Algorithms and Analysis

Lesson 5: *Use Arrays*

Variable length arrays, implementing stacks

Outline

1. **Why Arrays?**
2. Variable Length Arrays
3. Programming Language
4. Implementing Stacks

Use Arrays

- An array is a contiguous chunk of memory
- In C we can create arrays using
  ```
  int *array = new int[20]
  ```
- The array has an access time of $\Theta(1)$
- The constant factor is small (i.e. access time ≈ 1 time step)
- Arrays provide a very efficient use of memory
- 95% of the time using arrays is going to give you the best performance, although never use raw arrays!

Disadvantages of Arrays

- Arrays have a fixed length
- Very often we don't know how big an array we want
  - ⋆ E.g. reading words from a file
- Adding or deleting elements from the middle of an array is costly
- Sorted arrays are expensive to maintain
- Arrays don't know how big they are—annoying

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Variable Length Arrays

- We want a variable length array
- Initially a variable length array would have length zero
- We should be able to
  - Add an element to an array
  - Access any element in the array
  - Change an element
  - Delete elements
  - Know how many elements we have

ADT for a List

- What do we want of a list of `int`s?
  - `void push_back(int value)`
  - random access `array[i]`
  - `int size()`
- It would be useful if it resized
- It would be great to have some algorithms (e.g. sort) that can be run on a list

Implementation

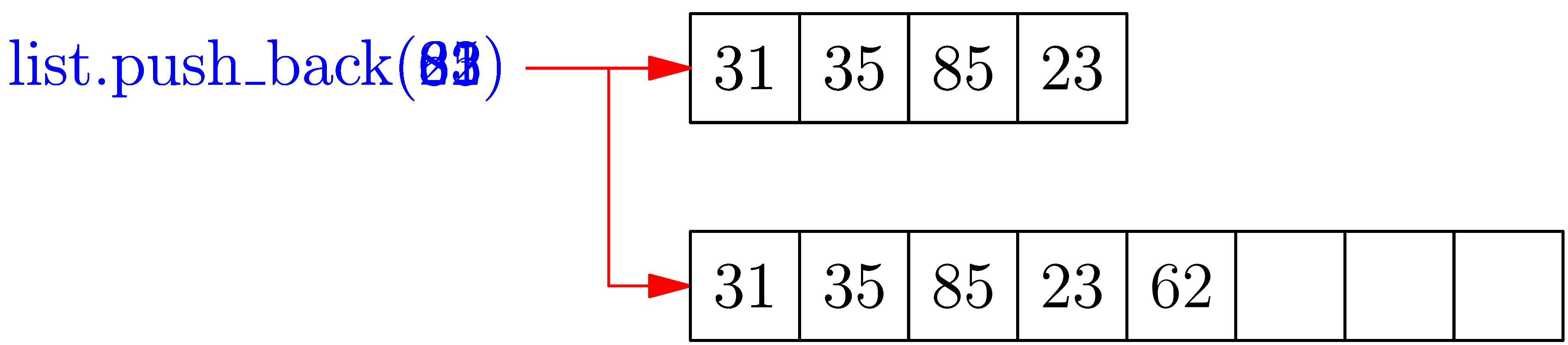
- How should we implement a list?
- Use an array, of course!
- We need to distinguish between
  - ⋆ the number of elements in the list `size()`
  - ⋆ the number of elements in the array `capacity()`
- If the number of elements grows larger than the capacity then we need to increase the capacity

Initial Capacity

- We could prevent resizing arrays by using a huge initial capacity
- However, how big is big enough?
- What happens when we have an array of arrays?
- Memory like time is resource we should care about
- In an analogy with **time complexity** we also care about **space complexity** (i.e. how much memory we need)
- If we want to store n elements it is reasonable to expect that we use $c\,n$ bits of memory where we want to keep c small

Resizing Memory

- We start with some reasonable capacity
- We can add elements until we reach the capacity
- A simple method for resizing memory is
  - ⋆ create a new array with double the capacity of the old array
  - ⋆ copy the existing elements from the old array to the new array



