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1 RANK

Cracking the **NEET**

NAVDEEP SINGH

NEET Class XI-XII | **JEE** ESSENTIALS

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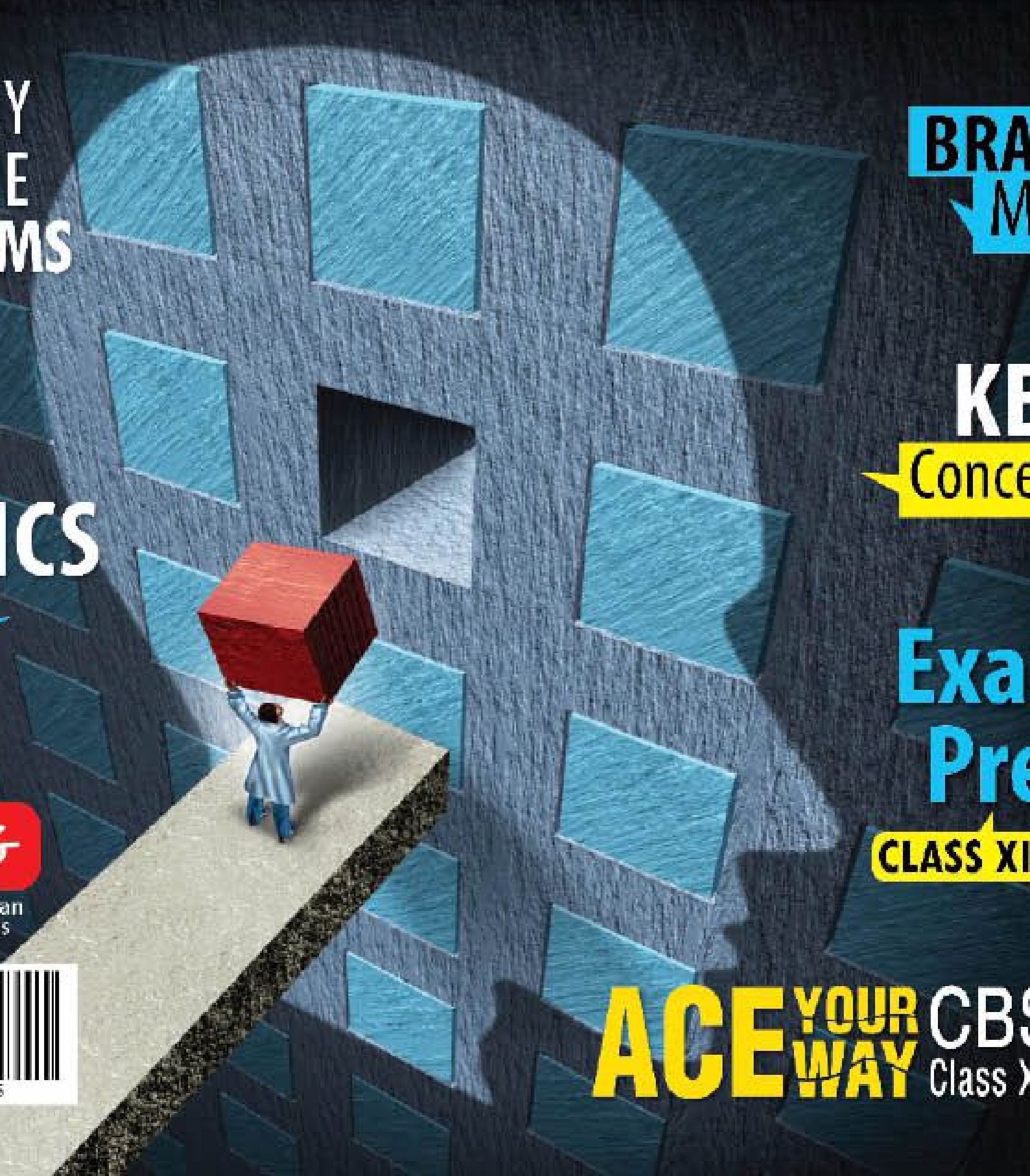

MONTHLY PRACTICE PROBLEMS (XI & XII)

BRAIN Map

KEY Concept

PHYSICS Musing

Exam Prep CLASS XI-XII

ACE YOUR WAY CBSE Class XI-XII

PHYSICS for you

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Cracking the NEET EXAM

Rank 1

Navdeep Singh

- **MTG : Why did you choose medical entrance?**

  Navdeep Singh : From the starting it was my dream to get admission in AIIMS.

- **MTG : What exams you have appeared for and what is your rank in these exams?**

  Navdeep : I also appeared for AIIMS examination and my rank is 46.

- **MTG : Any other achievements? (Please mention the name of exams and rank)**

  Navdeep : I got 56th rank in NTSE exam.

- **MTG : How did you prepare for NEET and other medical exams?**

  Navdeep : I have focused on my weak points and tried to study for 8 hours regularly.

- **MTG : What basic difference you found in various papers you cleared?**

  Navdeep : As far as NEET preparation is concerned it is more focused on NCERT. While one is preparing for AIIMS there has to be clarity of concepts.

- **MTG : How many hours in a day did you study to prepare for the examination?**

  Navdeep : I studied for 8-10 hours daily.

- **MTG : On which topic and chapters you laid more stress in each subject?**

  Navdeep : If you have set your target for Medical you have to study for each and every topic.

- **MTG : How much time does one require for serious preparation for this exam?**

  Navdeep : It depends on many things but I think minimum 6-8 hours are required.

- **MTG : Any extra coaching?**

  Navdeep : No.

- **MTG : Which subjects/topics were you strong/weak at?**

  Navdeep : Physics is a weak point of every medical student but I took Physics as a challenge and Biology was my favourite from the beginning.

- **MTG : Which Books/Magazines/Tutorial/Coaching classes you followed?**

  Navdeep : MTG Objective NCERT at Your Fingertips Biology, MTG Magazines - Physics For You, Chemistry Today, Biology Today, Tutorial - Helix Institute.

> **"MTG books were very helpful. All the medical books are of PMT entrance level and MTG Magazines also have unique and up to date questions. I would recommend MTG Books to all the PMT aspirants."**

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आप Intelligent हो, अच्छी बात है,
आप Confident हो, ये भी अच्छी बात है,
आप मेहनती हो, ये और भी अच्छी बात है
लेकिन IIT-JEE, NEET एवं सभी Eng./Med. की परीक्षा, बिना सही गुरू और सही मार्गदर्शन के सम्भव नहीं है।

About the mentor:

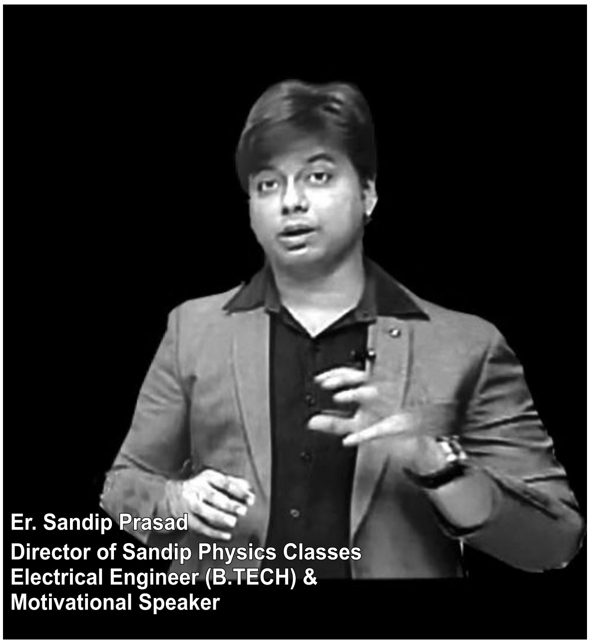
Er.Sandip Prasad is one of the most sought after and famous Physics teachers of India for IIT-JEE, Engineering and medical Entrance Examinations, who founded Sandip Physics Classes (SPC) 8 years ago. SPC which has several centres in Kolkata and patna has been guiding students, aspiring to be IITians and for all other medical and engineering entrance examinations. Many of his students have successfully cracked the IIT, AIIMS, AIPMT, WBJEE, and other exams.

His superhit show **"IIT Made Easy by Sandip Sir"**, is a unique initiative which stressed on the importance of motivation along with the knowledge of the subject, as an essential raw material to crack the exams. The 35-episodes long show, which he recently wrapped used to be telecasted on **Taaza tv (Eastern India's only Hindi news channel),** every Sunday. The show gained unprecedented popularity and viewership.

He is also a columnist of one of West Bengal's highest selling Hindi daily Prabhat Khabar ,where his career counseling articles are published every Saturday.(The e-paper of Kolkata Edition of Prabhat Khabar can be found at www.prabhatkhabar.com. You may also mail your career related queries to the given address).

An eminent speaker,he has conducted several motivational seminars in some of the most reputed schools of Kolkata. News about his seminars, results and contribution have also been printed in dailies like Sanmarg, Dainik Jagran, Chapte Chapte.

A man of absolute devotion, he leaves no stone unturned to help his students with his deep understanding of the subject and amazing problem-solving tricks. It is not surprising that the best and most brilliant of students hold him as their ideal.



Er. Sandip Prasad
Director of Sandip Physics Classes
Electrical Engineer (B.TECH) &
Motivational Speaker

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Near Haryana Bhawan

- **MTG : In your words what are the components of an ideal preparation plan?**

  Navdeep : The components are :

  (1) Hardwork (2) Dedication
  (3) Regularity (4) Focus on target

- **MTG : What role did the following play in your success:**
  (a) parents
  (b) teachers
  (c) school?

  Navdeep : Parents - They always motivated and supported me.

  Teachers - It was not possible for me to achieve my goals without the guidance of my teachers.

  School - I did my 12th from Shivalik Public School and all my school teachers were supportive and were always ready to help me out.

- **MTG : Your family background?**

  Navdeep : My father is physics lecturer and Principal at Government Senior Secondary School and my mother is employee of LIC.

- **MTG : What mistake you think you shouldn't have made?**

  Navdeep : I think I should have studied more in 11th class.

- **MTG : How did you de-stress yourself during the preparation? Share your hobbies and how often could you pursue them?**

  Navdeep : Listening to Gurbani and watching TV.

- **MTG : How have various MTG products like Explorer, Books and Magazines helped you in your preparation?**

  Navdeep : MTG books were very helpful. All the medical books are of PMT entrance level and MTG Magazines also have unique and up to date questions. I would recommend MTG Books to all the PMT aspirants.

- **MTG : Was this your first attempt?**

  Navdeep : Yes

- **MTG : Had you not been selected then what would have been your future plan?**

  Navdeep : I would have preferred research field.

- **MTG : What do you think is the secret of your success?**

  Navdeep : Hardwork

- **MTG : What advice would you like to give our readers who are PMT aspirants?**

  Navdeep : Make a strategy. Work on your weak points. Be focused. Maintain balance between your hobbies and studies.

All the Best!☺☺

NEET | JEE ESSENTIALS

Class XI

Maximize your chance of success, and high rank in NEET, JEE (Main and Advanced) by reading this column. This specially designed column is updated year after year by a panel of highly qualified teaching experts well-tuned to the requirements of these Entrance Tests.

Unit 2 KINEMATICS

Kinematics is the branch of mechanics which deals with the study of motion of a body without taking into account the factors which cause motion.

MOTION

- A body is said to be in motion if it changes its position with time, with respect to its surroundings. *e.g.* a bird flying in air.

Motion in one dimension

- The motion of a body is said to be one dimensional motion if only one out of the three coordinates specifying the position of the body changes with respect to time. In such a motion, the body moves along a straight line.

Motion in two dimensions

- The motion of a body is said to be two dimensional motion if two out of three coordinates specifying the position of the body change with respect to time. In such a motion, the body moves in a plane. *e.g.* motion of a body in a plane.

Distance and Displacement

- **Distance :** The length of the actual path traversed by a body during motion in a given interval of time is called distance travelled by that body.
- Distance is a scalar quantity.
- Distance covered by a moving body can not be zero or negative.
- **Displacement :** The displacement of a body in a given interval of time is defined as the shortest distance between the two positions of the body in a particular direction during that time and is given by the vector drawn from the initial position to its final position.
- Displacement is independent of the path.
- The value of displacement can never be greater than the distance travelled.

Speed and Velocity

- Average speed is defined for a time interval. Average speed of a trip

$$v_{av} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

- If a particular travels distances s_1, s_2, s_3 etc. with speeds v_1, v_2, v_3 etc. respectively, then total distance travelled $s = s_1 + s_2 + s_3 + ... + s_n$

$$\text{Total time taken} = \frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + ... + \frac{s_n}{v_n}$$

$$\text{Average speed of a trip} = \frac{s_1 + s_2 + s_3 + ... + s_n}{\left(\frac{s_1}{v_1} + \frac{s_2}{v_2} + ... + \frac{s_n}{v_n}\right)}$$

- The speed at a particular instant is defined as instantaneous speed (or speed).
- If Δt approaches zero, average speed becomes instantaneous speed. $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

i.e., instantaneous speed is the time derivative of distance.

Velocity

Average velocity : The ratio of net displacement to the time intervals.	$\bar{v}_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$
Instantaneous velocity : The rate of change of position with time at any instant is called instantaneous velocity.	$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

- If a particle moves 1st half of a distance with velocity v_1 and 2nd half with v_2, then $v_{avg} = \frac{2v_1v_2}{v_1 + v_2}$
- If a particle covers one third distance with velocity v_1, next one third with v_2 and last one third with v_3, then
  $v_{avg} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$

Acceleration

Average acceleration : The ratio of net change in velocity to the time taken for change is called the average acceleration.	$\bar{a}_{avg} = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}$
Instantaneous acceleration : The rate of change of velocity of an object is called acceleration of the object.	$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

- If a particle is accelerated for time t_1 with acceleration a_1 and for time t_2 with acceleration a_2, then $a_{avg} = \frac{a_1t_1 + a_2t_2}{t_1 + t_2}$.
- If a particle starts from rest and moves with uniform acceleration then distance covered by the body in t seconds is proportional to t^2.
  $\therefore$ Ratio of distances covered after 1 s, 2 s, 3 s .... is 1 : 4 : 9 ....
- If a body starts from rest and moves with uniform acceleration then distance covered by the body in n^{th} sec is proportional to $(2n - 1)$ (*i.e.*, $s_n \propto (2n - 1)$)
  $\therefore$ Ratio of distance covered in 1st, 2nd and 3rd second is 1 : 3 : 5.

Illustration 1 : A car moves from A to B with a speed of 30 km h^{-1} and from B to A with a speed of 20 km h^{-1}. What is the average speed and average velocity of the car.

Sol. : Let s be distance between A and B.
Let t_1 be time taken by the car to move from A to B with speed v_1 and t_2 be time taken by the car to move from B to A with speed v_2. Then

$t_1 = \frac{s}{v_1}$ and $t_2 = \frac{s}{v_2}$

Average speed of the car

$$v_{av} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{2s}{t_1 + t_2} = \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2v_1v_2}{v_2 + v_1}$$

Here, $v_1 = 30$ km h^{-1}, $v_2 = 20$ km h^{-1}

$$\therefore \quad v_{av} = \frac{2 \times 30 \times 20}{20 + 30} = 24 \text{ km h}^{-1}$$

Since the total displacement of the body through out the journey is zero.

As $\Delta x = x_f - x_i = 0$

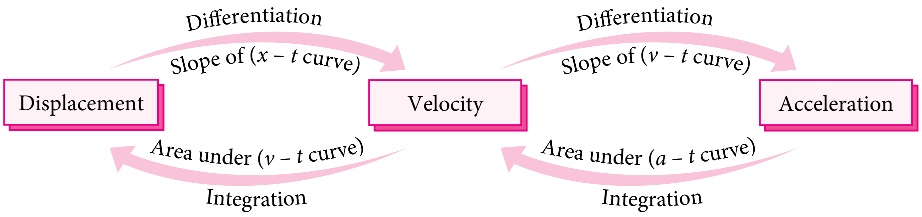
$$\therefore \quad \text{average velocity } \bar{v}_{av} = \frac{\Delta x}{\Delta t} = 0$$

KINEMATIC EQUATIONS

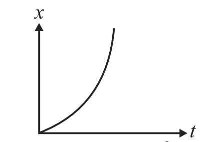
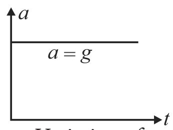
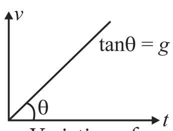
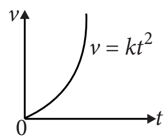
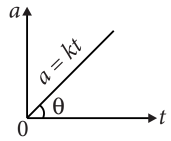
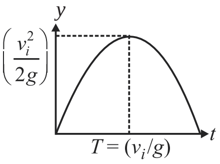
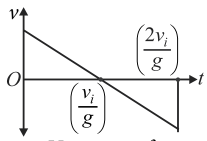
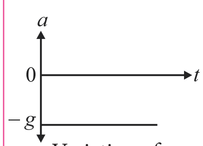
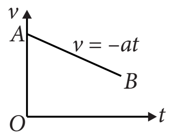
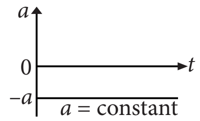
One-Dimensional Approach

Uniformly Accelerated Motion	Motion Under Gravity		Variable Acceleration	
	Vertically downward	Vertically upward	$a = f(t)$	$a = f(x)$
• $v = u + at$ • $s - ut = \frac{1}{2}at^2$ • $v^2 = u^2 + 2as$ • $S_n = u + \frac{a}{2}(2n - 1)$	• Free fall $u = 0$, $a = g$ • $v = gt$ • $h = (1/2)\,gt^2$ • $v^2 = 2gh$	• $v = 0, a = -g$ • $u = gt$ • $h = ut - (1/2)\,gt^2$ • $u^2 = 2gh$	• $v = u + \int_0^t f(t)dt$ • $s = ut + \int_0^t (f(t)dt)dt$	• $v^2 - u^2 = 2\int f(x)dx$

- If acceleration-time equation is given, we can get velocity-time equation by integration. From velocity-time equation, we can get displacement-time equation by integration.



Graphical Analysis

Case I : Motion of a freely falling body under garvity			Case III : Variable Acceleration	
 Variation of distance with time	 Variation of acceleration with time	 Variation of velocity with time	 Velocity time graph for increasing acceleration	 Acceleration time graph for constant increasing acceleration
Case II : Motion of a body projected vertically upward			**Case IV : Constant Retardation**	
 Variation of displacement with time	 Variation of velocity with time	 Variation of acceleration with time	 Velocity time graph for constant retardation	 Acceleration time graph for constant negative acceleration

- For a vertically upward projected body with initial speed v_i,

  Maximum height, $H_{max} = \frac{v_i^2}{2g}$

  Time of flight, $T = 2t = \frac{2v_i}{g}$

- A ball is dropped from a height and it reaches on the earth after t seconds. From the same height, if two balls are thrown one upwards and other downwards, with the same speed v_i and they reach the earth surface after t_1 and t_2 seconds respectively, then $t = \sqrt{t_1 t_2}$.

Illustration 2 : Two balls are projected simultaneously with the same speed from the top of a tower, one vertically upwards and the other vertically downwards. They reach the ground in 9 s and 4 s respectively. Find the height of the tower.
(Take $g = 10$ m s^{-2})

Sol.: Let the downward direction be positive. Initial speeds with which the balls are thrown be v_0 (say).
For the ball thrown upward, $u = -v_0$,
$t = 9$ s, $s = h$, $a = 10$ m s^{-2}

Using $s = ut + \frac{1}{2}at^2$

$h = (-v_0) \times 9 + \frac{1}{2} \times 10 \times 9^2$

$\Rightarrow \quad h = 405 - 9v_0$ ... (i)

For the ball thrown downward, $u = +\,v_0$
$t = 4$ s, $s = h$, $a = +\,10$ m s^{-2},

Using $s = ut + \frac{1}{2}at^2$

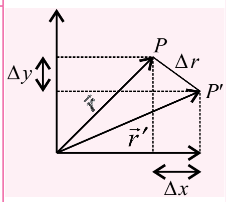
$h = (v_0)\cdot 4 + \left(\frac{10}{2}\right)\cdot 4^2 \Rightarrow h = 4v_0 + 80$ ... (ii)

Solving (i) and (ii), we get,
$v_0 = 25$ m s^{-1}, and $h = 180$ m

Two Dimensional Approach

Position vector : $\vec{r} = x\hat{i} + y\hat{j}$

Displacement vector : Suppose a body displaced from P to P' and $\vec{r}'$ be the position vector of P'.



Net displacement = $\Delta\vec{r} = \vec{r}' - \vec{r}$

$\therefore\ \Delta\vec{r} = (x'\hat{i} + y'\hat{j}) - (x\hat{i} + y\hat{j})$

or $\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j}$

Average velocity : $\vec{v}_{av} = \dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{\Delta x}{\Delta t}\hat{i} + \dfrac{\Delta y}{\Delta t}\hat{j} = \vec{v}_x\hat{i} + \vec{v}_y\hat{j}$

Instantaneous Velocity : $\vec{v} = \lim\limits_{\Delta t \to 0}\dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{d\vec{r}}{dt} = \dfrac{dx}{dt}\hat{i} + \dfrac{dy}{dt}\hat{j}$

Average acceleration

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta v_x\hat{i} + \Delta v_y\hat{j}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} = \vec{a}_x\hat{i} + \vec{a}_y\hat{j}$$

Instantaneous acceleration

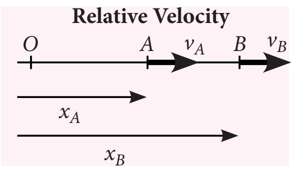
$$\vec{a} = \lim_{\Delta t \to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

Kinematic Equations

- $\vec{v} = \vec{u} + \vec{a}t$
  - along x-axis → $v_x = u_x + a_x t$
  - along y-axis → $v_y = u_y + a_y t$
- $\vec{r} = \vec{r}_0 + \vec{u}_0 t + \dfrac{1}{2}\vec{a}t^2$
  - along x-axis → $x = x_0 + u_x t + 1/2 a_x t^2$
  - along y-axis → $y = y_0 + u_y t + 1/2 a_y t^2$
- $\vec{v}\cdot\vec{v} = \vec{u}\cdot\vec{u} + 2\vec{a}\cdot\vec{r}$
  - along x-axis → $v_x^{\,2} = u_x^{\,2} + 2a_x x$
  - along y-axis → $v_y^{\,2} = u_y^{\,2} + 2a_y y$

RELATIVE MOTION

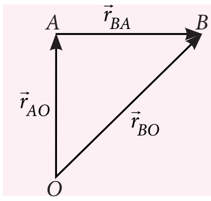
One Dimensional Approach



- Let two particles A and B move along the same straight line and at time t, their displacements measured from some fixed origin O on the line be x_A and x_B respectively. The velocities of A and B are, $v_A = \dfrac{dx_A}{dt}$ and $v_B = \dfrac{dx_B}{dt}$
- The relative velocities of A and B with respect to each other depends on their respective displacements.
- The displacement of B relative to A (*i.e.* displacement of B as measured from A) = $(x_B - x_A)$.
- The rate of change of this displacement is called the velocity of B relative to $A = \dfrac{d}{dt}\ (x_B - x_A)$.

  $\therefore$ The velocity of B relative to $A = \dfrac{dx_B}{dt} - \dfrac{dx_A}{dt}$
  $= v_B - v_A$.
- This is the velocity of B appears to have, when seen from A.

Two Dimensional Approach



- Let $\vec{r}_{AO}$ and $\vec{r}_{BO}$ be the position vectors at time t, of two moving particles with respect to fixed origin O. The velocities $\vec{v}_{AO}$ and $\vec{v}_{BO}$ are then given by,

  $\vec{v}_{AO} = \dfrac{d\vec{r}_{AO}}{dt}$ and $\vec{v}_{BO} = \dfrac{d\vec{r}_{BO}}{dt}$
- By the triangle law of vectors, $\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$

  $\therefore\ \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{r}_{BO} - \vec{r}_{AO} = \vec{r}_{BA}$
- $\overrightarrow{AB}$ is the displacement of A relative to B and $\vec{r}_{BA}$ is the position vector of B relative to A.
- The velocity of B relative to A is $\vec{v}_{BA} = \dfrac{d\vec{r}_{BA}}{dt}$

  $= \dfrac{d}{dt}(\vec{r}_{BO} - \vec{r}_{AO}) = \dfrac{d\vec{r}_{BO}}{dt} - \dfrac{d\vec{r}_{AO}}{dt}$

  $\therefore\ \vec{v}_{BA} = \vec{v}_{BO} - \vec{v}_{AO}$
- Relative velocity of a body A with respect to body B when the two bodies moving at an angle θ is given by

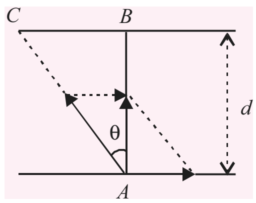
  $$v_{AB} = \sqrt{v_A^2 + v_B^2 + 2v_A v_B\cos(180° - \theta)}$$
  $$= \sqrt{v_A^2 + v_B^2 - 2v_A v_B\cos\theta}$$

  If $\vec{v}_{AB}$ makes an angle β with the direction of $\vec{v}_A$ then,

  $$\tan\beta = \frac{v_B\sin(180° - \theta)}{v_A + v_B\cos(180° - \theta)} = \frac{v_B\sin\theta}{v_A - v_B\cos\theta}$$

Boat-river problem

(a) To cross the river in the shortest path : For the shortest path, the boat should be rowed upstream making an angle θ with AB such that AB gives the direction of resultant velocity.



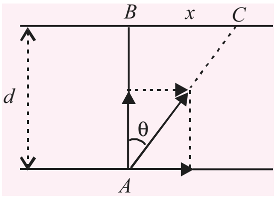
So, $\sin\theta = \dfrac{v_2}{v_1}$

and $v^2 = v_1^2 - v_2^2$

Also $t = \dfrac{d}{v} = \dfrac{d}{\sqrt{v_1^2 - v_2^2}}$

$\left\{\begin{array}{l} v_1 = \text{velocity of boat in still water} \\ v_2 = \text{velocity of flow of water in river} \\ d = \text{width of river} \end{array}\right.$

(b) To cross the river in the shortest time : For the boat to cross the river in shortest time, the boat should be directed along AB. Let v be the resultant velocity making an angle θ with AB. Then



$\tan\theta = \dfrac{v_2}{v_1}$ and $v^2 = v_1^2 + v_2^2$.

$\therefore$ Time of crossing, $t = d/v_1$.

Now the boat reaches the point C rather than reaching point B due to velocity of river. If $BC = x$, then

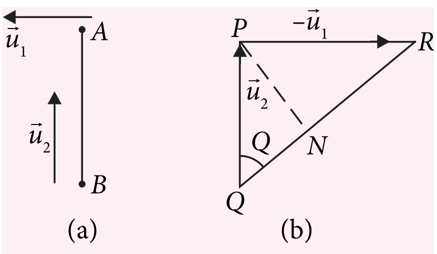
$$\tan\theta = \frac{v_2}{v_1} = \frac{x}{d} \quad \text{or} \quad x = d \times \left(\frac{v_2}{v_1}\right)$$

(c) If a man travels downstream in a river, time taken by the man to cover a distance d is $t_1 = \dfrac{d}{v_1 + v_2}$.

If a man swims upstream in a river, time taken by him to cover a distance d is $t_2 = \dfrac{d}{v_1 - v_2}$. So $\dfrac{t_1}{t_2} = \dfrac{v_1 - v_2}{v_1 + v_2}$.

Illustration 3 : Two ships are $10\sqrt{2}$ km apart on a line running south to north. The one further north is moving west with a speed of 25 km h^{-1}, while the other towards north with a speed of 25 km h^{-1}. What is their distance of closest approach and how long do they take to reach it?

Sol.: Let A and B be the initial positions of the two ships such that $AB = 10\sqrt{2}$ km.



The ship at A is moving with velocity u_1 (25 km h^{-1}) westward and the ship B northward with velocity u_2 (25 km h^{-1}).

The relative velocity of ship B with respect to ship A is given by $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = \vec{u}_2 - \vec{u}_1 = \vec{u}_2 + (-\vec{u}_1)$

In figure (b) $\overrightarrow{QP}$ and $\overrightarrow{PR}$ represent velocities $\vec{u}_2$ and $-\vec{u}_1$ respectively.

Then $\overrightarrow{QR}$ represents the relative velocity of B with respect to A.

Now, $|\overrightarrow{QR}| = \sqrt{PQ^2 + PR^2} = \sqrt{25^2 + 25^2} = 25\sqrt{2}$ km h^{-1}

and $\tan\theta = \dfrac{PR}{QP} = \dfrac{25}{25} = 1 \;\therefore\; \theta = 45°$.

Now, we can suppose that the ship A is at rest at P and the ship B is moving along QR with a velocity of $25\sqrt{2}$ km h^{-1}.

The closest distance between the ships will be PN, which is the perpendicular distance of P from QR.

Now, $\dfrac{PN}{PQ} = \sin 45°$; $PN = PQ \sin 45°$

or $PN = 10\sqrt{2} \times \dfrac{1}{\sqrt{2}} = 10$ km

$QN = PQ \cos 45° = 10\sqrt{2} \times \dfrac{1}{\sqrt{2}} = 10$ km

$\therefore$ Time required to reach the closest distance

$$= \frac{QN}{\text{relative velocity}} = \frac{10}{25\sqrt{2}}\ \text{h}$$

$$= \frac{10}{25\sqrt{2}} \times 60\ \text{min.} = 17\ \text{min}$$

PROJECTILE MOTION

Oblique Projectile Motion

Equation of Trajectory	$y = x\tan\theta - \dfrac{1}{2}\dfrac{gx^2}{u^2\cos^2\theta}$
Maximum Height	$H = \dfrac{u^2\sin^2\theta}{2g}$
Time of Flight	$T = \dfrac{2u\sin\theta}{g}$
Horizontal Range	$R = \dfrac{u^2\sin 2\theta}{g}$

Horizontal Projectile Motion

Equation of Trajectory	$y = \frac{1}{2}\frac{gx^2}{u^2}$
Time of Descent	$T = \sqrt{\frac{2h}{g}}$
Horizontal Range	$R = u\sqrt{\frac{2h}{g}}$
Instantaneous Velocity	$v = \sqrt{u^2 + 2gy} = \sqrt{u^2 + g^2t^2}$ $\tan\phi = \frac{v_y}{v_x}; \phi = \tan^{-1}\left(\frac{gt}{u}\right)$

Projectile Motion on an Inclined Plane of Inclination (α)

Time of Flight	$T = \frac{2u\sin\theta}{g\cos\alpha}$
Maximum Height	$H = \frac{u^2\sin^2\theta}{2g\cos\alpha}$
Horizontal Range	$R = \frac{2u^2}{g}\frac{\sin\theta\cos(\theta+\alpha)}{\cos^2\alpha}$
Maximum range when thrown upward	$R_{max} = \frac{u^2}{g(1+\sin\alpha)}$
Maximum range when thrown downward	$R_{max} = \frac{u^2}{g(1-\sin\alpha)}$

Some important facts of angular projection of projectile :

Item	Description
• Acceleration of projectile	It is constant throughout the motion of projectile and it acts vertically downwards.
• Velocity of projectile	It is different at different instants. It is maximum at the starting point O *i.e.* u and is minimum at the highest point *i.e.* $u\cos\theta$.
• Linear momentum at the highest point	$p_H = mu\cos\theta$
• Linear momentum at the lowest point	$p_0 = mu$
• Maximum horizontal range	$R_{max} = u^2/g$. It is so when $\theta = 45°$
• Horizontal range will be same	(i) if angle of projection is θ or $90° - \theta$ (ii) if angle of projection is $(45° + \theta)$ or $(45° - \theta)$
• Kinetic energy of projectile	It is maximum at the starting point O and is minimum at the highest point H.
• Angular momentum of projectile at H	$L = (mu\cos\theta) \times H$

Illustration 4 : A projectile A is thrown with velocity v_1 at an angle of 30° to the horizontal from point P. At the same time, another projectile B is thrown with velocity v_2 upwards from the point Q vertically below the highest point. For B to collide with A, $\frac{v_2}{v_1}$ should be

(a) 1 (b) 2

(c) $\frac{1}{2}$ (d) 4

Sol. (c) : The time of ascent for the projectile A is

$$\Rightarrow t_A = \frac{v_1\sin 30°}{g}$$

Maximum height of the projectile $A = \frac{v_1^2\sin^2 30°}{2g}$

Height attained by projectile $B = v_2t - \frac{1}{2}gt^2$.

For collision, both heights must be same at the time t_A,

$$\Rightarrow \frac{v_1^2\sin^2 30°}{2g} = v_2\left(\frac{v_1\sin 30°}{g}\right) - \frac{1}{2}g\left(\frac{v_1^2\sin^2 30°}{g^2}\right)$$

$$\text{or} \quad \frac{v_1^2\sin^2 30°}{g} = \frac{v_1\cdot v_2\sin 30°}{g} \Rightarrow \frac{v_2}{v_1} = \sin 30° = \frac{1}{2}$$

Illustration 5 : On an inclined plane of inclination 30°, a ball is thrown at an angle of 60° with the horizontal from the foot of the incline with a velocity of $10\sqrt{3}$ m s^{-1}. If g = 10 m s^{-2}, then the ball will hit the inclined plane in

(a) 1 s (b) 2 s

(c) $2\sqrt{3}$ s (d) $4\sqrt{3}$ s

Sol. (b) : Taking x-axis parallel to inclined plane and y-axis perpendicular to inclined plane,

$u_y = 10\sqrt{3}\ \sin(60° - 30°) = 5\sqrt{3}$ m s^{-1}

$a_y = -g\cos 30° = -\dfrac{g\sqrt{3}}{2}$

When the ball strikes back, $s_y = 0$

Using, $s_y = u_y t + \dfrac{1}{2} a_y t^2$

$$0 = (5\sqrt{3})t - \frac{1}{2}\frac{g\sqrt{3}}{2}t^2 \Rightarrow t = 0 \text{ or } t = \frac{5\times 4}{g} = \frac{20}{10} = 2 \text{ s.}$$

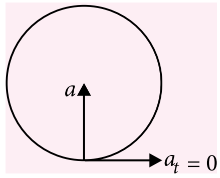
CIRCULAR MOTION

- When a particle moves in a plane such that its distance, from fixed (or moving) point remains constant then its motion is called as circular motion with respect to that fixed (or moving) point.

Angular displacement: Angle traced by position vector of a particle moving w.r.t. some fixed point.	$\text{Angle} = \dfrac{\text{Arc}}{\text{Radius}}$ $\Delta\theta = \dfrac{\text{Arc}}{r}$
Angular velocity : It is defined as the rate of change of angular displacement.	$\omega = \dfrac{\text{Angle traced}}{\text{Time taken}}$ $= \lim\limits_{\Delta t\to 0}\dfrac{\Delta\theta}{\Delta t} = \dfrac{d\theta}{dt}$
Relation between linear and angular velocity	$v = \omega r$ In vector form $\vec{v} = \vec{\omega}\times\vec{r}$
Average angular velocity (ω_{av})	$\omega_{av} = \dfrac{\theta_2 - \theta_1}{t_2 - t_1} = \dfrac{\Delta\theta}{\Delta t}$
Instantaneous angular velocity	$\omega = \lim\limits_{\Delta t\to 0}\dfrac{\Delta\theta}{\Delta t} = \dfrac{d\theta}{dt}$
Angular acceleration : Rate of change of angular velocity.	$\alpha = \lim\limits_{\Delta t\to 0}\dfrac{\Delta\omega}{\Delta t} = \dfrac{d\omega}{dt}; \vec{\alpha} = \dfrac{d\vec{\omega}}{dt}$

Uniform Circular Motion

- When a body moves along a circular path with uniform speed, its motion is said to be uniform circular motion.
  - Radius vector $\vec{r}$ is always perpendicular to the velocity vector $\vec{v}$ *i.e.* $\vec{r}\cdot\vec{v} = 0$
  - Velocity vector is always perpendicular to the *i.e.* $\vec{v}\cdot\vec{a} = 0$
- For circular motion, force towards centre (centripetal force) must act so that direction of $\vec{v}$ keeps on changing.



- The work done by centripetal force is always zero.
- Kinetic energy = constant
- Since $|\vec{v}|$ = constant, so tangential acceleration $a_t = 0$

Non-Uniform Circular Motion

- The magnitude of the velocity of the particle in horizontal circular motion changes with respect to time.
- The acceleration of particle is called tangential acceleration. It acts along the tangent to the circle at a point. It changes the magnitude of linear velocity of the particle.
- Centripetal acceleration $\vec{a}_c$ and tangential acceleration $\vec{a}_t$ act at right angles to each other.

$$\therefore\quad a^2 = a_c^2 + a_t^2 = \left(\frac{v^2}{r}\right)^2 + a_t^2.$$

$$\tan\phi = \frac{a_t}{a_c} = \frac{r\alpha}{v^2/r} = \frac{r^2\alpha}{v^2}.$$

Illustration 6 : A motor car is moving with a speed of 20 m s^{-1} on a circular track of radius 100 m. If its speed is increasing at the rate of 3 m s^{-2}, its resultant acceleration is

(a) 3 m s^{-2} (b) 5 m s^{-2}

(c) 2.5 m s^{-2} (d) 3.5 m s^{-2}

Sol. (b) : Here, v = 20 m s^{-1}, r = 100 m, a_t = 3 m s^{-2}

Centripetal acceleration, $a_c = \dfrac{v^2}{r} = \dfrac{20\times 20}{100} = 4$ m s^{-2}

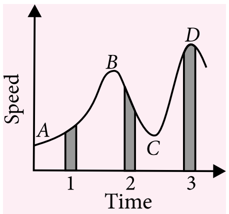
Here, a_t is acting tangential to the circular path and a_c is acting along the radius towards the centre of the circular path.

