

Yes,
we GAN.

VLC = $\int \int \int$ Vision
Learning
Control

Deep Generative Modelling

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Introduction

- What is generative modelling and why do we do it?
- Differentiable Generator Networks
- Variational Autoencoders
- Generative Adversarial Networks

Recap: Generative Models

- Learn models of the data: $p(x)$
- Learn *conditional* models of the data: $p(x|y = y)$
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 - The likelihood of data x is the weighted sum of the likelihood from each of the k Gaussians
 - Sampling can be achieved by sampling the categorical distribution of k weights followed by sampling a data point from the corresponding Gaussian

Why do generative modelling?

- Try to understand the processes through which the data was itself generated
 - Probabilistic latent variable models like VAEs or topic models (PLSA, LDA, ...) for text
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- Make 'new' data
 - Make 'fake' data to use to train large supervised models?
 - 'Imagine' new, but plausible, things?

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 - Generative adversarial networks - A way to train generative models by optimizing them to fool a classifier
- **Common thread in recent advances is that the loss functions are end-to-end differentiable.**

Differentiable Generator Networks: key idea

- We're interested in models that transform samples of latent variables z to
 - samples x , or,
 - distributions over samples x
- The model is a (differentiable) function $g(z, \theta)$
 - typically g is a neural network.

Example: drawing samples from $\mathcal{N}(\mu, \Sigma)$

- Consider a simple generator network with a single affine layer that maps samples $\mathcal{N}(\mathbf{0}, \mathbf{I})$ to $\mathcal{N}(\mu, \Sigma)$:

$$\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}) \longrightarrow \boxed{g_{\theta}(\mathbf{z})} \longrightarrow \mathbf{x} \sim \mathcal{N}(\mu, \Sigma)$$

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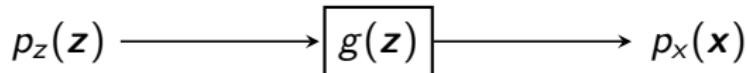
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- Note: Exact solution is $\mathbf{x} = g_{\theta}(\mathbf{z}) = \mu + \mathbf{L}\mathbf{z}$ where \mathbf{L} is the Cholesky decomposition of Σ : $\Sigma = \mathbf{L}\mathbf{L}^{\top}$, lower triangular \mathbf{L} .

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For any *invertible, differentiable, continuous* g :

$$p_z(\mathbf{z}) = p_x(g(\mathbf{z})) \left| \det \left(\frac{\partial g}{\partial \mathbf{z}} \right) \right|$$

Which implicitly imposes a probability distribution over \mathbf{x} :

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Note: usually use an indirect means of learning g rather than minimise $-\log(p(\mathbf{x}))$ directly

Generating distributions

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- The distribution over x is imposed by marginalising z :
$$p(x) = \mathbb{E}_z p(x|z)$$

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- Generating distributions:
 - + works for both continuous and discrete data
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- Generating samples:
 - + works for continuous data
 - + discrete data is recently possible - we need the STargmax
 - + don't need to specify the distribution in explicit form

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- Generative modelling is more complex than classification because
 - learning requires optimizing intractable criteria
 - data does not specify both input z and output x of the generator network
 - learning procedure needs to determine how to arrange z space in a useful way and how to map z to x

Variational Autoencoders (VAEs)

The Variational Autoencoder uses the following generative process to draw samples:

$$z \sim p_{\text{model}}(z) \rightarrow p_{\text{model}}(x|z; \theta) = p_{\text{model}}(x; g_\theta(z)) \quad x \sim p_{\text{model}}(x|z; \theta)$$

- The learning problem is to find θ that maximises the probability of each x in the training set under $p(x) = \int p(x|z; \theta)p(z)dz$
- $p_{\text{model}}(z)$ is most often chosen to be $\mathcal{N}(\mathbf{0}, I)$
- $p_{\text{model}}(x|z)$ is chosen according to the data; typically Gaussian for real-valued data (most often just predicting the means, with a fixed diagonal covariance) or Bernoulli for binary data.
 - Intuition: we don't exactly want to exactly create the training examples; we want to create things *like* the training examples

