Derivatives

DETERMINATION OF FORWARD AND FUTURES PRICES

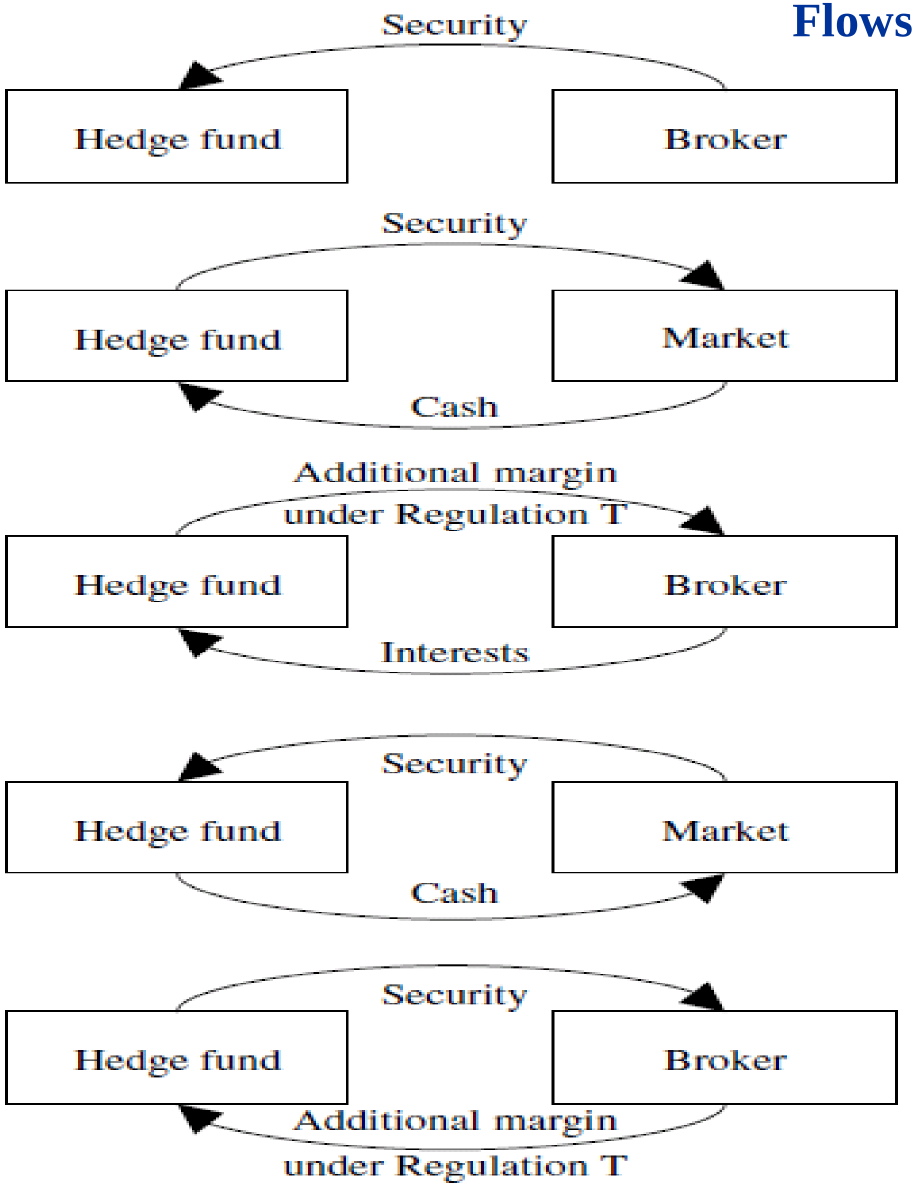
Consumption v/s Investment Assets

- **Investment Assets:** Investment assets are assets held by significant number of investors purely for investment purposes (Examples: stocks, bonds, gold, silver);
- **Consumption Assets:** Consumption assets are assets held primarily for consumption. It is not normally held for investment (Examples: copper, crude oil, corn);
- **Note:** We can use arbitrage arguments to determine the forward and futures prices of an investment asset from its spot price and other observable market variables. We cannot do this for consumption assets

Short Selling (Shorting) Revisited

Short Selling involves selling an asset that is not owned

Flows in the process of short selling



1. The hedge fund borrows a security from a broker.

2. The hedge fund sells the security back on the open market.

3. Under *Regulation T*, the Federal Reserve requires short sellers to make a cash deposit in their margin account of 50 % of the short sale value.

4. The hedge fund decides to close out the short position buying back the security on the open market.

4. Closing of the margin account.

Example: Short Selling

- Consider the position of an investor who shorts 500 shares in April when the price per share is $120 and closes out the position by buying them back in July when the price per share is $100. Suppose that a dividend of $1 per share is paid in May. Show the cash flows from the short sale.

Cash flows from purchase of shares and short sale

	Purchase of shares	
April:	Purchase 500 shares for \$120	–\$60,000
May:	Receive dividend	+\$500
July:	Sell 500 shares for \$100 per share	+\$50,000
		Net profit = –\$9,500
	Short sale of shares	
April:	Borrow 500 shares and sell them for \$120	+\$60,000
May:	Pay dividend	–\$500
July:	Buy 500 shares for \$100 per share	–\$50,000
	Replace borrowed shares to close short position	
		Net profit = +\$9,500

Assumptions for Valuing Forward and Futures Contracts:

1. The market participants are subject to no transaction costs when they trade;
2. The market participants are subject to the same tax rate on all net trading profits;
3. The market participants can borrow money at the same risk-free rate of interest as they can lend money;
4. The market participants take advantage of arbitrage opportunities as they occur

Notations for Valuing Forward and Futures Contracts

T: Time until delivery date in a forward or futures contract in years;

S_0: Spot price of the underlying asset;

F_0: Forward or futures price today;

r: Zero-coupon risk-free rate of interest per annum expressed with continuous compounding (e^{rt})for an investment maturing at the delivery date (in T years)

FORWARD PRICE FOR AN INVESTMENT ASSET: NO INCOME

1. An Arbitrage Opportunity?

- Suppose that:
  - The spot price of a non-dividend paying stock is \$40;
  - The 3-month forward price is \$43;
  - The 3-month US\$ risk-free interest rate is 5% per annum
- Is there an arbitrage opportunity?

Is there an arbitrage opportunity: Yes!

1. Borrow $40 at the risk-free interest rate of 5% per annum for 3-months [**Cost of Loan: $\$40e^{0.05*3/12}$ =$40.50 in 3-months];**
2. Buy one share @ $40 from the spot market;
3. Enter into a 3-month forward contract to sell one share @ $43;
4. At the end of 3-months, deliver (sell) the share and receive $43.

Profit (at the end of 3-months)= $43-$40.50 = $2.50

2. Another Arbitrage Opportunity?

- Suppose that:
  - The spot price of non-dividend-paying stock is $40;
  - The 3-month forward price is US$39;
  - The 1-year US$ risk-free interest rate is 5% per annum
- Is there an arbitrage opportunity?

Is there an arbitrage opportunity: Yes!

1. Short one share @ $40 in the spot market to realize $40;

2. Invest $40 at the risk-free interest rate of 5% per annum for 3-months **[Interest earned on Loan: $\$40e^{0.05*3/12}$=$40.50 in 3-months];**

3. Enter into a 3-month forward contract to buy one share @ $39;

4. At the end of 3-months, take the delivery (buy) the share and pay $39.

Profit (at the end of 3-months)= $40.50-$39 = $1.50

Summary: Arbitrage opportunities when forward price is out of line with spot price for asset providing no income. (Asset spot price = $40; risk-free interest rate = 5%; maturity of forward contract = 3 months)

Forward Price = $43	*Forward Price = $39*
Action now:	*Action now*:
Borrow $40 at 5% for 3 months	Short 1 unit of asset to realize $40
Buy one unit of asset	Invest $40 at 5% for 3 months
Enter into forward contract to sell asset in 3 months for $43	Enter into a forward contract to buy asset in 3 months for $39
Action in 3 months:	*Action in 3 months*:
Sell asset for $43	Buy asset for $39
Use $40.50 to repay loan with interest	Close short position
	Receive $40.50 from investment
Profit realized = $2.50	Profit realized = $1.50

Important Observations:

1. The first arbitrage works when the forward price is greater than $40.50;
2. The second arbitrage works when the forward price is less than $40.50;
3. Therefore, we deduce that for there to be no arbitrage opportunity the forward price must be exactly $40.50

A Generalization: A Theoretical Model

$$F_0 = S_0 e^{rT}$$

where…

S_0: The spot price of the underlying asset that provides no income;

F_0: The forward/futures price for a contract deliverable in T years;

r : The 1-year risk-free rate of interest compounded continuously;

T : The time to maturity

A Generalization: A Theoretical Model

$$F_0 = S_0 e^{rT}$$

Case I: If $F_0 > S_0 e^{rT}$, arbitrageurs can buy the underlying asset and short forward/futures contracts on the asset;

Case II: If $F_0 < S_0 e^{rT}$,arbitrageurs can short the underlying asset and enter into a long forward/futures contracts on the asset

In present example: S_0 =*$40, r=5%, T = 0.25 (3/12)*

$$F_0 = \$40e^{0.05*0.25} = \$40.50$$

Example

Consider a 4-month forward contract to buy a zero-coupon bond that will mature 1-year from today. The current price of the bond is $930. We assume that the 4-month risk-free rate of interest (continuously compounded) is 6% per annum. Calculate the forward price.

Example

Consider a 4-month forward contract to buy a zero-coupon bond that will mature 1 year from today. The current price of the bond is $930. We assume that the 4-month risk-free rate of interest (continuously compounded) is 6% per annum. Calculate the forward price

Solution:

$$F_0 = 930e^{0.06\times 4/12} = \$948.79$$

This would be the delivery price in a contract negotiated today

What If Short Sales Are Not Possible?

Short sales are not possible for all investment assets and sometimes a fee is charged for borrowing assets. As it happens, this does not matter. To derive the theoretical model, we need not be able to short the asset. All that we require is that ***there be market participants who hold the asset purely for investment***

Case I: If $\mathbf{F_0>S_0e^{rT}}$, an investor can adopt the following strategy (assume that the underlying investment asset gives rise to no storage costs or income):

1. Borrow S_0 dollars at an interest rate r for T years;
2. Buy 1 unit of the asset;
3. Take a short position in a forward contract on 1 unit of the asset

At time T, the asset is sold for F_0. An amount S_0e^{rT} is required to repay the loan (cost of loan) and the investor makes a profit of $F_0-S_0e^{rT}$.

What If Short Sales Are Not Possible?

