Externalities

Pure Exchange Economy with Externalities

Set of all Feasible allocations;

$$\mathcal{F} = \{((x_1, y_1), (x_2, y_2)) \in \mathbb{R}^2_+ \times \mathbb{R}^2_+ | x_1 + x_2 = \omega_1^X + \omega_2^X, y_1 + y_2 = \omega_1^Y + \omega_2^Y\}$$

Alternative Definition of feasible allocations;

$$\mathcal{F} = \{((x_1, y_1), (x_2, y_2)) \in \mathbb{R}^2_+ \times \mathbb{R}^2_+ | x_1 + x_2 \leq \omega_1^X + \omega_2^X, y_1 + y_2 \leq \omega_1^Y + \omega_2^Y\}$$

In an Environment with Externalities;

$$\begin{aligned} u_1 &: \mathbb{R}^2_+ \times \mathbb{R}^2_+ \quad u_1(x_1, y_1, x_2, y_2) \\ u_2 &: \mathbb{R}^2_+ \times \mathbb{R}^2_+ \quad u_2(x_1, y_1, x_2, y_2) \end{aligned}$$

Endowments of the two individuals are;

$$\left(\omega_1^X, \omega_1^Y\right) \quad \text{and} \quad \left(\omega_2^X, \omega_2^Y\right)$$

Question 1

Find the set of all Pareto Efficient allocations of the following economy;

$$\begin{aligned} u_1(x_1, y_1, x_2, y_2) &= 3x_1 + 2y_1 + 2x_2 \qquad (2, 0) \\ u_2(x_1, y_1, x_2, y_2) &= 2x_2 + 3y_2 + 2y_1 \qquad (0, 2) \end{aligned}$$

We first start with finding the adjusted utility functions using the feasability constraints;

$$\begin{aligned} u_1^{ADJ}(x_1, y_1) &= u_1(x_1, y_1, 2 - x_1, 2 - y_1) = x_1 + 2y_1 + 4 \\ u_2^{ADJ}(x_2, y_2) &= u_1(x_2, y_2, 2 - x_2, 2 - y_2) = 2x_2 + y_2 + 4 \end{aligned}$$

Now we can plot the indifference curves in the edgeworth box and find the set of all Pareto Efficient allocations;

$$PE = \{((x_1, y_1), (x_2, y_2)) \in \mathcal{F} |\ x_1 = 0 \vee y_1 = 2\}$$

NOTE: In this particular economy if we had the alternate version of the feasibility we will have the same set of Pareto efficient allocations because of the positive externalities in this particular question!

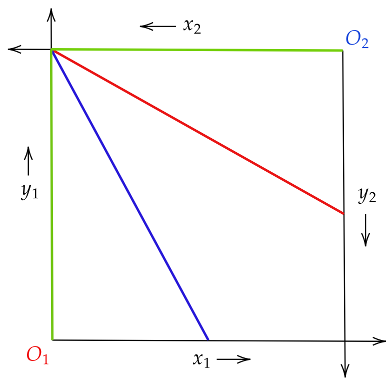

Figure 1: Edgeworth Box in Question 1

Competitive Equilibrium

It consists of $\left(p_x^*, p_y^*\right)$ and an allocation $((x_1^*, y_1^*), (x_2^*, y_2^*))$ such that

1. Given $\left(p_x^*, p_y^*\right)$,

   given, (x_2^*, y_2^*),

   (x_1^*, y_1^*) solves;

$$\max_{(x_1,y_1)\in\mathbb{R}^2_+} u_1(x_1, y_1, x_2^*, y_2^*)$$
$$s.t. p_x^* x_1 + p_y^* y_1 \leq p_x^* \omega_1^X + p_y^* \omega_1^Y$$

   and given (x_1^*, y_1^*),

   (x_2^*, y_2^*) solves;

$$\max_{(x_2,y_2)\in\mathbb{R}^2_+} u_2(x_2, y_2, x_1^*, y_1^*)$$
$$s.t. p_x^* x_2 + p_y^* y_2 \leq p_x^* \omega_2^X + p_y^* \omega_2^Y$$

   Alternatively, $((x_1^*, y_1^*), (x_2^*, y_2^*))$ is the Nash Equilibrium of the following demand game;

- Set of Players $\{1, 2\}$
- Action Sets;

$$A_1 = \{(x_1, y_1) \in \mathbb{R}^2_+ | p^*_x x_1 + p^*_y y_1 \leq p^*_x \omega^X_1 + \omega^Y_1\}$$
$$A_2 = \{(x_2, y_2) \in \mathbb{R}^2_+ | p^*_x x_2 + p^*_y y_2 \leq p^*_x \omega^X_2 + \omega^Y_2\}$$

- Payoffs;

$$u_1 : A_1 \times A_2 \to \mathbb{R} \qquad u_1(x_1, y_1, x_2, y_2)$$
$$u_2 : A_1 \times A_2 \to \mathbb{R} \qquad u_2(x_1, y_1, x_2, y_2)$$

