

Why do exchange rates overshoot? An exposition of the DD-AA Model

Dr. Kumar Aniket
9 March 2014

UNCOVERED INTEREST PARITY

The overshooting exchange rate model essentially just adds the uncovered interest parity relationship to the old standard IS-LM AD-AS framework.

- We assume that people form rational expectations about the future. Thus, $E(e)$ represent people's rational expectations about the exchange rate in the future. It also means that people cannot make systemic mistakes in forecasting the exchange rate and in the long run their expectations are realised. This means that $E_{t-1}[e_t] = e_t$ and $E[\hat{e}] = \frac{E_t[e_{t+1}] - e_t}{e_t} = 0$ in long run. In this, e_t is the exchange rate at time t and $E_t[e_{t+1}]$ is the expectation of e_{t+1} taken at one period earlier at t . In what follows, we drop the subscripts for ease of exposition. e represents the current exchange rate, $E[e]$ represent today's expectations of exchange rate one period from now and $E[\hat{e}]$ represents today's expectations of the rate at which exchange rate will change in one period.

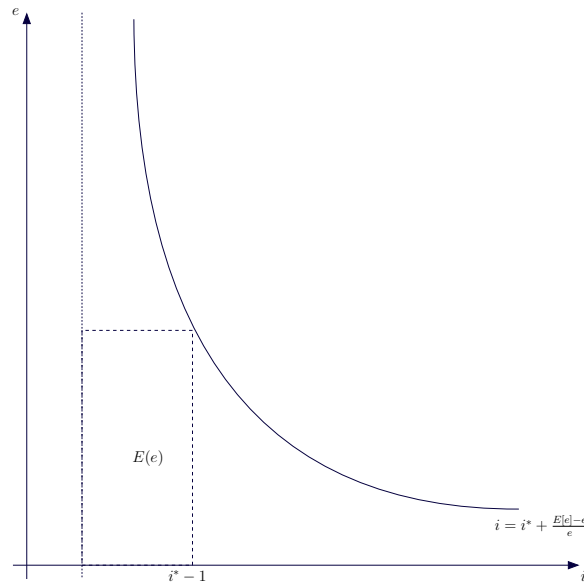


FIGURE 1. Uncovered Interest Rate Parity

- *Uncovered Interest Parity* essentially states that the returns from investing in the domestic bonds $(1 + i)$ are the same as the returns from investing in foreign bonds $(1 + i^*) \frac{E[e]}{e}$ where

investing in foreign bonds entails taking a risk on currency fluctuations which are not covered (insured) against by the investor.

$$(1 + i) = (1 + i^*) \frac{E_t[e_{t+1}]}{e_t} \quad (\text{UIP})$$

Using approximations, we can re-write this relationship down as follows.

$$\begin{aligned} i &= i^* + \frac{E_t[e_{t+1}] - e_t}{e_t} \\ &= i^* + E[\hat{e}_t] \end{aligned}$$

UIP can be written down as follows if we drop the time subscripts.

$$e [i - (i^* - 1)] = E[e]$$

DD-AA Model.

- The DD-AA model is essentially an open economy IS-LM or AD-AS framework that incorporates exchange rate and current accounts.

Like any other model, the equilibrium of the economy is given by the aggregate demand and aggregate supply of the economy. In the DD-AA model, the aggregate the supply is very similar to standard model. The aggregate demand of the economy is obtained in a slightly different way. We basically add exchange rate and current accounts to the IS-LM framework.

The model essentially has four endogenous variables, Y, i, P and e . To obtain the aggregate demand relationship between p and Y , we need three independent equations in Y, i, P and e . We set out the three equations that give us the aggregate demand relationship below. The aggregate demand and supply relationships thus give us the equilibrium and allow us to solve the model to obtain the equilibrium values of the endogenous variables.

