Platform Design: A Microeconomic Analysis
HSD700: Term Paper

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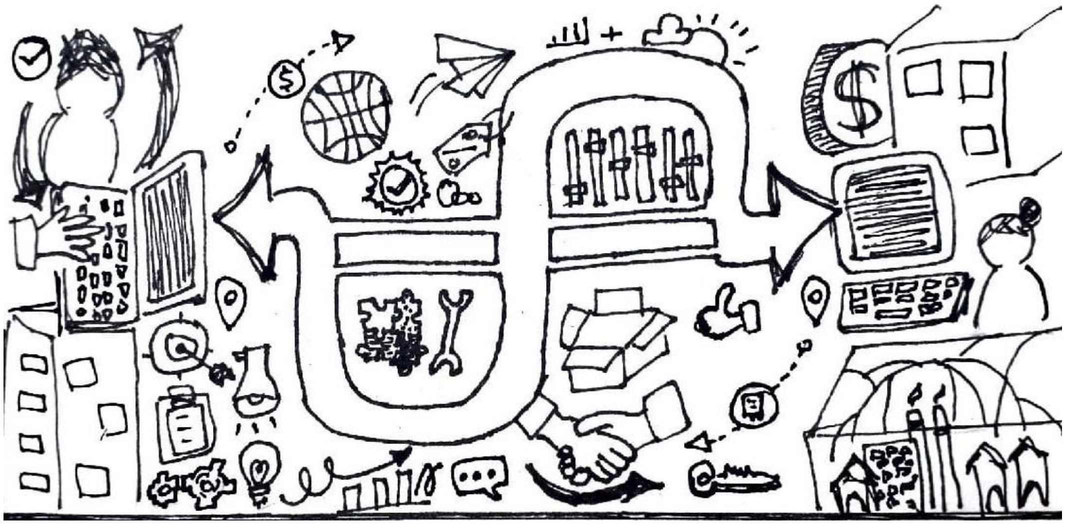

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1 Introduction

Rysman [2009] defines two sided market as the one where (i) two sets of agents interact with each other through an intermediary platform and (ii)the decisions of each set of agents affects the outcomes of the other set. In this perspective, an interesting evaluation is that of platform design and operation, and that is the theme of this paper.
Motivation for the choice of topic comes from our curiosity about the role that automation plays/can play in an interaction between agents, following which Saptarshi sir told us about two sided markets/networks, which exist when two groups of agents who could be buyers and sellers on an e-commerce platform, credit cards linking merchants and customers, or even potential suitors on a dating platform.
In the paper, we have first briefly discussed about the papers that have mathematical analysis on platform design. Most of the papers that we read discussed pricing as a strategy for platforms, and that we feel is the biggest caveat separating theory from practice. By including more elements that a platform can exercise as a strategy, such as differential pricing, matching policy, design and advertising, we have discussed them at length, and then presented mathematical models of the same. We begin with monopoly platform, and extend it to two platform competition. We discuss multi platform competition briefly. Lastly, we evaluate economic implications of these platforms, and means of operation and regulation for the same. For these, draw from some papers that we mention appropriately, and build upon the same.

Two-Sided Market	**Group-1**	**Group-2**	**Examples**
E-Commerce	Customers	Sellers	Amazon, Flipkart
Marriage	Potential Brides	Potential Bridegrooms	Shaadi.com
Shopping Malls	Shoppers	Retailers	Select Citywalk
Newspapers	Readers	Advertisers	Times Of India
Online Streaming	Viewers	Production Houses	Netflix, Prime
Mobile Applications	Users	Developers	PUBG, Instagram
Travel Aggregator	Travellers	Hospitality Industry	Trivago

Table 1: Some examples of two-sided markets.

2 Related Work

The papers that have extensive analysis relevant to our application are enumerated below:

1. Armstrong analyses platform pricing in a natural flow: monopoly platform, a model of competing platforms where agents join a single platform, and a model of "competitive bottlenecks" where one group joins all platforms. The determinants of equilibrium prices in this paper are (i) the magnitude of the cross group externalities, (ii) whether fees are levied on a lump-sum or per-transaction basis, and (iii) whether agents join one platform or several platforms.

2. Caillaud and Jullien have presented a fleshed out analysis of imperfect-competition, Bertrand game between two matchmakers in the presence of indirect network externalities with two pricing strategies- registration fees and transaction fees (with and without this), for exclusive and non-exclusive services, and equilibrium analysis of the same.

3. Rochet and Tirole is again a very thorough paper that begins with introducing network effects and modelling of membership and usage externalities, pricing structure and its detailed analysis. Lastly, it discusses principles of pricing and interaction regulation.

