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JEE Main SOLVED PAPER 2017

MONTHLY PRACTICE PROBLEMS (XI & XII)

KEY CONCEPT

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PHYSICS for you

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Physics Musing was started in August 2013 issue of Physics For You. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / AIIMS / NEET / Other PMTs with additional study material.

In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / various PMTs. The detailed solutions of these problems will be published in next issue of Physics For You.

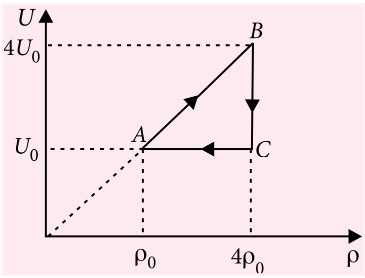
The readers who have solved five or more problems may send their detailed solutions with their names and complete address. The names of those who send atleast five correct solutions will be published in the next issue.

We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

PROBLEM Set 46

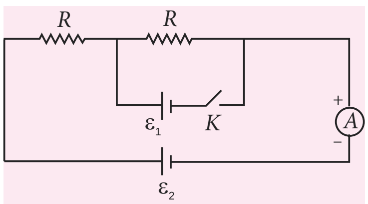
MORE THAN ONE OPTIONS CORRECT TYPE

1. A monatomic ideal gas is following the cyclic process $ABCA$. Then choose the correct option (s).



(a) Molar heat capacity for the process AB is $\frac{R}{2}$.
(b) Heat is rejected by the system in path BC.
(c) Molar heat capacity for the process BC is $\frac{2}{3}R$.
(d) Work done by the system in the process CA is $\frac{2U_0}{3}\ln 4$.

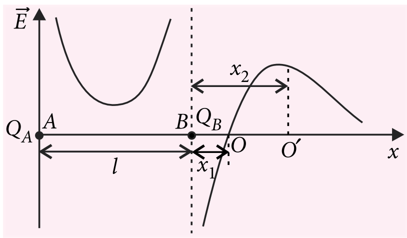
2. In the given circuit, when key K is open, reading of ammeter is I. Now key K is closed, then the correct statements are



(a) If $\varepsilon_1 = IR$, reading of the ammeter is I.
(b) If $IR < \varepsilon_1 < 2IR$, reading of the ammeter is greater than I.
(c) If $\varepsilon_1 = 2IR$, reading of the ammeter will be zero.
(d) Reading of the ammeter will not change.

3. The resistivity of a cylindrical conductor carrying steady current along its length varies linearly with the distance from the current carrying end as given by $\rho = \rho_0\left(1+\frac{x}{l}\right)$, where l is the length of the conductor and x is the distance from the current entry end and ρ_0 is a positive constant. Then,
(a) Electric field varies linearly with x
(b) Electric potential difference across the length varies linearly with x
(c) Volume charge density in the conductor is zero
(d) Volume charge density in the conductor is non zero.

4. Two point charges, Q_A and Q_B are positioned at points A and B. The electric field strength to the right of charge Q_B on the line that passes through the two charges varies according to a law that is represented schematically in the figure accompanying the problem without employing a definite scale. Assume electric field to be positive if its direction coincides with the positive direction on the x-axis. Distance between the charges is l. Then,



(a) Charge Q_A is negative and charge Q_B is positive.
(b) Charge Q_A is positive and charge Q_B is negative.
(c) $\left|\frac{Q_A}{Q_B}\right| = \left[\frac{l+x_1}{x_1}\right]^2$
(d) $x_2 = \frac{l}{(Q_A/Q_B)^{1/3}-1}$

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5. A conducting sphere of radius R, carrying charge Q lies inside an uncharged conducting shell of radius $2R$. If they are joined by a metal wire, then

(Here $k = \frac{1}{4\pi\varepsilon_0}$)

(a) $\frac{Q}{3}$ amount of charge will flow from the sphere to the shell.

(b) $\frac{2Q}{3}$ amount of charge will flow from the sphere to the shell.

(c) Q amount of charge will flow from the sphere to the shell.

(d) $k\frac{Q^2}{4R}$ amount of heat will be produced.

6. A metal cylinder of mass 0.5 kg is heated electrically by a 12 W heater in a room at 15 °C. The cylinder temperature rises to 25 °C in 5 min and finally becomes constant at 45 °C. Assuming that the rate of heat loss of the cylinder is proportional to the excess temperature over the surroundings. Then,

(a) The rate of loss of heat of the cylinder to surrounding at 20° C is 2 W.

(b) The rate of loss of heat of the cylinder to surrounding at 45° C is 12 W.

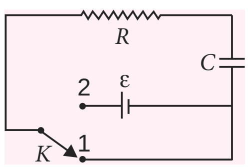
(c) Specific heat capacity of metal is

$$240\left[\ln\left(\frac{3}{2}\right)\right]^{-1} \text{J kg}^{-1}\,°\text{C}^{-1}.$$

(d) None of these.

PARAGRAPH BASED QUESTIONS

Paragraph I



The shown circuit involving a resistor of resistance R, capacitor of capacitance C and an ideal cell of emf ε, the capacitor is initially uncharged and the key is in position 1. At $t = 0$ s, the key is pushed to position 2 for $t_0 = RC$ and then key is pushed back to position 1 for $t = RC$. This process is repeated again and again. Assume the time taken to push key from position 1 to 2 and vice versa to be negligible.

7. The charge on capacitor at $t = 2RC$ is

(a) $C\varepsilon$

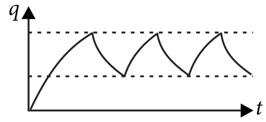
(b) $C\varepsilon\left(1-\frac{1}{e}\right)$

(c) $C\varepsilon\left(\frac{1}{e}-\frac{1}{e^2}\right)$

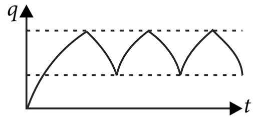
(d) $C\varepsilon\left(1-\frac{1}{e}+\frac{1}{e^2}\right)$

8. The variation of charge on capacitor with time is best represented by

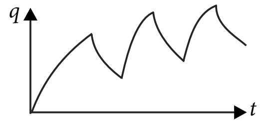
(a)



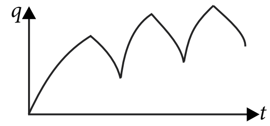
(b)



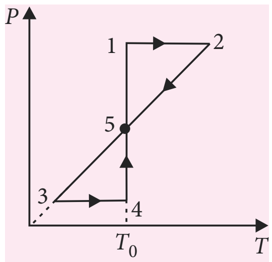
(c)



(d)



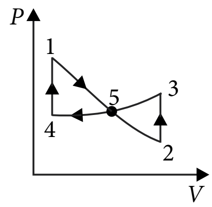
Paragraph II



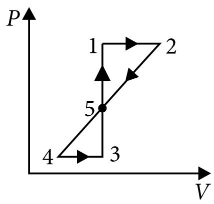
Consider P-T graph of cyclic process shown in the figure. Maximum pressure during the cycle is twice the minimum pressure. The heat received by the gas in the process 1-2 is equal to the heat received in the process 3-4. The process is done on one mole of monatomic gas.

9. Correct P-V diagram for the given process is

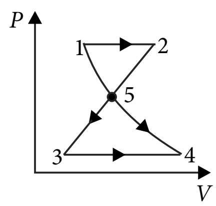
(a)



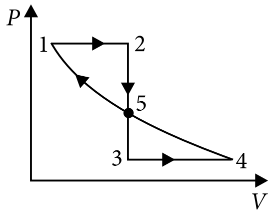
(b)



(c)



(d)



10. If the maximum pressure is P then what is the pressure at the point 5? (in P-T diagram)

(a) $\frac{2P}{3}$

(b) $\frac{4P}{5}$

(c) $\frac{3P}{4}$

(d) None of these.

◈◈

Exam on 21st May

JEE ADVANCED

PRACTICE PAPER 2017

PAPER-I

SECTION 1 (Maximum Marks : 15)

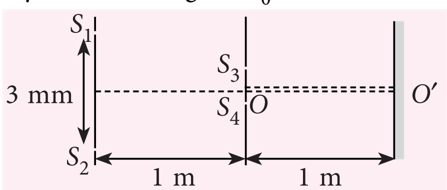
- *This section contains FIVE questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is correct.*
- *For each question, darken the bubble corresponding to the correct option in the ORS.*
- *For each question, marks will be awarded in one of the following categories :*

Full Marks : *+3 If only the bubble corresponding to the correct option is darkened.*

Zero Marks : *0 If none of the bubbles is darkened.*

Negative Marks : *–1 In all other cases.*

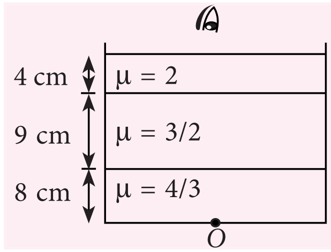
1. In the arrangement shown in figure, light of wavelength 6000 Å is incident on slits S_1 and S_2. Slits S_3 and S_4 have been opened such that S_3 is the position of first maximum above the central maximum and S_4 is the closest position, where intensity is same as that of the light used, below the central maximum. The point O is equidistant from S_1 and S_2 and O' is equidistant from S_3 and S_4. The intensity of incident light is I_0.



The intensity at O' (on the screen) and the intensity of the brightest fringe respectively be

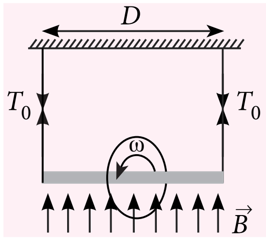
(a) $3I_0$ and $9I_0$ (b) $9I_0$ and $3I_0$
(c) $2I_0$ and $4I_0$ (d) $4I_0$ and $2I_0$

2. A tank contains three layers of immiscible liquids. The first layer is of water with refractive index 4/3 and thickness 8 cm. The second layer is an oil with refractive index 3/2 and thickness 9 cm while the third layer is of glycerine with refractive index 2 and thickness 4 cm. Find the apparent depth of the bottom of the container.



(a) 14 cm below the glycerine air interface
(b) 9 cm above the glycerine air interface
(c) 14 cm above the glycerine air interface
(d) 9 cm below the glycerine air interface

3. A ring of radius R having uniformly distributed charge Q is mounted on a rod suspended by two identical strings. The tension in strings in equilibrium is T_0. Now a vertical magnetic field is switched on and ring is rotated at constant angular velocity ω. The maximum ω with which the ring can be rotated if the strings can withstand a maximum tension of $3T_0/2$ is



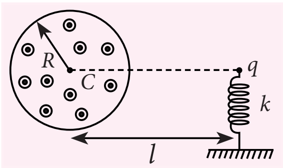
(a) $\dfrac{DT_0}{BQR}$ (b) $\dfrac{DT_0}{B^2QR}$ (c) $\dfrac{2DT_0}{QRB}$ (d) $\dfrac{DT_0}{QBR^2}$

4. From the surface of a wire of radius a carrying a direct current I, a positive charge q having mass m escapes with a velocity v_0 perpendicular to the surface. The maximum distance (x_{max}) of the electron from the axis of the wire before it turns back due to the action of the magnetic field generated by the current will be

(a) $x_{max} = ae^{\frac{2\pi m v_0}{q\mu_0 I}}$ (b) $x_{max} = ae^{-\frac{\pi m v_0}{q\mu_0 I}}$

(c) $x_{max} = ae^{\frac{m v_0}{qI}}$ (d) $x_{max} = ae^{\frac{\pi m v_0}{2q\mu_0 I}}$

5. There is a horizontal cylindrical uniform but time-varying magnetic field increasing at a constant rate dB/dt as shown in figure.



A charged particle having charge q and mass m is kept in equilibrium, at the top of a spring of spring constant k, in such a way that it is on the horizontal line passing through the center of the magnetic field as shown in the figure. The compression in the spring will be

(a) $\frac{1}{k}\left[mg - \frac{qR^2}{2l}\frac{dB}{dt}\right]$ (b) $\frac{1}{k}\left[mg + \frac{qR^2}{l}\frac{dB}{dt}\right]$

(c) $\frac{1}{k}\left[mg + \frac{2qR^2}{l}\frac{dB}{dt}\right]$ (d) $\frac{1}{k}\left[mg + \frac{qR^2}{2l}\frac{dB}{dt}\right]$

SECTION 2 (Maximum Marks : 32)

- *This section contains EIGHT questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is(are) correct.*
- *For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.*
- *For each question, marks will be awarded in one of the following categories :*

  Full Marks : +4 *If only the bubble(s) corresponding to the correct option(s) is(are) darkened.*

  Partial Marks : +1 *For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened.*

  Zero Marks : 0 *If none of the bubbles is darkened.*

  Negative Marks : –2 *In all other cases.*
- *For example, if (a), (c) and (d) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (a) and (d) will result in +2 marks; and darkening (a) and (b) will result in –2 marks, as a wrong option is also darkened.*

6. A 470 kg communication satellite is released from a space shuttle at a height of 280 km above the surface of the earth. From this height a rocket engine boosts it into a geosynchronous orbit. The correct statement(s) is/are

(Given that mass of the earth = 5.98×10^{24} kg and radius of the earth = 6400 km)

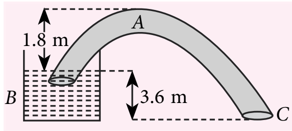
(a) The orbital radius of geosynchronous orbit of satellite is 4.23×10^7 m

(b) Energy supplied by engine to change the orbit is 1.18×10^{10} J

(c) Change in potential energy of the satellite is -2.36×10^{10} J

(d) Firing of engine results in decrease in potential energy and increase in kinetic energy of the satellite.

7. A siphon has a uniform circular base of diameter $8/\sqrt{\pi}$ cm with its crest A, 1.8 m above the water level. Vessel B is of large cross section as shown in figure (g = 10 m s^{-2} and atmospheric pressure $P_0 = 10^5$ N m^{-2}).



If water starts flowing from vessel to ground through siphon, then

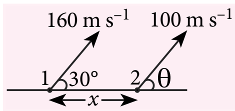
(a) velocity of flow of water through pipe is $6\sqrt{2}$ m s^{-1}.

(b) discharge rate of flow through pipe is $96\sqrt{2} \times 10^{-4}$ m^3 s^{-1}.

(c) velocity of flow of water through pipe is 6 m s^{-1}.

(d) pressure of A is 0.46×10^5 N m^{-2}.

8. Suppose two particles 1 and 2 are projected in vertical plane simultaneously. Their angles of projection are 30° and θ respectively with the horizontal as shown.



Suppose they collide after a time t in air. Then,

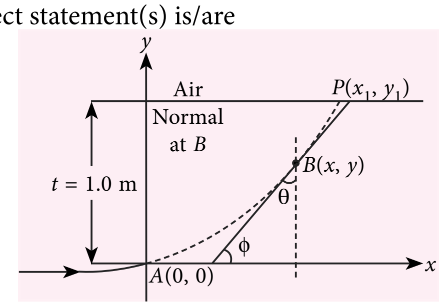
(a) $\theta = \sin^{-1}(4/5)$ and they will have same speed just before the collision

(b) $\theta = \sin^{-1}(4/5)$ and they will have different speed just before the collision

(c) $x < (1280\sqrt{3} - 960)$ m

(d) it is possible that the particles collide when both of them are at their highest point

9. A ray of light travelling in air is incident at grazing angle (angle of incidence $\simeq$ 90°) on a long rectangular slab of a transparent medium of thickness t = 1.0 m. The point of incidence is the origin $A(0, 0)$. The medium has a variable index of refraction given by $n(y) = (ky^{3/2} + 1)^{1/2}$, where k = 1.0 m$^{-3/2}$. The refractive index of air is 1. The correct statement(s) is/are



(a) Relation between the slope of the trajectory of the ray at a point $B(x, y)$ in the medium and the incidence angle at that point is $\cot\theta = \frac{dy}{dx}$.

(b) Equation for the trajectory $y(x)$ of the ray in the medium is $4y^{1/2} = \sqrt{k}\,x + C$.

(c) The coordinates (x_1, y_1) of the point P where the ray intersects the upper surface of the slab-air boundary is (4, 1).

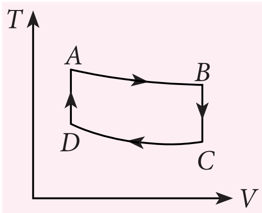
(d) The path of the ray will be perpendicular to the boundary.

10. One mole of a monatomic ideal gas is taken through the cycle shown in figure.

$A \rightarrow B$ adiabatic expansion

$B \rightarrow C$ cooling at constant volume

$C \rightarrow D$ adiabatic compression



$D \rightarrow A$ heating at constant volume

The pressures and temperatures at A, B, etc., are denoted by P_A, T_A, P_B, T_B, etc., respectively. Given $T_A = 1000$ K, $P_B = (2/3)P_A$ and $P_C = (1/3)P_A$.

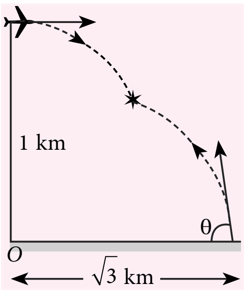
(a) The work done by the gas in the process $A \rightarrow B$ is 1869.75 J.

(b) The heat lost by the gas in the process $B \rightarrow C$ is – 5297.625 J.

(c) Temperature T_D is 500 K.

(d) Work done from $B \rightarrow C$ is 40 J.

11. An aircraft is flying horizontally with a constant velocity 200 m s^{-1}, at a height 1 km above the ground. At the moment shown, a bomb is released from the aircraft and the cannon-gun below fires a shell with initial speed 200 m s^{-1}, at some angle θ.



Both the bomb and the shell collide with each other in air

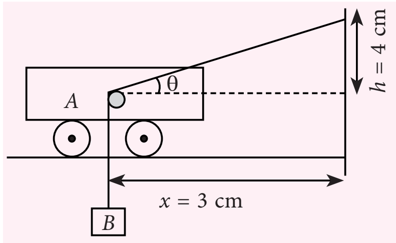
(a) The value of θ at which the projectile shell destroy the bomb in mid-air is 60°.

(b) Position of the collision w.r.t. O is $\left(\frac{2}{3}\sqrt{3}, -\frac{\sqrt{3}}{6}\right)$

(c) They will collide after 10 s

(d) The value of θ at which the projectile shell destroy the bomb in mid-air is 45°

12. The string shown in figure is passing over small smooth pulley rigidly attached to trolley A. If the speed of trolley is constant and equal to v_A towards right, speed and magnitude of acceleration of block B at the instant shown in figure are v_B and a_B, then



(a) $v_B = v_A$, $a_B = 0$ (b) $a_B = 0$

(c) $v_B = \frac{3}{5}v_A$ (d) $a_B = \frac{16v_A^2}{125}$

13. A thermostated chamber at a height h above earth's surface maintained at 30°C has a clock fitted with uncompensated pendulum. The maker of the clock for chamber mistakenly designed it to maintain correct time at 20°C. It is found that when the chamber is brought to earth's surface the clock in it clicked correct time. R_e is the radius of Earth. The coefficient of linear expansion of the material of pendulum is

(a) $\frac{h}{R_e}$ (b) $\frac{h}{5R_e}$ (c) $\frac{5R_e}{h}$ (d) $\frac{R_e}{h}$

SECTION 3 (Maximum Marks : 15)

- *This section contains FIVE questions.*
- *The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.*
- *For each question, darken the bubble corresponding to the correct integer in the ORS.*
- *For each question, marks will be awarded in one of the following categories :*

Full Marks : *+3 If only the bubble corresponding to the correct answer is darkened.*

Zero Marks : *0 In all other cases.*

14. In a certain hypothetical radioactive decay process, species A decays into species B and species B decays into species C according to the reactions :

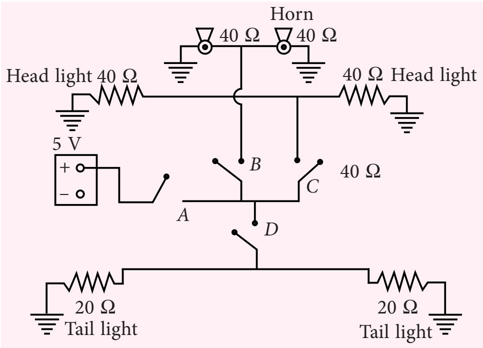
$A \rightarrow 2B$ + particles + energy

$B \rightarrow 3C$ + particles + energy

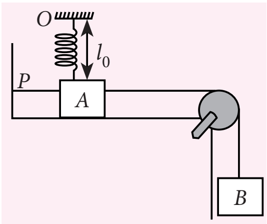
The decay constant for species A is λ_1 and that for species B is λ_2. Initial there was 10^x moles species A while there was none of B and C. It was found that species B reaches its maximum number at a time t_0. Find the value of x.

(Take λ_1 = 1 dps, λ_2 = 100 dps, N_B = 2 moles at $t_0 = 2 \ln 10$ s)

15. Figure shows an automobile circuit. How much power (in watt) is dissipated by the automobile circuit when switches A, B, C and D are all closed?



16. A small bar A resting on a smooth horizontal plane is attached by threads to a point P and by means of weightless pulley, to a weight B possessing the same mass as the bar itself.



The bar is also attached to a point O by means of a light non-deformed spring of length l_0 and stiffness $k = 5\ mg/l_0$, where m is the mass of the bar. Now the thread PA is burnt and the bar starts moving to the right. Its velocity at the moment when bar is breaking off the plane is given as $\sqrt{\dfrac{(17+n)gl_0}{16n}}$. The value of n is

17. End A of a rod AB of length L = 0.5 m and of uniform cross-sectional area is maintained at same constant temperature. The heat conductivity of the rod is $k = 17\ \text{J s}^{-1}\text{m}^{-1}\text{K}^{-1}$. The other end B of this rod is radiating energy into vacuum and the wavelength with maximum energy density emitted from this end is λ_0 = 75000 Å. If the emissivity of the end B is $e = 1$, the temperature of the end A is given as $2.11a \times 10^a$ K. (Assuming that except the ends, the rod is thermally insulated) Find the value of a.

18. When the soap bubble of radius R = 0.25 cm is charged, it experiences an outward electric pressure of magnitude $\dfrac{\sigma^2}{2\varepsilon_0}$ where its surface charge density $\sigma = 20\ \mu\text{C m}^{-2}$, If Q is the charge on the sphere so that the pressure inside and outside is same, then the surface tension of soap in terms of $\alpha \times 10^{-12}/8\varepsilon_0$ N m^{-1}. Find the value of α.

PAPER-II

SECTION 1 (Maximum Marks : 18)

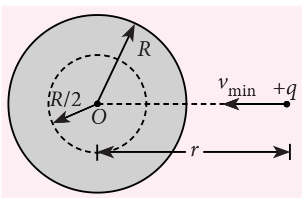
- *This section contains SIX questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is correct.*
- *For each question, darken the bubble corresponding to the correct option in the ORS.*
- *For each question, marks will be awarded in one of the following categories :*

Full Marks : *+3 If only the bubble corresponding to the correct option is darkened.*

Zero Marks : *0 If none of the bubbles is darkened.*

Negative Marks : *–1 In all other cases.*

1. A positive charge Q is uniformly distributed throughout the volume of a dielectric sphere of radius R. A point mass having charge $+q$ and mass m is fired toward the centre of the sphere with velocity v from a point which is at distance $r(r > R)$ from the centre of the sphere. The minimum velocity v so that it can penetrate $R/2$ distance of the sphere is



(Neglect any resistance other than electric interaction. Charge on the small mass remains constant throughout the motion.)

(a) $v = \sqrt{\dfrac{qQ}{2m\pi\varepsilon_0 R}\left[\dfrac{5}{8}-\dfrac{R}{r}\right]}$

(b) $v = \sqrt{\dfrac{qQ}{2m\pi\varepsilon_0 R}\left[1-\dfrac{R}{r}\right]}$

(c) $v = \sqrt{\dfrac{qQ}{2m\pi\varepsilon_0 R}\left[\dfrac{11}{8}-\dfrac{R}{r}\right]}$

(d) $v = \sqrt{\dfrac{qQ}{m\pi\varepsilon_0 R}\left[\dfrac{11}{8}-\dfrac{R}{r}\right]}$

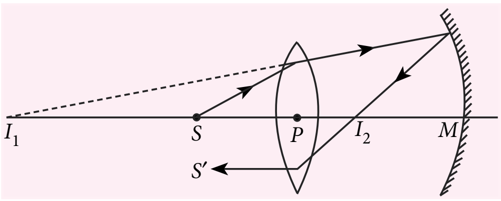
2. When a block of iron floats in mercury at 0°C, fraction k_1 of its volume is submerged, while at the temperature 60°C, a fraction k_2 is seen to be submerged. If the co-efficient of volume expansion of iron is γ_{Fe} and that of mercury is γ_{Hg}, then the ratio k_1/k_2 can be expressed as

(a) $\dfrac{1+60\gamma_{Fe}}{1+60\gamma_{Hg}}$ (b) $\dfrac{1-60\gamma_{Fe}}{1+60\gamma_{Hg}}$

(c) $\dfrac{1+60\gamma_{Fe}}{1-60\gamma_{Hg}}$ (d) $\dfrac{1+60\gamma_{Hg}}{1+60\gamma_{Fe}}$

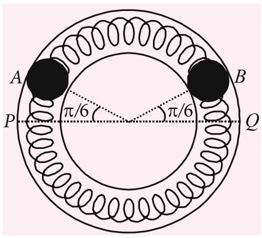
3. A potential barrier of 0.50 V exists across a p-n junction. If the depletion region is 5.0×10^{-7} m wide, an electron with speed 5.0×10^{5} m s^{-1} approaches the p-n junction from the n-side. The speed of electron enter the p-side and electric field intensity in the region is
(a) 2.7×10^{4} m s^{-1} and 2.0×10^{8} V m^{-1} respectively
(b) 5.0×10^{3} m s^{-1} and 1.0×10^{7} V m^{-1} respectively
(c) 2.7×10^{5} m s^{-1} and 1.0×10^{6} V m^{-1} respectively
(d) 5.0×10^{6} m s^{-1} and 2.0×10^{5} V m^{-1} respectively.

4. A converging lens of focal length 15 cm and a converging mirror of focal length 20 cm are placed with their principal axes coinciding. A point source S is placed on the principal axis at a distance of 12 cm from the lens as shown in figure. It is found that the final beam comes out parallel to the principle axis. The separation between the mirror and the lens is



(a) 50 cm (b) 30 cm (c) 25 cm (d) 40 cm

5. Two identical balls A and B each of mass 0.1 kg, are attached to two identical massless springs. The spring-mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in figure. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle of radius 0.06π m. Each spring has a natural length of 0.06 m and spring constant 0.1 N m^{-1}. Initially, both the balls are displaced by an angle $\theta = \pi/6$ radian with respect to the diameter PQ of the circle (as shown in figure) and released from rest. The frequency of oscillation of ball B and the total energy of the system respectively are



(a) $1/\pi$ s^{-1} and 4.76×10^{-4} J
(b) $1/\pi$ s^{-1} and 3.95×10^{-4} J
(c) $2/\pi$ s^{-1} and 2.72×10^{-4} J
(d) $2/\pi$ s^{-1} and 6.0×10^{-4} J

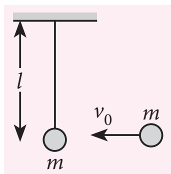
6. Two guns, situated on the top of a hill of height 10 m, fire one shot each with the same speed $5\sqrt{3}$ m s^{-1} at some interval of time. One gun fires horizontally and other fires upwards at an angle of 60° with the horizontal. The shots collide in air at a point P. Find the time - interval between the firings and the coordinates of the point P. (Take origin of the coordinate system at the foot of the hill right below the muzzle and trajectories in x-y plane).
(a) 1 s and (5, 5) m
(b) 1 s and $(5\sqrt{3}, 3)$ m
(c) 2 s and $(2\sqrt{3}, 2)$ m
(d) 2 s and $(2\sqrt{3}, 1)$ m

SECTION 2 (Maximum Marks : 32)

- *This section contains EIGHT questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is(are) correct.*
- *For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.*
- *For each question, marks will be awarded in one of the following categories :*
  Full Marks : *+4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.*
  Partial Marks : *+1 For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened.*
  Zero Marks : *0 If none of the bubbles is darkened.*
  Negative Marks : *–2 In all other cases.*
- *For example, if (a), (c) and (d) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (a) and (d) will result in +2 marks; and darkening (a) and (b) will result in –2 marks, as a wrong option is also darkened.*

7. n drops of a liquid each with surface energy E join to form a single drop. Then
(a) Some energy will be released in the process.
(b) Some energy will be absorbed in the process.
(c) The energy released will be $E(n - n^{2/3})$.
(d) The energy absorbed will be $nE(2^{2/3} - 1)$.

8. A simple pendulum consists of a bob of mass m and a light string of length l as shown in the figure. Another identical ball moving with the small velocity v_0 collides with the pendulum's bob and sticks to it. If the new pendulum has mass $2m$. Then



(a) Time period of the new pendulum is $2\pi\sqrt{\dfrac{l}{g}}$.
(b) The equation of motion for the new pendulum is $\theta_0 = \dfrac{v_0}{2\sqrt{gl}}$.
(c) The equation of motion for the new pendulum is $\theta = \dfrac{v_0}{2\sqrt{gl}}\cos\left[\sqrt{\dfrac{g}{l}}\,t\right]$.

(d) Time period of the new pendulum is $2\pi\sqrt{\dfrac{2l}{g}}$.

9. A light source, which emits two wavelengths λ_1 = 400 nm and λ_2 = 600 nm, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2, respectively, then

(a) $\beta_2 > \beta_1$
(b) $m_1 > m_2$
(c) From the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1
(d) The angular separation of fringes for λ_1 is greater than λ_2.

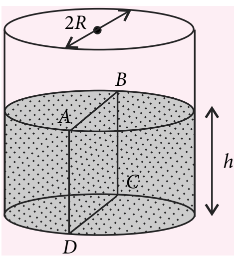
10. A sound wave of frequency υ travels horizontally to the right. It is reflected from a large vertical plane surface moving to left with a speed v. The speed of sound in medium is c. Then

(a) The number of wave striking the surface per second is $\upsilon\dfrac{(c+v)}{c}$
(b) The wavelength of reflected wave is $\dfrac{c(c-v)}{\upsilon(c+v)}$
(c) The frequency of the reflected wave is $\upsilon\dfrac{(c+v)}{(c-v)}$
(d) The number of beats heard by a stationary listener to the left of the reflecting surface is $\dfrac{v\upsilon}{c-v}$

11. A series R-C circuit is connected to an ac voltage source. Consider two cases : (A) when C is without a dielectric medium and (B) when C is filled with a dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?

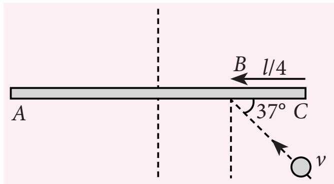
(a) $I_R^A > I_R^B$
(b) $I_R^A < I_R^B$
(c) $V_C^A > V_C^B$
(d) $V_C^A < V_C^B$

12. Water is filled up to a height h in a beaker of radius R as shown in the figure. The density of water is ρ, the surface tension of water is T and the atmospheric pressure is P_0. Consider a vertical section $ABCD$ of the water column through a diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude



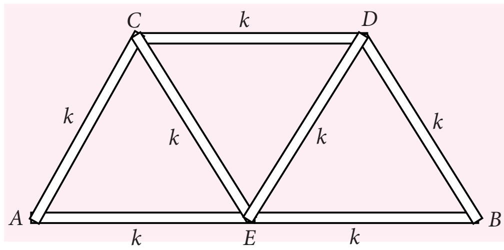
(a) $|2P_0Rh + \pi R^2\rho gh - 2RT|$
(b) $|2P_0Rh + R\rho gh^2 - 2RT|$
(c) $|P_0\pi R^2 + R\rho gh^2 - 2RT|$
(d) $|P_0\pi R^2 + R\rho gh^2 + 2RT|$.

13. A rod AC of length l and mass m is kept on a horizontal smooth plane. It is free to rotate and move. A particle of same mass m moving on the plane with velocity v strikes the rod at point B making angle 37° with the rod. The collision is elastic. After collision,



(a) The angular velocity of the rod will be $72/55\ v/l$
(b) The centre of the rod will travel a distance $\pi l/3$ in the time in which it makes half rotation
(c) Impulse of the impact force is $24mv/55$
(d) None of these.

14. Seven identical rods of material of thermal conductivity k are connected as shown in the figure. All the rods are of identical length l and cross-sectional area A. If the one end A is kept at 100°C and the other end B is kept at 0°C, what would be the temperatures of the junctions C, D and E (θ_C, θ_D and θ_E) in the steady state?



(a) $\theta_C > \theta_E > \theta_D$
(b) θ_C = 25°C, θ_D = 37.5°C and θ_E = 50°C
(c) θ_C = 62.5°C, θ_D = 37.5°C and θ_E = 50°C
(d) θ_C = 60°C, θ_D = 40°C and θ_E = 50°C

SECTION 3 (Maximum Marks : 12)

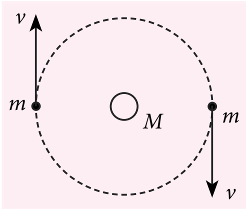
- *This section contains TWO paragraphs.*
- *Based on each paragraph, there are TWO questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is correct.*
- *For each question, darken the bubble corresponding to the correct option in the ORS.*
- *For each question, marks will be awarded in one of the following categories :*

Full Marks : +3 *If only the bubble corresponding to the correct option is darkened.*

Zero Marks : 0 *In all other cases.*

PARAGRAPH 1

A triple star system consists of two stars, each of mass m, in the same circular orbit about central star with mass $M = 2 \times 10^{30}$ kg. The two outer stars always lie at opposite ends of a diameter of their common circular orbit.



The radius of the circular orbit is $r = 10^{11}$ m and the orbital period of each star is 1.6×10^7 s.

[Take $\pi^2 = 10$ and $G = \frac{20}{3} \times 10^{-11}$ Nm^{2}kg^{-2}]

15. The mass m of the outer star is

(a) $\frac{16}{15} \times 10^{30}$ kg (b) $\frac{11}{8} \times 10^{30}$ kg

(c) $\frac{15}{16} \times 10^{30}$ kg (d) $\frac{8}{11} \times 10^{30}$ kg

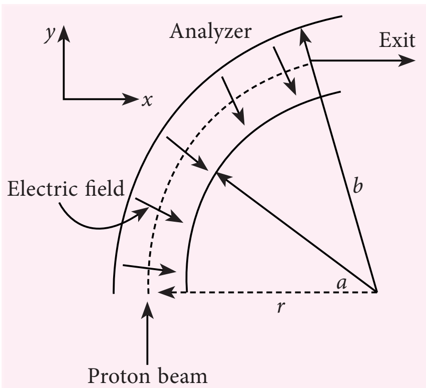
16. The total mechanical energy of the system is

(a) $-\frac{1375}{64} \times 10^{35}$ J (b) $-\frac{1375}{64} \times 10^{38}$ J

(c) $-\frac{1375}{64} \times 10^{34}$ J (d) $-\frac{1375}{64} \times 10^{37}$ J

PARAGRAPH 2

Figure shows an electrostatic analyzer. It can sort out charged particles by speed and charge to mass ratio. Space craft use such analyzers to characterize charged particles in interplanetary space. Two curved metal plates establish an electric field given by $E = E_0\left(\frac{b}{r}\right)$ where E_0 is positive constant with unit of electric field. The field points towards the centre of curvature and r is distance from centre. There is no influence of gravity. A beam of Proton (charge $+q$ and mass m) enters along y-axis and exits along x-axis while moving along a circular path.



17. Speed with which proton is to be projected is v and centripetal acceleration of electron is a_c. The correct statement is

(a) $v = \sqrt{\frac{qE_0b}{m}}$ and $a_c = \frac{2q}{m}E_0\left(\frac{b}{r}\right)$

(b) $v = \sqrt{\frac{2qE_0b}{m}}$ and $a_c = \frac{q}{2m}E_0\left(\frac{b}{r}\right)$

(c) $v = \sqrt{\frac{2E_0b}{2m}}$ and $a_c = \frac{2q}{m}E_0\left(\frac{b}{r}\right)$

(d) $v = \sqrt{\frac{qE_0b}{m}}$ and $a_c = \frac{q}{m}E_0\left(\frac{b}{r}\right)$

18. The incorrect option :

(a) Work done by electric field on proton is zero

(b) If $v = \sqrt{\frac{2qE_0b}{m}}$ proton may strike outer surface of analyzer.

(c) If $v = \sqrt{\frac{2qE_0b}{m}}$ proton may strike inner surface of analyzer.

(d) If proton is released with zero initial velocity from inner surface of analyzer it will strike outer surface with velocity

$$v = \sqrt{\frac{2qE_0b}{m}\ln\left(\frac{b}{a}\right)}.$$

Solutions

Paper-I

1. **(a) :** From the given condition,

$$OS_3 = \frac{D\lambda}{d} = \frac{1 \times 6 \times 10^{-7}}{3 \times 10^{-3}} = 2 \times 10^{-4}\text{ m}$$

Let light reaching from S_1 and S_2 to S_4 has phase difference ϕ and intensity of incident light is I_0.

Resultant intensity at S_4, $I = 4I_0\cos^2\frac{\phi}{2}$

As $I = I_0$, hence, $\frac{I_0}{4I_0} = \cos^2\frac{\phi}{2}$ or $\cos\frac{\phi}{2} = \frac{1}{2} = \cos 60°$

or $\phi = \frac{2\pi}{3}$

For $\phi = \frac{2\pi}{3}$, $OS_4 = \frac{D\lambda}{3d}$

Therefore, $S_3S_4 = OS_3 + OS_4 = \frac{4}{3}\frac{D\lambda}{d} = \frac{8}{3} \times 10^{-4}$ m

Now resultant wave coming out of S_3 has intensity $4I_0$ and waves coming out of S_4 have intensity I_0.

