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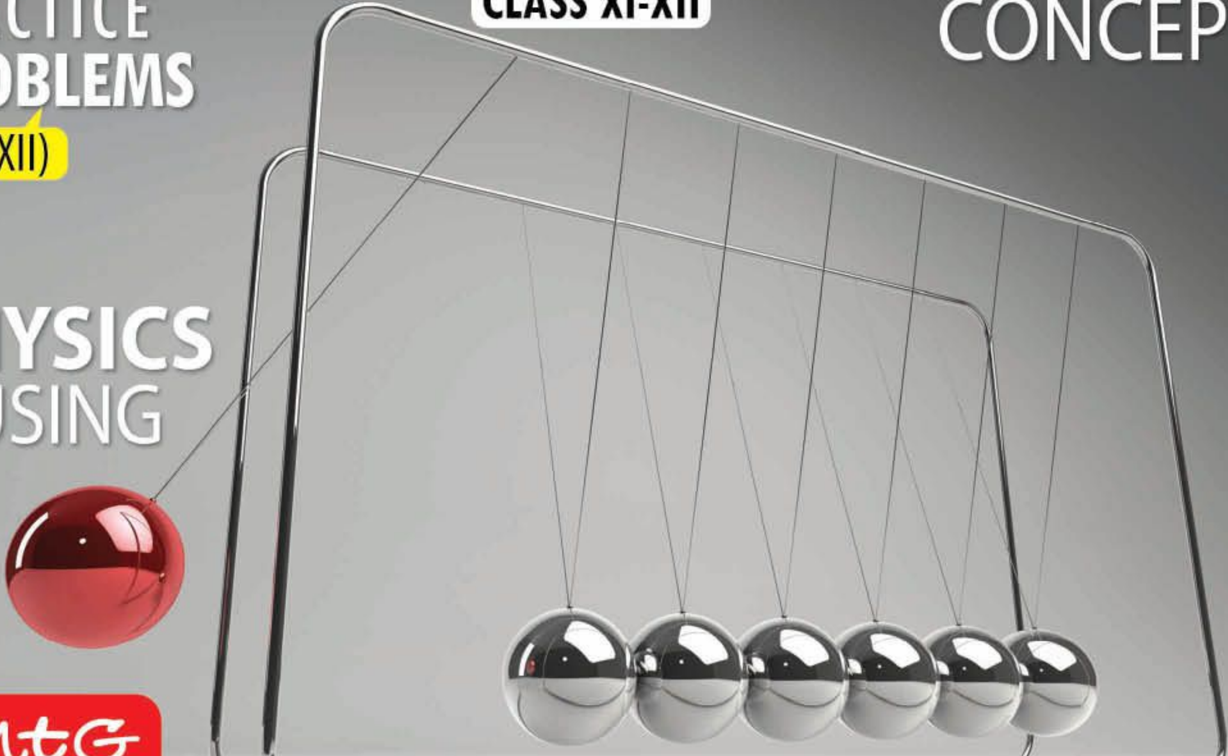

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PHYSICS for you

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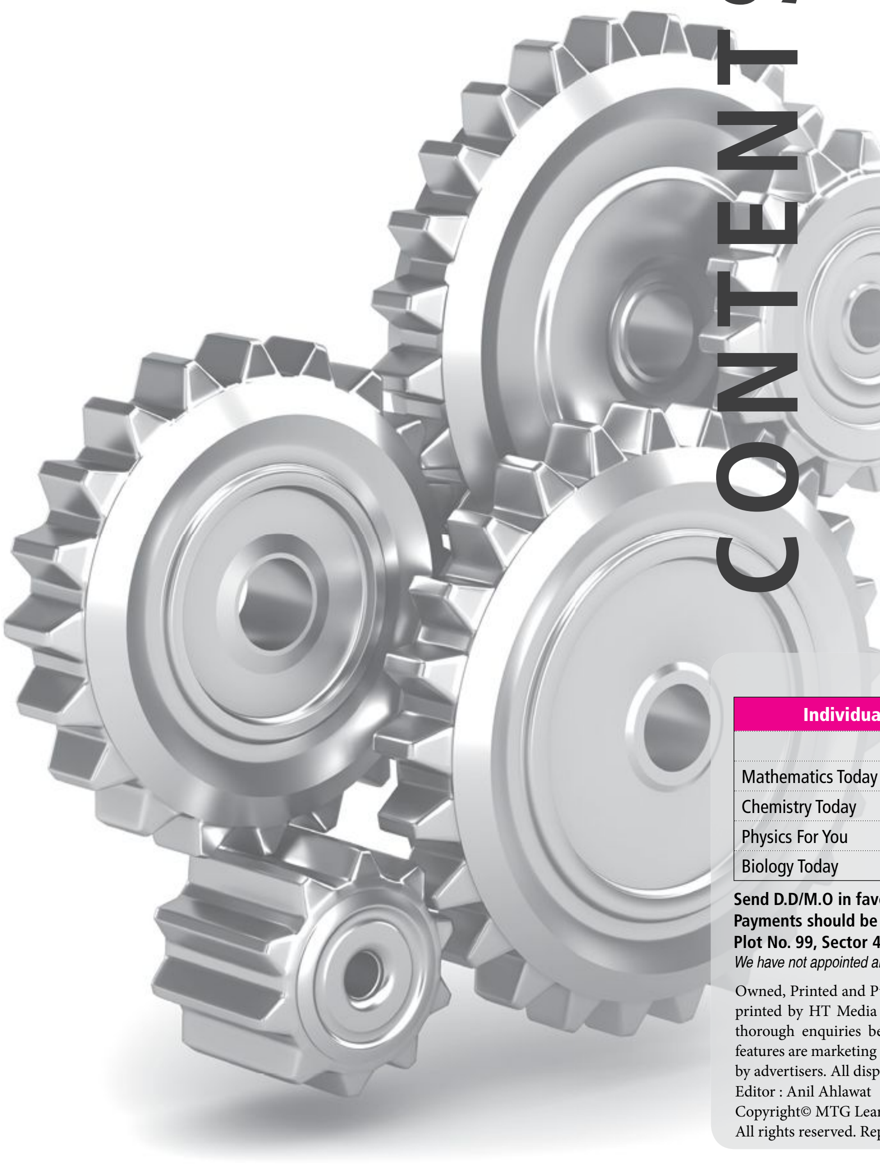
CONTENTS



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NEET | JEE ESSENTIALS

Class XI

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Unit 3 LAWS OF MOTION WORK, ENERGY AND POWER

NEWTON'S FIRST LAW OF MOTION

- A body continues to be in its state of rest or uniform motion along a straight line, unless it is acted upon by some external force to change the state.

State of uniform motion

State of rest (Special case of uniform motion with zero speed)

- Inertia is the property of the body to which body opposes the change of its state.
- Quantitatively, inertia of a body $\propto$ mass of the body.
- **Different types of inertia**

Inertia of rest	Inertia of motion	Inertia of direction
It is the inability of a body to change its state of rest.	It is the inability of a body to change its state of uniform motion.	It is the inability of a body to change its direction of motion.
When we shake a branch of a mango tree, the mangoes fall down.	When a bus or train stops suddenly, passengers sitting inside it tends to fall forward.	Rotating wheels of the vehicle throw out mud, mudguards over the wheels stop this mud.

NEWTON'S SECOND LAW OF MOTION

- The rate of change of linear momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts.
- Mathematically, $\vec{F} = \frac{d\vec{p}}{dt}$
- It can be shown experimentally that the acceleration of an object is
  - ➤ proportional to the net force $\vec{F}_{net}$ on the object and
  - ➤ inversely proportional to the mass m of the object.

  $a \propto F_{net}$ and $a \propto \frac{1}{m}$.

  Thus, $a \propto \frac{F_{net}}{m} \Rightarrow a = \frac{kF_{net}}{m}$,

  The value of constant k is 1 in SI unit.

  $\therefore\ a = \frac{F_{net}}{m}$, $F_{net} = ma$

Forces are Balanced

$a = 0\ \text{m s}^{-2}$

Object at rest	Object in Motion
($v = 0$ m s^{-1})	($v \neq 0$ m s^{-1})
$\Downarrow$	$\Downarrow$
Stays at rest	Stays in Motion (Same speed and direction)

Forces are unbalanced

There is an acceleration.

The acceleration depends directly upon the net force.	The acceleration depends inversely upon the mass of the object.

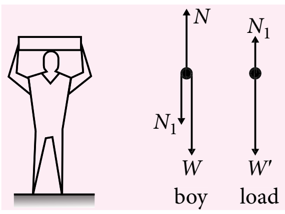
- More fundamental relationship

$$F_{\text{net external}} = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$$

NEWTON'S THIRD LAW OF MOTION

- It states that to every action, there is an equal (in magnitude) and opposite (in direction) reaction.
- If a body A exerts a force $\vec{F}$ on another body B then B exerts a force $-\vec{F}$ on A. The two forces act along the same line.
- Action and reaction never cancel each other *i.e.*, they cannot balance each other as they act on different bodies.
- The forces of action and reaction may appear due to actual physical contact of the two bodies or even from a distance.
- This law is applicable whether the bodies are at rest or in motion.
- Single isolated force is not possible, force is always possible in action reaction pair form.

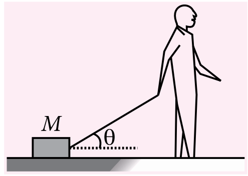
FREE BODY DIAGRAMS



- Body is represented by a dot and each external force is represented by a vector with its tails on the dot.
- Consider a system of a boy and a heavy load. The boy stands on the floor balancing a heavy load on his head.
- List of forces on boy and load

System	Force exerted by	Magnitude of the force	Direction of the force	Nature of the force
Boy	Earth Floor Load	W N N_1	Downward Upward Downward	Gravitational Electromagnetic Electromagnetic
Load	Earth Boy	W' N_1	Downward Upward	Gravitational Electromagnetic

Illustration 1 : A block of mass M is pulled on a smooth horizontal table by a string making an angle θ with the horizontal as shown in figure.

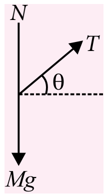


If the acceleration of the block is a, find the force applied by the string and by the table on the block.

Sol.: Let us consider the block as the system. The forces on the block are

(a) pull of the earth, Mg vertically downward,
(b) contact force by the table N, vertically upward,
(c) pull of the string, T, along the string.

The free body diagram for the block is shown in figure.



The acceleration of the block is horizontal and towards the right. Take this direction as the x-axis and vertically upward direction as the y-axis. We have,

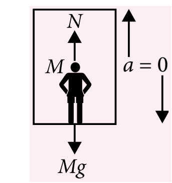
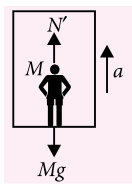
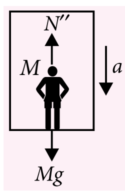
component of Mg along the x-axis = 0
component of N along the x-axis = 0
component of T along the x-axis = $T\cos\theta$.
Hence the total force along the x-axis = $T\cos\theta$.
Using Newton's second law of motion, $T\cos\theta = Ma$ ...(i)
Component of Mg along the y-axis = $-Mg$
Component of N along the y-axis = N
Component of T along the y-axis = $T\sin\theta$.
Total force along the Y-axis = $N + T\sin\theta - Mg$.
Using Newton's second law, $N + T\sin\theta - Mg = 0$ ...(ii)

From equation (i), $T = \dfrac{Ma}{\cos\theta}$.

Putting this value of T in equation (ii)

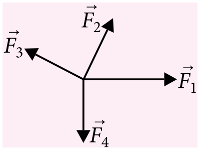
$N = Mg - Ma\tan\theta$.

APPARENT WEIGHT OF A BODY IN MOVING LIFT

Vertically upward or downward with uniform velocity ($a = 0$)	Vertically upward with uniform acceleration a	Vertically downward with uinform acceleration a
		
$N = Mg$ = Actual weight of body	$N' = M(g + a)$ $\Rightarrow N' > N$	$N'' = M(g - a)$ $\Rightarrow N'' < N$

EQUILIBRIUM OF FORCES

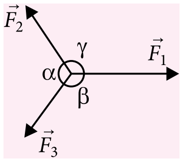
- **Concurrent coplanar forces :** If all forces are in equilibrium, or $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$, then we can write,



$$\Sigma F_x = 0 \text{ and } \Sigma F_y = 0$$

where x and y are any two mutually perpendicular directions.

- **Lami's Theorem :** If a body is in equilibrium under three concurrent forces as shown in figure. Then we can write,



$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

- **Non concurrent coplanar forces :** If a body is in equilibrium under non concurrent coplanar forces we can write, $\Sigma F_x = 0, \Sigma F_y = 0$ and Σ(moment about any point) = 0

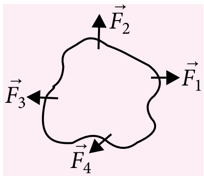
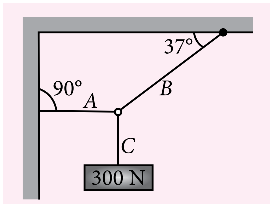


Illustration 2 : A block of mass 30 kg is suspended by three strings as shown in figure. Find the tension in each string.

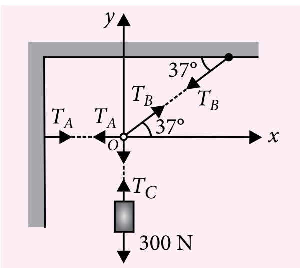


Sol.: Considering equilibrium of each part of system.

The whole system is in equilibrium; therefore, for each part $\Sigma\vec{F} = 0$.

From the free-body diagram of block C, $T_C = 300$ N.

Now consider the equilibrium of point O,



$\Sigma F_x = 0$

or $T_B \cos 37° - T_A = 0$

$\therefore \quad T_A = T_B \cos 37°$

$$T_A = T_B \cdot \frac{4}{5} \qquad \text{...(i)}$$

and $\Sigma F_y = 0$

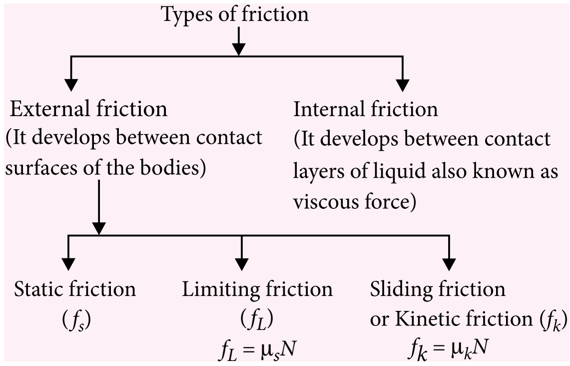
or $T_B \sin 37° - T_C = 0$ ...(ii)

$$\therefore \quad T_B = \frac{T_C}{\sin 37°} = \frac{300}{3/5} = 500 \text{ N}$$

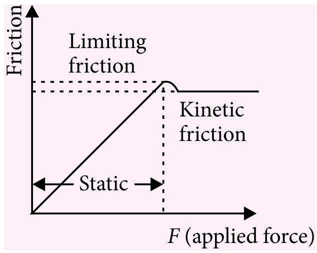
From Eq. (i), we get $T_A = \frac{4}{5}T_B = \frac{4}{5} \times 500 = 400$ N

FRICTION

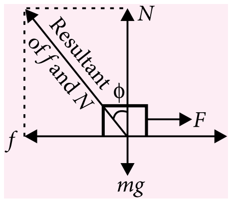
- It is a tangential component of net contact force between two bodies in contact.



- Variation of friction force with applied force

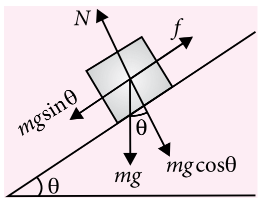


- **Angle of Friction (ϕ) :** The angle which the resultant of the force of limiting friction f and normal reaction N makes with the direction of normal reaction N.



$$\tan\phi = \frac{f}{N} = \mu \text{ or, } \phi = \tan^{-1}\mu$$

- **Angle of Repose :** It is defined as the maximum angle of inclination of a plane with the horizontal at which a body placed on it is at rest.



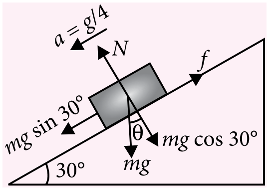
$$f = mg\sin\theta \text{ and } N = mg\cos\theta$$

$$\text{So, } \mu_s = \frac{f}{N} = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta \Rightarrow \theta = \tan^{-1}\mu_s$$

  - This fact is used for finding the coefficient of static friction in the laboratory.
  - Angle of repose (θ) = Angle of friction (ϕ)

Illustration 3 : A block slides down an incline of angle 30° with an acceleration $g/4$. Find the coefficient of kinetic friction.

Sol.: Let the mass of the block be m. The forces on the block are



(a) mg downward by the earth (gravity),
(b) N normal force by the incline and
(c) f up the plane, (friction) by the incline.

Taking components parallel to the incline and using Newton's second law,

$$mg \sin 30° - f = \frac{mg}{4} \quad \text{or} \quad f = \frac{mg}{4}$$

There is no acceleration perpendicular to the incline.

Hence, $N = mg \cos 30° = mg \cdot \frac{\sqrt{3}}{2}$.

As the block is slipping on the incline, friction is $f = \mu_k N$.

So, $\mu_k = \frac{f}{N} = \frac{mg}{4 mg\sqrt{3}/2} = \frac{1}{2\sqrt{3}}$

CIRCULAR TURNINGS AND BANKING OF ROADS

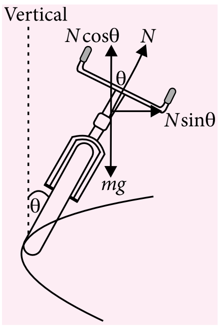
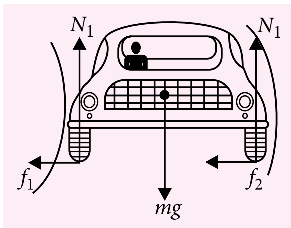
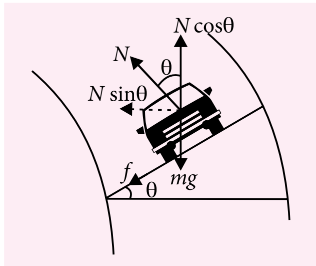
Bending of cyclist on circular turning for safe going	Motion of a car on a level road	Motion of a car on a banked circular road
		
• Component of normal reaction $N \sin\theta$ provides centripetal force. • Maximum speed of cyclist, $v = \sqrt{rg \tan\theta}$	• Friction force (μN) between tyres and road provides a suitable centripetal force. • Maximum speed of car for safe turning $v = \sqrt{\mu rg}$	• Component of friction force ($f \cos \theta$) and component of normal reaction ($N \sin \theta$) provide required centripetal force for safe turning. • The maximum permissible speed to avoid slipping $v_{max} = \left[\frac{rg(\mu + \tan\theta)}{1 - \mu \tan\theta}\right]^{1/2}$ • If road is smooth then $\mu = 0$ and $v_{max} = \sqrt{rg \tan\theta}$

Illustration 4 : A circular race track of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race car and the road is 0.2, what is the

(i) optimum speed of the race car to avoid wear and tear on its tyres, and

(ii) maximum permissible speed to avoid slipping?

Sol.: Given r = 300 m, θ = 15°, μ = 0.2

(i) The optimum speed of the race car will be

$$v_0 = \sqrt{rg \tan\theta} = \sqrt{300 \times 9.8 \times \tan 15°}$$
$$= \sqrt{300 \times 9.8 \times 0.2679} = 28.1 \text{ m s}^{-1}$$

(ii) The maximum permissible speed of race car will be

$$v_{max} = \sqrt{rg \frac{\mu + \tan\theta}{1 - \mu \tan\theta}}$$
$$= \sqrt{300 \times 9.8 \times \frac{0.2 + 0.2679}{1 - 0.2 \times 0.2679}} = 38.1 \text{ m s}^{-1}$$

Work	Energy	Power
refers to an activity involving a force and movement in the direction of the force. A force of 20 N pushing an object 5 m in the direction of the force does 100 J of work.	is the capacity for doing work. You must have energy to accomplish work. It is like the currency for performing work. To do 100 J of work, you must expend 100 J of energy.	is the rate of doing work or the rate of using energy, which are numerically the same. If you do 100 J of work in 1 s (using 100 J of energy), the power is 100 W.

WORK

- Work is said to be done on a body only if a force acts on the body and the point of application of the force moves in the direction of the force.

Work Done

- **By a constant force**

$$W = \vec{F}\cdot\vec{s} = \vec{F}\cdot\left(\vec{r}_f - \vec{r}_i\right) = Fs\cos\theta$$

$= \text{Force} \times \text{displacement in the direction of force.}$

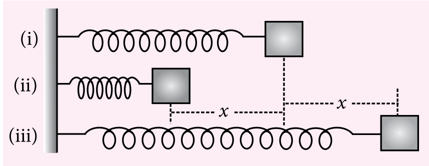
- **By a variable force**

$$W = \int_{x_i}^{x_f} F\cdot dx, \text{ where } F = f(x)$$

- **By area under *F*-*x* graph :** If force is a function of x, we can find work done by area under F-x graph with projection along x-axis. Work done can be obtained by area under F-x graph, but sign of work done should be decided by you. If force and displacement both are positive or negative, work done will be positive. If one is positive and other is negative then work done will be negative.

- **Work done by spring-force**
  - $W = \int_{x_1}^{x_2} -kx\,dx = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$

Initial state of the spring	Final state of the spring	x_1	x_2	W
Natural	Compressed	0	$-x$	$-\frac{1}{2}kx^2$
Natural	Elongated	0	x	$-\frac{1}{2}kx^2$
Elongated	Natural	x	0	$\frac{1}{2}kx^2$
Compressed	Natural	$-x$	0	$\frac{1}{2}kx^2$
Elongated	Compressed	x	$-x$	0
Compressed	Elongated	$-x$	x	0



(i) Spring is in its natural length.
(ii) Spring is elongated by an amount x.
(iii) Spring is compressed by an amount x.

Conservative and Non-conservative Forces

- All central forces are conservative but all conservative forces are not central forces.
- Forces acting along the line joining the centres of two bodies are called central forces. Gravitational and electrostatic forces are central forces.
- The concept of potential energy exists only in the case of conservative force.

Conservative force	Non-conservative force
• Work done does not depend on the path.	• Work done depends on the path.
• Work done in a round trip is zero.	• Work done in a round trip is not zero.
• When only a conservative force acts within a system, the kinetic energy and potential energy can change. However their sum, the mechanical energy of the system does not change.	• Work done against a non-conservative force may be dissipated as heat energy. Hence, the mechanical energy of the system changes.
• Work done is completely recoverable.	• Work done is not completely recoverable.

Illustration 5 : A particle of mass 20 g is thrown vertically upwards with a speed of 10 m s^{-1}. Find the work done by the force of gravity during the time the particle goes up.

Sol.: Suppose the particle reaches a maximum height h. As the velocity at the highest point is zero, we have

$$0 = u^2 - 2gh$$

or $$h = \frac{u^2}{2g}$$

The work done by the force of gravity is

$$-mgh = -mg\frac{u^2}{2g} = -\frac{1}{2}mu^2$$

$$= -\frac{1}{2}(0.02 \text{ kg}) \times (10 \text{ m s}^{-1}) = -1.0 \text{ J}$$

Kinetic Energy

- Kinetic energy (KE) is the capacity of a body to do work by virtue of its motion.
- If a body of mass m has a velocity v its kinetic energy is equivalent to the work which an external force would have to do to bring the body from rest upto its velocity v.

  $$\text{KE} = \frac{1}{2}mv^2$$

  Work done by the constant force = Fs

  $$W = (ma)\left(\frac{v^2}{2a}\right) = \frac{1}{2}mv^2$$
- Since, both m and v^2 are always positive. KE is always positive and does not depend on the direction of motion of the body.
- Kinetic energy depends on the frame of reference. For example, the kinetic energy of a person of mass m sitting in a train moving with speed v is zero in the frame of train but $\frac{1}{2}mv^2$ in the frame of earth.

Potential Energy

- Potential energy is defined only for conservative forces.
- In a conservative force field, difference in potential energy between two points is the negative of work done by conservative forces in displacing the body (or system) from some initial position to final position. Hence,

  $$\Delta U = -W \quad \text{or} \quad U_B - U_A = -W_{A \to B}$$
- Absolute potential energy at a point can be defined with respect to a reference point where potential energy is assumed to be zero. Reference point corresponding to gravitational potential energy and electrostatic potential energy is assumed at infinity. Reference point corresponding to spring potential energy is taken at a point at natural length of spring. Now, negative of work done in displacement of body from reference point (say O) to the point under consideration (say P) is called absolute potential energy at P. Thus,

  $$U_P = -W_{O \to P}$$
- Potential energy in different conservative fields
  - Gravitation potential energy of a body at height h from earth's surface, $U_g = -W_g = mgh$
  - Electrostatic potential energy between two point charges q_1 and q_2 is

    $$U_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r};$$

    r = separation between charge particles.
  - Elastic potential energy : Reference point is taken at natural length of the spring.

    If x is expansion / compression in spring then potential enerrgy of spring (Energy stored in spring) is $U = \frac{1}{2}kx^2$ and spring force $F = -kx$ (directed towards natural length).
- Relation between potential energy and conservative force
  - If U is function of only one variable, then

    $$F = -\frac{dU}{dr} = -\text{ slope of } U\text{-}r \text{ graph.}$$
  - If U is a function of three coordinate variables x, y and z, then

    $$\vec{F} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right]$$

Work-Energy Theorem

- It states that work done by all forces acting on a body is equal to the change in the kinetic energy of the body.
- This theorem is valid for a system in presence of all types of forces (external or internal, conservative or non-conservative).
- This theorem can be applied to non-inertial frame also.

Illustration 6 : Two charged particles A and B repel each other by a force $\frac{k}{r^2}$, where k is a constant and r is the separation between them. The particle A is clamped to a fixed point in the lab and the particle B which has a mass m, is released from rest with an initial separation r_0 from A. Find the change in the potential energy of the two-particle system as the separation increases to a large value. What will be the speed of the particle B in this situation?

Sol.: There are two internal forces; F_{AB} acting on A and F_{BA} acting on B.

$$W = \int \vec{F}\cdot d\vec{r} = \int_{r_0}^{\infty} \frac{k}{r^2}dr = \frac{k}{r_0}$$

The change in potential energy of the system is

$$U_f - U_i = -W = -\frac{k}{r_0}$$

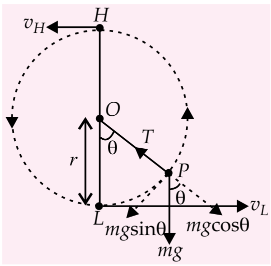
As the total mechanical energy is conserved,

$$K_f + U_f = K_i + U_i \quad \text{or} \quad K_f = K_i - (U_f - U_i)$$

or $\frac{1}{2}mv^2 = \frac{k}{r_0}$ or $v = \sqrt{\frac{2k}{mr_0}}$

Motion in Vertical Circle

When a small body of mass m is attached to an inextensible light string of length r and whirling in a vertical circle about a fixed point O to which the other end of the string is attached as shown in figure, then



- Tension at any position of angular displacement, (θ) along a vertical circle is given by
  $$T = \frac{mv^2}{r} + mg\cos\theta$$
- Thus, tension at the lowest point (θ = 0) is given by
  $$T_L = \frac{mv_L^2}{r} + mg$$
  and tension at the highest point (θ = 180°) is given by
  $$T_H = \frac{mv_H^2}{r} - mg$$
- Minimum velocity at the highest point, $v_H = \sqrt{gr}$
- Minimum velocity at the lowest point for looping the loop, $v_L = \sqrt{5gr}$.
- When the string is horizontal, θ = 90°, minimum velocity, $v = \sqrt{3gr}$.
- Height through which a body should fall for looping the vertical loop or radius r is, $h = 5r/2$.

POWER

Power of a body is defined as the rate at which the body can do the work.

Average power $(P_{av.}) = \frac{\Delta W}{\Delta t} = \frac{W}{t}$

Instantaneous power $(P_{int.}) = \frac{dW}{dt} = \frac{\vec{F}\cdot d\vec{s}}{dt}$

[As $dW = \vec{F}\cdot d\vec{s}$]

$P_{inst} = \vec{F}\cdot\vec{v}$

i.e., power is equal to the scalar product of force with velocity.

- The slope of work time curve gives the instantaneous power. As $P = dW/dt = \tan\theta$
- Area under power-time curve gives the work done as $P = \frac{dW}{dt}$

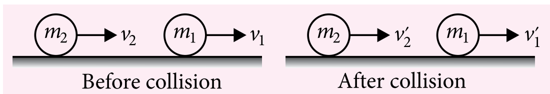
  ∴ $W = \int P\,dt$

  ∴ W = Area under P-t curve

COLLISION

Head on Elastic Collision

- In this case linear momentum and kinetic energy both are conserved. After solving two conservation equations, we get



$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2 \qquad \ldots\text{(i)}$$

and $$v_2' = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2 + \left(\frac{2m_1}{m_1 + m_2}\right)v_1 \qquad \ldots\text{(ii)}$$

Regarding the above two formulae following are three special cases.

- If $m_1 = m_2$, then $v_1' = v_2$ and $v_2' = v_1$ *i.e.* in case of equal masses bodies will exchange their velocities.
- If $m_1 >> m_2$ and $v_1 = 0$, then $v_1' \approx 0$ and $v_2' = -v_2$
- If $m_2, >> m_1$ and $v_1 = 0$, then, $v_1' = 2v_2$ and $v_2' = v_2$

Head on Inelastic Collision

In this type of collision only linear momentum remains constant.

Two unknowns are v_1' and v_2'. Make following two equations to solve them.

- Conservation of linear momentum

  $m_1v_1 + m_2v_2 = m_1v_1'$ and m_2v_2'
- Coefficient of restitution (e)

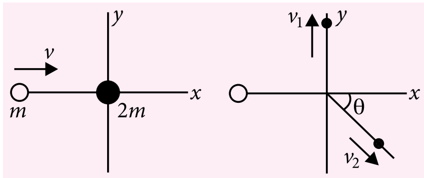
  $$e = \frac{|\text{Relative velocity of separation}|}{|\text{Relative velocity of approach}|} = \frac{v_1' - v_2'}{v_2 - v_1}$$

Oblique Collision (Both Elastic and Inelastic)

- Resolve the velocities along common normal and common tangent directions. Now,
  - velocity components along common tangent direction will remain unchanged.
  - along common normal direction theory of head on collision (elastic as well as inelastic) can be used.

Illustration 7 : A ball of mass m, moving with a velocity v along x-axis, strikes another ball of mass 2 m kept at rest. The first ball comes to rest after collision and the other breaks into two equal pieces. One of the pieces starts moving along y-axis with a speed v_1. What will be the velocity of the other pieces?

Sol.: The total linear momentum of the balls before the collision is mv along the x-axis. After the collision, momentum of the first ball = 0, momentum of the first piece = mv_1 along the Y-axis and momentum of the second piece = mv_2 along its direction of motion where v_2 is the speed of the second piece. These three should add to mv along the x-axis, which is the initial momentum of the system.



Taking components along the x-axis,

$mv_2 \cos\theta = mv$ ...(i)

and taking components along the y-axis,

$mv_2\sin\theta = mv_1$ ...(ii)

From eqn. (i) and (ii),

$$v_2 = \sqrt{v^2 + v_1^2} \text{ and } \tan\theta = \frac{v_1}{v}$$

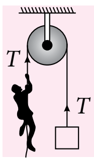
SPEED PRACTICE

1. Three forces are acting on a particle of mass m initially in equilibrium. If the first two forces (R_1 and R_2) are perpendicular to each other and suddenly the third force (R_3) is removed, then the acceleration of the particle is

   (a) $\frac{R_3}{m}$ (b) $\frac{R_1 + R_2}{m}$

   (c) $\frac{R_1 - R_2}{m}$ (d) $\frac{R_1}{m}$

2. In order to raise a mass of 100 kg, a man of mass 60 kg fastens a rope to it and passes the rope over a smooth pulley. He climbs the rope with acceleration $5g/4$ relative to the rope. The tension in the rope is (take $g = 10$ m s^{-2})



   (a) $\frac{4875}{8}$N (b) $\frac{4875}{2}$N

   (c) $\frac{4875}{4}$N (d) $\frac{4875}{6}$N

3. A 60 kg man stands on a spring scale in a lift. At some instant, he finds that the scale reading has changed from 60 kg to 50 kg for a while and then comes back to original mark. What should be concluded?

   (a) The lift was in constant motion upward
   (b) The lift was in constant motion downward.
   (c) The lift while in motion downward suddenly stopped.
   (d) None of these

4. A block of mass m is placed on a smooth wedge of inclination θ. The whole system is accelerated horizontally so that the block does not slip on the wedge. The force exerted by the wedge on the block has a magnitude

   (a) mg (b) $mg/\cos\theta$
   (c) $mg\cos\theta$ (d) $mg\tan\theta$

5. The force on a particle of mass 10 g is $(10\hat{i} + 5\hat{j})$N. If it starts from rest what would be its position at time $t = 5$ s?

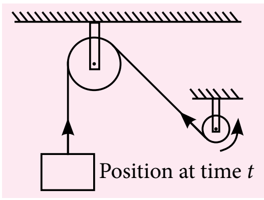
(a) $(6250\hat{i} + 12500\hat{j})$m

(b) $(12500\hat{i} + 6250\hat{j})$m

(c) $(12500\hat{i} - 6250\hat{j})$m

(d) $(6250\hat{i} - 12500\hat{j})$m

6. A body of mass 400 kg is suspended at the lower end of a light vertical chain and is being pulled up vertically (see figure). Initially the body is at rest and the pull on the chain is 6000 g N. The pull gets smaller uniformly at the rate of 360 g N per each meter through which the body is raised. What is the velocity of the body when it has been raised to 10 m?



(a) + 43.2 m s^{-1} (b) – 43.2 m s^{-1}

(c) + 4.32 m s^{-1} (d) – 4.32 m s^{-1}

7. The breaking strength of a steel cable is 20 kN. If one pulls horizontally with this cable, what is the maximum horizontal acceleration which can be given to an 8 ton (metric) body resting on a rough horizontal surface if the coefficient of kinetic friction is 0.15?

(a) 1.03 m s^{-2} (b) 4.02 m s^{-2}

(c) 2.98 m s^{-2} (d) 3.90 m s^{-2}

8. A moving truck crashes into a stationary car. The truck's mass is ten times that of the car. How does the magnitude of the force exerted by the truck on the car compare with that exerted by the car on the truck?

(a) The force the truck exerts on the car is ten times bigger.

(b) The force the truck exerts on the car is ten times smaller.

(c) The ratio depends on the speed of the truck.

(d) They are the same.

9. A certain system has potential energy given by the function $U(x) = -ax^2 + bx^4$ with constants, $a, b > 0$. Which of the following is an unstable equilibrium point?

(a) 0 (b) $\sqrt{a/2b}$

(c) $-\sqrt{a/2b}$ (d) $\sqrt{a/b}$

10. An engineer wants to design an improved elevator for a building. The original design used a motor that could lift 1000 kg through a distance of 20 m in 30 s. The engineer wants a motor that can lift 800 kg through a distance of 30 m in 20 s. Compared to the old motor, the new motor (In both cases elevator starts and stops at rest)

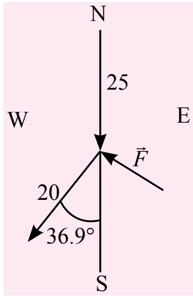
(a) should exert an average force of a larger magnitude, and must provide a larger average power output

(b) can exert an average force of a smaller magnitude, and can provide a smaller average power output

(c) can exert an average force of a smaller magnitude, but must provide the same average power output

(d) can exert an average force of small magnitude, but must provide a larger average power output.

11. A football of mass 0.42 kg is passed with a velocity of 25 m s^{-1} due south. A defending player lunges at the ball and deflects it so that the new velocity is 20 m s^{-1}, 36.9° west of south (fig). If the player is in contact with the ball for 0.05 s, what is the magnitude of the average force he exerts?



(a) 63 N (b) 126 N

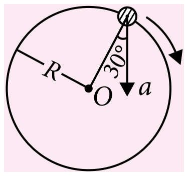
(c) 75 N (d) 105 N

12. In unloading grain from the hold of a ship, an elevator lifts the grain through a distance of 12 m. Grain is discharged at the top of the elevator at a rate of 2.0 kg each second and the discharge speed of each grain particle is 3.0 m s^{-1}. Find the minimum horsepower of motor that can elevate grain in this way.

