Indian Institute of Technology, Kharagpur

Date..... FN/AN Time: 2 Hrs Full Marks: 30 No. of Students: 176
Mid (Autumn) Semester 2023-24, Deptt: MA Sub. No. MA 61019/ MA 30227
Subject Name: Optimization Techniques

Instruction: Answer **all** questions.

Question 1 [$5 \times 2 = 10$ marks]

a) A firm wants to expand Rs. 10 lakhs for advertising one of its products on radio, TV and a newspaper. It can purchase radio time for Rs. 2000 per spot, TV time for Rs. 10,000 per spot and newspaper advertisement for Rs. 4000 per insertion. The payoff from the advertisement is a measure of the audience reached. Based on past experience, a radio spot is given 40 audience points, a TV spot 160 points and a newspaper insertion 300 points. The firm wishes to determine the money to allocate to each medium so that the number of total audience points is maximised. Based on subjective consideration, the firm decides not to spend more than Rs. 2.5 lakhs on radio and at least Rs. 4 lakhs on TV. Also, the firm wants to keep newspaper allocation to not exceed 50% of allocation to TV. Formulate the problem as a L.P.P.

b) Solve the following L.P.P. graphically:

$$\begin{aligned} &\text{Minimize } z = 4x_1 + 2x_2 \\ &\text{subject to } 3x_1 + x_2 \leq 27, \\ &\qquad -x_1 - x_2 \leq -213, \\ &\qquad x_1 + 2x_2 \geq 303 \\ &\text{and } x_1, x_2 \geq 0. \end{aligned}$$

c) Convert the following L.P.P. to standard form:

$$\begin{aligned} &\text{Minimize } z = |x_1| - 2|x_2| + |x_3| \\ &\text{subject to } x_1 + x_2 - x_3 \leq 10, \\ &\qquad x_1 - 3x_2 + 2x_3 \geq 5 \end{aligned}$$

d) Write down the dual of the following L.P.P. :

$$\begin{aligned} &\text{Maximize } z = 2x_1 + 3x_2 - 4x_3 \\ &\text{subject to } 3x_1 + x_2 + x_3 \leq 2, \\ &\qquad -4x_1 \qquad + 3x_3 \geq 4, \\ &\qquad x_1 - 5x_2 + x_3 = 5, \end{aligned}$$

$x_1 \geq 0, x_2 \geq 0$ and x_3 is unrestricted in sign.

——P.T.O.——

e) Find the extreme points of the convex set of the feasible solutions of the L.P.P.

$$\text{Minimize } z = 2x_1 + 3x_2 + 4x_3 + 5x_4$$

$$\text{subject to } 2x_1 + 3x_2 + 5x_3 + 6x_4 = 16,$$
$$x_1 + 2x_2 + 2x_3 + 3x_4 = 9$$
$$\text{and } x_1, x_2, x_3, x_4 \geq 0.$$

Question 2 [$3 \times 4 = 12$ marks]

a) The following incomplete table represents the second stage in the solution of an L.P.P. by the simplex method. All variables corresponding to zero coefficients of the objective function are slack variables. Complete the table. Also, state the original problem assuming it to be a maximization problem. (The notations have their usual meanings.)

		c_j					0	0	0
B	c_B	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7
a_5	0	8	-1	-13	20				-6
a_6	0	26	7	13	-11				5
a_4		2	1	2	-3				1
$z_j - c_j$		12	4	15	-16				6

b) $x_1 = 1, x_2 = 2, x_3 = 1, x_4 = 0$ is a feasible solution of the set of equations

$$11x_1 + 2x_2 - 9x_3 + 4x_4 = 6,$$
$$15x_1 + 3x_2 - 12x_3 + 5x_4 = 9$$

Reduce the feasible solution to two different basic feasible solutions. Clearly specify if there is any degenerate basic feasible solution.

c) Find a basic feasible solution, if there is any, of the following set of linearly independent equations and if such solution exists, taking that basis as an admissible basis, calculate all y_j, $z_j - c_j$ $[j = 1, 2, 3]$ and the value of the objective function corresponding to that B.F.S. (without using simplex method)

$$\text{Maximize } z = 2x_2 + x_3$$

$$\text{subject to } x_1 + x_2 - 2x_3 \leq 7,$$
$$-3x_1 + x_2 + 2x_3 \leq 3$$
$$\text{and } x_1, x_2, x_3 \geq 0.$$

d) Solve the following equations by simplex method:

$$3x_1 + x_2 = 7,$$
$$x_1 + x_2 = 3$$

——P.T.O.——

Question 3 [3 + 3 + 2 = 8 marks]

a) Mention two situations of degeneracy that may occur in solving a L.P.P. Prove that a situation of degeneracy occurs in solving the L.P.P. given by

$$\text{Maximize } z = 22x_1 + 30x_2 + 25x_3$$

$$\begin{aligned} \text{subject to } & 2x_1 + 2x_2 \leq 100, \\ & 2x_1 + x_2 + x_3 \leq 100, \\ & x_1 + 2x_2 + 2x_3 \leq 100 \\ \text{and } & x_1, x_2, x_3 \geq 0. \end{aligned}$$

Mention clearly the method to resolve the degeneracy you are applying in this problem.

b) Prove the conditions for existance of alternative non-basic optimal solutions for a maximization L.P.P.

c) What are the conditions for existance of alternative basic optimal solutions for a maximization L.P.P.

——-The End———