

# 1 Mixed Strategy Nash equilibrium

## 1.1 Battle of Sexes

	M	S	$(\frac{1}{2}, \frac{1}{2})$	$(q, 1 - q)$
M	(2,1)	(0,0)	$1, \frac{1}{2}$	$2q, q$
S	(0,0)	(1,2)	$\frac{1}{2}, 1$	$(1 - q), 2(1 - q)$
$(\frac{1}{2}, \frac{1}{2})$	$1, \frac{1}{2}$	$\frac{1}{2}, 1$	$\frac{3}{4}, \frac{3}{4}$	
$(p, 1 - p)$	$2p, p$	$(1 - p), 2(1 - p)$		$(2pq + (1 - p)(1 - q)), (pq + 2(1 - p)(1 - q))$

Row player(Wife) plays the mixed strategy  $(p, 1 - p)$  s.t.  $p \in [0, 1]$ .

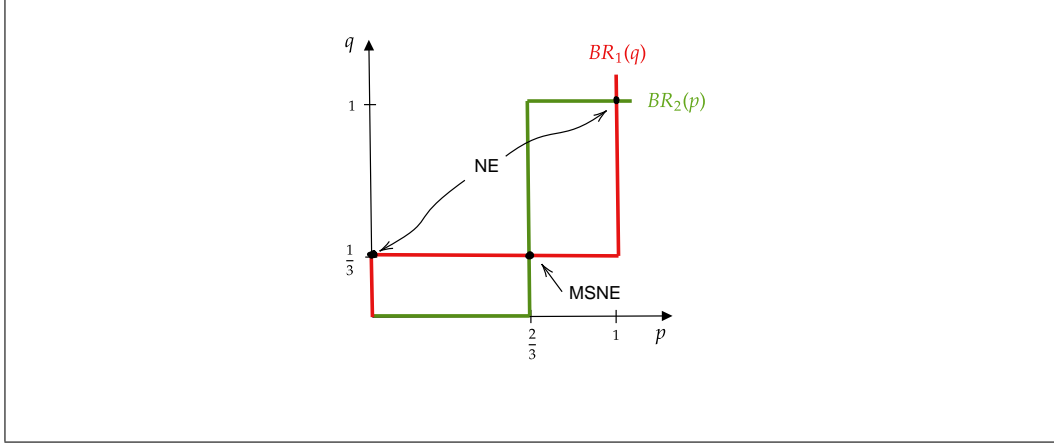
Column player(Husband) plays the mixed strategy  $(q, 1 - q)$  s.t.  $q \in [0, 1]$

$$\begin{aligned}
 A_1 &= [0, 1], A_2 = [0, 1] \\
 U_W(p, q) &= pq + 2(1 - p)(1 - q) = p(2q) + (1 - p)(1 - q) \\
 U_H(p, q) &= 2pq + (1 - p)(1 - q) = q(p) + (1 - q)(2(1 - p)) \\
 BR_W(q) &= \begin{cases} 1 & \text{if } 2q > 1 - q \leftrightarrow q > \frac{1}{3} \\ 0 & \text{if } 2q < 1 - q \leftrightarrow q < \frac{1}{3} \\ [0, 1] & \text{if } 2q = 1 - q \leftrightarrow q = \frac{1}{3} \end{cases} \\
 BR_H(p) &= \begin{cases} 1 & \text{if } p > 2(1 - p) \leftrightarrow p > \frac{2}{3} \\ 0 & \text{if } p < 2(1 - p) \leftrightarrow p < \frac{2}{3} \\ [0, 1] & \text{if } p = 2(-p) \leftrightarrow p = \frac{2}{3} \end{cases}
 \end{aligned}$$

So the mixed strategy nash equilibrium strategy in this game is  $(p, q) = (\frac{2}{3}, \frac{1}{3})$

## 1.2 Bargaining Game @3883

Two players negotiate over the price of a painting in which player 1 is the seller and quotes a price  $p \in [0, 100]$  and player 2 is the buyer with the following attributes;

**Figure 1:** MSNE in Battle of Sexes

$$N = \{1, 2\}$$

$$T = \{(p, D) \in \mathbb{R}_+ \times \{A, R\}\}$$

$$\mathcal{P}(\phi) = 1$$

$$\mathcal{P}(p) = 2 \quad \forall p \in \mathbb{R}_+$$

$$U_1(p, A) = p, U_2(p, A) = 100 - p$$

$$U_1(p, R) = 0, U_2(p, R) = 0$$

Now player 2 will accept is  $100 - p \geq 0$  that gives us the following best response function of player 2

$$BR_2(p) = \begin{cases} \text{Accept} & \text{if } p < 100 \\ \text{Reject} & \text{if } p > 100 \\ \text{Accept} & \text{if } p = 100 \end{cases}$$

and given this Best Response function of player 2, player 1 chooses  $p = 100$  to maximize his payoff, which gives us our SPE.

Now,

Strategy Set (or Choice Set) of Player 1 is  $S_1 = \mathbb{R}_+$  i.e. player 1 can offer any non-negative price  $s_1 \in S_1$ . Strategy Set (or Choice Set) of Player 2 is  $S_2 = \{s_2 \mid s_2 : \mathbb{R}_+ \rightarrow \{\text{Accept}, \text{Reject}\}\}$  i.e. player 2 chooses a function

$s_2 \in S_2$ . Payoffs are represented by  $u_i : S_1 \times S_2 \rightarrow \mathbb{R}$  :

$$u_1(s_1, s_2) = \begin{cases} s_1 & \text{if } s_2(s_1) = \text{Accept} \\ 0 & \text{otherwise} \end{cases}$$

$$u_2(s_1, s_2) = \begin{cases} 100 - s_1 & \text{if } s_2(s_1) = \text{Accept} \\ 0 & \text{otherwise} \end{cases}$$

In the strategic form of this game and there are infinite Nash equilibria in this game, two such NE strategy profiles are;

- $s_1 = 100, s_2(s_1) = \text{Accept}$  if  $s_1 \leq 100$
- $s_1 = 100, s_2(s_1) = \text{Accept}$  if  $s_1 = 100$

### 1.3 More games

Suppose there are  $n$  people in a locality and a crime has been committed in this locality, every person has the same action set and the players are symmetric;

$$A_i = \{\text{Report}, \text{Don't Report}\}$$

$$u_i(\text{Report}, a_2, a_3, \dots, a_n) = v - c \quad v > 0, 0 < c < v$$

$$u_i(\text{Don't Report}, a_2, a_3, \dots, a_n) = v \quad \text{if } \exists i \neq 1 \text{ s.t. } a_i = \text{Report}$$

$$u_i(\text{Don't Report}, \dots, \text{Don't Report}) = 0$$

Only one player reports and others do not report from the set of all players is a pure strategy Nash Equilibrium.

Now, let us assume that players randomize their actions by the probability  $(p, 1 - p)$  then, in order for any player to be indifferent between randomizing and not randomizing between reporting and not reporting the crime, the following must hold;

$$v - c = (1 - p)^{n-1}(0) + (1 - (1 - p)^{n-1})(v)$$

$$v - c = v - v(1 - p)^{n-1}$$

$$(1 - p)^{n-1} = \frac{c}{v}$$

$$1 - p = \left(\frac{c}{v}\right)^{\frac{1}{n-1}}$$

$$p = 1 - \left(\frac{c}{v}\right)^{\frac{1}{n-1}}$$