Normal forms

- a variable or negation of a variable → literal

  p or $\neg p$

- product of the variables & their negations in a formula → elementary product

  $\neg p \wedge q$, $p \wedge r \wedge \neg s$

- sum of the variables and their negations in a formula → elementary sum.

  $\neg p \vee q$, $q \vee p \vee s$, p

- Sum of elementary products → disjunctive normal form (DNF)

  ~~$p \vee q \vee \neg s$ → disjunction or sum~~

  $(p \wedge q) \vee (r \wedge q) \vee (p \wedge r) \vee (r \wedge r)$

- product of elementary sum. → Conjunctive normal form (CNF).

  $((p \vee q) \wedge (\neg p \vee q \vee r)) \wedge (\neg s \vee q)$.

- Well formed formula of propositional logic / propositional form / formula

  → a string consisting of propositional variables, connectives & parentheses used in the proper manner.

Example ~~Obtain the DNF of $\neg(p \wedge q) \Leftrightarrow (p \vee q)$~~

~~$\neg(p \wedge q) \Leftrightarrow (p \vee q) \Leftrightarrow [\neg(p \wedge q) \rightarrow (p \vee q)] \wedge [\neg(p \wedge q) \rightarrow$~~ ~~$\Leftrightarrow \neg(p \wedge q) \wedge (p \vee q)$~~

Note A NASC for an elementary product to be identically false (a contradiction) is that it contains at least one pair of literals in which one is the negation of the other.

$\cdots \wedge p \wedge \neg p \wedge \cdots \Leftrightarrow \cdots \wedge F \wedge \cdots \Leftrightarrow F$

Example:

Obtain DNF of $\neg(p\wedge q) \Leftrightarrow (p\vee q)$

$\neg(p\wedge q)\Leftrightarrow(p\vee q)$

$\Leftrightarrow [\neg(p\wedge q)\to(p\vee q)]\wedge[\neg\neg(p\wedge q)\to\neg(p\vee q)]$

$\Leftrightarrow [\neg\neg(p\wedge q)\vee(p\vee q)]$

$\wedge [\neg(p\wedge q)\vee\neg(p\vee q)]$

$\Leftrightarrow (p\wedge q)\vee(p$

$r\Leftrightarrow s$

$\Leftrightarrow (r\to s)\wedge(s\to r)$

$\Leftrightarrow (r\to s)\wedge(\neg r\to\neg s)$

$r\to s$

$\Leftrightarrow \neg r\vee s$

$r\Leftrightarrow s$

$\Leftrightarrow (r\wedge s)\vee(\neg r\wedge\neg s)$

Using $r\Leftrightarrow s \Leftrightarrow (r\wedge s)\vee(\neg r\wedge\neg s)$.

$\neg(p\wedge q)\Leftrightarrow(p\vee q)$

$\Leftrightarrow [\neg(p\wedge q)\wedge(p\vee q)]\vee[\neg(\neg(p\wedge q))\wedge\neg(p\vee q)]$

$\Leftrightarrow [(\neg p\vee\neg q)\wedge(p\vee q)]\vee[(p\wedge q)\wedge(\neg p\wedge\neg q)]$

$\cancel{\Leftrightarrow (\neg p\vee p\vee q)}$

$\cancel{\Leftrightarrow \neg p\vee(p\vee q}$

$\Leftrightarrow [(\neg p\vee\neg q)\wedge p]\vee[(\neg p\vee\neg q)\wedge q]$

$\vee [p\wedge q\wedge\neg p\wedge\neg q]$

$\Leftrightarrow (p\wedge\neg p)\vee(p\wedge\neg q)\vee(q\wedge\neg p)\vee(q\wedge\neg q)$

$\vee [p\wedge q\wedge\neg p\wedge\neg q]$.

Note: A NASC for an elementary sum to be identically true (a tautology) is that it contains at least one ~~literal~~ pair of literals in which one is the negation of the other.