(a) 3.3 (b) 0.33

(c) 1.5 (d) 0.75

13. In the given figure, a = 15 m s^{-2} represents the total acceleration of a particle moving in the clockwise direction in a circle of radius R = 2.5 m at a given instant of time. The speed of the particle is



(a) 4.5 m s^{-1} (b) 5.0 m s^{-1}

(c) 5.7 m s^{-1} (d) 6.2 m s^{-1}

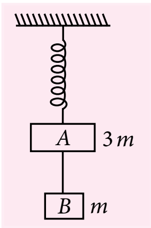
[NEET Phase II 2016]

14. One end of string of length l is connected to a particle of mass m and the other end is connected to a small peg on a smooth horizontal table. If the particle moves in circle with speed v, the net force on the particle (directed towards centre) will be (T represents the tension in the string)

(a) $T + \frac{mv^2}{l}$ (b) $T - \frac{mv^2}{l}$

(c) zero (d) T **[NEET 2017]**

15. Two blocks A and B of masses $3m$ and m respectively are connected by a massless and inextensible string. The whole system is suspended by a massless spring as shown in figure. The magnitudes of acceleration of A and B immediately after the string is cut, are respectively



(a) $\frac{g}{3}, g$ (b) g, g

(c) $\frac{g}{3}, \frac{g}{3}$ (d) $g, \frac{g}{3}$ **[NEET 2017]**

16. Consider a drop of rain water having mass 1 g falling from a height of 1 km. It hits the ground with a speed of 50 m s^{-1}. Take 'g' constant with a value 10 m s^{-2}. The work done by the (i) gravitational force and the (ii) resistive force of air is

(a) (i) 1.25 J (ii) −8.25 J

(b) (i) 100 J (ii) 8.75 J

(c) (i) 10 J (ii) −8.75 J

(d) (i) −10 J (ii) −8.25 J **[NEET 2017]**

17. A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10$ m s^{-1}. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be

(a) 10^{-3} kg m^{-1} (b) 10^{-3} kg s^{-1}

(c) 10^{-4} kg m^{-1} (d) 10^{-1} kg m^{-1} s^{-1}

[JEE Main Offline 2017]

18. A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 s will be

(a) 4.5 J (b) 22 J

(c) 9 J (d) 18 J

[JEE Main Offline 2017]

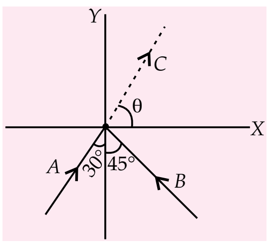
19. Two particles A and B of equal mass M are moving with the same speed v as shown in the figure. They collide completely inelastically and move as a single particle C. The angle θ that the path of C makes with the X-axis is given by

(a) $\tan\theta = \dfrac{\sqrt{3}+\sqrt{2}}{1-\sqrt{2}}$

(b) $\tan\theta = \dfrac{1-\sqrt{3}}{1+\sqrt{2}}$

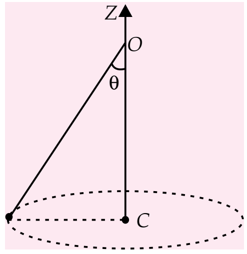
(c) $\tan\theta = \dfrac{\sqrt{3}-\sqrt{2}}{1-\sqrt{2}}$

(d) $\tan\theta = \dfrac{1-\sqrt{2}}{\sqrt{2}(1+\sqrt{3})}$



[JEE Main Online 2017]

20. A conical pendulum of length 1 m makes an angle $\theta = 45°$ w.r.t. Z-axis and moves in a circle in the XY plane. The radius of the circle is 0.4 m and its center is vertically below O. The speed of the pendulum, in its circular path, will be (Take $g = 10$ m s^{-2})



(a) 0.4 m s^{-1} (b) 2 m s^{-1}

(c) 0.2 m s^{-1} (d) 4 m s^{-1}

[JEE Main Online 2017]

SOLUTION

1. **(a) :** $a = \dfrac{\sqrt{R_1^2+R_2^2}}{m} = \dfrac{R_3}{m}$ $\left[\because R_3 = \sqrt{R_1^2+R_2^2}\right]$

2. **(c) :** Let T be the tension in the rope and a be the acceleration of rope. The absolute acceleration of man is therefore, $\left(\frac{5g}{4}-a\right)$. Equations of motion for mass and man gives

$$T - 100\,g = 100\,a \quad \text{...(i)}$$

$$T - 60\,g = 60\left(\frac{5g}{4}-a\right) \quad \text{...(ii)}$$

Solving (i) and (ii), we get $T = \left(\frac{4875}{4}\text{N}\right)$

3. **(c) :** The reading of the spring scale is the normal reaction between man and spring scale. If the lift starts accelerating downward, normal reaction becomes smaller. Once the lift is stopped sudden then reading of scale comes to original mark.

4. **(b)**

5. **(b):** We have, $F_x = 10$ N

$$\therefore \quad a_x = \frac{F_x}{m} = \frac{10\text{ N}}{0.01\text{ kg}} = 1000\text{ m s}^{-2}$$

As this is a case of constant acceleration in x-direction,

$$x = u_x t + \frac{1}{2}a_x t^2 = \frac{1}{2}\times 1000\text{ m s}^{-2}\times(5\text{ s})^2$$
$$= 12500\text{ m}$$

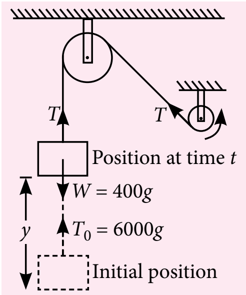
Similarly, $a_y = \dfrac{F_y}{m} = \dfrac{5\text{ N}}{0.01\text{ kg}} = 500\text{ m s}^{-2}$

and $y = 6250$ m

Thus, the position of the particle at $t = 5$ s is,

$$\vec{r} = (12500\hat{i} + 6250\hat{j})\text{ m}$$

6. **(a) :** At time t, let y be the height (in metres) of the body above its initial position. The pull in the chain is then $T = (6000 - 360y)g$ and Newton's second law gives



$$T - 400g = 400\frac{d^2y}{dt^2}$$

or $\quad (5600 - 360y)g = 400\frac{d^2y}{dt^2}$ ...(i)

This equation may be changed into one for $\frac{dy}{dt} = v$ (the velocity of the body), by use of the identity

$$2\frac{d^2y}{dt^2} = 2\frac{dv}{dt} = 2\frac{dv}{dy}\frac{dy}{dt} = 2v\frac{dv}{dy} = \frac{d}{dy}(v^2)$$

Thus from eqn. (i), $200\frac{d(v^2)}{dy} = (5600 - 360y)g$

or $\quad d(v^2) = g(28 - 1.8y)dy$

Let V be the velocity at height 10 m. Then, on integrating,

$$\int_0^{V^2} d\left(v^2\right) = g\int_0^{10}(28 - 1.8y)dy$$

$V^2 = g[28y - 0.9y^2]_0^{10} = g[28(10) - 0.9(100)] = 190g$

$V = +\sqrt{190g} = +43.2\ \text{m s}^{-1}$.

The choice of the + sign for V (upward motion) should be checked. For $0 \le y \le 10$, the net force, $(5600 - 360y)g$, is positive and so the acceleration is positive. Then, since the body started from rest, V must be positive.

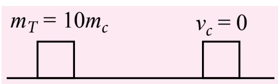
7. **(a) :** Let T be the cable force. Then $\Sigma F = ma$, becomes $T - \mu_k mg = ma$. For maximum acceleration, $T = 2.0 \times 10^4$ N, so that

$2.0 \times 10^4\ \text{N} - 0.15\ (8000\ \text{kg})\ (9.8\ \text{m s}^{-2}) = (8000\ \text{kg})a$.

Solving we get $a = 1.03\ \text{m s}^{-2}$

8. **(d):** Impulse imparted on car due to car, F_1t = change in momentum, Δp_c

Impulse imparted on truck, F_2t = change in momentum $= \Delta p_T$,



$\because \quad \Delta p_c = \Delta p_T$

$\therefore \quad F_1 = F_2$

9. **(a) :** $F = -\frac{dU}{dx} = 2ax - 4bx^3 = 0$

So possible values of x,

$$x = 0,\ x = +\sqrt{\frac{a}{2b}} \ \text{ or } \ x = -\sqrt{\frac{a}{2b}}$$

Now, $\frac{d^2U}{dx^2} = -2a + 12bx^2$; Use $\frac{d^2U}{dx^2} < 0$

$\therefore \quad$ At $x = 0, \frac{d^2U}{dx^2} < 0$,

At $x = \sqrt{\frac{a}{2b}}, \frac{d^2U}{dx^2} = -2a + 12b \times \frac{a}{2b} > 0$

$x = -\sqrt{\frac{a}{2b}} = -2a + 12b \times \frac{a}{2b} > 0$

Since maxima of curve $U(x)$ occurs at $x = 0$, so point $x = 0$ will be the point of unstable equilibrium.

10. **(d):** Power output of old motor,

$$\frac{W_m}{t} = \frac{mgh}{t} = \frac{10^3 \times 10 \times 20}{30} = 6.6 \times 10^3\ \text{W}$$

Power output of new motor,

$$\frac{W_{m'}}{t'} = \frac{800 \times 10 \times 30}{20} = 12 \times 10^3\ \text{W}$$

So, the new motor can exert an average force of small magnitude, but must provide larger power output.

11. **(b):** We choose south and west as positive. Final velocity has components 16 m s^{-1} south and 12 m s^{-1} west.

Initial momentum $= 0.42 \times 25 = 10.5$ kg m s^{-1} south

Final momentum $= (0.42 \times 16)$ south and (0.42×12) west

$[\Delta mv]_{south} = 0.42(16 - 25) = -3.78$ kg m s^{-1}

$[\Delta mv]_{west} = 0.42 \times 12 = 5.04$ kg m s^{-1}

$|\Delta mv| = [(5.04)^2 + (3.78)^2]^{1/2} = 6.3$ kg m s^{-1}

Impulse $= \Delta mv = 6.3$ N s

Impulse $= \vec{F}t = 0.05\vec{F} = 6.3$ N s

$\vec{F} = 6.3\ \text{N s}/0.05\ \text{s} = 126\ \text{N}$

12. **(b):** The work done by the motor each second is power $= mgh + 1/2mv^2$

where m is the mass of grain discharged (and lifted) each second. We are given that $m = 2.0$ kg, $v = 3.0$ m s^{-1}, and $h = 12$ m.

Hence, power $= 2 \times 9.8 \times 12 + \frac{1}{2} \times 2 \times 3^2$

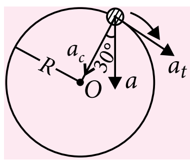
$= 244$ W $= 0.33$ hp

The motor must have an output of at least 0.33 hp.

13. **(c) :** Here, $a = 15$ m s^{-2}

$R = 2.5$ m

From figure,



$a_c = a \cos 30° = 15 \times \frac{\sqrt{3}}{2}$ m s^{-2}

As we know, $a_c = \dfrac{v^2}{R} \Rightarrow v = \sqrt{a_c R}$

$\therefore \quad v = \sqrt{15 \times \dfrac{\sqrt{3}}{2} \times 2.5} = 5.69 \simeq 5.7 \text{ m s}^{-1}$

14. (d) : Centripetal force (mv^2/l) is provided by tension so net force on the particle will be equal to tension T.

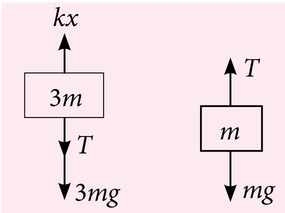
15. (a) : Before the string is cut,

$kx = T + 3mg$ ...(i)

$T = mg$ ...(ii)

From eqns. (i) and (ii)

$kx = 4mg$



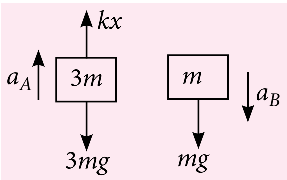
Before the string is cut

Just after the string is cut, $T = 0$

$a_A = \dfrac{kx - 3mg}{3m}$

$a_A = \dfrac{4mg - 3mg}{3m}$

$= \dfrac{mg}{3m} = \dfrac{g}{3}$



After the string is cut

and also $a_B = g$

16. (c) : Here, m = 1 g = 10^{-3} kg, h = 1 km = 1000 m, v = 50 m s^{-1}, g = 10 m s^{-2}.

(i) The work done by the gravitational force, $W_g = mgh = 10^{-3} \times 10 \times 1000 = 10$ J

(ii) The total work done by gravitational force (W_g) and the resistive force (W_r) of air is equal to change in kinetic energy of rain drop.

$\therefore \quad W_g + W_r = \dfrac{1}{2}mv^2 - 0$

$10 + W_r = \dfrac{1}{2} \times 10^{-3} \times 50 \times 50$ or $W_r = -8.75$ J

17. (c) : Initial K.E. of the body, $K_i = \dfrac{1}{2}mv_0^2$

Final K.E. of the body, $K_f = \dfrac{1}{8}mv_0^2$

Now, $\dfrac{K_i}{K_f} = 4$

Let initial velocity = v_i ; Final velocity = v_f

$\dfrac{v_i^2}{v_f^2} = \dfrac{4}{1}$ or $v_f = \dfrac{v_i}{2}$

$\Rightarrow \quad v_f = \dfrac{v_0}{2} = \dfrac{10}{2} = 5 \text{ m s}^{-1}$ (Given v_0 = 10 m s^{-1})

Also, $F = -kv^2$

$\Rightarrow \quad m\dfrac{dv}{dt} = -kv^2 \Rightarrow \dfrac{-m}{k}\dfrac{dv}{v^2} = dt$

Integrating both sides,

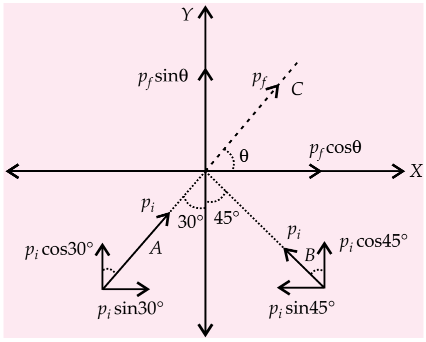
$\Rightarrow \quad \dfrac{-m}{k}\displaystyle\int_5^{10} \dfrac{dv}{v^2} = \int_0^{10} dt \Rightarrow \dfrac{-m}{k}\left[\dfrac{-1}{v}\right]_{10}^{5} = [t]_0^{10}$

$\Rightarrow \quad \dfrac{-10^{-2}}{k}\left(\dfrac{-1}{5} + \dfrac{1}{10}\right) = (10 - 0) \Rightarrow \dfrac{10^{-3}}{k} = 10$

$\therefore \quad k = 10^{-4}$ kg m^{-1}

18. (a)

19. (a) : During completely inelastic collision, both particles A and B stick together and move as a single particle C.



Here, p_i = initial momentum of each particle

p_f = final momentum of the system

Using the law of conservation of linear momentum, along X-axis,

$p_f\cos\theta = p_i\sin30° - p_i\sin45°$ ...(i)

along Y-axis,

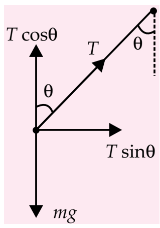
$p_f\sin\theta = p_i\cos30° + p_i\cos45°$ ...(ii)

Divide eqn. (ii) by eqn. (i), we get

$$\frac{\sin\theta}{\cos\theta} = \frac{\cos30° + \cos45°}{\sin30° - \sin45°} = \frac{\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}}{\frac{1}{2} - \frac{1}{\sqrt{2}}}$$

$$\therefore \quad \tan\theta = \frac{\sqrt{3} + \sqrt{2}}{1 - \sqrt{2}}$$

20. (b) : FBD of the pendulum is shown in the figure.



$T\sin\theta = \dfrac{mv^2}{r}$

$T\cos\theta = mg$

$\tan\theta = \dfrac{v^2}{rg}$

$\because$ θ = 45°, r = 0.4 m

$\therefore v^2 = rg$

$v = \sqrt{rg} = \sqrt{0.4 \times 10} = 2 \text{ m s}^{-1}$

EXAM PREP 2018

CLASS XI

Useful for Medical/Engg. Entrance Exams

CHAPTERWISE MCQs FOR PRACTICE

GRAVITATION

1. The kinetic energy of a satellite is 2 MJ. What is the total energy of the satellite?
(a) −2 MJ (b) −1 MJ
(c) $-\frac{1}{2}$ MJ (d) −4 MJ

2. A satellite is in a circular orbit very close to the surface of a planet. At some point it is given an impulse along its direction of motion, causing its velocity to increase n times. It now goes into an elliptical orbit, with the planet at the centre of the ellipse. The maximum possible value of n for this to occur is
(a) 2 (b) $\sqrt{2}$
(c) $\sqrt{2}+1$ (d) $\frac{1}{\sqrt{2}-1}$

3. The escape velocity corresponding to a planet of mass M and radius R is 50 km s^{-1}. If the planet's mass and radius were $4M$ and R respectively, then the corresponding escape velocity would be
(a) 100 km s^{-1} (b) 50 km s^{-1}
(c) 200 km s^{-1} (d) 25 km s^{-1}

4. There are two planets. The ratio of radii of the two planets is K, but ratio of acceleration due to gravity of both planets is g. What will be the ratio of their escape velocity?
(a) $(Kg)^{1/2}$ (b) $(Kg)^{-1/2}$
(c) $(Kg)^2$ (d) $(Kg)^{-2}$

5. Two stars each of mass m and radius R approach each other to collide head-on. Initially the stars are at a distance $r(>>R)$. Assuming their speeds to be negligible at this distance of separation, the speed with which the stars collide is
(a) $\sqrt{Gm\left(\frac{1}{R}-\frac{1}{r}\right)}$ (b) $\sqrt{Gm\left(\frac{1}{2R}-\frac{1}{r}\right)}$
(c) $\sqrt{Gm\left(\frac{1}{R}+\frac{1}{r}\right)}$ (d) $\sqrt{Gm\left(\frac{1}{2R}+\frac{1}{r}\right)}$

6. A system of binary stars of masses M_A and M_B are moving in circular orbits of radii R_A and R_B respectively. If T_A and T_B are the time periods of masses M_A and M_B respectively, then
(a) $\frac{T_A}{T_B}=\left(\frac{R_A}{R_B}\right)^{1/2}$
(b) $T_A > T_B$ (if $R_A > R_B$)
(c) $T_A > T_B$ (if $M_A > M_B$)
(d) $T_A = T_B$

7. Let V and E be the gravitational potential and gravitational field at a distance r from the centre of a uniform spherical shell. Consider the following two statements:
(A) The plot of V against r is discontinuous.
(B) The plot of E against r is discontinuous.
(a) Both A and B are correct.
(b) A is correct but B is wrong.
(c) B is correct but A is wrong.
(d) Both A and B are wrong.

8. The magnitude of gravitational potential energy of the moon-earth system is U with zero potential energy at infinite separation. The kinetic energy of the moon with respect to the earth is K. Then

(a) $U < K$ (b) $U > K$
(c) $U = K$ (d) $U = K/2$

9. The orbital time-period of a satellite very close to the surface of the earth is

(a) $\frac{\pi}{4}\sqrt{\frac{R_E}{g}}$ (b) $\frac{\pi}{2}\sqrt{\frac{R_E}{g}}$

(c) $\pi\sqrt{\frac{R_E}{g}}$ (d) $2\pi\sqrt{\frac{R_E}{g}}$

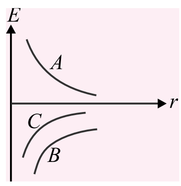
10. If the distance between the Earth and the Sun becomes half its present value, the number of days in a year would have been

(a) 64.5 (b) 129
(c) 182.5 (d) 730

11. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be

(a) 0.11 km s^{-1} (b) 1.1 km s^{-1}
(c) 11 km s^{-1} (d) 110 km s^{-1}

12. Figure shows the variation of energy E with the orbital radius r of a satellite in a circular motion. Mark the correct statement.



(a) A shows the kinetic energy, B shows the total energy and C the potential energy of the satellite.
(b) A and B are the kinetic energy and potential energy respectively and C the total energy of the satellite.
(c) A and B are the potential energy and kinetic energy respectively and C the total energy of the satellite.
(d) C and A are the kinetic and potential energies respectively and B the total energy of the satellite.

13. The magnitudes of the gravitational field at distances r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are F_1 and F_2 respectively. Then

(a) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$, if $r_1 < R$ and $r_2 < R$

(b) $\frac{F_1}{F_2} = \frac{r_2}{r_1}$, if $r_1 < R$ and $r_2 < R$

(c) $\frac{F_1}{F_2} = \frac{r_1^3}{r_2^3}$, if $r_1 > R$ and $r_2 > R$

(d) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$, if $r_1 > R$ and $r_2 > R$

14. The radii of two planets are respectively R_1 and R_2 and their densities are respectively ρ_1 and ρ_2. The ratio of the accelerations due to gravity (g_1/g_2) at their surfaces is

(a) $\frac{R_1\rho_2}{R_2\rho_1}$ (b) $\frac{R_1\rho_1}{R_2\rho_2}$ (c) $\frac{\rho_1 R_2^2}{\rho_2 R_1^2}$ (d) $\frac{R_1 R_2}{\rho_1\rho_2}$

15. Suppose the gravitational force varies inversely as the n^{th} power of distance. Then the time period of a planet in circular orbit of radius R around the sun will be proportional to

(a) $R^{\left(\frac{n+1}{2}\right)}$ (b) $R^{\left(\frac{n-1}{2}\right)}$

(c) R^n (d) $R^{\left(\frac{n-2}{2}\right)}$

MECHANICAL PROPERTIES OF SOLIDS

16. If the work done in stretching a wire by 1 mm is 2 J, the work necessary for stretching another wire of same material but with double radius of cross-section and half the length, by 1 mm is

(a) 16 J (b) 8 J (c) 4 J (d) $\frac{1}{4}$ J

17. For a given material, the Young's modulus is 2.4 times that of modulus of rigidity. Its Poisson's ratio is

(a) 2.4 (b) 1.2 (c) 0.4 (d) 0.2

18. The length of an elastic string is 'a' metre when the tension is 4 N, and 'b' metre when the tension is 5 N. The length in metre when the tension is 9 N is

(a) $5b - 4a$ (b) $4a - 5b$
(c) $a + b$ (d) $b - a$

19. The Poisson's ratio of a material is 0.4. If a force is applied to a wire of this material, there is a decrease of cross-sectional area by 2%. The percentage increase in its length is

(a) 3% (b) 2.5% (c) 1% (d) 0.5%

20. One end of uniform wire of length L and of weight W is attached rigidly to a point in the roof and a weight W_1 is suspended from its lower end. If s is the area of cross-section of the wire, the stress in the wire at a height $(3L/4)$ from its lower end is

(a) $\frac{W_1}{s}$ (b) $\left[W_1 + \frac{W}{4}\right]s$

(c) $\frac{\left[W_1 + \frac{3W}{4}\right]}{s}$ (d) $\frac{W_1 + W}{s}$

21. Consider a parallelopiped block of length a, breadth b and height c. The lower face of the block, bounded by length and breadth, is fixed to a horizontal surface. A force F is applied, in a vertical plane, perpendicular to the top face of the block at inclination θ to the vertical. The shearing stress on the top face is

(a) $\frac{F\cos\theta}{ab}$ (b) $\frac{F\sin\theta}{ab}$

(c) $\frac{F\cos\theta}{ca}$ (d) $\frac{F\cos\theta}{bc}$

22. A metallic rod of length l and cross - sectional area A is made of a material of Young's modulus Y. If the rod is elongated by an amount y, then the work done is proportional to

(a) y (b) $1/y$ (c) y^2 (d) $1/y^2$

23. For a perfectly rigid body,

(a) Young's modulus is infinite and bulk modulus is zero.

(b) Young's modulus is zero and bulk modulus is infinite.

(c) Young's modulus is infinite and bulk modulus is also infinite.

(d) Young's modulus is zero and bulk modulus is also zero.

24. A light rod of length 200 cm is suspended from the ceiling horizontally by means of two vertical wires of equal length tied to its ends. One of the wires is made of steel and is of cross-section 0.1 cm^2 and the other of brass of cross-section 0.2 cm^2. A distance along the rod at which a weight may be hung to produce equal stresses in both the wires?

(a) $\frac{4}{3}$ m from steel wire

(b) $\frac{4}{3}$ m from brass wire

(c) 1 m from steel wire

(d) $\frac{1}{4}$ m from brass wire

25. The length of a wire is L_1 when the tension in it is T_1 and is L_2 when the tension is T_2. The original length of the wire is

(a) $\frac{L_1 + L_2}{2}$ (b) $\frac{L_1T_2 + L_2T_1}{T_1 + T_2}$

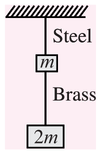
(c) $\frac{L_1T_2 - L_2T_1}{T_2 - T_1}$ (d) $\sqrt{T_1T_2L_1L_2}$

26. The relation between Y, η and B is

(a) $\frac{1}{Y} = \frac{1}{3\eta} + \frac{1}{9B}$ (b) $\frac{9}{Y} = \frac{1}{\eta} + \frac{3}{B}$

(c) $\frac{1}{\eta} = \frac{1}{B} + \frac{1}{Y}$ (d) $\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{B}$

27. If the ratio of lengths, radii and Young's moduli of steel and brass wires in the figure are a, b and c respectively. Then the corresponding ratio of increase in their lengths would be



(a) $\frac{2ac}{b^2}$ (b) $\frac{3a}{2b^2c}$ (c) $\frac{3c}{2ab^2}$ (d) $\frac{2a^2c}{b}$

28. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If length of wire 1 increased by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount?

(a) $4F$ (b) $6F$ (c) $9F$ (d) F

29. Four wires of the same material are stretched by the same load. The dimensions are given below. Which of them will elongate the most ?

(a) Length 100 cm, diameter 1 cm

(b) Length 200 cm, diameter 2 cm

(c) Length 300 cm, diameter 3 cm

(d) Length 400 cm, diameter 0.5 cm

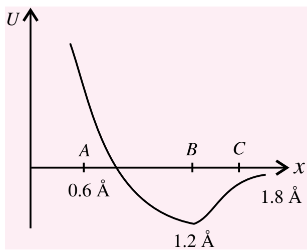
30. The potential energy U between the two molecules as a function of the distance x between them is shown in figure. A, B and C are points for which x = 0.6 Å, 1.2 Å and 1.8 Å. At A, B and C, force between the two molecules respectively is

(a) attractive, zero, repulsive

(b) zero, attractive, repulsive

(c) repulsive, zero, attractive

(d) zero, repulsive, attractive



SOLUTIONS

1. **(a) :** Total energy of satellite = – Kinetic energy of satellite
So, total energy of satellite = – 2 MJ
(∵ K.E. = 2 MJ(given))

2. **(b):** The initial velocity of the satellite is

$$v = \sqrt{\frac{GM}{R}}$$

The satellite will remain in the elliptical orbit as long as $v < v_e$, where v_e is the escape velocity.

Also, $v_e = \sqrt{\frac{2GM}{R}}$

$\therefore \quad nv \le v_e$ or $n\sqrt{\frac{GM}{R}} \le \sqrt{\frac{2GM}{R}}$

or $\quad n \le \sqrt{2}$ or $n_{max} = \sqrt{2}$

3. **(a) :** In first case, $v_e = \sqrt{\frac{2GM}{R}} = 50$

In second case, $v_e' = \left[\frac{2G(4M)}{R}\right]^{1/2} = 2\sqrt{\frac{2GM}{R}}$
$= 2 \times 50 = 100 \text{ km s}^{-1}$

4. **(a) :** Given, $\frac{R_1}{R_2} = K$ and $\frac{g_1}{g_2} = g$

Escape velocity, $v_e = \sqrt{2gR}$

$\therefore \quad \frac{v_{e_1}}{v_{e_2}} = \sqrt{\frac{R_1 g_1}{R_2 g_2}} = \sqrt{Kg} = (Kg)^{1/2}$

5. **(b):** Since the speeds of the stars are negligible when they are at a distance of r, the initial kinetic energy of the system is zero. Therefore, initial total energy of the system is

$$E_i = \text{KE} + \text{PE} = 0 + \left(-\frac{Gmm}{r}\right) \quad \text{...(i)}$$

where m represents the mass of each star and r the initial separation between them.
When two stars collide, their centres will be at a distance twice the radius of a star, *i.e.*, $2R$. Let v be speed with which two stars collide. Then total energy of the system at the instant of their collision is

$$E_f = \left(\frac{1}{2}mv^2\right) \times 2 + \left(-\frac{Gmm}{2R}\right) = mv^2 - \frac{Gmm}{2R}$$

According to law of conservation of energy
$E_f = E_i$

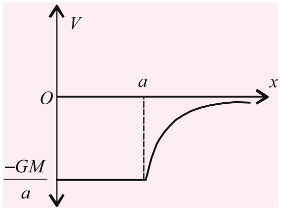
$\therefore \quad mv^2 - \frac{Gmm}{2R} = -\frac{Gmm}{r}$ or $v^2 = Gm\left(\frac{1}{2R} - \frac{1}{r}\right)$

or $\quad v = \sqrt{Gm\left(\frac{1}{2R} - \frac{1}{r}\right)}$

6. **(d):** In case of binary star system, both the stars rotate with same angular velocity ω about their centre of mass in their respective orbits.

$\therefore \quad \omega = \frac{2\pi}{T_A} = \frac{2\pi}{T_B}$ or $T_A = T_B$

7. **(c) :** Let the radius of the shell be a, and its mass be M, then

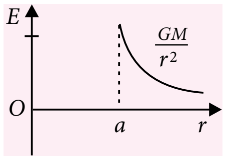


$V(r \le a) = \frac{-GM}{a}$

and $V(r > a) = \frac{-GM}{r}$

As we can see V is continuous.

Now, $E(r < a) = 0$



and $\quad E(r \ge a) = \frac{GM}{r^2}$

As we can see, E is discontinuous.
⇒ (c) is correct.

8. **(b):** $U(r) = \frac{-GMm}{r}$

Where M is mass of the earth, m is the mass of the moon and r is their distance of separation.

$$F = \frac{-dU(r)}{dr} = -GMm\left(\frac{-1}{r^2}\right) = \frac{GMm}{r^2}$$

The F provides the centripetal force for the moon to go around.

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow mv^2 = \frac{GMm}{r}$$

or $\quad K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$

If we take U as the magnitude of potential energy,

$\Rightarrow \quad U = \frac{GMm}{r} \Rightarrow K = \frac{U}{2}$

Hence, $U > K$.

9. **(d):** $F_{centripetal} = \frac{mv^2}{(R_E + h)}$ ...(i)

$F_{gravitational} = \frac{GmM_E}{(R_E + h)^2}$ ...(ii)

From eqns. (i) and (ii),

$$v^2 = \frac{GM_E}{(R_E + h)}$$

For $h \to 0$,

$$v^2 = \frac{GM_E}{R_E} = gR_E \Rightarrow v = \sqrt{gR_E};$$

Time period, $T = \frac{2\pi R_E}{v} = 2\pi\sqrt{\frac{R_E}{g}}$

10. (b): According to Kepler's 3rd law, $T^2 \propto R^3$

$$\therefore \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3 \text{ or } \left(\frac{T_2}{365}\right)^2 = \left(\frac{R/2}{R}\right)^3$$

$\Rightarrow \quad T_2 = 129$ days

11. (d) : $v_{escape} = \sqrt{\frac{2GM}{R}}$

For the earth, $v_e = 11$ km s^{-1}

Mass of the planet $M = 10\ M_e$

Radius of the planet $R = R_e/10$

$\therefore$ The escape velocity from the surface of the planet, $v_p = \sqrt{\frac{2GM_e \times 10}{R_e/10}} = 10 \times 11 = 110$ km s^{-1}

12. (b): $\text{KE} = \frac{GMm}{2r}$, $\text{PE} = -\frac{GMm}{r}$ and $\text{TE} = -\frac{GMm}{2r}$

KE is always positive and $\text{KE} \propto \frac{1}{r}$

PE is always negative and $\text{PE} \propto \frac{1}{r}$

TE is also negative and $\text{TE} \propto \frac{1}{r}$.

Also $|\text{TE}| < |\text{PE}|$

Thus the curve A represents KE, curve B represents PE and curve C represents TE of the satellite.

13. (a)

14. (b): $g = \frac{GM}{R^2} = \frac{G}{R^2}\frac{4}{3}\pi R^3\rho = \frac{4}{3}\pi GR\rho$

$$\therefore \quad \frac{g_1}{g_2} = \frac{R_1\rho_1}{R_2\rho_2}$$

15. (a): For motion of a planet in circular orbit,
Centripetal force = Gravitational force

$$\therefore \quad mR\omega^2 = \frac{GMm}{R^n} \quad \text{or} \quad \omega = \sqrt{\frac{GM}{R^{n+1}}}$$

$$\therefore \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R^{n+1}}{GM}} = \frac{2\pi}{\sqrt{GM}}R^{\left(\frac{n+1}{2}\right)}$$

$\therefore \quad T$ is proportional to $R^{\left(\frac{n+1}{2}\right)}$.

16. (a): Stretching force, $F = \frac{Y\pi r^2 \Delta l}{l}$

$$\therefore \quad F \propto \frac{r^2}{l} \Rightarrow \frac{F'}{F} = \frac{(2r)^2}{(l/2)} \times \frac{l}{r^2} = 8$$

or $\quad F' = 8F$ ...(i)

Work done in stretching a wire,

$$W = \frac{1}{2} \times F \times \Delta l$$

For same extension

$W \propto F$

$\therefore \quad \frac{W'}{W} = \frac{F'}{F} = 8$ (Using (i))

$W' = 8W = 8 \times 2$ J $= 16$ J

17. (d): As $Y = 2\eta(1 + \sigma)$

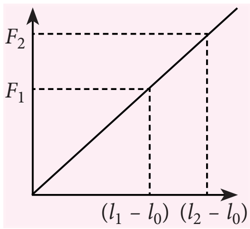
Given: $Y = 2.4\eta$

$\therefore \quad 2.4\eta = 2\eta(1 + \sigma)$

$1.2 = 1 + \sigma$ or $\sigma = 1.2 - 1 = 0.2$

18. (a): $Y = \frac{F \cdot L}{A \cdot x}$; Here, $\frac{F}{x}$ = constant

Let l_0 be original length, then



$$\frac{4}{a - l_0} = \frac{5}{b - l_0} = \frac{9}{l - l_0}$$

$$\Rightarrow \quad \frac{a - l_0}{4} = \frac{b - l_0}{5}$$

$\Rightarrow \quad l_0 = 5a - 4b$

Again, $\frac{l - l_0}{9} = \frac{b - l_0}{5}$

Substituting for l_0, $\quad l = 5b - 4a$

Here, the right choice is (a).

19. (b): Poisson's ratio, $\sigma = \frac{\Delta d/d}{\Delta l/l}$

Area, $A = \pi r^2 = \pi\frac{d^2}{4}$ $\quad (\because\ d = 2r)$

$$\therefore \quad \Delta A = \frac{2\pi d\Delta d}{4} = \frac{\pi}{2}d\Delta d$$

$$\therefore \quad \frac{\Delta A}{A} = \frac{\pi\frac{d}{2}\Delta d}{\pi\frac{d^2}{4}} = 2\frac{\Delta d}{d}$$

Given: $\frac{\Delta A}{A} \times 100 = 2\%$

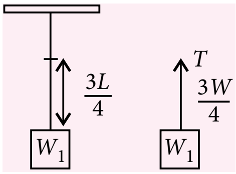
$\therefore \quad 2 = 2\frac{\Delta d}{d} \times 100$ or $\frac{\Delta d}{d} \times 100 = 1\%$ ...(i)

Given: $\sigma = \dfrac{\Delta d / d}{\Delta l / l} = 0.4$ or $\dfrac{\Delta d}{d} = 0.4\dfrac{\Delta l}{l}$

$\therefore \dfrac{\Delta l}{l} \times 100 = \dfrac{1}{0.4}\dfrac{\Delta d}{d} \times 100$

$= 2.5 \times 1\% = 2.5\%$ (Using (i))

20. (c) : $T = W_1 + \dfrac{3W}{4}$



$\text{Stress} = \dfrac{T}{A} = \dfrac{W_1 + \dfrac{3W}{4}}{s}$

21. (b)

22. (c) : Volume, V = cross-sectional area A × length l
or $V = Al$

$\text{Strain} = \dfrac{\text{Elongation}}{\text{Original length}} = \dfrac{y}{l}$

Young's modulus, $Y = \dfrac{\text{Stress}}{\text{Strain}}$

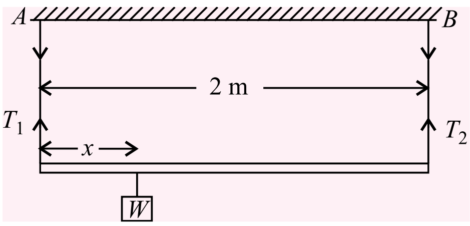
Work done, $W = \dfrac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$

or $W = \dfrac{1}{2} \times Y \times (\text{strain})^2 \times Al = \dfrac{1}{2} \times Y \times \left(\dfrac{y}{l}\right)^2 \times Al$

$= \dfrac{1}{2}\left(\dfrac{YA}{l}\right)y^2$ or $W \propto y^2$

23. (c) : For a perfectly rigid body, both Young's modulus and bulk modulus are infinite.

24. (a) :



As stresses are equal, so $\dfrac{T_1}{A_1} = \dfrac{T_2}{A_2}$

or $\dfrac{T_1}{T_2} = \dfrac{A_1}{A_2} = \dfrac{0.1}{0.2}$ or $T_2 = 2T_1$ ...(i)

Now for translatory equilibrium of the rod,

$T_1 + T_2 = W$ ...(ii)

From (i) and (ii), we get $T_1 = \dfrac{W}{3}$, $T_2 = \dfrac{2W}{3}$

Now, if x is the distance of weight W from steel wire, then for rotational equilibrium of rod,

$T_1 x = T_2(2 - x)$ or $\dfrac{W}{3}x = \dfrac{2W}{3}(2 - x)$

$\therefore x = \dfrac{4}{3}$ m

25. (c) : If L is the original length of wire, then change in length of first wire, $\Delta L_1 = (L_1 - L)$
change in length of second wire, $\Delta L_2 = (L_2 - L)$
Now, Young's modulus,

$Y = \dfrac{T_1}{A} \times \dfrac{L}{\Delta L_1} = \dfrac{T_2}{A} \times \dfrac{L}{\Delta L_2}$

or $\dfrac{T_1}{\Delta L_1} = \dfrac{T_2}{\Delta L_2}$ or $\dfrac{T_1}{L_1 - L} = \dfrac{T_2}{L_2 - L}$

or $T_1 L_2 - T_1 L = T_2 L_1 - L T_2$

or $L = \dfrac{T_2 L_1 - T_1 L_2}{T_2 - T_1}$

26. (d): The relation between Y, η and B is

$\dfrac{9}{Y} = \dfrac{1}{B} + \dfrac{3}{\eta}$

where Y is Young's modulus, B is bulk modulus and η modulus of rigidity.

