Forget to remember
Remember to forget

VLC = $\iiint$ Vision Learning Control $dV\,dL\,dC$

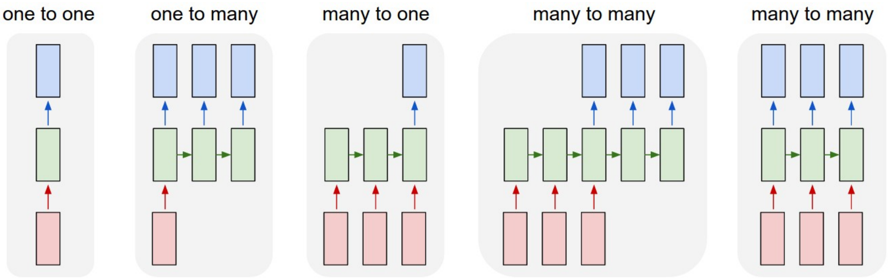
Long Short Term Memories and Gated Recurrent Units

Kate Farrahi

Vision, Learning and Control
University of Southampton

Some of the slides, images and animations used here were originally designed by Jonathon Hare and Adam Prügel-Bennett.

Recap: RNN

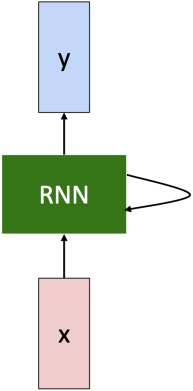


Example Applications: image classification, image captioning, video classification, machine translation, language modeling

Image from `http://karpathy.github.io/2015/05/21/rnn-effectiveness/`

Recap: Vanilla or Elman RNN

Excluding bias terms

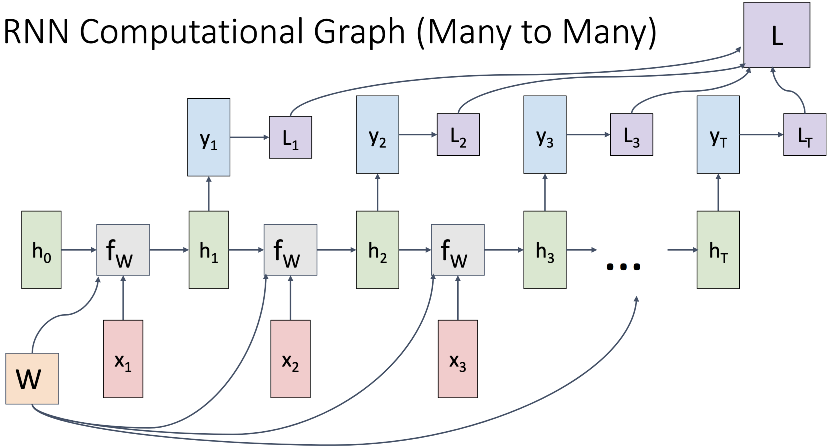


$$h_t = tanh(W_{hh} h_{t-1} + W_{xh} x_t)$$

$$y_t = W_{hy} h_t$$

Image from `https://web.eecs.umich.edu`

Recap: RNN Computational Graph (with Losses)



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Example: Character-level language modelling

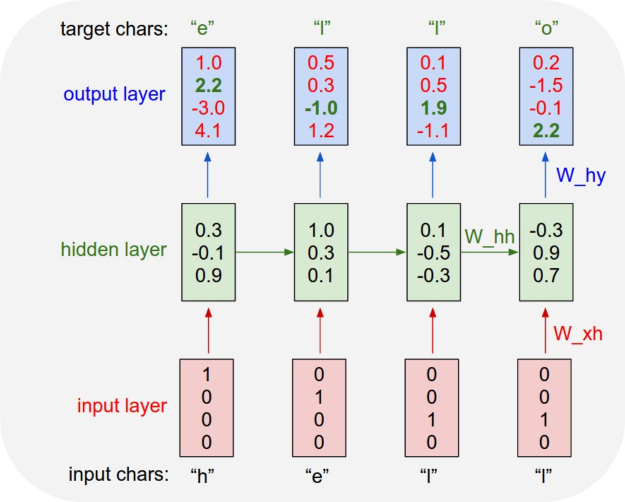
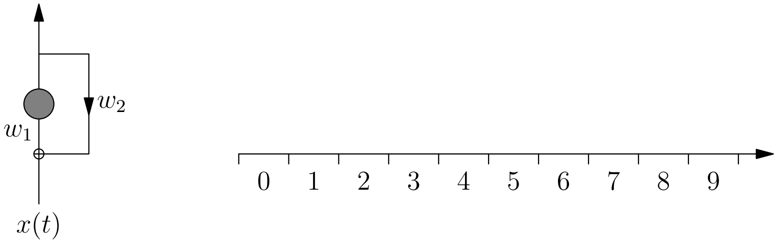


Image from http://karpathy.github.io/2015/05/21/rnn-effectiveness/

Vanishing and Exploding Gradients

$$y(t) = w_1 \left(x(t) + w_2\, y(t-1)\right)$$



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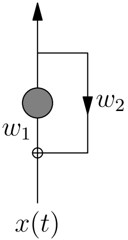
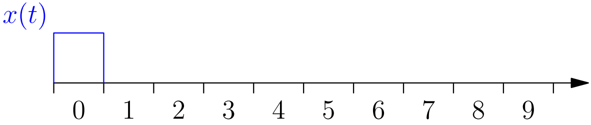


$$w_1 = w_2 = 0.9$$



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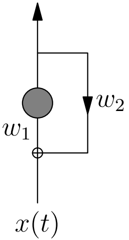