Phase difference at $S_3 = 2\pi$, phase difference at $S_4 = 2\pi/3$.
These phase differences are relative to the light incident on slits S_1 and S_2.
Now S_3 and S_4 are secondary sources of light.
Phase difference at $O' = \frac{4\pi}{3}$, equal to initial phase difference between the light reaching at O'

$$= 2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}.$$

Let intensity at O' be I'.

$$I' = I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0}\cos\frac{4\pi}{3} = 3I_0$$

For brightest fringe, $\phi = 2n\pi$, $n = 0, \pm1, \pm2,....$
Let I'' be the intensity of brightest fringe.

$$I'' = I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0}\cos\phi = 9I_0 \text{ (where } \cos\phi = 1)$$

2. (a): Required apparent depth from top,

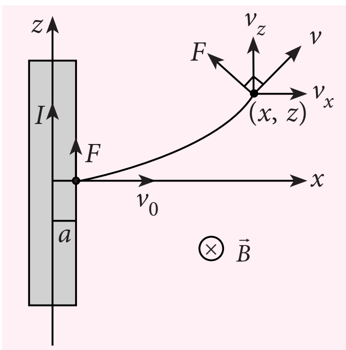
$$d' = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3} = \frac{8}{4/3} + \frac{9}{3/2} + \frac{4}{2} = 14 \text{ cm}$$

3. (d)

4. (a): Let at any instant velocity of charge q is v, then force on the charge due to magnetic field caused by wire is given by, $\vec{F} = q(\vec{v} \times \vec{B})$

$$ma_x\hat{i} + ma_z\hat{k} = q(v_x\hat{i} + v_z\hat{k}) \times \frac{\mu_0 I}{2\pi x}\hat{j}$$

$$\Rightarrow \quad ma_x\hat{i} + ma_z\hat{k} = qv_x\frac{\mu_0 I}{2\pi x}\hat{k} - qv_z\frac{\mu_0 I}{2\pi x}\hat{i}$$



$$\Rightarrow \quad ma_x = -\frac{q\mu_0 I}{2\pi}\frac{v_z}{x} \quad \Rightarrow \quad a_x = -\frac{q\mu_0 I}{2\pi m}\frac{v_z}{x}$$

[But $v_0^2 = v_x^2 + v_z^2$ or $v_z = \pm\sqrt{v_0^2 - v_x^2}$

Here we have to take $v_z = +\sqrt{v_0^2 - v_x^2}$ because a_x is negative]

$$\Rightarrow \quad v_x\frac{dv_x}{dx} = -\frac{q\mu_0 I}{2\pi m}\frac{\sqrt{v_0^2 - v_x^2}}{x}$$

$$\Rightarrow \quad \int_{v_0}^{0}\frac{v_x dv_x}{\sqrt{v_0^2 - v_x^2}} = -\frac{q\mu_0 I}{2\pi m}\int_{a}^{x_{max}}\frac{dx}{x}$$

$$\Rightarrow \quad \left[-\sqrt{v_0^2 - v_x^2}\right]_{v_0}^{0} = -\frac{q\mu_0 I}{2\pi m}\ln\left(\frac{x_{max}}{a}\right)$$

$$\Rightarrow \quad v_0 = \frac{q\mu_0 I}{2\pi m}\ln\left(\frac{x_{max}}{a}\right) \quad \Rightarrow \quad x_{max} = ae^{\frac{2\pi m v_0}{q\mu_0 I}}$$

5. (d): $E2\pi l = \pi R^2\left(\frac{dB}{dt}\right)$; $E = \frac{R^2}{2l}\left(\frac{dB}{dt}\right)$

$qE + mg = kx$

$$\Rightarrow \quad x = \frac{qR^2}{k2l}\left(\frac{dB}{dt}\right) + \frac{mg}{k}; x = \frac{1}{k}\left[mg + \frac{qR^2}{2l}\frac{dB}{dt}\right]$$

6. (a, b): The orbit period of a geosynchronous satellite is one day, *i.e.*, the satellite travels once around the earth in the same time that the earth spins once on its axis. For a satellite around earth,

$$\frac{GM_E m_S}{r^2} = \frac{m_S v^2}{r}$$

As orbital speed of planet is $2\pi r/T$, where T is time period of revolution,

$$\therefore \quad \frac{GM_E}{r^2} = \frac{(2\pi r/T)^2}{r} \quad \text{or} \quad T^2 = \left(\frac{4\pi^2}{GM_E}\right)r^3 = K_E r^3$$

Thus constant, $K_E = \frac{4\pi^2}{GM_E} = 9.89 \times 10^{-14}\ s^2\ m^{-3}$

$$r = \sqrt[3]{\frac{T^2}{K_E}} = \sqrt[3]{\frac{(86400)^2}{(9.89\times10^{-14})}} = 4.23 \times 10^7 \text{ m}$$

which is radius of the final orbit from the centre of the earth.
Radius of initial orbit, $R_i = R_E + 280 \times 10^3$ m
$= 6.68 \times 10^6$ m
Total energies in initial and final orbits are

$$E_i = -\frac{GM_E m_S}{2R_i}; \quad E_f = -\frac{GM_E m_S}{2R_f}$$

Thus the energy supplied by the booster to change the orbit is

$$\Delta E = E_f - E_i = -\frac{GM_E m_S}{2}\left[\frac{1}{R_f} - \frac{1}{R_i}\right] = 1.18 \times 10^{10} \text{ J}$$

The change in kinetic energy of satellite is

$$\Delta K = \frac{GM_E m_S}{2}\left[\frac{1}{R_f} - \frac{1}{R_i}\right] = -1.18 \times 10^{10} \text{ J}$$

The change in potential energy of the satellite

$$\Delta U = -GM_E m_S\left(\frac{1}{R_f}-\frac{1}{R_i}\right) = 2.36\times 10^{10}\text{ J}$$

Thus the firing of engine results in increase of potential energy and decrease of kinetic energy, but total mechanical energy is increased.

7. (a, b, d)

8. (b, c, d) : If they collide, their vertical components of velocities should be same *i.e.,*

$100\sin\theta = 160\sin 30° \Rightarrow \sin\theta = 4/5$

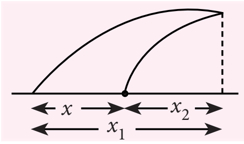
Their vertical components will always be same.

Horizontal components :

$160\cos 30° = 80\sqrt{3}\text{ m s}^{-1}$

and $100\cos\theta = 100\times 3/5$

$= 60\text{ m s}^{-1}$



They are not same, hence their velocities will not be same at any time. So (b) is correct.

$x = x_1 - x_2 = 160\cos 30° t - 100\cos\theta\, t$

$\Rightarrow x = (80\sqrt{3} - 60)t$

Time of flight, $T = \dfrac{2\times 160\times\sin 30°}{g} = 16\text{ s}$

Now $t < T$ as they to collide in air

$\Rightarrow \dfrac{x}{80\sqrt{3}-60} < 16 \Rightarrow x < (1280\sqrt{3} - 960)\text{ m}$

Since their times of flight are same, they will simultaneously reach their maximum height. So, it is possible to collide at highest point for certain values of x.

9. (a, c) : The slope of tangent at point B is, $\tan\phi = \dfrac{dy}{dx}$

The angle of incidence at B is $\theta = 90° - \phi$.

Hence, $\tan(90° - \theta) = \dfrac{dy}{dx} \Rightarrow \cot\theta = \dfrac{dy}{dx}$ ...(i)

From Snell's law at A and B, we have

$$1\sin 90° = n(y)\sin\theta, \sin\theta = \frac{1}{n(y)} = \frac{1}{(ky^{3/2}+1)^{1/2}}$$

$$\therefore \cot\theta = \frac{\sqrt{1-(1/ky^{3/2}+1)}}{1/(ky^{3/2}+1)^{1/2}} = k^{1/2}y^{3/4} \quad \text{...(ii)}$$

From eqns. (i) and (ii), $\dfrac{dy}{dx} = \sqrt{k}y^{3/4}$

$\Rightarrow 4y^{1/4} = \sqrt{k}x + C$ ...(iii)

Now we substitute boundary conditions in eqn. (iii) $x = 0$, $y = 0$, hence $C = 0$. The required trajectory is

$$y = k^2\left(\frac{x}{4}\right)^4$$

At point P, $y = 1$ m, $k = 1.0\text{ m}^{-3/2}$, we get $x = 4$ m

$\therefore$ The coordinates of P are (4, 1).

From Snell's law, $n_A\sin i_A = n_P\sin i_P$

As $n_A = n_P = 1$ so $i_P = i_A = 90°$

The ray will emerge parallel to boundary.

10. (a, b, c) : For $A \to B$, $T_A^\gamma P_A^{1-\gamma} = T_B^\gamma P_B^{1-\gamma}$

where $\gamma = 5/3$ for a monatomic gas.

$$T_B = T_A\left(\frac{P_B}{P_A}\right)^{1-\frac{1}{\gamma}} = 1000\left(\frac{2}{3}\right)^{2/5} = 850\text{ K}$$

Work done in an adiabatic process is given by

$$W = \frac{nR(T_A - T_B)}{\gamma - 1} = \frac{1\times 8.31(1000-850)}{(5/3)-1} = 1869.75\text{ J}$$

$\because$ Process $B \to C$ is isochoric.

Hence $W = 0$ and $\dfrac{P_C}{T_C} = \dfrac{P_B}{T_B}$

$$\therefore T_C = \frac{P_C}{P_B}\times T_B = \frac{(1/3)P_A}{(2/3)P_A}T_B = \frac{T_B}{2} = \frac{850}{2} = 425\text{ K}$$

From first law of thermodynamics,

$$Q = \Delta U + W = nC_V\Delta T + 0 = n\frac{3}{2}R(T_C - 850)$$

Hence, $Q = 1\times\dfrac{3}{2}\times 8.31(425 - 850) = -5297.625\text{ J}$

Negative sign implies that the system has lost heat.

Process $D \to A$ is isochoric

$$\frac{P_D}{P_A} = \frac{T_D}{T_A} \quad\text{or}\quad P_D = P_A\frac{T_D}{T_A}$$

Process $C \to D$ is adiabatic.

Therefore, $\left(\dfrac{T_D}{T_C}\right)^\gamma = \left(\dfrac{P_D}{P_C}\right)^{\gamma-1} = \left(\dfrac{P_AT_D}{P_CT_A}\right)^{\gamma-1}$

$$\therefore T_D^{1/\gamma} = T_C\left[\frac{P_A}{P_CT_A}\right]^{1-1/\gamma}$$

$$\Rightarrow T_D^{3/5} = T_C\left[\frac{P_A}{(1/3)P_A\times 1000}\right]^{2/5} = 425\left(\frac{3}{1000}\right)^{2/5}$$

$T_D = 500$ K

11. (a) : Suppose the shell destroys the bomb at time t. Then for horizontal motion,

$t(200 + 200\cos\theta) = \sqrt{3}\times 1000$

$\therefore t(1 + \cos\theta) = 5\sqrt{3}$ ...(i)

For vertical motion,

$$\frac{1}{2}gt^2 + (200\sin\theta)t - \frac{1}{2}gt^2 = 1000$$

$\therefore \sin\theta t = 5$ ...(ii)

From eqns. (i) and (ii), $\dfrac{\sin\theta}{1+\cos\theta} = \dfrac{1}{\sqrt{3}}$

On solving, we get $\theta = 60°$.

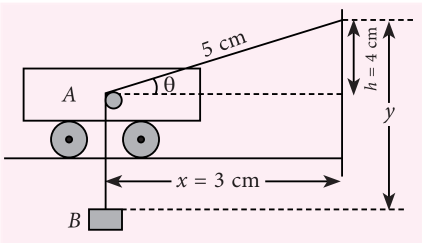
Putting value of θ in eqn. (ii), we get $t = \frac{10\sqrt{3}}{3}$ s

If (x, y) is the coordinate of point of collision,

$$x = 200\ t = \frac{2000\sqrt{3}}{3}\text{ m} = \frac{2\sqrt{3}}{3}\text{ km}$$

$$y = 200\sin\theta t - \frac{1}{2}gt^2 = 1000 - \frac{500\sqrt{3}}{3} = \left(1 - \frac{\sqrt{3}}{6}\right)\text{ km}$$

12. (c, d) : $(y - h) + \sqrt{x^2 + h^2} = l = \text{constant}$



or $\frac{dy}{dt} + \frac{x}{\sqrt{x^2+h^2}}\frac{dx}{dt} = 0$ or $\frac{dy}{dt} = -\frac{x}{\sqrt{x^2+h^2}}\frac{dx}{dt}$

$\therefore\ \frac{d^2y}{dt^2} = \frac{v_A^2h^2}{(x^2+h^2)^{3/2}}\ \therefore v_B = \frac{3}{5}v_A$ and $a_B = \frac{16}{125}v_A^2$

13. (b): Variation of acceleration due to gravity with altitude

$$g_h = g\left(1 - \frac{2h}{R_e}\right) \quad \text{or} \quad \Delta g = \frac{2hg}{R_e}$$

$$T_h = 2\pi\sqrt{\frac{l}{g - \Delta g}} = 2\pi\sqrt{\frac{l}{g}}\left(1 - \frac{\Delta g}{g}\right)^{-1/2} = T\left(1 + \frac{\Delta g}{2g}\right)$$

Variation of l with temperature $= \Delta l$

$$T_\theta = 2\pi\sqrt{\frac{l + \Delta l}{g}} = T\left(1 + \frac{\Delta l}{2l}\right)$$

The clock shows correct time if $T_h = T_\theta$

$$\therefore\ \frac{\Delta l}{2l} = \frac{\Delta g}{2g}$$

Linear expansivity $= \frac{\Delta l}{l(\Delta\theta)}$, where $\Delta\theta = 30 - 20 = 10°\text{C}$

$$\text{Linear expansivity} = \frac{\Delta l}{10l} = \frac{\Delta g}{10g} = \frac{2h}{10R_e} = \frac{h}{5R_e}$$

14. (4): $\frac{dN_A}{dt} = -\lambda_1 N_A$ and $\frac{dN_B}{dt} = -2\frac{dN_A}{dt} - \lambda_2 N_B$

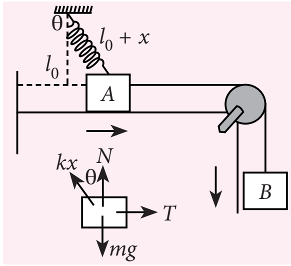
Now when N_B is maximum, at that time

$$\frac{dN_B}{dt} = 0 \Rightarrow 2\lambda_1 N_A - \lambda_2 N_{B\max} = 0$$

or $N_0 = \frac{\lambda_2 N_{B\max}}{2\lambda_1}e^{\lambda_1 t} = 10{,}000$ moles

15. (5)

16. (2): Let x be the elongation produced in spring and θ be the angle between spring and vertical at the instant when block A breaks off the plane.



$$\Rightarrow\ \cos\theta = \frac{l_0}{l_0 + x} \quad \text{...(i)}$$

$$N + kx\cos\theta = mg$$

$$\Rightarrow\ kx\cos\theta = mg \quad (\text{as } N = 0) \quad \text{...(ii)}$$

Let d is the distance covered by A and B and v is the speed acquired by them.

From eqns. (i) and (ii), using $k = \frac{5mg}{l_0}$

$$\Rightarrow\ \frac{5mg}{l_0}\times\frac{l_0 x}{l_0 + x} = mg$$

$$\Rightarrow\ x = \frac{1}{4}l_0 \quad \Rightarrow\ d = \sqrt{(l_0 + x)^2 - l_0^2} = \frac{3l_0}{4}$$

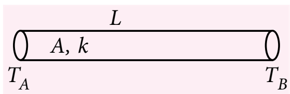
Using energy conservation,

$$mgd = 2\left(\frac{1}{2}mv^2\right) + \frac{1}{2}kx^2$$

$$\Rightarrow\ mg\frac{3l_0}{4} = mv^2 + \frac{1}{2}\frac{5mg}{l_0}\frac{l_0^2}{16} \quad \Rightarrow\ v = \sqrt{\frac{19gl_0}{32}}$$

$\therefore$ The value of n is 2

17. (2): $T_B\lambda_0 = b,\ T_B = \frac{b}{\lambda_0}$



$b = 2.89\times 10^{-3}$ m K,

$\lambda_0 = 75000\ \text{Å} = 75\times 10^{-7}\ \text{m},\ \sigma = 5.67\times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4}$

$$\frac{kA(T_A - T_B)}{L} = \sigma A T_B^4$$

$$T_A = T_B + \frac{L\sigma T_B^4}{k} \Rightarrow T_A = \frac{b}{\lambda_0} + \frac{L\sigma}{k}\times\frac{b^4}{\lambda_0^4}$$

Substituting values, we get $T_A = 422.1$ K $\therefore\ a = 2$

18. (1): The pressure inside and outside the charged soap bubble is same if $\frac{\sigma^2}{2\varepsilon_0} = \frac{4T}{R};\ T = \frac{\sigma^2 R}{8\varepsilon_0}$

$$T = \frac{\sigma^2 R}{8\varepsilon_0} = \frac{(2\times 10^{-5})^2\times 0.25\times 10^{-2}}{8\varepsilon_0} = \frac{1\times 10^{-12}}{8\varepsilon_0}\ \text{N m}^{-1}$$

$\therefore$ The value of α is 1.

Paper-II

1. **(c) :** Using conservation of mechanical energy $\Delta KE + \Delta PE = 0$,

$$\left(0 - \frac{1}{2}mv^2\right) + q(V_f - V_i) = 0$$

or $\quad \frac{1}{2}mv^2 = q(V_f - V_i) \quad$ ...(i)

$$V_i = \frac{Q}{4\pi\varepsilon_0 r} \text{ and } V_f = \frac{Q}{8\pi\varepsilon_0 R}\left[3 - \frac{r^2}{R^2}\right]$$

where $r = \frac{R}{2}$; hence $V_f = \frac{11Q}{32\pi\varepsilon_0 R}$

Putting the values of V_i and V_f in eqn. (i)

$$\frac{1}{2}mv^2 = \frac{11qQ}{32\pi\varepsilon_0 R} - \frac{qQ}{4\pi\varepsilon_0 r}$$

or $\quad v^2 = \frac{qQ}{2m\pi\varepsilon_0 R}\left[\frac{11}{8} - \frac{R}{r}\right]$

$\Rightarrow \quad v = \sqrt{\frac{qQ}{2m\pi\varepsilon_0 R}\left[\frac{11}{8} - \frac{R}{r}\right]}$

2. **(a) :** $\because$ Upthrust due to liquid = Weight of floating body

(Volume dipped) $\times \rho_L g$ = (Volume of body) $\times \rho_S \times g$

$\therefore \quad (k_1 V)\rho_L = V\rho_s \Rightarrow k_1 = \frac{\rho_S}{\rho_L}$ at temperature 0°C

Similarly $k_2 = \frac{\rho'_S}{\rho'_L}$ at temperature 60°C

$$\therefore \quad \frac{k_1}{k_2} = \frac{\rho_S}{\rho'_S} \times \frac{\rho'_L}{\rho_L}$$

or $\quad \frac{k_1}{k_2} = \frac{\rho'_S(1+\gamma_{Fe}\times 60)}{\rho'_S} \times \frac{\rho'_L}{\rho'_L(1+\gamma_{Hg}\times 60)}$

or $\quad \frac{k_1}{k_2} = \frac{1+60\gamma_{Fe}}{1+60\gamma_{Hg}}$

3. **(c) :** The electric field is $E = V/d$

$$\Rightarrow \quad E = \frac{0.50}{5.0\times 10^{-7}} = 1.0 \times 10^6 \text{ V m}^{-1}$$

Suppose the electron has a speed v_1 when it enters the depletion layer and v_2 when it comes out of it. As the potential energy increases by $q \times 0.50$ V, using principle of conservation of energy,

$$\frac{1}{2}mv_1^2 = q \times V + \frac{1}{2}mv_2^2$$

Solving this, $v_2 = 2.7 \times 10^5$ m s^{-1}.

4. **(d):** Let us first locate the image of S formed by the lens L. Here $u = -12$ cm and $f = 15$ cm. We have,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \text{ or } \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{15} - \frac{1}{12} \text{ or } v = -60 \text{ cm.}$$

The image I_1 acts as the source for the mirror. The mirror forms an image I_2 of the source I_1. This image I_2 then acts as the source for the lens and the final beam comes out parallel to the principal axis. Clearly I_2 must be at the focus of the lens. We have,

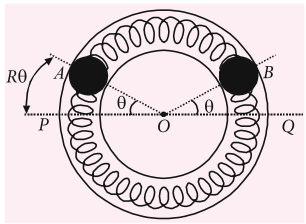
$$I_1I_2 = I_1P + PI_2 = 60 \text{ cm} + 15 \text{ cm} = 75 \text{ cm}$$

Suppose the distance of the mirror from I_2 is x cm.
For the reflection from the mirror,
$u = MI_1 = -(75 + x)$cm, $v = -x$ cm and $f = -20$ cm.

Using $\quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{x} + \frac{1}{75+x} = \frac{1}{20}$

This gives $x = 25$ or -60.
Taking $x = 25$, the separation between the lens and the mirror is (15 + 25) cm = 40 cm

5. **(b):** Initially, both the balls are displaced by an angle $\theta = \pi/6$ radian with respect to the diameter PQ of the circle and released from rest. It results into compression of spring in upper segment and an equal elongation of spring in lower segment. Let it be x. PA and QB denote x in the figure.



Compression = $R\theta$ = elongation = x

$\therefore$ Force exerted by each spring on each ball = $2kx$

$\therefore$ Total force on each ball due to two springs = $4kx$

$\therefore$ Restoring torque about origin $O = -(4kx)R$

$\therefore \quad \tau = -4k(R\theta)R$, where θ = Angular displacement

or $\quad \tau = -4kR^2\theta \quad$ ...(i)

Since torque τ is proportional to θ, each ball executes angular SHM about the centre O.

Again, $\quad \tau = -4kR^2\theta$

or $\quad I\alpha = -4kR^2\theta \quad$ where α = angular acceleration

or $\quad (mR^2)\alpha = -4kR^2\theta \quad$ or $\quad \alpha = -\left(\frac{4k}{m}\right)\theta$

$\therefore \quad$ Frequency $\upsilon = \frac{1}{2\pi}\sqrt{\left|\frac{\alpha}{\theta}\right|} = \frac{1}{2\pi}\sqrt{\frac{4k}{m}}$

$$= \frac{1}{2\pi}\sqrt{\frac{4\times 0.1}{0.1}} = \frac{1}{\pi} \text{ s}^{-1} \quad \text{...(ii)}$$

Let the velocity at mean position = v

In stretched position, PE = $2\left[\frac{1}{2}k(2x)^2\right]$

or $\quad$ PE = $4kx^2$

KE at mean position = $2\left[\frac{1}{2}mv^2\right]$ or KE = mv^2

Since the energy of the system is conserved, KE of system = PE of the system

or $mv^2 = 4kx^2 \Rightarrow v = 2x\sqrt{\frac{k}{m}} = 2R\theta\sqrt{\frac{k}{m}}$

or $v = 2\times(0.06)\left(\frac{\pi}{6}\right)\sqrt{\frac{0.1}{0.1}}$ or $v = 0.0628\ \text{m s}^{-1}$

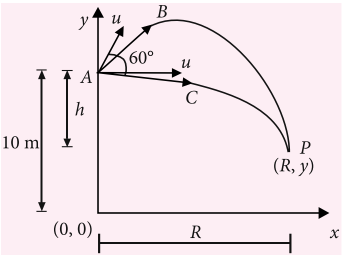
Total energy = KE at mean position

$\therefore \quad E = \left(\frac{mv^2}{2}+\frac{mv^2}{2}\right)$, for the system

or $E = mv^2$ or $E = (0.1)\times(0.0628)^2$

or $E = 3.95\times 10^{-4}$ J

6. **(b) :** Bullets fired by the two guns follow different parabolic path till they collide in air at point P. ACP and ABP denote their paths.



Let t_1 = Time taken by first bullet in reaching P.

t_2 = Time taken by second bullet in reaching P.

$R = (u\cos 60°)t_2$ for ABP

$R = (u\,t_1)$ for ACP

$\therefore \quad \frac{ut_2}{2} = ut_1$ or $t_2 = 2t_1$

$h = -(u\sin 60°)t_2 + \frac{1}{2}gt_2^2$ for ABP

$h = \frac{1}{2}gt_1^2$ for ACP

$\therefore \quad \frac{-u\times\sqrt{3}t_2}{2}+\frac{1}{2}gt_2^2 = \frac{1}{2}gt_1^2$

or $\frac{-5\sqrt{3}\times\sqrt{3}t_2}{2}+\frac{10t_2^2}{2} = \frac{10t_1^2}{2}$

or $t_1 = 1$ s or $t_2 = 2t_1 = 2$ s

$\therefore$ The time-interval between the firings

$= (t_2 - t_1) = (2-1) = 1$ s

Now $R = ut_1 = 5\sqrt{3}$ m for path ACP

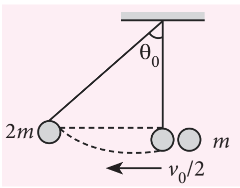
Again $h = \frac{1}{2}gt_1^2 = 5$ m for path ACP

$\therefore$ y-coordinate of $P = 10 - h = 10 - 5 = 5$ m

Hence, coordinates of $P = (R, y) = (5\sqrt{3}, 5)$ m

7. **(a, c)**

8. **(a, b) :** The time period of simple harmonic pendulum is independent of mass, so it would be same as that $T = 2\pi\sqrt{l/g}$. After collision, the combined mass acquires a velocity of $v_0/2$, as a result of this velocity, the mass $2m$ moves up and at an angel θ_0 with vertical, it stops, this is the extreme position of bob.



From work-energy theorem, $\Delta K = W_{\text{total}}$

$$0 - \frac{2m}{2}\left(\frac{v_0}{2}\right)^2 = -2mgl(1-\cos\theta_0)$$

$$\frac{v_0^2}{8gl} = 1-\cos\theta_0 = 2\sin^2\frac{\theta_0}{2}$$

$$\sin\frac{\theta_0}{2} = \frac{v_0}{4\sqrt{gl}}$$

If θ_0 is small, $\sin\frac{\theta_0}{2}\simeq\frac{\theta_0}{2} \Rightarrow \theta_0 = \frac{v_0}{2\sqrt{gl}}$

9. **(a, b, c) :** Fringe width, $\beta = \frac{\lambda D}{d}$

$\beta_1 = \frac{\lambda_1 D}{d}$ and $\beta_2 = \frac{\lambda_2 D}{d}$

$\therefore \quad \frac{\beta_2}{\beta_1} = \frac{\lambda_2}{\lambda_1}; \quad \because \lambda_2 > \lambda_1$ so $\beta_2 > \beta_1$

Number of fringes within a distance y on one side of the central maximum is $m = \frac{y}{\beta}$

Since y is same for λ_1 and λ_2

$\therefore \quad m_1\beta_1 = m_2\beta_2$ or $\frac{1}{2} \quad \frac{2}{1}$

$\because \quad \beta_2 > \beta_1$ so $m_1 > m_2$

3rd maxima of λ_2 is at $= \frac{3\lambda_2 D}{d} = (1800)\frac{D}{d}$

5th minima of λ_1 is at $= (2\times 5-1)\frac{\lambda_1 D}{2d} = (1800)\frac{D}{d}$

So, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1.

Angular separation of fringes, $\theta = \frac{\lambda}{d}$

The angular separation of fringes for λ_1 and λ_2 is

$\theta_1 = \frac{\lambda_1}{d}$ and $\theta_2 = \frac{\lambda_2}{d} \Rightarrow \frac{\theta_1}{\theta_2} = \frac{\lambda_1}{\lambda_2}$

As $\lambda_1 < \lambda_2 \quad \therefore \quad \theta_1 < \theta_2$

Thus the angular separation of fringes for λ_1 is lesser than λ_2.

10. **(a, b, c)**

11. **(b, c) :** Case (A) : $Z_A = \sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}$

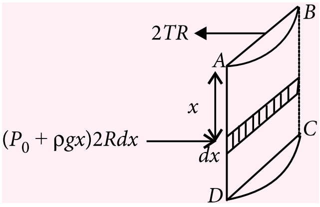
Case (B) : $Z_B = \sqrt{R^2+\left(\frac{1}{\omega KC}\right)^2}$

So $Z_B < Z_A$

$$I_R^A = \frac{V}{Z_A} \quad \text{and} \quad I_R^B = \frac{V}{Z_B} \quad \text{Clearly } I_R^A < I_R^B$$

Since current in case (B) is greater, so potential across R will increase in case (B) and thus across capacitor will decrease. Hence $V_C^A > V_C^B$ $\quad (\because V_R^2 + V_C^2 = V_0^2)$

12. (b):



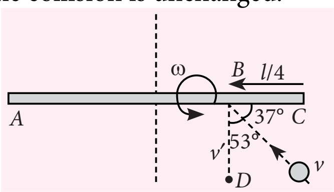
Consider water on one side of the vertical section $ABCD$ as the system. Note that this is the half cylinder. Draw the forces on this half cylinder by another half cylinder. At a depth below x from the top surface consider a strip of width dx. Force on this strip is $(P_0 + \rho gx)(2Rdx)$.

Total force on one half cylinder by the other half cylinder is

$$F = \int_0^h (P_0 + \rho gx)2Rdx - 2RT = 2P_0Rh + R\rho gh^2 - 2RT$$

13. (a, b, c) : The ball has v' component of its velocity perpendicular to the length of the rod immediately after the collision.

u is the velocity of CM of the rod and ω is angular velocity of the rod just after collision. The ball strikes the rod with a speed of $v \cos 53°$ in the perpendicular direction and its component along the length of the rod after the collision is unchanged.



At the point of collision,

velocity of separation = velocity of approach

$$\frac{3v}{5} = \left(\frac{\omega l}{4} + u\right) + v' \qquad \text{...(i)}$$

Conserving linear momentum (of rod + particle) in the direction perpendicular to the rod,

$$mv\frac{3}{5} = mu - mv' \qquad \text{...(ii)}$$

Conserving angular momentum about point D,

$$0 = 0 + \left[mu\frac{l}{4} - \frac{ml^2}{12}\omega\right] \Rightarrow u = \frac{\omega l}{3} \qquad \text{...(iii)}$$

From eqn. (i), (ii) and (iii)

$$\Rightarrow u = \frac{24v}{55},\ \omega = \frac{72v}{55l}$$

Time taken to rotate by π angle, $t = \frac{\pi}{\omega}$

In the same time, distance travelled = ut = —

Using impulse-momentum equation on the rod,

$$\text{Impact force} = \int N dt = mu = \frac{24mv}{55}$$

14. (a, c)

15. (b): F_{mm} = Gravitational force between two outer stars

$$= \frac{Gm^2}{4r^2}$$

F_{mM} = Gravitational force between central star and outer star $= \frac{GmM}{r^2}$

For circular motion of outer star,

$$\frac{mv^2}{r} = F_{mm} + F_{mM}$$

$$\therefore \quad v^2 = \frac{G(m+4M)}{4r}$$

$$T = \text{period of orbital motion} = \frac{2\pi r}{v}$$

$$m = \frac{16\pi^2 r^3}{GT^2} - 4M = \left(\frac{150}{16} - 8\right)10^{30}$$

$$= \frac{11}{8} \times 10^{30} \text{ kg}$$

16. (b): Total mechanical energy = KE + PE

$$= 2\left(\frac{1}{2}mv^2\right) - \frac{2GMm}{r} - \frac{Gm^2}{2r} = -\frac{Gm}{r}\left[M + \frac{m}{4}\right]$$

Substituting value we get,

$$E = -\frac{1375}{64} \times 10^{38} \text{ J}$$

17. (d): $qE = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{qE_0 b}{m}}$

$$a_c = \frac{qE}{m} = \frac{q}{m}E_0\left(\frac{b}{r}\right)$$

18. (c) : Centrifugal force = electrostatic force in the reference frame of proton.

$$\frac{1}{2}mv^2 = qE_0 b\int_a^b \frac{dr}{r} \Rightarrow v = \sqrt{\frac{2qE_0 b}{m}\ln\left(\frac{b}{a}\right)}$$

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PRACTICE PAPER
AIIMS

Exam on 28^{th} May

1. In a semiconducting material, $\left(\frac{1}{5}\right)^{th}$ of the total current is carried by the holes and the remaining is carried by the electrons. The drift speed of electrons is twice that of holes at this temperature, the ratio between the number densities of electrons and holes is
(a) 21 : 6 (b) 5 : 1 (c) 3 : 8 (d) 2 : 1

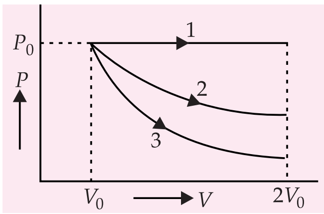
2. A ball is projected from the ground at an angle θ with the horizontal. After 1 s it is moving at an angle 45° with the horizontal and after 2 s, it is moving horizontally. The velocity of projection of the ball is ($g = 10$ m s^{-2})
(a) $5\sqrt{3}$ m s^{-1} (b) $10\sqrt{3}$ m s^{-1}
(c) $10\sqrt{5}$ m s^{-1} (d) $20\sqrt{5}$ m s^{-1}

3. Two identical mass M moving with velocity u_1 and u_2 collide perfectly inelastically. The loss in energy is
(a) $\frac{M}{2}(u_2 - u_1)^2$ (b) $\frac{M}{8}(u_1 - u_2)^2$
(c) $\frac{M}{4}(u_1 - u_2)^2$ (d) Zero

4. A car starts from rest and covers distance x along straight line before its speed becomes v. The speed v reached by a car of mass m driven with constant power P is given by
(a) $v = \frac{3xP}{m}$ (b) $v = \left(\frac{3xP}{m}\right)^{1/2}$
(c) $v = \left(\frac{3xP}{m}\right)^{1/3}$ (d) $v = \left(\frac{3xP}{m}\right)^{2}$

5. A heater coil is cut into two equal parts and only one part is now used in the heater. The heat generated will now be
(a) one fourth (b) halved
(c) doubled (d) four times

6. A gas is expanded from volume V_0 to $2V_0$ under three different processes, as shown in the figure. Then,



(a) Process 2 is isobaric
(b) Process 3 is isothermal
(c) Process 3 is adiabatic
(d) Process 1 is isothermal

7. Two particles of masses m_1 and m_2 are connected to a rigid massless rod of length r to constitute a dumbbell which is free to move in the plane. The moment of inertia of the dumb bell about an axis perpendicular to the plane passing through the centre of mass is
(a) $\frac{m_1 m_2 r^2}{m_1 + m_2}$ (b) $(m_1 + m_2)\, r^2$
(c) $\frac{m_1 m_2 r^2}{m_1 - m_2}$ (d) $(m_1 - m_2)\, r^2$

8. The half life of radium is 1620 years and its atomic weight is 226 g mol^{-1}. The number of atoms that will decay from its 1 g sample per second will be (Avogdro's number $N_A = 6.023 \times 10^{23}$)
(a) 3.61×10^{10} (b) 3.6×10^{12}
(c) 3.11×10^{15} (d) 31.1×10^{15}

9. A vector $\vec{Q}$ which has a magnitude of 8 is added to the vector $\vec{P}$ which lies along x-axis. The resultant of two vectors lies along y-axis and has magnitude twice that of $\vec{P}$. The magnitude of $\vec{P}$ is
(a) $\frac{6}{\sqrt{5}}$ (b) $\frac{8}{\sqrt{5}}$ (c) $\frac{12}{\sqrt{5}}$ (d) $\frac{16}{\sqrt{5}}$

10. The velocity of sound waves in air is 330 m s^{-1}. For sound of particular frequency in air, a path difference of 40 cm is equivalent to a phase difference of 1.6π. The frequency of the wave is
(a) 165 Hz (b) 150 Hz (c) 660 Hz (d) 330 Hz

11. The work done in placing a charge of 8×10^{-18} C on a condenser of capacity 100 μF is
(a) 16×10^{-32} J (b) 3.1×10^{-26} J
(c) 4×10^{-10} J (d) 32×10^{-32} J

12. A prism of refractive index 1.5 is placed in water of refractive index 1.33. The refracting angle of a prism is 60°. The angle of minimum deviation in water is (Given sin34° = 0.56)
(a) 4° (b) 8° (c) 12° (d) 16°

13. The ratio of minimum to maximum wavelength in Balmer series is
(a) 5 : 9 (b) 5 : 36 (c) 1 : 4 (d) 3 : 4

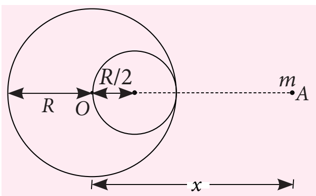
14. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to
(a) x^2 (b) e^x (c) x (d) $\log e^x$

15. In an experiment of simple pendulum, the errors in the measurement of length of the pendulum L and time period T are 3% and 2% respectively. The maximum percentage error in the value of $\frac{L}{T^2}$ is
(a) 5% (b) 7% (c) 8% (d) 1%

16. If velocity of light c, graviational constant G and Planck's constant h are chosen as fundamental units, the unit of mass would be
(a) $\sqrt{hc/G}$ (b) $\sqrt{hG/c}$
(c) $\sqrt{G/hc}$ (d) $\sqrt{c/Gh}$

17. A vessel of depth x is half filled with oil of refractive index μ_1 and the other half is filled with water of refractive index μ_2. The apparent depth of the vessel when viewed from above is
(a) $\frac{x(\mu_1+\mu_2)}{2\mu_1\mu_2}$ (b) $\frac{x_1\mu_2}{2(\mu_1+\mu_2)}$
(c) $\frac{x\mu_1\mu_2}{(\mu_1+\mu_2)}$ (d) $\frac{2x(\mu_1+\mu_2)}{\mu_1\mu_2}$

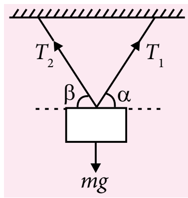
18. A spherical mass of radius $R/2$ is taken out from a uniform sphere of radius R and mass M as shown in figure. The force which this sphere having a cavity will exert on a mass m placed at distance x (where $x > R$) from the centre O is



(a) $\frac{GMmR}{x^3}$ (b) $\frac{GMm}{(R^2+x^2)}$
(c) $GMm\left[\frac{1}{x^2}-\frac{1}{2(2x-R)^2}\right]$
(d) $GMm\left[\frac{1}{x^2}-\frac{1}{(2x-R)^2}\right]$

19. Initial angular velocity of a circular disc of mass M is ω_1. Then two small spheres each of mass m are attached gently to two diametrically opposite points on the edge of the disc. The final angular velocity of the disc is
(a) $\left(\frac{M+m}{M}\right)\omega_1$ (b) $\left(\frac{M+m}{m}\right)\omega_1$
(c) $\left(\frac{M}{M+4m}\right)\omega_1$ (d) $\left(\frac{M}{M+2m}\right)\omega_1$

20. A body of mass m is suspended by two strings making angles α and β with the horizontal. Tensions in the two strings are



(a) $T_1 = \frac{mg\cos\beta}{\sin(\alpha+\beta)} = T_2$
(b) $T_1 = \frac{mg\sin\beta}{\sin(\alpha+\beta)} = T_2$
(c) $T_1 = \frac{mg\cos\beta}{\sin(\alpha+\beta)}$, $T_2 = \frac{mg\cos\alpha}{\sin(\alpha+\beta)}$
(d) None of these

21. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon
(a) the rates at which currents are changing in the two coils.
(b) relative position and orientation of the two coils.
(c) the materials of the wires of the coils.
(d) the currents in the two coils.