4. Rysman defines two sided markets, and elucidates its economic aspects in what is an interesting theoretical paper- pricing and openness, public policy and other strategies. It further discusses the scope of literature in this context

5. Vulkan, in his theoretical paper, has discussed applications of two sided markets, multi-agent systems, economic implications of trading using agents, and other implications.

3 Monopoly Platform Design

We begin with analysis as presented in Armstrong [2006]. We consider two groups of agents, 1 and 2. It is assumed that utility is derived from agents of the other group on the platform. If the platform hosts n_1 and n_2 people from each group and p_i is the platform price, u_i is the utility enjoyed by a group and α_i is the benefit enjoyed per agent on the other side for $i = 1, 2$, then utilities of a group 1 and 2 agent and number of people joining the platform as a function of utility ($\phi(.)$ is some increasing function), are respectively given by

$$u_1 = \alpha_1 n_2 - p_1; \quad u_2 = \alpha_2 n_1 - p_2 \tag{1}$$

$$n_1 = \phi_1(u_1); \quad n_2 = \phi_2(u_2)$$

If f_i is the cost incurred by the platform per agent of group i, then profit by platform is

$$\pi(p_1, p_2) = n_1(p_1 - f_1) + n_2(p_2 - f_2)$$

We assume that platforms provide utilities instead of prices to the agents, thus we convert all the quantities in terms of utilities to get

$$\pi(u_1, u_2) = \phi_1(u_1)[\alpha_1\phi_2(u_2) - u_1 - f_1] + \phi_2(u_2)[\alpha_2\phi_1(u_1) - u_2 - f_2] \tag{2}$$

Let $\nu_i(u_i)$ be the consumer surplus and satisfy $\nu_i'(u_i) = \phi_i(u_i)$, then the unweighted total welfare is given by

$$w(u_1, u_2) = \pi(u_1, u_2) + \nu_1(u_1) + \nu_2(u_2) \tag{3}$$

Maximising (3) gives

$$u_1 = (\alpha_1 + \alpha_2)n_2 - f_1; \quad u_2 = (\alpha_2 + \alpha_1)n_1 - f_2$$

Thus the socially optimal prices from (1) are

$$p_1 = f_1 - \alpha_2 n_2; \quad p_2 = f_2 - \alpha_1 n_1 \tag{4}$$

and profit maximising prices from (2) are

$$p_1 = f_1 - \alpha_2 n_2 + \frac{\phi_1(u_1)}{\phi_1'(u_1)}; \quad p_2 = f_2 - \alpha_1 n_1 + \frac{\phi_2(u_2)}{\phi_2'(u_2)} \tag{5}$$

In the subsequent sections we will see how addition of some features in the model changes the price levied by the platform.

3.1 Strategies other than pricing

Some strategies other than pricing that are in the hands of the platform are:

- **Tiered Pricing**: Flat rate pricing is simplistic, but does not enable the firm to extract maximum consumer surplus, and hence is mostly not used. A common mode of pricing that we see in any service/software or membership, including in platform pricing, is tiered- say in two tiers, standard and premium. This allows the customers to avail exclusive services by paying more, which means more income for the platform. Needless to say, the upgrade should be worth the price, else will be deemed worthless. A parallel analysis in this context is that of discounting some fraction of agents. There are three fundamental questions that thus arise in this context- what should the upgrade comprise of, how much should the platform charge (depending on the number of people who show interest in the same), how many agents/spots should be offered (especially relevant for discounting)? In Section 3.2, we formally analyse the question of tiered pricing, and how it affects platform profit.

- **Advertising**: Advertising is as essential as the implementation of any platform itself, for it serves the two fold purpose of informing potential audience about the platform and its objectives, in addition to informing them how it distinguishes itself from the alternatives/conventional means, which helps the brand build its identity. Investment in advertising is consequently a natural step for a platform. Mathematical manifestation of advertising is directly the increased number of agents that it is able to procure because of increased information outreach, which has been evaluated in Section 3.3. It is a strategy because even advertising will have diminishing returns and ultimately saturation, unless the platform reinvents itself to suit the potential audience from time to time.

- **Web page/app design**: The design aspect provides useful information about platform's services, and can even nudge consumers to make specific choices aligning with its goals. Like advertising, we have modelled design as an investment by the firm, although the returns yielded may not be in proportion of investment, and this argument is contestable. This is treated by the investment being modelled in the function which captures number of people joining the platform because of platform's utility (the ϕ function).

