Heterogeneous Firms, Variable Markups, and FDI*

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Abstract

This paper introduces *ad valorem* tariff and horizontal FDI into the Melitz and Ottaviano [2008] framework, producing the first framework in the trade policy literature that incorporates firm heterogeneity, variable markups, and multinational production. The model generates novel equilibrium implications. First, the presence of multinational production generates a competitive effect on the economy, and firms need to be more productive to survive the competition. Second, the *ad valorem* tariff and quadratic quasi-linear preference collectively result in an endogenous level of firm entry. Therefore, the impact of trade/tariff liberalization depends on the equilibrium number of firms. In the short-run, when the firm entry is prohibited, an increase in import tariff shields the domestic economy from the Foreign competition, making it easier for firms to survive. This result is overturned when firms can enter the market freely in the long-run. In the long-run, an increase in Home's import tariff will make the Home country a more desirable environment to do business, attracting more entrants in the Home market, making the Home market more competitive. Firms need to be more productive to survive. Home's tariff increase also makes it harder for the least productive Foreign exporters to survive, and triggers tariff-jumping FDI among the most productive exporters. Markups also respond to tariff change differently in the short-run vs. long-run, primarily due to the change of competitive environment associated with firm entry.

Keywords: Firm heterogeneity, Variable markups, Foreign direct investment, tariff liberalization, market structure

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1 Introduction

Over the last two decades, firm heterogeneity has been steadily transforming international trade theory. This transformation has been fueled by micro-level empirical studies. The two most robust empirical findings among these studies are: heterogeneous firms charge heterogeneous markups, and many firms are engaging in multinational production. Paradoxically, the workhorse model[1] in the heterogeneous firm trade policy literature assumes monopolistic competition and constant elasticity of substitution (henceforth, CES) preference, which implies a constant markup for all the firms. Furthermore, firms only access foreign markets through export. It goes without saying that such a framework cannot provide an adequate understanding of trade policy analysis.

The goal of this paper is to provide a theoretical framework that features heterogeneous firms, variable markups, and a particular form of multinational production, horizontal foreign direct investment (henceforth, FDI). To this end, this paper introduces variable markups through quadratic quasi-linear preference, as in Melitz and Ottaviano [2008], into a two-country (Home and Foreign) model with firm heterogeneity and FDI, as in Helpman et al. [2004]. In the current framework, the economy features two sectors: a perfectly competitive sector that produces homogeneous goods and a monopolistically competitive sector that produces differentiated varieties. Firm entry and exit only happen in the monopolistically competitive sector. A firm needs to pay a fixed cost and draw its marginal production cost (which is inversely related to the firm's productivity) to enter the market. Post-entry, firms produce with different marginal cost levels. Exporters encounter two types of costs, iceberg-type trade costs, an *ad valorem* tariff. As for multinationals, they face an iceberg-type of *efficiency loss* as in Keller and Yeaple [2008]. Before the individual firm's productivity is realized, firms formulate entry, export, and FDI decisions based on expected profit. The difference in marginal cost preserves the sorting of firms[2]: the most productive firms access the Foreign market through FDI, the less productive firms export, and the least productive firms only serve their domestic market. An increase in the Home country's import tariff affects the variable profit of Foreign exporters and multinationals, making FDI a more profitable entry mode for the most productive Foreign exporters,

[1] For example, see Demidova and Rodriguez-Clare [2009], Felbermayr et al. [2013], and Ossa [2016].

[2] In Helpman et al. [2004], the sorting of firms is preserved by the combination of fixed cost and variable cost. Here, with bounded marginal utility, high-cost firms will not survive, even without such fixed costs. The difference in marginal cost is sufficient to generate the sorting. Adding fixed cost will substantially degrade the tractability of the model, without generating additional insight.

inducing tariff-jumping FDI under the heterogeneous firm framework.

The analysis of the findings shows that the economy responds to a tariff change differently in the short-run versus long-run. In the short-run, where the entry has not taken place yet, and fixed costs are sunk, exiting the market is never optimal for firms. In this case, the economy is characterized by a fixed number and distribution of incumbents. These incumbents decide whether they should operate and produce or shut down. If they choose to shut down, they can restart production without incurring the entry cost again. In this short-run case, an increase in the Home import tariff makes it harder for the least productive Foreign exporters to export. These exporters will shut down their export department and only serve their domestic market. At the same time, an increase in Home's import tariff makes export a less desirable entry mode for the most productive Foreign exporters. These firms will switch to FDI simply because the variable profit of FDI is higher than that of export. In the current setup, if the level of tariff is low, the reduction of Foreign exporters will dominate the increase of Foreign multinationals, resulting in a reduction of the total number of Foreign firms in the Home country. In equilibrium, an increase in Home tariff creates an easier environment for Home firms to survive. Therefore, Home's tariff effectively shields its firms from Foreign competition.

In the long-run, due to the presence of the *ad valorem* tariff and quadratic quasi-linear preference, the number of entrants and the number of firms in the economy are endogenously determined by the tariff level. With the free-entry condition, an increase in Home's import tariff makes the Home country a more desirable place to do business, generating more domestic entry, intensifying competition in the Home market. On the one hand, the increase of Home's tariff makes it harder for the least productive Foreign exporters to export, reducing the number of Foreign exporters. On the other hand, the protection makes it more profitable for the most productive Foreign exporters to do FDI and increases the number of Foreign multinationals. However, the total impact on the number of firms in the Home market is dominated by the domestic entry. Different from the short-run equilibrium, an increase in Home tariff creates more entry, generates more competition in the Home market, and makes it harder for local producers to survive.

In contrast to the traditional combination of monopolistic competition and CES preference, where all the firms charge identical markups, in the current setting, markups also respond to tariff change differently in the short-run versus in the long-run. In the short-run, Home import protectionist tariff reduces the competition in the Home market, causing both Home's domestic firms and Foreign's FDI firms to charge higher markups than before. Foreign exporters' markups vary according to their productivity. For the least productive Foreign exporters, Home's protectionist tariff reduces their markups by giving a competitive edge to the Home firms. However, for the more productive Foreign exporters, Home's protection does not affect them much, and they benefit from the reduced competition by charging a higher markup. These responses are reversed in the long-run. Home's protectionist trade policy makes the Home market a more desirable environment to do business, attracting more firm entry into Home's market. This channel will substantially increase the competition level in the Home market, making it harder for firms to survive and reducing the markups for all the firms that operate in the Home market.

This paper is not the first to address questions regarding trade policy within the heterogeneous firm framework. For example, Demidova and Rodriguez-Clare [2009] use a Melitz-type model to study the trade policy implication in a small open economy. Felbermayr et al. [2013] study the bilateral trade policy implication in a two-country, asymmetric Melitz-type economy. Bagwell and Lee [2015] study trade policy in the Melitz and Ottaviano [2008] model and provide a rationale for the treatment of export subsidies within the World Trade Organization. Costinot et al. [2016] utilize a generalized Melitz model to study the trade policy implication both from a micro and macro perspective. Demidova [2017] studies the optimal tariff in the Melitz and Ottaviano [2008] environment without the numéraire good and finds that protection is always desirable, and reductions in cost-shifting trade barriers are welfare-improving. A common feature of the papers mentioned above is their exclusive focus on domestic producers and foreign exporters. A key message from the current analysis is that ignoring multinational production may provide an incomplete picture of protectionist trade policy.

A recent article by Cole and Davies [2011] is closely related to the current paper. The authors introduce an ad valorem tariff and heterogeneous fixed costs into Helpman et al. [2004], and find equilibria in which both pure exporters and multinationals coexist, resolving a well-known puzzle[3] in the strategic tariff literature in the presence of multinationals. Heterogeneous fixed costs for exporters and multinationals are the critical elements to generate their results. In contrast, the coexistence of exporters and multinationals in the current framework comes from the different

[3]In equilibrium, all foreign firms are either multinationals or exporters.

iceberg costs they face.

Despite the apparent similarity between the two frameworks, it should be clear that the two exercises are very different. First, Cole and Davies [2011] combine quasi-linear CES preference with monopolistic competition, which produces a constant markup for all the firms. Although being analytically tractable, the combination of CES and monopolistic competition has little merit, even as a first approximation, for welfare analysis. In contrast, the current framework utilizes quadratic quasi-linear preference to generate variable markups and incomplete pass-through for firms with different productivity levels. This attribute is more suitable for pricing and welfare analysis. Second, although Cole and Davies [2011]'s model features ad valorem tariff and multiple sectors, the policy implications from their paper are entirely independent[4] of firm's entry level. In the current model, tariff level endogenously affects the number of entrants, generating different implications for protectionist trade policy in both the short-run and the long-run.

The remainder of this paper is organized as follows. Section 2 presents the complete model and the analytical solution of the model. Section 3 introduces the short-run version of the model and studies the effect of a unilateral tariff change. Section 4 presents the long-run version of the model and investigates its equilibrium features. All the predictions regarding a tariff change are contrasted with those in Cole and Davies [2011]. Section 5 discusses the limitations of the current framework and then concludes. All the relevant graphs and proofs are included in the appendix.

2 A Model with Heterogeneous Firms, Variable Markups, and FDI

This section introduces quadratic quasi-linear preference into the Helpman et al. [2004] framework. There are two symmetric countries, Home (H) and Foreign (F). The markets are segmented, and international trade entails trade costs that take the form of transportation costs as well as *ad valorem* import tariffs. Tariff revenue is redistributed equally across consumers in the tariff-imposing country. Multinational firms engaging in FDI face an iceberg-type marginal cost

[4] To the best of my knowledge, this feature is contradicting with the implications from Balistreri et al. [2011], Arkolakis et al. [2012], and Caliendo et al. [2017].

(i.e., efficiency loss) in the spirit of Keller and Yeaple [2008]. Different from Cole and Davies [2011], where firms' sorting is induced by different fixed costs, the nonhomothetic preference here induces different productivity cutoffs through different marginal costs. The framework presented here is suitable for this analysis due to two reasons. First, it enables one to study trade policy in an environment that features firm heterogeneity, variable markups, and multinational production. Second, the model produces tractable analytical solutions due to the specific assumption regarding demand and production structure, yielding quite transparent comparative statics.

2.1 Consumers

Each country is endowed with one unit of consumers. In the H economy, each consumer supplies one unit of labor. Consumers in country H maximize their utility by consuming the numéraire good, q_0^H, and the differentiated varieties, q_i^H, subject to their income budget constraint:

$$U^H = q_0^H + \alpha \int_{i \in \Omega^H} q_i^H di - \frac{1}{2}\gamma \int_{i \in \Omega^H} \left(q_i^H\right)^2 di - \frac{1}{2}\eta \left(\int_{i \in \Omega^H} q_i^H di\right)^2$$

$$\text{subject to: } q_0^H + \int_{i \in \Omega^H} p_i^H q_i^H di \leq I^H \equiv w^H + TR^H + \Pi^H$$

where α and η indicate the substitutability between the differentiated varieties and numéraire good, and γ indicates the substitutability among the differentiated varieties. An increase in α and a decrease in η both shift out the demand for the differentiated varieties *relative* to the numéraire good. The degree of product differentiation increases as γ increases since consumers are paying more attention to the distribution of varieties that they consume. All these three demand parameters are positive. Notice that different from Melitz and Ottaviano [2008], the tariff revenue and aggregate profit will enter into consumer's budget constraint through government redistribution.