$\ldots\vee p\vee\neg p\vee\ldots \Leftrightarrow \ldots\vee T\vee\ldots \Rightarrow T$

- The DNF is not unique

<u>Example:</u> $(p \wedge q) \vee r$ is in DNF.

$$\underbrace{(p \wedge q) \vee r}_{\text{DNF}} \Leftrightarrow \cancel{p \wedge r}\ (r \vee p) \wedge (r \vee q)$$

$$\Leftrightarrow \underbrace{(r \wedge r) \vee (r \wedge q) \vee (p \wedge r) \vee (p \wedge q)}_{\text{DNF}}.$$

<u>Note</u> A ~~DNF~~ given formula is identically false (i.e. a contradiction) if every elementary product in its DNF is identically false.

<u>Example:</u> Bring to conjunctive normal form:

$$\neg(p \wedge q) \leftrightarrow (p \vee q)$$

$r \leftrightarrow s \Leftrightarrow$.

$(r \to s) \wedge (s \to r)$

$\neg r \vee s$

$$\Leftrightarrow [\neg(p \wedge q) \to (p \vee q)] \wedge [\cancel{\neg(p \wedge q)}\ (p \vee q) \to \neg(p \wedge q)]$$

$$\Leftrightarrow [\neg\neg(p \wedge q) \vee (p \vee q)] \wedge [\neg(p \vee q) \vee \neg(p \wedge q)]$$

$$\Leftrightarrow [(p \wedge q) \vee \underline{(p \vee q)}] \wedge [(\neg p \wedge \neg q) \vee \underline{(\neg p \vee \neg q)}]$$

$\cancel{\Leftrightarrow p \vee}$

$$\Leftrightarrow [p \vee (p \vee q)] \wedge [q \vee (p \vee q)]$$

$\cancel{\Leftrightarrow p \wedge}$

$$\wedge [(\neg p \vee (\neg p \vee \neg q)) \wedge (\neg q \vee (\neg p \vee \neg q))]$$

$$\Leftrightarrow (p \vee p \vee q) \wedge (q \vee p \vee q) \wedge (\neg p \vee \neg p \vee \neg q) \wedge (\neg q \vee \neg p \vee \neg q)$$

Note: A given formula is identically true (a tautology) if every elementary sum appearing in its CNF ~~has at least two literals~~ is identically true

~~i.e. has two literals~~

i.e. has at least two literals, of which one is the negation of the other.

~~Potenci~~

Def^n (DNF) A formula which is equivalent to a given formula and consists of sum of elementary products is called a disjunction normal form (DNF).

Def^n (CNF) A formula which is equivalent to a given formula and consists of a product of elementary sums is called a conjunctive normal form (CNF).

Minterms of p, q

$p \wedge q, \quad \neg p \wedge q, \quad p \wedge \neg q, \quad \neg p \wedge \neg q.$

· In general if there are n variables, we will have 2^n minterms.

Minterm :— each minterm is a conjunction in which ~~each~~ each variable occurs once either in the negated form or in the non negated form.

Principal Disjunctive Normal form

- for a given formula, an equivalent formula consisting of disjunctions of minterms only is known as its principal disjunctive normal form.
- Such a normal form is also called Sum-of-products cannonical form.

Two variable

p	q	$p \wedge q$ (m_3) $\to 11 \to$ binary of 3	$p \wedge \neg q$ (m_2) $\to 10 \to$ binary of 2	$\neg p \wedge q$ (m_1) $\to 01 \to$ binary of 1	$\neg p \wedge \neg q$ (m_0) $\to 00 \to$ binary of 0
T	T	T	F	F	F
T	F	F	T	F	F
F	T	F	F	T	F
F	F	F	F	F	T

each minterm $\to$ Corresponding column has exactly one entry T & all the other entries are F.

$p \to q$, $p \leftrightarrow q$

p	q	$p \to q$	$p \leftrightarrow q$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	T

Formula		principal DNF
$p \to q$	$\Leftrightarrow$	$(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$
$p \leftrightarrow q$	$\Leftrightarrow$	$(p \wedge q) \vee (\neg p \vee \neg q)$.

- A formula which is a tautology, will have all minterms
- A contradiction will have no minterm.

- Any formula which is not a contradiction, we can have a principal DNF.

- A formula in which minterms are rearranged is considered as an equivalent formula.