- Solving the following three equations gives us the aggregate demand relationship between p and Y .
 - *Foreign Exchange Market* or the *uncovered interest rate parity condition*.

$$i = i^* + \frac{E[e] - e}{e} \quad (\text{UIP})$$

- *Money Market equilibrium*.

$$\frac{M}{P} = L \left(\bar{i}, Y^+ \right) \quad (\text{LM})$$

- *Goods Market equilibrium* where CA , the current account (or net exports) is increasing in $\varepsilon = \frac{eP^*}{P}$, the real exchange rate.

$$Y = C(Y - \bar{T}) + \bar{I} + \bar{G} + CA \left(\frac{+}{\varepsilon}, Y - \bar{T} \right) \quad (\text{IS/DD})$$

- Aggregate supply curve:

$$Y^s = \bar{Y} + \alpha(p - p^e) \quad (\text{AS})$$

\bar{Y} is the productive capacity of the economy and $p^e = E(p)$ is people's rational expectations of prices. The aggregate supply curves implies that output deviates from \bar{Y} , the potential capacity of the economy, if prices deviate from the expectations. For our purposes, only two supply curves matter, the short-run supply curve,

$$P = P^e \quad (\text{SRAS})$$

when $\alpha \rightarrow \infty$ and the long-run supply curve

$$Y = \bar{Y} \quad (\text{LRAS})$$

when $\alpha = 0$

We can obtain the *aggregate demand curve* of the economy from the two equations below that have been obtained by rearranging the equations above.

- Goods Market Equilibrium:

$$Y = C(Y - \bar{T}) + \bar{I} + \bar{G} + CA \left(\left[\frac{eP^*}{P} \right], Y - \bar{T} \right) \quad (\text{IS/DD})$$

- Money market and foreign exchange market equilibrium:

$$\frac{\bar{M}}{P} = L \left(\left[i^* + \frac{E[e] - e}{e} \right], Y \right) \quad (\text{LM/AA})$$

Long Run in the DD-AA Model. Lets look at what will happen in the long run:

- The aggregate supply in the long run is given by:

$$Y = \bar{Y} \quad (\text{LRAS})$$

- In the long run, the goods market equilibrium just gives us the purchasing power parity. Notice that in the long run, when $Y = \bar{Y}$, the goods market equilibrium is just left with one variable ε , the real exchange rate.

$$\bar{Y} = C(\bar{Y} - \bar{T}) + \bar{I} + \bar{G} + CA \left(\left[\frac{eP^*}{P} \right], \bar{Y} - \bar{T} \right) \quad (\text{LR-IS/DD})$$

The goods market equilibrium condition solves and gives us the following where $\bar{\kappa}$ is a constant determined by the values of $\bar{Y}, \bar{T}, \bar{I}$ and \bar{G} .

