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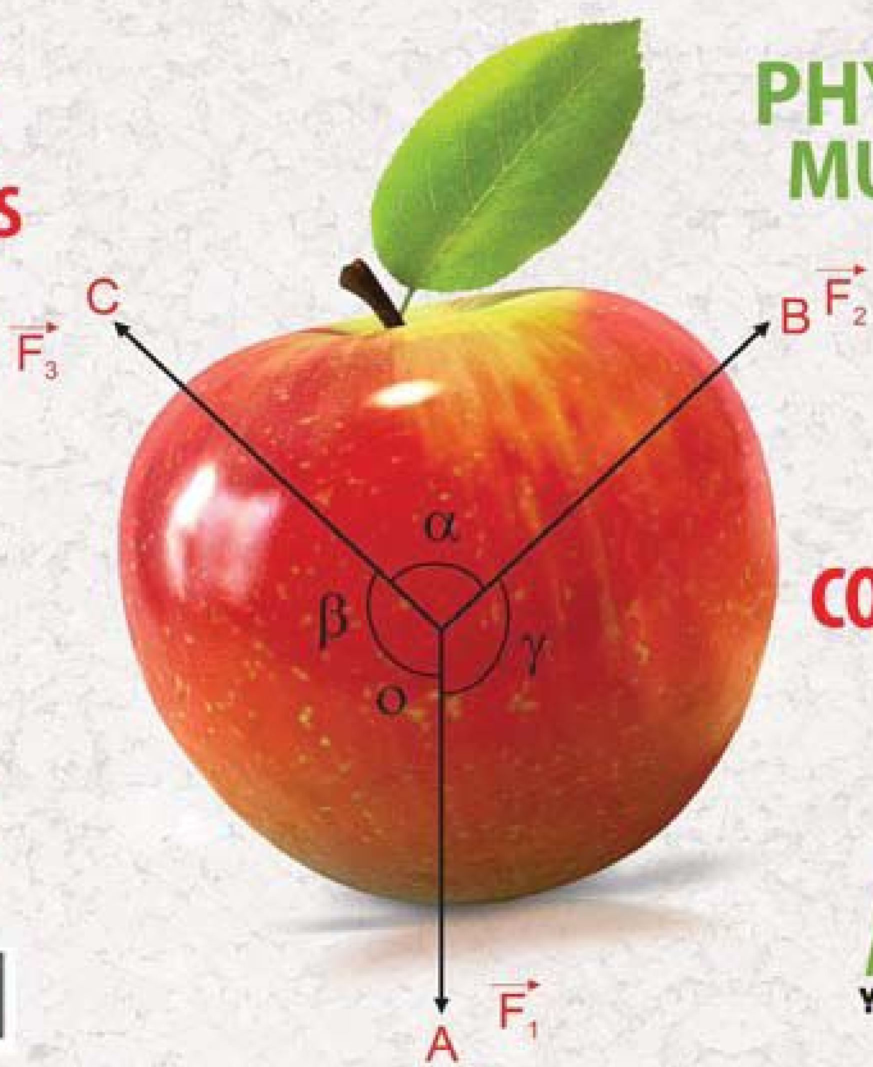

BRAIN MAP

CORE CONCEPT ON RELATIVE MOTION

ACE YOUR WAY CBSE CLASS XI-XII

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PHYSICS for you

Volume 24 No. 9 September 2016

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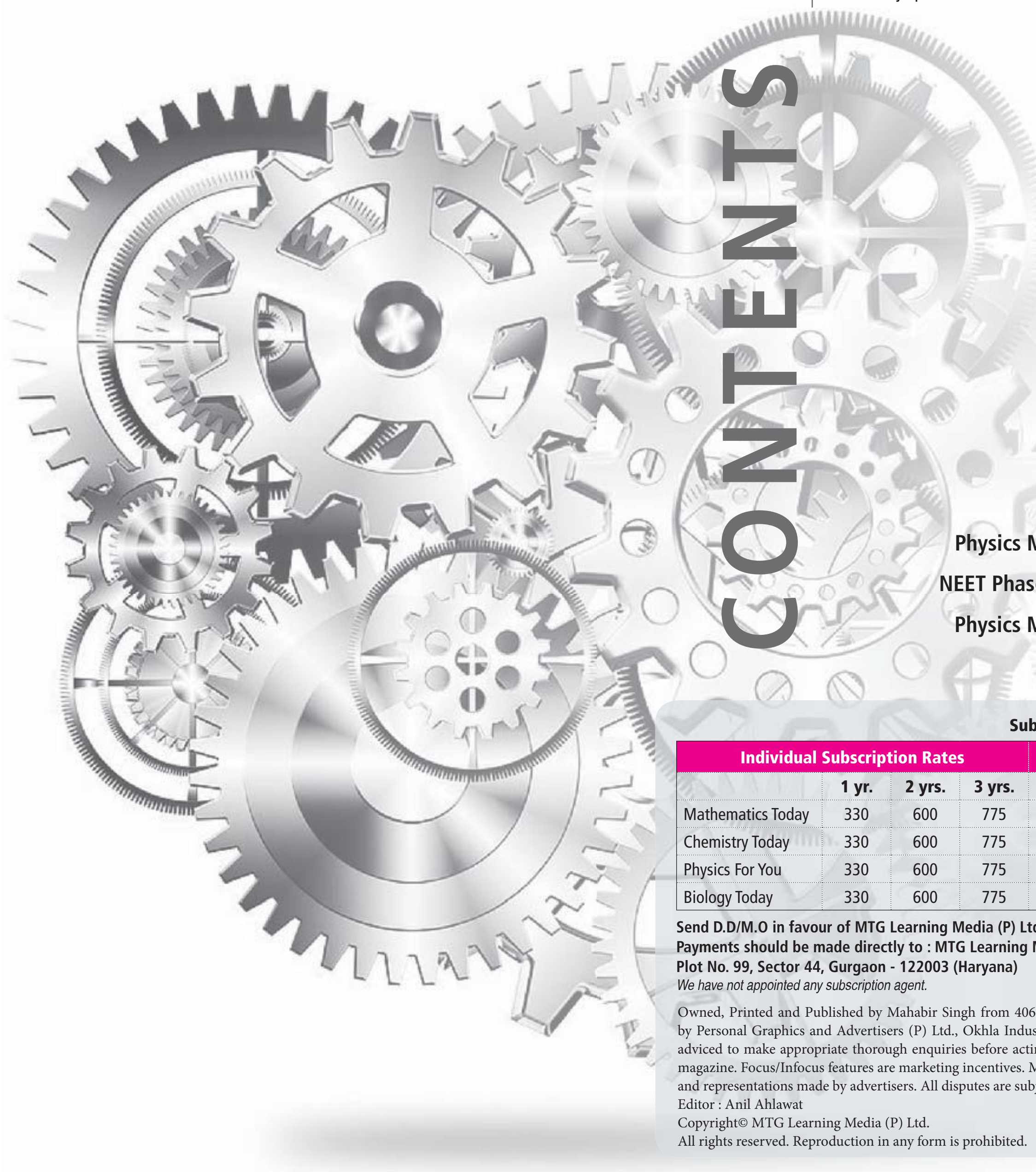

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NEET | JEE ESSENTIALS

Class XI

Maximize your chance of success, and high rank in NEET, JEE (Main and Advanced) by reading this column. This specially designed column is updated year after year by a panel of highly qualified teaching experts well-tuned to the requirements of these Entrance Tests.

Unit 3 LAWS OF MOTION WORK, ENERGY AND POWER

FIRST LAW OF MOTION

- A body in uniform motion remains in uniform motion and a body at rest remains at rest, unless acted on by a non zero net external force.

State of uniform motion

State of rest (Special case of uniform motion with zero speed)

- Inertia is the property of the body due to which body opposes the change of its state. Inertia of a body is measured by mass of the body.
  - Inertia ∝ mass
  - Heavier the body, greater is the force required to change its state and hence greater is inertia. The reverse is also true *i.e.*, lighter body has less inertia.
- **Different types of inertia**
  - **Inertia of rest :** It is the inability of a body to change its state of rest. E.g. When we shake a branch of a mango tree, the mangoes fall down.
  - **Inertia of motion :** It is the inability of a body to change its state of uniform motion. E.g. When a bus or train stops suddenly, a passenger sitting inside it tends to fall forward.
  - **Inertia of direction :** It is the inability of a body to change its direction of motion. E.g. Rotating wheels of vehicle throw out mud, mudguard over the wheels stop this mud.

MOMENTUM

- The total quantity of motion possessed by a moving body is known as the momentum of the body. It is the product of the mass and velocity of a body,

  i.e., $p = mv$
- The concept of momentum was introduced by Newton to measure the quantitative effect of force.

SECOND LAW OF MOTION

- Rate of change of momentum of a body is directly proportional to the external force applied on it and the change in momentum takes place in the direction of force.

$$\vec{F} \propto \frac{d\vec{p}}{dt} \quad \text{or ,} \quad \vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a} \quad \text{if } m = \text{constant}$$

$$\vec{F} = \vec{v}\frac{dm}{dt} \quad \text{(in case of conveyor belt or rocket)}$$

- This law gives magnitude, unit and dimension of force.

☞ THIRD LAW OF MOTION

- It states that to every action, there is an equal (in magnitude) and opposite (in direction) reaction.
- If a body A exerts a force $\vec{F}$ on another body B then B exerts a force $-\vec{F}$ on A. The two forces act along the same line.
- Action and reaction never cancel each other *i.e.*, they cannot balance each other as they act on different bodies.
- The forces of action and reaction may appear due to actual physical contact of the two bodies or even from a distance.
- This law is applicable whether the bodies are at rest or in motion.
- Single isolated force is not possible, force is always possible in action reaction pair form.

☞ CONSERVATION OF LINEAR MOMENTUM

- When no external force acts on a system of several interacting particles, the total linear momentum of the system is conserved.
- If $\vec{F}$ is the external force acting on the system, then according to Newton's second law

  $$\vec{F} = \frac{d\vec{p}}{dt}$$

  For an isolated system with no external force,

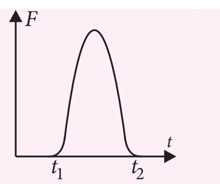
  i.e., $\vec{F} = 0$ or, $\frac{d\vec{p}}{dt} = 0$ or, $\vec{p}$ = constant

  or, $\vec{p}_1 + \vec{p}_2 + ...+ \vec{p}_n$ = constant

☞ IMPULSE

- When a large force act for an extremely short duration neither the magnitude of the force nor the time for which it acts is important. In such case the total effect of force is measured. The total effect of force is called impulse (measure of the action of force).
- If a large force acts on a body or particle for a small time then,
  impulse = product of force and time
  - ➢ If a force $\vec{F}$ acts for a short time dt then impulse $= \vec{F}dt$
  - ➢ If a force $\vec{F}$ acts for a finite interval of time from t_1 to t_2 then,

    impulse $= \int_{t_1}^{t_2} \vec{F}dt$

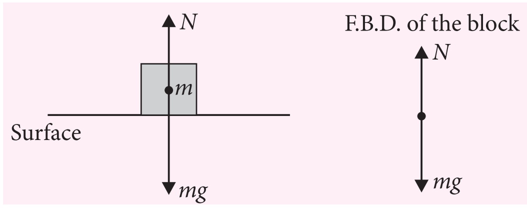


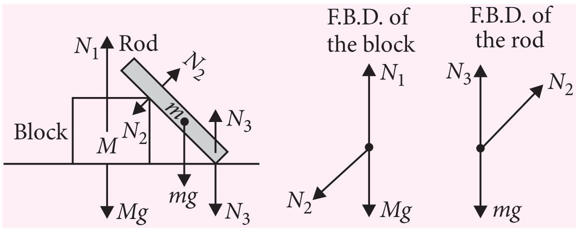
impulse = area under curve

  - ➢ If a constant force acts for an interval Δt then, impulse $= \vec{F}\Delta t$
- **Impulse - Momentum theorem :** Impulse of a force is equal to the change in momentum $\vec{F}\Delta t = \Delta\vec{p}$

☞ FREE BODY DIAGRAM (F.B.D.)

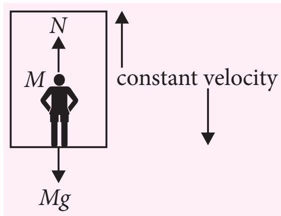
- Body is represented by a dot and each external force is represented by a vector with its tail on the dot.
- The body is isolated from the surrounding showing all the forces (contact as well as non-contact forces) acting on it.
- In F.B.D. internal forces are not shown.





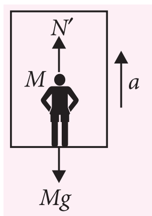
☞ APPARENT WEIGHT OF A BODY IN MOVING LIFT

- When lift is moving vertically upwards or downwards with uniform velocity



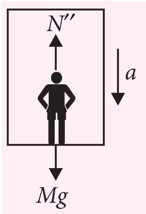
$N = Mg$ = Actual weight of man

- When lift is moving vertically upwards with uniform acceleration a.



$N' = M(g + a) \Rightarrow N' > N$

- When lift is moving vertically downwards with uniform acceleration a.

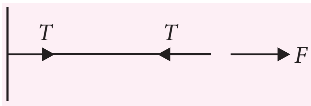


$N'' = M(g - a) \Rightarrow N'' < N$

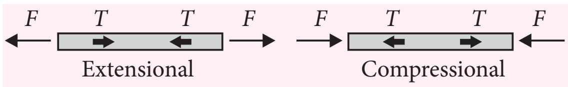
- When lift falls freely then man feels weightessness condition.

TENSION IN A STRING

- **Features of an ideal string**
  - Massless
  - Perfectly smooth
  - Inextensible (length of the string remains constant)
- **Tension :** When the two ends of a string are pulled in opposite directions then a force develops in the string called tension.



- **Rules for making tension in a string / chain / rod**
  - Tension in a string is always along the string and away from the object.
  - In an ideal string tension at all points of the string will be same.
  - In a string or in a chain tension is only extensional.
  - In a rod tension can be extensional or compressional.



  - Net force acting on a ideal string (massless) is always zero.

    $F = ma = 0 \times a = 0.$

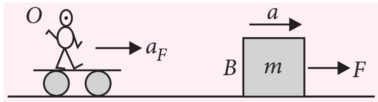
FRAME OF REFERENCE

- Observers in different frame may measure different displacements, velocities and accelerations.

  There are two kinds of frames of reference :

- **Inertial frame of reference**
  - Reference frame which is either at rest or moving with uniform velocity $a = 0$.
  - Newton's laws are applicable.
- **Non-inertial frame of reference**
  - Reference frames which have $a \neq 0$.
  - Newton's laws are not applicable.
  - To apply Newton's law we have to apply an imaginary force called pseudo force.
  - The magnitude of the pseudo force is the product of mass of the body and the acceleration of the reference frame.
  - The direction of pseudo force is opposite to the acceleration of the reference frame (not in the direction of motion of frame of reference, if a and v have opposite direction).
- Acceleration of block with respect to observer

  $\vec{a}_{BO} = \vec{a}_B - \vec{a}_O = \vec{a} - \vec{a}_F$



  Force on the block according to the observer

  $\vec{F}_{BO} = m(a - \vec{a}_F) = m\vec{a} - m\vec{a}_F$

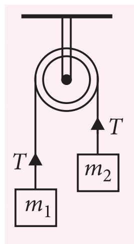
  $\because \quad \vec{F}_{\text{Pseudo}} = -m\vec{a}_F$

  $\therefore \quad \vec{F}_{BO} = \vec{F} + \vec{F}_{\text{Pseudo}}$

PULLEY

- Ideal pulley is considered to be weightless and frictionless.
- The pulley may change the direction of force in the string but not the tension.
- **Some cases of pulley and mass as a system**
  - Masses suspended over a pulley in a vertical plane with $m_1 = m_2 = m$
    - Tension in the string $T = mg$
    - Acceleration a = zero
    - Reaction at the suspension of the pulley or thrust or pressure force,

      $R = 2T = 2\,mg$



  - Masses suspended over a pulley in a vertical plane with $m_1 > m_2$

    Acceleration

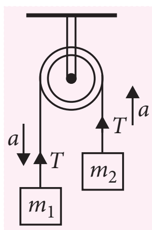
    $$= \frac{\text{net pulling force}}{\text{total mass to be pulled}} = \frac{(m_1 - m_2)}{m_1 + m_2} g$$

    $$\text{Tension} = \frac{2 \times \text{Product of masses}}{\text{Sum of two masses}} g$$

    $$= \frac{2m_1 m_2}{m_1 + m_2} g$$

    Reaction at the suspension of pulley or thrust or pressure force,

    $$R = 2T = \frac{4m_1 m_2}{(m_1 + m_2)} g$$

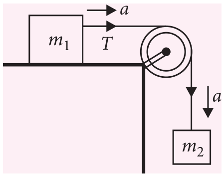


- Mass suspended over a pulley from another mass on the horizontal plane with $m_2 > m_1$

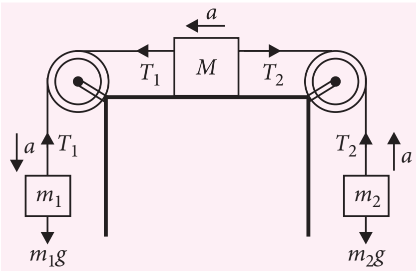
Acceleration, $a = \dfrac{m_2 g}{(m_1 + m_2)}$

Tension, $T = \dfrac{m_1 m_2}{(m_1 + m_2)} g$

Thrust on pulley $P = \sqrt{2}T$



- Masses suspended over pulley from another mass on the horizontal plane with $m_1 > m_2$

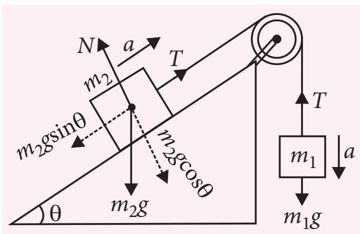


Acceleration, $a = \dfrac{(m_1 - m_2)}{(m_1 + m_2 + M)} g$

Tension, $T_1 = m_1(g - a)$

$T_2 = m_2(g + a)$

- Mass suspended over a pulley from another mass on an inclined plane with $m_1 > m_2$



For mass m_1 : $m_1g - T = m_1a$

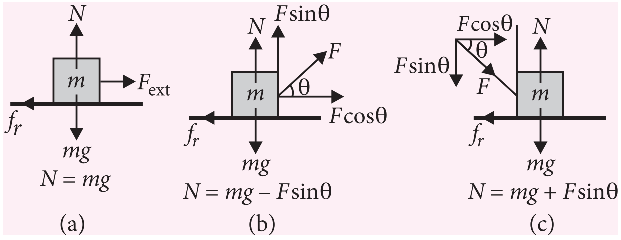
For mass m_2 : $T - m_2 g \sin\theta = m_2a$

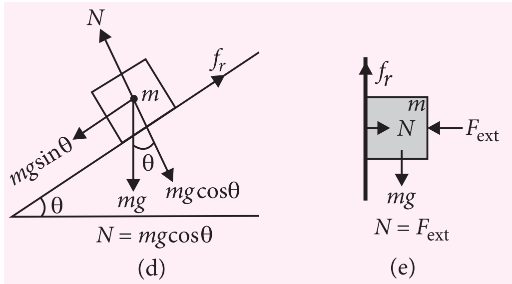
Acceleration $a = \dfrac{(m_1 - m_2 \sin\theta)}{(m_1 + m_2)} g$

Tension, $T = \dfrac{m_1 m_2 (1 + \sin\theta)}{(m_1 + m_2)} g$

FRICTION

- It is opposing force that is set up between the surface of contact, when one body slides or rolls or tends to do so on the surface of another body.
- Except static friction (f_s), friction force depends on normal reaction (N) $f_r \propto N$





Types of friction

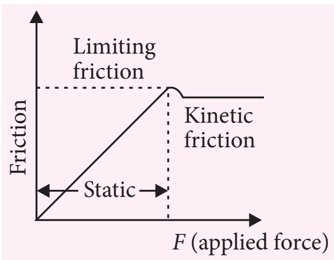
- External friction (It develops between contact surfaces of the bodies)
  - Static friction (f_s)
  - Limiting friction (f_{Lim}) $f_{Lim} = \mu_s N$
  - Sliding friction or Kinetic friction (f_k) $f_k = \mu_k N$
- Internal friction (It develops between contact layers of liquid also known as viscous force)

- **Static Friction**
  - Numerical value of static friction is equal to external force which creates the tendency of motion of body. So, its nature is self adjusting.
  - Maximum value of static friction is called limiting friction *i.e.*, $f_{Lim} = \mu_s N$.
- **Laws of Limiting Friction**
  - The magnitude of the force of limiting friction (f) between any two bodies in contact is directly proportional to the normal reaction (N) between them *i.e.*, $f \propto N$.
  - The direction of the force of limiting friction is always opposite to the direction in which one body is on the verge of moving over the other.
  - The force of limiting friction is independent of the apparent contact area, so long as normal reaction between the two bodies in contact remains the same.

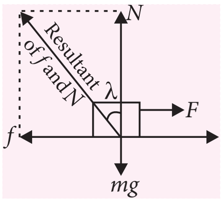
- Limiting friction between any two bodies in contact depends on the nature of material of the surfaces in contact and their roughness and smoothness.
- Its value is more than other types of friction force.

• **Kinetic Friction**
- If the body is in motion, the friction opposing its motion is called dynamic or kinetic friction.
- This is always slightly less than the limiting friction.
- Its numerical value is $f_k = \mu_k N$.

• Variation of friction force with applied force

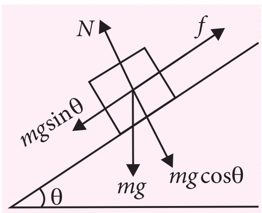


• **Angle of Friction (λ) :** The angle which the resultant of the force of limiting friction f and normal reaction N makes with the direction of normal reaction N.



$$\tan\lambda = \frac{f}{N} = \mu \text{ or, } \lambda = \tan^{-1}\mu$$

• **Angle of Repose or Angle of Sliding :** It is defined as the maximum angle of inclination of a plane with the horizontal at which a body placed on it just begins to sliding down.



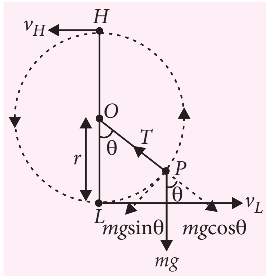
$$f = mg\sin\theta \text{ and } N = mg\cos\theta$$

$$\text{So, } \mu_s = \frac{f}{N} = \tan\theta = \frac{mg\sin\theta}{mg\cos\theta} \Rightarrow \theta = \tan^{-1}\mu_s$$

- This fact is used for finding the coefficient of static friction in the laboratory.
- Angle of repose (θ) = Angle of friction (λ)

VERTICAL CIRCULAR MOTION

• Tension at any position of angular displacement, (θ) along a vertical circle is given by



$$T = \frac{mv^2}{r} + mg\cos\theta$$

- Tension at the lowest point ($\theta = 0°$) is given by

$$T_L = \frac{mv_L^2}{r} + mg$$

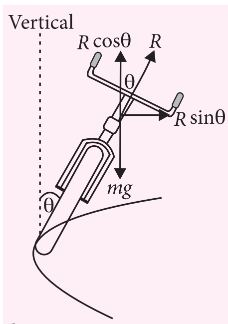
- Tension at the highest point ($\theta = 180°$) is given by

$$T_H = \frac{mv_H^2}{r} - mg$$

• For looping the loop
- Minimum velocity at the highest point, $v_H = \sqrt{gr}$
- Minimum velocity at the lowest point, $v_L = \sqrt{5gr}$.
- When the string is horizontal, $\theta = 90°$, minimum velocity, $v = \sqrt{3gr}$.
- Height through which a body should fall for looping the vertical loop of radius r is, $h = 5r/2$.

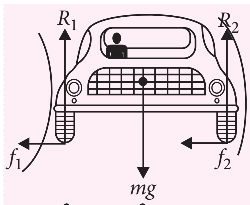
APPLICATIONS OF UNIFORM CIRCULAR MOTION

• **Bending of cyclist on circular turning for safe going**



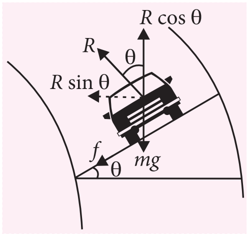
- Component of normal reaction $R\sin\theta$ provides centripetal force.
- Maximum speed of cyclist, $v = \sqrt{rg\tan\theta}$

• **Motion of a car on a level road**



- Friction force (μR) between tyres and road provides a suitable centripetal force.
- Maximum speed of car for safe turning $v = \sqrt{\mu_s rg}$

- **Motion of a car on a banked circular road**
  - Component of friction force (f cos θ) and component of normal reaction ($R\sin\theta$) provide required centripetal force for safe turning.
  - The maximum permissible speed to avoid slipping

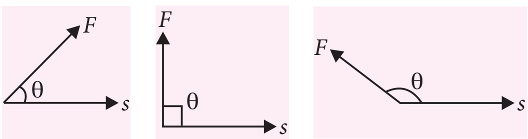


$$v_{max} = \left[\frac{rg(\mu_s + \tan\theta)}{1 - \mu_s \tan\theta}\right]^{1/2}$$

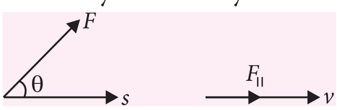
  - If road is smooth then $\mu_s = 0$ and

$$v_{max} = \sqrt{rg\tan\theta}$$

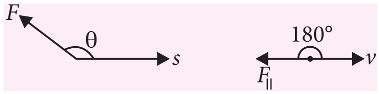
WORK

- Work is said to be done on a body only if the following conditions are satisfied
  - A force acts on the body.
  - The point of application of the force moves in the direction of the force.
- **Work done by a Constant Force**
  - Measurement of work done when the force acts along the direction of motion.
    Work done (W) = Force (F) × distance moved in the direction of force (s)
  - Measurement of work done when force and displacement are inclined to each other
    Work done (W) = component of force in the direction of displacement (Fcosθ) × magnitude of displacement (s) $= Fs\cos\theta = \vec{F}\cdot\vec{s}$
- **Positive and Negative Work**
  - Work done is a scalar quantity it can be positive or negative.
  - 

    θ < 90°, W is +ve   θ = 90°, W is zero   θ > 90°, W is –ve
- If the force applied to a body does a positive work, the velocity of the body increases. In this case the force, and hence the acceleration are directed along velocity and velocity of the body increases.



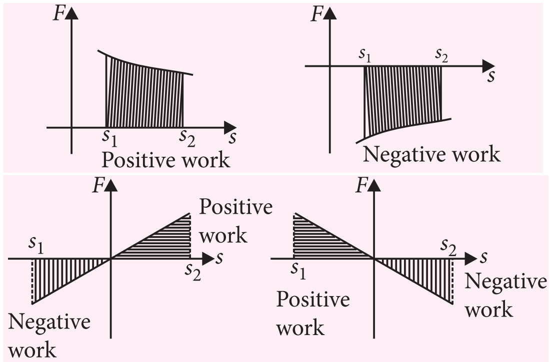
- If the force does a negative work the acceleration is directed against the velocity and the velocity of the body decreases.



- Work done by variable force

$$W = \int_{r_i}^{r_f} \vec{F}\cdot d\vec{r} = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$$

  - Geometrical Interpretation of work done
    Work done = Area under F versus s curve



CONSERVATIVE AND NON-CONSERVATIVE FORCE

- All central forces are conservative but all conservative forces are not central forces.
- Forces acting along the line joining the centres of two bodies are called central forces. Gravitational and electrostatic forces are central forces.
- The concept of potential energy exists only in the case of conservative force.

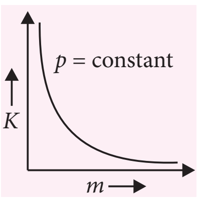
Conservative force	Non-conservative force
• Work done does not depend upon path.	• Work done depends upon path.
• Work done in a round trip is zero.	• Work done in a round trip is not zero.
• When only a conservative force acts within a system, the kinetic energy and potential energy can change. However their sum, the mechanical energy of the system does not change.	• Work done against a non-conservative force may be dissipated as heat energy.
• Work done is completely recoverable.	• Work done is not completely recoverable.

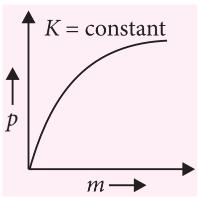
☞ ENERGY

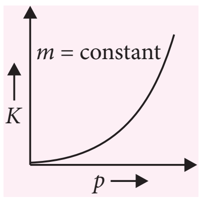
- It is defined as internal capacity to do work.
- When we say a body has energy it means it can do work.
- Different forms of energy are mechanical energy, electrical energy, optical (light) energy etc.
- **Kinetic energy**
  - Kinetic energy of a body can be calculated by the amount of work done in stopping the moving body or from the amount of work done in giving the present velocity to the body from the state of rest.
  - If a particle of mass m is moving with velocity v much less than the velocity of the light then the kinetic energy K.E. is given by K.E. = $\frac{1}{2}mv^2$
  - As mass m and v^2 or $\vec{v}\cdot\vec{v}$ are always positive so K.E. can never be negative.
  - The kinetic energy depends on the frame of reference.
  - Work done on the body is the measure of K.E. of the body.
  - The expression K.E. = $\frac{1}{2}mv^2$ holds even when the force applied varies in magnitude or in direction or in both.
  - Relation between K.E. (K) and linear momentum (p) :

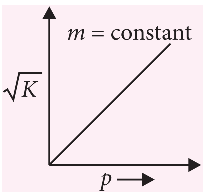
$$p = mv \text{ and } K = \frac{1}{2}mv^2 = \frac{1}{2m}(m^2v^2) = \frac{p^2}{2m}$$

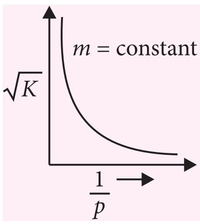
$$K = \frac{p^2}{2m} \quad \text{or} \quad p = \sqrt{2mK}$$











- **Work energy theorem :** According to this theorem, work done by net force on a body is equal to change in its kinetic energy.

$$W = \Delta K.E. \quad \text{or,} \quad W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

- **Potential energy**
  - It is the energy of body by virtue of its position or configuration in a conservative force field.
  - In case of conservative forces

$$dW = dU = -\vec{F}.d\vec{r}$$

$$\int_{U_1}^{U_2} dU = -\int_{r_1}^{r_2} \vec{F}.d\vec{r}$$

We take the reference point at infinity and assume potential energy to be zero at infinity.

If $r_1 = \infty$ and $U_1 = 0$ then

$$U_2 = U = -\int_{\infty}^{r} \vec{F}.d\vec{r}$$

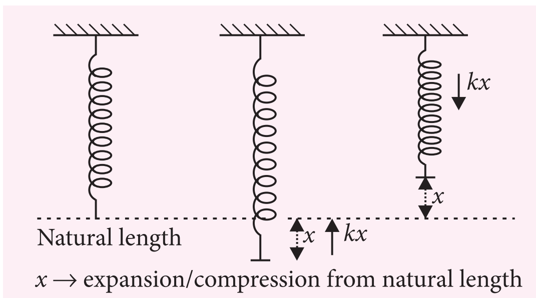
- Potential energy of a body in a conservative force field is defined as work done by an external agent against the action of conservative force in order to shift it from reference point (P.E. = 0) to the present position.
- Potential energy in different conservative fields
  - Gravitation potential energy of a body at height h from earth's surface, $U_g = -W_g = mgh$
  - Electrostatic potential energy between two point charges q_1 and q_2 is

$$U_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2};$$

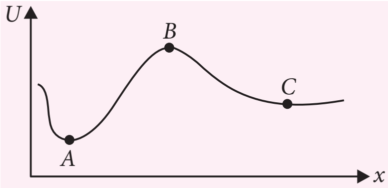
r = separation between charge particles.

  - Elastic potential energy : Reference point is taken at its natural length.

    If x is expansion / compression in spring then P.E. of spring (Energy stored in spring) is $U = \frac{1}{2}kx^2$ and spring force $F = -kx$ (directed towards natural length).



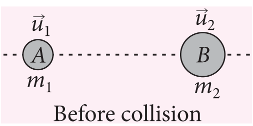
TRANSLATORY EQUILIBRIUM



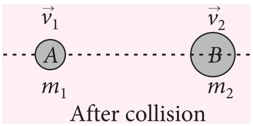
- At points where $\frac{dU}{dx} = 0$, $F = 0$, there the body will be in translatory equilibrium.
  (Here equilibrium is decided by conservative forces).
- In the shown figure $\frac{dU}{dx} = 0$, when P.E. is minimum (A), maximum (B) or constant (C).
  So, there are three types of equilibrium
  - Stable Equilibrium (Point A)
  - Unstable Equilibrium (Point B)
  - Neutral Equilibrium (Point C)

COLLISION OF BODIES

- A collision is said to take place when either two bodies physically collide against each other or when the path of one body is changed by the influence of the other body.
- As a result of collision, the momentum and kinetic energy of the interacting bodies change.
- Forces involved in a collision are action reaction forces *i.e.*, the internal forces of the system.
- The total momentum remains conserved in any type of collision.
- Types of collision according to the direction of collision :
  - **Head on collision :** Direction of velocities of bodies is similar to the direction of collision.

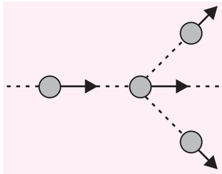


Before collision



After collision

  - **Oblique collision :** Direction of velocities of bodies is not similar to the direction of collision.



- Types of collision according to the conservation law of kinetic energy :
  - **Elastic collision :** Kinetic energy is conserved.
    K.E.$_{\text{before collision}}$ = K.E.$_{\text{after collision}}$
  - **Inelastic collision :** Kinetic energy is not conserved. Some energy is lost in collision.
    K.E.$_{\text{before collision}}$ > K.E.$_{\text{after collision}}$
  - **Perfect inelastic collision :** Two bodies stick together after the collision.
- Momentum remains conserved in all types of collisions.
- **Coefficient of restitution (e)**
  - $e = \dfrac{\text{velocity of separation along line of impact}}{\text{velocity of approach along line of impact}}$
  - Value of e is 1 for elastic collision, 0 for perfectly inelastic collision, $0 < e < 1$ for inelastic collision.

POWER

- Rate of doing work *i.e.*, work performed by a force per unit time is called power.
- Instantaneous power : If a force $\vec{F}$ performs the work $\vec{F} \cdot d\vec{r}$ during the time interval dt, the power developed by that force at a given moment of time is equal to,

$$P = \vec{F} \cdot \frac{d\vec{r}}{dt} \text{ and } \frac{d\vec{r}}{dt} = \vec{v}$$

$$P = \vec{F} \cdot \vec{v} \text{ (Instantaneous power developed)}$$

- Average Power $= \dfrac{\text{Total work done}}{\text{Total time taken}}$

MPP-3 CLASS XI ANSWER KEY

1.	(b)	**2.**	(b)	**3.**	(c)	**4.**	(a)	**5.**	(c)
6.	(a)	**7.**	(a)	**8.**	(a)	**9.**	(a)	**10.**	(a)
11.	(b)	**12.**	(b)	**13.**	(b)	**14.**	(a)	**15.**	(b)
16.	(b)	**17.**	(c)	**18.**	(a)	**19.**	(d)	**20.**	(a, b, d)
21.	(a, c)	**22.**	(a, d)	**23.**	(a, b)	**24.**	(8)	**25.**	(4)
26.	(2)	**27.**	(b)	**28.**	(b)	**29.**	(c)	**30.**	(a)

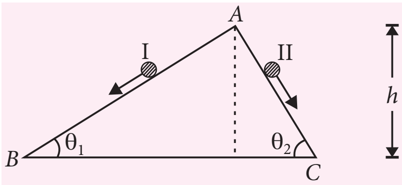
"A scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die and a new generation grows up that is familiar with it."

– Max Planck

SPEED PRACTICE

1. A body of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5\ \text{m}^{-1/2}\ \text{s}^{-1}$. The work done by the net force during its displacement from $x = 0$ to $x = 2$ m is
(a) 1.5 J (b) 50 J (c) 10 J (d) 100 J

2. Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track as shown in figure.



Which of the following statement is correct?
(a) Both the stones reach the bottom at the same time but not with the same speed.
(b) Both the stones reach the bottom with the same speed and stone I reaches the bottom earlier than stone II.
(c) Both the stones reach the bottom with the same speed and stone II reaches the bottom earlier than stone I.
(d) Both the stones reach the bottom at different times and with different speeds.

3. A block of mass $m = 1$ kg moving on a horizontal surface with speed $v_i = 2\ \text{m s}^{-1}$ enters a rough patch ranging from $x = 0.10$ m to $x = 2.01$ m. The retarding force F_r on the block in this range is inversely proportional to x over this range,

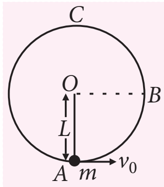
$$F_r = -\frac{k}{x} \text{ for } 0.1 < x < 2.01 \text{ m}$$

$$= 0 \text{ for } x < 0.1 \text{ m and } x > 2.01 \text{ m}$$

where $k = 0.5$ J. What is the final kinetic energy of the block as it crosses this patch?
(a) 0.5 J (b) 1.5 J (c) 2.0 J (d) 2.5 J

4. Two billiard balls A and B, each of mass 50 g and moving in opposite directions with speed of $5\ \text{m s}^{-1}$ each, collide and rebound with the same speed. The impulse imparted to each ball is
(a) $0.25\ \text{kg m s}^{-1}$ (b) $0.5\ \text{kg m s}^{-1}$
(c) $0.1\ \text{kg m s}^{-1}$ (d) $0.125\ \text{kg m s}^{-1}$

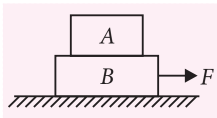
5. A bob of mass m is suspended by a light string of length L. It is imparted a horizontal velocity v_0 at the lowest point A such that it completes a semicircular trajectory in the vertical plane with the string becoming slack only on reaching the topmost point, C. What is the ratio of the kinetic energies of the mass at point B and point C?



