Optimal Tariffs with Firm Heterogeneity, Variable Markups, and FDI*

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Abstract

Variable markups and multinational production have gathered considerable attention in the trade literature because of their empirical prevalence and welfare implications. This paper studies the welfare implication of tariffs and optimal tariffs in an environment that features firm heterogeneity, variable markups, and FDI. I find: (i) Tariffs endogenously affect firm entry level, producing different comparative statics in the short-run versus long-run. (ii) Variable markups generate multiple externalities in this economy, causing market outcome to differ from the socially optimum outcome systematically. Permitting tariff-jumping FDI can lower the domestic cutoff levels and reduces the misallocation in the economy. (iii) Free trade is not always socially optimal. If the domestic marginal cost cutoff is sufficiently high, a positive tariff can be welfare-improving since it encourages firm entry. The Nash equilibrium tariff level will also be higher than the socially optimal tariff. (iv) The interaction of variable markup and FDI generates novel welfare implications that are absent if consumers possess CES preference.

Keywords: Optimal tariff, Firm heterogeneity, Misallocation, Variable markups, FDI
JEL Codes: F12, F13, F23, F60, R13

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1 Introduction

What is the optimal tariff in an environment that features variable markups and foreign direct investment (hereafter FDI)? On the one hand, protectionism may hurt consumer welfare in the presence of variable markups if protection results in higher market concentration. This has been a concern since Adam Smith, and it has received increasing attention in recent years[1]. On the other hand, in a highly-integrated global market, foreign firms can avoid import tariffs by locating production within the destination market. Such "tariff-jumping" activities[2] can diminish the market power of domestic producers, thereby substantially mitigate welfare consequences of the original trade protection policy.

The goal of this paper is to study the optimal tariff in the context of monopolistic competition, heterogeneous firms, variable markups, and FDI. To this end, this paper introduces variable markups through quadratic quasi-linear preference, as in Melitz and Ottaviano [2008] (hereafter **MO**), into a two-country (Home and Foreign) model with firm heterogeneity and FDI, as in Helpman et al. [2004] (hereafter **HMY**). In the current framework, the economy features two sectors: a perfectly competitive sector that produces homogeneous goods and a monopolistically competitive sector that produces differentiated varieties. Firm entry and exit only happen in the monopolistically competitive sector. A firm needs to pay a fixed cost and draw its marginal production cost (which is inversely related to the firm's productivity) to enter the market. Post-entry, firms produce with different marginal cost levels. Exporters encounter two types of costs, iceberg-type trade costs, an *ad valorem* tariff. As for multinationals, they face an iceberg-type of *efficiency loss* as in Keller and Yeaple [2008]. Before the individual firm's productivity is realized, firms formulate entry, export, and FDI decisions based on expected profit. The difference in marginal cost preserves the sorting of firms[3]: the most productive firms access the Foreign market through FDI, the less productive firms export, and the least productive firms only serve their domestic market. An increase in the Home country's import tariff affects the variable profit of Foreign exporters and multinationals, making FDI a more profitable entry mode for the most productive Foreign exporters, inducing tariff-jumping FDI under the heterogeneous firm framework.

The analysis of the findings shows that the economy responds to a tariff change differently in the short-run versus long-run. In the short-run, where the entry has not taken place yet, and fixed costs are sunk, exiting the market is never optimal for firms. In this case, the economy is characterized by a fixed number and distribution of incumbents. These incumbents decide whether they should operate and produce or shut down. If they choose to shut down, they can restart production without incurring the entry cost again. In this short-run case, an increase in the Home import tariff makes it harder for the least productive Foreign exporters to export. These exporters will shut down their export department and only serve their domestic

[1]Outside of the academic literature, increasing market concentration has received significant attention, e.g., *A lapse in concentration* (The Economist, September 2016), CEA [2016]. In the academic literature, see Asker et al. [2017], De Loecker and Eeckhout [2017] for recent evidence.

[2]With the improvement in micro-level data availability, tariff-jumping FDI has received increasing empirical support, see Blonigen [2002], Belderbos et al. [2004], Hijzen et al. [2008] and more recently, Pietrovito et al. [2013], Alfaro and Chen [2018, 2015].

[3]In **HMY**, the sorting of firms is preserved by the combination of fixed cost and variable cost. Here, with bounded marginal utility, high-cost firms will not survive, even without such fixed costs. The difference in marginal cost is sufficient to generate the sorting. Adding fixed cost will substantially degrade the tractability of the model, without generating additional insight.

market. At the same time, an increase in Home's import tariff makes export a less desirable entry mode for the most productive Foreign exporters. These firms will switch to FDI simply because the variable profit of FDI is higher than that of export. In the current setup, if the level of tariff is low, the reduction of Foreign exporters will dominate the increase of Foreign multinationals, resulting in a reduction of the total number of Foreign firms in the Home country. In equilibrium, an increase in Home tariff creates an easier environment for Home firms to survive. Therefore, Home's tariff effectively shields its firms from Foreign competition.

In the long-run, the number of entrants and the number of firms in the economy are endogenously determined by the tariff level. With the free-entry condition, an increase in Home's import tariff makes the Home country a more desirable place to do business, generating more domestic entry, intensifying competition in the Home market. On the one hand, the increase of Home's tariff makes it harder for the least productive Foreign exporters to export, reducing the number of Foreign exporters. On the other hand, the protection makes it more profitable for the most productive Foreign exporters to do FDI and increases the number of Foreign multinationals. However, the total impact on the number of firms in the Home market is dominated by the domestic entry. Different from short-run, an increase in Home tariff creates more entry, generates more competition in the Home market, and makes it harder for local producers to survive.

In contrast to the constant markup results from the combination of monopolistic competition and CES preference, markups here also respond to tariff change differently in the short-run versus in the long-run. In the short-run, Home import protectionist tariff reduces the competition in the Home market, causing both Home's domestic firms and Foreign's FDI firms to charge higher markups than before. Foreign exporters' markups vary according to their productivity: (i) For the least productive ones, Home's protectionist tariff reduces their markups by giving a competitive edge to the Home firms. (ii) For the more productive ones, Home's protection does not affect them much, and they benefit from the reduced competition by charging a higher markup. These responses are reversed in the long-run. Home's protectionist trade policy makes the Home market a more desirable environment to do business, attracting more firm entry into Home's market. This channel will substantially increase the competition level in the Home market, making it harder for firms to survive and reducing the markups for all the firms that operate in the Home market.

This paper is not the first to address questions regarding trade policy within the heterogeneous firm framework. For example, Demidova and Rodriguez-Clare [2009] use a Melitz-type model to study the trade policy implication in a small open economy. Felbermayr et al. [2013] study the bilateral trade policy implication in a two-country, asymmetric Melitz-type economy. Bagwell and Lee [2015] study trade policy in the **MO** model and provide a rationale for the treatment of export subsidies within the World Trade Organization. Costinot et al. [2016] utilize a generalized Melitz model to study the trade policy implication both from a micro and macro perspective. Demidova [2017] studies the optimal tariff in the **MO** environment without the numéraire good and finds that protection is always desirable, and reductions in cost-shifting trade barriers are welfare-improving. A common feature of the papers mentioned above is their exclusive focus on domestic producers and foreign exporters. A key message from the current analysis is that ignoring multinational production may provide an incomplete picture of the trade policy implication.

A recent article by Cole and Davies [2011] (hereafter **CD**) is closely related to the current paper. The authors introduce an ad valorem tariff and heterogeneous fixed costs into **HMY**, and find equilibria in which both pure exporters and multinationals coexist, resolving a well-known puzzle[4] in the strategic tariff literature in the presence of multinationals. Heterogeneous fixed costs for exporters and multinationals are the critical elements to generate their results. In contrast, the coexistence of exporters and multinationals in the current framework comes from the different effective marginal costs they face.

Despite the apparent similarity between the two frameworks, it should be clear that the two exercises are very different. First, **CD** combine quasi-linear CES preference with monopolistic competition, which produces a constant markup for all the firms. Although being analytically tractable, the combination of CES and monopolistic competition has little merit, even as a first approximation, for welfare analysis. In contrast, the current framework utilizes quadratic quasi-linear preference to generate variable markups and incomplete pass-through for firms with different productivity levels. This attribute is more suitable for pricing and welfare analysis. Second, although **CD**'s model features ad valorem tariff and multiple sectors, the policy implications from their paper are entirely independent[5] of firm's entry level. In the current model, tariff level endogenously affects the number of entrants, generating different implications for tariff in both the short-run and the long-run.

I then utilize this framework to study the welfare implication of tariff and optimal tariffs. I first follow Nocco et al. [2014] to compare the market allocation with the socially optimum allocation. I find with the free-entry condition, the market outcome in the monopolistically competitive sector[6] is not efficient in several dimensions: (i) The selection is too weak in domestic and export cutoff, but too strong in FDI cutoff. (ii) The market oversupplies high-cost varieties and undersupplies low-cost varieties. (iii) It may feature excessive (insufficient) entry and oversupply (undersupply) the total number of varieties. These market failures stem from several externalities[7]. If the market selection is too weak, then an increase in tariff can improve market selection, reducing the welfare gap between market allocation and the planner's allocation.

Several interesting policy implications[8] stand out from the analysis. First, I find that free trade is not always socially optimal. Whether imposing a tariff is socially optimal depends on the level of market selection. If the domestic cutoff is sufficiently high, which means the selection is too weak, then an additional firm entry can increase social welfare. In this case, a positive tariff is socially optimal because it encourages firm entry. **CD** find the socially optimal tariff is always to subsidize trade. They reach this conclusion because, in their model, trade liberalization can foster competition and eliminate the least productive firms, increasing aggregate productivity. In addition, I find that the degree of firm heterogeneity, which is governed

[4]In equilibrium, all foreign firms are either multinationals or exporters.

[5]To the best of my knowledge, this feature is contradicting with the implications from Balistreri et al. [2011], Arkolakis et al. [2012], and Caliendo et al. [2017].

[6]Since tariff is only imposed in this sector, here I focus exclusively on the policy implication in this sector. There is, indeed, an inter-sector misallocation between the numéraire good sector and the monopolistically competitive sector. Interesting readers should refer to Nocco et al. [2014].

[7](i) On the supply side, both the markup-pricing and business-stealing effect tend to create too many varieties in the economy. (ii) On the demand side, the 'love of variety' from the quadratic quasi-linear preference tends to create insufficient varieties in the economy. (iii) With variable markups, firm heterogeneity becomes another source of inefficiency: as the demand becomes more inelastic with consumption, low-cost firms charge higher markups, and do not fully transmit their cost advantage to prices. This behavior leaves inefficiently large room for entry and also allows high-cost firms to be inefficiently large. These externalities collectively result in inefficiencies in the market outcome.

[8]It should be noted that all these implications are based on the long-run analysis.

by the Pareto shape parameter k and the upper bound of cost draw c_M, affects the welfare implication of tariff. For example, when the domestic cutoff is sufficiently high, an increase in firm heterogeneity (through an increase in c_M or a decrease in k) is socially inefficient because it reduces the positive externality generated through the firm entry, dampening the welfare impact of tariff.

Second, the Nash tariff is lower than the socially optimal tariff. This result can be analyzed from two perspectives: (i) When Home country sets its uncooperative tariff level, it focuses exclusively on its own tariff revenue and consumer surplus, ignoring the impact on Foreign tariff revenue and consumers. Therefore, Home country will set a higher tariff than the one social planner would choose. (ii) A higher Nash tariff level could also arise from Home country's incentive to manipulate the terms of trade. Due to the presence of variable markup, Home's import price varies with its tariff level. Furthermore, the profits of Foreign exporters and multinationals are all affected by Home's tariff level. An increase in Home's import tariff thus generates a terms of trade gain at the cost of Foreign's terms of trade deterioration. This channel is absent in **CD** due to the constant markup implied by the CES preference.

Third, the interaction of variable markups with FDI yields novel insights on the pro-competitive effect of trade. In the current framework, a decrease in Home's import tariff makes it easier for the most productive Foreign domestic firms to export, increasing the number of Foreign exporters serving the Home market, and creating downward pressure on the Home average markup. At the same time, the reduction of tariffs also makes it less desirable for the least productive Foreign multinationals to pursue FDI, decreasing the number of Foreign FDI firms and generating upward pressure on the Home average markup. If the initial protection level is sufficiently high, the decrease of multinational firms can dominate the increase of exporters, driving up the average markup in the Home market, and generating a negative pro-competitive effect. Recently, Arkolakis et al. [2018] show that the under a large class of demand function, the non-homothetic preference dampens the pro-competitive effect of trade liberalization by increasing the degree of misallocation. Edmond et al. [2015] show that the size of the pro-competitive gain[9] can be quite large in the presence of significant misallocations and weak cross-country comparative advantage in individual sectors. Different from these two papers, the current framework shows that the pro-competitive effect of trade can be very different when FDI is incorporated.

Finally, the welfare implication of FDI is affected by variable markups. In **CD**, FDI is welfare-improving because it reduces the tariff base, mitigating the tariff competition between the two countries. In the current framework, FDI's impact on the tariff level depends on a particular parameter[10] that governs the variable markups: the shape of Pareto distribution, k. A decrease in k means an increase in the degree of firm heterogeneity, which has two impacts on the economy: changing the equilibrium cutoff levels and altering the relative distribution of firms with different marginal costs. When FDI is an option, the easiness of doing FDI (measured by φ) interacts with k, jointly affecting the Nash tariff level: if the degree of firm heterogeneity is big (smaller k), reducing the FDI barrier can lower the Nash tariff level; if the degree of firm heterogeneity is small (larger k), however, promoting FDI can increase the Nash tariff level. The welfare implication of FDI is an old topic in the field, see *e.g.,* Brecher and Alejandro [1977]. Some recent papers have revisited

[9] According to their setup, a pro-competitive gain is associated with a lower average markup.

[10] Another parameter that also impacts the degree of firm heterogeneity, c_M, only affects the equilibrium cutoffs proportionally, without shifting the relative distribution of firms. Therefore, its interaction with FDI does not change the Nash tariff level.

the welfare impact of FDI, either analytically or quantitatively. Ramondo and Rodríguez-Clare [2013] show that when taking account of the multinational production, the gains from openness are around twice the gains calculated in trade-only models. Irarrazabal et al. [2013] extend Helpman et al. [2004] to allow intra-firm trade and structurally estimate their model using firm-level data from Norwegian manufacturing sector. Their counterfactual analysis indicates that impeding FDI has substantial effects on trade flows but not on welfare. Different from their exercises, this paper studies explicitly the welfare implication of FDI through its interaction with tariff.

The remainder of this paper is organized as follows. Section 2 presents the complete model and its anlytical solution. Section 3 explores the equilibrium features of the model both in the short-run and long-run. Section 4 compares the market allocation with the socially optimum allocation in this economy. Section 5 investigates whether free trade is socially optimal in the current framework. Section 6 contrasts the socially optimal tariff level with the Nash tariff level and examines how FDI affects the Nash tariff level. Section 7 is dedicated to the role of variable markups. The last section concludes.

2 Model

This section introduces quadratic quasi-linear preference into the **HMY** framework. There are two symmetric countries, Home (H) and Foreign (F). The markets are segmented, and international trade entails trade costs that take the form of transportation costs as well as *ad valorem* import tariffs. Tariff revenue is redistributed equally across consumers in the tariff-imposing country. Multinational firms engaging in FDI face an iceberg-type marginal cost (i.e., efficiency loss) in the spirit of Keller and Yeaple [2008]. Different from **CD**, where firms' sorting is induced by different fixed costs, the nonhomothetic preference here induces different productivity cutoffs through different marginal costs. The framework presented here is suitable for this analysis due to two reasons. First, it enables one to study trade policy in an environment that features firm heterogeneity, variable markups, and multinational production. Second, the model produces tractable analytical solutions due to the specific assumption regarding demand and production structure, yielding quite transparent comparative statics.

2.1 Consumers

Each country is endowed with one unit of consumers. In the H economy, each consumer supplies one unit of labor. Consumers in country H maximize their utility by consuming the numéraire good, q_0^H, and the differentiated varieties, q_i^H, subject to their income budget constraint:

$$U^H = q_0^H + \alpha \int_{i\in\Omega^H} q_i^H di - \frac{1}{2}\gamma \int_{i\in\Omega^H} \left(q_i^H\right)^2 di - \frac{1}{2}\eta \left(\int_{i\in\Omega^H} q_i^H di\right)^2$$

$$\text{subject to: } q_0^H + \int_{i\in\Omega^H} p_i^H q_i^H di \leq I^H \equiv w^H + TR^H + \Pi^H$$

where α and η indicate the substitutability between the differentiated varieties and numéraire good, and γ indicates the substitutability among the differentiated varieties. An increase in α and a decrease in η both shift out the demand for the differentiated varieties *relative* to the numéraire good. The degree of product differentiation increases as γ increases since consumers are paying more attention to the distribution of varieties that they consume. All these three demand parameters are positive. Notice that different from **MO**, the tariff revenue and aggregate profit will enter into consumer's budget constraint through government redistribution.

Assuming consumers have positive demands for the numéraire good $\left(q_0^H > 0\right)$, utility maximization of the previous consumer problem leads to the following inverse demand for each variety i:

$$p_i^H = \alpha - \gamma q_i^H - \eta Q^H \tag{1}$$

where $Q^H \equiv \int_{i\in\Omega^H} q_i^H d_i$ is the aggregate consumption of these varieties, and Ω^H is the variety space that is available to the consumers. Invert equation (1) to obtain the linear market demand for these varieties:

$$q_i \equiv q_i^H = \frac{\alpha}{\eta N^H + \gamma} - \frac{1}{\gamma} p_i^H + \frac{\eta N^H}{\eta N^H + \gamma}\frac{1}{\gamma}\bar{p}^H = \frac{1}{\gamma}\left(p_{\max}^H - p_i^H\right) \tag{2}$$

where $p_{\max}^H = (\gamma\alpha + \eta N^H \bar{p}^H)/(\eta N^H + \gamma)$ represents the price at which demand for a variety is driven to 0, $\bar{p}^H \equiv (1/N^H)\int_{i\in\hat{\Omega}^H} p_i^H di$ is the average price of all consumed variety in country H, and $\hat{\Omega}^H$ is the consumed subset of Ω^H. Note that equation (1) also implies $p_{\max}^H \le \alpha$ for positive q_i^H and Q^H. Different from CES preference, where the elasticity of demand across varieties is constant, the price elasticity of demand here is given by:

$$\varepsilon_i^H \equiv \left|\frac{\partial q_i^H}{\partial p_i^H} \times \frac{p_i^H}{q_i^H}\right| = \frac{1}{p_{\max}^H / p_i^H - 1} \tag{3}$$

A lower average price $\bar{p}^H$ or a larger number of competing varieties N^H will induce a decrease in the choke price $p_{\max}^H$, and hence an increase in the price elasticity of demand ε_i^H at any given p_i^H. These movements all represent a *tougher* competitive environment, which has received strong empirical support in the industrial organization literature but can not be captured in an environment with constant elasticity of demand.

As in **MO**, welfare can be evaluated using the following indirect utility function:

$$U^H = I^H + \frac{1}{2}\left(\frac{N^H}{\eta N^H + \gamma}\right)\left(\alpha - \bar{p}^H\right)^2 + \frac{1}{2}\frac{N^H}{\gamma}\sigma_{p^H}^2 \tag{4}$$

where $\sigma_{p^H}^2 \equiv \left(1/N^H\right)\int_{i\in\hat{\Omega}^H}\left(p_i^H - \bar{p}^H\right)^2 di$ measures the variance of variety prices. To ensure positive demand levels for the numéraire, I assume that consumer's income is sufficiently high, so that $I^H > \int_{i\in\hat{\Omega}^H} p_i^H q_i^H di = \bar{p}^H Q^H - N^H \sigma_{p^H}^2/\gamma$. Consumer's utility will be higher when the average price $\bar{p}^H$ is lower, the variance of prices $\sigma_{p^H}^2$ is higher, and the number of varieties N^H is larger.

2.2 Firms

Production in the economy only utilizes labor, which is supplied in an inelastic fashion in a competitive market. The economy consists of two sectors, the traditional sector (produces q_0^H) and the modern sector (produces q_i^H). q_0^H is produced under a constant return to scale technology at a unitary cost. Thus the wage[11] in each country equals to one: $w^H = 1$. In the differentiated-good sector, firms are competing with each other in a monopolistically competitive fashion, and each firm produces a single variety.

Entry only happens in the modern sector. Different from Melitz [2003], here exit only happens at the moment when productivity is realized. There is no exogenous per-period death shock in the modern sector due to the one-period nature of the model[12]. To enter the market, a firm needs to pay a sunk entry cost $f_E > 0$ and draws its marginal production cost c, which indicates the unit labor requirement, from a Pareto distribution with cumulative distribution function $G(c) = (c/c_M)^k$, where $k \geq 1$ represents a shape parameter and $c_M > 0$ represents the upper bound of c. When $k = 1$, the marginal cost distribution is uniform on $[0, c_M]$. As k increases, the relative number of low productivity firms increases, and the distribution is more concentrated at these lower productivity levels. As k approaches infinity, the distribution of firm productivity approaches c_M. A lower c_M stands for a higher technology state in the economy. In this paper, I assume H and F share the same technology, hence the same upper bound c_M and the same f_E for both countries.

Depending on its productivity draw, a firm enters country H may exit, produce locally, export to country F or engage in the multinational activity. Following **MO**, since each firm's marginal cost of production does not vary with its production level, the decisions in each market can be made separately. Therefore, all the monopolistically competitive firms make separate decisions about their prices at each market, taking the total number of varieties and the average price in a market as given. In what follows, I analyze each type of producer's profit maximization problem.

Domestic Producers

A firm located in country H with cost level c selects its price in the domestic market, p_D^H, to maximize its domestic profit $\pi_D^H(c) = \left[p_D^H(c) - c\right] q_D^H(c)$. Together with equation (2), the optimal price, markup, quantity, and profit can be solved as:

$$p_D^H(c) = \frac{1}{2}\left(c_D^H + c\right) \tag{5}$$

$$m_D^H(c) = \frac{1}{2c}\left(c_D^H + c\right) \tag{6}$$

$$q_D^H(c) = \frac{1}{2\gamma}\left(c_D^H - c\right) \tag{7}$$

$$\pi_D^H(c) = \frac{1}{4\gamma}\left(c_D^H - c\right)^2 \tag{8}$$

Let $c_D^H \equiv \sup\left\{c : \pi_D^H(c) > 0\right\}$ represent the cost of the firm which is indifferent between exit and remaining in the market. This firm earns zero profit as its price is driven down to marginal cost. Together with equation (2), the firm's optimal price is equal to its marginal cost, $p_D^H\left(c_D^H\right) = c_D^H = p_{\max}^H$. Hence, a firm will only serve domestic market if $c \leq c_D^H$. As expected, lower cost firms set lower prices and earn higher profits. However, lower cost firms are also more productive, and have larger market power, therefore they do not pass all of

[11]One can attempt to drop the numéraire good, then wage will be endogenized and can be pinned down by trade balance condition.

[12]For a dynamic application of **MO**, see Moon [2015]. Notice the benchmark model in her work is without the numéraire good.

the cost differentials to the consumer and charge higher markups (which is defined as $m(c) = p(c)/MC(c)$[13], decreasing in c).

