# Work Games: Evaluating the Pandemood
## HUL311 Applied Game Theory

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**Abstract**

COVID-19 has thrust upon us the "new normal", and in this paper, I evaluate aspects of work from home with the lens of evolutionary game theory to characterise problems which threaten to affect the work routine, given the comfort of our beds. The three aspects that are analysed are procrastination and updating working based on interactions by learning from successful agents, contributing to a group project, and lying about whether or not a task is done and supervising the same based on the signal received by supervisor. The topic that I have chosen has not been discussed before, as I chose to characterise the problem based on current experience. But there are several useful tools and games from which parallels can be drawn, which help me solve them.

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# 1 The Coronial Problem

## 1.1 Overview


Figure 1: Word clouds- size of a word represents its usage frequency in context of COVID-19 and work [1]

The COVID-19 pandemic forced the world into what the WHO calls as the "new normal". Lockdown and social distancing have been perhaps the biggest mental and physical challenge that people are facing and adjusting to. With no people out of their houses, most businesses have either revamped or crumbled, with the exception of the likes of Zoom and Netflix (which successfully feed on people's current needs because of restraints, and well, boredom). Articles in The Economist [5] have talked about how the economy will shrink to 90% of its previous size because of businesses rendered obsolete by the lockdown. Work life integration has stolen the spotlight from work life balance, changing behaviours and expectations. As for online education and assessments, while accessibility is a big issue, the "sanctity of evaluation and the degree" (quoting the Director, Prof. Rao) is another obstacle that institutes are yet to adjust with.
The situation required an unprecedented social, political, professional and business response, and technology and innovation have changed ways for accommodation. Depending on how well we understand the situation and how proactively we work towards redefining self utility in these circumstances, we can either define the new normal, or watch it unfold.
Game theory and its evolutionary dynamics potentially offer an interesting perspective on this, helping us understand how social behaviour will change, further offering insight into what needs to be done to tap into efficient functioning of resources and in particular ourselves. Using relevant game formulation for some concepts, I attempt to uncover the mood of the pandemic, the pandemood.
This is a fairly interesting problem owing to the relevance it has in current times, and the novelty of the current times itself, which has been unprecedented and unpredictable. In such a situation, perhaps the only strategy set is our own behaviour, and efficient outcome is working as a society and not as an individual.

## 1.2 In this paper

**Work from home** was a concept that was limited to very few firms in India, and the past three months have forced most private firm and government functioning to be carried out from home. Education online was something no one had thought of, and no one was prepared for. This has forced people to readjust, bring some changes in place, and renew energy and motivation to work and learn fruitfully.
I have chosen to focus on behavioral aspects, in particular, those related to work and education. Relevant to both are procrastination, especially relevant in the premise of work/study from home. The other aspect relevant to this is how people contribute to a group project, and if there exist enough incentives which would drive people to do better. While group projects exist irrespective of work setting, at home implementation reduces micro supervision and accountability. Lastly, interaction between a supervisor and an agent assigned with some task or learning is evaluated, where the signal from the agent is whether or not the work has been done, and supervisor's response is to choose between assessing the agent, or doing nothing and moving on based on signal.

# 2 Solution

Since there is no work that directly addresses these problems in literature, I have instead picked some features of models that exhibit the behaviour I desire, and built upon them to address some problems relevant to behavioral game theory for work from home. I rely on multiple problems and their solutions in characterising work from home instead of a single well defined problem-solution.
Nature of work varies for different work roles, so I am evaluating some problems under this umbrella. I have chosen to evaluate two specific scenarios- procrastination and group project. Further, I have evaluated lying in interactions.

