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Modality Tests for Use in Applied Econometrics: Application to Macroeconomic Convergence

by

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Abstract

We examine three tests, widely employed in the statistics literature, that help determine the number of modes in an empirical nonparametric kernel density. The tests are described and applied to the Penn World Table (PWT) data set on world-wide per capita income to study convergence. Our results suggest that the distribution of world-wide per capita income, contrary to conventional wisdom based on informal inspection, may have been multimodal throughout the period covered by the PWT data set.

JEL classification codes: C1.

Key words: Modality tests, nonparametric kernel, convergence.

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1. Introduction.

For many reasons, nonparametric density estimation has become an increasingly useful and popular tool. One important reason is that such density estimates can help capture the stylized facts that need explanation and provide insights into how well a potential model is likely to fit the data. The flexibility of the estimates facilitates identification of the salient features of a distribution. As with photographs, however, different investigators looking at the same nonparametric density estimate can see markedly different features. It is often important, therefore, to employ more rigorous statistical methods in order to help determine the truth.

Many tests are available for testing hypotheses about densities. These include tests for comparisons between densities (*e.g.*, whether two unknown distributions are identical) and tests concerning a single density (*e.g.*, testing for symmetry). This paper is concerned with determining the number of modes in a single nonparametric density estimate. The question of multimodality arises in several areas of current research interests. For example, the question of the disappearance of the "middle class" can be formulated as a question about multimodality. Another area of current interest is whether worldwide per-capita incomes are converging—whether the poor are catching up with the rich. A literature initiated primarily by Quah (1996a, 1996b, 1997)[1] focuses on the observation that, while in 1965 there were many countries in the middle income group, in 1990 the world had become divided, as a stylized fact, into two categories: the rich and the poor. Quah refers to this emergent bidomality as "two-club," or "twin-peak," convergence. In this paper, we add to the contribution of Bianchi (1997) in tests of this stylized fact.

Throughout the paper, nonparametric kernel estimates are used. The best known of these estimators, they are better developed than others with respect to both their numerical calculation and their known analytical properties.

[1] See also Galor (1996), Jones (1997), and Kumar and Russell (2002).

The paper is organized as follows. Section 2 briefly discusses three methods available in the statistics literature to determine the number of modes in an empirical nonparametric density estimate. Section 3 uses each of the three methods on a sample of 126 countries in order to examine the world distribution of per capita income. Section 4 concludes.

2. Methods.

The kernel density estimate for univariate observations, $x = x_1, x_2, \ldots, x_N$, is defined by

$$\hat{f}(x) = (nh)^{-1} \sum_i K\Big(\frac{x_i - x}{h}\Big), \tag{2.1}$$

where $K(\cdot)$ is the standard normal kernel with window width (smoothing parameter or bandwidth) h.[2] The window width controls the degree to which the data are smoothed to produce the kernel estimate. The larger the value of h, the smoother the kernel distribution. For example, if the data possess two strong modes, a very large value of h will be needed to obtain a unimodal estimate. On the other hand, very small values of h produce additional modes in the kernel estimate that should instead be considered spurious artifacts of the sampling process. This section examines three methods of determining the number of modes in a distribution.

Silverman (1981) develops a method to test the null hypothesis that a density f has k modes against the alternative that f has more than k modes, where k is a non-negative integer. The test statistic in this case is the critical window width, defined by

$$h_{crit}(k) = \inf \big\{ h \mid \hat{f} \text{ has at most } k \text{ modes} \big\}. \tag{2.2}$$

For $h < h_{crit}(k)$, the estimated density has at least $k + 1$ modes. The value of $h_{crit}(k)$ is computed through a binary search algorithm, and its significance level can be assessed by the smoothed bootstrap procedure attributable to Efron (1979).

[2] See Pagan and Ullah (1999, especially pp. 9 and 23) for details.

The second method of determining the number of modes in an empirical distribution is a useful complement to the Silverman test. Chaudhuri and Marron (1997) and Marron and Chaudhuri (1998) present the SiZer (SIgnifiance of ZERo) approach, which they characterize as an extension of the mode tree of Minnotte and Scott (1993). The SiZer approach has two graphical components. The first is an overlaid family of empirical kernel distributions, each corresponding to a different window width. The idea is that no single window width can explain all the information available in the data; hence, various window widths reveal different features of the distribution. The second graphical component, the SiZer map, studies the significance of zero crossings to determine which features in the overlaid family of empirical kernel distributions are statistically significant for given window widths.

Although interpretation of the first component, analyzing the bumps and valleys of the kernel estimates, is straightforward, the analysis of the SiZer map requires a little more explanation. Each pixel in the map represents a point in scale space that is indexed by the location (on the horizontal axis) and the window width (on the vertical axis). For each location and window width, a simple confidence interval for the derivative is constructed. When the confidence interval is completely above zero, the pixel is shaded dark and the kernel distribution is considered to be significantly increasing at that location. Conversely, when the pixel is shaded light, the confidence interval is completely below zero and the kernel distribution is said to be significantly decreasing at that location. Finally, when the confidence interval contains zero and there is no statistically significant slope, the pixel is shaded an intermediate gray.[3] Thus, significant modes have a dark region on the left and a light region on the right. Similarly, significant valleys have light shading on the left and dark shading on the right. The smaller insignificant modes seen in some of the overlaid family empirical kernels appear in the gray region of the SiZer map.[4]

[3] These pixels are color coded in the full-color version.

[4] As stated previously, not all modes are seen at all window widths. Very large window widths lead to too few modes, while very small window widths lead to too many modes. Thus, it may be most appropiate to look at the middle third of the SiZer map.

The final (and most complicated) method is quite different from the first two. The Dip Test, developed by Hartigan and Hartigan (1985), is the maximal difference between the empirical kernel distribution function and the unimodal distribution function that minimizes this maximum difference. That is, the DIP is the distance between the "tightest fitting" unimodal distribution function and the empirical kernel distribution, where distance in this function space is given by the sup metric. For N observations, the statistic can be computed in order N operations, and it is consistent for testing any unimodal distribution against any multimodal distribution.