Case II: If $\mathbf{F_0<S_0e^{rT}}$, in this case, an investor who **owns** the asset can adopt the following strategy:

1. Sell the asset for S_0;

2. Invest the proceeds at interest rate r for time T;

3. Take a long position in a forward contract on 1 unit of the asset

At time T, the cash invested has grown to S_0e^{rT}. The asset is repurchased for F_0 and the investor makes a profit of $S_0e^{rT}-F_0$ relative to the position the investor would have been in if the asset had been kept

FORWARD PRICE FOR AN INVESTMENT ASSET: KNOWN INCOME

1. An Arbitrage Opportunity?

- Suppose that:
  - The spot price of a coupon bearing bond is $900;
  - The 9-month forward price is US$910;
  - Coupon payment of $40 is expected after 4 months;
  - 4-month and 9-month risk-free interest rates (continuously compounded) are, respectively, 3% and 4% per annum
- Is there an arbitrage opportunity?

2. An Arbitrage Opportunity?

- Suppose that:
  - The spot price of a coupon bearing bond is \$900;
  - The 9-month forward price is US\$870;
  - Coupon payment of \$40 is expected after 4 months;
  - 4-month and 9-month risk-free interest rates (continuously compounded) are, respectively, 3% and 4% per annum.
- Is there an arbitrage opportunity?

Summary: Arbitrage opportunities when 9-month forward price is out of line with spot price for asset providing known cash income. (Asset spot price= \$900; income of \$40 occurs at 4 months; 4-month and 9-month risk-free rates are, respectively, 3% and 4% per annum.)

Forward price = \$910	*Forward price = \$870*
Action now:	*Action now*:
Borrow \$900: \$39.60 for 4 months and \$860.40 for 9 months	Short 1 unit of asset to realize \$900
Buy 1 unit of asset	Invest \$39.60 for 4 months and \$860.40 for 9 months
Enter into forward contract to sell asset in 9 months for \$910	Enter into a forward contract to buy asset in 9 months for \$870
Action in 4 months:	*Action in 4 months*:
Receive \$40 of income on asset	Receive \$40 from 4-month investment
Use \$40 to repay first loan with interest	Pay income of \$40 on asset
Action in 9 months:	*Action in 9 months*:
Sell asset for \$910	Receive \$886.60 from 9-month investment
Use \$886.60 to repay second loan with interest	Buy asset for \$870
	Close out short position
Profit realized = \$23.40 **= \$910-\$886.60**	Profit realized = \$16.60 **= \$886.60-\$870**

Important Observations:

1. The first strategy produces a profit when the forward price is greater than $886.60;
2. The second strategy produces a profit when the forward price is less than $886.60;
3. Therefore, we deduce that for there to be no arbitrage opportunity the forward price must be exactly $886.60

A Generalization: A Theoretical Model

$$F_0 = (S_0 - I)e^{rT}$$

where…

S_0: The spot price of the underlying asset that provides income;

F_0 : The forwards/futures price for a contract deliverable in T years;

I: The ***present value of income*** from an investment asset during the life of a forward contract;

r : The 1-year risk-free rate of interest compounded continuously;

T: The time to maturity.

A Generalization: A Theoretical Model

$$F_0 = (S_0 - I)e^{rT}$$

Case I: If $F_0 > (S_0\text{-I})e^{rT}$, an arbitrageur can lock in a profit by buying the asset and taking a short position in a forward contract on the asset;

Case II: If $F_0 < (S_0\text{-I})e^{rT}$, arbitrageurs can lock in a profit by shorting the asset and taking a long position in a forward contract.

In present example: S_0 =*$900*, I = $\$40e^{-0.03*4/12}$ =\$39.60 *r=0.04, and T = 0.75 (9/12)*

$$\boldsymbol{F_0 = (\$900 - \$39.60)e^{0.04*0.75} = \$886.60}$$

Example

Consider a 10-month forward contract on a stock when the stock price is $50. We assume that the risk-free rate of interest (continuously compounded) is 8% per annum for all maturities. We also assume that dividends of $0.75 per share are expected after 3- months, 6-months, and 9-months. Calculate the forward price.

Example

Consider a 10-month forward contract on a stock when the stock price is \$50. We assume that the risk-free rate of interest (continuously compounded) is 8% per annum for all maturities. We also assume that dividends of \$0.75 per share are expected after 3-months, 6-months, and 9-months. Calculate the forward price.

Solution: The present value of the dividends, I:

$$I = 0.75e^{-0.08\times 3/12} + 0.75e^{-0.08\times 6/12} + 0.75e^{-0.08\times 9/12} = 2.162$$

$$F_0 = (50 - 2.162)e^{0.08\times 10/12} = \$51.14$$

If the forward price were greater than \$51.14, an arbitrageur would buy the stock in the spot market and short forward contracts. If the forward price were less than \$51.14, an arbitrageur would short the stock and buy forward contracts.

FORWARD PRICE FOR AN INVESTMENT ASSET: KNOWN YIELD

Now consider a situation where the asset underlying a forward contract provides a known yield (in %) rather than a known cash income. This means that the income is known and is expressed as a percentage of the asset's price at the time the income is paid. It can be shown that:

$$F_0 = S_0 e^{(r-q)T}$$

where q is the average yield during the life of the contract (expressed with continuous compounding)

Note: As we are measuring interest rates with continuous compounding, so we will also normally measure yields with continuous compounding

- Formula to convert a rate with a compounding frequency of m times per annum to a continuously compounded rate:

$$R_c = m \ln\left(1 + \frac{R_m}{m}\right)$$

- where R_c is a rate of interest (yield) with continuous compounding and R_m is the equivalent rate with compounding m times per annum

Example

Consider a 6-month forward contract on an asset that is expected to provide income yield equal to 4% per annum with semiannual compounding. The risk-free rate of interest with continuous compounding is 10% per annum. The asset price is $25. Calculate the forward price.

Example

Consider a 6-month forward contract on an asset that is expected to provide income (yield) equal to 4% per annum with semiannual compounding. The risk-free rate of interest with continuous compounding is 10% per annum. The asset price is $25. Calculate the forward price.

Solution: First calculate q (the semiannual compounding yield) = 3.96%

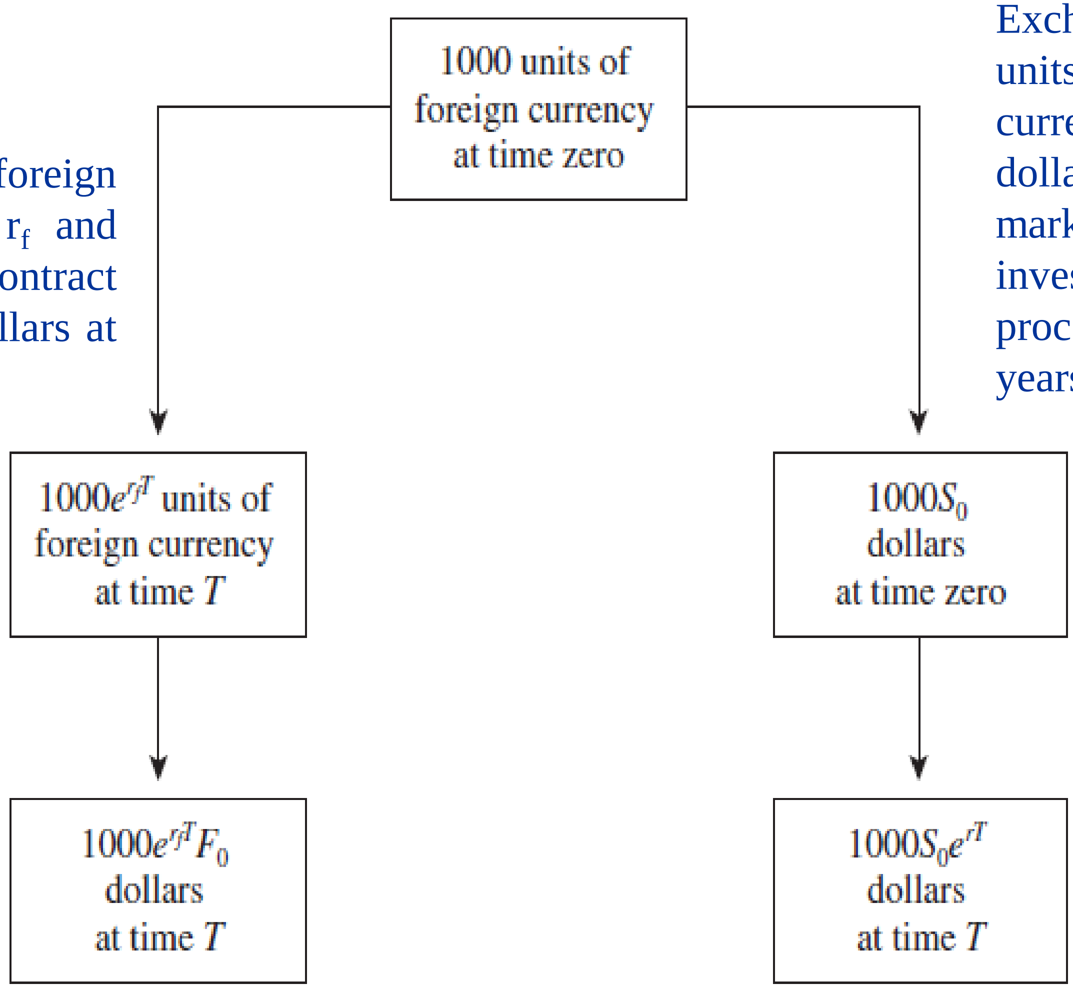
$$F_0 = 25e^{(0.10-0.0396)\times 0.5} = \$25.77$$

PRICING CURRENCY FORWARDS/FUTURES

US Investor: Two ways of converting 1,000 units of a foreign currency to dollars at time T. Here, S_0 is spot exchange rate, F_0 is forward exchange rate, and r and r_f are the dollar and foreign risk-free rates respectively.

Investing 1000 units of foreign currency for T years at r_f and entering into a forward contract to sell the proceeds for dollars at time T

Exchanging 1000 units of foreign currency for dollars in the spot market and investing the proceeds for T years at rate r



- In the absence of arbitrage opportunities, the two strategies must give the same result. Hence, the relationship between F_0 and S_0 is:

$$1{,}000e^{r_f T} F_0 = 1{,}000 S_0 e^{rT}$$

so that

$$F_0 = S_0 e^{(r-r_f)T}$$

Interest Rate Parity relationship: The spot exchange rate and the interest rate in two countries determine the forward exchange rate of their currencies. Thus, the relationship between the spot exchange rate, the interest rate in two countries and the forward rate is known as **Interest Rate Parity**

- This parity relationship implies that an investor by hedging in the forward exchange rate market realizes the same sure domestic return whether investing domestically or in a foreign country. The arbitrage process that forces interest rate parity is called ***covered interest arbitrage***.

- Suppose that the 2-year interest rates in Australia and the United States are 3% and 1%, respectively, and the spot exchange rate is 0.9800 USD per AUD. Determine the 2-year forward exchange rate?

