Q1 $f = 3x^3 + 2y^3 + x^2 - xy$

$g_1 = x^2 + 2y - 10$

$g_2 = x^3 + y^3 - 5$, $h = x + 2y - 3$

Q(i) Convexity : $\nabla^2 f(1,1) = \begin{pmatrix} 20 & -1 \\ -1 & 18 \end{pmatrix} \succ 0$

$\nabla^2 g_2(1,1) = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix} \succ 0$

$\nabla^2 g_1(1,1) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \succeq 0.$

h is linear, hence cx.

(2M)

(ii) Not regular

(iii) Feasible

(1M)

(iv) Normal : $2\lambda_1 + 3\lambda_2 + \mu = -7$

$2\lambda_1 + 3\lambda_2 + \mu = -5$ (*)

(1M)

(v) CS condn $\Rightarrow \lambda_1 = 0, \lambda_2 = 0$

(vi) Dual restriction does not hold as μ does not exist, (*) is inconsistant

(1M)

Q.2 $\sigma^2(\alpha) = (1-\alpha)^2\sigma_1^2 + \alpha^2\sigma_2^2 + 2\rho(\alpha)(1-\alpha)\sigma_1\sigma_2$

$0 \leq \alpha \leq 1, \quad -1 < \rho < 1$.

$$\frac{d\sigma^2(\alpha)}{d(\alpha)} = 0 \Rightarrow \alpha^* = \frac{\sigma_1^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$$ derive — $1\frac{1}{2}$

$$\frac{d\sigma^2(\alpha)}{d\alpha^2} > 0$$ prove using $-1 < \rho < 1$ — $1\frac{1}{2}$

$$\sigma^2_{min}(\alpha^*) = \text{Derive} = \frac{\sigma_1^2\sigma_2^2(1-\rho^2)}{\sigma_1^2 + \sigma_2^2 + 2\rho(\sigma_1\sigma_2)}$$ — 1

1M
$\mu(\alpha^*) = (1-\alpha^*)\mu_1 + \alpha^*(\mu_2)$.

Min Variance point $= (\sigma(\alpha^*), \mu(\alpha^*))$

Q.3

De

Q.3 • Derivation of all KKT Condns — (1)

• Derive $w^* = \frac{-\Omega^{-1}}{2}(\alpha e + \beta \mu)$ — (1)

• Solve $e^T w = 1, \mu^T w = 4$, derive w^* formula — (1)

• Derive $\alpha = 1/5$, $\beta = -1/5$ and $w^* = \begin{pmatrix}0\\0\\1\end{pmatrix}$ — (1)

• $w^* (A_1, A_2, A_3) = (0, 0,$ Rs 10000/-$)$ — (1)

Q5 $\Omega^{-1} = \frac{1}{10^3} \times \frac{1}{21}\begin{pmatrix}8 & -1 & -2\\ -1 & 8 & -5\\ -2 & -5 & 11\end{pmatrix}$ — (1) — (1)

$m - \mu_f e = \begin{pmatrix}-1\\0\\1\end{pmatrix}$, $\Omega^{-1}(m - \mu_f e) = \frac{1}{10^3 \times 21}\begin{pmatrix}-10\\-4\\13\end{pmatrix}$

$e^T \Omega^{-1}(m - \mu_f e) = \frac{1}{10^3 \times 21}(-1)$

(1) — $\mu_d = -20$, $\sigma_d \approx 6.95$ — (1) 9147

$w_d = \begin{pmatrix}10\\4\\-13\end{pmatrix}$ — (2M)

Correct formula
Wrong calculat.
5M
9147105221

Rest 1+1+1+1+1

(a) $\mu = -\frac{23}{6.95}\sigma + 3$ — (1)

(b) $(6.95, -20)$ — (1)

(c) $(16000, 64000, -208000)$ in ₹. — (1)

(d) $\mu = -23\beta + 3$, $\beta_1 = \frac{1}{23}$, $\beta_2 = 0$, $\beta_3 = -\frac{1}{23}$ — (1)

(e) S.R $= -3.31$

Q 4. (P): Min $w^T \Omega w$

s.to $e^T w = 1$

$3 \le \mu^T w \le 4$

$\equiv$ $\min_{w_1, w_2, w_3}$ $0.3 w_1^2 + 0.4 w_2^2 + 0.3 w_3^2 + 0.2 w_1 w_2 + 0.2 w_1 w_3 + 0.4 w_2 w_3$

sto $w_1 + w_2 + w_3 = 1$ .... λ_1

$2w_1 + 3w_3 + 4w_3 \le 4$ .... λ_2

$2w_1 + 3w_2 + 4w_3 \ge 3$ .... λ_3

Lagrange functn:

$L(w, \lambda_1, \lambda_2, \lambda_3) = w^T \Omega w + \lambda_1 (1 - e^T w) + \lambda_2 (4 - \mu^T w) + \lambda_3 (3 - \mu^T w)$

All condns

1 $\begin{cases} 2\Omega w - \lambda_1 e - \lambda_2 \mu + \lambda_3 \mu = 0 & \text{.... } a_1, a_2, a_3 \text{ are artificial variable} \\ e^T w = 1 & \text{.... } a_4 \\ \mu^T w - s_1 = 3 & \text{.... } a_5 \\ \mu^T w + s_2 = 4 & \text{.... No artificial var} \end{cases}$

C S condn, Dual restrictn.

Final LPP

Min $a_1 + a_2 + a_3 + a_4 + a_5$ — 1

sto

1 $\begin{cases} 0.6 w_1 + 0.2 w_2 + 0.2 w_3 - \lambda_1 - 2\lambda_2 + 2\lambda_3 + a_1 = 0 \\ 0.2 w_1 + 0.8 w_2 + 0.4 w_3 - \lambda_1 - 6\lambda_2 + 6\lambda_3 + a_2 = 0 \\ 0.2 w_1 + 0.4 w_2 + 0.6 w_3 - \lambda_1 - 4\lambda_2 + 4\lambda_3 + a_3 = 0 \end{cases}$

1 $\begin{cases} w_1 + w_2 + w_3 + a_4 = 1 \\ 2w_1 + 3w_2 + 4w_3 - s_1 + a_5 = 3 \\ 2w_1 + 3w_2 + 4w_3 + s_2 = 4 \end{cases}$

1 $\begin{cases} \lambda_1 \in \mathbb{R}, \lambda_2 \le 0, \lambda_3 \ge 0 \\ \lambda_2 s_1 = 0, \lambda_3 s_2 = 0 \end{cases}$

1 mark deducted if numerical expression is not provided