Resultant acceleration $= \sqrt{a_c^2 + a_t^2}$

$= \sqrt{(4)^2 + (3)^2} = \sqrt{25} = 5$ m s^{-2}

SPEED PRACTICE

1. Figure gives speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. Among the three intervals, the incorrect statement is

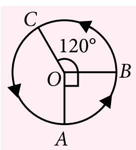


(a) The average acceleration in the second interval is greatest
(b) The average speed is greatest in the third interval
(c) The acceleration at *A*, *B*, *C*, *D* is zero.
(d) The distance travelled in the second interval is greatest.

2. An engineer works at a factory out of town. A car is sent for him from the factory every day and arrives at the railway station at the same time as the train. One day the engineer arrived at the station one hour before his usual time and without waiting for the car, started walking towards factory. On his way he met the car and reached his factory 10 min before the usual time. For how much time did the engineer walk before he met the car? The car moves with the same speed everyday.
(a) 105 min (b) 55 min
(c) 75 min (d) 45 min

3. A projectile is projected at an angle α with an initial velocity u. The time t at which its horizontal component will equal the vertical component is
(a) $\frac{u}{g}(\cos\alpha - \sin\alpha)$ (b) $\frac{u}{g}(\sin\alpha + \cos\alpha)$
(c) $\frac{u}{g}(\sin\alpha - \cos\alpha)$ (d) $\frac{u}{g}(\sin^2\alpha - \cos^2\alpha)$

4. In the given figure, a particle starting from point *A* travelling upto *B* with speed v, then upto point *C* with speed $2v$ and to *A* with speed $3v$. The average speed during the entire path is (Take radius = *R*.)



(a) $1.8v$
(b) $3v$
(c) $1.2v$
(d) $2v$

5. An armoured car 2 m long and 3 m wide is moving at 13 m s^{-1} when a bullet hits it in a direction making an angle $\tan^{-1}\left(\frac{3}{4}\right)$ with the car as seen from the street. The bullet enters one edge of the car at the corner and passes out at the diagonally opposite corner. Neglecting any interaction between bullet and the car, the time for the bullet to cross the car is
(a) 0.15 s (b) 0.23 s (c) 0.4 s (d) 0.12 s

6. A man can swim with a speed of 4 km h^{-1} in still water. He crosses a river 1 km wide that flows steadily at 3 km h^{-1}. If he makes his strokes normal to the river current, how far down the river does he go when he reaches the other bank?
(a) 500 m (b) 600 m (c) 750 m (d) 850 m

7. A particle has an initial velocity of 9 m s^{-1} due east and a constant acceleration of 2 m s^{-2} due west. The distance covered by the particle in the fifth second of its motion is
(a) zero (b) 0.5 m (c) 2 m (d) none

8. Two boys are standing at the ends *A* and *B* of a ground where $AB = a$. The boy at *B* starts running in a direction perpendicular to *AB* with velocity v_1. The boy at *A* starts running simultaneously with velocity v and catches the other boy in a time t, where t is
(a) $\frac{a}{\sqrt{v^2 + v_1^2}}$ (b) $\sqrt{\frac{a^2}{(v^2 - v_1^2)}}$
(c) $\frac{a}{(v - v_1)}$ (d) $\frac{a}{(v + v_1)}$

9. A gun is mounted on the top of a tower of height h. When a bullet is fired from the gun at angle θ with the horizontal, bullet moves with speed u to cover maximum range x_m. Then
(a) $\tan\theta = \frac{2u}{\sqrt{u^2 + 2gh}}$ (b) $x_m = \frac{u\sqrt{u^2 + 2gh}}{g}$
(c) $\tan\theta = \frac{u^2}{u^2 + 2gh}$ (d) $x_m = \frac{\sqrt{u^2 + 2gh}}{g}$

10. A train is moving at constant speed V. Its driver observes another train in front of him on the same track and moving in the same direction with a constant speed v. If the distance between the trains be x, then the minimum retardation of the train so as to avoid a collision is

(a) $(V+v)^2/x$ (b) $(V-v)^2/x$
(c) $(V+v)^2/2x$ (d) $(V-v)^2/2x$

11. Two particles A and B get 4 m closer each second while travelling in opposite direction. They get 0.4 m closer every second while travelling in same direction. The speeds of A and B are respectively

(a) 2.2 m s^{-1} and 0.4 m s^{-1}
(b) 2.2 m s^{-1} and 1.8 m s^{-1}
(c) 4 m s^{-1} and 0.4 m s^{-1}
(d) none of these

12. A person is walking at the rate of 3 km h^{-1}, the rain appears to fall vertically. When he increases his speed to 6 km h^{-1}, it appears to meet him at angle of 45° with the vertical. The speed of rain is

(a) $3\sqrt{2}$ km h^{-1} (b) $\frac{3}{\sqrt{2}}$ km h^{-1}
(c) $6\sqrt{2}$ km h^{-1} (d) $2\sqrt{3}$ km h^{-1}

13. A balloon starts rising from ground with an acceleration of 1.25 m s^{-2}. After 8 s a stone is released from the balloon. The maximum height from the ground reached by stone will be

(a) 40 m (b) 50 m (c) 45 m (d) 55 m

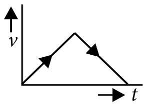
14. Preeti reached the metro station and found that the escalator was not working. She walked up the stationary escalator in time t_1. On other days, if she remains stationary on the moving escalator, then the escalator takes her up in time t_2. The time taken by her to walk up on the moving escalator will be

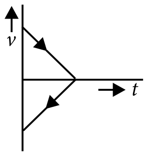
(a) $\frac{t_1 t_2}{t_2 - t_1}$ (b) $\frac{t_1 t_2}{t_2 + t_1}$
(c) $t_1 - t_2$ (d) $\frac{t_1 + t_2}{2}$ **[NEET 2017]**

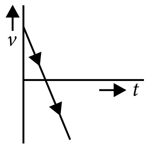
15. The x and y coordinates of the particle at any time are $x = 5t - 2t^2$ and $y = 10t$ respectively, where x and y are in m and t in s. The acceleration of the particle at $t = 2$ s is

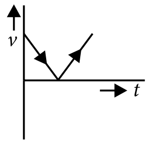
(a) 5 m s^{-2} (b) −4 m s^{-2}
(c) −8 m s^{-2} (d) 0 **[NEET 2017]**

16. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity versus time?

(a)


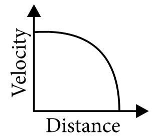
(b)


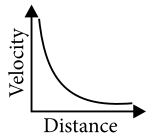
(c)


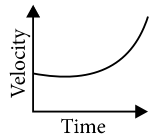
(d)


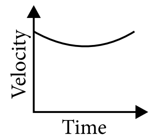
[JEE Main Offline 2017]

17. Which graph corresponds to an object moving with a constant negative acceleration and a positive velocity?

(a)


(b)


(c)


(d)


[JEE Main Online 2017]

18. A car is standing 200 m behind a bus, which is also at rest. The two start moving at the same instant but with different forward accelerations. The bus has acceleration 2 m s^{-2} and the car has acceleration 4 m s^{-2}. The car will catch up with the bus after a time of

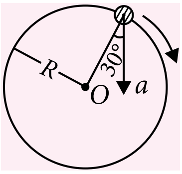
(a) $\sqrt{120}$ s (b) 15 s (c) $10\sqrt{2}$ s (d) $\sqrt{110}$ s
[JEE Main Online 2017]

19. Two cars P and Q start from a point at the same time in a straight line and their positions are represented by $x_P(t) = (at + bt^2)$ and $x_Q(t) = (ft - t^2)$. At what time do the cars have the same velocity ?

(a) $\frac{a-f}{1+b}$ (b) $\frac{a+f}{2(b-1)}$
(c) $\frac{a+f}{2(1+b)}$ (d) $\frac{f-a}{2(1+b)}$

[NEET Phase-II 2016]

20. In the given figure, $a = 15$ m s^{-2} represents the total acceleration of a particle moving in the clockwise direction in a circle of radius $R = 2.5$ m at a given instant of time. The speed of the particle is



(a) 4.5 m s^{-1} (b) 5.0 m s^{-1}
(c) 5.7 m s^{-1} (d) 6.2 m s^{-1}

[NEET Phase-II 2016]

SOLUTIONS

1. **(d) :** As area under the curve in the third interval is largest, so distance travelled in the third interval is greatest. Hence statement (d) is incorrect.

2. **(b)**

3. **(c) :** Horizontal component at any time t is

$v_x = u\cos\alpha$

Vertical component at any time t is

$v_y = u\sin\alpha - gt$

From given problem, $v_x = v_y$

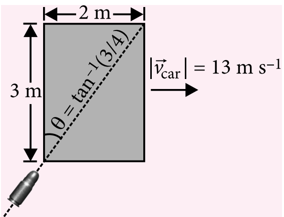
$\therefore \quad u\cos\alpha = u\sin\alpha - gt$ or $t = \frac{u}{g}(\sin\alpha - \cos\alpha)$

4. **(a) :** $t_{AB} = \frac{\pi R}{2v}, t_{BC} = \frac{2}{3}\left(\frac{\pi R}{2v}\right) = \frac{\pi R}{3v}$

$$t_{CA} = \frac{5}{12}\times\frac{2\pi R}{3v} = \frac{5\pi R}{18v}$$

$$v_{av} = \frac{2\pi R}{t_{AB}+t_{BC}+t_{CA}} = \frac{2}{\frac{1}{2v}+\frac{1}{3v}+\frac{5}{18v}} = 1.8v$$

5. **(a) :** Let the speed of the bullet be v. Velocity of bullet relative to car along x-axis = ($v\cos\theta$ – 13) and along y-axis = $v\sin\theta$. Since bullet appears from diagonally opposite corner, its displacement relative to car along x-axis and y-axis are 2 m and 3 m respectively,



i.e., 2 = ($v\cos\theta$ – 13)t and 3 = $v\sin\theta t$

On eliminating v from above eqns., we get

$$t = \frac{1}{13}\left(\frac{3}{\tan\theta} - 2\right) = \frac{2}{13} \approx 0.15 \text{ s}$$

6. **(c) :** To cross the river, the time taken by man

$$= \frac{1\text{ km}}{4\text{ km h}^{-1}} = \frac{1}{4}\text{h}$$

During this time, the river takes him down stream by

$$= \frac{3\text{ km}}{\text{h}}\times\frac{1}{4}\text{h} = \frac{3}{4}\text{km} = 750\text{ m} \quad (1\text{ km} = 1000\text{ m})$$

7. **(b) :** Initial velocity, u = 9 m s^{-1} (due east)
Acceleration, a = –2 m s^{-2} (due east)

$\therefore$ Velocity after any time t, $v = u + at$

= (9 –2t) m s^{-1} (due east)

$\therefore$ Distance travelled in fifth seconds

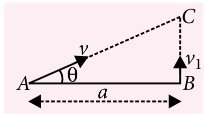
$$d = \int_{t_1}^{t_2}|v|dt = \int_4^5 |9-2t|dt$$

$$= \int_4^{4.5}(9-2t)dt + \int_{4.5}^5 (2t-9)dt = 0.5\text{ m}$$

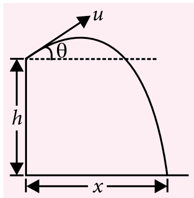
8. **(b) :** The two boys meet at C after time t.
Taking horizontal motion of boy A from A to C,

$a = v\cos\theta t$

or $t = \frac{a}{v\cos\theta} = \sqrt{\frac{a^2}{v^2 - v_1^2}}$



9. **(b) :** The equation of trajectory is



$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$-h = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \quad \ldots(i)$$

For x to be maximum, $\frac{dx}{d\theta} = 0$ and $\frac{d^2x}{d\theta^2}$ = negative

Differentiating eqn. (i) w.r.t. to θ

$$0 = \left(\tan\theta\frac{dx}{d\theta} + x\sec^2\theta\right) - \frac{g}{2u^2}\left(\sec^2\theta\cdot 2x\frac{dx}{d\theta} + x^2 2\sec^2\theta\tan\theta\right)$$

At $x = x_m$, $\frac{dx}{d\theta} = 0 \quad \therefore \quad \tan\theta = \frac{u^2}{gx_m} \quad \ldots(ii)$

Put $\sec^2\theta = 1 + \tan^2\theta$ in eqn. (i)

$$-h = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta)$$

$$-h = x_m\frac{u^2}{gx_m} - \frac{gx_m^2}{2u^2}\left(1+\frac{u^4}{g^2x_m^2}\right) \quad \text{[using eqn. (ii)]}$$

$$x_m = \frac{u\sqrt{u^2+2gh}}{g} \text{ and } \tan\theta = \frac{u}{\sqrt{u^2+2gh}}$$

10. **(d) :** Here relative velocity of the train w.r.t. other train = $V - v$

Hence, $0 - (V - v)^2 = 2ax$ or $a = -\frac{(V-v)^2}{2x}$

$\therefore$ Minimum retardation $= -\frac{(V-v)^2}{2x}$

11. **(b) :** If u and v are speeds of A and B respectively, then $u + v = 4$ and $u - v = 0.4$.

$\therefore \quad 2u = 4.4$ or $u = 2.2$ m s^{-1}; $v = 1.8$ m s^{-1}

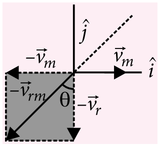
12. **(a) :** Let velocity of rain be

$\vec{v}_r = x\hat{i} + y\hat{j}$

Velocity of man $\vec{v}_m = 3\hat{i}$

Velocity of rain with respect to man,

$\vec{v}_{rm} = \vec{v}_r + (-\vec{v}_m) = (x-3)\hat{i} + y\hat{j}$



Rain seem to be vertical

$\therefore \quad x - 3 = 0 \quad$ or $\quad x = 3$

In second case, $\vec{v}_m = 6\hat{i}$

$\therefore \quad \vec{v}_{rm} = (x-6)\hat{i} + y\hat{j} = -3\hat{i} + y\hat{j} \qquad (\because x = 3)$

It seems to be at an angle 45° with vertical.

$\therefore \quad |x| = |y| = 3$

$\therefore \quad |v_r| = \sqrt{x^2 + y^2} = \sqrt{3^2 + 3^2} = 3\sqrt{2}$ km h^{-1}

13. **(c) :** When a particle separates from a moving body it retains the velocity of the body but not its acceleration. The distance travelled by the balloon when stone is released is $s_0 = \frac{1}{2} \times (1.25) \times 64 = 40$ m above the ground and is having an upward velocity = 1.25 × 8 = 10 m s^{-1}.

For the motion of stone from the balloon to the highest point, u = 10 m s^{-1}, $a = -g = -10$ m s^{-2} and v = 0. Using $v^2 - u^2 = 2a(s - s_0)$, where s is the displacement from the ground, we get

$0 - (10)^2 = -2\,(10)\,(s - 40)$ or $s = 45$ m

$\therefore$ Maximum height attained by stone from the ground = 45 m.

14. **(b):** Let v_1 is the velocity of Preeti on stationary escalator and d is the distance travelled by her,

$\therefore \; v_1 = d/t_1$

Again, let v_2 is the velocity of escalator

$\therefore \quad v_2 = d/t_2$

$\therefore$ Net velocity of Preeti on moving escalator with respect to the ground is

$$v = v_1 + v_2 = \frac{d}{t_1} + \frac{d}{t_2} = d\left(\frac{t_1 + t_2}{t_1 t_2}\right)$$

The time taken by her to walk up on the moving escalator will be

$$t = \frac{d}{v} = \frac{d}{d\left(\frac{t_1 + t_2}{t_1 t_2}\right)} = \frac{t_1 t_2}{t_1 + t_2}$$

15. **(b):** $x = 5t - 2t^2$, $y = 10t$

$$\frac{dx}{dt} = 5 - 4t, \; \frac{dy}{dt} = 10 \Rightarrow v_x = 5 - 4t, \; v_y = 10$$

$$\frac{dv_x}{dt} = -4, \; \frac{dv_y}{dt} = 0 \Rightarrow a_x = -4, \; a_y = 0$$

Acceleration, $\vec{a} = a_x\hat{i} + a_y\hat{j} = -4\hat{i}$, *i.e.*, $a = -4$ m s^{-2}

16. **(c) :** Velocity of the body going upwards is given by

$v = v_0 - gt \qquad (v_0 =$ initial velocity$)$

Hence, the graph between velocity and time should be a straight line with negative slope g and intercept v_0.

Also, during the whole motion, acceleration of the body is constant *i.e.,* slope should be constant.

17. **(a) :** Here, acceleration is given by, $a = -c$

$$\frac{dv}{dt} = -c \quad \text{or} \quad \frac{dx}{dt} \cdot \frac{dv}{dx} = -c$$

or $\quad v\,dv = -c\,dx \qquad \ldots\text{(i)} \qquad \left(\because \frac{dx}{dt} = v\right)$

On integrating eqn. (i) we get $\frac{v^2}{2} = -cx + k$

where k is a constant of integration.

$\therefore \quad x = -\frac{v^2}{2c} + \frac{k}{c}$, *i.e.*, graph (a) is correct.

18. **(c) :** Acceleration of car, a_C = 4 m s^{-2}

Acceleration of bus, a_B = 2 m s^{-2}

Initial separation between the bus and car, s_{CB} = 200 m

Acceleration of car with respect to bus,

$a_{CB} = a_C - a_B$ = 2 m s^{-2}

Initial velocity $u_{CB} = 0$, t = ?

As, $s_{CB} = u_{CB} \times t + \frac{1}{2} a_{CB} t^2$

$\therefore \quad 200 = 0 \times t + \frac{1}{2} \times 2 \times t^2 \Rightarrow t = 10\sqrt{2}$ s

The car will catch the bus after $10\sqrt{2}$ s.

19. **(d):** Position of the car P at any time t, is

$x_P(t) = at + bt^2$

$$v_P(t) = \frac{dx_P(t)}{dt} = a + 2bt$$

Similarly, for car Q,

$x_Q(t) = ft - t^2$

$$v_Q(t) = \frac{dx_Q(t)}{dt} = f - 2t$$

$\because \quad v_P(t) = v_Q(t) \qquad$ (Given)

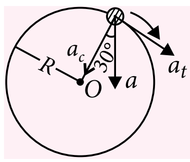
$\therefore \quad a + 2bt = f - 2t$ or $2t(b + 1) = f - a$

$\therefore \quad t = \frac{f - a}{2(1 + b)}$

20. **(c) :** Here, a = 15 m s^{-2}

R = 2.5 m

From figure,



$a_c = a \cos 30° = 15 \times \frac{\sqrt{3}}{2}$ m s^{-2}

As we know, $a_c = \frac{v^2}{R} \Rightarrow v = \sqrt{a_c R}$

$$\therefore \quad v = \sqrt{15 \times \frac{\sqrt{3}}{2} \times 2.5} = 5.69 \simeq 5.7 \text{ m s}^{-1}$$

EXAM PREP 2018

CLASS XI

Useful for Medical/Engg. Entrance Exams

CHAPTERWISE MCQs FOR PRACTICE

WORK, ENERGY AND POWER

1. When a body moves in a circle, the work done by the centripetal force is always
   (a) > 0 (b) < 0
   (c) Zero (d) None of these

2. Power applied to a particle varies with time as $P = (3t^2 - 2t + 1)$ W, where t is in second. Find the change in its kinetic energy between $t = 2$ s and $t = 4$ s
   (a) 32 J (b) 46 J (c) 61 J (d) 100 J

3. A bomb moving with velocity $(40\hat{i} + 50\hat{j} - 25\hat{k})$ m s^{-1} explode into two pieces of mass ratio 1 : 4. After explosion the smaller piece moves away with velocity $(200\hat{i} + 70\hat{j} + 15\hat{k})$ m s^{-1}. The velocity of larger piece after explosion is
   (a) $45\hat{j} - 35\hat{k}$ (b) $45\hat{i} - 35\hat{j}$
   (c) $45\hat{k} - 35\hat{j}$ (d) $-35\hat{i} + 45\hat{k}$

4. A body of mass 5 kg makes an elastic collision with another body at rest and continues to move in the original direction after collision with a velocity equal to $\left(\frac{1}{10}\right)^{th}$ of its original velocity. Then the mass of the second body is
   (a) 4.09 kg (b) 0.5 kg (c) 5 kg (d) 5.09 kg

5. A body of mass m strikes another body at rest of mass $\frac{m}{9}$. Assuming the impact to be inelastic, the fraction of the initial kinetic energy transformed into heat during the contact is
   (a) 0.1 (b) 0.2 (c) 0.5 (d) 0.64

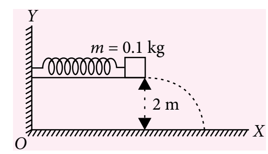
6. Which of the following is true for any collision?
   (a) Both linear momentum and kinetic energy are conserved.
   (b) Neither linear momentum nor kinetic energy may be conserved.
   (c) Linear momentum is always conserved, however, kinetic energy may or may not be conserved.
   (d) Kinetic energy is always conserved, but linear momentum may or may not be conserved.

7. The kinetic energy of particle moving along a circle of radius R depends upon the distance covered S and is given by $K = aS$, where a is a constant. Then the force acting on the particle is
   (a) $\frac{aS}{R}$ (b) $\frac{2(aS)^2}{R}$ (c) $\frac{aS^2}{R^2}$ (d) $\frac{2aS}{R}$

8. A body of mass M is dropped from a height h on a sand floor. If the body penetrates x cm into the sand, the average resistance offered by the sand to the body is
   (a) $Mg\left(\frac{h}{x}\right)$ (b) $Mg\left(1 + \frac{h}{x}\right)$
   (c) $Mgh + Mgx$ (d) $Mg\left(1 - \frac{h}{x}\right)$

9. The displacement x and time t for a particle are related to each other as $t = \sqrt{x} + 3$. What is work done in first six seconds of its motion.
   (a) 6 J (b) zero (c) 4 J (d) 2 J

10. A small block of mass 0.1 kg is pressed against a horizontal spring fixed at one end to compress the spring through 5.0 cm as shown in figure. The spring constant is 100 N m^{-1}. When released the block moves horizontally till it



leaves the spring, it will hit the ground 2 m below the spring

(a) At a horizontal distance of 1 m from free end of the spring.
(b) At a horizontal distance of 2 m from free end of the spring.
(c) Vertically below the edge on which the mass is resting.
(d) At a horizontal distance of $\sqrt{2}$ m from free end of the spring.

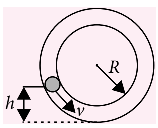
11. A proton is kept at rest. A positively charged particle is released from rest at a distance d in its field. Consider two experiments; one in which the charged particle is also a proton and in another, a positron. In the same time t, the work done on the two moving charged particles is

(a) Same as the force law is involved in the two experiments.
(b) Less for the case of a positron, as the positron moves away more rapidly and the force on it weakens.
(c) More in the case of positron, as the positron moves away a larger distance.
(d) Same as the work done by charged particle on the stationary proton.

12. A bullet losses 19% of its kinetic energy when passes through an obstacle. The percentage change in its speed is

(a) Reduced by 10% (b) Reduced by 19%
(c) Reduced by 9.5% (d) Reduced by 11%

13. With what minimum speed v must a small ball should be pushed inside a smooth vertical tube from a height h, so that it may reach the top of the tube? Radius of the tube is R. (Assume radius of cross-section of tube is negligible in comparison to R.)



(a) $\sqrt{2g(h+2R)}$ (b) $\frac{5}{2}R$
(c) $\sqrt{g(5R-2h)}$ (d) $\sqrt{2g(2R-h)}$

14. A ball is thrown from a height of h metre with an initial downward velocity v_0. It hits the ground, loses half of its kinetic energy and bounces back to the same height. The value of v_0 is

(a) $\sqrt{2gh}$ (b) $\sqrt{gh}$
(c) $\sqrt{3gh}$ (d) $\sqrt{2.5gh}$

15. A girl in a swing is 2.5 m above ground at the maximum height and at 1.5 m above the ground at the lowest point. Her maximum velocity in the swing is (g = 10 m s^{-2})

(a) $5\sqrt{2}$ m s^{-1} (b) $2\sqrt{5}$ m s^{-1}
(c) $2\sqrt{3}$ m s^{-1} (d) $3\sqrt{2}$ m s^{-1}

SYSTEM OF PARTICLES AND ROTATIONAL MOTION

16. A diver makes 2.5 revolution on the way from a 10 m high platform to the water. Assuming zero initial vertical velocity, the average angular velocity during the dive is

(a) $\frac{3\pi}{\sqrt{2}}$ rad s^{-1} (b) $\frac{5\pi}{\sqrt{2}}$ rad s^{-1}
(c) $\frac{5\pi}{\sqrt{3}}$ rad s^{-1} (d) $\frac{\pi}{\sqrt{2}}$ rad s^{-1}

17. The angular velocity of a rigid body about any point of that body is the same

(a) only in magnitude
(b) only in direction
(c) both in magnitude and direction necessarily
(d) both in magnitude and direction about some points but not about all points.

18. An ice skater starts a spin with her arms stretched out to the sides. She balance on the tip of one skate to turn without friction. She then pulls her arms in so that her moment of inertia decreases by a factor of 2. In the process of her doing so, what happens to her kinetic energy?

(a) It increases by a factor of 4.
(b) It increases by a factor of 2.
(c) It remains constant.
(d) It decreases by a factor of 2.

19. Two particles of equal mass have velocities $\vec{v}_1 = 4\hat{i}$ m s^{-1} and $\vec{v}_2 = 4\hat{j}$ m s^{-1}. First particles has an acceleration $\vec{a}_1 = (5i + 5j)$ m s^{-2} while the acceleration of the other particle is zero. The centre of mass of the two particles moves in a path of

(a) straight line (b) parabola
(c) circle (d) ellipse

20. Two particles of mass 1 kg and 3 kg have position vectors $2\hat{i}+3\hat{j}+4\hat{k}$ and $-2\hat{i}+3\hat{j}-4\hat{k}$ respectively. The centre of mass has a position vector

(a) $\hat{i}+3\hat{j}-2\hat{k}$ (b) $-\hat{i}-3\hat{j}-2\hat{k}$
(c) $-\hat{i}+3\hat{j}+2\hat{k}$ (d) $-\hat{i}+3\hat{j}-2\hat{k}$

21. Moment of inertia of ring about its diameter is I. Then, moment of inertia about an axis passing through centre perpendicular to its plane is

(a) $2I$ (b) $\frac{I}{2}$ (c) $\frac{3}{2}I$ (d) I

22. A flywheel rotates with a uniform angular acceleration. Its angular velocity increases from 20π rad s^{-1} to 40π rad s^{-1} in 10 s. How many rotations did it make in this period?

(a) 80 (b) 100 (c) 120 (d) 150

23. Circular disc of mass 2 kg and radius 1 m is rotating about an axis perpendicular to its plane and passing through its centre of mass with a rotational kinetic energy of 8 J. The angular momentum in (J s) is

(a) 8 (b) 4 (c) 2 (d) 1

24. From a circular ring of mass M and radius R, an arc corresponding to a 90° sector is removed. The moment of inertia of the remaining part of the ring about an axis passing through the centre of the ring and perpendicular to the plane of the ring is k times MR^2. Then the value of k is

(a) 3/4 (b) 7/8 (c) 1/4 (d) 1

25. Three identical spheres, each of mass M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 2 m each. Taking their point of intersection as the origin, the position vector of centre of mass is

(a) $\frac{1}{3}(\hat{i}-\hat{j})$ (b) $\frac{2}{3}(\hat{i}-\hat{j})$

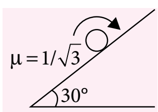
(c) $\frac{2}{3}(\hat{i}+\hat{j})$ (d) $\frac{1}{3}(\hat{i}+\hat{j})$

26. A thin circular ring of mass M and radius R is rotating in a horizontal plane about an axis vertical to its plane with a constant angular velocity ω. If two objects each of mass m be gently attached to the opposite ends of a diameter of the ring, the ring will then rotate with an angular velocity.

(a) $\frac{\omega M}{M+m}$ (b) $\frac{\omega(M-2m)}{M+2m}$

(c) $\frac{\omega M}{M+2m}$ (d) $\frac{\omega(M+2m)}{M}$

27. A cylinder of mass m is rotated about its axis by an angular velocity ω and lowered gently on an inclined plane as shown in figure. Then:



(a) it will start going upward
(b) it will first going upward and then downward
(c) it will go downward just after it is lowered
(d) it can never go upward

28. A thin uniform rod of length L is initially at rest with respect to an inertial frame of reference. The rod is given impulse at one end perpendicular to its length. How far does the centre of the mass translate while the rod completes one revolution about its centre of mass?

(a) $\frac{L\pi}{3}$ (b) $\frac{L\pi}{2}$ (c) $\frac{L\pi}{4}$ (d) $\frac{L\pi}{5}$

29. A body rolls without slipping. The radius of gyration of the body about an axis passing through its centre of mass is K. The radius of the body is R. The ratio of rotational kinetic energy to translational kinetic energy is

(a) $\frac{K^2}{R^2}$ (b) $\frac{R^2}{K^2+R^2}$

(c) $\frac{K^2}{K^2+R^2}$ (d) $K^2 + R^2$

30. A uniform rod of length L (in between the supports) and mass m is placed on two supports A and B. The rod breaks suddenly at length $L/10$ from the support B. Find the reaction at support A immediately after the rod breaks.

(a) $\frac{9}{40}mg$ (b) $\frac{19}{40}mg$ (c) $\frac{mg}{2}$ (d) $\frac{9}{20}mg$

SOLUTIONS

1. (c)

2. (b): $P = 3t^2 - 2t + 1 = \frac{dE}{dt}$

$\therefore \quad dE = (3t^2 - 2t + 1)\,dt$

$$E = \int_{t=2\,s}^{t=4\,s} (3t^2 - 2t + 1)dt = \left[\frac{3t^3}{3} - \frac{2t^2}{2} + t\right]_{t=2s}^{t=4s}$$

$= [(4^3 - 2^3) - (4^2 - 2^2) + (4 - 2)]$

$E = 56 - 12 + 2 = 46$ J

3. (a): Let the mass of the unexploded bomb be $5m$. It explodes into the two pieces of masses m and $4m$ respectively.

Initial momentum of the unexploded bomb

$= 5m(40\hat{i} + 50\hat{j} - 25\hat{k})$

After explosion, momentum of the smaller piece

$= m\vec{v}_1 = m(200\hat{i} + 70\hat{j} + 15\hat{k})$

and momentum of the larger piece $= 4m\vec{v}_2$

where $\vec{v}_1$ and $\vec{v}_2$ are the velocities of the two pieces respectively.
According to the law of conservation of momentum, we get

$$5m(40\hat{i}+50\hat{j}-25\hat{k}) = m(200\hat{i}+70\hat{j}+15\hat{k})+4m\vec{v}_2$$

$$4m\vec{v}_2 = 5m(40\hat{i}+50\hat{j}-25\hat{k})-m(200\hat{i}+70\hat{j}+15\hat{k})$$

$$\vec{v}_2 = \frac{1}{4}(180\hat{j}-140\hat{k}) = 45\hat{j}-35\hat{k}$$

4. **(a) :** Here, $m_1 = 5$ kg, $m_2 = ?$, $u_2 = 0$, $v_1 = \frac{u_1}{10}$

Velocity of the first body after collision is given by

$$v_1 = \frac{(m_1-m_2)u_1}{m_1+m_2}+\frac{2m_2u_2}{m_1+m_2}$$

$$\therefore \quad \frac{u_1}{10} = \frac{(5-m_2)}{5+m_2}u_1+0 \quad \text{or} \quad \frac{1}{10} = \frac{5-m_2}{5+m_2}$$

$$\Rightarrow \quad m_2 = \frac{45}{11} = 4.09 \text{ kg}$$

5. **(a)**

6. **(c) :** Linear momentum is conserved in all types of collision but kinetic energy is not conserved in all types of collision. Kinetic energy is conserved in elastic collision but not conserved in inelastic collision.

7. **(d):** Force on a particle moving on a circular path

$$= \text{Centripetal force} = \frac{mv^2}{R}$$

$$= \left(\frac{1}{2}mv^2\right)\frac{2}{R} = \frac{2K}{R} = \frac{2aS}{R}$$

8. **(b)**

9. **(b):** Given; $t = \sqrt{x}+3 \quad \Rightarrow \quad x = (t-3)^2$

Now, $v = \frac{dx}{dt} = 2(t-3)$

At $t = 0$, $v_1 = 2(-3) = -6$
At $t = 6$, $v_2 = 2(6-3) = 6$
Work done = Change in kinetic energy

$$= \frac{1}{2}m(v_2^2-v_1^2) = \text{zero}$$

10. **(a) :** Here; $m = 0.1$ kg, $h = 2$m, $k = 100$ N m^{-1},

$x = 5$ cm, $\frac{1}{2}kx^2 = mv^2$

$$\frac{1}{2}(100)\left(\frac{5}{100}\right)^2 = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{5}{2}}$$

Then, a horizontal distance from free and of spring,

$$x' = v\sqrt{\frac{2h}{g}} = \sqrt{\frac{5}{2}}\sqrt{\frac{2\times2}{10}} = 1 \text{ m}$$

11. **(c) :** Force between two protons is same as that of between proton and a positron.
As positron is much lighter than proton, it moves away through much larger distance compared to a proton.
We know that, work done = force × distance.
As forces are same in case of proton and positron but distance moved by positron is larger, hence, work done will be more.

12. **(a) :** The kinetic energy of a bullet is given by,

$$K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}}$$

When it losses 19% of its kinetic energy, then kinetic energy, $K' = \frac{81}{100}K$

$$\therefore \quad v' = \sqrt{\frac{2K'}{m}} = \sqrt{\frac{2\times0.81K}{m}} = 0.9\,v$$

$\therefore$ Percentage decrease in speed

$$= \frac{1-0.9}{1}\times100\% = 10\%$$

13. **(d):** For minimum v, velocity of ball at the topmost point will be zero.

By conservation of energy, $\frac{1}{2}mV^2 = mg(2R-h)$

$$V = \sqrt{2g(2R-h)}$$

14. **(a)**

15. **(b):** At the highest point, $v = 0$, $h_1 = 2.5$ m
$\therefore$ Total energy, $E_1 = mgh_1 + 0 = mgh_1$
At the lowest point, $v = v_{max} = ?$, $h_2 = 1.5$ m
$\therefore$ Total energy, $E_2 = mgh_2 + \frac{1}{2}mv^2$
According to the law of conservation of mechanical energy, $E_1 = E_2$

$$\therefore \quad mgh_1 = \frac{1}{2}mv^2 + mgh_2 \text{ or } v^2 = 2g(h_1-h_2)$$

$$\text{or} \quad v = \sqrt{2g(h_1-h_2)} = \sqrt{2\times10\times(2.5-1.5)}$$

$$= \sqrt{2\times10\times1} = \sqrt{20}\text{ m s}^{-1} = 2\sqrt{5}\text{ m s}^{-1}$$

16. **(b):** Let, the free-fall time be t.

$$\Delta y = v_{0y}t+\frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2\Delta y}{g}} = \sqrt{\frac{2(10\text{ m})}{10\text{ m s}^{-2}}} = \sqrt{2}\text{ s}$$

Thus, the magnitude of the average angular velocity

is $\omega_{avg} = \frac{(2.5\text{ rev})(2\pi\text{ rad rev}^{-1})}{\sqrt{2}\text{ s}} = \frac{5\pi}{\sqrt{2}}\text{rad s}^{-1}$

17. **(c)**

18. **(b):** As $L = I\omega \quad \because \quad L$ is constant $\quad \therefore \quad \omega \propto \frac{1}{I}$

$\because$ Inertia decreases by a factor of 2

$\therefore$ Angular velocity increases by a factor of 2.

$\therefore$ Her kinetic energy with be $\frac{1}{2}I\omega^2$ *i.e.*, become double.

