Supervision 4

For each question, if asked to describe a subgame-perfect equilibrium, or Nash equilibrium, you should describe it *in full*. You need not describe any mixed strategy equilibria.

1. A committee with three members, $\{1, 2, 3\}$, has to choose a new member of a club from a set of four candidates, $\{a, b, c, d\}$. Each member of the committee has veto power which is used in a successive way, starting with member 1, and finishing with member 3. Each member of the committee has to veto one and only one of the candidates that have not been vetoed yet. (a) Draw the extensive form of the game, writing in the terminal nodes the name of the elected candidate. (b) How many strategies does each player have? Do not try to write them (player 3 should have a lot).

2. (Bargaining over two indivisible objects). Two people are to allocate 2 identical desirable objects between them. One of them, A, makes a proposal to B as to how to allocate them. Only three proposals are allowed: both go to A, both go to B or each gets one object. B can either accept or refuse the proposal. If B refuses, neither receives either object; if she accepts, they get the proposed allocation. (i) Represent this as an extensive form game. (ii) Represent it as a strategic-form game. (iii) Find all the Nash equilibria of the game in pure strategies. (iv) Find all the subgame-perfect equilibria in pure strategies. Is there any outcome that is generated by a Nash equilibrium but not by any subgame-perfect equilibrium?

3. Consider the variant of Bertrand's duopoly game in which first firm 1 chooses a price, then firm 2 chooses a price. Assume that each firm's unit cost is constant and equal to c and that the monopoly profit is positive.

(i) Specify an extensive game with perfect information that models this situation.

(ii) Give an example of a strategy of firm 1 and an example of a strategy of firm 2.

(iii) Find the subgame perfect equilibria of the game. (You can assume for this purpose that for any price p it makes sense to speak of the "highest price below p").

4. Consider a Stackelberg game in which 3 firms move sequentially. Inverse demand is $p(q) = 1 - q$ and costs are zero. Find the subgame-perfect equilibrium. What happens if n firms move sequentially?

5. Consider a game in which first firm 1 chooses an output q_1, then firm 2 observes 1's output and chooses its output q_2, and finally firm 1, having observed q_2, is able, if it wants, to change its mind and choose a different output $q_1' \neq q_1$. What happens in subgame-perfect equilibrium?

6. Two players (1 and 2) play the following game. First player 1 chooses either l or r. Player 2 observes 1's move. If 1 chooses l they play the simultaneous-move game A below. If 1 chooses r they play the simultaneous-move game B.

Game A

	L	R
U	0,0	1,3
D	3,1	0,0

Game B

	L	R
U	1,1	1,0
D	2,0	-1,-1

(i) Draw the extensive form.

(ii) List the pure strategies available to each player in this game and describe the normal (strategic) form. [Hint: you should have found 8 strategies for player 1].

(iii) Find all the Nash equilibria in pure strategies.

(iv) Which of the equilibria in (iii) involve weakly dominated strategies?

(v) Find all the subgame-perfect equilibria in pure strategies.

7. The following game is repeated 1000 times and the payoffs are discounted using discount factor δ where $0 < \delta < 1$.

	L	R
U	2,2	5,1
D	1,5	4,4

(i) Can any outcome which Pareto-dominates the Nash equilibrium of the stage game be supported in a subgame-perfect equilibrium through a Nash-reversion strategy?

(ii) Suppose now that it is repeated infinitely many times. The players would like to play (D, R) in every period. Under what conditions can this outcome be supported in a subgame-perfect equilibrium through a Nash-reversion strategy?

8. Three firms play a Bertrand game repeatedly, infinitely many times. Each has constant marginal cost c. Market demand is $a - p$. They have a common discount factor δ.

(i) Find the monopoly price.

(ii) The firms want to charge the monopoly price each period and share the profits, using a Nash reversion strategy. For what discount factors can this be done in a subgame-perfect equilibrium? Explain why it does not work if the discount factor is outside the range that you have identified.

(iii) If there are n firms, is there a larger or smaller set of discount factors for which it can be done?