Assuming consumers have positive demands for the numéraire good $\left(q_0^H > 0\right)$, utility maximization of the previous consumer problem leads to the following inverse demand for each variety i:

$$p_i^H = \alpha - \gamma q_i^H - \eta Q^H \tag{1}$$

where $Q^H \equiv \int_{i \in \Omega^H} q_i^H d_i$ is the aggregate consumption of these varieties, and Ω^H is the variety space that is available to the consumers. Invert equation (1) to obtain the linear market demand for these varieties:

$$
\begin{aligned}
q_i \equiv q_i^H &= \frac{\alpha}{\eta N^H + \gamma} - \frac{1}{\gamma} p_i^H + \frac{\eta N^H}{\eta N^H + \gamma} \frac{1}{\gamma} \bar{p}^H \\
&= \frac{1}{\gamma} \left(p_{\max}^H - p_i^H \right)
\end{aligned} \tag{2}
$$

where $p_{\max}^H = (\gamma\alpha + \eta N^H \bar{p}^H)/(\eta N^H + \gamma)$ represents the price at which demand for a variety is driven to 0, $\bar{p}^H \equiv (1/N^H) \int_{i\in\hat{\Omega}^H} p_i^H di$ is the average price of all consumed variety in country H, and $\hat{\Omega}^H$ is the consumed subset of Ω^H. Note that equation (1) also implies $p_{\max}^H \leq \alpha$ for positive q_i^H and Q^H. Different from CES preference, where the elasticity of demand across varieties is constant, the price elasticity of demand here is given by:

$$
\varepsilon_i^H \equiv \left| \frac{\partial q_i^H}{\partial p_i^H} \times \frac{p_i^H}{q_i^H} \right| = \frac{1}{p_{\max}^H / p_i^H - 1} \tag{3}
$$

A lower average price $\bar{p}^H$ or a larger number of competing varieties N^H will induce a decrease in the choke price $p_{\max}^H$, and hence an increase in the price elasticity of demand ε_i^H at any given p_i^H. These movements all represent a *tougher* competitive environment, which has received strong empirical support in the industrial organization literature but can not be captured in an environment with constant elasticity of demand.

As in Melitz and Ottaviano [2008], welfare can be evaluated using the following indirect utility function:

$$
U^H = I^H + \frac{1}{2} \left(\frac{N^H}{\eta N^H + \gamma} \right) \left(\alpha - \bar{p}^H \right)^2 + \frac{1}{2} \frac{N^H}{\gamma} \sigma_{p^H}^2 \tag{4}
$$

where $\sigma_{p^H}^2 \equiv \left(1/N^H\right) \int_{i\in\hat{\Omega}^H} \left(p_i^H - \bar{p}^H\right)^2 di$ measures the variance of variety prices. To ensure positive demand levels for the numéraire, I assume that consumer's income is sufficiently high, so that $I^H > \int_{i\in\hat{\Omega}^H} p_i^H q_i^H di = \bar{p}^H Q^H - N^H \sigma_{p^H}^2 / \gamma$. Consumer's utility will be higher when the average price $\bar{p}^H$ is lower, the variance of prices $\sigma_{p^H}^2$ is higher, and the number of varieties N^H is larger.

2.2 Firms

Production in the economy only utilizes labor, which is supplied in an inelastic fashion in a competitive market. The economy consists of two sectors, the traditional sector (produces q_0^H) and the modern sector (produces q_i^H). q_0^H is produced under a constant return to scale technology at a unitary cost. Thus the wage[5] in each country equals to one: $w^H = 1$. In the differentiated-good sector, firms are competing with each other in a monopolistically competitive fashion, and each firm produces a single variety.

Entry only happens in the modern sector. Different from Melitz [2003], here exit only happens at the moment when productivity is realized. There is no exogenous per-period death shock in the modern sector due to the one-period nature of the model[6]. To enter the market, a firm needs to pay a sunk entry cost $f_E > 0$ and draws its marginal production cost c, which indicates the unit labor requirement, from a Pareto distribution with cumulative distribution function $G(c) = (c/c_M)^k$, where $k \geq 1$ represents a shape parameter and $c_M > 0$ represents the upper bound of c. When $k = 1$, the marginal cost distribution is uniform on $[0, c_M]$. As k increases, the relative number of low productivity firms increases, and the distribution is more concentrated at these lower productivity levels. As k approaches infinity, the distribution of firm productivity approaches c_M. A lower c_M stands for a higher technology state in the economy. In this paper, I assume H and F share the same technology, hence the same upper bound c_M and the same f_E for both countries.

Depending on its productivity draw, a firm enters country H may exit, produce locally, export to country F or engage in the multinational activity. Following Melitz and Ottaviano [2008], since each firm's marginal cost of production does not vary with its production level, the decisions in each market can be made separately. Therefore, all the monopolistically competitive firms make separate decisions about their prices at each market, taking the total number of varieties and the average price in a market as given. In what follows, I analyze each type of producer's profit maximization problem.

[5]One can attempt to drop the numéraire good, then wage will be endogenized and can be pinned down by trade balance condition.

[6]For a dynamic application of Melitz and Ottaviano [2008], see Moon [2015]. Notice the benchmark model in her work is without the numéraire good.

2.2.1 Domestic Producers

A firm located in country H with cost level c selects its price in the domestic market, p_D^H, to maximize its domestic profit $\pi_D^H(c) = \left[p_D^H(c) - c\right] q_D^H(c)$. Together with equation (2), the optimal price, markup, quantity, and profit can be solved as:

$$p_D^H(c) = \frac{1}{2}\left(c_D^H + c\right) \tag{5}$$

$$m_D^H(c) = \frac{1}{2c}\left(c_D^H + c\right) \tag{6}$$

$$q_D^H(c) = \frac{1}{2\gamma}\left(c_D^H - c\right) \tag{7}$$

$$\pi_D^H(c) = \frac{1}{4\gamma}\left(c_D^H - c\right)^2 \tag{8}$$

Let $c_D^H \equiv \sup\left\{c : \pi_D^H(c) > 0\right\}$ represent the cost of the firm which is indifferent between exit and remaining in the market. This firm earns zero profit as its price is driven down to marginal cost. Together with equation (2), the firm's optimal price is equal to its marginal cost, $p_D^H\left(c_D^H\right) = c_D^H = p_{\max}^H$. Hence, a firm will only serve domestic market if $c \le c_D^H$. As expected, lower cost firms set lower prices and earn higher profits. However, lower cost firms are also more productive, and have larger market power, therefore they do not pass all of the cost differentials to the consumer and charge higher markups (which is defined as $m(c) = p(c)/MC(c)$[7], decreasing in c).

2.2.2 Exporters

The exporter in country H will face an *ad valorem* import tariff imposed by country F, denoted as $t^F \ge 1$. On top of that, the exporter will also face a per-unit trade cost, denoted by τ^F. More specifically, the delivered cost of a unit cost c to country F is $\tau^F c$ where $\tau^F > 1$. An exporter takes t^F and τ^F as given and maximizes its profit $\pi_X^H(c) = \left[p_X^H(c)/t^F - \tau^F c\right] q_X^H(c)$ by choosing optimal price $p_X^H(c)$. Together with equation (2), the optimal price, markup, quantity, and profit are:

$$p_X^H(c) = \frac{t^F \tau^F}{2}\left(c_X^H + c\right) \tag{9}$$

[7]Different from the Lerner index, where markup is measured in a relative sense, here the markup is measured in absolute sense. These two have no qualitative difference. Here I adopt the latter simply due to analytical convenience.

$$m_X^H(c) = \frac{t^F}{2c}\left(c_X^H + c\right) \tag{10}$$

$$q_X^H(c) = \frac{t^F \tau^F}{2\gamma}\left(c_X^H - c\right) \tag{11}$$

$$\pi_X^H(c) = \frac{t^F \left(\tau^F\right)^2}{4\gamma}\left(c_X^H - c\right)^2 \tag{12}$$

Let $c_X^H \equiv \sup\left\{c : \pi_X^H(c) > 0\right\}$ denotes the marginal cost of the least productive exporter from H to F, which barely finds export profitable. Combine this threshold with the definition of c_D^F (parallel to c_D^H), this cutoff level then satisfies $c_X^H = c_D^F / t^F \tau^F$. Intuitively, tariffs and transportation cost make it harder for exporters to break even compared to the domestic market.

It should be noted that the presence of iceberg-type transportation cost ensures that when the net tariff is zero, there are still exporters in the economy. Following Melitz and Ottaviano [2008], I abstract from any fixed cost of exporting, which could substantially reduce the tractability of the model without adding additional insights. With the bounded marginal utility, different marginal costs are enough to induce the sorting of firms.

2.2.3 Multinational Firms

To engage in the multinational activity, a firm located in country H with cost level c sets its product price for consumers in country F, denoted as $p_{FDI}^H(c)$. Instead of serving the Foreign market through export, it directly serves locally in country F, but doing so will incur an *efficiency loss*, which effectively increases the marginal cost of production. Here, I assume that the efficiency loss φ^F is greater than τ^F to ensure there are still multinational firms in the economy even when the net tariff is zero. Multinational firm's profit function is as the following:

$$\pi_{FDI}^H(c) = \left[p_{FDI}^H(c) - \varphi^F c\right] q_{FDI}^H(c) \tag{13}$$

Together with equation (2), the optimal price, markup, quantity, and profit are:

$$p_{FDI}^H(c) = \frac{1}{2}\left(c_D^F + \varphi^F c\right) \tag{14}$$

$$m_{FDI}^H(c) = \frac{1}{2\varphi^F c}\left(c_D^F + \varphi^F c\right) \tag{15}$$

$$q_{FDI}^{H}(c) = \frac{1}{2\gamma}\left(c_D^F - \varphi^F c\right) \tag{16}$$

$$\pi_{FDI}^{H}(c) = \frac{1}{4\gamma}\left(c_D^F - \varphi^F c\right)^2 \tag{17}$$

Let $c_{FDI}^H = \sup\left\{c : \pi_{FDI}^H(c) > \pi_X^H(c)\right\}$ denote the marginal cost of the least productive multinational firm, which finds it indifferent between export and FDI. Combine this threshold with the definition of c_D^F, the cutoff then satisfies $c_{FDI}^H = \xi^F c_D^F$, where $\xi^F \equiv (1-\sqrt{t^F})/(t^F\tau^F - \sqrt{t^F}\varphi^F)$ is derived by setting $\pi_{FDI}^H(c) = \pi_X^H(c)$. Before moving on to the industry-level entry analysis, I want to discuss a few important modeling features of the multinational firms.

(i) *Efficiency loss* The efficiency loss feature is similar to Keller and Yeaple [2008], who demonstrate that when technologies are complex, it is more difficult for US-owned foreign affiliates to substitute local production with imports from the multinational headquarter. φ^F can also stand for the information costs of working abroad, transaction costs of dealing with FDI policy barriers[8], the costs of maintaining the affiliate, servicing network costs, and other costs associated with technology costs in offshore production. In a recent quantitative study by Head and Mayer [2019], the authors utilize highly disaggregated automotive industry data and find this type of variable distribution and marketing costs is higher than the conventional trade costs such as tariffs and freight, which is consistent with the assumption that I made here.

