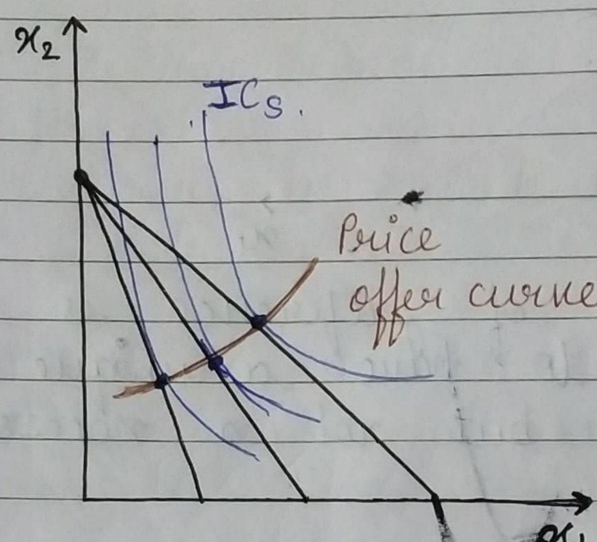
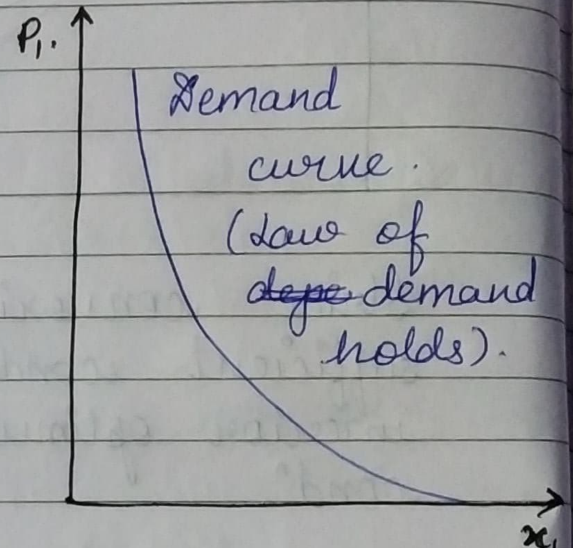
∴ As the price of x_1 decreases, the quantity of x_1 in the optimal bundle increases i.e. as price of x_1 decreases consumption of x_1 increases.





★ Price Consumption Curve (PCC)

PCC for x_1: It is the locus of the MPBs corresponding to different prices of x_1, given income and P_2.

Shape of PCC is related to price elasticity.

In terms of PCC we can define ordinary good & ~~Go~~ Giffen good.

If law of demand is violated, then the PCC for the good will be backward bending. ∴ A backward bending PCC corresponds to a ~~f~~ Giffen good.

Page No.
Date :

1) An upward rising PCC corresponds to inelastic demand.

Proof: We are considering the PCC of good 1. i.e. price of x_1 is changing.

Along any PCC budget constraint is satisfied.

$x_1' > x_1^0$, $x_2' > x_2^0$ (As PCC is upward rising ∴ quantity consumed of both goods will increase).

$$P_1^0 x_1^0 + P_2^0 x_2^0 = \overline{M} = P_1' x_1' + P_2^0 x_2'$$

Since P_2 is unchanged $x_2' > x_2^0$

$$\therefore \cancel{P_2^0 x_2^0} \quad P_2^0 x_2' > P_2^0 x_2^0$$

Since M is unchanged, $P_1' x_1' < P_1^0 x_1^0$
Since $P_1' < P_1^0$ ∴ $x_1^0 < x_1'$
∴ Change in price will be more than the change in quantity

$$\therefore \Delta x < \Delta P$$

$$\therefore \cancel{|e_P < 1|} \quad |e_P| < 1.$$

2) Downward sloping PCC corresponds to elastic demand.

3) Straight line PCC ~~core~~ corresponds to unitary elastic demand.

Proof 2:

$x_1' > x_1^0, \; x_2' < x_2^0$.

P_2^0 remains unchanged & P_1^0 is decreasing

$\therefore \; P_1' < P_1^0$.

$P_1^0 x_1^0 + P_2^0 x_2^0 = \overline{M} = P_1' x_1' + P_2^0 x_2'$

$P_2^0 x_2' < P_2^0 x_2^0$

$P_1' x_1' > P_1^0 x_1^0$

$P_1' < P_1^0 \qquad x_1' > x_1^0$.

$\therefore \; \Delta x > \Delta P$

$\therefore \; |e_p| > 1$.

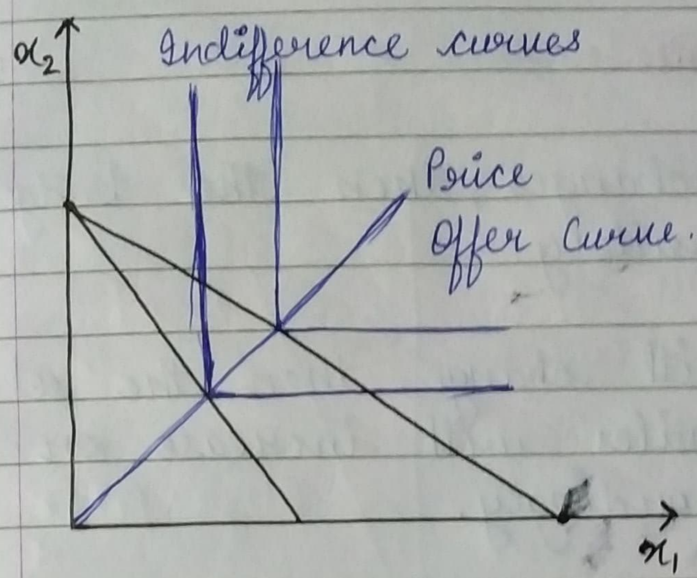
Perfect Complements

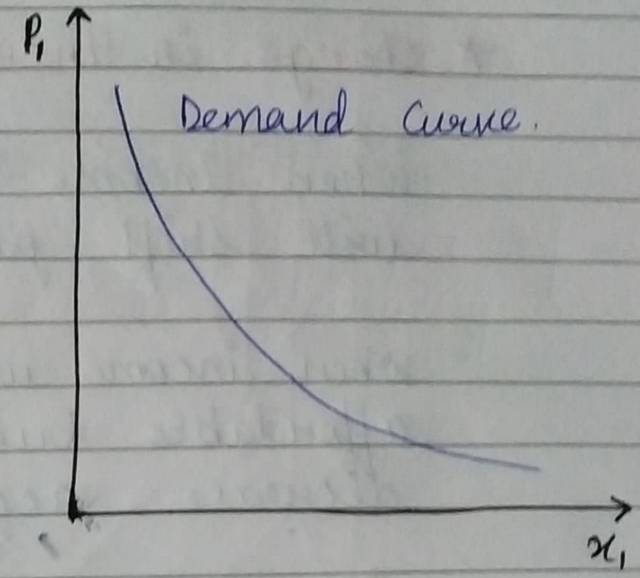
Let the two goods be consumed in the following proportion:

$$x_1 : x_2 = a : b$$

$$\overline{M} = P_1 x_1 + P_2 x_2 = P_1 x_1 + P_2 x_1 \frac{b}{a}$$

$$\therefore \; x_1 = \frac{\overline{M}}{P_1 + P_2 \frac{b}{a}}$$

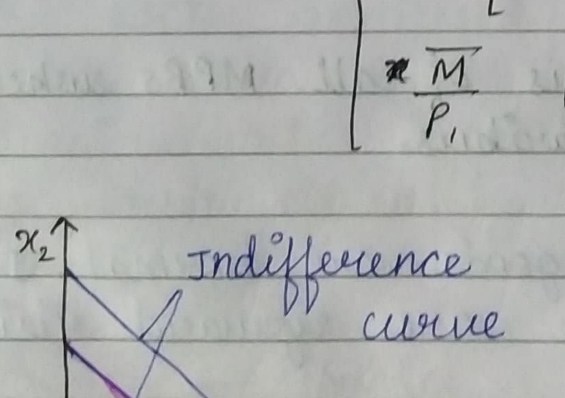


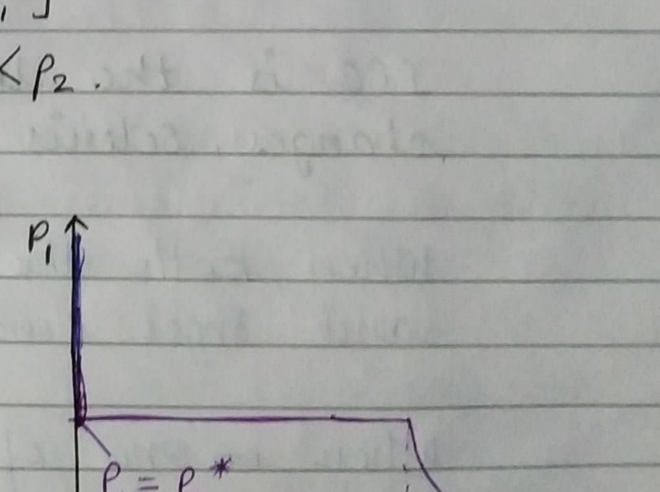


Perfect Substitutes

When price of both the goods are same consumer is indifferent between consuming the two goods; otherwise the consumer will consume the cheaper good.

$$\therefore \quad x_1 = \begin{cases} 0 & \forall \; p_1 > p_2 \\ x_1 \in \left[0, \frac{M}{P_1}\right] & \forall \; P_1 = P_2 \\ \frac{M}{P_1} & p_1 < p_2 . \end{cases}$$

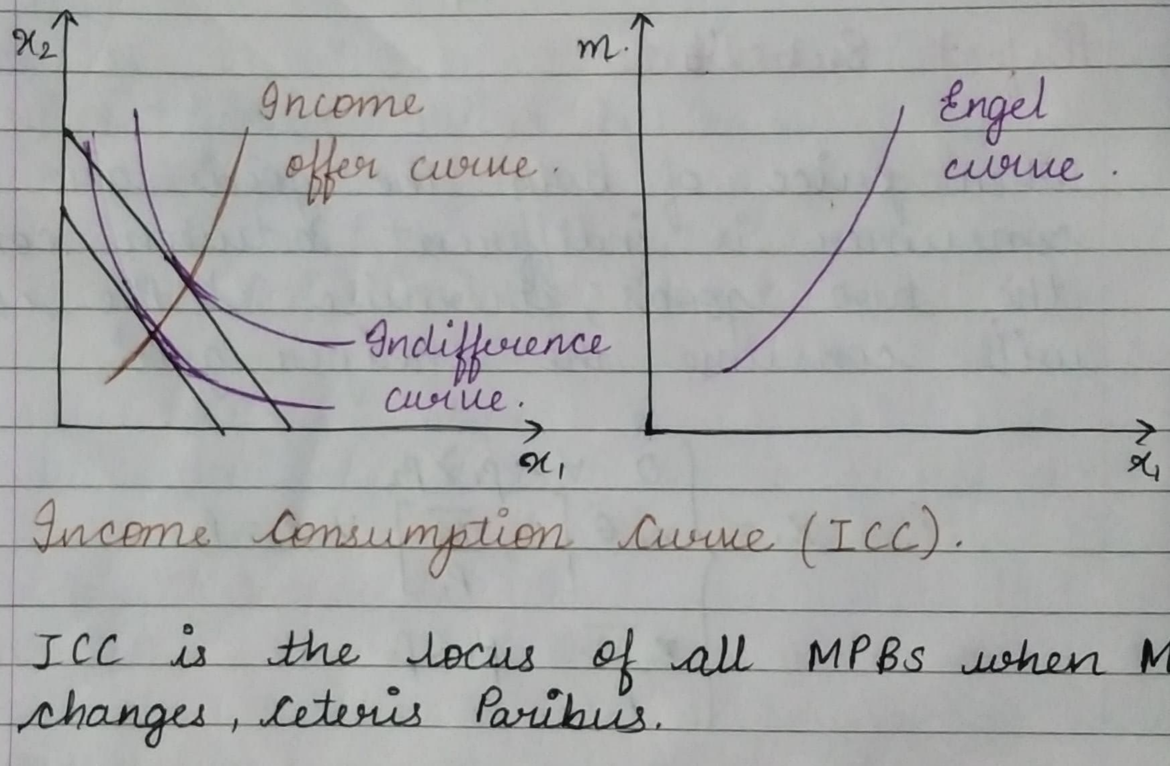




* Change in Income

When income changes, then the budget line will shift parallely.