- **Matching policy**: In Armstrong's model, it is assumed that matching is perfect and consistent, which means that if an agent joins the platform, a transaction is sure to follow as the desirable agent of the complementary group is always found. This, in fact, is not a good assumption, and we have modelled it using the probabilistic variable λ in Section 3.4. λ is in fact an indicator of the efficiency of platform's matching algorithm, and a useful tool to compare platform's performance in competition analysis.

- **Pricing breakdown**: Instead of charging a price p from every agent who decides to participate on the platform, irrespective of success of participation, is unreasonable in the context of many platforms. Instead, in Section 3.4, we incorporate p as the registration charge, and a transaction charge c, which is paid by both the participating agents in some ratio. c itself could be equivalent to some fixed price, or some percentage of total transaction value. For simplicity, we have modelled fixed c.

- **Return/exchange policy/after sales**: This may/may not be relevant, depending on the type of platform we decide to look at. For instance, it is relevant to e-commerce websites, but not so much to dating apps. For a platform, flexibility could potentially imply more cost but a larger market share. This not only depends on the platform's cost consideration, but also on a group of agents. So, platform's policy could either be variable, or depend on who it decides to assign more bargaining power/welfare to.

- **Personalisation, ease of registration and use**: This is intersects with design, but is addressed separately because of what it means in terms of mathematical modelling, because registered users who choose to be idle derive utility attached to this aspect without participation per se. This is also an investment by the firm, but the level or personalisation it offers is a decision variable, and eventually affects the utility of both idle and active members on the platform.

3.2 Tiered Membership

Consider that the platform provides two tiers of services to each side of the agent, say standard and premium. Extending the notations used above, let $n_i = n_{i1} + n_{i2}$ where n_{i1} and n_{i2} respectively denote the number of agents in group i who use standard and premium membership. It is assumed that standard members can only start an interaction with standard members on the other side, whereas premium members can start interactions with both kinds of members on the other side. Thus the four utility function will be

$$\begin{aligned} u_{11} &= \alpha_{11} n_{21} - p_{11}; \quad u_{12} = \alpha_{12}(n_{21} + n_{22}) - p_{12} \\ u_{21} &= \alpha_{21} n_{11} - p_{21}; \quad u_{22} = \alpha_{22}(n_{11} + n_{12}) - p_{22} \end{aligned} \tag{6}$$

The number of agents on each side are given by an increasing function $n_{ij} = \phi_{ij}(u_{ij})$. The fixed costs remain the same for a group irrespective of the type of membership and are given by f_i. The platform profit in terms of prices and utilities is given by equation 7 and equation 8 respectively

$$\pi(p_{11}, p_{12}, p_{21}, p_{22}) = n_{11}(p_{11} - f_1) + n_{12}(p_{12} - f_1) + n_{21}(p_{21} - f_2) + n_{22}(p_{22} - f_2) \tag{7}$$

$$\begin{aligned} \pi(u_{11}, u_{12}, u_{21}, u_{22}) = {} & \phi_{11}(u_{11})[\alpha_{11}\phi_{21}(u_{21}) - u_{11} - f_1] + \\ & \phi_{12}(u_{12})[\alpha_{12}(\phi_{21}(u_{21}) + \phi_{22}(u_{22})) - u_{12} - f_1] + \\ & \phi_{21}(u_{21})[\alpha_{21}\phi_{11}(u_{11}) - u_{21} - f_2] + \\ & \phi_{22}(u_{22})[\alpha_{22}(\phi_{11}(u_{11}) + \phi_{12}(u_{12})) - u_{22} - f_2] \end{aligned} \tag{8}$$

Let $\nu_{ij}(u_{ij})$ denote the consumer surplus and satisfy $\nu'_{ij}(u_{ij}) = \phi_{ij}(u_{ij})$, then the unweighted total welfare is given by

$$w(u_{11}, u_{12}, u_{21}, u_{22}) = \pi(u_{11}, u_{12}, u_{21}, u_{22}) + \nu_{11}(u_{11}) + \nu_{12}(u_{12}) + \nu_{21}(u_{21}) + \nu_{22}(u_{22})$$

Maximising total welfare and using equation 6 we get the platform prices as

$$\begin{aligned} p_{11} &= f_1 - \alpha_{21} n_{21} - \alpha_{22} n_{22}; \quad p_{12} = f_1 - \alpha_{22} n_{22} \\ p_{21} &= f_2 - \alpha_{11} n_{11} - \alpha_{12} n_{12}; \quad p_{22} = f_2 - \alpha_{12} n_{12} \end{aligned} \tag{9}$$