19. (a)

20. (d): Here, $m_1 = 1$ kg, $m_2 = 3$ kg

$\vec{r}_1 = 2i + 3\hat{j} + 4\hat{k}, \vec{r}_2 = -2\hat{i} + 3\hat{j} - 4\hat{k}$

The position vector of the center of mass is

$$\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

$$= \frac{(1)(2\hat{i} + 3\hat{j} + 4\hat{k}) + (3)(-2\hat{i} + 3\hat{j} - 4\hat{k})}{1+3}$$

$$= \frac{2\hat{i} + 3\hat{j} + 4\hat{k} - 6\hat{i} + 9\hat{j} - 12\hat{k}}{1}$$

$$= \frac{-4\hat{i} + 12\hat{j} - 8\hat{k}}{4} = -\hat{i} + 3\hat{j} - 2\hat{k}$$

21. (a)

22. (d): Here, $\omega_1 = 20\pi$ rad s^{-1} , $\omega_2 = 40\pi$ rad s^{-1}, $t = 10$ s

As $\omega_2 = \omega_1 + \alpha t$

$\therefore$ $40\pi = 20\pi + \alpha \times 10$ or $\alpha = 2\pi$ rad s^{-2}

From, $\omega_2^2 - \omega_1^2 = 2\alpha\theta$

$(40\pi)^2 - (20\pi)^2 = 2 \times 2\pi\theta$

or $\theta = \frac{1200\pi^2}{4\pi} = 300\pi$

Number of rotations completed

$$= \frac{\theta}{2\pi} = \frac{300\pi}{2\pi} = 150$$

23. (b): Here, Mass of the disc, $M = 2$ kg

Radius of the disc, $R = 1$ m

Moment of inertia of the disc about an axis perpendicular to, its plane and passing through its centre of mass is

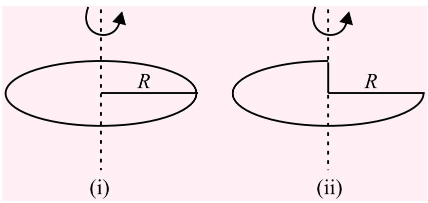
$$I = \frac{1}{2}MR^2 = \frac{1}{2}\times(2\text{ kg})(1)^2 = 1\text{ kg m}^2$$

Kinetic energy of rotation, $K_R = \frac{L^2}{2I}$

where L is the angular momentum

or $L = \sqrt{2K_R I} = \sqrt{2\times 8\times 1} = 4$ J s

24. (a) :



Moment of inertia of a ring about an axis passing through the centre and perpendicular to the plane of the ring is

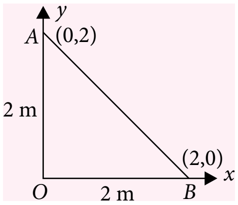
$I = MR^2$

Mass of the remaining portion of the ring as shown in figure (ii) is $M - \frac{M}{4} = \frac{3M}{4}$

Moment of inertia of the remaining portion of the ring about the given axis is $I' = \frac{3}{4}MR^2$

But $I' = kMR^2$ (given) $\therefore$ $k = 3/4$

25. (c) :



$$x_{CM} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{M\times 0 + M\times 0 + M\times 2}{M+M+M} = \frac{2}{3}$$

$$y_{CM} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{M\times 2 + M\times 0 + M\times 0}{M+M+M} = \frac{2}{3}$$

$\therefore$ Position vector of centre of mass is $\frac{2}{3}(\hat{i} + \hat{j})$

26. (c) : As no torque is being applied,

L = constant, *i.e*, $I_1, \omega_1 = I_2\,\omega_2$

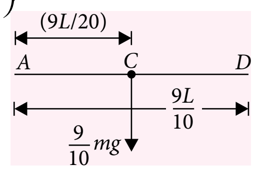
$$\omega_2 = \frac{I_1\omega_1}{I_2} = \frac{MR^2\omega}{(M+2m)R^2} = \frac{\omega M}{(M+2m)}$$

27. (d): Since net force along the incline is zero, so cylinder will remain in position till it stops rotating. After that it will start moving downwards.

28. (a)

29. (a) : $$\frac{\text{Rotational K.E.}}{\text{Translational K.E.}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2} = \frac{mK^2\frac{v^2}{R^2}}{mv^2} = \frac{K^2}{R^2}$$

30. (a) : Torque, $\tau = \frac{9}{10}mg\left(\frac{9}{20}L\right)$



Also, $\tau = I\alpha = \frac{m}{3}\left(\frac{9}{10}L\right)^2\alpha$

$\therefore$ $\alpha = \frac{3g}{2L}$

Acceleration, $a_{CM} = \alpha(AC) = \frac{3g}{2L}\left(\frac{9L}{20}\right) = \frac{27g}{40}$

Now, $\frac{9}{10}mg - N_A = ma_{CM} = m\cdot\frac{27g}{40}$

or $N_A = \frac{9}{40}mg$

ACE YOUR WAY CBSE XI

Series 2

CHAPTERWISE PRACTICE PAPER

MOTION IN A PLANE | LAWS OF MOTION

Time Allowed : 3 hours | Maximum Marks : 70

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Q.no.1 to 5 are very short answer questions and carry 1 mark each.
(iii) Q.no.6 to 10 are short answer questions and carry 2 marks each.
(iv) Q.no.11 to 22 are also short answer questions and carry 3 marks each.
(v) Q.no.23 is a value based question and carries 4 marks.
(vi) Q.no.24 to 26 are long answer questions and carry 5 marks each.
(vii) Use of log tables if necessary, use of calculators is not allowed.

SECTION - A

1. Air is thrown on a sail attached to a boat from an electric fan placed on the boat. Will the boat start moving?
2. Sand is thrown on tracks covered with snow. Why?
3. It is easy to catch a table tennis ball than a cricket ball even both are moving with same velocity. Why?
4. Can the magnitude of the resultant vector of the two given vectors be less than the magnitude of any of the given vectors?
5. Why are porcelain objects wrapped in paper or straw before packing for transportation?

SECTION - B

6. A skilled gunman always keeps his gun tilted above the line of sight while shooting, why?
7. When a ball is thrown upward from the surface of the earth, its momentum first decreases and then increases. Is the conservation of momentum violated in this process?
8. The driver of a three-wheeler moving with a speed of 36 km h^{-1} sees a child standing in the middle of the road and brings his vehicle to rest in 4.0 s just to save the child. What is the average retarding force on the vehicle? The mass of the three-wheeler is 400 kg and the mass of the driver is 65 kg.

OR

A nucleus is at rest in the laboratory frame of reference. Show that if it disintegrates into two smaller nuclei, the products must move in opposite directions.

9. A boy travelling in an open car moving on a levelled road with constant speed tosses a ball vertically up in the air and catches it back. Sketch the motion of the ball as observed by a boy standing on the footpath. Give explanation to support your diagram.
10. A person of mass 50 kg stands on a weighing scale on a lift. If the lift is descending with a downward acceleration of 9 m s^{-2}, what would be the reading of the weighing scale? (g = 10 m s^{-2})

SECTION - C

11. Define projectile. Show that a path of projectile projected with initial velocity u and making an angle θ with the horizontal is parabola.

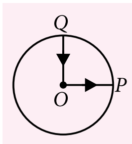
12. A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with a speed of 40 rev. per min in a horizontal plane. What is the tension in the string? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N?

13. Derive the relation between linear velocity and angular velocity.

14. State and prove the polygon law of vector addition.

15. Express Newton's second law of motion in component form. Give its significance.

16. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in figure. If the round trip takes 10 min, what is the (a) net displacement, (b) average velocity, and (c) average speed of the cyclist?



17. For ordinary terrestrial experiments, which of the following observers are inertial and which are non-inertial?
(a) A child rounding in a giant wheel.
(b) A driver in sports-car moving with a constant speed of 200 km h^{-1} on a straight road.
(c) The pilot of an aeroplane which is taking off.
(d) A cyclist negotiating a sharp turn.
(e) The guard of a train which is slowing down to stop at a station.

18. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball?

OR

The position of a particle is given by

$$\vec{r} = 3.0\, t\hat{i} - 2.0t^2\hat{j} + 4.0\hat{k} \text{ m}$$

where t is in seconds and the coefficients have the proper units for $\vec{r}$ to be in metres.
(a) Find the $\vec{v}$ and $\vec{a}$ of the particle?
(b) What is the magnitude and direction of velocity of the particle at $t = 2.0$ s?

19 Give some important implications of the third law of motion.

20. What are the laws of the limiting friction? Is the coefficient of kinetic friction less than or greater than the coefficient of static friction?

21. A helicopter of mass 1000 kg rises with a vertical acceleration of 15 m s^{-2}. The crew and the passengers weigh 300 kg. Give the magnitude and direction of the
(a) force on the floor by the crew and passengers,
(b) action of the rotor of the helicopter on the surrounding air,
(c) force on the helicopter due to the surrounding air.

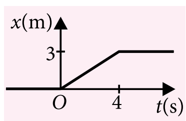
22. Define angle of repose. Deduce its relation with coefficient of static friction?

SECTION - D

23. From a school a group of boys went for a picnic in a village. They went through fields and enjoyed the beauty of nature. While walking, they saw a well which they had never seen in the city. They were very excited and started drawing water from well. They planned to have a competition in which they decided that the who would draw more water would become winner. A villager who was listening to them, went to them and told them about the importance of water. He also explained that they use the water of this for irrigating their fields and also for drinking.
(i) What values were possessed by the villager?
(ii) If the two boys raising the bucket, pull it an angle θ to each other and each exerts a force of 20 N then their effective pull is 30 N. What is the angle between their arms?
(iii) What are the other means of irrigation in villages?

SECTION - E

24. Figure shows the position-time graph of a particle of mass 4 kg. What is the (a) force on the particle for $t < 0$, $t > 4$ s, $0 < t < 4$ s? (b) Impulse at $t = 0$ and $t = 4$ s?



OR

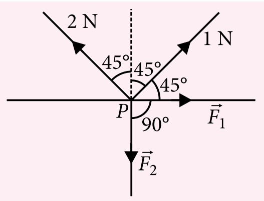
Read each statement carefully and state, with reasons and examples, if it is true or false : A scalar quantity is one that
(a) is conserved in a process
(b) can never take negative values
(c) must be dimensionless
(d) does not vary from one point to another in space
(e) has the same value for observers with different orientations of axes.

25. Give four examples from daily life which illustrate the law of conservation of linear momentum.

OR

(a) State and prove Lami's theorem.

(b) There are four forces acting at a point P produced by strings as shown in figure, which is at rest. Find the forces F_1 and F_2.



26. (a) Show that for a projectile the angle between the velocity and the x-axis as a function of time is given by

$$\theta(t) = \tan^{-1}\left(\frac{v_{oy} - gt}{v_{ox}}\right)$$

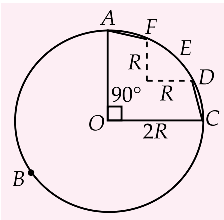
(b) Show that the projection angle θ_0 for a projectile launched from the origin is given by

$$\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right)$$

where the symbols have their usual meanings.

OR

A racing car travels (without banking) on a track $ABCDEFA$ (see figure). ABC is a circular arc of radius $2R$. CD and FA are straight paths of length R and DEF is a circular arc of radius $R = 100$ m. The coeffecient of friction on the road is $\mu = 0.1$. The maximum speed of the car is 50 m s^{-1}. Find the minimum time for completing one round.



SOLUTIONS

1. According to Newton's second law motion, a body moves only if an external force is applied to it. Since the fan is a part of the boat, the force exerted by the air thrown by it on the sail is an internal force. As such, the boat cannot be put into motion.

2. When tracks are covered with snow, there is considerable reduction of frictional force. So, the driving is not safe. When sand is thrown on the snow-covered tracks, the frictional force increases. So, safe driving is possible.

3. Due to its small mass, the momentum of the table tennis ball is much smaller than that of the cricket ball of same velocity. Less force is required to stop the table tennis ball than the cricket ball. Hence it is easy to catch the table tennis ball than the cricket ball.

4. This is possible if the angle between the two vectors is obtuse (> 90°).

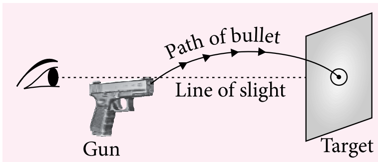
5. When porcelain objects are wrapped in paper or straw, the time of impact between themselves is very much increased during jerk in transportation.

$$F = \frac{\Delta P}{\Delta t}$$

As $\Delta t \rightarrow$ increased, $F \rightarrow$ decreased.

Hence, force on the porcelains is reduced during transportation, and saves them from breakage.

6. As soon as a bullet is fired from a gun, it also starts falling on account of acceleration due to gravity. Therefore, if the gun is along the line of sight, it will hit below the target. To avoid this, the gun is tilted towards the line of sight so that the bullet, after travelling along parabolic path, hits the target as shown in below figure.



7. No. The combined momentum of the ball and the earth is conserved. The ball attracts the earth by the same force as the earth attracts the ball. When the ball moves upward, its momentum decreases in the upward direction but simultaneously the momentum of the earth increases in the upward direction at the same rate. Similarly, when the ball falls down, its momentum increases in the downward direction but simultaneously the momentum of the earth increases in the upward direction at the same rate.

8. Here, $u = 36 \text{ km h}^{-1} = \dfrac{36 \times 1000}{60 \times 60} \text{ m s}^{-1} = 10 \text{ m s}^{-1}$

$v = 0;\ t = 4$ s

Now, $v = u + at$

$\therefore\quad 0 = 10 + a \times 4 \quad \text{or} \quad a = -2.5 \text{ m s}^{-2}$

$m = 65 + 400 = 465$ kg

Therefore, retarding force (in magnitude) on the vehicle,

$F = 465 \times 2.5 = 1162.5 \text{ N} \simeq 1.2 \times 10^3 \text{ N}$

OR

Let m = mass of the nucleus at rest.

$\vec{u}$ = its initial velocity = 0 as it is at rest.

Let m_1, m_2 be the masses of the two smaller nuclei also called product nuclei and v_1, v_2 be their respective velocities.

If $\vec{p}_i$ and $\vec{p}_f$ be the initial and final momentum of the nucleus and the two nuclei respectively, then

$$\vec{p}_i = m\vec{u} = 0 \quad \ldots\text{(i)}$$

and $\vec{p}_f = m_1\vec{v}_1 + m_2\vec{v}_2$ ...(ii)

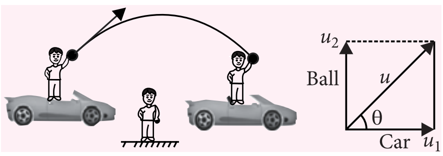
According to the law of conservation of linear momentum,

$$\vec{p}_i = \vec{p}_f \quad \text{or} \quad 0 = m_1\vec{v}_1 + m_2\vec{v}_2$$

or $m_2\vec{v}_2 = -m_1\vec{v}_1$ or $\vec{v}_2 = -\dfrac{m_1\vec{v}_1}{m_2}$ ...(iii)

The negative sign in equation (iii) shows that $\vec{v}_1$ and $\vec{v}_2$ are in opposite directions, *i.e.*, the two smaller nuclei are moved in opposite directions.

9.



With respect to the ground observer, ball is thrown with velocity u at angle θ with the horizontal. Hence it seems as a projectile. So path of the ball will be parabola.

u_1 = car speed, u_2 = initial vertical speed of ball.

$$u = \sqrt{u_1^2 + u_2^2}, \quad \theta = \tan^{-1}\left(\frac{u_2}{u_1}\right)$$

10. Given m = 50 kg, g = 10 m s^{-2}

Acceleration of lift, a = 9 m s^{-2} (downward)

N_1 = Reading of weighing scale = Normal on the man

Applying Newton's second law of motion,

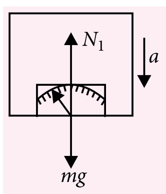
$mg - N_1 = ma$

$\Rightarrow N_1 = m(g - a) = 50(10 - 9)$

$\Rightarrow N_1 = 50$ N

Reading of weighing scale,

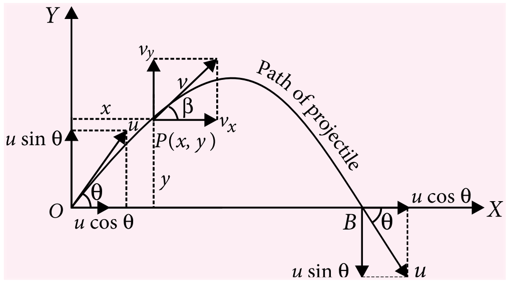
$$m' = \frac{50}{10} = 5 \text{ kg}$$



11. Any object launched in an arbitrary direction in space with an initial velocity and then allowed to move under the influence of gravity alone is called a projectile. Suppose a body is projected with initial velocity u, making an angle θ with the horizontal. The velocity u has two rectangular components :

(i) The horizontal component u cos θ, which remains constant throughout the motion.

(ii) The vertical component u sin θ, which changes with time under the effect of gravity.



Equation of trajectory of a projectile : Suppose the body reaches the point $P(x, y)$ after time t.

∴ The horizontal distance covered by the body in time t,

x = Horizontal velocity × time = u cos θ t

or $t = \dfrac{x}{u\cos\theta}$

For vertical motion, $u = u \sin\theta$, $a = -g$, so the vertical distance covered in time t is given by

$$s = ut + \frac{1}{2}at^2$$

or $y = u\sin\theta \cdot \dfrac{x}{u\cos\theta} - \dfrac{1}{2}g \cdot \dfrac{x^2}{u^2\cos^2\theta}$

or $y = x\tan\theta - \dfrac{g}{2u^2\cos^2\theta}x^2$

Thus y is a quadratic function of x. Hence the trajectory of a projectile is a parabola.

12. Frequency of revolution, $\upsilon = 40$ rev/min $= \dfrac{40}{60}$ rev/s

mass of stone, m = 0.25 kg;

radius of circle, r = 1.5 m;

angular speed of the stone, $\omega = 2\pi\upsilon$

$$= 2\pi \times \frac{40}{60} = \frac{4\pi}{3} \text{ rad s}^{-1}$$

T = tension in the string = ?

T_{max} = maximum tension in the string = 200 N

v_{max} = maximum speed of the stone = ?

The centripetal force is provided by the tension (T) in the string,

$$T = \frac{mv^2}{r} = mr\omega^2 \qquad (\because v = r\omega)$$

$$= 0.25 \times 1.5 \times \left(\frac{4\pi}{3}\right)^2 \text{ N} = 6.58 \text{ N} = 6.6 \text{ N}$$

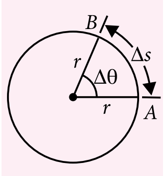
As the string can withstand a maximum tension of 200 N,

$$\therefore \quad T_{max} = \frac{mv_{max}^2}{r} \Rightarrow v_{max} = \sqrt{\frac{rT_{max}}{m}}$$

$$\text{or} \quad v_{max} = \sqrt{\frac{1.5 \times 200}{0.25}} = \sqrt{1200}$$

$$= 34.64 \text{ m s}^{-1} \approx 35.0 \text{ m s}^{-1}$$

13. Consider a particle moving along a circular path of radius r. As shown in figure, suppose the particle moves from A to B in time Δt covering distance Δs along the arc AB. Hence the angular displacement of the particle is



$$\Delta\theta = \frac{\Delta s}{r}$$

Dividing both sides by Δt, we get $\frac{\Delta\theta}{\Delta t} = \frac{1}{r}\frac{\Delta s}{\Delta t}$

Taking the limit $\Delta t \to 0$ on both sides,

$$\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{1}{r} \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

$$\text{But} \quad \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \omega$$

is the instantaneous angular velocity

$$\text{and} \quad \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{ds}{dt} = v,$$

is the instantaneous linear velocity.

$$\therefore \quad \omega = \frac{1}{r} \cdot v \quad \text{or} \quad v = \omega r$$

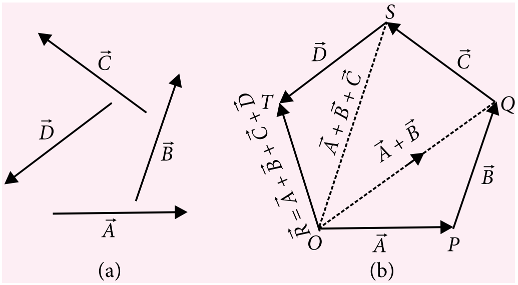
Linear velocity = Angular velocity × radius,

In vector notation, $\vec{v} = \vec{\omega} \times \vec{r}$

For a given angular velocity, the linear velocity of particle is directly proportional to its distance from the centre.

14. If a number of vectors are represented both in magnitude and direction by the sides of an open polygon taken in the same order, then their resultant is represented both in magnitude and direction by the closing side of the polygon taken in opposite order. Suppose we wish to add four vectors $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$, as shown in figures. Draw vector $\overrightarrow{OP} = \vec{A}$. Move vectors $\vec{B}$, $\vec{C}$ and $\vec{D}$ parallel to themselves so that the tail of $\vec{B}$ touches the head of $\vec{A}$, the tail of $\vec{C}$ touch the head of $\vec{B}$ and the tail of $\vec{D}$ touches the head of $\vec{C}$, as shown in figure (b). According to the polygon law, the closing side OT of the polygon taken in the reverse order represents the resultant $\vec{R}$. Thus

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$



We apply triangle law of vector addition to different triangles of the polygon shown in figure (b),

In ΔOPQ, $\quad \overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ} = \vec{A} + \vec{B}$

In ΔOQS, $\quad \overrightarrow{OS} = \overrightarrow{OQ} + \overrightarrow{QS} = \vec{A} + \vec{B} + \vec{C}$

In ΔOST, $\quad \overrightarrow{OT} = \overrightarrow{OS} + \overrightarrow{ST} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$

or $\quad \vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$

This proves the polygon law of vector addition.

15. In terms of their rectangular components, the force, momentum and acceleration vectors can be expressed as

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \; ; \vec{p} = p_x\hat{i} + p_y\hat{j} + p_z\hat{k}$$

and $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

The vector form of Newton's second law is

$$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$$

$$\therefore \quad F_x\hat{i} + F_y\hat{j} + F_z\hat{k} = \frac{d}{dt}(\vec{p}_x\hat{i} + p_y\hat{j} + p_z\hat{k})$$

$$= m(a_x\hat{i} + a_y\hat{j} + a_2\hat{k})$$

Equating the components along the three coordinate axes, we get

$$F_x = \frac{dp_x}{dt} = ma_x \; ; F_y = \frac{dp_y}{dt} = ma_y \; ; F_z = \frac{dp_z}{dt} = ma_z$$

The above equations express Newton's second law in component form.

The component form of Newton's second law indicates that if the applied force makes some angle with the velocity of the body, it changes the component along the direction of the force. The component of velocity normal to the force remains unchanged. For example, in the motion of a projectile under the vertical gravitational force, the horizontal component of velocity remains unchanged.

16. (a) Net displacement is zero as both initial and final positions are same.

(b) Average velocity $= \dfrac{\text{Displacement}}{\text{Time taken}}$

As displacement is zero, average velocity of the cyclist is also zero.

(c) Average speed $= \dfrac{\text{Distance}}{\text{Time taken}} = \dfrac{OP + PQ + QO}{t}$

Now, $OP = QO = 1$ km;

$PQ = \dfrac{1}{4}(2\pi r) = \dfrac{1}{4}(2\pi \times 1) = 1.571$ km and

$t = 10 \text{ min} = \dfrac{10}{60}\text{ h} = \dfrac{1}{6}\text{ h}$

$\therefore$ Average speed $= \dfrac{1 + 1.571 + 1}{1/6}$

$= 3.571 \times 6 = 21.43$ km h^{-1}

17. (a) A revolving wheel has acceleration so a child in it is non-inertial observer.

(b) A sports-car moving with constant speed on a straight road has no acceleration so its driver is an inertial observer.

(c) When the plane takes off, it has accelerated motion hence its pilot is a non-inertial observer.

(d) During sharp turn, a cyclist has centripetal acceleration. So, the cyclist is in non-inertial frame.

(e) A slowing down train has decelerated motion so its guard is a non-inertial observer.

18. Here, $R_{max} = 100$ m

Now, $R_{max} = \dfrac{u^2}{g}$

or $u = \sqrt{R_{max} \times g}$ or $u = \sqrt{100g}$

For vertical motion,

initial velocity, $u = \sqrt{100g}$; final velocity, $v = 0$

acceleration, $a = -g$

Now, $v^2 - u^2 = 2aS$

If H is the maximum height attained, then

$(0)^2 - (\sqrt{100g})^2 = 2(-g)H$

or $2gH = 100\,g$ or $H = \dfrac{100g}{2g} = 50$ m

OR

(a) Velocity,

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3.0\ t\hat{i} - 2.0\ t^2\hat{j} + 4.0\ \hat{k})$$

$$= (3.0\ \hat{i} - 4.0t\ \hat{j}) \text{ m s}^{-1}$$

Acceleration, $\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d}{dt}(3.0\hat{i} - 4.0\ t\hat{j})$

$= 0 - 4.0\hat{j} = -4.0\hat{j}$ m s^{-2}

(b) At time $t = 2$ s,

$\vec{v} = 3.0\hat{i} - 4.0 \times 2\hat{j} = 3.0\hat{i} - 8.0\hat{j}$

$\therefore \quad v = \sqrt{(3)^2 + (-8)^2} = \sqrt{73} = 8.54$ m s^{-1}

If θ is the angle which v makes with x-axis, then

$\tan\theta = \dfrac{v_y}{v_x} = \dfrac{-8}{3} = -2.667$

$\therefore \quad \theta = \tan^{-1}(-2.667) \simeq 70°$ with x-axis

19. Some important implications about the third law of motion :

(i) Newton's third law of motion is applicable irrespective of the nature of the forces. The forces of action and reaction may be mechanical, gravitational, electric or of any other nature.

(ii) Action and reaction always act on different bodies. If they act on the same body, the resultant force would be zero and there could never be accelerated motion.

(iii) The forces of action and reaction cannot cancel each other. This is because action and reaction, though equal and opposite, always act on different bodies and so cannot balance each other.

(iv) No reaction can occur in the an absence of an action. For example, in a tug-of-war, one team can pull the rope only if the other team is pulling the other end of the rope. No force can be exerted if the other end is free. One team exerts the force of action and the other team provides the force of reaction.

20. Laws of limiting friction are :

(i) The limiting friction depends on the nature of the surfaces in contact.

(ii) The limiting friction acts tangential to the two surfaces in contact and in a direction opposite to the direction of motion of the body.

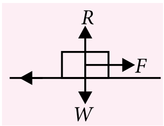
(iii) The value of limiting friction is independent of the area of the surface in contact so long as the normal reaction remains the same.

(iv) The limiting friction $(f_s)_{max}$ is directly proportional to the normal reaction R between the two surfaces.

i.e., $(f_s)_{max} \propto R$ or $(f_s)_{max} = \mu_s R$

$\mu_s = \dfrac{(f_s)_{max}}{R} = \dfrac{\text{Limiting friction}}{\text{Normal reaction}}$

The proportionality constant μ_s is called coefficient of static friction. It is defined as the ratio of limiting friction to the normal reaction.



As $f_k < (f_s)_{max}$ or $\mu_k R < \mu_s R$ $\therefore$ $\mu_k < \mu_s$

Thus the coefficient of kinetic friction is less than the coefficient of static friction.

21. Here, mass of the helicopter, $M = 1000$ kg

Mass of the crew and the passengers, $m = 300$ kg

Acceleration, $a = 15$ m s^{-2} (vertically upwards)

$g = 10$ m s^{-2}

(a) Force on the floor by the crew and the passengers will be equal to their apparent weight. If the helicopter is rising up with an acceleration a, then the apparent weight is $m(g + a)$ and hence the required force,

$F = m(g + a) = 300(10 + 15)$
$= 7500$ N (vertically downwards)

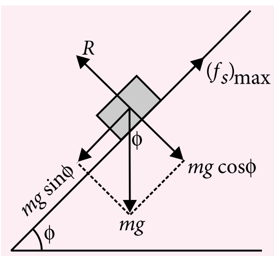
(b) Action of the rotor of the helicopter on the surrounding air,

$F = (M + m)(g + a) = (1000 + 300)(10 + 15)$
$= 32500$ N (vertically downwards)

(c) Force on the helicopter due to surrounding air will be equal and opposite to the action of the rotor of the helicopter on the surrounding air (third law of motion). Therefore, the required force is given by $F = 32500$ N (vertically upwards).

22. Angle of repose is the minimum angle that an inclined plane makes with the horizontal when a body placed on it just begins to slide down.

As shown in figure, consider a body of mass m placed on an inclined plane. The angle of inclination ϕ of the inclined plane is so adjusted that a body placed on it just begins to slide down. Thus ϕ is the angle of repose.



Various forces acting on the body are :

(i) Weight mg of the body acting vertically downwards.

(ii) The limiting friction $(f_s)_{max}$ in upward direction along the inclined plane. It balances the component $mg \sin\phi$ of the weight mg along the inclined plane. Thus

$(f_s)_{max} = mg \sin\phi$ ...(i)

(iii) The normal reaction R perpendicular to the inclined plane. It balances the component $mg \cos\phi$ of the weight mg perpendicular to the inclined plane. Thus

$R = mg \cos\phi$ ...(ii)

Dividing equation (i) and (ii), we get

$$\frac{(f_s)_{max}}{R} = \frac{mg \sin\phi}{mg \cos\phi}$$

or $\mu_s = \tan\phi$

Thus the coefficient of static friction is equal to the tangent of the angle of repose.

23. (i) The villager was nature loving and he knows the importance of natural resources.

(ii) Given, $A = 20$ N, $B = 20$ N, $R = 30$ N, $\theta = ?$

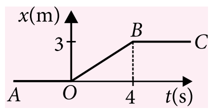
$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$30 = \sqrt{20^2 + 20^2 + 2 \times 20 \times 20 \cos\theta}$$

$$\cos\theta = \frac{30^2 - 20^2 - 20^2}{2 \times 20^2} = 0.125 \Rightarrow \theta = 82°49'$$

(iii) Canals, lakes and ponds are other means of irrigation in villages.

24. (a) (i) For $t < 0$, the position time graph is AO which means displacement of the particle is zero, *i.e.*, particle is at rest at the origin. Hence force on the particle must be zero.



(ii) For $t > 4$ s, the position time graph BC is parallel to time axis. Therefore, the particle remains at a distance of 3 m from the origin, *i.e.*, it is at rest. Hence force on the particle is zero.

(iii) For $0 < t < 4$ s, the position time graph OB has a constant slope. Therefore, velocity of the particle is constant in this interval *i.e.*, particle has zero acceleration. Hence force on the particle must be zero.

(b) Impulse at $t = 0$

Impulse = change in linear momentum.

Before $t = 0$, particle is at rest, *i.e.*, $u = 0$.

After $t = 0$, particle has a constant velocity.

$\therefore$ Impulse $= m(v - u) = 4(0.75 - 0) = 3$ kg m s^{-1}

Impulse at $t = 4$ s

Before $t = 4$ s, particle has a constant velocity $u = 0.75$ m s^{-1}

After $t = 4$ s, particle is at rest, *i.e.*, $v = 0$

$\therefore$ Impulse $= m(v - u) = 4(0 - 0.75) = -3$ kg m s^{-1}

OR

(a) The statement is false, as several scalar quantities are not conserved in a process.
For example energy being a scalar quantity is not conserved during inelastic collisions.

(b) The statement is false, because there are some scalar quantities which can be negative in a process.
For example, temperature being scalar quantity can be negative (–30 °C,–4 °C), charge being scalar can also be negative.

(c) The statement is false, there are large number of scalar quantities which may not be dimensionless.
For example, mass, density, charge etc. being scalar quantities have dimensions.

(d) The statement is false as there are some scalar quantities which vary from one point to another in space.
For example, temperature, gravitational potential, density of a fluid or anisotropic medium, charge density vary from point to point.

(e) The statement is true, orientation of axes does not change the value of a scalar quantity.
For example, mass is independent of the coordinate axes.

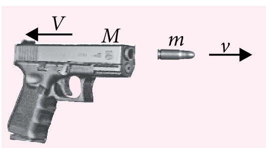
25. Practical applications based on the law of conservation of linear momentum :

(i) Recoil of a gun : Let M be the mass of the gun and m be the mass of the bullet. Before firing, both the gun and the bullet are at rest. After firing, the bullet moves with velocity $\vec{v}$ and the gun moves with velocity $\vec{V}$. As no external force acts on the system, so according to the principle of conservation of momentum,
total momentum before firing
= total momentum after firing

or $0 = m\vec{v} + M\vec{V}$

or $M\vec{V} = -m\vec{v}$

or $\vec{V} = -\dfrac{m}{M}\vec{v}$



The negative sign shows that $\vec{V}$ and $\vec{v}$ are in opposite directions, *i.e.*, the gun gives a kick in the backward direction or the gun recoils with velocity $\vec{V}$. Further, as $M >> m$, $V << v$ *i.e.*, the recoil velocity of the gun is much smaller than the forward velocity of the bullet.

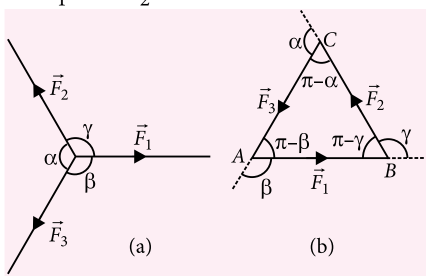
(ii) When a man jumps out of the boat to the shore, the boat slightly moves away from the shore : Initially, the total momentum of the boat and the man is zero. As the man jumps from the boat to the shore, he gains a momentum in the forward direction. To conserve the momentum, the boat also gains an equal momentum in the opposite direction. So the boat slightly moves backwards.

(iii) An astronaut in open space, who wants to return to the spaceship, throws some object in a direction opposite to the direction of motion of the spaceship : By doing so, he gains a momentum equal and opposite to that of the thrown object and so he moves towards the spaceship.

(iv) Rocket and jet planes work on the principle of conservation of momentum : Initially, both the rocket and its fuel at rest. Their total momentum is zero. For rocket propulsion, the fuel is exploded. The burnt gases are allowed to escape through a nozzle with a very high downward velocity. The gases carry a large momentum in the downward direction. To conserve momentum, the rocket also acquires an equal momentum in the upward direction and hence starts moving upwards.

OR

(a) Given figure (a) shows a particle O under the equilibrium of three concurrent force $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$. Let α be angle between $\vec{F}_2$ and $\vec{F}_3$, β between $\vec{F}_3$ and $\vec{F}_1$, and γ between $\vec{F}_1$ and $\vec{F}_2$.



As shown in figure (b), the forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ can be represented by the sides of ΔABC, taken in the same order.
Applying law of sines of ΔABC, we get

$$\frac{F_1}{\sin(\pi-\alpha)} = \frac{F_2}{\sin(\pi-\beta)} = \frac{F_3}{\sin(\pi-\gamma)}$$

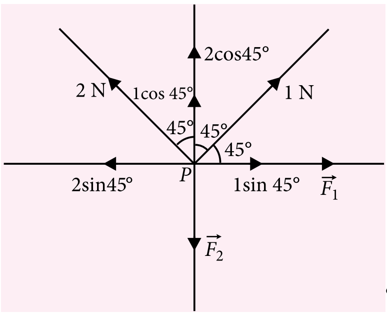
or $$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

This is Lami's theorem which states that if three forces acting on a particle keep it in equilibrium, then each force is proportional to the sine of the angle between the other two forces.

(b) As P is in equilibrium,
so, net force along horizontal direction = 0

$\Rightarrow F_1 + \sin45° = 2\sin45°$

$\Rightarrow F_1 = \sin45° = \dfrac{1}{\sqrt{2}}$ N

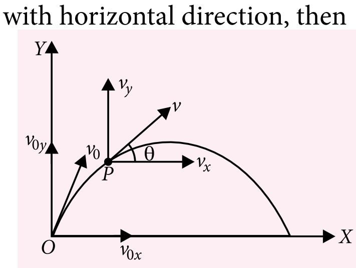


Net force along vertical direction = 0

$F_2 = \cos45° + 2\cos45° = 3\cos45° = \dfrac{3}{\sqrt{2}}$ N

26. (a) Let v_{ox} and v_{oy} be the initial component velocities of the projectile at O along OX direction and OY direction respectively, where OX is horizontal and the OY is vertical. Let the projectile go from O to P in time t and v_x, v_y be the component velocities of projectile at P along horizontal and vertical directions respectively. Then, $v_y = v_{oy} - gt$ and $v_x = v_{ox}$

If θ is the angle which the resultant velocity $\vec{v}$ makes with horizontal direction, then



$$\tan\theta = \frac{v_y}{v_x} = \frac{v_{oy} - gt}{v_{ox}}$$

or $\quad \theta = \tan^{-1}\left(\dfrac{v_{oy} - gt}{v_{ox}}\right)$

(b) In angular projection,
maximum vertical height,

$$h_m = \frac{u^2 \sin^2\theta_0}{2g}$$

Horizontal range,

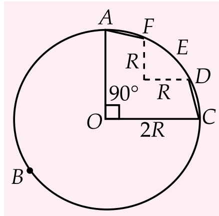
$$R = \frac{u^2 \sin 2\theta_0}{g} = \frac{u^2}{g} 2\sin\theta_0 \cos\theta_0$$

So, $\dfrac{h_m}{R} = \dfrac{\tan\theta_0}{4}$

or $\quad \tan\theta_0 = \dfrac{4h_m}{R}$ or $\theta_0 = \tan^{-1}\left(\dfrac{4h_m}{R}\right)$

OR

$\mu = 0.1$, $R = 100$ m, $v = 50$ m s^{-1}



(a) t_1 = time taken to cover the distance on arc DEF.

(v_m = Maximum possible speed on track DEF)
Centripetal force is provided by friction force.

$$\frac{mv_m^2}{R} = \mu mg$$

$\Rightarrow \quad v_m = \sqrt{\mu g R} = \sqrt{0.1 \times 10 \times 100} = 10$ m s^{-1}

Length of arc $DEF = R(\pi/2) = 50\pi$ m ($\because \theta = l/r$)

$\therefore \quad t_1 = \dfrac{50\pi}{10} = 5\pi \text{ s} = 15.7 \text{ s}$

(b) t_2 = time taken by the car to cover the distance on circular track ABC.
Again, centripetal force = friction force

$$\frac{mv'^2_m}{2R} = \mu\, mg$$

(v'_m = maximum possible speed on track ABC)

$\Rightarrow \quad v'_m = \sqrt{2\mu g R} = \sqrt{200} = 14.14$ m s^{-1}

Length of arc $= ABC = 2R\left(\dfrac{3\pi}{2}\right) = 3\pi R = 300\,\pi$ m

$$t_2 = \frac{300\pi}{14.14} = 66.6 \text{ s}$$

(c) $CD = FA = R = 100$ m, $v = 50$ m s^{-1}
t_3 = time taken to cover the path CD and FA

$$t_3 = \frac{2R}{v} = \frac{2 \times 100}{50} = 4 \text{ s}$$

$\therefore$ Total time taken, $t = t_1 + t_2 + t_3$
$= 15.7 + 66.6 + 4 = 86.3$ s

◈◈

MPP-4 MONTHLY Practice paper

Class XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.

Self check table given at the end will help you to check your readiness.