Exporters

The exporter in country H will face an *ad valorem* import tariff imposed by country F, denoted as $t^F \geq 1$. On top of that, the exporter will also face a per-unit trade cost, denoted by τ^F. More specifically, the delivered cost of a unit cost c to country F is $\tau^F c$ where $\tau^F > 1$. An exporter takes t^F and τ^F as given and maximizes its profit $\pi_X^H(c) = \left[p_X^H(c)/t^F - \tau^F c\right] q_X^H(c)$ by choosing optimal price $p_X^H(c)$. Together with equation (2), the optimal price, markup, quantity, and profit are:

$$p_X^H(c) = \frac{t^F \tau^F}{2}\left(c_X^H + c\right) \tag{9}$$

$$m_X^H(c) = \frac{t^F}{2c}\left(c_X^H + c\right) \tag{10}$$

$$q_X^H(c) = \frac{t^F \tau^F}{2\gamma}\left(c_X^H - c\right) \tag{11}$$

$$\pi_X^H(c) = \frac{t^F \left(\tau^F\right)^2}{4\gamma}\left(c_X^H - c\right)^2 \tag{12}$$

Let $c_X^H \equiv \sup\left\{c : \pi_X^H(c) > 0\right\}$ denotes the marginal cost of the least productive exporter from H to F, which barely finds export profitable. Combine this threshold with the definition of c_D^F (parallel to c_D^H), this cutoff level then satisfies $c_X^H = c_D^F / t^F \tau^F$. Intuitively, tariffs and transportation cost make it harder for exporters to break even compared to the domestic market.

It should be noted that the presence of iceberg-type transportation cost ensures that when the net tariff is zero, there are still exporters in the economy. Following **MO**, I abstract from any fixed cost of exporting, which could substantially reduce the tractability of the model without adding additional insights. With the bounded marginal utility, different marginal costs are enough to induce the sorting of firms.

Multinational Firms

To engage in the multinational activity, a firm located in country H with cost level c sets its product price for consumers in country F, denoted as $p_{FDI}^H(c)$. Instead of serving the Foreign market through export, it directly serves locally in country F, but doing so will incur an *efficiency loss*, which effectively increases the marginal cost of production. Here, I assume that the efficiency loss φ^F is greater than τ^F to ensure there are still multinational firms in the economy even when the net tariff is zero. Multinational firm's profit function is as the following:

$$\pi_{FDI}^H(c) = \left[p_{FDI}^H(c) - \varphi^F c\right] q_{FDI}^H(c) \tag{13}$$

[13]Different from the Lerner index, where markup is measured in a relative sense, here the markup is measured in absolute sense. These two have no qualitative difference. Here I adopt the latter simply due to analytical convenience.

Together with equation (2), the optimal price, markup, quantity, and profit are:

$$p_{FDI}^{H}(c) = \frac{1}{2}\left(c_D^F + \varphi^F c\right) \tag{14}$$

$$m_{FDI}^{H}(c) = \frac{1}{2\varphi^F c}\left(c_D^F + \varphi^F c\right) \tag{15}$$

$$q_{FDI}^{H}(c) = \frac{1}{2\gamma}\left(c_D^F - \varphi^F c\right) \tag{16}$$

$$\pi_{FDI}^{H}(c) = \frac{1}{4\gamma}\left(c_D^F - \varphi^F c\right)^2 \tag{17}$$

Let $c_{FDI}^H = \sup\left\{c : \pi_{FDI}^H(c) > \pi_X^H(c)\right\}$ denote the marginal cost of the least productive multinational firm, which finds it indifferent between export and FDI. Combine this threshold with the definition of c_D^F, the cutoff then satisfies $c_{FDI}^H = \xi^F c_D^F$, where $\xi^F \equiv (1-\sqrt{t^F})/(t^F\tau^F - \sqrt{t^F}\varphi^F)$ is derived by setting $\pi_{FDI}^H(c) = \pi_X^H(c)$. Before moving on to the industry-level entry analysis, I want to discuss a few important modeling features of the multinational firms.

(i) *Efficiency loss* The efficiency loss feature is similar to Keller and Yeaple [2008], who demonstrate that when technologies are complex, it is more difficult for US-owned foreign affiliates to substitute local production with imports from the multinational headquarter. φ^F can also stand for the information costs of working abroad, transaction costs of dealing with FDI policy barriers[14], the costs of maintaining the affiliate, servicing network costs, and other costs associated with technology costs in offshore production. In a recent quantitative study by Head and Mayer [2019], the authors utilize highly disaggregated automotive industry data and find this type of variable distribution and marketing costs is higher than the conventional trade costs such as tariffs and freight, which is consistent with the assumption that I made here.

(ii) *Productivity sorting* There are two possible answers to equation $\pi_{FDI}^H(c) = \pi_X^H(c)$:

$$c_{FDI}^H = (1 \pm \sqrt{t^F})/(t^F\tau^F \pm \sqrt{t^F}\varphi^F)c_D^F \tag{18}$$

However, only one of the answers is interesting and relevant here. From both theoretical and empirical point of view, among those firms that serve foreign markets, multinational firms that engage in FDI are the most productive ones[15]. In the current framework, this implies $c_{FDI}^H < c_X^H < c_D^H$. Compare the expression of c_X^H and c_{FDI}^H, together with the assumption of $\varphi^F > \tau^F$, one can easily verify that both solutions of c_{FDI}^H in equation (18) imply that $c_{FDI}^H < c_X^H < c_D^H$. However, for the case of $c_{FDI}^H = (1+\sqrt{t^F})/(t^F\tau^F + \sqrt{t^F}\varphi^F)c_D^F$, c_{FDI}^H will decrease in response to an increase in t^F, indicating the marginal multinationals will choose to become exporters when tariff increases. This is at odds with the empirical evidence in the literature[16]. Therefore, the other solution $c_{FDI}^H = (1-\sqrt{t^F})/(t^F\tau^F - \sqrt{t^F}\varphi^F)c_D^F$ is more relevant here since c_{FDI}^H will

[14] For example, according to Head and Mayer [2019], foreign car makers complained about the additional costs of daytime running lamps when Canada mandated them for new cars in 1990).

[15] For theoretical work, see **HMY**. For empirical evidence, see Doms and Jensen [1998] for the US and Conyon et al. [2002] for the U.K, for more recent evidence, see Mataloni [2011].

[16] For example, Blonigen [2002] finds that tariff-jumping is a realistic option for multinational firms from industrialized countries. Hijzen et al. [2008] find horizontal tariff-jumping M&A evidence for 23 OECD countries for the period 1990–2001. More recently, Alfaro and Chen [2018] also find strong empirical evidence of tariff-jumping FDI through Orbis manufacturing firm-level dataset(60 countries, 2002–2007).

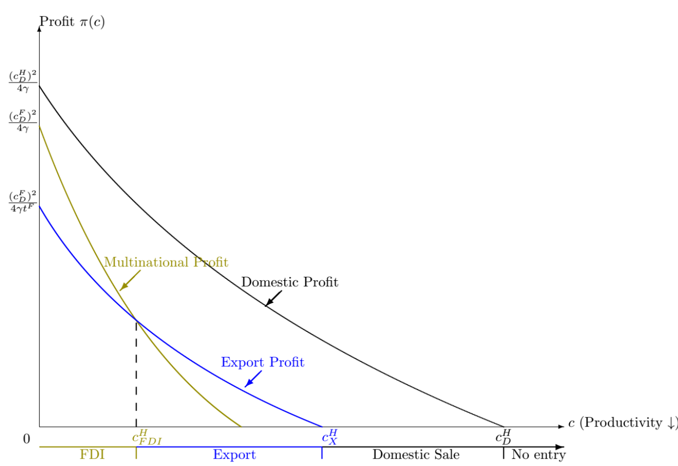
increase in response to an increase in t^F, which is in line with the empirical evidence of productivity sorting and the tariff-jumping FDI.

(iii) *FDI motivation* In **HMY**, the sorting of firms is induced by the assumption that $f_I > \tau^{\epsilon-1} f_X > f_D$: export incurs a fixed cost (f_X) and a higher marginal cost (τ), but as long as the fixed cost of FDI (f_I) is sufficiently high, the most productive firms are guaranteed to find FDI more desirable than export. This is a classic proximity-concentration trade-off in the spirit of Brainard [1997]. The similar trade-off is also present in **CD**, where the authors embed *ad valorem* tariff and *variable* fixed cost[17] into the **HMY** framework. They find that as the tariff increases, the exporter's variable profit decreases, while the differences in fixed cost remain the same. When the tariff level is sufficiently high, the gain from avoiding the tariff is higher than the fixed cost of becoming multinational, and a firm prefers FDI over export as an entry mode. In the current setup, this is no longer the case. Compare the profit function for an exporter and a multinational firm:

$$\pi^H_X(c) = \left[p^H_X(c)/t^F - \tau^F c\right] q^H_X(c) \tag{19}$$
$$\pi^H_{FDI}(c) = \left[p^H_{FDI}(c) - \varphi^F c\right] q^H_{FDI}(c) \tag{20}$$

As tariff increases, the revenue of the exporter will drop, making export a less desirable mode of accessing the Foreign market. Eventually, FDI becomes a more desirable entry mode. Although the marginal cost of FDI is higher than export (due to the assumption that $\varphi^F > \tau^F$), the operating profit of FDI exceeds the profit of export. The trade-off between export and FDI is no longer the conventional proximity-concentration trade-off, but a comparison of the profits. I also plot firms' profits as a function of marginal production cost, as in **Figure 1**. Notice that the presence of a positive net tariff ensures that the profit of FDI is strictly higher than the profit of exports, whereas in **HMY** and **CD**, a similar condition is obtained through the combination of fixed cost and variable costs.

Figure 1: Firms' Profit as a Function of Marginal Cost



Note: The graph plots the profit function of firms in each market according to the firm's serving mode. The black line represents the firm's profit function in the domestic market (in this case, it is Home). The blue line plots the exporter's profit function in the Foreign market, and the green line plots the profit function of multinational firms.

Given that the goal of this paper is to investigate the trade policy implications in an environment that

[17] It means firms with different productivity levels will face a different level of fixed cost when accessing the Foreign market.

features both export and multinational production, the selection into FDI (typically introduced through the fixed cost of FDI) margin is not of first-order interest here. A recent paper by Mrázová and Neary [2018] provides a justification for the current framework from a different perspective. They argue that statements like "Only the more productive firms select into the higher fixed-cost activity" are misleading: They are true given super-modularity[18] of the profit function, but otherwise may not hold. They discover that what matters for the direction of second-order selection effects (referring to the choice between export and FDI) is not a trade-off between fixed and variable costs, but whether there is a *complementarity between variable costs of production and trade*. In other words, if we allow FDI to be an equilibrium mode of accessing the Foreign market, then whether firms can afford them or not is independent of the fixed cost of FDI, but depends on the cross-effect of tariffs and production costs on profits. When super-modularity prevails, a more efficient firm has relatively higher operating profits in the FDI case, but when sub-modularity holds, the opposite may hold. The reason that the current setup can preserve the conventional sorting of firm productivity (i.e., second-order selection effect) is precisely due to the super-modularity of profit function[19] since there exists complementarity between the variable costs of production (in this case, the marginal cost of production c) and of trade (in this case, import tariff t).

2.3 Free Entry Condition

Entry is unrestricted in both countries. Firms choose a production location before entry and pay a sunk cost (f_E) to enter the market. To restrict the analysis on the effects of trade costs differences, I assume that countries share the same technology[20] (i.e., the same entry cost f_E and the same cost distribution $G(c)$). Free entry of domestic firms in country H implies zero expected profits in equilibrium, hence:

$$\int_0^{c_D^H} \pi_D^H(c)\, dG(c) + \int_{c_{FDI}^H}^{c_X^H} \pi_X^H(c)\, dG(c) + \int_0^{c_{FDI}^H} \pi_{FDI}^H(c)\, dG(c) = f_E \tag{21}$$

Given the Pareto assumption for cost distribution in both countries, the free entry condition for country H can be rewritten as:

$$\left(c_D^H\right)^{k+2} + \Phi_1^F \left(c_D^F\right)^{k+2} + \Phi_2^F \left(c_D^F\right)^{k+2} = \gamma\phi \tag{22}$$

where $\phi \equiv 2(k+1)(k+2)(c_M)^k f_E$ is a technology index that combines the effects of the better distribution of cost draws (lower c_M) and lower entry costs f_E. Moreover, Φ_1^F, Φ_2^F[21] are indices that combine the trade-off between tariff and higher marginal cost of FDI. The free entry condition is homogenous to degree $k+2$

[18]For example, super-modularity in $\Pi(t, c)$ means a higher tariff (t) reduces, in absolute value, the cost disadvantage of a higher-cost firm (larger c). For more details, please refer to Mrázová and Neary [2018].

[19]For a step-by-step verification, please refer to Mrázová and Neary [2018], their setup is a general preference, so they rely on the fixed cost to generate selection effects. For quadratic quasi-linear preference, they point out the first-order selection effect (according to their description, this is referring to whether serve the foreign market or not) needs the existence of choke price. The second-order effect is taken care of by the assumption that $\varphi > \tau$.

[20]For implications of Ricardian comparative advantage in the heterogeneous firm framework, please refer to the Appendix in **MO**.

[21]$\Phi_1^F \equiv \frac{(k+1)(k+2)t^F(\tau^F)^2}{2}\left\{\left(\frac{1}{t^F\tau^F}\right)^{k+2} - \left(\frac{1}{t^F\tau^F}\right)^2\left(\xi^F\right)^k - \frac{2k}{k+1}\left[\left(\frac{1}{t^F\tau^F}\right)^{k+2} - \left(\frac{1}{t^F\tau^F}\right)\left(\xi^F\right)^{k+1}\right] + \frac{k}{k+2}\left[\left(\frac{1}{t^F\tau^F}\right)^{k+2} - \left(\xi^F\right)^{k+2}\right]\right\}$

$\Phi_2^F \equiv \frac{(k+1)(k+2)\left(\xi^F\right)^k}{2}\left[1 - \frac{2k\varphi^F\xi^F}{k+1} + \frac{k\left(\varphi^F\xi^F\right)^2}{k+2}\right]$

regarding the cutoff cost level. This system (for H, F) can then be solved for the cutoffs in both countries:

$$c_D^H = \left[\gamma\phi \frac{1-\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}\right]^{\frac{1}{k+2}} \tag{23}$$

This equilibrium cutoff level differs from the one in **MO** in two aspects. First, this cutoff is lower[22] than the closed-economy cutoff in **MO**:

$$c_D^H < (\gamma\phi)^{\frac{1}{k+2}} \equiv \text{Closed-economy cutoff in } \textbf{MO}$$

indicating the opening up of an economy via export and multinational activity will increase the aggregate productivity by forcing the least productive firms to exit. This result is similar to Melitz [2003], but the operating channel is the product market competition, not the factor market competition, as argued in **MO**. Second, this cutoff is even lower[23]than the open economy cutoff in **MO**:

$$c_D^H < \left(\gamma\phi\frac{1-\rho^F}{1-\rho^H\rho^F}\right)^{\frac{1}{k+2}} \equiv \text{Open-economy cutoff in } \textbf{MO}$$

The intuition is straightforward: the presence of FDI, here the most productive firms in the distribution, intensifies the competitive environment in the economy, forcing the least productive firms to exit and hence further increases aggregate productivity[24]. Notice, the open economy cutoff stated above is slightly different from **MO** due to the presence of *ad valorem* tariffs[25].

2.4 Prices, Product Variety, Number of Entrants and Welfare

To see more features in the current setup, I first compute $\bar{p}^H$. Notice, the marginal cost of H's operating firms fall into the range $[0, c_D^H]$, which is also the range for delivered cost of exporters ($\tau^F c$), and the effective marginal cost of multinationals production ($\varphi^F c$). They all share identical distributions over the support given by $G^H(c) = (c/c_D^H)^k$. Therefore, the price distributions of H's domestic firms, $p_D^H(c)$, H's exporters producing in F, $p_X^F(c)$, and F's multinationals producing in H, $p_{FDI}^F(c)$, are all identical. The average price in country H is thus given by:

$$\bar{p}^H = \frac{\int_0^{c_D^H} p_D^H(c)\, dG(c)}{G\left(c_D^H\right)} = \frac{\int_{c_{FDI}^F}^{c_x^F} p_X^F(c)\, dG(c)}{G\left(c_X^F\right)} = \frac{\int_0^{c_{FDI}^F} p_{FDI}^F(c)\, dG(c)}{G\left(c_{FDI}^F\right)} = \frac{2k+1}{2k+2} c_D^H \tag{24}$$

Combining this with the definition of $p_{\max}^H$ and $p_{\max}^F$, the number of firms selling in country H is:

$$N^H = \frac{2\gamma\left(\alpha - c_D^H\right)(k+1)}{\eta c_D^H} \tag{25}$$

---

[22]This is based on Proposition 3, see Appendix B.3.

[24]This is consistent with the recent empirical evidence discovered in Fons-Rosen et al. [2013], although I abstract from the possibility of any spillover effect.

[25]Please see Appendix B.3 for more details.

From this expression, it must be the case that $\alpha > c_D^H$ so that the number of firms selling in country H is positive in equilibrium. The total number of product varieties in country H is composed of domestic producers, exporters, and multinationals from country F. Given a positive mass of entrants N_E in both countries, there are $G(c_D^H)N_E^H$ domestic producers, $[G(c_X^F) - G(c_{FDI}^F)]N_E^F$ Foreign exporters, and $G(c_{FDI}^F)N_E^F$ Foreign multinationals selling in H. Altogether they satisfy the following condition:

$$G\left(c_D^H\right) N_E^H + \left[G\left(c_X^F\right) - G\left(c_{FDI}^F\right)\right] N_E^F + G\left(c_{FDI}^F\right) N_E^F = N^H \tag{26}$$

Solving this system (for H and F) will give us the number of entrants in country H:

$$N_E^H = \frac{2\left(c_M\right)^k (k+1)\gamma}{\eta\left(1 - \delta^H \delta^F\right)} \left[\frac{\alpha - c_D^H}{\left(c_D^H\right)^{k+1}} - \delta^H \frac{\alpha - c_D^F}{\left(c_D^F\right)^{k+1}}\right] \tag{27}$$

where $\delta^l = (t^l \tau^l)^{-k}$, for $l \in \{H, F\}$. Notice, the condition that ensures positive equilibrium number of varieties (N^l) in the economy, $\alpha > c_D^l$, also guarantees the positive mass of entry (N_E^l) in the equilibrium.

Equation (27) marks a crucial difference between the current framework and **CD**: The number of entrants in the economy is endogenously affected by the tariff level. As first noted by Balistreri et al. [2011], the equilibrium level of firm entry in the Melitz-type model is no longer fixed if: (i) *ad valorem* tariffs are imposed rather than iceberg transport costs, or (ii) there are multiple sectors in the economy. Equilibrium entry-level becomes endogenous in the current framework because of not only the *ad valorem* tariff, but also the two-sector economy. However, no discussion of entry is mentioned in **CD** even though their framework also features these two aspects. It is true that in the basic Melitz model, firm entry is independent of trade costs. However, when analyzing the revenue-generating tariff, this perception is no longer valid[26]. This implication is crucial in understanding the equilibrium feature of the model. I will come back to this point in Section 3.2.

Following **MO**, combine equation (4), (24), (25) and the definition of $\sigma_{p^H}^2$, it is straightforward to show the consumer welfare in H equals to:

$$U^H = I^H + \frac{\alpha - c_D^H}{2\eta}\left(\alpha - \frac{k+1}{k+2} c_D^H\right) \tag{28}$$

Once again, consumer welfare changes monotonically with the domestic cutoff, which captures the effect of an increase in product variety and a decrease in the average price. Also notice, consumer surplus in country H is given by the second term in equation (28).

2.5 Tariff Revenue and National Welfare

This part of the model is very important for the analysis of socially optimal tariff and Nash tariff. Note that tariff revenue is also a component of consumer income I^H through the redistribution from the government.

[26] More specifically, according to Arkolakis et al. [2012], one of the 'macro' assumption (R2) is violated, therefore entry become endogenous.

I define the pre-tax value of country H's import as:

$$\begin{aligned} IM^H &= N_E^F \int_{c_{FDI}^F}^{c_X^F} \frac{p_X^F(c)}{t^H} q_X^F(c)\, dG(c) \\ &= N_E^F \frac{t^H \left(\tau^H\right)^2 \left(c_D^H\right)^{k+2}}{4\gamma (k+2) \left(c_M\right)^k} \left[2 \left(\frac{1}{t^H \tau^H} \right)^{k+2} - \frac{k+2}{\left(t^H \tau^H\right)^2} \left(\xi^H\right)^k + k \left(\xi^H\right)^{k+2} \right] \end{aligned} \tag{29}$$

Therefore, the total import tariff revenue of country H is defined as

$$\begin{aligned} TR^H &\equiv \left(t^H - 1\right) \times IM^H \\ &= N_E^F \frac{t^H - 1}{t^H} \frac{\left(c_D^H\right)^{k+2}}{4\gamma (k+2) \left(c_M\right)^k} \left[2 \left(\frac{1}{t^H \tau^H} \right)^k - (k+2) \left(\xi^H\right)^k + k \left(\xi^H\right)^{k+2} \left(t^H \tau^H\right)^2 \right] \end{aligned} \tag{30}$$

From the trade-policy perspective, the government will use its policy instrument to maximize consumer welfare:

$$U_n^H = \underbrace{w^H + \left(t^H - 1\right) \times IM^H + \Pi^H}_{\equiv I^H} + \underbrace{\frac{\alpha - c_D^H}{2\eta} \left(\alpha - \frac{k+1}{k+2} c_D^H \right)}_{\equiv CS^H} \tag{31}$$

Therefore tariff affects consumer welfare from two channels: (i) consumer surplus, which is directly affected by the change in c_D^H in response to tariff, and (ii) tariff revenue, which is affected by both the tariff level (t^H) and the tariff base (IM^H). Due to the free-entry condition, aggregate profit Π^H will be driven to zero in equilibrium. Notice due to the presence of numéraire good, $w^H = 1$, although consumers will not take t^H into consideration when maximizing their utility, the government does take consumers into consideratoin by choosing the utility maximizing tariff level.

With the model above, I will now discuss the equilibrium features of this economy. All the results are contrasted with **CD**: an economy that features heterogeneous firms, FDI, but constant markups.

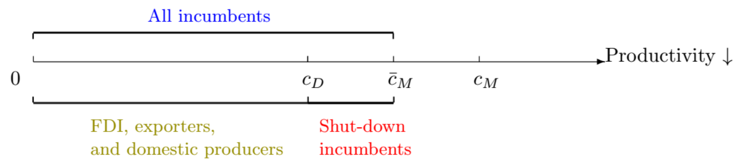
3 Equilibrium

This section studies the short-run and the long-run equilibrium features of the model developed in the previous section, with an emphasis on the comparative statics in response to a change in import tariff. As discussed in Section 2, due to the presence of *ad valorem* tariff and the quadratic quasi-linear preference (which results in multiple sectors in the current setup), the level of firm-entry become endogenous under the free-entry condition. To see how the model directly responds to the trade/tariff liberalization, it is, therefore, necessary to separate the short-run (when the entry is restricted) from the long-run (free entry).

3.1 Short-run Equilibrium

In this subsection, I introduce the short-run version of the model and discuss its equilibrium characteristics. In the short-run, when entry and exit are prohibited, the economy is characterized by a fixed number of incumbents, and they decide whether to produce or shut-down based on their profits. More specifically, Home country is characterized by a fixed number of incumbents, $\bar{N_I^H}$, with cost distribution $\bar{G^H}$ on $[0, \bar{c}_M]$, where $\bar{c}_M$ is within the long-run technology frontier, c_M. I keep the assumption that the productivity $1/c$ is distributed with Pareto shape k, implying $\bar{G}^H(c) = (c/\bar{c}_M)^k$. The distribution of firm's productivity in the short-run model is briefly displayed in **Figure 2**.

