AIFA Class Test 2

Model answers

Part A

1a. Deduction system D is sound: If a formula f (or null clause) is derived using D, then it is correctly derived.
Deduction system D is complete: If a formula f (or null clause) can be derived logically then it will be derived using D.

1b. [Sound] The resolution rule is a ~~valid rule~~ valid rule.
C1: $f \vee a$ and C2: $g \vee \sim a$
Then we can derive C3: $f \vee g$.
It can be proved that $C1 \wedge C2 \Rightarrow C3$ is a valid ~~formula~~ formula. Therefore all new clauses derived by the resolution rule are correctly derived. So, when NULL clause is derived, it is correctly derived.

[Complete] Because the number of propositions is finite, so eventually either NULL will be derived or no other new clause will be derived. If NULL is not derived, then we can show a satisfying assignment exists.

1c. [Sound] Resolution and unification is a valid rule.
<Same as 1b>

[Complete] Predicate logic is undecidable. Therefore, there cannot exist any complete method. Resolution refutation is partially complete – if NULL can be derived, then it will be derived, else the system may not terminate.

Part B

2a. p : Swapna wrote on paper.

t : Swapna typed the answers.

h : Swapna has a camera

c : Swapna completed in time.

2b.

F_1 : $(p \wedge \sim t) \vee (\sim p \wedge t)$

$\Rightarrow (p \vee t) \wedge (\sim p \vee \sim t)$

F_2 : $(p \vee \sim h) \rightarrow \sim c$

$\Rightarrow \sim(p \vee \sim h) \vee \sim c$

$\Rightarrow (\sim p \wedge h) \vee \sim c$

$\Rightarrow (\sim p \vee \sim c) \wedge (h \vee \sim c)$

F_3 : $\sim c$

G : p

$\sim G$: $\sim p$

2c. Using resolution refutation

C_1 : $p \vee t$

C_2 : $\sim p \vee \sim t$

C_3 : $\sim p \vee \sim c$

C_4 : $h \vee \sim c$

C_5 : $\sim c$

C_6 : $\sim p$

C_1 & $C_6 \Rightarrow t$ .... (C_7)

C_2 & $C_7 \Rightarrow \sim p$ ... (C_8)

From here, we cannot derive the NULL clause. So, the conclusion cannot be derived from the given clause.

Using tree method.

$F_1 \wedge F_2 \wedge F_3 \rightarrow G$

$(p \vee t) \wedge (\sim p \vee \sim t) \wedge (\sim p \vee \sim c) \wedge (h \vee \sim c) \wedge \sim c \rightarrow p$

$p = T$ / \ $p = F$

$t \wedge (h \vee \sim c) \wedge (\sim c) \rightarrow F$

$c = T$ / \ $c = F$

$t \rightarrow F$

$t = T$ / \ $t = F$

$\boxed{T \rightarrow F}$

Since $F_1 \wedge F_2 \wedge F_3 \rightarrow G$ is not consistent for at least one branch, the conclusion cannot be derived from the given facts.

Part C

3a. Predicates:

$q(x)$: x is a question
$c(x)$: x is a candidate
$ans(x,y)$: x answers y.

3b. Encoding:

$F1: \exists z[q(z) \land \forall x\{c(x) \to \{(\exists y\ ans(x,y)) \to ans(x,z)\}\}]$

$F2: \forall x\{c(x) \to \exists y\{q(y) \land ans(x,y)\}\}$

$G: \exists x\{q(x) \land \forall y\{c(y) \to ans(y,x)\}\}$

3c. Clausal Forms:

$F_1: \exists z[q(z) \land \forall x\{c(x) \to \{(\exists y\ ans(x,y)) \to ans(x,z)\}\}]$

$$\Rightarrow \underbrace{q(A)}_{C_1} \land \underbrace{(\sim C(x) \lor \sim ans(x,y) \lor ans(x,A))}_{C_2}$$

$F_2: \forall x\{c(x) \to \exists y\{q(y) \land ans(x,y)\}\}$

$$\Rightarrow \underbrace{\{\sim c(x) \lor q(f(x))\}}_{C_3} \land \underbrace{\{\sim c(x) \lor ans(x,f(x))\}}_{C_4}$$

$\sim G: \forall x\{\sim q(x) \lor \exists y\{c(y) \land \sim ans(y,x)\}\}$

$\Rightarrow \forall x\{\sim q(x) \lor \{c(g(x)) \land \sim ans(g(x),x)\}\}$

$$\Rightarrow \underbrace{\{\sim q(x) \lor c(g(x))\}}_{C_5} \land \underbrace{\{\sim q(x) \lor \sim ans(g(x),x)\}}_{C_6}$$

3d. Resolution Refutation:

$C_1: q(A)$
$C_2: \sim c(x) \lor \sim ans(x,y) \lor ans(x,A)$
$C_3: \sim c(x) \lor q(f(x))$
$C_4: \sim c(x) \lor ans(x,f(x))$
$C_5: \sim q(x) \lor c(g(x))$
$C_6: \sim q(x) \lor \sim ans(g(x),x)$

$c(g(A))$

$\sim ans(g(A),y) \lor ans(g(A),A)$

$\sim c(g(A)) \lor ans(g(A),A)$

$\sim q(A) \lor \sim c(g(A))$

$\sim c(g(A))$

NULL

Since NULL clause can be derived, the conclusion can be drawn from the facts.