(a) 1 : 2 (b) 2 : 1 (c) 3 : 1 (d) 1 : 3

6. An elevator can carry a maximum load of 1800 kg (elevator + passengers) is moving up with a constant speed of $2\ \text{m s}^{-1}$. The frictional force opposing the motion is 4000 N. What is minimum power delivered by the motor to the elevator?
(a) 22 kW (b) 44 kW (c) 88 kW (d) 8 kW

7. In figure, the coefficient of friction between the floor and the block B is 0.1. The coefficient of friction between the blocks B and A is 0.2. The mass of A is $m/2$ and that of B is m. What is the maximum horizontal force F can be applied to the block B so that two blocks move together?

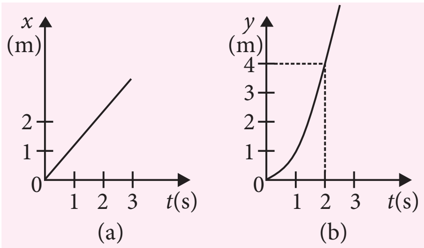


(a) $0.15mg$ (b) $0.05mg$ (c) $0.1mg$ (d) $0.45mg$

8. A person in an elevator accelerating upwards with an acceleration of $2\ \text{m s}^{-2}$, tosses a coin vertically upwards with a speed of $20\ \text{m s}^{-1}$. After how much time will the coin fall back into his hand? (Take $g = 10\ \text{m s}^{-2}$)
(a) 1.67 s (b) 2 s (c) 3.33 s (d) 5 s

9. A 100 kg gun fires a ball of 1 kg horizontally from a cliff of height 500 m. It falls on the ground at a distance of 400 m from the bottom of the cliff. Find the recoil velocity of the gun. (Take $g = 10\ \text{m s}^{-2}$)
(a) $0.2\ \text{m s}^{-1}$ (b) $0.4\ \text{m s}^{-1}$
(c) $0.6\ \text{m s}^{-1}$ (d) $0.8\ \text{m s}^{-1}$

10. Figure shows (x, t), (y, t) diagram of a particle moving in 2-dimensions.



If the particle has a mass of 500 g, the force acting on the particle is

(a) 1 N along y-axis
(b) 1 N along x-axis
(c) 0.5 N along x-axis
(d) 0.5 N along y-axis

11. A circular racetrack of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a racing car and the road is 0.2, what is the maximum permissible speed to avoid slipping? (Take tan15° = 0.27)

(a) 18.2 m s^{-1} (b) 28.2 m s^{-1}
(c) 38.2 m s^{-1} (d) 48.2 m s^{-1}

12. A block of mass 4 kg is kept on a rough horizontal surface. The coefficient of static friction is 0.8. If a force of 19 N is applied on the block parallel to the floor, then the force of friction between the block and floor is

(a) 32 N (b) 18 N (c) 19 N (d) 9.8 N

13. A particle of mass 10 g moves along a circle of radius 6.4 cm with a constant tangential acceleration. What is the magnitude of this acceleration if the kinetic energy of the particle becomes equal to 8×10^{-4} J by the end of the second revolution after the beginning of the motion?

(a) 0.18 m s^{-2} (b) 0.2 m s^{-2}
(c) 0.1 m s^{-2} (d) 0.15 m s^{-2}

[NEET Phase-I 2016]

14. A body of mass 1 kg begins to move under the action of a time dependent force $\vec{F} = (2t\hat{i} + 3t^2\hat{j})$ N, where $\hat{i}$ and $\hat{j}$ are unit vectors along x and y axis. What power will be developed by the force at the time t?

(a) $(2t^3 + 3t^4)$ W (b) $(2t^3 + 3t^5)$ W
(c) $(2t^2 + 3t^3)$ W (d) $(2t^2 + 4t^4)$ W

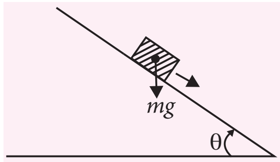
[NEET Phase-I 2016]

15. Two stones of masses m and $2m$ are whirled in horizontal circles, the heavier one in a radius $\frac{r}{2}$ and the lighter one in radius r. The tangential speed of lighter stone is n times that of the value of heavier stone when they experience equal centripetal force. The value of n is

(a) 4 (b) 1 (c) 2 (d) 3

[AIPMT 2015]

16. A plank with a box on it at one end is gradually raised about the other end. As the angle of inclination with the horizontal reaches 30°, the box starts to slip and slides 4.0 m down the plank in 4.0 s. The coefficients of static and kinetic friction between the box and the plank will be, respectively (Take $g = 10$ m s^{-2})



(a) 0.5 and 0.6 (b) 0.4 and 0.3
(c) 0.6 and 0.6 (d) 0.6 and 0.5

[AIPMT 2015]

17. A rocket is fired vertically from the earth with an acceleration of $2g$, where g is the gravitational acceleration. On an inclined plane inside the rocket, making an angle θ with the horizontal, a point object of mass m is kept. The minimum coefficient of friction μ_{min} between the mass and the inclined surface such that the mass does not move is

(a) $\tan 2\theta$ (b) $\tan\theta$ (c) $3\tan\theta$ (d) $2\tan\theta$

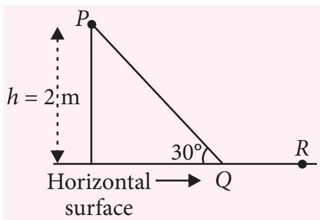
[JEE Main 2016]

18. A particle of mass m is acted upon by a force F given by the empirical law $F = \frac{R}{t^2}v(t)$. If this law is to be tested experimentally by observing the motion starting from rest, the best way is to plot

(a) $\log v(t)$ against $\frac{1}{t}$ (b) $v(t)$ against t^2
(c) $\log v(t)$ against $\frac{1}{t^2}$ (d) $\log v(t)$ against t

[JEE Main 2016]

19. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction between the particle



and the rough track is μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance x (= QR) are, respectively close to

(a) 0.2 and 6.5 m (b) 0.2 and 3.5 m
(c) 0.29 and 3.5 m (d) 0.29 and 6.5 m

[JEE Main 2016]

20. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate.

(a) 2.45×10^{-3} kg (b) 6.45×10^{-3} kg
(c) 9.89×10^{-3} kg (d) 12.89×10^{-3} kg

[JEE Main 2016]

SOLUTIONS

1. (b) : Given : $v = ax^{3/2}$ where, $a = 5\ \text{m}^{-1/2}\ \text{s}^{-1}$

$$\text{Accelrartion} = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = v\frac{dv}{dx}\quad\left(\because\ v = \frac{dx}{dt}\right)$$

As $v^2 = a^2x^3$

Differentiating both sides with respect to x, we get

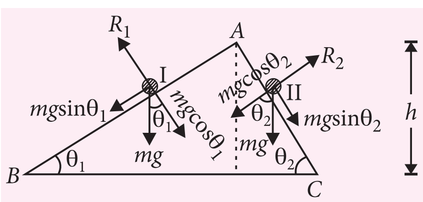
$$\therefore\quad 2v\frac{dv}{dx} = 3a^2x^2 \text{ or, Acceleration} = \frac{3}{2}a^2x^2$$

Force, F = Mass × Acceleration $= \frac{3}{2}ma^2x^2$

$$\text{Work done, } W = \int F dx = \int_0^2 \frac{3}{2}ma^2x^2 dx$$

$$W = \frac{3}{2}ma^2\left[\frac{x^3}{3}\right]_0^2 = \frac{3}{2}\times 0.5\times 5^2\times\frac{8}{3} = 50\text{ J}$$

2. (c) :



According to law of conservation of mechanical energy,

PE at the top = KE at the bottom

$$\therefore\quad mgh = \frac{1}{2}mv_1^2 \qquad \text{...(i)}$$

$$\text{and } mgh = \frac{1}{2}mv_2^2 \qquad \text{...(ii)}$$

From (i) and (ii), we get $v_1 = v_2$

As it is clear from the figure, acceleration of the two stones are $a_1 = g\sin\theta_1$ and $a_2 = g\sin\theta_2$.

As $\theta_2 > \theta_1\ \therefore\ a_2 > a_1$

From $v = u + at = 0 + at$ or $t = \frac{v}{a}$

As $t \propto \frac{1}{a}$, and $a_2 > a_1\ \therefore\ t_2 < t_1$

Stone II will take lesser time and reach the bottom earlier than stone I.

3. (a) : Here, $m = 1$ kg, $v_i = 2\ \text{m s}^{-1}$, $k = 0.5$ J

Initial kinetic energy,

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}\times(1\text{ kg})(2\text{ m s}^{-1})^2 = 2\text{ J}$$

Work done by retarding force

$$W = \int F_r dx = \int_{0.1}^{2.01} -\frac{k}{x}dx = -k[\ln x]_{0.1}^{2.01}$$

$$= -k\ln\left(\frac{2.01}{0.1}\right) = -0.5\ln(20.1) = -1.5\text{ J}$$

According to work-energy theorem

$W = K_f - K_i$

or $K_f = W + K_i = -1.5\text{ J} + 2\text{ J} = 0.5$ J

4. (b) : (A) → $5\ \text{m s}^{-1}$ $5\ \text{m s}^{-1}$ ← (B)

Before collision

$5\ \text{m s}^{-1}$ ← (A) (B) → $5\ \text{m s}^{-1}$

After collision

Initial momentum of ball A = $(0.05\text{ kg})(5\text{ m s}^{-1})$
$= 0.25\text{ kg m s}^{-1}$

As the speed is reversed on collision,

Final momentum of the ball A

$= (0.05\text{ kg})(-5\text{ m s}^{-1}) = -0.25\text{ kg m s}^{-1}$

Impulse imparted to the ball A

= Change in momentum of ball A

= Final momentum – Initial momentum

$= -0.25\text{ kg m s}^{-1} - 0.25\text{ kg m s}^{-1} = -0.5\text{ kg m s}^{-1}$

Similarly,

Impulse imparted to ball B

$= (0.25\text{ kg m s}^{-1}) - (-0.25\text{ kg m s}^{-1})$

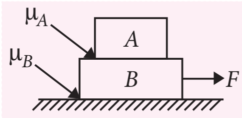
$= 0.5\text{ kg m s}^{-1}$

Impulse imparted to each ball is 0.5 kg m s^{-1} in magnitude. The two impulses are opposite in direction.

5. **(c)**

6. **(b) :** Here, m = 1800 kg
Frictional force, f = 4000 N
Uniform speed, $v = 2$ m s^{-1}
Downward force on elevator is
$F = mg + f$
$= (1800 \text{ kg} \times 10 \text{ m s}^{-2}) + 4000 \text{ N} = 22000 \text{ N}$
The motor must supply enough power to balance this force. Hence,
$P = Fv = (22000 \text{ N})(2 \text{ m s}^{-1})$
$= 44000 \text{ W} = 44 \times 10^3 \text{ W} = 44 \text{ kW}$

7. **(d) :** Here, $m_A = \dfrac{m}{2}$, $m_B = m$



$\mu_A = 0.2$, $\mu_B = 0.1$
Let both the blocks are moving with common acceleration a. Then,

$$a = \frac{\mu_A m_A g}{m_A} = \mu_A g = 0.2g$$

and $F - \mu_B(m_B + m_A)g = (m_B + m_A)a$
$F = (m_B + m_A)a + \mu_B(m_B + m_A)g$

$$= \left(m + \frac{m}{2}\right)(0.2g) + (0.1)\left(m + \frac{m}{2}\right)g$$

$$= \left(\frac{3}{2}m\right)(0.2g) + \left(\frac{3}{2}m\right)(0.1g) = \frac{0.9}{2}mg = 0.45mg$$

8. **(c) :** Here, $v = 20$ m s^{-1}, $a = 2$ m s^{-2}, $g = 10$ m s^{-2}
The coin will fall back into the person's hand after t s.

$$\therefore \quad t = \frac{2v}{a+g} = \frac{2 \times 20 \text{ m s}^{-1}}{(2+10) \text{ m s}^{-2}} = \frac{40}{12} \text{ s} = \frac{10}{3} \text{ s} = 3.33 \text{ s}$$

9. **(b) :** Here,
Mass of the gun, M = 100 kg
Mass of the ball, m = 1 kg
Height of the cliff, h = 500 m, g = 10 m s^{-2}
Time taken by the ball to reach the ground is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 500 \text{ m}}{10 \text{ m s}^{-2}}} = 10 \text{ s}$$

Horizontal distance covered = ut; where u is the velocity of the ball
$\therefore$ 400 = $u \times 10$
$u = 40$ m s^{-1}
According to law of conservation of linear momentum, we get
$0 = Mv + mu$

$$v = -\frac{mu}{M} = -\frac{(1 \text{ kg})(40 \text{ m s}^{-1})}{100 \text{ kg}} = -0.4 \text{ m s}^{-1}$$

Negative sign shows that the direction of recoil of the gun is opposite to that of the ball.

10. **(a) :** Since the graph between x and t is a straight line and passing through the origin.
$\therefore \quad x = t$
Since the graph between y and t is a parabola.
$\therefore \quad y = t^2$

$$\therefore \quad v_x = \frac{dx}{dt} = 1 \text{ and } a_x = \frac{dv_x}{dt} = 0$$

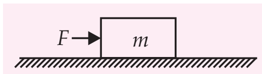
and $v_y = \dfrac{dy}{dt} = 2t$ and $a_y = 2$ m s^{-2}
The force acting on the particle is
$F = ma_y = (0.5 \text{ kg})(2 \text{ m s}^{-2})$
= 1 N along y-axis

11. **(c) :** Here R = 300 m, $\theta = 15°$, $g = 9.8$ m s^{-2}, $\mu = 0.2$
The maximum permissible speed is given by

$$v_{max} = \sqrt{\frac{Rg(\mu + \tan\theta)}{1 - \mu\tan\theta}} = \sqrt{\frac{300 \times 9.8 \times (0.2 + 0.27)}{1 - 0.2 \times 0.27}}$$

$= 38.2$ m s^{-1}

12. **(c) :**



Here, m = 4 kg, F = 19 N, $\mu_s = 0.8$
The limiting force of friction,
$f = \mu_s mg = 0.8 \times 4 \times 10 = 32$ N
As applied force $F < f$, therefore, block does not move. Further as static friction is self adjusting, therefore, frictional force = 19 N.

13. **(c) :** Here, $m = 10$ g $= 10^{-2}$ kg,
$R = 6.4$ cm $= 6.4 \times 10^{-2}$ m, $K_f = 8 \times 10^{-4}$ J
$K_i = 0$, a_t = ?
Using work energy theorem,
Work done by all the forces = Change in KE
$W_{\text{tangential force}} + W_{\text{centripetal force}} = K_f - K_i$
$\Rightarrow \; F_t \times s + 0 = K_f - 0 \Rightarrow ma_t \times (2 \times 2\pi R) = K_f$

$$\Rightarrow \quad a_t = \frac{K_f}{4\pi R m} = \frac{8 \times 10^{-4}}{4 \times \frac{22}{7} \times 6.4 \times 10^{-2} \times 10^{-2}}$$

$= 0.099 \approx 0.1$ m s^{-2}

14. **(b) :** Here, $\vec{F} = (2t\hat{i} + 3t^2\hat{j})$ N, $m = 1$ kg

Acceleration of the body, $\vec{a} = \dfrac{\vec{F}}{m} = \dfrac{(2t\hat{i} + 3t^2\hat{j}) \text{ N}}{1 \text{ kg}}$

Velocity of the body at time t,

$$\vec{v} = \int \vec{a}dt = \int\left(2t\hat{i} + 3t^2\hat{j}\right)dt = \left(t^2\hat{i} + t^3\hat{j}\right) \text{ m s}^{-1}$$

$\therefore$ Power developed by the force at time t,

$$P = \vec{F}\cdot\vec{v} = (2t\hat{i} + 3t^2\hat{j})\cdot(t^2\hat{i} + t^3\hat{j}) \text{ W}$$
$$= (2t^3 + 3t^5) \text{ W}$$

15. (c) : Let v be tangential speed of heavier stone. Then, centripetal force experienced by lighter stone is $(F_c)_{\text{lighter}} = \dfrac{m(nv)^2}{r}$

and that of heavier stone is $(F_c)_{\text{heavier}} = \dfrac{2mv^2}{(r/2)}$

But $(F_c)_{\text{lighter}} = (F_c)_{\text{heavier}}$ (given)

$$\therefore \quad \frac{m(nv)^2}{r} = \frac{2mv^2}{(r/2)};\ n^2\left(\frac{mv^2}{r}\right) = 4\left(\frac{mv^2}{r}\right)$$

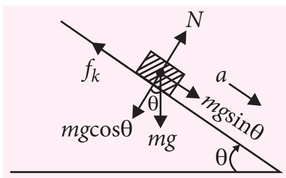
$$n^2 = 4 \quad \text{or} \quad n = 2$$

16. (d) : Let μ_s and μ_k be the coefficients of static and kinetic friction between the box and the plank respectively.

When the angle of inclination θ reaches 30°, the block just slides,

$$\therefore \quad \mu_s = \tan\theta = \tan 30° = \frac{1}{\sqrt{3}} = 0.6$$

If a is the acceleration produced in the block, then



$ma = mg\sin\theta - f_k$

$\quad = mg\sin\theta - \mu_k N$

(where f_k is force of kinetic friction as $f_k = \mu_k N$)

$a = g(\sin\theta - \mu_k\cos\theta)$ (as $N = mg\cos\theta$)

As $g = 10$ m s^{-2} and $\theta = 30°$

$\therefore\ a = (10 \text{ m s}^{-2})(\sin 30° - \mu_k \cos 30°)$ ...(i)

If s is the distance travelled by the block in time t, then

$$s = \frac{1}{2}at^2 \quad (\text{as } u = 0) \text{ or } a = \frac{2s}{t^2}$$

But $s = 4.0$ m and $t = 4.0$ s (given)

$$\therefore \quad a = \frac{2(4.0 \text{ m})}{(4.0 \text{ s})^2} = \frac{1}{2} \text{ m s}^{-2}$$

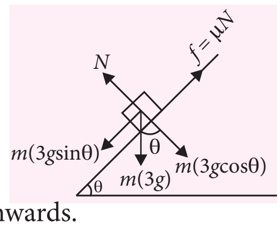
Substituting this value of a in eqn. (i), we get

$$\frac{1}{2} \text{ m s}^{-2} = (10 \text{ m s}^{-2})\left(\frac{1}{2} - \mu_k \frac{\sqrt{3}}{2}\right)$$

$$\frac{1}{10} = 1 - \sqrt{3}\mu_k \quad \text{or} \quad \sqrt{3}\mu_k = 1 - \frac{1}{10} = \frac{9}{10} = 0.9$$

$$\mu_k = \frac{0.9}{\sqrt{3}} = 0.5$$

17. (b) : Since the rocket is moving vertically upwards with acceleration $2g$, therefore the apparent acceleration experienced by the point object is $g + 2g = 3g$ vertically downwards.



From the figure, $N = 3mg\cos\theta$

Point object does not move on inclined surface,

$\mu N = 3mg\sin\theta$

or $\quad \mu\, 3mg\cos\theta = 3mg\sin\theta \quad$ or $\quad \mu = \tan\theta$

18. (a) : Here, $F = \dfrac{R}{t^2}v(t)$

$$\Rightarrow \quad m\frac{dv}{dt} = \frac{R}{t^2}v(t) \Rightarrow \frac{dv}{v(t)} = \frac{R}{m}\frac{dt}{t^2}$$

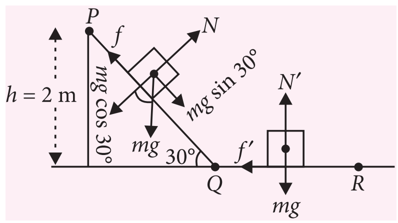
Integrating both sides,

$$\int\frac{dv}{v(t)} = \frac{R}{m}\int\frac{dt}{t^2} \Rightarrow \log v = -\left(\frac{R}{m}\right)\left(\frac{1}{t}\right) + C$$

Graph between $\log v$ and $\left(\dfrac{1}{t}\right)$ is a straight line.

19. (c) : Here, $PQ = \dfrac{h}{\sin 30°} = 2h = 4$ m

$QR = x = ?,\ \mu = ?$



Energy of the particle is lost only due to friction between the track and the particle.

According to the question,

Energy lost by the particle over the part PQ = Energy lost by the particle over the part QR

or, $\quad f \times PQ = f' \times QR$

or, $\quad \mu\, mg \cos 30° \times 4 = \mu\, mg\, x$

or, $x = 4 \cos 30° = 4\dfrac{\sqrt{3}}{2} = 2\sqrt{3}$ m ≈ 3.5 m

Using work energy theorem for the motion of the particle, $mgh - (f \times PQ) - f'(QR) = 0 - 0$

or $mgh - 2f'(QR) = 0$ or, $mgh - 2\mu mg\, x = 0$

$$\therefore \quad \mu = \frac{h}{2x} = \frac{2}{2\times 2\sqrt{3}} = 0.288 \approx 0.29$$

20. (d)

CLASS XI Series 3

ACE YOUR WAY CBSE

Work, Energy and Power
System of Particles and Rotational Motion

Time Allowed : 3 hours
Maximum Marks : 70

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Q. no. 1 to 5 are very short answer questions and carry 1 mark each.
(iii) Q. no. 6 to 10 are short answer questions and carry 2 marks each.
(iv) Q. no. 11 to 22 are also short answer questions and carry 3 marks each.
(v) Q. no. 23 is a value based question and carries 4 marks.
(vi) Q. no. 24 to 26 are long answer questions and carry 5 marks each.
(vii) Use log tables if necessary, use of calculators is not allowed.

SECTION-A

1. A body is moved along a closed loop. Is the work done in moving the body necessarily zero? If not, state the condition under which work done over a closed path is always zero.
2. Does the centre of mass of a solid necessarily lie within the body? If not, give an example.
3. Moment of a force and work done by a force have same units. What is the difference between them?
4. Why do we prefer to use a wrench of longer arm?
5. A rough inclined plane is placed on a cart moving with a constant velocity u on a horizontal ground. A block of mass M rests on the incline. Is there any work done by force of friction between the block and incline? Is there then a dissipation of energy?

SECTION-B

6. In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 Å (1 Å = 10^{-10} m). Find the approximate location of the centre of mass of the molecule. Given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.
7. A uniform rectangular parallelopiped of mass m having sides l, $2l$ and $4l$ is placed in turn on each of its three sides on a horizontal surface. What is the potential energy of the parallelopiped in each of these positions? Which position will be the most stable?
8. Define moment of inertia. On what factors does it depend?
9. Deduce the relation between torque and angular momentum.

OR

Show that the angular momentum of a satellite of mass M_s revolving around the earth having mass M_e in an orbit of radius r is

$$L = \sqrt{GM_e M_s^2 r}.$$

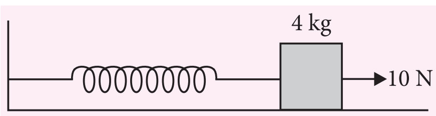
10. A massless pan is kept on a light elastic spring. When a sand particle of mass 0.1 kg is dropped on the pan from a height of 0.24 m, the particle strikes the pan, and the spring compresses by 0.01 m. From what height should the particle be dropped to cause a compression of 0.04 m?

SECTION-C

11. Explain how work done by a variable force may be measured.
12. What is a couple? What effect does it have on a body? Show that the moment of couple is same irrespective of the point of rotation of a body?
13. Suppose the rod is non-uniform and its mass per unit length (λ) varies linearly with x according to the expression $\lambda = \alpha x$, where α is a constant. Find the centre of mass as a fraction of L.
14. Find the components along the x, y, z axes of the angular momentum $\vec{L}$ of a particle, whose position vector is $\vec{r}$ with components x, y, z and momentum is $\vec{p}$ with components p_x, p_y and p_z. Show that if the particle moves only in the x-y plane the angular momentum has only a z-component.
15. Draw a plot of spring force F_s versus displacement x. Hence find an expression for the P.E. of an elastic stretched spring.

OR

The spring shown in figure has a force constant of 24 N m^{-1}. The mass of the block attached to the spring is 4 kg. Initially the block is at rest and spring is unstretched. The horizontal surface is frictionless. If a constant horizontal force of 10 N is applied on the block, then what is the speed of the block when it has been moved through a distance of 0.5 m?



16. Prove the work-energy theorem for a variable force.
17. A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump.
18. Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speeds ω_1 and ω_2 are brought into contact face to face with their axes of rotation coincident. Calculate (a) What is the angular speed of the two disc system? (b) Show that the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this loss in energy? Take $\omega_1 \neq \omega_2$.
19. Obtain an expression for the work done by a torque. Hence write the expression for power.
20. Show that the total mechanical energy of a freely falling body remains constant throughout its fall.
21. A bullet of mass 0.012 kg and horizontal speed 70 m s^{-1} strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.
22. Distinguish between stable, unstable and neutral equilibria of a body.

SECTION-D

23. Ravi came to stay in a multistoried building. He noticed that motor supplying water to the second floor has power rating X kW while of that supplying water to eighth floor has Y kW. He asked his father the reason behind the difference of the power ratings. His father explained him the reason.
(a) What values does Ravi possess?
(b) Which power rating is more X or Y?
(c) A motor pumps up 1000 kg of water through height of 10 m in 5 s. If the power of the motor is 36 kW, calculate the efficiency of the motor.

SECTION-E

24. (a) State and prove the theorem of perpendicular axes.
(b) A hoop of radius 2 m weighs 100 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 20 cm s^{-1}. How much work has to be done to stop it?

OR

(a) State and prove the theorem of parallel axes.
(b) What is the moment of inertia of a rod of mass M, length l about an axis perpendicular to it through one end?

25. Prove that in a one-dimensional elastic collision between two bodies, the relative velocity of approach

before collision is equal to the relative velocity of separation after the collision. Hence derive expressions for the final velocities of the two bodies in terms of their initial velocities before collision.

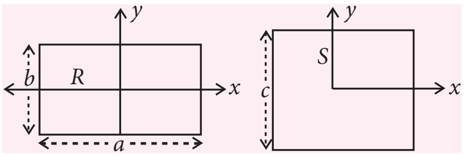
OR

What is a perfectly inelastic collision? Show that kinetic energy is invariably lost in such a collision.

26. Find the centre of mass of a uniform (a) half-disc, (b) quarter-disc.

OR

(a) Define radius of gyration of a body rotating about an axis. On what factors does it depend?
(b) A uniform square plate S (side c) and a uniform rectangular plate R (sides b, a) have identical areas and masses as shown in the given figure.



Show that

(i) $I_{xR}/I_{xS} < 1$ (ii) $I_{yR}/I_{yS} > 1$ (iii) $I_{zR}/I_{zS} > 1$

SOLUTIONS

1. The work done in moving the body along a closed loop is not necessarily zero. Work done by non conservative force like friction over a closed path is not zero.
Work done over any closed path is always zero if all the forces acting on the system is conservative.

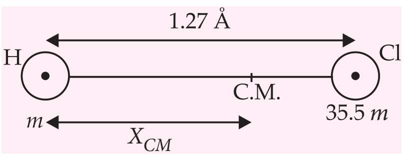
2. No. For example, the centre of mass of L-shaped rod lies in the region outside the rod.

3. Moment of force is a vector quantity while work done is a scalar quantity.

4. The torque applied on the nut by the wrench is equal to the force multiplied by the perpendicular distance from the axis of rotation. Hence to increase torque, a wrench of longer arm is preferred.

5. Work done by the force of friction between the block and the inclined plane is zero, because the block is at rest on the incline plane. Also due to this reason there is no dissipation of energy.

6.



Suppose hydrogen atom is at the origin *i.e.*, $x_1 = 0$

$$X_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$= \frac{m\times 0 + 35.5m\times(1.27\text{ Å})}{m + 35.5m} = 1.235\text{ Å}$$

7. When the parallelopiped is placed on its small, middle and large side, its centre of gravity will lie at heights $2l$, l and $l/2$ respectively above the horizontal surface. If U_1, U_2 and U_3 are the potential energies in the three cases, then

$U_1 = mg \times 2l = 2mgl$

$U_2 = mg \times l = mgl$

$U_3 = mg\times\frac{l}{2} = \frac{1}{2}mgl$

When the parallelopiped rests on its large side, its potential energy is minimum. So this position is the position of most stable.

8. The moment of inertia of a rigid body about an axis is the sum of the products of the masses of the particles constituting the body and the squares of their respective perpendicular distances from the axis of rotation.

$I = \sum m_i r_i^2$

The moment of inertia of a body depends on

(i) mass of the body
(ii) size and shape of the body
(iii) distribution of mass about the axis of rotation
(iv) position and orientation of the axis of rotation with respect to the body.

9. Torque, $\vec{\tau} = \vec{r}\times\vec{F}$

Angular momentum, $\vec{L} = \vec{r}\times\vec{p}$

Differentiating both sides w.r.t. time t, we get

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r}\times\vec{p}) = \frac{d\vec{r}}{dt}\times\vec{p} + \vec{r}\times\frac{d\vec{p}}{dt}$$

$$= \vec{v}\times\vec{p} + \vec{r}\times\vec{F} \quad \left[\because \frac{d\vec{p}}{dt} = \vec{F}\right]$$

$$= 0 + \vec{\tau} \quad [\because \vec{v}\times\vec{p} = \vec{v}\times m\vec{v} = 0]$$

$$\therefore \quad \vec{\tau} = \frac{d\vec{L}}{dt}$$

Thus the torque acting on a particle is equal to its rate of change of angular momentum.

OR

Let the satellite revolve around the earth with orbital speed v. Then, Centripetal force on the satellite = Gravitational force between the earth and the satellite

or $\frac{M_s v^2}{r} = G\frac{M_e M_s}{r^2}$ or $v^2 = \frac{GM_e}{r}$ or $v = \sqrt{\frac{GM_e}{r}}$

As the satellite is considered a point mass, its angular momentum is

$$L = M_s vr = M_s\sqrt{\frac{GM_e}{r}}\cdot r = \sqrt{GM_e M_s^2 r}$$

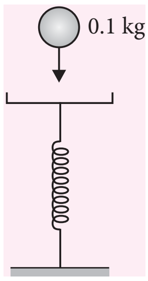
10. Here, m = 0.1 kg, h = 0.24 m, x = 0.01 m, x' = 0.04 m, h' = ?

As the particle is dropped from height h, it compresses the spring through distance x.

$\therefore$ Total loss in potential energy of the particle = Total gain in potential energy of the spring

or $mg(h+x) = \frac{1}{2}kx^2$ ...(i)

and $mg(h'+x') = \frac{1}{2}kx'^2$ ...(ii)



Dividing (ii) by (i), we get

$$\frac{h'+x'}{h+x} = \left(\frac{x'}{x}\right)^2$$

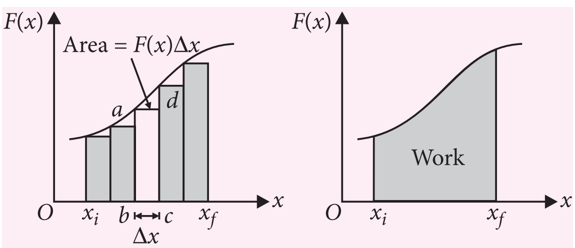
or $\frac{h'+0.04}{0.24+0.01} = \left(\frac{0.04}{0.01}\right)^2$

$\therefore$ h' = 3.96 m

11. Suppose a variable force F acts on a body along the fixed direction, say x-axis. The magnitude of the force F depends on x, as shown by force displacement graph in figure (a). Let us calculate the work done when the body moves from the initial position x_i to the final position x_f under the force F.

The displacement can be divided into a large number of small equal displacements Δx. During a small displacement Δx, the force F can be assumed to be constant. Then the work done is

$W \simeq F\,\Delta x$ = Area of rectangle $abcd$



(a) (b)

Adding areas of all the rectangles in figure (a), we get the total work done as

$$W \simeq \sum_{x_i}^{x_f} F\Delta x = \text{Sum of areas of all rectangles erected over all the small displacements}$$

In the limit when $\Delta x \to 0$, the number of rectangles tends to be infinite, but the above summation approaches a definite integral whose value is equal to the area under the curve. Thus the total work done is

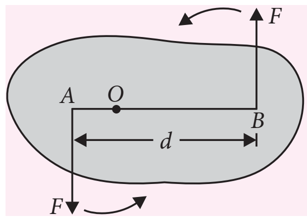
$$W = \lim_{\Delta x\to 0}\sum_{x_i}^{x_f} F\,\Delta x = \int_{x_i}^{x_f} F\,dx$$

= Area under the force-displacement curve

12. A pair of equal and opposite forces acting on a body along two different lines of action constitute a couple. A couple has a turning effect, but no resultant force acts on a body.

The moment of couple can be found by taking the moments of the two forces about any point and then adding them.

In figure, two opposite forces, each of magnitude F act at two points A and B of a rigid body, which can rotate about point O. The turning tendency of the two forces is anticlockwise.



Moment or torque of the couple about O is

$\tau = F \times AO + F \times OB = F(AO + OB) = F \times AB$

or $\tau = Fd$

Moment of a couple = Force × perpendicular distance between two forces.

Hence the moment of a couple is equal to the product of either of the forces and the perpendicular distance, called the arm of the couple, between their lines of action. Note that the torque exerted by couple about O does not depend on the position of O. Hence torque or moment of a couple is independent of the choice of the fulcrum or the point of rotation.

13. As we know, $x_{cm} = \frac{1}{M}\int_0^L x\,dm = \frac{1}{M}\int_0^L x(\lambda\,dx)$

$$= \frac{1}{M}\int_0^L x(\alpha x)dx = \frac{\alpha}{M}\int_0^L x^2 dx = \frac{\alpha}{M}\times\frac{L^3}{3} = \frac{\alpha L^3}{3M}$$

But $M = \int dm = \int_0^L \lambda\, dx = \int_0^L \alpha x\, dx = \frac{\alpha L^2}{2}$

Thus, $x_{cm} = \frac{\alpha L^3}{3(\alpha L^2/2)} = \frac{2}{3}L$ (from origin, $x = 0$)

14. (a) Angular momentum,

$\vec{L} = \vec{r} \times \vec{p}$ ... (i)

It is a vector quantity and its direction is given by right hand rule for vector product.

In cartesian co-ordinates,

$$\left.\begin{aligned} \vec{r} &= x\hat{i} + y\hat{j} + z\hat{k} \\ \text{and} \quad \vec{p} &= p_x\hat{i} + p_y\hat{j} + p_z\hat{k} \end{aligned}\right\} \quad ...(ii)$$

From (i) and (ii), we get

$$\vec{L} = (x\hat{i} + y\hat{j} + z\hat{k}) \times (p_x\hat{i} + p_y\hat{j} + p_z\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

or $L_x\hat{i} + L_y\hat{j} + L_z\hat{k}$

$$= \hat{i}(yp_z - zp_y) + \hat{j}(zp_x - xp_z) + \hat{k}(xp_y - yp_x)$$

On comparing, we get

$$\left.\begin{aligned} L_x &= yp_z - zp_y \\ L_y &= zp_x - xp_z \\ L_z &= xp_y - yp_x \end{aligned}\right\} \quad ...(iii)$$

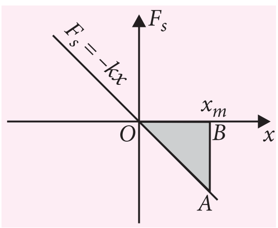
Equation (iii) gives the required components of L along x, y and z axes.

(b) As the particle moves in x-y plane, then $z = 0$ and $p_z = 0$

Hence, $\vec{L} = \hat{k}(xp_y - yp_x) = L_z\hat{k}$

Hence angular momentum has only z-component.

15. Figure shows the plot of spring force F_s versus displacement x of a block attached to the free end of the spring. According to Hook's law, the spring force for an extension x_m is



$F_s = -k\,x_m$

The work done by the spring force for an extension x_m is

$$W_s = \text{Area of } \Delta OBA = \frac{1}{2} AB \times OB$$

$$= \frac{1}{2} F_s \times x_m = \frac{1}{2}(-k\,x_m) \times x_m = -\frac{1}{2}kx_m^2$$

This work done is stored as the P.E. of the spring.

$\therefore \quad U = \frac{1}{2}kx_m^2$

OR

Here $k = 24$ N m^{-1}, $m = 4$ kg, $x = 0.5$ m, $F = 10$ N

By the law of conservation of energy,

Work done on the spring

= Gain in K.E. + Gain in P.E.

or $\quad Fx = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

or $\quad 10 \times 0.5 = \frac{1}{2} \times 4 \times v^2 + \frac{1}{2} \times 24 \times (0.5)^2$

or $\quad 5 = 2v^2 + 3$ or $v^2 = 1$

$\therefore \quad v = 1$ m s^{-1}

16. Suppose a variable force $\vec{F}$ acts on a body of mass m and produces displacement $\vec{ds}$ in its own direction ($\theta = 0°$). The small work done is

$$dW = \vec{F} \cdot \vec{ds} = Fds\cos 0° = F\,ds$$

According to Newton's second law of motion,

$$F = ma = m\frac{dv}{dt}$$

$$\therefore \quad dW = m\frac{dv}{dt} \cdot ds = mv\,dv \quad \left[\because \frac{ds}{dt} = v\right]$$

If the applied force increases the velocity from u to v, then the total work done on the body will be

$$W = \int dW = \int_u^v mv\,dv = m\int_u^v v\,dv$$

$$= m\left[\frac{v^2}{2}\right]_u^v = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$W = K_f - K_i$ = Change in K.E. of the body.

