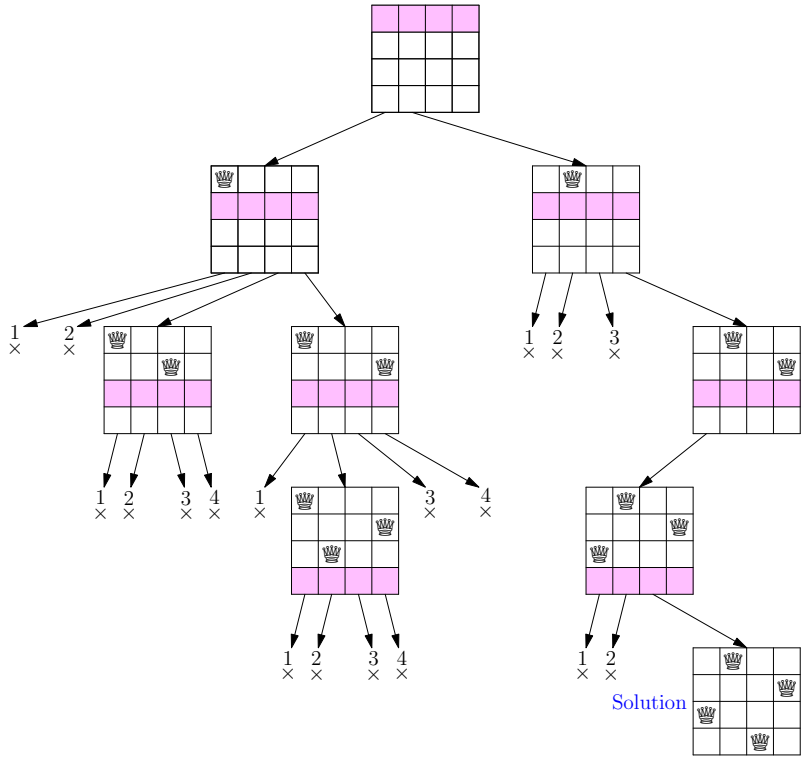
Algorithms and Analysis

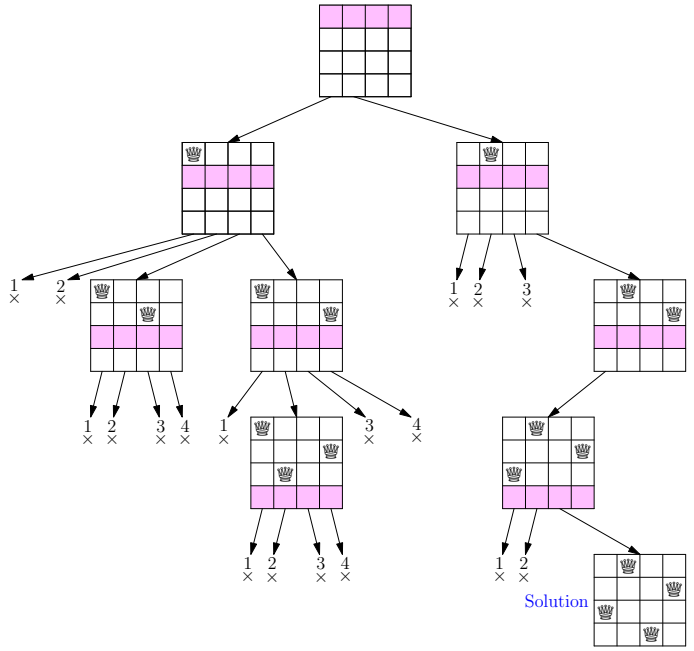
Lesson 22: *Know how to Search*



Backtracking, Branch and Bound

Outline

1. **Search Trees**
2. Backtracking
3. Branch and Bound
4. Search in AI



State Space Representation

- Many real world problems involve taking a series of actions to manipulate the state of the system
- This is the area of planning and search which sits within the domain of artificial intelligence
- One of the key props to help us develop algorithms is to think of the states as nodes of a graph which are linked if there exists an action taking us from one state to another
- This provides a **state space representation** of the problem (we saw this before when we derived a low bound on sorting)

State Space Representation

- Many real world problems involve taking a series of actions to manipulate the state of the system
- This is the area of planning and search which sits within the domain of artificial intelligence
- One of the key props to help us develop algorithms is to think of the states as nodes of a graph which are linked if there exists an action taking us from one state to another
- This provides a **state space representation** of the problem (we saw this before when we derived a low bound on sorting)

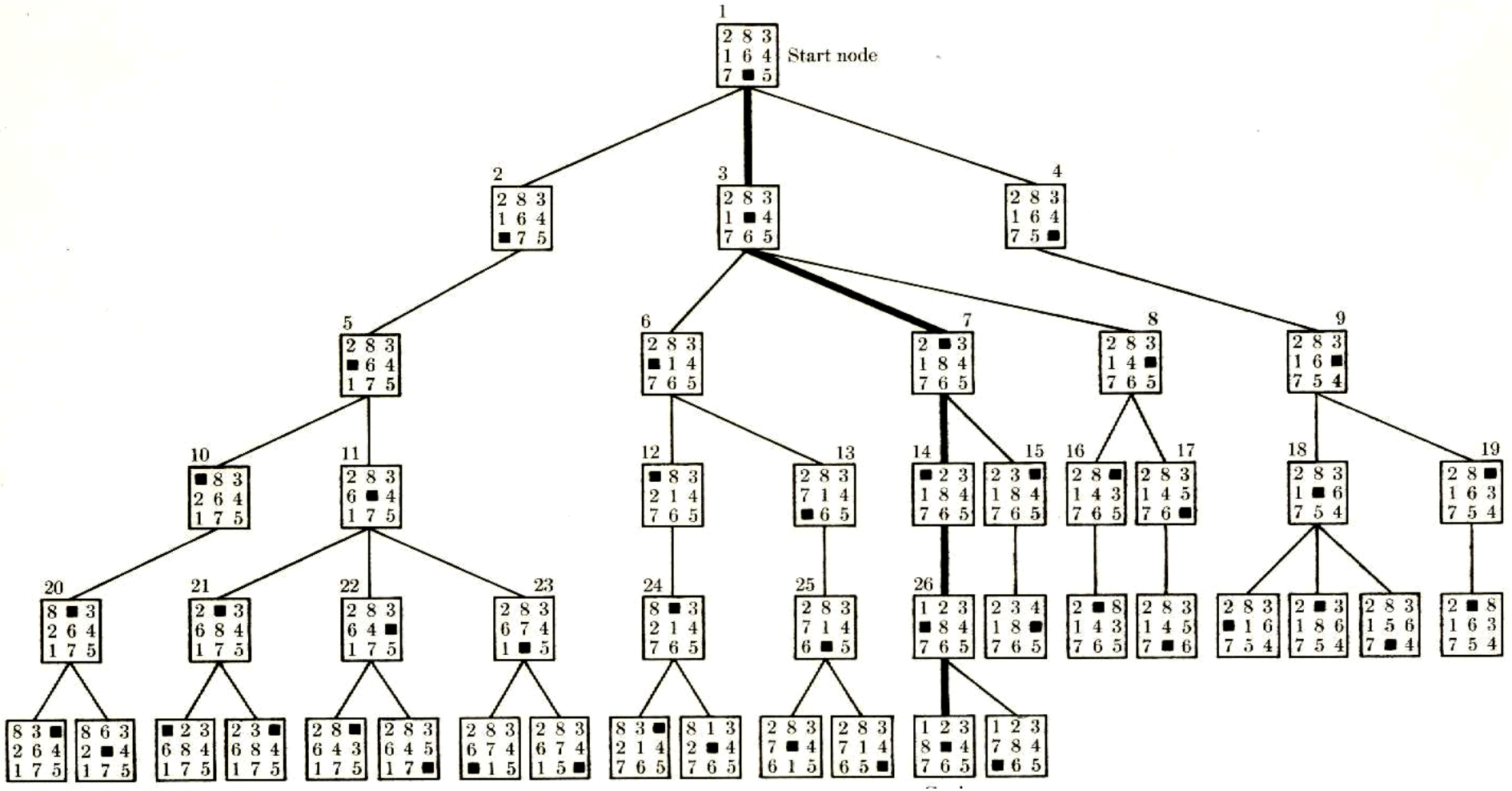
State Space Representation

- Many real world problems involve taking a series of actions to manipulate the state of the system
- This is the area of planning and search which sits within the domain of artificial intelligence
- One of the key props to help us develop algorithms is to think of the states as nodes of a graph which are linked if there exists an action taking us from one state to another
- This provides a **state space representation** of the problem (we saw this before when we derived a low bound on sorting)

State Space Representation

- Many real world problems involve taking a series of actions to manipulate the state of the system
- This is the area of planning and search which sits within the domain of artificial intelligence
- One of the key props to help us develop algorithms is to think of the states as nodes of a graph which are linked if there exists an action taking us from one state to another
- This provides a **state space representation** of the problem (we saw this before when we derived a low bound on sorting)

8-Puzzle Example



Large State Spaces

- The search space typically increase exponentially with the problem size
- We can find the quickest solution to the 8-puzzle (and the 15 puzzle) using breadth first search, but larger puzzles soon become intractable
- Nevertheless, a lot of important problems involve very large state spaces and we have to find algorithms to explore them

Large State Spaces

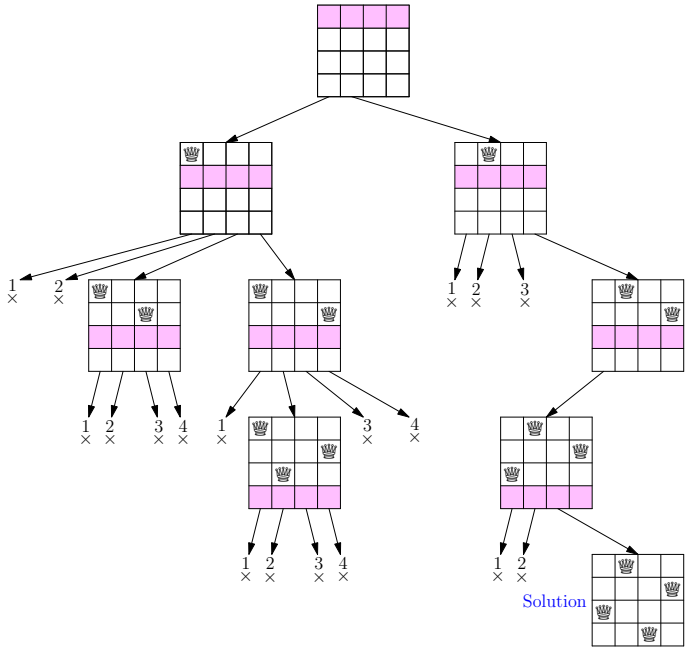
- The search space typically increase exponentially with the problem size
- We can find the quickest solution to the 8-puzzle (and the 15 puzzle) using breadth first search, but larger puzzles soon become intractable
- Nevertheless, a lot of important problems involve very large state spaces and we have to find algorithms to explore them

Large State Spaces

- The search space typically increase exponentially with the problem size
- We can find the quickest solution to the 8-puzzle (and the 15 puzzle) using breadth first search, but larger puzzles soon become intractable
- Nevertheless, a lot of important problems involve very large state spaces and we have to find algorithms to explore them

Outline

1. Search Trees
2. **Backtracking**
3. Branch and Bound
4. Search in AI



Backtracking

- Backtracking is used to find feasible solutions in large state spaces
- E.g. solving sudoku
- It works by growing partial solutions until either
  - ⋆ a feasible solution is found when we can finish
  - ⋆ no feasible solution is found when we backtrack
- We often search the state space using depth first search

Backtracking

- Backtracking is used to find feasible solutions in large state spaces
- E.g. solving sudoku
- It works by growing partial solutions until either
  - ⋆ a feasible solution is found when we can finish
  - ⋆ no feasible solution is found when we backtrack
- We often search the state space using depth first search

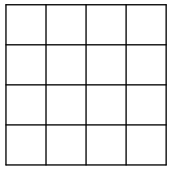
Backtracking

- Backtracking is used to find feasible solutions in large state spaces
- E.g. solving sudoku
- It works by growing partial solutions until either
  - ⋆ a feasible solution is found when we can finish
  - ⋆ no feasible solution is found when we backtrack
- We often search the state space using depth first search

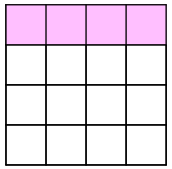
Backtracking

- Backtracking is used to find feasible solutions in large state spaces
- E.g. solving sudoku
- It works by growing partial solutions until either
  - ⋆ a feasible solution is found when we can finish
  - ⋆ no feasible solution is found when we backtrack
- We often search the state space using depth first search