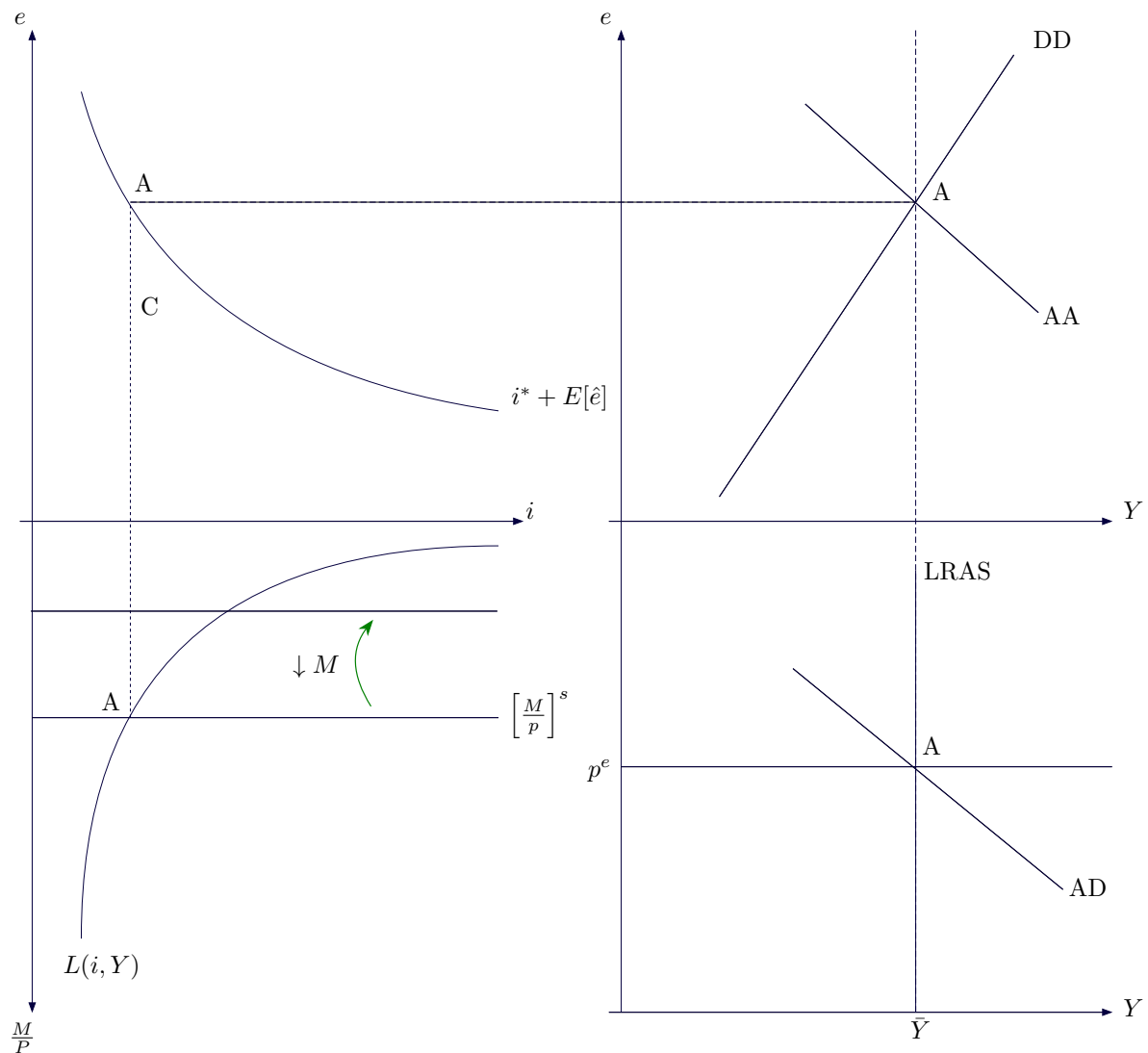
$$\frac{eP^*}{P} = \bar{\kappa}$$

We can make a simplifying assumption that $\bar{\kappa} = 1$, which implies purchasing power parity. *Purchasing power parity* implies

$$e = \frac{p}{p^*}, \quad (\text{PPP})$$

that is, a good cost the same at home or abroad. For example it would imply that a big mac will cost p at home and ep^* abroad and both prices are equal. Of course, purchasing power

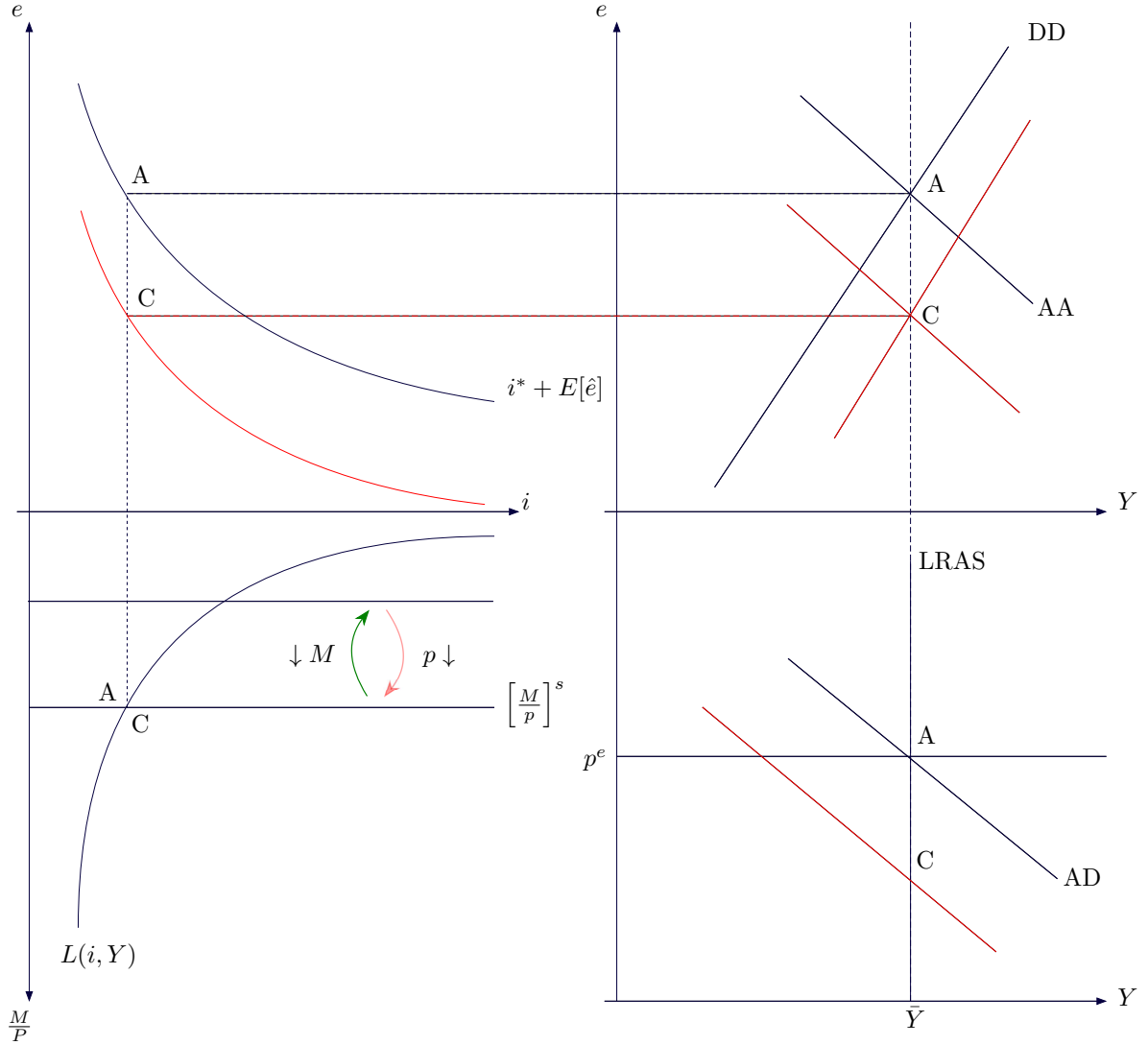
FIGURE 2. DD-AA Model Equilibrium: The economy is in a long run equilibrium at A when the money supply falls due to a monetary contraction.