Ans: The 2-year forward exchange rate should be:

$$F_0 = S_0 e^{(r - r_f)T}$$

$$0.9800 e^{(0.01 - 0.03) \times 2} = 0.9416$$

Case 1: Given the spot exchange rate of 0.9800 USD per AUD, suppose first that the 2-year forward exchange rate is LESS than (0.9416), say 0.9300. An arbitrageur can:

1. Borrow 1,000 AUD at 3% per annum for 2 years, convert to 980 USD @ current spot rate of 0.9800 USD per AUD and invest the USD at 1%. The principal and interest on the 1,000 AUD that are borrowed will be = $1000e^{0.03*2}$ = 1061.84;

2. Enter into a forward contract to buy 1,061.84 AUD for 1,061.84*0.93=987.51 USD.

- The 980 USD that are invested at 1% grow to $980e^{0.01*2}$ =999.80 USD in 2 years. Of this, 987.51 USD are used to purchase 1,061.84 AUD under the terms of the forward contract.
- **Risk-free Profit:** The strategy therefore gives rise to a riskless profit of 999.80-987.51=12.29 USD.

Case 2: Given the spot exchange rate of 0.9800 USD per AUD, suppose next that the 2-year forward exchange rate is GREATER than (0.9416), say 0.9600. An arbitrageur can:

1. Borrow 1,000 USD at 1% per annum for 2 years, convert to 1,000/0.9800=1,020.41 AUD, and invest the AUD at 3%. The principal and interest on the 1,000 USD that are borrowed will be = $1000e^{0.01*2}$ = 1020.20 USD; the 1,020.41 AUD that are invested at 3% grow to $1{,}020.41e^{0.03*2}$ =1,083.51 AUD in 2 years.

2. Enter into a forward contract to sell 1,083.51 AUD for 1,083.51*0.96=1,040.17 USD. The forward contract has the effect of converting this to 1,040.17 USD.

- **Risk-free Profit:** The strategy therefore gives rise to a riskless profit of 1,040.17- 1020.20=19.97 USD.

THE COST OF CARRY MODEL:
FUTURES ON CONSUMPTION ASSETS

Futures on Consumption Assets (Commodities)

Income and Storage Costs (Cost of Carrying): Commodities that are consumption assets rather than investment assets usually provide no income, but can be subject to significant storage costs. The cost of storing or carrying includes: storage costs; insurance costs; transportation costs; financing costs

Recall: We have seen that forward price for an investment asset with known income is given by:

$$F_0 = (S_0 - I)e^{rT}$$

Storage costs are generally or can be treated as *negative income*. If U is the ***present value of all the storage costs***, net of income, during the life of a forward contract, then it follows from the above equation that, the forward price is given by:

$$F_0 = (S_0 + U)e^{rT}$$

If the storage costs (net of income) incurred at any time are proportional to the price of the commodity, they can be treated as negative yield. We have seen that forward price for an investment asset with known yield is given by,

$$F_0 = S_0 e^{(r-q)T}$$

where q is the average yield during the life of the contract (expressed with continuous compounding). In this case, from above equation, we have,

$$F_0 = S_0 e^{(r+u)T}$$

where u denotes the storage costs per annum as a proportion of the spot price net of any yield earned on the asset

Example

Consider a 1-year futures contract on an investment asset that provides no income. It costs $2 per unit to store the asset, with the payment being made at the end of the year. Assume that the spot price is $450 per unit and the risk-free rate is 7% per annum for all maturities. What is the futures price.

The present value needs to calculated

Example

Consider a 1-year futures contract on an investment asset that provides no income. It costs \$2 per unit to store the asset, with the payment being made at the end of the year. Assume that the spot price is \$450 per unit and the risk-free rate is 7% per annum for all maturities. What is the futures price.

Solution: First calculate U (the present value of all the storage costs) = U = $\$2e^{-0.07*1}$ =\$1.865; the theoretical futures price, F_0, is given by:

$$F_0 = (450 + 1.865)e^{0.07\times 1} = \$484.63$$

Note: If the actual futures price is greater than \$484.63 (F_0>\$484.63), an arbitrageur can buy the asset and short 1-year futures contracts to lock in a profit. If the actual futures price is less than \$484.63 (F_0<\$484.63), an investor who already owns the asset can improve the return by selling the asset and buying futures contracts

Futures Price on Consumption Assets: Arbitrage Strategies used to determine futures prices from spot prices (The cost-and-carry arbitrage):

Suppose that, instead of equation , $F_0=(S_0+U)e^{rT}$, we have,

Case I: If $F_0>(S_0+U)e^{rT}$ (Futures Price>$484.63), an arbitrageur can implement the following strategy:

--Borrow an amount equal to S_0+U at the risk-free rate and use it to purchase one unit of the commodity and to pay storage costs;

--Buy commodity;

--Sell a futures contract on one unit of the commodity

Profit = $F_0-(S_0+U)e^{rT}$ at time T

The cost-and-carry arbitrage: An arbitrageur can borrow an amount equal to S_0+U at the risk-free rate, buy the underlying asset today for cash and sell a futures contract on the underlying asset. Carry the underlying asset till the expiration of the futures contract, then deliver the underlying against the futures contract and pay off the loan. **This strategy can be implemented for both investment and consumption assets**

0 | -- | 1

1. Borrow money
2. Buy commodity
3. Sell futures contract

4. Carry the underlying asset till the expiration and deliver the commodity against the futures contract
5. Recover money and payoff loan

The Reverse cost-and-carry arbitrage:

Suppose next that,

Case II: If $F_0 < (S_0 + U)e^{rT}$ (Futures Price<$484.63), an arbitrageur can implement the following strategy (**when the commodity is an investment asset, cannot be used for a commodity that is a consumption asset**):

--Sell the asset, (save the storage costs), and invest the proceeds at the risk-free interest rate;

--Take a long position in a futures contract and accept delivery from futures contract

Profit = $(S_0 + U)e^{rT} - F_0$ at time T

Reverse cost-and-carry arbitrage: An arbitrageur can sell the underlying asset; invest the proceeds at the risk-free interest rate and buy a futures contract. On expiration take the delivery of the underlying asset and use commodity received to cover the short sale

1. Short Sell the commodity
2. Lend/Invest money received from short sale at risk free rate
3. Buy futures contract

4. Accept delivery from futures contract
5. Use commodity received to cover the short sale

Transactions for Arbitrage Strategies

Market	Cash-and-Carry	Reverse Cash-and-Carry
Debt	Borrow funds	Lend short sale proceeds
Physical	Buy asset and store; deliver against futures	Sell asset short; secure proceeds from short sale
Futures	Sell futures	Buy futures; accept delivery; return physical asset to honor short sale commitment

Note: The case II argument $[\mathbf{F_0<(S_0+U)e^{rT}}]$ cannot be used for a commodity that is a consumption asset because individuals and companies who own a consumption commodity usually plan to use/consume it in some way. They are reluctant to sell the commodity in the spot market and buy forward or futures contracts, because forward and futures contracts cannot be used in a manufacturing process or consumed in some other way. There is therefore nothing to stop this equation, $\mathbf{F_0<(S_0+U)e^{rT}}$ from holding and all we can assert for a consumption commodity is: $\mathbf{F_0 \leq (S_0+U)e^{rT}}$ **(1)**

Accordingly, if storage costs (net of income) incurred at any time are proportional to the price of the commodity ($\mathbf{S_0}$), they can be treated as negative yield. In this case, similar to equation, $F_0 = S_0e^{(r-q)T}$ [the asset underlying a forward contract provides a known yield (in %) suggesting that the income is known and is expressed as a percentage of the asset's price at the time the income is paid where q is the average yield during the life of the contract (expressed with continuous compounding], the equivalent result of futures price from equation (1) can be expressed as: $\mathbf{F_0 \leq S_0e^{(r+u)T}}$ **(2)**

where 'u' denotes the storage costs per annum as a proportion of the spot price ($\mathbf{S_0}$) net of any yield earned on the asset

The users of a consumption commodity may feel that ownership of the physical commodity provides benefits that are not obtained by holders of futures contracts. ***So we do not necessarily have equality in equation (1) and (2)***

Convenience Yields

- Example: An oil refiner is unlikely to regard a futures contract on crude oil in the same way as crude oil held in inventory. The crude oil in inventory can be an input to the refining process, whereas a futures contract cannot be used for this purpose. In general, ownership of the physical asset enables a manufacturer to keep a production process running and perhaps profit from temporary local shortages. A futures contract does not do the same. *The benefits from holding the physical asset are sometimes* referred to as the ***convenience yield*** provided by the commodity.
- If the dollar amount of storage costs is known and has a present value U, then the ***convenience yield 'y'*** is defined such that:

$$F_0 e^{yT} = (S_0 + U)e^{rT} \qquad (3)$$

- If the storage costs per unit are a constant proportion, 'u' of the spot price [as defined earlier in equation (2)], then y is defined so that:

$$F_0 e^{yT} = S_0 e^{(r+u)T} \qquad (4)$$

Or

$$F_0 = S_0 e^{(r+u-y)T} \qquad (5)$$

The convenience yield simply measures the extent to which the left-hand side is less than the right-hand side in equation (1) and (2). For investment assets the convenience yield must be zero; otherwise, there are arbitrage opportunities.

Note: The convenience yield reflects the market's expectations regarding the future availability of the commodity. The greater the possibility that shortages will occur, the higher the convenience yield. If users of the commodity have high inventories, there is very little chance of shortages in the near future and the convenience yield tends to be low. If inventories are low, shortages are more likely and the convenience yield is usually higher.