(ii) *Productivity sorting* There are two possible answers to equation $\pi_{FDI}^H(c) = \pi_X^H(c)$:

$$c_{FDI}^H = (1 \pm \sqrt{t^F})/(t^F\tau^F \pm \sqrt{t^F}\varphi^F)c_D^F \tag{18}$$

However, only one of the answers is interesting and relevant here. From both theoretical and empirical point of view, among those firms that serve foreign markets, multinational firms that engage in FDI are the most productive ones[9]. In the current framework, this implies $c_{FDI}^H < c_X^H < c_D^H$. Compare the expression of c_X^H and c_{FDI}^H, together with the assumption of $\varphi^F > \tau^F$, one can easily verify that both solutions of c_{FDI}^H in equation (18) imply that $c_{FDI}^H < c_X^H < c_D^H$. However, for the case of $c_{FDI}^H = (1+\sqrt{t^F})/(t^F\tau^F + \sqrt{t^F}\varphi^F)c_D^F$, c_{FDI}^H will decrease in response to an increase in t^F, indicating

[8] For example, according to Head and Mayer [2019], foreign car makers complained about the additional costs of daytime running lamps when Canada mandated them for new cars in 1990).

[9] For theoretical work, see Helpman et al. [2004]. For empirical evidence, see Doms and Jensen [1998] for the US and Conyon et al. [2002] for the U.K, for more recent evidence, see Mataloni [2011].

the marginal multinationals will choose to become exporters when tariff increases. This is at odds with the empirical evidence in the literature. For example, Blonigen [2002] finds that tariff-jumping is a realistic option for multinational firms from industrialized countries. Hijzen et al. [2008] find horizontal tariff-jumping M&A evidence for 23 OECD countries for the period 1990–2001. More recently, Alfaro and Chen [2018] also find strong empirical evidence of tariff-jumping FDI through Orbis manufacturing firm-level dataset(60 countries, 2002–2007). Therefore, the other solution $c^H_{FDI} = (1 - \sqrt{t^F})/(t^F \tau^F - \sqrt{t^F}\varphi^F)c^F_D$ is more relevant here since c^H_{FDI} will increase in response to an increase in t^F, which is in line with the empirical evidence of productivity sorting and the tariff-jumping FDI.

(iii) *FDI motivation* Discussion on the multinational firm's FDI motivation here is important. In Helpman et al. [2004], the sorting of firms is induced by the assumption that $f_I > \tau^{\epsilon-1} f_X > f_D$: export incurs a fixed cost (f_X) and a higher marginal cost (τ), but as long as the fixed cost of FDI (f_I) is sufficiently high, the most productive firms are guaranteed to find FDI more desirable than export. This is a classic proximity-concentration trade-off in the spirit of Brainard [1997]. The similar trade-off is also present in Cole and Davies [2011], where the authors embed *ad valorem* tariff and *variable* fixed cost[10] into the Helpman et al. [2004] framework. They find that as the tariff increases, the exporter's variable profit decreases, while the differences in fixed cost remain the same. When the tariff level is sufficiently high, the gain from avoiding the tariff is higher than the fixed cost of becoming multinational, and a firm prefers FDI over export as an entry mode. In the current setup, this is no longer the case. Compare the profit function for an exporter and a multinational firm:

$$\pi^H_X(c) = \left[p^H_X(c)/t^F - \tau^F c\right] q^H_X(c) \tag{19}$$

$$\pi^H_{FDI}(c) = \left[p^H_{FDI}(c) - \varphi^F c\right] q^H_{FDI}(c) \tag{20}$$

As tariff increases, the revenue of the exporter will drop, making export a less desirable mode of accessing the Foreign market. Eventually, FDI becomes a more desirable entry mode. Although the marginal cost of FDI is higher than export (due to the assumption that $\varphi^F > \tau^F$), the operating

[10] It means firms with different productivity levels will face a different level of fixed cost when accessing the Foreign market.

profit of FDI exceeds the profit of export. The trade-off between export and FDI is no longer the conventional proximity-concentration trade-off, but a comparison of the profits. I also plot firms' profits as a function of marginal production cost, as in **Figure 1**. Notice that the presence of a positive net tariff ensures that the profit of FDI is strictly higher than the profit of exports, whereas in Helpman et al. [2004] and Cole and Davies [2011], a similar condition is obtained through the combination of fixed cost and variable costs.

Given that the goal of this paper is to investigate the trade policy implications in an environment that features both export and multinational production, the selection into FDI (typically introduced through the fixed cost of FDI) margin is not of first-order interest here. A recent paper by Mrázová and Neary [2018] provides a justification for the current framework from a different perspective. They argue that statements like "Only the more productive firms select into the higher fixed-cost activity" are misleading: They are true given super-modularity[11] of the profit function, but otherwise may not hold. They discover that what matters for the direction of second-order selection effects (referring to the choice between export and FDI) is not a trade-off between fixed and variable costs, but whether there is a *complementarity between variable costs of production and trade*. In other words, if we allow FDI to be an equilibrium mode of accessing the Foreign market, then whether firms can afford them or not is independent of the fixed cost of FDI, but depends on the cross-effect of tariffs and production costs on profits. When super-modularity prevails, a more efficient firm has relatively higher operating profits in the FDI case, but when sub-modularity holds, the opposite may hold. The reason that the current setup can preserve the conventional sorting of firm productivity (i.e., second-order selection effect) is precisely due to the super-modularity of profit function[12] since there exists complementarity between the variable costs of production (in this case, the marginal cost of production c) and of trade (in this case, import tariff t).

[11] For example, super-modularity in $\Pi(t, c)$ means a higher tariff (t) reduces, in absolute value, the cost disadvantage of a higher-cost firm (larger c). For more details, please refer to Mrázová and Neary [2018].

[12] For a step-by-step verification, please refer to Mrázová and Neary [2018], their setup is a general preference, so they rely on the fixed cost to generate selection effects. For quadratic quasi-linear preference, they point out the first-order selection effect (according to their description, this is referring to whether serve the foreign market or not) needs the existence of choke price. The second-order effect is taken care of through the assumption we make.

2.3 Free Entry Condition

Entry is unrestricted in both countries. Firms choose a production location before entry and pay a sunk cost (f_E) to enter the market. To restrict the analysis on the effects of trade costs differences, I assume that countries share the same technology[13] (i.e., the same entry cost f_E and the same cost distribution $G(c)$). Free entry of domestic firms in country H implies zero expected profits in equilibrium, hence:

$$\int_0^{c_D^H} \pi_D^H(c)\, dG(c) + \int_{c_{FDI}^H}^{c_X^H} \pi_X^H(c)\, dG(c) + \int_0^{c_{FDI}^H} \pi_{FDI}^H(c)\, dG(c) = f_E \tag{21}$$

Given the Pareto assumption for cost distribution in both countries, the free entry condition for country H can be rewritten as:

$$\left(c_D^H\right)^{k+2} + \Phi_1^F \left(c_D^F\right)^{k+2} + \Phi_2^F \left(c_D^F\right)^{k+2} = \gamma\phi \tag{22}$$

where $\phi \equiv 2(k+1)(k+2)(c_M)^k f_E$ is a technology index that combines the effects of the better distribution of cost draws (lower c_M) and lower entry costs f_E. Moreover,

$$\begin{aligned}
\Phi_1^F \equiv{}& \frac{(k+1)(k+2)\,t^F(\tau^F)^2}{2}\left\{\left(\frac{1}{t^F\tau^F}\right)^{k+2} - \left(\frac{1}{t^F\tau^F}\right)^2\left(\xi^F\right)^k\right.\\
&\left. - \frac{2k}{k+1}\left[\left(\frac{1}{t^F\tau^F}\right)^{k+2} - \left(\frac{1}{t^F\tau^F}\right)\left(\xi^F\right)^{k+1}\right] + \frac{k}{k+2}\left[\left(\frac{1}{t^F\tau^F}\right)^{k+2} - \left(\xi^F\right)^{k+2}\right]\right\}\\
\Phi_2^F \equiv{}& \frac{(k+1)(k+2)\left(\xi^F\right)^k}{2}\left[1 - \frac{2k\varphi^F\xi^F}{k+1} + \frac{k\left(\varphi^F\xi^F\right)^2}{k+2}\right]
\end{aligned}$$

are indices that combine the trade-off between tariff and higher marginal cost of FDI. The free entry condition is homogenous to degree $k+2$ regarding the cutoff cost level. This system (for H, F) can then be solved for the cutoffs in both countries:

$$c_D^H = \left[\gamma\phi\frac{1-\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}\right]^{\frac{1}{k+2}} \tag{23}$$

[13] For implications of Ricardian comparative advantage in the heterogeneous firm framework, please refer to the Appendix in Melitz and Ottaviano [2008].

This equilibrium cutoff level differs from the one in Melitz and Ottaviano [2008] in two aspects. First, this cutoff is lower[14] than the closed-economy cutoff in Melitz and Ottaviano [2008]:

$$c_D^H < (\gamma\phi)^{\frac{1}{k+2}} \equiv \text{Closed-economy cutoff in Melitz and Ottaviano [2008]}$$

indicating the opening up of an economy via export and multinational activity will increase the aggregate productivity by forcing the least productive firms to exit. This result is similar to Melitz [2003], but the operating channel is the product market competition, not the factor market competition, as argued in Melitz and Ottaviano [2008]. Second, this cutoff is even lower[15]than the open economy cutoff in Melitz and Ottaviano [2008]:

$$c_D^H < \left(\gamma\phi\frac{1-\rho^F}{1-\rho^H\rho^F}\right)^{\frac{1}{k+2}} \equiv \text{Open-economy cutoff in Melitz and Ottaviano [2008]}$$

The intuition is straightforward: the presence of FDI, here the most productive firms in the distribution, intensifies the competitive environment in the economy, forcing the least productive firms to exit and hence further increases aggregate productivity[16]. Notice, the open economy cutoff stated above is slightly different from Melitz and Ottaviano [2008] due to the presence of *ad valorem* tariffs[17].

2.4 Prices, Product Variety, Number of Entrants and Welfare

To see more features in the current setup, I first compute $\bar{p}^H$. Notice, the marginal cost of H's operating firms fall into the range $[0, c_D^H]$, which is also the range for delivered cost of exporters $(\tau^F c)$, and the effective marginal cost of multinationals production $(\varphi^F c)$. They all share identical distributions over the support given by $G^H(c) = (c/c_D^H)^k$. Therefore, the price distributions of H's domestic firms, $p_D^H(c)$, H's exporters producing in F, $p_X^F(c)$, and F's multinationals producing in

[14]This is based on Proposition 3, see Appendix B.3.

[16]This is consistent with the recent empirical evidence discovered in Fons-Rosen et al. [2013], although I abstract from the possibility of any spillover effect.

[17]Please see Appendix B.3 for more details.

