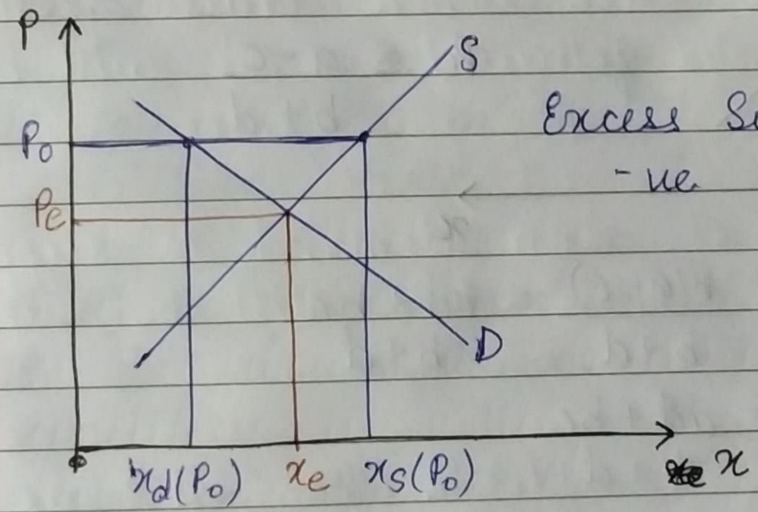
$$\Rightarrow p_e = \frac{a-c}{b+d}, \quad x_e = \frac{ad+bc}{b+d}$$

$$p_d = \frac{a}{b} - \frac{x}{b}$$

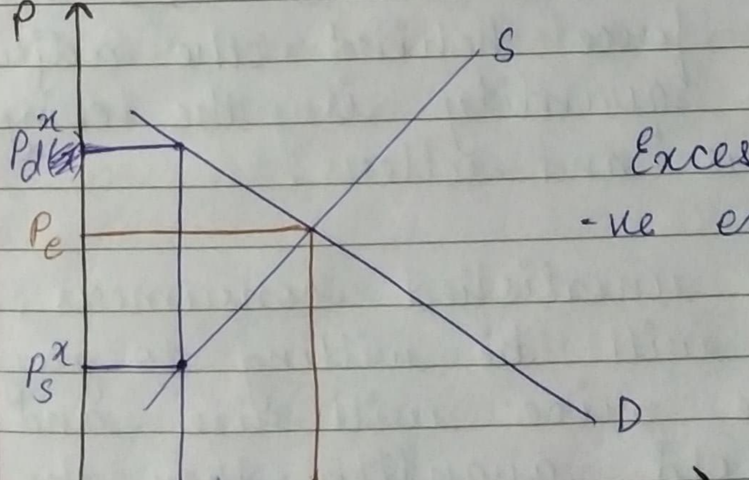
$$p_s = \frac{x}{d} - \frac{c}{d}$$

At eqm. in Marshallian approach:

$$p_d(x_e) = p_s(x_e) \Rightarrow x_e = \frac{ad+bc}{b+d}, \; p_e = \frac{a-c}{b+d}$$



Excess Supply
-ve Excess demand

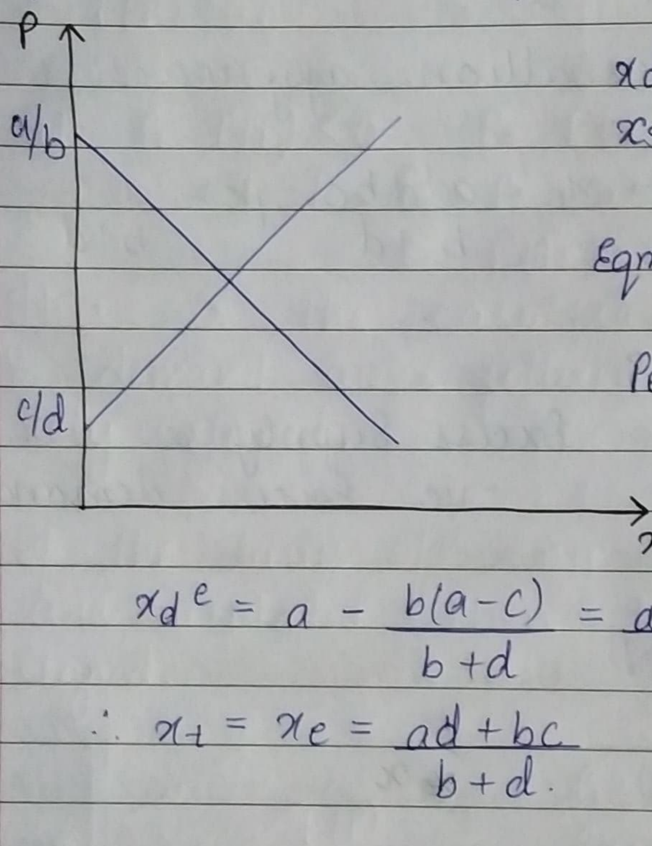


Excess demand
-ve excess supply

In disequilibrium, ~~mum~~ ~~x~~ minimum will be realized.

If $x_d < x_s \Rightarrow$ Plan of the buyer is realized.

If ~~x_d~~ $x_s < x_d \Rightarrow$ Plan of the seller is realized.



$$x_d = a - bp$$
$$x_s = c + dp.$$

$$\text{Eqm} \Rightarrow a - bp_e = c + dp_e$$

$$p_e = \frac{a-c}{b+d}$$

$$x_d^e = a - \frac{b(a-c)}{b+d} = \frac{ad+bc}{b+d}$$

$$\therefore x_t = x_e = \frac{ad+bc}{b+d}.$$

The driving force behind the adjustment of price & quantity is the competition among buyers and sellers.

If there are unsatisfied consumers, then the buyers will be willing to pay more so the price will rise and if there is unsold quantity then the sellers will ~~compete~~ engage competition among themselves & price will come down.