27. (b): As Young's modulus, $Y = \dfrac{FL}{A\Delta L} = \dfrac{FL}{\pi r^2 \Delta L}$

or $\Delta L = \dfrac{FL}{\pi r^2 Y}$

$\therefore \dfrac{\Delta L_S}{\Delta L_B} = \dfrac{F_S}{F_B} \times \dfrac{L_S}{L_B} \times \dfrac{r_B^2}{r_S^2} \times \dfrac{Y_B}{Y_S}$

or $\dfrac{\Delta L_S}{\Delta L_B} = \dfrac{3mg}{2mg} \times a \times \dfrac{1}{b^2} \times \dfrac{1}{c} = \dfrac{3a}{2\,b^2 c}$

28. (c) : As, $Y = \dfrac{FL}{A\Delta L}$

or $F = \dfrac{YA\Delta L}{L} = \dfrac{YA^2\Delta L}{AL} = \dfrac{YA^2\Delta L}{V} = \dfrac{YA^2\Delta x}{V}$

where $AL = V$ = Volume of wire, Young modulus is the same as both the wires are made of same material. It is given that both the wires have same volume and same extension in length.

$\therefore \dfrac{F'}{F} = \dfrac{A'^2}{A^2} = \dfrac{(3A)^2}{A^2} = 9$ or $F' = 9F$

29. (d)

30. (c) : $F = \dfrac{-dU}{dx}$

At A, B, C, $\dfrac{dU}{dx}$ is negative, 0 and positive respectively. Hence F is positive, 0 and negative respectively. Thus at A, B, C force is repulsive, zero and attractive respectively.

ACE YOUR WAY CBSE XI

Series 3

CHAPTERWISE PRACTICE PAPER

WORK ENERGY AND POWER | SYSTEM OF PARTICLES AND ROTATIONAL MOTION

Time Allowed : 3 hours | Maximum Marks : 70

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Q. no. 1 to 5 are very short answer questions and carry 1 mark each.
(iii) Q. no. 6 to 10 are short answer questions and carry 2 marks each.
(iv) Q. no. 11 to 22 are also short answer questions and carry 3 marks each.
(v) Q. no. 23 is a value based question and carries 4 marks.
(vi) Q. no. 24 to 26 are long answer questions and carry 5 marks each.
(vii) Use log tables if necessary, use of calculators is not allowed.

SECTION - A

1. When a constant force is applied to a body moving with constant acceleration, is the power of the force constant? If not, how would the force have to vary with speed for the power to be constant?

2. Why are spokes fitted in a cycle wheel?

3. The Earth moving around the Sun in the circular orbit is acted upon by a force and hence work must be done on the Earth by the force. Do you agree with this statement?

4. Mountain roads rarely go straight up but wind up gradually. Why?

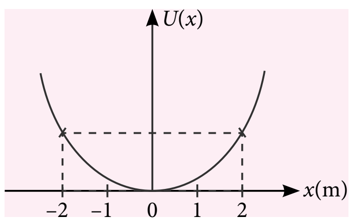
5. Assume that you are at the north pole holding a rapidly spinning gyroscope that has its angular momentum vector pointing straight up. Which way will it point if you transport it to the south pole without exerting any torque on it?

SECTION - B

6. Springs A and B are identical except that A is stiffer than B, *i.e.*, force constant $k_A > k_B$. In which spring does more work expended if :

(a) those are stretched by the same amount
(b) those are stretched by the same force?

7. The potential energy function for a particle executing linear simple harmonic motion is given by $U(x) = \frac{1}{2}kx^2$, where k is the force constant of the oscillator. For $k = \frac{1}{2}$ N m^{-1}, the graph of $U(x)$ versus x is shown in figure. Show that a particle of total energy 1 J is moving under this potential must turn back when it reaches $x = \pm 2$ m.



8. A sphere of radius 10 cm weighs 1 kg. Calculate its moment of inertia : (a) about the diameter (b) about the tangent.

9. A body of moment of inertia I is rotating with angular velocity ω_1 about a certain axis. Derive an expression for the work done by a torque in increasing its angular velocity to ω_2 about the same axis.

OR

A body of mass m accelerates uniformly from rest to velocity v_1 in time t_1. Find the expression for the instantaneous power delivered to the body as a function of time.

10. A 2 kg particle has a velocity of $(2\hat{i} - \hat{j})$ m s^{-1} and a 3 kg particle has a velocity of $(\hat{i} + 6\hat{j})$ m s^{-1}. Find (a) the velocity of the centre of mass and (b) the total momentum of the system.

SECTION - C

11. Show that there is a loss of kinetic energy during one dimensional perfectly inelastic collision. How will you account for this loss of energy?

12. What will be the duration of the day if the Earth suddenly shrinks to (1/64) of its original volume, mass remaining unchanged?

13. The blades of a windmill sweep out a circle of area A.

(a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t?

(b) What is the kinetic energy of the air?

(c) Assume that the windmill converts 25% of the wind's energy into electrical energy and that $A = 30$ m^2, $v = 36$ km h^{-1} and the density of air is 1.2 kg m^{-3}. What is the electrical power produced?

14. A large mass M moving with a velocity v collides head-on with a very small mass m at rest. If the collision is elastic, obtain an expression for the energy lost by the large mass M (Take $M + m \approx M$).

OR

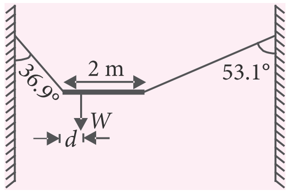
How much tangential force would be needed to stop the Earth in one year, if it were rotating with angular velocity of 7.3×10^{-5} rad s^{-1}? Given the moment of inertia of the Earth $= 9.3 \times 10^{37}$ kg m^2 and radius of the Earth $= 6.4 \times 10^6$ m.

15. A thin uniform circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity ω. Another disc of the same dimension but of mass $M/4$ is placed gently on the first disc coaxially. Show that the angular velocity of the system in $4\omega/5$.

16. A flywheel mass 500 kg and 1 m diameter makes 500 revolutions per minute. Assuming the mass to be concentrated along the rim, calculate : (a) the angular velocity (b) the moment of inertia and (c) energy of the flywheel.

17. A lighter body collides elastically with a much more massive body at rest. Show that the direction of lighter body is reversed and massive body remains at rest.

18. A non-uniform bar of weight W is suspended at rest by two strings of negligible weight as shown in figure. The angles made by the strings with the vertical are 36.9° and 53.1° respectively. The bar is 2 m long. Calculate the distance d of the centre of gravity of the bar from its left end.



(Given that tan 36.9° = 0.7508 and tan 53.1° = 1.3319)

19. Derive a relation between torque applied and angular acceleration produced in a rigid body and hence define moment of inertia.

20. State work-energy theorem. Prove it for a variable force.

21. A disc of radius R is rotating with an angular speed ω_0 about a horizontal axis. It is placed on a horizontal table. The coefficient of kinetic friction is μ_k.

(a) What was the velocity of its centre of mass before being brought in contact with the table?

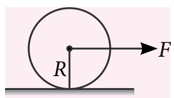
(b) What happens to the linear velocity of a point on its rim when placed in contact with the table?

(c) What happens to the linear speed of the centre of mass when disc is placed in contact with the table?

(d) Which force is responsible for the effects in (b) and (c)?

(e) What condition should be satisfied for rolling to begin?

22. A uniform disc of radius R, is resting on a table on its rim. The coefficient of friction between disc and table is μ. Now the disc is pulled with a force F as shown in figure. What is the maximum value of F for which the disc rolls without slipping?



SECTION - D

23. Puneet decides to use his bath tub water to generate electric power to run a 40 W bulb. The bath tub is located at a height of 10 m from the ground and it holds 200 litres of water. He installs a water driven wheel generator on the ground. Initially, he slowly drains water from the bath tub but the bulb does not glow. He gradually increases the rate of water drainage and finally succeeds in making the bulb glow.

(a) What are the values displayed by Puneet?

(b) What should be the minimum rate of water drainage from the bath tub for making the bulb glow? Given efficiency of generator = 90% and $g = 9.8$ m s^{-2}.

SECTION - E

24. Define perfectly elastic and perfectly inelastic collisions? Give examples. Also write the expression for final velocities of the bodies after collision.

A 5 kg body and a 10 kg body approach each other with equal speeds of 21 m s^{-1}. Determine their speeds after elastic collision.

OR

(a) State and prove the theorem of perpendicular axes.

(b) State and prove the theorem of parallel axes.

25. Prove the result that the velocity v of translation of a rolling body (like a ring, disc, cylinder or sphere) at the bottom of an inclined plane of a height h is given by

$$v^2 = \frac{2gh}{(1+k^2/R^2)}$$

Using dynamical consideration (*i.e.*, by consideration of forces and torques). Note k is the radius of gyration of the body about its symmetry axis, and R is the radius of the body. The body starts from rest at the top of the plane.

A slide cylinder of mass 10 kg and radius 15 cm is rolling perfectly on a plane of inclination 30°. The coefficient of static friction, $\mu_s = 0.25$.

(a) Find the force of friction acting on the cylinder.

(b) What is the work done against friction during rolling?

(c) If the inclination θ of the plane is increased, at what value of θ does the cylinder begin to skid, and not roll perfectly?

OR

Prove that the work done by a constant force in displacing a body through a certain distance measures the change in the kinetic energy of the body.

A running man has half the kinetic energy than that of a boy of half of his mass. The man speed up by 1 m s^{-1} and then has the same kinetic energy as the boy. What were the original speeds of the man and the boy?

26. A light string is wound round a cylinder and carries a mass tied to it at the free end. When the mass is released, calculate (a) the linear acceleration of the descending mass and (b) the angular acceleration of the cylinder and (c) the tension in the string. Show that the acceleration of mass is less than g.

OR

A body of mass m is released from the top of a tower of height h. Calculate its total energy at the top of the tower and then just before striking the ground. Show that it is in accordance with the law of conservation of mechanical energy.

SOLUTIONS

1. We know that power (P) = force (F) × velocity (v). Since the body is moving with acceleration, v changes and as a result of that P also changes, F being constant. For P to be constant, Fv = constant or $F \propto \frac{1}{v}$. Thus, as v increases, F should decrease to keep P constant.

2. The spokes of the cycle wheel increases the moment of inertia due to increase in the distribution of mass. This opposes the change in the rotatory motion of the wheel. Thus spokes fitted to the cycle wheel gives a steady motion.

3. The statement is wrong as the centripetal force $(\vec{F}_c)$ acting on Earth is at right angles to its direction of displacement $(\vec{s})$.

Thus, $W = \vec{F}_c \cdot \vec{s} = 0$ and as such work done by the centripetal force on the Earth is zero.

4. If the roads go straight up, the angle of slope θ would be large. In that case frictional force ($f = \mu\, mg \cos\theta$) would be less and the vehicles may skid. Moreover, while going up a large slope, a greater power would be required (as it will take less time).

5. Gyroscope is an instrument which is so constructed that the net torque on the axle is zero. As mentioned no torque is applied during transportation, total torque on the gyroscope is zero. Hence angular moment is conserved.
So it will point towards north direction always.
$\therefore$ When viewed at south pole, it will points towards ground.

6. (a) Work done in stretching a spring of force constant k through a distance x, *i.e.*,

$$W = \frac{1}{2}kx^2$$

Thus, $\dfrac{W_A}{W_B} = \dfrac{(1/2)k_A x^2}{(1/2)k_B x^2} = \dfrac{k_A}{k_B}$

As $k_A > k_B$, $W_A > W_B$

Spring A does more work.

(b) As $F = kx$, $x = \dfrac{F}{k}$

Further, as F is the same,

$$W_A = \frac{1}{2}k_A x^2 = \frac{1}{2}k_A\left(\frac{F}{k_A}\right)^2 = \frac{1}{2}\frac{F^2}{k_A}$$

Similarly, $W_B = \dfrac{1}{2}\dfrac{F^2}{k_B}$

Thus, $\dfrac{W_A}{W_B} = \dfrac{k_B}{k_A}$

As $k_A > k_B$, $W_A < W_B$

Spring B does more work.

7. At $x = \pm 2$ m,

Potential energy of the particle, $U = \dfrac{1}{2}kx^2$

$$= \frac{1}{2}\left(\frac{1}{2}\text{ N m}^{-1}\right)(\pm 2\text{ m})^2 = 1\text{ J} = \text{total energy}$$

Obviously, at these positions, kinetic energy is zero and the particle turns back, though it first comes to rest.

8. Here, mass of the sphere, $M = 1$ kg
radius of the sphere, $R = 10$ cm $= 0.1$ m

(a) We know that moment of inertia of the sphere about its diameter

$$= \frac{2}{5}MR^2 = \left[\frac{2}{5}\times 1\times(0.1)^2\right]\text{kg m}^2$$

$$= 4\times 10^{-3}\text{ kg m}^2$$

(b) Further, moment of inertia of the sphere about its tangent

$$= \frac{7}{5}MR^2 = \left[\frac{7}{5}\times 1\times(0.1)^2\right]\text{kg m}^2 = 1.4\times 10^{-2}\text{ kg m}^2$$

9. We know that $\tau = I\alpha$

Using the chain rule from calculus,

$$\tau = I\alpha = I\frac{d\omega}{dt} = I\frac{d\omega}{d\theta}\times\frac{d\theta}{dt} = I\omega\left(\frac{d\omega}{d\theta}\right)$$

(as $d\theta/dt = \omega$)

or $\tau d\theta = I\omega d\omega$

But as $\tau d\theta = dW$, $dW = I\omega d\omega$

If W is the total work done by the torque (τ) in changing its angular velocity from ω_1 to ω_2,

$$W = \int dW = \int_{\omega_1}^{\omega_2} I\omega d\omega$$

or $W = I\displaystyle\int_{\omega_1}^{\omega_2}\omega d\omega = I\left[\frac{\omega^2}{2}\right]_{\omega_1}^{\omega_2}$ *i.e.*, $W = \dfrac{1}{2}I(\omega_2^2 - \omega_1^2)$

OR

If a is the uniform acceleration of the body, then

$v_1 = 0 + at_1$ or $a = v_1/t_1$

The velocity of the body at instant t,

$$v = 0 + at = \frac{v_1}{t_1}\cdot t$$

Instantaneous power,

$$P = Fv = mav = m\times\frac{v_1}{t_1}\times\frac{v_1}{t_1}\cdot t = \frac{mv_1^2 t}{t_1^2}.$$

10. We are given that

$\vec{v}_1 = (2\hat{i} - \hat{j})\text{ m s}^{-1}$, $\vec{v}_2 = (\hat{i} + 6\hat{j})\text{ m s}^{-1}$,

$m_1 = 2$ kg, $m_2 = 3$ kg

(a) If $\vec{v}_{cm}$ is the velocity of the centre of mass,

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \frac{2(2\hat{i}-\hat{j}) + 3(\hat{i}+6\hat{j})}{2+3}\text{ m s}^{-1}$$

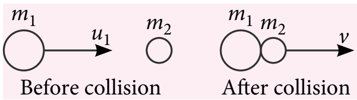
$$= (1.4\hat{i} + 3.2\hat{j})\text{ m s}^{-1}$$

(b) If $\vec{p}$ is the total momentum of the system of total mass M,

$$\vec{p} = M\vec{v}_{cm} = (2+3)(1.4\hat{i} + 3.2\hat{j})\text{ m s}^{-1}$$

$$= (7\hat{i} + 16\hat{j})\text{ kg m s}^{-1}$$

11. Perfectly inelastic collision : When the two colliding bodies stick together and move as a single body with a common velocity after the collision, the collision is perfectly inelastic.
As shown in figure, a body of mass m_1 moving with velocity u_1 collides head-on with another body of mass m_2 at rest. After the collision, the two bodies move together with a common velocity v.



As the linear momentum is conserved, so

$$m_1u_1 + m_2 \times 0 = (m_1 + m_2)v$$

or $v = \dfrac{m_1}{m_1+m_2}u_1$

The loss in kinetic energy on collision is

$$\Delta K = K_i - K_f = \frac{1}{2}m_1u_1^2 - \frac{1}{2}(m_1+m_2)v^2$$

$$= \frac{1}{2}m_1u_1^2 - \frac{1}{2}(m_1+m_2)\left[\frac{m_1}{m_1+m_2}u_1\right]^2$$

$$= \frac{1}{2}m_1u_1^2 - \frac{1}{2}\frac{m_1^2}{m_1+m_2}u_1^2 = \frac{1}{2}m_1u_1^2\left[1-\frac{m_1}{m_1+m_2}\right]$$

or $\Delta K = \dfrac{1}{2}\dfrac{m_1m_2}{m_1+m_2}u_1^2.$

This is a positive quantity. The kinetic energy is lost mainly in the form of heat and sound.

Moreover,

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}(m_1+m_2)v^2}{\frac{1}{2}m_1u_1^2} = \frac{m_1+m_2}{m_1}\cdot\frac{v^2}{u_1^2}$$

$$= \frac{m_1+m_2}{m_1}\left(\frac{m_1}{m_1+m_2}\right)^2 \text{ or } \frac{K_f}{K_i} = \frac{m_1}{m_1+m_2} < 1$$

This again shows that the kinetic energy after the collision is less than the kinetic energy before the collision.

If the target is massive, *i.e.*, $m_2 >> m_1$, then

$$\frac{K_f}{K_i} \simeq 0 \quad i.e., \quad K_f \simeq 0$$

Hence when a light moving body collides against any massive body at rest and sticks to it, practically all of its kinetic energy is lost.

12. Let V be the original volume of the Earth and V' be its volume after shrinking. If R and R' are the corresponding radii, then

$$\frac{V'}{V} = \frac{(4\pi/3)R'^3}{(4\pi/3)R^3} = \frac{1}{64}$$

or $\left(\dfrac{R'}{R}\right)^3 = \dfrac{1}{64}$ or $\dfrac{R'}{R} = \dfrac{1}{4}$ or $\dfrac{R}{R'} = 4$

Let I and I' be the moments of inertia of the Earth about its axis (diameter) before and after shrinking and ω and ω' be the corresponding angular velocities. According to the law of conservation of angular momentum,

$$I'\omega' = I\omega$$

or $\dfrac{\omega'}{\omega} = \dfrac{I}{I'} = \dfrac{(2/5)MR^2}{(2/5)MR'^2}$ or $\dfrac{\omega'}{\omega} = \left(\dfrac{R}{R'}\right)^2 = (4)^2 = 16$

If T and T' be the time periods of rotation of the Earth about its own axis,

$$T = \frac{2\pi}{\omega} \text{ and } T' = \frac{2\pi}{\omega'}$$

or $\dfrac{T}{T'} = \dfrac{2\pi/\omega}{2\pi/\omega'} = \dfrac{\omega'}{\omega} = 16$ or $T' = \dfrac{T}{16} = \dfrac{24\text{ h}}{16} = 1.5\text{ h}$

13. (a) Volume of the air passing through the windmill in time t

= Area of circle × distance covered by wind in time t

$= A \times vt = Avt$

Mass of the air passing through the windmill in time t,

$$m = \text{Density} \times \text{volume} = \rho Avt$$

(b) Kinetic energy of the air is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\rho Avt \times v^2 = \frac{1}{2}\rho Av^3t.$$

(c) Kinetic energy of air converted into electrical energy in time t,

$$K' = 25\% \text{ of } K = \frac{25}{100}\times\frac{1}{2}\rho Av^3t = \frac{1}{8}\rho Av^3t$$

Electrical power produced

$$= \frac{K'}{t} = \frac{1}{8}\rho Av^3 = \frac{1}{8}\times 1.2\times 30\times(10)^3$$

$[\because\ v = 36 \text{ km h}^{-1} = 10 \text{ m s}^{-1}]$

$= 4.5 \times 10^3$ W = 4.5 kW

14. Here $m_1 = M$, $u_1 = v$, $m_2 = m$, $u_2 = 0$

Velocity of M after collision,

$$v_1 = \frac{m_1-m_2}{m_1+m_2}u_1 + \frac{2m_2}{m_1+m_2}\cdot u_2$$

$$= \frac{M-m}{M+m}.v + \frac{2m}{M+m}\cdot 0$$

Kinetic energy lost by mass M,

$$= \frac{1}{2}Mv^2 - \frac{1}{2}M\left(\frac{M-m}{M+m}v\right)^2$$

$$= \frac{1}{2}Mv^2\left[1-\left(\frac{M-m}{M+m}\right)^2\right] = \frac{1}{2}Mv^2\cdot\frac{4Mm}{(M+m)^2}$$

$$= \frac{1}{2}Mv^2\cdot\frac{4Mm}{M^2} \qquad [M+m\approx M]$$

$= 2mv^2$

OR

Here $I = 9.3 \times 10^{37}$ kg m^2, $R = 6.4 \times 10^6$ m,
$\omega_0 = 7.3 \times 10^{-5}$ rad s^{-1},
t = 1 year = $365 \times 24 \times 3600$ s

As $\omega = \omega_0 + \alpha t$

$$\therefore \quad \alpha = \frac{\omega - \omega_0}{t} = \frac{0 - 7.3\times10^{-5}}{365\times24\times3600}$$

$$= -\frac{7.3\times10^{-5}}{365\times24\times3600} \text{ rad s}^{-2}$$

Torque, $\tau = I\alpha = 9.3 \times 10^{37} \times \dfrac{7.3\times10^{-5}}{365\times24\times3600}$ N m

(Omitting –ve sign)

Let F be the tangential force needed to stop the Earth. Then

$\tau = FR$

$$\text{or} \quad F = \frac{\tau}{R} = \frac{9.3\times10^{37}\times7.3\times10^{-5}}{365\times24\times3600\times6.4\times10^{6}}$$

$= 3.363 \times 10^{19}$ N

15. Moment of inertia of the disc about the given axis is

$$I = \frac{1}{2}MR^2$$

Moment of inertia of the system of two discs (masses M and $M/4$ and each having a radius R) placed coaxially, *i.e.*,

$$I' = \frac{1}{2}MR^2 + \frac{1}{2}\left(\frac{M}{4}\right)R^2 = \frac{5}{8}MR^2$$

According to the law of conservation of angular momentum,

$I\omega = I'\omega'$ or $\omega' = (I/I')\omega$

$$\text{or} \quad \omega' = \frac{(1/2)MR^2}{(5/8)MR^2}\omega = \frac{4}{5}\omega$$

16. (a) Here, mass of flywheel, M = 500 kg

Frequency of the flywheel, $\upsilon = \dfrac{500}{60} = 8.3$ rps

Thus, $\omega = 2\pi\upsilon = (2 \times 3.14 \times 8.3)$ rad s^{-1} = 52 rad s^{-1}

(b) Since the mass of the flywheel is supposed to be concentrated at the rim (edge), distance of the rim from the axis of rotation (passing through the centre) = radius of gyration (k)

$= \dfrac{1}{2}$(diameter of wheel) = 0.5 m

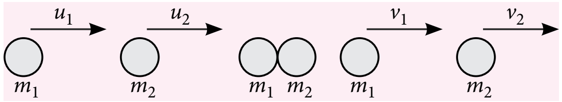
Moment of inertia of the flywheel about the axis of rotation, *i.e.*,

$I = Mk^2 = [500 \times (0.5)^2]$ kg m^2 = 125 kg m^2

(c) Kinetic energy of rotation of the flywheel

$$= \frac{1}{2}I\omega^2 = \left[\frac{1}{2}\times125\times(52)^2\right] = 1.7 \times 10^5 \text{ J}$$

17. Using conservation of momentum,



$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow \quad m_1(u_1 - v_1) = m_2(v_2 - u_2) \qquad \text{...(i)}$$

Now from conservation of kinetic energy,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\Rightarrow \quad m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \qquad \text{...(ii)}$$

Velocity after collision

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2}{m_1 + m_2}u_2 \text{ and} \qquad \text{...(iii)}$$

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1}{m_1 + m_2}u_1 \qquad \text{...(iv)}$$

When a light body collides against a massive stationary body. Here $m_1 << m_2$ and $u_2 = 0$. Neglecting m_1 in equation (iii), we get

$$v_1 = \frac{m_2u_1}{m_2} = -u_1$$

From eqn. (iv), $v_2 \simeq 0$.

Hence when a light body collides against a massive body at rest, the light body rebounds after the collision with an equal and opposite velocity while the massive body practically remains at rest.

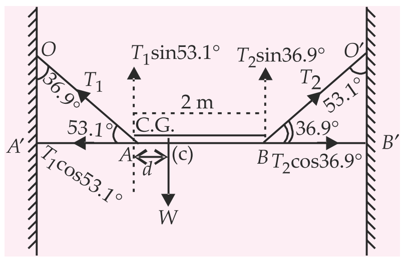
18. Let AB be the uniform bar of weight W suspended at rest by the two strings OA and $O'B$ which make angles 36.9° and 53.1° respectively with the vertical.

$\therefore \quad \angle OAA' = 90° - 36.9° = 53.1°$

Similarly $\angle O'BB' = 36.9°$

AB = 2 m, $AC = d$ m.

Let T_1 and T_2 be the tensions in the strings OA and $O'B$ respectively and their rectangular components are shown in the figure.



As the rod is at rest, so the vector sum of the forces acting along $A'B'$ axis and $\perp$ to it are zero *i.e.*

$$-T_1\cos53.1° + T_2\cos36.9° = 0 \qquad \text{...(i)}$$

$$\text{and } T_1\sin53.1° + T_2\sin36.9° - W = 0 \qquad \text{...(ii)}$$

Taking the torques about A and equating the sum of torques to zero, we get

$$-(T_2 \sin 36.9°) \times 2 + Wd = 0$$

or $T_2 = \dfrac{Wd}{2\sin 36.9°}$ ...(iii)

$\therefore$ From (ii) and (iii), we get

$$T_1 \sin 53.1° = W - T_2 \sin 36.9° = W - \frac{Wd}{2}$$

$\therefore$ $T_1 = \dfrac{W}{\sin 53.1°}\left(1 - \dfrac{d}{2}\right)$ ...(iv)

$\therefore$ From (i), (iii) and (iv), we get

$$T_1 \cos 53.1° = T_2 \cos 36.9°$$

or $\dfrac{W\left(1-\dfrac{d}{2}\right)}{\sin 53.1°}\cos 53.1° = \dfrac{Wd\cos 36.9°}{2\sin 36.9°}$

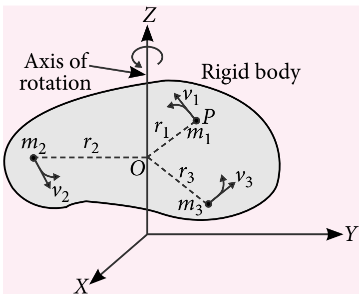
or $\dfrac{W\left(1-\dfrac{d}{2}\right)}{\tan 53.1°} = \dfrac{Wd}{2\tan 36.9°}$

or $1 - \dfrac{d}{2} = \dfrac{d}{2} \times \dfrac{1.3319}{0.7508} = \dfrac{d}{2} \times 1.7740 = 0.8870\, d$

or $0.5d + 0.8870d = 1$

or $d = \dfrac{1}{1.3870} = 0.721 \text{ m} = 72.1 \text{ cm}$

19. When a torque acts on a body capable of rotation about an axis, it produces an angular acceleration in the body. If the angular velocity of each particle is ω then the angular acceleration, $\alpha = d\omega/dt$ will be same for all particles of the body. The linear acceleration will depend on their distances $r_1, r_2, ..., r_n$ from the axis of the rotation.



As shown in figure, consider a particle P of mass m_1 at a distance r_1 from the axis of rotation. Let its linear velocity be v_1.

Linear acceleration of the first particle, $a_1 = r_1\alpha$

Force acting on the first particle, $F_1 = m_1 r_1 \alpha$

Moment of force F_1 about the axis of rotation is

$$\tau_1 = F_1 r_1 = m_1 r_1^2 \alpha$$

Total torque acting on the rigid body is

$$\tau = \tau_1 + \tau_2 + \tau_3 + ... + \tau_n$$
$$= m_1 r_1^2\alpha + m_2 r_2^2 \alpha + m_3 r_3^2 \alpha + \ldots\ldots + m_n r_n^2 \alpha$$
$$= (\Sigma mr^2)\alpha$$

But $\Sigma mr^2 = I$, moment of inertia of the body about the given axis

$\therefore$ $\tau = I\alpha$

Torque = Moment of inertia × Angular acceleration

When $\alpha = 1$, $\tau = 1$

Thus the moment of inertia of a rigid body about an axis of rotation is numerically equal to the external torque required to produce unit angular acceleration in the body about that axis.

20. Work-energy theorem states that the work done by net force acting on a body is equal to the change produced in the kinetic energy of the body.

Suppose a variable force $\vec{F}$ acts on a body of mass m and produces displacement $d\vec{s}$ in its own direction *i.e.*, $\theta = 0°$.

The small work done is

$$dW = \vec{F} \cdot d\vec{s} = F\,ds \cos 0° = Fds$$

According to Newton's second law of motion,

$$F = ma = m\frac{dv}{dt}$$

$\therefore$ $dW = m\dfrac{dv}{dt} \cdot ds = mv\,dv$ $\left[\because \dfrac{ds}{dt} = v\right]$

If the applied force increases, the velocity changes from u to v, then the total work done on the body will be

$$W = \int dW = \int_u^v mvdv = m\int_u^v vdv$$

$$= m\left[\frac{v^2}{2}\right]_u^v = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$W = K_f - K_i$

= Change in kinetic energy of the body.

This proves the work-energy theorem for a variable force.

21. (a) Zero as the disc does not move as a whole.

(b) Decreases due to friction between the disc and the table.

(c) Increases as acceleration is produced in the centre of mass due to friction ($a_{CM} = \mu_k g$).

(d) Friction force between table and disc

(e) Condition for rolling to begin, $v_{CM} = R\omega$

22. If f is the force of friction acting on the disc and a is the acceleration produced in it due to applied force F, then

$$Ma = F - f \quad ...(i)$$

When the disc rolls without slide, $a = R\alpha$ or $\alpha = \frac{a}{R}$

As $I\alpha = Rf$,

$$\left(\frac{1}{2}MR^2\right)\left(\frac{a}{R}\right) = Rf \text{ or } Ma = 2f \quad ...(ii)$$

From eqn. (i) and (ii), $2f = F - f$ or $F = 3f$

Since there is no sliding, $f \le \mu mg$ or $F \le 3\mu mg$.

$\therefore \quad F_{max} = 3\ \mu mg$

23. (a) Scientific thinking, determination and curiosity.

(b) Let V be the minimum volume of water flowing out per second from the tub kept at height h which makes the bulb glow.

Then input power = $(V\rho)gh$

($\because$ $V\rho$ = mass of water flowing per second)

Output power or required power = 40 W

$$\text{Efficiency, } \eta = \frac{\text{Output power}}{\text{Input power}} \Rightarrow \frac{90}{100} = \frac{40}{V\rho gh}$$

$$\therefore \quad V = \frac{40 \times 100}{90\rho gh} = \frac{40 \times 100}{90 \times 1000 \times 9.8 \times 10}$$

$= 0.453 \times 10^{-3}\ m^3\ s^{-1} = 0.453$ litre s^{-1}.

24. (a) Perfectly elastic collision : A collision in which the kinetic energy is conserved is said to be perfectly elastic.

Collision between two billiard balls is approximately elastic. Truly elastic collisions can occur only in atomic scale (these are not always elastic).

(b) Perfectly inelastic collision : A perfectly inelastic collision is one in which the two colliding bodies stick together after collision.

In such a collision, the loss of kinetic energy is maximum but not complete and is consistent with momentum conservation.

Following are the examples of perfectly inelastic collision.

(i) Collision between two clay spheres.

(ii) A man jumping into a moving trolley.

Velocities after elastic collision

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2}{m_1 + m_2}u_2 \text{ and}$$

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1}{m_1 + m_2}u_1$$

Velocities after inelastic collision,

$$v_1 = v_2 = \left(\frac{m_1}{m_1 + m_2}\right)u_1 \quad (u_2 = 0)$$

Here, $m_1 = 5$ kg, $m_2 = 10$ kg,
$u_1 = 21\ m\ s^{-1}$, $u_2 = -21 m\ s^{-1}$

If v_1 and v_2 are the velocities after collision,

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$= \left(\frac{5-10}{5+10}\right)21 + \left(\frac{2 \times 10}{5+10}\right)(-21) = -35\ m\ s^{-1}$$

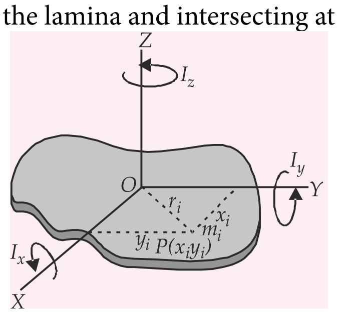
$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \left(\frac{2m_1}{m_1 + m_2}\right)u_1$$

$$= \left(\frac{10-5}{5+10}\right)(-21) + \left(\frac{2 \times 5}{5+10}\right)(21) = 7\ m\ s^{-1}$$

OR

(a) Theorem of perpendicular axes : It states that the moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the lamina about any two mutually perpendicular axes in its plane and intersecting each other at the point, where the perpendicular axis passes through it.

Let OZ be the axis perpendicular to the plane lamina and passing through the point O. Let OX and OY be two mutually perpendicular axes in the plane of the lamina and intersecting at the point O.



If I_x, I_y and I_z are the moments of inertia of the plane lamina about the axes OX, OY and OZ respectively, then according to theorem of perpendicular axes,

$$I_z = I_x + I_y \quad ...(i)$$

Proof : Suppose that the rigid body is made of n particles of masses m_1, m_2, ... m_n lying at distance r_1, r_2........, r_n from the axis of rotation OZ. Further suppose that the i^{th} particle of mass m_i lies at point $P(x_i, y_i)$, such that $OP = r_i$. Then,

$$r_i^2 = x_i^2 + y_i^2$$

Now, moment of inertia of the body about the axis OZ is given by

$$I_z = \sum_{i=1}^{n} m_i r_i^2 = \sum_{i=1}^{n} m_i (x_i^2 + y_i^2)$$

$$= \sum_{i=1}^{n} m_i x_i^2 + \sum_{i=1}^{n} m_i y_i^2 \quad \text{...(ii)}$$

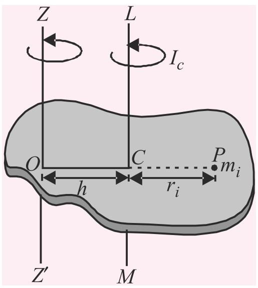
But $\sum_{i=1}^{n} m_i x_i^2 = I_y$, moment of inertia of the body about axis OY and $\sum_{i=1}^{n} m_i y_i^2 = I_x$, the moment of inertia of the body about axis OX

Therefore, the equation (ii) becomes

$$I_z = I_x + I_y$$

(b) Theorem of parallel axes : It states that the moment of inertia of a rigid body about any axis is equal to its moment of inertia about a parallel axis through its centre of mass plus the product of the mass of the body and the square of the perpendicular distance between the two axes.

Let I_c be the moment of inertia of a body of mass M about an axis LM passing through its centre of mass C. Let I be the moment of inertia of the body about an axis ZZ' parallel to the axis LM and at a distance h from it as shown in the figure.



Then, according to the theorem of parallel axes,

$$I = I_c + Mh^2 \quad \text{...(iii)}$$

Proof : Consider that i^{th} particle located at the point P in the body is of mass m_i and lies at a distance r_i from the axis LM. Then, the distance of i^{th} particle from axis ZZ' is $(r_i + h)$. The moment of inertia of the i^{th} particle about the axis LM is $m_i r_i^2$. Therefore, moment of inertia of the body about the axis LM is given by

$$I_c = \sum_{i=1}^{n} m_i r_i^2$$

Also, moment of inertia of the i^{th} particle about the axis ZZ' is $m_i(r_i + h)^2$, so that moment of inertia of the body about the axis ZZ' is given by

$$I = \sum_{i=1}^{n} m_i (r_i + h)^2 = \sum_{i=1}^{n} m_i (r_i^2 + h^2 + 2hr_i)$$

$$= \sum_{i=1}^{n} m_i r_i^2 + \sum_{i=1}^{n} m_i h^2 + \sum_{i=1}^{n} 2hm_i r_i$$

$$= I_c + \left(\sum_{i=1}^{n} m_i\right) h^2 + 2h \sum_{i=1}^{n} m_i r_i \quad \text{...(iv)}$$

Now, $\sum_{i=1}^{n} m_i = M$, mass of the body and $\sum_{i=1}^{n} m_i r_i =$ sum of the moments of the masses of the particles constituting the body about an axis through its centre of mass. Since sum of the moments of the masses of the particles constituting the body an axis through its centre of mass must be zero,

$$\sum_{i=1}^{n} m_i r_i = 0$$

In R.H.S. of the equation (iv), substituting the values of the two factors, we have

$$I = I_c + Mh^2$$

It proves the theorem of parallel axes for moment of inertia.