- If the literals in a minterm are rearranged, we get an equivalent minterm.

  (literal is a variable or negation of a variable).

- If two given formulas are equivalent, then both of them must have identical principal DNFs

Finding principal DNF without constructing its truth table

- replace $\rightarrow$ & $\leftrightarrow$ by using $\vee, \wedge$ & $\neg$.
- Use DeMorgan's laws & distributive laws & bring it to DNF.
- An elementary product which is a contradiction is dropped
- Obtain minterms in the disjunctions by introducing the missing factors.
- Avoid duplications

Example: Obtain principal DNF for $p \vee \neg q$.

$$p \vee \neg q = [p \wedge (q \vee \neg q)] \vee [\neg q \wedge (p \vee \neg p)]$$

$$= (p \wedge q) \vee (p \wedge \neg q) \vee (\neg q \wedge p)$$

$$\vee (\neg q \wedge \neg p).$$

$$= \Sigma(0, 2, 3) \quad \text{(Shortened form)}$$

Example: Obtain principal DNF for

$$(p \wedge q) \vee (\neg p \wedge r) \vee (q \wedge r).$$

$$p \wedge q \Leftrightarrow (p \wedge q) \wedge (r \vee \neg r).$$

$$\Leftrightarrow (p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r).$$

$$\neg p \wedge r \Leftrightarrow (\neg p \wedge r) \wedge (q \vee \neg q)$$

$$\Leftrightarrow (\neg p \wedge r \wedge q) \vee (\neg p \wedge r \wedge \neg q).$$

$$q \wedge r \Leftrightarrow (q \wedge r) \wedge (p \vee \neg p)$$

$$\cancel{\Leftrightarrow (q \wedge p)}$$

$$\Leftrightarrow (q \wedge r \wedge p) \vee (q \wedge r \wedge \neg p).$$

Avoiding duplication, the given expression is equivalent to the following principal DNF:

$$\underset{\substack{111 \\ \downarrow \\ 7}}{(p \wedge q \wedge r)} \vee \underset{110 \to 6}{(p \wedge q \wedge \neg r)} \underset{011 \to 3}{(\neg p \wedge r \wedge q)} \underset{001 \to 1}{(\neg p \wedge r \wedge \neg q)}$$

$$= \Sigma(1, 3, 6, 7). \quad \text{(Shortened form).}$$

Two variables p, q (minterms). 2^2 minterms $= 4$

i	binary i	minterm (m_i)
0	00	$\neg p \wedge \neg q$
1	01	$\neg p \wedge q$
2	10	$p \wedge \neg q$
3	11	$\neg p \wedge \neg q$.

Three variables p, q, r (minterm) $2^3 = 8$ minterms.

i	binary i	minterm (m_i)	
0	000	$\neg p \wedge \neg q \wedge \neg r$	→ minterm 0
1	001	$\neg p \wedge \neg q \wedge r$	→ minterm 1
2	010	$\neg p \wedge q \wedge \neg r$	
3	011	$\neg p \wedge q \wedge r$	
4	100	$p \wedge \neg q \wedge \neg r$	
5	101	$p \wedge \neg q \wedge r$	
6	110	$p \wedge q \wedge \neg r$	
7	111	$p \wedge q \wedge r$.	→ minterm 7.

Ordering Criterion

- In the minterm m_i, the variable p_j ~~occurs~~

  occurs as p_j if $b_j = 1$
  and occurs as $\neg p_j$ if $b_j = 0$

Where $b_1 b_2 \cdots b_n$ is the binary representation of i corresponding to the minterm m_i

$i = (b_1 b_2 \cdots b_n)_2$ in n-variables $p_1, p_2, \ldots, p_n$

i.e. 2^n minterms in $p_1 p_2, \ldots, p_n$.

assigned values 0 to $2^n - 1$.

each variable p_j, $1 \le j \le n$, comes either in p_j form or $\neg p_j$ form

- 00..0 corresponds to $\neg p_1 \wedge \neg p_2 \wedge \cdots \wedge \neg p_n$
- 11..1 Corresponds to $p_1 \wedge p_2 \wedge \cdots \wedge p_n$.
- A formula which is disjunction of minterms $m_{i_1}, m_{i_2}, \ldots, m_{i_r}$ is denoted by $\sum(i_1, i_2, \ldots, i_r)$

