1 Functions

Let A and B be any non-empty sets. A function from A to B is a rule that associates with each member of A a unique member of B.

The notation is $f : A \to B$, where input comes from the set A and output belongs to the set B.

If $a \in A$, we denote the unique element of B that the rule associates to a by $f(a)$ We refer to the element a of A as an argument of the function, and the corresponding element $f(a)$ of B as the value of the function at that argument (or sometimes the image of the point a under f).

Consider an example $f(x) = x^2 + x + 1$. The value of the function f at argument 2 is $f(2)$, which is further equal to 7.

1.1 Domain and Codomain

If $f : A \to B$, we refer the set A as the domain of f and the set B as the codomain.

1.2 Injective, or One-to-one

If a function $f : A \to B$ is such that it never happens that different arguments lead to the same value, we say that f is injective.

- Mathematically, $f : A \to B$ is injective iff $(\forall a, b \in A)[a \neq b \Rightarrow f(a) \neq f(b)]$
- Alternatively, we may express this condition using contrapositive: $f : A \to B$ is injective iff

$$(\forall a, b \in A)[f(a) = f(b) \Rightarrow a = b]$$

- The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ is not injective but the function $g : \mathbb{R}_+ \to \mathbb{R}$ defined by $g(x) = x^2$ is injective.

1.3 Surjective or onto

If every member of B is the value of the function at some argument, we say f is surjective.

- Mathematically, a function $f : A \to B$ is surjective iff $(\forall b \in B)(\exists a \in A)[f(a) = b]$.

- Note the order of the quantifiers in the above condition. For every b in B it must be possible to find an a in A such that $f(a) = b$.
- The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ is not surjective but the function $g : \mathbb{R} \to \mathbb{R}_+$defined by $g(x) = x^2$ is surjective.