This proves the work-energy theorem for a variable force.

17. As given that

Volume of water = 30 m^3

Time taken by pump to fill tank = 15 min

= 15 × 60 = 900 s

The height of tank = 40 m

The efficiency of pump = 30%

Consumption of power by pump = ?

Mass of water pumped = Volume × density

= 30 × 10^3 kg

(Density of water = 10^3 kg m^{-3})

Output power, $P_0 = \frac{W}{t} = \frac{mgh}{t}$

$= \frac{30 \times 10^3 \times 9.8 \times 40}{900} = 13067$ W

P_i is required power (input power), then efficiency

$\eta = \frac{P_0}{P_i}$

$P_i = \frac{P_0}{\eta} = \frac{13067}{30/100} \simeq 43557 \text{ W} \simeq 43.557 \text{ kW}$

18. Let I_1 and I_2 be the moments of inertia of two discs. When they are brought in contact, the moments of inertia of the two discs system will be $I_1 + I_2$. Let ω be the angular speed of the combined system.

(a) Total initial angular momentum of the two discs, $L_1 = I_1\omega_1 + I_2\omega_2$

Total final angular momentum of the combined system, $L_2 = (I_1 + I_2)\omega$

From the law of conservation of angular momentum,

$L_2 = L_1$ or $(I_1 + I_2)\omega = I_1\omega_1 + I_2\omega_2$

$$\text{or} \quad \omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} \quad \text{...(i)}$$

(b) Initial K.E. of the two disc,

$$K_1 = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 \quad \text{...(ii)}$$

Final K.E. of the combined system,

$$K_2 = \frac{1}{2}(I_1 + I_2)\omega^2 \quad \text{...(iii)}$$

$\therefore$ From (i) and (iii), we get

$$K_2 = \frac{1}{2}(I_1 + I_2)\left(\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2$$

$$\text{or} \quad K_2 = \frac{1}{2}\frac{(I_1\omega_1 + I_2\omega_2)^2}{(I_1 + I_2)} \quad \text{...(iv)}$$

Eqn. (ii) and eqn. (iv) gives,

$$K_1 - K_2 = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 - \frac{1}{2}\frac{(I_1\omega_1 + I_2\omega_2)^2}{(I_1 + I_2)}$$

$$= \frac{1}{2(I_1 + I_2)}[(I_1\omega_1^2 + I_2\omega_2^2)(I_1 + I_2) - (I_1\omega_1 + I_2\omega_2)^2]$$

$$= \frac{1}{2(I_1 + I_2)}\left[\begin{array}{c} I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_2 I_1\omega_2^2 + I_1 I_2\omega_1^2 \\ - I_1^2\omega_1^2 - I_2^2\omega_2^2 - 2I_1 I_2\omega_1\omega_2 \end{array}\right]$$

$$= \frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2$$

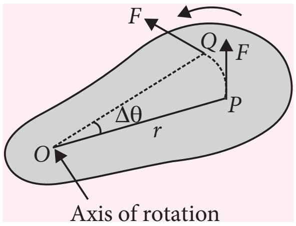
which is a positive quantity.

Hence, $K_1 - K_2 > 0$ or $K_1 > K_2$

or $K_2 < K_1$ *i.e.*, rotational K.E. of the combined system is less than the sum of the initial energies of the two discs.

Hence there occurs a loss of K.E. on combining the two discs and is the dissipation of energy because of the frictional forces between the faces of the two discs. These forces bring about a common angular speed of the two discs on combining. This however is an internal loss and angular momentum remains conserved.

19. As shown in the figure, suppose a body undergoes an angular displacement $\Delta\theta$ under the action of a tangential force F.



The work done in the rotational motion of the body,

$\Delta W = F \times$ distance along the arc PQ

But $\Delta\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{\text{Arc } PQ}{r}$

$\therefore$ Arc $PQ = r\Delta\theta$

Hence $\Delta W = F \times r\,\Delta\theta$

or $\Delta W = \tau\,\Delta\theta$

i.e., Work done by a torque

= Torque $\times$ angular displacement

If the torque applied is not constant, but variable, the total work done by the torque is given by

$$W = \int_{\theta_1}^{\theta_2} \tau\, d\theta$$

We know that

$\Delta W = \tau\,\Delta\theta$

Dividing both sides by Δt, we get

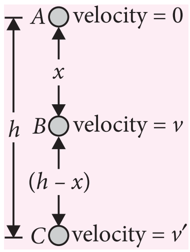
$$\frac{\Delta W}{\Delta t} = \tau\frac{\Delta\theta}{\Delta t}$$

or $P = \tau\omega$

i.e., Power = Torque $\times$ angular velocity

20. Consider a body of mass m lying at position A at a height h above the ground which is reference line. As the body falls, its kinetic energy increases at the expense of potential energy.

At point A : The body is at rest.
K.E. of the body, $K_A = 0$
P.E. of the body, $U_A = mgh$
Total mechanical energy,
$E_A = K_A + U_A = mgh$



At point B : Suppose the body falls freely through height x and reaches the point B with velocity v. Then

$v^2 - 0^2 = 2gx$ [Using $v^2 - u^2 = 2as$]

or $v^2 = 2gx$

$$\therefore \quad K_B = \frac{1}{2}mv^2 = \frac{1}{2}m \times 2gx = mgx$$

and $U_B = mg(h - x)$

Hence $E_B = K_B + U_B = mgx + mg(h - x) = mgh$

At point C : Suppose the body finally reaches at point C on the ground with velocity v'. Then considering motion from A to C,

$v'^2 - 0^2 = 2gh$ or $v'^2 = 2gh$

$$\therefore \quad K_C = \frac{1}{2}mv'^2 = \frac{1}{2}m \times 2gh = mgh$$

$U_C = mg \times 0 = 0$

$E_C = K_C + U_C = mgh$

Clearly, as the body falls, its P.E. decreases and K.E. increases by an equal amount. However, its total mechanical energy remains constant (= mgh) at all points. Thus total mechanical energy is conserved during free fall of a body.

21. Here, Mass of the bullet $m_1 = 0.012$ kg
Mass of the block $m_2 = 0.4$ kg
Initial velocity of the bullet, $u_1 = 70 \text{ m s}^{-1}$
Initial velocity of the block, $u_2 = 0$
Since on striking the wooden block, the bullet comes to rest w.r.t. the block of wood, the collision is inelastic in nature.
Let v be the velocity acquired by the combination.
Applying principle of conservation of linear momentum

$(m_1 + m_2)v = m_1u_1 + m_2u_2 = m_1u_1$

$$v = \frac{m_1u_1}{m_1 + m_2} = \frac{0.012 \times 70}{0.012 + 0.4} = \frac{0.84}{0.412} = 2.04 \text{ m s}^{-1}$$

Let the block rise to a height h.

$$\frac{1}{2}(m_1 + m_2)v^2 = (m_1 + m_2)gh$$

$$h = \frac{v^2}{2g} = \frac{(2.04)^2}{2 \times 9.8} = 21.2 \text{ cm}$$

The loss in kinetic energy of the system will appear as heat.

$\therefore$ Heat produced = Initial K.E. of bullet – final K.E. of combination

$$= \frac{1}{2}m_1u_1^2 - \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2} \times 0.012 \times (70)^2 - \frac{1}{2}(0.412)(2.04)^2$$

$= 29.4 - 0.86 = 28.54$ J

22. Stable equilibrium : A body is said to be in stable equilibrium if it tends to regain its equilibrium position after being slightly displaced and released. In stable equilibrium, a body has minimum potential energy and its centre of mass goes higher when it is slightly displaced.
Unstable equilibrium : A body is said to be in unstable equilibrium if it gets further displaced from its equilibrium position after being slightly displaced and released. In unstable equilibrium, a body possesses maximum potential energy and its centre of mass goes lower on being slightly displaced.
Neutral equilibrium : If a body stays in equilibrium position even after being slightly displaced and released, it is said to be in neutral equilibrium. When a body is slightly displaced from its position of neutral equilibrium, its centre of mass is neither raised nor lowered and its potential energy remains constant.

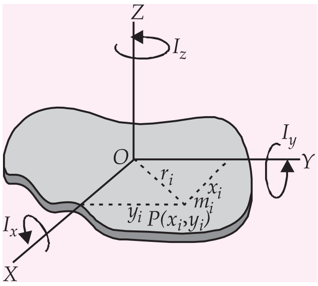
23. (a) Keen observer, curiosity and quest for knowledge.
(b) Y kW power rating is more than X kW power rating.
(c) Output power

$$= \frac{mgh}{t} = \frac{1000 \times 9.8 \times 10}{5} = 19600 \text{ W}$$

As $\eta = \dfrac{\text{Output power}}{\text{Input power}}$ $\quad \therefore \quad \eta = \dfrac{19600}{36000}$

$\Rightarrow \quad \eta = 0.54$ or $\eta = 0.54 \times 100 = 54\%$

24. (a) Theorem of perpendicular axes : It states that the moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the lamina about any two mutually perpendicular axes in its plane and intersecting each other at the point, where the perpendicular axis passes through it.



If I_x, I_y and I_z are the moments of inertia of the plane lamina about the axes OX, OY and OZ respectively, then according to theorem of perpendicular axes,

$I_z = I_x + I_y$ ...(i)

Suppose that the rigid body is made of n particles of masses $m_1, m_2, ... m_n$ lying at distance $r_1, r_2........, r_n$ from the axis of rotation OZ. Further suppose that the i^{th} particle of mass m_i lies at point $P(x_i, y_i)$, such that $OP = r_i$. Then,

$$r_i^2 = x_i^2 + y_i^2$$

Now, moment of inertia of the body about the axis OZ is given by

$$I_z = \sum_{i=1}^{n} m_i r_i^2 = \sum_{i=1}^{n} m_i (x_i^2 + y_i^2)$$

$$= \sum_{i=1}^{n} m_i x_i^2 + \sum_{i=1}^{n} m_i y_i^2 \quad ...(ii)$$

But $\sum_{i=1}^{n} m_i x_i^2 = I_y$, moment of inertia of the body about axis OY and $\sum_{i=1}^{n} m_i y_i^2 = I_x$, the moment of inertia of the body about axis OX.

Therefore, the equation (ii) becomes

$I_z = I_x + I_y$

(b) Radius of hoop, $R = 2$ m

Mass of hoop, $M = 100$ kg

Velocity of centre of mass = 20 cm s^{-1} = 0.2 m s^{-1}

The total kinetic energy of the hoop

$$= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}MR^2\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = Mv^2$$

$= 100 \times (0.2)^2 = 4$ J

By work-energy theorem,

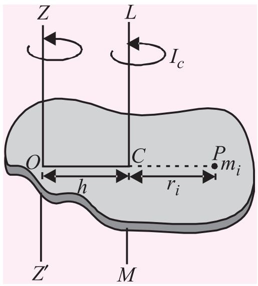
Work required to stop the hoop

= Total kinetic energy of hoop = 4 J

OR

(a) Theorem of parallel axes : It states that the moment of inertia of a rigid body about any axis is equal to its moment of inertia about a parallel axis through its centre of mass plus the product of the mass of the body and the square of the perpendicular distance between the two axes.

Let I_c be the moment of inertia of a body of mass M about an axis LM passing through its centre of mass C. Let I be the moment of inertia of the body about an axis ZZ' parallel to the axis LM and at a distance h from it as shown in the figure.



Then, according to the theorem of parallel axes,

$I = I_c + Mh^2$ ...(i)

Consider that i^{th} particle located at the point P in the body is of mass m_i and lies at a distance r_i from the axis LM. Then, the distance of i^{th} particle from axis ZZ' is $(r_i + h)$. The moment of inertia of the i^{th} particle about the axis LM is $m_i r_i^2$. Therefore, moment of inertia of the body about the axis LM is given by

$$I_c = \sum_{i=1}^{n} m_i r_i^2$$

Also, moment of inertia of the i^{th} particle about the axis ZZ' is $m_i(r_i + h)^2$, so that moment of inertia of the body about the axis ZZ' is given by

$$I = \sum_{i=1}^{n} m_i (r_i + h)^2 = \sum_{i=1}^{n} m_i (r_i^2 + h^2 + 2hr_i)$$

$$= \sum_{i=1}^{n} m_i r_i^2 + \sum_{i=1}^{n} m_i h^2 + \sum_{i=1}^{n} 2hm_i r_i$$

$$= I_c + \left(\sum_{i=1}^{n} m_i\right)h^2 + 2h\sum_{i=1}^{n} m_i r_i \quad ...(ii)$$

Now, $\sum_{i=1}^{n} m_i = M$, mass of the body and

$\sum_{i=1}^{n} m_i r_i$ = sum of the moments of the masses of the particles constituting the body about an axis through its centre of mass. Since sum of the moments of the masses of the particles constituting the body an axis through its centre of mass must be zero.

$$\sum_{i=1}^{n} m_i r_i = 0$$

In R.H.S. of the equation (ii), substituting the values of the two factors, we have

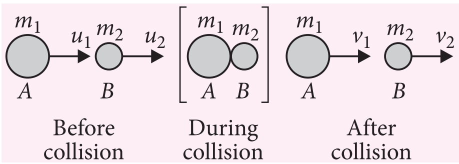
$I = I_c + Mh^2$

It proves the theorem of parallel axes for moment of inertia.

(b) For the rod of mass M and length l, $I = Ml^2/12$. Using the parallel axes theorem, $I' = I + Ma^2$ with $a = l/2$ we get,

$$I' = M\frac{l^2}{12} + M\left(\frac{l}{2}\right)^2 = \frac{Ml^2}{3}$$

25. Consider two perfectly elastic bodies A and B of masses m_1 and m_2 moving along the same straight line with velocities u_1 and u_2 respectively. Let $u_1 > u_2$. After sometime, the two bodies collide head-on and continue moving in the same direction with velocities v_1 and v_2 respectively. The two bodies will separate after the collision if $v_2 > v_1$.



As linear momentum is conserved in any collision, so

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ ...(i)

or $m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$

or $m_1(u_1 - v_1) = m_2(v_2 - u_2)$ ...(ii)

Since K.E. is also conserved in an elastic collision, so

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

or $m_1u_1^2 - m_1v_1^2 = m_2v_2^2 - m_2u_2^2$

or $m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2)$ ...(iii)

Dividing (iii) by (ii), we get

$u_1 + v_1 = v_2 + u_2$

or $u_1 - u_2 = v_2 - v_1$ ...(iv)

or Relative velocity of A w.r.t. B before collision

= Relative velocity of B w.r.t. A after collision

or Relative velocity of approach

= Relative velocity of separation

Thus, in a one-dimensional elastic collision, the relative velocity of approach before collision is equal to the relative velocity of separation after the collision.

Velocities of the bodies after the collision : from equation (iv), we get

$v_2 = u_1 - u_2 + v_1$

Putting this value of v_2 in equation (i), we get

$m_1u_1 + m_2u_2 = m_1v_1 + m_2(u_1 - u_2 + v_1)$

$= m_1v_1 + m_2u_1 - m_2u_2 + m_2v_1$

or $(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$

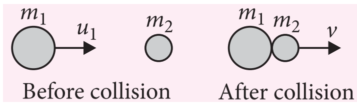
or $$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

Similarly, $$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \left(\frac{2m_1}{m_1 + m_2}\right)u_1$$

OR

When the two colliding bodies stick together and move as a single body with a common velocity after the collision, the collision is perfectly inelastic.

A body of mass m_1 moving with velocity u_1 collides head-on with another body of mass m_2 at rest. After the collision, the two bodies move together with a common velocity v.



As the linear momentum is conserved, so

$m_1u_1 + m_2 \times 0 = (m_1 + m_2)v$

or $$v = \frac{m_1}{(m_1 + m_2)}u_1$$

The loss in kinetic energy on collision is

$$\Delta K = K_i - K_f = \frac{1}{2}m_1u_1^2 - \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2}m_1u_1^2 - \frac{1}{2}(m_1 + m_2)\left[\frac{m_1u_1}{m_1 + m_2}\right]^2$$

$$= \frac{1}{2}m_1u_1^2 - \frac{1}{2}\frac{m_1^2}{(m_1 + m_2)}u_1^2 = \frac{1}{2}m_1u_1^2\left[1 - \frac{m_1}{(m_1 + m_2)}\right]$$

or $$\Delta K = \frac{1}{2}\frac{m_1m_2}{(m_1 + m_2)}u_1^2$$

This is a positive quantity. The kinetic energy is lost mainly in the form of heat and sound.

Moreover,

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}(m_1 + m_2)v^2}{\frac{1}{2}m_1u_1^2} = \frac{(m_1 + m_2)}{m_1}\cdot\frac{v^2}{u_1^2}$$

$$= \frac{(m_1 + m_2)}{m_1}\left(\frac{m_1}{m_1 + m_2}\right)^2$$

or $\dfrac{K_f}{K_i} = \dfrac{m_1}{m_1 + m_2}$

which is less than 1. This again shows that the kinetic energy after the collision is less than the kinetic energy before the collision.

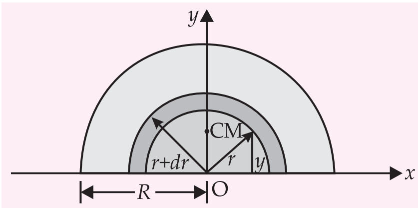
If the target is massive, *i.e.*, $m_2 >> m_1$, then

$\dfrac{K_f}{K_i} \simeq 0$ *i.e.*, $K_f \simeq 0$

Hence, when a light moving body collides against any massive body at rest and sticks to it, practically all of its kinetic energy is lost.

26. (a) If M is the mass of the half-disc, of radius R, then mass per unit area of the half-disc

$$\sigma = \frac{M}{\pi R^2/2} = \frac{2M}{\pi R^2}$$



Let us consider an element (which is a semi-circular wire) of this disc between r and $(r + dr)$.

Area of the element $= \dfrac{1}{2}[\pi(r+dr)^2 - \pi r^2]$

$= \dfrac{\pi}{2}[r^2 + (dr)^2 + 2rdr - r^2] \approx \pi r dr \qquad [\because (dr)^2 << 0]$

Mass of the element,

$$dm = (\pi r dr)\,\sigma = (\pi r dr)\left(\frac{2M}{\pi R^2}\right) = \frac{2Mr}{R^2}dr$$

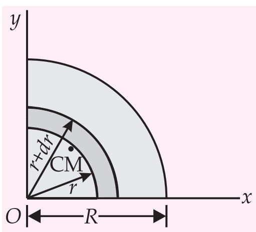
(i) $x_{CM} = 0$ (due to symmetry)

(ii) $y_{CM} = \dfrac{1}{M}\displaystyle\int y\,dm = \dfrac{1}{M}\int_0^R \left(\dfrac{2r}{\pi}\right)\left(\dfrac{2Mr}{R^2}dr\right)$ [as $y = 2r/\pi$]

$= \dfrac{4}{\pi R^2}\displaystyle\int_0^R r^2 dr = \dfrac{4}{\pi R^2}\left(\dfrac{R^3}{3}\right) = \dfrac{4R}{3\pi}$

Thus, the centre of mass of the half-disc is $(0, 4R/3\pi)$.

(b)



For quarter-disc,

$$\sigma = \frac{M}{\pi R^2/4} = \frac{4M}{\pi R^2},$$

Area of the element $= \dfrac{1}{2}\pi r dr$,

$$dm = \left(\frac{1}{2}\pi r dr\right)\sigma = \frac{2Mr}{R^2}dr$$

$$x_{CM} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^R \left(\frac{2r}{\pi}\right)\left(\frac{2Mr}{R^2}dr\right) = \frac{4R}{3\pi}$$

[as $x = y = \dfrac{2r}{\pi}$]

Similarly, $y_{CM} = \dfrac{1}{M}\displaystyle\int y\,dm = \dfrac{4R}{3\pi}$

Thus, the centre of mass of the quarter disc is $\left(\dfrac{4R}{3\pi}, \dfrac{4R}{3\pi}\right)$.

OR

(a) The radius of gyration of a body about its axis of rotation may be defined as the distance from the axis of rotation at which, if the whole mass of the body were concentrated, its moments of inertia about the given axis would be the same as with the actual distribution of mass.

The radius of gyration of a body depends on

(i) the position and orientation of the axis of rotation.

(ii) the distribution of mass of the body about the axis of rotation.

(b) Given, area of square = area of rectangle

$\Rightarrow \quad c^2 = ab$

Also, $a > b$

(i) $\dfrac{I_{xR}}{I_{xS}} = \dfrac{Mb^2/12}{Mc^2/12} = \dfrac{b^2}{c^2} = \dfrac{b^2}{ab} = \dfrac{b}{a} < 1$

(ii) $\dfrac{I_{yR}}{I_{yS}} = \dfrac{Ma^2/12}{Mc^2/12} = \dfrac{a^2}{c^2} = \dfrac{a^2}{ab} = \dfrac{a}{b} > 1$

(iii) $I_{zR} = \dfrac{1}{12}M(a^2 + b^2)$

$I_{zS} = \dfrac{1}{12}M(c^2 + c^2)$

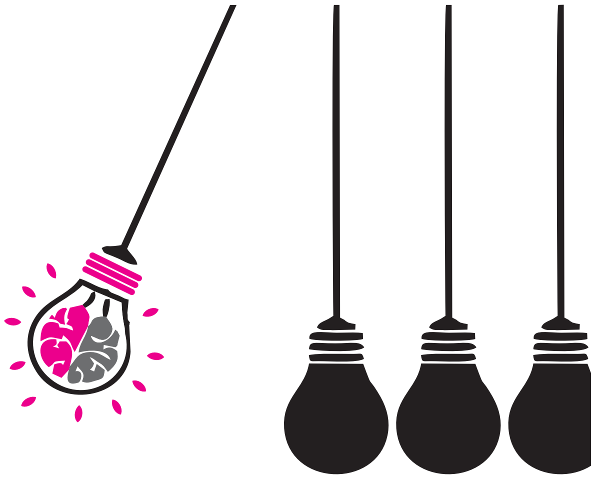
Now, $I_{zR} - I_{zS} = \dfrac{1}{12}M[a^2 + b^2 - 2c^2]$

$= \dfrac{1}{12}M(a^2 + b^2 - 2ab) \qquad (\because c^2 = ab)$

$\Rightarrow \quad I_{zR} - I_{zS} = \dfrac{1}{12}M(a-b)^2 > 0$

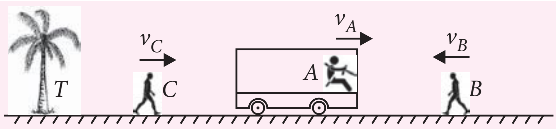
$\therefore \quad I_{zR} > I_{zS} \Rightarrow \dfrac{I_{zR}}{I_{zS}} > 1$

CORE CONCEPT



Relative Motion

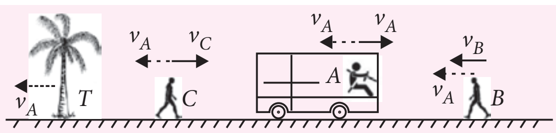
Consider a situation where we have multiple objects being observed from ground reference frame and their velocity have also been marked as shown-



A, B and C are 3 persons and T is a tree.

Now let us change our reference frame and situate ourselves into A's frame. When we fix our frame on A, obviously A won't appear to move now. Velocity being a vector quantity, how does it appear zero? Only when you add another vector of same magnitude but opposite in direction to it. So basically what we did was we reverted and added the velocity of our reference frame. But this cannot be done alone for us and need to be done to all other objects being observed from the frame, *i.e.* we "REVERT AND ADD" the velocity and acceleration of the frame in which we are situated to the velocity and acceleration of all other objects being observed.

For the sake of understanding, let us make a convention that we put a big **+** symbol on the person/object that we situate our reference frame in. So in A's reference frame -



$\therefore$ Velocity of B with respect to A becomes,

$v_{BA} = v_A + v_B$ towards left

Velocity of C with respect to A becomes,

$$v_{CA} = \begin{cases} v_C - v_A & \text{towards right} \\ v_A - v_C & \text{towards left} \end{cases}$$

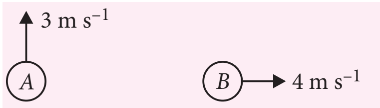
Velocity of T with respect to A becomes,

$v_{TA} = v_A$ towards left

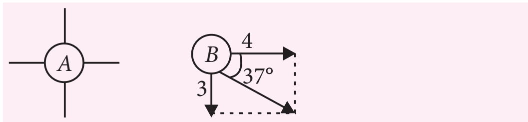
In general we can say, velocity of B with respect to A when the motion is not constrained along one dimension can be written as,

$$\underbrace{\vec{v}_{BA} = \vec{v}_B}_{\text{adding velocity of } B} + \underbrace{(-\vec{v}_A)}_{\text{reverting velocity of } A}$$

For example if A is moving towards north with 3 m s^{-1} and B towards east with 4 m s^{-1} with respect to ground then,



(With respect to ground)



$v_{BA} = \sqrt{(3)^2 + (4)^2} = 5$ ms^{-1}

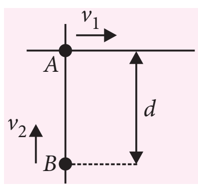
(With respect to A)

$\therefore$ Velocity of B with respect to A becomes 5 m s^{-1} at 37° south of east.

Remember one thing as a thumb rule, the frame in which we situate our reference frame becomes stationary for us while all other objects surrounding it appear to move. To make you understand what I am trying to convey, consider a situation where you are driving a car and you hit a person standing on road. In your frame what you see is that the person came and struck your car since you don't see your moving car!

Now, let us try to use these in some examples-

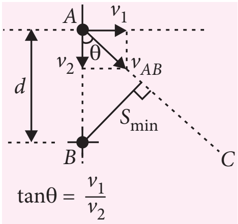
Q1. A and B start moving from different points in mutually perpendicular directions as shown. Find the minimum separation between them.



Contributed By: Bishwajit Barnwal, Aakash Institute, Kolkata

Sol.: Let us fix our frame on B.

A appears to be moving along the line AC with respect to B as shown. The minimum separation clearly is the length of perpendicular drawn on AC.

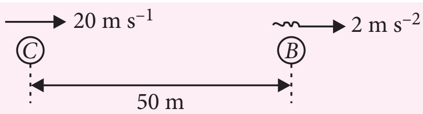


Using geometry,

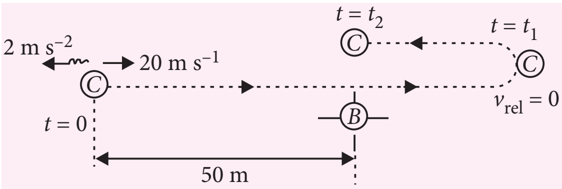
$$S_{min} = d\sin\theta = d\cdot\frac{v_1}{\sqrt{v_1^2+v_2^2}}$$

Q2. A car moving with a constant velocity of 20 m s^{-1} is behind a bus at a separation of 50 m. The bus starts accelerating from rest at 2 m s^{-2}. Find the time at which the car is farthest from the bus ahead of bus. Also, find the time after which the bus overtakes the car.

Sol.: With respect to ground



With respect to bus B



With respect to B, the motion of C is decelerated uniformly. So, B would be farthest when its relative velocity becomes zero.

$\therefore$ $v_{rel} = u_{rel} - a_{rel}\, t_1$

$\Rightarrow$ $0 = 20 - 2t_1$ $\Rightarrow$ $t_1 = 10$ s

At the instant B overtakes C, their positions are identical and for B, C appears moving backward which is shown at time $t = t_2$.

$\therefore$ Between $t = 0$ and $t = t_2$,

$$S_{rel} = u_{rel}\, t_2 - \frac{1}{2}a_{rel}\, t^2$$

$$\Rightarrow \quad 50 = 20t_2 - \frac{1}{2}\times 2t_2^2 \quad \Rightarrow \quad t_2^2 - 20t_2 + 50 = 0$$

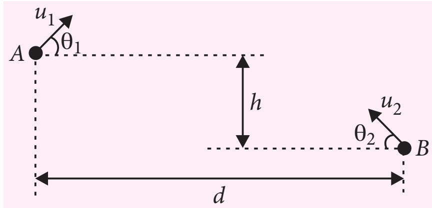
$$\Rightarrow \quad t_2 = \frac{20 \pm \sqrt{400-200}}{2}$$

$$= \frac{20 \pm 10\sqrt{2}}{2} = \left(10 \pm 5\sqrt{2}\right)\text{ s}$$

$\left(10 - 5\sqrt{2}\right)$ s is the time when C overtakes B.

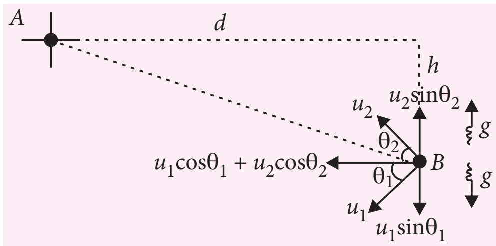
$\therefore$ Our required time is $\left(10 + 5\sqrt{2}\right)$ s.

Q3. Two stones A and B are fired from air with velocities as shown in the figure.



If they collide in mid air, find the condition of collision.

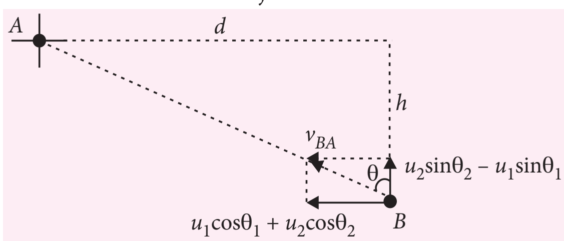
Sol.: Both the stones are under the action of gravity, hence both have the same acceleration irrespective of their direction of projection. Hence if we fix our frame in one of them (say A).



$\therefore$ Their relative acceleration is zero.

So, it means their relative velocity remains constant in both magnitude and direction.

$\therefore$ If an object travels with a constant velocity and strikes a stationary object, its velocity should be directed towards the object.



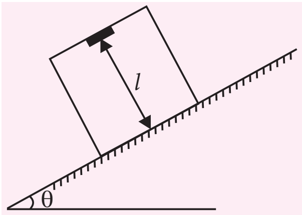
Considering the components,

$$\therefore \quad \tan\theta = \frac{u_1\cos\theta_1 + u_2\cos\theta_2}{u_2\sin\theta_2 - u_1\sin\theta_1} = \frac{d}{h}$$

Had time of collision been asked, it could have been found from either direction.

$$\Delta t = \frac{d}{u_1\cos\theta_1 + u_2\cos\theta_2} = \frac{h}{u_2\sin\theta_2 - u_1\sin\theta_1}$$

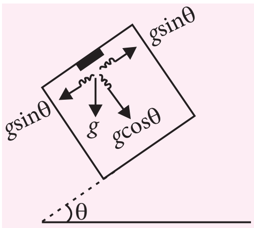
Q4. A box is kept on a smooth inclined plane of inclination θ and released to fall from rest. Sometime later, a nut-bolt attached to the ceiling of box gets detached. Find the time after which the bolt strikes the floor of box if the floor to ceiling distance is l.



Sol.: Since the surface is smooth, the acceleration of box is $g\sin\theta$ onwards. Let us say after sometime the bolt gets detached. Since the box attained a non-zero velocity and bolt being a part of the box had the same velocity. Hence their relative velocity was zero at the instant of dropping of bolt.

$\therefore$ Considering with respect to box,

The bolt had a free fall and hence we reverted the acceleration of box $g\sin\theta$ and added to g vectorially. Breaking the components of g we see that $g\sin\theta$ gets cancelled out. We are left over only with $g\cos\theta$ which can be treated to be g_{eff}.

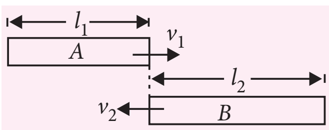


$\therefore$ Applying,

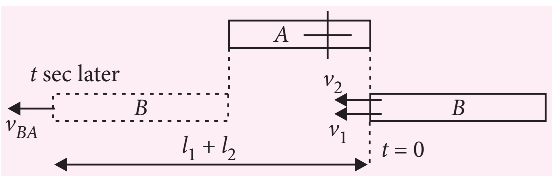
$$l = \frac{1}{2} g_{\text{eff}}\, t^2 \quad \therefore \quad t = \sqrt{\frac{2l}{g\cos\theta}}$$

Q4. Two trains of lengths l_1 and l_2 are moving with speed v_1 and v_2 in opposite directions on parallel tracks. Find the distance travelled by each train till they cross each other.

Sol.: Crossing starts when their head are parallel to each other.



We can easily find distance travelled by them $x_1 = v_1 t$ and $x_2 = v_2 t$ if the time of crossing is known. For this, let us fix our reference frame on A.



$\therefore$ $v_{BA} = v_1 + v_2$

Do we observe one interesting fact here? It does not matter who moves faster, when moving from opposite directions, the other speed will always appear to be faster to us since the magnitude of our velocity is getting added to theirs.

$\therefore$ Relative distance covered,

$S_{BA} = v_{BA}\, t$

$$\Rightarrow \quad t = \frac{l_1 + l_2}{v_1 + v_2}$$

Q5. Consider a hot air balloon starting from rest from surface of earth accelerating upwards with a constant acceleration of 5 m s^{-2}. After 2 s a stone is thrown from the balloon vertically upwards with a relative velocity of 10 m s^{-1}. Find

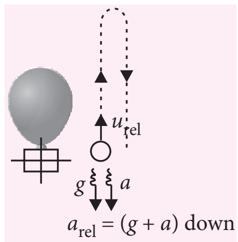
(i) the time after which the stone again reaches the balloon.

(ii) distance travelled by the stone after being thrown till it lands on the ground.

Sol.: In the 1st part it does not matter how far has the ballon reached or how fast it was moving at the instant of throw, since the relative velocity of throw is already given.

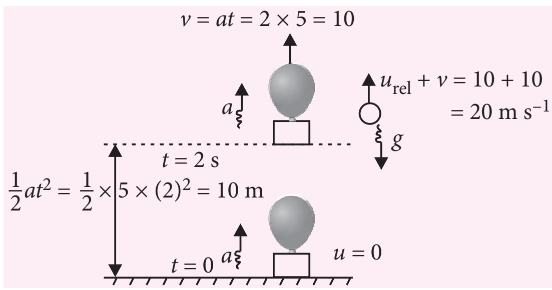
Whenever an object is thrown from a moving frame, it carries the inertia, *i.e*, velocity of the frame but not the acceleration since it is no longer a part of frame.

For 1st part, we fix our frame on balloon, so we revert and add the acceleration of balloon to the acceleration of the stone. Time after which the stone reaches the balloon.

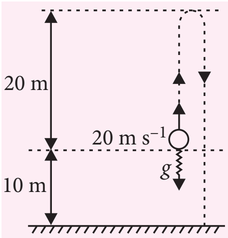


$$\Delta t = \frac{2u_{\text{rel}}}{a_{\text{rel}}} = \frac{2\times 10}{10+5} = \frac{4}{3}\text{ s}$$

For the second part we need the velocity as well as acceleration of the balloon.



So, basically we have a stone projected upwards with a velocity 20 m s^{-1} with respect to ground from a height 10 m above the ground which means it moves to a maximum height of H above point of projection where



$$H=\frac{(20)^2}{2\times 10}=20\text{ m}$$

$\therefore$ Total distance travelled

= 20 + 20 + 10 = 50 m

Let us now move to two more applications of relative motion.

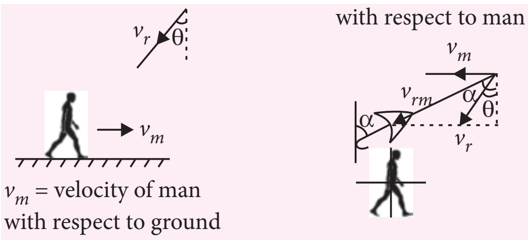
1. Rain-man-umbrella problems
2. Crossing of river problems

Rain-man-unbrella problems

Here rain falling at a certain speed v_r with respect to ground at an angle θ with respect to vertical would be given and a person standing/running in a particular direction would be needed to be protected by properly directing the axis of the umbrella.

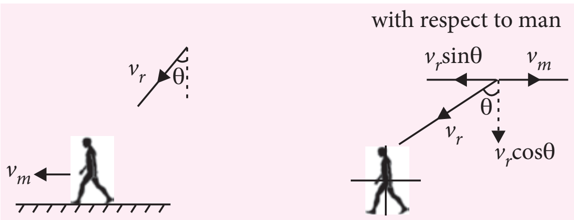
In such situation, since the man is supposed to hold the umbrella so, we would fix our reference frame into the person.