Work, Energy and Power

Total Marks : 120 **Time Taken : 60 min**

NEET / AIIMS

Only One Option Correct Type

1. Two spheres A and B of masses m_1 and m_2 respectively collide. A is at rest initially and B is moving with velocity v along x-axis. After collision B has a velocity $\frac{v}{2}$ in a direction perpendicular to the original direction. The mass A moves after collision in the direction

(a) same as that of B (b) opposite to that of B

(c) $\theta = \tan^{-1}\left(\frac{1}{2}\right)$ to the x-axis

(d) $\theta = \tan^{-1}\left(-\frac{1}{2}\right)$ to the x-axis

2. A girl in a swing is 2.5 m above ground at the maximum height and at 1.5 m above the ground at the lowest point. Her maximum velocity in the swing is ($g = 10$ m s^{-2})

(a) $5\sqrt{2}$ m s^{-1} (b) $2\sqrt{5}$ m s^{-1}

(c) $2\sqrt{3}$ m s^{-1} (d) $3\sqrt{2}$ m s^{-1}

3. An open watertight railway wagon of mass 5×10^3 kg is moving with an initial velocity of 1.2 m s^{-1} without friction on a railway track. Rain falls vertically downwards into the wagon. What change will occur in the kinetic energy of wagon, when it has collected 10^3 kg of water?

(a) 1200 J (b) 300 J (c) 600 J (d) 900 J

4. A body of mass 4 kg is moving with momentum of 8 kg m s^{-1}. A force of 0.2 N acts on it in the direction of motion of the body for 10 s. The increase in kinetic energy in joules is

(a) 10 (b) 8.5 (c) 4.5 (d) 4

5. One man takes 1 min to raise a box to a height of 1 m and another man takes $\frac{1}{2}$ min to do so. The energy of the two men is

(a) different (b) same

(c) energy of the first is more

(d) energy of the second is more

6. When a long spring is stretched by 2 cm, its potential energy is U. If the spring is stretched by 10 cm, the potential energy in it will be

(a) $10U$ (b) $25U$ (c) $\frac{U}{5}$ (d) $5U$

7. The displacement of a body of mass 2 kg varies with time t as $S = t^2 + 2t$, where S is in m and t is in s. The work done by all the forces acting on the body during the time interval $t = 2$ s to $t = 4$ s is

(a) 36 J (b) 64 J (c) 100 J (d) 120 J

8. The work done by external agent in stretching a spring of force constant k from length l_1 to l_2 is

(a) $k(l_2^2 - l_1^2)$ (b) $\frac{1}{2}k(l_2^2 - l_1^2)$

(c) $k(l_2 - l_1)$ (d) $\frac{1}{2}k(l_2 + l_1)$

9. One end of an unstretched vertical spring is attached to the ceiling and an object attached to the other end is slowly lowered to its equilibrium position. If S be gain in spring energy and G be loss in gravitational potential energy in the process, then

(a) $S = G$ (b) $S = 2G$

(c) $G = 2S$ (d) None of these

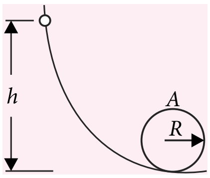
10. Concrete blocks of masses m_A and m_B are balanced on two identical vertical springs. $m_A = 2m_B$. The gravitational potential energy of each system is zero at the equilibrium position of the springs. Which statement is true for the total mechanical energy of the systems when the blocks are balanced on the springs?

(a) $E_A = E_B$ (b) $E_A = 2E_B$
(c) $E_A = 4E_B$ (d) $E_A = -2E_B$

11. A pendulum bob has a speed of 3 m s^{-1} at its lowest position. The pendulum is 0.5 m long. The speed of the bob, when the length makes an angle of 60° to the verticle, will be

(a) 3 m s^{-1} (b) 1/3 m s^{-1} (c) 1/2 m s^{-1} (d) 2 m s^{-1}

12. A bead slides without friction around a loop the loop as shown in the figure. The bead is released from rest at a height $h = 3.50\ R$. How large is the normal force on the bead at point (A) if its mass is 50 g?



(a) 0.10 N downward (b) 0.10 N upward
(c) 1.0 N downward (d) 1.0 N upward

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :

(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. Assertion : A quick collision between two bodies is more violent than a slow collision, even when initial and final velocities are identical.

Reason : The rate of change of momentum which determines the force is greater in the first case.

14. Assertion : Potential energy of a stretched or compressed spring is proportional to square of extension or compression.

Reason : Graph between potential energy of a spring versus the extension or compression of the spring is a straight line.

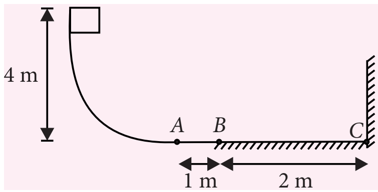
15. Assertion : The mass equivalent of 1000 kW h energy is 40 μg.

Reason : This follows from $E = mc^2$, where $c = 3 \times 10^8$ m s^{-1}.

JEE MAIN / JEE ADVANCED

Only One Option Correct Type

16. A block of mass $m = 0.1$ kg is released from a height of 4 m on a curved smooth surface. On the horizontal surface, path AB is smooth and path BC offers coefficient of friction $\mu = 0.1$. If the impact of block with the vertical wall at C be perfectly elastic, the total distance covered by the block on the horizontal surface before coming to rest will be (take $g = 10$ m s^{-2})



(a) 29 m (b) 49 m
(c) 59 m (d) 109 m

17. A locomotive of mass m starts moving so that its velocity varies as $v = \alpha s^{2/3}$, where α is a constant and s is the distance traversed. The total work done by all the forces acting on the locomotive during the first t s after the starts of motion is

(a) $\frac{1}{8}m\alpha^4t^2$ (b) $\frac{m\alpha^6t^4}{162}$

(c) $\frac{m\alpha^6t^4}{81}$ (d) $\frac{m\alpha^4t^2}{2}$

18. A uniform chain has a mass m and length L. It is placed on a frictionless table with length l_0 hanging over the edge. The chain begins to slide down. The speed v with which the chain slides away from the edge is given by

(a) $v = \sqrt{\frac{gl_0}{L}(L+l_0)}$ (b) $v = \sqrt{\frac{gl_0}{L}(L-l_0)}$

(c) $v = \sqrt{\frac{g}{L}(L^2-l_0^2)}$ (d) $v = \sqrt{2g(L-l_0)}$

19. A block of mass m is pulled by a constant power P placed on a rough horizontal plane. The friction co-efficient between the block and surface varies with its speed v as $\mu = \frac{1}{\sqrt{1+v}}$. The acceleration of the block when its speed is 3 m s^{-1} will be

(a) $\frac{P}{3m} - \frac{g}{2}$ (b) $\frac{P}{3m} + \frac{g}{2}$

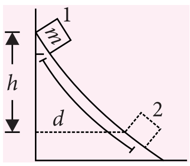
(c) $\frac{P}{3m}$ (d) $\frac{g}{2}$

More than One Options Correct Type

20. One of the forces acting on a particle is conservative then which of the following statement(s) are true about this conservative force?
(a) Its work is zero when the particle moves exactly once around any closed path.
(b) Its work equals the change in the kinetic energy of the particle.
(c) Then that particular force must be constant.
(d) Its work depends on the end points of the motion, not on the path between.

21. A man standing on the edge of the terrace of a high rise building throws a stone, vertically up with a speed of 20 m s^{-1}. After 2 s, an identical stone is thrown vertically downwards with the same speed of 20 m s^{-1}. Then
(a) the relative velocity between the two stones remains constant till one hits the ground
(b) both will have the same kinetic energy, when they hit the ground
(c) the time interval between their hitting the ground is 2 s
(d) if the collisions on the ground are perfectly elastic, both will rise to the same height above the ground.

22. A box of mass m is released from rest at position 1 on the frictionless curved track as shown in the figure. It slides a distance d along the track in time t to reach position 2, dropping a vertical height h. Let v and a be the instantaneous speed and instantaneous acceleration respectively, of the box at position 2. Which of the following equations is not valid for this situation?

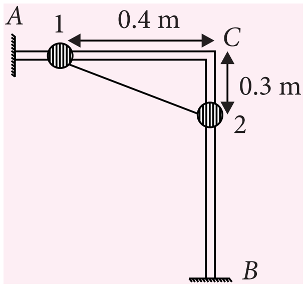


(a) $h = vt$
(b) $h = (1/2)\, gt^2$
(c) $d = (1/2)\, at^2$
(d) $mgh = (1/2)mv^2$

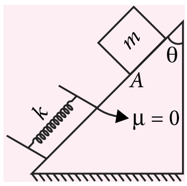
23. One end of a light spring of spring constant k is fixed to a wall and the other end is tied to a block placed on a smooth horizontal surface. For a displacement of block, the work done by the spring is $\frac{1}{2}kx^2$. The possible cases are
(a) the spring was initially compressed by a distance x and was finally in its natural length.
(b) it was initially stretched by a distance x and finally was in its natural length.
(c) it was initially in its natural length and finally in a compressed position.
(d) it was initially in its natural length and finally in a stretched position.

Integer Answer Type

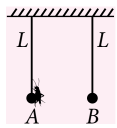
24. Two identical beads of m = 100 g are connected by an inextensible massless string that can slide along the two arms AC and BC of a rigid smooth wireframe in a vertical plane. If the system is released from rest, the kinetic energy of the first particle when they have moved by a distance of 0.1 m is $16x \times 10^{-3}$ J. Find the value of x. (g = 10 m s^{-2})



25. A block of mass m is released from rest at point A. The compression in spring (force constant k) when the speed of block is maximum is found to be $\frac{nmg\cos\theta}{4k}$. What should be the value of n?



26. An insects jumps from ball A onto ball B, which are suspended from inextensible light strings each of length L = 8 cm. The mass of each ball and insect is same. What should be the minimum relative velocity (in m s^{-1}) of jump of insect with respect to ball A, if both the balls manage to complete the full circle?



Comprehension Type

We generally ignore the kinetic energy of the moving coil of a spring but consider a spring of mass m, equilibrium length L and spring constant k. Consider a spring, as described above, that has one end fixed and the other end moving with speed v. Assume that the speed of points along the length of the spring varies linearly with distance L from the fixed end. Assume also that the mass m of the spring is distributed uniformly along the length of the spring. Assume further that the force applied by the spring is spring constant times its deformation.

27. The speed of small length (dx) at a distance x from fixed end is

(a) $\frac{x}{L}v$ (b) v (c) $\frac{L}{x}v$ (d) xv

28. Kinetic energy of the spring

(a) $\frac{1}{2}mv^2$ (b) $\frac{1}{6}mv^2$

(c) mv^2 (d) $\frac{1}{4}mv^2$

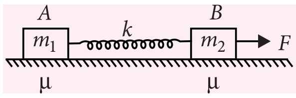
Matrix Match Type

29. A chain of length l and mass m lies on the surface of a smooth sphere of radius $R > l$ with one end tied to the top of the sphere.

Column I	Column II
(A) Gravitational potential energy with respect to centre of the sphere	(P) $\frac{Rg}{l}\left[1-\cos\left(\frac{l}{R}\right)\right]$
(B) The chain is released and slides down, its kinetic energy when it has slide by θ	(Q) $\frac{2Rg}{l}\left[\sin\left(\frac{l}{R}\right)+\sin\theta-\sin\left(\theta+\frac{l}{R}\right)\right]$
(C) The initial tangential acceleration	(R) $\frac{MR^2g}{l}\sin\left(\frac{l}{R}\right)$
(D) The radial acceleration	(S) $\frac{MR^2g}{l}\left[\sin\left(\frac{l}{R}\right)+\sin\theta-\sin\left(\theta+\frac{l}{R}\right)\right]$

	A	B	C	D
(a)	Q	R	S	P
(b)	R	S	P	Q
(c)	Q	P	S	R
(d)	P	Q	S	R

30. As shown in figure, $m_1 = 8$ kg, $m_2 = 16$ kg, $k = 100$ N m^{-1}, $\mu = 0.2$



Column I	Column II
(A) The minimum value of F (in N) in order to shift the block of mass m_1	(P) 5.12
(B) Negative of work done by friction (in J) on block B till this moment	(Q) 40
(C) Work done by F till this moment	(R) Zero
(D) The minimum value of F in order to shift the block of mass m_2 if it is applied on A	(S) 32
	(T) 6.4

	A	B	C	D
(a)	Q	T	R, S	P
(b)	P	T	S	Q
(c)	Q	P	T	S
(d)	S, R	Q	P	T

◈◈

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Class XII

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Unit 2 CURRENT ELECTRICITY

ELECTRIC CURRENT

Electric Current

- The rate of flow of charge through a cross-section of some region of a metallic wire (or an electrolyte) is called the current through that region.
- Current at any instant is given by $I = dq/dt$. If a charge q flows through the circuit for time t, then $I = q/t$.
- SI unit of current is ampere (A) or (C s^{-1}).
- In metallic conductors, the current is due to the motion of free electrons whereas in electrolytes and ionized gases, both positive and negative ions are responsible for current flow.

Drift Velocity

- When an electric field is applied across a conductor, the free electrons are accelerated opposite to the direction of the field and therefore they have a net drift in that direction. Due to frequent collisions of electrons with the atoms, their average velocity is very small. This average velocity with which the electrons move in a conductor under an applied potential difference is called the drift velocity.
- The acceleration of the electron is $a = e\,E/m$
- The drift velocity must be

$$v_d = \frac{eE}{m}\tau \qquad \begin{cases} E = \text{applied electric field} \\ m = \text{mass of electron} \\ \tau = \text{relaxation time} \end{cases}$$

- **Current :** $I = neAv_d$

  {n = Number density of electrons}

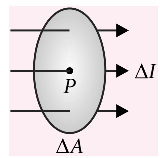
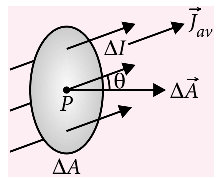
- **Mobility :** The drift velocity per unit electric field.

  $\mu = v_d/E = e\tau/m$

Current Density

- Current density at a point is defined as the amount of current following per unit area around that point, provided the area is normal to the direction of current.

  $J = I/A$, where I is current and A is area.

Current is perpendicular to area : The average current density is given by $J_{av} = \Delta I/\Delta A$.	
Current not perpendicular to area : The average current density is given by $J_{av} = \dfrac{\Delta I}{\Delta A\cos\theta}$	

OHM'S LAW AND RESISTIVITY

Ohm's Law

- The current flowing through a conductor is proportional to the potential difference across its ends, only when physical conditions (such as temperature) remain unchanged,

  i.e., $V \propto I$ or $V = RI$.

  The constant of proportionality R is called the resistance of the conductor.

- **Resistance :** The resistance of a conductor is a measure of the opposition offered by the conductor to the flow of current.
- The resistance of a conductor is directly proportional to its length l and inversely proportional to the area of cross-section A, *i.e.*,

  $$R = \rho\frac{l}{A}$$

  where the constant ρ depends on the nature of the material, is called the resistivity.

  In vector form

  $$J = \frac{I}{A} = \frac{V}{RA} = \frac{El}{(\rho l/A)A} = \frac{E}{\rho} = \sigma E$$

  where $\sigma = \frac{1}{\rho}$ is called conductivity.

  $$\vec{J} = \sigma\vec{E}$$
- **Resistivity :** The resistivity of material of a conductor is given by

  $$\rho = \frac{m}{ne^2\tau}$$

  where n is number of free electrons per unit volume and τ is the relaxation time of the free electron.

Temperature Dependence of Resistivity

- If ρ_0 and ρ_T are the values of resistivity at 0° C and T° C respectively then over a temperature range that is not too large, we have approximately,

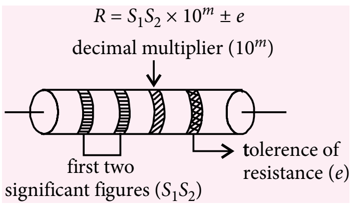
  $$\rho_T = \rho_0(1 + \alpha T)$$

  Consequently, for resistance we have, approximately

  $$R_T = R_0(1 + \alpha T).$$

Colour Code of Resistors

Colour	Figure
black	0
brown	1
red	2
orange	3
yellow	4
green	5
blue	6
violet	7
grey	8
white	9



Colour	Tolerence
gold	5%
silver	10%
no colour	20%

Illustration 1. The resistance of a silver wire at 0 °C is 1.25 Ω. Upto what temperature it must be heated so that its resistance is doubled? [α_{silver} = 0.00375 °C^{-1}]

Sol. : As $\Delta R = \alpha R\Delta T$ and in this case, $\Delta R = R$

$\therefore \quad \Delta T = 1/\alpha = 267$ °C

$T = T + \Delta T = 0$ °C + 267 °C = 267 °C

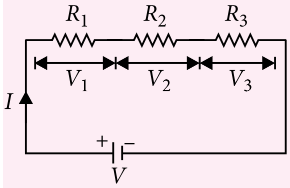
ELECTRICAL RESISTANCES

Combination of Resistance

Combination of Resistance

Series combination

- Current is same across each resistance.



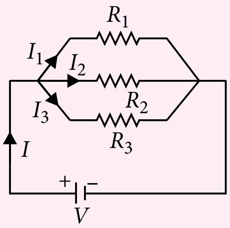
$$V = V_1 + V_2 + V_3$$

$$IR_S = IR_1 + IR_2 + IR_3$$

- $R_S = R_1 + R_2 + R_3$
- $R_S > \max(R_1 / R_2 / R_3)$

Parallel combination

- Potential drop across each reasistance is same.



$$I = I_1 + I_2 + I_3$$

$$\frac{V}{R_P} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

- $\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$
- $R_P < \min(R_1 / R_2 / R_3)$

Current and Potential Divider

- **Voltage divider :** In a series circuit, current through each resistor is the same.

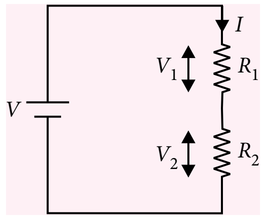
  $$V = V_1 + V_2 = IR_1 + IR_2$$

  $$I = \frac{V}{R_1 + R_2}$$

  Voltage is divided as

  $$V_1 = \frac{VR_1}{R_1 + R_2}$$

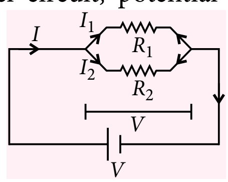
  $$V_2 = \frac{VR_2}{R_1 + R_2}$$



- **Current divider :** In a parallel circuit, potential drop across each resistance is same.

  $$I = I_1 + I_2 \qquad \text{.. (i)}$$

  $$I_1R_1 = I_2R_2 \qquad \text{... (ii)}$$



On solving eqns. (i) and (ii), we get

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right)I;\ I_2 = \left(\frac{R_1}{R_1 + R_2}\right)I$$

Note : Current through any resistance is

$$I = I_{total} \times \left[\frac{\text{Resistance of opposite branch}}{\text{Total resistance}}\right]$$

Resistances in special cases

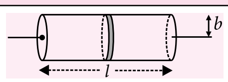
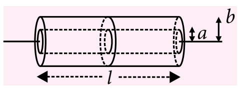
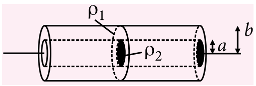
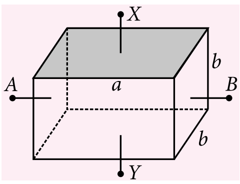
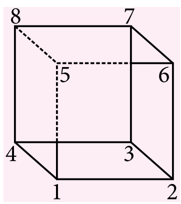
Geometry of object	Object description	Resistance of object
A solid cylinder		$R = \frac{\rho l}{\pi b^2}$
A hollow cylinder		$R = \frac{\rho l}{\pi(b^2 - a^2)}$
Solid cylinder with different densities ρ_1 and ρ_2		$R_1 = \frac{\rho_1 l}{\pi(b^2 - a^2)}$ $R_2 = \frac{\rho_2 l}{\pi a^2}$ Net $R = \frac{R_1 R_2}{R_1 + R_2}$
Rectangular slab		$R_{AB} = \rho\frac{a}{b^2}$ $R_{XY} = \rho\frac{b}{a \cdot b} = \frac{\rho}{a}$
A cubical frame with equal resistance r on edges		$R_{12} = \frac{7}{12}r$ $R_{13} = \frac{3}{4}r$ $R_{17} = \frac{5}{6}r$

Illustration 2. Let there be n resistors R_1 ... R_n with R_{max} = max $\{R_1 ... R_n\}$ and R_{min} = min $\{R_1 ... R_n\}$. Show that when they are connected in parallel, the resultant resistance $R_P < R_{min}$ and when they are connected in series, the resultant resistance $R_S > R_{max}$. Interpret the result physically.

Sol. : When n resistors are connected in parallel, the effective resistance R_P is

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}$$

$$\frac{R_{min}}{R_P} = \frac{R_{min}}{R_1} + \frac{R_{min}}{R_2} + ... + \frac{R_{min}}{R_n}$$

Since $\frac{R_{min}}{R_n} \leq 1$; for n = 1, 2, 3, ..., n

$\therefore \quad \frac{R_{min}}{R_P} > 1$ or $R_{min} > R_P$

Due to one resistor of resistance R_{min}, there is only one route for the current in circuit.

When n resistors are in parallel, there are additional $(n - 1)$ routes for the current in circuit.

When n resistors are in series, then

$R_S = R_1 + R_2 + ... + R_n$

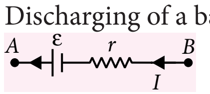
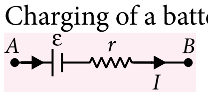
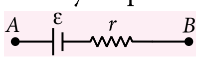
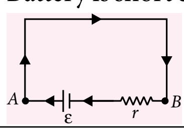
$\therefore \quad R_S > R_{max}$

Due to one resistor of resistance R_{max}, the current in circuit is more than the current due to n resistors in series of effective resistance R_S.

CELLS, EMF AND INTERNAL RESISTANCES

EMF (Electro-Motive Force)

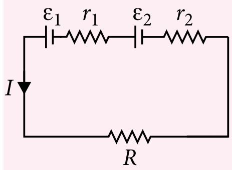
- The potential difference across the terminals of a cell when it is not producing any current is called emf of the cell.
- **Emf depends on**
  - nature of electrolyte
  - metal of electrodes
- **Emf does not depend on**
  - area of plates
  - distance between the electrodes
  - quantity of eletrolyte
  - size of cell
- **Terminal voltage :** When current drawn through the cell or current is supplied to cell, the potential difference across its terminals is its terminal voltage.

Situation	Potential difference (V) across the terminals of cell
Discharging of a battery 	$V_A - \varepsilon + Ir = V_B$ or $V_A - V_B = \varepsilon - Ir$ $V_{AB} = \varepsilon - Ir$ or $V_{AB} < \varepsilon$
Charging of a battery 	$V_A - \varepsilon - Ir = V_B$ or $V_A - V_B = \varepsilon + Ir$ $V_{AB} = \varepsilon + Ir$ or $V_{AB} > \varepsilon$
Battery is open circuited 	$V_{AB} = \varepsilon$ as $I = 0$
Battery is short circuited 	$I = \frac{\varepsilon}{r}$ or $\varepsilon = Ir$ $\therefore \ \varepsilon - Ir = 0$ or $V_{AB} = 0$

Combination of Cells (Battery)

Combination of Cells (Battery)

Series Combination



Equivalent emf, $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 = \sum_i \varepsilon_i$

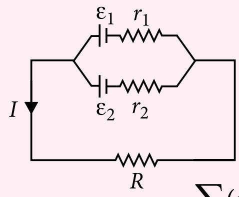
Equivalent resistance,

$r_{eq} = r_1 + r_2 = \sum_i r_i$

Current in the circuit :

$$I = \frac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2 + R}$$

Parallel Combination

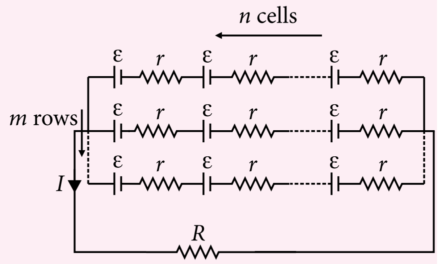


$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} = \frac{\sum_i (\varepsilon_i / r_i)}{\sum_i (1/r_i)}$$

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \Rightarrow \frac{1}{r_{eq}} = \sum_i \left(\frac{1}{r_i}\right)$$

$$I = \frac{\varepsilon_{eq}}{R + r_{eq}}$$

Mixed Combination (Identical cells)



$$\varepsilon_{eq} = n\varepsilon, \quad r_{eq} = \frac{nr}{m}, \quad I = \frac{n\varepsilon}{R + \frac{nr}{m}}$$

Illustration 3. 20 cells, each of 2 V emf and 0.5 Ω internal resistance are given. How will you arrange them in a mixed grouping so as to send maximum current through a 2.5 Ω external resistance? Calculate the value of the current.

Sol. : Since the external and the internal resistances are of the same order, the cells are to be placed in mixed grouping.

With usual notation,

$N = 20$, $\varepsilon = 2$ V, $r = 0.5\ \Omega$, $R = 2.5\ \Omega$

Let there be m rows of n cells in parallel. Clearly,

$N = nm = 20$ ... (i)

For maximum current,

$R = \frac{nr}{m} \Rightarrow 2.5 = \frac{n \times 0.5}{m}$ or $n = 5m$ ... (ii)

From eqns. (i) and (ii) we get $m \times 5m = 20$

or $m^2 = 4$ or $m = 2$

From eqn. (ii), $n = 5 \times 2$ or $n = 10$

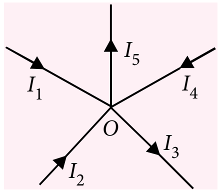
$$I = \frac{mn\varepsilon}{mR + nr} = \frac{20 \times 2\ \text{V}}{(2 \times 2.5 + 10 \times 0.5)\ \Omega} = \frac{40\ \text{V}}{10\ \Omega} = 4\ \text{A}$$

KIRCHHOFF'S LAWS FOR ELECTRICAL NETWORK

Junction Rule

- The algebraic sum of currents meeting at any junction in a circuit is zero, *i.e.*, $\sum I = 0$.

Conventionally the incoming currents at the point are taken as positive while those outgoing are taken as negative. In figure for junction *O*,



$\sum I = I_1 + I_2 - I_3 + I_4 - I_5 = 0$

This law is based on conservation of charge.

Loop Rule

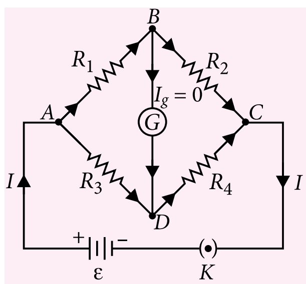
- According to this law in any closed part of an electrical circuit, the algebraic sum of the emfs is equal to the algebraic sum of the products of the resistance and current flowing through them.

$\Rightarrow \sum \varepsilon = \sum IR$

This law follows from the law of conservation of energy.

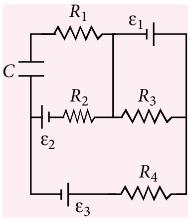
Wheatstone's Bridge

- In a balanced condition even though current flows in the rest of the circuit, galvanometer will not show any deflection (*i.e.* $I_g = 0$).



Also, $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

Illustration 4 : In the given circuit,



$\varepsilon_1 = 3\varepsilon_2 = 2\varepsilon_3 = 6$ V

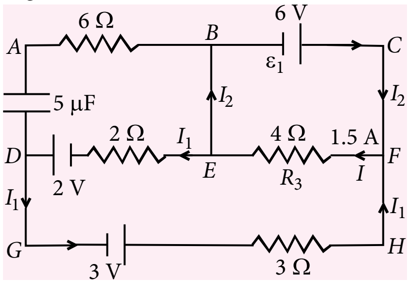
$R_1 = 2R_4 = 6\ \Omega$

$R_3 = 2R_2 = 4\ \Omega$

$C = 5\ \mu F$

Find the current in R_3 and the energy stored in the capacitor.

Sol. : In steady state condition, no current flows through the capacitor $C(5\ \mu F)$. Obviously no current flows through R_1 ($6\ \Omega$).



Current through $R_3 = \varepsilon_1/R_3$

$\therefore \quad I = \frac{6}{4} = \frac{3}{2} = 1.5$ A

Assume this I (1.5 A) divides as I_1 and I_2 as shown.

Apply Kirchhoff's law to closed loop $DGHFED$,

$3 - 3I_1 - (4 \times 1.5) - 2I_1 + 2 = 0$ or $5I_1 = -1$

or $\quad I_1 = -\frac{1}{5} = -0.2$ A

The negative sign of I_1 shows that its direction is reverse of the direction shown in the figure.

$V_A - 2I_1 + 2 - V_D = 0$ along passage $ABED$

or $\quad V_A - V_D = 2I_1 - 2 = 2(-0.2) - 2 = -2.4$ V

or $\quad V_D - V_A = 2.4$ V

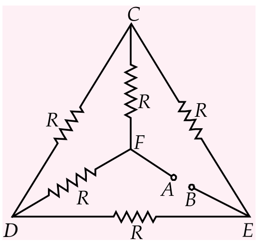
$\therefore$ Potential difference across capacitor = 2.4 V

$\therefore$ Energy stored in capacitor $= \frac{1}{2}CV^2$

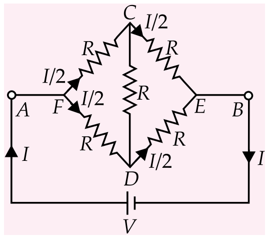
or $\quad U = \frac{1}{2} \times (5 \times 10^{-6}) \times (2.4)^2$

or $\quad U = \frac{1}{2} \times 5 \times 10^{-6} \times 5.76 = 1.44 \times 10^{-5}$ J

Illustration 5 : Five equal resistances, each of resistance R, are connected as shown in figure below. A battery of V volt is connected between A and B. Find the current flowing in FC.



Sol. : The equivalent circuit diagram is as shown in figure.



It is a balanced Wheatstone bridge, hence no current flows in arm CD. Therefore resistance of arm CD becomes ineffective.

Resistance of upper arm FCE $= R + R = 2R$

Resistance of lower arm $FDE = R + R = 2R$

The equivalent resistance between A and B is

$$R_{eq} = \frac{(2R)(2R)}{2R + 2R} = R$$

Current in the circuit is $I = \frac{V}{R}$

$\therefore$ Current in $FC = \frac{I}{2} = \frac{V}{2R}$

HEATING EFFECT OF ELECTRIC CURRENT

Joule's Law of Heating

- The amount of heat produced H in a conductor of resistance R, carrying current I for time t is

$$H = I^2Rt \text{ (in joule)} \qquad \text{or} \quad H = \frac{I^2Rt}{J} \text{ (in calorie)}$$

where J is Joule's mechanical equivalent of heat (= 4.2 J cal^{-1}).

Electric Power

- Electric power, $P = \dfrac{\text{electric work done}}{\text{time taken}}$

$$P = VI = I^2R = \frac{V^2}{R}$$

- The SI unit of power is watt (W).
- The practical unit of power is kilowatt (kW) and horse power (hp).
- 1 kilowatt = 1000 watt.
- Power loss in transmission cable,

$$P_c = I^2R_c = \frac{P^2R_c}{V_c^2} \qquad \left\{\begin{array}{l} P = \text{power delivered} \\ R_c = \text{Resistance of cable} \end{array}\right\}$$

Illustration 6 : A copper wire having cross-sectional area of 0.5 mm^2 and a length of 0.1 m is initially at 25 °C and is thermally insulated from the surrounding. If a current of 10 A is set up in this wire, (i) find the time in which the wire will start melting. The change of resistance with the temperature of the wire may be neglected. (ii) What will this time be, if the length of the wire is doubled ?

Melting point of copper = 1075 °C.

Specific resistance of copper $= 1.6 \times 10^{-8}\ \Omega$ m

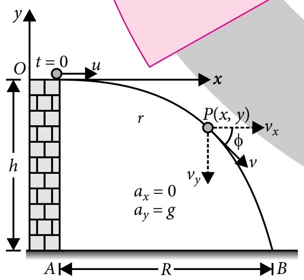
BRAIN MAP CLASS XI

PROJECTILE MOTION

PROJECTILE Motion

A body which is in flight through the atmosphere under the effect of gravity alone and is not being propelled by any fuel is called projectile and its motion is called projectile motion.

Horizontal Projectile Motion



Equation of Trajectory

$$y = \frac{1}{2}\frac{gx^2}{u^2}$$

Time of Descent

$$T = \sqrt{\frac{2h}{g}}$$

Horizontal Range

$$R = u\sqrt{\frac{2h}{g}}$$

Instantaneous Velocity

$$v = \sqrt{u^2 + 2gy} = \sqrt{u^2 + g^2t^2}$$

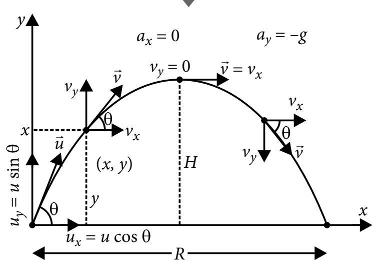
$$\tan\phi = \frac{v_y}{v_x} = \tan^{-1}\left(\frac{gt}{u}\right)$$

Oblique Projectile Motion

Equation of Trajectory

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$$

This represents the parabolic path.



Maximum Height

$$H = \frac{u^2\sin^2\theta}{2g}$$

Time of Flight

$$T = \frac{2u\sin\theta}{g}$$

Horizontal Range

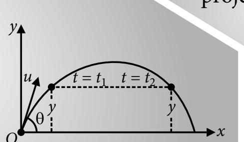
$$R = \frac{u^2\sin 2\theta}{g}$$

- Projectile passing through two different points on same height at time t_1 and t_2

$$y = \frac{gt_1t_2}{2}$$

$$t_2 = \frac{u\sin\theta}{g}\left[1 + \sqrt{1 - \left[\frac{2gy}{u\sin\theta}\right]^2}\right]$$

$$t_1 = \frac{u\sin\theta}{g}\left[1 - \sqrt{1 - \left[\frac{2gy}{u\sin\theta}\right]^2}\right]$$



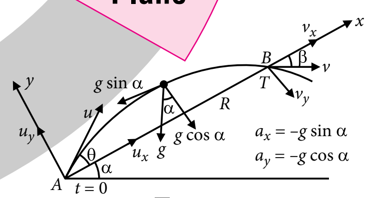
- Ratio of time of flights for projectiles at complimentary angles θ and $90 - \theta$

$$\frac{T_\theta}{T_{90-\theta}} = \tan\theta$$

- Range R is n times the maximum height H: $R = nH; \theta = \tan^{-1}[4/n]$

- For complimentary angles θ and $(90 - \theta)$ range remains unchanged
- Relation between horizontal range and maximum height: $R = 4H\cot\theta$
- If $R = H$ then $\theta = \tan^{-1}(4)$ or $\theta = 76°$
- If $R = 4H$ then $\theta = \tan^{-1}(1)$ or $\theta = 45°$

Projectile Motion on an Inclined Plane



Time of Flight

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$

Maximum Height

$$H = \frac{u^2\sin^2\theta}{2g\cos\alpha}$$

Horizontal Range

$$R = \frac{2u^2}{g}\frac{\sin\theta\cos(\theta + \alpha)}{\cos^2\alpha}$$

- Maximum range occurs when $\theta = \frac{\pi}{4} + \frac{\alpha}{2}$
- Maximum range along the incline when projectile is thrown upwards $R_{max} = \frac{u^2}{g(1 + \sin\alpha)}$
- Maximum range along incline when the projectile thrown downwards $R_{max} = \frac{u^2}{g(1 - \sin\alpha)}$

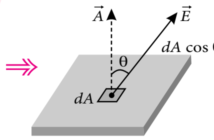
ELECTRIC FLUX AND GAUSS'S LAW

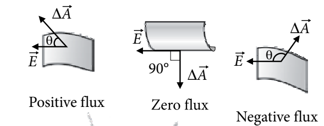
BRAIN MAP CLASS XII

ELECTRIC FLUX

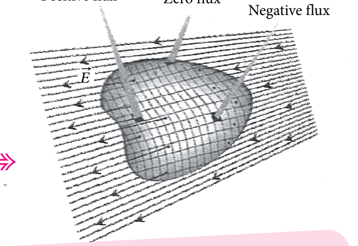
Electric flux is a measure of flow of electric field through a surface. It is equal to the dot product of an area vector and electric field

- Flux of electric field E through any area A: $\phi = EA\cos\theta$ or $\phi = \vec{E}\cdot\vec{A}$
- For variable electric field or curved area $\phi = \int \vec{E}\cdot d\vec{A}$





- For a closed surface outward flux is taken to be positive while inward flux is taken to be negative.



To get a direct connection between the electric flux through closed surface and the total charge inside it.