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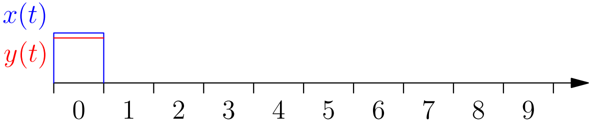

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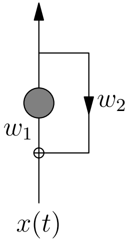

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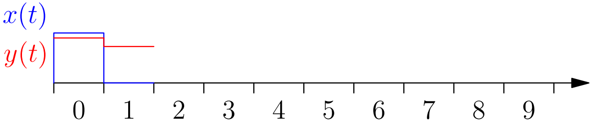

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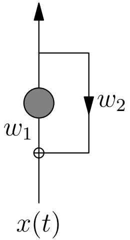

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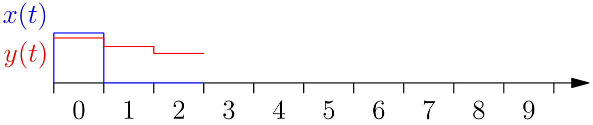

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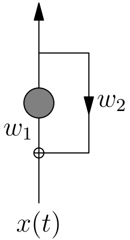

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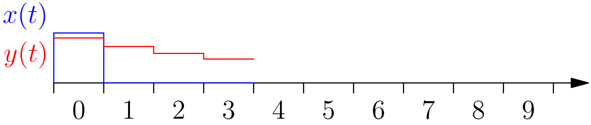

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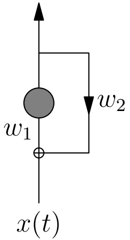

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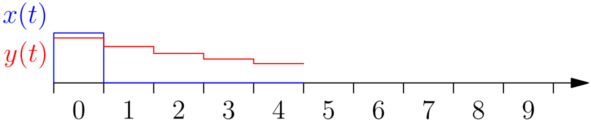

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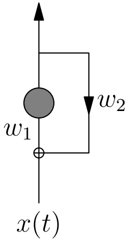

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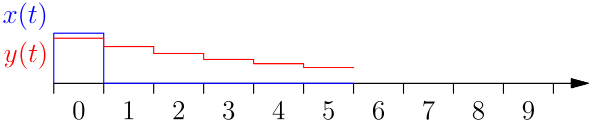

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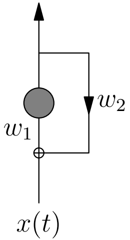

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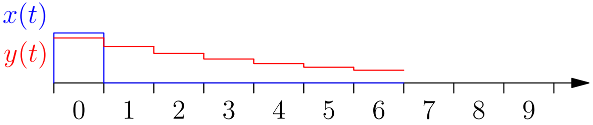

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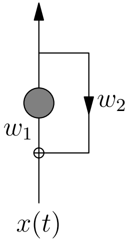

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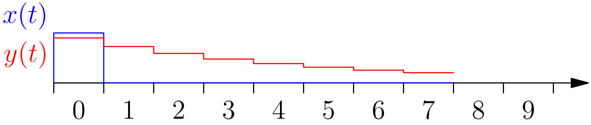

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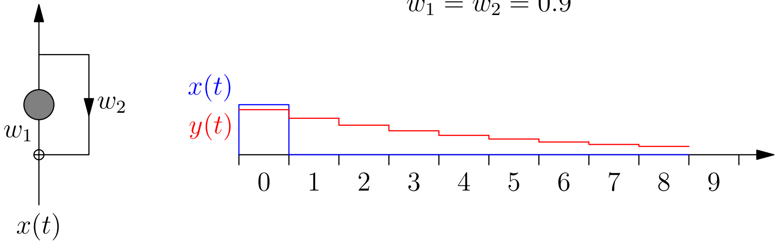

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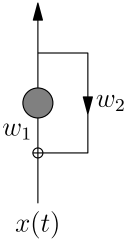

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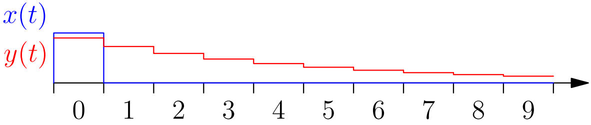

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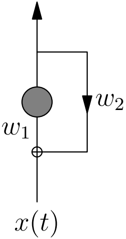

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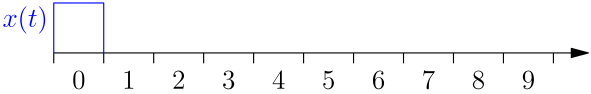

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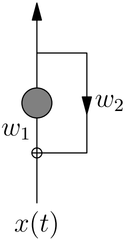

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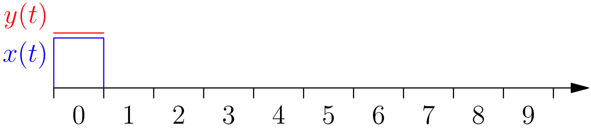

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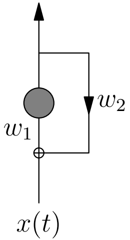

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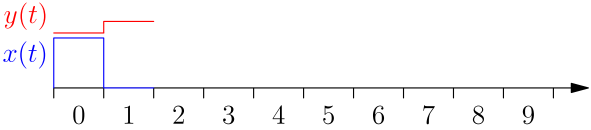

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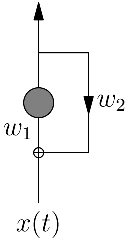

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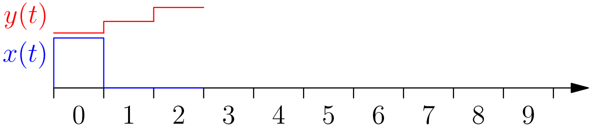

