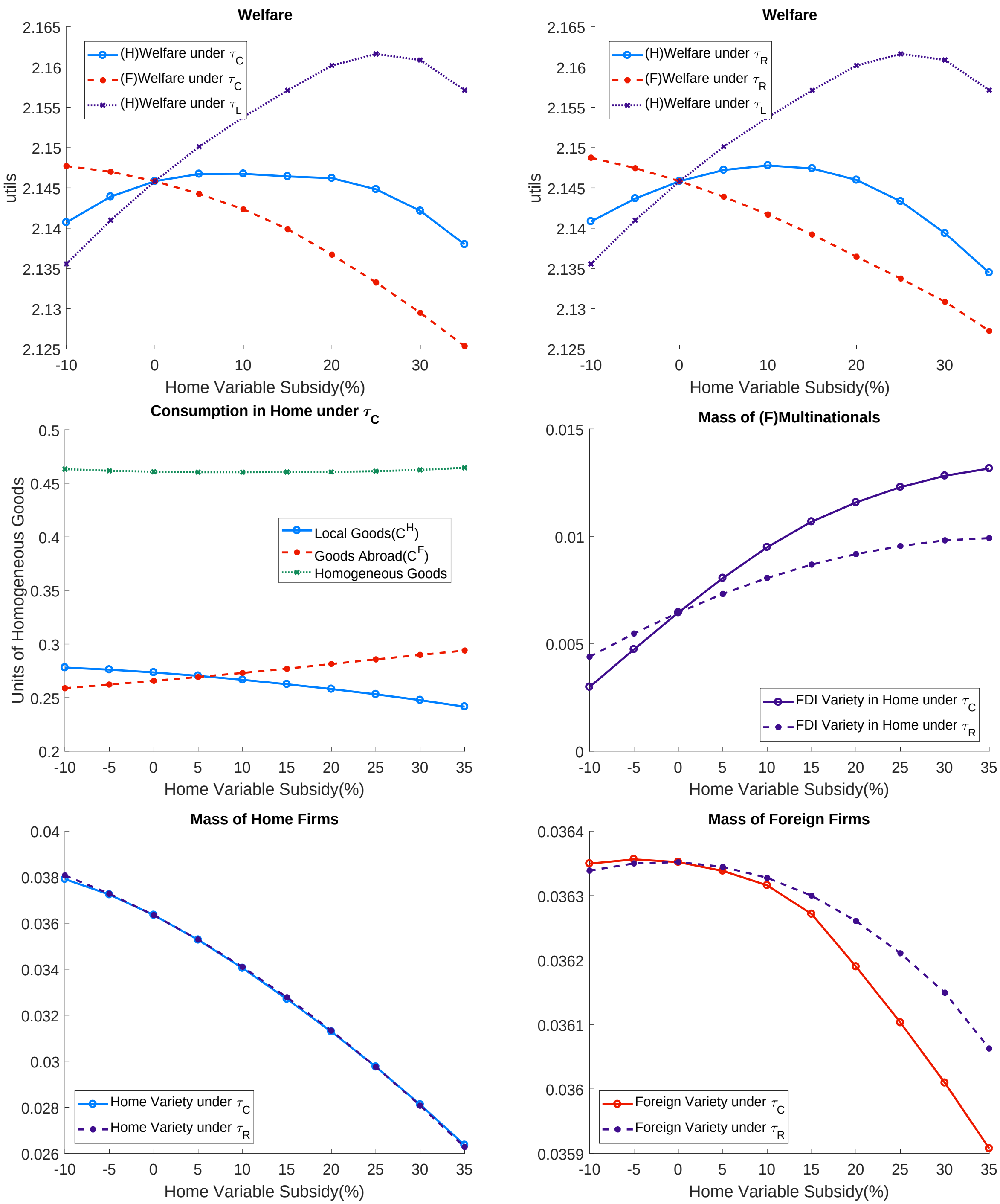
Note – The horizontal axis represents different tax schemes to finance FDI subsidies in the Home country. τ_L denotes tax on labor income; τ_C stands for tax on consumption; τ_R represents tax on firm revenue. Hybrid τ shows the outcome when the subsidy bill is financed by all three tax schemes: 33% from τ_L, 41% from τ_C, and 26% from τ_R. All welfare gain measures are relative to the welfare levels when there is no FDI subsidy.

Figure 3: Equilibrium Allocations under Different FDI Subsidy Rates and Tax on Labor Income



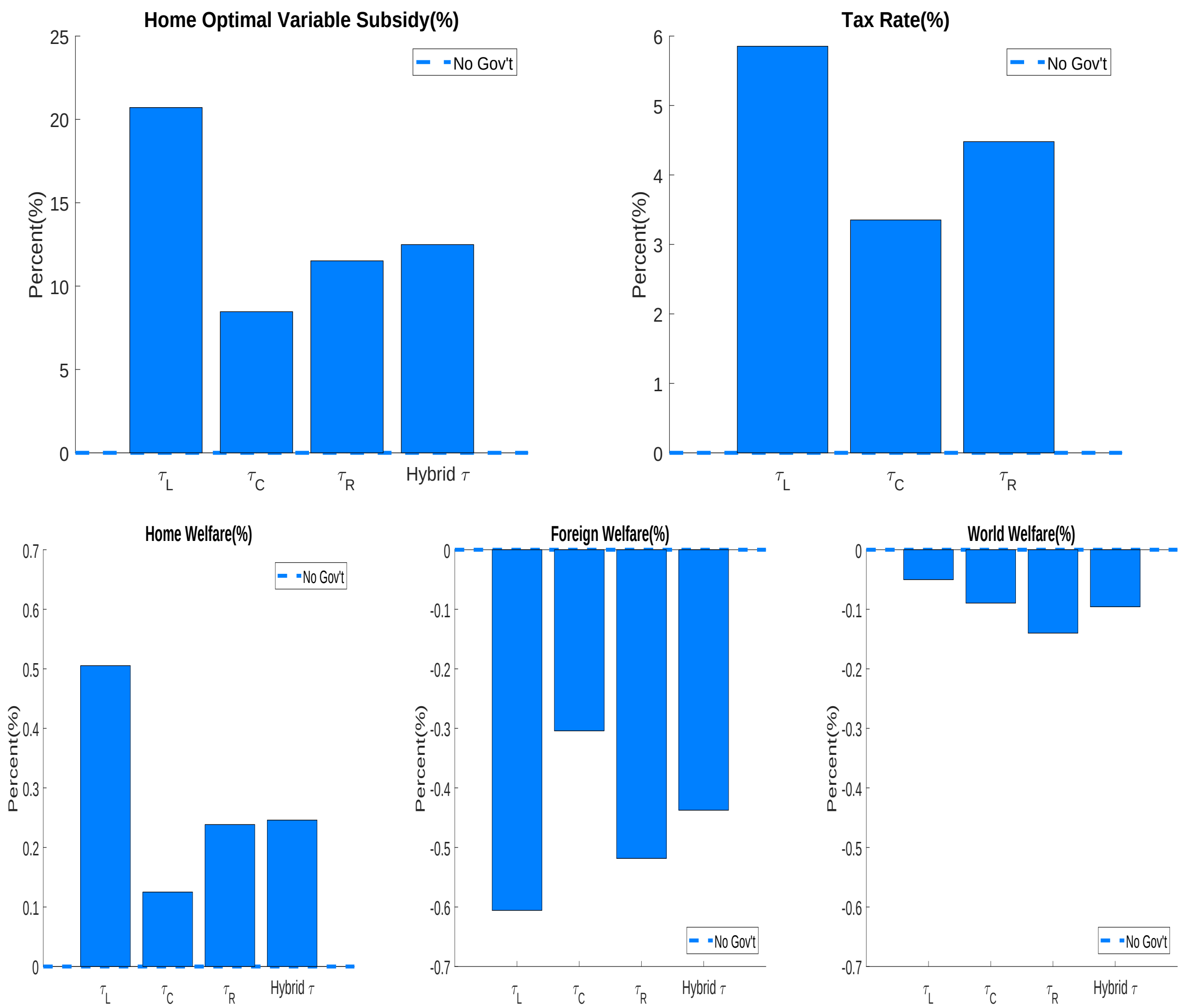
Note – **H** denotes the Home country; **F** denotes the Foreign country. The horizontal axis represents different FDI subsidy rates offered by the Home government and the vertical axis shows the corresponding equilibrium allocation of each variable. For each level of the subsidy rate, FDI subsidies are funded by a tax on labor income.

Figure 4: Equilibrium Allocations under Different FDI Subsidy Rates and Distortionary Taxes



