consumed, marginal utility will decrease,
∴ Eventually $\frac{MU_X(X,Y)}{P_X}$ will become equal to $\frac{MU_Y(X,Y)}{P_Y}$.

If there are n no. of goods.

Utility : $U = U(X_1, X_2, X_3, \ldots, X_n)$.

$M = P_{X_1} X_1 + P_{X_2} X_2 + \ldots + P_{X_n} X_n$.

Equal marginal principle :

$$\lambda = \frac{MU_{X_1}(X_1, \ldots X_n)}{P_{X_1}} = \frac{MU_{X_2}(X_1, X_2, \ldots, X_n)}{P_{X_2}} = \ldots\ldots = \frac{MU_{X_n}(X_1, X_2, \ldots, X_n)}{P_{X_n}}$$

$$\frac{MU_X(X,Y)}{MU_Y(X,Y)} = \frac{P_X}{P_Y}$$

Duality in Consumer Theory.

Rather than choosing the highest indifference curve, given a budget constraint, the consumer chooses the lowest budget line that touches a given indifference curve.

Minimizing the cost of ~~each~~ achieving a particular level of utility:

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Minimize $P_X X + P_Y Y$ subject to the constraint that

$$U(X, Y) = U^*$$

The corresponding Lagrangian is given by

$$L = P_X X + P_Y Y - \mu(U(X, Y) - U^*)$$

Differentiating wrt X, Y and μ & setting the derivatives ~~to~~ equal to zero, we find the following necessary conditions for expenditure minimization

$$P_X - \mu MU_X(X, Y) = 0$$
$$P_Y - \mu MU_Y(X, Y) = 0.$$
$$U(X, Y) = U^*$$

Direct & Indirect Utility function.

Direct utility func^n $u = u(x_1, x_2)$

At optimum : $x_i = f(M, P_1, P_2)$.

$$\boxed{U = u\left[x_1(M, P_1, P_2),\ x_2(M, P_1, P_2)\right] = u\left[M, P_1, P_2\right]}$$

$\therefore$ ~~Utd~~ Ultimately utility func^n is a func^n of income & prices! ~~beca~~

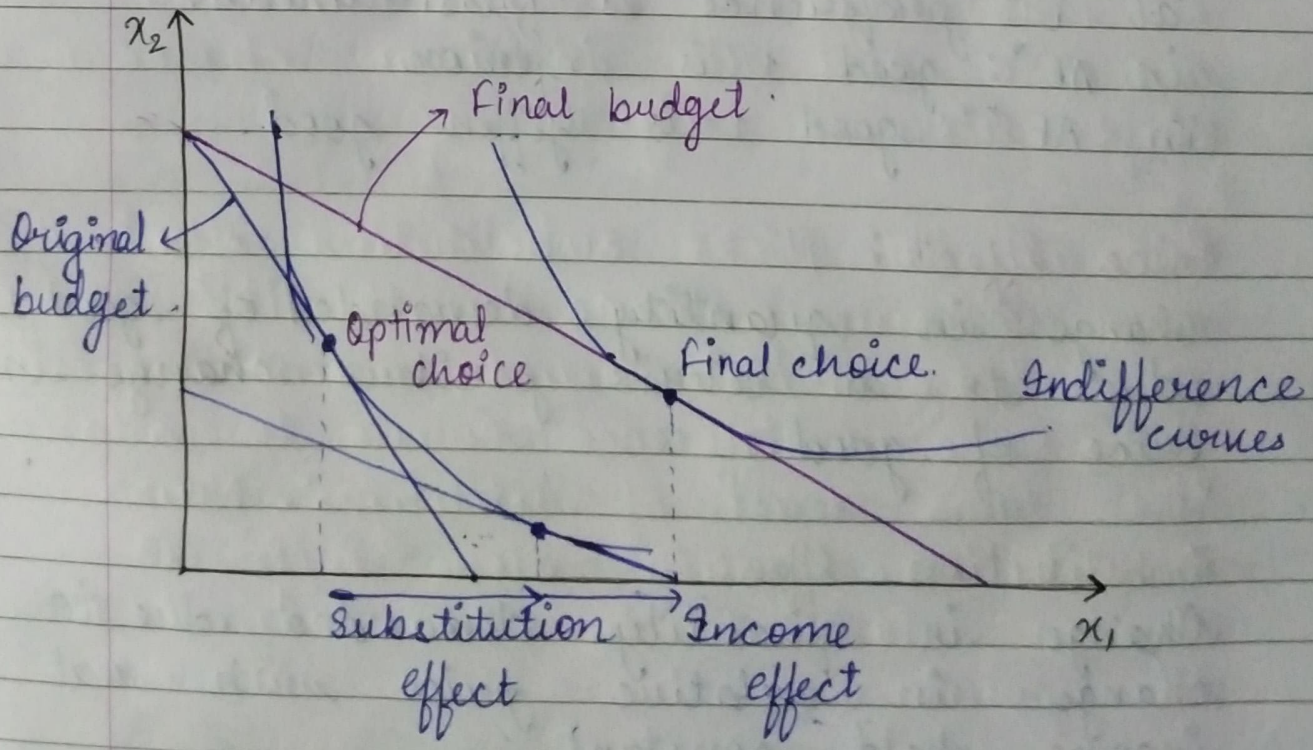
~~U~~ Utility is a function of quantity of consumption which is itself a func^n of income & prices $\therefore$ utility is

