Two Country Model

There are two countries, Country 1 and Country 2, and a single input labor$l$, and there are two goods, good X and good Y which are produced in both countries separately using their own labor input using the technology

$$\begin{aligned} x_i &= f_i^X(l_i^X) \\ y_i &= f_i^Y(l_i^Y) \quad \forall i \in \{1, 2\} \end{aligned}$$

There is an international market for X and Y where X and Y can be traded. The are single prices for these two in the whole world p_X p_Y. There are separate labor markets in both countries and the labor can not work in each other's countries, threfore there are also wage rates w_1 and w_2 in country 1 and country 2 respectively.

There is also an autarchy case in which the goods as well as the labor can not be traded and therefore it is like the standard Crusore economy type case.

The competitive equilibrium with internation trade

The competitive equilibrium with internation trade consists of $(p_X^*, p_Y^*, \omega_1^*, \omega_2^*) \in \mathbb{R}_+^4$ and $\left(l_1^{X^*}, l_1^{Y^*}\right), (x_1^*, y_1^*), \left(x_1^{c^*}, y_1^{c^*}\right), \left(l_2^{X^*}, l_2^{Y^*}\right), (x_2^*, y_2^*), \left(x_2^{c^*}, y_2^{c^*}\right)$, such that,

1. $$(l_i^{X^*}, x_i^*) \text{solves} \max_{l_i^X, x_i} p_X^* x_i - \omega_i^* l_i^X \text{s.t.} x_i \leq f_i^X(l_i^X), \text{Let} \pi_i^* = p_X^* x_i^* - \omega_i^* l_i^{X^*} \forall i \in \{1, 2\}$$

2. $$(l_i^{Y^*}, y_i^*) \text{ solves } \max_{l_i^Y, y_i} p_Y^* y_i - \omega_i^* l_i^Y \text{ s.t. } x_i \leq f_i^Y(l_i^Y), \text{ Let } \pi_i^* = p_Y^* y_i^* - \omega_i^* l_i^{Y^*} \ \forall i \in \{1, 2\}$$

3. $$(x_i^{c^*}, y_i^{c^*}) \text{ solves } \max_{x_i^c, y_i^c} u_i(x_i^c, y_i^c) \text{ s.t.}$$