The Cost of Carry Model

- **The Cost of Carry Model:** The relationship between futures prices and spot prices can be summarized in terms of the *cost of carry*. The cost of carry, *c*, measures ***the storage cost plus the interest that is paid to finance the asset less the income earned on the asset***
- For a non-dividend-paying stock, the cost of carry, c, is **'r'**, because there are no storage costs and no income is earned;
- For a dividend-paying stock, the cost of carry, c, is **'r-q',** because there are no storage costs but income is earned at rate q on the asset;
- For a commodity that provides income at rate q and requires storage costs at rate u, the cost of carry, c, is **r–q+u**

Define the cost of carry as c, the futures price:

- **For an investment asset:** $F_0 = S_0 e^{cT}$
- **For a consumption asset:** $F_0 = S_0 e^{(c-y)T}$ (where y is convenience yield)

OPTIONS

Options

- An option is a derivative contract between a buyer and a seller of a option, where one party (seller/writer) gives to the other, the right, but not the obligation, to buy or to sell the underlying asset on or before a specific day at an agreed-upon price;
- The agreed-upon price is known as strike price/expiration/exercise price;
- The settlement date in the contract is known as expiration date/exercise date/maturity;
- The party granting the option (right) collects a payment from the other party called as **“premium”** or price of the option

Counterparties of an Option:

- **Buyer of an Option (Option Holder):** The buyer of an option is the one who by paying the option premium has the right to buy or sell but not an obligation to exercise his option on the seller/writer. The right to buy or sell is held by the "option buyer";
- **Seller of an Option (Option Writer):** The seller of an option is the one who receives the option premium and is thereby obliged to buy/sell the asset if the buyer of the option exercises on him

Types of Options: Call and Put Options

Call Option: A call option is an option granting the right to the buyer of the option to buy the underlying asset on a specific day at an agreed upon price, but not an obligation to do so. The buyer of the call option will exercise his option to buy the underlying asset if and only if the SPOT PRICE of the underlying asset is MORE than the STRIKE PRICE (SPOT PRICE>STRIKE PRICE);

Put Option: A put option is an option granting the right to the buyer of the option to sell the underlying asset on or before a specific day at an agreed upon price, but not an obligation to do so. The buyer of the put option will exercise his option to sell the underlying asset if and only if the SPOT PRICE of the underlying asset is LESS than the STRIKE PRICE (SPOT PRICE<STRIKE PRICE)

Option Positions

Two sides to every option contract:

1. The investor who has taken the long position (i.e., has bought the option-call/put);
2. The investor who has taken a short position (i.e., has sold or written the option-call/put)

Positions in Call Options:

1. A long position in a call option (Long Call)
2. A short position in a call option (Short Call)

Positions in Put Options:

1. A long position in a put option (Long Put)
2. A short position in a put option (Short Put)

Exercising an Option

There are two exercising styles of Options:

European Option and American Option

1. European options: European options are options that can be exercised only on the expiration date

For example: All options based on indices such as Nifty, Mini Nifty, Bank Nifty, CNX IT traded at the NSE are European options which can be exercised by the buyer (of the option) only on the final settlement date or the expiry date.

2. American options: American options are options that can be exercised on any day on or before the expiry date

For example: All options on individual stocks like Reliance, SBI, and Infosys traded at the NSE are American options. They can be exercised by the buyer on any day on or before the final settlement date or the expiry date.

OTC v/s Exchange Traded Options

- Options are traded on the stock exchange as well as in over-the-counter (OTC) markets;
- Options traded on the exchanges are backed by the clearing corporation thereby minimizing the risk arising due to default by the counter parties involved;
- Options traded in the OTC market, however are not backed by the clearing corporation

Differences between Forwards/Futures and Options

Futures/Forwards	Options
Obligation: Both the buyer and the seller are under an obligation to buy or sell the underlying asset as per the contract.	The buyer of the option (call/put) has the right but not an obligation whereas the seller of the option (call/put) is under the obligation to buy/sell when the buyer exercises his right to buy/sell.
Costs: It costs nothing to enter into a forward or futures contract.	There is a cost acquiring an option which is price of the option.
Loss: The buyer and the seller are subject to unlimited risk of loss.	The seller of the option is subjected to unlimited risk of losing whereas the buyer has limited potential to lose (which is the option premium).
Profit: The buyer and the seller have potential to make unlimited gain.	The buyer of the option has potential to make unlimited gain where as the seller of the option has an limited potential to gain (which is the price of an option).

Moneyness of an Option

- “Moneyness” of an option indicates whether an option is worth exercising or not…;
- “Moneyness” of an option at any given time depends on where the SPOT PRICE of the underlying asset is at that point in time relative to the STRIKE PRICE;
- While determining moneyness of an option, the premium paid is not taken into consideration…Because the premium once paid is a sunk cost…The decision whether to exercise the option or not is NOT affected by the size of the premium;
- Moneyness of an option should not be confused with the profit and loss arising from exercising/holding an option contract;
- It should be noted that while *moneyness of an option does not depend on the premium paid but the profit/loss of a option buyer does depend on premium paid*;
- Thus, a holder of an option after exercising his option need not always make profit as the profitability also depends on the premium paid

Moneyness of Options

The three terms used to define the moneyness of an option are:

1. In-the-money;
2. Out-of-the-money;
3. At-the-money;

1. In-the-money (ITM): An option is said to be in-the-money if on exercising the option, it would produce a cash inflow for the buyer of the option:

Call Option: SPOT PRICE>STRIKE PRICE

Put Option: SPOT PRICE<STRIKE PRICE

2. Out-of-the-money (OTM): An out-of-the-money option is opposite of an in-the-money option and there won't be any cash inflow:

Call Option: SPOT PRICE<STRIKE PRICE

Put Option: SPOT PRICE>STRIKE PRICE

3. At-the-money (ATM): An at-the-money option is one in which the spot price of the underlying is equal to the strike price:

Call Option: SPOT PRICE=STRIKE PRICE

Put Option: SPOT PRICE=STRIKE PRICE

PREMIUM

Premium (Price of the option): The premium of the option has two components viz., **Intrinsic Value and Time Value**

Intrinsic Value of an option: Intrinsic value of an option at a given point in time is the amount the holder of an option will get if he exercises the option at that time. In other words, the intrinsic value of an option is the amount the option is in-the-money (ITM). Thus, if an option is out-of-the-money (OTM) or at-the-money (ATM) its intrinsic value will be zero

Intrinsic Value:

Call Option: Max [0, (Spot Price- Strike Price)]

Put Option: Max [0, (Strike Price-Spot Price)]

Time value of an option: The seller/writer of an option also charges a ‘time value’ from the buyers of the option. This is because the more the time there is for the option to expire, the greater the probability/chance that the buyer of an option will exercise his right. This is a risk for the seller and he seeks compensation for it by demanding a ‘time value’. The time value of an option is calculated by taking the difference between its premium and its intrinsic value

Note: An option that is Out-of-the-money (OTM) or At-the-money (ATM) has only time value and no intrinsic value

Example: Calculation of Intrinsic and Time Value for Call Options

Spot Price ($)	Strike price ($)	Premium ($)	Intrinsic Value ($) (Spot Price-Strike Price)	Time Value ($) (Premium - Intrinsic Value)
100	90	12	10	2
101	90	13	11	2
103	90	14		
88	90	1		
95	90	5.50		

Example: Calculations of Intrinsic and Time Value for Call Options

Spot Price ($)	Strike price **($)**	Premium **($)**	Intrinsic Value **($)** (Spot Price-Strike Price)	Time Value **($)** (Premium - Intrinsic Value)
100	**90**	**12**	**10**	**2**
101	**90**	**13**	**11**	**2**
103	**90**	**14**	**13**	**1**
88	**90**	**1**	**0**	**1**
95	**90**	**5.50**	**5**	**0.50**

Example: Calculation of Intrinsic and Time Value for Put Options

Spot Price ($)	Strike price **($)**	Premium **($)**	Intrinsic Value **($)** (Strike Price-Spot Price)	Time Value **($)** (Premium - Intrinsic Value)
100	**110**	**12**	**10**	**2**
99	**110**	**13**	**11**	**2**
97	**110**	**14**		
112	**110**	**1**		
105	**110**	**5.50**		

Example: Calculation of Intrinsic and Time Value for Put Options

Spot Price ($)	Strike price **($)**	Premium **($)**	Intrinsic Value **($)** (Strike Price- Spot Price)	Time Value **($)** (Premium - Intrinsic Value)
100	**110**	**12**	**10**	**2**
99	**110**	**13**	**11**	**2**
97	**110**	**14**	**13**	**1**
112	**110**	**1**	**0**	**1**
105	**110**	**5.50**	**5**	**0.50**

PROFIT EQUATIONS

TERMINOLOGY AND NOTATION

$$C = \text{current call price}$$
$$P = \text{current put price}$$
$$S_0 = \text{current stock price}$$
$$T = \text{time to expiration as a fraction of a year}$$
$$X = \text{exercise price}$$
$$S_T = \text{stock price at option's expiration}$$
$$\Pi = \text{profit from the strategy}$$

The following symbols indicate the number of calls, puts, or shares of stock:

$$N_C = \text{number of calls}$$
$$N_P = \text{number of puts}$$
$$N_S = \text{number of shares of stock}$$

Profit Equations

One of the powerful features of the N_C, N_P, and N_S notation is that these numbers' signs indicate whether the position is long or short. For example,

$N_C > (<) \ 0$, the investor is buying (writing) calls.

$N_P > (<) \ 0$, the investor is buying (writing) puts.

$N_S > (<) \ 0$, the investor is buying (selling short) stock.

Profit Equations: Call Option

- The Profit Equations for Call Option: Profit Equation for Calls (both buyer and writer) held to Expiration:

$$\prod = N_C[\mathrm{Max}(0, S_T - X) - C] \quad \ldots(1)$$

[Note: The sign of N_C allows the equation (1) to give the profit for both the call buyer and the call writer].

LONG CALL OPTION (BUYER/HOLDER)

I. For Buyer of One Call ($N_C = 1$), Profit:

$$\prod = [\mathrm{Max}(0, S_T - X) - C]; \text{ given } N_C > 0$$

If $S_T > X$ on expiration, the call option is in-the-money:

$$\prod = S_T - X - C$$

If $S_T \leq X$ on expiration, the call option is out-of-the-money:

$$\prod = -C$$

[Note: 1 means that 1 call option has been bought]

II. Breakeven stock Price (BEP, S_T^*) on Expiration: BEP at expiration is found by setting the profit equation equal to zero, when $S_T > X$. Next for S_T^* (BEP) the equation is solved:

So when $S_T > X$ on expiration (i.e, call option is in-the-money):

$$\prod = S_T - X - C$$

$$\prod = S_T^* - X - C = 0$$

$$\textbf{BEP} = S_T^* = X + C$$

Profit Equations: Call Option

- Profit Equation for Calls (both buyer and writer) held to Expiration:

$$\prod = N_C[Max(0, S_T - X) - C] \quad \ldots(1)$$

SHORT CALL OPTION (SELLER/WRITER)

I. For Writer of One Call ($N_C = -1$), Profit:

$$\prod = -Max(0, S_T - X) + C; \text{ given } N_C<0$$

If $S_T>X$ on expiration, the call option is in-the-money:

$$\prod = - S_T + X + C$$

If $S_T \leq X$ on expiration, the call option is out-of-the-money:

$$\prod = C$$

[Note: – 1 means that 1 call option has been sold]

II. Breakeven stock Price (BEP) on Expiration: BEP at expiration is found by setting the profit equation equal to zero, when $S_T > X$. Next for S_T^* (BEP) the equation is solved:

So when $S_T > X$ on expiration (i.e, call option is in-the-money):

$$\prod = -S_T + X + C$$

$$\prod = -S_T^* + X + C = 0$$

$$\text{BEP} = S_T^* = X + C$$

Profit Equations: Put Option

- The Profit Equations for Put Option: Profit Equation for Put (both buyer and writer) held to Expiration:

$$\prod = \mathbf{N_P[Max(0, X - S_T) - P]} \quad \mathbf{\ldots(2)}$$

<u>LONG PUT OPTION (BUYER/HOLDER)</u>

I. For Buyer of One Put ($N_P = 1$), Profit:

$$\prod = \mathbf{[Max(0, X - S_T) - P]};\ \text{given } N_P > 0$$

If $S_T < X$ on expiration, the Put option is in-the-money:

$$\prod = \mathbf{X - S_T - P}$$

If $S_T \geq X$ on expiration, the Put option is out-of-the-money:

$$\prod = \mathbf{-P}$$

[Note: 1 means that 1 put option has been bought]

II. Breakeven stock Price (BEP, S_T^*) on Expiration: BEP at expiration is found by setting the profit equation equal to zero, when $S_T < X$. Next, for S_T^* (BEP) the equation is solved:

So when $S_T < X$ on expiration (i.e, Put option is in-the-money):

$$\prod = X - S_T - P$$

$$\prod = X - S_T^* - P = 0$$

$$\mathbf{BEP} = S_T^* = X - P$$

The Profit Equations for Put Option:

- Profit Equation for Put (both buyer and writer) held to Expiration:

$$\prod = \mathbf{N_P[Max(0, X - S_T) - P]} \quad \ldots(2)$$

SHORT PUT OPTION (SELLER/WRITER)

I. For Writer of One Put ($N_P = -1$), Profit:

$$\prod = -\mathbf{Max(0, X - S_T) + P} \text{ ; given } \mathbf{N_P<0}$$

If $S_T<X$ on expiration, the Put option is in-the-money:

$$\prod = \mathbf{- X + S_T + P}$$

If $S_T \geq X$ on expiration, the Put option is out-of-the-money:

$$\prod = \mathbf{P}$$

[Note: **–** 1 means that 1 put option has been sold]

II. Breakeven stock Price (BEP) on Expiration: BEP at expiration is found by setting the profit equation equal to zero, when $S_T<X$. Next for S_T^* the equation is solved:

So when $S_T<X$ on expiration (i.e, Put option is in-the-money):

$$\prod = -X + S_T + P$$

$$\prod = -X + S_T^* + P = 0$$

$$BEP = S_T^* = X - P$$

Stock Transactions: Profit Equations

(Combining stocks with options is an attractive strategy)

BUY STOCK

I. The simplest transaction is the purchase of stock (Let $N_S = 1$). The profit equation is:

$$\prod = N_S(S_T - S_0);\ \text{given } N_S>0$$

[Note: Let $N_S = 1$, means that 1 share of stock has been bought]

SELL SHORT STOCK

II. The short sale of stock is the mirror image of the purchase of stock (Let $N_S = -1$). The profit equation is:

$$\prod = -(S_T - S_0);\ \text{given } N_S<0$$

$$\prod = -S_T + S_0$$

[Note: Let $N_S = -1$, means that 1 share has been sold short]

STOCK TRANSACTIONS

DCRB OPTION DATA, MAY 14

	Calls			Puts		
Exercise Price	May	June	July	May	June	July
120	8.75	15.40	20.90	2.75	9.25	13.65
125	5.75	13.50	18.60	4.60	11.50	16.60
130	3.60	11.35	16.40	7.35	14.25	19.65

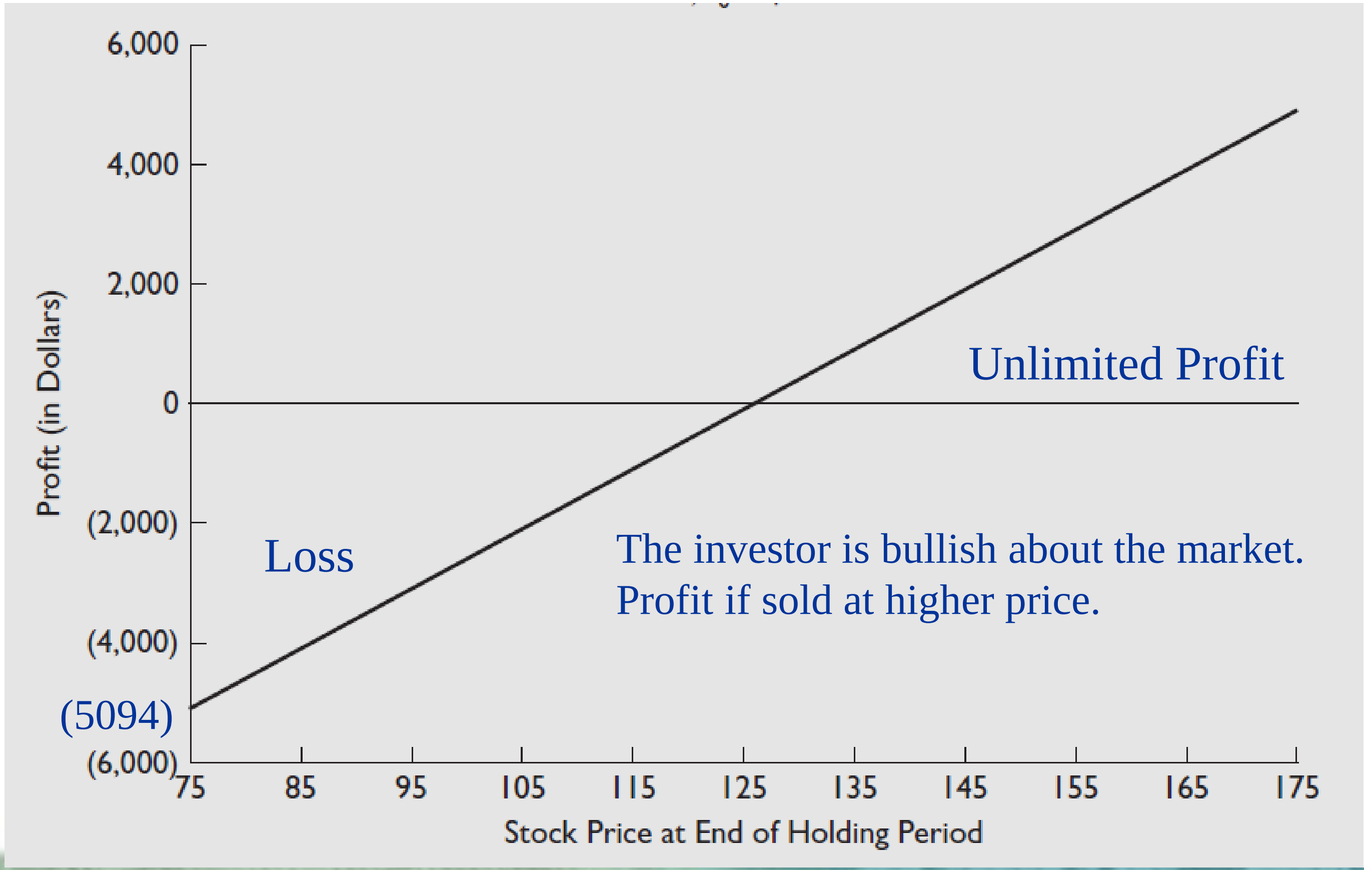
Current stock price: 125.94
Expirations: May 21, June 18, July 16
Risk-free rates (continuously compounded): 0.0447 (May); 0.0446 (June); 0.0453 (July)

BUY STOCK

I. The simplest transaction is the purchase (buy) of stock The profit equation is:

$$\prod = N_S(S_T - S_0);\ \text{given } N_S>0$$

S_0=$125.94; N_S = 100 (which means that 100 shares have been bought)



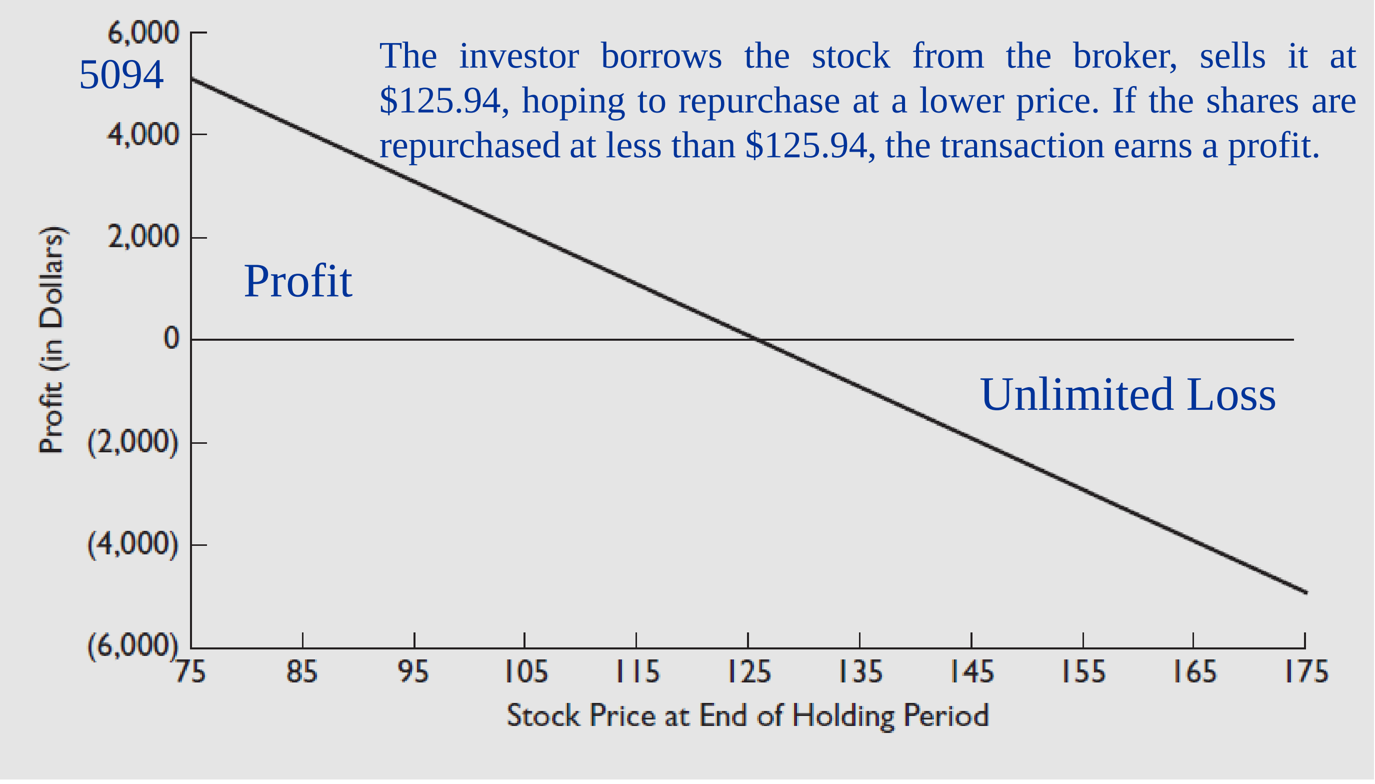
SELL SHORT STOCK

II. The short sale of stock is the mirror image of the purchase of stock (Let $N_S = -1$). The profit equation is:

$$\prod = -N_S(S_T - S_0);\ \text{given } N_S<0$$

$$\prod = -S_T + S_0$$

S_0=\$125.94; N_S = -100 (which means that 100 shares have been sold short)



