Derivatives

Speculating with Options: Advanced Options Strategies

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By using options in combinations/spreads, traders can take low-risk speculative positions. Speculators use several techniques for combining options to generate profit profiles that are not available with positions in single options

Combinations: A combination trading strategy involves taking either a long position in a call and a put simultaneously or taking a short position in a call and a put simultaneously on the same underlying with **SAME expiration date** but with **SAME or DIFFERENT exercise prices**. Examples include *Straddles*, *Strangles*, *Strips*, *Straps and Collars*

Spreads: A spread trading strategy involves taking a long position and short position using either calls or puts on the same underlying with **SAME (DIFFERENT) expiration date** simultaneously with **DIFFERENT (SAME) exercise prices.** Spreads can be executed using either calls or puts (two or more calls or two or more puts). Examples include *Bull and Bear Spreads*, *Butterfly Spreads*, *Box Spreads*, *Calendar Spreads*

1. STRADDLES:

- A straddle is an option position simultaneously involving a put and a call option on the same stock with the SAME expiration date and with SAME strike prices;
- **Long Straddle:** To BUY a straddle (bottom straddle), an investor will buy both ONE put and ONE call that have the SAME expiration and the SAME strike price. The buyer of this straddle would be making a bet on a large movement in the stock price in some direction (up/down);
- **Short Straddle:** To SELL a straddle (top straddle), a trader sells both ONE call and ONE put that have the SAME expiration and the SAME strike price. The seller would be betting that the stock price remains reasonably close to the exercise price;
- The profits and losses for the long/short straddle depends on where the stock price is at expiration

Payoff from Long Straddle: Consider the PURCHASE OF A STRADDLE with the call and put having an exercise price of X (CALL and PUT exercise price is SAME) and an expiration of T (SAME expiration). Then $N_C = 1$ and $N_P = 1$ and the profit from this transaction if held to expiration is:

$$\Pi = \text{Max}(0, S_T - X) - C + \text{Max}(0, X - S_T) - P.$$

Since there is only one exercise price involved, there are only two ranges of the stock price at expiration. The profits are as follows:

$$\Pi = S_T - X - C - P \qquad \text{if } S_T \geq X.$$

$$\Pi = X - S_T - C - P \qquad \text{if } S_T < X.$$

Upper/Upside BEP: For the case in which the stock price exceeds the exercise price, $S_T > X$, set the profit equal to zero:

$$S_T^* - X - C - P = 0.$$

and solving for S^*_T gives a breakeven of

$$S_T^* = X + C + P.$$

Lower/Downside BEP: For the case in which the stock price is below the exercise price, $S_T < X$, set the profit equal to zero:

$$X - S_T^* - C - P = 0.$$

and solving for S^*_T gives a breakeven of

$$S_T^* = X - C - P.$$

Important Observations: Long Straddle

1. When the stock price equals or exceeds the exercise price ($S_T \geq X$), the call expires in-the-money. It is exercised for a gain of $S_T - X$ while the put expires out-of- the-money. The profit is the gain on the call minus the premiums paid on the call and the put. For the range of stock prices above the exercise price, the profit increases dollar for dollar with the stock price at expiration;
2. When the stock price is less than the exercise price ($S_T < X$), the put expires in-the-money and is exercised for a gain of $X - S_T$ while the call expires out-of-the-money. The profit is the gain on the put minus the premiums paid for the put and the call. For the range of stock prices below the exercise price the profit decreases dollar for dollar with the stock price at expiration;
3. When the options expire with the stock price at the exercise price ($S_T = X$), both options are at-the-money and essentially expire worthless. The profit then equals the premiums paid, which, of course, makes it a loss;
4. The breakeven stock prices are simply the exercise price plus or minus the premiums paid for the call and the put. On the **upside**, the call is exercised for a gain equal to the difference between the stock price and the exercise price. For the investor to profit, the stock price must exceed the exercise price by enough that the gain from exercising the call will cover the premiums paid for the call and the put. On the **downside**, the put is exercised for a gain equal to the difference between the exercise price and the stock price. To generate a profit, the stock price must be sufficiently below the exercise price that the gain on the put will cover the premiums on the call and the put;

Important Observations: Long Straddle

5. The worst-case outcome for a straddle is for the stock price to end up equal to the exercise price where neither the call nor the put can be exercised for a gain. The straddle trader will lose the premiums on the call and the put limiting the loss to the premiums paid on call and put;

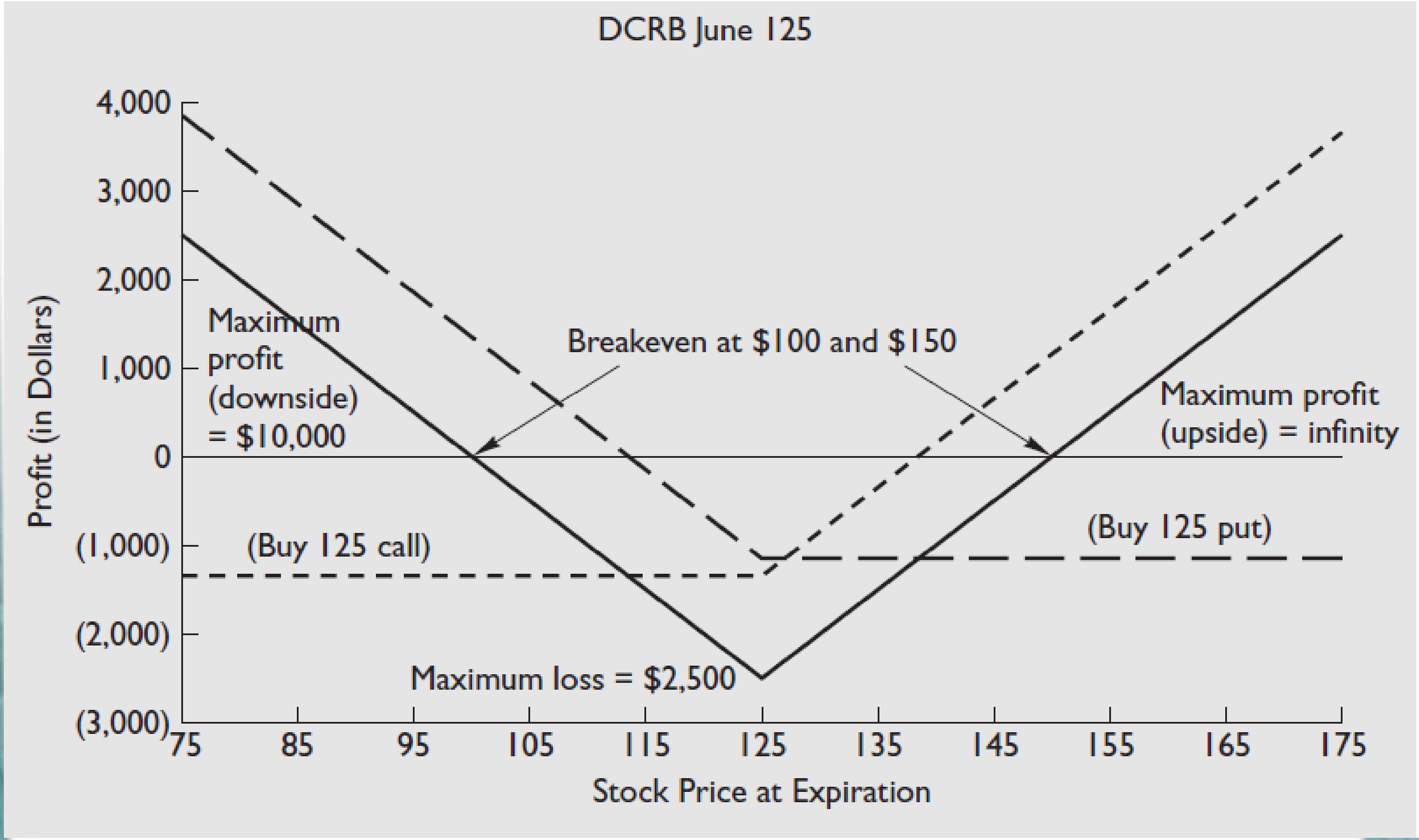
6. The profit potential on a long straddle is unlimited. On the upside, the stock price can rise infinitely, and the long straddle will earn profits dollar for dollar with the stock price in excess of the exercise price. On the downside, the profit is limited simply because the stock price can go no lower than zero.

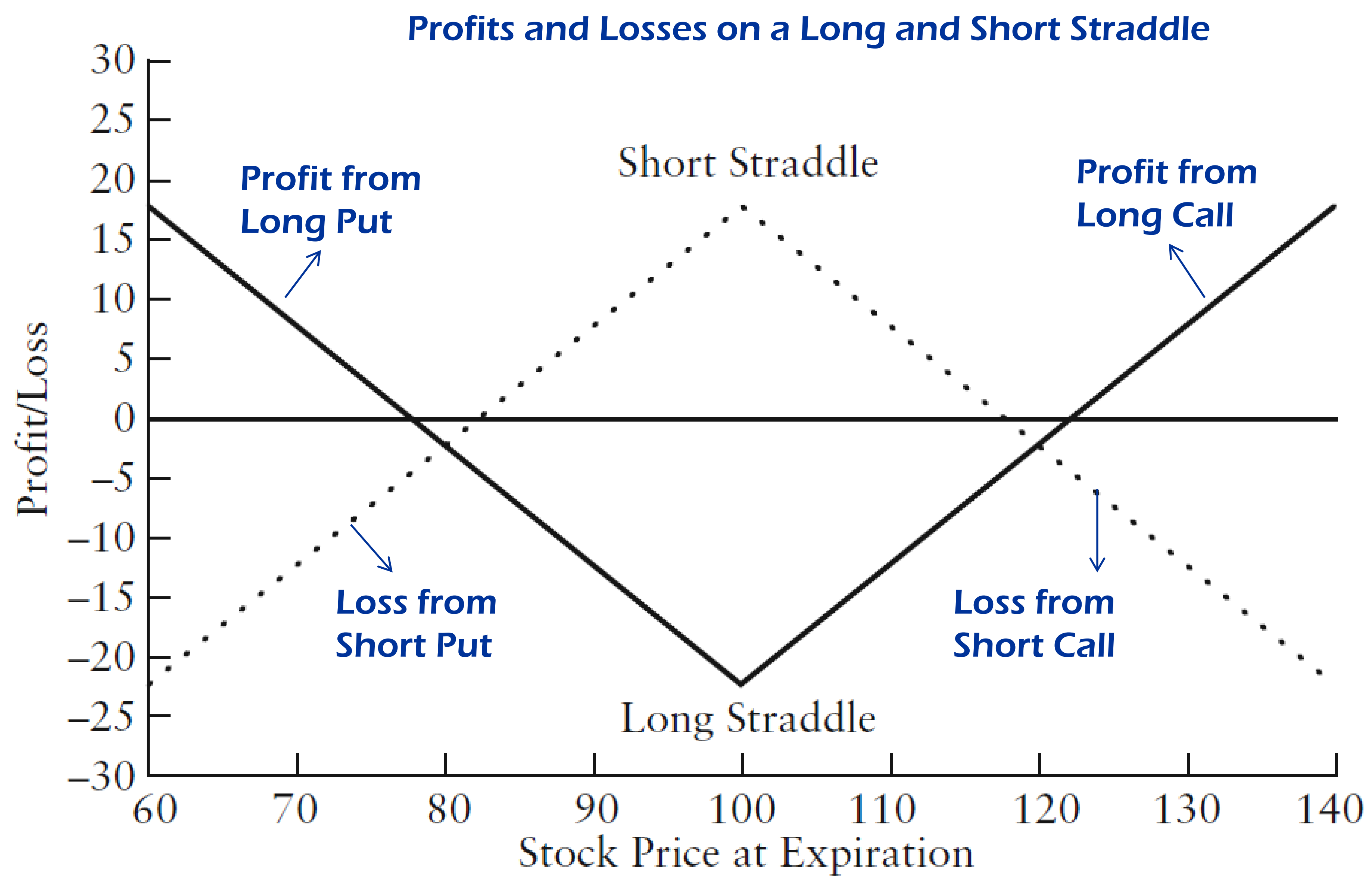
DCRB OPTION DATA, MAY 14

	Calls			Puts		
Exercise Price	May	June	July	May	June	July
120	8.75	15.40	20.90	2.75	9.25	13.65
125	5.75	13.50	18.60	4.60	11.50	16.60
130	3.60	11.35	16.40	7.35	14.25	19.65

Current stock price: 125.94
Expirations: May 21, June 18, July 16
Risk-free rates (continuously compounded): 0.0447 (May); 0.0446 (June); 0.0453 (July)

Profits and Losses on a Long Straddle




Profits and Losses on a Long and Short Straddle
Short Straddle
Profit from Long Put
Profit from Long Call
Loss from Short Put
Loss from Short Call
Long Straddle
Profit/Loss
30
25
20
15
10
5
0
−5
−10
−15
−20
−25
−30
60
70
80
90
100
110
120
130
140
Stock Price at Expiration

2. STRANGLES:

- A strangle is an option position simultaneously involving a put and a call option on the same stock with the SAME expiration date but with DIFFERENT strike prices;
- **Long Strangle:** To BUY a strangle (bottom vertical combination), an investor will buy both ONE put and ONE call that have the same expiration date BUT with the call having a HIGHER exercise price than the put [BUYING ONE slightly out-of-the-money (OTM) call and ONE slightly out-of-the-money (OTM) put]. Since OTM buyer pays less premium. The buyer of this strangle would be making a bet on a large movement in the stock price in some direction (up/down);
- **Short Strangle:** To SELL a strangle (top vertical combination), a trader sells both ONE call and ONE put that have the same expiration BUT with the call having a HIGHER exercise price than the put [SELLING ONE slightly out-of-the-money (OTM) call and ONE slightly out-of-the-money (OTM) put]. The seller would be betting that the stock price remains reasonably close to the exercise price;
- The profits and losses for the long/short strangle depends on where the stock price is at expiration

Payoff from Long Strangle: Consider the PURCHASE OF A STRANGLE with the call having an exercise price of X_1 and put having an exercise price of X_2 (CALL and PUT exercise price is DIFFERENT) and an expiration of T (SAME expiration). Then $N_C = 1$ and $N_P = 1$ and the profit from this transaction if held to expiration is:

$$\prod = \mathbf{Max}(0, S_T - X_1) - C + \mathbf{Max}(0, X_2 - S_T) - P$$

Since two exercise prices are involved, there will be three ranges of the stock price at expiration. The profits are as follows:

$$\prod = (-C) + (X_2 - S_T - P) \quad \text{if } S_T \leq X_2 < X_1$$

$$\prod = (-C) + (-P) \quad \text{if } X_2 < S_T \leq X_1$$

$$\prod = (S_T - X_1 - C) + (-P) \quad \text{if } X_2 < X_1 < S_T$$

Lower/Downside BEP: Take the profit equation for the first case in which the stock price (S_T) is less than or equal to X_2, $S_T \leq X_2$, and set it equal to zero and solve it for S^*_T :

$$\prod = (-C) + (X_2 - S_T - P) = 0$$

$$\mathbf{S^*_T = X_2 - P - C}$$

Upper/Upside BEP: Take the profit equation for the third case in which the stock price (S_T) is greater X_1, $S_T > X_1$ and set it equal to zero and solve it for S^*_T :

$$\prod = (S_T - X_1 - C) + (-P) = 0$$

$$\mathbf{S^*_T = X_1 + C + P}$$

Example: Long Strangle		
	Exercise Price ($)	Option Premium ($)
BUY 1 CALL (OTM)	$110 ($X_1$)	$3
BUY 1 PUT (OTM)	$100 ($X_2$)	$7
CURRENT STOCK PRICE	$105	
EXPIRATION	26 JUNE	
Lower Breakeven; $S_T < X$; [$S_T * = X_2 - C - P$]	L-BEP = 100-3-7=$90	
Upper Breakeven $S_T \geq X$; [$S_T * = X_1 + C + P$]	U-BEP = 110+3+7=$120	

Profits and Losses for a Long Call and Long Put: Long Strangle

Stock Price at Expiration (S_T)	Long Call X_1 =\$110 C =\$3 If $S_T > X$; $\prod_1 = S_T - X - C$ If $S_T \leq X$; $\prod_1 = -C$	Long Put X_2 =\$100 P =\$7 If $S_T < X$; $\prod_2 = X - S_T - P$ If $S_T \geq X$; $\prod_2 = -P$	Long Strangle C+P =\$10 $\prod = \prod_1 + \prod_2$
\$50	\$−3	+\$43	+\$40
80	−3	+13	+10
83	−3	+10	+7
85	−3	+8	+5
90	−3	+3	0
95	−3	−2	−5
100	−3	−7	−10
105	−3	−7	−10
110	-3	−7	−10
115	+2	−7	−5
117	+5	−7	-2
120	+7	−7	0
125	+12	-7	+5
150	+37	−7	+30

Important Observations: Long Strangle

1. Long Strangle has two exercise prices (call exercise price (X_1) and put exercise price (X_2). Accordingly, given (X_1)>(X_2) there will be three ranges of stock price at expiration: stock price at expiration can be less than or equal to put exercise price (X_2); stock price can be greater than put exercise price (X_2) but less than or equal call exercise price (X_1) and greater than call exercise price (X_1);
2. When the stock price at expiration is less than or equal to put exercise price ($S_T \leq X_2 < X_1$) the put expires in-the-money and is exercised for a gain of $X_2 - S_T$ while the call expire out-of-the-money. The profit is the gain on the put minus the premiums paid for the put and the call. For the range of stock prices below the put exercise price the profit decreases dollar for dollar with the stock price at expiration;
3. When the stock price at expiration is greater than put exercise price (X_2) but less than or equal to call exercise price (X_1), ($X_2 < S_T \leq X_1$) both options are out-of-the-money and essentially expire worthless. The profit then equals the premiums paid, which, of course, makes it a loss;
4. When the stock price at expiration is greater than call exercise price ($X_2 < X_1 < S_T$) the call expires in-the-money and is exercised for a gain of $S_T - X_1$ while the put expires out-of- the-money. The profit is the gain on the call minus the premiums paid on the call and the put. For the range of stock prices above the exercise price, the profit increases dollar for dollar with the stock price at expiration;

Important Observations: Long Strangle

5. The breakeven stock prices are simply the exercise price plus or minus the premiums paid for the call and the put. On the **upside**, the call is exercised for a gain equal to the difference between the stock price and the exercise price. For the investor to profit, the stock price must exceed the call exercise price by enough that the gain from exercising the call will cover the premiums paid for the call and the put. On the **downside**, the put is exercised for a gain equal to the difference between the exercise price and the stock price. To generate a profit, the stock price must be sufficiently below the put exercise price that the gain on the put will cover the premiums on the call and the put;

6. The worst-case outcome for a strangle is for the stock price to end up less than or equal to call exercise price (X_1) but greater than call exercise price (X_2) where neither the call nor the put can be exercised for a gain. The strangle trader will lose the premiums on the call and the put limiting the loss to the premiums paid on call and put;

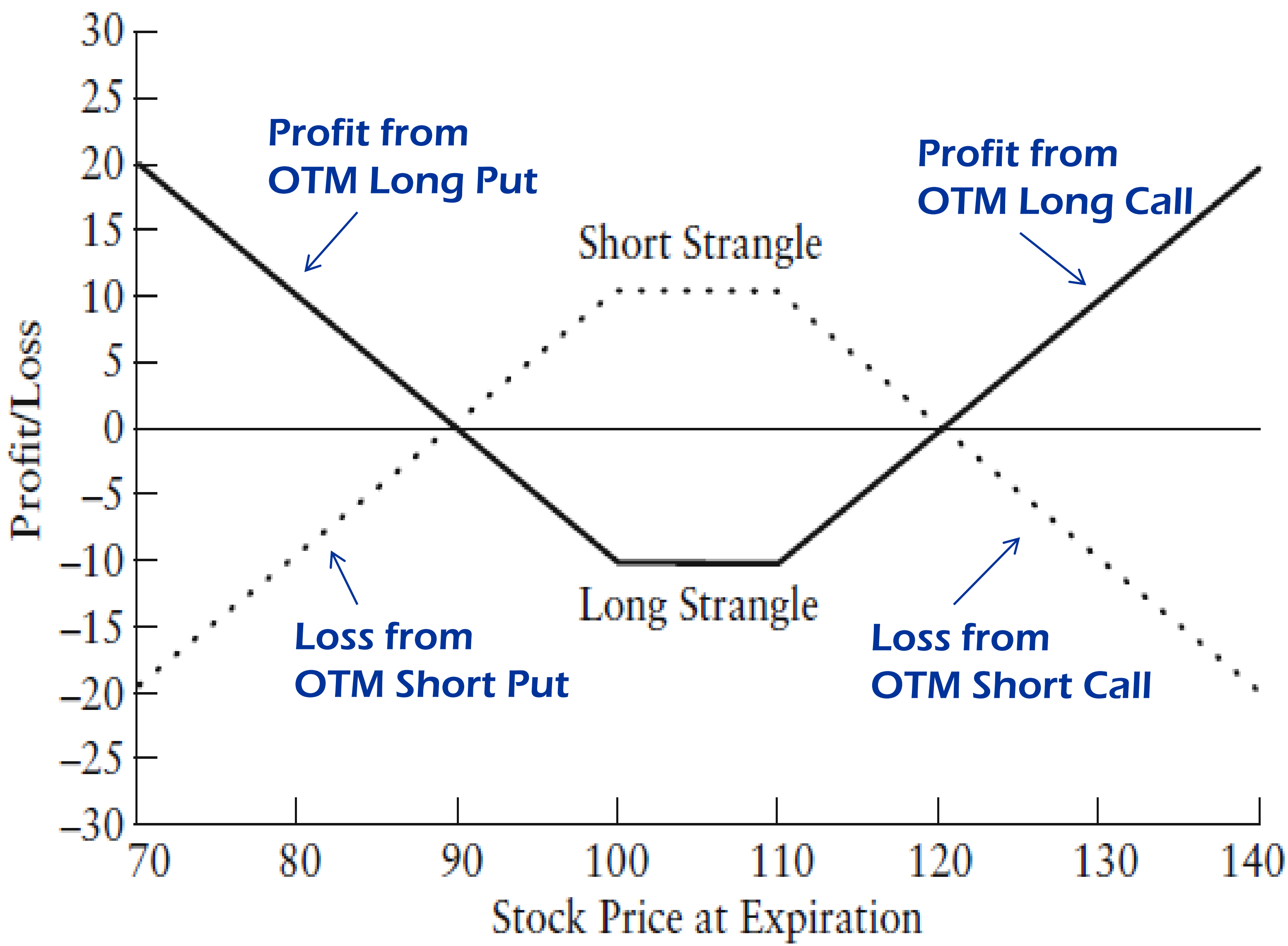
7. The profit potential on a long strangle is unlimited. The stock price can rise infinitely, and the long strangle will earn profits dollar for dollar with the stock price in excess of the call exercise price (X_1). On the downside, the profit is limited simply because the stock price can go no lower than zero.

Example: Short Strangle		
	Exercise Price ($)	Option Premium ($)
SELL 1 CALL (OTM)	$110 ($X_1$)	$3
SELL 1 PUT (OTM)	$100 ($X_2$)	$7
Lower Breakeven; $S_T < X$; [$S_T * = X_2 - C - P$]	L-BEP = 100-3-7=$90	
Upper Breakeven $S_T \geq X$; [$S_T * = X_1 + C + P$]	U-BEP = 110+3+7=$120	

Profits and Losses for a Short Call and Short Put: Short Strangle

Stock Price at Expiration (S_T)	Short Call X_1 =\$110 C =\$3 If $S_T>X$; $\prod_1 = -S_T + X + C$ If $S_T \le X$; $\prod_1 = C$	Short Put X_2 =\$100 P =\$7 If $S_T<X$; $\prod_2 = -X + S_T + P$ If $S_T \ge X$; $\prod_2 = +P$	Short Strangle C+P =\$10 $\prod = \prod_1 + \prod_2$
\$50	\$+3	-\$43	-\$40
80	+3	-13	-10
83	+3	-10	-7
85	+3	-8	-5
90	+3	-3	0
95	+3	+2	+5
100	+3	+7	+10
105	+3	+7	+10
110	+3	+7	+10
115	-2	+7	+5
117	-5	+7	+2
120	-7	+7	0
125	-12	+7	-5
150	-37	+7	-30

Profits and Losses on a Long and Short Strangle



3. STRIPS

- A strip is an option position simultaneously involving put and call options on the same stock with the SAME strike price and SAME expiration date;
- **Long Strip:** To BUY a strip, an investor will buy ONE call and TWO puts with the SAME strike price and SAME expiration date. The buyer of this strip would be betting that there will be a big stock price move and considers a ***decrease*** in the stock price to be more likely than an increase;
- **Short Strip:** To SELL a strip, a trader will sell ONE call and TWO puts with the SAME strike price and expiration date. The seller would be betting that the stock price remains reasonably close to the exercise price;
- The profits and losses for the long/short strip depends on where the stock price is at expiration.

Payoff from Long Strip: Consider the PURCHASE OF A STRIP with the call having an exercise price of X_1 and put having an exercise price of X_2 (CALL and PUT exercise price is SAME, $X_1=X_2=X$) and an expiration of T (SAME expiration). Then $N_C = 1$ and $N_P = 2$ and the profit from this transaction if held to expiration is:

$$\prod = 1[\mathrm{Max}(0, S_T - X_1) - C] + 2[\mathrm{Max}(0, X_2 - S_T) - P]$$

Since there is only one exercise price involved, there are only two ranges of the stock price at expiration. The profits are as follows:

$$\prod = (S_T - X_1 - C) + 2(-P) \quad \text{if } S_T \geq X$$

$$\prod = (-C) + 2(X_2 - S_T - P) \quad \text{if } S_T < X$$

Lower/Downside BEP: Take the profit equation in which the stock price, $S_T<X$, and set it equal to zero and solve it for S^*_T :

$$\prod = (-C) + 2(X_2 - S^*_T - P) = 0$$

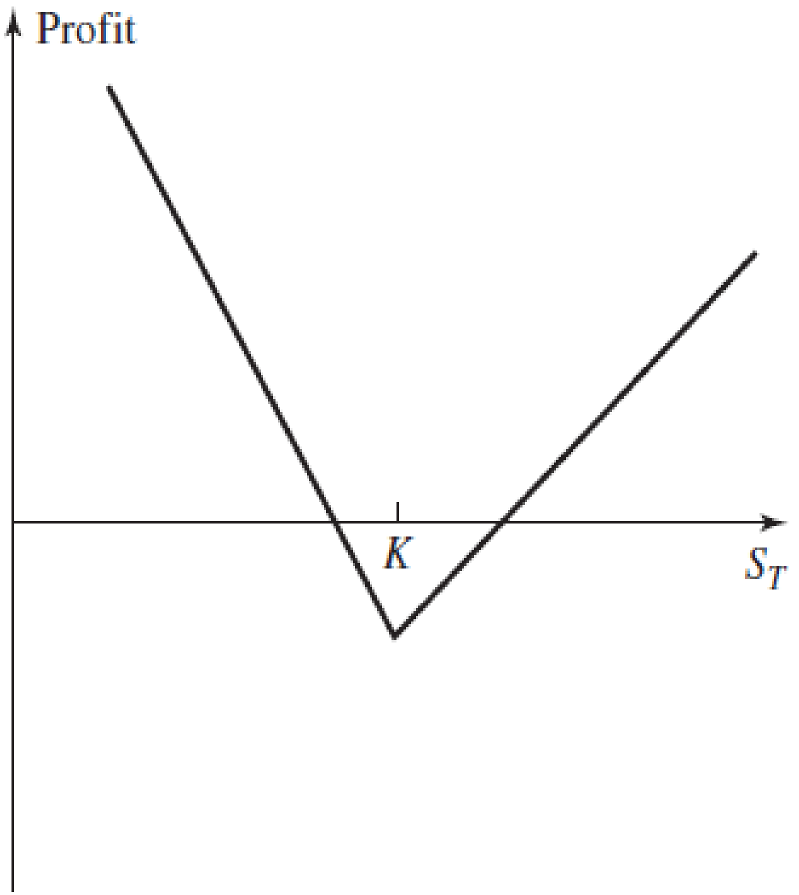
$$\mathbf{S^*_T = X_2 - \left(\frac{C+2P}{2}\right)}$$

Upper/Upside BEP: Take the profit equation in which the stock price $S_T \geq X$, and set it equal to zero and solve it for S^*_T :

$$\prod = (S^*_T - X_1 - C) + 2(-P) = 0$$

$$\mathbf{S^*_T = X_1 + C + 2P}$$

Profits and Losses on a Long Strip



Strip (one call + two puts)

3. STRAPS

- A straps is an option position simultaneously involving put and call options on the same stock with the SAME strike price and SAME expiration date;
- **Long Strap:** To BUY a strap, an investor will buy TWO calls and ONE put with the SAME strike price and SAME expiration date. The buyer of this strap would be betting that there will be a big stock price move and considers an ***increase*** in the stock price to be more likely than a decrease;
- **Short Strap:** To SELL a strap, a trader will sell TWO calls and ONE put with the SAME strike price and SAME expiration date. The seller would be betting that the stock price remains reasonably close to the exercise price;
- The profits and losses for the long/short strap depends on the where the stock price is at expiration.

Payoff from Long Strap: Consider the PURCHASE OF A STRAP with the call having an exercise price of X_1 and put having an exercise price of X_2 (CALL and PUT exercise price is SAME, $X_1 = X_2 = X$) and an expiration of T (SAME expiration). Then $N_C = 2$ and $N_P = 1$ and the profit from this transaction if held to expiration is:

$$\prod = 2[Max(0, S_T - X_1) - C] + 1[Max(0, X_2 - S_T) - P]$$

Since there is only one exercise price involved, there are only two ranges of the stock price at expiration. The profits are as follows:

$$\prod = 2(S_T - X_1 - C) + 1(-P) \quad \text{if } S_T \geq X$$
$$\prod = 2(-C) + 1(X_2 - S_T - P) \quad \text{if } S_T < X$$

Lower/Downside BEP: Take the profit equation in which the stock price, $S_T<X$ and set it equal to zero and solve it for S^*_T:

$$\prod = 2(-C) + 1(X_2 - S^*_T - P) = 0$$

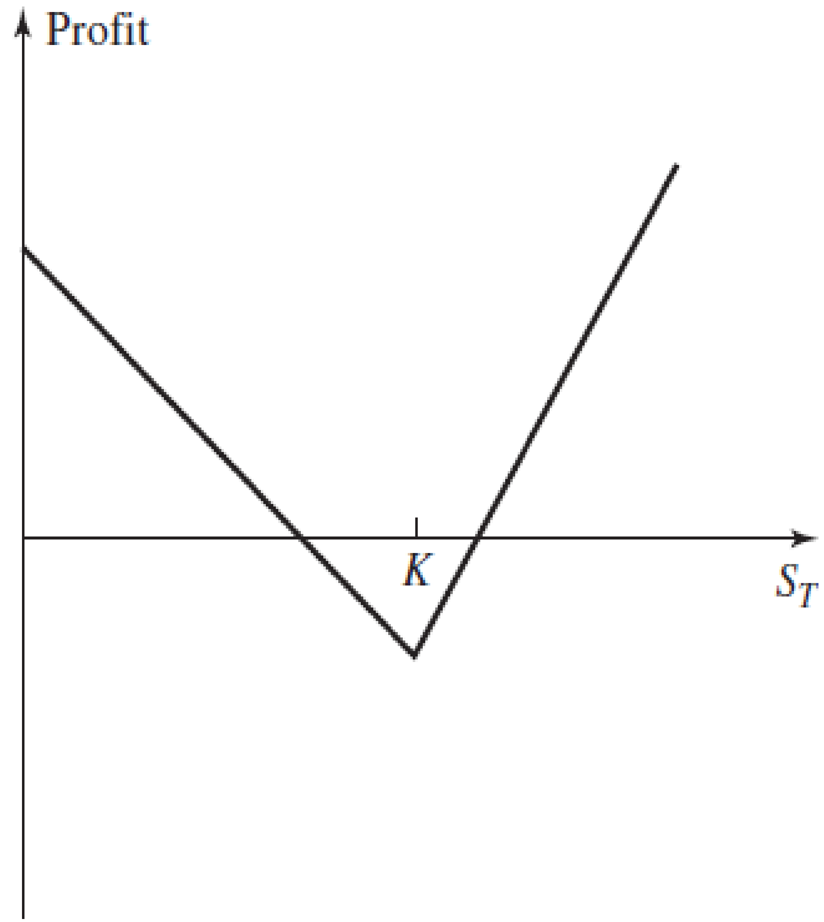
$$\mathbf{S^*_T = X_2 - P - 2C}$$

Upper/Upside BEP: Take the profit equation in which the stock price $S_T \geq X$, and set it equal to zero and solve it for S^*_T:

$$\prod = 2(S^*_T - X_1 - C) + (-P) = 0$$

$$\mathbf{S^*_T = X_1 + \frac{(P+2C)}{2}}$$

Profits and Losses on a Strap



Strap (two calls + one put)

4. COLLARS:

BUY 1 STOCK (S_0)**; BUY 1 PUT (X_1); SELL 1 CALL (X_2)**

- Suppose that an investor buys a stock and wish to protect it against a loss at the same time participate in any gains;
- An obvious strategy is the protective put [**BUY STOCK AND LONG PUT].** However, if the investor buys a put, he will have to pay out cash for the price of the put;
- The collar reduces the cost of the put by adding a short position in a call, where the exercise price of call is HIGHER than the exercise price of the put;
- Although a call with any exercise price can be chosen, there is in fact one particular call that tends to be preferred: the one whose price (premium) is the same as that of the put the investor is buying. Therefore, it is common to set X_2 ($X_2 > X_1$) such that $C_1 = P_1$ and the expiration date of call and put is SAME;
- However, it is not necessary that we choose the call such that its price offsets the price paid for the put;
- The potential loss and gain on the stock are fixed and limited respectively.

