Introduction to

Information Retrieval

Lecture 5: Index Compression

Inverted index

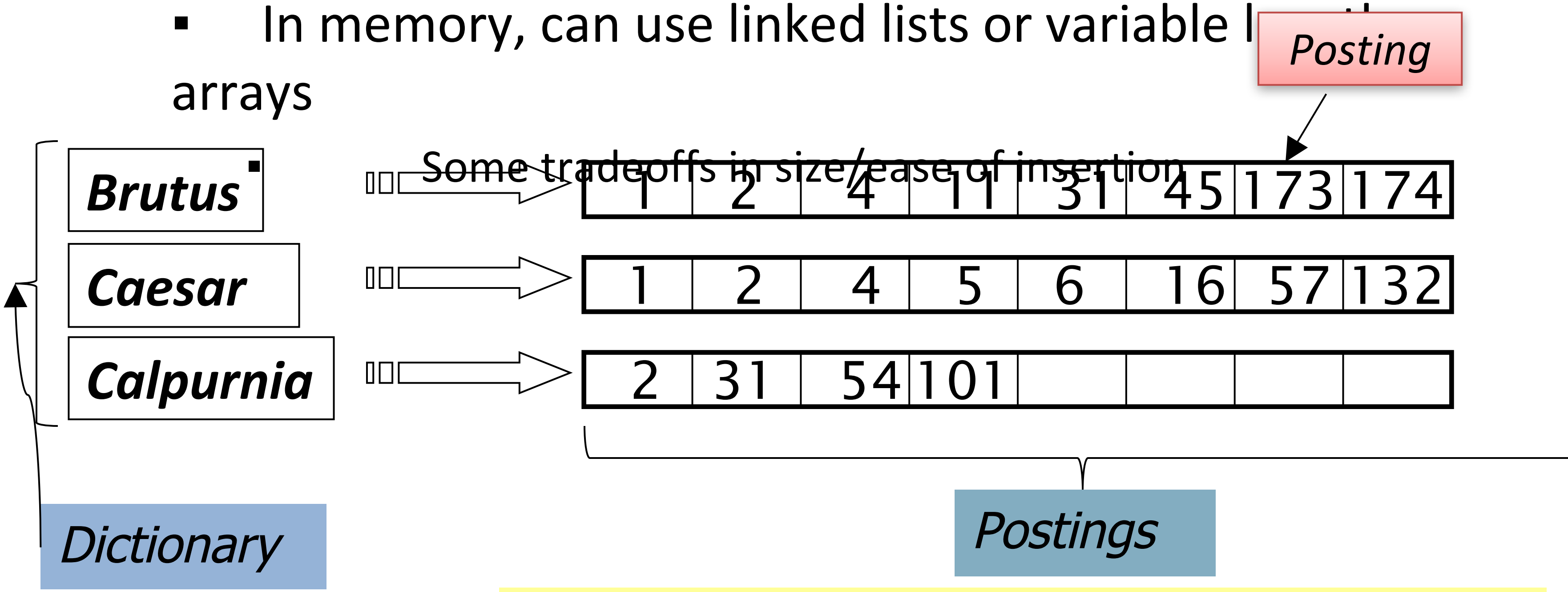
- For each term *t*, we must store a list of all documents that contain *t*.
  - Identify each doc by a **docID**, a document serial number
- Can we used fixed-size arrays for this?

Brutus →	1	2	4	11	31	45	173	174
Caesar →	1	2	4	5	6	16	57	132
Calpurnia →	2	31	54	101				

What happens if the word ***Caesar*** is added to document 14?

