Supervision 4 (end of week 8)

Public Goods

1. There are 10 people in a community and each is willing to pay 5 pounds for each extra unit of a public good regardless of how much of the public good there is. The cost of providing x units of the good is $c(x) = 0.5x^2$. Each has the same utility function, which is quasi-linear in wealth.

(i) Write down the utility function.

(ii) Find the Pareto-efficient quantity of the public good by using the Samuelson Rule.

(iii) Find the Pareto-efficient quantity of the public good by maximizing the sum of the agents' utilities. [This is a valid way of finding Pareto-efficient allocations but it will identify them all only if utilities are quasi-linear].

(iv) Suppose that each agent can unilaterally increase or decrease x, paying the full extra cost for an increase and receiving the full saving for a decrease. Find the equilibrium value of x (i.e, the value such that nobody wants to change it).

(v) Suppose that each agent can unilaterally increase or decrease x but pays (or, if a decrease, receives) one-tenth of the change in cost. What is the equilibrium value of x now?

(vi) If instead half of the agents valued a unit of the good at 3 pounds and the other half at 7 pounds, what variation of the scheme in (v) would give an efficient quantity of the public good?

(vii) Comment.

2. Consider the following variation on the Groves-Clarke scheme for preference revelation. There are three agents (A, B, C) and a discrete public good which will either be produced or not. If it is produced each pays one third of the cost c. The net value of agent i $(i = A, B, C)$, denoted by n_i, is the difference between i's reservation value and $c/3$. Reservation values are private information. The scheme is:

(i) Each agent announces her net value. Let $\hat{n}_i$ denote i's announcement.

(ii) Produce the good if and only if $\hat{n}_A + \hat{n}_B + \hat{n}_C \geq 0$.

(iii) If the good is not produced, nobody pays anything.

(iv) If the good is produced, each agent pays, in addition to their share of the cost, a tax equal to $-$(sum of the other two agents' announcements).

From A's point of view, there are four possible configurations: (1) $\hat{n}_B + \hat{n}_C < 0$, $n_A + \hat{n}_B + \hat{n}_C < 0$, (2) $\hat{n}_B + \hat{n}_C < 0$, $n_A + \hat{n}_B + \hat{n}_C > 0$, (3) $\hat{n}_B + \hat{n}_C > 0$, $n_A + \hat{n}_B + \hat{n}_C < 0$, (4) $\hat{n}_B + \hat{n}_C > 0$, $n_A + \hat{n}_B + \hat{n}_C > 0$. [Ignore cases of equality].

In each of these four cases, find A's utility if she tells the truth and the highest other utility she can get by lying. Deduce that telling the truth is optimal. Explain why the scheme is efficient. Give an intuition for why this scheme works.

3. "In practice, many goods that are non-excludable and non-rival in consumption can be provided by the private sector as well as or better than by government". Discuss.

Reading

Varian, chap. 34.

David van Zandt, "The Lessons of the Lighthouse: 'Government' or 'Private' Provision of Goods", *The Journal of Legal Studies*, Jan, 1993, pp. 47-72.

Michael Montgomery and Richard Bean, "Market Failure, Government Failure, and the Private Supply of Public Goods: the Case of Climate-Controlled Walkway Networks", *Public Choice*, June 1999, pp. 403-37.

Tyler Cowen, "Public Goods",
http://www.econlib.org/library/Enc/PublicGoods.html.