Payoff from Collar: Consider the construction of a COLLAR with ONE LONG STOCK (S_0), ONE LONG PUT (X_1) and ONE SHORT CALL (X_2). Generally, CALL exercise price is greater than PUT exercise price ($X_2 > X_1$) and PUT exercise price is less than current stock price, S_0. Then $N_S = 1$, $N_P = 1$ and $N_C = -1$ and P_1 is the put price and C_1 is the call price (where $P_1 = C_1$) and the profit from this transaction if held to expiration is:

$$\prod = S_T - S_0 + \text{Max}(0, X_1 - S_T) - P_1 - \text{Max}(0, S_T - X_2) + C_1$$

Since two exercise prices are involved, there will be three ranges of the stock price at expiration. The profits are as follows:

$$\prod = S_T - S_0 + X_1 - S_T - P_1 + C_1 \quad \text{if } S_T \leq X_1 < X_2$$

$$\prod = X_1 - S_0 - P_1 + C_1$$

$$\prod = S_T - S_0 - P_1 + C_1 \quad \text{if } X_1 < S_T < X_2$$

$$\prod = S_T - S_0 - P_1 - S_T + X_2 + C_1 \quad \text{if } X_1 < X_2 \leq S_T$$

$$\prod = X_2 - S_0 - P_1 + C_1$$

BEP: Take the profit equation for the second case in which the stock price (S_T) is less than X_2, $\mathbf{X_1 < S_T < X_2}$, and set it equal to zero and solve it for S^*_T:

$$\prod = S_T - S_0 - P_1 + C_1 = 0$$

$$S^*_T = S_0 \quad (\text{since } P_1 = C_1)$$

DCRB OPTION DATA, MAY 14

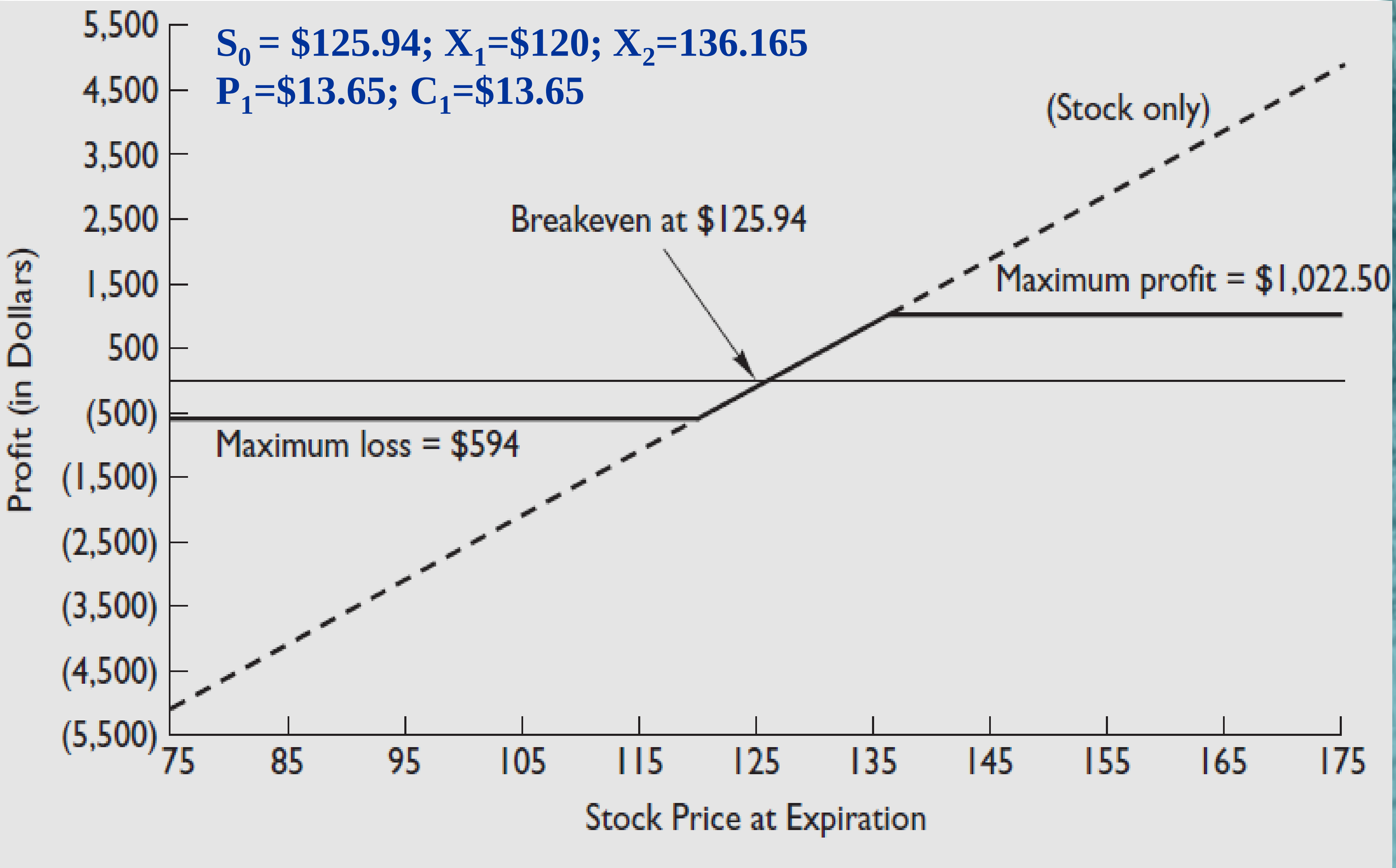
	Calls			Puts		
Exercise Price	May	June	July	May	June	July
120	8.75	15.40	20.90	2.75	9.25	13.65
125	5.75	13.50	18.60	4.60	11.50	16.60
130	3.60	11.35	16.40	7.35	14.25	19.65

Current stock price: 125.94
Expirations: May 21, June 18, July 16
Risk-free rates (continuously compounded): 0.0447 (May); 0.0446 (June); 0.0453 (July)

Profits and Losses on Collar

S_0 = \$125.94; X_1=\$120; X_2=136.165

P_1=\$13.65; C_1=\$13.65



OPTION SPREADS

OPTION SPREADS

Option Spreads: A spread trading strategy involves taking a long position and short position using either calls or puts on the same underlying with SAME (different) expiration date simultaneously with DIFFERENT (same) exercise prices. Spreads can be executed using either calls or puts (two or more calls or two or more puts)

Types of Spreads:

1. Vertical, strike, or money spread: This strategy involves the purchase of an option with a particular exercise price and the sale of another option DIFFERING only by exercise price. The expiration month of long and short option is SAME (EXPIRATION DATE IS SAME, EXERCISE PRICE IS DIFFERENT). For example, one might purchase an option on DCRB expiring in June with an exercise price of 120 and sell an option on DCRB also expiring in June but with an exercise price of 125;

2. Horizontal, time, or calendar spread: This strategy involves the purchase of an option with an expiration of a given month and sale of an otherwise identical option with a DIFFERENT expiration month. The exercise price of long and short option is SAME (EXPIRATION DATE IS DIFFERENT, EXERCISE PRICE IS SAME). For example, one might purchase a DCRB June 120 call and sell a DCRB July 120 call.

DCRB OPTION DATA, MAY 14

	Calls			Puts		
Exercise Price	May	June	July	May	June	July
120	8.75	15.40	20.90	2.75	9.25	13.65
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130	3.60	11.35	16.40	7.35	14.25	19.65

Current stock price: 125.94
Expirations: May 21, June 18, July 16
Risk-free rates (continuously compounded): 0.0447 (May); 0.0446 (June); 0.0453 (July)

Why Investors Use Option Spreads:

- Spreads offer the potential for a small profit while limiting the risk. Spreads can be very useful in modifying risk while allowing profits if market forecasts prove accurate;
- Risk reduction is achieved by being long in one option and short in another. For example, if the stock price decreases, the loss on a long call will be somewhat offset by a gain on a short call. Whether the gain outweighs the loss depends on the volatility of each call.

BULL AND BEAR SPREADS

I. BULL AND BEAR SPREADS:

- An investor who enters into a bull spread is hoping that the stock price will increase. By contrast, an investor who enters into a bear spread is hoping that the stock price will decline;
- Consider a money spread held to expiration: Assume that the trader buys a call with the low strike price (ITM) and sells a call with the high strike price (OTM). In a **bull market**, trader will make money, because the low-exercise-price call will bring a higher payoff at expiration than will the high-exercise-price call. In a **bear market**, both calls will probably expire worthless and trader will lose money;
- **Call Bull spread:** For that reason, the spread involving the PURCHASE of the low-exercise price call and SALE of the high exercise price call is referred to as a bull spread;
- **Call Bear Spread**: Similarly, in a bear market investor makes money if the investor is LONG the high exercise price call and SHORT the low-exercise-price call. This is called a bear spread;
- **Put Bull and Bear Spreads:** Opposite rules apply for puts. A position of long (short) the low exercise price put and short (long) the high exercise-price put is a put bull (bear) spread;
- In general, a bull spread should profit in a bull market and a bear spread should profit in a bear market;
- Time spreads are not classified into bull and bear spreads. They profit by either increased or decreased volatility.

1. BULL CALL SPREAD:

LONG CALL (ITM), SHORT CALL (OTM)

- A bull call spread consists of the PURCHASE of an ITM call [For ITM calls STOCK PRICE will be greater than EXCERCISE PRICE ($S_0>X_1$)] and the SALE of an OTM call [For OTM calls STOCK PRICE will be less than EXERCISE PRICE ($S_0<X_2$)] and given this $X_2>X_1$;
- Naturally, given the current stock price, the long ITM call with the lower exercise price (X_1) will be in-the-money while the short OTM call with the higher exercise price (X_2) will be out-of-the-money;
- Both calls have the SAME underlying asset and SAME expiration month;
- The net effect of the strategy is to bring down the cost and BEP on a long call option strategy;
- This strategy is exercised when investor is moderately bullish to bullish, and the investor will make a profit only when the stock price rises;
- If the stock price on expiration FALLS to the LOWER (LONG CALL) exercise price, the investor makes the maximum LOSS (net cost of the trade/net premium) and if the stock price RISES on expiration to the HIGHER (SHORT CALL) exercise price, the investor makes the maximum PROFIT;
- The spread is a “bull” spread because the trader hopes to profit from a price rise in the stock. The trade is a “spread” because it involves buying one option and selling a related option

Payoff from Bull Call Spread: Consider a BULL CALL SPREAD involving purchase of an ITM call and the SALE of an OTM call with the ITM call having an exercise price of X_1 and OTM call having an exercise price of X_2 [ITM CALL and OTM CALL exercise price is DIFFERENT, ($X_1<X_2$) and an expiration of T (SAME expiration)]. Then $N_{C1} = 1$ and $N_{C2} = -1$, C_1 ITM call price, C_2 OTM call price ($C_1>C_2$) and the profit from this transaction if held to expiration is:

$$\prod = \mathrm{Max}(0, S_T - X_1) - C_1 - \mathrm{Max}(0, S_T - X_2) + C_2$$

Since two exercise prices are involved, there will be three ranges of the stock price at expiration. The profits are as follows:

$$\prod = - C_1 + C_2 \quad \text{if } S_T \leq X_1 < X_2$$

$$\prod = S_T - X_1 - C_1 + C_2 \quad \text{if } X_1 < S_T \leq X_2$$

$$\prod = S_T - X_1 - C_1 - S_T + X_2 + C_2 \quad \text{if } X_1 < X_2 < S_T$$

$$\prod = X_2 - X_1 - C_1 + C_2$$

BEP: The BEP at expiration is between the two exercise prices, X_1, X_2. To calculate the BEP take the profit equation when $\mathbf{X_1 < S_T \leq X_2}$ and set it equal to zero and solve it for S^*_T :

$$\prod = \mathbf{S_T - X_1 - C_1 + C_2} = 0$$

$$\mathbf{S^*_T = X_1 + C_1 - C_2}$$

DCRB OPTION DATA, MAY 14

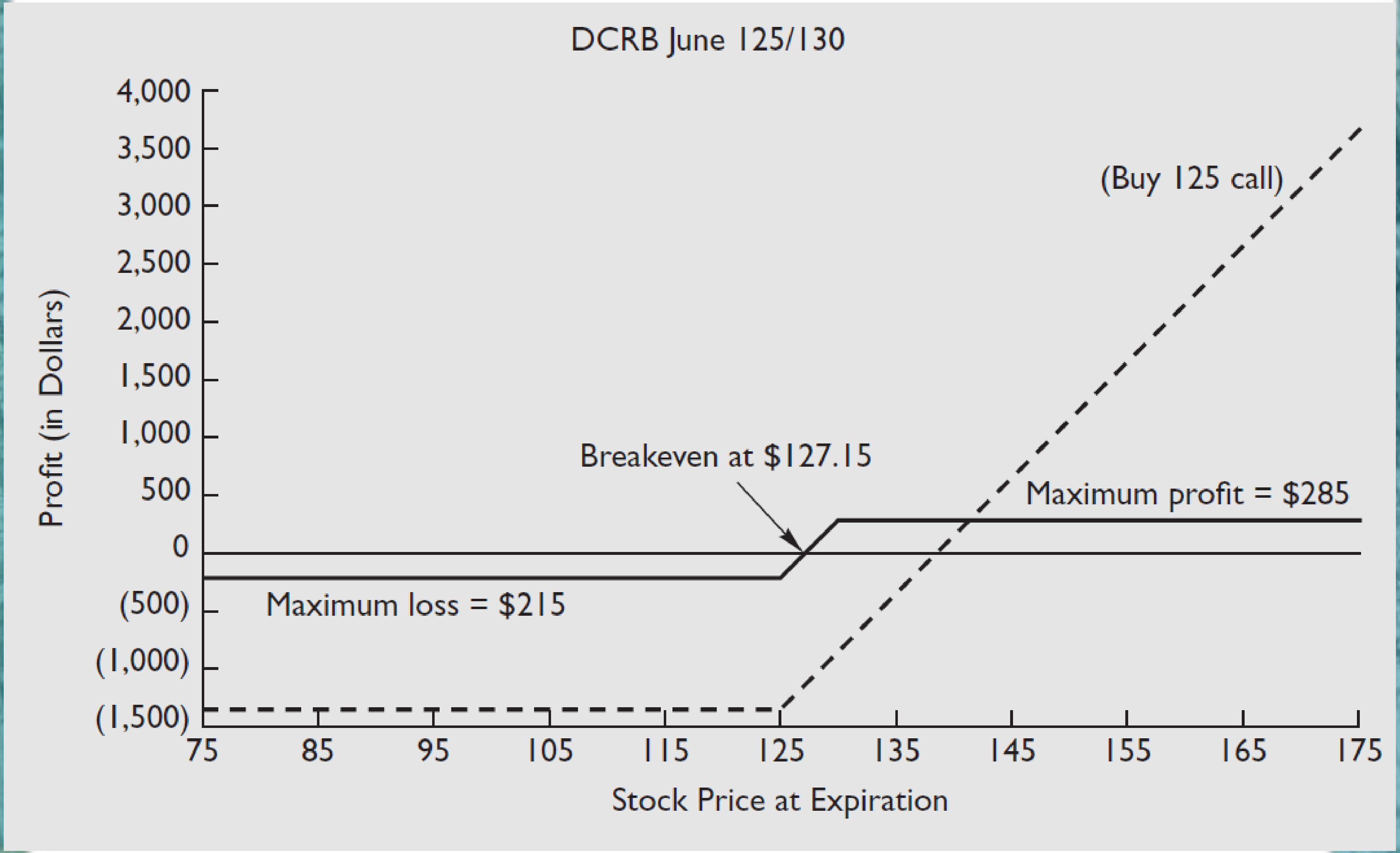
	Calls			Puts		
Exercise Price	May	June	July	May	June	July
120	8.75	15.40	20.90	2.75	9.25	13.65
125	5.75	13.50	18.60	4.60	11.50	16.60
130	3.60	11.35	16.40	7.35	14.25	19.65

Current stock price: 125.94
Expirations: May 21, June 18, July 16
Risk-free rates (continuously compounded): 0.0447 (May); 0.0446 (June); 0.0453 (July)

Important Observations: Call Bull Spread

1. The long CALL starts making profits if the stock price moves above $127.15. The short CALL profits mostly if the stock price does not exceed $130;
2. The CALL Bull Spread Strategy has brought the breakeven stock price down to $127.15 (if only the $125 exercise price call was purchased the breakeven stock price would have been $138.5);
3. Reduced the cost of the trade to $2.15 (if only the $125 exercise price call was purchased the cost of the trade would have been $13.5);
4. Reduced the loss on the trade to $2.15 (if only the $125 exercise price call was purchased the loss would have been $13.5 i.e. the premium of the call purchased);
5. However, the strategy also has limited gains ($2.85) and is therefore ideal when markets are moderately bullish.

Call Bull Spread

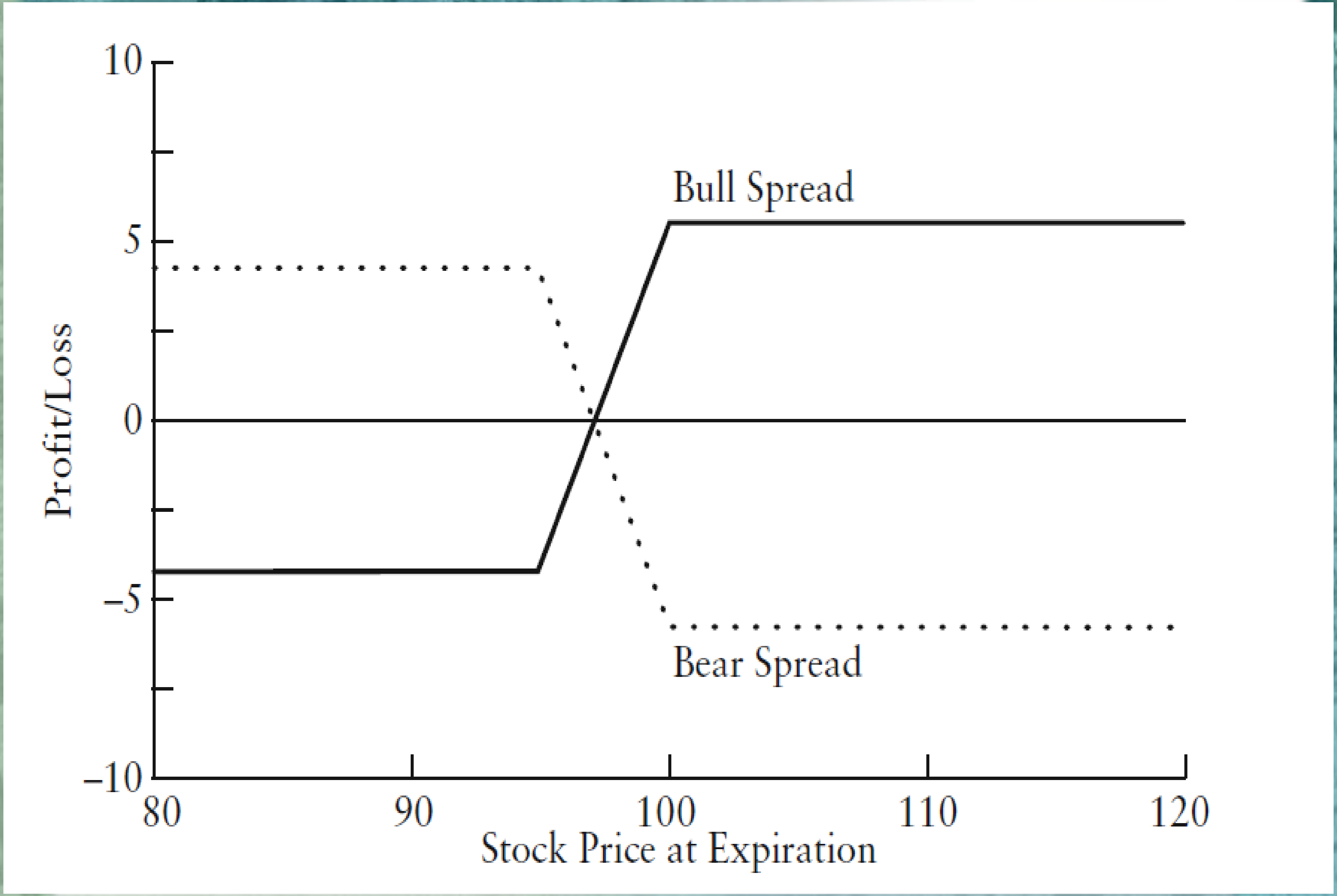


2. BEAR CALL SPREAD:

SHORT CALL (ITM), LONG CALL (OTM)

- A call bear spread consists of the SALE of slightly ITM call [For ITM calls STOCK PRICE will be greater than EXCERCISE PRICE ($S_0 > X_1$)] and the PURCHASE of OTM call [For OTM calls STOCK PRICE will be less than EXERCISE PRICE ($S_0 < X_2$)] and given this $X_2 > X_1$;
- Naturally, given the current stock price, the SHORT ITM call with the lower exercise price (X_1) will be in-the-money while LONG OTM call with higher exercise price (X_2) will be out-of-the-money;
- Both calls have the SAME underlying asset and SAME expiration month;
- This strategy is exercised when investor is moderately bearish to bearish, and the investor will make a profit only when the stock price falls;
- The strategy is to protect the downside of a call sold by buying a call of higher exercise price to insure the call sold;
- If the stock price on expiration FALLS to the lower (SHORT CALL) exercise price, the investor makes the maximum PROFIT (net premium received from the trade) and if the stock price on expiration RISES to the higher (LONG CALL) exercise price, the investor makes the maximum LOSS;
- The spread is a “bear” spread because the trader hopes to profit from a price fall in the stock. The trade is a “spread” because it involves selling one option and buying a related option.

Profits and Losses on a Bull and Bear Call Spread



3. BULL PUT SPREAD:

LONG PUT (OTM), SHORT PUT (ITM)

(Buying a put with a lower strike price and selling a put with higher strike price)

- A bull put spread consists of the PURCHASE of an OTM put [For OTM puts STOCK PRICE will be greater than EXCERCISE PRICE ($S_0 > X_1$)] and the SALE of an ITM put [For ITM puts STOCK PRICE will be less than EXERCISE PRICE ($S_0 < X_2$)] and given this $X_1 < X_2$;

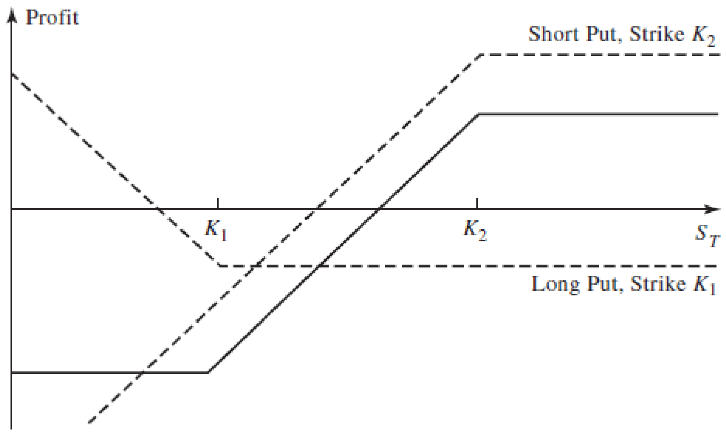
4. BEAR PUT SPREAD:

SHORT PUT (OTM), LONG PUT (ITM)

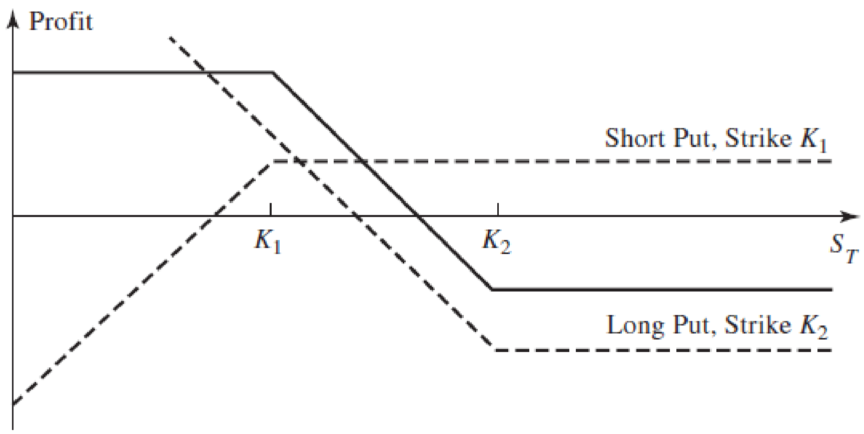
(Buying a Put with a higher strike price and selling a put with lower strike price)

- A bull put spread consists of the SALE of an OTM put [For OTM puts STOCK PRICE will be greater than EXCERCISE PRICE ($S_0 > X_1$)] and the PURCHASE of an ITM put [For ITM puts STOCK PRICE will be less than EXERCISE PRICE ($S_0 < X_2$)] and given this $X_1 < X_2$;

Profit from bull spread created using put options



Profit from bear spread created using put options



BUTTERFLY SPREADS

1. LONG CALL BUTTERFLY SPREAD:

BUY 1 ITM CALL; SELL 2 ATM CALL; BUY 1 OTM CALL

- A long call butterfly spread strategy consists of the purchase of 1 call option with the HIGHER STOCK PRICE compared to exercise price (ITM) ($S_0>X_1$); the sale of 2 call option with (ATM or slightly ITM) $S_0=X_2$ and the purchase of 1 call option with the LOWER STOCK PRICE compared to exercise price (OTM) ($S_0<X_3$);
- A butterfly spread is a combination of a bull spread and a bear spread and this strategy involves three exercise prices: X_1, X_2, and X_3, where X_2 is between X_1 and X_3;
- **Call bull spread:** Suppose we construct a call bull spread by purchasing the call with the low exercise price (ITM), X_1, and writing the call with the middle exercise price (ATM), X_2;
- **Call bear spread:** Next, we also construct a call bear spread by purchasing the call with the high exercise price (OTM), X_3, and writing the call with the middle exercise price (ATM), X_2;
- Combining these positions shows that we are long ONE each of the low and high-exercise-price options and short TWO middle-exercise-price options ($N_{C1} = 1$, $N_{C2}= -2$, and $N_{C3} = 1$).

- A long call butterfly is adopted when the investor is expecting very LITTLE movement in the price of the underlying. The investor is looking to gain from low volatility at a low cost;
- The result is positive incase the stock price remains range bound. The maximum reward in this strategy is however restricted and takes place when the stock price is at the middle exercise price at expiration. The maximum losses are also limited

Payoff from Long Call Butterfly Spread: A combination of bull call spread and bear call spread. It involves three exercise prices, X_1, X_2, X_3, where X_2 is between X_1 and X_3. LONG ONE CALL (ITM), X_1; SHORT TWO CALLS (ATM), X_2 ; LONG ONE CALL (OTM), X_3.

BULL CALL SPREAD = Long Call with X_1 + Short Call with X_2

BEAR CALL SPREAD = Long Call with X_3 + Short Call with X_2

Then $N_{C1} = 1$; $N_{C2} = -2$, and $N_{C3} = 1$ (C_1 ITM call price; C_2 ATM call price and C_3 OTM call price) and the profit from this transaction if held to expiration is:

$$\prod = \text{Max}(0, S_T - X_1) - C_1 - 2\text{Max}(0, S_T - X_2) + 2C_2 + \text{Max}(0, S_T - X_3) - C_3$$

Since three exercise prices are involved, there will be FOUR ranges of the stock price at expiration. The profits are as follows:

$$\prod = -C_1 + 2C_2 - C_3 \text{ if } S_T \leq X_1 < X_2 < X_3$$

$$\prod = S_T - X_1 - C_1 + 2C_2 - C_3 \text{ if } X_1 < S_T \leq X_2 < X_3$$

$$\prod = S_T - X_1 - C_1 - 2S_T + 2X_2 + 2C_2 - C_3 \text{ if } X_1 < X_2 < S_T \leq X_3$$
$$\prod = -S_T + 2X_2 - X_1 - C_1 + 2C_2 - C_3$$

$$\prod = S_T - X_1 - C_1 - 2S_T + 2X_2 + 2C_2 + S_T - X_3 - C_3 \text{ if } X_1 < X_2 < X_3 < S_T$$
$$\prod = 2X_2 - X_1 - X_3 - C_1 + 2C_2 - C_3$$

Lower/Downside BEP: To calculate the lower BEP, take the profit equation when $X_1 < S_T \le X_2 < X_3$ and set it equal to zero and solve it for S^*_T :

$$\prod = S_T - X_1 - C_1 + 2C_2 - C_3 = 0$$
$$S^*_T = X_1 + C_1 - 2C_2 + C_3$$

Upper/Upside BEP: To calculate the upper BEP, take the profit equation when $X_1 < X_2 < S_T \le X_3$ and set it equal to zero and solve it for S^*_T :

$$\prod = - S_T + 2X_2 - X_1 - C_1 + 2C_2 - C_3 = 0$$
$$S^*_T = 2X_2 - X_1 - C_1 + 2C_2 - C_3$$

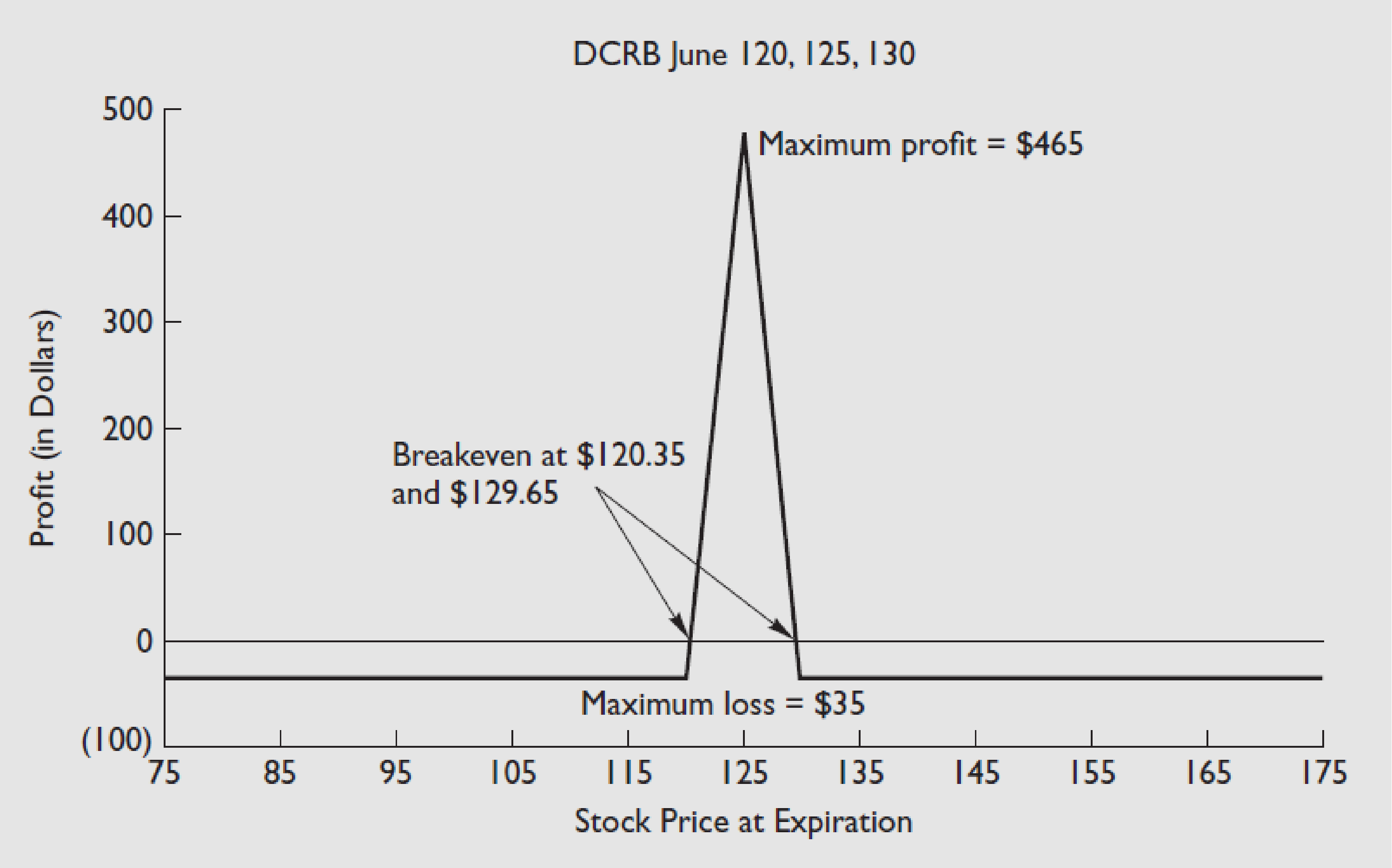
DCRB OPTION DATA, MAY 14

	Calls			Puts		
Exercise Price	May	June	July	May	June	July
120	8.75	15.40	20.90	2.75	9.25	13.65
125	5.75	13.50	18.60	4.60	11.50	16.60
130	3.60	11.35	16.40	7.35	14.25	19.65

Current stock price: 125.94
Expiration: May 21, June 18, July 16
Risk-free rates (continuously compounded): 0.0447 (May); 0.0446 (June); 0.0453 (July)

Notation: X_1, X_2, X_3 = exercise prices of calls where $X_1 < X_2 < X_3$
C_1, C_2, C_3 = prices of calls with exercise prices X_1, X_2, X_3
N_1, N_2, N_3 = quantity held of each option.

