- Negligible Capital Depreciation
- Types of Treasury Bills
- 91-Day, 182-Day and 364-Day Treasury Bills: Marketable: Can be issued by RBI & again marketable by the commercial banks.
- 14-Day Intermediate Treasury Bills
  It is ~~only~~ sold only to State governments in order to replenish shortfall in the daily minimum cash balances of State Governments.

→ Mostly these are used for investments.

- Investors and Sale of T-bills is
- Banks, Primary Dealers, State Governments, Provident Funds, Financial Institutions, Insurance companies, NBFCs, FIIs (as per prescribed norms), NRIs & OCBs can invest in T-Bills.
- Treasury Bills are available for a minimum amount of Rs. 25000 and multiples of Rs. 25000.
- T-bills are sold through auction.

* Auction Process

- Auction is a process of calling of bids with an objective of arriving at the market price.
- A **yield based auction** is generally conducted when a new Government security is issued.
- Investors bid in yield terms up to 2 decimal places (for example, 7.49%, 8.21%, etc).
- Bids are arranged in ascending order and the cut-off yield is arrived at the yield corresponding to the notified amount of the auction.
- The cut-off yield is taken as the coupon rate for the security.
- Successful bidders are those who have bid at or below the cut-off yield. Bids that are higher than the cut-off yield are rejected.
- A price based auction is conducted when Government of India re-issues securities issued earlier.
- Bidders quote in terms of price per Rs.100 of face value of the security (e.g. Rs 102.00, Rs 101.00, Rs 100.00, Rs 99.00, etc, per Rs 100/-)

- Bids are arranged in descending order and the successful bidders are those who have bid at or above the cut-off price.
- Bids which are below the cut-off price are rejected.
- Two types of bidders:

(i) Competitive bidders
(ii) Non-competitive bidders.

- Banks and other financial institutions are the competitive bidders, so there is no limit on the money they can bid.
- ~~Non~~ Some ~~20~~ % of the notified amount is reserved for non-competitive bidders, but they cannot participate in the bidding process. The yield decided in the ~~final~~ auction is also applicable for non-competitive bidders.

Bid No.	Bid yield	Amount of Bid (₹ Crore)	Cumulative amount (₹ Cr)	Price with coupon at 8.22%
1	8.19%	300	300	100.19
2	8.20%	200	500	100.14
3	8.20%	250	750	100.13
4	8.21%	150	900	100.09
5	8.22%	100	→ 1000	100.00
6	8.22%	100	1000	100.00
7	8.23%	150	1250	~~99~~ 99.93
8	8.24%	100	1350	99.87

- Amount of bid is the amount that the bidder is willing to lend to the government.
- Government will go up to bid number 5 at which the notified amount is completed.
- Notified amount is the amount that the central bank is willing to borrow.
- The issuer would get the notified amount by accepting bids upto 5. Since the bid number 6 also is at the same yield, bid numbers 5 & 6 would get allotment pro-rata so that the notified amount is not exceeded. In the above case each would get ₹50 crore. Bid numbers 7 & 8 are rejected as the yields are higher than the cut-off yield.

Bid No.	Price of bid	Amount of bid (₹ Crore)	Implicit yield	Cumulative Amount
1	100.31	300	8.1912%	300
2	100.26	200	8.1987%	500
3	100.25	250	8.2002%	750
4	100.21	150	8.2062%	900
5	100.20	100	8.2077%	1000
6	100.20	100	8.2077%	1100
7	100.16	150	8.2136%	1250
8	100.15	100	8.2151%	1350

* Uniform Price Vs. Multiple Price based Auction

- In a Uniform Price auction, all the successful bidders are required to pay for the allotted ~~quat~~ quantity of securities at the same rate, i.e., at the auction cut-off rate, irrespective of the rate quoted by them.

- Multiple Price auction, the successful bidders are required to pay for the allotted quantity of securities at the respective price/yield at which they have bid.

* Competitive Vs. Non-Competitive Bidding

* Competitive bids

Made by ~~not~~ well informed investors such as banks, financial institutions, primary dealers, mutual funds and insurance companies.

- The minimum bid amount is Rs 10000 and in multiples of Rs 10000 thereafter.

- Multiple bidding is also allowed, i.e., an investor may put in several bids at various price/yield levels.

* Non-competitive bidding

- Open to individuals, HUFs, RRBs, co-operative banks, firms, companies, corporate bodies,

institutions, provident funds, and trusts.

- Under the scheme, eligible investors apply for a certain amount of securities in an auction without mentioning a specific price/yield. Such bidders are allotted securities at the weighted average price/yield of the auction.

* Trading Platform

- T-bills auctions are held on the Negotiated Dealing System (NDS).

- The 91-day T-bills are auctioned on every Wednesday.

- The Treasury bills of 182 days and 364 days tenure are auctioned on alternate Wednesdays.

- Treasury Bill Rate

- Treasury bill rate is the rate of interest at which treasury bills are sold by the RBI.

- The effective return on treasury bills is the discount at which they are sold, and is based on the difference between the price at which they are sold and their redemption value.