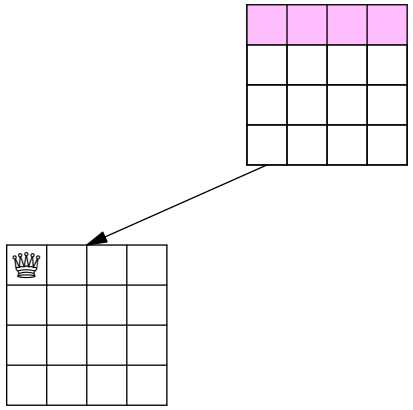
4-Queens Problem



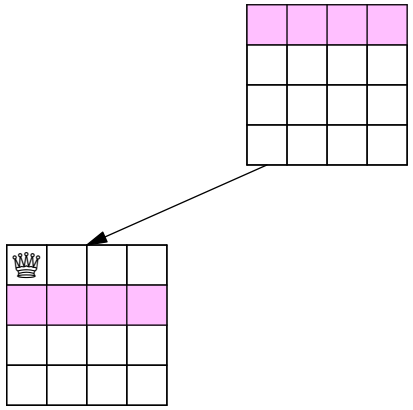
4-Queens Problem



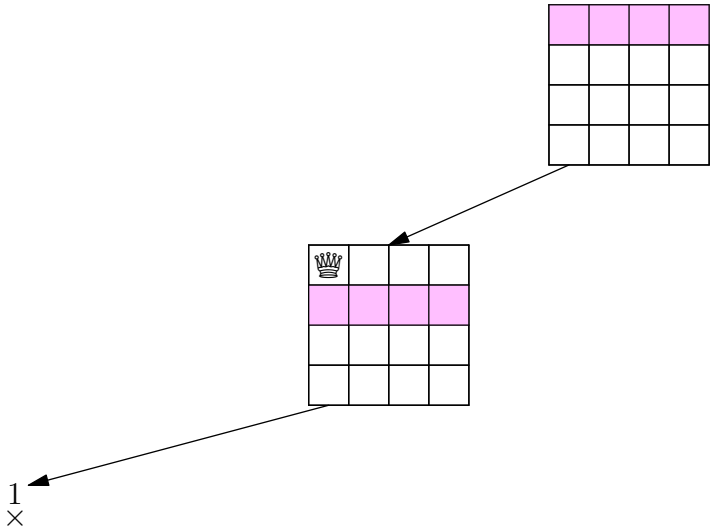
4-Queens Problem



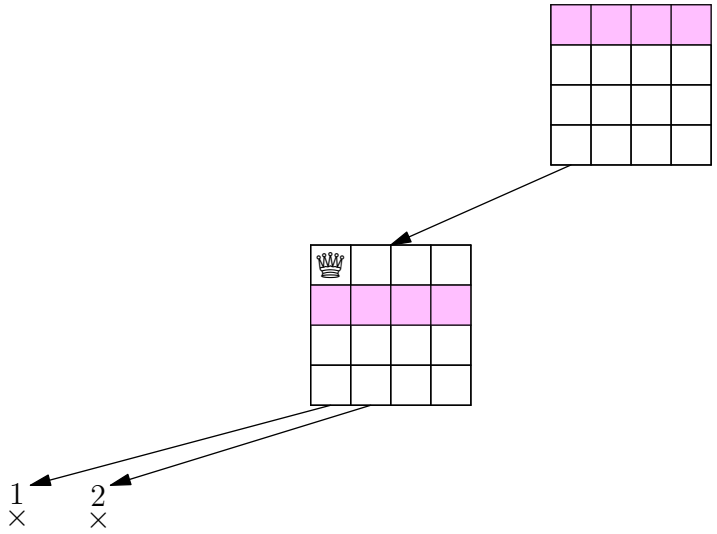
4-Queens Problem



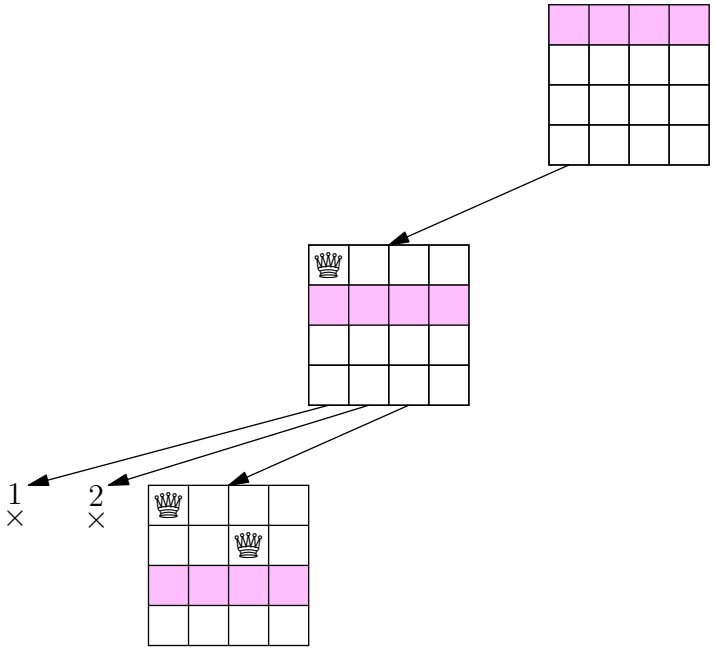
4-Queens Problem



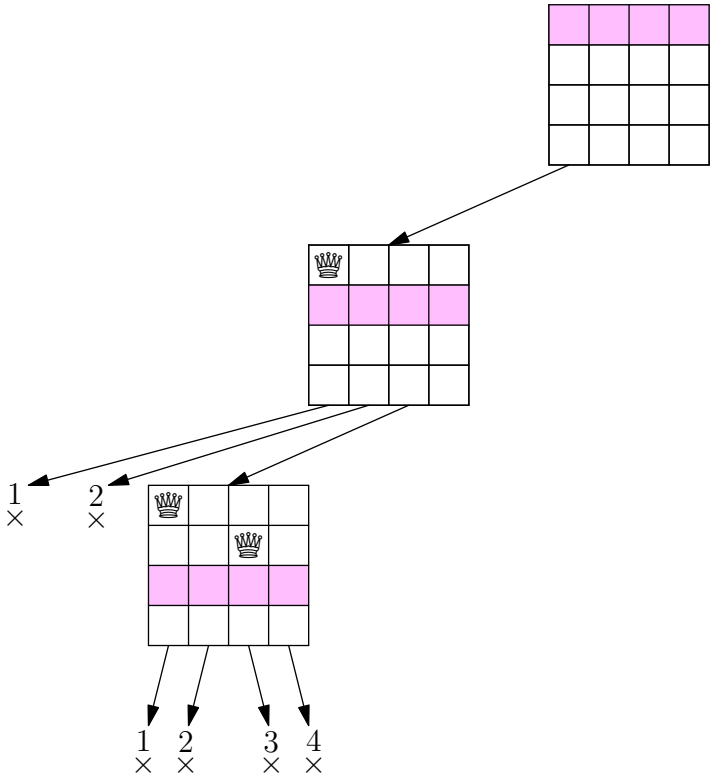
4-Queens Problem



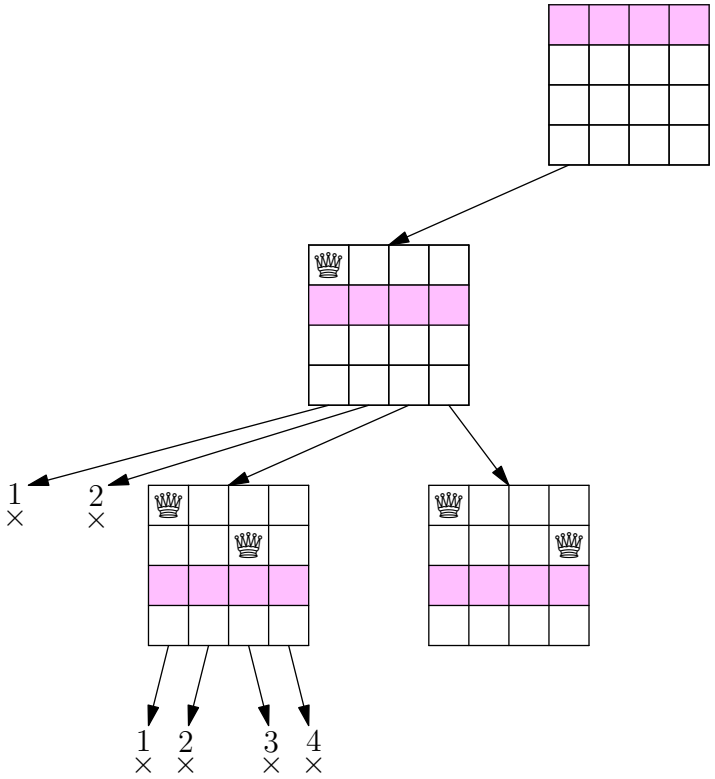
4-Queens Problem



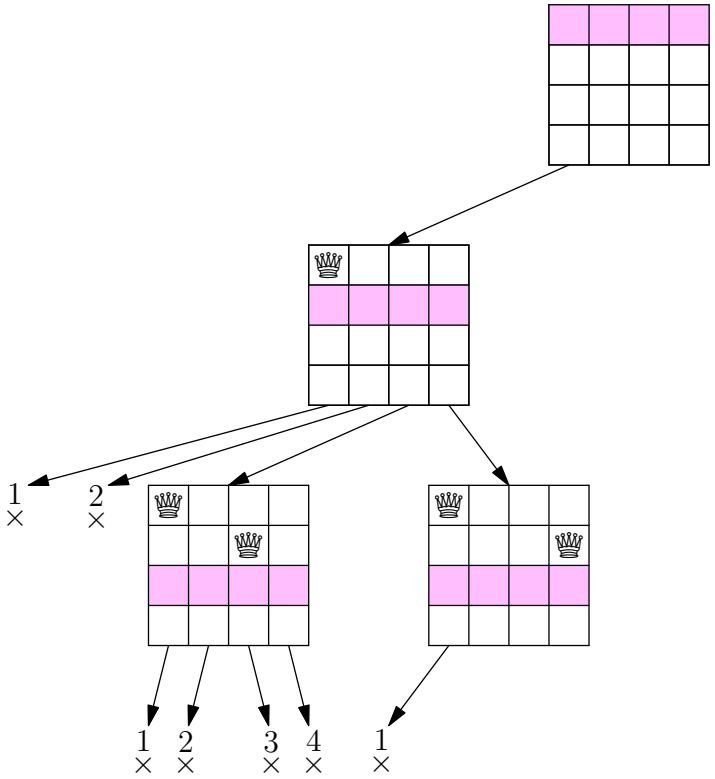
4-Queens Problem



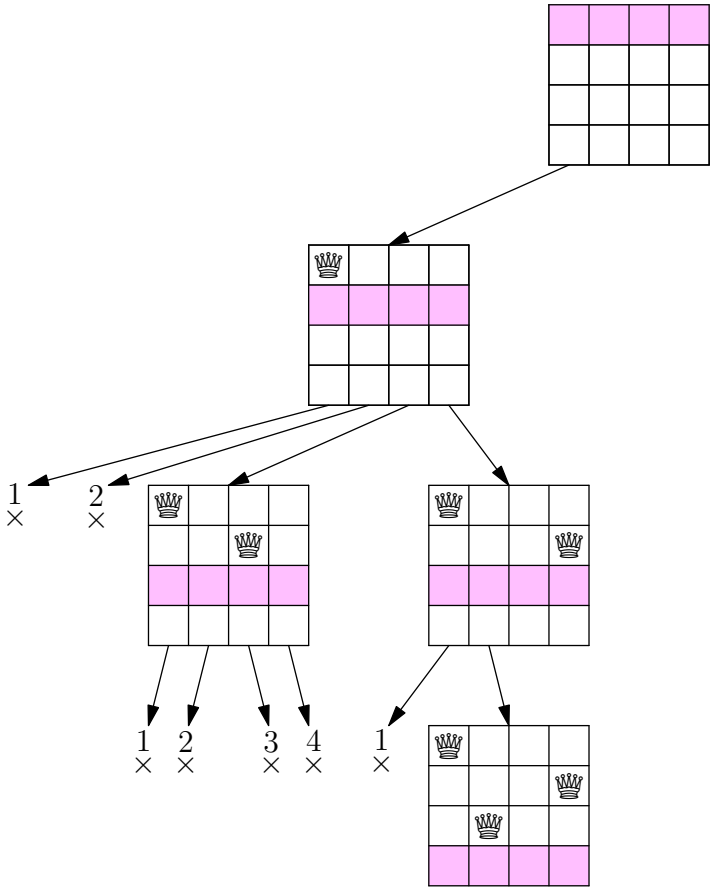
4-Queens Problem



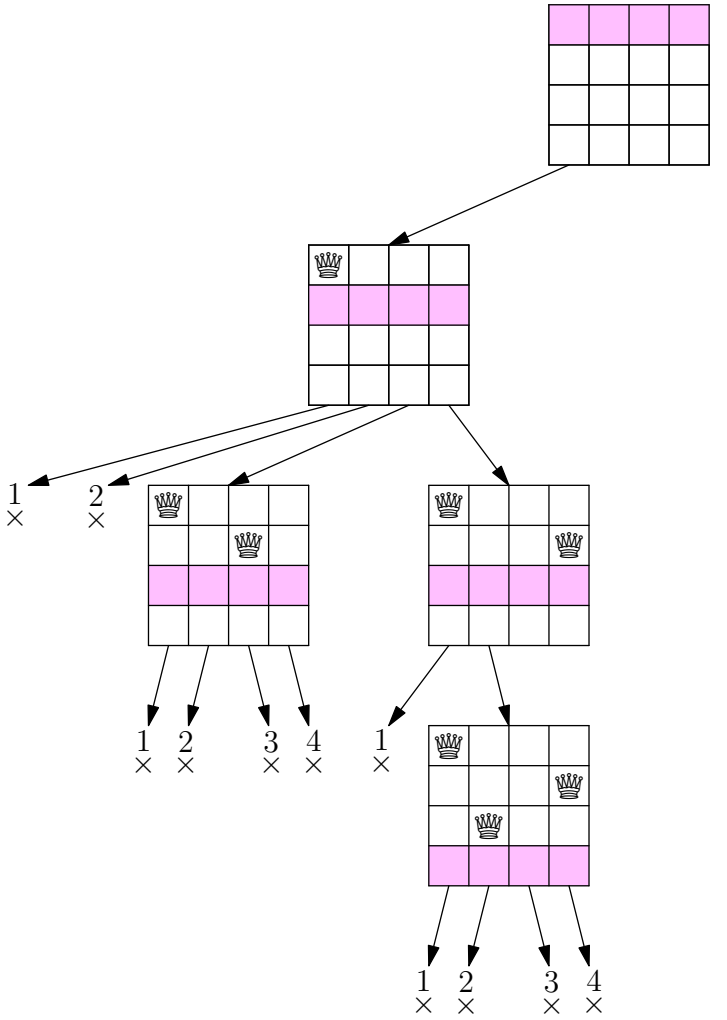
4-Queens Problem



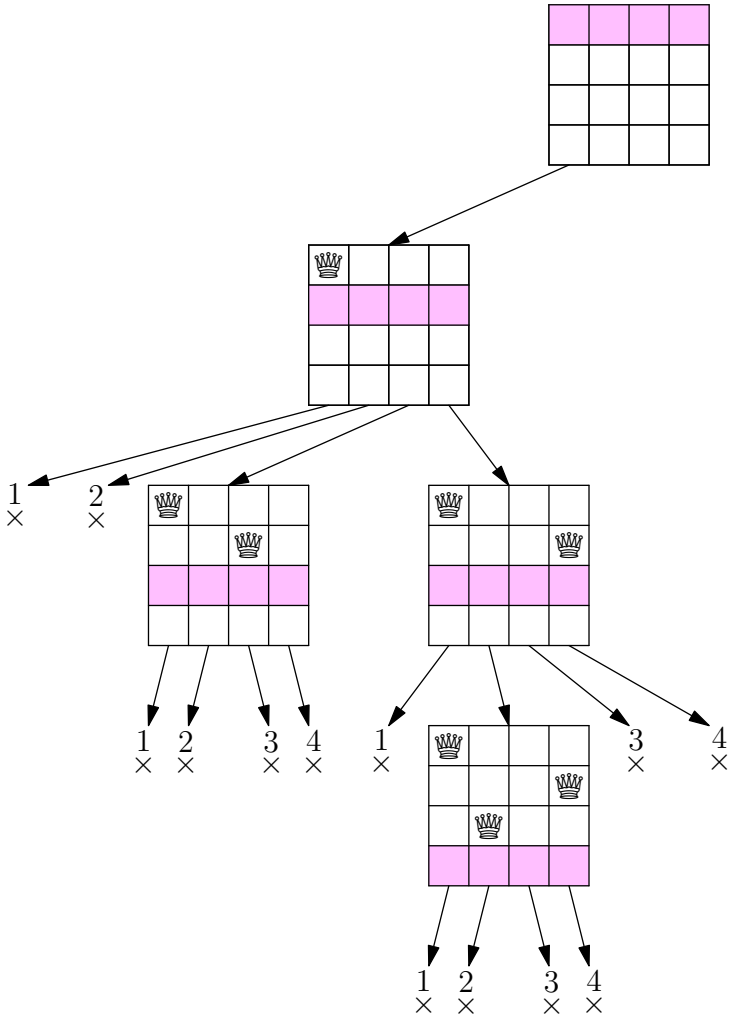
4-Queens Problem



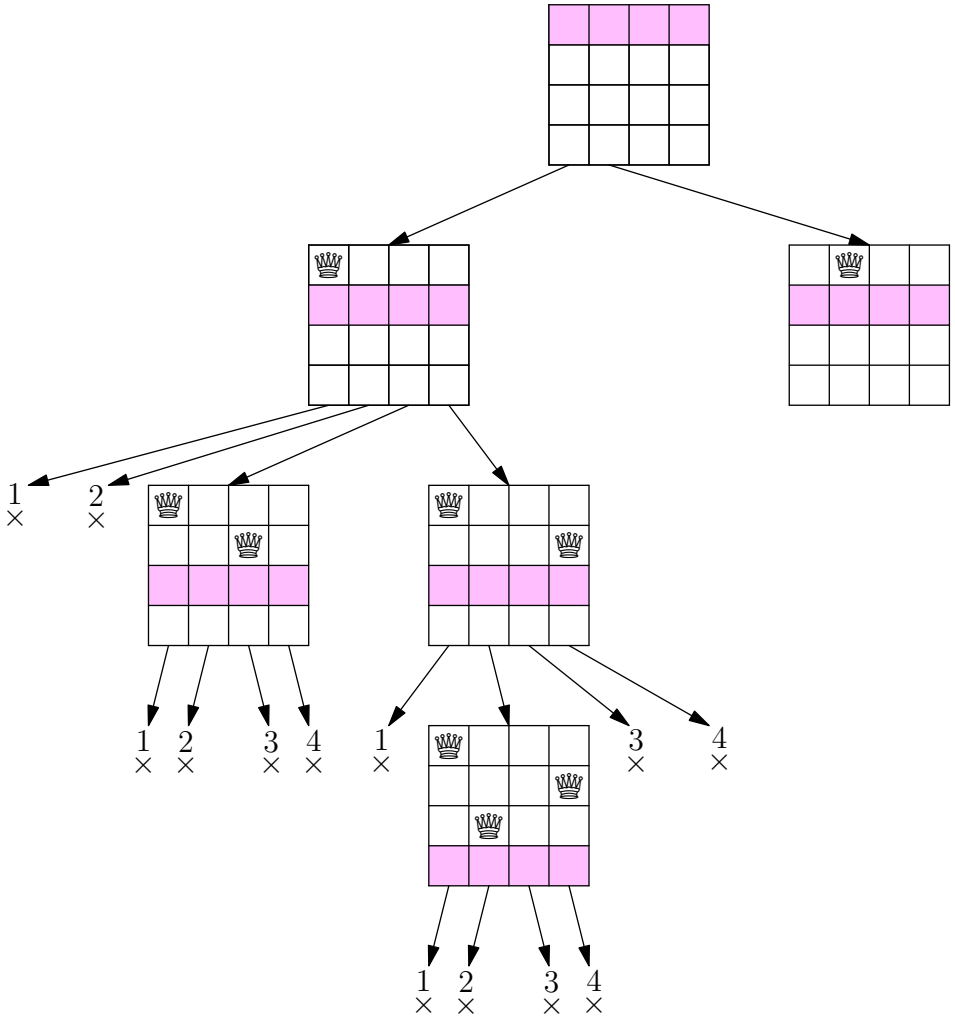
4-Queens Problem



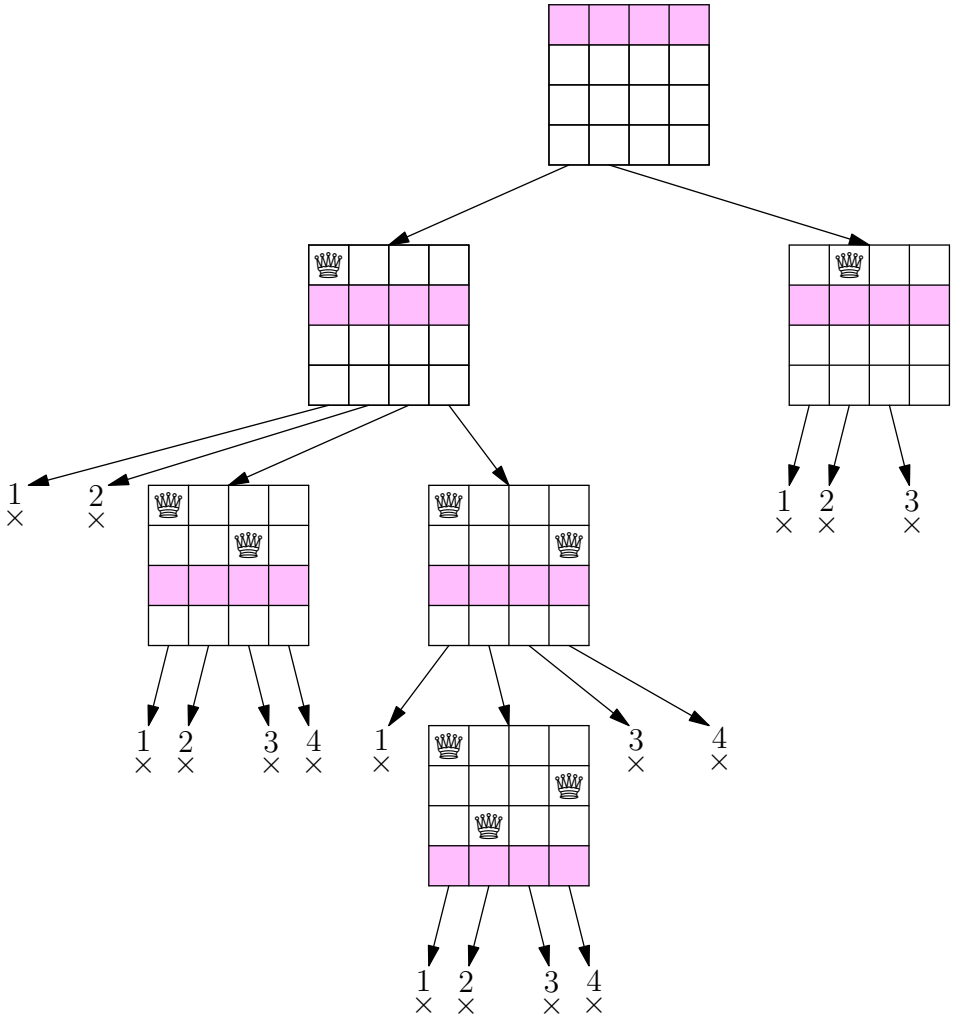
4-Queens Problem



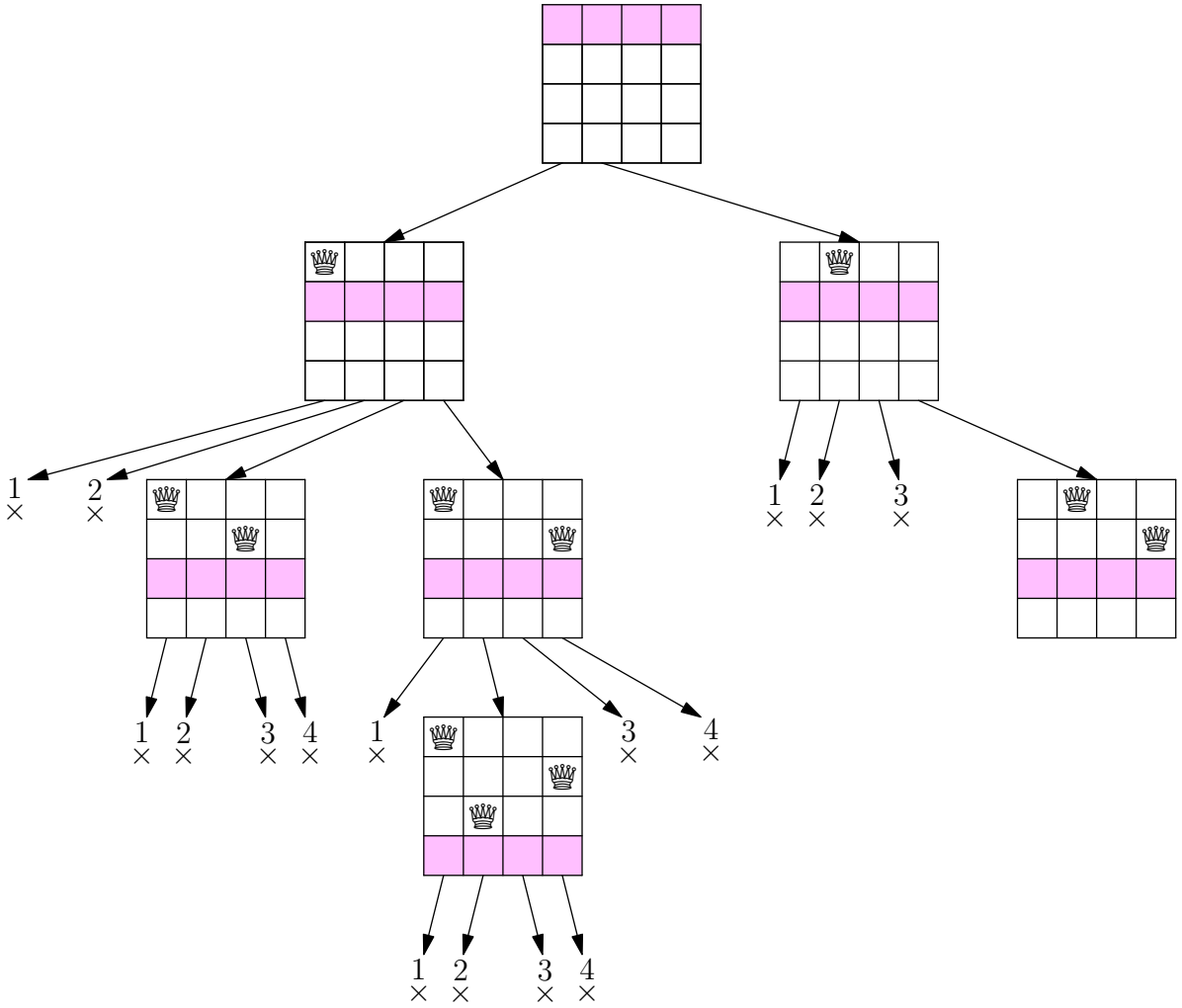
4-Queens Problem



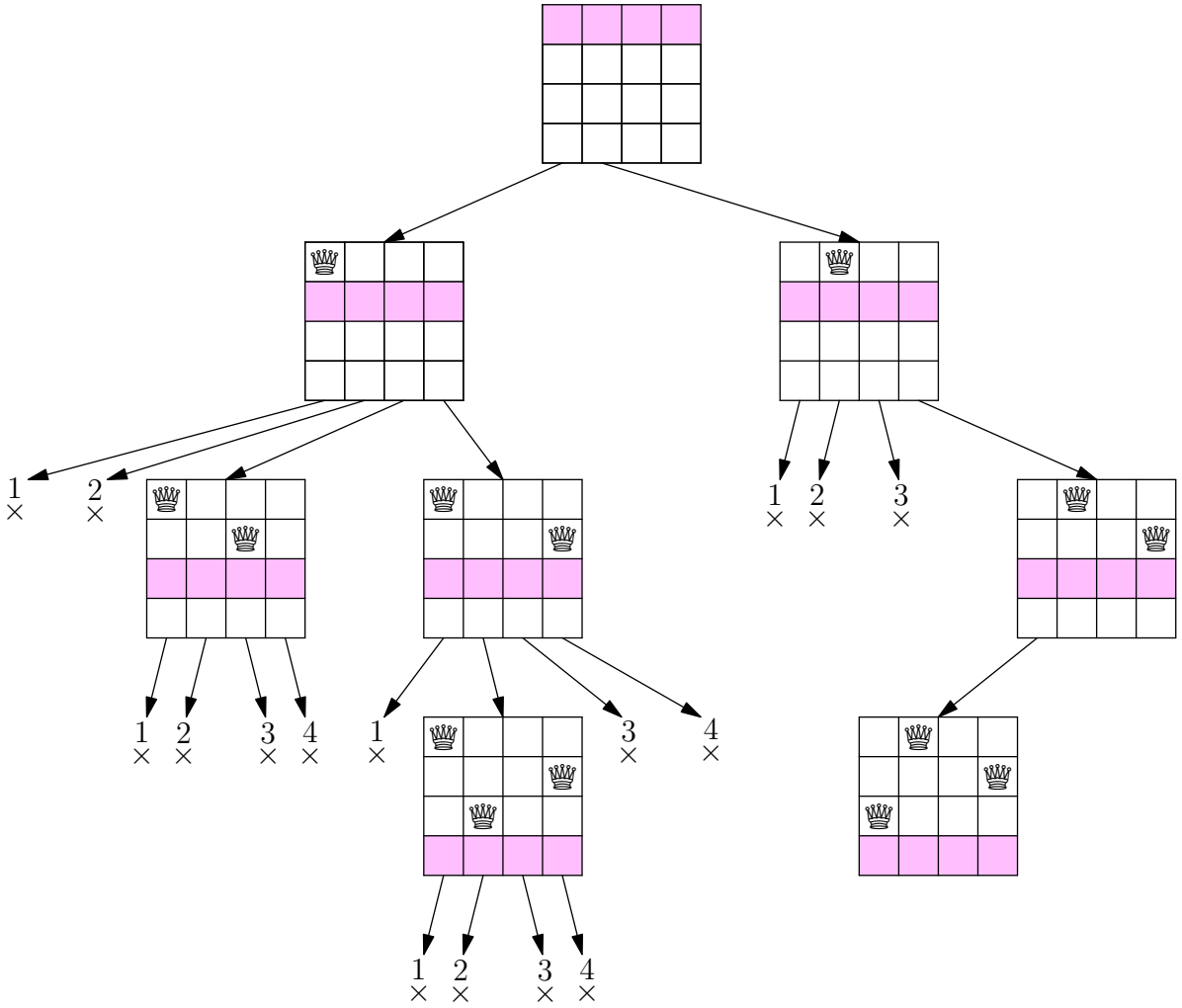
4-Queens Problem



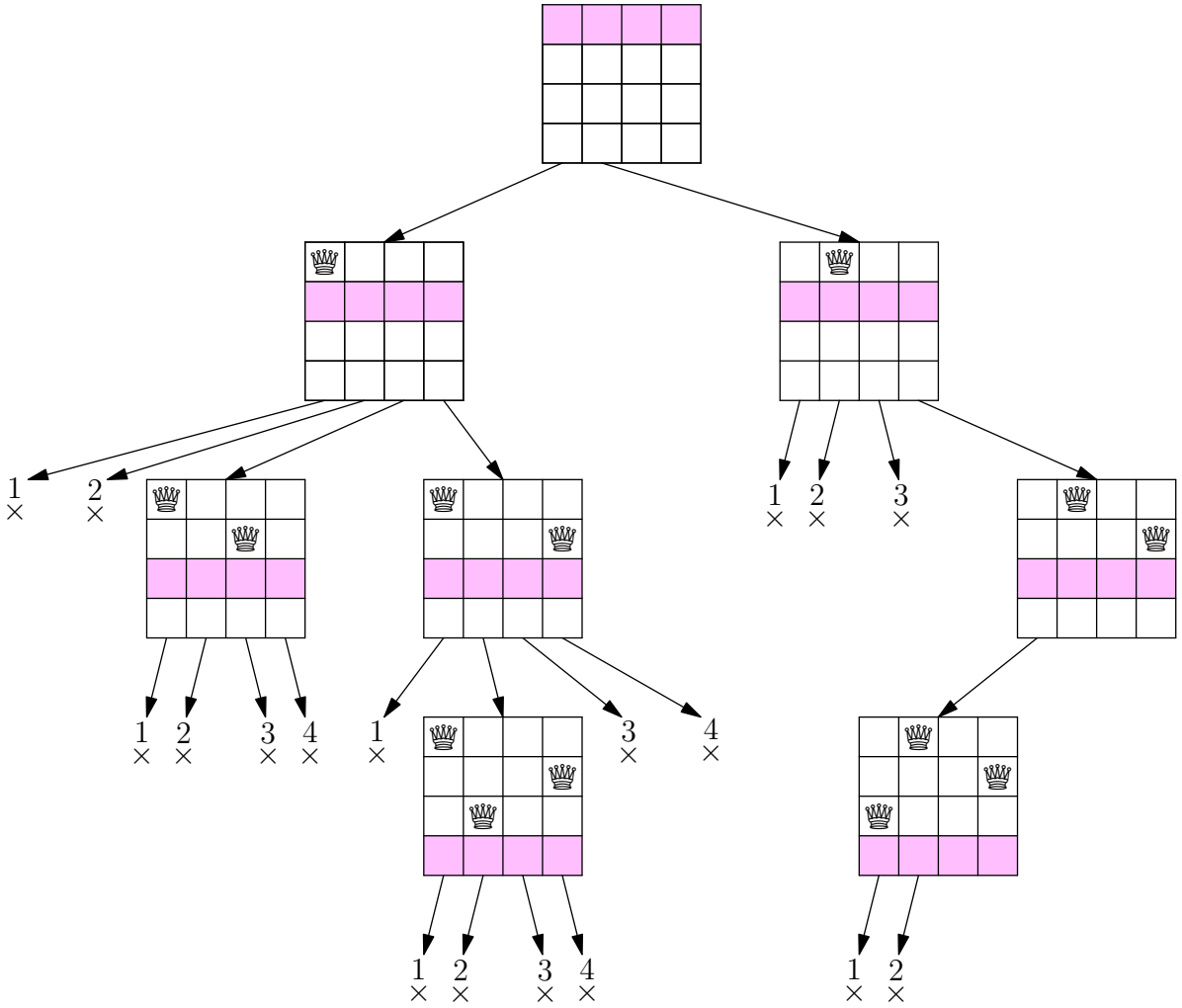
4-Queens Problem



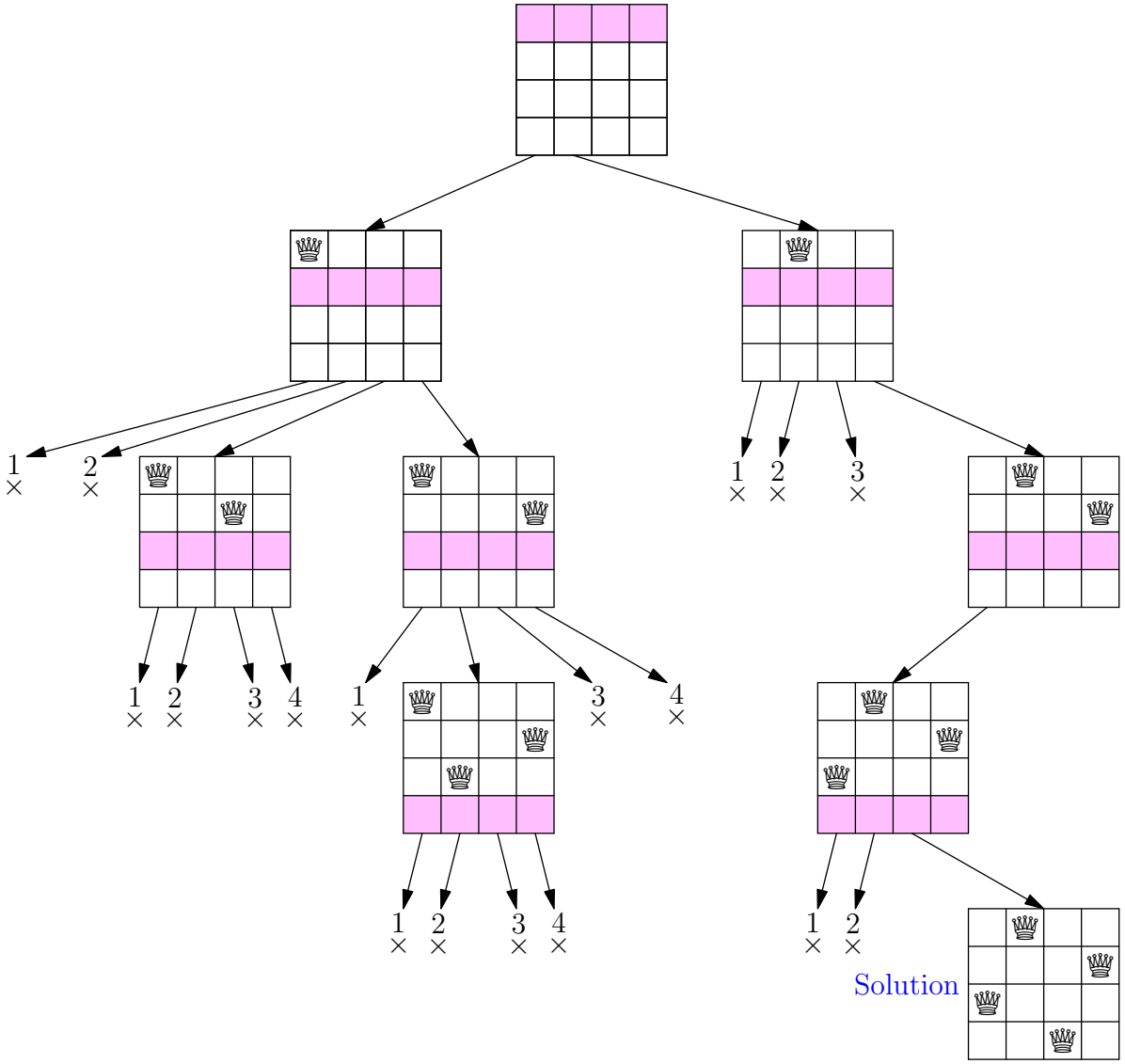
4-Queens Problem



4-Queens Problem

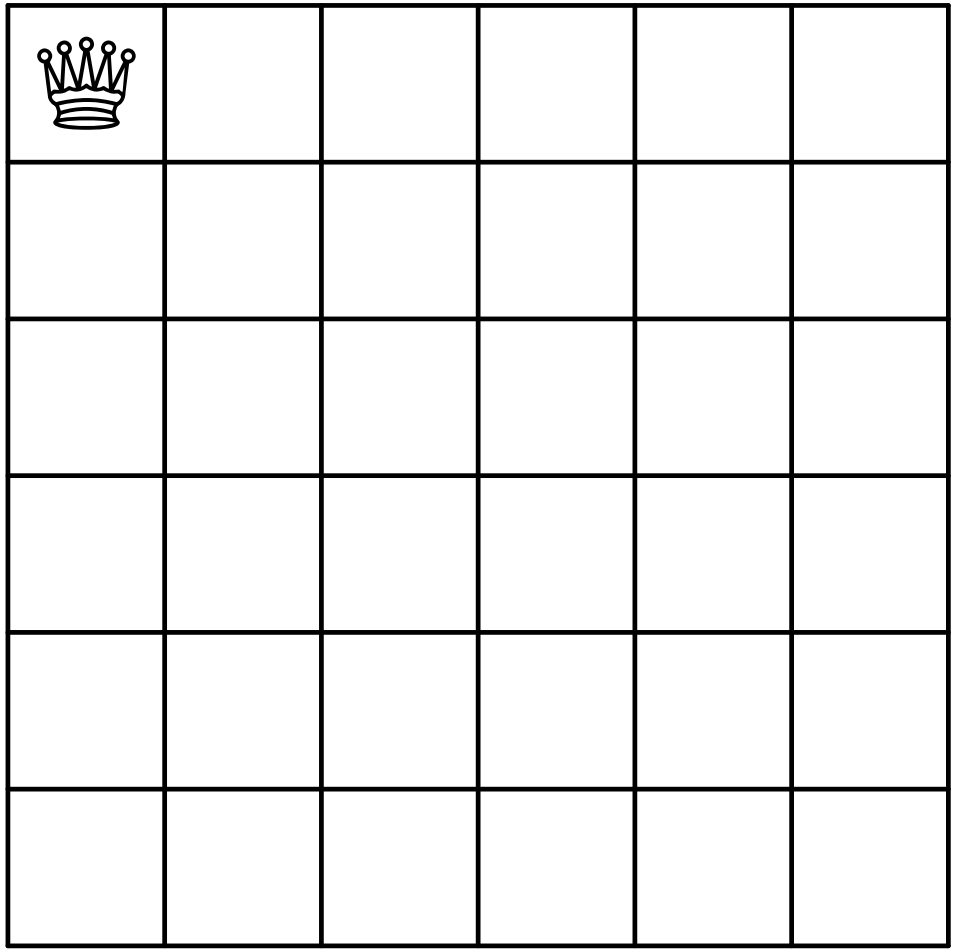


4-Queens Problem

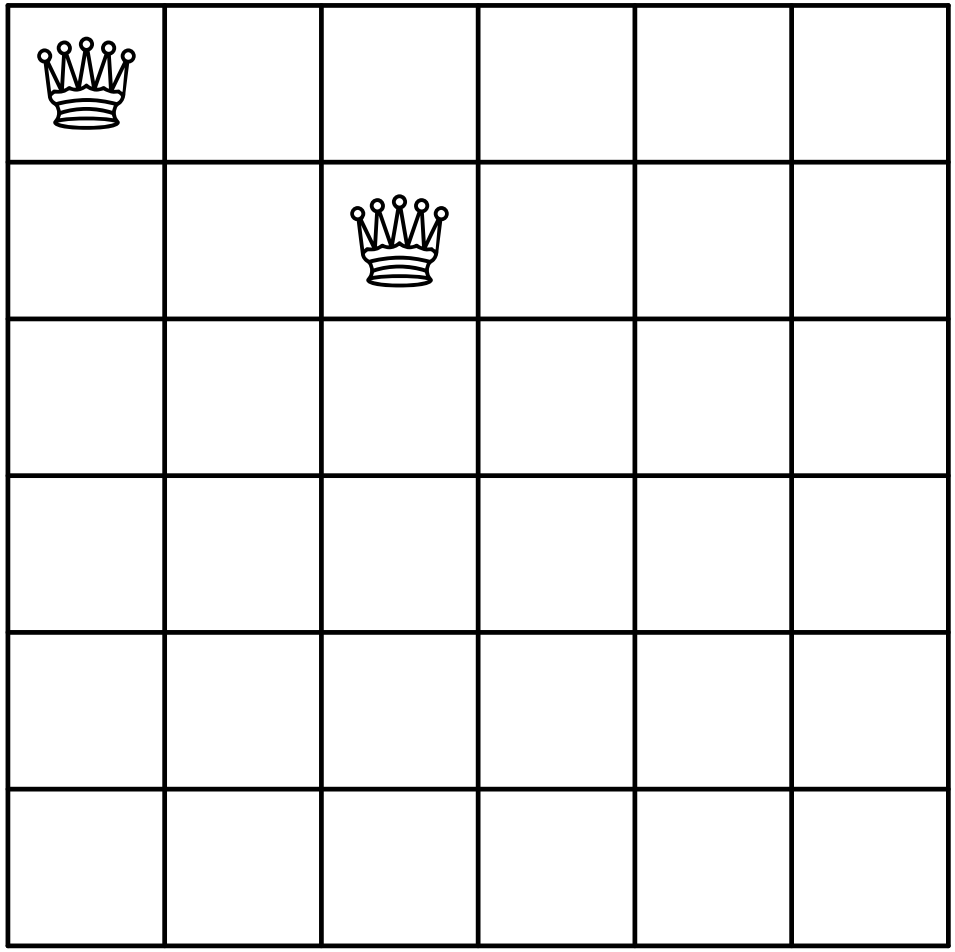


6-Queens Problem

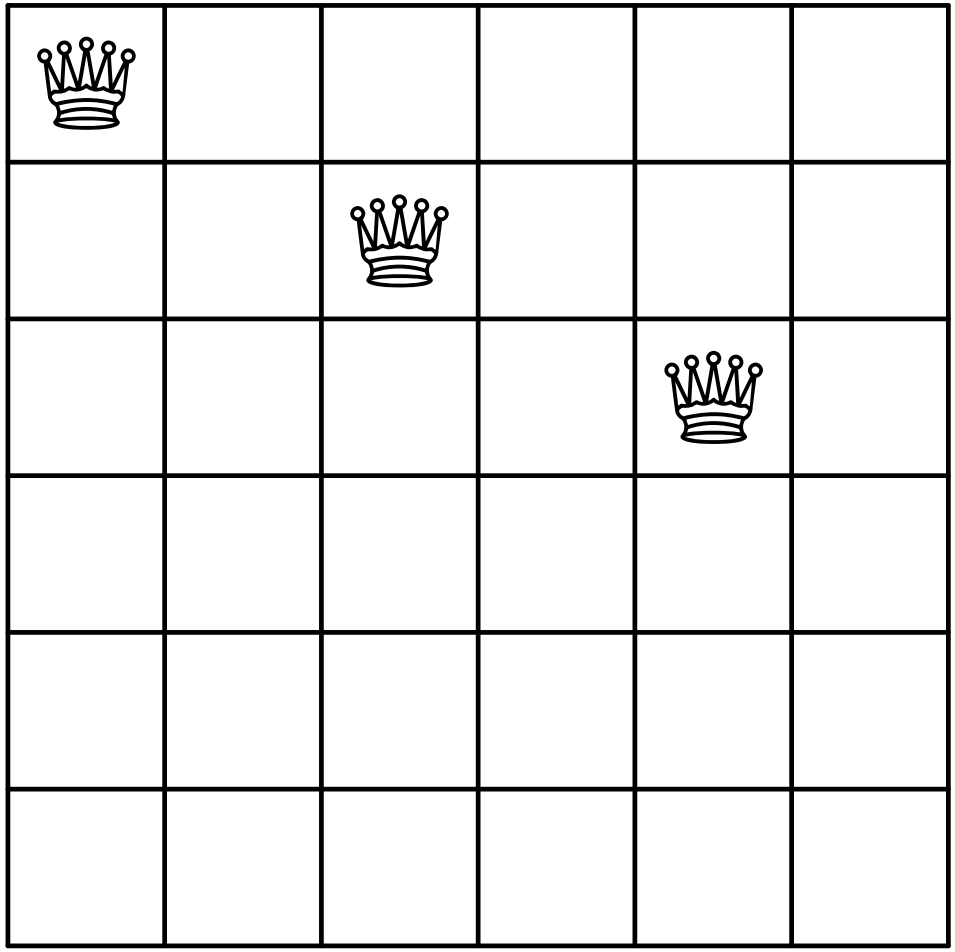
6-Queens Problem



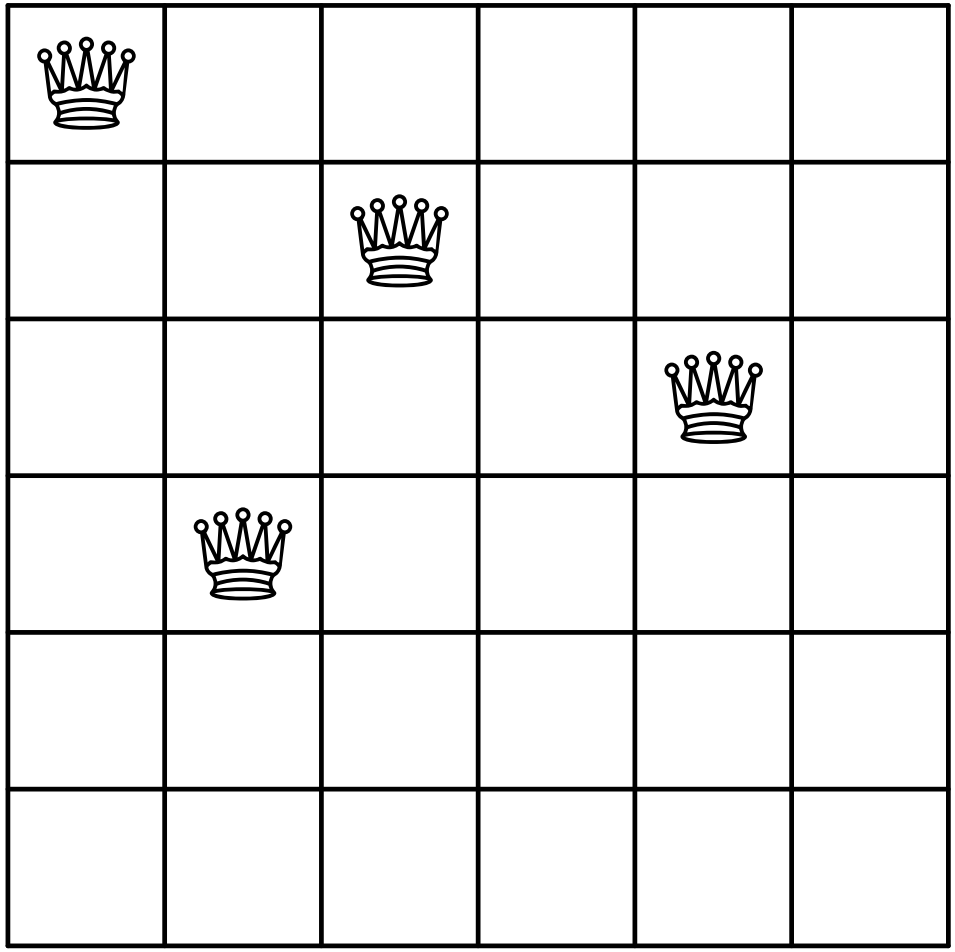
6-Queens Problem



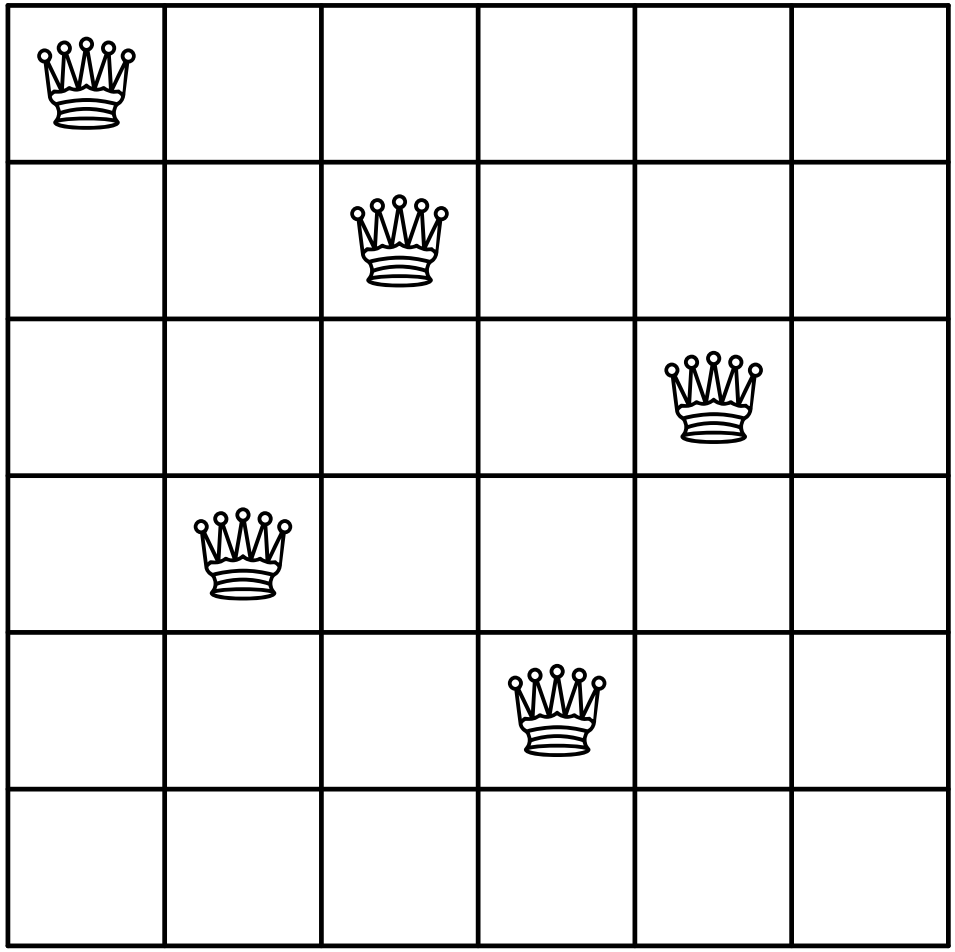
6-Queens Problem



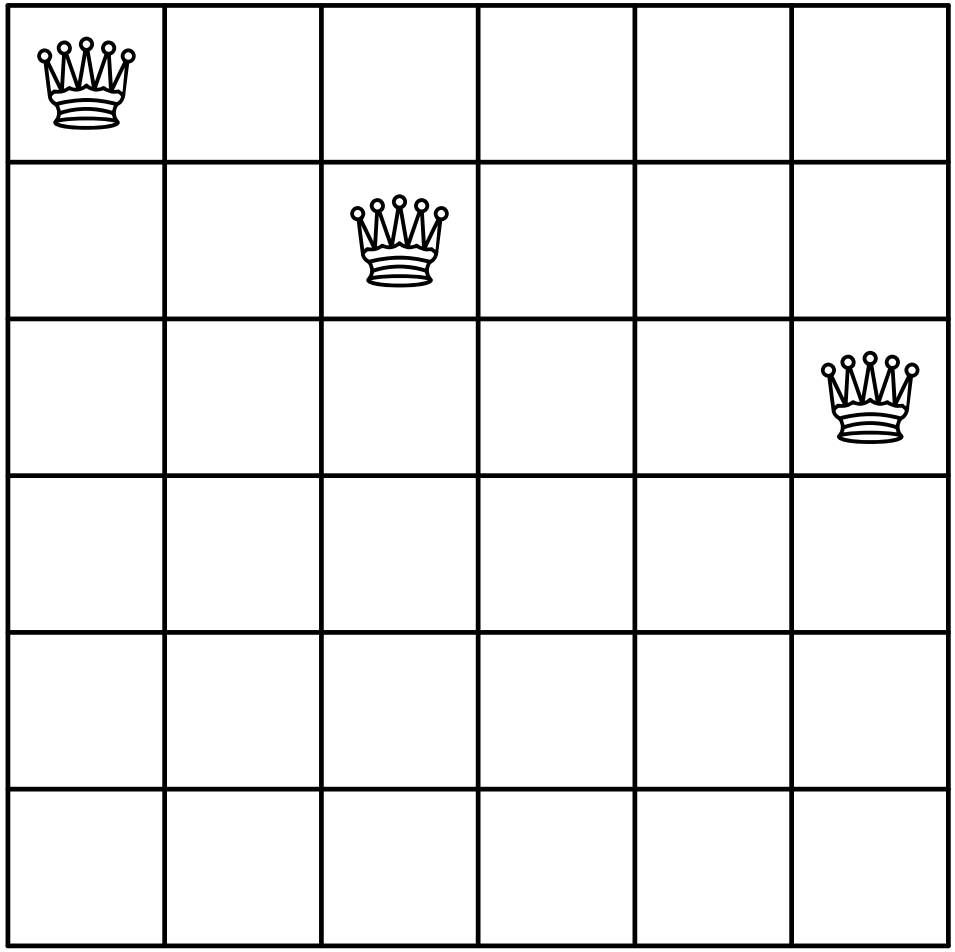
6-Queens Problem



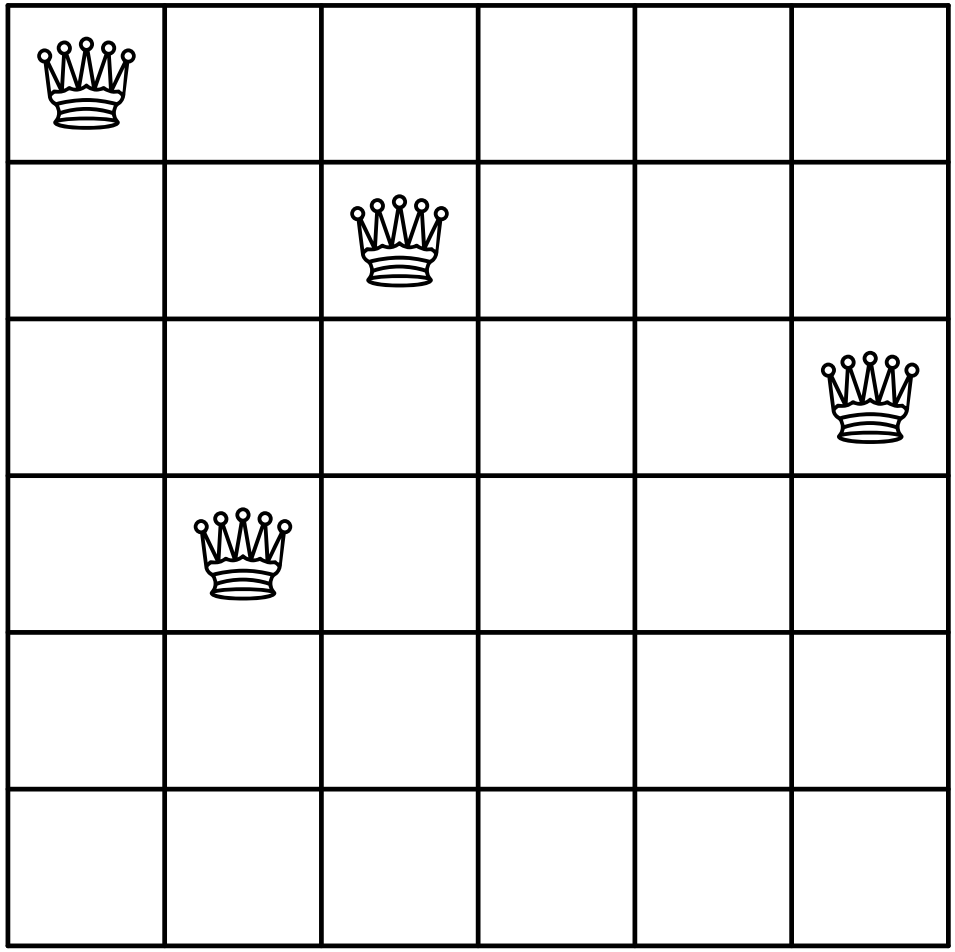
6-Queens Problem



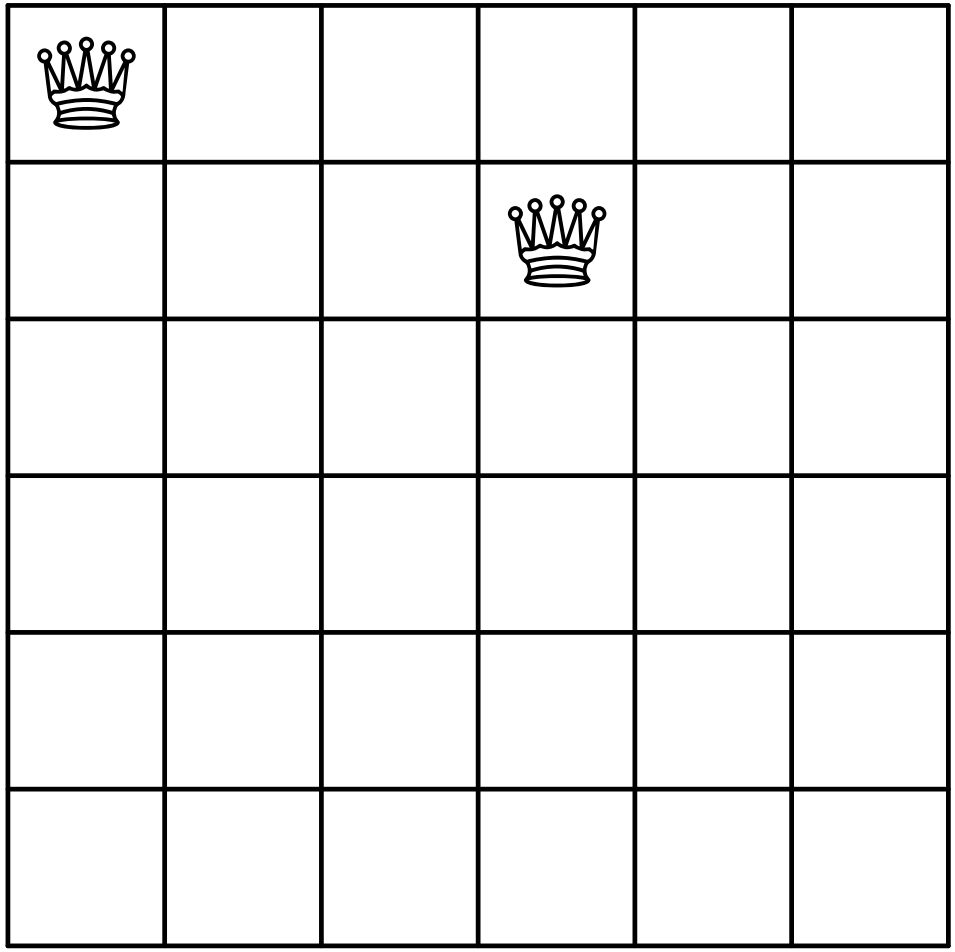
6-Queens Problem



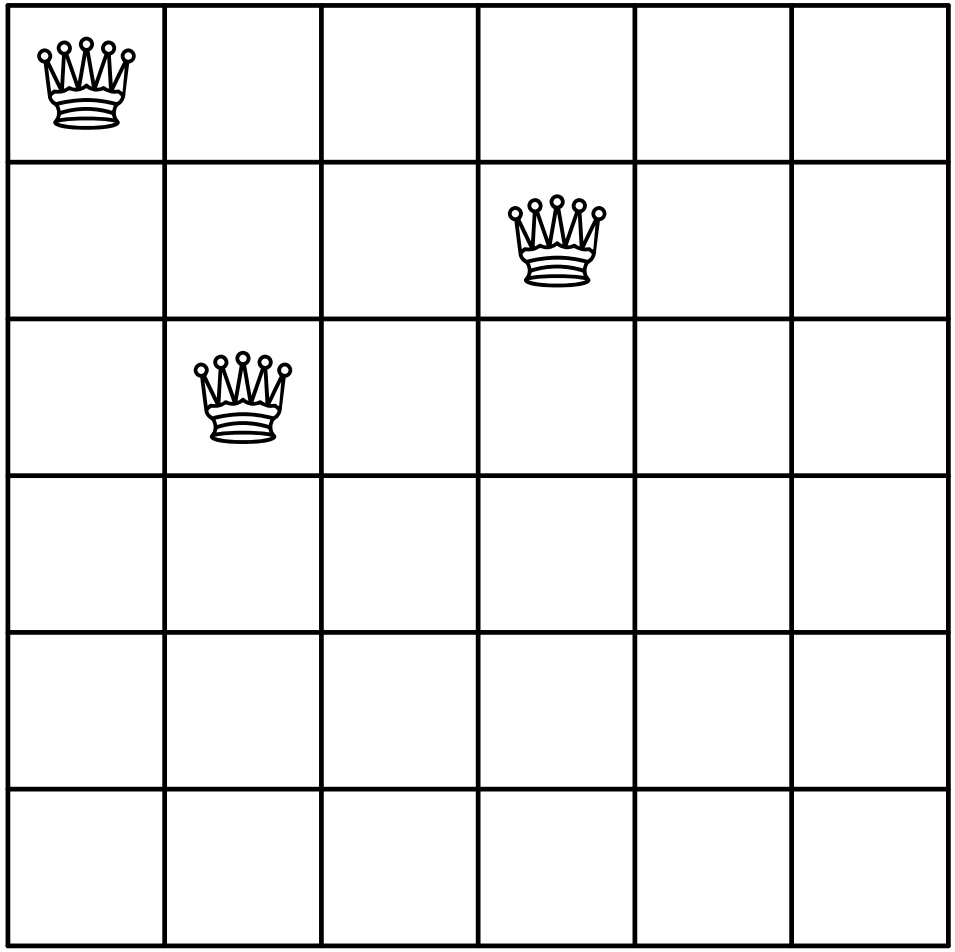
6-Queens Problem



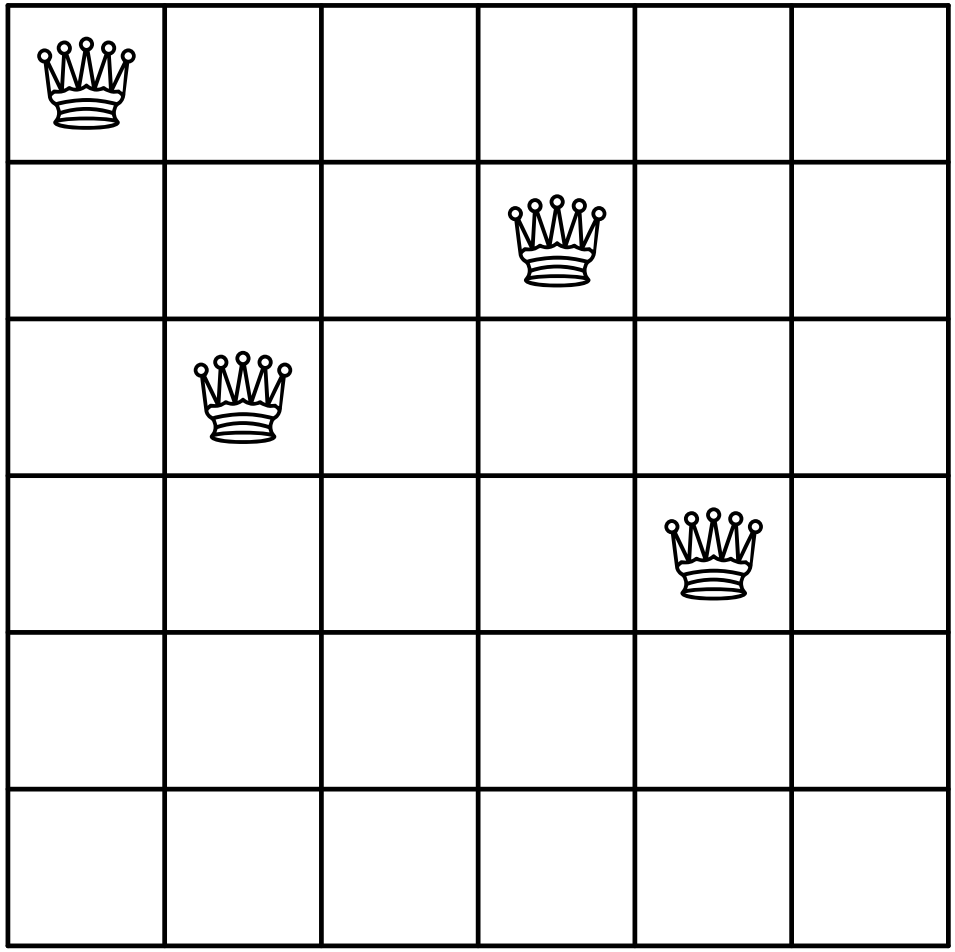
6-Queens Problem



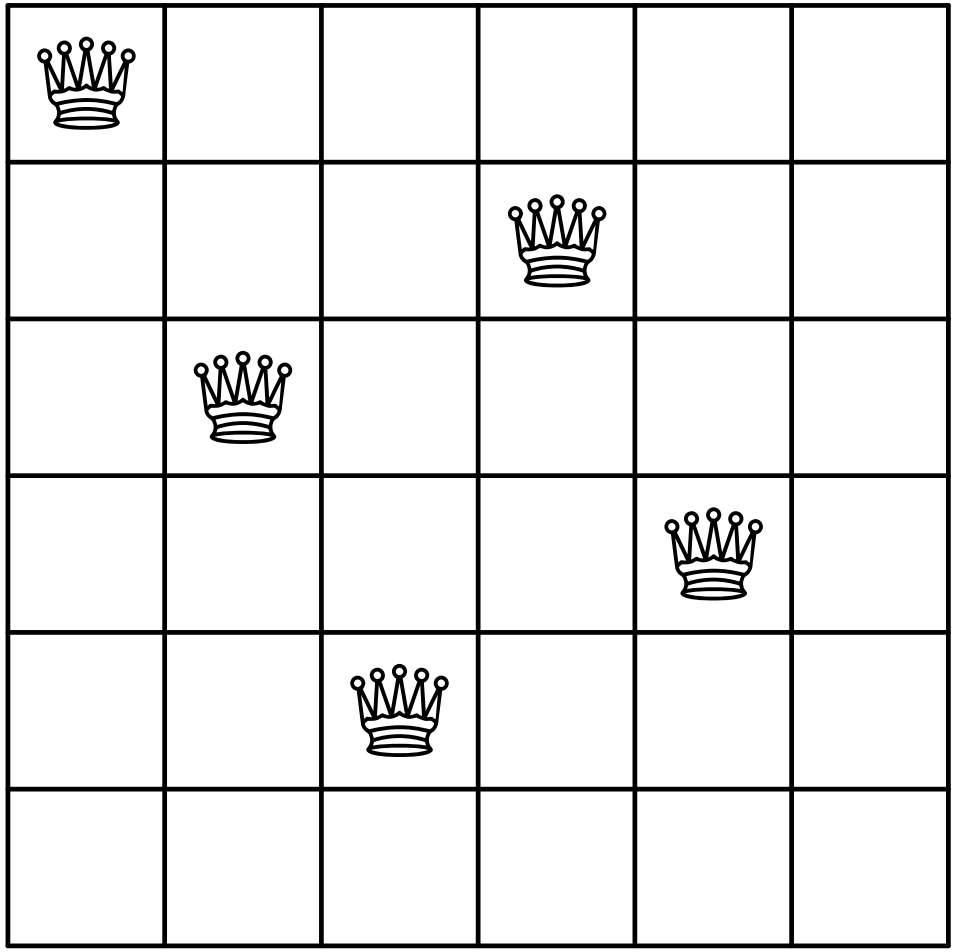
6-Queens Problem



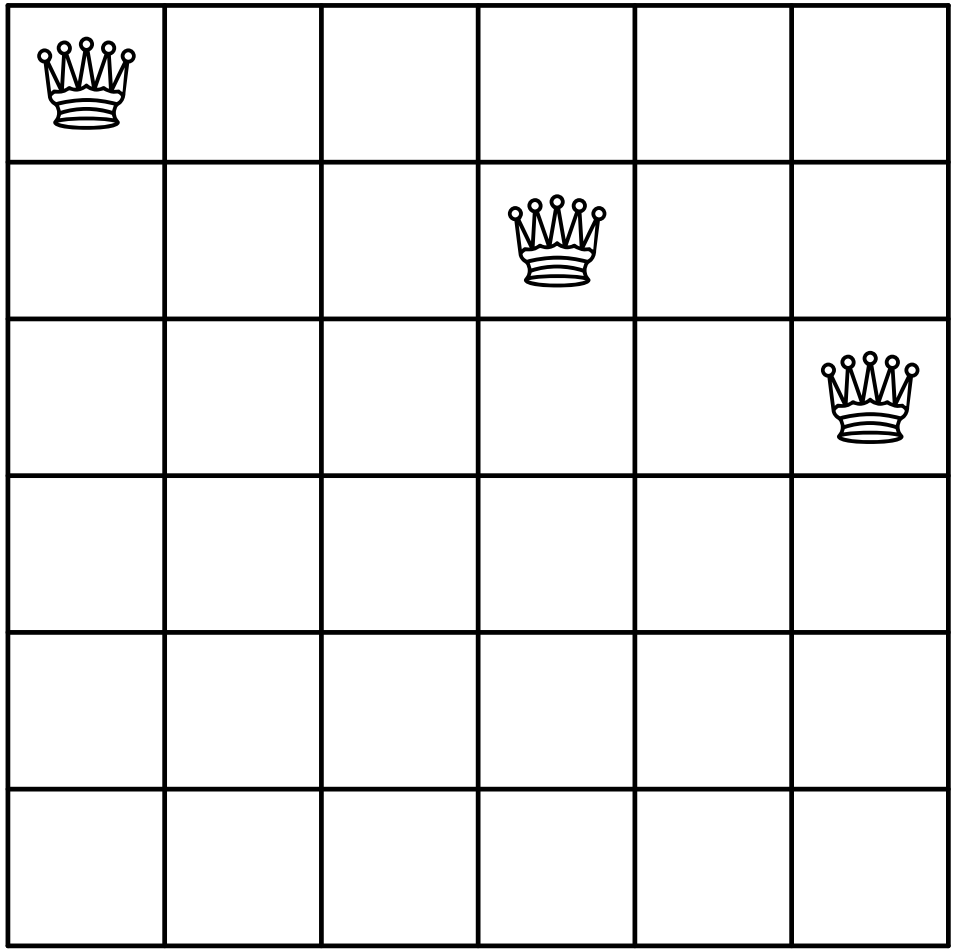
6-Queens Problem



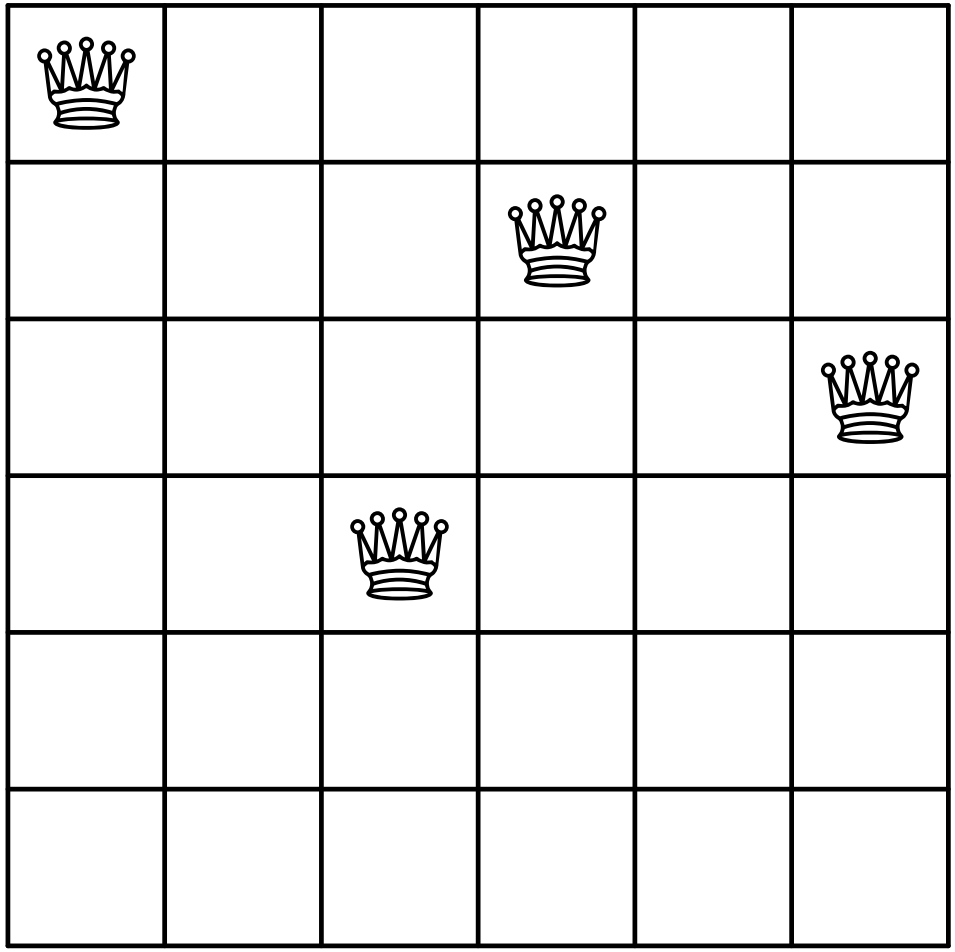
6-Queens Problem



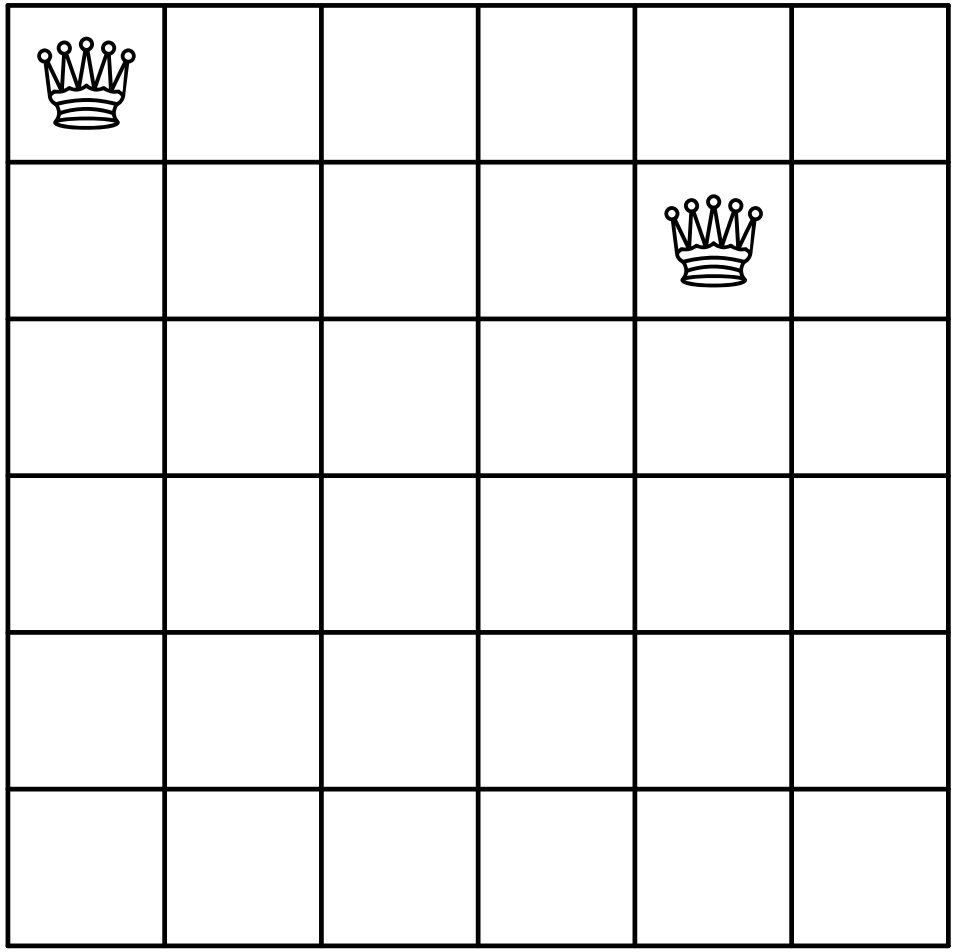
6-Queens Problem



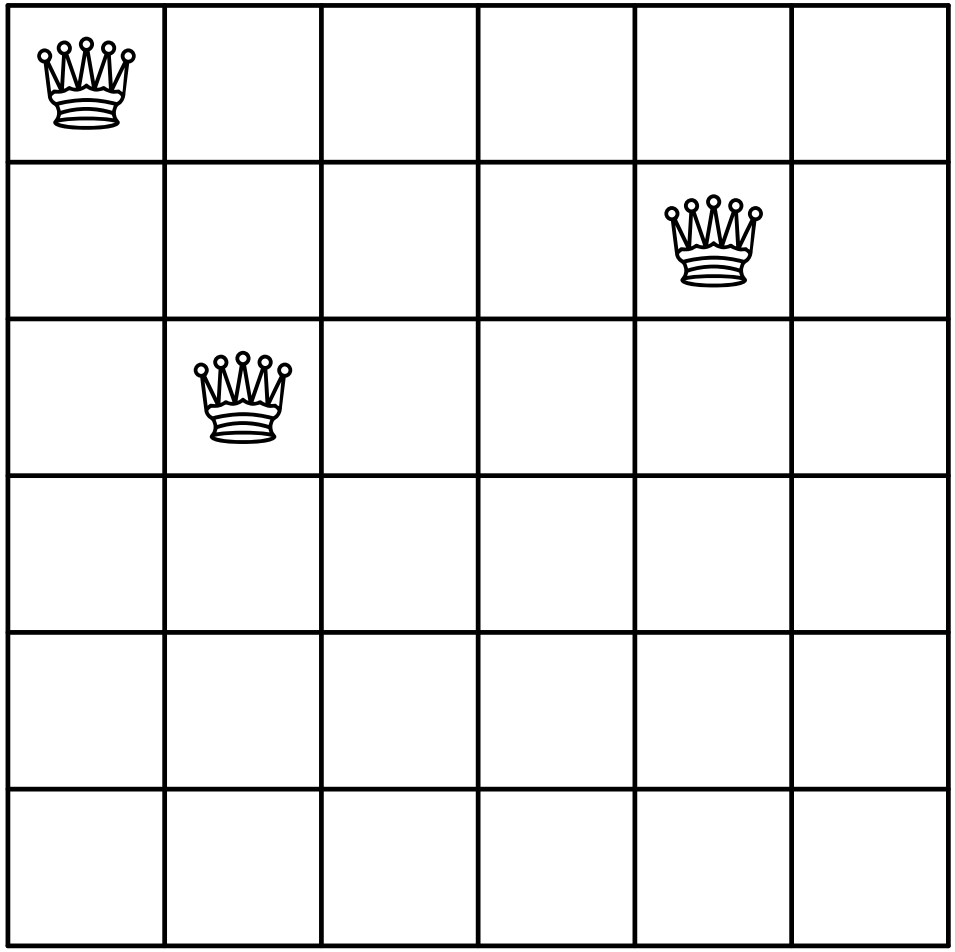
6-Queens Problem



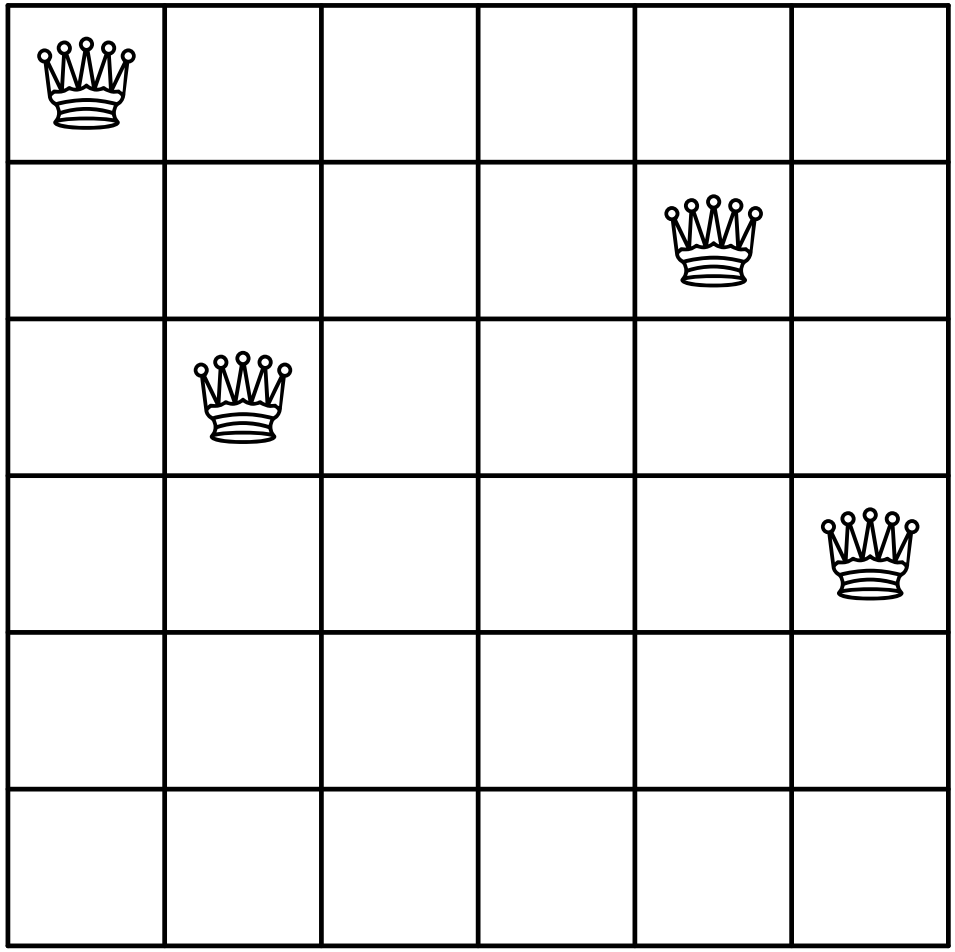
6-Queens Problem



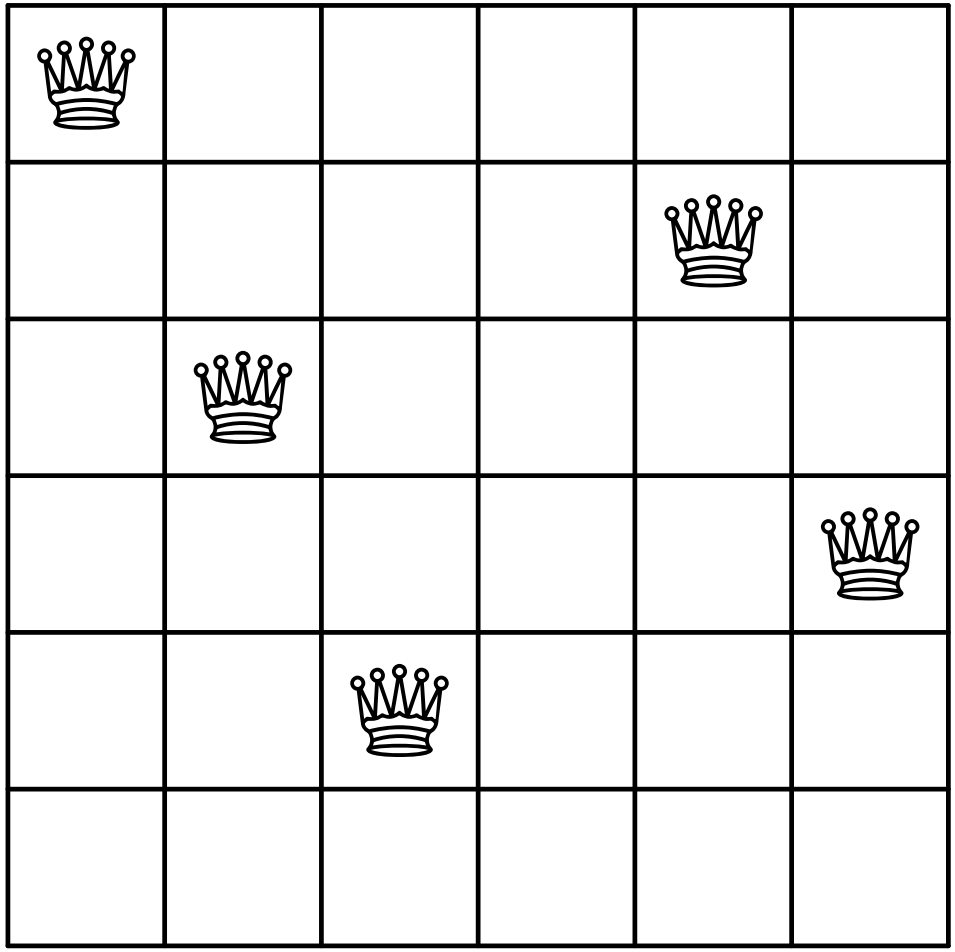
6-Queens Problem



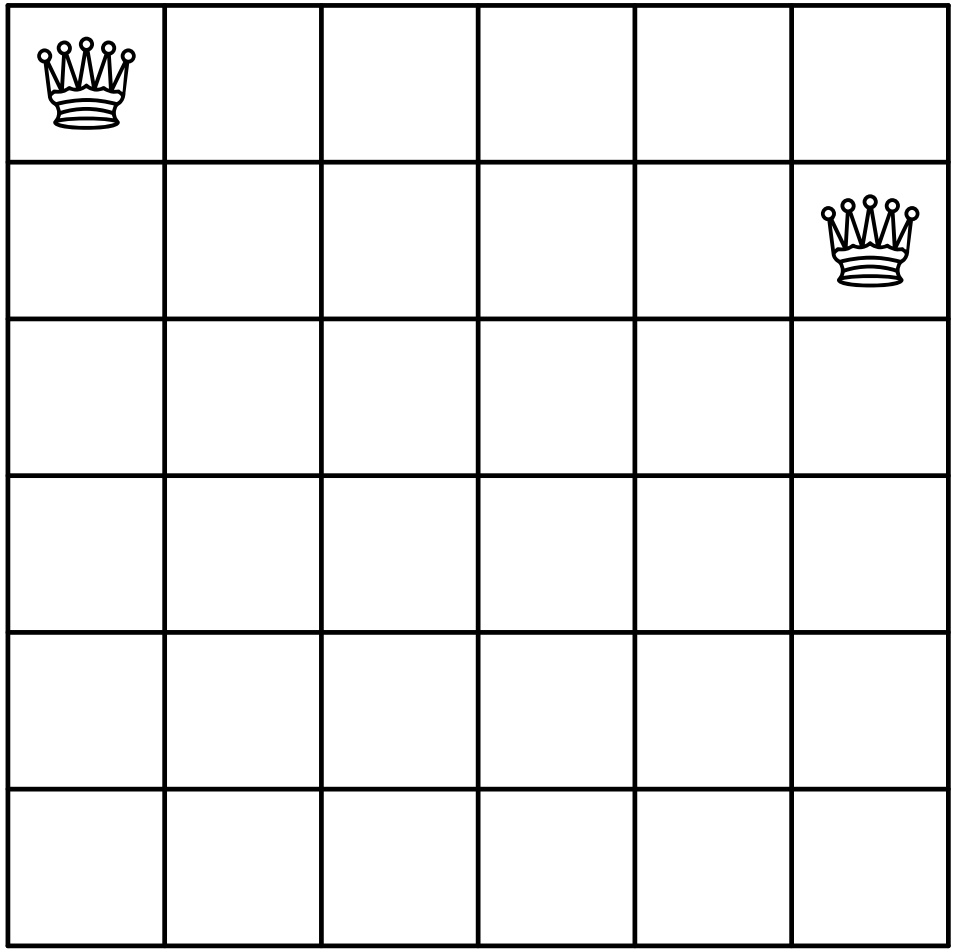
6-Queens Problem



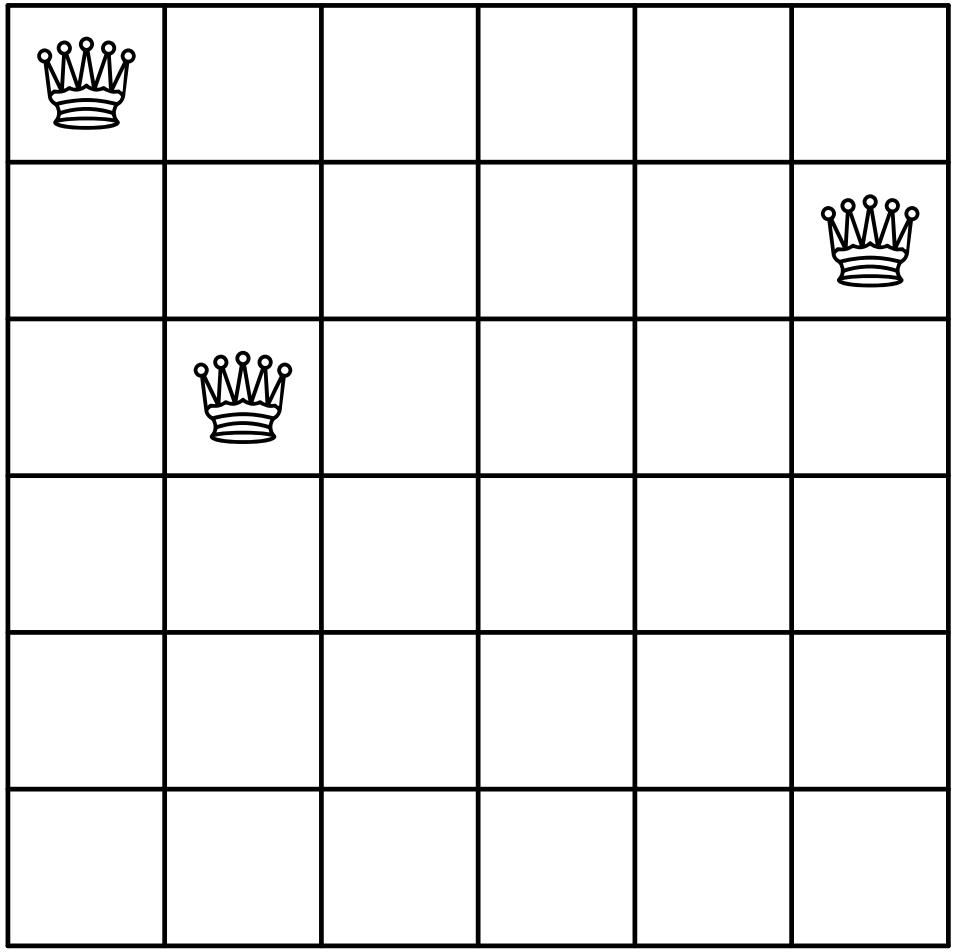
6-Queens Problem



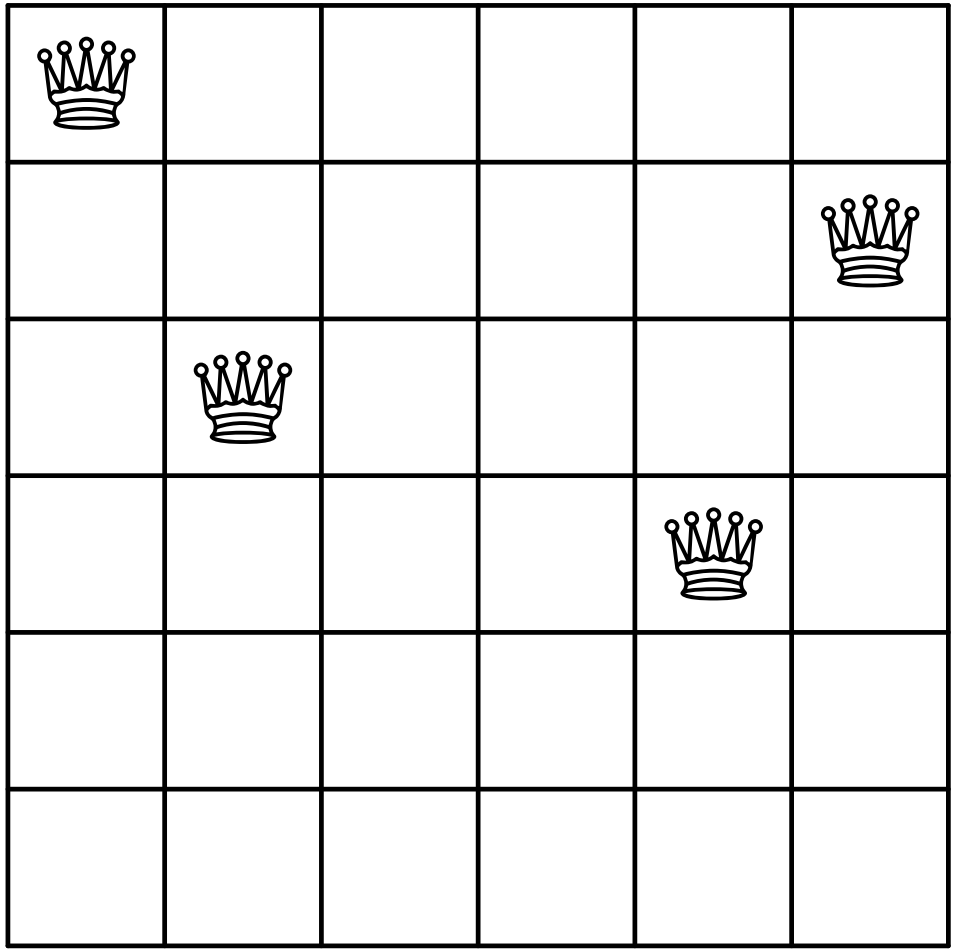
6-Queens Problem



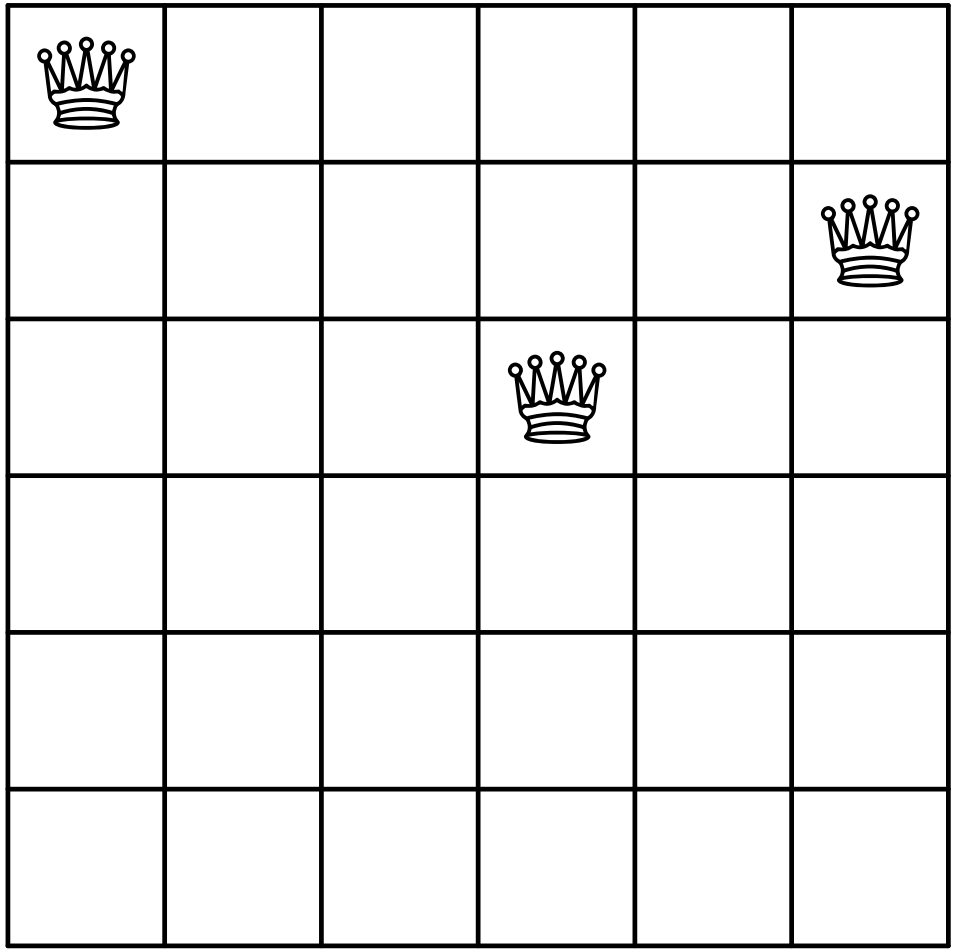
6-Queens Problem



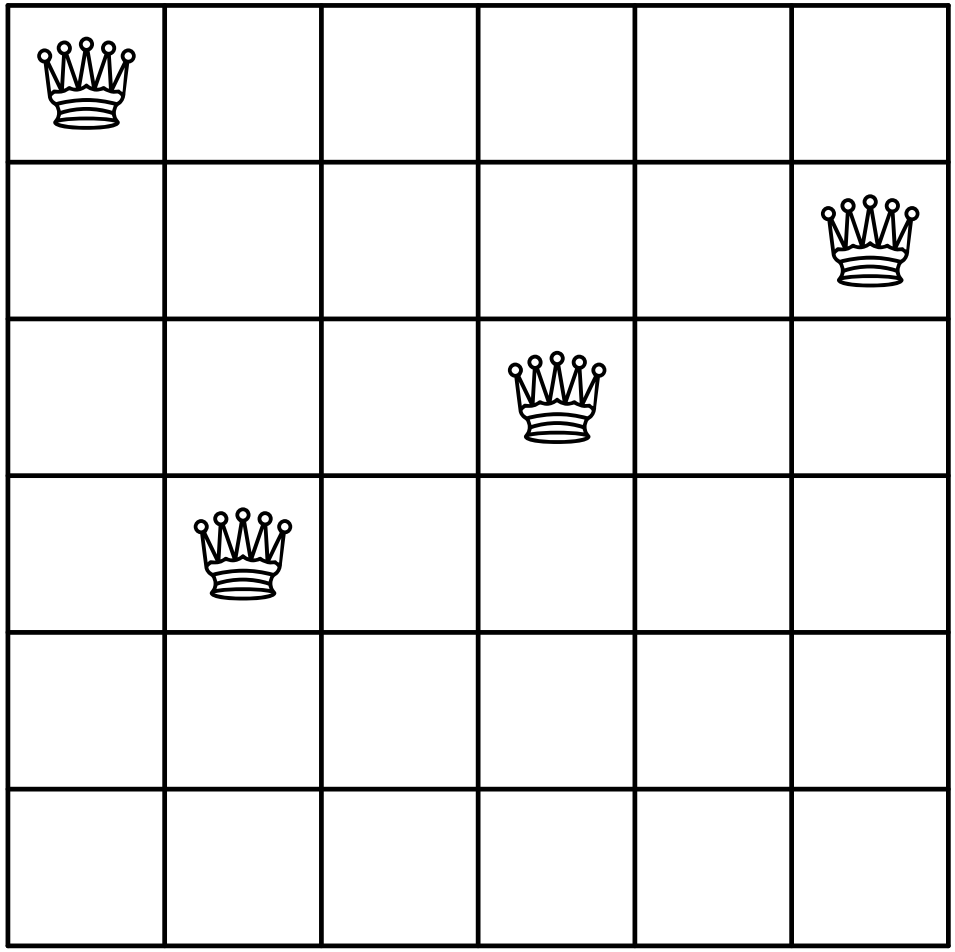
6-Queens Problem



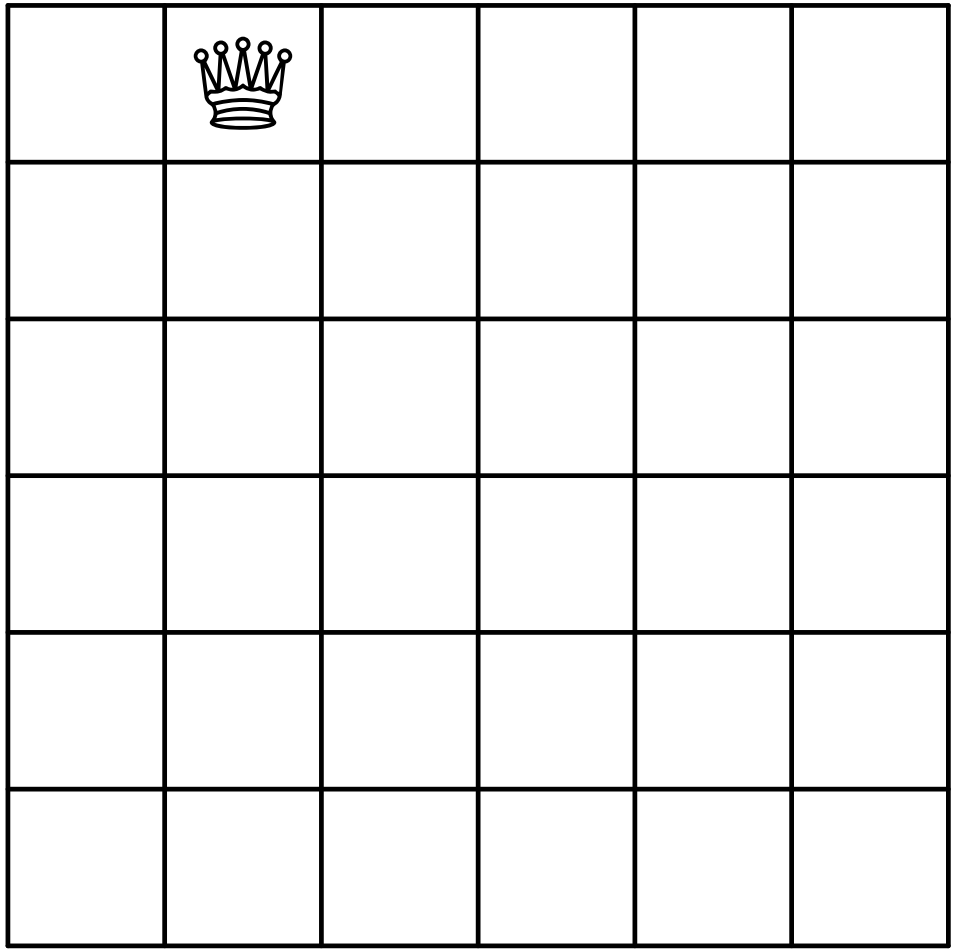
6-Queens Problem



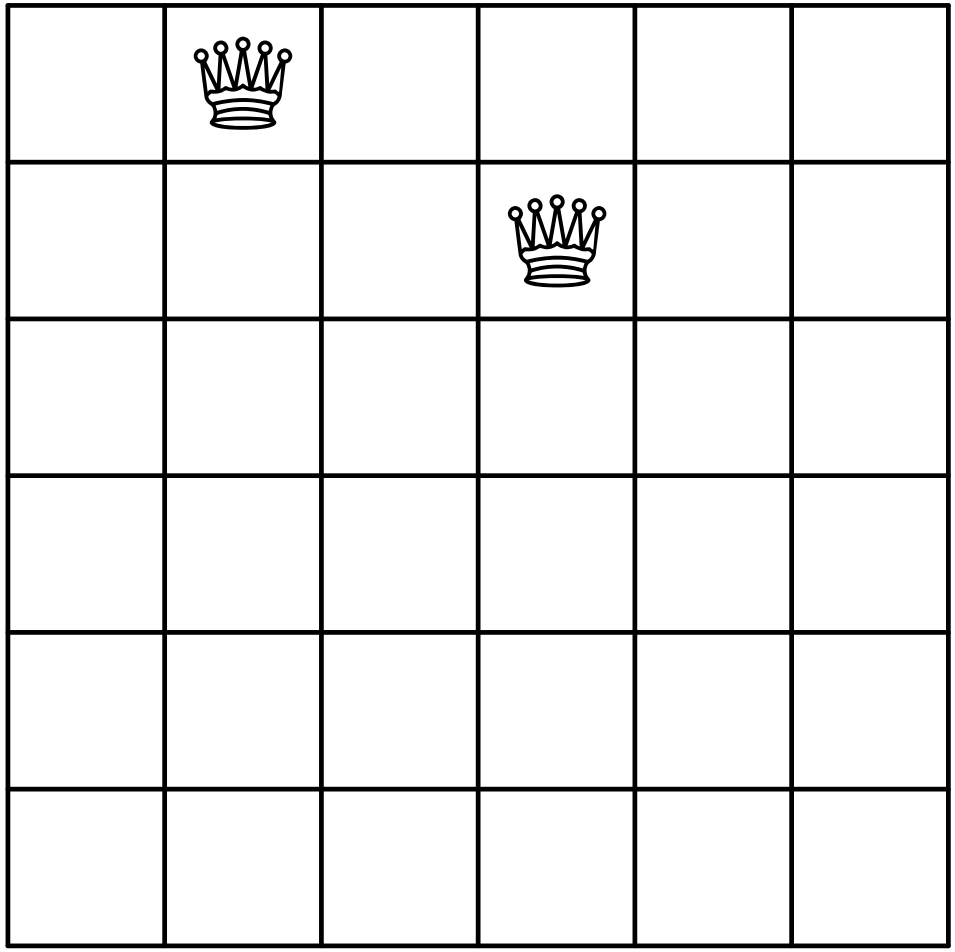
6-Queens Problem



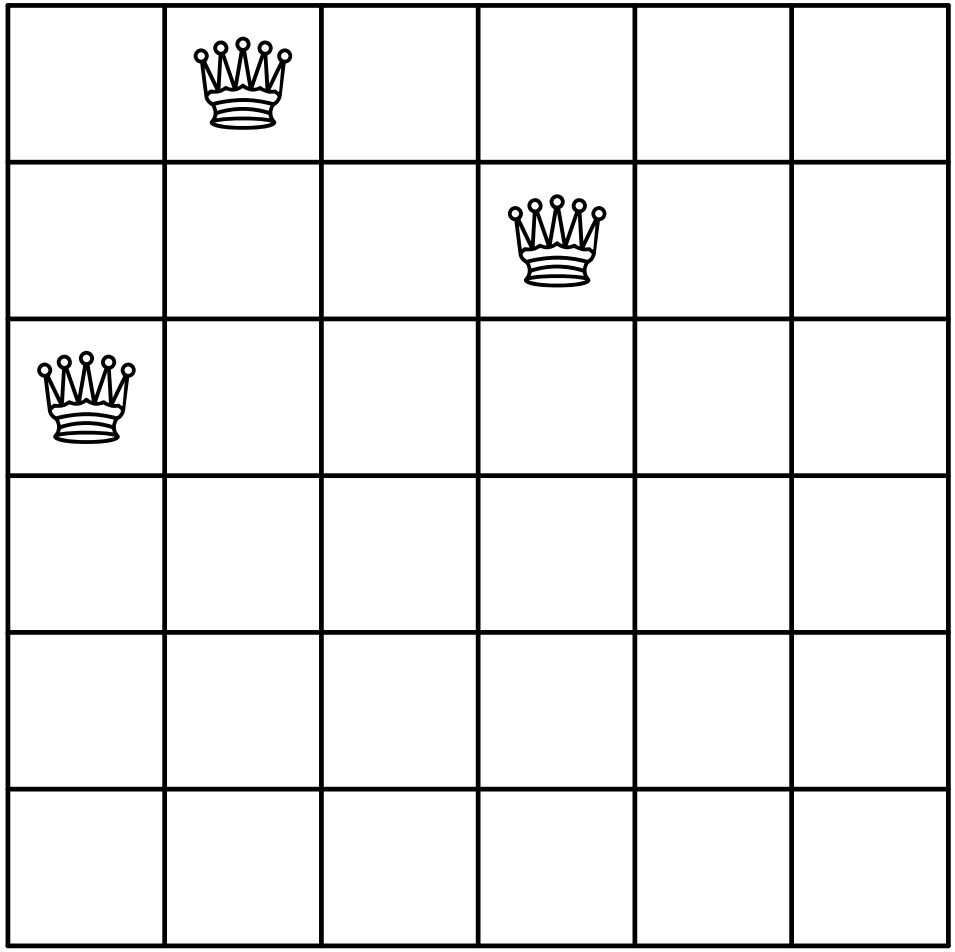
6-Queens Problem



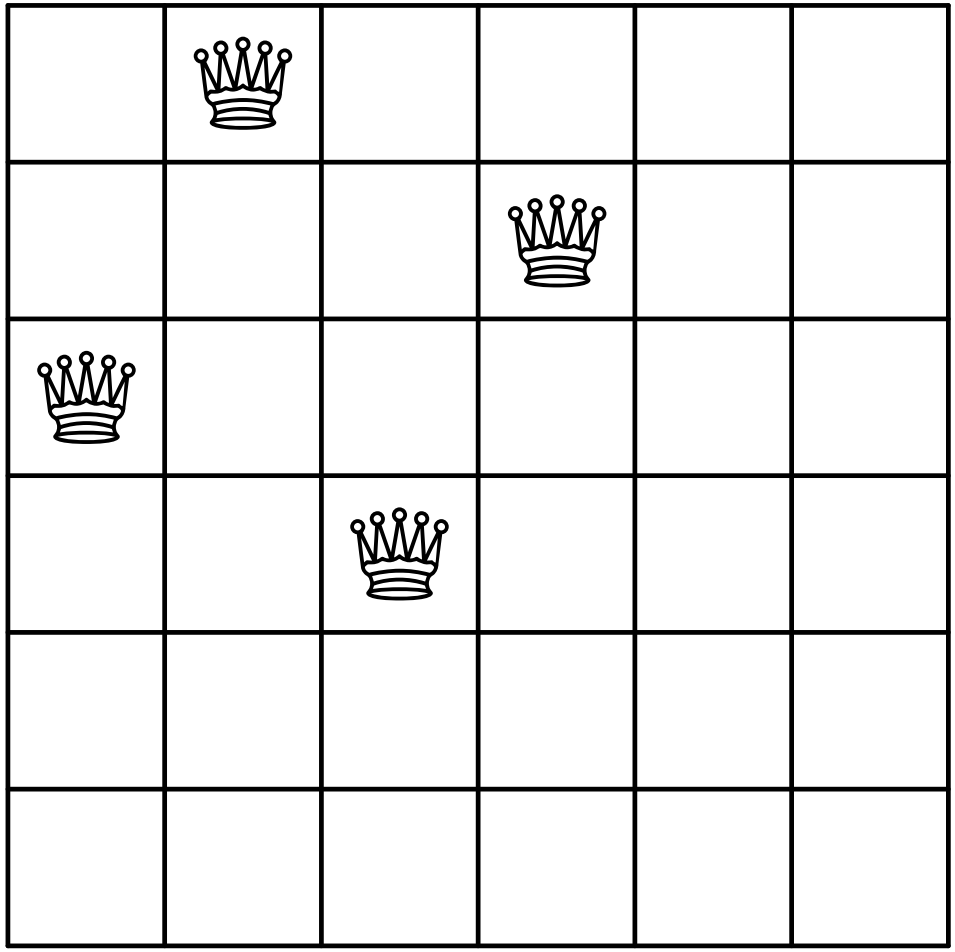
6-Queens Problem



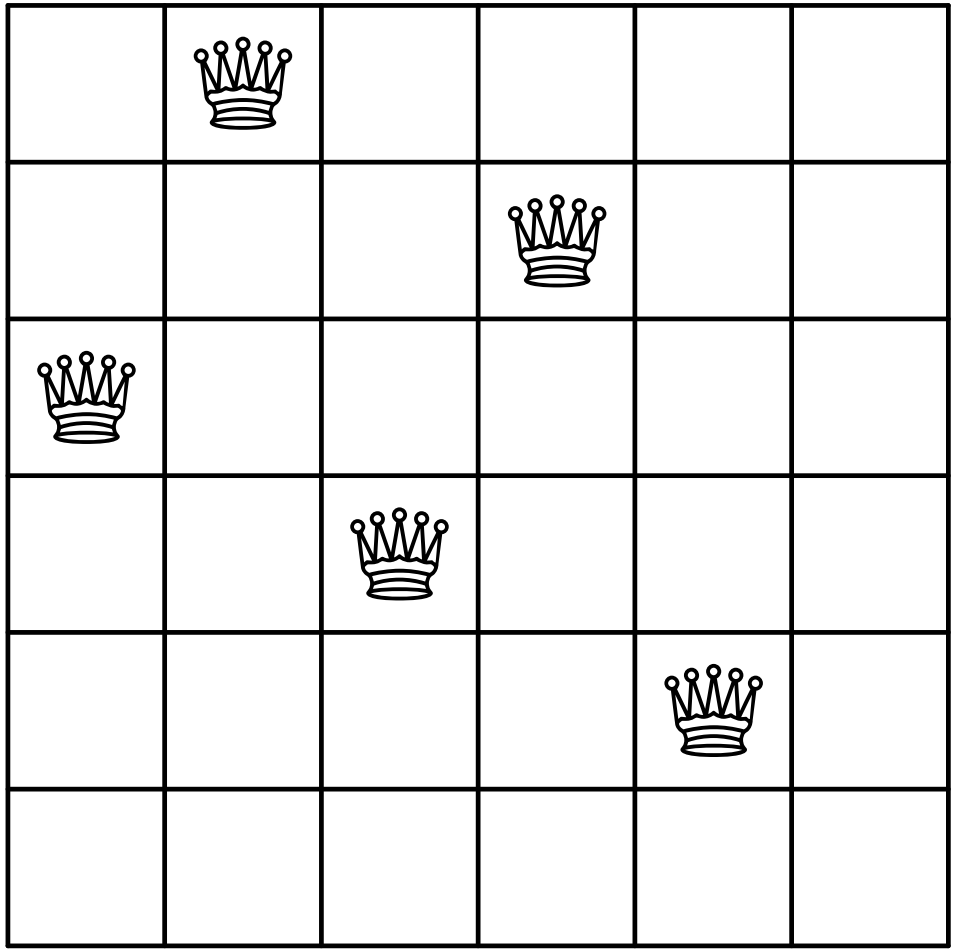
6-Queens Problem



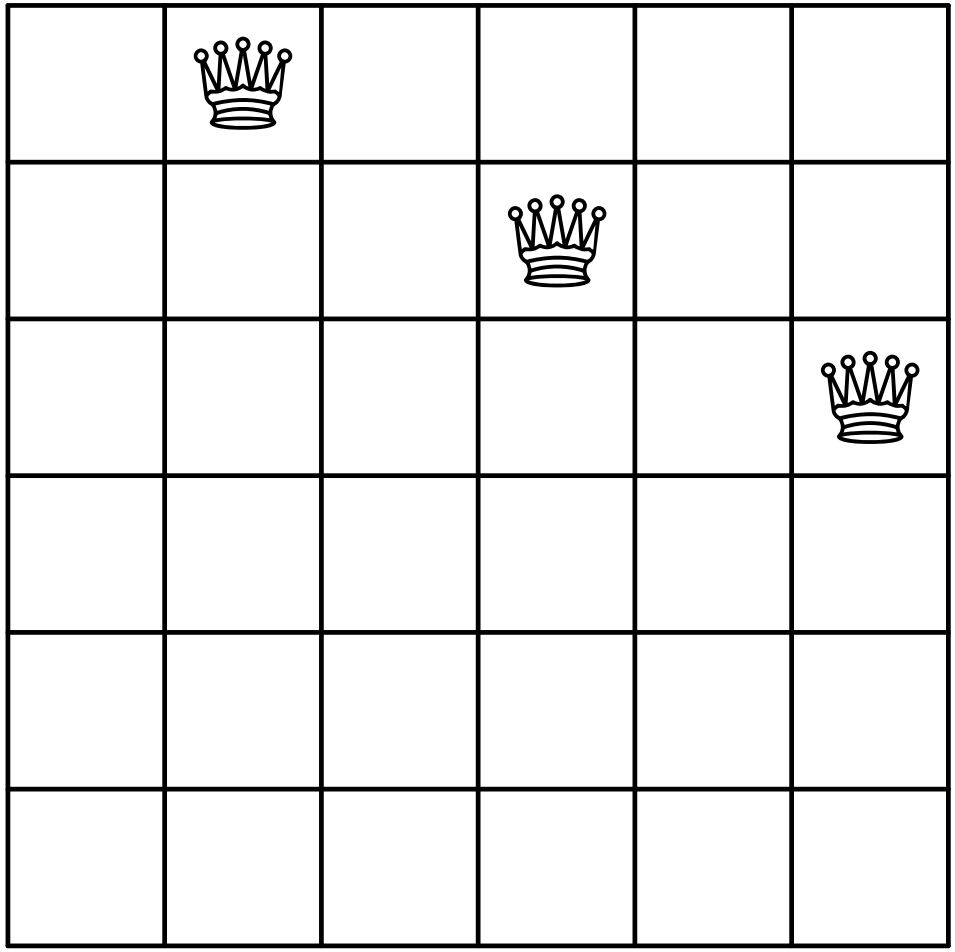
6-Queens Problem



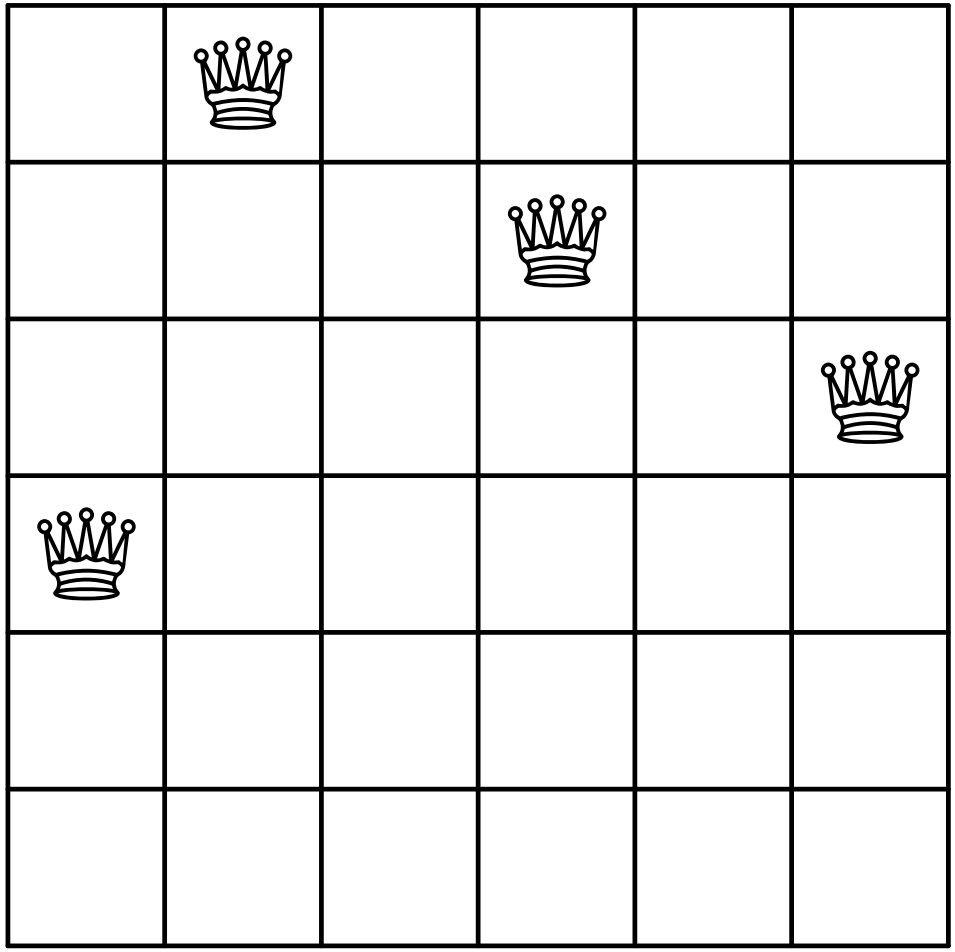
6-Queens Problem



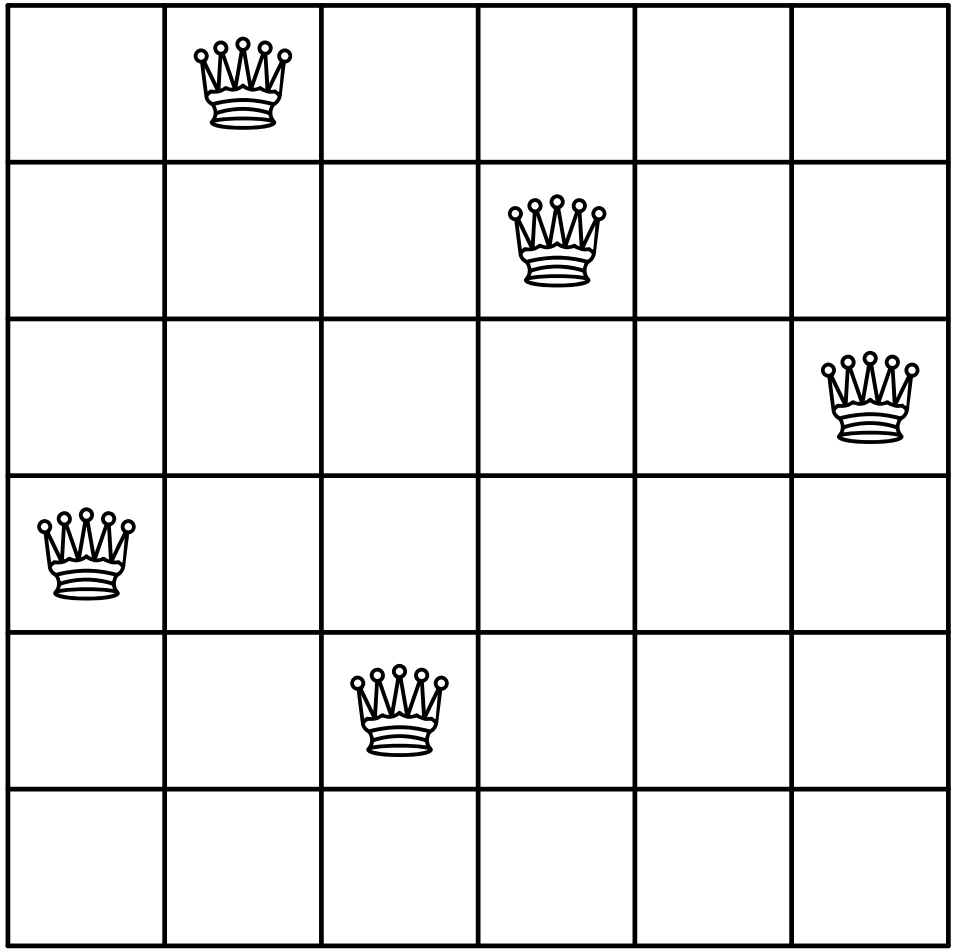
6-Queens Problem



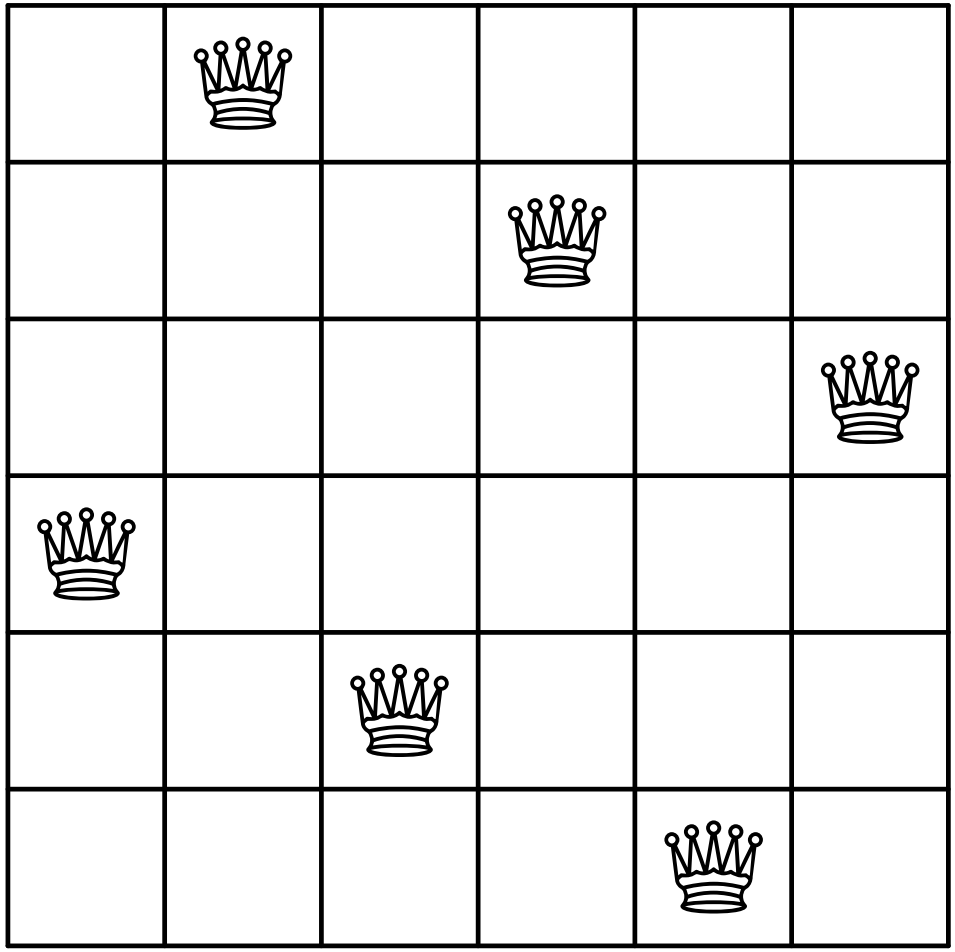
6-Queens Problem



6-Queens Problem



6-Queens Problem



Implementing n-Queens

- Implementing backtracking is easily done using recursion
- Recall depth-first search is easily implemented using recursion
- We just need a recursive function `next(n, row, sol)` which for a n-Queens problem searches new solutions in `row` given queens in previous rows given in `sol`
- Run: `List sol = nextRow(6, 0, new List());`

Implementing n-Queens

- Implementing backtracking is easily done using recursion
- Recall depth-first search is easily implemented using recursion
- We just need a recursive function `next(n, row, sol)` which for a n-Queens problem searches new solutions in `row` given queens in previous rows given in `sol`
- Run: `List sol = nextRow(6, 0, new List());`

Implementing n-Queens

- Implementing backtracking is easily done using recursion
- Recall depth-first search is easily implemented using recursion
- We just need a recursive function `next(n, row, sol)` which for a n-Queens problem searches new solutions in `row` given queens in previous rows given in `sol`
- Run: `List sol = nextRow(6, 0, new List());`

Implementing n-Queens

- Implementing backtracking is easily done using recursion
- Recall depth-first search is easily implemented using recursion
- We just need a recursive function `next(n, row, sol)` which for a n-Queens problem searches new solutions in `row` given queens in previous rows given in `sol`
- Run: `List sol = nextRow(6, 0, new List());`

Code

```
List nextRow(int noRows, int row, List queenPositions) {
  if (row==noRows) {return queenPositions;}
  for (int col=0; col<noRows; ++col) {
    if (legalQueen(col, row, queenPositions)) {
      queenPositions.add(col);
      List solution = nextRow(noRows, row+1, queenPositions);
      if (solution!=null)
        return solution;
    }
  }
  return null;
}

bool LegalQueen(int col, int row, List sol) {