Profits and Losses on a Long Call Butterfly



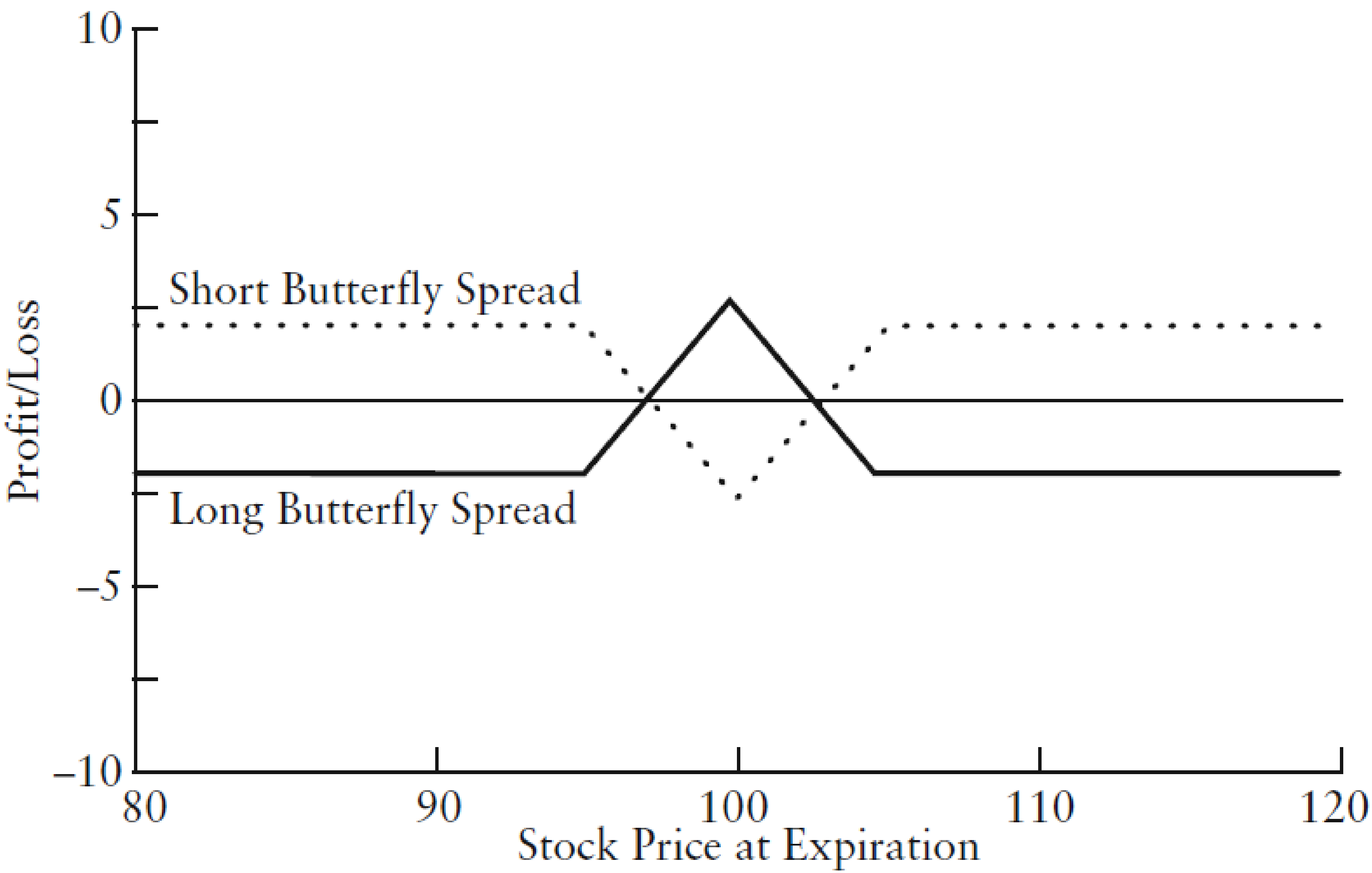
The LONG CALLL butterfly spread strategy assumes that the stock price will fluctuate very little. In this example, the trader is betting that the stock price will stay within the range of $120.35, a downward move of 4.4 percent, to $129.65, an upward move of 2.9 percent. If this prediction of low stock price volatility proves incorrect, however, the potential loss will be limited—in this case, to $35. Thus, the LONG CALL butterfly spread is a low-risk transaction.

2. SHORT CALL BUTTERFLY SPREAD:

SELL 1 ITM CALL; BUY 2 ATM CALL; SELL 1 OTM CALL

- A Short Call Butterfly is a strategy for volatile markets. It is the opposite of Long Call Butterfly, which is a range bound strategy;
- The short call butterfly can be constructed by selling one lower striking in-the-money call, buying two at-the-money calls and selling another higher strike out-of-the-money call, giving the investor a net credit (therefore it is an income strategy);
- A trader who believes that the stock price will be extremely volatile and will fall outside of the two breakeven stock prices might want to short or write a butterfly spread. This will involve one short position in each of the X_1 and X_3 calls and two long positions in the X_2 call;
- The resulting position will be profitable in case there is a big move in the stock;
- The maximum risk occurs if the stock is at the middle strike at expiration;
- The maximum profit occurs if the stock finishes on either side of the upper and lower strike prices at expiration

Profits and Losses on a Long and Short Call Butterfly



BOX SPREADS

Box Spreads

- A box spread is a combination of a bull call money spread and a bear put money spread. The box spread is a low-risk in fact riskless strategy;
- Consider a group of options with two exercise prices, X_1 and X_2, and the SAME expiration;
- **A bull call spread** would involve the purchase of the (ITM) call with (lower) exercise price X_1 at a premium of C_1 and the sale of the (OTM) call with (higher) exercise price X_2 at a premium of C_2 where $X_2>X_1$ and $C_1>C_2$;
- **A bear put spread** would require the sale of the (OTM) put with (lower) exercise price X_1 at a premium of P_1 and purchase of the (ITM) put with (higher) exercise price X_2 at a premium of P_2 where $X_2>X_1$ and $P_2> P_1$

DCRB OPTION DATA, MAY 14

	Calls			Puts		
Exercise Price	May	June	July	May	June	July
120	8.75	15.40	20.90	2.75	9.25	13.65
125	5.75	13.50	18.60	4.60	11.50	16.60
130	3.60	11.35	16.40	7.35	14.25	19.65

Current stock price: 125.94
Expirations: May 21, June 18, July 16
Risk-free rates (continuously compounded): 0.0447 (May); 0.0446 (June); 0.0453 (July)

Payoff from BOX Spread: A bull call spread would involve the purchase of the call with exercise price X_1 (\$125) at a premium of C_1 (\$13.5) and the sale of the call with exercise price X_2 (\$130) at a premium of C_2 (\$11.35). A bear put spread would require the purchase of the put with exercise price X_2 (\$130) at a premium of P_2 (\$14.25) and the sale of the put with exercise price X_1 (\$125) at a premium of P_1 (\$11.50). Under the rules for the effect of exercise price on put and call prices, both the call and put spread would involve an initial cash outflow, because $C_1 > C_2$ and $P_2 > P_1$. Thus, the box spread would have a net cash outflow at the initiation of the strategy. The profit at expiration is:

$$\begin{aligned}\Pi = {} & \text{Max}(0, S_T - X_1) - C_1 - \text{Max}(0, S_T - X_2) + C_2 \\ & + \text{Max}(0, X_2 - S_T) - P_2 - \text{Max}(0, X_1 - S_T) + P_1\end{aligned}$$

Since two exercise prices are involved, there will be three ranges of the stock price at expiration. The profits are as follows:

$$\begin{aligned}\Pi &= -C_1 + C_2 + X_2 - S_T - P_2 - X_1 + S_T + P_1 \\ &= X_2 - X_1 - C_1 + C_2 - P_2 + P_1 && \text{if } S_T \le X_1 < X_2. \\ \Pi &= S_T - X_1 - C_1 + C_2 + X_2 - S_T - P_2 + P_1 \\ &= X_2 - X_1 - C_1 + C_2 - P_2 + P_1 && \text{if } X_1 < S_T \le X_2. \\ \Pi &= S_T - X_1 - C_1 + X_2 - S_T + C_2 - P_2 + P_1 \\ &= X_2 - X_1 - C_1 + C_2 - P_2 + P_1 && \text{if } X_1 < X_2 < S_T.\end{aligned}$$

Note: Notice that the profit is the same in each case. The box spread will be worth $X_2 - X_1$ at expiration, and the profit will be $X_2 - X_1$ minus the premiums paid, $C_1 - C_2 + P_2 - P_1$. The box spread is thus a riskless strategy

Why Box Spread: Why would anyone want to execute a box spread if one can more easily earn the risk-free rate by purchasing Treasury bills? The reason is that the box spread may prove to be incorrectly priced, as a valuation analysis can reveal. Because the box spread is a riskless transaction that pays off the difference in the exercise prices at expiration, it should be easy to determine whether it is correctly priced

Payoff: The payoff is discounted at the risk-free rate. The present value of this amount is then compared to the cost of obtaining the box spread, which is the net premiums paid. This procedure is like analyzing a capital budgeting problem. ***The present value of the payoff at expiration minus the net premiums is a net present value (NPV).*** Since the objective of any investment decision is to maximize NPV, an investor should undertake all box spreads in which the NPV is positive. On those spreads with a negative NPV, one should execute a reverse (short) box spread;

The net present value of a box spread is:

$$NPV = (X_2 - X_1)(1 + r)^{-T} - C_1 + C_2 - P_2 + P_1,$$

where r is the risk-free rate and T is the time to expiration. If NPV is positive, the present value of the payoff at expiration will exceed the net premiums paid. If NPV is negative, the total amount of the premiums paid will exceed the present value of the payoff at expiration

The Box Spread

The Box Spread

$-(C_1 - C_2 + P_2 - P_1)$ $\qquad$ $(X_2 - X_1)$

Today $\qquad$ Expiration

(Find the present value of the future payoff)

$$NPV = (X_2 - X_1)(1 + r)^{-T} - (C_1 - C_2 + P_2 - P_1)$$

If NPV > 0, execute the box spread.
If NPV < 0, execute a reverse box spread.

Calendar Spreads (Time Spread/Horizontal Spread)

1. Calendar Spreads (Time Spread/Horizontal Spread)

- Up to now the options used to create a spread all expire at the same time. We now move on to calendar spreads in which the options have the SAME strike price but DIFFERENT expiration dates;
- Calendar spread involves the purchase of an option with one expiration date and the sale of an otherwise identical option with a different expiration date. Both options have the same exercise prices;
- Because it is not possible to hold both options until expiration, analyzing a calendar spread is more complicated;
- Since one option expires before the other, the longest possible holding period would be to hold the position until the shorter-maturity option's expiration. Then, the other option would have some remaining time value that must be estimated;
- Because both options have the same exercise prices, they will have the same intrinsic values; thus, the profitability of the calendar spread will be determined solely by the difference in their time values

Calendar Spreads with Calls and Puts

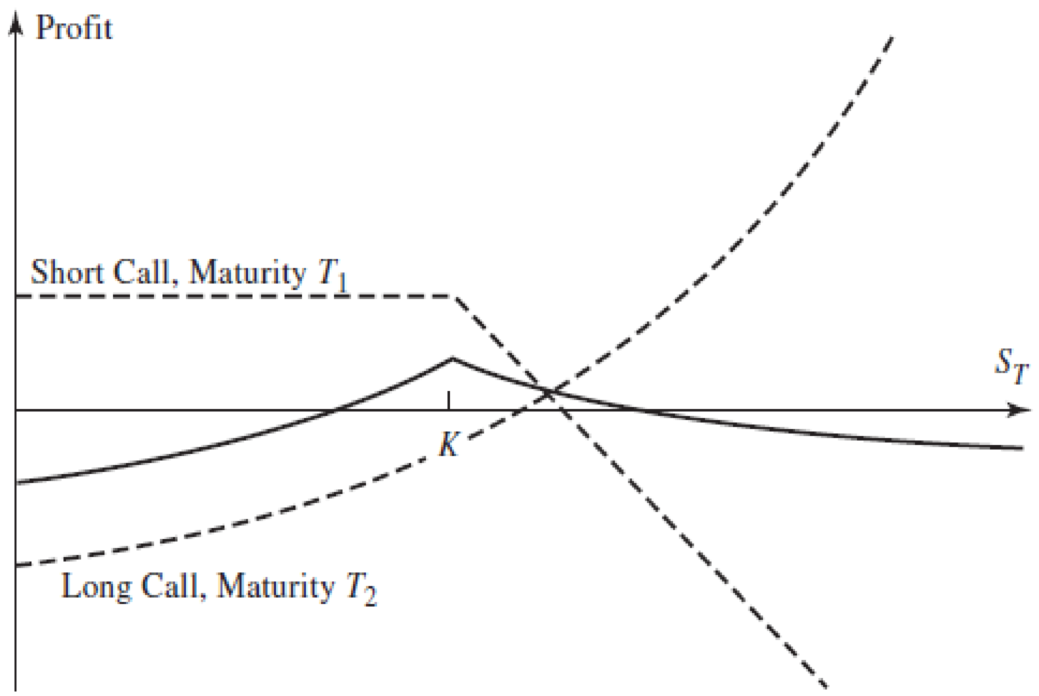
1. Calendar Spread with Call Options: A calendar spread using calls can be created by BUYING a longer-maturity call option with a certain strike price and SELLING a short maturity call option with the SAME strike price. The longer the maturity of an option, the more expensive it usually is. A calendar spread therefore usually requires an initial investment. Here investor may construct a *Neutral calendar spread*, where a strike price close to the current stock price is chosen; *Bullish calendar spread* involving a higher strike price; *Bearish calendar spread* involving a lower strike price

2. Calendar Spread with Put Options: The investor BUYS a long-maturity put option and SELLS a short-maturity put option

3. Reverse Calendar Spreads: Opposite of above calendar spreads involving calls and puts. The investor BUYS a short-maturity option and SELLS a long-maturity option. A small profit arises if the stock price at the expiration of the short-maturity option is well above or well below the strike price of the short-maturity option. However, a loss results if it is close to the strike price

Payoffs: Profit diagrams for calendar spreads are usually produced so that they show the profit when the short-maturity option expires on the assumption that the long-maturity option is closed out at that time.

Profit from calendar spread created using two call options, calculated at the time when the short-maturity call option expires



The investor makes a profit if the stock price at the expiration of the short-maturity option is close to the strike price of the short-maturity option. However, a loss is incurred when the stock price is significantly above or significantly below this strike price

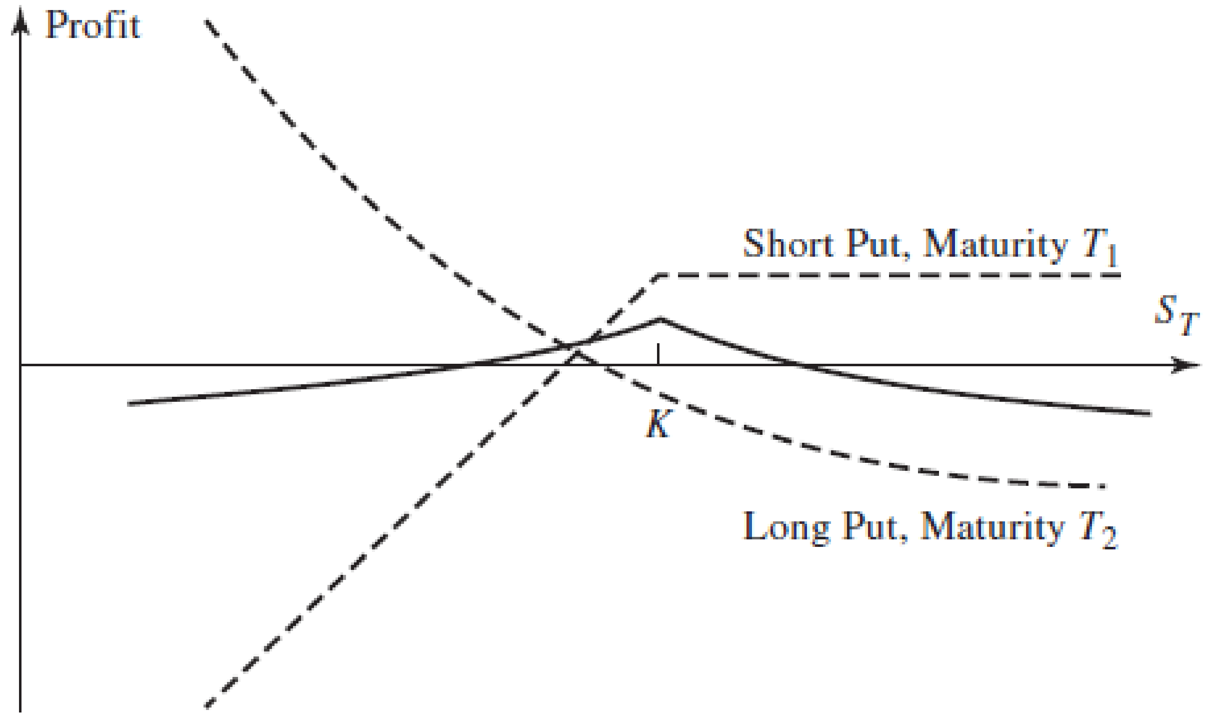
Profit from calendar spread created using two call options, calculated at the time when the short-maturity call option expires

Case I: **If S_T (stock price at expiration) is very low from strike price when the short-maturity option expires** (both the calls will be out of the money). The short-maturity option is worthless and the value of the long-maturity option is close to zero. The investor therefore incurs a loss that is close to the cost of setting up the spread initially;

Case II: **If S_T (stock price at expiration) is very high from strike price when the short-maturity option expires** (both the calls will be in the money). The short-maturity option costs the investor S_T -X, and the long maturity option is worth close to S_T -X, where X is the strike price of the options. Again, the investor makes a net loss that is close to the cost of setting up the spread initially;

Case III: **If S_T (stock price at expiration) is very close to strike price when the short-maturity option expires**. The short-maturity option costs the investor either a small amount or nothing at all. However, the long-maturity option is still quite valuable. In this case a significant net profit is made.

Profit from calendar spread created using two put options, calculated at the time when the short-maturity put option expires



2. Reverse Calendar Spread: A reverse calendar spread is the opposite of calendar spread. The investor buys a short-maturity option and sells a long-maturity option. A small profit arises if the stock price at the expiration of the short-maturity option is well above or well below the strike price of the short-maturity option. However, a loss results if it is close to the strike price

SWAPS

What is a Swap?

A **swap** is a financial contract/agreement whereby two **counterparties** agree to exchange periodic payments

- The amount of the payments exchanged is based on a pre-determined principal which is known as **notional amount** or **notional principal**
- The amount each counterparty pays to the other is the agreed upon periodic rate times the notional principal
- In a swap both parties are exposed to **counterparty risk**

Swap banks

It is difficult and time-consuming for two end users to arrange a swap directly. A more efficient structure for them is to obtain a financial intermediary that serves as counterparty to both end users. This counterparty is called a swap bank. A **swap bank** is a generic term used to describe a financial institution that assists in the completion of a swap. The swap bank profits from the bid–ask spread it imposes on the swap coupon.

The swap bank serves as either a broker or a dealer. A **swap broker** is a swap bank that acts strictly as an agent without taking any financial position in the swap transaction. In other words, the swap broker matches counterparties but does not assume any risk of the swap. The broker receives a commission for this service. A **swap dealer** is a swap bank that actually transacts for its own account to help complete the swap. In this capacity, the swap dealer assumes a position in the swap and thus assumes certain risks.

Why Swaps?

1. MNCs use swaps to fund their foreign investments and manage their interest rate risk and currency risk;
2. Swaps present opportunities to MNCs to reduce financing/borrowing cost (All-in-Cost);
3. Some borrowers have different preferences for debt service payment schedule

All-in-Cost

The counterparties to a swap will be concerned about all-in-cost (**Interest expense, transaction cost and other service charges)** that is ***the effective interest rate*** on the money they have raised/borrowed.

This interest rate is calculated as the discount rate that equates the present value of the future payments and principal payment to the net proceeds received by the issuer.

Types of Swaps

1. Interest Rate Swap
2. Interest-Rate Equity Swap
3. Equity Swap
4. Currency Swap
5. Credit Default Swap (Credit Derivatives)

Types of Swaps

1. Interest Rate Swap: The two counterparties agree to exchange *interest payments* in the *same currency* based on an interest rate with *no exchange of principal* for a specific maturity

Variations:

1. The payments made by both parties can be based on fixed interest rates;

2. **Coupon Swaps:** One party can pay a fixed interest rate and the other party a floating interest rate (reference rate);

3. **Basis Swaps:** The payments made by both parties can be based on different reference rates (floating interest rate)

Market Microstructure: Plain Vanilla Swap or Generic Swap

- The party who pays fixed interest and receives floating is called the ***fixed-rate payer***; the other party (who pays floating and receives fixed interest) is called the ***floating-rate payer***;
- The fixed-rate payer is also called the ***floating-rate receiver*** and is often referred to as having bought the swap or having a long position; the floating rate payer is also called the **fixed-rate receiver** and is referred to as having sold the swap and being short;
- The interest rate paid by the fixed payer often is specified in terms of the yield to maturity (YTM) on a T-note plus basis points;
- The rate paid by the floating payer on a generic swap is the LIBOR;
- Swap payments on a generic swap are made semiannually and the maturities typically range from three to 10 years;

Market Microstructure: Plain Vanilla Swap or Generic Swap

- In the swap contract, a trade date, effective date, settlement date, and maturity date are specified;
- The ***trade date*** is the day the parties agree to commit to the swap;
- The ***effective date*** is the date when interest begins to accrue;
- The ***settlement or payment date*** is when interest payments are made (interest is paid in arrears six months after the effective date);
- The ***maturity date*** is the last payment date;
- On the payment date, only the interest differential between the counterparties is paid. That is, generic swap payments are based on a net settlement basis: The counterparty owing the greater amount pays the difference between what is owed and what is received

Generic Swap Terms

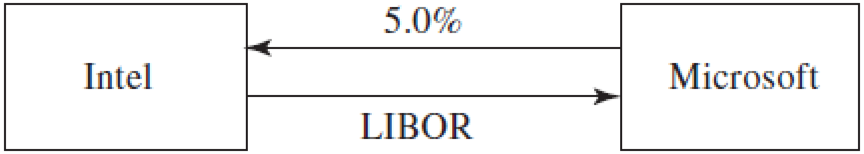
- *Rates*:
  - Fixed rate is usually a T-note rate plus BP
  - Floating rate is a benchmark rate: LIBOR.
- *Reset Frequency*: Semi-annual
- *Notional Principal*: Interest is applied to a notional principal; the NP is used for calculating the swap payments.
- *Maturity* ranges between 3 and 10 years.
- *Dates*: Payments are made in arrears on a semi-annual basis:
  - *Effective Date* is the date interest begins to accrue
  - *Payment Date* is the date interest payments are made.
- *Net Settlement Basis*: The counterparty owing the greater amount pays the difference between what is owed and what is received.
- *Documentation*: Most swaps use document forms suggested by the International Swap and Derivatives Association (ISDA) or the British Bankers' Association. The ISDA publishes a book of definitions and terms to help standardize swap contracts.

Reasons for using Interest Rate Swap:

1. To obtain a cost saving in raising funds;
2. Better match of cash inflows and outflows;
3. Changes in financial markets may cause interest rates to change resulting in interest rate risk;
4. Borrowers have different credit ratings in different countries

Illustration Fixed for floating (Coupon Swap)

Consider a hypothetical 3-year swap initiated on March 5, 2014, between Microsoft and Intel. Suppose Microsoft agrees to pay Intel an interest rate of 5% per annum on a notional principal of $100 million, and in return Intel agrees to pay Microsoft the 6-month LIBOR rate on the same notional principal. The agreement specifies that payments are to be exchanged every 6 months and that the 5% interest rate is quoted with semiannual compounding.



Interest rate swap: 5%/LIBOR swap with NP=$100M

Date (1)	6 LIBOR (%) (2)	Floating rate payers payment* (Intel) (3)	Fixed rate payer's payment** (Microsoft) (4)	Net interest received by fixed rate payer (3)-(4)	Net interest received by floating rate payer (3)-(4)
Mar. 5, 2014	4.20				
Sept. 5, 2014	4.80	2.10	2.50	-0.40	+0.40
Mar. 5, 2015	5.30	2.40	2.50	-0.10	+0.10
Sept. 5, 2015	5.50	2.65	2.50	+0.15	-0.15
Mar. 5, 2016	5.60	2.75	2.50	+0.25	-0.25
Sept. 5, 2016	5.90	2.80	2.50	+0.30	-0.30
Mar. 5, 2017		2.95	2.50	+0.45	-0.45

*(LIBOR/2)($100,000,000)

**(0.05/2)($100,000,000)

Microsoft: Using the Swap to transform a floating rate liability

Microsoft (fixed-rate payer): The swap is used to transform a floating-rate loan into a fixed-rate loan. Suppose, that initially Microsoft has arranged to borrow $100 million at LIBOR+10bp. After Microsoft has entered into the swap, it has the following three sets of cash flows:

1. It pays LIBOR plus 0.1% to its outside lenders;
2. It receives LIBOR under the terms of the swap from Intel;
3. It pays 5% under the terms of the swap

- These three sets of cash flows net out to an interest rate payment of 5.1%
- For Microsoft, the swap has the effect of transforming borrowings at a floating rate of LIBOR+10bp (LIBOR+0.10%) into borrowings at a fixed rate of 5.1%

Intel: Using the Swap to transform a fixed rate liability

Intel (floating-rate payer): The swap is used to transform a fixed-rate loan into a floating-rate loan. Suppose that initially Intel has arranged to borrow $100 million at a fixed-rate loan of 5.2%. After Intel has entered into the swap, it has the following three sets of cash flows:

1. It pays 5.2% to its outside lenders;
2. It pays LIBOR under the terms of the swap;
3. It receives 5% under the terms of the swap from Microsoft.

- These three sets of cash flows net out to an interest rate payment of LIBOR+20bp (LIBOR+0.20%)
- For Intel, the swap has the effect of transforming borrowings at a fixed rate of 5.2% into borrowings at a floating rate of LIBOR+20b.p

Synthetic Loans

- One of the important uses of swaps is in creating a synthetic fixed or floating-rate liability that yields a better rate than the conventional one

Illustration: Synthetic fixed-rate loan

Synthetic fixed-rate loan: A synthetic fixed-rate loan is formed by combining a ***fixed-rate payer's position*** with a ***floating-rate loan***. This loan then can be used as an alternative to a fixed-rate loan

- Suppose a corporation with an AAA credit rating wants a three-year, $10 million fixed-rate loan starting on March 23, Y1. Suppose one possibility available to the company is to borrow $10 million from a bank at a fixed rate of 6% (assume semiannual payments) with a loan maturity of three years. Suppose, that the bank is also willing to provide the company with a three-year floating-rate loan, with the rate set equal to the LIBOR on March 23 and September 23 each year for three years.

Fixed Rate	**6%**
Floating Rate	LIBOR

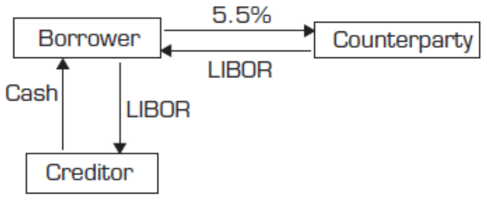
- If a swap agreement identical to the one described above were available, then instead of a direct fixed-rate loan, the company alternatively could obtain a fixed-rate loan by borrowing $10 million on the ***floating-rate loan***, then fix the interest rate by taking a fixed-rate payer's position on the swap as follows:

Conventional floating-rate loan	Pay floating rate
Swap: Fixed-rate payer position	Pay fixed rate
Swap: Fixed-rate payer position	Receive floating rate
Synthetic fixed rate	Pay fixed rate

- If the floating-rate loan is hedged with a swap, any change in the LIBOR would be offset by an opposite change in the net receipts on the swap position. In this example, the company (as shown in the table) would end up paying a constant $275,000 every sixth month, which equates to an annualized borrowing rate of 5.5%: $R = 2(\$275{,}000)/\$10{,}000{,}000 = .055$. Thus, the corporation would be better off combining the swap position as a fixed-rate payer with the floating-rate loan to create a **synthetic fixed-rate** loan than simply taking the straight fixed-rate loan (the synthetic fixed-rate loan yields a 0.5% lower interest rate each period (annualized rate) than a fixed-rate loan): **Synthetic Fixed-Rate Loan: Floating-Rate Loan Set at LIBOR and Fixed-Payer Position on 5.5%/LIBOR Swap**

		Swap				Synthetic loan	
(1) Effective dates	(2) LIBOR	(3) Floating-rate payer's payment*	(4) Fixed-rate payer's payment**	(5) Net interest received by fixed-rate payer (column 3 – column 4)	(6) Loan interest paid on floating-rate loan*	(7) Payment on swap and loan (column 6 – column 5)	(8) Effective annualized rate***
3/1/03	0.045						
9/1/03	0.05	225,000	275,000	−50,000	225,000	275,000	0.055
3/1/04	0.055	250,000	275,000	−25,000	250,000	275,000	0.055
9/1/04	0.06	275,000	275,000	0	275,000	275,000	0.055
3/1/05	0.065	300,000	275,000	25,000	300,000	275,000	0.055
9/1/05	0.07	325,000	275,000	50,000	325,000	275,000	0.055
3/1/06		350,000	275,000	75,000	350,000	275,000	0.055

* (LIBOR/2)($10,000,000)
** (0.055/2)*($10,000,000)
*** 2(Payment on swap and loan)/$10,000,000



Synthetic Loans

Illustration: Synthetic floating-rate loan

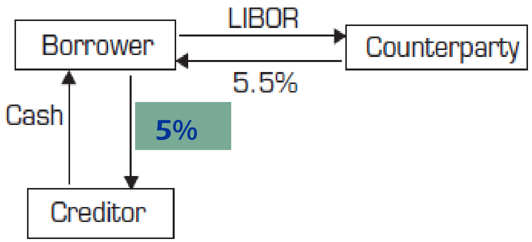
Synthetic floating-rate loan: A synthetic floating-rate loan is formed by combining a floating-rate payer's position with a fixed-rate loan. This loan then can be used as an alternative to a floating-rate loan:

Conventional fixed-rate loan	Pay fixed rate
Swap: Floating-rate payer position	Pay floating rate
Swap: Floating-rate payer position	Receive fixed rate
Synthetic floating rate	Pay floating rate

- An example of a synthetic floating-rate loan is shown below. The synthetic loan is formed with a 5% fixed-rate loan (semiannual payments) and the floating rate payer's position on our illustrate swap. As shown in the table, the synthetic floating-rate loan yields a 0.5% lower interest rate each period (annualized rate) than a floating-rate loan tied to the LIBOR: **Synthetic Floating-Rate Loan: 5% Fixed-Rate Loan and Floating-Payer Position on 5.5%/LIBOR Swap**

		Swap				Synthetic loan	
(1) Effective dates	(2) LIBOR	(3) Floating-rate payer's payment*	(4) Fixed-rate payer's payment**	(5) Net interest received by floating-rate payer (column 4 – column 3)	(6) Loan interest paid on 5% fixed-rate loan	(7) Payment on swap and loan (column 6 – column 5)	(8) Effective annualized rate***
3/1/03	0.045						
9/1/03	0.05	225,000	275,000	50,000	250,000	200,000	0.04
3/1/04	0.055	250,000	275,000	25,000	250,000	225,000	0.045
9/1/04	0.06	275,000	275,000	0	250,000	250,000	0.05
3/1/05	0.065	300,000	275,000	–25,000	250,000	275,000	0.055
9/1/05	0.07	325,000	275,000	–50,000	250,000	300,000	0.06
3/1/06		350,000	275,000	–75,000	250,000	325,000	0.065

* (LIBOR/2)($10,000,000)
** (0.055/2)*($10,000,000)
*** 2 (Payment on swap and loan)/$10,000,000



FORMAL ARGUMENTS: COMPARATIVE ADVANTAGE

Swaps are often used by financial and nonfinancial corporations to take advantage of apparent arbitrage opportunities resulting from capital market inefficiencies. To see this, consider the case of the Star Chemical Company who wants to raise $300 million with a five-year loan to finance an expansion of one of its production plants. Based on its moderate credit ratings, suppose Star can borrow five-year funds at a 10.5% fixed rate or at a floating rate equal to LIBOR+75bp. Given the choice of financing, Star prefers the fixed-rate loan. Suppose the treasurer of the Star Company contacts his investment banker for suggestions on how to finance the acquisition. The investment banker knows that the Moon Development Company is also looking for five-year funding to finance its proposed $300 million office park development. Given its high credit rating, suppose Moon can borrow the funds for 5 years at a fixed rate of 9.5% or at a floating rate equal to the LIBOR + 25 bp. Given the choice, Moon prefers a floating-rate loan. In summary, Star and Moon have the following fixed and floating rate loan opportunities:

	Fixed Rate	Floating Rate	Preference	Comparative Advantage
Star Chemical Company	10.50%	LIBOR+75 bp	Fixed	Floating
Moon Company	9.50%	LIBOR+25 bp	Floating	Fixed
Credit Spread (Differential)	100 bp	50 bp	**QSD:** 100bp-50bp = 50bp	

In this case, the Moon Company has an absolute advantage in both the fixed and floating markets because of its higher quality rating. However, after looking at the credit spreads of the borrowers in each market, ***the investment banker realizes that there is a comparative advantage for Moon in the fixed market and a comparative advantage for Star in the floating market.*** That is, *Moon has a relative advantage in the fixed market where it gets 100 basis points less than Star*; *Star, in turn, has a relative advantage (or relatively less disadvantage) in the floating rate market where it only pays 50 basis points more than Moon*.