When income will change, then the set of affordable bundles will increase or decrease accordingly.



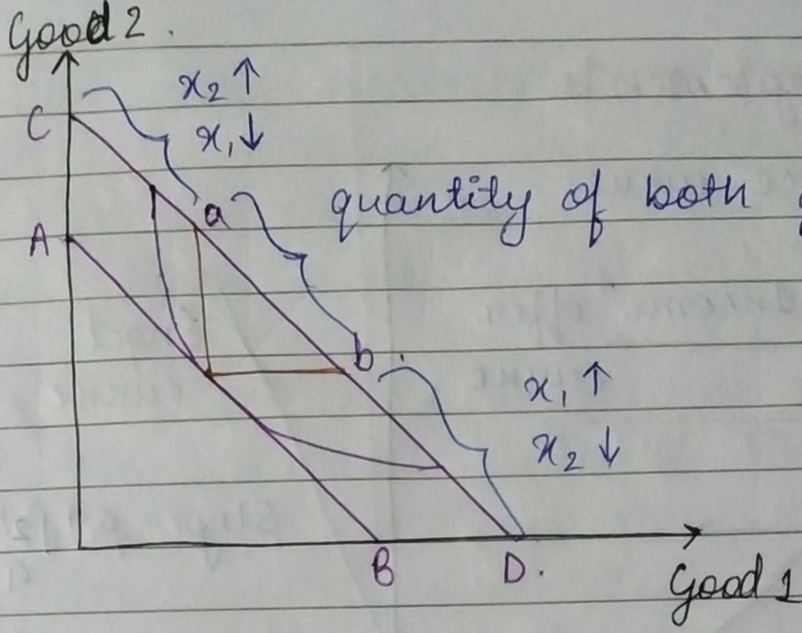
Income Consumption Curve (ICC).

ICC is the locus of all MPBs when M changes, ceteris Paribus.

When both the goods are normal, ICC and Engel curve are upward rising.

When one of the goods is normal, ICC and Engel curve are backward bending

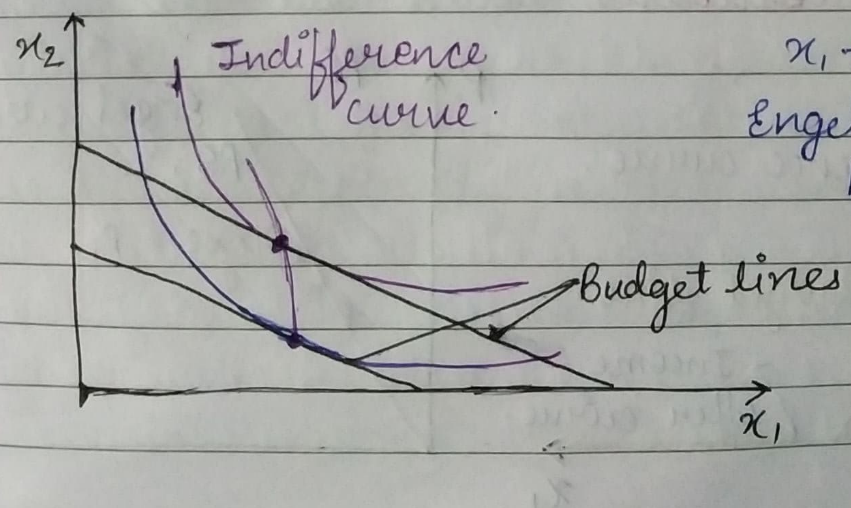
Both goods cannot be inferior.



If the optimum is: ~~in the~~
In the range ab, both are normal.
In Ca, good 1 inferior, In bd, good 2 inferior.

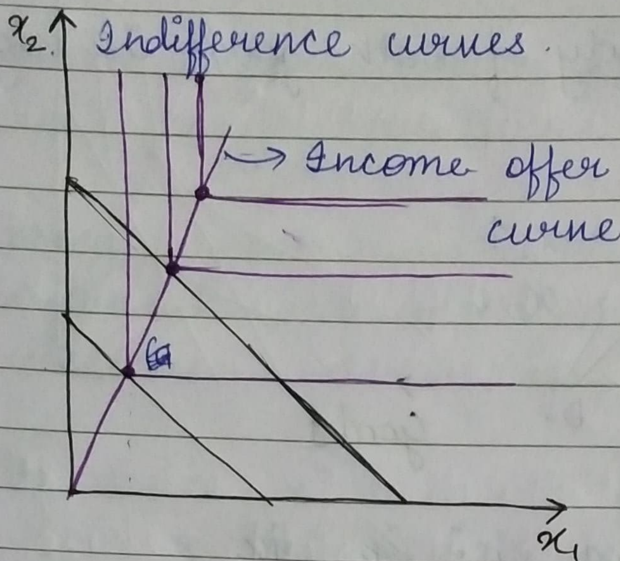
∴ If income increases & the quantity consumed of good decreases then it is inferior & if the quantity consumed of good increases then it is normal.

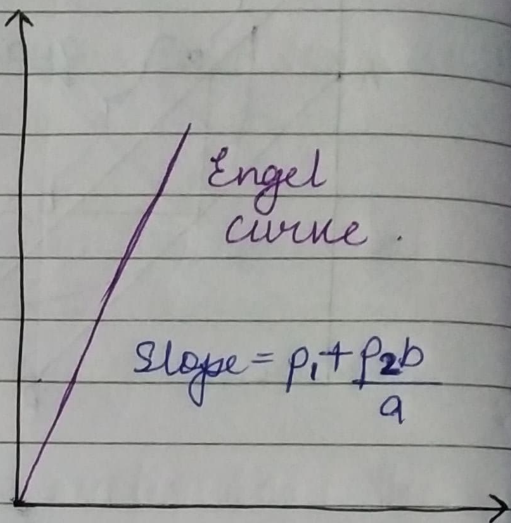
Engel curve is the ~~cur~~ graph between income & quantity consumed of a good when price remains constant.