BASIC OPTION STRATEGIES

Basic Options Strategies

An option strategy is implemented to try and maximize gains from the movement in the underlying price of an asset:

1. **Long Options Strategy;**
2. **Short Options Strategy**

1. **Long Options Strategy:** A long options strategy is a strategy where options are bought (call/put) to make money with a view that the options will expire in-the-money at the expiry date:

- An investor with a BULLISH opinion on the underlying will buy a call option in the hope that prices will rise and he will exercise the option leading to profit
- An investor with a BEARISH opinion on the underlying will buy a put option in the hope that prices will fall and he will exercise the option leading to profit

2. **Short Option Strategy:** A short options strategy is a strategy where options are sold (call/put) to make money upfront with a view that the options will expire out-of-money at the expiry date:

- An investor with a BEARISH view on the underlying will sell a call option in the hope that prices will fall and the buyer will not exercise the option leading to profit for the seller
- An investor with a BULLISH opinion on the underlying will sell a put option in the hope that prices will rise and the buyer will not exercise the option leading to profit for the seller

Summary of Basic Options Strategies used to maximize returns:

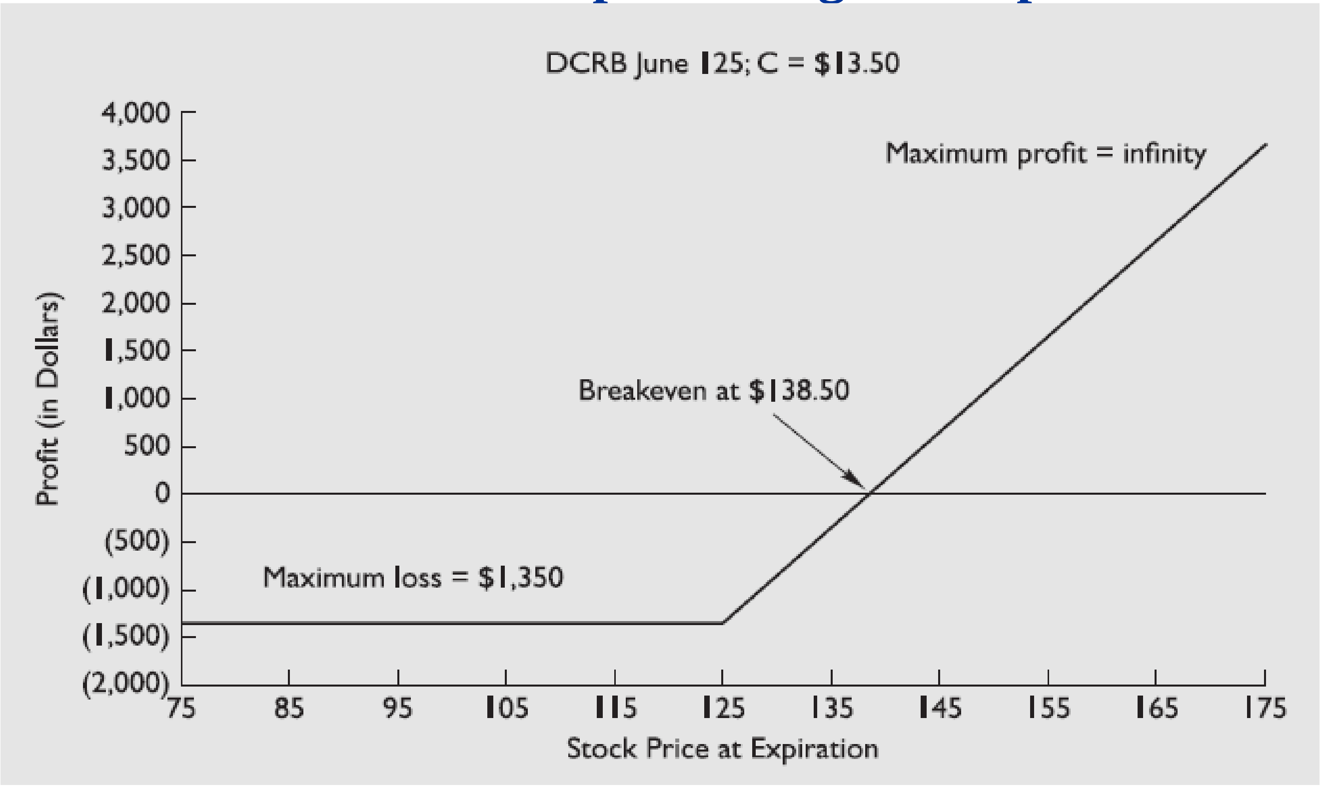
Long Option Strategy	Short Option Strategy
Bullish Opinion : Buy a Call Option	Bearish Opinion: Sell a Call Option
Bearish Opinion: Buy a Put Option	Bullish Opinion : Sell a Put Option
Limited Risk and Unlimited Profits (call option) and Limited Profits-full strike price if the price of the underlying falls to zero (put options)	Unlimited Risk (call option) and Limited Loss-full strike price if the price of the underlying falls to zero (put options) and Limited Profits

DCRB OPTION DATA, MAY 14

	Calls			Puts		
Exercise Price	May	June	July	May	June	July
120	8.75	15.40	20.90	2.75	9.25	13.65
125	5.75	13.50	18.60	4.60	11.50	16.60
130	3.60	11.35	16.40	7.35	14.25	19.65

Current stock price: 125.94
Expirations: May 21, June 18, July 16
Risk-free rates (continuously compounded): 0.0447 (May); 0.0446 (June); 0.0453 (July)

Profit from a European Long Call Option



The profit from a call option purchase is

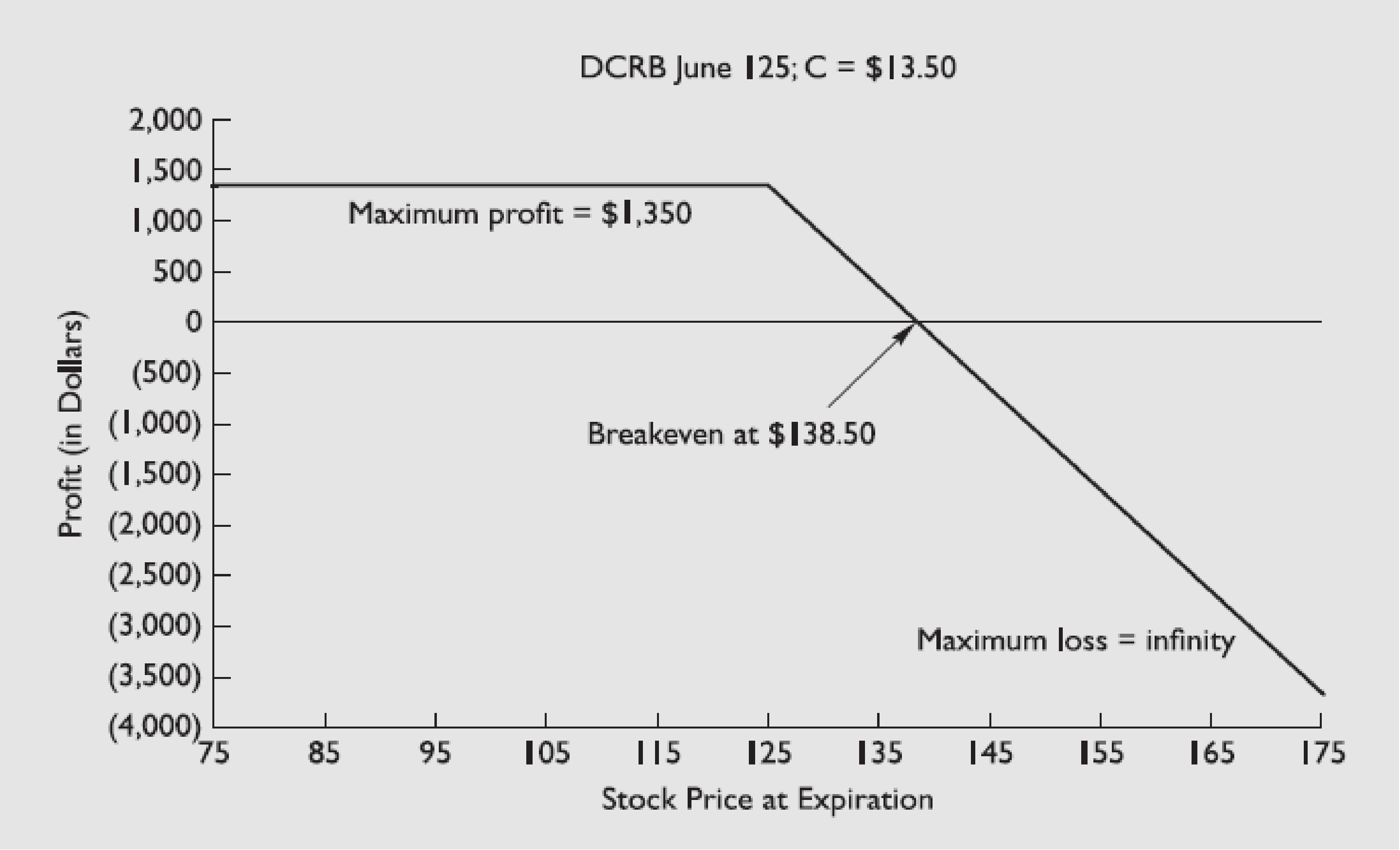
$$\Pi = N_C[\text{Max}(0, S_T - X) - C], \text{ given that } N_C > 0.$$

