# ECON 3350/7350: Applied Econometrics for Macroeconomics and Finance

## Tutorial 5: Deterministic and Stochastic Trends

The specification for a general ARIMA($p, d, q$) model is

$$\Delta^d y_t = \sum_{i=1}^{p} \pi_i \Delta^d y_{t-i} + \sum_{j=0}^{q} \alpha_j \epsilon_t + \delta_t$$

where $\alpha_0 = 1$.

- If you decide there is a constant only then

$$\delta_t = a_0$$

- If you decide there is a constant and a trend then

$$\delta_t = a_0 + a_2 t$$

- If you decide there is a constant, a trend and a quadratic trend, then

$$\delta_t = a_0 + a_2 t + a_3 t^2$$

- If you decide there are no deterministic trend, then

$$\delta_t = 0$$

**Testing for Trends** Lecture slides provide a procedure to test for unit roots which involves the three testing equations (constant and trend, constant, and no constant or trend). However, this procedure is a bit complicated. We will be using a simplified version.

1. We conduct the ADF tests ($\tau_\tau$,$\tau_\mu$, $\tau$) using the three test equations, and ERS tests.

   (a) ADF

$$\triangle y_t = a_o + \gamma y_{t-1} + a_2 t + \sum \beta \triangle y_{t-1} + \varepsilon_t \quad (1)$$

$$\triangle y_t = a_o + \gamma y_{t-1} + \sum \beta \triangle y_{t-1} + \varepsilon_t \quad (2)$$

$$\triangle y_t = \gamma y_{t-1} + \sum \beta \triangle y_{t-1} + \varepsilon_t \quad (3)$$

(b) ERS

$$
\begin{aligned}
y_t^d &= y_t - \hat{a}_0 - \hat{a}_2 t && (4)\\
&or\\
y_t^d &= y_t - \hat{a}_0 && (5)\\
\Delta y_t^d &= \gamma y_{t-1}^d + \sum_{i=1}^{p} c_i \Delta y_{t-i}^d + \epsilon_t
\end{aligned}
$$

2. If we reject $\gamma = 0$ in all cases, we conclude the series has no unit roots.

3. If we reject $\gamma = 0$ using equations (1) and (4) we can conclude the series has no unit roots. In this case we do not consider the results from the other equations and proceed to test for a deterministic trend.

4. If we fail to reject $\gamma = 0$ in all cases, we can conclude the series has at least one unit root.

5. If the tests on equations (1), (2), (4) and (5) fail to reject $\gamma = 0$, we test further for the option of a deterministic trend in addition to the stochastic trend.

We use the following diagram to guide the testing procedure (modified from Enders').

