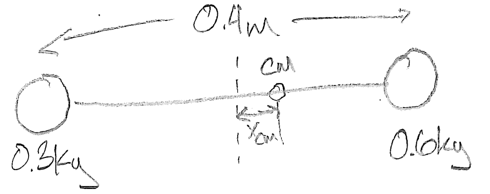
Chapter 12 EOC Solutions

50.



$$X_{CM} = \frac{1}{0.9kg}\left((-0.2)(0.3) + (0.2)(0.6)\right)$$

$$X_{CM} = 0.067m$$

$$I = (0.3)(0.267)^2 + (0.6)(0.133)^2$$

$$I = 0.032 \text{ kg m}^2$$

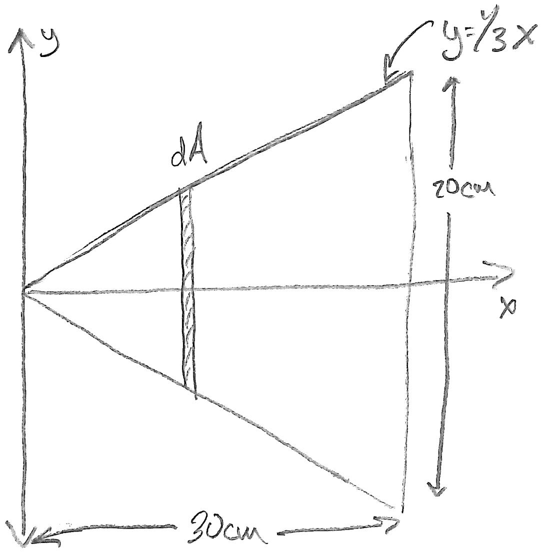
$$\left(\frac{100 \cancel{rot}}{1 \cancel{min}}\right)\left(\frac{1 \cancel{min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \cancel{rot}}\right) = 10.5 \text{ rad/s} = \omega$$

$$K = \tfrac{1}{2} I \omega^2$$

$$K = \tfrac{1}{2}(0.032)(10.5)^2$$

$$\boxed{K = 1.75 \text{ J}}$$

52.



$$x_{cm} = \frac{1}{M}\int x\,dm$$

$$\sigma = \frac{M}{A} = \frac{dm}{dA}$$

$$dA = 2y\,dx$$

$$dA = 2(\tfrac{1}{3}x)\,dx$$

$$dm = \sigma \tfrac{2}{3} x\,dx$$

$$x_{cm} = \frac{1}{M}\int_0^{30} (x)(\sigma \tfrac{2}{3}x)\,dx$$

$$x_{cm} = \frac{2\sigma}{3M}\int_0^{30} x^2\,dx$$

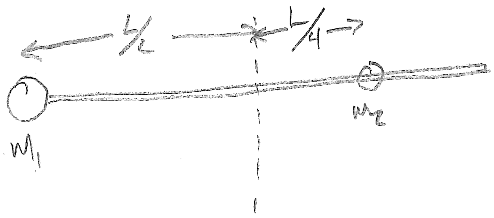
$$x_{cm} = \frac{2}{3A}\,\frac{x^3}{3}\Big|_0^{30}$$

$$x_{cm} = \frac{2(30)^3}{9\left(\frac{1}{2}(20)(30)\right)}$$

$$x_{cm} = 20\text{cm}$$

$$\boxed{\text{Center of mass} = (20, 0)}$$

54.



$$I = I_{rod} + I_1 + I_2$$

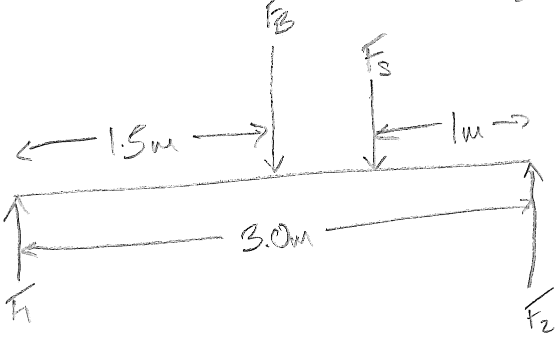
$$I = \frac{ML^2}{12} + m_1\left(\frac{L}{2}\right)^2 + m_2\left(\frac{L}{4}\right)^2$$

$$I = \frac{ML^2}{12} + \frac{m_1 L^2}{4} + \frac{m_2 L^2}{16}$$

$$I = \frac{4ML^2}{48} + \frac{12m_1L^2}{48} + \frac{3m_2L^2}{48}$$

$$I = \frac{4ML^2 + 12m_1L^2 + 3m_2L^2}{48}$$

61.



