SLOW-SWAN MODEL (PART I)

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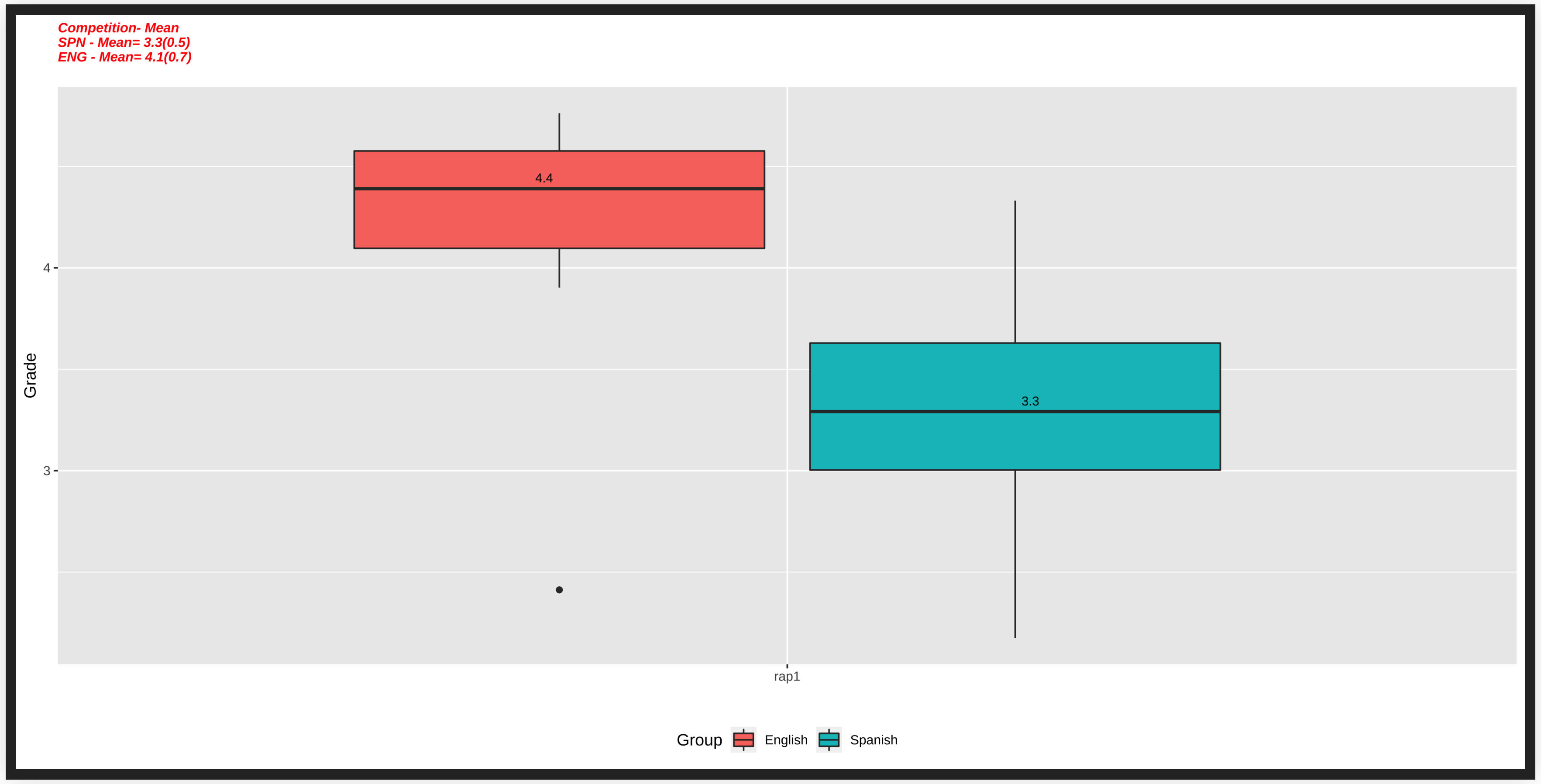
COMPETITION

(proposal)

SPANISH VS ENGLISH

- Winner +0.06 in the overall RAP
- Loser -0.01 in the overall RAP



Competition- Mean
SPN - Mean= 3.3(0.5)
ENG - Mean= 4.1(0.7)
4.4
3.3
Grade
4
3
rap1
Group
English
Spanish

LEARNING BY CONTEXT - POSTWAR ECONOMY


MEDIA RICH

- Productivity
- Technology
- Consumption
- Labor!!

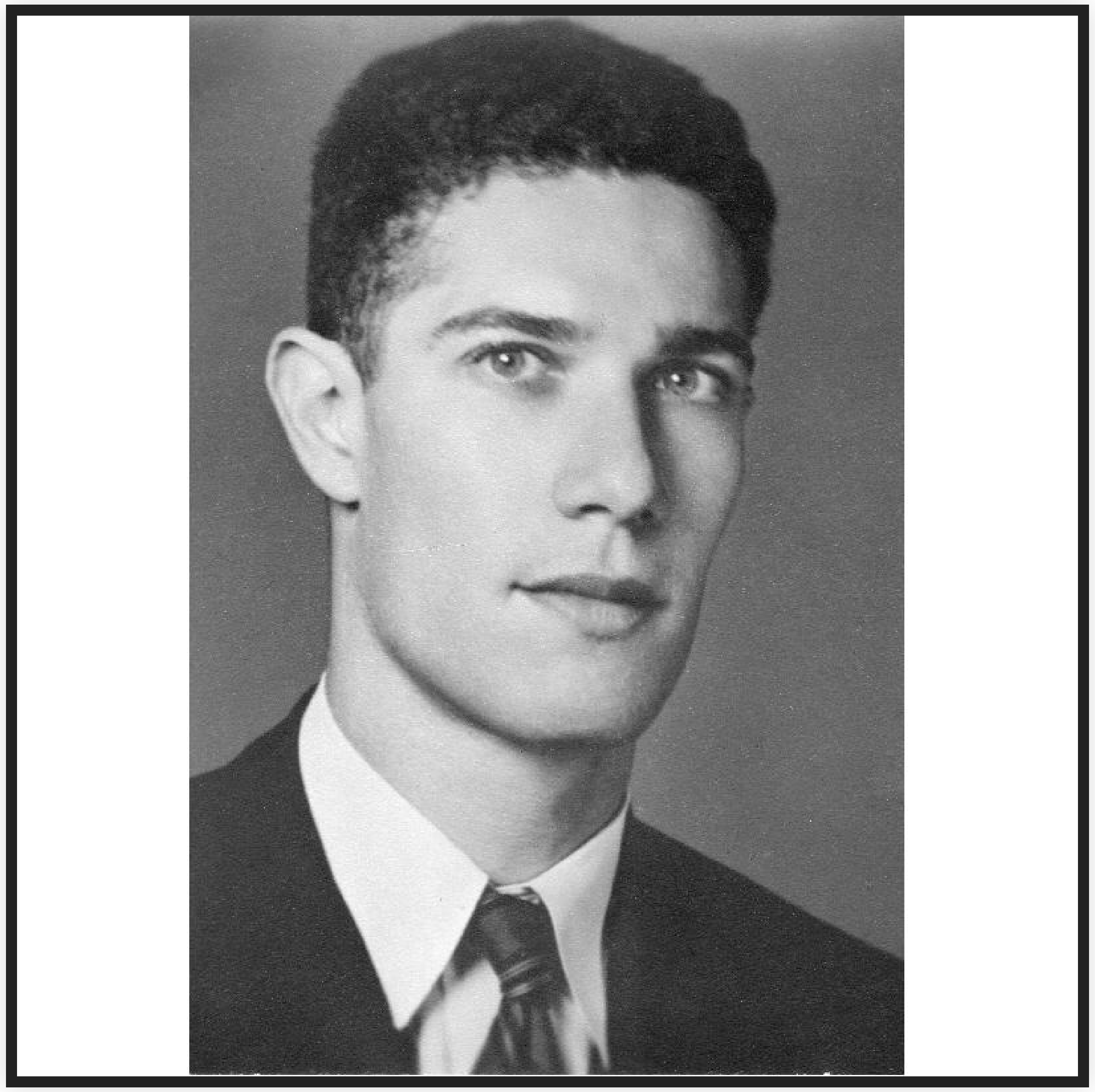
WHAT DID WE KNOW ABOUT LONG TERM GROWTH AT THAT TIME?

- The main model was the HD model?
  - Fixed Output-Labor ratio
  - No role of knowledge
  - No productivity
- Believe or not, Ramsey already had the model (no used).
- It was not enough to explain what was happening

SOME UNASWERED QUESTIONS USING THE HD

- What was the role of technological change?
- What is the role of productivity?
- Where is the knowledge accomulation?
- Population growth?

THE ANSWER



Robert Solow (1924)

- 1941, Solow left the university and joined the U.S. Army (until - August 1945)
- RA of Leontief

- A very simple model of long-term growth. Robert Solow and Trevor Swan in 1956
- Fits pretty well the US data at that point

THE BASICS

ROBISON CROUSE ECONOMY

- Closed Economy
- No government purchases of goods and services.
- One sector economy (eat or produce)
- Production = Income

PRODUCTION FUNCTION - TREE MAIN INGREDIENTS:

- Capital $K(t)$
  - such as machines, buildings, pencils, and so on (Rival good)
- Labor $L(t)$
  - the number of workers and the amount of time they work, as well as their physical strength, skills, and health (Rival Good)
  - *Population* (growths) $\rightarrow L(t) = e^{nt}$
- knowledge or technology $T(t)$
  - Workers and machines cannot produce anything without a formula or blueprint that shows them how to do it. (Rival Good)

WHAT DOES THIS ECONOMY PRODUCE FOR? $\mathbf{Y(t)}$

- Consumption $C(t)$
- Investment $I(t)$
  - Investment is used to create new units of pyshical capital $\uparrow K(t)$
  - Replace old, depreciated capital (How does it happen? - next)

$$Y(t) = C(t) + I(t)$$

Moreover, we know that $S(t) \equiv Y(t) - C(t)$

- $S(t) \equiv Y(t) - C(t) = I(t)$

ENDOGENOUS SAVING RATE/INVESTMET RATE

- $s(.)$ the fraction of production that is save
  $\rightarrow\ s(.)Y(t) = S(t)$
- $\Rightarrow (1 - s(.))$ is the fraction of consumed
- $Y(t) = C(t) + I(t) = (1 - s(.))Y(t) + s(.)Y(t) = Y(t)$

HOW THIS SAVING RATE IS DERIVED?

> *Rational households choose the saving rate by comparing the costs and benefits of consuming today rather than tomorrow; this comparison involves preference parameters and variables that describe the state of the economy, such as the level of wealth and the interest rate.*

It is assumed exogenous.

WHAT ABOUT $\mathbf{K(t)}$

As all machines, buildings, pencils, and so on might get older we need to introduce depreciation.

- it depreciates at the constant $\delta > 0$
- $\rightarrow$ a constant fraction of the capital stock wears out and, hence, can no longer be used for production.

Then,

$$\frac{\delta K}{\delta t} = \dot{K}(t) = I(t) - \delta K(t) = sF[K(t), L(t), T(t)] - \delta K(t)$$

Let's make a diagram of the basic model!

WHAT DO WE NEED TO MAKE THIS MODEL WORK?

a "well behaved" production function

what does it means that $F(K, L, T)$ is a 'well behaved function'?

0 - CONSTANT RETURNS TO SCALE

$$F(\lambda K, \lambda L, T) = \lambda F(K, L, T)$$

homogeneous function $\rightarrow f(\alpha v) = \alpha^k f(\alpha v)$ where $k = 1$

- Intuition: an increase in inputs (capital and labour) cause the same proportional increase in output.
- Lets make a graph ($45 degree$)

1 - POSITIVE AND DIMINISHING RETURNS TO PRIVATE INPUTS.

$\forall K > 0$ and $L > 0$, F(.) exhibits positive and diminishing marginal products with respect to each input:

$$\frac{\delta F}{\delta K} > 0 \text{ and } \frac{\delta^2 F}{\delta K^2} < 0$$

$$\frac{\delta F}{\delta L} > 0 \text{ and } \frac{\delta^2 F}{\delta L^2} < 0$$

Intuition

2 - INADA CONDITIONS

marginal product of capital (or labor) approaches infinity as capital (or labor) goes to 0 and approaches 0 as capital (or labor) goes to infinity:

$$lim_{k \to 0}(\frac{\delta F}{\delta K}) = \infty$$

$$lim_{l \to 0}(\frac{\delta F}{\delta L}) = \infty$$

$$lim_{k \to \infty}(\frac{\delta F}{\delta K}) = 0$$

$$lim_{l \to \infty}(\frac{\delta F}{\delta L}) = 0$$

Intuition

3 - ESSENTIALITY

Each input is essential for production

$$F(0, L) = F(K, 0) = 0$$

Intuition

LAST PRELIMINARIES

INFORMATION PER-CAPITA

$Y = F(K, L, T)$ if we multiplicate $\frac{1}{L}$ (per capita)

$$\frac{Y}{L} = y = \frac{1}{L}F(K, L, T) = F(\frac{K}{L}, 1, T) = f(k)$$

or, $Y = Lf(k)$

intensive form (that is, in per worker or per-capita form) of production function.

WHAT DOES IT IMPLY FOR THE CAPITAL ACCUMULATION?

We know that:

$$\frac{\delta K}{\delta t} = \dot{K}(t) = I(t) - \delta K(t) = sF[K(t), L(t), T(t)] - \delta K(t)$$

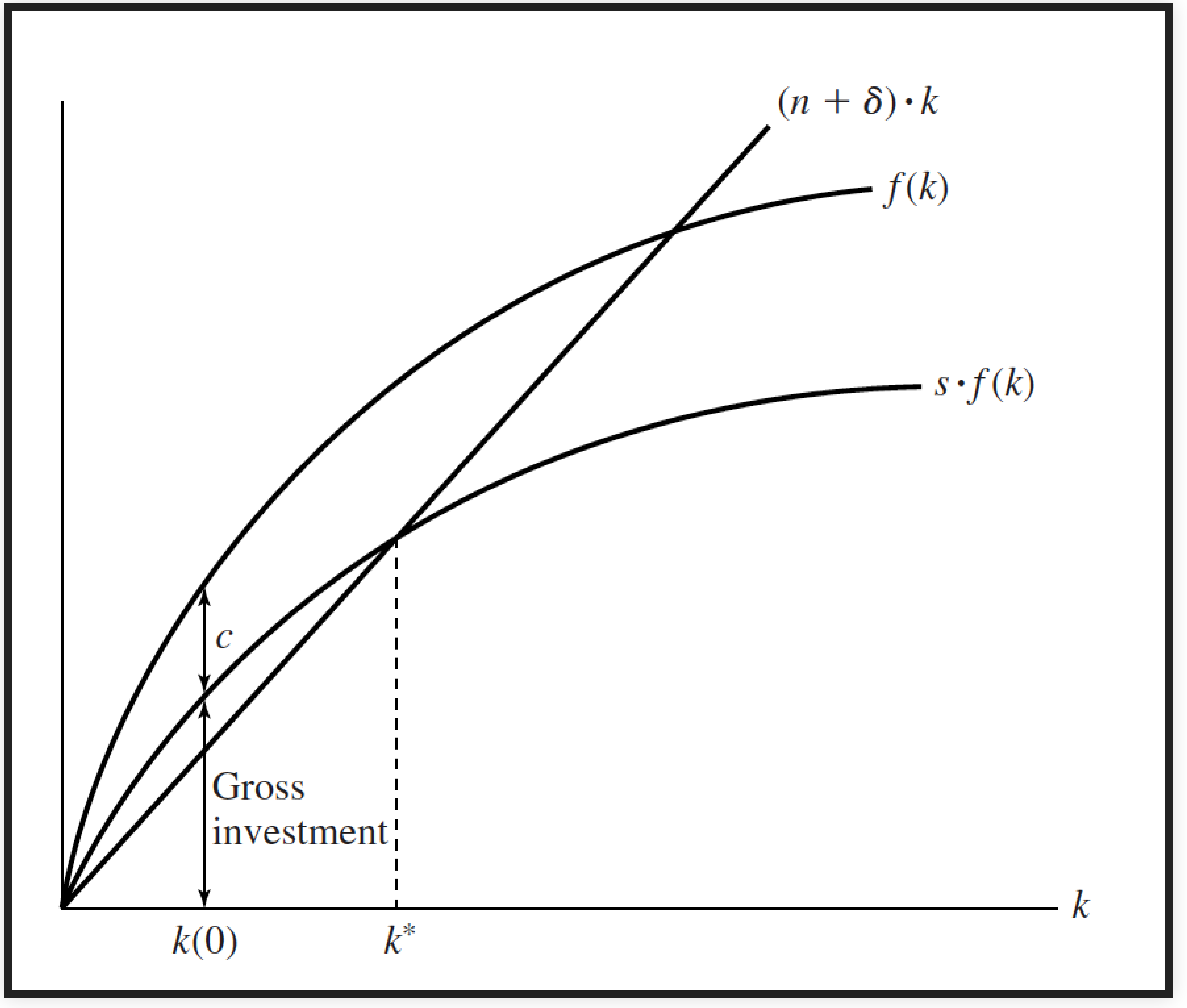
Then,

- $\dot{k} = \frac{\delta k}{\delta t} = \frac{\delta \frac{K}{L}}{\delta t} = sf(k) - (\delta + n)k$

NOW, WE ARE READY!

THE SOLOW–SWAN MODEL

(in action)

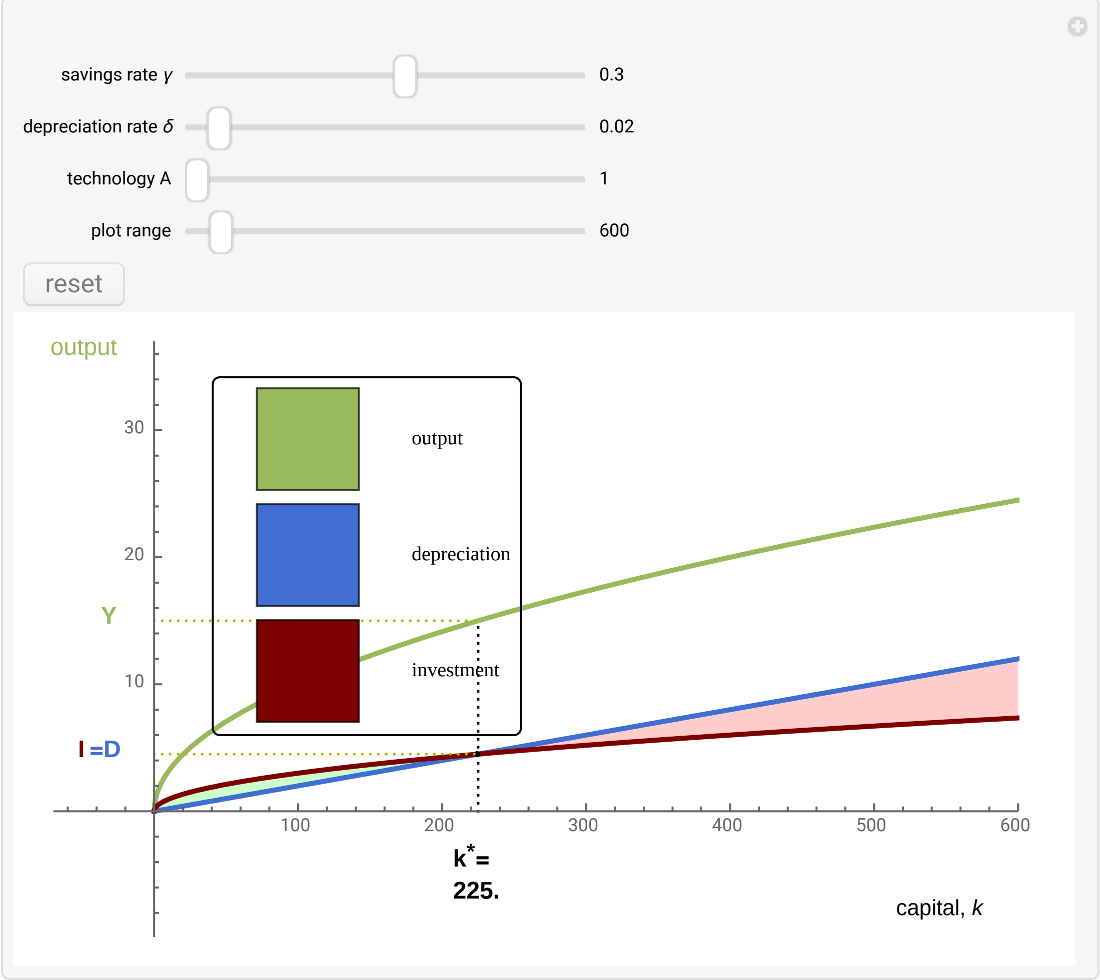


(n + δ)·k
f(k)
s·f(k)
c
Gross
investment
k
k(0)
k*

STEADY STATE

- a situation in which the various quantities grow at constant (perhaps zero) rates.
- Slow-Swan Model $\rightarrow sf(k^*) = (n + \delta)k^*$ --> Intuition?
- the per capita quantities k, yy, and c do not grow in the steady state.