Case-I Man running against rain

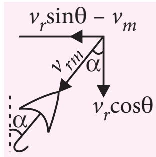
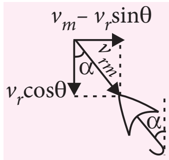
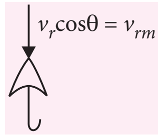


Clearly, $\tan\alpha=\dfrac{v_m+v_r\sin\theta}{v_r\cos\theta}$

Case-II Man running with rain



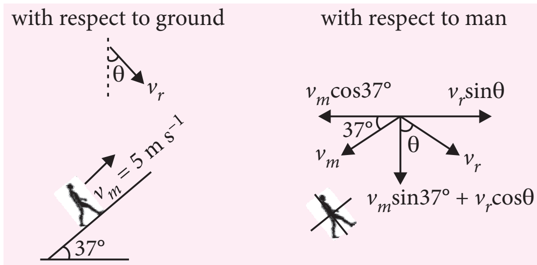
Here again 3 situations may arise-

Case-I	Case-II	Case-III
$v_r\sin\theta > v_m$ ⇓  $\tan\alpha=\dfrac{v_r\sin\theta-v_m}{v_r\cos\theta}$	$v_r\sin\theta < v_m$ ⇓  $\tan\alpha=\dfrac{v_m-v_r\sin\theta}{v_r\cos\theta}$	$v_r\sin\theta = v_m$ ⇓  In this case rain appears to fall vertically, which can happen only if the horizontal velocity of rain and man match.

Let us solve an example.

Q6. To a man running up an inclined plane of inclination 37° with horizontal at the speed of 5 m s^{-1}, rain appears, to be falling at the speed of 6 m s^{-1} vertically downwards. Find the actual magnitude and direction of velocity of rain.

Sol.: Let rain be falling at speed v_r at angle θ with respect to ground.



$$\therefore\quad v_r\sin\theta = v_m\cos 37° = 5\times\frac{4}{5}=4\text{ m s}^{-1}$$

and $v_m\sin 37° + v_r\cos\theta = 6$ (given)

$$\therefore\quad 5\times\frac{3}{5}+v_r\cos\theta=6 \quad\Rightarrow\quad v_r\cos\theta=3$$

$$\tan\theta=\frac{4}{3}\quad\Rightarrow\quad\theta=53°$$

$$\text{and }\sqrt{(v_r\sin\theta)^2+(v_r\cos\theta)^2}=\sqrt{(4)^2+(3)^2}$$

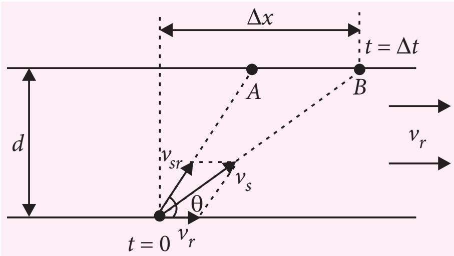
$$\Rightarrow\quad v_r=5\text{ m s}^{-1}$$

$\therefore$ Rain is falling at 5 m s^{-1} at 53° with respect to vertical with respect to ground.

Crossing of river problems

Let d be the width of a river which is flowing at speed v_r and v_{sr} be the velocity of a swimmer with respect to river which basically means, had the river been stationary, the swimmer would have moved with this

velocity but since the river is flowing, the actual veocity of swimmer will be a vector sum of $\vec{v}_{sr} + \vec{v}_r$.

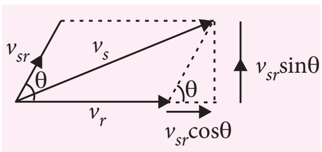


So basically, if the person tries to reach A, he reaches B, due to the drift further created by the river flow. The motion along the two mutually perpendicular directions take place independently, *i.e.*, the motion along x-axis is not influenced by forces acting along y-axis. So we split the motion into two directions.

(i) along the length of river

(ii) perpendicular to the length of river

As clear from the velocity diagram, the component $v_{sr}\sin\theta$ will enable the person to cross the river, hence time of crossing



$$\Delta t = \frac{d}{v_{sr}\sin\theta} \qquad \text{... (i)}$$

While the component $(v_r + v_{sr}\cos\theta)$ will enable the person to drift along the length of river. Hence drift

$$\Delta x = (v_r + v_{sr}\cos\theta)\Delta t \qquad \text{... (ii)}$$

Using these two equations whatever is asked can be found out.

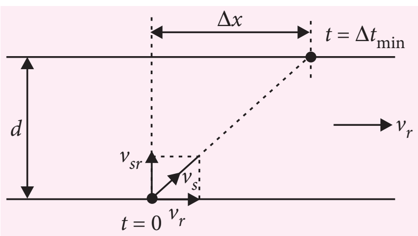
Special cases:

I Minimum time of crossing

Since, $\Delta t = \dfrac{d}{v_{sr}\sin\theta}$, so to keep Δt_{min},

$\sin\theta$ = maximum $\Rightarrow$ $\theta = 90°$

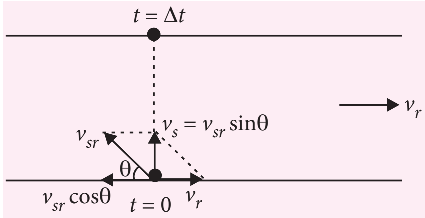
So, the person should try to swim perpendicular to the direction of the river flow.



$$\therefore \quad \Delta t_{min} = \frac{d}{v_{sr}} \text{ and hence drift } \Delta x = v_r \frac{d}{v_{sr}}$$

II Shortest path

The person should try to swim such that the resultant velocity becomes perpendicular to the river flow.



$$\therefore \quad v_r = v_{sr}\cos\theta \quad \Rightarrow \quad \cos\theta = \frac{v_r}{v_{sr}}$$

$$\therefore \quad v_s = v_{sr}\sin\theta = v_{sr}\cdot\frac{\sqrt{v_{sr}^2 - v_r^2}}{v_{sr}} = \sqrt{v_{sr}^2 - v_r^2}$$

$$\therefore \quad \Delta t = \frac{d}{\sqrt{v_{sr}^2 - v_r^2}}$$

SOLUTION OF AUGUST 2016 CROSSWORD

[1]S				[2]P	H	O	T	O	C	A	T	H	O	D	E		[3]H			[4]H
I				L			[5]D						[6]S	T	A	T	O	R		O
G			[7]M	A	G	N	O	X									W			O
M				N			U						[8]B	A	[9]F	F	L	E		K
A				C			B								R					U
P		[10]S		K		[11]F	L	U	I	[12]D	I	T	Y		E					P
A		U					E			I					S				[13]L	
R		R					T			[14]L	E	W	I	S	N	U	M	B	E	R
T		G	[15]C		[16]P					A					E				E	
I	[17]C	E	R	M	E	T				T			[18]P		L		[19]G		S	
C			Y		W		[20]E			O			E				L		R	
L		[21]C	O	N	T	I	N	U	U	M			R				I		U	
E			T		E		D			E			M				D		L	
[22]C			R		R		O			T		[23]B	E	C	Q	U	E	R	E	L
[24]A	X	I	O	M			E			E			A							
L			N		[25]B		R		[26]B	R	O	W	N	D	W	A	R	F		
X					A		G						C							
				[27]C	U	R	I	U	M		[28]R	H	E	O	L	O	G	Y		
[29]T	A	U	O	N	D		C					[30]F	L	U	X	M	E	T	E	R

Winner (August 2016)

- Aparna Shankar, Chennai

Solution Sender (July 2016)

- Ramesh Jha, Patna
- Devjit Acharjee, Kolkata

Solution Sender of Physics Musing

Set - 37

- Sachin Vyas, Indore (M.P.)
- Shayan Ghatak, Kolkata (W.B.)
- Anish Chandra, Kota (Rajasthan)

MPP-3 MONTHLY Practice Problems

Class XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Laws of Motion | Work, Energy and Power

Total Marks : 120 **Time Taken : 60 min**

NEET / AIIMS / PMTs

Only One Option Correct Type

1. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time as $a_c = k^2rt^2$, where k is a constant. The power delivered to the particle by the forces acting on it is

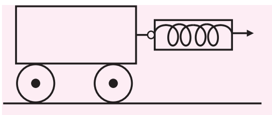
(a) $2\pi mk^2r^2t$ (b) mk^2r^2t

(c) $\frac{1}{3}mk^4r^2t^5$ (d) 0

2. A particle of mass m is tied to a light string of length l and rotated along a vertical circular path. What should be the minimum speed at the highest point of its path so that the string does not become slack at any position?

(a) $\sqrt{2gl}$ (b) $\sqrt{gl}$ (c) zero (d) $\sqrt{gl/2}$

3. A massless spring balance is attached to 2 kg trolley and is used to pull the trolley along a flat smooth surface as shown in the figure. The reading on the spring balance remains at 10 kg during the motion. The acceleration of the trolley is (Use $g = 9.8$ m s^{-2})



(a) 4.9 m s^{-2} (b) 9.8 m s^{-2}

(c) 49 m s^{-2} (d) 98 m s^{-2}

4. A curved section of a road is banked for a speed v. If there is no friction between the road and the tyres then,

(a) a car moving with speed v will not slip on the road

(b) a car is more likely to slip on the road at speeds higher than v, than at speeds lower than v

(c) a car is more likely to slip on the road at speeds lower than v, than at speeds higher than v

(d) a car can remain stationary on the road without slipping.

5. In a simple pendulum, the breaking strength of the string is double the weight of the bob. The bob is released from rest when the string is horizontal. The string breaks when it makes an angle θ with the vertical. Then,

(a) $\theta = \cos^{-1}(1/3)$ (b) $\theta = 60°$

(c) $\theta = \cos^{-1}(2/3)$ (d) $\theta = 75°$

6. A horizontal force of 12 N pushes a 0.5 kg book against a vertical wall. The book is initially at rest. If the coefficients of friction are $\mu_k = 0.6$ and $\mu_s = 0.8$, which of the following statements is true?

(a) The magnitude of the frictional force is 5 N.

(b) The magnitude of the frictional force is 7.2 N.

(c) The normal force is 5 N.

(d) The book will start moving and accelerate.

7. For a given force $F = 2x^2 - 3x - 2$, choose the correct option.

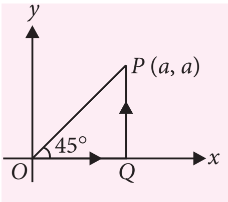
(a) $x = -1/2$ is position of stable equilibrium.

(b) $x = 2$ is position of stable equilibrium.

(c) $x = -1/2$ is position of unstable equilibrium.

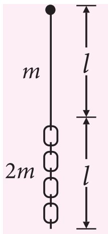
(d) $x = 2$ is position of neutral equilibrium.

8. A particle is moved from (0, 0) to (a, a) under a force $\vec{F} = (3\hat{i} + 4\hat{j})$ from two paths. Path 1 is OP and path 2 is OQP. Let W_1 and W_2 be the work done by this force in these two paths. Then,



(a) $W_1 = W_2$ (b) $W_1 = 2W_2$
(c) $W_2 = 2W_1$ (d) $W_2 = 4W_1$

9. A rope of length l and mass m is connected to a chain of length l and mass 2 m, and hung vertically as shown in figure. What is the change in gravitational potential energy if the system is inverted and hung from same point?



(a) mgl (b) 1.5 mgl (c) 0.5 mgl (d) 2 mgl

10. The ratio of momentum and kinetic energy of particle is inversely proportional to the time. Then, this is the case of a
(a) uniformly accelerated motion
(b) uniform motion
(c) uniformly retarted motion
(d) simple harmonic motion.

11. In a ballistics demonstration a police officer fires a bullet of mass 50.0 g with speed 200 m s^{-1} on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet?
(a) $2\sqrt{10}$ m s^{-1} (b) $20\sqrt{10}$ m s^{-1}
(c) $10\sqrt{2}$ m s^{-1} (d) $10\sqrt{20}$ m s^{-1}

12. A large force is acting on a body for a short time. The impulse imparted is equal to the change in
(a) acceleration (b) momentum
(c) energy (d) velocity.

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :
(a) If both assertion and reason are true and reason is the correct explanation of assertion
(b) If both assertion and reason are true but reason is not the correct explanation of assertion
(c) If assertion is true but reason is false
(d) If both assertion and reason are false.

13. **Assertion :** Pseudo force is an imaginary force which is recognised only by a non-inertial observer to explain the physical situation according to Newton's laws.
Reason : Pseudo force has no physical origin, *i.e.*, it is not caused by one of the basic interactions in nature. It does not exist in the action-reaction pair.

14. **Assertion :** A quick collision between two bodies is more violent than a slow collision, even when the initial and the final velocities are identical.
Reason : The rate of change of momentum determines the force is greater in a quick collision between two bodies.

15. **Assertion :** All central forces which follow the inverse square law are conservative forces.
Reason : Work done by the force or against the force does not depend on path, then force is called conservative force.

JEE MAIN / JEE ADVANCED / PETs

Only One Option Correct Type

16. A railway track is banked for a speed v, by making the height of the outer rail h higher than that of the inner rail. The distance between the rails is d. The radius of curvature of the track is r. Then,
(a) $\frac{h}{d} = \frac{v^2}{rg}$ (b) $\tan\left(\sin^{-1}\frac{h}{d}\right) = \frac{v^2}{rg}$
(c) $\tan^{-1}\left(\frac{h}{d}\right) = \frac{v^2}{rg}$ (d) $\frac{h}{r} = \frac{v^2}{dg}$

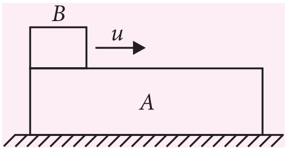
17. A car of mass m is driven from rest, with constant power P. After travelling a distance x, speed of the car becomes v. Then v is
(a) $\frac{P}{m}$ (b) $\frac{m}{P}$ (c) $\left(\frac{3xP}{m}\right)^{1/3}$ (d) $\left(\frac{2xP}{m}\right)^{1/3}$

18. An electron, mass m, collides head-on with an atom, mass M, initially at rest. As a result of the collision, a characteristic amount of energy E is stored internally in the atom. What is the minimum initial speed v_0 that the electron must have?
(a) $\sqrt{2(M+m)E/(mM)}$ (b) $\sqrt{(M+m)E/(mM)}$
(c) $\frac{1}{M}\sqrt{(M+m)E}$ (d) $\frac{1}{m}\sqrt{(M+m)E}$

19. A stiff spring has a force law given by $F = -kx^3$. The work required to stretch the spring from the relaxed state $x = 0$ to the stretched length $x = l$ is W_0. In terms of W_0, how much work is required to extend the spring from the stretched length l to the length $2l$?
(a) W_0 (b) $4\ W_0$ (c) $10\ W_0$ (d) $15\ W_0$

More than One Options Correct Type

20. A long block A is at rest on a smooth horizontal surface. A small block B, whose mass is half of A, is placed on A at one end and projected along A with some velocity u. The coefficient of friction between the blocks is μ. Then,

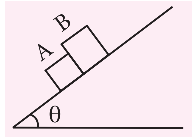


(a) the blocks will reach a final common velocity $\frac{u}{3}$
(b) the work done against friction is two-third of the initial kinetic energy of B
(c) before the blocks reach a common velocity, the acceleration of A relative to B is $\frac{2}{3}\mu g$
(d) before the blocks reach a common velocity the acceleration of A relative to B is $\frac{3}{2}\mu g$.

21. A stone of weight W is thrown vertically upward into the air with an initial speed v_0. Suppose that the air drag force f dissipates an amount fy of mechanical energy as the stone travels a distance y. Then,
(a) the maximum height reached by the stone is $\frac{v_0^2}{2g(1+f/W)}$
(b) the maximum height reached by stone is $\frac{v_0^2}{2g\left(4+f/W\right)}$
(c) the speed of the stone upon impact with the ground is $v_0\left(\frac{W-f}{W+f}\right)^{1/2}$
(d) the speed of the stone upon impact with the ground is $v_0\left(\frac{W+f}{W-f}\right)^{1/2}$.

22. Two balls, having linear momenta $\vec{p}_1 = p\hat{i}$ and $\vec{p}_2 = -p\hat{i}$, undergo a collision in free space. There is no external force acting on the balls. Let $\vec{p}_1'$ and $\vec{p}_2'$ be their final momenta. The following options are not allowed for any non-zero value of p, a_1, a_2, b_1, b_2, c_1 and c_2.
(a) $\vec{p}_1' = a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$, $\vec{p}_2' = a_2\hat{i}+b_2\hat{j}$
(b) $\vec{p}_1' = c_1\hat{k}$, $\vec{p}_2' = c_2\hat{k}$
(c) $\vec{p}_1' = a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$, $\vec{p}_2' = a_2\hat{i}+b_2\hat{j}-c_1\hat{k}$
(d) $\vec{p}_1' = a_1\hat{i}+b_1\hat{j}$, $\vec{p}_2' = a_2\hat{i}+b_1\hat{j}$

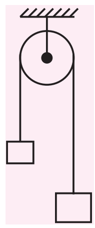
23. The two blocks A and B of equal mass are initially in contact when released from rest on the inclined plane. The coefficients of friction between the inclined plane and A and B are μ_1 and μ_2 respectively. Then,



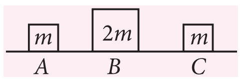
(a) if $\mu_1 > \mu_2$, the blocks will always remain in contact
(b) if $\mu_1 < \mu_2$, the blocks will slide down with different accelerations
(c) if $\mu_1 > \mu_2$, the blocks will have a common acceleration $\frac{1}{2}(\mu_1+\mu_2)g\sin\theta$
(d) if $\mu_1 < \mu_2$, the blocks will have a common acceleration $\frac{\mu_1\mu_2 g}{\mu_1+\mu_2}\sin\theta$.

Integer Answer Type

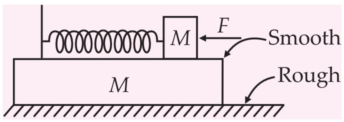
24. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking $g = 10\ \text{m s}^{-2}$, find the work done (in J) by the string on the block of mass 0.36 kg during the first second after the system is released from rest.



25. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, $2m$ and m, respectively. The object A moves towards B with a speed $9\ \text{m s}^{-1}$ and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in m s^{-1}) of the object C.

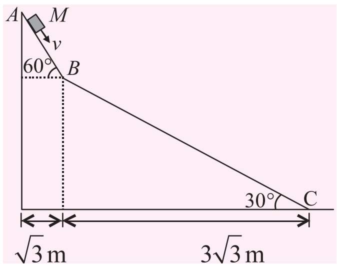


26. A constant force F is aplied on block, the block is placed on a plank as shown in figure. Block and plank are connected with a spring. There is no friction between block and plank but friction exists between plank and ground. Find minimum constant force (in N) by external agent so that plank just begins to move. (Take : M = 1 kg, $k = 1\ \text{N m}^{-1}$, $\mu = 0.2$)



Comprehension Type

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in the figure. The angle of the incline suddenly changes from 60° to 30° at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic ($g = 10\ \text{m s}^{-2}$).



27. The speed of the block at point B immediately after it strikes the second incline is

(a) $\sqrt{60}\ \text{m s}^{-1}$ (b) $\sqrt{45}\ \text{m s}^{-1}$

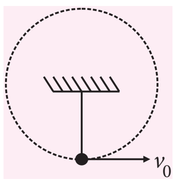
(c) $\sqrt{30}\ \text{m s}^{-1}$ (d) $\sqrt{15}\ \text{m s}^{-1}$

28. The speed of the block at point C, immediately before it leaves the second incline is

(a) $\sqrt{120}\ \text{m s}^{-1}$ (b) $\sqrt{105}\ \text{m s}^{-1}$

(c) $\sqrt{90}\ \text{m s}^{-1}$ (d) $\sqrt{75}\ \text{m s}^{-1}$

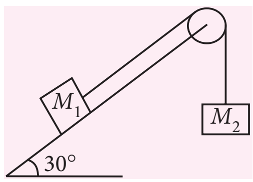
29. A bob of mass 1 kg is hanging by an inextensible string of length 1 m as shown in the figure. The bob is given a velocity v_0 at lowest point as shown in the figure. Match the entries of column I with the statements in column II.



Column I	Column II
(A) $v_0 = 2\ \text{m s}^{-1}$	(P) Tension becomes zero at some points in motion.
(B) $v_0 = 4\ \text{m s}^{-1}$	(Q) The path is always circular.
(C) $v_0 = 6\ \text{m s}^{-1}$	(R) The string may be horizontal at some points in its motion.
(D) $v_0 = 8\ \text{m s}^{-1}$	(S) The bob completes vertical circle.

	A	B	C	D
(a)	Q	P,S	R	Q,R
(b)	S	Q	P,R	Q,R,S
(c)	Q	Q	P,R	Q,R,S
(d)	P	R	P,Q	R,S

30. For the system shown in the figure, the incline is frictionless and the string is massless and inextensible, pully is light and frictionless. As the system is released from rest, the possible situations are given in the two columns. Match the entries of column I with entries of column II.



Column I	Column II
(A) $M_1 > M_2$	(P) M_2 accelerates down
(B) $M_2 > M_1$	(Q) M_2 accelerates up
(C) $M_1 = M_2$	(R) M_1 and M_2 are in equilibrium
(D) $M_1 >> M_2$	(S) M_1 accelerates up the incline

	A	B	C	D
(a)	P,Q,R,S	P,S	P,S	Q
(b)	P,Q,S	P,R,T	P	S
(c)	P,Q	P,R	P,S	Q
(d)	R,S	P,Q	R,S	P,S

◈◈

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ......

No. of questions correct ......

Marks scored in percentage ......

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

BRAIN MAP

MOTION IN A PLANE

CLASS XI

VECTOR

Physical quantities having both magnitude and direction

Addition of Vectors

Triangle Law

If two vectors are represented by two sides of a triangle in same order, the resultant will be the third side but in opposite order. $\vec{C} = \vec{A} + \vec{B}$

Parallelogram Law

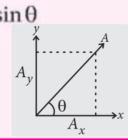
For two coinitial vectors represented by the two adjacent side of a parallelogram, the diagonal of the parallelogram so formed will be the resultant. $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

Resolution of Vector

Horizontal component of $\vec{A}$, $A_x = A\cos\theta$

Vertical component of $\vec{A}$, $A_y = A\sin\theta$

$A = \sqrt{A_x^2 + A_y^2}$, $\tan\theta = \left(\frac{A_y}{A_x}\right)$



Multiplication by a Scalar Quantity

Multiplication of a vector by a scalar ϕ changes its magnitude by a factor of ϕ.

KINEMATICS IN TWO DIMENSIONS

Kinematic Equations in 2-Dimensions

- $\vec{v} = \vec{u} + \vec{a}t$
  - $v_x = u_x + a_x t$ (along x-axis)
  - $v_y = u_y + a_y t$ (along y-axis)
- $\vec{r} = \vec{r}_0 + \vec{u}_0 t + (1/2)\vec{a}t^2$
  - $x = x_0 + u_x t + (1/2)a_x t^2$ (along x-axis)
  - $y = y_0 + u_y t + (1/2)a_y t^2$ (along y-axis)

Relative Velocity

According to vector law of addition

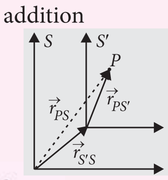
$\vec{r}_{PS} = \vec{r}_{PS'} + \vec{r}_{S'S}$

$\frac{d\vec{r}_{PS}}{dt} = \frac{d\vec{r}_{PS'}}{dt} + \frac{d\vec{r}_{S'S}}{dt}$

So, $\vec{v}_{PS} = \vec{v}_{PS'} + \vec{v}_{S'S}$

i.e., $\vec{v}_{\text{actual}} = \vec{v}_{\text{relative}} + \vec{v}_{\text{reference}}$

$\vec{v}_{\text{relative}} = \vec{v}_{\text{actual}} - \vec{v}_{\text{reference}}$



Projectile Motion

Projectile Fired Horizontally

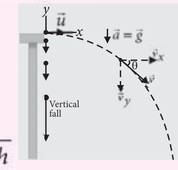
Equation of trajectory: $y = \frac{g}{2u^2}.x^2$

Velocity after time t: $v = \sqrt{u^2 + g^2t^2}$

$\beta = \tan^{-1}\frac{gt}{u}$

Time of flight: $T = \sqrt{\frac{2h}{g}}$

Horizontal range: $R = u\sqrt{\frac{2h}{g}}$



Projectile Fired at an Angle with Horizontal

Position after time t:

$x = (u\cos\theta)t$, $y = (u\sin\theta)t - \frac{1}{2}gt^2$

Equation of trajectory:

$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}.x^2$

Maximum height: $H = \frac{u^2\sin^2\theta}{2g}$

Time of flight: $T = \frac{2u\sin\theta}{g}$

Horizontal range: $R = \frac{u^2\sin 2\theta}{g}$

Circular Motion

Uniform Circular Motion

A particle moves along a circular path with a constant speed.

Angular displacement, $\theta = \frac{s}{r}$

Angular velocity, $\omega = \frac{\theta}{t}$

Also, $\omega = \frac{2\pi}{T} = 2\pi\upsilon$

Linear velocity, $v = r\omega$

Centripetal acceleration, $a = \frac{v^2}{r} = r\omega^2$

Linear acceleration, $a = r\alpha$

Non Uniform Circular Motion

Circular motion with variable speed

Resultant acceleration of the particle

$a = \sqrt{a_t^2 + a_r^2}$

$a_r = \frac{v^2}{r}$, a_t = tangential acceleration

CURRENT ELECTRICITY

CLASS XII

Current Electricity

Electric Current

- $I = \frac{q}{t} = \frac{ne}{t}$
- In case of an electron revolving in a circle of radius r with speed v, period of revolution is $T = \frac{2\pi r}{v}$
- Frequency of revolution $\upsilon = \frac{v}{2\pi r}$
- Current at any point of the orbit is $I = \frac{e}{T} = \frac{ev}{2\pi r}$

Electric Power

$P = VI = I^2R = \frac{V^2}{R}$

Variation of Resistance with Temperature

- Temperature coefficient of resistance, $\alpha = \frac{R_2 - R_1}{R_1(T_2 - T_1)}$

If T_1 = 0°C and T_2 = T °C then $\alpha = \frac{R_T - R_0}{R_0 \times T}$ or $R_T = R_0(1 + \alpha T)$

Ohm's Law, Resistance and Resistivity

- $V = IR$
- Resistance of uniform conductor of length l and cross sectional area A, $R = \frac{\rho l}{A}$
- Resistivity or specific resistance, $\rho = \frac{RA}{l}$
- Effective specific resistance in series combination is $\frac{\rho_1 l_1 + \rho_2 l_2}{l_1 + l_2}$ (A is same).
- Effective specific resistance in parallel combination is $\frac{(A_1 + A_2)\rho_1\rho_2}{A_1\rho_2 + A_2\rho_1}$ (l is same).

Kirchhoff's Laws

- Law of conservation of charge applied at a junction, i.e., $\Sigma I = 0$
- Law of conservation of energy applied in closed loop, i.e., $\Sigma\varepsilon = \Sigma IR$

Combination of Resistances

- In series, equivalent resistance, $R_s = R_1 + R_2 + R_3 + ...$
- In parallel, equivalent resistance, $\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$
- For two resistances in parallel current through the two resistances will be, $I_1 = \frac{R_2 I}{R_1 + R_2}$, $I_2 = \frac{R_1 I}{R_1 + R_2}$
- When resistances are connected in series, the current through each resistance is same. In parallel combination voltage is same.

Current Density, Conductance and Conductivity

- Conductance, $G = \frac{1}{R}$
- Conductivity, $\sigma = \frac{1}{\rho} = \frac{l}{RA}$
- Current density, $J = \frac{I}{A} = \sigma E = nev_d$

Drift Velocity and Mobility of Charge

- Drift speed, $v_d = \frac{eE}{m}\tau$
- Mobility, $\mu_e = \frac{v_d}{E}$
- Current in terms of drift velocity,

$I = neAv_d = \frac{ne^2A\tau E}{m} = neA\mu_e E = neA\mu_e \frac{V}{l}$

- In terms of relaxation time τ,

$R = \frac{ml}{ne^2\tau A}$ and $\rho = \frac{m}{ne^2\tau}$

Emf, Internal Resistance, Current in case of Grouping of Cells

- Emf of a cell, $\varepsilon = \frac{W}{q}$
- Terminal potential difference where current is being drawn from the cell, $V = \varepsilon - Ir$
- Terminal potential difference when the cell is being charged $V = \varepsilon + Ir$
- Internal resistance of a cell, $r = R\left[\frac{\varepsilon - V}{V}\right]$
- Grouping of identical cells:
  - Cells in series, $I = \frac{n\varepsilon}{R + nr}$ (n cells)
  - Cells in parallel, $I = \frac{m\varepsilon}{mR + r}$ (m cells)
  - Cells in mixed grouping, $I = \frac{mn\varepsilon}{mR + nr}$

NEET | JEE ESSENTIALS

Class XII

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Unit 3 MAGNETIC EFFECT OF CURRENT AND MAGNETISM

Magnetic Effect of Current

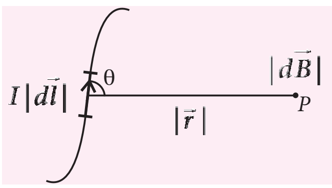
- Moving electric charge $\longrightarrow$ Magnetic field $\longrightarrow$ Moving electric charge
- Magnetic field is established by moving electric charges (current) and in turn the field can exert a force (magnetic) on other moving electric charges (current element).
- **Biot - Savart Law :** Magnetic field at a point P due to a current element Idl depends as

$dB \propto I$

$dB \propto dl$

$dB \propto \sin\theta$

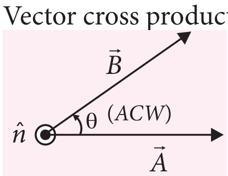
$dB \propto \dfrac{1}{r^2}$



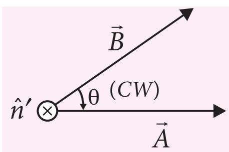
On combining, $dB \propto \dfrac{Idl\ \sin\theta}{r^2}$

$$dB = \frac{kIdl\ \sin\theta}{r^2},\ \text{where } k = \begin{cases} \dfrac{\mu_0}{4\pi} = 10^{-7}\ (\text{S.I.}) \\ = 1\ (\text{C.G.S.}) \end{cases}$$

- Vector cross product



$\vec{C} = \vec{A} \times \vec{B}$



$\vec{C}' = \vec{B} \times \vec{A}$

$\vec{C} = AB\ \sin\theta\ \hat{n}$ $\qquad$ $\vec{C}' = BA\ \sin\theta\ \hat{n}'$

$\hat{n} \longrightarrow$ out of paper $\qquad$ $\hat{n}' \longrightarrow$ into the paper

- Vector form of Biot-Savart Law

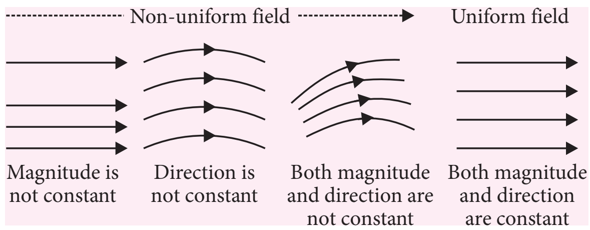
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$$

- Magnetic force on a moving charge

$\vec{F}_m = q(\vec{v} \times \vec{B})$

Magnetic Field Lines

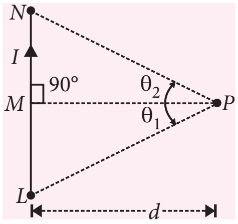
- Magnetic field lines are closed curves.
- Tangent drawn at any point on field line represents direction of the field at that point.
- Field lines never intersect to each other.
- At any place crowded lines represents stronger field while distant lines represents weaker field.
- In any region, if field lines are equidistant and straight the field is uniform otherwise not.



- Magnetic field lines exist inside every magnetised material.
- Magnetic field lines can be mapped by using iron dust or using small magnetic needle.

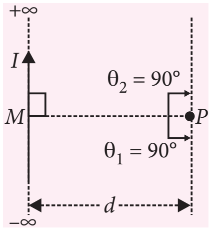
☞ Application of Biot - Savart's Law

- Magnetic field due to current carrying straight conductor or wire
  - Finite length wire : Magnetic field due to finite length wire at point P



$$B_P = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)$$

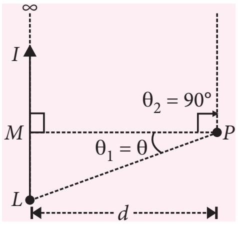
  - Infinite length wire : Magnetic field due to semi infinite length wire at point P



$$B_P = \frac{\mu_0 I}{4\pi d}(\sin 90° + \sin 90°)$$

$$B_P = \frac{\mu_0 I}{2\pi d}$$

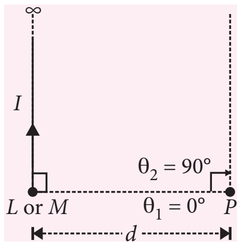
  - Semi infinite length wire: Magnetic field due to semi infinite length wire at point P



$$B_P = \frac{\mu_0 I}{4\pi d}(\sin\theta + \sin 90°)$$

$$B_P = \frac{\mu_0 I}{4\pi d}(\sin\theta + 1)$$

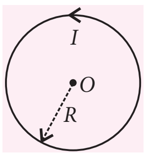
  - Special semi infinite wire: Magnetic field due to special semi infinite length wire at point P



$$B_P = \frac{\mu_0 I}{4\pi d}(\sin 0° + \sin 90°)$$

$$B_P = \frac{\mu_0 I}{4\pi d}$$

- Magnetic field at the centre of current carrying circular loop and coil
  - Circular loop ($N = 1$)

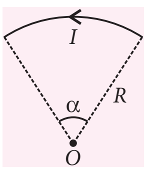


$$B_0 = \frac{\mu_0 I}{2R}$$

  - Circular coil ($N > 1$)

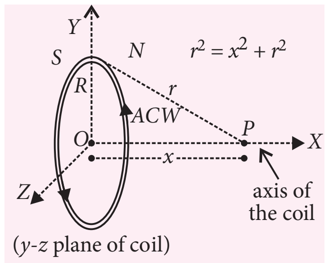
$$B = \frac{\mu_0 NI}{2R}, \; N = \text{number of turns in coil}$$

- Magnetic field at the centre of current carrying circular arc



$$B_{0\alpha} = \frac{\mu_0 I\alpha}{4\pi R} \Rightarrow B_{0\alpha} = \left(\frac{B_0}{2\pi}\right)\alpha$$

- Magnetic field at axial point of the current carrying circular coil



$$B_x = \frac{\mu_0 NIR^2}{2(x^2 + R^2)^{3/2}}$$

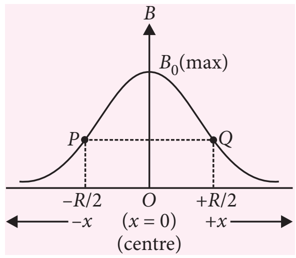
  - If $x = 0$ then

$$B_{max} = B_0 = \frac{\mu_0 NI}{2R}$$

  - If $x \to \pm\infty$ then $B_{min} = 0$
  - If $x >>> R$ (very far points) then $B_x \simeq \dfrac{\mu_0 NIR^2}{2x^3}$
  - If $x <<< R$ (very near points) then $B_x \neq B_0$ (maximum value) but by using Binomial theorem and neglecting higher power of (x^2/R^2)

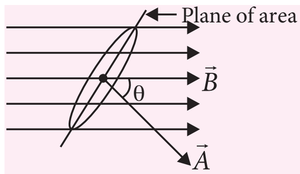
$$\Rightarrow B_x \simeq B_0\left(1 - \frac{3x^2}{2R^2}\right)$$

  - B-x curve : It is symmetrical about the centre of coil ($x = 0$). Points of inflection are $R/2$ distance from the centre of the coil.



☞ Magnetic Flux

- The number of magnetic field lines which are crossing through given area of cross section is called magnetic flux of that area.
- Magnetic flux through given area of cross section



$$\phi = \vec{B} \cdot \vec{A}$$

$$\phi = BA\cos\theta$$

Where $\vec{B} \to$ External magnetic field vector

$\vec{A} \to$ Area vector which is directed perpendicular to the plane of area of cross section.

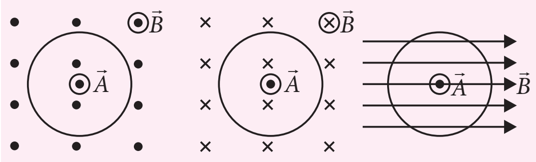
$\theta \to$ Angle between $\vec{B}$ and $\vec{A}$.

- Magnetic flux through a coil of N turns and A area of cross section

$$\phi = N(\vec{B} \cdot \vec{A}) = NBA\cos\theta$$

- When a plane of coil is perpendicular to the external magnetic field direction then field is called transverse for the coil. In transverse magnetic field, magnetic flux through the coil is always maximum.

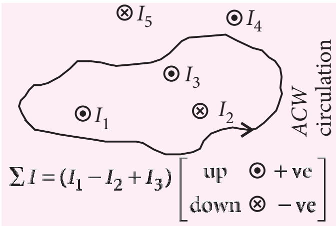
- When plane of coil is parallel to the external magnetic field direction then field is called longitudinal for the coil. In longitudinal magnetic field, magnetic flux through the coil is always zero.



$\theta = 0° \Rightarrow \phi = +NBA$ (out flux) Transverse field; $\theta = 180° \Rightarrow \phi = -NBA$ (in flux) Transverse field; $\theta = 90° \Rightarrow \phi = 0$ (zero flux) Transverse field

Ampere's Circuital Law

- It states that line integral of the magnetic field along any closed path in free space is equal to μ_0 times of net current, which rossing through area bounded by the closed path. Mathematically



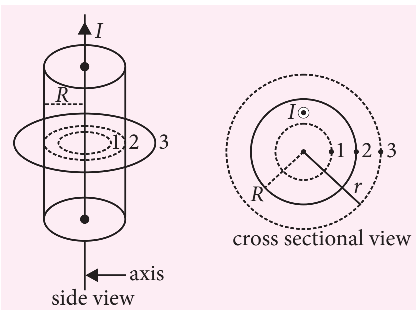
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \Sigma I$$

- **Limitations**
  - This law is suitable for infinite long and symmetrical current distribution.
  - Radius of cross section of thick cylinderical wire and current density must be given to apply this law.
  - Current density in a thick cylinderical wire must be constant.