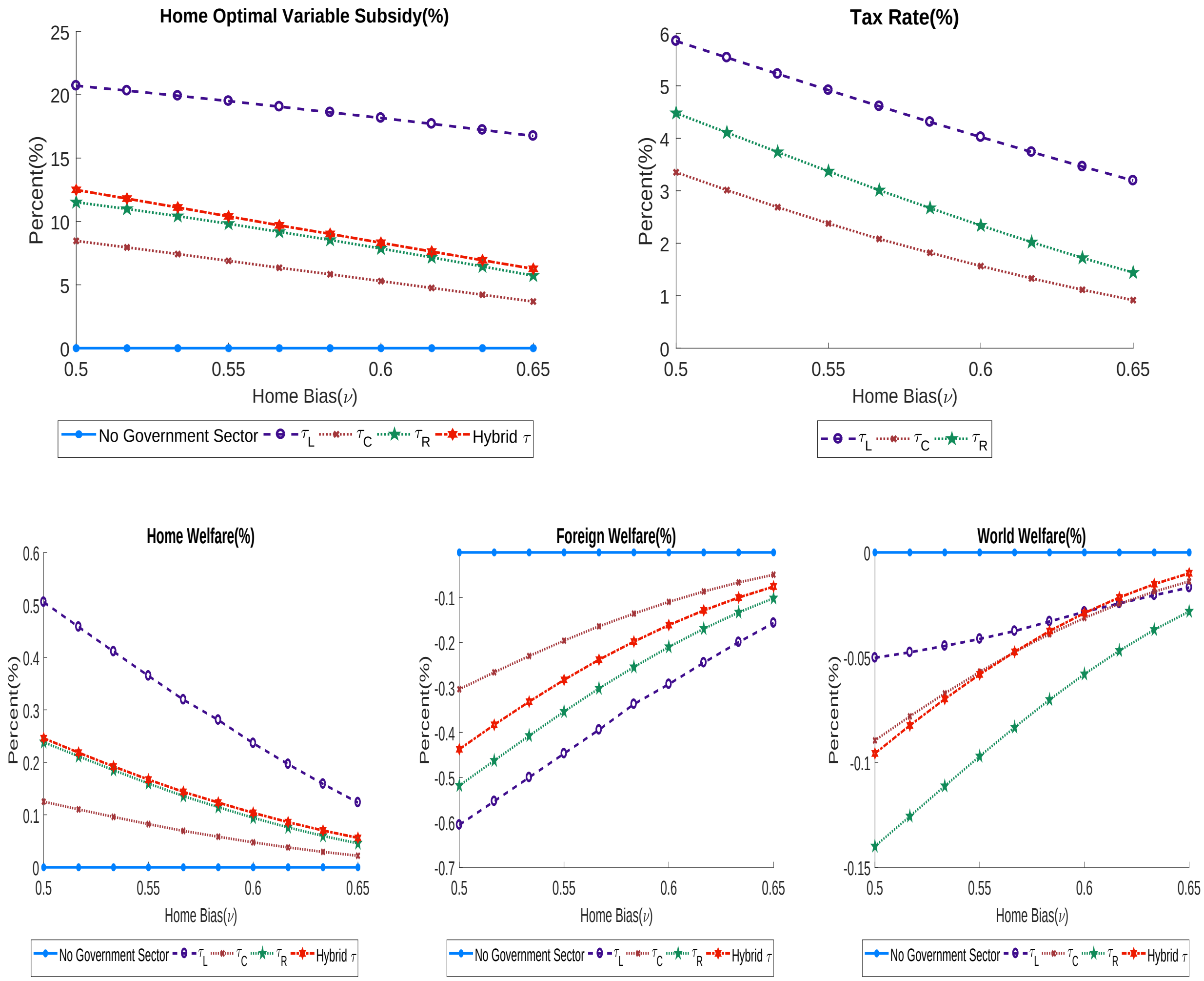
Note – **H** denotes the Home country; **F** denotes the Foreign country. The horizontal axis represents different FDI subsidy rates offered by the Home government and the vertical axis shows the corresponding equilibrium allocation of each variable. For each level of the subsidy rate, FDI subsidies are funded by either a tax on labor income (τ_L) or a tax on consumption (τ_C), or a tax on corporate revenue (τ_R).

Figure 5: Results under the CES Preference



Note – The horizontal axis represents different tax schemes to finance FDI subsidies in the Home country. τ_L denotes tax on labor income; τ_C stands for tax on consumption; τ_R represents tax on firm revenue. Hybrid τ shows the outcome when the subsidy bill is financed by all three tax schemes: 33% from τ_L, 41% from τ_C, and 26% from τ_R. All welfare gain measures are relative to the welfare levels when there is no FDI subsidy.

Figure 6: Results for Home Bias in Consumption under the CES Preference



Note – The horizontal axis represents different degrees of home bias ranging from $\nu = 0.50$ (no bias) to $\nu = 0.65$ (bias toward C^H). τ_L denotes tax on labor income; τ_C stands for tax on consumption; τ_R represents tax on firm revenue. Hybrid τ shows the outcome when the subsidy bill is financed by all three tax schemes: 33% from τ_L, 41% from τ_C, and 26% from τ_R. All welfare gain measures are relative to the welfare levels when there is no FDI subsidy.

Appendix

A.1 Sources of Government Revenue

Table A.1: Sources of 2017 Government Revenues in Emerging Markets

Country	Consumption tax	Labor income tax	Corporate tax	Other
Chile	53.3	10.2	25.6	11.0
Colombia	42.8	11.2	30.3	15.7
Czech Republic	32.1	34.3	31.9	1.8
Estonia	41.3	34.0	23.5	1.2
Hungary	44.0	30.8	19.8	5.5
Latvia	44.5	33.8	18.0	3.6
Lithuania	37.5	34.5	26.1	1.9
Poland	35.1	33.9	24.8	6.2
Slovenia	38.0	34.6	25.3	2.1
Turkey	40.5	30.4	23.7	5.5
Average	40.9	28.8	24.9	5.4
Median	40.9	33.9	25.0	4.5

– Data source is OECD Global Revenue Statistics Database, and the choice of emerging markets in this table is determined by data availability.
– Consumption tax is from the tax on goods and services.
– Labor income tax is the sum of tax on income & profits of individuals and half of the social security contributions.
– Corporate tax includes tax on income and profits of corporations and half of the social security contributions.

A.2 Proof of Propositions

This section provides formal proofs for propositions 1 and 2 in the main text. Note that all tax variables in this appendix A.2 are expressed as wedge terms for algebraic simplicity. The following table A.2 displays the mapping of tax variables from the main text to this appendix A.2.

Table A.2: Notations for tax variables in the appendix A.2.

Main Text	Appendix A.2
Labor-income tax rate τ_L	Labor-income tax wedge $\tau'_L = 1 - \tau_L$
Consumption tax rate τ_C	Consumption tax wedge $\tau'_C = 1 + \tau_C$
Firm-revenue tax rate τ_R	Firm-revenue tax wedge $\tau'_R = 1 - \tau_R$

In what follows, we use τ'_L, τ'_C, and τ'_R and suppress the superscript '$'$' in all equilibrium conditions. By contrast, the notation for a FDI subsidy, s_V, indicates a rate as in the main text.

In addition to this appendix A.2, we also provides the online technical appendix[1] which shows that all measures for consumption, market demand, and cutoff productivity can be decomposed into mass of firms and additional multiplicative terms. In the whole appendix, we use the notation Ξ and Ψ to denote those multiplicative terms attached to mass of firms in each variable. In what follows, we list consumption, market demand, and cutoff productivity without their derivations. The details are available in the online technical appendix which can be found in the corresponding author's homepage.

Home-good Consumption, Demand, and Cutoff in Home:

$$
\begin{aligned}
C^H &= \Xi_{C^H} \left(M\right)^{\frac{1}{\theta}} \\
\Xi_{C^H} &= \left(\tau_C\right)^{\frac{1}{\theta}} \left(\tau_R\right)^{\frac{-1}{\theta}} \left(\sigma\right)^{\frac{1}{\theta}} \left(f^D\right)^{\frac{1}{\theta}} \left(\frac{\eta}{\eta-\sigma+1}\right)^{\frac{1}{\theta}} \\
A^H &= \Xi_{A^H} \left(M\right)^{\kappa(\sigma-1)} \\
\Xi_{A^H} &= \left(\tau_C\right)^{-\frac{(\sigma-1)}{\theta}} \left(\tau_R\right)^{-\kappa(\sigma-1)} \left(\sigma\right)^{\kappa(\sigma-1)} \left(f^D\right)^{\kappa(\sigma-1)} \left(\frac{\eta}{\eta-\sigma+1}\right)^{\kappa(\sigma-1)} \\
Z^D &= \Xi_{Z^D} \left(M\right)^{-\kappa} \\
\Xi_{Z^D} &= \left(\tau_C\right)^{\frac{1}{\theta}} \left(\tau_R\right)^{\frac{-1}{\theta}} \left(\rho\right)^{-1} \left(\sigma\right)^{\frac{1-\theta}{\theta}} \left(f^D\right)^{\frac{1-\theta}{\theta}} \left(\frac{\eta}{\eta-\sigma+1}\right)^{-\kappa}
\end{aligned}
\tag{A.2.1}
$$

where $\theta\sigma + 1 - \sigma < 0$ and $\sigma > 1$. We define $\kappa \equiv \frac{\theta\sigma+1-\sigma}{\theta(\sigma-1)} < 0$. Here C^H denotes Home consumption on Home goods; A^H Home market demand on Home goods; Z^D cutoff productivity for Home local firms.

[1] The online technical appendix is available from the corresponding author's homepage: https://sites.google.com/site/econhanwt/

Foreign-good Consumption, Demand, and Cutoff in Foreign:

$$\begin{aligned}
C^{F*} &= \Xi_{C^{F*}} \left(M^*\right)^{\frac{1}{\theta}} \\
\Xi_{C^{F*}} &= (\sigma)^{\frac{1}{\theta}} \left(f^{D*}\right)^{\frac{1}{\theta}} \left(\frac{\eta^*}{\eta^*-\sigma+1}\right)^{\frac{1}{\theta}} \\
A^{F*} &= \Xi_{A^{F*}} \left(M^*\right)^{\kappa(\sigma-1)} \\
\Xi_{A^{F*}} &= (\sigma)^{\kappa(\sigma-1)} \left(f^{D*}\right)^{\kappa(\sigma-1)} \left(\frac{\eta^*}{\eta^*-\sigma+1}\right)^{\kappa(\sigma-1)} \\
Z^{D*} &= \Xi_{Z^{D*}} \left(M^*\right)^{-\kappa} \\
\Xi_{Z^{D*}} &= (\rho)^{-1} (\sigma)^{\frac{1-\theta}{\theta}} \left(f^{D*}\right)^{\frac{1-\theta}{\theta}} \left(\frac{\eta^*}{\eta^*-\sigma+1}\right)^{-\kappa}
\end{aligned} \tag{A.2.2}$$

where $\theta\sigma+1-\sigma<0$ and $\sigma>1$. We define $\kappa \equiv \frac{\theta\sigma+1-\sigma}{\theta(\sigma-1)}<0$. Here C^{F*} denotes Foreign consumption on Foreign goods; A^{F*} Foreign market demand on Foreign goods; Z^{D*} cutoff productivity for Foreign local firms.