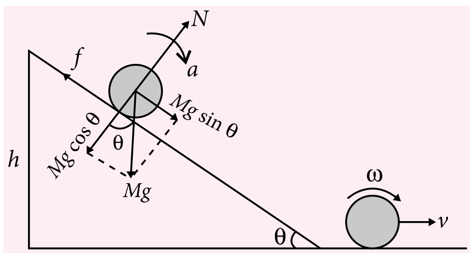
25. Consider a body of mass M and radius R rolling down a plane inclined at an angle θ with the horizontal, as shown in figure. It is only due to friction at the line of contact that body can roll without slipping. The centre of mass of the body moves in a straight line parallel to the inclined plane.

The external forces on the body are

(i) The weight Mg acting vertically downwards.

(ii) The normal reaction N of the inclined plane.

(iii) The force of friction acting up the inclined plane.



Let a be the downward acceleration of the body. The equation of motion for the body can be written as

$N - Mg\cos\theta = 0$

$F = Ma = Mg\sin\theta - f$

As the force of friction f provides the necessary torque for rolling, so

$$\tau = f \times R = I\alpha = Mk^2\left(\frac{a}{R}\right) \text{ or } f = M\frac{k^2}{R^2}\cdot a$$

where k is the radius of gyration of the body about its axis of rotation. Clearly

$$Ma = Mg\sin\theta - M\frac{k^2}{R^2}\cdot a \text{ or } a = \frac{g\sin\theta}{(1+k^2/R^2)}$$

Let h be height of the inclined plane and s the distance travelled by the body down the plane. The velocity v attained by the body at the bottom of the inclined plane can be obtained as follow :

$$v^2 - u^2 = 2as$$

or $v^2 - 0^2 = \dfrac{2g\sin\theta}{(1+k^2/R^2)}\cdot s$

or $v^2 = \dfrac{2gh}{1+k^2/R^2}$ $\qquad \left[\because \dfrac{h}{s} = \sin\theta\right]$

or $v = \sqrt{\dfrac{2gh}{(1+k^2/R^2)}}$

Here $M = 10$ kg, $R = 15$ cm $= 0.15$ m, $\mu_s = 0.25$, $\theta = 30°$

(a) Force of friction, $f = M\dfrac{k^2}{R^2}a$

For cylinder, $k = \dfrac{R}{\sqrt{2}}$

$\therefore \quad a = \dfrac{2}{3}g\sin\theta$

$$f = \frac{M}{2}\left(\frac{2}{3}g\sin\theta\right) = \frac{M}{3}g\sin\theta$$

$$f = \frac{1}{3}Mg\sin\theta = \frac{1}{3}\times 10\times 9.8\times \sin 30°$$

$$= \frac{1}{3}\times 10\times 9.8\times 0.5 = 16.33 \text{ N}$$

(b) Work done against friction during rolling = 0 J

(c) Condition for skidding (or no rolling) is $\dfrac{f}{N} \le \mu_s$

or $\dfrac{\frac{1}{3}Mg\sin\theta}{Mg\cos\theta} \le \mu_s$ or $\dfrac{1}{3}\tan\theta \le \mu_s$

Thus the cylinder will start skidding at an angle of inclination θ given by

$\tan\theta = 3\,\mu_s = 3\times 0.25 = 0.75$ or $\theta = 36°52'$.

OR

For a body moving under constant force, acceleration a is constant.

$\therefore \quad v^2 - u^2 = 2as$

where u and v are the initial and final speeds and s is the distance traversed.

Multiplying both sides by $m/2$, we have

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = ma\,s = Fs$$

The left side of the equation is the difference is the kinetic energies at two different time and right side is the product of displacement and the component of force, *i.e.*, work.

$\therefore \quad K_f - K_i = W$

This relationship is called the work-energy theorem. In words, it states that :

The net work done by the external forces acting on a body is equal to the change in kinetic energy of the body.

Kinetic energy of the man, $K_m = \dfrac{1}{2}mv_m^2$

Kinetic energy of the boy, $K_b = \dfrac{1}{2}\left(\dfrac{m}{2}\right)v_b^2 = \dfrac{1}{4}mv_b^2$

As $K_m = \dfrac{1}{2}K_b \Rightarrow \dfrac{1}{2}mv_m^2 = \dfrac{1}{2}\left(\dfrac{1}{4}mv_b^2\right)$

or $v_m = \dfrac{1}{2}v_b$ or $v_b = 2v_m$ ...(i)

Increased speed of man, *i.e.*, $v'_m = (v_m + 1)$m s^{-1}

Increased kinetic energy of man, *i.e.*,

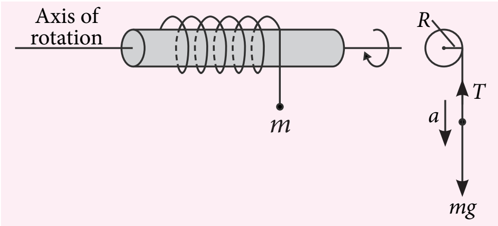
$$K'_m = \frac{1}{2}m(v_m+1)^2$$

As per the given condition, $K'_m = K_b$

or $\dfrac{1}{2}m(v_m+1)^2 = \dfrac{1}{4}mv_b^2 = \dfrac{1}{4}m(2v_m)^2$ (using (i))

or $v_m = 2.4$ m s^{-1}, $v_b = 4.8$ m s^{-1}

26. As shown in figure, consider a solid cylinder of mass m and radius R. It is mounted on a frictionless horizontal axle so that it can freely rotate about its axis. A light string is wound round the cylinder and mass m is suspended from it. When the mass m is released from rest, it moves down with acceleration a. Let T be the tension in the string.



(a) The forces acting on the point mass are

(i) Its weight mg acting vertically downwards.

(ii) Tension T in the string acting upwards. According to newton's second law, the net downward force on the point mass is

$$ma = mg - T \qquad \text{...(i)}$$

The tension T in the string acts tangentially on the cylinder and produces a torque τ given by

$$\tau = \text{force} \times \text{lever arm} = T \cdot R \qquad \text{...(ii)}$$

If I is the moment of inertia of the cylinder and α, the angular acceleration produced in it, then

$$\tau = I\alpha \qquad \text{...(iii)}$$

From equations (ii) and (iii),

$$TR = I\alpha$$

or $$T = \frac{I}{R}\alpha = \frac{Ia}{R^2} \qquad \left[\because \alpha = \frac{a}{R}\right] \qquad \text{...(iv)}$$

From equation (i), we have

$$ma = mg - \frac{Ia}{R^2} \quad \text{or} \quad ma + \frac{Ia}{R^2} = mg$$

or $$ma\left(1 + \frac{I}{mR^2}\right) = mg \text{ or } a = \frac{g}{1 + \frac{I}{mR^2}} \qquad \text{...(v)}$$

This gives the linear downward acceleration of the point mass.

As I, m and R are positive quantities, so a is always less than g.

(b) Angular acceleration,

$$\alpha = \frac{a}{R} = \frac{g/R}{1 + \frac{I}{mR^2}} \qquad \text{(Using eqn. (v))}$$

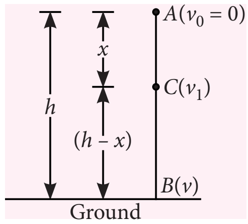
(c) From equations (iv) and (v), we have

$$T = \frac{Ia}{R^2} = \frac{Ig}{R^2\left(1 + \frac{I}{mR^2}\right)} = \frac{Ig}{R^2 \cdot \frac{I}{mR^2}\left(\frac{mR^2}{I} + 1\right)}$$

or $$T = \frac{mg}{1 + \frac{mR^2}{I}}$$

Clearly, T is less than the weight mg of the point mass.

OR



Let us consider a body of mass m placed at A as shown in figure.

$h = AB$ = height of the body above the ground

x = distance of any point C from A

g = acceleration due to gravity at the place

v_1 = velocity of the body at C

v = velocity of the body of B, a point just above the ground.

The velocity at the point A is zero.

At A potential energy $U_1 = mgh$ and kinetic energy $K_1 = 0$

Total mechanical energy, *i.e.*, $E_1 = mgh + 0 = mgh$ ...(i)

At C, as the body covers a distance x when it moves from A to C and v_1 is the velocity at C, then from $v^2 - u_0^2 = 2as$, we get

$v_1^2 - 0 = 2gx$ (as $v = v_1$, $u = 0$, $a = g$, $s = x$)

or $v_1^2 = 2gx$

Kinetic energy at C, $K_2 = \frac{1}{2}mv_1^2 = \frac{1}{2}m \times 2gx = mgx$

Potential energy at C, $U_2 = mg(h - x)$

Total mechanical energy at C, $E_2 = mgx + mg(h - x)$
$= mgh$ ...(ii)

At B,

From $v^2 - u_0^2 = 2as$,

$v^2 - 0 = 2gh$ (as $v = v$, $u = 0$, $a = g$, $s = h$)

or $v^2 = 2gh$

Kinetic energy at B, $K_3 = \frac{1}{2}mv^2 = \frac{1}{2}m \times 2gh = mgh$

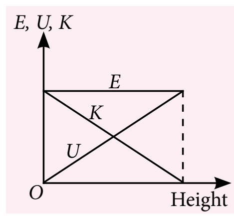
Potential energy at B, $U_3 = 0$

Total mechanical energy at B,

$$E_3 = mgh + 0 = mgh \qquad \text{...(iii)}$$

From eqns. (i), (ii) and (iii), $E_1 = E_2 = E_3$

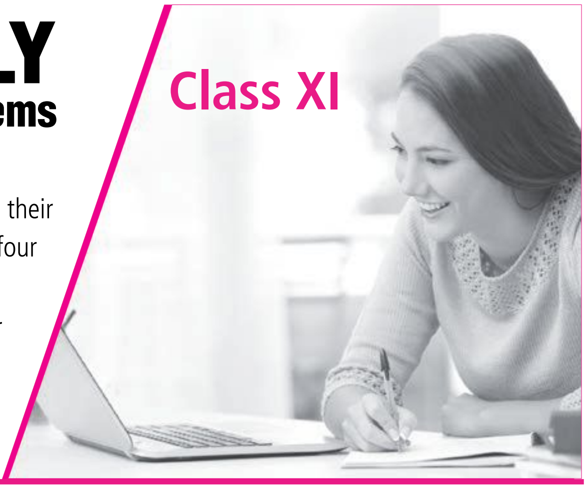
Clearly, total mechanical energy of the body at the points A, B and C (also at any other point) is the same. Thus, we find the total mechanical energy throughout the free fall is conserved.



As the body falls down, the potential energy goes on decreasing whereas the kinetic energy goes on increasing. The variation of kinetic energy (K) and potential energy (U) and the mechanical energy (E) have been represented graphically in figure.

MPP-5 MONTHLY Practice Problems

Class XI



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.

Self check table given at the end will help you to check your readiness.

System of Particles and Rotational Motion

Total Marks : 120 **Time Taken : 60 min**

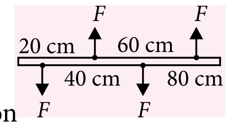
NEET / AIIMS

Only One Option Correct Type

1. The centre of mass of a system of three particles of masses 1 g, 2 g and 3 g is taken as the origin of a coordinate system. The position vector of a fourth particle of mass 4 g such that the centre of mass of the four particle system lies at the point (1, 2, 3) is $\alpha(\hat{i} + 2\hat{j} + 3\hat{k})$, where α is a constant. The value of α is

(a) $\frac{10}{3}$ (b) $\frac{5}{2}$ (c) $\frac{1}{2}$ (d) $\frac{2}{5}$

2. Four equal and parallel forces are acting on a rod of length 100 cm, as shown in figure, at distances of 20 cm, 40 cm, 60 cm and 80 cm respectively from one end of the rod. Under the influence of these forces, the rod (neglecting its weight)



(a) experiences no torque
(b) experiences torque
(c) experiences a linear motion
(d) experiences torque and also a linear motion

3. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius $4R$ is made from an iron plate of thickness $\frac{t}{4}$. Then the relation between the moment of inertia I_X and I_Y is

(a) $I_Y = 32I_X$ (b) $I_Y = 16I_X$
(c) $I_Y = I_X$ (d) $I_Y = 64I_X$

4. If I_1 is the moment of inertia of a thin rod about an axis perpendicular to its length and passing through the centre of mass and I_2 is the moment of inertia of the ring formed by bending this rod in the form of a ring, then $\frac{I_1}{I_2}$ is

(a) 1 : 1 (b) $\pi^2 : 3$ (c) $\pi : 4$ (d) 3 : 5

5. A solid body rotates about a stationary axis so that its angular velocity depends on the rotational angle ϕ as $\omega = \omega_0 - k\phi$, where ω_0 and k are positive constants. At the moment $t = 0$, $\phi = 0$, the time dependence of rotation angle is

(a) $k\omega_0 e^{-kt}$ (b) $\frac{\omega_0}{k} e^{-kt}$
(c) $\frac{\omega_0}{k}(1 - e^{-kt})$ (d) $\frac{k}{\omega_0}(e^{-kt} - 1)$

6. A string is wound round the rim of a mounted flywheel of mass 20 kg and radius 20 cm. A steady pull of 25 N is applied on the cord. Neglecting friction and mass of the string, the angular acceleration of the wheel is

(a) 50 rad s^{2} (b) 25 rad s^{2}
(c) 12.5 rad s^{2} (d) 6.25 rad s^{2}

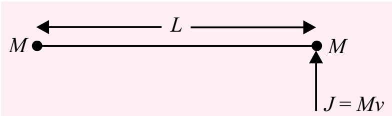
7. A particle with position vector $\vec{r}$ has a linear momentum $\vec{p}$. Which of the following statements is true in respect of its angular momentum $\vec{L}$ about the origin?

(a) $\vec{L}$ acts along $\vec{p}$. (b) $\vec{L}$ acts along $\vec{r}$.
(c) $\vec{L}$ is maximum when $\vec{p}$ and $\vec{r}$ are parallel.
(d) $\vec{L}$ is maximum when $\vec{p}$ is perpendicular to $\vec{r}$.

8. The angular speed of a body changes from ω_1 to ω_2 without applying a torque, but due to changes in moment of inertia. The ratio of radii of gyration in the two cases is

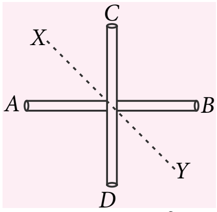
(a) $\omega_2 : \omega_1$ (b) $\sqrt{\omega_1} : \sqrt{\omega_2}$
(c) $\sqrt{\omega_2} : \sqrt{\omega_1}$ (d) $\omega_1 : \omega_2$

9. Consider a body shown in figure consisting two identical balls, each of mass M connected by a light rigid rod. If an impulse $J = Mv$ is imparted to the body at one of its ends, what would be its angular velocity ?



(a) $\frac{v}{L}$ (b) $\frac{2v}{L}$ (c) $\frac{v}{3L}$ (d) $\frac{v}{4L}$

10. AB and CD are two identical rods each of length l and mass m joined to form a cross. The moment of inertia of these two rods about a bisector (XY) of angle between the rods is



(a) $\frac{ml^2}{12}$ (b) $\frac{ml^2}{3}$ (c) $\frac{2ml^2}{3}$ (d) $\frac{ml^2}{6}$

11. A solid sphere, a hollow sphere and a disc, all having same mass and radius, are placed at the top of an incline and released. The friction coefficients between the objects and the incline are same and not sufficient to allow pure rolling. Least time will be taken in reaching the bottom by

(a) the solid sphere (b) the hollow sphere
(c) the disc (d) all will take same time

12. A wheel of radius r and mass m stands in front of a step of height h. The least horizontal force which should be applied to the axle of the wheel to allow it to raise on to the step is

(a) $\frac{mgh(2r-h)}{r-h}$ (b) $mgh\,(r-h)$

(c) $\frac{mg\sqrt{h(2r-h)}}{r-h}$ (d) none of these

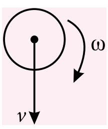
Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :

(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. Assertion : Angular velocity is a vector quantity.
Reason : Angular velocity may be different for different particles of rigid body about axis of rotation.

14. Assertion : A ball rotating with angular velocity ω strikes a rough horizontal surface as shown in figure. Then path of ball after collision is parabolic towards right.



Reason : A force of friction will act towards right during collision.

15. Assertion : Moment of inertia for any rigid body about z axis is $I_z = I_x + I_y$.
Reason : Perpendicular axis theorem is valid for all rigid bodies.

JEE MAIN / JEE ADVANCED

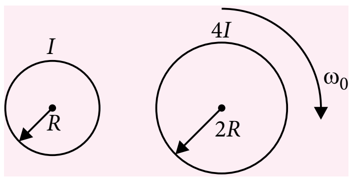
Only One Option Correct Type

16. A uniform rod of length l is free to rotate in a vertical plane about a fixed horizontal axis through its centre. The rod begins rotation from rest from its unstable equilibrium position. When it has turned through an angle θ, its angular velocity ω is given as

(a) $\sqrt{\frac{6g}{l}}\sin\theta$ (b) $\sqrt{\frac{6g}{l}}\sin\frac{\theta}{2}$

(c) $\sqrt{\frac{6g}{l}}\cos\frac{\theta}{2}$ (d) $\sqrt{\frac{6g}{l}}\cos\theta$

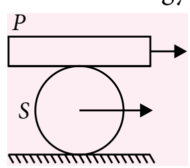
17. Two cylinders having radii $2R$ and R and moment of inertia $4I$ and I about their central axes are supported by axles perpendicular to their planes. The large cylinder is initially rotating clockwise with angular velocity ω_0. The small cylinder is moved to the right until it touches the large cylinder and is caused to rotate by the frictional force between the two. Eventually slipping ceases and the two cylinders rotate at constant rates in opposite directions. During this



(a) angular momentum of system is conserved
(b) kinetic energy is conserved
(c) neither the angular momentum nor the kinetic energy is conserved
(d) both the angular momentum and kinetic energy are conserved

18. A plank P is placed on a solid cylinder S, which rolls on a horizontal surface. The two are of equal mass. There is no slipping at any of the surfaces in contact. The ratio of kinetic energy of P to the kinetic energy of S is

(a) 1 : 1
(b) 2 : 1
(c) 8 : 3
(d) 11 : 8



19. A ring rolls without slipping on the ground. Its centre C moves with a constant speed u. P is any point on the ring. The speed of P with respect to the ground is v. Then

(a) $\frac{1}{2} \le v \le 2u$

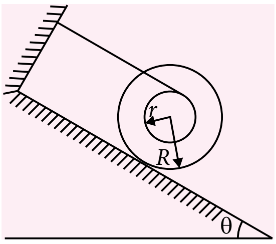
(b) $v = u$, if CP is horizontal

(c) $v = u$, if CP makes an angle of 30° with the horizontal and P is below the horizontal level of C.

(d) $v = \sqrt{3}u$, if CP is horizontal

More than One Options Correct Type

20. Figure shows a spool with thread wound on it placed on a smooth plane inclined at angle θ with horizontal. The spool has mass m, edge radius R and is wound up to a radius r. Its moment of inertia about its own axis is I. The free end of the thread is attached as shown in the figure. So that the thread is parallel to the inclined plane. T is the tension in the thread. Which of the following statements are correct?



(a) The linear acceleration of the spool axis down the slope is $\frac{mg\sin\theta - T}{m}$.

(b) Angular acceleration is $\frac{Tr}{2I}$.

(c) The linear acceleration of the spool axis down the plane is $\frac{Tr^2}{I}$.

(d) The acceleration of the spool axis down the slope is $\frac{g\sin\theta}{1+\frac{I}{mr^2}}$.

21. A uniform rod is resting freely over a smooth horizontal plane. A particle moving horizontally strikes at one end of the rod normally and gets stuck. Then

(a) the momentum of the particle is shared between the particle and the rod and remains conserved

(b) the angular momentum about the mid-point of the rod before and after the collision is equal

(c) the angular momentum about the centre of mass of the combination before and after the collision is equal

(d) the centre of mass of the rod particle system starts to move translationally with the original momentum of the particle

22. A particle of mass m is projected with a velocity v making an angle of 45° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle at its maximum height h is

(a) zero (b) $\frac{mv^3}{4\sqrt{2}g}$

(c) $\frac{mv^3}{\sqrt{2}g}$ (d) $m\sqrt{2gh^3}$

23. A solid sphere is released from height h from the top of an incline making an angle θ with the horizontal. When the sphere reaches the bottom of the incline

(a) the speed of the sphere in the case that it rolls without slipping is $\sqrt{\frac{10}{7}gh}$

(b) the speed of the sphere in the case that it slides frictionlessly without rolling is $\sqrt{2gh}$

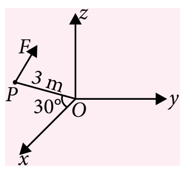
(c) the time required for the sphere to reach the bottom when it rolls is $t = \left(\frac{2h}{\sin\theta}\right)\sqrt{\frac{10}{7}gh}$

(d) the time required for the sphere to reach the bottom when it slides is $t = \left(\frac{2h}{\sin\theta}\right)\sqrt{\frac{1}{2}gh}$

Integer Answer Type

24. A solid sphere rolls on a smooth horizontal surface at 10 m s^{-1} and then rolls up a smooth inclined plane of inclination 30° with horizontal. The mass of the sphere is 2 kg. Find the height (in m) attained by the sphere before it stops.

25. A force F = 2.0 N acts on a particle P in the xz-plane. The force F is parallel to x-axis. The particle P (as shown in the figure) is at a distance 3 m and the line joining P with the origin makes angle 30° with the x-axis. What is the magnitude of torque on P with respect to origin O (in N m).



26. A wheel rotates clockwise about its central axis with an angular momentum of 600 kg m^2 s^{-1}. At time t = 0, a torque of magnitude 50 N m is applied to the wheel to reverse the rotation. If the angular speed becomes zero at time $t = 2x$ s. What is the value of x?

Comprehension Type

A small block of mass 4 kg is attached to a cord passing through a hole in a horizontal frictionless surface. The block is originally revolving in a circle of radius of 5 m about the hole, with a tangential velocity of 4 m s^{-1}. The cord is then pulled slowly from below, shortening the radius of the circle in which the block revolves. The breaking strength of the cord is 200 N.

27. What will be the radius of the circle when the cord breaks?

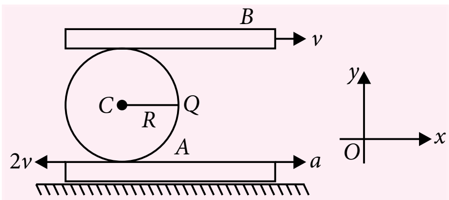
(a) 4.0 m (b) 1.0 m
(c) 3.0 m (d) 2.0 m

28. Velocity of the block at the time of breaking of the string

(a) 10 m s^{-1} (b) 20 m s^{-1}
(c) 7.5 m s^{-1} (d) 12.5 m s^{-1}

Matrix Match Type

29. A cylinder of radius 1 m rolls on the planks A and B without relative sliding. If the planks move with velocities $-4\hat{i}$ m s^{-1} and $2\hat{i}$ m s^{-1} respectively and the plank A has acceleration $\vec{a}\,(=2\hat{i}$ m s$^{-2})$, find the

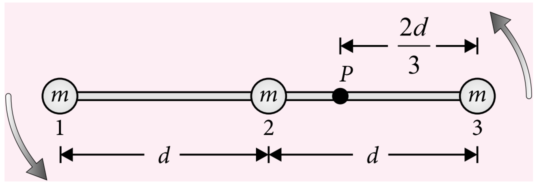


Column I	Column II
(A) v_C (in m s^{-1})	(P) $-\hat{i}-3\hat{j}$
(B) v_Q (in m s^{-1})	(Q) $-\hat{i}$
(C) a_C (in m s^{-2})	(R) $\hat{i}$
(D) a_Q (in m s^{-2})	(S) $-7\hat{i}+2\hat{j}$

Now match the given column and select the correct option from the codes given below.

	A	B	C	D
(a)	P	Q	R	S
(b)	Q	P	R	S
(c)	S	P	Q	R
(d)	P	R	Q	S

30. A rigid, massless rod has three particles with equal masses attached to it as shown in figure. The rod is free to rotate in a vertical plane about a frictionless axle perpendicular to the rod through the point P and is released from rest in the horizontal position at $t = 0$. Assume $m = 1$ kg and $d = 1$ m



Column I	Column II
(A) The angular acceleration of the system at $t = 0$ (in rad s^{-2})	(P) 10
(B) The maximum kinetic energy of the system (in J)	(Q) $2\sqrt{\dfrac{5}{21}}$
(C) The maximum angular moment of the system (in kg m^2 s^{-1})	(R) $\dfrac{30}{7}$
(D) The maximum speed reached by the particle labeled 2 (in m s^{-1})	(S) $2\sqrt{\dfrac{35}{3}}$

Now match the given columns and select the correct option from the codes given below.

	A	B	C	D
(a)	P	Q	R	S
(b)	Q	P	S	R
(c)	R	P	S	Q
(d)	P	R	Q	S

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

NEET | JEE ESSENTIALS

Class XII

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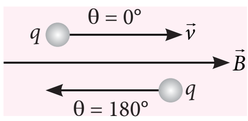
Unit 3 MAGNETIC EFFECT OF CURRENT AND MAGNETISM

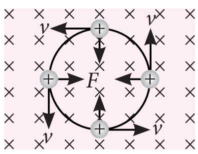
Here our aim is to develope an understanding about how magnetic field exerts forces on moving charged particles, like electrons, protons, and current-carrying wires. We shall see how particles can be accelerated to very high energies in a cyclotron. We shall study how currents and voltages are detected by a galvanometer.

MAGNETIC FORCE ON A MOVING CHARGE

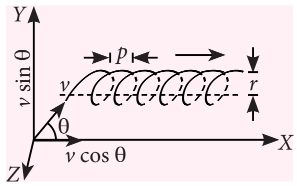
Trajectory of a Charged Particle in a Magnetic Field



- **Straight line** : If the direction of a particle moving with velocity $\vec{v}$ is parallel or antiparallel to magnetic field $\vec{B}$, *i.e.*, θ = 0 or θ = 180° then $F = 0$. Hence the trajectory of the particle is a straight line.



- **Circular path** : If $\vec{v}$ is perpendicular to $\vec{B}$, *i.e.*, θ = 90°, then particle will experience a maximum magnetic force *i.e.*, $F_{max} = qvB$.
- Radius of path $r = \dfrac{mv}{qB} = \dfrac{p}{qB} = \dfrac{\sqrt{2mK}}{qB} = \dfrac{1}{B}\sqrt{\dfrac{2mV}{q}}$, where K is kinetic energy and V is potential.
- The time period of the particle $T = \dfrac{2\pi m}{qB}$



- **Helical path** : When the charged particle is moving at an angle to the field (other than 0°, 90° or 180°), particle describes a path called helix.
- Radius of path $r = \dfrac{m(v\sin\theta)}{qB}$
- Time period and frequency $T = \dfrac{2\pi m}{qB}$ and $\upsilon = \dfrac{qB}{2\pi m}$
- The pitch of the helix,

$$p = T(v\cos\theta) = 2\pi\frac{m}{qB}(v\cos\theta) = \frac{2\pi r}{\tan\theta}$$

Lorentz Force

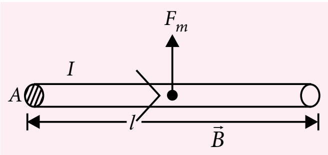
- When the moving charged particle is subjected simultaneously to both electric field $\vec{E}$ and magnetic field $\vec{B}$, it experiences both electric force, $\vec{F}_e = q\vec{E}$ and magnetic force, $\vec{F}_m = q(\vec{v}\times\vec{B})$; so the net force on it will be $\vec{F} = q[\vec{E} + (\vec{v}\times\vec{B})]$.

Cyclotron

- Cyclotron is a device used to accelerate positive (like, α-particles, deutrons etc.) to acquire enough energy to carry out nuclear disintegration etc.
- Cyclotron frequency : $\upsilon = \dfrac{1}{T} = \dfrac{Bq}{2\pi m}$
- Maximum energy of particle : $E_{max} = \left(\dfrac{q^2B^2}{2m}\right)r^2$

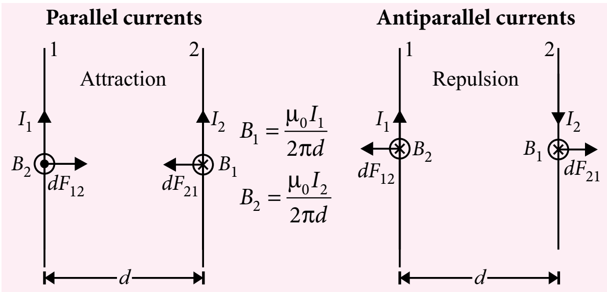
Force on a Current Carrying Conductor

- Net mobile charge carriers in a conductor, $q = nAle$
- Average drift velocity of electron = v_d
- Magnetic force on charged conductor



$$\vec{F}_m = (nAl)e(\vec{v}_d \times \vec{B}) = [(ne\vec{v}_d)Al] \times \vec{B}$$
$$= (\vec{J}Al) \times \vec{B} = I(\vec{l} \times \vec{B})$$

Magnetic Force between Two Long Parallel Current Carrying Wires



- Magnetic force per unit length of each wire is given as

$$\frac{dF_{12}}{dl} = \frac{dF_{21}}{dl} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

Illustration 1 : A particle of charge -16×10^{-18} C moving with velocity 10 m s^{-1} along the x-axis enters a region where a magnetic field of induction B is along the y-axis, and an electric field of magnitude 10^4 V m^{-1} is along the negative z-axis. If the charged particle continues moving along the x-axis, what is the magnitude of magnetic field?

Soln : Particle travels along x-axis. Hence $v_y = v_z = 0$

Field of induction B is along y-axis. $\therefore B_x = B_z = 0$

Electric field is along the negative z-axis. $\therefore E_x = E_y = 0$

$\therefore$ Net force on particle is $F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

Resolve the motion along the three coordinate axis

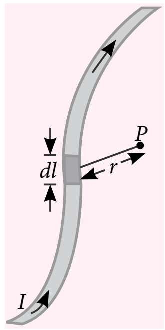
$$\therefore \quad a_x = \frac{F_x}{m} = \frac{q}{m}(E_x + v_y B_z - v_z B_y)$$
$$a_y = \frac{F_y}{m} = \frac{q}{m}(E_y + v_z B_x - v_x B_z)$$
$$a_z = \frac{F_z}{m} = \frac{q}{m}(E_z + v_x B_y - v_y B_x)$$

Since $E_x = E_y = 0$, $v_y = v_z = 0$, $B_x = B_z = 0$ $\therefore$ $a_x = a_y = 0$,

Again $a_z = 0$ as the particle traverse through the region undeflected

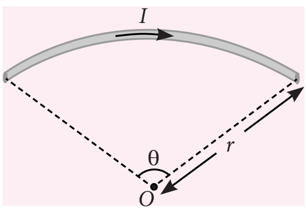
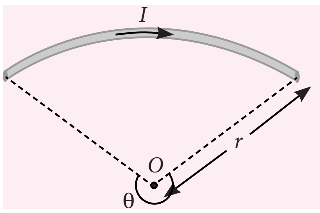
$$\therefore \quad E_z = v_x B_y \text{ or } B_y = \frac{E_z}{v_x} = \frac{10^4}{10} = 10^3 \text{ Wb m}^{-2}.$$

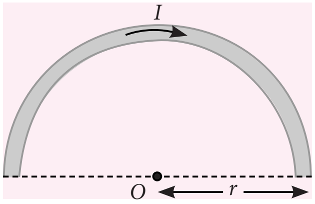
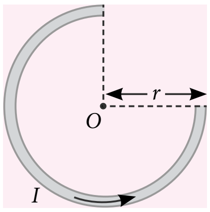
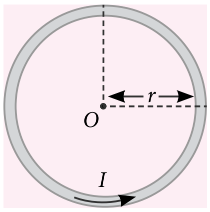
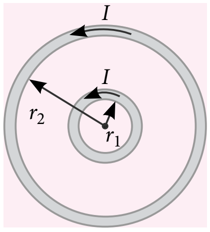
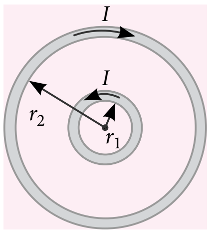
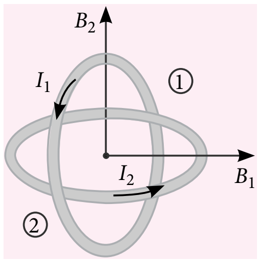
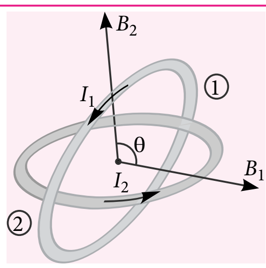
BIOT-SAVART'S LAW



- Biot-Savart's law is used to determine the magnetic field at any point due to a current carrying conductor.
- According to Biot-Savart's law, magnetic field at point P due to the current element $I\vec{dl}$ is given by the expression $d\vec{B} = K\frac{I\,dl\sin\theta}{r^2}\hat{n}$
- Vectorially, $d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I(\vec{dl} \times \hat{r})}{r^2} = \frac{\mu_0}{4\pi} \cdot \frac{I\vec{dl} \times \vec{r}}{r^3}$

Application of Biot-Savart's Law

Condition	Figure	Magnetic field at the centre
Arc subtends angle θ at the centre		$B = \frac{\mu_0}{4\pi} \cdot \frac{\theta I}{r}$
Arc subtends angle (2π – θ) at the centre		$B = \frac{\mu_0}{4\pi} \cdot \frac{(2\pi - \theta)I}{r}$

Semi-circular arc		$B = \frac{\mu_0}{4\pi}\cdot\frac{\pi I}{r} = \frac{\mu_0 I}{4r}$
Three quarter semi-circular current carrying arc		$B = \frac{\mu_0}{4\pi}\cdot\frac{\left(2\pi - \frac{\pi}{2}\right)I}{r}$ $= \frac{3\mu_0 I}{8r}$
Circular current carrying arc		$B = \frac{\mu_0}{4\pi}\frac{2\pi I}{r} = \frac{\mu_0 I}{2r}$
Concentric coplanar circular loops carrying current in the same direction		(i) If the number of turns is same $B_1 = \frac{\mu_0}{2}I\left[\frac{1}{r_1}+\frac{1}{r_2}\right]\cdot n$ (ii) If number of turns are different $B_1 = \frac{\mu_0 I}{2}\left[\frac{n_1}{r_1}+\frac{n_2}{r_2}\right]$
Concentric coplanar circular loops carrying current in the opposite direction		(i) If the number of turns is same $B_2 = \frac{\mu_0}{2}I\left[\frac{1}{r_1}-\frac{1}{r_2}\right]\cdot n$ (ii) If number of turns are different $B_2 = \frac{\mu_0 I}{2}\left[\frac{n_1}{r_1}-\frac{n_2}{r_2}\right]$
Concentric loops but their planes are perpendicular to each other		$B = \sqrt{B_1^2 + B_2^2}$ $= \frac{\mu_0}{2r}\sqrt{I_1^2 + I_2^2}$
Concentric loops but their planes are at an angle θ with each other		$B = \sqrt{B_1^2 + B_2^2 + 2B_1B_2\cos\theta}$ $= \frac{\mu_0}{2r}\sqrt{I_1^2 + I_2^2 + 2I_1I_2\cos\theta}$

CIRCULAR MOTION

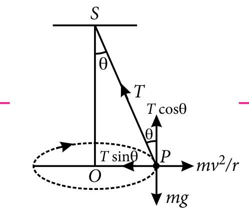
CAPACITOR AND CAPACITANCE

CIRCULAR MOTION

Dynamics of Circular Motion

Conical Pendulum : Tension in the string

$T = mg\sqrt{1+\left(\frac{v^2}{rg}\right)^2}$



Time period $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

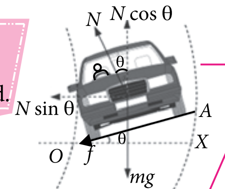
Linear velocity $v = \sqrt{gr\tan\theta}$

Angular velocity $\omega = \sqrt{\frac{g}{l\cos\theta}}$

- As $F = \frac{mv^2}{r} \neq 0$, Particle is not in equilibrium
- $\tau_{net} = 0$ as central force is involved. *i.e.*, $p \neq$ constant but L = constant

Banking of a road : Optimum speed of a vehicle on a banked road.

$v = \sqrt{rg\tan\theta}$



Maximum safe speed on a banked frictional road

$v = \sqrt{\frac{rg(\mu+\tan\theta)}{1-\mu\tan\theta}}$

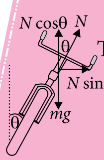
Forces

Centripetal force

$F_c = ma_c = \frac{mv^2}{r} = mr\omega^2$

Tangential force, $F_t = ma_t$

Net force, $F_{net} = m\sqrt{a_c^2 + a_t^2}$



Bending of a Cyclist : The cyclist should bend through an angle

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

Kinematics of Non-uniform Circular Motion

Tangential acceleration : $\vec{a}_t = \vec{\alpha}\times\vec{r}$

Centripetal acceleration : $\vec{a}_c = \vec{\omega}\times\vec{v}$

Non-uniform Circular Motion

Speed of the particle in a horizontal circular motion changes with respect to time.

Kinematics of Uniform Circular Motion

Uniform Circular Motion

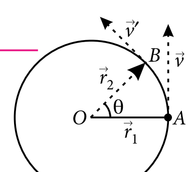
A particle moves in a circle at a constant speed.