parity is more likely to hold in tradable goods than non-tradable goods, so it is more likely to hold for small countries that trade a lot.

Purchasing power parity, if it holds in the long run also implies that rate the changes in exchange rate is determined by the way in which the domestic prices and and foreign prices are moving. Put another way the exchange rate changes are determined by the inflation differentials between two countries in the long run. The (PPP) equation above can be

FIGURE 3. Anticipated Monetary contraction in the DD-AA Model: The central bank announces that it will reduce money supply some time in the future. People anticipate that the reduced money supply would lead to lower prices and adjust their price expectations accordingly. The exchange falls from A to C immediately and there is no change in exchange rate when the monetary contraction actually takes places.



written down as $\frac{1}{e} \frac{de}{dt} = \frac{1}{p} \frac{dp}{dt} - \frac{1}{p^*} \frac{dp^*}{dt} = \pi - \pi^*$ if we take logs and differentiate with respect to t .

- We also know that in the long-run, people's expectations should be realised. If the expectations have not been realised, it would naturally give rise to short-run dynamics. Thus, in

the long-run the exchange rate should be at the level people expected it to be at.

$$e = E(e)$$

This would naturally imply that $E[\hat{e}] = \frac{E[e]-e}{e} = 0$.

- The money market equilibrium can be written in the following way if we substitute $E[\hat{e}] = 0$ and $Y = \bar{Y}$.

$$p = \frac{M}{L(i^*, \bar{Y})} \quad (\text{LR-LM/AA})$$

- Substituting the long run money market condition (LR-IS/LM) in the goods market equilibrium (PPP) gives us the following:

$$e = \frac{p}{p^*} = \frac{\frac{M}{L(i^*, \bar{Y})}}{p^*} \quad (\text{LR-LM/AA})$$

This implies that in the long-run e, p and M change at the same rate, that is

$$\frac{1}{e} \frac{de}{dt} = \frac{1}{p} \frac{dp}{dt} = \frac{1}{M} \frac{dM}{dt} \quad (1)$$

A change in money supply would lead to a proportional change in the prices and exchange rate in the long-run. We can see this if we compare Figure 2 and Figure 3. Figure 2 represent the economy at A in an long run equilibrium. The Central Bank announces that it will reduce the money supply in a particular point in the future. The exchange rate expectations are immediately formed and the UIP curve shifts down in the $e - i$ space. Prices expectations are slower to adjust but given that the monetary policy is anticipated, price expectations decrease proportionally exactly at the point when the nominal money supply decreases (1). That is the percentage change in prices is the exactly the same as the percentage change in the nominal money supply and as a result the real money supply does not change. As we can see in Figure 3 and know from (1), the percentage change in the exchange rate is exactly the same as the percentage change in the nominal money supply and price.