Problem Solving Strategies

- *Select a symmetric gaussian surface as per the charge distribution.*
- *Calculate total electric charge inside the gaussian surface.*
- *For uniform charge density, simply multiply it by length, area and volume of surface.*
- *For non uniform charge density integrate it over the region enclosed the surface.*
- *Calculate electric field on the gaussian surface as per the given uniform charge distribution.*

GAUSS'S LAW

The total flux linked with a closed Gaussian surface is $(1/\varepsilon_0)$ times the charge enclosed by the closed surface *i.e.*,

$$\phi = \oint \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0}(Q_{enc})$$

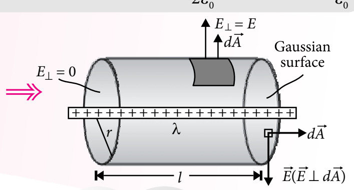
APPLICATIONS OF GAUSS'S LAW

Flux across some definite symmetric closed surfaces

- **Hemispherical body**
  In uniform electric field $\phi_{circular} = -\phi_{curved}; |\phi_{circular}| = \pi R^2 E$
  In non uniform electric field $\phi_{circular} = -\phi_{curved}; |\phi_{curved}| = 2\pi R^2 E$
- **Gaussian Cube**
  Charge kept at corner $\phi_{cube} = \frac{Q}{8\varepsilon_0}$ $\phi_{face} = \frac{Q}{24\varepsilon_0}$ $\phi_{total} = \frac{Q}{\varepsilon_0}$
  Charge kept at centre of face $\phi_{cube} = \frac{Q}{2\varepsilon_0}(5\ faces)$ $\phi_{total} = \frac{Q}{\varepsilon_0}$

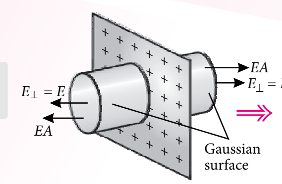
Field of a line charge

$$E_{net} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$



Field of an infinite plane sheet of charge

$$E = \frac{\sigma}{2\varepsilon_0}$$



Result in field due to a sheet depends only on total charge of the sheet and independent of distribution of charge.

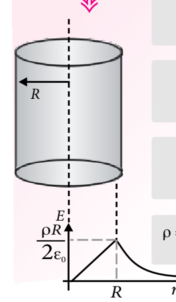
Field due to a long uniformly charged solid cylinder

$$E_{in} = \frac{\rho r}{2\varepsilon_0}$$

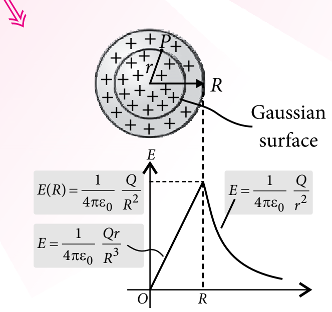
$$E_s = \frac{\rho R}{2\varepsilon_0}$$

$$E_{out} = \frac{\rho R^2}{2\varepsilon_0 r}$$

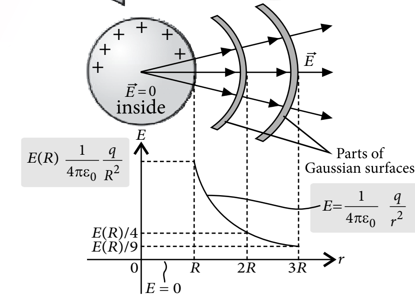
ρ = Volume charge density



Field of a uniformly charged solid sphere and charged conducting sphere



Uniformly charged sphere



Charged conducting sphere

Density of copper = 9×10^3 kg m^{-3}
Specific heat of copper = 9×10^{-2} cal kg^{-8} °C^{-1}

Sol. : Mass of wire (m) = area × length × density
= $0.5 \times 10^{-6} \times 0.1 \times 9 \times 10^3 = 45 \times 10^{-5}$ kg

Rise in temperature (ΔT) = 1075 °C – 25 °C = 1050 °C

(i) If a current I flows in a wire of resistance R for a time t, the heat energy generated is

$$Q = \frac{I^2Rt}{4.2} \text{ cal}$$

If s is the specific heat of copper, the wire will melt in time t given by

$$\frac{I^2Rt}{4.2} = ms\Delta T \qquad .....(i)$$

The resistance R is given by

$$R = \frac{\rho L}{A}$$

where ρ is the specific resistance. Using this in the equation (i), we have

$$\frac{I^2\rho Lt}{4.2\,A} = ms\Delta T \text{ or } t = \frac{4.2\,ms\Delta TA}{I^2\rho L}$$

$$t = \frac{4.2 \times (45 \times 10^{-5}) \times (9 \times 10^{-2}) \times 1050 \times (0.5 \times 10^{-6})}{(10)^2 \times (1.6 \times 10^{-8}) \times 0.1}$$

= 0.59 s

(ii) When the length of the wire is doubled, its resistance R as well as its mass m are doubled. From equation (i)

$$t = \frac{4.2\,ms\,\Delta T}{I^2R}$$

As t is proportional to m/R and as both m and R are doubled, it follows that t will remain the same, *i.e.*, the wire will start melting in the same time.

CHARGING AND DISCHARGING OF CAPACITORS

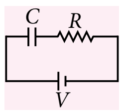
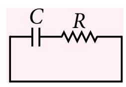
Basic parameters	Charging a capacitor	Discharging of capacitor
Charge on capacitor at time t	$q = CV(1 - e^{-t/RC})$  $q_{max} = q_0 = CV$	$q = q_0\,e^{-t/RC}$  $q_0 = CV$ = initial charge
Current in the circuit at time t	$I = \frac{V}{R}e^{(-t/RC)}$ $I_{max} = I_0 = V/R$	$I = -I_0e^{-t/RC}$ $I_0 = \frac{V}{R}$ = initial current at $t = 0$
Total voltage	$V_C + V_R = V$	$V_C + V_R = 0$; $\lvert V_C\rvert = \lvert V_R\rvert$
Voltage and current dependency over charge	As q increases $\Rightarrow V_C$ increases $\therefore$ V_R decreases and I decreases	As q decreases $\Rightarrow V_C$ decreases $\therefore$ V_R decreases and I decreases

Illustration 7 : An uncharged capacitor is connected to a 15 V battery through a resistance of 10 Ω. It is found that in a time of 2 μs, potential difference across the capacitor becomes 5 V. Find the capacitance of the capacitor. Take ln (1.5) = 0.4.

Sol. : We know that charge on the capacitor at any time is given by $q = q_0\,(1 - e^{-t/\tau})$ where $q_0 = \varepsilon C = 15C$. Here charge q at any time is given by $q = VC$ where V is potential difference across capacitor at that time. Here $V = 5$ V, so $q = 5C$. Putting the values, we get

$$5C = 15C(1 - e^{-t/\tau}) \text{ or } e^{-t/\tau} = 2/3$$

$$\text{or} \quad \frac{t}{\tau} = \ln\left(\frac{3}{2}\right) \text{ or } \frac{t}{RC} = \ln\left(\frac{3}{2}\right)$$

$$\text{or} \quad C = \frac{t}{R\ln(3/2)} = \frac{2 \times 10^{-6}}{10\ln(3/2)} = 0.5\,\mu\text{F}$$

ELECTRICAL DEVICES

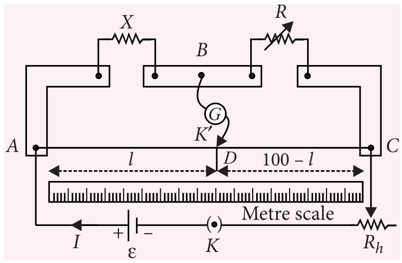
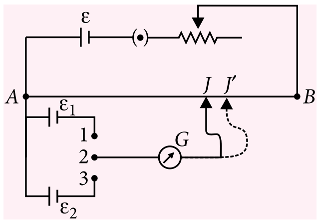
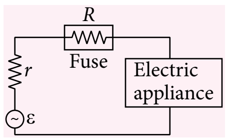
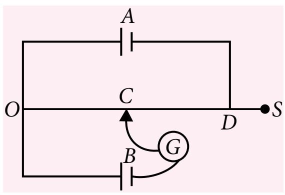
Devices and their principle	Application	Circuit diagram
Meter Bridge Principle : It is based on Wheatstone bridge.	(i) To measure unknown resistance, $X = \frac{R(100-l)}{l}$. (ii) To compare two unknown resistances $\frac{R_1}{R_2} = \frac{l_2}{l_1}\frac{(100-l_1)}{(100-l_2)}$ (iii) To measure the unknown temperature $\theta = \frac{R - R_0}{R_{100} - R_\theta} \times 100$	
Potentiometer Principle : Potential difference across any segments ∝ Length of the segment	(i) Comparision of emf of two cells $\therefore \frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$ (ii) Internal resistance of given primary cell $r = \left(\frac{l_1 - l_2}{l_2}\right)R$	
Fuse Principle : Heat produced by electric current in wire $H = I^2 Rt$	It is a safety device that protects the appliance from getting damaged, by melting and opening the circuit.	

Illustration 8 : Cells A and B and a galvanometer G are connected to a slide wire OS by two sliding contacts C and D as shown in figure. The slide wire is 100 cm long and has a resistance of 12 Ω. With OD = 75 cm, the galvanometer gives no deflection when OC is 50 cm. If D is moved to touch the end of wire S, the value of OC for which the galvanometer shows no deflection is 62.5 cm. The emf of cell B is 1.0 V. Calculate

(i) the potential difference across O and D when D is at 75 cm mark from O

(ii) the potential difference across OS when D touches S



(iii) internal resistance and emf of cell A

Sol. : Resistance of wire OD is $\frac{12}{100} \times 75 = 9\ \Omega$

Let ε and r be the emf and internal resistance of cell A, respectively.

(i) Since 1.0 V is balanced across 50 cm, so potential gradient of wire is 1/50 Vcm^{-1}. Therefore, voltage drop across wire OD of length 75 cm is (1/50) × 75 = 1.5 V.

(ii) Now potential gradient of wire is 1/62.5 V cm^{-1}. Therefore, voltage drop across wire OS of length 100 cm is (1/62.5) × 100 = 1.6 V.

(iii) For first case,

$$\left(\frac{\varepsilon}{9+r}\right) \times 9 = 1.5 \quad \text{... (i)}$$

For second case,

$$\left(\frac{\varepsilon}{12+r}\right) \times 12 = 1.6 \quad \text{... (ii)}$$

On solving eqns. (i) and (ii), we get r = 3 Ω and ε = 2 V.

MPP-4 CLASS XI ANSWER KEY

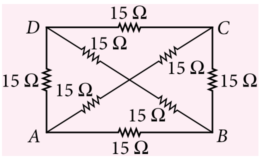
1. (d)	2. (b)	3. (c)	4. (c)	5. (b)
6. (b)	7. (b)	8. (b)	9. (c)	10. (c)
11. (d)	12. (c)	13. (a)	14. (c)	15. (a)
16. (c)	17. (b)	18. (c)	19. (a)	20. (a, d)
21. (a, b, c, d)		22. (a, b, c)	23. (a, b)	24. (4)
25. (4)	26. (8)	27. (a)	28. (b)	29. (b)
30. (c)				

SPEED PRACTICE

1. The number density of free electrons in a copper conductor is 8.5×10^{28} m^{-3}. The area of cross-section of the wire is 2.0×10^{-6} m^2 and it is carrying a current of 3.0 A. How long does an electron take to drift from one end of a wire 3.0 m long to its other end?
(a) 8.1×10^4 s (b) 2.7×10^4 s
(c) 9×10^3 s (d) 3×10^3 s

2. If voltage across a bulb rated 220 V-100 W drops by 2.5% of its rated value, the percentage of the rated value by which the power would decrease is
(a) 20% (b) 2.5% (c) 5% (d) 10%

3. The equivalent resistance between the points A and B will be (each resistance is 15 Ω)
(a) 10 Ω
(b) 40 Ω
(c) 30 Ω
(d) 8 Ω

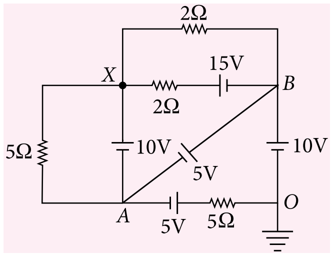


4. The resistance of the wire in the platinum resistance thermometer at ice point is 5 Ω and at steam point is 5.25 Ω. When the thermometer is inserted in an unknown hot bath its resistance is found to be 5.5 Ω. The temperature of the hot bath is
(a) 100°C (b) 200°C (c) 300°C (d) 350°C

5. A resistance of 2 Ω is connected across the gap of a metre-bridge (the length of the wire is 100 cm) and an unknown resistance, greater than 2 Ω, is connected across the other gap. When these resistances are interchanged, the balance point shifts by 20 cm. Neglecting any correction, the unknown resistance is
(a) 3 Ω (b) 4 Ω (c) 5 Ω (d) 6 Ω

6. In a potentiometer experiment, when three cells A, B and C are connected in series the balancing length is found to be 740 cm. If A and B are connected in series balancing length is 440 cm and for B and C connected in series that is 540 cm. Then the emf of ε_A, ε_B and ε_C are respectively (in V)
(a) 1, 1.2 and 1.5 (b) 1, 2 and 3
(c) 1.5, 2 and 3 (d) 1.5, 2.5 and 3.5

7. In the circuit shown, if point O is earthed, the potential of point X is equal to
(a) 10 V
(b) 15 V
(c) 25 V
(d) 12.5 V



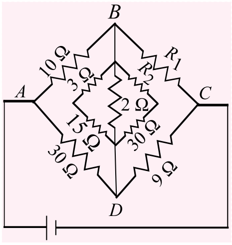
8. Two cells of equal emf having internal resistances r_1 and r_2 $(r_1 > r_2)$ are connected in series. On connecting the combination to an external resistance R, it is observed that the potential difference across the first cell becomes zero. The value of R will be
(a) $(r_1 - r_2)$ (b) $(r_1 + r_2)$
(c) $\left(\frac{r_1 + r_2}{2}\right)$ (d) $\left(\frac{r_1 - r_2}{2}\right)$

9. A voltmeter of resistance R_V and an ammeter of resistance R_A are connected in series across a battery of emf ε and of negligible internal resistance. When a resistance R is connected in parallel to voltmeter, reading of ammeter increases to three times while that of voltmeter reduces to one-third. Then,
(a) $R_A = \frac{R_V}{3}$ (b) $R_A = R_V$
(c) $R_A = 2R$ (d) $R_V = 6R$

10. The cross-section area and length of a cylindrical conductor are A and l, respectively. The specific conductivity varies as $\sigma(x) = \sigma_0 \frac{l}{\sqrt{x}}$, where x is the distance along the axis of the cylinder from one of its ends.
(a) The resistance of the system along the cylindrical axis is $\sqrt{l}/3A\sigma_0$.
(b) The current density if the potential drop along the cylinder is V_0 is $\sigma_0 V_0$.
(c) The electric field at each point in the cylinder is $3V_0\sqrt{x}/2l^{3/2}$.
(d) None of these.

11. Two wires A and B of the same material have their lengths in the ratio 1 : 2 and radii in the ratio 2 : 1. The two wires are connected in parallel across a battery. The ratio of the heat produced in A to the heat produced in B for the same time is
(a) 1 : 2 (b) 2 : 1 (c) 1 : 8 (d) 8 : 1

12. In the Wheatstone's bridge shown in the figure, in order to balance the bridge we must have

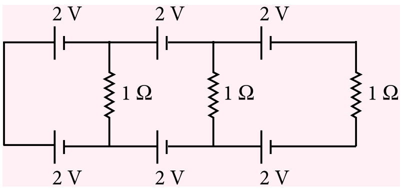


(a) $R_1 = 3\ \Omega, R_2 = 3\ \Omega$
(b) $R_1 = 6\ \Omega, R_2 = 1.5\ \Omega$
(c) $R_1 = 1.5\ \Omega$, R_2 = any finite value
(d) $R_1 = 3\ \Omega$, R_2 = any finite value

13. The resistance of a wire is R. If it is melted and stretched to n times its original length, its new resistance will be
(a) R/n (b) n^2R (c) R/n^2 (d) nR
[NEET 2017]

14. A potentiometer is an accurate and versatile device to make electrical measurements of emf because the method involves
(a) potential gradients
(b) a condition of no current flow through the galvanometer
(c) a combination of cells, galvanometer and resistances
(d) cells **[NEET 2017]**

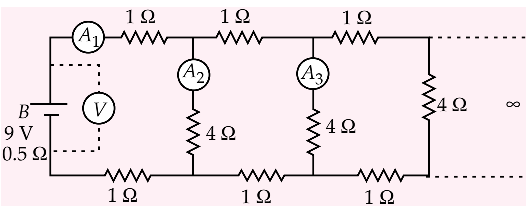
15.



In the above circuit the current in each resistance is
(a) 1 A (b) 0.25 A (c) 0.5 A (d) 0 A
[JEE Main Offline 2017]

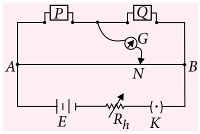
16. Which of the following statements is false ?
(a) Wheatstone's bridge is the most sensitive when all the four resistances are of the same order of magnitude.
(b) In a balanced Wheatstone's bridge, if the cell and the galvanometer are exchanged, the null point is disturbed.
(c) A rheostat can be used as a potential divider.
(d) Kirchhoff's second law represents energy conservation.
[JEE Main Offline 2017]

17.



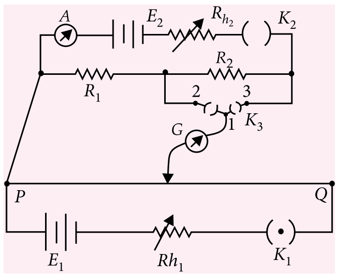
A 9 V battery with internal resistance of 0.5 Ω is connected across an infinite network as shown in the figure. All ammeters A_1, A_2, A_3 and voltmeter V are ideal. Choose correct statement.
(a) Reading of V is 9 V.
(b) Reading of A_1 is 2 A.
(c) Reading of V is 7 V.
(d) Reading of A_1 is 18 A.
[JEE Main Online 2017]

18. In a meter bridge experiment resistances are connected as shown in the figure. Initially resistance $P = 4\ \Omega$ and the neutral point N is at 60 cm from A. Now an unknown resistance R is connected in series to P and the new position of the neutral point is at 80 cm from A. The value of unknown resistance R is



(a) $\frac{20}{3}\ \Omega$ (b) $\frac{33}{5}\ \Omega$
(c) $6\ \Omega$ (d) $7\ \Omega$
[JEE Main Online 2017]

19. A potentiometer PQ is set up to compare two resistances as shown in the figure. The ammeter A in the circuit reads 1.0 A when two way key K_3 is open. The balance point is at a length l_1 cm from P when two way key K_3 is plugged in between 2 and 1, while the balance points is at a length l_2 cm from P when key K_3 is plugged in between 3 and 1. The ratio of two resistances $\frac{R_1}{R_2}$, is found to be



(a) $\frac{l_2}{l_2 - l_1}$
(b) $\frac{l_1}{l_2 - l_1}$
(c) $\frac{l_1}{l_1 + l_2}$
(d) $\frac{l_1}{l_1 - l_2}$
[JEE Main Online 2017]

20. A uniform wire of length l and radius r has a resistance of 100 Ω. It is recast into a wire of radius $\frac{r}{2}$. The resistance of new wire will be
(a) 400 Ω (b) 100 Ω
(c) 200 Ω (d) 1600 Ω
[JEE Main Online 2017]

SOLUTIONS

1. (b) : Here,
Number density of free electrons, $n = 8.5 \times 10^{8}\ \ m^{3}$
Area of cross-section of a wire, $A = 2.0 \times 10^{6}\ \ m^2$
Length of the wire, $l = 3.0$ m
Current, $I = 3.0$ A

The drift velocity of an electron is $v_d = \dfrac{I}{neA}$ ...(i)

The time taken by the electron to drift from one end to other end of the wire is $t = \dfrac{l}{v_d} = \dfrac{lneA}{I}$ (Using (i))

$$= \frac{(3.0\,\text{m})(8.5\times10^{28}\,\text{m}^{-3})(1.6\times10^{-19}\,\text{C})(2.0\times10^{-6}\,\text{m}^2)}{(3.0\text{ A})}$$

$= 2.7 \times 10^4$ s

2. (c) : Power, $P = \dfrac{V^2}{R}$

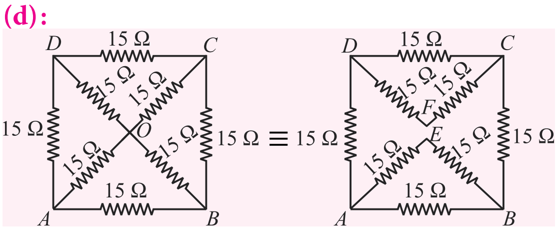
As the resistance of the bulb is constant

$\therefore\ \dfrac{\Delta P}{P} = \dfrac{2\Delta V}{V}$

Percentage decrease in power,

$$\frac{\Delta P}{P}\times 100 = \frac{2\Delta V}{V}\times 100 = 2 \times 2.5\% = 5\%$$

3. (d) :



By symmetry,
$R_{\text{effective}}$ of arm $(AE + EB)$ and arm $AB = 30\ \Omega$ and $15\ \Omega$ are in parallel *i.e.*, $\dfrac{1}{30}+\dfrac{1}{15}=\dfrac{1}{10}$ or $R_1 = 10\ \Omega$.

Same for CFD and CD. Therefore 15 Ω, 10 Ω, 15 Ω are in series and their resultant are in parallel with $AB_{\text{effective}}$, R_1 *i.e.*, $\dfrac{1}{40}+\dfrac{1}{10}=\dfrac{5}{40} \Rightarrow R_{\text{total}} = 8\ \Omega$

4. (b) : Here, $R_0 = 5\ \Omega$, $R_{10} = 5.25\ \Omega$, $R_t = 5\ \ \ \Omega$
As $R_t = R_0(1+\ \alpha t)\ \therefore\ R_{10} = R_0(1+\ \alpha 0)$

$\alpha = \dfrac{R_{100} - R_0}{R_0 \times 100}$ ...(i)

Let the temperature of hot bath be t °C

$R_t = R_0(1 + \alpha t)$ or $\alpha = \dfrac{R_t - R_0}{R_0 \times t}$ ...(ii)

Equating equations (i) and (ii), we get

$$\frac{R_{100}-R_0}{R_0\times100} = \frac{R_t - R_0}{R_0 \times t} \text{ or } t = \frac{R_t - R_0}{R_{100}-R_0}\times 100 = 200\ °\text{C}$$

5. (a) : Let x be the unknown resistance.
In first case, when Wheatstone's bridge is balanced, we get $\dfrac{2}{x} = \dfrac{l}{(100-l)}$ ...(i)

In second case, when Wheatstone's bridge is balanced, then $\dfrac{x}{2} = \dfrac{l+20}{80-l}$ ...(ii)

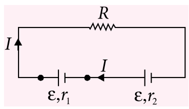
Solving equations (i) and (ii), we get $x = 3\ \Omega$

6. (a) : Let ε_A, ε_B and ε_C be the emf of three cells A, B and C respectively. As per question,
$\varepsilon_A + \varepsilon_B + \varepsilon_C = kl_1 = k \times 740$ ...(i)
$\varepsilon_A + \varepsilon_B = kl_2 = k \times 440$ ...(ii)
$\varepsilon_B + \varepsilon_C = kl_3 = k \times 540$ ...(iii)
Inserting the value of $\varepsilon_A + \varepsilon_B$ from (ii) and $(\varepsilon_B + \varepsilon_C)$ from (iii) into (i), we get $\varepsilon_C = 300k$ and $\varepsilon_A = 200k$
Inserting this value of ε_A into (ii), we get $\varepsilon_B = 240k$
$\therefore\ \ \varepsilon_A : \varepsilon_B : \varepsilon_C = 200k : 240k : 300k = 1 : 1.2 : 1.5$
$\Rightarrow\ \ \varepsilon_A = 1$ V, $\varepsilon_B = 1.2$ V, $\varepsilon_C = 1.5$ V

7. (b) : In open loop $XABO$,
$V_X - 10 + 5 - 10 = 0 \Rightarrow V_X = 15$ V

8. (a) :

$I = \dfrac{2\varepsilon}{R + r_1 + r_2}$

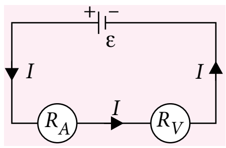


Across 1st cell, $V = \varepsilon - Ir_1$

$$\Rightarrow\ V = \varepsilon - \frac{2\varepsilon}{R+r_1+r_2}\cdot r_1 \text{ or } 0 = \varepsilon - \frac{2\varepsilon r_1}{(R+r_1+r_2)}$$

$\Rightarrow 2r_1 = R + r_1 + r_2 \Rightarrow R = (r_1 - r_2)$

9. (a) : Let initially a current I flow through the circuit as shown in figure.
Applying Kirchhoff's voltage law on the circuit,
$IR_A + IR_V - \varepsilon = 0$ or $\varepsilon = IR_A + IR_V$ ...(i)

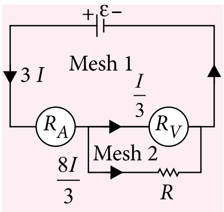


Initial reading of ammeter is I and that of voltmeter is IR_V.
When resistance R is connected in parallel with voltmeter, reading of ammeter increases to three times, it means current $3I$ flows through ammeter but reading of voltmeter decreases to one third, it means current $I/3$ flows through the voltmeter.

Hence, remaining $\left(3I - \dfrac{I}{3} = \dfrac{8I}{3}\right)$ current passes through R as shown in figure.
First, applying KVL on mesh 2.

$$\frac{I}{3}R_V - \frac{8I}{3}R = 0 \text{ or } R_V = 8R$$

Now applying KVL on mesh 1



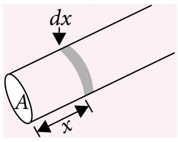
$$\varepsilon - \frac{I}{3}R_V - 3IR_A = 0$$

or $\quad \varepsilon = \frac{1}{3}IR_V + 3IR_A \quad$ ...(ii)

From equations (i) and (ii),

$$\frac{1}{3}IR_V + 3IR_A = IR_A + IR_V \text{ or } R_A = \frac{R_V}{3}; \therefore \; R_A = \frac{8}{3}R$$

10. (c) : Consider the cylinder as composed of thin discs of width dx connected in series. The resistance of a disc at a distance x away from the cylinder end is



$$dR = \frac{1}{\sigma(x)}\frac{dx}{A} = \frac{\sqrt{x}dx}{Al\sigma_0}$$

where A is the cross-section area of the disc and dx is its width. Since the discs are connected in series, the total resistance is $R = \int_0^l dR = \frac{1}{Al\sigma_0}\int_0^l \sqrt{x}dx = \frac{2\sqrt{l}}{3A\sigma_0}$

From Ohm's law, we deduce that the current flowing across the cylinder is given by $I = \frac{V_0}{R} = \frac{3A\sigma_0 V_0}{2\sqrt{l}}$

The current density is therefore, $J = \frac{I}{A} = \frac{3\sigma_0 V_0}{2\sqrt{l}}$

The electric field in the cylinder may be found by using Ohm's law, $E(x) = \frac{J}{\sigma(x)} = \frac{J}{\sigma_0 \frac{l}{\sqrt{x}}} = \frac{3V_0\sqrt{x}}{2l^{3/2}}$

11. (d): Given $\frac{l_1}{l_2} = \frac{1}{2}$ and $\frac{r_1}{r_2} = \frac{2}{1}$ or $\frac{r_2}{r_1} = \frac{1}{2}$

Since, $R_1 = \rho\frac{l_1}{A_1}$ and $R_2 = \rho\frac{l_2}{A_2}$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{l_1}{l_2}\cdot\frac{A_2}{A_1} = \frac{l_1}{l_2}\left(\frac{r_2}{r_1}\right)^2 = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2 = \frac{1}{8}$$

$$\therefore \quad \text{Ratio of heat} = \frac{H_1}{H_2} = \frac{V^2/R_1}{V^2/R_2} = \frac{R_2}{R_1} = \frac{8}{1}$$

12. (d): For $ABCD$ network, balance condition is

$$\frac{10}{30} = \frac{R_1}{9}; \quad \therefore \; R_1 = 3\ \Omega$$

The resistance of the inner network should have some finite value for having the bridge balanced. Hence R_2 should have any finite value.

13. (b): The resistance of a wire of length l and area A and resistivity ρ is given as $R = \rho l/A$, $l' = nl$ (Given)

As the volume of the wire remains constant,

$$\therefore \quad A'l' = Al \Rightarrow A' = \frac{Al}{l'} = \frac{Al}{nl} = \frac{A}{n}$$

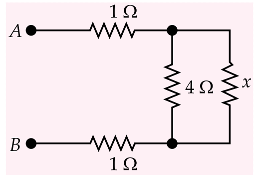
$$\therefore \quad R' = \frac{\rho l'}{A'} = \frac{n^2\rho l}{A} = n^2 R$$

14. (b): A potentiometer is an accurate and versatile device to make electrical measurements of emf because the method involves a condition of no current flow through the galvanometer, the device can be used to measure potential difference, internal resistance of a cell and compare emfs of two sources.

15. (d): The potential difference across each loop is zero. Therefore no current will flow in the circuit.

16. (b): In a balanced Wheatstone bridge if the cell and the galvanometer are interchanged, the null point remains unchanged.

17. (b): Let equivalent resistance of the infinite network be x. Equivalent resistance between points A and B,



$$x = \frac{4x}{4+x} + 2 \text{ or } x^2 - 2x - 8 = 0$$

$x = 4\ \Omega$ ($\because$ –ve value not accepted) ; $\therefore I_1 = \frac{9}{4+0.5} = 2$ A

$\Rightarrow$ Reading of A_1 is 2 A.

18. (a): For $P = 4\ \Omega$, $l_1 = 60$ cm

$$\therefore \quad \frac{P}{Q} = \frac{l_1}{100-l_1} = \frac{60}{40} = \frac{3}{2} \text{ or } Q = \frac{2}{3}P = \frac{8}{3}\ \Omega$$

Now, $P' = P + R$, $l_1' = 80$ cm; $\therefore \frac{P'}{Q} = \frac{l_1'}{100-l_1'} = \frac{80}{20} = 4$

$$\frac{P+R}{Q} = 4 \Rightarrow \frac{4+R}{\frac{8}{3}} = 4 \; ; \; 4+R = \frac{32}{3}$$

$$\therefore \quad R = \frac{32}{3} - 4 = \frac{20}{3}\ \Omega$$

19. (b): When key is plugged between 2 and 1,

$V_1 = iR_1 = Xl_1$ ...(i)

When key is plugged between 3 and 1,

$V_2 = i(R_1 + R_2) = Xl_2$ ...(ii)

On dividing eqn. (ii) by eqn. (i),

$$\frac{V_1}{V_2} = \frac{R_1}{R_1+R_2} = \frac{l_1}{l_2} \text{ or } \frac{R_1}{R_2} = \frac{l_1}{l_2 - l_1}$$

20. (d): Resistance of a wire of length l and radius r is given by $R = \frac{\rho l}{A} = \frac{\rho l}{A}\times\frac{A}{A} = \frac{\rho V}{A^2} = \frac{\rho V}{\pi^2 r^4}$ $\quad (\because V = Al)$

i.e., $R \propto \frac{1}{r^4} \quad \therefore \quad \frac{R_1}{R_2} = \left(\frac{r_2}{r_1}\right)^4$

Here, $R_1 = 100\ \Omega$, $r_1 = r$, $r_2 = \frac{r}{2}$, $R_2 = ?$

$$\therefore \quad R_2 = R_1\left(\frac{r_1}{r_2}\right)^4 = 16R_1 = 1600\ \Omega$$

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EXAM

CLASS XII

PREP 2018

Useful for Medical/Engg. Entrance Exams

CHAPTERWISE MCQs FOR PRACTICE

ELECTRIC CHARGES AND FIELDS

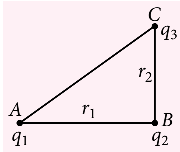
1. Charge q_1 = + 6.0 nC is on y-axis at y = + 3 cm and charge q_2 = – 6.0 nC is on y-axis at y = – 3 cm. Calculate force on a test charge q_0 = 2 nC placed on x-axis at x = 4 cm.
(a) $-51\cdot8\hat{j}\,\mu N$ (b) $51\cdot8\hat{j}\,\mu N$
(c) $-5\cdot18\hat{j}\,\mu N$ (d) $5\cdot18\hat{j}\,\mu N$

2. Three charges Q_1, Q_2 and q are placed on a straight line such that q is somewhere in between Q_1 and Q_2. If this system of charges is in equilibrium, what should be the magnitude and sign of charge q?
(a) $\dfrac{Q_1Q_2}{\left(\sqrt{Q_1}+\sqrt{Q_2}\right)^2}$, positive
(b) $\dfrac{Q_1+Q_2}{2}$, positive
(c) $\dfrac{Q_1Q_2}{\left(\sqrt{Q_1}+\sqrt{Q_2}\right)^2}$, negative
(d) $\dfrac{Q_1+Q_2}{2}$, negative

3. The electric field in a region is given by $\vec{E} = \alpha x\hat{i}$, where α is a constant of proper dimension. Find the total flux passing through a cube bounded by surfaces, $x = l$, $x = 2l$, $y = 0$, $y = l$, $z = 0$, $z = l$.
(a) αl^3 (b) αl^2
(c) $\alpha^2 l$ (d) $\alpha^2 l^2$

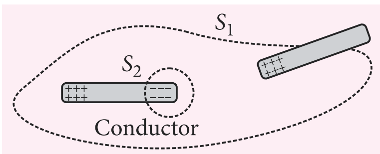
4. A pendulum bob of mass 80 μg carrying a charge of 2×10^{-8} C is at rest in a uniform horizontal electric field E = 20,000 V m^{-1}. Find the tension in the thread of the pendulum.
(a) 2.2×10^{-4} N (b) 4.4×10^{-4} N
(c) 6.6×10^{-4} N (d) 8.9×10^{-4} N

5. Charges q_1 = 1.5 mC, q_2 = 0.2 mC and q_3 = – 0.5 mC are placed at the points A, B and C respectively, as shown in figure. If r_1 = 1.2 m and r_2 = 0.6 m, calculate the magnitude of resultant force on q_2.



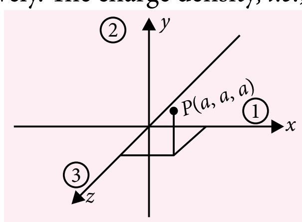
(a) 3.125 kN (b) 2.475 kN
(c) 4.231 kN (d) 6.541 kN

6. Charge on an originally uncharged conductor is separated by holding a positively charged rod very close nearby, as shown in figure. Assume that the induced negative charge on the conductor is equal to the positive charge q on the rod. Then the flux through surface S_1 is



(a) zero (b) q/ε_0
(c) $-q/\varepsilon_0$ (d) none of these.

7. Find the electric field vector at $P(a, a, a)$ due to three infinitely long lines of charges along the x, y and z-axes, respectively. The charge density, *i.e.*, charge per unit length of each wire is λ.



(a) $\dfrac{\lambda}{3\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$
(b) $\dfrac{\lambda}{2\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$

(c) $\frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$

(d) $\frac{\sqrt{2}\lambda}{\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$

8. A large sheet carries uniform surface charge density σ. A rod of length $2l$ has a linear charge density λ on one half and $-\lambda$ on the other half. The rod is hinged at midpoint O and makes angle θ with the normal to the sheet. The torque experienced by the rod is

(a) $\frac{\sigma\lambda l^2}{2\varepsilon_0}\cos\theta$ (b) $\frac{\sigma\lambda l}{\varepsilon_0}\cos^2\theta$

(c) $\frac{\sigma\lambda l^2}{2\varepsilon_0}\sin\theta$ (d) $\frac{\sigma\lambda l}{\varepsilon_0}\sin^2\theta$

9. Two spheres carrying charges +6 μC and +9 μC, separated by a distance d, experiences a force of repulsion F. When a charge of –3 μC is given to both the sphere and kept at the same distance as before, the new force of repulsion is

(a) $3F$ (b) $\frac{F}{9}$ (c) F (d) $\frac{F}{3}$

10. Three identical spheres, each having a charge q and radius R, are kept in such a way that each touches the other two. The magnitude of the electric force on any sphere due to the other two is

(a) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$ (b) $\frac{\sqrt{3}}{4\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$

(c) $\frac{\sqrt{3}}{16\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$ (d) $\frac{\sqrt{5}}{16\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$

11. A particle of mass m and charge q is released from rest in a uniform electric field of intensity E. Find the kinetic energy it attains after moving a distance x in the field.

(a) qEx^2 (b) qEx^3 (c) qEx (d) qEm

12. Two charges, each equal to q are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement ($y << a$) along the y-axis, the net force acting on the particle is proportional to

(a) y (b) $-y$ (c) $\frac{1}{y}$ (d) $-\frac{1}{y}$

13. An electron moves a distance of 6 cm when accelerated from rest by an electric field of strength 2×10^4 N C^{-1}. Calculate the time of travel. The mass and charge of electron are 9×10^{-31} kg and 1.6×10^{-19} C respectively.

(a) 4 ns (b) 2.2 ns

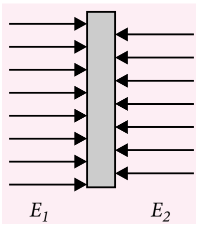
(c) 5.85 ns (d) 7.2 ns

14. A spherical shell of radius R = 1.5 cm has a charge q = 20 μC uniformly distributed over it. The force exerted by one half over the other half is

(a) zero (b) 10^{-2} N

(c) 500 N (d) 2000 N

15. A large charged metal sheet is placed in a uniform electric field, perpendicularly to the electric field lines. After placing the sheet into the field, the electric field on the left side of the sheet is $E_1 = 5 \times 10^5$ V m^{-1} and on the right it is $E_2 = 3 \times 10^5$ V m^{-1}. The sheet experiences a net electric force of 0.08 N. Find the area of one face of the sheet. Assume the external field to remain constant after introducing the large sheet.



(a) $3.6\,\pi \times 10^{-2}$ m^2 (b) $0.9\,\pi \times 10^{-2}$ m^2

(c) $1.8\,\pi \times 10^{-2}$ m^2 (d) none of these

ELECTROSTATIC POTENTIAL AND CAPACITANCE

16. A charge Q is distributed over two concentric hollow spheres of radii r and $R > r$ such that their surface charge densities are equal. Find the potential at the common centre.

