A multiscaling intensity-duration-frequency model for extreme precipitation

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INTRODUCTION

IDF curves

Rainfall intensity-duration-frequency (IDF) curves are one of the most commonly used tools in water resources engineering and management. They are particularly useful for the design of sewer systems and the management of water supply. IDF curves express how return levels of extreme rainfall intensity vary with duration d. Typically, d ranges from a few minutes to several hours or few days.

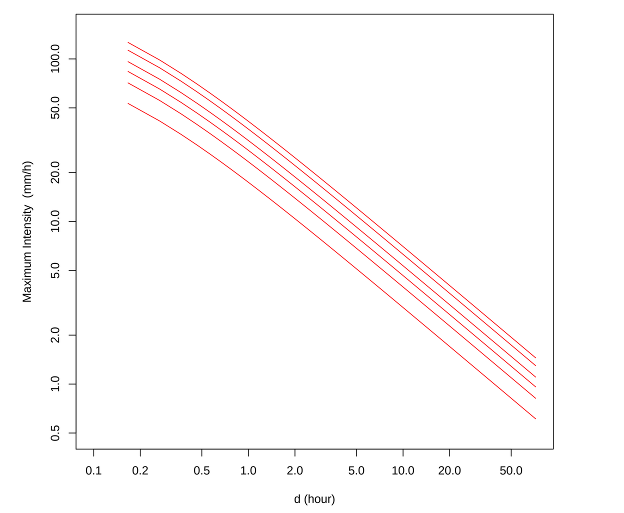

Figure: IDF curves at Uccle (Belgium) for return periods $T = 2, 5, 10, 20, 50$ and 100 year.

- Define by $I(d)$: the annual maximum average rainfall intensity of duration d.
- $i_T(d)$: the return level of $I(d)$ with return period T.
- The general IDF model of [KKM98] is taken to be a separable function of d and T,

$$i_T(d) = \frac{a(T)}{(d+\theta)^{\eta}}, \qquad \text{with } \theta \geq 0, \text{ and } 0 < \eta < 1, \qquad (1)$$

  with $a(T)$, the T-year return level of the scaled intensity: $Y = I(d)\,(d+\theta)^{\eta}$.
- We assume that Y follows the GEV-distribution.
- For durations of interest $d \geq 1$ h, we may put $\theta = 0$ in Eq. (1).

Goal of the research

Make an improvement of the existing IDF model, Eq. (1), based on (i) the multifractal properties of temporal rainfall, and (ii) Extreme Value Theory.

SCALING PROPERTIES OF PRECIPITATION

- **Multifractal theory** provides a framework to analyze statistical rainfall characteristics across a range of temporal or spatial scales.
- The theory typically applies for rainfall durations $d = 1$ h, …, 72 h.
- **Simple scaling.**
  - *Strict sense:*
  $$I(\lambda\, d) \stackrel{distr}{=} \lambda^{-\eta}\, I(d), \qquad \text{with } 0 < \eta < 1.$$
  - *Wide sense:*
  $$E[I^q(\lambda\, d)] = \lambda^{-q\eta}\, E[I^q(d)].$$
- **Multiscaling** [GW90]:
$$E[I^q(\lambda\, d)] = \lambda^{-\alpha_q}\, E[I^q(d)], \qquad \text{with } \alpha_q = q\,\varphi_q\,\eta, \qquad \text{and } \varphi_q \geq 1.$$

GEV SCALING MODELS FOR IDF CURVES

- We assume
$$I(d) \sim \mathrm{GEV}[\mu(d), \sigma(d), \gamma].$$
- *Simple scaling hypothesis.* One easily proves that:
$$\mu(d) = d^{-\eta}\,\mu, \qquad \sigma(d) = d^{-\eta}\sigma, \qquad \gamma(d) = \gamma,$$
  with $0 < \eta < 1$. $\Rightarrow$ Equivalent with IDF model, Eq. (1), in case of $\theta = 0$.
- *Multiscaling hypothesis.* We showed that this is equivalent to [dV18]:
$$\mu(d) = d^{-\eta_1}\,\mu, \qquad \sigma(d) = d^{-\eta_2}\sigma, \qquad \gamma(d) = \gamma,$$
  with $0 < \eta_1 \leq \eta_2 < 1$. $\Rightarrow$ Extension of the simple scaling GEV-model.
- Extreme intensity return levels:
$$i_T(d) = d^{-\eta_1}\,\mu - \frac{d^{-\eta_2}\,\sigma}{\xi}\left\{1 - \left[-\log\left(1 - \frac{1}{T}\right)\right]^{-\xi}\right\}, \text{with } \begin{cases} \eta_1 = \eta_2, & (\textit{simple sc.}) \\ \eta_1 < \eta_2, & (\textit{multisc.}) \end{cases}$$

