ECON 3350/7350
Cointegration

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Tutorial 6

Spurious Regression vs Cointegration

- What are the implications for empirical economic research of having I(1) variables?
  - Spurious Regressions or Cointegration
    It is generally true that any combination of two $I(1)$ variables will also be $I(1)$.
- Spurious Regression
  - Conclude there is a significant relationship when there is none.
- Cointegration
  - Linear combinations of $I(1)$ variables are $I(0)$.

Spurious Regression (cont.)

- Indications of a spurious regression:
  Signicant t-values; respectable (sometimes high) R^2; low Durbin-Watson (DW) statistics.
- The signicant t-values occur because the random walks tend to wander, and this wandering looks like a trend.
- If they wander in the same direction for a while (say for the time of the observed sample), there appears to be a relationship.
- In:
  - $y_t = \alpha + \beta x_t + \varepsilon_t$, $\varepsilon_t \sim I(1)$ so the regression is meaningless.
  - This explains why DW is low.

Cointegration and Equilibrium

- The economic interpretation and signicance of cointegration
- We may regard the cointegrating relation

$$z_t = x_t - ay_t$$

  as a stable equilibrium relation.
- Although x_t and y_t are themselves unstable as they are $I(1)$, they are attracted to a stable relationship that exists between them—i.e., $z_t \sim I(0)$.
- For example, there is strong evidence that interest rates are $I(1)$. But the spread between two rates of different maturities, within the same market, appear to be $I(0)$.

Cointegration Order

- If $w_t = (w_{1,t}, w_{2,t}, ..., w_{n,t})' \sim I(1)$ but

$$w_t'\beta \sim I(0)$$

  - where

$$\begin{aligned}\beta' w_t &= w_{1,t}\beta_1 + w_{2,t}\beta_2 + ...w_{n,t}\beta_n \\ &= (\beta_1, \beta_2, ..., \beta_n)' \begin{pmatrix} w_{1,t} \\ w_{2,t} \\ \vdots \\ w_{n,t} \end{pmatrix}\end{aligned}$$

  - Then we say that components of the vector w_t are cointegrated of order $(1, 1)$, denoted CI$(1, 1)$.

Testing for Cointegration

- Consider the case of three variables: $x_t, y_t,$ and z_t
  - Estimate: $x_t = \widehat{\alpha}_t + \widehat{\beta}_1 y_t + \widehat{\beta}_2 z_t + e_t$ (by OLS)
  - Test the residual, e_t, for a unit root. If $e_t \sim I(0)$, then $x_t, y_t,$ and z_t cointegrate.
- There are a number of ways we could perform this test. We will look at using the Dickey-Fuller test statistic (Engle-Granger) and the Durbin-Watson statistic.

Testing for Cointegration (cont.)

- Because the residual e_t comes from a potential cointegrating relation, the test statistics will not have the usual distributions so we cannot use the same critical values.
- The Augmented Dickey-Fuller test to test for cointegration. We proceed as usual but use critical values from Table C in Enders.
  - We estimate the ADF equation: $\Delta e_t = \gamma e_{t-1} + \nu_t$ (ν_t is WN) and test $H_0 : \gamma = 0$.
- The Durbin-Watson test to test for cointegration (CRDW). We proceed as usual but use critical values from Table 9.3 in Verbeek. CRDW is not widely used as it is applicable only for AR(1) processes.