## 2.1 Procrastination

On a personal front, without supervision and setting, procrastination is something that quite a few of us deal with. This is a problem that I wish to study, the energy to deal with which would have resulted in a more fleshed out study than the one that you are currently reading. This ends up affecting not just work quality, but also time utilisation and eventually mental health of a person. I have drawn the rudimentary model from Applications of Present Biased Preferences in the book by Dhami [2], and built upon it for evolutionary analysis.
Given a task for the individual do be done once over $T$ time periods (say days) wherein the task has to be done once, let the reward and cost schedule be $v$ ($v$ could have been a vector too, but for simplicity, it is assumed to be a constant), $c = (c_1, c_2, \ldots, c_T),\ c_i > c_j \forall i > j$. Consider $T$ replicas of me sitting on every time period with quasi hyperbolic preferences $(\beta, 1)$, and the beliefs of each self are that any future self has preference $(\hat{\beta}, 1)$. The parameter $\beta$ captures a player's bias for the present, for it implies that more relative weight is assigned to $t$'s payoff at $t$ than at any time before $t$. Utility is given by

$$U_t(t) = \beta v - c_t$$

For the work to be done,

$$U_T(T) = \beta v - c_T > 0 \implies v > \frac{c_T}{\beta}$$

Dhami [2] describes three kinds of typical behaviours:

1. Time-consistents: $\beta = \hat{\beta} = 1$, since $min(c) = c_1$, the work will be done in the beginning.

2. Sophisticates: $\beta = \hat{\beta} < 1$, since self in $T$ will do the work, self in $(T-1)$ anticipates it and does not do the work. $(T-2)$ anticipating this, does the work but $(T-3)$ does not, and so on. For even $T$, work is done in the 2nd period, one period later than time consistents. For odd $T$, work is done in first period.

3. Naifs: $\beta < \hat{\beta} = 1$, naifs believe, incorrectly, that their future selves are time-consistent, and have no self-control problems. In particular, self $t$ believes, incorrectly, that self $(t+1)$ is a time consistent and will do the work. The work is then in time period $T$.

The work will not be done altogether if the reward is estimated to be less than $c_T/\beta$ (possibly denial). Laibson [3] qualitatively comments on the scenario when a person decides to commit to work.
I now build the evolutionary game perspective on this- does a person learns to do better based on interactions? For this, I append replication by imitation model in Weibull [6], suited to this scenario. Perhaps self learning can also be done by updating the belief $\beta$ based on one's own history of payoffs, but the model that I could think on this was based on a numerical method wherein the belief will converge to true value, which did not fall into the game theory framework.
Now, imitation can be of:

- Agents present most in number/herd mentality- would result in spread of inefficiency when most of the population consists of naif agents

- Successful agents- depending on work's payoff, the strategy which is optimal is imitated by whoever encounters the same $-->$ this is the method of imitation I choose to demonstrate and discuss

- Dissatisfaction- strategy of whoever is drawn first from the population probabilistically is adopted

Let there be three populations playing a pure strategy, time-consistents ($i = 1$, strategy is $\beta = \hat{\beta}$=1), sophisticates ($i = 2$, strategy is $\beta = \hat{\beta} = 1/2$) and naifs ($i = 3$, strategy is $\beta = 1/2, b\hat{e}ta = 1$). The numeric values can and would differ, but I have chosen these numbers for simplicity. Let $T = 4$, cost structure $c = (3, 5, 8, 13)$, and reward $v > 26$, so let $v = 30$. Then $u_1 = 27$, $u_2 = 10$ and $u_3 = 2$.

Let $r_i$ be the average review rate of an agent who uses pure strategy $i \in K$. Probability that $i$-strategist shifts to strategy $j$ is $p_i^j(x), p_i(x) = (p_1^1(x), p_i^2(x), p_1^3(x))$. $p_i^i$ is the probability that strategist does not revise. Then the replicator dynamics is given by

$$\dot{x_i} = \sum_{j \in K} x_j r_j(x) p_j^i(x) - r_i(x) x_i$$

Now for simplicity we assume that review rates $r_i(x) = 1$. In the model described above, probability that strategist-$i$ shifts to $j$ is given by $p_i^j(x) = x_j \phi(u_j - u_i, x), j \neq i$ and $p_i^i = 1 - \sum_{j \neq i} x_j \phi(u_j - u_i, x)$, where $\phi$ is a monotonically increasing function for $x > 0$. This gives

$$\dot{x_i} = [\sum_{j \in K} x_j (\phi(u_i - u_j, x) - \phi(u_j - u_i, x))] x_i \tag{1}$$

We use equation 1 to observe the dynamics with a quadratic $\phi$ function.

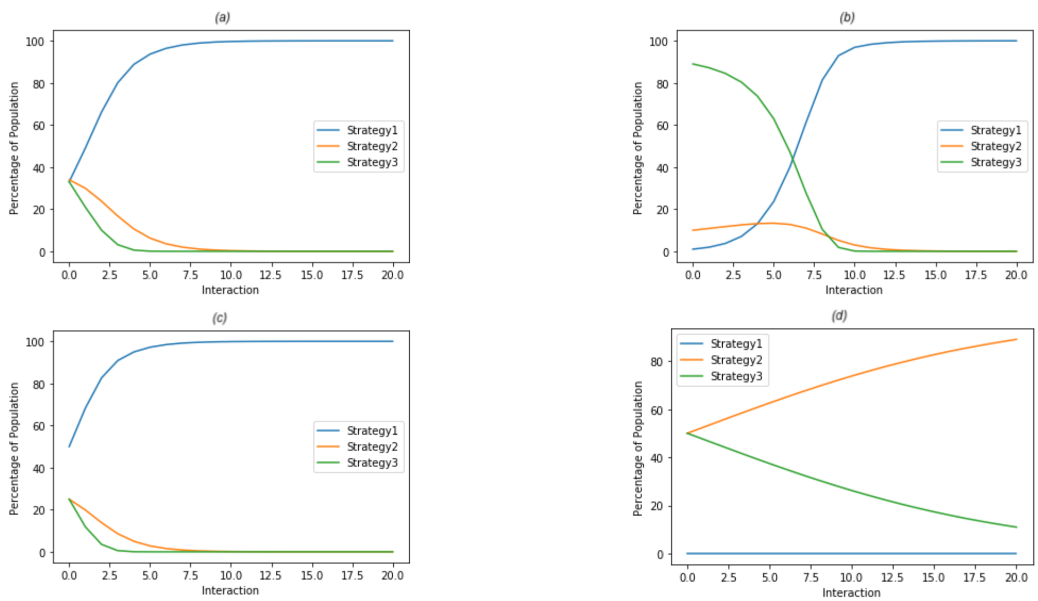


Figure 2: Replicator Dynamics for the game when a) initial population of all kinds are equal, b) initial population of time-consistents is very low (0.1%) and that of naifs is very high (89%), c) time-consistents comprise of 50% of the population, while sophisticates and naifs are 25% each and d) there are no time-consistents, and half sophisticates and half naifs

Purely by revising one's strategy imitation, we see that people become time-consistents, which is the perfect manifestation of setting a good example so that others learn. This highlights the importance of positive interaction at work, which benefits the entire population. The catch is that the strategy revision rates for people might be far from the ideal value of 1. The more is the number of efficient people, more quickly would we expect convergence to the only ESS of $(100, 0, 0)$. Additionally, to identify the type of any person, progress monitoring at time intervals would help in building perspective.

## 2.2 The Group Projectblem

The public goods game is the beginning point of this analysis. In the basic game, subjects choose how many of their private tokens to put into a public pot. The tokens in this pot are multiplied by a factor (greater than

one and less than the number of players, $N$) and this "public good" payoff is evenly divided among players. Each subject also keeps the tokens they do not contribute. Naturally, for human subjects, rewards and/or punishments are required to induce cooperation. I employ basic formulation from Sasaki and Unemi [4] and build on it. This is not a public good, but just as in that case, the whole group is rewarded. Additionally, I do not get rid of the pool reward, because a rewarder can be seen as someone who identifies as a stakeholder in the project/company and hence is entitled to it, and can choose to reap collective rewards with team mates. Punishment is simply loss of that reward.
Consider the random formation of a group of N players from the population of $N \geq 2$. Each player is asked to contribute $c_1 > 0$ . The contribution, $c_1$, will be multiplied by $r_1 > 1$ and then equally shared among $N$ players in the group. The pool rewarding mechanism is as follows- each player is first asked to contribute $c_2 > 0$ to a fund to reward cooperative behavior. The integrated contribution to the reward fund is multiplied by $r_2 > 1$, and distributed equally to those who have contributed to the project.
We have three strategies: rewarders (R, population share $x_R$) who contribute both to the project and to the reward fund, cooperators (C, $x_C$) who contribute to the project but not to the reward fund, and defectors (D, $x_D$) who contribute neither to the project nor to the reward and free ride. Say $S$ is the number of people who contribute to the game. Replicator dynamics are given by