INFERENCE

- Annual maximum intensity rainfall process, $I = (I(d_1), \ldots, I(d_M))$
- IDF data: N year of observed annual maximum intensities:
$$\mathbf{i} = \underbrace{\begin{pmatrix} i_{11} & \cdots & i_{1M} \\ \vdots & \ddots & \vdots \\ i_{N1} & \cdots & i_{NM} \end{pmatrix}}_{M \text{ durations}}.$$
- **Independence likelihood:** density function for independent IDF data, which is
$$L_{ind}(\mathbf{i}; \psi) = \prod_{j=1}^{N}\prod_{k=1}^{M} g(i_{jk}; \psi),$$
  with $g(;\psi)$ the GEV-density function, and marginal parameters:
$$\psi = \begin{cases} (\mu, \sigma, \gamma, \eta), & (\textit{simple-scaling GEV model}), \\ (\mu, \sigma, \gamma, \eta_1, \eta_2), & (\textit{multiscaling GEV model}). \end{cases}$$
- Independence likelihoods are an example of composite likelihoods.
- Bayesian rule for the independence likelihood:
$$\pi_{ind}(\psi|\mathbf{i}) \propto L_{ind}(\mathbf{i}; \psi)\,\pi(\psi),$$
  with $\pi(\psi)$, the prior distribution, and $\pi_{ind}(\psi\,|\,\mathbf{i})$, the posterior distribution.
- **Adjusted posterior distribution**, $\pi_{adj}(\psi|\mathbf{i})$, to take account for dependence:
$$\pi_{adj}(\psi|\mathbf{i}) \propto L_{adj}(\mathbf{i}; \psi)\,\pi(\psi),$$
  with $L_{adj}(\mathbf{i}; \psi)$, the **adjusted independence likelihood**. For details, see [RCD12].

MODEL SELECTION

- Model selection (*simple scaling versus multiscaling*) with AIC and BIC.
- For Bayesian independence likelihoods, we get (cfr. [VV05]):
$$\begin{aligned} \mathrm{AIC} &= -2\,\log L_{ind}(\mathbf{i}; \bar{\psi}_{adj}) + 2\,p_D, \\ \mathrm{BIC} &= -2\,\log L_{ind}(\mathbf{i}; \bar{\psi}_{adj}) + p_D\,\log N, \end{aligned}$$
  where:
- $\bar{\psi}_{adj}$ is the mean of the adjusted posterior, and
- p_D is a measure for the number of effective parameters (model complexity) [SBCvdL02]. For adjusted Bayesian inference, this extends to:
$$p_D = \mathrm{E}_{\psi_{adj}|\mathbf{i}}[-2\,\log L_{adj}(\mathbf{i}; \psi_{adj}) + 2\,\log L_{adj}(\mathbf{i}; \bar{\psi}_{adj})].$$

DATA

- We considered 10-min observations provided by the Royal Meteorological Institute of Belgium.
- Measurement device: Hellmann-Fuess pluviograph.
- Uccle: 110-year time series (1898–2007).
- 15 other locations: 38-year time series (1967–2004).
- We extracted the annual maxima of sliding 10-min aggregated precipitation depths over 7 durations:

  $d = 1, 2, 6, 12, 24, 48$, and 72 h.

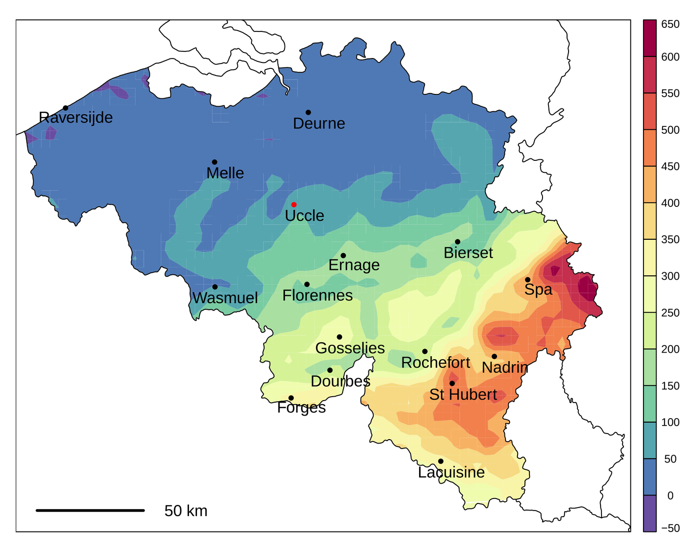

Figure: Location of the 10-min pluviograph stations.