$x_1 \rightarrow$ Inferior good.
Engel curve $\rightarrow$ backward bending

Perfect complements





$x_1 : x_2 = a : b.$

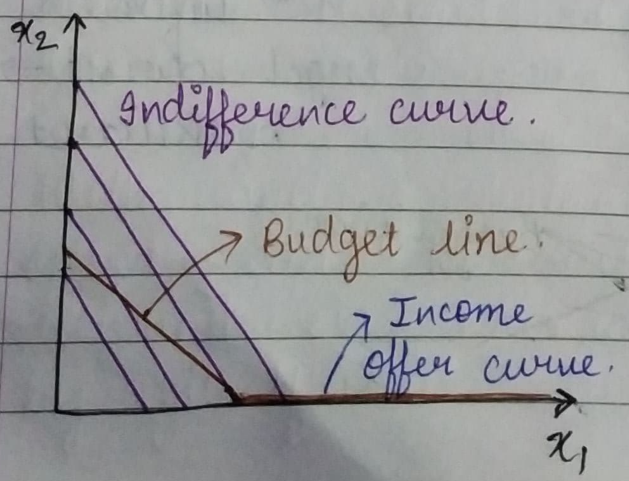
$$\overline{M} = P_1 x_1 + P_2 x_2 = P_1 x_1 + P_2 x_1 \frac{b}{a}$$

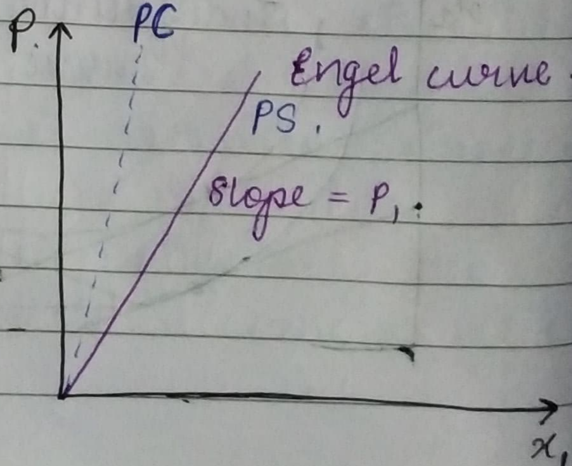
$$x_1 = \frac{\overline{M}}{P_1 + P_2 \frac{b}{a}}.$$

$$\overline{M} = \left(P_1 + P_2 \frac{b}{a}\right) \overline{x}$$

$\frac{b}{a} = 1$ Slope $= p_1 + p_2.$

Perfect Substitutes





We can draw the ICC only when $P_1 < P_2$ because for $P_1 > P_2$ $x_1^* = 0$ ∴ ICC will coincide with the horizontal axis in that case.

If $P_1 = P_2$, then budget line is coinciding with the IC ∴ there is no unique optimal ∴ We cannot derive the ICC in this case. because when income will change, the new budget line will again coincide with another indifference curve, ∴ we cant trace out the locus of the optimal bundle.

Slope of Engel curve for perfect complement $= P_1 + P_2$

Slope for perfect substitute $= P_1$.

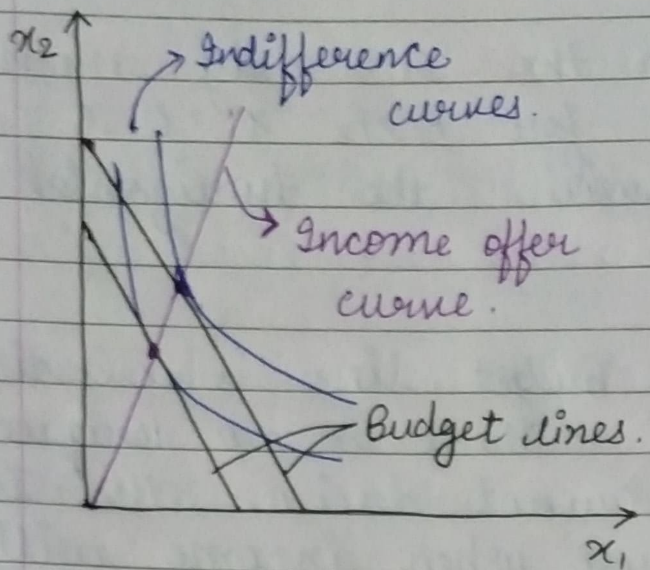
∴ Engel curve for perfect complement is always steeper than the Engel curve for perfect substitute.

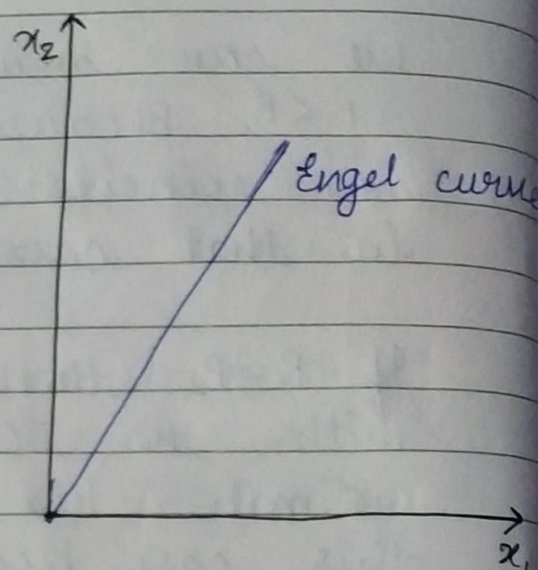
Homothetic Preference.

A preference is said to be homothetic if it depends only on the ratio in which the two goods are consumed, e.g., perfect substitutes, and perfect complements.

- If $(x_1, x_2) P (y_1, y_2)$ then $(tx_1, tx_2) P (ty_1, ty_2)$ for all $t > 0$.

∴ $MRS(x_1, x_2) = MRS(tx_1, tx_2) \ \forall \ t > 0$.





In case of homothetic preference, the goods have to ~~t~~ be normal.

When x increases with income, then it is luxury good.

