1. A farmer's available owns $\bar{l}$ acres of land. She has the option of growing crop 1 (*soya*) or crop 2 (*wheat*) on the land. For a crop i, output is denoted by y_i and land allocated to the crop is denoted by l_i. The net profit[1] per unit of output for crop i is r_i. The production function for the crops are given by

$$y_i = l_i^{a_i} \qquad a_i \in (0, 1),\ i = 1, 2.$$

   What is the optimal land allocation?

2. A firm produces output y and pays £10 per unit for input x_1 and £8 for input x_2 used. The production function is given by

$$y = \left(0.4x_1^{-2} + 0.6x_2^{-2}\right)^{-1/2}$$

   What combination of inputs x_1 and x_2 should the firm use if it wants to produce one unit of output y at minimum cost?

3. Solve the following problem:

$$\begin{aligned} \text{Minimise} \quad & (rK + wL) \\ \text{subject to} \quad & F(K, L) = \bar{q}. \end{aligned}$$

   for a production function of the form

$$F(K, L) = \left(aK^{-2} + bL^{-2}\right)^{-1/2}$$

   where r is the price of capital K, w is the price of labour L and $\bar{q}$ is a constant.

4. A consumer's utility function is given by

$$U(x_1, x_2) = b_1 \ln(x_1 - c_1) + b_2 \ln(x_2 - c_2)$$

   and her income m is such that

$$p_1 c_1 + p_2 c_2 < m.$$

   where p_1 and p_2 are the prices for goods x_1 and x_2 respectively and b_1, b_2, c_1 and c_2 are positive constants.

   (a) Interpret the constants the c_1 and c_2.

   (b) Draw the indifference curve for the consumer

[1] price minus variable costs

(c) Obtain the demand function for x_1 and x_2.

5. Obtain the demand function for goods x_1 and x_2 by solving the following problem:

$$\begin{array}{rll} \text{Maximise} & u = x_1^{\alpha} x_2^{1-\alpha} & 0 < \alpha < 1 \\ \text{subject to} & \bar{m} = p_1 x_1 + p_2 x_2 & \end{array}$$

where $\bar{m}$ is a constant denoting consumer's income and p_1 and p_2 are the price of goods x_1 and x_2.

6. Obtain the demand function for goods x_1 and x_2 by solving the following problem:

$$\begin{array}{rl} \text{Minimise} & e = p_1 x_1 + p_2 x_2 \\ \text{subject to} & \bar{u} = x_1^{\alpha} x_2^{1-\alpha} \end{array}$$

where e is the expenditure and $\bar{u}$ is the constant required level of subsistence utility and p_1 and p_2 are the price of goods x_1 and x_2.

Readings

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