(a) $\frac{QR}{4\pi\varepsilon_0(r^2+R^2)}$ (b) $\frac{Qr}{4\pi\varepsilon_0(r^2+R^2)}$

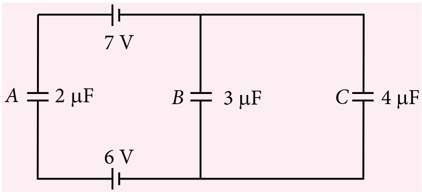
(c) $\frac{Q(r+R)}{4\pi\varepsilon_0(r^2+R^2)}$ (d) $\frac{QRr}{4\pi\varepsilon_0(r^2+R^2)}$

17. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is similarly charged to a potential difference $2V$. The charging battery is then disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is

(a) zero (b) $\frac{3}{2}CV^2$

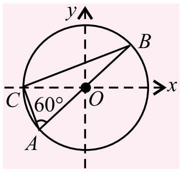
(c) $\frac{25}{6}CV^2$ (d) $\frac{9}{2}CV^2$

18. Three capacitors *A*, *B* and *C* are connected in a circuit as shown in figure. What is the charge in μC on the capacitor *B*?



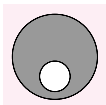
(a) 1/3 (b) 2/3 (c) 1 (d) 4/3

19. Consider a system of three charges $\frac{q}{3}, \frac{q}{3}$ and $-\frac{2q}{3}$ placed at points *A*, *B* and *C*, respectively, as shown in the figure. The centre of the circle is *O* and radius is *R* and angle $CAB = 60°$. Then



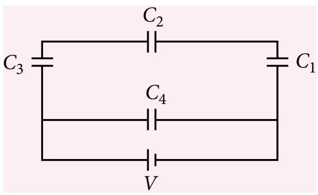
(a) the electric field at point *O* is $\frac{q}{8\pi\varepsilon_0 R^2}$ directed along the negative *x*-axis
(b) the potential energy of the system is zero
(c) the magnitude of the force between the charges at *C* and *B* is $\frac{q^2}{54\pi\varepsilon_0 R^2}$
(d) the potential at point *O* is $\frac{q}{12\pi\varepsilon_0 R}$

20. A spherical portion has been removed from a solid sphere having a charge distributed uniformly in its volume as shown in the figure. The electric field inside the emptied space is



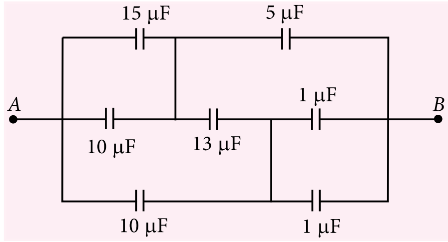
(a) zero everywhere
(b) non-zero and uniform
(c) non-uniform
(d) zero only at its center

21. A network of four capacitors of capacitances equal to $C_1 = C$, $C_2 = 2C$, $C_3 = 3C$ and $C_4 = 4C$ are connected to a battery as shown in the figure. The ratio of the charges on C_2 and C_4 is



(a) $\frac{4}{7}$ (b) $\frac{3}{22}$ (c) $\frac{7}{4}$ (d) $\frac{22}{3}$

22. Find the equivalent capacitance across *A* and *B*.



(a) $\frac{35}{6}$ μF (b) $\frac{25}{6}$ μF
(c) 15 μF (d) none of these

23. A parallel plate capacitor has area of each plate *A*, the separation between the plates is *d*. It is charged to a potential *V* and then disconnected from the battery. How much work will be done in filling the capacitor completely with a dielectric of constant *K*?

(a) $\frac{1}{2}\frac{\varepsilon_0 AV^2}{d}\left(1-\frac{1}{K^2}\right)$ (b) $\frac{1}{2}\frac{V^2\varepsilon_0 A}{Kd}$

(c) $\frac{1}{2}\frac{V^2\varepsilon_0 A}{K^2 d}$ (d) $\frac{1}{2}\frac{\varepsilon_0 AV^2}{d}\left(1-\frac{1}{K}\right)$

24. Two isolated metallic solid spheres of radii *R* and 2*R* are charged such that both of these have same charge density σ. The spheres are located far away from each other and connected by a thin wire. Find the new charge density on the bigger sphere.

(a) $\frac{3\sigma}{6}$ (b) $\frac{5\sigma}{6}$ (c) $\frac{7\sigma}{6}$ (d) $\frac{5\sigma}{7}$

25. Two thin wire rings each having a radius *R* are placed at a distance *d* apart with their axes coinciding. The charges on the two rings are $+q$ and $-q$. The potential difference between the centres of the two rings is

(a) $\frac{q}{4\pi\varepsilon_0}\frac{R}{d^2}$ (b) $\frac{q}{2\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right)$

(c) zero (d) $\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right)$

26. Mark the correct statement:
(a) An electron and a proton when released from rest in a uniform electric field experience the same force and the same acceleration.
(b) Two equipotential surfaces may intersect.
(c) A solid conducting sphere holds more charge than a hollow conducting sphere of the same radius.
(d) No work is done in taking a positive charge from one point to another inside a negatively charged metallic sphere.

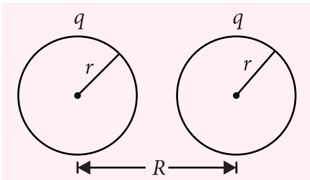
27. A uniformly charged solid sphere of radius R has potential V_0 is measured with respect to infinity on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then

(a) $R_1 = 0$ and $R_2 < (R_4 - R_3)$
(b) $8R_1 < R_4$ and $2R_1 > R_3$
(c) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
(d) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

28. A parallel plate air capacitor has a capacitance C. When it is half filled with a dielectric of dielectric constant 5, the percentage increase in the capacitance will be

(a) 400% (b) 66.6%
(c) 33.3% (d) 200%

29. Two identical charged spheres, each of radius r having charge q are held at a distance R between their centres as shown in figure. The potential energy of the system is

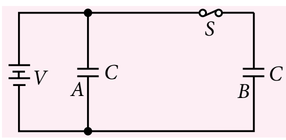


(a) $\frac{q^2}{4\pi\varepsilon_0(R+r)}$ (b) $\frac{q^2}{4\pi\varepsilon_0(R+2r)}$

(c) $\frac{q^2}{4\pi\varepsilon_0}\left[\frac{2}{R}+\frac{1}{r}\right]$ (d) none of these

30. Figure shows two identical parallel plate capacitors connected to a battery with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant 3. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric.

(a) 3 : 4
(b) 2 : 3
(c) 3 : 5
(d) 5 : 3



SOLUTIONS

1. **(a) :** Here, $q = \pm 6.0$ nC $= \pm 6.0 \times 10^{-9}$C
$2a = 6$ cm $= 6 \times 10^{-2}$ m
$r = 4$ cm (on equatorial line) $= 4 \times 10^{-2}$ m
$q_0 = 2$ nC $= 2 \times 10^{-9}$ C, $\vec{F} = ?$

$$\therefore \quad F = q_0 E = \frac{q_0(q\times 2a)}{4\pi\varepsilon_0(r^2+a^2)^{3/2}}$$

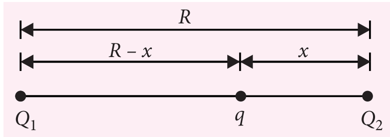
$$= \frac{2\times10^{-9}(6.0\times10^{-9}\times6\times10^{-2})\times9\times10^{9}}{[(4\times10^{-2})^2+(3\times10^{-2})^2]^{3/2}}$$

$= 51.8 \times 10^{-6}$ N

$\vec{F} = 51.8\ \mu$N, along negative direction of y-axis

$$\therefore \quad \vec{F} = -51\cdot 8\hat{j}\ \mu\text{N}$$

2. **(c) :** Since, the system is in equilibrium, net force on each charge is zero so q should be negative.



$$\frac{kQ_1Q_2}{R^2} = \frac{kqQ_2}{x^2} \text{ or } \frac{x}{R} = \sqrt{\frac{q}{Q_1}}$$

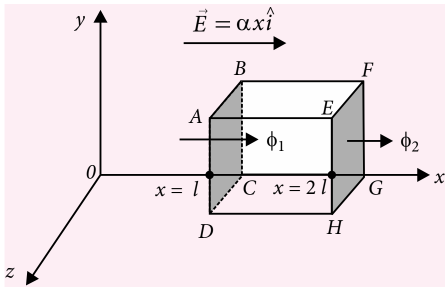
$\therefore$ Net force on charge q is also zero.

$$\therefore \quad \frac{kQ_1q}{(R-x)^2} = \frac{kqQ_2}{x^2} \text{ or } \frac{R-x}{x} = \frac{\sqrt{Q_1}}{\sqrt{Q_2}}$$

$$\text{or} \quad \frac{R}{x} = \frac{\sqrt{Q_1}+\sqrt{Q_2}}{\sqrt{Q_2}} \text{ or } \sqrt{\frac{Q_1}{q}} = \frac{\sqrt{Q_1}+\sqrt{Q_2}}{\sqrt{Q_2}}$$

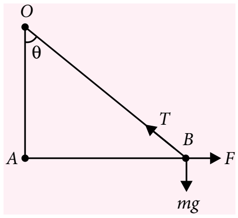
$$\text{or} \quad q = \frac{Q_1Q_2}{(\sqrt{Q_1}+\sqrt{Q_2})^2}$$

3. **(a) :** $\vec{E}$ is perpendicular to the faces $ABCD$ and $EFGH$, whereas is tangential to all other faces.



Flux entering the cube at face $ABCD$, *i.e.*,
$\phi_1 = E_1S = (\alpha l)(l^2) = \alpha l^3$ [as $E_1 = \alpha l$]
Flux leaving the cube at the face $EFGH$, *i.e.*,
$\phi_2 = E_2S = [\alpha(2l)](l^2) = 2\alpha l^3$ [as $E_2 = \alpha(2l)$]
Total flux passing through the cube, *i.e.*,
$\phi_c = \phi_2 - \phi_1 = 2\alpha l^3 - \alpha l^3 = \alpha l^3$

4. (d): Here ; $m = 80\ \mu g$, $q = 2 \times 10^{-8}$ C
$E = 20{,}000\ \text{V m}^{-1}$
At B, three forces mg, T and $F = qE$ are in equilibrium as shown in figure.



Applying Lami's theorem,

$$\frac{T}{OB} = \frac{mg}{OA} = \frac{F}{AB}$$

$$\therefore \quad F = mg \times \frac{AB}{OA} = mg \tan\theta$$

$$\tan\theta = \frac{F}{mg} = \frac{qE}{mg} = \frac{2\times10^{-8}\times20000}{80\times10^{-6}\times10} = 0.5$$

$$\therefore \cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}} = \frac{1}{\sqrt{1.25}} = 0.894$$

$$T = mg \times \frac{OB}{OA} = \frac{mg}{\frac{OA}{OB}} = \frac{mg}{\cos\theta}$$

$$T = \frac{80\times10^{-6}\times10}{0.894} = 8.9\times10^{-4}\,\text{N}$$

5. (a): Force on q_2 due to q_1

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_1^2} = \frac{9\times10^9\times1.5\times10^{-3}\times0.2\times10^{-3}}{(1.2)^2}$$

$= 1.875 \times 10^3$ N, along AB

Force on q_2 due to q_3

$$\vec{F}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2q_3}{r_2^2} = \frac{9\times10^9\times0.2\times10^{-3}\times0.5\times10^{-3}}{(0.6)^2}$$

$= -2.5 \times 10^3$ N, along BC

As $\vec{F}_1$ is perpendicular to $\vec{F}_2$, so the resultant force on q_2 is

$$F = \sqrt{F_1^2 + F_2^2} = 3.125\times10^3\ \text{N} = 3.125\ \text{kN}$$

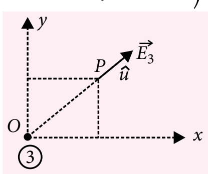
6. (b): Net charge on the conductor will be zero. So, net charge inside S_1 will be due to the charge on the positively charged rod. Hence, flux through S_1 is q/ε_0.

7. (b): Let us consider the electric field due to wire 3 only.

$$\vec{E}_3 = E\hat{u} = \frac{\lambda}{2\pi\varepsilon_0(a^2+a^2)^{1/2}}\left(\hat{i}\cos45^\circ + \hat{j}\cos45^\circ\right)$$

$$= \frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 a}\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$$

$$= \frac{\lambda}{4\pi\varepsilon_0 a}\left(\hat{i}+\hat{j}\right)$$

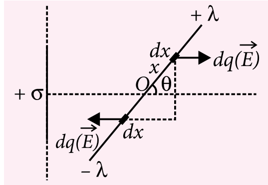


Similarly, electric field due to wires 1 and 2

$$\vec{E}_1 = \frac{\lambda}{4\pi\varepsilon_0 a}\left(\hat{j}+\hat{k}\right) \text{ and } \vec{E}_2 = \frac{\lambda}{4\pi\varepsilon_0 a}\left(\hat{i}+\hat{k}\right)$$

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{\lambda}{2\pi\varepsilon_0 a}\left(\hat{i}+\hat{j}+\hat{k}\right)$$

8. (c): Consider a pair of small elements of rod, each of length dx, at a distance x from O.



Charge on each element, $dq = \lambda\, dx$

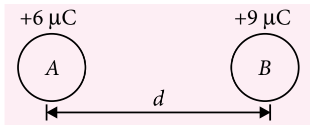
Force on each element $= dq\,(E) = \lambda\, dx\left(\frac{\sigma}{2\varepsilon_0}\right)$

Perpendicular distance between the forces $= 2\,x \sin\theta$

$$\therefore \quad d\tau = \lambda dx\left(\frac{\sigma}{2\varepsilon_0}\right)2\,x\sin\theta$$

$$\tau = \int_0^l \frac{\sigma\lambda\sin\theta}{\varepsilon_0}x\,dx = \frac{\lambda\sigma\sin\theta}{2\varepsilon_0}l^2$$

9. (d): Let us assume spheres as A and B.



According to Coulomb's law, the force of repulsion between A and B separated by a distance d is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{(+6\ \mu\text{C})(+9\ \mu\text{C})}{d^2} \quad \text{...(i)}$$

When a charge of $-3\ \mu$C is given to both the spheres, then charge on $A = +6\ \mu\text{C} - 3\ \mu\text{C} = +3\ \mu\text{C}$
and on $B = +9\ \mu\text{C} - 3\ \mu\text{C} = +6\ \mu\text{C}$
Again by Coulomb's law, the new force of repulsion between A and B separated by the same distance d is

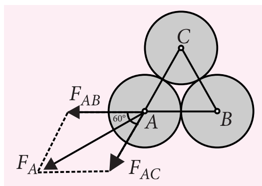
$$F' = \frac{1}{4\pi\varepsilon_0}\frac{(+3\ \mu\text{C})(+6\ \mu\text{C})}{d^2} \quad \text{...(ii)}$$

Dividing eqn. (ii) by eqn. (i), we get

$$\frac{F'}{F} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{(+3\ \mu\text{C})(+6\ \mu\text{C})}{d^2}}{\frac{1}{4\pi\varepsilon_0}\frac{(+6\ \mu\text{C})(+9\ \mu\text{C})}{d^2}} = \frac{1}{3}$$

$$\therefore \quad F' = \frac{F}{3}$$

10. (c): For external points, a charged sphere behaves as a point charge at its centre. Since both B and C carry equal charge and are at equal distance from A



$\therefore \quad F_{AB} = F_{AC} = F$

where, $F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2R)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4R^2}$

Now as angle between BA and CA is 60°

$\therefore \quad F_A = \sqrt{F^2 + F^2 + 2FF\cos 60°} = \frac{\sqrt{3}}{16\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$

11. (c) : Force acting on the charge q in the electric field of intensity E, *i.e.*, $F = qE$

If v is the velocity acquired by the particle in moving through a distance x after starting from rest $v_0 = 0$, then from $v^2 - v_0^2 = 2ax$, we get $v^2 = 2ax$

or $\quad v^2 = 2\left(\frac{qE}{m}\right)x \qquad \left(\because a = \frac{F}{m} = \frac{qE}{m}\right)$

Thus, kinetic energy of the particle, *i.e*,

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{2qEx}{m}\right) = qEx$$

12. (a)

13. (c) : Force exerted on the electron by the electric field, $F = eE$

$\therefore$ Acceleration of the electron,

$$a = \frac{F}{m} = \frac{eE}{m} = \frac{1.6\times10^{-19}\times2\times10^4}{9\times10^{-31}}$$

$= 0.35 \times 10^{16}$ m s^{-2}

Now $u = 0$, $s = 6.0$ cm $= 0.06$ m, $a = 0.35 \times 10^{16}$ m s^{-2}

As $\quad s = ut + \frac{1}{2}at^2$

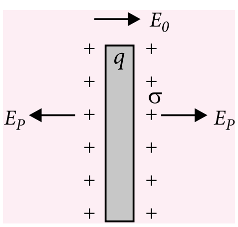
$\therefore \quad 0.06 = 0 + \frac{1}{2} \times 0.35 \times 10^{16} \times t^2$

or $\quad t = \sqrt{\frac{0.06\times2}{0.35\times10^{16}}} = 5.85 \times 10^{-9}$ s $= 5.85$ ns

14. (d)

15. (a) : Here, $E_1 = 5 \times 10^5$ V m^{-1}, $E_2 = 3 \times 10^5$ V m^{-1}, $F = 0.08$ N.

Let external field be E_0 and surface charge density of sheet be σ (including both surfaces). So, $E_P = \frac{\sigma}{2\varepsilon_0}$



Electric field due to sheet

$E_1 = E_0 - E_P$ ...(i) $\qquad -E_2 = E_0 + E_P$ ...(ii)

From eqns. (i) and (ii),

$$E_0 = \frac{E_1 - E_2}{2} = \frac{5\times10^5 - 3\times10^5}{2} = 10^5 \text{ V m}^{-1}$$

and $\quad E_P = \frac{-E_1 - E_2}{2} = \frac{-5\times10^5 - 3\times10^5}{2}$

$= -4 \times 10^5$ V m^{-1}

$\because \quad E_P = \frac{\sigma}{2\varepsilon_0} = -4\times10^5 \therefore \sigma = -8\varepsilon_0\times10^5$

Force on sheet,

$|\vec{F}| = |q\vec{E}_0|$ or $0.08 = |\sigma A\vec{E}_0| \qquad [\because \sigma = q/A]$

or $\quad 0.08 = 8\varepsilon_0 \times 10^5 \times A \times 10^5$

or $\quad A = \frac{10^{-12}}{\varepsilon_0} = 10^{-12}\times36\pi\times10^9 = 3.6\pi\times10^{-2}$ m^2

16. (c) : If q_1 and q_2 are the charges on the spheres of radii r and R, respectively, then as per the given conditions,

$q_1 + q_2 = Q$ ...(i)

Since their surface charge densities are equal

$\therefore \quad \frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2}$ or $\frac{q_1}{r^2} = \frac{q_2}{R^2}$ ...(ii)

From eqns. (i) and (ii),

$q_1 = \frac{Qr^2}{r^2 + R^2}$ and $q_2 = \frac{QR^2}{r^2 + R^2}$

Potential at the centre, $V = k\left[\frac{q_1}{r} + \frac{q_2}{R}\right]$ ...(iii)

Substituting values of q_1 and q_2 in eqn. (iii),

$$V = \frac{kQ(r+R)}{(r^2+R^2)} = \frac{Q(r+R)}{4\pi\varepsilon_0(r^2+R^2)}$$

17. (b): Here $Q_1 = CV$, $Q_2 = 2\,C \times 2V = 4\,CV$

As the two capacitors are connected with opposite polarity, the common potential is

$$V' = \frac{Q_2 - Q_1}{C_1 + C_2} = \frac{4CV - CV}{C + 2C} = V$$

Equivalent capacitance, $C' = C + 2C = 3C$

Final energy of the configuration is

$$U' = \frac{1}{2}C'V'^2 = \frac{1}{2}\times3C\times V^2 = \frac{3}{2}CV^2$$

18. (b): Capacitors B and C are in parallel, then A is in series

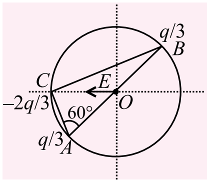
$$C_{eq} = \frac{2\times(3+4)}{2+(3+4)} = \frac{14}{9}\mu\text{F} = \frac{14}{9}\times10^{-6}\text{ F}$$

$$Q = C_{eq}V = \frac{14}{9}\times10^{-6}(7-6)\text{C} = \frac{14}{9}\mu\text{C}$$

Charge will be divided between the capacitors B and C. So charge on the capacitor B is

$$q = \frac{3\times\left(\frac{14}{9}\right)}{3+4} = \frac{2}{3}\mu\text{C}$$

19. (c) : The electric field at O due to the two charges $q/3$ will get cancelled. Electric field due to $\left(\frac{-2q}{3}\right)$ will be directed towards C.



$$E = \frac{-1}{4\pi\varepsilon_0}\frac{(2q/3)}{R^2} = \frac{-q}{6\pi\varepsilon_0 R^2}$$

Potential energy of the system

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{\left(\frac{q}{3}\right)^2}{2R} + \frac{\frac{q}{3}\left(-\frac{2q}{3}\right)}{2R\sin 60°} + \frac{\frac{q}{3}\left(-\frac{2q}{3}\right)}{2R\cos 60°}\right] \neq 0$$

The magnitude of the force between B and C,

$$F = \frac{1}{4\pi\varepsilon_0}\left[\frac{\left(\frac{2q}{3}\right)\left(\frac{q}{3}\right)}{(2R\sin 60°)^2}\right] = \frac{2q^2}{9\times 3\times 4\pi\varepsilon_0 R^2}$$

$$= \frac{1}{54}\frac{q^2}{\pi\varepsilon_0 R^2}$$

The electric potential at O,

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{\left(\frac{q}{3}+\frac{q}{3}-\frac{2q}{3}\right)}{R}\right] = 0$$

20. (b)

21. (b): The series combination of C_1, C_2 and C_3 is in parallel with C_4. The equivalent capacitance C_{123} of the series combination is given by

$$\frac{1}{C_{123}} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3} = \frac{1}{C}+\frac{1}{2C}+\frac{1}{3C} = \frac{11}{6C}$$

or $\quad C_{123} = \frac{6C}{11}$

Let q_1, q_2, q_3 and q_4 be the charges on the respective capacitor. Since in a series combination, charge on all the capacitors is same,

$$q_1 = q_2 = q_3 = C_{123}\times V = \frac{6CV}{11}$$

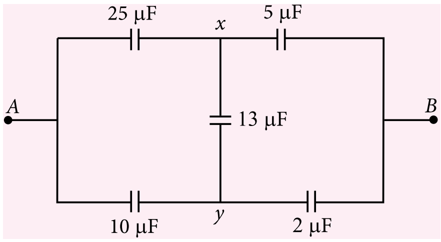
Also, the charge on the capacitor C_4,

$q_4 = C_4 V = 4CV$

$\therefore$ The ratio of the charges on C_2 and C_4 is,

$$\frac{q_2}{q_4} = \frac{\frac{6CV}{11}}{4CV} = \frac{3}{22}$$

22. (a) : Circuit can be redrawn as follows:



Now, $\frac{25\ \mu F}{5\ \mu F} = \frac{10\ \mu F}{2\ \mu F}$ $\therefore V_x = V_y$

$$C_{eq} = \left(\frac{5\times 25}{5+25}\right)+\left(\frac{10\times 2}{10+2}\right) = \frac{35}{6}\ \mu F$$

23. (d): Before filling with dielectric, energy stored is

$$U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2$$

On filling with dielectric, capacity will be

$$C_2 = KC_1 = \frac{K\varepsilon_0 A}{d} \text{ and } V_2 = \frac{V_1}{K} = \frac{V}{K}$$

$\therefore$ Energy, $U_2 = \frac{1}{2}C_2V_2^2$

$$= \frac{1}{2}K\frac{\varepsilon_0 A}{d}\frac{V^2}{K^2} = \frac{1}{2K}\frac{\varepsilon_0 AV^2}{d}$$

Work done = decrease in energy

$$= U_1 - U_2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2\left(1-\frac{1}{K}\right)$$

24. (b): $Q_1 = \sigma\,(4\pi R^2) = 4\pi R^2\sigma$

$Q_2 = \sigma[4\pi(2R)^2] = 16\pi R^2\sigma$

Total charge of both the spheres, $Q = Q_1 + Q_2$
$= 20\pi R^2\sigma$

Total capacitance of both the spheres,

$C = C_1 + C_2 = 4\pi\varepsilon_0 R + 4\pi\varepsilon_0 (2R) = 12\pi\varepsilon_0 R$

Common potential, $V = \frac{Q}{C} = \frac{20\pi R^2\sigma}{12\pi\varepsilon_0 R} = \frac{5\sigma R}{3\varepsilon_0}$

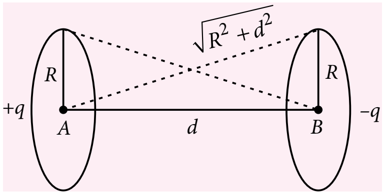
If Q_2' is charge on the bigger sphere after both the spheres are connected by a thin wire,

$$Q_2' = C_2V = (8\pi\varepsilon_0 R)\left(\frac{5\sigma R}{3\varepsilon_0}\right) = \left(\frac{40}{3}\pi R^2\right)\sigma$$

New charge density on bigger sphere, *i.e.*,

$$\sigma' = \frac{Q_2'}{4\pi(2R)^2} = \frac{\left(\frac{40}{3}\pi R^2\right)\sigma}{16\pi R^2} = \frac{5\sigma}{6}$$

25. (b): The two charged rings are shown in the figure.



$$V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{R} + \frac{-q}{\sqrt{R^2+d^2}}\right]$$

$$V_B = \frac{1}{4\pi\varepsilon_0}\left[-\frac{q}{R} + \frac{q}{\sqrt{R^2+d^2}}\right]$$

$\therefore \quad V_A - V_B$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{R} - \frac{q}{\sqrt{R^2+d^2}} + \frac{q}{R} - \frac{q}{\sqrt{R^2+d^2}}\right]$$

$$= \frac{q}{2\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$$

26. (d): Option (a) is wrong because magnitude of force will be the same, but acceleration will be different because masses of electrons and protons are different.
Option (b) is wrong because at a point there can be only one potential.
Option (c) is wrong because charge always lies on the outer surface of a conductor.
Option (d) is correct because the whole conductor will be an equipotential body.

27. (a): Spherical surface of radius r inside sphere will be equipotential surface with potential ($> V_0$)

$$V = \frac{kq}{2R^3}(3R^2 - r^2) = \frac{V_0}{2R^2}(3R^2 - r^2)\left[\because V_0 = \frac{kq}{R}\right]$$

For $V = \dfrac{3V_0}{2}$; $\dfrac{3V_0}{2} = \dfrac{V_0}{2R^2}(3R^2 - R_1^2) \Rightarrow R_1 = 0$

For $V = \dfrac{5V_0}{4}$, $\dfrac{5V_0}{4} = \dfrac{V_0}{2R^2}(3R^2 - R_2^2) \Rightarrow R_2 = \dfrac{R}{\sqrt{2}}$

Spherical surface of radius r' outside this sphere will be equipotential surface with potential $V'(< V_0)$

$$V' = \frac{kq}{r'} = \frac{V_0 R}{r'}$$

$\therefore \quad$ For $V' = \dfrac{3V_0}{4}$; $\dfrac{3V_0}{4} = \dfrac{V_0 R}{R_3} \Rightarrow R_3 = \dfrac{4R}{3}$

For $V' = \dfrac{V_0}{4}$; $\dfrac{V_0}{4} = \dfrac{V_0 R}{R_4} \Rightarrow R_4 = 4R$

Here $R_1 = 0$, $R_2 < (R_4 - R_3)$, and $(R_2 - R_1) < (R_4 - R_3)$
Hence only option (a) is correct.

28. (b): Capacitance of the given capacitor, $C = \dfrac{\varepsilon_0 A}{d}$.

When the capacitor is half-filled with dielectric constant $K = 5$, capacitance is given by

$$C' = \frac{\varepsilon_0 A}{\left(\frac{d}{2}\right) + \left(\frac{d}{2K}\right)} = \frac{5}{3}\frac{\varepsilon_0 A}{d} = \frac{5}{3}C$$

Increase in capacitance $= \dfrac{\frac{5}{3}C - C}{C} \times 100 = 66.6\%$

29. (c) : Each sphere possesses potential energy due to its own charge

$$U_1 = U_2 = \frac{1}{2}\frac{q^2}{C} = \frac{q^2}{8\pi\varepsilon_0 r}$$

Mutual potential energy of each sphere due to electric field of other

$$U_1' = U_2' = \frac{q^2}{4\pi\varepsilon_0 R}$$

$\therefore \quad$ Total potential energy of the system of two spheres,

$$U = (U_1 + U_2) + (U'_1 + U'_2) = \frac{2q^2}{8\pi\varepsilon_0 r} + \frac{2q^2}{4\pi\varepsilon_0 R}$$

$$U = \frac{q^2}{4\pi\varepsilon_0}\left[\frac{1}{r} + \frac{2}{R}\right]$$

30. (c) : Initially when the switch S is closed, both the capacitors have same potential difference V across them. Therefore, initial energy stored in both the capacitors is

$$U_i = U_A + U_B = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$$

When the dielectric of dielectric constant, $K = 3$ is inserted, the capacitance of each capacitor becomes $3C$. The potential difference across A is still V as it is still connected across the battery. With switch S open, the potential difference on B attains a new value V' but charge $q = CV$does not change.

$$\therefore \quad CV = 3C \times V' \text{ or } V' = \frac{V}{3}$$

The final total energy stored in both capacitors is

$$U_f = U_A' + U_B'$$

$$= \frac{1}{2} \times 3C \times V^2 + \frac{1}{2} \times 3C \times \left(\frac{V}{3}\right)^2 = \frac{5}{3}CV^2$$

$\therefore \quad$ The required ratio will be,

$$\frac{U_i}{U_f} = \frac{CV^2}{\frac{5}{3}CV^2} = \frac{3}{5} = 3:5$$

◆◆

ACE YOUR WAY CBSE XII

Series 3

CHAPTERWISE PRACTICE PAPER

Moving Charges and Magnetism | Magnetism and Matter

Time Allowed : 3 hours | Maximum Marks : 70

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Q.no. 1 to 5 are very short answer questions and carry 1 mark each.
(iii) Q.no. 6 to 10 are short answer questions and carry 2 marks each.
(iv) Q.no. 11 to 22 are also short answer questions and carry 3 marks each.
(v) Q.no. 23 is a value based question and carries 4 marks.
(vi) Q.no. 24 to 26 are long answer questions and carry 5 marks each.
(vii) Use log tables if necessary, use of calculators is not allowed.

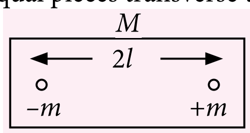
SECTION - A

1. Show that a force that does no work must be a velocity dependent force.

2. If the ratio of the horizontal component of the Earth's magnetic field to the resultant magnetic field at a place is $1/\sqrt{2}$, what is the angle of dip at that place?

3. A proton has spin and magnetic moment just like an electron. Why then its effect is neglected in magnetism of materials?

4. How does the (a) pole strength and (b) magnetic moment of each part of a bar magnet change if it is cut into two equal pieces transverse to length?



5. Soft iron is used to make electromagnets. Why?

SECTION - B

6. A current of 2.00 A exists in a square loop of edge 10.0 cm. Find the magnetic field at the centre of the square loop.

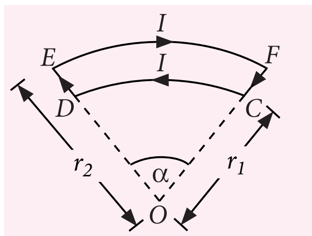
7. In a chamber, a uniform magnetic field of 6.5 G (1 G = 10^{-4} T) is maintained. An electron is shot into the field with a speed of 4.8×10^{6} m s^{-1} normal to the field. Explain why the path of the electron is a circle. Determine the radius of the circular orbit. ($e = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg)

OR

What torque acts on a 40 turn coil of a 100 cm^2 area carrying a current of 10 A held : (a) parallel to and (b) at right angles to a magnetic field of flux density 0.2 T?

8. What is meant by magnetic screening or shielding?

9. Find magnetic induction at centre O due to current I through the circuit shown in figure.



10. Draw diagrams to depict the behaviour of magnetic field lines near a bar of

(a) Copper

(b) Aluminium

(c) Mercury, cooled to a very low temperature (4.2 K)

SECTION - C

11. Assume the dipole model for earth's magnetic field $\vec{B}$ which is given by

B_V = vertical component of magnetic field

$$= \frac{\mu_0}{4\pi}\frac{2m\cos\theta}{r^3}$$

B_H = Horizontal component of magnetic field

$$= \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^3}$$

θ = (90° – latitude measured from magnetic equator).

Find loci of points for which (i) $|\vec{B}|$ is minimum; (ii) dip angle is zero; and (iii) dip angle is ± 45°.

12. Verify the Gauss's law for magnetic field of a point dipole of dipole moment $\vec{m}$ at the origin for the surface which is a sphere of radius R.

13. A straight wire of length (π/2) m is bent into a circular shape. If the wire were to carry a current of 5 A, calculate the magnetic field due to it, before bending, at a point distant 0.01 times the radius of the circle formed from it. Also, calculate the magnetic field at the centre of the circular loop formed, for the same value of current.

14. Two identical circular coils of radius 0.1 m, each having 20 turns are mounted co-axially 0.1 m apart. A current of 0.5 A is passed through both of them (i) in the same direction, (ii) in the opposite directions. Find the magnetic field at the centre of each coil.

OR

A solenoid of length 0.4 m and having 500 turns of wire carries a current of 3 A. A thin coil having 10 turns of wire and of radius 0.01 m carries a current of 0.4 A. Calculate the torque required to hold the coil in the middle of the solenoid with its axis perpendicular to the axis of the solenoid ($\mu_0 = 4\pi \times 10^{-7}$ Vs A^{-1}m^{-1}).

15. A bar magnet of magnetic moment M and moment of inertia I (about centre, perpendicular to length) is cut into two equal pieces, perpendicular to length. Let T be the period of oscillations of the original magnet about an axis through the mid point, perpendicular to length, in a magnetic field . What would be the similar period T' for each piece?

16. A moving coil galvanometer when shunted with a resistance of 5 Ω gives a full scale deflection for 250 mA and when a resistance of 1050 Ω is connected in series, it gives a full scale deflection for 25 V. Find the resistance of the galvanometer and the current required to produce a full scale deflection when it is used alone.

17. (a) Why should a solenoid tend to contract when a current passes through it?

(b) A solenoid of length 1.0 m and 3.0 cm diameter has five layers of windings of 850 turns each and carries a current of 5 A. What is the magnetic field at the centre of the solenoid? Also calculate the magnetic flux for a cross-section of the solenoid at the center of the solenoid.

18. Distinguish between Biot Savart's law and Ampere's circuital law.

19. A combination of electric and magnetic fields is used as velocity selector for charged particles. Obtain the conditions required and also explain what happens to the particles of higher and lower velocities than one selected.

20. A long horizontal wire P carries a current of 50 A. It is rigidly fixed. Another fine wire Q is placed directly above and parallel to P. The weight of the wire Q is 0.075 N m^{-1} and it carries a current of 25 A. Find the position of the wire Q from the wire P so that Q remains suspended due to the magnetic repulsion. Also indicate the direction of current in Q with respect to P.

21. A compass needle free to turn in a horizontal plane is placed at the centre of a circular coil of 30 turns and radius 12 cm. The coil is in a vertical plane making an angle of 45° with the magnetic meridian when the current in the coil is 0.35 A. the needle points west to east.

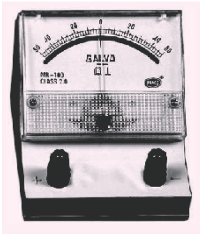
(a) Determine the horizontal component of earth's magnetic field at the location.

(b) The current in the coil is reversed and the coil is rotated about its vertical axis by an angle of 90° in the anticlockwise sense looking from above. Predict the direction of the needle. Take the magnetic declination at the places to be zero.

22. A beam of protons with a velocity of 4×10^5 m s^{-1} enters a uniform magnetic field of 0.3 T. The velocity makes an angle of 60° with the magnetic field. Find the radius of the helical path taken by the proton beam and the pitch of the helix.

SECTION - D

23. Manak was doing the activities in Physics lab , while using galvanometer he requested his Physics teacher to help him performing a new activity of conversion of galvanometer into voltmeter and ammeter using standard resistances.



He also observed that the reading of ammeter was less than actual current in circuit and reading of voltmeter was less than actual value of potential difference between the two points where voltmeter was attached.

(a) What values are shown by Manak and his Physics teacher?

(b) How could Manak convert galvanometer into ammeter and voltmeter?

(c) Why were the measured readings less than the actual value of current and potential difference?

SECTION - E

24. Derive an expression for the force per unit length between two long straight parallel current carrying conductors. Hence define one ampere.

OR

(a) With the help of a diagram. Explain the principle and working of a moving coil galvanometer.

(b) Two moving coil galvanometers M_1 and M_2 have the following particulars :

$R_1 = 10\ \Omega, N_1 = 30, A_1 = 3.6 \times 10^{-3}\ \text{m}^2,$

$B_1 = 0.25$ T

$R_2 = 14\ \Omega, N_2 = 42, A_2 = 1.8 \times 10^{-3}\ \text{m}^2,$

$B_2 = 0.50$ T.

The spring constant are identical for two springs. Determine ratio of (i) current sensitivity (ii) voltage sensitivity of M_2 and M_1.

25. Answer the following questions :

(a) Explain qualitatively on the basis of domain picture the irreversibility in the magnetisation curve of a ferromagnet.