22. The coordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by
(a) $3t\sqrt{\alpha^2+\beta^2}$ (b) $3t^2\sqrt{\alpha^2+\beta^2}$
(c) $t^2\sqrt{\alpha^2+\beta^2}$ (d) $\sqrt{\alpha^2+\beta^2}$

23. The energy of radiation is 207 eV. The type of radiation is [Given, hc = 1242 eV nm]
(a) Infrared (b) Ultraviolet
(c) Visible (d) Microwaves

24. The two slits at a distance of 1 mm are illuminated by the light of wavelength 6.5×10^{-7} m. The interference fringes are observed on a screen placed at a distance of 1 m. The distance between third dark fringe and fifth bright fringe will be

(a) 0.65 cm (b) 4.8 mm
(c) 1.63 mm (d) 3.25 cm

25. 14 g of a gas occupy a volume of 4×10^{-3} m^3 at a temperature 27°C. After the gas is heated at constant pressure, its density becomes 7×10^{-4} g cm^{-3}. The temperature to which the gas was heated is
(a) 1127 °C (b) 1227 °C
(c) 1287 °C (d) 1327 °C

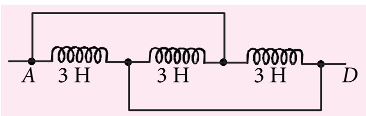
26. A galvanometer of resistance 25 Ω measures 10^{-3} A. Shunt required to increase range upto 2 A is
(a) 12.5 Ω (b) 0.0125 Ω
(c) 0.125 Ω (d) 1.25 Ω

27. A particle is released from the top of two inclined rough surfaces of height h each. The angle of inclination of the two planes are 30° and 60° respectively. All other factors (*e.g.*, coefficient of friction, mass of block etc.) are same in both the cases. Let K_1 and K_2 be the kinetic energies of the particle at the bottom of the plane in two cases. Then
(a) $K_1 = K_2$ (b) $K_1 > K_2$
(c) $K_1 < K_2$ (d) Data insufficient

28. If a charge q is placed at the centre of the line joining two equal charges Q such that the system is in equilibrium then the value of q is
(a) $Q/2$ (b) $-Q/2$ (c) $Q/4$ (d) $-Q/4$

29. A mass M is attached to a horizontal spring, executes SHM with an amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is
(a) $\frac{M}{M+m}$ (b) $\frac{M+m}{M}$
(c) $\left(\frac{M}{M+m}\right)^{1/2}$ (d) $\left(\frac{M+m}{M}\right)^{1/2}$

30. The inductance between points A and D is



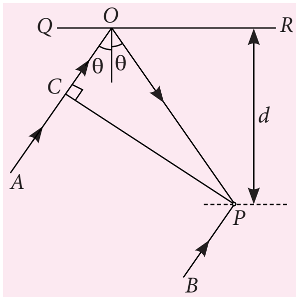
(a) 3.66 H (b) 9 H (c) 0.66 H (d) 1 H.

31. For a given liquid, if h is the height of capillary rise and r be the radius of capillary tube, then which of the following relation will be correct?
(a) hr = constant (b) h/r^2 = constant
(c) hr^2 = constant (d) h/r = constant

32. The plane face of a planoconvex lens is silvered. If μ be the refractive index and R is the radius of curvature of curved surface, then the system will behave like a concave mirror of radius of curvature
(a) μR (b) $\frac{R}{(\mu-1)}$
(c) $\frac{R^2}{\mu}$ (d) $\left[\frac{(\mu+1)}{(\mu-1)}R\right]$

33. A body has a time period T_1 under the action of one force and T_2 under the action of another force, the square of the time period when both the forces are acting in the same direction simultaneously is
(a) $T_1^2\, T_2^2$ (b) T_1^2 / T_2^2
(c) $T_1^2 + T_2^2$ (d) $T_1^2 T_2^2 / (T_1^2 + T_2^2)$

34. In the figure, CP represents a wavefront and AO and BP, the corresponding two rays. The condition on θ for constructive interference at P between the ray BP and the reflected ray OP.



(a) $\cos\theta = 3\lambda/2d$
(b) $\cos\theta = \lambda/4d$
(c) $\sec\theta - \cos\theta = \lambda/d$
(d) $\sec\theta - \cos\theta = 4\lambda/d$

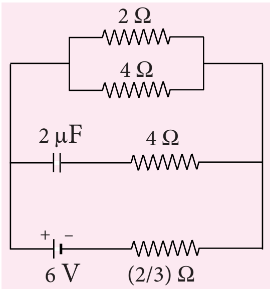
35. The radius of gyration of a solid cylinder of mass M and radius R about its own axis is
(a) $\frac{R}{\sqrt{2}}$ (b) $\frac{R}{2}$ (c) $\frac{R}{\sqrt{3}}$ (d) $\frac{R}{3}$

36. When p-n junction diode is forward biased, then
(a) the depletion region is reduced and barrier height is increased
(b) the depletion region is widened and barrier height is reduced.
(c) both the depletion region and barrier height are reduced
(d) both the depletion region and barrier height are increased.

37. Pick out the correct statements
(i) Susceptibility of a diamagnetic substance is high and positive.
(ii) In paramagnetic substance, the intrinsic magnetic moment is not zero.
(iii) When a paramagnetic substance is heated, it becomes ferromagnetic.
(iv) Spin exchange interaction is present in the absence of external magnetic field.

(a) (i) and (iii) (b) (iii) and (iv)
(c) (ii) and (iii) (d) (ii) and (iv)

38. In the given figure, the current in 2 Ω resistor is:



(a) 1 A (b) 2 A (c) 3/2 A (d) 4/3 A

39. Two wires *A* and *B* of the same material have their lengths in the ratio of 1 : 2 and their diameters in the ratio of 2 : 1. If they are stretched with the same force, the ratio of the increase in the length of *A* to that of *B* will be

(a) 1 : 2 (b) 4 : 1 (c) 1 : 8 (d) 1 : 4

40. According to Einstein's photoelectric equation, the plot of the kinetic energy of the emitted photoelectrons from a metal versus the frequency, of the incident radiation gives a straight line whose slope (Assume photoelectrons are emitted.)

(a) depends on the nature of the metal used
(b) depends on the intensity of the radiation
(c) depends both on the intensity of the radiation and the metal used
(d) is the same for all metals and independent of the intensity of the radiation.

Directions : In the following questions (41-60), a statement of assertion is followed by a statement of reason. Mark the correct choice as :

(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

41. Assertion : At a fixed temperature, silicon will have a minimum conductivity when it has a smaller acceptor doping.
Reason: The conductivity of an intrinsic semiconductor is slightly higher than that of a lightly doped *p*-type semiconductor.

42. Assertion : Photoelectric effect demonstrates the wave nature of light.
Reason: The number of photoelectrons is proportional to the frequency of light.

43. Assertion : Parallax method is used for measuring distances of nearby stars only.
Reason: With increase of distance of star parallactic angle becomes too small to be measured accurately.

44. Assertion : The restoring couple in a moving coil galvanometer is because of current in the coil.
Reason: Current in the moving coil galvanometer produces a resultant force which is responsible for the restoring couple.

45. Assertion : Current through an inductor lags behind the applied alternating emf across it by $\pi/2$.
Reason: If frequency of ac is doubled, resistance offered by inductor becomes twice.

46. Assertion : Two balls of different masses are thrown vertically upward with same speed. They will pass through their point of projection in the downward direction with the same speed.
Reason: The maximum height and downward velocity attained at the point of projection are independent of the mass of the ball.

47. Assertion : For angular projection, when angle of projection $\theta = \tan^{-1}(1)$, the horizontal range is four times the maximum height.
Reason: The horizontal range of projectile is directly proportional to square of velocity and inversely proportional to acceleration due to gravity.

48. Assertion : A current flows in a conductor only when there is an electric field within the conductor.
Reason: The drift velocity of electron in presence of electric field decreases.

49. Assertion : If a pendulum is suspended in a lift and lift is falling freely, then its time period becomes infinite.
Reason: Free falling body has acceleration equal to acceleration due to gravity.

50. Assertion : If objective and eye lenses of a microscope are interchanged, then it can work as telescope.
Reason: The objective lens of telescope has small focal length.

51. Assertion : In Young's double slit experiment, interference pattern disappears when one of the slits is closed.
Reason: Interference occurs due to superposition of light waves from two coherent sources.

52. Assertion : Colours are seen in thin layers of oil on the surface of water.

Reason: White light is composed of several colours.

53. **Assertion :** When a stationary bomb exploded into two pieces, their speeds are in the inverse ratio of their masses.
Reason: Explosion does not violate law of conservation of linear momentum.

54. **Assertion :** Light can travel in vacuum whereas sound cannot do so.
Reason: Light is an electromagnetic wave whereas sound is mechanical wave.

55. **Assertion :** When a solid sphere is heated, increase in its surface area is maximum.
Reason: Surface area involves expansion in three dimensions.

56. **Assertion :** When two coils are wound on each other, the mutual induction between the coils is maximum.
Reason: Mutual induction does not depend on the orientation of the coils.

57. **Assertion :** Static crashes are heard on radio, when lightning flash occurs in the sky.
Reason: Electromagnetic waves having frequency of radio wave range interfere with radio waves.

58. **Assertion :** The angular velocity of a planet orbiting around the Sun increases when it is nearest to the Sun.
Reason: Angular momentum of a body is proportional to angular velocity.

59. **Assertion :** When radius of circular wire carrying current is doubled, its magnetic moment becomes four times.
Reason: Magnetic moment depends on area of the loop.

60. **Assertion :** When a spring is cut into two equal parts, the spring constant of each part of spring is doubled.
Reason: Spring constant is inversely proportional to length of spring.

SOLUTIONS

1. **(d):** Current carried by the holes, $I_h = \frac{1}{5}I$
where I is the total current

Current carried by the electrons, $I_e = \frac{4}{5}I$

As $v_e = 2v_h$
where v_e and v_h are the drift speeds of electrons and holes respectively.
$\therefore\ I_e = n_eAe\,v_e\ ;\quad I_h = n_hAe\,v_h$
where n_e and n_h are the number densities of electrons and holes respectively.

$$\therefore\ \frac{I_e}{I_h} = \frac{n_eAev_e}{n_hAev_h} \quad \text{or} \quad \frac{n_e}{n_h} = \frac{I_e}{I_h} \times \frac{v_h}{v_e}$$

$$\frac{n_e}{n_h} = \frac{\frac{4}{5}I}{\left(\frac{1}{5}I\right)} \times \left(\frac{v_h}{2v_h}\right) = 2 \quad \Rightarrow \quad \frac{n_e}{n_h} = \frac{2}{1}$$

2. **(c) :** Let α be the angle which the instantaneous velocity of a projectile makes with the horizontal at time t during its flight. Then

$$\tan\alpha = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta} \quad \text{...(i)}$$

When $\alpha = 45°$ and $t = 1$ s, then from eqn. (i),
$u\cos\theta = u\sin\theta - g \times 1$ ...(ii)
When $\alpha = 0°$ and $t = 2$ s. Then,

$$u\sin\theta - g \times 2 = 0 \text{ or } u\sin\theta = 2g$$

From eqn. (ii), $u\cos\theta = 2g - g = g$

$$\therefore\ u^2(\cos^2\theta + \sin^2\theta) = g^2 + (2g)^2 = 5g^2$$

or $u = \sqrt{5}g = \sqrt{5} \times 10 = 10\sqrt{5}$ m s^{-1}

3. **(c) :** Loss of kinetic energy in a perfectly inelastic collision is

$$\Delta K = \frac{1}{2}\frac{m_1m_2}{(m_1+m_2)}(u_1-u_2)^2$$

Here, $m_1 = m_2 = M$

$$\therefore\ \Delta K = \frac{1}{2}\frac{M \times M}{(M+M)}(u_1-u_2)^2 = \frac{M}{4}(u_1-u_2)^2$$

4. **(c) :** $P = Fv = \left(m\frac{dv}{dt}\right)v$ = constant

$$\text{or } v\left(\frac{dv}{dx}\right)\left(\frac{dx}{dt}\right) = \frac{P}{m}; \quad v^2\frac{dv}{dx} = \frac{P}{m}$$

$$v^2dv = \frac{P}{m}dx$$

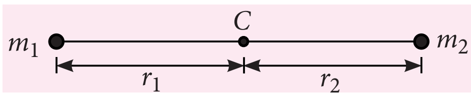
Integrating both sides, we get

$$\frac{v^3}{3} = \frac{P}{m}x\ ; \quad v = \left(\frac{3xP}{m}\right)^{1/3}$$

5. **(c)**

6. **(c)**

7. **(a) :** Suppose C is centre of mass of the dumb-bell. r_1 and r_2 are distances of m_1 and m_2 from C as shown in figure therefore, moment of inertia of dumbbell about the given axis is



$$I = m_1r_1^2 + m_2r_2^2 \qquad ...(i)$$

Also $\quad r = r_1 + r_2$

and $\quad m_1r_1 = m_2r_2 = m_2(r - r_1)$

$$(m_1 + m_2)r_1 = m_2 r; \quad r_1 = \frac{m_2 r}{m_1 + m_2}$$

Similarly, $\quad r_2 = \dfrac{m_1 r}{m_1 + m_2}$

From eqn. (i), $I = \dfrac{m_1 m_2 r^2}{m_1 + m_2}$

8. (a) : Number of atoms in 1 g of sample,

$$N = \frac{6.023 \times 10^{23}}{226}$$

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1620 \text{ years}} = \frac{0.693}{1620 \times 3.1536 \times 10^7 \text{ s}}$$

$$R = \lambda N = \frac{0.693 \times 6.023 \times 10^{23}}{1620 \times 3.1536 \times 10^7 \text{ s} \times 226}$$

$$= 3.61 \times 10^{10} \text{ dps.}$$

9. (b) : Let θ be the angle between $\vec{P}$ and $\vec{Q}$ and β be the angle between the resultant of $\vec{P}$ and $\vec{Q}$ and $\vec{P}$. Then $\beta = 90°$. $Q = 8$ and $R = 2P$

$$\tan\beta = \frac{Q \sin\theta}{P + Q\cos\theta} \quad \text{or} \quad \infty = \frac{8 \sin\theta}{P + 8\cos\theta}$$

or $\quad P + 8\cos\theta = 0 \;$ or $\; \cos\theta = -P/8$

As $\quad R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$\therefore \quad 4P^2 = P^2 + 8^2 + 2P \times 8 \times (-P/8)$

or $\quad 5P^2 = 64 \quad$ or $\quad P = \dfrac{8}{\sqrt{5}}$

10. (c) : Here, velocity of sound, $v = 330 \text{ m s}^{-1}$

$$\text{Phase difference} = \frac{2\pi}{\lambda} \times \text{Path difference}$$

$$1.6\pi = \frac{2\pi}{\lambda} \times 40\,; \quad \lambda = 50 \text{ cm} = 0.5 \text{ m}$$

$$\upsilon = \frac{v}{\lambda} = \frac{330}{0.5} = 660 \text{ Hz}$$

11. (d)

12. (b) : Here, ${}^a\mu_g = 1.5 = \dfrac{3}{2}$, ${}^a\mu_w = 1.33 = \dfrac{4}{3}$

$A = 60°$

As $\; {}^a\mu_w \times {}^w\mu_g = {}^a\mu_g$

$$\text{or} \quad {}^w\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{(3/2)}{(4/3)} = \frac{9}{8}$$

$$ {}^w\mu_g = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}; \quad \frac{9}{8} = \frac{\sin\left(\frac{60° + \delta_m}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$$

$$\text{or} \quad \sin\left(\frac{60° + \delta_m}{2}\right) = \frac{9}{8} \times \sin 30° = \frac{9}{8} \times \frac{1}{2} = \frac{9}{16} = 0.56$$

$$\text{or} \quad \frac{60° + \delta_m}{2} = \sin^{-1}(0.56) = 34°$$

or $\; \delta_m = 68° - 60° = 8°$

13. (a)

14. (a) : Given : Retardation $\propto$ displacement

$$\text{or} \quad \frac{dv}{dt} = -kx$$

$$\text{or} \quad \left(\frac{dv}{dx}\right)\left(\frac{dx}{dt}\right) = -kx \quad \text{or} \quad dv\,(v) = -kx\,dx$$

$$\text{or} \quad \int_{v_1}^{v_2} v\,dv = -k\int_0^x x\,dx \quad \text{or} \quad \frac{v_2^2}{2} - \frac{v_1^2}{2} = -\frac{kx^2}{2}$$

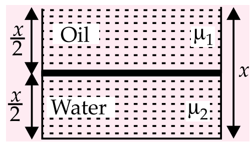
$$\text{or} \quad \frac{mv_2^2}{2} - \frac{mv_1^2}{2} = -\frac{mkx^2}{2}$$

$$\text{or} \quad (K_2 - K_1) = -\frac{mk}{2}x^2$$

i.e., Loss of kinetic energy is proportional to x^2.

15. (b) **16. (a)**

17. (a) : As refractive index,



$$\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$$

$\therefore \quad$ Apparent depth of the vessel when viewed from above is

$$d_{\text{apparent}} = \frac{x}{2\mu_1} + \frac{x}{2\mu_2} = \frac{x}{2}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right) = \frac{x(\mu_1 + \mu_2)}{2\mu_1\mu_2}$$

18. (c) : Mass of the sphere, $M = \dfrac{4}{3}\pi R^3 \rho$

$$\text{Mass of cavity } M' = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \rho = \frac{M}{8}$$

Gravitational force on the particle at A due to sphere of mass M radius R is

$$F_1 = \frac{GMm}{x^2}$$

Gravitational force on the particle at A due to cavity is

$$F_2 = \frac{G(M/8)m}{(x - R/2)^2} = \frac{GMm}{8(x - R/2)^2}$$

Force on the particle at A due to sphere with cavity is

$$F = F_1 - F_2 = \frac{GMm}{x^2} - \frac{GMm}{8(x - R/2)^2}$$
$$= GMm\left[\frac{1}{x^2} - \frac{1}{2(2x - R)^2}\right]$$

19. (c) : Angular momentum of the system is conserved

$\therefore \quad \frac{1}{2}MR^2\omega_1 = 2mR^2\omega + \frac{1}{2}MR^2\omega$

or $\quad M\omega_1 = (4m + M)\omega$

or $\quad \omega = \left(\frac{M}{M + 4m}\right)\omega_1$

20. (c)

21. (b): Mutual inductance of the pair of coils depends on relative position and orientation of the two coils.

22. (b): $x = \alpha t^3 \quad \therefore \frac{dx}{dt} = v_x = 3\alpha t^2$

Again $y = \beta t^3 \quad \therefore \frac{dy}{dt} = v_y = 3\beta t^2 \quad \because v^2 = v_x^2 + v_y^2$

$\therefore \quad v^2 = (3\alpha t^2)^2 + (3\beta t^2)^2 = (3t^2)^2(\alpha^2 + \beta^2)$

or $\quad v = 3t^2\sqrt{\alpha^2 + \beta^2}$

23. (b)

24. (c) : Position of fifth bright fringe, $x_5 = \frac{5D\lambda}{d}$

Position of third dark fringe,

$$x_3 = (2\times 3 - 1)\frac{D\lambda}{2d} = \frac{5D\lambda}{2d}$$

Required distance,

$$x_5 - x_3 = \left(5 - \frac{5}{2}\right)\frac{D\lambda}{d} = \frac{5}{2}\frac{D\lambda}{d}$$
$$= \frac{5\times 1\times 6.5\times 10^{-7}}{2\times 1\times 10^{-3}} \approx 1.63 \text{ mm.}$$

25. (b): Volume at 27°C (= 300 K) = 4×10^{-3} m^3. Let it be heated to temperature x K.

Volume at x K = $\frac{14}{7\times 10^{-4}}$ cm^3

$= 2 \times 10^4$ cm^3 = 2×10^{-2} m^3

Since pressure is constant, so $\frac{V}{T}$ = constant

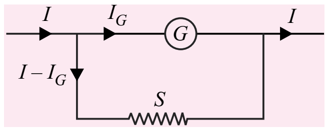
$\therefore \quad \frac{4\times 10^{-3}}{300} = \frac{2\times 10^{-2}}{x}$

or $\quad x = \frac{(2\times 10^{-2})\times 300}{(4\times 10^{-3})} = 1500$ K or 1227 °C

26. (b): Here, $I_G = 10^{-3}$ A, $I = 2$ A

Galvanometer resistance $G = 25\ \Omega$

Shunt resistance S = ?

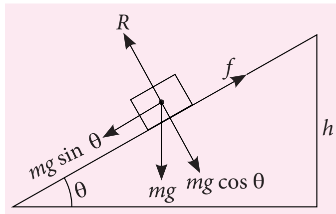


From figure, $I_G G = (I - I_G)S$ or $S = \frac{GI_G}{I - I_G}$

Substituting the values, we get

$$S = \frac{(25\ \Omega)\times(10^{-3}\text{ A})}{2\text{ A} - 10^{-3}\text{ A}} = \frac{25\times 10^{-3}}{1.999}$$
$$= 12.5 \times 10^{-3}\ \Omega = 0.0125\ \Omega.$$

27. (c) : As is known, on sliding down a rough inclined plane, work done against friction,



$W = \mu R \times s = \mu\, mg\cos\theta \times \left(\frac{h}{\sin\theta}\right)$

$W = \mu\, mgh\cot\theta$ ...(i)

$\because \cot\theta_1 = \cot 30° = \sqrt{3}$ and $\cot\theta_2 = \cot 60° = \frac{1}{\sqrt{3}}$

From eqn. (i), $W_1 > W_2$

Now $\quad K = mgh - W \quad \therefore K_1 < K_2$

28. (d)

29. (d): As $T_1 = 2\pi\sqrt{\frac{M}{k}}$ ...(i)

When a mass m is placed on mass M, the new system is of mass $(M + m)$ attached to the spring. New time period of oscillations

$$T_2 = 2\pi\sqrt{\frac{(m + M)}{k}} \quad \text{...(ii)}$$

Consider v_1 is the velocity of mass M passing through mean position and v_2 velocity of mass $(m + M)$ passing through mean position.

Using, law of conservation of linear momentum

$Mv_1 = (m + M)v_2$

$M(A_1\omega_1) = (m + M)(A_2\omega_2)$

($\because v_1 = A_1\omega_1$ and $v_2 = A_2\omega_2$)

$\therefore \quad \frac{A_1}{A_2} = \frac{(m + M)}{M}\frac{\omega_2}{\omega_1}$

$$= \left(\frac{m+M}{M}\right) \times \frac{T_1}{T_2} \qquad \left(\because\ \omega_1 = \frac{2\pi}{T_1} \text{ and } \omega_2 = \frac{2\pi}{T_2}\right)$$

$$\frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}} \qquad \text{(Using eqn. (i) and (ii))}$$

30. (d)

31. (a) : The height h to which the liquid rises in a capillary tube is given by $h = \frac{2S\cos\theta}{r\rho g}$

Since S, $\cos\theta$, ρ and g are constants,

$\therefore \quad hr = \text{constant}$

32. (b) : Focal of the plano-convex lens is given by

$$\frac{1}{f_L} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{\mu - 1}{R}$$

When the plane surface of the plano-convex lens is silvered, the system acts as a combination a plane mirror and a plano-convex lens. Its effective focal length F_L is given by

$$\frac{1}{F_L} = \frac{1}{f_L} + \frac{1}{f_m} + \frac{1}{f_L} = \frac{2}{f_L} + \frac{1}{\infty} = \frac{2(\mu-1)}{R} + 0$$

or $\quad F_L = \dfrac{R}{2(\mu - 1)} \qquad [\because f_m = \infty]$

Radius of curvature $= 2F_L = \dfrac{R}{(\mu - 1)}$

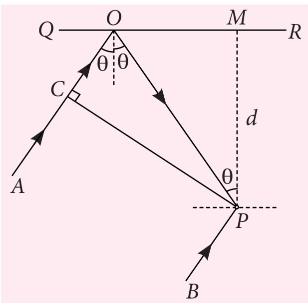
33. (d) : $F_1 = \dfrac{m4\pi^2 a}{T_1^2}$; $F_2 = \dfrac{m4\pi^2 a}{T_2^2}$

$$F = F_1 + F_2 = \frac{4\pi^2 ma}{T_1^2} + \frac{4\pi^2 ma}{T_2^2} = 4\pi^2 ma\left(\frac{1}{T_1^2} + \frac{1}{T_2^2}\right)$$

or $\quad \dfrac{4\pi^2 ma}{T^2} = 4\pi^2 ma\left[\dfrac{1}{T_1^2} + \dfrac{1}{T_2^2}\right]$

or $\quad \dfrac{1}{T^2} = \dfrac{1}{T_1^2} + \dfrac{1}{T_2^2}$ or $T^2 = \dfrac{T_1^2 T_2^2}{T_1^2 + T_2^2}$

34. (b) : In ΔOPM, $\dfrac{PM}{OP} = \cos\theta$ or $OP = \dfrac{d}{\cos\theta}$



In ΔCOP, $\quad \cos 2\theta = \dfrac{OC}{OP} = \dfrac{OC}{d/\cos\theta}$

or $\quad OC = \dfrac{d\cos 2\theta}{\cos\theta}$

The ray after reflection from mirror QR suffers extra path difference of $\frac{\lambda}{2}$. Therefore, the net path difference between two rays reaching point P is,

$$= CO + OP + \frac{\lambda}{2} = \frac{d\cos 2\theta}{\cos\theta} + \frac{d}{\cos\theta} + \frac{\lambda}{2}$$

$$= \frac{d}{\cos\theta}(\cos 2\theta + 1) + \frac{\lambda}{2}$$

$$= \frac{d}{\cos\theta} \times 2\cos^2\theta + \frac{\lambda}{2} = 2d\cos\theta + \frac{\lambda}{2}$$

For constructive interference

$$2d\cos\theta + \frac{\lambda}{2} = n\lambda \text{ or } 2d\cos\theta = (2n-1)\frac{\lambda}{2}$$

or $\cos\theta = (2n-1)\dfrac{\lambda}{4d}$

For $n = 1$, $\cos\theta = \dfrac{\lambda}{4d}$

35. (a) **36. (c)**

37. (d) : In paramagnetic substances, intrinsic magnetic moment is not zero. Further, in the absence of external magnetic field, spin exchange interaction is present.

38. (b) : In steady state, there will be no current through the capacitor branch of network. Total resistance of the circuit is

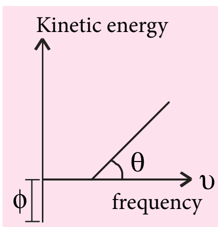
$$R = \frac{2\times 4}{2+4} + \frac{2}{3} = 2\ \Omega$$

Current in the circuit, $I = \dfrac{6}{2} = 3\text{ A}$

Current through 2 Ω resistor $= 3 \times \dfrac{4}{(2+4)} = 2\text{ A}$

39. (c)

40. (d) :



According to Einstein's equation,

Kinetic energy = $h\upsilon - \phi$

where kinetic energy and υ (frequency) are variables, compare it with equation, $y = mx + c$.

$\therefore$ slope of line = h

h is Planck's constant.

Hence the slope is same for all metals and independent of the intensity of radiation.

41. (c)

42. (d) The number of photoelectrons depends on the intensity and not on frequency of light.

43. (a) : As distance of star from the earth increases, parallactic angle decreases and it may become too small to be measured accurately, when star is far off.

44. (d) : The restoring couple in moving coil galvanometer is due to twist produced in the suspension wire when the coil is rotated on passing the current in it. On passing the current in the coil of moving coil galvanometer, resultant force on coil is zero but a torque acts on it which rotates the coil.

45. (b)

46. (a) : $h = ut - \frac{1}{2}gt^2$ and $v^2 = u^2 - 2gh$.

The above equations are independent of mass.

47. (b)

48. (c) : Before the presence of electric field, the free electrons move randomly in the conductor, so their drift velocity is zero and therefore there is no current in the conductor. In the presence of electric field, each electron in the conductor experiences a force in a direction opposite to the electric field. Now the free electrons are accelerated from negative end to the positive end of the conductor and hence a current starts to flow from the conductor.

49. (b) : If a pendulum is suspended in a lift and lift is moving downward with some acceleration a, then time period of pendulum is given by, $T = 2\pi\sqrt{\frac{l}{g-a}}$

In the case of free fall, $a = g$ then $T = \infty$

i.e., the time period of pendulum becomes infinite.

50. (d) : A microscope cannot work as a telescope by interchanging its two lenses. These two lenses have short focal lengths and so the difference $(f_e - f_o)$ is small. In a telescope, the objective lens has much larger focal length than the eyepiece.

51. (a) : When one of the slits is closed, there appears general illumination from a single source. Interference does not take place.

52. (b) : Colours are seen in thin layer of oil on the surface of water, because of interference of light. When white light falls on a thin film of a liquid it will appear bright having colour whose wavelength satisfies the relation $2\mu t \cos r = (2n + 1)\lambda/2$. The colour of the film will depend upon the thickness of the film and the angle of refraction.

53. (a) :

54. (a) : Light being electromagnetic wave do not require any material medium for its propagation. Hence light can travel in vacuum. On the other hand sound is a mechanical wave and requires a material medium for its propagation. Hence sound cannot travel in vacuum.

55. (d) : On heating, increase in volume is maximum as it involves three dimensional expansion.

56. (c) : The manner in which the two coils are oriented, determines the coefficient of coupling between them *i.e.*, $K = \sqrt{\frac{M}{L_1 L_2}}$,

where L_1 and L_2 are self inductance of two coils. When the two coils are wound on each other, the coefficient of coupling is maximum and hence mutual inductance between the coil is maximum.

57. (a)

58. (b) : According to law of conservation of angular momentum, $L = I\omega$ = constant.

If a planet while orbiting comes close to sun moment of inertia of planet I decreases. To conserve angular momentum L, ω increases.

59. (a) : Magnetic dipole moment of the current loop

= Ampere turns × Area of the coil

So $M = NI\pi r^2$

$M_1 = NI\pi(2r)^2 = 4NI\pi r^2 = 4M$

So magnetic moment becomes four times when radius is doubled.

60. (a) : For a given force, as spring constant $k = F/x$ or $k \propto 1/x$.

When $x' = x/2$, $k' = F/x/2 = 2\,F/x = 2k$.

FULL LENGTH PRACTICE PAPER

BITSAT

Exam date : 16^{th} to 30^{th} May 2017

SECTION-I (PHYSICS)

1. The diffusivity is given by $K/S\rho$, where K is coefficient of thermal conductivity, S is specific heat and ρ is density. Then dimensional formula of diffusivity is
(a) $[L^2T^{-1}]$ (b) $[LT^{-1}]$ (c) $[L^2T^2]$ (d) $[LT^{-3}]$

2. Sound from two identical sources S_1 and S_2 reach a point P. When the sounds reach directly, and in the same phase, the intensity at P is I_0. The power of S_1 is now reduced by 60%, and the phase difference between S_1 and S_2 is varied continuously. The maximum and minimum intensities recorded at P are now I_{max} and I_{min}. Then,
(a) $I_{max} = 0.64I_0$ (b) $I_{min} = 0.36I_0$
(c) $I_{max}/I_{min} = 16$ (d) $I_{max}/I_{min} = 1.64/0.36$

3. A deutron of atomic mass 2.0147 amu and negligible kinetic energy is absorbed by a Li^6 nucleus of mass 6.0169 amu, the intermediate nucleus disintegrates spontaneously into two α-particles, each of mass 4.0039 amu. The energy transferred to each α-particle is
(a) 12.08 MeV (b) 11.08 MeV
(c) 6.04 MeV (d) 5.54 MeV

4. A block of mass 0.50 kg is moving with a speed of 2.00 m s^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then move together as a single body. The energy loss during the collision is
(a) 0.16 J (b) 1.00 J (c) 0.67 J (d) 0.34 J

5. A plate of thickness t made of a material of refractive index μ is placed in front of one of the slits in a double slit experiment. What should be the minimum thickness t which will make the intensity at the centre of the fringe pattern zero ?
(a) $(\mu - 1)\frac{\lambda}{2}$ (b) $(\mu - 1)\lambda$
(c) $\frac{\lambda}{2(\mu - 1)}$ (d) $\frac{\lambda}{(\mu - 1)}$

6. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 m s^{-2}. He reaches the ground with a speed of 3 m s^{-1}. At what height, did he bail out?
(a) 293 m (b) 111 m (c) 91 m (d) 182 m

7. The kinetic energy given to a body is K so that it moves from the surface of earth to infinity. If only 20% of this kinetic energy is given to the same body on the surface of earth, it rises to a height nR, where R is the radius of the earth. Then n is equal to
(a) 1 (b) 3/4 (c) 1/2 (d) 1/4

8. A particle moves on a straight line as such its product of acceleration and velocity is constant. The distance moved by particle in time t is proportional to
(a) t (b) $\sqrt{t}$ (c) $t^{3/2}$ (d) t^2

9. A Carnot engine whose sink is at 300 K has an efficiency of 40%. By how much should the temperature of source be increased so as to increase its efficiency by 50% of original efficiency ?
(a) 380 K (b) 275 K (c) 325 K (d) 250 K

10. If two balls are projected at an angle of 60° and 45° and the total heights reached are same, then their initial velocities are in the ratio of
(a) $\sqrt{3}:\sqrt{2}$ (b) $\sqrt{2}:\sqrt{3}$
(c) 3 : 2 (d) 2 : 3

11. The half life of radioactive radon is 3.8 day. The time at the end of which $(1/20)^{th}$ of the radon sample will remain undecayed is (given $\log_{10}e = 0.4343$)
(a) 3.8 days (b) 16.5 days
(c) 33 days (d) 76 days.

12. The plate separation in a parallel plate condenser is d and plate area is A. If it is charged to V volt and battery is disconnected then the work done in increasing the plate separation to $2d$ will be
(a) $\frac{3}{2}\frac{\varepsilon_0 AV^2}{d}$ (b) $\frac{\varepsilon_0 AV^2}{d}$
(c) $\frac{2\varepsilon_0 AV^2}{d}$ (d) $\frac{\varepsilon_0 AV^2}{2d}$

13. Two radioactive sources A and B of half lives 1 hour and 2 hours respectively initially contain the same number of radioactive atoms. At the end of 2 hours, their rates of disintegration are in the ratio of

(a) 1 : 4 (b) 1 : 3 (c) 1 : 2 (d) 1 : 1

14. A bullet of mass 0.01 kg is travelling at a speed of 500 m s^{-1} strikes a block of 2 kg, which is suspended by a string of length 5 m. The centre of the gravity of the block is found to rise through a vertical height of 0.1 m. The speed of the bullet after it emerges from the block is

(a) 200 m s^{-1} (b) 240 m s^{-1}
(c) 220 m s^{-1} (d) 280 m s^{-1}

15. A convex lens is in contact with concave lens. The magnitude of the ratio of their focal length is 2/3. Their equivalent focal length is 30 cm. Their individual focal lengths in cm are

(a) 50, – 75 (b) 15, – 10
(c) 50, 75 (d) 10, – 15

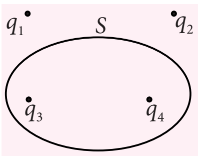
16. A resistor and a capacitor are connected to an AC supply of 200 V-50 Hz in series. The current in the circuit is 2 A. If the power consumed in the circuit is 100 W, then the resistance in the circuit is

(a) 100 Ω (b) 25 Ω (c) 75 Ω (d) 400 Ω

17. A circular disc X of radius R is made from an iron plate of thickness t and another disc Y of radius 4 R is made from an iron plate of thickness $t/4$. Then the relation between the moment of inertia I_X and I_Y is

(a) $I_Y = I_X$ (b) $I_Y = 16\, I_X$
(c) $I_Y = 32\, I_X$ (d) $I_Y = 64\, I_X$

18. Figure shows four charges q_1, q_2, q_3 and q_4 fixed in space Then the total flux of the electric field through a closed surface S, due to all the charges, is

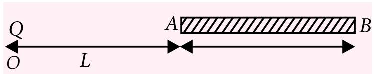


(a) equal to the total flux through S due to q_3 and q_4
(b) not equal to the total flux through S due to q_3 and q_4
(c) zero if $q_1 + q_2 = q_3 + q_4$
(d) twice the total flux through s due to q_3 and q_4 if $q_1 + q_2 = q_3 + q_4$.

19. Two conducting circular loops of radii R_1 and R_2 are placed in the same plane with their centres coinciding. If $R_1 > R_2$, the mutual inductance M between them will be directly proportional to

(a) $\dfrac{R_1}{R_2}$ (b) $\dfrac{R_2}{R_1}$ (c) $\dfrac{R_1^2}{R_2}$ (d) $\dfrac{R_2^2}{R_1}$

20. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at a distance L from the end A is



(a) $\dfrac{Q\ln 2}{4\pi\epsilon_0 L}$ (b) $\dfrac{Q\ln 2}{8\pi\epsilon_0 L}$
(c) $\dfrac{3Q}{4\pi\epsilon_0 L}$ (d) $\dfrac{Q}{4\pi\epsilon_0 L\,\ln 2}$

21. A geostationary satellite is orbiting the earth at a height of 6R above the surface of the earth, R being the radius of the earth. The time period of another satellite at a height of 2.5R from the surface of the earth is

(a) $6\sqrt{2}$ hours (b) 6 hours
(c) $6\sqrt{3}$ hours (d) 10 hours

22. A block of mass m is placed on a smooth wedge of inclination θ. The whole system is accelerated horizontally so that the block does not slip on the wedge. The force exerted by the wedge on the block (g is acceleration due to gravity) will be

(a) $mg\cos\theta$ (b) $mg\sin\theta$
(c) mg (d) $mg/\cos\theta$

23. A 60 V-10 W bulb is operated at 100 V-60 Hz AC. The inductance required is

(a) 2.56 H (b) 0.32 H (c) 0.64 H (d) 1.28 H

24. A locomotive of mass m starts moving so that its velocity varies according to the law $v = \alpha\sqrt{s}$, where α is a constant and s is the distance covered. Find the total work done by all the forces acting on the locomotive during the first t s after the beginning of motion.

(a) $\frac{1}{6}m\alpha^4t^2$ (b) $\frac{1}{4}m\alpha^4t$
(c) $\frac{1}{4}m\alpha^4t^2$ (d) $\frac{1}{8}m\alpha^4t^2$

25. The lower end of a capillary tube of radius r is placed vertically in water of density ρ, then with the rise of water in the capillary to height h, heat evolved is

(a) $\dfrac{\pi r^2h^2\rho g}{J}$ (b) $\dfrac{\pi r^2h^2\rho g}{2J}$
(c) $-\dfrac{\pi r^2h^2\rho g}{J}$ (d) $-\dfrac{\pi r^2h^2\rho g}{2J}$

26. A voltage applied to an X-ray tube being increased by 1.5 times, the short wavelength limit of a continuous X-ray spectrum shifts by 26 pm. The initial voltage applied to the tube is
(a) 1.6 kV (b) 3.2 kV
(c) 15.9 kV (d) 19.2 kV

27. A small metallic ball is charged positively and negatively in a sinusoidal manner at a frequency of 10^6 cps. The maximum charge on the ball is 10^{-6} C. The displacement current due to this alternating current is
(a) 6.28 A (b) 3.8 A
(c) 3.75×10^{-4} A (d) 122.56 A

28. Imagine river-basin of width w with slope α. If the height of the river water is h and the water is flowing with a terminal velocity of v. The value of η/ρ (coefficient of viscosity/density) is
(a) $\dfrac{h^2 g \sin\alpha}{v}$ (b) $\dfrac{w^2 g \sin\alpha}{v}$
(c) $\dfrac{h^2 g \sin\alpha}{w/v}$ (d) $\dfrac{whg \sin\alpha}{v}$

29. A transistor is connected in common emitter configuration. The collector supply is 8 V and the voltage drop across a resistor of 800 Ω in the collector circuit is 0.5 V. If the current transfer ratio is 0.96, the base current is
(a) 13 μA (b) 18 μA (c) 26 μA (d) 30 μA

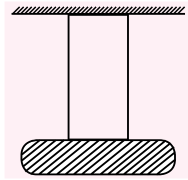
30. A magnet makes 40 oscillations per minute at a place having magnetic field intensity of 0.1×10^{-5} T. At another place, it takes 2.5 s to complete one vibration. The value of earth's horizontal field at that place is
(a) 0.25×10^{-6} T (b) 0.66×10^{-8}T
(c) 0.36×10^{-6} T (d) 1.2×10^{-6} T.