Variational Autoencoders (VAEs)

- Conceptually we can compute $p(\mathbf{x}) \approx \frac{1}{n} \sum_i^n p(\mathbf{x}|\mathbf{z}_i; \boldsymbol{\theta})$ for n samples of \mathbf{z} , $\{\mathbf{z}_1, \dots, \mathbf{z}_n\}$ and just use gradient ascent to do the optimisation

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 - We can now compute $\mathbb{E}_{\mathbf{z} \sim q_\phi} p(\mathbf{x}|\mathbf{z}; \theta)$ easily
 - if the PDF $q(\mathbf{z})$, is not $\mathcal{N}(\mathbf{0}, \mathbf{I})$, then how does that help us optimize $p(\mathbf{x})$?
 - and how does this expectation relate to $p(\mathbf{x})$?

Variational Inference

Log-probability $\log p(x) = \log \int p(x|z)p(z)dz$

Jensen's inequality: $\log \int p(x)g(x)dx \geq \int p(x) \log g(x)dx$

Log product rule: $\log(a \cdot b) = \log a + \log b$

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ELBO $\log p(x) \geq \mathbb{E}_{z \sim q(z|x)} \log p(x|z) - D_{\text{KL}}(q(z|x)||p(z))$

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The Evidence LOwer Bound (ELBO) / variational lower bound

The ELBO expression we just derived is a cornerstone of variational inference:

$$\begin{aligned}\mathcal{L}(q) &= \mathbb{E}_{z \sim q(z|x)} \log p_{\text{model}}(x|z) - D_{\text{KL}}(q(z|x) || p_{\text{model}}(z)) \\ &\leq \log p_{\text{model}}(x)\end{aligned}$$

- The expectation term looks just like a reconstruction log-likelihood found in normal autoencoders
 - If $p_{\text{model}}(x|z)$ is Gaussian, then this is MSE between the true training x and a generated sample computed from z , averaged across many z 's (each a function of x)
- The KL term is forcing the approximate posterior $q(z|x)$ towards the prior $p_{\text{model}}(z)$.

Why is it called an autoencoder?

- $q(z|x)$ is referred to as an encoder; it's used to take x and turn it into a z

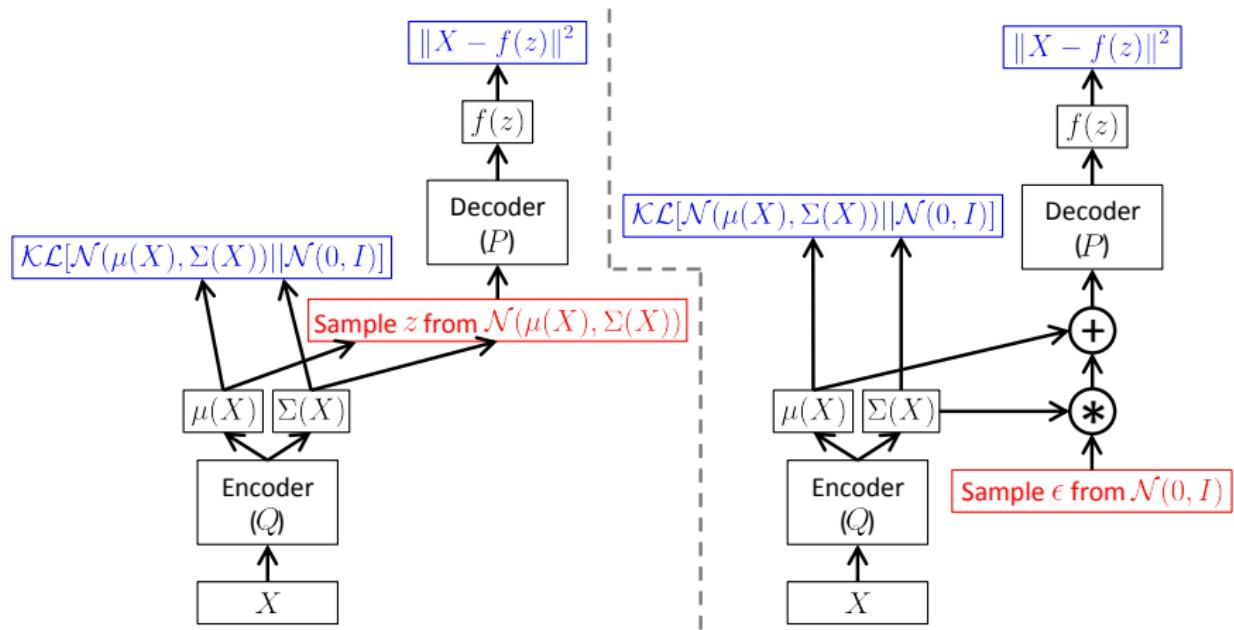
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- $p_{\text{model}}(x; g_{\theta}(z))$ is referred to as a decoder network; it takes a z and decodes it into a target x
- From a practical standpoint, a VAE is a normal autoencoder with two key differences:
 - the encoder generates a distribution that must be sampled
 - the network produces the sufficient statistics of the distribution (e.g. means and diagonal co-variances for a typical VAE with Gaussian $q(z|x)$)
 - the decoder generates a distribution, which, during training the NLL of the true data x is compared against