Amortised Time Analysis

- How efficient is resizing?
- Most `push_back(elem)` operations are $\Theta(1)$
- When we are at full capacity we have to copy all elements
- Adding to a full array is slow but it is **amortised** by other quick adds

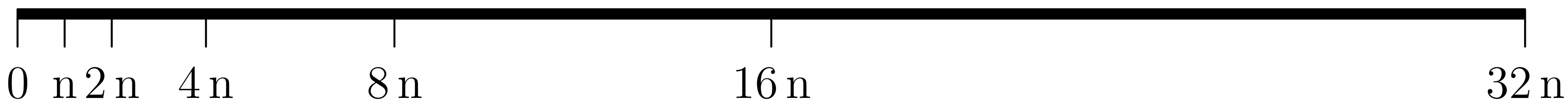
  amortised: effect of a single operations 'deadened' by other operations

Example

- If we have an initial capacity of 10 and add 100 elements then the number of operations needed is
  - adds: 100
  - copies: 10+20+40+80=150
  - `new int[]`: 4
- 250 adds and copies operations + 4 `new` operations

General Time Analysis

- If we perform N adds with an initial capacity of n
- We must perform m copies where



$$n \times 2^{m-1} < N \leq n \times 2^m \qquad \text{i.e.} \quad m = \left\lceil \log_2 \left(\frac{N}{n} \right) \right\rceil$$

- The number of elements copied is

$$n + 2n + 4n + \cdots + 2^{m-1}\, n = n(1 + 2 + \cdots + 2^{m-1}) = n\left(2^m - 1\right)$$

- Total number of operations is (using $\lceil \log(a) \rceil < \log(a) + 1$)

$$N + n\left(2^m - 1\right) = N + n 2^{\left\lceil \log_2 \left(\frac{N}{n}\right) \right\rceil} - n < N + 2N - n < 3N$$

Insertion and Deletion

- `vector<T>` is very useful and very fast for lots of things
- But if you try to insert or delete an element anywhere other than the end then you have to shove all the subsequent elements one space forward
- This is not the right data structure if you want to keep elements in order (binary trees will do that for you much more efficiently)
- Linked lists allow you to splice in a sublist into a list in constant time although linked lists have a lot of drawbacks

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Computer Languages

- Different computer languages are designed for different roles and have different advantages and disadvantages
- **C++** was designed to be fast (as fast as C), it pays the price of allowing bugs that hard to detect
- **Java** was designed to be vary safe (avoiding lots of bugs), but is not fast and a bit long winded
- **Python** was designed so you can rapidly write powerful programmes with a small amount of code, but it is not fast or safe

Problems with C++

- Amongst a number of issues that make C++ dangerous are
  - ⋆ Memory management
  - ⋆ Writing to parts of memory that you should not
  - ⋆ Multiple inheritance, although you seldom need to do this
- However, by using existing data structures (STL) and following established programming patterns these don't have to be an issue

Memory Management

- Most programming languages have two types of memory

  The Stack: is the area of memory controlled by compiler for local variables, function calls, etc.

  The Heap: is area that the programmer (you) can request, which is nice

- In C++ you are given the **right** to ask for memory

```
int *storage = new int[n];
```

- You have **responsibility** to free the memory

```
delete[] storage;
```

Trouble with Memory Management

- If you don't release memory acquired with `new` using `delete` you cause a **memory leak**
- Often memory leaks are no concern, but in large programs memory leaks will rapid exhaust the computer's memory, slowing down the code and eventually leading to the programme crashing
- To release a block of memory we can use:
  ```
  delete[] storage;
  ```
- Now `storage` is a **dangling pointer** and must not be used as it is no longer valid
- If we accidentally delete the storage twice we get an *undefined behaviour*, but often the programme will crash

Resource Acquistion is Initialisation (RAII)

- Java and Python use garbage collectors which automatically checks whether memory can be accessed and if not it is removed
- In C++ this is your responsibility
- But there is a standard **programming pattern** to elevate the problem known as **Resource Acquistion is Initialisation (RAII)**

  Wrap all resources in classes. Request the resources in the constructor and release the resource in the destructor