- Yield of the T-bill

$$\text{Yield} = \frac{100-P}{P} \times \frac{365}{D} \times 100$$

P = purchase price
D = days to maturity
Day Count: For treasury bills,

$$D = \frac{\text{Actual number of days to maturity}}{365}$$

- Example: Assuming that the price of a 91-day treasury bill at issue is ₹98.20, the yield on the same would be as follows:

$$\text{Yield} = \frac{100-98.20}{98.20} \times \frac{365}{91} \times 100$$

$$= 7.3521\%$$

- After say, 41 days, if the same Treasury bill is trading at price of ₹99, the yield would then be

$$\text{Yield} = \frac{100-99}{99} \times \frac{365}{50} \times 100$$

$$= 7.3737\%$$

- The remaining maturity of the treasury bill is 50 days (91-41).

* other short-term Central Government securities

- Cash Management Bills (CMBs)
- Issued to meet the temporary mismatches in the cash flow of the Government.
- Maturities less than 91 days.
- Issued at a discount and redeemed at face value at maturity
- Tenure, notified amount and date of issue of the CMBs depend upon the temporary cash requirement of the Government.
- The settlement of the auction is on T+1 basis. i.e. all the transactions are done on T+1 th day.
- Ways and Means Advances (WMA)
- Issued to help the states to tide over temporary mismatches in the cash flow of their receipts and payments
- Normal WMA are clean advances, special WMA are secured advances provided against the pledge of Government of India dated securities.

★ Commercial Paper

- CP is an unsecured money market instrument issued in the form of a promissory note by a corporation with high credit ratings to finance its short-term needs.

- CPs can be issued in a wide range of denominations, can be either discounted or interest-bearing, and usually have a limited or nonexistent secondary market.

- Characteristics of CPs.

- CPs can be issued on discount to face value basis or on a fixed interest basis.

- CPs are unsecured, negotiable by endorsement and normally have a buy-back facility.

- CPs as a source of short-term debt regarded as highly safe, simple, flexible, and quality liquid instrument.

- CPs in India

- It was introduced in India in 1990 with a view to enabling highly rated corporate borrowers to diversify their sources of short-term borrowings and to provide an additional instrument to investors.

- Subsequently, primary dealers and all-India financial institutions were also permitted to issue CP to enable them to meet their short-term funding requirements for their operations.

- Corporates, Primary Dealers (PDs) and the All-India Financial Institutions (FIs) are eligible to issue CP.

- The tangible net worth of the company, as per the latest audited balance sheet, is not less than Rs. 4 crore.

- CP can be issued in denominations of Rs. 5 lakh or multiples thereof.

- The fund based working capital of the company should not be less than 4 crore.

- Every issue of CP, including renewal, should be treated as a fresh issue.

- There is no lock in period for CPs.

- CPs can be issued for maturities between a minimum of 7 days and a maximum up to one year, from the date of issue (since October 2004).

- Individuals, banking companies, other corporate

bodies (registered or incorporated in India) and unincorporated bodies, Non-Resident Indians (NRIs) and Foreign Institutional Investors (FII) etc can invest in CPs.

- The total amount of CP proposed to be issued should be raised within a period of 2 weeks from the date on which the issuer opens the issue for subscription.
- Mandatory credit rating for issuance of Commercial Paper. The minimum credit rating shall be P-2 of CRISIL and A2 for ICRA.
- Role and responsibilities of the Issuer.
- Every issuer must appoint an issuing and paying agent (IPA) for issuance of CP.
- The issuer should disclose to the potential investors its financial position as per the standard market practice.
- After the exchange of deal confirmation between the investor and the issuer, issuing company shall issue physical certificates to the investor or arrange for crediting the CP to the investor's account with a depository.

- Depository: National Securities Depository Limited (NSDL) or Central Depository Services (India) Limited (CSDL).

- Role and responsibilities of IPA.

- IPA would ensure that issuer has the minimum credit rating as stipulated by the RBI and amount mobilised through issuance of CP is within the quantum indicated by CRA for the specified rating or as approved by its Board of Directors, whichever is lower.

- IPA has to verify all the documents submitted by the ~~issuer~~ issuer viz, copy of board resolution, signatures of authorised executants (when CP in physical form) and issue a certificate that documents are in order. It should also certify that it has a valid agreement with the issuer.

- Certified copies of original documents verified by the IPA should be held in the custody of IPA.

- Factors affecting CP Market Development

- Credit quality of CP issuer: A minimum credit rating is required to be an eligible issuer of CP.

- Market liquidity: Depends on cost beared by the

investor by investing in the CP. If the market is highly liquid, i.e. transaction cost is low, then the CP market can develop.

- Cost of other alternatives assets: There are many other assets such as Bank loans, other debts from some other institutions which can serve as alternatives to CP. If the cost of the other assets is less than the cost incurred from issuing CPs, then the company can go to adopting other ~~alt~~ alternatives.

- Financial market infrastructure

- Working capital limit: Based on the working capital of the company, there is a certain limit upto which a company can borrow credit. If the limit of a company upto which it can borrow from a bank is less & the requirement is more than the company relies on other sources for credit and they will rely on CPs but if the working capital limit is high then reliance on the ~~sec~~ CPs will be less.

- The CP market is underdeveloped in comparison to other money markets because there is no secondary market for these issues.

★ Certificate of Deposit (CD)

- CDs represent bank deposit accounts which are transferable from one party to another.
- Marketable nor negotiable short-term instruments in bearer from and are known as Negotiable Certificate of Deposit (NCDs).
- liquidity and marketability as its hallmark.
- CDs are issued by banks for attracting large corporate deposits rather mobilising individual savings.
- The introduction of CDs in an economy has usually preceded the introduction of CPs.
- CDs in India
- In initial years (1980-1987) feasibility of CDs in India is subject to various constraints like lack of secondary money market, administered interest rates (not market-driven, decided by the RBI), lack of proper regulatory system.
- Introduction of CDs in 1989: recommendation of RBI working group on money market (Vaghul working groups, 1987).
- Broad objective is to further widen the

range of money market instruments and to give investors greater flexibility in the deployment of short term surplus funds.

- CDs can be issued by
(i) Scheduled Commercial Banks (excluding Regional Rural Banks) and Local area Banks); and select All-India Financial Institutions (FIs)

- Banks have the freedom to issue CDs depending on their funding requirements.

- Minimum amount of a CD should be Rs. 1 lakh, i.e. the minimum cash deposit that could be accepted from a single subscriber should not be less than Rs. 1 lakh, and in multiples of Rs. 1 lakh thereafter.

- CDs can be issued to individuals, corporations, companies (including banks and PDs), trust; funds associations, etc. Non-Resident Indians (NRI) may also subscribe to CDs.

- The maturity period of CDs issued by banks should not be less than 7 days and not more than one year, from the date of issue.

- The FIs can issue CDs for a period of not less than 1 year and not exceeding

3 years from the date of issue.

- CDs may be issued at a discount on face value.

- All CDs were subject to Cash Reserve Ratio (CRR) and Statutory Liquidity Ratio (SLR) requirement on the issues price of the CDs.

- Banks/FIs cannot grant loans against CDs and cannot buy-back their own CDs before maturity.

- CDs are freely transferable by endorsement and delivery.

- RBI guidelines for Issue of CDs wrt the maturity period, minimum size of issue and denomination modified from time-to-time.

- Mutual funds are allowed to invest in CDs with certain limit stipulated by Securities Exchange Board of India (SEBI).

LONG-TERM BOND MARKET

★ Basic features of a Bond

- Pays a fixed amount of interest periodically to the holder of record.
- Repay a fixed amount of principal at the date of maturity.
- Bond characteristics
  Intrinsic features
  Types of issues
  Indenture provisions
- Intrinsic features
- Coupon:
  Indicates the income that the bond investor will receive over the life of the issue. It depends upon the par value of the bond.
- Term to maturity
  The date or the number of years before a bond matures.
  Term bond: Single maturity date
  Serial Obligation bond: Series of maturity dates
- Principal or Par Value
  It is the original value of obligation

- Types of ownership
Bearer Bond
Registered Bond.

- Serial Obligation Bond: If the maturity of the bond is 20-25 years, and it consists of 5 bonds, one is matured in 5 years, another is matured in 10 years and so on.

- Bearer Bond: The holder or bearer is the owner of the bond, the ~~issuer~~ issuer doesn't keep any record of ownership, of the bond. ~~So~~ and therefore they issue coupons to the bond-holders which are used for payments to the bond-holder.

- Registered Bond: ~~The~~ The record of the investor, coupon rate, maturity period is kept by the bond issuer and the periodical payments are also the responsibility of the bond issuer.

- Types of Issues

- Secured (Senior) Bond
Backed by a legal claim on some specified property of the issuer in the case of default.

- If the issuer defaults on the payments, then the payment is made through the specified property.

- Unsecured Bond (Debentures)
  Backed by the promise of the issuer to pay interest and principal on a timely basis.

- Indentures

- It is the contract between the issuer and the bond holders specifying the issuer's legal requirements.

- A trustee acting on behalf of the bond holders ensures that all the indenture provisions are met including the timely payment of interest and principal.

- Price Quotes

- Many traders quote bond prices as a % of their par value.

- For example, if a bond is selling at par, it would be quoted at 100 (100% of par).

- A bond with a face value of Rs. 10000 and quoted at 80.125 would be selling at
  $(0.80125)(10000) = Rs\ 8012.50$

- When a bond's price is quoted as a % of its par, the quote is usually expressed ~~as~~ in points and fractions of a point, with each point equal to Rs. 1.

- Thus, a quote of 97 points means that the bond is selling for Rs. 97 for each Rs. 100 at par.

- Bid and ask price / Yield.

- The bid price is the price the dealer is willing to pay for the bond.

- The ask price is the price the dealer is willing to sell the bond.

- The bid yield is the return expressed as a % of the par value that the dealer wants if she buys the bills; this ~~yield~~ yield is often annualized.

- The ask yield is the rate that the dealer is offering to sell bills.

- For Treasury bills and some other securities, bid and ask yields are quoted as discount yield.

$$\text{Annual Discount Yield} = R_D = \frac{F - P_0}{F} \times \frac{360}{\text{Days to maturity}}$$

- Bond Value

- The value of a bond is the present value of its future cash flow.

$$V_0^B = \sum_{t=1}^{M} \frac{CF_t}{(1+R)^t} = \frac{CF_1}{(1+R)} + \frac{CF_2}{(1+R)^2} + \cdots + \frac{CF_M}{(1+R)^M}$$

where

CF_t = cash flow at t; principal and/or coupon

R = required return

M = term to maturity

- Assume the bond makes fixed coupon payments each year and principal at maturity

$$V_0^B = \sum_{t=1}^{M} \frac{C}{(1+R)^t} + \frac{F}{(1+R)^M}$$

C = annual coupon

F = principal

$$V_0^B = C\sum_{t=1}^{M} \frac{1}{(1+R)^t} + \frac{F}{(1+R)^M}$$

$$V_0^B = C[PV\ IF\ (R, M)] + \frac{F}{(1+R)^M}$$

↓

Present value of interest rate factor

$$V_0^B = \frac{C}{R}\left[1 - \frac{1}{(1+R)^M}\right] + \frac{F}{(1+R)^M}$$

$$\boxed{V_0^B = \frac{C}{R}\left[1 - \frac{1}{(1+R)^M}\right] + \frac{F}{(1+R)^M}}$$

Example: Maturity period : 10 years
Annual Coupon : 9 %
Par Value : Rs 1000
Market interest rate : 10%

$$V_B = \frac{90}{0.1}\left[1 - \frac{1}{1.1^{10}}\right] + \frac{1000}{1.1^{10}}$$

$$= 553.011 + 385.543 = 938.554$$

- Let coupon is payed semi-anually:

$$V_B^{\circ} = \frac{C/2}{R/2}\left[1 - \frac{1}{(1+R/2)^{2M}}\right] + \frac{F}{(1+R/2)^{2M}}$$

$$= \frac{90}{0.1}\left[1 - \frac{1}{1.05^{20}}\right] + \frac{1000}{1.05^{20}}$$

$$= 560.799 + 376.889 = 937.688$$

- n - Coupon payments per year

$$V_B = \frac{C/n}{R/n}\left[1 - \frac{1}{(1+R/n)^{nM}}\right] + \frac{F}{(1+R/n)^{nM}}$$

- Compounding frequency:

10% → Rs. 100 → Rs. 110

Semi-annual ~~company~~ → 5% in every compounding six months.

100 → Rs 105 → Rs. 110.25
(6 months) (after 1 year).

Monthly compounding → 10/12% every month.

$$100\left(1+\frac{10}{120}\right)^{12} = 110.47$$

- Compounding frequency is the number of times, the coupons are payed in a year.

- With the change in compounding frequency the price of the bond also changes because in case of compound interest, the interest ~~payed~~ paid in the present time depends on the past cash flows which change with the compounding frequency, ∴ value of bond also changes.

- Effective Rate $\left[\left[1+\frac{R_A}{n}\right]^n - 1\right]$

$R_A \rightarrow$ Simple Annual Rate.

- Continuous Compounding

$$FV = \lim_{n\to\infty} A\left[1+\frac{R}{n}\right]^{nM}$$

$A \rightarrow$ Amount invested
$R \rightarrow$ Simple Annual Rate.

$$\boxed{FV = Ae^{RM}}$$

10%, Rs.100
Continuously compounded
$FV = 100e^{0.1\times1} = 110.52$

Present value of future receipts with continuous compounding.

$$FV = Ae^{RM}$$

$$\boxed{A = FVe^{-RM}}$$

Eg: If $R = 0.1$, Rs 100, 2 years from now. What is the current value of that bond?

$$PV = 100e^{-0.1 \times 2} = 81.87$$

Eg: Let a bond paying Rs 100 each year for 2 years with $R = 0.1$, continuous compounding. Find the current worth of the bond.

$$PV = 100e^{-0.1 \times 1} + 100e^{-0.1 \times 2}$$
$$= 90.48 + 81.87 = 172.35$$

Eg: Continuous Compounding, Discount rate = 10%, Maturity period = 10 yrs, coupon = 9%, ~~par value~~ par value = Rs 1000. Find the value of the bond.

$$PV = \sum_{t=1}^{10} 90e^{-0.1 \times t} + 1000e^{-0.1 \times 10}$$

$$= 90e^{-0.1}\frac{(1 - e^{-0.1 \times 10})}{1 - e^{-0.1}} + 1000e^{-1}$$

$$= \frac{81.44 \times 0.632}{0.095} + 367.879$$

$$= 908.82$$

Zero-Coupon Bond: No period coupon payments

Eg: Maturity period = 10 years.
Rs 1000/- Discount rate = 10%.

$$PV = \frac{1000}{1.1^{10}} = 385.54$$

Semi-annual compounding

$$PV = \frac{1000}{1.05^{20}} = 376.89$$

Eg: March 1: A Zero Coupon bond is sold with Rs 1000 with maturity on Sept. 1.
Annual Rate = 8%.

March : 31
April : 30
May : 31 ⇒ 184 days } Day-Count Convention
June : 30
July : 31
August : 31

~~PV = 1000 / (1 + 8 × 365 / 184 × 100)~~

$$PV = \frac{1000}{(1.08)^{184/365}} = 961.95$$

$\frac{184}{365}$: $\frac{\text{Actual}}{\text{Actual}}$

And in some cases, every month is ~~consid~~ considered of 30 days ⇒ $\frac{180}{360}$.

- Four factors are used for the valuation of bonds: Coupon, Discount Rate, Maturity period, Par value.

- Three types of bonds:
1) Discount Bond
2) Par Bond
3) Premium Bond

Coupon (C), Discount Rate (R), Maturity period (M), Par Value (F).

Discount Bond:
$V < F \Rightarrow C < R$

Par Bond:
$V = F \Rightarrow C = R$

Premium Bond: $V > F \Rightarrow C > R$.

1) Bond price moves inversely with the yield.
2) For a given change in interest rate, longer maturity bonds experience larger price change.

Example: par value = Rs 1000
Coupon: 8%
~~Discount rate~~
Maturity periods: 1 yr ~~10 yr~~ ~~20 yr~~ ~~30 yr~~.

Discount rate →	7%	10%
P.V. of Interest pay:	75	73
P.V. of Principal:	934	907
	1009	980

% change in total value -2.9%

10 years.

7%	10%
569	498
505	377
1074	875

% change = -18.5%.

Similarly for 20 years ⇒ -28.7%

- If coupon rate is same, then for some change in discount rate, the change in bond price is higher for longer maturity bond.

- Bond price volatility is higher in longer maturity bonds. i.e. long maturity bonds show higher fluctuation in price if discount rate changes.

- Bond price volatility increases at a diminishing rate as term to maturity increases.

Discount rate: 7% → 10%

-2.9% → -18.5% → -25.7% → -28.7%

1 yr 10 yr 20 yr 30 yr.

∴ Change in bond price is increasing but at a diminishing rate.

- Effect of Coupon on Bond Price Change

M = 20 yrs F = Rs 1000

Coupon Rate	3%		8%		12%
Discount rate	7% → 10%		7% → 10%		7% → 10%
P.V. Interest	322	257	858	686	
P.V. Principal	257	142	257	142	
Total value	579	399	1115	828	
% Change in Total value	-31.3%		-25.7%		-24.1%

- If there is an interest rate change, then bonds having low coupon rate have higher change in bond price.
- Bond price volatility increases as coupon rate falls.

Example: M = 20 yrs, Coupon = 4%, F = 1000

	Low Yield		Intermediate		High	
Discount:	3% → 4%		6% → 8%		9% → 12%	9% → 18%
P.V. Interest	602	547				
P.V. Principal	562	453				
Total	1164	1000	769	604	545	398
% Change	-14.1%		-21.5%		-27.0%	-11%

- ∴ If the change in the yields are different like 3% → 4% (1%), 6% → 8% (2%), 9% → 12% (3%) then the change in price levels may be different.

- ∴ Bond price volatility depends upon the term to maturity, coupon rate and the yield to maturity.
- Term to maturity & coupon rate is decided by the bond issuer but yield or the interest rate is completely determined by the market.

★ Yield.

Types of yield:

1) Nominal yield = Coupon rate

$$= \frac{\text{Annual Coupon payment}}{\text{Face value of the Bond}}$$

e.g. Par value = Rs 1000

Rs 90 → Coupon / year.

$\frac{90}{1000}$ → 9% → Coupon (Nominal yield).

2) Current yield = $\frac{\text{Annual Coupon payment}}{\text{Market price of the bond}}$

e.g. $\frac{90}{938.55} \times 100 = 9.59\%$ → Current yield

3) Yield to Maturity (YTM)

It is the yield which equates the price of the bond to the present value of the bond's cash flow.

E.g. M = 10 years, Coupon = 9%, Price = Rs 938.55.

$$P.V. = \sum_{t=1}^{10} \frac{90}{(1+R)^t} + \frac{1000}{(1+R)^{10}}$$

$$P.V. = \frac{90}{R}\left[1 - \frac{1}{(1+R)^{10}}\right] + \frac{1000}{(1+R)^{10}}$$

$\Rightarrow R = 10\%$

Semi-Annual Compounding, Price = Rs 937.68.

$$937.68 = \sum_{t=1}^{20} \frac{45}{(1+R)^t} + \frac{1000}{(1+R)^{20}}$$

$R = 0.05$

$\Rightarrow R' = 0.05 \times 2 = 0.1$

Effective Annual rate $= (1.05)^2 - 1 = 10.25\%$

- Yield to maturity has 3 components:

(i) Coupons
(ii) Capital Gains and Losses in the cashflows.
(iii) Reinvestment of the Coupons

* Average Rate to Maturity

Approximation for the yield to maturity

$$ARTM = \frac{C + \left(\frac{F - P_B}{M}\right)}{\left(\frac{F + P_B}{2}\right)}$$

Eg.: $ARTM = \frac{90 + (1000 - 938.55)/10}{(1000 + 938.55)/2} = \frac{96.145}{969.275} = 0.0992$

$\approx 9.92\%$

YTM on zero-coupon bond

$$P = \frac{F}{(1+R)^M} \Rightarrow (1+R)^M = \frac{F}{P}$$

$$\boxed{YTM = \left(\frac{F}{P}\right)^{1/M} - 1}$$

E.g. $P = 800$, $F = 1000$, $M = 3$ yrs.

$$YTM = \left(\frac{1000}{800}\right)^{1/3} - 1 = 0.772 \approx 7.72\%$$

$M = 182$ days, $P = 96$, $F = 100$

$$P = \frac{F}{(1+R)^{182/365}}$$

$$R = \left(\frac{100}{96}\right)^{365/182} - 1 = 0.085 \approx 8.5\%$$

- Yield to Call
- Call feature: It allows the issuer to buy back the bond at a specific price in a particular time period.
- The call price is decided and specified at the time of issuance of bond.
- The yield to maturity of this type of bond is called yield to call.

$$P.V. = \sum_{t=1}^{M} \frac{CF_t}{(1+r)^t} + \frac{F}{(1+r)^M}$$

$$P.V. = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} + \frac{CP}{(1+r)^n}$$

$n \rightarrow$ no. of periods to the identified call rate.

e.g. $M = 10$ years, Coupon = 9%, Callable in 5 years.
CP = Rs 1100, F = Rs 1000, Semi-Annual Compounding

$P = 937.68$ (Semi-Annually)

$$937.68 = \sum_{t=1}^{10} \frac{45}{(1+r)^t} + \frac{1100}{(1+r)^{10}} = 0.6106 \approx 6.106$$

$r' \approx 12.21\%$

$\downarrow$

$r = YTC.$

Annual Compounding: $938.55 = \sum_{t=1}^{5} \frac{90}{(1+r)^5} + \frac{1100}{(1+r)^5}$

$$\boxed{P = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} + \frac{CP}{(1+r)^n}} \rightarrow YTC.$$

- 12.21% is higher than the normal YTM in the same case because risk is higher. & whenever risk is higher, Call Price should be higher than the normal face value.. or par value.

- Yield to put

- Put feature: It gives the investor or the bond-holder to sell back the bond to the issuer at a pre-specified price.

- The price is specified by the issuer at the time of issuance of the bond.

- Yield to put is the yield that equates the present value of the cash flows till the put date and the present value of the put price to the price of the bond.

$$P = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} + \frac{PP}{(1+r)^n}$$

$n \rightarrow$ no. of periods till the identified put date.

- Put price is generally lower than the face value because in this case, investor is taking the risk.

E.g. Put price = Rs 950, Price = 937.69.
Semi-Annual Compounding.

$$937.69 = \sum_{t=1}^{10} \frac{45}{(1+r)^t} + \frac{950}{(1+r)^{10}}$$

$r = 0.04904 \approx 4.9\%$

$r' \approx 9.8\%$.

- Realised Yield or the Total Return.

- If an investor wants to sell the bond before the yield to maturity, then the yield that the investor gets is the realised yield.

E.g. Buys the bond : Coupon = 10%
Maturity period = 4 years
Bond is selling at its par value.
Par Value = Rs 1000.
Market price of the bond is equal to the par value ∴ C = R.
Holding period = 3 years
Investor sells the bond after 3 years.

$$P_B = \frac{100}{(1+r)} + \frac{1000}{(1+r)} = \frac{1100}{(1+r)} = \frac{1100}{1.1} = 1000$$

$$r = 0.1$$

Reinvestment of the coupons.

End of first year → Rs 100 → $100(1.1)^2 = 121$
End of second year → Rs 100 → $100(1.1) = 110$
End of third year → Rs 100.

Rs 331

↓

Total money after 3 years.

300 → Coupons payment
31 → Interest from reinvesting the coupons.

Total money generated from the bond = Rs 1331
Buying price = Rs 1000

Holding period return = ~~1331~~
30

$$= \left[\frac{\text{Ending value}}{\text{Beginning value}}\right]^{1/3} - 1$$

$$= \left(\frac{1331}{1000}\right)^{1/3} - 1 = 10\% \text{ (Total return)}$$

- If discount rate changes in between then accordingly reinvested coupon payment will change & the total return will change.

$$\boxed{\text{Holding period return} = \left[\frac{\text{Ending value}}{\text{Beginning value}}\right]^{1/T} - 1}$$

Or realised return or Total return.

- Interest Rate Determination

$$r = f(\text{Economic factors, Bond characteristics})$$

- Economic factors includes risk free rate which comes from the treasury bill rate, government bonds/securities rate, etc & the inflation.
- Economic factors ⇒ RFR + I
- Bond characteristics includes the ~~exter~~ risk premium that is given by a certain type of bond.

$$\therefore r = f((RFR, I), RP)$$

→ What causes overall market interest rate to rise or fall?
→ Why the different types of bonds have different interest rates?
→ What causes the different interest rates of the bonds to change?

