Risk Aversion

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1. Risk Aversion

1.1. **Absolute Risk Aversion.** For a $u(c)$ utility function that depends on consumption c, the Arrow-Pratt measure of absolute risk-aversion (ARA) is given by:

$$r(w) = \left[\frac{u''(c)}{u'(c)}\right] \tag{1}$$

Decreasing Absolute Risk Aversion (DARA): e.g.,

$$u(w) = \ln c; \quad r_u = \frac{1}{c} \tag{2}$$

monotonically decreasing in consumption.
Constant Absolute Risk Aversion (CARA): e.g.,

$$u(c) = -e^{-\eta c} \tag{3}$$

constant in consumption.
Increasing Absolute Risk Aversion (IARA): e.g.,

$$u = c - \eta c^2; \quad r_u = \frac{2\eta}{1 - 2\eta c} \tag{4}$$

monotonic increasing in consumption. The marginal utility function is positive only over the range $(0, \frac{c}{2\eta})$.

1.2. **Relative Risk Aversion.** The Arrow-Pratt measure of relative risk-aversion (RRA) or coefficient of relative risk aversion is defined as:

$$R_u(c) = cr_u(c) = -\left[\frac{cu''(c)}{u'(c)}\right] \tag{5}$$

This measure has the advantage that it is still a valid measure of risk aversion, even if it changes from risk-averse to risk-loving, i.e. is not strictly convex /concave over all c.
A constant RRA implies a decreasing ARA, but the reverse is not always true.

e.g., the expected utility function $u(c) = \ln(c)$ does imply RRA = 1.

In intertemporal choice problems, it is not possible to disentangle the the elasticity of intertemporal substitution from the coefficient of relative risk aversion. The following isoelastic

utility function exhibits constant relative risk aversion with $Ru(c) = \rho$ and the elasticity of intertemporal substitution $\epsilon = \frac{1}{\rho}$.

$$u(c) = \frac{c^{1-\rho}}{1-\rho} \tag{6}$$

As $\rho \to 1$, this simplifies to the case of log utility, and the income effect and substitution effect on saving exactly offset each other.

$$\lim_{\rho \to 1} u(c) = \ln(c) \tag{7}$$

2. Understanding the CARA preferences

Constant Absolute Risk Aversion (CARA) preferences:

$$u(w, a) = -e^{-\eta[w - \psi(a)]} \tag{8}$$

w wages
a costly action or effort
$\psi(a)$ cost of action or effort, assume convex in effort, e.g. $\psi(a) = \frac{1}{2}a^2$
η coefficient of absolute risk aversion *and*
c is given by $c = [w - \psi(a)]$, income net of effort cost.

The above curve can be broken down into components:

$$u(w, a) = -\left[\frac{e^{\psi(a)}}{e^{\eta w}}\right] \tag{9}$$

By putting $\eta w = x$ and $e^{\psi(a)} = K(a)$, $K(a)$ is decreasing in a, we can write this the preferences as

$$u(w, a) = -\left[\frac{K(a)}{e^{x}}\right] \tag{10}$$

It is an increasing, concave function in x or in turn in w. Its shape is determined by η, i.e., as η increases, the function becomes more and more concave. $K(a)$ should be looked at just a constant, which reduced u as a increases.