Principal CNF

- for a given formula, an equivalent formula consisting of conjunction of maxterms only is known as its principal ~~CNF~~ conjunctive normal form.
- This normal form is also called the product of sum canonical form.
- $\boxed{\text{maxterm} \rightarrow \text{dual of minterm.}}$
  - → each ~~max~~ term is a ~~conjunction~~ disjunction in which each variable or its negation, but not both, appears only once.
  - → each maxterm has truth value F for exactly one combination of the truth value of the variables.
- the column corresponding to a maxterm has exactly one entry F & all the other entries are T.

Two variable

p	q	(M_0) 00 $p \vee q$	(M_1) 01 $\neg p \vee q$	(M_2) 10 $p \vee \neg q$	(M_3) 11 $\neg p \vee \neg q$
T	T	T	T	T	F
T	F	T	F	T	T
F	T	T	T	F	T
F	F	F	T	T	T

- Every ~~maxterm~~ formula which is not a tautology has an equivalent principal CNF which is unique except for the rearrangement of the factors in the maxterm as well as rearrangement of the maxterms.
- If two given formulas are equivalent, then both of them must have identical principal CNFs.
- Finding principal CNF without constructing its truth table

→ Similar to the one described earlier for the principal DNF.

→ In each disjunction, missing variable is provided in the negated and non-negated forms.

Example: find principal CNF for $p \leftrightarrow q$.

$$p \leftrightarrow q \iff (p \to q) \wedge (q \to p)$$

$$\iff \underbrace{(\neg p \vee q)}_{1\ 0\ \to\ \text{binary of } 2} \wedge \underbrace{(\neg q \vee p)}_{0\ 1\ \to\ \text{binary of } 1} = \Pi(2,1)$$

Example: find principal CNF for $[(p \vee q) \wedge \neg p] \to \neg q$

$$[(p \vee q) \wedge \neg p] \to \neg q \quad \cancel{\iff (p \vee \neg p) \wedge}$$

$$\iff [(p \wedge \neg p) \vee (q \wedge \neg p)] \to \neg q$$

$$\iff (q \wedge \neg p) \to \neg q$$

$$\iff \neg(q \wedge \neg p) \vee \neg q$$

$$\iff (\neg q \vee p) \vee \neg q \iff \neg q \vee p \vee \neg q \iff p \vee \neg q$$

binary of 1

0 1

$$= \Pi(1)$$

Two variables p, q (maxterm) 2^2 maxterms $= 4$.

i	binary i	maxterm (M_i)
0	00	$p \vee q$
1	01	$p \vee \neg q$
2	10	$\neg p \vee q$
3	11	$\neg p \vee \neg q$.

Three variables p, q, r (maxterms) $2^3 = 8$ maxterms.

i	binary i	Maxterm (M_i)
0	000	$p \vee q \vee r$
1	001	$p \vee q \vee \neg r$
2	010	$p \vee \neg q \vee r$
3	011	$p \vee \neg q \vee \neg r$
4	100	$p \vee \neg q \vee \neg r$
5	101	$\neg p \vee q \vee \neg r$
6	110	$\neg p \vee \neg q \vee r$
7	111	$\neg p \vee \neg q \vee \neg r$.

Ordering criterion

- n variables $p_1, p_2, \ldots, p_n$
- # maxterms 2^n.
- maxterm M_i, $i = 0, 1, \ldots, 2^n - 1$ $\rightarrow$ each variable or its negation should appear in it.
- $(i)_{10} = (b_1 b_2 \ldots b_n)_2$

  if $b_j = 0$, then variable p_j appears in M_i as p_j

  if $b_j = 1$, then variable p_j appears in M_i as $\neg p_j$

- A formula which is the conjunction of maxterms $M_{i_1}, M_{i_2}, \ldots, M_{i_r}$ is denoted by $\prod(i_1, i_2, \ldots, i_r)$

- To show to formulas are equivalent, we can bring them to principal conjunctive normal form (or principal disjunctive normal form).