2. Total demand equals total supply in the economy;

$$x^*_1 + x^*_2 = \omega^X_1 + \omega^X_2$$
$$y^*_1 + y^*_2 = \omega^Y_1 + \omega^Y_2$$

Question 2

Find the set of all Pareto Efficient allocations and the Competitive equilibrium;

$$\mathcal{F}_1 = \{((x_1, y_1), (x_2, y_2)) \in \mathbb{R}^2_+ \times \mathbb{R}^2_+ | x_1 + x_2 = 2, y_1 + y_2 = 2\}$$
$$\mathcal{F}_2 = \{((x_1, y_1), (x_2, y_2)) \in \mathbb{R}^2_+ \times \mathbb{R}^2_+ | x_1 + x_2 \leq 2, y_1 + y_2 \leq 2\}$$

$$u_1 = (x_1, y_1, x_2, y_2) = x_1 + y_1 - x_2 \qquad (2, 0)$$
$$u_2 = (x_1, y_1, x_2, y_2) = x_2 \qquad (0, 2)$$

The set of all Pareto efficient allocations is; $PE_1 = PE_2$;

$$PE_1 = \{((x_1, y_1)(x_2, y_2)) \in \mathcal{F}_1 | y_2 = 0\}$$

$$PE_2 = \{((x_1, y_1)(x_2, y_2)) \in \mathcal{F}_2 | y_2 = 0, y_1 = 2, x_1 + x_2 = 2\}$$

The Competitive equilibrium is; $CE = ((0, 2), (2, 0))$ at $p_x = p_y = 1$

Now is there any allocation like $((x_1, 2), (x_2, 0))$ such that $x_1 + x_2 < 2$ and $((x_1, 2), (x_2, 0))$ is Pareto Efficient in then $\mathcal{F}_2$?

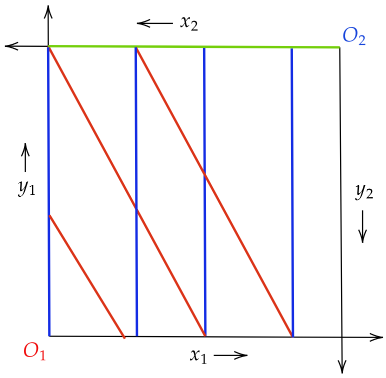

Figure 2: Edgeworth Box in Question 2

NO! Because if we define $\epsilon = 2 - x_1 - x_2$ and then if we increase the amount of x_1 by $\frac{2\epsilon}{3}$ and x_2 by $\frac{\epsilon}{3}$ Then utilities of both individuals will increase and therefore any such allocation where $x_1 + x_2 < 2$ will not be Pareto Efficient.

Question 3

Find the set of all Pareto Efficient allocations and also find the Competitve equilibrium;

$$\begin{aligned} u_1(x_1, y_1, x_2, y_2) &= x_1 + 2\sqrt{y_1} && (2,0) \\ u_2(x_1, x_2, y_1, y_2) &= x_2 + 2\sqrt{\max(0, 2 - y_1)} && (0,2) \\ u_1^{ADJ}(x_1, y_1) &= x_1 + 2\sqrt{y_1} \\ u_2^{ADJ}(x_2, y_2) &= x_2 + 2\sqrt{y_2} \end{aligned}$$

So the set of all Pareto Efficient allocations is;

$PE = \{((x_1, y_1)(x_2, y_2)) \in \mathcal{F} | (y_1 = y_2 = 1) \vee (x_1 = 0 \wedge y_1 \leq 1) \vee (x_2 = 0 \wedge y_2 \leq 1)\}$

and the Competitive equilibrium is

$$CE = \left(\left(\frac{P_X}{P_Y} = \sqrt{2}, \right), \left((2 - \sqrt{2}, 2), (\sqrt{2}, 0) \right) \right)$$

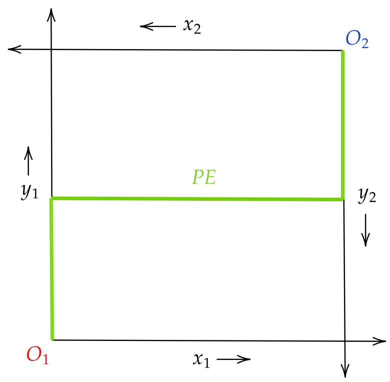

Figure 3: Edgeworth Box In Question 3

Question 4

$$u_1(x_1, y_1, x_2, y_2) = \max(x_1, y_1, x_2, y_2) \qquad (3,3)$$
$$u_2(x_1, y_1, x_2, y_2) = x_2 + y_2 \qquad (1,1)$$

$PE = ((0,0),(4,4))$

$CE = \text{DNE}$

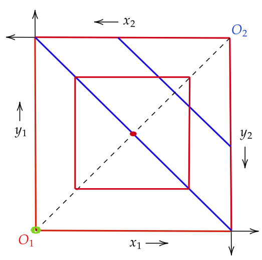

Figure 4: Edgewoth Box in Question 4