Foreign-good Consumption, Demand, and Cutoff in Home:

$$\begin{aligned}
C^{F} &= \left(\Xi_{C^F}\right)^{\frac{1}{\theta-\theta\kappa\eta^*}} \left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{\theta-\theta\kappa\eta^*}} \left(M^*\right)^{\frac{1}{\theta}} \\
\Xi_{C^F} &= \begin{pmatrix}
+ & (\tau^*)^{-(\sigma-1)} (\rho)^{\sigma-1} \left(\frac{\eta^*}{\eta^*-\sigma+1}\right) \left(\Psi_{Z^{X*}}\right)^{\frac{-\eta^*+\sigma-1}{\sigma-1}} \\
- & (\tau^*)^{-(\sigma-1)} (\rho)^{\sigma-1} \left(\frac{\eta^*}{\eta^*-\sigma+1}\right) \left(\Psi_{Z^{I*}}\right)^{\frac{-\eta^*+\sigma-1}{\sigma-1}} \\
+ & (1-s_V)^{-(\sigma-1)} (\rho)^{\sigma-1} \left(\frac{\eta^*}{\eta^*-\sigma+1}\right) \left(\Psi_{Z^{I*}}\right)^{\frac{-\eta^*+\sigma-1}{\sigma-1}}
\end{pmatrix} \\
A^{F} &= \Xi_{A^F} \left(M^*\right)^{\kappa(\sigma-1)} \\
\Xi_{A^F} &= \left(\Xi_{C^F}\right)^{\frac{\kappa(\sigma-1)}{1-\kappa\eta^*}} \left(\Xi_{Z^{D*}}\right)^{\frac{\kappa(\sigma-1)\eta^*}{1-\kappa\eta^*}} \\
Z^{X*} &= \Xi_{Z^{X*}} \left(M^*\right)^{-\kappa} \\
\Xi_{Z^{X*}} &= \left(\Psi_{Z^{X*}}\right)^{\frac{1}{\sigma-1}} \left(\Xi_{C^F}\right)^{\frac{-\kappa}{1-\kappa\eta^*}} \left(\Xi_{Z^{D*}}\right)^{\frac{-\kappa\eta^*}{1-\kappa\eta^*}} \\
Z^{I*} &= \Xi_{Z^{I*}} \left(M^*\right)^{-\kappa} \\
\Xi_{Z^{I*}} &= \left(\Psi_{Z^{I*}}\right)^{\frac{1}{\sigma-1}} \left(\Xi_{C^F}\right)^{\frac{-\kappa}{1-\kappa\eta^*}} \left(\Xi_{Z^{D*}}\right)^{\frac{-\kappa\eta^*}{1-\kappa\eta^*}}
\end{aligned} \tag{A.2.3}$$

where $\theta\sigma+1-\sigma<0$ and $\sigma>1$. We define $\kappa \equiv \frac{\theta\sigma+1-\sigma}{\theta(\sigma-1)}<0$. We also define

$$\begin{aligned}
\Psi_{Z^{X*}} &= \frac{f^{X*}}{(\tau^*)^{-(\sigma-1)}\left(\frac{1}{\sigma}\right)(\rho)^{\sigma-1}} \\
\Psi_{Z^{I*}} &= \frac{f^{I}-f^{X*}}{\left[(1-s_V)^{-(\sigma-1)}-(\tau^*)^{-(\sigma-1)}\right]\left(\frac{1}{\sigma}\right)(\rho)^{\sigma-1}}
\end{aligned} \tag{A.2.4}$$

The rank of cutoff productivity measures satisfies $Z^{D*} < Z^{X*} < Z^{I*}$, implying $\Xi_{Z^{D*}} < \Xi_{Z^{X*}} < \Xi_{Z^{I*}}$ and

$$\Xi_{Z^{D*}} < \left(\Psi_{Z^{X*}}\right)^{\frac{1}{\sigma-1}} \left(\Xi_{C^F}\right)^{\frac{-\kappa}{1-\kappa\eta^*}} \left(\Xi_{Z^{D*}}\right)^{\frac{-\kappa\eta^*}{1-\kappa\eta^*}} < \left(\Psi_{Z^{I*}}\right)^{\frac{1}{\sigma-1}} \left(\Xi_{C^F}\right)^{\frac{-\kappa}{1-\kappa\eta^*}} \left(\Xi_{Z^{D*}}\right)^{\frac{-\kappa\eta^*}{1-\kappa\eta^*}}$$

$$\therefore\ \Psi_{Z^{X*}} < \Psi_{Z^{I*}} \quad \text{implies} \quad \left(f^{X*}\right)\left(\tau^*\right)^{\sigma-1} < \left(f^I - f^{X*}\right)\left(\left[(1-s_V)^{-(\sigma-1)} - (\tau^*)^{-(\sigma-1)}\right]^{(-1)}\right)$$

Here C^F denotes Home consumption on Foreign goods; A^F Home market demand on Foreign goods; Z^{X*} and Z^{I*} cutoff productivity for Foreign exporters and multinationals.

Home-good Consumption, Demand, and Cutoff in Foreign:

$$\begin{aligned}
C^{H*} &= \left(\Xi_{C^{H*}}\right)^{\frac{1}{\theta-\theta\kappa\eta}} \left(\Xi_{Z^D}\right)^{\frac{\eta}{\theta-\theta\kappa\eta}} (M)^{\frac{1}{\theta}} \\
\Xi_{C^{H*}} &= \begin{pmatrix} + & (\tau)^{-(\sigma-1)}(\rho)^{\sigma-1}\left(\frac{\eta}{\eta-\sigma+1}\right)\left(\Psi_{Z^X}\right)^{\frac{-\eta+\sigma-1}{\sigma-1}} \\ - & (\tau)^{-(\sigma-1)}(\rho)^{\sigma-1}\left(\frac{\eta}{\eta-\sigma+1}\right)\left(\Psi_{Z^I}\right)^{\frac{-\eta+\sigma-1}{\sigma-1}} \\ + & (\rho)^{\sigma-1}\left(\frac{\eta}{\eta-\sigma+1}\right)\left(\Psi_{Z^I}\right)^{\frac{-\eta+\sigma-1}{\sigma-1}} \end{pmatrix} \\
A^{H*} &= \Xi_{A^{H*}} (M)^{\kappa(\sigma-1)} \\
\Xi_{A^{H*}} &= \left(\Xi_{C^{H*}}\right)^{\frac{\kappa(\sigma-1)}{1-\kappa\eta}} \left(\Xi_{Z^D}\right)^{\frac{\kappa(\sigma-1)\eta}{1-\kappa\eta}} \\
Z^X &= \Xi_{Z^X} (M)^{-\kappa} \\
\Xi_{Z^X} &= \left(\Psi_{Z^X}\right)^{\frac{1}{\sigma-1}} \left(\Xi_{C^{H*}}\right)^{\frac{-\kappa}{1-\kappa\eta}} \left(\Xi_{Z^D}\right)^{\frac{-\kappa\eta}{1-\kappa\eta}} \\
Z^I &= \Xi_{Z^I} (M)^{-\kappa} \\
\Xi_{Z^I} &= \left(\Psi_{Z^I}\right)^{\frac{1}{\sigma-1}} \left(\Xi_{C^{H*}}\right)^{\frac{-\kappa}{1-\kappa\eta}} \left(\Xi_{Z^D}\right)^{\frac{-\kappa\eta}{1-\kappa\eta}}
\end{aligned} \tag{A.2.5}$$

where $\theta\sigma + 1 - \sigma < 0$ and $\sigma > 1$. We define $\kappa \equiv \frac{\theta\sigma+1-\sigma}{\theta(\sigma-1)} < 0$. We also define

$$\begin{aligned}
\Psi_{Z^X} &= \frac{f^X}{(\tau)^{-(\sigma-1)}\left(\frac{1}{\sigma}\right)(\rho)^{\sigma-1}} \\
\Psi_{Z^I} &= \frac{f^{I*} - f^X}{\left[1-(\tau)^{-(\sigma-1)}\right]\left(\frac{1}{\sigma}\right)(\rho)^{\sigma-1}}
\end{aligned} \tag{A.2.6}$$

The rank of cutoff productivity measures satisfies $Z^D < Z^X < Z^I$, implying $\Xi_{Z^D} < \Xi_{Z^X} < \Xi_{Z^I}$ and

$$\Xi_{Z^D} < \left(\Psi_{Z^X}\right)^{\frac{1}{\sigma-1}} \left(\Xi_{C^{H*}}\right)^{\frac{-\kappa}{1-\kappa\eta}} \left(\Xi_{Z^D}\right)^{\frac{-\kappa\eta}{1-\kappa\eta}} < \left(\Psi_{Z^I}\right)^{\frac{1}{\sigma-1}} \left(\Xi_{C^{H*}}\right)^{\frac{-\kappa}{1-\kappa\eta}} \left(\Xi_{Z^D}\right)^{\frac{-\kappa\eta}{1-\kappa\eta}}$$

$$\therefore\ \Psi_{Z^X} < \Psi_{Z^I} \quad \text{implies} \quad \left(f^X\right)(\tau)^{\sigma-1} < \left(f^{I*} - f^X\right)\left(\left[1-(\tau)^{-(\sigma-1)}\right]^{(-1)}\right)$$

Here C^{H*} denotes Foreign consumption on Home goods; A^{H*} Foreign market demand on Home goods; Z^X and Z^I cutoff productivity for Home exporters and multinationals.