indirectly a func^n of income & prices. This is called the Indirect utility function.

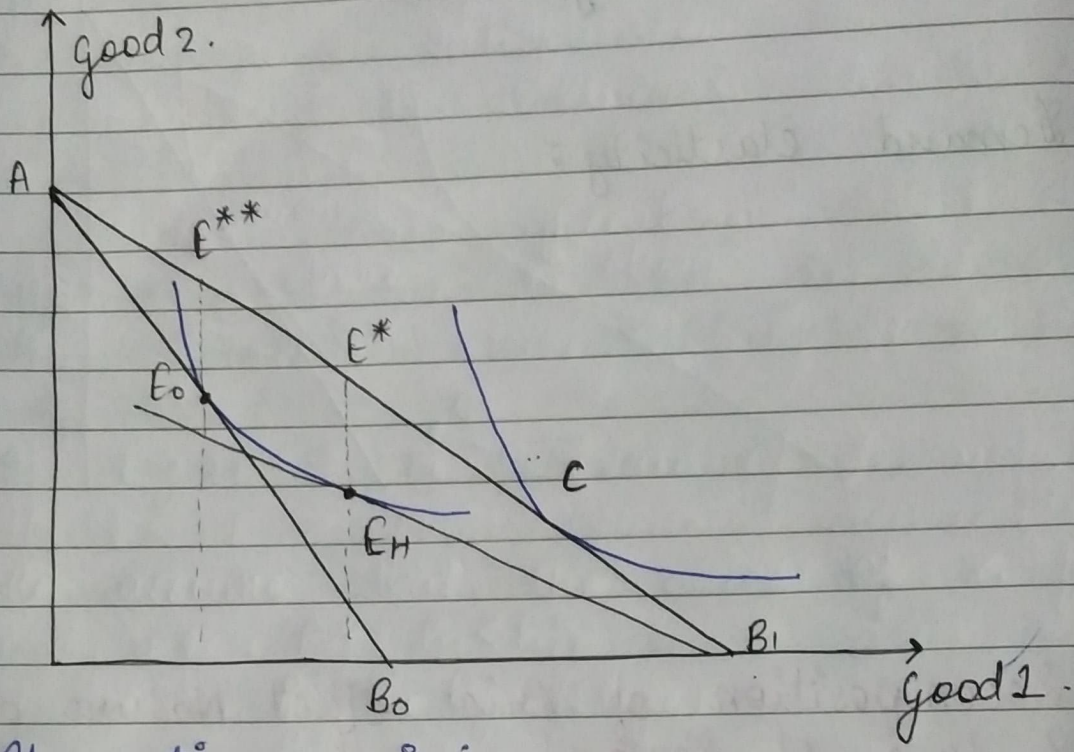
* Income elasticity:

Demand elasticity:

Decomposition of Price Effect, Nature of Good & Law of Demand



∴ When price of x falls, the purchasing power of the consumer will increase as budget set will expand.



If optimum is:
at E^* preference is Quasi-linear.
in AE^*: good 1 is inferior
in AE^{**}: good 1 is Giffen good.

Price Effect:
Change in quantity demanded of good 1 due to a ceteris paribus change in price of good 1.

Substitution Effect:
Change in quantity demanded due to change in relative price with real income held constant.

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Relative price for good 1 = $\frac{P_1}{P_2}$

∴ when P_1 falls, relative price for good 1 falls ∴ relative quantity demanded of good 1 will increase i.e. $\frac{x_1}{x_2}$ will rise.

Income effect:
Change in quantity demanded ~~or~~ when only real income changes.

when $\frac{M}{P_1}$ i.e. real income rises due to fall in P_1, purchasing power of the consumer rises wrt good 1. Thus if real income rises, quantity demanded of that good increases.