More formally, define

$$\partial(F, G) \equiv \sup_x |F(x) - G(x)| \tag{2.3}$$

for any bounded functions F and G. Further,

$$\partial(F, A) \equiv \inf_{G \in A} \partial(F, G) \tag{2.4}$$

for any class A of bounded functions. Let U be the class of unimodal cumulative distribution functions. The dip of a distribution function F is then defined by

$$D(F) \equiv \partial(F, U). \tag{2.5}$$

Note that

$$D(F_1) \leq D(F_2) + \partial(F_1, F_2), \tag{2.6}$$

$$D(F) = 0 \text{ for } F \in U, \tag{2.7}$$

and

$$D(F) > 0 \text{ for } F \notin U, \tag{2.8}$$

so that the DIP is a measure of departure from unimodality.

The algorithm of Hartigan and Hartigan for calculating the "tightest fitting" unimodal distribution and the DIP statistic exploits the fact that a cumulative unimodal distribution on the real line is concave over $(\infty, a]$ and convex over $[a, \infty)$, where a is the mode. The greatest convex minorant (gcm) of F on $(-\infty, a]$ is the supremum over all real-valued convex functions mapping from $(-\infty, a]$ and satisfying $G(x) \leq F(x)$ for all $x \in (-\infty, a]$. The least concave majorant (lcm) of F in $[a, \infty)$ is the infimum over all concave functions satisfying $L(x) \geq F(x)$ for all $x \in [a, \infty)$.

The algorithm of Hartigan and Hartigan for constructing the "tightest fitting" unimodal cumulative distribution function and the concomitant DIP statistic is instructive. Let $x = x_1, x_2, \ldots, x_N$ be the atoms of F where the x's are listed in ascending order. To compute minorant and majorant functions and the DIP statistic, execute the following steps:[5]

(i) Begin with $x_L = x_1$, $x_U = x_N$, and $D = 0$.

(ii) Compute the gcm, say G, and the lcm, say L, for F in $[x_L, x_U]$; denote the points of contact with F by $g_1, g_2, \ldots, g_K$ and $l_1, l_2, \ldots, l_J$, respectively.

(iii) Suppose $d := \sup |G(g_i) - L(g_i)| > \sup |G(l_i) - L(l_i)|$ and that the supremum occurs at g_i satisfying $l_j \leq g_i \leq l_{j+1}$. Define $x_L^o = g_i$ and $x_U^o = l_{j+1}$.

(iv) Suppose $d := \sup |G(l_i) - L(l_i)| > \sup |G(g_i) - L(g_i)|$ and that the supremum occurs at $g_j \leq l_i \leq g_{j+1}$. Define $x_L^o = g_i$ and $x_U^o = l_j$.

(v) If $d \leq D$, stop and set $D(F) = D$.

(vi) If $d > D$, set

$$D = \sup \left\{ D, \sup_{x_L \leq x \leq x_L^o} |G(x) - F(x)|, \sup_{x_U^o \leq x \leq x_U} |L(x) - F(x)| \right\}.$$

(vii) Set $x_U = x_U^o, x_L = x_L^o$ and return to (ii).

Hartigan and Hartigan argue that the appropiate null distribution is the uniform distribution. They show that the DIP is asymptotically larger for the uniform distribution than for any distribution in a wide class of unimodal distributions (those with exponentially decreasing tails). They therefore determine the distribution of the Dip statistic by sampling from the space of uniform distributions. The basic idea is that, as the sample size grows, the

[5] See page 79 of Hartigan and Hartigan [1985]; we have made only minor notational changes in their description.

DIP for a unimodal distribution approaches zero while the DIP of a multimodal distribution approaches a positive constant.

As compared to the Silverman test, the Dip test has the disadvantage of testing only for multimodality against the null of a unimodal distribution. The Silverman method tests the null hypothesis that k modes exist against the alternative that more than k modes exist. Although the Silverman test is more flexible in its hypotheses, it does have the disadvantage of not being a nested test. For example, it could fail to reject the null hypothesis of having m modes, but reject the null of having $m-p$ modes, where $m-p \geq 0$. An advantage of the SiZer approach is that it looks at a wide variety of window widths and examines how changes in the window widths affect a particular location in the empirical distribution. Although a reject or fail-to-reject conclusion is not immediate, the SiZer brings out more information on the empirical distribution that the researcher can use in his or her analysis. Thus, none of these three methods strictly dominates the others.[6] It may therefore be wise to use all three in determining the number of modes in an empirical distribution.

3. Empirical Example.

The only economics application of these methods that we have been able to find is Bianchi's (1997) use of the Silverman method to test for macroeconomic convergence. He examines "worldwide" cross-sectional distributions of real Gross Domestic Product (GDP) per capita of 119 industrialized and developing countries in 1970, 1980, and 1989. If the countries are tending to converge towards one another over time, the tests should show that distributions are tending towards a unimodal distribution. The data for real GDP per capita (in constant

[6] Earlier methods of testing for modality would seem to be dominated by one or more of the three tests considered here; see the discussion of these tests in Hartigan and Hartigan (1985).

1985 international dollars) are taken from the Penn World Table (PWT) (Summers and Heston (1991)).[7]

The main results of Bianchi's paper are as follows:

(1) In 1970 the hypothesis of a unimodal distribution cannot be rejected at the 5% level of significance (p-value = 0.07).

(2) In 1980 the hypothesis of a unimodal distribution is strongly rejected at the 5% level of significance (p-value = 0.01).

(3) In 1989 the hypothesis of a unimodal distribution is strongly rejected at the 5% level of significance (p-value = 0.00).

These results suggest that the distribution of world income may be transforming from a unimodal to a bimodal distribution, a finding that is consistent with the bipolarization theories of Quah (1996a, 1996b, 1997), among others. Bianchi further suggests that the gap between less and more developed countries (the distance between the modes) widened significantly during the 1980's.

In our application of the three types of modality tests, we employ the same PWT data as Bianchi but extend the number of years and countries. Specifically, the data set includes 126 countries for the years of 1961-1986.[8] Thus, the data set goes back nine years before the start of Bianchi, enabling one to examine the period leading up to 1970. Unfortunately, enhancement of the number of countries forces a three-year earlier cutoff date (1986).

We execute three different Silverman tests for each year, with null hypotheses of one, two, and three modes (hence alternative hypotheses of more than one, more than two, and

[7] Bianchi also examines modality under two transformations of the data. To alleviate potential problems of income cross-correlation among countries, he considers the distribution of each country's per capita income relative to the aggregate; this does not change the shape of the distribution, and the results are nearly identical. Motivated by the notion of β-convergence, he also examines the distribution of the log of real GDP per capita. This transformation does change the shape of the distribution, leading to different conclusions about multimodality. While this test is relevant to the simplest form of the convergence hypothesis, we believe that the more-complicated modality question should be answered using data measured in the original units.

