Supervision 3

1. Each of 4 people chooses whether to contribute a fixed amount toward the provision of a public good. The good is provided if and only if at least 2 people contribute; if it is not provided, contributions are not refunded. Each person ranks outcomes from best to worst as follows: (i) any outcome in which the good is provided and she does not contribute, (ii) any outcome in which the good is provided and she does contribute, (iii) any outcome in which the good is not provided and she does not contribute, (iv) any outcome in which the good is not provided and she contributes. Formulate this situation as a strategic game and find its Nash equilibria.

2. A park is to be built in a town with a population of 100. Voluntary contributions are being solicited to cover the cost. Each citizen is asked to give 100 pounds. The more people contribute, the larger will be the park and the greater the benefit to each citizen. It is not possible to keep out the non-contributors; they get their share of the benefit anyway. Suppose that when there are n contributors, where n can be any whole number between 0 and 100, the benefit to each citizen in monetary units is n^2 pounds.

(a) Suppose that initially no one is contributing. You are the mayor. You would like everyone to contribute, and can use persuasion on some people. What is the number you need to persuade before everyone else will join in voluntarily?

(b) Find the Nash equilibria of the game in which each citizen decides independently whether or not to contribute 100 pounds.

3. Three firms compete in a market in which the inverse demand function is $p(q) = a - q$ for $q \leq a$ and $p(q) = 0$ otherwise. Each has unit cost $c < a$ and zero fixed cost. They each simultaneously choose a quantity. Find the Nash equilibrium quantities, price and profits. What would happen if two of the firms merged and there were two firms? Is there an incentive for two firms to do this?

4. Find all the Nash equilibria if costs and market demand are as in question 3, but the three firms play a Bertrand game rather than a Cournot game.

5. Find the Nash equilibrium of the Cournot game when there are two firms, the inverse demand function is $p(q) = a - q$ unless $q > a$, in which case $p(q) = 0$, and the cost function of each firm is $C(q_i) = c_i q_i$, where $c_1 > c_2$ and $c_1 < a$. Which firm produces more output in equilibrium? What is the effect of technical change that lowers firm 2's cost parameter c_2 (while not affecting c_1) on the firms' equilibrium outputs, the total output, and the price?

6. Find the Nash equilibrium or equilibria of a variant of the Cournot duopoly game (with linear demand, constant unit cost) which differs from the standard one only in that one firm, instead of maximizing profits, wants to maximize its market share subject to not making a loss.

7. For the following game, assuming that the players may use mixed strategies, draw the best response functions and hence find all the mixed strategy Nash equilibria.

	H	T
H	2,-1	-1,1
T	-1,1	3,-1

8. Players 1 and 2 each choose a positive integer up to K. If they choose the same number then player 2 pays 1 pound to player 1; otherwise no payment is made. Each player's preferences are represented by her expected monetary payoff.

Show that the game has a mixed strategy Nash equilibrium in which each player chooses each positive integer up to K with probability $1/K$.

9. Suppose that there are 3 possible political positions, A, B and C, and 3 citizens, one of whom prefers A to B to C, one of whom prefers B to C to

A, and one of whom prefers C to A to B. Two candidates simultaneously choose a position. If the candidates choose different positions, each citizen votes for the candidate whose position he prefers; if the candidates choose the same position they tie for first place. Each candidate prefers to win the election outright than to tie and prefers to tie than to lose.

(a) Formulate as a strategic game and find a mixed strategy equilibrium.

(b) Find all the mixed strategy equilibria in the modified game in which candidate 1 cannot choose position C.