

1. Tools of the trade:

- matrix-matrix multiplication in different ways
- BLAS
- vector and matrix norms, $\|\cdot\|_1, \|\cdot\|_2, \|\cdot\|_F, \|\cdot\|_\infty$.

2. Floating-point arithmetic

- Floating point representation of numbers
- IEEE floating-point numbers, single and double precision
- Rounding modes and errors
- Floating-point arithmetic
- Catastrophic cancellation

3. Lower and upper triangular linear systems

row-major or column-major
componentwise or vectorized

4. Gaussian Elimination = LU factorization with pivoting

$$PA = LU$$

The need of pivoting, *mathematically and numerically*

Cholesky decomposition of a symmetric positive definite matrix

Applications:

- solve $Ax = b$
- solve $AX = B$, where B is a $n \times p$ matrix
- compute A^{-1}

5. Error analysis for a computed solution of $Ax = b$.

Rule of thumb:

$$\text{relative forward error} \leq \text{condition number} \times \text{relative error in } b$$