To find out Substitution effect:

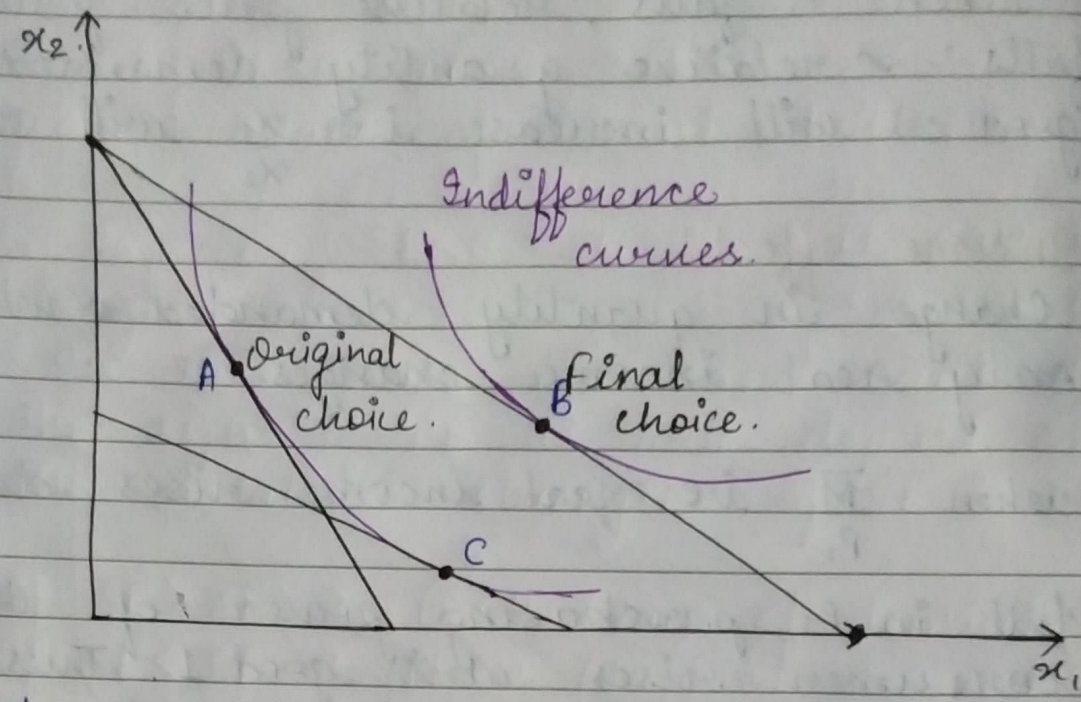
Have to hypothetically keep real income constant and allow for only a change in relative price.

Two approaches to keep real income constant:

Hicksian definition: enabling the buyer to attain the same ~~price~~ ~~at~~ pre-price change utility level.

Slutsky definition: When the buyer is

able to purchase the earlier MPB.



We assume that the price of x_1 falls
i.e. $P_1' < P_1^0$.

$\therefore \frac{P_1'}{P_2^0} < \frac{P_1^0}{P_2^0}$ (Slope)

i.e. the slope of budget line will decrease and the optimum choice will shift towards right.

But a/c to Hicks substitution effect, the new optimum choice with the new slope i.e. P_1'/P_2^0 should again lie on the same indifference curve to attain the same level of utility. ∴ New budget line with slope P_1'/P_2^0 is drawn tangent to the original IC at point C.

∴ As relative price of good 1 has fallen, relative price of good 1 has increased.

Price Effect (PE) / Total Effect (TE).
= Substitution Effect + Income Effect

SE is always < 0.
Because when relative price of a good falls, its quantity demanded increases and vice-versa ∴ they are inversely related.

~~ex~~ except for perfect complements but IE depends on the nature of the good.

If Good 1 is normal, law of demand holds as both SE & IE are -ve implying PE < 0.

If good ~~is~~ 1 is inferior SE & IE move in opposite direction.

PE > = < 0

↓ ↓ → Law of demand holds.

Giffen good — Vertical demand curve

Normal good ⇒ Quantity demanded increases when income increases.

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Inferior good: Quantity demanded decreases when income increases.

Normal good: { SE < 0 IE < 0 ⇒ PE < 0
→ Law of demand holds.