31. 2 kg of ice at –20°C is mixed with 5 kg of hot water at 20°C in an insulating vessel having negligible heat capacity. Calculate the final mass of water left in the container. Given specific heats of water and ice are 1 kcal kg^{-1} $°C^{-1}$ and 0.5 kcal kg^{-1} $°C^{-1}$ and latent heat of fusion of ice is 80 kcal kg^{-1}.
(a) 2 kg (b) 4 kg (c) 6 kg (d) 7 kg

32. A thin lens has focal length f and its aperture has diameter d. It forms an image of intensity I. Now the central part of the aperture upto diameter $d/2$ is blocked by an opaque paper. The focal length and the image intensity will change to
(a) $\dfrac{f}{2}$ and $\dfrac{I}{2}$ (b) f and $\dfrac{I}{4}$
(c) $\dfrac{3f}{4}$ and $\dfrac{I}{2}$ (d) f and $\dfrac{3}{4}I$

33. Two straight long conductors AOB and COD are perpendicular to each other and carry currents I_1 and I_2 respectively. Find the magnitude of the magnetic field at a point P at a distance a from the point O in a direction perpendicular to the plane $ABCD$?
(a) $\dfrac{\mu_0}{2\pi a}(I_1^2 + I_2^2)^{1/2}$ (b) $\dfrac{\mu_0}{4\pi a}(I_1^2 + I_2^2)$
(c) $\dfrac{\mu_0}{2\pi a}(I_1^2 + I_2^2)^2$ (d) $\dfrac{\mu_0}{2\pi a}(I_1^2 + I_2^2)$

34. Two wires of equal length and cross-section are suspended as shown. Their Young's moduli are Y_1 and Y_2 respectively. The equivalent Young's modulus will be



(a) $Y_1 + Y_2$ (b) $\dfrac{Y_1 + Y_2}{2}$
(c) $\dfrac{Y_1 Y_2}{Y_1 + Y_2}$ (d) $\sqrt{Y_1 Y_2}$

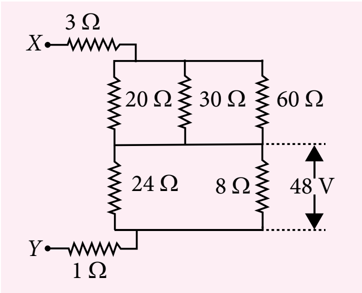
35. The path difference between two interfering waves at a point on the screen is $\dfrac{\lambda}{8}$. The ratio of intensity at this point and that at the central fringe will be
(a) 0.853 (b) 8.53 (c) 85.3 (d) 853

36. A particle moves along a circle of radius $\left(\dfrac{20}{\pi}\right)$ m with constant tangential acceleration. If the velocity of the particle is 80 m s^{-1} at the end of the second revolution after motion has begun, the tangential acceleration is
(a) 40 m s^{-2} (b) 640 π m s^{-2}
(c) 160 π m s^{-2} (d) 40 π m s^{-2}

37. In a spring block system, length of the spring is reduced by 2%, the time period will
(a) increase by 2% (b) increase by 1%
(c) decrease by 2% (d) decrease by 1%

38. A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is k. If radius of the ball be R, the ratio of its rotational energy to its total energy will be
(a) $\dfrac{k^2 + R^2}{R^2}$ (b) $\dfrac{k^2}{R^2}$
(c) $\dfrac{k^2}{k^2 + R^2}$ (d) $\dfrac{R^2}{k^2 + R^2}$

39. The potential difference across 8 Ω resistance is 48 V as shown in the figure. The value of potential difference across X and Y points will be



(a) 160 V (b) 128 V (c) 80 V (d) 62 V

40. Two particles P and Q describe S.H.M. of same amplitude a and frequency υ along the same straight line. The maximum distance between two particles is $a\sqrt{2}$. The initial phase difference between the particle is

(a) $\pi/3$ (b) $\pi/6$ (c) $\pi/2$ (d) zero

SECTION-II (CHEMISTRY)

41. For adsorption of gas in solid, $\log\left(\frac{x}{m}\right)$ versus $\log p$ is linear with the slope equal to

(a) k (b) $\log k$ (c) $\frac{1}{n}$ (d) n

42. Which one of the following is not correctly matched?

(a) $>C=O \xrightarrow{\text{Clemmensen reduction}} >CH_2$

(b) $>C=O \xrightarrow{\text{Wolff–Kishner reduction}} >CHOH$

(c) $—COCl \xrightarrow{\text{Rosenmund reduction}} —CHO$

(d) $—C\equiv N \xrightarrow{\text{Stephen reduction}} —CHO$

43. Which of the following sets of quantum numbers represents the highest energy of an atom?

(a) $n = 3, l = 0, m = 0, s = +\frac{1}{2}$

(b) $n = 3, l = 1, m = 1, s = +\frac{1}{2}$

(c) $n = 3, l = 2, m = 1, s = +\frac{1}{2}$

(d) $n = 4, l = 0, m = 0, s = +\frac{1}{2}$

44. An organic compound with the formula $C_6H_{12}O_6$ forms a yellow crystalline solid with phenylhydrazine and gives a mixture of sorbitol and mannitol when reduced with sodium. Which among the following could be the compound?

(a) Fructose (b) Glucose

(c) Mannose (d) Sucrose

45. Which of the following will give maximum number of isomers?

(a) $[Co(NH_3)_4Cl_2]$ (b) $[Ni(en)(NH_3)_4]^{2+}$

(c) $[Ni(C_2O_4)(en)_2]$ (d) $[Cr(SCN)_2(NH_3)_4]^{2+}$

46. Which one of the following sequences represents the correct increasing order of bond angles in the given molecules?

(a) $H_2O < OF_2 < OCl_2 < ClO_2$

(b) $OCl_2 < ClO_2 < H_2O < OF_2$

(c) $OF_2 < H_2O < OCl_2 < ClO_2$

(d) $ClO_2 < OF_2 < OCl_2 < H_2O$

47. An ideal gas with pressure P, volume V and temperature T is expanded isothermally to a volume $2V$ and a final pressure is P_I. If the same gas is expanded adiabatically to a volume $2V$, and the final pressure is P_{II} and the ratio of specific heats for the gas is 1.67, then the ratio of P_{II}/P_I is

(a) $\frac{1}{(2)^{0.67}}$ (b) $(2)^{0.67}$

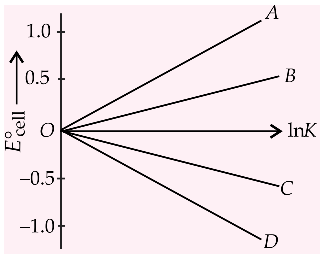
(c) $\left(-\frac{1}{2}\right)^{-0.67}$ (d) $\frac{1}{2^{-0.67}}$

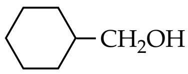
48. Given: $\Delta G° = -nFE°_{cell}$ and $\Delta G° = -RT\ln K$

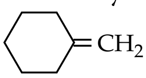
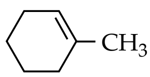
The value of $n = 2$ will be given by the slope of which line in the given figure?

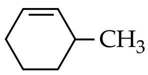
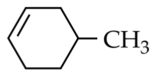
(a) OA (c) OC

(b) OB (d) OD



49.  on dehydration with conc. H_2SO_4 predominantly forms

(a)  (b) 

(c)  (d) 

50. Three elements A, B and O crystallise in ccp lattice with atoms A at corners, atoms B at body centre and atoms O at the edge centre. Chemical formula of solid is

(a) AB_2O_4 (b) ABO_4 (c) A_2BO_3 (d) ABO_3

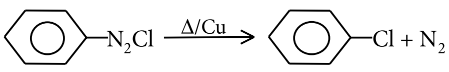
51. During the formation of the N_2O_4 dimer from two molecules of NO_2, the odd electrons, one in each of the nitrogen atoms of the NO_2 molecules, get paired to form a

(a) weak N–N bond, two N–O bonds become equivalent and the other two N–O bonds become non-equivalent.

(b) weak N–N bond and all the four N–O bonds become equivalent.
(c) weak N–N bond and all the four N–O bonds become non-equivalent.
(d) strong N–N bond and all the four N–O bonds become non-equivalent.

52. The emf of the three galvanic cells given below are represented by E_1, E_2 and E_3.
I. $Zn \mid Zn^{2+}$ (1 M) || Cu^{2+} (1 M) | Cu
II. $Zn \mid Zn^{2+}$ (0.1 M) || Cu^{2+} (1 M) | Cu
III. $Zn \mid Zn^{2+}$ (1 M) || Cu^{2+} (0.1 M) | Cu
Which of the following is true?
(a) $E_1 > E_2 > E_3$ (b) $E_3 > E_2 > E_1$
(c) $E_3 > E_1 > E_2$ (d) $E_2 > E_1 > E_3$

53. The aqueous solution of which of the following salts will have the lowest pH?
(a) NaClO (b) $NaClO_2$
(c) $NaClO_3$ (d) $NaClO_4$

54. 

(*A*)

Half-life is independent of concentration of *A*. After 10 minutes volume of N_2 gas is 10 L and after complete reaction 50 L. Hence, rate constant is
(a) $\frac{2.303}{10}\log 5\ \text{min}^{-1}$ (b) $\frac{2.303}{10}\log 1.25\ \text{min}^{-1}$
(c) $\frac{2.303}{10}\log 2\ \text{min}^{-1}$ (d) $\frac{2.303}{10}\log 4\ \text{min}^{-1}$

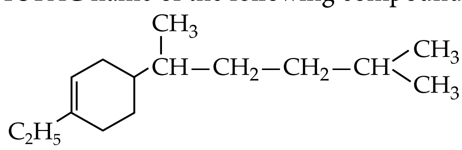
55. An ester (*A*) with molecular formula, $C_9H_{10}O_2$ was treated with excess of CH_3MgBr and the complex so formed was treated with H_2SO_4 to give an olefin (*B*). Ozonolysis of (*B*) gave a ketone with molecular formula C_8H_8O which shows positive iodoform test. The structure of (*A*) is
(a) $C_6H_5COOC_2H_5$
(b) $C_6H_5COOC_6H_5$
(c) $CH_3OCH_2COC_6H_5$
(d) *p*-$CH_3OCOC_6H_4COCH_3$

56. Consider the following statements :
1. Atomic hydrogen is obtained by passing hydrogen through an electric arc.
2. Hydrogen gas will not reduce heated aluminium oxide.
3. Finely divided palladium absorbs large volume of hydrogen gas.
4. Pure nascent hydrogen is best obtained by reacting Na with C_2H_5OH.

Which of the given statements is/are correct?
(a) 1 only (b) 2 only
(c) 1, 2 and 3 (d) 2, 3 and 4

57. If *M* is the element of actinide series, the degree of complex formation decreases in the order
(a) $M^{4+} > M^{3+} > MO_2^{2+} > MO_2^{+}$
(b) $MO_2^{+} > MO_2^{2+} > M^{3+} > M^{4+}$
(c) $M^{4+} > MO_2^{2+} > M^{3+} > MO_2^{+}$
(d) $MO_2^{2+} > MO_2^{+} > M^{4+} > M^{3+}$

58. Which of the following solutions will have pH close to 1.0?
(a) 100 mL of M/100 HCl + 100 mL of M/100 NaOH
(b) 55 mL of M/10 HCl + 45 mL of M/10 NaOH
(c) 10 mL of M/10 HCl + 90 mL of M/10 NaOH
(d) 75 mL of M/5 HCl + 25 mL of M/5 NaOH

59. The IUPAC name of the following compound is


(a) 1-ethyl-4-(5-methylhexyl)cyclohex-1- ene
(b) 1-ethyl-4-(1, 4-dimethylpentyl)cyclohex-1-ene
(c) 5-(1, 4-dimethylpentyl)-2-ethylcyclohex-1-ene
(d) 4-(1,4-dimethylpentyl)-1-ethylcyclohex-1-ene.

60. Heat of neutralisation of CsOH with all strong acids is 13.4 kcal mol^{-1}. The heat released on neutralisation of CsOH with HF (weak acid) is 16.4 kcal mol^{-1}. $\Delta H°$ of ionisation of HF is
(Heat of neutralisation of strong acid with strong base = – 13.7 kcal)
(a) 3.0 kcal (b) – 3.0 kcal
(c) 6.0 kcal (d) 5.0 kcal

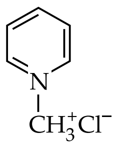
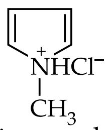
61. Three separate samples of a solution of a single salt gave these results. One formed a white precipitate with excess ammonia solution, one formed a white precipitate with NaCl solution and one formed a black precipitate with H_2S. The salt would be
(a) $AgNO_3$ (b) $Pb(NO_3)_2$
(c) $Hg(NO_3)_2$ (d) $MnSO_4$

62. A hexapeptide has the composition Ala, Gly, Phe, Val. Both the N-terminal and C-terminal units are Val. Cleavage of the hexapeptide by chemotrypsin gives two different tripeptides, both having Val as the

N-terminal group. Among the products of random hydrolysis, one is Ala-Val dipeptide fragment. What is the primary structure of the hexapeptide?
(a) Val-Gly-Phe-Val-Ala-Val
(b) Val-Ala-Phe-Val-Gly-Val
(c) Val-Gly-Ala-Val-Phe-Val
(d) Val-Phe-Val-Ala-Gly-Val

63. The electron gain enthalpies (in kJ mol^{-1}) of halogens X, Y and Z are respectively –349, –333 and –325. Then X, Y and Z respectively are
(a) F, Cl and Br (b) Cl, F and Br
(c) Cl, Br and F (d) Br, Cl and F

64. A gas 'X' is passed through water to form a saturated solution. The aqueous solution on treatment with silver nitrate gives a white precipitate. The saturated aqueous solution also dissolves magnesium ribbon with evolution of a colourless gas 'Y'.
'X' and 'Y' respectively are
(a) CO_2, Cl_2 (b) Cl_2, CO_2
(c) Cl_2, H_2 (d) H_2, Cl_2

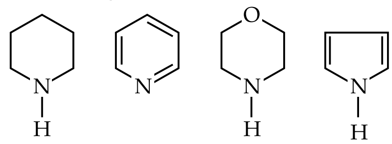
65. Which one of the following is a quaternary salt?
(a) 
(b) $(CH_3)_3N^+HCl^-$
(c) 
(d) $(CH_3)_4N^+Cl^-$

66. N_2 is passed through overheated CaC_2. Which of the following statements are correct for the product formed?
I. State of hybridisation of C is *sp*.
II. Urea is an intermediate formed during hydrolysis of the above product.
III. Anion present in the product is not a pseudohalide ion.
IV. Hydrolysis of the product gives rise to NH_3 gas slowly.
(a) I, II and III (b) III and IV
(c) I, II and IV (d) None of these.

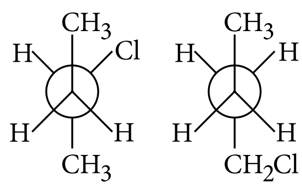
67. If λ_0 is the threshold wavelength for photoelectric emission, λ the wavelength of light falling on the surface of a metal and m the mass of the electron, then the velocity of ejected electron is given by
(a) $\left[\frac{2h}{m}(\lambda_0-\lambda)\right]^{1/2}$ (b) $\left[\frac{2hc}{m}(\lambda_0-\lambda)\right]^{1/2}$
(c) $\left[\frac{2hc}{m}\left(\frac{\lambda_0-\lambda}{\lambda_0\lambda}\right)\right]^{1/2}$ (d) $\left[\frac{2h}{m}\left(\frac{1}{\lambda_0}-\frac{1}{\lambda}\right)\right]^{1/2}$

68. An organic compound 'X' when treated with CH_3—MgI followed by acidification gives 'Y' ($C_5H_{12}O$) which gives C_5H_{10} on heating with copper. The structure of 'X' will be
(a) $CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-C_2H_5$ (b) $CH_3-\overset{\overset{\displaystyle CH_3}{|}}{CH}-CH{=}O$
(c) $CH_3CH_2CH_2-CH{=}O$
(d) $(CH_3)_3C-CH{=}O$

69. The order of the magnitude of ionic radii of ions N^{3-}, O^{2-} and F^- is
(a) $N^{3-} > O^{2-} > F^-$ (b) $N^{3-} < O^{2-} < F^-$
(c) $N^{3-} > O^{2-} < F^-$ (d) $N^{3-} < O^{2-} > F^-$

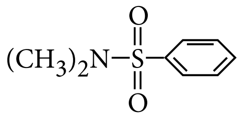
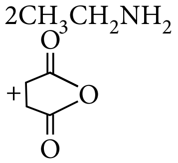
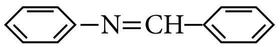
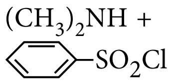
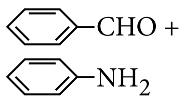
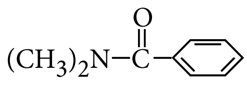
70. In the following compounds:

I II III IV
the order of basicity is
(a) IV > I > III > II (b) III > I > IV > II
(c) II > I > III > IV (d) I > III > II > IV

71. An alkane with the formula C_6H_{14} can be prepared by the hydrogenation of only two alkenes (C_6H_{12}). IUPAC name of the alkane is
(a) 2,2-dimethylbutane (b) 2,3-dimethylbutane
(c) 2-methylpentane (d) *n*-hexane.

72. The pair of structures given below represent

(a) enantiomers
(b) diastereomers
(c) structural isomers
(d) two molecules of the same compound.

73. Lemon juice normally has a pH of 2. If all the acid in the lemon juice is citric acid and there are no citrate salts present, then what will be the citric acid concentration (H.Cit) in the lemon juice? (Assume that only the first hydrogen of citric acid is important).
(H.Cit $\rightleftharpoons$ H^+ + Cit^-; $K_a = 8.4 \times 10^{-4}$ mol L^{-1})
(a) 8.4×10^{-4} M (b) 4.2×10^{-4} M
(c) 16.8×10^{-4} M (d) 12.0×10^{-2} M

74. Match list I with list II and select the correct answer using the given codes :

List I (Reactions)		List II (Products)	
(P)	$(CH_3)_2NH + C_6H_5COCl$	I.	
(Q)		II.	
(R)		III.	$C_2H_5NHCO(CH_2)_2COO^- \; C_2H_5\overset{+}{N}H_3$
(S)		IV.	

Codes :

	P	Q	R	S
(a)	III	IV	I	II
(b)	IV	III	II	I
(c)	I	II	III	IV
(d)	IV	III	I	II

75. HCl cannot be used in place of H_2SO_4 in $KMnO_4$ titration, because

(a) HCl will reduce Mohr's salt or oxalic acid
(b) HCl will oxidise Mohr's salt or oxalic acid
(c) HCl will reduce $KMnO_4$
(d) none of these.

76. The plot of $1/x_A$ versus $1/y_A$ (where x_A and y_A are the mole fractions of A in liquid and vapour phases respectively) is linear whose slope and intercept respectively are given as

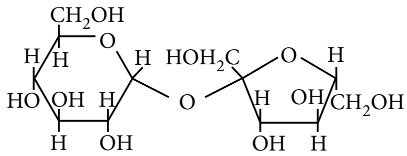
(a) $\dfrac{p_A^\circ}{p_B^\circ}, \dfrac{(p_A^\circ - p_B^\circ)}{p_B^\circ}$ (b) $\dfrac{p_A^\circ}{p_B^\circ}, \dfrac{(p_B^\circ - p_A^\circ)}{p_B^\circ}$

(c) $\dfrac{p_B^\circ}{p_A^\circ}, \dfrac{(p_A^\circ - p_B^\circ)}{p_B^\circ}$ (d) $\dfrac{p_B^\circ}{p_A^\circ}, \dfrac{(p_B^\circ - p_A^\circ)}{p_B^\circ}$

77. Carbylamine test is performed in alcoholic KOH by heating a mixture of

(a) chloroform and silver powder
(b) trihalogenated methane and a primary amine
(c) an alkyl halide and a primary amine
(d) an alkyl cyanide and a primary amine.

78. The correct statement about the following disaccharide is



(P) (Q)

(a) ring (P) is pyranose with α-glycosidic linkage
(b) ring (P) is furanose with α-glycosidic linkage
(c) ring (Q) is furanose with α-glycosidic linkage
(d) ring (Q) is pyranose with β-glycosidic linkage.

79. The best way to prepare polyisobutylene is through

(a) coordination polymerisation
(b) cationic polymerisation
(c) anionic polymerisation
(d) free radical polymerisation.

80. Aqueous sodium hydroxide reacts with a metal ion producing a coloured precipitate. This precipitate changes colour on standing. Identify the ion present.

(a) Fe^{2+} (b) Fe^{3+} (c) Cu^+ (d) Cu^{2+}

SECTION-III (ENGLISH AND LOGICAL REASONING)

Direction (Questions 81 to 84): Read the passage and answer the following questions.

The strength of the electronics industry in Japan is the Japanese ability to organise production and marketing rather than their achievements in original research. The British are generally recognised as a far more inventive collection of individuals, but never seem able to exploit what they invent. There are many examples, from the TSR Z hovercraft, high speed train and Sinclair scooter to the Triumph, BSA and Norton motorcycle which all prove this sad rule. The Japanese were able to exploit their strengths in marketing and development many years ago, and their success was at first either not understood in the West or was dismissed as something which could have been produced only at their low price. They were sold because they were cheap copies of other people's ideas churned out of a workhouse which was dedicated to hard grind above anything else.

81. The main theme of this passage is ________.

(a) the importance of original research in industry
(b) the role of marketing efficiency in industrial prosperity
(c) electronics industry in Japan
(d) industrial comparison between Japan and Britain

82. The TSR Z hovercraft, high speed train, Sinclair Scooter, etc. are the symbols of ________.

(a) British failure (b) British success
(c) Japanese failure (d) Japanese success.

83. The sad rule mentioned in this passage refers to the ________.
(a) poorer marketing ability of the British
(b) inability of the British to be industrious like the Japanese
(c) lack of variety in Japanese inventions
(d) inability of the Japanese to be inventive like the British

84. According to the passage, prosperity in industry depends upon ________.
(a) official patronage (b) inventiveness
(c) marketing ability (d) productivity

Direction (Questions 85 and 86): In each of the following, a word has been written in four different ways out of which only one is correctly spelt. Choose the correctly spelt word.

85. (a) Conscintious (b) Conscientious
(c) Consceintious (d) Conscientous

86. (a) Occurance (b) Occurence
(c) Occurrance (d) Occurrence

Direction (Questions 87 to 88): Choose the correct synonym for each of the following.

87. Emulate
(a) Likely to be late (b) Inspired to win
(c) Trying to do as well (d) Enable

88. Lapidary
(a) Engraver (b) Harmful
(c) High-sounding (d) Abusive

Direction (Questions 89 to 93): Rearrange the given six sentences A, B, C, D, E, F in the proper sequence so as to form a meaningful paragraph and then answer the given questions.

A. Miss Sullivan arrived at the Keller home when Helen was seven.
B. The deaf and blind Helen learned to communicate verbally.
C. But eventually, Miss Sullivan's effort was rewarded.
D. Before Helen Keller was two years old, she lost her sight and her hearing.
E. Miss Sullivan worked closely with Helen, her new student.
F. At times, the teacher became frustrated.

89. Which of the following should be the first sentence in the passage?
(a) A (b) B (c) C (d) D

90. Which of the following should be the last sentence in the passage?
(a) A (b) B (c) C (d) D

91. Which of the following should be the fourth sentence in the passage?
(a) A (b) C (c) F (d) E

92. Which of the following should be the third sentence in the passage?
(a) A (b) E (c) C (d) D

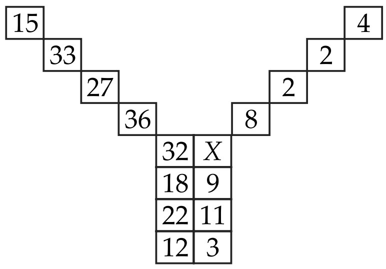
93. Which of the following should be the second sentence in the passage?
(a) A (b) C (c) D (d) E

Direction (Questions 94 and 95): In each of the following, an idiomatic expression/a proverb has been given, followed by four alternatives. Choose the one which best expresses the meaning of the given idiom/proverb.

94. Hard-pressed
(a) Hard discipline (b) In difficulties
(c) Bewildered (d) Insulted

95. To take the wind out of another's sails
(a) To defeat the motives of another
(b) To anticipate another and to gain advantage over him
(c) To manoeuvre to mislead another on the high seas
(d) To cause harm to another

96. Find the value of X in the following figure.

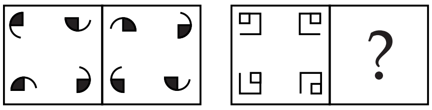


(a) 3 (b) 4 (c) 8 (d) 12

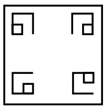
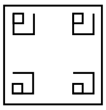
97. If PALAM could be given the code number 43, what code number can be given to SANTACRUZ?
(a) 75 (b) 85 (c) 120 (d) 123

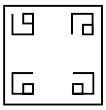
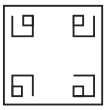
98. There are five persons P, Q, R, S and T. One is football player, one is chess player and one is hockey player. P and S are unmarried ladies and do not participate in any game. None of the ladies plays chess or football. There is a married couple in which T is the husband. Q is the brother of R and is neither a chess player nor a hockey player.
Who is the football player?
(a) P (b) Q (c) R (d) S

99. There is a definite relationship between figures A and B. Establish a similar relationship between figures C and D by selecting a suitable figure from the answer set that would replace the question (?) in figure D.



A B C D

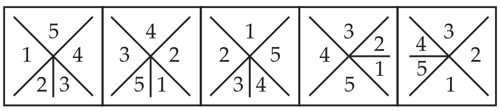
(a)  (b) 

(c)  (d) 

100. Find the next term in the given letter series.
LXF, MTJ, NPN, OLR, ?
(a) PHV (b) PIU
(c) PJW (d) PKX

101. In a certain code language
(A) 'pit na som' means 'bring me water';
(B) 'na jo tod' means 'water is life';
(C) 'tube od pit' means 'give me toy';
(D) 'jo lin kot' means 'life and death';
Which of the following represents 'is' in that language ?
(a) jo (b) na (c) tod (d) lin

102. There are five figures marked (1), (2), (3), (4) and (5), four are similar in a certain manner. However, one figure is not like the other four. Choose the figure which is different from the rest.



(1) (2) (3) (4) (5)

(a) 1 (b) 3 (c) 2 (d) 4

103. Which of the following options be the folded and form of Fig. (X)?

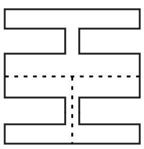
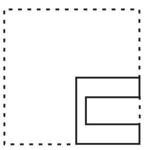
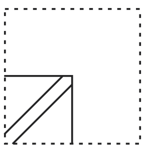
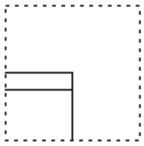
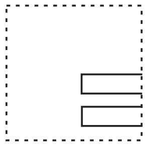
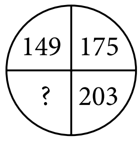


Fig. (X)

(a)  (b) 

(c)  (d) 

104. Find the missing character in the given figure.



(a) 148
(b) 208
(c) 213
(d) 233

105. There is a certain relation between pair of word on both sides of : :. Identify the relationship to find the missing word.
Hermit : Solitude : : Intruder : ?
(a) Theif (b) Privacy (c) Burglar (d) Alm

SECTION-IV (MATHEMATICS)

106. The domain of the function $f(x)=\dfrac{\sqrt{9-x^2}}{\sin^{-1}(3-x)}$ is
(a) $(2, 3)$ (b) $[2, 3)$
(c) $(2, 3]$ (d) none of these

107. If $x_1 > x_2 > x_3$ and x_1, x_2, x_3 are roots of
$$\frac{x-a}{b}+\frac{x-b}{a}=\frac{b}{x-a}+\frac{a}{x-b};\ (a, b>0)$$
and $x_1 - x_2 - x_3 = c$, then a, c, b are in
(a) A.P. (b) G.P.
(c) H.P. (d) none of these

108. $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+....\infty}}}$, then $\dfrac{dy}{dx}=$
(a) $\dfrac{2y-1}{\cos x}$ (b) $\dfrac{\cos x}{2y-1}$ (c) $\dfrac{2x-1}{\cos y}$ (d) $\dfrac{\cos y}{2y-1}$

109. If $f(x)=\begin{cases}\displaystyle\int_0^x (1+|1-t|)\,dt, & x>2\\ 5x-7, & x\le 2\end{cases}$, then
(a) $f(x)$ is not continuous at $x = 2$
(b) f is differentiable everywhere
(c) RHL at $x = 2$ doesn't exist
(d) f is continuous but not differentiable at $x = 2$

110. If each observation of a raw data, whose variance is σ^2, is multiplied by λ, then the variance of the new set is
(a) σ^2 (b) $\lambda^2\sigma^2$ (c) $\lambda + \sigma^2$ (d) $\lambda^2 + \sigma^2$

111. The general solution of $e^{-1/\sqrt{2}}(e^{\sin x}+e^{\cos x})=2$ is
(a) $x = m\pi$ (b) $x=\dfrac{(4m+1)\pi}{4}$
(c) $x=\dfrac{(4m+1)\pi}{2}$ (d) none of these

112. $\int \frac{2a\sin x + b\sin 2x}{(b + a\cos x)^3}\,dx$ is equal to

[where $t = (b + a\cos x)$]

(a) $\frac{1}{a^2}\frac{(a^2-b^2)}{t^2} + \frac{2b}{a^2 t} + c$

(b) $\frac{2}{a^2}\frac{(a^2-b^2)}{t^2} + \frac{2b}{a^2 t} + c$

(c) $\frac{2}{a^2}\frac{(a^2-b^2)}{t^2} + \frac{b}{a^2 t} + c$

(d) $\frac{2}{a^2}\frac{(a^2-b^2)}{t^3} + \frac{2b}{a^2 t} + c$

113. Given z is a complex number with modulus 1. Then the equation $\left(\frac{1+ia}{1-ia}\right)^4 = z$ has

(a) all roots are real and distinct
(b) two real and two imaginary roots
(c) three real roots and one imaginary
(d) one real root and three imaginary

114. $\frac{\frac{1}{2}\cdot\frac{2}{2}}{1^3} + \frac{\frac{2}{2}\cdot\frac{3}{2}}{1^3+2^3} + \frac{\frac{3}{2}\cdot\frac{4}{2}}{1^3+2^3+4^3} + \dots$ up to n terms =

(a) $\frac{n^2}{(n+1)^2}$ (b) $\frac{n^3}{(n+1)^3}$

(c) $\frac{n}{n+1}$ (d) $\frac{1}{n+1}$

115. Six papers are set in an examination, 2 of them in mathematics. The number of ways the papers can be arranged provided the 2 mathematics papers are not successive is

(a) 480 (b) 440 (c) 460 (d) 420

116. The locus of chords of contact of perpendicular tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ touch another fixed ellipse is a/an

(a) circle (b) straight line
(c) ellipse (d) hyperbola

117. The period of the function

$f(x) = \frac{\sin x + \sin 2x + \sin 4x + \sin 5x}{\cos x + \cos 2x + \cos 4x + \cos 5x}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) π (d) none of these

118. The equation of the plane containing the lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}$ is

(a) $\vec{r}\cdot(\vec{a}_1 - \vec{a}_2)\times\vec{b} = [\vec{a}_1\vec{a}_2\vec{b}]$

(b) $\vec{r}\cdot(\vec{a}_2 - \vec{a}_1)\times\vec{b} = [\vec{a}_1\,\vec{a}_2\,\vec{b}]$

(c) $\vec{r}\cdot(\vec{a}_1 + \vec{b}_2)\times\vec{b} = [\vec{a}_2\,\vec{a}_1\,\vec{b}]$

(d) none of these

119. The value of

$\int e^{\sec x}\cdot \sec^3 x(\sin^2 x + \cos x + \sin x + \sin x\cos x)dx$ is

(a) $e^{\sec x}(\sec^2 x + \sec x\tan x) + C$
(b) $e^{\sec x} + C$
(c) $e^{\sec x}(\sec x + \tan x) + C$
(d) none of these.

120. Let $F(x) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ where $\alpha \in R$

then $[F(\alpha)]^{-1}$ is equal to

(a) $F(-\alpha)$ (b) $F(\alpha^{-1})$
(c) $F(2\alpha)$ (d) none of these

121. If $[x]$ denotes the greatest integer less than or equal to x, then the value of

$\lim_{x\to 0}(1 - x + [x-1] + [1-x])$ is

(a) 0 (b) 1
(c) doesn't exist (d) -1

122. Area between the curves $y = x^3$ and $y = \sqrt{x}$ is

(a) $\frac{5}{12}$ (b) $\frac{5}{3}$

(c) $\frac{5}{4}$ (d) none of these

123. If A, B, C are the angles which a directed line makes with the positive directions of the coordinate axes, then $\sin^2 A + \sin^2 B + \sin^2 C =$

(a) 1 (b) 2
(c) 3 (d) none of these

124. The roots of $\begin{vmatrix} x & a & b & 1 \\ \lambda & x & b & 1 \\ \lambda & \mu & x & 1 \\ \lambda & \mu & v & 1 \end{vmatrix} = 0$ are independent of

(a) λ, μ, v (b) a, b
(c) λ, μ, v, a, b (d) none of these

125. $\int \tan^{-1}\left(\frac{2\cos^2\theta}{2-\sin 2\theta}\right)\sec^2\theta\, d\theta = \theta\tan\theta + \log\cos\theta - A + \frac{1}{2}\log(2 - 2\tan\theta + \tan^2\theta) + c$

BRAIN MAP

KINETIC THEORY

Class XI

Class XII

ATOMS AND NUCLEI

BRAIN MAP

KINETIC THEORY

Kinetic Theory of Ideal Gases

Relation between v_{rms}, v_{av} and v_{mp}

$v_{rms} : v_{av} : v_{mp}$

$= \sqrt{\frac{3RT}{M}} : \sqrt{\frac{8RT}{\pi M}} : \sqrt{\frac{2RT}{M}}$

$= \sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2}$; $(v_{rms} > v_{av} > v_{mp})$

Maxwell's Law of Distribution of Velocities

The distribution of molecules at different speed is given as,

$dN = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} dv$

Mean Free Path

The average distance travelled between successive collisions of molecules of a gas is called mean free path (λ).

$\lambda = \frac{1}{\sqrt{2}n\pi d^2}$; where n is the number density and d is the diameter of the molecule.

Kinetic Interpretation of Temperature

$KE_{avg} = E = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$

$KE/\text{mole} = \left(\frac{3}{2}kT\right)N_A = \frac{3}{2}RT$

Pressure Exerted by a Gas

$P = \frac{1}{3}\frac{mN}{V}v_{rms}^2 = \frac{1}{3}\rho v_{rms}^2 = \frac{2}{3}E'$

E' = Average KE per unit volume

Specific Heat Capacity

Specific Heat of a Gas

At constant pressure (C_P) :

$C_P = \frac{(\Delta Q)_P}{n\Delta T}$ or $C_P = \left(1+\frac{f}{2}\right)R$

At constant volume (C_V) :

$C_V = \frac{(\Delta Q)_V}{n\Delta T}$ or $C_V = \frac{1}{2}fR$

Mayer's relation : $C_P - C_V = R$

(f = degree of freedom)

Monoatomic Gas ($f = 3$)

$U = \frac{3}{2}RT, C_V = \frac{3}{2}R, C_P = \frac{5}{2}R, \gamma = \frac{5}{3}$

Diatomic Gas ($f = 5$)

$U = \frac{5}{2}RT, C_V = \frac{5}{2}R, C_P = \frac{7}{2}R, \gamma = \frac{7}{5}$

Polyatomic Gas

$U = (3+f')RT$

$C_V = (3+f')R$

$C_P = (4+f')R$

$\gamma = (4+f')/(3+f')$

f' = a certain number of vibrational mode

Law of Equipartition of Energy

For any system in thermal equilibrium, the total energy is equally distributed among its various degrees of freedom and each degree of freedom is associated with energy $\frac{1}{2}kT$.

Behaviour of Gases

Vander Waal's Equation

For n moles of a gas, $[a] = [ML^5T^{-2}]$, $[b] = [L^3]$

$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$

- **Critical Temperature :** $T_c = \frac{8a}{27Rb}$
- **Critical Pressure :** $P_c = \frac{a}{27b^2}$
- **Critical Volume :** $V_c = 3b$

Graham's Law of Diffusion

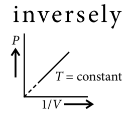
For given temperature and pressure, the rate of diffusion of gas is inversely proportional to the square root of the density of the gas. $r \propto \frac{1}{\sqrt{\rho}} \propto \frac{1}{\sqrt{M}}$

Gas Laws

Boyle's Law

At constant temperature, volume of a fixed mass of a gas is inversely proportional to its pressure.

$P \propto \frac{1}{V}$ or PV = constant

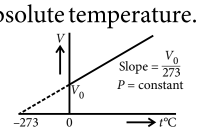


Charle's Law

The volume of the gas is directly proportional to its absolute temperature.

$V \propto T$ (at constant P)

$V_t = V_0\left(1 + \frac{t}{273}\right)$



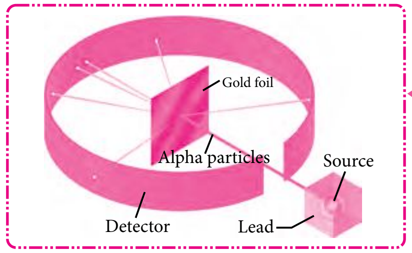
Gay-Lussac's Law

Pressure of the gas varies directly with the temperature at constant volume.

$P \propto T$ (at constant volume)

$P_t = P_0\left[1 + \frac{t}{273}\right]$

ATOMS & NUCLEI



Rutherford's Model of Atom

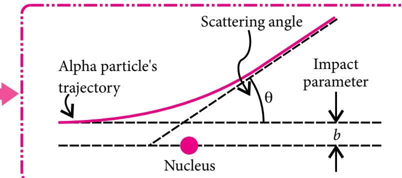
- K.E. of α–particles, $K = \frac{1}{2}mv^2$
- Distance of closest approach,

$r_0 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{K} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4Ze^2}{mv^2}$

- Impact parameter,

$b = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2\cot\frac{\theta}{2}}{K} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2\cot\frac{\theta}{2}}{\frac{1}{2}mv^2}$

- **Conclusion :** An atom consists of a small and massive central core in which entire positive charge and whole mass of atom is concentrated.
- **Drawback :** The revolving electron continuously loses its energy due to centripetal acceleration and finally it should collapse into the nucleus.



Bohr's Atomic Model

Electron orbits and their energy

- Radius of permitted n^{th} orbits,

$r_n = \frac{n^2h^2}{4\pi^2mkZe^2} \Rightarrow r_n \propto n^2$

- Velocity of electron in n^{th} orbit,

$v_n = \frac{2\pi kZe^2}{nh} \Rightarrow v_n \propto \frac{1}{n}$

- Energy of electron in n^{th} orbit

$E_n = \frac{-2\pi^2mk^2Z^2e^4}{n^2h^2} \Rightarrow E_n \propto \frac{1}{n^2}$

where the symbols have their usual meanings.