Thus, lenders in the fixed-rate market supposedly assess the difference between the two creditors to be worth 100 basis points, whereas lenders in the floating-rate market assess the difference to be only 50 basis points. *Whenever a comparative advantage exists, arbitrage opportunities can be realized by each firm borrowing in the market where it has a* **comparative advantage** *and then swapping loans or having a swap bank set up a swap.*

The key, or necessary condition, giving rise to the swap is that a **quality spread differential** (QSD) exists. A QSD is the difference between the default-risk premium differential on the fixed rate debt and the default-risk premium differential on the floating-rate debt.

For the swap to work, the two companies cannot just pass on their respective costs: Star swaps a floating rate at LIBOR+75bp for a 10.5% fixed; Moon swaps a 9.5% fixed for a floating at LIBOR+25bp. Typically, the companies divide the differences in credit spreads, with the most creditworthy company taking the most savings.

Star Chemical Company:

In this case, suppose the investment banker arranges a five-year, 9.5%/LIBOR generic swap with an NP of $300 million in which Star takes the fixed-rate payer position and Moon takes the floating-rate payer position. The Star Company would then issue a $300 million FRN paying LIBOR+75 bp. This loan, combined with the fixed-rate position on the 9.5%/LIBOR swap would give Star a synthetic fixed-rate loan paying 10.25%, 25 basis points less than its direct fixed-rate loan:

Star Company's Synthetic Fixed-Rate Loan

Issue FRN	Pay LIBOR + 75bp	−LIBOR − .75%
Swap: Fixed-rate payer's position	Pay 9.5%	−9.5%
Swap: Fixed-rate payer's position	Receive LIBOR	+LIBOR
Synthetic fixed rate	Pay 9.5% + .75%	−10.25%
Direct fixed rate	Pay 10.5%	−10.5%

The Moon Company:

The Moon Company, on the other hand, would issue a $300 million, 9.5% fixed rate bond that, when combined with its floating-rate position on the 9.5%/LIBOR swap, would give Moon a synthetic floating-rate loan paying LIBOR, which is 25 basis points less than the rates paid on the direct floating-rate loan of LIBOR plus 25 bp:

Moon Company's Synthetic Floating-Rate Loan

Issue 9.5% fixed-rate bond	Pay 9.5%	−9.5%
Swap: Floating-rate payer's position	Pay LIBOR	−LIBOR
Swap: Floating-rate payer's position	Receive 9.5%	+9.5%
Synthetic Floating Rate	Pay LIBOR	−LIBOR
Direct Floating Rate	Pay LIBOR + 25bp	−LIBOR − .25%

- Thus, the swap makes it possible for both the companies to create synthetic loans with better rates than direct ones. As a rule, for a swap to provide arbitrage opportunities, at least one of the counterparties must have a comparative advantage in one market. The total arbitrage gain available to each party depends on whether one party has an absolute advantage in both markets or each has an absolute advantage in one market. If one party has an absolute advantage in both markets (as in this case), then the arbitrage gain is the difference in the comparative advantages in each market: 50bp = 100bp-50bp. In this case, Star and Moon split the difference in the 50bp gain. In contrast, if each party has an absolute advantage in one market, then the arbitrage gain is equal to the sum of the comparative advantages.

Synthetic Loans and Comparative Advantage

Illustration: Fixed-for-floating Rate Swap

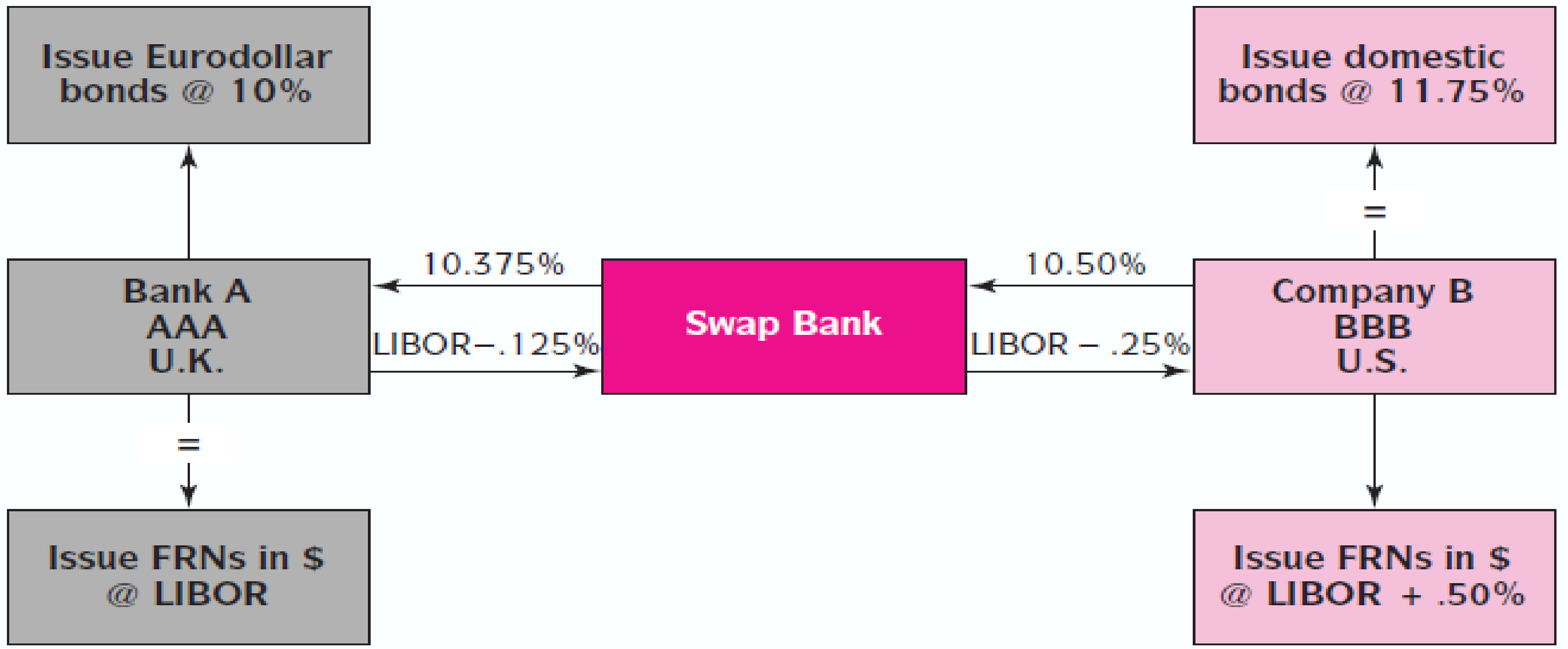
Bank A is a **AAA-rated** international bank located in the United Kingdom. The bank needs $10,000,000 to finance floating-rate Eurodollar term loans to its clients. It is considering issuing five-year floating-rate notes indexed to LIBOR. Alternatively, the bank could issue five-year fixed-rate Eurodollar bonds at 10%. The FRNs make the most sense for Bank A since it would be using a floating-rate liability to finance a floating-rate asset. In this manner, the bank avoids the interest rate risk associated with a fixed-rate issue. Without this hedge, Bank A could end up paying a higher rate than it is receiving on its loans should LIBOR fall substantially

Company B is a **BBB-rated** U.S. company. It needs $10,000,000 to finance a capital expenditure with a five-year economic life. It can issue five-year fixed-rate bonds at a rate of 11.75% in the U.S. bond market. Alternatively, it can issue five-year FRNs at LIBOR+0.50%. The fixed-rate debt makes the most sense for Company B because it locks its financing cost. The FRN alternative could prove very unwise should LIBOR increase substantially over the life of the note, and could possibly result in the project being unprofitable

	Company B	**Bank A**	**Credit Spread (Differential)**	**Comparative Advantage**
Fixed Rate	11.75%	10%	1.75%	**Bank A: Fixed**
Floating Rate	LIBOR+0.50%	LIBOR	0.50%	**Company B: Floating**
Preference	Fixed	Floating	**QSD =1.25%**	

A swap bank familiar with the financing needs of Bank A and Company B has the opportunity to set up a fixed-for-floating interest rate swap that will benefit each counterparty and the swap bank. The key, or necessary condition, giving rise to the swap is that a **quality spread differential** (QSD) exists. A QSD is the difference between the default-risk premium differential on the fixed rate debt and the default-risk premium differential on the floating-rate debt

	Company B	**Bank A**	**Credit Spread (Differential)**
Fixed Rate	11.75%	10%	1.75%
Floating Rate	LIBOR+0.50%	LIBOR	0.50%
		QSD	**1.75%-0.50%=1.25%**



Net Cash Out Flows **Synthetic floating-rate loan** **Synthetic fixed-rate loan**

	Bank A	Swap Bank	Company B
Pays	LIBOR – .125%	10.375%	10.50%
	10%	LIBOR – .25%	LIBOR + .50%
Receives	–10.375%	–10.50%	–(LIBOR – .25%)
		–(LIBOR – .125%)	
Net	LIBOR – .50%	–.25%	11.25%

*Debt service expressed as a percentage of $10,000,000 notional value.

Company B:

The swap bank has instructed Company B to issue FRNs at LIBOR+0.50% rather than the more suitable fixed-rate debt at 11.75 percent. Company B passes through to the swap bank 10.50 percent (on the notional principal of $10,000,000) and receives LIBOR-0.25% in return. In total, Company B pays 10.50 percent (to the swap bank) plus LIBOR+0.50% (to the floating-rate bondholders) and receives LIBOR-0.25% (from the swap bank) for an all-in cost (interest expense, transaction costs, and service charges) of 11.25%. Thus, through the swap, Company B has converted floating-rate debt into fixed-rate debt at an all-in cost 0.50% lower than the 11.75% fixed rate it could arrange on its own.

Bank A:

Similarly, Bank A was instructed to issue fixed-rate debt at 10% rather than the more suitable FRNs. Bank A passes through to the swap bank LIBOR-0.125% and receives 10.375% in return. In total, Bank A pays 10% (to the fixed rate Eurodollar bondholders) plus LIBOR-0.125% (to the swap bank) and receives 10.375 percent (from the swap bank) for an all-in cost of LIBOR-0.50 percent. Through the swap, Bank A has converted fixed-rate debt into floating-rate debt at an all in cost 0.50 percent lower than the floating rate of LIBOR it could arrange on its own.

Swap Bank:

The swap bank also benefits because it pays out less than it receives from each counterparty to the other counterparty. It receives 10.50 percent (from Company B) plus LIBOR-0.125% (from Bank A) and pays 10.375% (to Bank A) and LIBOR-0.25% (to Company B). The net inflow to the swap bank is 0.25% per annum on the notional principal of $10,000,000.

In sum, Bank A has saved 0.50%, Company B has saved 0.50%, and the swap bank has earned 0.25%. This totals 1.25%, which equals the QSD. Thus, if a QSD exists, it can be split in some fashion among the swap parties resulting in lower all-in costs for the counterparties.

CURRENCY SWAPS

2. Currency Swap

- Currency swaps constitute an exchange of principal and interest payments in one currency for principal and interest payments in another currency;
- Both principal and interest is exchanged;
- Currency swap helps to hedge both interest rate risk as well as currency risk. It also helps to reduce the all-in-cost

Variations:

(1) Borrow in foreign currency and swap for home currency;

(2) Borrow in home currency and swap for foreign currency

Motivations/Benefits:

Why issue Foreign Denominated Debt

1. In a world with capital market imperfections, it may be possible for an issuer to reduce its borrowing cost by borrowing funds denominated in a foreign currency;
2. Some companies seek to raise funds in foreign countries as a means of increasing their recognition by foreign investors, despite the fact that the cost of funding may be similar in the home or domestic country

Case-1: Borrow in foreign currency and Swap for home currency

Assume that two MNCs; a US corporation and a Swiss corporation each seek to borrow for 10 years in their respective *domestic currencies*. The US company seeks $100 million US$-denominated debt and the Swiss company seeks debt in the amount of SFr127 million. However, the investment bankers of these companies have advised them that if they issue foreign currency denominated 10-year bonds in the foreign bond market the *all-in-cost* would be LESS. Specifically, the US company is advised to issue SFr-denominated bonds equivalent of $100 million in Switzerland, and the Swiss company is advised to issue US$-denominated bonds equivalent of SFr127 million in the United States having a maturity of 10 years. At the time when both companies want to issue their 10-year bonds, the spot exchange rate between US dollars and Swiss francs is SFr1.27/US$

	Fixed Rate
US Company	**(SFr) 6.00%**
Swiss Company	**(US$) 11.00%**

The cash outflows that the companies must make for the next 10 years are as follows:

Year	US Corporation	Swiss Corporation
1-10 (Interest Payments)	SFr76,20,000 (SFr7.62 M)	US$11,000,000 (US$11 M)
10 (on Maturity)	SFr127,000,000 (SFr127 M)	US$100,000,000 (US$100 M)

Each corporation faces the risk; that at the time a payment on its liability must be made, its domestic currency will have depreciated relative to the other currency exposing the corporation to the FOREIGN CURRENCY RISK

In order to hedge this foreign currency risk their investment banker arranges the following three stage *currency swap deal*:

First Stage: The two parties exchange the proceeds received from the sale of the bonds (exchange of principal);

Second Stage: The two parties make the coupon payments to service the debt of the other party (exchange of interest payments);

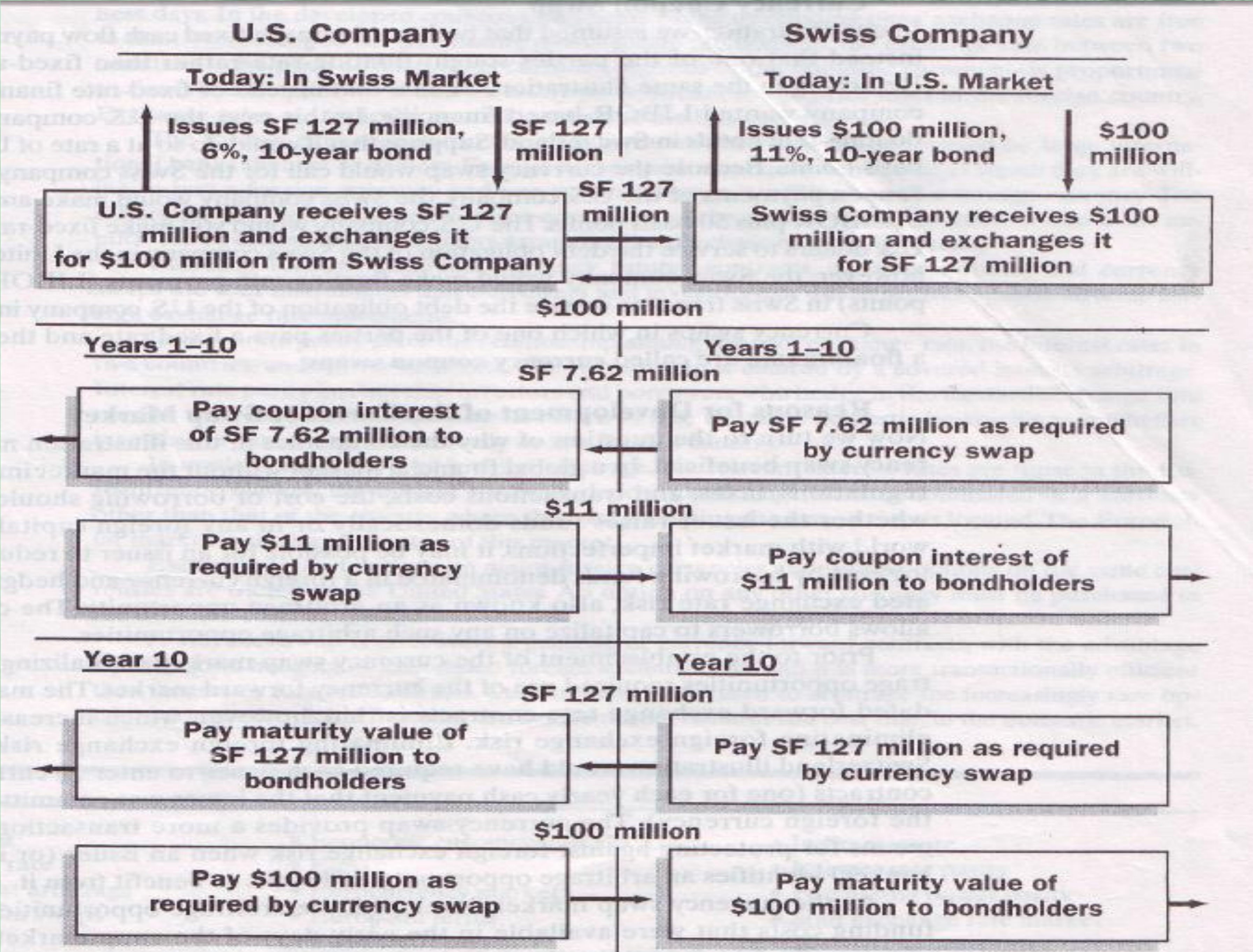
Third Stage: At the termination date of the currency swap both parties agree to exchange the par value of the bonds (re-exchange of principal)

In our illustration, these arrangements result in the following:

Stage 1: The US corporation issues 10-year, 6% coupon bonds with a par value of SFr127 million in Switzerland and gives the proceeds to the Swiss company. At the same time, the Swiss company issues 10-year, 11% bonds with a par value of $100 million in the US and give the proceeds to the US company;

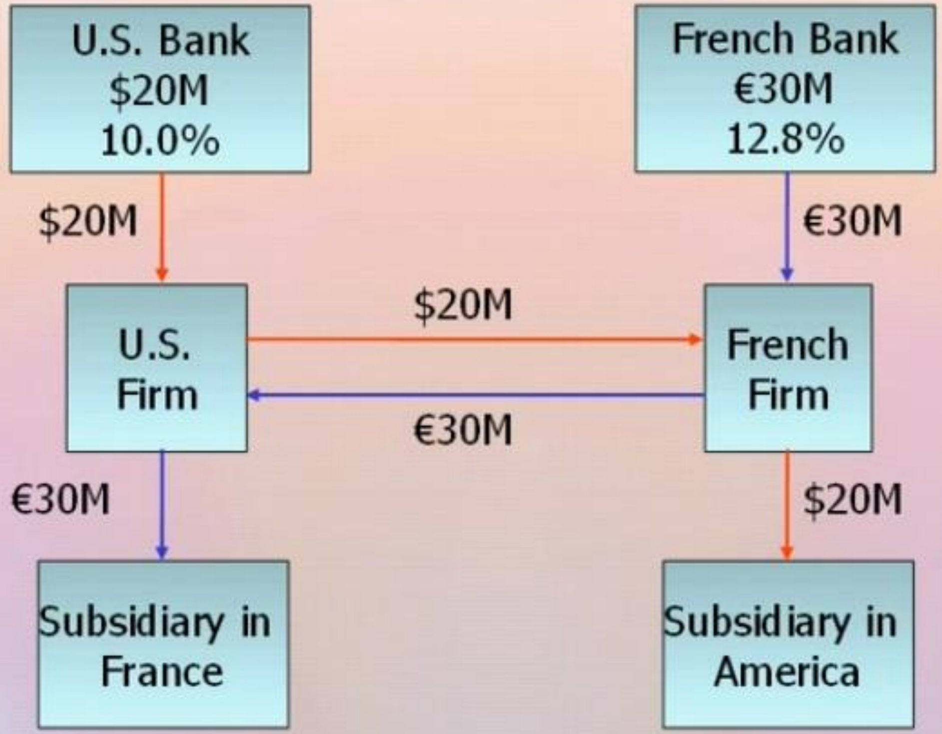
Stage 2: The US corporation agrees to service the coupon payments to the Swiss company by paying the $11,000,000 per year for the next 10 years to the Swiss company; the Swiss corporation agrees to service the coupon payments of the US company by paying the SF76,20,000 per year for the next 10 years to the US company;

Stage 3: At the end of 10 years, the US company would pay $100 million to the Swiss company and the Swiss company would pay SFr127 million to the US company


U.S. Company
Swiss Company
Today: In Swiss Market
Issues SF 127 million, 6%, 10-year bond
SF 127 million
U.S. Company receives SF 127 million and exchanges it for $100 million from Swiss Company
SF 127 million
$100 million
Today: In U.S. Market
Issues $100 million, 11%, 10-year bond
$100 million
Swiss Company receives $100 million and exchanges it for SF 127 million
Years 1–10
SF 7.62 million
Pay coupon interest of SF 7.62 million to bondholders
Pay SF 7.62 million as required by currency swap
$11 million
Pay $11 million as required by currency swap
Pay coupon interest of $11 million to bondholders
Year 10
SF 127 million
Pay maturity value of SF 127 million to bondholders
Pay SF 127 million as required by currency swap
$100 million
Pay $100 million as required by currency swap
Pay maturity value of $100 million to bondholders

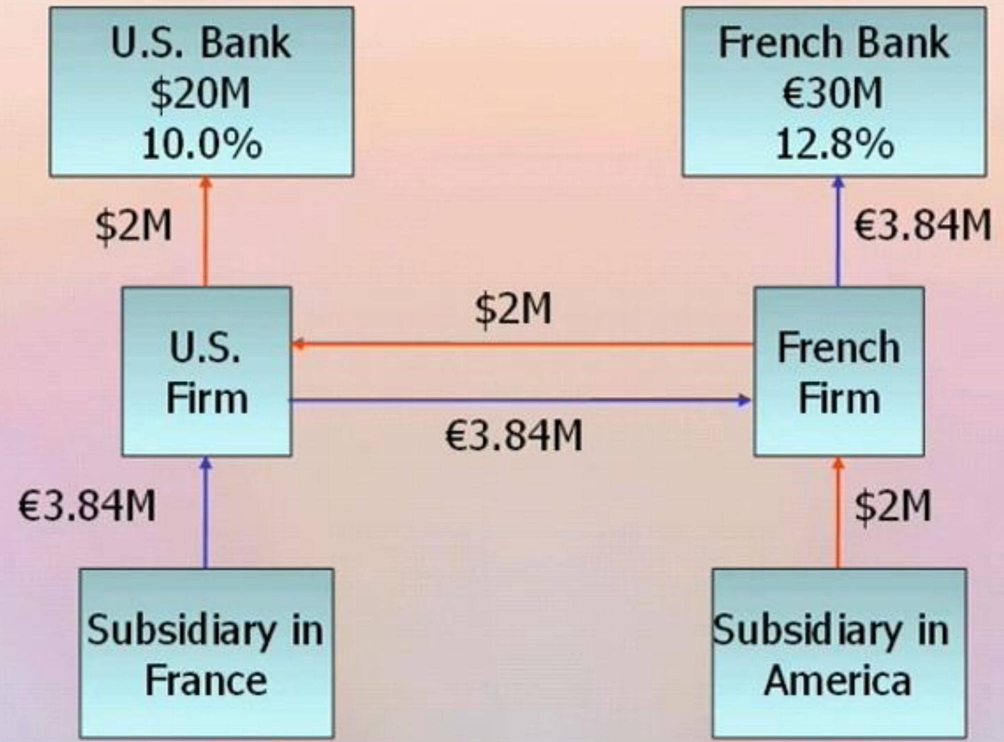
Borrow in home currency and swap for foreign currency

Initial Phase

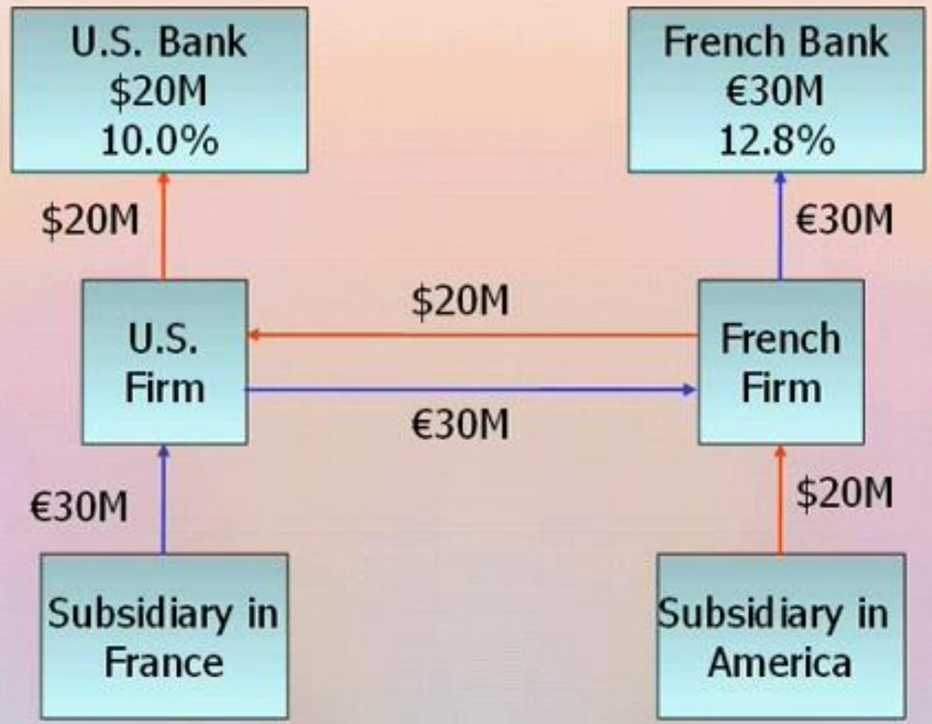


Direct Cost of Borrowing	
US Firm	(€) 13.80%
French Firm	($) 11.00%

Interest Payments



Final Phase


U.S. Bank
$20M
10.0%
French Bank
€30M
12.8%
$20M
€30M
$20M
U.S. Firm
French Firm
€30M
€30M
$20M
Subsidiary in France
Subsidiary in America

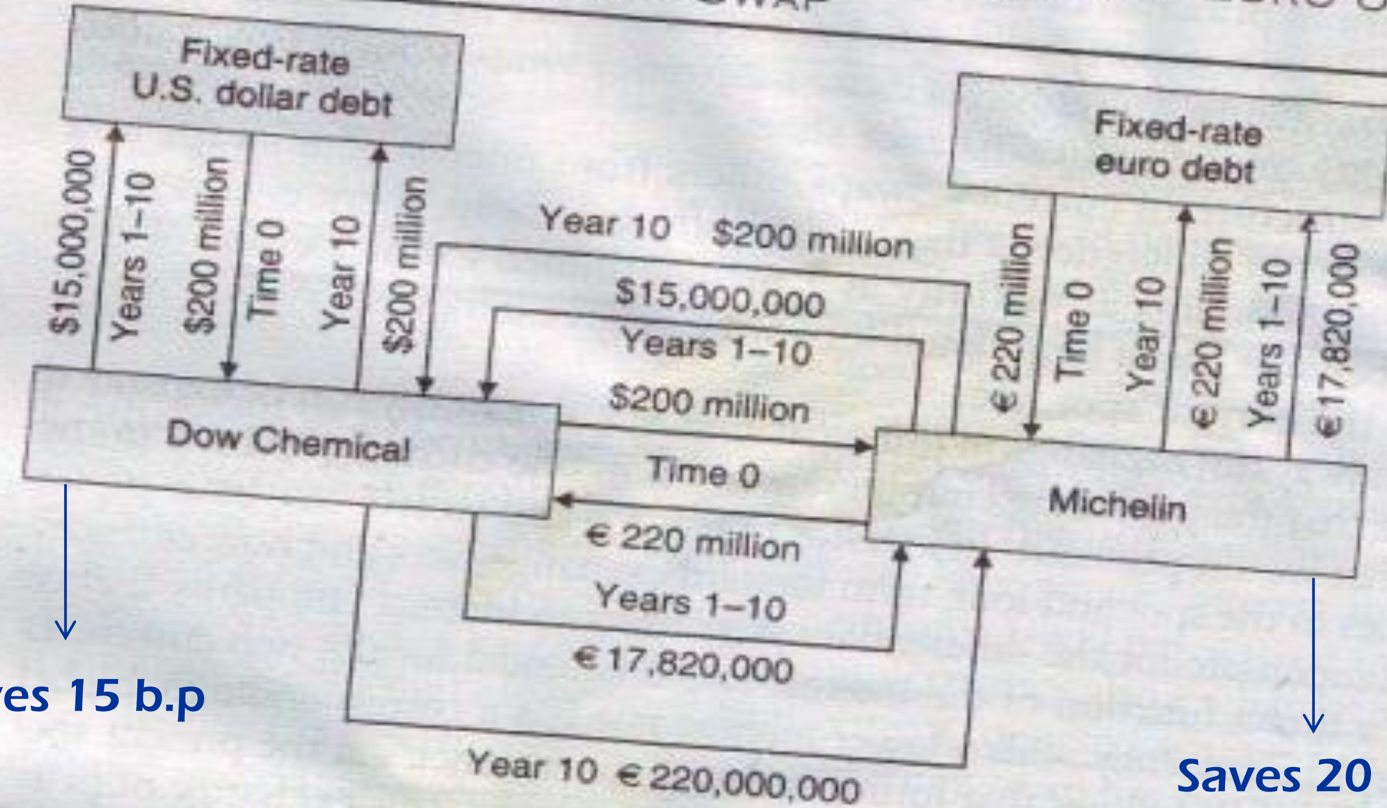
Case-2: Borrow in home currency and swap for foreign currency

Dow Chemical (DC) swaps fixed-for-fixed with Michelin (M)

Dow Chemical is looking to hedge some of its Euro exposure by borrowing in Euros. At the same time French tire manufacturer Michelin is seeking dollars to finance additional investment in the U.S market. Both want the equivalent of $200 million in fixed-rate financing for 10 years. At the time of the contract, the spot rate is €1.1/US$.

	Fixed Rate	
DC	**($) 7.50%**	**(€) 8.25%**
M	**($) 7.70%**	**(€) 8.10%**
Differential Spread	0.20 or 20bp	0.15 or 15bp

EXHIBIT 9.4 EXAMPLE OF A FIXED-FOR-FIXED EURO U.S. DOLLAR CURRENCY SWAP

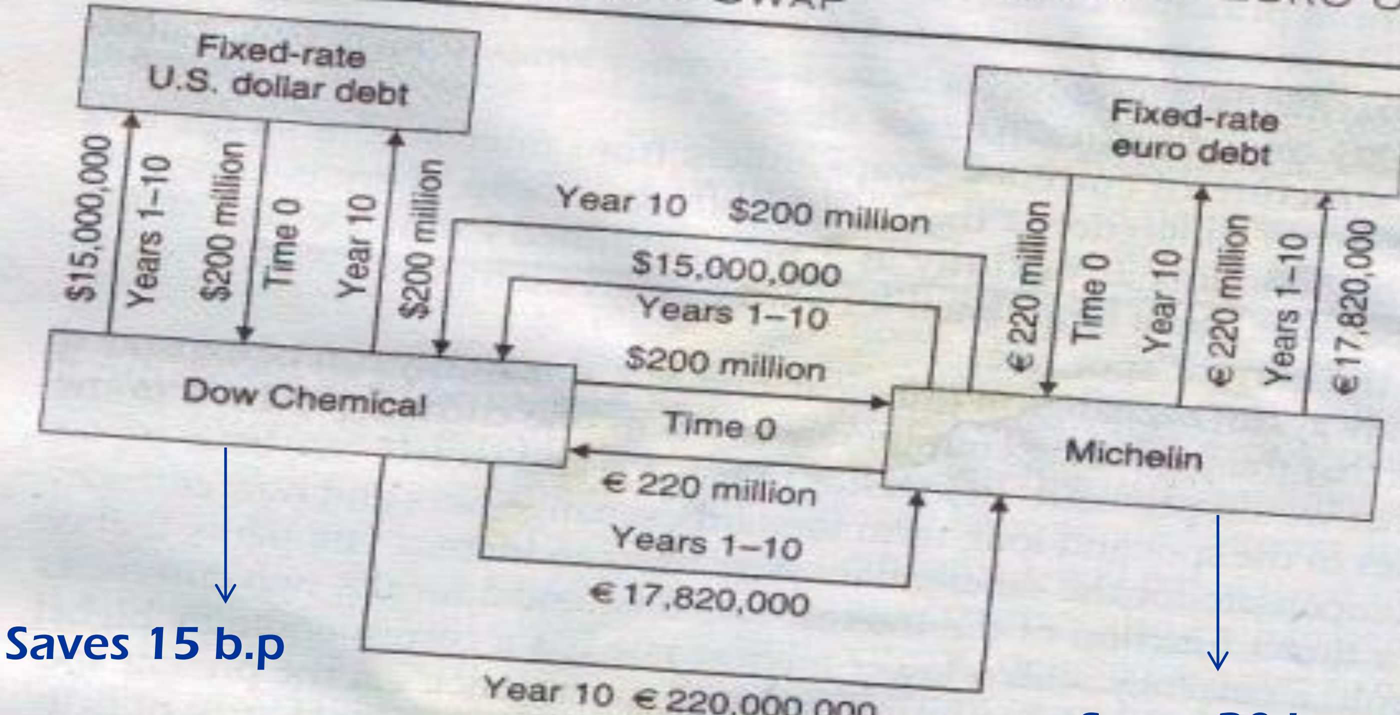


Saves 15 b.p

Saves 20 b.p

Cost of Borrowing Euros	Cost of Borrowing U.S. Dollars
8.1%	7.5%

EXHIBIT 9.4 EXAMPLE OF A FIXED-FOR-FIXED EURO U.S. DOLLAR CURRENCY SWAP



Cost of Borrowing Euros	Cost of Borrowing U.S. Dollars
8.1%	7.5%

After swapping the proceeds at time 0 (now), Dow Chemical winds up with €220 million in euro debt and Michelin has $200 million in dollar debt to service. In subsequent years, they would exchange coupon payments and the principal amounts at repayment. The cash inflows and outflows for both parties are summarized in Exhibit 9.4. The net result is that the swap enables Dow to borrow fixed-rate euros indirectly at 8.1%, saving 15 basis points relative to its 8.25% cost of borrowing euros directly, and Michelin can borrow dollars at 7.5%, saving 20 basis points relative to its direct cost of 7.7%.

