UNIVERSIDAD EAFIT

Problem Set

Growth&Development

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Slow-Swan Model

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Instructions: .

- Please, read carefully each point before answering. Make sure you understand!
- One PS per team
- Intuition, intuition, intuition! (be concise, yet do not forget the intuition)
- Why not in LaTeX? ☺

Question1 (25 points)

Consider the production function $Y = AK + BL$ where A and B are positive constants:

(a) Is this production function neoclassical? Which of the neoclassical conditions does it satisfy and which ones does it not?

(b) Write the output per person as a function of capital per person. What is the marginal product of $k = \frac{K}{L}$? What is the average product of k?

Now let's assume that population grows at the constant rate n and that capital depreciates at the constant rate δ.

(c) Write down the fundamental equation of the Solow-Swam model.

(d) Under which conditions does this model have a steady state with no growth of per capita capital?

(e) If $s = 0.4$, $A = 1$, $B = 2$, $\delta = 0.08$ and $n = 0.02$, what is the long-run growth rate of this economy? what if B=5? Explain the diferences

Question2 (25 points)

Let us introduce government spending in the basic Solow-Swan Model. Consider the basic model without technological change and suppose that:

$$Y(t) = C(t) + I(t) + G(t)$$

with $G(t)$ denoting government spending at time t. Image that government spending is given by $G(t) = \sigma Y(t)$

(a) Discuss how the relationship between income and consumption should be changed. Is it reasonable to assume that $C(t) = sY(t)$

(b) Suppose that government spending partly comes out of private consumption, so that $C(t) = (s - \lambda\sigma)Y(t)$, where $\lambda \in [0, 1]$. What is the effect of higher government spending (in the form of higher σ) on the equilibrium of the Solow model?

(c) Now suppose that a fraction ϕ of $G(t)$ is invested in the capital, so that total investment at time t is given by

$$I(t) = (1 - s - (1 - \lambda)\sigma + \phi\sigma)Y(t)$$

Show that if ϕ is is sufficiently high, the steady-state level of capital-labor ratio will increase as a result of higher government spending (corresponding to higher σ) Is this reasonable? How would you alternatively introduce public investments in this model?

Question3 (20 points)

Let us consider that a economy follows this production function:

$$Y = AK^{\lambda}H^{\eta}[T(t)L]^{1-\alpha-\eta}$$

(a) Is this production function neoclassical?

(b) What the growth rate of the physical capita?

(c) what is the main steady-state condition?

(d) what is the the convergence coefficient in the steady state?

Question4 (20 points)

Let us consider the standard Solow model introducing technology:

$$Y = F(K, AL) = K^{\alpha}(AL)^{1-\alpha}$$

Where A is a technology variable.

(a) Why this model differs from the Solow–Swan Model? Do we need any extra assumption on how A is growing?

(b) Solve the steady state (make a graph)

(c) Draw the solow diagram with technological progress

Question5 (10 points)

Suppose the U.S. Congress enacts legislation that discourages saving and investment, such as the elimination of the investment tax credit that occurred in 1990. As a result, suppose the investment rate falls permanently from s' to s''. (i.e. $s' > s''$). Examine this policy change in the Solow model with technological progress, assuming that the economy begins in steady state. Sketch a graph of how (the natural log of) output per worker evolves over time with and without the policy change. Make a similar graph for the growth rate of output per worker. Does the policy change permanently reduce the level or the growth rate of output per worker?