Line Spectra of Hydrogen

- While transition between different atomic levels, light radiated in various discrete frequencies are called spectral series of hydrogen atom.
- Rydberg formula :

Wave number $\bar{\upsilon} = \frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$

R = Rydberg's constant $= 1.097 \times 10^7\ m^{-1}$

Radioactivity

- **Law of radioactive decay**

$\frac{dN}{dt} = -\lambda N(t)$ or $N(t) = N_0e^{-\lambda t}$

- **Half-life**

$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$

- **Mean life or Average life**

$\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693} = 1.44\ T_{1/2}$

- **Fraction of nuclei left undecayed after n half lives is**

$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{t/T_{1/2}}$, where $t = nT_{1/2}$

Decay Schemes

- α-Decay :

${}^A_ZX \xrightarrow{\alpha\text{–decay}} {}^{A-4}_{Z-2}Y + {}^4_2\text{He} + Q$ (Energy released)

- β-Decay :

${}^A_ZX \xrightarrow{\beta^+} {}^A_{Z-1}Y + {}^0_{+1}e + \upsilon$

${}^A_ZX \xrightarrow{\beta^-} {}^A_{Z+1}Y + {}^0_{-1}e + \bar{\upsilon}$

- γ-Decay :

${}^A_ZX^*$ (Excited state) $\xrightarrow{\gamma\text{–decay}}$ ${}^A_ZX$ (Ground state) $+ {}^0_0\gamma$ + Energy

Composition and Size of Nucleus

- Nucleus of an atom consists of protons and neutrons collectively called nucleons.
- Radius of a nucleus is proportional to its mass number as $R = R_0A^{(1/3)}$. ($R_0 = 1.2$ fm)

Concept of Binding Energy

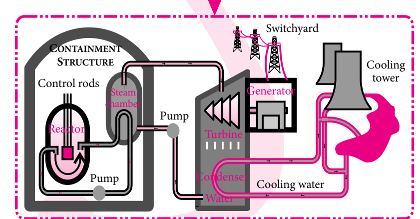
- The binding energy is defined as the surplus energy which the nucleons give up by virtue of their attractions when they bound together to form a nucleus.

$\Delta E_b = [Zm_p + (A-Z)m_n - M_N]c^2$

- **Binding energy per nucleon :** $\therefore E_{bn} = \frac{E_b}{A}$

Nuclear Reactions

- **Nuclear fission :** It is the phenomenon of splitting a heavy nucleus into two or more smaller nuclei of nearly comparable masses.
- **Nuclear fusion :** It is the phenomenon of fusing two or more lighter nuclei to form a single heavy nucleus.



Application of Nuclear Reactions

A. Fission

- Uncontrolled chain reaction: Principle of atomic bombs.
- Controlled chain reaction: Principle of nuclear reactors.

B. Fusion

Nuclear fusion is the source of energy in the Sun and stars.

Then A is equal to

(a) $(1 - \tan^{-1}\theta)(1 - \tan\theta)$

(b) $(1 - \tan\theta)\tan^{-1}(1 - \tan\theta)$

(c) $\tan^{-1}(1 - \tan\theta)$ (d) $\frac{1}{2}\tan^{-1}(1-\tan\theta)$

126. Four fair dice D_1, D_2, D_3, D_4 each having six faces numbered 1, 2, 3, 4, 5 and 6 are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is

(a) $\frac{91}{216}$ (b) $\frac{108}{216}$ (c) $\frac{125}{216}$ (d) $\frac{127}{216}$

127. Let $\vec{v} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{w} = \hat{i} + 3\hat{k}$. If $\vec{u}$ is a unit vector, then the maximum value of the scalar triple product $[\vec{u}\ \vec{v}\ \vec{w}]$ is

(a) -1 (b) $\sqrt{10}+\sqrt{6}$

(c) $\sqrt{59}$ (d) $\sqrt{60}$

128. The equation of the circle having its centre on the line $x + 2y - 3 = 0$ and passing through the point of intersection of the circles $x^2 + y^2 - 2x - 4y + 1 = 0$ and $x^2 + y^2 - 4x - 2y + 4 = 0$ is

(a) $x^2 + y^2 - 6x + 7 = 0$

(b) $x^2 + y^2 - 3x + 4 = 0$

(c) $x^2 + y^2 - 2x - 2y + 1 = 0$

(d) $x^2 + y^2 + 2x - 4y + 4 = 0$

129. The objective function $Z = 4x + 3y$ can be maximized subjected the constraints : $3x + 4y \le 24$, $8x + 6y \le 48$, $x \le 5$, $y \le 6$; $x, y \ge 0$.

(a) at only one point (b) at two points only

(c) at an infinite number of points

(d) none of these

130. The greatest and least values of $(\sin^{-1}x)^3 + (\cos^{-1}x)^3$ are

(a) $-\frac{\pi}{2}, \frac{\pi}{2}$ (b) $-\frac{\pi^3}{8}, \frac{\pi^3}{8}$

(c) $-\frac{\pi^3}{32}, \frac{7\pi^3}{8}$ (d) none of these

131. The mid points of the sides of a triangle are (5, 0), (5, 12) and (0, 12). The orthocentre of this triangle is

(a) (0, 0) (b) (10, 0)

(c) (0, 24) (d) (13/3, 8)

132. If $y = \int_0^x f(t)\sin\{k(x-t)\}dt$, then $\frac{d^2y}{dx^2} + k^2y$ equals

(a) 0 (b) y (c) $kf(x)$ (d) $k^2f(x)$

133. If $z = \cos\theta + i\sin\theta$, then $\frac{z^{2n}-1}{z^{2n}+1} = ?$ (where n is an integer)

(a) $i\cot n\theta$ (b) $i\tan n\theta$ (c) $\tan n\theta$ (d) $\cot n\theta$

134. The fractional part of $\frac{2^{4n}}{15}$ is

(a) $\frac{1}{15}$ (b) $\frac{2}{15}$

(c) $\frac{4}{15}$ (d) none of these

135. The tangent at any point on the curve $x^4 + y^4 = a^4$ cuts off intercepts p and q on the coordinate axes then the value of $p^{-4/3} + q^{-4/3}$ is equal to

(a) $a^{-4/3}$ (b) $a^{-1/2}$

(c) $a^{1/2}$ (d) none of these

136. The equation of the directrix of the parabola $x^2 - 4x - 3y + 10 = 0$ is

(a) $y = -\frac{5}{4}$ (b) $y = \frac{5}{4}$

(c) $y = -\frac{3}{4}$ (d) $y = \frac{3}{4}$

137. $\int \frac{1-\cos x}{\cos x(1+\cos x)}dx =$

(a) $\log|\sec x + \tan x| - 2\tan\frac{x}{2} + C$

(b) $\log|\sec x + \tan x| + 2\tan\frac{x}{2} + C$

(c) $\log|\sec x - \tan x| - \tan\frac{x}{2} + C$

(d) none of these

138. If $f(x) = \begin{vmatrix} 5+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 5+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 5+4\sin 2x \end{vmatrix}$, then

(a) domain of function $f(x) \in (0, \infty)$

(b) range of function $f(x) \in (0, \infty)$

(c) period of function $f(x)$ is 2π

(d) $\lim_{x\to 0}\frac{f(x)-150}{x} = 200$

139. Six cards and six envelopes are numbered 1, 2, 3, 4, 5 and 6. Cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is

(a) 264 (b) 265 (c) 53 (d) 67

140. If $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$, then

(a) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs and $|\vec{a}| = |\vec{c}|$ and $|\vec{b}| = 1$.

(b) $\vec{a}, \vec{b}, \vec{c}$ are not orthogonal to each other.

(c) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs but $|\vec{a}| \neq |\vec{c}|$.

(d) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal but $|\vec{b}| \neq 1$.

141. The equation of the one of the tangents to the curve $y = \cos(x+y)$, $-2\pi \le x \le 2\pi$ that is parallel to the line $x + 2y = 0$, is

(a) $x + 2y = 1$ (b) $x+2y=\dfrac{\pi}{2}$

(c) $x+2y=\dfrac{\pi}{4}$ (d) none of these

142. If $|x| < 1$ and $|y| < 1$, the sum to infinity of the series $x + y$, $(x^2 + xy + y^2)$, $(x^3 + x^2y + xy^2 + y^3)$, ... is

(a) $\dfrac{x+y-xy}{1-x-y+xy}$ (b) $\dfrac{x+y+xy}{1-x-y+xy}$

(c) $\dfrac{x}{1-x}+\dfrac{y}{1-y}$ (d) $\dfrac{(x-y)(x+y-xy)}{1-x-y+xy}$

143. If $4n\alpha = \pi$, then the value of $\tan\alpha \tan 2\alpha \tan 3\alpha \tan 4\alpha \ldots \tan(2n-2)\alpha \tan(2n-1)\alpha$ is

(a) 0 (b) 1

(c) -1 (d) none of these

144. If $x \in R$, then $\dfrac{x^2-x+1}{x^2+x+1}$ takes values in the interval

(a) $\left(\dfrac{1}{3}, 3\right)$ (b) $\left[\dfrac{1}{3}, 3\right]$

(c) $(0, 3)$ (d) none of these

145. The equation of curve, whose slope at any point different from origin is $y+\dfrac{y}{x}$, is

(a) $xy = e^x$ (b) $y = cxe^x (c \neq 0)$

(c) $y = x e^x$ (d) $y + x e^x = c$

146. The eccentric angle of a point on the ellipse $\dfrac{x^2}{6}+\dfrac{y^2}{2}=1$ whose distance from the centre of the ellipse is 2 units, is

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{3\pi}{2}$ (c) $\dfrac{5\pi}{3}$ (d) $\dfrac{7\pi}{6}$

147. A line meets the coordinate axes in A and B. A circle is circumscribed about the ΔAOB. The distance from the end points of the side AB to the line touching the circle at origin O are equal to p and q respectively. The diameter of the circle is

(a) $p(p + q)$ (b) $q(p + q)$

(c) $p + q$ (d) $\dfrac{p+q}{2}$

148. If $f(a) = 2$, $f'(a) = 1$, $g(a) = -1$, $g'(a) = 2$, then the value of $\lim\limits_{x\to a}\dfrac{g(x)f(a)-g(a)f(x)}{x-a}$ is

(a) -5 (b) $\dfrac{1}{5}$ (c) 5 (d) 0

149. If t_1, t_2 and t_3 are distinct, the points $(t_1, 2at_1 + at_1^3)$, $(t_2, 2at_2 + at_2^3)$, $(t_3, 2at_3 + at_3^3)$ are collinear if

(a) $t_1t_2t_3 = 1$ (b) $t_1 + t_2 + t_3 = t_1t_2t_3$

(c) $t_1 + t_2 + t_3 = 0$ (d) $t_1 + t_2 + t_3 = -1$

150. A and B are two independent events. The probability that both A and B occur is 1/6 and the probability that neither of them occurs is 1/3. Then, the probability of the two events are respectively,

(a) 1/2, 1/3 (b) 1/5, 1/6

(c) 1/2, 1/6 (d) 2/3, 1/4

SOLUTIONS

1. (a) : For coefficient of thermal conductivity, K we use

$$\frac{\Delta Q}{\Delta t}=KA\left(\frac{\Delta T}{\Delta x}\right)$$

$$K=\frac{\Delta Q/\Delta t}{A(\Delta T/\Delta x)}=\frac{[ML^2T^{-3}]}{[L^2\theta L^{-1}]}\Rightarrow K=[M^1L^1T^{-3}\theta^{-1}]$$

From $Q = S\,m\theta$

$$S=\frac{Q}{m\theta}=\frac{[ML^2T^{-2}]}{[M\theta]}=[L^2T^{-2}\theta^{-1}]$$

Density, $\rho = [ML^{-3}]$

$$\text{Diffusivity}=\frac{K}{S\rho}=\frac{[M^1L^1T^{-3}\theta^{-1}]}{[L^2T^{-2}\theta^{-1}]\times[ML^{-3}]}=[L^2T^{-1}]$$

2. (c) : Let a = Initial amplitude due to S_1 and S_2 each.
$I_0 = k(4a^2)$, where k is a constant.
After reduction of power of S_1, amplitude due to $S_1 = 0.6a$.
Due to superposition, $a_{max} = a + 0.6a = 1.6\,a$ and
$a_{min} = a - 0.6a = 0.4a$
$I_{max}/I_{min} = (a_{max}/a_{min})^2 = (1.6a/0.4a)^2 = 16.$

3. (b)

4. (c) : Momentum after collision
= Momentum before collision
$= (m_1 + m_2)v = m_1u_1 + m_2u_2$

$(0.50 + 1.00)v = 0.50 \times 2.00 + 1.00 \times 0$

$$v=\frac{1}{1.5}=\frac{2}{3}\text{ m s}^{-1}$$

$$\text{Loss of energy}=\frac{1}{2}m_1u_1^2-\frac{1}{2}(m_1+m_2)v^2$$

$$=\frac{1}{2}\times0.5\times2^2-\frac{1}{2}(0.50+1.0)\left(\frac{2}{3}\right)^2=1-\frac{1}{3}=0.67\text{ J}$$

5. (c) : Intensity at the centre will be zero if path difference $=\dfrac{\lambda}{(\mu-1)}$ or $(\mu-1)t=\dfrac{\lambda}{2}$ or $t=\dfrac{\lambda}{2(\mu-1)}$

6. (a) : Velocity attained after falling freely through first 50 m,

$$v=\sqrt{2gh}=\sqrt{2\times 9.8\times 50}=\sqrt{980}\text{ m s}^{-1}$$

After the parachute opens,

$$u=\sqrt{980}\text{ m s}^{-1},\ v=3\text{ m s}^{-1},\ a=2\text{ m s}^{-2}$$

As $v^2 - u^2 = 2gh$

$$\therefore\quad 9-980=2\times -2\times h \quad\text{or}\quad h=\frac{971}{4}\approx 243\text{ m}$$

Total height = 50 + 243 = 293 m.

7. (d) : Kinetic energy of the escaping body from surface of earth

$$K=\frac{1}{2}mv_e^2=\frac{1}{2}m\times\frac{2GM}{R}=\frac{GMm}{R}$$

Kinetic energy to be given to the body on the surface of the earth $=\dfrac{20}{100}K=\dfrac{1}{5}\dfrac{GMm}{R}$

According to law of conservation of mechanical energy, we have

$$-\frac{GMm}{R}+\frac{1}{5}\frac{GMm}{R}=\frac{GMm}{(nR+R)}$$

or $\dfrac{4}{5R}=\dfrac{1}{(n+1)R}$ or $4n+4=5$ or $n=1/4$

8. (c) : $\because$ av = constant = c

or $v\dfrac{dv}{dt}=c$ or $\displaystyle\int_0^v v\,dv=\int_0^t c\,dt$

or $\dfrac{v^2}{2}=ct$ or $v^2=2ct$ or $v=\sqrt{2ct}$ or $\dfrac{ds}{dt}=\sqrt{2ct}$

or $\displaystyle\int_0^s ds=\sqrt{2c}\int_0^t t^{\frac{1}{2}}dt$ or $s=\sqrt{2c}\left[\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_0^t=\frac{2\sqrt{2c}}{3}\times t^{3/2}$

$\therefore\quad s\propto t^{3/2}$

9. (d)

10. (b) : Given : $H_1 = H_2$

or $\dfrac{u_1^2\sin^2 60^\circ}{2g}=\dfrac{u_2^2\sin^2 45^\circ}{2g}$ or $\dfrac{u_1}{u_2}=\dfrac{\sin 45^\circ}{\sin 60^\circ}$

$$=\frac{1/\sqrt{2}}{\sqrt{3}/2}=\frac{\sqrt{2}}{\sqrt{3}}$$

11. (b) : $N=N_0\,e^{-\lambda/t}$ where $\lambda=\dfrac{0.693}{3.8}=0.18$

$\dfrac{N_0}{20}=N_0e^{-0.18t}$ or $\log_{10}20=0.18\times t\times\log_{10}e$

or $1.3 = 1.18 \times 0.4343 \times t$

or $t=\dfrac{1.3}{0.18\times 0.4343}$ or t = 16.5 days

12. (d) : $q=CV=\left(\dfrac{\varepsilon_0 A}{d}\right)V$

After charging, q will become constant.

$$W=U_f-U_i$$

$$=\frac{q^2}{2C_f}-\frac{q^2}{2C_i}=\frac{q^2}{2}\left(\frac{1}{C_f}-\frac{1}{C_i}\right)$$

$$=\frac{(\varepsilon_0AV/d)^2}{2}\left(\frac{2d}{\varepsilon_0A}-\frac{d}{\varepsilon_0A}\right)=\frac{\varepsilon_0AV^2}{2d}$$

13. (c)

14. (c) : Here, m_1 = 0.01 kg, u_1 = 500 m s^{-1}

m_2 = 2 kg, u_2 = 0, l = 5 m, v_1 = ?, h = 0.1 m

$\therefore$ Velocity of block

$$v_2=\sqrt{2gh}=\sqrt{2\times 10\times 0.1}=1.4\text{ m s}^{-1}$$

Applying principle of conservation of linear momentum, we get

$$m_1v_1+m_2v_2=m_1u_1+m_2u_2$$

$$(0.01)v_1+2\times 1.4=0.01\times 500+0$$

$$v_1=\frac{5-2.8}{0.01}=220\text{ m s}^{-1}$$

15. (d) : Focal length of convex lens, $f_1=f$ (say)

Focal length of concave lens, $f_2=-\dfrac{3f}{2}$

Equivalent focal length, F = 30 cm

As, $\dfrac{1}{F}=\dfrac{1}{f_1}+\dfrac{1}{f_2}$

$\therefore\quad \dfrac{1}{30}=\dfrac{1}{f}-\dfrac{2}{3f}=\dfrac{1}{3f}$ or $f=10$ cm

$\therefore\quad f_1$ = + 10 cm and $f_2=-\dfrac{3}{2}\times 10=-15$ cm

16. (b) : $P=E_vI_v\cos\phi$

$100=200\times 2\cos\phi \Rightarrow \cos\phi=\dfrac{100}{400}=\dfrac{1}{4}$

$$Z=\frac{E_v}{I_v}=\frac{200}{2}=100$$

From $\cos\phi=\dfrac{R}{Z}$, $R=Z\cos\phi$

$$R=100\times\frac{1}{4}=25\,\Omega$$

17. (d)

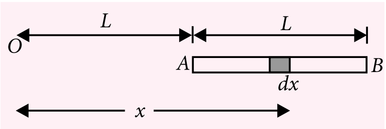
18. (a) : Net flux is due to charges inside surface S only.

19. (d) : Magnetic field at the centre of primary coil $B=\mu_0I_1/2R_1$. Considering it to be uniform, magnetic flux passing through secondary coil is

$$\phi_2 = BA = \frac{\mu_0 I_1}{2R_1}(\pi R_2^2)$$

Now, $M = \frac{\phi_2}{I_1} = \frac{\mu_0 \pi R_2^2}{2R_1}$ $\therefore$ $M \propto \frac{R_2^2}{R_1}$

20. (a) :



Consider a small element of length dx at a distance x from O.

Charge on the element, $dQ = \frac{Q}{L}dx$

Potential at O due to the element is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dQ}{x} = \frac{1}{4\pi\epsilon_0}\frac{Q}{Lx}dx$$

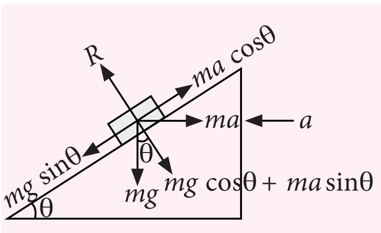
Potential at O due to the rod is

$$V = \int dV = \int_L^{2L} \frac{1}{4\pi\epsilon_0}\frac{Q}{Lx}dx$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{L}[\ln x]_L^{2L} = \frac{Q\ln 2}{4\pi\epsilon_0 L}$$

21. (a)

22. (d): Refer to the figure, $ma\cos\theta = mg\sin\theta$



$a = g\tan\theta$

Total reaction of the wedge on the block is

$R = mg\cos\theta + ma\sin\theta$

$$= mg\cos\theta + m\cdot\frac{g\sin\theta}{\cos\theta}\cdot\sin\theta$$

$$= \frac{mg(\cos^2\theta + \sin^2\theta)}{\cos\theta} = \frac{mg}{\cos\theta}$$

23. (d): $I = \frac{P}{V} = \frac{10}{60} = \frac{1}{6}$ A, $R = \frac{V^2}{P} = \frac{60\times 60}{10} = 360\ \Omega$

$$Z = \frac{V}{I} = \frac{100}{1/6} = 600\ \Omega$$

$X_L^2 = Z^2 - R^2 = 600^2 - 360^2 = (600 + 360)(600 - 360)$

$X_L = \sqrt{960\times 240} = 240\times 2 = 480\ \Omega$

$\omega L = 2\pi\upsilon L = X_L = 480$

$$\therefore\ L = \frac{480}{2\pi\upsilon} = \frac{480}{120\pi} = 1.28\ \text{H}$$

24. (d): Velocity, $v = \alpha\sqrt{s} = \alpha\, s^{1/2}$

Acceleration,

$$a = \frac{dv}{dt} = \frac{1}{2}\alpha s^{-1/2}\cdot\frac{ds}{dt} = \frac{1}{2}\alpha s^{-1/2}\cdot v$$

$$= \frac{1}{2}\alpha s^{-1/2}\cdot\alpha s^{1/2} = \frac{1}{2}\alpha^2$$

Force, $F = ma = \frac{1}{2}m\alpha^2$

Distance covered by the locomotive during first t s,

$$\therefore\ s = ut + \frac{1}{2}at^2 = 0\times t + \frac{1}{2}\times\frac{1}{2}\alpha^2 t^2 = \frac{1}{4}\alpha^2 t^2$$

$\therefore$ Work done,

$$W = Fs = \frac{1}{2}m\alpha^2 \times \frac{1}{4}\alpha^2 t^2 = \frac{1}{8}m\alpha^4 t^2$$

25. (b)

26. (c) : We know that, $\lambda_{min} = \frac{hc}{eV}$

Change in wavelength, $\lambda_2 - \lambda_1 = \frac{hc}{e}\left[\frac{1}{V_1} - \frac{1}{V_2}\right]$

Here, $\lambda_2 - \lambda_1 = 26$ pm $= 26\times 10^{-12}$ m and $V_2 = 1.5\ V_1$

$$\therefore\ 26\times 10^{-12} = \frac{(6.62\times 10^{-34})\times(3\times 10^8)}{1\cdot 6\times 10^{-19}}\left[\frac{1}{V_1} - \frac{1}{1\cdot 5V_1}\right]$$

On solving, $V_1 = 15.9\times 10^3$ V $= 15{\cdot}9$ kV

27. (a) : Charge oscillating sinusoidally is given by

$q = q_0\sin\omega t$

Displacement current, $I_d = \frac{dq}{dt} = q_0\omega\cos\omega t$

$(I_d)_{max} = q_0\omega = q_0\times 2\pi\upsilon = 10^{-6}\times 2\times 3.14\times 10^6 = 6.28$ A

28. (a) : Let l be the length of the river basin with width w and height h. The mass of water flowing in river basin, $m = (wlh)\,\rho$. Downward tangential force on the river water is given as

$F = mg\sin\alpha = wlh\rho g\sin\alpha$

As, $F = \eta A\frac{v}{h} = \eta(wl)\frac{v}{h}$

so $wlh\rho g\sin\alpha = \eta\frac{wlv}{h}$ or $\frac{\eta}{\rho} = \frac{h^2 g\sin\alpha}{v}$

29. (c) : $I_C = \frac{V_C}{R_L} = \frac{0.5\ \text{V}}{800\ \Omega} = 0.625\times 10^{-3}$ A

$$\alpha = \frac{I_C}{I_E} = \frac{I_C}{I_C + I_B}\quad\therefore\quad 0.96 = \frac{0.625\times 10^{-3}}{0.625\times 10^{-3} + I_B}$$

On solving we get, $I_B = 26\ \mu$A

30. (c)

31. (c) : Total heat available in hot water

$= mC\,\Delta\theta = 5 \times 1 \times 20 = 100$ kcal

Heat spent in raising temperature of ice from – 20°C to 0°C

$= m'\,C'\,\Delta\theta = 2 \times 0.5 \times (20) = 20$ kcal.

Heat available for melting ice at 0°C

$= 100 - 20 = 80$ kcal

Mass of ice melted, $m_{ice} = \dfrac{80\text{ kcal}}{80\text{ kcal}} = 1\text{ kg}$

$\therefore$ Mass of ice left = 2 – 1 = 1 kg

Final mass of water left in the container = 5 + 1 = 6 kg

32. (d): On blocking the central part, f is not affected. Only the intensity decreases. Original area of lens,

$$A = \pi\frac{d^2}{4}$$

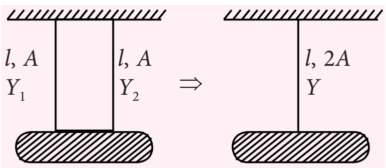
New area of lens that transmits light

$$A_1 = \pi\frac{d^2}{4} - \frac{\pi(d/2)^2}{4} = \frac{3\pi d^2}{16}$$

$$\frac{A_1}{A} = \frac{3\pi d^2}{16}\cdot\frac{4}{\pi d^2} = \frac{3}{4} \quad \therefore \quad \frac{I_1}{I} = \frac{3}{4}\ i.e.,\ I_1 = \frac{3}{4}I$$

33. (a)

34. (b): Equivalent spring constant of a wire is given by



$$k = \frac{YA}{l}$$

$$k_{eq} = k_1 + k_2$$

or $\dfrac{Y(2A)}{l} = \dfrac{Y_1A}{l} + \dfrac{Y_2A}{l}$ or $Y = \dfrac{Y_1 + Y_2}{2}$

35. (a)

36. (a) : Here $r = \dfrac{20}{\pi}$ m, $v = 80$ m s^{-1}, $\omega_0 = 0$

$\theta = 2$ rev $= 4\pi$ rad

As $\omega^2 = \omega_0^2 + 2\alpha\theta \quad \therefore \quad \left(\dfrac{v}{r}\right)^2 = 0 + 2\left(\dfrac{a}{r}\right)\theta$

or $a = \dfrac{v^2}{2r\theta} = \dfrac{(80)^2}{2\times 20/\pi\times 4\pi} = 40\text{ m s}^{-2}$.

37. (d) **38. (c)**

39. (a): In this circuit 20 Ω, 30 Ω and 60 Ω are in parallel. Their effective resistance R_1 is

$$\frac{1}{R_1} = \frac{1}{20} + \frac{1}{30} + \frac{1}{60} = \frac{3+2+1}{60} \text{ or } R_1 = 10\,\Omega$$

Here, 24 Ω and 8 Ω are also in parallel, their effective resistance R_2 will be

$$\frac{1}{R_2} = \frac{1}{24} + \frac{1}{8} = \frac{1+3}{24} = \frac{4}{24} \text{ or } R_2 = 6\,\Omega$$

Net resistance between X and Y = 3 + 10 + 6 + 1 = 20 Ω

Current through $8\,\Omega = \dfrac{48}{8} = 6$ A

Current through $24\,\Omega = \dfrac{48}{24} = 2$ A

$\therefore$ Total current of circuit = 6 + 2 = 8 A

Therefore, potential difference across X and Y

$= 8 \times 20 = 160$ V

40. (c) : Let $y_1 = a\sin 2\pi\upsilon t$

$y_2 = a\sin(2\pi\upsilon t + \phi)$

Distance between the two particles at time t is

$y = y_2 - y_1 = a[\sin(2\pi\upsilon t + \phi) - \sin 2\pi\upsilon t]$

$y = 2a\sin\dfrac{\phi}{2}\cos[2\pi\upsilon t + \phi/2]$

$y_{max} = 2a\sin\dfrac{\phi}{2}\times 1 = a\sqrt{2}$

$\sin\dfrac{\phi}{2} = \dfrac{1}{\sqrt{2}},\ \dfrac{\phi}{2} = \dfrac{\pi}{4}$ or $\phi = \dfrac{\pi}{2}$

41. (c)

42. (b): $>C{=}O \xrightarrow{\text{Wolff–Kishner reduction}} >CH_2$

43. (c) : $n = 3, l = 0$ represents $3s$ orbital.

$n = 3, l = 1$ represents $3p$ orbital.

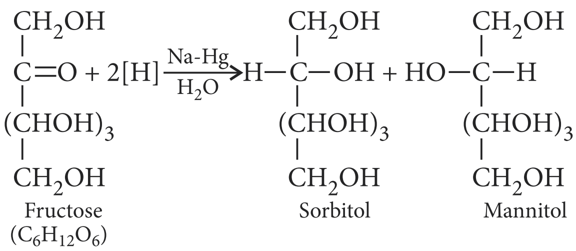
$n = 3, l = 2$ represents $3d$ orbital.

$n = 4, l = 0$ represents $4s$ orbital.

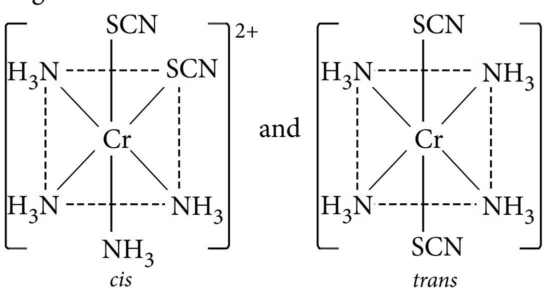
The order of increasing energy of the orbitals is :

$3s < 3p < 4s < 3d$

44. (a) :

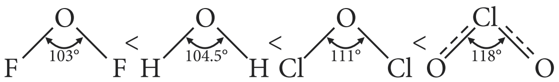


45. (d): $[Cr(SCN)_2(NH_3)_4]^{2+}$ shows geometrical and linkage isomerism.



$[Cr(SCN)_2(NH_3)_4]^{2+}$ and $[Cr(NCS)_2(NH_3)_4]^{2+}$ are linkage isomers. *cis*- and *trans*-forms are possible for both linkage isomers.

46. (c) : H_2O is sp^3 hybridised with bond angle 104.5° due to presence of two lone pairs. OF_2 has structure similar to H_2O with bond angle 103° due to higher electronegativity of fluorine. OCl_2 also has similar structure with bond angle 111° because of steric crowding of two chlorine atoms. However, ClO_2 has π-bond character with an odd electron so that bond angle is 118°. Thus, four compounds can be arranged in order of their bond angles as



47. (a) : For an isothermal process

PV = constant

$P_1V_1 = P_2V_2$

$PV = P_I \times 2V \quad \text{or} \quad \frac{P}{P_I} = 2$ ...(i)

For an adiabatic process

PV^γ = constant ($\gamma = 1.67$)

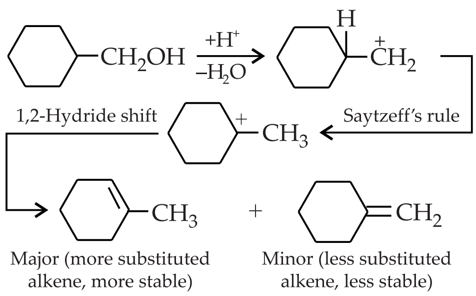
$PV^\gamma = P_{II}(2V)^\gamma \text{ or } \frac{P}{P_{II}} = 2^\lambda$ ...(ii)

Dividing (i) by (ii)

$$\frac{P_{II}}{P_I} = \frac{2}{2^\gamma} \quad \text{or} \quad \frac{P_{II}}{P_I} = \frac{1}{2^{\gamma-1}} = \frac{1}{(2)^{0.67}}$$

48. (b)

49. (b) :



50. (d) : Number of atoms $A = \frac{1}{8}\times 8 = 1$

Number of atoms B at body centre = 1

Number of atoms O at edge centre $= \frac{1}{4}\times 12 = 3$

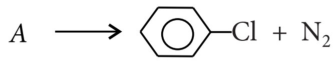
$\therefore$ Formula of compound = ABO_3

51. (b) : In N_2O_4 dimer, N–N bond is formed by pairing of odd electrons on each nitrogen atom in NO_2 and all four N–O bonds become equivalent.

52. (d)

53. (d) : $NaClO_4$ on hydrolysis gives NaOH and strongest $HClO_4$ acid as compared to other salts. Therefore pH of the solution will be lowest in this case.

54. (b) : It follows first order kinetics since half-life is independent of concentration.

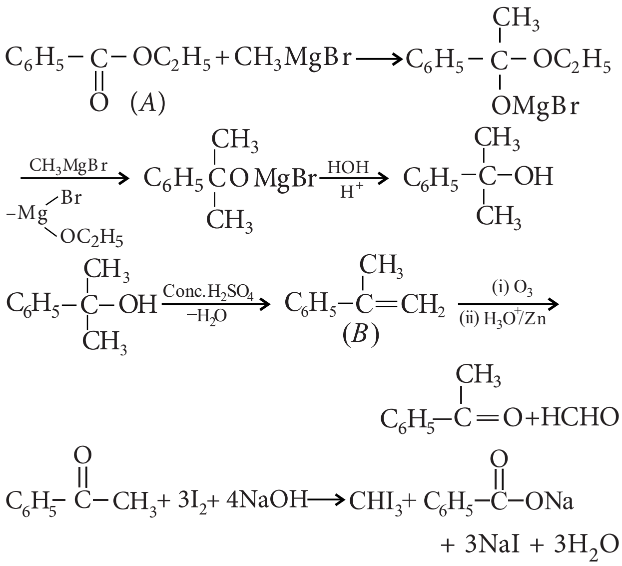


	A		N_2
$t = 0$	a	0	0
t = 10 min	$(a - x)$	x	x
complete	$(a - a)$	a	a

Hence, x = 10 L, a = 50 L

$$\therefore\ k = \frac{2.303}{10}\log\frac{50}{40}\ \text{min}^{-1} = \frac{2.303}{10}\log 1.25\,\text{min}^{-1}$$

55. (a) :



56. (c) : Pure hydrogen is evolved by reacting absolute alcohol and Na.

$C_2H_5OH + Na \longrightarrow C_2H_5ONa + 1/2H_2$

Other statements are correct.

57. (c) : The higher the charge on the metal ion, smaller is the ionic size and more is the complex forming ability. Thus, the degree of complex formation decreases in the order :

$$M^{4+} > MO_2^{2+} > M^{3+} > MO_2^+$$

The higher tendency of complex formation of MO_2^{2+} as compared to M^{3+} is due to high concentration of charge on metal atom M in MO_2^{2+}.

58. (d) : (a) Mixture of 100 mL of M/100 HCl and 100 mL of M/100 NaOH is an exact neutralisation. Hence pH = 7.

(b) After neutralisation, M/10 HCl left = 10 mL

Total volume = 100 mL, dilution = 10 times

$\therefore$ $[H^+] = 10^{-2}$ or pH = 2

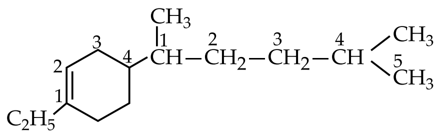
(c) After neutralisation, M/10 NaOH left = 80 mL

Total volume = 100 mL ; pH > 7

(d) After neutralisation, M/5 HCl left = 50 mL

Total volume = 100 mL, dilution = 2 times

$\therefore$ $[H^+] = \frac{1}{10} = 10^{-1}$ or pH = 1

59. (b): 

1-Ethyl-4-(1, 4-dimethylpentyl)cyclohex-1-ene

60. (b): $CsOH + H^+ \longrightarrow Cs^+ + H_2O$; $\Delta H = -13.4$ kcal

Heat of ionisation of CsOH = +0.3 kcal

$CsOH + HF \rightleftharpoons CsF + H_2O$; $\Delta H = -16.4$ kcal

Heat of ionisation of HF = x kcal

Heat of ionisation of CsOH = 0.3 kcal

Heat of neutralisation = –13.7 kcal

$-13.7 + x + 0.3 = -16.4$ or, $x = -3.0$ kcal

61. (b): $\underset{}{Pb(NO_3)_2} + 2NH_4OH \rightarrow \underset{\text{White ppt.}}{Pb(OH)_2} + 2NH_4NO_3$

$Pb(NO_3)_2 + 2NaCl \longrightarrow \underset{\text{White ppt.}}{PbCl_2} + 2NaNO_3$

$Pb(NO_3)_2 + H_2S \longrightarrow \underset{\text{Black ppt.}}{PbS} + 2HNO_3$

62. (a): Val-Gly-Phe-Val-Ala-Val, its random hydrolysis gives Ala-Val dipeptide fragment which indicates the connected presence of Ala and Val.

63. (b): Amongst halogens, chlorine has most negative electron gain enthalpy. So, the correct order is :

Cl(–349) > F(–333) > Br(–325) > I(–296)

64. (c): $\underset{(X)}{Cl_2\uparrow} + H_2O \longrightarrow HClO + HCl$

$AgNO_3 + HCl \longrightarrow \underset{\text{White ppt.}}{AgCl\downarrow} + HNO_3$

$Mg + 2HCl \longrightarrow MgCl_2 + \underset{(Y)}{H_2\uparrow}$

65. (d): N is attached to four carbon atoms.

66. (c): $CaC_2 + N_2 \xrightarrow{1100°C} \underset{\text{Calcium cyanamide}}{CaNCN} + C$

$Ca^{2+}[N = C = N]^{2-}$

linear shape (sp-hybridised C- atom)

On hydrolysis, calcium cyanamide gets converted into urea which then decomposes into ammonia.

CN_2^{2-} is pseudohalide ion.

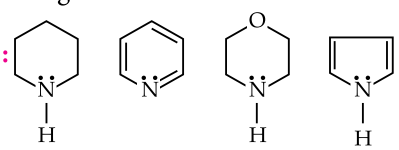
67. (c): $\frac{1}{2}mv^2 = h(\upsilon - \upsilon_0) = h\left(\frac{c}{\lambda} - \frac{c}{\lambda_0}\right)$

$$= hv\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = hc\left(\frac{\lambda_0 - \lambda}{\lambda_0\lambda}\right)$$

or $v^2 = \frac{2hc}{m}\left(\frac{\lambda_0 - \lambda}{\lambda_0\lambda}\right)$ or $v = \left[\frac{2hc}{m}\left(\frac{\lambda_0 - \lambda}{\lambda_0\lambda}\right)\right]^{1/2}$

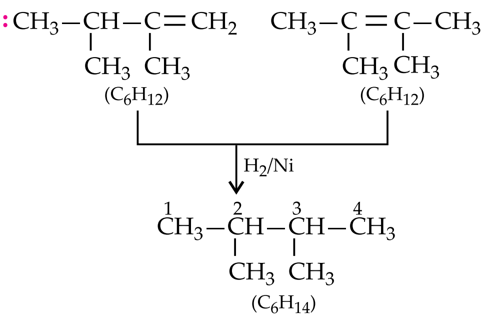
68. (a): From the preparation and molecular formula of Y, it is clear that Y is an alcohol. As Y on heating with copper gives hydrocarbon C_5H_{10}, hence Y must be 3° alcohol. This 3° alcohol having 5 carbon atoms can be obtained only when 4 carbon atoms ketone (X) is treated with CH_3—MgI.

69. (a): In case of isoelectronic species, ionic radii decrease with the increase in the magnitude of the nuclear charge.

70. (d): 

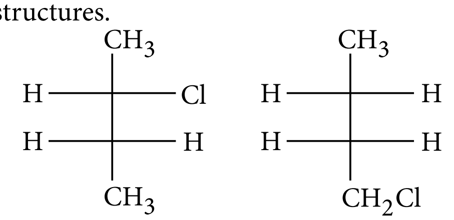
I II III IV

I is most basic as the lone pair occupies an sp^3-hybrid orbital therefore, it is available for donation. The next is the basicity of III as the lone pair is again in sp^3-hybrid orbital but presence of electronegative oxygen atom reduces the electron density on N. In case of IV, the lone pair is a part of aromatic sextet and in II, lone pair occupies an sp^2-hybrid orbital and is not a part of the aromatic sextet. Thus, the order of basicity is I > III > II > IV.