Figure 2: Productivity Distribution in the Short-run Equilibrium



Note: c_M together with f_E stand for the state of the technology in the long-run equilibrium. $\bar{c}_M$ together with f_E stands for the state of the technology in the short-run equilibrium. c_D stands for the domestic cutoff level in the short-run equilibrium. Exporter and FDI cutoffs distributed on the left-hand side of c_D.

A Home firm produces if it can earn nonnegative profits from either its domestic market, export market, or FDI market. These decisions based on profits lead to the following short-run cutoff conditions:

$$
\begin{aligned}
c_D^H &= \sup\{c : \pi_D^H(c) \geq 0 \text{ and } c \leq \bar{c}_M\} \\
c_X^H &= \sup\{c : \pi_X^H(c) \geq 0 \text{ and } c \leq \bar{c}_M\} \\
c_{FDI}^H &= \sup\{c : \pi_{FDI}^H(c) \geq \pi_X^H(c) \text{ and } c \leq \bar{c}_M\}
\end{aligned}
$$

Firms with marginal cost $c > c_D^H$ will shut-down. Utilizing the zero-profit conditions, one can establish the following relations between cutoff levels and the number of operating firms in Home and Foreign:

$$
N^H = \frac{2\,(k+1)\,\gamma}{\eta} \times \frac{\alpha - c_D^H}{c_D^H}, \qquad N^F = \frac{2\,(k+1)\,\gamma}{\eta} \times \frac{\alpha - t^F \tau^F c_X^H}{t^F \tau^F c_X^H}
$$

where N^H and N^F represent the endogenous number of sellers in country H and F in the short-run. Notice that the different cutoffs satisfy the same condition as in the long-run. There are $\bar{N}_I^H \bar{G}\left(c_D^H\right)$ producers in H who sell in their domestic market, $\bar{N}_I^F\left[\bar{G}\left(c_X^F\right) - \bar{G}\left(c_{FDI}^F\right)\right]$ Foreign exporters, and $\bar{N}_I^F \bar{G}\left(c_{FDI}^F\right)$ Foreign FDI firms in H. These numbers must add up to the total number of producers in country H. Similar equation also holds for country F:

$$
\begin{aligned}
N^H &= \bar{N}_I^H \bar{G}\left(c_D^H\right) + \bar{N}_I^F\left[\bar{G}\left(c_X^F\right) - \bar{G}\left(c_{FDI}^F\right)\right] + \bar{N}_I^F \bar{G}\left(c_{FDI}^F\right) \\
N^F &= \bar{N}_I^F \bar{G}\left(c_D^F\right) + \bar{N}_I^H\left[\bar{G}\left(c_X^H\right) - \bar{G}\left(c_{FDI}^H\right)\right] + \bar{N}_I^H \bar{G}\left(c_{FDI}^H\right)
\end{aligned}
$$

Combining these two equations with the threshold price conditions yield expressions for the cost cutoffs in both countries:

$$
\frac{\alpha - c_D^H}{\left(c_D^H\right)^{k+1}} = \frac{\eta}{2\,(k+1)\,\gamma}\left\{\frac{\bar{N}_I^H}{\bar{c}_M^k} + \left[\left(\frac{1}{t^H \tau^H}\right)^k - \left(\xi^H\right)^k\right]\bar{N}_I^F + \left(\xi^H\right)^k \bar{N}_I^F\right\} \tag{32}
$$

$$
\frac{\alpha - c_D^F}{\left(c_D^F\right)^{k+1}} = \frac{\eta}{2\,(k+1)\,\gamma}\left\{\frac{\bar{N}_I^F}{\bar{c}_M^k} + \left[\left(\frac{1}{t^F \tau^F}\right)^k - \left(\xi^F\right)^k\right]\bar{N}_I^H + \left(\xi^F\right)^k \bar{N}_I^H\right\} \tag{33}
$$

Note, these two conditions uniquely identify the short-run cutoff levels(c_D^H, c_D^F) with the number of producing firms in each country (N^H, N^F).

Equation (32) and (33) also clearly highlight the protection role played by import tariff in the short-run. Based on these two equations, we have the following proposition for the short-run equilibrium.

Proposition 1. *In the short-run equilibrium, an increase in Home country's import tariff (t^H) can protect Home producers from Foreign competition, increasing the domestic cost cutoff:*

$$\frac{\partial c_D^H}{\partial t^H}|_{\textit{short-run}} > 0$$

Proof. See Appendix B.1. □

Intuitively, an increase in H's tariff will make it harder for the Foreign exporters to access the Home market, so the number of exporters from F to H will decrease. At the same time, an increase in H's tariff will induce tariff-jumping FDI among the Foreign exporters, so the number of Foreign firms that access the Home market through FDI will increase. In the current setup, the decrease of exporters surpasses the increase of FDI firms, so the right-hand side of the equation (32) is decreasing in t^H, indicating an increase in H's domestic cost cutoff (c_D^H). Therefore, an increase in H's tariff reduces the total number of Foreign firms (exporters and FDI firms) accessing the Home market, making it easier for Home producers to survive.

In other words, import tariff, in the short-run, can effectively shield Home from Foreign competition. This result is similar to the result in Section 3.7 of **MO**. However, they obtain the result of an increase in cutoff level through an exogenous variation of trading partner industrial size (N^H or N^F). In the current framework, a change in tariff level alters the relative size of Home and Foreign firms, affecting the cutoff levels. This finding might seem to confirm the findings in **CD**[27], but if we allow firms to enter freely, then the result will be quite different. As we will see in the next section, the classic 'delocation'[28] result will arise.

Based on equation (6), (10) and (15), one can also obtain the following proposition regarding markups in respond to a tariff change:

Proposition 2. *In the short-run equilibrium, an increase in Home country's import tariff (t^H) can increase domestic producer's markup, may decrease or increase Foreign exporter's markup, and increase Foreign FDI firm's markup.*

$$\frac{\partial m_D^H}{\partial t^H}|_{\textit{short-run}} > 0, \frac{\partial m_X^F}{\partial t^H}|_{\textit{short-run}} \gtrless 0, \frac{\partial m_{FDI}^F}{\partial t^H}|_{\textit{short-run}} > 0$$

Proof. See Appendix B.2. □

Intuitively, for an increase in t^H, tariff affects *Home domestic producer's* markup $m_D^H(c)$ through the equilibrium effect on c_D^H. Protection makes it easier for Home producers to survive and results in a higher c_D^H, meaning a higher markup for all the domestic sellers. For *Foreign exporters*, their markup $m_X^F(c)$ is affected by tariff from two aspects: (i) direct effect—-an increase in t^H directly raises m_X^F, meaning Foreign exporters will pass the tariff burden to the consumers by increasing markup, and (ii) indirect effect—-the tariff indirectly affects m_X^F through the equilibrium effect on c_D^H. With restricted entry, these two effects are in

[27]Specifically, their justification of their equation (11). Similar results are qualitatively identical to those of other similar models in the heterogeneous firm literature, such as Melitz [2003] and **HMY**.

[28]The delocation effect has been studied in previous work(see, for example, Venables [1985], Helpman and Krugman [1989], Baldwin et al. [2003]) and here is also confirmed in the heterogeneous firm framework with FDI.

the opposite direction. Which effect is more dominant depends on the individual exporter's productivity. For the least productive Foreign exporters (large c), the direct effect will dominate the indirect effect, resulting in a drop in Foreign exporter's markup, indicating Home's protection will reduce the market power of Foreign exporters. Nevertheless, for the most productive Foreign exporters (small c), the indirect effect will dominate the direct effect, resulting in the bigger market power of Foreign exporters. For *Foreign FDI firms*, tariff affects $m^F_{FDI}(c)$ through the equilibrium effect on c^H_D. Protectionism results in a less competitive Home environment, which benefits the more productive Foreign FDI firms, and allows them to charge higher markups.

In this short-run equilibrium, where additional entry of firms is restricted, the findings in the current framework confirm the previous results in the literature on unilateral trade/tariff liberalization. As we will see in the next section, these results will be reversed with an endogenous level of firm entry in the long-run.

3.2 Long-run Equilibrium

In this subsection, I analyze the equilibrium features of the model when firms can enter and exit freely. As mentioned in Section 2, firm entry, in the long-run, is endogenously affected by the tariff level. In what follows, I will illustrate the comparative statics of the model and contrast its benchmark results with **CD**.

First of all, following the discussion of equation (23), the presence of FDI in the **MO** world will deliver a different equilibrium domestic cutoff in the economy, which can be summarized by the following proposition:

Proposition 3. *The presence of FDI makes the economy more competitive, and the domestic cutoff is lower compared to the case when there is no FDI:*

$$c^H_D|_{\textit{With FDI}} = \left[\gamma\phi \frac{1-\left(\Phi^F_1+\Phi^F_2\right)}{1-\left(\Phi^F_1+\Phi^F_2\right)\left(\Phi^H_1+\Phi^H_2\right)}\right]^{\frac{1}{k+2}} < c^H_D|_{\textit{Without FDI}} = \left[\gamma\phi\frac{1-\psi^F}{1-\psi^F\psi^H}\right]^{\frac{1}{k+2}}$$

Proof. See Appendix B.3. □

Different from the short-run equilibrium case, here the domestic cost cutoff is pinned down through the long-run free-entry condition (equation (22)). In the Appendix B.3, I show $\Phi^l_1+\Phi^l_2>\psi^l$ for $l\in\{H,F\}$. The sum of Φs can be viewed as a measure of 'openness'. Intuitively, the presence of FDI makes the country more 'open' compared to the case when FDI is not an option. In **MO**, ψ^l [29]measures the 'freeness' of trade. The presence of Foreign FDI intensifies the Home country's competitive environment, making it harder for Home producers to survive. The marginally surviving firm needs to be more productive. Openness (either through export or FDI) increases competition[30] in the domestic product market, shifting up residual demand price elasticities for all firms at any given demand level. Therefore, the least productive firms are forced to exit. This effect is very similar to an increase in market size in the closed economy: the increased competition induces a downward shift in the distribution of markups across firms. Although only relatively

[29]More precisely, the freeness of trade is measured by τ^{-k} in **MO**. Here, due to the presence of tariff, this term is augmented to incorporate tariff, $\tau^{-k}t^{-(k+1)}$.

[30]Compared to the case when export is the only option to access Foreign market.

more productive firms survive (with higher markups than the less productive firms who exit), the average markup is reduced.

With free-entry, firms can freely enter and exit the market in the long-run. Due to the presence of *ad valorem* tariff and the quadratic quasi-linear preference, the number of entrants in the monopolistically competitive sector is endogenously affected by the level of tariff. As we will see soon, this feature has important implications for trade/tariff liberalization. For example, a unilateral change in Home country's import tariff has quite different impacts on the domestic productivity cutoffs of both countries, as can be seen from the following proposition:

Proposition 4. *An increase in country H's import tariff results in a decrease in the cutoff cost level in country H's domestic market, and an increase in the cutoff cost level in country F's domestic market:*

$$\frac{\partial c_D^H}{\partial t^H} < 0 < \frac{\partial c_D^F}{\partial t^H}$$

Proof. See Appendix B.4. □

Different from the short-run outcome, where the H's domestic cost cutoff would increase as a result of tariff protection, in the long-run, H's domestic cost cutoff would decrease in response to tariff protection. Intuitively, although an increase in the import tariff raises the protection level in country H in the short-run, it also fosters a more extensive entry from domestic firms over time. Proposition 7 near the end of this section further demonstrates this point. Protection makes the Home country a more desirable environment for firms to do business in the long-run. With the free-entry condition, the larger entry will generate a higher competition in the domestic market, driving out the least productive firms, and forcing the marginally surviving firms to be more productive.

This result is quite different from **CD**. They find an increase in the import tariff in country H will raise the protection level in country F, shield country H's firm from the competition, and make the domestic surviving firms less productive, i.e., $\partial c_D^H / \partial t^H > 0$ (their equation (11)). This result is primarily due to the fact that the presence of quadratic quasi-linear preference affects firm-entry, which is entirely absent in **CD**.

A unilateral change in Home's import tariff also affects the exporters in both countries. The impact can be summarized as the following:

Proposition 5. *An increase in country H's import tariff results in an increase in the export cutoff cost level in country H and a decrease in the export cutoff cost level in country F:*

$$\frac{\partial c_X^H}{\partial t^H} > 0 > \frac{\partial c_X^F}{\partial t^H}$$

Proof. See Appendix B.5. □

An increase in import tariff in country H will cause the least productive exporters from F to quit exporting, and only serve their domestic market. The reason is that the increase in tariff reduces exporter's revenue (hence profit), making it less desirable for the least productive exporters to serve H's market. With their exit, the marginally surviving exporters are more productive, hence a lower c_X^F. This result is the same as the one obtained in **CD**. Moreover, although protection generates more entry of Home firms in the long-run, giving a competitive pressure for Home's export market, this impact is dominated by the protective effect of the tariff on exporters, resulting in a higher Home exporter cutoff, and making it easier for Home country to export.

Proposition 6. *An increase in country H's import tariff results in an increase in the FDI cutoff cost level in country F:*

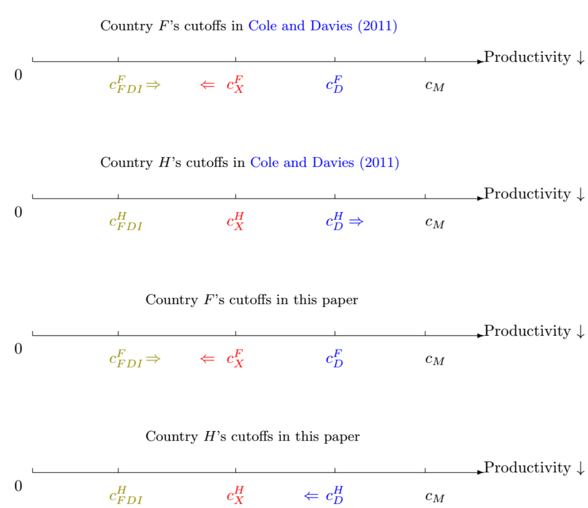
$$\frac{\partial c_{FDI}^F}{\partial t^H} > 0$$

Proof. See Appendix B.6. □

The intuition here is straightforward: the most productive exporters from F, when facing an increase in import tariff in H, will find it less desirable to access H's market through export, and hence choose FDI as the entry mode. This result is due to the profit of FDI outweighing the profit of export when t^H increases, as can be seen from equation (19) and (20). Hence the marginally surviving multinationals from country F are now less productive since previously they are exporters, resulting in a higher c_{FDI}^F. This result is similar to the findings in **CD**.

To sum up these results and contrast them with **CD**, I plot the productivity cutoffs and their responses toward Home country's unilateral change in tariff, which is shown in **Figure 3**. When t^H increases, from equation (11)–(13) in **CD**, the least productive Foreign exporters exit the domestic market (c_X^F decreases) and the most productive Foreign exporters become multinationals (c_{FDI}^F increases). This change makes the composition of domestic Foreign firms (including F's exporters and multinationals) more productive. Due to the protection, the domestic market is shielded from the Foreign competition. Hence domestic firms find it easier to survive (c_D^H increases).

Figure 3: Productivity Cutoffs Responses Comparison in the Long-run Equilibrium



Note: Each axis represents the distribution of productivity within the monopolistically competitive industry. The first two panels are based on equation (11)-(13) from **CD**. The last two panels are based on Proposition 4 through Proposition 6.

In the current setup, an increase in t^H will similarly lead the least productive Foreign exporters to exit the domestic market (c_X^F decreases) and the most productive Foreign exporters to become multinationals (c_{FDI}^F increases), making the composition of domestic Foreign firms more productive. However, Home's protection will attract more Home firms to enter the domestic market (N_E^H increases), making the Home country's environment more competitive. Therefore, domestic firms need to be more productive to survive (c_D^H decreases).

In both cases, we have tariff-jumping FDI in response to the increase of t^H. In **CD**, tariff-jumping intensifies the competitive environment in the domestic market of H, but this effect is dominated by the protection effect raised through tariff. So the outcome is an easier-to-survive environment. In the current setup, the tariff-jumping FDI intensifies the competitive environment in the domestic market. The excessive entry generated by protection also makes the domestic environment more competitive. These two effects together result in a tougher environment in Home's domestic market, making it harder for firms to survive.

Here, it is worthwhile to mention that the domestic cutoff's response to tariff depends on the presence of FDI, which will be an important feature when it comes to the discussion of optimal tariff. I summarize it in the following corollary:

Corollary 1. *Under the assumption that $\varphi^H > \tau^H$ [31], an increase in H's import tariff results in a tougher competitive environment in the domestic market over time, and this effect is exacerbated by the presence of FDI:*

$$\left| \frac{\partial c_D^H}{\partial t^H}\Big|_{\textit{without FDI}} \Big/ \frac{\partial c_D^H}{\partial t^H}\Big|_{\textit{with FDI}} \right| < 1$$

Proof. See Appendix B.7. □

Finally, the protection also affects the number of entrants and eventually the number of products available in each country. I summarize this result in the following proposition:

Proposition 7. *An increase in H's import tariff results in an increase in the number of entrants in H and a decrease in the number of entrants in F. Over time, this effect contributes to an increase in the number of varieties in H and a decrease in the number of varieties in F:*

$$\frac{\partial N_E^H}{\partial t^H} > 0 > \frac{\partial N_E^F}{\partial t^H}, \qquad \frac{\partial N^H}{\partial t^H} > 0 > \frac{\partial N^F}{\partial t^H}$$

Proof. See Appendix B.8. □

The intuition is obvious: Home's tariff protection makes the Home country a more desirable environment to do business for the firms. In the long-run, more firms would choose to enter Home's market, resulting in a larger number of products available in the equilibrium. The opposite condition will hold for the Foreign market. Clearly, this result crucially depends on the fact that tariff can affect the number of entrants in this economy. No similar results are discussed in **CD**.

[31] The domestic cutoff without FDI but with ad valorem tariff is $c_D^H = \left[\gamma\phi\left(1-\rho^F\right)/\left(1-\rho^H\rho^F\right)\right]^{1/(k+2)}$, where $\rho^H = (\tau^H)^{-k}(t^H)^{-(k+1)}$. The domestic cutoff with FDI is defined in equation (23).

Proposition 8. *In the long-run equilibrium, an increase in Home country's import tariff (t^H) can decrease domestic producer's markup, decrease Foreign exporter's markup, and decrease Foreign FDI firm's markup.*

$$\frac{\partial m_D^H}{\partial t^H}|_{\textit{long-run}} < 0, \frac{\partial m_X^F}{\partial t^H}|_{\textit{long-run}} < 0, \frac{\partial m_{FDI}^F}{\partial t^H}|_{\textit{long-run}} < 0$$

Proof. See Appendix B.9. □

The intuition follows right after the previous proposition. Due to the increase of protectionist tariff, Home becomes a more favorable environment to do business, attracting more firm to enter in the long-run. This effect will increase the competition in the Home market, reducing the markups for all kinds of producers that serve the Home market.

4 Social Optimum vs. Market Outcome

In this section, I first derive the socially optimum outcome in the framework introduced in Section 2. Then I compare its socially optimum outcome with the market outcome. Then I analyze the forces in the economy that result in the departure of market outcome from socially optimum outcome.

The social planner's problem can be described as the following. Since the quadratic quasi-linear utility implies transferable utility, social welfare can be expressed as the sum of all the representative consumers' utilities. Following Nocco et al. [2014], the planner chooses the number of entrants(N_E^H, N_E^F), and production level for homogeneous good and heterogeneous good($q_0^H, q_0^F, q_i^H, q_i^F$) to maximize social welfare subject to aggregate resource budget constraint:

$$\max_{\{N_E^H, q_0^H, q_i^H, N_E^F, q_0^F, q_i^F\}} \mathbb{W} \equiv \mathbb{U}_{\mathbb{H}} + \mathbb{U}_{\mathbb{F}}$$

$$\begin{aligned} \text{s.t. } q_0^H + q_0^F + f\left(N_E^H + N_E^F\right) + N_E^H \int_0^{c_M} \left[c q_D^H(c) + \tau^F c q_X^H(c) + \varphi^F c q_{FDI}^H(c)\right] dG(c) \\ + N_E^F \int_0^{c_M} \left[c q_D^F(c) + \tau^H c q_X^F(c) + \varphi^H c q_{FDI}^F(c)\right] dG(c) = 2 + \bar{q}_0^H + \bar{q}_0^F \end{aligned}$$

where $q_0^H + q_0^F$ stands for the supply of homogeneous good in both countries, $f\left(N_E^H + N_E^F\right)$ the sunk entry cost in the monopolistically competitive sector in H and F, $N_E^H \int_0^{c_M} [c q_D^H(c) + \tau^F c q_X^H(c) + \varphi^F c q_{FDI}^H(c)] dG(c)$ the supply of differentiated varieties in the Home country, and the last term on the left-hand side of the constraint gives the supply for differentiated varieties in the Foreign country. On the right-hand side, we have the endowment of labor and homogeneous good in both countries. The differences between socially optimum outcome and market outcome can be summarized in the following proposition:

Proposition 9. *In the current framework, compared to the socially optimum, the market outcome differs in several dimensions:*
(A) Marginal cost cutoffs
(i) the Home domestic market selection is weaker than the socially optimum selection

$$c_D^{HM} > c_D^{HO}$$

(ii) the Home exporter market selection is weaker than the socially optimum selection

$$c_X^{HM} > c_X^{HO}$$

(ii) the Home FDI market selection is stronger than the socially optimum selection

$$c_{FDI}^{HM} < c_{FDI}^{HO}$$

(B) Intensive margin
(i) Home's domestic producers undersupply varieties with low marginal production cost[32]*,*

$$q_D^{HM} < q_D^{HO} \text{ if } c < \left[2 - (2/\triangle_F)^{1/(k+2)}\right] c_D^{HO}$$

(ii) Home's exporters undersupply varieties with low marginal cost

$$q_X^{HM} < q_X^{HO} \text{ if } c < \left[2 - (2/\triangle_F)^{1/(k+2)}\right] \frac{c_D^{FO}}{\tau^F(2-t^F)}$$

(iii) Home's FDI firms also undersupply varieties with low marginal production cost

$$q_{FDI}^{HM} < q_{FDI}^{HO} \text{ if } c < \left[2 - (2/\triangle_F)^{1/(k+2)}\right] \frac{c_D^{FO}}{\varphi^F}$$

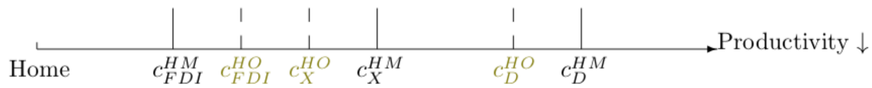
(C) Extensive margin
Depending on the of domestic cutoff levels (c_D^{HO},c_D^{FO}) in the socially optimum, the market outcome does not always yield the same level of the total number of varieties (N^H, N^F) and the number of entrants (N_E^H, N_E^F) as those in the socially optimum.

Proof. See Appendix B.10. □

This proposition says that the market outcome differs from that of the socially optimum in several dimensions. For the Home country, the market selection is weaker than the planner's selection in both the domestic producers' and exporters' market, but is stronger than the planner's selection in the FDI margin, as illustrated in **Figure 4**. Intuitively, compared to the planner's choice, Home country produces too many varieties in the domestic market, imports too many varieties from abroad, and does not have enough varieties from Foreign FDI firms. On the intensive margin, the more productive domestic producers (i.e. firms with low marginal costs) produce less than the socially optimal level. The same results hold true among the exporters and FDI firms, indicating the distribution of products is skewed too much toward high cost varieties. On the extensive margin, the number of entrants and the equilibrium number of varieties are also different from those of the socially optimum level. The market produces too many products from firms with large marginal costs, but not enough from firms with low marginal costs.