[8] Our data set and the programming codes used in our tests are available at the UCR Working Paper website.

more than three modes). The results are listed in Table 1. The first row for any given year represents the values of $h_{crit}(k)$, while the second row is the p-value associated with the corresponding critical window width.

Although the results for 1970, 1980 and 1986 are similar to those of Bianchi for for these three years, it is interesting tonote that, for our expanded data set, the hypothesis that the distribution is unimodal in 1970 is rejected at the 5% level of significance. Thus, according to the Silverman Test, the distribution of world per capita income in 1970 was already bimodal. In order to determine whether the distribution was orginally unimodal, it is necessary to look at pre-1970 distributions. Our results indicate that the hypothesis of unimodality cannot be rejected before 1970 at the 5% level of significance. In the case of small samples such as this, however, it is instructive to consider a lower level of significance. Testing at the 10% level of significance, we can reject the hypothesis that the distribution was unimodal, against the alternative of multimodality as early as 1963 (and also in 1967 and 1969). Further, all but two of the remaining years have p-values less than or equal to 0.16. This finding suggests that, even though it cannot be concluded that the distributions during those years are bimodal, other methods should be used to further analyze that possibility. An intriguing result of Table 1 is that, at the 10% level of significance, more than three modes exist in 1971 and 1972. This result is possible because the Silverman tests for higher values of k are not nested. If this result is not an anomaly, it raises the possibility that countries were moving between the two modes in the earlier 1970's.

The inconclusiveness of many of the Silverman tests underscores the potential value of executing more than one test of modality. The SiZer approach is reflected in Figures 1 and 2. Figure 1 plots the overlaid family of empirical kernel distributions and Figure 2 displays the SiZer maps for each year. Figure 1 shows that the higher mode (at higher levels of real GDP per capita), although quite small and perhaps insignificant at first, grows over time and separates itself from the lower mode. Still, the question of whether or not the distribution of real GDP per capita has always been bimodal is unclear.

The first map in Figure 2 (1961) shows bimodalism for a reasonable window width of 500. The first mode appears near 1000 while the second sits between five and six thousand (on the horizontal axis). Although the size and shape of the second mode fluctuates, the map indicates that during the 1960's the two modes remain prominent. Further, from 1971 to 1974 another mode appears. This suggests that countries may be moving between the modes, a finding that is consistent with the results of the Silverman Test for 1971 and 1972. From 1975 on, the higher mode becomes more pronounced. This again is consistent with the Silverman Test results. Although not conclusive, the SiZer maps support the results of the Silverman tests suggesting that the second mode may have existed during the entire time period.

Table 2 displays the calculated values of the Dip Test statistic. We also show, for $N = 126$, the critical probability levels obtained by interpolation from the first table in Hartigan and Hartigan (1985, p. 80). The tests suggest that, for a majority of the years (at the 5the distribution is multimodal. These results are consistent with the hypothesis that the distribution of real GDP per capita has been multimodal for the entire period covered by the Penn World Table. During the years in which the test does not yield multimodality (1970–1975), the other two methods suggest more than two modes. If the Dip Test requires more mass (relative to the other tests) to signify a mode, then the additional modes seen during the multimodal period may have gone unnoticed by the test. Although the Dip Test alone cannot prove that the distribution was multimodal during the entire period, along with the other tests it provides convincing evidence of the veracity of this finding.

4. Conclusion

We have examined three methods of determining the number of modes in an empirical nonparametric kernel distribution, methods that appear to to be the dominant ones in the statistics literature. The Silverman Test calculates a critical window width and rejects or fails to reject the null hypothesis that an empirical distribution has k modes against the alternative of more than k modes. The graphical analysis of the SiZer approach examines the empirical distribution at various window widths to determine the strength and location of the modes of a distribution. The Dip Test measures the maximum difference between the empirical kernel distribution function and the unimodal distribution function that minimizes this maximum difference.

We have applied these methods to the question of international economic convergence, first analyzed using the Silverman test by Bianchi (1997). The Silverman test points toward the possibility of bimodalism throughout the sample period. This conclusion is supported by the SiZer map, which shows that, for a reasonable window width, at least two modes exist during the entire time period. The Dip Test provides additional evidence of perennial multimodality.

The possibility that the international distribution of real GDP per capita has always been multimodal might provide empirical support to the theoretical macroeconomic research on multiple equilibria.[9] This may be an interesting idea for macroeconomic researchers to pursue.

[9] See, *e.g.*, Azariadis (1981), Cass and Shell (1983), Benhabib and Farmer (1994), and Guo and Farmer (1994).

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Table 1 - Silverman Test

	k				k		
Year	1	2	3	Year	1	2	3
1961	25085	1587.8	1327.7	1974	4896.7	1794.1	1699.1
	0.12	0.36	0.18		0.01	0.76	0.73
1962	2365.5	1746.6	1402.8	1975	4776.7	1719.1	1708.8
	0.24	0.35	0.28		0.01	1.00	0.67
1963	2727.9	1865.7	1328.6	1976	4894.1	1830.0	1627.8
	0.07	0.18	0.43		0.01	0.80	0.73
1964	2755.7	1847.1	1415.1	1977	5173.3	1821.4	1390.3
	0.16	0.20	0.33		0.01	1.00	1.00
1965	2569.2	1795.4	1764.4	1978	5230.2	2083.2	1695.4
	0.26	0.46	0.08		0.01	0.84	0.89
1966	2966.5	1665.7	1554.7	1979	5514.5	1792.2	1436.1
	0.11	0.73	0.33		0.01	0.84	0.89
1967	3337.7	1957.7	1444.2	1980	5927.2	1811.1	1646.4
	0.06	0.40	0.50		0.00	0.89	0.57
1968	3324.8	1579.3	1554.3	1981	5887.7	1816.2	1543.1
	0.13	0.94	0.46		0.00	0.89	0.89
1969	3282.9	1948.7	1790.7	1982	6215.8	1829.1	1383.6
	0.09	0.52	0.25		0.00	0.89	0.89
1970	4020.8	2028.7	1845.9	1983	6215.8	1684.4	1396.4
	0.04	0.39	0.25		0.00	0.84	0.87
1971	4104.2	2428.6	2409.8	1984	6066.3	1504.4	1479.8
	0.04	0.18	0.01		0.00	1.00	0.84
1972	4179.7	2464.5	2385.6	1985	6329.3	1343.7	1280.9
	0.04	0.16	0.04		0.00	1.00	1.00
1973	4545.4	2231.0	2053.8	1986	6753.6	8246.8	1701.4
	0.03	0.42	0.20		0.00	0.84	0.84