Existence

Can we have at least one

$$P_e \geq 0 \ni f(P_e) = 0$$

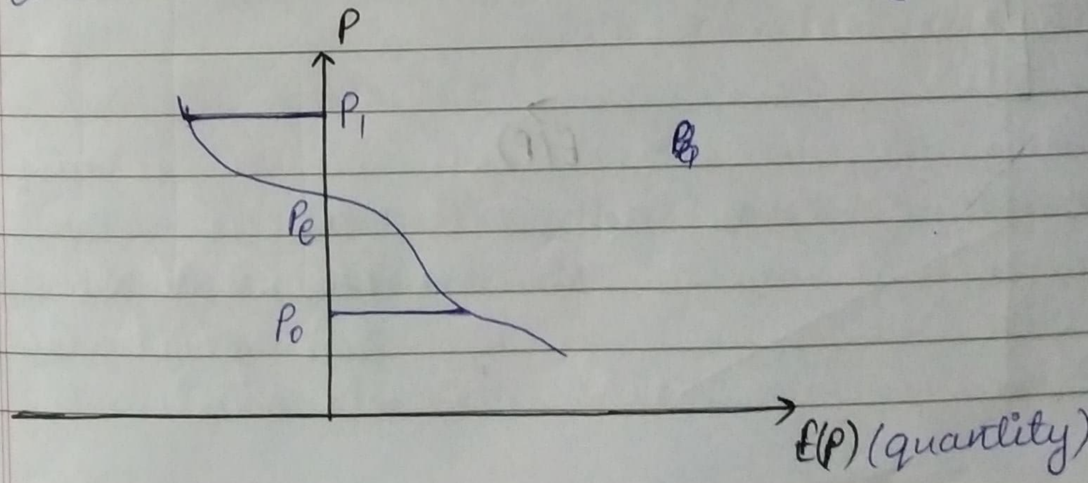
Or in Marshallian sense one

$$x_e \geq 0 \ni f(x_e) = 0$$

Condition for Existence of Walrasian equilibrium

A price $P_e \geq 0$ exists if the following conditions hold:

(i) **E(P) is continous in P (SC, not NC).**
It is a sufficient but not necessary condition.
We need the E(P) func^n continuous only near the eqm price.
If its +ve at some point & -ve at some other, then there will be a point b/w them at which it will be zero.



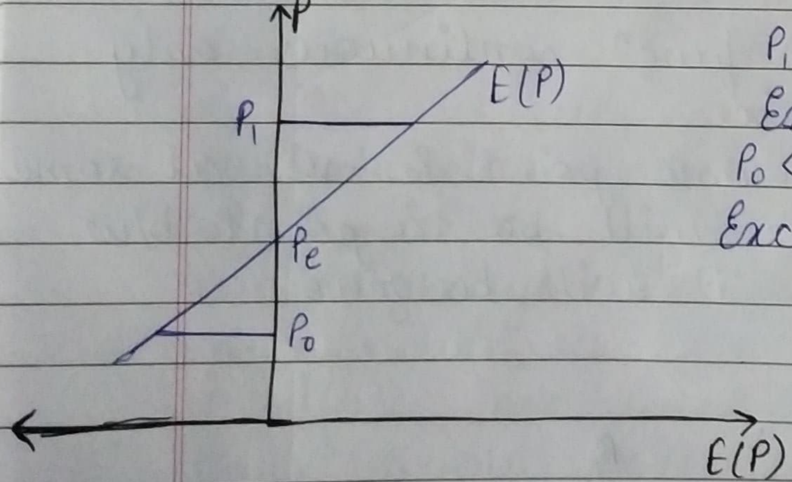
$P_1 > P_e$ (Supply > demand)
+ve excess supply or -ve excess demand. $E(P) < 0$

$P_0 < P_e$ (Demand > Supply)
+ve excess demand or -ve excess supply
$E(P) > 0$.

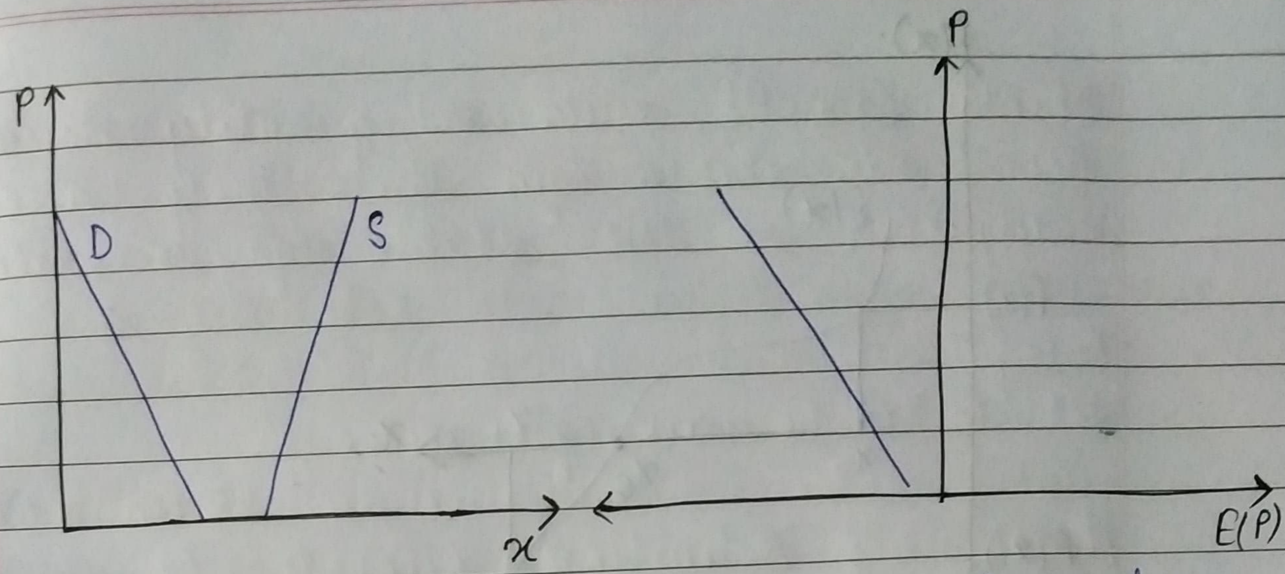
at $P_1 : E(P) < 0$ $P_0 : E(P) > 0$.
$\therefore \exists \; P_0 < P_e < P_1$ at which ~~$P_e = 0$~~
$E(P_e) = 0$.

This is the case when laws of supply & demand were followed $\therefore E(P)$ will be -vely sloped.

If the laws of ~~dem~~ demand & supply are violated then E(P) will be +vely sloped.



$P_1 > P_e$ (Demand > Supply)
Excess demand, $E(P) > 0$
$P_0 < P_e$ (Demand < Supply)
Excess Supply, $E(P) < 0$.



Supply > Demand $\forall P$ $\therefore$ Excess demand <0.
$\therefore$ Equilibrium doesnt exist. $\because$ NO P for which $E(P) > 0$.