Short Run DD-AA Model.

- As we saw above, in the long run, a change in M would lead p and e to change by exactly the same proportion. The prices tend to be sticky in the short run where as exchange rate tends to be extremely flexible or even to some extent hyper-flexible in the short run. Thus, in the short run, while prices are sticky, the exchange rate absorbs any exogenous an shock to the economy and overshoots beyond its new long run value as a consequence. As prices adjust to the shock and converge to their new long run value, the exchange rate returns to its long run value. Let's set out the model for short-run analysis.
- We obtain the demand curve of the economy by solving the following equations and obtaining a relationship between p and Y .

- *Money Market equilibrium* that incorporates the foreign exchange market. The resulting (AA) condition is downward sloping in the $e - Y$ space.

$$\frac{M}{P} = L(i^* + E[\hat{e}], Y) \quad (\text{LM/AA})$$

Note that (AA) shifts down in the $e - Y$ space if the $E[e]$ decreases.

- Goods market equilibrium. The resulting (DD) condition is upward sloping in $e - Y$ space.

$$Y = C(Y - \bar{T}) + \bar{I} + \bar{G} + CA\left(\frac{ep^*}{p}, Y - \bar{T}\right) \quad (\text{IS/DD})$$

Note that that (DD) shifts down in the $e - Y$ space if the p decreases.

- In the short run, the supply curve of the economy is given by the following equation:

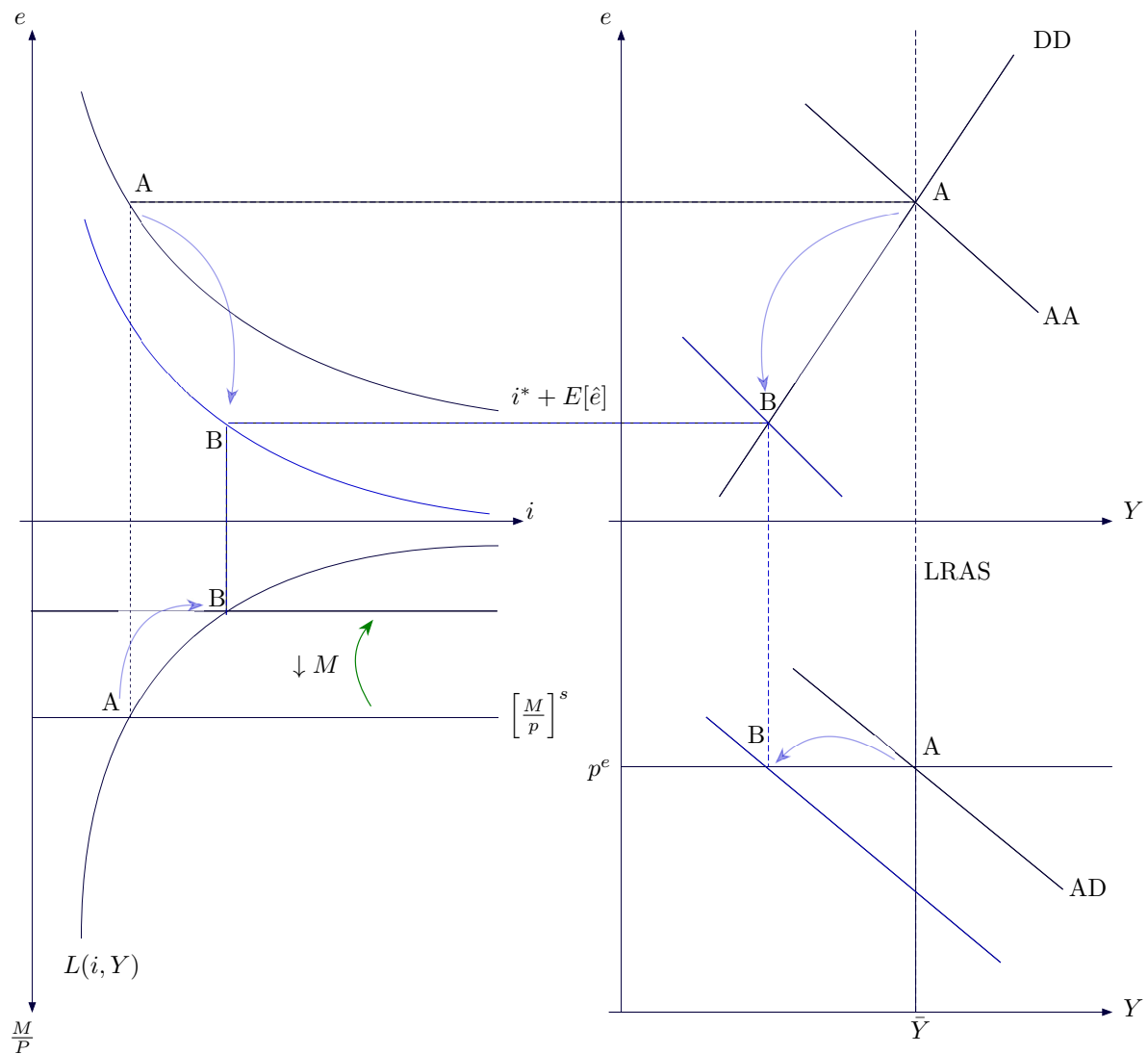
$$p = p^e \quad (\text{SRAS})$$

This means that prices are stuck at people's expectation of the prices change. If there is an anticipated shock to the economy, people would have factored the shock and formed appropriate expectations in advance. Thus, the prices would move smoothly to the new long run prices exactly when the anticipated shock materialises. Conversely, if there is an unanticipated shock to the economy, it would take people some time to form new expectations and in the mean while the prices will be stuck at the old price expectations.

Implicit in the overshooting exchange rate model is the idea that expectations on exchange rate reacts almost instantaneously where as expectations on price change more slowly, i.e., they are stickier. This is not an entirely unreasonable assumption given that exchange rate is one simple value where as prices are far more complicated. There are millions of prices and it is less clear exactly how each of these prices react to a monetary contraction. Naturally some are stickier than others and some prices depend on other prices. Thus with a shock to the economy, it is much easier to determine how the exchange rate will change but it more complicated and takes a lot longer to for everyone to form expectations on how the prices of bread, milk, bus tickets, houses, cars etc. etc. will change.