A.2.1 Proof of Proposition 1

A.2.1.1 Tax on Consumption

Suppose the Home government finances FDI subsidies only by tax on consumption. For the clear analytical derivation, we reduce the government tax base (8) to $G_C = M \int_{Z^D}^{\infty} p^D(z) y^D(z) \frac{dG(z)}{1-G(Z^D)}$; otherwise, an analytical characterization is infeasible under endogenous mass of firms. Then the government budget balance (7) can be derived as

$$\begin{aligned}
(\tau_C - 1)G_C &= G_S \\
G_C &= M\left(\frac{\eta}{\eta-\sigma+1}\right)(\rho)^{\sigma-1}\left(\Xi_{Z^D}\right)^{\sigma-1}\left(\Xi_{C^H}\right)^{\kappa\theta(\sigma-1)} \\
&= M\left(\frac{\eta}{\eta-\sigma+1}\right)(\sigma)\left(f^D\right)(\tau_C)^{\sigma} \\
&= M\left(\Xi_{C^H}\right)^{\theta}(\tau_C)^{\sigma-1} \\
G_S &= M^*\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)(\rho)^{\sigma}\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}} s_V\left(1-s_V\right)^{-\sigma}\left(\Psi_{Z^{I*}}\right)^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}}\left(\Xi_{C^F}\right)^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}
\end{aligned} \tag{A.2.7}$$

where G_S denotes the government expenditure: $G_S \equiv M^{I*}\int_{Z^{I*}}^{\infty} s_V W l^I(z) \frac{dG^*(z)}{1-G^*(Z^{I*})}$. $\eta^* - \sigma + 1 > 0$ and $\Psi_{Z^{X*}} < \Psi_{Z^{I*}}$ and we make use of $\left(\Xi_{Z^D}\right)^{\sigma-1}\left(\Xi_{C^H}\right)^{\theta\sigma+1-\sigma} = (\tau_C)^{\sigma}(\rho)^{-(\sigma-1)}(\sigma)\left(f^D\right)$.

The derivative of the Home welfare (10) with respect to subsidy is given by

$$\begin{aligned}
\frac{\partial V}{\partial s_V} &= (1-\theta)\left(C^F\right)^{\theta}\left(\frac{1}{C^F}\frac{\partial C^F}{\partial s_V}\right) + (1-\theta)\left(C^H\right)^{\theta}\left(\frac{1}{C^H}\frac{\partial C^H}{\partial s_V}\right) \\
&= \begin{pmatrix}
+\left(\frac{1}{\theta}-1\right)\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}}\left(\frac{1}{1-\kappa\eta^*}\right)\left(\Xi_{C^F}\right)^{\frac{1}{1-\kappa\eta^*}}\left(\frac{1}{\Xi_{C^F}}\frac{\partial \Xi_{C^F}}{\partial s_V}\right)M^* \\
+\left(\frac{1}{\theta}-1\right)\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}}\left(\Xi_{C^F}\right)^{\frac{1}{1-\kappa\eta^*}}\frac{\partial M^*}{\partial s_V} \\
+(1-\theta)\left(\Xi_{C^H}\right)^{\theta}\left(\frac{1}{\Xi_{C^H}}\frac{\partial \Xi_{C^H}}{\partial s_V}\right)M + \left(\frac{1}{\theta}-1\right)\left(\Xi_{C^H}\right)^{\theta}\frac{\partial M}{\partial s_V}
\end{pmatrix}
\end{aligned} \tag{A.2.8}$$

Therefore, in order to establish the sign of $\frac{\partial V}{\partial s_V}$ at $s_V = 0$ and $\tau_C = 1$, we need to derive all derivatives of Ξ and Ψ terms with respect to s_V. From equations (A.2.1), (A.2.2), (A.2.3), (A.2.4), (A.2.5), and (A.2.6), we can derive the first-order derivatives of Ξ and Ψ terms as follows.

$$\begin{aligned}
&\frac{1}{\Xi_{C^H}}\frac{\partial \Xi_{C^H}}{\partial s_V} = \frac{1}{\theta}\frac{1}{\tau_C}\frac{\partial \tau_C}{\partial s_V}, && \frac{1}{\Xi_{Z^D}}\frac{\partial \Xi_{Z^D}}{\partial s_V} = \frac{1}{\theta}\frac{1}{\tau_C}\frac{\partial \tau_C}{\partial s_V}, \\
&\frac{1}{\Psi_{Z^{I*}}}\frac{\partial \Psi_{Z^{I*}}}{\partial s_V} = \frac{-(\sigma-1)(1-s_V)^{-\sigma}}{(1-s_V)^{-(\sigma-1)}-(\tau^*)^{-(\sigma-1)}}, && \frac{\partial \Xi_{C^F}}{\partial s_V} = (\rho)^{\sigma-1}\left(\frac{(\eta^*)^2}{\eta^*-\sigma+1}\right)(1-s_V)^{-\sigma}\left(\Psi_{Z^{I*}}\right)^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}},
\end{aligned} \tag{A.2.9}$$

where the other terms have no dependence on s_V: $0 = \frac{\partial \Psi_{Z^X}}{\partial s_V} = \frac{\partial \Psi_{Z^I}}{\partial s_V} = \frac{\partial \Xi_{C^{H*}}}{\partial s_V} = \frac{\partial \Xi_{C^{F*}}}{\partial s_V} = \frac{\partial \Xi_{Z^{D*}}}{\partial s_V} = \frac{\partial \Psi_{Z^{X*}}}{\partial s_V}$.

From the Home and Foreign free entry conditions derived in the online technical appendix, given by

$$\delta F^{D}\left(M\right)^{-\kappa\eta} = \left(\begin{array}{ll} + & \left(f^{D}\right)\left(\frac{\sigma-1}{\eta-\sigma+1}\right)\left(z_{min}\right)^{\eta}\left(\Xi_{Z^{D}}\right)^{-\eta} \\ + & \left(f^{X}\right)\left(\frac{\sigma-1}{\eta-\sigma+1}\right)\left(z_{min}\right)^{\eta}\left(\Psi_{Z^{X}}\right)^{\frac{-\eta}{\sigma-1}}\left(\Xi_{C^{H*}}\right)^{\frac{\kappa\eta}{1-\kappa\eta}}\left(\Xi_{Z^{D}}\right)^{\frac{\kappa\eta\eta}{1-\kappa\eta}} \\ + & \left(f^{I*}-f^{X}\right)\left(\frac{\sigma-1}{\eta-\sigma+1}\right)\left(z_{min}\right)^{\eta}\left(\Psi_{Z^{I}}\right)^{\frac{-\eta}{\sigma-1}}\left(\Xi_{C^{H*}}\right)^{\frac{\kappa\eta}{1-\kappa\eta}}\left(\Xi_{Z^{D}}\right)^{\frac{\kappa\eta\eta}{1-\kappa\eta}} \end{array}\right),$$

$$\delta F^{D*}\left(M^{*}\right)^{-\kappa\eta^{*}} = \left(\begin{array}{ll} + & \left(f^{D*}\right)\left(\frac{\sigma-1}{\eta^{*}-\sigma+1}\right)\left(z^{*}_{min}\right)^{\eta^{*}}\left(\Xi_{Z^{D*}}\right)^{-\eta^{*}} \\ + & \left(f^{X*}\right)\left(\frac{\sigma-1}{\eta^{*}-\sigma+1}\right)\left(z^{*}_{min}\right)^{\eta^{*}}\left(\Psi_{Z^{X*}}\right)^{\frac{-\eta^{*}}{\sigma-1}}\left(\Xi_{C^{F}}\right)^{\frac{\kappa\eta^{*}}{1-\kappa\eta^{*}}}\left(\Xi_{Z^{D*}}\right)^{\frac{\kappa\eta^{*}\eta^{*}}{1-\kappa\eta^{*}}} \\ + & \left(f^{I}-f^{X*}\right)\left(\frac{\sigma-1}{\eta^{*}-\sigma+1}\right)\left(z^{*}_{min}\right)^{\eta^{*}}\left(\Psi_{Z^{I*}}\right)^{\frac{-\eta^{*}}{\sigma-1}}\left(\Xi_{C^{F}}\right)^{\frac{\kappa\eta^{*}}{1-\kappa\eta^{*}}}\left(\Xi_{Z^{D*}}\right)^{\frac{\kappa\eta^{*}\eta^{*}}{1-\kappa\eta^{*}}} \end{array}\right),$$

we can derive their derivatives as

$$\left(\frac{1}{M}\frac{\partial M}{\partial s_V}\right) = -\frac{\Xi_M}{\theta}\left(\frac{1}{\tau_C}\frac{\partial \tau_C}{\partial s_V}\right), \qquad \left(\frac{1}{M^{*}}\frac{\partial M^{*}}{\partial s_V}\right) = \frac{\Xi_{M_2^{*}}}{\Xi_{M_1^{*}}}, \tag{A.2.10}$$

where coefficients are defined as

$$\Xi_M \equiv \frac{1}{(-\kappa)\delta F^{D}(M)^{-\kappa\eta}}\left(\begin{array}{ll} + & \left(f^{D}\right)\left(\frac{\sigma-1}{\eta-\sigma+1}\right)\left(z_{min}\right)^{\eta}\left(\Xi_{Z^{D}}\right)^{-\eta} \\ + & \left(f^{X}\right)\left(\frac{\sigma-1}{\eta-\sigma+1}\right)\left(z_{min}\right)^{\eta}\left(\Psi_{Z^{X}}\right)^{\frac{-\eta}{\sigma-1}}\left(\Xi_{C^{H*}}\right)^{\frac{\kappa\eta}{1-\kappa\eta}}\left(\Xi_{Z^{D}}\right)^{\frac{\kappa\eta\eta}{1-\kappa\eta}}\left(\frac{-\kappa\eta}{1-\kappa\eta}\right) \\ + & \left(f^{I*}-f^{X}\right)\left(\frac{\sigma-1}{\eta-\sigma+1}\right)\left(z_{min}\right)^{\eta}\left(\Psi_{Z^{I}}\right)^{\frac{-\eta}{\sigma-1}}\left(\Xi_{C^{H*}}\right)^{\frac{\kappa\eta}{1-\kappa\eta}}\left(\Xi_{Z^{D}}\right)^{\frac{\kappa\eta\eta}{1-\kappa\eta}}\left(\frac{-\kappa\eta}{1-\kappa\eta}\right) \end{array}\right),$$