$$\Pi = S_T - X - C \qquad \text{if } S_T > X.$$

$$\Pi = -C \qquad \text{if } S_T \leq X.$$

$$S_T^* = X + C.$$

Profit from a European Short Call Option



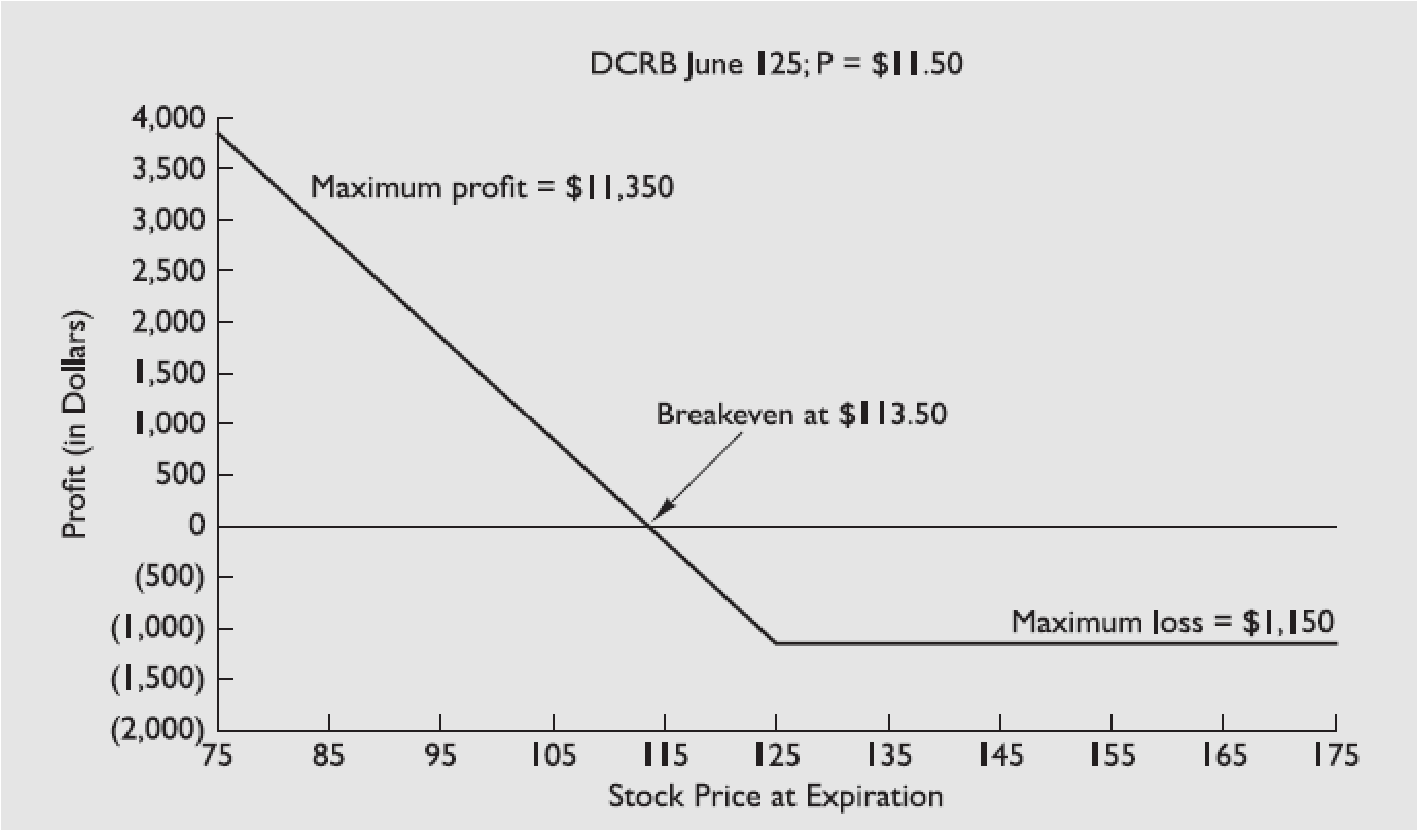
$$\Pi = N_C[\text{Max}(0, S_T - X) - C], \text{ given that } N_C < 0.$$

$$\Pi = C \qquad \text{if } S_T \leq X.$$

$$\Pi = -S_T + X + C \qquad \text{if } S_T > X.$$

$$S_T^* = X + C.$$

Profit from a European Long Put Option

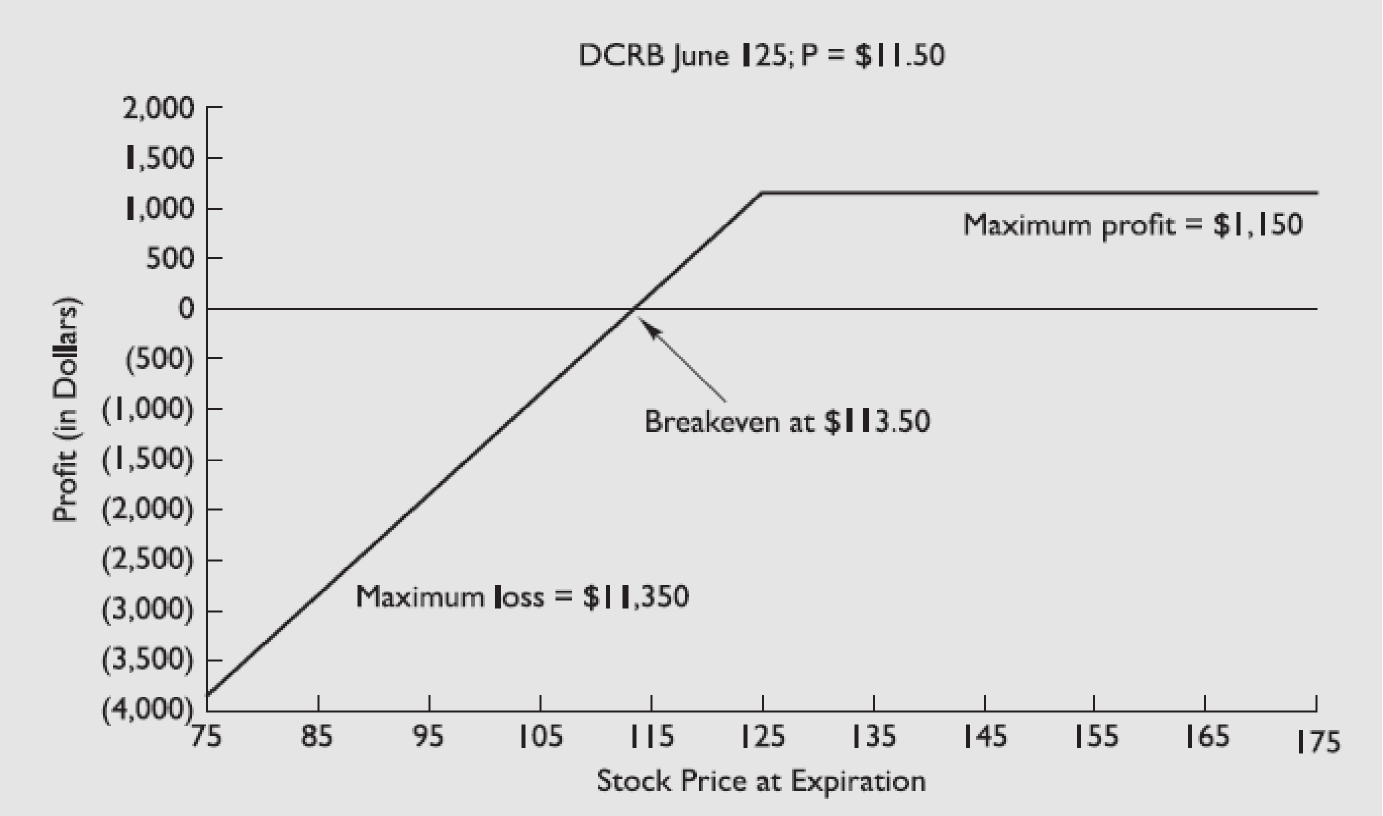


$$\Pi = N_P[\mathrm{Max}(0, X - S_T) - P], \text{ given that } N_P > 0.$$

$$\Pi = X - S_T - P \qquad \text{if } S_T < X.$$
$$\Pi = -P \qquad \text{if } S_T \geq X.$$

$$S_T^* = X - P.$$

Profit from a European Short Put Option



$$\Pi = N_P[\text{Max}(0, X - S_T) - P], \text{ given that } N_P < 0.$$

$$\Pi = -X + S_T + P \qquad \text{if } S_T < X.$$

$$\Pi = P \qquad \text{if } S_T \geq X.$$

$$S_T^* = X - P.$$

SPECULATING WITH OPTIONS:
BASIC SPECULATING OPTIONS STRATEGIES

BASIC SPECULATING OPTIONS STRATEGIES

CALLS AND STOCK	PUTS AND STOCK
THE COVERED CALL: BUY STOCK AND SHORT CALL	THE COVERED PUT: SELL SHORT STOCK AND SHORT PUT
THE PROTECTIVE CALL: SELL SHORT STOCK AND LONG CALL	**PROTECTIVE PUT: BUY STOCK AND LONG PUT**

CALLS AND STOCK: THE COVERED CALL

I. THE COVERED CALL: BUY STOCK AND SHORT CALL

- A covered call is a strategy simultaneously involving buying (owns) a stock and writing a call on the same stock;
- If an investor is BULLISH about the stock price he may purchase it and write a call option. If the call is not exercised, the investor pockets the premium. If the call is exercised, the investor simply delivers the stock;
- If the option writer owns the stock there is no risk of buying it in the market at a potentially high price compared to uncovered call, in which the investor writes a call on a stock not owned;
- Also, the holder of stock with no options written thereon is exposed to substantial risk of the stock price moving down. By writing a call against that stock, the investor reduces the downside risk. Because if the stock price falls substantially, the loss will be mitigated by the premium received for writing the call;
- The profits and losses for the covered call depends on the where the stock price is at expiration.

DCRB OPTION DATA, MAY 14

	Calls			Puts		
Exercise Price	May	June	July	May	June	July
120	8.75	15.40	20.90	2.75	9.25	13.65
125	5.75	13.50	18.60	4.60	11.50	16.60
130	3.60	11.35	16.40	7.35	14.25	19.65

Current stock price: 125.94
Expirations: May 21, June 18, July 16
Risk-free rates (continuously compounded): 0.0447 (May); 0.0446 (June); 0.0453 (July)

Payoffs: Determining the profits from the COVERED CALL strategy is to add the profit equations from the strategy of BUY STOCK AND SHORT CALL:

$$\Pi = N_S(S_T - S_0) + N_C[\text{Max}(0, S_T - X) - C]$$
given that $N_S > 0, N_C < 0$, and $N_S = -N_C$.