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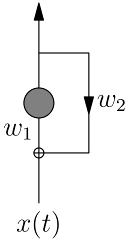

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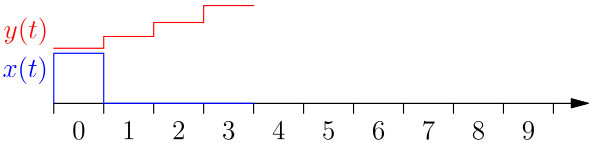

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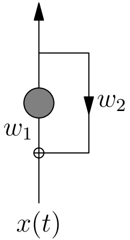

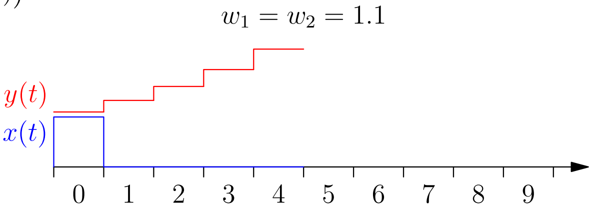

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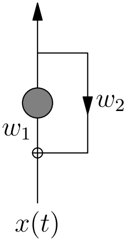

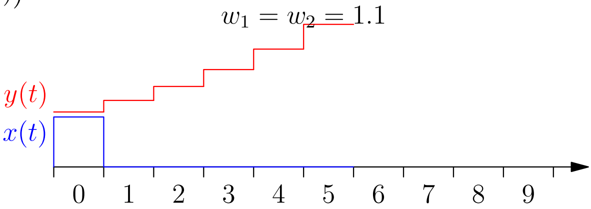

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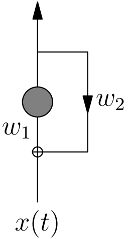

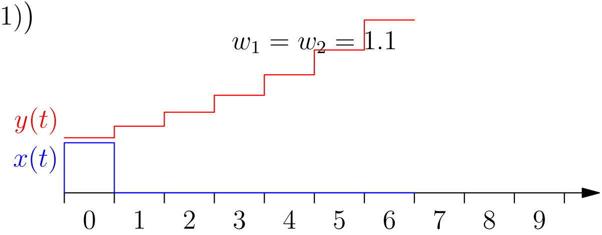

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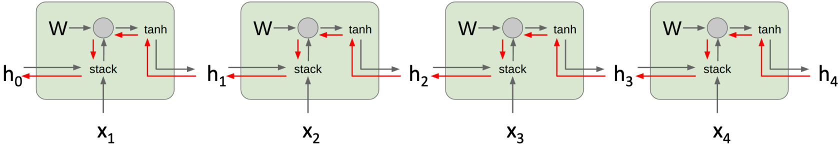

Image from `https://web.eecs.umich.edu`

LSTM Architecture

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- Sigmoid functions naturally saturate at 0 and 1

LSTM Architecture

LSTM

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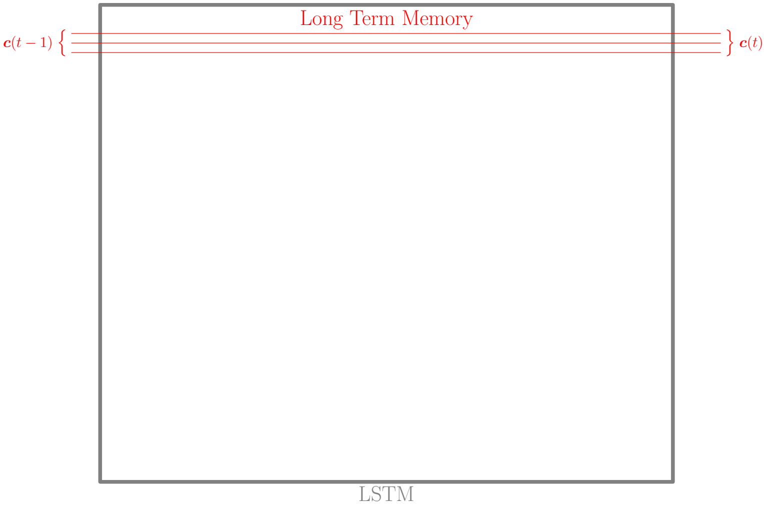

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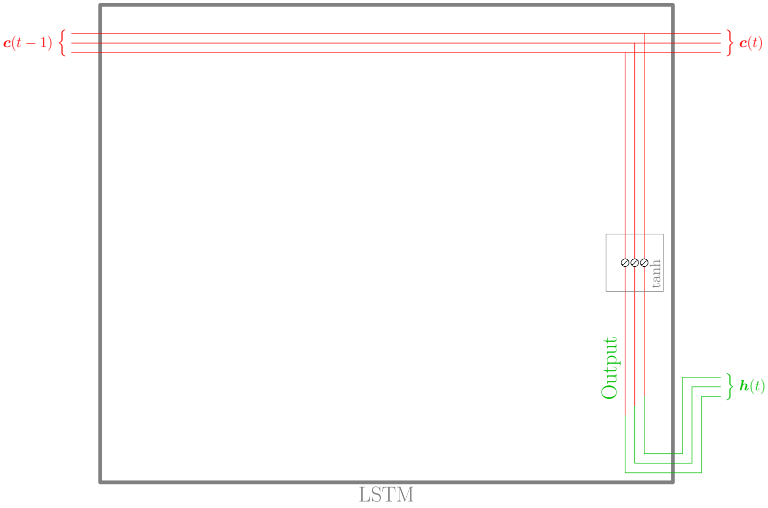