H, $p^F_{FDI}(c)$, are all identical. The average price in country H is thus given by:

$$\bar{p}^H = \frac{1}{G\left(c^H_D\right)} \int_0^{c^H_D} p^H_D(c)\, dG(c) = \frac{1}{G\left(c^F_X\right)} \int_{c^F_{FDI}}^{c^F_x} p^F_X(c)\, dG(c)$$
$$= \frac{1}{G\left(c^F_{FDI}\right)} \int_0^{c^F_{FDI}} p^F_{FDI}(c)\, dG(c) = \frac{2k+1}{2k+2} c^H_D \tag{24}$$

Combining this with the definition of $p^H_{\max}$ and $p^F_{\max}$, the number of firms selling in country H is:

$$N^H = \frac{2\gamma\left(\alpha - c^H_D\right)(k+1)}{\eta c^H_D} \tag{25}$$

From this expression, it must be the case that $\alpha > c^H_D$ so that the number of firms selling in country H is positive in equilibrium. The total number of product varieties in country H is composed of domestic producers, exporters, and multinationals from country F. Given a positive mass of entrants N_E in both countries, there are $G(c^H_D)N^H_E$ domestic producers, $[G(c^F_X) - G(c^F_{FDI})]N^F_E$ Foreign exporters, and $G(c^F_{FDI})N^F_E$ Foreign multinationals selling in H. Altogether they satisfy the following condition:

$$G\left(c^H_D\right) N^H_E + \left[G\left(c^F_X\right) - G\left(c^F_{FDI}\right)\right] N^F_E + G\left(c^F_{FDI}\right) N^F_E = N^H \tag{26}$$

Solving this system (for H and F) will give us the number of entrants in country H:

$$N^H_E = \frac{2\left(c_M\right)^k (k+1)\gamma}{\eta\left(1 - \delta^H \delta^F\right)} \left[\frac{\alpha - c^H_D}{\left(c^H_D\right)^{k+1}} - \delta^H \frac{\alpha - c^F_D}{\left(c^F_D\right)^{k+1}}\right] \tag{27}$$

where $\delta^l = (t^l \tau^l)^{-k}$, for $l \in \{H, F\}$. Notice, the condition that ensures positive equilibrium number of varieties (N^l) in the economy, $\alpha > c^l_D$, also guarantees the positive mass of entry (N^l_E) in the equilibrium.

Equation (27) marks a crucial difference between the current framework and Cole and Davies [2011]: The number of entrants in the economy is endogenously affected by the tariff level. As first noted by Balistreri et al. [2011], the equilibrium level of firm entry in the Melitz-type model is no longer fixed if: (i) *ad valorem* tariffs are imposed rather than iceberg transport costs, or (ii) there are multiple sectors in the economy. Equilibrium entry-level becomes endogenous in the current framework because of not only the *ad valorem* tariff, but also the two-sector economy. However, it is surprising to note that Cole and Davies [2011] ignore this dimension in their discussion, although their framework also features these two aspects. The authors might have got the impression from the basic Melitz model that trade costs are independent of firm entry, which is true in Melitz [2003]. However, when analyzing the revenue-generating tariff, this perception is no longer valid[18]. This implication is crucial in understanding the equilibrium feature of the model. I will come back to this point in Section 4.

Following Melitz and Ottaviano [2008], combine equation (4), (24), (25) and the definition of $\sigma^2_{p^H}$, it is straightforward to show the consumer welfare in H equals to:

$$U^H = I^H + \frac{\alpha - c_D^H}{2\eta}\left(\alpha - \frac{k+1}{k+2}c_D^H\right) \tag{28}$$

Once again, consumer welfare changes monotonically with the domestic cutoff, which captures the effect of an increase in product variety and a decrease in the average price. Also notice, consumer surplus in country H is given by the second term in equation (28).

2.5 Tariff Revenue and National Welfare

This part of the model is very important for the analysis of socially optimal tariff and Nash tariff. Note that tariff revenue is also a component of consumer income I^H through the redistribution from the government. I define the pre-tax value of country H's import as:

$$\begin{aligned} IM^H &= N_E^F \int_{c_{FDI}^F}^{c_X^F} \frac{p_X^F(c)}{t^H} q_X^F(c)\, dG(c) \\ &= N_E^F \frac{t^H \left(\tau^H\right)^2 \left(c_D^H\right)^{k+2}}{4\gamma (k+2)(c_M)^k}\left[2\left(\frac{1}{t^H\tau^H}\right)^{k+2} - \frac{k+2}{\left(t^H\tau^H\right)^2}\left(\xi^H\right)^k + k\left(\xi^H\right)^{k+2}\right] \end{aligned} \tag{29}$$

[18]More specifically, according to Arkolakis et al. [2012], one of the 'macro' assumption (R2) is violated, therefore entry become endogenous.

Therefore, the total import tariff revenue of country H is defined as

$$\begin{aligned} TR^H &\equiv (t^H - 1) \times IM^H \\ &= N_E^F \frac{t^H - 1}{t^H} \frac{\left(c_D^H\right)^{k+2}}{4\gamma (k+2) (c_M)^k} \left[2 \left(\frac{1}{t^H \tau^H} \right)^k - (k+2) \left(\xi^H \right)^k + k \left(\xi^H \right)^{k+2} \left(t^H \tau^H \right)^2 \right] \end{aligned} \tag{30}$$

From the trade-policy perspective, the government will use its policy instrument to maximize consumer welfare:

$$U_n^H = \underbrace{w^H + (t^H - 1) \times IM^H + \Pi^H}_{\equiv I^H} + \underbrace{\frac{\alpha - c_D^H}{2\eta} \left(\alpha - \frac{k+1}{k+2} c_D^H \right)}_{\equiv CS^H} \tag{31}$$

Therefore tariff affects consumer welfare from two channels: (i) consumer surplus, which is directly affected by the change in c_D^H in response to tariff, and (ii) tariff revenue, which is affected by both the tariff level (t^H) and the tariff base (IM^H). Due to the free-entry condition, aggregate profit Π^H will be driven to zero in equilibrium. Notice due to the presence of numéraire good, $w^H = 1$, although consumers will not take t^H into consideration when maximizing their utility, the government does take consumers into consideratoin by choosing the utility maximizing tariff level.

With the model above, I will now discuss the equilibrium features of this economy. All the results will be contrasted with an economy that features heterogeneous firms, FDI, but constant markups, i.e., the one in Cole and Davies [2011].

3 Short-run Equilibrium

As discussed in Section 2, due to the presence of *ad valorem* tariff and the quadratic quasi-linear preference (which results in multiple sectors in the current setup), the level of firm-entry become endogenous under the free-entry condition. To see how the model directly responds to the trade/tariff liberalization, it is, therefore, necessary to separate the short-run (when the entry is restricted) from the long-run (free entry). In this section, I introduce the short-run version of the model and discuss its equilibrium characteristics.

In the short-run, when entry and exit are prohibited, the economy is characterized by a fixed number of incumbents, and they decide whether to produce or shut-down based on their profits. More specifically, Home country is characterized by a fixed number of incumbents, $\bar{N_I^H}$, with cost distribution $\bar{G^H}$ on $[0, \bar{c}_M]$, where $\bar{c}_M$ is within the long-run technology frontier, c_M. I keep the assumption that the productivity $1/c$ is distributed with Pareto shape k, implying $\bar{G}^H(c) = (c/\bar{c}_M)^k$. The distribution of firm's productivity in the short-run model is briefly displayed in **Figure 2**.

A Home firm produces if it can earn nonnegative profits from either its domestic market, export market, or FDI market. These decisions based on profits lead to the following short-run cutoff conditions:

$$c_D^H = \sup\{c : \pi_D^H(c) \geq 0 \text{ and } c \leq \bar{c}_M\}$$
$$c_X^H = \sup\{c : \pi_X^H(c) \geq 0 \text{ and } c \leq \bar{c}_M\}$$
$$c_{FDI}^H = \sup\{c : \pi_{FDI}^H(c) \geq \pi_X^H(c) \text{ and } c \leq \bar{c}_M\}$$

Firms with marginal cost $c > c_D^H$ will shut-down. Utilizing the zero-profit conditions, one can establish the following relations between cutoff levels and the number of operating firms in Home and Foreign:

$$N^H = \frac{2\,(k+1)\,\gamma}{\eta} \times \frac{\alpha - c_D^H}{c_D^H}$$

$$N^F = \frac{2\,(k+1)\,\gamma}{\eta} \times \frac{\alpha - t^F \tau^F c_X^H}{t^F \tau^F c_X^H}$$

where N^H and N^F represent the endogenous number of sellers in country H and F in the short-run. Notice that the different cutoffs satisfy the same condition as in the long-run. There are $\bar{N}_I^H \bar{G}\left(c_D^H\right)$ producers in H who sell in their domestic market, $\bar{N}_I^F \left[\bar{G}\left(c_X^F\right) - \bar{G}\left(c_{FDI}^F\right)\right]$ Foreign exporters, and $\bar{N}_I^F \bar{G}\left(c_{FDI}^F\right)$ Foreign FDI firms in H. These numbers must add up to the total number of producers in country H. Similar equation also holds for country F:

$$N^H = \bar{N}_I^H \bar{G}\left(c_D^H\right) + \bar{N}_I^F \left[\bar{G}\left(c_X^F\right) - \bar{G}\left(c_{FDI}^F\right)\right] + \bar{N}_I^F \bar{G}\left(c_{FDI}^F\right)$$

$$N^F = \bar{N}_I^F \bar{G}\left(c_D^F\right) + \bar{N}_I^H \left[\bar{G}\left(c_X^H\right) - \bar{G}\left(c_{FDI}^H\right)\right] + \bar{N}_I^H \bar{G}\left(c_{FDI}^H\right)$$

Combining these two equations with the threshold price conditions yield expressions for the cost cutoffs in both countries:

$$\frac{\alpha - c_D^H}{\left(c_D^H\right)^{k+1}} = \frac{\eta}{2\left(k+1\right)\gamma}\left\{\frac{\bar{N}_I^H}{\bar{c}_M^k} + \left[\left(\frac{1}{t^H \tau^H}\right)^k - \left(\xi^H\right)^k\right]\bar{N}_I^F + \left(\xi^H\right)^k \bar{N}_I^F\right\} \tag{32}$$

$$\frac{\alpha - c_D^F}{\left(c_D^F\right)^{k+1}} = \frac{\eta}{2\left(k+1\right)\gamma}\left\{\frac{\bar{N}_I^F}{\bar{c}_M^k} + \left[\left(\frac{1}{t^F \tau^F}\right)^k - \left(\xi^F\right)^k\right]\bar{N}_I^H + \left(\xi^F\right)^k \bar{N}_I^H\right\} \tag{33}$$

Note, these two conditions uniquely identify the short-run cutoff levels(c_D^H, c_D^F) with the number of producing firms in each country (N^H, N^F).

Equation (32) and (33) also clearly highlight the protection role played by import tariff in the short-run. Based on these two equations, we have the following proposition for the short-run equilibrium.

