Whale Song Unit Classification

Some Preliminary Exploration
Using Linear Prediction Vector Quantization
and Hidden Markov Modeling

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The Problem

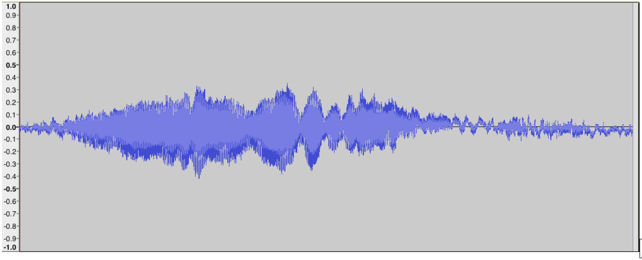
Following Mitchell (1997), we state the problem as follows:

- Task:
  Classify whale song unit instances
  according to a given vocabulary of whale song unit types

- Performance measure:
  Percent of instances correctly classified

- Training experience:
  A database of labelled whale song unit instances

Whale Song Units



Play

A "modulated cry" instance, from `HBSe_20151207T070326.wav, 124.5sec-126.5sec`

- Acoustic signal:

$$\mathbf{x} = \langle x_1, x_2, \ldots, x_N \rangle \qquad (N = 66,283)$$

Linear Predictive Coding

- Acoustic signal:

$$\mathbf{x} = \langle x_1, x_2, \ldots, x_N \rangle$$

- Transformed into a sequence of predictor vectors:

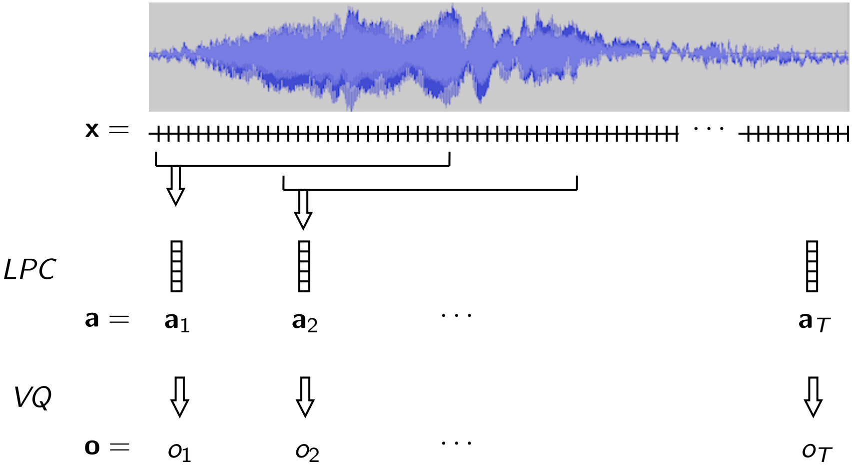
$$\mathbf{a} = \langle \mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_T \rangle$$

- Transformed into a sequence of symbols:

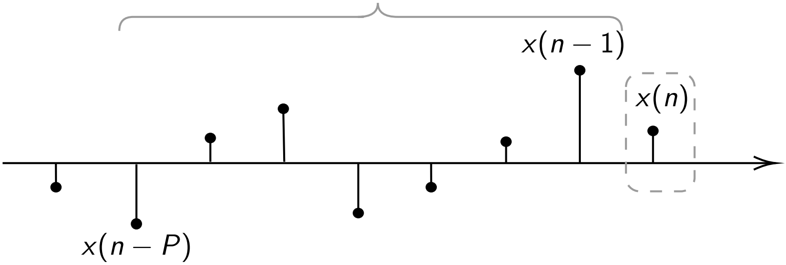
$$\mathbf{o} = \langle o_1, o_2, \ldots, o_T \rangle$$

where $o_t \in \{1, 2, \ldots, M\}$

Linear Predictive Coding



Linear Prediction



- Estimate $x(n)$ as a linear combination of P previous samples:

$$\hat{x}(n) = -\sum_{i=1}^{P} a(i)x(n-i)$$

Linear Prediction

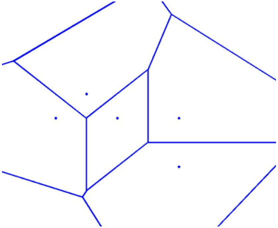
- Error:

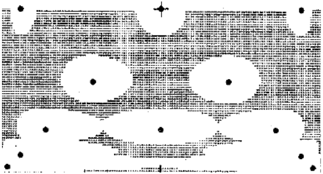
$$\begin{aligned} e(n) &= x(n) - \hat{x}(n) \\ &= \sum_{i=0}^{P} a(i)x(n-i) \end{aligned}$$

- Find prediction coefficients by minimizing the sum of the squared error over the signal interval:

$$\begin{aligned} E &= \sum_{n} e^2(n) \\ &= \sum_{n} \left(\sum_{i=0}^{P} a(i)x(n-i) \right)^2 \end{aligned}$$

Vector Quantization





- Partition the P-dimensional space into M regions that "best" represent the predictor vectors arising from song units instances
- "Best" in terms of minimizing some overall distortion measure
- Let $\mathbf{C} \equiv \langle C_1, C_2, \ldots, C_M \rangle$ be the set of centroids that best represent the space

Vector Quantization

- Quantization:
  With $\mathbf{C}$, we can map each $\mathbf{a}^{(t)}$ to a symbol o_t:

$$o_t \leftarrow \operatorname*{argmin}_{k=1}^{M} d(\mathbf{a}^{(t)}, C_k)$$

Hidden Markov Model

- π, initial state probability distribution:

$$\pi = (\pi_1, \pi_2, \ldots, \pi_N)$$

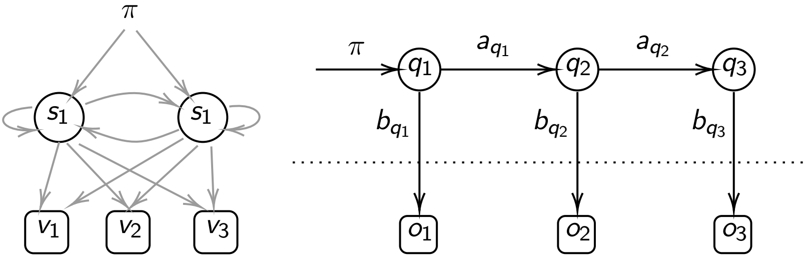
- A, state transition distributions:

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1N} \\ a_{21} & a_{22} & \ldots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \ldots & a_{NN} \end{bmatrix}$$

- B, observation symbol probabilities:

$$B = \begin{bmatrix} b_{11} & b_{12} & \ldots & b_{1M} \\ b_{21} & b_{22} & \ldots & b_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ b_{N1} & b_{N2} & \ldots & b_{NM} \end{bmatrix}$$

Hidden Markov Modeling



- $\mathbf{q} = \langle q_1, q_2, \ldots, q_T \rangle$: hidden state sequence
- $\mathbf{o} = \langle o_1, o_2, \ldots, o_T \rangle$: our acoustic signal

HMM Operations

With $\lambda = (\pi, A, B)$ denoting an HMM

- For given $\mathbf{o}$ and λ, compute $P[\mathbf{o}|\lambda]$
- For given $\mathbf{o}$ and λ, estimate a most likely state sequence $\mathbf{q}^*$
- Learn a model $\lambda^* \equiv (\pi^*, A^*, B^*)$
  such that $P[\mathbf{o}|\lambda^*]$ is maximized for a given observation sequence $\mathbf{o}$

Statistical Pattern Classification

- Vocabulary of R song unit types:

$$U = \{u_1, u_2, \ldots, u_R\}$$

- Assume we have a model h_r for each unit type u_r.
  We are interested in computing:

$$P[h_r|\mathbf{x}] \equiv \text{Probability of } h_r \text{ given } \mathbf{x}$$

- Classification of an observation sequence $\mathbf{x}$
  is based on a maximum likelihood model selection:

$$h^* \equiv \underset{h \in H}{\operatorname{argmax}} \, P[h|\mathbf{x}]$$

  where $H = \{h_1, \ldots, h_R\}$.

Maximum Likelihood model selection

$$
\begin{aligned}
h^* &\equiv \operatorname*{argmax}_{h \in H} P[h|\mathbf{x}] \\
&= \operatorname*{argmax}_{h \in H} \frac{P[\mathbf{x}|h]P[h]}{P[\mathbf{x}]} && \text{(Bayes' theorem)} \\
&= \operatorname*{argmax}_{h \in H} P[\mathbf{x}|h]P[h] && \text{(}P[\mathbf{x}]\text{ is constant wrt }h\text{)} \\
&= \operatorname*{argmax}_{h \in H} P[\mathbf{x}|h] && \text{(Assuming models are equiprobable)}
\end{aligned}
$$