- **Angular displacement :** It is denoted by θ. SI unit : rad or degree
- **Angular velocity (ω) :**

$\omega = \underset{\Delta t\to 0}{Lt}\frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$

Relations between :

- s and $\theta \Rightarrow s = r\,\theta$
- v and $\omega \Rightarrow \vec{v} = \vec{\omega}\times\vec{r}$



Motion in a Vertical Circle

Body is under the influence of gravity of earth and total mechanical energy is conserved.

Condition of Leaving the Circle

Tension in the string becomes zero before reaching the highest point if $\sqrt{2gL} < u < \sqrt{5gL}$

Tension at any point on vertical loop :

$T - mg\cos\theta = \frac{mv^2}{L}$

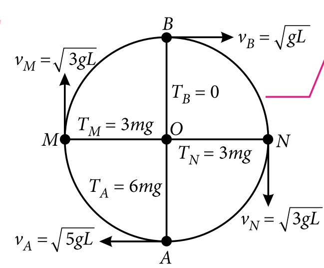
Condition of Oscillations

Particle will oscillate if velocity becomes zero the tension in the string is not zero.

$0 < u \leq \sqrt{2gL}$

Condition of Looping the Loop

The particle will complete the circle if $u \geq \sqrt{5gL}$



CAPACITOR AND CAPACITANCE

CAPACITOR

Capacitor is a passive electronic component that stores energy in the form of an electrostatic field. The ratio of charge and the potential it raised is called capacitance.

Dielectric and its Properties

Dielectric Constant

$K = \frac{\varepsilon\text{ (Permitivity of medium)}}{\varepsilon_0\text{(Permitivity of free space)}}$

Dielectric is an insulator which transmits electric effect without conducting. If field exceeds the dielectric strength, the dielectric begins to conduct.

Types of Dielectric

Polar dielectric
Permanent dipole moment exist in absence of electric field also.

Non-polar dielectric
Each molecule has zero dipole moment in its normal state.

For linear isotropic dielectric
$\vec{P} = \chi_e\vec{E}$ (χ_e = electrical susceptibility)

Energy Storage in a Capacitor

Parallel Plate Capacitor

Energy Stored

$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$

Energy Density

$u = \frac{1}{2}\varepsilon_0 E^2$

Spherical Conductor of Radius a

Energy Stored

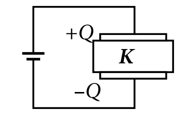
$U = \frac{1}{2}QV = \frac{Q^2}{8\pi\varepsilon_0 a}$

Energy Density

$u = \frac{Q^2}{32\pi^2\varepsilon_0 r^4}$

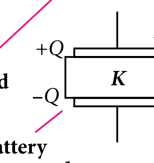
Capacitors with Dielectrics

Battery Connected

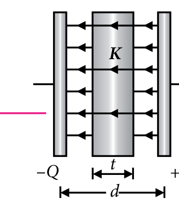


- $C' = KC$
- $U = KU_0$
- $Q = KQ_0$
- $E = E_0$
- $V = V_0$

Battery Disconnected



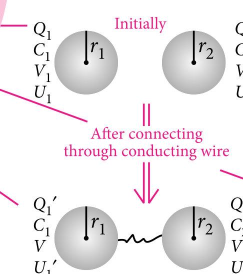
- $C' = KC$
- $Q = Q_0$
- $V = \frac{V_0}{K}$
- $U = \frac{U_0}{K}$
- $E = \frac{E_0}{K}$



- **Partially filled dielectric**

$C' = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$

Charge Sharing Between Conductors



- **New charge**

$Q_1' = Q\left[\frac{r_1}{r_1 + r_2}\right]$

$Q_2' = Q\left[\frac{r_2}{r_1 + r_2}\right]$

- **Common potential**

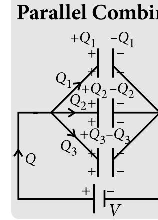
$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

- **Energy loss**

$\Delta U = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$

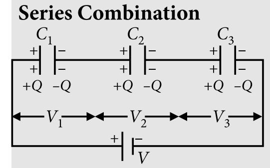
Combination of Capacitors

Parallel Combination



- $Q = Q_1 + Q_2 + Q_3$
- $C_{eq} = C_1 + C_2 + C_3$
- For n identical capacitor $C_{eq} = nC_1$ and $Q' = Q/n$
- Used when high capacity is required at low potential

Series Combination



- $V = V_1 + V_2 + V_3$
- $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$
- For n identical capacitor $C_{eq} = C/n$ and $V' = V/n$

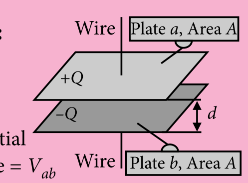
Types of Capacitors

Parallel Plate Capacitor

It consists of two large plates placed parallel to each other with a separation d.

Capacitance:

$C = \frac{\varepsilon_0 A}{d}$



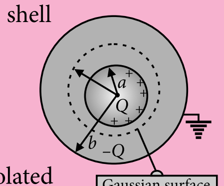
Spherical Capacitor

It consists of two concentric spherical conducting shells of radii a and b, say $b > a$. The outer shell is earthed.

Capacitance:

$C = \frac{4\pi\varepsilon_0 ab}{b - a}$

For a single isolated spherical conductor of radius R, $C = 4\pi\varepsilon_0 R$

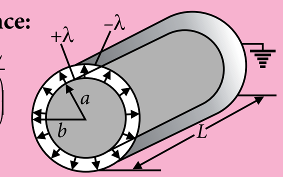


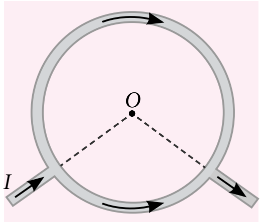
Cylindrical Capacitor

It consists of two coaxial cylinders of radii a and b, say $b > a$ and length L. The outer one is earthed.

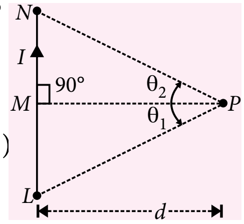
Capacitance:

$C = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$



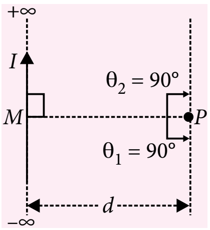
Distribution of current between any two points on the circumference		$B = 0$

- **Magnetic field due to current carrying straight conductor or wire at point P**
  - Finite length wire :



$$B_P = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)$$

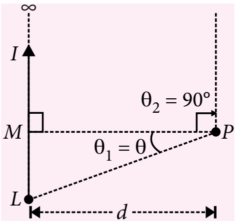
  - Infinite length wire :



$$B_P = \frac{\mu_0 I}{4\pi d}(\sin 90° + \sin 90°)$$

$$B_P = \frac{\mu_0 I}{2\pi d}$$

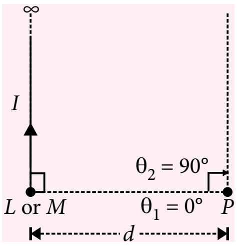
  - Semi infinite length wire:



$$B_P = \frac{\mu_0 I}{4\pi d}(\sin\theta + \sin 90°)$$

$$B_P = \frac{\mu_0 I}{4\pi d}(\sin\theta + 1)$$

  - Special semi infinite wire :

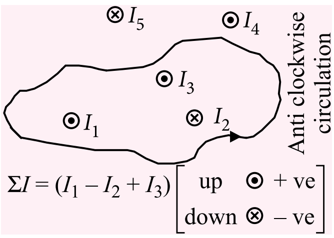


$$B_P = \frac{\mu_0 I}{4\pi d}(\sin 0° + \sin 90°)$$

$$B_P = \frac{\mu_0 I}{4\pi d}$$

AMPERE'S CIRCUITAL LAW

- It states that line integral of the magnetic field along any closed path in free space is equal to μ_0 times of net current, which crossing through area bounded by the closed path. Mathematically $\oint \vec{B}\cdot d\vec{l} = \mu_0 \Sigma I$



Application of Ampere's Law

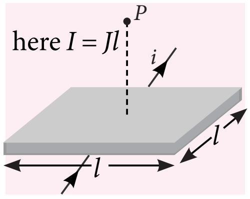
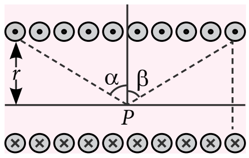
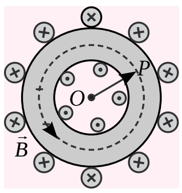
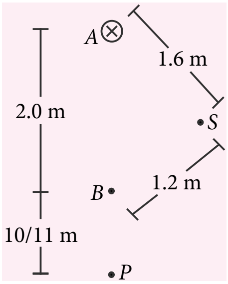
Magnetic field due to an infinite sheet carrying current	
• An infinite sheet of current with linear current density J According to Ampere's law, $2Bl = \mu_0(Jl)$ or $B = \dfrac{\mu_0 J}{2}$	
Solenoid	
• Finite length solenoid : $B = \dfrac{\mu_0}{4\pi}(2\pi nI)[\sin\alpha + \sin\beta]$ • Infinite length solenoid : If the solenoid is of infinite length *i.e.*, $\alpha = \beta = (\pi/2)$. So $B_{in} = \mu_0 nI$. • If the solenoid is of infinite length and the point is near one end *i.e.*, $\alpha = 0$ and $\beta = (\pi/2)$ so $B_{end} = \dfrac{1}{2}(\mu_0 nI)$ $\left(B_{end} = \dfrac{1}{2}B_{in}\right)$	
Toroid	
A toroid can be considered as a ring shaped closed solenoid. $B = \dfrac{\mu_0 NI}{2\pi r} = \mu_0 nI,$ where $n = \dfrac{N}{2\pi r}$	

Illustration 2 : Two long straight wires A and B are 2.0 m apart, perpendicular to the plane of the paper as shown in figure. The wire A carries a current of 9.6 A directed into the plane of the paper. The magnetic field at the point P at a distance of 10/11 m from the wire B is zero. Find (a) the magnitude and direction of the current in B, (b) the magnitude of the magnetic field at the point S and (c) the force per unit length on the wire B.



Soln : (a) For the magnetic field at P to be zero, the current in the wire B should be coming out of the plane of the paper so that the fields due to A and B may be opposite at P and the magnitudes of these fields should be equal.

$$\therefore \quad \frac{\mu_0(9\cdot 6\text{ A})}{2\pi\left(2+\frac{10}{11}\right)\text{m}}=\frac{\mu_0 I}{2\pi\left(\frac{10}{11}\right)\text{m}}$$

or $\quad I = 3.0$ A

(b) $(AB)^2 = 4\text{ m}^2$, $(AS)^2 = 2.56\text{ m}^2$ and $(BS)^2 = 1.44\text{ m}^2$

so that $(AB)^2 = (AS)^2 + (BS)^2$ and $\angle ASB = 90°$

The field at S due to the wire A

$$B_1=\frac{\mu_0(9\cdot 6\text{ A})}{2\pi\times 1.6\text{ m}}=\frac{\mu_0}{2\pi}\times 6\text{ A m}^{-1}$$

and that due to the wire B

$$B_2=\frac{\mu_0}{2\pi}\frac{3\text{A}}{1.2\text{ m}}=\frac{\mu_0}{2\pi}\times 2.5\text{ A m}^{-1}$$

These fields are at 90° to each other so that their resultant will have a magnitude

$$B=\sqrt{\left(\frac{\mu_0}{2\pi}\times 6\text{ A m}^{-1}\right)^2+\left(\frac{\mu_0}{2\pi}\times 2.5\text{A m}^{-1}\right)^2}$$

$$=\frac{\mu_0}{2\pi}\sqrt{36+6.25}\text{ A m}^{-1}$$

$$= 2\times 10^{-7}\text{ T m A}^{-1}\times 6.5\text{ A m}^{-1} = 1.3\times 10^{-6}\text{ T}$$

(c) The force per unit length on the wire B

$$\frac{F}{l}=\frac{\mu_0 i_1 i_2}{2\pi d}=(2\times 10^{-7}\text{T m A}^{-1})\times\frac{(9.6\text{ A})(3\text{ A})}{2.0\text{ m}}$$

$$=2.9\times 10^{-6}\text{N m}^{-1}$$

TORQUE ON A CURRENT LOOP

Torque on a Rectangular Current Loop

- $\vec{\tau}=\vec{m}\times\vec{B}$
  $= mB\sin\theta\,\hat{n}$,

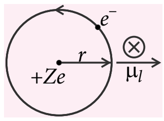
  where, $\vec{m} = NI\vec{A} = NI(ab)\hat{A}$ = magnetic moment of current in loop,
  $A = ab$ = area of rectangular loop ;
  θ = angle between $\vec{m}$ and $\vec{B}$

Circular Current Loop as a Magnetic Dipole

- Magnetic moment of circular current loop, $m = IA = I\pi R^2$
- Magnetic field perpendicular to the loop at any point at a distance x from the centre, for ($x >> R$),
  $$B_\perp=\frac{\mu_0 I R^2}{2x^3}=\frac{\mu_0}{4\pi}\cdot\frac{2m}{x^3}$$
- Magnetic field in the plane of the loop at any point at a distance x from the centre, (for $x >> R$),
  $$B_{||}=\frac{\mu_0}{4\pi}\frac{m}{x^3}.\text{ Also, } B_\perp = 2\times B_{||}$$

Magnetic Dipole Moment of a Revolving Electron

- Orbital current, $I=\dfrac{e}{T}=\dfrac{ev}{2\pi r}$
- Magnetic moment associated with the circulating current, $\mu_l = I\pi r^2=\dfrac{evr}{2}=\dfrac{e}{2m_e}L$,
- Vectorially, $\vec{\mu}_l=-\dfrac{e}{2m_e}\vec{L}$



- Gyromagnetic ratio, $\dfrac{\mu_l}{L}=\dfrac{e}{2m_e}$
- Bohr magneton,
  $$\mu_B=(\mu_l)_{\min}=\frac{e}{4\pi m_e}h=9.27\times 10^{-24}\text{ A m}^2$$

Moving Coil Galvanometer

- **Principle :** When a current carrying coil is placed in a magnetic field, it experiences a torque. In moving coil galvanometer, the current I passing through the galvanometer is directly proportional to its deflection (ϕ).

  $I \propto \phi \quad$ or $\quad I = G\phi$,

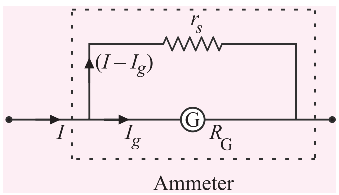
  where $G=\dfrac{k}{NAB}$ = galvanometer constant

  A = area of the coil, N = number of turns in the coil, B = strength of magnetic field, k = torsional constant of the spring *i.e.*, restoring torqe per unit twist.
- Current sensitivity, $\dfrac{\phi}{I}=\dfrac{NAB}{k}$
- Voltage sensitivity, $\dfrac{\phi}{V}=\left(\dfrac{NAB}{k}\right)\dfrac{I}{V}=\left(\dfrac{NAB}{k}\right)\dfrac{1}{R}$
- Conversion of galvanometer into ammeter :

  r_s is a very small resistance ($r_s << R_G$)

  $$\frac{r_s}{R_G}=\frac{I_g}{I-I_g}$$

  $$\therefore\quad r_s=\left(\frac{I_g}{I-I_g}\right)R_G$$

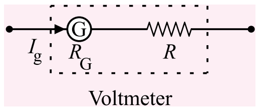


  (Shunt (r_s) is connected in parallel)

- Conversion of galvanometer into voltmeter : R is a large resistance ($R >> R_G$)

$V = I_g (R_G + R)$

$\therefore \quad R = \frac{V}{I_g} - R_G$



(A large resistance (R) is connected in series)

Illustration 3 : A rectangular coil of sides 8 cm and 6 cm having 2000 turns and carrying a current of 200 mA is placed in a uniform magnetic field of 0.2 T directed along the positive x-axis.

(a) What is the maximum torque the coil can experience? In which orientation does it experience the maximum torque?

(b) For which orientations of the coil is the torque zero? When is the equilibrium stable and when is it unstable?

Soln : (a) We are given that,

area of the coil, $A = (8 \text{ cm})(6 \text{ cm}) = 48 \text{ cm}^2 = 48 \times 10^{-4} \text{ m}^2$,

number of turns in the coil, $N = 2000$

current in the coil, $I = 200 \text{ mA} = 200 \times 10^{-3}$ A

magnetic field, $B = 0.2$ T

maximum torque acting on the coil, *i.e.*,

$\tau_{max} = NIAB = 2000(200 \times 10^{-3})(48 \times 10^{-4})0.2 \text{ N m}$
$= 0.384 \text{ N m}$

τ_{max} is experienced whenever the x-axis lies in the plane of the coil.

(b) τ is zero whenever the coil lies in the y-z plane.

For stable equilibrium, the direction of current circulation through the loop should be such that the area vector $(\vec{A})$ is parallel to $\vec{B}$.

For unstable equilibrium, $\vec{A}$ is antiparallel to $\vec{B}$.

MAGNETISM

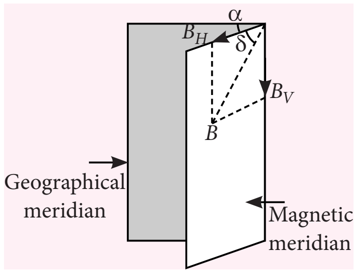
Various Term Related to Magnetism

Magnetic field and magnetic lines of force	Space around a magnetic pole or magnet or current carrying wire within which it's effect can be experienced.
Magnetic flux (ϕ) and flux density (B)	The number of magnetic lines of force passing normally through a surface is defined as magnetic flux (ϕ). S.I. unit : weber (Wb) and CGS unit is maxwell.
Magnetic permeability	It is the degree or extent to which magnetic lines of force can enter a substance and is denoted by μ.
Intensity of magnetising field ($\vec{H}$)	It is the degree of extent to which a magnetic field can magnetise a substance. Also $H = \frac{B}{\mu}$ SI unit : $\frac{\text{J}}{\text{m} \times \text{Wb}}$ or J m^{-1} Wb^{-1}
Intensity of magnetisation (I)	It can also be defined as the pole strength per unit cross sectional area of the substance or the induced dipole moment per unit volume. $I = \frac{m}{A} = \frac{M}{V}$ S.I. unit is A m^{-1}.
Magnetic susceptibility (χ_m)	It is the property of the substance which shows how easily a substance can be magnetised. $\chi_m = \frac{I}{H}$.
Relation between permeability and susceptibility	$B = B_0 + B_m = \mu_0 H + \mu_0 I$ $= \mu_0(H + I) = \mu_0 H (1 + \chi_m) = \mu_r H$ where $\mu_r = (1 + \chi_m)$.

EARTH'S MAGNETISM

Magnetic Declination (α)

- It is the angle between geographic and the magnetic meridian planes.



Angle of Inclination or Dip (δ)

- It is the angle between the direction of intensity of total magnetic field of earth and a horizontal line in the magnetic meridian.

Earth's Magnetic Field

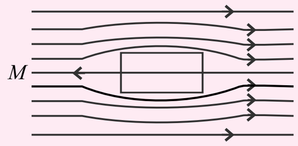
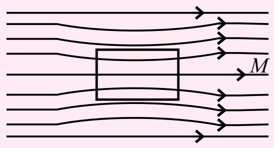
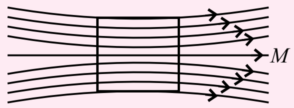
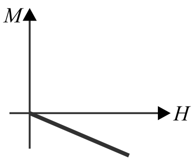
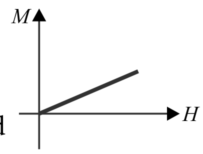
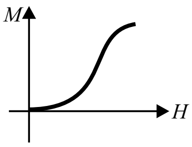
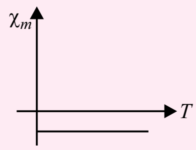
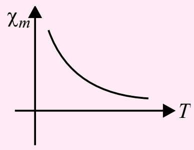
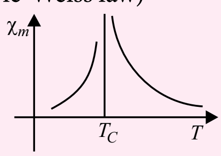
- $B_H = B \cos \delta$ ...(i) and $B_V = B \sin \delta$ ...(ii)

By squaring and adding equations (i) and (ii),

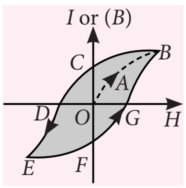
$B = \sqrt{B_H^2 + B_V^2}$

Dividing equation (ii) by equation (i), $\tan \phi = \frac{B_V}{B_H}$

CLASSIFICATION OF MAGNETIC MATERIALS

Properties	Diamagnetic	Paramagnetic	Ferromagnetic
Cause of magnetism	Orbital motion of electrons	Spin motion of electrons	Formation of domains
Substance placed in uniform magnetic field.	Poor magnetisation in opposite direction. Here $B_m < B_0$ 	Poor magnetisation in same direction. Here $B_m > B_0$ 	Strong magnetisation in same direction Here $B_m >>> B_0$ 
$M - H$ curve	$M \rightarrow$ Small, negative, varies linearly with field 	$M \rightarrow$ Small, positive, varies linearly with field 	$M \rightarrow$ Very large, positive and varies non-linearly with field 
$\chi_m - T$ curve	$\chi_m \rightarrow$ Small, negative and temperature independent $\chi_m \propto T^0$ 	$\chi_m \rightarrow$ Small, positive and varies inversely with temperature $\chi_m \propto \frac{1}{T}$ (Curie law) 	$\chi_m \rightarrow$ Very large, positive and temperature dependent $\chi_m \propto \frac{1}{T - T_C}$ (for $T > T_C$) (Curie-Weiss law)  $T_{C\,(\text{iron})}$ = 770 °C or 1043 K
μ_r (Relative Permeability)	$1 > \mu_r > 0$ ($\mu < \mu_0$)	$1 + \varepsilon > \mu_r > 1$ ($\mu > \mu_0$)	$\mu_r >>> 1$ ($\mu >>> \mu_0$)

Magnetic Hysteresis



- The lack of retracibility as shown in figure is called hysteresis and the curve is known as hysteresis loop.
- **Retentivity :** The property by virtue of which the magnetism (I) remains in a material even on the removal of magnetising field is called retentivity or residual magnetism.
- **Coercivity or coercive force :** When magnetic field H is reversed, the magnetisation decreases and for a particular value of H, denoted by H_c, it becomes zero *i.e.*, $H_c = OD$ when $I = 0$. This value of H is called the coercivity.
  Magnetic hard substance (steel) $\rightarrow$ High coercivity
  Magnetic soft substance (soft iron) $\rightarrow$ Low coercivity

Illustration 4 : Find (*a*) the magnetisation I, (*b*) the magnetic intensity H and (*c*) the magnetic field B at the centre of a bar magnet having pole strength 3.6 A m, magnetic length 12 cm and cross-sectional area 0.90 cm^2.

Soln : (a) Magnetisation, $I = \frac{m}{A} = \frac{3.6\text{A m}}{0.90 \times 10^{-4}\,\text{m}^2}$

$= 4 \times 10^4$ A m^{-1}

The direction will be from the south pole to the north pole at the centre of the magnet.

(b) Magnetic intensity H_n due to the north pole is

$$H_n = \frac{1}{4\pi}\frac{m}{d^2} = \frac{3.6\text{ A m}}{4\pi \times (6 \times 10^{-2}\text{m})^2} = 79.6\text{ A m}^{-1}$$

The direction will be towards the south pole. The magnetic intensity H_s at this point due to the south pole is also 79.6 A m^{-1} in the same direction. The resultant magnetic intensity is $H = H_n + H_s = 159.2$ A m^{-1} towards the south pole.

(c) The magnetic field at the centre is $\vec{B} = \mu_0(\vec{I} + \vec{H})$

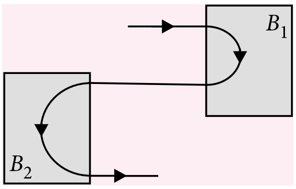
$B = (4\pi \times 10^{-7}\text{ T m A}^{-1})(4 \times 10^4 - 159.2)\text{A m}^{-1}$

$= 5.0 \times 10^{-2}$ T

The field is towards the north pole.

SPEED PRACTICE

1. Following figure shows the path of an electron that passes through two regions containing uniform magnetic fields of magnitudes B_1 and B_2. Its path in each region is a half circle, choose the correct option.

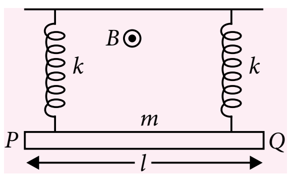


(a) B_1 is into the page and it is stronger than B_2.
(b) B_1 is into the page and it is weaker than B_2.
(c) B_1 is out of the page and it is weaker than B_2.
(d) B_1 is out of the page and it is stronger than B_2.

2. Two long conductors, separated by a distance d, carry current I_1 and I_2 in the same direction. They exert a force F on each other. Now the current in one of them is doubled and its direction is reversed. The distance is also increased to $3d$. The new value of the force between them is

(a) $-\dfrac{2F}{3}$ (b) $\dfrac{F}{3}$ (c) $-2F$ (d) $-\dfrac{F}{3}$

3. A conducting rod of mass m and length l is connected by two identical springs as shown in figure. Initially, the system is in equilibrium. A uniform magnetic field of magnitude B directed perpendicular to the plane of the paper outwards also exists in the region. If a current I is switched on that passes from P to Q through the rod. Further maximum elongation in the spring is
[Given : $|mg| = |BIl|$]



(a) $\dfrac{BIl}{k}$ (b) $\dfrac{BIl}{4k}$
(c) $\dfrac{BIl}{8k}$ (d) $\dfrac{BIl}{16k}$

4. A magnetic dipole $\vec{M} = (A\hat{i} + B\hat{j})$ J Wb^{-1} is placed in magnetic field $\vec{B} = (Cx^2\hat{i} + Dy^2\hat{j})$ Wb in xy plane at $\vec{r} = (E\hat{i} + F\hat{j})$ m. The force experienced (in N) by the bar magnet is

(a) $2ACE\hat{i} + 2BDF\hat{j}$ (b) $2ACE\hat{i}$
(c) 0 (d) $ACE\hat{i} + BDF\hat{j}$

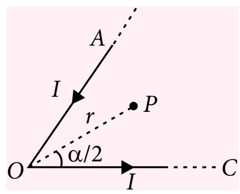
5. There exist uniform magnetic and electric fields of magnitudes 1 T and 1 V m^{-1} respectively, along positive y-axis. A charged particle of mass 1 kg and charge 1C is having velocity 1 m s^{-1} along x-axis and is at origin at $t = 0$. Then, the coordinates of the particle at time π s will be

(a) (0, 1, 2) m (b) $\left(0, \dfrac{-\pi}{2}, 0\right)$ m
(c) $\left(2, \dfrac{\pi^2}{2}, 2\right)$m (d) $\left(0, \dfrac{\pi^2}{2}, 2\right)$m

6. An insulating rod of length l carriers a charge q distributed uniformly on it. The rod is pivoted at its mid-point and is rotated at a frequency υ about a fixed axis perpendicular to the rod and passing through the pivot. The magnetic moment of the rod system is

(a) $\dfrac{1}{12}\pi q \upsilon l^2$ (b) $\pi q \upsilon l^2$
(c) $\dfrac{1}{6}\pi q \upsilon l^2$ (d) $\dfrac{1}{3}\pi q \upsilon l^2$

7. Two wires AO and OC carry equal currents I as shown in figure. One end of both the wires extends to infinity. Angle AOC is α.



The magnitude of magnetic field at point P on the bisector of these two wires at a distance r from point O is

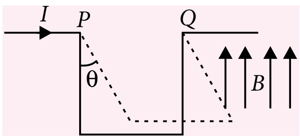
(a) $\dfrac{\mu_0}{2\pi}\dfrac{I}{r}\cot\left(\dfrac{\alpha}{2}\right)$ (b) $\dfrac{\mu_0}{4\pi}\dfrac{I}{r}\cot\left(\dfrac{\alpha}{2}\right)$
(c) $\dfrac{\mu_0}{2\pi}\dfrac{I}{r}\left(\dfrac{1+\cos\dfrac{\alpha}{2}}{\sin\left(\dfrac{\alpha}{2}\right)}\right)$ (d) $\dfrac{\mu_0}{4\pi}\dfrac{I}{r}\left(\dfrac{\alpha}{2}\right)$

8. A spherical shell of radius R and uniformly charged with charge Q is rotating about its axis with frequency ω. Find the magnetic moment of the sphere.

(a) $\dfrac{QR^2\omega}{2}$ (b) $\dfrac{2QR^2\omega}{3}$

(c) $\dfrac{QR^2\omega}{4}$ (d) $\dfrac{QR^2\omega}{3}$

9. As shown in figure, a three-sided frame is pivoted at P and Q and hangs vertically. Its sides are of same length and have a linear density of $\sqrt{3}$ kg m^{-1}. A current of $10\sqrt{3}$ A is sent through the frame, which is in a uniform magnetic field of 2 T directed upwards as shown. Then angle through which the frame will be deflected in equilibrium is
(Take $g = 10$ m s^{-2}).



(a) 30° (b) 45° (c) 60° (d) 90°

10. The area of hysteresis loop of a material is equivalent to 250 J. When 10 kg material is magnetised by an alternating field of 50 Hz then energy lost in 1 h will be (if the density of material is 7.5 g cm^{-3})

(a) 6×10^4 J (b) 6×10^4 erg

(c) 3×10^2 J (d) 3×10^2 erg

11. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2. The magnetic field on a point lying at a distance d from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by

(a) $\dfrac{\mu_0}{2\pi d}(I_1^2 + I_2^2)$ (b) $\dfrac{\mu_0}{2\pi}\left(\dfrac{I_1 + I_2}{d}\right)^{1/2}$

(c) $\dfrac{\mu_0}{2\pi d}\left(I_1^2 + I_2^2\right)^{1/2}$ (d) $\dfrac{\mu_0}{2\pi d}(I_1 + I_2)$

12. A cylindrical conductor of radius R carries a current I. The value of magnetic field at a point which is $\dfrac{R}{4}$ distance inside from the surface is 10 T. Find the value of magnetic field at point which is $4R$ distance outside from the surface.

(a) $\dfrac{4}{3}$T (b) $\dfrac{8}{3}$T (c) $\dfrac{40}{3}$T (d) $\dfrac{80}{3}$T

13. A long solenoid of diameter 0.1 m has 2×10^4 turns per meter. At the centre of the solenoid, a coil of 100 turns and radius 0.01 m is placed with its axis coinciding with the solenoid axis. The current in the solenoid reduces at a constant rate to 0 A from 4 A in 0.05 s. If the resistance of the coil is $10\,\pi^2\,\Omega$, the total charge flowing through the coil during this time is

(a) 16 μC (b) 32 μC

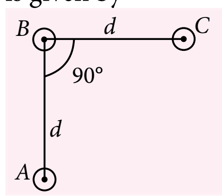
(c) 16 π μC (d) 32 π μC **[NEET 2017]**

14. A 250-turn rectangular coil of length 2.1 cm and width 1.25 cm carries a current of 85 μA and subjected to a magnetic field of strength 0.85 T. Work done for rotating the coil by 180° against the torque is

(a) 4.55 μJ (b) 2.3 μJ

(c) 1.15 μJ (d) 9.1 μJ **[NEET 2017]**

15. An arrangement of three parallel straight wires placed perpendicular to plane of paper carrying same current 'I' along the same direction as shown in figure. Magnitude of force per unit length on the middle wire 'B' is given by



(a) $\dfrac{2\mu_0 I^2}{\pi d}$ (b) $\dfrac{\sqrt{2}\mu_0 I^2}{\pi d}$ (c) $\dfrac{\mu_0 I^2}{\sqrt{2}\pi d}$ (d) $\dfrac{\mu_0 I^2}{2\pi d}$

[NEET 2017]

16. A long wire carrying a steady current is bent into a circular loop of one turn. The magnetic field at the centre of the loop is B. It is then bent into a circular coil of n turns. The magnetic field at the centre of this coil of n turns will be

(a) nB (b) n^2B (c) $2nB$ (d) $2n^2B$

[NEET Phase-II 2016]

17. A bar magnet is hung by a thin cotton thread in a uniform horizontal magnetic field and is in equilibrium state. The energy required to rotate it by 60° is W. Now the torque required to keep the magnet in this new position is

(a) $\dfrac{W}{\sqrt{3}}$ (b) $\sqrt{3}W$ (c) $\dfrac{\sqrt{3}W}{2}$ (d) $\dfrac{2W}{\sqrt{3}}$

[NEET Phase-II 2016]

18. An electron is moving in a circular path under the influence of a transverse magnetic field of 3.57×10^{-2} T. If the value of e/m is 1.76×10^{11} C kg^{-1}, the frequency of revolution of the electron is

(a) 1 GHz (b) 100 MHz

(c) 62.8 MHz (d) 6.28 MHz

[NEET Phase-II 2016]

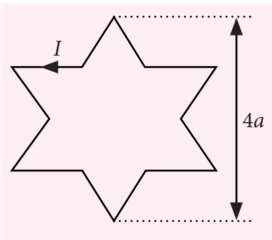
19. A magnetic needle of magnetic moment 6.7×10^{-2} A m^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is

(a) 6.65 s (b) 8.89 s

(c) 6.98 s (d) 8.76 s

[JEE Main Offline 2017]

20. A symmetric star shaped conducting wire loop is carrying a steady state current I as shown in the figure. The distance between the diametrically opposite vertices of the star is $4a$. The magnitude of the magnetic field at the center of the loop is



(a) $\frac{\mu_0 I}{4\pi a}3[2-\sqrt{3}]$ (b) $\frac{\mu_0 I}{4\pi a}6[\sqrt{3}-1]$

(c) $\frac{\mu_0 I}{4\pi a}3[\sqrt{3}-1]$ (d) $\frac{\mu_0 I}{4\pi a}6[\sqrt{3}+1]$

[JEE Advanced 2017]

SOLUTION

1. (a) : Direction of field can be find using Fleming left-hand rule and $r \propto \frac{1}{B}$.

2. (a) : Initially, $F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} l$ ...(i)

Finally, $F' = \frac{\mu_0}{2\pi}\frac{(-2I_1)(I_2)}{3d} l = \frac{-2}{3}F$ (using (i))

3. (a) : Let z be the maximum elongation in spring. When the system is in equilibrium,

$mg = 2kx$ ...(i)

By work-energy theorem,

$mzg + BIlz - \int 2k(x+z)\,dz = 0$

$$2mgz = 2k\left[\int_0^z x\,dz + \int_0^z z\,dz\right] \quad (\because mg = BIl)$$

(where x is elongation in the equilibrium position)

$\therefore$ $2mgz = 2kxz + kz^2$

$\Rightarrow$ $2mg = mg + kz$ (using eqn. (i))

$z = \frac{mg}{k} = \frac{BIl}{k}$ $(\because mg = BIl)$

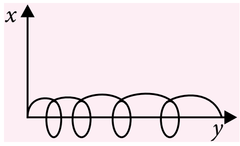
4. (a) : Since $U = -\vec{M}.\vec{B}$

$\vec{F} = -\vec{\nabla}U = M_x \frac{\partial B_x}{\partial x}\hat{i} + M_y \frac{\partial B_y}{\partial y}\hat{j}$

$\Rightarrow$ $\vec{F} = A(2Cx)\hat{i} + B(2Dy)\hat{j}$

At $\vec{r} = (E\hat{i} + F\hat{j})$, $\vec{F} = 2AEC\hat{i} + 2BDF\hat{j}$

5. (d) : The particle will move in a non-uniform helical path with increasing pitch as shown in figure.



Its time period will be $T = \frac{2\pi m}{qB} = 2\pi$ s

Changing the view, the particle seems to move in a circular path in (x-z) plane as shown in figure.

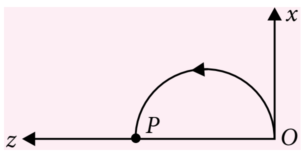
After π s, the particle will be at point P (after completing half circle), hence x-coordinate of P will be 0.

Using, $s = ut + \frac{1}{2}at^2$ for y-coordinates,

$$y(\pi) = 0(\pi) + \frac{1}{2}\frac{Eq}{m}(\pi)^2 = \frac{\pi^2}{2}\text{ m}$$

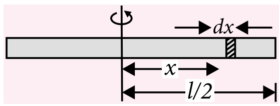
For z-coordinates,

$z = 2r = \frac{2mv}{qB} = 2$ m



Hence, the coordinates of the particle are $\left(0, \frac{\pi^2}{2}, 2\right)$.

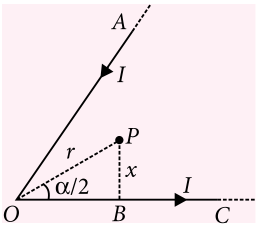
6. (a) : At a distance x, consider a small element of width dx. Magnetic moment of the small element is



$$dM = IA = (dq)\upsilon\pi x^2 = \frac{q}{l}dx\upsilon\pi x^2$$

$$M = \frac{q\upsilon\pi}{l}\int_{-l/2}^{l/2} x^2\,dx = \frac{q\pi\upsilon l^2}{12}$$

7. (c) : Here $BP = x = r\sin\frac{\alpha}{2}$

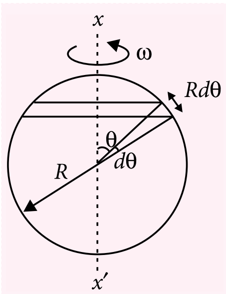


Magnetic field at P (B_P) due to OA and OC wires is in same direction and has same magnitude.

$$\therefore\quad B_P = 2\left(\frac{\mu_0}{4\pi}\right)\left(\frac{I}{x}\right)\sin\left[\left(90^\circ - \frac{\alpha}{2}\right) + \sin 90^\circ\right]$$

$$= \frac{\mu_0}{2\pi}\frac{I}{r}\frac{\left(1+\cos\frac{\alpha}{2}\right)}{\sin\frac{\alpha}{2}}$$

8. (d): The solid sphere rotates about axis xx' as shown in figure with angular speed ω.
We choose an elementary ring of radius $R\sin\theta$ and width $R\,d\theta$. The charge on ring is



$$dq = \sigma(2\pi R\sin\theta)\,(R\,d\theta) \qquad ...(i)$$

where $\sigma = \frac{Q}{4\pi R^2} = \frac{\text{Charge}}{\text{Area}}$ on the sphere.