VAE: Diagram



From Carl Doersch's Tutorial on VAEs - <https://arxiv.org/pdf/1606.05908.pdf>

VAE Models and Performance

- VAEs can be used with any kind of data
 - the distributions and network architecture just needs to be set accordingly
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- VAEs have nice learning dynamics; they tend to be easy to optimise with stable convergence
- VAEs have a reputation for producing blurry reconstructions of images
 - Not fully understood why, but most likely related to a side effect of maximum-likelihood training

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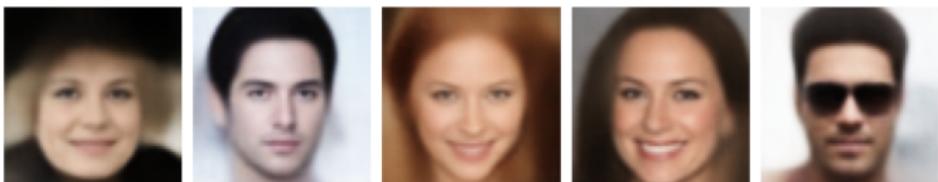
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- VAEs have nice learning dynamics; they tend to be easy to optimise with stable convergence
- VAEs have a reputation for producing blurry reconstructions of images
 - Not fully understood why, but most likely related to a side effect of maximum-likelihood training
- VAEs tend to only utilise a small subset of the dimensions of z
 - Pro: automatic latent variable selection
 - Con: better reconstructions should be possible given the available code-space

Reconstructions Example

Input



VAE



VAE_{Dis_i}



VAE/GAN



Sampling Example

VAE



VAE_{Dis_l}



VAE/GAN



GAN



Generative Adversarial Networks (GANs)

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¹c.f. Schmidhuber

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- LeCun quote 'GANs, the most interesting idea in the last ten years in machine learning'

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Aside: Adversarial Learning vs. Adversarial Examples

The approach of GANs is called adversarial since the two networks have *antagonistic* objectives.