[32]Please refer to the Appendix B.10 for a detailed expression of $\triangle_F$.

Figure 4: Productivity Cutoffs Comparison between Market Outcome and Socially Optimum Outcome



Note: This graph shows the marginal cost distributions of all the domestic firms in Home country. The letter M stands for the market outcome, which are marked with the black color. The letter O stands for the socially optimum outcome, which are marked with the olive color. Note, the indexes here are different from those in **Figure 8**.

Since the trade policy study in this paper exclusively focuses on the monopolistically competitive sector, the discussion of inter-sector inefficiency is omitted here[33]. The inefficiencies in the market outcome originate from multiple externalities in this economy. Two inefficiencies occur with quadratic quasi-linear preference: (i) The consumers display the 'love of variety' feature, which, however, firms do not consider when making entry decisions. As a result, there are not enough varieties in the economy. (ii) Firms can charge variable markups, and thus firm heterogeneity becomes another source of inefficiency in this economy. Since markup decreases in marginal cost, the low marginal cost firms (more productive) are inefficiently small and high marginal cost firms (less productive) are inefficiently large in the market outcome. The monopoly power in the differentiated-good sector allows a firm to price over its marginal cost. Under the free-entry condition, this externality tends to create too many varieties. The new entrant will take up the market share of existing firms, and this business-stealing effect also tends to create too many varieties. All these externalities work together to generate market failures in the current economy.

According to Dhingra and Morrow [2019], when monopolistic competition is combined with CES preference, the market outcome coincides with the socially optimum outcome. The externalities mentioned above exactly cancel each other out[34]. The current economy, however, deviates from this benchmark due to the quadratic quasi-linear preference. The forces that generate externalities do not cancel each other out, and firm heterogeneity becomes a new source of inefficiency in the economy. The market outcome hence differs from the first-best outcome systematically. As discussed in Section 3, tariff can affect not only the cutoff levels, entrants, and number of varieties in the equilibrium, but also social welfare. I will come back to this issue again in Section 6.

5 Free Trade and Its Welfare Implication

In this section, I study the classical question in the trade policy literature: is free trade socially optimal in the current economy? Here the free trade is referring to zero net tariff value. To answer this question, I set $t^H = t^F = 1$ and study the joint welfare in H and F:

$$\mathbb{W} \equiv \mathbb{U}^H|_{t^H=1} + \mathbb{U}^F|_{t^F=1}$$

The results can be summarized in the following proposition:

[33]Interesting reader could refer to Nocco et al. [2014] for further discussion.

[34]Note, firm heterogeneity does not create externality since all firms charge identical markup under CES preference.

Proposition 10. *Free trade is, in general, not socially optimal. If H and F start with free trade ($t^H = t^F = 1$), then a small symmetric increment in import tariff raises social welfare if and only if $\tilde{c}_D > \alpha/2$, lowers social welfare if and only if $\tilde{c}_D < \alpha/2$ and has no effect on social welfare if and only if $\tilde{c}_D = \alpha/2$, where $\tilde{c}_D$ is the domestic cutoff under symmetry when net tariff is zero.*

Proof. See Appendix B.11. □

Interestingly, in this economy, free trade is not always socially optimal. On the one hand, if $\tilde{c}_D$, the domestic marginal cost cutoff level under symmetry with free trade, is sufficiently high, then a small increase in import tariff will raise the social welfare. This result means that if domestic cutoff is sufficiently high, the market selection too weak, then there are not enough firms competing in the economy. Therefore, the social planner should increase tariff to encourage firm entry. On the other hand, if the domestic cutoff is too low, the market selection too strong, there will be too many firms competing in this economy. In this situation, the social planner should discourage entry by reducing tariff (i.e., subsidize trade). If the domestic marginal cost cutoff level, however, is exactly equal to the threshold value, then free trade is socially optimal. The parameter α here stands for the relative demand of consumers toward differentiated varieties. So whether free trade is socially optimal crucially depends on whether the market outcome could meet consumer's demand on the varieties.

To gain more intuition of this proposition, and to elaborate on the externalities mentioned in Proposition 9, I rewrite the social planner problem as the following:

$$\mathbb{W} = \max_{\{N_E^H, N_E^F\}} I^H + \frac{\alpha - c_D^H}{2\eta}\left(\alpha - \frac{k+1}{k+2}c_D^H\right) + I^F + \frac{\alpha - c_D^F}{2\eta}\left(\alpha - \frac{k+1}{k+2}c_D^F\right)$$

Following Mankiw and Whinston [1986], here I consider a constrained-optimum problem faced by a social planner, who cannot affect the market outcome for any given number of firms. This is particularly relevant under the current heterogeneous firm framework since the first-best outcome cannot be reached due to the presence of externalities in this economy, as discussed in Section 4. On top of that, one should also keep in mind that the presence of a numéraire good adds an extra distortion to the model. There is no markup in the numéraire-good sector, but in the differentiated-good sector, producers charge prices above their marginal costs due to their monopoly power. As pointed out by Bhagwati [1969], the presence of distortions can result in the breakdown of Pareto-optimality of laissez-faire.

The planner chooses the optimal level of entry to maximize social welfare. Under free entry condition, firms make entry decisions irrespective of the externalities that they generate on consumers and other firms, so the market entry level might not be socially desirable. Notice, since wage in the economy equals to one, and tariff revenue equals to zero at free trade, maximizing the income is equivalent to maximizing aggregate profits. After imposing symmetry and $t = 1$, the above social welfare function $\mathbb{W}$ can be rewritten as:

$$\max_{\{N_E\}} \mathbb{W} \equiv \Pi + \underbrace{\frac{\alpha - c_D}{2\eta}\left(\alpha - \frac{k+1}{k+2}c_D\right)}_{\text{Consumer Surplus}}$$

where $\Pi = N_E \times (\bar{\pi} - f_E)$. Consumers take the number of entrants as given and maximize their utilities.

The above consumer surplus can be rewritten in terms of the optimized choice of variety i as follow:

$$CS = \frac{1}{2}\gamma \int_{i\in\Omega^H} (\hat{q}_i)^2 \, di + \frac{1}{2}\eta \left(\int_{i\in\Omega^H} \hat{q}_i di \right)^2$$

According to Ottaviano et al. [2002], the first term corresponds to the sum of consumer surplus at each variety i, and the second term reflects the variety effect that brings to consumer surplus. To understand the role of the entry in this economy and its impact on the welfare, I follow Bagwell and Lee [2015] to rewrite the above equation in the following way:

$$CS = \underbrace{N_E \times \frac{\gamma}{2} \int_0^{\tilde{c}_D} (q_D(c))^2 \, dG(c)}_{N_E \times \text{Average CS for a variety}} + \underbrace{\frac{(\alpha - \tilde{c}_D)}{2\eta} \left[\alpha - \frac{(k+1)\left(1+\tau^{-k}\right)+1}{(k+2)\left(1+\tau^{-k}\right)} \tilde{c}_D \right]}_{\text{Variety Effect (VE)}}$$

where equation (31) is utilized to express CS in terms of c_D^H to obtain the exact expression of variety effect. The $\tilde{c}_D$ is the domestic cutoff level under symmetry. Therefore the social planner's problem can be further rewritten as:

$$\max_{\{N_E\}} \mathbb{W} \equiv N_E \times \text{Avg.CS} + \text{VE} + \Pi$$

The first order condition related to entry will generate the following condition:

$$\underbrace{\text{Avg. CS} + N_E \frac{\partial \text{Avg. CS}}{\partial N_E} + \frac{\partial \text{VE}}{\partial N_E} + N_E \frac{\partial \bar{\pi}}{\partial N_E}}_{\neq 0 \text{ due quadratic quasi-linear preference}} + \underbrace{\bar{\pi} - f_E}_{\text{Free entry}} = 0 \tag{34}$$

The free entry will only take care of the last item, and that is why it is not guaranteed to deliver the socially desirable level of entry. According to the seminal work[35] by Spence [1976] and Mankiw and Whinston [1986], the first term is positive, representing the average consumer surplus gain from a new variety following entry. The second term is negative, representing the average consumer surplus loss for existing varieties when a new variety becomes available (substitution effect). The third item is positive, representing the variety effect/benefit from a new variety. Lastly, the fourth item is negative, which represents the business-stealing effect since it measures how the new entrant affects the average profit of existing firms. These four items added up together give the externality of firms' entry. In the Appendix B.12, I show that this externality effect is positive when $\tilde{c}_D > \alpha/2$, which means firm entry increases social welfare. In this case, positive import tariff will increase social welfare by encouraging entry. When $\tilde{c}_D < \alpha/2$, however, the sum of these four terms is negative, indicating that firm entry decreases social welfare. In this case, positive import tariff decreases social welfare by introducing more entry. The optimal thing to do in this case is to subsidize trade and discourage firm entry. Only when $\tilde{c}_D = \alpha/2$, the market entry level coincides with the socially optimum entry level.

The result here is different from that in **CD**, where the authors find that the socially optimal tariff in their setting is always a subsidy. The intuition is that opening up to trade will expose domestic firms to the Foreign competition, driving out the least productive firms and reallocating resources to the more

[35]For recent related discussions under heterogeneous firms framework, see Dhingra and Morrow [2019], Weinberger [2015], Bagwell and Lee [2015] and Behrens et al. [2018].

productive firms. When trade barrier is a choice variable, the social planner will have an additional incentive to promote trade since trade-liberalization can boost aggregate productivity. In the current setup, their conclusion only holds when $\tilde{c}_D < \alpha/2$. The fundamental reason for this difference is the deviation from CES preference. Under CES preference, the first four terms in equation (34) always add up to zero. As demonstrated in Dhingra and Morrow [2019], free-entry delivers the first-best outcome. In the current framework, the presence of quadratic quasi-linear preference creates variable elasticity across the varieties, generating multiple externalities in the economy, causing the sum of those four terms to deviate from zero.

More importantly, firm heterogeneity, which does not produce any externality under the CES preference, now becomes a source of inefficiency in this economy. To be more specific, firm heterogeneity is governed by two parameters here: c_M and k. c_M represents the upper bound of the marginal cost distribution. Larger c_M indicates larger region that marginal cost can be drawn from, leading to an increase in firm heterogeneity. k governs the shape of Pareto distribution. When k equals to 1, the marginal cost follows uniform distribution, and different marginal costs can be drawn with equal probability. As k approaches infinity, the marginal cost distribution becomes degenerate at c_M. Therefore, an increase in k means the distribution of firms is skewed toward less productive firms, reducing the degree of firm heterogeneity.

For example, suppose $\tilde{c}_D > \alpha/2$, which means the sum of the first four terms in equation (34) is positive, then an additional entry creates positive externality to the society. In this situation, an increase in c_M or a decrease in k (both represent an increase in firm heterogeneity) will reduce the aggregate externality of firm entry. Therefore, an increase in firm heterogeneity is socially inefficient because it holds back the positive externality of firm entry. However, these two dimensions work quite differently. An increase in c_M will reduce the average consumer surplus, the absolute value of substitution effect, the variety effect, and the absolute value of business-stealing effect. The positive terms (average CS and VE) dominate the negative terms (substitution and business-stealing effect), and therefore externality decreases as c_M increases. A decrease in k has the exact opposite impacts on these four terms. The negative terms dominate the positive terms, and therefore externality decreases as k decreases. The fundamental reason behind this observation can be seen from the solutions of cutoffs and the expressions of these four terms[36]. c_M only affects the share of valid varieties on the market ($\tilde{c}_D/c_M$), but k affects both the share of valid varieties and the average profits. As we will see in the next section, this impact on cutoffs generates important welfare implications of tariff.

6 Socially Optimum Tariff and Nash Tariff

In this section, I first follow the discussion in Section 4 to study the welfare implication of tariff, with an emphasis on the comparison between market outcome and socially optimum outcome. Then I derive the socially optimal tariff and the Nash tariff in this economy, and investigate the forces that result in the differences between them. Toward the end of this section, I compare the Nash tariff with FDI with the Nash tariff without FDI.

[36] Please refer to Appendix B.12.

6.1 Welfare Implication of Tariff

Imposing symmetry and utilizing the setup in Appendix B.10 and the model setup in Section 2, one can write the social welfare in market outcome and socially optimum outcome as functions of the corresponding cutoffs:

$$\mathbb{W}^M = 1 + \bar{q}_0 + \frac{\alpha - c_D^M}{2\eta}\left(\alpha - \frac{k+1}{k+2}c_D^M\right), \qquad \mathbb{W}^O = 1 + \bar{q}_0 + \frac{1}{2\eta}\left(\alpha - c_D^O\right)^2$$

Therefore, the welfare implication of a tariff change can be summarized in the following proposition:

Proposition 11. *(i) Imposing symmetry, if the domestic cutoff level is sufficiently high, i.e., $c_D^M > \alpha(A+B)/2B$[37], then a bilateral increase of tariff is welfare-improving; if the domestic cutoff level is sufficiently low, i.e., $c_D^M < \alpha(A+B)/2B$, then a bilateral increase of tariff is welfare-deteriorating. (ii) Removing symmetry, then a unilateral increase of Home's import tariff is welfare-improving to the Home country, but welfare-deteriorating to the Foreign country.*

Proof. See Appendix B.13. □

The first part of this proposition is directly built on Proposition 9 and 10. On the one hand, when the domestic cutoff is sufficiently high, the market selection is too weak. In this case, the market outcome does not have enough varieties. According to Proposition 7, an increase in import tariff can increase the level of entry, foster market selection, reduce the equilibrium domestic cutoff level, and therefore improve the social welfare. On the other hand, if the domestic cutoff is too low, which means the market selection is too strong, then an increase in import tariff will only make the market selection stronger, deteriorating the social welfare. From another perspective, the reduction in domestic cutoff reduces the prices charged by all the firms, but more so for the more productive firms that charge higher markups. This effect reduces the distortions created by all the externalities mentioned in Proposition 9, thereby improving social welfare.

The second part of this proposition is more straightforward. Based on Proposition 2 and 7, an increase in Home's import tariff increases firm entry, resulting in more equilibrium varieties and lower domestic cutoff. These impacts raise the welfare in the Home country. For the Foreign country, Home's import tariff decreases firm entry, reducing the equilibrium number of varieties and producing a higher domestic cutoff. These impacts deteriorate Foreign country's welfare. With these understandings about the welfare implications of tariff in mind, now I turn to the study of optimal tariffs in this economy.

6.2 Socially Optimal Tariff vs. Nash Tariff

In this subsection, I derive both the socially optimum tariff and Nash tariff. For the socially optimum tariff, due to the symmetric nature of Home and Foreign, I assume the social planner puts identical weight on the welfare of each country. Thus, the socially optimal tariff maximizes the sum of the two countries' consumer utilities:

$$\max_{\{t^H, t^F\}} \mathbb{W} \equiv \mathbb{U}^H + \mathbb{U}^F = I^H + CS^H + I^F + CS^F$$

[37]Please refer Appendix B.13 for the exact expression of A and B.

Recall that the income $I^l \equiv w^l + (t^l - 1) \times IM^l + \Pi^l$ for $l \in \{H, F\}$, and the equilibrium wages in both countries are equal to one. It is straightforward to verify that the optimal level of t^H satisfies the following condition:

$$\frac{\partial \mathbb{W}}{\partial t^H} = \underbrace{IM^H + (t^H - 1) \times \frac{\partial IM^H}{\partial t^H}}_{\text{Effect on } H\text{'s tariff revenue}} + \underbrace{\left(t^F - 1\right) \times \frac{\partial IM^F}{\partial t^H}}_{\text{Effect on } F\text{'s tariff revenue}} + \underbrace{\frac{\partial CS^H}{\partial t^H} + \frac{\partial CS^F}{\partial t^H}}_{\text{Effect on } CS^H \text{ and } CS^F} = 0 \tag{35}$$

From the socially optimum perspective, the social planner needs to consider Home import tariff's impact on the tariff revenue and consumer surplus in both countries. In contrast, the Nash tariff level for H only focuses on the tariff revenue and consumer surplus of its own country. It is defined as follow:

$$\max_{\{t^H\}} \mathbb{U}^H = I^H + CS^H$$

It is easy to show that the optimal non-cooperative tariff level should satisfy:

$$\frac{\partial \mathbb{U}^H}{\partial t^H} = \underbrace{IM^H + (t^H - 1) \times \frac{\partial IM^H}{\partial t^H}}_{\text{Effect on } H\text{'s tariff revenue}} + \underbrace{\frac{\partial CS^H}{\partial t^H}}_{\text{Effect on } CS^H} = 0 \tag{36}$$

Combining equation (35) and (36), and if we focus on one of the cases in Proposition 10, $\tilde{c}_D > \alpha/2$, which implies the socially optimal import tariff is greater than one, it then can be shown that the Nash tariff level for country H satisfies the following proposition:

Proposition 12. *When the symmetric domestic cost cutoff is sufficiently high ($\tilde{c}_D > \alpha/2$), the Nash tariff (t_N) is higher than the socially optimal tariff (t_S).*

Proof. See Appendix B.14. □

This finding is similar to the Proposition 2 in **CD**, but it is established in the **MO** framework with the presence of FDI and under a particular equilibrium outcome. We can obtain the intuitions from several different angles. To understand the incentive of Home's import tariff, let us investigate F's free-entry condition, which can be expressed in a similar fashion as in equation (22):

$$\underbrace{\left(c_D^F\right)^{k+2}}_{\uparrow \text{in } t^H} + \underbrace{\Phi_1^H}_{\downarrow \text{in } t^H} \underbrace{\left(c_D^H\right)^{k+2}}_{\downarrow \text{in } t^H} + \underbrace{\Phi_2^H}_{\uparrow \text{in } t^H} \underbrace{\left(c_D^H\right)^{k+2}}_{\downarrow \text{in } t^H} = \gamma\phi$$

where the first term on the left represents the expected profit of being a domestic producer in F, the second term is the expected profit of being an exporter in F, and the third term is the expected profit of being multinational firm in F. When H country sets its tariff, according to Proposition 4–6, F's exporter cutoff level decreases in t^H. Based on the earlier proofs, Φ_1^H decreases in t^H, and Φ_2^H increases in t^H. So if t^H increases, the expected profit of an exporter in F goes down. In fact, the sum of the expected profit of exporter and multinational in F also goes down. When H sets its unilateral optimal tariff, it ignores the impact of its tariff on F's exporter and multinational. Therefore, the tariff level that solves (36) will be negative when evaluated at (35), implying H will set a higher tariff than the social planner would choose.

In **CD**, the terms of trade effect is not present for two reasons: (i) Pre-tariff import prices do not change due to the fixed markup over a constant wage. (ii) The quasi-linear utility pushes domestic and overseas income changes onto the numéraire good, leaving the profits from Home exporters or multinationals unaffected by Home's import tariff. In the current setting, neither of these two reasons is valid: (i) Pre-tariff import prices do change due to the variable markups responding to tariff change. (ii) Profits from the Foreign exporters or multinationals do depend on the relevant cutoffs, which are all affected by Home's import tariff level. To understand the terms of trade effect in the current setup, one can easily obtain the following conditions between average prices and their corresponding cutoffs:

$$\bar{p}^H = \frac{2k+1}{2k+2} c_D^H, \;\; \bar{p}^F = \frac{2k+1}{2k+2} c_D^F$$

Also notice the IM and CS in equation (35) and (36) are all functions of c_D^H and c_D^F, so we can rewrite the welfare function and the first-order conditions in terms of $\bar{p}^H$ and $\bar{p}^F$. Based on Proposition 4, it is easy to verify that an increase in Home's import tariff generates a terms of trade gain for itself at the cost of Foreign's terms of trade deterioration. It is evident that the incentive to manipulate the terms of trade also results in the inefficiency of Nash tariff.

6.3 Nash Tariff with and without FDI

In this subsection, I compare the symmetric Nash tariff when FDI is an option with the case when it is not. Due to the quadratic quasi-linear preference and the numéraire good, there is no closed-form analytical solution for the socially optimal tariff and the Nash tariff. Therefore, I numerically compute the tariff levels based on equations (35) and (36) in Mathematica. I follow Behrens et al. [2011] in choosing the parameter values, which are listed in **Table 1**. To focus on the role of FDI, here I fix the parameters that affect the degree of firm heterogeneity (k and c_M)[38].

Table 1: Paramaterization

α	12	Relative preferences toward the differentiated varieties
η	0.1	Substitutability among the varieties
c_M	5	Upper bound of marginal cost draw in Pareto distribution
γ	0.6	Degree of love for variety
φ	1.9	Iceberg-type efficiency loss of FDI
τ	1.1	Iceberg-type transportation cost
f_E	0.1	Fixed cost of entry

Note: These parameters are the baseline values in Behrens et al. [2011]. All of the computations performed in this paper are based on this table. For the comparative statics in Section 6 and 7, some parameters are varied around the benchmark value listed above.

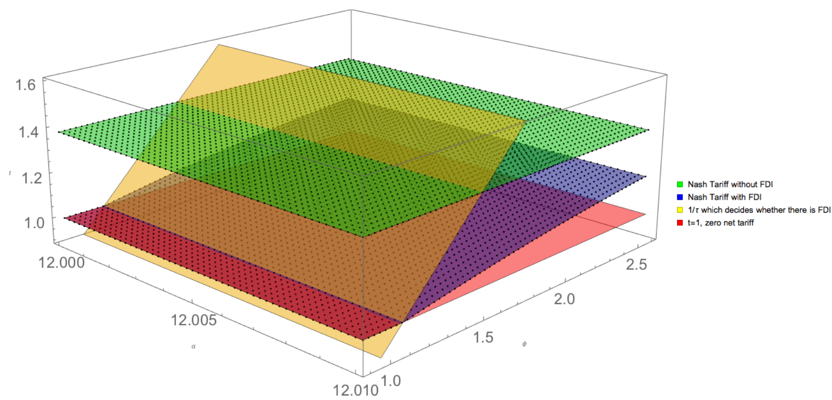
For illustration purpose, I plot the computed Nash tariff level as a function of φ and α[39]. When the degree of firm heterogeneity is fixed, the Nash tariff levels are plotted in **Figure 5**. The yellow plane separates the space: the area above indicates no FDI activity, and the area below indicates where FDI occurs. Since α is

[38] I will discuss the role of heterogeneity and its interaction with FDI in Section 7. In that case, k and c_M will be varied.

[39] To focus on one case, here α is chosen to be small enough so that the optimal tariff level will be greater than 1.

chosen such that the optimal tariff when FDI occurs is greater than one, it then makes sense that the blue plane is in-between the yellow plane and the red plane.

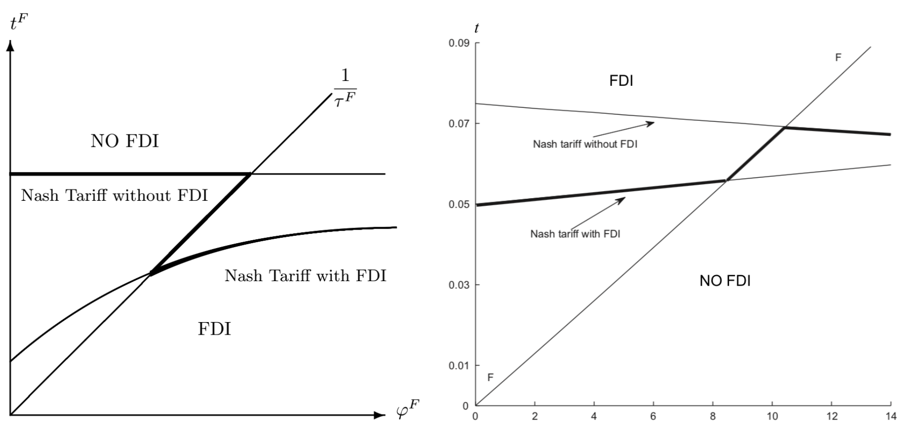
Figure 5: Three-dimensional Nash Tariff with and without FDI



Note: The graph is computed based on the parameter values in **Table 1**. The green plane plots the Nash tariff without FDI and the blue plane plots the Nash tariff with FDI. The yellow plane separates the space into FDI region and no FDI region. The red plane indicates zero net tariff.