On the other hand, the profit maximising prices on maximising equation 8 are

$$\begin{aligned} p_{11} &= f_1 - \alpha_{21} n_{21} - \alpha_{22} n_{22} + \frac{\phi_{11}(u_{11})}{\phi'_{11}(u_{11})}; \quad p_{12} = f_1 - \alpha_{22} n_{22} + \frac{\phi_{12}(u_{12})}{\phi'_{12}(u_{12})} \\ p_{21} &= f_2 - \alpha_{11} n_{11} - \alpha_{12} n_{12} + \frac{\phi_{21}(u_{21})}{\phi'_{21}(u_{21})}; \quad p_{22} = f_2 - \alpha_{12} n_{12} + \frac{\phi_{22}(u_{22})}{\phi'_{22}(u_{22})} \end{aligned} \tag{10}$$

The prices change because of the interaction terms α_{ab} and n_{ij}. $p_{i1} > p_{i2}$ as $\alpha n > 0$ and considerably greater than ϕ/ϕ'. This means that premium pricing will be higher, or, unlocking more interaction means paying a higher price.

By making the platform price $p_{i1} < p_{i2}$, we can model tiered services such that the first group receives some discount on price whereas the second group is expected to pay the full price. Further, the number of discounts can be capped. Thus, this analysis helps evaluate differential pricing which can be exercised by platforms to lure more customers and to extract surplus.
The welfare, which has been considered unweighted, need not be so. Depending on platform's aims, market power and service, it can decide to assign more weightage to a certain component like profit or a certain group of agents (say customers between sellers and customers).

3.3 Advertising and User Experience

Say the platform decides to invest on making services better and reaching out to more people by investing in advertising or personalisation by offering a more customised and easy to navigate account, ease of registration or use etc. To include this in the model, we include an additional price incurred by platform for both groups of agents in the form of k_i. Additionally, we assume that while this attracts more customers, the effect of k is captured in ϕ alone, and u and k are independent. Then we get

$$n_1 = \phi_1(u_1, k_1); \quad n_2 = \phi_2(u_2, k_2)$$

$$\begin{aligned} \pi(u_1, u_2, k_1, k_2) &= n_1(p_1 - f_1) + n_2(p_2 - f_2) - (k_1 + k_2) \\ &= \phi_1(u_1, k_1)[\alpha_1 \phi_2(u_2, k_2) - u_1 - f_1] \\ &+ \phi_2(u_2, k_2)[\alpha_2 \phi_1(u_1, k_1) - u_2 - f_2] - (k_1 + k_2) \end{aligned} \tag{11}$$

We had assumed $\phi(.)$ to be an increasing function in u. We conjecture that it is also increasing in k, which is the spending on advertising and personalisation by the platform. Thus profit maximising prices and expenditure equations from equation 11 are

$$\begin{aligned} p_1 &= f_1 - \alpha_2 n_2 + \frac{\phi_1(u_1, k_1)}{\partial \phi_1(u_1, k_1)/\partial u_1} \\ &= f_1 - \alpha_2 n_2 - \frac{1}{\partial \phi_1(u_1, k_1)/\partial k_1} \\ p_2 &= f_2 - \alpha_1 n_1 + \frac{\phi_2(u_2, k_2)}{\partial \phi_2(u_2, k_2)/\partial u_2} \\ &= f_2 - \alpha_1 n_1 - \frac{1}{\partial \phi_2(u_2, k_2)/\partial k_2} \end{aligned} \tag{12}$$

Since we do not know the functional form of ϕ here, we refrain from further solving the implicit system of equations 12, but we make some comments based on what we get.

The first thing that we see is that unlike the per agent cost f that directly appears in price equation, the cost incurred by platform is not born by the agents as k directly does not appear in the equations. We can interpret it as a cost that is injected to increase n. Further, we see that the value of ϕ function is equal to the negative of marginal values of variables of the function.