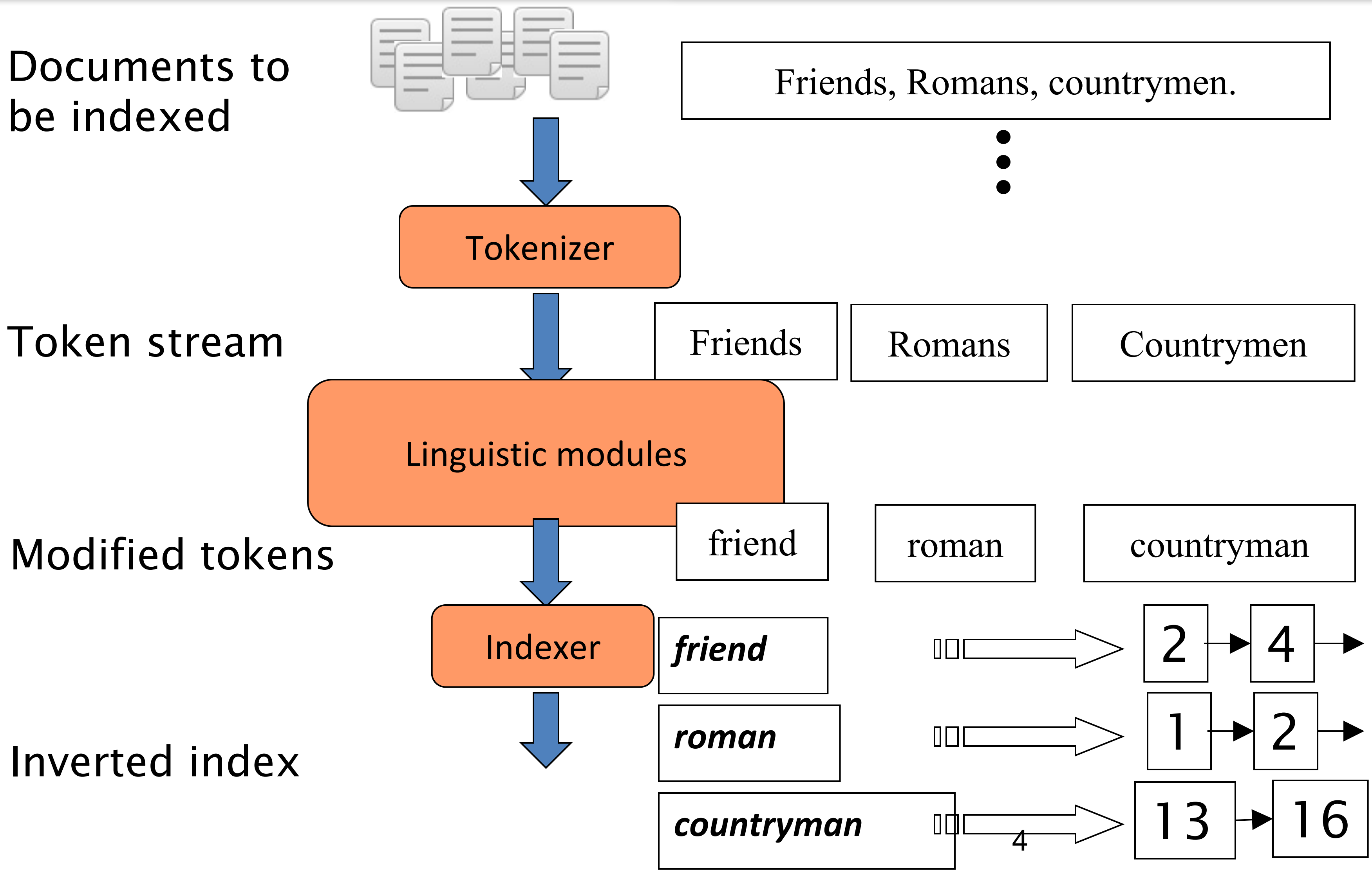
Inverted index

- We need variable-size postings lists
  - On disk, a continuous run of postings is normal and best
  - In memory, can use linked lists or variable length arrays
    - Some tradeoffs in size/ease of insertion



Sorted by docID (more later on why).

Inverted index construction



Where do we pay in storage?

term	doc. freq.	→	postings lists
ambitious	1	→	2
be	1	→	2
brutus	2	→	1 → 2
capitol	1	→	1
caesar	2	→	1 → 2
did	1	→	1
enact	1	→	1
hath	1	→	2
i	1	→	1
i'	1	→	1
it	1	→	2
julius	1	→	1
killed	1	→	1
let	1	→	2
me	1	→	1
noble	1	→	2
so	1	→	2
the	2	→	1 → 2
told	1	→	2
you	1	→	2
was	2	→	1 → 2
with	1	→	2

Terms and counts

Lists of docIDs

Pointers

Today

BRUTUS	$\longrightarrow$	1	2	4	11	31	45	173	174	
CAESAR	$\longrightarrow$	1	2	4	5	6	16	57	132	...
CALPURNIA	$\longrightarrow$	2	31	54	101					

- Why compression?
- Collection statistics in more detail (with RCV1)
  - How big will the dictionary and postings be?
- Dictionary compression
- Postings compression

Why compression (in general)?

- Use less disk space
  - Saves a little money
- Keep more stuff in memory
  - Increases speed
- Increase speed of data transfer from disk to memory
  - [read compressed data | decompress] is faster than [read uncompressed data]
  - Premise: Decompression algorithms are fast
    - True of the decompression algorithms we use

Why compression for inverted indexes?

- Dictionary
  - Make it small enough to keep in main memory
  - Make it so small that you can keep some postings lists in main memory too
- Postings file(s)
  - Reduce disk space needed
  - Decrease time needed to read postings lists from disk
  - Large search engines keep a significant part of the postings in memory.
    - Compression lets you keep more in memory
- We will devise various IR-specific compression schemes

RCV1: Our collection for this lecture

- Shakespeare's collected works definitely aren't large enough for demonstrating many of the points in this course.
- The collection we'll use isn't really large enough either, but it's publicly available and is at least a more plausible example.
- As an example for applying scalable index construction algorithms, we will use the Reuters RCV1 collection.
- This is one year of Reuters newswire (part of 1995 and 1996)

Reuters RCV1 statistics

symbol	value	statistic	
N	800,000	documents	
L	200	avg. # tokens per doc	
M	~400,000	terms (= word types)	
		avg. # bytes per token (incl. spaces/punct.)	6
		avg. # bytes per token (without spaces/punct.)	4.5
		avg. # bytes per term	7.5

Index parameters vs. what we index

(details *IIR* Table 5.1, p.80)

size of	word types (terms)			non-positional postings			positional postings		
	dictionary			non-positional index			positional index		
	Size (K)	∆%	cumul %	Size (K)	∆ %	cumul %	Size (K)	∆ %	cumul %
Unfiltered	484			109,971			197,879		
No numbers	474	-2	-2	100,680	-8	-8	179,158	-9	-9
Case folding	392	-17	-19	96,969	-3	-12	179,158	0	-9
30 stopwords	391	-0	-19	83,390	-14	-24	121,858	-31	-38
150 stopwords	391	-0	-19	67,002	-30	-39	94,517	-47	-52
stemming	322	-17	-33	63,812	-4	-42	94,517	0	-52

Exercise: give intuitions for all the ‘0’ entries. Why do some zero entries correspond to big deltas in other columns?

