

1. Consider the following cubic polynomial

$$p(x) = 816x^3 - 3835x^2 + 6000x - 3125.$$

It has three three closely spaced roots: $25/15, 25/16, 25/17$

- (a) Plot $p(x)$ for $1.43 \leq x \leq 1.71$. Show the location of the three roots.
 - (b) Starting with the interval $[1, 2]$, what does the bisection method do?
 - (b) Starting with $x_0 = 1.5$, what does Newton's method do?
 - (c) Starting with $x_0 = 1$ and $x_1 = 2$, what does the secant method do?
2. Investigate the behavior of the secant method on the function

$$f(x) = \text{sign}(x - 2)\sqrt{|x - 2|}$$

Hint: start from "Example Newton 2" from our class website.

3. Let

$$f(x_1, x_2) = \frac{1}{2}(x_1^2 - x_2)^2 + \frac{1}{2}(1 - x_1)^2$$

- (a) What is the minimizer of $f(x_1, x_2)$?
- (b) Compute one iteration of Newton's method for minimizing $f(x_1, x_2)$ starting from the point $(2, 2)$. Is this a good step?

4. Let

$$f(x_1, x_2) = \frac{1}{2}x_1^2 + \frac{9}{2}x_2^2.$$

It's easy to see that the minimizer is $x_* = (0, 0)$

- (a) Derive the steepest descent method for finding the minimizer of $f(x)$.
- (b) Compute the first four iterations starting from the point $(9, 1)$.