محمدرضا میرشمسی ۹۷۵۲۲۰۷۶

۱) الف) $r=\sqrt{4+9}=\sqrt{13} \qquad \theta=\tan^{-1}\left(\frac{3}{2}\right)\simeq 56 \Rightarrow \sqrt{13}\,(\cos 56 + j\sin 56)$

ب) $r=\sqrt{1+9}=\sqrt{10} \qquad \theta=\tan^{-1}\left(\frac{-3}{1}\right)\simeq -71 \Rightarrow \sqrt{10}\,\big(\cos(-71) - j\sin(71)\big)$

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۲) الف) $\underline{2e^{j\frac{\pi}{3}}}=2\left(\cos\frac{\pi}{3}+j\sin\frac{\pi}{3}\right)=2\left(\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)=1+\sqrt{3}\,j$

ب) $\underline{4e^{-j\frac{3\pi}{4}}}=4\left(\cos\left(-\frac{3\pi}{4}\right)+j\sin\left(-\frac{3\pi}{4}\right)\right)=4\left(\frac{-\sqrt{2}}{2}-j\frac{\sqrt{2}}{2}\right)=-2\sqrt{2}-2\sqrt{2}\,j$

ج) $\underline{2e^{j4\pi}}=2(\cos 4\pi + j\sin 4\pi)=2(1+0)=2$

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۳) الف) $\underline{2z_1-z_2}=4\left(\cos\frac{\pi}{4}+j\sin\frac{\pi}{4}\right)-8\left(\cos\frac{\pi}{3}+j\sin\frac{\pi}{3}\right)$

$4\left(\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2}\right)-8\left(\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)=(2\sqrt{2}-4)+(2\sqrt{2}-4\sqrt{3})\,j$

ب) $\underline{\frac{1}{z_1}}=\frac{1}{2e^{j\frac{\pi}{4}}}=\frac{1}{2}e^{-j\frac{\pi}{4}}=\frac{1}{2}\left(\frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{2}}{4}-\frac{\sqrt{2}}{4}\,j$

ج) $\underline{\frac{z_1}{z_2}}=\frac{2}{8}\,e^{\left(\frac{\pi}{4}-\frac{\pi}{3}\right)j}=\frac{1}{4}\,e^{-\frac{\pi}{12}j}=\dots$

د) $\sqrt[3]{z_2}=z_2^{\frac{1}{3}}=2e^{j\frac{\pi}{9}}=\dots$

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۴) الف) $\frac{2+jw}{3+j4w}\times\frac{3-j4w}{3-j4w}=\frac{6-8wj+3wj+4w^2}{9+16w^2}$

$\Rightarrow$ حقیقی: $\frac{6+4w^2}{9+16w^2}$ ، موهومی: $\frac{-5wj}{9+16w^2}$

ب) $r^2 = \dfrac{(6+4w^2)^2}{(9+16w^2)^2} + \dfrac{25w^2}{(9+16w^2)^2} \Rightarrow r = \dfrac{16w^4 + 73w^2 + 36}{9+16w^2}$

$\cancel{\theta = \tan^{-1}\left(\dfrac{\frac{6+4w^2}{9+16w^2}}{-}\right)}$ $\quad \theta = \tan^{-1}\left(\dfrac{\frac{-5w}{9+16w^2}}{\frac{6+4w^2}{9+16w^2}}\right) = \tan^{-1}\left(\dfrac{-5w}{6+4w^2}\right)$

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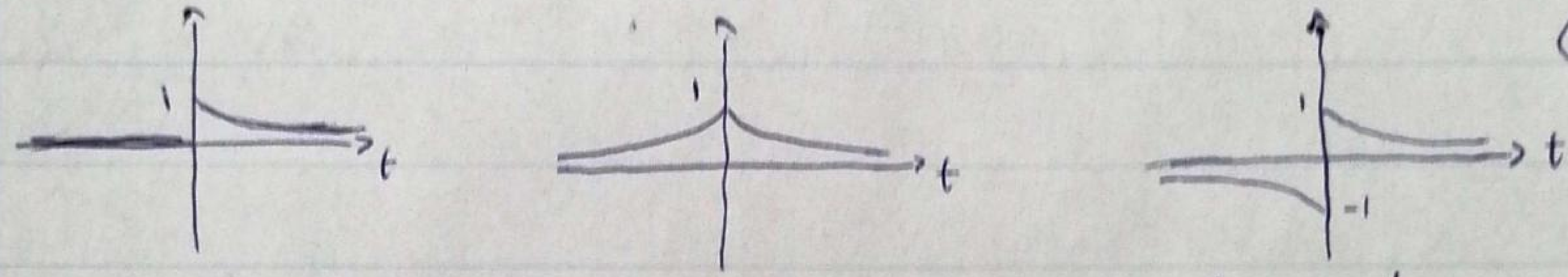
۴)