Lossless vs. lossy compression

- Lossless compression: All information is preserved.
  - What we mostly do in IR.
- Lossy compression: Discard some information
- Several of the preprocessing steps can be viewed as lossy compression: case folding, stop words, stemming, number elimination.
- Chap/Lecture 7: Prune postings entries that are unlikely to turn up in the top *k* list for any query.
  - Almost no loss quality for top *k* list.

Vocabulary vs. collection size

- How big is the term vocabulary?
  - That is, how many distinct words are there?
- Can we assume an upper bound?
  - Not really: At least $70^{20} = 10^{37}$ different words of length 20
- In practice, the vocabulary will keep growing with the collection size
  - Especially with Unicode ☺

Vocabulary vs. collection size

- Heaps' law: $M = kT^b$
- M is the size of the vocabulary, T is the number of tokens in the collection
- Typical values: $30 \leq k \leq 100$ and $b \approx 0.5$
- In a log-log plot of vocabulary size M vs. T, Heaps' law predicts a line with slope about ½
  - It is the simplest possible relationship between the two in log-log space
  - An empirical finding ("empirical law")

Heaps' Law

Fig 5.1 p81

For RCV1, the dashed line

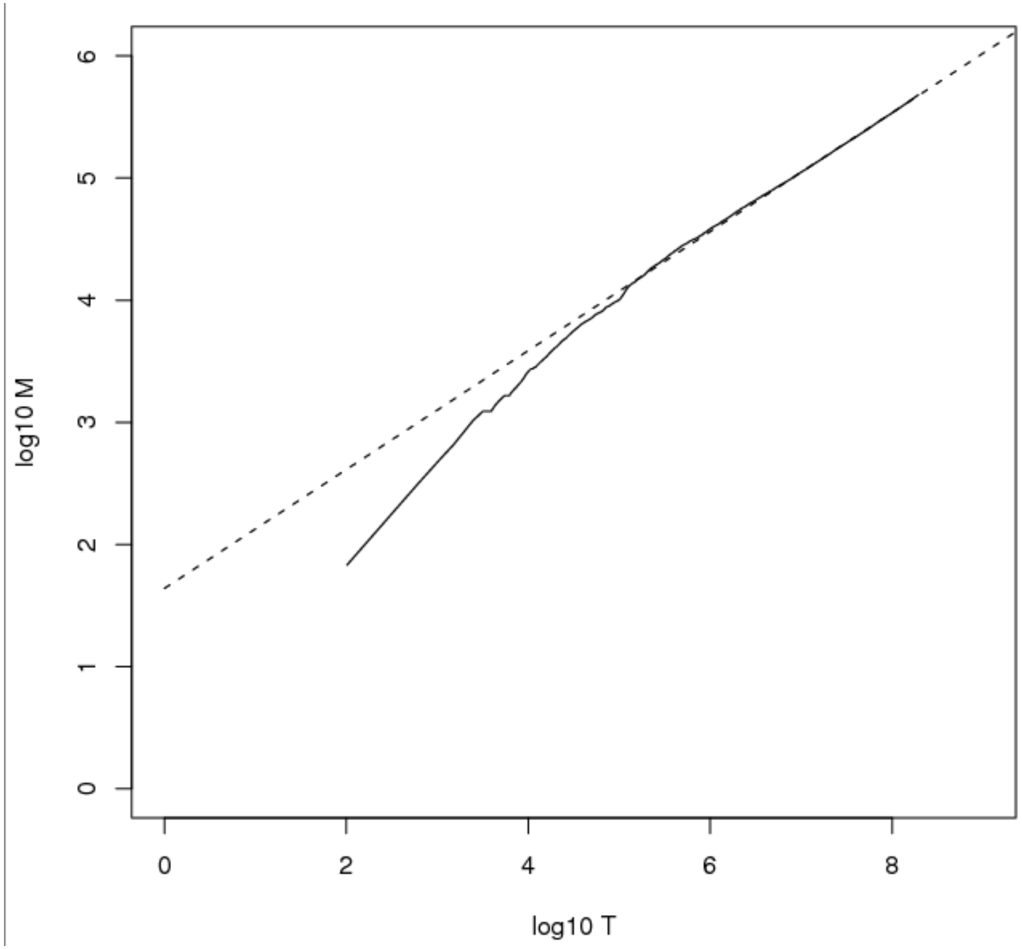
$\log_{10} M = 0.49 \log_{10} T + 1.64$

is the best least squares fit.

Thus, $M = 10^{1.64} T^{0.49}$ so $k = 10^{1.64} \approx 44$ and $b = 0.49$.

Good empirical fit for Reuters RCV1 !

For first 1,000,020 tokens, law predicts 38,323 terms; actually, 38,365 terms



Exercises

- What is the effect of including spelling errors, vs. automatically correcting spelling errors on Heaps' law?
- Compute the vocabulary size *M* for this scenario:
  - Looking at a collection of web pages, you find that there are 3000 different terms in the first 10,000 tokens and 30,000 different terms in the first 1,000,000 tokens.
  - Assume a search engine indexes a total of 20,000,000,000 (2×10^{10}) pages, containing 200 tokens on average
  - What is the size of the vocabulary of the indexed collection as predicted by Heaps' law?

Zipf's law

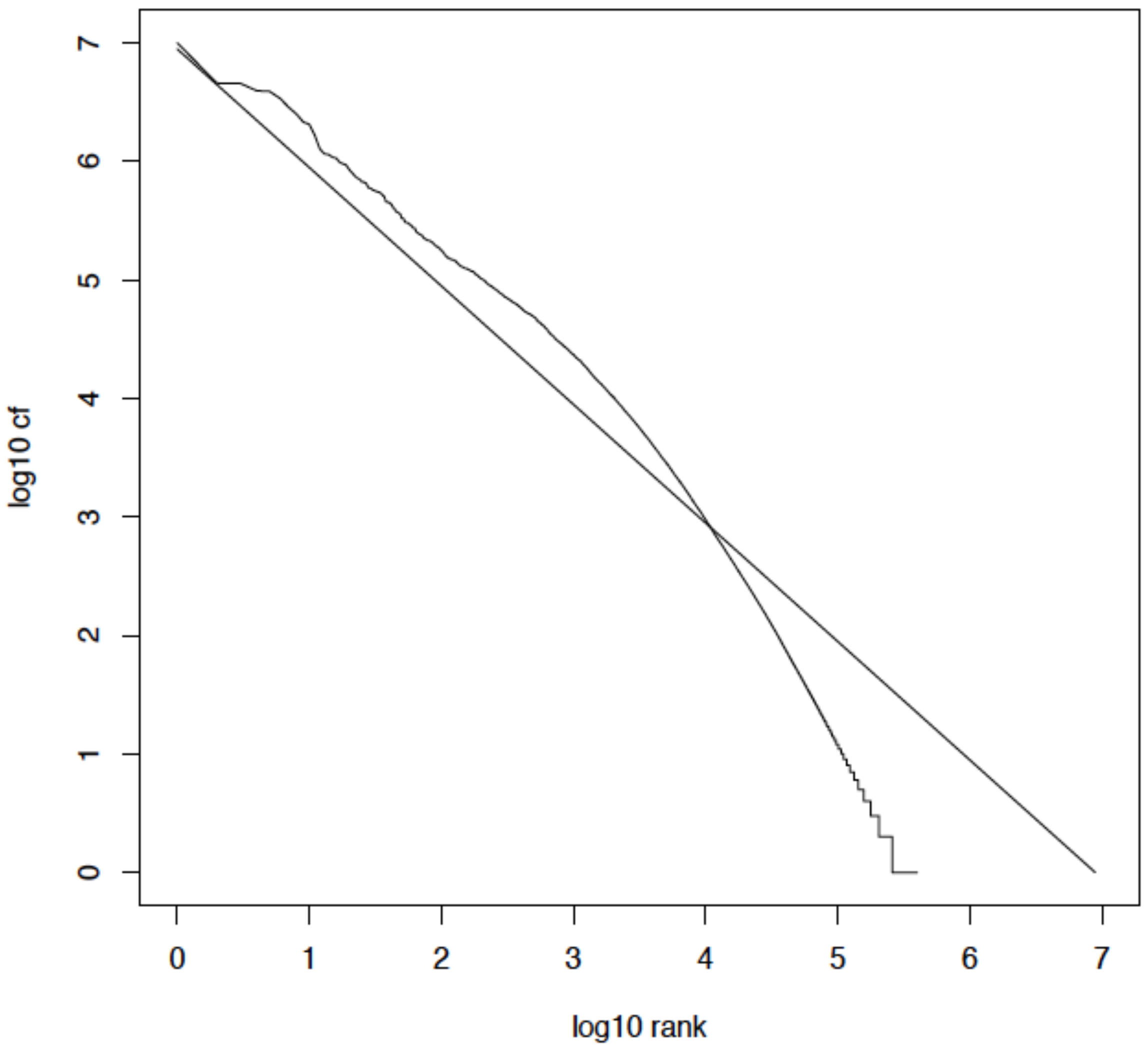
- Heaps' law gives the vocabulary size in collections.
- We also study the relative frequencies of terms.
- In natural language, there are a few very frequent terms and very many very rare terms.
- Zipf's law: The *i*th most frequent term has frequency proportional to $1/i$.
- $\mathrm{cf}_i \propto 1/i = K/i$ where K is a normalizing constant
- cf_i is collection frequency: the number of occurrences of the term t_i in the collection.

Zipf consequences

- If the most frequent term (*the*) occurs cf_1 times
  - then the second most frequent term (*of*) occurs $cf_1/2$ times
  - the third most frequent term (*and*) occurs $cf_1/3$ times …
- Equivalent: $cf_i = K/i$ where K is a normalizing factor, so
  - $\log cf_i = \log K - \log i$
  - Linear relationship between $\log cf_i$ and $\log i$

- Another power law relationship

Zipf's law for Reuters RCV1



Compression

- Now, we will consider compressing the space for the dictionary and postings
  - Basic Boolean index only
  - No study of positional indexes, etc.
  - We will consider compression schemes

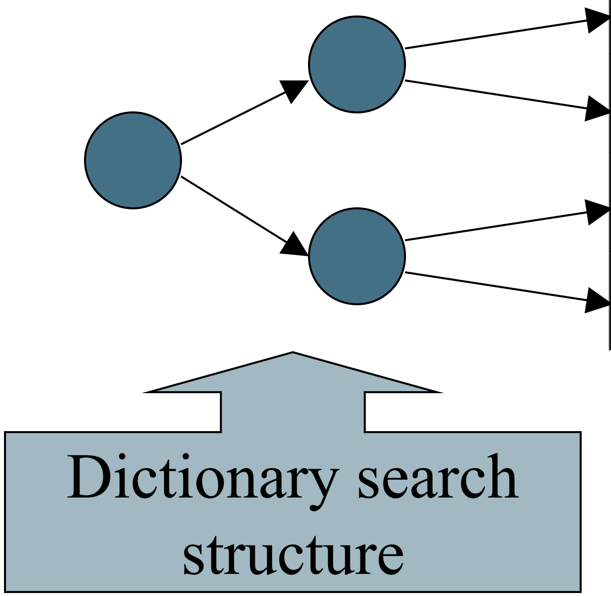
DICTIONARY COMPRESSION

Why compress the dictionary?

- Search begins with the dictionary
- We want to keep it in memory
- Memory footprint competition with other applications
- Embedded/mobile devices may have very little memory
- Even if the dictionary isn't in memory, we want it to be small for a fast search startup time
- So, compressing the dictionary is important

Dictionary storage - first cut

- Array of fixed-width entries
  - ~400,000 terms; 28 bytes/term = 11.2 MB.



Terms	Freq.	Postings ptr.
a	656,265	
aachen	65	
….	….	
zulu	221	

20 bytes

4 bytes each

Fixed-width terms are wasteful

- Most of the bytes in the **Term** column are wasted – we allot 20 bytes for 1 letter terms.
  - And we still can't handle *supercalifragilisticexpialidocious* or *hydrochlorofluorocarbons.*
- Written English averages ~4.5 characters/word.
  - Exercise: Why is/isn't this the number to use for estimating the dictionary size?
- Ave. dictionary word in English: ~8 characters
  - How do we use ~8 characters per dictionary term?
- Short words dominate token counts but not type average.

Compressing the term list: Dictionary-as-a-String

- Store dictionary as a (long) string of characters:
  - Pointer to next word shows end of current word
  - Hope to save up to 60% of dictionary space.

….systilesyzygeticsyzygialsyzygyszaibelyiteszczecinszomo….

Freq.	Postings ptr.	Term ptr.
33		
29		
44		
126		

Total string length = 400K x 8B = 3.2MB

Pointers resolve 3.2M positions: $\log_2 3.2M$ = 22bits = 3bytes

Space for dictionary as a string

- 4 bytes per term for Freq.
- 4 bytes per term for pointer to Postings.
- 3 bytes per term pointer
- Avg. 8 bytes per term in term string
- 400K terms x 19 → 7.6 MB (against 11.2MB for fixed width)

Now avg. 19 bytes/term, not 28.

Blocking

- Store pointers to every *k*th term string.
  - Example below: *k*=4.
- Need to store term lengths (1 extra byte)

….*7systile9syzygetic8syzygial6syzygy11szaibelyite8szczecin9szomo*….

Freq.	Postings ptr.	Term ptr.
33		
29		
44		
126		
7		

Save 9 bytes on 3 pointers.

Lose 4 bytes on term lengths.

Net

- Example for block size $k = 4$
- Where we used 3 bytes/pointer without blocking
  - 3 x 4 = 12 bytes,

now we use 3 + 4 = 7 bytes.

Shaved another ~0.5MB. This reduces the size of the dictionary from 7.6 MB to 7.1 MB.
We can save more with larger *k*.

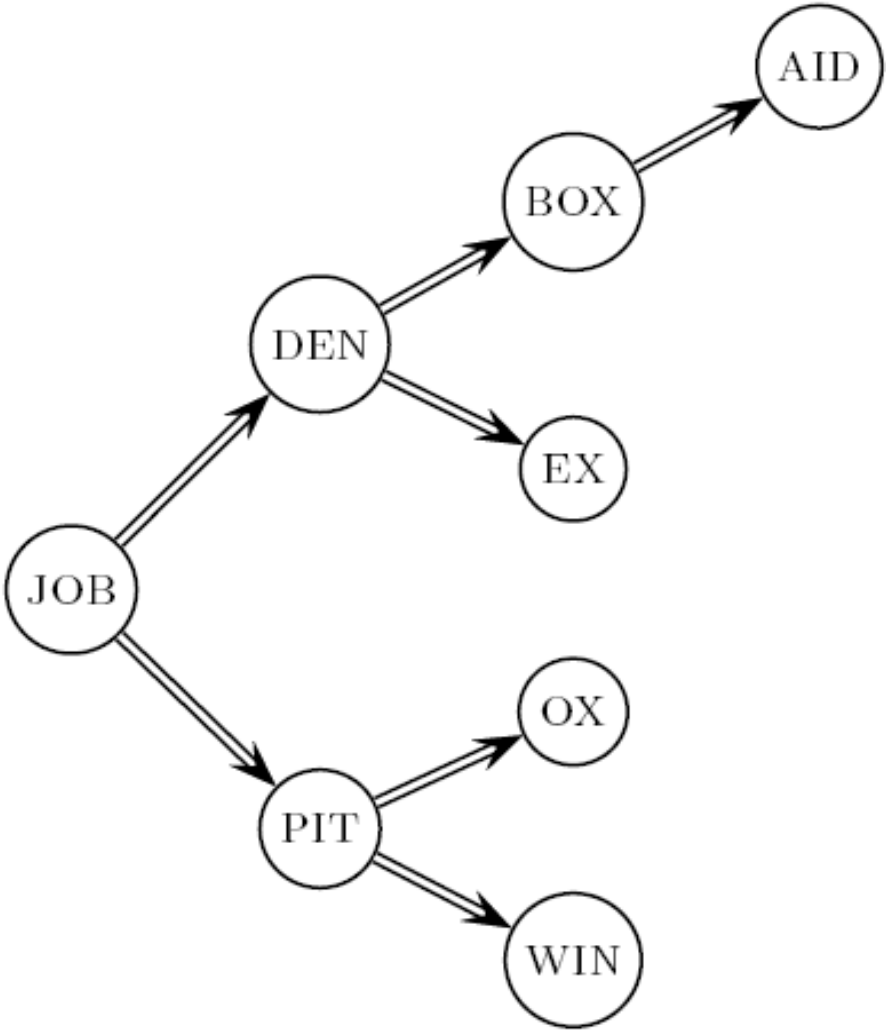
Why not go with larger *k*?

Exercise

- Estimate the space usage (and savings compared to 7.6 MB) with blocking, for block sizes of $k = 4, 8$ and 16.

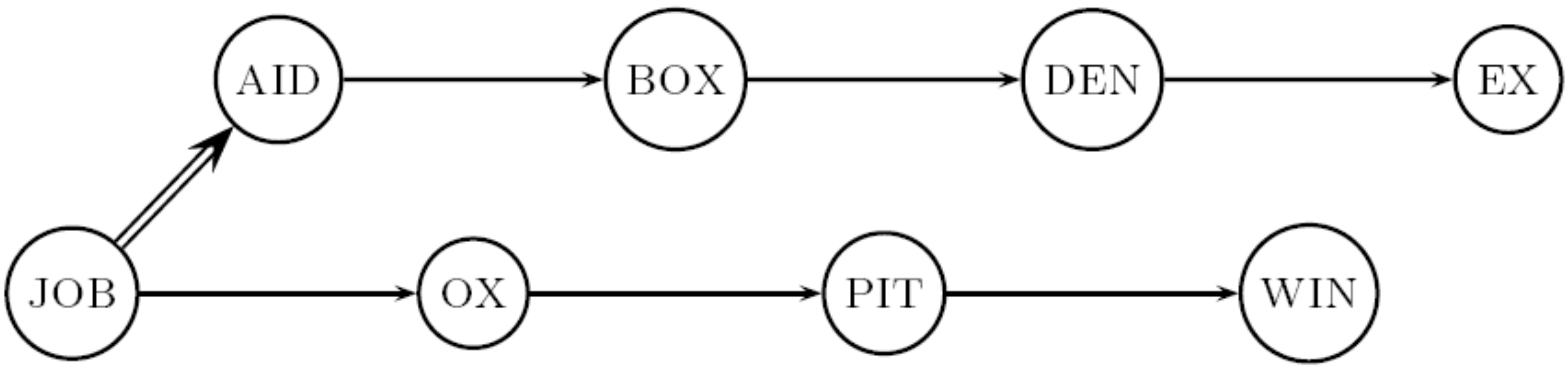
Dictionary search without blocking

- Assuming each dictionary term equally likely in query (not really so in practice!), average number of comparisons = $(1+2\cdot2+4\cdot3+4)/8 \sim 2.6$



Exercise: what if the frequencies of query terms were non-uniform but known, how would you structure the dictionary search tree?

Dictionary search with blocking



- Binary search down to 4-term block;
  - Then linear search through terms in block.
- Blocks of 4 (binary tree), avg. = $(1+2\cdot2+2\cdot3+2\cdot4+5)/8 = 3$ compares

Exercise

- Estimate the impact on search performance (and slowdown compared to k=1) with blocking, for block sizes of k = *4, 8* and *16.*

Front coding

- Front-coding:
  - Sorted words commonly have long common prefix – store differences only
  - (for last *k-1* in a block of *k*)

8***automata***8***automate***9***automatic***10***automation***

◊8***automat****a*1◊*e*2◊*ic*3◊*ion*

Encodes ***automat***

Extra length beyond ***automat.***

Begins to resemble general string compression.

Front Encoding [Witten, Moffat, Bell]

- Sorted words commonly have long common prefix
  - store differences only
- Complete front encoding
  - (prefix-len, suffix-len, suffix)
- Partial 3-in-4 front encoding
  - No encoding/compression for the first string in a block
  - Enables binary search

Assume previous string is “auto”

String	Complete Front Encoding	Partial 3-in-4 Front Encoding
8, automata	4, 4, mata	, 8, automata
8, automate	7, 1, e	7, 1, e
9, automatic	7, 2, ic	7, 2, ic
10, automation	8, 2, on	8, , on

RCV1 dictionary compression summary

Technique	Size in MB
Fixed width	11.2
Dictionary-as-String with pointers to every term	7.6
Also, blocking k = 4	7.1
Also, Blocking + front coding	5.9

POSTINGS COMPRESSION

Postings compression

- The postings file is much larger than the dictionary, factor of at least 10.
- Key desideratum: store each posting compactly.
- A posting for our purposes is a docID.
- For Reuters (800,000 documents), we would use 32 bits per docID when using 4-byte integers.
- Alternatively, we can use $\log_2 800{,}000 \approx 20$ bits per docID.
- Our goal: use a lot less than 20 bits per docID.

Postings: two conflicting forces

- A term like ***arachnocentric*** occurs in maybe one doc out of a million – we would like to store this posting using $\log_2$ 1M ~ 20 bits.
- A term like ***the*** occurs in virtually every doc, so 20 bits/posting is too expensive.
  - Prefer 0/1 bitmap vector in this case

Postings file entry

- We store the list of docs containing a term in increasing order of docID.
  - ***computer***: 33,47,154,159,202 …
- Consequence: it suffices to store *gaps*.
  - 33,14,107,5,43 …
- Hope: most gaps can be encoded/stored with far fewer than 20 bits.