When x does not change with increase in income, then it is necessary good.

When ~~x~~ ~~doe~~ x increases with fall in income, then it is inferior good.

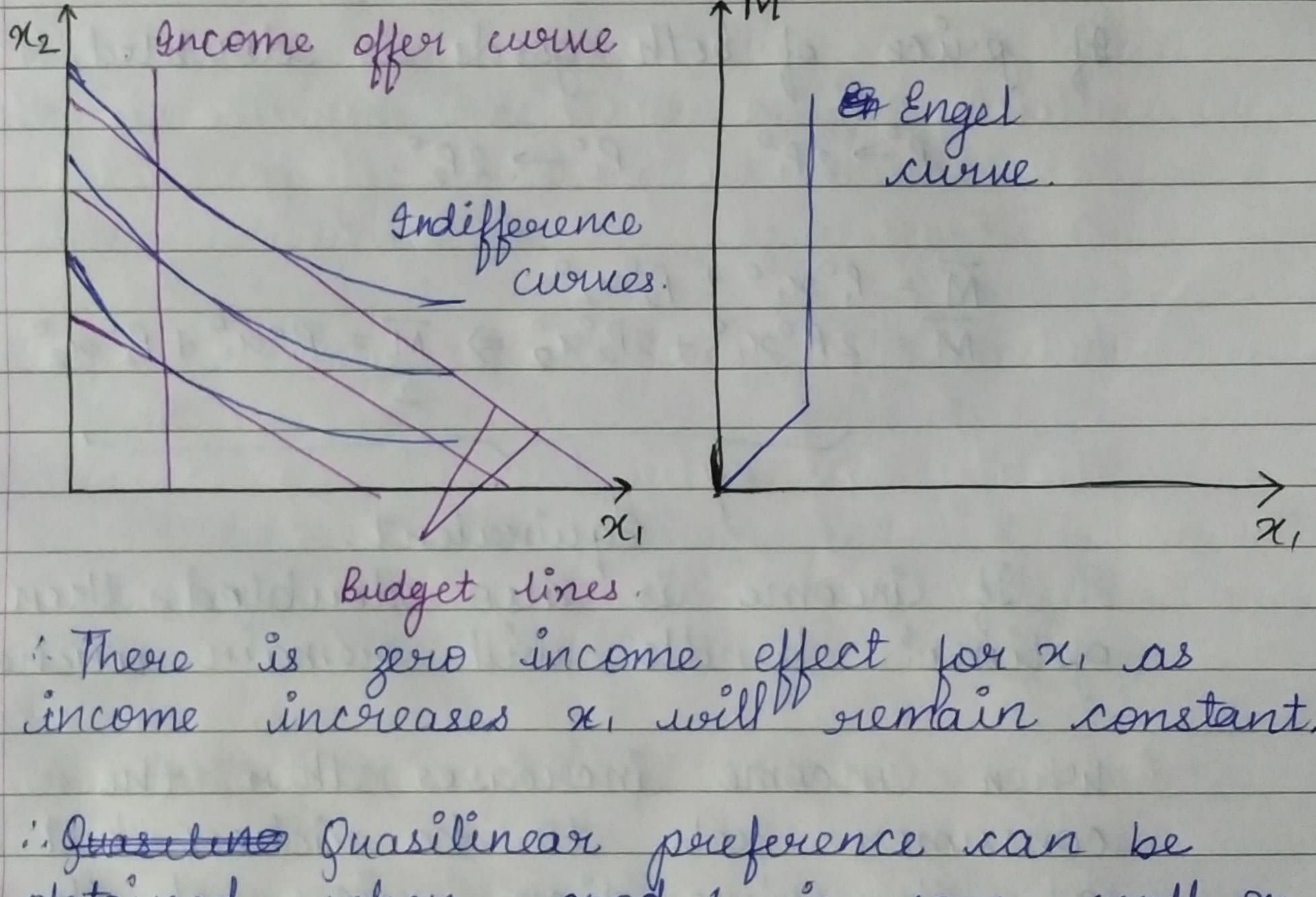
If income is scaled up or down by some $t > 0$, then the optimum bundle is also scaled up or down by the same amount. ∴ Engel curve is a straight line.

With applying homothetic preference, income effect can be ruled out as the demand side ~~not~~ would become sorted by itself.

Page No. :
Date :

Quasilinear Preferences (Semilinear).

Indifference curves are vertically shifted forms of each other for different levels of utility i.e. the amount of consumption of good 1 is not changing, ∴ as income increases, only consumption of good 2 is increasing.



∴ There is zero income effect for x_1 as income increases x_1 will remain constant.

∴ ~~Quasiline~~ Quasilinear preference can be obtained when good 1 is some small or necessary good.

Optimum Choice.

A proportionate increase in all prices is the same ~~is~~ in effect as an equi-proportionate fall in money in money income of the consumer.

Equi-proportionate increase in M & P will leave the MPB ~~un~~ unchanged.

Demand function ~~of~~ is homogenous of degree 0.
Consumer is free of money illusion.

If price of both goods is doubled.

$$P_1^0 \rightarrow 2P_1^0 \qquad P_2^0 \rightarrow 2P_2^0.$$

$$\overline{M} = P_1^0 x_1^0 + P_2^0 x_2^0.$$

$$\underbrace{\overline{M} = 2P_1^0 x_1^0 + 2P_2^0 x_2^0 \Rightarrow \frac{\overline{M}}{2} = P_1^0 x_1^0 + P_2^0 x_2^0}_{\text{Equivalent.}}$$

∴ If income is also doubled, then optimal bundle will remain unchanged

When income increases, then the consumer feels to be richer but if at the same time prices of the goods also increase, then there is an overall inflationary pressure on the economy, i.e. even if ~~income~~ money income i.e. nominal income has increased ~~consumption~~ purchasing capacity has not increased. ∴ The consumer is only having money illusion that nominal income has increased, but actually, real income remains unchanged.

Homogenous function

A function is said to be homogenous of degree 'r' if increase in all its argument by the rate λ increases the function value by the rate λ^r.

$$\boxed{f(\lambda x, \lambda z) = \lambda^r f(x, z) = \lambda^r y}$$

Example: $y = x + z$.