RESULTS

Station	Simple scaling $\bar{\eta}$	p_D	AIC	BIC	Multiscaling $\bar{\eta}_1$	$\bar{\eta}_2$	p_D	AIC	BIC
Forges	0.706	3.41	813.5	819.1	0.721	0.873	4.64	**792.3**	**799.9**
Nadrin	0.689	3.81	752.7	759.0	0.693	0.822	4.77	**734.2**	**742.0**
Uccle	0.722	3.91	2182.4	2193.2	0.723	0.842	4.89	**2138.1**	**2151.6**
Lacuisine	0.635	3.66	844.2	850.2	0.641	0.785	4.67	**825.4**	**833.1**
St Hubert	0.649	3.64	693.9	699.9	0.655	0.787	4.67	**675.0**	**682.7**

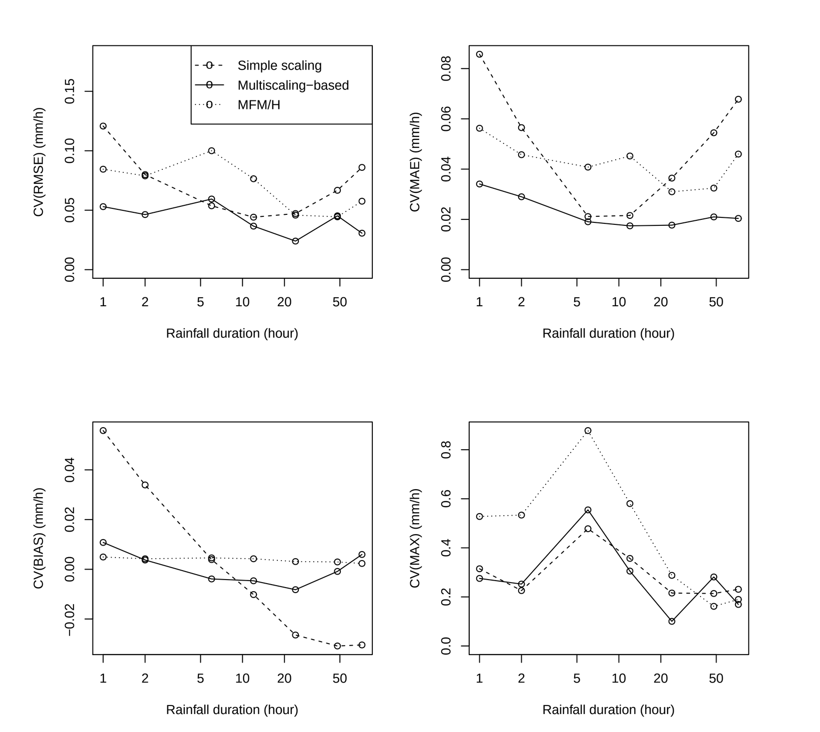

Figure: Scores of the scaling IDF models, against rainfall duration d. (Station Uccle).