(ii) $\exists$ a $P_0 > 0 \ni E(P_0) > 0$
(iii) $\exists$ a $P_1 > 0 \ni E(P_1) < 0$

$\Big\}$ $\exists$ P_e $E(P_e) = 0$. $P_e \in (P_0, P_1)$

Condition for Existence of Marshallian equilibrium:

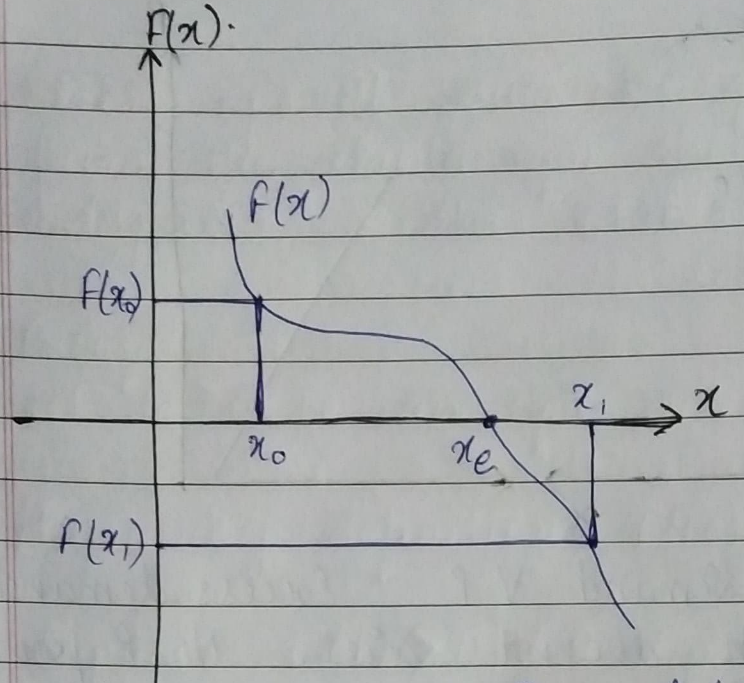
A quantity $x_e \geq 0$ exists if the following conditions hold:

(i) $f(x)$ is continuous in x

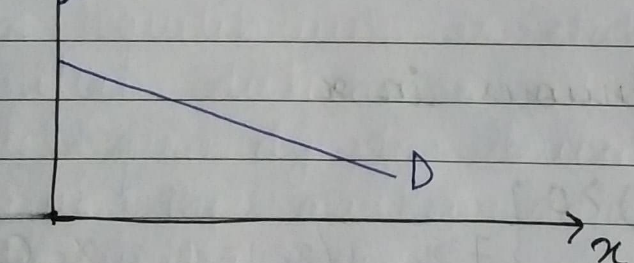
(ii) $\exists$ a $x_0 > 0 \ni f(x_0) > 0$
(iii) $\exists$ a $x_1 > 0 \ni f(x_1) < 0$

$\Big\}$ $\exists$ x_e $f(x_e) = 0$ $x_e \in (x_0, x_1)$.

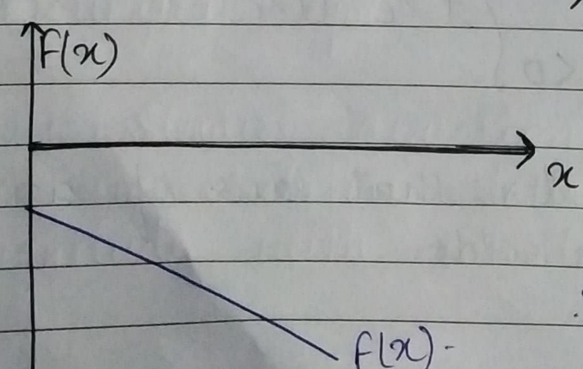
First, we consider that law of demand & supply ~~hon~~ holds.



x_1 : Demand < Supply
$f(x) < 0$ Excess supply
x_0 : Demand > Supply
$f(x) > 0$ Excess demand.
$\therefore$ ~~x_e~~ $x_0 < x_e < x_1 \Rightarrow f(x_e) = 0$.



Demand < Supply



$\therefore f(x) < 0 \;\forall\, x$.
$\therefore$ Eqm point does not exist.

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* Uniqueness

Can we have only one $P_e \geq 0 \ni E(P_e) = 0$ or in Marshallian sense one $x_e \geq 0 \ni F(x_e) = 0$.

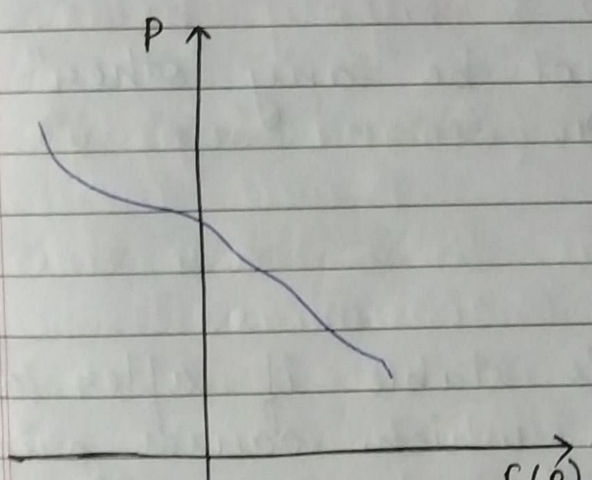
Condition:

If the eqm exists, it will be unique iff

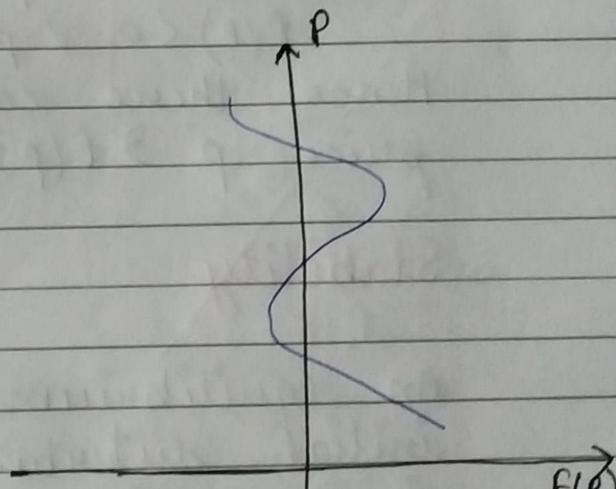
either $E'(P) > 0 \ \forall p$

or $F'(x) > 0 \ \forall x$.

i.e., $E(P)/F(x)$ should be monotonic.



Unique Equilibrium



Multiple Equilibria

Linearity of dd and ss doesn't necessarily imply existence and uniqueness of eqm.

If there are multiple equilibria, then all may not be stable eqm.

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Stable eqm: If price or quantity deviates from the stable eqm then they are readjusted so the stable eqm is maintained.

Proof:
Let $P_e > 0$ be an eqm price.

Hence, by definition, $E(P_e) = 0$. — ①

Suppose $E'(P) > 0 \ \forall\ 0\ \forall\ p$ — ②

Combining ① & ②,

$E(P) > 0 \ \forall\ p > p_e$
$E(P) < 0 \ \forall\ p < p$

Hence, there cannot be any "other" price $p' \ni E(p') = 0$.

Stability

An equilibrium is stable if after an initial disturbance which causes price or quantity to deviate from its eqm value, the market adjusts itself towards the initial eqm.

Static: Studies direction of adjustment
Dynamic stability: Tells about speed & magnitude with which eqm is established.

Local: Eqm is reestablished only when the disturbance is within a small range.

Global stability: Eqm is stable for any initial disturbance however large it is. There can only be a unique global stable eqm.

Whereas there can be multiple local stable eqm.

$\therefore$ Global stability implies local stability but the converse is not true.

Walrasian Static Stability

Adjustment mechanism: price

$$\boxed{E(P) = D(P) - S(P)}$$

Condition:

Rule 1: If at a price P_0, $E(P_0) > 0$, price increases further.
If there is +ve excess demand, there is competition among the buyers, therefore, price will go up.

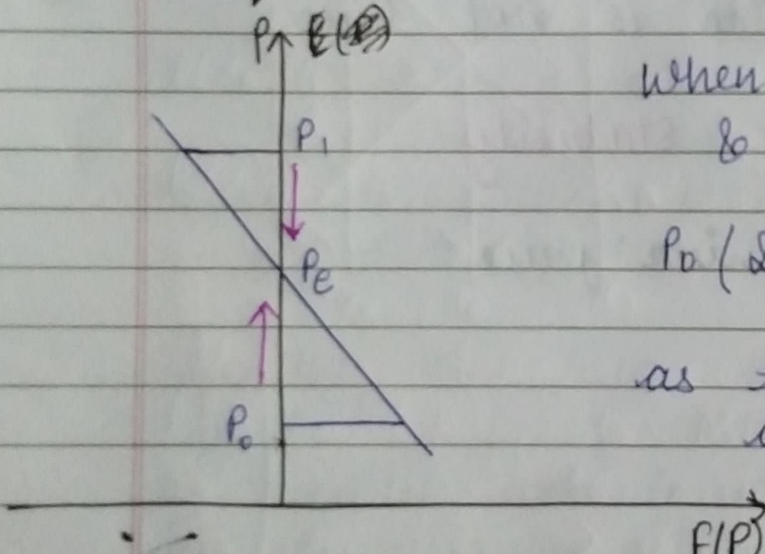
Rule 2: If at a price P_1, $E(P_1) < 0$, price decreases further.
Then there is -ve excess demand i.e. +ve

excess supply, producers are willing to sell but there are not enough buyers, producers will engage competition among themselves to sell the good. price will go down.

Given rules 1 & 2, Walrasian Static stability requires that

$$\boxed{E'(P) < 0 \ \forall P}$$

<u>Proof</u>:

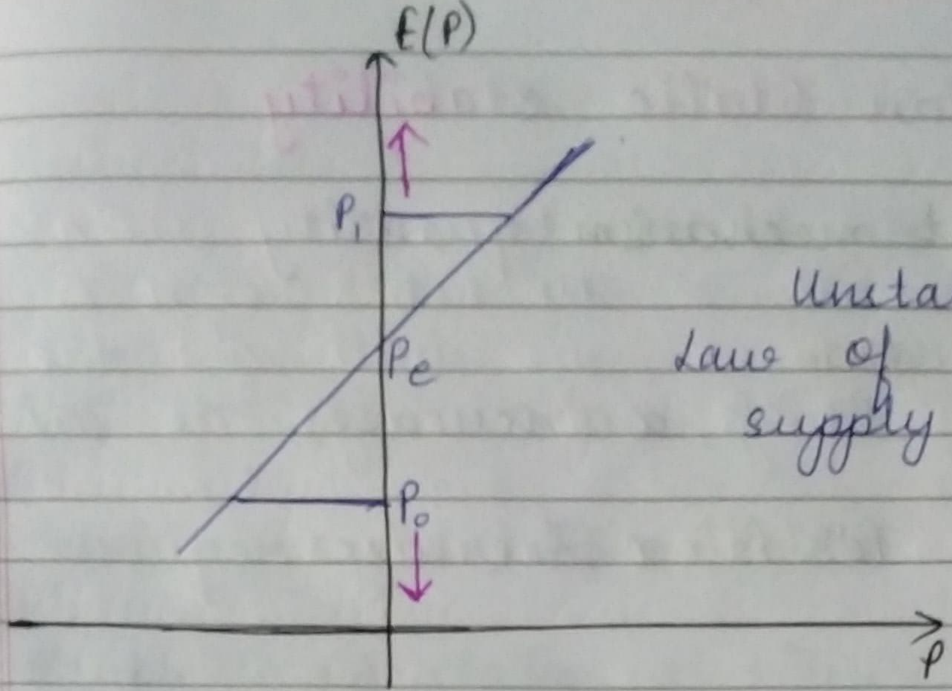


When law of demand & Supply holds.

P_0 (Demand > Supply)
∴ P will Increase as there are unsatisfied consumers.

P_1 (Demand < Supply)
∴ P will decrease as there is unsold quantity.

∴ Stable eqm.



Unstable eqm.
Law of demand & supply does not hold.

$P_1 \to$ +ve excess demand $\therefore$ P will increase.
$P_0 \to$ -ve excess demand $\therefore$ P will decrease.

If $E'(P) < 0$

then
a) $E(P) < 0 \ \forall \ p > P_e$
b) $E(P) > 0 \ \forall \ p < p_e$.

If initially price deviates below P_e, there arises ED for the good which in turn increases price towards P_e.

If initially price increases above P_e, there arises ES for the good which in turn decreases price towards P_e.

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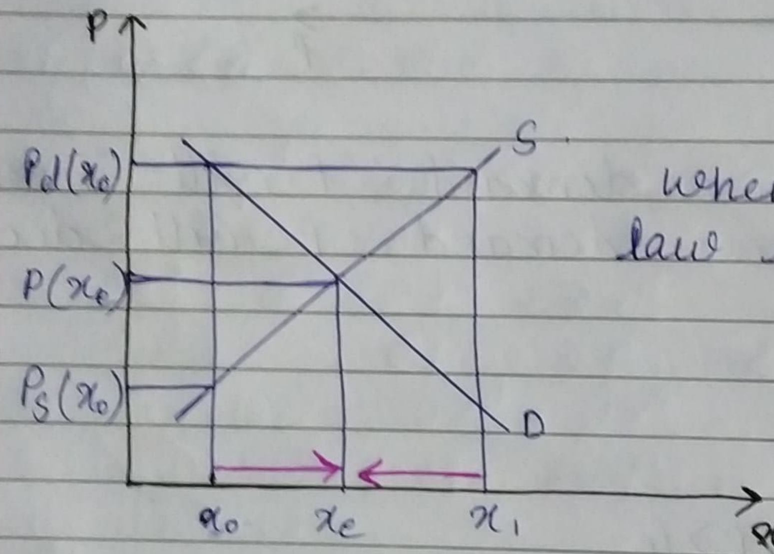
Marshallian Static Stability

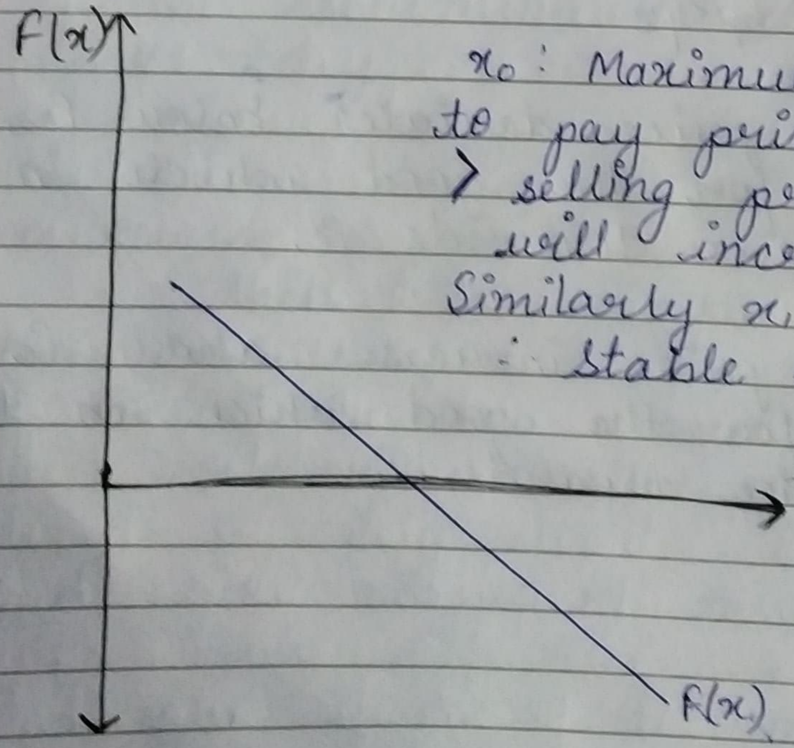
Adjustment mechanism: quantity

Condition: ~~Rule~~
Rule 1: If $P_d > P_s$, x increases.

Rule 2: If $P_d < P_s$, x decreases.



when supply & demand law are followed.



x_0: Maximum willingness to pay price by consumer > selling price ∴ quantity will increase.
Similarly x_1 will decrease
∴ Stable eqm.

Given rules 1 & 2, a market is Marshallian Static stable.

i) $p^d > p^s \;\forall\; x < x_e$

ii) $p^d < p^s \;\forall\; x > x_e$

Combining (i) & (ii) $f'(x) < 0$

Marshallian static stability requires:

a) $f(x) < 0 \;\forall\; x > x_e$

b) $f(x) > 0 \;\forall\; x < x_e$.

Global stability:

$f'(x) < 0 \;\forall\; x \geq 0.$

Local stability:

$f'(x) < 0 \;\forall\; x \in [x_e - \varepsilon, x_e + \varepsilon].$

Difference between the two approaches

When dd and ss curves ~~do~~ slope in opposite direction the two concepts give us identical results.

When dd and ss curves slope ~~in~~ ~~same~~ in same direction the two concepts give us polar opposite results.

Proof:

If both dd & ss law are violated, then it will be unstable eqm & from both the processes, we will get the same result as they are in opposite direction

Marshallian Static stability requires

$F'(x) < 0$

i.e. $\frac{\partial p^d}{\partial x} - \frac{\partial p^s}{\partial x} < 0$

$$\frac{\frac{\partial x^s}{\partial p} - \frac{\partial x^d}{\partial p}}{\frac{\partial x^d}{\partial p} \cdot \frac{\partial x^s}{\partial p}} < 0.$$

If the denominator is > 0. (which happens when the dd and ss curves slope in the same direction)

then $(F') < 0$ implies $(E') > 0$.

If one of the law is violated then both dd & ss curves will either both be +vely sloped or -vely sloped. ∴ Both approaches will give different result.

And also if one of the ~~two~~ laws is

violated then the market will become stable in one approach & unstable in the other approach.

$\cancel{\partial x^d}$ $\frac{\partial x^d}{\partial p} \cdot \frac{\partial x^s}{\partial p}$ ⇒ (Slope of dd) * (Slope of ss)

∴ If $\frac{\partial x^d}{\partial p} \cdot \frac{\partial x^s}{\partial p} > 0$ ⇒ Both slope in the same direction

∴ One of the laws is violated

i.e. if $(F') < 0$ then $(E') > 0$

& if $(F') > 0$ then $(E') < 0$.

And if $\frac{\partial x^d}{\partial p} \cdot \frac{\partial x^s}{\partial p} < 0$ ⇒ Then slopes of dd & ss are in oppo. direction

∴ Either both laws are valid or both are violated.

∴ It will be a stable or unstable eqm for both approaches.

i.e. $(F') < 0$ ⇒ $(E') < 0$ (stable eqm)

or $(F') > 0$ ⇒ $(E') > 0$. (Unstable eqm).

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* Optimum Choice

Most preferred bundle (MPB) amongst the feasible bundles.

It is the point at which slope of indifference curve & budget line is equated.

Optimum choice is characterized by the commodity bundle for which

$\boxed{MRS = \frac{P_1}{P_2}}$ and $\boxed{\overline{M} = PX}$

It is the point of tangency b/w IC & budget line. The optimum should lie on the budget line.

$\overline{M} = PX$ i.e. ~~Total income = Total expendi~~
Total income = Total expenditure.
i.e. the consumer is neither saving nor borrowing.

Any commodity bundle on the budget line is strictly preferred over any commodity bundle inside the budget line (or in the budget set).
Budget set : Set of all commodity bundles which are affordable by the consumer

Budget line is the boundary or upper limit of the budget set.

~~X ∋~~ T.P.: Any commodity bundle, say, $X \ni \overline{M} = PX$ will be strictly preferred to $X' \ni \overline{M} > PX'$

Proof:

Let $X = \{x_1, x_2\} \rightarrow$ on the budget line
$X' = \{x_1', x_2'\} \rightarrow$ inside the budget set.

By presumption, atleast for one i, $x_i' < x_i$

$$P_1 x_1 + P_2 x_2 = \overline{M}$$
$$P_1 x_1' + P_2 x_2' < \overline{M} \quad (\text{Expenditure} < \text{Income})$$

∴ The consumer is saving a part of his income.

Now if

(a) $x_i' < x_i \ \forall \ i \Rightarrow XPX'$

(b) (~~x₁' < x₁~~ $x_1' < x_1$, $x_2' = x_2$) or ($x_1' = x_1$, $x_2' < x_2$)

(c) $x_1' < x_1$ but $x_2' > x_2$ such that $\Delta x_1 > \Delta x_2$.

Define $X'' = \{x_1'', x_2''\} \ni X'' I X'$ & ~~x₁~~ $x_1'' < x_1$, $x_2'' < x_2$

∴ XPX'' & $X'' I X' \Rightarrow XPX'$ (By Transitivity)

∴ Optimum will also be on budget line because all bundles on budget line are strictly preferred over all bundles in budget set.

Optimum choice will be the point at which the indifference curve is tangent to the budget line.

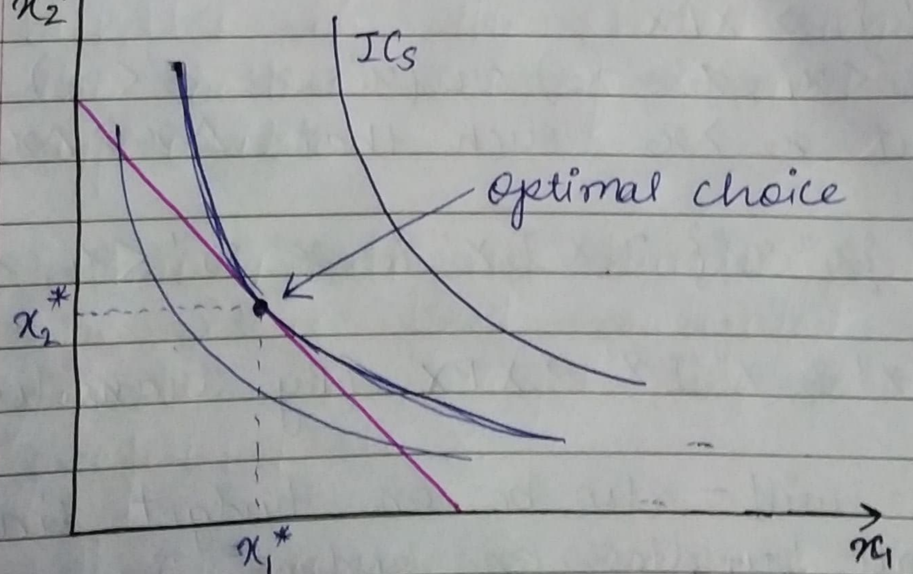
Proof: Any commodity bundle $X \ni \bar{M} = PX$ will be strictly preferred to any other bundle if at X an IC is tangent to the budget line, i.e., XPY.

By the axioms of strict convexity of preferences and transitivity at any other bundle Y (on the budget line)

$$Y \ni \bar{M} = PY.$$

the ICs must be cutting the budget line.

X lies in the better set to Y.



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There will be only one point at which the indifference curve will be tangential with the budget line because of transitivity. i.e. indifference curves cannot cut each other.

At any other point Y on the budget some IC must be cutting the budget line at that point and as ICs cannot intersect ∴ point X & Y should lie on different ICs.

∴ By axiom of convexity & monotonicity, X will lie in the better set of Y.

Remark 1:

As long as preference is monotonic and strictly convex, the optimum choice is the one where IC is tangent to the budget line, that is,

(a) $MRS = \frac{P_1}{P_2}$

(b) $\overline{M} = PX$.

Remark 2:

The consumer doesn't save at the optimum. Because the optimum always occurs on the budget line.

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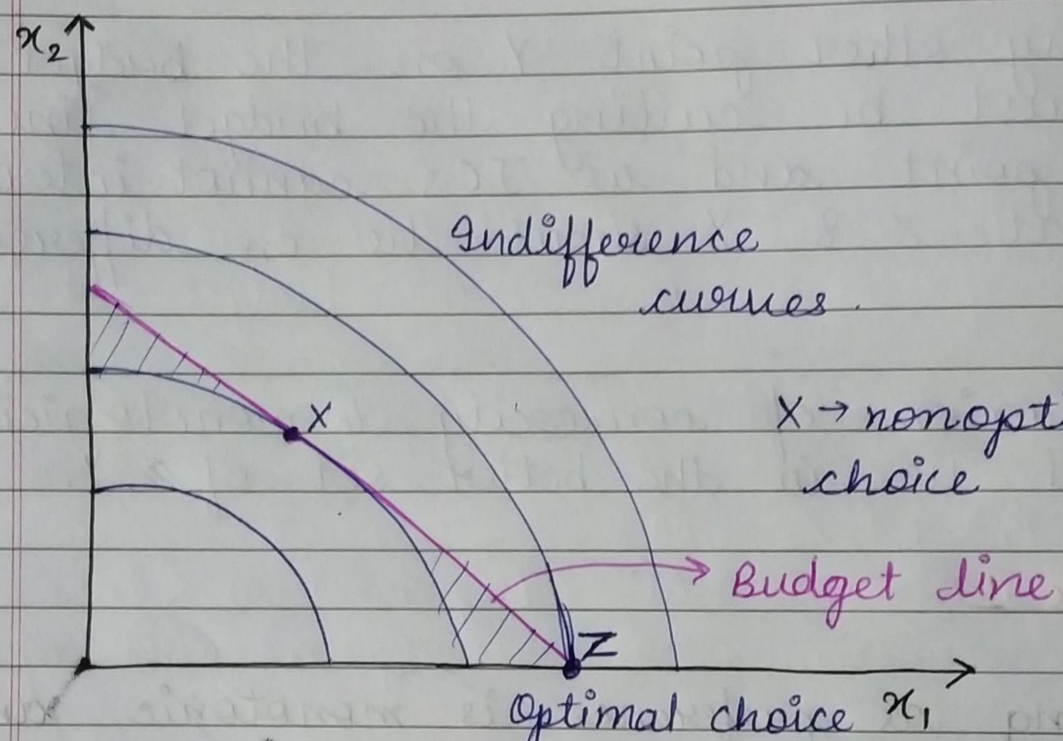
Concave preference.