Three postings entries

	encoding	postings list									
THE	docIDs	...		283042		283043		283044		283045	...
	gaps				1		1		1		...
COMPUTER	docIDs	...		283047		283154		283159		283202	...
	gaps				107		5		43		...
ARACHNOCENTRIC	docIDs	252000		500100							
	gaps	252000	248100								

Variable length encoding

- Aim:
  - For ***arachnocentric***, we will use ~20 bits/gap entry.
  - For ***the***, we will use ~1 bit/gap entry.
- If the average gap for a term is *G*, we want to use ~$\log_2 G$ bits/gap entry.
- Key challenge: encode every integer (gap) with about as few bits as needed for that integer.
- This requires a *variable length encoding*
- Variable length codes achieve this by using short codes for small numbers

Variable Byte (VB) codes

- For a gap value *G*, we want to use close to the fewest bytes needed to hold $\log_2 G$ bits
- Begin with one byte to store *G* and dedicate 1 bit in it to be a continuation bit *c*
- If $G \leq 127$, binary-encode it in the 7 available bits and set $c = 1$
- Else encode *G*'s lower-order 7 bits and then use additional bytes to encode the higher order bits using the same algorithm
- At the end set the continuation bit of the last byte to 1 ($c = 1$) – and for the other bytes $c = 0$.

Hex(824)=0x0338
Hex(214577)=0x00034631

Example

docIDs	824	829	215406
gaps		5	214577
VB code	00000110 10111000	10000101	00001101 00001100 10110001

Postings stored as the byte concatenation
000001101011100010000101000011010000110010110001

Key property: VB-encoded postings are uniquely prefix-decodable.

For a small gap (5), VB uses a whole byte.

Other variable unit codes

- Instead of bytes, we can also use a different "unit of alignment": 32 bits (words), 16 bits, 4 bits (nibbles).
- Variable byte alignment wastes space if you have many small gaps – nibbles do better in such cases.
- Variable byte codes:
  - Used by many commercial/research systems
  - Good low-tech blend of variable-length coding and sensitivity to computer memory alignment matches (vs. bit-level codes, which we look at next).
- There is also recent work on word-aligned codes that pack a variable number of gaps into one word (e.g., simple9)

Simple9

- Encodes as many gaps as possible in one DWORD
- 4 bit selector + 28 bit data bits
  - Encodes 9 possible ways to "use" the data bits

Selector	# of gaps encoded	Len of each gap encoded	Wasted bits
0000	28	1	0
0001	14	2	0
0010	9	3	1
0011	7	4	0
0100	5	5	3
0101	4	7	0
0110	3	9	1
0111	2	14	0
1000	1	28	0

Unary code

- Represent *n* as *n* 1s with a final 0.
- Unary code for 3 is 1110.
- Unary code for 40 is
  11111111111111111111111111111111111111110 .
- Unary code for 80 is:
  111111111111111111111111111111111111111111
  111111111111111111111111111111111111110

- This doesn't look promising, but....

Bit-Aligned Codes

- Breaks between encoded numbers can occur after any bit position
- *Unary* code
  - Encode *k* by *k* 1s followed by 0
  - 0 at end makes code unambiguous

Number	Code
0	0
1	10
2	110
3	1110
4	11110
5	111110

Unary and Binary Codes

- Unary is very efficient for small numbers such as 0 and 1, but quickly becomes very expensive
  - 1023 can be represented in 10 binary bits, but requires 1024 bits in unary
- Binary is more efficient for large numbers, but it may be ambiguous

Elias-γ Code

- To encode a number *k*, compute
  - $k_d = \lfloor \log_2 k \rfloor$
  - $k_r = k - 2^{\lfloor \log_2 k \rfloor}$
  - k_d is number of binary digits, encoded in unary

unary

binary

Number (k)	k_d	k_r	Code
1	0	0	0
2	1	0	10 0
3	1	1	10 1
6	2	2	110 10
15	3	7	1110 111
16	4	0	11110 0000
255	7	127	11111110 1111111
1023	9	511	1111111110 111111111

Elias-δ Code

- Elias-γ code uses no more bits than unary, many fewer for $k > 2$
  - 1023 takes 19 bits instead of 1024 bits using unary
- In general, takes $2\lfloor \log_2 k \rfloor + 1$ bits
- To improve coding of large numbers, use Elias-δ code
  - Instead of encoding k_d in unary, we encode $k_d + 1$ using Elias-γ
  - Takes approximately $2 \log_2 \log_2 k + \log_2 k$ bits

Elias-δ Code

- Split $(k_d + 1)$ into:

$$k_{dd} = \lfloor \log_2(k_d + 1) \rfloor$$

$$k_{dr} = (k_d + 1) - 2^{\lfloor \log_2(k_d + 1) \rfloor}$$

  - encode k_{dd} in unary, k_{dr} in binary, and k_r in binary

Number (k)	k_d	k_r	k_{dd}	k_{dr}	Code
1	0	0	0	0	0
2	1	0	1	0	10 0 0
3	1	1	1	0	10 0 1
6	2	2	1	1	10 1 10
15	3	7	2	0	110 00 111
16	4	0	2	1	110 01 0000
255	7	127	3	0	1110 000 1111111
1023	9	511	3	2	1110 010 111111111

```
#
# Generating Elias-gamma and Elias-delta codes in Python
#

import math

def unary_encode(n):
 return "1" * n + "0"

def binary_encode(n, width):
    r = ""
    for i in range(0,width):
     if ((1<<i) & n) > 0:
     r = "1" + r
     else:
     r = "0" + r
    return r

def gamma_encode(n):
    logn = int(math.log(n,2))
    return unary_encode( logn ) + " " + binary_encode(n, logn)

def delta_encode(n):
 logn = int(math.log(n,2))
if n == 1:
 return "0"
 else:
 loglog = int(math.log(logn+1,2))
 residual = logn+1 - int(math.pow(2, loglog))
        return unary_encode( loglog ) + " " + binary_encode( residual, loglog ) + " " + binary_encode(n, logn)

if __name__ == "__main__":
    for n in [1,2,3, 6, 15,16,255,1023]:
        logn = int(math.log(n,2))
        loglogn = int(math.log(logn+1,2))
        print n, "d_r", logn
        print n, "d_dd", loglogn
        print n, "d_dr", logn + 1 - int(math.pow(2,loglogn))
        print n, "delta", delta_encode(n)
        #print n, "gamma", gamma_encode(n)
        #print n, "binary", binary_encode(n)
```