• Term Structure of Interest rate

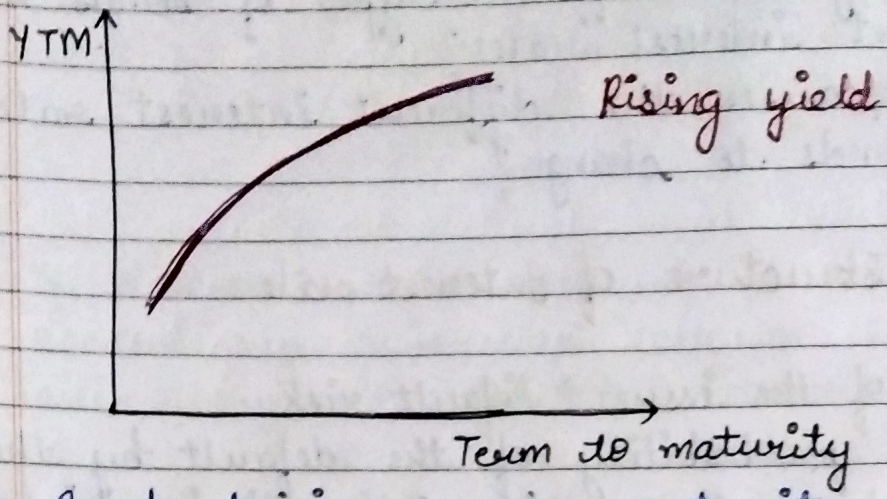
1) Quality of the issue ⇒ Default risk
If the probability of the default by the issuer is higher then this risk will be higher & the issuer should pay more.

2) Term to maturity
If the maturity period is longer, price volatility of the bond is higher & it is riskier to invest in that bond. ∴ It should pay higher return in comparison the short term bond.

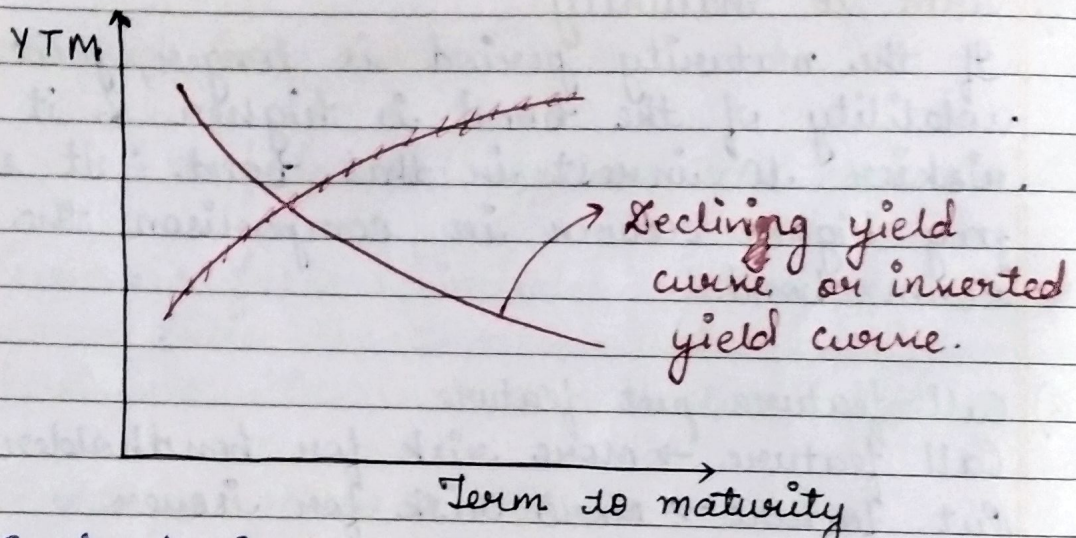
3) Call feature, put feature.
Call feature → More risk for bondholder
Put feature → More risk for issuer

4) Exchange rate risk, Country risk.
If someone is holding an international bond, then these risks come into existence.

* Yield curve: function or relation b/w the term to maturity and the yield to maturity.



- Bonds having longer maturity are giving more yield compared to bonds having shorter maturity.



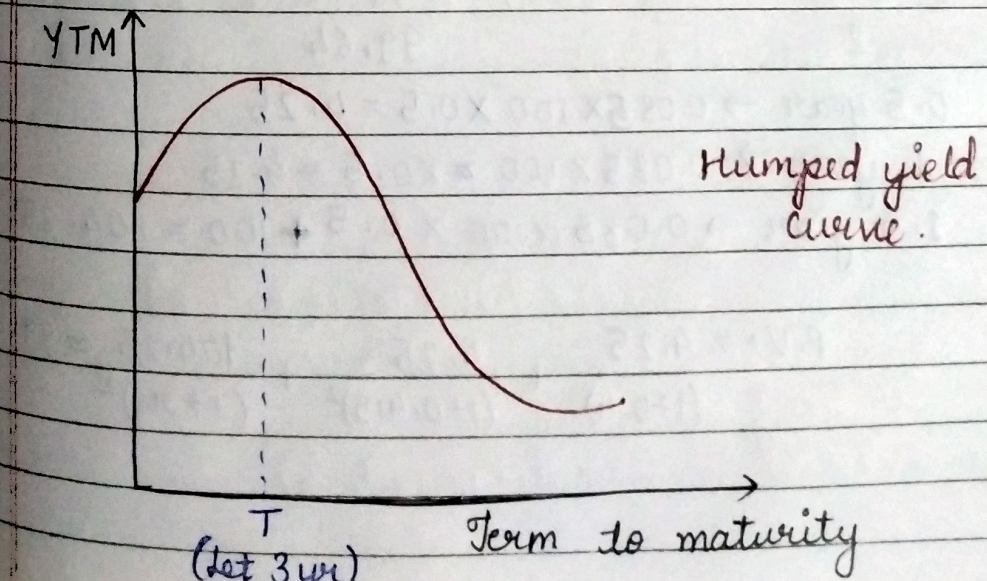
- Bonds having shorter maturity are giving more yield compared to bonds having longer maturity.

