1 Mixed Strategy Nash equilibrium

1.1 Battle of Sexes

	M	S	$\left(\frac{1}{2}, \frac{1}{2}\right)$	$(q, 1-q)$
M	(2,1)	(0,0)	$1, \frac{1}{2}$	$2q, q$
S	(0,0)	(1,2)	$\frac{1}{2}, 1$	$(1-q), 2(1-q)$
$\left(\frac{1}{2}, \frac{1}{2}\right)$	$1, \frac{1}{2}$	$\frac{1}{2}, 1$	$\frac{3}{4}, \frac{3}{4}$	
$(p, 1-p)$	$2p, p$	$(1-p), 2(1-p)$		$(2pq+(1-p)(1-q)), (pq+2(1-p)(1-q))$

Row player(Wife) plays the mixed strategy $(p, 1-p)$ s.t. $\quad p \in [0, 1]$.

Column player(Husband) plays the mixed strategy $(q, 1-q)$ s.t. $\quad q \in [0, 1]$

$$
\begin{aligned}
A_1 &= [0,1]\,, A_2 = [0,1] \\
U_W(p,q) &= pq + 2(1-p)(1-q) = p(2q) + (1-p)(1-q) \\
U_H(p,q) &= 2pq + (1-p)(1-q) = q(p) + (1-q)(2(1-p)) \\
BR_W(q) &= \begin{cases} 1 & \text{if} \quad 2q > 1-q \leftrightarrow q > \frac{1}{3} \\ 0 & \text{if} \quad 2q < 1-q \leftrightarrow q < \frac{1}{3} \\ [0,1] & \text{if} \quad 2q = 1-q \leftrightarrow q = \frac{1}{3} \end{cases} \\
BR_H(p) &= \begin{cases} 1 & \text{if} \quad p > 2(1-p) \leftrightarrow p > \frac{2}{3} \\ 0 & \text{if} \quad p < 2(1-p) \leftrightarrow p < \frac{2}{3} \\ [0,1] & \text{if} \quad p = 2(-p) \leftrightarrow p = \frac{2}{3} \end{cases}
\end{aligned}
$$

So the mixed strategy nash equilibrium strategy in this game is $(p, q) = (\frac{2}{3}, \frac{1}{3})$

1.2 Bargaining Game @3883

Two players negotiate over the price of a painting in which player 1 is the seller and quotes a price $p \in [0, 100]$ and player 2 is the buyer with the following attributes;

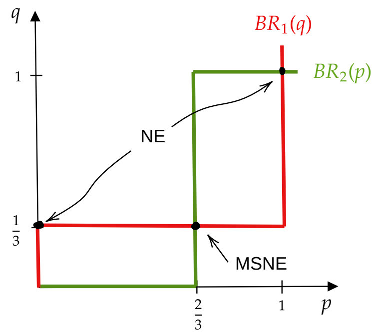

Figure 1: MSNE in Battle of Sexes

$$
\begin{aligned}
N &= \{1,2\} \\
T &= \{(p,D) \in \mathbb{R}_+ \times \{A,R\}\} \\
\mathcal{P}(\phi) &= 1 \\
\mathcal{P}(p) &= 2 \quad \forall p \in \mathbb{R}_+ \\
U_1(p,A) &= p, U_2(p,A) = 100 - p \\
U_1(p,R) &= 0, U_2(p,R) = 0
\end{aligned}
$$

Now player 2 will accept is $100 - p \geq 0$ that gives us the following best response function of player 2

$$
BR_2(p) = \begin{cases} \text{Accept} & \text{if} \quad p < 100 \\ \text{Reject} & \text{if} \quad p > 100 \\ \text{Accept} & \text{if} \quad p = 100 \end{cases}
$$

and given this Best Response function of player 2, player 1 chooses $p = 100$ to maximize his payoff, which gives us our SPE.

Now,

Strategy Set (or Choice Set) of Player 1 is $S_1 = \mathbb{R}+$ i.e. player 1 can offer any non-negative price $s_1 \in S_1$. Strategy Set (or Choice Set) of Player 2 is $S_2 = \{s_2 \mid s_2 : \mathbb{R}_+ \to \{ \text{ Accept, Reject } \}\}$ i.e. player 2 chooses a function

$s_2 \in S_2$. Payoffs are represented by $u_i : S_1 \times S_2 \to \mathbb{R}$:

$$u_1(s_1, s_2) = \begin{cases} s_1 & \text{if } s_2(s_1) = \text{ Accept} \\ 0 & \text{otherwise} \end{cases}$$

$$u_2(s_1, s_2) = \begin{cases} 100 - s_1 & \text{if } s_2(s_1) = \text{ Accept} \\ 0 & \text{otherwise} \end{cases}$$

In the strategic form of this game and there are infite Nash equilibria in this game, two such NE strategy profiles are;

- $s_1 = 100, s_2(s_1) = \text{Accept} \quad \text{if} \quad s_1 \leq 100$
- $s_1 = 100, s_2(s_1) = \text{Accept} \quad \text{if} \quad s_1 = 100$

1.3 More games

Suppose there are n people in a locality and a crime has been committed in this locality, every person has the same action set and the players are symmetric;

$$\begin{aligned} A_i &= \{\text{Report}, \text{Don't Report}\} \\ u_i(\text{Report}, a_2, a_3, \ldots, a_n) &= v - c \quad v > 0, 0 < c < v \\ u_i(\text{Don't Report}, \mathrm{a}_2, \mathrm{a}_3, \ldots, a_n) &= v \quad \text{if} \quad \exists \quad i \neq 1 \quad \text{s.t. } a_i = \text{Report} \\ u_i(\text{Don't Report}, \ldots, \text{Don't Report}) &= 0 \end{aligned}$$

Only one player reports and others do not report from the set of al players is a pure starategy Nash Equilibrium.

Now, let us assume that players randomize their actions by the probability $(p, 1-p)$ then, in order for any player to be indeifferent between randomizing and not randomizing between reporting and not reporting the crime, the following must hold;

$$\begin{aligned} v - c &= (1-p)^{n-1}(0) + (1 - (1-p)^{n-1})(v) \\ v - c &= v - v(1-p)^{n-1} \\ (1-p)^{n-1} &= \frac{c}{v} \\ 1 - p &= \left(\frac{c}{v}\right)^{\frac{1}{n-1}} \\ p &= 1 - \left(\frac{c}{v}\right)^{\frac{1}{n-1}} \end{aligned}$$