Proposition 1. *In the short-run equilibrium, an increase in Home country's import tariff (t^H) can protect Home producers from Foreign competition, increasing the domestic cost cutoff:*

$$\frac{\partial c_D^H}{\partial t^H}\Big|_{short\text{-}run} > 0$$

Proof. See Appendix B.1. ◻

Intuitively, an increase in H's tariff will make it harder for the Foreign exporters to access the Home market, so the number of exporters from F to H will decrease. At the same time, an increase in H's tariff will induce tariff-jumping FDI among the Foreign exporters, so the number of Foreign firms that access the Home market through FDI will increase. In the current setup, the decrease of exporters surpasses the increase of FDI firms, so the right-hand side of the equation (32) is decreasing in t^H, indicating an increase in H's domestic cost cutoff (c_D^H). Therefore, an increase in

H's tariff reduces the total number of Foreign firms (exporters and FDI firms) accessing the Home market, making it easier for Home producers to survive.

In other words, import tariff, in the short-run, can effectively shield Home from Foreign competition. This result is similar to the result in Section 3.7 of Melitz and Ottaviano [2008]. However, they obtain the result of an increase in cutoff level through an exogenous variation of trading partner industrial size (N^H or N^F). In the current framework, a change in tariff level alters the relative size of Home and Foreign firms, affecting the cutoff levels. This finding might seem to confirm the findings in Cole and Davies [2011][19], but if we allow firms to enter freely, then the result will be quite different. As we will see in the next section, the classic 'delocation'[20] result will arise.

Based on equation (6), (10) and (15), we can also obtain the following proposition regarding markups in respond to a tariff change:

Proposition 2. *In the short-run equilibrium, an increase in Home country's import tariff (t^H) can increase domestic producer's markup, may decrease or increase Foreign exporter's markup, and increase Foreign FDI firm's markup.*

$$\frac{\partial m_D^H}{\partial t^H}|_{short\text{-}run} > 0, \frac{\partial m_X^F}{\partial t^H}|_{short\text{-}run}?0, \frac{\partial m_{FDI}^F}{\partial t^H}|_{short\text{-}run} > 0$$

Proof. See Appendix B.2. □

Intuitively, for an increase in t^H, tariff affects *Home domestic producer's* markup $m_D^H(c)$ through the equilibrium effect on c_D^H. Protection makes it easier for Home producers to survive and results in a higher c_D^H, meaning a higher markup for all the domestic sellers. For *Foreign exporters*, their markup $m_X^F(c)$ is affected by tariff from two aspects: (i) direct effect—-an increase in t^H directly raises m_X^F, meaning Foreign exporters will pass the tariff burden to the consumers by increasing markup, and (ii) indirect effect—-the tariff indirectly affects m_X^F through the equilibrium effect on c_D^H. With restricted entry, these two effects are in the opposite direction. Which effect is more dominant depends on the individual exporter's productivity. For the least productive Foreign exporters (large c), the direct effect will dominate the indirect effect, resulting in a drop in Foreign exporter's markup,

[19] Specifically, their justification of their equation (11). Similar results are qualitatively identical to those of other similar models in the heterogeneous firm literature, such as Melitz [2003] and Helpman et al. [2004].

[20] The delocation effect has been studied in previous work(see, for example, Venables [1985], Helpman and Krugman [1989], Baldwin et al. [2003]) and here is also confirmed in the heterogeneous firm framework with FDI.

indicating Home's protection will reduce the market power of Foreign exporters. Nevertheless, for the most productive Foreign exporters (small c), the indirect effect will dominate the direct effect, resulting in the bigger market power of Foreign exporters. For *Foreign FDI firms*, tariff affects $m^F_{FDI}(c)$ through the equilibrium effect on c^H_D. Protectionism results in a less competitive Home environment, which benefits the more productive Foreign FDI firms, and allows them to charge higher markups.

In this short-run equilibrium, where additional entry of firms is restricted, the findings in the current framework confirm the previous results in the literature on unilateral trade/tariff liberalization. As we will see in the next section, these results will be reversed with an endogenous level of firm entry in the long-run.

4 Long-run Equilibrium

In this section, I analyze the equilibrium features of the model when firms can enter and exit freely. As mentioned in Section 2, firm entry, in the long-run, is endogenously affected by the tariff level. This will have crucial implications for trade/tariff liberalization. In what follows, I will illustrate the comparative statics of the model and contrast its benchmark results with Cole and Davies [2011].

First of all, following the discussion of equation (23), the presence of FDI in the Melitz and Ottaviano [2008] world will deliver a different equilibrium domestic cutoff in the economy, which can be summarized by the following proposition:

Proposition 3. *The presence of FDI makes the economy more competitive, and the domestic cutoff is lower compared to the case when there is no FDI:*

$$c^H_{D1} = \left[\gamma\phi\frac{1-\left(\Phi^F_1+\Phi^F_2\right)}{1-\left(\Phi^F_1+\Phi^F_2\right)\left(\Phi^H_1+\Phi^H_2\right)}\right]^{\frac{1}{k+2}} < c^H_{D2} = \left[\gamma\phi\frac{1-\psi^F}{1-\psi^F\psi^H}\right]^{\frac{1}{k+2}}$$

Proof. See Appendix B.3. □

Different from the short-run equilibrium case, here the domestic cost cutoff is pinned down through the long-run free-entry condition (equation (22)). In the Appendix B.3, I show $\Phi^l_1+\Phi^l_2 > \psi^l$

for $l \in \{H, F\}$. The sum of Φs can be viewed as a measure of 'openness'. Intuitively, the presence of FDI makes the country more 'open' compared to the case when FDI is not an option. In Melitz and Ottaviano [2008], ψ^l [21]measures the 'freeness' of trade. The presence of Foreign FDI intensifies the Home country's competitive environment, making it harder for Home producers to survive. The marginally surviving firm needs to be more productive. Openness (either through export or FDI) increases competition[22] in the domestic product market, shifting up residual demand price elasticities for all firms at any given demand level. Therefore, the least productive firms are forced to exit. This effect is very similar to an increase in market size in the closed economy: the increased competition induces a downward shift in the distribution of markups across firms. Although only relatively more productive firms survive (with higher markups than the less productive firms who exit), the average markup is reduced.

With free-entry, firms can freely enter and exit the market in the long-run. Due to the presence of *ad valorem* tariff and the quadratic quasi-linear preference, the number of entrants in the monopolistically competitive sector is endogenously affected by the level of tariff. As we will see soon, this feature has important implications for trade/tariff liberalization. For example, a unilateral change in Home country's import tariff has quite different impacts on the domestic productivity cutoffs of both countries, as can be seen from the following proposition:

Proposition 4. *An increase in country H's import tariff results in a decrease in the cutoff cost level in country H's domestic market, and an increase in the cutoff cost level in country F's domestic market:*

$$\frac{\partial c_D^H}{\partial t^H} < 0 < \frac{\partial c_D^F}{\partial t^H}$$

Proof. See Appendix B.4. □

Different from the short-run outcome, where the H's domestic cost cutoff would increase as a result of tariff protection, in the long-run, H's domestic cost cutoff would decrease in response to tariff protection. Intuitively, although an increase in the import tariff raises the protection level in country H in the short-run, it also fosters a more extensive entry from domestic firms over time.

[21]More precisely, the freeness of trade is measured by τ^{-k} in Melitz and Ottaviano [2008]. Here, due to the presence of tariff, this term is augmented to incorporate tariff, $\tau^{-k} t^{-(k+1)}$.

[22]Compared to the case when export is the only option to access Foreign market.

Proposition 7 near the end of this section further demonstrates this point. Protection makes the Home country a more desirable environment for firms to do business in the long-run. With the free-entry condition, the larger entry will generate a higher competition in the domestic market, driving out the least productive firms, and forcing the marginally surviving firms to be more productive.

This result is quite different from Cole and Davies [2011]. They find an increase in the import tariff in country H will raise the protection level in country F, shield country H's firm from the competition, and make the domestic surviving firms less productive, i.e., $\partial c_D^H/\partial t^H > 0$ (their equation (11)). This result is primarily due to the fact that the presence of quadratic quasi-linear preference affects firm-entry, which is entirely absent in Cole and Davies [2011].

A unilateral change in Home's import tariff also affects the exporters in both countries. The impact can be summarized as the following:

Proposition 5. *An increase in country H's import tariff results in an increase in the export cutoff cost level in country H and a decrease in the export cutoff cost level in country F:*

$$\frac{\partial c_X^H}{\partial t^H} > 0 > \frac{\partial c_X^F}{\partial t^H}$$

Proof. See Appendix B.5. □

An increase in import tariff in country H will cause the least productive exporters from F to quit exporting, and only serve their domestic market. The reason is that the increase in tariff reduces exporter's revenue (hence profit), making it less desirable for the least productive exporters to serve H's market. With their exit, the marginally surviving exporters are more productive, hence a lower c_X^F. This result is the same as the one obtained in Cole and Davies [2011]. Moreover, although protection generates more entry of Home firms in the long-run, giving a competitive pressure for Home's export market, this impact is dominated by the protective effect of the tariff on exporters, resulting in a higher Home exporter cutoff, and making it easier for Home country to export.

Proposition 6. *An increase in country H's import tariff results in an increase in the FDI cutoff cost level in country F:*

$$\frac{\partial c_{FDI}^F}{\partial t^H} > 0$$

Proof. See Appendix B.6. □

The intuition here is straightforward: the most productive exporters from F, when facing an increase in import tariff in H, will find it less desirable to access H's market through export, and hence choose FDI as the entry mode. This result is due to the profit of FDI outweighing the profit of export when t^H increases, as can be seen from equation (19) and (20). Hence the marginally surviving multinationals from country F are now less productive since previously they are exporters, resulting in a higher c^F_{FDI}. This result is similar to the findings in Cole and Davies [2011].

To sum up these results and contrast them with Cole and Davies [2011], I plot the productivity cutoffs and their responses toward Home country's unilateral change in tariff, which is shown in **Figure 3**. When t^H increases, from equation (11)–(13) in Cole and Davies [2011], the least productive Foreign exporters exit the domestic market (c^F_X decreases) and the most productive Foreign exporters become multinationals (c^F_{FDI} increases). This change makes the composition of domestic Foreign firms (including F's exporters and multinationals) more productive. Due to the protection, the domestic market is shielded from the Foreign competition. Hence domestic firms find it easier to survive (c^H_D increases).

In the current setup, an increase in t^H will similarly lead the least productive Foreign exporters to exit the domestic market (c^F_X decreases) and the most productive Foreign exporters to become multinationals (c^F_{FDI} increases), making the composition of domestic Foreign firms more productive. However, Home's protection will attract more Home firms to enter the domestic market (N^H_E increases), making the Home country's environment more competitive. Therefore, domestic firms need to be more productive to survive (c^H_D decreases).

In both cases, we have tariff-jumping FDI in response to the increase of t^H. In Cole and Davies [2011], tariff-jumping intensifies the competitive environment in the domestic market of H, but this effect is dominated by the protection effect raised through tariff. So the outcome is an easier-to-survive environment. In the current setup, the tariff-jumping FDI intensifies the competitive environment in the domestic market. The excessive entry generated by protection also makes the domestic environment more competitive. These two effects together result in a tougher environment in Home's domestic market, making it harder for firms to survive.