In **Figure 6**, I plot a two-dimensional version of **Figure 5**, and contrast it with the Figure 4 in **CD**. First of all, given the current parameter choice, the Nash tariff without FDI is always higher than the one with FDI. This confirms the finding in **CD**. On the one hand, the gain from implementing tariff is smaller due to the tariff-jumping multinationals. On the other hand, based on Corollary 1, the presence of FDI causes the domestic cutoff to have a bigger response to tariff change, which affects the consumer surplus component in equation (36). These two channels collectively result in the lower Nash tariff level when FDI is present.

Figure 6: Two-dimensional Nash Tariff with and without FDI



Note: The left panel is a two-dimensional graph of **Figure 5**. The vertical axis indicates the tariff level, the horizontal axis indicates the easiness of doing FDI, and the $1/\tau^F$ separates the plane into No FDI region and FDI region. The right panel is taken from **CD**. The vertical axis stands for net tariff, the horizontal axis represents the exogeneous component of the fixed cost for all modes of production, and the FF line represents the combination of τ and λ that induces FDI. The bold lines indicate the path of corner solution.

Second, φ is in a similar position as the fixed cost parameter (λ) in **CD**, but they have notable differences. φ does not affect the Nash tariff without FDI because it does not have any impact on exporters when FDI is not an option. The fixed cost parameter affects the Nash tariff level, regardless of the presence of FDI. The reason is that both the fixed costs of export and FDI are affected by λ. Hence, the change of λ will have a direct impact on the tariff level. In the current framework, the change in φ will affect the tariff level only when FDI occurs.

Third, as φ increases, the Nash tariff with FDI increases, and gets closer to the Nash Tariff without FDI. One the one hand, if φ approaches infinity, then the FDI cutoff will be zero, indicating Foreign firms only access Home country through exports. Hence, the Nash tariff level returns to the Nash tariff without FDI case. On the other hand, when FDI is an option, Nash tariff level increases in φ. This is similar to the results in **CD** regarding λ: a higher φ reduces the cutoff of multinational (c_{FDI}), causing the least productive multinationals to switch to export, increasing the tariff base, and hence increasing the incentive of imposing a higher tariff. If φ is sufficiently high, FDI will occur in the equilibrium, confirming the corner solution finding in **CD**. As we will see in the next subsection, the interaction of φ and the degree of firm heterogeneity generates very interesting policy implications that would have been missing in the CES framework.

7 Role of Variable Markup

This section is dedicated to the discussion of variable markups. Under quadratic quasi-linear preference, firms with different marginal costs can charge different markups[40]. This feature not only enables the tariff to affect the entire distribution of markups, but also generates misallocation in the economy. As we will see toward the end of this section, the interaction between variable markups and FDI unveils important trade policy implications, which would be otherwise absent in the CES framework.

7.1 Average Markup

In the current setup, a movement in iceberg trade cost (τ) does not affect the average markup[41] due to the assumption of Pareto cost distribution. However, the *ad valorem* tariff does have the ability to affect the average markup. Note that all the operating firms (N_D^H) serve their domestic market, and on top of that, there are Foreign exporters (N_X^F) and multinationals (N_{FDI}^F). The average markup of all the firms in country H can be expressed as the following:

$$\bar{m}^H = \frac{1}{N_D^H + N_X^F + N_{FDI}^F}\left[N_D^H \int_0^{c_D^H} \frac{m_D^H(c)\, dG(c)}{G\left(c_D^H\right)} + N_X^F \int_{c_{FDI}^F}^{c_X^F} \frac{m_X^F(c)\, dG(c)}{G\left(c_X^F\right)} + N_{FDI}^F \int_0^{c_{FDI}^F} \frac{m_{FDI}^F(c)\, dG(c)}{G\left(c_{FDI}^F\right)}\right] \tag{37}$$

To simplify the analysis, here I focus on the symmetric case. This is similar to the bilateral liberalization

[40]There are other ways to generate variable markups, as discussed in Ding [2020], so the implications of variable markups discussed here might not be universal. For example, if variable markups is generated through firms engaging in Bertrand competition, then the implication will be certainly different from what we obtain here.

[41]See **MO** Section 3.2.

studied in Section 4.1[42] in **MO**. After imposing symmetry, the average markup can be rewritten as follow (for detailed derivation, see Appendix B.15):

$$\bar{m} = \underbrace{\frac{1}{1+(t\tau)^{-k}} \times \frac{2k-1}{2k-2}}_{\text{weighted expected markup in domestic}} + \underbrace{\underbrace{\frac{(t\tau)^{-k}-\xi^k}{1+(t\tau)^{-k}}}_{\text{share of Foreign exporters}} \times t \underbrace{\left\{\frac{1}{2}\left[1-(t\tau\xi)^k\right] + \frac{k}{2k-2}\left[1-(t\tau\xi)^{k-1}\right]\right\}}_{\text{expected markup of Foreign exporter}}}_{\text{weighted expected markup of Foreign exporters}}$$
$$+ \underbrace{\underbrace{\frac{\xi^k}{1+(t\tau)^{-k}}}_{\text{share of foreign FDI}} \times \underbrace{\left(\frac{k}{2k-2}\frac{1}{\varphi\xi} + \frac{1}{2}\right)}_{\text{expected markup of Foreign FDI}}}_{\text{weighted expected markup of Foreign FDI}}$$

Based on this expression, I obtain the following proposition regarding the impact of a tariff change on the average markup:

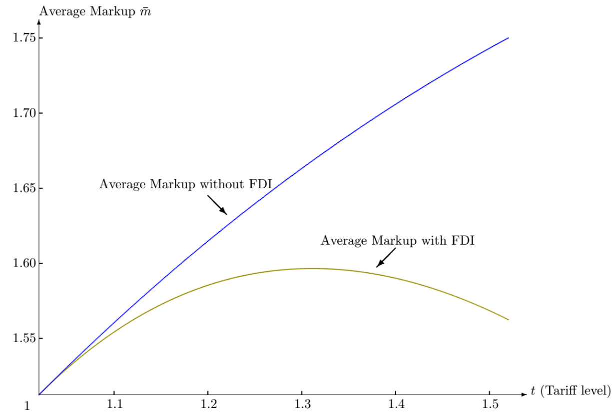
Proposition 13. *If the level of protection is high, the increase of tariff-jumping Foreign multinational firms, which creates downward pressure on average markup, can dominate the decrease of Foreign exporter firms, which creates upward pressure on average markup. The average markup in the economy decreases as protection level increases. Therefore, protectionist trade policy can end up reducing Home market's average markup.*

Proof. See Appendix B.15. □

As t increases, the weighted expected markup from domestic firms (the first term) increases. This relation is due to the fact that protection reduces the degree of competition and makes it easier for domestic firms to survive. As a result, the expected markup will increase. The weighted expected markup from Foreign exporters (the second term) will decrease as t increases. This effect is due to two channels: the decreasing share of Foreign exporters (extensive margin, based on Proposition 5 and 6, and the expected markup, which increases first and then decreases as t increases (intensive margin, based on Proposition 2). The weighted expected markup of Foreign FDI (the third term) will increase as t increases. This relation also comes from two channels, the increasing share of Foreign FDI (extensive margin, based on Proposition 6) and the increasing expected markup (intensive margin, based on Proposition 2). The first and third term will dominate the second term at the beginning, but as t increases, the second term will eventually dominate the other two terms, dragging down the average markup. For illustration purpose, the average markup without FDI and the average markup with FDI are plotted in **Figure 7**.

[42]Note, different from the long-run results established in Section 3.2, bilateral reduction in tariff delivers the same results as in the short-run case: liberalization increases competition and decreases the domestic cutoff level, making it harder for a firm to survive.

Figure 7: Average Markup with and without FDI



Note: All the other parameter values are taken from **Table 1**. The blue curve indicates the weighted average markup without FDI, whereas the green curve indicates the weighted average markup with FDI.

Edmond et al. [2015] suggest that under certain conditions, a reduction in trade barriers (iceberg-type trade costs) can lead to lower domestic markups (as Home producers lose their market share). Combined with higher markups on imported goods (as Foreign producers gain market share), the overall markup dispersion increases and the misallocation in the economy becomes worse. In this case, the pro-competitive gains from trade would be negative. In the current framework, a similar result is found when FDI is an option: the average markup can go up when the tariff level reduces. As the tariff level drops, although the number of imported varieties increases (hence exerting a downward pressure on average markup), the exiting of multinationals (which reduces the competition in the domestic market and exerts an upward pressure on average markup) also contributes to the increase in average markup.

7.2 Misallocation

Misallocation is a byproduct of all the inefficiencies discussed in Section 4. To gain more insight on the role of variable markups, in this subsection, I follow the approaches developed in Arkolakis et al. [2018] and Hsieh and Klenow [2009] to explore the misallocation in the current economy[43].

According to Arkolakis et al. [2018], variable markups can create a new source of gain or loss from trade liberalization, depending on whether low-cost firms, which charge high markups and under-supply their varieties, end up growing in size. According to their Appendix A.4, the effect of trade liberalization on the welfare of country j depends on two things: (i) the sign of the covariance of the markup, charged by a firm in country j that produces the variety for market i, and (ii) a change in its labor share that is needed to produce the variety for that market. These two forces can be expressed through the following term:

$$\operatorname{cov}\left(m^{i}\left(\omega\right), \frac{dl^{i}\left(\omega\right)}{L^{j}}\right) \tag{38}$$

where $l^i(\omega)$ is the total employment associated with a production of variety ω in country j for sales in

[43]These are the two most prominent papers on misallocation.

country i. In other words, if this covariance is positive, then trade liberalization has an additional positive effect on welfare in country j through a reduction in misallocation. In their setup, without considering the choice of FDI, equation (38) becomes:

$$\text{cov}\left(m^i(\omega), \frac{dl^i(\omega)}{L^j}\right) = N_D^H \int_0^{c_D^H} \frac{p_D^H(c)}{c} \frac{d\left[cq_D^H(c)\right]}{L^H} \frac{dG(c)}{G\left(c_D^H\right)} + N_X^H \int_0^{c_X^H} \frac{p_X^H(c)}{\tau^F c} \frac{d\left[c\tau^F q_X^H(c)\right]}{L^H} \frac{dG(c)}{G\left(c_X^H\right)}$$

It is important to notice that this covariance is at the firm-level. Therefore, it relates to not only firm's domestic production decision but also its export decision. In their setting, this covariance is negative, so the presence of variable markups reduces the welfare gain from trade. This negative effect is present because a decrease in trade costs makes exporting firms relatively more productive, leading to changes in markups. When demand is log-concave, as in Krugman [1979], higher markups imply incomplete pass-through of changes in marginal costs to prices, lowering the welfare gains from trade.

The covariance term in the current setup is:

$$\begin{aligned}\text{cov}\left(m^i(\omega), \frac{dl^i(\omega)}{L^j}\right) = {} & N_D^H \int_0^{c_D^H} \frac{p_D^H(c)}{c} \frac{d\left[cq_D^H(c)\right]}{L^H} \frac{dG(c)}{G\left(c_D^H\right)} + N_X^H \int_{c_{FDI}^H}^{c_X^H} \frac{p_X^H(c)}{\tau^F c} \frac{d\left[c\tau^F q_X^H(c)\right]}{L^H} \frac{dG(c)}{G\left(c_X^H\right)} \\ & + N_{FDI}^H \int_0^{c_{FDI}^H} \frac{p_{FDI}^H(c)}{\varphi^F c} \frac{d\left[c\varphi^F q_{FDI}^H(c)\right]}{L^H} \frac{dG(c)}{G\left(c_{FDI}^H\right)}\end{aligned}$$

which is different from the one in Arkolakis et al. [2018]. First, trade liberalization takes the form of a tariff reduction in this economy. Second, the covariance has an additional item due to the choice of FDI, which means now the welfare implication of a change in tariff also depends on firm's FDI activity. Third, as discussed in their June 2012 working paper[44], with quadratic quasi-linear preference, the change in welfare depends on the substitutability between the homogeneous good and the differentiated goods. In the current framework, the substitutability is also affected by FDI. In Appendix B.16, I analytically derive the covariance term under symmetry:

$$\begin{aligned}\text{cov}\left(m^i(\omega), \frac{dl^i(\omega)}{L^j}\right) = {} & \frac{\left(\alpha - c_D^M\right) dc_D^M}{2\eta\left(1 + t^{-k}\tau^{-k}\right)} \left\{2k + 1 + \left[(t\tau)^{-k} - \xi^k\right]\right. \\ & \left. \times \left[2k + 1 - k(1 - t\tau\xi)(t\tau\xi)^k - (t\tau\xi)^k\right] + \xi^k (k + k\varphi + 1)\right\}\end{aligned}$$

And I show that the covariance term is *positive*, indicating a reduction in misallocation through protection. As discussed earlier, when c_D^M is sufficiently high, an increase in tariff is welfare-improving. The presence of variable markup and FDI results in a positive covariance term between the firm-level markup and change in firm-level employment share. This effect means the welfare gain from protection is even larger due to the reduction in misallocation. Intuitively, an increase in tariff will decrease the relative demand for high-cost varieties, and labor will be reallocated toward the low-cost varieties, which include those produced by Foreign FDI firms. Therefore, misallocation is reduced since the market becomes more concentrated, generating a positive correlation between markups and the labor share, and hence increasing the gains from the change in tariff.

Alternatively, we can also follow the way that Hsieh and Klenow [2009] introduce misallocation. For

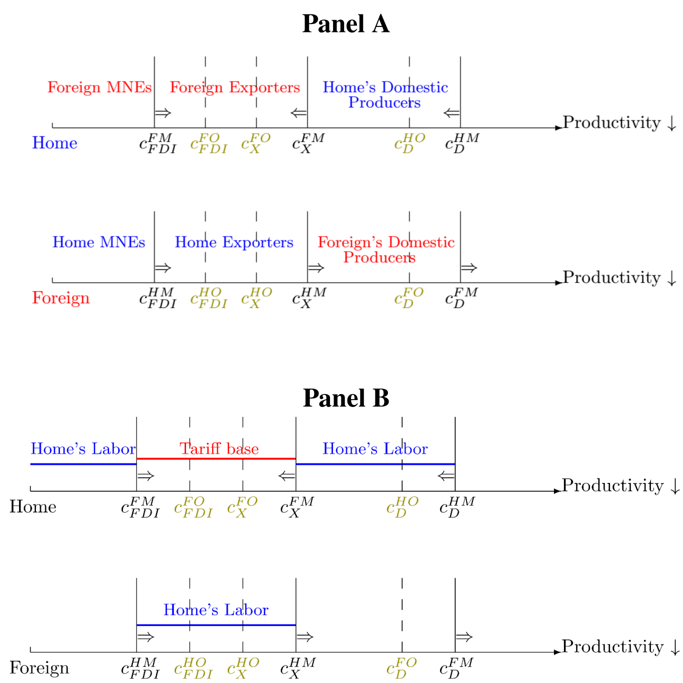
[44] http://www.econ.uzh.ch/dam/jcr:00000000-0db7-f8ad-0000-00005b2a6145/Arkolakis_Costas_The_Elusive_Pro_Competitive_Effects_of_Trade.pdf.

illustration purpose, here I focus on the exporters. A Home exporter with marginal cost c has the following corresponding $TFPR$:

$$TFPR_X^{HO}(c) \equiv \frac{p_X^{HO}(c)}{c} = \frac{\tau^F c}{c} = \tau^F, \quad TFPR_X^{HM}(c) \equiv \frac{p_X^{HM}(c)}{c} = \frac{c_D^{FM}/c + \tau^F t^F}{2}$$

In the planner's economy, $TFPR_X^{HO}$ is the same for all the exporters, and there is no misallocation in this case. However, in the market outcome, an exporter with lower marginal cost will have a bigger $TFPR$, implying that the low cost firms are allocated with too little labor. This is consistent with the conclusion in Proposition 9. Hence, misallocation also exists in the market outcome according to the definition in Hsieh and Klenow [2009]. Based on Proposition 4, an increase in t^H will increase $TFPR_X^{HM}$, exacerbating the misallocation among Home's exporters.

Figure 8: Productivity Cutoffs Comparison between Market Outcome and Socially Optimum Outcome when Home's Tariff Increases



Note: Panel A and Panel B are the same plots with different notations. Each axis indicates the varieties that are available within that country. For example, the first axis indicates Home country has varieties from Foreign MNEs, Foreign exporters, and all the Home firms that serve their domestic market. Panel A focuses on the composition of firms, whereas Panel B focuses on the utilization of labor.

Figure 8 is an attempt to present all the misallocation concepts on the graph. Both panels are demonstrating the responses of cutoffs to an increase in Home's import tariff. The differences between socially optimum outcome and market outcome in Proposition 9 are revealed in the cutoff levels. The positive covariance term in equation (38) can be seen from Panel B: following the increase of t^H, c_{FDI}^{FM} and c_D^{HM} both move toward their socially optimum level, indicating that Home's labor are reallocated toward the more productive firms, and hence misallocation is reduced. Finally, the increase of $TFPR_X^{HM}$ can be seen from the widening gap between c_X^{HO} and c_X^{HM}.

7.3 Interaction of Variable Markup and FDI

In this subsection, I focus on the interaction of variable markups and FDI through the lens of firm heterogeneity. For the same reason in Section 6, the analysis in this subsection is based on numerical computation. All the relevant parameter values are chosen from **Table 1**. As discussed in Section 4, the quadratic quasi-linear preference makes firm heterogeneity an additional source of inefficiency in this economy. To see how the interaction of variable markup and FDI affects the Nash tariff level, I focus on the two parameter values that govern the degree of firm heterogeneity: k and c_M. It is also helpful to keep in mind that a country cares for tariff revenue and its consumer surplus, as can be seen from the following objective function:

$$\max_{\{t^H\}} \mathbb{U}^H = I^H + CS^H = 1 + TR^H + CS^H$$

The Nash tariff level is implied by the following first order condition:

$$\frac{\partial TR^H}{\partial t^H} + \frac{\partial CS^H}{\partial t^H} = 0 \tag{39}$$

The Role of k

The impact of k can be seen from **Figure 9** and **Figure 10** in the Appendix. k governs the shape of Pareto distribution. An increase in k means the distribution of firms is skewed toward less productive firms, reducing the degree of firm heterogeneity.

With the current parameter values, the Nash tariff without FDI decreases in k, which means it increases in the degree of firm heterogeneity. As k increases, two things are happening at the same time: (i) The domestic cutoff level is increasing. (ii) The probability of getting the low marginal cost draws are shrinking. The first channel will affect all cutoff levels proportionally, without changing the relative distribution of firms. The second channel changes the relative distribution of firms, and this channel is particularly important when multinational firms are present.

When FDI is an option, the impact of k on Nash tariff level depends on the size of φ, the parameter that measures the efficiency loss of FDI. First, an increase in k increases domestic cutoff and lowers the consumer surplus, so the second term in equation (39) is positive. This implies that the first term in equation (39) must be negative. When φ is small, the Nash tariff increases in k, i.e., decreases in firm heterogeneity. Intuitively, when φ is small, many firms access the Foreign market through FDI, so the tariff base is relatively small. In this case, an increase in k lowers the average probability of getting a low marginal cost draw. To maintain the first order condition in equation (39), the country needs to charge a higher tariff. This effect can be seen from the bottom panel in **Figure 10**: when φ is small, the blue plane (bigger k) is above the green plane (smaller k).

When φ is large, however, the Nash tariff decreases in k, i.e., increases in firm heterogeneity. This is because when φ is large, many firms will choose to access the Foreign market through export, so the tariff base is relatively big. In this case, although an increase in k lowers the average probability of getting a low marginal cost draw, the sizable tariff base is big enough to maintain the first order condition in equation (39). Therefore, the Nash tariff level is lower. As shown in the bottom panel in **Figure 10**: when φ is big, the blue plane (bigger k) is below the green plane (smaller k).

The Role of c_M

The impact of k can be seen from **Figure 11** and **Figure 12** in the Appendix. c_M represents the lower bound of the marginal cost distribution. An increase in c_M expands the region that marginal cost can be drawn from, raising up the degree of firm heterogeneity.

With the current parameter values, when FDI is an option, the Nash tariff decreases in c_M, which means it decreases in the degree of firm heterogeneity. To understand the intuition, recall that an increase in c_M expands the lower bound of cost draws, which eventually results in higher cutoff levels[45] The impact is uniform to all the firms, without changing the relative distributions of firms. This is why in **Figure 12**, the green plane (smaller c_M) is always above the blue plane (bigger c_M), i.e., the relative position of Nash tariff levels under different c_M remains the same. There is no interaction between variable markup (induced by c_M) and FDI (measured by the size of φ).

In summary, both k and c_M affect the degree of firm heterogeneity in the economy. While both of them affect the equilibrium cutoff levels, k also alters the relative distribution of firms with different marginal costs. When FDI is an option, the freeness of doing FDI interacts with k, generating a novel implication for trade policy. If the degree of firm heterogeneity is big (smaller k), reducing the FDI barrier (smaller φ) can effectively lower the Nash tariff level. If the degree of firm heterogeneity is small (bigger k), increasing the FDI barrier (bigger φ) can effectively lower the Nash tariff level.

8 Concluding Remarks

In this paper, I introduce *ad valorem* tariff and horizontal FDI into the Melitz and Ottaviano [2008] model, and study the welfare implication of tariff and optimal tariffs in this framework. The conclusions can be broadly summarized as follows. First, firm entry level is endogenously affected by tariff, hence producing different equilibrium features in the short-run versus long-run. Second, quadratic quasi-linear preference generates multiple externalities in this economy, causing market outcome to differ from the socially optimum outcome systematically. Permitting FDI lowers the domestic cutoff levels and reduces the misallocation in the economy. Third, free trade is not always socially optimal. If the domestic cutoff is sufficiently high, an additional firm entry can improve social welfare. In this case, a positive import tariff is welfare-improving because it encourages firm entry. Lastly, I find that the interaction of variable markup and FDI generates novel trade policy insights that are absent if consumers are under CES preference.

Given these results, there are several interesting questions to ask. First of all, do the trade policy results still hold under an alternative demand or supply structure that generates variable markups? I suspect that the alternative demand structures will produce similar results, but different supply-side structures may generate different outcomes. Second, the presence of numéraire good is a blessing and a curse. It would

[45] This can be easily verified through the closed-form solution of c_D, c_X, and c_{FDI} in Section 2.

be interesting to drop the numéraire good and endogenize the wage as in Arkolakis [2008]. It is possible to obtain a closed-form solution of the Nash tariff, as shown by Demidova [2017], and investigate the trade policy implication regarding the labor market in the presence of FDI. Lastly, the trade policy implications here primarily focus on the import tariff. It would be interesting and relevant to study other forms of trade policy, such as export subsidy or corporate taxes.

This paper provides evidence that the interaction of variable markups and FDI generates interesting trade policy implications. The steady-state analysis here, however, might produce very different tariff levels than the actual tariff levels observed in the data. In the light of Larch and Lechthaler [2013], long-run and short-run effects of tariffs may run in opposite directions, implying that an exclusive focus on the steady-state could lead to biased policy conclusions. Carefully disentangling the dynamic effects of tariffs is undoubtedly a fruitful area for future research in the era of globalization .

