Demand Curve Cheat Sheet

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Perfect Compliments: $u(x_1, x_2) = \min(\alpha x_1, \beta x_2)$

$$\max_{x_1,x_2} \Big[\min(\alpha x_1, \beta x_2)\Big] \qquad \text{s.t.} \qquad p_1x_1 + p_2x_2 = M$$

- Solve:

$$\alpha x_1 x_1 = \beta x_2$$
$$p_1x_1 + p_2x_2 = M$$

To get the demand Curves:

$$x_1^d = \frac{\beta M}{\beta p_1 + \alpha p_2}$$
$$x_2^d = \frac{\alpha M}{\beta p_1 + \alpha p_2}$$

Perfect Substitutes: $u(x_1, x_2) = \alpha x_1 + \beta x_2$

$$\max_{x_1,x_2} \Big[\alpha x_1 + \beta x_2\Big] \qquad \text{s.t.} \qquad p_1x_1 + p_2x_2 = M$$

- Demand Curves:
  - If $\frac{\alpha}{\beta} > \frac{p_1}{p_2}$,

$$x_1^d = 0$$
$$x_2^d = \frac{M}{p_2}$$

  - If $\frac{\alpha}{\beta} < \frac{p_1}{p_2}$

$$x_1^d = \frac{M}{p_1}$$
$$x_2^d = 0$$

  - If $\frac{\alpha}{\beta} = \frac{p_1}{p_2}$, any point on the budget constraint can be the demands for x_1 and x_2.

- Uncompensated demand from Cobb Douglas: $u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$

$$\max_{x_1, x_2} \quad \left[x_1^{\alpha} x_2^{1-\alpha}\right] \qquad \text{s.t.} \qquad p_1 x_1 + p_2 x_2 = M$$

  - Solve:

$$\frac{p_2 x_2}{p_1 x_1} = \frac{1-\alpha}{\alpha}$$
$$p_1 x_1 + p_2 x_2 = M$$

  - To get demand curves:

$$x_1^d = \frac{\alpha M}{p_1}$$
$$x_2^d = \frac{(1-\alpha) M}{p_2}$$

- Compensated demand from Cobb Douglas: $u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$

$$\min_{x_1, x_2} \quad \left[e = p_1 x_1 + p_2 x_2\right] \qquad \text{s.t.} \qquad \bar{u} = x_1^{\alpha} x_2^{1-\alpha}$$

  - Solve:

$$\frac{p_2 x_2}{p_1 x_1} = \frac{1-\alpha}{\alpha}$$
$$\bar{u} = x_1^{\alpha} x_2^{1-\alpha}$$

  - To get demand curves:

$$x_1^d = \left[\frac{p_2}{p_1} \frac{\alpha}{(1-\alpha)}\right]^{(1-\alpha)} \bar{u}$$
$$x_2^d = \left[\frac{p_1}{p_2} \frac{(1-\alpha)}{\alpha}\right]^{\alpha} \bar{u}$$