3.4 Pricing breakdown and matching policy

In the basic model, we include two more aspects that we saw was used in the competition model in Caillaud and Jullien [2003]. If p is considered as a fixed registration fee that every agent pays to participate on the platform, then we include in this model a transaction fee c (considered to be a constant in this model, so this relevant only for homogeneous product/service transactions). Further, we assume that group 1 agents bear $kc, 0 \leq k \leq 1$ fraction of transaction charge, and the other side bears $(1-k)c$ Additionally, we include a parameter λ, which represents the concept of matching policy. If there are n_2 agents, probability of matching with an agent for group 1 is given by $\lambda n_2 \leq 1$.
Model specifications are thus given by:

$$u_1 = \lambda\alpha_1 n_2 - p_1 - \lambda n_2 kc; \quad u_2 = \lambda\alpha_2 n_1 - p_2 - \lambda n_1(1-k)c$$

$$n_1 = \phi_1(u_1); \quad n_2 = \phi_2(u_2)$$

$$\begin{aligned} \pi &= n_1(p_1 - f_1) + n_2(p_2 - f_2) + \lambda n_1 n_2 c \\ &= \phi_1(u_1)[\lambda\alpha_1\phi_2(u_2) - u_1 - \lambda\phi_2(u_2)kc - f_1] \\ &+ \phi_2(u_2)[\lambda\alpha_2\phi_1(u_1) - u_2 - \lambda\phi_1(u_1)(1-k)c - f_2] + \lambda n_1 n_2 c \end{aligned} \tag{13}$$

We maximise Equation 13 with respect to u_1, u_2.

$$u_1 = \lambda(\alpha_1+\alpha_2)n_2 - \lambda n_2 c - f_1 - \frac{\phi_1(u_1)}{\phi_1'(u_1)}; \quad u_2 = \lambda(\alpha_1+\alpha_2)n_1 - \lambda n_1 c - f_2 - \frac{\phi_2(u_2)}{\phi_2'(u_2)}$$

Consequently, profit maximising platform prices are

$$p_1 = f_1 + \lambda n_2(1-k)c - \lambda\alpha_2 n_2 + \frac{\phi_1(u_1)}{\phi_1'(u_1)}$$

$$p_2 = f_2 + \lambda n_1 kc - \lambda\alpha_1 n_1 + \frac{\phi_2(u_2)}{\phi_2'(u_2)}$$

4 Platform Competition

4.1 Two platform single-homing, hotelling specification

We extend the platform to two platforms competing for market share, with single homing- each agent is only associated with one platform. Consider two platforms, A and B, and two groups of agents 1 and 2 which get utilities $\{u_1^i, u_2^i\}$ respectively.

$$u_1^i = \alpha_1 n_2^i - p_1^i; \quad u_2^i = \alpha_2 n_1^i - p_2^i \tag{14}$$

Agents in a group are assumed to be uniformly located along a unit platforms located at the two endpoints, and $t_1, t_2 > 0$ are the product differentiation parameters for the two groups that describe the competitiveness of the market. The Hotelling specification is given by Equation 15.

$$n_1^i = \frac{1}{2} + \frac{u_1^i - u_1^j}{2t_1}; \quad n_2^i = \frac{1}{2} + \frac{u_2^i - u_2^j}{2t_2} \tag{15}$$

Using Equation 14 and the identity that $n_k^i + n_k^j = 1 \; \forall k \in \{1, 2\}$ we get

$$n_1^i = \frac{1}{2} + \frac{\alpha_1(2n_2^i - 1) - (p_1^i - p_1^j)}{2t_1}$$

$$n_2^i = \frac{1}{2} + \frac{\alpha_2(2n_1^i - 1) - (p_1^i - p_1^j)}{2t_2}$$

We solve the above two equations to get

$$n_1^i = \frac{1}{2} + \frac{1}{2}\frac{\alpha_1(p_2^j - p_2^i) + t_2(p_1^j - p_1^i)}{t_1t_2 - \alpha_1\alpha_2}$$

$$n_2^i = \frac{1}{2} + \frac{1}{2}\frac{\alpha_2(p_1^j - p_1^i) + t_1(p_2^j - p_2^i)}{t_1t_2 - \alpha_1\alpha_2}$$

Platform i's profit is thus

$$(p_1^i - f_1)(\frac{1}{2} + \frac{1}{2}\frac{\alpha_1(p_2^j - p_2^i) + t_2(p_1^j - p_1^i)}{t_1t_2 - \alpha_1\alpha_2}) + (p_2^i - f_2)(\frac{1}{2} + \frac{1}{2}\frac{\alpha_2(p_1^j - p_1^i) + t_1(p_2^j - p_2^i)}{t_1t_2 - \alpha_1\alpha_2})$$

For the case of a symmetric equilibrium where each platform offers the same price pair $(p1, p2)$, the first-order conditions for equilibrium prices are

$$p_1 = f_1 + t_1 - \frac{\alpha_2}{t_2}(\alpha_1 + p_2 - f_2); \quad p_2 = f_2 + t_2 - \frac{\alpha_1}{t_1}(\alpha_2 + p_1 - f_1)$$