- If both the formula given rise to the same principal ~~conj~~ CNF (or principal DNF), they are equivalent.

- Ordering of variables & representation of minterms & maxterms by integers in the range 0 to $2^n - 1_s$ is done following different convention for minterms & maxterms.

  - $(i)_{10} = (b_1 b_2 \cdots b_n)$

    p_j present in minterm m_i if $b_j = 1$

    $\neg p_j$ present in minterm m_i if $b_j = 0$.

  - in maxterm, it is the otherway round.

Note. Same formula $\to$ p.dnf $\to \sum i_1, i_2, \ldots, i_r$
$\to$ p.cnf $\to \prod j_1, j_2, \ldots, j_s$

$i_1, i_2, \ldots, i_r \in \{0, \ldots, 2^n - 1\} \setminus \{j_1, j_2, \ldots, j_s\}$
$j_1, j_2, \ldots, j_s \in \{0, \ldots, 2^n - 1\} \setminus \{i_1, i_2, \ldots, i_r\}$.

nos. appearing in $\sum$ will not appear in $\prod$, and nos. appearing in $\prod$ do not appear in $\sum$

Example:

$$(p \to q) \to (q \to p)$$

$$\Leftrightarrow (\neg p \vee q) \to (\neg q \vee p)$$

$$\Leftrightarrow \neg(\neg p \vee q) \vee (\neg q \vee p)$$

$$\Leftrightarrow (p \wedge \neg q) \vee (\neg q \vee p)$$

$$\cancel{\Leftrightarrow (p \wedge \neg q \vee p) \vee \neg q}$$

$$\Leftrightarrow (p \vee \neg q \vee p) \wedge (\neg q \vee \neg q \vee p)$$

$$\Leftrightarrow (p \vee \neg q) \wedge (\neg q \vee p)$$

$$\Leftrightarrow (\underset{0}{p} \vee \underset{1}{\neg q}) = \Pi(1), \text{ principal CNF.}$$

$$\Leftrightarrow (p \wedge \neg q) \vee (\neg q) \vee p$$

$$\Leftrightarrow (p \wedge \neg q) \vee [\neg q \wedge (p \vee \neg p)] \vee [p \wedge (q \vee \neg q)]$$

$$\Leftrightarrow \underset{10}{(p \wedge \neg q)} \vee [(\neg q \wedge p) \vee (\neg q \wedge \neg p)] \vee [(p \wedge q) \vee (p \wedge \neg q)]$$

$$\Leftrightarrow \underset{10}{(p \wedge \neg q)} \vee \underset{00}{(\neg q \wedge \neg p)} \vee \underset{11}{(p \wedge q)} \vee \cancel{(p \wedge \neg q)}$$

$$= \Sigma(2, 0, 3)$$

1st. order logic → quantifies only variables that range over individuals

2nd. order logic → quantifies over sets.

3rd. order → quantifies over sets of sets.

& so on.

Higher order → union of 1st, 2nd, 3rd, .., nth order logic.

→ admits quantification over sets that are nested arbitrarily deeply.

Example: (the principle of mathematical induction)

P is a sets

$\forall P\ \Big(0 \in P \wedge \forall i\ (i \in P \to i+1 \in P)\Big) \to \forall n\ (n \in P$

1st. order logic.

- Every Wff can be converted to DNF / CNF (not unique)
- # of terms in some cases may be exponential.
- Every Wff has unique p-DNF or p-CNF form.

upto rearrangement of literals or minterms.

Normal forms for first order logic (involved quantified statements)

Ex. Socrates is a man (proposition)

1st order logic → There exists X s.t. X is Socrates & X is a man

Def^n (prenex normal form)

A formula F in the first order logic is said to be in a <u>prenex normal form</u> if and only if the formula F is in the form of

$$(Q_1 x_1)(Q_2 x_2)\cdots(Q_n x_n)(M)$$

where every $(Q_i x_i)$, $i=1,2,\dots,n$ is either $(\forall x_i)$ or $(\exists x_i)$, and M is a formula containing no quantifiers. $(Q_1 x_1)\dots(Q_n x_n)$ is called the prefix and M is called the matrix of the formula F.