- When the object goes out of scope (you leave a `for` loop, function call, etc.) the destructor is called and the resource is safely released

RAII

```
template <typename T>
class container {
  private:
    T* data;

  public:
    container(unsigned n) {data = new T[n];}
    ~container(unsigned n) {delete[] data;}
};


  main() {

    for (int i=0; i<1000; ++i) {
      container<int> my_container(10000);
      // do something
    }

  }
```

Writing over Memory

- In C++ the following will compile and run

```
int *array = new int[4];
int *a = new int[2];
double *darray = new double[4];
array[4] = 4;
```

- However `array[4]` has not been assigned (unlike `array[0]`, `array[1]`, `array[2]` and `array[3]`)

- The memory on the heap corresponding to the address of `array[4]` might have been assigned to `a[0]` in which case you may inadvertently have set `a[0]` to 4 leading to the program not doing what you want

- It might be that you have put an `int` into `darray[0]` which will then crash the system when you read `darray[0]`

Guarding Against Mistakes

- These are really hard problems to debug because where the program goes wrong or crashes can be very far from the assignment that caused the error
- Java takes the approach that it always tests whether you are writing in valid memory
- By default C++ doesn't even for data structures—making this check slows down random access
- Checks can also make pipeline optimisations harder to make
- The onus is on the user to use the memory correctly

Follow Programming Idioms

- Using common data structures and following common idioms will prevent most errors

```
int n = 5;
vector<int> array(n);

for(int i=0; i<array.size(); ++i) {
  array[i] = i;
}

for(auto pt=array.begin(); pt != array.end(); ++pt){
  *pt *= 2
}

for(int& element: array) {
  element += 2;
}
```

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Stacks

- Lets look at implementing a stack
- Remember a stack has methods
  - `push(Object )`
  - `pop()`
  - `top()`
  - `empty()`

Implementation of Stack

```
template <typename T>
class MyStack
{
private:
  std::vector<T> stack;

public:
  void push(const T& obj) {stack.push_back(obj);}

  T top() const {return stack.back();}

  T pop() {
    T tmp = stack.back();
    stack.pop_back();
    return tmp;
  }

  T empty() {return stack.size()==0;}
};
```

Notes on Implementation

- I don't need to write a constructor as C++ generates a default constructor that will initialise the stack correctly
- I don't need to write a desctuctor because by default the destructor for `vector<T>` will be called which releases memory
- I've written the `pop` command, that I like, but if I run

```
stack<Widget> widget_stack;
Widget w;
widget_stack.push(w);
Widget w1(widget.pop());
```

  if the last command throws an exception then the last term on the stack is lost for ever

Why not use a `vector`

- Surely it is mad to use `MyStack<T>` as I could just use the more powerful `vector<T>`
- I can make `MyStack<T>` as efficient as `vector<T>` by inlining function calls
- But why would I want to lose functional?
- By using `MyStack<T>` I am **declaring my intention** of using this data structure as a stack
- I'm not going to do something weird like modify an element inside the stack
- My code becomes self-explanatory—I don't need to write comments as it is clear what I am doing

Using `MyStack`

- Implementing a stack using a dynamically re-sizable array is trivial
- Stacks have many applications
- E.g. suppose we want to write a program to reverse the order of strings in a file

you can cage a swallow can't **you**

you can't swallow a cage can you

you
can't
swallow
a
cage
can
you

Reversing Strings in File

```
#include <stack>
#include <iostream>
#include <fstream>
using namespace std;

int main(int argc, char *argv[]) {
  ifstream in(argv[1]);

  stack<string> stack;

  string word;
  while (in >> word)
    stack.push(word);

  while(!stack.empty()) {
      cout << stack.top() << ' ';
      stack.pop();
  }
}
```

Lessons

- Arrays are very efficient both in space (memory) and access time
- Resizing an array is not that costly
- insertion and deletion from the middle of an array are expensive, $O(n)$
- Arrays are often the simplest way to implement many other data structures, e.g. stacks
- Use (dynamically re-sizable) arrays (`vector<T>`) frequently!
- Stop using raw arrays