  for(int r=0; r<row: ++r) {
    rf (sol[r] == col || sol[r]-row+r == col || sol[r]+row-r==col) {
      return false;
    }
  }
  return true;
}
```

Code

```
List nextRow(int noRows, int row, List queenPositions) {
  if (row==noRows) {return queenPositions;}
  for (int col=0; col<noRows; ++col) {
    if (legalQueen(col, row, queenPositions)) {
      queenPositions.add(col);
      List solution = nextRow(noRows, row+1, queenPositions);
      if (solution!=null)
        return solution;
    }
  }
  return null;
}

bool LegalQueen(int col, int row, List sol) {
  for(int r=0; r<row: ++r) {
    rf (sol[r] == col || sol[r]-row+r == col || sol[r]+row-r==col) {
      return false;
    }
  }
  return true;
}
```

Code

```
List nextRow(int noRows, int row, List queenPositions) {
  if (row==noRows) {return queenPositions;}
  for (int col=0; col<noRows; ++col) {
    if (legalQueen(col, row, queenPositions)) {
      queenPositions.add(col);
      List solution = nextRow(noRows, row+1, queenPositions);
      if (solution!=null)
        return solution;
    }
  }
  return null;
}

bool LegalQueen(int col, int row, List sol) {
  for(int r=0; r<row: ++r) {
    rf (sol[r] == col || sol[r]-row+r == col || sol[r]+row-r==col) {
      return false;
    }
  }
  return true;
}
```

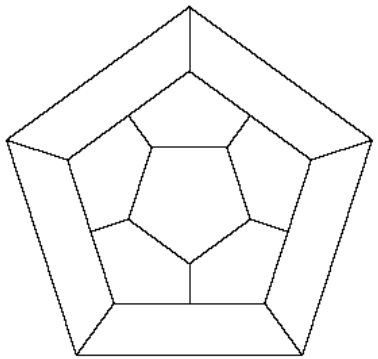
Code

```
List nextRow(int noRows, int row, List queenPositions) {
  if (row==noRows) {return queenPositions;}
  for (int col=0; col<noRows; ++col) {
    if (legalQueen(col, row, queenPositions)) {
      queenPositions.add(col);
      List solution = nextRow(noRows, row+1, queenPositions);
      if (solution!=null)
        return solution;
    }
  }
  return null;
}

bool LegalQueen(int col, int row, List sol) {
  for(int r=0; r<row: ++r) {
    rf (sol[r] == col || sol[r]-row+r == col || sol[r]+row-r==col) {
      return false;
    }
  }
  return true;
}
```

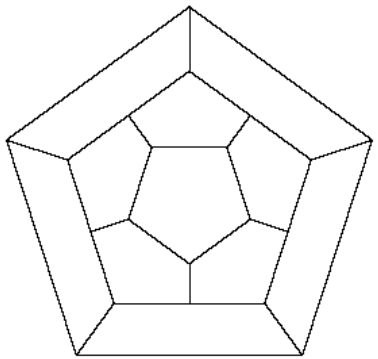
Hamiltonian Circuit

- A Hamiltonian cycle is a tour through a graph which visits every vertex once only and returns to the start
- It is a hard problem in that there are no known algorithms that are guaranteed to find a Hamiltonian cycle in polynomial time
- For many graphs it is not too hard



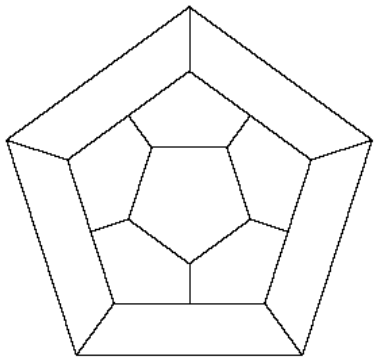
Hamiltonian Circuit

- A Hamiltonian cycle is a tour through a graph which visits every vertex once only and returns to the start
- It is a hard problem in that there are no known algorithms that are guaranteed to find a Hamiltonian cycle in polynomial time
- For many graphs it is not too hard

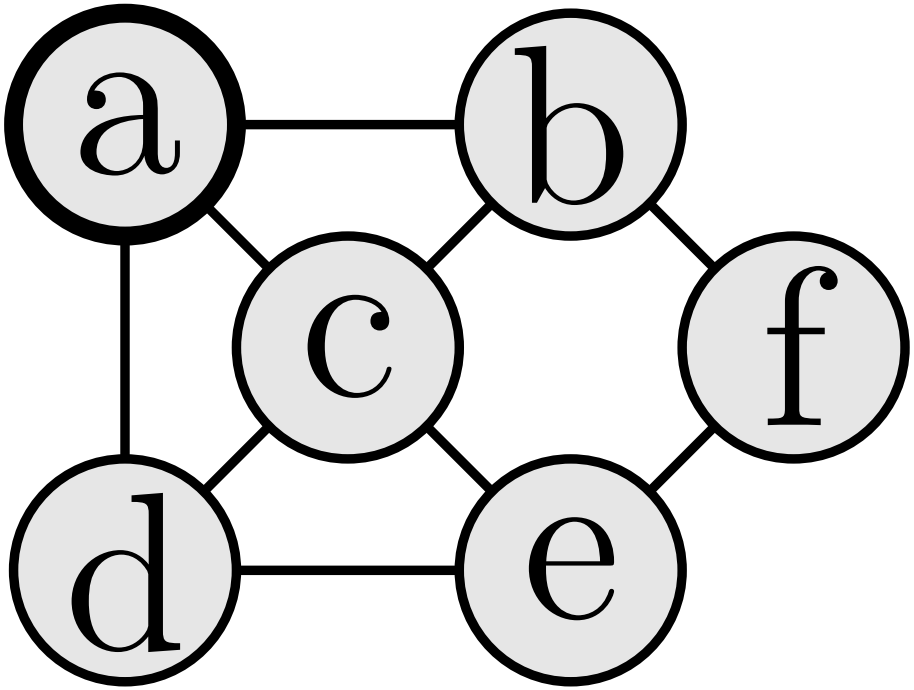


Hamiltonian Circuit

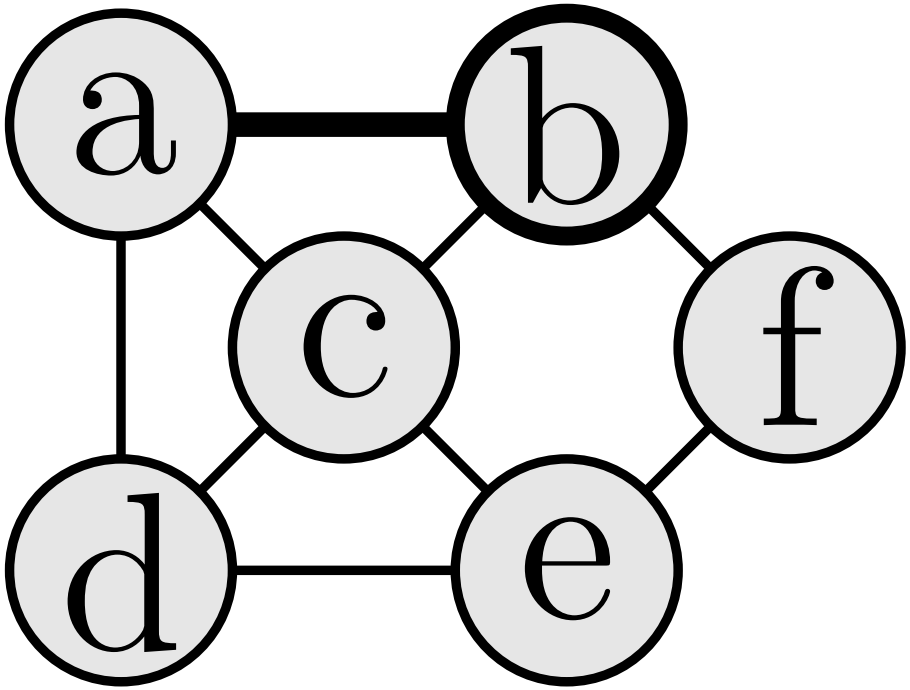
- A Hamiltonian cycle is a tour through a graph which visits every vertex once only and returns to the start
- It is a hard problem in that there are no known algorithms that are guaranteed to find a Hamiltonian cycle in polynomial time
- For many graphs it is not too hard



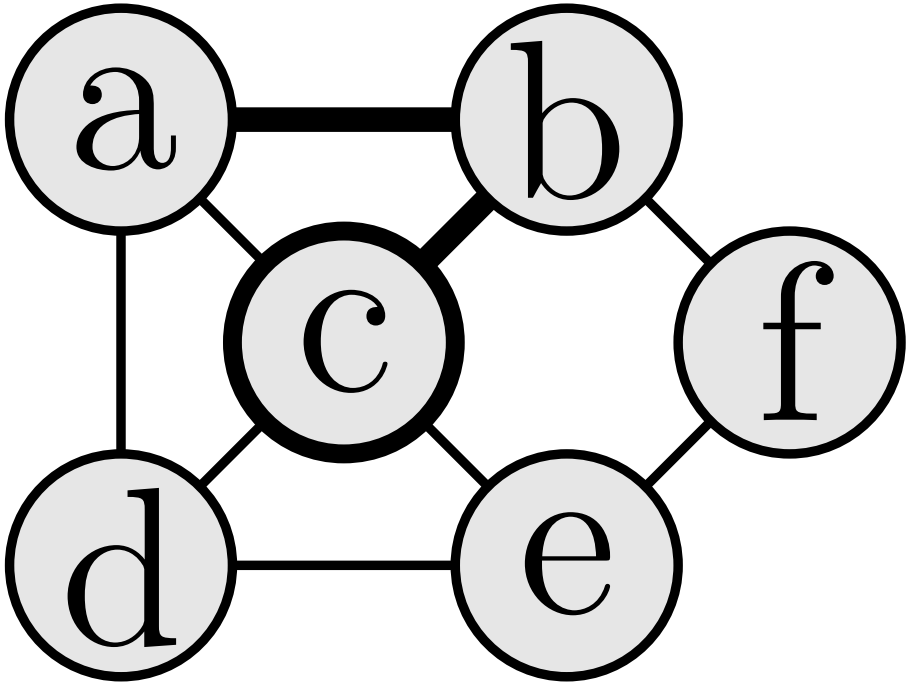
Hamiltonian Circuit Example



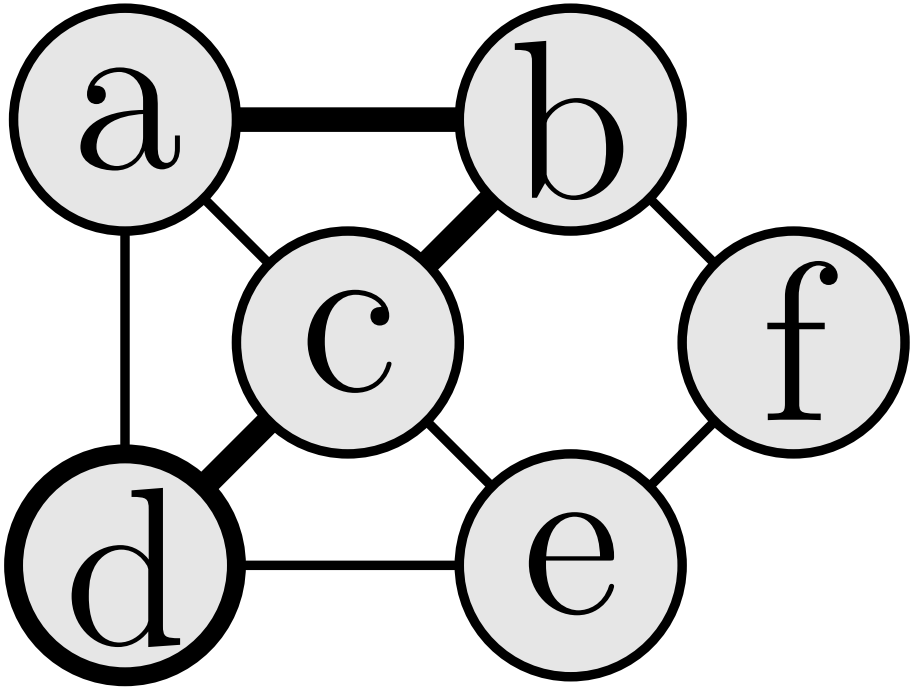
Hamiltonian Circuit Example



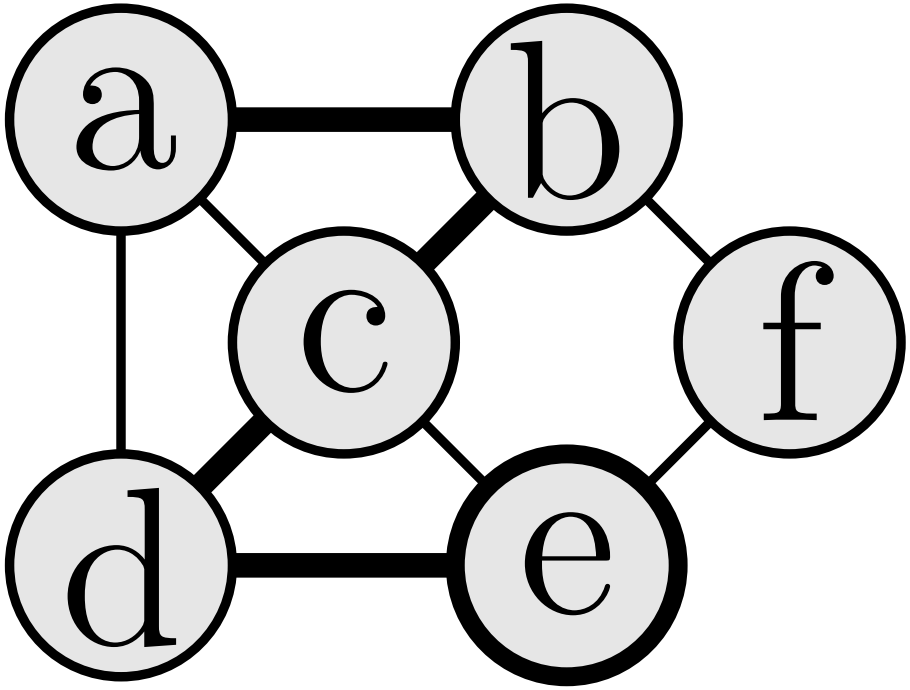
Hamiltonian Circuit Example



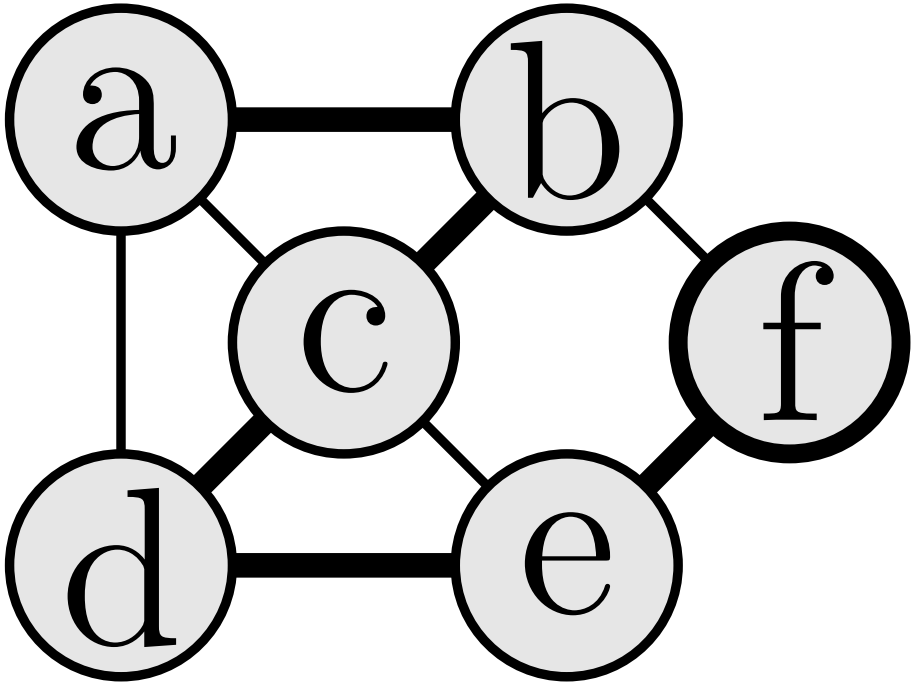
Hamiltonian Circuit Example



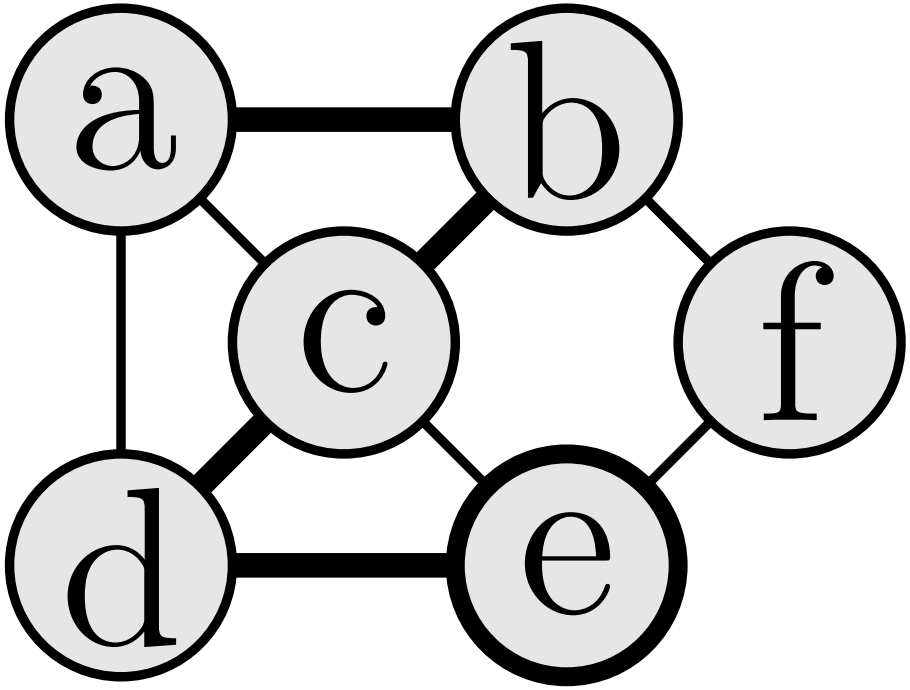
Hamiltonian Circuit Example



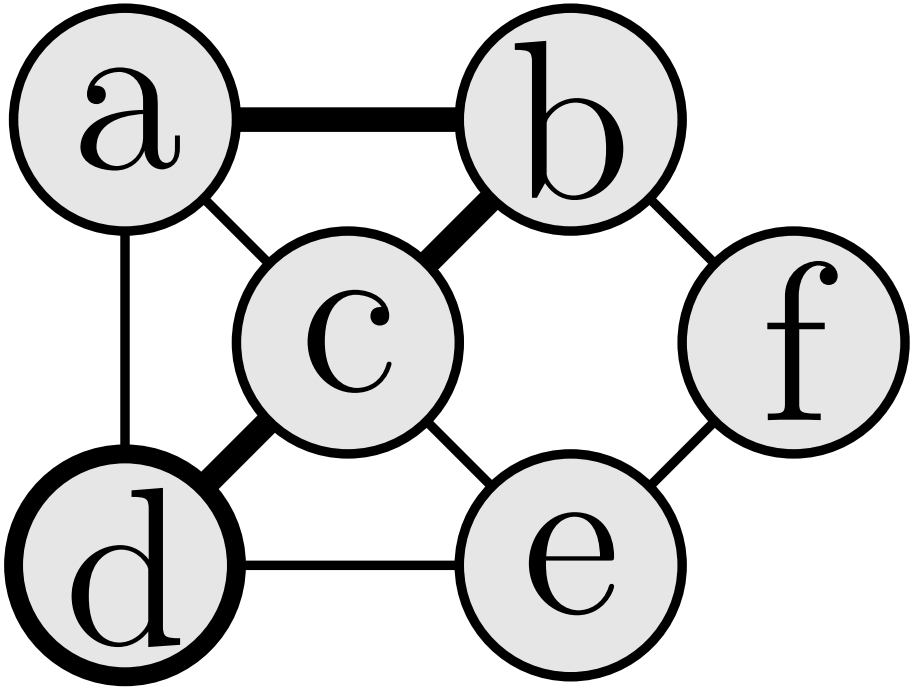
Hamiltonian Circuit Example



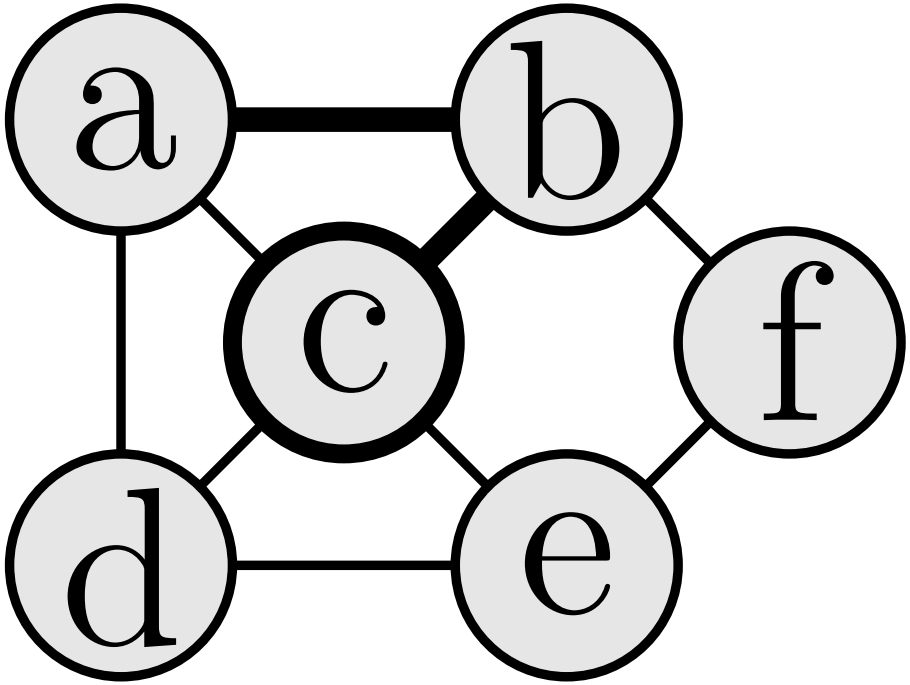
Hamiltonian Circuit Example



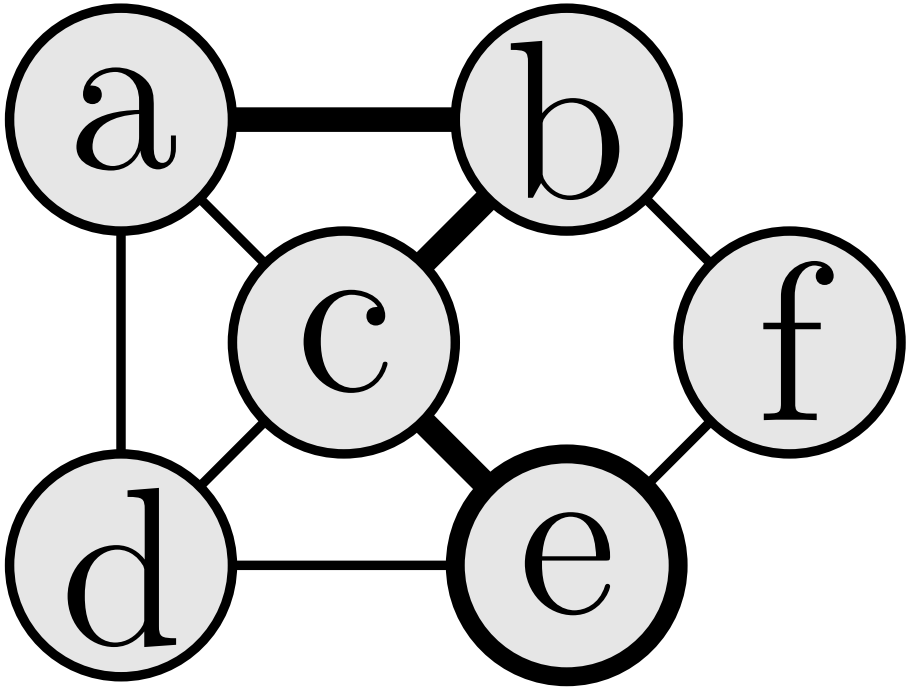
Hamiltonian Circuit Example



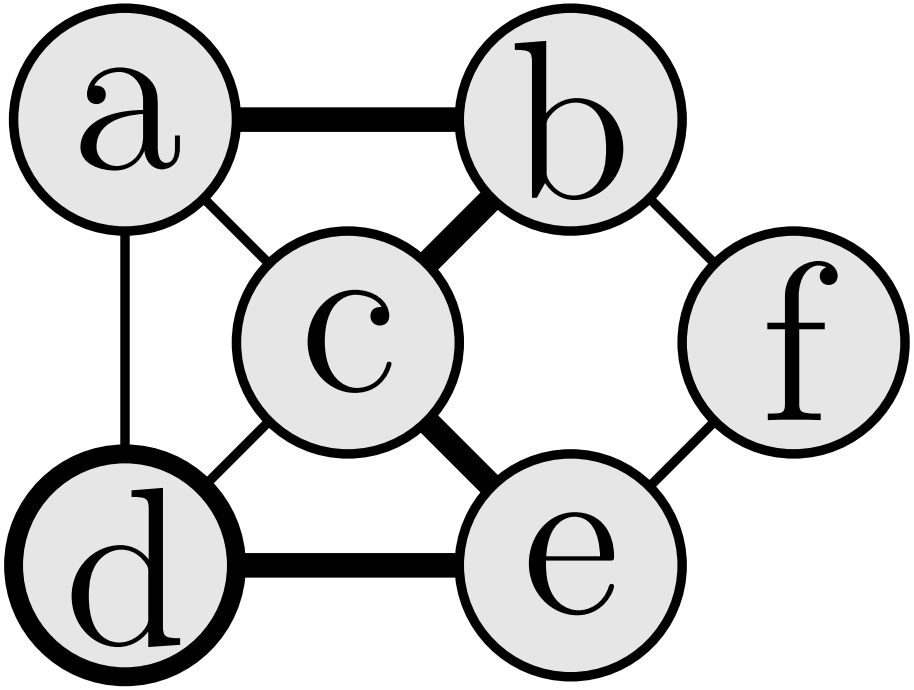
Hamiltonian Circuit Example



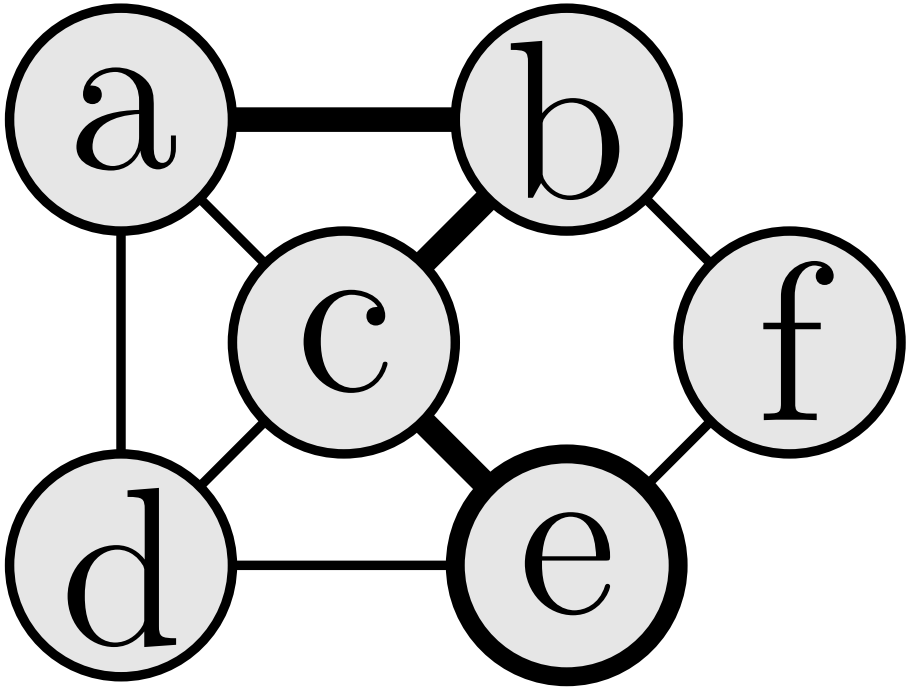
Hamiltonian Circuit Example



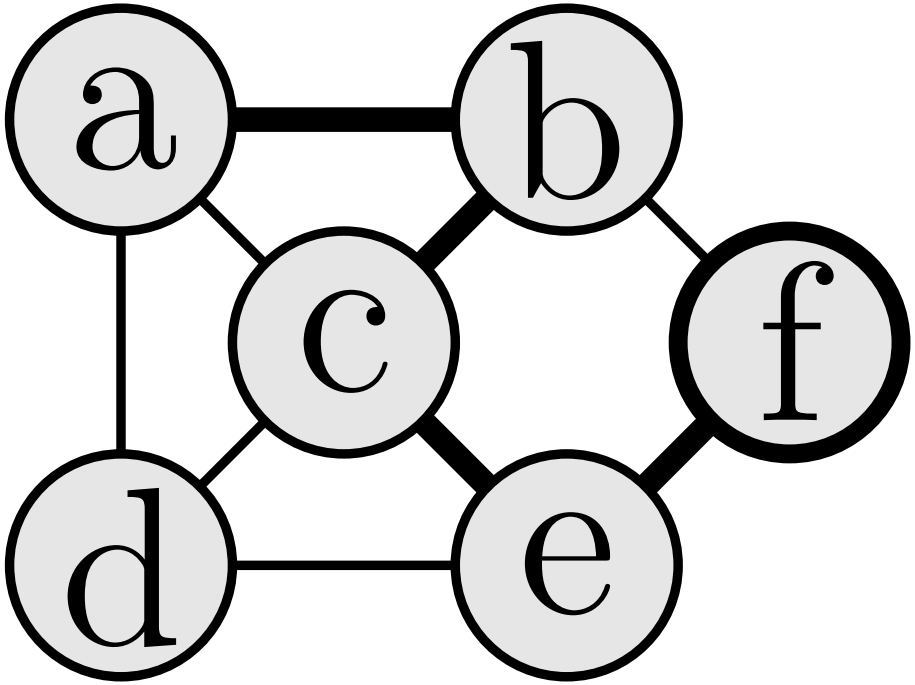
Hamiltonian Circuit Example



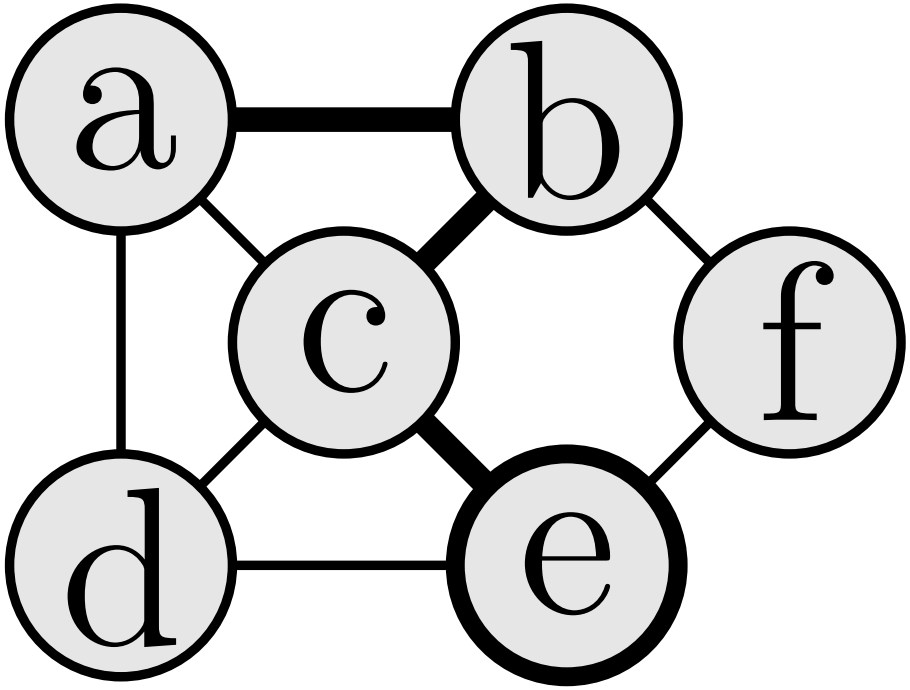
Hamiltonian Circuit Example



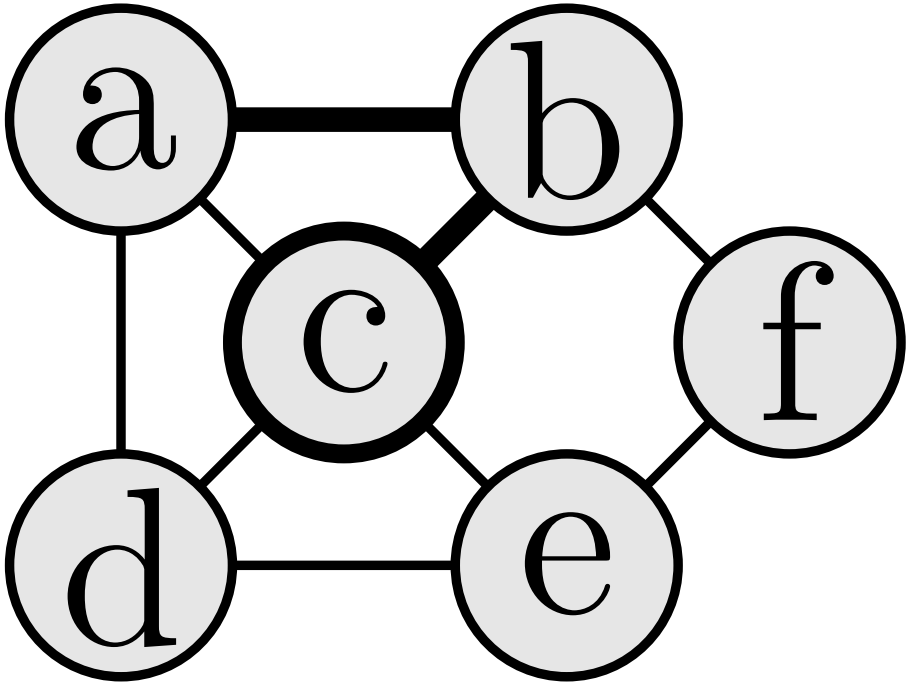
Hamiltonian Circuit Example



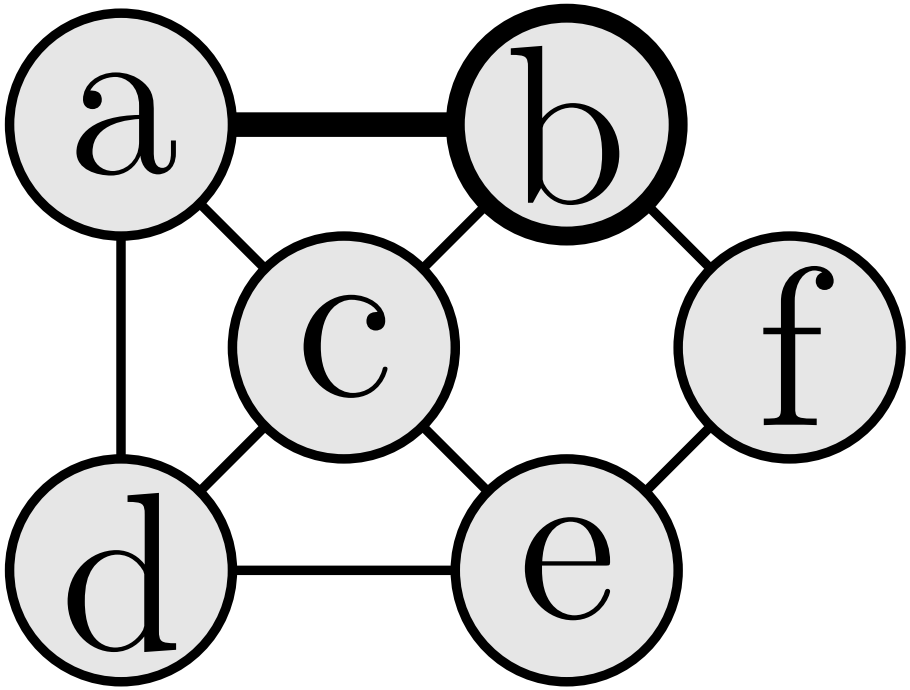
Hamiltonian Circuit Example



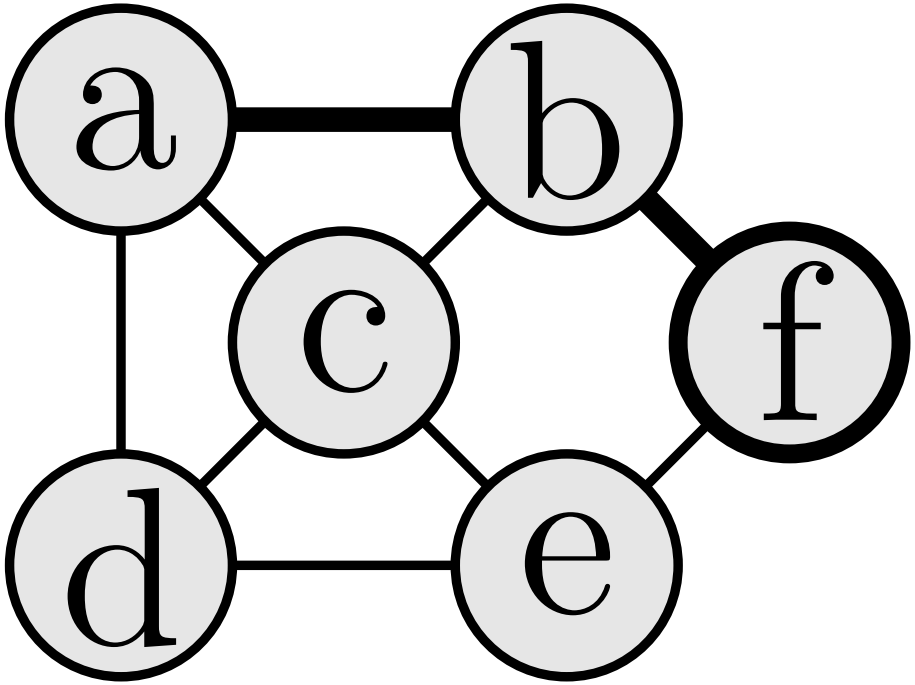
Hamiltonian Circuit Example



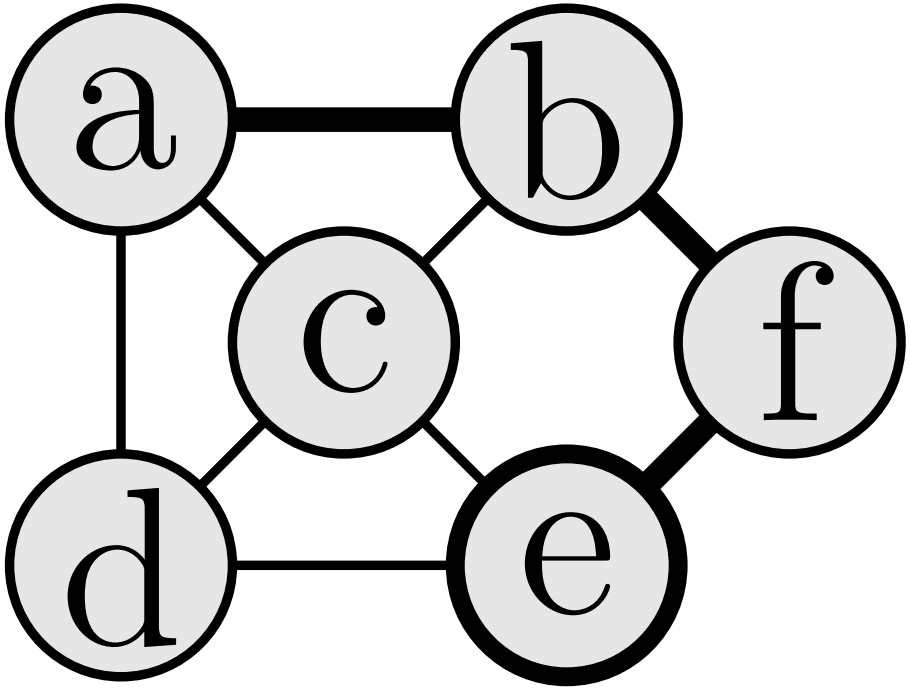
Hamiltonian Circuit Example



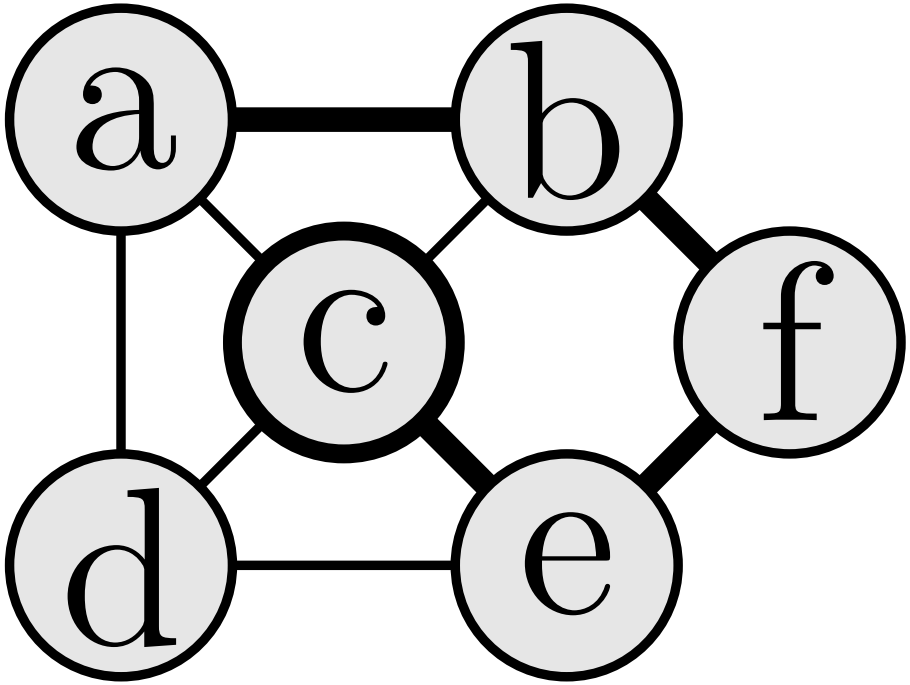
Hamiltonian Circuit Example



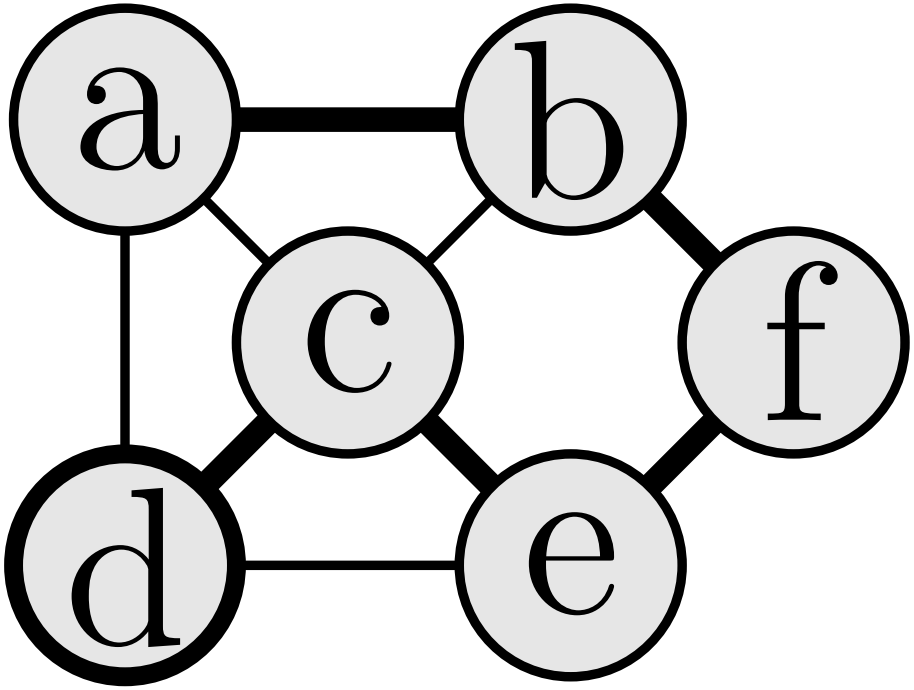
Hamiltonian Circuit Example



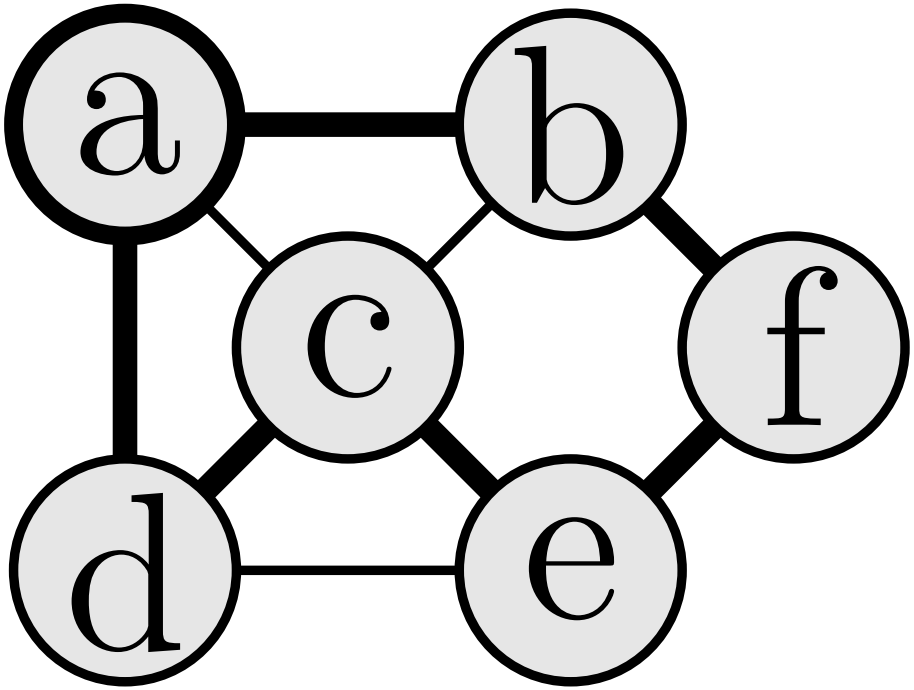
Hamiltonian Circuit Example



Hamiltonian Circuit Example



Hamiltonian Circuit Example



Backtracking

- Backtracking is a standard algorithm for solving constraint problems with large search spaces
- It can take exponential amount of time, however with many constraints it will often find solutions relatively quickly
- A backtracking algorithm does not solve, for example, sudoku in the same way as a human
- We can often speed up backtracking by adding more constraints (although, this can make writing the program longer)