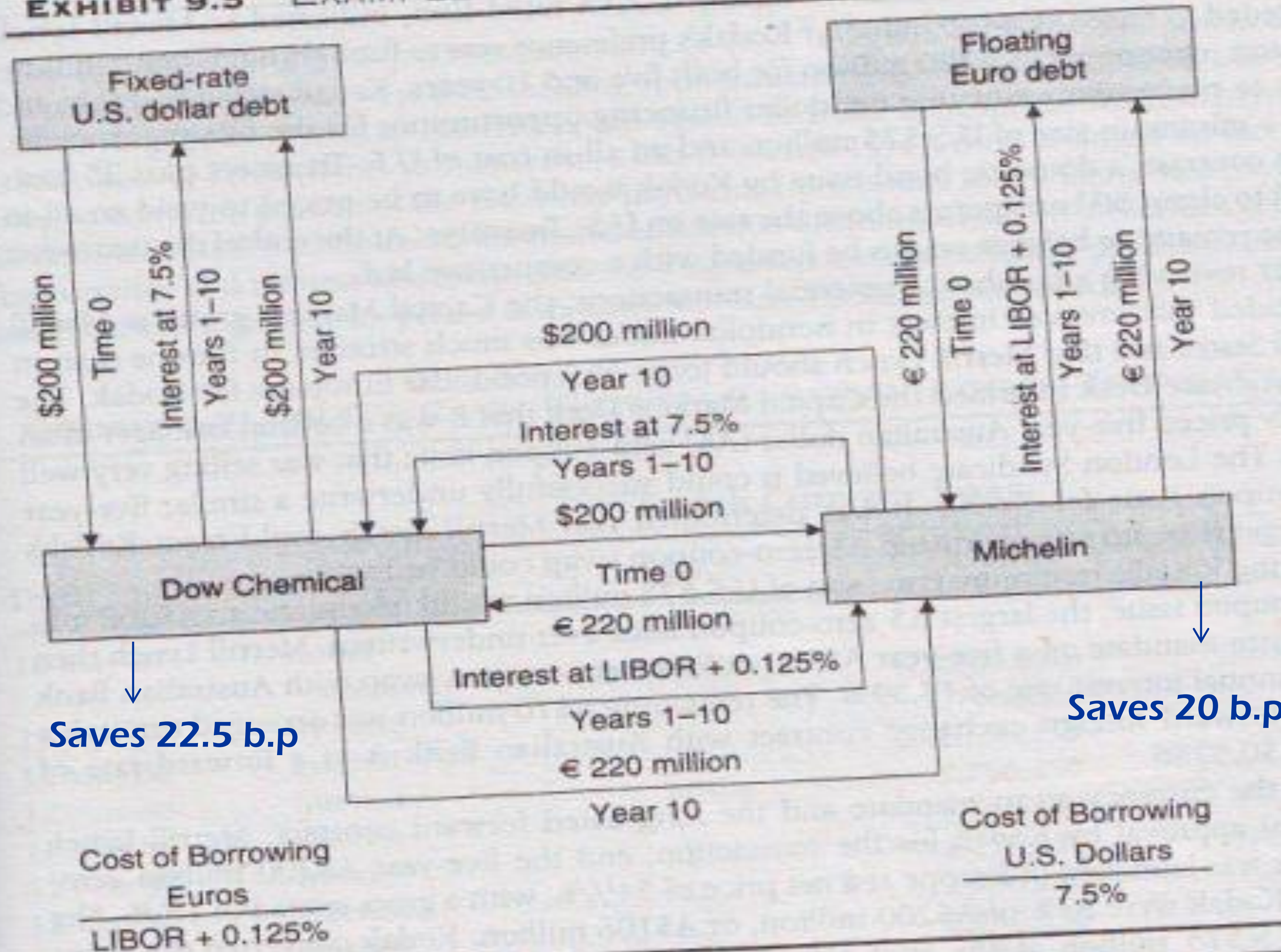
Case-3: Borrow in home currency and swap for foreign currency

Dow Chemical swaps fixed-for-floating with Michelin

Dow chemicals decides to borrow floating-rate Euros instead of fixed-rate Euros where as Michelin maintains its preference for fixed-rate dollars. Both want the equivalent of $200 million financing for 10 years. The spot rate is €1.1/$.

	Fixed Rate	**Floating Rate**
DC	**($) 7.50%**	**(€)** LIBOR+0.350%
M	**($) 7.70%**	**(€)** LIBOR+0.125%
Differential Spread	0.20 or 20bp	0.225 or 22.5bp

EXHIBIT 9.5 EXAMPLE OF A FIXED-FOR-FLOATING CURRENCY SWAP



CREDIT DERIVATIVES

1. Credit Default Swap (CDS): Counterparties can buy or sell protection against particular types of credit events that can adversely affect the credit quality of a debt obligation

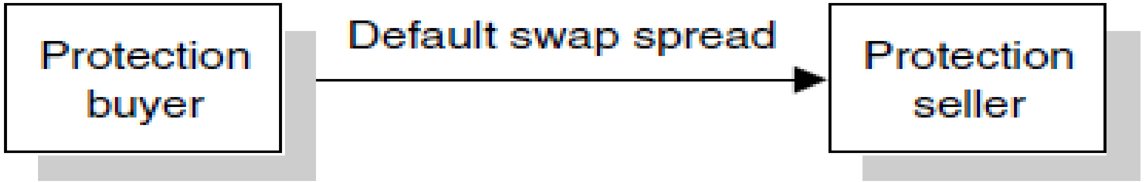
- The two parties are called **protection buyer** and **protection seller**
- The specific credit-related events are identified in the contract is called the **credit event**

Reference Entity and Reference Obligation:

- The **reference entity** is the issuer of the debt instrument and hence is also referred to as the **reference issuer**
- The **reference obligation**, also called the **reference asset**, is the particular debt issue for which credit protection is sought

Mechanics of a Credit Default Swap

Between trade initiation and default or maturity, protection buyer makes regular payments of default swap spread to protection seller



Credit Events:

Credit default products have a payout that is contingent upon a **credit event** occurring:

1. Bankruptcy is defined as a variety of acts that are associated with bankruptcy or insolvency laws.

2. A failure to pay one or more payments results in a default.

3. Credit event upon merger.

4. Downgrade.

5. Obligation acceleration occurs when, upon default, the obligation becomes due and payable prior to the scheduled due date

6. Restructuring occurs when the terms of the obligation are altered so as to make the new terms less attractive to the debt holder than the original terms.

Mechanism of Settlement:

Credit default swaps can be settled in cash or physically: If the credit event occurs, two things happen:

1. There are NO further payments of the swap premium (premium leg) by the protection buyer to the protection seller;
2. A **termination value** is determined for the swap. (Computation of the terminal value depends on the settlement terms provided in the swap deal)

Physical Settlement

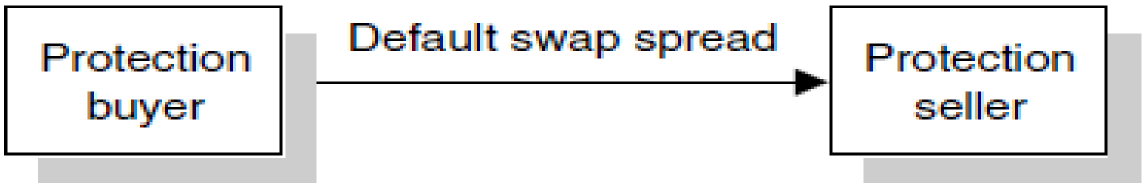
With physical settlement the protection buyer has two options:

- First, the default swap (protection) buyer delivers the defaulted bond to the default swap (protection) seller and receives the ***reference price (quoted in %) of the bond, which is typically the par value***;
- Second, protection buyer has the right (option) to deliver a bond from a pre-specified basket of bonds of the reference entity to the protection seller. The protection seller pays the protection buyer the face value of the bonds

[**Deliverable Obligations:** The reference entity that is subject of credit default swaps may/will have many issues outstanding, there will be a number of alternative issues of the reference entity that the protection buyer can deliver to the protection seller which are known as DELIVERABLE OBLIGATIONS. From the list of deliverable obligations, the protection buyer will select for delivery to the protection seller the ***cheapest*** to deliver issue]

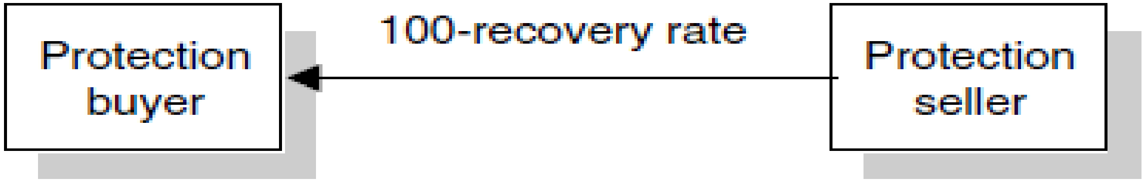
Mechanics of a Credit Default Swap

Between trade initiation and default or maturity, protection buyer makes regular payments of default swap spread to protection seller



Following the credit event one of the following will take place:

Cash settlement



Physical settlement

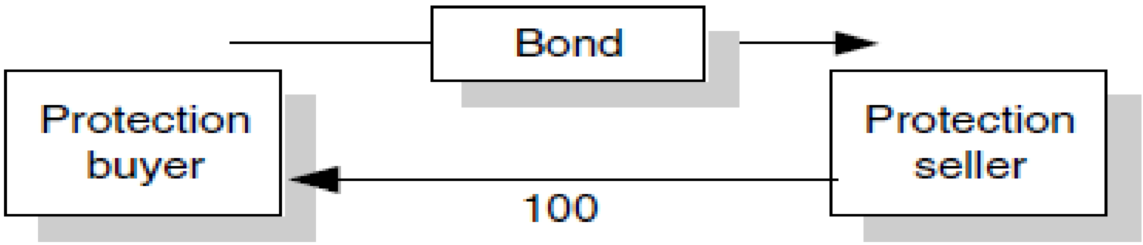
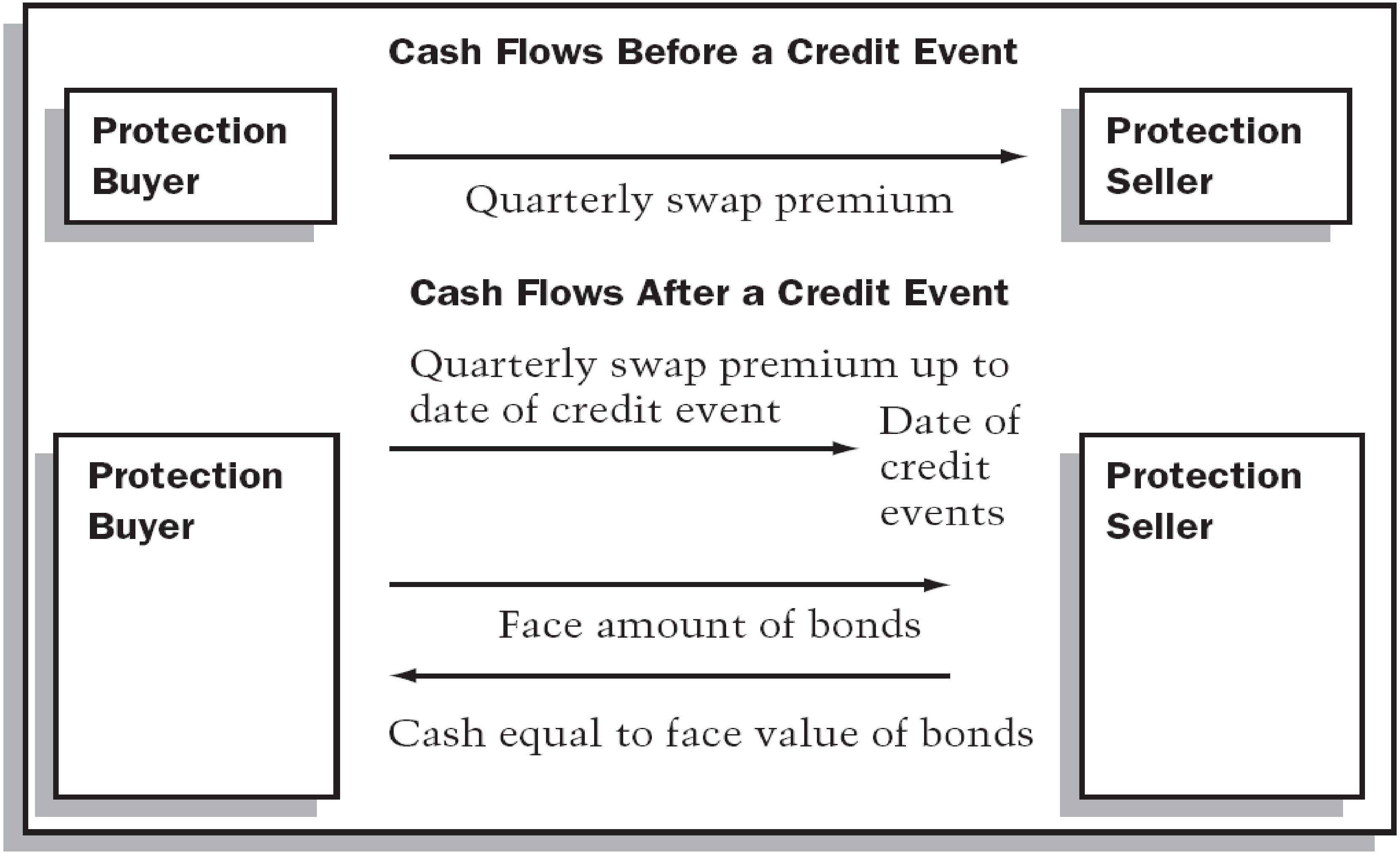


FIGURE 32-2 Mechanics of a Single-Name Credit Default Swap with Physical Delivery



Example: Physical Settlement

The physical settlement amount paid by the default swap seller to the default swap buyer is simply calculated as

$$N \times \text{Reference price}$$

where N is the notional amount of the default swap. The reference price is quoted in percent.

Example 2.2: The notional amount of a default swap is \$10,000,000 and the reference price is 100%. What is the physical settlement amount paid by the default swap seller in case of default?

$$\$10{,}000{,}000 \times 100\% = \$10{,}000{,}000$$

Cash Settlement:

In cash settlement, a cash payment is made by the protection seller to the protection buyer equal to par value of the bond times reference price minus the recovery price (rate)/final price (market price/dealer poll at the time of credit event) of the cheapest to deliver reference asset/reference obligation:

Cash Settlement: In the case of cash settlement, the cash paid from the default swap seller to the default swap buyer in case of default is usually determined as

$$N \times [\text{Reference price} - (\text{Final price} + \text{Accrued interest on reference obligation})]$$

where N is the notional amount, also called calculation amount or principal amount of the default swap. The reference price is determined at the inception of the default swap and is typically the par value of 100. The final price, also called recovery rate, is determined at the time of default. The reference price as well as the final price are quoted in percent.

Example: Cash Settlement

Example 2.1: The notional amount of a default swap is \$50,000,000. The reference price is 100%, and a dealer poll determines the final price of the reference bond as 35.00%. The last coupon payment was 45 days ago and the bond has an annual coupon of 9%. What is the cash settlement amount in case of default of the reference bond? It is:

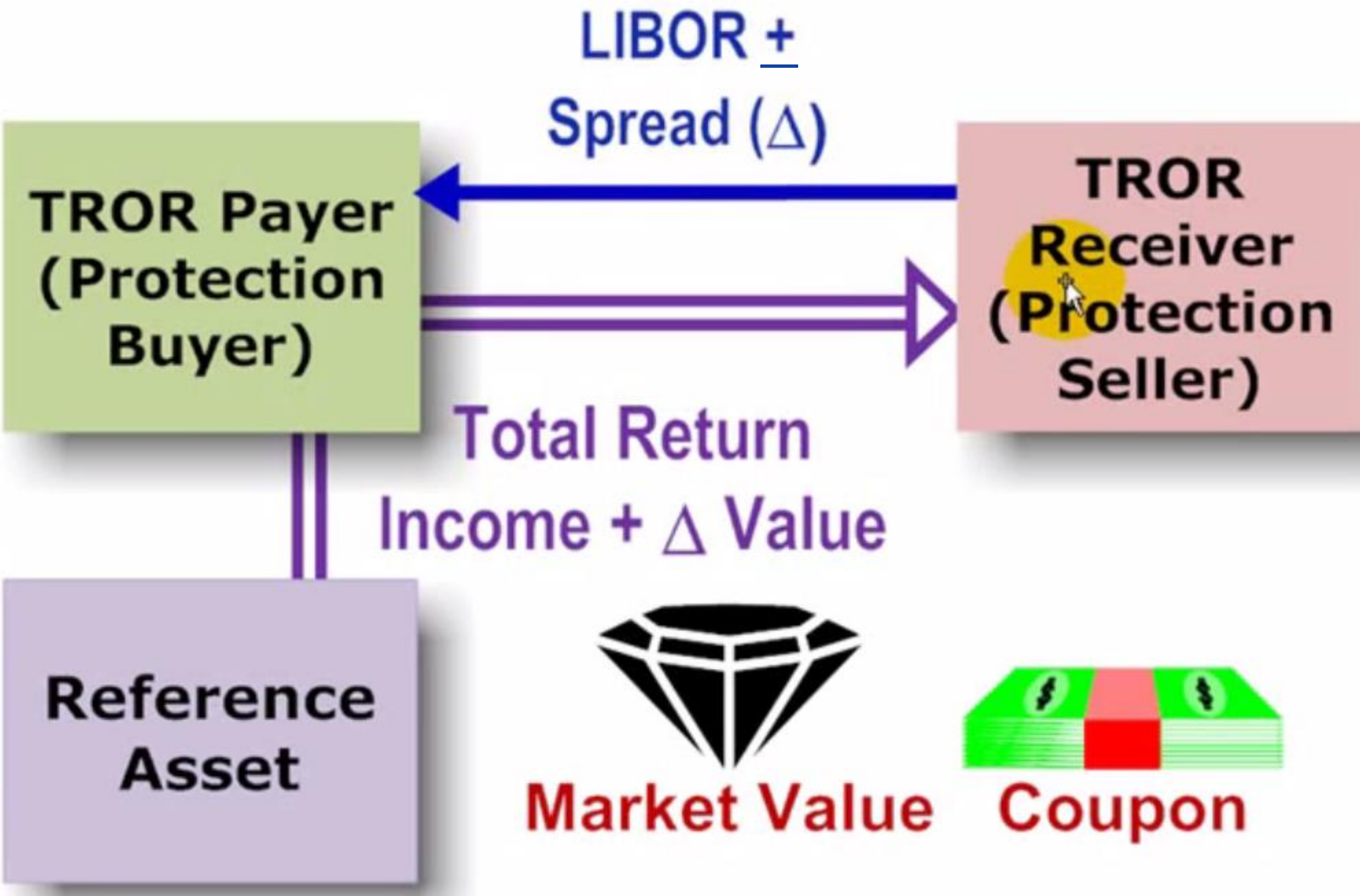
$$\$50{,}000{,}000[100\% - (35\% + 9\% \times 45/360)] = \$31{,}937{,}500.$$

2. Total Rate of Return Swaps (TROR):

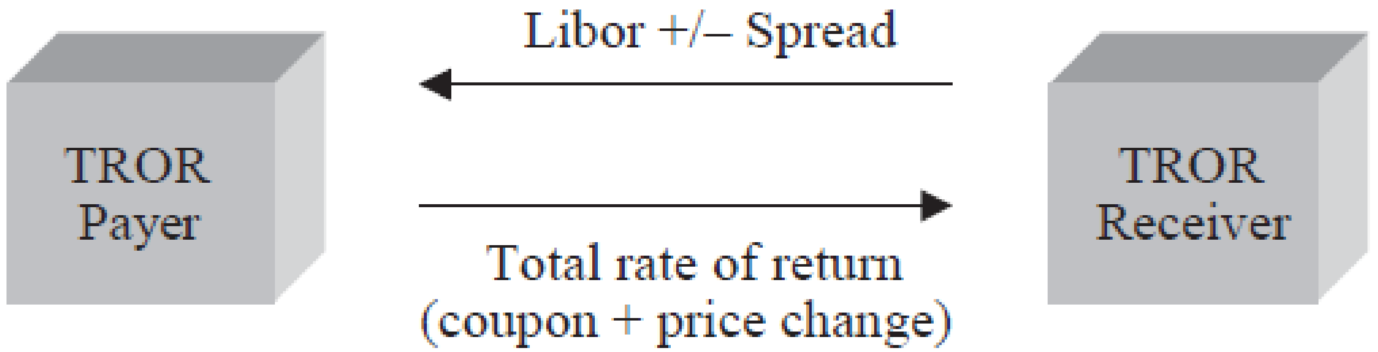
- A **total return swap** is a swap in which one party makes periodic floating payments to a counterparty in exchange for the total return realized on an individual reference asset (obligation) or a basket of reference assets (obligations);
- The total return payment includes all cash flows that flow from the reference assets as well as any capital appreciation or depreciation;
- The party that agrees to make the floating payments and receive the total return is referred to as the **total return receiver (protection seller)**; the party that agrees to receive the floating payments and pay the total return is referred to as the **total return payer (protection buyer);**
- The total return receiver/protection seller is exposed to credit risk, credit downgrade and interest rate risk;

- The total return payer owns the reference asset and pays the absolute return (income plus any capital appreciation or depreciation) of the asset to the total return receiver in exchange for a periodic payment (LIBOR +/- a fixed spread). By doing this he is hedging credit risk, credit downgrade and interest rate risk;
- The total return swap receiver/protection seller is synthetically *long the obligation*, which means he will benefit if the price of the obligation increases;
- The total return swap payer/protection buyer is synthetically *short the obligation*, which means he will benefit if the price of the obligation decreases;
- *A total return swap enables an investor (total return swap receiver-protection seller) to obtain the cash flow benefits of owning an asset without having to actually hold the physical asset on their balance sheet*

Total Rate of Return (TROR) Swaps



Total Rate of Return (TROR) Swaps



Receiver (Protection Seller): Why TRORs?

- Since the TROR receiver has a long position in the reference obligation, which is usually a bond or a loan, the question arises: Why does the TROR receiver not simply buy the asset in the market?

The reason arises due to several issues:

- First, the TROR receiver has no need for funding, i.e. he does not have to borrow cash to buy the bond. This is especially advantageous if the credit rating of the TROR receiver is poor and his potential funding costs are high;
- Second, due to the lack/difficulty of funding, the leverage for the TROR receiver will be extremely high;
- Third, TRORs are currently off-balance-sheet investments. Thus, the TROR receiver does not have to set aside regulatory capital for the investment;
- Fourth, the TROR market might be more liquid than the market for the underlying asset

Payer (Protection Buyer): Why TRORs?

For the TROR payer, who has a short position in the asset, the motives are:

- First, he can conveniently short the asset, which is often difficult in the rather illiquid secondary bond market (For loans the situation is even worse; hardly any secondary market exists);
- Second, as is the case for the TROR receiver, the TROR payer has an off-balance-sheet position;
- Third, most importantly, the TROR payer hedges the default risk and price deterioration risk, as well as the market risk of the asset

Example: Total Return Swap

Consider a portfolio manager who believes that the fortunes of XYZ Mobile Corporation will improve over the next year, and that the company's credit spread relative to U.S. Treasury securities will decline. The company has issued a 10-year bond at par with a coupon rate of 8.5% and therefore the yield is 8.5%. Suppose at the time of issuance, the 10-year Treasury yield is 5.5%. This means that the credit spread is 300 basis points and the portfolio manager believes it will decrease over the year to less than this amount.

The portfolio manager can express this view by entering into a total return swap that matures in one year as a total return receiver with the reference obligation being the 10-year, 8.5% XYZ Mobile Corporate bond issue. Suppose (1) the swap calls for an exchange of payments semiannually and (2) the terms of the swap are such that the total return receiver pays the 6-month Treasury rate plus 140 basis points in order to receive the total return on the reference obligation. The notional amount for the contract is $10 million.

Assume that over the one year, the following occurs:

- the 6-month Treasury rate is 4.6% initially
- the 6-month Treasury rate for computing the second semiannual payment is 5.6%
- at the end of one year the 9-year Treasury rate is 7%
- at the end of one year the credit spread for the reference obligation is 200 basis points

OPTION PRICING MODELS

PROPERTIES OF STOCK OPTIONS

PREMIUM: REVISITED

Premium (Price of the option): The premium of the option has two components viz., **Intrinsic Value and Time Value**

Intrinsic Value of an option: Intrinsic value of an option at a given point in time is the amount the holder of an option will get if he exercises the option at that time. In other words, the intrinsic value of an option is the amount the option is in-the-money (ITM). Thus, if an option is out-of-the-money (OTM) or at-the-money (ATM) its intrinsic value will be zero.

Intrinsic Value:

Call Option: Max [0, (Spot Price- Strike Price)]

Put Option: Max [0, (Strike Price-Spot Price)]

Time value of an option: The seller/writer of an option also charges a 'time value' from the buyers of the option. This is because the more the time there is for the option to expire, the greater the probability/chance that the buyer of an option will exercise his right. This is a risk for the seller and he seeks compensation for it by demanding a 'time value'. The time value of an option is calculated by taking the *difference between its premium and its intrinsic value*.

Note: An option that is out-of-the-money (OTM) or at-the-money (ATM) has only time value and no intrinsic value.

FACTORS AFFECTING OPTION PRICES

1. The Current/Spot Price (S_0);
2. The Exercise Price (X, K);
3. The Time to Expiration (T);
4. The Volatility of the Stock Price (σ);
5. The Risk-free Interest Rate (r);
6. The dividends that are expected to be paid

FACTORS AFFECTING OPTION PRICES

1. The Current/Spot Price (S_0);
2. The Exercise Price (X, K);
3. The Time to Expiration (T);
4. The Volatility of the Stock Price (σ);
5. The Risk-free Interest Rate (r);
6. The dividends that are expected to be paid

We consider what happens to option prices when there is a change to one of these factors, with all the other factors remaining fixed. The results are summarized below:

Summary of the effect on the price of a stock option of increasing one variable while keeping all others fixed:

Variable	*European call*	*European put*
Current stock price	+	−
Strike price	−	+
Time to expiration	?	?
Volatility	+	+
Risk-free rate	+	−
Amount of future dividends	−	+

+ indicates that an increase in the variable causes the option price to increase or stay the same;

- indicates that an increase in the variable causes the option price to decrease or stay the same;

? indicates that the relationship is uncertain.

1. The Current/Spot Price (S_0):

Given the exercise price (X=$50), as the current stock price increases, the intrinsic value (**S_0-X)** of a call increases where as the intrinsic value (**X-S_0**) of a put decreases

or

Given the exercise price, as the current stock price decreases the intrinsic value of a call decreases where as the intrinsic value of a put increases

Call Option (Direct Relationship): Higher (lower) the current spot price of the underlying asset from the exercise price, the higher (lower) will be the value (premium) of the call option

Put Option (Indirect Relationship): The higher (lower) the current spot price of the underlying asset from the exercise price, the lower (higher) will be the value (premium) of the put option

2. The Exercise Price (X):

Given the current stock price (S_0=$50), as the exercise price increases, the intrinsic value **(S_0-X)** of a call decreases where as the intrinsic value (**X-S_0**) of a put increases

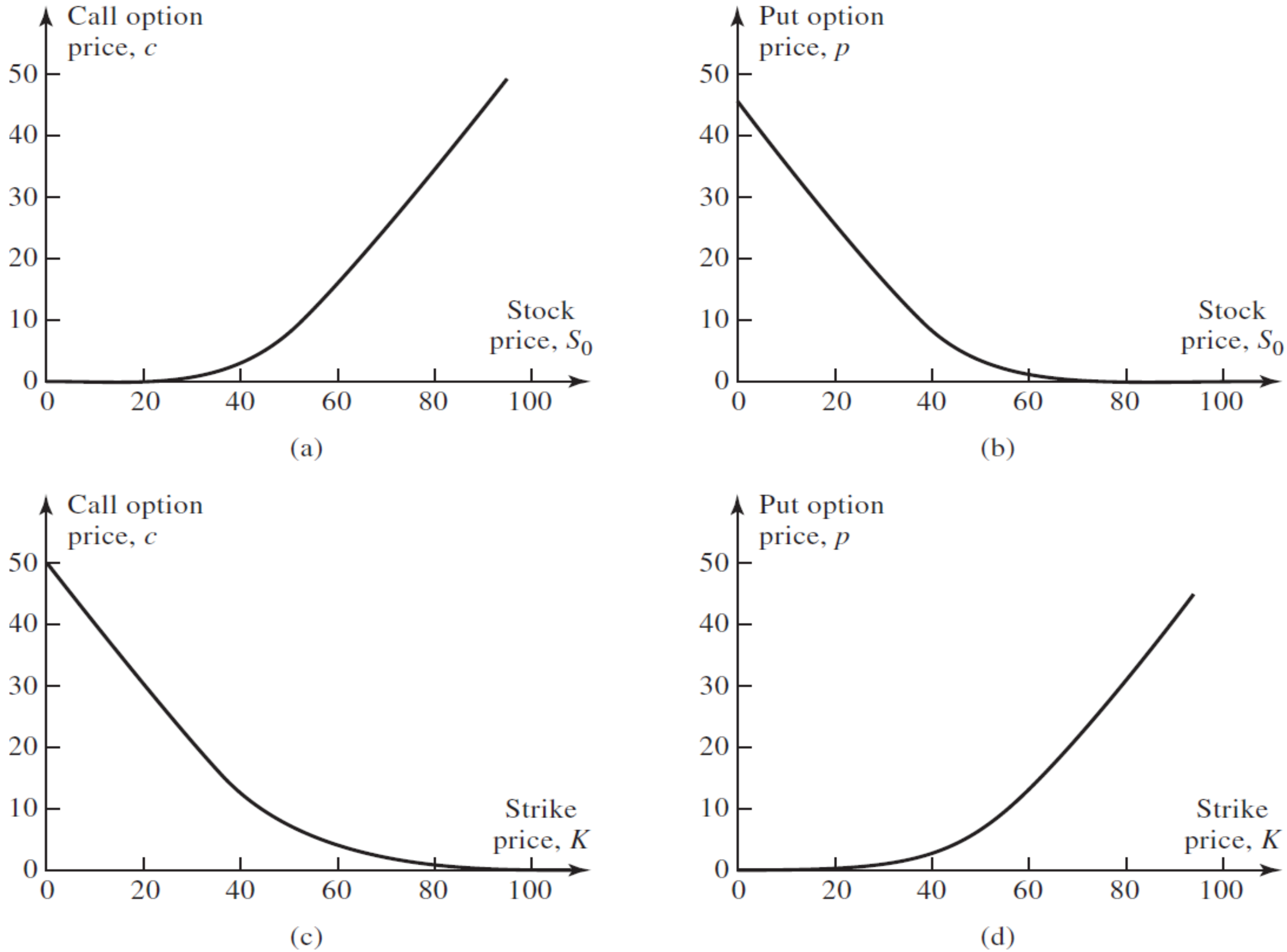
or

As the exercise price decreases the intrinsic value of a call increases where as the intrinsic value of a put decreases

Call Option (Indirect Relationship): Higher (lower) the exercise price of the underlying asset from the current stock price, the lower (higher) will be the value (premium) of the call option.