$$\Sigma F_y = F_1 + F_2 - F_B - F_S = 0$$

$$\Sigma \tau = F_2 l - F_B \frac{l}{2} - F_S(l-1) = 0$$

$$F_2 = \frac{F_B \frac{l}{2} + F_S(l-1)}{l}$$

$$F_2 = \frac{(100)(9.8)\left(\frac{3}{2}\right) + (80)(9.8)(3-1)}{(3)}$$

$m_B = 100\text{kg}$

$m_S = 80\text{kg}$

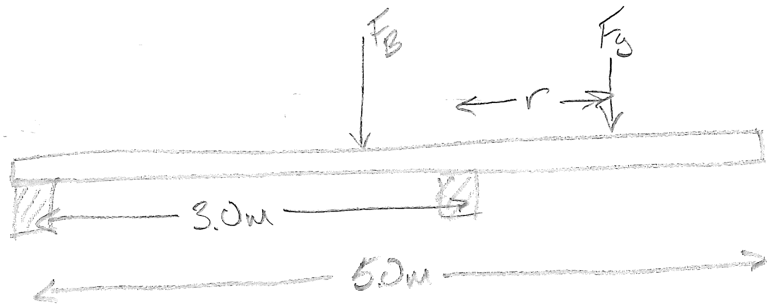
$$\boxed{F_2 = 1013\text{N}}$$

$$F_1 = F_S + F_B - F_2$$

$$F_1 = (80)(9.8) + (100)(9.8) - (1013)$$

$$\boxed{F_1 = 751\text{N}}$$

63.



$m_B = 40kg$

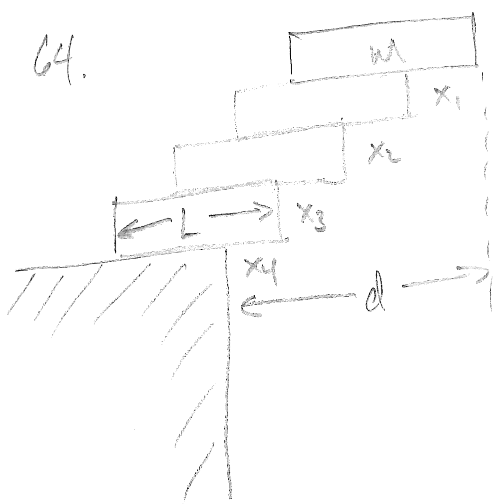
$m_s = 80kg$

$\Sigma \tau = F_B(0.5) - F_g r = 0$

$F_B(0.5) = F_g r$

$r = \frac{m_B \cancel{g}(0.5)}{m_s \cancel{g}} = \boxed{1\,m}$

64.



$x_1 = L/2$

$x_2 = \frac{1}{2\cancel{m}}\left(\cancel{m(0)}^{0} - \cancel{m}(L/2)\right)$

$x_2 = L/4$

$x_3 = \frac{1}{3\cancel{m}}\left(\cancel{2m(0)}^{0} - \cancel{m}(L/2)\right)$

$x_3 = L/6$

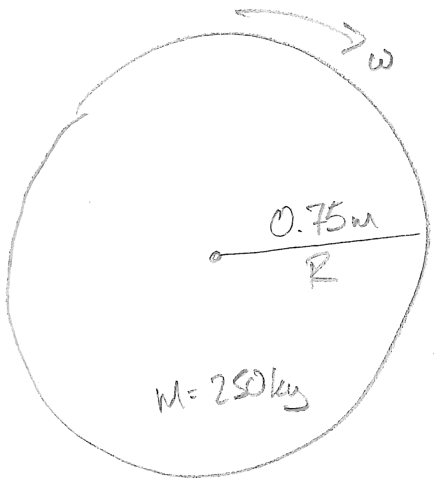
$x_4 = \frac{1}{4\cancel{m}}\left(\cancel{3m(0)}^{0} - \cancel{m}(L/2)\right)$

$x_4 = L/8$

$d = \frac{L}{2} + \frac{L}{4} + \frac{L}{6} + \frac{L}{8}$

$\boxed{d = \frac{25}{24} L}$

68.



$$I_{disc} = \frac{1}{2}MR^2 = 70.3\ kg\,m^2$$

$$\omega_f = \left(\frac{1200\ \cancel{rev}}{1\ \cancel{min}}\right)\left(\frac{1\ \cancel{min}}{60\ s}\right)\left(\frac{2\pi\ rad}{1\ \cancel{rev}}\right) = 125.7\ rad/s$$

a. $\tau = 50\ Nm$ $\quad \alpha = \frac{\tau}{I}$ $\quad \omega_f = \cancel{\omega_0}^{\,0} + \alpha t$ $\quad \omega_f = \frac{\tau}{I}t$