Backtracking

- Backtracking is a standard algorithm for solving constraint problems with large search spaces
- It can take exponential amount of time, however with many constraints it will often find solutions relatively quickly
- A backtracking algorithm does not solve, for example, sudoku in the same way as a human
- We can often speed up backtracking by adding more constraints (although, this can make writing the program longer)

Backtracking

- Backtracking is a standard algorithm for solving constraint problems with large search spaces
- It can take exponential amount of time, however with many constraints it will often find solutions relatively quickly
- A backtracking algorithm does not solve, for example, sudoku in the same way as a human
- We can often speed up backtracking by adding more constraints (although, this can make writing the program longer)

Backtracking

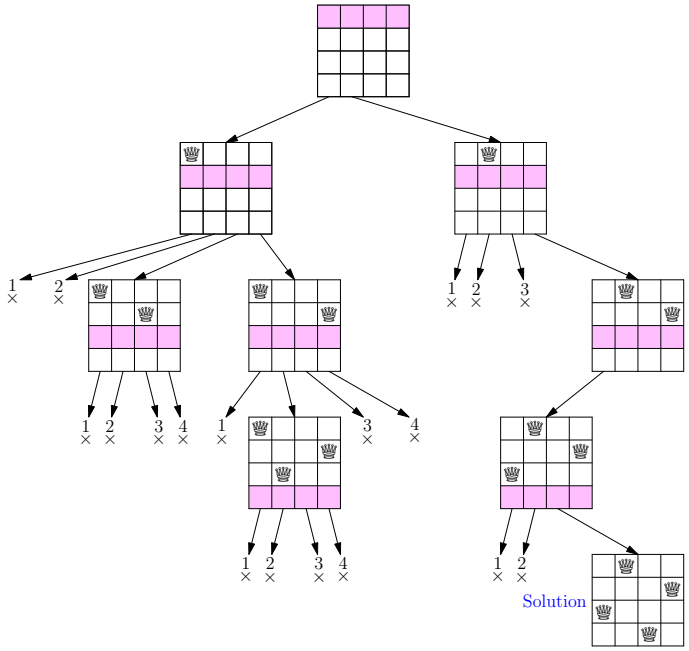
- Backtracking is a standard algorithm for solving constraint problems with large search spaces
- It can take exponential amount of time, however with many constraints it will often find solutions relatively quickly
- A backtracking algorithm does not solve, for example, sudoku in the same way as a human—it uses speed rather than brains
- We can often speed up backtracking by adding more constraints (although, this can make writing the program longer)

Backtracking

- Backtracking is a standard algorithm for solving constraint problems with large search spaces
- It can take exponential amount of time, however with many constraints it will often find solutions relatively quickly
- A backtracking algorithm does not solve, for example, sudoku in the same way as a human—it uses speed rather than brains
- We can often speed up backtracking by adding more constraints (although, this can make writing the program longer)

Outline

1. Search Trees
2. Backtracking
3. **Branch and Bound**
4. Search in AI



Optimisation Problems

- In many optimisation problems (TSP, Graph-colouring, etc.) we again have a huge search space ($n!$, k^n)
- However, we don't have hard constraints
- If we are interested in finding the optimal then we can use the cost as a constraint

  any partial solution has to have a lower cost than the best solution we have found so far

- This allows us to develop a backtracking strategy known as branch and bound

Optimisation Problems

- In many optimisation problems (TSP, Graph-colouring, etc.) we again have a huge search space ($n!$, k^n)
- However, we don't have hard constraints
- If we are interested in finding the optimal then we can use the cost as a constraint

  any partial solution has to have a lower cost than the best solution we have found so far

- This allows us to develop a backtracking strategy known as branch and bound

Optimisation Problems

- In many optimisation problems (TSP, Graph-colouring, etc.) we again have a huge search space ($n!$, k^n)
- However, we don't have hard constraints
- If we are interested in finding the optimal then we can use the cost as a constraint

  any partial solution has to have a lower cost than the best solution we have found so far

- This allows us to develop a backtracking strategy known as branch and bound

Optimisation Problems

- In many optimisation problems (TSP, Graph-colouring, etc.) we again have a huge search space ($n!$, k^n)
- However, we don't have hard constraints
- If we are interested in finding the optimal then we can use the cost as a constraint

  any partial solution has to have a lower cost than the best solution we have found so far

- This allows us to develop a backtracking strategy known as branch and bound

Branch and Bound

- Branch and bound is used on optimisation problems where efficient strategies just don't work
- It beats exhaustive enumeration by eliminate many possible solutions without having to enumerate them all
- Branch and bound can be slow as the constraints aren't necessarily very strong
- By working harder we can sometimes strengthen the constraints thus eliminating much of the search space
- This strategy works quite well on smallish problems, but usually fails on large problems

Branch and Bound

- Branch and bound is used on optimisation problems where efficient strategies just don't work
- It beats exhaustive enumeration by eliminate many possible solutions without having to enumerate them all
- Branch and bound can be slow as the constraints aren't necessarily very strong
- By working harder we can sometimes strengthen the constraints thus eliminating much of the search space
- This strategy works quite well on smallish problems, but usually fails on large problems

Branch and Bound

- Branch and bound is used on optimisation problems where efficient strategies just don't work
- It beats exhaustive enumeration by eliminate many possible solutions without having to enumerate them all
- Branch and bound can be slow as the constraints aren't necessarily very strong
- By working harder we can sometimes strengthen the constraints thus eliminating much of the search space
- This strategy works quite well on smallish problems, but usually fails on large problems

Branch and Bound

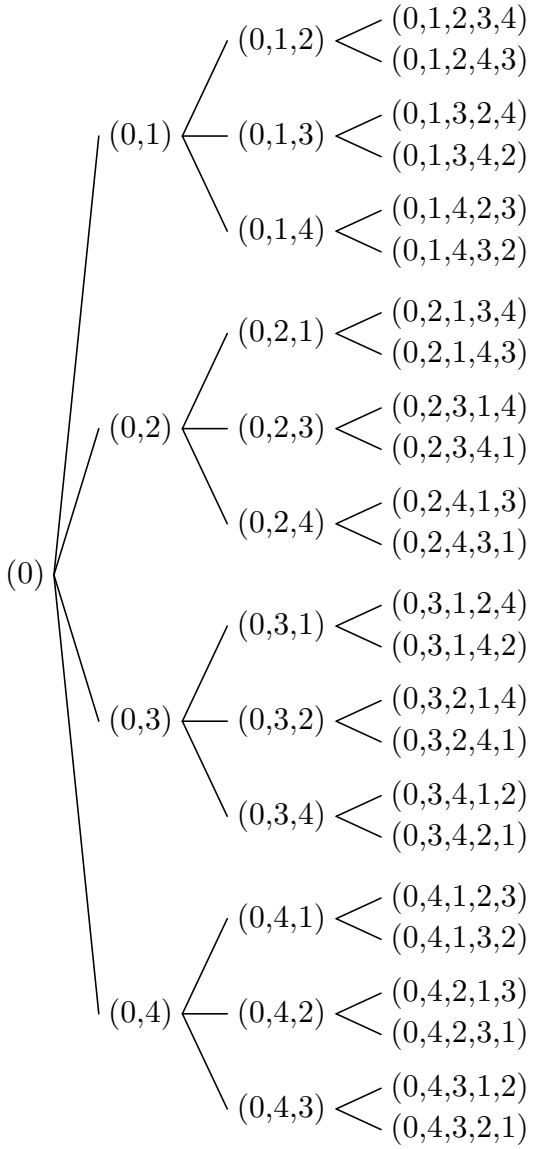
- Branch and bound is used on optimisation problems where efficient strategies just don't work
- It beats exhaustive enumeration by eliminate many possible solutions without having to enumerate them all
- Branch and bound can be slow as the constraints aren't necessarily very strong
- By working harder we can sometimes strengthen the constraints thus eliminating much of the search space
- This strategy works quite well on smallish problems, but usually fails on large problems

Branch and Bound

- Branch and bound is used on optimisation problems where efficient strategies just don't work
- It beats exhaustive enumeration by eliminate many possible solutions without having to enumerate them all
- Branch and bound can be slow as the constraints aren't necessarily very strong
- By working harder we can sometimes strengthen the constraints thus eliminating much of the search space
- This strategy works quite well on smallish problems, but usually fails on large problems

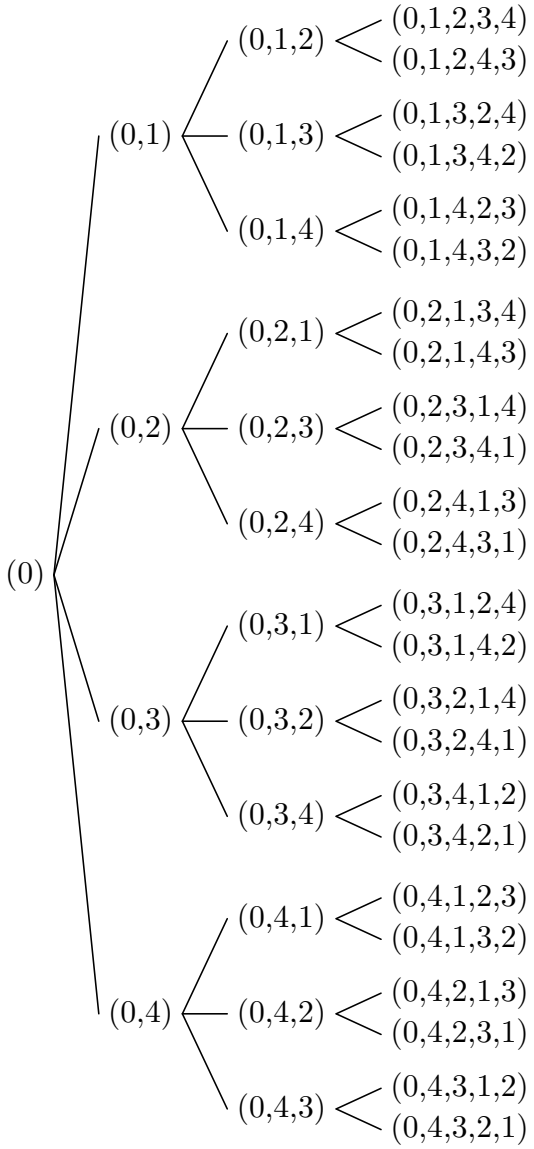
Cutting the Search Tree

- We can think of exact enumeration as exploring a giant search tree
- If we know a partial solution is worse than our bound we cut the search tree
- The earlier we cut the tree the more we can save



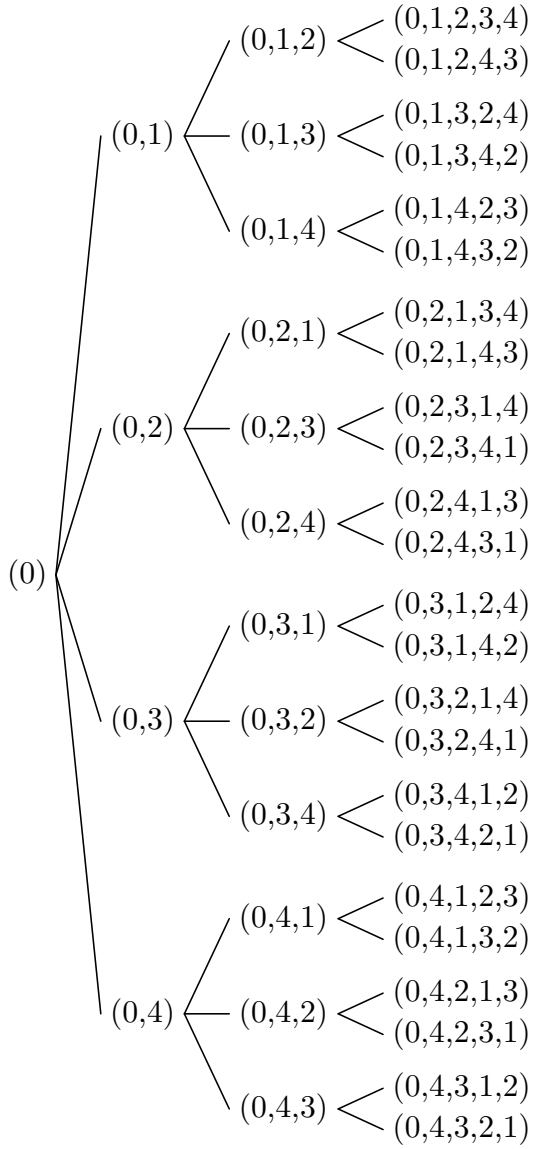
Cutting the Search Tree

- We can think of exact enumeration as exploring a giant search tree
- If we know a partial solution is worse than our bound we cut the search tree
- The earlier we cut the tree the more we can save

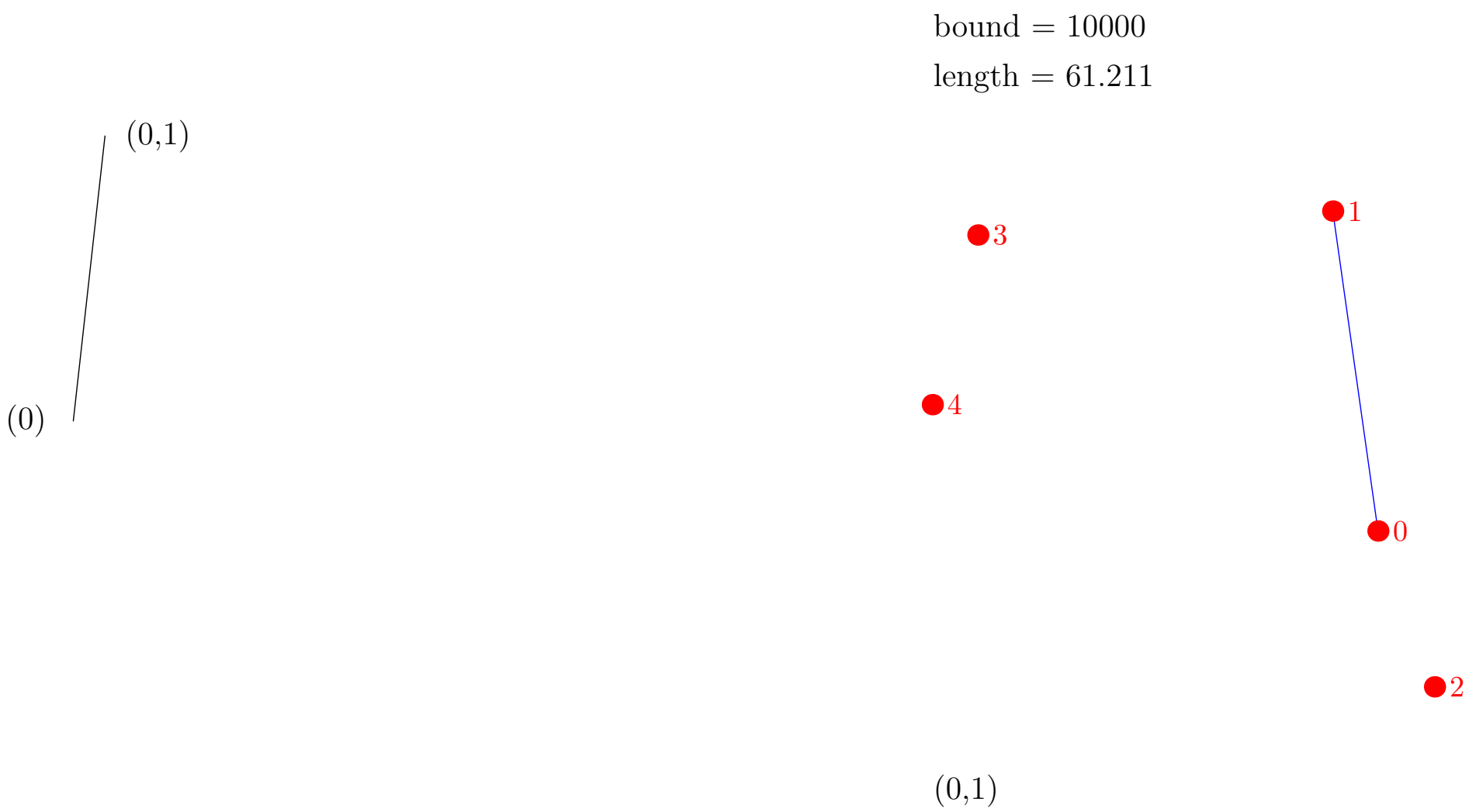


Cutting the Search Tree

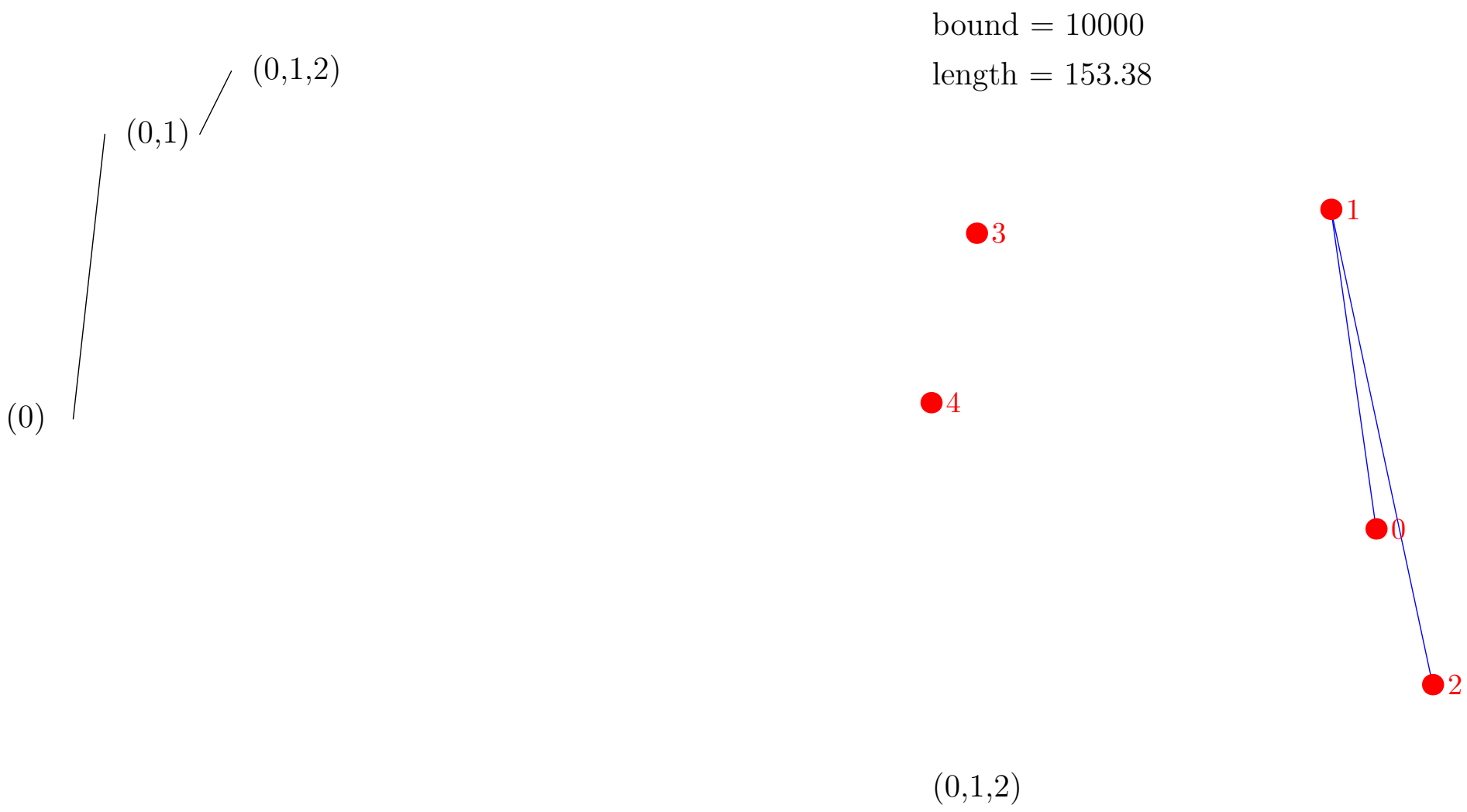
- We can think of exact enumeration as exploring a giant search tree
- If we know a partial solution is worse than our bound we cut the search tree
- The earlier we cut the tree the more we can save



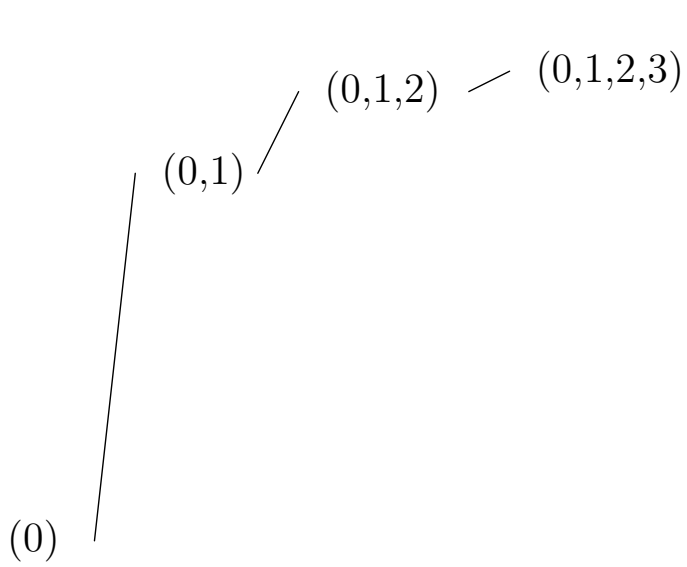
Branch and Bound in Action



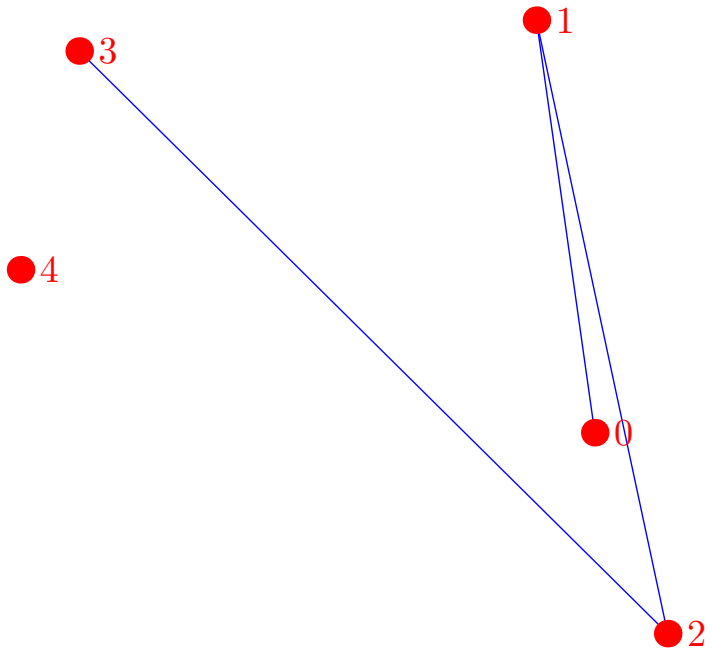
Branch and Bound in Action



Branch and Bound in Action

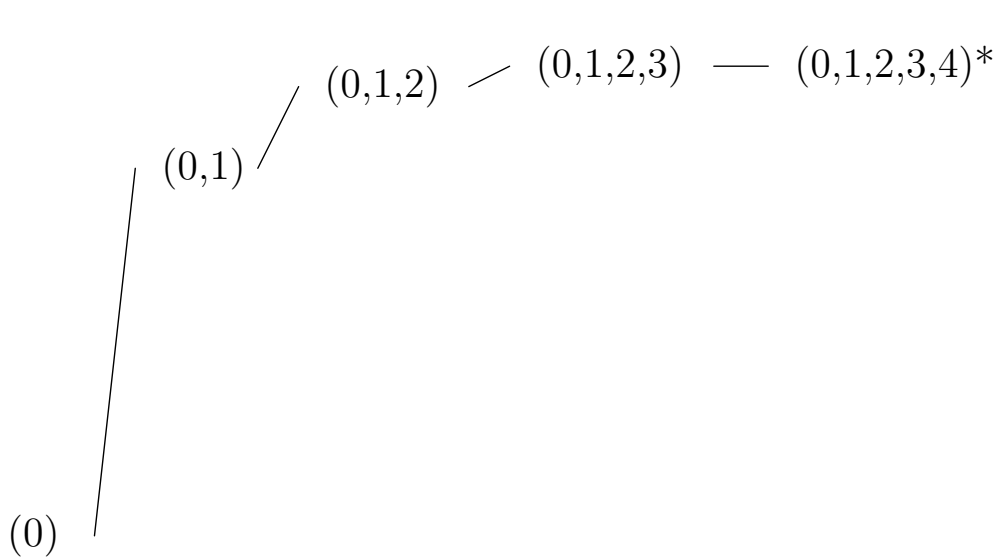


bound = 10000
length = 275.05



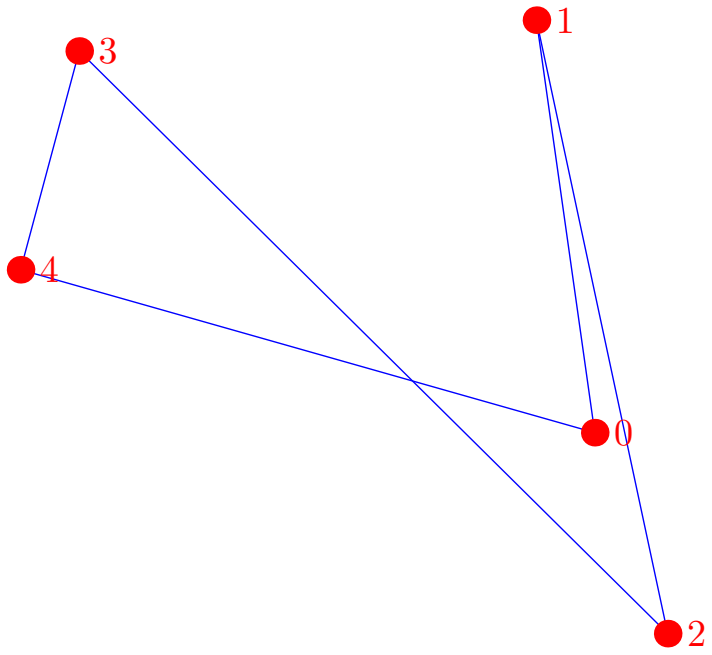
(0,1,2,3)

Branch and Bound in Action



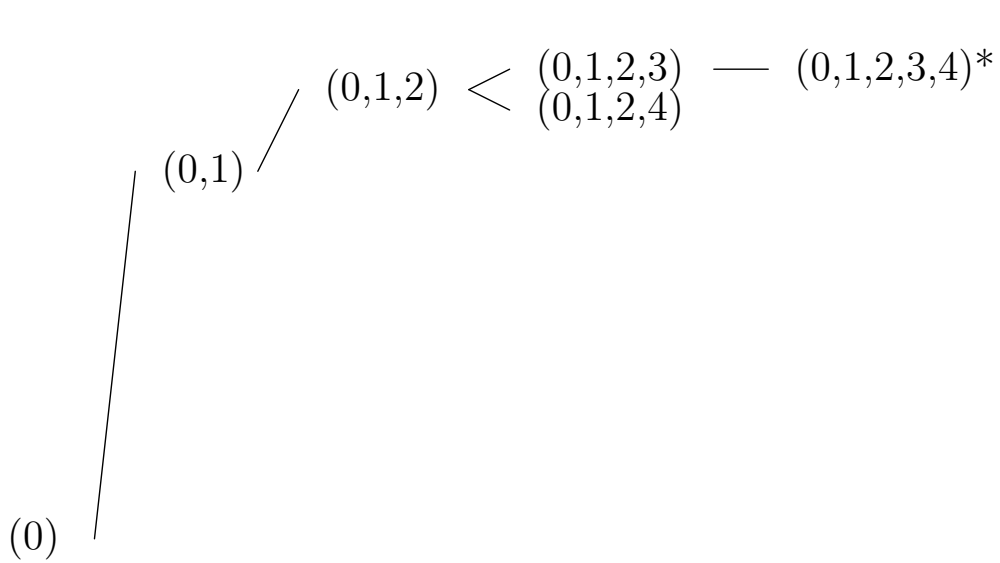
bound = 396.02

length = 396.02



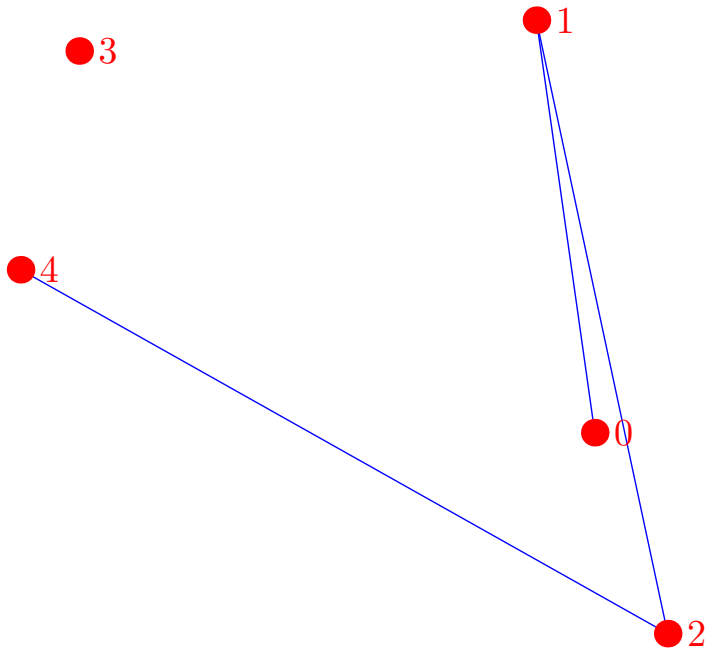
(0,1,2,3,4)

Branch and Bound in Action



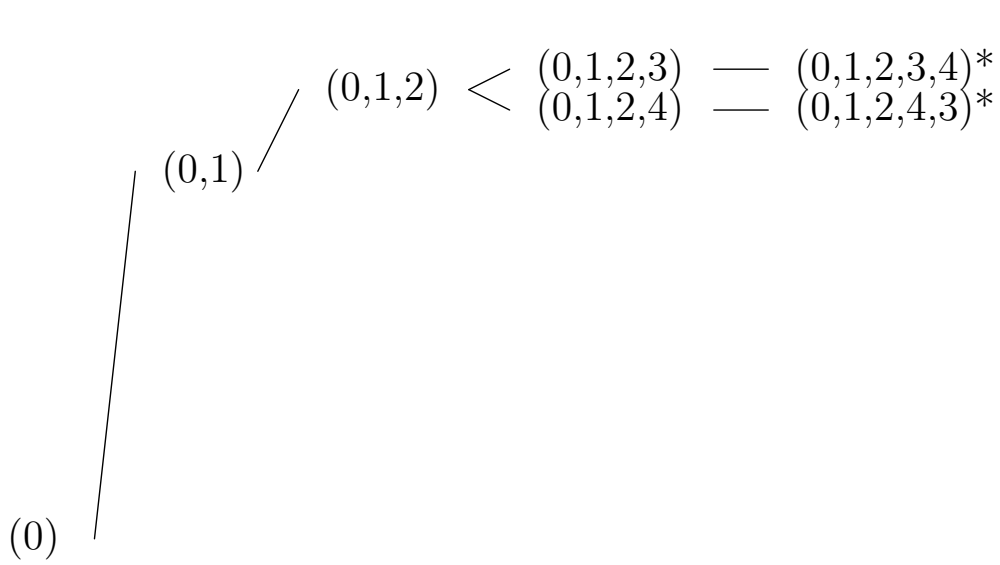
bound = 396.02

length = 262.46



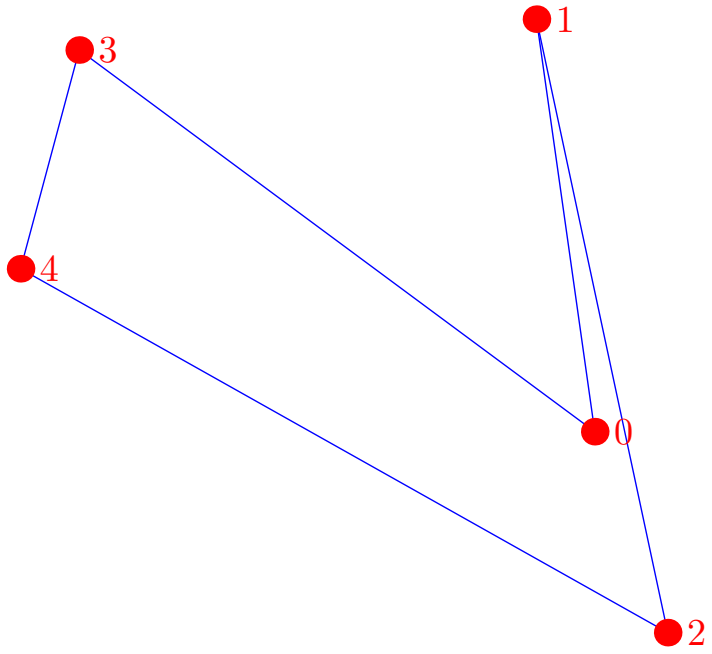
(0,1,2,4)

Branch and Bound in Action



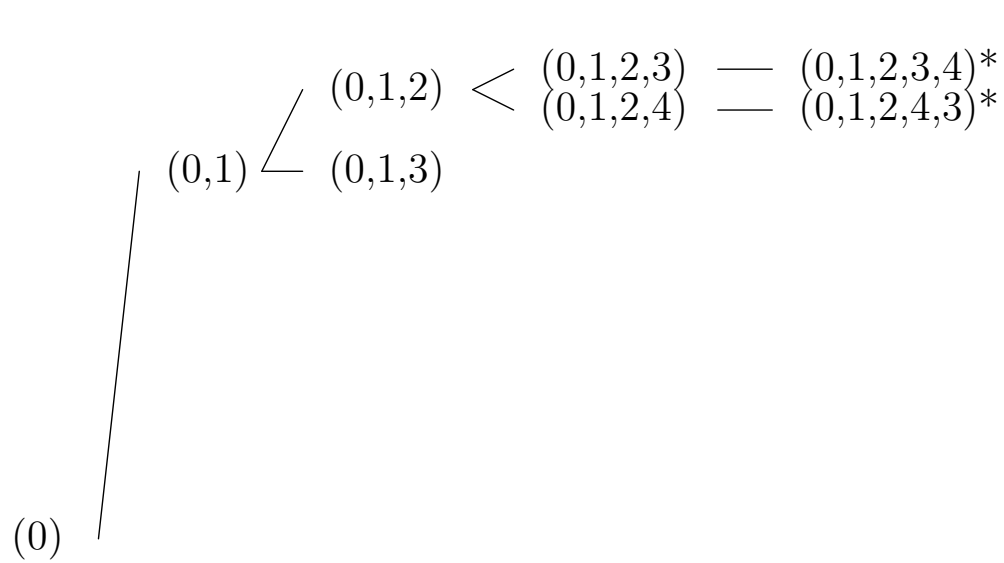
bound = 389.98

length = 389.98



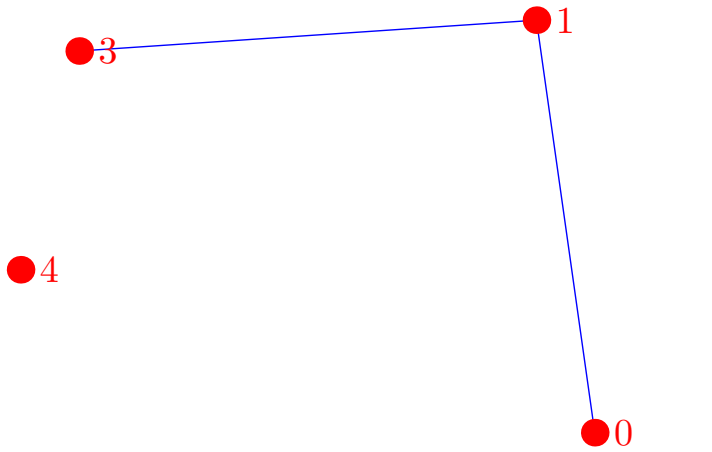
(0,1,2,4,3)

Branch and Bound in Action



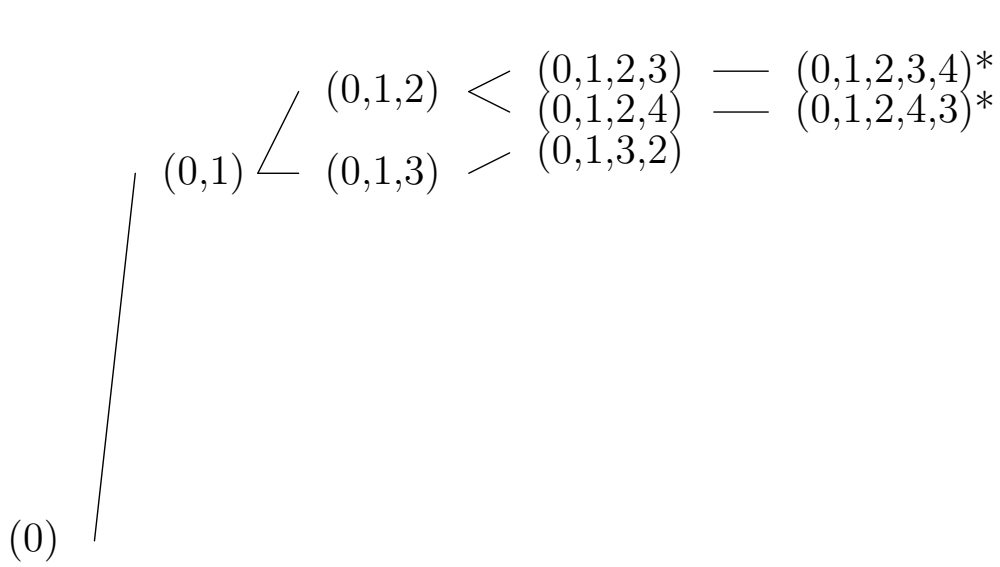
bound = 389.98

length = 128.54



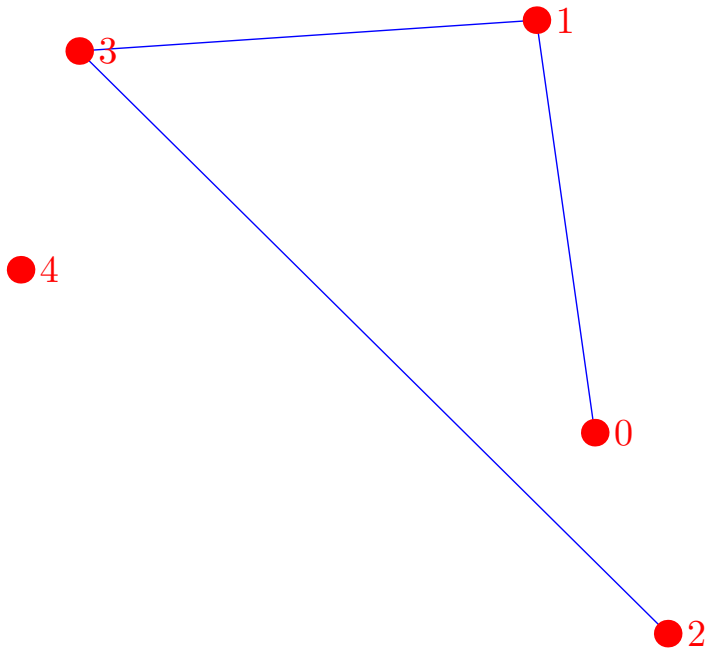
(0,1,3)

Branch and Bound in Action



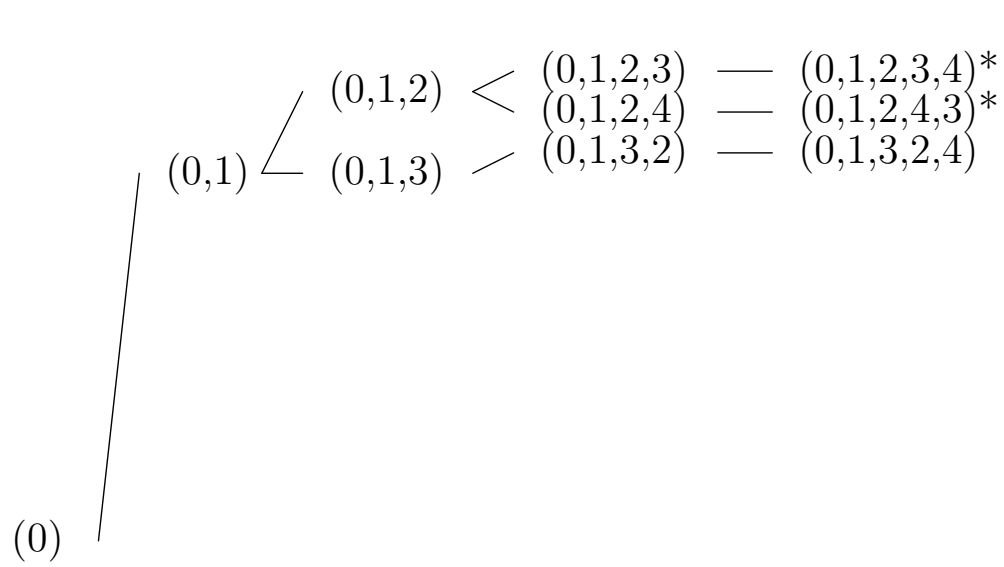
bound = 389.98

length = 250.21



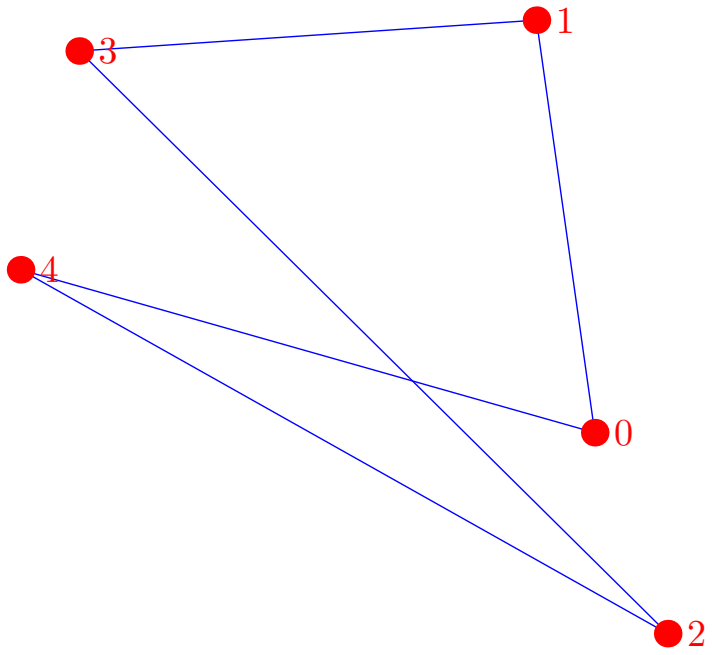
(0,1,3,2)

Branch and Bound in Action



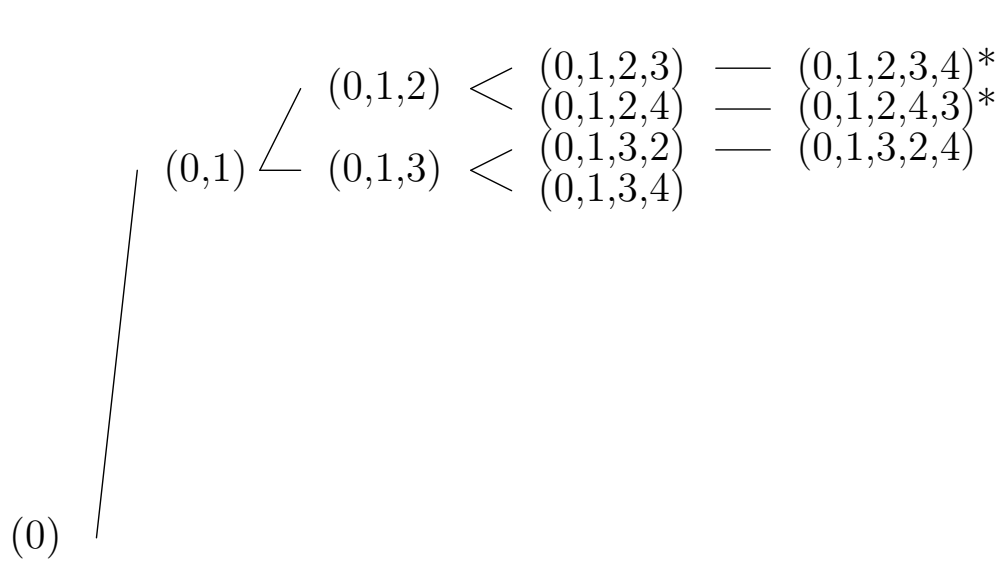
bound = 389.98

length = 446.99



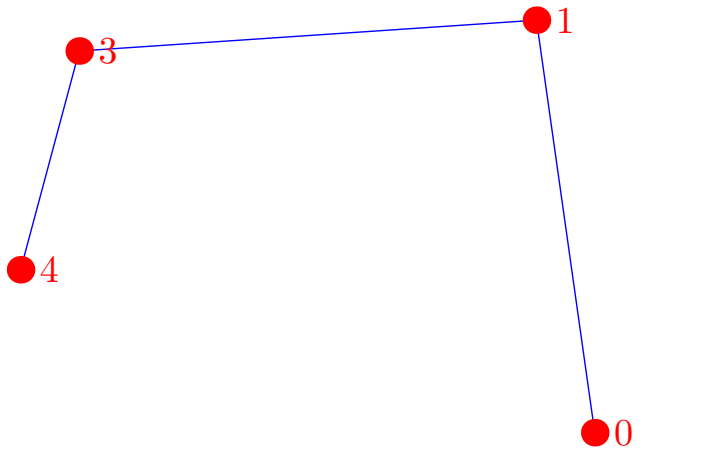
(0,1,3,2,4)

Branch and Bound in Action



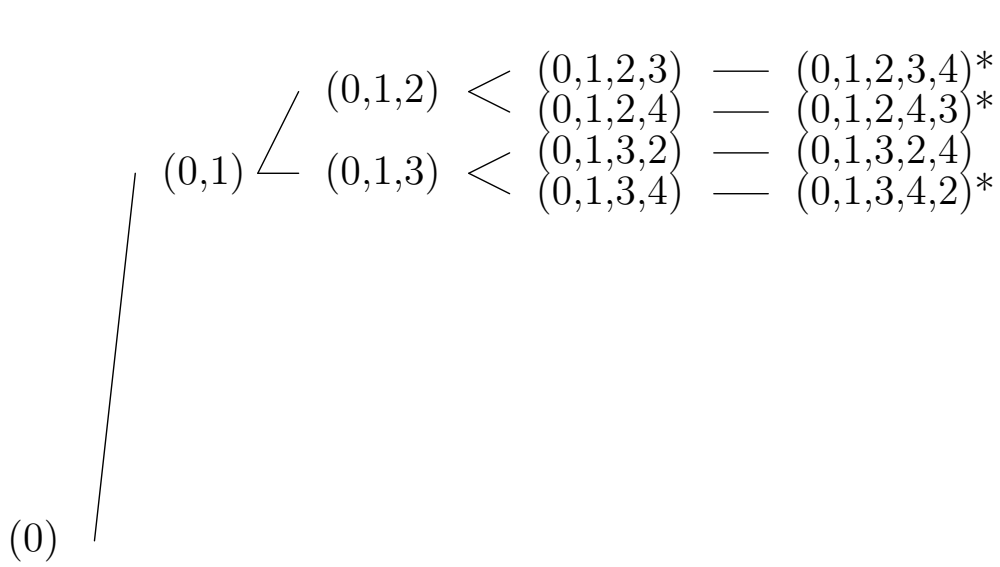
bound = 389.98

length = 161.82



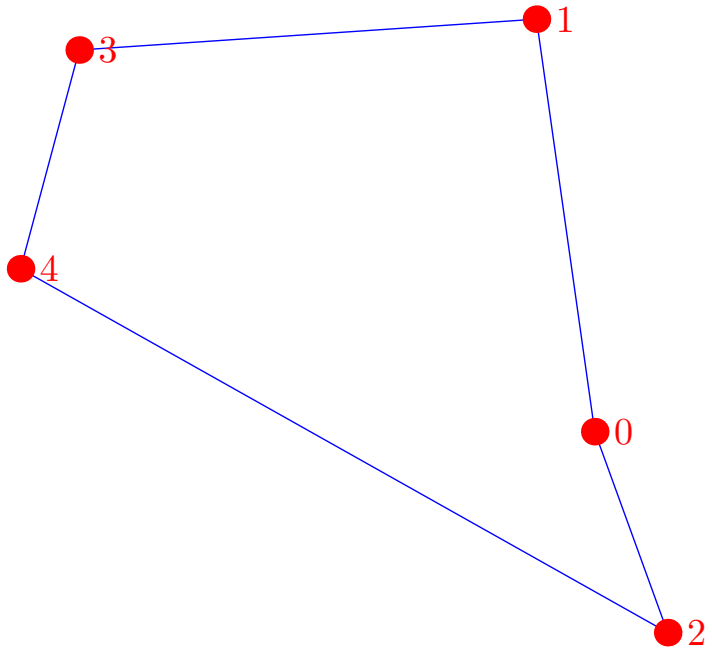
(0,1,3,4)

Branch and Bound in Action



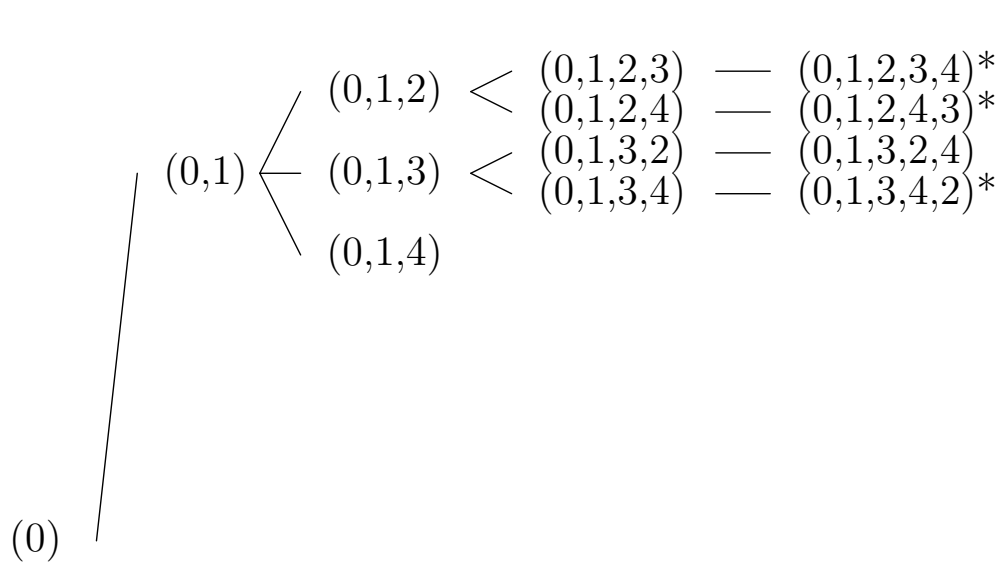
bound = 302.31

length = 302.31



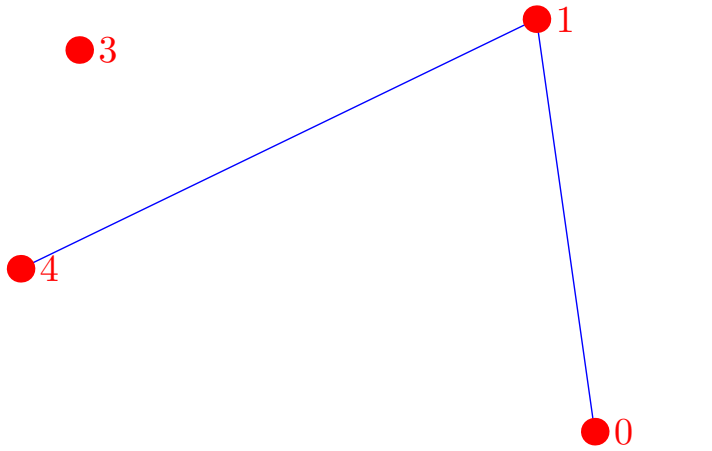
(0,1,3,4,2)

Branch and Bound in Action



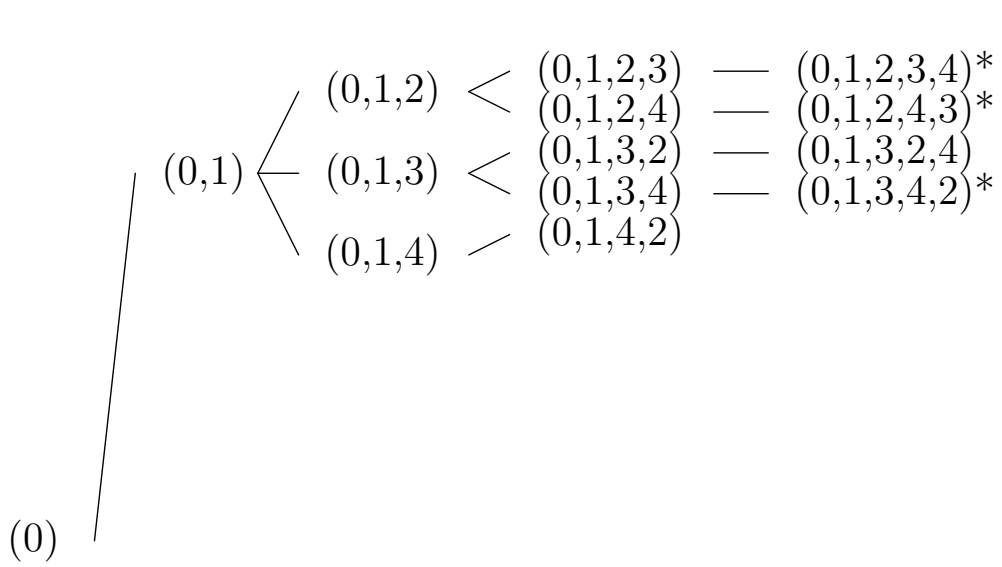
bound = 302.31

length = 145.41



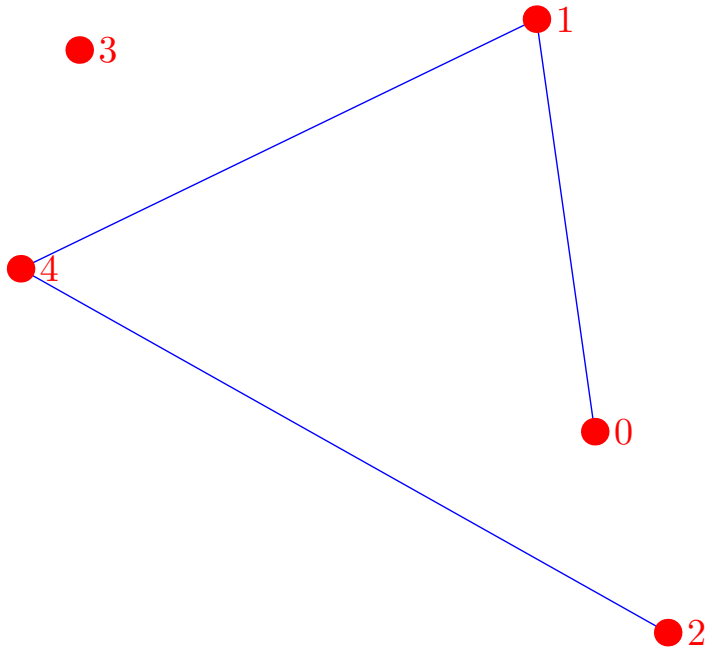
(0,1,4)

