ECON 3350/7350
Deterministic and Stochastic Trends

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Tutorial 5

Trend Stationary

$$y_t = a_0 + a_2 t + a_1 y_{t-1} + \varepsilon_t; \; |a_1| < 1$$

- y_t is **trend stationary** because if we take the trend out the new process is stationary.
- Implies a_2 is the average or long run growth rate.

Difference Stationary

- An alternative form of trend comes from $a_2 = 0$ and $a_1 = 1$

$$y_t = a_0 + a_2 t + a_1 y_{t-1} + \varepsilon_t$$

$$y_t = a_0 + y_{t-1} + \varepsilon_t \text{ or } \Delta y_t = a_0 + \varepsilon_t$$

a_0 is a trend and can be interpreted as the average or long run growth rate..

Consider the stationary AR(1)

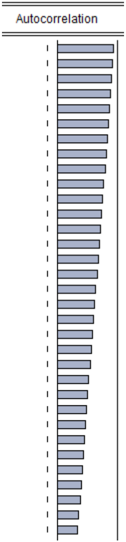
$$\begin{aligned} y_t &= a_1 y_{t-1} + \varepsilon_t \text{ with } |a_1| < 1 \\ &= a_1^t y_0 + \varepsilon_t + a_1 \varepsilon_{t-1} + \cdots + a_1^{t-1} \varepsilon_1 \end{aligned}$$

The effect of shocks or innovations dies out since $a_1^t \to 0$ as t increases. Contrast this with the **Random Walk Model**

$$\begin{aligned} y_t &= y_{t-1} + \varepsilon_t \text{ or } \Delta y_t = \varepsilon_t \\ &= y_0 + \sum_{i=0}^{t} \varepsilon_{t-i} \end{aligned}$$

All past shocks have a permanent effect.

ACF when Stochastic Trends in the data



Random Walk with Drift

The original model

$$y_t = a_0 + y_{t-1} + \varepsilon_t \text{ or } \Delta y_t = a_0 + \varepsilon_t$$

The expected change is $E(\Delta y_t) = a_0$; If $a_0 > 0$, $y_t$will tend to drift upwards. Hence this is called the **Random Walk with Drift**.

Substituting back

$$\begin{aligned} y_t &= a_0 + y_{t-1} + \varepsilon_t \\ &= a_0 + \mu + y_{t-2} + \varepsilon_{t-1} + \varepsilon_t \\ &\vdots \\ &= a_0 t + y_0 + \varepsilon_t + \varepsilon_{t-1} + \cdots \varepsilon_1 \end{aligned}$$

Thus

$$E(y_t | y_0) = y_0 + a_0 t$$

And, the variance also has a trend in it.

Difference Stationary or Integrated Process

- $y_t \sim I(1)$ integrated of order 1
- An $AR(p)$ for $y_t \sim I(1)$ implies an $AR(p-1)$ for $\Delta y_t \sim I(0)$.
- Difference of y_t, $\Delta y_t \sim I(0)$. Thus y_t is **difference stationary**.
- Generally, if y_t has d unit roots,

$$\begin{aligned} y_t &\sim I(d), \\ \Delta y_t &\sim I(d-1), \\ &\vdots \\ \Delta^d y_t &\sim I(0) \end{aligned}$$

The Dickey-Fuller Test Equations

$$\Delta y_t = a_0 + a_2 t + \gamma y_{t-1} + \sum_{i=1}^{p-1} \beta_i \Delta y_{t-i} + \varepsilon_t \tag{1}$$

$$\Delta y_t = a_0 + \gamma y_{t-1} + \sum_{i=1}^{p-1} \beta_i \Delta y_{t-i} + \varepsilon_t \tag{2}$$

$$\Delta y_t = \gamma y_{t-1} + \sum_{i=1}^{p-1} \beta_i \Delta y_{t-i} + \varepsilon_t \tag{3}$$

- Use testing procedure in the next slide and Tables A and B pp 488-489 Enders

Testing Procedure

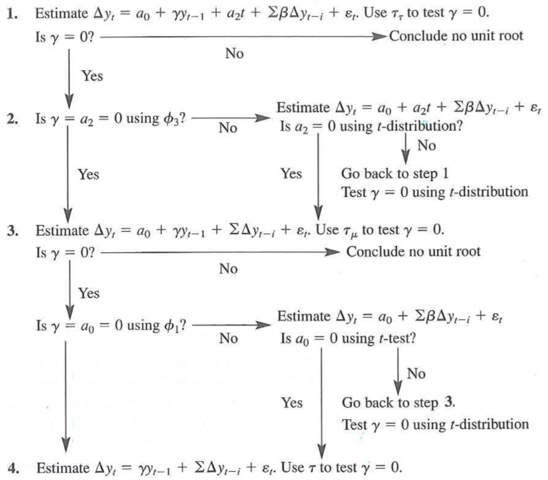


FIGURE A2.1 A Procedure to Test for Unit Roots

KPSS Test

- Tests of unit roots, e.g., $H_0: \; y_t \sim I(1)$ against $H_1: \; y_t \sim I(0)$; often suffer from low power. That is, the probability they lead us to reject the null is low. This may lead us to conclude there are unit roots where there are not.
- One way to circumvent this problem is to test the null there is no unit root against the alternative that there is a unit root.

That is, test

$$H_0: \; y_t \sim I(0) \text{ against } H_1: \; y_t \sim I(1)$$

KPSS test (cont.)

The Kwiatkowski–Phillips–Schmidt–Shin (KPSS) tests test is one such test. The steps are:

1. Estimate $y_t = a_0 + \varepsilon_t$ and save the residuals, e_t
2. Compute $S_t = \sum_{s=1}^{t} e_s$ for $t = 1, 2, ..., T$.
3. Compute $KPSS = T^{-2} \sum_{t=1}^{T} \frac{S_t^2}{\widehat{\sigma}^2}$
4. Compare this to the critical values. - See Table

KPSS trend stationary

If we assume the process is trend stationary under the null, the test becomes:

$$H_0 : \; y_t \sim \text{trend stationary against } H_1 : \; y_t \sim I(1)$$

The steps are:

1. Estimate $y_t = a_0 + a_2 t + \varepsilon_t$ and save the residuals, e_t
2. Compute $S_t = \sum_{s=1}^{t} e_s$ for $t = 1, 2, ..., T$.
3. Compute $KPSS = T^{-2} \sum_{t=1}^{T} \frac{S_t^2}{\widehat{\sigma}^2}$
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Identification of $ARIMA(p, d, q)$

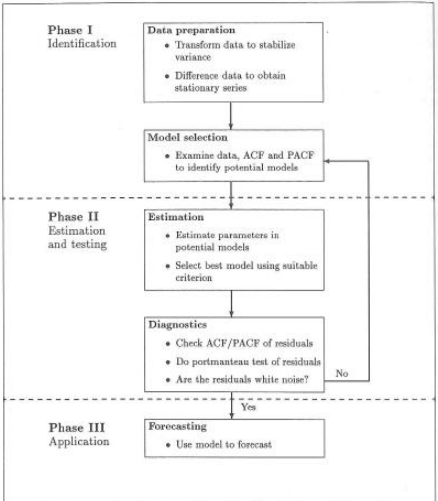

Figure 7-1: *Schematic representation of the Box-Jenkins methodology for time series modeling.*