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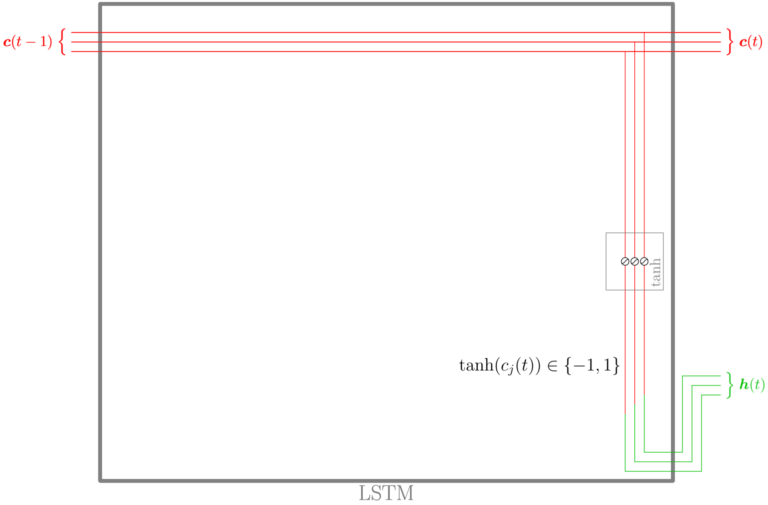

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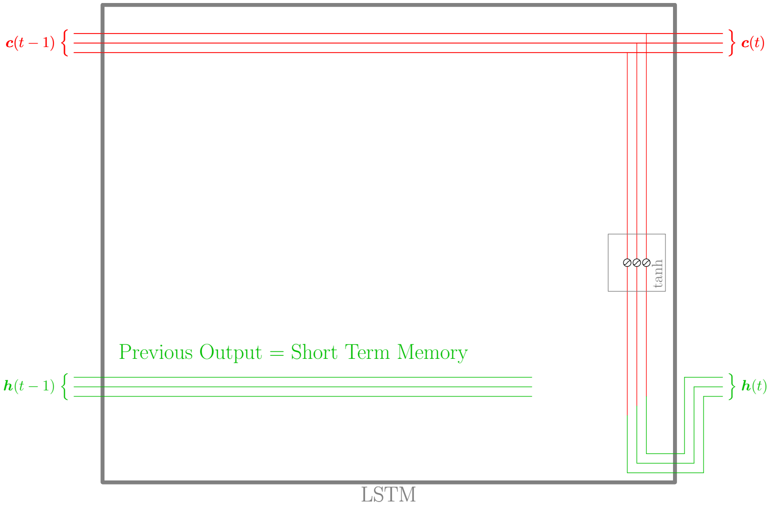

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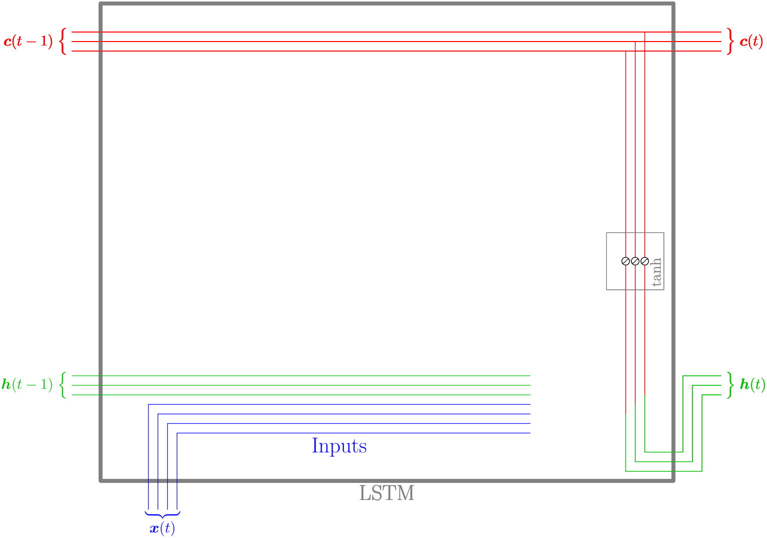

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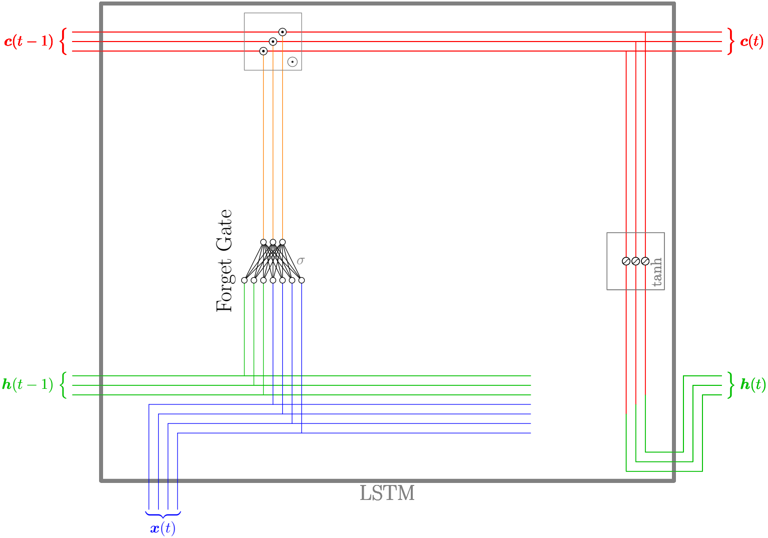

