ECON 3350/7350
Univariate Time Series - I

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Tutorial 2

Stationarity

When a single realisation of observations is available, the aggregation of observations over time is important.

Definition

A stochastic process is **stationary** if the data generating process is such that the mean, variance and covariances are independent of time.

$E(y_t) = \mu$

$Var(y_t) = E[(y_t - \mu)^2] = \sigma_y^2 = \gamma_0 \; Cov(y_t, y_{t-k}) = E[(y_t - \mu)((y_{t-k} - \mu)] = \gamma_k$

$k = 1, 2, ...$

These conditions must be satisfied for all values of t.

At this initial stage we will only consider stationary processes and we will relax this assumption later in the course.

Autoregressive (AR) Models

To model the dependence in y_t upon its own past behaviour.
AR(1)

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_t$$

AR(2)

$$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t$$

AR(p)

$$y_t = a_0 + a_1 y_{t-1} + ... + a_p y_{t-p} + \varepsilon_t$$

$$y_t = a_0 + \sum_{i=1}^{p} a_i y_{t-i} + \varepsilon_t$$

Moving Average (MA) Models

- MA(1)

$$y_t = \mu + \varepsilon_t + \beta_1 \varepsilon_{t-1}$$

- MA(2)

$$y_t = \mu + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2}$$

- MA(q)

$$y_t = \mu + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \ldots + \beta_q \varepsilon_{t-q}$$

The Autoregressive Moving Average Model (ARMA)

- ARMA$(1,1)$

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_t + \beta_1 \varepsilon_{t-1}$$

- ARMA$(2,2)$

$$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2}$$

- ARMA$(3,1)$

$$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + a_3 y_{t-3} + \varepsilon_t + \beta_1 \varepsilon_{t-1}$$

Autocovariance and Autocorrelation

Definitions

- $Var(y_t) = E[(y_t - \mu)^2] = \gamma_0$
- $Cov(y_t, y_{t-k}) = E[(y_t - \mu)(y_{t-k} - \mu)] = \gamma_k \; k = 1, 2, ...$

Definitions

Autocovariance Function: γ_k, $k = 1, 2, ...$ If the process is stationary $\gamma_k = \gamma_{-k}$.

Autocorrelation Function (ACF): $\rho_k = \frac{\gamma_k}{\gamma_0}$, $k = 1, 2, ...$

Correlogram or **SACF**: Plot of the sample autocorrelation function, r_k, against k.

Partial Autocorrelation Function (PACF)

Definition

The **partial autocorrelation function** (ϕ_{kk}) is given by the kth coefficients in the corresponding $AR(k)$ system of autoregressions.

$AR(1)$ $y_t = a_0 + a_1 y_{t-1}$; $\phi_{11} = a_1$

$AR(2)$ $y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2}$; $\phi_{22} = a_2$

$\vdots$

$AR(k)$ $y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + ... + a_k y_{t-k}$; $\phi_{kk} = a_k$

We identify the DGP by plotting the sample ACF (SACF) and sample PACF (SPACF) together.

PACF (cont.)

Using the Yule-Walker equations, we can work out the PACF from the ACF,

$$
\begin{aligned}
\phi_{11} &= \rho_1 \\
\phi_{22} &= (\rho_2 - \rho_1^2)/(1-\rho_1^2) \\
&\vdots \\
\phi_{kk} &= \frac{\rho_k - \sum_{j=1}^{k-1} \phi_{k-1,j}\rho_{k-j}}{1 - \sum_{j=1}^{k-1} \phi_{k-1,j}\rho_j}, k = 3, 4, 5, \ldots
\end{aligned}
$$

where, $\phi_{k,j} = \phi_{k-1,j} - \phi_{kk}\phi_{k-1,k-j}, j = 1, 2, 3, \ldots, k-1$