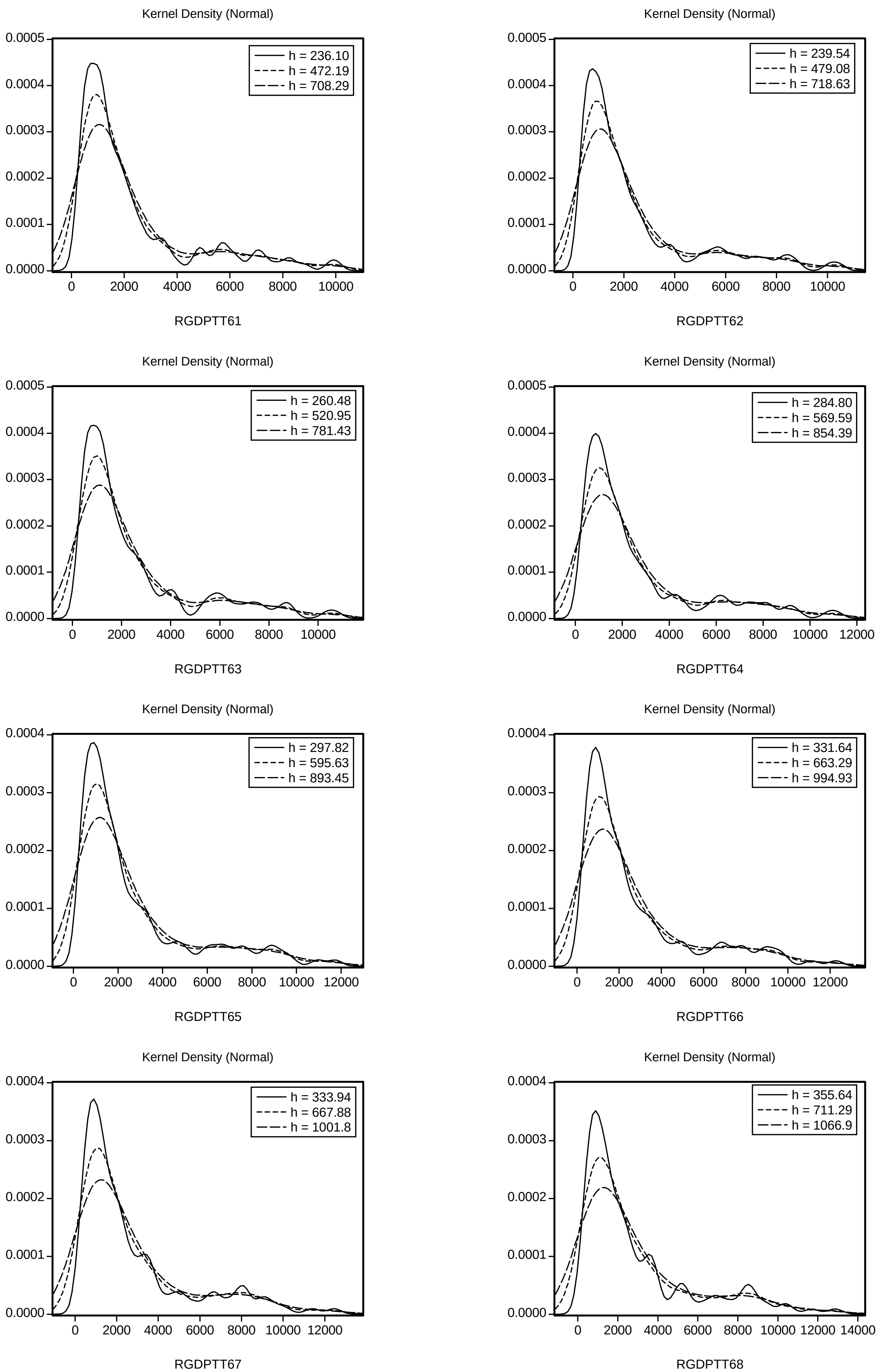
Table 2 - Dip Test

Year	DIP	x_L	x_U	Year	DIP	x_L	x_U
1961	0.0310	467	614	1974	0.0203	382	1284
1962	0.0249	353	602	1975	0.0215	524	963
1963	0.0251	389	603	1976	0.0241	434	1206
1964	0.0279	368	702	1977	0.0268	450	1162
1965	0.0331	351	618	1978	0.0242	435	1118
1966	0.0264	403	701	1979	0.0253	444	1208
1967	0.0261	379	770	1980	0.0305	438	1207
1968	0.0233	393	798	1981	0.0288	456	1140
1969	0.0261	399	850	1982	0.0363	464	960
1970	0.0211	608	862	1983	0.0287	460	1063
1971	0.0193	638	912	1984	0.0257	454	1063
1972	0.0196	625	914	1985	0.0243	473	824
1973	0.0212	727	751	1986	0.0318	498	926

probability of DIP less than tabled value is

sample size	0.01	0.05	0.10	0.50	0.90	0.95	0.99	0.995	0.999
126	0.0207	0.02332	0.02483	0.03434	0.04296	0.04662	0.05464	0.0565	0.0624

Figure 1



Kernel Density (Normal)
h = 236.10
h = 472.19
h = 708.29
0.0005
0.0004
0.0003
0.0002
0.0001
0.0000
0
2000
4000
6000
8000
10000
RGDPTT61
Kernel Density (Normal)
h = 239.54
h = 479.08
h = 718.63
0.0005
0.0004
0.0003
0.0002
0.0001
0.0000
0
2000
4000
6000
8000
10000
RGDPTT62
Kernel Density (Normal)
h = 260.48
h = 520.95
h = 781.43
0.0005
0.0004
0.0003
0.0002
0.0001
0.0000
0
2000
4000
6000
8000
10000
RGDPTT63
Kernel Density (Normal)
h = 284.80
h = 569.59
h = 854.39
0.0005
0.0004
0.0003
0.0002
0.0001
0.0000
0
2000
4000
6000
8000
10000
12000
RGDPTT64
Kernel Density (Normal)
h = 297.82
h = 595.63
h = 893.45
0.0004
0.0003
0.0002
0.0001
0.0000
0
2000
4000
6000
8000
10000
12000
RGDPTT65
Kernel Density (Normal)
h = 331.64
h = 663.29
h = 994.93
0.0004
0.0003
0.0002
0.0001
0.0000
0
2000
4000
6000
8000
10000
12000
RGDPTT66
Kernel Density (Normal)
h = 333.94
h = 667.88
h = 1001.8
0.0004
0.0003
0.0002
0.0001
0.0000
0
2000
4000
6000
8000
10000
12000
RGDPTT67
Kernel Density (Normal)
h = 355.64
h = 711.29
h = 1066.9
0.0004
0.0003
0.0002
0.0001
0.0000
0
2000
4000
6000
8000
10000
12000
14000
RGDPTT68