Branch and Bound in Action



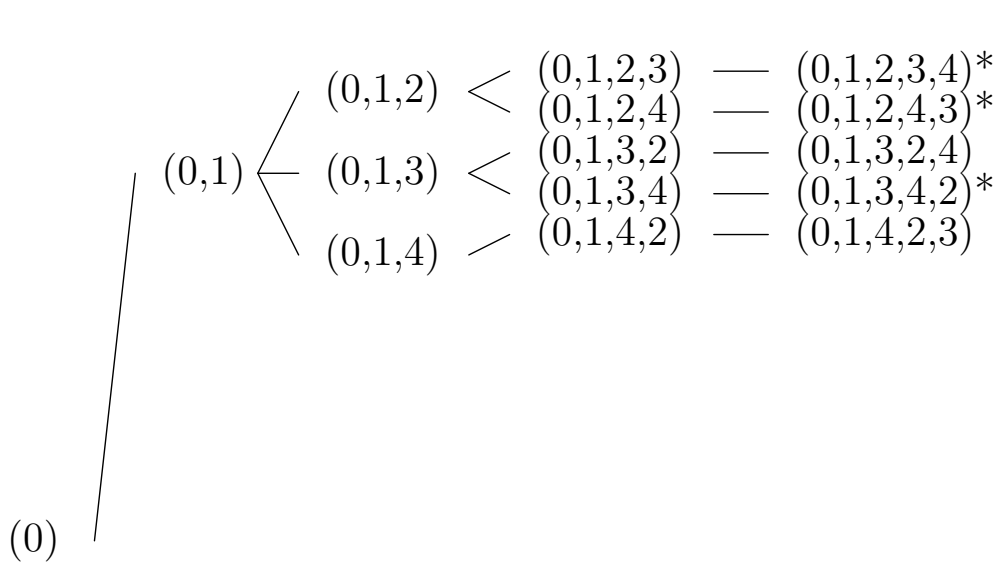
bound = 302.31

length = 254.49



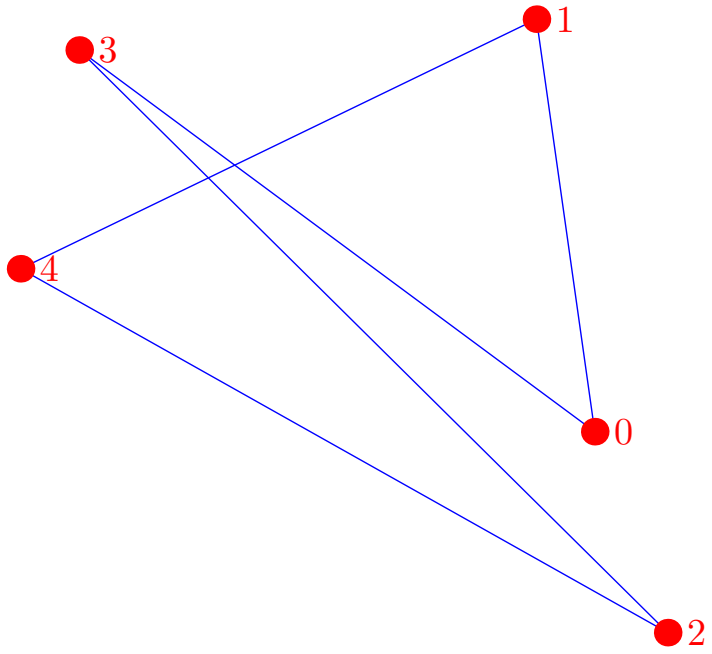
(0,1,4,2)

Branch and Bound in Action



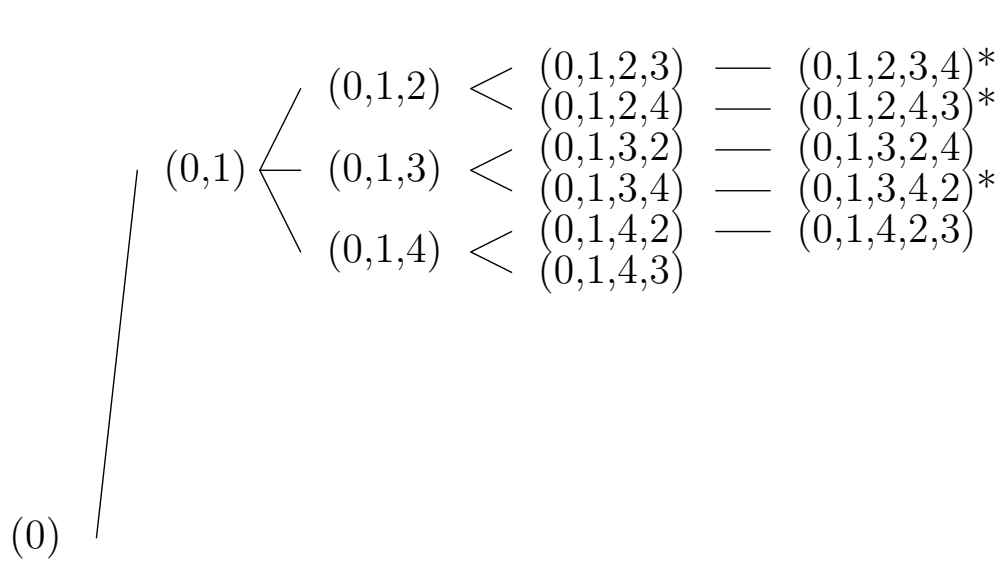
bound = 302.31

length = 470.41



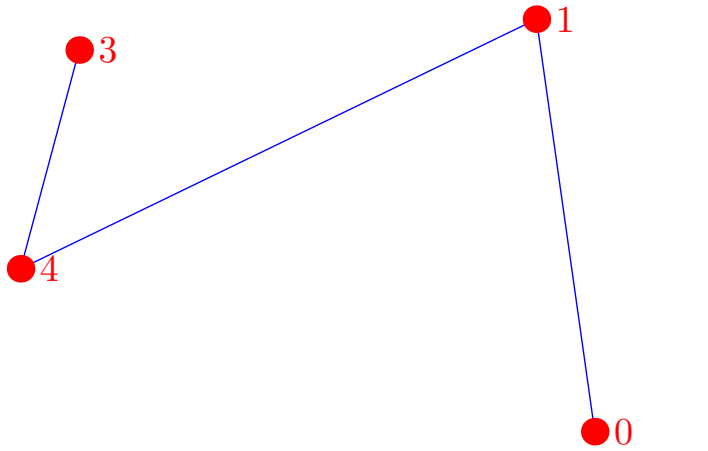
(0,1,4,2,3)

Branch and Bound in Action



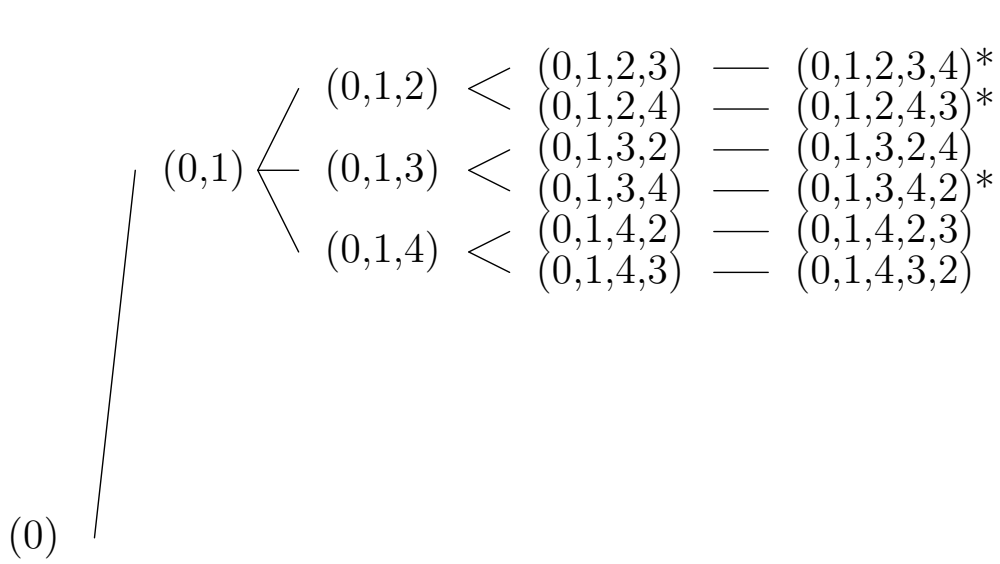
bound = 302.31

length = 178.69



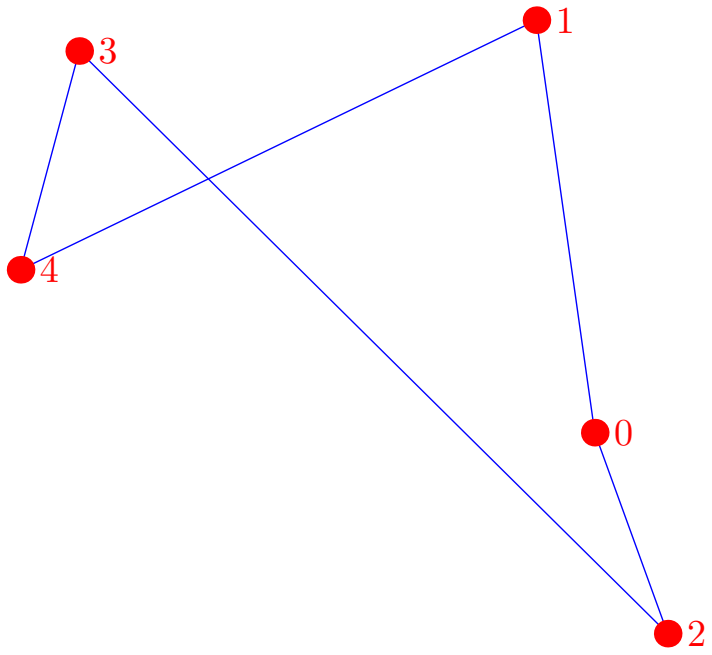
(0,1,4,3)

Branch and Bound in Action



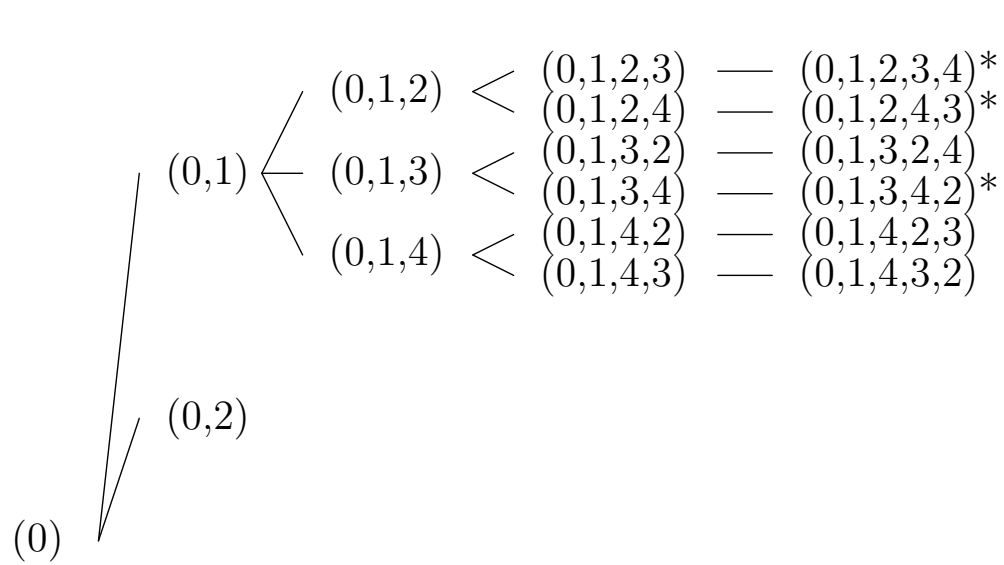
bound = 302.31

length = 331.77



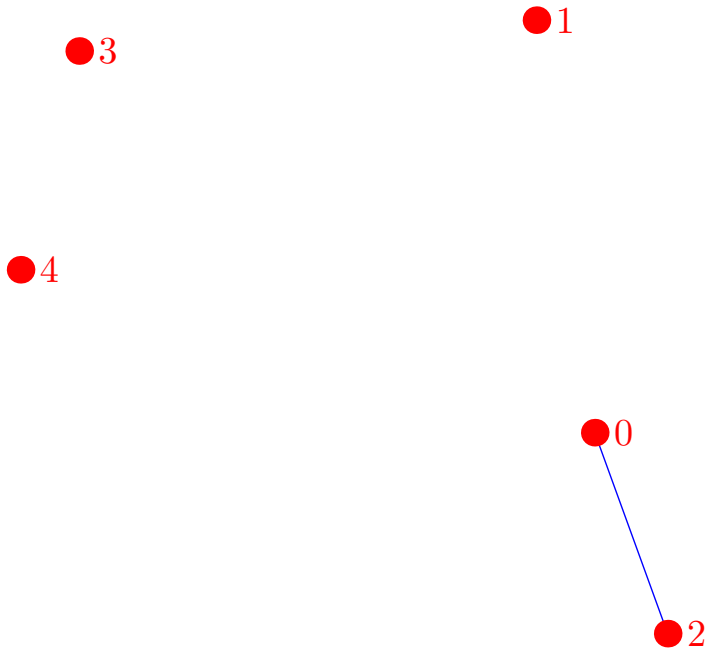
(0,1,4,3,2)

Branch and Bound in Action



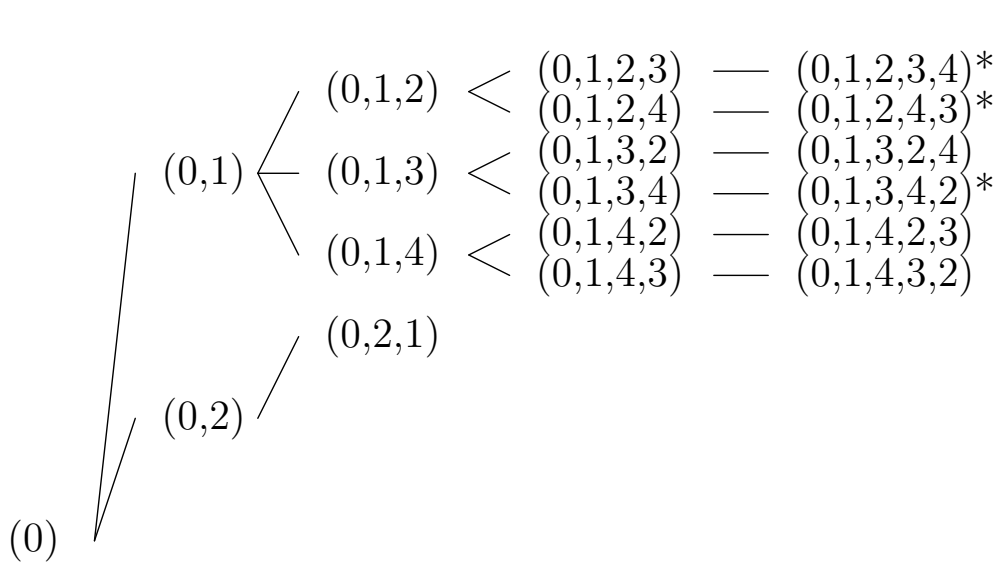
bound = 302.31

length = 31.41



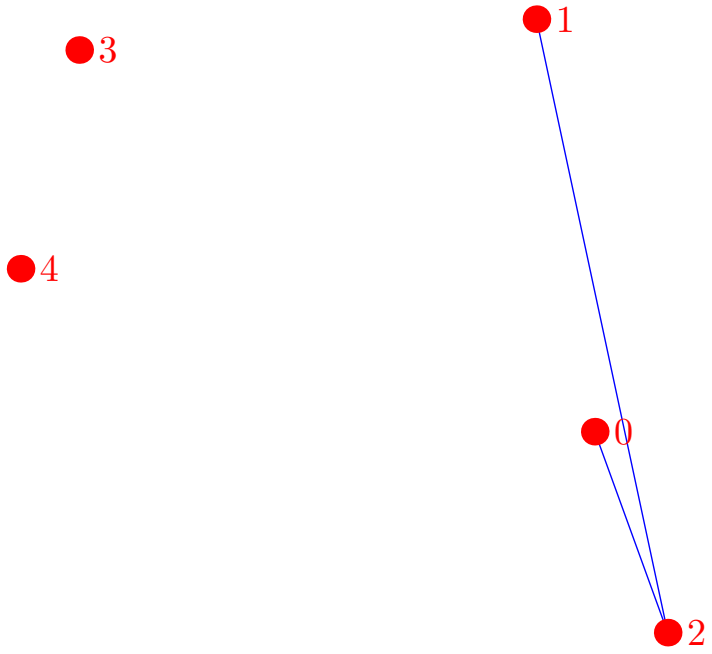
(0,2)

Branch and Bound in Action



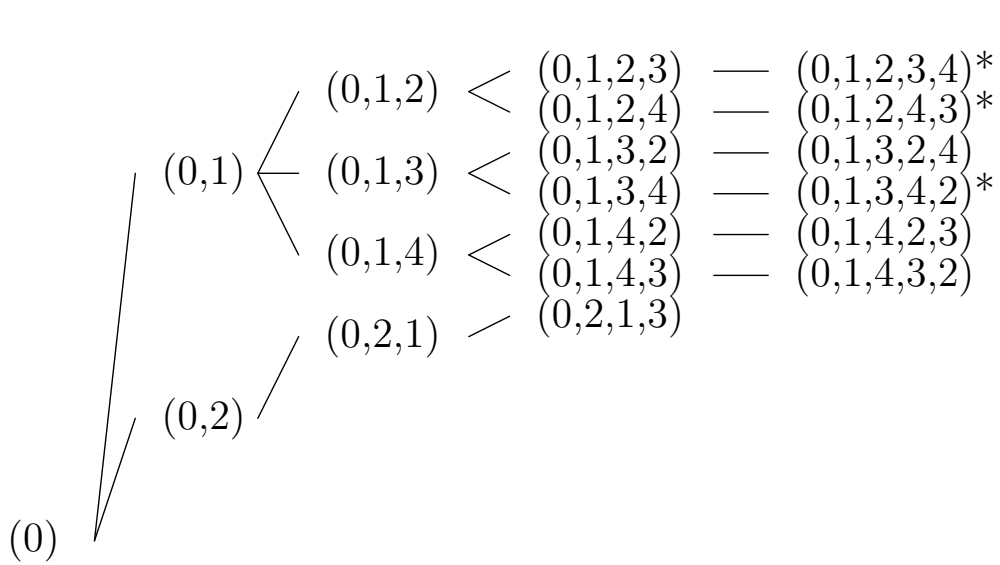
bound = 302.31

length = 123.58



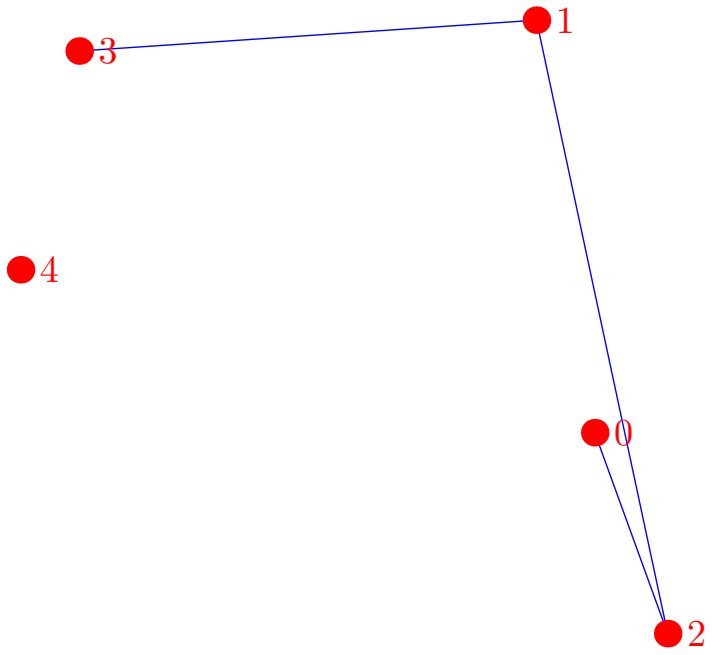
(0,2,1)

Branch and Bound in Action



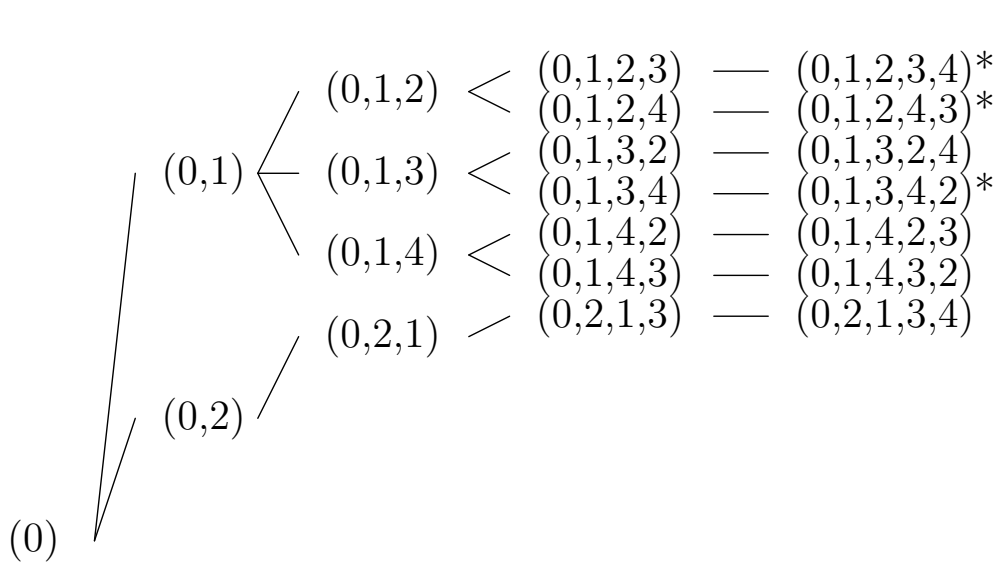
bound = 302.31

length = 190.92



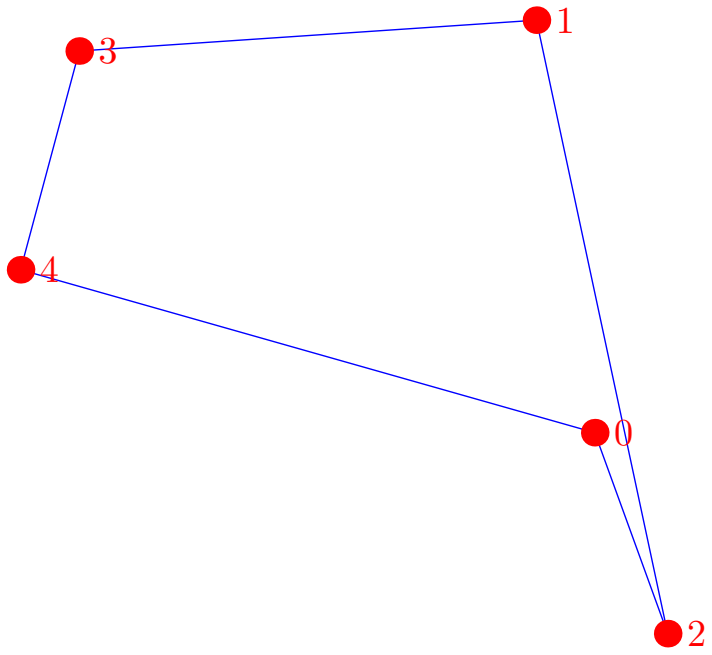
(0,2,1,3)

Branch and Bound in Action



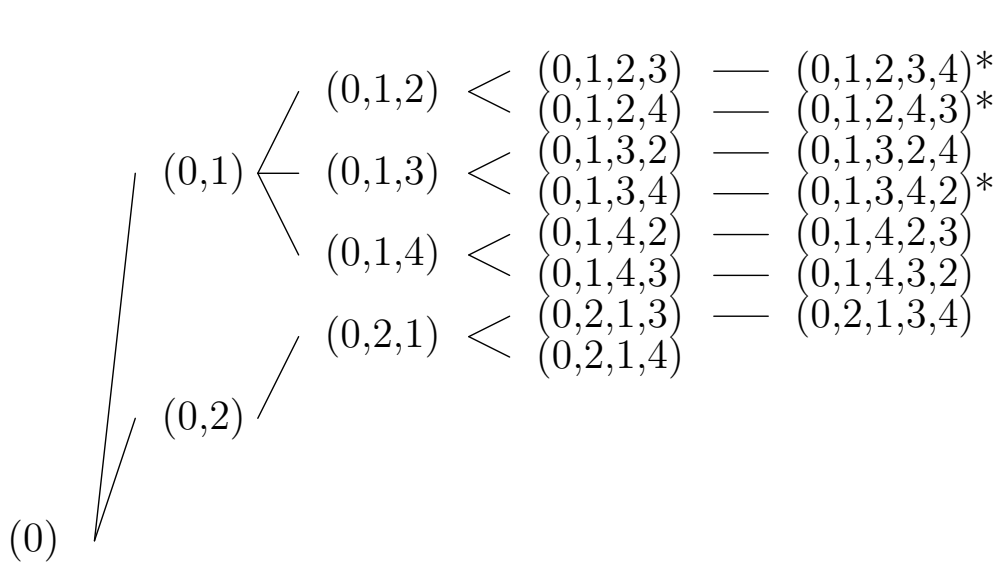
bound = 302.31

length = 311.88



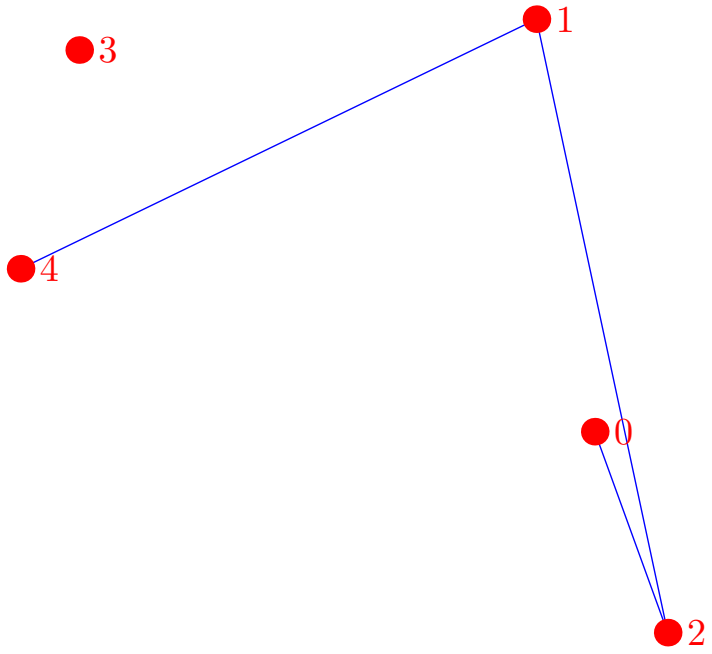
(0,2,1,3,4)

Branch and Bound in Action



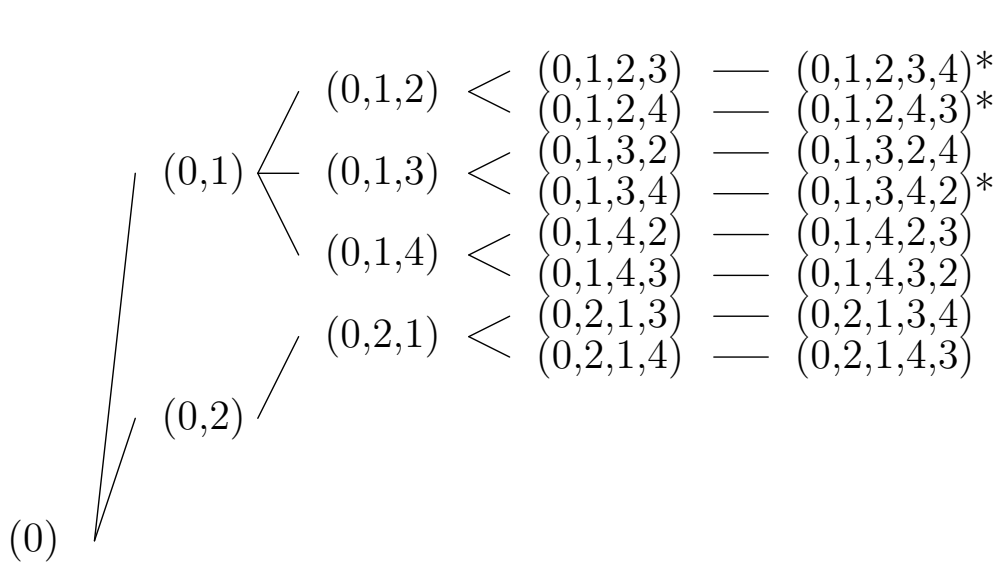
bound = 302.31

length = 207.79



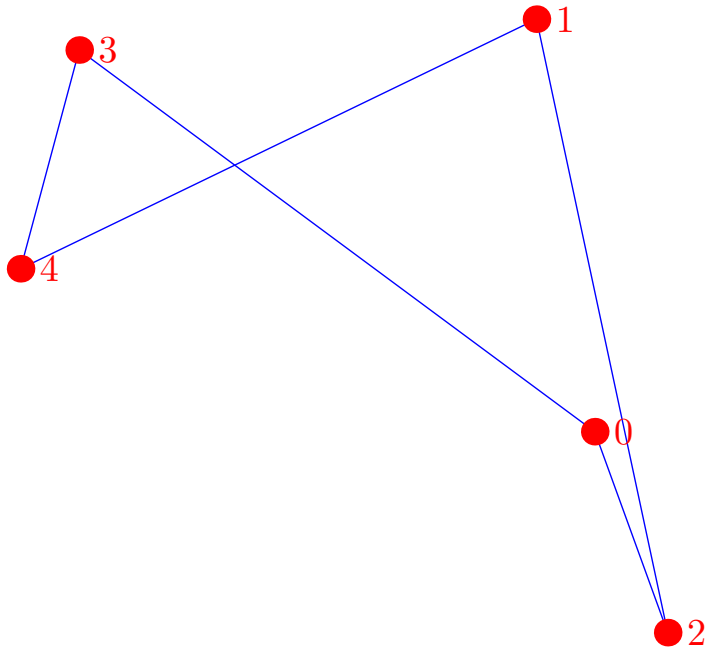
(0,2,1,4)

Branch and Bound in Action



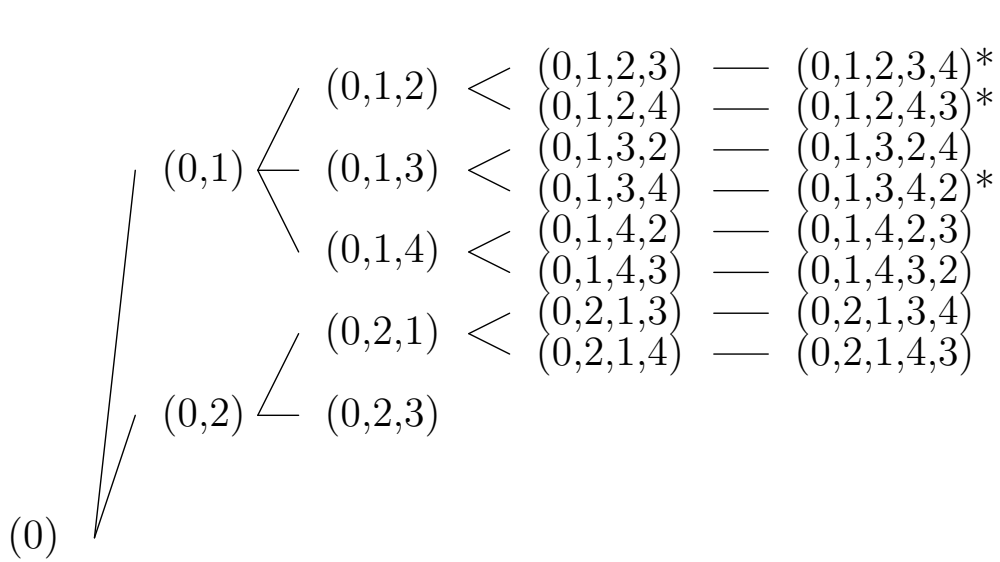
bound = 302.31

length = 335.3



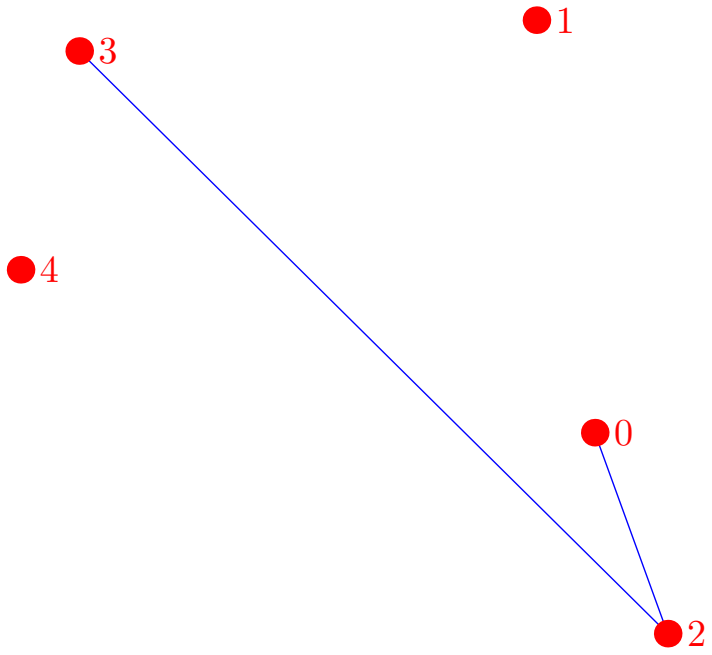
(0,2,1,4,3)

Branch and Bound in Action



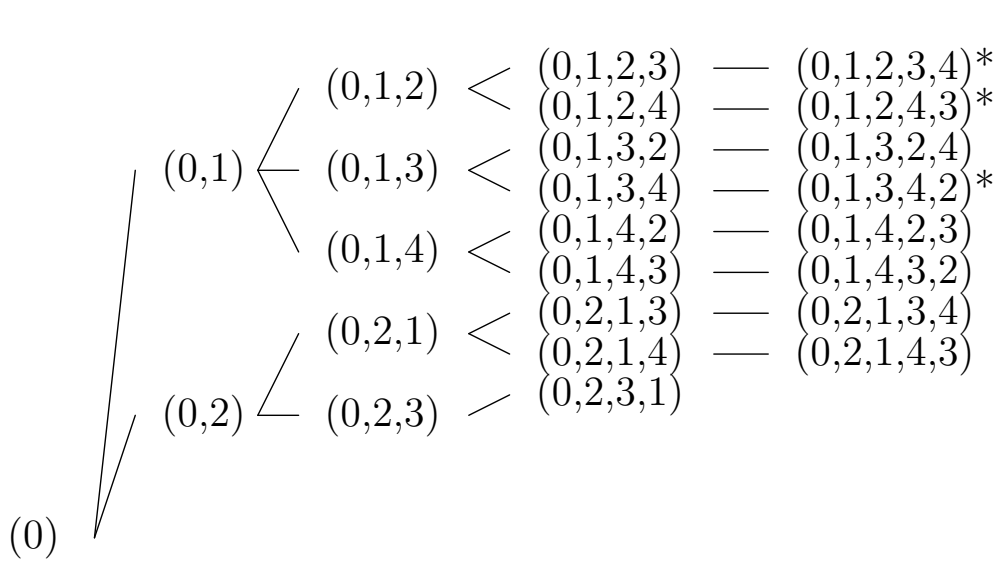
bound = 302.31

length = 153.08



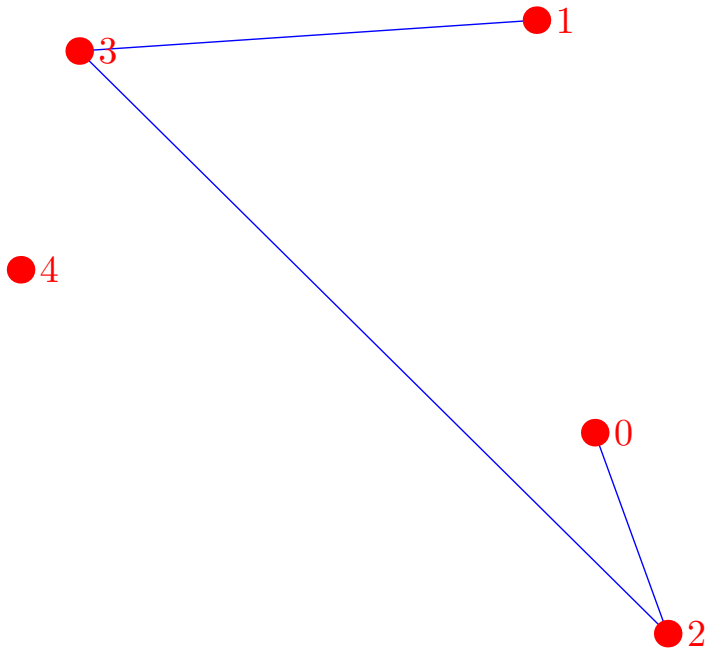
(0,2,3)

Branch and Bound in Action



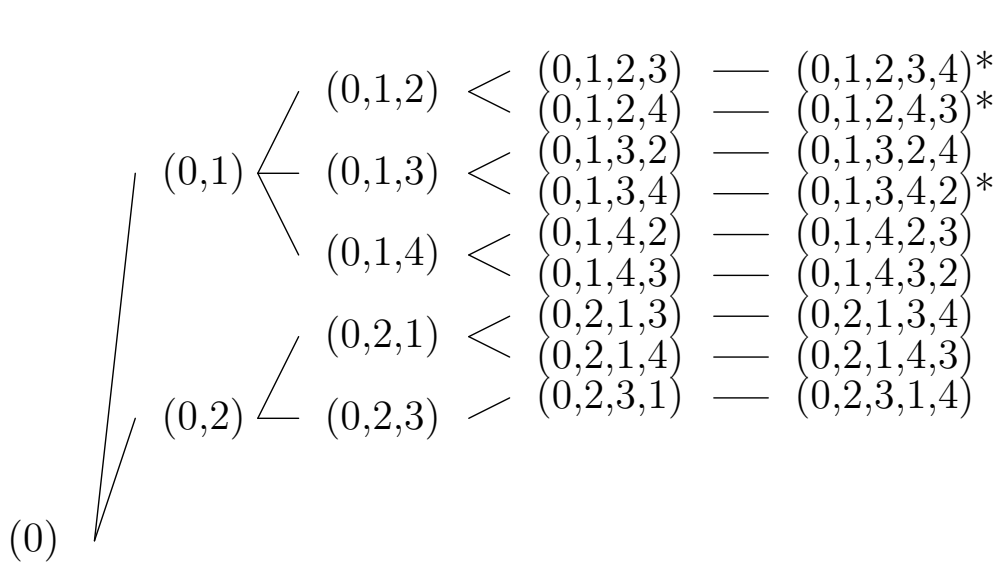
bound = 302.31

length = 220.41



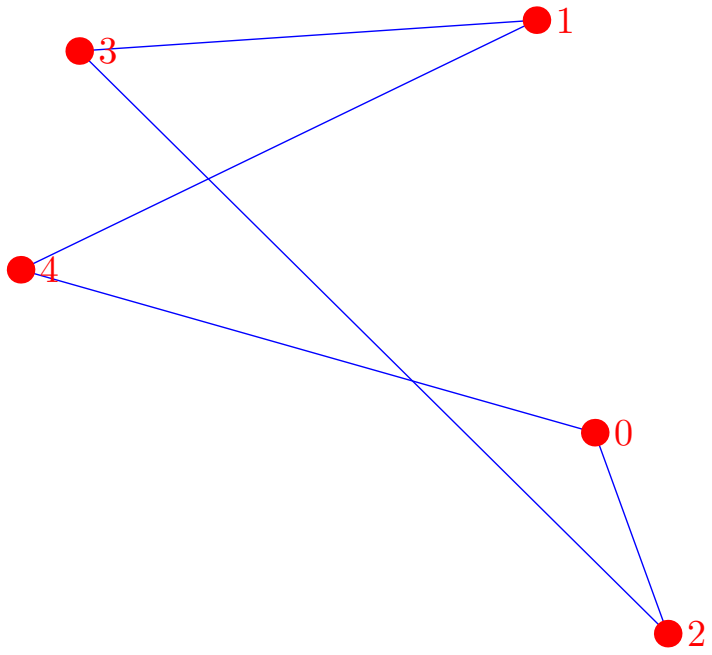
(0,2,3,1)

Branch and Bound in Action



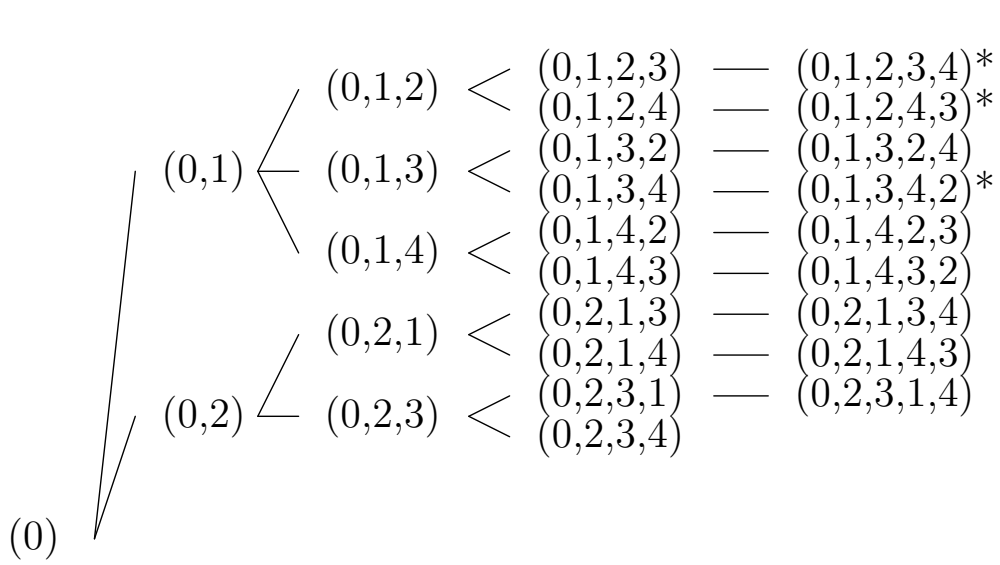
bound = 302.31

length = 392.31



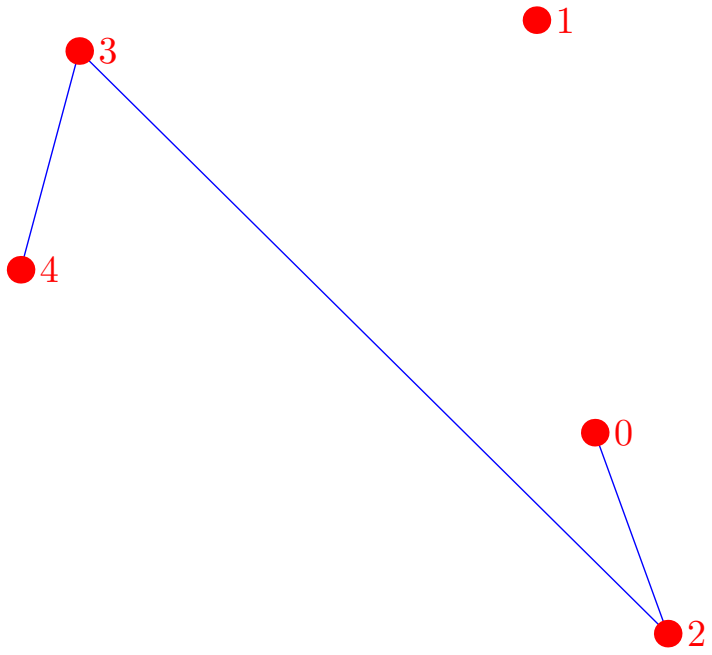
(0,2,3,1,4)

Branch and Bound in Action



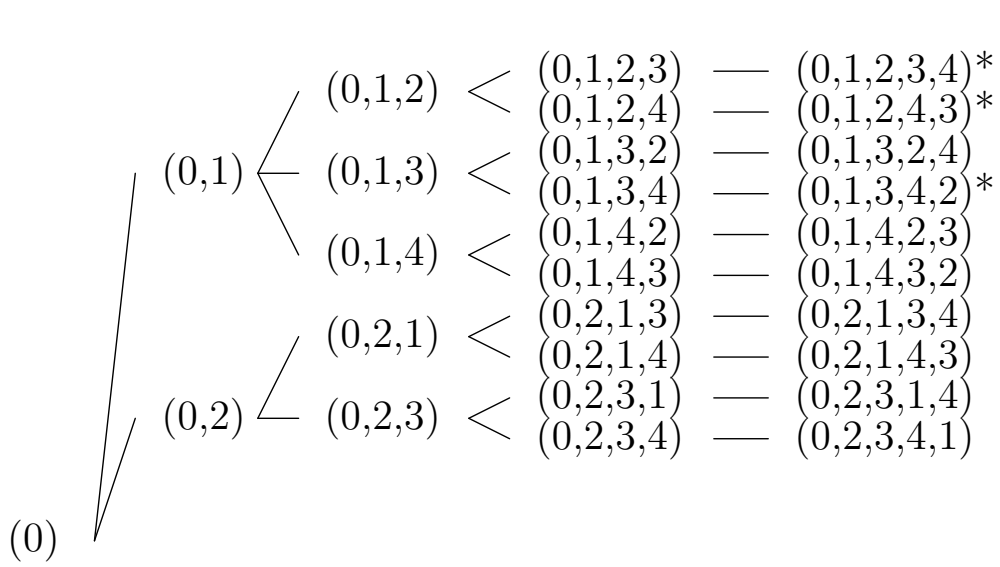
bound = 302.31

length = 186.35



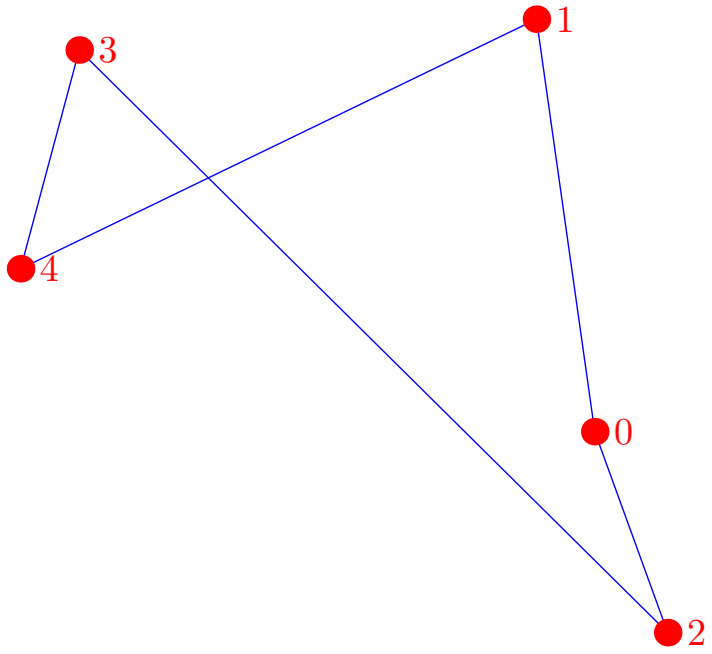
(0,2,3,4)

Branch and Bound in Action



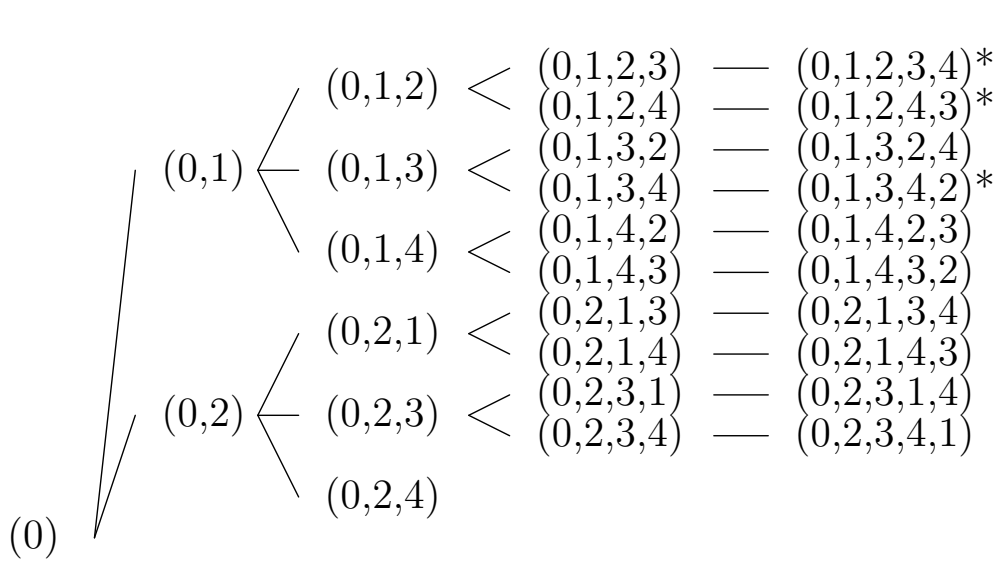
bound = 302.31

length = 331.77



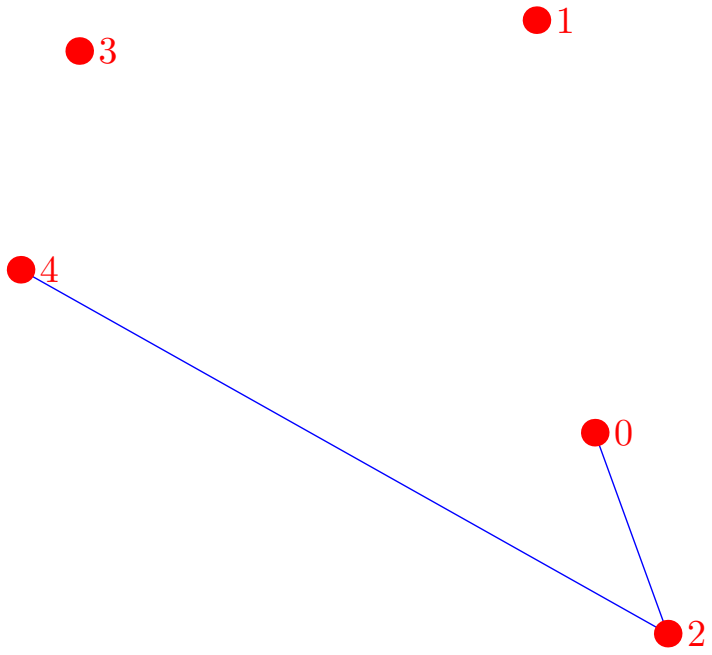
(0,2,3,4,1)

Branch and Bound in Action



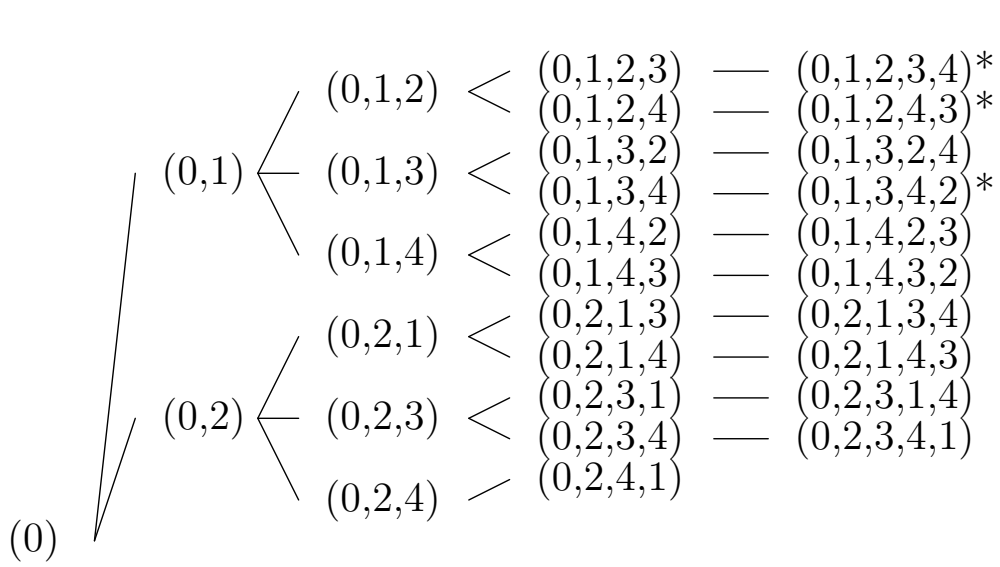
bound = 302.31

length = 140.49



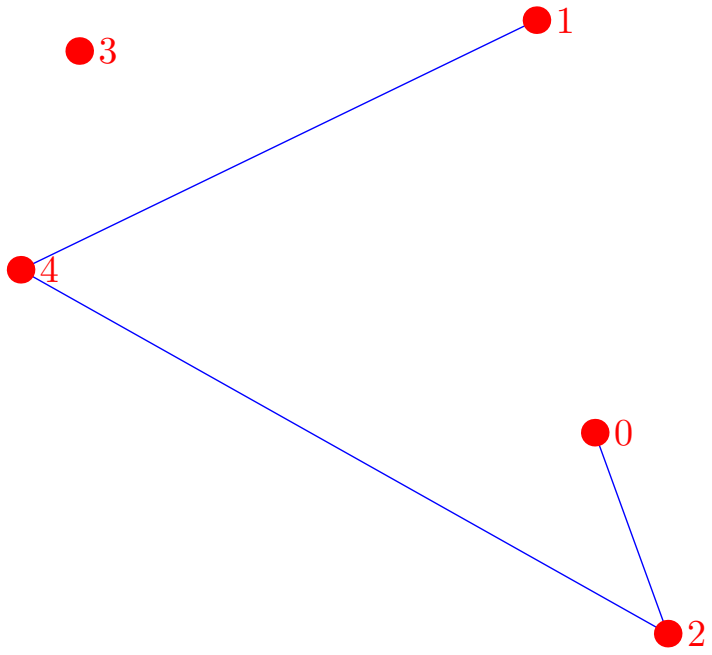
(0,2,4)

Branch and Bound in Action



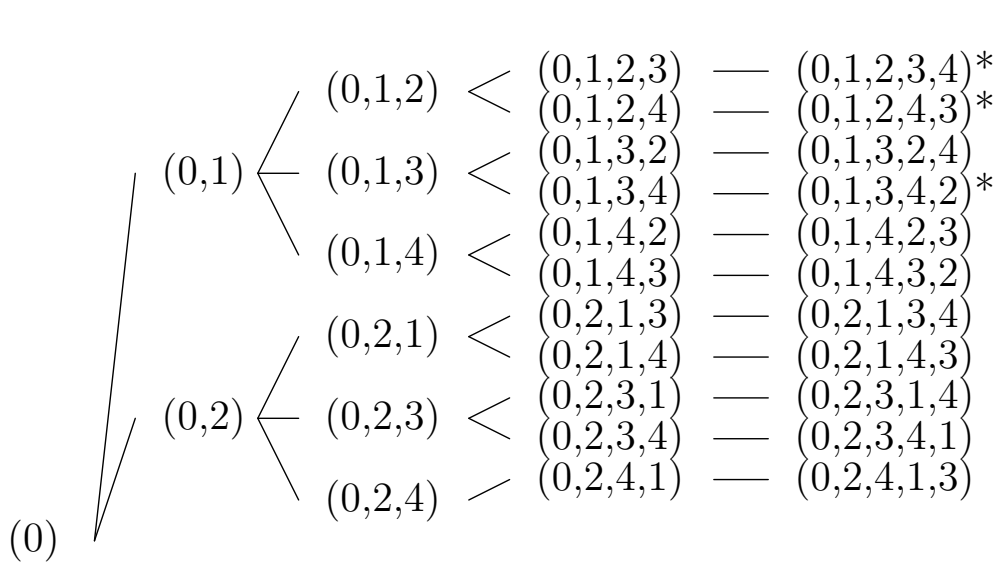
bound = 302.31

length = 224.69



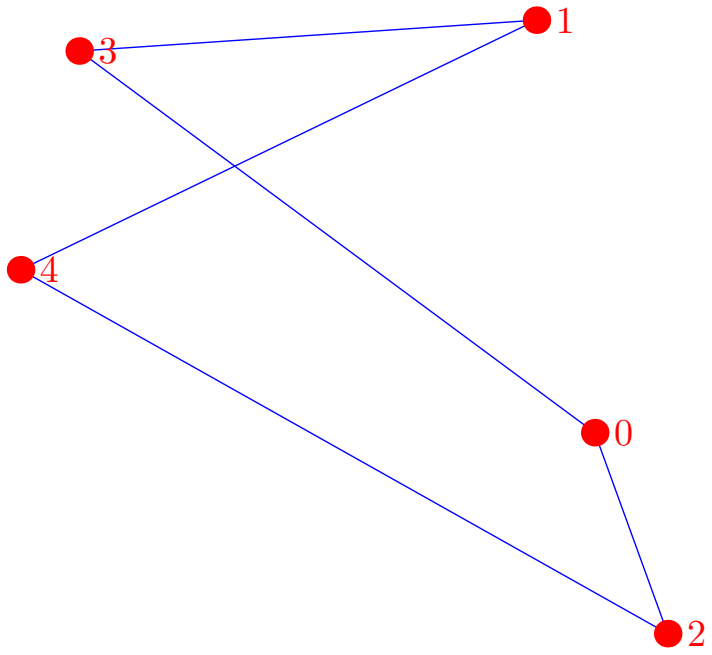
(0,2,4,1)

Branch and Bound in Action



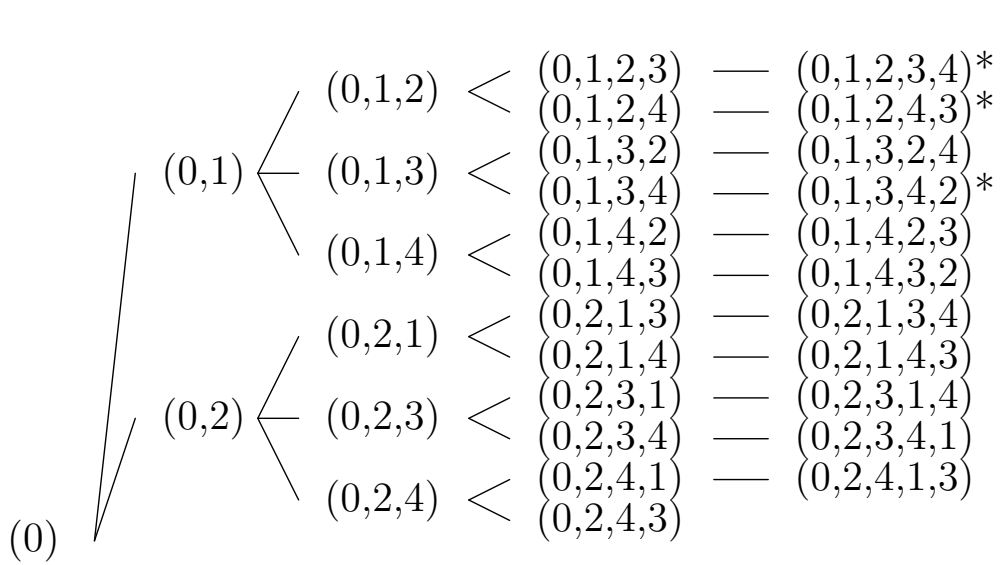
bound = 302.31

length = 386.27



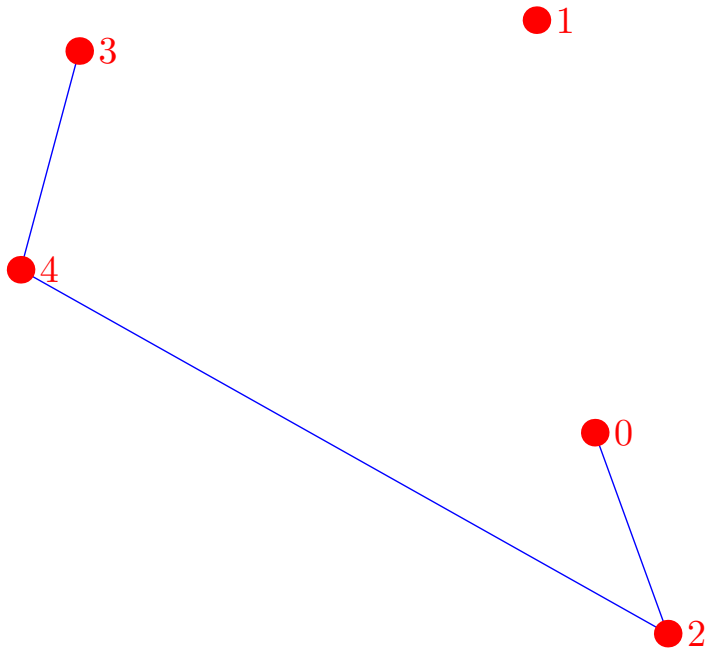
(0,2,4,1,3)