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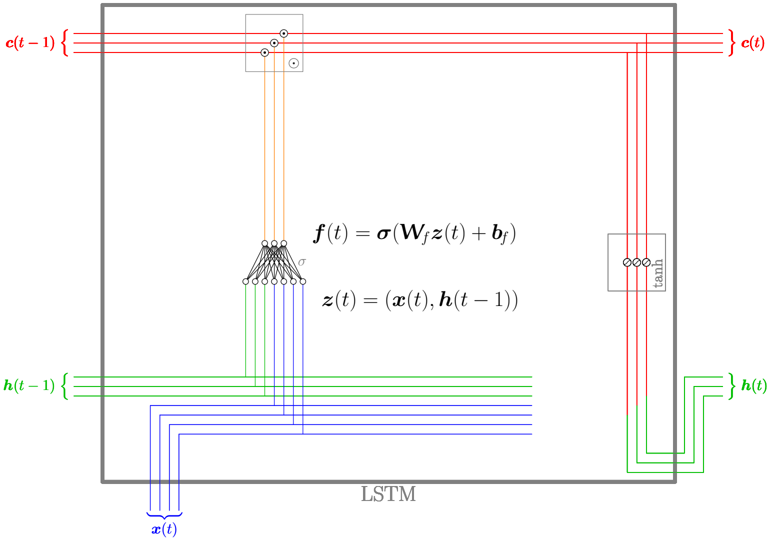
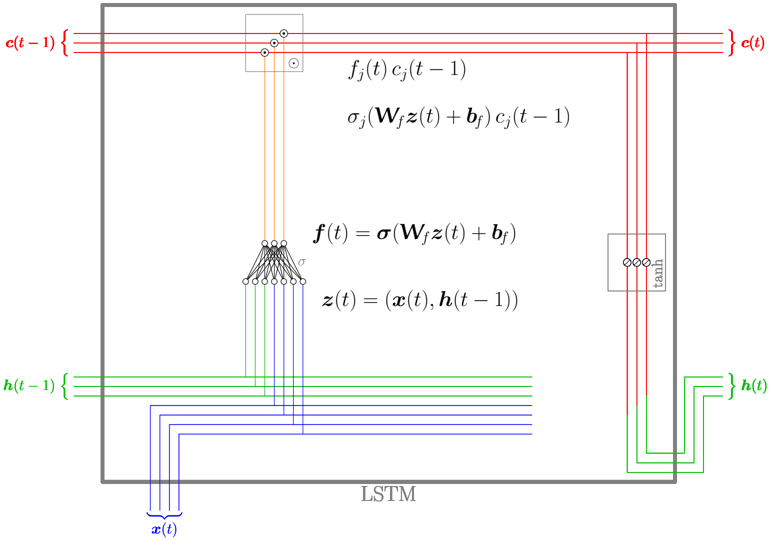


$\boldsymbol{c}(t-1)$
$\boldsymbol{c}(t)$
$\odot$
σ
$\boldsymbol{f}(t) = \boldsymbol{\sigma}(\boldsymbol{W}_f \boldsymbol{z}(t) + \boldsymbol{b}_f)$
$\boldsymbol{z}(t) = (\boldsymbol{x}(t), \boldsymbol{h}(t-1))$
tanh
$\boldsymbol{h}(t-1)$
$\boldsymbol{h}(t)$
LSTM
$\boldsymbol{x}(t)$

LSTM Architecture


$\boldsymbol{c}(t-1)$
$\boldsymbol{c}(t)$
$\odot$
$f_j(t)\,c_j(t-1)$
$\sigma_j(\boldsymbol{W}_f\boldsymbol{z}(t)+\boldsymbol{b}_f)\,c_j(t-1)$
$\boldsymbol{f}(t)=\boldsymbol{\sigma}(\boldsymbol{W}_f\boldsymbol{z}(t)+\boldsymbol{b}_f)$
σ
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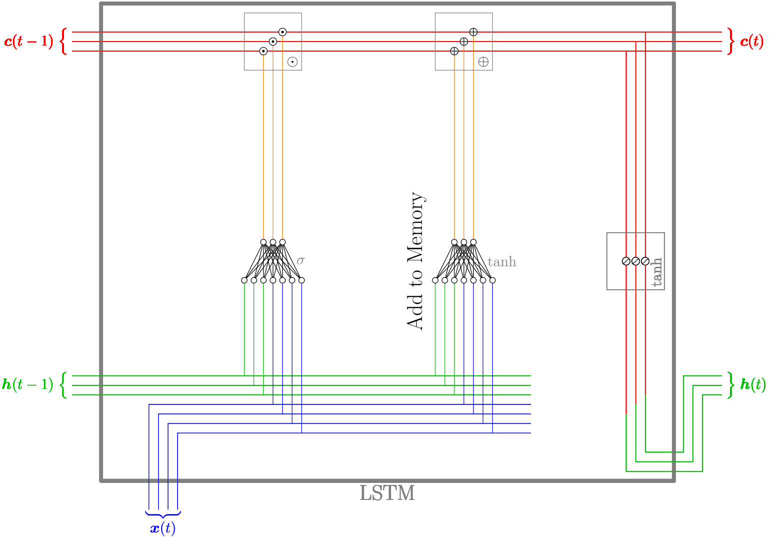

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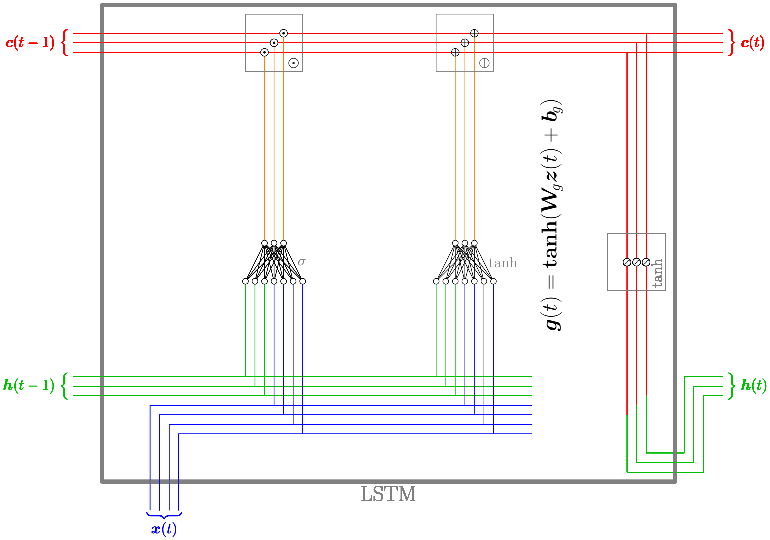