The symmetric equilibrium prices are given by Equation 16, which is obtained by solving the pair of linear equations above.

$$p_1 = f_1 + t_1 - \alpha_2; \quad p_2 = f_2 + t_2 - \alpha_1 \tag{16}$$

4.2 Single-homing with transaction fees and matching, hotelling spec.

We extend the model presented in Section 4.1 to account for transaction costs and efficiency of matching. We assume that a fixed transaction cost of c is levied by the platform on each transaction and group-1 agents pay kc, while group-2 agents pay $(1 - k)c$ of it, with $0 \le k \le 1$. We introduce a parameter λ which gives the probability of transaction between two agents in a matching pair. Hence, if the number of agents in group-2 are n_2, then the probability of transaction of a group-1 agent is λn_2. Let i and j denote the two platforms, which the agents can join. We assume that an agent joins either one of the platforms, and that the matching policies and the per transaction costs are same for both the platforms. The utilities of each group are given by Equation 17.

$$\begin{aligned} u_1^i &= \lambda\alpha_1 n_2^i - p_1^i - k\lambda n_2^i c; \quad u_2^i = \lambda\alpha_2 n_1^i - p_2^i - (1-k)\lambda n_1^i c \\ u_1^j &= \lambda\alpha_1 n_2^j - p_1^j - k\lambda n_2^j c; \quad u_2^j = \lambda\alpha_2 n_1^j - p_2^j - (1-k)\lambda n_1^j c \end{aligned} \tag{17}$$

We follow the Hotelling specification of number of agents on each side of the platform, which is given by Equation 15. We now get the number of agents which are present on each platform,

$$n_1^i = \frac{1}{2} + \frac{\lambda(2n_2^i - 1)(\alpha_1 - kc) - (p_1^i - p_1^j)}{2t_1}$$

$$n_2^i = \frac{1}{2} + \frac{\lambda(2n_1^i - 1)(\alpha_2 - (1-k)c) - (p_2^i - p_2^j)}{2t_2}$$

On solving the above equations and the similar counterpart for platform j, we get the number of agents on each side of each platform. This function is described in Equation 18.

$$\begin{aligned} n_1^i &= \frac{1}{2} + \frac{1}{2}\frac{\lambda(\alpha_1 - kc)(p_2^j - p_2^i) + t_2(p_1^j - p_1^i)}{t_1t_2 - \lambda^2(\alpha_1 - kc)(\alpha_2 - (1-k)c)} \\ n_2^i &= \frac{1}{2} + \frac{1}{2}\frac{\lambda(\alpha_2 - (1-k)c)(p_1^j - p_1^i) + t_1(p_2^j - p_2^i)}{t_1t_2 - \lambda^2(\alpha_1 - kc)(\alpha_2 - (1-k)c)} \end{aligned} \tag{18}$$

Platform i's profit $\pi(p_1^i, p_2^i, p_1^j, p_2^j)$ is given by Equation 19, where n_1^i and n_2^j are described in Equation 18. This profit is a quadratic in p_1^i and p_2^i.

$$(p_1^i - f_1)n_1^i + (p_2^i - f_2)n_2^i + \lambda n_1^i n_2^i c \tag{19}$$

We will study about symmetric equilibrium, where $p_1^i = p_1^j = p_1$ and $p_2^i = p_2^j = p_2$. The symmetric equilibrium prices satisfy the following first order conditions

$$p_1 = t_1 + f_1 - \frac{\lambda c}{2} - \frac{\lambda(\alpha_2 - (1-k)c)}{t_1}(\lambda(\alpha_1 - kc) + p_2 - f_2 + \frac{\lambda c}{2})$$

$$p_2 = t_2 + f_2 - \frac{\lambda c}{2} - \frac{\lambda(\alpha_1 - kc)}{t_1}(\lambda(\alpha_2 - (1-k)c) + p_1 - f_1 + \frac{\lambda c}{2})$$

On solving the above equations, we get a the symmetric equilibrium prices for the agents on the both sides of the platform. These are given by Equation 20.

$$\begin{aligned} p_1 &= f_1 + t_1 - \lambda(\alpha_2 - (1-k)c) - \frac{\lambda c}{2} \\ p_2 &= f_2 + t_2 - \lambda(\alpha_1 - kc) - \frac{\lambda c}{2} \end{aligned} \tag{20}$$

The prices and maths described in this equation can also be arrived directly by doing the substitutions mentioned in Equation 21 in Section 4.1. We use starred variables in the equations in Section 4.1 and do the following substitutions.

$$\begin{aligned} \alpha_1^* &\longleftarrow \lambda(\alpha_1 - kc) \\ \alpha_2^* &\longleftarrow \lambda(\alpha_2 - (1-k)c) \\ f_1^* &\longleftarrow f_1 - \frac{\lambda c}{2} \\ f_2^* &\longleftarrow f_2 - \frac{\lambda c}{2} \end{aligned} \tag{21}$$

4.3 Two platform generalised single-homing

We get rid of the hotelling specification assumption in this analysis. So we extend the model in Section 3.4 to analysis competition.