Branch and Bound in Action



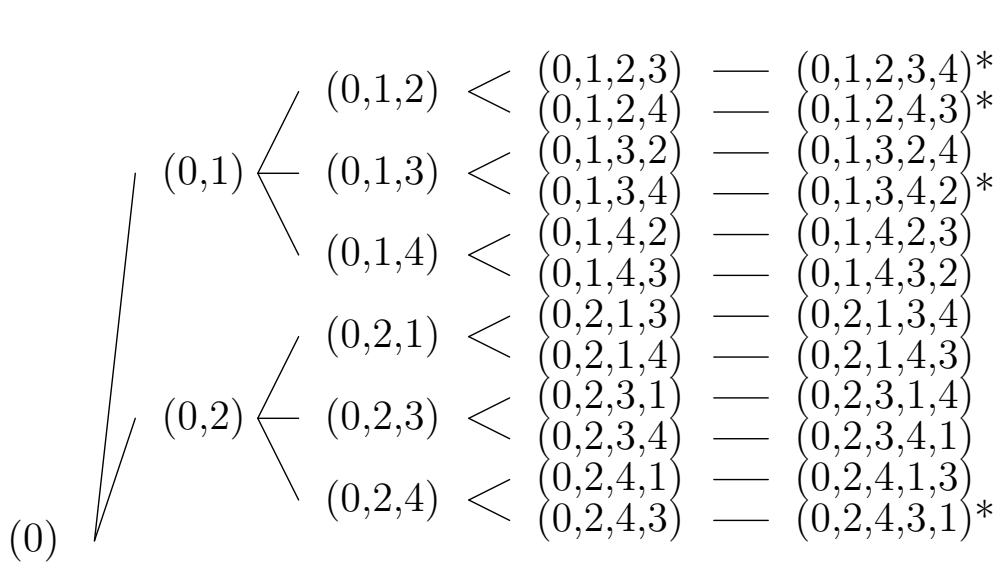
bound = 302.31

length = 173.77



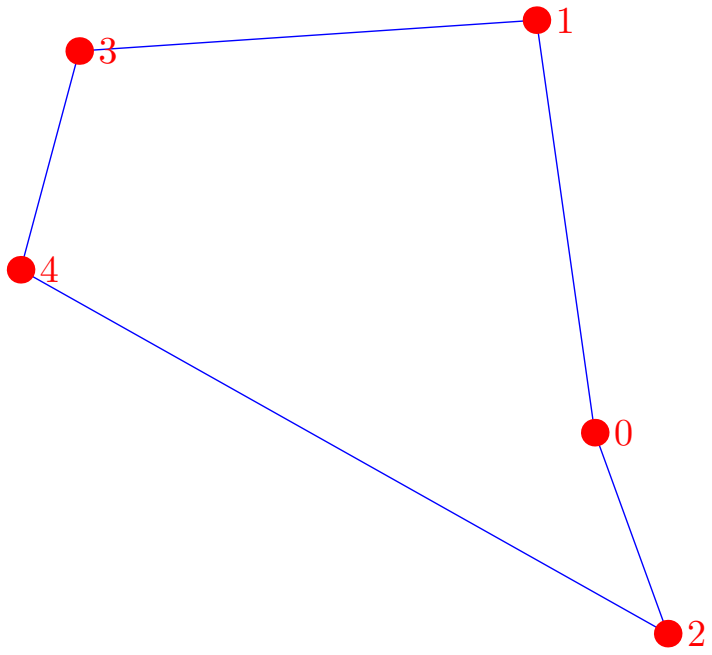
(0,2,4,3)

Branch and Bound in Action



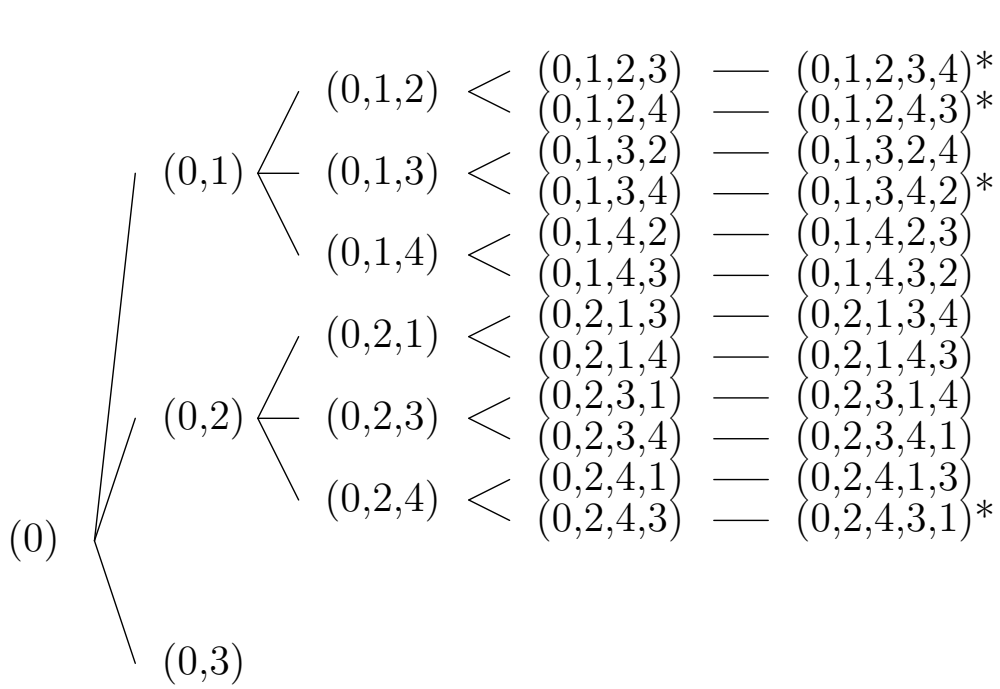
bound = 302.31

length = 302.31



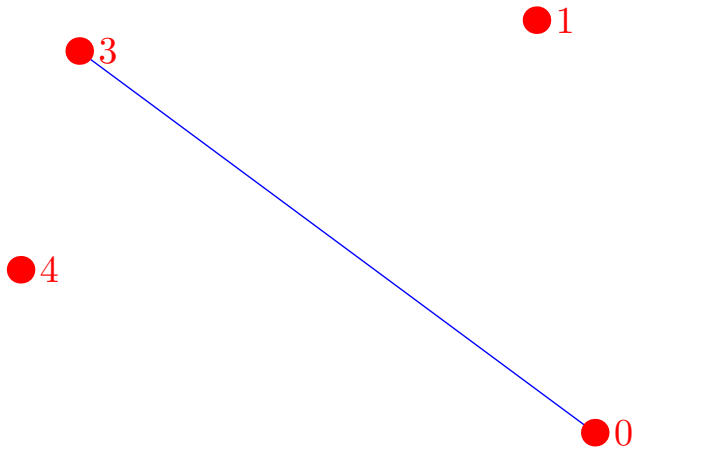
(0,2,4,3,1)

Branch and Bound in Action



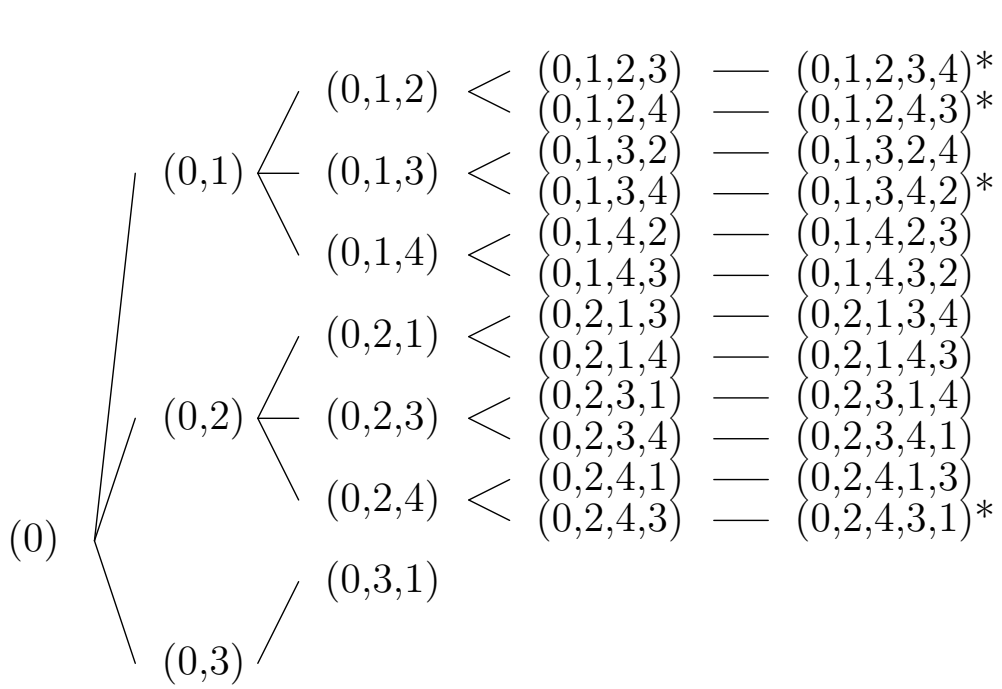
bound = 302.31

length = 94.244



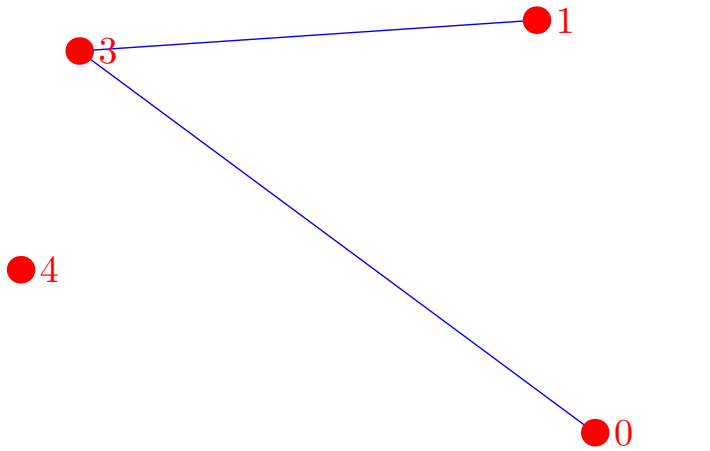
(0,3)

Branch and Bound in Action



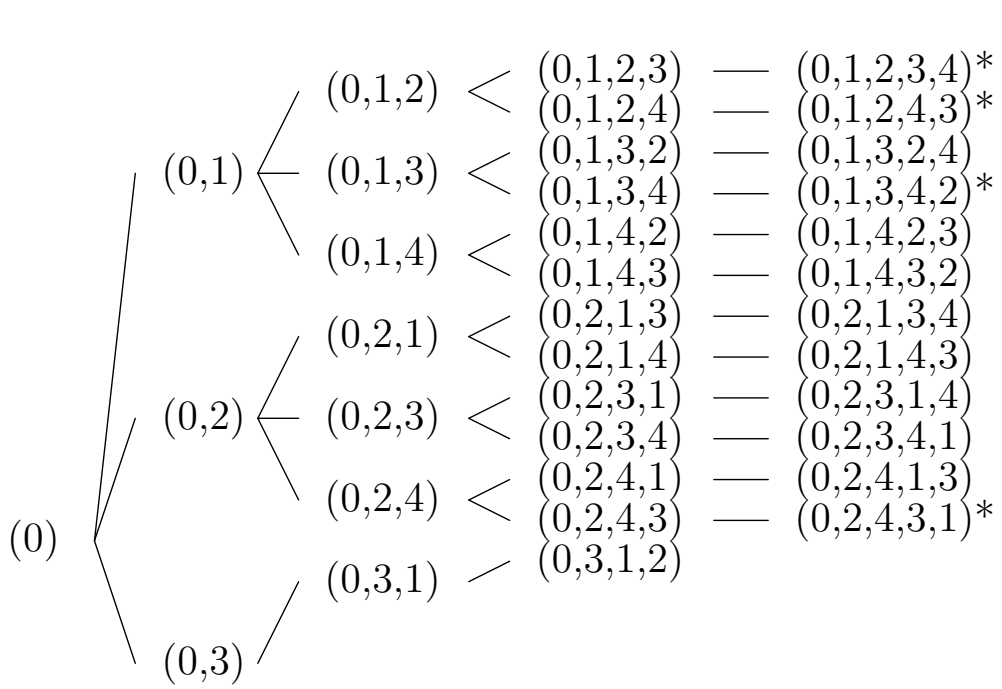
bound = 302.31

length = 161.58



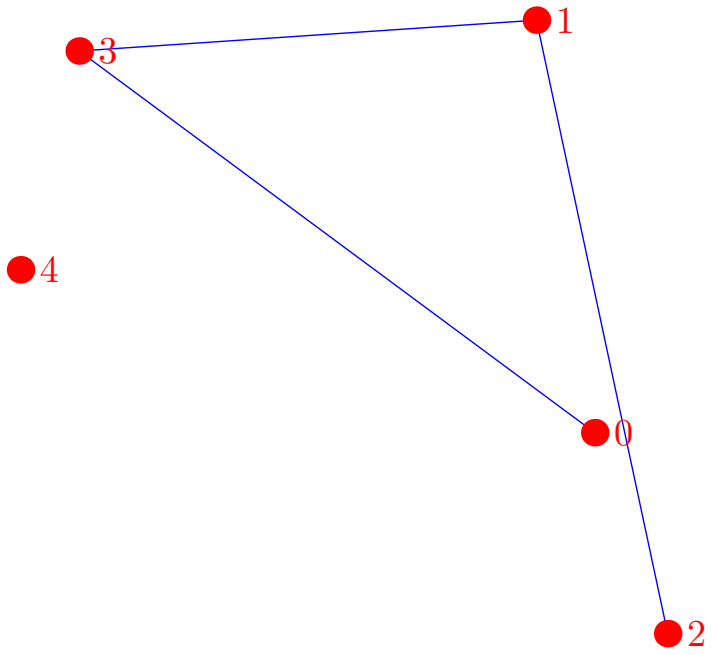
(0,3,1)

Branch and Bound in Action



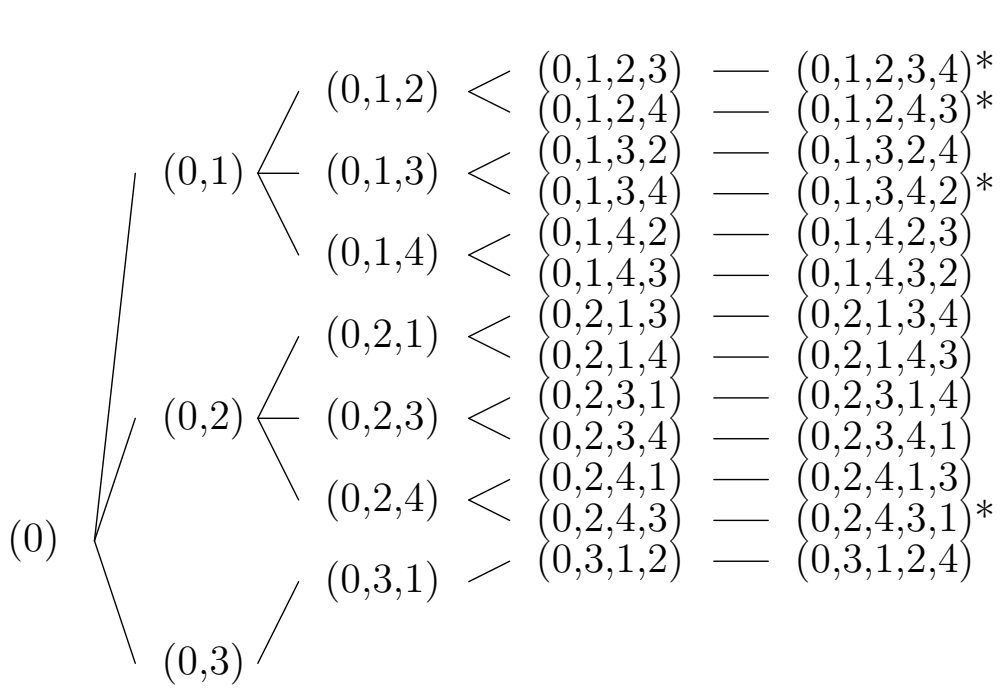
bound = 302.31

length = 253.75



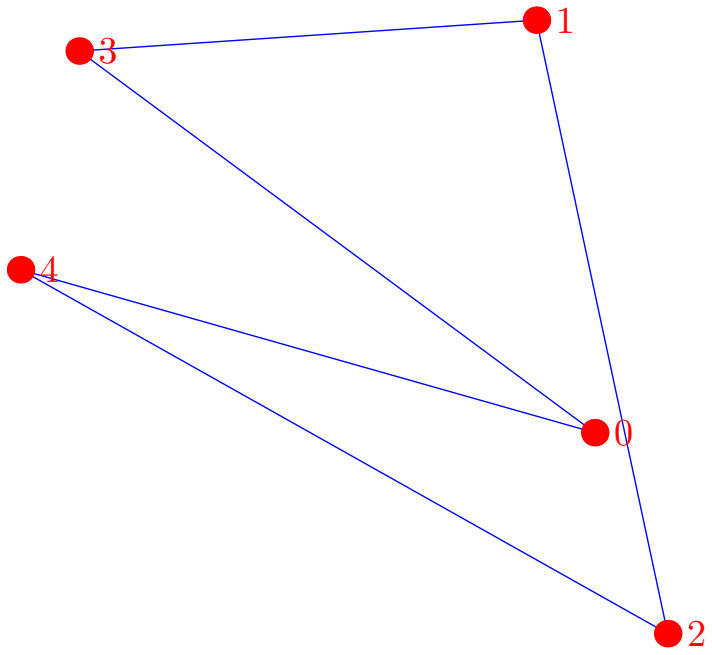
(0,3,1,2)

Branch and Bound in Action



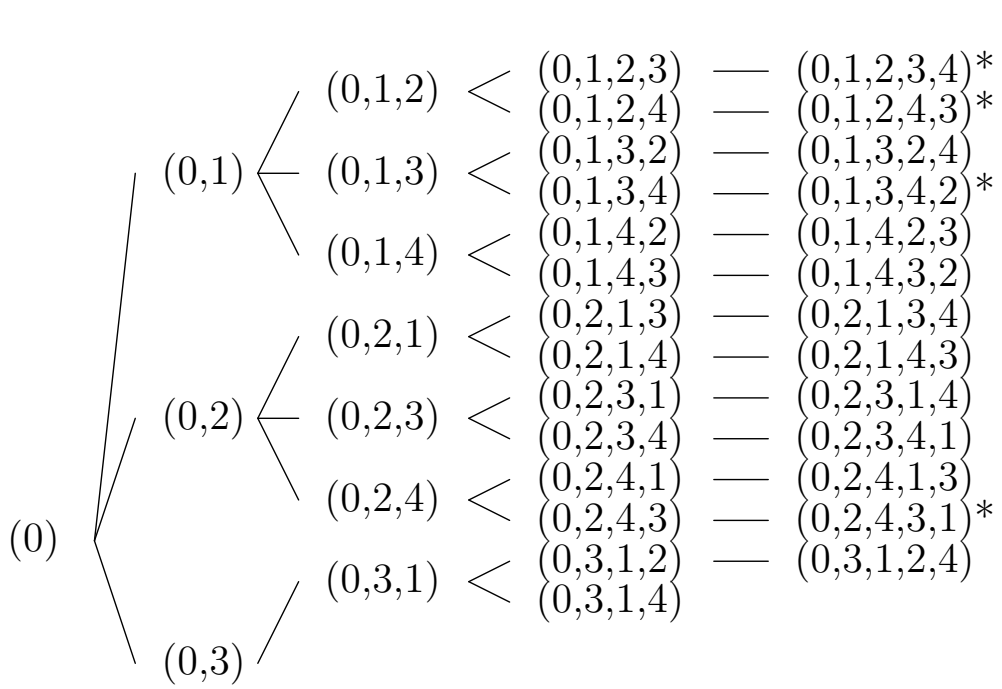
bound = 302.31

length = 450.52



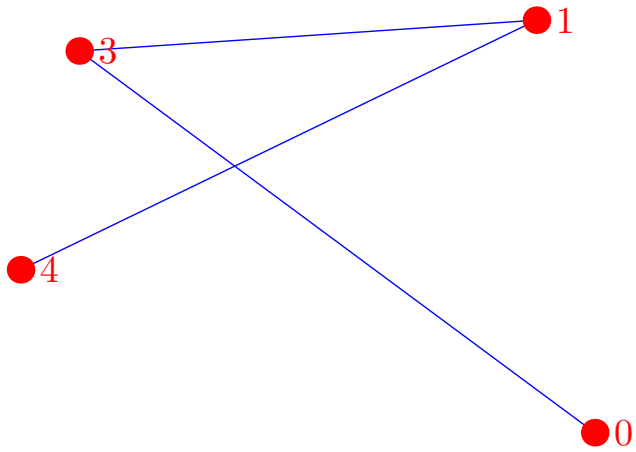
(0,3,1,2,4)

Branch and Bound in Action



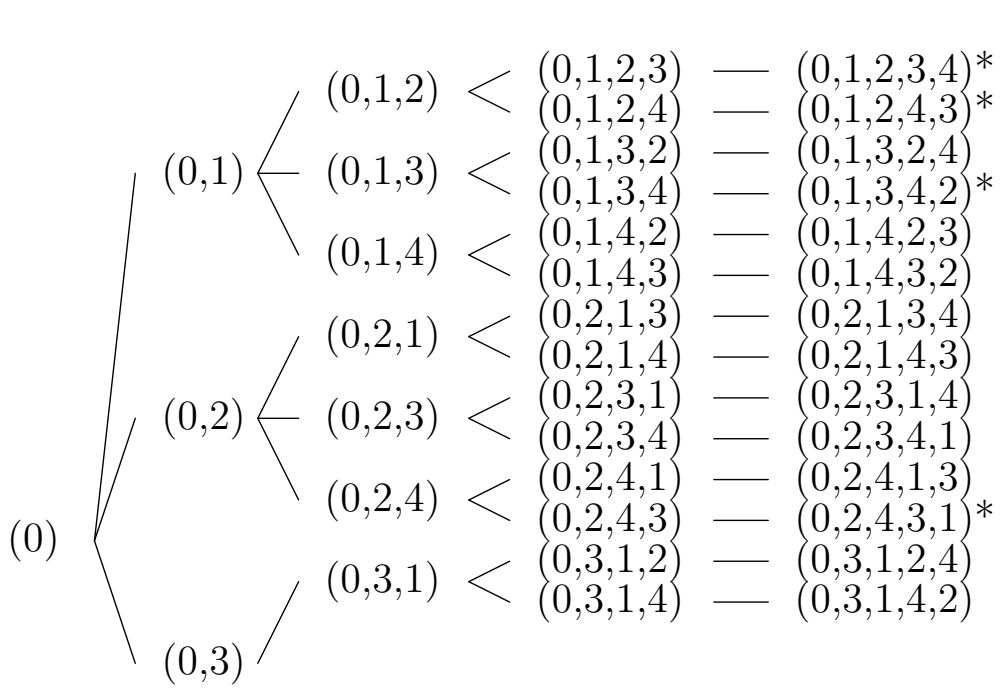
bound = 302.31

length = 245.78



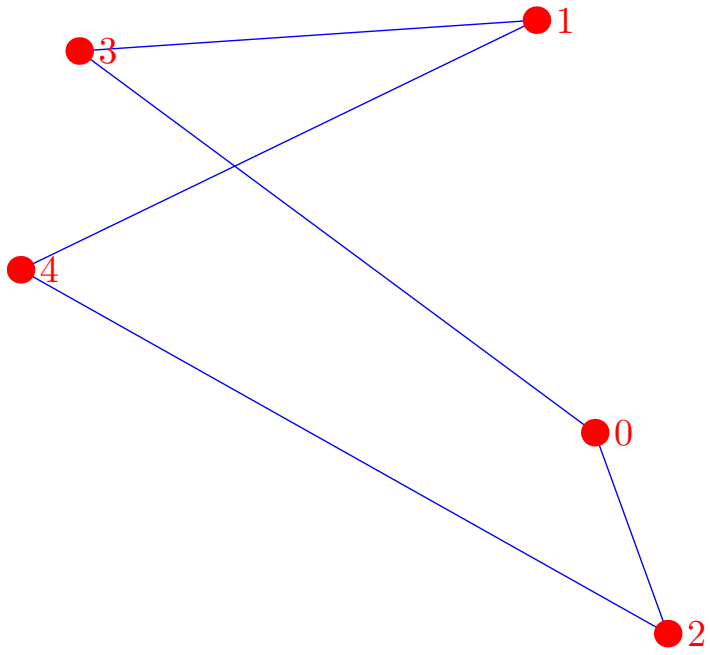
(0,3,1,4)

Branch and Bound in Action



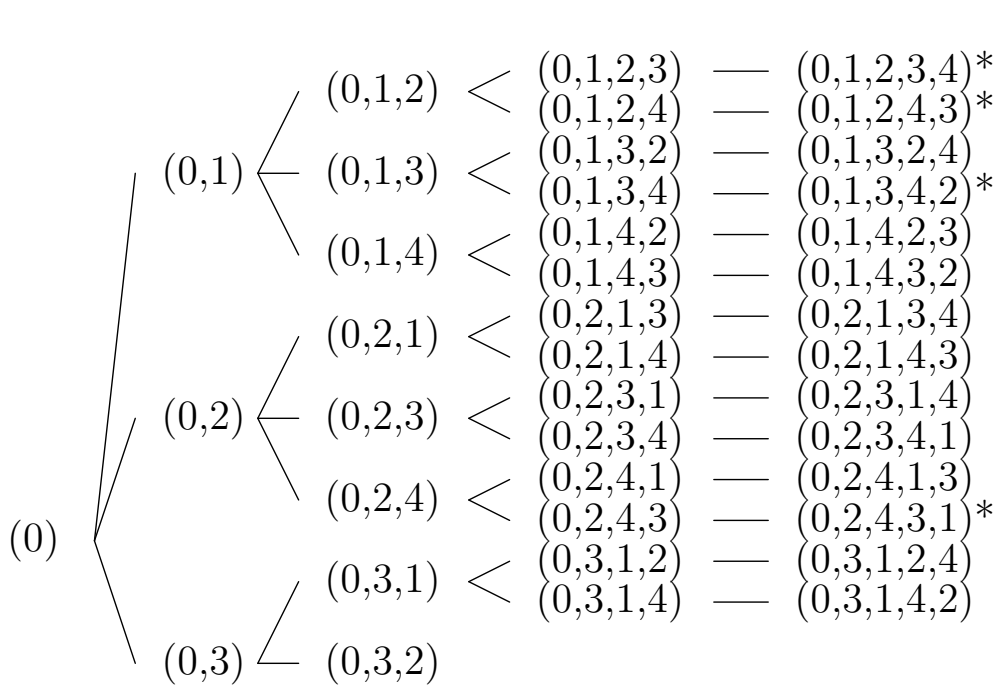
bound = 302.31

length = 386.27



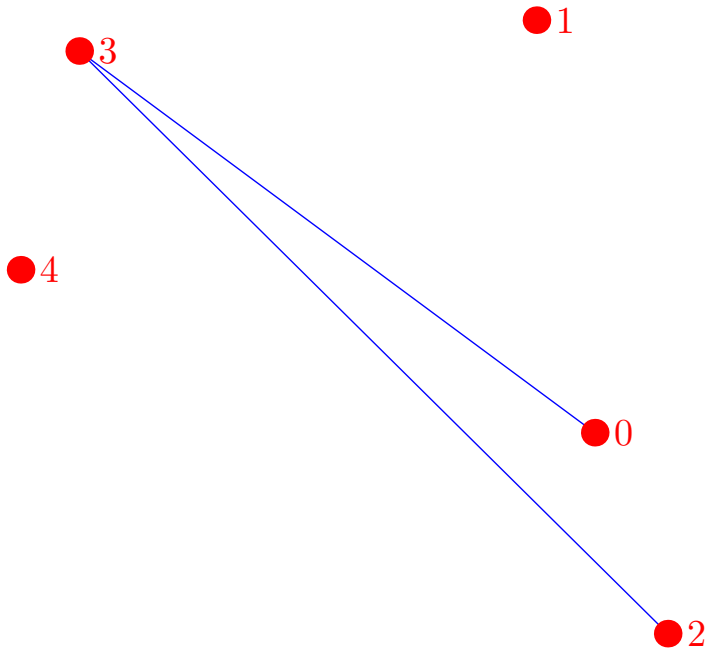
(0,3,1,4,2)

Branch and Bound in Action



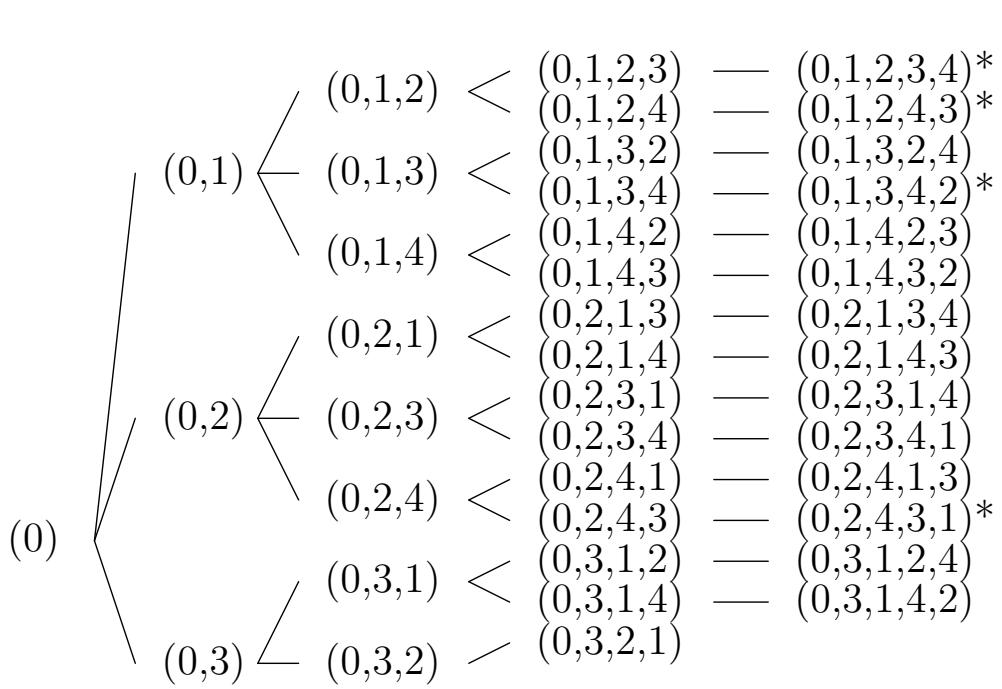
bound = 302.31

length = 215.91



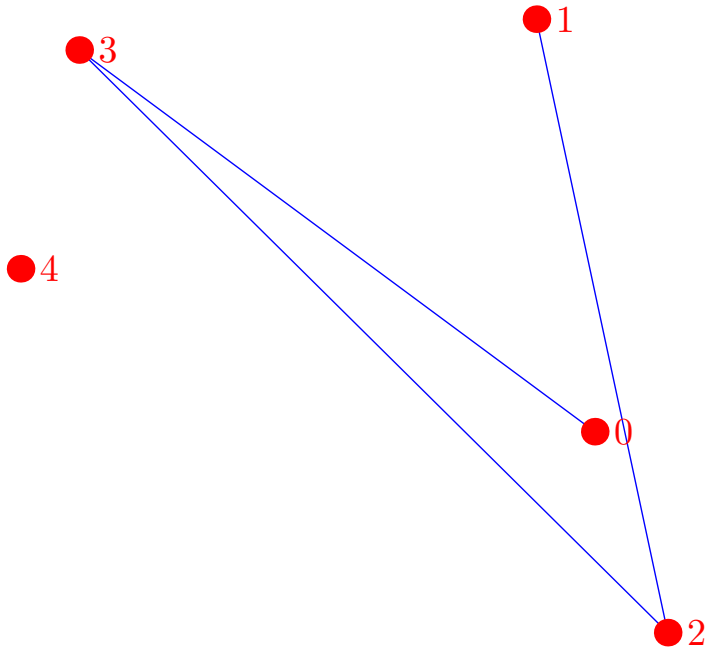
(0,3,2)

Branch and Bound in Action



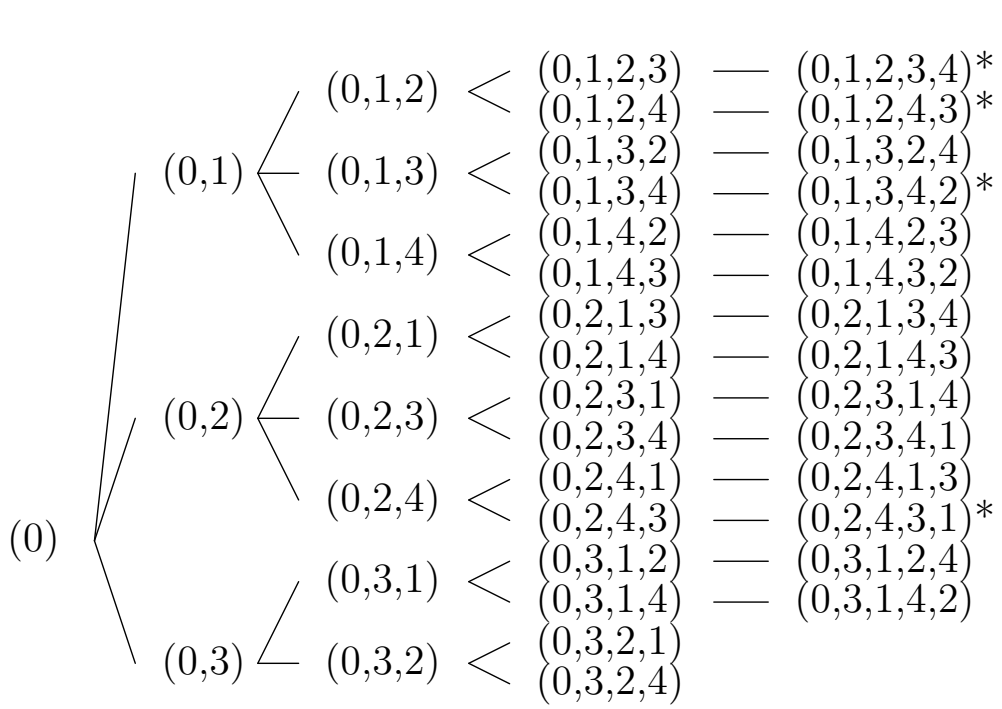
bound = 302.31

length = 308.09



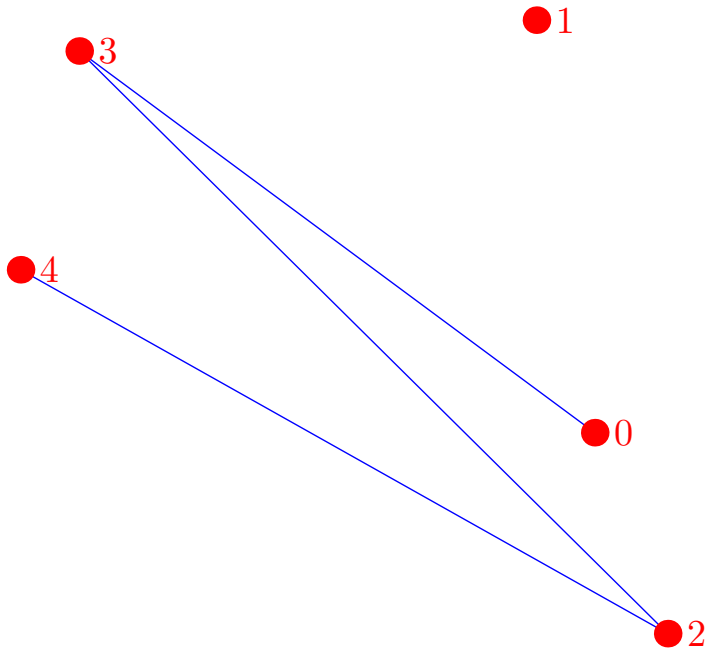
(0,3,2,1)

Branch and Bound in Action



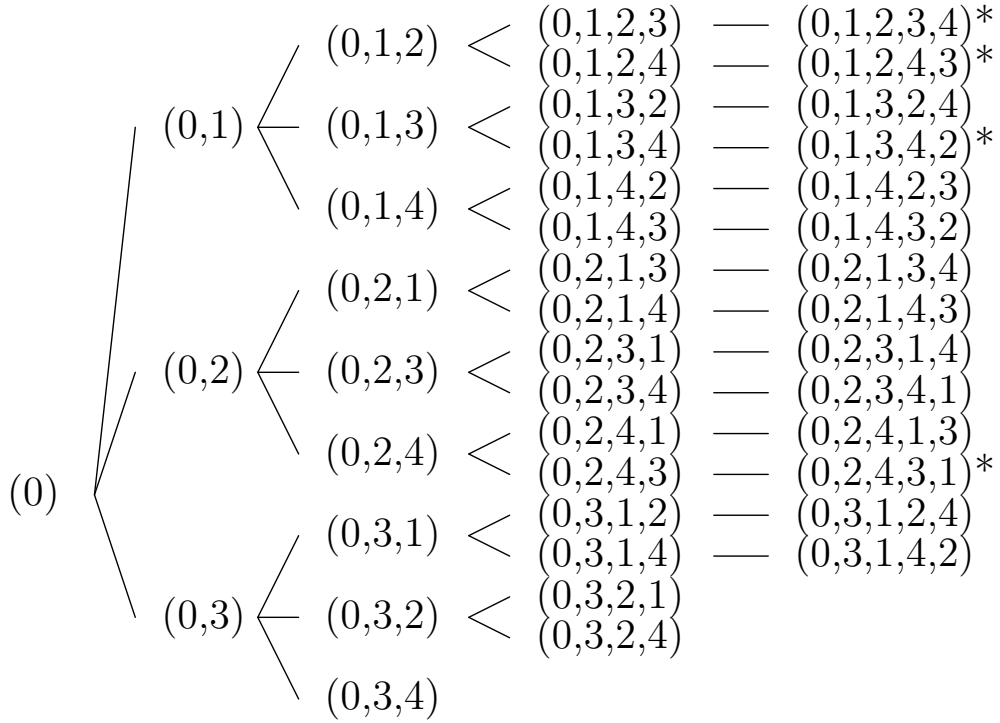
bound = 302.31

length = 324.99



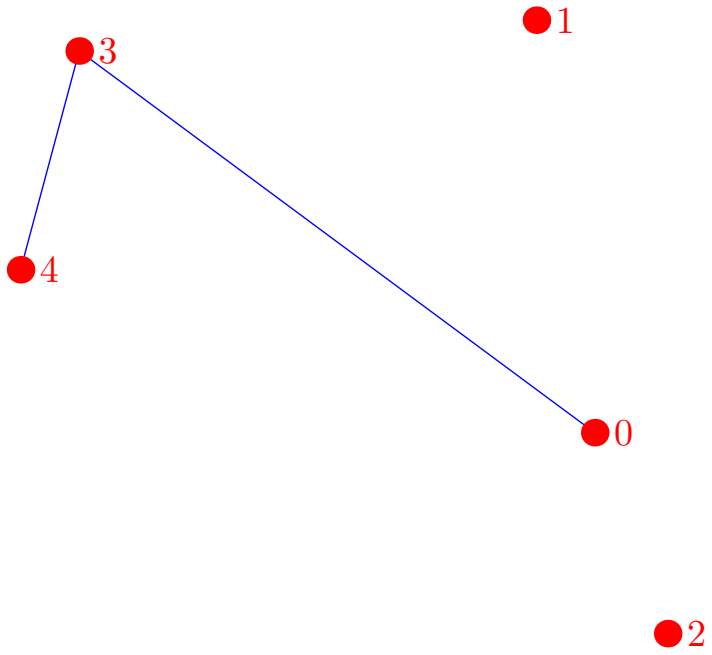
(0,3,2,4)

Branch and Bound in Action



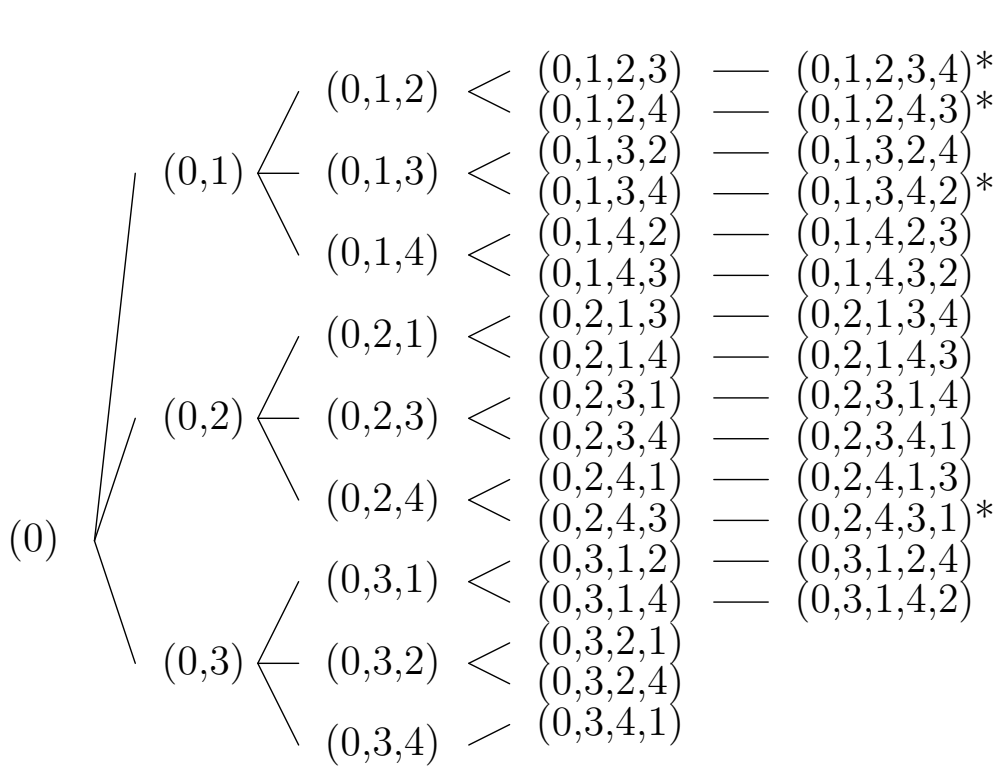
bound = 302.31

length = 127.52



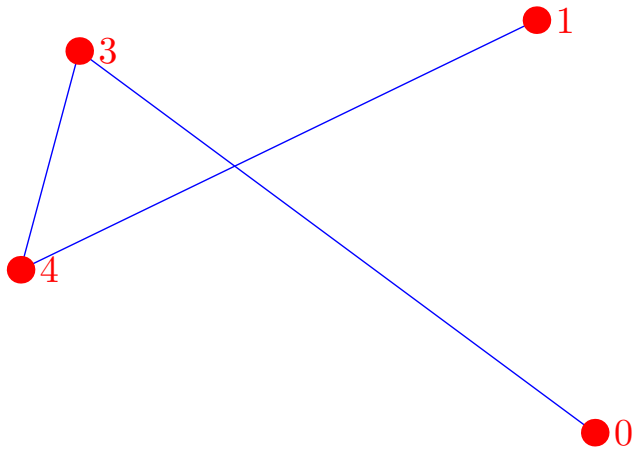
(0,3,4)

Branch and Bound in Action



bound = 302.31

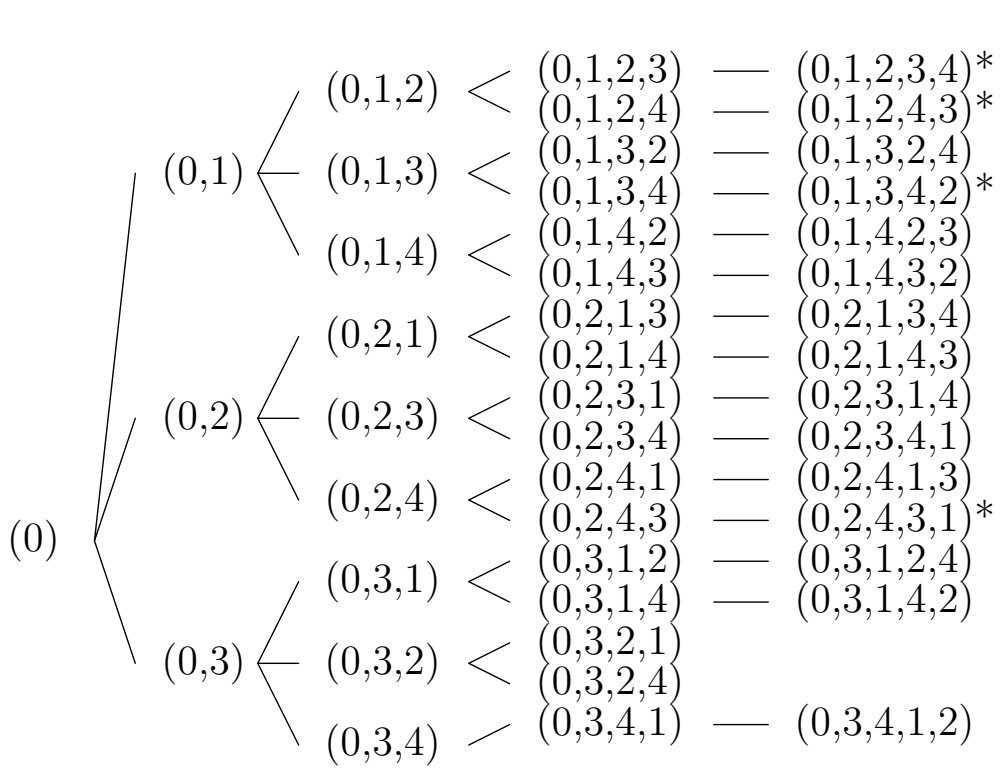
length = 211.72



2

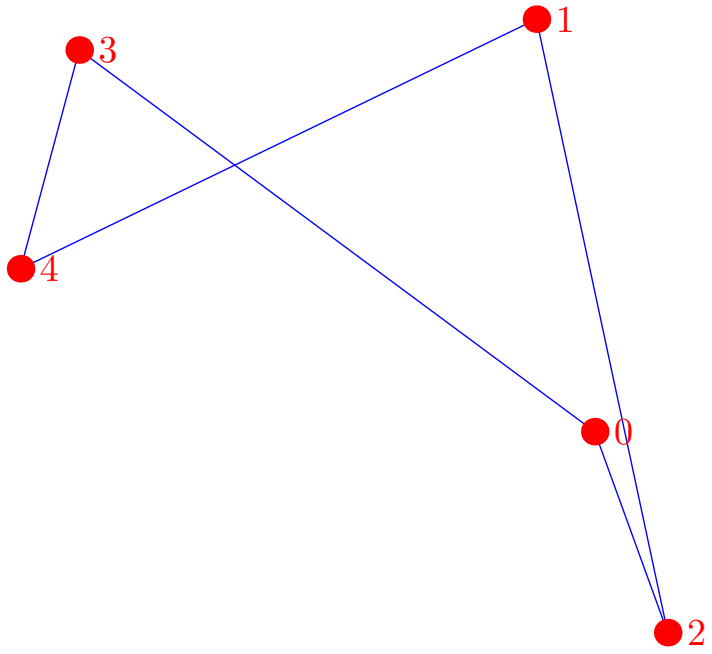
(0,3,4,1)

Branch and Bound in Action



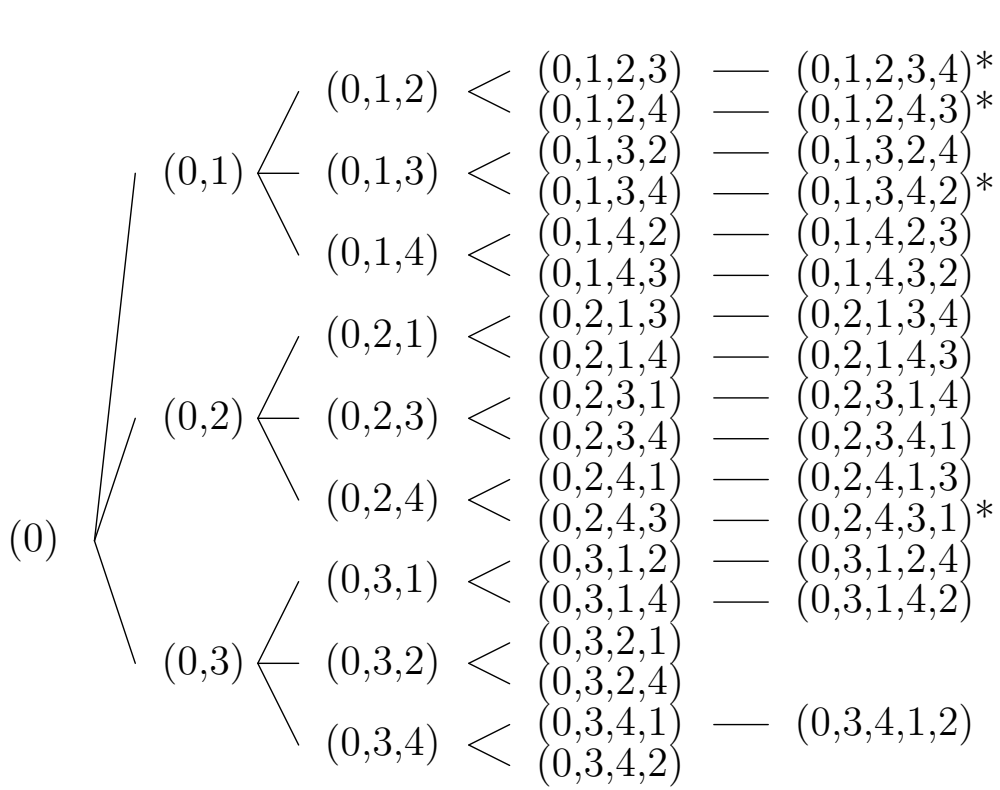
bound = 302.31

length = 335.3



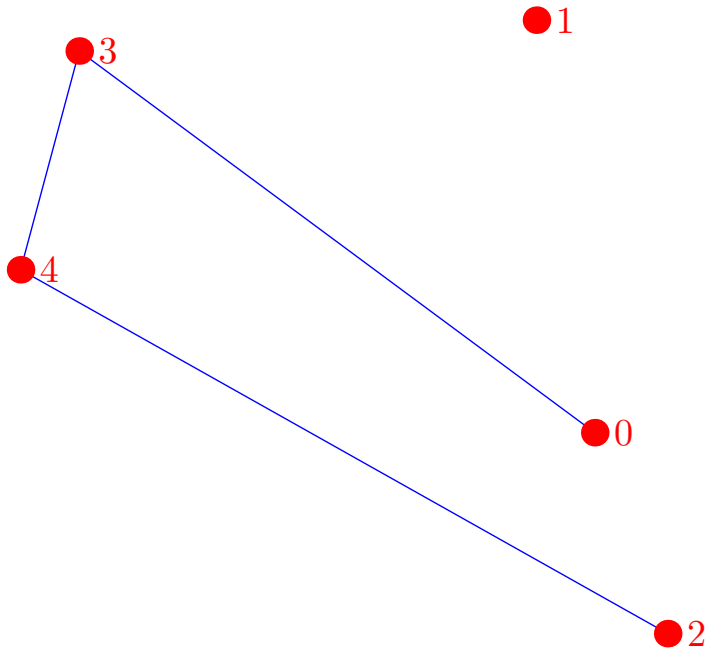
(0,3,4,1,2)

Branch and Bound in Action



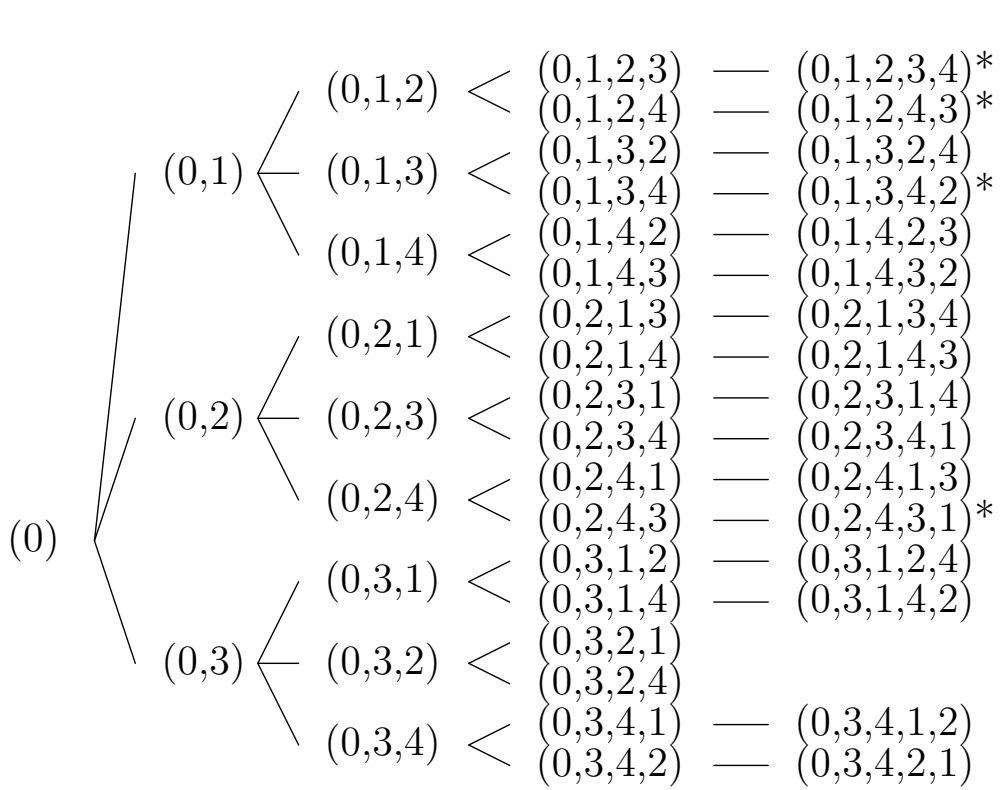
bound = 302.31

length = 236.6



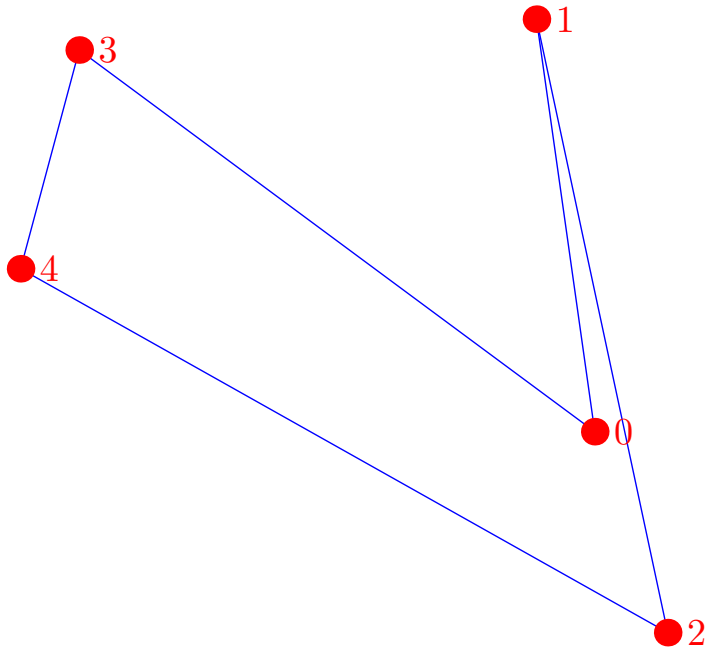
(0,3,4,2)

Branch and Bound in Action



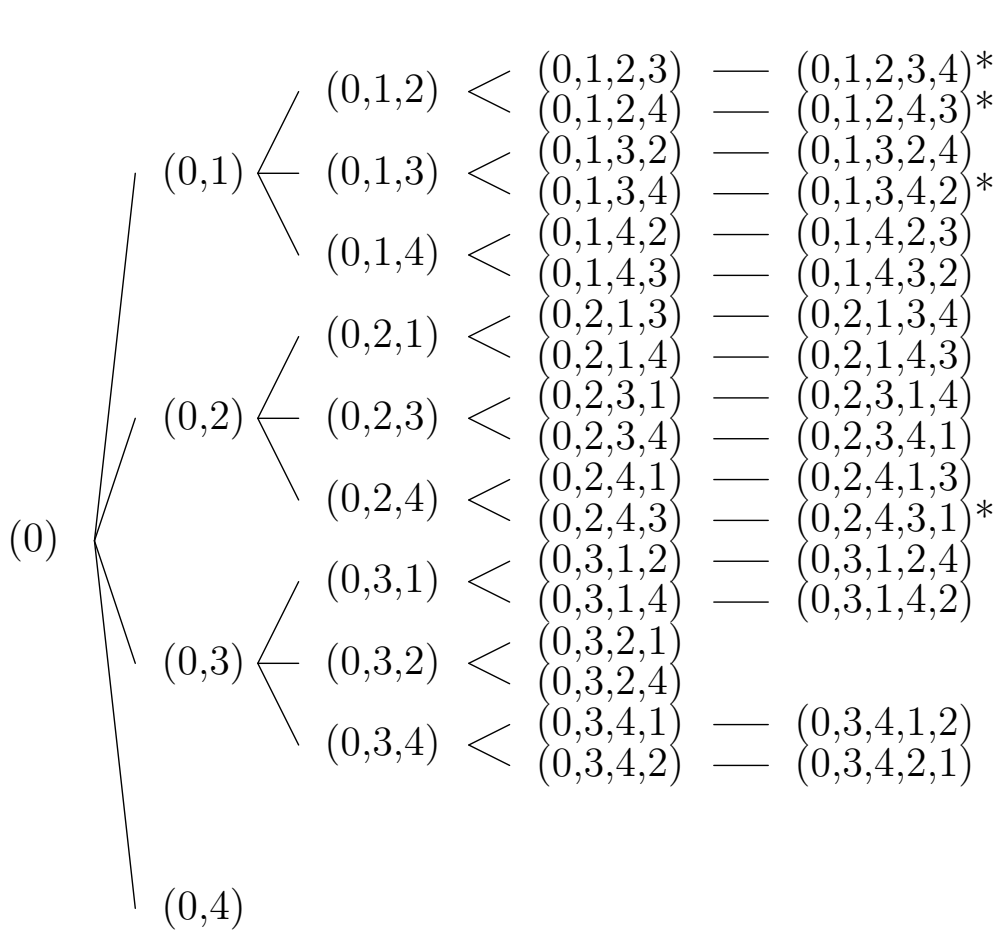
bound = 302.31

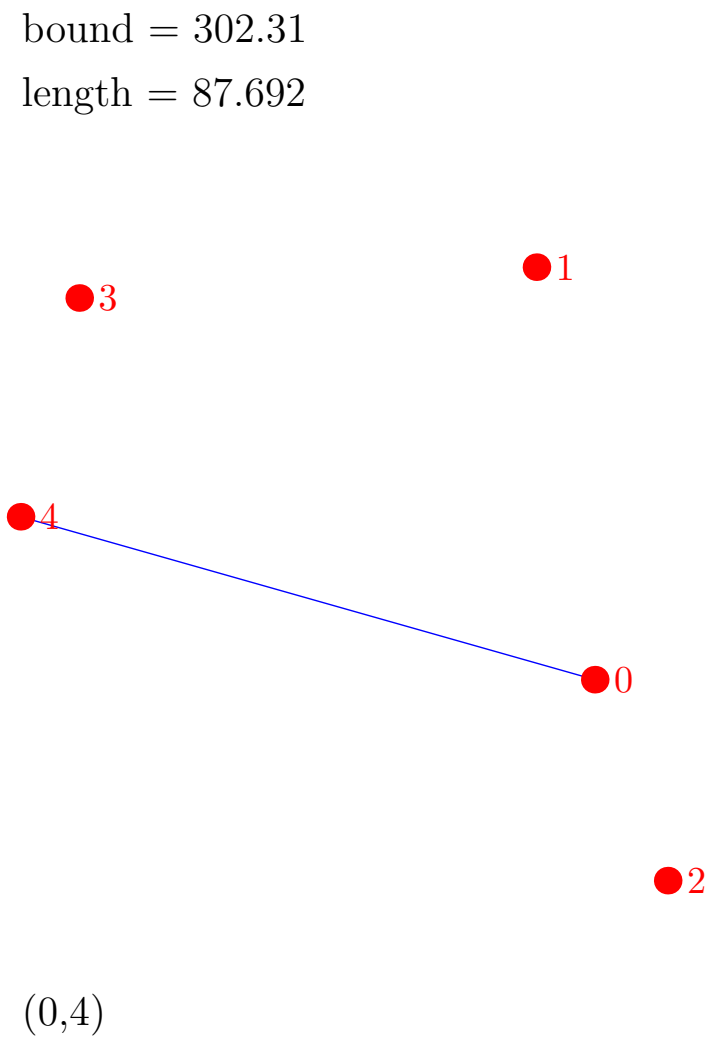
length = 389.98



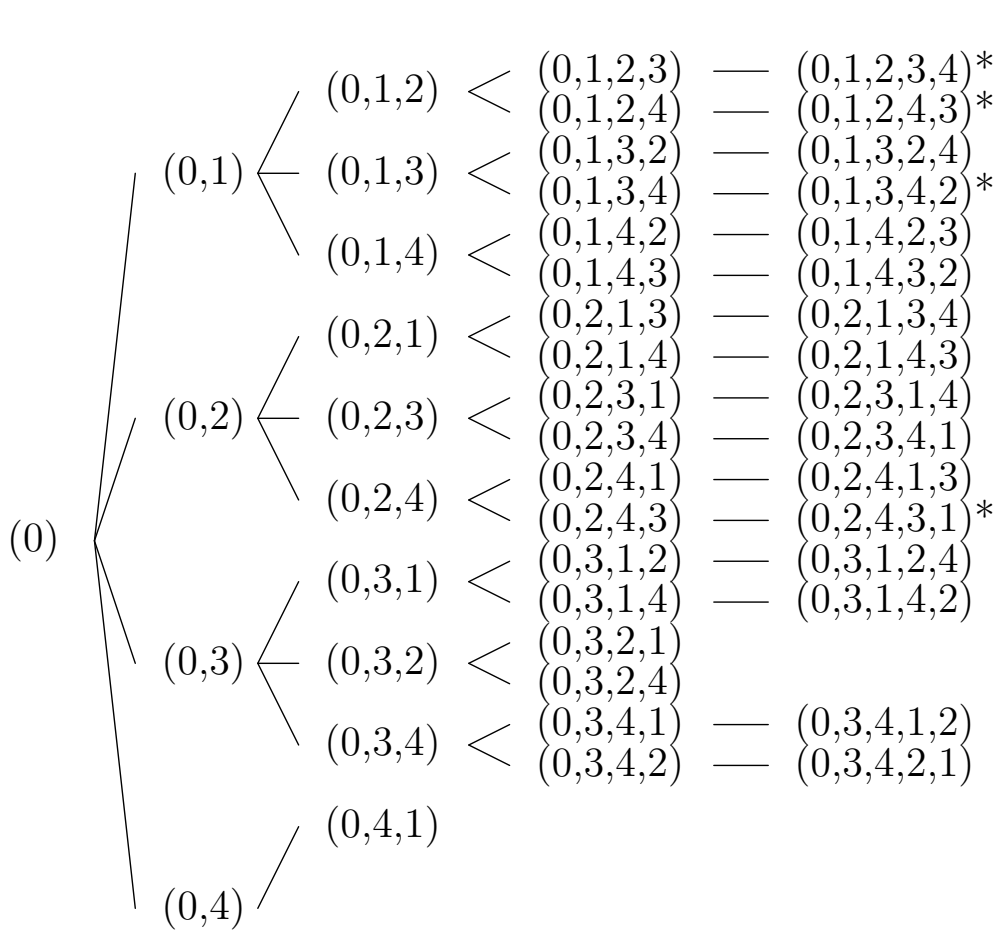
(0,3,4,2,1)

Branch and Bound in Action



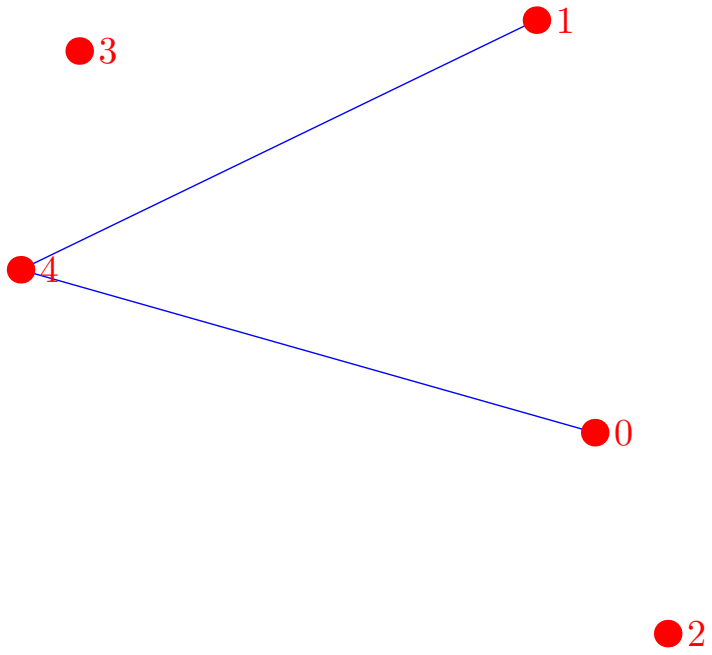


Branch and Bound in Action



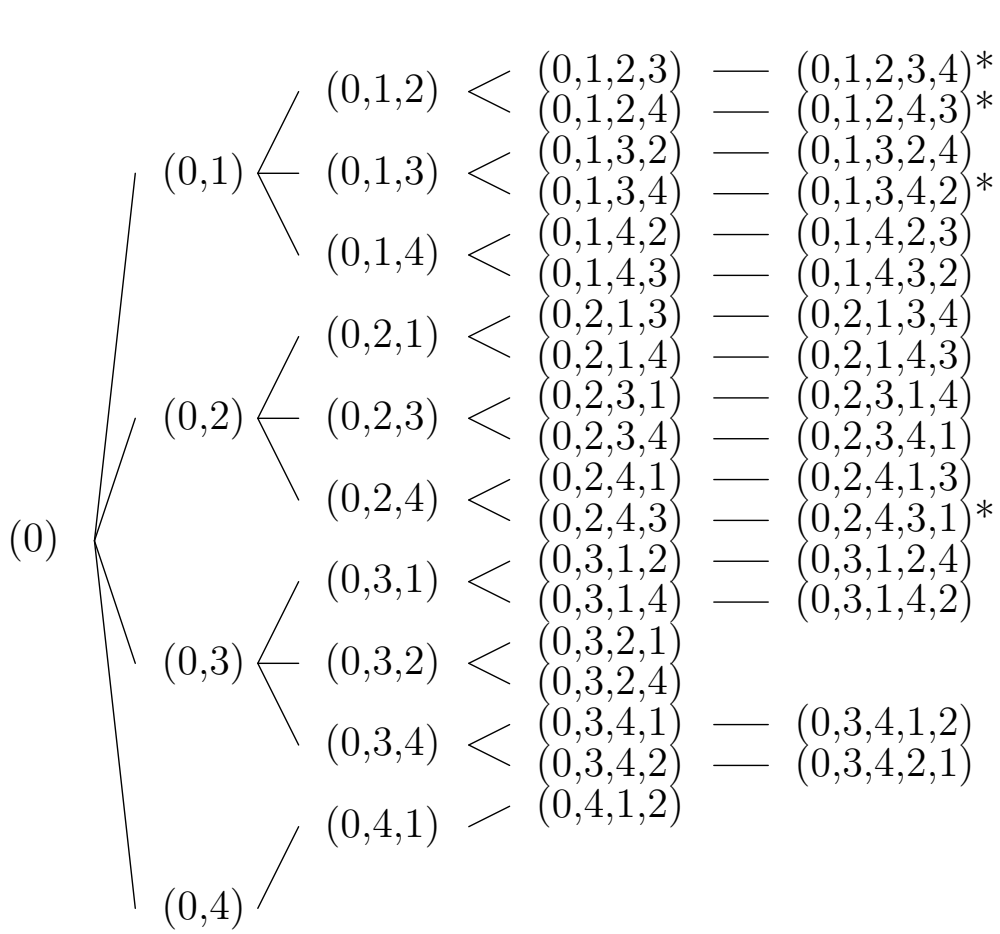
bound = 302.31

length = 171.9



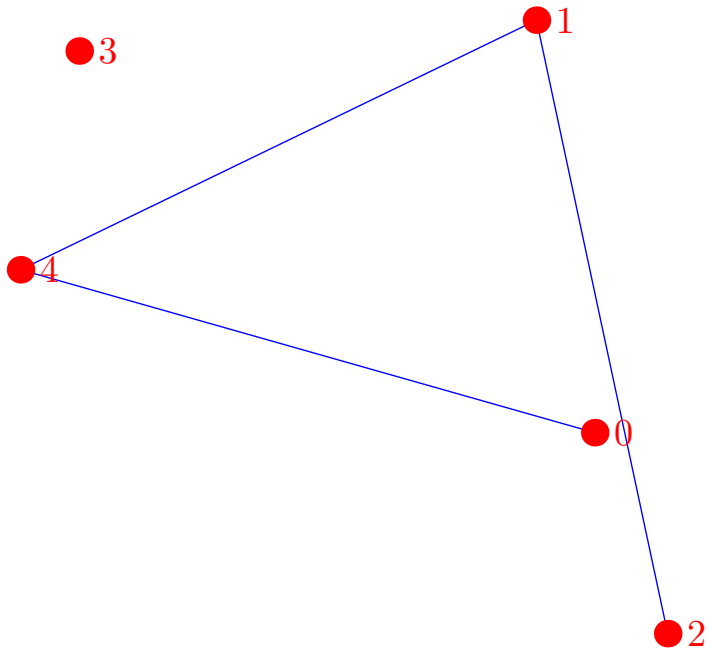
(0,4,1)

Branch and Bound in Action



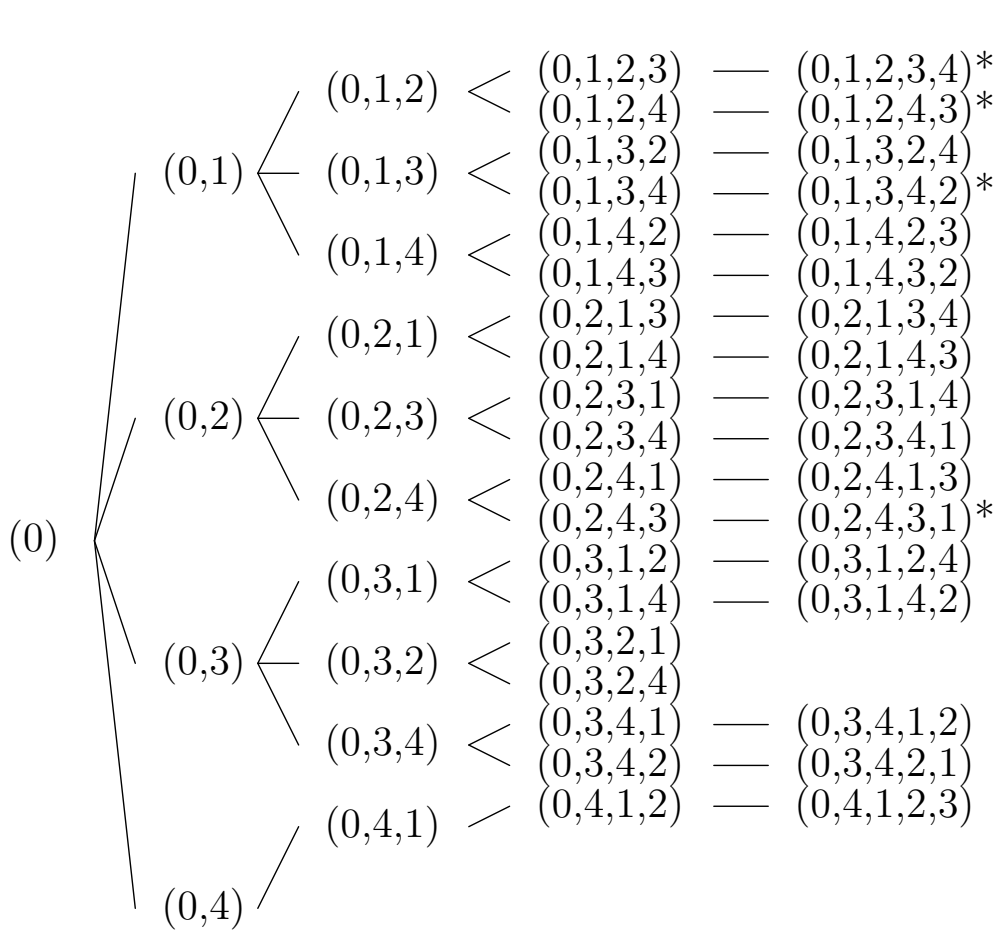
bound = 302.31

length = 264.07



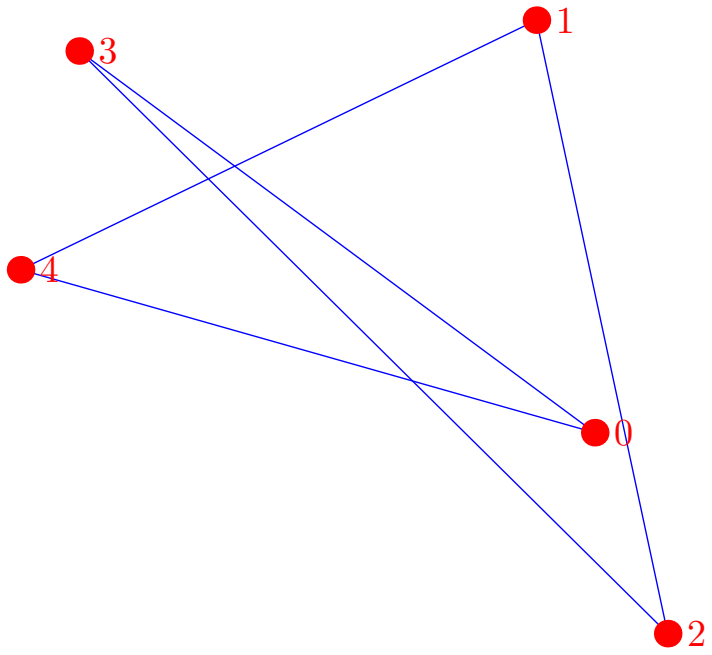
(0,4,1,2)

Branch and Bound in Action



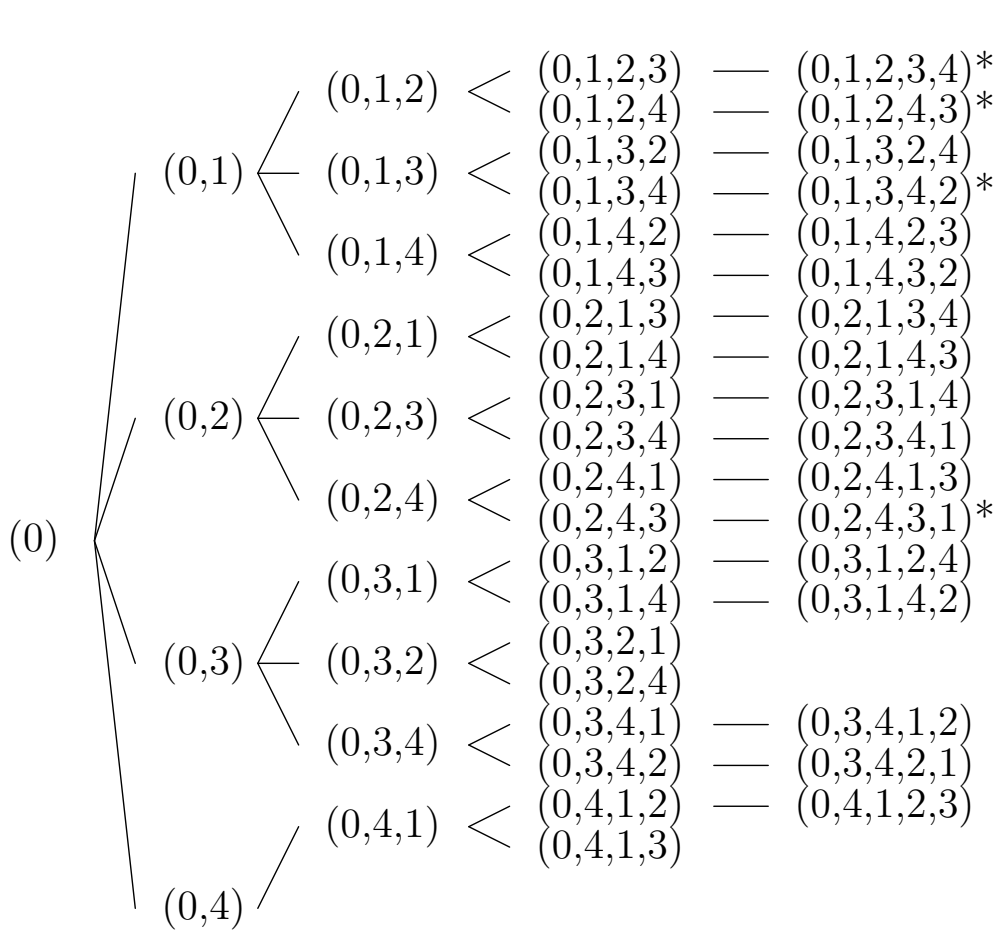
bound = 302.31

length = 479.98



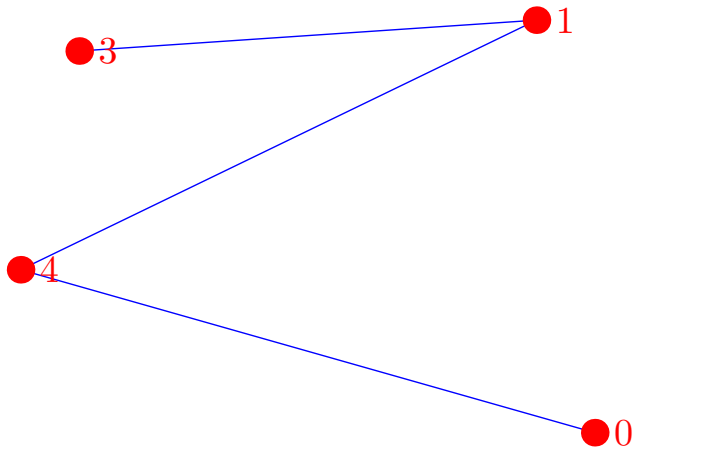
(0,4,1,2,3)

Branch and Bound in Action



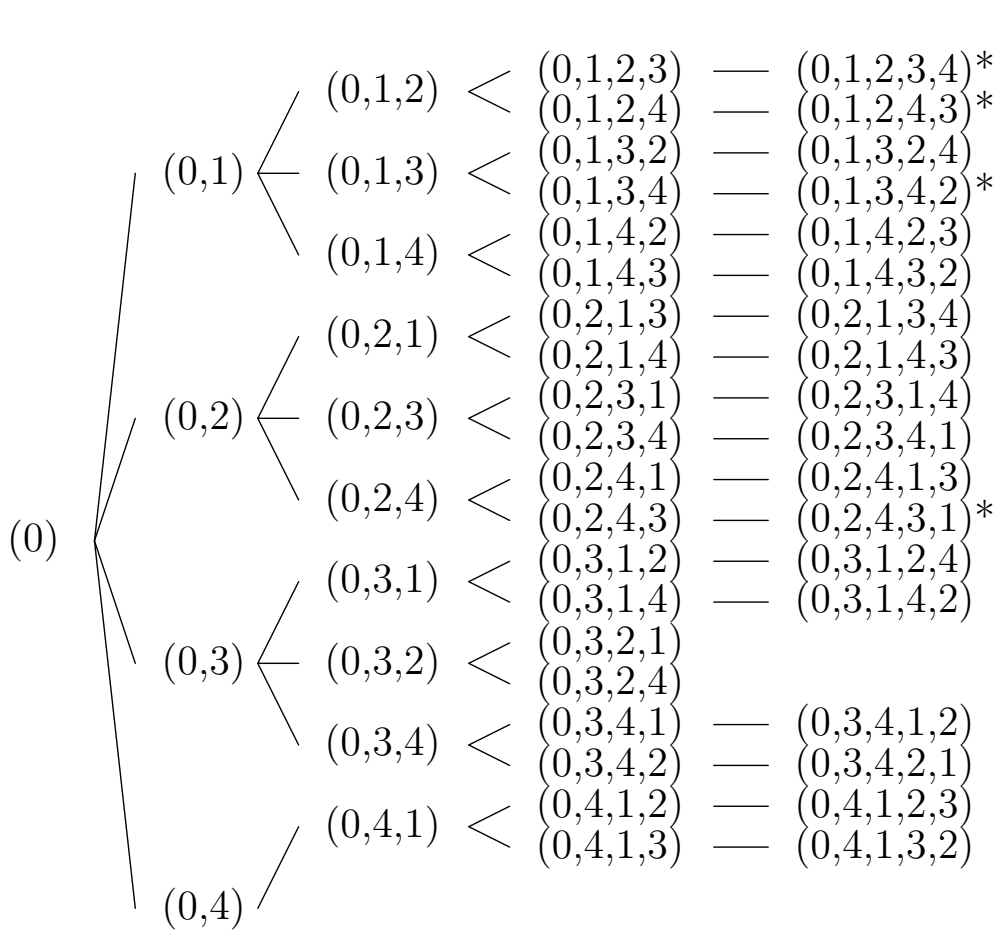
bound = 302.31

length = 239.23



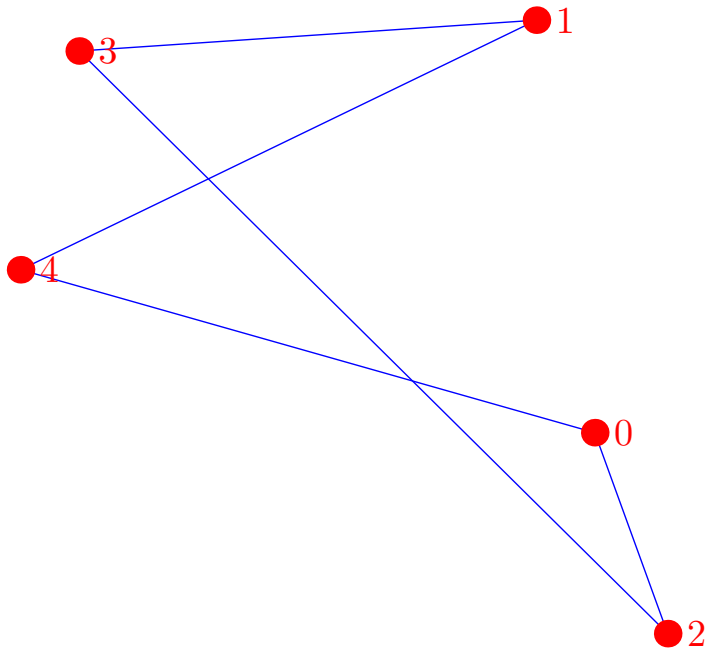
(0,4,1,3)

Branch and Bound in Action



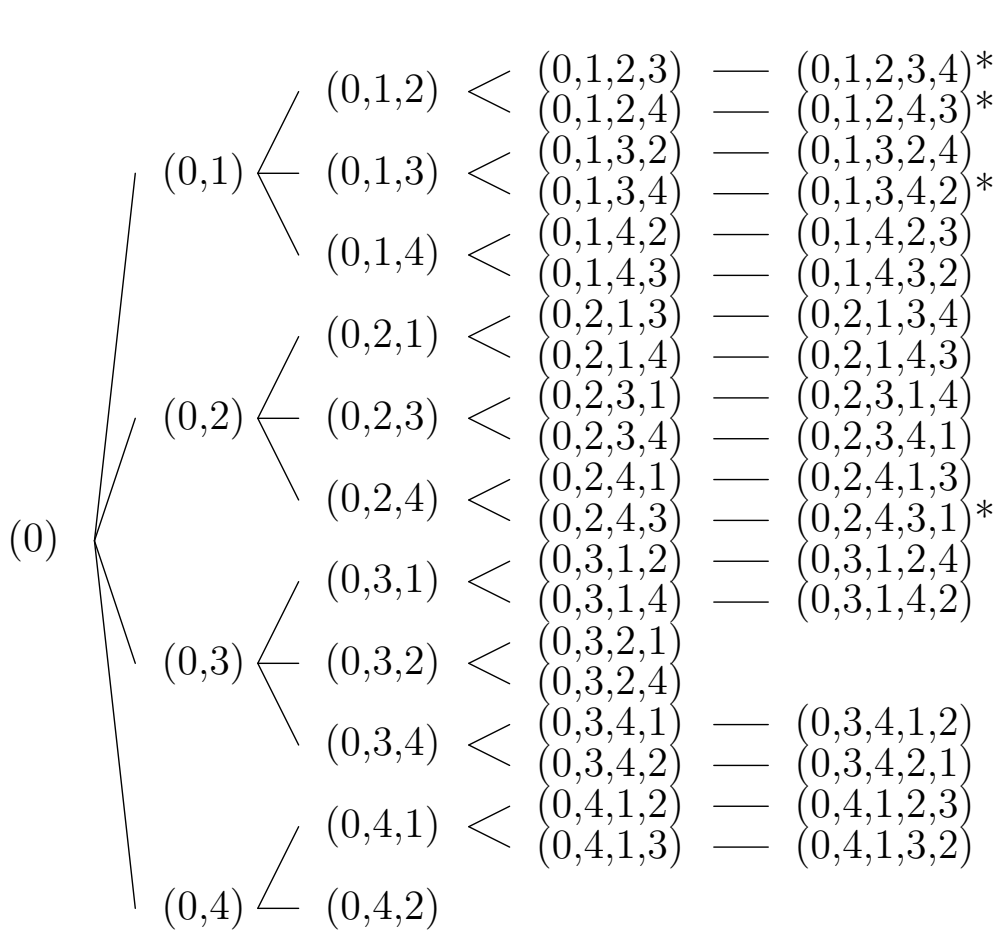
bound = 302.31

length = 392.31



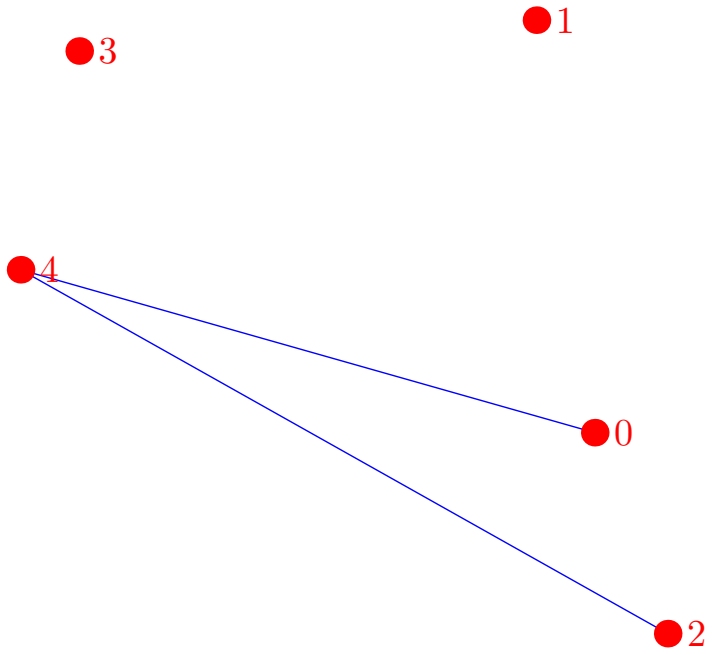
(0,4,1,3,2)

Branch and Bound in Action



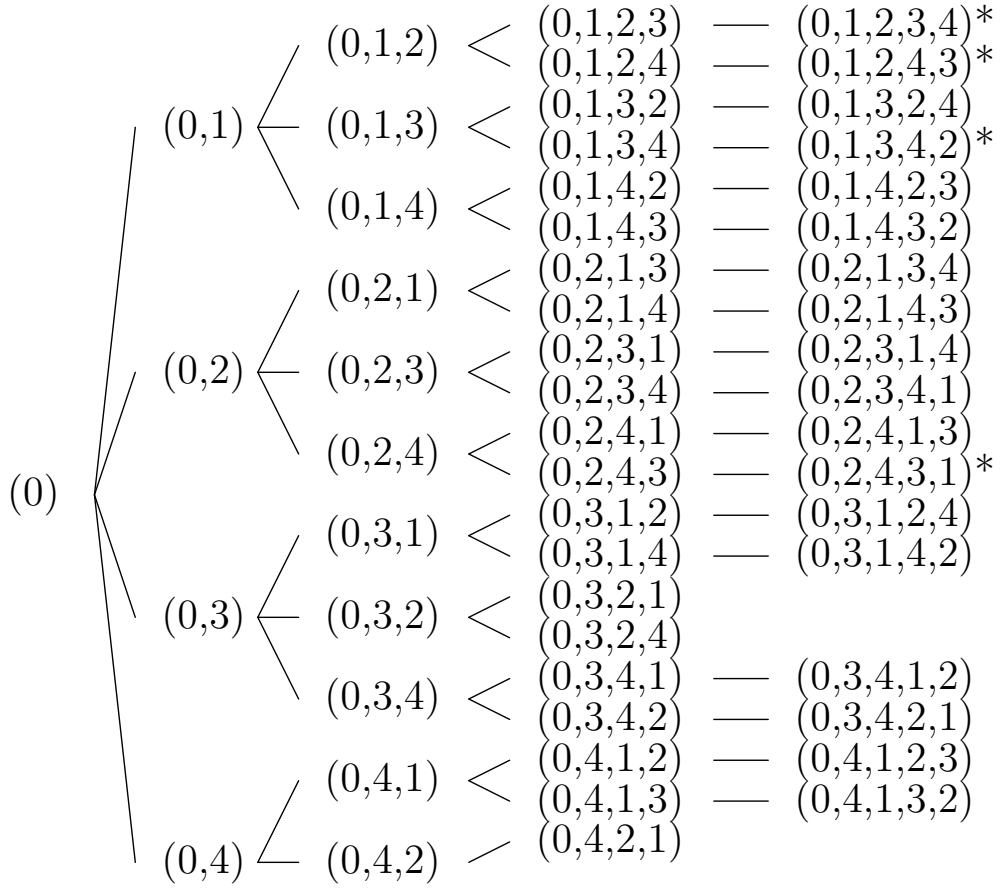
bound = 302.31

length = 196.77



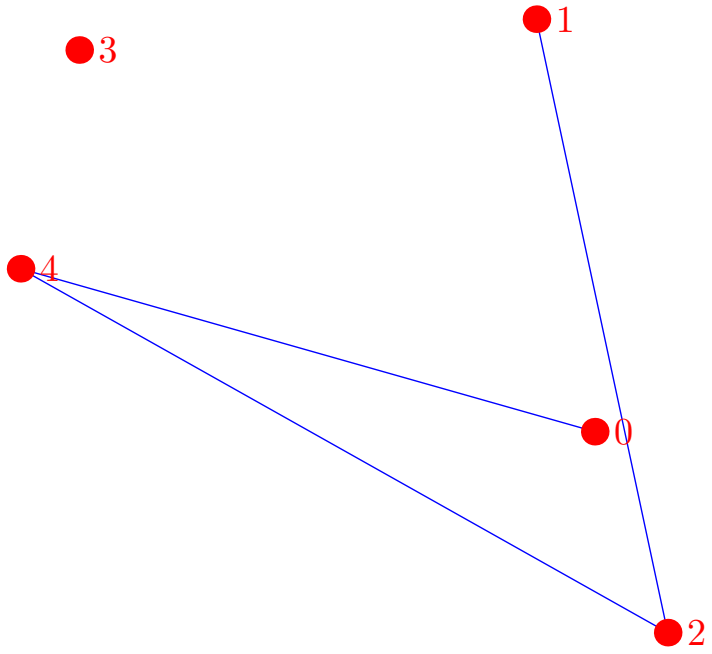
(0,4,2)

Branch and Bound in Action



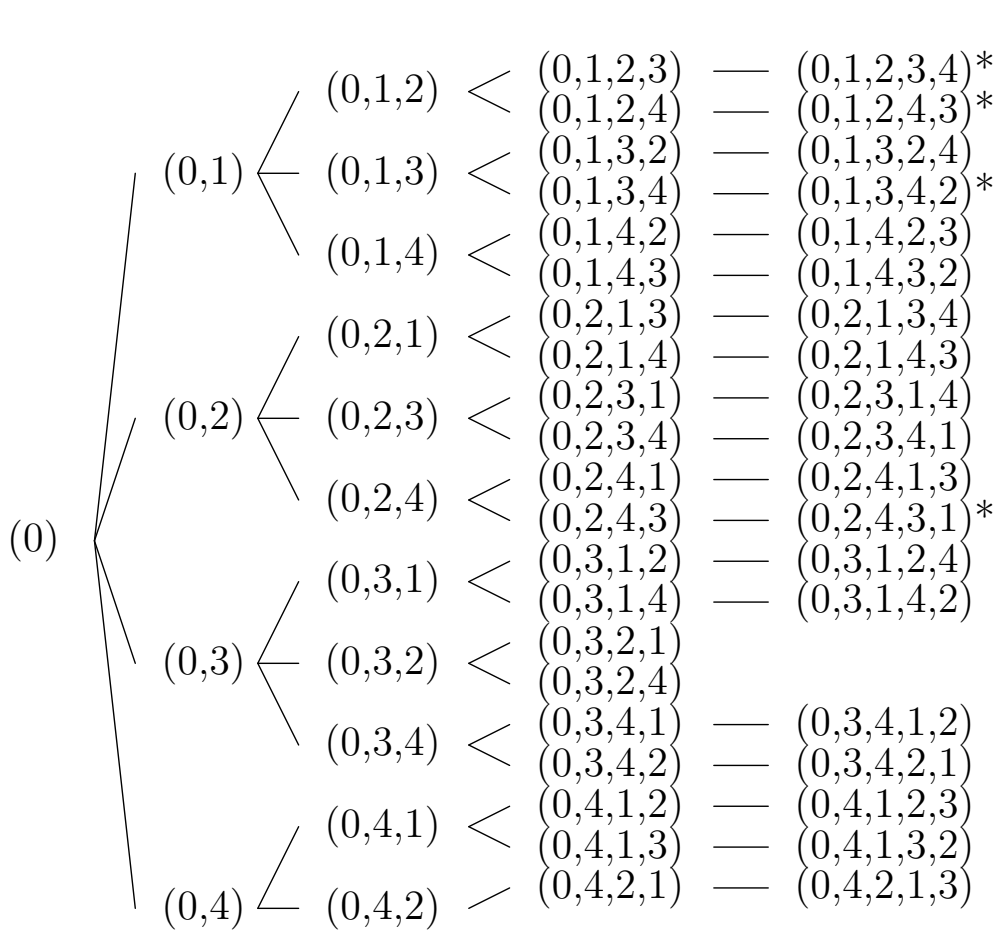
bound = 302.31

length = 288.95



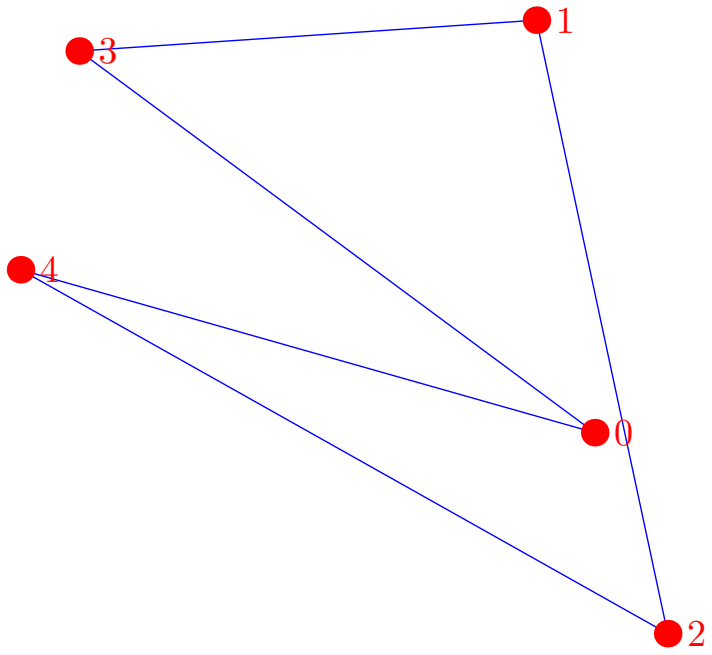
(0,4,2,1)

Branch and Bound in Action



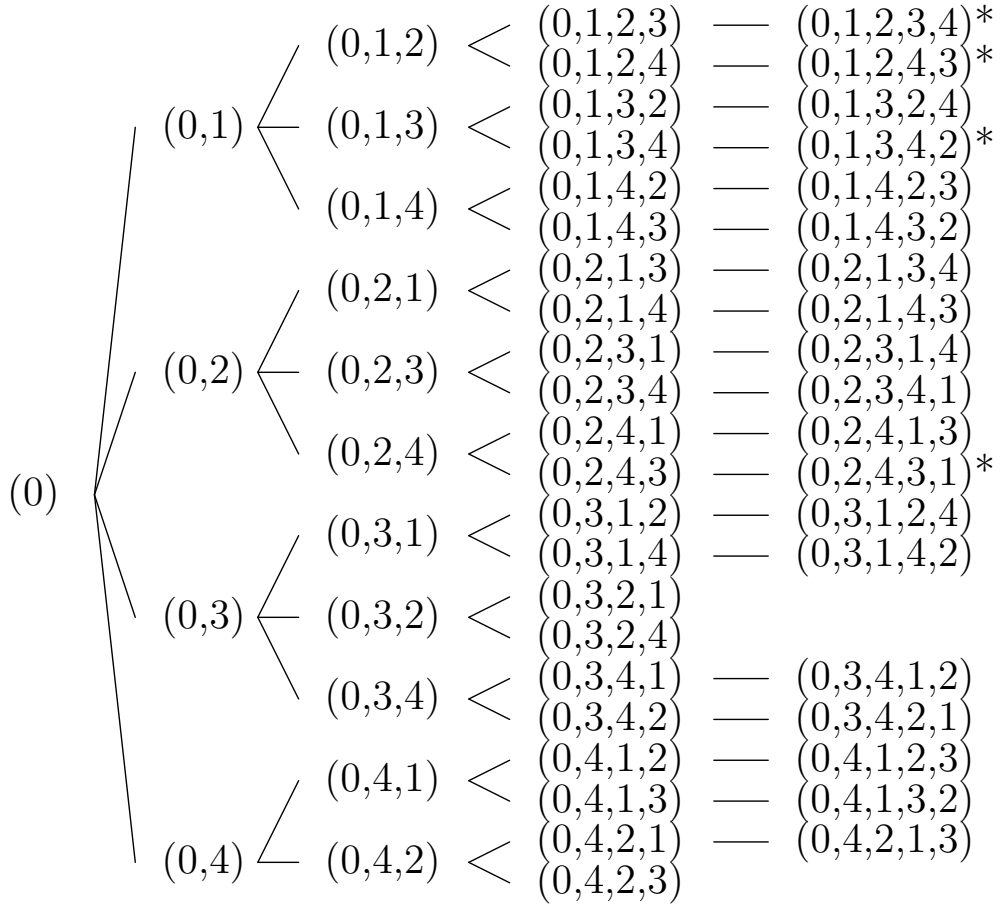
bound = 302.31

length = 450.52



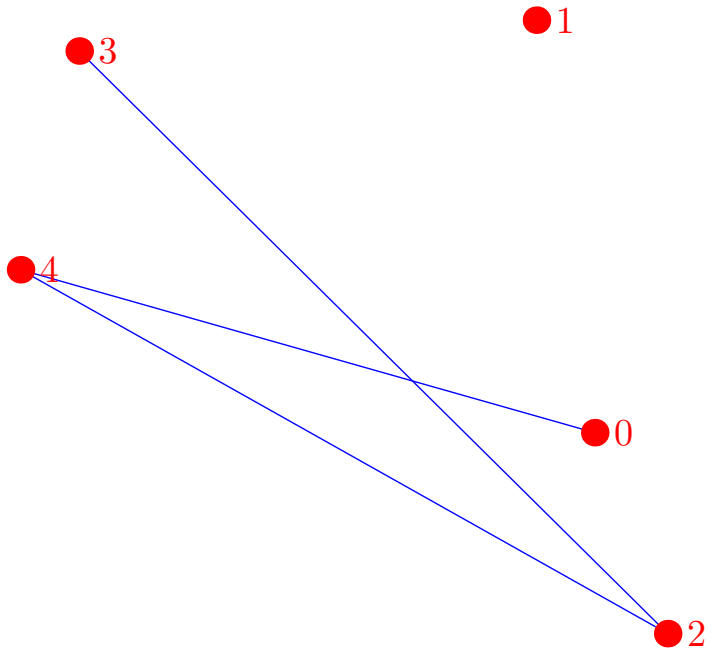
(0,4,2,1,3)

Branch and Bound in Action



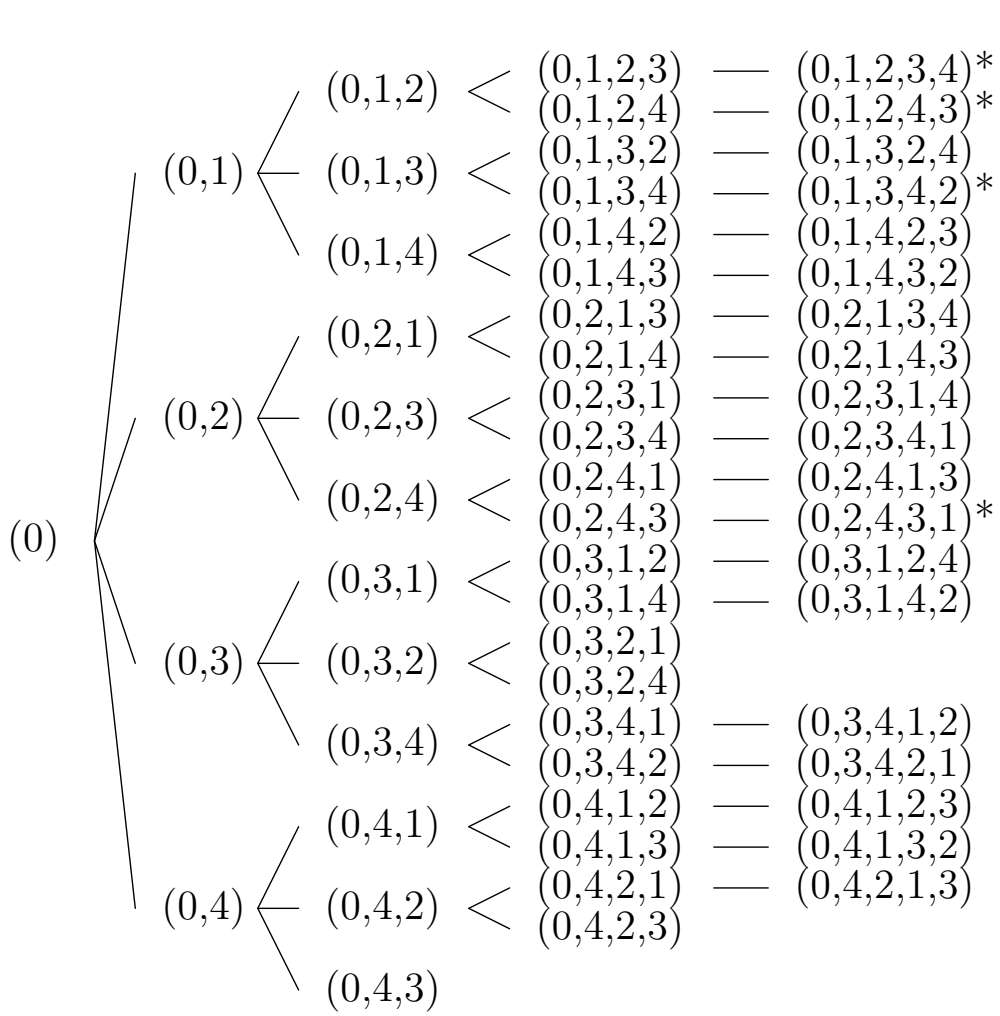
bound = 302.31

length = 318.44



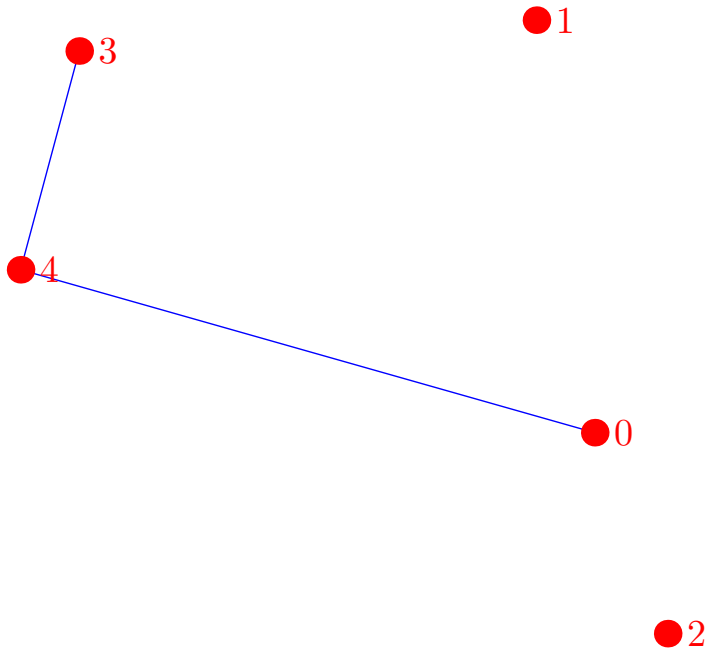
(0,4,2,3)

Branch and Bound in Action



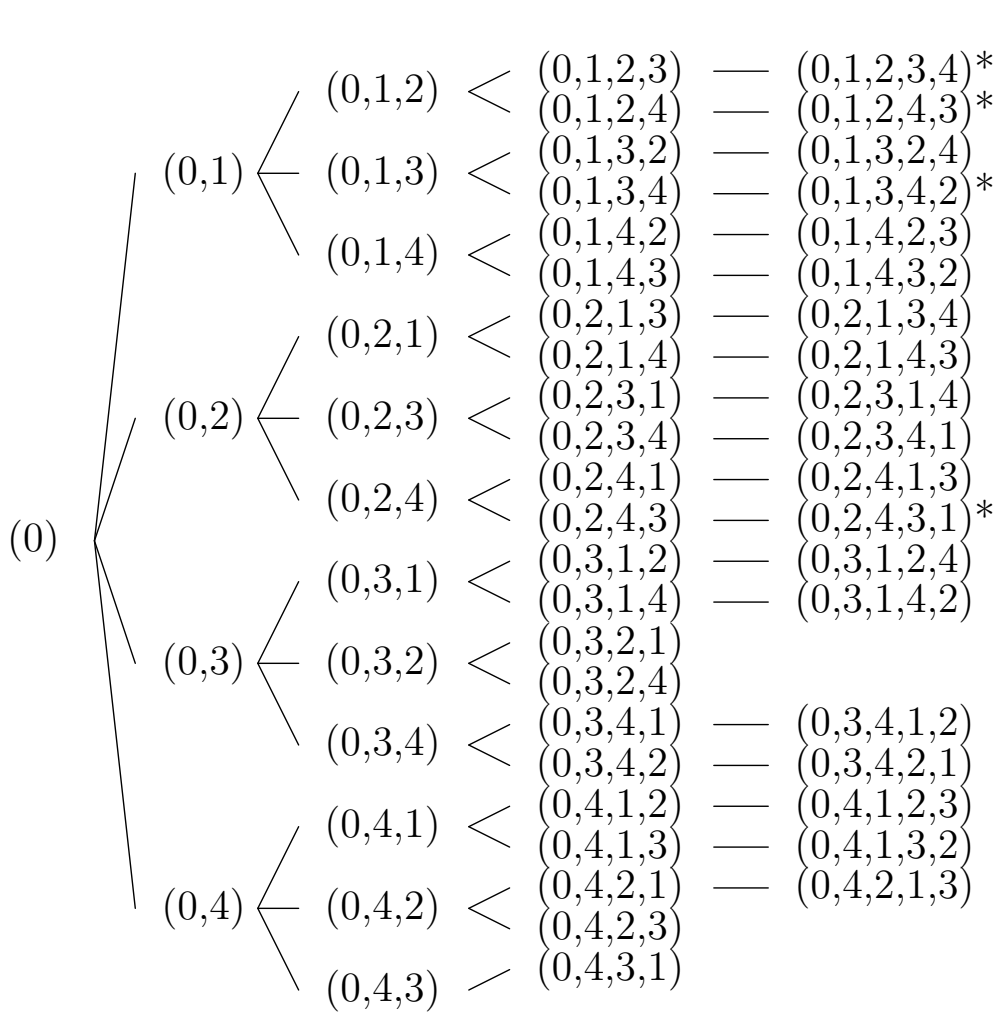
bound = 302.31

length = 120.97



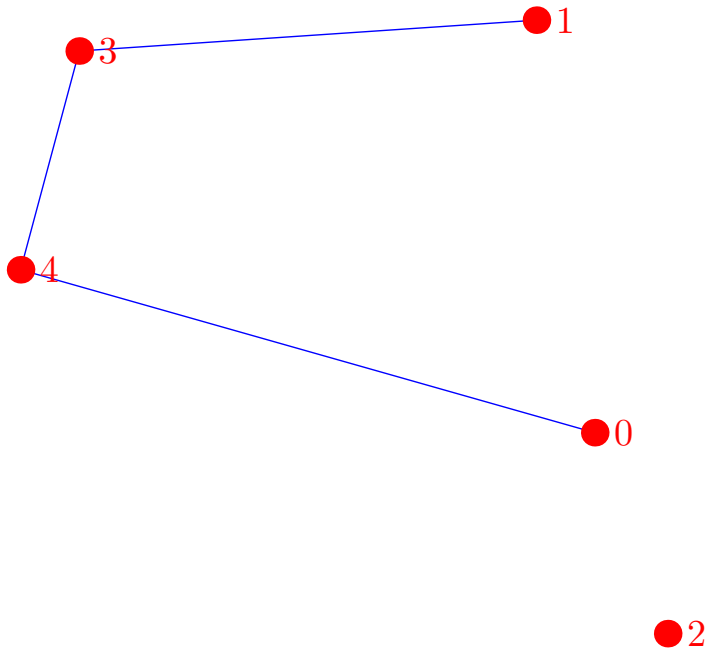
(0,4,3)

Branch and Bound in Action



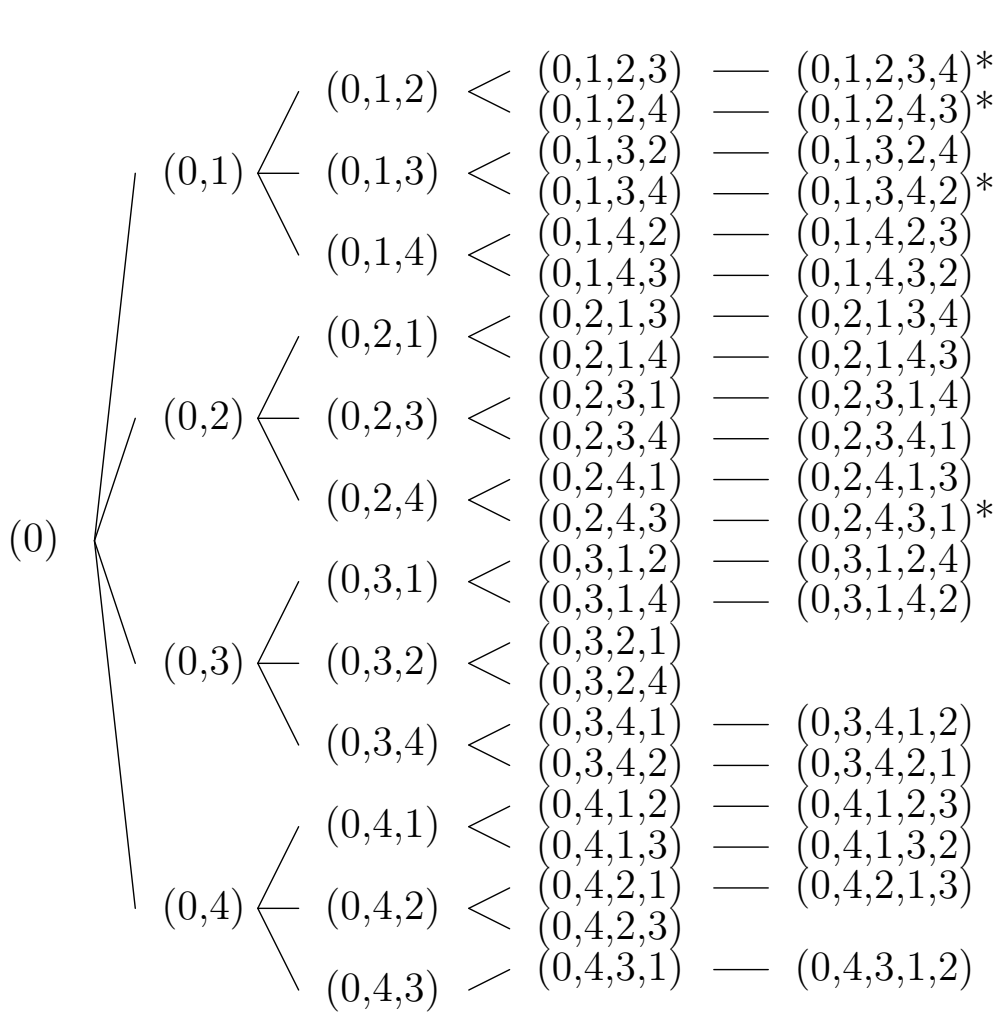
bound = 302.31

length = 188.3



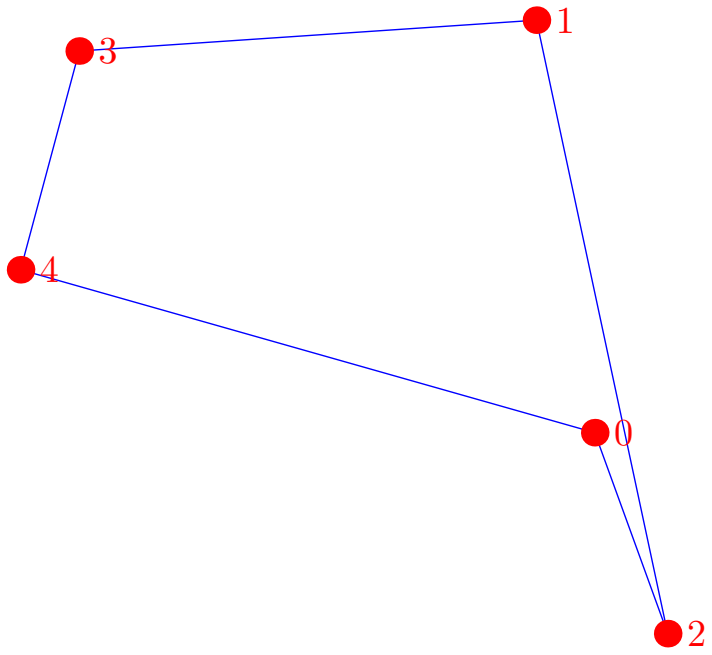
(0,4,3,1)

Branch and Bound in Action



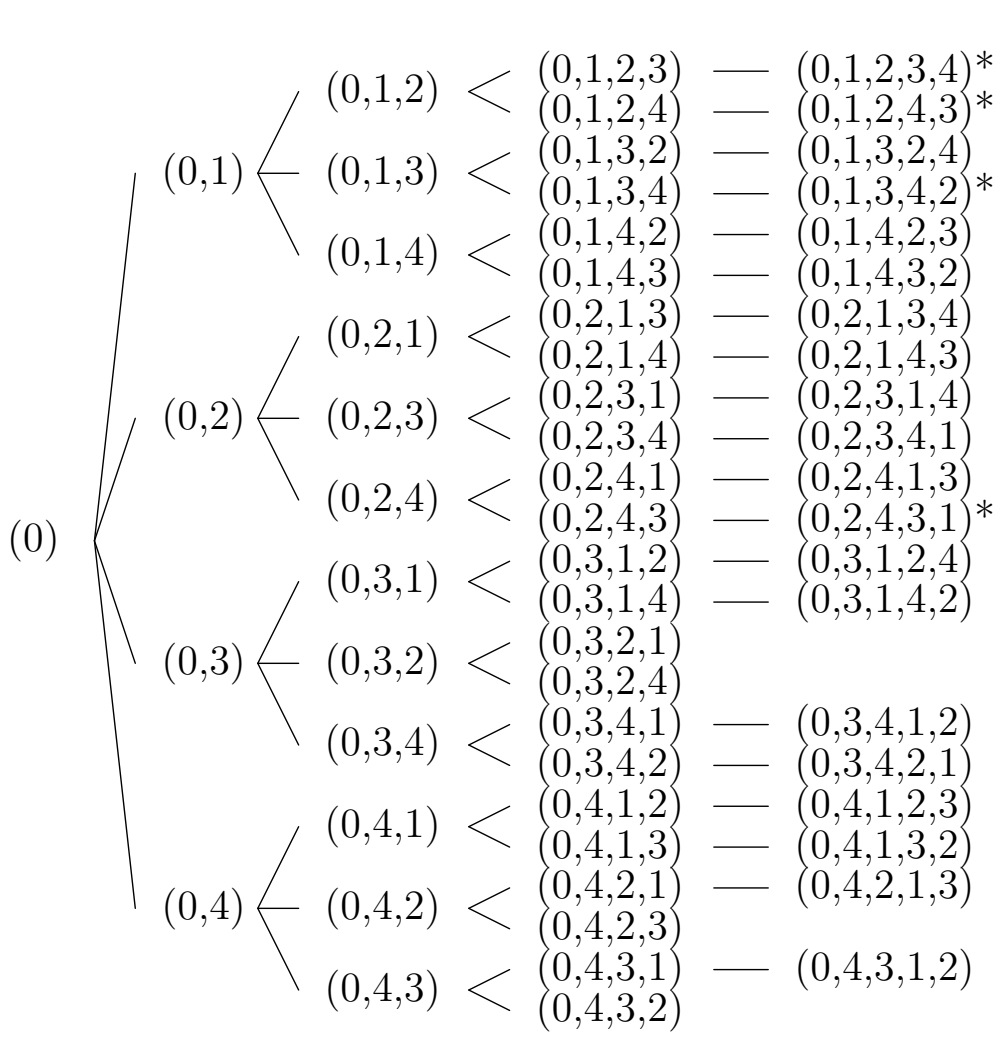
bound = 302.31

length = 311.88



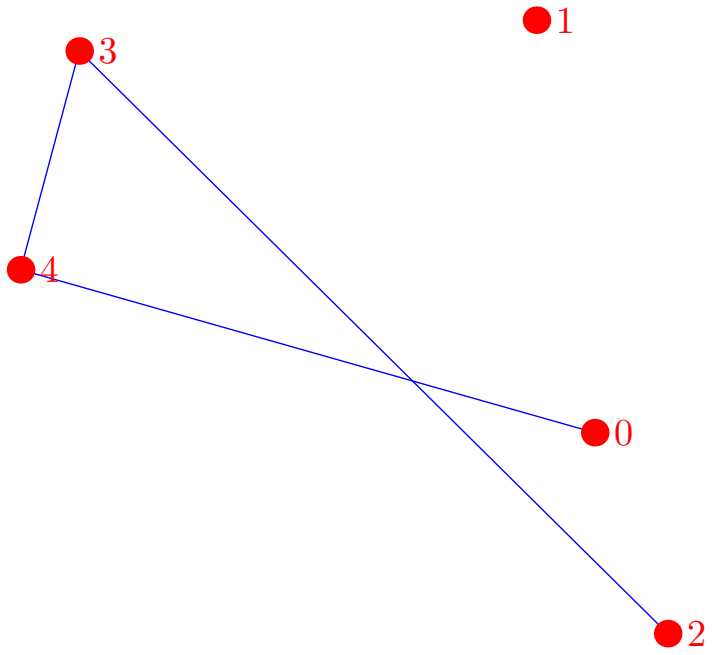
(0,4,3,1,2)