Figure 1 cont

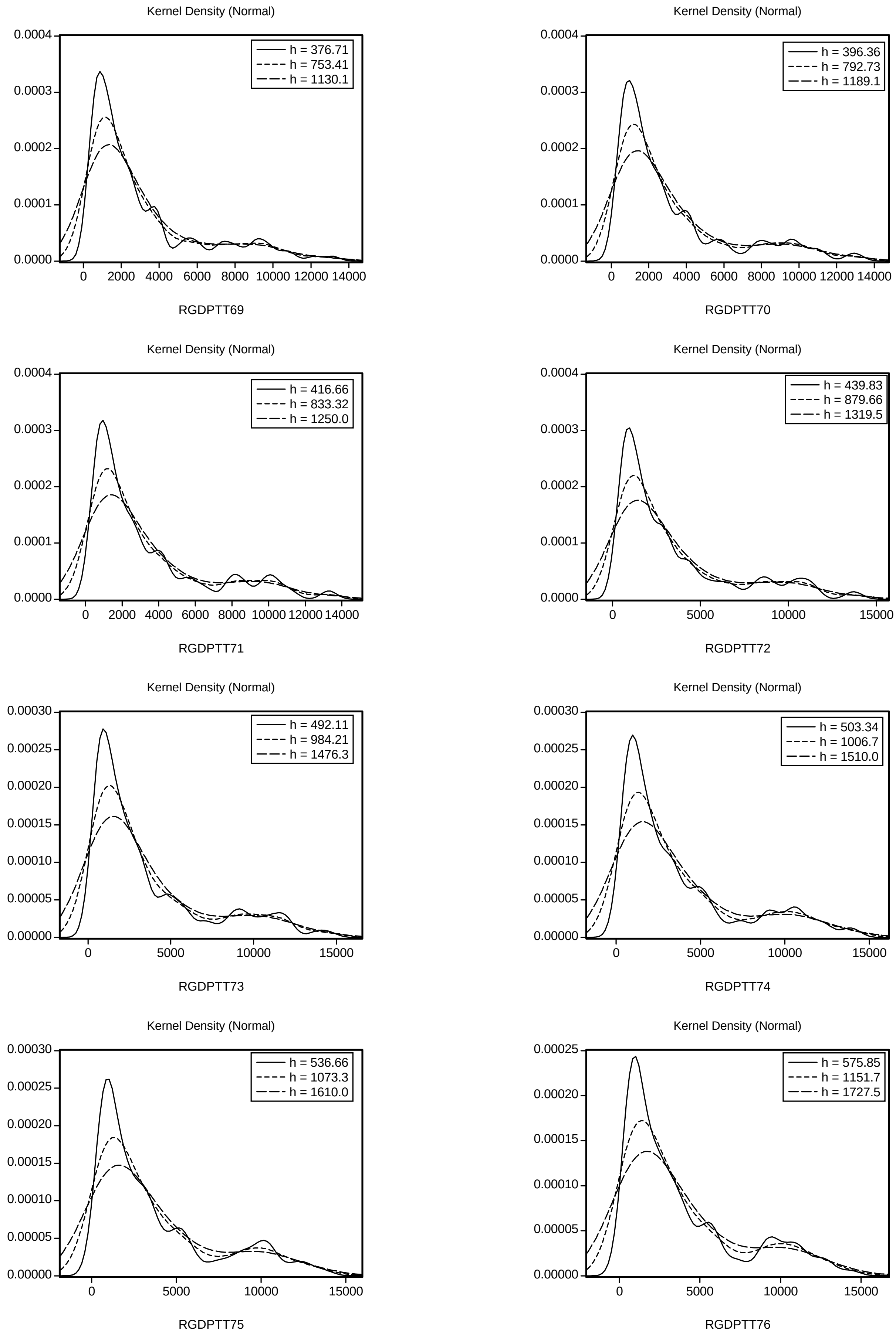


Figure 1 cont

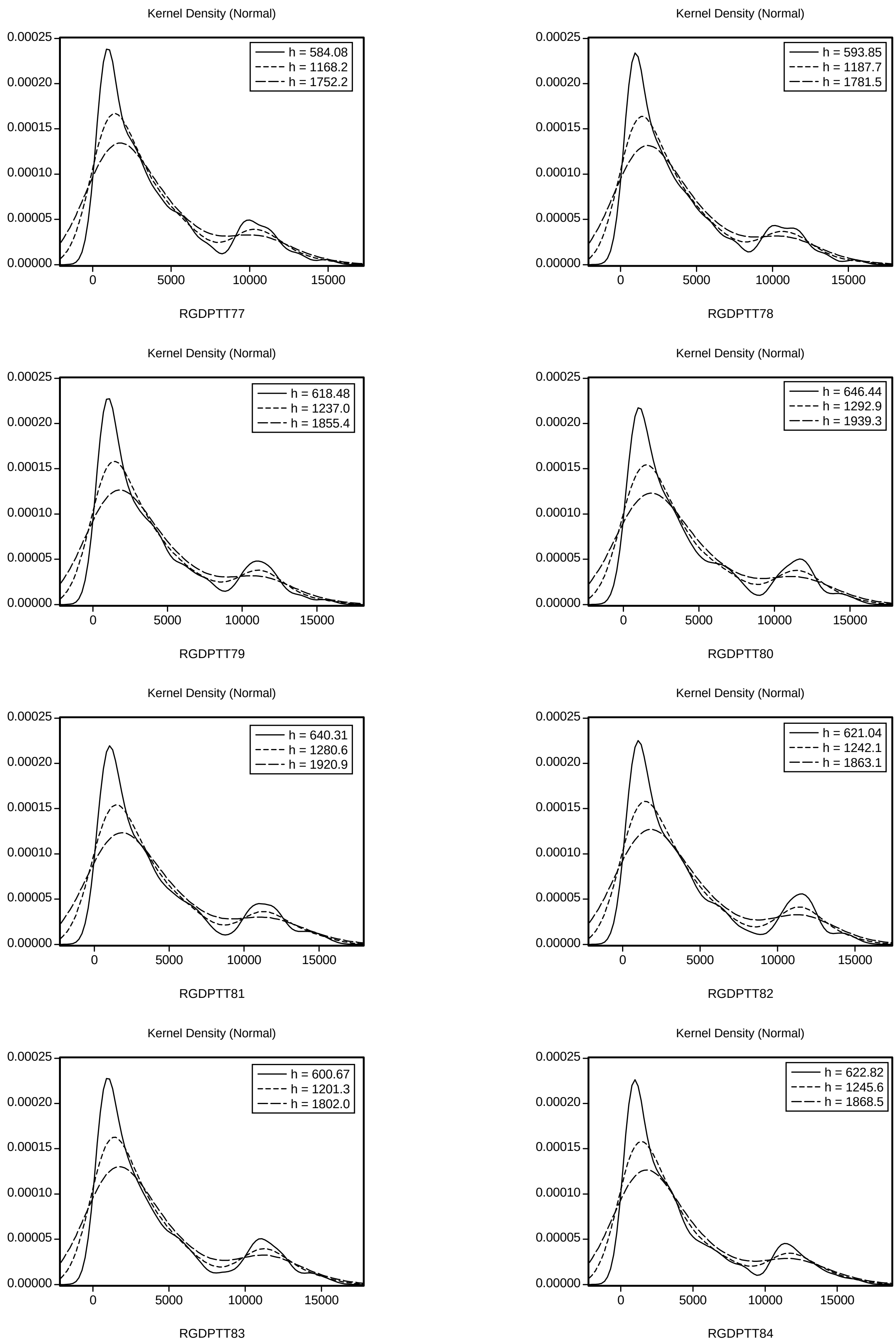
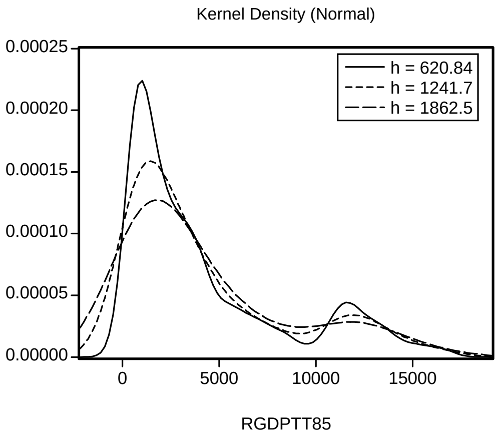


Figure 1 cont



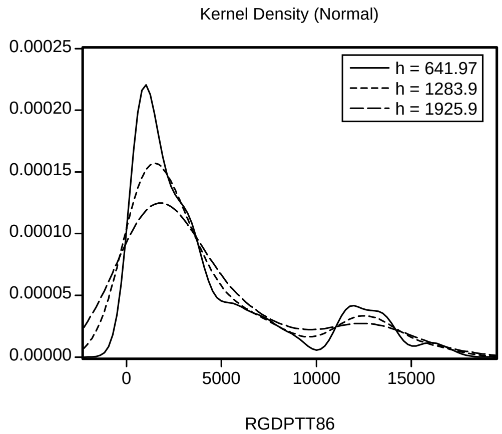
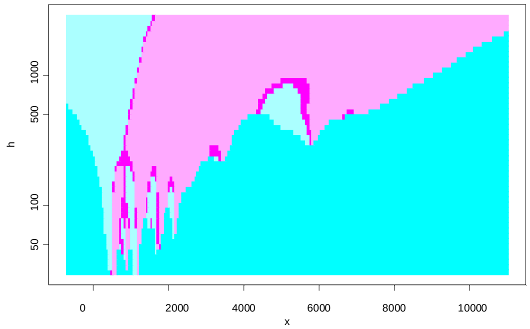


Figure 2



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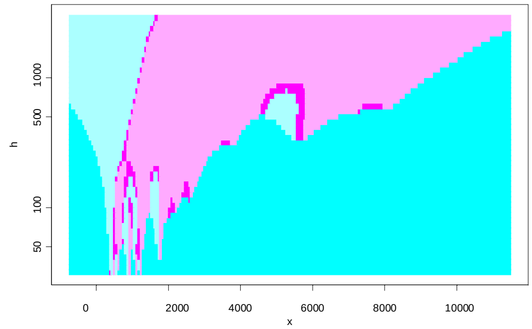