Consider the case of one share of stock and one short call, $N_S = 1$, $N_C = -1$, the profit equation is:

$$\Pi = S_T - S_0 - \text{Max}(0, S_T - X) + C.$$

On expiration,

$$\Pi = S_T - S_0 + C \qquad \text{if } S_T \leq X.$$

$$\Pi = S_T - S_0 - S_T + X + C = X - S_0 + C \quad \text{if } S_T > X.$$

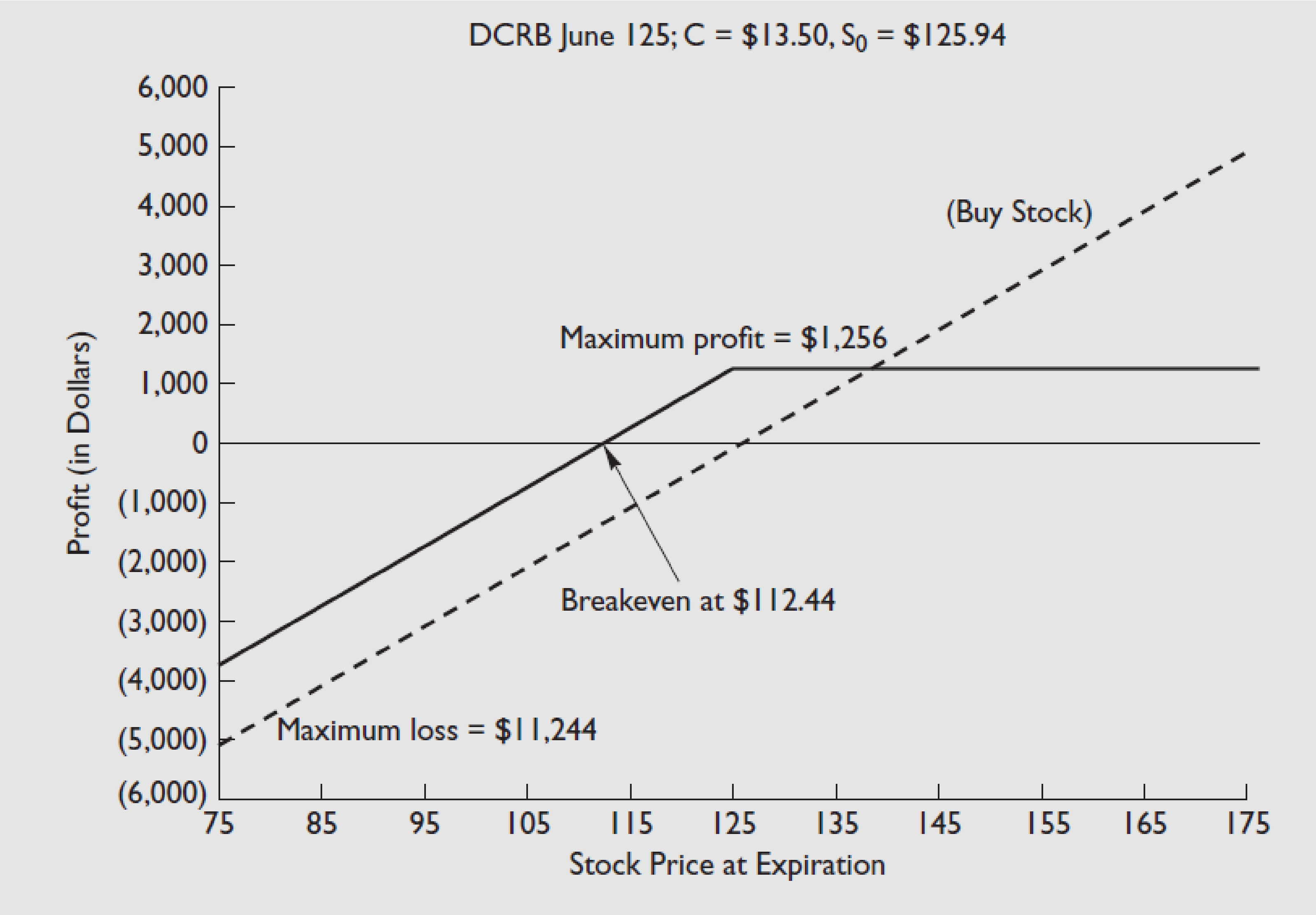
Breakeven stock price (BEP): BEP occurs where the profit is zero. This happens when the call ends up out-of-the-money, ($S_T \leq X$). Setting the profit equal to zero for the case where the call is out-of-the-money,

$$\Pi = S_T^* - S_0 + C = 0,$$

and solving for S^*_T gives a breakeven of

$$S_T^* = S_0 - C.$$

THE COVERED CALL: BUY STOCK AND SHORT CALL



PUTS AND STOCK: THE PROTECTIVE PUT

II. PROTECTIVE PUT: BUY STOCK AND LONG PUT

- A protective put is a strategy simultaneously involving buying a stock and a long put on the same stock. The put provides a minimum selling price for the stock;
- If an investor is bullish about the stock price he may purchase it. However, the investor faces the downside risk (the price of the stock may fall). Therefore, to protect himself from this downside price risk, he simultaneously buys a put option (long put) on the same stock;
- The profits and losses for the protective put depends on the where the stock price is at expiration.

DCRB OPTION DATA, MAY 14

	Calls			Puts		
Exercise Price	May	June	July	May	June	July
120	8.75	15.40	20.90	2.75	9.25	13.65
125	5.75	13.50	18.60	4.60	11.50	16.60
130	3.60	11.35	16.40	7.35	14.25	19.65

Current stock price: 125.94
Expirations: May 21, June 18, July 16
Risk-free rates (continuously compounded): 0.0447 (May); 0.0446 (June); 0.0453 (July)

Payoffs: Determining the profits from the PROTECTIVE PUT strategy is to add the profit equations from the strategy of buying stock and buying a put:

$$\Pi = N_S(S_T - S_0) + N_P[\text{Max}(0, X - S_T) - P]$$
$$\text{given that } N_S > 0, N_P > 0, \text{ and } N_S = N_P.$$

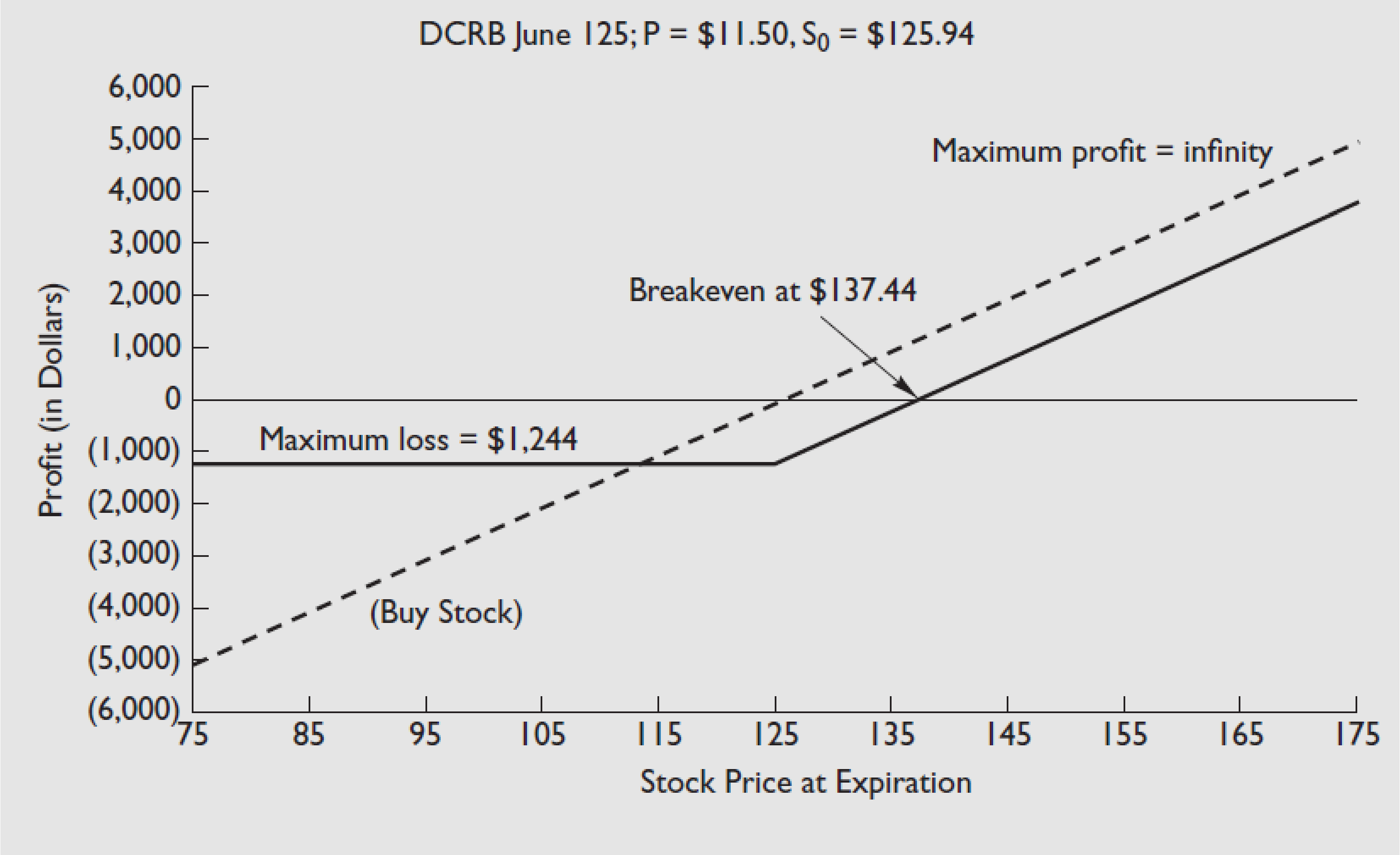
Consider the case of one share of stock and one long put, $N_S = 1$, $N_P = 1$, the profit equation on expiration is:

$$\Pi = S_T - S_0 - P \qquad \text{if } S_T \geq X.$$
$$\Pi = S_T - S_0 + X - S_T - P = X - S_0 - P \qquad \text{if } S_T < X.$$

Breakeven stock price (BEP): BEP occurs where the profit is zero. This happens when the stock price at expiration exceeds the exercise price, $S_T > X$. Setting this profit to zero and solving for the breakeven stock price, S^*_T gives:

$$\Pi = S^*_T - S_0 - P = 0,$$
$$S^*_T = P + S_0.$$

PROTECTIVE PUT: BUY STOCK AND LONG PUT



III. THE COVERED PUT:

SELL SHORT STOCK AND SHORT PUT

- The investor shorts a stock because he is bearish about it. In writing a covered put the investor expects the stock price will decline relative to the strike price and thus enhance his income by receiving the put option's premium;
- If the stock price falls below the exercise price, the holder of put will exercise his right to sell the stock and covered put investor will have to buy the stock at the strike price (which is anyway his target price to repurchase the stock);
- The investor is covered here because he shorted the stock in the first place;
- However, if the stock price is much greater than the strike price at expiry then the writer will lose money;
- Risk is unlimited if the price of the stock rises substantially.

IV. THE PROTECTIVE CALL:

SELL SHORT STOCK AND LONG CALL

- The investor shorts a stock because he is bearish about it. In buying a protective call the investor strategy is to protect profits from the rising stock price with respect to the strike price;
- If the stock price falls the investor gains in the downward fall in the price. Thus, if the stock price is less than the strike at expiry then the option will not be exercised and the call buyer will only lose the premium paid;
- However, in case if the stock price rises the investor limits his loss by exercising his call option.