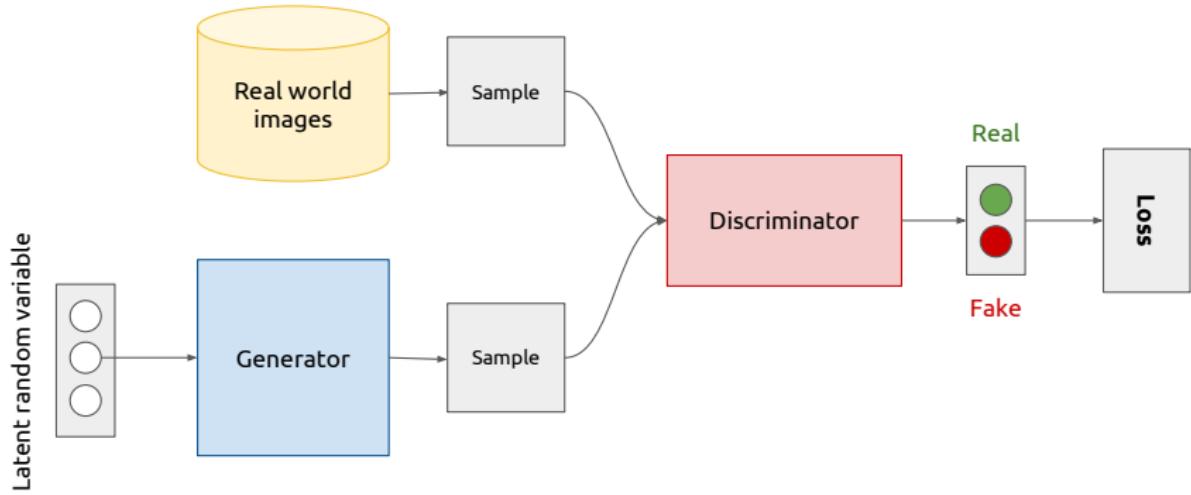
This is not to be confused with *adversarial examples* in machine learning.

See these two papers for more details:

<https://arxiv.org/pdf/1412.6572.pdf>

<https://arxiv.org/pdf/1312.6199.pdf>

Generative adversarial networks (conceptual)



Picture Credit: Xavier Giro-i-Nieto

More Formally

- The **generator**

$$\mathbf{x} = g(\mathbf{z}) \quad (1)$$

is trained so that it gets a random input $\mathbf{z} \in \mathbb{R}^n$ from a distribution (typically $\mathcal{N}(\mathbf{0}, \mathbf{I})$ or $\mathcal{U}(\mathbf{0}, \mathbf{I})$) and produces a sample $\mathbf{x} \in \mathbb{R}^d$ following the data distribution as output (ideally). Usually $n \ll d$.

- The **discriminator**

$$y = d(\mathbf{x}) \quad (2)$$

gets a sample \mathbf{x} as input and predicts a probability $y \in [0, 1]$ (or logit of a Bernoulli distribution in \mathbb{R}) determining if it is real or fake.

More Practically

- Training a standard GAN is difficult and often results in two undesirable behaviours
 - ① Oscillations without convergence. Contrary to standard loss minimisation, we have no guarantee that the loss will actually decrease.
 - It has been shown that a GAN has saddle-point solution, rather than a local minima.
 - ② The **mode collapse** problem, when the generator models very well a small sub-population, concentrating on a few modes.
- Additionally, performance is hard to assess and often boils down to heuristic observations.

Deep Convolutional Generative Adversarial Networks (DCGANs)

- Motivates the use of GANS to learn reusable feature representations from large unlabelled datasets.
- GANs known to be unstable to train, often resulting in generators that produce “nonsensical outputs”.
- Model exploration to identify architectures that result in **stable** training across datasets with higher resolution and deeper models.



Architecture Guidelines for Stable DCGAN

- Replace pooling layers with strided convolutions in the discriminator and fractional-strided (transpose) convolutions in the generator.
 - This will allow the network to learn its own spatial downsampling.

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- Use LeakyReLU activation in the discriminator for all layers.

Summary

- Generative modelling is a massive field with a long history
- Differentiable generators have had a profound impact in making models that work with real data at scale
- VAEs and GANs are currently the most popular approaches to training generators for spatial data
- We've only scratched the surface of generative modelling
 - Auto-regressive approaches are popular for sequences (e.g. language modelling).
 - But also for images (e.g. PixelRNN, PixelCNN)
 - typically RNN-based
 - but not necessarily - e.g. WaveNet is a convolutional auto-regressive generative model