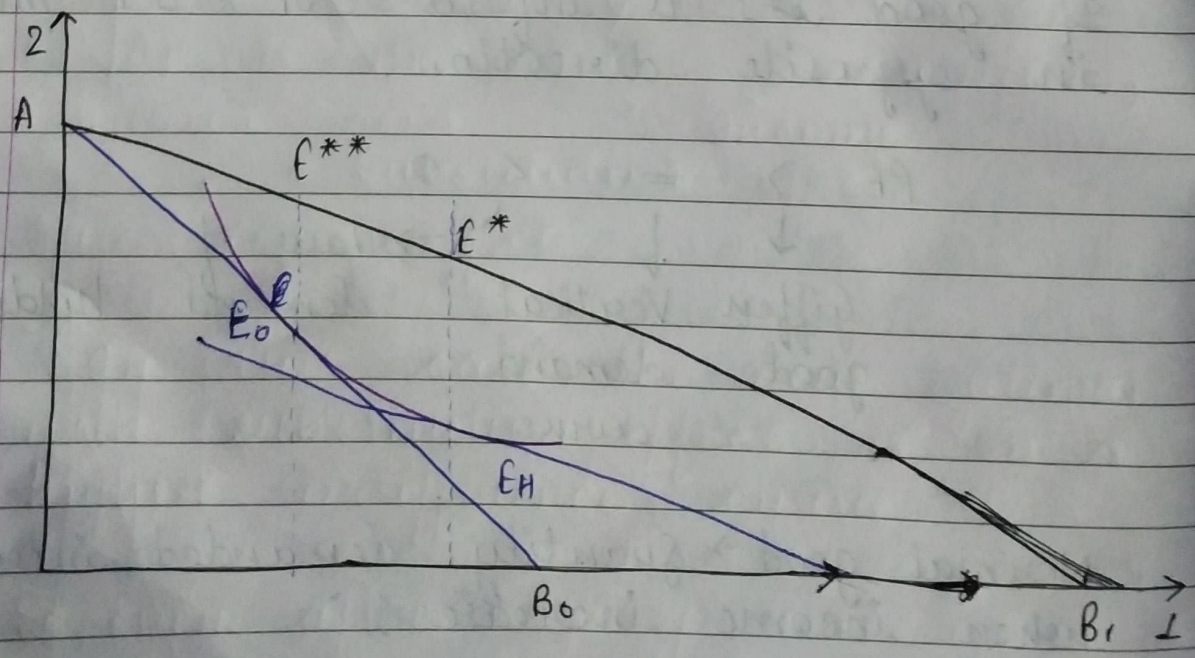
~~Gf~~ Inferior good { SE < 0 IE > 0 ⇒ PE can be +ve, -ve, 0.

∴ In this case, we ~~cant~~ cant decide the sign of PE, it will depend on the relative strengths of SE & IE.

If SE > IE ⇒ PE < 0 (Law of demand holds)
If SE < IE ⇒ PE > 0 (Law of demand is violated)

Only for Giffen inferior good, law of demand is violated.

If SE = IE ⇒ PE = 0 (Vertical demand curve).



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New optimum : E^*B, then good is normal as both SE & IE are -ve.

In the range E^*, E^{**}, the quantity of consumption of good 1 is increasing but not as much as the substitution effect ∴ it is the case of Non-giffen inferior good. Because as price of good 1 has fallen, consumption of good 1 has increased slightly, ∴ law of demand still holds, ∴ it is the case of Non-giffen inferior good.

But if optimum is in the range AE^{**}, quantity demanded of x_1 has fallen when price has fallen ∴ It is Giffen good.

If the optimum is at E^{**}, then it is the case of vertical demand curve.

If the optimum is at E^*, then it is entirely substitution effect & no income effect, ∴ it is the case of quasilinear preference. i.e. as income increases, expenditure on composite good increases.