71. (b): 

(2,3-Dimethylbutane)

72. (c): Convert these Newmann projections into open chain structures.



Both structures have same molecular formula C_4H_9Cl, thus these are isomers. However, the two have different groups, *viz* CH_3 and CH_2Cl, so these are neither enantiomers nor diastereomers. Hence, these are structural isomers.

73. (d): $K_a = \frac{[H^+][cit^-]}{[H.\ cit]}$

$8.4 \times 10^{-4} = \frac{[10^{-2}][10^{-2}]}{[H.\ cit]} \Rightarrow 8.4 \times 10^{-4} = \frac{10^{-4}}{[H.\ cit]}$

$[H.\ cit] = \frac{10^{-4}}{8.4 \times 10^{-4}} \Rightarrow 12 \times 10^{-2}$ M

74. (d)

75. (c) : If HCl is used, it is oxidised to Cl_2.

$$2K\overset{+7}{Mn}O_4 + 16HCl \longrightarrow 2KCl + 2\overset{+2}{Mn}Cl_2 + 8H_2O + 5Cl_2$$

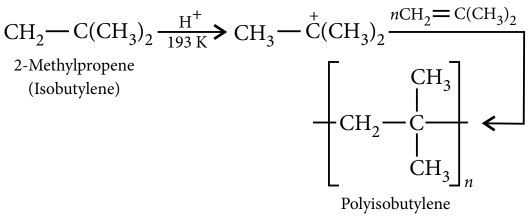
76. (b)

77. (b) : Trihalogenated methane (CHX_3) and a primary amine, *i.e.*;

$$CHX_3 + RNH_2 + 3KOH \longrightarrow RNC + 3KX + 3H_2O$$

78. (a) : The disaccharide is sucrose, with α-glycosidic linkage between C_1 of glucose present in the pyranose form (ring *P*) and C_2 of fructose present in the furanose form (ring *Q*).

79. (b) : Since 3° carbocations are most stable, the best way to obtain polyisobutylene is through cationic polymerisation in the presence of Lewis acid or protonic acid.



80. (a) : Both Fe^{2+} and Fe^{3+} ions form coloured precipitates of metal hydroxides *i.e.*, dirty green $Fe(OH)_2$ and reddish brown $Fe(OH)_3$. Ion with the lower oxidation number, Fe^{2+} changes the colour on standing.

81. (b)	82. (a)	83. (a)	84. (c)
85. (b)	86. (d)	87. (c)	88. (a)
89. (d)	90. (b)	91. (c)	92. (b)
93. (a)	94. (b)	95. (b)	96. (b)
97. (d)	98. (b)	99. (d)	100. (a)
101. (c)	102. (c)	103. (a)	104. (d)
105. (c)			

106. (b) : $\sqrt{9-x^2}$ is defined for $9 - x^2 \geq 0$

$\Rightarrow\ (3 - x)(3 + x) \geq 0$ ... (i)

$\Rightarrow\ -3 \leq x \leq 3$

$\sin^{-1}(3 - x)$ defined for $-1 \leq 3 - x \leq 1$

$\Rightarrow\ -4 \leq -x \leq -2$

$\Rightarrow\ 2 \leq x \leq 4$ ... (ii)

Also, $\sin^{-1}(3 - x) \neq 0 \Rightarrow 3 - x \neq 0$ or $x \neq 3$ ... (iii)

From (i), (ii) and (iii), we get

the domain of $f = ([-3, 3] \cap [2, 4]) - \{3\} = [2, 3)$.

107. (c)

108. (b) : $y = \sqrt{\sin x + y} \Rightarrow y^2 = \sin x + y$

$$2y\cdot\frac{dy}{dx} = \cos x + \frac{dy}{dx} \qquad \therefore\ \frac{dy}{dx} = \frac{\cos x}{2y-1}$$

109. (d) : The function for $x > 2$ can be redefined as

$$f(x) = \int_0^1 1 + (1-t)dt + \int_1^x 1 + (t-1)dt$$

$$= 2(1) - \frac{1}{2} + \frac{x^2}{2} - \frac{1}{2} = \frac{x^2}{2} + 1$$

$$\text{R.H.L.} = \lim_{h\to 0}\left\{\frac{(2+h)^2}{2} + 1\right\} = 3$$

$$\text{L.H.L.} = \lim_{h\to 0}\{5(2-h) - 7\} = 3$$

$\because$ L.H.L = R.H.L $\Rightarrow$ f is continuous at $x = 2$

$$\text{Now, LHD} = \lim_{h\to 0}\frac{f(2) - f(2-h)}{h}$$

$$= \lim_{h\to 0}\frac{3 - 5(2-h) + 7}{h} = 5$$

$$\text{RHD} = \lim_{h\to 0}\frac{f(2+h) - f(2)}{h} = \lim_{h\to 0}\frac{\frac{(2+h)^2}{2} + 1 - 3}{h}$$

$$= \lim_{h\to 0}\left\{\frac{h^2 + 4h + 4 + 2 - 6}{2h}\right\} = \lim_{h\to 0}\left[\frac{h}{2} + 2\right] = 2$$

$\because$ LHD $\neq$ RHD, f is non-differentiable at $x = 2$.

110. (b) : We know that $\text{Var}(aX) = a^2\,\text{Var}(X)$.

So, $\text{Var}(\lambda X) = \lambda^2\,\text{Var}(X) = \lambda^2\sigma^2$.

111. (b) : $e^{\sin x} + e^{\cos x} = 2e^{1/\sqrt{2}}$ ...(i)

$e^{\sin x} > 0$ and $e^{\cos x} > 0$.

Therefore A.M. $\geq$ G.M.

$\Rightarrow\ e^{\sin x} + e^{\cos x} \geq 2\sqrt{e^{\sin x + \cos x}}$

$e^{\sin x} + e^{\cos x} \geq 2e^{1/\sqrt{2}}$ ...(ii)

Since equality holds $\Rightarrow e^{\sin x} = e^{\cos x}$

$\Rightarrow\ \sin x = \cos x$ (since e^x is an increasing function)

$\Rightarrow\ \tan x = 1 \Rightarrow x = m\pi + \frac{\pi}{4}$

Hence, $x = \frac{(4m+1)\pi}{4}$

112. (a) : We have, $\int \frac{2a\sin x + b\sin 2x}{(b + a\cos x)^3}dx$

$$= 2\int\frac{\sin x(a + b\cos x)}{(b + a\cos x)^3}dx = -\frac{2}{a}\int\frac{a + b\left(\frac{t-b}{a}\right)}{t^3}dt$$

[On putting $b + a\cos x = t$ and $-a\sin x\, dx = dt$]

$$= -\frac{2}{a^2}\int\frac{(a^2 - b^2) + bt}{t^3}dt$$

$$= -\frac{2}{a^2}(a^2 - b^2)\int\frac{1}{t^3}dt - \frac{2b}{a^2}\int\frac{1}{t^2}dt$$

$$= \frac{1}{a^2}\frac{(a^2 - b^2)}{t^2} + \frac{2b}{a^2 t} + c$$

113. (a) : We have, $\left(\frac{1+ia}{1-ia}\right)^4 = z$

$$\Rightarrow \left(\frac{1+ia}{1-ia}\right)^4 = \cos\theta + i\sin\theta$$

$$[\because |z| = 1 \therefore z = \cos\theta + i\sin\theta]$$

$$\Rightarrow \frac{1+ia}{1-ia} = (\cos\theta + i\sin\theta)^{1/4}$$

$$\Rightarrow \frac{1+ia}{1-ia} = [\cos(2r\pi+\theta) + i\sin(2r\pi+\theta)]^{1/4},$$

where $r \in Z$

$$\Rightarrow \frac{1+ia}{1-ia} = \cos\left(\frac{2r\pi+\theta}{4}\right) + i\sin\left(\frac{2r\pi+\theta}{4}\right),$$

where $r = 0, 1, 2, 3$

$$\Rightarrow \frac{1+ia}{1-ia} = \cos\alpha + i\sin\alpha, \text{ where } \alpha = \frac{2r\pi+\theta}{4}$$

$$\Rightarrow \frac{2}{2ia} = \frac{\cos\alpha + i\sin\alpha + 1}{\cos\alpha + i\sin\alpha - 1}$$

$$\Rightarrow \frac{1}{ia} = \frac{\cot\frac{\alpha}{2}}{i} \Rightarrow a = \tan\frac{\alpha}{2}$$

$$\Rightarrow a = \tan\left(\frac{2r\pi+\theta}{8}\right), \text{ where } r = 0, 1, 2, 3$$

$$\Rightarrow a = \tan\frac{\theta}{8}, \tan\left(\frac{\pi}{4}+\frac{\theta}{8}\right), -\cot\frac{\pi}{8}, -\cot\left(\frac{\pi}{4}+\frac{\theta}{8}\right)$$

$\Rightarrow$ All roots are real and distinct.

114. (c) : If $n = 1$, then

$$\frac{n^2}{(n+1)^2} = \frac{1}{4}, \frac{n^3}{(n+1)^3} = \frac{1}{8}, \frac{n}{n+1} = \frac{1}{2}, \frac{1}{n+1} = \frac{1}{2}$$

If $n = 2$, then $\frac{n}{n+1} = \frac{2}{3}$ and $\frac{1}{n+1} = \frac{1}{3}$.

So, sum is $\frac{n}{n+1}$.

115. (a) : The non mathematics papers can be arranged in 4! = 24 ways

The mathematics papers can be arranged in ${}^5P_2 = 20$ ways.

The desired number is 20 × 24 = 480.

116. (c) : Locus of the point of intersection of perpendicular tangents to the given ellipse is

$x^2 + y^2 = a^2 + b^2$

Any point on this circle can be taken as

$$P \equiv \left(\sqrt{a^2+b^2}\cos\theta, \sqrt{a^2+b^2}\sin\theta\right)$$

The equation of the chord of contact of tangents from P is

$$\frac{x}{a^2}\sqrt{a^2+b^2}\cos\theta + \frac{y}{b^2}\sqrt{a^2+b^2}\sin\theta = 1$$

Let this line be a tangent to the fixed ellipse $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$

$$\Rightarrow \frac{x}{A}\cos\theta + \frac{y}{B}\sin\theta = 1, \text{ where } A = \frac{a^2}{\sqrt{a^2+b^2}},$$

$$B = \frac{b^2}{\sqrt{a^2+b^2}}$$

$$\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{(a^2+b^2)} \qquad \therefore \text{ An ellipse.}$$

117. (a) : We have,

$$f(x) = \frac{(\sin 5x + \sin x) + (\sin 4x + \sin 2x)}{(\cos 5x + \cos x) + (\cos 4x + \cos 2x)}$$

$$= \frac{2\sin 3x\cos 2x + 2\sin 3x\cos x}{2\cos 3x\cos 2x + 2\cos 3x\cos x}$$

$$= \frac{2\sin 3x(\cos 2x + \cos x)}{2\cos 3x(\cos 2x + \cos x)} = \tan 3x$$

which is a periodic with period $\frac{\pi}{3}$.

118. (b) : The required plane passes through the points having position vectors $\vec{a}_1$ and $\vec{a}_2$ and is parallel to the vector $\vec{b}$. Therefore, if $\vec{r}$ is the position vector of any variable point on the plane, then the vector $\vec{r} - \vec{a}_1$, $\vec{a}_2 - \vec{a}_1$ and $\vec{b}$ are coplanar.

$$\therefore (\vec{r} - \vec{a}_1)\cdot((\vec{a}_2 - \vec{a}_1)\times\vec{b}) = 0$$

$$\Rightarrow \vec{r}\cdot(\vec{a}_2 - \vec{a}_1)\times\vec{b} - \vec{a}_1\cdot(\vec{a}_2\times\vec{b}) = 0$$

$$\Rightarrow \vec{r}\cdot(\vec{a}_2 - \vec{a}_1)\times\vec{b} = [\vec{a}_1\,\vec{a}_2\,\vec{b}]$$

119. (c) : $\int e^{\sec x}(\tan^2 x\sec x + \sec^2 x$

$$+ \tan x\sec^2 x + \tan x\sec x)dx$$

$$= \int e^{\sec x}[(\sec x\tan x(\sec x + \tan x) + (\sec x\tan x + \sec^2 x)]dx$$

$$= \int e^{\sec x}\sec x\tan x\cdot(\sec x + \tan x)dx + \int e^{\sec x}(\sec x\tan x + \sec^2 x)\,dx$$

$$= (\sec x + \tan x)e^{\sec x} - \int(\sec x\tan x + \sec^2 x)e^{\sec x}dx + \int e^{\sec x}(\sec x\tan x + \sec^2 x)dx$$

$$= e^{\sec x}(\sec x + \tan x) + C$$

120. (a) : We have,

$$F(\alpha)F(-\alpha) = \begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \qquad \therefore \quad F(-\alpha) = [F(\alpha)]^{-1}$$

121. (a) : L.H.L. $= \lim_{h\to 0} \{1-(-h)+[-h-1]+[1+h]\}$

$= 1 + (-2) + (1) = 0$

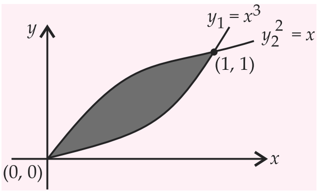
R.H.L. $= \lim_{h\to 0} \{1-h+[h-1]+[1-h]\}$

$= 1 + (-1) + (0) = 0$

122. (a) : Required area

$$= \int_0^1 (y_2 - y_1)\, dx$$

$$= \int_0^1 (\sqrt{x} - x^3)dx = \frac{5}{12}$$



123. (b) : $\Sigma\cos^2 A = 1 \Rightarrow \Sigma(1-\sin^2 A) = 1 \Rightarrow \Sigma\sin^2 A = 2$

124. (b) : $\Delta = \begin{vmatrix} x & a & b & 1 \\ \lambda & x & b & 1 \\ \lambda & \mu & x & 1 \\ \lambda & \mu & \nu & 1 \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$, $R_3 \to R_3 - R_4$.

Then $(x-\lambda)(x-\mu)(x-\nu) = 0$

$\therefore$ x is independent of a and b

125. (b) **126. (a)**

127. (c) : $[\vec{u}\ \vec{v}\ \vec{w}] = \vec{u}\cdot(\vec{v}\times\vec{w}) \le |\vec{u}||\vec{v}\times\vec{w}| = |\vec{v}\times\vec{w}|$

But $\vec{v}\times\vec{w} = 3\hat{i} - 7\hat{j} - \hat{k} \Rightarrow |\vec{v}\times\vec{w}| = \sqrt{59}$

$\therefore \ [\vec{u}\ \vec{v}\ \vec{w}] \le \sqrt{59}$

128. (a) : The equation of a circle passing through the intersection of two given circles is

$(x^2 + y^2 - 2x - 4y + 1) + \lambda(x^2 + y^2 - 4x - 2y + 4) = 0$

$$\Rightarrow x^2 + y^2 - 2x\left(\frac{1+2\lambda}{1+\lambda}\right) - 2y\frac{(2+\lambda)}{1+\lambda} + \left(\frac{1+4\lambda}{1+\lambda}\right) = 0 \quad \text{...(i)}$$

Coordinates of the centre are $\left(\frac{1+2\lambda}{1+\lambda}, \frac{2+\lambda}{1+\lambda}\right)$.

Since the centre lies on $x + 2y - 3 = 0$.

$\therefore \quad 1 + 2\lambda + 4 + 2\lambda - 3 - 3\lambda = 0 \Rightarrow \lambda = -2$

Putting $\lambda = -2$ in (i), we obtain the required circle $x^2 + y^2 - 6x + 7 = 0$.

129. (c)

130. (c) : We have, $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$

$= (\sin^{-1}x + \cos^{-1}x)^3 - 3\sin^{-1}x\cos^{-1}x(\sin^{-1}x + \cos^{-1}x)$

$$= \frac{\pi^3}{8} - 3(\sin^{-1}x\cos^{-1}x)\frac{\pi}{2}$$

$$= \frac{\pi^3}{8} - \frac{3\pi}{2}\sin^{-1}x\left(\frac{\pi}{2} - \sin^{-1}x\right)$$

$$= \frac{\pi^3}{8} - \frac{3\pi^2}{4}\sin^{-1}x + \frac{3\pi}{2}(\sin^{-1}x)^2$$

$$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[(\sin^{-1}x)^2 - \frac{\pi}{2}\sin^{-1}x\right]$$

$$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[\left(\sin^{-1}x - \frac{\pi}{4}\right)^2\right] - \frac{3\pi^3}{32}$$

$$= \frac{\pi^3}{32} + \frac{3\pi}{2}\left(\sin^{-1}x - \frac{\pi}{4}\right)^2$$

So, the least value is $= \frac{\pi^3}{32}$

Since, $\left(\sin^{-1}x - \frac{\pi}{4}\right)^2 \le \left(\frac{3\pi}{4}\right)^2$

Therefore, the greatest value is $\frac{\pi^3}{32} + \frac{9\pi^2}{16}\times\frac{3\pi}{2} = \frac{7\pi^3}{8}$

131. (a)

132. (c) : We have, $\frac{d}{dt}\left(\int_a^b f(x,\ t)dx\right)$

$$= \int_a^b \frac{\partial f}{\partial t}dx + \frac{\partial b}{\partial t}f(b,t) - \frac{\partial a}{\partial t}f(a,t)$$

$\therefore \quad y = \int_0^x f(t)\sin\{k(x-t)\}dt$

$\Rightarrow \frac{dy}{dx} = \int_0^x kf(t)\cos\{k(x-t)\}dt$

Differentiating again w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \int_0^x -k^2 f(t)\sin\{k(x-t)\}dt$$

$$+ 1\cdot kf(x)\{\cos\{k(x-x)\} - 0$$

$$\frac{d^2y}{dx^2} = -k^2 y + kf(x) \Rightarrow \frac{d^2y}{dx^2} + k^2 y = kf(x)$$

133. (b) : We have,

$$\frac{z^{2n}-1}{z^{2n}+1} = \frac{(\cos\theta + i\sin\theta)^{2n} - 1}{(\cos\theta + i\sin\theta)^{2n} + 1}$$

$$= \frac{\cos 2n\theta + i\sin 2n\theta - 1}{\cos 2n\theta + i\sin 2n\theta + 1}$$

(Using De Moivre's theorem)

$$= \frac{(1 - 2\sin^2 n\theta) + 2i\sin n\theta\cos n\theta - 1}{(2\cos^2 n\theta - 1) + 2i\sin n\theta\cos n\theta + 1}$$

$$= \frac{i\sin n\theta\cos n\theta + i^2\sin^2 n\theta}{\cos^2 n\theta + i\sin n\theta\cos n\theta} \quad (\because\ i^2 = -1)$$

$$= \frac{i\sin n\theta(\cos n\theta + i\sin n\theta)}{\cos n\theta(\cos n\theta + i\sin n\theta)} = i\tan n\theta$$

134. (a) : $\frac{2^{4n}}{15} = \frac{16^n}{15} = \frac{(1+15)^n}{15}$

$$= \frac{1 + {}^nC_1 15 + {}^nC_2 15^2 + ... + {}^nC_n 15^n}{15}$$

$$= \frac{1+15k}{15}, \text{ where } k \in N, = \frac{1}{15} + k$$

$$\therefore \left\{\frac{2^{4n}}{15}\right\} = \left\{\frac{1}{15} + k\right\} = \frac{1}{15}$$

135. (a) : $(X - x)\left(\frac{\partial f}{\partial x}\right) + (Y - y)\left(\frac{\partial f}{\partial y}\right) = 0$

$(X - x)(4x^3) + (Y - y)4y^3 = 0$

or, $Xx^3 + Yy^3 = x^4 + y^4 = a^4$ ... (i)

If intercepts are p and q, then $(p, 0)$ and $(0, q)$ satisfy (i)

$\therefore \quad px^3 = a^4,\ qy^3 = a^4$

$p^{-4/3} + q^{-4/3} = (a^4)^{-4/3}(x^4 + y^4) = a^{-4/3}$

136. (b) : The given equation can be written as,

$(x - 2)^2 = 3(y - 2)$

Shifting the origin at (2, 2), this equation reduces to $X^2 = 3Y$, where, $x = X + 2$, $y = Y + 2$.

Thus directrix of this parabola with reference to new axes is $Y = -a$, where $a = \frac{3}{4}$

$$\Rightarrow \quad y - 2 = -\frac{3}{4} \Rightarrow y = \frac{5}{4}$$

137. (a) : $\frac{1-\cos x}{(\cos x)(1+\cos x)} = \frac{1-t}{t(1+t)}$ (where $\cos x = t$)

(Resolving into partial fractions)

$$= \frac{1}{t} - \frac{2}{1+t} = \frac{1}{\cos x} - \frac{2}{1+\cos x}$$

$$\therefore \quad I = \int\left(\frac{1}{\cos x} - \frac{2}{1+\cos x}\right)dx$$

$$= \int\left(\sec x - \sec^2\frac{x}{2}\right)dx$$

$$= \log|\sec x + \tan x| - 2\tan\frac{x}{2} + C$$

138. (d) : Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$f(x) = \begin{vmatrix} 5 & -5 & 0 \\ 0 & 5 & -5 \\ \sin^2 x & \cos^2 x & 5 + 4\sin 2x \end{vmatrix}$$

$$= 25\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \sin^2 x & \cos^2 x & 5 + 4\sin 2x \end{vmatrix}$$

$\Rightarrow f(x) = 150 + 100\sin 2x$.

Clearly,

(a) domain $(-\infty, \infty)$ (b) range [50, 250]

(c) period π (d) $\lim_{x\to 0}\frac{f(x) - 150}{x} = 200$

139. (c)

140. (a) : $\vec{c} = \vec{a} \times \vec{b} \Rightarrow \vec{c} \perp \vec{a}$ and $\vec{c} \perp \vec{b}$;

$\vec{a} = \vec{b} \times \vec{c} \Rightarrow \vec{a} \perp \vec{b}$ and $\vec{a} \perp \vec{c}$

$\Rightarrow \vec{a} \perp \vec{b} \perp \vec{c}$

Now, $\vec{a} \times \vec{b} = \vec{c} \Rightarrow (\vec{b} \times \vec{c}) \times \vec{b} = \vec{c}$ $\quad [\because\ \vec{a} = \vec{b} \times \vec{c}]$

$\Rightarrow (\vec{b}\cdot\vec{b})\vec{c} - (\vec{b}\cdot\vec{c})\vec{b} = \vec{c}$

$\Rightarrow |\vec{b}|^2\ \vec{c} = \vec{c}$ $\quad \left[\because\ |\vec{b}| \perp \vec{c}\ \therefore\ \vec{b}\cdot\vec{c} = 0\right]$

$\Rightarrow |\vec{b}| = 1$

Also, $\vec{c} = \vec{a} \times \vec{b} \Rightarrow |\vec{c}| = |\vec{a} \times \vec{b}|$

$\Rightarrow |\vec{c}| = |\vec{a}||\vec{b}|\sin(\pi/2)$

$\Rightarrow |\vec{c}| = |\vec{a}|$ $\quad [\because\ |\vec{b}| = 1]$

141. (b) : We have, $y = \cos(x + y)$.

Differentiating with respect to x, we get

$$\frac{dy}{dx} = -\sin(x+y)\left(1 + \frac{dy}{dx}\right) \quad ...(i)$$

Since the tangent is parallel to $x + 2y = 0$

$\therefore$ Slope of the tangent $= -\frac{1}{2} \Rightarrow \frac{dy}{dx} = -\frac{1}{2}$

Putting $\frac{dy}{dx} = -\frac{1}{2}$ in (i), we get

$$-\frac{1}{2} = -\sin(x+y)\left(1 - \frac{1}{2}\right) \Rightarrow \sin(x+y) = 1 \quad ...(ii)$$

Squaring and adding, $y = \cos(x + y)$ and $1 = \sin(x + y)$, we get

$y^2 + 1 = 1 \Rightarrow y^2 = 0 \Rightarrow y = 0$

Putting $y = 0$ in $y = \cos(x + y)$ and $\sin(x + y) = 1$, we get

$\sin x = 1$ and $\cos x = 0 \Rightarrow x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$

Thus, the points on the curve $y = \cos(x + y)$ where tangents are parallel to $x + 2y = 0$ are

$$\left(\frac{\pi}{2}, 0\right), \left(-\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{2}, 0\right), \left(-\frac{3\pi}{2}, 0\right)$$

$\therefore$ The equation of the tangent at $\left(\frac{\pi}{2}, 0\right)$ is

$$y - 0 = -\frac{1}{2}\left(x - \frac{\pi}{2}\right) \Rightarrow x + 2y = \frac{\pi}{2}.$$

142. (a) : The given series is

$$\frac{x^2 - y^2}{x - y} + \frac{x^3 - y^3}{x - y} + \frac{x^4 - y^4}{x - y} + \ldots \text{ to } \infty$$

$$= \frac{1}{x - y}\{(x^2 + x^3 + x^4 + \ldots) - (y^2 + y^3 + y^4 + \ldots)\}$$

$$= \frac{1}{x - y}\left\{\frac{x^2}{1 - x} - \frac{y^2}{1 - y}\right\} = \frac{x + y - xy}{1 - x - y + xy}$$

143. (b) : We have,

$\tan\alpha \tan 2\alpha \tan 3\alpha \ldots \tan(2n - 2)\alpha \tan(2n - 1)\alpha$
$= \{\tan\alpha \tan(2n - 1)\alpha\}\{\tan 2\alpha \tan(2n - 2)\alpha\}\ldots$
$\{\tan(n - 1)\alpha \tan(n + 1)\alpha\} \cdot \tan n\alpha$

$$= \left\{\tan\alpha \tan\left(\frac{\pi}{2} - \alpha\right)\right\}\left\{\tan 2\alpha \tan\left(\frac{\pi}{2} - 2\alpha\right)\right\}$$

$$\ldots \tan\frac{\pi}{4} = 1$$

144. (b) : Let, $\dfrac{x^2 - x + 1}{x^2 + x + 1} = m$

Discriminant ≥ 0

$\Rightarrow (1 + m)^2 - 4(1 - m)^2 \geq 0$

$$\Rightarrow (m - 3)\left(m - \frac{1}{3}\right) \leq 0 \Rightarrow m \in \left[\frac{1}{3}, 3\right]$$

145. (b) : By given condition

$$\frac{dy}{dx} = y + \frac{y}{x} \Rightarrow \frac{dy}{dx} = y\left(1 + \frac{1}{x}\right)$$

$$\Rightarrow \frac{dy}{y} = \left(1 + \frac{1}{x}\right)dx \Rightarrow \log y = x + \log x + \log c$$

$$\Rightarrow \log\left(\frac{y}{cx}\right) = x \Rightarrow y = cxe^x \ (c \neq 0)$$

146. (a) : Let θ be the eccentric angle of the point P. Then the coordinates of P are $(\sqrt{6}\cos\theta, \sqrt{2}\sin\theta)$.

The centre of the ellipse is at the origin. It is given that $OP = 2$ units

$\therefore$ $OP = 2$ units

$$\Rightarrow \sqrt{6\cos^2\theta + 2\sin^2\theta} = 2;$$

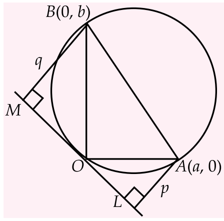
$\Rightarrow 6\cos^2\theta + 2\sin^2\theta = 4 \Rightarrow 3\cos^2\theta + \sin^2\theta = 2$

$$\Rightarrow 2\sin^2\theta = 1 \Rightarrow \sin^2\theta = \frac{1}{2} \Rightarrow \sin\theta = \pm\frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \pm\frac{\pi}{4}$$

147. (c) : Equation of the circle is $x^2 + y^2 - ax - by = 0$

Tangent at (0, 0) is $ax + by = 0$



$$\Rightarrow p = \frac{a^2}{\sqrt{a^2 + b^2}}, \quad q = \frac{b^2}{\sqrt{a^2 + b^2}}$$

Diameter $AB = \sqrt{a^2 + b^2} = p + q$

148. (c) : $\displaystyle\lim_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{x - a}$

$$= \lim_{x \to a} \frac{(g(x) - g(a))f(a) - g(a)(f(x) - f(a))}{x - a}$$

$$= f(a)\lim_{x \to a} \frac{g(x) - g(a)}{x - a} - g(a)\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$= f(a)g'(a) - g(a)f'(a)$
$= (2)(2) - (-1)(1) = 5$

149. (c) : The given points are collinear, if

$$\begin{vmatrix} t_1 & 2at_1 + at_1^3 & 1 \\ t_2 & 2at_2 + at_2^3 & 1 \\ t_3 & 2at_3 + at_3^3 & 1 \end{vmatrix} = 0 \Rightarrow a\begin{vmatrix} t_1 & 2t_1 + t_1^3 & 1 \\ t_2 & 2t_2 + t_2^3 & 1 \\ t_3 & 2t_3 + t_3^3 & 1 \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} t_1 & 2t_1 + t_1^3 & 1 \\ t_2 - t_1 & 2(t_2 - t_1) + (t_2^3 - t_1^3) & 0 \\ t_3 - t_1 & 2(t_3 - t_1) + (t_3^3 - t_1^3) & 0 \end{vmatrix} = 0$$

$$\Rightarrow (t_2 - t_1)(t_3 - t_1)\begin{vmatrix} t_1 & 2t_1 + t_1^3 & 1 \\ 1 & 2 + t_2^2 + t_1^2 + t_2t_1 & 0 \\ 1 & 2 + t_3^2 + t_1^2 + t_3t_1 & 0 \end{vmatrix} = 0$$

$$\Rightarrow (t_2 - t_1)(t_3 - t_1)(t_3 - t_2)(t_3 + t_2 + t_1) = 0$$

$$\Rightarrow t_1 + t_2 + t_3 = 0 \qquad [\because t_1 \neq t_2 \neq t_3]$$

150. (a) : $P(A \cap B) = \dfrac{1}{6}$, $P(\bar{A} \cap \bar{B}) = \dfrac{1}{3}$

$$\Rightarrow P(A)P(B) = \frac{1}{6} \qquad \ldots\text{(i)}$$

$$\text{and } P(\overline{A \cup B}) = \frac{1}{3} \Rightarrow P(A \cup B) = \frac{2}{3}$$

$$\therefore P(A) + P(B) - P(A \cap B) = \frac{2}{3}$$

$$\Rightarrow P(A) + P(B) = \frac{5}{6} \qquad \ldots\text{(ii)}$$

Solving (i) and (ii), $P(A) = \dfrac{1}{2}$, $P(B) = \dfrac{1}{3}$

$\diamond\diamond$

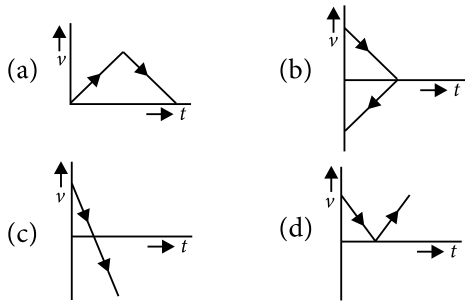
JEE MAIN

SOLVED PAPER 2017

1. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of

   (a) 9 (b) $\frac{1}{9}$ (c) 81 (d) $\frac{1}{81}$

2. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity versus time?



3. A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -k\,v^2$. Its initial speed is $v_0 = 10$ m s^{-1}. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be

   (a) 10^{-3} kg m^{-1} (b) 10^{-3} kg s^{-1}
   (c) 10^{-4} kg m^{-1} (d) 10^{-1} kg m^{-1} s^{-1}

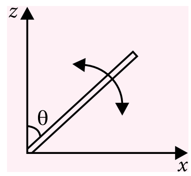
4. A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec will be

   (a) 4.5 J (b) 22 J (c) 9 J (d) 18 J

5. The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I. What is the ratio l/R such that the moment of inertia is minimum ?

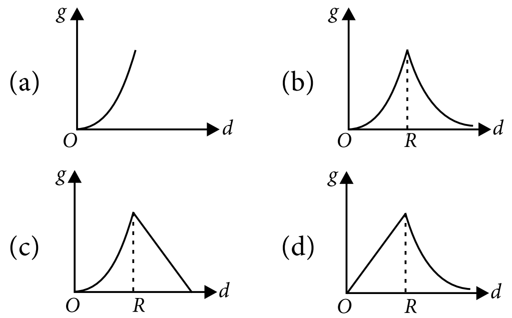
   (a) $\sqrt{\frac{3}{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\frac{3}{\sqrt{2}}$

6. A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is



   (a) $\frac{3g}{2l}\sin\theta$ (b) $\frac{2g}{3l}\sin\theta$
   (c) $\frac{3g}{2l}\cos\theta$ (d) $\frac{2g}{3l}\cos\theta$

7. The variation of acceleration due to gravity g with distance d from centre of the Earth is best represented by (R = Earth's radius)



8. A copper ball of mass 100 g is at a temperature T. It is dropped in a copper calorimeter of mass 100 g, filled with 170 g of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by
   (Given : room temperature = 30°C, specific heat of copper = 0.1 cal g^{-1} °C^{-1})

   (a) 800°C (b) 885°C
   (c) 1250°C (d) 825°C

9. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is

the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by

(a) $\frac{P}{3\alpha K}$ (b) $\frac{P}{\alpha K}$ (c) $\frac{3\alpha}{PK}$ (d) $3PK\alpha$

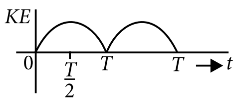
10. C_P and C_V are specific heats at constant pressure and constant volume respectively. It is observed that
$C_P - C_V = a$ for hydrogen gas
$C_P - C_V = b$ for nitrogen gas
The correct relation between a and b is

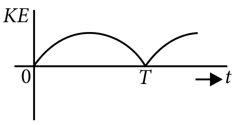
(a) $a = \frac{1}{14}b$ (b) $a = b$
(c) $a = 14b$ (d) $a = 28b$

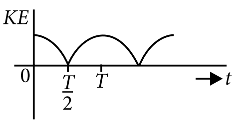
11. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to the sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If N_i and N_f are the number of molecules in the room before and after heating, then $N_f - N_i$ will be

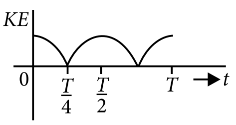
(a) -1.61×10^{23} (b) 1.38×10^{23}
(c) 2.5×10^{25} (d) -2.5×10^{25}

12. A particle is executing simple harmonic motion with a time period T. At time $t = 0$, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like

(a)


(b)


(c)


(d)


13. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer ?
(speed of light = 3×10^8 m s^{-1})

(a) 10.1 GHz (b) 12.1 GHz
(c) 17.3 GHz (d) 15.3 GHz

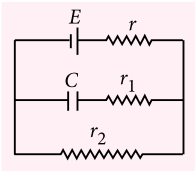
14. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\hat{j}$ it experiences a torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is

(a) 30° (b) 45° (c) 60° (d) 90°

15. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is

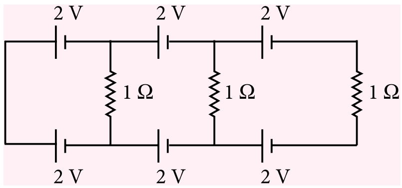
(a) 2 (b) 16 (c) 24 (d) 32

16. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be



(a) CE (b) $CE\frac{r_1}{(r_2+r)}$

(c) $CE\frac{r_2}{(r+r_2)}$ (d) $CE\frac{r_1}{(r_1+r)}$

17.


In the above circuit the current in each resistance is

(a) 1 A (b) 0.25 A (c) 0.5 A (d) 0 A

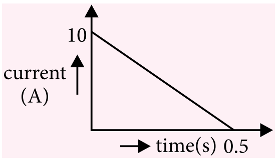
18. A magnetic needle of magnetic moment 6.7×10^{-2} A m^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is

(a) 6.65 s (b) 8.89 s
(c) 6.98 s (d) 8.76 s

19. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0-10 V is

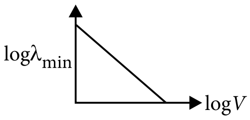
(a) 1.985×10^3 Ω (b) 2.045×10^3 Ω
(c) 2.535×10^3 Ω (d) 4.005×10^3 Ω

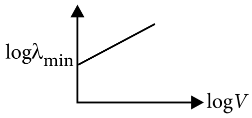
20. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is

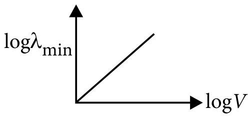


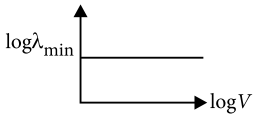
(a) 200 Wb
(b) 225 Wb
(c) 250 Wb
(d) 275 Wb

21. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-ray in the spectrum, the variation of $\log\lambda_{min}$ with $\log V$ is correctly represented in

(a) 

(b) 

(c) 

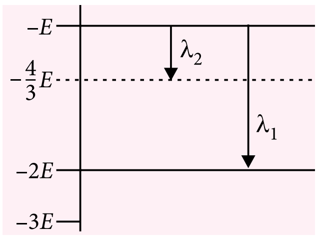
(d) 

22. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is
(a) real and at a distance of 40 cm from convergent lens.
(b) virtual and at a distance of 40 cm from convergent lens.
(c) real and at a distance of 40 cm from the divergent lens.
(d) real and at a distance of 6 cm from the convergent lens.

23. In a Young's double slit experiment, slits are separated by 0.5 mm and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is
(a) 1.56 mm (b) 7.8 mm
(c) 9.75 mm (d) 15.6 mm

24. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is
(a) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$ (b) $\frac{\lambda_A}{\lambda_B} = 2$
(c) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$ (d) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$

25. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \frac{\lambda_1}{\lambda_2}$ is given by



(a) $r = \frac{4}{3}$ (b) $r = \frac{2}{3}$ (c) $r = \frac{3}{4}$ (d) $r = \frac{1}{3}$

26. A radioactive nucleus A with a half-life T, decays into a nucleus B. At $t = 0$, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then t is given by
(a) $t = \frac{T}{2}\frac{\log 2}{\log(1.3)}$ (b) $t = T\frac{\log(1.3)}{\log 2}$
(c) $t = T\log(1.3)$ (d) $t = \frac{T}{\log(1.3)}$

27. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be
(a) 45° (b) 90° (c) 135° (d) 180°

28. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth $(\Delta\omega_m)$ of the signal is such that $\Delta\omega_m << \omega_c$. Which of the following frequencies is not contained in the modulated wave?
(a) ω_m (b) ω_c
(c) $\omega_m + \omega_c$ (d) $\omega_c - \omega_m$

29. Which of the following statement is false ?
(a) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude.

(b) In a balanced Wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.
(c) A rheostat can be used as a potential divider.
(d) Kirchhoff's second law represents energy conservation.

30. The following observations were taken for determining surface tension T of water by capillary method:
Diameter of capillary, $D = 1.25 \times 10^{-2}$ m
rise of water, $h = 1.45 \times 10^{-2}$ m
Using $g = 9.80$ m s^{-2} and the simplified relation, $T = \frac{rhg}{2} \times 10^3$ N m^{-1}, the possible error in surface tension is closest to
(a) 0.15% (b) 1.5% (c) 2.4% (d) 10%

SOLUTIONS

1. (a) : We know stress is given by

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{mg}{A} = \frac{\rho V g}{A} \qquad \left(\because \rho = \frac{m}{V}\right)$$

i.e., stress $\propto \frac{L^3}{L^2}$ (L is the linear dimension.)