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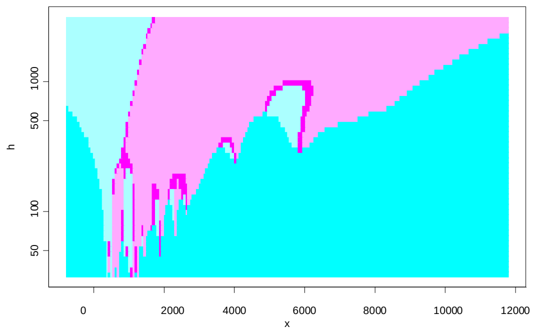

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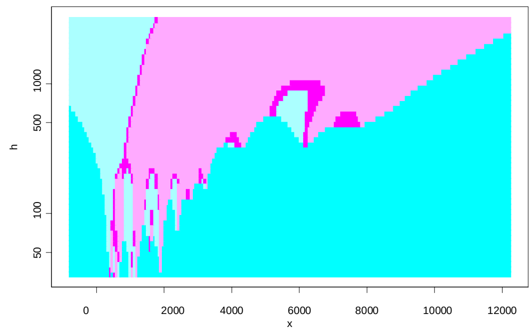

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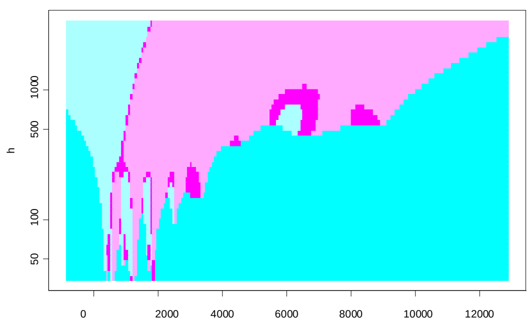