Magnetic Field due to Infinite Long Solid Cylinderical Conductor or wire

- Current density, $J = \dfrac{I}{\pi R^2} =$ constant; $A = \pi R^2$
- Magnetic field at specific positions :
  - For $r < R$, $B_{in}(2\pi r) = \mu_0 I_{enc} = \mu_0 \dfrac{Ir^2}{R^2}$

$$B_{in} = \frac{\mu_0 I r}{2\pi R^2} \Rightarrow B_{in} \propto r$$



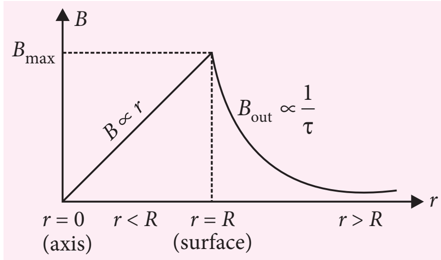
  - For $r = R$, $B_s(2\pi R) = \mu_0 I$

$$B_s = \frac{\mu_0 I}{2\pi R} \quad \text{(maximum)}$$

  - For $r > R$, $B_{out}(2\pi r) = \mu_0 I$

$$B_{out} = \frac{\mu_0 I}{2\pi r} \Rightarrow B_{out} \propto \frac{1}{r}$$

  - For $r = 0$, $B_{axis} = 0$



Magnetic Field due to Infinite Long Hollow Cylindrical Conductor

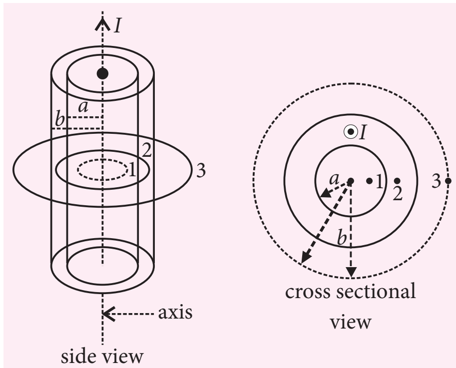
- Current density, $J = \dfrac{I}{\pi(b^2 - a^2)} =$ constant, where $A = \pi(b^2 - a^2)$
- Magnetic field at specific positions
  - $r < a$, $B_1 = 0$
  - $a < r < b$, $B_2(2\pi r) = \mu_0 I_{enc}$

$$B_2(2\pi r) = \mu_0 I \left(\frac{r^2 - a^2}{b^2 - a^2}\right)$$

$$B_2 = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - a^2}{b^2 - a^2}\right)$$

$r = a$ (inner surface) $\Rightarrow B_{is} = 0$ (min)

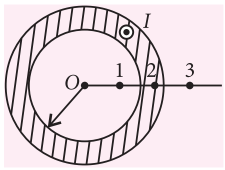
$r = b$ (outer surface) $\Rightarrow B_{os} = \dfrac{\mu_0 I}{2\pi b}$ (maximum)



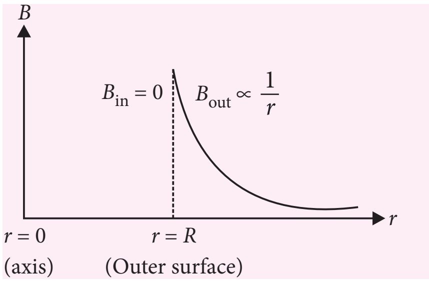
  - $r > b > a$, $B_3(2\pi r) = \mu_0 I$

$$B_3 = \frac{\mu_0 I}{2\pi r}$$

- For thin hollow cylinder, the area of cross section of conductor is negligible so there is no concept of current density.



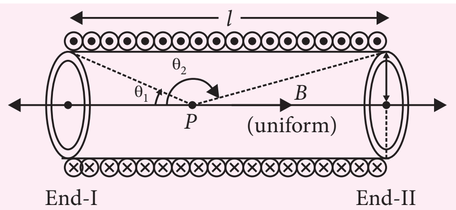
- $B_1 = 0$
- $B_2 = \dfrac{\mu_0 I}{2\pi R}$ [outer surface]
  and $B_2 = 0$ [inner surface]
- $B_3 = \dfrac{\mu_0 I}{2\pi r}$
- $B_{axis} = 0$



The Solenoid

- It is a coil which has length and used to produce uniform magnetic field of long range.
- **Type** → Finite length solenoid (by Biot-Savart Law)
  → Infinite length solenoid (by Biot-Savart Law and by Ampere Circuital Law)
- Magnetic field on the axial point inside of solenoid of finite length

$$B = \frac{\mu_0 nI}{2}(\cos\theta_1 - \cos\theta_2)$$



where $n \to$ turn density (N/l)
$N \to$ total number of turns
$l \to$ length of solenoid
$r \to$ radius of solenoid

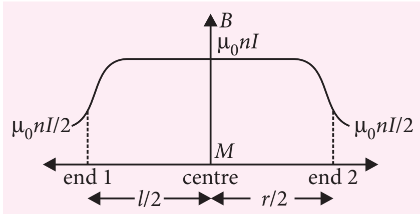
- If solenoid length is infinite then

$$B = \frac{\mu_0 nI}{2}(\cos 0° - \cos 180°) = \mu_0 nI \quad \text{(unifrom)}$$

- Magnetic field at end point of infinite solenoid

$$B = \frac{\mu_0 nI}{2}(\cos 90° - \cos 180°) = \frac{\mu_0 nI}{2}$$

- Magnetic field outside the volume of the infinite solenoid approaches to zero.
- If magnetic permeability of solenoid frame is μ_r then field produced by the solenoid $B = \mu_0 \mu_r nI$



Toroid

- A toroid can be considered as a ring shaped closed solenoid.
- Magnetic field inside a toroid by Ampere's Circuital Law given as

$B = \mu_0 nI$

where n = turn density = $\left(\dfrac{N}{2\pi R_m}\right)$

N = total number of turns

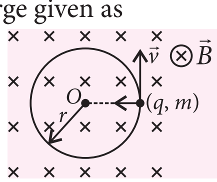
R_m = mean radius of toroid = $\left(\dfrac{R_1 + R_2}{2}\right)$

R_1 and R_2 are internal and external radius of toroid respectively.

Motion of Charge in Uniform Transverse Magnetic Field

- When a charge $+q$ projected in uniform transverse magnetic field ($\theta = 90°$, $\vec{v} \perp \vec{B}$) then maximum magnetic force of constant magnitude always acts perpendicular to its direction of motion so the charge moves along circular path and required centripetal force provided by the magnetic force.
- Magnetic force on moving charge given as

$$F_m = qvB = \frac{mv^2}{r}$$



- Radius of circular path

$$r = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2mE_K}}{qB} = \frac{\sqrt{2mqV}}{qB}$$

$$r \propto v \propto p \propto \sqrt{E_K} \propto \sqrt{V}$$

- Area bounded by the charge in uniform transverse magnetic field is directly proportional to r^2 ($A = \pi r^2$)

$\therefore \quad A \propto r^2 \propto v^2 \propto p^2 \propto E_K \propto V$

- Time period of charge in circular motion

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

- Frequency $\nu = \dfrac{1}{T} = \dfrac{qB}{2\pi m}$

Angular frequency $\omega = \frac{qB}{m}$

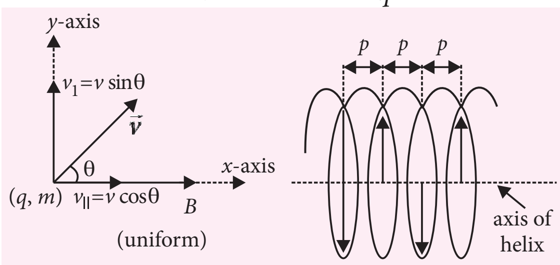
- Kinetic energy of charge

$$E_K = \frac{p^2}{2m} \Rightarrow E_K = \frac{(qBr)^2}{2m}$$

Motion of Charge in Uniform Magnetic Field at an Acute Angle (0° < θ < 90°)

- When a charge particle projected in uniform magnetic field neither perpendicular norparallel then trajectory of charge is called helix.
- Axis of helix : It is a straight line along external magnetic field which joins all the centres of circular turns.
- Radius of circular path

$$q(v\sin\theta)B = \frac{m(v\sin\theta)^2}{r} \Rightarrow r = \frac{mv\sin\theta}{qB}$$



- Time period of circular motion

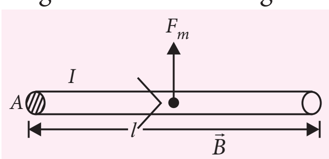
$$T = \frac{2\pi r}{v\sin\theta} = \frac{2\pi m}{qB}$$

  - It does not depends on angle of projection with the field direction.
- Pitch of helix (p) : The linear distance travelled by the charge particle in one rotation along external magnetic field direction is called pitch of helix.

$$p = (v\cos\theta)T = \frac{2\pi mv\cos\theta}{qB}$$

Magnetic Force on a Current Carrying Conductor

- Net mobile charge carries in a conductor, $q = nAle$
  Average drift velocity of electron = v_d
- Magnetic force on charge



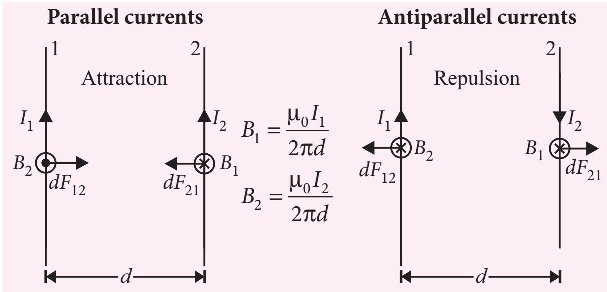
$$\vec{F}_m = (nAl)e(\vec{v}_d \times \vec{B}) = [(ne\vec{v}_d)Al] \times \vec{B}$$
$$= (\vec{J}Al) \times \vec{B} = I(\vec{l} \times \vec{B})$$

- For an arbitrary shape of wire.

$$\vec{F} = \sum_i I(d\vec{l}_i \times \vec{B})$$

- This force acts through the centre of the rod.

Magnetic Force between Two Long Parallel Current Carrying Wires



- Consider a small element of length dl on each wire. Each element is at right angle to the external magnetic field produced by other wire, so magnetic force experienced by each element of the wire is given as

$$dF_{12} = B_2 I_1 dl = \left(\frac{\mu_0 I_2}{2\pi d}\right) I_1 dl \quad (\text{where } I_1 d\vec{l} \perp \vec{B}_2)$$

$$dF_{21} = B_1 I_2 dl = \left(\frac{\mu_0 I_1}{2\pi d}\right) I_2 dl \quad (\text{where } I_2 d\vec{l} \perp \vec{B}_1)$$

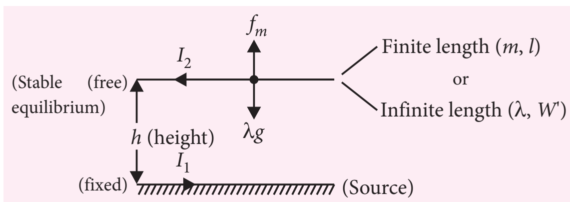
- Magnetic force per unit length of each wire is given as

$$\frac{dF_{12}}{dl} = \frac{dF_{21}}{dl} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

- Application : Equilibrium of free wire
  - Upper wire is free

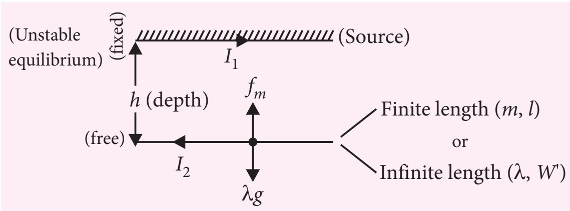
$$\frac{\mu_0 I_1 I_2}{2\pi h} = \lambda g = \left(\frac{m}{l}\right) g$$

  (stable equilibrium condition),
  Where λ = mass per unit length of free wire.
  W' = weight per unit length of free wire.
  - If free wire is slightly displaced and released then it will execute S.H.M. in vertical plane. The time period of S.H.M. is given by, $T = 2\pi\sqrt{\frac{h}{g}}$



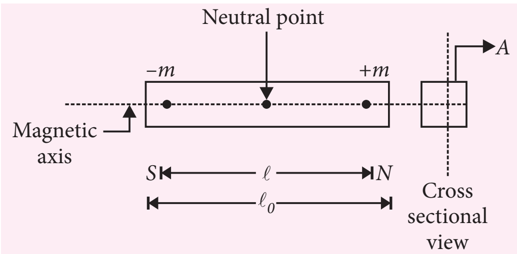
- Lower wire is free

$$\frac{\mu_0 I_1 I_2}{2\pi d} = \lambda g = \left(\frac{m}{l}\right) g \quad \text{(Unstable equilibrium)}$$

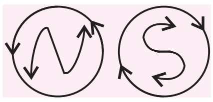


Magnetic Dipole Moment

- A magnetic dipole consists of a pair of magnetic poles of equal and opposite strength separated by small distance. *e.g.* Magnetic needle, bar magnet, current carrying solenoid, current carrying coil or loop.
- Magnetic moment of bar magnet



- Magnetic moment of current carrying coil (or loop)
  - Current carrying coil (or loop) behaves like magnetic dipole. The face of coil in which current appears to flow *ACW* acts as north pole while face of coil in which current appears to flow *CW* acts as south pole.



  - A loop of geometrical area A, carries a current I then its magnetic moment is given as $M = IA$
  - For a coil of turns N, $M = NIA$

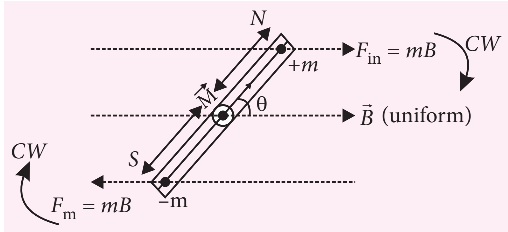
Magnetic Dipole in Magnetic Field

- Torque on magnetic dipole
  - Bar magnet

τ = force × perpendicular distance between coupled force

$\tau = (mB)(l\sin\theta) = MB\sin\theta$, where $M = ml$

$$\begin{cases} \theta = 90^\circ & \Rightarrow \quad \tau = MB(\max) \\ \theta = 0^\circ & \Rightarrow \quad \tau = 0(\min) \end{cases}$$



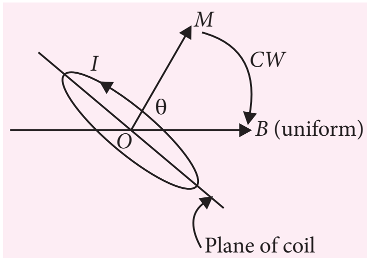
Vector form, $\vec{\tau} = \vec{M} \times \vec{B}$

- Coil or loop

$\vec{\tau} = \vec{M} \times \vec{B}$, here $\vec{M} = NI\vec{A}$,

$\vec{\tau} = NI(\vec{A} \times \vec{B})$, $\tau = BINA\sin\theta$

$$\begin{cases} \theta = 90^\circ & \Rightarrow \quad \tau = BINA(\max) \\ \theta = 0^\circ \text{ or } 180^\circ & \Rightarrow \quad \tau = 0(\min) \end{cases}$$



- Work done in rotating a magnetic dipole in a uniform magnetic field

$$W = \int_{\theta_1}^{\theta_2} dW = \int_{\theta_1}^{\theta_2} \tau d\theta = \int_{\theta_1}^{\theta_2} MB\sin\theta d\theta$$

$W = MB(\cos\theta_1 - \cos\theta_2)$

- Potential energy of magnetic dipole
It is the work done in rotating the dipole through an angle θ with respect to a direction perpendicular to the field.

$U = -MB\cos = -\vec{M} \cdot \vec{B}$

Moving Coil Galvanometer

- It is an instrument used for the detection of small current.
- Principle : When a current carrying coil is placed in a magnetic field, it experiences a torque.
In moving coil galvanometer the current I passing through the galvanometer is directly proportional to its deflection (θ).

$I \propto \theta$ or, $I = G\theta$.

where $G = \frac{k}{NAB}$ = galvanometer constant

A = area of a coil, N = number of turns in the coil, B = strength of magnetic field, k = torsional constant of the spring *i.e.* restoring torque per unit twist.

- Current sensitivity : It is defined as the deflection produced in the galvanometer, when unit current flows through it.

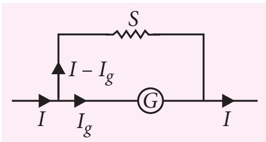
$$I_s = \frac{\theta}{I} = \frac{NAB}{k}$$

- Voltage sensitivity : It is defined as the deflection produced in the galvanometer when a unit voltage is applied across the two terminals of the galvanometer.

$$V_s = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{I_s}{R} = \frac{NAB}{kR}$$

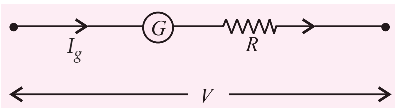
- Ammeter : It is an instrument used to measure current in an electrical circuit.
  - Conversion of galvanometer into a ammeter

$$S = \left(\frac{I_g}{I - I_g}\right)G$$



  - Ammeter is a low resistance instrument and it is always connected in series to the circuit. An ideal ammeter has zero resistance.
  - In order to increase the range of an ammeter n times, the value of shunt resistance to be connected in parallel is $S = G/(n - 1)$.
- Voltmeter : It is an instrument used to measure potential difference across any element in an electrical circuit.
  - Conversion of galvanometer into voltmeter

$$R = \frac{V}{I_g} - G$$



  - Voltmeter is a high resistance instrument and it is always connected in parallel with the circuit element across which potential difference is to be measured. An ideal voltmeter has infinite resistance.
  - In order to increase the range of voltmeter n times the value of resistance to be connected in series with galvanometer is $R = (n - 1)G$.

Vibration Magnetometer

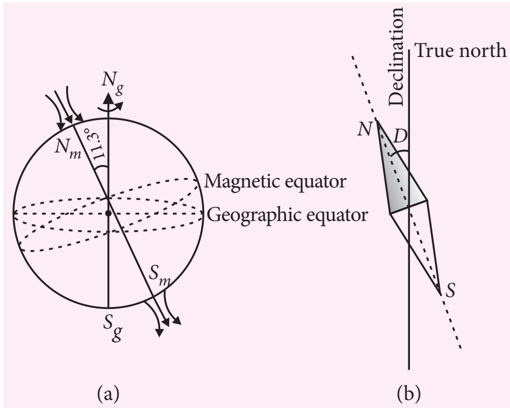
- It is an instrument used to compare the horizontal compoment of earth's magnetic field at two different places and magnetic moment of two bar magnets.
- Principle : Whenever a freely suspended bar magnet in earth's horizontal magnetic field (B_H) is slightly disturbed from its equilibrium position then it will experience a torque and excutes angular S.H.M.
- Time period of angular S.H.M.

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{MB_H}}$$

I = moment of interia of bar magnet
M = magnetic moment of bar magnet

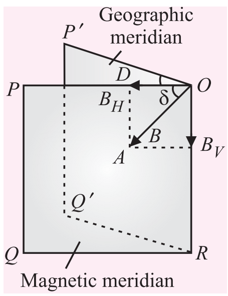
The Earth's Magnetism

- Magnetic declination : Magnetic declination at a place is defined as the angle between the geographic meridian and magnetic meridian.
- Magnetic dip or inclination : Magnetic dip at a place is defined as the angle made by the earth's magnetic field with the horizontal in the magnetic meridian. It is denoted by δ.



(a) The earth as a giant magnetic dipole
(b) A magnetic needle free to move in horizontal plane, points toward the magnetic north-south direction.

- Horizontal component of earth's magnetic field (B_H)
At a given place horizontal component of earth's magnetic field is the component of resultant magnetic field of the earth along the horizontal line in magnetic meridian.
Horizontal component along OP, $B_H = B\cos\delta$
Vertical component along OR, $B_V = B\sin\delta$



$$\tan\delta = \frac{B_V}{B_H},\ B = \sqrt{B_H^2 + B_V^2}$$

Magnetisation and Magnetic Intensity

- Net magnetic moment per unit volume of a sample known as its magnetisation (M).

$$M = \frac{m_{\text{net}}}{V},$$

- For current in coils and conductors, magnetic intensity H is defined as

$$H = \frac{B_0}{\mu_0} = nI$$

- When a magnetic field is applied to a material, the material gets magnetised. The actual magnetic field (B) inside the material is the sum of the applied magnetic field (B_0) and the magnetic field due to magnetisation (M).

$$\vec{B} = \vec{B}_0 + \mu_0 \vec{M}$$

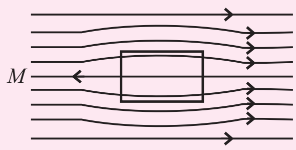
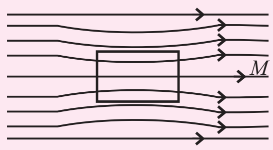
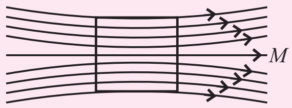
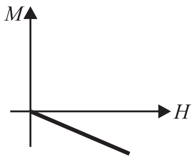
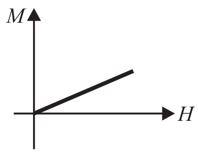
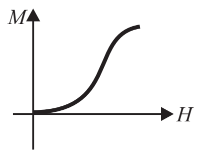
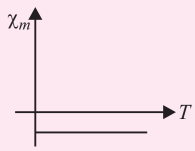
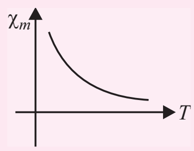
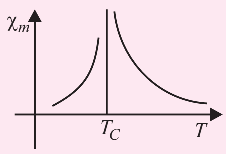
$$\vec{B} = \mu_0(\vec{H} + \vec{M}) \qquad (\because \vec{B}_0 = \mu_0 \vec{H})$$

$$\vec{B} = \mu_0(1+\chi_m)\vec{H} = \mu\vec{H} \qquad (\because \vec{M} = \chi_m \vec{H})$$

$$\mu = \mu_0\,\mu_r = \mu_0\,(1 + \chi_m) \Rightarrow \mu_r = 1 + \chi_m$$

☞ Classification of Magnetic Materials

- On the basis of magnetic properties of the material those are divided in three classes

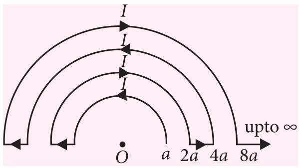
Properties	Diamagnetic	Paramagnetic	Ferromagnetic
Cause of magnetism	Orbital motion of electrons	Spin motion of electrons	Formation of domains
Substance placed in uniform magnetic field.	Poor magnetisation in opposite direction. Here $B_m < B_0$ 	Poor magnetisation in same direction. Here $B_m > B_0$ 	Strong magnetisation in same direction Here $B_m >>> B_0$ 
$M - H$ curve	$M \rightarrow$ Small, negative, varies linearly with field 	$M \rightarrow$ Small, positive, varies linearly with field 	$M \rightarrow$ Very large, positive and varies non-linearly with field 
$\chi_m - T$ curve	$\chi_m \rightarrow$ Small, negative and temperature independent $\chi_m \propto T^0$ 	$\chi_m \rightarrow$ Small, positive and varies inversely with temperature $\chi_m \propto \frac{1}{T}$ (Curie law) 	$\chi_m \rightarrow$ Very large, positive and temperature dependent $\chi_m \propto \frac{1}{T - T_C}$ (Curie Weiss law) (for $T > T_C$)  $T_{C\,(\text{iron})}$ = 770 °C or 1043 K
μ_r	$(\mu < \mu_0)$ $1 > \mu_r > 0$	$1 + \varepsilon > \mu_r > 1$ $(\mu > \mu_0)$	$\mu_r >>> 1$ $(\mu >>> \mu_0)$

☞ Hysteresis

- The phenomenon of lagging of the magnetic induction behind the magnetising field in a ferromagnetic material is called hysteresis. The area of the hysteresis (B-H) loop gives the energy wasted in a sample when it is taken through a cycle of magnetisation.
- The magnetic induction left behind in the sample after the magnetising field has been removed is called retentivity.
- The value of the reverse magnetising field required to make the residual magnetism of a sample equal to zero is called coercivity.

SPEED PRACTICE

1. A conductor is bent in the form of concentric semicircles as shown in the figure. The magnetic field at the point O is (Distances a, $2a$, $4a$, … are not to scale)



(a) zero (b) $\frac{\mu_0 I}{6a}$ (c) $\frac{\mu_0 I}{a}$ (d) $\frac{\mu_0 I}{4a}$

2. A thin rod is bent in the shape of a small circle of radius r. If the charge per unit length of the rod is λ, and if the circle is rotated about its axis at a rate of υ rotations per second, the magnetic field at a point on the axis at a large distance y from the centre is

(a) $\frac{\mu_0 \pi r^3 \upsilon \lambda}{y^3}$ (b) $\frac{2\mu_0 \pi r^3 \upsilon \lambda}{y^3}$

(c) $\frac{\mu_0 r^3 \upsilon \lambda}{4\pi y^3}$ (d) $\frac{\mu_0 r^3 \upsilon \lambda}{2\pi y^3}$

3. A uniform electric field $\vec{E}$ in the y-direction and uniform magnetic field $\vec{B}$ in the x-direction exist in free space. A particle of mass m and carrying charge q is projected from the origin with speed v_0 along the y axis. The speed of the particle as a function of its y coordinate will be

(a) $\sqrt{v_0^2 + \frac{2qEy}{m}}$ (b) $\sqrt{v_0^2 - \frac{4qEy}{m}}$

(c) $\sqrt{v_0^2 + \frac{qEy}{m}}$ (d) v_0

4. When a positively charged particle enters into a uniform magnetic field with uniform velocity, its trajectory can be (i) a straight line (ii) a circle (iii) a helix.

(a) (i) only (b) (i) or (ii)
(c) (i) or (iii) (d) Any one of (i), (ii) and (iii)

5. A straight horizontal conducting rod of length 50 cm and mass 50 g is suspended by two vertical wires at its ends. A current of 5.0 A is set up in the rod through the wires. What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero? (Take $g = 10$ m s^{-2})

(a) 0.05 T (b) 0.1 T (c) 0.2 T (d) 0.5 T

6. The magnetic induction and the intensity of magnetic field inside an iron core of an electromagnet are 1 Wb m^{-2} and 150 A m^{-1} respectively. The relative permeability of iron is ($\mu_0 = 4\pi \times 10^{-7}$ henry m^{-1})

(a) $\frac{10^6}{4\pi}$ (b) $\frac{10^5}{6\pi}$ (c) $\frac{10^3}{4\pi}$ (d) $\frac{10^3}{6\pi}$

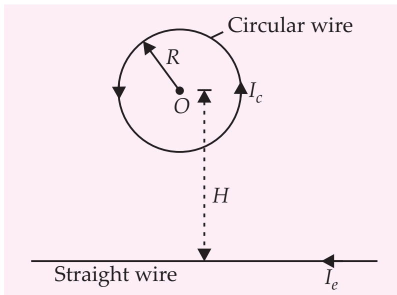
7. A bar magnet of magnetic moment 1.5 J T^{-1} lies aligned with the direction of a uniform magnetic field of 1 T. What is the amount of work required by an external torque to turn the magnet so as to align its magnetic moment, opposite to the field direction?

(a) 2 J (b) 3 J (c) 4 J (d) 6 J

8. A vibration magnetometer placed in magnetic meridian has a small bar magnet. The magnet executes oscillations with a time period of 2 s in earth's horizontal magnetic field of 24 μT. When a horizontal field of 18 μT is produced opposite to the earth's field by placing a current carrying wire, the new time period of the magnet will be

(a) 1 s (b) 2 s (c) 3 s (d) 4 s

9. Circular loop of a wire and a long straight wire carry currents I_c and I_e respectively as shown in figure. Assuming that these are placed in the same plane, the magnetic fields will be zero at the centre of the loop when separation H is



(a) $\frac{I_e R}{I_c \pi}$ (b) $\frac{I_c R}{I_e \pi}$ (c) $\frac{\pi I_e}{I_c R}$ (d) $\frac{\pi I_c}{I_e R}$

10. An electron having momentum 2.4×10^{-23} kg m s^{-1} enters a region of uniform magnetic field of 0.15 T. The field vector makes an angle of 30° with the initial velocity vector of the electron. The radius of the helical path of the electron in the field shall be

(a) 2 mm (b) 1 mm (c) 1.5 mm (d) 0.5 mm

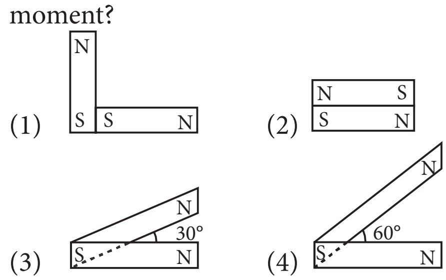
11. A vertical circular coil of radius 0.1 m and having 10 turns carries a steady current. When the plane of the coil is normal to magnetic meridian, a neutral point is observed at the centre of the coil. If $B_H = 0.314 \times 10^{-4}$ T, then current in the coil is
(a) 0.25 A (b) 0.5 A (c) 1 A (d) 2 A

12. A rectangular coil of length 0.12 m and width 0.1 m having 50 turns of wire is suspended vertically in a uniform magnetic field of strength 0.2 Wb m^{-2}. The coil carries a current of 2 A. If the plane of the coil is inclined at an angle of 30° with the direction of the field, the torque required to keep the coil in stable equilibrium will be
(a) 0.24 N m (b) 0.12 N m
(c) 0.15 N m (d) 0.20 N m
(AIPMT 2015)

13. A proton and an alpha particle both enter a region of uniform magnetic field B, moving at right angles to the field B. If the radius of circular orbits for both the particles is equal and the kinetic energy acquired by proton is 1 MeV, the energy acquired by the alpha particle will be
(a) 1.5 MeV (b) 1 MeV
(c) 4 MeV (d) 0.5 MeV **(AIPMT 2015)**

14. In an ammeter 0.2% of main current passes through the galvanometer. If resistance of galvanometer is G, the resistance of ammeter will be
(a) $\frac{1}{499}G$ (b) $\frac{499}{500}G$ (c) $\frac{1}{500}G$ (d) $\frac{500}{499}G$
(AIPMT 2014)

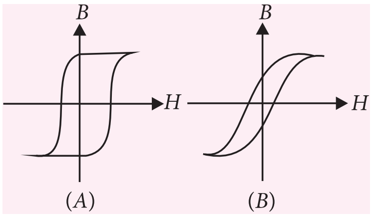
15. Following figures show the arrangement of bar magnets in different configurations. Each magnet has magnetic dipole moment $\vec{m}$. Which configuration has highest net magnetic dipole moment?



(a) (1) (b) (2) (c) (3) (d) (4)
(AIPMT 2014)

16. Two identical wires A and B, each of length l, carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side a. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is
(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{16\sqrt{2}}$ (c) $\frac{\pi^2}{16}$ (d) $\frac{\pi^2}{8\sqrt{2}}$
(JEE Main 2016)

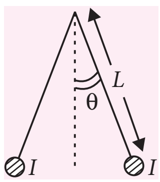
17. Hysteresis loops for two magnetic materials A and B are given below



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use
(a) A for electric generators and transformers.
(b) A for electromagnets and B for electric generators.
(c) A for transformers and B for electric generators.
(d) B for electromagnets and transformers.
(JEE Main 2016)

18. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection, when a current of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is
(a) 0.01 Ω (b) 2 Ω
(c) 0.1 Ω (d) 3 Ω **(JEE Main 2016)**

19. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle θ with the vertical. If wires have mass λ per unit length then the value of I is (g = gravitational acceleration)



(a) $2\sqrt{\frac{\pi gL}{\mu_0}}\tan\theta$ (b) $\sqrt{\frac{\pi\lambda gL}{\mu_0}}\tan\theta$
(c) $\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$ (d) $2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$
(JEE Main 2016)

20. A long straight wire of radius a carries a steady current I. The current is uniformly distributed over its cross-section. The ratio of the magnetic fields B and B', at radial distances $\frac{a}{2}$ and $2a$ respectively, from the axis of the wire is

(a) 1 (b) 4 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

(NEET Phase-I 2016)

SOLUTIONS

1. (b): Magnetic field at the centre of a semicircular current carrying conductor is given by

$$B = \frac{\mu_0 I\pi}{4\pi a} = \frac{\mu_0 I}{4a}$$

where a is the radius of the first semicircle.

The current in all the semicircles is the same but its sense is alternately opposite and the radii are in the proportion 1 : 2 : 4 : 8 : 16 : ....

Net magnetic field at the point O is

$$B = \frac{\mu_0 I}{4a}\left[1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + ...\right]$$

$$= \frac{\mu_0 I}{4a}\left[\left(1 + \frac{1}{4} + \frac{1}{16} + ...\right) - \left(\frac{1}{2} + \frac{1}{8} + \frac{1}{32} + ...\right)\right]$$

$$= \frac{\mu_0 I}{4a}\left[\left(1 + \frac{1}{2^2} + \frac{1}{2^4} + ...\right) - \left(\frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^5} ......\right)\right]$$

$$= \frac{\mu_0 I}{4a}\left[\left(\frac{1}{1-\frac{1}{4}}\right) - \left(\frac{1/2}{1-\frac{1}{4}}\right)\right] = \frac{\mu_0 I}{4a}\left[\left(\frac{4}{3}\right) - \left(\frac{2}{3}\right)\right]$$

$$= \frac{\mu_0 I}{4a}\left(\frac{2}{3}\right) = \frac{\mu_0 I}{6a}$$

2. (a): Current in the rod, $I = \dfrac{2\pi r\lambda}{(1/\upsilon)} = 2\pi r\lambda\upsilon$

Magnetic field at a point on the axis at a distance y from the centre of current carrying circular rod is

$$B = \frac{\mu_0}{4\pi}\frac{2\pi I r^2}{(r^2 + y^2)^{3/2}}$$

If $y >> r$, then

$$B = \frac{\mu_0 I r^2}{2y^3} = \frac{\mu_0 r^2}{2y^3}(2\pi r\lambda\upsilon) = \frac{\mu_0 \pi r^3 \lambda\upsilon}{y^3}$$

3. (a): Here, $\vec{E} = E\hat{j}$; $\vec{B} = B\hat{i}$; $\vec{v} = v_0\hat{j}$

The Lorentz force acting on a charged particle of charge q and mass m is

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$= q(E\hat{j} + v_0\hat{j} \times B\hat{i}) = q(E\hat{j} - v_0 B\hat{k})$$

$$\therefore a_y = \frac{qE}{m} \quad ....(i)$$

and $$a_z = -\frac{qv_0 B}{m} \quad ...(ii)$$

From equation (i), we get

$$\frac{dv_y}{dt} = \frac{qE}{m} \Rightarrow \frac{dv_y}{dy}\frac{dy}{dt} = \frac{qE}{m}$$

$$v_y dv_y = \frac{qEdy}{m}$$

Integrate both sides of the above equation, we get

$$\int_{v_0}^{v_y} v_y dv_y = \int_0^y \frac{qEdy}{m} \Rightarrow v_y = \sqrt{v_0^2 + \frac{2qEy}{m}}$$

4. (d)

5. (c) : Here, $l = 50$ cm $= 50 \times 10^{-2}$ m

$m = 50$ g $= 50 \times 10^{-3}$ kg; $I = 5.0$ A

Tension in the wires is zero if the force on the rod due to magnetic field is equal and opposite to the weight of the rod.

i.e., $mg = BIl \Rightarrow B = \dfrac{mg}{Il}$

Substituting the given values, we get

$$B = \frac{50 \times 10^{-3} \times 10}{5 \times 50 \times 10^{-2}} = 0.2 \text{ T}$$

6. (b): Here, $B = 1$ Wb m^{-2}; $H = 150$ A m^{-1}

$$B = \mu_0\mu_r H \Rightarrow \mu_r = \frac{B}{\mu_0 H}$$

$$\mu_r = \frac{1}{4\pi \times 10^{-7} \times 150} = \frac{10^5}{6\pi}$$

7. (b): The amount of work done by the external torque is given by

$$W = \int_{\theta_1}^{\theta_2} \tau_{\text{ext}}\, d\theta$$

$$= \int_{\theta_1}^{\theta_2} MB \sin\theta\, d\theta = MB(\cos\theta_1 - \cos\theta_2)$$

Here, $\theta_1 = 0°$; $\theta_2 = 180°$; $M = 1.5$ J T^{-1}; $B = 1$ T

Substituting the given values, we get

$W = (1.5 \text{ J T}^{-1})(1 \text{ T})[\cos 0° - \cos 180°] = 3$ J

8. (d): The time period T of oscillation of a magnet is given by

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

As I and M remain the same,

$$\therefore \quad T \propto \frac{1}{\sqrt{B}} \text{ or } \frac{T_2}{T_1} = \sqrt{\frac{B_1}{B_2}}$$

According to given problem,

$B_1 = 24\ \mu T$, $B_2 = 24\ \mu T - 18\ \mu T = 6\ \mu T$, $T_1 = 2$ s

$$\therefore \quad T_2 = (2\text{ s})\sqrt{\frac{24\ \mu T}{6\ \mu T}} = 4 \text{ s}$$

9. (a) : Magnetic field at the centre O of the circular current loop is $\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{2\pi I_c}{R} \odot$

Magnetic field at the centre O of the circular loop due to current through long straight wire is

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2I_e}{H} \otimes$$

As the resultant magnetic field at O is zero (given), so $B_1 = B_2$

$$\frac{\mu_0}{4\pi}\frac{2\pi I_c}{R} = \frac{\mu_0}{4\pi}\frac{2I_e}{H} \quad \text{or} \quad H = \frac{I_e R}{\pi I_c}$$

10. (d) : The radius of the helical path of the electron in the uniform magnetic field is

$$r = \frac{mv_\perp}{eB} = \frac{mv\sin\theta}{eB} = \frac{p\sin\theta}{eB}$$

$$= \frac{2.4\times10^{-23}\times\sin 30°}{1.6\times10^{-19}\times0.15}$$

$= 5 \times 10^{-4}$ m $= 0.5 \times 10^{-3}$ m $= 0.5$ mm

11. (b) : Given: $r = 0.1$ m, $N = 10$, $B_H = 0.314 \times 10^{-4}$ T

Magnetic field at the centre of current carrying circular coil

$$B = \frac{\mu_0 NI}{2r}$$

Since at neutral point, the magnetic field due to circular coil is completely cancelled by the horizontal component of earth's magnetic field. Therefore,

$$B = B_H \quad \text{or} \quad \frac{\mu_0 NI}{2r} = B_H$$

$$\therefore \quad I = \frac{2rB_H}{\mu_0 N} = \frac{2\times0.1\times0.314\times10^{-4}}{(4\pi\times10^{-7})\times10} = 0.5 \text{ A}$$

12. (d) : The required torque is

$\tau = NIAB\sin\theta$

Here, $N = 50$, $I = 2$ A, $A = 0.12$ m $\times$ 0.1 m $= 0.012$ m^2

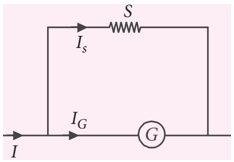
$B = 0.2$ Wb m^{-2} and $\theta = 90° - 30° = 60°$

$\therefore \quad \tau = (50)(2\text{ A})(0.012\text{ m}^2)(0.2\text{ Wb m}^{-2})\sin 60°$

$= 0.20$ N m

13. (b)

14. (c) : Here, resistance of the galvanometer $= G$ Current through the galvanometer,



$$I_G = 0.2\% \text{ of } I = \frac{0.2}{100}I = \frac{1}{500}I$$

As shunt and galvanometer are in parallel

$\therefore \quad I_G G = I_S S \quad$ or $\quad I_G G = (I - I_G)S$

$$\left(\frac{1}{500}I\right)G = \left(\frac{499}{500}I\right)S \text{ or } S = \frac{G}{499}$$

Resistance of the ammeter R_A is given by

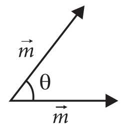
$$\frac{1}{R_A} = \frac{1}{G} + \frac{1}{S} = \frac{1}{G} + \frac{499}{G} = \frac{500}{G} \quad \Rightarrow \quad R_A = \frac{1}{500}G$$

15. (c) : The direction of magnetic dipole moment is from south to north pole of the magnet.