Consider the case when there are two platforms A and B, offering pricing (p_i^A, p_i^B). Consider an agent required to register with only one out of the two platforms- the agent registers with the one offering higher utility/ lower price. We know that network externality induces concentration, so by induction, every subsequent agent chooses the same platform as the first agent. This means that no agent joins one platform, say B. Then

$$\lambda^A\alpha_i^A n_j^A - p_i^A - \lambda^A n_j^A k_i^A c^A \geq \lambda^B\alpha_i^B n_j^B - p_i^B - \lambda^B n_j^B k_i^B c^B, \ \ i,j = 1,2, \ i \neq j, \ k_i + k_j = 1$$

Since we know that no agents join platform B,

$$\lambda^A \alpha_i^A n_j^A - p_i^A - \lambda^A n_j^A k_i^A c^A \geq -p_i^B, \;\; i,j = 1,2, \; i \neq j$$

This is sort of the result and analysis that has been done in Caillaud and Jullien [2003], and we get result similar to theirs, differing only in notation and parameters.
Now, as has been done in Caillaud and Jullien [2003], we evaluate the Divide and Conquer (referred hereafter as DC) policy that B must adopt to gain market share. B must subsidise pricing for one group so that

$$p_i^B < p_i^A - \lambda^A \alpha_i^A n_j^A + \lambda^A n_j^A k_i^A c^A \leq 0 \tag{22}$$

Further, group j users, who expect group i users to now register with B, loose the externality benefits, where $n_i^B = 1$ since all j-users expect all i-users to register with B now, given the subsidy.

$$p_j^B + \lambda^B (1 - k^B) c^B < \lambda^B c^B + inf\{p_j^A, 0\}$$

It is thus optimal for B to set the transaction fee at its maximum, $k = 0$.
How this model can now be transformed is that instead of offering a subsidy to all users such that $n_i^B = 1$, platform B can subsidize a subset of users to share market space with A. To deter new entrants in the market, platform A must have a pricing policy such that under no condition, Equation 22 is profitable for a platform, in this case, B.
Caillaud and Jullien [2003] proposed that with exclusive intermediation services, the only equilibria are dominant-firm equilibria, where one intermediary captures all users, charges the maximal transaction fee, subsidizes registration, and makes zero profit.

4.4 Two platform multi-homing

Consider that there are two platforms available in the market offering the exact same service but have different price structures. Examples like Amazon and Flipkart in e-commerce industry, or Tinder and Bumble in dating industry. An agent is said to be multi-homing when she is a member of more than one platforms. We model the case of multi-homing adapted from Armstrong [2006]. We consider the case when only one group of agents multi-home to get the benefit of interacting with all possible agents on the other side. If both the groups multi-home, dominant strategy suggests that agents from one group should leave one platform, because they will still be able to interact with all the agents on the other side. Thus in the model presented we assume that group-1 agents single-home while group-2 agents multi-home.
The basic idea of differential pricing is the weight that group-2 agents put on the benefit received on interaction with group-1 agents, which forces them to join both the platforms. This nature of group-2 agents is exploited by the platforms to make profits from these interactions. Let the two platforms be i and j and we have n_1^i and n_1^j group-1 agents on them respectively. Let p_1^i and p_2^i be the membership charges charged by platform i to group-1 and group-2 agents respectively. Now, the number of group-2 agents joining platform i is given by

$$n_2^i = \Phi^i(n_1^i, p_2^i)$$

Φ^i is a function which is increasing in n_1^i, as more group-1 agents should lead to more participation by group-2 and decreasing in p_2^i as higher prices lead to a decrease in participation. Also, as group-2 agents multi-home so the decision of joining one platform is independent of the other, hence $n_2^i + n_2^j \geq 1$. Utility of a group-1 agent joining platform i is given by

$$u_1^i = \alpha_1 n_2^i - p_1^i$$

We have assumed in the model that group-1 agents single-home. Thus, the number of agents joining one platform depends not only on the utility she derives from this platform, but on the utilities she derives from both the platforms. This is given by Equation 23.