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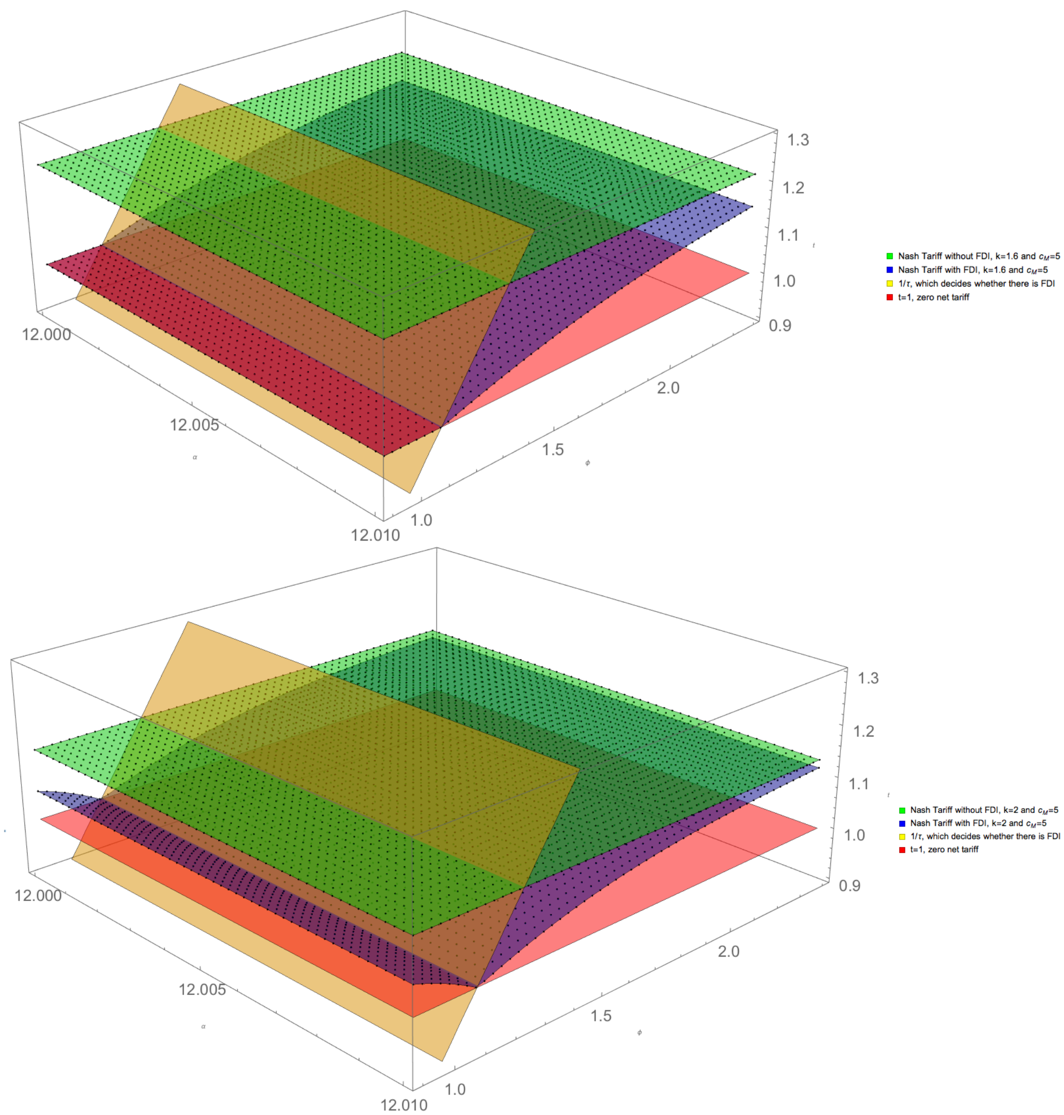
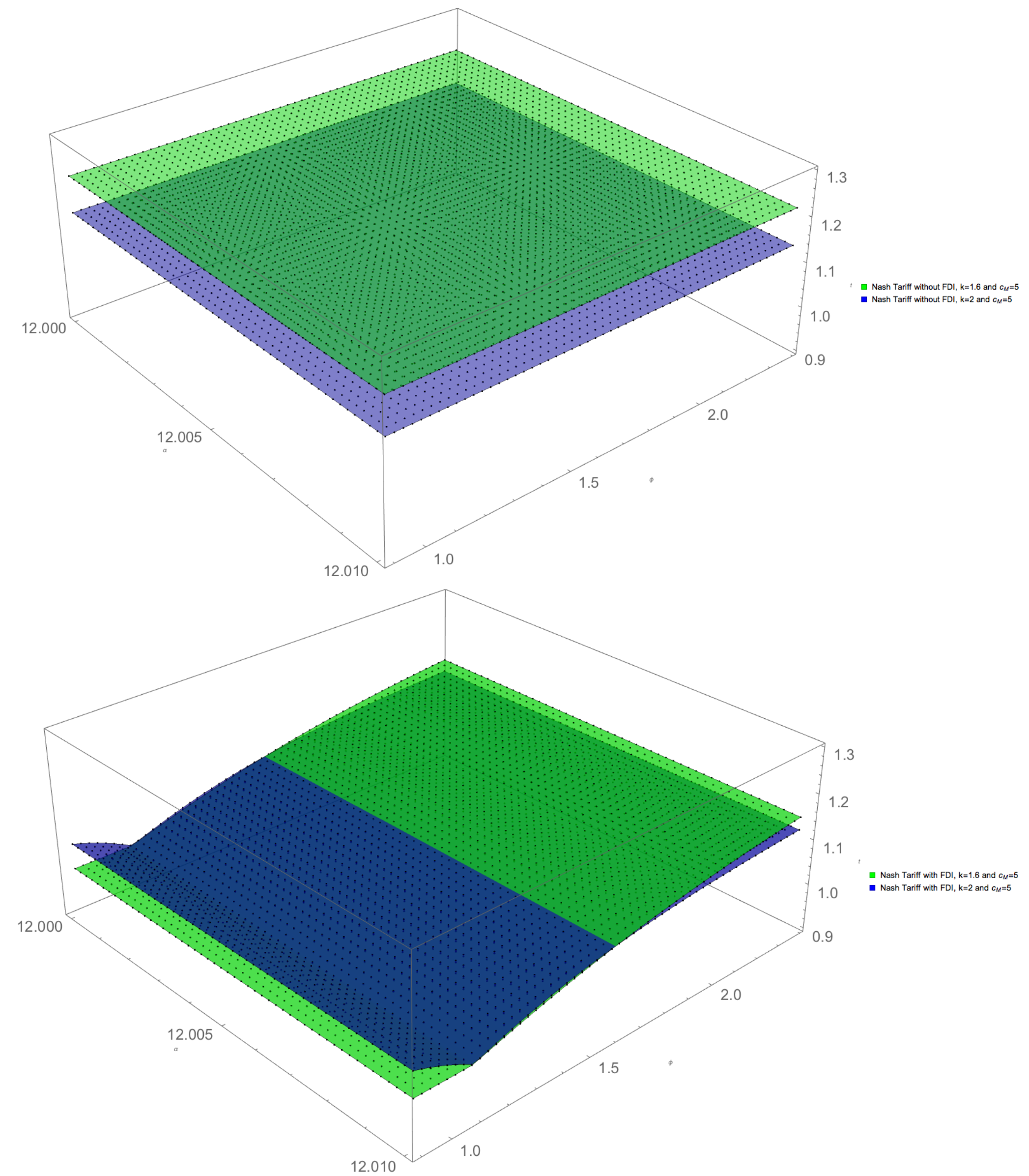
A Figures

Figure 9: The Impact of k on the Nash Tariff: Part I



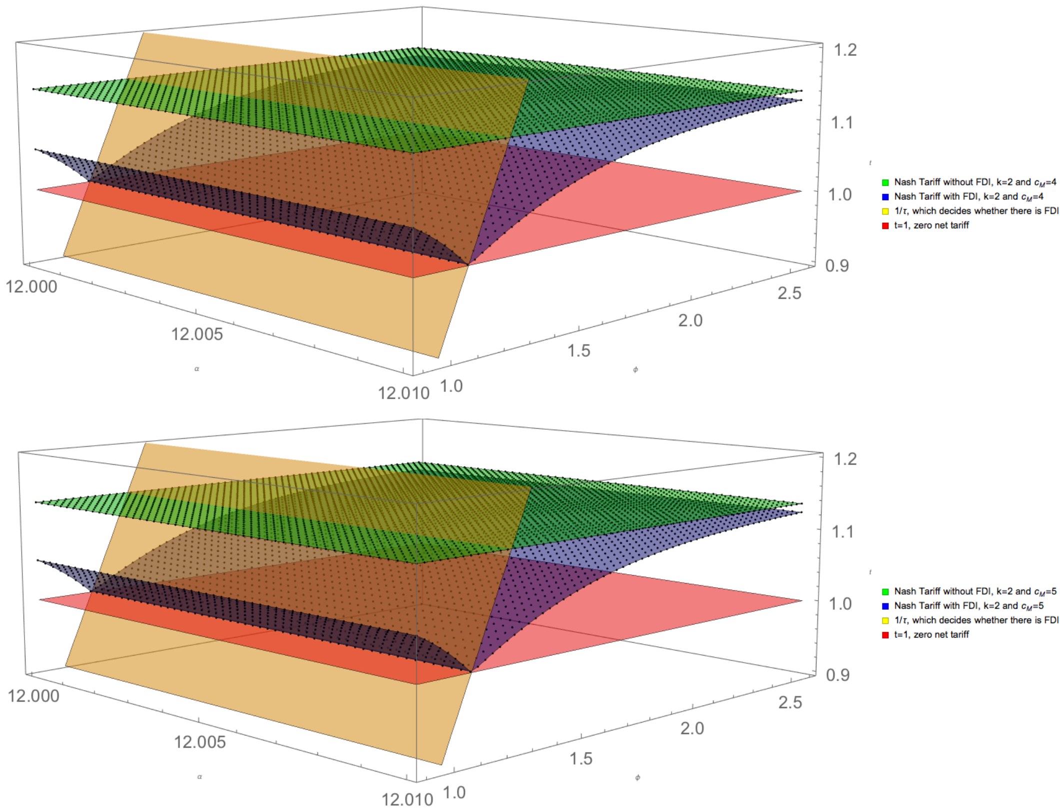
Note: The top panel plots the Nash tariff without FDI and the Nash tariff with FDI when $k = 1.6$. The bottom panel plots the Nash tariff without FDI and the Nash tariff with FDI when $k = 2$. The yellow plane separates the space into FDI region and no FDI region. The red plane indicates zero net tariff.

Figure 10: The Impact of k on the Nash Tariff: Part II



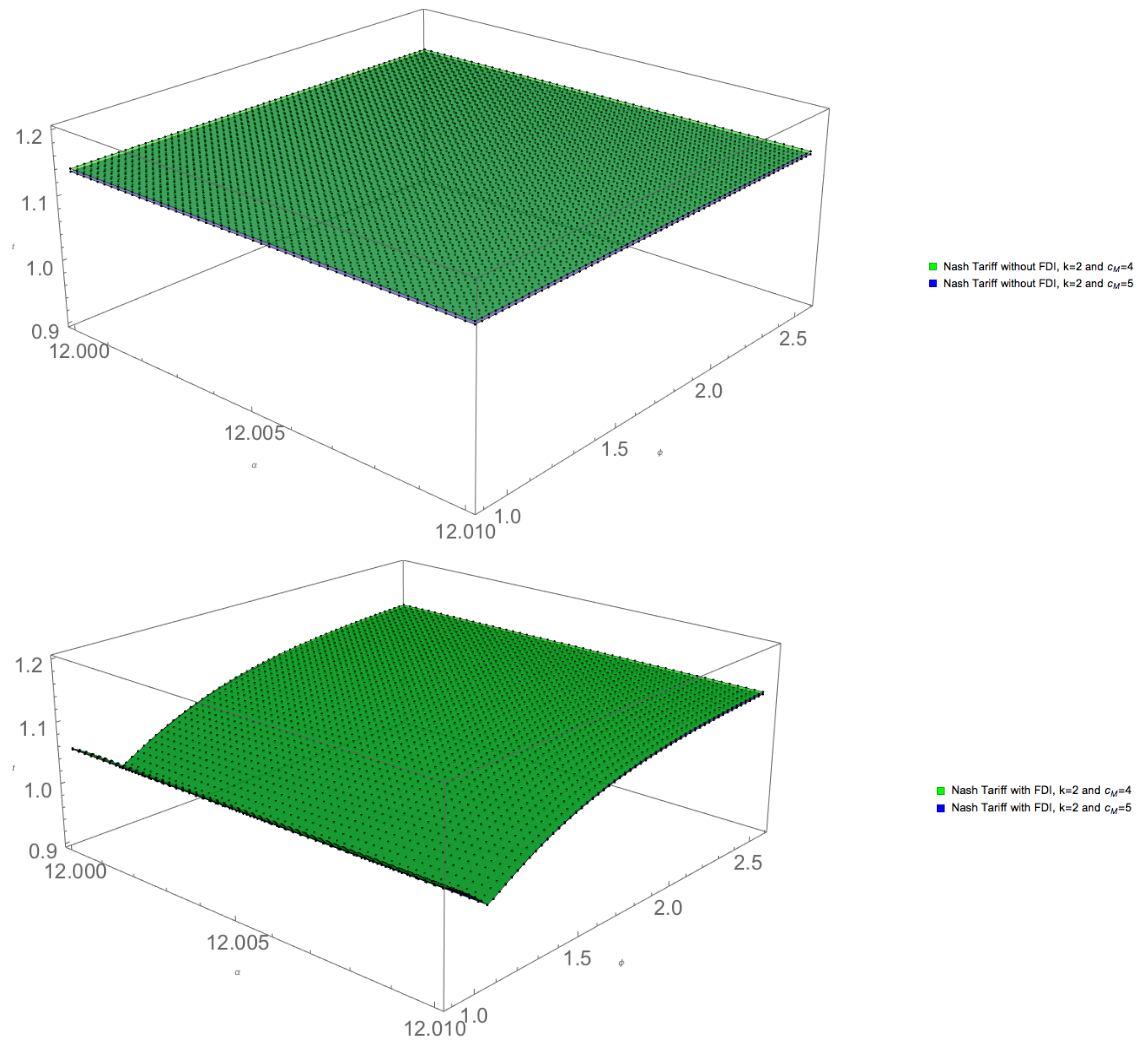
Note: These graphs are extracted from **Figure 9**. The top panel plots the Nash tariff level without FDI. The green plane, which has a lower k value, stays entirely above the blue plane. The bottom panel plots the Nash tariff level with FDI. In the region where FDI occurs, the green plane, which has a lower k value, stays above the blue plane when φ is big, but stays below the blue plane when φ is small. This graph shows the interaction of FDI and k does affect the Nash tariff level.

Figure 11: The Impact of c_M on the Nash Tariff: Part I



Note: The top panel plots the Nash tariff without FDI and the Nash tariff with FDI when $c_M = 4$. The bottom panel plots the Nash tariff without FDI and the Nash tariff with FDI when $c_M = 5$. The yellow plane separates the space into FDI region and no FDI region. The red plane indicates zero net tariff.

Figure 12: The Impact of c_M on the Nash Tariff: Part II



Note: These graphs are extracted from **Figure 11**. The top panel plots the Nash tariff level without FDI. The green plane, which has a lower c_M value, stays entirely above the blue plane. The bottom panel plots the Nash tariff level with FDI. In the region where FDI occurs, the green plane, which has a lower c_M value, also stays entirely above the blue plane. This graph shows the interaction of FDI and c_M does not change the relative position of Nash tariff.

B Proofs

B.1 Proof of Proposition 1

Proof: The proof mainly comes from the following equation:

$$\frac{\alpha - c_D^H}{\left(c_D^H\right)^{k+1}} = \frac{\eta}{2\left(k+1\right)\gamma}\left\{\frac{\bar{N}_I^H}{\bar{c}_M^k} + \left[\left(\frac{1}{t^H \tau^H}\right)^k - \left(\xi^H\right)^k\right]\bar{N}_I^F + \left(\xi^H\right)^k \bar{N}_I^F\right\}$$

It is straightforward to show that $\partial \xi^H / \partial t^H > 0$, therefore the whole expression on the right-hand side will decrease as t^H increases. It follows from the equation that it must be true that c_D^H will increase, hence $\partial c_D^H / \partial t^H > 0$ in the short-run. □

B.2 Proof of Proposition 2

Proof: To prove this proposition, it is helpful to rewrite the markups as follows:

$$\begin{aligned} m_D^H\left(c\right) &= \frac{1}{2c}\left(c_D^H + c\right) \\ m_X^F\left(c\right) &= \frac{t^H}{2c}\left(c_X^F + c\right) = \frac{t^H}{2c}\left(c_D^H / t^H \tau^H + c\right) = \frac{1}{2c}\left(c_D^H / \tau^H + c/t^H\right) \\ m_{FDI}^F\left(c\right) &= \frac{1}{2\varphi^H c}\left(c_D^H + \varphi^H c\right) \end{aligned}$$

It follows from Proposition 1 that $\partial m_D^H / \partial t^H > 0$ and $\partial m_{FDI}^F / \partial t^H > 0$. The responses of m_X^F is less transparent. On the one hand, c_D^H / τ^H increases as t^H increases. On the other hand, c/t^H decreases as t^H increases. The total impact on m_X^F is therefore ambiguous. If c is small, then the first effect will dominate the second effect, $\partial m_X^F / \partial t^H > 0$. If c is big, then the second effect will dominate the first effect, $\partial m_X^F / \partial t^H < 0$. □

B.3 Proof of Proposition 3

Proof: To prove this proposition, I first prove the following condition:

$$\Phi_1^l + \Phi_2^l > \psi^l \in (0, 1) \text{ for } l \in \{H, F\}$$

Given $\psi^l \equiv \left(\tau^l\right)^{-k}\left(t^l\right)^{-(k+1)}$ and:

$$\begin{aligned} \Phi_1^l \equiv &\frac{\left(k+1\right)\left(k+2\right) t^l (\tau^l)^2}{2}\left\{\left(\frac{1}{t^l \tau^l}\right)^{k+2} - \left(\frac{1}{t^l \tau^l}\right)^2 \left(\xi^l\right)^k\right. \\ &\left. - \frac{2k}{k+1}\left[\left(\frac{1}{t^l \tau^l}\right)^{k+2} - \left(\frac{1}{t^l \tau^l}\right)\left(\xi^l\right)^{k+1}\right] + \frac{k}{k+2}\left[\left(\frac{1}{t^l \tau^l}\right)^{k+2} - \left(\xi^l\right)^{k+2}\right]\right\} \\ \Phi_2^l \equiv &\frac{\left(k+1\right)\left(k+2\right)\left(\xi^l\right)^k}{2}\left[1 - \frac{2k\varphi^l \xi^l}{k+1} + \frac{k\left(\varphi^l \xi^l\right)^2}{k+2}\right] \end{aligned}$$

It is then straightforward to show

$$\Phi_1^l + \Phi_2^l = \psi^l + \frac{(k+1)(k+2)\left(\xi^l\right)^k}{2}\left\{\left(1-\frac{1}{t^l}\right)\right.$$
$$\left.-\frac{2k}{k+1}\xi^l\left(\varphi^l-\tau^l\right)+\frac{k}{k+2}\left(\xi^l\right)^2\left(\left(\varphi^l\right)^2-t^l\left(\tau^l\right)^2\right)\right\}$$

To show that $\Phi_1^l + \Phi_2^l > \psi^l$, it is equivalent to show that

$$\left(1-\frac{1}{t^l}\right)-\frac{2k}{k+1}\xi^l\left(\varphi^l-\tau^l\right)+\frac{k}{k+2}\left(\xi^l\right)^2\left(\left(\varphi^l\right)^2-t^l\left(\tau^l\right)^2\right)>0$$

Based on the definition of $\xi^l \equiv \left(\sqrt{t^l}-1\right)/\left(\sqrt{t^l}\varphi^l - t^l\tau^l\right)$, the above equation becomes:

$$\left(1-\frac{1}{t^l}\right)-\frac{2k}{k+1}\frac{\sqrt{t^l}-1}{\sqrt{t^l}\varphi^l-t^l\tau^l}\left(\varphi^l-\tau^l\right)+\frac{k}{k+2}\left(\frac{\sqrt{t^l}-1}{\sqrt{t^l}\varphi^l-t^l\tau^l}\right)^2\left(\left(\varphi^l\right)^2-t^l\left(\tau^l\right)^2\right)>0$$
$$\Leftrightarrow\left(1-\frac{1}{t^l}\right)+\frac{k}{k+2}\left(\frac{\sqrt{t^l}-1}{\sqrt{t^l}\varphi^l-t^l\tau^l}\right)^2\left(\left(\varphi^l\right)^2-t^l\left(\tau^l\right)^2\right)>\frac{2k}{k+1}\frac{\sqrt{t^l}-1}{\sqrt{t^l}\varphi^l-t^l\tau^l}\left(\varphi^l-\tau^l\right)$$
$$\Leftrightarrow\frac{t^l-1}{t^l}+\frac{k}{k+2}\frac{\left(\sqrt{t^l}-1\right)^2}{t^l}\frac{\varphi^l+\sqrt{t^l}\tau^l}{\varphi^l-\sqrt{t^l}\tau^l}>\frac{2k}{k+1}\frac{\sqrt{t^l}-1}{\sqrt{t^l}}\frac{\varphi^l-\tau^l}{\varphi^l-\sqrt{t^l}\tau^l}$$
$$\Leftrightarrow\frac{t^l-1}{t^l}+\frac{k}{k+2}\frac{\left(\sqrt{t^l}-1\right)^2}{t^l}\frac{\varphi^l+\sqrt{t^l}\tau^l}{\varphi^l-\sqrt{t^l}\tau^l}>\frac{2k}{k+1}\frac{\sqrt{t^l}-1}{\sqrt{t^l}}\frac{\varphi^l-\tau^l}{\varphi^l-\sqrt{t^l}\tau^l}$$

Multiply both sides by $(k+1)(k+2)t^l\left(\varphi^l-\sqrt{t^l}\tau^l\right)$, I have

$$\left(k^2+3k+2\right)\left(\sqrt{t^l}+1\right)\left(\varphi^l-\sqrt{t^l}\tau^l\right)+$$
$$\left(k^2+k\right)\left(\sqrt{t^l}-1\right)\left(\varphi^l+\sqrt{t^l}\tau^l\right)>\left(2k^2+4k\right)\sqrt{t^l}\left(\varphi^l-\tau^l\right)$$
$$\Leftrightarrow 2\sqrt{t^l}\varphi^l-2\sqrt{t^l}\tau^l+2(k+1)\left(\varphi^l-t^l\tau^l\right)>0$$
$$\Leftrightarrow 2\sqrt{t^l}\left(\varphi^l-\tau^l\right)+2(k+1)\left(\varphi^l-t^l\tau^l\right)>0$$

This is obviously true when $\varphi^l > t^l\tau^l$ (note $t^l > 1$), which is the assumption we made to guarantee the existence of tariff-jumping FDI. Compare the cutoff expressions, for $l \in \{H, F\}$

$$\text{Open economy, with tariff, export and FDI: } c_D^H|_{\text{with FDI}} = \left[\gamma\phi\frac{1-\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}\right]^{\frac{1}{k+2}}$$

$$\text{Open economy, with tariff and export: } c_D^H|_{\text{Without FDI}} = \left(\gamma\phi\frac{1-\psi^F}{1-\psi^F\psi^H}\right)^{\frac{1}{k+2}}, \psi^l = \left(\tau^l\right)^{-k}\left(t^l\right)^{-(k+1)}$$