Corollary 1. *Under the assumption that $\varphi^H > \tau^H$ [23], an increase in H's import tariff results in a*

[23] The domestic cutoff without FDI but with ad valorem tariff is $c^H_D = \left[\gamma\phi\left(1-\rho^F\right)/\left(1-\rho^H\rho^F\right)\right]^{1/(k+2)}$, where $\rho^H = (\tau^H)^{-k}(t^H)^{-(k+1)}$. The domestic cutoff with FDI is defined in equation (23).

tougher competitive environment in the domestic market over time, and this effect is exacerbated by the presence of FDI:

$$\left| \frac{\frac{\partial c_D^H}{\partial t^H}|_{without\ FDI}}{\frac{\partial c_D^H}{\partial t^H}|_{with\ FDI}} \right| < 1$$

Proof. See Appendix B.7. □

Finally, the protection also affects the number of entrants and eventually the number of products available in each country. I summarize this result in the following proposition:

Proposition 7. *An increase in H's import tariff results in an increase in the number of entrants in H and a decrease in the number of entrants in F. Over time, this effect contributes to an increase in the number of varieties in H and a decrease in the number of varieties in F:*

$$\frac{\partial N_E^H}{\partial t^H} > 0 > \frac{\partial N_E^F}{\partial t^H}$$
$$\frac{\partial N^H}{\partial t^H} > 0 > \frac{\partial N^F}{\partial t^H}$$

Proof. See Appendix B.8. □

The intuition is obvious: Home's tariff protection makes the Home country a more desirable environment to do business for the firms. In the long-run, more firms would choose to enter Home's market, resulting in a larger number of products available in the equilibrium. The opposite condition will hold for the Foreign market. Clearly, this result crucially depends on the fact that tariff can affect the number of entrants in this economy. No similar results are discussed in Cole and Davies [2011].

Proposition 8. *In the long-run equilibrium, an increase in Home country's import tariff (t^H) can decrease domestic producer's markup, decrease Foreign exporter's markup, and decrease Foreign FDI firm's markup.*

$$\frac{\partial m_D^H}{\partial t^H}|_{long\text{-}run} < 0, \frac{\partial m_X^F}{\partial t^H}|_{long\text{-}run} < 0, \frac{\partial m_{FDI}^F}{\partial t^H}|_{long\text{-}run} < 0$$

Proof. See Appendix B.9. □

The intuition follows right after the previous proposition. Due to the increase of protectionist tariff, Home becomes a more favorable environment to do business, attracting more firm to enter in the long-run. This effect will increase the competition in the Home market, reducing the markups for all kinds of producers that serve the Home market.

5 Concluding Remarks

This paper introduces *ad valorem* tariff and horizontal FDI into the Melitz and Ottaviano [2008] framework. To the best of my knowledge, this is the first paper in the trade policy literature that incorporates firm heterogeneity, variable markups, and multinational production. The results can be broadly summarized as follows.

First, I find that the presence of multinational production generates a competitive effect on the economy, and firms need to be more productive to survive the competition. Second, I find that the *ad valorem* tariff and quadratic quasi-linear preference collectively result in an endogenous level of firm entry. Therefore, the impact of trade/tariff liberalization will depend on the equilibrium number of firms. In the short-run, when the firm entry is prohibited, an increase in import tariff shields the domestic economy from the Foreign competition, making it easier for firms to survive. This result is overturned when I allow firms to enter the market freely in the long-run. In the long-run, an increase in Home's import tariff will make the Home country a more desirable environment to do business, attracting more entrants in the Home market, making the Home market more competitive. Firms need to be more productive to survive. Home's tariff increase also makes it harder for the least productive Foreign exporters to survive, and triggers tariff-jumping FDI among the most productive exporters. Markups also respond to tariff change differently in the short-run vs. long-run, primarily due to the change of competitive environment brought in through firm entry.

Given these results, there are several potential avenues for future theoretical research. One the one hand, one can deviate from the Pareto distribution assumption. As discussed by Feenstra [2018], if the support of cost distribution becomes bounded, other channels that affect the pro-competitive effect of trade will begin to work, delivering a different welfare implication of trade. On the other hand, the combination of Melitz and Pareto implies that trade costs will only affect export through extensive margin, but this implication is at odds with the empirical fact that most of the adjustments happen along the intensive margin. This phenomenon can be reconciled by introducing log-normal distribution[24].

Another exciting path will be to investigate if the current results on trade/tariff liberalization are robust under alternative demand/supply structures that generate variable markups. Several approaches are readily available. (i) Deviating from the *quadratic quasi-linear preference*. Many preferences that are surveyed in Ding [2020] are great alternatives. I have investigated *translog expenditure function* as in Rodriguez-Lopez [2011], and find that most of the claims in this paper will still be valid. (ii) Dropping the *monopolistic competition* assumption by introducing oligopolistic competition. I expect this option to be particularly interesting, given the recent increased attention on oligopoly[25]. (iii) Deviating from *quadratic quasi-linear preference* and *monopolistic competition*. However, based on the survey in Ding [2020], all the existing works adopt quadratic quasi-linear preference, so (iii) is identical to (ii).

Lastly, the implications of trade/tariff liberalization here primarily focus on the monopolistically competitive sector. It would be interesting and relevant to see how a multi-sector framework would affect the results. For example, Spearot [2016] extends Melitz and Ottaviano [2008] by incorporating multiple countries and multiple industries, with heterogeneity in the country-by-industry shape parameters of the Pareto cost distributions, and provides a quantitative implication for unilateral tariff liberalization. The excess entry in the current framework might have different implications in his model since the mass of entrants in a particular sector now depends on the relative expenses on goods produced in that sector. I leave these questions for future work.

[24]See Fernandes et al. [2017] for a detailed discussion.

[25]See Head and Spencer [2017] for more discussion.

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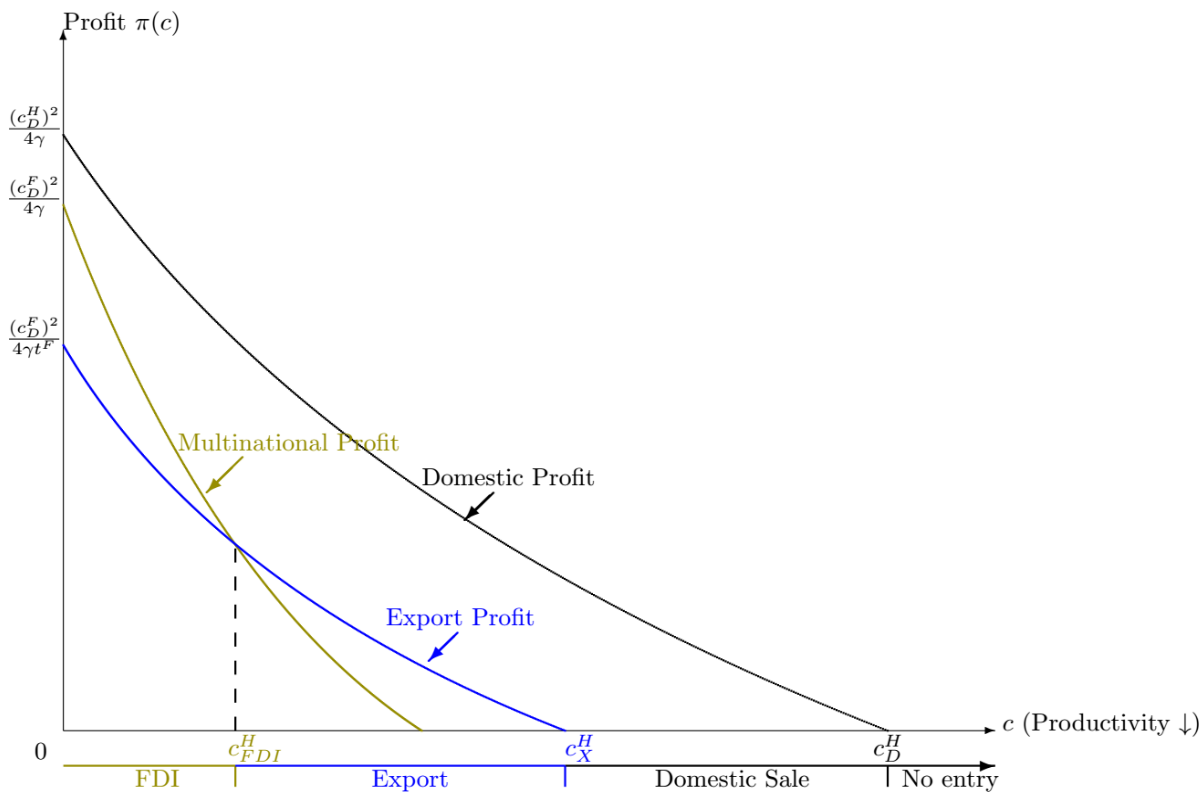
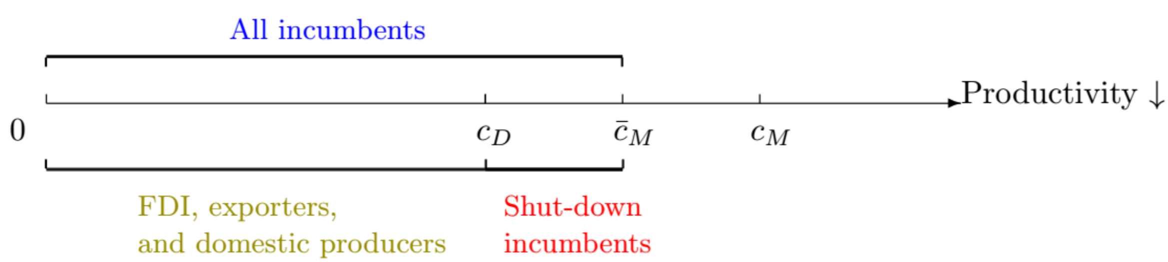
A Figures

Figure 1: Firms' Profit as a Function of Marginal Cost



Note: The graph plots the profit function of firms in each market according to the firm's serving mode. The black line represents the firm's profit function in the domestic market (in this case, it is Home). The blue line plots the exporter's profit function in the Foreign market, and the green line plots the profit function of multinational firms.

Figure 2: Productivity Distribution in the Short-run Equilibrium



Note: c_M together with f_E stand for the state of the technology in the long-run equilibrium. $\bar{c}_M$ together with f_E stands for the state of the technology in the short-run equilibrium. c_D stands for the domestic cutoff level in the short-run equilibrium. Exporter and FDI cutoffs distributed on the left-hand side of c_D.

Figure 3: Productivity Cutoffs Responses Comparison in the Long-run Equilibrium

Note: Each axis represents the distribution of productivity within the monopolistically competitive industry. The first two panels are based on equation (11)-(13) from Cole and Davies [2011]. The last two panels are based on Proposition 4 through Proposition 6.