$$\dot{x_R} = (U_R - \bar{U})x_R; \;\; \dot{x_C} = (U_C - \bar{U})x_C; \;\; \dot{x_D} = (U_D - \bar{U})x_D; \;\; \bar{U} = x_R U_R + x_C U_C + x_D U_D$$

Expected utilities are given by

$$U_D = \sum_{S=0}^{N-1} \binom{N-1}{S} (1-x_D)^S x_D^{N-S-1} \frac{r_1 c_1 S}{N} = r_1 c_1 (1-x_D) \frac{N-1}{N}$$

and similarly for $D$ and $R$, except that for them, reward is reduced by the contribution $c_1(1 - r_1/N)$. Calculating rewards,

$$U'_C(S) = \sum_{n_R=0}^{S-1} (\frac{x_R}{1-x_D})^{n_R} (\frac{x_C}{1-x_D})^{S-n_R-1} \frac{r_2 c_2 n_R}{S} = r_2 c_2 \frac{S-1}{S} \frac{x_R}{1-x_D}$$

This reward is summed over values of S and hence

$$U'_C = \sum_{S=1}^{N} U'_C(S) = r_2 c_2 (1 - \frac{1-x_D^N}{N(1-x_D)})(\frac{x_R}{1-x_D})$$

$$U'_R = U'_C - c_2(1 - \frac{r_2}{N}\frac{1-x_D^N}{1-x_D})$$

Further solving (the paper [4] calculations help a lot),

$$U_R = r_1 c_1 (1 - \frac{1}{N})(1-x_D) - c_1(1-\frac{r_1}{N}) + r_2 c_2 (1 - \frac{1-x_D^N}{N(1-x_D)})(\frac{x_R}{1-x_D}) - c_2(1 - \frac{r_2}{N}\frac{1-x_D^N}{1-x_D})$$

$$U_C = r_1 c_1 (1 - \frac{1}{N})(1-x_D) - c_1(1-\frac{r_1}{N}) + r_2 c_2 (1 - \frac{1-x_D^N}{N(1-x_D)})(\frac{x_R}{1-x_D})$$

$$U_D = r_1 c_1 (1 - \frac{1}{N})(1-x_D)$$

$$\bar{U} = c_1(r_1 - 1)(1-x_D) + c_2(r_2-1)x_R$$

Finally, plug in values of $U$ in the replicator dynamics we began with (which is too huge and complex, but easier to deal with in a code and hence is done that way), and simulate the same.
I present some specific results where evolutionary dynamics vary greatly.