$u(t)e^{-t}$ $\qquad$ زوج: $\dfrac{e^{-t}u(t) + e^{t}u(-t)}{2}$ $\qquad$ فرد: $\dfrac{e^{-t}u(t) - e^{t}u(-t)}{2}$

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۵) الف) $2\left(\frac{1}{2}\cos w_0 t - \frac{\sqrt{3}}{2}\sin w_0 t\right) = 2\cos\left(w_0 t - \frac{\pi}{3}\right)$

ب) $f(t) = \dfrac{-1}{\sqrt{13}}\left(\dfrac{2}{\sqrt{13}}\cos(w_0 t) + \dfrac{3}{\sqrt{13}}\sin(w_0 t)\right)$, $\quad \theta = \tan^{-1}\left(\dfrac{\frac{3}{\sqrt{13}}}{\frac{2}{\sqrt{13}}}\right)$

$$\frac{-1}{\sqrt{13}}\left(\cos\theta\cos(w_0 t) + \sin\theta\sin(w_0 t)\right) = \frac{-1}{\sqrt{13}}\cos(w_0 t - \theta)$$

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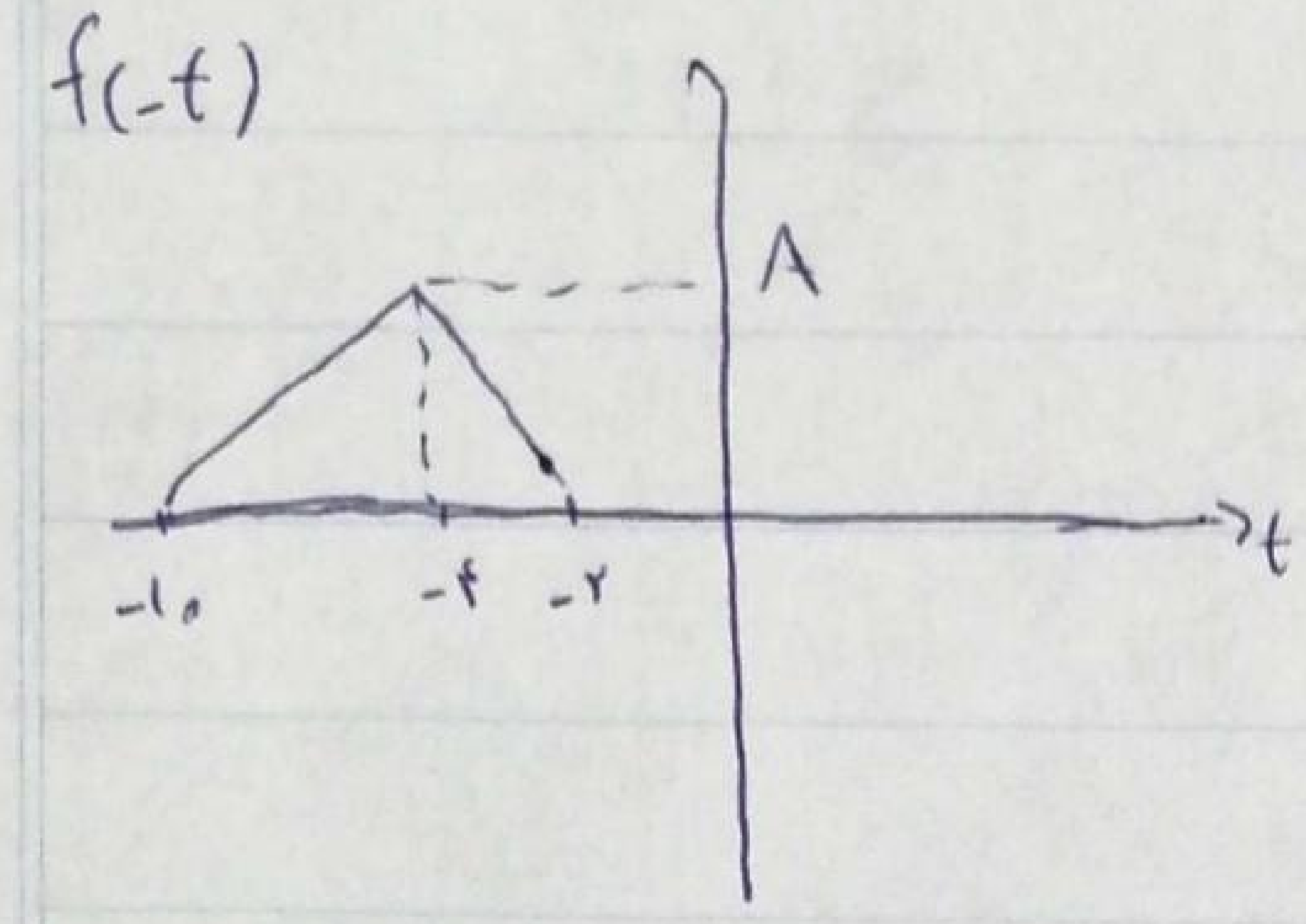
