Supervision 2 (end of week 4 or later)

Production, Welfare, Trade

1. Robinson Crusoe lives alone on an island. He has T hours that he can either devote to work, producing coconuts, or consume as leisure. His utility function is given by $u(l, c) = \alpha[ln(l)] + c$, where l is number of hours of leisure, c is amount of coconuts consumed, $\alpha > 0$ and ln refers to the natural logarithm. c and l are both continuous variables. The production function for coconuts is $y = ln(L + 1)$, where L is the amount of hours he works and y is quantity of coconuts. There is one firm, which Robinson owns. There is a market price for coconuts and a market wage. Robinson takes these as given and separately decides (i) as owner of the firm, how much labour to employ and how many coconuts to produce and sell; and (ii) as a consumer, how much labour to supply and how many coconuts to buy. We assume that the price and the wage have adjusted so that both markets clear, i.e., labour demand by the firm equals labour supplied by the consumer and supply of coconuts by the firm equals demand for coconuts by the consumer. Set the price of coconuts equal to 1 and denote the wage rate by w.

(a) Sketch the production function. If the firm sells an amount y of coconuts and employs L hours of labour, what is its profit? Sketch some iso-profit lines. Derive the firm's profit-maximizing labour demand function $L^d(w)$ and output supply function $y(w)$. Note: your answer will depend on whether $w \leq 1$ or $w > 1$; provide a diagrammatic illustration for each case.

(b) Derive the firm's profit function $\pi(w)$.

(c) Write down Robinson's budget constraint as a consumer, giving the combinations of l and c that he can afford. Derive his coconut demand function $c(w)$ and labour supply function $L^s(w)$.

(d) Assuming $w \leq 1$, write down the market clearing condition for the labour market and hence find the market-clearing wage w^e. Show that this equilibrium exists if $\alpha \leq T$. Show that at this wage the coconut market also clears. What is the allocation in this equilibrium?

(f) If a central planner interested in achieving efficiency were to organize Robinson's activities, how many coconuts would he ask Robinson to produce and how much leisure to consume? Explain your answer. Show the planner's solution in a diagram.

2. (a) State the Second Welfare Theorem.

(b) Show, by means of a diagram, why it is true for an exchange economy.

(c) Suppose, in question 1, Robinson Crusoe's production function is zero if $L < \underline{L}$ and

$$y = A(L - \underline{L})^{0.5}$$

if $L \geq \underline{L}$. Suppose his utility function is $u(c, l) = c^{0.5} l^{0.5}$. (i) Find his optimal labour input L^* and coconut output c^*. (ii) Provide a suitable sketch. (iii) Let $\alpha = y(L^*) - L^* y'(L^*)$ be the intercept of the tangent to the production function at (L^*, c^*). Show that $\alpha < 0$ if $\underline{L} > T/4$. (iv) For what parameters does the conclusion of the Second Welfare Theorem apply?

(d) Discuss conditions, both for exchange and for production economies, under which the conclusions of the Second Welfare Theorem do not apply. How relevant do you think the theorem is in practice?

Reading

Varian, Chaps. 28, 29.