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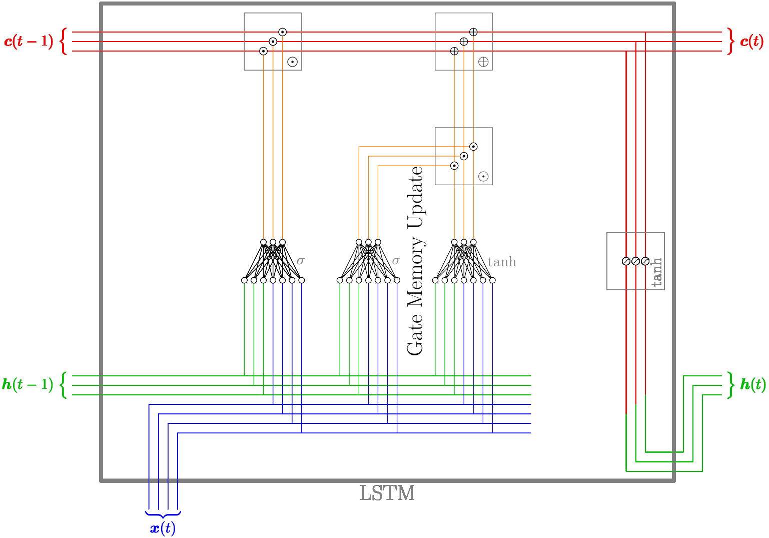

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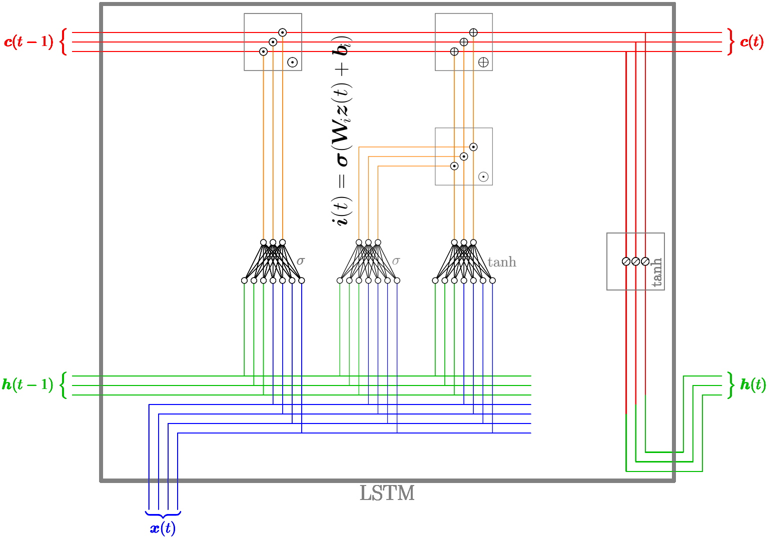
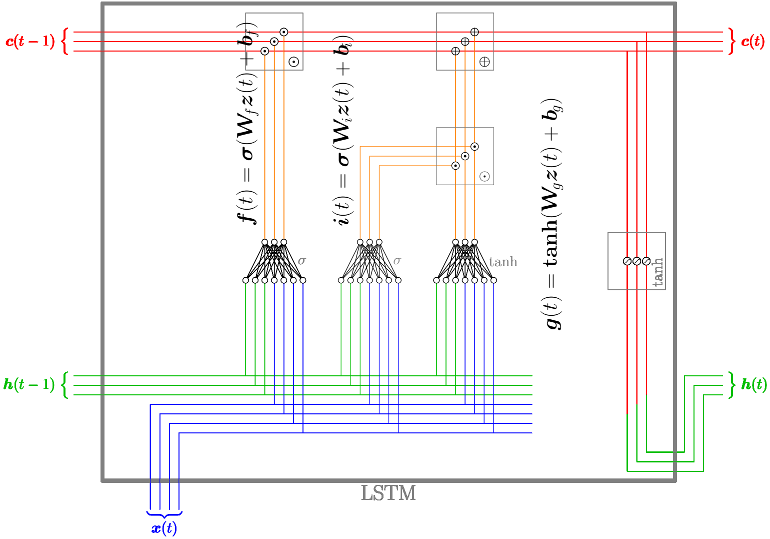