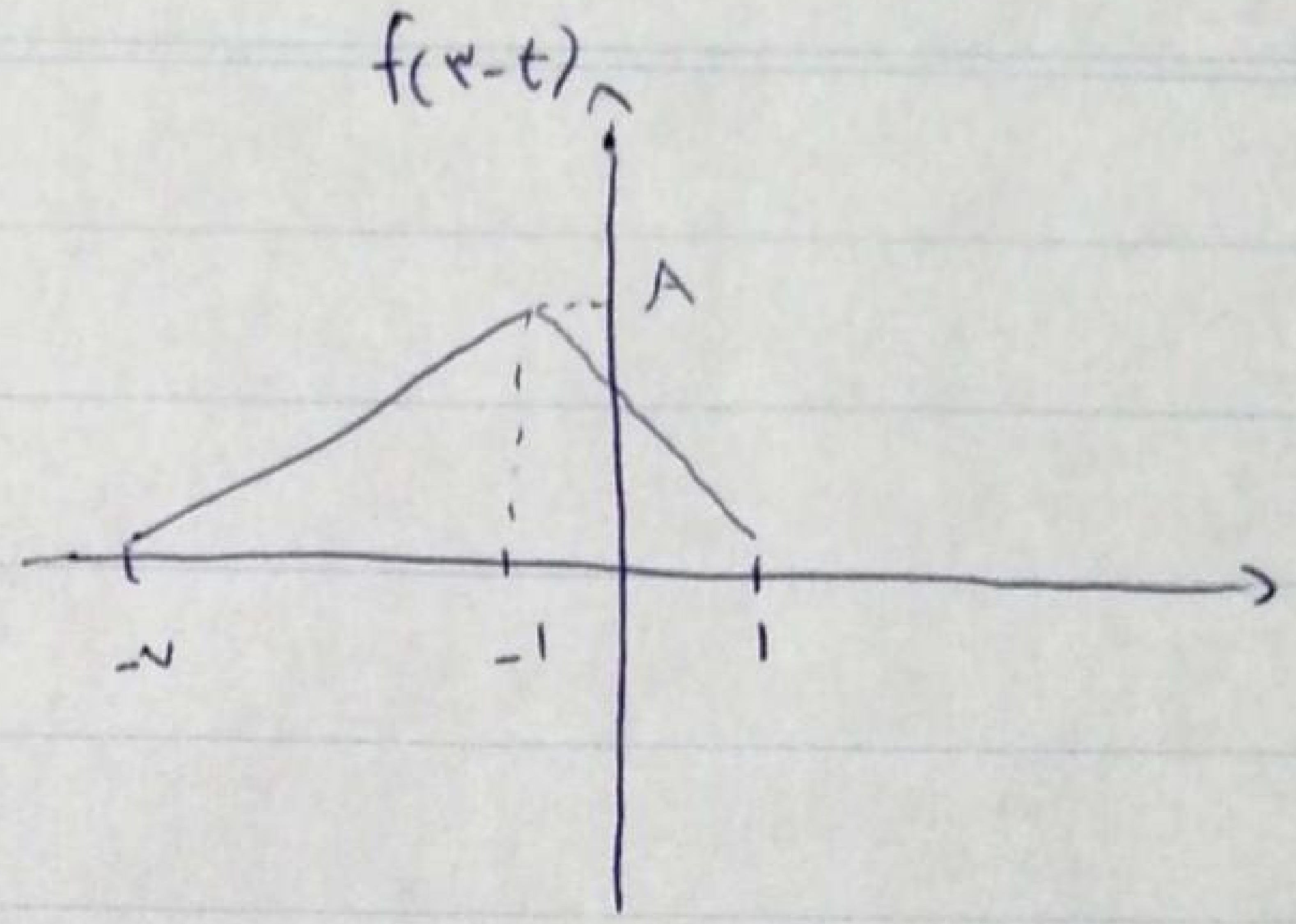
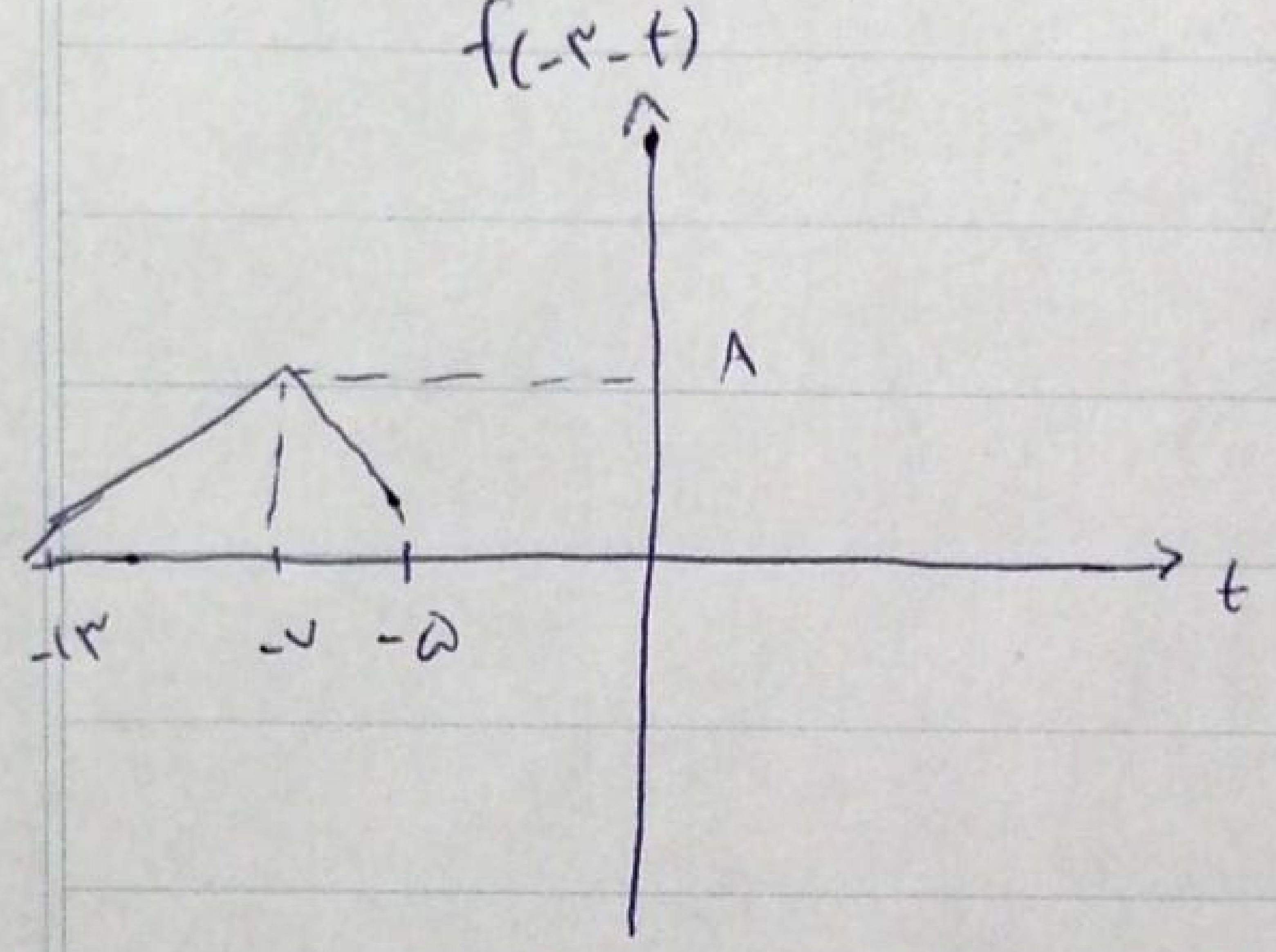
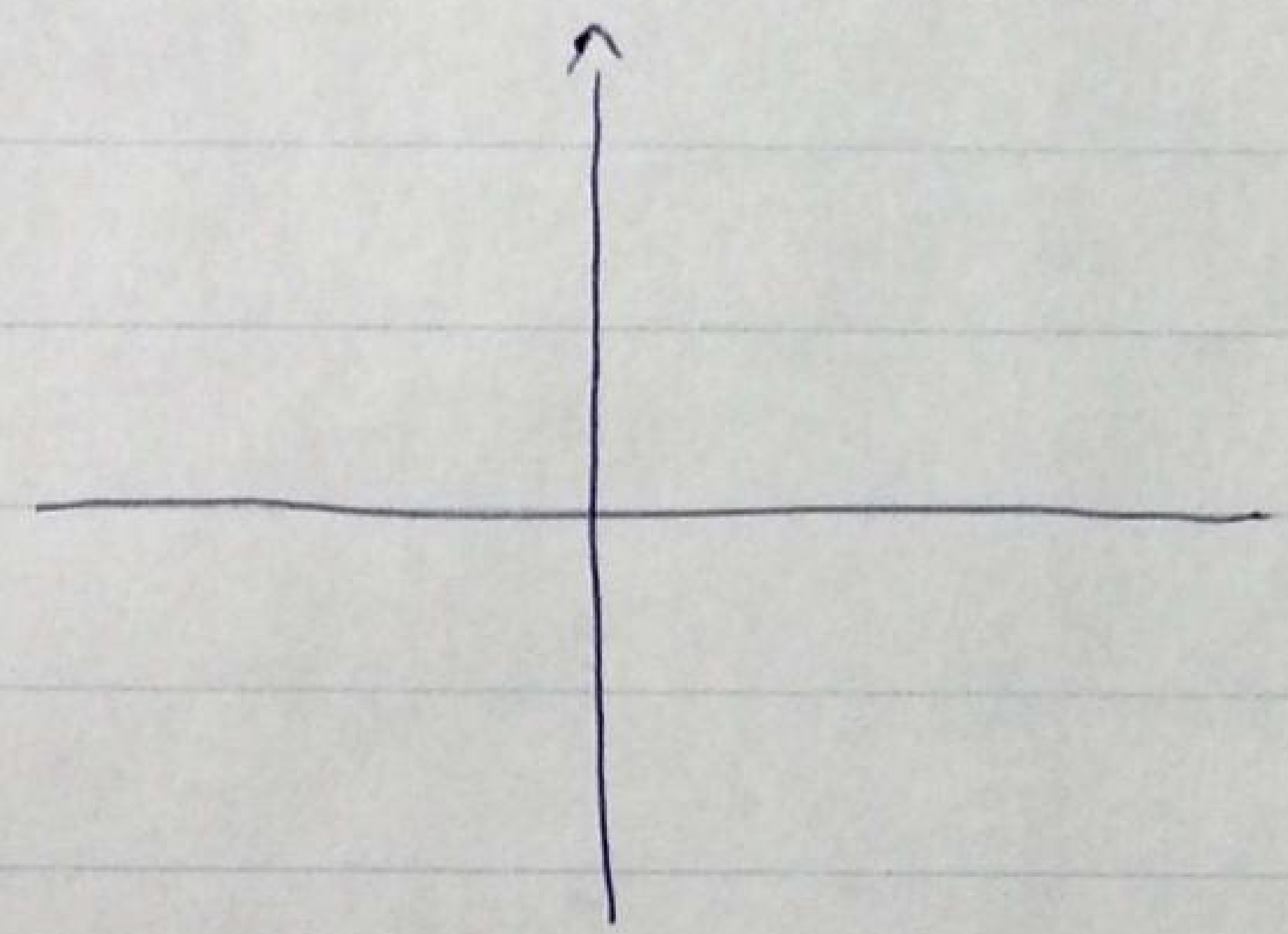
۷) زوج: $\dfrac{e^{dt} + e^{-dt}}{2}$ $\qquad$ فرد: $\dfrac{e^{dt} - e^{-dt}}{2}$