The bundle for which (a) and (b) are satisfied is in fact the least preferred bundle among those for which $M = PX$.



Monotonicity still holds, therefore the bundles in the shaded region are in the better set of X and are also affordable.

∴ Among all the bundles on the budget line, X is least preferred i.e. all other points on budget line are strictly preferred to X.

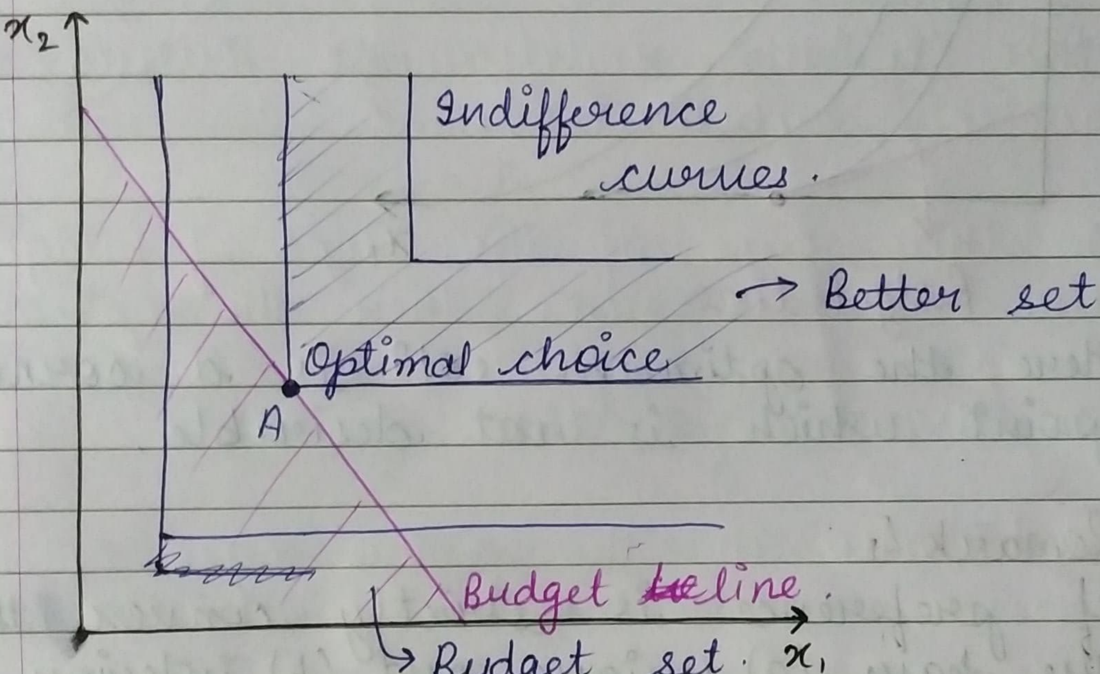
∴ If preference is concave, the point of tangency is the least preferred bundle.

∴ Point of tangency will be the optimum choice only when the preference ~~is~~ is strictly convex and monotonic.

Remark 3:

A MPB is the one where the better set and budget sets are non-overlapping.

Upper set is better for monotonic preference



A is the optimal choice even though it is not the point of tangeny of an IC and budget line.

But at point A, better set & ~~but~~ budget set are non-overlapping, ∴ it is the optimum solⁿ ∴ the optimum solⁿ need not be the point of tangency.

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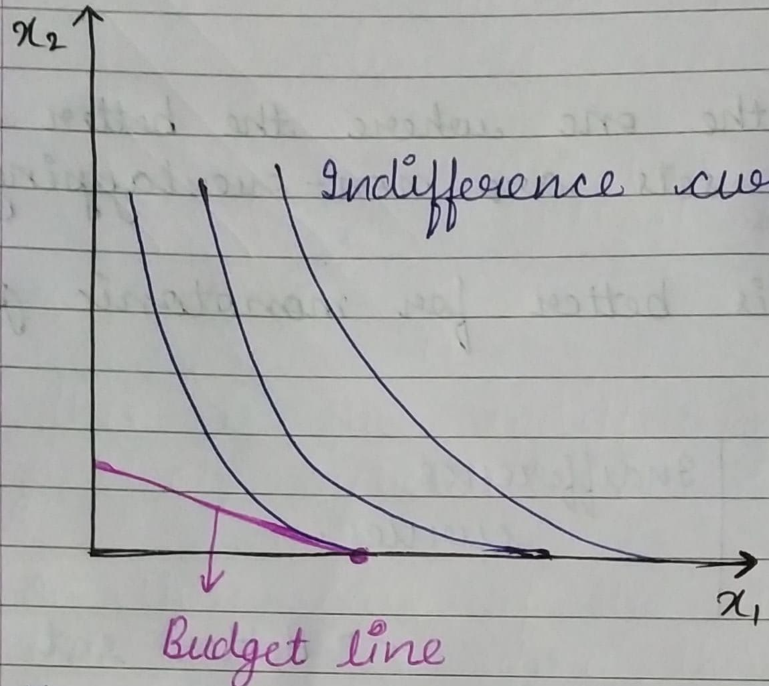
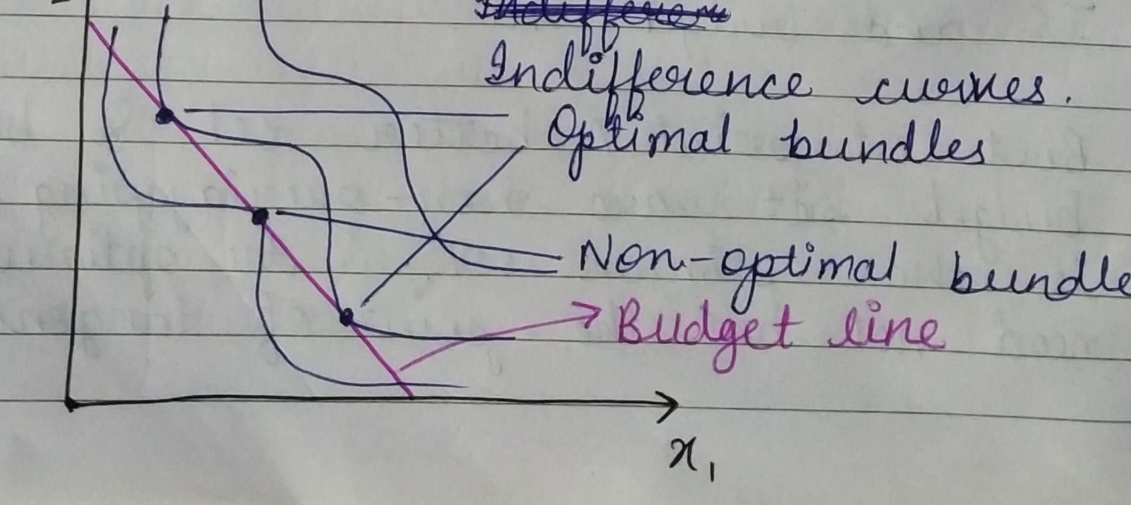
Also, in this case, $MRS \neq \frac{P_1}{P_2}$

as at the optimum sol^n, ~~sol~~ slope of the IC is not defined.