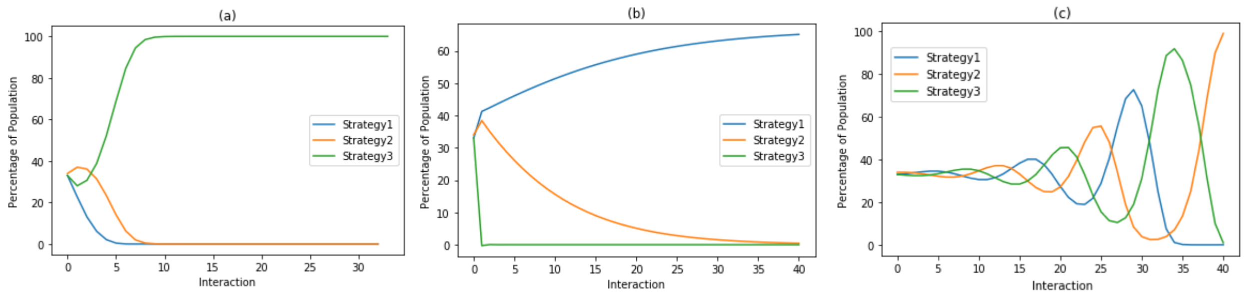


Figure 3: Replicator Dynamics for the game when $c_1 = 1, c_2 = 1, N = 5$ a) defectors/free riders prevail for $r_1 = 1.2, r_2 = 3$, b) rewarders prevail for $r_1 = 1.2, r_2 = 5.5$, c) oscillations like those in rock paper scissors, when $r_1 = 1.44, r_2 = 4.7$

Naturally, if a company can ensure that $r_1 > N$, everyone will necessarily contribute. But by varying rates of return and introducing stakeholders in the model, wherein people are more invested (this investment and return does not have to be monetary, it can be in the form of appreciation, freedom to take decisions, being heard, promotion or identifying performers), it can be guaranteed that people contribute not just to the project, but also to the pool rewarding mechanism which increases overall returns for everyone.

## 2.3 Lies and lows

Lack of direct management means that an agent can lie about progress/learning, which may be detected but the process of tracking is costly for the supervisor. So I construct a simple game to evaluate this. Since there are multiple strategies for both supervisor and agent, I construct something similar to Beer-Quiche game with changed strategies, combined with evolutionary game theoretical multipopulation model [6].
The agent does work with probability $p$ (say 0.9, this probability would vary from agent to agent), and then signals to the supervisor $y$, meaning yes the work is done, or $n$, meaning work isn't done. True signal is costless, while lying costs some $l$ (say 10). Upon receiving this signal, the supervisor decides to either assess the agent, or do nothing/move on to next task. Depending on whether or not the work is done, supervisor will lose or gain $s$ (say $10->$ gains because inefficiency in the system is spotted, loss is because supervisor invested time without any purpose). Agent loses some $m$ (say 20, because agent will have to sit for a viva/test/make presentation) whenever supervisor decides to assess. Each player position thus has four pure strategies. For agent: "always y," "y when done, n when not", "y when not, n when done", and "always n". For supervisor: "nothing when y, assess when n", "always nothing", "always assess", and "assess when y, retreat when n". Payoff matrix is given in Equation 2 by $A$ for agent and $S$ for supervisor.

$$\boldsymbol{A} = \begin{pmatrix} -1 & -1 & -21 & -21 \\ -2 & 0 & -20 & -18 \\ -28 & -10 & -30 & -12 \\ -29 & -9 & -29 & -9 \end{pmatrix}; \quad \boldsymbol{S} = \begin{pmatrix} 0 & 0 & -8 & -8 \\ 1 & 0 & -8 & -9 \\ -9 & 0 & -8 & 1 \\ -8 & 0 & -8 & 0 \end{pmatrix} \tag{2}$$

We can easily figure that there are two components of Nash Equilibrium$\Theta = (C, C')$ :

1. Agent always says that the work is done, and supervisor does nothing when hearing this; otherwise, assesses with a probability of at least $\frac{1}{2}$. If $x_i$ is $i$th pure strategy of agent as listed, and $y_i$ is $i$th pure strategy of supervisor, then

$$C = \{(x, y) \in \Theta : x_1 = 1, y_1 + y_2 = 1, y_1 \geq \frac{1}{2}\}$$

2. Agent always says work is not done, and supervisor does nothing when hearing this, otherwise, assesses with a probability of at least $\frac{1}{2}$.

$$C' = \{(x, y) \in \Theta : x_4 = 1, y_2 + y_4 = 1, y_4 \geq \frac{1}{2}\}$$

Both $C, C'$ are perfect and proper.
While agents' actions are intuitive, supervisor seems to be defensive as an option, always. This is because there is a cost attached to assessment, but the supervisor gets no benefit if the work is done.
Replicator dynamics for $i$th side ($i = A, S$) for $h$ strategy, where $u_i(x)$ is the average population payoff, is given by

$$\dot{x_{ih}} = \frac{1}{u_i(x)}[u_i(e_i^h, x_{-1}) - u_i(x)]x_i^h \tag{3}$$

Simulation of Equation 3 gives the result as in Figure 2.3.