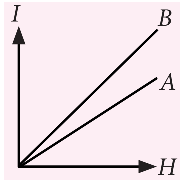
(b) The hysteresis loop of a soft iron piece has a much smaller area than that of a carbon steel piece. If the material is to go through repeated cycles of magnetisation, which piece will dissipate greater heat energy?

(c) A system displaying a hysteresis loop such as a ferromagnet, is a device for storing memory? Explain the meaning of this statement.

OR

(a) Define 'intensity of magnetisation' of a magnetic material. How does it vary with temperature for paramagnetic material?

(b) The given figure shows the variation of intensity of magnetisation I versus the applied magnetic field intensity H, for two magnetic materials A and B:



(i) Identify the materials A and B.

(ii) Why does the material B, have a larger susceptibility than A, for a given field at constant temperature?

26. (a) What is the relationship between the current and the magnetic moment of a current carrying circular loop? Use the expression to derive the relation between the magnetic moment of an electron moving in a circle and its related angular momentum?

(b) A muon is a particle that has the same charge as an electron but is 200 times heavier than it. If we had an atom in which the muon revolves around a proton instead of an electron, what would be the magnetic moment of the muon in the ground state of such an atom?

OR

(a) Magnetic field lines show the direction (at every point) which a small magnetised needle takes up (at that point). Do the magnetic field lines also represent the lines of force of a moving charged particle at every point?

(b) If magnetic monopoles existed, how would Gauss's law of magnetism be modified?

(c) Does a bar magnet exert a torque on itself due to its own field? Does one element of a current carrying wire exert a force on another element of the same wire?

(d) Magnetic field arises due to charges in motion. Can a system have magnetic moment even though its net charge is zero?

(e) Magnetic force is always normal to the velocity of a charge and therefore does no work. An iron nail held near a magnet, when released, increases its kinetic energy as it moves to cling to the magnet. What agency is responsible for this increase in kinetic energy if not the magnetic field?

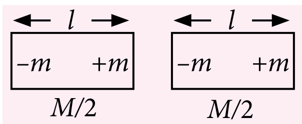
SOLUTIONS

1. We know that, work done, $dW = \vec{F}\cdot\vec{ds} = \vec{F}\cdot(\vec{v}\,dt)$
When a force does no work *ie*., $dW = 0$
$\vec{F}\cdot(\vec{v}\,dt) = 0 \Rightarrow \vec{F}\cdot\vec{v} = 0$
$\therefore$ $\vec{F}$ is perpendicular to $\vec{v}$.
If direction of $\vec{v}$ changes then direction of $\vec{F}$ should also change so that condition $\vec{F}\cdot\vec{v} = 0$ is satisfied.

2. As $B_H = B\cos\delta$ or $\cos\delta = B_H/B = 1/\sqrt{2}$
$\delta = 45°$.

3. Magnetic moment of proton, $\mu_p \approx \dfrac{e\hbar}{2m_p}$
Magnetic moment of electron, $\mu_e \approx \dfrac{e\hbar}{2m_e}$
$\therefore \quad \dfrac{\mu_e}{\mu_p} = \dfrac{m_p}{m_e} \approx 1837 >> 1$
or $\quad \mu_e >> \mu_p$

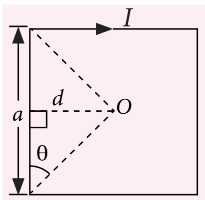
4. When a bar magnet of magnetic moment $(\vec{M} = m2\vec{l})$ is cut into two equal pieces transverse to length,



(a) the pole strength remains unchanged, since pole strength depends on number of atoms in cross-sectional area.
(b) the magnetic moment is reduced to half (since $M \propto$ length and here length is halved).

5. An electromagnet should be a strong but temporary magnet and for that its material should have: (i) high permeability and (ii) low retentivity. Soft iron possesses both the properties.

6. The magnetic fields at the centre due to the four sides will be equal in magnitude and direction. The field due to one side will be



$$B_1 = \frac{\mu_0 Ia}{2\pi d\sqrt{a^2+4d^2}}$$

Here, a = 10 cm and
$d = a/2 = 5$ cm
Thus,

$$B_I = \frac{\mu_0(2\text{A})}{2\pi(5\text{ cm})}\left[\frac{10\text{ cm}}{\sqrt{(10\text{ cm})^2+4(5\text{ cm})^2}}\right]$$

$= \quad 2\times10^{-7}\text{ T m A}^{-1}\times 2\text{ A}\times\dfrac{1}{5\sqrt{2}\times10^{-2}\text{ m}}$

$= \quad 5.66\times10^{-6}$ T

Hence, the net field at the centre of the loop will be $4\times5.66\times10^{-6}\text{ T} = 22.6\times10^{-6}$ T

7. The magnetic force $F = qvB$ acts normal to the direction of motion, thus provide the necessary centripetal force to follow the circular path.

$qvB = \dfrac{mv^2}{r}$

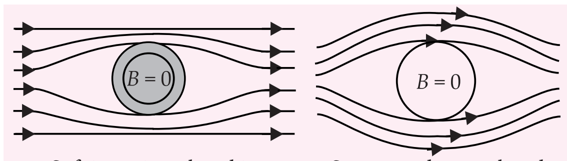
$r = \dfrac{mv}{qB} = \dfrac{9.1\times10^{-31}\times4.8\times10^{6}}{1.6\times10^{-19}\times6.5\times10^{-4}}$

or $\quad r = 4.2\times10^{-2} = 42$ mm

OR

(a) With usual notation, we are given that $N = 40$, $A = 100\text{ cm}^2 = 0.01\text{ m}^2$, $I = 10$ A, $B = 0.2$ T, $\theta = 90°$
Thus, $\tau = NIAB\sin\theta$
$= [40\times10\times0.01\times0.2\,(\sin 90°)]$N m $= 0.8$ N m
(b) In this case, $\theta = 0°$ and as such $\tau = 0$.

8. Magnetic screening or shielding is the phenomenon of protection of a region against any external magnetic effects. For example, when a soft iron ring is placed in a magnetic field, most of the lines are found to pass through the ring and no lines pass through the space inside the ring. Thus space inside the ring is shielded as shown in figure.



Soft iron ring placed in a magnetic field

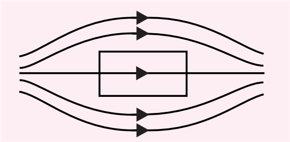
Super conductor placed in a magnetic field

Superconductors also provide perfect magnetic screening as no magnetic lines of force pass through the superconductor as shown in figure.

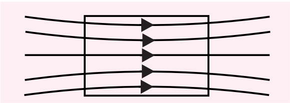
9. Magnetic field induction at O due to current through an arc CD is $\vec{B}_1 = \dfrac{\mu_0 I\alpha}{4\pi r_1}$ upwards.
Magnetic field induction due to current through an arc EF is $\vec{B}_2 = \dfrac{\mu_0 I\alpha}{4\pi r_2}$ downwards.
The current though DE and FC will not contribute the magnetic field induction at O. Therefore, total magnetic field induction at O due to current through the entire circuit $CDEF$ will be

$$\vec{B} = \vec{B}_1 - \vec{B}_2 = \frac{\mu_0 I\alpha}{4\pi}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \text{upwards.}$$

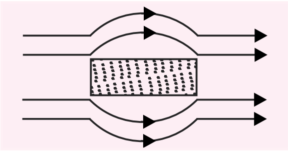
10. (a) Copper is diamagnetic.



(b) Aluminium is paramagnetic.



(c) Mercury cooled to low temperature (4.2 K) is a superconductor. It behaves as a perfect diamagnet, so no lines of force pass through it.



11. (i) $B = \sqrt{B_V^2 + B_H^2} = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$

$= k\sqrt{3\cos^2\theta + 1}$

$\left[\text{where } \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^3} = k \text{ and } \cos^2\theta + \sin^2\theta = 1\right]$

Now B is minimum when $\cos\theta$ = minimum = 0

or $\theta = \frac{\pi}{2}$

which corresponds to magnetic equator. Thus, magnetic equator is the locus of points for which B is minimum.

(ii) If δ denotes dip angle, then

$\tan\delta = \frac{B_V}{B_H} = \frac{2\cos\theta}{\sin\theta} = \frac{2}{\tan\theta}$

Now $\delta = 0°$ if $\tan\theta = \infty$ or $\theta = \frac{\pi}{2}$

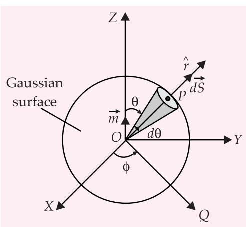
Thus, it is again the magnetic equator which is the locus of points with zero dip.

(iii) When $\delta = \pm 45°, \left|\frac{B_V}{B_H}\right| = 1$

or $\frac{2}{\tan\theta} = 1$ or $\tan\theta = 2$

or $\theta = \tan^{-1}(2)$ which is the required locus of points for which $\delta = \pm 45°$.

12. Consider a Gaussian surface of radius r around a point dipole of dipole moment $\vec{m}$, pointing along Z-axis.



$\therefore \quad \vec{m} = m\hat{k}$

Let $d\vec{s}$ be the surface area of an element around P.

Magnetic field at P along $\hat{r}$ due to point dipole, *ie* ., $\vec{B}_r = \frac{\mu_0}{4\pi}\left(\frac{3m\cos\theta}{r^3}\right)\hat{r}$

Also, $d\vec{s} = (r^2\sin\theta\, d\theta\, d\phi)\,\hat{r}$.

Thus, $\oint \vec{B}\cdot d\vec{s} = \oint \vec{B}_r \cdot d\vec{s}$

$= \oint \frac{\mu_0}{4\pi}\left(\frac{3m\cos\theta}{r^3}\right)\hat{r}\cdot(r^2\sin\theta\, d\theta\, d\phi)\,\hat{r}$

$= \frac{3\mu_0 m}{4\pi r}\int_0^{\pi}\sin\theta\cos\theta\, d\theta\, d\phi = 0$

$\left(\text{as } \oint_0^{\pi}\sin\theta\cos\theta\, d\theta = 0\right)$

13. Since $2\pi r = \left(\frac{\pi}{2}\right)$ m, $r = 0.25$ m

Magnetic field due to straight wire at a distance $a = 0.01r = 0.01 \times 0.25$ m $= 2.5 \times 10^{-3}$ m, *i.e.*,

$B = k_m\frac{2I}{a} = \left(10^{-7}\frac{\text{T m}}{\text{A}}\right)\left(\frac{2\times 5\text{ A}}{2.5\times 10^{-3}\text{ m}}\right)$

$= 4 \times 10^{-4}$ T

Magnetic field at the centre of the circular loop,

$B = k_m\left(\frac{2\pi I}{r}\right) = \left(10^{-7}\frac{\text{T m}}{\text{A}}\right)\left(\frac{2\times 3.14\times 5\text{ A}}{0.25\text{ m}}\right)$

$= 1.256 \times 10^{-5}$ T

14. Here; $a = 0.1$ m, $N = 20$, $r = 0.1$ m, $I = 0.5$ A

Magnetic field at the center of each coil due to its own current is

$B_1 = \frac{\mu_0 NI}{2a} = \frac{4\pi\times 10^{-7}\times 20\times 0.5}{2\times 0.1} = 6.28\times 10^{-5}\text{T}$

Magnetic field at the centre of one coil due to the current in the other coil is

$B_2 = \frac{\mu_0 NI r^2}{2(a^2+r^2)^{3/2}}$

$= \frac{4\pi\times 10^{-7}\times 20\times 0.5\times (0.1)^2}{2[(0.1)^2+(0.1)^2]^{3/2}} = 2.22 \times 10^{-5}$ T

(i) When the currents are in the same direction, the resultant field at the centre of each coil is

$B = B_1 + B_2 = 6.28 \times 10^{-5} + 2.22 \times 10^{-5}$

$= 8.50 \times 10^{-5}$ T

(ii) When the currents are in opposite directions, the resultant field is

$B = B_1 - B_2 = 6.28 \times 10^{-5} - 2.22 \times 10^{-5}$

$= 4.06 \times 10^{-5}$ T

OR

For solenoid, $l = 0.4$ m, $N_1 = 500$, $I_1 = 3$ A

For coil, $N_2 = 10$, $r = 0.01$ m, $I_2 = 0.4$ A

Field inside the solenoid,

$B = \frac{\mu_0 N_1 I_1}{l}$ along the axis of solenoid.

Magnetic moment of coil,
$m = N_2 I_2 A = N_2 I_2 \pi r^2$, along the axis of coil.
As the axis of the coil is perpendicular to the axis of solenoid, $\vec{m}$ and $\vec{B}$ will be perpendicular to each other.

Required torque,
$\tau = m B \sin\theta = N_2 I_2 \pi r^2 . \frac{\mu_0 N_1 I_1}{l} . \sin 90°$

$= 10 \times 0.4 \times \pi \times (0.01)^2 \times \frac{4\pi \times 10^{-7} \times 500 \times 3}{0.4} \times 1$

$= 6\pi^2 \times 10^{-7} = 6 \times 9.87 = 5.92 \times 10^{-6}$ N m

15. The moment of inertia of a bar magnet of mass m, length l about an axis passing through its centre and perpendicular to its length is $I = \frac{ml^2}{12}$. Let M be the magnetic moment of the magnet, B is the uniform magnetic field in which the magnet is oscillating, then time period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

When magnet is cut into two equal pieces, perpendicular to length, then moment of inertia of each piece of magnet about an axis perpendicular to length passing through its centre is

$$I' = \frac{m}{2}\frac{(l/2)^2}{12} = \frac{ml^2}{12} \times \frac{1}{8} = \frac{I}{8}$$

Magnetic dipole moment, $M' = M/2$

Its time period of oscillation is

$$T' = 2\pi\sqrt{\frac{I'}{M'B}} = 2\pi\sqrt{\frac{I/8}{(M/2)B}} = \frac{2\pi}{2}\sqrt{\frac{I}{MB}} = \frac{T}{2}$$

16. With shunt, current required to produce full scale deflection is given by

$$I_g = \frac{R_s}{R_g + R_s} \cdot I$$

$$\therefore \quad I_g = \frac{5 \times 250 \times 10^{-3}}{R_g + 5} \qquad \text{...(i)}$$

When a resistance R is connected in series,

$$I_g = \frac{V}{R_g + R} = \frac{25}{R_g + 1050} \qquad \text{...(ii)}$$

From equations (i) and (ii), $\frac{1.25}{R_g + 5} = \frac{25}{R_g + 1050}$

or $1.25\,R_g + 1312.5 = 25\,R_g + 125$ or $23.75\,R_g = 1187.5$

or $R_g = \frac{1187.5}{23.75} = 50\ \Omega$

17. (a) We know that two parallel conductors carrying currents, in the same direction attract each other and in opposite directions repel each other. Therefore, when current is passed through the coil of a solenoid, the parallel currents in the various turns of solenoid flow in the same direction. As a result, the various turns start attracting one another and solenoid tends to contract.

(b) Number of turns, $N = 850 \times 5$

Area of cross section,

$$A = \pi r^2 = \frac{22}{7}\left(\frac{3}{2} \times 10^{-2}\right)^2 \text{ m}^2;$$

$l = 1$ m, $I = 5$ A

Magnetic field induction at the centre of solenoid is
$B = \mu_0 NI/l = 4\pi \times 10^{-7} \times (850 \times 5)\ 5/1 = 2.671 \times 10^{-2}$ T

$$\text{Magnetic flux} = BA = 2.671 \times 10^{-2} \times \frac{22}{7} \times \left(\frac{3}{2} \times 10^{-2}\right)^2$$

$$= 1.89 \times 10^{-5} \text{ Wb}$$

18.

S. No.	Biot Savart's law	Ampere's circuital law
1.	This law is based on the principle of magnetism.	This law is based on the principle of electromagnetism.
2.	This law is valid for asymmetrical current distribution.	This law is valid for symmetrical current distributions.
3.	This law is the differential form of magnetic induction $\vec{B}$, *i.e.*, $\lvert d\vec{B}\rvert = \frac{\mu_0 I dl \sin\theta}{4\pi r^2}$	This law is the integral form of $\vec{B}$, *i.e.*, $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

19. Let $\vec{E}$ and $\vec{B}$ denote the electric field and magnetic field respectively. If $\vec{v}$ is the velocity of the particles to be selected, force acting on a charged particle due to electric field, $\vec{F}_e = q\vec{E}$ and the force acting on a charged particle due to magnetic field, $\vec{F}_m = q(\vec{v} \times \vec{B})$.

For the particles with velocity $\vec{v}$ to remain undeflected, $\vec{F}_e + \vec{F}_m = \vec{0}$

or $\vec{F}_e = -\vec{F}_m$

or $q\vec{E} = -q(\vec{v} \times \vec{B})$

or $\vec{E} = -(\vec{v} \times \vec{B}) = (\vec{B} \times \vec{v})$

Thus : (i) The direction of $\vec{E}$ should be that of $(\vec{B} \times \vec{v})$, q is being taken as positive.

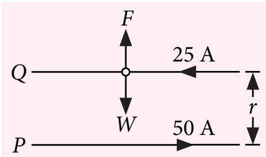
(ii) $|\vec{E}| = |\vec{v} \times \vec{B}|$.

These are the required conditions.

For the particles with velocities higher than $\vec{v}, (\vec{v} \times \vec{B})$ dominates and hence the particles get deflected towards the direction of $-\vec{E}$. But particles with velocities less than $\vec{v}$, get deflected in the direction of $+\vec{E}$.

20. The magnetic force per unit length on the wire Q due to the current in wire P is $F = \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2}{r}$

The currents in P and Q must have opposite directions, only then Q will experience a repulsive force which would balance the weight of Q.



$$\therefore \quad F = \frac{\mu_0 I_1 I_2}{2\pi r} \Rightarrow W = \frac{\mu_0 I_1 I_2}{2\pi r}$$

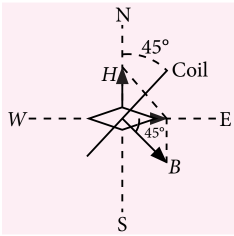
$$\text{or} \quad r = \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2}{W} = \frac{2 \times 10^{-7} \times 50 \times 25}{0.075}$$

$= 3.33 \times 10^{-3}$ m $= 3.33$ mm

21. Here, $N = 30$, $r = 12$ cm $= 12 \times 10^{-2}$ m

$I = 0.35$ A, $H = ?$

It is clear from figure the needle can point west to east only when $H = B \sin 45°$, where, B = magnetic field strength due to current in coil



$$B = \frac{\mu_0}{4\pi} \frac{2\pi NI}{r}$$

$$\therefore \quad H = \frac{\mu_0}{4\pi} \frac{2\pi NI}{r} \sin 45° = 10^{-7} \times \frac{2\pi \times 30 \times 0.35}{12 \times 10^{-2}} \cdot \frac{1}{\sqrt{2}}$$

$= 3.9 \times 10^{-5}$ T

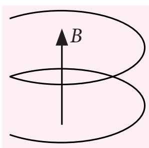
(b) When current in coil is reversed and coil is turned through 90° anticlockwise, the direction of needle will reverse (*i.e.*, it will point from east to west).

22. The components of the proton's velocity along and perpendicular to the magnetic field are

$v_{||} = (4 \times 10^5 \text{ m s}^{-1}) \cos 60° = 2 \times 10^5 \text{ m s}^{-1}$

and $v_{\perp} = (4 \times 10^5 \text{ m s}^{-1}) \sin 60° = 2\sqrt{3} \times 10^5 \text{ m s}^{-1}$

As the force $q\vec{v} \times \vec{B}$ is perpendicular to the magnetic field, the component $v_{||}$ will remain constant. In the plane perpendicular to the field, the proton will describe a circle whose radius is obtained from the equation



$$qv_{\perp} B = \frac{mv_{\perp}^2}{r}$$

$$\text{or} \quad r = \frac{mv_{\perp}}{qB} = \frac{(1.67 \times 10^{-27} \text{ kg})(2\sqrt{3} \times 10^5 \text{ m s}^{-1})}{(1.6 \times 10^{-19} \text{ C}) \times (0.3 \text{ T})}$$

≈ 0.012 m $= 1.2$ cm

The time taken in one complete revolution in the plane perpendicular to B is

$$T = \frac{2\pi r}{v_{\perp}} = \frac{2 \times 3.14 \times 0.012 \text{ m}}{2\sqrt{3} \times 10^5 \text{ m s}^{-1}}$$

The distance moved along the field during this period, *i.e.*,

$$\text{the pitch} = \frac{(2 \times 10^5 \text{ m s}^{-1}) \times 2 \times 3.14 \times 0.012 \text{ m}}{2\sqrt{3} \times 10^5 \text{ m s}^{-1}}$$

$= 0.044$ m $= 4.4$ cm

The qualitative nature of the path of the protons is shown in figure.

23. (a) Manak had shown curiosity, research oriented mind, ambitious, maturity. His Physics teacher has also shown helping nature, ability to implement theory into practical , involvement with student and readiness to motivate student.

(b) Galvanometer was converted into ammeter by connecting suitable small resistance in parallel and into a voltmeter by connecting suitable high resistance in series.

(c) An ammeter shows the reading less than actual as some current passes through shunt. Similarly, a voltmeter shows less reading as some potential drops across high resistance connected in series.

24. Refer to point 3.3 (9), page no. 174 (MTG Excel in Physics)

OR

(a) Refer to point 3.4 (2), page no. 175 (MTG Excel in Physics)

(b) Current sensitivity of a moving coil galvanometer is defined as $C.S. = \frac{\phi}{I} = \frac{NAB}{k}$ and voltage sensitivity, $V.S. = \frac{NAB}{kR}$

(i) Ratio of current sensitivity,

$$\frac{C.S._1}{C.S._2} = \frac{N_1 B_1 A_1 k_2}{k_1 N_2 B_2 A_2}$$

$$\frac{C.S._1}{C.S._2} = \frac{30 \times 0.25 \times 3.6 \times 10^{-3} \times k}{42 \times 0.5 \times 1.8 \times 10^{-3} \times k} = \frac{5}{7}$$

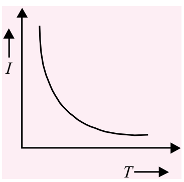
(ii) Ratio of voltage sensitivity,

$$\frac{V.S._1}{V.S._2} = \frac{C.S_1}{C.S_2} \times \frac{R_2}{R_1} = \frac{5}{7} \times \frac{14}{10} = \frac{1}{1}$$

25. (a) In a specimen of a ferromagnets, the atomic dipoles are grouped together in domains. All the dipoles of a domain are aligned in the same direction and have net magnetic moment. In an unmagnetised substance these domains are randomly distributed so that the resultant magnetisation is zero. When the substance is placed in an external magnetic field, these domains align themselves in the direction of the field. Some energy is spent in the process of alignment. When the external field is removed, these domains do not come back into their random positions completely. The substance retains some magnetisation. The energy spent in the process of magnetisation is not fully recovered. The balance of energy is lost as heat. This is the basic cause for irreversibility of the magnetisation curve of a ferromagnetic substance.
(b) Carbon steel piece because the heat produced in complete cycle of magnetisation is directly proportional to the area under the hysteresis loop.
(c) Magnetisation of a ferromagnet is not a single valued function of the magnetising field. Its value for a particular field depends both on the magnetising field and on the history of its magnetisation *i.e.*, how many cycles of magnetisation it has gone through etc. So, the value of magnetisation is a record or memory of its cycles of magnetisation. If information bits can be made to correspond to these cycles, the system displaying such a hysteresis loop can act as a device for storing information.

OR

(a) Refer to point 3.8 (2) page no. 180 (MTG Excel in Physics)
In paramagnetic, intensity of magnetisation I varies inversely with temperature of the given sample.



(b) (i) A is paramagnetic and B is ferromagnetic.
(ii) Material B has larger susceptibility than A, because in ferromagnetic the domains easily orient themselves along the direction of magnetising field and hence, it can be easily magnetised.

26. (a) Magnetic moment of current carrying loop is $M = IA$. For an electron revolving in a circular orbit of radius r with speed v, effective current is

$$I = \upsilon e = \frac{ve}{2\pi r}$$

$\therefore$ Associated magnetic moment of electron revolving in orbit is

$$M = IA = \frac{ve}{2\pi r} \times \pi r^2 \text{ or } M = \frac{evr}{2}$$

Related angular momentum is $\vec{L} = \vec{r} \times \vec{p}$.
$L = rp \sin 90° = rmv \times 1$ or $L = mvr$

$$\Rightarrow L = m \times \frac{2M}{e} \qquad \left(\because vr = \frac{2M}{e}\right)$$

$$\Rightarrow L = \frac{2m}{e} M$$

(b) By Bohr's condition of quantisation, $mvr = \dfrac{nh}{2\pi}$
For ground state $n = 1$, so

$$mvr = \frac{h}{2\pi} \text{ or } vr = \frac{h}{2\pi m}$$

$$\text{or } \frac{evr}{2} = \frac{eh}{4\pi m} \text{ or } M = \frac{eh}{4\pi m}$$

$$\text{For muon, } M = \frac{1.6 \times 10^{-19}\,\text{C} \times 6.6 \times 10^{-34}\,\text{J s}}{4 \times 3.14 \times 200 \times 9.1 \times 10^{-31}\,\text{kg}}$$

or $M = 4.6 \times 10^{-26}$ A m^2

OR

(a) Force on a charge q moving with a velocity $\vec{v}$ in a uniform magnetic field of strength $\vec{B}$ is $\vec{F} = q(\vec{v} \times \vec{B})$
$\therefore$ Magnetic force is always normal to $\vec{B}$.
Magnetic field lines of B cannot represent the lines of force of moving charged particle.
(b) According to Gauss's law in magnetism, magnetic flux over any closed surface is always zero.

$$\oint_S \vec{B} \times d\vec{S} = 0$$

If monopoles existed, the magnetic flux would no longer be zero, but equal to μ_0 times the pole strength enclosed by the surface.
(c) No, there is no force or torque on an element due to the field produced by that element itself. But there is a force (or torque) on an element of the same wire. However, for the special case of a straight wire, this force is zero.
(d) Yes, a system can have magnetic moment even if its net charge is zero. For example, every atom of paramagnetic and ferromagnetic materials has a magnetic moment, though every atom is electrically neutral. Again, a neutron has no charge, but it does have some magnetic moment.
(e) An iron nail is made up of a large number of atoms, in which so many electronic charges are in motion. All these charges in motion experience a magnetic force when held near a magnet. The magnetic forces do not change speed of the charges, but they do change their velocity. The velocity of centre of mass may increase at the expense of nail's internal energy. Thus, internal energy of the nail is responsible for increase in kinetic energy of the nail, as a whole.

MPP-4 MONTHLY Practice Problems

Class XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.

Self check table given at the end will help you to check your readiness.

Magnetic Effects of Current and Magnetism

Total Marks : 120 | **Time Taken : 60 min**

NEET / AIIMS

Only One Option Correct Type

1. The charge on a particle Y is double the charge on another particle X. These two particles X and Y, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2 respectively. The ratio of the mass of X to that of Y is

   (a) $\left(\frac{2R_1}{R_2}\right)^2$ (b) $\left(\frac{R_1}{2R_2}\right)^2$

   (c) $\frac{1}{2}\left(\frac{R_1}{R_2}\right)^2$ (d) $2\left(\frac{R_1}{R_2}\right)^2$

2. A triangular loop of side l carries a current I. It is placed in a magnetic field B such that the plane of the loop is in the direction of B. The torque on the loop is

   (a) IBl (b) I^2Bl (c) $\frac{\sqrt{3}}{4}BIl^2$ (d) infinity

3. Temperature above which a ferromagnetic substance becomes paramagnetic is called

   (a) Critical temperature (b) Boyle temperature
   (c) Debye's temperature (d) Curie temperature.

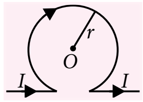
4. A solenoid has a core of a material with relative permeability 400. The windings of the solenoid are insulated from the core and carry a current of 2 A. If the number of turns is 1000 per metre, the magnetisation at centre of solenoid is

   (a) 4×10^5 A m^{-1} (b) 6×10^5 A m^{-1}
   (c) 7×10^5 A m^{-1} (d) 8×10^5 A m^{-1}

5. A short magnet of moment 6.75 A m^2 produces a neutral point on its axis. If horizontal component of earth's magnetic field is 5×10^{-5} Wb m^{-2}, then the distance of the neutral point should be

   (a) 10 cm (b) 20 cm (c) 30 cm (d) 40 cm

6. An infinitely long straight conductor is bent into the shape as shown in the figure. It carries a current I and the radius of the circular loop is r. Then the magnetic induction at its centre will be



   (a) $\frac{\mu_0}{4\pi}\frac{2I}{r}(\pi+1)$ (b) $\frac{\mu_0}{4\pi}\frac{2I}{r}(\pi-1)$

   (c) zero (d) infinite

7. A frog can be levitated in a magnetic field produced by a current in a vertical solenoid placed below the frog. This is possible because the body of the frog behaves as

   (a) paramagnetic (b) diamagnetic
   (c) ferromagnetic (d) antiferromagnetic.

8. An electron experiences a force $\vec{F} = (4.0\hat{i} + 3.0\hat{j}) \times 10^{-13}$ N in a uniform magnetic field when its velocity is $(2.5 \times 10^7)\hat{k}$ m s^{-1}. When the velocity is redirected and becomes $(1.5\hat{i} - 2.0\hat{j}) \times 10^7$ m s^{-1}, the magnetic force on the electron is zero. The magnetic field vector $\vec{B}$ is

   (a) $(0.1\hat{i} - 0.075\hat{j})$ (b) $(0.1\hat{i} + 0.075\hat{j} + \hat{k})$
   (c) $(0.075\hat{i} - 0.1\hat{j})$ (d) $(0.1\hat{i} - 0.075\hat{j} + \hat{k})$.

9. A particle of mass m and charge q is projected from origin with initial velocity $(u\hat{i} - v\hat{j})$. Uniform electric and magnetic field exist in the region along $+y$ direction of magnitudes E and B, respectively. Then particle will definitively return to the origin if

   (a) $\frac{vB}{2\pi E}$ is an integer

   (b) $\frac{(u^2+v^2)^{1/2}}{[B/\pi E]}$ is an integer

(c) $\frac{vB}{\pi E}$ is an integer (d) $\frac{vB}{3\pi E}$ is an integer

10. A proton of mass 1.67×10^{-27} kg and charge 1.6×10^{-19} C is projected with a speed of 2×10^6 m s^{-1} at an angle of 60° to the x-axis. If a uniform magnetic field of 0.104 T is applied along y-axis. The path of proton is

(a) a circle of radius = 0.2 m and time period $\pi \times 10^{-7}$ s
(b) a circle of radius = 0.1 m and time period $2\pi \times 10^{-7}$ s
(c) a helix of radius = 0.1 m and time period $2\pi \times 10^{-7}$ s
(d) a helix of radius = 0.2 m and time period $4\pi \times 10^{-7}$ s

11. A circular current carrying loop of a radius R, carries a current I. The magnetic field at a point on the axis of coil is $\frac{1}{\sqrt{8}}$ times the value of magnetic field at the centre. Distance of point from centre is

(a) $\frac{R}{\sqrt{2}}$ (b) $\frac{R}{\sqrt{3}}$ (c) $R\sqrt{2}$ (d) R

12. A cyclotron's oscillator frequency is 10 MHz. If the radius of the dees is 60 cm, the kinetic energy of the proton beam produced by the accelerator is
($e = 1.6 \times 10^{-19}$ C, $m_P = 1.6 \times 10^{-27}$ kg)

(a) 5 MeV (b) 6 MeV (c) 7 MeV (d) 8 MeV

Assertion & Reason Type

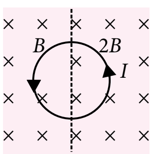
Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :

(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. **Assertion :** A rectangular current loop is in an arbitrary orientation in an external uniform magnetic field. No work is required to rotate the loop about an axis perpendicular to its plane.
Reason : All positions represent the same level of energy.

14. **Assertion :** When radius of a circular loop carrying current is doubled, its magnetic moment becomes four times.
Reason : Magnetic moment depends on the area of the loop.

15. **Assertion :** The magnetic force on the closed loop in given figure is zero.



Reason : Force (magnetic) on the wire is $\int d\vec{F} = \int \vec{B} \times \vec{dl}$

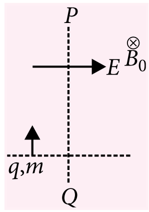
JEE MAIN / JEE ADVANCED

Only One Option Correct Type

16. A particle of charge per unit mass α is released from origin with a velocity $\vec{v} = v_0\hat{i}$ in a uniform magnetic $\vec{B} = -B_0\hat{k}$. If the particle passes through $(0, y, 0)$, then y is equal to

(a) $-\frac{2v_0}{B_0\alpha}$ (b) $\frac{v_0}{B_0\alpha}$ (c) $\frac{2v_0}{B_0\alpha}$ (d) $-\frac{v_0}{B_0\alpha}$

17. A uniform electric field E is present horizontally along the paper throughout the region but uniform magnetic field B_0 is present horizontally (perpendicular to plane of paper in inward direction) right to the line PQ as shown. A charge particle having charge q and mass m is projected vertically upward and crossed the line PQ after time t_0. Find the speed of projection, if particle moves with constant velocity after t_0 (given $qE = mg$).

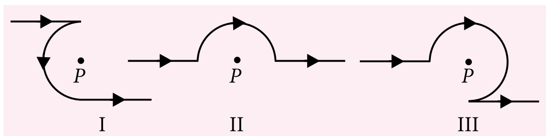


(a) gt_0 (b) $2gt_0$ (c) $\frac{gt_0}{2}$
(d) particle can't move with constant velocity after crossing PQ

18. A particle of specific charge $\frac{q}{m} = \pi$ C kg^{-1} is projected from the origin toward positive x-axis with a velocity of 10 m s^{-1} in a uniform magnetic field $\vec{B} = -2\hat{k}$ T. The velocity $\vec{v}$ of particle after time $t = \frac{1}{12}$ s will be (in m s^{-1})

(a) $5(\hat{i} + \sqrt{3}\hat{j})$ (b) $5(\sqrt{3}\hat{i} + \hat{j})$
(c) $5(\sqrt{3}\hat{i} - \hat{j})$ (d) $5(\hat{i} + \hat{j})$

19. The magnetic field B at the centre of a circular coil of radius r is π times that due to a long straight wire at a distance r from it, for equal currents. Figure here shows three cases. In all case the circular part has radius r and straight ones are infinitely long. For same current, the magnetic field B at the centre P in cases I, II, III have the ratio



(a) $\left(\frac{-\pi}{2}\right) : \frac{\pi}{2} : \left(\frac{3\pi}{4} - \frac{1}{2}\right)$

(b) $\left(\frac{-\pi}{2} + 1\right) : \left(\frac{\pi}{2} + 1\right) : \left(\frac{3\pi}{4} + \frac{1}{2}\right)$

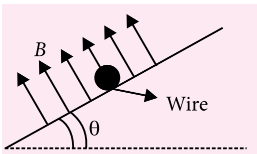
(c) $\frac{-\pi}{2} : \frac{\pi}{2} : \frac{3\pi}{4} +$

(d) $\left(\frac{-\pi}{2} - 1\right) : \left(\frac{\pi}{2} - \frac{1}{4}\right) : \left(\frac{3\pi}{4} + \frac{1}{2}\right).$

More than One Options Correct Type

20. A charged particle with velocity $\vec{v} = x\hat{i} + y\hat{j}$ moves in a magnetic field $\vec{B} = y\hat{i} + x\hat{j}$. The force acting on the particle has magnitude F. Which one of the following statements is/are correct?
(a) No force will act on charged particle if $x = y$
(b) If $x > y$, $F \propto (x^2 - y^2)$
(c) If $x > y$, the force will act along z-axis
(d) If $y > x$, the force will act along y-axis

21. A wire of mass m and length l is placed on a smooth incline making an angle θ with the horizontal, whose front view is shown in figure. When a finite amount of charge is passed through it for an infinitesimal time, the wire immediately acquires some velocity and then ascends the incline by a distance s. For this small duration, we can neglect the gravitational force because the current can be considered very large due to small time duration. The amount of charge passed through the wire is



(a) $\frac{m\sqrt{2gs\sin\theta}}{Bl}$ (b) $\frac{mv}{Bl}$

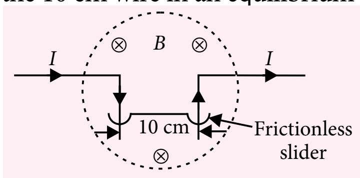
(c) $\frac{m\sqrt{2gs\sin\theta}}{Bl\cos\theta}$ (d) information insufficient

22. A charged particle of specific charge α moves with a velocity $\vec{v} = v_0\hat{i}$ in a magnetic field $\vec{B} = \frac{B_0}{\sqrt{2}}(\hat{j} + \hat{k})$.
Then (specific charge = charge per unit mass)
(a) path of the particle is a helix
(b) path of the particle is circle
(c) distance moved by the particle in time $t = \frac{\pi}{B_0\alpha}$ is $\frac{\pi v_0}{B_0\alpha}$
(d) velocity of the particle after time $t = \frac{\pi}{B_0\alpha}$ is $\left(\frac{v_0}{2}i + \frac{v_0}{2}\hat{j}\right)$

23. A charged particle is fired at an angle θ to a uniform magnetic field directed along the x-axis. During its motion along a helical path, the particle will
(a) never move parallel to the x-axis
(b) move parallel to the x-axis once during every rotation for all value of θ
(c) move parallel to the x-axis at least once during every rotation if $\theta = 45°$
(d) never move perpendicular to the x-direction

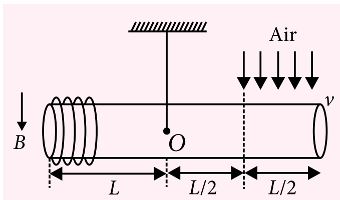
Integer Answer Type

24. A 10 cm length of wire with a mass of 20 g is attached frictionlessly to the vertical segments of a wire in which a current I flows. The surrounding has uniform horizontal field $B = 10^4$ G and the direction is shown in figure. What must be the current I (in A) to maintain the 10 cm wire in an equilibrium position?