Ring rotates about its axis with angular speed ω and hence is equivalent to a current carrying ring.

$\therefore$ Magnetic dipole moment of the ring is

$$dM = \left(\frac{dq}{2\pi}\omega\right)\pi(R\sin\theta)^2 = \omega\sigma R^4\pi\sin^3\theta\,d\theta$$

(Using (i))

$\therefore$ Magnetic dipole moment of sphere is

$$M = \int_0^\pi \omega\sigma R^4\pi\sin^3\theta\,d\theta = \frac{4}{3}\omega\sigma R^4\pi = \frac{QR^2\omega}{3}$$

$$\left(\text{Using } \int_0^\pi \sin^3\theta = \left.\frac{\cos^3\theta}{3} - \cos\theta\right|_0^\pi = \frac{4}{3}\right)$$

9. (b): Torque of magnetic force about PQ

$\tau_m = (ILB)\,L\cos\theta = IL^2B\cos\theta$

Torque of gravitational force about PQ

$$\tau_g = \left[(\lambda L)gL\sin\theta + 2(\lambda L)g\left(\frac{1}{2}\right)L\sin\theta\right] = 2\lambda L^2 g\sin\theta$$

$$\text{As } \tau_m = \tau_g \Rightarrow \tan\theta = \frac{IB}{2\lambda g} = \frac{10\sqrt{3}\times 2}{2\times\sqrt{3}\times 10} = 1$$

$\Rightarrow\quad \theta = 45°$

10. (a): Given : $\upsilon = 50$ Hz, m = 10 kg, t = 1 h = 3600 s, $\rho = 7.5$ g cm^{-3} = 7.5×10^{-3} kg m^{-3}
The area of hysteresis loop of a material is equivalent magnetic loss per unit volume.

$$\therefore\quad E = \upsilon AVt = \upsilon A\frac{m}{\rho}t = \frac{50\times 250\times 10\times 3600}{7.5\times 10^3}$$

$$= 6\times 10^4\text{ J}$$

11. (c) : The field at the same point at the same distance from the mutually perpendicular wires carrying currents I_1 and I_2, in perpendicular direction will be

$$B = \sqrt{B_1^2 + B_2^2}$$

$$\text{Here, } B_1 = \frac{\mu_0}{2\pi d}I_1 \text{ and } B_2 = \frac{\mu_0}{2\pi d}I_2$$

$$\therefore\quad B = \frac{\mu_0}{2\pi d}(I_1^2 + I_2^2)^{1/2}$$

12. (b): Magnetic field inside the cylindrical conductor

$B_{in} = \frac{\mu_0}{4\pi}\cdot\frac{2Ir}{R^2}$ (R = Radius of cylinder, r = distance of observation point from axis of cylinder)

Magnetic field outside the cylinder at a distance r' from its axis, $B_{out} = \frac{\mu_0}{4\pi}\cdot\frac{2I}{r'}$

$$\Rightarrow \frac{B_{in}}{B_{out}} = \frac{rr'}{R^2} \Rightarrow \frac{10}{B_{out}} = \frac{\left(R-\frac{R}{4}\right)(R+4R)}{R^2} = \frac{15}{4}$$

$$\Rightarrow\quad B_{out} = \frac{8}{3}\text{T}$$

13. (b): Given $n = 2 \times 10^4$; $I = 4$ A

Initially $\quad I = 0$ A

$\therefore \quad B_i = 0$ or $\phi_i = 0$

Finally, the magnetic field at the centre of the solenoid is given as

$B_f = \mu_o nI = 4\pi \times 10^{-7} \times 2 \times 10^4 \times 4 = 32\pi \times 10^{-3}$ T

Final magnetic flux through the coil is given as

$\phi_f = NBA = 100 \times 32\,\pi \times 10^{-3} \times \pi \times (0.01)^2$

$\phi_f = 32\pi^2 \times 10^{-5}$ T m^2

Induced charge, $q = \dfrac{|\Delta\phi|}{R} = \dfrac{|\phi_f - \phi_i|}{R} = \dfrac{32\pi^2 \times 10^{-5}}{10\pi^2}$

$= 32 \times 10^{-6}$ C $= 32$ μC

14. (d): Work done in a coil

$$W = mB(\cos\theta_1 - \cos\theta_2)$$

When it is rotated by angle 180° then

$$W = 2mB = 2\,(NIA)B \qquad \text{...(i)}$$

Given: $\quad N = 250$, $I = 85$ μA $= 85 \times 10^{-6}$ A

$A = 1.25 \times 2.1 \times 10^{-4}$ m$^2 \approx 2.5 \times 10^{-4}$ m^2

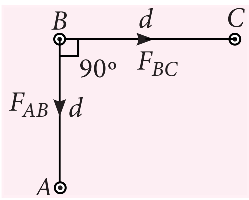
$B = 0.85$ T

Putting these values in eqn. (i), we get

$W = 2 \times 250 \times 85 \times 10^{-6} \times 2.5 \times 10^{-4} \times 0.85$

$\approx 9.1 \times 10^{-6}$ J $= 9.1$ μJ

15. (c) : Force between wires A and B = force between wires B and C



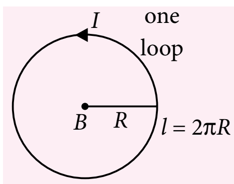
$\therefore\ F_{BC} = F_{AB} = \dfrac{\mu_o I^2 l}{2\pi d}$

As $\vec{F}_{AB} \perp \vec{F}_{BC}$, net force on wire B,

$$F_{net} = \sqrt{2}F_{BC} = \frac{\sqrt{2}\,\mu_o I^2 l}{2\pi d}$$

$$F_{net} = \frac{\mu_o I^2 l}{\sqrt{2}\,\pi d} \text{ or } \frac{F_{net}}{l} = \frac{\mu_o I^2}{\sqrt{2}\,\pi d}$$

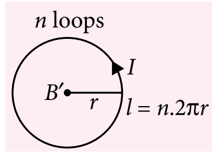
16. (b): Let l be the length of thewire. Magnetic field at the centre of the loop is



$$B = \frac{\mu_0 I}{2R}$$

$$\therefore \quad B = \frac{\mu_0 \pi I}{l} \quad (\because\ l = 2\pi R) \qquad \text{...(i)}$$

$$B' = \frac{\mu_0 nI}{2r} = \frac{\mu_0 nI}{2\left(\dfrac{l}{2n\pi}\right)}$$



$$B' = \frac{\mu_0 n^2 \pi I}{l} \qquad \text{...(ii)}$$

From eqns. (i) and (ii), we get, $B' = n^2 B$

17. (b): At equilibrium, initial potential energy of dipole

$U_i = -MB_H$

Final potential energy of dipole,

$$U_f = -MB_H\cos 60° = -\frac{MB_H}{2}$$

$$W = U_f - U_i = -\frac{MB_H}{2} - (-MB_H) = \frac{MB_H}{2} \qquad \text{...(i)}$$

Required torque, $\tau = MB_H \sin 60°$

$$\tau = 2W \times \frac{\sqrt{3}}{2} \qquad \text{[Using eqn. (i)]}$$

$$= \sqrt{3}W$$

18. (a) : Here, $B = 3.57 \times 10^{-2}$ T, $\dfrac{e}{m} = 1.76 \times 10^{11}$ C kg^{-1}

Frequency of revolution of the electron,

$$\upsilon = \frac{1}{T} = \frac{v}{2\pi r} \qquad \text{...(i)}$$

Also, $\dfrac{mv^2}{r} = evB \Rightarrow \dfrac{v}{r} = \dfrac{eB}{m}$ $\qquad$ ...(ii)

From eqns. (i) and (ii)

$$\upsilon = \frac{1}{2\pi} \times \frac{eB}{m} = \frac{1}{2 \times 3.14} \times 1.76 \times 10^{11} \times 3.57 \times 10^{-2}$$

$= 10^9$ Hz $= 1$ GHz

19. (a) : Time period of magnetic needle oscillating simple harmonically is given by

$$T = 2\pi\sqrt{\frac{I}{MB}} \Rightarrow T = 2\pi\sqrt{\frac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 0.01}}$$

$$\Rightarrow \quad T = \frac{2\pi}{10} \times 1.05 \text{ s}$$

For 10 oscillations, total time taken

$T' = 10T = 2\pi \times 1.05 \approx 6.65$ s

20. (b)

MPP-5 CLASS XI ANSWER KEY

1. (b)	2. (b)	3. (d)	4. (b)	5. (c)
6. (c)	7. (d)	8. (c)	9. (a)	10. (a)
11. (d)	12. (c)	13. (b)	14. (a)	15. (d)
16. (b)	17. (c)	18. (c)	19. (c)	20. (a, c, d)
21. (a, b, c, d)		22. (b, d)	23. (a, b)	24. (7)
25. (3)	26. (6)	27. (d)	28. (a)	29. (b)
30. (c)				

EXAM PREP 2018

CLASS XII

Useful for Medical/Engg. Entrance Exams

CHAPTERWISE MCQs FOR PRACTICE

CURRENT ELECTRICITY

1. A uniform wire is cut into 10 segments increasing in length in equal steps, the resistance of the shortest segment is R and the resistances of the other segments increase in steps of 8 Ω. If the resistance of the longest segments is $2R$, find the resistance of the original wire.

(a) 1080 Ω (b) 72 Ω
(c) 144 Ω (d) 720 Ω

2. An electric motor operating on a 50 V dc supply draws a current of 12 A. If the efficiency of the motor is 30%, estimate the resistance of the windings of the motor.

(a) 1.9 Ω (b) 2.9 Ω
(c) 3.9 Ω (d) 4.9 Ω

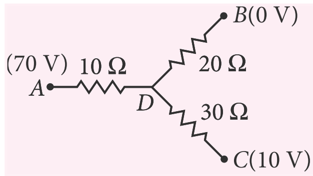
3. Assume that each atom of copper contributes one electron. If the current flowing through a copper wire of 1 mm diameter is 1.1 A, the drift velocity of electrons will be

(a) 0.3 mm s^{-1} (b) 0.5 mm s^{-1}
(c) 0.1 mm s^{-1} (d) 0.2 mm s^{-1}

4. A resistance R is to be measured using a meter bridge, student chooses the standard resistance S to be 100 Ω. He finds the null point at l_1 = 2.9 cm. He is told to attempt to improve the accuracy. Which of the following is a useful way?

(a) He should measure l_1 more accurately.
(b) He should change S to 1000 Ω and repeat the experiment.
(c) He should change S to 3 Ω and repeat the experiment.
(d) He should give up hope of a more accurate measurement with a meter bridge.

5. In the network shown, points A, B and C are at potentials of 70 V, zero and 10 V respectively.



(a) Point D is at a potential of 200 V.
(b) The currents in the sections AD, DB, DC are in the ratio 3 : 2 : 1.
(c) The currents in the sections AD, DB, DC are in the ratio 1 : 2 : 3.
(d) The network draws a total power of 500 W.

6. A carbon filament has a resistance of 100 Ω at 0°C. What must be the resistance of a copper filament placed in series with carbon so that the combination has the same resistance at all temperatures? Temperature coefficient of resistance of carbon is –0.0007°C^{-1} and that of copper is 0.004°C^{-1}.

(a) 17.5 Ω (b) 1.75 Ω
(c) 175 Ω (d) 0.175 Ω

7. The potential difference across the terminals of a battery is 50 V when 11 A current is drawn and 60 V when 1 A current is drawn. The emf and the internal resistance of the battery are

(a) 62 V, 2 Ω (b) 63 V, 1 Ω
(c) 61 V, 1 Ω (d) 64 V, 2 Ω

8. The charge flowing through a resistance R varies with time t as $Q = at - bt^2$. The total heat produced in R is

(a) $\frac{a^3R}{6b}$ (b) $\frac{a^3R}{3b}$ (c) $\frac{a^3R}{2b}$ (d) $\frac{a^3R}{b}$

9. Two identical 1 Ω wires are laid side-by-side and soldered together so that they touch each other for half of their lengths. What is the equivalent resistance of this combinations ?

(a) 2.5 Ω (b) 1.25 Ω
(c) 0.5 Ω (d) 5 Ω

10. A uniform wire of resistance R is shaped into a regular n sided polygon, where n is even. Find the equivalent resistance between adjacent corners of polygon.

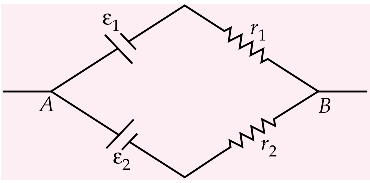
(a) $\frac{(n-1)R}{n^2}$ (b) $\frac{R}{n^2}$

(c) $\frac{(n^2-1)R}{n}$ (d) $\frac{(n+1)R}{n^2}$

11. An electric toaster uses nichrome for its heating element. When a negligibly small current passes through it, its resistance at room temperature (27.0°C) is found to be 75.3 Ω. When the toaster is connected to a 230 V supply, the current settles, after a few seconds to a steady value of 2.68 A. The temperature coefficient of resistance is 1.7×10^{-4} °C^{-1}. The steady temperature of the nichrome element is

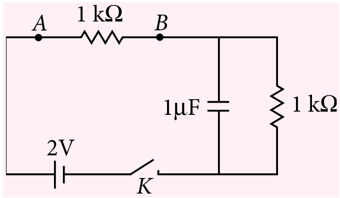
(a) 747°C (b) 847°C
(c) 897°C (d) 927°C

12. Two batteries of emf ε_1 and ε_2 ($\varepsilon_2 > \varepsilon_1$) and internal resistances r_1 and r_2 respectively are connected in parallel as shown in figure.



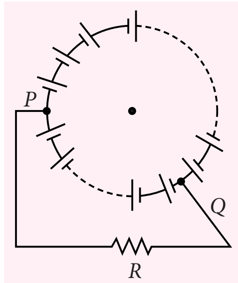
(a) The equivalent emf ε_{eq} of the two cells is between ε_1 and ε_2, *i.e.*, $\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$.
(b) The equivalent emf ε_{eq} is smaller than ε_1.
(c) The ε_{eq} is given by $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$ always.
(d) ε_{eq} is independent of internal resistances r_1 and r_2.

13. When the key K is pressed at time $t = 0$, then which of the following statements about the current I in the resistor AB of the given circuit figure shown is true?



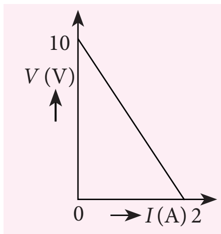
(a) $I = 1$ mA at all t.
(b) $I = 2$ mA at all t.
(c) I oscillates between 0.5 mA and 2 mA.
(d) At $t = 0$, $I = 2$ mA and with time, it goes to 1 mA.

14. n cells, each of emf ε and internal resistance r are connected in a closed circuit so that the positive terminal of a cell is joined to the negative terminal of the next, as shown in figure. Any two points of the circuit are connected by an external resistance R. Find the current in R.



(a) zero (b) 1 A (c) 2 A (d) 3 A

15. A battery of emf ε and internal resistance r is connected across an external resistance R which can be adjusted to have any value. A graph is plotted between the current I flowing through the resistance and potential difference V across it. The graph is as shown in the following figure. Select the incorrect alternative.



(a) Internal resistance of the battery is 5 Ω.
(b) The emf of the battery is 10 V.
(c) Maximum power developed across external resistance is 5 W.
(d) V-I graph cannot be a straight line graph as shown in the figure.

MOVING CHARGES AND MAGNETISM

16. A circular current carrying coil has a radius R. The distance from the centre of the coil on the axis of the coil, where the magnetic induction is $1/8^{th}$ of its value at the centre of the coil is

(a) $\sqrt{3}\,R$ (b) $\frac{R}{\sqrt{3}}$

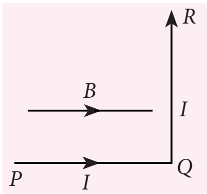
(c) $\left(\frac{2}{\sqrt{3}}\right)R$ (d) $\frac{R}{2\sqrt{3}}$

17. An electric field acts along positive x-axis. A charged particle of charge q and mass m is released from origin and moves with velocity $\vec{v} = v_0\hat{j}$ under the action of electric field and magnetic field, $\vec{B} = B_0\hat{i}$. The velocity of particle becomes $2v_0$ after time $\frac{\sqrt{3}\,mv_0}{\sqrt{2}\,qE_0}$. Find the magnitude of electric field.

(a) $\frac{\sqrt{2}}{\sqrt{3}}E_0$ (b) $\frac{\sqrt{3}}{\sqrt{2}}E_0$

(c) $\sqrt{3}\,E_0$ (d) $\sqrt{2}\,E_0$

18. A wire PQR is bent as shown in figure and is placed in a region of uniform magnetic field B. The length of $PQ = QR = l$. A current I ampere flows through the wire as shown. The magnitude of the force on PQ and QR will be



(a) BIl, 0
(b) 2 BIl, 0
(c) 0, BIl
(d) 0, 0

19. Two charged particles having charges Q and $-Q$ and masses m and $4m$ respectively, enter in uniform magnetic field B at an angle θ with magnetic field from same point with speed v. The displacement from starting point, where they will meet again, is

(a) $\frac{2\pi m}{QB}v\sin\theta$ (b) $\frac{2\pi m}{QB}v\cos\theta$

(c) $\frac{8\pi m}{QB}v\cos\theta$ (d) $\frac{12\pi m}{QB}v\cos\theta$

20. Moment of inertia of a magnetic needle is 40 g cm^2 has time period 3 s in earth's horizontal field having value 3.6×10^{-5} Wb m^{-2}. Its magnetic moment will be

(a) 0.5 A m^2 (b) 5 A m^2
(c) 0.250 A m^2 (d) 5×10^2 A m^2

21. In an ammeter, 10% of main current is passing through the galvanometer. If the resistance of the galvanometer is G, then the shunt resistance, (in Ω) is

(a) 9 G (b) G/9 (c) 90 G (d) G/90

22. A cyclotron, in which the magnetic field is 1.4 Wb m^{-2}, is used to accelerate protons. How rapidly should the electric field between the dees be reversed?

(a) 0.2 ns (b) 0.3 ns
(c) 0.2 μs (d) 0.4 ms

23. When a galvanometer having 30 divisions scale and 100 Ω resistance is connected in series to a battery of emf 3 V through a resistance of 200 Ω, it shows full scale deflection. Find the figure of merit of the galvanometer in μA.

(a) 333.3 (b) 3.333
(c) 33.33 (d) 333

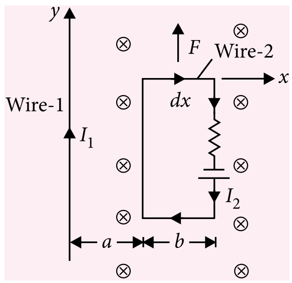
24. A toroidal winding has a total of 400 turns on a core with inner radius 8 cm and outer radius 10 cm. Calculate the magnitude of the magnetic field at a point midway between the inner and outer walls of the core when there is a current of 0.75 A maintained in the windings.

(a) 6 mT (b) 4 mT
(c) 2 mT (d) 5 mT

25. The wires that supply current to a 120 V, 2 kW electric heater are 2 mm apart. What is the force per metre between the wires?

(a) 0.028 N m^{-1} (b) 16.7 N m^{-1}
(c) 5.57 N m^{-1} (d) 0.033 N m^{-1}

26. Wire-1 in figure shown is oriented along the y-axis and carries a steady current I_1. A rectangular circuit located to the right of the wire carries a current I_2. Find the force experienced by the top of the horizontal wire (wire-2) of the rectangular circuit.



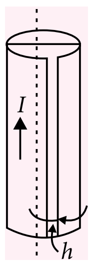
(a) $\frac{\mu_0 I_1 I_2}{2}\ln\left(1+\frac{a}{b}\right)\hat{j}$

(b) $\frac{\mu_0 I_1 I_2}{2}\ln\left[1-\frac{a}{b}\right]\hat{j}$

(c) $\frac{\mu_0 I_1 I_2}{2\pi}\ln\left[1-\frac{b}{a}\right]\hat{j}$

(d) $\frac{\mu_0 I_1 I_2}{2\pi}\ln\left[1+\frac{b}{a}\right]\hat{j}$

27. A current I flows along, thin-walled tube of radius R with a long longitudinal slit of width h. Find the induction of the magnetic field at a distance $r < R$ from the axis of the tube under the condition $h << R$.



(a) $\frac{\mu_0 Ih}{4\pi R(r-R)}$ (b) $\frac{\mu_0 Ir}{4\pi R(r-h)}$

(c) $\frac{\mu_0 Ih}{4\pi R(R-r)}$ (d) $\frac{\mu_0 IR}{4\pi r(R-r)}$

28. Two protons move parallel to each other with an equal velocity $v = 3 \times 10^5$ m s^{-1}. Find the ratio of forces of magnetic and electrical interaction of the protons.
(a) 10^6 (b) 10^{-5} (c) 10^5 (d) 10^{-6}

29. The coil of a galvanometer has 500 turns and each turn is of average area 3×10^{-4} m^2. When a current of 0.5 A is passed through it. If a torque of 1.5 N m is required to set this coil carrying same current parallel to a magnetic field, calculate the strength of the magnetic field.
(a) 10 T (b) 15 T (c) 20 T (d) 25 T

30. A coil in the shape of an equilateral triangle of side 0.02 m is suspended from a vertex such that it is hanging in a vertical plane between the pole pieces of permanent magnet producing a horizontal magnetic field 5×10^{-2} T. Find the couple acting on the coil when a current of 0.1 A is passed through it and the magnetic field is parallel to its plane.
(a) $5\sqrt{3} \times 10^{-7}$ N m (b) $5\sqrt{3} \times 10^{-5}$ N m
(c) 5×10^{-5} N m (d) $\sqrt{3} \times 10^{-5}$ N m

SOLUTIONS

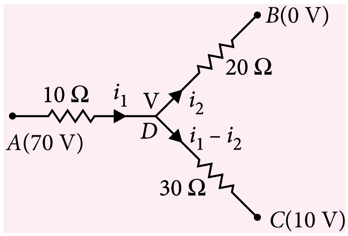
1. **(a) :** Resistance of first or shortest segment = $R\ \Omega$
Resistance of second segment = $R + 8 \times 1 = (R + 8)\ \Omega$
Resistance of third segment = $R + 8 \times 2 = (R + 16)\ \Omega$
$\therefore$ Resistance of tenth segment
$= R + 8 \times 9 = (R + 72)\ \Omega$
But resistance of longest segment = $2R$
$\therefore\ 2R = R + 72$ or $R = 72\ \Omega$
Resistance of the original wire
$= R + (R + 8) + (R + 16) + ... + (R + 72)$
$= 10R + 8(1 + 2 + 3 + ... + 9) = 1080\ \Omega$

2. **(b) :** We are given that $V = 50$ V and $I = 12$ A
Total power consumption of the motor, *i.e.*,
$P = VI = (50\text{ V})(12\text{ A}) = 600$ W
Since the efficiency of the motor is 30%, power dissipated as heat in the motor windings
$= (100\% - 30\%) = 70\%$
Total power dissipated as heat in the windings
$$= \left(\frac{70}{100}\right) \times 600\text{ W} = 420\text{ W}$$
If R is the resistance of the windings of the motor, total power dissipated = I^2R
Thus, $I^2R = 420$ W
$$\text{or}\quad R = \frac{420\text{ W}}{(12\text{ A})^2} = 2.9\ \Omega$$

3. **(c) :** Atomic mass of copper, $m = 63.5 \times 10^{-3}$ kg
Density of copper, $\rho = 9 \times 10^3$ kg m^{-3}
Number density of electrons,
$$n = \frac{N_A}{m/\rho} = \frac{6 \times 10^{23}}{(63.5 \times 10^{-3})/(9 \times 10^3)}$$
$= 8.5 \times 10^{28}$ m^{-3}
$A = \pi r^2 = \pi(0.5 \times 10^{-3}\text{m})^2 = 0.25\,\pi \times 10^{-6}$ m^2
$$v_d = \frac{I}{neA}$$
$$= \frac{1.1\text{ A}}{[8.5 \times 10^{28}\text{ m}^{-3}](1.6 \times 10^{-19}\text{ C})(0.25\pi \times 10^{-6}\text{ m}^2)}$$
$= 0.1 \times 10^{-3}$ m s^{-1} = 0.1 mm s^{-1}

4. **(c) :** The percentage error in R can be minimised by adjusting the balance point near the middle of the bridge, *i.e.*, when l_1 is close to 50 cm. This requires a suitable choice of S.
$$\text{Since}\quad \frac{R}{S} = \frac{l_1}{100 - l_1}$$
Since here, $R : S :: 2.9 : 97.1$ imply that the S is nearly 33 times to that of R. In order to make this ratio 1 : 1, it is necessary to reduce the value of S nearly $\frac{1}{33}$ times *i.e.*, nearly 3 Ω.

5. **(b) :** Let V be the potential at D.



$70\text{ V} - V = i_1 \times 10\ \Omega$ ...(i)
$V - 0\text{ V} = i_2 \times 20\ \Omega$ ...(ii)
$V - 10\text{ V} = (i_1 - i_2) \times 30\ \Omega$ ...(iii)
By solving eqns. (i), (ii) and (iii), we get point D is at potential of 40 V. The values of currents i_1, i_2 and i_3 are 3A, 2A and 1A respectively. And the network draws a total power of 200 W.

6. **(a) :** As $R = R_0(1 + \alpha T)$
$\therefore$ Change in resistance = $\alpha R_0 T$
where R_0 is the resistance at 0°C
Let the resistance of copper placed in series with carbon at 0°C be R, so that the combination has the same resistance at all temperatures. Then at any temperature T,

Increase in resistance of copper per °C

= Decrease in resistance of carbon per °C

($\alpha R_0 T$) for copper = ($\alpha R_0 T$) for carbon

$\therefore\ 0.004 \times R \times T = 0.0007 \times 100 \times T \Rightarrow R = 17.5\ \Omega$

7. **(c) :** As per relation, $V = \varepsilon - Ir$, we have
$50 = \varepsilon - 11r$ and $60 = \varepsilon - 1r$
On solving these equations, we get
$\varepsilon = 61$ V and $r = 1\ \Omega$

8. **(a) :** $Q = at - bt^2$

$$i = \frac{dQ}{dt} = a - 2bt$$

$i = 0$ for $t = t_0 = \frac{a}{2b}$, *i.e.*, current flow from $t = 0$ to $t = t_0$.

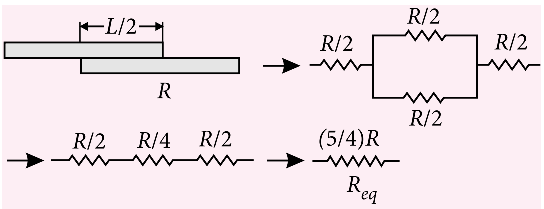
$$\text{The heat produced} = \int_0^{t_0} i^2 R\,dt = \int_0^{t_0} (a-2bt)^2 R\,dt$$

$$= R\left[\int_0^{t_0} (a^2 dt + 4b^2t^2 dt - 4ab\,t\,dt)\right]$$

$$= R\left[a^2 t + \frac{4b^2t^3}{3} - \frac{4ab\,t^2}{2}\right]_0^{t_0 = \frac{a}{2b}}$$

$$= R\left[\frac{a^3}{2b} + \frac{a^3}{6b} - \frac{a^3}{2b}\right] = \frac{a^3 R}{6b}$$

9. **(b) :** Since $R \propto L$, the system will behave as shown in figure



$$R_{eq} = \left(\frac{5}{4}\right)R = \left(\frac{5}{4}\right)(1\,\Omega) = 1.25\,\Omega$$

10. **(a) :** Resistance of each side of polygon $= \frac{R}{n}$

R_1 = Resistance of one side $= \frac{R}{n}$

R_2 = Resistance of remaining $(n-1)$ sides connected in series $= \frac{(n-1)R}{n}$

These two parts form a parallel combination. So the equivalent resistance of the polygon between two adjacent corners will be

$$R_{eq} = \frac{\frac{R}{n} \times \frac{(n-1)R}{n}}{\frac{R}{n} + \frac{(n-1)R}{n}} = \frac{(n-1)R}{n^2}.$$

11. **(b)**

12. **(a) :** For the given circuit, $\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$

Since $\varepsilon_2 > \varepsilon_1$, ε_{eq} lies between ε_1 and ε_2,
i.e. $\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$.

13. **(d) :** At $t = 0$, 1 μF capacitor connected in parallel with 1 kΩ by passes it (*i.e.*, current flows only through it and AB and not through 1 kΩ). Thus, only resistance AB (1 kΩ) is effective. As such,

$$I = \frac{2\,\text{V}}{1000\,\Omega} = 2 \times 10^{-3}\ \text{A} = 2\ \text{mA}$$

With time, when the capacitor gets fully charged, no current flows through it. Now it flows through AB and the other resistance.

$$\text{Thus, } I' = \frac{2\,\text{V}}{1000\,\Omega + 1000\,\Omega} = 1 \times 10^{-3}\ \text{A} = 1\,\text{mA}$$

14. **(a)**

15. **(d) :** Maximum voltage across battery terminals in open circuit (*i.e.*, when $I = 0$) = emf of battery = 10 V.

Slope of $V - I$ curve $= -r = -\frac{10}{2} = -5$

$\Rightarrow\ r = 5\ \Omega$

For maximum power dissipation,
$R = r = 5\ \Omega$

and then current flowing, $I = \frac{\varepsilon}{R+r} = \frac{10}{5+5} = 1$ A

Maximum power, $P_{max} = I^2 R = (1)^2 \times 5 = 5$ W

16. **(a) :** $B_{\text{axial}} = \frac{1}{8} B_{\text{centre}}$

$$\Rightarrow \frac{\mu_0 I R^2}{2(R^2 + r^2)^{3/2}} = \frac{1}{8}\frac{\mu_0 I}{2R} \quad \text{or} \quad \frac{R^2}{(R^2+r^2)^{3/2}} = \frac{1}{8R}$$

or $\quad (R^2 + r^2)^{3/2} = 8R^3$ or $r = \sqrt{3}\,R$

17. **(d) :** Path of particle is helix with increasing pitch

$$v = (v_x^2 + v_y^2 + v_z^2)^{1/2}$$

Here $v_x^2 = \left(\frac{qE}{m}t\right)^2$ and $v_y^2 + v_z^2 = v_0^2$

Also $v = 2v_0$

$$(2v_0)^2 = \frac{q^2E^2t^2}{m^2} + v_0^2$$

$$t = \frac{\sqrt{3}\,mv_0}{qE} = \frac{\sqrt{3}\,mv_0}{\sqrt{2}\,qE_0} \quad \text{(given)}$$

$$\vec{E} = \sqrt{2}\,E_0\,\hat{i}$$

18. (c) : $F_{PQ} = IlB \sin 0° = 0$

$F_{QR} = IlB \sin 90° = IlB.$

19. (c) : Time period for first $= \dfrac{2\pi m}{QB}$

Time period for second $= \dfrac{8\pi m}{QB}$

They will meet again after $\dfrac{8\pi m}{QB}$

So, displacement in this time interval $= \dfrac{8\pi m}{QB} v\cos\theta$

20. (a) : $T = 2\pi\sqrt{\dfrac{I}{MB_H}}$

$I = 40 \text{ g cm}^2 = 400 \times 10^{-8} \text{ kg m}^2$

$$\therefore \quad 3 = 2\pi\sqrt{\frac{400\times10^{-8}}{36\times10^{-6}\times M}}$$

$$\Rightarrow \quad \frac{1}{M} = \frac{9}{4\pi^2}\times\frac{36}{4} \Rightarrow M \approx 0.5 \text{ A m}^2$$

21. (b) : $I_g = 10\%$ of $I = 0.1\ I$

$$S = \frac{I_g G}{I - I_g} = \frac{0.1IG}{I - 0.1I} = \frac{G}{9}$$

22. (a) : Frequency of cyclotron,

$$\upsilon_c = \frac{qB}{2\pi m} = \frac{(1.6\times10^{-19})1.4}{2\times3.14\times(1.67\times10^{-27})} \text{ Hz}$$

$$= 2.136 \times 10^7 \text{ Hz} = 21.36 \text{ MHz}$$

$$T = \frac{1}{\upsilon_c} = \frac{1}{21.36\times10^6} \text{ s}$$

$$\text{or } t = \frac{T}{2} = \frac{1}{2\times21.36\times10^6} = 2.34\times10^{-8} \text{ s} \simeq 0.2 \text{ ns}$$

23. (a) : Current flowing through galvanometer.

$$I_g = \frac{\varepsilon}{R+G} = \frac{3\text{ V}}{(200+100)\,\Omega} = 10^{-2} \text{ A}$$

Figure of merit, $k = \dfrac{I_g}{n} = \dfrac{10^{-2}\text{ A}}{30} = 333.3\ \mu\text{A}$

24. (c) : Magnetic field at a point midway, *i.e.*, at a distance R,

$$B = \frac{\mu_0 NI}{2R} = \frac{(4\pi\times10^{-7})(400)(0.75)}{2(9\times10^{-2})} \text{ T}$$

$$= 2.09 \times 10^{-3}\text{ T} \simeq 2 \text{ mT}$$

$$\left(\text{as } R = \frac{8\text{ cm} + 10\text{ cm}}{2} = 9\text{ cm} = 9\times10^{-2}\text{ m}\right)$$

25. (a) : Since $P = VI$, the current in the wires is

$$I = I_1 = I_2 = \frac{P}{V} = \frac{2000\text{ W}}{120\text{ V}} = 16.7\text{ A}$$

Since $s = 2$ mm $= 2 \times 10^{-3}$ m, the force between the wires is

$$\frac{F}{L} = \left(\frac{\mu_0}{2\pi}\right)\left(\frac{I_1 I_2}{s}\right)$$

$$= \frac{(4\pi\times10^{-7}\text{ T m A}^{-1})(16.7\text{ A})^2}{(2\pi)(2\times10^{-3}\text{ m})} = 0.028 \text{ N m}^{-1}$$

The currents are in opposite directions, so the force is repulsive.

26. (d) : Magnetic field $(\vec{B})$ created by wire-1 at a distance x from it, *i.e.*, $\vec{B} = \dfrac{\mu_0 I_1}{2\pi x}(-\hat{k})$

Force acting on a small element dx of wire-2, *i.e*, $d\vec{F} = I_2 d\vec{x} \times \vec{B}$

$$\text{or} \quad d\vec{F} = I_2 d\vec{x} \times \left(\frac{\mu_0 I_1}{2\pi x}(-\hat{k})\right)$$

$$= \frac{\mu_0 I_1 I_2}{2\pi x} dx[\hat{i}\times(-\hat{k})] \qquad (\text{as } d\vec{x} = dx\,\hat{i})$$

$$= \left(\frac{\mu_0 I_1 I_2}{2\pi x} dx\right)\hat{j}$$

$$\vec{F} = \int d\vec{F} = \left[\frac{\mu_0 I_1 I_2}{2\pi}\int_a^{a+b}\frac{1}{x}dx\right]\hat{j}$$

$$= \left[\frac{\mu_0 I_1 I_2}{2\pi}\ln\left(1+\frac{b}{a}\right)\right]\hat{j}$$

$\vec{F}$ points upward as indicated by $\hat{j}$.

27. (c) : By the principle of superposition, the field due to entire conductor = field due to strip in place of the slit + field due to the remainder.
Field due to the entire conductor = 0
(because the point lies inside)

Field due to strip $= \frac{\mu_0}{4\pi} \times \frac{2I'}{R-r}$

(where I' = current through strip)

$$I' = \frac{I}{2\pi R - h} \times h = \frac{Ih}{2\pi R}$$

$$\therefore \quad 0 = \frac{\mu_0}{4\pi} \times \frac{2}{R-r} \times \frac{Ih}{2\pi R} - B$$

(The negative sign in used because the two fields are opposite)

$$\therefore \quad B = \frac{\mu_0 Ih}{4\pi R(R-r)}$$

28. (d) : $F_{mag} = \frac{\mu_0}{4\pi} \frac{(qv)(qv)\sin 90°}{r^2} = \frac{\mu_0}{4\pi} \times \frac{q^2v^2}{r^2}$

$$F_{elec} = \frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{r^2}$$

$$\therefore \quad \frac{F_{mag}}{F_{elec}} = \mu_0\varepsilon_0 \times v^2$$

$$= 4\pi \times 10^{-7} \times 8.85 \times 10^{-12} \times 9 \times 10^{10} = 10^{-6}$$

29. (c) : We have $\tau = IAB \sin\theta$,
where τ = torque, I = current in ampere, A = area, B = magnetic field and θ = angle between B and normal to A. Here $\theta = 90°$

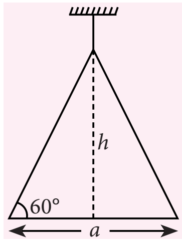
$\therefore \quad 1.5 = 0.5(3 \times 10^{-4} \times 500)\, B \sin 90°$

or $\quad B = \frac{1.5}{0.5 \times 3 \times 5 \times 10^{-2}} = 20$ T

30. (a) : Area of the triangle,

$$A = \frac{1}{2} \text{ base} \times \text{height} = \frac{1}{2} a \times a \sin 60°$$

$$= \frac{1}{2} \times 0.02 \times 0.02 \times \frac{\sqrt{3}}{2} = \sqrt{3} \times 10^{-4} \text{ m}^2$$



Magnetic dipole moment of the coil,

$m = IA = 0.1 \times \sqrt{3} \times 10^{-4}$

$= \sqrt{3} \times 10^{-5}$ A m^2

Magnetic moment m acts perpendicular to the plane of the loop and the field B. Therefore, torque acting on the coil is

$\tau = mB \sin\theta = \sqrt{3} \times 10^{-5} \times 5 \times 10^{-2} \times \sin 90°$

$= 5\sqrt{3} \times 10^{-7}$ N m

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ACE YOUR WAY CBSE XII

Series 4

CHAPTERWISE PRACTICE PAPER
Electromagnetic Induction | Alternating Current

Time Allowed : 3 hours | Maximum Marks : 70

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Q. no. 1 to 5 are very short answer questions and carry 1 mark each.
(iii) Q. no. 6 to 10 are short answer questions and carry 2 marks each.
(iv) Q. no. 11 to 22 are also short answer questions and carry 3 marks each.
(v) Q. no. 23 is a value based question and carries 4 marks.
(vi) Q. no. 24 to 26 are long answer questions and carry 5 marks each.
(vii) Use log tables if necessary, use of calculators is not allowed.