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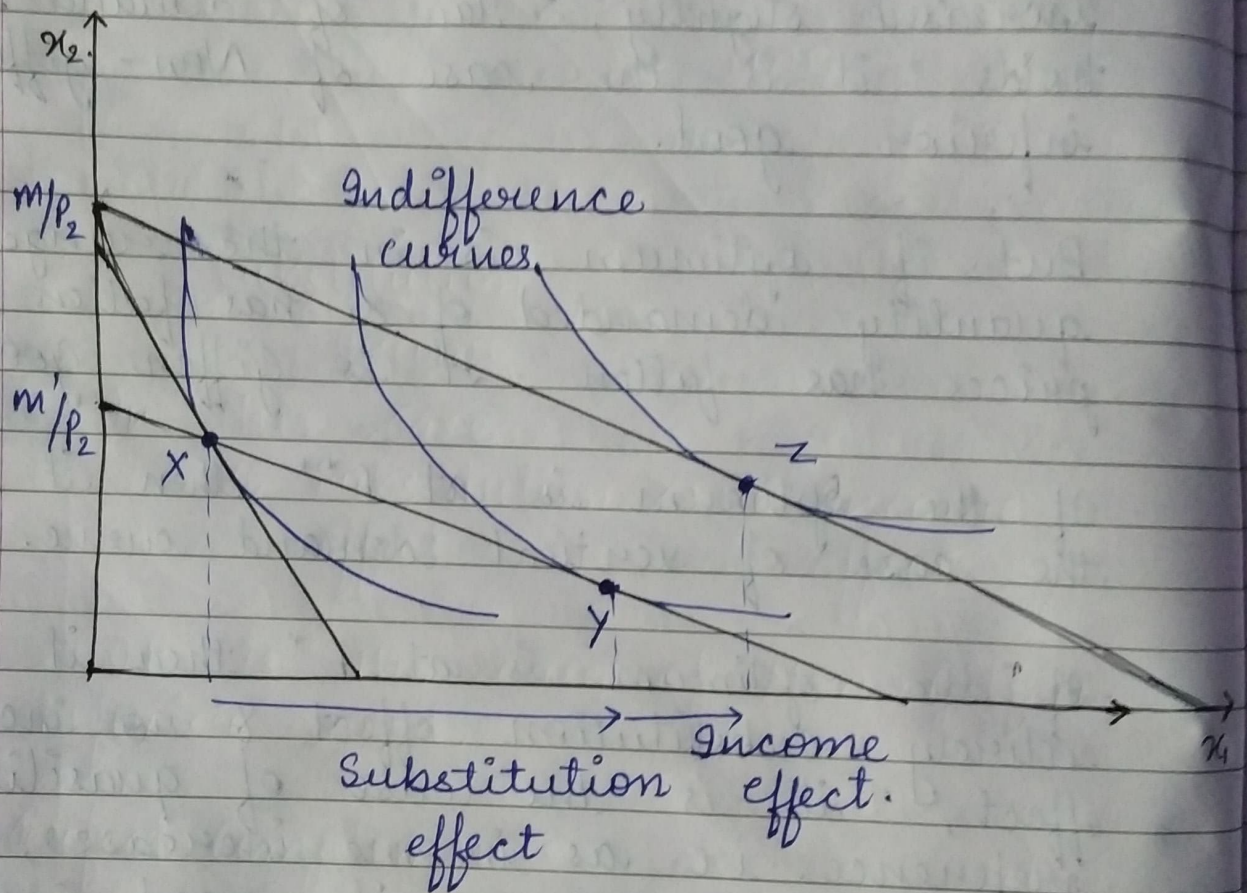
Slutsky definition:
When the buyer is able to purchase the earlier MPB.

∴ the new budget line with slope $\frac{P_1'}{P_2^0}$ is passing through the initial optimal bundel.