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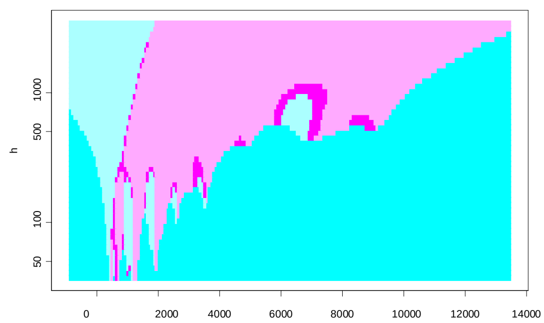

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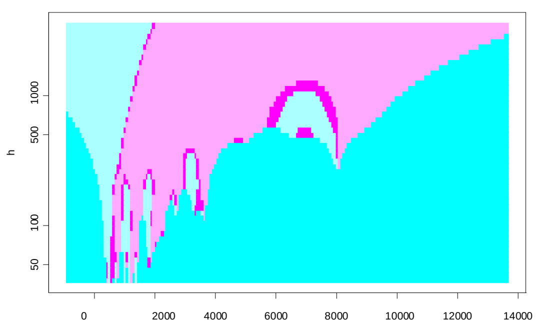

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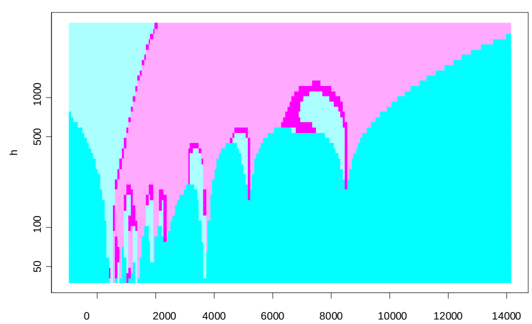

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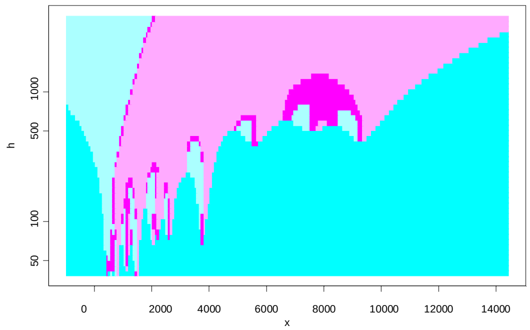

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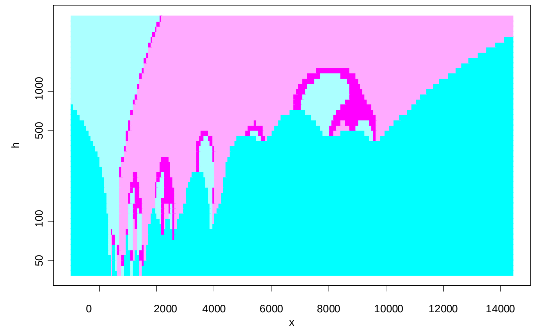

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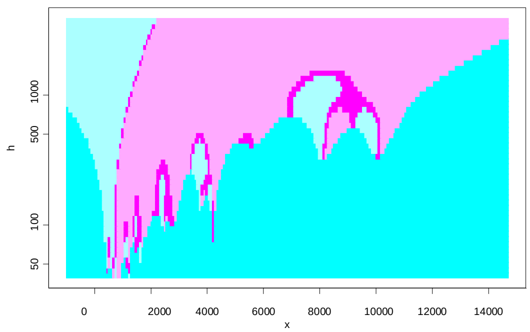

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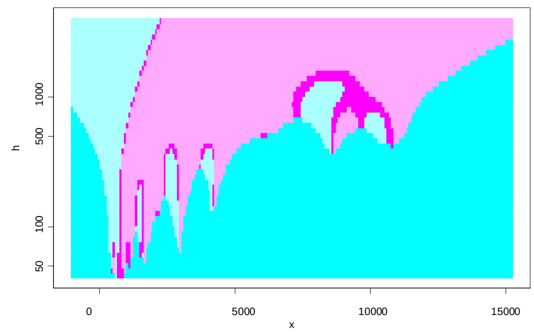

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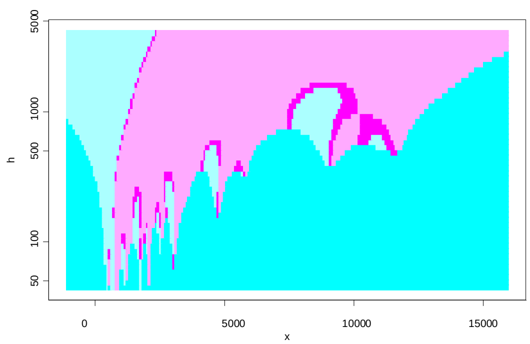

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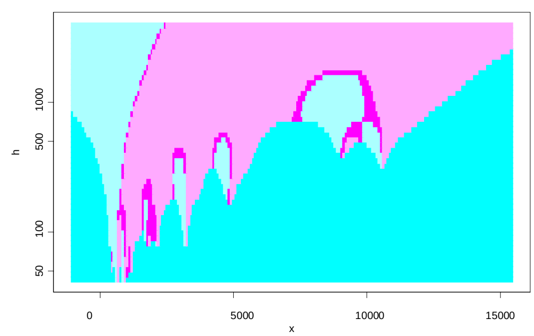

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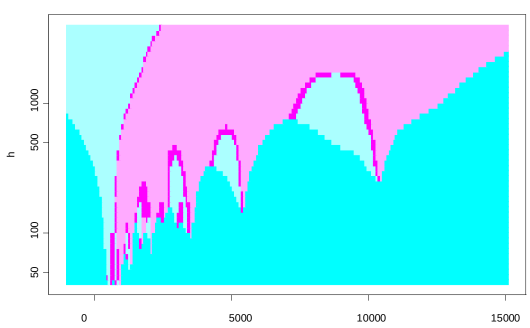

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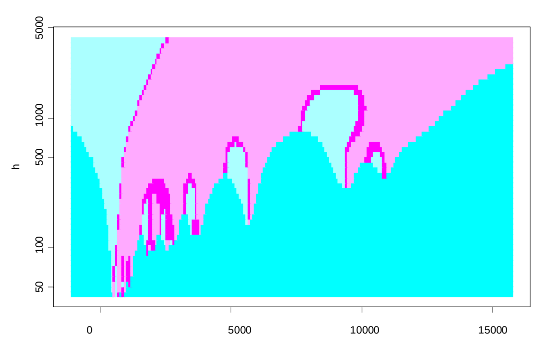