For the configuration shown in figure,

$$m_{\text{net}} = \sqrt{m^2 + m^2 + 2mm\cos\theta} = \sqrt{2m^2(1+\cos\theta)}$$

From the given values of θ, $\cos\theta$ is maximum for $\theta = 30°$. Hence configuration (3) has maximum value of m_{net}.

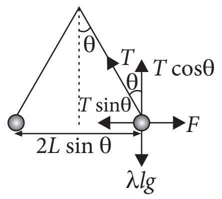


16. (d)

17. (d) : For both, the electromagnet and transformer, the magnetic field changes with time. Hence the energy loss must be less in both devices. Hysteresis loop represented in B has less area which means it dissipates less energy.

18. (a)

19. (d) : Let the length of right wire be l, then its mass is λl. Forces acting on this wire are tension (T), weight (λlg) and force of repulsion due to other wire (F).



From figure, $T\cos\theta = \lambda lg$ ...(i)

$T\sin\theta = F$ ...(ii)

$$\text{Here, } F = \frac{\mu_0}{2\pi}\frac{I^2 l}{(2L\sin\theta)}$$

$$\text{or} \quad T\sin\theta = \frac{\mu_0}{2\pi}\frac{I^2 l}{(2L\sin\theta)} \quad \text{(Using (ii))}$$

$$\text{or} \quad \frac{\lambda lg}{\cos\theta}\sin\theta = \frac{\mu_0}{2\pi}\frac{I^2 l}{(2L\sin\theta)} \quad \text{(Using (i))}$$

$$\Rightarrow \quad I = \sqrt{\frac{4\pi L\lambda g\sin^2\theta}{\mu_0\cos\theta}} = 2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$$

20. (a)

CLASS XII Series 4

ACE YOUR WAY CBSE

Electromagnetic Induction and AC Electromagnetic Waves

Time Allowed : 3 hours
Maximum Marks : 70

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Q. no. 1 to 5 are very short answer questions and carry 1 mark each.
(iii) Q. no. 6 to 10 are short answer questions and carry 2 marks each.
(iv) Q. no. 11 to 22 are also short answer questions and carry 3 marks each.
(v) Q. no. 23 is a value based question and carries 4 marks.
(vi) Q. no. 24 to 26 are long answer questions and carry 5 marks each.
(vii) Use log tables if necessary, use of calculators is not allowed.

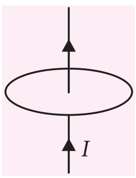
Previous Years Analysis

	2016		2015		2014	
	Delhi	AI	Delhi	AI	Delhi	AI
VSA	2	–	1	1	–	3
SA-I	–	–	–	–	1	1
SA-II	2	2	2	2	1	2
VBQ	–	1	1	1	–	1
LA	1	1	–	–	1	–

SECTION-A

1. A light metal disc on the top of an electromagnet is thrown up as the current is switched on. Why? Give reason.

2. The current I in a wire passing normally through the centre of a conducting loop is increasing at a constant rate. Will any current be induced in the loop?



3. The frequency of a.c. is doubled. How do R, X_L and X_C get affected?

4. Which part of electromagnetic spectrum is used for LASIK eye surgery, and why?

5. How does a charge oscillating at certain frequency produce electromagnetic waves?

SECTION-B

6. State Faraday's laws of electromagnetic induction. Express them mathematically.

7. Considering the case of a parallel plate capacitor being charged, show how one is required to generalize Ampere's circuital law to include the term due to displacement current.

8. A coil with an average diameter of 0.02 m is placed perpendicular to a magnetic field of 6000 T. If the induced emf is 11 V when the magnetic field is changed to 1000 T in 4 s, what is the number of turns in the coil?

9. An a.c. source of frequency 50 Hz is connected to a 50 mH inductor and a bulb. The bulb glows with some brightness. Calculate the capacitance of the capacitor to be connected in series with the circuit so that the bulb glows with maximum brightness.

OR

A coil offers an inductive reactance of 18.0 Ω at a frequency of 50 Hz. What is the self-inductance of the coil? What reactance will the coil offer at a frequency of 20 kHz?

10. The oscillating magnetic field in a plane electromagnetic wave is given by

$B_y = 8 \times 10^{-6} \sin[2 \times 10^{11} t + 300 \pi x]$ (in T)

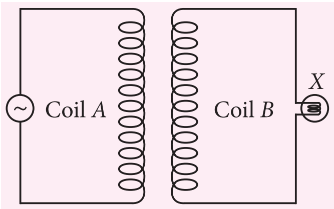
(a) Calculate the wavelength of the wave?

(b) Write down the expression for the oscillating electric field?

SECTION-C

11. Name the SI unit of magnetic flux and show that it equals volt-sec. Give three possible ways of producing an induced emf in a coil giving an example in each case.

12. Figure given below shows an arrangement by which current flows through the bulb (X) connected with coil B, when a.c. is passed through coil A.
Explain the following observations



(a) Bulb lights up.

(b) Bulb gets dimmer if the coil 'B' is moved upwards.

(c) If a copper sheet is inserted in the gap between the coils how the brightness of the bulb would change?

13. Calculate the total energy U_B and the energy density u_B of the magnetic field stored in a solenoid 0.5 m long, having 5000 turns and a current 10 A. The radius of the solenoid is 4 cm.

14. Define self inductance and give its units. Derive an expression for self inductance of a long, air-cored solenoid of length l, radius r having N number of turns.

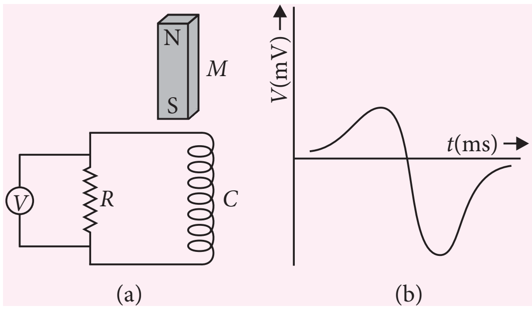
15. Which constituent radiation of the electromagnetic spectrum is used : (i) in radar (ii) to photograph internal parts of a human body and (iii) for taking photographs of sky during night and foggy conditions?
Give one reason for your answer in each case.

16. Derive the relation between electric field, magnetic field and the speed of an electromagnetic wave.

17. A bar magnet M is dropped so that it falls vertically through the coil C figure (a). The graph obtained for voltage produced across the coil versus time is shown in figure (b).

(a) Explain the shape of the graph.

(b) Why is the negative peak longer than the positive peak?



18. A 750 Hz, 20 V source is connected to a resistance of 100 Ω, an inductance of 0.1803 H and a capacitance of 10 μF all in series. Calculate the time in which the resistance (thermal capacity = 2 J °C^{-1}) will get heated by 10 °C.

19. (a) An electric heater is connected, turn by turn, to DC and AC sources of equal voltages. Will the rate of heat production be same in the two cases? Explain.

(b) What are the disadvantages of AC as compared to DC?

20. An inductive circuit draws a power 550 W from a 220 V – 50 Hz source. The power factor of the circuit is 0.8. The current in the circuit lags behind the voltage. Show that a capacitor of about $\frac{1}{42\pi}\times 10^{-2}$ F will have to be connected in the circuit to bring its power factor to unity.

OR

A radio can tune over the frequency range of a portion of MW broadcast band (800 kHz to 1200 kHz). If its LC circuit has an effective inductance of 200 μH, what must be the range of its variable capacitor?

21. When a circuit element X is conneted across an a.c. source, a current of $\sqrt{2}$ A flows through it and this current is in phase with the applied voltage. When another element Y is connected across the same a.c. source, the same current flows in the circuit but it leads the voltage by $\pi/2$ radians.

(a) Name the circuit elements X and Y.

(b) Find the current that flows in the circuit when the series combination of X and Y is connected across the same a.c. voltage.

(c) Plot a graph showing variation of the net impedance of this series combination of X and Y as a function of the angular frequency of the applied voltage.

22. A 25.0 μF capacitor, a 0.10 H inductor and a 25.0 Ω resistor are connected in series with an a.c. source whose emf is given by $\varepsilon = 310 \sin 314\, t$ V.

(a) What is the frequency of the emf?

(b) What is the reactance of the circuit?

(c) What is the impedance of the circuit?

(d) What is the current in the circuit?

(e) What is the phase angle of the current by which it leads or lags the applied emf?

SECTION-D

23. Two friends were passing through the market. They saw two welders using welding machines. One welder was using the goggles and face masks with window in order to protect his face. The other one was welding with naked eyes. They went to the welder who was not using face mask and explained him the advantages of goggles and masks. Next day, the welder bought a set of goggles and began to do his work fearlessly.

(a) What values were displayed by two friends?

(b) Why do welders wear glass goggles or face masks with glass windows while carrying out welding?

SECTION-E

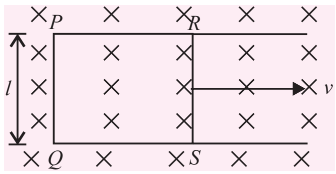
24. (a) Draw a schematic diagram of an ac generator describing its basic elements. State briefly its working principle. Draw a plot of variation of alternating emf versus time generated by a loop of wire rotating in a magnetic field.

(b) Why is choke coil needed in the use of fluorescent tubes with ac mains?

OR

(a) What are eddy currents? Write their two applications.

(b) Figure shows a rectangular conducting loop $PQSR$ in which arm RS of length l is movable. The loop is kept in a uniform magnetic field B directed downward perpendicular to the plane of the loop. The arm RS is moved with a uniform speed v.



Deduce expressions for

(i) the emf induced across the arm RS,

(ii) the external force required to move the arm, and

(iii) the power dissipated as heat.

25. Draw a schematic diagram of a step-up transformer. Explain its working principle. Deduce the expression for the radio of secondary to primary voltage in terms of the number of turns in the two coils. In an ideal transformer, how is this ratio related to the currents in the two coils?
How is the transformer used in large scale transmission and distribution of electrical energy over long distances?

OR

Derive an expression for the impedance of a series LCR circuit connected to an a.c. supply of variable frequency.
Plot a graph showing variation of current with the frequency of the applied voltage.
Explain briefly how the phenomenon of resonance in the circuit can be used in tuning mechanism of a radio or a TV set.

26. (a) Derive the relationship between the peak and the rms value of current in an a.c. circuit.

(b) Show that in an a.c. circuit containing a pure inductor, the voltage is ahead of current by $\pi/2$ in phase.

OR

(a) Define mutual inductance. Deduce an expression for the mutual inductance of two long coaxial solenoids having different radii and different number of turns.

(b) State Lenz's law. Give one example to illustrate this law. Lenz's law is a consequence of the law of conservation of energy. Justify this statement.

SOLUTIONS

1. As the electric current is switched on, the increasing magnetic flux sets up eddy current in the disc and convert it into a small magnet. This disc has same polarity as that on the top of the electromagnet and hence it is thrown up due to repulsion.

2. The magnetic lines of force due to the current I are parallel to the plane of the loop. The flux linked with the loop is zero. Hence no current is induced in the loop.
3. (i) R remains unaffected.
(ii) X_L gets doubled, because $X_L \propto \upsilon$
(iii) X_C becomes one-half of the original value, because $X_C \propto \frac{1}{\upsilon}$
4. Ultraviolet rays are generally used for LASIK eye surgery because being of shorter wavelengths, these can be easily focussed into very narrow beam for high precision required in the surgery.
5. The oscillating charge produces an oscillating electric field in space. It produces an oscillating magnetic field, which, in turn, is a source of oscillating electric field and so on. In this way, the oscillating electric and magnetic fields regenerate each other and as a result, an electromagnetic wave propagates through free space.
6. Faraday's laws of electromagnetic induction can be stated as follows
First law : Whenever the magnetic flux linked with a closed circuit changes, an emf (and hence a current) is induced in it which lasts only so long as the change in flux is taking place. This phenomenon is called electromagnetic induction.
Second law : The magnitude of the induced emf is equal to the rate of change of magnetic flux linked with the closed circuit.
Mathematically, according to the Faraday's flux rule,
Magnitude of induced emf
= Rate of change of magnetic flux

or $|\varepsilon| = \frac{d\phi}{dt}$

or $\varepsilon = -\frac{d\phi}{dt}$

The negative sign indicates that the direction of induced emf is such that it opposes the change in magnetic flux.
If the coil consists of N tightly wound turns, then total induced emf will be

$$\varepsilon = -N\frac{d\phi}{dt}$$

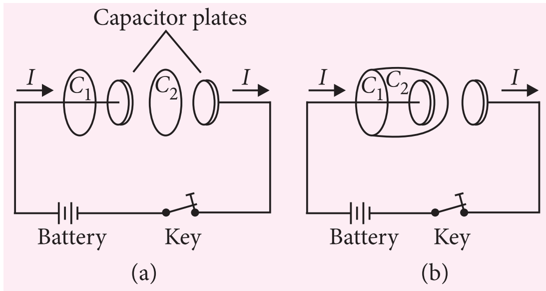
If the flux changes from ϕ_1 to ϕ_2 in time t, then the average induced emf will be

$\varepsilon = -N\frac{\phi_2 - \phi_1}{t}$, where ϕ is in webers, t in seconds, and ε in volts.

7. According to Ampere's circuital law,

$$\oint_C \vec{B}\cdot\vec{dl} = \mu_0 I \quad \text{...(i)}$$

As the current I flows across the area bounded by loop C_1, so



$$\oint_{C_1} \vec{B}\cdot\vec{dl} = \mu_0 I \quad \text{...(ii)}$$

But the area bounded by C_2 lies in the region between the capacitor plates, so no current flows across it.

$$\therefore \quad \oint_{C_2} \vec{B}\cdot\vec{dl} = 0 \quad \text{...(iii)}$$

Imagine the loops C_1 and C_2 to be infinitesimally close to each other, as shown in figure (b). Then we must have

$$\oint_{C_1} \vec{B}\cdot\vec{dl} = \oint_{C_2} \vec{B}\cdot\vec{dl}$$

This result is inconsistent with the eqns. (ii) and (iii). To remove this inconsistency, Maxwell argued that a changing electric field between the capacitor plates must induce a magnetic field which, in turn, must be associated with a current. Maxwell called this current as the displacement current. It is given by

$$I_D = \varepsilon_0 \frac{d\phi_E}{dt}$$

The total current must be

$$I = I_{\text{conduction}} + I_{\text{displacement}} = I_C + \varepsilon_0 \frac{d\phi_E}{dt}$$

Hence, the generalised from of the Ampere's circuital law is

$$\oint_C \vec{B}\cdot\vec{dl} = \mu_0 \left[I_C + \varepsilon_0 \frac{d\phi_E}{dt} \right]$$

8. Here, $r = \frac{0.02}{2} = 0.01$ m
$B_1 = 6000$ T, $B_2 = 1000$ T, $t = 4$ s, $\varepsilon = 11$ V

Now $\varepsilon = N\frac{|\phi_2 - \phi_1|}{t} = NA\frac{|B_2 - B_1|}{t}$

$= N \cdot \pi r^2 \cdot \frac{|B_2 - B_1|}{t}$

$\therefore \quad 11 = N \times \frac{22}{7} \times (0.01)^2 \times \frac{|1000 - 6000|}{4}$

Number of turns, $N = \frac{11 \times 7 \times 4}{22 \times (0.01)^2 \times 5000} = 28$

9. As per question, frequency, υ = 50 Hz,

L = 50 mH = 0.05 H

The bulb glows with maximum brightness when its impedance is minimum or equal to the resistance of the bulb filament and for this,

$X_L - X_C = 0$ or $X_L = X_C$

or $\quad \omega L = \frac{1}{\omega C}$

$\Rightarrow \quad C = \frac{1}{\omega^2 L} = \frac{1}{4\pi^2\upsilon^2 L} = \frac{1}{4 \times (3.14)^2 \times (50)^2 \times 0.05}$

$= 2.03 \times 10^{-4}\ \text{F} = 203\ \mu\text{F}$

OR

Here, initial frequency, υ = 50 Hz and inductive reactance, $X_L = 18.0\ \Omega$

As per relation, $X_L = \omega L = 2\pi\upsilon L$, we have

$\therefore$ Inductance,

$L = \frac{X_L}{2\pi\upsilon} = \frac{18.0}{2 \times 3.14 \times 50} = 0.0573\ \text{H} \simeq 57\ \text{mH}$

If frequency is changed to $\upsilon' = 20$ kHz $= 20 \times 10^3$ Hz, the new value of reactance,

$X'_L = 2\pi\upsilon' L = 2 \times 3.14 \times 20 \times 10^3 \times 0.0573$

$= 7.2 \times 10^3\ \Omega$

10. Here, $B_y = 8 \times 10^{-6} \sin[2 \times 10^{11}\, t + 300\,\pi\, x]$

(a) The Y-component of the magnetic field is given by

$B_y = B_0 \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$

Comparing the given equation with the above equation

$\frac{2\pi}{\lambda} = 300\pi$

or $\quad \lambda = \frac{2}{300} = 6.67 \times 10^{-3}$ m

(b) Since the direction of propagation of the e.m. wave is along negative X-axis and the magnetic field is along negative Y-axis. Hence, the electric field is along negative Z-axis and expression for it is given by

$E_z = E_0 \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$

Here, $E_0 = B_0 c = 8 \times 10^{-6} \times 3 \times 10^8 = 2400$ V m^{-1}

Since $\frac{2\pi}{\lambda} = 300\ \pi$ and $\frac{2\pi}{T} = 2 \times 10^{11}$, we have

$E_z = 2400 \sin [2 \times 10^{11}\, t + 300\,\pi\, x]$ (in V m^{-1})

11. SI unit of magnetic flux is weber (Wb). By Faraday's law

$|\varepsilon| = \frac{d\phi}{dt}$

$\therefore \quad 1\ \text{volt} = \frac{1\ \text{weber}}{1\ \text{second}}$ or 1 weber = 1 volt – 1 sec.

Magnetic flux linked with a closed coil is given by $\phi = BA \cos\theta$

Whenever ϕ changes, emf is induced. Hence emf may be produced by three methods,

(i) by changing the magnetic field B, *e.g.*, by changing the relative separation between a closed coil and magnet.

(ii) by changing the area A of the closed coil, *e.g.*, by moving a closed loop into and out of a region of magnetic field.

(iii) by changing the relative orientation θ between a closed coil and a magnetic field, *e.g.*, by rotating a closed coil about an axis perpendicular to the magnetic field.

12. (a) Bulb lights up due to the induced current set up in coil B because of alternating current in coil A.

(b) Bulb gets dimmer when the coil B is moved upwards because the flux linked with coil B decreases and induced current also decreases.

(c) When the copper sheet is inserted, eddy currents are set up in it which oppose the passage of magnetic flux. The induced emf in coil B decreases. This decreases the brightness of the bulb.

13. Here, l = 0.5 m, N = 5000, I = 10 A,

r = 4 cm = 4×10^{-2} m

As $u_B = \frac{B^2}{2\mu_0}$ and $B = \mu_0 nI$,

$u_B = \frac{(\mu_0 nI)^2}{2\mu_0} = \frac{1}{2}\mu_0 n^2 I^2$

$= \frac{1}{2}(4\pi \times 10^{-7})\left(\frac{5000}{0.5\ \text{m}}\right)^2 (10)^2$

$= 2\pi \times 10^3$ J m^{-3} $= 6.28 \times 10^3$ J m^{-3}

Volume of the solenoid,

$V = lA = (0.5)(3.14)(4 \times 10^{-2})^2 = 25 \times 10^{-4}$ m^3

Thus, $U_B = u_B V = (6.28 \times 10^3)(25 \times 10^{-4}) = 15.7$ J

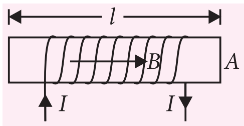
14. Self inductance of a coil may be defined as the induced emf set up in the coil due to a unit rate of change of current through it.
In SI, the unit of L is henry (H).
From relation

$$\varepsilon = L\frac{dI}{dt} \text{ or } L = \frac{\varepsilon}{dI/dt}$$

If ε = 1 V and dI/dt = 1 A s^{-1} then L = 1 H.
The self inductance of a coil is said to be one henry if an induced emf of one volt is set up in it when the current in it changes at the rate of one ampere per second.

$$\text{Thus, } 1\text{ H} = \frac{1\text{ V}}{1\text{ A s}^{-1}} = 1\text{ Vs A}^{-1} = 1\text{ Wb A}^{-1}$$

Let us consider a solenoid which is so long that its radius can be neglected as compared to its length. When a current flows through such a solenoid, the magnetic field inside it can be taken to be uniform. As shown in figure, let



A = cross-sectional area of the solenoid,
l = length of the solenoid,
N = total number of turns of the solenoid,
$n = N/l$ = number of turns per unit length of the solenoid.
Clearly, $B = \mu_0 nI$
Flux linked with each turn of the solenoid

$$= BA = \mu_0 nIA$$

Total flux linked with entire solenoid, *i.e.*,

$$\phi_B = (BA)N = (\mu_0 nIA)(nl) = \mu_0 n^2 AlI \quad \text{...(i)}$$

Also, $\phi_B = LI$ ...(ii)
where L is the inductance of the solenoid.
From eqns. (i) and (ii)
$LI = \mu_0 n^2 AlI$

$$\text{or} \quad L = \mu_0 n^2 Al = \frac{\mu_0 N^2 A}{l} \quad \text{...(iii)}$$

It is clear from eqn. (iii) that inductance
(i) depends upon the geometric factors (l and A) of the solenoid and
(ii) is proportional to the square of the number of turns.

15. (i) Microwaves : Due to their short wavelength these waves are not diffracted much by objects of normal dimensions. So they can be transmitted as a beam in a particular direction.
(ii) X-rays : Due to their high penetrating power, they pass through the human body. As they pass through the body, more X-rays are absorbed or scattered by bone than by tissue. If the transmitted radiation is directed onto a photographic film, the exposed areas show variations in intensity-a picture of internal structures.
(iii) IR radiations : Special photographic films sensitive to IR are used. They enable the picture to be taking in the dark and foggy conditions since infrared is scattered less than the visible light by particles in the atmosphere (because of its longer wavelength).

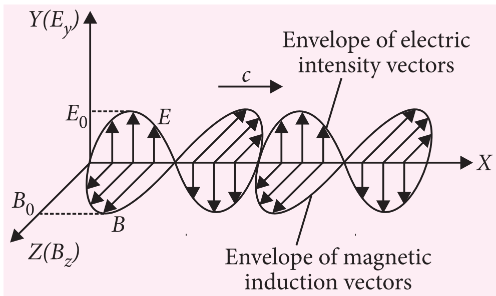
16. The sinusoidally varying electric and magnetic fields of a plane electromagnetic wave travelling along X-axis are given as

$$E_y = E_0 \sin(kx - \omega t) \quad \text{...(i)}$$

$$\text{and } B_z = B_0 \sin(kx - \omega t) \quad \text{...(ii)}$$

Here, E_0 and B_0 are the maximum values of the electric and magnetic fields associated with the electromagnetic wave.
Also, $k = 2\pi/\lambda$
where λ is the wavelength of the electromagnetic wave and
$\omega = 2\pi\upsilon$
where υ is the frequency of the wave. Clearly,

$$\frac{\omega}{k} = \frac{2\pi\upsilon}{2\pi/\lambda} = \upsilon\lambda = c \quad \text{...(iii)}$$



Taking partial derivatives of eqns. (i) and (ii), we get

$$\frac{\partial E_y}{\partial x} = kE_0\cos(kx - \omega t) \text{ and } \frac{\partial B_z}{\partial t} = -\omega B_0 \cos(kx - \omega t)$$

$$\text{Since } \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

$kE_0 \cos(kx - \omega t) = \omega B_0 \cos(kx - \omega t)$
or $\quad kE_0 = \omega B_0$

$$\text{or} \quad \frac{E_0}{B_0} = \frac{\omega}{k} \quad \text{...(iv)}$$

From eqns. (iii) and (iv),

$$\frac{E_0}{B_0} = c \quad \text{...(v)}$$

Further, from eqns. (i) and (ii),

$$\frac{E_y}{B_z} = \frac{E_0}{B_0} = c$$

Hence, at every instant, the ratio of the electric field to the magnetic field of an electromagnetic wave is equal to the speed of light.

17. (a) Initially, the magnet is at rest and the rate of change of magnetic flux through the coil is zero. Likewise, initially the induced emf produced in the coil is zero. As the magnet falls freely under gravity, its speed increases. As a result, the rate of change of magnetic flux through the coil also increases. Likewise, the induced emf also goes on increasing. However, in accordance with Lenz's law, the induced emf produced opposes the fall of magnet. Therefore, after the induced emf becomes maximum, it starts decreasing. When the magnet is well inside the coil, the magnetic flux linked with it becomes constant and hence induced emf becomes zero. As the magnet moves out of the coil, the induced emf increases from zero to a maximum value and then again falls to zero as shown in figure (b). The induced emf is of opposite sign, when the magnet moves out of the coil. It is because, the magnetic flux linked with the coil is now decreasing.

(b) The rate of change of magnetic flux, when the magnet moves out of the emf is greater than that when it moves towards it. Due to this, the negative peak is longer than the positive peak.

18. Here, $R = 100\ \Omega$; $L = 0.1803$ H;
$C = 10\ \mu F = 10^{-5}$ F; $E_v = 20$ V; $\upsilon = 750$ Hz

$$\text{Now, } Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{(100)^2 + \left(2\pi\times750\times0.1803 - \frac{1}{2\pi\times750\times10^{-5}}\right)^2}$$

$$= \sqrt{(100)^2 + (849.64 - 21.22)^2} = 834.43\ \Omega$$

Average power,

$$P = E_v I_v \cos\phi = E_v\left(\frac{E_v}{Z}\right)\frac{R}{Z} = E_v^2 \times \frac{R}{Z^2}$$

$$= \frac{(20)^2 \times 100}{(834.43)^2} = 0.0574\ \text{W}$$

If t is the required time, then

$P \times t$ = thermal capacity × rise in temperature

or $\quad 0.0574 \times t = 2 \times 10$ or $t = \dfrac{2\times10}{0.0574} = 348.4$ s

19. (a) The heating element of the electric heater is a coil of nichrome wire which has a resistance R and an inductance L. When the heater is connected to DC source of voltage V, the current,

$$I_{DC} = \frac{V}{R}$$

and rate of heat production $= I_{DC}^2 R = \dfrac{V^2}{R}$

When the same heater is connected to an AC supply of rms voltage V, the impedance of the circuit,

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$$

and hence, the current,

$$I_{AC} = \frac{V}{Z}$$

Therefore, the rate of heat production

$$= I_{AC}^2 R = \frac{V^2 R}{Z^2} = \frac{V^2 R}{(R^2 + \omega^2 L^2)}$$

From the above, it is clear that rate of heat production in AC circuit is less than that in DC circuit of same voltage.

(b) Main disadvantages of AC as compared to DC are as follows

(i) AC is more dangerous than DC having same effective voltage.

(ii) For a given effective voltage, the peak voltage in AC is $\sqrt{2}$ times the effective voltage, hence better insulation is required for AC line.

(iii) AC supply cannot be used as such for chemical and magnetic effects of current and in electronic devices.

20. Circuit is inductive, *i.e.*, there is no capacitor in the circuit. Power factor,

$$\cos\phi = \frac{R}{\sqrt{R^2 + X_L^2}} = 0.8 \Rightarrow X_L = \frac{3}{4}R$$

$$\text{Now power, } P = \frac{E_v^2}{Z^2}R$$

$$550 = \frac{(220)^2 R}{R^2 + \left(\frac{3}{4}R\right)^2} \Rightarrow R = 56.32\ \Omega$$

Now we have to bring the power factor to unity. For this

$$X_C = X_L \Rightarrow \frac{1}{\omega C} = \frac{3}{4}R \Rightarrow C = \frac{4}{3\omega R}$$

$$\Rightarrow \quad C = \frac{4}{3\times2\pi\times50\times56.32} = \frac{1}{4224\pi}\ \text{F}$$

OR

For tuning, the natural frequency *i.e.*, the frequency of LC oscillations should be equal to frequency of radio waves received by the antenna in the form of same frequency current in the LC circuit. For tuning at 800 kHz, required capacitance (C_1) is given by

$$\upsilon_1 = \frac{1}{\pi\sqrt{LC_1}}$$

$$C_1 = \frac{1}{4\pi^2 L\upsilon_1^2} = \frac{1}{4\pi^2(200\times10^{-6})(800\times10^3)^2}$$

C_1 = 197.9 pF

For tuning of 1200 kHz, required capacitance (C_2) is given by

$$\upsilon_2 = \frac{1}{2\pi\sqrt{LC_2}}$$

$$C_2 = \frac{1}{4\pi^2 L\upsilon_2^2} = \frac{1}{4\pi^2(200\times10^{-6})(1200\times10^3)^2}$$

C_2 = 87.9 pF

So, the variable capacitor should have a range between 87.9 pF to 197.9 pF.

21. (a) The circuit element X is a resistor and Y is a capacitor.

(b) Here, $R = X_C; V_{eff} = \sqrt{2}R$

When X and Y are connected in series, the impedance becomes

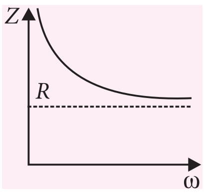
$$Z = \sqrt{R^2 + X_C^2} = \sqrt{2R^2} = \sqrt{2}R$$

Current, $I_{eff} = \frac{V_{eff}}{Z} = \frac{V_{eff}}{\sqrt{2}R} = \frac{\sqrt{2}R}{\sqrt{2}R} = 1\text{ A}$

(c) Impedance,

$$Z = \sqrt{R^2 + X_C^2}$$

$$= \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$



22. (a) Given ε = 310 sin 314 t V

Comparing it with $\varepsilon = \varepsilon_0 \sin 2\pi\upsilon t$, we get

$$2\pi\upsilon = 314 \text{ or } \upsilon = \frac{314}{2\pi} = \frac{314}{2\times3.14} = 50\text{ Hz}$$

(b) $X_C = \frac{1}{2\pi\upsilon C} = \frac{1\times7}{2\times22\times50\times25\times10^{-6}} = 127.3\ \Omega$

$$X_L = 2\pi\upsilon L = 2\times\frac{22}{7}\times50\times0.1 = 31.4\ \Omega$$

As X_L and X_C are out of phase by 180°, therefore, Net reactance = $X_C - X_L$ = 127.3 – 31.4 = 95.9 Ω and it is capacitive.

(c) Impedance,

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{(25)^2 + (95.9)^2}$$

$$= \sqrt{625 + 9196.81} = \sqrt{9821.81} = 99.1\ \Omega$$

(d) Effective current, $I_{eff} = \frac{\varepsilon_{eff}}{Z}$

But $\varepsilon_{eff} = \frac{\varepsilon_0}{\sqrt{2}} = \frac{310}{\sqrt{2}} \simeq 220\text{ V}$

$\therefore \quad I_{eff} = \frac{220}{99.1} = 2.22\text{ A}$

(e) The phase angle ϕ is given by

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{95.9}{25} = 3.84$$

Hence ϕ = 75.4°

As the circuit is capacitive, the current leads the voltage by 75.4°.

23. (a) Knowledge, creating awareness and social responsibility.

(b) Welders wear special glass goggles or face masks with glass windows to protect their eyes from large amount of harmful UV radiation produced by welding arc.

24. (a) Refer to point 4.8(2), page no. 275-276 (MTG Excel in Physics)

(b) Refer to point 4.8(3), page no. 276 (MTG Excel in Physics)

OR

(a) Refer to point 4.3(1) & (4), page no. 252-253 (MTG Excel in Physics)

(b) Refer to point 4.1(7e), page no. 248-249 (MTG Excel in Physics)

25. Refer to point 4.8(1), page no. 274-275 (MTG Excel in Physics)

OR

Refer to point 4.6(6, 7, 8), page no. 269-270 (MTG Excel in Physics)

26. (a) Refer to point 4.5(4), page no. 267 (MTG Excel in Physics)

(b) Refer to point 4.6(2), page no. 268 (MTG Excel in Physics)

OR

(a) Refer to point 4.2(2), page no. 250-251 (MTG Excel in Physics)

(a) Refer to point 4.1(6), page no. 247 (MTG Excel in Physics)

MPP-3 MONTHLY Practice Problems

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.

Self check table given at the end will help you to check your readiness.

Current Electricity

Total Marks : 120 **Time Taken : 60 min**

NEET / AIIMS / PMTs

Only One Option Correct Type

1. Three similar cells, each of emf 2 V and internal resistance r Ω send the same current through an external resistance of 2 Ω, when connected in series or in parallel. The strength of the current flowing through the external resistance is

(a) 1 A (b) 1.5 A (c) 2 A (d) 0.75 A

2. Which of the following quantities do not change when a metallic resistor connected to a battery is heated due to the current?

(a) Drift speed (b) Resistivity
(c) Resistance (d) Number of free electrons

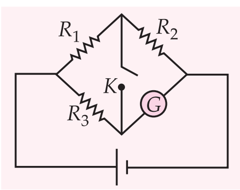
3. Two electric bulbs rated at 25 W, 220 V and 100 W, 220 V are connected in series across a 220 V source. The 25 W and 100 W bulbs now draw powers P_1 and P_2 respectively. Then

(a) $P_1 = 4$ W, $P_2 = 16$ W
(b) $P_1 = 16$ W, $P_2 = 8$ W
(c) $P_1 = 16$ W, $P_2 = 4$ W
(d) $P_1 = 4$ W, $P_2 = 8$ W

4. The resistance of the wire in the platinum resistance thermometer at ice point is 5 Ω and at steam point is 5.25 Ω. When the thermometer is inserted in an unknown hot bath its resistance is found to be 5.5 Ω. The temperature of the hot bath is

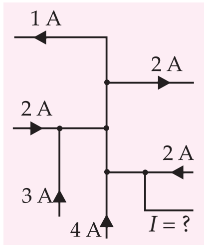
(a) 100°C (b) 200°C (c) 300°C (d) 350°C

5. In the above circuit, the values of resistances are such that $\frac{R_1}{R_2} = \frac{R_3}{R_G}$, where R_G is the internal resistance of the galvanometer. The reading of the galvanometer



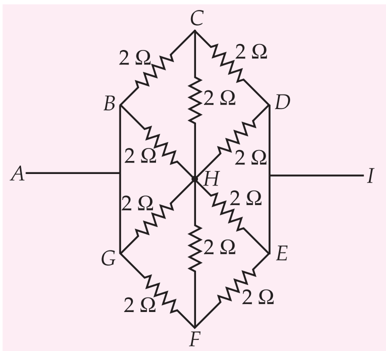
(a) remains constant whether the switch K is ON or OFF
(b) increases when the switch K is ON
(c) increases when the switch condition is changed from ON to OFF
(d) always shows zero value.

