1a $E\ln(w_i^U | x_i) - E\ln(w_i^N | x_i)$

$E\ln(w_i^U | U_i^* > 0) - E(\ln w_i^N | U_i^* > 0)$

$U_i = \delta_0 + \delta_1(\ln w_i^U - \ln w_i^N) + x_i'\delta_2 + z_i'\delta_3 - v_i$

$E\ln(w_i^U | U_i > 0) = E(x_i'\beta + u_i^U | U_i > 0)$

$E(x_i'\beta + u_i^U | \delta_0 + \delta_1(\ln w_i^U - \ln w_i^N) + x_i'\delta_2 + z_i'\delta_3 - v_i > 0)$

$x_i'\beta^U + E(u_i^U | \delta_0 + \delta_1(x_i\beta^U - x_i\beta^N - u_i^U - u_i^N) + x_i'\delta_2 + z_i\delta_3 - v_i > 0)$

$x_i'\beta^U + E(u_i^U | \delta_1(u_i^U - u_i^N) - v_i > -\delta_0 - \delta_1(x_i\beta^U - x_i\beta^N) - x_i\delta_2 - z_i\delta_3$ $= c$

regress u_i^U on $\delta_1(u_i^U - u_i^N) - v_i$ (probit)

$u_i^U = \alpha_1(\delta_1(u_i^U - u_i^N) - v_i) + \varepsilon$

$$\alpha_1 = \frac{Cov(u_i^U, \delta_1(u_i^U - u_i^N) - v_i)}{Var(\delta_1(u_i^U - u_i^N) - v_i}$$

$$\alpha_1 = \frac{\delta\sigma_U^2}{\delta^2(\sigma_U^2 + \sigma_N^2) + \sigma_v^2}$$

b) The estimates can not be obtained because of selection bias

c) Take 3 equations

$$\log w_{ui} = \theta_{u0} + x_{ui}\,\theta_{u1} + z_{ui}\,\theta_{u2} + \varepsilon_{ui}$$

$$\log w_{ni} = \theta_{n0} + x_{ni}\,\theta_{n1} + z_{ni}\,\theta_{n2} + \varepsilon_{ni}$$

$$I_i = \delta_0 + \delta_1(\log w_{ui} - \log w_{ni}) + \delta_2 x_i + \delta_3 z_i - \varepsilon_i$$

1c) ~~γ_1~~ = 3 equations

$\ln w_i^U = x_{1i}\beta^U + v_1^U$

$\ln w_i^N = x_{2i}\beta^N + v_i^N$

and dummy

$l_i = 1$ if $\underbrace{\delta_0 + \delta_1(\ln w_i^U - \ln w_i^N) + x_i\delta_2 + z_i\delta_3}_{\gamma' z_i} > v_i$

assume v_i^U, v_i^N and v_i are trivariate normal with mean zero and

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{1v} \\ & \sigma_2^2 & \sigma_{2v} \\ & & 1 \end{bmatrix}$$

normalize -> parameter identified up to scale

Likelihood is $L(\beta^U, \beta^N, \sigma_1^2, \sigma_2^2, \sigma_{1v}, \sigma_{2v})$

$$\prod \left[\int_{-\infty}^{\gamma' z_i} g(y_i - \beta^U x_{1i}, v_i)\,dv_i\right]^{l_i} \left[\int_{\gamma' z_i}^{\infty} f(y_i - \beta^N x_{2i}, v_i)\,dv_i\right]^{1-l_i}$$

bivariate normal density of (v_1^U, v_i)

bivariate normal of (v_i^N, v_i)

One can take log of C and estimate coefficients of β^0, β^U, σ_{1U} and σ_{2U} by ~~ML~~ MLE.

(11) ∗ two stage Heckmann estimation

→ Run probit model of being member of a union on c (defined as before) to estimate corresponding coefficients.