SECTION - A

1. If the frequency of the a.c. source in a *RLC* series a.c. circuit is increased, how does the current in the circuit change?
2. What are eddy currents? Why is their magnitude so high?
3. Self-induction is called the inertia of electricity. Why?
4. State fleming's right hand rule.
5. In a step-up transformer, a high voltage a.c. is obtained by supplying a low voltage a.c.. Does it contradict the principle of conservation of energy?

SECTION - B

6. Distinguish between self induction and mutual induction.
7. An induced emf has no direction of its own. Comment.
8. Duranto Express runs from Howrah to New Delhi in east-west direction with an average speed of 120 km h^{-1}. Calculate the induced emf between the ends of an axle of the train. Given that length of axle l = 1.68 m, the Earth's magnetic field = 0.60 G and angle of dip $\delta = 45°$.

OR

A metal disc of radius 20.0 cm is rotated at a constant angular speed of 60 rad s^{-1} in a plane at right angles to an external uniform field of 0.05 T. Find the emf induced between the centre of disc and a point on its rim.

9. Ordinary moving coil galvanometer used for d.c. cannot be used to measure an alternating current even if its frequency is low. Explain, why.
10. A lamp is connected in series with a capacitor. Predict your observations for d.c. and a.c. connections. What happens in each case if the capacitance of the capacitor is reduced?

SECTION - C

11. What is a choke coil? Why is it preferred over resistance in a.c. circuits?
12. How does the mutual inductance of a pair of coils change when

(a) the distance between the coil is increased?
(b) the number of turns in each coil is decreased?
(c) a thin iron sheet is placed between the two coils, other factors remaining the same?

Justify your answer in each case.

13. Describe the use of a series resonant circuit in the tuning of a radio receiver.

14. A long solenoid *S* has *n* turns per metre, with diameter *a*. At the centre of this coil, we place a smaller coil of *N* turns and diameter *b* (where $b < a$). If the current in the solenoid increases linearly, with time, what is the induced emf appearing in the smaller coil. Plot graph showing nature of variation in emf, if current varies as $(mt^2 + C)$.

15. Show that in the free oscillations of an *LC* circuit, the sum of energies stored in the capacitor and the inductor is

OR

A series *LCR* circuit is made by taking $R = 100\ \Omega$, $L = 2/\pi$ H, $C = 100/\pi\ \mu$F. The series combination is connected across an a.c. source of 220 V, 50 Hz. Calculate :
(a) the impedance of the circuit,
(b) the peak value of the current flowing in the circuit.

16. Two inductors of self-inductances L_1 and L_2 are connected in parallel. The inductors are so far apart that their mutual inductance is negligible. Derive the equivalent inductance of the combination.

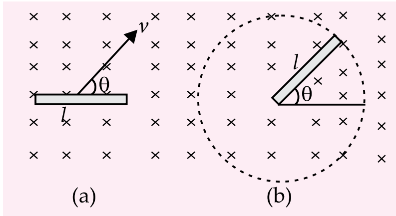
17. An alternating emf is applied across a capacitor. Show mathematically that current in it leads the applied emf by a phase angle of $\pi/2$. What is its capacitive reactance? Draw a graph showing the variation of capacitive reactance with the frequency of the a.c. source.

18. Define average value of a.c. over half a cycle. Establish the relationship between the average value and the peak value of an alternating current.

19. A radio can tune over the frequency range (800 kHz to 1200 kHz) of a portion of medium wave broadcast band. If its *LC* circuit has an effective inductance of 200 μH, what must be the range of its variable capacitor?

20. A 60 W load is connected to the secondary coil of a transformer whose primary coil draws line voltage. If a current of 0.54 A flows in the load, what is the current in the primary coil? Comment on the type of transformer being used.

21. Calculate the rate at which the flux linked with the generated area changes with time when a rod of length *l* is (a) translated (b) rotated in a uniform magnetic field of induction *B* as shown in figure.



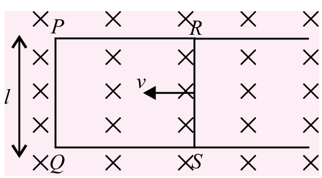
22. A heating coil of 100 Ω resistance, a choke coil of 0.50 H inductance and a capacitor of 15 μF capacitance are connected in series across a 220 V, 50 Hz a.c. supply. Find the power factor of the circuit.

SECTION - D

23. One day Priyanka went to the market with her mother in a metro rail. At the metro station, they were made to walk through a doorway of a metal detector for security reasons. Priyanka passed through it and started waiting for her mother to come. She heard a long beep when her mother passed through metal detector. Priyanka was surprised why the metal detector beeped in case of her mother. She asked the duty staff, who told her that it was due to the bunch of metal keys lying in the purse of her mother. Both Priyanka and her mother were satisfied with the security system.
(a) What values were displayed by Priyanka?
(b) What is the cause of sound through the metal detector?
(c) On what principle does a metal detector work?

SECTION - E

24. Figure shows a rectangular conducting loop *PQSR* in which arm *RS* of length *l* is movable. The loop is kept in a uniform magnetic field *B* directed downward perpendicular to the plane of the loop. The arm *RS* is moved with a uniform speed *v*.



Deduce an expression for
(i) the emf induced across the arm RS,
(ii) the external force required to move the arm
(iii) the power dissipated as heat.

OR

(a) State Lenz's law. Give one example to illustrate this law. "The Lenz's law is a consequence of the principle of conservation of energy". Justify this statement.
(b) Deduce an expression for the mutual inductance of two long coaxial solenoids but having different radii and different number of turns.

25. Name and explain three different methods of inducing emf with suitable example.

OR

Explain with the help of a labeled diagram, the principle, construction and working of a transformer. Why is the core of transformer laminated?
Give few energy losses in transformer. Why is the transformer used in long distance transmission of electrical energy.

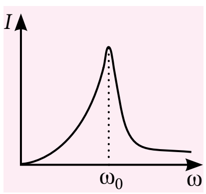
26. Explain with the help of a labelled diagram, the principle, construction and working of an a.c. generator.

OR

(a) Derive an expression for the average power consumed in a series LCR circuit connected to a.c. source in which the phase difference between the voltage and the current in the circuit is ϕ.
(b) Define the quality factor in an a.c. circuit. Why should the quality factor have high value in receiving circuits? Name the factors on which it depends.

SOLUTIONS

1. As frequency of an a.c. source applied across a RLC series circuit is gradually increased, the current initially increases with increase in frequency, then it acquires a maximum value for a frequency of a.c. equal to the resonant frequency. As the frequency is further increased, the current again begins to fall.



2. Eddy currents are the currents induced in a bulk conductor whenever magnetic flux linked with the conductor changes. Magnitude of eddy currents is generally high because electrical resistance offered by a bulk conductor is extremely small.

3. Self-induction of a coil is the property by virtue of which it tends to maintain the magnetic flux linked with it and opposes any change in the flux by inducing current in it. This property of a coil is analogous to mechanical inertia. That is why self-induction is called inertia of electricity.

4. If we stretch the thumb and the first two fingers of our right hand in mutually perpendicular directions and if the forefinger points in the direction of the magnetic field, thumb in the direction of motion of the conductor then the central finger points in the direction of current induced in the conductor.

5. Although a step-up transformer changes low a.c. voltage into high a.c. voltage output signal, yet it is strictly in accordance with the principle of conservation of energy. It is because a step-up transformer increases the magnitude of voltage in secondary but the current in the secondary is less so that even for an ideal transformer, the output power is just equal to the input power. Mathematically, $V_P I_P = V_S I_S$. Thus, total energy of the system remains conserved.

6.

Self Induction	Mutual Induction
It is the phenomenon of production of induced emf in a coil when a changing current passes through it.	It is the phenomenon of production of induced emf in one coil due to a change of current in the neighbouring coil.
It depends upon the size, shape and the number of turns of the coil. Larger the number of turns and area of cross-section, larger is the self inductance.	The mutual inductance of two coils depends on the number of turns in the two coils, their geometrical shape and their relative separation.

7. According to Lenz's law, the direction of induced emf in a circuit is always such as to oppose the change in magnetic flux which produces it. Thus, if the magnetic flux linked with a closed circuit increases, the induced current flows in such a direction so as to create a magnetic flux in the opposite direction of the original magnetic flux. If the magnetic flux linked with the closed circuit decreases, the induced current flows in such a direction so as to create a magnetic flux in the direction of the original flux. So we can say that the induced emf has no direction of its own.

8. As the train runs horizontally along east-west direction, hence $\vec{v}$ is along east-west and naturally, $\vec{l}$ is along north-south direction. Thus, emf is induced due to vertical component B_V of the Earth's magnetic field where $B_V = B_E \sin\delta$

$\therefore$ Induced emf,

$$|\varepsilon| = B_V \cdot lv = B_E \sin\delta \cdot lv$$

where, $B_E = 0.60\text{ G} = 6.0 \times 10^{-5}\text{ T}, \delta = 45°, l = 1.68\text{ m}$

and $v = 120\text{ km h}^{-1} = 120 \times \frac{5}{18}\text{ m s}^{-1}$

$$\therefore |\varepsilon| = 6.0 \times 10^{-5} \times \sin 45° \times 1.68 \times 120 \times \frac{5}{18}$$

$$= 2.4 \times 10^{-3}\text{ V} = 2.4\text{ mV}$$

OR

Given : radius of a disc, $r = 20.0\text{ cm} = 0.20\text{ m}$; angular speed, $\omega = 60\text{ rad s}^{-1}$ and uniform magnetic field acting normally to the plane of disc, $B = 0.05\text{ T}$.

$\therefore$ Induced emf between the centre of disc and a point on its rim,

$$\varepsilon = \frac{1}{2}Br^2\omega = \frac{1}{2} \times 0.05 \times (0.20)^2 \times 60 = 0.06\text{ V}$$

9. Ordinary moving coil galvanometer cannot be used to measure a.c. Ordinary moving coil galvanometer is based on magnetic effect of current which, in turn, depends on direction of current. So it cannot be used to measure a.c. During one half cycle of a.c., its pointer moves in one direction and during next half cycle, it will move in the opposite direction. Now the average value of a.c. over a complete cycle is zero. Even if we measure an alternating current of low frequency, the pointer will appear to be stationary at the zero position due to persistence of vision.

10. Here, combination of lamp and capacitor is a series *LCR* circuit. Capacitor offers infinite resistance for d.c.

For d.c.,

$$X_C = \frac{1}{2\pi\upsilon C} = 0, X_C = \infty$$

On switching d.c. on after a fraction of second, the capacitor gets charged fully and then no current flows. For a.c. circuit, capacitor offers finite resistance.

$$X_C = \frac{1}{2\pi\upsilon C}, X_L = 2\pi\upsilon L, Z = \sqrt{R^2 + (X_L - X_C)^2}$$

reducing C will increase Z and thus reduce the current.

11. A choke coil is simply an inductor with large inductance which is used to reduce current in a.c. circuits without much loss of energy.

Preference of choke coil over the ohmic resistance: A choke coil reduces current in a.c. circuit without consuming any power. When an ohmic resistance is used, current reduces but energy losses occur due to heating. So a choke coil is preferred.

Average power dissipated per cycle in the *LR* circuit

is $P_{av} = V_{eff} I_{eff} \cos\phi = V_{eff} I_{eff} \cdot \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$.

Inductance L of the choke coil is very large so that $R << \omega L$. Then

Power factor, $\cos\phi = \frac{R}{\omega L} \simeq 0$

12. (a) The mutual inductance of two coils, decreases when the distance between them is increased. This is because the flux passing from one coil to another decreases.

(b) Mutual inductance, $M = \frac{\mu_0 N_1 N_2 A}{l}$

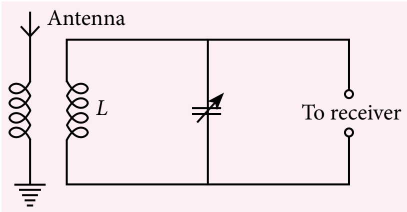
i.e., $M \propto N_1 N_2$

Clearly, when the number of turns N_1 and N_2 in the two coils is decreased, the mutual inductance decreases.

(c) When an iron sheet is placed between the two coils, the mutual inductance increases, because $M \propto$ permeability (μ).

13. The tuning circuit of a radio or TV is an example of *LCR* resonant circuit. Signals are transmitted by different stations at different frequencies. These frequencies are picked up by the antenna and corresponding to these frequencies, a number of voltages appear across the series *LCR*-circuit. But maximum current flows through the circuit for that a.c. voltage which has frequency equal to

$$\upsilon_r = \frac{1}{2\pi\sqrt{LC}}.$$



Series Resonant Circuit

If Q-value of the circuit is large, the signals of the other stations will be very weak. By changing the value of the adjustable capacitor C, the signal from the desired station can be tuned in.

14. Magnetic field due to a solenoid S,

$B = \mu_0 nI$

Magnetic flux in smaller coil, $\phi = NBA$,

where $A = \pi b^2$

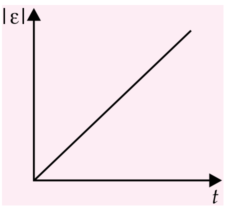
$$\text{So, } \varepsilon = \frac{-d\phi}{dt} = \frac{-d}{dt}(NBA)$$

$$= -N\pi b^2\left(\frac{dB}{dt}\right) = -N\pi b^2 \frac{d}{dt}(\mu_0 nI)$$

$$= -N\pi b^2 \mu_0 n \frac{dI}{dt}$$

$$= -Nn\pi\mu_0 b^2 \frac{d}{dt}(mt^2 + C) = -\mu_0 Nn\pi b^2\, 2mt$$

Negative sign signifies opposite nature of induced emf. The magnitude of emf varies linearly with time as shown in the figure.

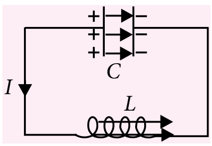


15. A charged capacitor introduced in the LC circuit discharges through inductor L.

Let q_0 is the initial charge on the capacitor, at any instant t the charge on capacitor is given by

$q = q_0 \cos \omega t$

current in the circuit,



$$I = \frac{dq}{dt} = -q_0 \omega \sin \omega t$$

Energy stored in capacitor plates at any time t as electric field is

$$U_E = \frac{1}{2}\frac{q^2}{C} = \frac{q_0^2 \cos^2 \omega t}{2C}$$

Energy stored in the inductor at time t, as magnetic field

$$U_B = \frac{1}{2}LI^2 = \frac{1}{2}L[-q_0\omega \sin \omega t]^2$$

$$\text{or } \quad U_B = \frac{1}{2}Lq_0^2\omega^2 \sin^2 \omega t,$$

$$\text{where } \quad \omega = \frac{1}{\sqrt{LC}}$$

$$\therefore \quad U_B = \frac{q_0^2}{2C}\sin^2 \omega t$$

$\therefore$ Total energy,

$$U = U_E + U_B = \frac{q_0^2}{2C}[\sin^2 \omega t + \cos^2 \omega t]$$

$$U = \frac{q_0^2}{2C}$$

The sum of energy is constant in time as both q_0 and C are time independent

OR

As per question, $R = 100\ \Omega$, $L = 2/\pi$ H

$$C = \frac{100}{\pi}\ \mu\text{F} = \frac{100}{\pi} \times 10^{-6}\ \text{F} = \frac{1}{\pi} \times 10^{-4}\ \text{F}$$

$V_{rms} = 220$ V and $\upsilon = 50$ Hz

(a) Inductive reactance, $X_L = \omega L = 2\pi\upsilon \times L$

$$= 2\pi \times 50 \times \frac{2}{\pi} = 200\ \Omega$$

$$\text{Capacitive reactance, } X_C = \frac{1}{\omega C} = \frac{1}{2\pi\upsilon C}$$

$$= \frac{1}{2\pi \times 50 \times \left(\frac{10^{-4}}{\pi}\right)} = 100\ \Omega$$

$$\therefore \quad \text{Impedance, } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

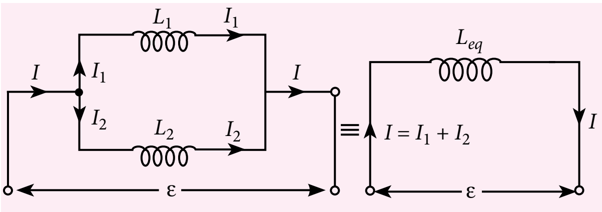
$$= \sqrt{(100)^2 + (200-100)^2} = 100\sqrt{2}\ \Omega$$

(b) Peak value of current, $I_0 = \sqrt{2}\, I_{rms}$

$$= \frac{\sqrt{2} \times V_{rms}}{Z} = \frac{\sqrt{2} \times 220}{100\sqrt{2}} = 2.2\ \text{A}$$

16. Inductances in parallel : For the parallel combination, the total current I divides up through the two coils as $I = I_1 + I_2$.

$$\therefore \quad \frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$



For parallel combination, induced emf across the combination is equal to the induced emf across each inductance. Thus

$$\varepsilon = -L_1 \frac{dI_1}{dt} \text{ or } \frac{\varepsilon}{L_1} = -\frac{dI_1}{dt}$$

$$\varepsilon = -L_2 \frac{dI_2}{dt} \text{ or } \frac{\varepsilon}{L_2} = -\frac{dI_2}{dt}$$

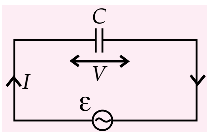
This is because the mutual inductance M is negligible. If L_{eq} is the equivalent inductance of the parallel combination, then

$$\varepsilon = -L_{eq} \cdot \frac{dI}{dt} = -L_{eq}\left[\frac{dI_1}{dt} + \frac{dI_2}{dt}\right] = L_{eq}\left[\frac{dI_1}{dt} - \frac{dI_2}{dt}\right]$$

$$\text{or} \quad \frac{\varepsilon}{L_{eq}} = \left[\frac{\varepsilon}{L_1} + \frac{\varepsilon}{L_2}\right] \text{ or } \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$\text{or} \quad \frac{\varepsilon}{L_{eq}} = \frac{L_1 L_2}{L_1 + L_2}$$

17. Alternating emf applied to the capacitor is



$$\varepsilon = \varepsilon_0 \sin\omega t \qquad \text{...(i)}$$

Suppose at any instant of time t, charge on the capacitor is q.

Therefore, potential difference across the plates of capacitor is $V = \dfrac{q}{C}$

At any instant, the V must be equal to the emf applied.

$$\text{So} \quad V = \frac{q}{C} = \varepsilon = \varepsilon_0 \sin\omega t$$

i.e., $q = C\varepsilon_0 \sin \omega t$

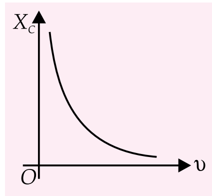
$$\frac{dq}{dt} = C\omega\varepsilon_0 \cos\omega t \text{ or } I = \frac{\varepsilon_0}{1/C\omega}\cos\omega t$$

$$I = \frac{\varepsilon_0}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right), \text{ where } X_C = \frac{1}{C\omega}$$

The alternating current at any instant t is given by

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \text{...(ii)}$$

From equations (i) and (ii), we conclude that current leads the emf by a phase angle of $\pi/2$.



Opposition offered by the capacitor towards the flow of current is known as capacitive reactance (X_C).

18. Average value of a.c. is defined as that value of direct current which sends the same charge in a circuit in the same time as is sent by the given alternating current in its half time period. It is denoted by I_{av} or I_m.

The value of alternating current at any instant t is given by

$$I = I_0 \sin \omega t$$

This current can be assumed to remain constant for a small time dt. Then the amount of charge that flows through the circuit in small time dt is given by

$$dq = I \cdot dt = I_0 \sin \omega t \cdot dt$$

The total charge that flows through the circuit, say in the first half cycle, *i.e.*, from $t = 0$ to $t = T/2$ is given by

$$q = \int_0^{T/2} dq = \int_0^{T/2} I_0 \sin \omega t \, dt = I_0\left[-\frac{\cos \omega t}{\omega}\right]_0^{T/2}$$

$$= -\frac{I_0}{2\pi/T}\left[\cos\frac{2\pi}{T}t\right]_0^{T/2} \qquad \left[\because \omega = \frac{2\pi}{T}\right]$$

$$= -\frac{I_0 T}{2\pi}[\cos \pi - \cos 0] = -\frac{I_0 T}{2\pi}[-1 - 1] = \frac{I_0 T}{\pi}$$

$\therefore$ The average value of a.c. over the first half cycle is

$$I_{av} = \frac{\text{Charge}}{\text{Time}} = \frac{q}{T/2} = \frac{2q}{T} = \frac{2}{T}\cdot\frac{I_0 T}{\pi}$$

$$\text{or} \quad I_{av} = \frac{2}{\pi} I_0 = 0.637\, I_0$$

Thus the mean or average value of an alternating current is $2/\pi$ or 0.637 times its peak value.

19. For tuning, the natural frequency *i.e.*, the frequency of LC oscillations should be equal to frequency of radio waves received by the antenna in the form of same frequency current in the LC circuit. For tuning at 800 kHz, required capacitance,

$$C_1 = \frac{1}{4\pi^2 L\upsilon_1^2} \qquad \left(\because \upsilon_1 = \frac{1}{2\pi\sqrt{LC_1}}\right)$$

$$= \frac{1}{4\pi^2(200\times10^{-6})(800\times10^3)^2}$$

$= 1.978 \times 10^{-10}$ F = 197.8 pF

For tuning of 1200 kHz, required capacitance

$$C_2 = \frac{1}{4\pi^2 L\upsilon_2^2} \qquad \left(\because \upsilon_2 = \frac{1}{2\pi\sqrt{LC_2}}\right)$$

$$= \frac{1}{4\pi^2(200\times10^{-6})(1200\times10^3)^2}$$

$= 8.795 \times 10^{-11}$ F = 87.95 pF

So, the variable capacitor should have a range between 87.9 pF to 197.8 pF.

20. Let V_1, I_1 and V_2, I_2 are respectively voltages and currents of secondary and primary coils of a transformer.

$P = 60$ W, $I_1 = 0.54$ A, $V_2 = 220$ V

$$P = V_1 I_1 \Rightarrow V_1 = \frac{60}{0.54} \approx 110 \text{ V}.$$

$$\frac{V_1}{V_2} = \frac{110}{220} = \frac{1}{2}$$

Transformer is step down.

$\because \quad V_2 I_2 = V_1 I_1$

$$\therefore \quad I_2 = \frac{V_1 I_1}{V_2} = \frac{110}{220} \times 0.54 = 0.27 \text{ A}$$

Current in the primary coil is 0.27 A.

21. (a) Component of velocity perpendicular to the rod = $v \sin\theta$.

Therefore, in time t, area traversed = $l \times v \sin\theta \times t$

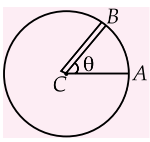
$\therefore \quad \phi = B(l \times v \sin\theta \times t)\cos 0° = Blvt \sin\theta$

$$\frac{d\phi}{dt} = Blv\sin\theta$$

(b) If θ is the angle between the rod and the radius of the circle CA at time t, the area of the sector ACB is given by

$$\therefore \quad \text{Area swept, } A = \pi l^2 \times \left(\frac{\theta}{2\pi}\right) = \frac{1}{2} l^2\theta$$

$$\phi = B\left(\frac{1}{2} l^2\theta\right)\cos 0° = \frac{1}{2} Bl^2\theta$$



$$\frac{d\phi}{dt} = \frac{1}{2} Bl^2 \frac{d\theta}{dt} = \frac{1}{2} Bl^2\omega$$

22. Given : $R = 100\ \Omega$, $L = 0.50$ H and $C = 15\ \mu$F $= 15 \times 10^{-6}$ F

Moreover, $V_{rms} = 220$ V and $\upsilon = 50$ Hz

Hence, $\quad \omega = 2\pi\upsilon = 2 \times 3.14 \times 50 = 314$ rad s^{-1}

$\therefore$ Inductive reactance,

$X_L = \omega L = 314 \times 0.50 = 157\ \Omega$

Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{314\times15\times10^{-6}} = 212\ \Omega$$

$\therefore$ Net reactance,

$X = (X_C - X_L) = (212 - 157)\Omega$

$= 55\ \Omega$ (capacitive)

$$\therefore \quad \text{Impedance, } Z = \sqrt{R^2 + X^2} = \sqrt{(100)^2 + (55)^2} \simeq 114\ \Omega$$

$\therefore$ Power factor,

$$\cos\phi = \frac{R}{Z} = \frac{100}{114} = 0.88$$

23. (a) Keen observer, curiosity and quest for knowledge.

(b) When we walk through the doorway of a metal detector carrying some metallic object, the impedance of the circuit changes, changing the current significantly. This change in current is detected as sounds in an alarm.

(c) A metal detector works on the principle of resonance in a.c. circuits.

24. Refer to point 4.1(7), page no. 248 (MTG Excel in Physics)

OR

(a) Refer to point 4.1(6), page no. 247 (MTG Excel in Physics)

(b) Refer to point 4.2(2), page no. 250 (MTG Excel in Physics)

25. Refer to point 4.1 (7), page no. 247 (MTG Excel in Physics)

OR

Refer to point 4.8(1), page no. 274 (MTG Excel in Physics)

26. Refer to point 4.8(2), page no. 275 (MTG Excel in Physics)

OR

(a) Refer to point 4.6(9), page no. 271 (MTG Excel in Physics)

(b) Refer to point 4.6(8), page no. 270 (MTG Excel in Physics)

MPP-5 MONTHLY Practice Problems

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.

Self check table given at the end will help you to check your readiness.

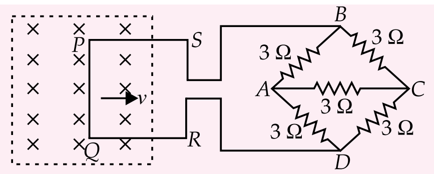
Electromagnetic Induction and Alternating Currents

Total Marks : 120 **Time Taken : 60 min**

NEET / AIIMS

Only One Option Correct Type

1. A square metal wire loop $PQRS$ of side 10 cm and resistance 1 Ω is moved with a constant velocity v in a uniform magnetic field of $B = 2$ Wb m^{-2}, as shown in the figure. The magnetic field lines are perpendicular to the plane of the loop (directed into the paper). The loop is connected to network $ABCD$ of resistors each of value 3 Ω. The resistance of the lead wires SB and RD are negligible. The speed of the loop so as to have a steady current of 1 mA in the loop is



(a) 2 m s^{-1} (b) 2×10^{-2} m s^{-1}
(c) 20 m s^{-1} (d) 200 m s^{-1}

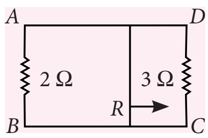
2. Whenever there is a relative motion between a coil and a magnet, the magnitude of induced emf set up in the coil does not depend upon the
(a) relative speed between the coil and magnet
(b) magnetic moment of the coil
(c) resistance of the coil
(d) number of turns in the coil

3. Two alternating currents are given by
$I_1 = I_0 \sin\omega t$ and $I_2 = I_0 \cos(\omega t + \phi)$
The ratio of rms values is
(a) 1 : 1 (b) $1 : \sqrt{2}$
(c) 1 : 2 (d) 2 : 1

4. In a series resonant RLC circuit, the voltage across R is 100 V and the value of R = 1000 Ω. The capacitance of the capacitor is 2×10^{-6} F; angular frequency of ac is 200 rad s^{-1}. Then the potential difference across the inductance coil is
(a) 100 V (b) 40 V (c) 250 V (d) 400 V

5. A pure resistive circuit element X when connected to an ac supply of peak voltage 200 V gives a peak current of 5 A which is in phase with the voltage. A second circuit element Y, when connected to the same ac supply also gives the same value of peak current but the current lags behind by 90°. If the series combination of X and Y is connected to the same supply, what will be the rms value of current?
(a) $\frac{10}{\sqrt{2}}$ A (b) $\frac{5}{\sqrt{2}}$ A (c) (5/2) A (d) 5 A

6. A rectangular loop with a sliding connector of length 10 cm is situated in uniform magnetic field perpendicular to plane of loop. The magnetic induction is 0.1 T and resistance of connector (R) is 1 Ω. The sides AB and CD have resistance 2 Ω and 3 Ω respectively. Find the current in the connector during its motion with constant velocity 1 m s^{-1}.

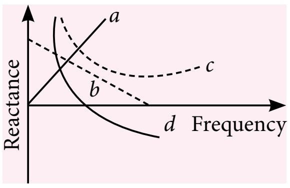


(a) $\frac{1}{110}$ A (b) $\frac{1}{220}$ A (c) $\frac{1}{55}$ A (d) $\frac{1}{440}$ A

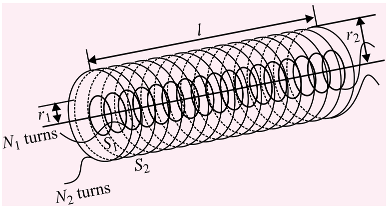
7. A step down transformer reduces the voltage of a transmission line from 2200 V to 220 V. The power delivered by it is 880 W and its efficiency is 88%. The input current is
(a) 4.65 mA (b) 0.0465 A
(c) 0.4545 A (d) 4.65 A

8. A power transmission line feeds input power at 2300 V to a step down transformer, with its primary windings having 4000 turns. What should be the number of turns in the secondary windings in order to get output power at 230 V?
(a) 200 (b) 400 (c) 600 (d) 800

9. Which of the following plots may represent the reactance of a series LC combination?
(a) a (b) b
(c) c (d) d



10. A circular coil with a cross-sectional area of 4 cm^2 has 10 turns. It is placed at the centre of a long solenoid that has 15 turns cm^{-1} and a cross-sectional area of 10 cm^2, as shown in the figure. The axis of the coil coincides with the axis of the solenoid. What is their mutual inductance?
(a) 7.54 μH
(b) 8.54 μH
(c) 9.54 μH
(d) 10.54 μH



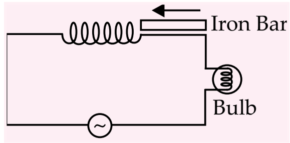
11. The power is transmitted from a power house on high voltage ac because
(a) electric current travels faster at higher volts
(b) it is more economical due to less power wastage
(c) it is difficult to generate power at low voltage
(d) chances of stealing transmission lines are minimized

12. A circuit area 0.01 m^2 is kept inside a magnetic field which is normal to its plane. The magnetic field changes from 2 T to 1 T in 1 ms. If the resistance of the circuit is 2 Ω. The rate of heat evolved is
(a) 5 J s^{-1} (b) 50 J s^{-1}
(c) 0.05 J s^{-1} (d) 0.5 J s^{-1}

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :
(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. **Assertion :** A light bulb is powered by an ac source with an inductor in circuit.
When the iron bar is inserted into the coil which increases magnetic field inside coil, the brightness of bulb will increase.



Reason : Introduction of iron bar decreases self inductance of inductor.

14. **Assertion :** Two concentric conducting rings of different radii are placed in space. The mutual inductance of the pair of rings is maximum if the rings are coplanar.
Reason : For two concentric conducting rings of different radii, the magnitude of magnetic flux developed in one ring due to the current flowing through other ring is maximum when the two rings are coplanar.

15. **Assertion :** Inductance coils are usually made of thick copper wire.
Reason: Induced current is more in wire having less resistance.

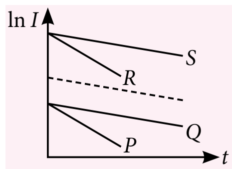
JEE MAIN / JEE ADVANCED

Only One Option Correct Type

16. A dc ammeter and a hot wire ammeter are connected to a circuit in series. When a direct current is passed through circuit, the dc ammeter shows 6 A. When ac current flows through circuit, the ac ammeter shows 8 A. What will be reading of each ammeter if dc and ac currents flow simultaneously through the circuit?
(a) dc = 6 A, ac = 10 A (b) dc = 3 A, ac = 5 A
(c) dc = 5 A, ac = 8 A (d) dc = 2 A, ac = 3 A

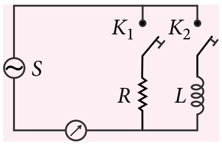
17. A 4μF capacitor and a resistance of 2.5 MΩ are in series with 12 V battery. Find the time after which the potential difference across the capacitor is 3 times the potential difference across the resistor. [Given ln(2) = 0.693].
(a) 13.86 s (b) 6.93 s (c) 7 s (d) 14 s

18. In an RC circuit while charging, the graph of ln I versus time is as shown by the dotted line in the adjoining diagram where I is the current. When the value of the resistance is doubled, which of the solid curves best represents the variation of ln I versus time?



(a) P (b) Q (c) R (d) S

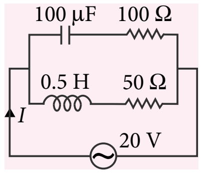
19. In the circuit shown in figure, R is a pure resistor, L is an inductor of negligible resistance (as compared to R,) S is a 100 V, 50 Hz ac source of negligible resistance. With either key K_1 alone or K_2 alone closed, the current is I_0. If the source is changed to 100 V, 100 Hz the current with K_1 alone closed and with K_2 alone closed will be, respectively,



(a) $I_0, \dfrac{I_0}{2}$ (b) $I_0, 2I_0$ (c) $2I_0, I_0$ (d) $2I_0, \dfrac{I_0}{2}$

More than One Options Correct Type

20. In the given circuit, the ac source has ω = 100 rad s^{-1}. Considering the inductor and capacitor to be ideal, the correct choice(s) is/are



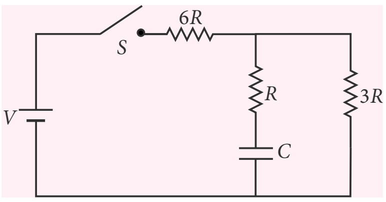
(a) the current through the circuit, I is 0.3 A.

(b) the current through the circuit, I is $0.3\sqrt{2}$ A.

(c) the voltage across 100 Ω resistor is $10\sqrt{2}$ V.

(d) the voltage across 50 Ω resistor is 10 V.

21. The circuit shown in figure switch S is closed at time t = 0. If the current I is flowing through resistance $6R$ and charge q is stored on the capacitor at any time t,

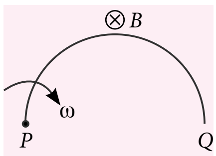


(a) $I = \dfrac{V}{3R} + \dfrac{V}{27R}e^{-t/6RC}$

(b) $q = \dfrac{CV}{3}\left(1 - e^{-t/3RC}\right)$ (c) $I = \dfrac{V}{9R}\left(1 + \dfrac{1}{3}e^{-t/3RC}\right)$

(d) $q = \dfrac{CV}{3}e^{-t/2RC}$

22. A wire is bent to form a semi-circle of radius a. The wire rotates about its one end with angular velocity ω. Axis of rotation is perpendicular to plane of the semicircle.



In the space, a uniform magnetic field of induction B exists along the axis of rotation as shown in figure. Then

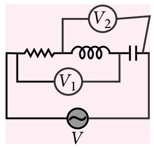
(a) potential difference between P and Q is equal to $2\,B\,\omega a^2$

(b) potential difference between P and Q is equal to $2\,\pi^2\,B\,\omega a^2$

(c) P is at higher potential than Q

(d) P is at lower potential than Q

23. In an RLC series circuit shown in figure, the readings of voltmeters V_1 and V_2 are 100 V and 120 V, respectively. The source voltage is 130 V. For this situation, mark out the correct statements(s).



(a) Voltage across resistor, inductor and capacitor are 50 V, $50\sqrt{3}$ V, and $(120 + 50\sqrt{3})$ V, respectively

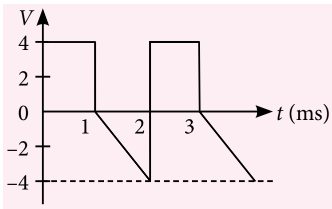
(b) Voltage across resistor, inductor, and capacitor are 50 V, $50\sqrt{3}$ V, and $(120 - 50\sqrt{3})$ V, respectively

(c) Power factor of the circuit is $\dfrac{5}{13}$

(d) The circuit is capacitive in nature

Integer Answer Type

24. Variation of voltage with time is shown in figure.



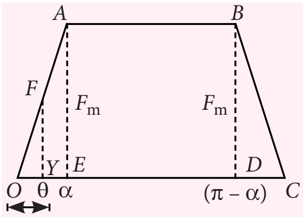
The rms voltage is found to be $N\sqrt{\dfrac{2}{3}}$ V . Find N?