λy = RHS $= \lambda x + \lambda z = \lambda^1 y$. (Order = 1).

$y = \frac{x}{z}$ RHS $= \frac{\lambda x}{\lambda z} = \lambda^0 y$ (Order = 0).

$y = 10 + xz \Rightarrow$ Non-homogenous.

$y = xz \Rightarrow$ RHS $= \lambda x \cdot \lambda z = \lambda^2 y$ (Order = 2).

Demand func^n: $x_1^d = f(P_1, P_2, M)$.

RHS $= f(\lambda P_1, \lambda P_2, \lambda M)$.
$= \lambda^0 x^d$

$\therefore$ Demand func^n is homogenous func^n with order 0.

Page No. :
Date :

* Utility ~~Fun~~ Function

Working with preference relations is not always convenient.

Economists like to work with Utility Functions which are simple and easy way of summarizing preferences.

"Utility" is want-satisfying power. The utility of a good or service is the satisfaction or pleasure it provides to a consumer.

Not a synonym for usefulness.

Utility is subjective.

Difficult to quantify (unit of measurement is called "utils").

* The cardinal utility analysis

The consumer is able to measure the utility in cardinal numbers.
Goods are comparable.

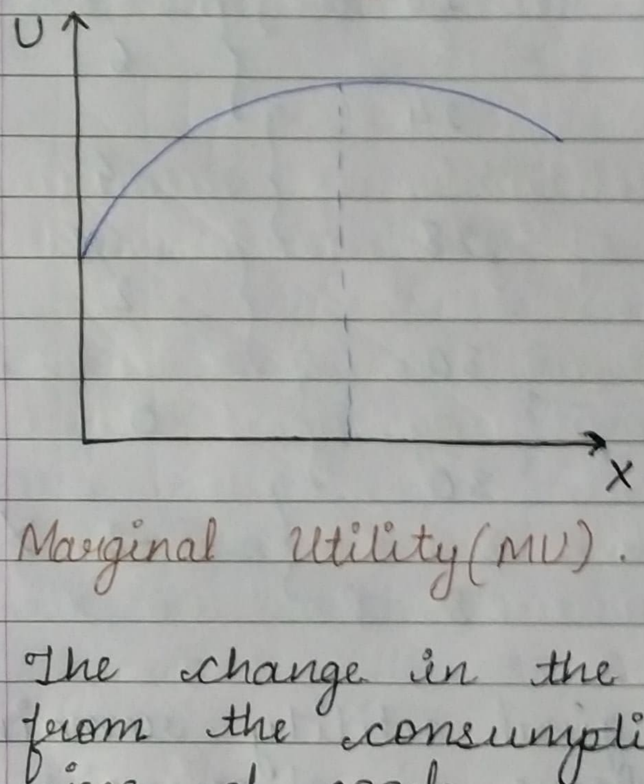
Concepts:

Total ~~utility~~ utility

Marginal utility- extra, additional, ~~incre~~ incremental.

~~Law~~ ~~of~~ Law of diminishing Marginal utility- Beyond some point of consumption, utility will decline.

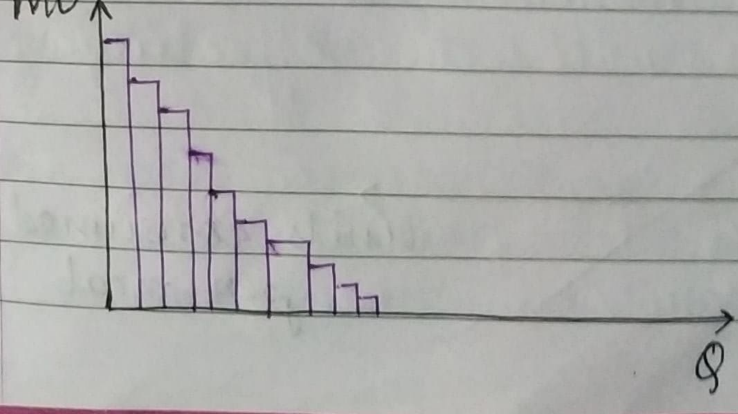
Total utility



Marginal Utility (MU).

The change in the utility resulting from the consumption of a subsequent piece of goods.

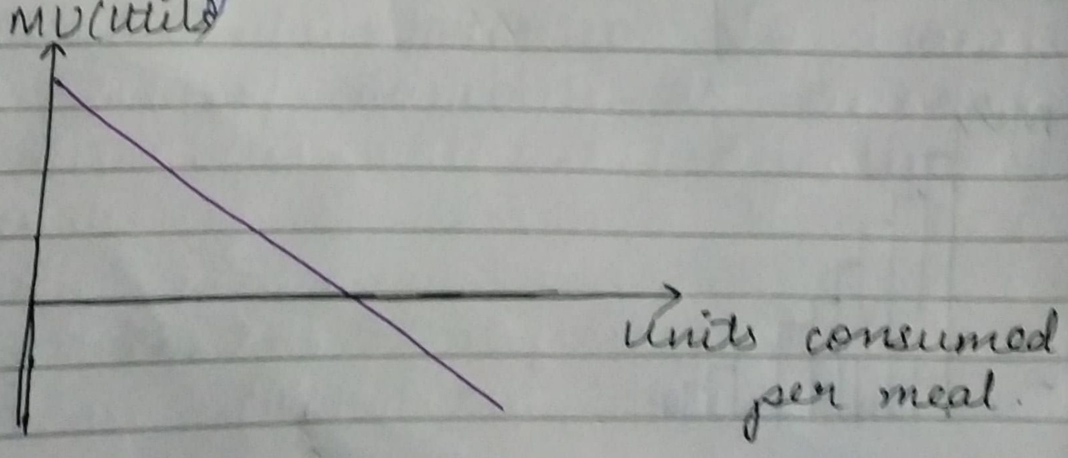
$$\boxed{MU = \frac{\Delta TU}{\Delta Q}}$$



Example.

Chips consumed per meal.	Total Utility, Utils	Marginal Utility, Utils
0	0	
		10
1	10	
		8
2	18	
		6
3	24	
		4
4	28	
		2
5	30	
		0
6	30	
		-2. X
7	28	

∴ ~~As~~ The marginal utility remains +ve but it keeps on decreasing as the quantity consumed increases gradually.



