

## Correctness of Algorithm:

Let  $T$  be set containing Terminals,  $V$  be set of all vertices,  $E$  be edges of the given graph.

Steiner-Tree ( $G, T$ )

{

for each  $t \in T$  do

for each  $v \in V$

$$ST[t][v] = \text{dist}(t, v)$$

for ( $m=2$  to  $m \leq |T|$ ) do — (A)

{ for each subset  $X$  of size  $m$  do — (B)

for each  $v \in V$

$$\{ ST[X][v] = \infty$$

for each  $u \in V$  do

(C) ← { for each non disjoint non empty subset combination of  $X$  do  
{ let  $x', x''$  disjoint subset.  
Sum =  $\min(\text{sum}, ST[x'][u] + ST[x''][u])$ .  
}

$$ST[X][v] = \min(ST[X][v], \text{sum} + \text{dist}(v, t))$$

}

if  $|X| = |T|$

return  $ST[X][v]$  last terminal.

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{ let  $x', x''$  disjoint subset.  
Sum =  $\min(\text{sum}, ST[x'][v] + ST[x''][v])$ .  
}

$$ST[X][v] = \min(ST[X][v], \text{sum} + \text{dist}(v, \text{last terminal}))$$

if  $|X| = |T|$

return  $ST[X][v]$  last terminal.

Loop Invariant: at the start of each iteration of A we have ~~an~~ minimum stener wright for all terminal ~~prose~~ subset of size  $|m-1|$ .

Initialization: just before first iteration of loop (A) we have all one size terminal set stenerer wright ~~and~~ value in  $ST[][]$  array [from base case].

Maintenance: In order to <sup>prove</sup> loop invariant, we need to take a look inside each loops inside loop (A). In loop (B) we are generating all 'm' size subset, so All ~~at~~ <sup>'m'</sup> size subset are generated. Inside that loop (B) we are generating All disjoint nonempty subset of  $X$  [ $x'$  and  $x''$ ,  $x' \cap x'' = \phi$ ,  $x' \cup x'' = X$ ],  $x', x'' \neq \phi$ . since by optimal substructure property the  $ST[x'][v]$  and  $ST[x''] [v]$  will be already filled in table  $\therefore |x'| \& |x''| < m$ . so in loop (C) we will take minimum value from all subset and fill the  $ST[x][v]$  from most optimum value with help of All pair shortest path graph

Termination "  $\therefore$  after each <sup>i<sup>th</sup></sup> iteration of loop (A) we have filled up ~~out~~  $(i+2-1)^{th}$  size subset. value ~~in~~ in  $ST[][]$  ~~graph~~ matrix.

Termination: when  $m = |T| + 1$  loop (A) All <sup>value</sup> ~~and~~ <sup>in</sup> ~~subset~~  $ST[][]$  graph has been filled and  $ST[\text{root}][T.\text{least}()]$  will return minimum value of stener tree.



Loop Invariant: at the start of each iteration of A we have ~~an~~ minimum steiner weight for all terminal ~~prose~~ subset of size  $|m|-1$ .

Initialization: just before first iteration of loop (A) we have all one size terminal set steiner weight ~~one~~ value in  $ST[][]$  array [from base case].

Maintenance: In order to <sup>prove</sup> loop invariant, we need to take a look inside each loops inside loop (A). In loop (B) we are generating all ' $m$ ' size subset, so All ~~at~~ <sup>'m'</sup> size subset are generated. Inside that loop (B) we are generating All disjoint nonempty subset of  $X$  [ $x'$  and  $x''$ ,  $x' \cap x'' = \emptyset$ ,  $x' \cup x'' = X$ ,  $x', x'' \neq \emptyset$ ]. since by optimal substructure property the  $ST[x'] [v]$  and  $ST[x''] [v]$  will be already filled in table  $\therefore |x'| \& |x''| < m$ . so in loop (C) we will take minimum value from all subset and fill the  $ST[x] [v]$  from most optimum value with help of All pair shortest path graph

Termination "  $\therefore$  after each  <sup>$i^{th}$</sup>  iteration of loop (A) we have filled up ~~up to~~  $(i+1)^{th}$  size subset value in  $ST[][]$  ~~graph~~ matrix.

Termination: when  $m = |T| + 1$  loop (A) All <sup>value</sup> ~~in~~ <sup>in</sup>  $ST[][]$  graph has been filled and  $ST[\bar{t}] [T.least(c)]$  will return minimum value of steiner tree.

In short,

loop (A) will emulate all 'm' sized subset of T.

But due to optimal substructure property we have  
Stones table filled for all subset of T of size less  
than m.

loop (B) will generate all subset of taken size m.  
thus loop (B) ensure filling of all  $T \mid C_m$  subset in  
ST[][] table.

loop (C) will enumerate all subset of subset  
taken from loop (B) and filled ~~table~~ ST[][] table  
from all ~~given~~ <sup>possible</sup> ~~subset~~ distinct non empty subset part  
tion of given input X. minimum value is taken  
from all possible enumeration.

- Filling of DP table is bottom up by using fact of optimal decomposition property of given problem.