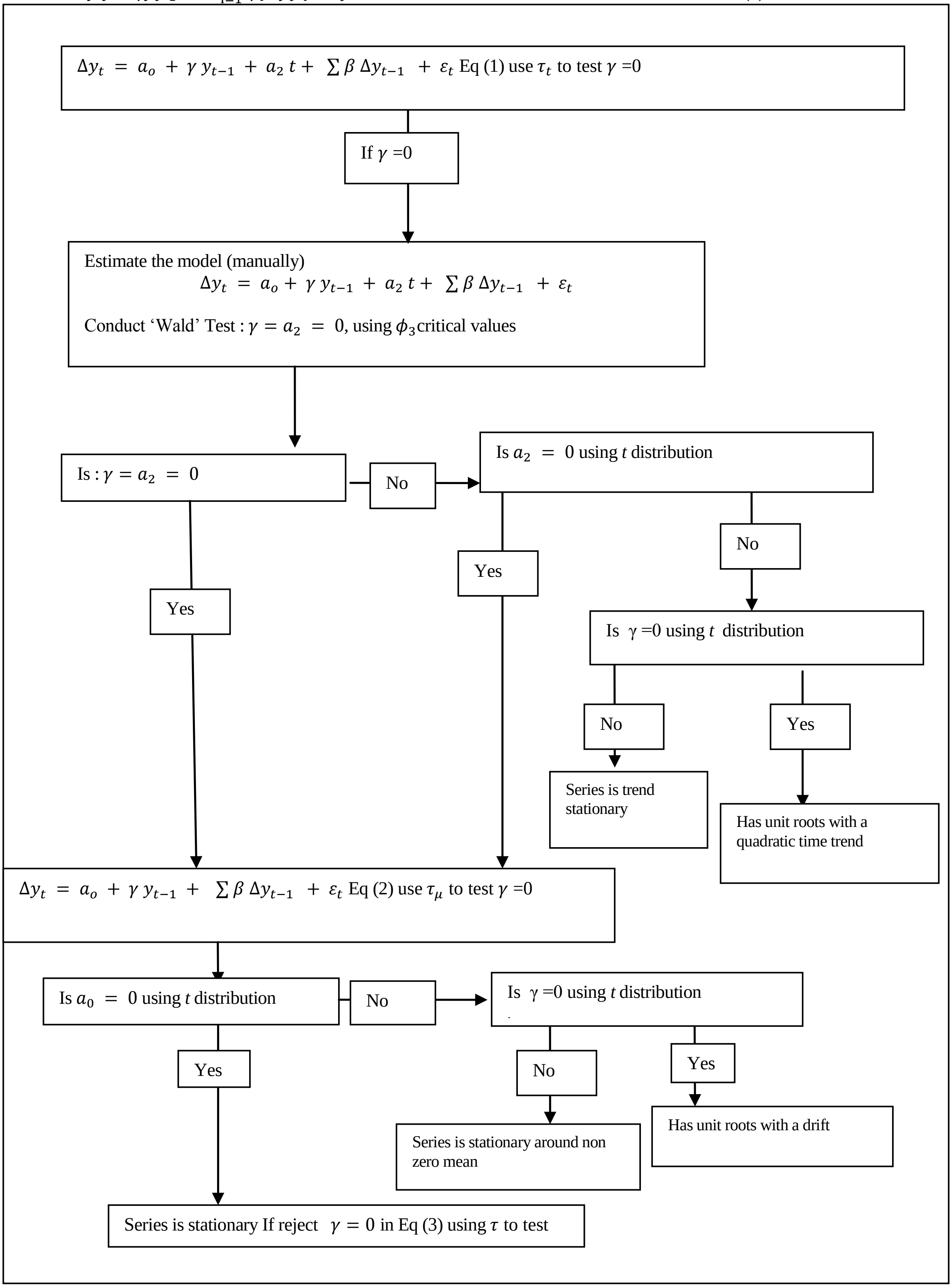
# Practical approach to test for trends

Testing Equations:

$$\Delta y_t = a_0 + a_2 t + \gamma y_{t-1} + \sum_{i=1}^{p-1} \beta_i \Delta y_{t-i} + \epsilon_t \qquad (1)$$

$$\Delta y_t = a_0 + \gamma y_{t-1} + \sum_{i=1}^{p-1} \beta_i \Delta y_{t-i} + \epsilon_t \qquad (2)$$

$$\Delta y_t = \gamma y_{t-1} + \sum_{i=1}^{p-1} \beta_i \Delta y_{t-i} + \epsilon_t \qquad (3)$$



**Tutorial Questions** The file `usdata.csv` contains 209 observations on:

- $r_t$ = the overnight Federal Funds Rate for the US (*ffr*);
- $y_t$ = log real per capita GDP (*gdp*); and
- $p_t$ = the natural log of the CPI for the US (*cpi*).

1. For $r_t$
   (a) Plot the time series plot.
   (b) Conduct the appropriate tests for deterministic and stochastic components using ADF (equations (1), (2), and (3)). Transform the series as needed before continuing.
   (c) Plot the ACF and PACF to suggest AR and MA terms needed.
   (d) Propose three possible models.
   (e) Estimate the proposed models.
   (f) Which model do you choose? Why?
2. Repeat for $y_t$.
3. Repeat for $p_t$.