LSTM Architecture

$$c_j(t) = f_j(t)\, c_j(t-1) + b_f + g_j(t)\, i_j(t) + b_i$$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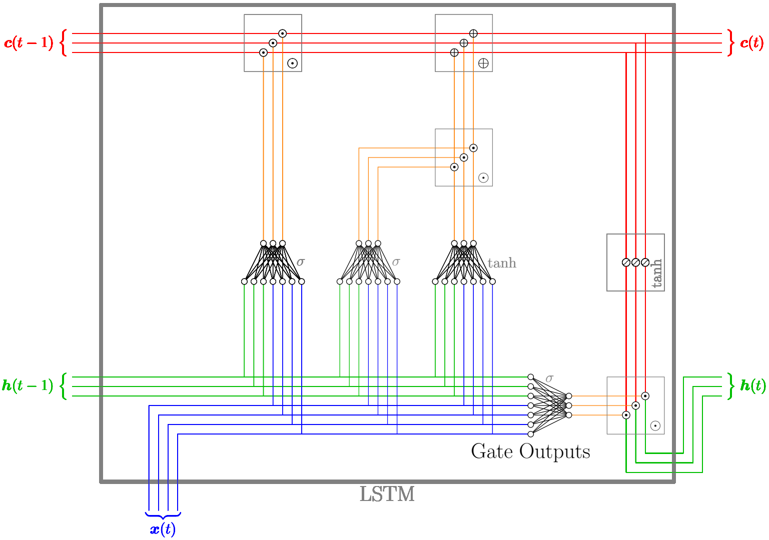
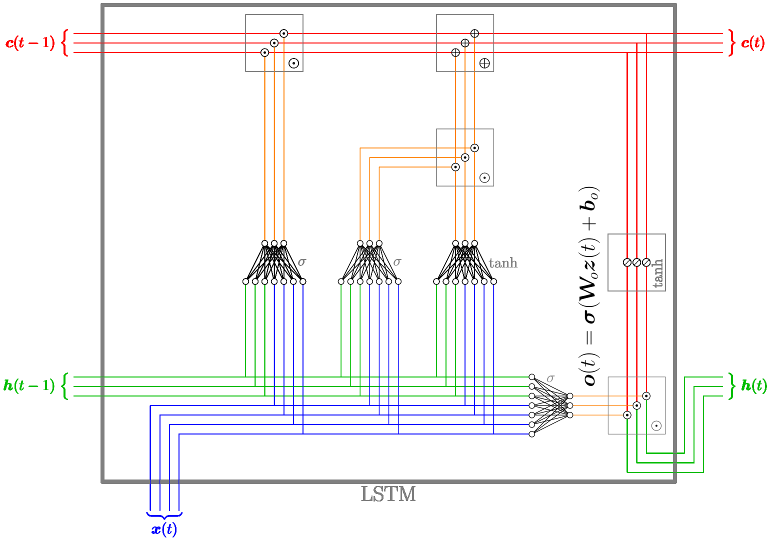
$\boldsymbol{i}(t) = \boldsymbol{\sigma}(\mathbf{W}_i \boldsymbol{z}(t) + \boldsymbol{b}_i)$
$\boldsymbol{g}(t) = \tanh(\mathbf{W}_g \boldsymbol{z}(t) + \boldsymbol{b}_g)$
$\odot$
$\oplus$
$\odot$
σ
σ
tanh
tanh
$\boldsymbol{h}(t-1)$
$\boldsymbol{h}(t)$
LSTM
$\boldsymbol{x}(t)$

LSTM Architecture



LSTM Architecture



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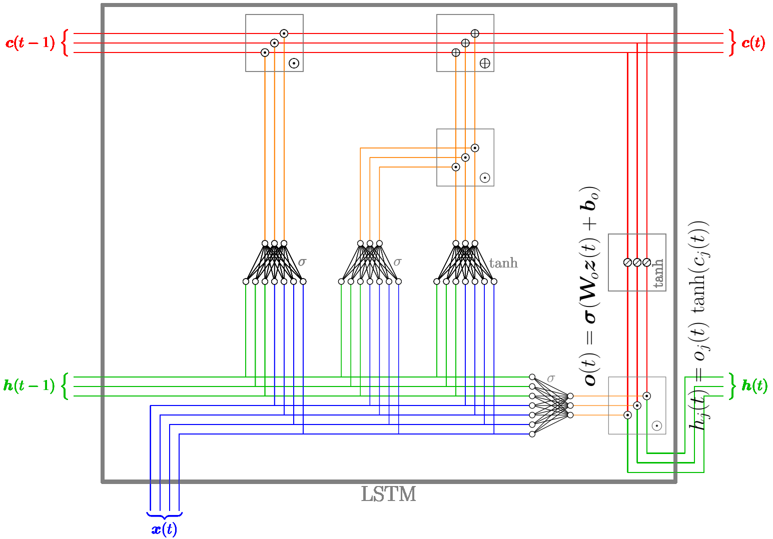

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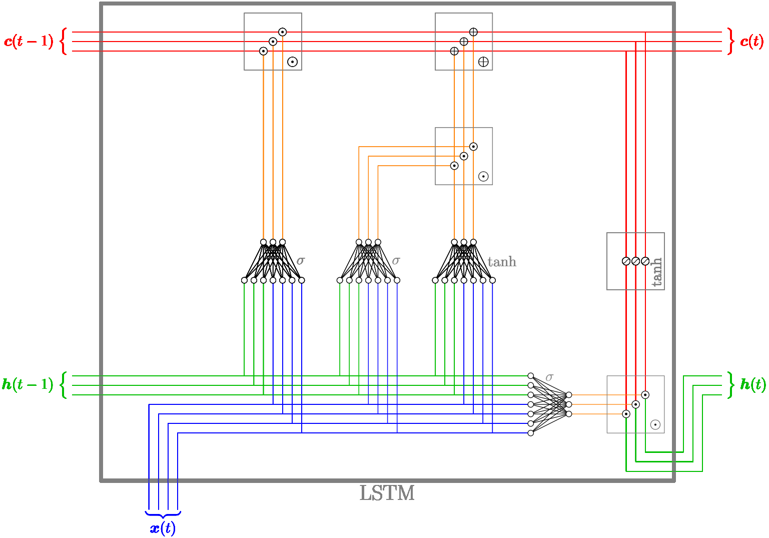