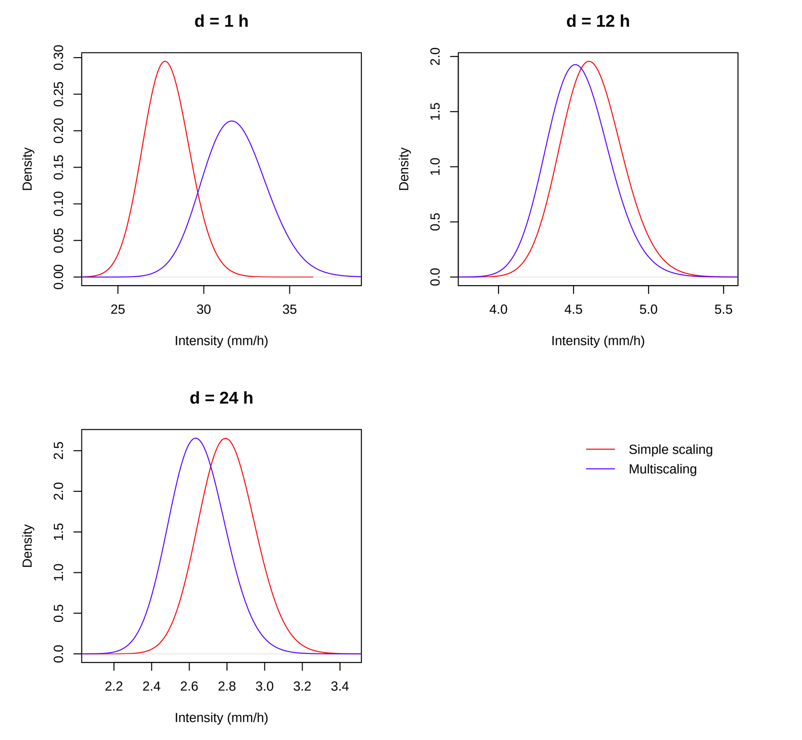

Figure: Posterior of 20-year return levels of the maximum intensity. (Station Uccle).

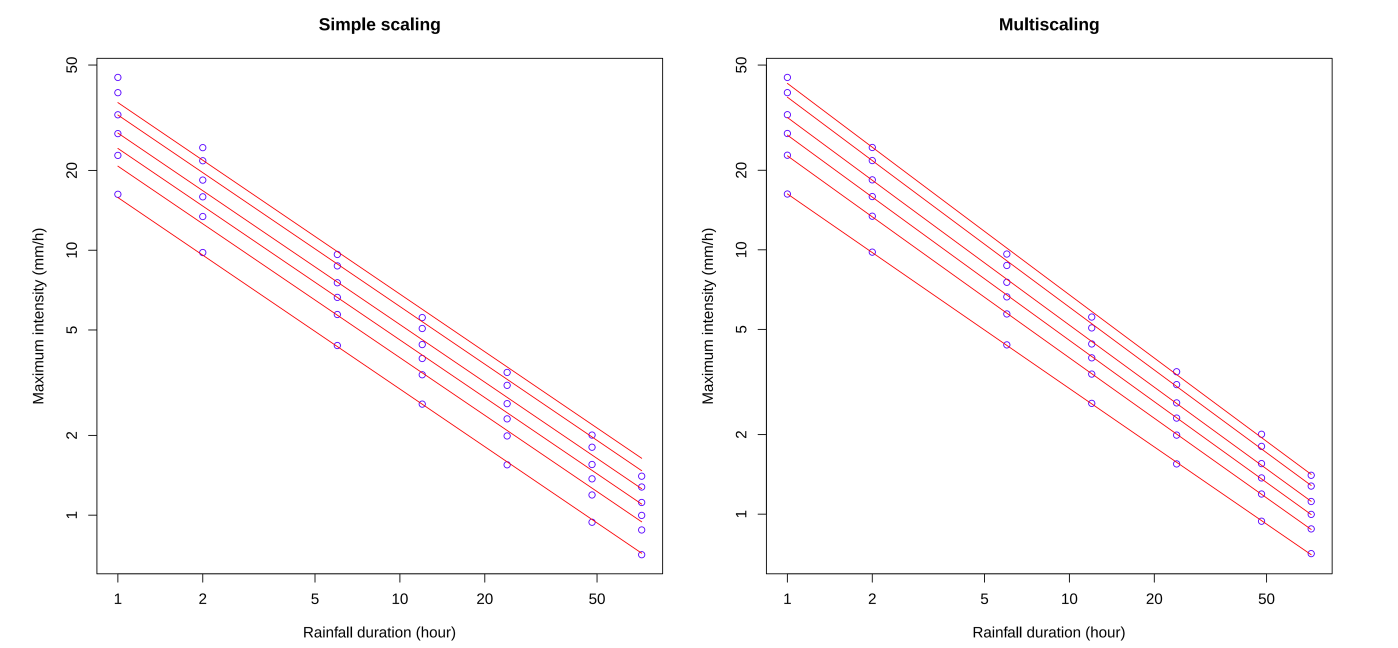

Figure: *Red lines.* IDF curves (Simple scaling versus multiscaling). *Blue dots.* Intensities obtained directly from the individually-fitted GEV-distribution for each duration. (Station Uccle).

Conclusions

- Strong evidence that the multiscaling IDF model is better than the simple scaling IDF model in Western Europe.
- **Open question:** IDF characteristics worldwide?

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