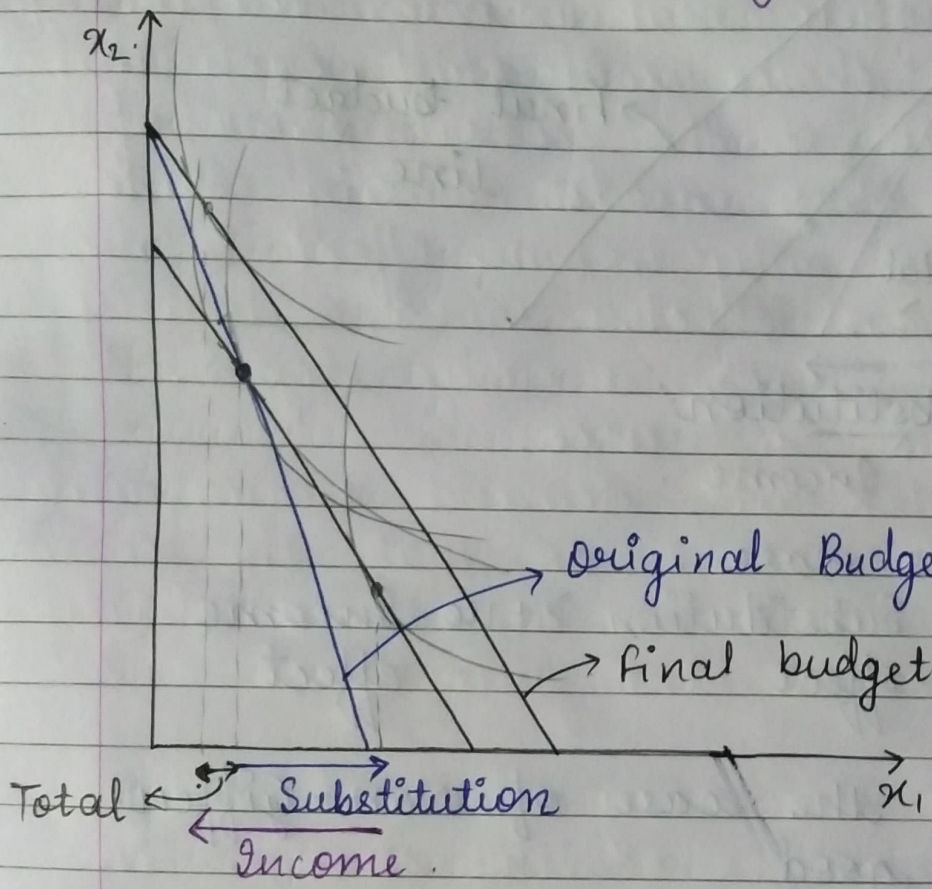


∴ In this case, the new optimal with substitution effect is on a higher indifference curve.

So, even after keeping real income constant, on decreasing the price of

one good, the utility is increasing.

Consumers are over-compensated in case of Slutsky substitution effect because even after keeping real income constant, utility corresponding to MPB is increasing.



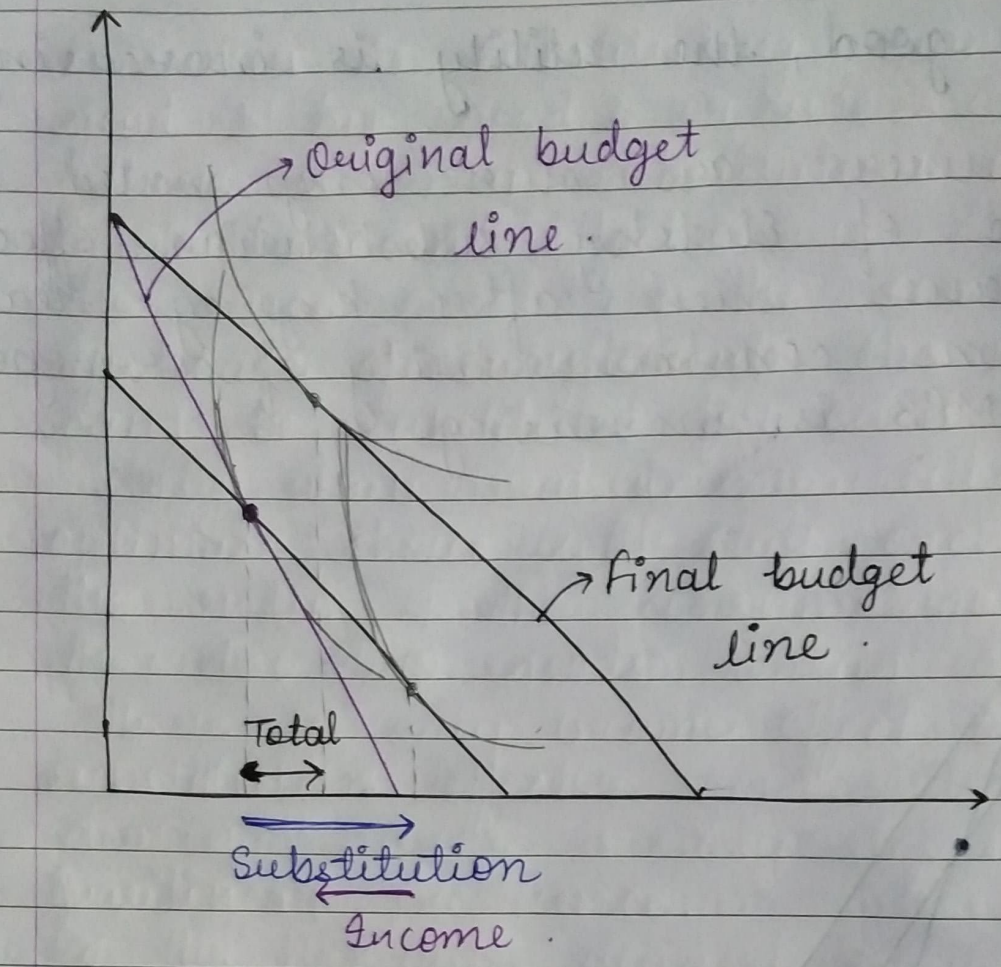
Giffen case.