B Proofs

B.1 Proof of Proposition 1

The proof mainly comes from the following equation:

$$\frac{\alpha - c_D^H}{\left(c_D^H\right)^{k+1}} = \frac{\eta}{2\left(k+1\right)\gamma}\left\{\frac{\bar{N}_I^H}{\bar{c}_M^k} + \left[\left(\frac{1}{t^H\tau^H}\right)^k - \left(\xi^H\right)^k\right]\bar{N}_I^F + \left(\xi^H\right)^k\bar{N}_I^F\right\}$$

It is straightforward to show that $\partial\xi^H/\partial t^H > 0$, therefore the whole expression on the right-hand side will decrease as t^H increases. It follows from the equation that it must be true that c_D^H will increase, hence $\partial c_D^H/\partial t^H > 0$ in the short-run. □

B.2 Proof of Proposition 2

To prove this proposition, it is helpful to rewrite the markups as follows:

$$\begin{aligned}
m_D^H\left(c\right) &= \frac{1}{2c}\left(c_D^H + c\right)\\
m_X^F\left(c\right) &= \frac{t^H}{2c}\left(c_X^F + c\right) = \frac{t^H}{2c}\left(c_D^H/t^H\tau^H + c\right) = \frac{1}{2c}\left(c_D^H/\tau^H + c/t^H\right)\\
m_{FDI}^F\left(c\right) &= \frac{1}{2\varphi^H c}\left(c_D^H + \varphi^H c\right)
\end{aligned}$$

It follows from Proposition 1 that $\partial m_D^H/\partial t^H > 0$ and $\partial m_{FDI}^F/\partial t^H > 0$. The responses of m_X^F is less transparent. On the one hand, c_D^H/τ^H increases as t^H increases. On the other hand, c/t^H decreases as t^H increases. The total impact on m_X^F is therefore ambiguous. If c is small, then the first effect will dominate the second effect, $\partial m_X^F/\partial t^H > 0$. If c is big, then the second effect will dominate the first effect, $\partial m_X^F/\partial t^H < 0$. □

B.3 Proof of Proposition 3

To prove this proposition, I first prove the following condition:

$$\Phi_1^l + \Phi_2^l > \psi^l \in (0,1) \text{ for } l \in \{H, F\}$$

Given $\psi^l \equiv \left(\tau^l\right)^{-k} \left(t^l\right)^{-(k+1)}$ and

$$\Phi_1^l \equiv \frac{(k+1)\,(k+2)\,t^l(\tau^l)^2}{2}\left\{\left(\frac{1}{t^l\tau^l}\right)^{k+2} - \left(\frac{1}{t^l\tau^l}\right)^2 \left(\xi^l\right)^k \right.$$
$$\left. -\frac{2k}{k+1}\left[\left(\frac{1}{t^l\tau^l}\right)^{k+2} - \left(\frac{1}{t^l\tau^l}\right)\left(\xi^l\right)^{k+1}\right] + \frac{k}{k+2}\left[\left(\frac{1}{t^l\tau^l}\right)^{k+2} - \left(\xi^l\right)^{k+2}\right]\right\}$$
$$\Phi_2^l \equiv \frac{(k+1)\,(k+2)\,\left(\xi^l\right)^k}{2}\left[1 - \frac{2k\varphi^l\xi^l}{k+1} + \frac{k\left(\varphi^l\xi^l\right)^2}{k+2}\right]$$

It is then straightforward to show

$$\Phi_1^l + \Phi_2^l = \psi^l + \frac{(k+1)\,(k+2)\,\left(\xi^l\right)^k}{2}\left\{\left(1-\frac{1}{t^l}\right)\right.$$
$$\left. -\frac{2k}{k+1}\xi^l\left(\varphi^l - \tau^l\right) + \frac{k}{k+2}\left(\xi^l\right)^2\left(\left(\varphi^l\right)^2 - t^l\left(\tau^l\right)^2\right)\right\}$$

To show that $\Phi_1^l + \Phi_2^l > \psi^l$, it is equivalent to show that

$$\left(1-\frac{1}{t^l}\right) - \frac{2k}{k+1}\xi^l\left(\varphi^l - \tau^l\right) + \frac{k}{k+2}\left(\xi^l\right)^2\left(\left(\varphi^l\right)^2 - t^l\left(\tau^l\right)^2\right) > 0$$

Based on the definition of $\xi^l \equiv \left(\sqrt{t^l}-1\right)/\left(\sqrt{t^l}\varphi^l - t^l\tau^l\right)$, the above equation becomes:

$$\left(1-\frac{1}{t^l}\right) - \frac{2k}{k+1}\frac{\sqrt{t^l}-1}{\sqrt{t^l}\varphi^l - t^l\tau^l}\left(\varphi^l - \tau^l\right) + \frac{k}{k+2}\left(\frac{\sqrt{t^l}-1}{\sqrt{t^l}\varphi^l - t^l\tau^l}\right)^2\left(\left(\varphi^l\right)^2 - t^l\left(\tau^l\right)^2\right) > 0$$
$$\Leftrightarrow \left(1-\frac{1}{t^l}\right) + \frac{k}{k+2}\left(\frac{\sqrt{t^l}-1}{\sqrt{t^l}\varphi^l - t^l\tau^l}\right)^2\left(\left(\varphi^l\right)^2 - t^l\left(\tau^l\right)^2\right) > \frac{2k}{k+1}\frac{\sqrt{t^l}-1}{\sqrt{t^l}\varphi^l - t^l\tau^l}\left(\varphi^l - \tau^l\right)$$
$$\Leftrightarrow \frac{t^l-1}{t^l} + \frac{k}{k+2}\frac{\left(\sqrt{t^l}-1\right)^2}{t^l}\frac{\varphi^l + \sqrt{t^l}\tau^l}{\varphi^l - \sqrt{t^l}\tau^l} > \frac{2k}{k+1}\frac{\sqrt{t^l}-1}{\sqrt{t^l}}\frac{\varphi^l - \tau^l}{\varphi^l - \sqrt{t^l}\tau^l}$$
$$\Leftrightarrow \frac{t^l-1}{t^l} + \frac{k}{k+2}\frac{\left(\sqrt{t^l}-1\right)^2}{t^l}\frac{\varphi^l + \sqrt{t^l}\tau^l}{\varphi^l - \sqrt{t^l}\tau^l} > \frac{2k}{k+1}\frac{\sqrt{t^l}-1}{\sqrt{t^l}}\frac{\varphi^l - \tau^l}{\varphi^l - \sqrt{t^l}\tau^l}$$

Multiply both sides by $(k+1)(k+2)\,t^l\left(\varphi^l - \sqrt{t^l}\tau^l\right)$, I have

$$\begin{aligned}
&\left(k^2+3k+2\right)\left(\sqrt{t^l}+1\right)\left(\varphi^l-\sqrt{t^l}\tau^l\right)+\\
&\quad\left(k^2+k\right)\left(\sqrt{t^l}-1\right)\left(\varphi^l+\sqrt{t^l}\tau^l\right)>\left(2k^2+4k\right)\sqrt{t^l}\left(\varphi^l-\tau^l\right)\\
&\qquad\Leftrightarrow 2\sqrt{t^l}\varphi^l-2\sqrt{t^l}\tau^l+2(k+1)\left(\varphi^l-t^l\tau^l\right)>0\\
&\qquad\Leftrightarrow 2\sqrt{t^l}\left(\varphi^l-\tau^l\right)+2(k+1)\left(\varphi^l-t^l\tau^l\right)>0
\end{aligned}$$

This is obviously true when $\varphi^l > t^l\tau^l$ (note $t^l > 1$), which is the assumption we made to guarantee the existence of tariff-jumping FDI.

Compare the cutoff expressions, for $l \in \{H, F\}$

$$\text{Open economy, with tariff, export and FDI: } c_{D1}^H = \left[\gamma\phi\frac{1-\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}\right]^{\frac{1}{k+2}}$$

$$\text{Open economy, with tariff and export: } c_{D2}^H = \left(\gamma\phi\frac{1-\psi^F}{1-\psi^F\psi^H}\right)^{\frac{1}{k+2}},\ \psi^l = \left(\tau^l\right)^{-k}\left(t^l\right)^{-(k+1)}$$

$$\text{Closed economy: } c_{D3}^H = (\gamma\phi)^{\frac{1}{k+2}},\ \text{as in MO(2008) Section 2}$$

With the proved condition, it is straightforward to show that

$$c_{D3}^H > c_{D2}^H > c_{D1}^H$$

□

B.4 Proof of Proposition 4

Based on the solution of $c_D^H = \left[\gamma\phi \frac{1-\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}\right]^{\frac{1}{k+2}}$, I have

$$\frac{\partial c_D^H}{\partial t^H} = \frac{\gamma\phi}{k+2}\left[\gamma\phi\frac{1-\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}\right]^{-\frac{k+1}{k+2}} \frac{\left(\Phi_1^F+\Phi_2^F\right)\left[1-\left(\Phi_1^F+\Phi_2^F\right)\right]}{\left[1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)\right]^2}\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}$$

$$= \underbrace{\frac{1}{k+2}c_D^H\frac{\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}}_{>0}\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}$$

So the sign crucially depends on $\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}$. It is straightforward to show that

$$\begin{aligned}
\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H} &= -\frac{k+1}{2t^H\left(\sqrt{t^H}\varphi^H-t^H\tau^H\right)^2}\left\{\frac{2}{\left(t^H\tau^H\right)^k}\left[t^H\left(\tau^H\right)^2-2\varphi^H\sqrt{t^H}\tau^H+\left(\varphi^H\right)^2\right]\right.\\
&+\left(\xi^H\right)^k\left[-2t^H\left(\tau^H\right)^2-2k\sqrt{t^H}t^H\left(\tau^H\right)^2+k\left(t^H\tau^H\right)^2\right.\\
&\left.\left.+2\left(k+2\right)\sqrt{t^H}\tau^H\varphi^H-\left(k+2\right)\left(\varphi^H\right)^2\right]\right\}\\
&= \underbrace{-\frac{k+1}{2t^H\left(\sqrt{t^H}\varphi^H-t^H\tau^H\right)^2}}_{<0}\left\{2\left(\varphi^H-\sqrt{t^H}\tau^H\right)^2\left[\left(t^H\tau^H\right)^{-k}-\left(\xi^H\right)^k\right]\right.\\
&\left.-\left(\xi^H\right)^k k\left(\varphi^H-t^H\tau^H\right)\left(\varphi^H+t^H\tau^H-2\sqrt{t^H}\tau^H\right)\right\}
\end{aligned}$$

The expression within the big bracket is greater than zero for all $k\in[1,+\infty)$ when $\varphi^H > t^H\tau^H$, to see this, it is equivalent to show

$$2\left(\varphi^H-\sqrt{t^H}\tau^H\right)^2\left[\left(t^H\tau^H\right)^{-k}-\left(\xi^H\right)^k\right] > \left(\xi^H\right)^k k\left(\varphi^H-t^H\tau^H\right)\left(\varphi^H+t^H\tau^H-2\sqrt{t^H}\tau^H\right)$$

For $k = 1$, the expression become

$$
\begin{aligned}
&2\left(\varphi^H - \sqrt{t^H}\tau^H\right)^2\left[\frac{1}{\left(t^H\tau^H\right)} - \xi^H\right] > \xi^H\left(\varphi^H - t^H\tau^H\right)\left(\varphi^H + t^H\tau^H - 2\sqrt{t^H}\tau^H\right)\\
\Leftrightarrow &2\left(\varphi^H - \sqrt{t^H}\tau^H\right)^2\left(\frac{1}{\xi^H t^H\tau^H} - 1\right) > \left(\varphi^H - t^H\tau^H\right)\left(\varphi^H + t^H\tau^H - 2\sqrt{t^H}\tau^H\right)\\
\Leftrightarrow &2\left(\varphi^H - \sqrt{t^H}\tau^H\right)^2\frac{\varphi^H - t^H\tau^H}{\sqrt{t^H}\tau^H\left(\sqrt{t^H} - 1\right)} > \left(\varphi^H - t^H\tau^H\right)\left(\varphi^H + t^H\tau^H - 2\sqrt{t^H}\tau^H\right)\\
\Leftrightarrow &2\left(\varphi^H - \sqrt{t^H}\tau^H\right)^2 > \sqrt{t^H}\tau^H\left(\sqrt{t^H} - 1\right)\left(\varphi^H + t^H\tau^H - 2\sqrt{t^H}\tau^H\right)\\
\Leftrightarrow &2\left(\varphi^H\right)^2 - 3\sqrt{t^H}\tau^H\varphi^H + 3\sqrt{t^H}t^H\left(\tau^H\right)^2 - t^H\tau^H\varphi^H - \left(t^H\tau^H\right)^2 > 0\\
\Leftrightarrow &\left(\varphi^H - t^H\tau^H\right)\left(2\varphi^H + t^H\tau^H - 3\sqrt{t^H}\tau^H\right) > 0
\end{aligned}
$$

which is obviously true.