→ Use these estimates to find inverse Mills ratio $\left(\frac{\hat{\phi}(c)}{1-\Phi(c)}\right) = \hat{\lambda}(c)$

Run OLS regression of wages on x_i and $\hat{\lambda}(d)$ to get consistent coefficients for β^U and β^N

Use predicted values for $\ln \hat{\omega}_i^U$ and $\ln \hat{\omega}_i^N$ to run probit regression for

$Pr(U_i > 0)$ on ~~const.~~ $\delta_0 + \delta_1(\ln \hat{\omega}_i^U - \ln \hat{\omega}_i^N) + x_i \delta_2 + \varepsilon_i \delta_3$

to get consistent estimates for δ.

f) The reference value for education is ~~8-12~~ 9-11 ~~years of education~~ grades completed.

One can see that returns to education is higher for union members with low level of education (level 1 and 2) and higher for non union members with high levels of education

Experience has a more positive effect for union members

Females have less negative effect for non unions

and race has less of an influence when being a union member

g) $\frac{-f(\psi_1)}{F(\psi_1)}$ is negative and therefor the effect of selectivity variable still positive in the union equation

For both union and Non union ~~eq~~ equation the selectivity ~~is~~ is positive associated with wage so this confirms that individuals indeed self select in to higher wages.

2) keep in mind that marginal effects ~~coefficients~~ ~~are~~ are non linear in probit model

$$\frac{\partial P(y_i=1 \mid x_{1i} \ldots x_{ki}; \beta_0 \ldots \beta_K)}{\partial x_{ki}} = \beta_K \phi\Big(\beta_0 + \sum_{k=1}^{K} \beta_k x_{ki}\Big)$$

non linear

but relative effects of these $\dfrac{\dfrac{\partial P}{\partial x_{ki}}}{\dfrac{\partial P}{\partial x_{ji}}} = \dfrac{\beta_k}{\beta_j}$

are constant

Table 6) The wage difference is ~~[illegible]~~ statistically significant and has high relative magnitude, → very important

Table 6 gives result of probit estimation on ~~the~~ wage differences and other relevant variables

Table 7 → reduced form so the ~~ag~~ wage difference is replaced by the underlying equation for the wage process

→ Table 7 gives net effects

Union firms tend to select more highly educated workers but the higher ~~rate~~ rate of return on education in the non-union sector will induce higher educated workers in to the non-union sector

nonwhites might be discriminated but benefits would attract non-whites

[illegible]

$$\Pr(inlf=1) = \delta_0 + \delta_1\, nwifeinc + \delta_2\, educ + \delta_3\, exper + \delta_4\, expersq + \delta_5\, age + \varepsilon_1$$

$$\Pr(kidslt6=1) = \beta_0 + \beta_1\, nwifeinc + \beta_2\, educ + \beta_3\, exper + \beta_4\, expersq + \beta_5\, age + \varepsilon_2$$

a bivariate probit allows for ε_1 and ε_2 being jointly distributed and correlation does not assume independence

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} \sim N\left(\begin{matrix} 0 \\ 0 \end{matrix} , \begin{matrix} 1 & \rho \\ \rho & 1 \end{matrix} \right)$$

If $\rho \neq 0$ this model is a better fit because it takes into account the right distribution the data is drawn from you assume that the error term associated to kids has an additional impact on the probability of being in the labor force but the error associated to labor force participation does not impact the likelihood of having kids.

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d) In the paper they basically simulate how ~~so~~ robust the results are to the introduction to some unobservable effect C for different magnitudes of the correlation among the error term. Applied to our scenario this means what the would be the effect of not controlling for the likelihood of having kids on the likelihood of being in the labor force.

If there is a correlation of -0.15 of unobservables in co and the error related to labor force participation then the coefficient of leaving kids on labor force participation becomes insignificant. ~~[illegible]~~

This is expected as ~~[illegible]~~ strong negative correlation of the unobservables in leaving kids that influence the decision of being ~~[illegible]~~ in the labor force dominate the relationship and now it seems like leaving kids is insignificant