In this case

+ve Income effect > -ve Substitution effect

∴ Total effect → +ve

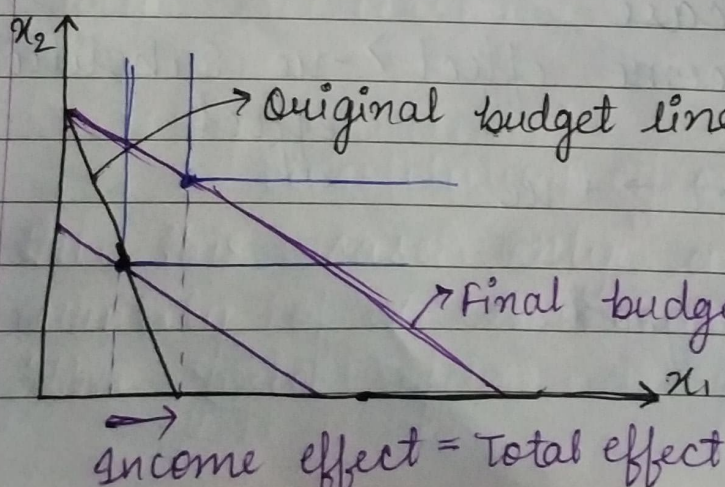
∴ Giffen Good.



In this case
-ve Substitution effect > +ve Income effect

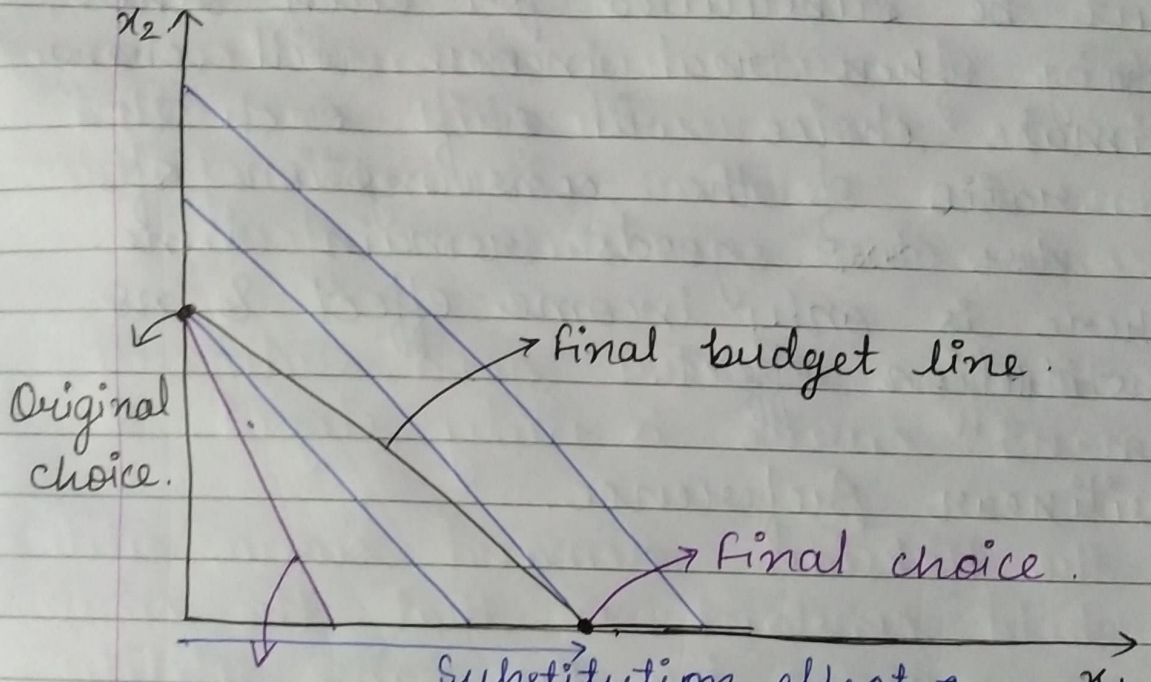
∴ It is the case of Non-Giffen inferior good.

Perfect Complements



In this case, there is no substitution effect as one good cant be substituted for other.
e.g. right shoe & left shoe.

