Monotone Likelihood Ration Property

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1. Monotone Likelihood Ratio Property

- n output levels such that $q_1 < \ldots < q_i < \ldots < q_n$.
- two effort levels $e_k = \{0, 1\}$ that influence the output level
- π_{ik} is probability of output level i when effort level k has been exerted.

2. FOSD and MLRP

- *FOSD just show that higher effort produces more output:*

  First order stochastic dominance shows that *increasing effort is either socially valuable or good for the principal*, i.e., a principal with utility function is increasing in output favours a higher effort. (FOSD just ensures that high effort leads to higher output or high principal's utility.)
- *MRLP show that*

  Monotone Ratio Likelihood property shows that higher output implies that the agents has put in higher effort.

FOSD: Example with 2 output levels. Output i such that $q_1 < q_2$. Effort $e_k = \{0, 1\}$. Probabilities π_{ik}.

$$\pi_{10} \geqslant \pi_{11}$$

$$\pi_{20} + \pi_{10} = \pi_{11} + \pi_{21} = 1$$

With FOSD, the probabilities of low output levels is higher with low effort, thus it pushes up (increases) the probabilities with high effort of high output levels.

If $\pi_{10} \uparrow \implies \pi_{20} \downarrow$ since $\pi_{10} + \pi_{20} = 1$.

π_{10}	$>$	π_{11}
π_{20}	$<$	π_{21}
$\pi_{10}+\pi_{20}$	$=$	$\pi_{11}+\pi_{21}$

A project with two outcomes - success of failure. Let π_1 be probability of success when effort is high and π_0 when effort is low.

$1-\pi_0$	$>$	$1-\pi_1$
π_0	$<$	π_1
$(1-\pi_0)+\pi_0$	$=$	$(1-\pi_1)+\pi_1$

If there is an increase in π_1 probability of success due to high effort, then it automatically lead to decreases in $(1-\pi_1)$, probability of failure due to high effort.

In stochastic dominance, all comparisons are at the binary level (cumulative distribution function). *At every output level, the dominant project has a lower cumulative probability.*

P(Low output, *low effort*)	$>$	P(Low output, *high effort*)
P(all output, *low effort*)	$=$	P(all output, *high effort*)
P(High output, *low effort*)	$<$	P(High output, *high effort*)

FOSD: Example with 3 output levels. Output i such that $q_1 < q_2 < q_3$. Effort $e_k = \{0,1\}$. Probabilities π_{ik}.

$$\pi_{10} \geqslant \pi_{11}$$

$$\pi_{20}+\pi_{10} \geqslant \pi_{11}+\pi_{21}$$

$$\pi_{30}+\pi_{20}+\pi_{10} = \pi_{11}+\pi_{21}+\pi_{31} = 1$$

With FOSD, the probabilities of low output levels is higher with low effort, thus it pushes up (increases) the probabilities with high effort of high output levels.

MRLP: Example with three outcomes. Output i such that $q_1 < q_2 < q_3$. Effort $e_k = \{0,1\}$. Probabilities π_{ik}.

- *Likelihood Ratio:* State i occurs if the output is q_i. The likelihood ratio for state i is $\frac{\pi_{i1}-\pi_{i0}}{\pi_{i1}} = \frac{\Delta\pi_i}{\pi_{i1}}$. It can also be written as:

$\left(\frac{\Delta\pi_i}{\pi_{i1}}\right)$: proportional change in probabilities brought about by change in effort.

$\left(1 - \frac{\pi_{i0}}{\pi_{i1}}\right)$:

The likelihood ratio is increasing in i if $\frac{\pi_{i0}}{\pi_{i1}}$ is decreasing in i. That means that π_{i1} is growing faster than π_{i0} with i.

If $\pi_{i1} > \pi_{i0}$, this would imply the $\frac{\pi_{i0}}{i1} < 1$ and $1-\frac{\pi_{i0}}{i1} > 0$. Positive likelihood ratio implies high effort is more probable than low effort for that state. The higher the ratio, the more likely that high effort was exerted.

- For a risk neutral agent, you only have to pay them in the state of the world where the likelihood ratio is the highest. This is called the *bong-bang* contract. (See Innes (1990)).
- *Monotone Likelihood Ratio Property:* The likelihood ratio monotonically increases with i. Higher the i, (the lower the $\frac{\pi_{i0}}{\pi_{i1}}$) the higher the likelihood that high effort was exerted.