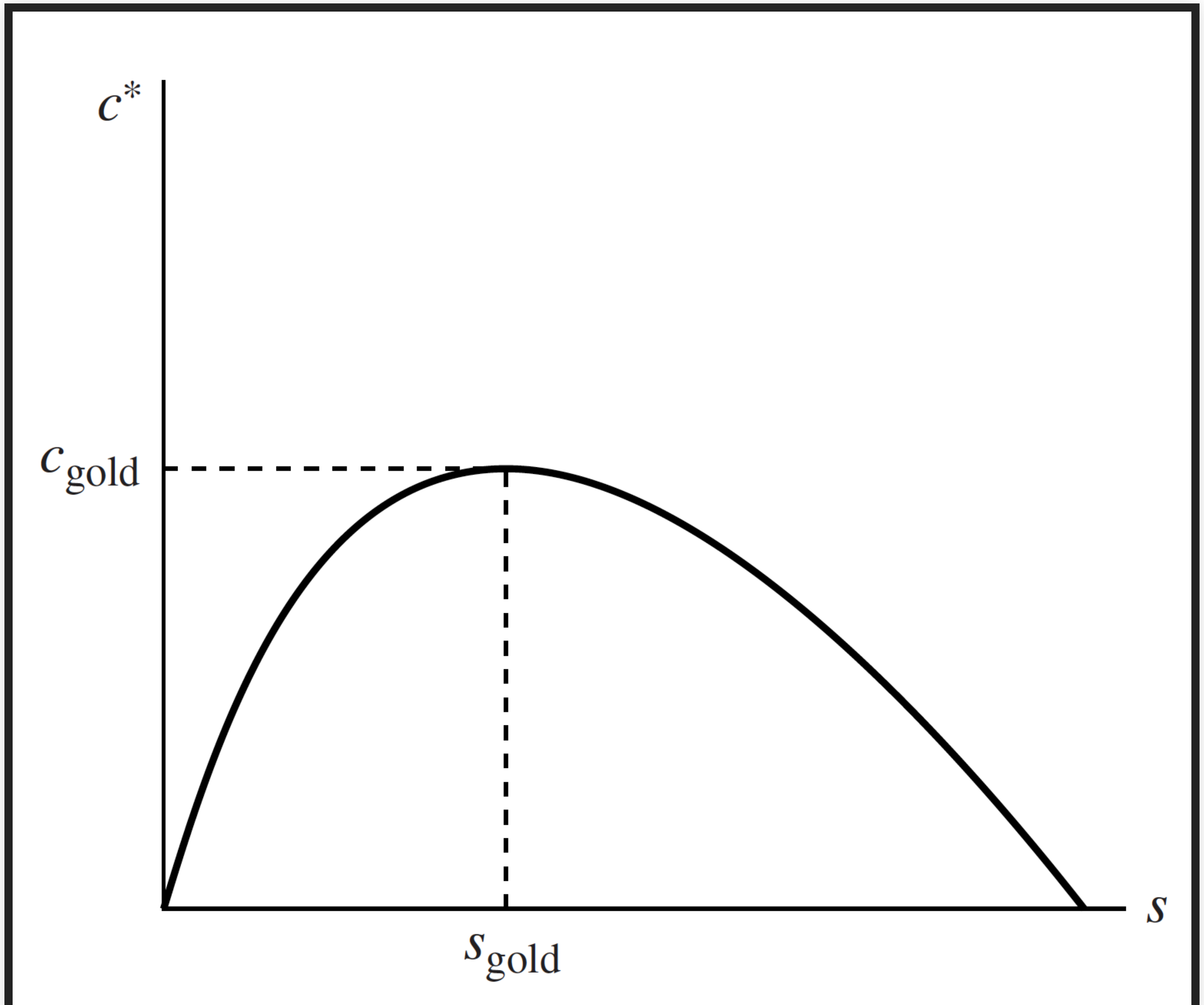
BASIC COMPARATIVE STATICS



THE GOLDEN RULE OF CAPITAL ACCUMULATION

- At steady-state $c^* = (1 - s)f(k^*)$
- we know that $sf(k^*) = (n + \delta)k^*$
- Then, $c^* = f(k^*) - (n + \delta)k^*$
- $max(c^*)$

THE GOLDEN RULE OF CAPITAL ACCUMULATION.



Next Class - Readings

** Reading Chapter 2

- Cobb-douglas
- 1.2.12 - Technological Progress