Law of diminishing MU
As the quantity consumed increases, marginal utility falls gradually i.e. rate of change of utility falls.

Total utility : $u = u(x)$.

$$MU = \frac{\partial U(x)}{\partial x_i} > 0 \text{ (good)}$$

$$\frac{\partial U(x)}{\partial x_i} = 0 \text{ (neutral)}$$

$$\frac{\partial U(x)}{\partial x_i} < 0 \text{ (bad)}.$$

$$\boxed{\frac{\partial}{\partial x_i}\left(\frac{\partial U}{\partial x_i}\right) < 0} \text{ (Law of diminishing MU)}.$$

Utility can vary from person to person as well as time to time for the same person, therefore, it is unrealistic to quantify utility.

★. Preference Ordering & Ordinal Utility.

Sometimes we attach numerical numbers - construction of a utility function.

In this approach, the utility depends on the order in which the consumer will rank the bundles.

Page No. :
Date :

		Rank.	Utility (Utils)		
Four bundles	X	1	30	100	As rank decrease
	X'	2	20	70	Utility decreases
	X''	3	10	50	
	X'''	4	5	2	↓

In ordinal utility theory, the ~~differ~~ difference b/w the value of utilities of one good & the other is immaterial as long as the order is maintained.

An ordinal utility fn is numerical representation of preference ordering.

A utility fn $u(x)$ will be a ~~a~~ representation of preference ordering if:

(i) $u(X') = u(X'')$ when $X' \, I \, X''$.

(ii) $u(X''') > u(X'''')$ when $X''' \, P \, X''''$ i.e. X''' is ranked above X''''.

Monotonic transformations of utility functions

Monotonic transformation: To transfer one set of numbers to another set of numbers without affecting their order.

E.g. $x_1 > x_2 > x_3 \xrightarrow{f(x)} f(x_1) > f(x_2) > f(x_3)$
(Monotonic Transformation)

An ordinal utility fn allows ~~as~~ any positive monotonic transformation as they preserve the ordering.

$$V = \phi[u(x)], \ \phi' > 0.$$

Any monotonic transformation of a utility function will represent the same preferences.

Examples: $\log(U(x_1, x_2, \cdots, x_n))$, $\exp(U(x_1, x_2, \cdots, x_n))$, $\sqrt{U(x_1, x_2, \cdots, x_n)}$

Indifference curves are utility contours

Proof:

Consider a particular level of utility u_0, for which the utility contour

$$u(X) = u_0.$$

Consider any two bundles X' & X'' such that $X' I X''$.

$\therefore$, by construction of utility fn

$$u(X') = u(X'') = u_0$$

That ~~xx~~ means X' & X'' lies on the same IC.

By definition of utility contour,
$u(X') = u(X'')$ is true.

Contour of a func^n ~~f~~ $y = f(x_1, x_2)$ will be all the values of x_1 & x_2 for which y attains a particular value.

$\therefore$ X' & X'' are on the same utility contour. This will be true for all such X' & X''.

$\therefore$ All indifference curves are utility contours.

An ordinal utility fn is a rule by which we assign a real number to each particular commodity bundle.

Assume $X' I X''$

$\therefore$ $u(X') = u(X'') = 3$.

But ~~XPX~~ $X' P X'''$

$u(X') > u(X''')$.

By ~~mono~~ monotonic preference, a higher IC should be assigned a higher utility index.

In case of ordinal utility, the ~~gad~~ gap

Page No.
Date

b/w two ICs is immaterial as long as order is maintained while cardinal utility gives more emphasis to the ~~diff~~ ~~to~~ magnitude of difference b/w utilities i.e. magnitude of distance b/w two ICs.

MU & Interdependence of MRS.

Let $u = u(x_1, x_2)$.

$$\left.\frac{dx_2}{dx_1}\right|_{U_0} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2}$$

$$\boxed{MRS = \frac{MU_1}{MU_2}}$$

$u_i = u_i(x_i)$

Let ~~of~~ $u = u(x_1) + u(x_2)$.

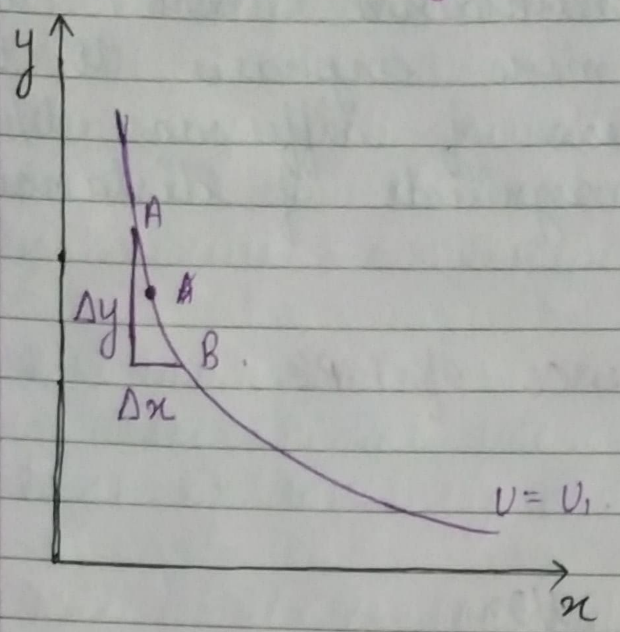
If additively seperable

$$\frac{\partial}{\partial x_2}\left(\frac{\partial u}{\partial x_1}\right) = 0$$

$$\frac{\partial}{\partial x_2}\left(\frac{\partial u}{\partial x_1}\right) \neq 0$$ ~~Addit~~ Additively non-seperable.

If $\frac{\partial}{\partial x_2}\left(\frac{\partial u}{\partial x_1}\right) = 0$, then marginal utilities of the 2 goods are independent i.e. MU_1 & MU_2 are independent.

Marginal Utility



From point A to point B, the consumer loses Δy & gains Δx. We also know that both A & B give the consumer the same utility U_1.

Marginal utility lost from less y must be offset by marginal utility gained from more x.