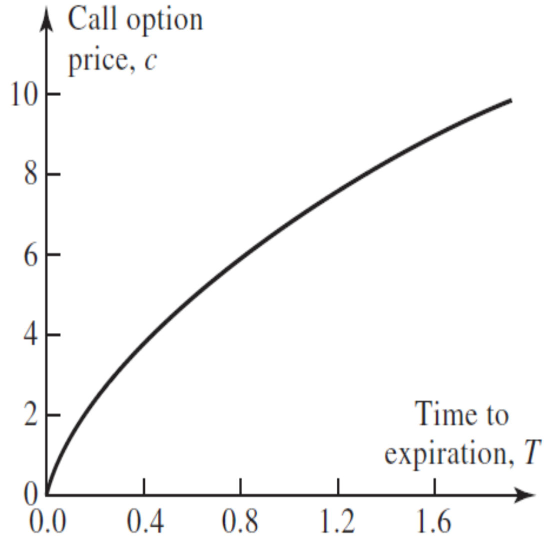
Put Option (Direct Relationship): The higher (lower) the exercise price of the underlying asset from the current stock price, the higher (lower) will be the value (premium) of the put option

Effect of changes in stock price, exercise price, and expiration date on option prices when S_0 =\$50, X= \$50, r = 5%, σ =30%, and T =1.



3. Time to Expiration: Time to expiration is the time remaining for the option to expire. Both call and put option becomes more valuable as its time to maturity increases. The longer the time to expiry, the greater the chance that the underlying price can move significantly in favour of the holder of the option before expiry. So the value of an option will increase with term to maturity.

Effect of changes in stock price, exercise price, and expiration date on option prices when S_0 =$50, X= $50, r = 5%, σ =30%, and T =1:



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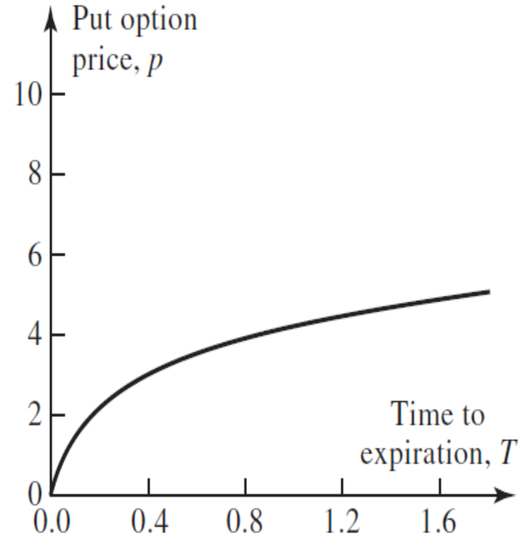
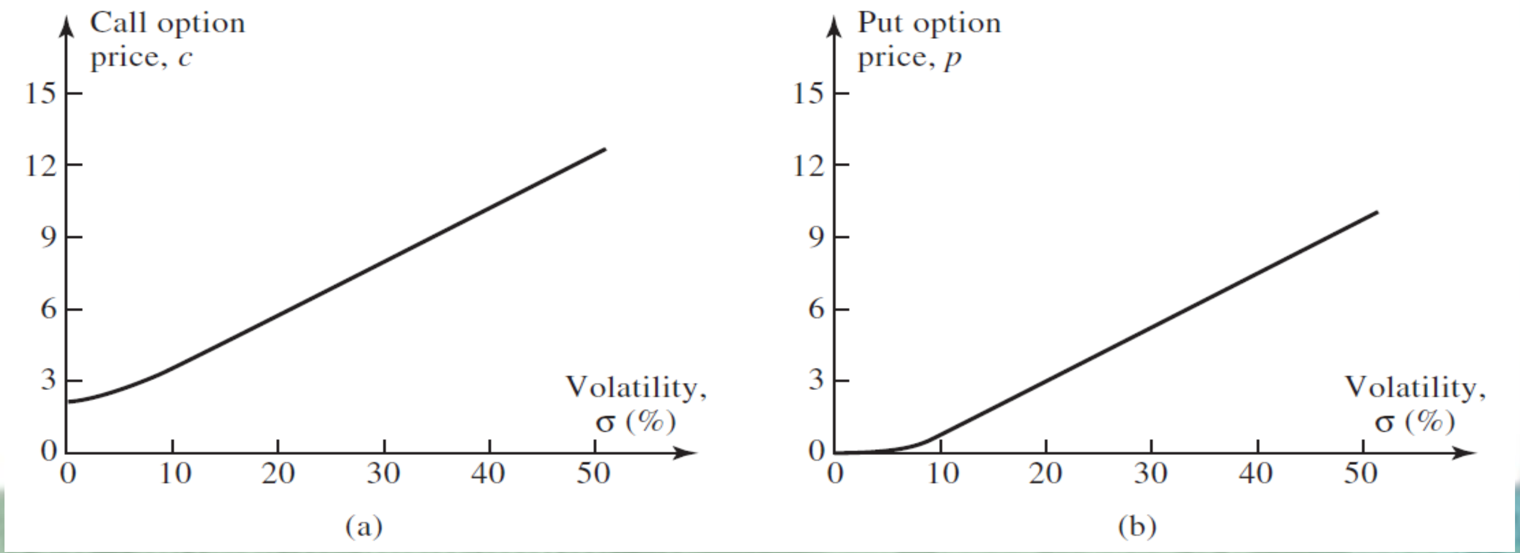


(f)

4. Volatility: Volatility is defined as the uncertainty of returns. The more volatile the underlying, higher is the price of the option on the underlying. As volatility increases, the chance that the stock will do very well or very poorly increases. For a call or a put option, this relationship remains the same. The higher the volatility of the underlying, the greater the chance that the underlying price can move significantly in favour of the holder of the option before expiry. So the value of an option will increase with the volatility of the underlying

Effect of changes in stock price, exercise price, and expiration date on option prices when S_0 =\$50, X= \$50, r = 5%, σ =30%, and T =1:

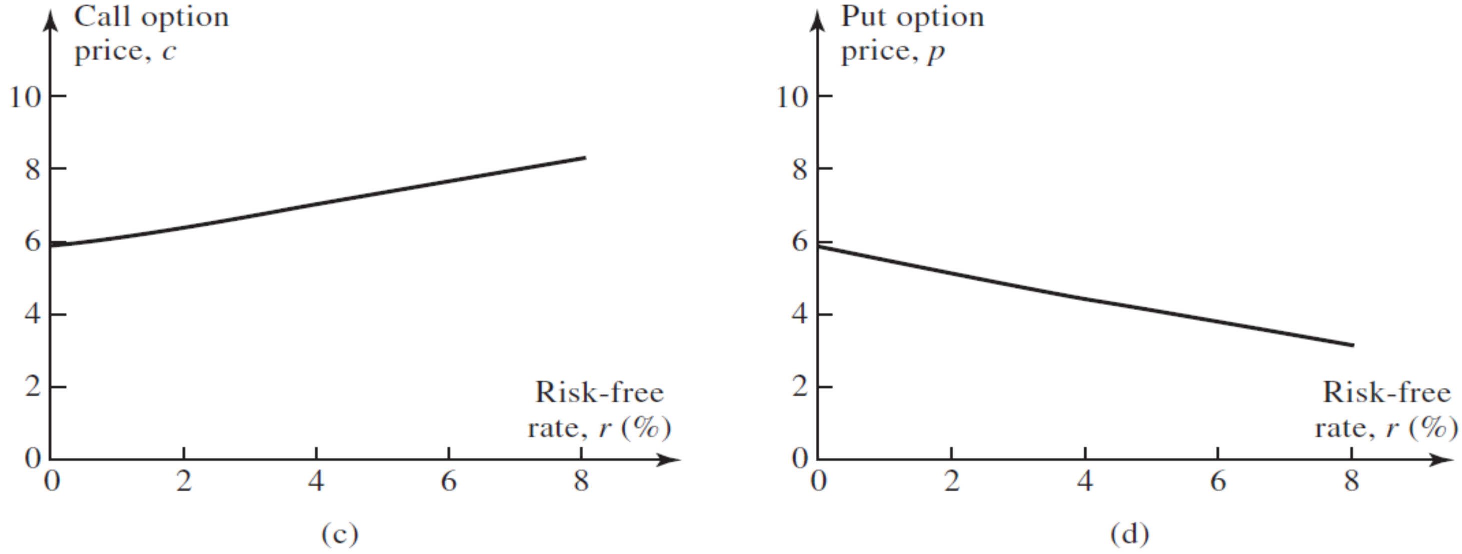


5. The risk-free interest rate: As the risk- free rate of interest increases, the expected return required by investors from the stock tends to increase. Also, the PV of any expected future cash flow received by the holder of the option decreases. This expectation increases the value of a call option and decreases the value of a put option

Call Option: A call option can be looked at as the right to delay a purchase. Therefore, the money saved by purchasing the option rather than purchasing the underlying asset can be invested at a higher rate of interest, thus increasing the value of the call option

Put Option: A put option can be looked at as the right to delay a sale. The higher interest rate you can earn on the cash generated from that sale, the less desirable it is to delay that sale, thus decreasing the value of the put option.

Effect of changes in stock price, exercise price, and expiration date on option prices when S_0 =$50, X= $50, r = 5%, σ =30%, and T =1:



6. Expected Future Dividends: Dividends have the effect of reducing the stock price on the ex-dividend date. This is bad news for the value of call options and good news for the value of put options. Consider a stock whose ex-dividend date is during the life of an option. The value of the option is negatively related to the size of the dividend if the option is a call and positively related to the size of the dividend if the option is a put.

UPPER AND LOWER BOUNDS FOR OPTION PRICES:

ASSUMPTIONS AND NOTATION

ASSUMPTIONS: We assume that there are some market participants, such as large investment banks, for which the following statements are true:

1. There are no transaction costs;
2. All trading profits (net of trading losses) are subject to the same tax rate;
3. Borrowing and lending are possible at the risk-free interest rate;
4. These market participants are prepared to take advantage of arbitrage opportunities as they arise suggesting that any available arbitrage opportunities disappear very quickly. For the purposes of our analysis, it is therefore reasonable to assume that there are no arbitrage opportunities

NOTATION

S_0: Current stock price

K: Strike price of option

T: Time to expiration of option

S_T: Stock price on the expiration date

r: Continuously compounded risk-free rate of interest for an investment maturing in time T

C: Value of American call option to buy one share

P: Value of American put option to sell one share

c: Value of European call option to buy one share

p: Value of European put option to sell one share

Upper Bounds (for puts): An American put option gives the holder the right to sell one share of a stock for K. *No matter how* ***low the stock price becomes****, the option can never be worth more than K*. Hence, the **exercise price** is an upper bound to the put option price:

$$P \leqslant K$$

For European options, we know that at maturity the option cannot be worth more than K. It follows that it cannot be worth more than the present value of K today:

$$p \leqslant Ke^{-rT}$$

Note: If this were not true, an arbitrageur could make a riskless profit by writing the put option and investing the proceeds of the sale at the risk-free interest rate

Upper Bounds (for calls): An American or European call option gives the holder the right to buy one share of a stock for a given exercise price. *No matter how* ***low the stock price becomes****, the value of the call option can never be worth more than the* ***stock price****.* Hence, the **stock price** is an upper bound to the call option price:

$$c \leqslant S_0 \quad \text{and} \quad C \leqslant S_0$$

If an option is at expiration and the stock price is less than or equal to the exercise price, the call option has no value

Note: If these relationships were not true, an arbitrageur could easily make a riskless profit by buying the stock and selling the call option

Lower Bound for Calls Non-Dividend-Paying Stocks: A lower bound for the price of a European call option on a non-dividend-paying stock is:

$$S_0 - Ke^{-rT}$$

- Suppose that S_0= \$20, K =\$18, r =10% per annum, and T =1 year:

$$c = S_0\text{-}Ke^{-rT} = \$20\text{-}\$18e^{-0.1*1} = \$3.71$$

Lower Bound for Calls on Non-Dividend-Paying Stocks

A lower bound for the price of a European call option on a non-dividend-paying stock is

$$S_0 - Ke^{-rT}$$

We first look at a numerical example and then consider a more formal argument.

Suppose that $S_0 = \$20$, $K = \$18$, $r = 10\%$ per annum, and $T = 1$ year. In this case,

$$S_0 - Ke^{-rT} = 20 - 18e^{-0.1} = 3.71$$

or \$3.71. Consider the situation where the European call price is \$3.00, which is less than the theoretical minimum of \$3.71. An arbitrageur can short the stock and buy the call to provide a cash inflow of \$20.00 – \$3.00 = \$17.00. If invested for 1 year at 10% per annum, the \$17.00 grows to $17e^{0.1} = \$18.79$. At the end of the year, the option expires. If the stock price is greater than \$18.00, the arbitrageur exercises the option for \$18.00, closes out the short position, and makes a profit of

$$\$18.79 - \$18.00 = \$0.79$$

If the stock price is less than \$18.00, the stock is bought in the market and the short position is closed out. The arbitrageur then makes an even greater profit. For example, if the stock price is \$17.00, the arbitrageur's profit is

$$\$18.79 - \$17.00 = \$1.79$$

For a more formal argument, consider the following two portfolios:

Portfolio A: One European call option plus a zero-coupon bond that provides a pay off of K at time T;

Portfolio B: One share of the stock

Payoff from Portfolio A:

1. The zero-coupon bond will be worth K at time T;

2. At time T, if $S_T>K$, the call option is exercised at maturity and portfolio A is worth $(S_T-K)+K=S_T$; At time T, if $S_T<K$, the call option expires worthless and the portfolio is worth (0)+K=K. Hence, at time T, portfolio A is worth:

$$\max(S_T,\ K)$$

Payoff from Portfolio B:

1. Portfolio B is worth S_T at time T. Hence, portfolio A is always worth as much as, and can be worth more than, portfolio B at the option's maturity. It follows that in the absence of arbitrage opportunities this must also be true today. Specifically, it follows that in the absence of arbitrage opportunities, portfolio A must be worth at least as much as worth of portfolio B **TODAY** (The worth of zero-coupon bond is (PV of K) Ke^{-rT} today. Hence,

$$c + Ke^{-rT} \geqslant S_0$$

or

$$c \geqslant S_0 - Ke^{-rT}$$

Because the worst that can happen to a call option is that it expires worthless, its value cannot be negative. This means that $c \geq 0$ and therefore:

$$c \geqslant \max(S_0 - Ke^{-rT},\ 0)$$

Lower Bound for European Puts Stocks: For a European put option on a non-dividend-paying stock, a lower bound for the price is:

$$Ke^{-rT} - S_0$$

- Suppose that S_0= \$37, K =\$40, r =5% per annum, and T =0.5 year:

$$p = Ke^{-rT}-S_0=\$40e^{-0.05*0.5}-\$37=\$2.01$$

Lower Bound for European Puts on Non-Dividend-Paying Stocks

For a European put option on a non-dividend-paying stock, a lower bound for the price is

$$Ke^{-rT} - S_0$$

Again, we first consider a numerical example and then look at a more formal argument.

Suppose that $S_0 = \$37$, $K = \$40$, $r = 5\%$ per annum, and $T = 0.5$ years. In this case,

$$Ke^{-rT} - S_0 = 40e^{-0.05\times 0.5} - 37 = \$2.01$$

Consider the situation where the European put price is \$1.00, which is less than the theoretical minimum of \$2.01. An arbitrageur can borrow \$38.00 for 6 months to buy both the put and the stock. At the end of the 6 months, the arbitrageur will be required to repay $38e^{0.05\times 0.5} = \$38.96$. If the stock price is below \$40.00, the arbitrageur exercises the option to sell the stock for \$40.00, repays the loan, and makes a profit of

$$\$40.00 - \$38.96 = \$1.04$$

If the stock price is greater than \$40.00, the arbitrageur discards the option, sells the stock, and repays the loan for an even greater profit. For example, if the stock price is \$42.00, the arbitrageur's profit is

$$\$42.00 - \$38.96 = \$3.04$$

For a more formal argument, consider the following two portfolios:

Portfolio C: One European put option plus one share;

Portfolio D: Zero-coupon bond paying off K at time T

Payoff from Portfolio C:

1. The share will be worth S_T at time T

2. At time T, if $S_T<K$, the put option is exercised at maturity and portfolio C is worth $(K-S_T)+S_T = K$ at time T; At time T, if $S_T>K$, the put option expires worthless and the portfolio is worth $(0)+S_T=S_T$. Hence, at time T, portfolio C is worth:

$$\max(S_T,\ K)$$

Payoff from Portfolio D:

1. Portfolio D is worth K at time T. Hence, portfolio C is always worth as much as, and can be worth more than, portfolio D at the option's maturity (at time T). It follows that in the absence of arbitrage opportunities, portfolio C must be worth at least as much as portfolio D **TODAY**. Hence,

$$p + S_0 \geqslant Ke^{-rT}$$

or

$$p \geqslant Ke^{-rT} - S_0$$

Because the worst that can happen to a put option is that it expires worthless, its value cannot be negative. This means that that p ≥ 0 and therefore:

$$p \geqslant \max(Ke^{-rT} - S_0,\ 0)$$

PUT–CALL PARITY

Relationship between the prices of European put and call options that have the SAME strike price and time to maturity:

- **Portfolio A:** One European call option plus a zero-coupon bond that provides a payoff of K at time T;
- **Portfolio C**: One European put option plus one share of the stock;
- It is assumed that the stock pays no dividends. The call and put options have the SAME strike price K and the SAME time to maturity T

Payoff from Portfolio A:

1. The zero-coupon bond will be worth K at time T;

2. At time T, if $S_T>K$, the call option is exercised at maturity and portfolio A is worth $(S_T-K)+K=S_T$; At time T, if $S_T<K$, the call option expires worthless and the portfolio is worth 0+K =K.

Payoff from Portfolio C:

1. The share will be worth S_T at time T

2. At time T, if S_T<K, the put option is exercised at maturity and portfolio C is worth $(K-S_T)+S_T = K$; At time T, if S_T>K, the put option expires worthless and the portfolio is worth $(0)+S_T=S_T$.

Values of Portfolio A and Portfolio C at time T:

		$S_T > K$	$S_T < K$
Portfolio A	Call option	$S_T - K$	0
	Zero-coupon bond	K	K
	Total	S_T	K
Portfolio C	Put Option	0	$K - S_T$
	Share	S_T	S_T
	Total	S_T	K

Important Observations:

1. At time T, if $S_T > K$, both portfolios are worth S_T;
2. At time T, if $S_T < K$, both portfolios are worth K

Thus, when the options expire at time T both the portfolios are worth:

$$\max(S_T, K)$$

- Since the portfolios have identical values at time T, they must have identical values today. If this were not the case, an arbitrageur could buy the less expensive portfolio and sell the more expensive one;
- The components of portfolio A are worth c and Ke^{-rT} today;
- The components of portfolio C are worth p and S_0 today. Hence,

$$c + Ke^{-rT} = p + S_0$$

- **Put Call Parity:** This relationship is known as put–call parity. ***It shows that the value of a European call with a certain exercise price and exercise date can be deduced from the value of a European put with the SAME exercise price and exercise date, and vice versa***

Case I: Arbitrage opportunities when put–call parity does not hold. Suppose Stock price=$31; interest rate=10%; call price=$3; put price =$2.25. Both put and call have strike price of $30 and three months to maturity:

- **Payoff from A:** $c + Ke^{-rT} = 3 + 30e^{-0.1\times3/12} = \$32.26$
- **Payoff from C:** $p + S_0 = 2.25 + 31 = \$33.25$

Portfolio C is overpriced relative to portfolio A

Case II: Arbitrage opportunities when put–call parity does not hold. Suppose Stock price=$31; interest rate=10%; call price=$3; put price =$1. Both put and call have strike price of $30 and three months to maturity:

- **Pay-off from A:** $c + Ke^{-rT} = 3 + 30e^{-0.1\times3/12} = \$32.26$
- **Pay-off from C:** $p + S_0 = 1 + 31 = \$32.00$

Portfolio A is overpriced relative to portfolio C

Arbitrage opportunities when put–call parity does not hold. Stock price=$31; interest rate=10%; call price=$3; put price =$2.2 and $1. Both put and call have strike price of $30 and three months to maturity:

Three-month put price = $2.25	***Three-month put price = $1***
Action now:	*Action now:*
Buy call for $3	Borrow $29 for 3 months
Short put to realize $2.25	Short call to realize $3
Short the stock to realize $31	Buy put for $1
Invest $30.25 for 3 months	Buy the stock for $31
Action in 3 months if $S_T > 30$:	*Action in 3 months if* $S_T > 30$:
Receive $31.02 from investment	Call exercised: sell stock for $30
Exercise call to buy stock for $30	Use $29.73 to repay loan
Net profit = $1.02	Net profit = $0.27
Action in 3 months if $S_T < 30$:	*Action in 3 months if* $S_T < 30$:
Receive $31.02 from investment	Exercise put to sell stock for $30
Put exercised: buy stock for $30	Use $29.73 to repay loan
Net profit = $1.02	Net profit = $0.27

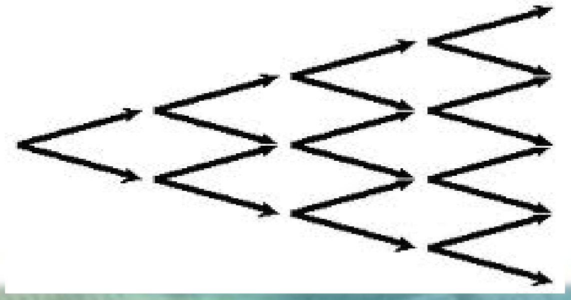
BINOMIAL OPTION PRICING MODEL (BOPM)

Binomial Option Pricing Model (BOPM) and Binomial Trees

- A useful and very popular technique for pricing an option involves constructing a binomial tree. This is a diagram representing different possible paths that might be followed by the stock price over the life of an option;

- The **BOPM** is essentially a tree that is constructed to show possible values that an underlying asset can take and the resulting value of the option at these values;

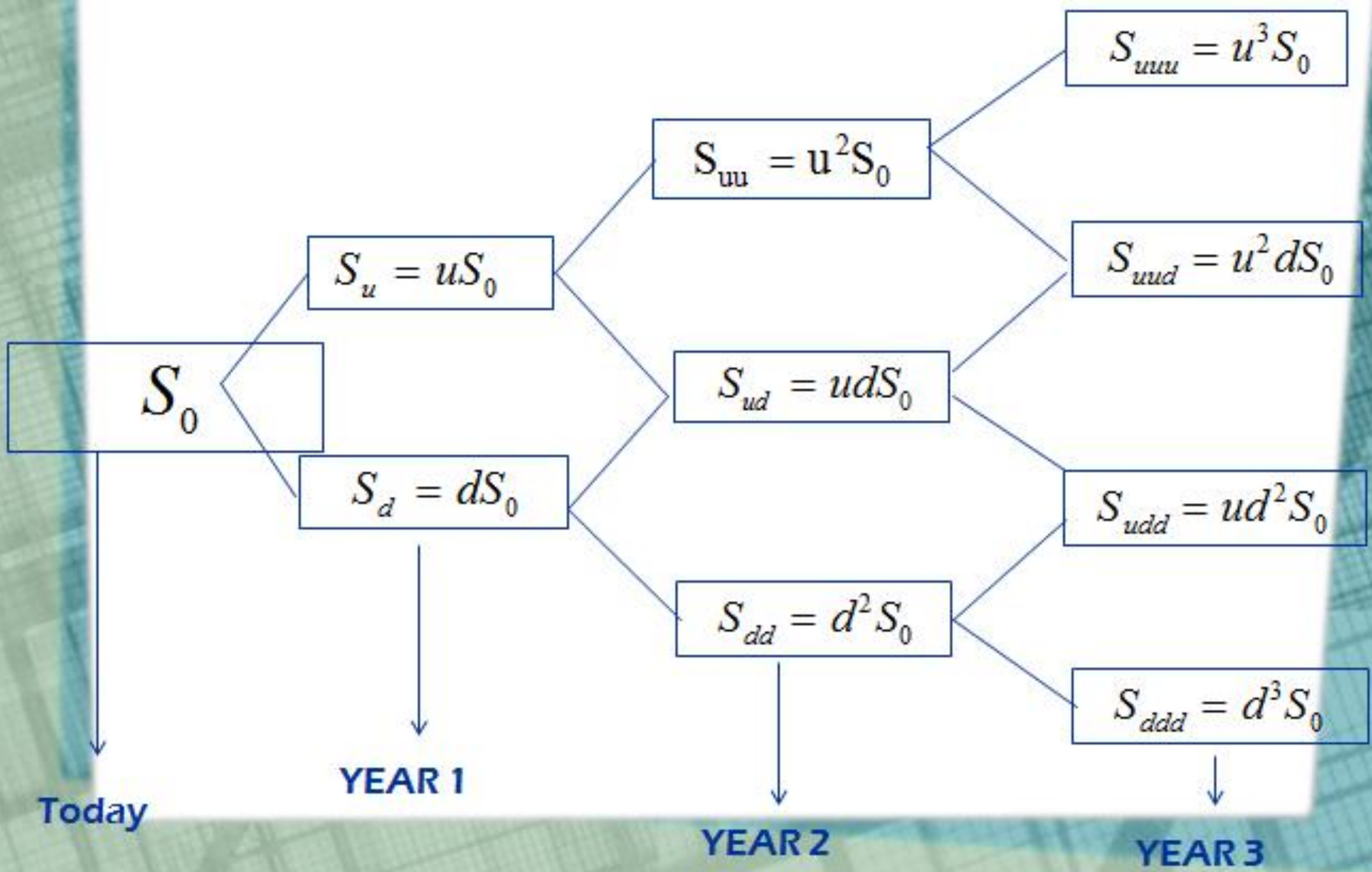
- The underlying assumption is that the stock price follows a ***random walk***. In each time step, it has a certain probability of moving up by a certain percentage amount and a certain probability of moving down by a certain percentage amount



Binomial Interest Rate Model

- Model:
  - Assume a one-period, stock price (S_0) follows a process in which in each period the price is equal to a proportion **u** times its beginning-of-the-period value or a proportion **d** times its initial value, where u is greater than d (u>d);
  - After one period, there would be two possible one-period stock prices: $\mathbf{S_u = uS_0}$ and $\mathbf{S_d = dS_0}$;
  - If the proportions u and d are constant over different periods, then after two periods there would be three possible stock prices: $S_{uu} = u^2S_0$, $S_{ud} = udS_0$, and $S_{dd} = d^2S_0$.

BINOMIAL TREE



A One Step Binomial Model and No-Arbitrage Argument

One-Step Binominal Model: Assumptions of the BOPM

1. The stock price follows a random walk;
2. There are two and only two possible prices for the underlying asset on the next date. The underlying price will either:
   - Increase by a factor of u% (an upward movement)
   - Decrease by a factor of d% (a downward movement)

3. The uncertainty is that we do not know which of the two prices will be realized;
4. No dividends;
5. There are no risk-free arbitrage opportunities;
6. The one-period risk-free interest rate, r, is constant over the life of the option;
7. Markets are perfect (no commissions, transaction cost and taxes);
8. The underlying asset can be traded continuously and divisible in small numbers of units

A One Step Binomial Model and No-Arbitrage Argument: Pricing an European Call Option

Valuing a European call option to buy the stock for $21 in 3 months:

Spot Price: $20

Strike Price: $21

u:1.1

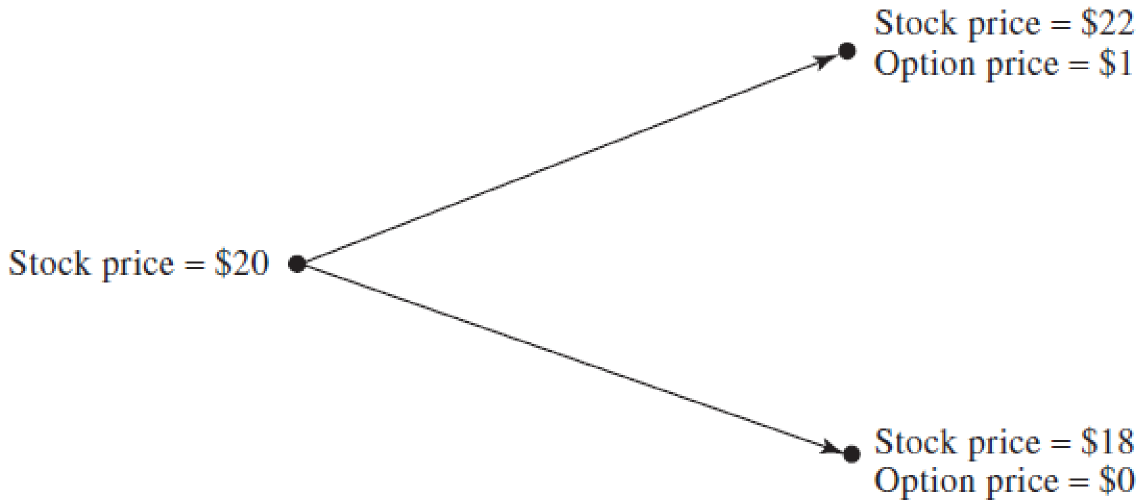
d:0.9

Risk-free rate: 12% (0.12)

t: 3 months (0.25= 3/12)

A One Step Binomial Model

- A stock price is currently $20;
- In 3 months it will be either u= $22 ($20*1.1) or d= $18 ($20*0.9);
- This option will have one of two values at the end of the 3 months. If the stock price turns out to be $22, the value of the option will be $1; if the stock price turns out to be $18, the value of the option will be zero



One-Step Binominal Model

1. It turns out that a relatively simple argument can be used to price the option in this example. The only assumption needed is that ***arbitrage opportunities do not exist***;
2. We set up a ***portfolio of the stock and the option*** in such a way that there is no uncertainty about the value of the portfolio at the end of the 3 months;
3. We then argue that, because the portfolio has NO RISK, the return it earns must EQUAL the ***risk-free interest rate***. This enables us to work out the COST of setting up the portfolio and therefore the option's price;
4. Because there are two securities (the stock and the stock option) and only two possible outcomes, it is always possible to set up the riskless portfolio

A One Step Binomial Model

1. Define a portfolio that has no uncertainty and therefore no risk;
2. Since this portfolio has no risk, the return it earns must equal risk-free interest rate

Consider a risk-less portfolio consisting of a *long position in* Δ *shares of the stock and a short position in one call option*:

Risk-less Portfolio:
Long: Δ Shares
Short: One Call Option

We calculate the value of Δ that makes the portfolio riskless

Pay-off from the riskless portfolio:
Long: Δ Shares
Short: One Call Option

1. Payoff (total value of the portfolio) if the stock price goes up ($22) = $22Δ-1
(The value of the shares is $22Δ and the value of the option is $1. We are deducting 1 from $22Δ because the call option is in-the-money and the holder of the call option will exercise his right to buy the underlying, therefore a loss of $1);

2. Payoff (total value of the portfolio) if the stock price goes down ($18) = $18Δ- $0
(The value of the shares is $18Δ and the value of the option is zero ($0). We are deducting $0 from $18Δ because the call option is out-of-the-money and the holder of the call option will not exercise his right to buy the underlying)

The portfolio is riskless if the value of Δ is chosen so that the final value of the portfolio is the SAME for BOTH alternatives. Thus, the portfolio to be riskless, the above two pay-offs should be EQUAL to each other:

$22Δ-$1=$18Δ-$0
Δ = 0.25 shares

A riskless portfolio is therefore:
Long: 0.25 Shares
Short: One Call Option

1. If the stock price moves up to \$22, the value of the portfolio in 3 months is $\$22\Delta-\$1= 22*0.25-1=\$4.5$;

2. If the stock price moves down to \$18, the value of the portfolio in 3 months is $\$18\Delta-\$0 = 18*0.25=\$4.5$

Note: Regardless of whether the stock price moves up or down, the value of the portfolio is always \$4.5 at the end of the life of the option. Therefore, since this is a riskless portfolio, it must, in the absence of arbitrage opportunities, earn the ***risk-free rate of interest***

Calculation of Present Value (PV) of the Portfolio:

Given, the risk-free rate of interest of 12% per annum, it follows that the value of the portfolio **TODAY** must be the ***present value*** of $4.5, calculated as follows:

Present Value (PV) of the Portfolio: $\$4.5e^{-0.12*3/12} = \$4.3670$

Calculation of Option Price:

The present value of the option = PV of the shares today-PV of the portfolio today: $5.000-$4.367= **$0.633**]

[Given the spot price of $20, the present value of the shares today (with Δ shares) @ $20 = $20*0.25 = $5

Alternatively,

The value of the stock price today is known to be $20. Suppose the option price is denoted by *f*. The value (or the COST setting up) of the portfolio today is:

$$\mathbf{20*0.25-}f$$
$$\mathbf{= 5-}f$$

It follows that:

$5-f=\$4.367$ or

$f = \mathbf{\$0.633}$

Important Observations:

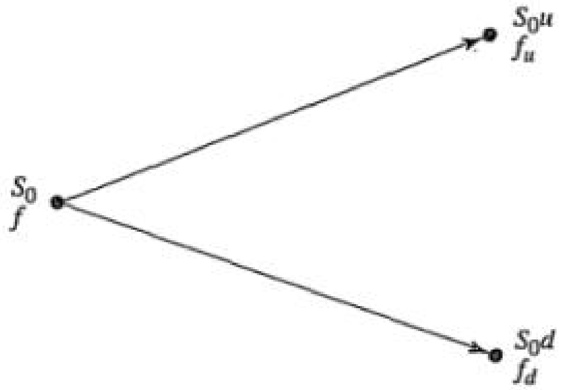
1. This shows that, in the absence of arbitrage opportunities, the current value of the option must be $0.633;

2. If the value of the option is more than $0.633, the portfolio would cost LESS than $4.3670 to set up and would earn more than the risk-free rate;

3. If the value of the option is less than $0.633, shorting the portfolio would provide a way of borrowing money at less than the risk-free rate