Gamma code properties

- G is encoded using $2 \lfloor \log G \rfloor + 1$ bits
  - Length of offset is $\lfloor \log G \rfloor$ bits
  - Length of length is $\lfloor \log G \rfloor + 1$ bits
- All gamma codes have an odd number of bits
- Almost within a factor of 2 of best possible, $\log_2 G$

- Gamma code is uniquely prefix-decodable, like VB
- Gamma code can be used for any distribution
- Gamma code is parameter-free

Gamma seldom used in practice

- Machines have word boundaries – 8, 16, 32, 64 bits
  - Operations that cross word boundaries are slower
- Compressing and manipulating at the granularity of bits can be slow
- Variable byte encoding is aligned and thus potentially more efficient
- Regardless of efficiency, variable byte is conceptually simpler at little additional space cost

Shannon Limit

- Is it possible to derive codes that are optimal (under certain assumptions)?
- What is the optimal average code length for a code that encodes each integer (gap lengths) independently?
- Lower bounds on average code length: Shannon entropy
  - $H(X) = - \Sigma_{x=1}^{n} \Pr[X=x] \log \Pr[X=x]$
- Asymptotically optimal codes (finite alphabets): arithmetic coding, Huffman codes

How to design an optimal code for geometric distribution?

Global Bernoulli Model

- Assumption: term occurrence are Bernoulli events
- Notation:
  - n: # of documents, m: # of terms in vocabulary
  - N: total # of (unique) occurrences
- Probability of a term t_j occurring in document d_i: p = N/nm
- Each term-document occurrence is an independent event
- Probability of a gap of length x is given by the geometric distribution $\Pr[X = x] = (1-p)^{x-1} \cdot p$

It can also be deemed as a generalization of the unary code.

Golomb Code

- Golomb Code (Golomb 1966): highly efficient way to design optimal Huffman-style code for geometric distribution
  - Parameter b
  - For given x ≥ 1, computer integer quotient
  - and remainder

$$q = \lfloor (x-1)/b \rfloor$$
$$r = (x-1) - q \cdot b$$

- Assume b = 2^k
  - Encode q in unary, followed by r coded in binary
  - A bit complicated if b != 2^k. See wikipedia.
- First step: (q+1) bits
- Second step: log(b) bits

Golomb Code & Rice Code

- How to determine optimal b*?
- Select minimal b such that

$$(1-p)^{b} + (1-p)^{b+1} \leq 1$$

- Result due to Gallager and Van Voorhis 1975: generates an optimal prefix code for geometric distribution
- Small p approximation:

$$b^{*} \approx \ln 2/p = 0.69 \cdot avg\_val$$

- Rice code: only allow b = 2^k

Local Bernoulli Model

- If length of posting lists is known, then a Bernoulli model on each individual inverted list can be used
- Frequent words are coded with smaller b, infrequent words with larger b
- Term frequency need to be encoded (use gamma-code)
- Local Bernoulli outperforms global Bernoulli model in practice (method of practice!)

RCV1 compression

Data structure	Size in MB
dictionary, fixed-width	11.2
dictionary, term pointers into string	7.6
with blocking, k = 4	7.1
with blocking & front coding	5.9
collection (text, xml markup etc)	3,600.0
collection (text)	960.0
Term-doc incidence matrix	40,000.0
postings, uncompressed (32-bit words)	400.0
postings, uncompressed (20 bits)	250.0
postings, variable byte encoded	116.0
postings, γ–encoded	101.0

Google's Indexing Choice

- Index shards partition by doc, multiple replicates
- Disk-resident index
  - Use outer parts of the disk
  - Use different compression methods for different fields: Rice_k (a special kind of Golomb code) for gaps, and Gamma for positions.
- In-memory index
  - All positions; No docid
    - Keep track of document boundaries
  - Group-variant encoding
    - Fast to decode

Source: [Jeff Dean's WSDM 2009 Keynote]

Other details

- Gap = $docid_n$- $docid_{n-1}$ **- 1**
- Freq = freq – 1
- Pos_Gap = pos_n- pos_{n-1} - 1

- C.f., Jiangong Zhang, Xiaohui Long and Torsten Suel: Performance of Compressed Inverted List Caching in Search Engines. WWW 2008.

Index compression summary

- We can now create an index for highly efficient Boolean retrieval that is very space efficient
- Only 4% of the total size of the collection
- Only 10-15% of the total size of the text in the collection
- However, we've ignored positional information
- Hence, space savings are less for indexes used in practice
  - But techniques substantially the same.

Resources for today's lecture

- *IIR* 5
- *MG* 3.3, 3.4.
- F. Scholer, H.E. Williams and J. Zobel. 2002. Compression of Inverted Indexes For Fast Query Evaluation. *Proc. ACM-SIGIR 2002*.
  - Variable byte codes
- V. N. Anh and A. Moffat. 2005. Inverted Index Compression Using Word-Aligned Binary Codes. *Information Retrieval* 8: 151–166.
  - Word aligned codes