$$\Xi_{M_1^{*}} \equiv \left(\frac{\eta^{*}-\sigma+1}{\sigma-1}\right)\left(z^{*}_{min}\right)^{-\eta^{*}}\left(\Xi_{C^{F}}\right)\left(\Xi_{C^{F}}\right)^{\frac{-\kappa\eta^{*}}{1-\kappa\eta^{*}}}\left(\Xi_{Z^{D*}}\right)^{\frac{-\kappa\eta^{*}\eta^{*}}{1-\kappa\eta^{*}}}\delta F^{D*}(-\kappa)\left(M^{*}\right)^{-\kappa\eta^{*}},$$

$$\Xi_{M_2^{*}} \equiv \left(\begin{array}{l} +\frac{1}{\sigma}\left(\rho\right)^{(\sigma-1)}\left(\Psi_{Z^{X*}}\right)^{\frac{-(\eta^{*}-\sigma+1)}{\sigma-1}}\left(\Psi_{Z^{I*}}\right)^{\frac{-(\eta^{*}-\sigma+1)}{\sigma-1}}\left(1-s_V\right)^{-\sigma}\left(\tau^{*}\right)^{-(\sigma-1)}\left(\rho\right)^{\sigma-1}\left(\frac{\eta^{*}}{\eta^{*}-\sigma+1}\right)\left(\frac{1}{1-\kappa\eta^{*}}\right) \\ +\left(f^{I}-f^{X*}\right)\left(\Psi_{Z^{I*}}\right)^{\frac{-\eta^{*}}{\sigma-1}}\left(\rho\right)^{\sigma-1}\left(\frac{\eta^{*}}{\eta^{*}-\sigma+1}\right)\left(1-s_V\right)^{-\sigma}\left(\Psi_{Z^{I*}}\right)^{\frac{-(\eta^{*}-\sigma+1)}{\sigma-1}}\left(\frac{1}{1-\kappa\eta^{*}}\right) \end{array}\right).$$

Note that $0 < \frac{-\kappa\eta}{1-\kappa\eta} < 1$ implies $\theta < \frac{\eta}{1-\kappa\eta} < \Xi_M < \frac{1}{(-\kappa)}$ due to $\frac{\eta}{1-\kappa\eta} - \theta = \frac{\theta(\sigma\eta-\sigma+1)}{(\sigma-1)(1-\kappa\eta)} > \frac{\theta(\eta-\sigma+1)}{(\sigma-1)(1-\kappa\eta)} > 0$. We also have $\tau_C \geq 1$ and $\frac{2\sigma\eta^{*}}{\sigma-1} = \frac{(\sigma+\sigma)\eta^{*}}{\sigma-1} > 1$.

By substituting out terms $\Psi_{Z^{X*}}$, $\Psi_{Z^{I*}}$, and $\Xi_{C^{F}}$, we can rewrite $\Xi_{M_2^{*}}$ as

$$\Xi_{M_2^{*}} = \Xi_{C^{F}}\frac{1}{\sigma}\left(\Psi_{Z^{I*}}\right)^{\frac{-(\eta^{*}-\sigma+1)}{\sigma-1}}\left(1-s_V\right)^{-\sigma}\left(\rho\right)^{\sigma-1}\left(\frac{1}{1-\kappa\eta^{*}}\right).$$

Therefore we can also rewrite $\left(\frac{1}{M^{*}}\frac{\partial M^{*}}{\partial s_V}\right) = \frac{\Xi_{M_2^{*}}}{\Xi_{M_1^{*}}}$ as

$$\begin{aligned} \left(\frac{1}{M^{*}}\frac{\partial M^{*}}{\partial s_V}\right) &= \frac{\left(\frac{\sigma-1}{\eta^{*}-\sigma+1}\right)\left(z^{*}_{min}\right)^{\eta^{*}}\left(\Psi_{Z^{I*}}\right)^{\frac{-\eta^{*}}{\sigma-1}}\left(\Xi_{C^{F}}\right)^{\frac{\kappa\eta^{*}}{1-\kappa\eta^{*}}}\left(\Xi_{Z^{D*}}\right)^{\frac{\kappa\eta^{*}\eta^{*}}{1-\kappa\eta^{*}}}\left(\frac{1}{\sigma}\right)\left(\Psi_{Z^{I*}}\right)\left(\rho\right)^{\sigma-1}\left(1-s_V\right)^{-\sigma}\left(\frac{1}{1-\kappa\eta^{*}}\right)}{\delta F^{D*}(-\kappa)\left(M^{*}\right)^{-\kappa\eta^{*}}} \\ &= \frac{\left[\left(f^{I}-f^{X*}\right)\left(\frac{\sigma-1}{\eta^{*}-\sigma+1}\right)\left(z^{*}_{min}\right)^{\eta^{*}}\left(\Psi_{Z^{I*}}\right)^{\frac{-\eta^{*}}{\sigma-1}}\left(\Xi_{C^{F}}\right)^{\frac{\kappa\eta^{*}}{1-\kappa\eta^{*}}}\left(\Xi_{Z^{D*}}\right)^{\frac{\kappa\eta^{*}\eta^{*}}{1-\kappa\eta^{*}}}\right]\left(\frac{(1-s_V)^{-\sigma}}{(1-s_V)^{-(\sigma-1)}-(\tau^{*})^{-(\sigma-1)}}\right)\left(\frac{1}{(-\kappa)(1-\kappa\eta^{*})}\right)}{\delta F^{D*}\left(M^{*}\right)^{-\kappa\eta^{*}}}. \end{aligned} \tag{A.2.11}$$

Therefore, $\left(\frac{1}{M^*}\frac{\partial M^*}{\partial s_V}\right)$ is strictly less than $\left(\frac{(1-s_V)^{-\sigma}}{(1-s_V)^{-(\sigma-1)}-(\tau^*)^{-(\sigma-1)}}\right)\left(\frac{1}{(-\kappa)(1-\kappa\eta^*)}\right)$ due to the Foreign free entry condition.

Finally, from the government budget balance (A.2.7), we derive the marginal change of the consumption tax wedge τ_C with respect to the change in the subsidy s_V. The differentiation of (A.2.7) implies

$$\frac{\partial \tau_C}{\partial s_V}G_C+(\tau_C-1)\frac{\partial G_C}{\partial s_V}=\frac{\partial G_S}{\partial s_V},$$

where we can derive

$$\begin{aligned}
\frac{\partial G_C}{\partial s_V} &= G_C\left(-\frac{\Xi_M}{\theta}\left(\frac{1}{\tau_C}\frac{\partial \tau_C}{\partial s_V}\right)\right)+G_C(\sigma)\left(\frac{1}{\tau_C}\frac{\partial \tau_C}{\partial s_V}\right)=-G_C\left(\frac{\Xi_M}{\theta}-\sigma\right)\left(\frac{1}{\tau_C}\frac{\partial \tau_C}{\partial s_V}\right),\\
\frac{\partial G_S}{\partial s_V} &= G_S\left[\ \left(\frac{1}{M^*}\frac{\partial M^*}{\partial s_V}\right)+\left(\frac{1}{s_V}\right)+\left(\frac{\sigma}{1-s_V}\right)+\left(\frac{-(\eta^*-\sigma+1)}{\sigma-1}\right)\left(\frac{1}{\Psi_{Z^{I*}}}\frac{\partial \Psi_{Z^{I*}}}{\partial s_V}\right)+\left(\frac{\kappa\eta^*}{1-\kappa\eta^*}\right)\left(\frac{1}{\Xi_{C^F}}\frac{\partial \Xi_{C^F}}{\partial s_V}\right)\ \right]\\
&= G_S\left[\begin{array}{l}
+\frac{\Xi_{M_2^*}}{\Xi_{M_1^*}}\\
+\left(\frac{1}{s_V}\right)+\left(\frac{\sigma}{1-s_V}\right)+\left(\frac{-(\eta^*-\sigma+1)}{\sigma-1}\right)\left(\frac{-(\sigma-1)(1-s_V)^{-\sigma}}{(1-s_V)^{-(\sigma-1)}-(\tau^*)^{-(\sigma-1)}}\right)\\
+\left(\frac{\kappa\eta^*}{1-\kappa\eta^*}\right)\left(\frac{1}{\Xi_{C^F}}\right)(\rho)^{\sigma-1}\left(\frac{(\eta^*)^2}{\eta^*-\sigma+1}\right)(1-s_V)^{-\sigma}\left(\Psi_{Z^{I*}}\right)^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}}
\end{array}\right],
\end{aligned}$$

where $\kappa\equiv\frac{\theta\sigma+1-\sigma}{\theta(\sigma-1)}<0$. Feeding $\frac{\partial G_C}{\partial s_V}$ and $\frac{\partial G_S}{\partial s_V}$ and substituting out G_C and G_S, we obtain $\frac{\partial \tau_C}{\partial s_V}$:

$$\begin{aligned}
&\left(\frac{1}{\tau_C}\frac{\partial \tau_C}{\partial s_V}\right)\left((\Xi_{C^H})^\theta M\right)\left(\ (\tau_C)^\sigma-\left((\tau_C)^\sigma-(\tau_C)^{\sigma-1}\right)\left(\frac{\Xi_M}{\theta}-\sigma\right)\ \right)=\\
&\left(\begin{array}{l}
+M^*\left(\frac{(\rho)^\sigma\eta^*}{\eta^*-\sigma+1}\right)(\Xi_{Z^{D*}})^{\frac{\eta^*}{1-\kappa\eta^*}}(s_V)(1-s_V)^{-\sigma}(\Psi_{Z^{I*}})^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}}(\Xi_{C^F})^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}\left(\frac{\Xi_{M_2^*}}{\Xi_{M_1^*}}\right)\\
+M^*\left(\frac{(\rho)^\sigma\eta^*}{\eta^*-\sigma+1}\right)(\Xi_{Z^{D*}})^{\frac{\eta^*}{1-\kappa\eta^*}}(s_V)(1-s_V)^{-\sigma}(\Psi_{Z^{I*}})^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}}(\Xi_{C^F})^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}\left(\frac{1}{s_V}\right)\\
+M^*\left(\frac{(\rho)^\sigma\eta^*}{\eta^*-\sigma+1}\right)(\Xi_{Z^{D*}})^{\frac{\eta^*}{1-\kappa\eta^*}}(s_V)(1-s_V)^{-\sigma}(\Psi_{Z^{I*}})^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}}(\Xi_{C^F})^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}\left(\frac{\sigma}{1-s_V}\right)\\
+M^*\left(\frac{(\rho)^\sigma\eta^*}{\eta^*-\sigma+1}\right)(\Xi_{Z^{D*}})^{\frac{\eta^*}{1-\kappa\eta^*}}(s_V)(1-s_V)^{-\sigma}(\Psi_{Z^{I*}})^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}}(\Xi_{C^F})^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}\left(\frac{(\eta^*-\sigma+1)(1-s_V)^{-\sigma}}{(1-s_V)^{-(\sigma-1)}-(\tau^*)^{-(\sigma-1)}}\right)\\
+M^*\left(\frac{(\rho)^\sigma\eta^*}{\eta^*-\sigma+1}\right)(\Xi_{Z^{D*}})^{\frac{\eta^*}{1-\kappa\eta^*}}(s_V)(1-s_V)^{-\sigma}(\Psi_{Z^{I*}})^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}}(\Xi_{C^F})^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}\left(\frac{\kappa\eta^*(1-s_V)^{-\sigma}}{(1-\kappa\eta^*)\Xi_{C^F}}\right)\left(\frac{(\rho)^{\sigma-1}(\eta^*)^2}{\eta^*-\sigma+1}\right)(\Psi_{Z^{I*}})^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}}
\end{array}\right)
\end{aligned}\tag{A.2.12}$$

where $\Psi_{\tau_C}\equiv\frac{1}{(\tau_C)^\sigma-\left((\tau_C)^\sigma-(\tau_C)^{\sigma-1}\right)\left(\frac{\Xi_M}{\theta}-\sigma\right)}$. Now we are ready to characterize $\frac{\partial V}{\partial s_V}$. Together (A.2.8), (A.2.9), and (A.2.10) imply

$$\begin{aligned}
\frac{\partial V}{\partial s_V} &= (1-\theta)\left(C^F\right)^{\theta-1}\frac{\partial C^F}{\partial s_V}+(1-\theta)\left(C^H\right)^{\theta-1}\frac{\partial C^H}{\partial s_V}\\
&=\left(\begin{array}{l}
+\left(\frac{1}{\theta}-1\right)(\Xi_{Z^{D*}})^{\frac{\eta^*}{1-\kappa\eta^*}}\left(\frac{1}{1-\kappa\eta^*}\right)(\Xi_{C^F})^{\frac{1}{1-\kappa\eta^*}}\left(\frac{1}{\Xi_{C^F}}\frac{\partial \Xi_{C^F}}{\partial s_V}\right)M^*+\left(\frac{1}{\theta}-1\right)(\Xi_{Z^{D*}})^{\frac{\eta^*}{1-\kappa\eta^*}}(\Xi_{C^F})^{\frac{1}{1-\kappa\eta^*}}\frac{\partial M^*}{\partial s_V}\\
+(1-\theta)(\Xi_{C^H})^\theta\left(\frac{1}{\Xi_{C^H}}\frac{\partial \Xi_{C^H}}{\partial s_V}\right)M+\left(\frac{1}{\theta}-1\right)(\Xi_{C^H})^\theta\frac{\partial M}{\partial s_V}
\end{array}\right)\\
&=\left(\begin{array}{l}
+\left(\frac{1}{\theta}-1\right)(\Xi_{Z^{D*}})^{\frac{\eta^*}{1-\kappa\eta^*}}(\Xi_{C^F})^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}(\rho)^{\sigma-1}\left(\frac{1}{1-\kappa\eta^*}\right)\left(\frac{(\eta^*)^2}{\eta^*-\sigma+1}\right)(1-s_V)^{-\sigma}(\Psi_{Z^{I*}})^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}}M^*\\
+\left(\frac{1}{\theta}-1\right)(\Xi_{Z^{D*}})^{\frac{\eta^*}{1-\kappa\eta^*}}(\Xi_{C^F})^{\frac{1}{1-\kappa\eta^*}}(M^*)\frac{\Xi_{M_2^*}}{\Xi_{M_1^*}}-\left(\frac{1}{\theta}-1\right)(\Xi_{C^H})^\theta(M)\left(\frac{1}{\tau_C}\frac{\partial \tau_C}{\partial s_V}\right)\left(\frac{\Xi_M}{\theta}-1\right)
\end{array}\right).
\end{aligned}$$

Making use of (A.2.12), we remove $\left(\frac{1}{\tau_C}\frac{\partial \tau_C}{\partial s_V}\right)$ in the above equation to get

$$\frac{\partial V}{\partial s_V} =$$
$$\left(\begin{array}{l}
-\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}}\left(\Xi_{C^F}\right)^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}\left(\frac{(\rho)^\sigma\eta^*}{\eta^*-\sigma+1}\right)M^*\left(\Psi_{Z^{I*}}\right)^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}}\frac{(1-\theta)\Psi_{\tau_C}\left(\frac{\Xi_M}{\theta}-1\right)}{\theta}\left(1-s_V\right)^{-\sigma} \\
-\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}}\left(\Xi_{C^F}\right)^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}\left(\frac{(\rho)^\sigma\eta^*}{\eta^*-\sigma+1}\right)M^*\left(\Psi_{Z^{I*}}\right)^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}}\frac{(1-\theta)\Psi_{\tau_C}\left(\frac{\Xi_M}{\theta}-1\right)}{\theta}\left(1-s_V\right)^{-\sigma}\left(\frac{\sigma(s_V)}{1-s_V}\right) \\
-\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}}\left(\Xi_{C^F}\right)^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}\left(\frac{(\rho)^\sigma\eta^*}{\eta^*-\sigma+1}\right)M^*\left(\Psi_{Z^{I*}}\right)^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}}\frac{(1-\theta)\Psi_{\tau_C}\left(\frac{\Xi_M}{\theta}-1\right)}{\theta}\left(1-s_V\right)^{-\sigma}\left(\frac{(\eta^*-\sigma+1)(1-s_V)^{-\sigma}}{(1-s_V)^{-(\sigma-1)}-(\tau^*)^{-(\sigma-1)}}\right)(s_V) \\
-\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}}\left(\Xi_{C^F}\right)^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}\left(\frac{(\rho)^\sigma\eta^*}{\eta^*-\sigma+1}\right)M^*\left(\Psi_{Z^{I*}}\right)^{\frac{-2(\eta^*-\sigma+1)}{\sigma-1}}\frac{(1-\theta)\Psi_{\tau_C}\left(\frac{\Xi_M}{\theta}-1\right)}{\theta}\left(\frac{\kappa\eta^*}{(1-\kappa\eta^*)}\right)\frac{(\rho)^{\sigma-1}}{\Xi_{C^F}}\left(\frac{(\eta^*)^2}{\eta^*-\sigma+1}\right)(s_V)\left(1-s_V\right)^{-2\sigma} \\
-\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}}\left(\Xi_{C^F}\right)^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}\left(\frac{(\rho)^\sigma\eta^*}{\eta^*-\sigma+1}\right)M^*\left(\Psi_{Z^{I*}}\right)^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}}\frac{(1-\theta)\Psi_{\tau_C}\left(\frac{\Xi_M}{\theta}-1\right)}{\theta}(s_V)\left(1-s_V\right)^{-\sigma}\left(\frac{\Xi_{M_2^*}}{\Xi_{M_1^*}}\right) \\
+\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}}\left(\Xi_{C^F}\right)^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}\left(\frac{(\rho)^\sigma\eta^*}{\eta^*-\sigma+1}\right)M^*\left(\Psi_{Z^{I*}}\right)^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}}\left(1-s_V\right)^{-\sigma}\left(\frac{(1-\theta)\eta^*}{\theta(1-\kappa\eta^*)\rho}\right) \\
+\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}}\left(\Xi_{C^F}\right)^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}\Xi_{C^F}\frac{(1-\theta)}{\theta}\left(M^*\right)\frac{\Xi_{M_2^*}}{\Xi_{M_1^*}}
\end{array}\right)$$

where $\left(\frac{1}{M}\frac{\partial M}{\partial s_V}\right) = -\frac{\Xi_M}{\theta}\left(\frac{1}{\tau_C}\frac{\partial \tau_C}{\partial s_V}\right)$ and $\Psi_{\tau_C} \equiv \frac{1}{(\tau_C)^\sigma - \left((\tau_C)^\sigma-(\tau_C)^{\sigma-1}\right)\left(\frac{\Xi_M}{\theta}-\sigma\right)} > 0$ is positive in the reasonable parametrization. Therefore $\frac{\partial V}{\partial s_V}|_{s_V=0}$ under $s_V = 0$ and $\tau_C = 1$ is derived as

$$\frac{\partial V}{\partial s_V}|_{s_V=0}$$
$$= \left(\begin{array}{l}
-\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}}\left(\Xi_{C^F}\right)^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}\left(\frac{(\rho)^\sigma\eta^*}{\eta^*-\sigma+1}\right)M^*\left(\Psi_{Z^{I*}}\right)^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}}\frac{(1-\theta)\Psi_{\tau_C}\left(\frac{\Xi_M}{\theta}-1\right)}{\theta} \\
+\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}}\left(\Xi_{C^F}\right)^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}\left(\frac{(\rho)^\sigma\eta^*}{\eta^*-\sigma+1}\right)M^*\left(\Psi_{Z^{I*}}\right)^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}}\left(\frac{(1-\theta)\eta^*}{\theta(1-\kappa\eta^*)\rho}\right) \\
+\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}}\left(\Xi_{C^F}\right)^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}\Xi_{C^F}\frac{(1-\theta)}{\theta}\left(M^*\right)\frac{\Xi_{M_2^*}}{\Xi_{M_1^*}}
\end{array}\right)$$
$$= \left(\begin{array}{l}
+\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}}\left(\Xi_{C^F}\right)^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}\left(\frac{(\rho)^\sigma\eta^*}{\eta^*-\sigma+1}\right)M^*\left(\Psi_{Z^{I*}}\right)^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}}\left[\frac{(1-\theta)\eta^*}{\theta(1-\kappa\eta^*)\rho}-\frac{(1-\theta)\Psi_{\tau_C}\left(\frac{\Xi_M}{\theta}-1\right)}{\theta}\right] \\
+\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}}\left(\Xi_{C^F}\right)^{\frac{1}{1-\kappa\eta^*}}\frac{(1-\theta)}{\theta}\left(\frac{\partial M^*}{\partial s_V}\right)
\end{array}\right)$$