One–Step: A Generalization

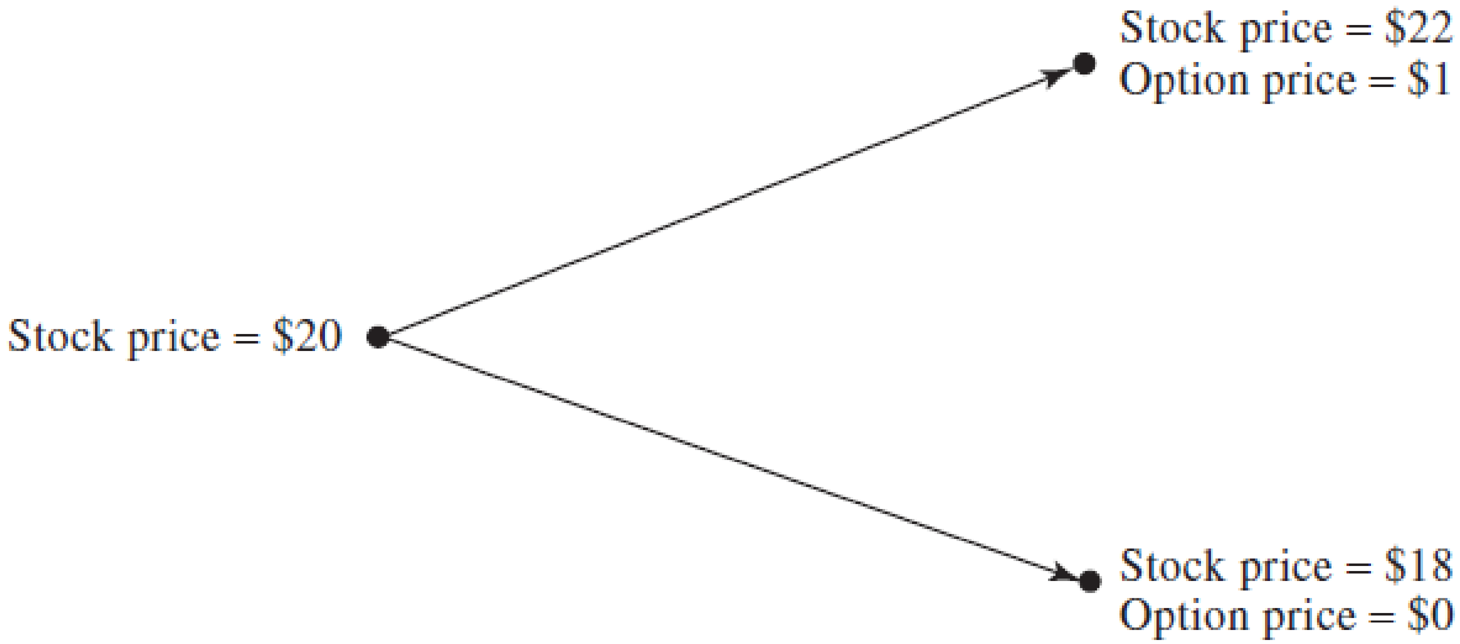
1. The BOPM makes an assumption that over a certain time period, the underlying can only do one of two things: go up, or go down:



where, S_0 is the spot price of the stock, *f is* the price of an option on the stock, S_0u *(where u>1)* is upward movement of the stock price, S_0d *(where d<1)* is downward movement of the stock price, [the percentage increase in the stock price when there is an up movement is u-1, the percentage decrease when there is a down movement is 1-d] if the stock price moves up to S_0u the payoff from the option is to f_u; if the stock price moves down to S_0d the payoff from the option is f_d

Stock price movements for numerical example:

Spot Price=$20; Strike Price=$21; u=1.1; d=0.9; r=12%; t=3 months (0.25= 3/12)



2. Consider the portfolio that is long Δ shares and short position in one call option. We calculate the value of the Δ that makes the portfolio riskless:

Long: Δ Shares
Short: One Call Option

$S_0 u\Delta - f_u$

This is the value of the portfolio at the end of the life of the option if there is an up movement in the stock price

$S_0 d\Delta - f_d$

This is the value of the portfolio at the end of the life of the option if there is a down movement in the stock price

- The portfolio is riskless when, $S_0 u\Delta - f_u = S_0 d\Delta - f_d$ or

$$\Delta = \frac{f_u - f_d}{S_0 u - S_0 d} \qquad \ldots(1)$$

- Equation (1) shows that Δ is the ratio of the change in the option price to the change in the stock price as we move between the nodes at time T

3. In this case, the portfolio is riskless and, for there to be no arbitrage opportunities, it must earn the risk-free interest rate. Denoting the risk-free interest rate by r, the present value of the risk-less portfolio today: $\boldsymbol{(S_0 u\Delta - f_u)e^{-rt}}$

4. The cost of setting up the portfolio today: $\boldsymbol{S_0\Delta - f}$

$$\text{It follows that, } S_0\Delta - f = (S_0 u\Delta - f_u)e^{-rT}$$

$$\text{Hence, } f = S_0\Delta(1 - ue^{-rT}) + f_u e^{-rT}$$

- Substituting for Δ from Equation (1) and simplifying, we obtain:

$$f = [\, p f_u + (1-p) f_d \,]e^{-rT} \qquad \ldots(2)$$

$$\text{where} \qquad p = \frac{e^{rT} - d}{u - d} \qquad \ldots(3)$$

Equations (2) and (3) enable an option to be priced when stock price movements are given by a one-step binomial tree. The only assumption needed for the equation is that there are no arbitrage opportunities in the market

A Generalization

If we denote the risk-free interest rate by r, the present value of the portfolio is

$$(S_0 u\Delta - f_u)e^{-rT}$$

The cost of setting up the portfolio is

$$S_0\Delta - f$$

It follows that

$$S_0\Delta - f = (S_0 u\Delta - f_u)e^{-rT}$$

or

$$f = S_0\Delta(1 - ue^{-rT}) + f_u e^{-rT}$$

Substituting from equation (13.1) for Δ, we obtain

$$f = S_0\left(\frac{f_u - f_d}{S_0 u - S_0 d}\right)(1 - ue^{-rT}) + f_u e^{-rT}$$

or

$$f = \frac{f_u(1 - de^{-rT}) + f_d(ue^{-rT} - 1)}{u - d}$$

or

$$f = e^{-rT}[pf_u + (1 - p)f_d] \tag{13.2}$$

where

$$p = \frac{e^{rT} - d}{u - d} \tag{13.3}$$

- In the numerical example considered previously u=1.1, d=0.9, r = 0.12, T = 0.25, f_u = 1, and f_d = 0;

$$p = \frac{e^{0.12\times 3/12} - 0.9}{1.1 - 0.9} = 0.6523$$

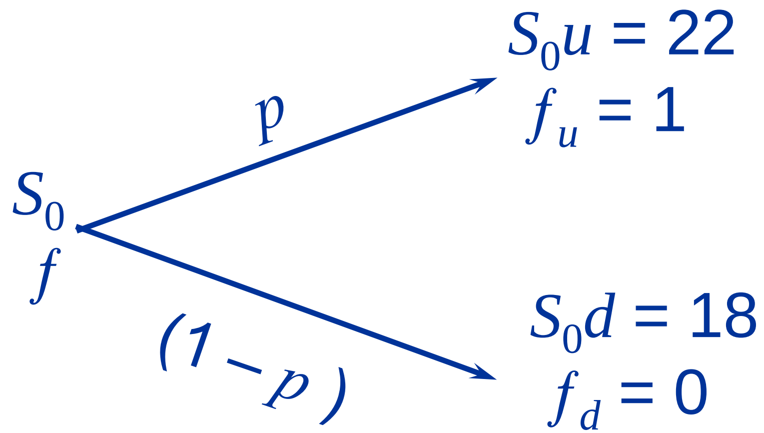
$$f = e^{-0.12\times 0.25}(0.6523 \times 1 + 0.3477 \times 0) = 0.633$$

p as a Probability

- It is natural to interpret p and 1-p as probabilities of up and down movements;
- The value of an option is then its expected payoff in a **risk-neutral** (all individuals are indifferent to risk) world discounted at the risk-free rate:

$$\begin{tikzcd} & \begin{array}{c} S_0u \\ f_u \end{array} \\ \begin{array}{c} S_0 \\ f \end{array} \arrow[ur, "p"] \arrow[dr, "(1-p)"'] & \\ & \begin{array}{c} S_0d \\ f_d \end{array} \end{tikzcd}$$

Original Example Revisited



- Since p is the probability that gives a return on the stock equal to the risk-free rate. We can find it from:

$$20e^{0.12 \times 0.25} = 22p + 18(1-p)$$

which gives $p = 0.6523$

- Alternatively, we can use the formula:

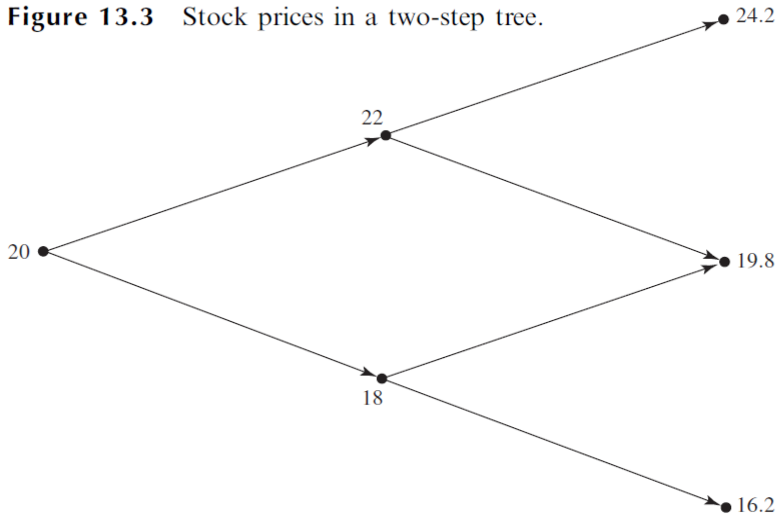
$$p = \frac{e^{rT} - d}{u - d} = \frac{e^{0.12 \times 0.25} - 0.9}{1.1 - 0.9} = 0.6523$$

TWO-STEP BINOMIAL TREES

We can extend the analysis to a two-step binomial tree such as that shown in the following figure:

Stock prices in a two-step tree:

Figure 13.3 Stock prices in a two-step tree.

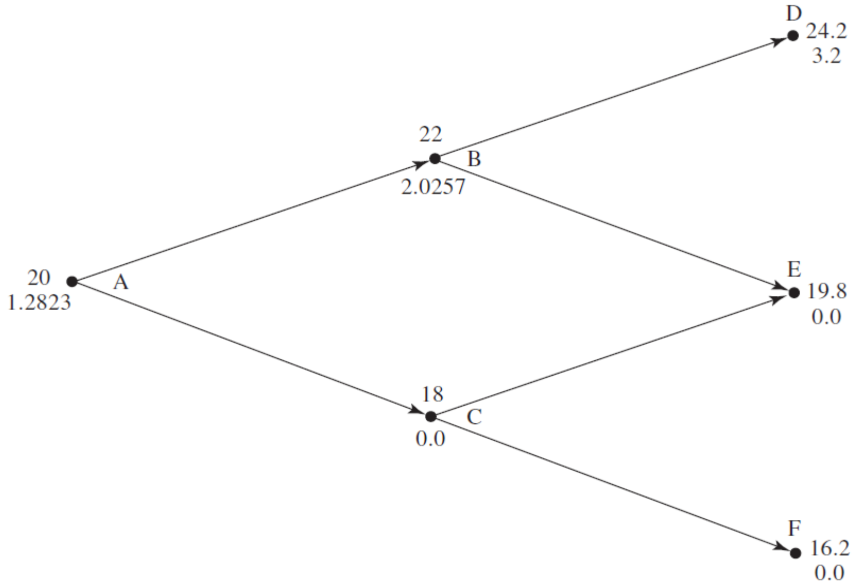


Here the stock price starts at \$20 and in each of two time steps may go up by 10% or down by 10%. Each time step is 3 months long and the risk-free interest rate is 12% per annum. We consider a 6-month option with a strike price of \$21. The objective of the analysis is to calculate the option price at the initial node of the tree. This can be done by repeatedly applying the principles established earlier

TWO-STEP BINOMIAL TREES

Valuing a Call Option:

Stock and option prices in a two-step tree. The upper number at each node is the stock price and the lower number is the option price



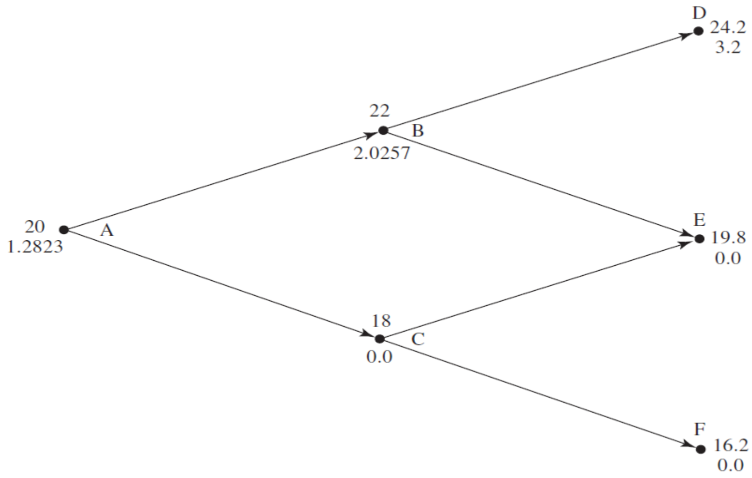
Each time step is 3 months: K=21, r=12%

At node D the stock price is 24.2 and the option price is 24.2-21=3.2; at nodes E and F the option is out of the money and its value is zero. At node C the option price is zero, because node C leads to either node E or node F and at both of those nodes the option price is zero

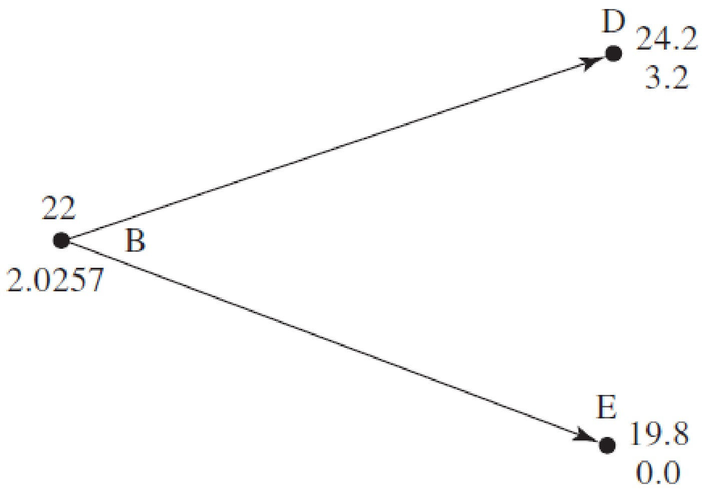
TWO-STEP BINOMIAL TREES

Valuing a Call Option:

We calculate the option price at node B by focusing our attention on the part of the tree shown in the following figure:



Evaluation of option price at node B of Figure



- **Value at node B:**

$e^{-0.12\times0.25}(0.6523\times3.2 + 0.3477\times0) = \$2.0257$

- **Value at node C:**

$e^{-0.12\times0.25}(0.6523\times0 + 0.3477\times0) = \$0$

- **Next, value at node A:**

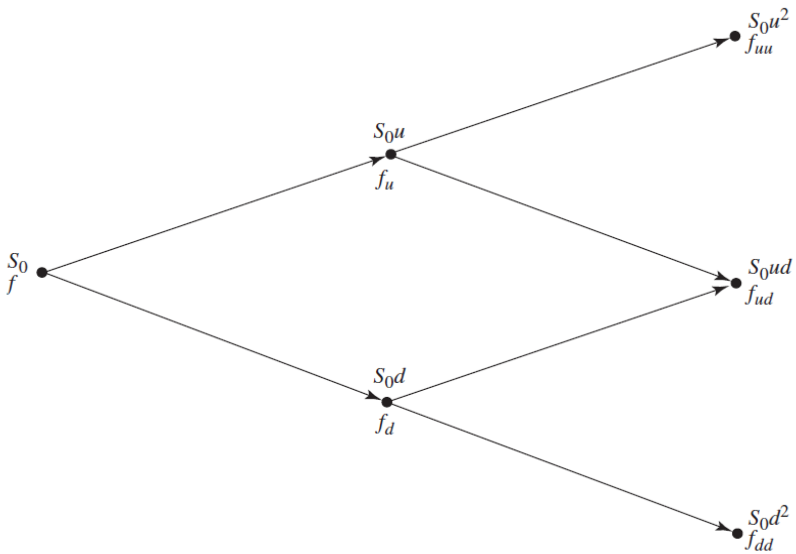
$e^{-0.12\times0.25}(0.6523\times2.0257 + 0.3477\times0) = \$1.2823$

The value of the option is $1.2823

A Generalization

We can generalize the case of two time steps by considering the situation given in the following figure:

Stock and option prices in general two-step tree



The stock price is initially S_0. During each time step, it either moves up to u times its initial value or moves down to d times its initial value. The notation for the value of the option is shown on the tree. For example, after two up movements the value of the option is fuu. We suppose that the risk-free interest rate is r and the length of the time step is Δt years

A Generalization

Because the length of a time step is now Δt rather than T, equations (13.2) and (13.3) become

$$f = e^{-r\Delta t}[pf_u + (1-p)f_d] \tag{13.5}$$

$$p = \frac{e^{r\Delta t} - d}{u - d} \tag{13.6}$$

Repeated application of equation (13.5) gives

$$f_u = e^{-r\Delta t}[pf_{uu} + (1-p)f_{ud}] \tag{13.7}$$

$$f_d = e^{-r\Delta t}[pf_{ud} + (1-p)f_{dd}] \tag{13.8}$$

$$f = e^{-r\Delta t}[pf_u + (1-p)f_d] \tag{13.9}$$

Substituting from equations (13.7) and (13.8) into (13.9), we get

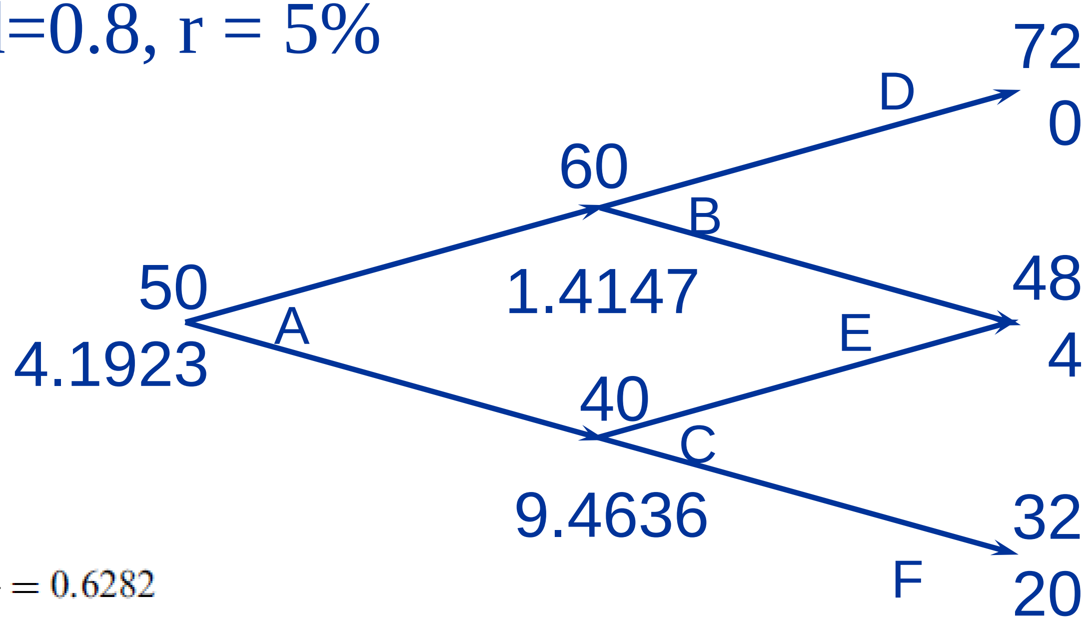
$$f = e^{-2r\Delta t}[p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd}] \tag{13.10}$$

- The variables p^2, $2p(1-p)$ and $(1-p)^2$ are the probabilities that the upper, middle, and lower final nodes will be reached

A Put Option Example

S_O =*\$50*, *K* = \$52, time step =1yr,

u= 1.2, d=0.8, r = 5%



$$p = \frac{e^{0.05\times 1} - 0.8}{1.2 - 0.8} = 0.6282$$

RISK-NEUTRAL VALUATION

Principle of risk-neutral valuation: A very important principle in the pricing of derivatives is known as **risk-neutral valuation**. It states that, when valuing a derivative, we can make the assumption that investors are **risk-neutral**. This assumption means investors **do not** increase the expected return they require from an investment to compensate for increased risk. A world where investors are **risk-neutral** is referred to as a **risk-neutral world**. Thus, in a risk neutral world all the individuals are indifferent to risk. In such a world, investors require no compensation for risk and the expected return on all securities is the risk free rate of interest.

Real World: The world we live in is, of course, not a risk-neutral world. The higher the risks investors take, the higher the expected returns they require. However, it turns out that assuming a risk-neutral world gives us the *right option price for the world we live in*, as well as *for a risk-neutral world*. Almost miraculously, it finesses the problem that we know hardly anything about the ***risk aversion*** of the buyers and sellers of options.

Risk-neutral valuation seems a surprising result when it is first encountered. Options are risky investments. Should not a person's risk preferences affect how they are priced?

The answer is that, when we are pricing an option in terms of the price of the underlying stock, ***risk preferences*** are unimportant. As investors become more risk averse, stock prices decline, but the formulas relating option prices to stock prices remain the same.

A risk-neutral world has two features that simplify the pricing of derivatives:

1. The expected return on a stock is the risk-free rate;

2. The discount rate used for the expected payoff on an option is the risk-free rate

Recall…

$$f = e^{-rT}[p f_u + (1 - p) f_d] \quad \textbf{(13.2)}$$

where

$$p = \frac{e^{rT} - d}{u - d} \quad \textbf{(13.3)}$$

Equations (13.2) and (13.3) enable an option to be priced when stock price movements are given by a one-step binomial tree. The only assumption needed for the equation is that there are no arbitrage opportunities in the market.

- Returning to equation (13.2), the parameter p should be interpreted as the probability of an up movement in a risk-neutral world, so that 1-p is the probability of a down movement in this world;
- We assume $u > e^{rT}$, so that $0 < p < 1$. The expression $\boldsymbol{pf_u+(1-p)f_d}$ is the expected future payoff from the option in a risk-neutral world and equation (13.2) states that the value of the option today is its expected future payoff in a risk-neutral world discounted at the risk-free rate

The validity of our interpretation of p

To prove the validity of our interpretation of p, we note that, when p is the probability of an up movement, the expected stock price $E(S_T)$ at time T is given by

$$E(S_T) = pS_0u + (1 - p)S_0d$$

or

$$E(S_T) = pS_0(u - d) + S_0d$$

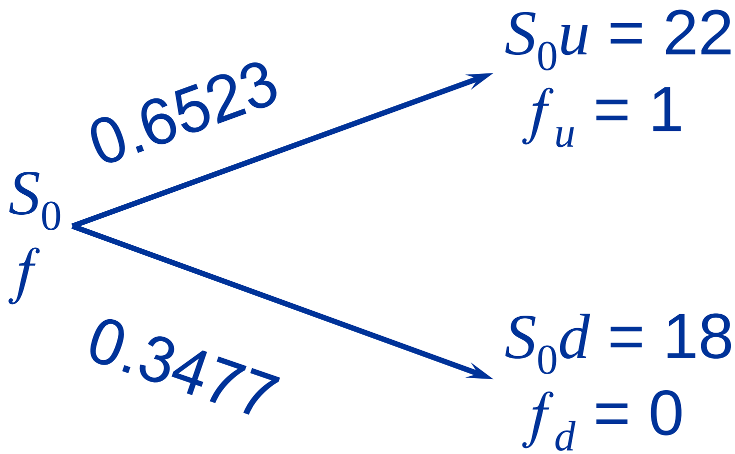
Substituting from equation (13.3) for p gives

$$E(S_T) = S_0e^{rT} \tag{13.4}$$

This shows that the stock price grows, on average, at the risk-free rate when p is the probability of an up movement. In other words, the stock price behaves exactly as we would expect it to behave in a risk-neutral world when p is the probability of an up movement.

To apply risk-neutral valuation to the pricing of a derivative, we first calculate what the probabilities of different outcomes would be if the world were risk-neutral. We then calculate the expected payoff from the derivative and discount that expected payoff at the risk-free rate of interest. Binomial trees illustrate the general result that to value a derivative we can assume that the expected return on the underlying asset is the risk-free rate and discount at the risk-free rate. This is known as risk-neutral valuation.

Valuing the Option Using Risk-Neutral Valuation



The value of the option is

$$e^{-0.12\times 0.25}(0.6523\times 1 + 0.3477\times 0)$$

$$= 0.633$$

Black-Scholes Model of Pricing Stock Options
(Black-Scholes-Merton Model)

1. The Pricing of Options and Corporate Liabilities by Fischer Black and Myron Scholes, Journal of Political Economy Vol. 81, No. 3 (May - Jun., 1973), pp. 637-654.
2. Theory of Rational Option Pricing by Robert C. Merton, The Bell Journal of Economics and Management Science, Vol. 4, No. 1 (Spring, 1973),pp. 141-183.

Continuous Time Option Pricing Model

- Assumptions of the Black-Scholes Option Pricing Model:
  - No taxes;
  - No transactions costs;
  - Unrestricted short-selling of stock, with full use of short-sale proceeds;
  - Shares are infinitely divisible;
  - Constant riskless interest rate for borrowing/lending;
  - No dividends;
  - European options (or American calls on non-dividend paying stocks);
  - Continuous trading;
  - The stock prices behave randomly and evolve according to a lognormal distribution.

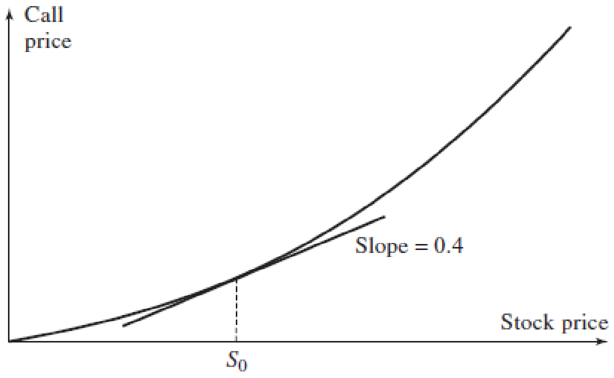
The Nobel idea underlying the Black–Scholes–Merton Differential Equation

- The Black–Scholes–Merton differential equation is an equation *that must be satisfied by the price of any derivative dependent on a non-dividend-paying stock.* The nature of the arguments are similar to the no-arbitrage arguments we used to value stock options for the situation where stock price movements were assumed to be binomial. They involve setting up a riskless portfolio consisting of a position in the derivative and a position in the stock. In the absence of arbitrage opportunities, the return from the portfolio must be the risk-free interest rate, r. This leads to the Black-Scholes-Merton differential equation;
- The reason a riskless portfolio can be set up is that the stock price and the derivative price are both affected by the same underlying source of uncertainty: **stock price movements**;
- In any short period of time, the price of the derivative is *perfectly correlated* with the price of the underlying stock. When an appropriate portfolio of the stock and the derivative is established, the gain or loss from the stock position always offsets the gain or loss from the derivative position so that the overall value of the portfolio at the end of the short period of time is known with certainty

Suppose, for example, that at a particular point in time the relationship between a small change ΔS in the stock price and the resultant small change Δc in the price of a European call option is given by:

$$\Delta c = 0.4 \Delta S$$

This means that the slope of the line representing the relationship between c and S is 0.4, as indicated in Figure:



A riskless portfolio would consist of:

1. A short position in 100 call options
2. A long position in 40 shares;

Suppose, for example, that the stock price increases by 10 cents. The option price will increase by 4 cents (10 cents*0.4) and the 40*$0.1 = $4 gain on the shares is equal to the 100*$0.04=$4 loss on the short option position

($1=100 cents = 10 cents/100 cents =$0.1 and 4 cents/100 cents =$0.04)

(100*0.4 = 40 shares)

The Difference: But there is one important difference between the Black–Scholes–Merton analysis and analysis using a binomial model. In Black–Scholes–Merton, the position in the stock and the derivative is riskless for only a very short period of time. (Theoretically, it remains riskless only for an instantaneously short period of time);

However, to remain riskless, it must be adjusted, or rebalanced, frequently. For example, the relationship between Δc and ΔS in our example might change from **$\Delta c = 0.4\Delta S$** today to **$\Delta c = 0.5\Delta S$** tomorrow. This would mean that, in order to maintain the riskless position, an additional/extra 10 shares (100*0.5=50 shares-40 shares=10 shares) would have to be purchased for each 100 call options sold. It is nevertheless true that the return from the riskless portfolio in any very short period of time must be the risk-free interest rate. This is the key element in the Black–Scholes–Merton analysis and leads to their pricing formulas;

Therefore, in the Black-Scholes-Merton model, trading occurs continuously, but the general idea is the same. A hedge portfolio is established and maintained by constantly adjusting the relative proportions of stock and options, a process called dynamic trading. The end result is obtained through complex mathematics, but the formula is straightforward

Black–Scholes–Merton differential equation and Black–Scholes–Merton Pricing Formulas:

Black–Scholes–Merton differential equation:

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \tfrac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$

The most famous solutions to the differential equation are the Black–Scholes–Merton formulas for the prices of European call and put options:

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

$$p = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$$

where

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

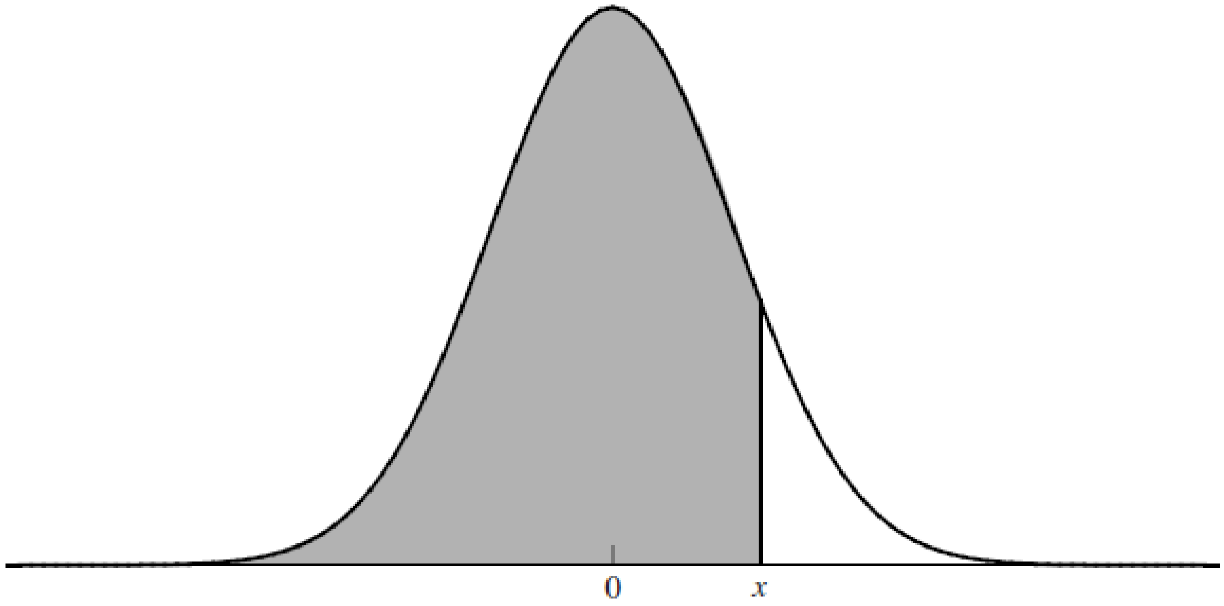
$$d_2 = \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

N(d) = Probability that a standardized, normally distributed, random variable will be less than or equal to d. The variables c and p are the European call and European put price, S_0 is the stock price at time zero, K is the strike price, r is the continuously compounded risk-free rate, σ is the annualized stock price volatility of the continuously compounded (log) return on the stock, and T is the time to maturity of the option.

The $N(x)$ Function

The $N(x)$ Function: The function $N(x)$ is the cumulative probability distribution function for a variable with a standard normal distribution. In other words, the *function* $N(x)$ is the probability that a normally distributed variable with a mean of zero and a standard deviation of 1 is less than x;

Shaded area represents N(x)



Understanding $N(d_1)$ and $N(d_2)$

To price the call option, it is assumed that investors are risk neutral and then, the expected payoff is found which is discounted at the risk-free rate. The expected payoff is the expected value of Max(0, $\mathbf{S_T - X}$) or, in other words, the expected value of:

$$\begin{array}{ll} \mathbf{0} & \textbf{if } \mathbf{S_T \le X} \\ \mathbf{S_T - X} & \textbf{if } \mathbf{S_T > X} \end{array}$$

To ignore the outcomes where $\mathbf{S_T \le X}$, we must calculate the expected value of $\mathbf{S_T - X}$ given that $S_T>X$. Let us multiply the first term on the right-hand side of the Black–Scholes–Merton formula by e^{rT},

$$\mathbf{S_0 N(d_1) e^{rT}}$$

This entire expression is the expected value of the stock price at expiration (time T) given that the stock price exceeds the exercise price times the probability that the stock price exceeds the exercise price at expiration; however, $N(d_1)$ is not that probability (it has a probability of $N(d_2)$ explained below). It is just a component of the entire expression.