$\tau = I\alpha$

$$t = \frac{\omega_f I}{\tau} = \frac{(125.7)(70.3)}{(50)}$$

$$\boxed{t = 177\ s}$$

b. $K = \frac{1}{2}I\omega^2$ $\quad \boxed{K = 5.55\times10^5\ J}$

$$K = \frac{1}{2}(70.3)(125.7)^2$$

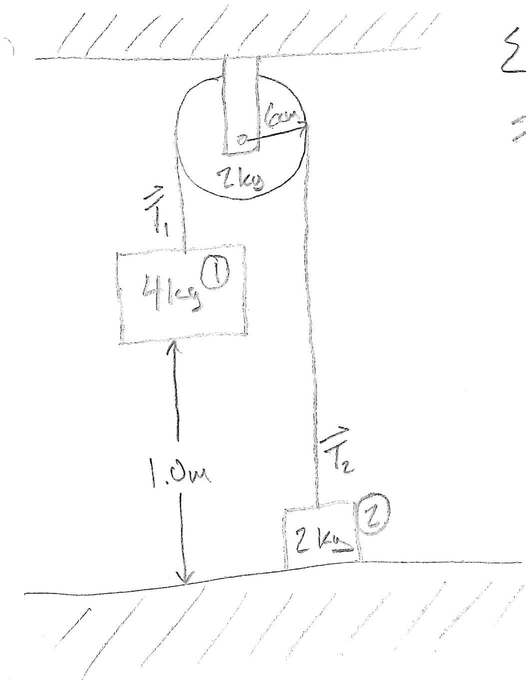
c. $P = \frac{K/2}{t}$ $\quad P = \frac{K}{4} = \boxed{1.39\times10^5\ W}$

d. $K_f = \frac{K_i}{2}$ $\quad \omega = \sqrt{\frac{2K}{I}}$ $\quad \omega = \sqrt{\frac{K}{I}} = 88.9\ rad/s$

$$\alpha = \frac{\Delta\omega}{t} = \frac{125.7 - 88.9}{2s} = 18.4\ rad/s^2$$

$$\tau = I\alpha = (70.3)(18.4) = \boxed{1300\ N\,m}$$

69.



$$\Sigma F_{y_1} = m_1 g - \vec{T}_1 = m_1 a \qquad \Sigma F_{y_2} = \vec{T}_2 - m_2 g = m_2 a$$

$$\vec{T}_1 = m_1 g - m_1 a \qquad \vec{T}_2 = m_2 a + m_2 g$$

$$\Sigma \tau = T_1 R - T_2 R - T_f = I\alpha \quad a = R\alpha$$

$$T_1 R - T_2 R - T_f = \tfrac{1}{2} m_p R^2 \left(\frac{a}{R}\right)$$

$$T_1 R - T_2 R - T_f = \tfrac{1}{2} m_p R a$$

$$(m_1 g - m_1 a)R - (m_2 a + m_2 g)R - T_f = \tfrac{1}{2} m_p R a$$

$$m_1 g R - m_1 a R - m_2 a R - m_2 g R - T_f = \tfrac{1}{2} m_p R a$$

$$m_1 g R - m_2 g R - T_f = \tfrac{1}{2} m_p R a + m_1 a R + m_2 a R$$

$$a = \frac{m_1 g R - m_2 g R - T_f}{\tfrac{1}{2} m_p R + m_1 R + m_2 R}$$

$$a = \frac{(4)(9.8)(0.06) - (1)(9.8)(0.06) - (0.5)}{\tfrac{1}{2}(2)(0.06) + (4)(0.06) + (2)(0.06)}$$

$$a = 3.0\ \text{m/s}^2$$

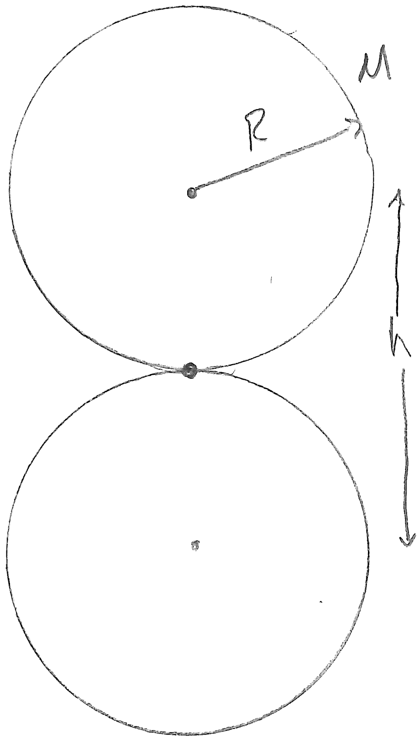
$$\Delta y = \cancel{v_i t}^{\,0} + \tfrac{1}{2} a t^2$$

$$t = \sqrt{\frac{2\Delta y}{a}}$$

$$t = \sqrt{\frac{2(1)}{(3)}}$$

$$\boxed{t = 0.82\ \text{s}}$$

75.