Example:

$$\left.\begin{array}{l}(\forall x)(\forall y)(P(x,y)\wedge Q(y))\\ \forall x\,\exists y\,\forall z\,(Q(x,y)\rightarrow R(z))\end{array}\right\}\text{ in prenex normal form.}$$

<u>Converting a given formula in first order logic in prenex normal form.</u>

- Two formulas F, G are equivalent, denoted by $F \Leftrightarrow G$, if and only if F and G are the same under every interpretation.

- $$\boxed{\begin{array}{l}\neg\forall x\,P(x) \leftrightarrow \exists x\,\neg P(x)\\ \neg\exists P(x) \leftrightarrow \forall x\,\neg P(x)\end{array}} \quad \text{— (1)}$$

- $\forall x$ distributes over $\wedge$ and $\exists$ over $\vee$

- if F has a variable x and G does not contain x, then

$$\boxed{\begin{array}{l}(Qx)F(x)\vee G \leftrightarrow Q(x)(F(x)\vee G)\\ (Qx)F(x)\wedge G \leftrightarrow Q(x)(F(x)\wedge G).\end{array}} \quad \text{— (2)}$$

- ~~if F_1, F_2 have variables x,~~

$$\cancel{\forall x\, F_1(x) \vee F_2(x) \neq \forall x (F_1(x) \vee F_2(x))}$$

- $\forall x$ does not distribute over $\vee$ and $\exists$ over $\wedge$.
- if F_1, F_2 have variable x,

$$\forall x\, F_1(x) \vee \forall x\, F_2(x) \neq \forall x (F_1(x) \vee F_2(x))$$

- Rename the variable x in $F_2(x)$ by z. Then

$$\forall x\, F_1(x) \vee \forall z\, F_2(z) = \forall x \forall z (F_1(x) \vee F_2(z))$$

as F_1 does not contain z and F_2 does not contain x.

- $\exists x\, F_1(x) \wedge \exists x\, F_2(x)$ can be ~~baught~~ brought to the following form by renaming of variable x as z in F_2.

$$\exists x F_1(x) \wedge \exists x\, F_2(x)$$
$$\exists x\, F_1(x) \wedge \exists z\, F_2(z)$$
$$= \exists x\, F_1(x) \wedge \exists z\, F_2(z)$$
$$= \exists x \exists z (F_1(x) \wedge F_2(z)).$$

- Hence it is possible to bring the quantifiers to the left of the formula. (Converting to prenex normal form)

Algm.
Step 1. Replace $\leftrightarrow$ and $\rightarrow$ using $\wedge, \vee, \neg$
Step 2. Use double negation & DeMorgan's laws repeatedly and the laws in 1
Step 3. Rename variables if necessary.
Step 4. Use rules in 3 to bring the quantifiers to the left.

Example: Transform the formula into prenex normal form

$\forall x P(x) \rightarrow \exists x Q(x)$

$\neg \forall x P(x) \vee \exists x Q(x)$.

$\exists x \neg P(x) \vee \exists x Q(x)$.

$\exists x \left(\neg P(x) \vee Q(x) \right)$ as $\exists$ distributes over $\vee$

Example: Obtain prenex normal form for the formula

$(\forall x)(\forall y)((\exists z)(P(x,z) \wedge P(y,z)) \rightarrow (\exists u) Q(x,y,u)$

$\neg (\forall x)(\forall y)((\exists z) P(x,z) \wedge P(y,z)) \vee (\exists u) Q(x,y,u)$.

$(\exists x)(\exists y)(\forall z)(\neg P(x,z) \vee \neg P(y,z)) \vee (\exists u) Q(x,y,u)$.

$(\exists x)(\exists y)(\forall z)(\exists u) \left(\neg P(x,z) \vee \neg P(y,z) \right) \vee Q(x,y,u)$

as $\exists$ distributes over $\vee$.