Closed economy: $c_D^H|_{\text{Autarky}} = (\gamma\phi)^{\frac{1}{k+2}}$, as in **MO** Section 2

With the proved condition, it is straightforward to show that

$$c_D^H|_{\text{Autarky}} > c_D^H|_{\text{Without FDI}} > c_D^H|_{\text{With FDI}}$$

$\square$

B.4 Proof of Proposition 4

Proof: Based on the solution of $c_D^H = \left[\gamma\phi \frac{1-\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}\right]^{\frac{1}{k+2}}$, I have

$$\begin{aligned}
\frac{\partial c_D^H}{\partial t^H} &= \frac{\gamma\phi}{k+2}\left[\gamma\phi\frac{1-\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}\right]^{-\frac{k+1}{k+2}} \frac{\left(\Phi_1^F+\Phi_2^F\right)\left[1-\left(\Phi_1^F+\Phi_2^F\right)\right]}{\left[1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)\right]^2}\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H} \\
&= \underbrace{\frac{1}{k+2}c_D^H\frac{\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}}_{>0}\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}
\end{aligned}$$

So the sign crucially depends on $\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}$. It is straightforward to show that

$$\begin{aligned}
\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H} =& -\frac{k+1}{2t^H\left(\sqrt{t^H}\varphi^H-t^H\tau^H\right)^2}\left\{\frac{2}{\left(t^H\tau^H\right)^k}\left[t^H\left(\tau^H\right)^2-2\varphi^H\sqrt{t^H}\tau^H+\left(\varphi^H\right)^2\right]\right. \\
&+\left(\xi^H\right)^k\left[-2t^H\left(\tau^H\right)^2-2k\sqrt{t^H}t^H\left(\tau^H\right)^2+k\left(t^H\tau^H\right)^2\right. \\
&\left.\left.+2\left(k+2\right)\sqrt{t^H}\tau^H\varphi^H-\left(k+2\right)\left(\varphi^H\right)^2\right]\right\} \\
=& \underbrace{-\frac{k+1}{2t^H\left(\sqrt{t^H}\varphi^H-t^H\tau^H\right)^2}}_{<0}\left\{2\left(\varphi^H-\sqrt{t^H}\tau^H\right)^2\left[\left(t^H\tau^H\right)^{-k}-\left(\xi^H\right)^k\right]\right. \\
&\left.-\left(\xi^H\right)^k k\left(\varphi^H-t^H\tau^H\right)\left(\varphi^H+t^H\tau^H-2\sqrt{t^H}\tau^H\right)\right\}
\end{aligned}$$

The expression within the big bracket is greater than zero for all $k\in[1,+\infty)$ when $\varphi^H > t^H\tau^H$, to see this, it is equivalent to show

$$2\left(\varphi^H-\sqrt{t^H}\tau^H\right)^2\left[\left(t^H\tau^H\right)^{-k}-\left(\xi^H\right)^k\right] > \left(\xi^H\right)^k k\left(\varphi^H-t^H\tau^H\right)\left(\varphi^H+t^H\tau^H-2\sqrt{t^H}\tau^H\right)$$

For $k=1$, the expression becomes

$$\begin{aligned}
&2\left(\varphi^H-\sqrt{t^H}\tau^H\right)^2\left[\frac{1}{\left(t^H\tau^H\right)}-\xi^H\right] > \xi^H\left(\varphi^H-t^H\tau^H\right)\left(\varphi^H+t^H\tau^H-2\sqrt{t^H}\tau^H\right) \\
\Leftrightarrow & 2\left(\varphi^H-\sqrt{t^H}\tau^H\right)^2\left(\frac{1}{\xi^H t^H\tau^H}-1\right) > \left(\varphi^H-t^H\tau^H\right)\left(\varphi^H+t^H\tau^H-2\sqrt{t^H}\tau^H\right) \\
\Leftrightarrow & 2\left(\varphi^H-\sqrt{t^H}\tau^H\right)^2\frac{\varphi^H-t^H\tau^H}{\sqrt{t^H}\tau^H\left(\sqrt{t^H}-1\right)} > \left(\varphi^H-t^H\tau^H\right)\left(\varphi^H+t^H\tau^H-2\sqrt{t^H}\tau^H\right) \\
\Leftrightarrow & 2\left(\varphi^H-\sqrt{t^H}\tau^H\right)^2 > \sqrt{t^H}\tau^H\left(\sqrt{t^H}-1\right)\left(\varphi^H+t^H\tau^H-2\sqrt{t^H}\tau^H\right) \\
\Leftrightarrow & 2\left(\varphi^H\right)^2-3\sqrt{t^H}\tau^H\varphi^H+3\sqrt{t^H}t^H\left(\tau^H\right)^2-t^H\tau^H\varphi^H-\left(t^H\tau^H\right)^2>0 \\
\Leftrightarrow & \left(\varphi^H-t^H\tau^H\right)\left(2\varphi^H+t^H\tau^H-3\sqrt{t^H}\tau^H\right)>0
\end{aligned}$$

which is obviously true. For k approach infinity, to prove the equation, it is equivalent to show

$$\frac{2\left(\varphi^H-\sqrt{t^H}\tau^H\right)^2}{\left(\varphi^H-t^H\tau^H\right)\left(\varphi^H+t^H\tau^H-2\sqrt{t^H}\tau^H\right)}>\frac{k}{\left(\xi^H t^H\tau^H\right)^{-k}-1}$$

As $k\to\infty$, the limit of right-hand side is 0. It means as long as the left-hand side is positive, the equation is true for $k\to\infty$. The left hand side is obviously positive given $\varphi^H>t^H\tau^H$. Therefore,

$$\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}<0\Rightarrow\frac{\partial c_D^H}{\partial t^H}<0$$

To show $\frac{\partial c_D^F}{\partial t^H}$ is easier, note that

$$\begin{aligned}
\frac{\partial c_D^F}{\partial t^H}&=\left[\gamma\phi\frac{1-\left(\Phi_1^H+\Phi_2^H\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}\right]^{-\frac{k+1}{k+2}}\frac{\gamma\phi\left[1-\left(\Phi_1^F+\Phi_2^F\right)\right]/\left(k+2\right)}{\left[1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)\right]^2}\frac{-\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}\\
&=\underbrace{\frac{c_D^F}{k+2}\frac{\left[1-\left(\Phi_1^F+\Phi_2^F\right)\right]}{\left[1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)\right]\left[1-\left(\Phi_1^H+\Phi_2^H\right)\right]}}_{>0}\underbrace{\frac{-\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}}_{>0}>0
\end{aligned}$$

□

B.5 Proof of Proposition 5

Proof: Based on the cutoff relations, it is straightforward to show

$$\begin{aligned}
\frac{\partial c_X^H}{\partial t^H}&=\frac{\partial\left(c_D^F/t^F\tau^F\right)}{\partial t^H}=\frac{1}{t^F\tau^F}\frac{\partial c_D^F}{\partial t^H}>0\\
\frac{\partial c_X^F}{\partial t^H}&=\frac{\partial\left(c_D^H/t^H\tau^H\right)}{\partial t^H}=\frac{1}{\left(t^H\right)^2\tau^H}\left(\frac{\partial c_D^H}{\partial t^H}t^H-c_D^H\right)
\end{aligned}$$

To show $\frac{\partial c_D^H}{\partial t^H}t^H-c_D^H<0$ is equivalent to show

$$\begin{aligned}
&\frac{1}{k+2}c_D^H\frac{\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}t^H<c_D^H\\
\Leftrightarrow&\underbrace{\frac{1}{k+2}\frac{\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}}_{>0}\underbrace{\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}t^H}_{<0}<1
\end{aligned}$$

which is obviously true. Therefore, Proposition 5 is proved. □

B.6 Proof of Proposition 6

Proof: This part is relatively easy to prove, notice

$$\begin{aligned}\frac{\partial c^F_{FDI}}{\partial t^H} &= \frac{\partial\left(c^H_D \xi^H\right)}{\partial t^H} = \frac{\partial c^H_D}{\partial t^H}\xi^H + \frac{\partial \xi^H}{\partial t^H}c^H_D \\ &= \underbrace{\frac{c^H_D\xi^H}{k+2}\frac{\left(\Phi^F_1+\Phi^F_2\right)}{1-\left(\Phi^F_1+\Phi^F_2\right)\left(\Phi^H_1+\Phi^H_2\right)}}_{>0}\underbrace{\frac{\partial\left(\Phi^H_1+\Phi^H_2\right)}{\partial t^H}}_{<0} + \underbrace{\frac{\partial \xi^H}{\partial t^H}c^H_D}_{>0}\end{aligned}$$

It is straightforward to verify that under the parameter choice in Section 2.3, the second term dominates the first term, so Foreign country's FDI cutoff level (c^F_{FDI})is strictly increasing as Home country's tariff (t^H) increases. Do notice, the other choice of ξ, which is $\xi = \left(\sqrt{t}+1\right)/\left(\sqrt{t}\varphi + t\tau\right)$ will make the second item negative, thereby making c^F_{FDI} decreasing in response to t^H's increase, hence no tariff-jumping FDI. □

B.7 Proof of Corollary 1

Proof: The proof is straightforward. Utilizing Proposition 3 and 4:

$$\begin{aligned}\left|\frac{\frac{\partial c^H_D}{\partial t^H}|_{\text{without FDI}}}{\frac{\partial c^H_D}{\partial t^H}|_{\text{with FDI}}}\right| &= \left|\frac{\frac{\gamma\phi(c^H_D|_{\text{without FDI}})^{-(k+1)}}{k+2}\frac{\psi^F\left[1-\psi^F\right]}{\left[1-\psi^F\psi^H\right]^2}\frac{\partial\psi^F}{\partial t^H}}{\frac{\gamma\phi(c^H_D|\text{with FDI})^{-(k+1)}}{k+2}\frac{\left(\Phi^F_1+\Phi^F_2\right)\left[1-\left(\Phi^F_1+\Phi^F_2\right)\right]}{\left[1-\left(\Phi^F_1+\Phi^F_2\right)\left(\Phi^H_1+\Phi^H_2\right)\right]^2}\frac{\partial\left(\Phi^H_1+\Phi^H_2\right)}{\partial t^H}}\right| \\ &= \left|\frac{(c^H_D|_{\text{with FDI}})^{k+1}\frac{\psi^F\left[1-\psi^F\right]}{\left[1-\psi^F\psi^H\right]^2}\frac{\partial\psi^F}{\partial t^H}}{(c^H_D|_{\text{without FDI}})^{k+1}\frac{\left(\Phi^F_1+\Phi^F_2\right)\left[1-\left(\Phi^F_1+\Phi^F_2\right)\right]}{\left[1-\left(\Phi^F_1+\Phi^F_2\right)\left(\Phi^H_1+\Phi^H_2\right)\right]^2}\frac{\partial\left(\Phi^H_1+\Phi^H_2\right)}{\partial t^H}}\right| \\ &= \left|\left(\frac{c^H_D|_{\text{with FDI}}}{c^H_D|_{\text{without FDI}}}\right)^{k+1} \times \frac{\psi^F}{\left(\Phi^F_1+\Phi^F_2\right)} \times \frac{\left[1-\left(\Phi^F_1+\Phi^F_2\right)\left(\Phi^H_1+\Phi^H_2\right)\right]^2}{\left[1-\psi^F\psi^H\right]^2} \times \frac{\frac{\partial\psi^F}{\partial t^H}}{\frac{\partial\left(\Phi^H_1+\Phi^H_2\right)}{\partial t^H}}\right|\end{aligned}$$

Based on the proof of Proposition 3 and 4, it is obvious that the 1st, 2nd and 3rd term in the above equation are all smaller than 1. The 4th term, however, is greater than 1. It is straightforward to verify that the 4th term will be dominated by the first three terms, therefore, the whole expression is smaller than 1. □

B.8 Proof of Proposition 7

Proof: Based on equation (24), it's straightforward to show

$$\begin{aligned}\frac{\partial N^H}{\partial t^H} &= \frac{2\gamma(k+1)}{\eta}\frac{-\frac{\partial c^H_D}{\partial t^H}c^H_D - \frac{\partial c^H_D}{\partial t^H}\left(\alpha - c^H_D\right)}{\left(c^H_D\right)^2} = -\frac{2\gamma\alpha(k+1)}{\eta\left(c^H_D\right)^2}\frac{\partial c^H_D}{\partial t^H} > 0 \\ \frac{\partial N^F}{\partial t^H} &= \frac{2\gamma(k+1)}{\eta}\frac{-\frac{\partial c^F_D}{\partial t^H}c^F_D - \frac{\partial c^F_D}{\partial t^H}\left(\alpha - c^F_D\right)}{\left(c^F_D\right)^2} = -\frac{2\gamma\alpha(k+1)}{\eta\left(c^F_D\right)^2}\frac{\partial c^F_D}{\partial t^H} < 0\end{aligned}$$

Now based on equation (26), as t^H increases

$$N_E^F = \frac{2\left(c_M\right)^k (k+1)\gamma}{\eta\left(1-\delta^H\delta^F\right)}\left[\frac{\alpha - c_D^F}{\left(c_D^F\right)^{k+1}} - \delta^F\frac{\alpha - c_D^H}{\left(c_D^H\right)^{k+1}}\right]$$

δ^H decreases (hence the coefficient in front of the bracket decreases), c_D^F increases (the first item in the bracket decreases), c_D^H decreases (the second item in the bracket increases). Hence the whole expression on the right decreases, therefore $\partial N_E^F/\partial t^H < 0$. Now utilizing the free-entry condition, which is equation (25)

$$G\left(c_D^H\right)N_E^H + G\left(c_X^F\right)N_E^F = N^H$$

As t^H increases, N^H increases, it means the left-side also needs to increase. Notice N_E^F decreases, c_X^F decreases, c_D^H decreases, it then must be true that N_E^H increases. Hence, $\partial N_E^H/\partial t^H > 0$. □

B.9 Proof of Proposition 8

Proof: To prove this proposition, it is helpful to rewrite the markups as follows:

$$\begin{aligned}
m_D^H(c) &= \frac{1}{2c}\left(c_D^H + c\right)\\
m_X^F(c) &= \frac{t^H}{2c}\left(c_X^F + c\right) = \frac{t^H}{2c}\left(c_D^H/t^H\tau^H + c\right) = \frac{1}{2c}\left(c_D^H/\tau^H + c/t^H\right)\\
m_{FDI}^F(c) &= \frac{1}{2\varphi^H c}\left(c_D^H + \varphi^H c\right)
\end{aligned}$$

It follows from Proposition 4 that $\partial m_D^H/\partial t^H < 0$ and $\partial m_{FDI}^F/\partial t^H < 0$. The responses of m_X^F now is different from what we see in Proposition 2. On the one hand, c_D^H/τ^H decreases as t^H increases. On the other hand, c/t^H decreases as t^H increases. The total impact on m_X^F is therefore negative. Hence, $\partial m_X^F/\partial t^H < 0$. □

B.10 Proof of Proposition 9

Proof: The social planner solves the following problem

$$\max_{\{N_E^H, q_0^H, q_i^H, N_E^F, q_0^F, q_i^F\}} \mathbb{W} \equiv \mathbb{U}_H + \mathbb{U}_F$$

$$\begin{aligned}
\text{s.t. } & q_0^H + q_0^F + f\left(N_E^H + N_E^F\right) + N_E^H\int_0^{c_M}\left[cq_D^H(c) + \tau^F cq_X^H(c) + \varphi^F cq_{FDI}^H(c)\right]dG(c)\\
&+ N_E^F\int_0^{c_M}\left[cq_D^F(c) + \tau^H cq_X^F(c) + \varphi^H cq_{FDI}^F(c)\right]dG(c) = 2 + \bar{q}_0^H + \bar{q}_0^F
\end{aligned}$$

Notice, $\mathbb{W} \equiv \mathbb{U}^H + \mathbb{U}^F$ and since labor has been normalized to 1, $\mathbb{U}^H$ is defined as follow

$$\begin{aligned}\mathbb{U}^H \equiv\ & q_0 + \alpha N_E^H \left\{ \int \left[q_D^H(c) + q_X^H(c) + q_{FDI}^H(c) \right] dG(c) \right\} \\ & - \frac{\gamma}{2} \left\{ N_E^H \int \left(q_D^H(c) \right)^2 dG(c) + N_E^F \int \left[\left(q_X^F(c) \right)^2 + \left(q_{FDI}^F(c) \right)^2 \right] dG(c) \right\} \\ & - \frac{\eta}{2} \left\{ N_E^H \int q_D^H(c)\, dG(c) + N_E^F \int \left[q_X^F(c) + q_{FDI}^F(c) \right] dG(c) \right\}\end{aligned}$$

The first order conditions with respect to q_D, q_X, q_{FDI} deliver the following results for the Home country:

$$\begin{aligned} q_D^H(c) &= \frac{c_D^{HO} - c}{\gamma}, c_D^{HO} = \alpha - \eta Q^{HO} \\ q_X^H(c) &= \frac{c_X^{HO} - c}{\gamma/\tau^F}, c_X^{HO} = \frac{\alpha - \eta Q^{FO}}{\tau^F} \\ q_{FDI}^H(c) &= \frac{c_{FDI}^{HO} - c}{\gamma/\varphi^F}, c_{FDI}^{HO} = \frac{\alpha - \eta Q^{FO}}{\varphi^F} \end{aligned}$$

The first order condition with respect to N_E delivers the following results

$$Q^{HO} = \frac{N^{HO} + 2N^{FO}}{\gamma + \eta\left(N^{HO} + 2N^{FO}\right)} \left(\alpha - \frac{k}{k+1} \left[\frac{N^{HO} + \left(\tau^H/\varphi^H\right)^{k+1} N^{FO}}{N^{HO} + 2N^{FO}} \right] c_D^{HO} \right)$$

Combine these with the corresponding results for the Foreign country, it's straightforward to obtain Home's domestic cutoff level under the planner's problem

$$c_D^{HO} = \left[\gamma (k+1)(k+2) f c_M^k \frac{1 - \Omega_F}{1 - \Omega_F \Omega_H} \right]^{\frac{1}{k+2}}$$

where $\Omega_F \equiv \left(\varphi^F\right)^{-k} + \frac{(k+1)(k+2)}{2} \left[\left(\tau^F\right)^{-k} - \left(\varphi^F\right)^{-k} \right] - k(k+2)\tau^F \left[\left(\tau^F\right)^{-(k+1)} - \left(\varphi^F\right)^{-(k+1)} \right] + \frac{k(k+1)\left(\tau^F\right)^2}{2} \left[\left(\tau^F\right)^{-(k+2)} - \left(\varphi^F\right)^{-(k+2)} \right]$. All the rest of the equilibrium variables, such as N_E^{HO}, N^{HO}, etc, can be expressed as a function of c_D^{HO} and other parameters. Compare the domestic cutoff from equation (23) with the socially optimum cutoff, it is straightforward to show that:

$$\left(\frac{c_D^{HM}}{c_D^{HO}} \right)^{k+2} = \frac{2}{\frac{1-\Omega_F}{1-\Omega_F\Omega_H} / \frac{1 - \Phi_1^F - \Phi_2^F}{1 - \left(\Phi_1^F + \Phi_2^F\right)\left(\Phi_1^H + \Phi_2^H\right)}} \equiv \frac{2}{\triangle_F}$$

The term in the denominator of the above expression is defined as $\triangle_F$. For displaying purpose, I then focus on the comparison of all the market outcomes and socially optimum outcomes in the Home market. It follows from the definition of $c_X^{HM}, c_X^{HO}, c_{FDI}^{HM}, c_{FDI}^{HO}$ that the following three equations must hold:

$$\begin{aligned} c_D^{HM} - c_D^{HO} &= \left[\left(\frac{2}{\triangle_F} \right)^{\frac{1}{k+2}} - 1 \right] c_D^{HO} \\ c_X^{HM} - c_X^{HO} &= \left[\left(\frac{2}{\triangle_F} \right)^{\frac{1}{k+2}} - t^F \right] \frac{c_D^{FO}}{\tau^F t^F} \end{aligned}$$

$$c_{FDI}^{HM} - c_{FDI}^{HO} = \left[\left(\frac{2}{\triangle_F} \right)^{\frac{1}{k+2}} - \frac{1}{\varphi^F \xi^F} \right] c_D^{FO} \xi^F$$

It then can be verified that if the tariff level is not sufficiently high, $\triangle_F$ will be less than 1, hence both $\left[(2/\triangle_F)^{\frac{1}{k+2}} - 1\right]$ and $\left[(2/\triangle_F)^{\frac{1}{k+2}} - t^F\right]$ are greater than zero. Notice, $\varphi^F \xi^F < 1$, with the assumption that $\varphi^F > t^F \tau^F$, it is straightforward to show that $\left[(2/\triangle_F)^{\frac{1}{k+2}} - 1/\varphi^F \xi^F\right] < 0$. Therefore, part (A) of proposition 9 is proved.

For part (B), we can just follow the definition of production levels. For example, we know

$$q_D^{HM} = \frac{1}{2\gamma}(c_D^{HM} - c), q_D^{HO} = \frac{1}{\gamma}(c_D^{HO} - c)$$

It can be easily verified that $q_D^{HM} < q_D^{HO}$ if $c < \left[2 - (2/\triangle_F)^{1/(k+2)}\right] c_D^{HO}$. The production levels of q_X and q_{FDI} also follow directly from the comparison of market outcome and socially optimum outcome.