25. A non-conducting non-magnetic rod having circular cross-section of radius R is suspended from a rigid support as shown in figure. A light and small coil of 300 turns is wrapped tightly at the left end of the rod where uniform magnetic field B exists in vertically downward direction. Air of density ρ hits one half of the right part of the rod with velocity v as shown in figure. The current must be in clockwise direction (as seen from O) in the coil so that rod remains horizontal. The magnitude of the current will be $x \times 10^{-2x}$ A. Value of x will be

$\left(\text{Given, } \frac{2}{Lv}\sqrt{\frac{\pi RB}{\rho}} = 1\,\text{A}^{-1/2}\right)$



26. Three infinitely long thin wires, each carrying current I in the same direction, are in the x-y plane of a gravity free space. The central wire is along the y-axis while the other two are along $x = \pm d$. The locus of the points for which the magnetic field B is zero is $x = \pm\frac{d}{\sqrt{\alpha}}$. The value of α is

Comprehension Type

In a certain region of space, there exists a uniform and constant electric field of magnitude E along the positive y-axis of a coordinate system. A charged particle of mass m and charge $-q(q > 0)$ is projected from the origin with speed $2v$ at an angle of 60° with the positive x-axis in x-y plane. When the x-coordinate of particle becomes $\frac{\sqrt{3}mv^2}{qE}$ a uniform and constant magnetic field of strength B is also switched on along positive y-axis.

27. Velocity of the particle just before the magnetic field is switched on is

(a) $v\hat{i}$ (b) $v\hat{i}+\frac{\sqrt{3}v}{2}\hat{j}$

(c) $v\hat{i}-\frac{\sqrt{3}v}{2}\hat{j}$ (d) $2v\hat{i}-\frac{\sqrt{3}v}{2}\hat{j}$

28. z-coordinate of the particle as a function of time after the magnetic field is switched on is

(a) $\frac{mv}{qB}\left[1-\cos\left(\frac{qB}{m}t\right)\right]$ (b) $-\frac{mv}{qB}\left[1+\cos\left(\frac{qB}{m}t\right)\right]$

(c) $-\frac{mv}{qB}\left[1-\cos\left(\frac{qB}{m}t\right)\right]$ (d) $\frac{mv}{qB}\left[1+\cos\left(\frac{qB}{m}t\right)\right]$

Matrix Match Type

29. A charged particle with some initial velocity is projected in a region where non-zero electric and/or magnetic fields are present. In Column I, information about the existence of electric and/or magnetic field and direction of initial velocity of charged particle are given, while in Column II the probable path of the charged particle is mentioned. Match Column I with Column II.

Column I	Column II
(A) $\vec{E}=0, \vec{B}\neq 0$, and initial velocity may be at any angle with $\vec{B}$	(P) Straight line
(B) $\vec{E}\neq 0, \vec{B}=0$ and initial velocity may be at any angle with $\vec{E}$	(Q) Parabola
(C) $\vec{E}\neq 0, \vec{B}\neq 0, \vec{E}\,\|\,\vec{B}$ and initial velocity is perpendicular to both	(R) Circular
(D) $\vec{E}\neq 0, \vec{B}\neq 0, \vec{E}$ is perpendicular to $\vec{B}$ and non-uniform pitch $\vec{v}$ perpendicular to both $\vec{E}$ and $\vec{B}$	(S) Helical path

	A	B	C	D
(a)	P, R	P, Q	S	P
(b)	P, Q	P, R	S	R
(c)	P, S	Q	R	S
(d)	P, S	R	Q	S

30. A charged particle passes through a region that could have electric field only or magnetic field only or both electric and magnetic fields or none of the field. Match Column I with Column II.

Column I	Column II
(A) Kinetic energy of the particle remains constant	(P) Under special conditions, this is possible when both electric and magnetic fields are present
(B) Acceleration of the particle is zero	(Q) The region has electric field only
(C) Kinetic energy of the particle changes and it also suffers deflection	(R) The region has magnetic field only
(D) Kinetic energy of the particle changes but it suffer no deflection	(S) The region contains no field

	A	B	C	D
(a)	P	Q	R, S	P, Q
(b)	P, R, S	P, S	P, Q	P, Q
(c)	P, R, S	Q, R	S	P
(d)	Q, R	P	S	R

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

PHYSICS MUSING

Physics Musing was started in August 2013 issue of Physics For You. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / NEET / AIIMS / JIPMER with additional study material.
In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / NEET. The detailed solutions of these problems will be published in next issue of Physics For You.
The readers who have solved five or more problems may send their detailed solutions with their names and complete address. The names of those who send atleast five correct solutions will be published in the next issue.
We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

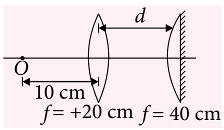
PROBLEM Set 49

INTEGER TYPE

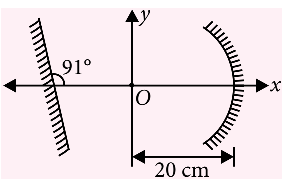
1. In a slow reaction, heat is being evolved at a rate about 10 mW in a liquid. If the heat were being generated by the decay of ^{32}P, a radioactive isotope of phosphorus that has half-life of 14 days and emits only beta-particles with a mean energy of 700 keV, estimate the number of ^{32}P atoms in the liquid. Express your answer in the form of $A \times 26 \times 10^{15}$ and find A to nearest integer.

[Take : ln 2 = 0.7]

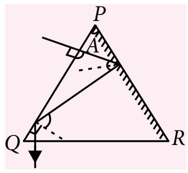
2. A convex lens of focal length 20 cm and another plano convex lens of focal length 40 cm are placed co-axially as shown in figure. The plano convex lens is silvered on plane surface. What should be the distance d (in dm) so that final image of the object O is formed on O itself.



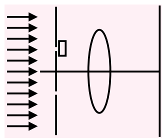
3. A point object is placed at the centre of curvature of a concave mirror (taken as origin). A plane mirror is also placed at a distance of 10 cm from the object as shown in figure. Consider two reflection first at plane mirror and then at concave mirror. The coordinates of the image thus formed are (x_0, y_0). Then find $\dfrac{\pi x_0}{20 y_0}$



4. The principal section of glass prism is an isosceles ΔPQR with $PQ = PR$. The face PR is silvered. A ray is incident perpendicularly on face PQ and after two reflections it emerges from base QR, normal to it as shown in figure. The angle (in degree) of the prism is $9x$. Find x.



5. The intensity received at the focus of the lens is I when no glass slab has been placed in front of the slit. Both the slits are of the same dimension and the plane wavefront incident perpendicularly on them, has wavelength λ. On placing the glass slab, the intensity reduces to $3I/4$ at the focus. The minimum thickness of the glass slab is $(2310 + x)$Å if its refractive index is 3/2. Given λ = 6933Å.



6. A sound source is located somewhere along the x-axis. Experiments show that the same wavefront simultaneously reaches listeners at x = –8 m and x = +2.0 m. A third listener is positioned along the positive y-axis. What is y-coordinate (in m) of third listener if the same wavefront reaches her at the same instant as it does the first two listeners ?

Solution Senders of Physics Musing

SET-48

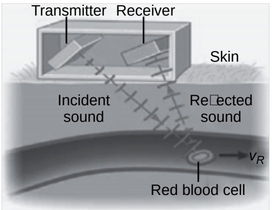
1. Nikita Verma, Kanpur (U.P.)
2. Prashant Pandey, Nagpur (Maharashtra)
3. Saumya Vyas, Surat (Gujarat)

By **Akhil Tewari,** Author Foundation of Physics for JEE Main & Advanced, Professor, IITians PACE, Mumbai.

COMPREHENSION TYPE

For questions 7, 8 and 9 :

The Doppler flow meter is a particularly interesting medical application of the Doppler's effect. This device measures the speed of blood flow, using transmitting and receiving elements that are placed directly on the skin, as shown in figure. The transmitter emits a continuous sound whose frequency is typically about 5 MHz. When the sound is reflected from the red blood cells, its frequency is changed in a kind of Doppler effect because the cells are moving with the same velocity as the blood. The receiving element detects the reflected sound and an electronic counter measures its frequency, which is Doppler-shifted relative to the transmitter frequency. From the change in frequency the speed of the blood flow can be determined. Typically, the change in frequency is around 600 Hz for flow speeds of about 0.1 m s^{-1}.



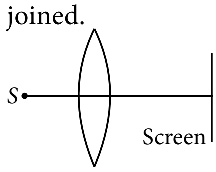
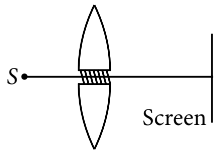
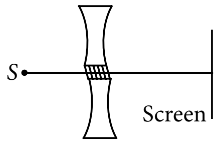
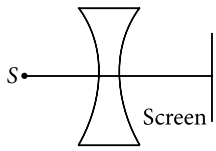
7. Assume that the red blood cell is directly moving away from the source and the receiver. What is the (approx.) speed of the sound wave in the blood?
   (a) 1700 m s^{-1} (b) 330 m s^{-1}
   (c) 5000 m s^{-1} (d) 3000 m s^{-1}

8. An abnormal segment of the artery is narrowed down by an arteriosclerotic plaque to one-fourth the normal cross-sectional area. What will be the change in frequency (approx.) due to reflection from the red blood cell in that region?
   (a) 150 Hz (b) 300 Hz (c) 600 Hz (d) 2400 Hz

9. At what extra rate does the heart have to work due to this narrowing down of the artery? Assume the density to be 1.5 g cm^{-3} and the area of cross-section of the normal artery to be 0.1 cm^2.
   (a) 1.125×10^{-4} W (b) 2.5×10^{-4} W
   (c) 6.25×10^{-5} W (d) 5.625×10^{-5} W

MATRIX MATCH

10. Light from source S ($|u| < |f|$) falls on lens and screen is placed on the other side. The lens is formed by cutting it along principal axis into two equal parts and are joined as indicated in column II. Match the column I with column II.

Column I	Column II
(A) Plane of image move towards screen if $|f|$ is increased	(P) Small portion of each part near pole is removed. The remaining parts are joined. 
(B) Images formed will be virtual	(Q) The two parts are separated slightly. The gap is filled by opaque material. 
(C) Separation between images increase if $|u|$ decreases	(R) The two parts are separated slightly. The gap is filled by opaque material. 
(D) Interference pattern can be obtained if screen is suitably positioned.	(S) Small portion of each part near pole is removed. The remaining parts are joined. 

	A	B	C	D
(a)	P, Q	R, S	R, S	P, Q
(b)	P, Q	P, Q, R, S	R, S	P
(c)	Q, S	P, R, S	Q	P
(d)	P	Q, R	R	P, S

MPP-4 CLASS XII ANSWER KEY

1. (c)	2. (c)	3. (d)	4. (d)	5. (c)
6. (b)	7. (b)	8. (c)	9. (c)	10. (c)
11. (d)	12. (c)	13. (a)	14. (a)	15. (d)
16. (c)	17. (b)	18. (b)	19. (a)	20. (a,b,c)
21. (a,b)	22. (b,c)	23. (a,d)	24. (2)	25. (1)
26. (3)	27. (a)	28. (c)	29. (a)	30. (b)

Scientists teleport first-ever object to space Star Trek-style

A team of Chinese scientists has teleported a photon over 480 km from the Earth into a special receiver orbiting the planet.

We're many years away from sending people across galaxies Star Trek-style, but it still marks a huge leap in quantum mechanics. The researchers brought together several disciplines, including advanced physics and rocket science, for the experiment.

Using a process called quantum entanglement, they teleported the photon to a satellite Micius, which was launched last year. On board the satellite is a sensitive photon receiver capable of picking up the elementary particle when it is sent from Earth.

Quantum entanglement occurs when two objects are formed at the same instant and point in space—existing in two separate places at once. Although they are separated, the two objects are immediately influenced by each other regardless of the distance between them. This is known as "wave function". Thus, one photon in space and one photon on Earth can be created at the same time with a common quantum link. Scientists said this method could be used to transmit data associated with one object over to the other instantaneously . By altering the state of one they can alter the state of the other.

"Long-distance teleportation has been recognised as a fundamental element in protocols such as large-scale quantum networks and distributed quantum computation," the team said.

Robot lawyer can save you from parking fines

After helping overturn parking tickets worth $10 million in the UK and the US, the world's first robot lawyer is ready to offer free legal aid in 1,000 different areas, including tackling rogue landlords, harassment at work and credit card fraud, and getting a refund for malfunctioning electronics.

The online tool was developed by 20-year-old British student Joshua Browder, currently enrolled at Stanford University, two years ago after he received several parking tickets and had to fight them. According to tech website `Mashable', he said the episodes helped him witness how many lawyers "exploit human misery" for profit, especially impacting low-income people.

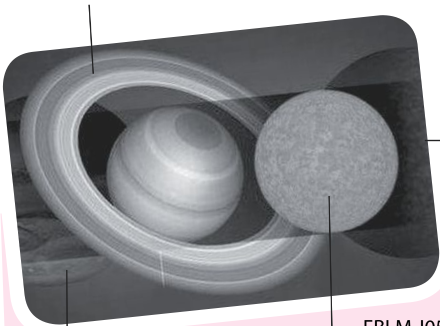
Explaining the tool's expansion, Browder said, "There's so much exploitation going on where landlords aren't behaving properly. I'm really excited about how it can help people."

To seek the robot lawyer's counsel and assistance, users need to log into `DoNotPay.com', where they will be asked to type in their problem. The website, which is currently only available in the US and the UK, then directs users to a chat bot that can solve their legal issue.

The bot can draft letters and offer advice on problems ranging from credit card fraud to airline compensation, British daily `The Telegraph' reported.

With problems around parental leave and harassment at work it provides options with different levels of formality. It can send a casual letter, an official one, and file a complaint to the regulator.

At present the robot lawyer can solve problems that involve a single document. Browder is planning to develop the tool so it can handle more complicated processes, but insists it will always be free.



Smallest-ever star discovered

Scientists have discovered the smallest known star in the universe — slightly larger than Saturn in size — which may possibly have Earth-sized planets with liquid water in its orbit.

Researchers from University of Cambridge in the UK identified the star located about 600 light years away, called EBLM J0555-57Ab as it passed in front of its much larger companion.

The star is likely as small as stars can possibly become, as it has just enough mass to enable the fusion of hydrogen nuclei into helium. The gravitational pull at its stellar surface is about 300 times stronger than what humans feel on Earth.

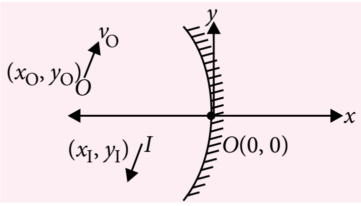
Courtesy : The Times of India

KEY CONCEPT

on VELOCITY OF THE IMAGE FOR SPHERICAL MIRROR

Er. Sandip Prasad*

When the position of the object changes with time on the principle axis relative to the mirror, the image position also changes with time relative to the mirror. Hence to know the relation between object and image speed we use the mirror equation.



Let us consider an object is moving with speed v_O in front of a concave mirror the position of the object at this instant is (x_O, y_O). An image is formed at location I whose coordinate is (x_I, y_I) at this instant. As usual, we take x-axis to be the principle axis.

Case I : Component of velocity of image along x-axis

As, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ or $\frac{1}{x_I}+\frac{1}{x_O}=\frac{1}{f}$

Differentiating both sides w.r.t. time,

$$-\frac{1}{x_I^2}\frac{dx_I}{dt}-\frac{1}{x_O^2}\frac{dx_O}{dt}=0 \quad \text{or} \quad \frac{dx_I}{dt}=-\frac{x_I^2}{x_O^2}\cdot\frac{dx_O}{dt}$$

or $v_{Im} = -m^2 \cdot v_{Om}$

where, $m=-\frac{x_I}{x_O}=-\frac{v}{u}=$ magnification

Negative sign indicates the object and image are always moving opposite to each other.

In concave mirror, depending on the position of the object, image speed may be greater or lesser or equal to the object speed.

(a) $R < u < \infty$; $|m| < 1$ $\therefore$ $v_I < v_O$

(b) $u = R$; $|m| = 1$ $\therefore$ $v_I = v_O$

(c) $f < u < R$; $|m| > 1$ $\therefore$ $v_I > v_O$

(d) $u < f$; $|m| > 1$ $\therefore$ $v_I > v_O$

Relation between object and image velocity is also valid for convex mirror. In convex mirror speed of image is slower than that of the object whatever the position of the object may be. This relation is not true in terms of acceleration of object and image.

Case II: Component of velocity of image along y-axis

As we know that transverse magnification.

$$m=\frac{I}{O}=\frac{\text{size of the image}}{\text{size of the object}}=\frac{y_I}{y_O}$$

$$y_I=\left(\frac{f}{f-u}\right)y_O$$

Differentiating both sides w.r.t. time,

$$\therefore \quad \frac{dy_I}{dt}=\frac{d}{dt}\left\{\left(\frac{f}{f-u}\right)y_O\right\}$$

$$\text{or} \quad \frac{dy_I}{dt}=\left(\frac{f}{f-u}\right)\frac{dy_O}{dt}+\frac{y_O f}{(f-u)^2}\left(\frac{du}{dt}\right)$$

$$\text{or} \quad v_{Im_y}=\left(\frac{f}{f-u}\right)v_{Om_y}+\frac{y_O f}{(f-u)^2}v_{Om_x}$$

Where,

v_{Im_y} = Velocity of image w.r.t. mirror along y-axis

v_{Om_y} = Velocity of object w.r.t. mirror along y-axis

y_O = y coordinate or height of the image from y-axis at that instant

v_{Om_x} = velocity of object w.r.t. mirror along x-axis

Note : In kinematics, we have studied that the velocity of the objects in x and y directions are independent of each other. But here, the velocity of the image in y direction will depend on its x coordinate and velocity in x direction also. Remember that these velocities are with sign convention. So the velocity along positive x direction will be positive and so on.

*Author is Director of Sandip Physics Classes and motivational speaker

Some special cases regarding motion of object

Case I : When the object is on the principle axis and moving along principle axis

Component of velocity along x-axis

$$v_{\mathrm{Im}_x} = -m^2 v_{\mathrm{Om}_x}$$

$$(v_{\mathrm{I}} - v_{\mathrm{m}})_x = -m^2 (v_{\mathrm{O}} - v_{\mathrm{m}})_x$$

Component of velocity along y-axis

$$v_{\mathrm{Im}_y} = \left(\frac{f}{f-u}\right) v_{\mathrm{Om}_y} + \frac{y_{\mathrm{O}} f}{(f-u)^2} v_{\mathrm{Om}_x}$$

Here,

$v_{\mathrm{Om}_y} = 0$, since object is only moving along x-axis

$y_{\mathrm{O}} = 0$, since object is lie on the principle axis

$\therefore \quad v_{\mathrm{Im}_y} = 0$

Case II : When the object is above the principle axis and moving parallel to principle axis

Component of velocity along x-axis

$$v_{\mathrm{Im}_x} = -m^2 v_{\mathrm{Om}_x}$$

$$(v_{\mathrm{I}} - v_{\mathrm{m}})_x = -m^2 (v_{\mathrm{O}} - v_{\mathrm{m}})_x$$

Component of velocity along y-axis

$$v_{\mathrm{Im}_y} = \left(\frac{f}{f-u}\right) v_{\mathrm{Om}_y} + \frac{y_{\mathrm{O}} f}{(f-u)^2} v_{\mathrm{Om}_x}$$

Here,

$v_{\mathrm{Om}_y} = 0$, since object is only moving along x-axis

$y_{\mathrm{O}} \neq 0$, since object is lie above the principle axis

$$\therefore \quad v_{\mathrm{Im}_y} = \frac{y_{\mathrm{O}} f}{(f-u)^2} v_{\mathrm{Om}_x}$$

Case III : When the object is on the principle axis and starts moving perpendicular to the principle axis

Component of velocity along x-axis

$$v_{\mathrm{Im}_x} = -m^2\, v_{\mathrm{Om}_x}$$

$$(v_{\mathrm{I}} - v_{\mathrm{m}})_x = -m^2 (v_{\mathrm{O}} - v_{\mathrm{m}})_x = -\, m^2\,(0 - 0)$$

$\therefore \quad v_{\mathrm{I}_x} = 0$

Component of velocity along y-axis

$$v_{\mathrm{Im}_y} = \left(\frac{f}{f-u}\right) v_{\mathrm{Om}_y} + \frac{y_{\mathrm{O}} f}{(f-u)^2} v_{\mathrm{Om}_x}$$

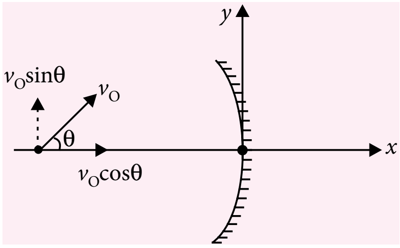
Here,

$v_{\mathrm{Om}_y} = v_{\mathrm{O}}$, since object is only moving along x-axis

$y_{\mathrm{O}} = 0$, since object is lie on the principle axis

$$\therefore \quad v_{\mathrm{Im}_y} = \frac{f}{f-u} \cdot v_{\mathrm{Om}_y} + 0 = \left(\frac{f}{f-u}\right) \cdot v_{\mathrm{Om}_y}$$

Case IV : When the object is on the principle axis and moving with a velocity which makes an angle θ with the principle axis



Component of velocity along x-axis

$$v_{\mathrm{Im}_x} = -m^2\, v_{\mathrm{Om}_x}$$

$$\Rightarrow \quad (v_{\mathrm{I}} - v_{\mathrm{m}})_x = -\left(\frac{f}{f-u}\right)^2 \cdot (v_{\mathrm{O}} - v_{\mathrm{m}})_x$$

Here, $(v_{\mathrm{O}})_x = v_{\mathrm{O}} \cos\theta$ and $(v_{\mathrm{m}})_x = 0$

Component of velocity along y-axis:

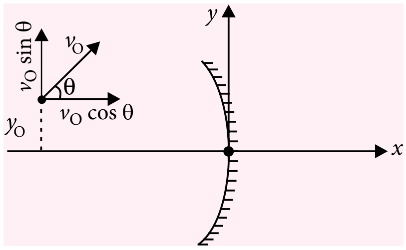
$$v_{\mathrm{Im}_y} = \left(\frac{f}{f-u}\right) v_{\mathrm{Om}_y} + \frac{y_{\mathrm{O}} f}{(f-u)^2} v_{\mathrm{Om}_x}$$

Here,

$v_{\mathrm{Om}_y} = v_{\mathrm{O}} \sin\theta$, $y_{\mathrm{O}} = 0$, since object is lie on the principle axis

$$\therefore \quad v_{\mathrm{Im}_y} = \frac{f}{f-u} \cdot v_{\mathrm{Om}_y} - 0 = \left(\frac{f}{f-u}\right) v_{\mathrm{Om}_y}$$

Case V : When the object is above the principle axis and moving at an angle θ with the horizontal



Component of velocity along x-axis

$$v_{\mathrm{Im}_x} = -m^2\, v_{\mathrm{Om}_x}$$

$$(v_{\mathrm{I}} - v_{\mathrm{m}})_x = -\left(\frac{f}{f-u}\right)^2 (v_{\mathrm{O}} - v_{\mathrm{m}})_x$$

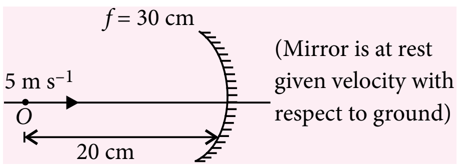
Here, $(v_{\mathrm{O}})_x = v_{\mathrm{O}} \cos\theta$ and $(v_{\mathrm{m}})_x = 0$

Component of velocity along y-axis

$$v_{\mathrm{Im}_y} = \left(\frac{f}{f-u}\right) v_{\mathrm{Om}_y} + \frac{y_{\mathrm{O}} f}{(f-u)^2} v_{\mathrm{Om}_x}$$

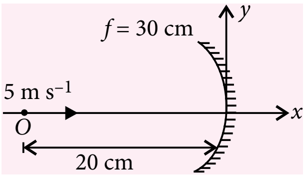
Here, $v_{\mathrm{Om}_y} = v_{\mathrm{O}} \sin\theta$ and y_{O} = constant

EXAMPLE 1 : The velocity of image w.r.t. ground in the given figure is



(a) 45 m s^{-1} and approaches the mirror
(b) 45 m s^{-1} and moves away from the mirror
(c) 60 m s^{-1} and approaches the mirror
(d) 60 m s^{-1} and moves away from the mirror

Soln.: (a) Step-I: First of all we need to draw the situation.



Step-II: Place the origin of a coordinate system at the pole .

Step-III: Find the component of the velocity of the object along x and y axis.

Here, $(v_O)_x = 5$ m s^{-1}, $(v_O)_y = 0$

Step-IV: Finally we have to apply the formula of velocity of the image along both the axes.

Component of velocity along x-axis relative to mirror

$v_{Im_x} = -m^2 v_{Om_x}$ .... (i)

Where, $m = -\dfrac{v}{u} = \dfrac{f}{f-u}$

Eqn. (i) can be written as

$$\Rightarrow (v_I - v_m)_x = -\left(\frac{f}{f-u}\right)^2 (v_O - v_m)_x$$

$$\Rightarrow (v_I - 0)_x = -\left(\frac{-30}{-30-(-20)}\right)^2 (+5-0)_x$$

$v_{I_x} = -45$ m s^{-1}

Component of velocity along y-axis

$$v_{Im_y} = \left(\frac{f}{f-u}\right) v_{Om_y} + \frac{y_O f}{(f-u)^2} v_{Om_x}$$

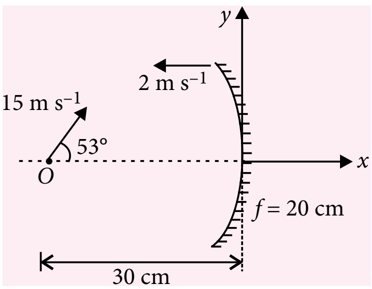
Here,

$v_{Om_y} = 0$, since object is only moving along x-axis

$y_O = 0$, since object is lie on the principle axis

$v_{Im_y} = 0 + 0$; $(v_I - v_m)_y = 0$ $\quad\therefore\; v_{Im_y} = 0$

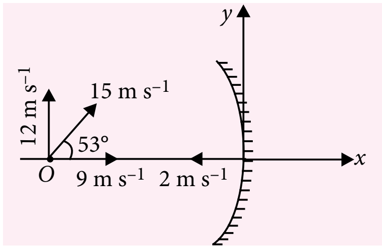
EXAMPLE 2 : Find the velocity of image w.r.t. ground in situation as shown in figure.



Soln.: Step-I: First of all we need to draw the situation.

Step-II: Place the origin of a co-ordinate system at the pole.

Step-III: Find the component of the velocity of the object along x and y axis.



Here, $(v_O)_x = 15 \cos 53° = 9$ m s^{-1}

$(v_O)_y = 15 \sin 53° = 12$ m s^{-1}

Step-IV: Finally we have to apply the formula of velocity of the image along both the axes.

Component of velocity along x-axis relative to mirror

$v_{Im_x} = -m^2 v_{Om_x}$ ...(i)

Where, $m = -\dfrac{v}{u} = \dfrac{f}{f-u}$

Eqn. (i) can be written as

$$(v_I - v_m)_x = -\left(\frac{f}{f-u}\right)^2 (v_O - v_m)_x$$

$$\Rightarrow \{v_I - (-2)\}_x = -\left(\frac{-20}{-20-(-30)}\right)^2 \{+9-(-2)\}_x$$

$= -(-2)^2 \times 11 = -44$

$v_{I_x} = -46$ m s^{-1}

Component of velocity along y-axis

$$v_{Im_y} = \left(\frac{f}{f-u}\right) v_{Om_y} + \frac{y_O f}{(f-u)^2} v_{Om_x}$$

Here,

$v_{O_y} = 12$ m s^{-1} , since object is only moving along x-axis

$y_O = 0$, since object is lie on the principle axis

$$v_{Im_y} = \left(\frac{-20}{-20-(-30)}\right)(v_O - v_m)_y - 0$$

$$(v_I - v_m)_y = \left(\frac{-20}{-20-(-30)}\right)(+12-0)_y$$

$(v_I - 0)_y = -24$

$v_{I_y} = -24$ m s^{-1}

$\vec{v}_{Ig}$ = Velocity of image w.r.t.ground

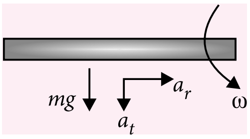
$$= (\vec{v}_{Ig})\,\hat{i} + (\vec{v}_{Ig})\,\hat{j} = (-46)\,\hat{i} + (-24)\,\hat{j}$$

PHYSICS MUSING

SOLUTION SET-48

1. **(c):** The angular velocity of the rod about the pivot when it passes through the horizontal is given by

$$mg \times \frac{L}{2}\sin 30^\circ = \frac{mL^2}{3} \times \frac{\omega^2}{2}$$

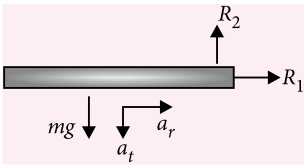


$$\omega = \sqrt{\frac{3g}{2L}}$$

Radial acceleration of the centre of mass (as centre of mass is moving in a circle of radius $L/2$) is given by $a_r^2 = \omega^2 \frac{L}{2} = \frac{3g}{4}$. Torque about pivot, in the horizontal position, is $\tau = mg\frac{L}{2} = I\alpha$

$$\alpha = \frac{mgL/2}{mL^2/3} = \frac{3g}{2L}$$

Tangential acceleration of the centre of mass, $a_t = \frac{L}{2}\alpha = \frac{3g}{4}$. From Newton's second law of equation.



$$R_1 = ma_r = \frac{3mg}{4}$$

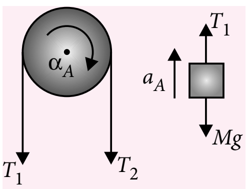
$$mg - R_2 = m \times a_t = \frac{3mg}{4} \quad \text{or} \quad R_2 = \frac{mg}{4}$$

So, reaction force by the pivot on the rod, $R = R_1^2 + R_2^2 = \sqrt{10}\, mg/4$ at an angle of $\tan^{-1} R_2/R_1$ [$= \tan^{-1}(1/3)$]with the horizontal.

2. **(b) :** In figure (a) $(T_2 - T_1) \times R = \frac{MR^2}{2}\alpha_A$

$T_1 - Mg = Ma_A$
$T_2 = 2Mg$
$a_A = R\alpha_A$

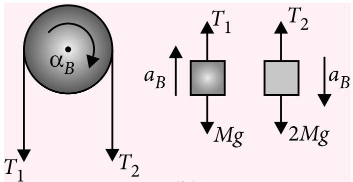
$$\alpha_A = \frac{2g}{3R}$$



(a)

In figure (b)

$$(T_2 - T_1) \times R = \frac{MR^2}{2}\alpha_B$$



(b)

$T_1 - Mg = Ma_B$
$2Mg - T_2 = 2Ma_B$

$a_B = R\alpha_B$, $\alpha_B = \frac{2g}{7R}$

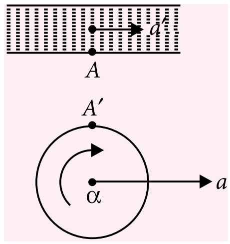
So, $\alpha_A > \alpha_B$

3. **(c):** $\frac{1}{b} + \frac{1}{a} = \frac{1}{f}$; $f = \frac{ab}{a+b}$

$AC^2 + BC^2 = AB^2$
$(a^2 + c^2) + (b^2 + c^2) = (a + b)^2$
$\Rightarrow\ c^2 = ab$

$$\therefore\ f = \frac{c^2}{a+b}$$

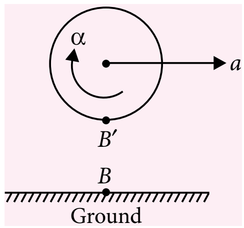
4. **(d) :** Let acceleration of tube is a' and radius of cylinder is R. There is no slipping between tube and cylinders. So, points A and A' should have same acceleration.



$\therefore\ a' = a + R\alpha$ ... (i)

Similarly, there is no slipping between cylinder and the ground.

So, points B and B' should have same acceleration.



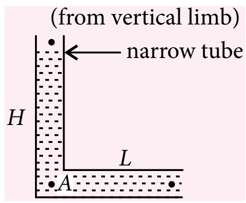
$\therefore\ 0 = a - R\alpha$ ... (ii)

From eqn. (i) and (ii), we get

$a' = 2a$

Pressure at A is

$p_A = p_{atm} + \rho g H$



Also, pressure at A is

$p_A' = p_{atm} + \rho a' L$ (from horizontal limb)
$= p_{atm} + \rho(2a)L$

For no spilling from any opening,

$p_A = p'_A$

$\therefore \; p_{atm} + \rho g H = p_{atm} + \rho(2a)L$

$$a = \frac{gH}{2L}$$

5. **(b) :** Time period for half part

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1}{g}} = \frac{2\pi}{\pi} = 2\,\text{s}$$

So 2° part will be covered in a time $t = \frac{T}{2} = 1\,\text{s}$

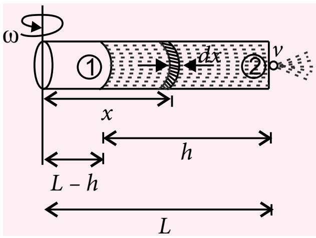
For the left 1° part : $\theta = \theta_0 \sin(\omega t)$

$$\Rightarrow \; 1° = 2° \sin\left(\frac{2\pi}{T} \times t\right) \;\Rightarrow\; \frac{1}{2} = \sin\left(\frac{2\pi}{2} \times t\right)$$

$$\Rightarrow \; \frac{\pi}{6} = \pi \times t \;\Rightarrow\; t = \frac{1}{6}\,\text{s}$$

Total time $= \frac{T}{2} + 2t = 1 + 2 \times \frac{1}{6} = 1 + \frac{1}{3} = \frac{4}{3}\,\text{s}$

6. **(b) :** From the non-inertial frame of tube, force on the liquid element is zero.



$(dp)A + (A\rho dx)\omega^2 x = 0$ ($A\rho dx$ = mass of element)

$dp = -\rho\omega^2 x dx$

$$\Rightarrow \; \int_{p_1}^{p_2} dp = -\rho\omega^2 \int_{L-h}^{L} x dx$$

$$p_2 - p_1 = -\frac{\rho\omega^2}{2}[L^2 - (L-h)^2]$$

$$= -\frac{\rho\omega^2}{2}(L^2 - L^2 - h^2 + 2Lh)$$

$$p_1 - p_2 = \frac{\rho\omega^2}{2}\left(2Lh - h^2\right) \qquad \text{... (i)}$$

On applying Bernoulli's theorem between points (1) and (2) we have

$$p_1 + \frac{1}{2}\rho(0)^2 = p_2 + \frac{1}{2}\rho v^2$$

$$p_1 - p_2 = \frac{1}{2}\rho v^2 \qquad \text{...(ii)}$$

From eqn. (i) and (ii), we get

$$\frac{1}{2}\rho v^2 = \frac{\rho\omega^2}{2}h^2\left(\frac{2L}{h} - 1\right)$$

$$v = \omega h\sqrt{\frac{2L}{h} - 1}$$

7. **(d) :** Here,

$T = 75$ dyn cm^{-1}

$= 75 \times 10^{-3}$ N m^{-1}

$mg = 0.1$ N, $2\pi R = 20$ cm

$= 0.2$ m

$F_{req} = mg + 2[T(2\pi R)]$

$= 0.1 + 2[75 \times 10^{-3}(0.2)] = 0.130$ N

8. **(b) :** With respect to the belt, pseudo force ma acts on cylinder at COM as shown in figure.

Here, $v = a + bt^2$

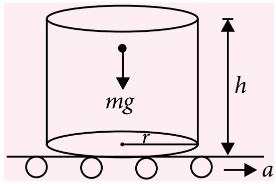
$$\frac{dv}{dt} = 2bt$$

Cylinder will be about to topple if

$\tau_{\text{pseudo force}} = \tau_{mg}$

$$m \cdot 2bt \cdot \frac{h}{2} = mg \cdot r$$

$$t = \frac{rg}{bh}$$



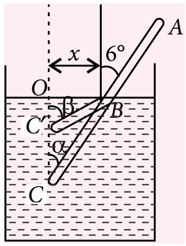
9. **(a) :** For small angle $\tan\theta = \sin\theta \approx \theta$

$$\alpha = \frac{x}{OC}, \; \beta = \frac{x}{OC'}$$

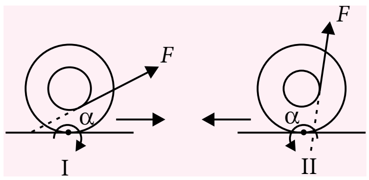
$$\Rightarrow \; \frac{\alpha}{\beta} = \frac{OC'}{OC} = \frac{1}{\mu}$$

$$\beta = \mu\alpha = \left(\frac{4}{3}\right)(6°) = 8°$$

Bending angle = $\beta - \alpha = 2°$



10. **(b) :**



Pure rotation about instantaneous point of contact.

Note: If line of action passes through point of contact, it does not rotate or translate.

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