Here, the optimum sol^n is a corner point which is not desirable.

Remark 4 :
If preference is strictly convex then we have (a) unique and (b) interior optimum.



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∴ In this, ~~indiff~~ indifference curve is not strictly convex, i.e. there are multiple points of tangencies ∴ multiple optimum sol^n's.

Proof:

If preference is unique then
$MRS(X) \neq MRS(X') \ \forall \ X \ \& \ X'$
and $\overline{M} = PX$ & $\overline{M} = PX'$
i.e. X & X' lies on the budget line.
If X is the MPB such that $MRS(X) = P_1/P_2$, then we cannot have
$$MRS(X') = \frac{P_1}{P_2}.$$

Let both X & Y be the MPBs.

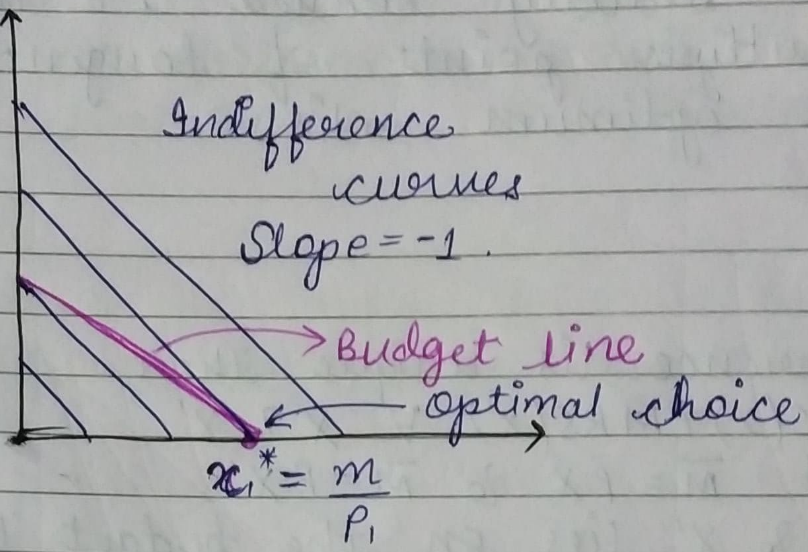
By tangency $MRS(X) = \frac{P_1}{P_2} = MRS(Y)$.

which implies $MRS(X) = MRS(Y)$
which is not possible.
∴ X is unique.

Remark 5:

For non convex and concave preference most preferred bundle "may" not be unique and interior one.

But the converse may not be true because unique & interior optimum doesnt necessarily imply preference is convex

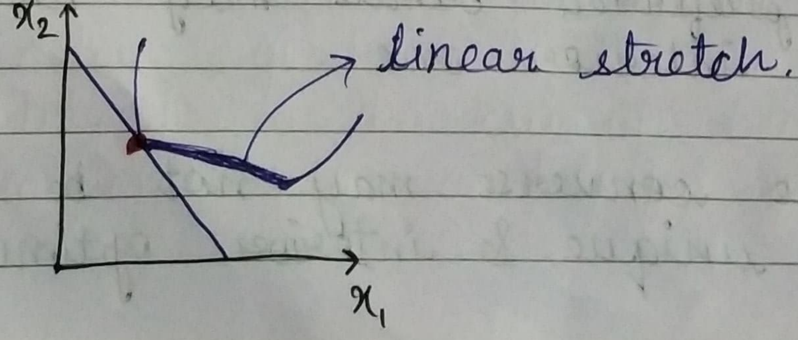


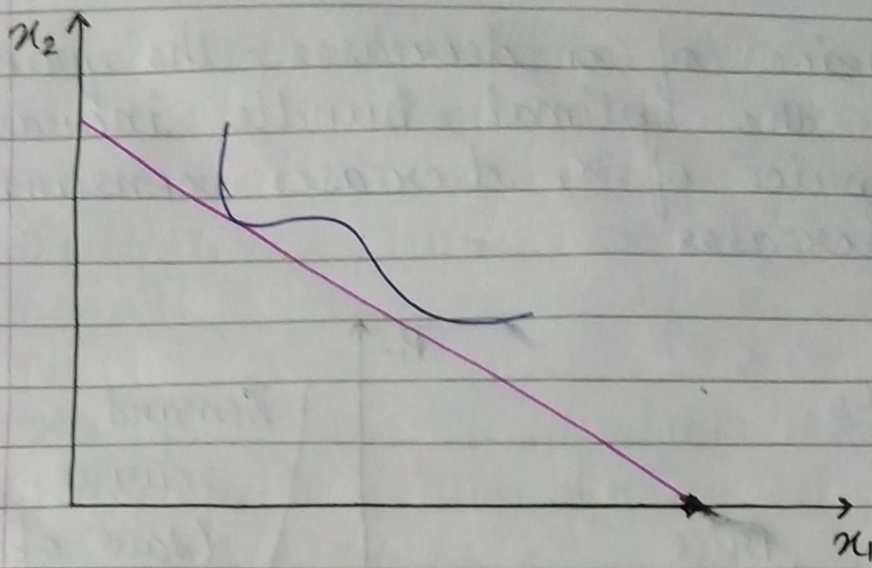
This is the indifference curve with perfect substitutes.

In this case, if the slope of budget line & IC is same, then the optimal choices are infinite i.e. all present on the budget line.

And also, in the above case, optimal choice is a corner point i.e. not interior.

It is also possible for a non convex preference to have unique & interior optimum.





Strict convexity of preference is a sufficient condⁿ to have a unique & interior optimum but not a necessary condⁿ.

* Change in Price

We consider that given income $\overline{M}$ & price of good 2 i.e. P_2, P_1 ~~increas~~ falls.

∴ When price of good 1 falls, purchasing power wrt good 1 will rise.