$I_{hoop@cm} = MR^2$ $I_{hoop@edge} = MR^2 + MR^2$

$I_{hoop@edge} = 2MR^2$

a. $U_g = K_R$

$mgh = \frac{1}{2} I \omega^2$

$\cancel{m}g\,2\cancel{R} = \cancel{\frac{1}{2}}(\cancel{2}\cancel{M}R^{\cancel{2}})\omega^2$

$2g = R\omega^2$

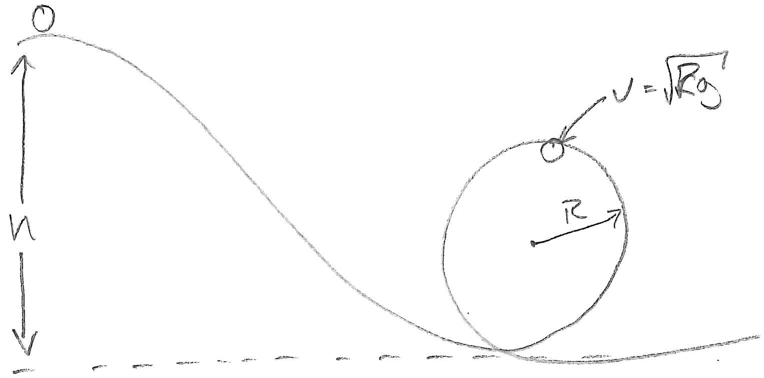
$\boxed{\omega = \sqrt{\frac{2g}{R}}}$

b. $V = \omega(2R)$

$V = \sqrt{\frac{2g}{R}}(2R)$

$\boxed{V = \sqrt{8gR}}$

84.



$I_{sphere} = \frac{2}{5} M R^2$

$v = r\omega$

$$mgh = mg2R + \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

$$\cancel{m}gh = \cancel{m}g2R + \tfrac{1}{2}\cancel{m}Rg + \tfrac{1}{2}\left(\tfrac{2}{5}\cancel{m}\cancel{r^2}\right)\left(\frac{v}{\cancel{r}}\right)^2$$

$$gh = 2gR + \tfrac{1}{2}gR + \tfrac{1}{5}v^2$$

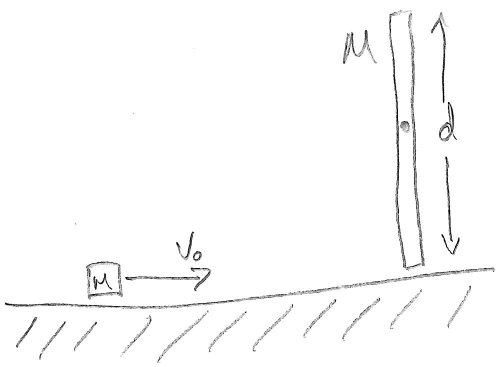
$$gh = 2gR + \tfrac{1}{2}gR + \tfrac{1}{5}gR$$

$$gh = \frac{20gR}{10} + \frac{5gR}{10} + \frac{2gR}{10}$$

$$\cancel{g}h = \frac{27\cancel{g}R}{10}$$

$$\boxed{h = \frac{27R}{10}}$$

86.



$I_{rod\,@cm} = \frac{1}{12} M d^2$

$M = 2m$

$\cancel{\tfrac{1}{2}} m v_0^2 = \cancel{\tfrac{1}{2}} m v^2 + \cancel{\tfrac{1}{2}} I\omega^2$

$m v_0 (d/2) = m v (d/2) + I\omega$

$\omega = \frac{md(v_0 - v)}{2I}$

$m v_0^2 = m v^2 + I\left(\frac{md(v_0 - v)}{2I}\right)^2$

$m v_0^2 = m v^2 + \frac{\cancel{I} m^2 d^2 (v_0 - v)^2}{4I^{\cancel{2}}}$

$\cancel{m} v_0^2 = \cancel{m} v^2 + \frac{\cancel{m^2} \cancel{d^2} (v_0 - v)^2}{4\left(\frac{1}{12}(2\cancel{m})\cancel{d^2}\right)}$

$v_0^2 = v^2 + \frac{3}{2}(v_0 - v)^2$

$0 = v^2 - v_0^2 + \frac{3}{2}v_0^2 - 3v_0 v + \frac{3}{2}v^2$

$0 = \frac{5}{2}v^2 - 3v_0 v + \frac{1}{2}v_0^2$

$v = \frac{3v_0 \pm \sqrt{9v_0 - \cancel{4}\left(\frac{5}{\cancel{2}}\right)\left(\frac{1}{\cancel{2}}v_0^2\right)}}{\cancel{2}\left(\frac{5}{\cancel{2}}\right)}$

$v = \frac{3v_0 \pm \sqrt{4v_0^2}}{5}$ $\boxed{v = \frac{v_0}{5}}$, v_0 trivial solution