The second term on the right-hand side of the Black–Scholes–Merton formula,

$$\mathbf{-XN(d_2)}$$

is the expected value of the payment of the exercise price at expiration. Specifically, here $N(d_2)$ is the probability for risk-neutral investors that X will be paid at expiration. That is $N(d_2)$ is the probability that stock price will be at or above the exercise price when the call option expires. In other words, it is the probability that a call option will be exercised in a risk-neutral world. Therefore, $-XN(d_2)$ is the expected payoff of the exercise price at expiration.

The expected payoff in a risk-neutral world at time T is therefore:

$S_0N(d_1)e^{rT}\text{-}XN(d_2)$

Calculating the present value (discounting the above expression at the continuously compounded risk-free rate—that is, multiplying by e^{rT}) of the this from time T to time zero gives the Black–Scholes–Merton equation for a European call option:

$$[S_0N(d_1)e^{r_cT} - XN(d_2)]e^{-r_cT} = S_0N(d_1) - Xe^{-r_cT}N(d_2)$$

Numerical Example:

The inputs are a stock price of $125.94, an exercise price of $125, and a time to expiration of 0.0959. The continuously compounded risk-free rate is 4.46%. The annualized volatility is 0.83:

The computation of the Black-Scholes-Merton price is a five-step process:

1. Calculate the value of d_1;

2. Calculate the value of d_2;

3. Look up $N(d_1)$ in the normal probability table;

4. Look up $N(d_2)$ in the normal probability table;

3. Plug the values into the formula for c.

Numerical Example: Calculating the Black-Scholes-Merton Price

$S_0 = 125.94 \qquad X = 125 \qquad r_c = 0.0446 \qquad \sigma = 0.83 \qquad T = 0.0959$

1. Compute d_1

$$d_1 = \frac{\ln(125.94/125) + (0.0446 + (0.83)^2/2)0.0959}{0.83\sqrt{0.0959}} = 0.1743.$$

2. Compute d_2

$$d_2 = 0.1743 - 0.83\sqrt{0.0959} = -0.0827.$$

3. Look up $N(d_1)$

$$N(0.17) = 0.5675.$$

4. Look up $N(d_2)$

$$N(-0.08) = 1 - N(0.08) = 1 - 0.5319 = 0.4681.$$

5. Plug into formula for C

$$C = 125.94(0.5675) - 125e^{-0.0446(0.0959)}(0.4681) = 13.21.$$

STANDARD NORMAL PROBABILITIES

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8860	0.8888	0.8907	0.8925	0.8943	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

VOLATILITY

VOLATILITY: The volatility, σ, of a stock is a measure of our uncertainty about the returns provided by the stock. Stocks typically have a volatility between 15% and 60%.

The volatility of a stock price can be defined as the standard deviation of the return provided by the stock in 1 year when the return is expressed using continuous compounding.

The standard deviation of the return in time Δt is $\sigma\sqrt{\Delta t}$

- This means that $\sigma\sqrt{\Delta t}$ is approximately equal to the standard deviation of the percentage change in the stock price in time Δt.
- Suppose that σ = 0.3, or 30%, per annum and the current stock price is $50. The standard deviation of the percentage change in the stock price in 1 week is approximately:

$$30 \times \sqrt{\frac{1}{52}} = 4.16\%$$

- A 1-standard-deviation move in the stock price in 1 week is therefore 50* 0.0416=2.08.

Estimating Volatility from Historical Data

To estimate the volatility of a stock price empirically, the stock price is usually observed at fixed intervals of time (e.g., every day, week, or month). Define:

$n+1$: Number of observations

S_i: Stock price at end of ith interval, with $i = 0, 1, \ldots, n$

τ: Length of time interval in years

and let

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right) \quad \text{for } i = 1, 2, \ldots, n$$

The usual estimate, s, of the standard deviation of the u_i is given by

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2}$$

or

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} u_i^2 - \frac{1}{n(n-1)}\left(\sum_{i=1}^{n} u_i\right)^2}$$

where $\bar{u}$ is the mean of the u_i.[4]

Calculation of Volatility

- The closing stock prices during 21 consecutive trading days of a company are as follows:

Day	Closing Price ($)
21	20
20	20.1
19	19.9
18	20
17	20.5
16	20.25
15	20.9
14	20.9
13	20.9
12	20.75
11	20.75
10	21
9	21.1
8	20.9
7	20.9
6	21.25
5	21.4
4	21.4
3	21.25
2	21.75
1	22

- Example:

$$\sum_{i=1}^{n} u_i = 0.09531 \quad \text{and} \quad \sum_{i=1}^{n} u_i^2 = 0.00326$$

and the estimate of the standard deviation of the daily return is

$$\sqrt{\frac{0.00326}{19} - \frac{0.09531^2}{20 \times 19}} = 0.01216$$

or 1.216%. Assuming that there are 252 trading days per year, $\tau = 1/252$ and the data give an estimate for the volatility per annum of $0.01216\sqrt{252} = 0.193$, or 19.3%. The standard error of this estimate is

$$\frac{0.193}{\sqrt{2 \times 20}} = 0.031$$

or 3.1% per annum.

- The volatility per annum is calculated from the volatility per trading day using the following formula:

$$\begin{array}{c}\text{Volatility}\\\text{per annum}\end{array} = \begin{array}{c}\text{Volatility}\\\text{per trading day}\end{array} \times \sqrt{\begin{array}{c}\text{Number of trading days}\\\text{per annum}\end{array}}$$

THE GREEKS (LETTERS)

The Greeks: The Context

- A financial institution that sells an option to a client in the over-the-counter markets is faced with the problem of managing its risk;
- If the option happens to be the same as one that is traded on an exchange, the financial institution can neutralize its exposure by buying on the exchange the same option as it has sold;
- However, when the option has been tailored to the needs of a client and does not correspond to the standardized products traded by exchanges, hedging the exposure is far more difficult;
- This problem is approached using ‘‘Greek letters’ or simply the ‘‘Greeks’’. Each Greek letter measures a different dimension to the risk in an option position and the aim of a trader is to manage the Greeks so that all risks are acceptable;
- The analysis is applicable to market makers in options on an exchange as well as to traders working in the ***over-the-counter market*** for financial institutions

Illustration

- A financial institution has sold $300,000 European call options on 100,000 shares of a non-dividend paying stock. The stock price is $49, the strike price is $50, the risk-free interest rate is 5% per annum, the stock price volatility is 20% per annum, the time to maturity is 20 weeks (0.3846 years), and the expected return from the stock is 13% per annum:

$$S_0=\$49;\ K=\$50;\ r=0.05;\ \sigma=0.20;\ T=0.3846$$

- The Black–Scholes–Merton value of an option to buy one share is $2.40.
- Thus, the Black–Scholes–Merton price of the option is about $2,40,000. The financial institution has therefore sold a product for $60,000 more than its theoretical value. But it is faced with the problem of hedging the risks.

Naked and Covered Positions

Naked Position: One strategy open to the financial institution is to do nothing (referred to as a naked position). It is a strategy that works well if the stock price is below $50 at the end of the 20 weeks. The option then costs the financial institution nothing and it makes a profit of $300,000. However, a naked position works LESS well if the call is exercised because the financial institution then has to buy 100,000 shares at the market price prevailing in 20 weeks to cover the call. The cost to the financial institution is 100,000 times the amount by which the stock price exceeds the strike price. For example, if after 20 weeks the stock price is $60, the option costs the financial institution $1,000,000 [($60-$50)*100000]. This is considerably greater than the $300,000 charged for the option

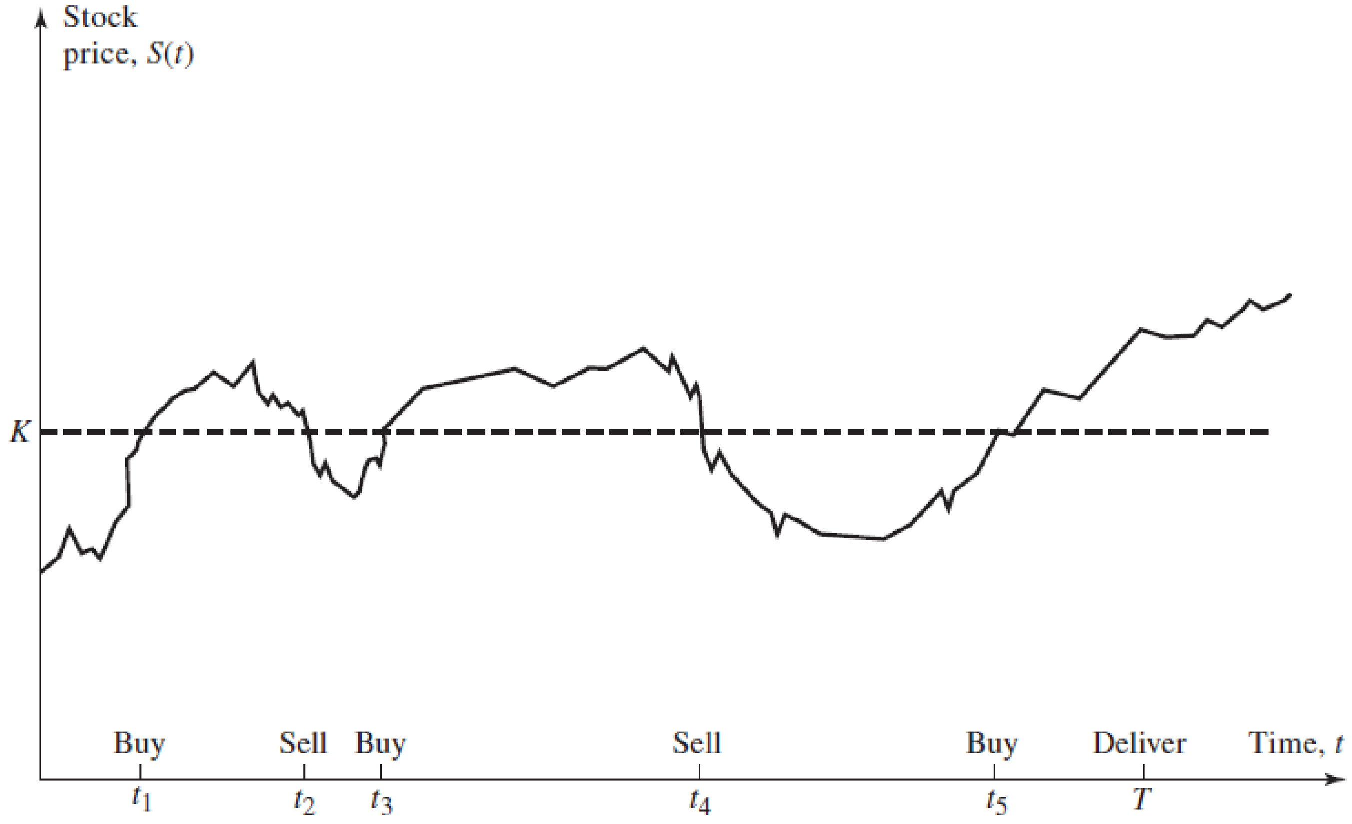
Naked and Covered Positions

Covered Position: As an alternative to a naked position, the financial institution can adopt a covered position. This involves buying 100,000 shares as soon as the option has been sold. If the option is exercised, this strategy works well, but in other circumstances it could lead to a significant loss. For example, if the stock price drops to $40, the financial institution loses $900,000 [($40-$49)*100000] on its stock position. This is considerably greater than the $300,000 charged for the option. Therefore, neither a naked position nor a covered position provides a good hedge. If the assumptions underlying the Black–Scholes–Merton formula hold, the cost to the financial institution should always be $240,000 on average for both approaches. But on any one occasion the cost is liable to range from zero to over $1,000,000. A good hedge would ensure that the cost is always close to $240,000.

A Stop-Loss Strategy

Stop-Loss Strategy: One interesting hedging procedure that is sometimes proposed involves a stop-loss strategy. To illustrate the basic idea, consider an institution that has *written a call option with strike price K to buy one unit of a stock*. The hedging procedure involves *buying one unit of the stock as soon as its price rises above K* and *selling it as soon as its price falls below K*. The objective is to hold a naked position whenever the stock price is less than K and a covered position whenever the stock price is greater than K. The procedure is designed to ensure that at time T the institution owns the stock if the option closes in the money and does not own it if the option closes out of the money.

A Stop-Loss Strategy



- It involves buying the stock at time t_1, selling it at time t_2, buying it at time t_3, selling it at time t_4, buying it at time t_5, and delivering it at time T.

A Stop-Loss Strategy

As usual, we denote the initial stock price by S_0. To illustrate the basic idea, consider an institution that has written a call. The cost of setting up the hedge initially is S_0 if $S_0>K$ and zero otherwise. It seems as though the total cost, Q, of writing and hedging the option is the option's initial intrinsic value:

$$Q = \max(S_0 - K,\ 0)$$

This is because all purchases and sales subsequent to time 0 are made at price K. If this were in fact correct, the hedging procedure would work perfectly in the absence of transaction costs.

However, there are two key reasons above equation is incorrect. The first is that the cash flows to the hedger occur at different times and must be discounted. The second is that purchases and sales cannot be made at exactly the same price K. This second point is critical. If we assume a risk-neutral world with zero interest rates, we can justify ignoring the time value of money. But we cannot legitimately assume that both purchases and sales are made at the same price. If markets are efficient, the hedger cannot know whether, when the stock price equals K, it will continue above or below K.

The Greeks

- The Option Pricing Model (OPM) shows how option prices are related to the key underlying variables S_o, X, T, σ, and r;
- The OPM isolates the independent effect of each underlying variable on the option's value. These isolated, independent effects measure the sensitivity of the option's value (price) to changes in the key underlying variables;
- In the jargon of the derivatives industry, these sensitivity measures are called *the Greeks*, because Greek letters are used to denote them. The measures are derived by applying basic calculus tools to the OPM. In particular, the first derivative of the OPM with respect to each of the underlying variables while holding the value of the other variables constant is taken;
- There are five Greeks used for hedging portfolios: Delta (Δ); Theta (Θ), Gamma (Γ), Vega (ν) and Rho (ρ)

Option Sensitivities (i.e, The Greeks) for the OPM

$\dfrac{\text{Change in option price}}{\text{Change in stock price}}$	Delta
$\dfrac{\text{Change in option price}}{\text{Change in volatilty}}$	Vega
$\dfrac{\text{Change in option price}}{\text{Change in time to expiration}}$	Theta
$\dfrac{\text{Change in option price}}{\text{Change in interest rate}}$	Rho
$\dfrac{\text{Change in Delta}}{\text{Change in stock price}}$	Gamma

DELTA HEDGING

Figure 13.1 Stock price movements for numerical example in Section 13.1.

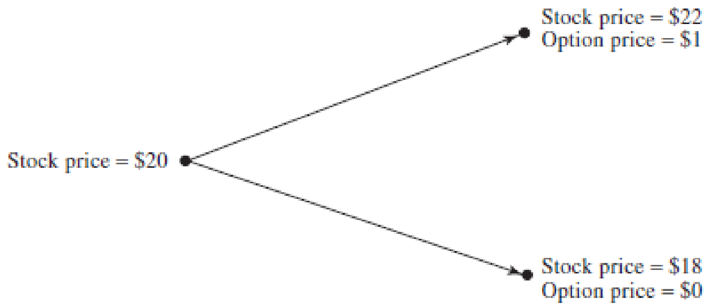


Figure 13.4 Stock and option prices in a two-step tree. The upper number at eac node is the stock price and the lower number is the option price.

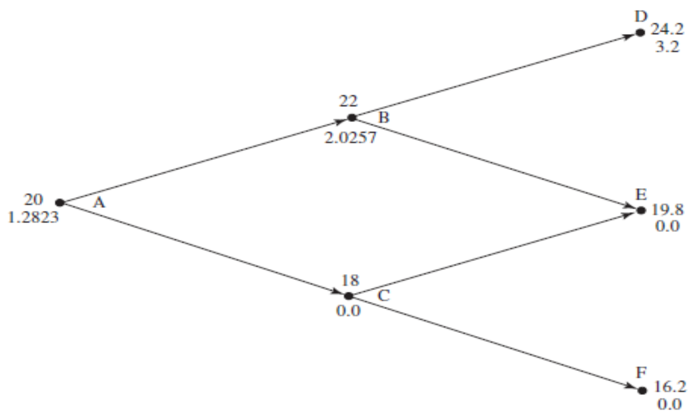
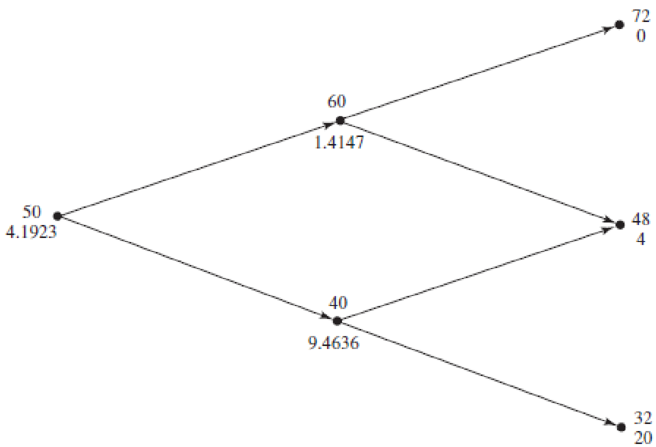


Figure 13.7 Using a two-step tree to value a European put option. At each node, the upper number is the stock price and the lower number is the option price.



Delta Hedging: Calculating Delta, Gamma, and Vega

- Delta (Δ): It is defined as the *rate of change of the option price with respect to the price of the underlying asset*. It is the slope of the curve that relates the option price to the underlying asset price. In other words, *Delta describes how sensitive the option value is to changes in the underlying stock price*. It is defined as the rate of change of the option price with respect to the price of the underlying asset:

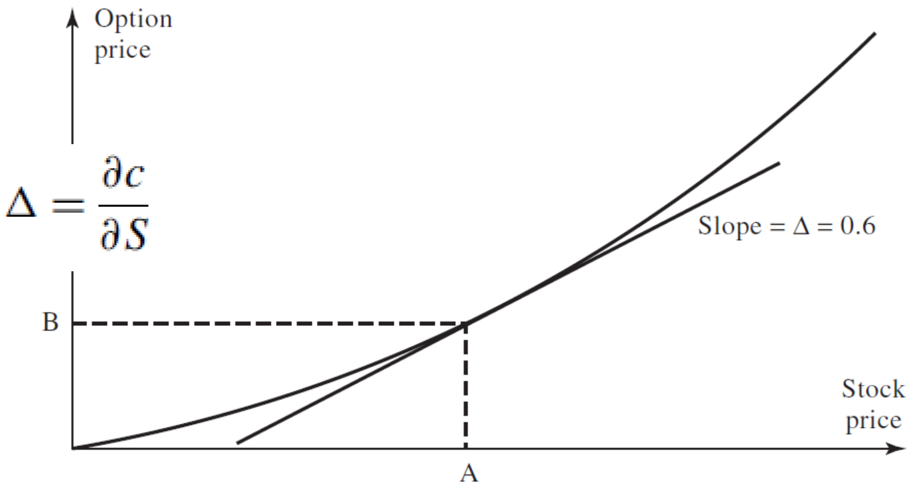
$$\Delta = \frac{\partial c}{\partial S}$$

- where c is the price of the call option and S is the stock price
- Example: Suppose that the delta of a call option on a stock is 0.6. This means that when the stock price changes by a small amount, the option price changes by about 60% of that amount;

Or

- Suppose that the delta on a call is 0.8944. It means two things: First, it means that if the stock price rises by $1, then the call price rises by $0.8944. Second, it implies a hedge ratio between stock and stock options that will leave a portfolio consisting of stock and options ***invariant*** to (small) changes in the underlying stock price. For example, a delta of 0.8944 means that one short call option for every 0.8944 shares of stock will hedge a portfolio consisting of this pair against small changes in the underlying stock price.

Figure below shows the relationship between a call price and the underlying stock price. When the stock price corresponds to point A, the option price corresponds to point B, and Δ is the slope of the line indicated:



Suppose that, in Figure, the stock price is $100 and the option price is $10. Imagine an investor who has sold 20 call option contracts, that is, options to buy 2000 shares and the Delta is given as 0.6.

Hedging the Position: The investor’s position could be hedged by buying 0.6*2,000=1,200 shares. The gain (loss) on the stock position would then tend to offset the loss (gain) on the option position

Gain/Loss: If the stock price goes up by $1 (producing a gain of $1,200 on the shares purchased), the option price will tend to go up by 0.6*$1=$0.60 (producing a loss of $1,200 on the options written); if the stock price goes down by $1 (producing a loss of $1,200 on the shares purchased), the option price will tend to go down by $0.60 (producing a gain of $1,200 on the options written)

- In this example, the delta of the investor's short option position is:

0.6*(-2,000)=-1,200

- This means that the trader loses 1,200ΔS on the short option position when the stock price increases by ΔS. The delta of one share of the stock is by definition 1.0, so that the long position in 1,200 shares has a delta of +1,200. The delta of the trader’s overall position is, therefore, zero. The delta of the stock position offsets the delta of the option position. A position with a delta of zero is referred to as delta neutral.

- **Important:** The delta of an option does not remain constant, the trader's position remains delta hedged for only a relatively short period of time. The hedge has to be adjusted periodically. This is known as *rebalancing*
- For example, by the end of 1 day the stock price might have increased to $110. As indicated in Figure, an increase in the stock price leads to an increase in delta. Suppose that delta rises from 0.60 to 0.65. An extra 0.05*2,000=100 shares would then have to be purchased to maintain the hedge.
- A procedure such as this, where the hedge is adjusted on a regular basis, is referred to as ***dynamic hedging***. It can be contrasted with static hedging, where a hedge is set up initially and never adjusted. Static hedging is sometimes also referred to as ‘‘hedge-and-forget.’’

Behavior of Delta:

The delta of a call option depends upon the moneyness of the option:

- For deep-in-the-money calls, the call value moves one for one with the stock price implying that delta equals one. For deep out-of-the-money calls, the call value hardly changes at all as the stock price moves implying a delta of zero. For call options that are near the money, the delta value is approximately one-half:

Moneyness of the Call Option	**Value of Delta**
Deep out-of-the-money	Close to Zero (0)
Deep in-the-money	One (1)
At-the-money	Close to 0.5

Behavior of Delta:

The delta of a put option depends upon the moneyness of the option:

- For deep-in-the-money puts, the put price increases one dollar for each one dollar decline in the stock price implying a delta of minus one. For a deep out-of-the-money put, the put price does not change at all when the stock price changes implying a delta of zero. And for near-the-money puts, the delta value is approximately minus one-half.

Moneyness of the Put Option	Value of Delta
Deep out-of-the-money	Close to Zero
Deep in-the-money	-1
At-the-money	Close to 0.5

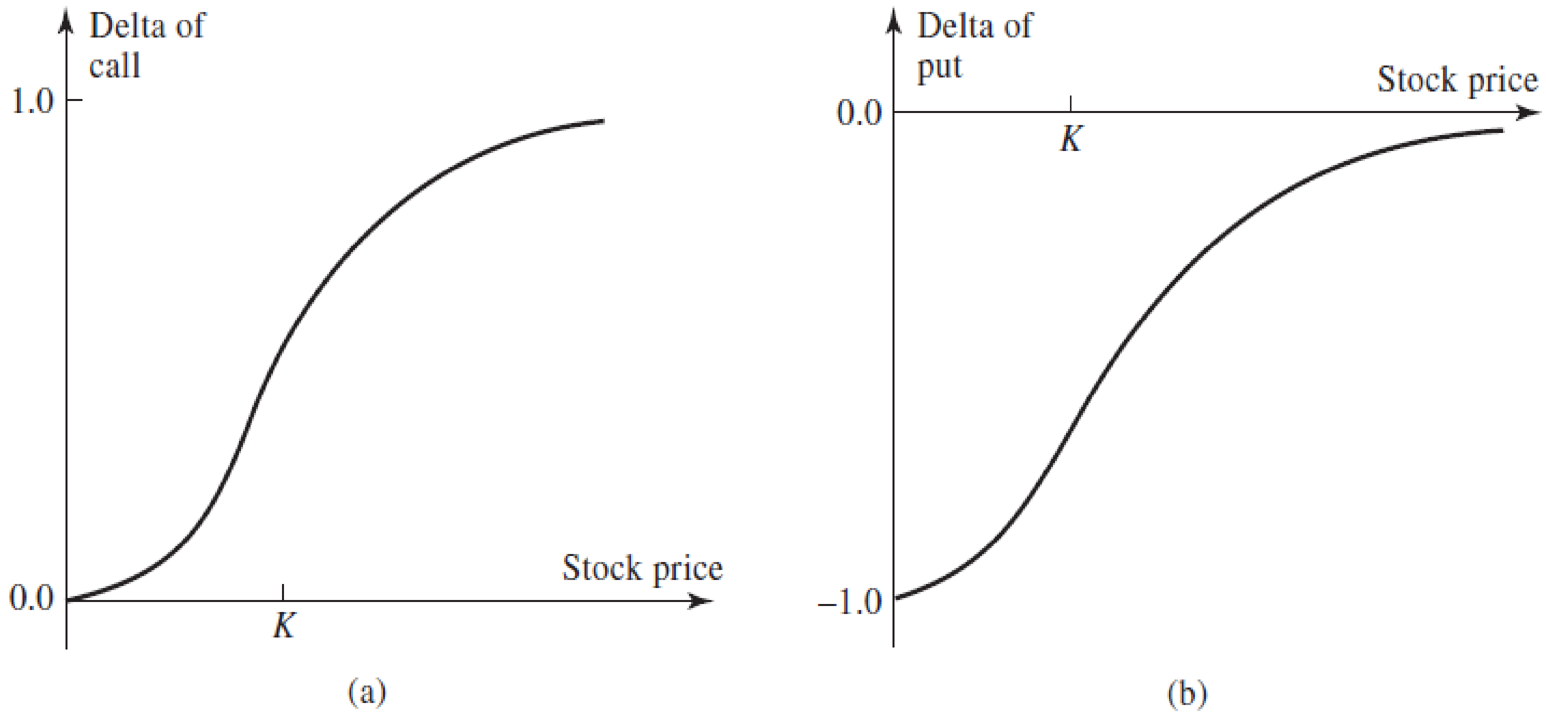
Summary:

- Call: Delta takes on values between 0 and 1.
- Put: Delta takes on values between 0 and −1.

Behavior of Delta:

- Call: Delta takes on values between 0 and 1.
- Put: Delta takes on values between 0 and −1.

Variation of delta with stock price for (a) a call option and (b) a put option on a non-dividend-paying stock:



Delta of European Stock Options:

European call option: For a European call option on a non-dividend-paying stock the delta for a long position in one call option is given by:

$$\Delta(\text{call}) = N(d_1)$$

- where d_1 is defined as per Black-Scholes-Merton formula and N(x) is the cumulative distribution function for a standard normal distribution
- The delta of a short position in one call option is given by **$-N(d_1)$.**

Long Position in option: Using delta hedging for a long position in a European call option involves maintaining a short position of **$N(d_1)$** shares for each option purchased

Short Potion in Option: Using delta hedging for a short position in a European call option involves maintaining a long position of **$N(d_1)$** for each option sold.

European put option: For a European put option on a non-dividend-paying stock, delta is given by:

$$\Delta(\text{put}) = N(d_1) - 1$$

- Delta is negative, which means that a long position in a put option should be hedged with a long position in the underlying stock, and a short position in a put option should be hedged with a short position in the underlying stock.

Example:

- Consider a call option on a non-dividend-paying stock where the stock price is \$49, the strike price is \$50, the risk-free rate is 5%, the time to maturity is 20 weeks (=0.3846 years), and the volatility is 20%. Calculate the options delta.
- Solution:

$$d_1 = \frac{\ln(49/50) + (0.05 + 0.2^2/2) \times 0.3846}{0.2 \times \sqrt{0.3846}} = 0.0542$$

- Delta $\mathbf{N(d_1)}$ **=** 0.522. When the stock price changes by ΔS, the option price changes by $0.522\Delta S$.

2. Theta (Q): The term used to measure the *time decay* of an option, or a portfolio that includes options. Theta measures the rate of change of the value of the portfolio with respect to the passage of time with all else remaining the same. Option values decline at an accelerating rate as they approach expiration. Theta provides an exact analytical tool for estimating this effect on option value;

Theta is almost always negative meaning that as the option's expiration draws near, the option price falls. That is as time passes with all else remaining the same, the option tends to become less valuable. This is true for both puts and calls;

If time is measured in years, and value in dollars, then a theta value of −10 means that as time to option expiration declines by 0.1 years, option value falls by $1.

- For a European call option on a non-dividend-paying stock theta is given by:

$$\Theta(\text{call}) = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2)$$

where d_1 and d_2 are defined as in Black-Scholes-Merton formula and

$$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

is the probability density function for a standard normal distribution

- For a European put option on a non-dividend-paying stock theta is given by:

$$\Theta(\text{put}) = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} + rKe^{-rT}N(-d_2)$$

- In these formulas, time is measured in years. Usually, when theta is quoted, time is measured in days, so that theta is the change in the portfolio value when 1 day passes with all else remaining the same. We can measure theta either ‘‘per calendar day’’ or ‘‘per trading day’’. To obtain the theta per calendar day, the formula for theta must be divided by 365; to obtain theta per trading day, it must be divided by 252.

Example:

- Consider a call option on a non-dividend-paying stock where the stock price is \$49, the strike price is \$50, the risk-free rate is 5%, the time to maturity is 20 weeks (=0.3846 years), and the volatility is 20%. Calculate the option's Theta.
- Solution:

The option's theta is

$$-\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2) = -4.31$$

The theta is $-4.31/365 = -0.0118$ per calendar day, or $-4.31/252 = -0.0171$ per trading day.

3. Gamma (Γ): *Gamma*, measures the changes to delta resulting from changes in the stock price (i.e., the sensitivity of delta to changes in the stock price). The gamma of an option tells the extent to which the value of the delta would change;

Unlike the other sensitivity measures, gamma does not measure the sensitivity of an option to one of the underlying variables. In terms of calculus, gamma is the first derivative of delta with respect to the stock price, or equivalently, it is the second derivative, evaluated for particular values, of the OPM with respect to the stock price;

Gamma can be either positive or negative depending on whether delta is increasing or decreasing for given changes in the underlying stock price;

For a European call or put option on a non-dividend-paying stock, the gamma is given by:

$$\Gamma = \frac{N'(d_1)}{S_0 \sigma \sqrt{T}}$$

Example:

- Consider a call option on a non-dividend-paying stock where the stock price is \$49, the strike price is \$50, the risk-free rate is 5%, the time to maturity is 20 weeks (=0.3846 years), and the volatility is 20%. Calculate the options Gamma.
- Solution:

The option's gamma is

$$\frac{N'(d_1)}{S_0\sigma\sqrt{T}} = 0.066$$

When the stock price changes by ΔS, the delta of the option changes by $0.066\Delta S$.

4. Vega (ν): *Vega*, sometimes called *kappa*, measures the change in an option's value due to changes in volatility. Vega is the first derivative of an option's price with respect to the volatility of the underlying stock. In practice, volatilities change over time. This means that the value of a derivative is liable to change because of movements in volatility as well as because of changes in the asset price and the passage of time.

If vega is highly positive or highly negative, the portfolio's value is very sensitive to small changes in volatility. If it is close to zero, volatility changes have relatively little impact on the value of the portfolio.

For a European call or put option on a non-dividend-paying stock, vega is given by:

$$\mathcal{V} = S_0\sqrt{T}\,N'(d_1)$$

Example:

- Consider a call option on a non-dividend-paying stock where the stock price is \$49, the strike price is \$50, the risk-free rate is 5%, the time to maturity is 20 weeks (=0.3846 years), and the volatility is 20%. Calculate the options Vega.
- Solution:

The option's vega is

$$S_0\sqrt{T}N'(d_1) = 12.1$$

Thus a 1% (0.01) increase in the volatility from (20% to 21%) increases the value of the option by approximately $0.01 \times 12.1 = 0.121$.

5. Rho (ρ): *Rho* measures the sensitivity of an option's value to changes in the interest rates. It is the first derivative of an option's price with respect to the interest rate. Rho for calls is always positive, whereas rho for puts is always negative. A rho of 25 means that a 1 percent increase in the interest rate would increase the value of a call by $.25 and decrease the value of a put by $.25. In general, option prices are not very sensitive to changes in interest rates.

For a European call option on a non-dividend-paying stock:

$$\text{rho (call)} = KTe^{-rT}N(d_2)$$

For a European put option on a non-dividend-paying stock:

$$\text{rho (put)} = -KTe^{-rT}N(-d_2)$$

Example:

- Consider a call option on a non-dividend-paying stock where the stock price is $49, the strike price is $50, the risk-free rate is 5%, the time to maturity is 20 weeks (=0.3846 years), and the volatility is 20%. Calculate the options Rho.
- Solution:

The option's rho is

$$KTe^{-rT}N(d_2) = 8.91$$

This means that a 1% (0.01) increase in the risk-free rate (from 5% to 6%) increases the value of the option by approximately $0.01 \times 8.91 = 0.0891$.