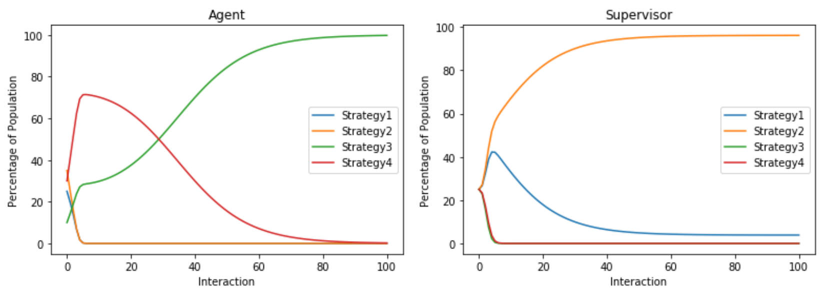


Figure 4: Replicator Dynamics for the game for agent and supervisor

For the supervisor, it again comes out be that doing nothing whether or not work is done is the result. The second best is to assess when the agent says work is done, and do nothing when work has not been done. This is clearly a window for the agent to lie.
The action of agent is a bit surprising but can be interpreted in the following manner- initially the agent always says work is not done and is assessed. But the agent is good, and work is done 90% of the time. Further down the road, the agent always lies. In this manner, whenever it is said that the work is not done, upon assessment, agent is proven to be industrious (and humble?). On the other hand, the agent lies when work is not done, which is 10% of the times, which supervisor may choose to believe, given the record.
Now we look at the scope for improvement in the model. The first is that the supervisor should be incentivised for assessment. Additionally, we can add to the model that if the agent does work, some personal reward $r$ (say 10) is added to the payoff, so that

$$\boldsymbol{A} = \begin{pmatrix} 8 & 8 & -12 & -12 \\ 7 & 9 & -11 & -9 \\ -19 & -1 & -21 & -3 \\ -20 & 0 & -20 & 0 \end{pmatrix}$$

Further, the model can be used for different kinds of students by varying $p$, the concept of carrot and stick can be integrated to reward work done when assessed or punished for lying wherein cooperation between interactors may emerge. Here, I have demonstrated only the basic model that I feel is suitable to this application.

# 3 Summary and Conclusions

I tried to use the concepts studied in evolutionary game theory to expand the scope of one shot interactions specifically relevant to my chosen problem, as people working in an office or studying interact not once or twice, but multiple times over the course of the completion of a task/tenure. By looking at work from home behavioral game theory, we can talk about whether work from home is feasible or not. This also lays a quantitative foundation of resolution of conflict between an employer desiring the comfort of work from home and an employee desiring micro supervision on the basis of possibility of increased/decreased efficiency. Ensuring meaningful interaction means that firms can actually ensure better productivity and mental health of employees, as people learn to improve their beliefs, thereby cutting on procrastination. Ample rates of return and stakeholder structuring in the form of reward pooling induces people to cooperate in group projects possibly because they get a sense of ownership and higher return based on their input. Both of these are also valid in an educational setting, wherein a student is expected to work and stick to deadlines at home. Lastly, the problem of lying to avoid assessments is evaluated, and we see that there exists a huge window for lying. The supervisors are benefited in either taking the agent's word for whether or not the work is done, or in doing nothing and moving ahead with other tasks. Also, agents tend to exploit this fact to make the supervisor believe that the work is mostly done.

The problem of work/learn from home is very interesting, though not very well defined. It is relevant to the current times especially in the Indian context, so attention to it is justified. There could have been more aspects to every model, as I have listed for each. More problems can be assessed under this topic, like the managerial/administrative structuring for increased accountability, utility of tracking progress for increased supervision and consequently efficiency, and how people feel the work load has changed. One shot interactions have been modelled extensively, and evolutionary games provide a handy tool for the extension of these interactions, as is suitable to this application.

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