Perfect ~~complements~~ substitutes



If initially ~~$P_1^0 < P_2^0$~~ $\therefore x_1 = 0$.
$P_1^0 > P_2^0$

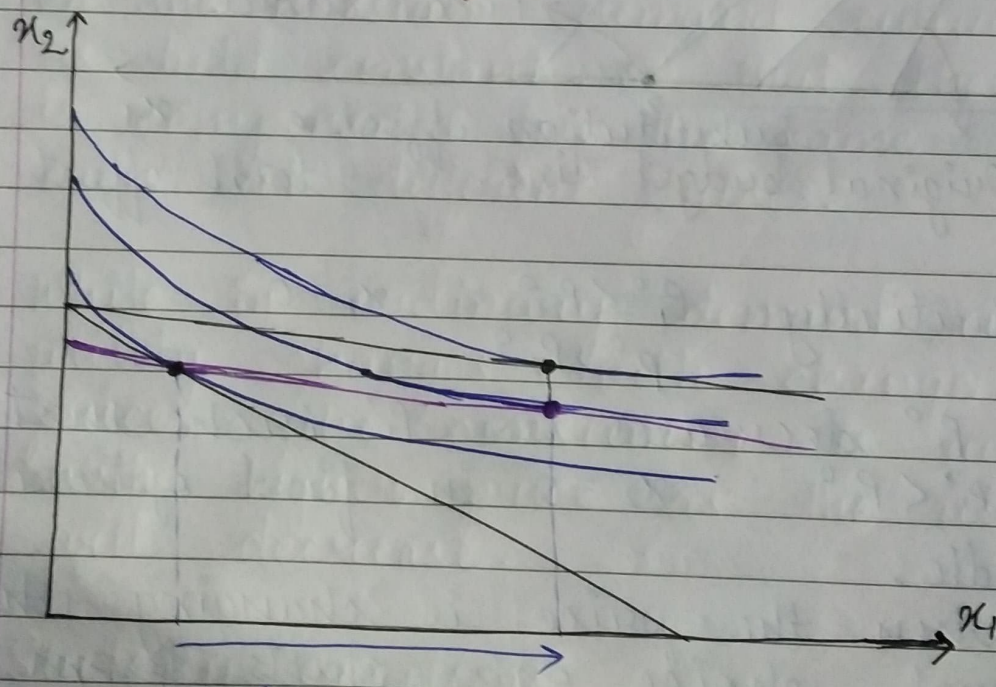
If P_1^0 decreases to P_1' such that $P_1' \leq P_2^0$ $\therefore x_2 = 0$ in final optimal bundle.

$\therefore$ When the price is changing, the optimal choice only shifts from vertical axis to horizontal axis.
$\therefore$ The total effect is due to the substitution effect & income effect = 0.
i.e. when price is changing, the

consumer is substituting completely good 1 by good 2 or vice-versa.
∴ There is only substitution effect & no income effect.

In case of complements, the goods cant be substituted for each other, ∴ only when real income will change optimal choice will shift such that the ratio of the consumption of the ~~two~~ two goods remain same.
∴ There is only income effect & no substitution effect.

Quasilinear Preference.



Substitution effect = total effect
In this case only a very small quantity of x_1 is needed, so even when the price of x_1 decreases, the entire

real income goes in the consumption of x_2. ∴ ~~Sub~~ there is no income effect & only substitution effect.

Comparison between Hicksian & Slutsky SE.

By Hicksian definition Slutsky buyer is over compensated in the sense that even if his real income remains the same, he can attain higher IC, & is thus better off.

If good is normal, Hicksian SE < Slutsky SE.

Compensated demand curve (Hicksian demand curve).

It is the relationship b/w own price and quantity demanded of a good when "real" income remains constant, i.e., we can measure SE along the compensated demand curve.

As the compensated demand curve measures substitution effect, it is -vely sloped for all types of goods.

And we generally consider the Hicksian substitution effect.

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Ordinary demand curve is called the Marshallian demand curve.

Marshallian demand curve captures the price effect i.e. the combined effect of substitution & income effect.

∴ Slope of ordinary demand curve can be +ve, -ve or zero depending upon the relative strength of the income and substitution effect.

For a Giffen inferior good, ordinary demand curve is +vely sloped.

On the Hicksian demand curve, utility is held constant, so that the consumer is compensated for the same utility even at different prices.

While in ordinary demand curve, utility changes with change in price, ∴ the consumer is worse off with higher price as the utility will decrease.

Slutsky equation

Relation b/w P_1 & x_2.
When P_1 is changing, due to the price effect, demand for x_2 is also

changing.

$$\frac{\partial x_j}{\partial p_i} = \frac{\partial x_j}{\partial p_i}\bigg|_{\bar{U}} - x_i \cdot \frac{\partial x_j}{\partial y}$$

↓ Cross effect / net-effect / Hicks-Allen effect.

↓ Income effect.

$\frac{\partial x_j}{\partial p_i}\big|_{\bar{U}}$ → Change in consumption of x_j with change in p_i with utility held constant ∴ it is the Hicksian substitution effect.

$\frac{\partial x_j}{\partial y}$ → Change in consumption of x_j with change in real income.

$y = \frac{M}{p_i}$ (Real income).