UNIVERSIDAD EAFIT	**Growth&Development** Juan Carlos Muñoz-Mora	First Exam (25%)	Name: ______ Student Code: ______ Team: ______ Date: September 11, 2019

Instructions: .

- **Duration: 1.5 hours**
- Please, read carefully the exam before answering. Make sure you understand!
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.
- Make a plan!

Question	Points	Score
1	10	
2	5	
3	10	
4	5	
5	8	
6	5	
7	10	
8	10	
9	0	
10	5	
11	12	
12	20	
Total:	100	

Table 1: Grading Table

1 General Motivation - Economic Growth

1. (10 points) According to figure 1, we can conclude that:

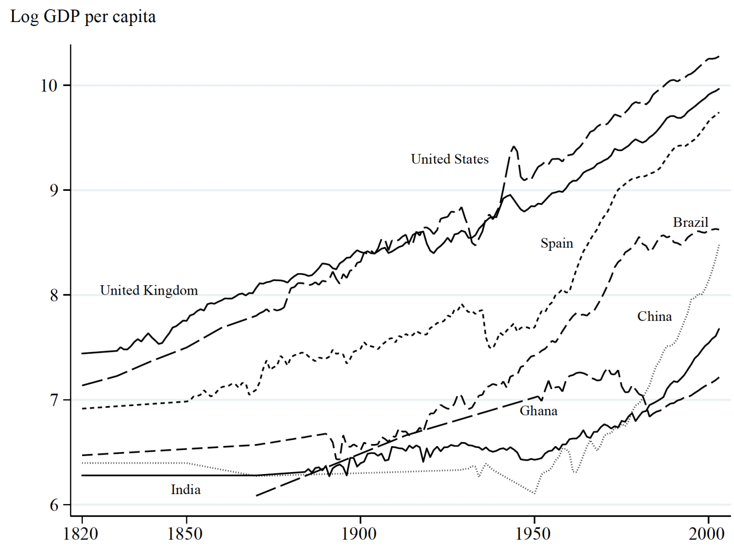

Figure 1: The evolution of income per capita in the United States, the United Kindgom, Spain, Brazil, China, India, and Ghana, 1820–2000.

- √ **The United States, the United Kingdom, and Spain were growing much faster than India and Ghana**
- √ **The origins of the current cross-country differences in income per capita are in the nineteenth and early twentieth centuries (or perhaps even during the late eighteenth century).**
- √ **Despite the starting point, a number of countries in the world "took off" and achieved sustained economic growth during 1820-2000**
- ☐ There is a clear convergence of all countries to get to the same growth rate.

2. (5 points) (Match) Please match each theory with the explanation of the evolution of income per capita US (1820–2000) (figure 1):

Classical Theory	Harrod-Domar Model	Marx Theory
Keynesian Theory	Neo-classical Theory	

(a) **Harrod-Domar Model** — Growth is related to the capital stock by the capital-output (c/o) ratio, which has a *fixed* technical or behavioural relationship.

(b) **Classical Theory** — A continuous increase of wages, that leads to an increasing expansion of population.

(c) **Keynesian Theory** — The demand for and supply of money ($MV = PT$)

2 Slow-Swan Model

Let's build together the essential settings for the Solow Model.

3. (10 points) The neoclassical growth model emphasizes the role of **capital accumulation** as the main driving force of long term growth. In a basic set up, the model start with a simplified setup that excludes **markets and firm** (which is know as **household/producer like Robinson Crusoe**. Under this context, production take place with three factor: $Y(t) = ($ **K(t)**, **L(t)**, **T(t)** $)$. Please give a very brief explanation, of each factor:

Solution:

- Capital $K(t)$
  - such as machines
  - buildings, pencils, and so on
  - Rival good
- Capital $L(t)$
  - the number of workers and the amount of time they work, as well as their physical strength, skills, and health
  - Rival good
  - *Population* (growths) $\rightarrow L(t) = e^{nt}$
- knowledge or technology $T(t)$
  - Workers and machines cannot produce anything without a formula or blueprint that shows them how to do it.
  - Non rival good

The basic version of Solow-Swan model considered a closed economy with no public spending, so it implies that $Y(t) = C(t) + I(t)$. By subtracting $C(t)$ from both sides and realizing that output equals income, we get that, in this simple economy, the amount saved, $S(t)Y(t)C(t)$, equals the amount invested, $I(t)$.

4. (5 points) What is the main assumption of the Solow-Swan Model about $S(t)$? How households calcuate this $S(t)$? What are the main implication of this assumption?

Solution: Let $s()$ be the fraction of output that is saved—that is, the saving rate—so that $1s()$ is the fraction of output that is consumed. Rational households choose the saving rate by comparing the costs and benefits of consuming today rather than tomorrow; this comparison involves preference parameters and variables that describe the state of the economy, such as the level of wealth and the interest rate. we assume that $s()$ is given exogenously. The simplest function, the one assumed by Solow (1956) and Swan (1956) in their classic articles, is a constant, $0 <= s()1$. We use this constant-saving-rate specification because it brings out a large number of results in a clear way. Given that saving must equal investment, $S(t) = I(t)$, it follows that the saving rate equals the investment rate. In other words, the saving rate of a closed economy represents the fraction of GDP that an economy devotes to investment.

Thus, the net increase in the stock of physical capital at a point in time equals gross investment less depreciation (δ):

$$\frac{\delta K}{\delta t} = \dot{K}(t) = I(t) - \delta K(t) = sY(t) - \delta K(t)$$

5. (8 points) Where $Y(t)$ is a neoclassical production function that satisfied the following properties:

(a) Constant returns to scale $\surd$ $F(\lambda K, \lambda L, T) = \lambda F(K, L, T)$ $\square$ $lim_{k\rightarrow\infty}(\frac{\delta F}{\delta K}) = 0$ $\square$ $F(\lambda K, \lambda L, T) = \lambda F(K, L,)$
Intuition:

Solution: To get some intuition on why our assumption makes economic sense, we can use the following replication argument. Imagine that plant 1 produces Y units of output using the production function F and combining K and L units of capital and labor, respectively, and using formula T . It makes sense to assume that if we create an identical plant somewhere else (that is, if we replicate the plant), we should be able to produce the same amount of output. In order to replicate the plant, however, we need a new set of machines and workers, but we can use the same formula in both plants. The reason is that, while capital and labor are rival goods, the formula is a nonrival good and can be used in both plants at the same time. Hence, because technology is a nonrival input, our definition of returns to scale makes sense.

(b) Positive and diminishing returns to private inputs $\surd$ $\frac{\delta F}{\delta K} > 0$ **and** $\frac{\delta^2 F}{\delta K^2} < 0$ $\square$ $\frac{\delta F}{\delta L} > 0$ or $\frac{\delta^2 F}{\delta L^2} < 0$ $\surd$ $\frac{\delta F}{\delta L} > 0$ **and** $\frac{\delta^2 F}{\delta L^2} < 0$ $\square$ $\frac{\delta F}{\delta L} > 0$ or $\frac{\delta^2 F}{\delta L^2} < 0$
Intuition:

Solution: The neoclassical technology assumes that, holding constant the levels of technology and labor, each additional unit of capital delivers positive additions to output, but these additions decrease as the number of machines rises. The same property is assumed for labor.

(c) Inada conditions $\surd$ $im_{k\rightarrow 0}(\frac{\delta F}{\delta K}) = \infty$ $\surd$ $lim_{l\rightarrow 0}(\frac{\delta F}{\delta L}) = \infty$ $\surd$ $lim_{k\rightarrow\infty}(\frac{\delta F}{\delta K}) = 0$ $\surd$ $lim_{l\rightarrow\infty}(\frac{\delta F}{\delta L}) = 0$
Intuition:

Solution: It follows the former property, yet at the infinity. It says that, at limit, the marginal contribution of a extra unit of capital or labor varies on the number of the stock.

(d) Essentiality $\surd$ $F(\lambda K, \lambda L, T) = \lambda F(K, L, T)$ ☐ $F(0, L) = F(K, 0) = K$ $\surd$ $F(0, L) = F(K, 0) = 0$
Intuition:

Solution: It implies that each input is essential for production. There is not production with only one factor.

6. (5 points) In order to take the scale effect of population, we construct the model in per capita terms and study primarily the dynamic behavior of the per capita quantities of GDP, consumption, and capital.
   (a) Thanks to the property of __constant returns to scale__ which applies $\forall\lambda$, especially for $\lambda = \frac{1}{L}$, we can re-write the production function as follows: ☐ $Y = f(k)$ ☐ $Y = f(k)$ $\surd$ $Y = Lf(k)$
   (b) where k is: $\surd$ $k = \frac{K}{L}$ ☐ $k = L^n K$ ☐ $k = k*$
   (c) Thus, the marginal products of the factor inputs are given by: $\surd$ $\frac{\partial Y}{\partial K} = f'(k)$ ☐ $\frac{\partial y}{\partial K} = Lf'(k)$ $\surd$ $\frac{\partial y}{\partial L} = f(k) - kf'(k)$ ☐ $\frac{\partial y}{\partial L} = f(k) - kLf'(k)$

7. (10 points) Now, we know that

$$\frac{\partial K}{\partial t} = \dot{K}(t) = I(t) - \delta K(t) = sF[K(t), L(t), T(t)] - \delta K(t)$$

Assuming that $\frac{\dot{L}}{L} = n$, then

   (a) we can re-write in accomolation capital-per capita: $\frac{\dot{K}}{L}$ as follow: $\surd$ $sf(k) - (\delta + n)k$ ☐ $f(k) - (\delta + n)k$ ☐ $f(k) - (\delta + n + g)k$ ☐ $f(K) - (\delta + n)K$
   because (true/false),
      i. [T] $\frac{\dot{K}}{L} = \frac{\partial \frac{K}{L}}{\partial t} =$
      ii. [F] $\frac{\dot{K}}{L} = \frac{\partial K}{\partial t}$
      iii. [T] $\frac{\partial \frac{K}{L}}{\partial t} = \frac{K'L - L'K}{L^2}$
      iv. [F] $\frac{\partial K}{\partial t} = f(K) - (\delta + n)K$
   (b) We can graphically represent this condition as follows:

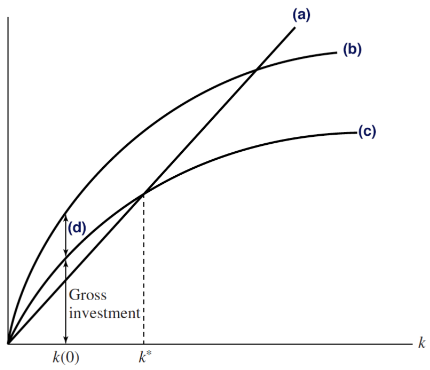

Figure 2: The Solow-Swam Model

where,

i. (a) $(n+\delta)k$
ii. (b) $f(k)$
iii. (c) $sf(k)$
iv. (d) **c**

8. (10 points) Please define what does "the steady state" mean in this context? Which conditions are satisfied at the steady stage? What are the value of k at the steady stage?

9. (10 points (bonus)) The steady state impliest that: $\frac{\partial \frac{\dot{k}}{k}}{\partial k} = [f'(k)] - f(k)/k]/k$. Does result mean that economies with lower capital per person tend to grow faster in per capita terms? In other words, does there tend to be convergence across economies? Based on those questions explain the differences among absolute convergence, conditional convergence, and a reduction in the dispersion of real per capita income across groups.

3 The Cass-Koopsman-Ramsey Model

10. (5 points) List the main differences between Solow Model vs Cass-Koopman-Ramsey model

Solution:

- There is evidence that the assumption of fixed saving rate
- Permanent Income
- Life-cycle -¿ Saving hypothesis
- Preferences for consumption

11. (12 points) The CKR solves the following maximiation problem:

$maxW$ s.a. $K_{t+1} = K_t + F(K_t) - C_t - \delta K_t$, $K_{T+1} > 0$, $K_0 > 0$.

where, $u(c_t)$ is the utility function whose the marginal utility is positive and decreasing $u'(c_t) > 0$ and $u''(c_t) < 0 \forall c_t > 0$.

(a) Please, resume very briefly the intuition behind the optimization problem in the CKR.

(b) In order to solve this problem, we define the Lagragian which is define.

$$L = \sum_{t=0}^{T} \beta^t u(c_t) + \sum \mu_{t+1}[K_t + F(K_t) - C_t - \delta K_t - K_{t+1}]$$

where, μ_{t+1} is the **Shadow price of utility** .

(c) Therefore, the First Order Condition (FOC) of this problem is given by:

√ $\frac{\partial L}{\partial c_t} = \beta^t u'(C_t) - \mu_{t+1} = 0$ ☐ $\frac{\partial L}{\partial k_t} = \mu_{t+1}(1 + F'(K) - \delta) = 0$ √ $\frac{\partial L}{\partial k_t} = \mu_{t+1}(1 + F'(K) - \delta) - \mu_t = 0$
☐ $\frac{\partial L}{\partial c_t} = \beta u'(C_t) - \mu_{t+1} = 0$

(d) Do we need any other conditions?

12. (20 points) If we define $\lambda_t = \beta^{-(t-1)}\mu_t$, then we can re-write the FOC as follows:

$$U'(c_t) - \lambda_{t+1} = 0$$

and,

$$\beta(1 - F'(k) - \delta) - \frac{\lambda_t}{\lambda_{t+1}} = 0$$

$$\forall t = 0, 1, ..., T$$

Moreover, if we define $r(t) = F'(k) - \delta$. Then, we can find the condition for the optimal consumption smoothing over time:

$$1 + r(k) = \frac{U'(c_t)}{U'(c_{t+1})}$$

(a) What is the intuition this expression:

(b) Now, we if assume $\beta = \frac{1}{1+\rho}$ where ρ is:

(c) Then, using the porperties of the marginal change we can re-write the optimal consumption smoothing over time:

$$\frac{1+\rho}{1+r(k)} = \frac{U''(c_t)}{U'(c_{t+1})}\dot{c}_t + 1$$

Moreover, as we need that $\frac{1+\rho}{1+r(k)}$ tend to zero because:

we can use the taylor expansion around zero, and we can re-write the equation as:

$$\frac{U''(c_t)}{U'(c_{t+1})}\dot{c}_t = \rho - r$$

This equation es known as "The Canonnical Euler Equation", which main intuition is:

(d) Now assuming that $u(c_t) = \frac{c_t^{1-\xi}-1}{1-\xi}$, we can re-write the "The Cannonical Euler Equation" as

$$r = \rho + \xi q$$

where $g = \frac{\dot{c}_t}{c_t}$. The main intuition of this equation is the following:

(e) What is the main prediction at the steady state of this model? what does it imply?