6. The magnitude and direction of current I (in A) indicated in the following circuit is



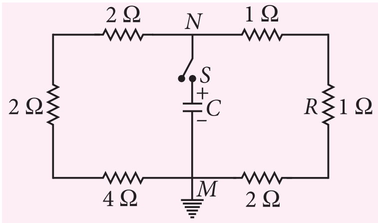
(a) 14⟶
(b) 8⟶
(c) ⟵4
(d) ⟵8

7. The effective resistance across the points A and I is



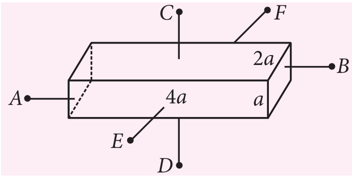
(a) 2 Ω (b) 1 Ω (c) 0.5 Ω (d) 5 Ω

8. A capacitor of capacity 6 μF and initial charge 160 μC is connected with a key S and different resistors as shown in figure. Point M is earthed. If key is closed at $t = 0$; then the current through resistance $R(= 1\ \Omega)$ at $t = 16$ μs is



(a) $\frac{10}{3e}$ A (b) $\frac{10}{e}$ A (c) $\frac{20}{3e}$ A (d) $\frac{5}{e}$ A

9. A conductor with rectangular cross-section has dimensions ($a \times 2a \times 4a$) as shown in figure. Resistance across AB is x, across CD is y and across EF is z. Then

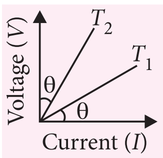


(a) $x = y = z$
(b) $x > y > z$
(c) $y > z > x$
(d) $x > z > y$

10. The number density of free electrons in a copper conductor is 8.5×10^{28} m^{-3}. How long does an electron take to drift from one end of a wire 3.0 m long to its other end? The area of cross-section of the wire is 2.0×10^{-6} m^2 and it is carrying a current of 3.0 A.

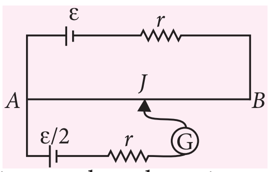
(a) 8.1×10^4 s (b) 2.7×10^4 s
(c) 9×10^3 s (d) 3×10^3 s

11. The $V - I$ graph for a conductor at temperature T_1 and T_2 are as shown in the figure. The term $(T_2 - T_1)$ is inversely proportional to



(a) cos 2θ (b) sin 2θ
(c) cot 2θ (d) tan 2θ

12. A potentiometer wire AB is 600 cm long and of resistance 15 r. At what length from end A of potentiometer wire should the jockey J touch the wire so that there is no deflection in the galvanometer?



(a) 20 cm (b) 120 cm
(c) 320 cm (d) 450 cm

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :

(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. Assertion : Voltmeter is much better than a potentiometer for measuring emf of cell.
Reason : A potentiometer draws small current while measuring emf of a cell.

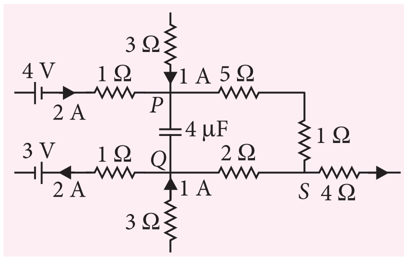
14. Assertion : Current is passed through a metallic wire so that it becomes red hot. When cold water is poured on half of its portion, the rest of the half portion becomes more hot.
Reason : Resistance of metallic wire decreases due to decrease in temperature.

15. Assertion : In a metre-bridge experiment, null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance.
Reason : Resistance of a metal remains constant with increase in temperature.

JEE MAIN / JEE ADVANCED / PETs

Only One Option Correct Type

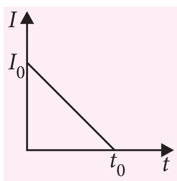
16. A part of a circuit in steady state along with the currents flowing in the branches, the values of resistances, etc., is shown in figure. The energy stored in the capacitor is



(a) 100 μJ (b) 300 μJ (c) 400 μJ (d) 800 μJ

17. A heating element is made by maintaining a potential difference of 75 V along the length of a nichrome wire with a 2.6 mm^2 cross-section and a resistivity of 5.0×10^{-7} Ω m. If the element dissipates 4.8 kW, what is its length?
(a) 6.1 m (b) 12.2 m (c) 3.5 m (d) 2.7 m

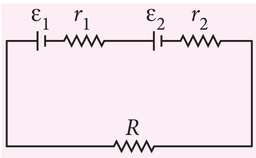
18. Current passing through a coil of resistance R decreases uniformly as shown in the figure. If total amount of charge q passing through the coil during a time interval t_0 then what will be the total amount of heat generated through it?



(a) $\frac{3}{4}\frac{q^2R}{t_0}$ (b) $\frac{4}{3}\frac{q^2R}{t_0}$

(c) $\frac{q^2}{t_0R}$ (d) $\frac{q^2R^2}{t_0^2}$

19. Under what condition current passing through the resistance R can be increased by short circuiting the battery of emf ε_2. The internal resistances of the two batteries are r_1 and r_2 as shown in the figure.



(a) $\varepsilon_2 r_1 > \varepsilon_1(R + r_2)$
(b) $\varepsilon_1 r_2 > \varepsilon_2(R + r_1)$
(c) $\varepsilon_2 r_2 < \varepsilon_1(R + r_2)$
(d) $\varepsilon_1 r_1 > \varepsilon_2(R + r_1)$

More than One Options Correct Type

20. Two bulbs consume same power when operated at 200 V and 300 V respectively. When the two bulbs are connected in series across a DC source of 500 V then ratio of
(a) potential difference across them is 3 : 2
(b) potential difference across them is 4 : 9
(c) power consumed across them is 4 : 9
(d) power consumed across them is 2 : 3.

21. A long round conductor of cross-sectional area S is made of material whose resistivity depends only on a distance r from the axis of the conductor as $\rho = \alpha/r^2$, where α is a constant. Then,
(a) the resistance per unit length of the conductor is $\frac{2\pi\alpha}{S^2}$
(b) the resistance per unit length of the conductor is $\frac{\pi\alpha}{S^2}$
(c) the electric field strength in the conductor due to which a current I flows through it is $\frac{2\pi\alpha I}{S^2}$
(d) the electric field strength in the conductor due to which a current I flows through it is $\frac{\pi\alpha I}{S^2}$.

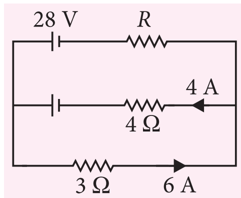
22. In a potentiometer wire experiment, the emf of a battery in the primary circuit is 20 V and its internal resistance is 5 Ω. There is a resistance box (in series with the battery and the potentiometer wire) whose resistance can be varied from 120 Ω to 170 Ω. Resistance of the potentiometer wire is 75 Ω. The potential difference can be measured using this potentiometer.
(a) 5 V (b) 6 V (c) 7 V (d) 8 V

23. A capacitor C_1 of capacitance 1 μF and a capacitor C_2 of capacitance 2 μF are separately charged by a common battery for a long time. The two capacitors are then separately discharged through equal resistors. Both the discharge circuits are connected at $t = 0$.
(a) The current in each of the two discharging circuits is zero at $t = 0$.
(b) The currents in the two discharging circuits at $t = 0$ are equal but not zero.
(c) The currents in the two discharging circuits at $t = 0$ are unequal.
(d) C_1 loses 50% of its initial charge sooner than C_2 loses 50% of its initial charge.

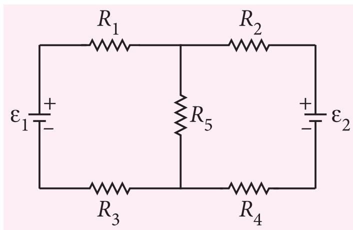
Integer Answer Type

24. A length of wire has a resistance of 120 Ω. The wire is cut into pieces that have the same length, and then, the wires are connected in parallel. The resistance of the parallel combination is 1.88 Ω. The number of pieces into which the wire was cut is

25. In the circuit shown in the figure, the resistance (in Ω) of resistor R is

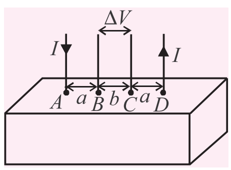


26. In the circuit shown in figure, $R_1 = 2$ Ω, $R_2 = 3$ Ω, $R_3 = 3$ Ω, $R_4 = 2$ Ω, $R_5 = 2$ Ω, $\varepsilon_1 = 16$ V and $\varepsilon_2 = 11$ V. The current (in A) through resistance R_5 is



Comprehension Type

Consider a block of conducting material of resistivity ρ shown in the figure. Current I enters at A and leaves from D. We apply superposition principle to find



voltage ΔV developed between B and C. The calculation is done in the following steps:

(i) Take current I entering from A and assume it to spread over a hemispherical surface in the block.

(ii) Calculate field $E(r)$ at distance r from A by using Ohm's law $E = \rho j$, where j is the current per unit area at r.

(iii) From the r dependence of $E(r)$, obtain the potential $V(r)$ at r.

(iv) Repeat (i), (ii) and (iii) for current I leaving D and superpose results for A and D.

27. ΔV measured between B and C is

(a) $\dfrac{\rho I}{2\pi(a-b)}$ (b) $\dfrac{\rho I}{\pi a} - \dfrac{\rho I}{\pi(a+b)}$

(c) $\dfrac{\rho I}{a} - \dfrac{\rho I}{(a+b)}$ (d) $\dfrac{\rho I}{2\pi a} - \dfrac{\rho I}{2\pi(a+b)}$

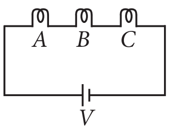
28. For current entering at A, the electric field at a distance r from A is

(a) $\dfrac{\rho I}{4\pi r^2}$ (b) $\dfrac{\rho I}{8\pi r^2}$ (c) $\dfrac{\rho I}{r^2}$ (d) $\dfrac{\rho I}{2\pi r^2}$

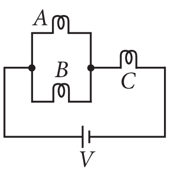
Matrix Match Type

29. Three bulbs A, B and C are having rated power P_A, P_B and P_C respectively, each bulb is designed to operate at rated voltage V. It is given that $P_A > P_B > P_C$. In column I, the three bulbs are arranged in different configurations, while in column II, the information about intensities of bulbs are mentioned. Match the column I with column II. Neglect the variation in resistance due to change in temperature.

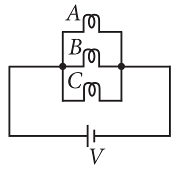
Column I | **Column II**

(A) 

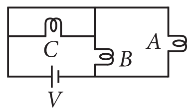
(P) A is glowing with minimum brightness

(B) 

(Q) C is glowing with maximum brightness

(C) 

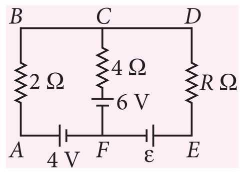
(R) B is glowing with minimum brightness

(D) 

(S) A is glowing with maximum brightness

	A	B	C	D
(a)	P, Q	Q, R	S	R, S
(b)	P, R	Q	S, R	P
(c)	P, Q	R	S, Q	S
(d)	P, Q	P	S	R

30. A circuit is shown in figure. R is a non-zero variable with finite resistance. ε is some unknown emf with polarities as shown. Match the columns.



Column I	Column II
(A) Current passing through $4\,\Omega$ resistance can be zero	(P) Possible if $\varepsilon = (R+6)$ V
(B) Current passing through $4\,\Omega$ resistance can be from F to C direction	(Q) Possible if $\varepsilon > (R+6)$ V
(C) Current passing through $4\,\Omega$ resistance can be from C to F direction	(R) Possible if $\varepsilon < (R+6)$ V
(D) Current passing through $2\,\Omega$ resistance can be zero	(S) Possible if $2\varepsilon = 8 - R$

	A	B	C	D
(a)	P	R	Q	S
(b)	S	R	Q	P
(c)	S	Q	R	P
(d)	P	Q	R	S

◇◇

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……

No. of questions correct ……

Marks scored in percentage ……

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

PHYSICS MUSING

Physics Musing was started in August 2013 issue of Physics For You with the suggestion of Shri Mahabir Singh. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / AIIMS / Other PMTs with additional study material.
In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / various PMTs. The detailed solutions of these problems will be published in next issue of Physics For You.
The readers who have solved five or more problems may send their detailed solutions with their names and complete address. The names of those who send atleast five correct solutions will be published in the next issue.
We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

PROBLEM Set 38

SINGLE OPTION CORRECT TYPE

1. A loaded vertical spring executes simple harmonic oscillations with period of 4 s. The difference between the kinetic energy and potential energy of this system oscillates with a period of

(a) 8 s (b) 1 s
(c) 2 s (d) 4 s

2. From an atom of mass number 220, initially at rest, α-decay takes place. If the total kinetic energy of system after decay is 5.5 MeV, the most probable K.E. of α-particle is

(a) 4.4 eV (b) 5.4 eV
(c) 5.6 eV (d) 6.5 eV.

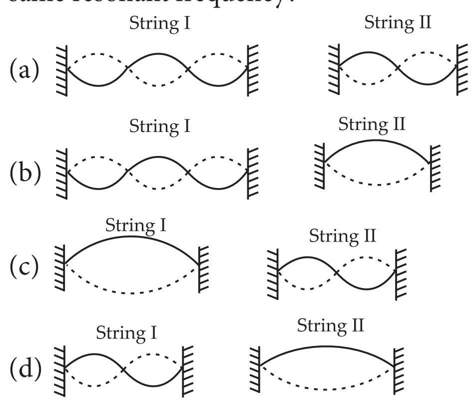
3. A substance of mass M kg requires a power input of P W (joules per second) to remain in the molten state at its melting point. When the power source is turned off, the sample completely solidifies in t s. The latent heat of fusion of the substance is

(a) $\frac{P}{Mt}$ (b) $\frac{Mt}{P}$
(c) $\frac{Pt}{2M}$ (d) $\frac{Pt}{M}$

4. White light is used to illuminate the two slits in a Young's double slit experiment. The separation between slits is b and the screen is at a distance $d(>>b)$ from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing. One of these missing wavelengths is

(a) $\lambda = \frac{b^2}{2d}$ (b) $\lambda = \frac{2b^2}{d}$
(c) $\lambda = \frac{b^2}{3d}$ (d) $\lambda = \frac{2b^2}{3d}$

5. String I and II have identical lengths and linear mass densities, but string I is under greater tension than string II. The accompanying figure shows four different situations, in which standing wave patterns exist on the two strings. In which situation is it possible that strings I and II oscillating at the same resonant frequency?



6. A straight rod of length L extends from $x = a$ to $x = L + a$. The gravitational force exerted on a point mass m at $x = 0$ if the mass per unit length of the rod is $A + Bx^2$, is

(a) $Gm\left[\frac{A}{a+L} - \frac{A}{a} + BL\right]$

(b) $Gm\left[\frac{A}{a} - \frac{A}{a+L} + BL\right]$

(c) $Gm\left[\frac{A}{a+L} - \frac{A}{a} - BL\right]$

(d) $Gm\left[\frac{A}{a} - \frac{A}{a+L} - BL\right]$

Contd. on Page no. 83

By Akhil Tewari, Author Foundation of Physics for JEE Main & Advanced, Professor, IITians PACE, Mumbai.

NEET

Phase - II held on 24th July

SOLVED PAPER 2016

GOOD NEWS

We are happy to inform our readers that more than 60% questions asked in NEET (Phase-II) 2016 Exam are very similar to the problems given in **MTG Physics Books.**

GOOD NEWS

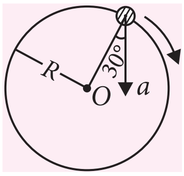
1. Planck's constant (h), speed of light in vacuum (c) and Newton's gravitational constant (G) are three fundamental constants. Which of the following combinations of these has the dimension of length ?

(a) $\frac{\sqrt{hG}}{c^{3/2}}$ (b) $\frac{\sqrt{hG}}{c^{5/2}}$ (c) $\sqrt{\frac{hc}{G}}$ (d) $\sqrt{\frac{Gc}{h^{3/2}}}$

2. Two cars P and Q start from a point at the same time in a straight line and their positions are represented by $x_P(t) = (at + bt^2)$ and $x_Q(t) = (ft - t^2)$. At what time do the cars have the same velocity ?

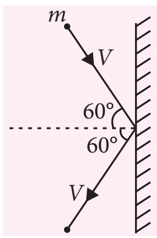
(a) $\frac{a-f}{1+b}$ (b) $\frac{a+f}{2(b-1)}$ (c) $\frac{a+f}{2(1+b)}$ (d) $\frac{f-a}{2(1+b)}$

3. In the given figure, a = 15 m s^{-2} represents the total acceleration of a particle moving in the clockwise direction in a circle of radius R = 2.5 m at a given instant of time. The speed of the particle is



(a) 4.5 m s^{-1} (b) 5.0 m s^{-1}
(c) 5.7 m s^{-1} (d) 6.2 m s^{-1}

4. A rigid ball of mass m strikes a rigid wall at 60° and gets reflected without loss of speed as shown in the figure. The value of impulse imparted by the wall on the ball will be



(a) mV (b) $2\,mV$
(c) $\frac{mV}{2}$ (d) $\frac{mV}{3}$

5. A bullet of mass 10 g moving horizontally with a velocity of 400 m s^{-1} strikes a wood block of mass 2 kg which is suspended by light inextensible string of length 5 m. As a result, the centre of gravity of the block found to rise a vertical distance of 10 cm. The speed of the bullet after it emerges out horizontally from the block will be

(a) 100 m s^{-1} (b) 80 m s^{-1}
(c) 120 m s^{-1} (d) 160 m s^{-1}

6. Two identical balls A and B having velocities of 0.5 m s^{-1} and –0.3 m s^{-1} respectively collide elastically in one dimension. The velocities of B and A after the collision respectively will be

(a) –0.5 m s^{-1} and 0.3 m s^{-1}
(b) 0.5 m s^{-1} and –0.3 m s^{-1}
(c) –0.3 m s^{-1} and 0.5 m s^{-1}
(d) 0.3 m s^{-1} and 0.5 m s^{-1}

7. A particle moves from a point $(-2\hat{i}+5\hat{j})$ to $(4\hat{j}+3\hat{k})$ when a force of $(4\hat{i}+3\hat{j})$ N is applied. How much work has been done by the force ?

(a) 8 J (b) 11 J (c) 5 J (d) 2 J

8. Two rotating bodies A and B of masses m and $2m$ with moments of inertia I_A and I_B ($I_B > I_A$) have equal kinetic energy of rotation. If L_A and L_B be their angular momenta respectively, then

(a) $L_A = \frac{L_B}{2}$ (b) $L_A = 2L_B$
(c) $L_B > L_A$ (d) $L_A > L_B$

9. A solid sphere of mass m and radius R is rotating about its diameter. A solid cylinder of the same mass and same radius is also rotating about its geometrical axis with an angular speed twice that of the sphere. The ratio of their kinetic energies of rotation (E_{sphere} / $E_{cylinder}$) will be

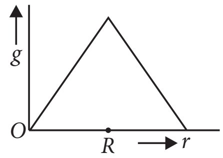
(a) 2 : 3 (b) 1 : 5 (c) 1 : 4 (d) 3 : 1

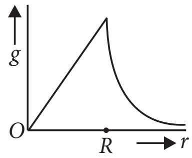
10. A light rod of length l has two masses m_1 and m_2 attached to its two ends. The moment of inertia of the system about an axis perpendicular to the rod and passing through the centre of mass is

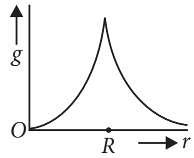
(a) $\dfrac{m_1 m_2}{m_1+m_2} l^2$ (b) $\dfrac{m_1+m_2}{m_1 m_2} l^2$

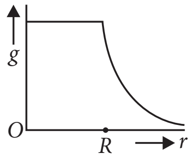
(c) $(m_1+m_2)l^2$ (d) $\sqrt{m_1 m_2}\, l^2$

11. Starting from the centre of the earth having radius R, the variation of g(acceleration due to gravity) is shown by

(a) 

(b) 

(c) 

(d) 

12. A satellite of mass m is orbiting the earth (of radius R) at a height h from its surface. The total energy of the satellite in terms of g_0, the value of acceleration due to gravity at the earth's surface, is

(a) $\dfrac{mg_0R^2}{2(R+h)}$ (b) $-\dfrac{mg_0R^2}{2(R+h)}$

(c) $\dfrac{2mg_0R^2}{R+h}$ (d) $-\dfrac{2mg_0R^2}{R+h}$

13. A rectangular film of liquid is extended from (4 cm × 2 cm) to (5 cm × 4 cm). If the work done is 3×10^{-4} J, the value of the surface tension of the liquid is

(a) 0.250 N m^{-1} (b) 0.125 N m^{-1}
(c) 0.2 N m^{-1} (d) 8.0 N m^{-1}

14. Three liquids of densities ρ_1, ρ_2 and ρ_3 (with $\rho_1 > \rho_2 > \rho_3$), having the same value of surface tension T, rise to the same height in three identical capillaries. The angles of contact θ_1, θ_2 and θ_3 obey

(a) $\dfrac{\pi}{2} > \theta_1 > \theta_2 > \theta_3 \geq 0$ (b) $0 \leq \theta_1 < \theta_2 < \theta_3 < \dfrac{\pi}{2}$

(c) $\dfrac{\pi}{2} < \theta_1 < \theta_2 < \theta_3 < \pi$ (d) $\pi > \theta_1 > \theta_2 > \theta_3 > \dfrac{\pi}{2}$

15. Two identical bodies are made of a material for which the heat capacity increases with temperature. One of these is at 100 °C, while the other one is at 0 °C. If the two bodies are brought into contact, then, assuming no heat loss, the final common temperature is

(a) 50 °C
(b) more than 50 °C
(c) less than 50 °C but greater than 0 °C
(d) 0 °C

16. A body cools from a temperature $3T$ to $2T$ in 10 minutes. The room temperature is T. Assume that Newton's law of cooling is applicable. The temperature of the body at the end of next 10 minutes will be

(a) $\dfrac{7}{4}T$ (b) $\dfrac{3}{2}T$ (c) $\dfrac{4}{3}T$ (d) T

17. One mole of an ideal monoatomic gas undergoes a process described by the equation PV^3 = constant. The heat capacity of the gas during this process is

(a) $\dfrac{3}{2}R$ (b) $\dfrac{5}{2}R$ (c) $2R$ (d) R

18. The temperature inside a refrigerator is t_2 °C and the room temperature is t_1 °C. The amount of heat delivered to the room for each joule of electrical energy consumed ideally will be

(a) $\dfrac{t_1}{t_1-t_2}$ (b) $\dfrac{t_1+273}{t_1-t_2}$

(c) $\dfrac{t_2+273}{t_1-t_2}$ (d) $\dfrac{t_1+t_2}{t_1+273}$

19. A given sample of an ideal gas occupies a volume V at a pressure P and absolute temperature T. The mass of each molecule of the gas is m. Which of the following gives the density of the gas ?

(a) $P/(kT)$ (b) $Pm/(kT)$
(c) $P/(kTV)$ (d) mkT

20. A body of mass m is attached to the lower end of a spring whose upper end is fixed. The spring has negligible mass. When the mass m is slightly pulled down and released, it oscillates with a time period of 3 s. When the mass m is increased by 1 kg, the time period of oscillations becomes 5 s. The value of m in kg is

(a) $\dfrac{3}{4}$ (b) $\dfrac{4}{3}$ (c) $\dfrac{16}{9}$ (d) $\dfrac{9}{16}$

21. The second overtone of an open organ pipe has the same frequency as the first overtone of a closed pipe L metre long. The length of the open pipe will be

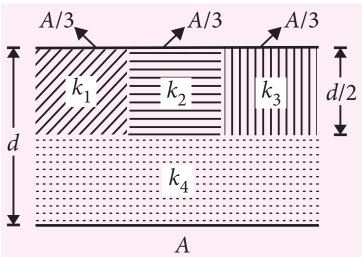
(a) L (b) $2L$ (c) $\dfrac{L}{2}$ (d) $4L$

22. Three sound waves of equal amplitudes have frequencies $(n-1)$, n, $(n+1)$. They superimpose to give beats. The number of beats produced per second will be

(a) 1 (b) 4 (c) 3 (d) 2

23. An electric dipole is placed at an angle of 30° with an electric field intensity 2×10^5 N C^{-1}. It experiences a torque equal to 4 N m. The charge on the dipole, if the dipole length is 2 cm, is
(a) 8 mC (b) 2 mC (c) 5 mC (d) 7 μC

24. A parallel-plate capacitor of area A, plate separation d and capacitance C is filled with four dielectric materials having dielectric constants k_1, k_2, k_3 and k_4 as shown in the figure. If a single dielectric material is to be used to have the same capacitance C in this capacitor, then its dielectric constant k is given by



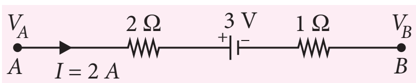
(a) $k = k_1 + k_2 + k_3 + 3k_4$

(b) $k = \frac{2}{3}(k_1 + k_2 + k_3) + 2k_4$

(c) $\frac{2}{k} = \frac{3}{k_1 + k_2 + k_3} + \frac{1}{k_4}$

(d) $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{3}{2k_4}$

25. The potential difference $(V_A - V_B)$ between the points A and B in the given figure is



(a) –3 V (b) +3 V (c) +6 V (d) +9 V

26. A filament bulb (500 W, 100 V) is to be used in a 230 V main supply. When a resistance R is connected in series, it works perfectly and the bulb consumes 500 W. The value of R is
(a) 230 Ω (b) 46 Ω (c) 26 Ω (d) 13 Ω

27. A long wire carrying a steady current is bent into a circular loop of one turn. The magnetic field at the centre of the loop is B. It is then bent into a circular coil of n turns. The magnetic field at the centre of this coil of n turns will be
(a) nB (b) n^2B (c) $2nB$ (d) $2n^2B$

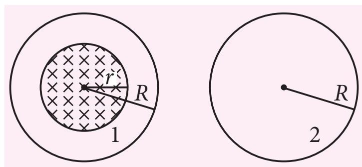
28. A bar magnet is hung by a thin cotton thread in a uniform horizontal magnetic field and is in equilibrium state. The energy required to rotate it by 60° is W. Now the torque required to keep the magnet in this new position is

(a) $\frac{W}{\sqrt{3}}$ (b) $\sqrt{3}W$ (c) $\frac{\sqrt{3}W}{2}$ (d) $\frac{2W}{\sqrt{3}}$

29. An electron is moving in a circular path under the influence of a transverse magnetic field of 3.57×10^{-2} T. If the value of e/m is 1.76×10^{11} C kg^{-1}, the frequency of revolution of the electron is
(a) 1 GHz (b) 100 MHz
(c) 62.8 MHz (d) 6.28 MHz

30. Which of the following combinations should be selected for better tuning of an L-C-R circuit used for communication ?
(a) R = 20 Ω, L = 1.5 H, C = 35 μF
(b) R = 25 Ω, L = 2.5 H, C = 45 μF
(c) R = 15 Ω, L = 3.5 H, C = 30 μF
(d) R = 25 Ω, L = 1.5 H, C = 45 μF

31. A uniform magnetic field is restricted within a region of radius r. The magnetic field changes with time at a rate $\frac{d\vec{B}}{dt}$. Loop 1 of radius $R > r$ encloses the region r and loop 2 of radius R is outside the region of magnetic field as shown in the figure. Then the e.m.f. generated is



(a) zero in loop 1 and zero in loop 2

(b) $-\frac{d\vec{B}}{dt}\pi r^2$ in loop 1 and $-\frac{d\vec{B}}{dt}\pi r^2$ in loop 2

(c) $-\frac{d\vec{B}}{dt}\pi R^2$ in loop 1 and zero in loop 2

(d) $-\frac{d\vec{B}}{dt}\pi r^2$ in loop 1 and zero in loop 2

32. The potential differences across the resistance, capacitance and inductance are 80 V, 40 V and 100 V respectively in an L-C-R circuit. The power factor of this circuit is
(a) 0.4 (b) 0.5 (c) 0.8 (d) 1.0

33. A 100 Ω resistance and a capacitor of 100 Ω reactance are connected in series across a 220 V source. When the capacitor is 50% charged, the peak value of the displacement current is
(a) 2.2 A (b) 11 A (c) 4.4 A (d) $11\sqrt{2}$ A

34. Two identical glass (μ_g = 3/2) equiconvex lenses of focal length f each are kept in contact. The space between the two lenses is filled with water (μ_w = 4/3). The focal length of the combination is
(a) $f/3$ (b) f (c) $4f/3$ (d) $3f/4$

35. An air bubble in a glass slab with refractive index 1.5 (near normal incidence) is 5 cm deep when viewed from one surface and 3 cm deep when viewed from the opposite face. The thickness (in cm) of the slab is
(a) 8 (b) 10 (c) 12 (d) 16

36. The interference pattern is obtained with two coherent light sources of intensity ratio n. In the interference pattern, the ratio $\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$ will be

(a) $\frac{\sqrt{n}}{n+1}$ (b) $\frac{2\sqrt{n}}{n+1}$

(c) $\frac{\sqrt{n}}{(n+1)^2}$ (d) $\frac{2\sqrt{n}}{(n+1)^2}$

37. A person can see clearly objects only when they lie between 50 cm and 400 cm from his eyes. In order to increase the maximum distance of distinct vision to infinity, the type and power of the correcting lens, the person has to use, will be
(a) convex, +2.25 diopter
(b) concave, –0.25 diopter
(c) concave, –0.2 diopter
(d) convex, +0.15 diopter

38. A linear aperture whose width is 0.02 cm is placed immediately in front of a lens of focal length 60 cm. The aperture is illuminated normally by a parallel beam of wavelength 5×10^{-5} cm. The distance of the first dark band of the diffraction pattern from the centre of the screen is
(a) 0.10 cm (b) 0.25 cm
(c) 0.20 cm (d) 0.15 cm

39. Electrons of mass m with de-Broglie wavelength λ fall on the target in an X-ray tube. The cutoff wavelength (λ_0) of the emitted X-ray is

(a) $\lambda_0 = \frac{2mc\lambda^2}{h}$ (b) $\lambda_0 = \frac{2h}{mc}$

(c) $\lambda_0 = \frac{2m^2c^2\lambda^3}{h^2}$ (d) $\lambda_0 = \lambda$

40. Photons with energy 5 eV are incident on a cathode C in a photoelectric cell. The maximum energy of emitted photoelectrons is 2 eV. When photons of energy 6 eV are incident on C, no photoelectrons will reach the anode A, if the stopping potential of A relative to C is
(a) +3 V (b) +4 V
(c) –1 V (d) –3 V

41. If an electron in a hydrogen atom jumps from the 3rd orbit to the 2nd orbit, it emits a photon of wavelength λ. When it jumps from the 4th orbit to the 3rd orbit, the corresponding wavelength of the photon will be

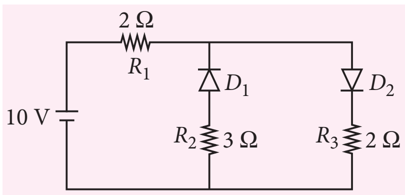
(a) $\frac{16}{25}\lambda$ (b) $\frac{9}{16}\lambda$

(c) $\frac{20}{7}\lambda$ (d) $\frac{20}{13}\lambda$

42. The half-life of a radioactive substance is 30 minutes. The time (in minutes) taken between 40% decay and 85% decay of the same radioactive substance is
(a) 15 (b) 30 (c) 45 (d) 60

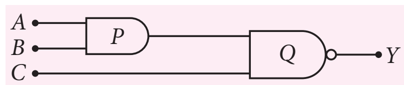
43. For CE transistor amplifier, the audio signal voltage across the collector resistance of 2 kΩ is 4 V. If the current amplification factor of the transistor is 100 and the base resistance is 1 kΩ, then the input signal voltage is
(a) 10 mV (b) 20 mV (c) 30 mV (d) 15 mV

44. The given circuit has two ideal diodes connected as shown in the figure. The current flowing through the resistance R_1 will be



(a) 2.5 A (b) 10.0 A
(c) 1.43 A (d) 3.13 A

45. What is the output Y in the following circuit, when all the three inputs A, B, C are first 0 and then 1 ?



(a) 0, 1 (b) 0, 0 (c) 1, 0 (d) 1, 1

SOLUTIONS

1. (a): According to question, length depends on h, c and G so,

$$l \propto h^p c^q G^r$$

$$l = k h^p c^q G^r \quad ...(i)$$

Writting dimensions of physical quantities on both sides,

$$[M^0LT^0] = [ML^2T^{-1}]^p [LT^{-1}]^q [M^{-1}L^3T^{-2}]^r$$

Applying the principle of homogeneity of dimensions, we get

$$p - r = 0 \quad ...(ii)$$

$$2p + q + 3r = 1 \quad ...(iii)$$

$$-p - q - 2r = 0 \quad ...(iv)$$

Solving eqns. (ii), (iii) and (iv), we get

$$p = r = \frac{1}{2},\ q = -\frac{3}{2}$$

From eqn. (i), dimension of $l = \dfrac{\sqrt{hG}}{c^{3/2}}$ $\quad (\because [k] = 1)$

2. (d): Position of the car P at any time t, is

$$x_P(t) = at + bt^2$$

$$v_P(t) = \frac{dx_P(t)}{dt} = a + 2bt$$

Similarly, for car Q,

$$x_Q(t) = ft - t^2$$

$$v_Q(t) = \frac{dx_Q(t)}{dt} = f - 2t$$

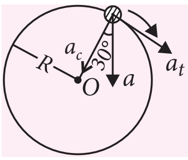
$\because \quad v_P(t) = v_Q(t) \qquad$ (Given)

$\therefore \quad a + 2bt = f - 2t$ or, $2t(b + 1) = f - a$

$$\therefore \quad t = \frac{f-a}{2(1+b)}$$

3. (c): Here, $a = 15$ m s^{-2}

$R = 2.5$ m



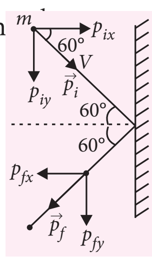
From figure,

$$a_c = a \cos 30° = 15 \times \frac{\sqrt{3}}{2} \text{ m s}^{-2}$$

As we know, $a_c = \dfrac{v^2}{R} \Rightarrow v = \sqrt{a_c R}$

$$\therefore \quad v = \sqrt{15 \times \frac{\sqrt{3}}{2} \times 2.5} = 5.69 \simeq 5.7 \text{ m s}^{-1}$$

4. (a): Given, $p_i = p_f = mV$



Change in momentum of the ball

$$= \vec{p}_f - \vec{p}_i$$

$$= (-p_{fx}\hat{i} - p_{fy}\hat{j}) - (p_{ix}\hat{i} - p_{iy}\hat{j})$$

$$= -\hat{i}(p_{fx} + p_{ix}) - \hat{j}(p_{fy} - p_{iy})$$

$$= -2p_{ix}\hat{i} = -mV\hat{i} \quad [\because p_{fy} - p_{iy} = 0]$$

Here, $p_{ix} = p_{fx} = p_i \cos 60° = \dfrac{mV}{2}$

$\because$ Impulse imparted by the wall = | change in the momentum of the ball | = mV

5. (c): Mass of bullet, $m = 10$ g $= 0.01$ kg

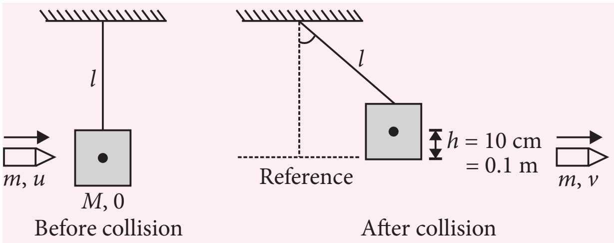
Initial speed of bullet, $u = 400$ m s^{-1}

Mass of block, $M = 2$ kg

Length of string, $l = 5$ m

Speed of the block after collision = v_1

Speed of the bullet on emerging from block, v = ?



Using energy conservation principle for the block,

$$(KE + PE)_{\text{Reference}} = (KE + PE)_h$$

$$\Rightarrow \quad \frac{1}{2}Mv_1^2 = Mgh \quad \text{or,} \quad v_1 = \sqrt{2gh}$$

$$v_1 = \sqrt{2 \times 10 \times 0.1} = \sqrt{2} \text{ m s}^{-1}$$

Using momentum conservation principle for block and bullet system,

$$(M \times 0 + mu)_{\text{Before collision}} = (M \times v_1 + mv)_{\text{After collision}}$$

$$\Rightarrow \quad 0.01 \times 400 = 2\sqrt{2} + 0.01 \times v$$

$$\Rightarrow \quad v = \frac{4 - 2\sqrt{2}}{0.01} = 117.15 \text{ m s}^{-1} \approx 120 \text{ m s}^{-1}$$

6. (b): Masses of the balls are same and collision is elastic, so their velocity will be interchanged after collision.