For k approach infinity, to prove the equation, it is equivalent to show

$$
\frac{2\left(\varphi^H - \sqrt{t^H}\tau^H\right)^2}{\left(\varphi^H - t^H\tau^H\right)\left(\varphi^H + t^H\tau^H - 2\sqrt{t^H}\tau^H\right)} > \frac{k}{\left(\xi^H t^H\tau^H\right)^{-k} - 1}
$$

As $k \to \infty$, the limit of right-hand side is 0. It means as long as the left-hand side is positive, the equation is true for $k \to \infty$. The left hand side is obviously positive given $\varphi^H > t^H\tau^H$. Therefore,

$$
\frac{\partial\left(\Phi_1^H + \Phi_2^H\right)}{\partial t^H} < 0 \Rightarrow \frac{\partial c_D^H}{\partial t^H} < 0
$$

To show $\dfrac{\partial c_D^F}{\partial t^H}$ is easier, note that

$$
\begin{aligned}
\frac{\partial c_D^F}{\partial t^H} &= \left[\gamma\phi\frac{1 - \left(\Phi_1^H + \Phi_2^H\right)}{1 - \left(\Phi_1^F + \Phi_2^F\right)\left(\Phi_1^H + \Phi_2^H\right)}\right]^{-\frac{k+1}{k+2}} \frac{\gamma\phi\left[1 - \left(\Phi_1^F + \Phi_2^F\right)\right]/(k+2)}{\left[1 - \left(\Phi_1^F + \Phi_2^F\right)\left(\Phi_1^H + \Phi_2^H\right)\right]^2}\frac{-\partial\left(\Phi_1^H + \Phi_2^H\right)}{\partial t^H}\\
&= \underbrace{\frac{c_D^F}{k+2}\frac{\left[1 - \left(\Phi_1^F + \Phi_2^F\right)\right]}{\left[1 - \left(\Phi_1^F + \Phi_2^F\right)\left(\Phi_1^H + \Phi_2^H\right)\right]\left[1 - \left(\Phi_1^H + \Phi_2^H\right)\right]}}_{>0}\underbrace{\frac{-\partial\left(\Phi_1^H + \Phi_2^H\right)}{\partial t^H}}_{>0} > 0
\end{aligned}
$$

$\square$

B.5 Proof of Proposition 5

Based on the cutoff relations, it is straightforward to show

$$\begin{aligned}
\frac{\partial c_X^H}{\partial t^H} &= \frac{\partial\left(c_D^F/t^F\tau^F\right)}{\partial t^H} = \frac{1}{t^F\tau^F}\frac{\partial c_D^F}{\partial t^H} > 0 \\
\frac{\partial c_X^F}{\partial t^H} &= \frac{\partial\left(c_D^H/t^H\tau^H\right)}{\partial t^H} = \frac{1}{\left(t^H\right)^2\tau^H}\left(\frac{\partial c_D^H}{\partial t^H}t^H - c_D^H\right)
\end{aligned}$$

To show $\frac{\partial c_D^H}{\partial t^H}t^H - c_D^H < 0$ is equivalent to show

$$\begin{aligned}
&\frac{1}{k+2}c_D^H\frac{\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}t^H < c_D^H \\
\Leftrightarrow\ &\underbrace{\frac{1}{k+2}\frac{\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}}_{>0}\underbrace{\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}}_{<0}t^H < 1
\end{aligned}$$

Which is obviously true. Therefore, Proposition 5 is proved. $\square$

B.6 Proof of Proposition 6

This part is relatively easy to prove, notice

$$\begin{aligned}
\frac{\partial c_{FDI}^F}{\partial t^H} &= \frac{\partial\left(c_D^H\xi^H\right)}{\partial t^H} = \frac{\partial c_D^H}{\partial t^H}\xi^H + \frac{\partial \xi^H}{\partial t^H}c_D^H \\
&= \underbrace{\frac{c_D^H\xi^H}{k+2}\frac{\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}}_{>0}\underbrace{\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}}_{<0} + \underbrace{\frac{\partial \xi^H}{\partial t^H}c_D^H}_{>0}
\end{aligned}$$

It is straightforward to verify that under the parameter choice in Section 2.3, the second term dominates the first term, so Foreign country's FDI cutoff level (c_{FDI}^F)is strictly increasing as Home country's tariff (t^H) increases. Do notice, the other choice of ξ, which is $\xi = \left(\sqrt{t}+1\right)/\left(\sqrt{t}\varphi + t\tau\right)$

will make the second item negative, thereby making c_{FDI}^{F} decreasing in response to t^{H}'s increase, hence no tariff-jumping FDI. □

B.7 Proof of Corollary 1

The proof is straightforward. Utilizing Proposition 3 and 4:

$$
\begin{aligned}
\left|\frac{\frac{\partial c_D^H}{\partial t^H}|_{\text{without FDI}}}{\frac{\partial c_D^H}{\partial t^H}|_{\text{with FDI}}}\right| &= \left|\frac{\frac{\gamma\phi(c_{D2}^H)^{-(k+1)}}{k+2}\frac{\psi^F\left[1-\psi^F\right]}{\left[1-\psi^F\psi^H\right]^2}\frac{\partial\psi^F}{\partial t^H}}{\frac{\gamma\phi(c_{D1}^H)^{-(k+1)}}{k+2}\frac{\left(\Phi_1^F+\Phi_2^F\right)\left[1-\left(\Phi_1^F+\Phi_2^F\right)\right]}{\left[1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)\right]^2}\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}}\right| \\
&= \left|\frac{(c_{D1}^H)^{k+1}\frac{\psi^F\left[1-\psi^F\right]}{\left[1-\psi^F\psi^H\right]^2}\frac{\partial\psi^F}{\partial t^H}}{(c_{D2}^H)^{k+1}\frac{\left(\Phi_1^F+\Phi_2^F\right)\left[1-\left(\Phi_1^F+\Phi_2^F\right)\right]}{\left[1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)\right]^2}\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}}\right| \\
&= \left|\left(\frac{c_{D1}^H}{c_{D2}^H}\right)^{k+1}\times\frac{\psi^F}{\left(\Phi_1^F+\Phi_2^F\right)}\times\frac{\left[1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)\right]^2}{\left[1-\psi^F\psi^H\right]^2}\times\frac{\frac{\partial\psi^F}{\partial t^H}}{\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}}\right|
\end{aligned}
$$

Based on the proof of Proposition 3 and 4, it is obvious that the 1st, 2nd and 3rd term in the above equation are all smaller than 1. The 4th term, however, is greater than 1. It is straightforward to verify that the 4th term will be dominated by the first three terms, therefore, the whole expression is smaller than 1. □

B.8 Proof of Proposition 7

Based on equation (24), it's straightforward to show

$$\frac{\partial N^H}{\partial t^H} = \frac{2\gamma(k+1)}{\eta} \frac{-\frac{\partial c_D^H}{\partial t^H} c_D^H - \frac{\partial c_D^H}{\partial t^H}\left(\alpha - c_D^H\right)}{\left(c_D^H\right)^2} = -\frac{2\gamma\alpha(k+1)}{\eta\left(c_D^H\right)^2} \frac{\partial c_D^H}{\partial t^H} > 0$$

$$\frac{\partial N^F}{\partial t^H} = \frac{2\gamma(k+1)}{\eta} \frac{-\frac{\partial c_D^F}{\partial t^H} c_D^F - \frac{\partial c_D^F}{\partial t^H}\left(\alpha - c_D^F\right)}{\left(c_D^F\right)^2} = -\frac{2\gamma\alpha(k+1)}{\eta\left(c_D^F\right)^2} \frac{\partial c_D^F}{\partial t^H} < 0$$

Now based on equation (26), as t^H increases

$$N_E^F = \frac{2\left(c_M\right)^k (k+1)\gamma}{\eta\left(1-\delta^H\delta^F\right)} \left[\frac{\alpha - c_D^F}{\left(c_D^F\right)^{k+1}} - \delta^F \frac{\alpha - c_D^H}{\left(c_D^H\right)^{k+1}}\right]$$

δ^H decreases (hence the coefficient in front of the bracket decreases), c_D^F increases (the first item in the bracket decreases), c_D^H decreases (the second item in the bracket increases). Hence the whole expression on the right decreases, therefore $\partial N_E^F/\partial t^H < 0$. Now utilizing the free-entry condition, which is equation (25)

$$G\left(c_D^H\right) N_E^H + G\left(c_X^F\right) N_E^F = N^H$$

As t^H increases, N^H increases, it means the left-side also needs to increase. Notice N_E^F decreases, c_X^F decreases, c_D^H decreases, it then must be true that N_E^H increases. Hence, $\partial N_E^H/\partial t^H > 0$. □

B.9 Proof of Proposition 8

To prove this proposition, it is helpful to rewrite the markups as follows:

$$m_D^H(c) = \frac{1}{2c}\left(c_D^H + c\right)$$

$$m_X^F(c) = \frac{t^H}{2c}\left(c_X^F + c\right) = \frac{t^H}{2c}\left(c_D^H/t^H\tau^H + c\right) = \frac{1}{2c}\left(c_D^H/\tau^H + c/t^H\right)$$

$$m_{FDI}^F(c) = \frac{1}{2\varphi^H c}\left(c_D^H + \varphi^H c\right)$$

It follows from Proposition 4 that $\partial m_D^H/\partial t^H < 0$ and $\partial m_{FDI}^F/\partial t^H < 0$. The responses of m_X^F now is different from what we see in Proposition 2. On the one hand, c_D^H/τ^H decreases as t^H increases. On the other hand, c/t^H decreases as t^H increases. The total impact on m_X^F is therefore negative. Hence, $\partial m_X^F/\partial t^H < 0$. $\square$