$\Rightarrow$ Stress $\propto L$

Since linear dimension increases by a factor of 9, stress also increases by a factor of 9.

2. (c) : Velocity of the body going upwards is given by

$v = v_0 - gt$ (v_0 = initial velocity)

Hence, the graph between velocity and time should be a straight line with negative slope (g) and intercept v_0.
Also, during the whole motion, acceleration of the body is constant *i.e.,* slope should be constant. Hence option (c) is correct.

3. (c) : Initial K.E. of the body, $K_i = \frac{1}{2}mv_0^2$

Final K.E. of the body, $K_f = \frac{1}{8}mv_0^2$

Now, $\frac{K_i}{K_f} = 4$

Let initial velocity = v_i
Final velocity = v_f

$$\frac{v_i^2}{v_f^2} = \frac{4}{1} \quad \text{or} \quad v_f = \frac{v_i}{2}$$

$$\Rightarrow \quad v_f = \frac{v_0}{2} = \frac{10}{2} = 5 \text{ m s}^{-1} \text{(Given } v_0 = 10 \text{ m s}^{-1})$$

Also, $F = -kv^2$

$$\Rightarrow \quad m\frac{dv}{dt} = -kv^2; \ \frac{-m}{k}\frac{dv}{v^2} = dt$$

Integrating both sides

$$\Rightarrow \quad \frac{-m}{k}\int_5^{10}\frac{dv}{v^2} = \int_0^{10} dt; \ \frac{-m}{k}\left[\frac{-1}{v}\right]_{10}^{5} = [t]_0^{10}$$

$$\Rightarrow \quad \frac{-10^{-2}}{k}\left(\frac{-1}{5} + \frac{1}{10}\right) = (10-0); \ \frac{10^{-3}}{k} = 10$$

$\therefore$ $k = 10^{-4}$ kg m^{-1}

4. (a) : We have been given, $F = 6t$

or $m\frac{dv}{dt} = 6t$

Rearranging and integrating both sides

$$\Rightarrow \quad \int_0^v dv = 6\int_0^1 t\,dt \qquad (\because m = 1 \text{ kg})$$

$$\Rightarrow \quad v = 6\left[\frac{t^2}{2}\right]_0^1 \ \Rightarrow \ v = \frac{6}{2} = 3 \text{ m s}^{-1}$$

Work done by the force during the first 1 s is given by the change in the kinetic energy of the object.

$$W = \Delta K.E. = \frac{1}{2}mv^2 \ \Rightarrow \ W = \frac{1}{2} \times 1 \times (3)^2 = 4.5 \text{ J}$$

5. (a) : Moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is given by

$$I = \frac{1}{12}ml^2 + \frac{mR^2}{4}$$

$$\text{or} \quad I = \frac{m}{4}\left(\frac{1}{3}l^2 + R^2\right) \qquad \text{... (i)}$$

Also, $m = \rho V = \rho\pi R^2 l$ or $R^2 = \frac{m}{\rho\pi l}$

Substitute R^2 in eqn. (i), we get

$$I = \frac{m}{4}\left(\frac{l^2}{3} + \frac{m}{\rho\pi l}\right)$$

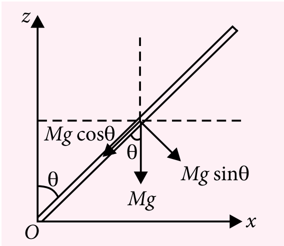
For moment of inertia to be maximum or minimum,

$$\frac{dI}{dl} = 0 \ \Rightarrow \ \frac{m}{4}\left(\frac{2l}{3} - \frac{m}{\rho\pi l^2}\right) = 0$$

$$\Rightarrow \quad \frac{2l}{3} = \frac{R^2}{l} \ \Rightarrow \ \frac{l}{R} = \sqrt{\frac{3}{2}} \quad \left(\text{Using } \frac{R^2}{l} = \frac{m}{\rho\pi l^2}\right)$$

6. (a) : The torque of the weight Mg of the rod about the pivot O is given by

$$\tau = Mg\sin\theta \times \left(\frac{l}{2}\right) \qquad \text{...(i)}$$



($Mg\cos\theta$ is passing through the pivot O. Hence, its contribution to the torque will be zero.)
Also,

$$\tau = I\alpha \quad \text{...(ii)}$$

$$\therefore \quad I\alpha = Mg\sin\theta \times \left(\frac{l}{2}\right) \quad \text{(Using (i) and (ii))}$$

Now, moment of inertia of the rod about the pivot O is

$$I = \frac{1}{3}Ml^2$$

$$\therefore \quad \frac{1}{3}Ml^2\alpha = Mg\sin\theta\left(\frac{l}{2}\right) \Rightarrow \alpha = \frac{3}{2}\frac{g}{l}\sin\theta$$

7. **(d):** Variation of g inside the earth's surface at depth h is given by

$$g' = g\left(1 - \frac{h}{R}\right) = g\left(\frac{R-h}{R}\right) = \frac{gd}{R}$$

where d is the distance from the centre of the Earth.
i.e., $g \propto d$ (inside the earth's surface)
Acceleration due to gravity outside the Earth's surface at height h is

$$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} = \frac{gR^2}{d^2} \quad \textit{i.e.,} \quad g' \propto \frac{1}{d^2}$$

Hence, option (d) is correct.

8. **(b):** Heat lost by the copper ball,
$Q = ms\Delta T = 100(0.1)(T - 75)$ cal
Heat gained by the water,
$Q_1 = 170(1)(75 - 30) = 7650$ cal
Heat gained by the copper calorimeter,
$Q_2 = 100(0.1)45 = 450$ cal
Now, $Q = Q_1 + Q_2$
$100(0.1)(T - 75) = 7650 + 450$
$10(T - 75) = 8100 \Rightarrow T = 885°C$

9. **(a):** Bulk modulus of the gas is given by

$$K = \frac{-P}{\left(\frac{\Delta V}{V_0}\right)}$$

(Here negative sign indicates the decrease in volume with pressure)

$$\text{or} \quad \frac{\Delta V}{V_0} = \frac{P}{K} \quad \text{... (i)}$$

Also, $V = V_0(1 + \gamma\Delta T)$

$$\frac{\Delta V}{V_0} = \gamma\Delta T \quad \text{... (ii)}$$

Comparing eq. (i) and (ii), we get

$$\frac{P}{K} = \gamma\Delta T \Rightarrow \Delta T = \frac{P}{K\gamma}$$

$$\Rightarrow \Delta T = \frac{P}{3\alpha K} \quad (\because \gamma = 3\alpha)$$

10. **(c) :** For an ideal gas, $c_P - c_V = R$
where c_P and c_V are the molar heat capacities.
$MC_P - MC_V = R$
($c_P = MC_P$ and $c_V = MC_V$)
Here, C_P and C_V are specific heats and M is the molar mass.

$$\therefore \quad C_P - C_V = \frac{R}{M}$$

$$\text{For hydrogen gas, } C_P - C_V = \frac{R}{2} = a \quad \text{... (i)}$$

$$\text{For nitrogen gas, } C_P - C_V = \frac{R}{28} = b \quad \text{... (ii)}$$

Dividing eqn. (i) by (ii), we get

$$\frac{a}{b} = 14 \quad \text{or} \quad a = 14b$$

11. **(d):** Initially, the gas equation can be written as
$P_0V_0 = n_i RT_0$
$P_0V_0 = n_i R\,(290)$ ... (i)
After heating the gas equation will be
$P_0V_0 = n_f R(300)$ ... (ii)

$$n_f - n_i = \frac{P_0V_0}{R(300)} - \frac{P_0V_0}{R(290)}$$

$$\Rightarrow \quad n_f - n_i = -\frac{P_0V_0}{R}\left(\frac{10}{290\times300}\right)$$

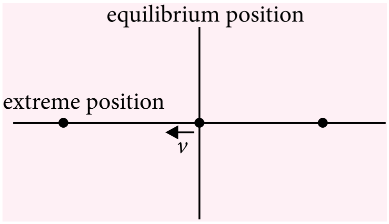
$$\Rightarrow \quad N_f - N_i = -\frac{P_0V_0}{R}\left(\frac{10}{290\times300}\right)N_A$$

where N_f and N_i are the number of molecules and N_A is Avagadro number.

$$N_f - N_i = \frac{10^5\times30\times10\times6.023\times10^{23}}{8.3\times290\times300}$$

$$\Rightarrow \quad N_f - N_i = -2.5\times10^{25}$$

12. **(d):** In a simple harmonic motion, velocity of the body is maximum at the equilibrium position.



Now, time taken by a particle executing simple harmonic motion to reach extreme position (where velocity of the body is zero) from equilibrium position is $T/4$.
Hence, option (d) is correct.

13. (c) : Frequency of the microwave measured by the observer will be given by Doppler's effect of light.

$$\frac{\upsilon'}{\upsilon}=\sqrt{\frac{1+\beta}{1-\beta}} \qquad \left(\text{Here } \beta=\frac{v}{c}\right)$$

$$\Rightarrow \frac{\upsilon'}{\upsilon}=\sqrt{\frac{1+v/c}{1-v/c}}=\sqrt{\frac{c+v}{c-v}}$$

$$\upsilon'=\upsilon\sqrt{\frac{\left(c+\frac{c}{2}\right)}{\left(c-\frac{c}{2}\right)}}$$

$$\upsilon'=10\sqrt{3}\text{ GHz} \quad \text{or} \quad \upsilon'=17.3\text{ GHz}$$

14. (c) : Dipole moment of fixed dipole can be written as

$$\vec{p}=p\cos\theta\,\hat{i}+p\sin\theta\,\hat{j}$$

For electric field

$$\vec{E}_1=E\hat{i}$$

Torque on the dipole

$$\vec{T}_1=(\vec{p}\times\vec{E}_1);\ \vec{T}_1=(p\cos\theta\,\hat{i}+p\sin\theta\,\hat{j})\times(E\hat{i})$$

$$\vec{T}_1=pE\sin\theta(-\hat{k}) \qquad \text{... (i)}$$

Now for $\vec{E}_2=\sqrt{3}E_1\hat{j}=\sqrt{3}E\hat{j}$

In this case, torque on the dipole

$$\vec{T}_2=(p\cos\theta\,\hat{i}+p\sin\theta\,\hat{j})\times(\sqrt{3}E\hat{j})$$

$$\vec{T}_2=\sqrt{3}pE\cos\theta(\hat{k}) \qquad \text{... (ii)}$$

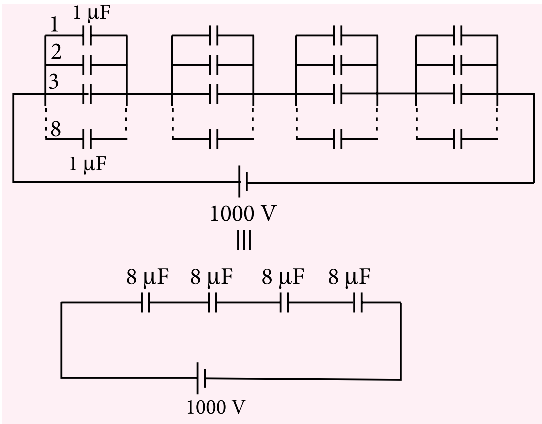
Now give, $\vec{T}_2=-\vec{T}_1$

$$\sqrt{3}pE\cos\theta(\hat{k})=-pE\sin\theta(-\hat{k})$$

$$\sqrt{3}\cos\theta=\sin\theta$$

or $\dfrac{\sin\theta}{\cos\theta}=\sqrt{3}$; $\tan\theta=\sqrt{3} \Rightarrow \theta = 60°$

15. (d): We have to get equivalent capacitance of 2 μF across 1000 V using 1 μF capacitor.
To obtain the desired capacitance, 8 capacitors of 1 μF should be connected in parallel with four such branches in series as shown in the figure.

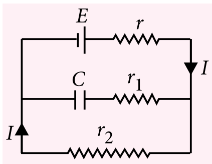


$$\frac{1}{C_{eq}}=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{4}{8}=\frac{1}{2}$$

∴ C_{eq} = 2 μF

∴ Total number of capacitor used = 8 × 4 = 32

16. (c) : In the steady state current in the capacitor becomes zero. Therefore, current in the circuit can be shown as below.



Current in the circuit, $I=\dfrac{E}{r+r_2}$

Charge on the capacitor will be

$Q = CV$ or $Q = (Ir_2)C$

or $Q=\dfrac{Er_2}{r+r_2}C$ or $Q=CE\dfrac{r_2}{r+r_2}$

17. (d): The potential difference across each loop is zero. Therefore no current will flow in the circuit.

18. (a) : Time period of magnetic needle oscillating simple harmonically is given by

$$T=2\pi\sqrt{\frac{I}{MB}}$$

$$\Rightarrow T=2\pi\sqrt{\frac{7.5\times10^{-6}}{6.7\times10^{-2}\times0.01}}$$

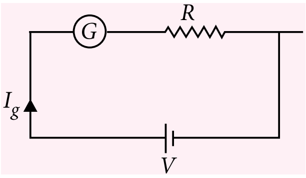
$$\Rightarrow T=\frac{2\pi}{10}\times1.05\text{ s}$$

For 10 oscillations, total time taken

$T' = 10T = 2\pi \times 1.05 \approx 6.65$ s

19. (a) : Given : $I_g = 5$ mA, $G = 15\ \Omega$

Let R be the resistance put in series with the galvanometer as shown in figure.



Now, $V = I_g(R + G)$

$10 = 5 \times 10^{-3}(R + 15)$; $2000 = R + 15$

$\Rightarrow R = 1985\ \Omega = 1.985 \times 10^3\ \Omega$

20. (c) : We know, induced emf (ε) is

$$|\varepsilon| = \frac{d\phi}{dt};\ iR = \frac{d\phi}{dt}$$

Now, $d\phi = R\,idt$ or $\int d\phi = R\int idt$

$\therefore$ Change in magnetic flux = R × area under the current-time graph

$$\Delta\phi = R\times\frac{1}{2}\times 10\times 0.5 = 100\times\frac{1}{2}\times 10\times 0.5 = 250 \text{ Wb}$$

21. (a) : Minimum possible wavelengthof X-rays is

$$\lambda_{min} = \frac{hc}{eV}$$

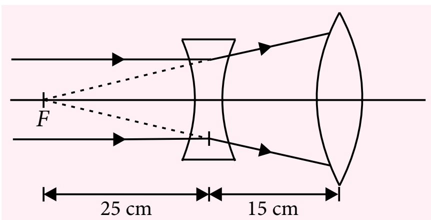
$$\log(\lambda_{min}) = \log\left(\frac{hc}{e}\right) - \log V$$

This is the equation of a straight line with negative slope and positive intercept on the y-axis ($\log\lambda_{min}$).

22. (a) : Given : focal length of concave lens, $f = -25$ cm

Focal length of convex lens, $f' = 20$ cm

The formation of image is shown here.



The image for diverging lens will form at F. *i.e.* at focal length of concave lens.

Now, this image will serve as the object for convex lens. It is at twice the focal length of the convex lens (*i.e.* $2f$). Hence, the final image will also form at $2f$, which is at a distance of 40 cm from the convergent lens. Also, the image formed is real.

23. (b) : Let y be the distance from the central maximum to the point where the bright fringes due to both the wavelengths coincides.

Now, for λ_1, $y = \frac{m\lambda_1 D}{d}$

For λ_2, $y = \frac{n\lambda_2 D}{d}$

$\therefore\ m\lambda_1 = n\lambda_2$

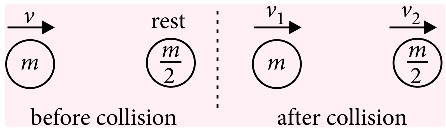
$$\Rightarrow \frac{m}{n} = \frac{\lambda_2}{\lambda_1} = \frac{520}{650} = \frac{4}{5}$$

i.e. with respect to central maximum 4th bright fringe of λ_1 coincides with 5th bright fringe of λ_2

$$\text{Now, } y = \frac{4\times 650\times 10^{-9}\times 1.5}{0.5\times 10^{-3}} \text{ m}$$

$\Rightarrow y = 7.8 \times 10^{-3}$ m or $y = 7.8$ mm

24. (b) : Velocity of the particle before and after the collision is shown in figure.



Applying momentum conservation, we get

$$mv = mv_1 + \frac{m}{2}v_2;\ v = v_1 + \frac{v_2}{2}$$

$2v = 2v_1 + v_2$ ... (i)

Also, coefficient of restitution,

$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}} = \frac{v_2 - v_1}{v - 0}$$

For elastic collision, $e = 1$

$v = v_2 - v_1$ ... (ii)

Solving eqns. (i) and (ii), we get

$$v_1 = \frac{v}{3},\ v_2 = \frac{4v}{3}$$

Now, $\lambda = \frac{h}{p} = \frac{h}{mv}$ i.e. $\lambda \propto \frac{1}{mv}$

$$\therefore \quad \frac{\lambda_A}{\lambda_B} = \frac{m_B v_B}{m_A v_A} = \frac{m_B v_2}{m_A v_1}$$

$$\frac{\lambda_A}{\lambda_B} = \frac{m}{2m}\left(\frac{4v}{3}\right)\left(\frac{3}{v}\right) = 2$$

25. (d): We know,

$$\lambda = \frac{hc}{E} \quad \text{i.e.} \quad \lambda \propto \frac{1}{\text{energy difference}}$$

Now, $\lambda_1 = \frac{hc}{-E-(-2E)} = \frac{hc}{E}$ ... (i)

$$\lambda_2 = \frac{hc}{-E-\left(-\frac{4}{3}E\right)} = \frac{hc}{\left(\frac{E}{3}\right)} \quad \text{... (ii)}$$

Dividing eqn. (i) by eq. (ii), we get

$$\frac{\lambda_1}{\lambda_2} = \frac{1}{3}$$

26. (b): Let N_A and N_B be the number of molecules of A and B after time t.

Also, after time t, $\frac{N_B}{N_A} = 0.3$

Also, let N_0 be the total number of nucleus initially.

After time t,

$N_A + N_B = N_0$

$N_A + 0.3N_A = N_0$

$$\therefore \quad N_A = \frac{N_0}{1.3}$$

Also, rate of disintegration of A

$N_A = N_0\, e^{-\lambda t}$

$$\Rightarrow \quad \frac{N_0}{1.3} = N_0 e^{-\lambda t}; \quad \frac{1}{1.3} = e^{-\lambda t} \quad \text{or} \quad \ln(1.3) = \lambda t$$

$$\text{or} \quad t = \frac{\ln(1.3)}{\lambda}$$

$$\therefore \quad t = \frac{T\ln(1.3)}{\ln(2)} = \frac{T\log(1.3)}{\log 2} \left(\because \text{ half-life } T = \frac{\ln 2}{\lambda}\right)$$

27. (d)

28. (a): Let $x(t) = A_c \sin\omega_c t$ represents carrier wave.

$y(t) = A_m \sin\omega_m t$ represents the modulating signal.

The modulated signal $x_m(t)$ can be written as

$x_m(t) = (A_c + A_m \sin\omega_m t)\sin\omega_c t$

$$= A_c\left(1 + \frac{A_m}{A_c}\sin\omega_m t\right)\sin\omega_c t$$

$x_m(t) = A_c\sin\omega_c t + \mu A_c \sin\omega_m t \sin\omega_c t$

Here $\mu = \frac{A_m}{A_c}$ is the modulation index.

Also, $x_m(t) = A_c \sin\omega_c t$

$$+ \mu A_c \frac{1}{2}[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]$$

[Using $2\sin A \sin B = \cos(A - B) - \cos(A + B)$]

$$\Rightarrow \quad x_m(t) = A_c \sin\omega_c t + \frac{\mu A_c}{2}\cos(\omega_c - \omega_m)t - \frac{\mu A_c}{2}\cdot\cos(\omega_c + \omega_m)t$$

Amplitude modulated wave contains the frequencies ω_c, $\omega_c - \omega_m$ and $\omega_c + \omega_m$.

So, the frequency ω_m is not contained in the amplitude modulated wave.

29. (b): In a balanced Wheatstone bridge if the cell and the galvanometer are interchanged the null point remains unchanged.

30. (b): Surface tension is given by

$$T = \frac{rhg}{2}\times 10^3 \text{ N m}^{-1} = \frac{Dhg}{4}\times 10^3 \text{N m}^{-1}$$

Possible error in the surface tension is

$$\frac{\Delta T}{T}\times 100 = \frac{\Delta D}{D}\times 100 + \frac{\Delta h}{h}\times 100 + 0$$

$$= \left(\frac{0.01\times 10^{-2}}{1.25\times 10^{-2}} + \frac{0.01\times 10^{-2}}{1.45\times 10^{-2}}\right)\times 100$$

(Permissible error in D and h is the place value of the last digit.)

$$\frac{\Delta T}{T}\times 100 = \left(\frac{100}{125} + \frac{100}{145}\right)$$

$$\frac{\Delta T}{T}\times 100 = 0.8 + 0.689 = 1.489 \approx 1.5\%$$

MPP-1 CLASS XI ANSWER KEY

1.	(c)	2.	(a)	3.	(a)	4.	(d)	5.	(c)
6.	(a)	7.	(d)	8.	(b)	9.	(d)	10.	(b)
11.	(c)	12.	(a)	13.	(c)	14.	(a)	15.	(d)
16.	(c)	17.	(b)	18.	(d)	19.	(b)	20.	(b, c)
21.	(a, b, c, d)			22.	(a, b, c)			23.	(a,b)
24.	(3)	25.	(2)	26.	(1)	27.	(d)	28.	(c)
29.	(a)	30.	(c)						

KEY CONCEPT on VARIABLE REFRACTIVE INDEX

Er. Sandip Prasad*

Concept of Refraction in a medium of variable refractive index

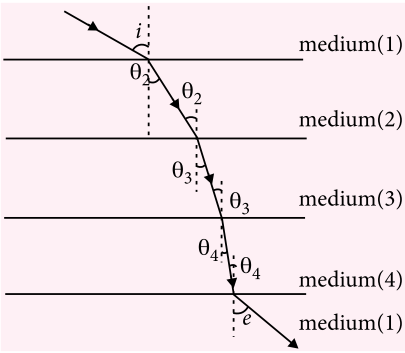
The variation of refractive index in a medium can be divided into two types:

(1) Refractive index varies in discrete form (or ray of light passes through successive plano parallel slabs)

(2) Refractive index varies in continuous form along horizontal and vertical direction

(1) Refractive index varies in discrete form:

Let us consider a ray of light travelling in situation as shown in the figure. Figure shows a light ray passing through successive thin plano parallel plates of different refractive indices.



According to Snell's law,

$\mu_1 \sin i = \mu_2 \sin\theta_2 = \mu_3 \sin\theta_3 = \mu_4 \sin\theta_4 = \mu_1 \sin e$

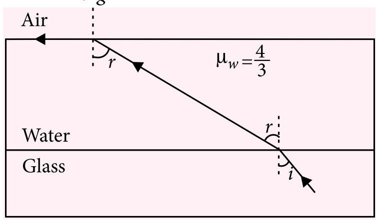
The product of the sine of the angle of incidence and the absolute refractive index has the same value at all interfaces, *i.e.*, $\mu \sin i$ is constant along the light ray's trajectory. This is valid only when all refractory surfaces are simple plane and parallel to each other.

$\therefore \quad \mu_1 \sin i = \mu_1 \sin e \Rightarrow i = e$

If all refractory surfaces are simple plane and parallel and nature of initial and final medium are same, then initial incident and final emergent rays are parallel to each other irrespective of nature and number of medium present between them. Final emergent ray may have some lateral shift.

This relationship is also valid for a medium whose refractive index continuously changes in one direction, since the medium can be considered as consisting of thin plano parallel plates.

EXAMPLE 1 : A ray of light is incident at the glass-water interface at an angle i as shown in figure, it emerges finally parallel to the surface of water, then find the value of μ_g



Soln : Applying Snell's law ($\mu \sin i$ =constant) at first and second interfaces, we have

At first interface (Glass-water interface),

$$\frac{\sin i}{\sin r} = \frac{\mu_w}{\mu_g}$$

$$\Rightarrow \mu_g \sin i = \mu_w \sin r \text{ or } \mu_g \sin i = \frac{4}{3}\sin r \qquad \text{...(i)}$$

At second interface (Water-air interface)

$$\frac{\sin r}{\sin e} = \frac{\mu_{air}}{\mu_w} \Rightarrow \mu_w \sin r = 1$$

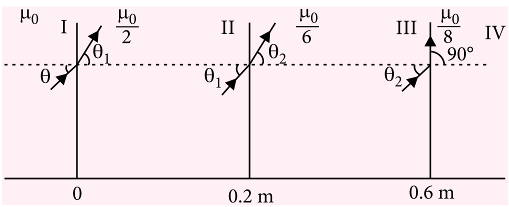
$$\text{or } \sin r = \frac{1}{\mu_w} = \frac{1}{\frac{4}{3}} \Rightarrow \sin r = \frac{3}{4} \qquad \text{...(ii)}$$

Put the value of $\sin r$ in eqn. (i)

$\mu_g \sin i = 1$

$$\therefore \quad \mu_g = \frac{1}{\sin i}$$

EXAMPLE 2 : A light beam is travelling from region I to region IV as shown in figure. The refractive index in regions I, II, III and IV are $\mu_0, \frac{\mu_0}{2}, \frac{\mu_0}{6}$ and $\frac{\mu_0}{8}$ respectively. Find the angle of incidence θ for which the beam just misses entering region IV?



*Author is Director of Sandip Physics Classes and motivational speaker

Soln : As the beam just misses entering the region IV, the angle of refraction in the region IV must be 90°. Application of Snell's law successively at different interfaces gives

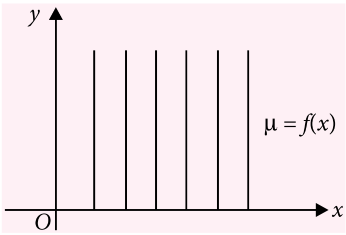
$$\mu_0 \sin\theta = \frac{\mu_0}{2}\sin\theta_1 = \frac{\mu_0}{6}\sin\theta_2 = \frac{\mu_0}{8}\sin 90^\circ$$

$$\Rightarrow \sin\theta = \frac{1}{8} \quad \text{or} \quad \theta = \sin^{-1}\left(\frac{1}{8}\right)$$

(2) Refractive index varies in continuous form along horizontal and vertical direction:

An example for a medium with variable refractive index is atmosphere. The atmosphere becomes thinner as we go up. The refractive index of air is highest close to earth and decreases as we move upward. But in hot areas like deserts surface of earth is very hot. So, air in the lower regions of atmosphere is hot as compared to that in higher regions. As a result it increases. In a situation where refractive index changes in one direction, medium can be considered as a collection of large number of thin layers. If refractive index is a function of y. *i.e.*, $\mu = f(y)$ then medium can be considered as to be made up of large number of thin slabs placed parallel to x-axis and optical normal at any interface is parallel to y-axis.

Similarly, if $\mu = f(x)$, then slabs are parallel to y-axis and optical normal at any interface is parallel to x-axis.



Steps for drawing the path of the ray :

(1) First of all we need to consider the medium as a collection of infinite number of differential slab having different refractive index.

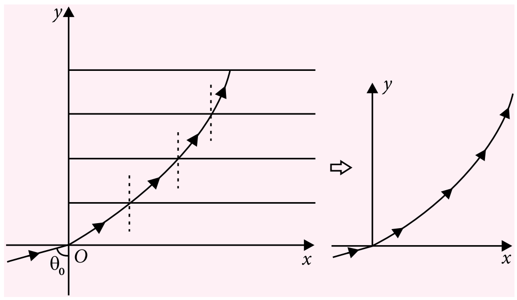
(2) Orientation of selected differential slabs depends on the variation of refractive index of the medium. It is always perpendicular to direction of variation of refractive index.

Let us consider the following situations of variable refractive index.

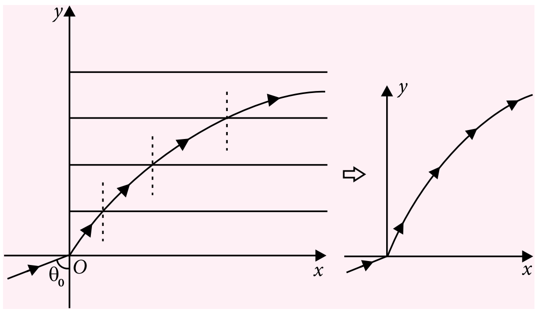
Case-I : When $\mu = f(y)$

Let us consider the situation where refractive index of medium changes in the direction of y-axis, *i.e.*, $\mu = f(y)$. Consider a glass slab with a medium where the refractive index varies with y. Assume that the outside medium is air. Let light be incident on the glass slab at an angle θ_0. The light ray strikes the glass slab at the point O. Now, the medium can be divided into slices perpendicular to y-axis. If the refractive index is considered constant in each slice, any given ray can be approximated by a number of short straight segments. Hence, for finding the nature of trajectory of the ray of light travelling inside the medium, we need to split the whole medium into series of differential elementary parallel layers to x-axis as shown in the figure. This indicates that each layer has different refractive index having value according to a given expression. Here μ is a function of y.

(a) When the refractive index of the medium increases with the increase with height

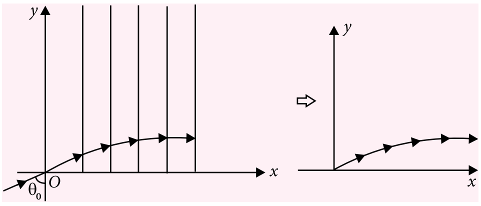


(b) When the refractive index of the medium decreases with the increase with height

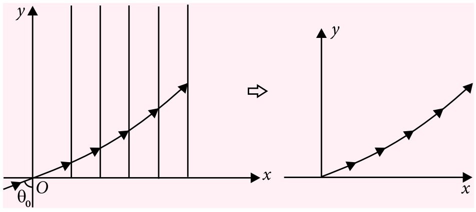


Case-II: When $\mu = f(x)$.

(a) When the refractive index of the medium increases with the increase in distance along positive x-direction:



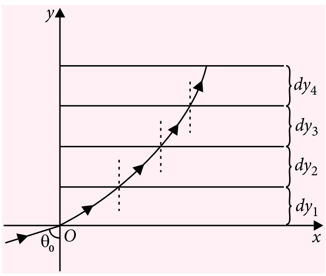
(b) When the refractive index of the medium decreases with the increase in distance along positive x-direction:



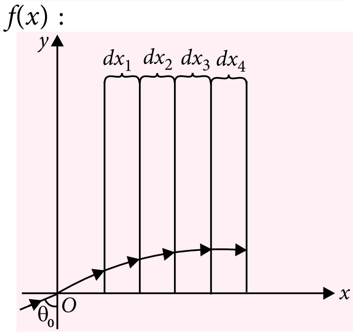
Steps for solving the problems related with variable refractive index:

Step-I : Divide the medium into various differential layers of thickness dy_1, dy_2 etc. This indicates that each layer has different refractive index. Orientation of selected differential slabs depends on the variation of refractive index of the medium .It is always perpendicular to the direction of variation of refractive index. If refractive index is a function of y. *i.e.*, $\mu = f(y)$ then medium can be considered as to be made up of large number of thin layers placed parallel to x-axis and optical normal at any interface is parallel to y-axis. Similarly, if $\mu = f(x)$, then layers are parallel to y-axis and optical normal at any interface is parallel to x-axis.

When $\mu = f(y)$:



When $\mu = f(x)$:

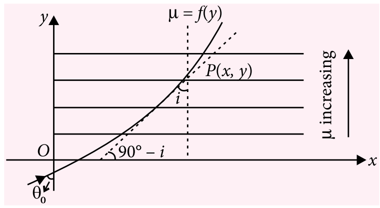


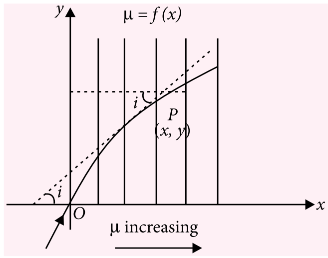
Step-II : Place the origin of a coordinate system at the point where the light ray enters the medium. From this we can get the value of angle of incidence at the first differential layer.

Step-III : Next we have to consider a layer at some distance x or y from origin according to variation of refractive index. Let the angle of incidence at this layer be i. Then we have to apply snell's law for this differential layer and starting differential layer.

$\mu_0 \times \sin\theta_0 = \mu \sin i$ ...(i)

where θ_0 is the angle made by the ray of light at starting layer.



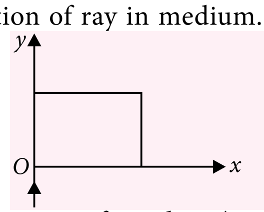


Step-IV : Draw a tangent at any point $P(x, y)$ on the trajectory which makes an angle i with optical normal parallel to y-axis if $\mu = f(y)$ or parallel to x-axis if $\mu = f(x)$. Then the slope of the tangent at this point $P(x, y)$ will be $\tan i$. Geometrically, relate the slope of this tangent to the angle i.

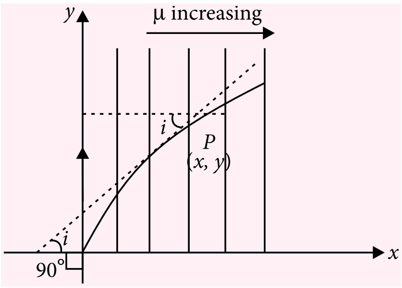
Step-V : Substitute for i from equation (i) and determine $\frac{dy}{dx}$ as a function of x and y.

Step-VI : Integrate and obtain an expression of y as a function of x.

Example 3 : The refractive index of an isotropic medium varies as $\mu = \mu_0\sqrt{x+1}$ where $0 \leq x \leq a$. A ray of light is incident at, the origin just along y-axis. Find the equation of ray in medium.



Soln : Between point of incident (*i.e.*, origin) and an arbitrary point inside the medium, optical normal at a point is parallel to x-axis.



We can use Snell's law [$\because$ incidence is grazing]

$1 \sin 90° = \mu \sin i$

$$\sin i = \frac{1}{\mu}$$

$$\therefore \cos i = \sqrt{1-\frac{1}{\mu^2}} = \sqrt{\frac{\mu^2-1}{\mu^2}} \quad \text{so, } \tan i = \frac{1}{\sqrt{\mu^2-1}}$$

Geometrically, relate the slope of the tangent to the angle i

$$\text{i.e., } \frac{dy}{dx} = \tan i \qquad [\because \mu = f(x)]$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{\mu^2-1}} = \frac{1}{\sqrt{\mu_0^2(x+1)-1}}$$

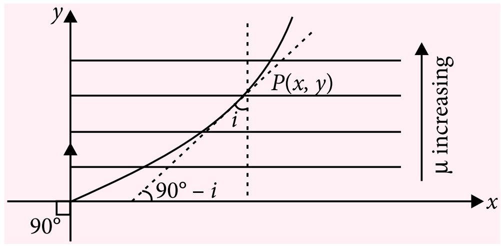
$$= \frac{1}{\sqrt{x}} \qquad [\because \mu_0 = 1]$$

$$dy = \frac{1}{\sqrt{x}}dx \Rightarrow \int_0^y dy = \int_0^x \frac{1}{\sqrt{x}}dx \Rightarrow y = 2\sqrt{x}$$

EXAMPLE 4 : A ray of light is incident on a glass slab at grazing incidence. The refractive index of the material of the slab is given by $\mu = \sqrt{(1+y)}$. If the thickness of the slab is d, determine the equation of the trajectory of the ray inside the slab and the coordinates of the point where the ray exits from the slab. Take the origin to be at the point of entry of the ray.

Soln : In this situation the value of refractive index of the medium is variable and the function of y *i.e.*, $\mu = f(y)$.

Now the medium can be divided into thin slices parallel to x-axis and optical normal parallel to y axis. Here θ_0 be the angle of incident in the variable medium at point (0, 0) whose value is equal to 90°. Draw a tangent at any point $P(x, y)$ on the trajectory which makes an angle i with optical normal parallel to y-axis. Then the slope of the tangent at this point $P(x, y)$ will be $\tan i$.



From the Snell's law :

$1 \times \sin 90° = \mu \sin i$ ...(i)

$$\therefore \mu = \frac{1}{\sin i} \quad \therefore \cos i = \sqrt{1-\frac{1}{\mu^2}} = \frac{\sqrt{\mu^2-1}}{\mu}$$

Geometrically, relate the slope of the tangent to the angle i. *i.e.*, $\frac{dy}{dx} = \tan(90-i) \Rightarrow \frac{dy}{dx} = \cot i$ ...(ii)

$$\cot i = \frac{\cos i}{\sin i} = \sqrt{\mu^2-1} = \sqrt{1+y-1} = \sqrt{y} \qquad \text{...(iii)}$$

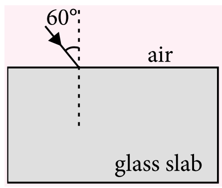
From equations (ii) and (iii),

$$\frac{dy}{dx} = y^{1/2} \Rightarrow \int_0^y \frac{dy}{y^{1/2}} = \int_0^x dx \quad \therefore y = \frac{x^2}{4}$$

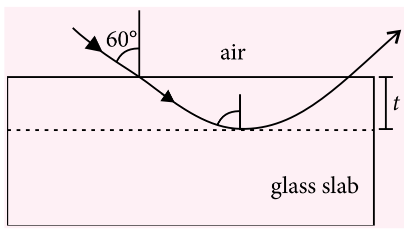
$\because$ $y = d$, the thickness of the slab

$\Rightarrow x = 2\sqrt{y} = 2\sqrt{d}$

EXAMPLE 5 : A ray of light enters a given glass slab from air as shown. The refractive index of glass slab is given by $\mu = A - Bt$ where A and B are constants and the thickness (t) of slab measured from top surface. Find the maximum depth travelled by ray slab. Assume depth travelled by ray in slab. Assume thickness of slab to be sufficiently large.



Soln :



$\mu_{air} \sin 60° = (A - Bt) \cdot \sin 90°$

$$\text{or } 1 \times \frac{\sqrt{3}}{2} = (A-Bt) \text{ or } t = \left(A - \frac{\sqrt{3}}{2}\right) \times \frac{1}{B}$$

MPP-1 MONTHLY Practice Problems

Class XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Units and Measurements

Total Marks : 120 **Time Taken : 60 min**

NEET / AIIMS

Only One Option Correct Type

1. Which of the following numbers has least number of significant figures ?
(a) 0.80760 (b) 0.80200
(c) 0.08076 (d) 80.267

2. A hypothetical experiment conducted to find Young's modulus $Y = \dfrac{\cos\theta \cdot T^x \tau}{l^3}$, where τ is torque, T is time and l is length. The value of x is
(a) 0 (b) 1
(c) 2 (d) –1

3. Which of the following physical quantities has the SI unit kg m^2 s^{-3} A^{-2} ?
(a) Resistance (b) Inductance
(c) Capacitance (d) Magnetic flux

4. The volume V of the liquid crossing through a tube is related to the area of cross section A, velocity v and time t as : $V \propto A^\alpha v^\beta t^\gamma$. Choose the correct alternative.
(a) $\alpha \neq \beta \neq \gamma$ (b) $\alpha \neq \beta = \gamma$
(c) $\alpha = \beta \neq \gamma$ (d) $\alpha = \beta = \gamma$

5. Given that T stands for time period and l stands for the length of simple pendulum. If g is the acceleration due to gravity, then which of the following statements about the relation $T^2 = \left(\dfrac{l}{g}\right)$ is correct ?
(a) It is correct both dimensionally as well as numerically.
(b) It is neither dimensionally correct nor numerically.
(c) It is dimensionally correct but not numerically.
(d) It is numerically correct but not dimensionally.

