Question

1. For a production $Y = AK^{\alpha}L^{1-\alpha}$, where K is the capital, L is the labour, w is the wage and r is the interest rate on capital, find the cost function in terms of Y, w and r.

2. Consider the following production function:

$$y = 2K^{1/2}L^{1/2}.$$

   (a) Comment on the returns to scale of this function.

   (b) Represent the isoquants for $y = 4$ and $y = 8$.

   (c) Suppose that one unit of L costs 1, and one unit of K costs 1. What is the cheapest way to produce 8 units of output?

   (d) What is the equation for the long run output expansion path when factors' prices are both equal to 1? Show it on your isoquant graph. Show the short run output expansion path, and state your assumptions about the availability of factors.

   (e) Calculate the long-run total cost, average cost, and marginal cost functions when factors' prices are both equal to 1. Show the curves on the graph.

   (f) Explain how the total cost, average cost, and marginal cost functions would change if the firm's technology were instead given by $y = (4KL)^{1/3}$, and show the new curves graphically.

3. Let a firm's production function be given by $f(x, y) = (3\sqrt{x} + \sqrt{y})^2$. The price of input x is £3 and the price of input y is £1. What is the cheapest way to produce 16 units of output?

4. Suppose that a firm's production function has the Leontief form $q = \min\{\frac{x}{2}, \frac{y}{3}\}$ where x and y are the two inputs.

   (a) Draw the isoquants.

   (b) For a given level of output, identify the cost-minimising combination(s) of inputs on the diagram.

   (c) Write down the cost function. Explain why in this example the slope of the isoquant is not relevant.

5. A firm which uses two factors x_1 and x_2 has a production of the following form:

$$\bar{y} = x_1 + 3x_2$$

   (i.) Draw the isoquants

   (ii.) Write down the cost function

   (iii.) What are the factors that determine the ratio in which the factors would be used?

6. Consider the production technology

$$y = \min\{x_1 + 3x_2, 2x_3\}$$

   where x_1 is skilled labour, x_2 is unskilled labour and x_3 is capital. The wages of skilled and unskilled labour are w_1 and w_2 respectively, and the rental rate of capital is r. Identify the cost-minimising combination(s) of inputs, and write down the cost function(s).