where $\Psi_{\tau_C} = 1$ if $s_V = 0$ and $\tau_C = 1$. Note that $0 < \frac{-\kappa\eta}{1-\kappa\eta} < 1$ and $\theta < \frac{\eta}{1-\kappa\eta} < \Xi_M < \frac{1}{(-\kappa)}$, where $\frac{\eta}{1-\kappa\eta} - \theta = \frac{\theta(\sigma\eta-\sigma+1)}{(\sigma-1)(1-\kappa\eta)} > \frac{\theta(\eta-\sigma+1)}{(\sigma-1)(1-\kappa\eta)} > 0$ due to

$$\Xi_M = \frac{1}{(-\kappa)\delta F^D(M)^{-\kappa\eta}}\left(\begin{array}{ll}
+ & \left(f^D\right)\left(\frac{\sigma-1}{\eta-\sigma+1}\right)\left(z_{min}\right)^\eta\left(\Xi_{Z^D}\right)^{-\eta} \\
+ & \left(f^X\right)\left(\frac{\sigma-1}{\eta-\sigma+1}\right)\left(z_{min}\right)^\eta\left(\Psi_{Z^X}\right)^{\frac{-\eta}{\sigma-1}}\left(\Xi_{C^{H*}}\right)^{\frac{\kappa\eta}{1-\kappa\eta}}\left(\Xi_{Z^D}\right)^{\frac{\kappa\eta\eta}{1-\kappa\eta}}\left(\frac{-\kappa\eta}{1-\kappa\eta}\right) \\
+ & \left(f^{I*}-f^X\right)\left(\frac{\sigma-1}{\eta-\sigma+1}\right)\left(z_{min}\right)^\eta\left(\Psi_{Z^I}\right)^{\frac{-\eta}{\sigma-1}}\left(\Xi_{C^{H*}}\right)^{\frac{\kappa\eta}{1-\kappa\eta}}\left(\Xi_{Z^D}\right)^{\frac{\kappa\eta\eta}{1-\kappa\eta}}\left(\frac{-\kappa\eta}{1-\kappa\eta}\right)
\end{array}\right),$$

$$\left(\frac{1}{M^*}\frac{\partial M^*}{\partial s_V}\right) = \frac{\left[\left(f^I-f^{X*}\right)\left(\frac{\sigma-1}{\eta^*-\sigma+1}\right)\left(z^*_{min}\right)^{\eta^*}\left(\Psi_{Z^{I*}}\right)^{\frac{-\eta^*}{\sigma-1}}\left(\Xi_{C^F}\right)^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}\left(\Xi_{Z^{D*}}\right)^{\frac{\kappa\eta^*\eta^*}{1-\kappa\eta^*}}\right]\left(\frac{(1-s_V)^{-\sigma}}{(1-s_V)^{-(\sigma-1)}-(\tau^*)^{-(\sigma-1)}}\right)\left(\frac{1}{(-\kappa)(1-\kappa\eta^*)}\right)}{\delta F^{D*}(M^*)^{-\kappa\eta^*}}.$$

Note that we have

$$\left(\frac{(1-\theta)}{\theta}\right)\left(\frac{\eta}{(1-\kappa\eta)\theta}-1\right) < \frac{(1-\theta)\Psi_{\tau_C}\left(\frac{\Xi_M}{\theta}-1\right)}{\theta} = \left(\frac{(1-\theta)}{\theta}\right)\left(\frac{\Xi_M}{\theta}-1\right) < \left(\frac{(1-\theta)}{\theta}\right)\left(\frac{1}{(-\kappa)\theta}-1\right)$$

where $\kappa \equiv \frac{\theta\sigma+1-\sigma}{\theta(\sigma-1)} < 0$, $\sigma > 1$, $\theta < 1$, and $0 < \rho = \frac{\sigma-1}{\sigma} < 1$. We assume $\theta < \frac{1}{2}$, $\theta < \rho = \frac{\sigma-1}{\sigma}$, and $\eta^* = \eta$. The assumption of $\theta < \rho = \frac{\sigma-1}{\sigma}$ is equivalent to $\theta\sigma + 1 - \sigma < 0$. A simple parameter relation for the positivity of $\frac{\partial V}{\partial s_V}|_{s_V=0} > 0$ is not available. However, we can establish $\frac{\partial V}{\partial s_V}|_{s_V=0}$ is lower under consumption tax than under labor income tax as follows. $\frac{\partial V}{\partial s_V}|_{s_V=0}$ is given by

$$\frac{\partial V}{\partial s_V}|_{s_V=0} = \left(\begin{array}{l} +\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}}\left(\Xi_{C^F}\right)^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}\left(\frac{(\rho)^\sigma\eta^*}{\eta^*-\sigma+1}\right)M^*\left(\Psi_{Z^{I*}}\right)^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}}\left[\frac{(1-\theta)\eta^*}{\theta(1-\kappa\eta^*)\rho}-\frac{(1-\theta)\Psi_{\tau_C}\left(\frac{\Xi_M}{\theta}-1\right)}{\theta}\right] \\ +\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}}\left(\Xi_{C^F}\right)^{\frac{1}{1-\kappa\eta^*}}\frac{(1-\theta)}{\theta}\left(\frac{\partial M^*}{\partial s_V}\right) \end{array}\right).$$

In this equation, the term $\frac{(1-\theta)\Psi_{\tau_C}\left(\frac{\Xi_M}{\theta}-1\right)}{\theta}$ can be shown to be greater than unity:

$$\begin{aligned}\frac{(1-\theta)\Psi_{\tau_C}\left(\frac{\Xi_M}{\theta}-1\right)}{\theta} = \left(\frac{(1-\theta)}{\theta}\right)\left(\frac{\Xi_M}{\theta}-1\right) &> \left(\frac{(1-\theta)}{\theta}\right)\left(\frac{\eta}{(1-\kappa\eta)\theta}-1\right) \\ &= \left(\frac{(1-\theta)}{\theta}\right)\left(\frac{\eta(\sigma-1)}{\theta(\sigma-1)-(\theta\sigma+1-\sigma)\eta}-1\right) \\ &= \left(\frac{(1-\theta)}{\theta}\right)\left(\frac{\eta(\sigma-1)-\theta(\sigma-1)+(\theta\sigma+1-\sigma)\eta}{\theta(\sigma-1)-(\theta\sigma+1-\sigma)\eta}\right) \\ &= 1+\left(\frac{\eta-\sigma+1}{\theta(\sigma-1)+(\sigma-1-\theta\sigma)\eta}\right) > 1,\end{aligned}$$

where we utilize $\eta > \sigma - 1$, $\sigma > 1$, and $\sigma - 1 - \theta\sigma > 0$ from the assumption of $\rho > \theta$. As a result, the following inequalities hold true:

$$\left[\frac{(1-\theta)\eta^*}{\theta(1-\kappa\eta^*)\rho}-\frac{(1-\theta)\Psi_{\tau_C}\left(\frac{\Xi_M}{\theta}-1\right)}{\theta}\right] < \left[\frac{(1-\theta)\eta^*}{\theta(1-\kappa\eta^*)\rho}-\left(\frac{(1-\theta)}{\theta}\right)\left(\frac{\eta}{(1-\kappa\eta)\theta}-1\right)\right] < \left[\frac{(1-\theta)\eta^*}{\theta(1-\kappa\eta^*)\rho}-1\right].$$

Therefore, the increase in Home welfare at $s_V = 0$ is smaller under the consumption tax than under the labor income tax.[2] This completes the proof of the proposition 1 under the consumption tax.

[2] The online technical appendix provides the derivation of $\frac{\partial V}{\partial s_V}|_{s_V=0}$ under the labor income tax which is reproduced here:

$$\frac{\partial V}{\partial s_V}|_{s_V=0} = \left(\begin{array}{l} +\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}}\left(\Xi_{C^F}\right)^{\frac{\kappa\eta^*}{1-\kappa\eta^*}}\left(\frac{(\rho)^\sigma\eta^*}{\eta^*-\sigma+1}\right)M^*\left(\Psi_{Z^{I*}}\right)^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}}\left[\frac{(1-\theta)\eta^*}{\theta(1-\kappa\eta^*)\rho}-1\right] \\ +\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}}\left(\Xi_{C^F}\right)^{\frac{1}{1-\kappa\eta^*}}\frac{(1-\theta)}{\theta}\left(\frac{\partial M^*}{\partial s_V}\right) \end{array}\right).$$

Note that $\frac{\partial V}{\partial s_V}|_{s_V=0}$ under the consumption tax is different from that under the labor income tax only due to the presence of the term $\frac{(1-\theta)\Psi_{\tau_C}\left(\frac{\Xi_M}{\theta}-1\right)}{\theta}$.