$$n_1^i = \phi^i(u_1^i, u_1^j) \tag{23}$$

Hotelling specification described in Section 4.1 is an example of ϕ^i function. ϕ^i is increasing in u_1^i as more utility on platform i translates to more agents joining it, and decreasing in u_1^j as more utility in the other platform translates to less agents joining platform i. Let $f_1^i n_1^i + f_2^i n_2^i$ denote the total cost incurred by the platform to serve the two sides of the agents, where f_1^i and f_2^i are the per agent fixed cost. The profit for platform i is given by Equation 24.

$$\pi^i(n_1^i, n_2^i) = n_1^i p_1^i + n_2^i p_2^i + f_1^i n_1^i + f_2^i n_2^i \tag{24}$$

Let us assume, like in Section 3, that platform provides utility to the agents which single-home. Also, as utilities given to agents in group-1 are known ex ante $(\hat{u}_1^i, \hat{u}_1^j)$, then we also know the number of group-1 agents on each platform at equilibrium using Equation 23. Let $\hat{n}_1^i$ be the number of agents of group-1 on platform i at equilibrium. We rewrite the Equation 24 as only a function of p_2^i, using the other equations given in this Section. The equilibrium profit for platform i is given by Equation 25.

$$\pi^i(p_2^i) = \hat{n}_1^i(\alpha_1 \Phi^i(\hat{n}_1^i, p_2^i) - \hat{u}_1^i) + \Phi^i(\hat{n}_1^i, p_2^i)p_2^i + f_1^i \hat{n}_1^i + f_2^i \Phi^i(\hat{n}_1^i, p_2^i \tag{25}$$

Let $\hat{p}_2^i$ maximise Equation 25, then $\hat{p}_2^i$ is the membership price levied by the platform from group-2 agents. The equilibrium number of group-2 agents joining platform i can now be derived as $\hat{n}_2^i = \Phi^i(\hat{n}_1^i, \hat{p}_2^i)$. Also, the membership price charged by the platform, from group-1 will be given as $\hat{p}_1^i = \alpha_1 \hat{n}_2^i - \hat{u}_1^i$.

The above procedure provides a simple algorithm on how a platform should charge the single-homing and the multi-homing group of agents.

4.5 Multiple Platforms

This has not been evaluated in any paper so far, which we feel is a big caveat, given the prevalence of increased access to technology, making not only joining more platforms easy, but also simplifying platform inception, with several start-ups and companies investing in the service industry by the means of a platform. We intend to work on this, but feel that there will be a gap in theory and practice because of the given reasons:

1. Often, platforms cut across audiences and do not cater to exactly and exclusively the same commodity/service.

2. Platforms hold a finite capacity, and additionally, the capacity of agents is also finite. So an agent may find it optimal to avail service from a platform, but would have to switch if the platform is unable to accommodate. In this perspective, the question is not just of optimality or profit and welfare maximisation, but that of finite horizon decision making and matching problem.

5 Social Perspective of Intermediaries

It is not always possible for platforms to charge monopoly prices because governments across the world function to protect the customers from big multinational companies and their policies. Also, in a competitive market, laws exist from stopping platforms to indulge in the activity of predatory pricing, which can potentially lead to other competitors leaving

the market. Thus, in most cases, the prices levied by the platforms have an upper as well as a lower bound which are regulated by central authorities.
It has been shown by Rysman [2009] that in case of perfect competition, the network effects in two-sided markets lead to convergence of various platforms to a single platform. This idea is further strengthened by the model presented in Section 4.3, where we see that at equilibrium all the customers and sellers are attracted towards a single platform. These kinds of situations often lead to merger between two platforms. The competition commissions of the country then needs to look into the merger such that the final platform does not charge monopoly prices from the consumers.

6 Future Work

Some ideas that we would like to work on are listed below:

- Multi homing competition modelling, inclusion of industry specifications in the mathematical model, having a more concrete evaluation of the ϕ function and pricing breakdown, contrasting active and idle user behaviour and effect on platform design.
- Study on matching policy, its real world algorithms, how does it award bargaining power to agents, and how is lost business identified and penalised in the system.
- A financial projection model that bridges the gap between theory and practice with the inclusion of strategies as discussed in Section 3.1.
- Something like a dashboard that enables exploration of strategies, simulation, and/or financial projection. This would help us test our models, and bring out the utility of modelling itself.
- The question that we initially were intrigued by pertained to automation and subsequently e-commerce. In the end, we covered a broader topic, but we would also like to attempt modelling specific to e-commerce, and possibly eventually the role of automation between agents in some other context.

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