INDIAN INSTITUTE OF TECHNOLOGY,
KHARAGPUR

Dept:Mathematics, Mid Autumn Semester-2023,
Time: 2 Hrs. Full Marks:30 No. of Students 82, Sub: No.MA61061/MA60269,
Sub. Name: Optimization Methods in Finance

Instruction: Answer all questions. No queries will be entertained during the examination.

1. Consider the following optimization problem,

$$\min \quad 3x^3 + 2y^3 + x^2 - xy$$

$$s.to\ x^2 + 2y \leq 10,\ x^3 + y^3 \leq 5,\ x + 2y = 3.$$

Verify all the necessary and sufficient optimality conditions at $(1, 1)$ for the above problem. [5]

2. A portfolio has two risky assets A_1 and A_2 with expected returns μ_1 and μ_2 respectively, Ω the corresponding covariance matrix. Derive the minimum variance point if their correlation lies in $(-1, 1)$. [5]

3. A portfolio P has three risky assets A_1 and A_2 and A_3, with expected returns 2% and 3% and 4% respectively. Short selling is allowed. The investor wants to invest Rs 10000 in this portfolio. Given that $\sigma_{12} = \sigma_{13} = 0.1$, $\sigma_{23} = 0.2$, $\sigma_1^2 = 0.3, \sigma_2^2 = 0.4, \sigma_3^2 = 0.3$.

Using KKT optimality conditions find the amount of investment in these assets at minimum variance point if the investor wants to achieve exactly 4% return of the portfolio. [5]

4. Construct the Markowitz model for the portfolio P with data from Question 3 if the investor wants to achieve an expected return between 3% to 4%. Convert this model to a linear programming problem with a restricted basis using KKT optimality conditions. [5]

5. Suppose the investor wants to add a risk-free asset A_f with fixed return 3% in the portfolio P of Question 3 and the investor wants to invest a total of Rs 20000/ in the new portfolio (A_1, A_2, A_3, A_f), out of which 80% will be invested in A_1, A_2, A_3 and the rest part will be invested in A_f. In this case, determine(do not derive the theory)
   (a) equation of the capital market line,
   (b) market portfolio,
   (c) amount of investment in each risky asset at market point,
   (d) equation of security market line and the Beta ratio of each risky asset,
   (e) maximum Sharpe ratio.

[2+2+2+2+2]

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