$\therefore \quad v_A = +u_B = -0.3$ m s^{-1} and $v_B = u_A = 0.5$ m s^{-1}

7. (c): Here $\vec{r}_1 = (-2\hat{i} + 5\hat{j})$ m, $\vec{r}_2 = (4\hat{j} + 3\hat{k})$ m

$$\vec{F} = (4\hat{i} + 3\hat{j}) \text{ N},\ W = ?$$

Work done by force F in moving from $\vec{r}_1$ to $\vec{r}_2$,

$$W = \vec{F} \cdot (\vec{r}_2 - \vec{r}_1)$$

$$W = (4\hat{i} + 3\hat{j}) \cdot (4\hat{j} + 3\hat{k} + 2\hat{i} - 5\hat{j})$$

$$= (4\hat{i} + 3\hat{j}) \cdot (2\hat{i} - \hat{j} + 3\hat{k})$$

$$= 8 + (-3) = 5 \text{ J}$$

8. (c)

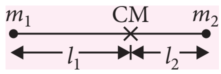
9. (b): $\dfrac{E_{\text{Sphere}}}{E_{\text{Cylinder}}}=\dfrac{\frac{1}{2}I_s\omega_s^2}{\frac{1}{2}I_c\omega_c^2}=\dfrac{I_s\omega_s^2}{I_c\omega_c^2}$

Here, $I_s=\dfrac{2}{5}mR^2,\ I_c=\dfrac{1}{2}mR^2,\ \omega_c=2\omega_s$

$$\frac{E_{\text{Sphere}}}{E_{\text{Cylinder}}}=\frac{\frac{2}{5}mR^2\times\omega_s^2}{\frac{1}{2}mR^2\times(2\omega_s)^2}=\frac{1}{5}$$

10. (a): Here, $l_1+l_2=l$

Centre of mass of the system,



$$l_1=\frac{m_1\times 0+m_2\times l}{m_1+m_2}=\frac{m_2 l}{m_1+m_2}$$

$$l_2=l-l_1=\frac{m_1 l}{m_1+m_2}$$

Required moment of inertia of the system,

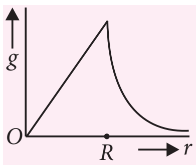
$$I=m_1l_1^2+m_2l_2^2$$

$$=(m_1m_2^2+m_2m_1^2)\frac{l^2}{(m_1+m_2)^2}$$

$$=\frac{m_1m_2(m_1+m_2)l^2}{(m_1+m_2)^2}=\frac{m_1m_2}{m_1+m_2}l^2$$

11. (b): Acceleration due to gravity is given by

$$g=\begin{cases}\dfrac{4}{3}\pi\rho Gr & ;\ 0<r\le R\\[2mm] \dfrac{4}{3}\dfrac{\pi\rho R^3G}{r^2} & ;\ r>R\end{cases}$$



12. (b)

13. (b): Work done = Surface tension of film × Change in area of the film

or, $W=T\times\Delta A$

Here, $A_1=4\text{ cm}\times 2\text{ cm}=8\text{ cm}^2$

$A_2=5\text{ cm}\times 4\text{ cm}=20\text{ cm}^2$

$\Delta A=2(A_2-A_1)=24\text{ cm}^2=24\times10^{-4}\text{ m}^2$

$W=3\times10^{-4}\text{ J},\ T=?$

$$\therefore\quad T=\frac{W}{\Delta A}=\frac{3\times10^{-4}}{24\times10^{-4}}=\frac{1}{8}=0.125\text{ N m}^{-1}$$

14. (b): Capillary rise, $h=\dfrac{2T\cos\theta}{r\rho g}$

For given value of T and r, $h\propto\dfrac{\cos\theta}{\rho}$

Also, $h_1=h_2=h_3$

or, $\dfrac{\cos\theta_1}{\rho_1}=\dfrac{\cos\theta_2}{\rho_2}=\dfrac{\cos\theta_3}{\rho_3}$

Since, $\rho_1>\rho_2>\rho_3$, so for positive value of $\cos\theta$

$\cos\theta_1>\cos\theta_2>\cos\theta_3$

For $0\le\theta<\dfrac{\pi}{2}$, $\theta_1<\theta_2<\theta_3$

Hence, option is (b) *i.e.* $0\le\theta_1<\theta_2<\theta_3<\dfrac{\pi}{2}$

15. (b): Since, heat capacity of material increases with increase in temperature so, body at 100 °C has more heat capacity than body at 0 °C. Hence, final common temperature of the system will be closer to 100 °C.

$\therefore\quad T_c>50\ °\text{C}$

16. (b): According to Newton's law of cooling,

$$\frac{dT}{dt}=K(T-T_s)$$

For two cases, $\dfrac{dT_1}{dt}=K(T_1-T_s)$ and $\dfrac{dT_2}{dt}=K(T_2-T_s)$

Here, $T_s=T,\ T_1=\dfrac{3T+2T}{2}=2.5\ T$

and $\dfrac{dT_1}{dt}=\dfrac{3T-2T}{10}=\dfrac{T}{10}$

$$T_2=\frac{2T+T'}{2}\ \text{and}\ \frac{dT_2}{dt}=\frac{2T-T'}{10}$$

So, $\dfrac{T}{10}=K(2.5T-T)$ ...(i)

$\dfrac{2T-T'}{10}=K\left(\dfrac{2T+T'}{2}-T\right)$ ...(ii)

Dividing eqn. (i) by eqn. (ii), we get

$$\frac{T}{2T-T'}=\frac{(2.5T-T)}{\left(\frac{2T+T'}{2}-T\right)}$$

$$\frac{2T+T'}{2}-T=(2T-T')\times\frac{3}{2}$$

$T'=3(2T-T')$ or, $4T'=6T$

$$\therefore\quad T'=\frac{3}{2}T$$

17. (d): For given process

$PV^3=\text{constant}$

For a polytropic process, $PV^{\alpha}=\text{constant}$

Heat capacity in this process is given by,

$$C=C_V+\frac{R}{1-\alpha}=\frac{3}{2}R+\frac{R}{1-3}=R\qquad(\because\ \alpha=3)$$

18. (b): Temperature inside refrigerator = t_2 °C

Room temperature = t_1 °C

For refrigerator,

$$\frac{\text{Heat given to higher temperature } (Q_1)}{\text{Heat taken from lower temperature } (Q_2)} = \frac{T_1}{T_2}$$

$$\frac{Q_1}{Q_2} = \frac{t_1 + 273}{t_2 + 273}$$

$$\Rightarrow \quad \frac{Q_1}{Q_1 - W} = \frac{t_1 + 273}{t_2 + 273} \text{ or, } 1 - \frac{W}{Q_1} = \frac{t_2 + 273}{t_1 + 273}$$

or $\quad \dfrac{W}{Q_1} = \dfrac{t_1 - t_2}{t_1 + 273}$

The amount of heat delivered to the room for each joule of electrical energy (W = 1 J)

$$Q_1 = \frac{t_1 + 273}{t_1 - t_2}$$

19. (b): As $PV = nRT$ or, $n = \dfrac{PV}{RT} = \dfrac{\text{mass}}{\text{molar mass}}$ ...(i)

$$\text{Density } (\rho) = \frac{\text{mass}}{\text{volume}} = \frac{(\text{molar mass})P}{RT} = \frac{(mN_A)P}{RT}$$

{From eqn. (i)}

$\rho = \dfrac{mP}{kT}$ $\quad (\because R = N_A k)$

20. (d): Time period of spring - block system,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

For a given spring, $T \propto \sqrt{m}$ $\therefore \dfrac{T_1}{T_2} = \sqrt{\dfrac{m_1}{m_2}}$

Here, T_1 = 3 s, $m_1 = m$, T_2 = 5 s, $m_2 = m + 1$

m = ?

$$\frac{3}{5} = \sqrt{\frac{m}{m+1}} \text{ or, } \frac{9}{25} = \frac{m}{m+1}$$

$$25m = 9m + 9 \Rightarrow 16m = 9 \therefore m = \frac{9}{16} \text{ kg}$$

21. (b): Second overtone of an open organ pipe

$$= \text{Third harmonic} = 3 \times \upsilon_o = 3 \times \frac{v}{2L_o}$$

First overtone of a closed organ pipe

$$= \text{Third harmonic} = 3 \times \upsilon_c = 3 \times \frac{v}{4L_c}$$

According to question,

$$3\upsilon_o = 3\upsilon_c \Rightarrow 3 \times \frac{v}{2L_o} = 3 \times \frac{v}{4L_c}$$

$\therefore \quad L_o = 2L_c = 2L$

22. (d)

23. (b): Here, $\theta = 30°$, $E = 2 \times 10^5$ N C^{-1}

τ = 4 N m, l = 2 cm = 0.02 m

q = ?

$\tau = pE\sin\theta = (ql)E\sin\theta$

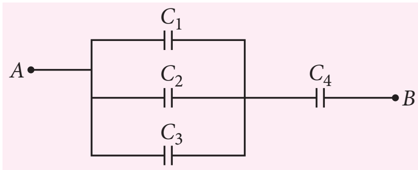
$$\therefore \quad q = \frac{\tau}{El\sin\theta} = \frac{4}{2 \times 10^5 \times 0.02 \times \frac{1}{2}}$$

$$= \frac{4}{2 \times 10^3} = 2 \times 10^{-3}\,\text{C} = 2\,\text{mC}$$

24. (c): Here, $C_1 = \dfrac{2\varepsilon_0 k_1 A}{3d}$, $C_2 = \dfrac{2\varepsilon_0 k_2 A}{3d}$

$$C_3 = \frac{2\varepsilon_0 k_3 A}{3d}, \; C_4 = \frac{2\varepsilon_0 k_4 A}{d}$$

Given system of capacitors C_1, C_2, C_3 and C_4 can be simplified as



$$\therefore \quad \frac{1}{C_{AB}} = \frac{1}{C_1 + C_2 + C_3} + \frac{1}{C_4}$$

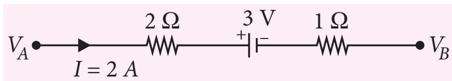
Suppose, $C_{AB} = \dfrac{k\varepsilon_0 A}{d}$

$$\frac{1}{k\left(\frac{\varepsilon_0 A}{d}\right)} = \frac{1}{\frac{2}{3}\frac{\varepsilon_0 A}{d}(k_1 + k_2 + k_3)} + \frac{1}{\frac{2\varepsilon_0 A}{d}k_4}$$

$$\Rightarrow \quad \frac{1}{k} = \frac{3}{2(k_1 + k_2 + k_3)} + \frac{1}{2k_4}$$

$$\therefore \quad \frac{2}{k} = \frac{3}{k_1 + k_2 + k_3} + \frac{1}{k_4}$$

25. (d):

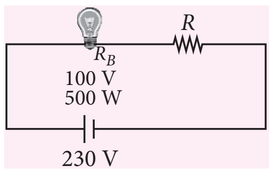


$V_{AB} = V_A - V_B = 2 \times 2 + 3 + 1 \times 2 = 9$ V

26. (c): Power of the bulb in the circuit,

$P = VI$

$$I = \frac{P}{V_B}$$

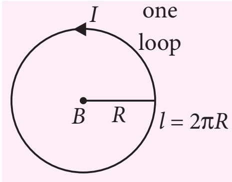


$$= \frac{500}{100} = 5 \text{ A}$$

$V_R = IR \Rightarrow (230 - 100) = 5 \times R$

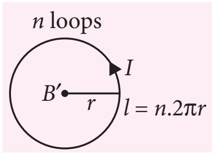
$\therefore \quad R = 26\ \Omega$

27. (b): Let l be the length of the wire. Magnetic field at the centre of the loop is



$$B = \frac{\mu_0 I}{2R}$$

$$\therefore \quad B = \frac{\mu_0 \pi I}{l} \qquad (\because l = 2\pi R) \qquad ...(i)$$



$$B' = \frac{\mu_0 n I}{2r} = \frac{\mu_0 n I}{2\left(\frac{l}{2n\pi}\right)}$$

$$B' = \frac{\mu_0 n^2 \pi I}{l} \qquad ...(ii)$$

From eqns. (i) and (ii), we get, $B' = n^2 B$

28. (b): At equilibrium, initial potential energy of dipole

$U_i = -MB_H$

Final potential energy of dipole,

$$U_f = -MB_H \cos 60° = -\frac{MB_H}{2}$$

$$W = U_f - U_i = -\frac{MB_H}{2} - (-MB_H) = \frac{MB_H}{2} \quad ...(i)$$

Required torque, $\tau = MB_H \sin 60°$

$$\tau = 2W \times \frac{\sqrt{3}}{2} \qquad \text{[Using eqn. (i)]}$$

$$= \sqrt{3}W$$

29. (a): Here, $B = 3.57 \times 10^{-2}$ T, $\frac{e}{m} = 1.76 \times 10^{11}$ C kg^{-1}

Frequency of revolution of the electron,

$$\upsilon = \frac{1}{T} = \frac{v}{2\pi r} \qquad ...(i)$$

Also, $\frac{mv^2}{r} = evB \Rightarrow \frac{v}{r} = \frac{eB}{m}$ ...(ii)

From eqns. (i) and (ii)

$$\upsilon = \frac{1}{2\pi} \times \frac{eB}{m}$$

$$= \frac{1}{2 \times 3.14} \times 1.76 \times 10^{11} \times 3.57 \times 10^{-2}$$

$= 10^9$ Hz $= 1$ GHz

30. (c): Quality factor of an L-C-R circuit is given by,

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$Q_1 = 10.35$, $Q_2 = 9.43$, $Q_3 = 22.77$, $Q_4 = 7.30$

Clearly Q_3 is maximum of Q_1, Q_2, Q_3, and Q_4.

Hence, option (c) should be selected for better tuning of an L-C-R circuit.

31. (d): Emf generated in loop 1,

$$\varepsilon_1 = -\frac{d\phi}{dt} = -\frac{d}{dt}(\vec{B}\cdot\vec{A}) = -\frac{d}{dt}(BA) = -A \times \frac{dB}{dt}$$

$$\varepsilon_1 = -\left(\pi r^2 \frac{dB}{dt}\right)$$

($\because$ $A = \pi r^2$ because $\frac{dB}{dt}$ is restricted upto radius r.)

Emf generated in loop 2,

$$\varepsilon_2 = -\frac{d}{dt}(BA) = -\frac{d}{dt}(0 \times A) = 0$$

32. (c): Here, $V_R = 80$ V, $V_C = 40$ V, $V_L = 100$ V

Power factor, $\cos\phi = \frac{R}{Z} = \frac{V_R}{V} = \frac{V_R}{\sqrt{V_R^2 + (V_L - V_C)^2}}$

$$= \frac{80}{\sqrt{(80)^2 + (100-40)^2}} = \frac{80}{100} = 0.8$$

33. (a): Here, $R = 100\ \Omega$, $X_c = 100\ \Omega$

Net impedance, $Z = \sqrt{R^2 + X_L^2} = 100\sqrt{2}\ \Omega$

Peak value of displacement current

= Maximum conduction current in the circuit

$$= \frac{\varepsilon_0}{Z} = \frac{220\sqrt{2}}{100\sqrt{2}} = 2.2 \text{ A}$$

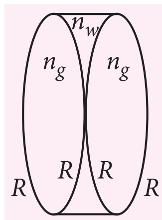
34. (d): Here, $\mu_g = \frac{3}{2}$, $\mu_w = \frac{4}{3}$

Focal length (f) of glass convex lens is given by

$$\frac{1}{f} = (\mu_g - 1)\left(\frac{2}{R}\right) \text{ or, } \frac{1}{f} = \left(\frac{3}{2} - 1\right)\frac{2}{R} = \frac{1}{R}$$

or, $f = R$ ...(i)

Focal length (f') of water filled concave lens is given by



$$\frac{1}{f'} = (\mu_w - 1)\left(-\frac{2}{R}\right)$$

$$\text{or, } \frac{1}{f'} = \left(\frac{4}{3} - 1\right)\left(-\frac{2}{R}\right)$$

$$= -\frac{2}{3R} = -\frac{2}{3f} \qquad \text{[Using eqn. (i)]}$$

Equivalent focal length (f_{eq}) of lens system

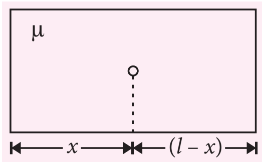
$$\frac{1}{f_{eq}} = \frac{1}{f} - \frac{2}{3f} + \frac{1}{f} = \frac{3-2+3}{3f} = \frac{4}{3f}$$

$$\therefore \quad f_{eq} = \frac{3f}{4}$$

35. (c) : Here $\mu = 1.5$

l = length of the slab

x = position of air bubble from one side



Here, $\frac{x}{\mu} = 5$ cm, $\frac{(l-x)}{\mu} = 3$ cm

$\therefore \quad \frac{x}{\mu} + \frac{(l-x)}{\mu} = 8$ or, $\frac{l}{\mu} = 8$

$\because \quad l = 8\mu = 8 \times 1.5 = 12$ cm

36. (b): Here, $\frac{I_1}{I_2} = n$

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{I_1/I_2}+1}{\sqrt{I_1/I_2}-1}\right)^2 = \left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$$

$$\frac{I_{max}-I_{min}}{I_{max}+I_{min}} = \frac{\frac{I_{max}}{I_{min}}-1}{\frac{I_{max}}{I_{min}}+1} = \frac{\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2-1}{\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2+1}$$

$$= \frac{(\sqrt{n}+1)^2-(\sqrt{n}-1)^2}{(\sqrt{n}+1)^2+(\sqrt{n}-1)^2} = \frac{4\sqrt{n}}{2(n+1)} = \frac{2\sqrt{n}}{n+1}$$

37. (b): Here, $u = 400$ cm $= 4$ m, $v = \infty$, $f = ?$

Using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

or, $\frac{1}{\infty} - \frac{1}{4} = \frac{1}{f}$ or, $f = -4$ m

Lens should be concave.

Power of lens $= \frac{1}{f} = \frac{1}{-4} = -0.25$ D

38. (d): Here, $a = 0.02$ cm $= 2 \times 10^{-4}$ m

$\lambda = 5 \times 10^{-5}$ cm $= 5 \times 10^{-7}$ m

$f = D = 60$ cm $= 0.6$ m

Position of first minima on the diffraction pattern,

$$y_1 = \frac{D\lambda}{a} = \frac{0.6 \times 5 \times 10^{-7}}{2 \times 10^{-4}}$$

$= 15 \times 10^{-4}$ m $= 0.15$ cm

39. (a) : Kinetic energy of electrons

$$K = \frac{p^2}{2m} = \frac{(h/\lambda)^2}{2m} = \frac{h^2}{2m\lambda^2}$$

So, maximum energy of emitted X-ray photon $= K$

$$\frac{hc}{\lambda_0} = \frac{h^2}{2m\lambda^2} \quad \therefore \lambda_0 = \frac{2mc\lambda^2}{h}$$

40. (d)

41. (c) : When electron jumps from higher orbit to lower orbit then, wavelength of emitted photon is given by,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Transition : $3 \longrightarrow 2$, wavelength $= \lambda$

Transition : $4 \longrightarrow 3$, wavelength $= \lambda' = ?$

so, $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$

and $\frac{1}{\lambda'} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{7R}{144}$

$\therefore \quad \lambda' = \frac{144}{7} \times \frac{5\lambda}{36} = \frac{20\lambda}{7}$

42. (d): N_0 = Nuclei at time $t = 0$

N_1 = Remaining nuclei after 40% decay

$= (1 - 0.4)\, N_0 = 0.6\, N_0$

N_2 = Remaining nuclei after 85% decay

$= (1 - 0.85)\, N_0 = 0.15\, N_0$

$\therefore \quad \frac{N_2}{N_1} = \frac{0.15 N_0}{0.6 N_0} = \frac{1}{4} = \left(\frac{1}{2}\right)^2$

Hence, two half life is required between 40% decay and 85% decay of a radioactive substance.

$\therefore$ Time taken $= 2\tau_{1/2} = 2 \times 30$ min $= 60$ min

43. (b): Here, $R_C = 2$ kΩ $= 2000$ Ω, $V_0 = 4$ V

$\beta = 100$, $R_B = 1$ kΩ $= 1000$ Ω, $V_i = ?$

Voltage gain, $A = \beta \frac{R_C}{R_B} = 100 \times \frac{2000}{1000} = 200$

Also, $A = \frac{V_0}{V_i}$ or, $V_i = \frac{V_0}{A} = \frac{4}{200} = \frac{2}{100}$ V $= 20$ mV

44. (a) **45. (c)**

MPP-3 CLASS XII ANSWER KEY

1.	(d)	**2.**	(d)	**3.**	(c)	**4.**	(b)	**5.**	(a)
6.	(b)	**7.**	(b)	**8.**	(c)	**9.**	(d)	**10.**	(b)
11.	(d)	**12.**	(c)	**13.**	(d)	**14.**	(a)	**15.**	(d)
16.	(d)	**17.**	(a)	**18.**	(b)	**19.**	(b)	**20.**	(b,c)
21.	(a,c)	**22.**	(a,b,c)	**23.**	(b,d)	**24.**	(8)	**25.**	(5)
26.	(3)	**27.**	(d)	**28.**	(d)	**29.**	(a)	**30.**	(a)

PHYSICS MUSING

SOLUTION SET-37

1. (b): Instantaneously after F_2 is withdrawn, block of mass m_2 experiences only spring force F_{sp}. Spring force does not change instantaneously.

Thus for m_1; $a_1 = a_0$

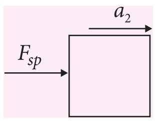
for m_2 $\quad F_{sp} = m_2 a_2$ ...(i)

Initially, $F_{sp} - F_2 = m_2 a_0$

$\Rightarrow F_{sp} = F_2 + m_2 a_0$ ...(ii)

From (i) and (ii),

$$a_2 = \frac{F_2}{m_2} + a_0$$



2. (d): Here given that,

The target m_2 is at rest

$\therefore \quad u_1 = v_0$, $u_2 = 0$

For maximum v_2, collision will be elastic and $m_1 >> m_2$

$$v_{2\,max} = \frac{2m_1u_1}{m_1+m_2} \quad \therefore v_{2\,max} = 2v_0 \ \{\text{for } m_1 >> m_2\}$$

For minimum v_2, collision must be perfectly inelastic.

$$v_2 = \frac{m_1u_1 + m_2u_2}{(m_1+m_2)} \quad \therefore \ v_2 = \frac{v_0 m_1}{(m_1+m_2)}$$

$\{v_2$ is minimum for $m_1 = m_2$ and $u_2 = 0\}$

Thus, $\frac{v_0}{2} < v_2 < 2v_0$

Hence option (d) is correct.

3. (b): Initially the acceleration at the ball of the surface would be

$$g = \frac{GM}{R^2}$$

but the accelerating would be progressively smaller as it approached the centre which is given by

$$g_{effective} = \frac{GM}{R^3}x \text{ or } a = -\frac{GM}{R^3}x, \text{ also } a = -\omega^2 x$$

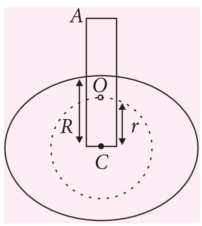
$$\text{So, } \omega = \sqrt{\frac{GM}{R^3}}, v_C = \omega A = \sqrt{\frac{GM}{R^3}} \times R$$

After collision velocity of ball towards A, is

$$v_C' = ev_C = 0.2\sqrt{\frac{GM}{R}} = \frac{1}{5}\sqrt{\frac{GM}{R}}$$

Let amplitude be A', then

$$A' = \frac{v_C'}{\omega} = \frac{\frac{1}{5}\sqrt{\frac{GM}{R}}}{\sqrt{\frac{GM}{R^3}}} = \frac{R}{5}$$



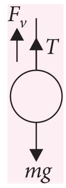
Net distance $= R + (R/5) + (R/5) = \frac{7R}{5}$

4. (d): When terminal velocity is attained, the whole system becomes in equillibrium

$\therefore \ mg = F_v$ (viscous force)

$$\frac{4}{3}\pi\left(r_1^3 + r_2^3\right)(\sigma - \rho)g = 6\pi\eta\,(r_1 + r_2)v \quad \text{...(i)}$$

From free body diagram of ball of radius r_2



$\therefore \ mg = T + F_v$

we get, $\frac{4}{3}\pi r_2^3\,(\sigma - \rho)g = T + 6\pi\eta r_1 v$ ...(ii)

substituting value of $6\pi\eta v$ from eqn. (i) in eqn. (ii)

$$T = \frac{4}{3}\pi\frac{\left(r_2^4 - r_1^4\right)(\sigma-\rho)g}{(r_1+r_2)}$$

5. (b): $\Delta Q = dU + pdV \Rightarrow \frac{\alpha}{T}dT = C_v dT + PdV$

$$\Rightarrow \frac{\alpha}{T}dT = C_v dT + \frac{RT}{V}dV$$

$$\alpha\frac{dT}{T} = \frac{R}{(\gamma-1)}dT + RT\frac{dV}{V} \Rightarrow \frac{\alpha}{R}\frac{dT}{T^2} = \frac{1}{(\gamma-1)}\frac{dT}{T} + \frac{dV}{V}$$

On integrating and solving

$$VT^{1/(\gamma-1)}e^{\alpha/RT} = \text{constant}$$

6. (c): $p - p_0 = \frac{4S}{r} \Rightarrow dp = \frac{4S}{r^2}dr$

$$C = C_V + \frac{dW}{ndT} \Rightarrow C = \frac{R}{\gamma-1} + \frac{PdV + VdP}{ndT}$$

$$= \frac{R}{\frac{5}{3}-1} + \frac{nRdT}{ndT} + \left(\frac{4}{3}\pi r^3\right)\left(\frac{4S}{r^2}dr\right)\frac{1}{ndT}$$

$$= \frac{3}{2}R + R + \frac{16S\pi}{3}\frac{rdr}{ndT} = \frac{5}{2}R + \frac{2S}{3n} \times \frac{8\pi rdr}{dT}$$

$$= \frac{5}{2}R + \frac{2S}{3n} \times \frac{3nR}{2S} = \frac{7}{2}R$$

7. (a, b): Since, equillibrium is maintained

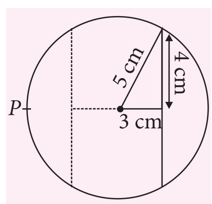
Applied force = Total surface tension

$16 \times 10^{-3} = 2T \times l$

$$l = \frac{16 \times 10^{-3}}{2 \times 0.1} = 8 \text{ cm}$$

$\Rightarrow$ Distance of the slider from the centre = 3 cm

Required distances of the slider from point P = 5 + 3 cm and 5 – 3 cm = 8 cm and 2 cm



8. (a, d) : Since the time period is given by, $T = 2\pi\sqrt{(m/k)}$, so quadrupling the mass will double the period. However, frequency and amplitude are independent of one another. Doubling the amplitude means energy = $\frac{1}{2}kA^2$ goes up by four times.

9. (c, d) : For total internal reflection to occur,
Angle of incidence, (i) > critical angle, (C)
or, $\sin i > \sin C$

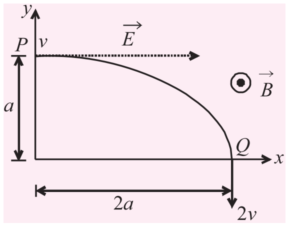
or, $\sin 45° > \frac{1}{\mu}$ or, $\frac{1}{\sqrt{2}} > \frac{1}{\mu}$ or, $\mu > \sqrt{2}$ or, $\mu > 1.414$

Possible values of μ can be 1.5 or 1.6.

10. (a, b, d) : Magnetic force does not perform any work as it is at right angle to the direction of motion of the charged particle. From work – energy theorem,

$W_{Fe} = \Delta KE$

$$qE \quad a \quad \frac{1}{\ }m \quad v \; - \; v \qquad E \quad \left(\frac{mv}{qa}\right)$$



Option (a) is correct.

At P, rate of work done = $(qE)v = q\left(\frac{mv}{qa}\right)v = \left(\frac{mv}{a}\right)$

Option (b) is correct and option (c) is incorrect.
Rate of work done at Q :
Rate of work done by electric field

$$= (qE) \times (2v) \times \cos 90° = 0$$

So, rate of work done by both the fields at Q is zero.

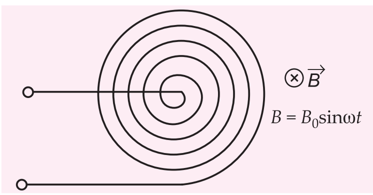
Contd. from Page no. 72

PHYSICS MUSING QUESTIONS SET-38

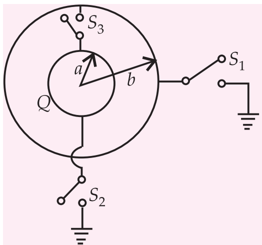
SUBJECTIVE TYPE

7. The pitch of a screw gauge is 1 mm and there are 50 divisions on its cap. When nothing is put in between the studs, the zero of the circular scale lies 6 division below the line of graduation. When a wire is placed between the studs, 3 linear scale divisions are clearly visible while 31 divisions on the circular scale coincide with the reference line. Determine the diameter of the wire.

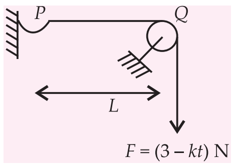
8. A plane spiral with a large number N of turns wound tightly to one another is located in a uniform magnetic field perpendicular to the plane of the spiral. The outside radius of the spiral's turns is equal to a. The magnetic induction varies with time as $B = B_0 \sin\omega t$, where B_0 and ω are constants. The amplitude of induced emf in the spiral is given by $\frac{\pi N a^2 B_0 \omega}{n}$. Find the value of n.



9. The figure shows a conducting sphere A of radius a which is surrounded by a neutral conducting spherical shell B of radius b (> a). Initially switches S_1, S_2 and S_3 are open and sphere A carries a charge Q. First the switch S_1 is closed to connect the shell B with the ground and then opened. Now the switch S_2 is closed so that the sphere A is grounded and then S_2 is opened. Finally, the switch S_3 is closed to connect the spheres together. Find the heat (in J) which is produced after closing the switch S_3. [Consider b = 4 cm, a = 2 cm and Q = 8 μC]



10. In the given figure, a string of linear mass density 3×10^{-2} kg m^{-1} and length L = 1 m, is stretched by a force $F = (3 - kt)$ N, where k is a constant and t is time in sec. At the time $t = 0$, a pulse is generated at the end P of the string. Find the value of k (in N s^{-1}) if the value of force becomes zero as the pulse reaches point Q.



20 THINGS YOU DIDN'T KNOW ABOUT NOTHING

1 There is vastly more nothing than something. Roughly 74 percent of the universe is "nothing," or what physicists call dark energy; 22 percent is dark matter, particles we cannot see. Only 4 percent is baryonic matter, the stuff we call something.

2 And even something is mostly nothing. Atoms overwhelmingly consist of empty space. Matter's solidity is an illusion caused by the electric fields caused by subatomic particles.

3 There is more and more nothing every second. In 1998 astronomers measuring the expansion of the universe determined that dark energy is pushing apart the universe at an ever-accelerating speed. The discovery of nothing – and its ability to influence the fate of the cosmos – is considered the most important astronomical finding of the past decade.

4 But even nothing has a weight. The energy in dark matter is equivalent to a tiny mass; there is about one pound of dark energy in a cube of empty space 250,000 miles on each side.

5 In space, no one can hear you scream: Sound, a mechanical wave, cannot travel through a vacuum. Without matter to vibrate through, there is only silence.

6 So what if Kramer falls in a forest? Luckily, electromagnetic waves, including light and radio waves, need no medium to travel through, letting TV stations broadcast endless reruns of Seinfeld, the show about nothing.

7 Light can travel through a vacuum, but there is nothing to refract it. Alas for extraterrestrial romantics, stars do not twinkle in outer space.

8 Black holes are not holes or voids; they are the exact opposite of nothing, being the densest concentration of mass known in the universe.

9 "Zero" was first seen in cuneiform tablets written around 300 B.C. by Babylonians who used it as a placeholder (to distinguish 36 from 306 or 360, for example). The concept of zero in its mathematical sense was developed in India in the fifth century.

10 Any number divided by zero is .... nothing, not even zero. The equation is mathematically impossible.

11 It is said that Abdülhamid II, sultan of the Ottoman Empire in the early 1900s, had censors expunge references to H_2O from chemistry books because he was sure it stood for "Hamid the Second is nothing."

12 Medieval art was mostly flat and two dimensional until the 15th century, when the Florentine architect Filippo Brunelleschi conceived of the vanishing point, the place where parallel lines coverage into nothingness. This alowed for the development of perspective in art.

13 Aristotle once wrote, "Nature abhors a vacuum," and so did he. His complete rejection of vacuums and voids and his subsequent influence on centuries of learning prevented the adoption of the concept of zero in the Western world until around the 13th century, when Italian bankers found it to be extraordinarily useful in financial transactions.

14 Vacuums do not such things. They create spaces into which the surrounding atmosphere pushes matter.

15 Creatio ex nihilo, the belief that the world was created out of nothing, is one of the most common themes in ancient myths and religions.

16 Current theories suggest that the universe was created out of a state of vacuum energy, that is, nothing.

17 But to a physicist there is no such thing as nothing. Empty space is instead filled with pairs of particles and antiparticles, called virtual particles, that quickly form and then, in accordance with the law of energy conservation, annihilate each other in about 10^{-25} second.

18 So Aristotle was right all along.

19 These virtual particles popping in and out of existence create energy. In fact, according to quantum mechanics, the energy contained in all the power plants and nuclear weapons in the world doesn't equal the theoretical energy contained in the empty spaces between these words.

20 In other words, nothing could be the key to the theory of everything.

LeeAundra Temescu

CROSSWORD

Readers can send their responses at editor@mtg.in or post us with complete address by 25th of every month to win exciting prizes. Winners' name with their valuable feedback will be published in next issue.

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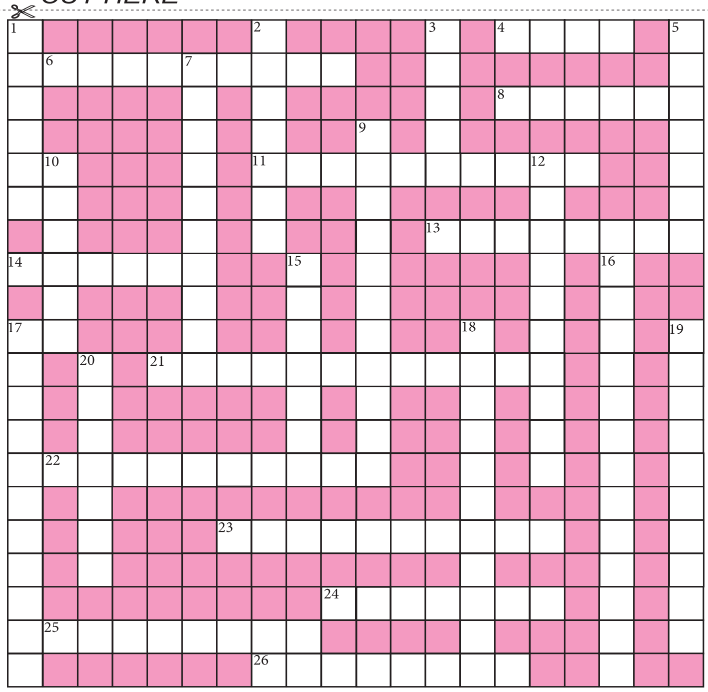

ACROSS

4. A unit used to measure the departure of a nuclear reactor from its critical condition. [4]

6. It is a disordered magnet, where the magnetic spin of the component atoms are not aligned in a regular pattern. [4, 5]

8. A place where highly radioactive products from nuclear reactors may be safely deposited and stored. [6]

11. A directional aerial array widely used for television and radio telescopes. [4, 6]

13. A limit on the energy of cosmic rays imposed by the interaction between the cosmic rays and the cosmic microwave. [3, 5]

14. An electrode in an electron tube, whose primary function is to provide secondary emission of electrons. [6]

21. In fluid dynamics, it is a dimensionless number that characterizes laminar flow in a conduit. [6, 6]

22. A concave lens placed between the objective and eyepiece in a telescope to increase magnification. [6, 4]

23. A device for calibrating thermometers at the boiling point of water at a known height above sea level. [10]

24. An astronomical event in which the plane of earth's equator passes through the centre of the sun, which occurs twice each year. [7]

25. Part of a signal-wave envelope between the steady-state value and the end of the envelope. [8]

26. A permanently electrified substance exhibiting electric charges of opposite sign at its extremities. [8]

DOWN

1. A proprietary glass-ceramic material that alters very little in size or shape when subjected to normal temperature changes. [6]

2. A hypothetical particle that has a speed greater than the speed of light. [7]

3. A small telemetry system in a balloon, rocket or satellite, used in meteorology, astronomy etc. [5]

5. Radioactive materials that fall to earth following a nuclear-bomb explosion. [7]

7. An instrument for measuring the difference in the force of gravity from one place to another. [10]

9. Units based on the metre, kilogram and second as fundamental mechanical units, together with one electrical unit of practical size. [6, 5]

10. An alignment of three celestial objects, as the sun, the earth and either the moon or a Planet. [6]

12. Physical phenomenon in which the wavelength of the fluorescent light emitted is different from that of the absorbed light. [10]

15. It is an atomic unit of energy in atomic physics and computational chemistry. [7]

16. An instrument for measuring the difference in intensity between radiation entering and leaving the earth's surface. [3, 10]

17. A reactive load in which the inductive reactance exceeds the capacitive reactance. [7, 4]

18. A measure of ability of a medium to absorb radiation. [11]

19. A nuclear reaction in which several nucleons are released from the nucleus of an atom. [10]

20. A device that uses a powerful magnetic field to confine plasma in the shape of a torus. [7]

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