Microeconomics
Part I, Paper 1
Consumer and Producer Theory; Rational Choice
Michaelmas Term 2016

Supervision set 4

(To be completed after Lecture 18)

1. A perfectly competitive firm has the following short-run cost function:

$$c(y) = y^3 - 8y^2 + 30y.$$

   (a) What is the firm's short-run marginal cost function?
   (b) What is the firm's average variable cost function?
   (c) Sketch the graph of marginal and average variable cost functions.
   (d) Find the minimum price at which the firm is willing to supply output.
   (e) What is the smallest (but strictly positive) amount the firm will ever supply?
   (f) At what price would the firm supply 6 units?

2. Suppose that a firm has two machines with different production technologies:

$$f(x_1, x_2) = 3\sqrt{x_1} + 2\sqrt{x_2} \quad \text{and} \quad g(x_1, x_2) = x_1^{0.6} x_2^{0.4}.$$

   The factor prices are $w_1 = 3$ and $w_2 = 2$.

   (a) Derive the cost functions c_f and c_g for machines f and g, respectively.
   (b) If the firm can use only one of the machines at a time, which machine will be used?
   (c) If the firm can run both machines at the same time, how will it organise its production? Derive the firm's cost function c. (Hint: Express the firm's cost as $k(m, n) = c_f(m) + c_g(n)$ when it is producing $q = m + n$ units, where f is used to produce m units, and g is used to produce n units.)
   (d) Suppose now that the second technology has a fixed cost of 20, so the production function is actually $g(x_1, x_2) = 20 + x_1^{0.6} x_2^{0.4}$. Answer the questions above for this case.

3. We have explained three different ways of expressing producer's surplus: using the supply curve, the MC, or the AVC curve. When studying the change in PS due to a change in the economic environment, one of those ways may be better than the others in illustrating PS on a graph. For example, we have a cleaner exposition if the area that corresponds to PS before the change lies within the area that corresponds to PS after the change.

   In each case below, which representation of PS gives the cleanest way of illustrating the change in PS? Show in a graph where you shade the area that corresponds to the change in PS. Write the algebraic expression for this area.

(a) Suppose the output price p goes up, while all other parameters stay constant.

(b) Suppose one of the inputs the firm uses gets cheaper, while all other parameters stay constant. (You need to think *what* would change in response to a decrease in the price of that input, and *how* would that change the graph.)

4. (2011 Tripos) Consider a monopolist who faces the following demand curve

$$z = a - 3p$$

where z is demand, p is price, and $a > 0$ is a constant.

(a) What is the monopolist's revenue? How does it differ from that of a firm in a perfectly competitive market?

(b) Assume that *per unit* cost of production is $c(z) = \lambda z$, where $\lambda > 0$ is a constant. What is the monopolist's profit? Derive its optimal choice of z.

(c) Explain why the monopolist's choice generates a deadweight loss.

(d) In a graph, show the deadweight loss generated by the monopolist. Comment on your graph.

(e) Now suppose that the monopolist can engage in perfect price discrimination. Discuss how this affects the size of the deadweight loss.

Additional questions

It is subject to the individual supervisor's discretion to assign any of the additional questions for their supervisions.

1. Explain and show graphically why economists say that a firm's supply curve in the short run is the upward sloping part of the marginal cost curve that lies above the average variable cost curve. Comment on the difference between the short-run and the long-run supply curves. Will a firm that operates at $p = MC$ always make a non-negative profit?

2. Assume that a monopoly has the following technology $q = L^{2/5}K^{3/5}$. The cost of labour is $w = 10$ and the cost of capital is $r = 8$. Answer the following questions.

(a) Find the cost function in the short-run when capital K is fixed at some level $\overline{K}$.

(b) Find the cost function in the long-run.

(c) Sketch the average cost curves in the short-run and in the long-run. Do they touch or cross?

(d) Suppose the demand for the monopolist's product is given by $q = 200 - 5p$. Find the monopolist's output q in the long run. Write down the first order condition that the monopolist's output q must satisfy in the short-run (do not solve for the exact value of q).