1. Define the *chain rule*. Using the chain rule, differentiate the following expressions with respect to x.
   (a) $f\Big(g\big(h(x)\big)\Big)$
   (b) $\Big(\ln(x^2)+2\Big)^2$
2. Define the *product rule*. Using the product rule, differentiate the following expressions with respect to x.
   (a) $(x^4-3x^2)(5x+1)$
   (b) $\dfrac{1+2x}{1-2x}$
   (c) $\ln(x)\cdot e^{2x}$
3. Graph the following function for $a>0$ and $a<0$.
$$f(x)=\frac{a}{x}$$
   (a) What the does a represent on the graph?
   (b) Find the point(s) where the function has a maxima or a minima.
   (c) Obtain $x\dfrac{f'(x)}{f(x)}$ try to interpret the result.
4. Graph the following function:
$$f(x)=ax^2+bx+c$$
   (a) Does the shape of the function depend on the value of a?
   (b) Find the point(s) where the function has a maxima or a minima.
   (c) Find the conditions under which $f(x)$ expression has
      i. one real root
      ii. two real roots
      iii. no real roots

5.

$$f(x) = 2x^3 - 3x^2 - 12x + 12 \qquad\qquad \text{where } -3 \leqslant x \leqslant 3.$$

(a) Find the *extrema*[1] of the function.

(b) Would the answer to 5a change if the range was $-\infty \leqslant x \leqslant \infty$.

(c) Does the function have a *point of inflection*.

`(Question 1, Tripos 2003)`

6. (a) Find the derivative $\frac{dy}{dx}$ of the following function:

$$y = x^{24} \ln\left(\frac{e^{2x}}{x^2 + e^x}\right)$$

(b) Find the partial derivatives, $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, of the following function:

$$z = f(x, y) = (x^{\frac{1}{3}} + y^{\frac{1}{3}})^{\frac{1}{2}}$$

`(Question 6, Tripos 2007)`

7. Show that the function

$$f(h) = \frac{1 - \lambda^h}{h}$$

is downward sloping for $h > 0$ where λ is a positive constant with $0 < \lambda < 1$.

`Hint:` You can use the fact that $\lambda^h = e^{h \ln(\lambda)}$ and $\ln(x) \leqslant x - 1$.

`(Question 1, Tripos 2008)`

Readings

Bradley, T., and P. Patton (2002). *Essential Mathematics for Economics and Business*. Chichester, West Sussex, England: Wiley.

Pemberton, M., and N. Rau (2007). *Mathematics For Economists: An Introductory Textbook.* Manchester University Press.

Chiang, A. C. (1984) *Fundamental Methods of Mathematical Economics*. 3rd edition. McGraw-Hill Publishing Co.

[1] the points of maxima and minima are collectively known as the extrema