ECS130 Homework Assignment #3 **Due: 4:00pm, Feburary 17, 2017**

1. Prove that interpolating polynomial is unique. That is $P_n(x)$ and $Q_n(x)$ are two polynomials withe degree less than n that agree at n distinct points, then they agree at all points.

2. (a) Interpolate the following data by each of the interpolants `polyinterp`, `piecelin`, `pchiptx` and `splinetx`. Plot the results for $-1 \le x \le 1$:

```
    x        y
-1.00    -1.0000
-0.96    -0.1512
-0.65     0.3860
 0.10     0.4802
 0.40     0.8838
 1.00     1.0000
```

   (b) What are values of each of the four interpolants at $x = -0.3$? Which of these values do you prefer? Why?

   (c) The data were actually generated from a low-degree polynomial with integer coefficient. What is that polynomial?

3. Make a plot of your favorite object. Start with

```
figure('position', get(0,'screensize'))
axis('position',[0 0 1 1])
[x,y] = ginput;
```

   Place your favorite object on the computer screen. Use the mouse to select a few dozen points outlining your object. Terminate the `ginput` with a carriage return.

   Now think of `x` and `y` as two functions of an independent variable that goes from one to the number of points you collected. You can interpolate both functions no finer grid and plot the result with

```
n = length(x);
s = (1:n)';
t = (1:0.05:n)';
u = splinetx(s,x,t);
v = splinetx(s,y,t);
clf reset
plot(x,y,'.',u,v,'-');
```

   Do the same thing with `pchiptx`. Which do you prefer?

4. The M-file `rungeinterp.m` provides an experiment with a famous polynomial interpolation problem due to Carl Runge. Let

$$f(x) = \frac{1}{1 + 25x^2},$$

   and let $P_n(x)$ denote the polynomial of degree $n-1$ that interpolates $f(x)$ at n equally spaced points on the interview $-1 \le x \le 1$. Runge asked whether, as n increases, $P_n(x)$ converges to $f(x)$. The answer is yes for some x, but no for others. Find for what x, does $P_n(x) \to f(x)$ as $n \to \infty$?

5. Ranking sport teams. Suppose we have four college teams, call T1, T2, T3 and T4. These four teams play each other with the following outcomes:

   - T1 beats T2 by 4 points: 21 to 17.
   - T3 beats T1 by 9 points: 27 to 18.
   - T1 beats T4 by 6 points: 16 to 10.
   - T3 beats T4 by 3 points: 10 to 7.
   - T2 beats T4 by 7 points: 17 to 10.

   To determine ranking points r_1, r_2, r_3, r_4 for each team, we do a least squares fit to the overdetermined system:

$$\begin{aligned} r_1 - r_2 &= 4, \\ r_3 - r_1 &= 9, \\ r_1 - r_4 &= 6, \\ r_3 - r_4 &= 3, \\ r_2 - r_4 &= 7. \end{aligned}$$

   In addition, we fix the total number of ranking points, i.e., $r_1 + r_2 + r_3 + r_4 = 100$. Find the values of r_1, r_2, r_3, r_4 that most closely satisfy these equations, and based on your results rank the four teams.[1]

6. Find the polynomial of degree 10

$$p(t) = \beta_1 t^{10} + \beta_2 t^9 + \cdots + \beta_{10} t + \beta_{11}$$

   that best fits the function $f(t) = \cos(4t)$ at equally-spaced point t between 0 and 1. Set up the design matrix X and right-hand side vector y, and determine the polynomial coefficients $\beta = (\beta_1, \ldots, \beta_{11})$ in two different ways:

   (a) By solving the normal equation $X^T X \beta = X^T y$. This can be done in MATLAB by typing `beta = (X' * X) \ (X' * y)`

   (b) By using the MATLAB backslash command `beta = X\y` (which uses a QR decomposition).

   Print the results to 16 digits (using `format long e`) and comment on the difference you see. *Note: you can compute the condition number using MATLAB built-in function* `cond`.

7. In `censusgui.m`, change the 1950 population from 150.697 million to 50.697 million. The produces an extreme `outliner` in the data. Which models are the most affected by this outliner? which models are least affected?

8. Let $x = [9, 2, 6]^T$.

   (a) Find the Householder reflection H that transforms x into

$$Hx = \begin{bmatrix} -11 \\ 0 \\ 0 \end{bmatrix}.$$

   (b) Find nonzero vectors u and v that satisfy

$$\begin{aligned} Hu &= -u \\ Hv &= v \end{aligned}$$

[1] This method of ranking sport teams is a simplification of one introduced by Ke Massey in 1997. It has evolved into a part of the famous BCS (Bowl Championship Series) model for ranking college football teams and is one factor in determining which teams play in bowl games.