6. A bus travels distance x_1 when accelerates from rest at constant rate a_1 for some time and after that travels a distance x_2 when decelerates at a constant rate a_2 to come to rest. A student established a relation, $x_1 + x_2 = \dfrac{a_1 a_2 t^2}{2(a_1 + a_2)}$.
(t = total time of travel)
Choose the correct option.
(a) The relation is dimensionally correct.
(b) The relation is dimensionally incorrect.
(c) The relation may be dimensionally correct.
(d) None of the above.

7. A highly rigid cubical block A of small mass M and side L is fixed rigidly onto another cubical block of same dimensions and low modulus of rigidity η, such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the side faces of A. After the force is withdrawn, block A executes small oscillations, the time period of which is given by
(a) $2\pi\sqrt{M\eta L}$ (b) $2\pi\sqrt{\dfrac{M\eta}{L}}$
(c) $2\pi\sqrt{\dfrac{ML}{\eta}}$ (d) $2\pi\sqrt{\dfrac{M}{\eta L}}$

8. Given that K = kinetic energy, V = velocity, T = time. If they are chosen as the fundamental units, then what is the dimensional formula for surface tension ?
(a) $[KV^2 T^2]$ (b) $[KV^{-2} T^{-2}]$
(c) $[K^2V^2T^{-2}]$ (d) $[K^2 V^{-2} T^{-2}]$

9. The dimensions of electromotive force in terms of current (A) are
(a) $[ML^0 T^{-1} A^{-2}]$ (b) $[ML^2 T^{-2} A^2]$
(c) $[ML^2 T^{-2} A^{-1}]$ (d) $[ML^2 T^{-3} A^{-1}]$

10. What is the maximum percentage error in density of a body if percentage error in measurement of mass is 4% and percentage error in measurement of side is 5% ?
(a) 18% (b) 19% (c) 20% (d) 11%

11. The heat generated in a circuit is dependent upon the resistance, current and time for which the current is flown. If the errors in measuring the resistance, current and time are 1%, 2% and 1%, respectively, the maximum error in calculating heat will be
(a) 2% (b) 3% (c) 6% (d) 1%

12. Specific heat of capacity of a substance is 29 J kg^{-1} K^{-1}. If unit of length is changed to 50 cm, unit of time is changed to 2 s and unit of temperature is changed to 2 K, keeping unit of mass and amount of substance unchanged, the value of specific heat of hydrogen would be
(a) 928 new units (b) 58 new units
(c) 92.8 new units (d) 9280 new units

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :
(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. **Assertion :** The watches having hour hand, minute hand and second hand have least count as 1 s.
Reason : Least count is the maximum measurement that can be measured accurately by an instrument.

14. **Assertion :** Surface tension and spring constant have the same dimensions.
Reason : Surface tension and spring constant are equivalent to force per unit length.

15. **Assertion :** If two physical quantities have same dimensions, then that can be certainly added or subtracted.
Reason : If the dimensions of both the quantities are same then both the physical quantities should be similar.

JEE MAIN / JEE ADVANCED / PETS

Only One Option Correct Type

16. The quantity $\left(\frac{nh}{(2\pi qB)}\right)^{1/2}$ where n is a positive integer, h is Planck's constant, q is charge and B is magnetic field, has the dimensions of
(a) area (b) speed
(c) length (d) acceleration

17. 1 cm of main scale of a vernier calliper is divided into 10 divisions. The least count of the calliper is 0.05 cm, then the vernier scale must have

SOLUTION OF APRIL 2017 CROSSWORD

[1]R	E	D	S	H	I	F	T								[2]I	C	E	L	I	N	[3]E
A					[4]P	A	S	C	A	L		[5]B	A	R	N						L
D												U							[6]A		L
I				[7]C	O	R	I	O	L	I	S	F	O	R	C	E			E		I
A		[8]M		H								F			[9]G				R		P
N		U		A			[10]D		[11]T			E			E				O		S
[12]C	O	L	O	R	I	M	E	T	R	Y		R			O				D		O
R		T		M			C		A						C		[13]S		Y		M
Y		I			[14]A		O		N			[15]B	O	R	E		C		N		E
O		V			S		U		S	[16]M					N		O		A		T
G		I			D		P		P	E					T		T		M		E
E		B			I		L		O	[17]G	E	T	T	E	R		O		I		R
N		R			C		I		N	A					I		P		C		
I		A		[18]R			N		D	P					C		H		S		
C		T		O			G	[19]H	E	H					M		O				
[20]S		O		D				U	R	O		[21]H	O	L	O	G	R	A	P	H	Y
C		R		S				M		N					D						
A			[19]F	A	L	L	T	I	M	E					E						[23]E
L								D			[24]P	E	A	K	L	O	A	D			L
E			[25]A	I	R	Y	D	I	S	K											V
R		[26]R	E	G	E	L	A	T	I	O	N						[27]P	U	L	S	E
							[28]H	Y	P	E	R	O	P	I	A						S

Solution Senders (April 2017)

- Anju Karnatak, Jammu
- Reva Singh, Delhi

Solution Senders of Physics Musing

SET-45

1. Ravindra Taneja, Kota
2. Mukesh Ahuja, Dhanbad
3. Ramendra Pillai, Chennai

(a) 10 divisions (b) 20 divisions
(c) 25 divisions (d) 50 divisions

18. The number of particles crossing a unit area perpendicular to the x-axis in a unit time is given by $N = -D\left(\frac{n_2 - n_1}{x_2 - x_1}\right)$, where n_1 and n_2 are the number of particles per unit volume at $x = x_1$ and $x = x_2$, respectively, and D is the diffusion constant. The dimensions of D are
(a) $[M^0 LT^{-2}]$ (b) $[M^0 L^2T^{-4}]$
(c) $[M^0 L^2 T^{-2}]$ (d) $[M^0 L^2 T^{-1}]$

19. The units of velocity, acceleration and force in two systems are related as $v_2 = \frac{\alpha^2}{\beta}v_1; a_2 = (\alpha\beta)a_1$ and $F_2 = \left(\frac{1}{\alpha\beta}\right)F_1$, where α and β are dimensionless constants. How are momentum units of the system related ?
(a) $\frac{p_2}{p_1} = \frac{1}{\alpha^3}$ (b) $\frac{p_2}{p_1} = \frac{1}{\beta^3}$
(c) $\frac{p_2}{p_1} = \frac{\beta}{\alpha}$ (d) $\frac{p_2}{p_1} = \frac{\alpha}{\beta}$

More than One Options Correct Type

20. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of the vacuum and $[\mu_0]$ that of the permeability of the vacuum. If M = mass, L = length, T = time and I = electric current, then
(a) $[\varepsilon_0] = [M^{-1} L^{-3} T^2 I]$
(b) $[\varepsilon_0] = [M^{-1} L^{-3} T^4 I^2]$
(c) $[\mu_0] = [MLT^{-2} I^{-2}]$
(d) $[\mu_0] = [ML^2T^{-1} I]$

21. If E_0 represents gravitational field, the dimensions of $\frac{E_0^2}{G}$ are same as,
ε_0 = electric permittivity, E = electric field,
B = magnetic field, μ = magnetic permeability
ρ = density, A = amplitude, G = gravitational constant
(a) $\varepsilon_0 E^2$ (b) $\frac{B^2}{2\mu}$
(c) $\rho A^2\omega^2$ (d) pressure

22. Consider three quantities, $x = \frac{E}{p}$, $y = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$, and $z = \frac{l}{CR}$. Here, l is the length of a wire, C is the capacitance, R is resistance, E is energy and p is momentum. Then,
(a) x and y have the same dimensions.
(b) x and z have the same dimensions.
(c) y and z have the same dimensions.
(d) None of the above three pairs have the same dimensions.

23. If U, m, L and G denote energy, mass, angular momentum and universal gravitational constant, respectively. The dimensions of $\frac{m^5G^2}{UL^2}$ are same as

(a) strain (b) dielectric constant
(c) wavelength (d) distance

Integer Answer Type

24. To find the distance d over which a signal can be seen clearly in foggy conditions, a railway engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. Find the value of n.

25. A gas bubble, from an explosion under water oscillates with a time period T. If energy of explosion, E is proportional to $T^a \rho^b p^c$ where p, ρ and E denote pressure, density and total energy of the explosion, respectively then find the value of $\frac{a^2c}{5b^2}$.

26. If the unit of velocity is run, the unit of time is second and unit of force is strength in a hypothetical system of unit. In this system of unit, the unit of mass is (strength)^x (second)^y (run)^z. Find the value of $\frac{y}{x}$.

Comprehension Type

Dimensional methods provide three major advantages in verification, derivation, and changing the system of units. Any empirical formula that is derived based on this method has to be verified and proportionality constant found by experimental means. The presence or absence of certain factors, non dimensional constants or variables cannot be identified by this method. So every dimensionally correct relation cannot be taken as perfectly correct.

27. If α kilogram, β meter, and γ second are the fundamental units, 1 cal can be expressed in new units as [1 cal = 4.2 J]

(a) $\alpha^{-1}\beta^2\gamma$ (b) $\alpha^{-1}\beta^{-2}\gamma$
(c) $4.2\,\alpha^{-1}\beta$ (d) $4.2\,\alpha^{-1}\beta^{-2}\gamma^2$

28. The time period of oscillation of a drop depends on surface tension σ, density of the liquid ρ, and radius r. The required time period is proportional to

(a) $\sqrt{\frac{\rho r^2}{\sigma}}$ (b) $\sqrt{\frac{r^2}{\rho\sigma}}$
(c) $\sqrt{\frac{r^3\rho}{\sigma}}$ (d) $\sqrt{\frac{\rho\sigma}{r^3}}$

Matrix Match Type

29. Match the physical quantities given in Column I with dimensions expressed in terms of mass (M), length (L), time (T), and current (A) given in Column II.

Column-I	Column-II
(A) Angular momentum	(P) $[ML^2T^{-1}]$
(B) Latent heat	(Q) $[M^{-1}L^{-2}T^4A^2]$
(C) Capacitance	(R) $[ML^3T^{-3}A^{-2}]$
(D) Resistivity	(S) $[L^2T^{-2}]$

	A	B	C	D
(a)	P	S	Q	R
(b)	S	Q	R	P
(c)	P	S	R	Q
(d)	R	P	S	Q

30. Match the column I with the column II and choose the correct option.

Column-I	Column-II
(A) Same positive dimensions of mass	(P) Energy density
(B) Same negative dimensions of length	(Q) Surface tension
(C) Same negative dimension of time	(R) Gravitational constant
(D) Sum of dimensions of length is 3	(S) Pressure

	A	B	C	D
(a)	P, R	Q, S	Q, R, S	P, Q, S
(b)	R, S	P, S	R, S	Q, P, R
(c)	P, Q, S	P, S	P, Q, R, S	R
(d)	P, Q	P, Q, R	S, R, Q	Q, P, R

◈◈

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

MPP-1 MONTHLY Practice Problems

Class XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Electric Charges and Fields

Total Marks : 120 **Time Taken : 60 min**

NEET / AIIMS

Only One Option Correct Type

1. An oil drop of 12 excess electrons is held stationary under a constant electric field of 2.55×10^4 V m^{-1} in Millikan's oil drop experiment. The density of the oil is 1.26 g cm^{-3}. Estimate the radius of the drop. ($g = 9.81$ m s^{-2}; $e = 1.60 \times 10^{-19}$ C)
(a) 9.81×10^{-4} mm (b) 9.81×10^{-2} mm
(c) 6.84×10^{-4} mm (d) 6.84×10^{-2} mm

2. An oil drop carrying a charge of 2 electrons has a mass of 3.2×10^{-17} kg. It is falling freely in air with terminal speed. The electric field required to make the drop move upwards with the same speed is
(a) 2×10^3 V m^{-1} (b) 4×10^3 V m^{-1}
(c) 3×10^3 V m^{-1} (d) 8×10^3 V m^{-1}

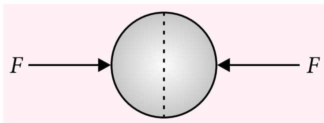
3. A charge q is placed at the centre of the line joining two equal charges Q. The system of the three charges will be in equilibrium if q is equal to
(a) $-\frac{Q}{2}$ (b) $-\frac{Q}{4}$ (c) $+\frac{Q}{4}$ (d) $+\frac{Q}{2}$

4. Five balls numbered 1, 2, 3, 4 and 5 are suspended using separate insulating threads. The balls (1, 2), (2, 4) and (4, 1) show electrostatic attraction, while balls (2, 3) and (4, 5) show repulsion. Therefore, ball 1 must be
(a) negatively charged (b) positively charge
(c) neutral (d) None of these

5. The ratio of the force between two small conducting spheres with constant charge in air to a medium of dielectric constant K is
(a) $1 : K$ (b) $K : 1$ (c) $1 : K^2$ (d) $K^2 : 1$

6. Assume that the charges of the Earth and the Sun are not quite neutralized and the net charges are of the same sign (negative) and equal in magnitude. What must be charge on each be in order that the coulomb force will just cancel the gravitational force? (mass of the Sun = 2×10^{30} kg, mass of the Earth = 6×10^{24} kg)
(a) 20.4×10^{16} C (b) 24.2×10^{16} C
(c) 29.8×10^{16} C (d) 26.8×10^{16} C

7. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F, as shown in figure then, F is proportional to



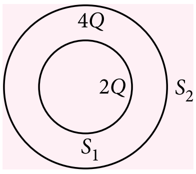
(a) $\frac{1}{\varepsilon_0}\sigma^2 R^2$ (b) $\frac{1}{\varepsilon_0}\sigma^2 R$
(c) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (d) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

8. If the flux of the electric field through a closed surface is zero, then
(a) the electric field must be zero everywhere on the surface.
(b) the net charge inside the surface must be zero
(c) the electric field must be uniform throughout the closed surface
(d) the charge outside the surface must be zero.

9. A large plane sheet of charge having surface charge density 5.0×10^{-16} C m^{-2} lies in the X-Y plane. Find the electric flux through a circular area of radius 0.1 m, if the normal to the circular area makes an angle of 60° with the Z-axis.
(a) 4.44×10^{-7} N m^2 C^{-1}
(b) 6.54×10^{-7} N m^2 C^{-1}
(c) 5.64×10^{-7} N m^2 C^{-1}
(d) 3.94×10^{-7} N m^2 C^{-1}

10. A spherical rubber balloon carries a charge that is uniformly distributed over its surface. As the balloon is blown up, electric field on the surface of the balloon
(a) increases
(b) remains same
(c) decreases
(d) either increases or decreases.

11. Consider two hollow concentric spheres, S_1 and S_2, enclosing charges $2Q$ and $4Q$ respectively as shown in figure. Find out the ratio of the electric flux through S_1 to S_2.



(a) 1 : 3
(b) 3 : 2
(c) 2 : 3
(d) 3 : 1

12. Two positive charges q_1 and q_2 are fixed at A and B. A unit positive charge is taken from A to B along the line joining AB. The force experienced by unit charge
(a) decreases continuously
(b) increases continuously
(c) increases first and then decreases
(d) decreases first and then increases.

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :
(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. **Assertion :** Electric field is inversely proportional to square of distance from a point charge.
Reason : On going away from a point charge or a small electric dipole, electric field decreases at the same rate in both the cases.

14. **Assertion :** Three equal charges are situated on a circle of radius r such that they form an equilateral triangle, then the electric field intensity at the centre is zero.
Reason : A charged particle free to move in an electric field always moves along an electric line of force.

15. **Assertion :** Electric field is zero inside a charged conductor.
Reason : Sharper is the curvature of spot on a charged body lesser will be the surface density of charge at that point.

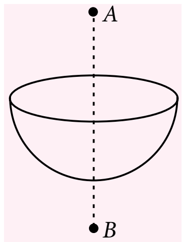
JEE MAIN / JEE ADVANCED

Only One Option Correct Type

16. Three charges (each Q) are placed at the three corners of an equilateral triangle. A fourth charge q is placed at the centre of the triangle. The ratio $\left|\frac{q}{Q}\right|$ so as to make the system in equilibrium is
(a) $1:3$ (b) $1:\sqrt{3}$ (c) $\sqrt{3}:1$ (d) $2:\sqrt{3}$

17. A pithball covered with tinfoil having a mass of m kg hangs by a fine silk thread l metre long in an electric field E. When the ball is given an electric charge of q coulomb, it stands out d metre from the vertical line. Then the electric field is
(a) $\dfrac{mgd}{ql^2}$ (b) $\dfrac{mgl}{qd^2}$
(c) $\dfrac{mgl}{q\sqrt{l^2-d^2}}$ (d) $\dfrac{mgd}{q\sqrt{l^2-d^2}}$

18. Figure shows a uniformly charged hemisphere of radius R. It has a volume charge density ρ. If the electric field at a point $2R$, above its centre is E, then what is the electric field at the point $2R$ below its centre?



(a) $\dfrac{\rho R}{6\varepsilon_0}+E$ (b) $\dfrac{\rho R}{12\varepsilon_0}-E$
(c) $\dfrac{-\rho R}{6\varepsilon_0}+E$ (d) $\dfrac{\rho R}{12\varepsilon_0}+E$

19. A and B are two points on the axis and the perpendicular bisector, respectively, of an electric dipole. A and B are far away from the dipole and at equal distances from it. The fields at A and B are $\vec{E}_A$ and $\vec{E}_B$. Then

(a) $\vec{E}_A = \vec{E}_B$

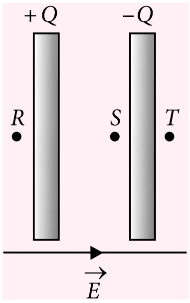
(b) $\vec{E}_A = 2\vec{E}_B$

(c) $\vec{E}_A = -2\vec{E}_B$

(d) $|\vec{E}_B| = \frac{1}{2}|\vec{E}_A|$, and $\vec{E}_A$ is perpendicular to $\vec{E}_B$.

More than One Option Correct Type

20. Two large thin conducting plates with a small gap in between are placed in a uniform electric field E (perpendicular to the plates). The area of each plate is A, and charges $+Q$ and $-Q$ are given to these plates as shown in figure. If R, S and T are three points in space, then the net electric field



(a) at point R is E

(b) at point S is E

(c) at point T is $\left(E + \frac{Q}{\varepsilon_0 A}\right)$

(d) at point S is $\left(E + \frac{Q}{A\varepsilon_0}\right)$

21. A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its centre

(a) increases as r increases for $r < R$

(b) decreases as r increases for $0 < r < \infty$

(c) decreases as r increases for $R < r < \infty$

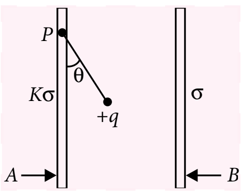
(d) is discontinuous at $r = R$.

22. A point charge q is placed at origin. Let $\vec{E}_A$, $\vec{E}_B$ and $\vec{E}_C$ be the electric fields at three points $A(1, 2, 3)$, $B(1, 1, -1)$ and $C(2, 2, 2)$ due to charge q. Then

(a) $\vec{E}_A \perp \vec{E}_B$ (b) $\vec{E}_A \,||\, \vec{E}_C$

(c) $|\vec{E}_B| = 4|\vec{E}_C|$ (d) $|\vec{E}_B| = 8|\vec{E}_C|$

23. Two infinite plane sheets of charge A and B of positive charge have surface charge densities $K\sigma$ and σ, respectively. A metallic ball of mass m and charge $+q$ is attached to a thread and the thread is tied at point P on sheet A. Initially ball is in equilibrium at plate P when sheets are uncharged. Find the correct options (K is a constant).



(a) If $K > 1$, the angle $\theta = \tan^{-1}\left(\frac{K\sigma q}{2\varepsilon_0 mg}\right)$.

(b) If $K < 1$, the ball will not leave its equilibrium.

(c) If $K = 1$, the angle $\theta = 45°$.

(d) If $K > 1$, the angle $\theta = \tan^{-1}\left(\frac{(K-1)\sigma q}{2\varepsilon_0 mg}\right)$.

Integer Answer Type

24. In Millikan's experiment, a drop of radius 1.64 μm and density 0.851 g cm^{-3} is balanced when an electric field of 1.92×10^5 N C^{-1} is applied. Find the charge on the drop, in terms of e.

25. A solid sphere of radius R has a charge Q distributed in its volume with charge density $\rho = kr^a$, where k and a are constants and r is the distance from its centre. If the electric field at $r = \frac{R}{2}$ is $\left(\frac{1}{8}\right)$ times that at $r = R$, find the value of a.

26. Four point charges q, $-q$, $2Q$ and Q are placed in order at the corners A, B, C and D of a square. If the field at the mid-point of CD is zero then the value of $\frac{q}{Q}$ is $\frac{5\sqrt{5}}{x}$. Find the value of x.

Comprehension Type

Four equal positive charges, each of value Q, are arranged at the four corners of a square of diagonal $2a$. A small body of mass m carrying a unit positive charge is placed at a height h above the centre of the square.

27. What should be the value of Q in order that this body may be in equilibrium?

(a) $\pi\varepsilon_0 \frac{mg}{2h}(h^2+2a^2)^{3/2}$

(b) $\pi\varepsilon_0 \frac{mg}{h}(h^2+a^2)^{3/2}$

(c) $\pi\varepsilon_0 \frac{2mg}{h}(h^2+2a^2)^{3/2}$

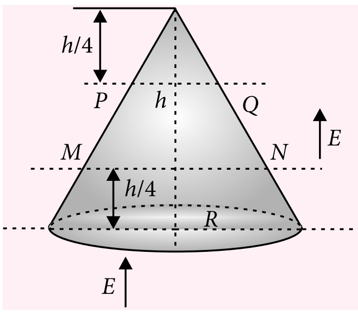
(d) $\pi\varepsilon_0 \frac{mg}{2h}(h^2-a^2)^{3/2}$

28. The type of equilibrium of the point mass is (consider only vertical displacement)

(a) stable equilibrium (b) unstable equilibrium
(c) neutral equilibrium
(d) cannot be determined.

Matrix Match Type

29. The axis of a hollow cone as shown in figure is vertical. Its base radius is R. It is kept in a uniform electric field E parallel to its axis.



	Column-I		Column-II
(A)	Magnitude of flux through base of cone	(P)	$\pi R^2 E$
(B)	Magnitude of flux through curved part of cone	(Q)	$\frac{\pi R^2 E}{2}$
(C)	Magnitude of flux through curved part $MNQP$ of cone	(R)	zero
(D)	Net flux through the entire cone	(S)	non zero

	A	B	C	D
(a)	P, S	P, S	Q, S	R
(b)	R	Q, S	P, S	P, S
(c)	Q, S	Q, S	P, R	S
(d)	R	Q, S	P	R

30. Match the column I with column II and select the correct option.

	Column-I		Column-II
(A)	Spherical charged conductor	(P)	At the surface electric field is continuous and maximum.
(B)	Sphere having uniform volume distribution of charge	(Q)	At the surface electric field is discontinuous.
(C)	Charged ring	(R)	Electric field is uniform.
(D)	Infinite sheet of charge	(S)	At the centre electric field is zero.

	A	B	C	D
(a)	R, S	Q	S	R
(b)	Q, S	P, S	S	R
(c)	P, R	R, Q	R, S	P
(d)	Q, P	R	S	Q

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SELF CHECK

No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

PHYSICS MUSING

SOLUTION SET-45

1. (a) : As we know, magnetic field due to a straight thin current carrying wire at a distance x is $B = \frac{\mu_0 I}{2\pi x}$.

Induced emf in resistance R due to motion of rod

$$EF = \int_a^b Bv\,dx = \int_a^b \frac{\mu_0 I}{2\pi x} v\,dx$$

$$\Rightarrow \quad \varepsilon = \frac{\mu_0 I v}{2\pi} \ln\left(\frac{b}{a}\right)$$

Power dissipated across resistance $(R) = \frac{\varepsilon^2}{R}$

Also, power required to maintain the motion of rod $= F.v \quad \Rightarrow \quad F = \frac{\varepsilon^2}{vR}$

$$\Rightarrow \quad F = \frac{1}{vR}\left[\frac{\mu_0 I v}{2\pi} \ln\left(\frac{b}{a}\right)\right]^2$$

2. (a) : Electric field due to plate S is $\frac{\sigma}{2\varepsilon_0}$.

Electrostatic force on plate S is

$$= \frac{\sigma}{2\varepsilon_0} \times \sigma A = \frac{Q^2}{2A\varepsilon_0} \qquad \left(\because \ \sigma = \frac{Q}{A}\right)$$

For equilibrium, $\frac{Q^2}{2A\varepsilon_0} = mg$

$$\Rightarrow \quad Q = \sqrt{2Amg\varepsilon_0}$$

3. (b) : Choosing a cylindrical Gaussian surface concentric and axial with ring of radius x and length l $(l << R)$.

$$\oint \vec{E} \cdot d\vec{s} = -E \cdot 2\pi x l + \left(\frac{1}{4\pi\varepsilon_0} \frac{Q\frac{l}{2}}{R^3}\right) 2\pi x^2 = 0$$

$$\Rightarrow \quad E\, 2\pi x l = \frac{Ql\pi x^2}{4\pi\varepsilon_0 R^3} \quad \therefore \quad E = \frac{Qx}{8\pi\varepsilon_0 R^3}$$

4. (a) : Consider N photons in the box. Each having energy $h\upsilon = pc$, here p = momentum.

Energy density, $u = \frac{Nh\upsilon}{V}$, here V = volume

If a photon strikes at an angle θ momentum absorbed will be $p\cos\theta$ and total number of photons striking in time dt will be

$$dN = ndV = nAcdt\cos\theta \ ; \ A = \text{Area of wall}$$

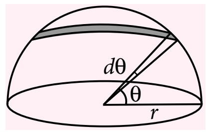
So momentum, $p\cos\theta dN = pnAcdt\cos^2\theta$

$$\therefore \quad \text{Pressure, } dP = \frac{1}{A}\frac{p\cos\theta dN}{dt} = pcn\cos^2\theta$$

$$= u\cos^2\theta$$

To find total pressure we need the fraction of photons striking the wall at angle θ. Since photons are random in direction we can represent their arrival by the hemisphere shown in figure in which the fraction arriving from angle θ is proportional to the surface area of hemisphere subtended by angle θ divided by total surface area.

$$f(\theta)\,d\theta = \frac{2\pi r^2 \sin\theta d\theta}{2\pi r^2} = \sin\theta d\theta$$



Total pressure is then

$$P = \int_0^{\pi/2} dP\, f(\theta)\, d\theta = \int_0^{\pi/2} u\cos^2\theta \sin\theta\, d\theta$$

$$P = \frac{1}{3}u$$

5. (b) : Induced emf due to motion of rod is inductor,

$$Blv = L\frac{di}{dt} \qquad \text{...(i)}$$

Force exerted on rod,

$$m\frac{dv}{dt} = -ilB \qquad \text{...(ii)}$$

Differentiating eqn (ii) w.r.t t,

$$m\frac{d^2v}{dt^2} = -Bl\frac{di}{dt} = -(Bl)\frac{(Bl)}{L}v$$

$$\Rightarrow \quad \frac{d^2v}{dt^2} = -\left(\frac{Bl}{\sqrt{mL}}\right)^2 v$$

$$\therefore \quad v = v_0 \cos\left(\frac{Bl}{\sqrt{mL}}\right)t \qquad \text{...(iii)}$$

Substituting eqn. (iii) in eqn. (i),

$$\text{So} \quad \frac{di}{dt} = \frac{Bl}{L} v_0 \cos\left(\frac{Bl}{\sqrt{mL}}\right)t$$

$$\therefore \quad i = \frac{Bl}{L} v_0 \sin\left(\frac{Bl}{\sqrt{mL}}\right) t \frac{1}{(Bl/\sqrt{mL})}$$

$$= \sqrt{\frac{m}{L}}\, v_0 \sin\left(\frac{Bl}{\sqrt{mL}}\right) t \ \Rightarrow\ i_{max} = \sqrt{\frac{m}{L}} v_0$$

6. (a, b, c) : $W_{A \to B}$ = Area of $ABED$

$$= \frac{1}{2}(BC \times AC) + (CD \times DE)$$

$$= \frac{1}{2}(6 \times 10^{-3}) \times 4 + 4 \times 6 \times 10^{-3}$$

$= 0.012 + 0.024 = 0.036$ J

$W_{B \to C}$ = Area of $BCDE$ = – 0.024 J

The negative sign shows that the work is done on the gas.

$W_{C \to A} = P\Delta V = 0 \qquad (\because \Delta V = 0)$

Work done in cycle $ABCA$ = Area of ΔABC = 0.012 J

7. (a, c): Initial state is same for all three processes (say initial internal energy = U_0)

In the final state, $V_A = V_B = V_C$

and $P_A > P_B > P_C$

$\Rightarrow P_AV_A > P_BV_B > P_CV_C$

$\Rightarrow U_A > U_B > U_C$

If $T_1 > T_2$ then $U_0 > U_f$ for all three processes.

Hence, $(U_0 - U_A) < (U_0 - U_B) < (U_0 - U_C)$

$\Rightarrow |\Delta U_A| < |\Delta U_B| < |\Delta U_C|$

If $T_1 < T_2$, then $U_0 < U_f$ for all three processes and hence $(U_A - U_0) > (U_B - U_0) > (U_C - U_0)$

$\Rightarrow |\Delta U_A| > |\Delta U_B| > |\Delta U_C|$

8. (a, b, c, d) : In the equilibrium position, the net force on the partition will be zero.

Hence pressure on both sides are same.

Hence, (a) is correct.

Initially, $PV = nRT$

$$n_1 = \frac{P_1V_1}{RT_1} = \frac{PV}{RT}$$

$$n_2 = \frac{(2P)(2V)}{RT} = 4\frac{PV}{RT}$$

$\Rightarrow n_2 = 4n_1$

Moles remain conserved.

Finally, pressure becomes equal in both parts.

Using, $P_1V_1 = n_1RT_1$ and $P_2V_2 = n_2RT_2$

$\because P_1 = P_2$ and $T_1 = T_2$

$$\because \quad \frac{V_1}{V_2} = \frac{n_1}{n_2} = \frac{1}{4}$$

$\Rightarrow V_2 = 4V_1$

Also $V_1 + V_2 = 3V$

$\Rightarrow V_1 + 4V_1 = 3V \quad \therefore V_1 = \frac{3}{5}V$

and $V_2 = \frac{12}{5}V$

Hence (b) and (c) are correct.

In compartment (I) :

$P_1' V_1 = n_1RT_1$

$$P_1'\left(\frac{3V}{5}\right) = \left(\frac{PV}{RT}\right)RT$$

$$P_1' = \frac{5PV}{3V} = \frac{5}{3}P$$

Hence (d) is also correct.

9. (3) : Heat lost by liquid flowing through the tube = Heat gained by liquid in bath

$\Rightarrow 1 \times s \times 40 = 2 \times 2s \times (T - 20)$

$\Rightarrow T = 30°C = 10x \quad \therefore x = 3°C$

10. (4) : Let ω be the heat generated per unit volume per unit time

$$\omega \frac{4}{3}\pi r^3 = K 4\pi r^2\left(-\frac{dT}{dr}\right)$$

$$-\frac{dT}{dr} = \frac{\omega r}{3K}$$

$$T = -\frac{\omega r^2}{6K} + C$$

At centre, $r = 0$, $C = T_C$

At $r = R$, $T = 20°C$, $\omega = 20$ W m^{-3},

$$K = \frac{1}{6}\text{W m}^{-1}\,°\text{C}^{-1}$$

$$\therefore \quad T_C = 20 + \frac{\omega R^2}{6K} = 40°\text{C} = 10x$$

$\therefore \quad x = 4°C$

MPP-1 CLASS XII ANSWER KEY

1. (a)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (b)
6. (c)	**7.** (a)	**8.** (b)	**9.** (a)	**10.** (c)
11. (a)	**12.** (d)	**13.** (c)	**14.** (c)	**15.** (c)
16. (b)	**17.** (d)	**18.** (b)	**19.** (c)	**20.** (a, d)
21. (a, c)	**22.** (a, c)	**23.** (b, d)	**24.** (5)	**25.** (2)
26. (2)	**27.** (b)	**28.** (a)	**29.** (a)	**30.** (b)

ANDHRA PRADESH TOPS LIST OF APPLICANTS

OVER 2.23 LAKH CANDIDATES SET TO APPEAR FOR VITEEE-2017

VELLORE: VIT University has yet again created history with a whopping 2.23 lakh set to appear for VIT Engineering Entrance Examination 2017.

This year, the university has registered 2,23,081 students for VITEEE, whereas the number was 2,12,238 last year. In addition, this year the university also received 10,843 more applications, which show the brand's reputation among students across the nation.

Announcing the results at a press conference in VIT University Founder & Chancellor, Dr. G. Viswanathan said that the university's record placement this year and its thrust to create innovation in academics has propelled the increase in patronage among students, especially from the Northern part of India and non-resident Indians.

Among the prominent states in India, Andhra Pradesh tops the chart with 34,068 registrations, which includes 25,011 male applicants, 9,054 female applicants. Second on the list is Uttar Pradesh with 23,360 registrations followed by Telangana with 19,847, Maharashtra with 19,684 and Rajasthan with 16,304 and Tamil Nadu with 16,173 registration.

As per the centre-wise calculations, while Hyderabad registered 16,856, Delhi has registered 15,079 candidates, Vijayawada has registered 13,209, Kota has registered 8,877 candidates and Chennai has registered 7,687. Patna has registered 7,321 candidates and Vellore 2,789.

About VIT Engineering Entrance Examination (VITEEE)

VIT Engineering Entrance Examination (VITEEE) is to be held as Computer-Based Test from 5th April to 16th April 2017, in 119 cities, with 167 centres across India, including Dubai, Kuwait and Muscat for admission to B.Tech. programmes offered by VIT University in Vellore, Chennai, Bhopal (MP) and Amaravathi (AP).

Dr. G. Viswanathan also said that the university would announce VITEEE results on or before 24th April (tentatively) in www.vit.ac.in

The counseling for admissions will begin from 10th to 13th May, 2017, with each day divided by the ranks in ascending order. The counseling for those with ranks upto 8,000 will be held on 10th May followed by those with ranks upto 12,000 on the 11th, Ranks upto 16,000 will sit for counseling on 12th May and those with ranks upto 20,000 will sit on 13th May 2017.

Scholarship under GV School Development programme

To help deserving students get high quality education at VIT, the university has instituted special Scholarships. Dr. Viswanathan said that Central and state board toppers would get 100 percent fee waiver for all four years.

Performance	Scholarship*
Toppers of each State Board and Central Board	100% Tuition fee waiver for all the four years.
VITEEE rank holders of 1 to 50	75% Tuition fee waiver for all the four years.
VITEEE rank holders of 51 to 100	50% Tuition fee waiver for all the four years.
VITEEE rank holders of 101 to 1000	25% Tuition fee waiver for all the four years.

*Terms and Conditions Apply

STARS (Supporting the advancement of Rural Students)

To encourage more Tamil Nadu students to study in VIT, the university has been offering 100 percent waiver in tuition fees and exemption from hostel fee for two plus two toppers from each district in the state under the Supporting the Advancement of Rural Students (STARS) scheme.

"This effort has been made to ensure that high scorers from each district who have poor economic background get their due and are given the best of educations", said the Chancellor.

Speaking about the need for reforms in higher education, Dr. G. Viswanathan.

☺☺

CROSS WORD

Readers can send their responses at editor@mtg.in or post us with complete address by 25th of every month to win exciting prizes. Winners' name with their valuable feedback will be published in next issue.

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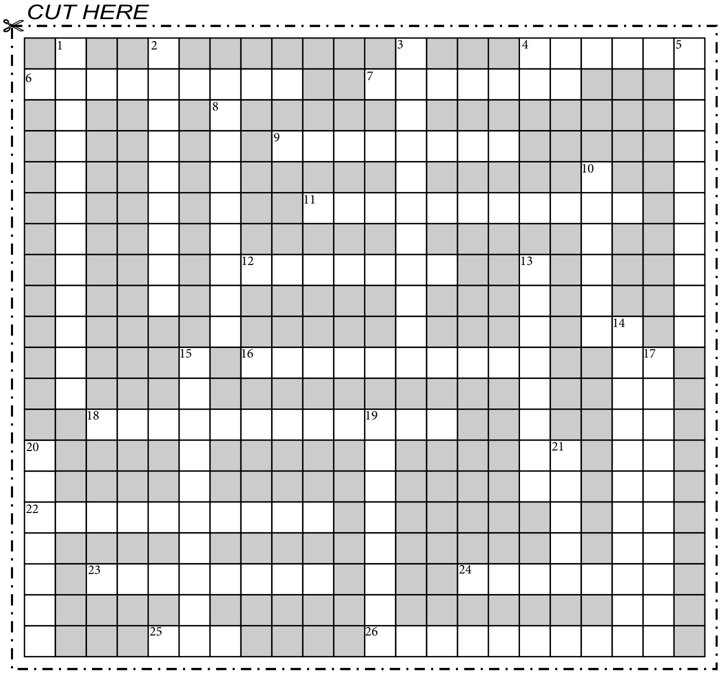

ACROSS

4. The SI unit of capacitance. (5)

6. The study of the origin, evolution and eventual fate of the universe. (9)

7. The point or support on which a lever pivots. (7)

9. The spreading of light beyond its proper boundaries in a developed photographic image. (8)

11. The lowest layer of the Earth's atmosphere. (11)

12. A mission "MOM" carried out by ISRO in 2014. Here 'O' stands for. (7)

16. Electromagnetic radiations emitted by nuclei. (5, 4)

18. Won the Nobel Prize for Physics in 1904. (4,8)

22. The science of measurement of light intensity and amount of illumination. (10)

23. The nuclides having the same number of neutrons. (8)

24. A marine navigation instrument which is used to measure the angle between two objects. (7)

25. CGS unit of energy. (3)

26. A phenomenon that occurs when two objects naturally vibrate at the same frequency. (9)

DOWN

1. Phenomenon that can be exhibited by transverse wave but not by longitudinal wave. (12)

2. In geology, this refers to repetitive layering in metamorphic rocks. (9)

3. A device that permits two or more data sources to share common transmission medium. (11)

5. The minimum distance to which large satellite can approach its primary body without being torn apart by tidal force. (5, 5)

8. The number of disintegrations occurring in a radioactive substance per second. (8)

10. A point on the wave, where the displacement of particles of the medium is minimum. (6)

13. A private network accessible only to an organisation's staff. (8)

14. Reciprocal of the reactance and is measured in siemens. (11)

15. An instrument for measuring the total energy or power received from a body in the form of electromagnetic radiation. (10)

17. Process used to measure the total amount of available motion at a specific joint of the body. (10)

19. A two - port device that transmits microwave or radio frequency power in one direction only. (8)

20. An unstable subatomic particle classified as a baryon, heavier than the neutron and proton. (7)

21. An electric conductor used to divert current. (5)