Update Equations

Initially, for $t = 0$, $\boldsymbol{h}(0) = \mathbf{0}$

- Inputs $\boldsymbol{z}(t) = \big(\boldsymbol{x}(t), \boldsymbol{h}(t-1)\big)$

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$$\boldsymbol{f}(t) = \boldsymbol{\sigma}(\boldsymbol{W}_f\, \boldsymbol{z}(t) + \boldsymbol{b}_f) \qquad \boldsymbol{i}(t) = \boldsymbol{\sigma}(\boldsymbol{W}_i\, \boldsymbol{z}(t) + \boldsymbol{b}_i)$$

$$\boldsymbol{g}(t) = \mathbf{tanh}(\boldsymbol{W}_g\, \boldsymbol{z}(t) + \boldsymbol{b}_g) \qquad \boldsymbol{o}(t) = \boldsymbol{\sigma}(\boldsymbol{W}_o\, \boldsymbol{z}(t) + \boldsymbol{b}_o)$$

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- Long-term memory update

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$$\boldsymbol{c}(t) = \boldsymbol{f}(t) \odot \boldsymbol{c}(t-1) + \boldsymbol{g}(t) \odot \boldsymbol{i}(t)$$

- Output $\boldsymbol{h}(t) = \boldsymbol{o}(t) \odot \mathbf{tanh}(\boldsymbol{c}(t))$

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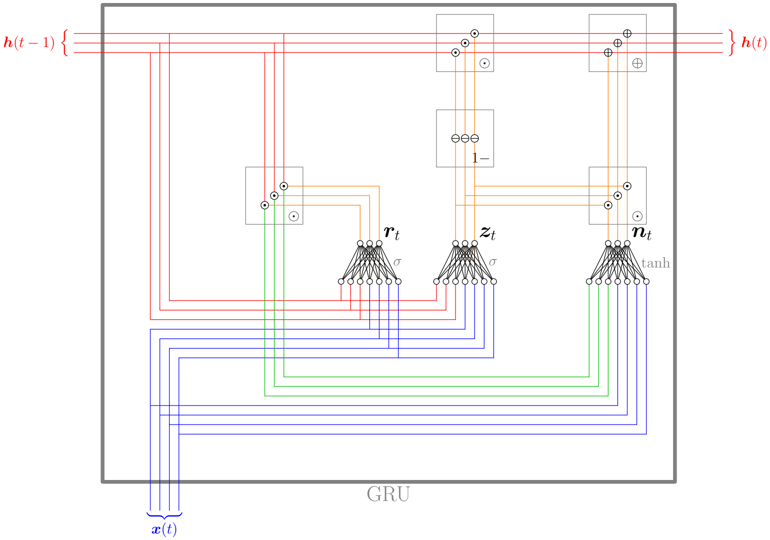
- We can train an LSTM by unwrapping it in time.
- Note that it involves four dense layers with sigmoidal (or tanh) outputs.
- This means that typically it is very slow to train.
- There are a few variants of LSTMs, but all are very similar. The most popular is probably the Gated Recurrent Unit (GRU).

LSTM Success Stories

- LSTMs have been used to win many competitions in speech and handwriting recognition.
- Major technology companies including Google, Apple, and Microsoft are using LSTMs as fundamental components in products.
- Google used LSTM for speech recognition on the smartphone, for Google Translate.
- Apple uses LSTM for the "Quicktype" function on the iPhone and for Siri.
- Amazon uses LSTM for Amazon Alexa.
- In 2017, Facebook performed some 4.5 billion automatic translations every day using long short-term memory networks[1].

[1] https://en.wikipedia.org/wiki/Long_short-term_memory

Gated Recurrent Unit (GRU)



Gated Recurrent Unit (GRU)

- $\boldsymbol{x}(t)$: input vector
- $\boldsymbol{h}(t)$: output vector (and 'hidden state')
- $\boldsymbol{r}(t)$: reset gate vector
- $\boldsymbol{z}(t)$: update gate vector
- $\boldsymbol{n}(t)$: new state vector (before update is applied)
- $\boldsymbol{W}$ and $\boldsymbol{b}$: parameter matrices and biases

Gated Recurrent Unit (GRU)

Initially, for $t = 0$, $\boldsymbol{h}(0) = \mathbf{0}$

$$\begin{aligned}
\boldsymbol{z}(t) &= \sigma(\boldsymbol{W}_z(\boldsymbol{x}(t), \boldsymbol{h}(t-1)) + \boldsymbol{b}_z) \\
\boldsymbol{r}(t) &= \sigma(\boldsymbol{W}_r(\boldsymbol{x}(t), \boldsymbol{h}(t-1)) + \boldsymbol{b}_r) \\
\boldsymbol{n}(t) &= \tanh(\boldsymbol{W}_n(\boldsymbol{x}(t), r(t) \odot h(t-1)) + \boldsymbol{b}_h) \\
\boldsymbol{h}(t) &= (1 - \boldsymbol{z}(t)) \odot \boldsymbol{h}(t-1) + \boldsymbol{z}(t) \odot \boldsymbol{n}(t)
\end{aligned}$$

Most implementations follow the original paper and swap $(1 - \boldsymbol{z}(t))$ and $(\boldsymbol{z}(t))$ in the $\boldsymbol{h}(t)$ update; this doesn't change the operation of the network, but does change the interpretation of the update gate, as the gate would have to produce a 0 when an update was to occur, and a 1 when no update is to happen (which is somewhat counter-intuitive)!

GRU or LSTM?

- GRUs have two gates (reset and update) whereas LSTM has three gates (input/output/forget)
- GRU performance on par with LSTM but computationally more efficient (less operations & weights).
- In general, if you have a very large dataset then LSTMs will likely perform slightly better.
- GRUs are a good choice for smaller datasets.