For part (C), one can show that

$$N_H^M = \frac{2\gamma(k+1)}{\eta} \times \frac{\alpha - c_D^{HM}}{c_D^{HM}}, N_H^O = \frac{\gamma(k+1)}{\eta} \times \frac{\alpha - c_D^{HO}}{c_D^{HO}}$$

One can then show that $N_H^M > N_H^O$ if

$$c_D^{HO} < \left[\frac{2}{(2/\triangle_F)^{\frac{1}{k+2}}} - 1 \right] \alpha$$

For the level of entrants, one can solve them through the following system of equations:

$$N_H^O = N_E^{HO} \left(\frac{c_D^{HO}}{c_M} \right)^k + N_E^{FO} \left(\frac{c_D^{HO}}{c_M} \right)^k (\tau^H)^{-k}, \quad N_F^O = N_E^{FO} \left(\frac{c_D^{FO}}{c_M} \right)^k + N_E^{HO} \left(\frac{c_D^{FO}}{c_M} \right)^k (\tau^F)^{-k}$$

And obtain the socially optimum entrant level:

$$N_E^{HO} = \frac{\gamma(k+1)c_M^k}{\eta[1 - (\tau^H \tau^F)^{-k}]} \times \left[\frac{\alpha - c_D^{HO}}{\left(c_D^{HO}\right)^{k+1}} - (\tau^H)^{-k} \frac{\alpha - c_D^{FO}}{\left(c_D^{FO}\right)^{k+1}} \right]$$

Comparing it with the entrant level in the market outcome:

$$N_E^{HM} = \frac{2\,(c_M)^k\,(k+1)\,\gamma}{\eta\,(1 - \delta^H \delta^F)} \times \left[\frac{\alpha - c_D^{HM}}{\left(c_D^{HM}\right)^{k+1}} - \delta^H \frac{\alpha - c_D^{FM}}{\left(c_D^{FM}\right)^{k+1}} \right]$$

where $\delta^l = (t^l \tau^l)^{-k}$, for $l \in \{H, F\}$. Together with the fact that $c_D^{HO} = c_D^{FO}[(1 - \Omega_F)/(1 - \Omega_F \Omega_H)]^{\frac{1}{k+2}}$, one can then find a similar threshold of c_D^{HO} where N_E^{HO} differs from N_E^{HM}. That completes the proof of part (C). $\square$

B.11 Proof of Proposition 10

Proof: Based on equation (28), the social welfare can be rewritten as follow:

$$\mathbb{U}^H + \mathbb{U}^F = I^H + I^F + \frac{\alpha - c_D^H}{2\eta}\left(\alpha - \frac{k+1}{k+2}c_D^H\right) + \frac{\alpha - c_D^F}{2\eta}\left(\alpha - \frac{k+1}{k+2}c_D^F\right)$$

Since consumer receive income from wage (which is equal to 1) and tariff revenue, so the above equation can be rewritten as:

$$\mathbb{U}^H + \mathbb{U}^F = 2 + \left(t^H - 1\right) IM^H + \left(t^F - 1\right) IM^F + \frac{\alpha - c_D^H}{2\eta}\left(\alpha - \frac{k+1}{k+2}c_D^H\right) + \frac{\alpha - c_D^F}{2\eta}\left(\alpha - \frac{k+1}{k+2}c_D^F\right)$$

To see the welfare implication of free trade, I evaluate the first order condition of the above expression with respect to tariff under symmetry when $t^H = t^F = 1$. Since symmetry implies $IM^F = IM^H, \partial IM^F/\partial t^H = \partial IM^H/\partial t^F, c_D^F = c_D^H, \partial c_D^F/\partial t^H = \partial c_D^H/\partial t^F$, therefore :

$$\frac{\partial\left(\mathbb{U}^H + \mathbb{U}^F\right)}{\partial t}|_{t^H=t^F=1} = \underbrace{(t-1)\left(\frac{\partial IM^H}{\partial t^H} + \frac{\partial IM^F}{\partial t^H}\right)}_{\circledast} + IM^H + \frac{2(k+1)c_D - (2k+3)\alpha}{2\eta(k+2)}\left(\frac{\partial c_D^H}{\partial t^H} + \frac{\partial c_D^F}{\partial t^H}\right)$$

Notice, when $t^H = t^F = 1$, $\circledast = 0$. Based on equation (27) and (29),

$$\begin{aligned} IM^H|_{t^H=1} &= N_E|_{t^H=1} \times \frac{(c_D)^{k+2}\tau^{-k}}{2\gamma(k+2)(c_M)^k}|_{t=1} \\ &= \frac{2(c_M)^k(k+1)\gamma\left(1-\tau^{-k}\right)}{\eta\left(1-\tau^{-2k}\right)}\frac{(c_D)^{k+2}\tau^{-k}}{2\gamma(k+2)(c_M)^k}|_{t=1} \\ &= \frac{\tau^{-k}(k+1)}{\eta\left(1+\tau^{-k}\right)(k+2)}(\alpha - c_D)\, c_D|_{t=1} \end{aligned}$$

Based on the definition of Φ^H and Φ^F, it is straightforward to show

$$\Phi_1 + \Phi_2 = \tau^{-k}$$

Based on the proof of Proposition 4, it is straightforward to show

$$\frac{\partial c_D^H}{\partial t^H} + \frac{\partial c_D^F}{\partial t^H}|_{t=1} = \frac{-(k+1)c_D^H\tau^{-2k}}{(k+2)\left(1-\tau^{-2k}\right)} + \frac{c_D^F}{k+2}\frac{-(k+1)\tau^{-k}}{\left(1-\tau^{-2k}\right)} = \frac{\tau^{-k}(k+1)}{\left(1+\tau^{-k}\right)(k+2)}c_D|_{t=1}$$

Therefore, the original first order condition can be rewritten as

$$\begin{aligned} \frac{\partial\left(\mathbb{U}^H + \mathbb{U}^F\right)}{\partial t}|_{t^H=t^F=1} &= \frac{\tau^{-k}(k+1)}{\eta\left(1+\tau^{-k}\right)(k+2)}(\alpha - c_D)\, c_D|_{t=1} \\ &+ \frac{2(k+1)c_D - (2k+3)\alpha}{2\eta(k+2)}\frac{\tau^{-k}(k+1)}{\left(1+\tau^{-k}\right)(k+2)}c_D|_{t=1} \\ &= \frac{\tau^{-k}(k+1)c_D}{2\eta(k+2)^2\left(1+\tau^{-k}\right)}\left[2(k+2)(\alpha - c_D) + 2(k+1)c_D - (2k+3)\alpha\right] \\ &= \frac{\tau^{-k}(k+1)c_D}{2\eta(k+2)^2\left(1+\tau^{-k}\right)}\left(-2c_D + \alpha\right)|_{t=1} \end{aligned}$$

Define $\tilde{c}_D \equiv c_D|_{t=1}$, then this completes the proof of proposition 10. $\square$

B.12 Proof of Second-Best Social Planner Problem

Proof: Based on the definition of average consumer surplus, it can be rewritten in terms of $\tilde{c}_D$:

$$\text{Avg. CS} \equiv \frac{\gamma}{2}\int_0^{\tilde{c}_D} (q_D(c))^2 \, dG(c) = \frac{(\tilde{c}_D)^{k+2}}{4\gamma c_M^k (k+1)(k+2)} > 0$$

Based on equation (26), under symmetry and $t = 1$,

$$N_E = \frac{2\gamma c_M^k (k+1)(\alpha - \tilde{c}_D)}{\eta (\tilde{c}_D)^{k+1} (1+\tau^{-k})}$$

Then, the variety effect can be defined as the difference between consumer surplus and the sum of average surplus at each variety:

$$\begin{aligned}
\text{VE} &\equiv \text{CS} - N_E \times \text{Avg. CS} \\
&= \frac{\alpha - \tilde{c}_D}{2\eta}\left(\alpha - \frac{k+1}{k+2}\tilde{c}_D\right) - \frac{2\gamma c_M^k (k+1)(\alpha - \tilde{c}_D)}{\eta (\tilde{c}_D)^{k+1}(1+\tau^{-k})} \times \frac{(\tilde{c}_D)^{k+2}}{4\gamma c_M^k (k+1)(k+2)} \\
&= \frac{(\alpha - \tilde{c}_D)}{2\eta}\left[\alpha - \frac{(k+1)\left(1+\tau^{-k}\right)+1}{(k+2)(1+\tau^{-k})}\tilde{c}_D\right]
\end{aligned}$$

The expected profit of a firm can be derived from equation (20):

$$\begin{aligned}
\bar{\pi} &= \int_0^{\tilde{c}_D} \pi_D(c)\, dG(c) + \int_0^{\tilde{c}_X} \pi_X(c)\, dG(c) \\
&= \frac{(\tilde{c}_D)^{k+2}}{2\gamma c_M^k (k+1)(k+2)} + \frac{\tau^2 (\tilde{c}_X)^{k+2}}{2\gamma c_M^k (k+1)(k+2)} = \frac{(\tilde{c}_D)^{k+2}\left(1+\tau^{-k}\right)}{2\gamma c_M^k (k+1)(k+2)}
\end{aligned}$$

Notice when $t = 1, c_{FDI} = 0$ and $\tilde{c}_X = \tilde{c}_D/\tau$. With all these components and the fact that $\tilde{c}_D < \alpha$, equation (34) can now be properly signed:

$$\begin{aligned}
\text{Avg. CS} &= \frac{(\tilde{c}_D)^{k+2}}{4\gamma c_M^k (k+1)(k+2)} > 0 \\
N_E \frac{\partial \text{Avg. CS}}{\partial N_E} &= \frac{(\alpha - \tilde{c}_D)(\tilde{c}_D)^{k+2}}{4\gamma c_M^k (k+1)\left[k\tilde{c}_D - \alpha(k+1)\right]} < 0 \\
\frac{\partial \text{VE}}{\partial N_E} &= \frac{(\tilde{c}_D)^{k+2}\left\{\alpha(k+2)(1+\tau^{-k}) + (\alpha - 2\tilde{c}_D)\left[(k+1)(1+\tau^{-k})+1\right]\right\}}{4\gamma c_M^k (k+1)(k+2)\left[\alpha(k+1) - k\tilde{c}_D\right]} > 0 \\
N_E \frac{\partial \bar{\pi}}{\partial N_E} &= \frac{(\tilde{c}_D)^{k+2}\left(1+\tau^{-k}\right)(\alpha - \tilde{c}_D)}{2\gamma c_M^k (k+1)\left[k\tilde{c}_D - (k+1)\alpha\right]} < 0
\end{aligned}$$

Therefore, the externality of entry equals to

$$\text{Avg. CS} + N_E \frac{\partial \text{Avg. CS}}{\partial N_E} + \frac{\partial \text{VE}}{\partial N_E} + N_E \frac{\partial \bar{\pi}}{\partial N_E} = \frac{\left(1+\tau^{-k}\right)(\tilde{c}_D)^{k+2}}{4\gamma c_M^k (k+1)(k+2)\underbrace{\left[k\tilde{c}_D - (k+1)\alpha\right]}_{<0}} \times (\alpha - 2\tilde{c}_D)$$

Therefore, the externality will be negative if $\tilde{c}_D < \alpha/2$, will be positive if $\tilde{c}_D > \alpha/2$. □

B.13 Proof of Proposition 11

Proof: Under symmetry, it is straightforward to derive the following welfare expression for market outcome and socially optimum outcome:

$$\mathbb{W}^M = 1 + \bar{q}_0 + \frac{\alpha - c_D^M}{2\eta}\left(\alpha - \frac{k+1}{k+2}c_D^M\right), \qquad \mathbb{W}^O = 1 + \bar{q}_0 + \frac{1}{2\eta}\left(\alpha - c_D^O\right)^2$$

And based on the proof of Proposition 9 that $c_D^M = (2/\triangle)^{\frac{1}{k+2}} c_D^O$, where the $\triangle$ is the symmetric version of $\triangle_F$ and $\triangle_H$. To simplify the expression, here I define:

$$A \equiv \left(\frac{2}{\triangle}\right)^{\frac{1}{k+2}}, \qquad B \equiv \frac{k+1}{k+2}$$

Therefore, the welfare gap between market outcome and socially optimum is:

$$\mathbb{W}^M - \mathbb{W}^O = \frac{1}{2\eta}\left[c_D^O(2\alpha - AB\alpha - A\alpha) + (c_D^O)^2(A^2B - 1)\right]$$

It is obvious that the market welfare is smaller than the socially optimum welfare. One can then show the gap is decreasing in t:

$$\frac{\partial(\mathbb{W}^O - \mathbb{W}^M)}{\partial t} = c_D^O \frac{\partial A}{\partial t}\left[\alpha(A+B) - 2ABc_D^O\right]$$

Given that $\partial A/\partial t > 0$, then the whole expression will be negative if $c_D^O > \alpha(A+B)/2AB$, which is equivalent to $c_D^M > \alpha(A+B)/2B$. Therefore part *(i)* of the proposition is proved.

The second part of the proposition is easy. Notice that:

$$\mathbb{W}^H = 1 + \bar{q}_0^H + \frac{\alpha - c_D^H}{2\eta}\left(\alpha - \frac{k+1}{k+2}c_D^H\right), \qquad \mathbb{W}^F = 1 + \bar{q}_0^F + \frac{\alpha - c_D^F}{2\eta}\left(\alpha - \frac{k+1}{k+2}c_D^F\right)$$

The rest of the proof directly follows from the Proposition 4. □

B.14 Proof of Proposition 12

Proof: First, based on the definition of IM in equation (29), and equation (35) and (36), one can show that under symmetry and when $\tilde{c}_D > \alpha/2$, the socially optimal tariff must satisfy:

$$\frac{\partial \mathbb{W}}{\partial t_s} = \frac{2\partial}{\partial t_s}\left[(t_s - 1) \times IM + CS\right] = 0$$

$$\Leftrightarrow \frac{2\partial}{\partial t_s}\left[\frac{t_s - 1}{t_s}\underbrace{\frac{N_E\left(c_D\right)^{k+2}}{4\gamma\left(k+2\right)\left(c_M\right)^k}}_{A(t_s)}\underbrace{\left[2\left(\frac{1}{t_s\tau}\right)^k - (k+2)\left(\xi_s\right)^k + k\left(\xi_s\right)^{k+2}\left(t_s\tau\right)^2\right]}_{B(t_s)} + CS\right] = 0$$

Notice, N_E, c_D, ξ_s and CS are all functions of t_s, therefore, the socially optimal tariff implies:

$$2\left[\frac{t_s-1}{t_s}\frac{\partial A(t_s)B(t_s)}{\partial t_s}+\frac{A(t_s)}{t_s^2}+\frac{\partial CS(t_s)}{\partial t_s}\right]=0$$

Now let's turn to the Nash tariff level under symmetry, one can show that for Home country

$$\frac{\partial \mathbb{U}^H}{\partial t^H}=\frac{\partial}{\partial t^H}\left[\left(t^H-1\right)\times IM^H+CS^H\right]=0$$

$$\Leftrightarrow \frac{\partial}{\partial t^H}\left[\frac{t^H-1}{t^H}\underbrace{\frac{N_E^F\left(c_D^H\right)^{k+2}}{4\gamma\left(k+2\right)\left(c_M\right)^k}}_{A(t^H,t^F)}\underbrace{\left[2\left(\frac{1}{t^H\tau^H}\right)^k-\left(k+2\right)\left(\xi^H\right)^k+k\left(\xi^H\right)^{k+2}\left(t^H\tau^H\right)^2\right]}_{B(t^H)}+CS^H\right]=0$$

Notice, N_E^F, c_D^H, CS^H are all functions of t^H, t^F, therefore, the Nash tariff level of Home country implies:

$$\left[\frac{t^H-1}{t^H}\frac{\partial A(t^H,t^F)B(t^H)}{\partial t^H}+\frac{A(t^H,t^F)B(t^H)}{(t^H)^2}+\frac{\partial CS^H}{\partial t^H}\right]=0$$

Similarly, the following equation will hold for Foreign country:

$$\left[\frac{t^F-1}{t^F}\frac{\partial A(t^F,t^H)B(t^F)}{\partial t^F}+\frac{A(t^F,t^H)B(t^F)}{(t^F)^2}+\frac{\partial CS^F}{\partial t^F}\right]=0$$

Combine the above two first-order conditions and apply symmetry:

$$\left[\frac{t_N-1}{t_N}\frac{\partial A(t_N,t_N)B(t_N)}{\partial t_N}+\frac{A(t_N,t_N)B(t_N)}{(t_N)^2}+\frac{\partial CS}{\partial t_N}\right]=0$$

One should notice that $\partial A(t_s)B(t_s)/\partial t_s$ has a different expression compared to $\partial A(t_N,t_N)B(t_N)/\partial t_N$ and the reason is that the social planner evaluates the joint welfare at the same time, whereas the Nash tariff is chosen to maximize the unilateral welfare. Evaluate the first-order condition of the social planner at the symmetric Nash tariff level (t_N), it is evident that $\partial \mathbb{W}/\partial t|_{t=t_N}<0$. Hence it must be the case that $t_N>t_s$. $\square$

B.15 Proof of Proposition 13

Proof: Once again, to simplify the proof, I assume symmetry, following Appendix B.16, $c_D=c_D^H=c_D^F$ and

$$N_E=\frac{2\gamma c_M^k\left(k+1\right)\left(\alpha-c_D\right)}{\eta\left(c_D\right)^{k+1}\left(1+t^{-k}\tau^{-k}\right)},\quad N_D=\frac{2\gamma\left(k+1\right)\left(\alpha-c_D\right)}{\eta\left(1+t^{-k}\tau^{-k}\right)c_D}$$

$$N_X=\frac{2\gamma\left(k+1\right)\left(\alpha-c_D\right)}{\eta\left(1+t^{-k}\tau^{-k}\right)c_D}\left[\left(t\tau\right)^{-k}-\xi^k\right],\quad N_{FDI}=\frac{2\gamma\left(k+1\right)\left(\alpha-c_D\right)}{\eta\left(1+t^{-k}\tau^{-k}\right)c_D}\xi^k$$

Together with equation (6), (10) and (15), the average markup in (36) can be written as follow:

$$\begin{aligned}
\bar{m} = & \frac{1}{N_D + N_X + N_{FDI}} \left[N_D \int_0^{c_D} m_D(c) \frac{dG(c)}{G(c_D)} \right. \\
& \left. + N_X \int_{c_{FDI}}^{c_X} m_X(c) \frac{dG(c)}{G(c_X)} + N_{FDI} \int_0^{c_{FDI}} m_{FDI}(c) \frac{dG(c)}{G(c_{FDI})} \right] \\
= & \frac{1}{N_D + N_X + N_{FDI}} \left[N_D \int_0^{c_D} \frac{c_D + c}{2c} \frac{kc^{k-1}}{c_D^k} dc \right. \\
& \left. + N_X \int_{c_{FDI}}^{c_X} \frac{t(c_X + c)}{2c} \frac{kc^{k-1}}{c_X^k} dc + N_{FDI} \int_0^{c_{FDI}} \frac{c_D + \varphi^H c}{2\varphi^H c} \frac{kc^{k-1}}{c_{FDI}^k} dc \right] \\
= & \frac{1}{N_D + N_X + N_{FDI}} \left[N_D \times \frac{2k-1}{2k-2} + N_X \times t \right. \\
& \left. \times \left(\frac{2k-1}{2k-2} - \frac{k}{2k-2} (t\tau\xi)^{k-1} - \frac{1}{2} (t\tau\xi)^k \right) + N_{FDI} \times \left(\frac{k}{2k-2} \frac{1}{\varphi\xi} + \frac{1}{2} \right) \right] \\
= & \frac{1}{1 + (t\tau)^{-k}} \times \frac{2k-1}{2k-2} + \frac{t \left[(t\tau)^{-k} - \xi^k \right]}{1 + (t\tau)^{-k}} \times \left[\frac{2k-1}{2k-2} - \frac{k}{2k-2} (t\tau\xi)^{k-1} - \frac{1}{2} (t\tau\xi)^k \right] \\
& + \frac{\xi^k}{1 + (t\tau)^{-k}} \times \left(\frac{k}{2k-2} \frac{1}{\varphi\xi} + \frac{1}{2} \right) \\
= & \underbrace{\frac{1}{1 + (t\tau)^{-k}} \times \frac{2k-1}{2k-2}}_{\text{weighted expected markup in domestic}} \\
& + \underbrace{\frac{1 - (t\tau\xi)^k}{1 + (t\tau)^{-k}} \times \frac{1}{t^{k-1}\tau^k} \left\{ \frac{1}{2} \left[1 - (t\tau\xi)^k \right] + \frac{k}{2k-2} \left[1 - (t\tau\xi)^{k-1} \right] \right\}}_{\text{weighted expected markup from Foreign exporters}} \\
& + \underbrace{\frac{\xi^k}{1 + (t\tau)^{-k}} \times \left(\frac{k}{2k-2} \frac{1}{\varphi\xi} + \frac{1}{2} \right)}_{\text{weighted expected markup from Foreign FDI}}
\end{aligned}$$

B.16 Proof of Covariance Term

Proof: Once again, to simplify the analysis, I imposed symmetry. It's clear from equation (26) that

$$N_E = \frac{2\gamma c_M^k (k+1)(\alpha - c_D)}{\eta (c_D)^{k+1} (1 + t^{-k}\tau^{-k})}$$

where $c_D = c_D^H = c_D^F$. Given equation (7), (11) and (16), and the following expression for the mass of firms:

$$\begin{aligned}
N_D &= N_E \times G(c_D) = \frac{2\gamma (k+1)(\alpha - c_D)}{\eta (1 + t^{-k}\tau^{-k}) c_D} \\
N_X &= N_E \times [G(c_X) - G(c_{FDI})] = \frac{2\gamma (k+1)(\alpha - c_D)}{\eta (1 + t^{-k}\tau^{-k}) c_D} \left[(t\tau)^{-k} - \xi^k \right] \\
N_{FDI} &= N_E \times G(c_{FDI}) = \frac{2\gamma (k+1)(\alpha - c_D)}{\eta (1 + t^{-k}\tau^{-k}) c_D} \xi^k
\end{aligned}$$

the covariance term can be derived as follow:

$$
\begin{aligned}
\operatorname{cov}\left(m^i(\omega), \frac{dl^i(\omega)}{L^j}\right) &= N_D \int_0^{c_D} p_D(c)\, d\left[q_D(c)\right] \frac{dG(c)}{G(c_D)} + N_X \int_{c_{FDI}}^{c_X} p_X(c)\, d\left[q_X(c)\right] \frac{dG(c)}{G(c_X)} \\
&+ N_{FDI} \int_0^{c_{FDI}} p_{FDI}(c)\, d\left[q_{FDI}(c)\right] \frac{dG(c)}{G(c_{FDI})} \\
&= \frac{2\gamma(k+1)(\alpha-c_D)}{\eta(1+t^{-k}\tau^{-k})c_D} \int_0^{c_D} \frac{kdc_D(c_D+c)c^{k-1}}{4\gamma c_D^k} dc \\
&+ \frac{2\gamma(k+1)(\alpha-c_D)}{\eta(1+t^{-k}\tau^{-k})c_D} \left[(t\tau)^{-k} - \xi^k\right] \int_{c_{FDI}}^{c_X} \frac{t^2\tau^2 kdc_X(c_X+c)c^{k-1}}{4\gamma c_X^k} dc \\
&+ \frac{2\gamma(k+1)(\alpha-c_D)}{\eta(1+t^{-k}\tau^{-k})c_D} \xi^k \int_0^{c_{FDI}} \frac{kdc_D(c_D+\varphi c)c^{k-1}}{4\gamma c_F^k} dc \\
&= \frac{(\alpha-c_D)\,dc_D}{2\eta(1+t^{-k}\tau^{-k})} \Big\{2k+1+\left[(t\tau)^{-k} - \xi^k\right] \\
&\times \left[2k+1-k(1-t\tau\xi)(t\tau\xi)^k - (t\tau\xi)^k\right] + \xi^k(k+k\varphi+1)\Big\}
\end{aligned}
$$

When $\varphi > t\tau$, it is straightforward to show that $t\tau\xi < 1$. Therefore, the covariance term can be rewritten as

$$
\begin{aligned}
\operatorname{cov}\left(m^i(\omega), \frac{dl^i(\omega)}{L^j}\right) = \frac{(\alpha-c_D)\,dc_D}{2\eta(1+t^{-k}\tau^{-k})} &\Bigg\{ 2k+1+(t\tau)^{-k} \underbrace{\left[1-(t\tau\xi)^k\right]}_{>0} \\
&\times \Bigg[k+k(t\tau\xi)^{k+1} + \underbrace{k-k(t\tau\xi)^k}_{>0} + \underbrace{1-(t\tau\xi)^k}_{>0}\Bigg] + \xi^k(k+k\varphi+1)\Bigg\}
\end{aligned}
$$

Notice, under symmetry

$$
\begin{aligned}
\frac{dc_D}{dt} &= \frac{d}{dt}\left[\frac{\gamma\phi}{1+\Phi_1+\Phi_2}\right]^{\frac{1}{k+2}} = \frac{1}{k+2}\left[\frac{\gamma\phi}{1+\Phi_1+\Phi_2}\right]^{-\frac{k+1}{k+2}} \times \frac{-\gamma\phi}{(1+\Phi_1+\Phi_2)^2} \frac{d(\Phi_1+\Phi_2)}{dt} \\
&= -\frac{1}{k+2} \times \frac{c_D}{1+\Phi_1+\Phi_2} \times \frac{d(\Phi_1+\Phi_2)}{dt}
\end{aligned}
$$

According to the proof of Proposition 4, $d(\Phi_1+\Phi_2)/dt < 0$, hence $dc_D/dt > 0$, hence the covariance term is positive for $dt > 0$. $\square$