(۸)



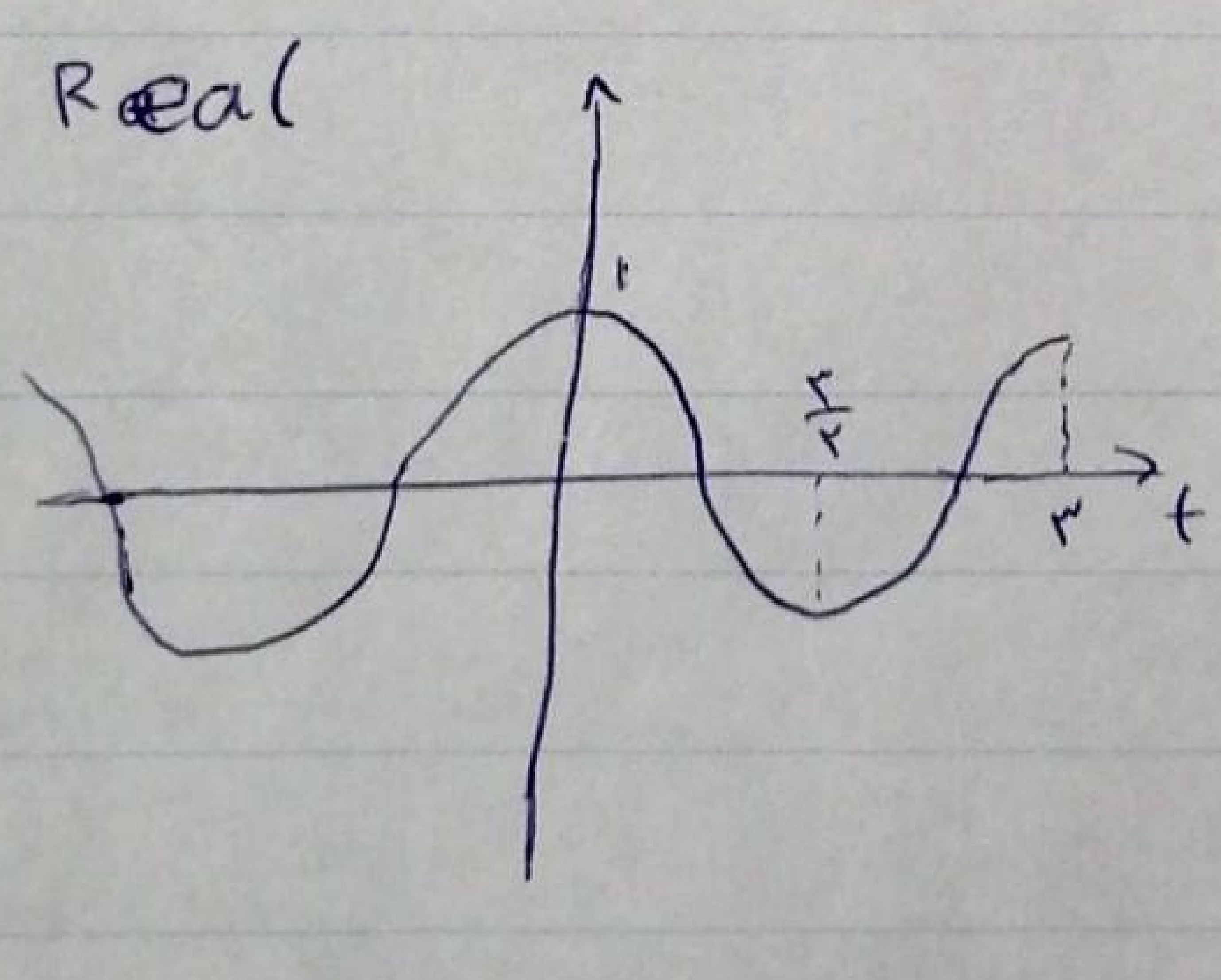


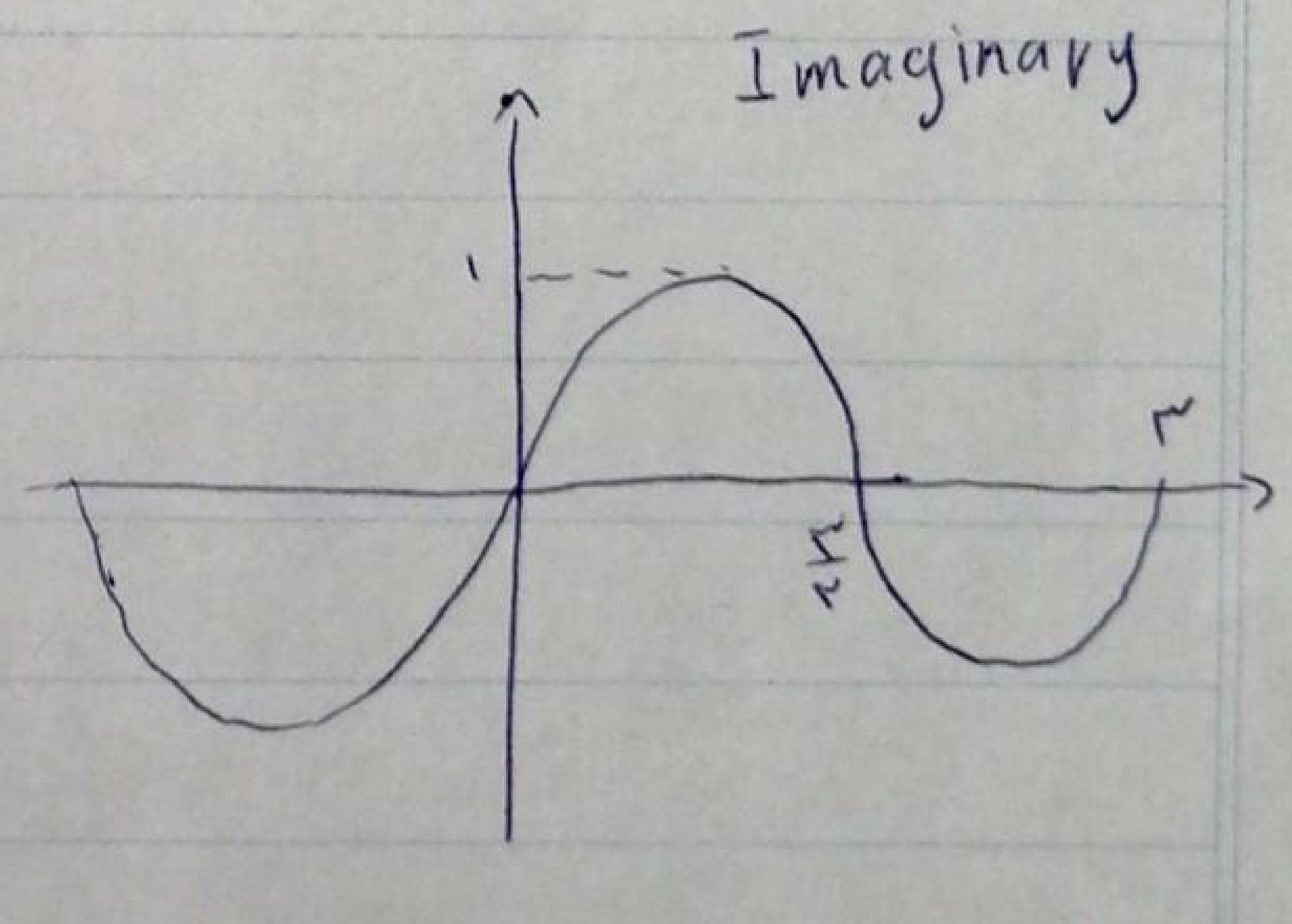




(۹)

$$f(t) = e^{j\omega_0 t} = \cos \frac{2\pi}{3} t + j \sin \frac{2\pi}{3} t$$





تابع $e^{(\sigma+j\omega)t}$ در صورت $\sigma=0$ بودن پریودیک است پس تابع بالا پریودیک

است و دوره تناوب آن برابر ۳ می باشد