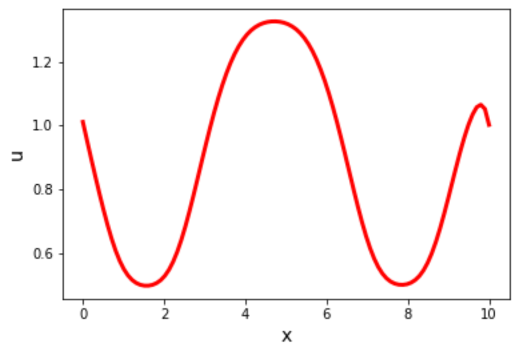
1. Use finite differences to solve the equation

$$0.1\frac{d^2u}{dx^2} + u - u^3 = \sin(x)$$

subject to the boundary conditions:

$$u(0) = 1, \quad u(10) = 1$$

The solution should look like:



2. The following differential equation can be used to model diffusion within a catalyst

$$\frac{d^2c(x)}{dx^2} = \Phi^2 c^2(x)$$

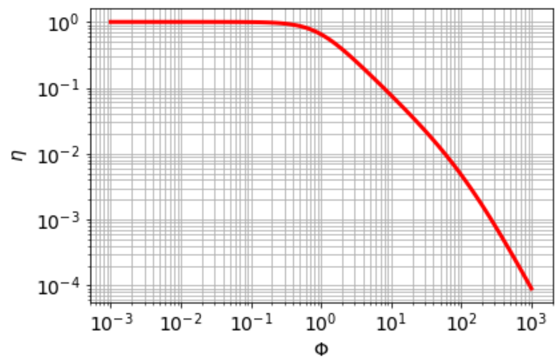
subject to the boundary conditions:

$$c(0) = 1, \quad \frac{d}{dx}c(1) = 0$$

The equation models the reaction $2A \rightarrow B$. The dimensionless parameter Φ is known as the Thiele modulus and captures the relative rates of reaction versus diffusion. Use finite differences to determine the effectiveness factor (η) as a function of the Thiele modulus Φ. Recall, the effectiveness factor is defined as

$$\eta = -\frac{1}{\Phi^2}\frac{d}{dx}c(0)$$

The solution should look like:



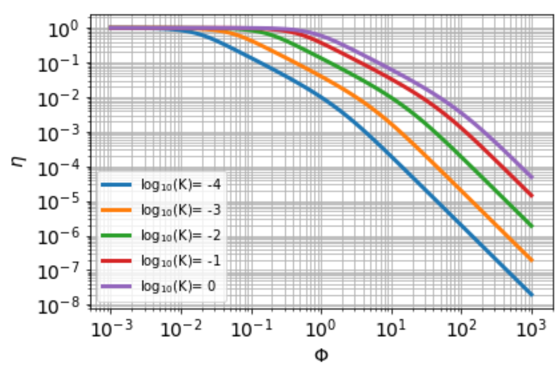
3. Suppose now that the reaction is reversible: $2A \leftrightarrow B$. The corresponding equations are

$$\frac{d^2 c_A(x)}{dx^2} = -\Phi^2 c_A^2(x) + \frac{1}{K}\Phi c_B(x), \quad \frac{d^2 c_B(x)}{dx^2} = \frac{1}{2}\Phi^2 c_A^2(x) - \frac{1}{2K}\Phi c_B(x),$$

subject to the boundary conditions:

$$c_A(0) = 1, \quad \frac{d}{dx}c_A(1) = 0, \quad c_B(0) = 1, \quad \frac{d}{dx}c_B(1) = 0,$$

Determine how the effectiveness factor (using the same definition for $c_A(x)$) varies as a function of Φ and K. You should generate the following plot:



4. Solve the partial differential equation

$$\frac{\partial u(t,x)}{\partial t} = \frac{\partial^2 u(t,x)}{\partial x^2} + u(1-u)$$

subject to the boundary conditions:

$$u(t,0) = 1, \qquad u(t,60) = 0,$$

and the initial condition

$$u(0,x) = 0, \qquad 0 < x < 60.$$

This equation is known as Fisher's equation. You should generate the following plots: