Indian Institute of Technology Kharagpur

Mid-Autumn Semester Examination 2023-24

Advanced Macroeconomics (HS60243)

Full Marks: 30

Answer All Questions

1. **Consider, an Overlapping Generation (OLG) model discussed in the class, where population grows at the rate, $0 < n < 1$; and capital fully depreciates in each period; $\delta = 1$. Production function of the economy is, $Y_t = K_t^{\alpha} N_t^{1-\alpha}; 0 < \alpha < 1$. K_t is the amount of capital at time, t; and N_t is the amount of labour at time, t. An individual is endowed with 1 unit of labour when young, and dies next period as old. Old does not have any endowment of labour. As a result, N_t is also the number of population at time, t. Suppose, the consumption of an individual when young at time t is, c_t^1; and when old at time $t+1$ is, c_{t+1}^2. Lifetime utility function of an individual is, $u(c_t^1, c_{t+1}^2) = \log(c_t^1) + \beta\log(c_{t+1}^2); 0 < \beta < 1$. β is the discount factor. The wage rate of the economy at time, t is w_t; and the rental rate at time t is, r_t. (25)**

   a) Write down the budget constraint of an individual when young, and also when old. Also, write down the budget constraint in the present discounted value format. (1+1+2)

   b) Using the profit maximization of the firm show that, $w_t = (1-\alpha)k_t^{\alpha}$, and $r_t = \alpha k_t^{\alpha-1}$ (1+1)

   c) Set, the relevant Lagrangian and derive the Euler Equation. Derive the demand function for, c_t^1, c_{t+1}^2. Also derive the optimal savings scheme of the young at time, t, s_t. (2+1+1+1)

   d) Derive the difference equation of the percapita capital stock at time; k_{t+1}, and calculate the steady state percpita capital stock, k_{ss} when $\alpha = \frac{1}{3}; n = 0.05; \beta = 0.99$. (3+2)

   e) Derive the resource constraint of the economy at the steady state, and calculate the golden rule level of percapita capital stock, k_g when $\alpha = \frac{1}{3}; n = 0.05; \beta = 0.99$. (3+2)

   f) Comment on the dynamic efficiency of the competitive equilibrium when, $\alpha = \frac{1}{3}; n = 0.05; \beta = 0.99$. (4)

2. **Consider the infinite horizon Neo-classical growth model discussed in the class. Suppose, the utility function is, $u(c_t) = log(c_t)$; where c_t is the percapita consumption. Suppose, the production function in the percapita form is, $y_t =$**

$f(k_t) = k_t^{\alpha}$; $\alpha = \frac{1}{3}$. **Suppose, the depreciation of capital stock, $\delta = 1$, and the discount factor, $\beta = 0.99$. Also assume that, the rate of growth of population is, $n = 0.05$. (5)**

1. Calculate the steady state real interest rate (1)
2. Calculate, the steady state percapita consumption and investment. Calculate the Golden rule level of percapita capital stock, k_g (1+1+2)