Supervision 2

1. Formulate a strategic game (in bimatrix form) which is the same as the joint project game (Example 2 in the lectures) except that each person prefers to work than to shirk when the other person works. Find the Nash equilibrium or equilibria.

2. Give an example of a 2-player game in which each player has 2 actions and the players have the same preferences.

3. Two people enter a bus. Two adjacent cramped seats are free. Each person must decide whether to sit or stand. Sitting alone is more comfortable than sitting next to the other person, which is more comfortable than standing.

(a) Suppose that each person only cares about her own comfort. Model the situation as a strategic game. Is it the same as the Prisoners' Dilemma? Find its Nash equilibrium or equilibria.

(b) Suppose that each person is altruistic and ranks outcomes according to the other person's comfort, but, out of politeness, prefers to stand if the other person stands. Model the situation as a strategic game. Find its Nash equilibrium or equilibria. How do the equilibrium levels of comfort compare in the two games?

4. (Hawk-Dove game). Two animals are fighting over some prey. Each can be passive or aggressive. Each prefers to be passive if the other is aggressive, and aggressive if the other is passive. Given its own stance, it prefers the outcome in which the other is passive to the outcome in which the other is aggressive. Formulate as a strategic game and find the Nash equilibria.

5. Find the best-response functions in the Prisoners' Dilemma, Battle of the Sexes, Matching Pennies and Stag Hunt (examples in the lectures). Use them to verify the Nash equilibria of each game.

6. Find the best-response functions, and hence the Nash equilibria, in the game:

	L	M	R
U	2,2	1,3	0,1
C	3,1	0,0	0,0
D	1,0	0,0	0,0

7. Two people have 10 pounds to divide between them. They use the following procedure. Each person names a number of pounds (a non-negative integer), at most equal to 10. If the sum of the amounts named is at most 10, then each person receives the amount of money she named (and the remainder is destroyed). If the sum of the amounts that the people name exceeds 10 and the amounts named are different, then the person who named the smaller amount receives that amount and the other person receives the remaining money. If the sum of the amounts that the people name exceeds 10 and the amounts named are the same, then each receives 5 pounds. Determine the best response functions, plot them in a diagram (use noughts to mark one best response function and crosses to mark the other) and thus find the Nash equilibria.

8. Find the Nash equilibria of the two-player game (sketch the best-response functions) in which each player's set of actions is the set of non-negative numbers and the players' payoff functions are

$$u_1(a_1, a_2) = a_1(a_2 - a_1)$$

and

$$u_2(a_1, a_2) = a_2(1 - a_1 - a_2).$$

9. Two people are engaged in a joint project. If each person i puts in the effort x_i, a non-negative number equal to at most 1, which costs her x_i^2, the outcome of the project is worth $3x_1x_2$. The worth of the project is split equally between the two people, regardless of their effort levels. Find the Nash equilibria of the game. Is there a pair of effort levels that yields higher payoffs for both players than do the Nash equilibrium effort levels?

10. For the following game, determine, for each player, whether any action is strictly dominated or weakly dominated. Find the Nash equilibria of the game; determine whether any equilibrium is strict.

	L	M	R
U	0,0	1,0	1,1
C	1,1	1,1	3,0
D	1,1	2,1	2,2

11. In the following game (i) solve by iterated deletion of strictly dominated strategies; (ii) find the Nash equilibria.

	L	M	R
U	1,0	7,1	4,4
C	-1,6	0,2	1,8
D	2,3	4,0	5,2

12. Firm 1 has profit function

$$a_1 x^{\frac{1}{2}} - c_1 x$$

and firm 2 has profit function

$$a_2 y^{\frac{1}{2}} - c_2 y - d_2 x,$$

where x is 1's output level, y is 2's output level, and a_1, a_2, c_1, c_2 and d_2 are positive constants. What might account for the form of these functions? Find the best response functions (sketch them) and the Nash equilibrium or equilibria. If the firms merged and then maximized the sum of profits, what would x and y be? Find a tax which would bring about efficient production levels if they do not merge.

13. There are 2 routes for driving from A to B. One is a motorway, and the other consists of local roads. The benefit of using the motorway is constant and equal to 1.8, irrespective of the number of people using it. Local roads get congested when too many people use them, but if too few people use them, the few isolated drivers run the risk of becoming victims of crime. Suppose that when a fraction x of the population uses the local roads, the benefit of this mode to each driver is given by

$$1 + 9x - 10x^2.$$

(a) Draw a graph showing the benefits of the two driving routes as functions of x, regarding x as a continuous variable that can range from 0 to 1.

(b) Identify all possible equilibrium traffic patterns from your graph. Which equilibria are stable and which unstable? Explain.

(c) What value of x maximizes the total benefit to the whole population? How might this value be achieved? Do you think that

Optional extra question: Exercise 34.3 in Osborne (p.34). For the second part, you can assume that in any Nash equilibrium exactly one car takes route AYB.