Supervision 1

1. 'In a Pareto-efficient allocation, a given agent's utility must be higher than in a Pareto-inefficient allocation'. Comment.

2. Two consumers, Alan and Betty, have the same utility functions, $u(x, y) = x^{\frac{1}{4}} y^{\frac{1}{4}}$. Alan has an endowment of $(1, 15)$ (i.e. 1 unit of x and 15 of y) and Betty has an endowment of $(15, 1)$. Sketch the set of Pareto-improving trades available to them. Show that if they agree to trade 7 units of x for 7 units of y they will reach an efficient outcome.

3. In the setting of the previous question, find the net demand functions (i.e. as functions of the two prices) of the two agents. Verify that Walras' Law holds. Find the competitive equilibrium prices and quantities.

4. Agent A has 1 unit of corn and none of wool. B has no corn and 1 unit of wool. For A the two goods are perfect substitutes, i.e.

$$u_A(x_c^A, x_w^A) = x_c^A + x_w^A,$$

where x_c^A and x_w^A are A's consumption of corn and wool respectively, whereas B's utility function is Cobb-Douglas:

$$u_B(x_c^B, x_w^B) = (x_c^B)^{\frac{1}{2}} (x_w^B)^{\frac{1}{2}}.$$

(a) Draw the Edgeworth box.

(b) Find an expression for the MRS of each agent and hence find the equation of the contract curve.

(c) Deduce, from the diagram, the equilibrium price ratio and equilibrium allocation (final consumptions).

(d) Suppose that A's utility function were $u_A(x_c^A, x_w^A) = 2x_c^A + x_w^A$. What would the equilibrium prices be? From the Edgeworth box, how does the final allocation of each good differ (qualitatively) from the one in (c)? Give an economic explanation of the change from (c).

5. [2006] 'The fundamental theorems of welfare economics demonstrate that equity is compatible with efficiency'. Discuss.

Reading: Varian, Chap 31.