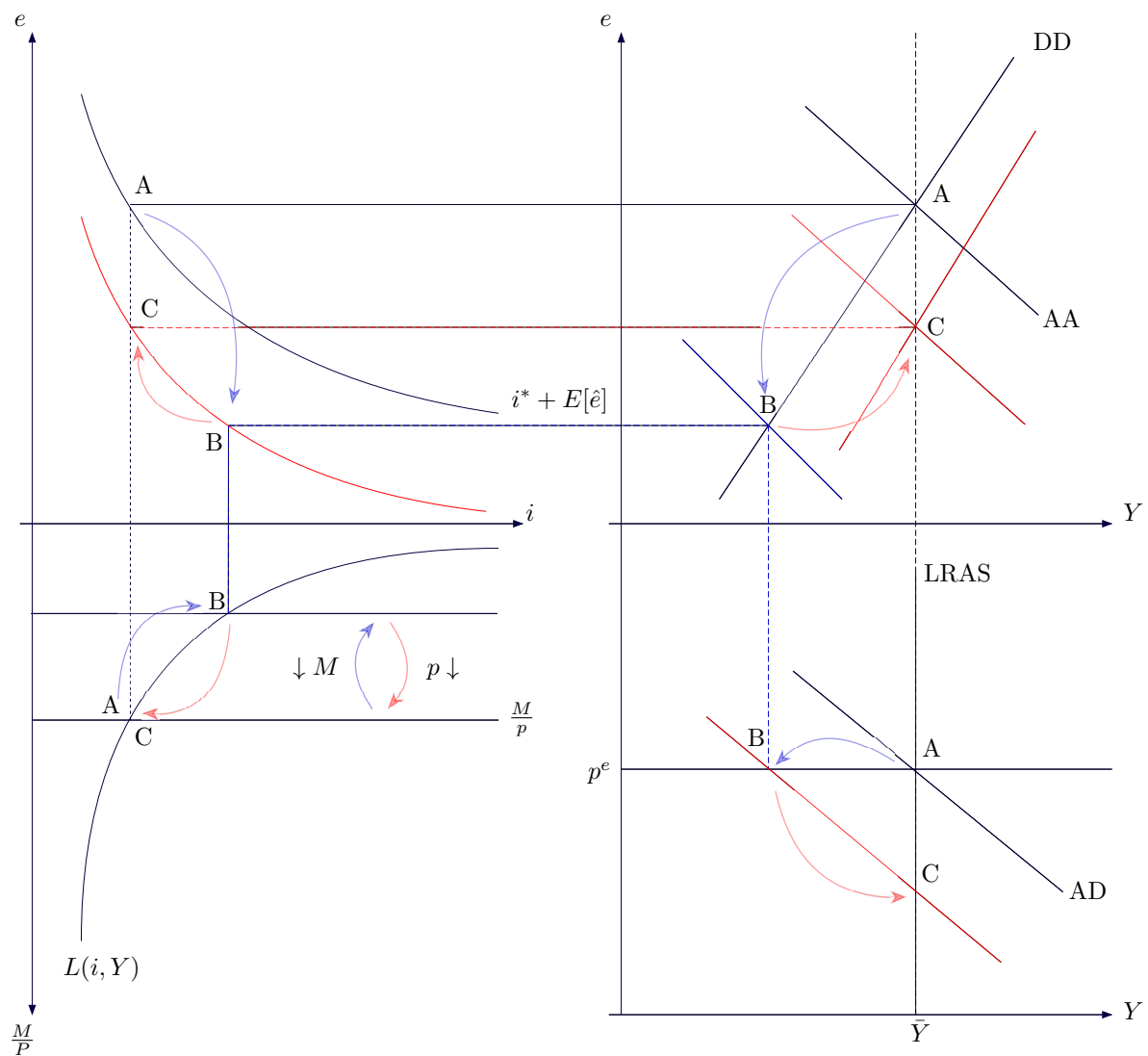
- Figure 5 shows the full effect of an unanticipated monetary contraction in the DD-AA model. The black lines represent the original equilibrium, the red lines represent the new long equilibrium that the economy settles into. Figure 4 depicts the economy just after an unanticipated monetary contraction. The economy was in a long run equilibrium at A. The economy moves to B moments after the shock hits the economy. The blue lines represent the moment after people realise that a monetary contraction has taken place. The blue lines represent the intermediate position or the intermediate dynamics as the economy converges to the new equilibrium.
- We can break down the effect of the monetary contraction ($\downarrow M$) into two distinct effects.
 - The interest rate rise as the real money supply falls.
 - The exchange rate appreciates (falls) due to two factors. The first factor is that people update their expectation about the new long run exchange rate and realise that it is

FIGURE 4. The immediately aftermath of an unanticipated monetary contraction in the DD-AA Model: The central bank reduces the money supply without any prior warning. People are taken by surprise. It takes them time to work out the exact effect on the price level but the effect on exchange rate is obvious immediately. Consequently, the expectations of future exchange fall immediately and UIP shifts down in the $e - i$ space. The price expectations remain where they are and the output of the economy drops immediately.



going to be much lower. Thus, the UIP shifts down and the exchange rate falls (to C in the $e - i$ space). Further, the exchange rate falls as due to the fall in the interest rate (to B in the $e - i$ space).

FIGURE 5. The fully dynamics of an unanticipated monetary contraction in a DD-AA Model: The economy is in a long run equilibrium at A before the monetary contraction. After the monetary contraction, the economy moves to B in the short run while prices are sticky. As prices adjust, the economy moves to the new long run equilibrium at C.



- The falls in $E(e)$, people's expectations about the interest rate leads to the AA curve shifting down and the intersecting with the DD at B.
- The aggregate demand curve shifts back and with sticky prices intersects with the flat short term aggregate supply curve at B.

- Output being below \bar{Y} the productive capacity of the economy puts a downward pressure on the prices ($p \downarrow$).
- As prices fall, the economy moves to C in the $p - Y$ space. This leads the DD curve to shift down. Further, as prices fall, the real money supply falls. The prices keep falling till the real money supply goes back to the pre monetary contraction levels. As interest rates rise, the exchange rate depreciates as the economy moves to C in the $e - i$ space. Concomitantly, the AA curve shifts up due to falling prices and the new AA curve intersects at C in $e - Y$ space. Thus, the economy comes to a rest in at C.
- The exchange rate falls further in the short run than it does in the long run and in the process overshoots its long run equilibrium value.