Different types of utility functions:

1. $u = u(x_1, x_2)$, $u_1 > 0$, $u_2 > 0$. (MUs > 0 ⇒ good comodities)

$$\frac{\partial^2 U}{\partial x_i \partial x_j} = \frac{\partial^2 U}{\partial x_j \partial x_i}$$

2. $U = \min(ax_1, bx_2)$ ⇒ Perfect complements

$$x_1 : x_2 = a : b.$$

3. $U = x_1^{\alpha} x_1^{\beta}$ (Coup Douglas utility func^n).

4. $\ln U = \alpha \ln x_1 + \beta \ln x_2$.

MRS of 3. & 4. is same as it is monotonic transformation of coup Douglas utility func^n. But in this case it is additively seperable.

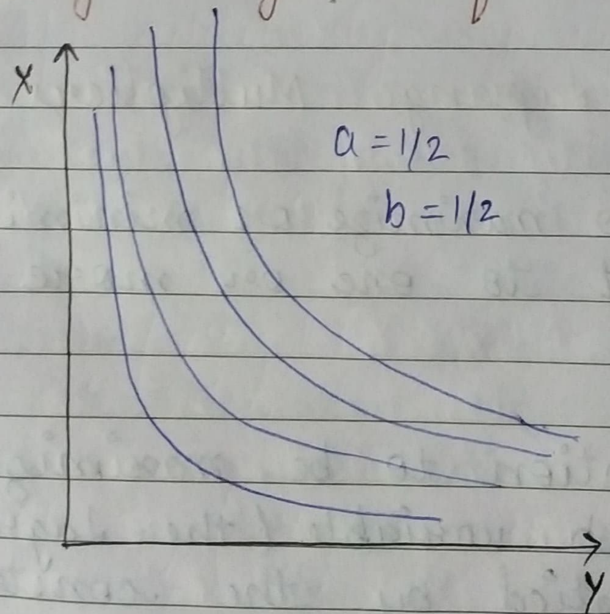
5. ~~$u = x$~~ $u = ax_1 + bx_2$ (Perfect substitutes).

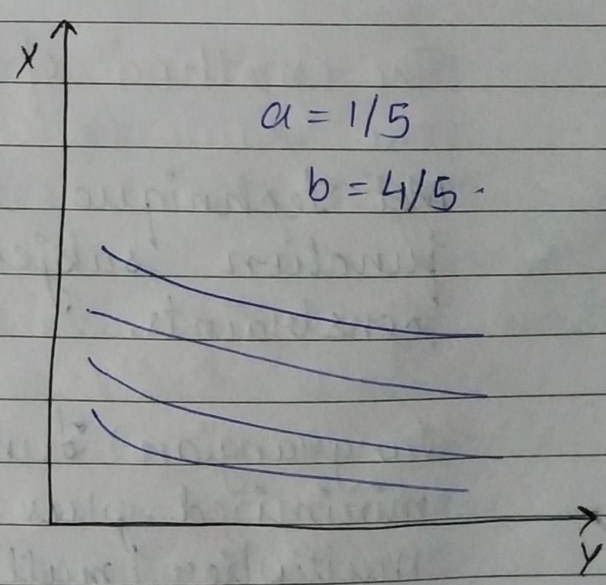
6. $u = x_1 + x_2^{\beta}$
7. $u = x_1^{a} + x_2$

$\left.\begin{matrix} 6. \\ 7. \end{matrix}\right\}$ Quasilinear preference (Semi-linear).

6. ⇒ MRS is independent ~~on~~ of x_1 &
7. ⇒ MRS is independent ~~on~~ of x_2.

Coup Douglas Preferences





Optimum Choice of a Consumer : Problem of Utility Maximization

Tangency will correspond to the most preferred bundle only when strict convexity & monotonicity holds.

~~*~~ Optimum Choice:
i) On the Budget line.
ii) Point of tangency b/w IC & Budget line.

Max $U = u(X, Y)$. (ordinal utility func^n).

Subject to $M = P_X X + P_Y Y$.

The consumer's aim is to maximize utility but he/she is constrained by the budget $\therefore$ it is a constraint maximization problem.

The method of Lagrange Multipliers

Technique to maximize or minimize a function subject to one or more constraints.

Langrangian: Function to be maximized or minimized, plus a variable (the Lagrange multiplier) multiplied by the constraint.

Lagrangian for the problem:

$$L = U(x,y) - \lambda(P_X X + P_Y Y - M).$$

Budget constraint : $P_X X + P_Y Y - M = 0.$

$\lambda \rightarrow$ Marginal utility of money income
$\lambda > 0.$

Dual of the problem:

Minimize : Expenditure

$$E = P_X X + P_Y Y.$$

Subject to : attaining a certain level of utility

$$\bar{U} = U(X, Y).$$

Sol^n of primal & dual will be the same.

Differentiating the Lagrangian:
We choose values of X & Y that satisfy the budget constraint, then the second term in L will be zero. By diff. wrt X, Y, M & then equating the derivatives to zero, we obtain the necessary cond^{n}s. for a maximum.

$$\frac{\partial L}{\partial X} = \underbrace{MU_X(X,Y)}_{\text{Marginal utility of } X.} - \lambda P_X = 0$$

$$\frac{\partial L}{\partial Y} = MU_Y(X, Y) - \lambda P_Y = 0$$

$$\frac{\partial L}{\partial \lambda} = M - P_X X - P_Y Y = 0. \text{ (Budget constraint)}$$

Solving the Resulting equations

$$MU_X = \lambda P_X$$
$$MU_Y = \lambda P_Y$$
$$P_X X + P_Y Y = M.$$

The Equal Marginal Principle:

We combine the first two ~~co~~ condition above to obtain the equal marginal principle:

$$\lambda = \frac{MU_X(X, Y)}{P_X} = \frac{MU_Y(X, Y)}{P_Y}.$$

To optimize, the consumer must get the same utility from the last rupee spent by consuming either X or Y.

If $\frac{MU_X(X, Y)}{P_X} > \frac{MU_Y(X, Y)}{P_Y}$.

∴ The consumer is getting more utility by consuming X, ∴ the consumer will consume more of good ∴ by the law of ~~margin~~ diminishing marginal utility, as more of good X will be