Validity problem for propositional & predicate logic

- A formula (/well formed formula) of the propositional logic is an assertion involving propositional variables by using connectives $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$ in a proper manner.

e.g. $p \wedge q$

- When particular propositions are assigned to p & q, we are giving an interpretation for the formula.

e.g. p denotes "I have exam tomorrow",
q denotes "I am going to study",

then $p \wedge q$ gets the interpretation
"I have exam tomorrow and I am going to study"

- A formula is a tautology if it always takes the value true whatever interpretation is given to it.

e.g. $p \vee \neg p$ is a tautology.

- A formula is a contradiction if it is always false.

- A formula is a contingency if it is neither a tautology nor a contradiction.

Validity problem ~~is~~ for the propositional logic

- Given a formula of propositional logic, does there exist an interpretation which ~~makes~~ will make the formula take the value true.

- Considering the propositional variables as Boolean variables, this is called the Boolean Satisfiability problem.

- This is decidable problem in the sense that we can give an algm. to find out if the given formula is a contingency or tautology.

  (Use truth table)

First order logic

- a predicates has n arguments ~~$c_1, c_2, \ldots, c_n$~~.
- if P is a n-place predicate constant and values $c_1, c_2, \ldots c_n$ are assigned to each of the individual variables, the result is a proposition.
- let V be the domain.
- if $P(c_1, c_2, \ldots, c_n)$ is true for every choice of arguments $c_1, c_2, \ldots, c_n$ selected from V, then P is said to be valid in the domain V.

- If $P(c_1, c_2, \ldots, c_n)$ is true for some (but not for all) choices of arguments selected from U, then P is said to be satisfiable in the domain U, & the values $c_1, c_2, \ldots, c_n$ which make $P(c_1, c_2, \ldots, c_n)$ true are said to satisfy P.

- If P is not satisfiable in the domain U, then P is unsatisfiable in U.

formula of the first-order logic

→ an expression involving predicate variables, quantifiers & connectives included in a proper manner.

e.g. $\forall x\, P(x) \vee \forall x\, Q(x)$.

- When particular predicates are assigned to P & Q, it is called an interpretation.

Imp

- A formula is valid if it is true for every domain no matter how the predicate variables are interpreted.
- A formula is said to be satisfiable if there exists ~~some~~ a domain and some interpretation of the predicate variables which makes it true.

- If a formula is not true for any domain or interpretation, it is <u>unsatisfiable</u>.

e.g. 1) $\forall x\, P(x) \rightarrow \exists x\, P(x)$ is true for all domains of interpretation & hence is a valid formula

2) $\exists x\, P(x) \rightarrow \exists x\, Q(x)$ may be true for some interpretation but may not be true for some other interpretation.

e.g. $P(x)$ = 'x is a third Semester B.Tech Student'.

- $Q(x)$ = 'x has taken a couse on Discrete Mathematics'.

If Discrete Mathematics is a compulsory course, ~~then~~ $\exists$ for third semester student, $\exists x\, P(x) \rightarrow \exists x\, Q(x)$ will be true.

- $Q(x)$ = 'x is female student'

$\exists x\, P(x) \rightarrow \exists x\, Q(x)$ may not be true, if the class has no female candidates.

3) $\forall x\, (P(x) \wedge \neg P(x))$ is unsatisfiable.

Validity problem of first order logic

Given a formula of first order logic, find if ~~is~~ it is <u>valid or not</u>.

- This is an undecidable problem.

Formal system

→ has ~~ex~~ axioms & rules of inferences explicitly stated.

Theorem

→ an assertion which ~~is~~ can be derived from the axioms using the rules ~~of~~ of inference.

Argument or proof

→ a step-by-step derivation of the theorem from axioms using rules of inferences.

Advantages of a formal system

- a characterization of axioms & rules of inferences implicitly defines the set of theorems.
- possible to distinguish between assertion which are true & those which are provable.
- an assertion is <u>provable</u> if it is a theorem.

- Some assertions are true in some domain, but not in others.

Example

- Set of integers with predicates corresponding to arithmetic identities.
  - One can prove all valid assertions ~~can be proved~~ regardless of domain or interpretation
  - One can not prove all assertions which are true of some ~~interpretation~~ particular universe with a specified interpretation of predicate symbols.

    e.g. Godel's Incompleteness Theorem