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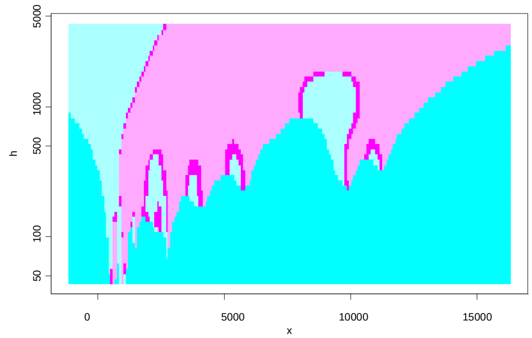

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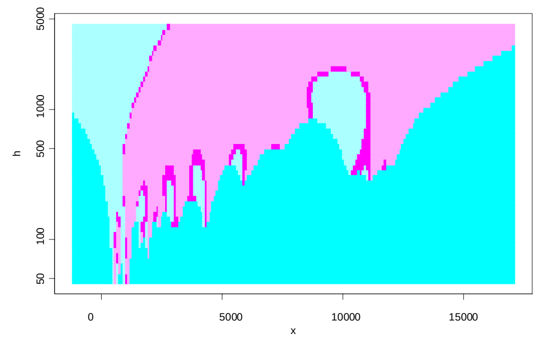

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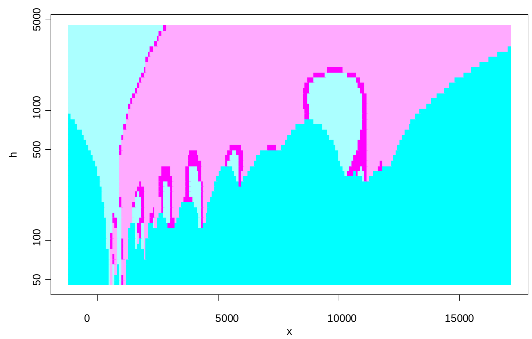

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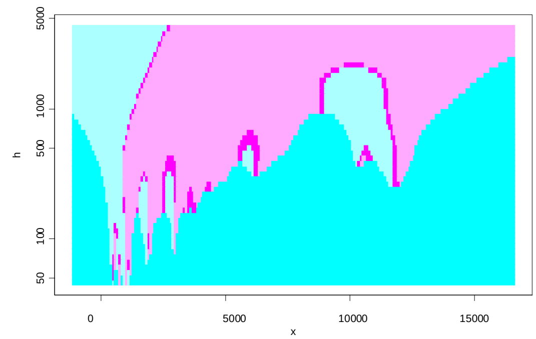

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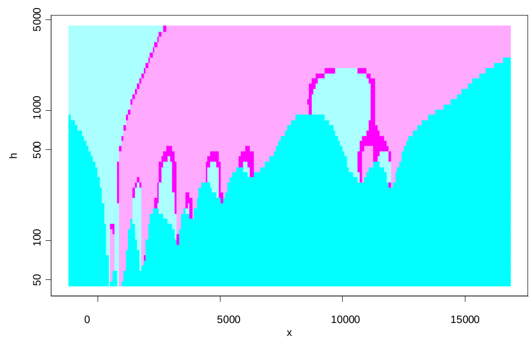

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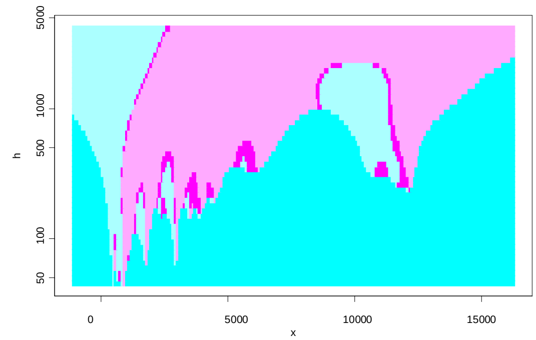

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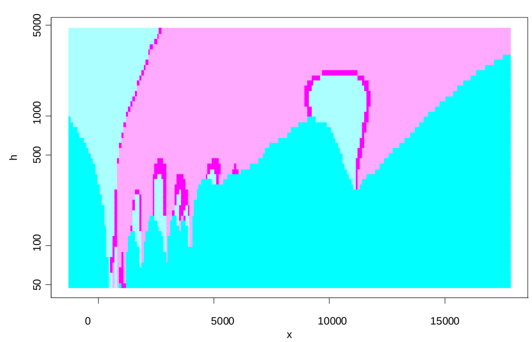

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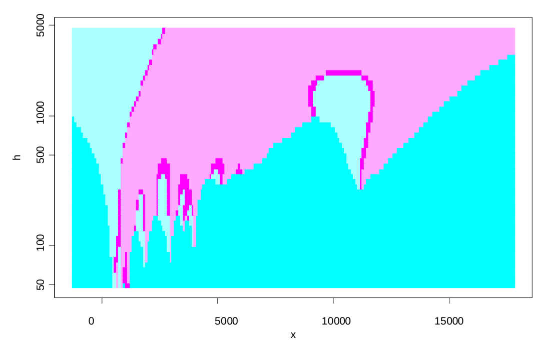
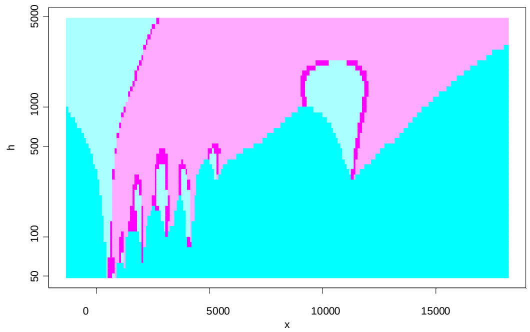


1984

Figure 2 cont



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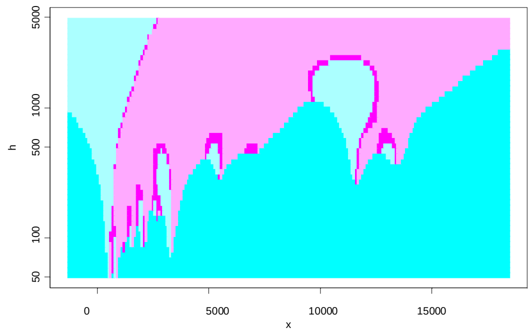

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