### **Computational Finance**

## [35 marks:]

Please write a C++ programme that performs the pricing of the following structured note. It is a note dependent on two interest rates, *IR1* and *IR2* (that could correspond to two different points on eg the GBP curve), and an exchange rate (eg EUR/USD) denoted *G*. The note pays, at the close of its maturity day *T*:

$$[G_T-K]^+$$
 .  $\tau/T$ 

where  $\tau$ .360 is the number of days where, at the close, the difference (in absolute value) between *IR1* and *IR2* is (strictly) lower than: 25% \* min{IR1,IR2}. (We follow the convention *ACTUAL/360* for time measurement.) You may assume that all three stochastic processes follow geometric Brownian motions. Without loss of generality you may assume that the two IRs have equal initial values.

In addition to providing basic pricing, your programme should facilitate sensitivity analysis.

Please write a report (and please make it as concise and to-the-point as possible, and please make it use itemising/enumerating whenever possible) that:

#### [35 marks:]

#### Part One:

- 0) Starts by a brief introduction focusing in particular on any differences/divergences from the inclass approach; and then
- 1) Presents a decent illustration of the effect on the price of both:
  - The correlation between the two IR rates; and
  - o The volatilities of the IR rates

And please comment on the intuition behind your findings.

- 2) Lists, and briefly explains, the (top three) value-adding elements (eg in providing extra sophistication/accuracy) of your code, compared to the in-class programme;
- 3) Lists, and briefly explains, the main (say, three) approximations to (or simplifications of) reality, that you have resorted to;
- 4) Lists, and briefly explains, the main (say, three) opportunities for future work that remain.

# [30 marks:]

#### Part Two:

Then please answer the following questions. They touch on the broad area you have worked on above. Please give succinct (rather than lengthy) answers.

a) In your coding above, have you identified a shortcut, that provides a great efficiency compared to a brute force method? If so, what is it?

- b) What are the main disadvantages of using geometric Brownian motion to model interest rates?
- c) Can you mention a related note (to the one above) that happens to have the opposite correlation profile? (Please give the corresponding payoff.)

For the remaining questions, please indicate whether True or False. (Importantly, please include a convincing and targeted explanation of why.)

- d) If your implementation of the explicit scheme of finite difference methods is currently stable, it then follows if you halve your time step (*delta\_t*), your scheme will definitely remain stable.
- e) If you double the number of paths (simulations) in your (pseudorandom number) Monte Carlo, you have halved the likely Monte Carlo error (ie you have doubled the accuracy).
- f) Let  $Z_1$  and  $Z_2$  be two independent N(0,1) random variables. It then follows that  $2Z_1-3Z_2$  and  $3Z_1+2Z_2$  are identically distributed.

(Remember that notation N(a, b) means normal with mean a and variance b.)

#### NB:

- Please submit both the code (source file) and the report (the latter should preferably be in Word format) plus any spreadsheet you used. You should also include the code (source file) from your C++ project as an appendix (preserving the default C++ Editor colours) to your report.
- In your report's answers, please use the same numbering used in the questions above.
- Please make sure that any (academic or other) sources are properly referenced.
- [To be confirmed:] Please note that the Keats submission facility is likely to permit you to submit only a maximum of three files.
- And finally: Please submit well ahead of the deadline. (Problems that arose for late submitters in the past include, among many other incidents, discovering that their login/password had expired. It only took them a few minutes to revive them, but because they left it so close to the deadline, they ended up missing the cut-off time. There are plenty of other incidents reported by later submitters an unexpected loss of internet connection, etc, etc so please do not treat the deadline as a target. Rather, it is a cliff edge to stay well clear of.)