- This curve occurs when the current interest rates are already high and are expected to go down in future.

- Interest rate in the market is high when inflation is high. When money supply is high, inflation becomes high & then interest rate is increased to counteract inflation. Policy rates, risky free rates, etc. are increased. ∴ market determined interest rates will rise.

- Whenever interest rate is high, it is expected to fall in the future. ∴ Investors will prefer short term bonds as they will get higher return because if in future interest rate goes down, then the return from bonds will go down. ∴ Short term bonds give more yield than the long term bonds.

- If a person has taken a loan for 10 years, and he could not repay it, and then he takes another short term loan to repay this long term loan. In this case, the short-term interest rate will be higher than the long-term interest rate.



- This curve maybe possible in certain scenarios when there is some change in the market & the issuer may not be willing to give high returns for a long period.

- Intermediate or Medium term bonds (3-5 years) will give highest returns & long term bonds give even less return than the short term bonds.

- **Spot Rate**

It is a discount rate for a particular cash flow at a specific maturity.

Let there is a bond

F = Rs 100, M = 1.5 years, Coupon = 8.5%

Maturity	Coupon	Price	YTM.
0.5		96.15	8%
1		92.19	8.3%
1.5		99.45	
2		99.64	

0.5 year → $0.085 \times 100 \times 0.5 = 4.25$

1 year → $0.085 \times 100 \times 0.5 = 4.25$

1.5 year → $0.085 \times 100 \times 0.5 + 100 = 104.25$

$$P.V^{\circ} = \frac{4.25}{(1+0.4)} + \frac{4.25}{(1+0.415)^2} + \frac{104.25}{(1+r_3)^2} = 99.45$$

$$99.45 = 4.08654 + 3.91805 + \frac{104.25}{(1+r_3)^3}$$

$$91.44591 = \frac{104.25}{(1+r_3)^3}$$

$$(1+r_3)^3 = 1.140024.$$

$$r_3 = 0.04465 \times 2$$

$$\approx 8.93\% \text{ (Theoretical spot rate).}$$

$$99.64 = \frac{4.25}{(1.04)} + \frac{4.25}{(1+0.415)^2} + \frac{4.25}{(1+0.0446)^3} + \frac{104.25}{(1+r_4)^4}$$

$$\frac{104.25}{(1+r_4)^4} = 99.64 - 4.0865 - 3.918 - 3.895 = 87.7405$$

$$1+r_4 = 1.0448 \Rightarrow r_4 = 0.044$$

$$\approx 8.8\%$$

Theoretical spot rate can be calculated by using the already available spot rates.

Generally, spot rates can be calculated from the treasury bill rates available in the market.

- Forward rate

It tells about the market expectations of the future short term rates.

Let an investment horizon period be 1 year.

Alternative - I: Buy a 1 year bond.

Alternative II: Buy a six month bond and when it matures in six month buy another six month bond.

The investor will be indifferent to both alternatives if the return is same.

Determine a forward rate that makes the investor indifferent to the two alternatives.

Investment in one year bond:

$$\text{Price} = \frac{100}{(1+r_2)^2} \qquad \text{Face value} = 100$$

$r_2 \Rightarrow$ Half of the theoretical spot rate.

Invest: X in six month bond.

$\Rightarrow X(1+r_1)$

$r_1 = \frac{1}{2}$ of the bond yield of the six month spot rate.

Let $t+0.5\, r_{0.5} \rightarrow \frac{1}{2}$ of the forward rate on a six month bond available six months from now $(t+0.5)$

$$X(1+r_1)(1+t+0.5\, r_{0.5}) = 100$$

$$X = \frac{100}{(1+r_1)(1+t+0.5\, r_{0.5})}$$

$$\frac{100}{(1+r_2)^2} = \frac{100}{(1+r_1)(1+t+0.5\, r_{0.5})}$$

Six months spot rate = 0.08 = 0.04
One year spot rate = 0.083 = 0.0415.

$$\frac{100}{(1+r_2)^2} = \frac{100}{(1+r_1)(1+t+0.5r_{0.5})}$$

$$t+0.5r_{0.5} = \frac{(1+r_2^2)}{(1+r_1)} - 1$$

$$= \frac{1.0415^2}{1.004} - 1 = 0.043$$

≈ 8.6% (Forward rate).

$$\frac{100}{1.0415^2} = 92.19$$

(92.19)(1.04) = 95.8776
(95.8776)(1.043) = 100.

∴ Both alternatives will give the same return & the investor will be indifferent if the yield or interest rate after 6 months for the next 6 months is 4.3%.

Implied forward rate

$t+1r_1$ = 1-year forward rate 1 year from now. (t+1)

$t+2r_1$ = 1-year forward rate 2 years from now (t+2)

$t+3r_1$ = 1-year forward rate 3 years from now (t+3).