Leibniz integral Rule

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1. Integrals

1.1. Leibniz integral Rule.

- Differentiation under the integral sign with constant limits.

$$\frac{d}{dx}\int_{y_0}^{y_1} f(x,y)\,dy = \int_{y_0}^{y_1} \frac{\partial}{\partial x} f(x,y)\,dy$$

for $x \in (x_0, x_1)$ provided that f and $\frac{\partial f}{\partial x}$ are continuous over a region in the form $[x_0, x_1] \times [y_0, y_1]$.

- Differentiation under the integral sign with variable limits that are a function of the variable used for differentiation.

$$\frac{d}{d\alpha}\int_{a(\alpha)}^{b(\alpha)} f(x,\alpha)\,dx = b'(\alpha)\cdot f(b(\alpha),\alpha) - a'(\alpha)\cdot f(a(\alpha),\alpha) + \int_{a(\alpha)}^{b(\alpha)} \frac{\partial}{\partial x} f(x,\alpha)\,dx$$

1.1.1. *Application to Consumer Demand.*

- Consumer demand for product X varied and determined by each consumer's $t \in [0,1]$
- Consumer Demand:

$$\begin{cases} 0 & p > P(t) \\ h(t,p) & p < P(t) \end{cases}$$

$P(t)$ Reservation price: given t the price has to be low enough for the consumer to buy.

$h(t, P(t)) > 0$, $h_t(t,p) > 0$, $h_p(t,p) < 0$, $P'(t) > 0$

$T(p)$ is the inverse function of $P(t)$: for a given p the taste t has to be high enough for the consumer to buy

Distribution of t: $f(t)$ is the distribution of t with the following properties:

i. Number of people in $[a, b]$ where $0 < a < b < 1$ is given by

$$\int_a^b f(t)dt$$

ii.

$$\int_0^1 f(t)dt = N$$

iii. For a small ε, there would be $\varepsilon f(t)$ consumers[1] with $t \in (t, t+\varepsilon)$

For a given price p, consumers with positive demand are $t \in (T(p), 1)$

Consumer demand $(t \in (T(p), 1))$:

$$x = \int_{T(p)}^1 f(t)h(t,p)dt$$

Using Leibinz formula

$$\frac{dx}{dp} = -f\Big(T(p)\Big)h\Big(T(p), p\Big) \cdot T'(p) + \int_{T(p)}^1 f(t)\frac{\partial h(t,p)}{\partial p}dt$$

– Both terms on the RHS are negative since $h_P(t,p) < 0$

1.1.2. *Integrating over a Stochastic Distribution.*

This is what I always get stuck on:

$$\frac{dG(w)}{dw} = g(w)$$
$$G(w) = \int_0^w g(w)dw$$

1

$$\int_t^{t+\varepsilon} f(t)dt \approx \epsilon f(t)$$

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w	random variable
$g(w)$	probability distribution
$G(w)$	cumulative distribution