25. A series LCR circuit with R = 120 Ω has an angular frequency 4 × 10^5 rad s^{-1}. At resonance, voltages across resistance and inductor are 60 V and 40 V respectively. If frequency at which the circuit current lags behind the voltage by a phase of $\pi/4$ radian is $k \times 10^5$ rad s^{-1}, then find the value of k.

26. If series combination of two inductors has a value 10 H and parallel combination 2.4 H, what will be the difference between values of two inductors (in H)?

Comprehension Type

The half cycle of an alternating signal is shown in figure. It increases uniformly from zero at 0° to F_m at $\alpha°$, it remains constant from $\alpha°$ to $(\pi - \alpha)°$, and decreases uniformly from F_m to zero at π.



27. The effective values of the signal is

(a) $F_m\sqrt{\left(1-\frac{4\alpha}{3\pi}\right)}$ (b) $F_m\sqrt{\left(1+\frac{4\alpha}{3\pi}\right)}$

(c) $F_m\sqrt{\left(1-\frac{3\alpha}{4\pi}\right)}$ (d) $F_m\sqrt{\left(1+\frac{3\alpha}{4\pi}\right)}$

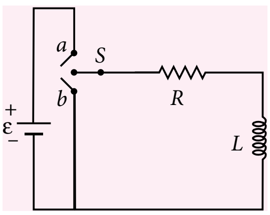
28. The average values of the signal is

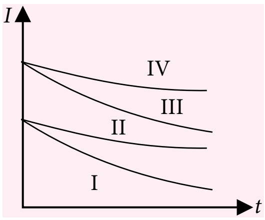
(a) $\frac{(\pi+\alpha)F_m}{\pi}$ (b) $\frac{(\pi-\alpha)F_m}{\pi}$

(c) $\left(\frac{\pi+\alpha}{3\pi}\right)F_m$ (d) $\left(\frac{2\pi+\alpha}{2\pi}\right)F_m$

Matrix Match Type

29. The switch S in the circuit is connected with the point a for a very long time, then it is shifted to position b. The resulting current through the inductor is shown by curves in the graph for four sets of values for the resistance R and inductance L (given in column I). Match the entries of column I with that of column II.

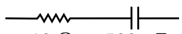
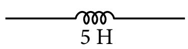
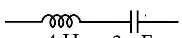
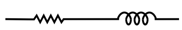




	Column I		**Column II**
(A)	R_0 and L_0	(P)	I
(B)	$2R_0$ and L_0	(Q)	II
(C)	R_0 and $2L_0$	(R)	III
(D)	$2R_0$ and $2L_0$	(S)	IV

	A	B	C	D
(a)	P	Q	R	S
(b)	P	S	Q	R
(c)	R	P	S	Q
(d)	R	S	P	Q

30. Four different circuit components are given in each situation of column I and all the components are connected across an ac source of same angular frequency $\omega = 200$ rad s^{-1}. The information of phase difference between the current and source voltage in each situation of column I is given in column II. Match the circuit components in column I with corresponding results in column II.

	Column I		**Column II**
(A)	 10 Ω 500 μF	(P)	the magnitude of required phase difference is π/2
(B)	 5 H	(Q)	the magnitude of required phase difference is π/4
(C)	 4 H 3 μF	(R)	the current leads in phase of source voltage
(D)	 4 kΩ 3 H	(S)	the current lags in phase to source voltage

	A	B	C	D
(a)	*Q, R*	*P, S*	*P, R*	*Q, S*
(b)	*R, S*	*R*	*S*	*Q*
(c)	*S, R*	*Q, S, R*	*R, S*	*Q*
(d)	*R, S*	*S, R*	*Q*	*R*

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Marks scored in percentage ......

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

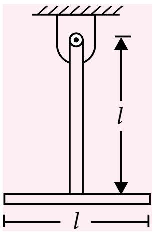
PHYSICS MUSING

Physics Musing was started in August 2013 issue of Physics For You. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / NEET / AIIMS / JIPMER with additional study material.
In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / NEET. The detailed solutions of these problems will be published in next issue of Physics For You.
The readers who have solved five or more problems may send their detailed solutions with their names and complete address. The names of those who send atleast five correct solutions will be published in the next issue.
We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

PROBLEM Set 50

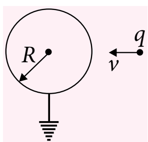
MULTIPLE OPTIONS CORRECT TYPE

1. A pendulum is constructed from two identical uniform thin rods each of length l and mass m connected at right angle forming T shape as shown in figure. It is suspended from free end and swings in vertical plane.



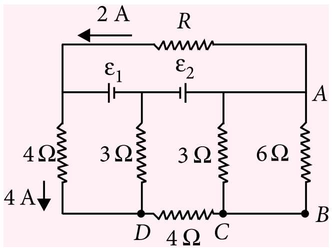
(a) Moment of inertia of the pendulum about axis of rotation is $\frac{17}{12}ml^2$.
(b) Moment of inertia of pendulum about axis of rotation is $(13/12)\ ml^2$
(c) Time period of small angular oscillation of pendulum is $2\pi\sqrt{\frac{17\,l}{18g}}$
(d) Angular frequency of pendulum for small oscillation is $\sqrt{\frac{18g}{19l}}$

2. A charged particle having a positive charge q approaches a grounded metallic sphere of radius R with a constant speed v as shown in figure. As the charged particle draws nearer to the surface of the sphere,



(a) a current flows into the ground.
(b) a current flows out of the ground into the sphere.
(c) the magnitude of current flowing in the connector joining the sphere to the ground increases.
(d) the magnitude of current flowing in the connector joining the sphere to the ground decreases.

3. In the circuit shown in figure, ε_1, and ε_2 are two ideal sources of unknown emfs. Some currents are shown. Potential difference appearing across 6 Ω resistance is $V_A - V_B = 10$ V,



(a) the current in the 4 Ω resistor in between C and D is 5 A.
(b) the unknown emf ε_1 is 43 V.
(c) the unknown emf ε_2 is 57 V.
(d) the resistance R is equal to 7 Ω

4. Two particles A and B executing SHM along same straight line, with same amplitude and same mean position. A starts its motion from mean position and moves towards positive extreme while B starts from negative extreme position. Angular frequency of A is ω_A, and that of B is ω_B, choose the correct statement (s).
(a) If $\omega_B = 2\omega_A$. then when they meet first their velocity will be zero.

Solution Senders of Physics Musing

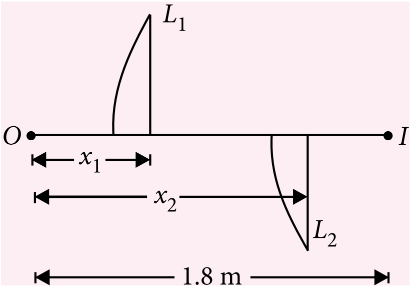
SET-49

1. Ankur Vashishth, Lucknow (U.P.)
2. Sonika Reddy, (Bangalore)
3. Nikhil Gaikwad, Ahmednagar (Maharashtra)

By Akhil Tewari, Author Foundation of Physics for JEE Main & Advanced, Professor, IITians PACE, Mumbai.

(b) If $\omega_B > 2\omega_A$ then when they meet first their velocity are in same direction.
(c) If $\omega_B < 2\omega_A$ then when they meet first their velocity will be in opposite direction.
(d) Their velocity when they meet does not depend on ω.

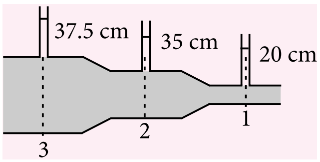
5. A thin plano-convex lens of focal length f is split into two halves, one is placed at certain distance to other. Magnification of image by one half is 2 and object and image are at a distance of 1.8 m, then (for both L_1 and L_2, O is object and I is image)



(a) $x_1 = 0.6$ m (b) $x_2 = 1.2$ m
(c) $f = 0.4$ m (d) none of these

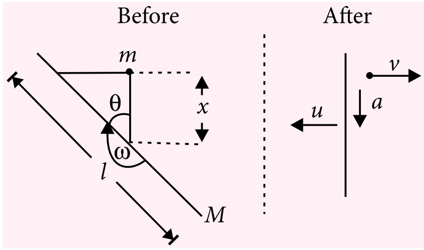
INTEGER ANSWER TYPE

6. Water is flowing in a horizontal pipe of varying cross-section. The areas of cross-sections 1, 2 and 3 are 1 cm², 2 cm² and A cm² respectively. Water levels are shown in different vertical tubes. The speed of water at cross-section 3 is $\frac{1}{\sqrt{x}}$ m s^{-1}. Find the value of x.



7. If the K_α radiation of Mo($Z = 42$) has a wavelength of 0.75 Å, calculate wavelength of the corresponding radiation of Fe, *i.e.*, K_α for Fe ($Z = 26$). Find the value of nearest integer value (in Å) of wavelength.

8. A long thin rod of length $l = 90$ cm and mass 600 g, is rotating clockwise on a frictionless horizontal table about its centre of mass, which is stationary. A small ball of mass 300 g is placed on the table at distance x from the centre of mass of the rod. The rod collides elastically with the ball. After the collision, the rod is translating to the left without rotating and the ball moves to the right. What is the distance $\left(\frac{x}{5}\right)$ (in cm)?



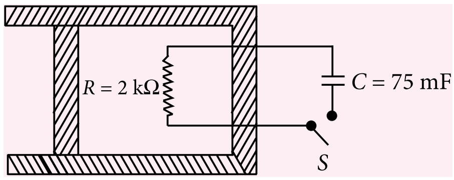
COMPREHENSION TYPE

For questions 9 and 10 :

A cylinder fitted with a piston which can slide without friction contains one mole of an ideal gas. The walls of the cylinder and piston are adiabatic. The cylinder contains a resistor of resistance $R = 2$ kΩ which is connected to a capacitor of capacity $C = 75$ mF.

Initially, potential difference across capacitor is $\left(\frac{640}{3}\right)$V and switch is open.

When switch is closed for (2.5 ln 4) min, then gas expands isobarically and its temperature increases by 72 K. Heat loss through the wires is negligible ($R = 8.3$ J mol^{-1}K^{-1}).



9. Heat supplied to the gas is approximately
(a) 0.2 kJ (b) 0.4 kJ
(c) 1.6 kJ (d) 0.8 kJ

10. The increment in internal energy of gas is
(a) 1 kJ (b) 1.2 kJ
(c) 3 kJ (d) 4 kJ

MPP-5 CLASS XII ANSWER KEY

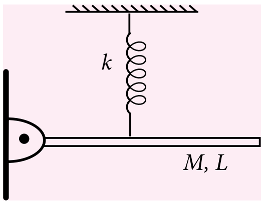
1. (b)	2. (c)	3. (a)	4. (c)	5. (c)
6. (b)	7. (c)	8. (b)	9. (d)	10. (a)
11. (b)	12. (b)	13. (d)	14. (a)	15. (a)
16. (a)	17. (a)	18. (b)	19. (a)	20. (a,c)
21. (b,c)	22. (a,d)	23. (a,c)	24. (4)	25. (8)
26. (2)	27. (a)	28. (b)	29. (c)	30. (a)

KEY CONCEPT

on TIME PERIOD OF OSCILLATING SYSTEM INVOLVING ROTATION

Er. Sandip Prasad*

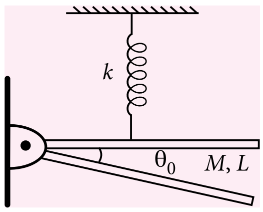
Let us consider a spring of spring constant k is attached at the centre of a rod as shown in the figure.



For the calculation of time period of the rod, first of all we have to provide an amplitude to the system. For this we need to displace the rod by a small angle and then it should be released at this position. Because of restoring torque about hinged point of the rod, it oscillates about its mean position. During the oscillation rod shows pure rotational motion about its centre of rotation.

Method-I:

In the equilibrium position, torque of spring force and weight balance each other.

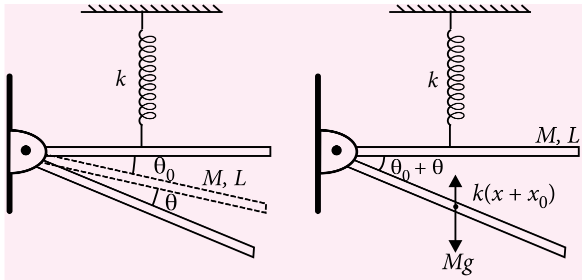


$$kx_0\frac{L}{2} = Mg\frac{L}{2} \qquad \text{... (i)}$$

Note: Here, equation of translational equilibrium can not be written because of a hinge force acting at the point where rod is hinged. Its value and exact direction is unknown to us.

Hence we can say that rod has attained an equilibrium position at angle θ_0, due to which weight of the rod has no role as a restoring torque during the oscillation of the rod. If weight of the body does not shift the position of the equilibrium of the rod then it would act as a restoring torque on the rod. So be careful about weight of the body.

Consider a position of the rod, displaced by an angle θ as shown in the figure. In this displaced position at any time t, the equation of motion of the particle for the rod is following,



$$(\tau_{net}) = I\alpha \;\Rightarrow\; k(x+x_0)\frac{L}{2} - Mg\frac{L}{2} = -I\alpha$$

$$kx\frac{L}{2} = -\frac{ML^2}{3}\alpha \qquad \text{(Using eqn. (i))}$$

$$k\frac{L}{2}\theta\frac{L}{2} = -\frac{ML^2}{3}\alpha \qquad (\because\; x = \theta L/2)$$

$$\Rightarrow \alpha = -\left(\frac{3}{4}\frac{k}{M}\right)\theta$$

Here, negative sign shows that the restoring force is opposite to the displacement.

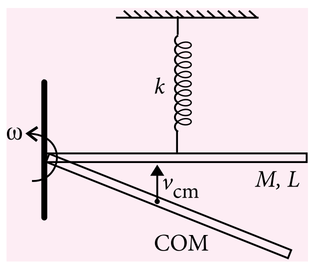
Comparing the above equation with the standard equation of S.H.M *i.e.*, with $\alpha = -\omega^2\theta$,

we get, $\omega^2 = \frac{3}{4}\frac{k}{M}$

$$\therefore\; T = 2\pi\sqrt{\frac{4M}{3k}}$$

Method-II

The total mechanical energy of oscillation of the system executing simple harmonic motion is always constant during the oscillation.



Total mechanical energy of oscillation at time t,

E = K.E. + P. E.

$$= \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2$$

*Author is Director of Sandip Physics Classes and motivational speaker

$$= \frac{1}{2} I \frac{v^2}{\frac{L^2}{4}} + \frac{1}{2}kx^2$$

$$= \frac{4}{2}\frac{ML^2}{3}\frac{v^2}{L^2} + \frac{1}{2}kx^2$$

$$\therefore \quad E = \frac{2}{3}Mv^2 + \frac{1}{2}kx^2 \qquad \text{...(ii)}$$

Since, the total mechanical energy (E) of the system remains constant during oscillation, hence

$$\therefore \quad \frac{dE}{dt} = 0$$

$$\frac{d}{dt}\left(\frac{2}{3}Mv^2 + \frac{1}{2}kx^2\right) = 0$$

$$\frac{2}{3}M\left(2v\frac{dv}{dt}\right) + \frac{1}{2}k\left(2x\frac{dx}{dt}\right) = 0$$

$$\frac{2}{3}M(2va) + \frac{1}{2}k(2xv) = 0 \quad \text{or} \quad \frac{4}{3}Ma + kx = 0$$

$$\therefore \quad a = -\frac{3k}{4M}x$$

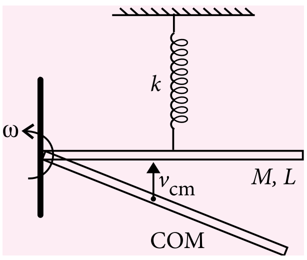
Comparing this equation with the standard equation of S.H.M *i.e.*, with $a = -\omega^2 x$,

we get, $\omega^2 = \frac{3}{4}\frac{k}{M}$

$$\therefore \quad T = 2\pi\sqrt{\frac{4M}{3k}}$$

Method-III:

The total mechanical energy of oscillation of the system executing simple harmonic motion is always constant during the oscillation.



Total mechanical energy of oscillation at time t,

$$E = \frac{2}{3}Mv^2 + \frac{1}{2}kx^2 \qquad \text{(From eqn. (ii))}$$

Since, the total mechanical energy (E) of the system remains constant during oscillation, hence

$\therefore$ E = constant

or $\frac{2}{3}mv^2 + \frac{1}{2}kx^2 = \text{constant} = C$

After this step, we need to make the coefficient of v^2 equal to 1 by taking common from L.H.S

$$\frac{2}{3}M\left(v^2 + \frac{1}{2}\frac{3k}{2M}x^2\right) = C$$

$$\therefore \quad \left(v^2 + \frac{3k}{4M}x^2\right) = C' \qquad \text{... (iii)}$$

Comparing the equation (iii) with the equation of S.H.M., $v^2 + \omega^2 x^2$ = constant (since $v = \omega\sqrt{a^2 - x^2}$)

we get, $\omega^2 = \frac{3}{4}\frac{k}{M}$

$$\therefore \quad T = 2\pi\sqrt{\frac{4M}{3k}}$$

Method-IV: Shortcut Method

For the application of this method, we have to find equivalent mass and equivalent force constant.

$$\because \quad T = 2\pi\sqrt{\frac{m_{\text{eff}}}{k_{\text{eff}}}}$$

Calculation of effective spring constant (k_{eff}):

(a) When the axis of rotation is not the centre of mass:

First of all, shift the end of the spring which is connected to any point of the rod to the center of mass of the body for finding n-factor of the spring.

$$n = \frac{\text{elongation in the spring}}{\text{displacement of the COM at that instant}}$$

If spring is connected to the centre of mass, then there is no need to calculate n-factor of the spring and in this situation k_{eff} becomes equal to k. Hence we can say that value of n is 1.

In this situation spring is connected to the centre of mass of the rod. So

$k_{\text{eff}} = (1)^2 k = k$

(b) When the axis of rotation is the centre of mass:

If there are more than one spring is connected to the rigid body, then we have to bring all the springs to a single point by relating the displacements of the springs.

n-factor of the spring which has to be shifted,

$$n = \frac{\text{elongation in the spring which has not be shifted}}{\text{elongation of the spring which has to be shifted}}$$

Calculation of effective mass (m_{eff}):

Next we need to find the equivalent translational mass of the rigid body. The equivalent mass can be calculated by following equation,

$$m_{\text{eff}} = \frac{I}{r^2}$$

where, I is the moment of inertia of the rigid body about the centre of rotation and r is the distance of the center of mass from the point of rotation. This value of r is only applicable if the axis of rotation is not the centre of mass.

In this case, $r = \frac{L}{2}$. Hence, $m_{eff} = \frac{ML^2/3}{L^2/4} = \frac{4}{3}M$

Note:

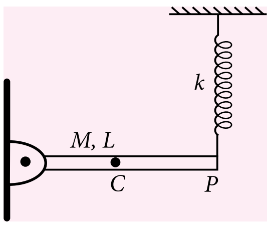
(1) When the body rotates about centre of mass, then the value of r which was earlier taken as the distance between the COM and COR, is now taken as the distance between the COM and the point at which the free end of the springs are attached.

(2) If the oscillating body have only translational motion during the oscillation then there is no need of finding the equivalent mass.

Hence, time period of oscillation is

$$T = 2\pi\sqrt{\frac{m_{eff}}{k_{eff}}} = 2\pi\sqrt{\frac{\frac{4}{3}M}{k}} = 2\pi\sqrt{\frac{4M}{3k}}$$

Problem-1: A rod of mass M and length L is hinged at one end and another end is attached to a massless spring, of spring constant k. Find the time period of the oscillation of the rod.



Solution : Using shortcut method,

n-factor of the spring,

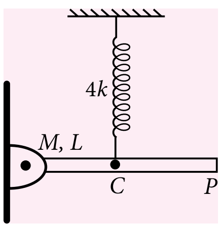
$$n = \frac{\text{elongation in the spring}}{\text{displacement of the COM at that instant}}$$

$$n = \frac{x'}{x} = \frac{L\theta}{\frac{L}{2}\theta} = 2$$

$\therefore\ k_{eff} = n^2k = 4k$

Hence, the spring can be replaced by a spring of spring constant $4k$ connected at position C.

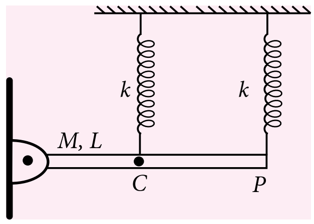
In this case, $r = \frac{L}{2}$



Hence, $m_{eff} = \frac{I}{r^2} = \frac{\frac{ML^2}{3}}{\frac{L^2}{4}} = \frac{4}{3}M$

$$\therefore\quad T = 2\pi\sqrt{\frac{\frac{4}{3}M}{4k}} = 2\pi\sqrt{\frac{M}{3k}}$$

Problem-2 : A rod of mass M and length L is hinged at one end. Two massless, identical springs, each of spring constant k are attached to a rod, one at the free end and other at centre as shown. Find the time period of the oscillation of the rod.



Solution : n-factor of the spring connected to the free end of the rod,

$$n = \frac{\text{elongation in the spring}}{\text{displacement of the COM at that instant}}$$

$$= \frac{x'}{x} = \frac{L\theta}{\frac{L}{2}\theta} = 2$$

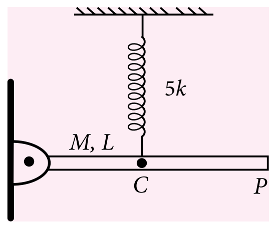
$\therefore\ k_{eff} = n^2k = 4k$

Hence, the spring can be replaced by a spring of spring constant $4k$ connected at position C.

Again in this case, we need to find the equivalent spring constant because there are two springs. Here both the springs are connected in parallel.

$\therefore\ k_{eq} = 4k + k = 5k$

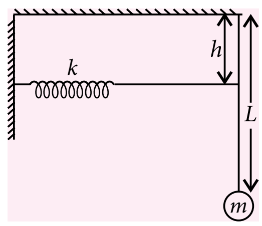
Here, $r = \frac{L}{2}$



$$\therefore\quad m_{eff} = \frac{I}{r^2} = \frac{\frac{ML^2}{3}}{\frac{L^2}{4}} = \frac{4}{3}M$$

$$\therefore\quad T = 2\pi\sqrt{\frac{\frac{4}{3}M}{5k}} = 2\pi\sqrt{\frac{4M}{15k}}$$

Problem-3 : A pendulum of mass m and length L is connected to a spring as shown in figure. If the bob is displaced slightly from its mean position and released, it performs simple harmonic motion. Find the time period of the oscillation of the bob ?



(Bob is connected to the light rod.)

Solution : n-factor of the spring connected to pendulum

$$n = \frac{\text{elongation in the spring}}{\text{displacement of the COM at that instant}}$$

$$n=\frac{x'}{x}=\frac{h\theta}{L\theta}=\frac{h}{L}$$

$$\therefore\ k_{eff}=\left(\frac{h}{L}\right)^2 k=\frac{h^2}{L^2}k$$

Hence, the spring can be replaced by a spring of spring constant k_{eff} connected at position of the bob .

Here, weight of the bob also provides as a restoring torque because it does not shift the equilibrium position of the arrangement.

Again the restoring effect of the weight can be converted into equivalent spring which can obtained by the following relation.

Time period of the simple pendulum = Time period of the spring mass system

$$2\pi\sqrt{\frac{L}{g}}=2\pi\sqrt{\frac{m}{k}}$$

$$\therefore\ k=\frac{mg}{L}$$

Hence, equivalent spring constant,

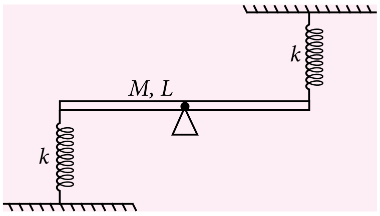
$$\therefore\ k_{eq}=\frac{h^2}{L^2}k+\frac{mg}{L}$$

Here, $r = L$

$$\therefore\ m_{eff}=\frac{I}{r^2}=\frac{mL^2}{L^2}=m$$

$$\therefore\quad T=2\pi\sqrt{\frac{m}{\frac{h^2}{L^2}k+\frac{mg}{L}}}=2\pi\sqrt{\frac{mL^2}{kh^2+mgL}}$$

Problem-4 : A uniform rod of length L and mass M is pivoted at its centre. It is held in a position by a system of springs as shown in figure. Show that when turned through a small angle θ and released the rod undergoes S.H.M. with time period, $T=2\pi\sqrt{\frac{M}{6k}}$.

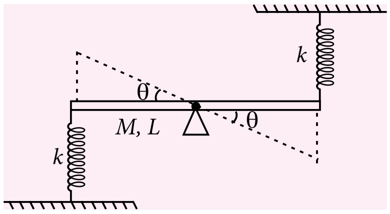


Solution : If there are more than one spring is connected to the rigid body, then we have to bring all the springs to a single point by relating the displacements of the springs. n-factor of the spring of the spring which has to be shifted,

$$n=\frac{\text{elongation in the spring which has not be shifted}}{\text{elongation of the spring which has to be shifted}}$$

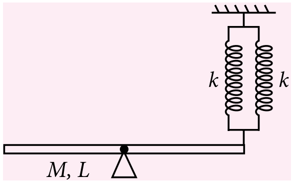
$$=\frac{x'}{x}$$

$$=\frac{\frac{L}{2}\theta}{\frac{L}{2}\theta}=1$$



$k_{eff} = n^2k = k$

Hence, one of the spring can be shifted to a position of another spring. Again in this case, we need to find the equivalent spring constant because there are two springs. Here both the springs are connected in parallel.



$\therefore\ k_{eq} = k + k = 2k$

When the body rotates about centre of mass, then the value of r which was earlier taken as the distance between the COM and COR, is now taken as the distance between the COM and the point at which the free end of the springs are attached.

Here, $r=\frac{L}{2}$

$$\therefore\ m_{eff}=\frac{I}{r^2}=\frac{\frac{ML^2}{12}}{\frac{L^2}{4}}=\frac{M}{3}$$

$$\therefore\quad T=2\pi\sqrt{\frac{\frac{M}{3}}{2k}}=2\pi\sqrt{\frac{M}{6k}}$$

PHYSICS MUSING

SOLUTION SET-49

1. (6) : Power evolved during reaction, $P = 10$ mW $= 10^{-2}$ W

Mean energy of one beta particle, $E = 700$ keV
$= 700 \times 10^3 \times 1.6 \times 10^{-19}$ J $= 1.12 \times 10^{-13}$ J

As $\frac{dN}{dt}$ is the decay rate of phosphorus

$$\therefore\ P = E \times \frac{dN}{dt}$$

or $$\frac{dN}{dt} = \frac{P}{E} = \frac{10^{12}}{11.2}\text{dps} \quad ...(i)$$

Decay law, $$\frac{dN}{dt} = \lambda N_0 \quad ...(ii)$$

where $$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{14 \times 86400}\, s^{-1}$$

Solving eqns. (i) and (ii), we get

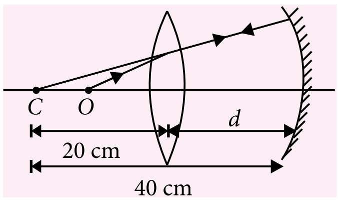
$$N_0 = \frac{14 \times 86400 \times 10^{12}}{11.2 \ln 2} = 1.54 \times 10^{17}$$

As given, $N_0 = A \times 26 \times 10^{15}$
$\therefore\ A = 6$

2. (2) : The silvered lens can be replaced by a mirror of focal length given as

$$\frac{1}{F_M} = \frac{1}{f_m} - \frac{2}{f_\ell}$$

$$\frac{1}{F_M} = 0 - \frac{2}{40}$$



$F_M = -20$ cm

For lens, object distance, $u = -10$ cm,

focal length, $f = 20$ cm

Using lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \text{ or } v = \frac{uf}{u+f} = \frac{-10 \times 20}{-10+20} = -20 \text{ cm}$$

Image is formed at point C as shown in figure.

The position of image formed by lens has to be centre of curvature of mirror so that the ray retraces its path.

$\therefore\ d = 40 - 20 = 20$ cm $= 2$ dm

3. (9) : For image formed by plane mirror,
$u' = 10\cos1° \simeq 10$ cm
$v' = 10\cos1°$
$\therefore\ OB = 2 \times 10\cos1° = 20$ cm

$OA = OB\cos1° = 20\cos^2 1° = 20$ cm
and $AB = OB\sin1° = 20\sin1°\cos1° = 10\sin2°$

$$= 10\sin\left(2 \times \frac{\pi}{180}\right) = 10\sin\left(\frac{\pi}{90}\right)\text{cm}$$

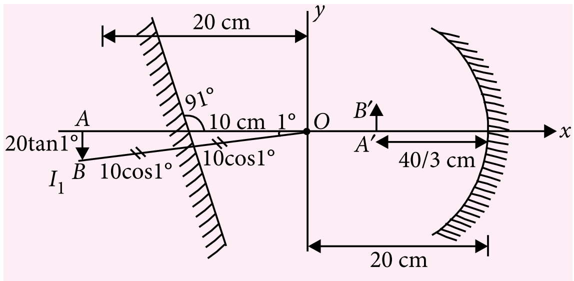
For small θ, $\sin\theta \approx \theta$

$$\therefore\ AB = 10 \times \frac{\pi}{90} = \frac{\pi}{9}\text{ cm}$$

For reflection from curved mirror,
$u = -(OA + 20) = -40$ cm, $f = -10$ cm
Using mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{-10} = \frac{1}{v} + \frac{1}{-40} \Rightarrow \frac{1}{v} = \frac{1}{40} - \frac{1}{10}$$

$$\Rightarrow \frac{1}{v} = \frac{1-4}{40} \Rightarrow v = -40/3\text{ cm}$$



Using magnification formula

$$m = \frac{A'B'}{AB} = -\frac{v}{u} \Rightarrow \frac{A'B'}{\pi/9} = -\frac{-40/3}{-40}$$

$$\Rightarrow A'B' = \frac{-\pi}{27}\text{cm} \Rightarrow y_0 = \frac{\pi}{27}\text{cm},$$

$$x_0 = 20 - \frac{40}{3} = \frac{20}{3}\text{cm} \quad \therefore \frac{\pi x_0}{20 y_0} = \frac{\pi(20/3)}{20(\pi/27)} = 9$$

4. (4) : At M, the ray of light passes through surface undeviated and reflected at N due to silvered surface.

Angle of incidence on plane mirror $= 90° - \angle MNP$
$= 90° - (90° - A) = A$

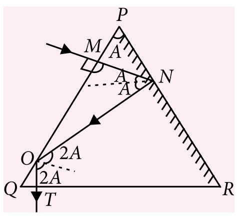
At O, light is totally internally reflected (as given in question)

Since $PQ = PR$, $\angle PQR = \angle PRQ$

and $$\angle PQR = \frac{\pi - \angle QPR}{2} = \frac{\pi}{2} - \frac{A}{2}$$

In ΔOQT, $\angle QOT = \frac{\pi}{2} - 2A$, $\angle OQT = \frac{\pi}{2} - \frac{A}{2}$ and

$\angle OTQ = \frac{\pi}{2}$ (given)



$$\therefore\ \frac{\pi}{2} - \frac{A}{2} + \frac{\pi}{2} - 2A = \frac{\pi}{2}$$

$$\frac{5A}{2} = \frac{\pi}{2} \text{ or } A = \frac{\pi}{5} = 36°$$

$x = 4$

5. **(1)** **:** Intensity of light after interference,

$$I = 4I_0\cos^2\frac{\phi}{2},$$

When there is no glass slab, $\phi = 0 \Rightarrow I = 4I_0$

When glass slab is placed in front of first slit,

intensity $= \frac{3I}{4} = 4I_0\cos^2\frac{\phi}{2} \Rightarrow \cos^2\frac{\phi}{2} = \frac{3}{4}$

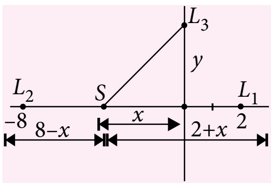
or $\cos\frac{\phi}{2} = \frac{\pm\sqrt{3}}{2} \Rightarrow \frac{\phi}{2} = \frac{\pi}{6}$ or $\phi = \frac{\pi}{3}$

Now, $\phi = \frac{(\mu-1)t\times 2\pi}{\lambda}$, $\frac{\pi}{3} = \frac{(\mu-1)t\times 2\pi}{\lambda}$

$\therefore\ t = \frac{\lambda}{6(\mu-1)} = \frac{6933}{3} = 2311$ Å

So, $x = 1$ Å

6. **(4)** **:** Let S be the position of the source and L_1, L_2 and L_3 are three listeners as shown in the figure. As the wavefront reaches listeners L_1 and L_2 simultaneously,



$\therefore\ \frac{8-x}{c} = \frac{2+x}{c}$

$\Rightarrow 6 = 2x$

$\therefore\ x = 3$ m

Sound reaches third listener at same time as L_1 and L_2.

$\therefore\ \frac{8-x}{c} = \frac{SL_3}{c}$ or $SL_3 = 8 - x = 8 - 3 = 5$ m

$\therefore\ 5 = \sqrt{3^2 + y^2} = 5 \Rightarrow y = 4$

7. **(a)** **:** Frequency of sound wave emitted by transmitter, $\upsilon_0 = 5$ MHz $= 5\times 10^6$ Hz

Frequency of sound wave observed by receiver,

$$\upsilon = \frac{c - v_R}{c + v_R}\upsilon_0,$$

where c is speed of sound and v_R is flow speed

Change in frequency, $\Delta\upsilon = \upsilon_0 - \upsilon$

$\therefore\ \frac{\Delta\upsilon}{\upsilon_0} = \frac{2v_R}{c+v_R} \approx \frac{2v_R}{c}$ (as $v_R <<< c$)

$\Rightarrow\ c = \frac{2\times 0.1}{600}\times 5\times 10^6 \approx 1700\ \text{m s}^{-1}$

8. **(d)** **:** Let area of cross-section of normal artery $= A$

$\therefore$ area of cross-section of narrowed artery $A' = \frac{A}{4}$

According to equation of continuity

$v_R A = v'_R\ A' \Rightarrow v'_R = 4v_R$

Since, $\frac{\Delta\upsilon}{\upsilon_0} = \frac{2\upsilon'_R}{c+\upsilon'_R}$

$\therefore$ For $c = 1700$ m s^{-1}, $v_R = 0.1$ m s^{-1}, $\upsilon_0 = 5\times 10^6$ Hz

we have $v'_R = 4v_R = (4\times 0.1)$ m s^{-1} and

$$\Delta\upsilon = \frac{5\times 10^6\times 2\times(0.1\times 4)}{1700} \approx 2400\ \text{Hz}$$

9. **(a):** Density of blood, $\rho = 1.5$ g cm^{-3} $= 1.5\times 10^3$ kg m^{-3}

Area of cross-section of normal artery, $A = 0.1$ cm^2 $= 10^{-5}$ m^2

Normal flow speed, $v_R = 0.1$ m s^{-1}

$\therefore$ Mass of blood flowing per second $= \rho A v$

$= (1.5\times 10^3\times 10^{-5}\times 0.1)$ kg $= 1.5\times 10^{-3}$ kg

Work done due to narrowing =

Change in kinetic energy $= \frac{1}{2}m\left(v_R'^2 - v_R^2\right)$

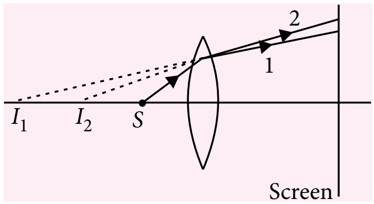
Required of rate of doing work

$= \frac{1}{2}\times 1.5\times 10^{-3}[(4\times 0.1)^2 - (0.1)^2] = 1.125\times 10^{-4}$ W

10. **(b)** **:** (A)– P,Q; (B)– P,Q,R,S; (C) – RS; (D) –P

(A) For convex lens,

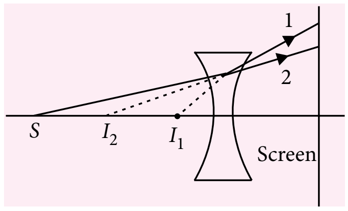
$|f|$ increases, converging power reduces : virtual image is formed closer to lens hence closer to screen



For concave lens $|f|$ increases, diverging power decreases

$\therefore$ I moves away

Hence (P), (Q)



(B) $|u| < |f|$ so virtual image in all cases *i.e.*, (P), (Q), (R), (S)

(C) Say $u = -u_0$

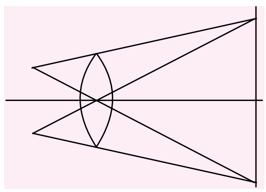
(i) converging lens, $m = \frac{f}{f+(-u_0)} = \frac{f}{f-u_0}$

If u decreases, m decreases thus distance between images decreases.

(ii) diverging lens, $m = \frac{-f_0}{-f_0+(-u_0)} = \frac{f_0}{f_0+u_0}$

If u_0 decreases, m increases thus distance between images increases.

(D)



In rest of cases no common region.

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