Let y be the output, z the capital. $\bar{y}$ and $\bar{z}$ denote a particular level of output and capital.

Short-run

Short run cost curve for a given level of capital $\bar{z}$ is given by

$$c(y, \bar{z}). \tag{Short-run cost-curve}$$

Long-run

In the long run, the capital used is the optimal amount of capital for the level of output. The long-run cost-curve thus ultimately just depends on output.

$$c\big(y, k(y)\big) \tag{Long-run cost-curve}$$

That is, it depends directly on output and indirectly on output through the cost-minimising amount of capital used. $k(y)$ the cost minimising level of capital for a given y is obtained by as follows.

$$\begin{aligned} \frac{dc(y,k)}{dk} &= 0 & \Rightarrow & & k = k(y) \\ \frac{d^2c}{dk^2} &> 0 \end{aligned}$$

$k(y)$ is the path along which the cost function is always minimised.[1]

Comparing the Long-run and Short-run

The slope of the long-run cost curve is given by the total differential of the cost function.

$$\frac{dc}{dy}$$

The slope of the short-run cost curve is given by the partial differential of the cost function. Remember that the partial differential of c with respect to y takes k as given.

$$\frac{\partial c}{\partial y}$$

The slope of the long run curve of the cost function is given by

$$\frac{dc}{dy} = \frac{dc\big(y, k(y)\big)}{dy} = \frac{\partial c}{\partial y} + \frac{\partial c}{\partial k} \cdot \frac{\partial k(y)}{\partial y} \tag{1}$$

[1] A more detailed interpretation is that $k(y)$ is the capital demanded by the firm is supply due to the fixed factor prices or perfectly elastic supply of capital.

$\frac{\partial c}{\partial y}$ is the direct effect of change in output on cost, taking capital as given. It is also the slope of the short-run cost curve.
$\frac{\partial c}{\partial k} \cdot \frac{\partial k(y)}{\partial y}$ is the indirect effect of output on capital through cost-minimising capital level where

$\frac{\partial k(y)}{\partial y}$ is the change in optimal capital employed due to change in output.
$\frac{\partial c}{\partial k}$ is the change in cost due to increase in capital.

$\frac{\partial c}{\partial k} = 0$ if the optimal[2] level of capital is being used.

This implies that the long-run and short-run cost curve will be tangent for a particular output level y^* where $\bar{k} = k(y^*)$. That is, $\frac{dc}{dy} = \frac{\partial c}{\partial y}$ at a point where $\bar{k} = k(y)$ or the amount of output produced is such that $\bar{k}$ is the optimal amount of capital used. It is useful to note that there is nothing to suggest that where the long-run and short-run cost curves are tangent, the curves have slope zero. All it says is that the long-run and short-run cost curves will be tangent at y^* where $\bar{k} = k(y^*)$ or the capital in the short-run is the optimal amount.

Envelope Theorem

(1) is just the restatement of the envelope theorem if you interpret k as just a parameter in the function $c(y, k)$.

[2] cost-minimising level of capital