ECON 3350/7350
Single Equation Models of Multiple Time Series

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Tutorial 4

$ARMA(p, q)$ with deterministic trend

$$y_t = a_o + a_2 t + a_1 y_{t-1} + \varepsilon_t;\ |a_1| < 1$$

- y_t is **trend stationary** because if we take the trend out the new process is stationary. We return to deterministic and stochastic trends next week.
- De-trending

$$\begin{aligned} y_t - a_2 t &= a_0 + a_1 y_{t-1} + \varepsilon_t \\ \widetilde{y}_t &= a_0 + a_1 y_{t-1} + \varepsilon_t \end{aligned}$$

  - $\widetilde{y}_t$ is an $ARMA(1, 0)$

$ARDL(p, l, s)$ with trend

- For c_t, a_t, and y_t we could have an $ARDL(p, q, m)$:

$$\theta(L)c_t = \delta + \gamma(L)\mathsf{a}_t + \lambda(L)y_t + \varepsilon_t$$

  Where,

$$\theta(L) = (1 - \theta_1 L - \theta_2 L^2 - ... - \theta_p L^p) = \sum_{i=0}^{p} \theta_i L^i$$

$$\gamma(L) = (\gamma_0 + \gamma_1 L + \gamma_2 L^2 - ... + \gamma_q L^q) = \sum_{j=0}^{q} \gamma_j L^j$$

$$\lambda(L) = (\lambda_0 + \lambda_1 L + \lambda_2 L^2 - ... + \lambda_m L^m) = \sum_{j=0}^{m} \lambda_j L^j$$

- Adding a deterministic trend

$$\theta(L)c_t = \delta_0 + \delta_1 t + \gamma(L)\mathsf{a}_t + \lambda(L)y_t + \varepsilon_t$$

The ARDL Family of Models

Using $ARDL(1,1)$

$y_t = \delta + a_1 y_{t-1} + \theta_0 x_t + \theta_1 x_{t-1} + \varepsilon_t$	
1. Static Regression: $y_t = \delta + \theta_0 x_t + \varepsilon_t$; Restrictions: $a_1 = 0; \quad \theta_1 = 0$	2. First order autoregressive process: $y_t = \delta + a_1 y_{t-1} + \varepsilon_t$; Restrictions: $\theta_0 = 0; \quad \theta_1 = 0$
3. Leading indicator equation: $y_t = \delta + \theta_1 x_{t-1} + \varepsilon_t$; Restrictions: $a_1 = 0; \quad \theta_0 = 0$	4. Equation in first differences: $\Delta y_t = \delta + \theta_0 \Delta x_t + \varepsilon_t$; Restrictions: $a_1 = 1, \theta_0 = -\theta_1$

The ARDL Family of Models-II

$y_t = \delta + a_1 y_{t-1} + \theta_0 x_t + \theta_1 x_{t-1} + \varepsilon_t$	
5. First order distributed lag model: $y_t = \delta + \theta_0 x_t + \theta_1 x_{t-1} + \varepsilon_t$ Restrictions: $a_1 = 0$	6. Partial adjustment model: $y_t = \delta + a_1 y_{t-1} + \theta_0 x_t + \varepsilon_t$ Restrictions: $\theta_1 = 0$
7. Dead Start model (lagged information only): $y_t = \delta + a_1 y_{t-1} + \theta_1 x_{t-1} + \varepsilon_t$ Restrictions: $\theta_0 = 0$	8. Proportional Response Model: $y_t = \delta + a_1(y_{t-1} - x_{t-1}) + \theta_0 x_t + \varepsilon_t$ Restrictions: $\theta_1 = -a_1$
9. Error Correction Mechanism: $\Delta y_t = \delta + \alpha(y_{t-1} - \beta x_{t-1}) + \theta_0 \Delta x_t + \varepsilon_t$ where, $\beta = \frac{(\theta_1 + \theta_0)}{(1 - a_1)}$; $\alpha = a_1 - 1$ This is a re-arrangement of the ARDL equation.	

Multipliers

1. Immediate Response or Impact Multiplier

$$\frac{\partial c_t}{\partial \mathsf{a}_t} = \gamma_0$$

2. The Effect after one period, two periods, ...

$$\frac{\partial c_{t+1}}{\partial \mathsf{a}_t} = \theta_1 \frac{\partial c_t}{\partial \mathsf{a}_t} + \gamma_1 = \theta_1\gamma_0 + \gamma_1$$

$$\frac{\partial c_{t+2}}{\partial \mathsf{a}_t} = \theta_1 \frac{\partial c_{t+1}}{\partial \mathsf{a}_t} = \theta_1(\theta_1\gamma_0 + \gamma_1)$$

3. Long-run multiplier

$$LRM = \frac{\gamma(1)}{\theta(1)} = \frac{(\gamma_0 + \gamma_1 + \gamma_2 + ... + \gamma_p)}{(1 - \theta_1 - \theta_2 - ... - \theta_p)}$$