A.2.1.2 Tax on Firm Revenues

Notice that the equilibrium conditions under tax on firm revenues are exactly symmetric to those under tax on consumption. Therefore, we follow the exactly same procedure to derive

$$\frac{\partial V}{\partial s_V}|_{s_V=0} = \left(\begin{array}{l} +\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}} \left(\Xi_{C^F}\right)^{\frac{\kappa\eta^*}{1-\kappa\eta^*}} \left(\frac{(\rho)^\sigma \eta^*}{\eta^*-\sigma+1}\right) M^* \left(\Psi_{Z^{I*}}\right)^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}} \left[\frac{(1-\theta)\eta^*}{\theta(1-\kappa\eta^*)\rho} - \frac{(1-\theta)\Psi_{\tau_R}\left(\frac{\Xi_M}{\theta}-1\right)}{\theta}\right] \\ +\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}} \left(\Xi_{C^F}\right)^{\frac{1}{1-\kappa\eta^*}} \frac{(1-\theta)}{\theta} \left(\frac{\partial M^*}{\partial s_V}\right) \end{array} \right),$$

where $\Psi_{\tau_R} \equiv \frac{1}{(\tau_R)^{-\sigma} - \left((\tau_R)^{-\sigma} - (\tau_R)^{1-\sigma}\right)\left(\frac{\Xi_M}{\theta} - \sigma\right)}$. As in the case with consumption tax, we can show the term $\frac{(1-\theta)\Psi_{\tau_R}\left(\frac{\Xi_M}{\theta}-1\right)}{\theta}$ is greater than unity:

$$\begin{aligned} \frac{(1-\theta)\Psi_{\tau_R}\left(\frac{\Xi_M}{\theta}-1\right)}{\theta} = \left(\frac{(1-\theta)}{\theta}\right)\left(\frac{\Xi_M}{\theta}-1\right) \quad &> \left(\frac{(1-\theta)}{\theta}\right)\left(\frac{\eta}{(1-\kappa\eta)\theta}-1\right) \\ &= \left(\frac{(1-\theta)}{\theta}\right)\left(\frac{\eta(\sigma-1)}{\theta(\sigma-1)-(\theta\sigma+1-\sigma)\eta}-1\right) \\ &= \left(\frac{(1-\theta)}{\theta}\right)\left(\frac{\eta(\sigma-1)-\theta(\sigma-1)+(\theta\sigma+1-\sigma)\eta}{\theta(\sigma-1)-(\theta\sigma+1-\sigma)\eta}\right) \\ &= 1 + \left(\frac{\eta-\sigma+1}{\theta(\sigma-1)+(\sigma-1-\theta\sigma)\eta}\right) > 1, \end{aligned}$$

where we utilize $\eta > \sigma - 1$, $\sigma > 1$, and $\sigma - 1 - \theta\sigma > 0$ from $\rho > \theta$. As a result, the following inequalities hold true:

$$\left[\frac{(1-\theta)\eta^*}{\theta(1-\kappa\eta^*)\rho} - \frac{(1-\theta)\Psi_{\tau_C}\left(\frac{\Xi_M}{\theta}-1\right)}{\theta}\right] < \left[\frac{(1-\theta)\eta^*}{\theta(1-\kappa\eta^*)\rho} - \left(\frac{(1-\theta)}{\theta}\right)\left(\frac{\eta}{(1-\kappa\eta)\theta}-1\right)\right] < \left[\frac{(1-\theta)\eta^*}{\theta(1-\kappa\eta^*)\rho} - 1\right].$$

Therefore, the increase in Home welfare at $s_V = 0$ is smaller under corporate revenue tax than under labor income tax.[3] This completes the proof of the proposition 1 under the corporate revenue tax.

[3] The online technical appendix provides the derivation of $\frac{\partial V}{\partial s_V}|_{s_V=0}$ under the labor income tax which is reproduced here:

$$\frac{\partial V}{\partial s_V}|_{s_V=0} = \left(\begin{array}{l} +\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}} \left(\Xi_{C^F}\right)^{\frac{\kappa\eta^*}{1-\kappa\eta^*}} \left(\frac{(\rho)^\sigma \eta^*}{\eta^*-\sigma+1}\right) M^* \left(\Psi_{Z^{I*}}\right)^{\frac{-(\eta^*-\sigma+1)}{\sigma-1}} \left[\frac{(1-\theta)\eta^*}{\theta(1-\kappa\eta^*)\rho} - 1\right] \\ +\left(\Xi_{Z^{D*}}\right)^{\frac{\eta^*}{1-\kappa\eta^*}} \left(\Xi_{C^F}\right)^{\frac{1}{1-\kappa\eta^*}} \frac{(1-\theta)}{\theta} \left(\frac{\partial M^*}{\partial s_V}\right) \end{array} \right).$$

Note that $\frac{\partial V}{\partial s_V}|_{s_V=0}$ under the corporate revenue tax is different from that under the labor income tax only due to the presence of the term $\frac{(1-\theta)\Psi_{\tau_R}\left(\frac{\Xi_M}{\theta}-1\right)}{\theta}$.

A.2.2 Proof of Proposition 2

As in the proof of proposition 1 in the previous section, the proof under the corporate revenue tax exhibits the exactly same procedure with that under the consumption tax. Hence, in this appendix we only provide the proof of proposition 2 under the consumption tax. The Foreign indirect utility is given by

$$V^* = L + \left(\tfrac{1}{\theta} - 1\right)\left(C^{F*}\right)^{\theta} + \left(\tfrac{1}{\theta} - 1\right)\left(C^{H*}\right)^{\theta},$$

which can be differentiated with respect to s_V as

$$\frac{\partial V^*}{\partial s_V} = (1-\theta)\left(C^{F*}\right)^{\theta}\left(\frac{1}{C^{F*}}\frac{\partial C^{F*}}{\partial s_V}\right) + (1-\theta)\left(C^{H*}\right)^{\theta}\left(\frac{1}{C^{H*}}\frac{\partial C^{H*}}{\partial s_V}\right).$$

From relations (A.2.2) and (A.2.9), we obtain

$$\frac{1}{C^{F*}}\frac{\partial C^{F*}}{\partial s_V} = \frac{1}{\theta}\left(\frac{1}{M^*}\frac{\partial M^*}{\partial s_V}\right),$$

where $\theta\sigma + 1 - \sigma < 0$ and $\sigma > 1$. We define $\kappa \equiv \frac{\theta\sigma+1-\sigma}{\theta(\sigma-1)} < 0$. Since we have shown $\frac{\partial M^*}{\partial s_V} > 0$ in the previous section, $\frac{\partial C^{F*}}{\partial s_V} > 0$ holds. In addition, from (A.2.5) and (A.2.9), we can derive

$$\left(\frac{1}{C^{H*}}\frac{\partial C^{H*}}{\partial s_V}\right) = \frac{\eta}{\theta-\theta\kappa\eta}\left(\frac{1}{\Xi_{ZD}}\frac{\partial \Xi_{ZD}}{\partial s_V}\right) + \frac{1}{\theta}\left(\frac{1}{M}\frac{\partial M}{\partial s_V}\right).$$

Using these two relations, we rewrite $\frac{\partial V^*}{\partial s_V}$ as

$$\begin{aligned}
\frac{\partial V^*}{\partial s_V} &= (1-\theta)\left(C^{F*}\right)^{\theta}\left(\frac{1}{C^{F*}}\frac{\partial C^{F*}}{\partial s_V}\right) + (1-\theta)\left(C^{H*}\right)^{\theta}\left(\frac{1}{C^{H*}}\frac{\partial C^{H*}}{\partial s_V}\right) \\
&= \begin{pmatrix} +(1-\theta)\left(C^{F*}\right)^{\theta}\left(\frac{1}{\theta}\left(\frac{1}{M^*}\frac{\partial M^*}{\partial s_V}\right)\right) \\ +(1-\theta)\left(C^{H*}\right)^{\theta}\frac{\eta}{\theta-\theta\kappa\eta}\left(\frac{1}{\Xi_{ZD}}\frac{\partial \Xi_{ZD}}{\partial s_V}\right) \\ +(1-\theta)\left(C^{H*}\right)^{\theta}\frac{1}{\theta}\left(\frac{1}{M}\frac{\partial M}{\partial s_V}\right) \end{pmatrix} = \begin{pmatrix} +\left(\frac{1}{\theta}-1\right)\left(C^{F*}\right)^{\theta}\left(\frac{1}{M^*}\frac{\partial M^*}{\partial s_V}\right) \\ +\left(\frac{1}{\theta}-1\right)\left(C^{H*}\right)^{\theta}\frac{\eta}{\theta-\theta\kappa\eta}\left(\frac{1}{\tau_C}\frac{\partial \tau_C}{\partial s_V}\right) \\ +\left(\frac{1}{\theta}-1\right)\left(C^{H*}\right)^{\theta}\left(-\frac{\Xi_M}{\theta}\right)\left(\frac{1}{\tau_C}\frac{\partial \tau_C}{\partial s_V}\right) \end{pmatrix} \\
&= \begin{pmatrix} +\left(\frac{1}{\theta}-1\right)\left(C^{F*}\right)^{\theta}\left(\frac{1}{M^*}\frac{\partial M^*}{\partial s_V}\right) \\ -\left(\frac{1}{\theta}-1\right)\left(C^{H*}\right)^{\theta}\left(\frac{\Xi_M}{\theta}-\frac{\eta}{\theta-\theta\kappa\eta}\right)\left(\frac{1}{\tau_C}\frac{\partial \tau_C}{\partial s_V}\right) \end{pmatrix},
\end{aligned}$$

where $\theta \in (0,1)$ and the third equality holds from (A.2.9). Note that $1 < \frac{\eta}{\theta-\theta\kappa\eta} < \frac{\Xi_M}{\theta} < \frac{1}{(-\kappa)\theta}$ and $\left(\frac{1}{\tau_C}\frac{\partial \tau_C}{\partial s_V}\right) > 0$ hold true whenever $s_V > 0$. Therefore, $\frac{\partial V^*}{\partial s_V}$ under the consumption tax is always smaller than that under the labor income tax since $\frac{\partial V^*}{\partial s_V}$ reduces to $\left(\frac{1}{\theta}-1\right)\left(C^{F*}\right)^{\theta}\left(\frac{1}{M^*}\frac{\partial M^*}{\partial s_V}\right)$ under the labor income tax as shown in the online technical appendix. This completes the proof of proposition 2 under the consumption tax.