Branch and Bound in Action



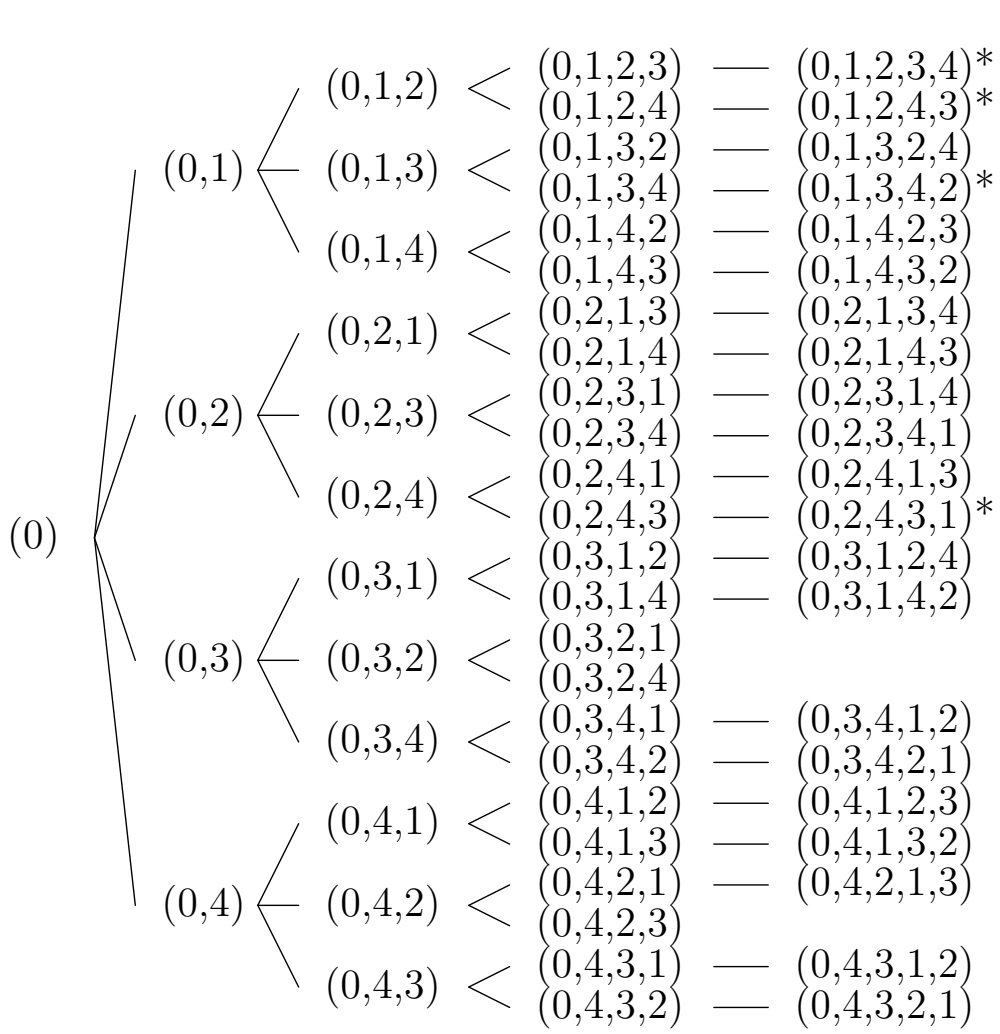
bound = 302.31

length = 242.64



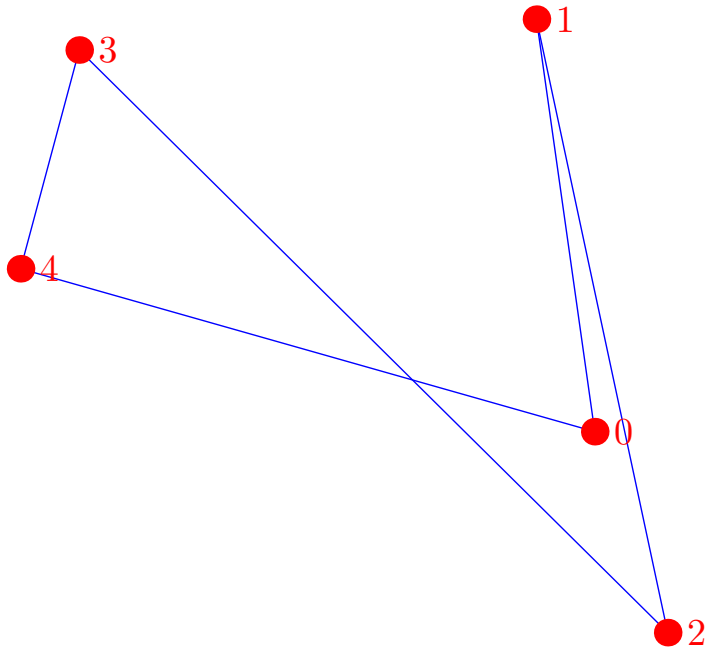
(0,4,3,2)

Branch and Bound in Action



bound = 302.31

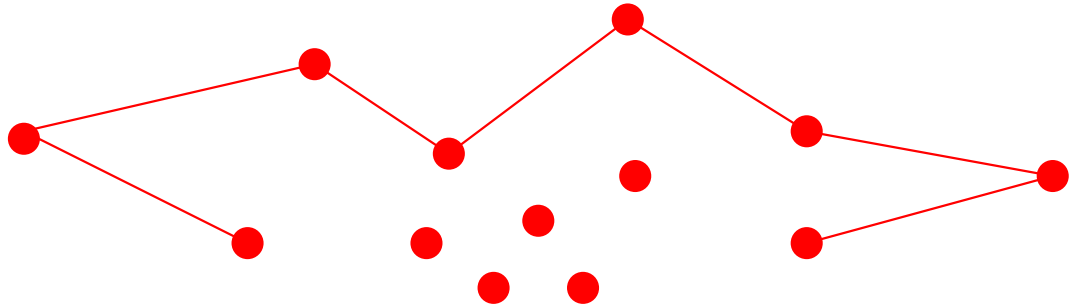
length = 396.02



(0,4,3,2,1)

Bound on Partial Solution

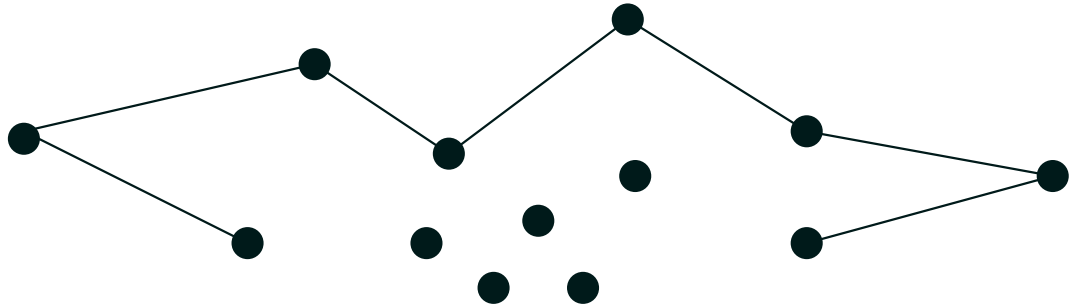
- We know that the partial solution has to include all the remaining cities



- We can use this to obtain a lower bound on the partial solution
- We know the remaining tour will go through each of the unvisited cities and the two edge cities
- In fact the remaining part of the tour is a spanning tree of these vertices (it connects all the vertices once and has no cycles)
- But we know a lower bound for this

Bound on Partial Solution

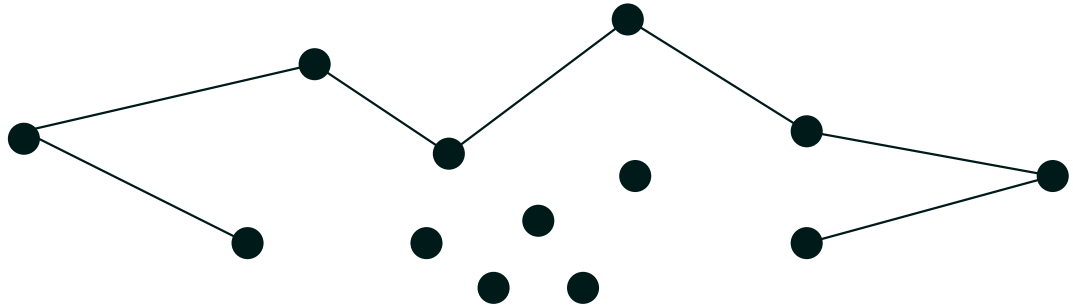
- We know that the partial solution has to include all the remaining cities



- We can use this to obtain a lower bound on the partial solution
- We know the remaining tour will go through each of the unvisited cities and the two edge cities
- In fact the remaining part of the tour is a spanning tree of these vertices (it connects all the vertices once and has no cycles)
- But we know a lower bound for this

Bound on Partial Solution

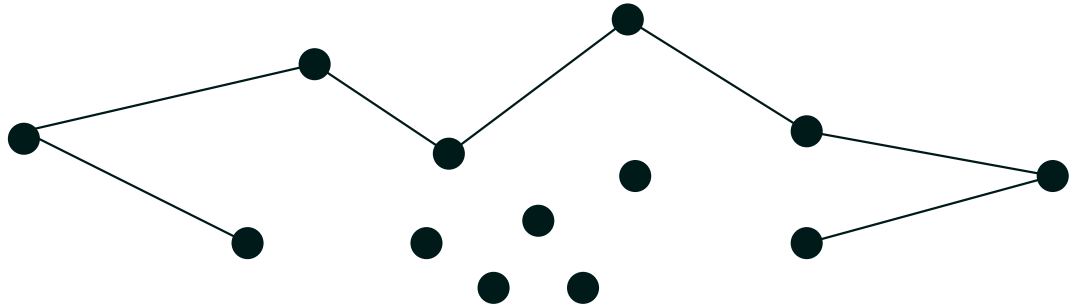
- We know that the partial solution has to include all the remaining cities



- We can use this to obtain a lower bound on the partial solution
- We know the remaining tour will go through each of the unvisited cities and the two edge cities
- In fact the remaining part of the tour is a spanning tree of these vertices (it connects all the vertices once and has no cycles)
- But we know a lower bound for this

Bound on Partial Solution

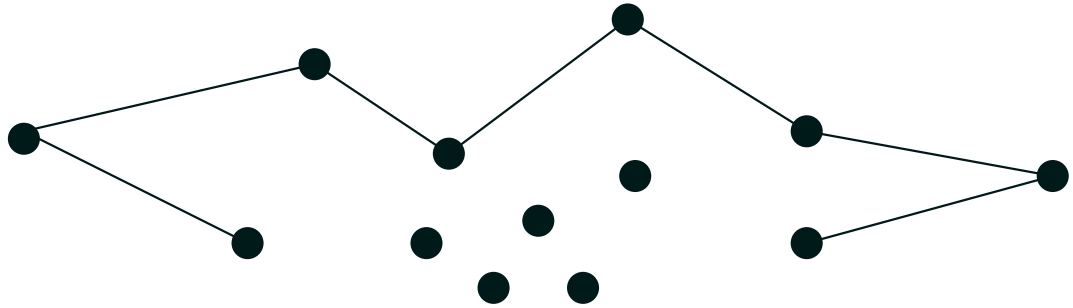
- We know that the partial solution has to include all the remaining cities



- We can use this to obtain a lower bound on the partial solution
- We know the remaining tour will go through each of the unvisited cities and the two edge cities
- In fact the remaining part of the tour is a spanning tree of these vertices (it connects all the vertices once and has no cycles)
- But we know a lower bound for this

Bound on Partial Solution

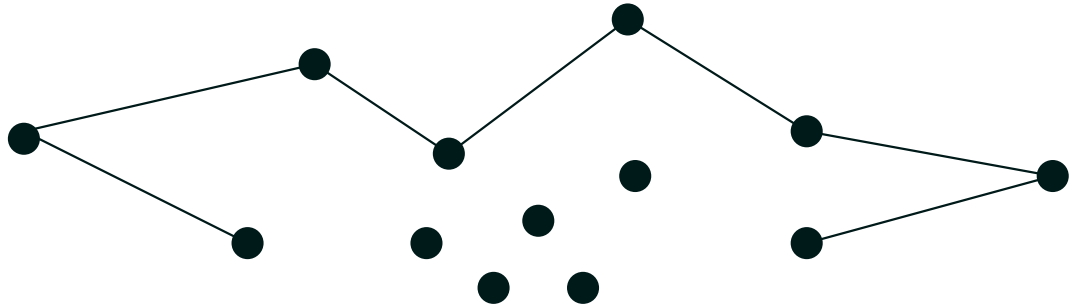
- We know that the partial solution has to include all the remaining cities



- We can use this to obtain a lower bound on the partial solution
- We know the remaining tour will go through each of the unvisited cities and the two edge cities
- In fact the remaining part of the tour is a spanning tree of these vertices (it connects all the vertices once and has no cycles)
- But we know a lower bound for this

Bound on Partial Solution

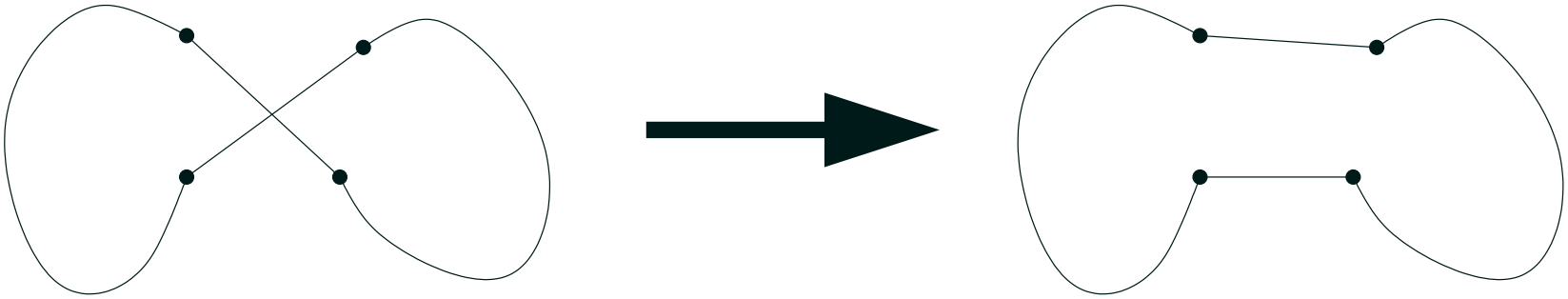
- We know that the partial solution has to include all the remaining cities



- We can use this to obtain a lower bound on the partial solution
- We know the remaining tour will go through each of the unvisited cities and the two edge cities
- In fact the remaining part of the tour is a spanning tree of these vertices (it connects all the vertices once and has no cycles)
- But we know a lower bound for this—**the minimum spanning tree**

Other Cuts

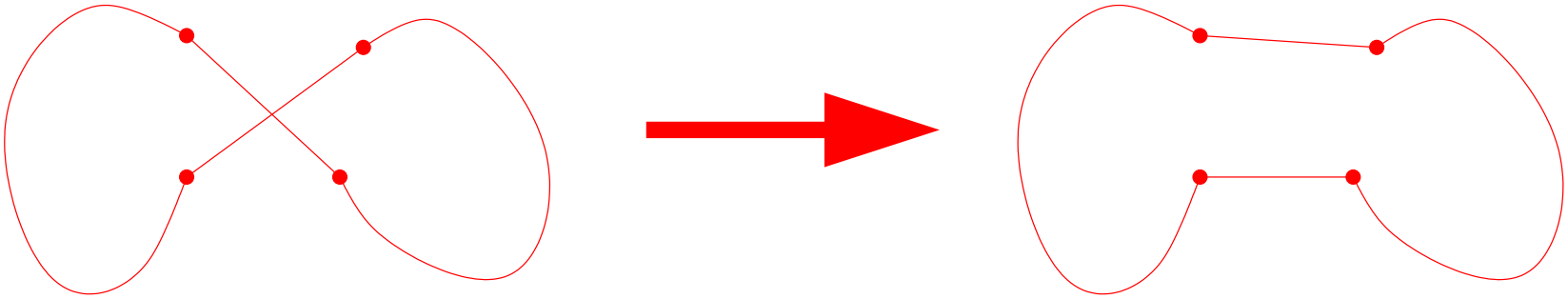
- For 2-D Euclidean TSPs edges should never cross



- In fact we can check that we cannot perform a 2-opt move
- We can also halve the search by considering only one direction—for example, by insisting we visit city 1 before city 2

Other Cuts

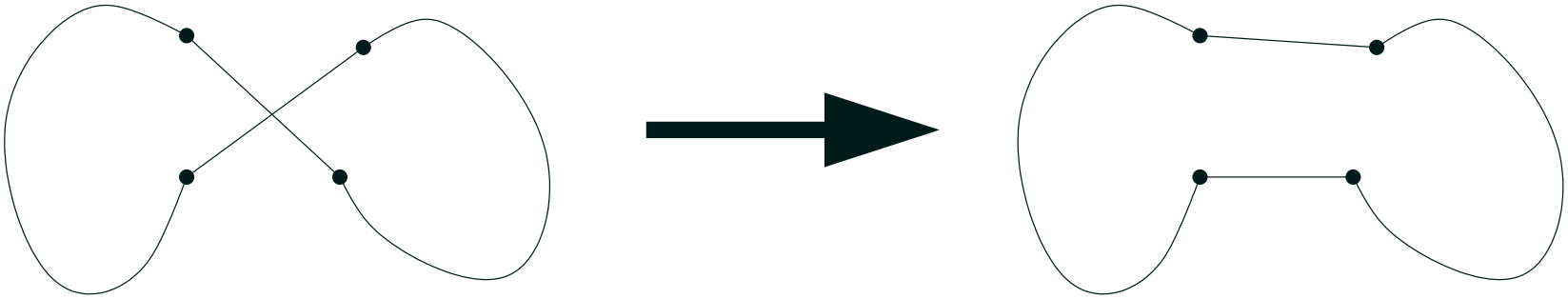
- For 2-D Euclidean TSPs edges should never cross



- In fact we can check that we cannot perform a 2-opt move
- We can also halve the search by considering only one direction—for example, by insisting we visit city 1 before city 2

Other Cuts

- For 2-D Euclidean TSPs edges should never cross

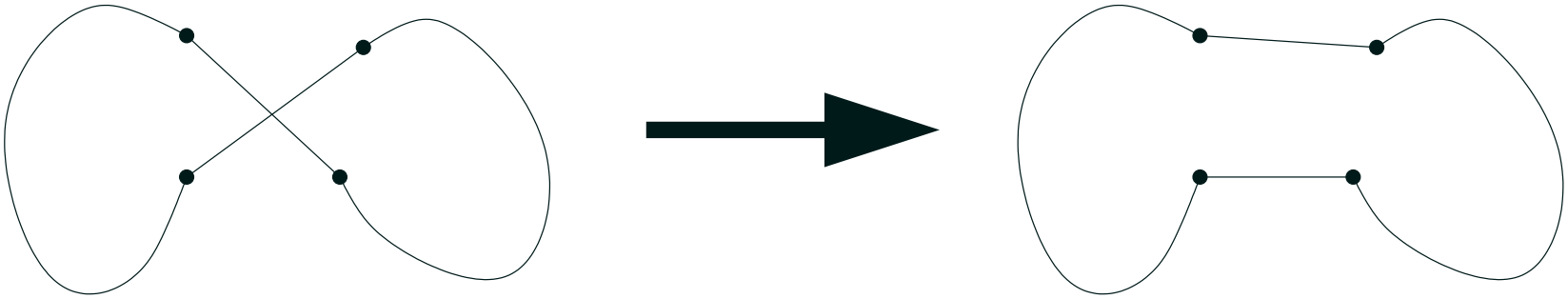


- In fact we can check that we cannot perform a 2-opt move

- We can also halve the search by considering only one direction—for example, by insisting we visit city 1 before city 2

Other Cuts

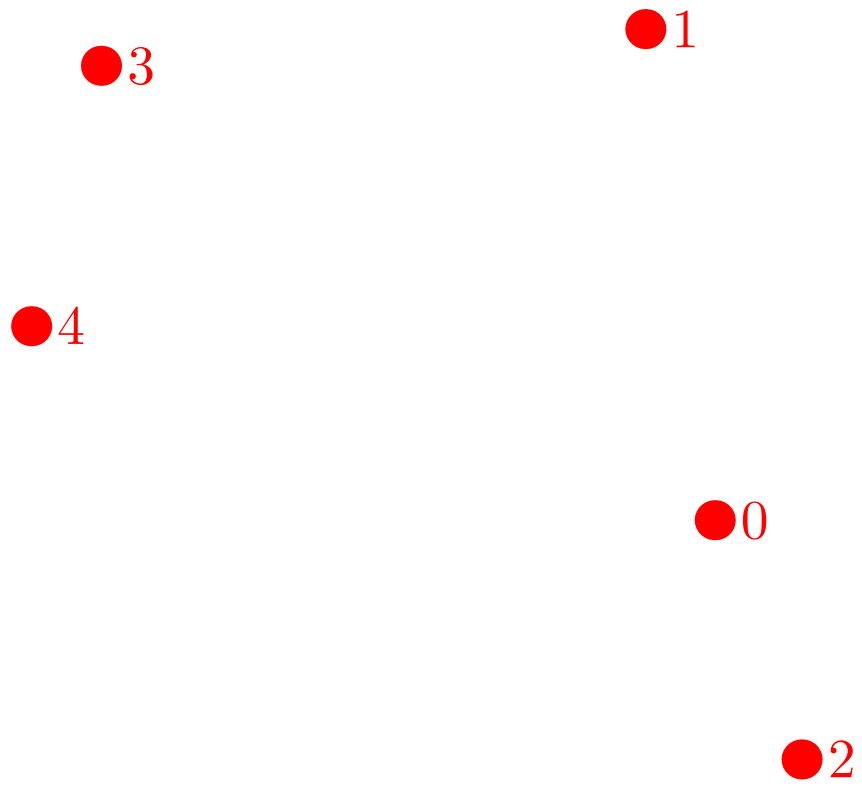
- For 2-D Euclidean TSPs edges should never cross



- In fact we can check that we cannot perform a 2-opt move
- We can also halve the search by considering only one direction—for example, by insisting we visit city 1 before city 2

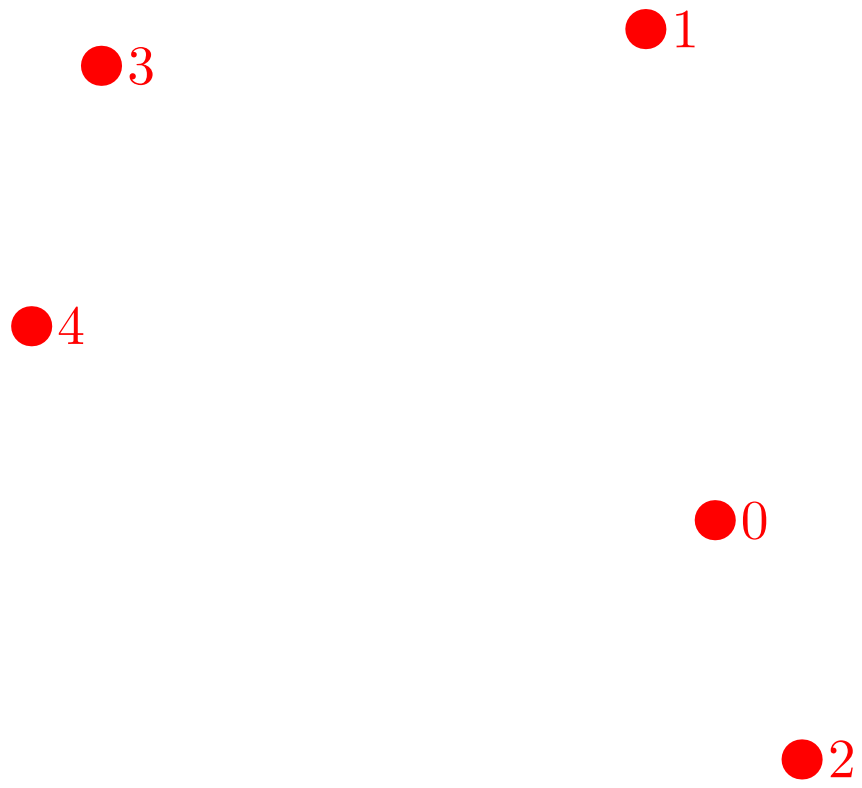
Good Starting Bound

- It helps to start with a good bound
- We can use an *incomplete heuristic algorithm* to find a good solution which will act as a starting bound
- One very simple heuristic is a greedy algorithm



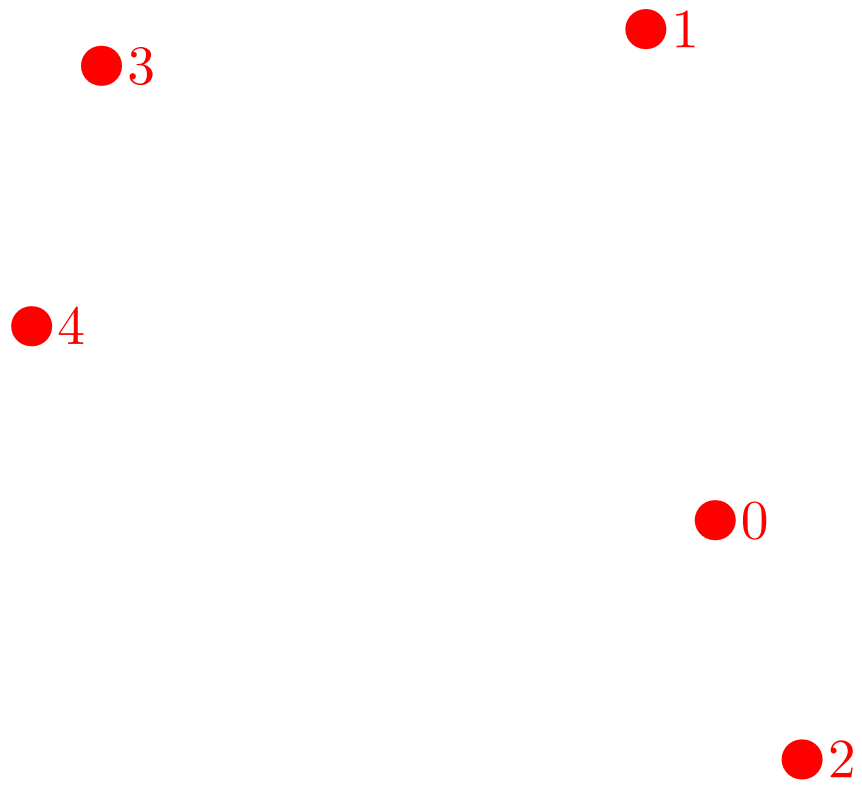
Good Starting Bound

- It helps to start with a good bound
- We can use an *incomplete heuristic algorithm* to find a good solution which will act as a starting bound
- One very simple heuristic is a greedy algorithm



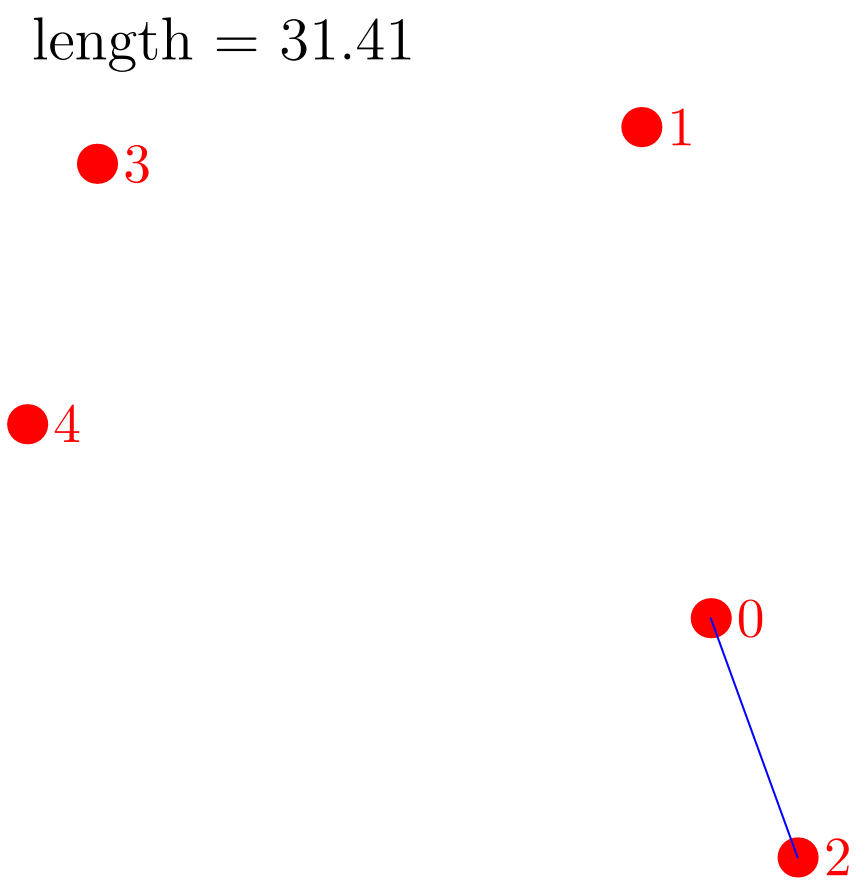
Good Starting Bound

- It helps to start with a good bound
- We can use an *incomplete heuristic algorithm* to find a good solution which will act as a starting bound
- One very simple heuristic is a greedy algorithm



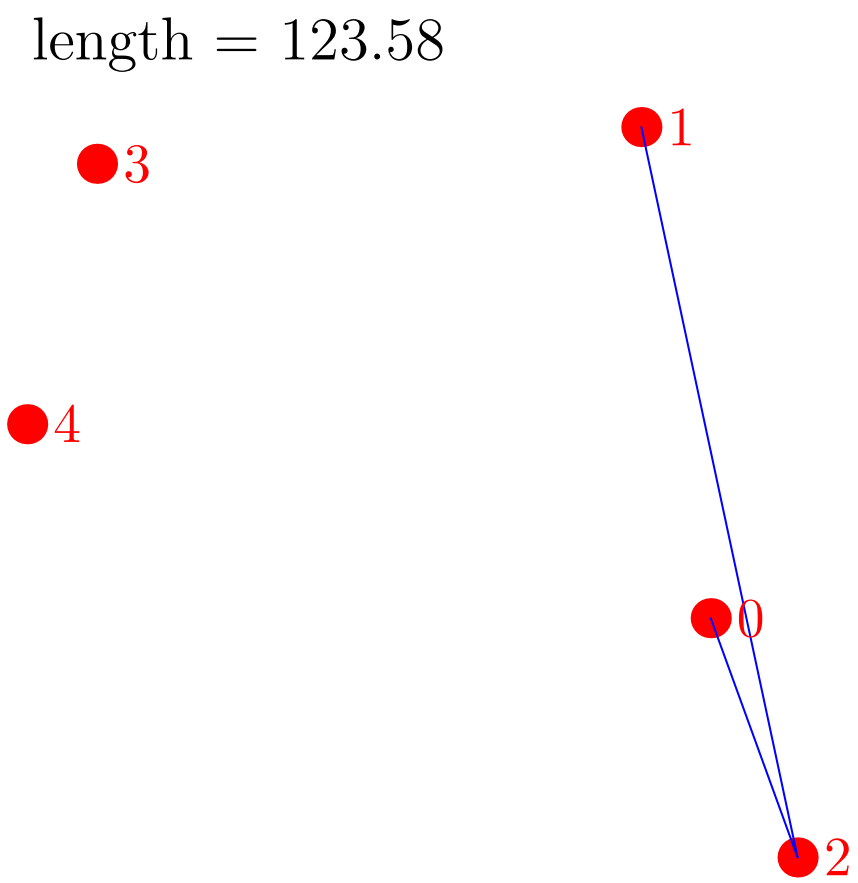
Good Starting Bound

- It helps to start with a good bound
- We can use an *incomplete heuristic algorithm* to find a good solution which will act as a starting bound
- One very simple heuristic is a greedy algorithm



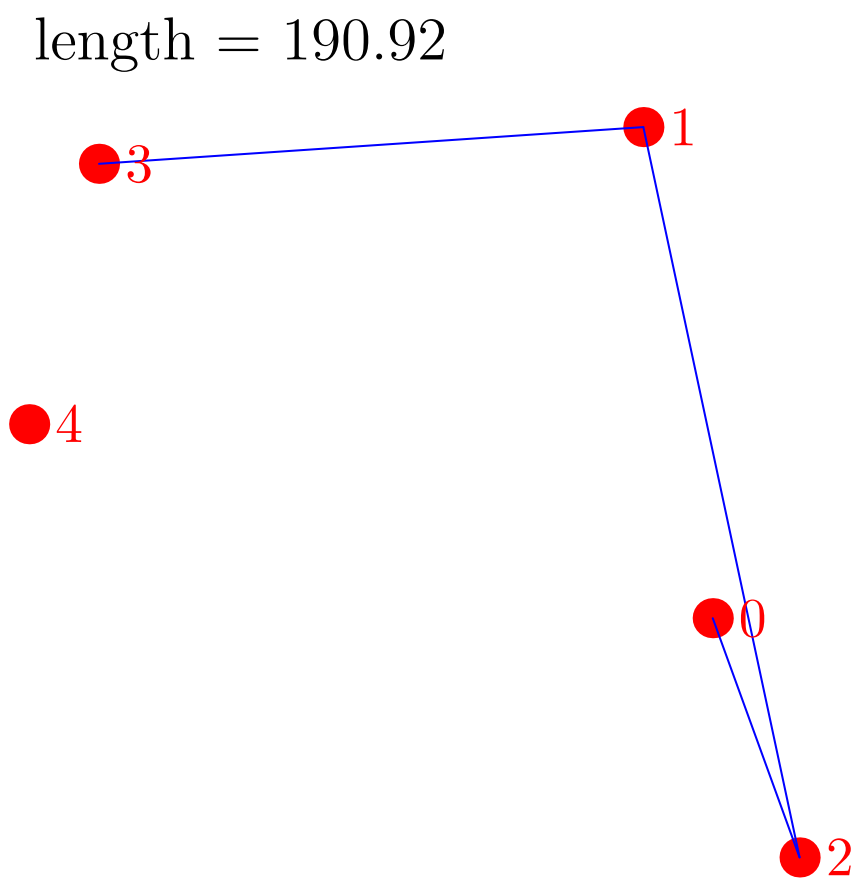
Good Starting Bound

- It helps to start with a good bound
- We can use an *incomplete heuristic algorithm* to find a good solution which will act as a starting bound
- One very simple heuristic is a greedy algorithm



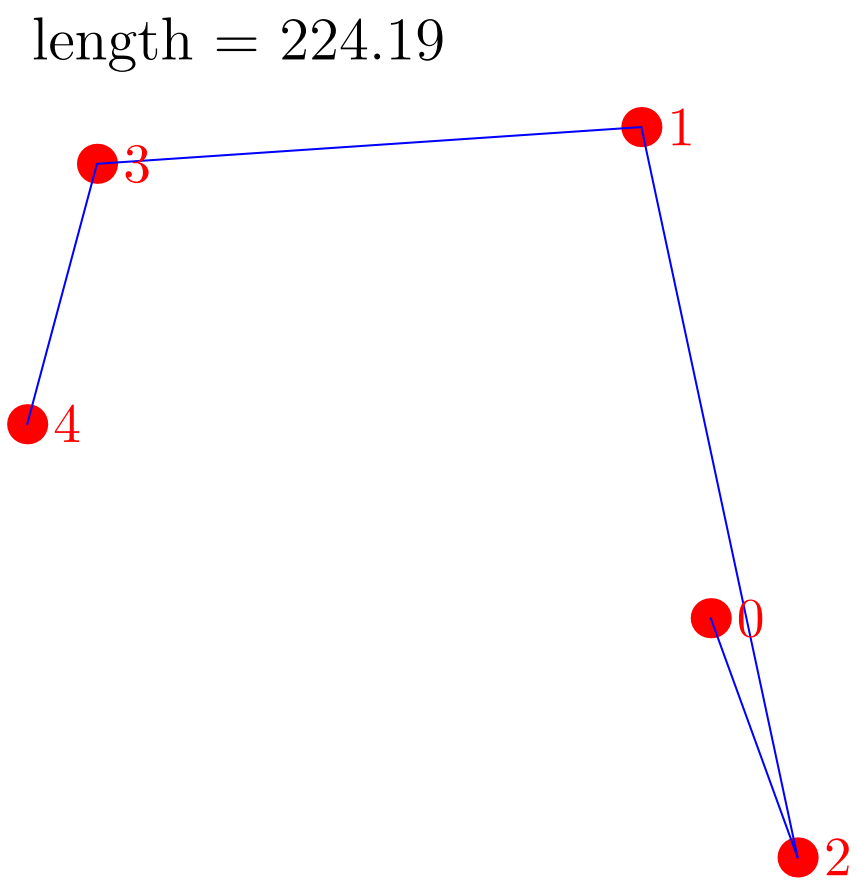
Good Starting Bound

- It helps to start with a good bound
- We can use an *incomplete heuristic algorithm* to find a good solution which will act as a starting bound
- One very simple heuristic is a greedy algorithm



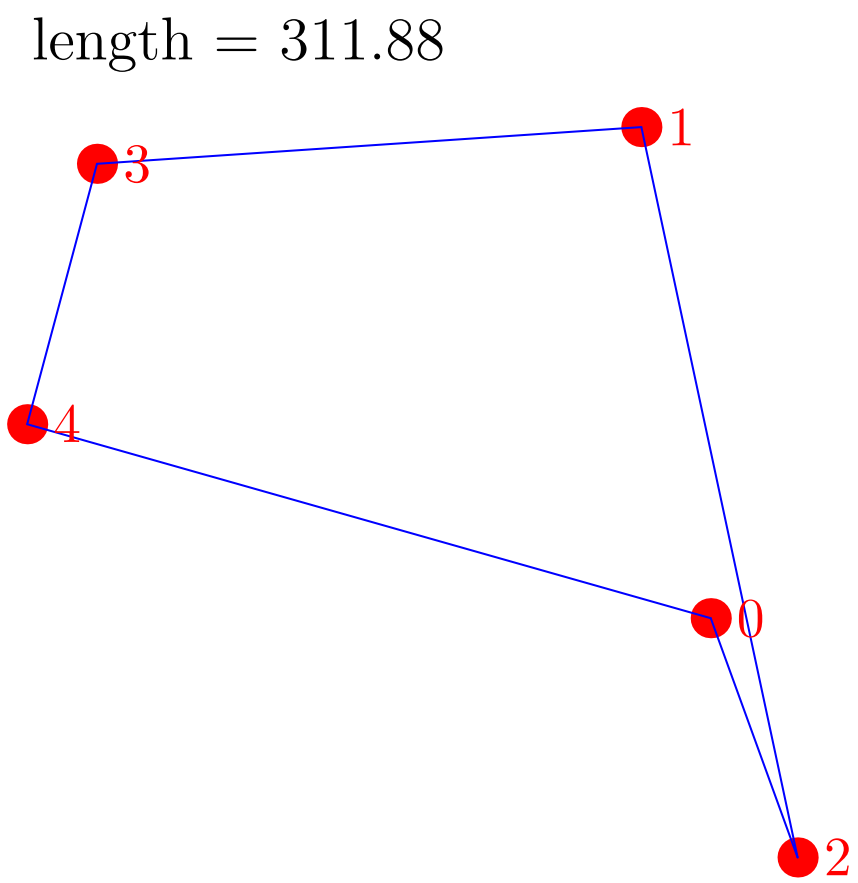
Good Starting Bound

- It helps to start with a good bound
- We can use an *incomplete heuristic algorithm* to find a good solution which will act as a starting bound
- One very simple heuristic is a greedy algorithm

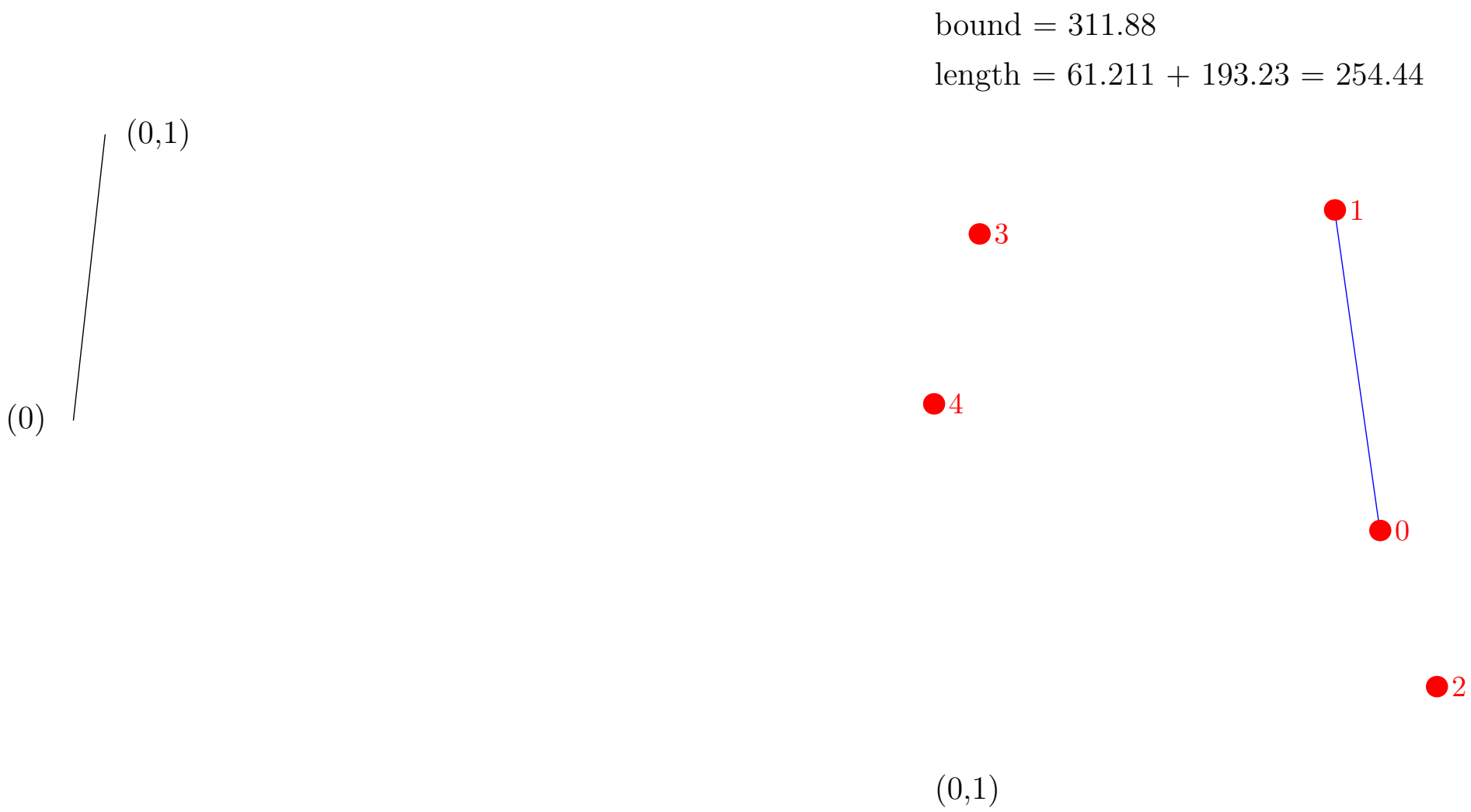


Good Starting Bound

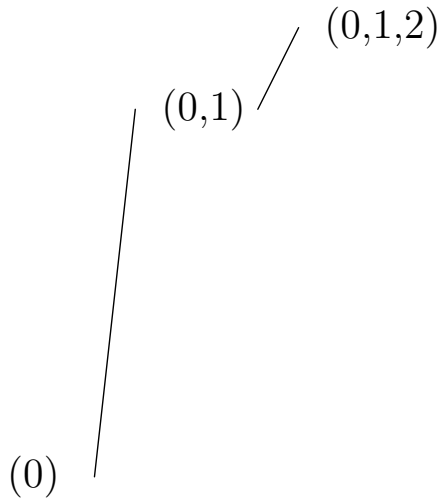
- It helps to start with a good bound
- We can use an *incomplete heuristic algorithm* to find a good solution which will act as a starting bound
- One very simple heuristic is a greedy algorithm



Branch and Bound after Pruning

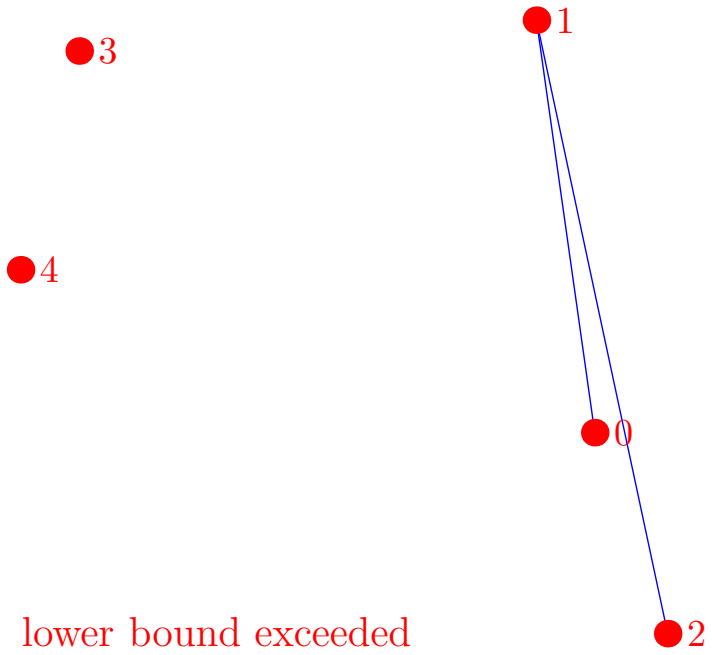


Branch and Bound after Pruning



bound = 311.88

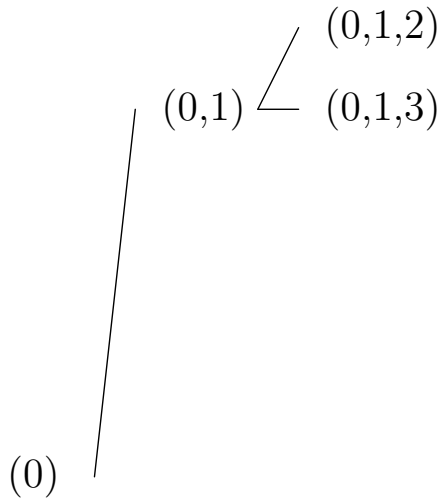
length = 153.38 + 159.95 = 313.34



lower bound exceeded

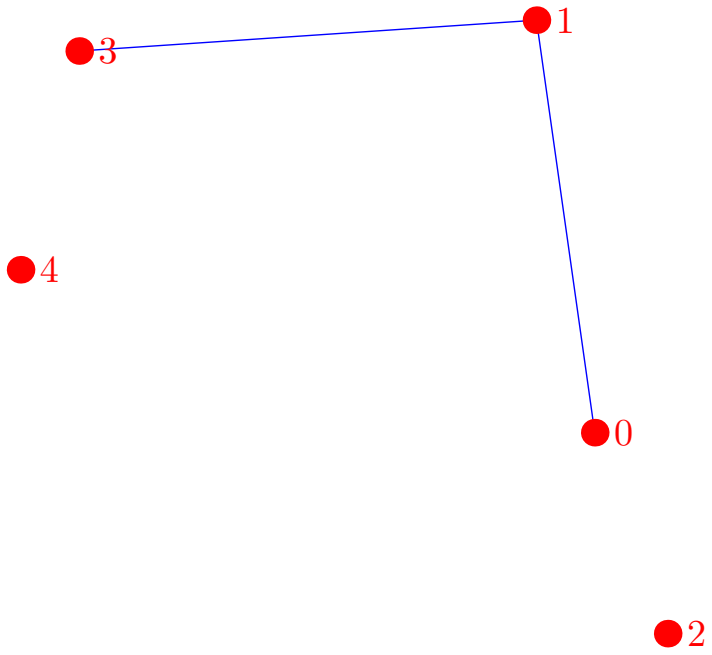
(0,1,2)

Branch and Bound after Pruning



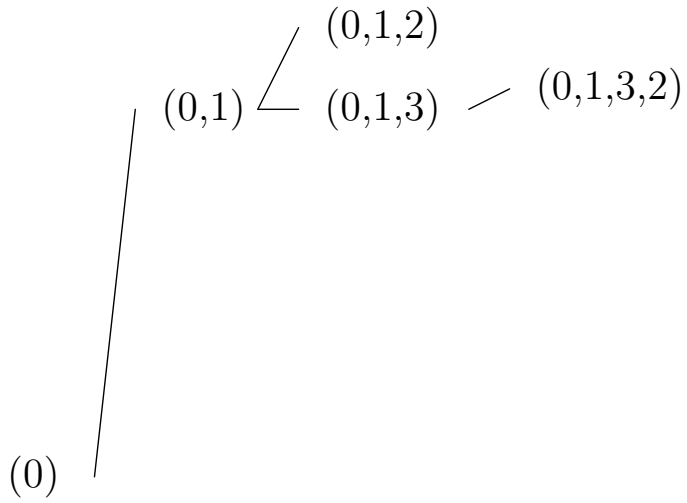
bound = 311.88

length = 128.54 + 159.95 = 288.5



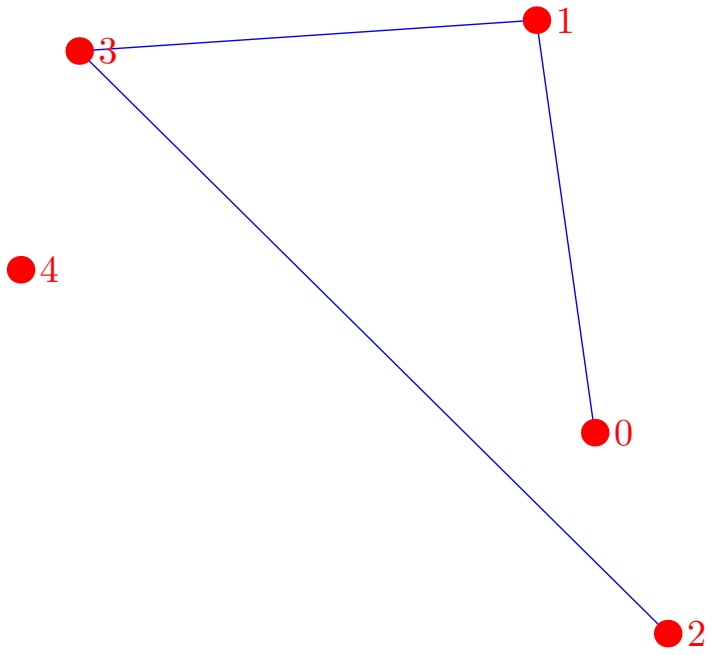
(0,1,3)

Branch and Bound after Pruning



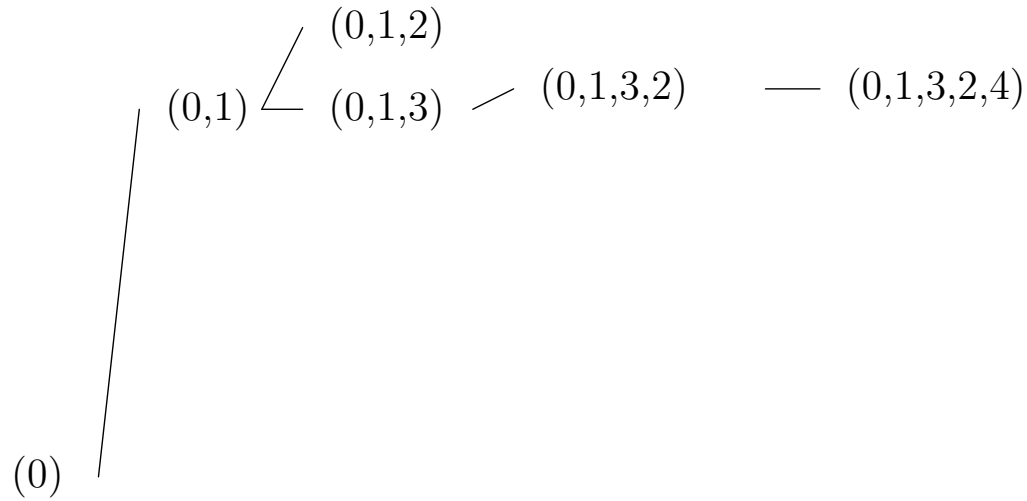
bound = 311.88

length = 250.21 + 31.41 = 281.63



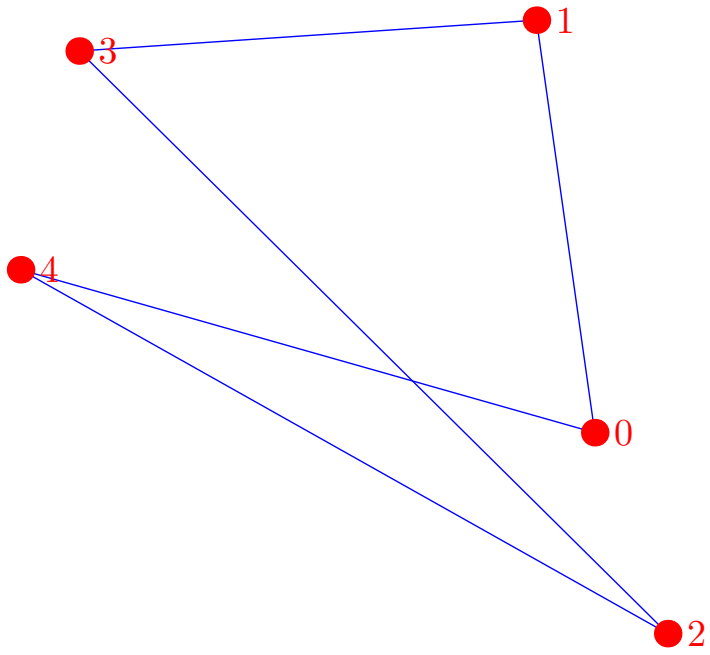
(0,1,3,2)

Branch and Bound after Pruning



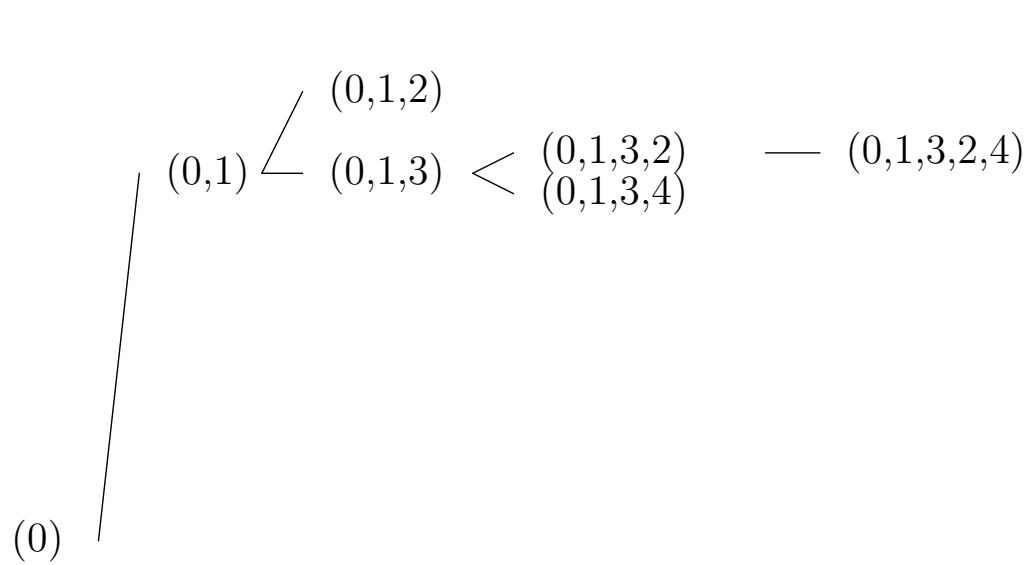
bound = 311.88

length = 446.99 + 0 = 446.99



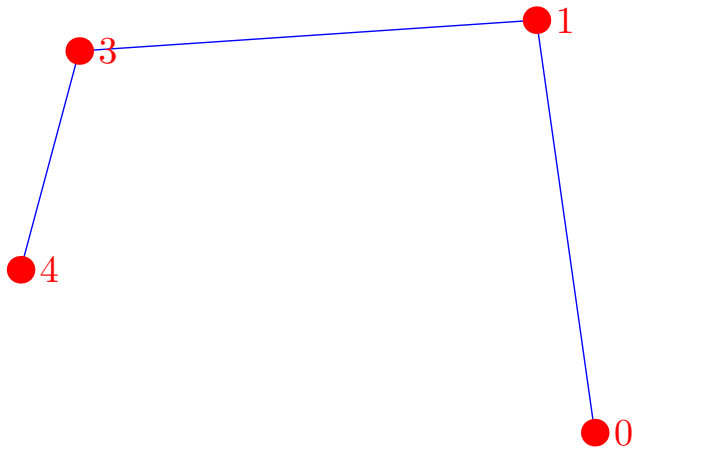
(0,1,3,2,4)

Branch and Bound after Pruning



bound = 311.88

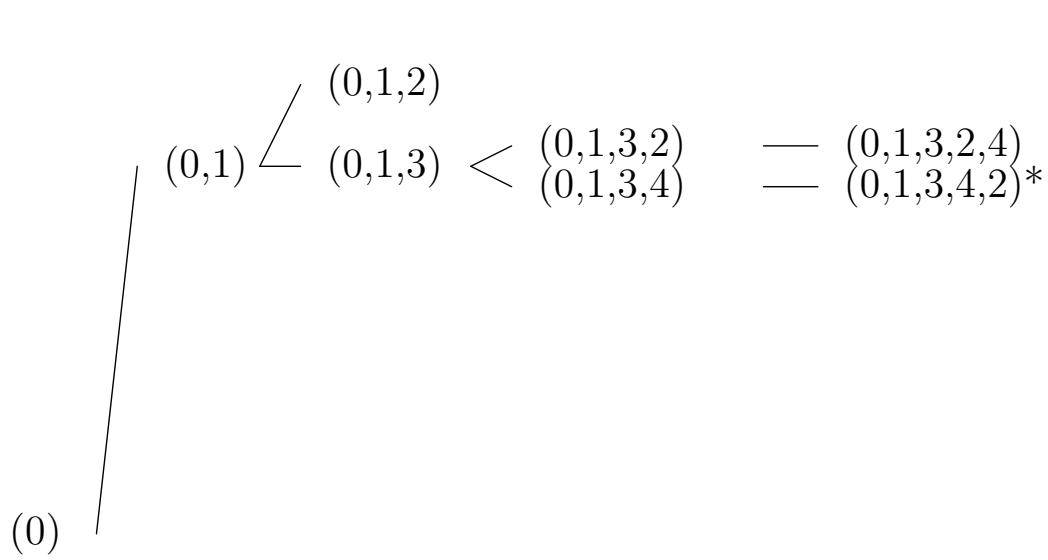
length = 161.82 + 31.41 = 193.23



2

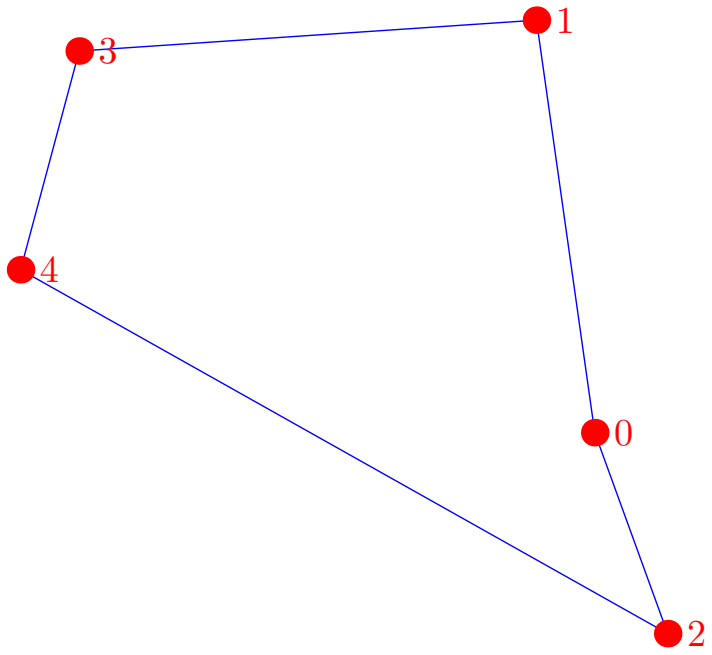
(0,1,3,4)

Branch and Bound after Pruning



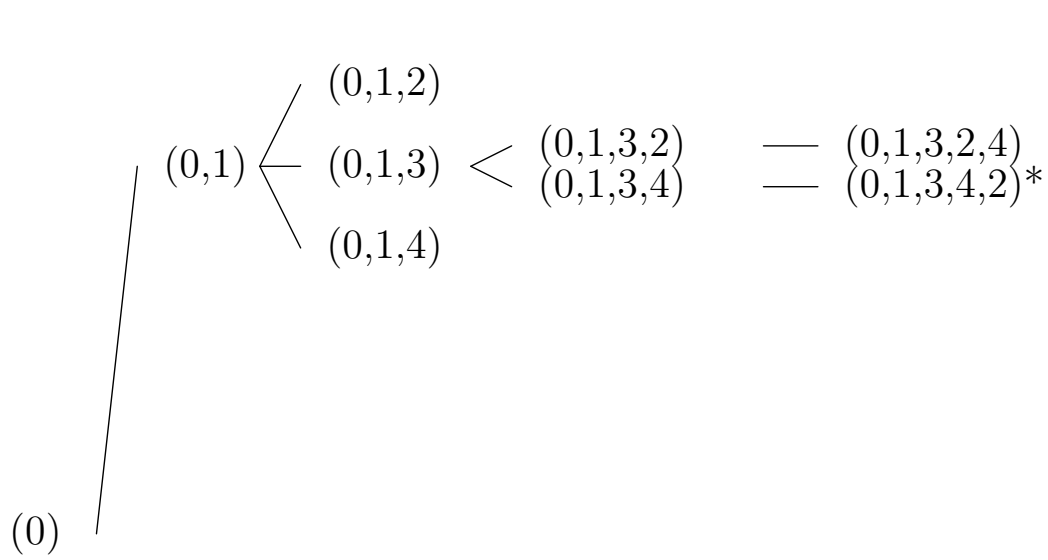
bound = 302.31

length = 302.31 + 0 = 302.31



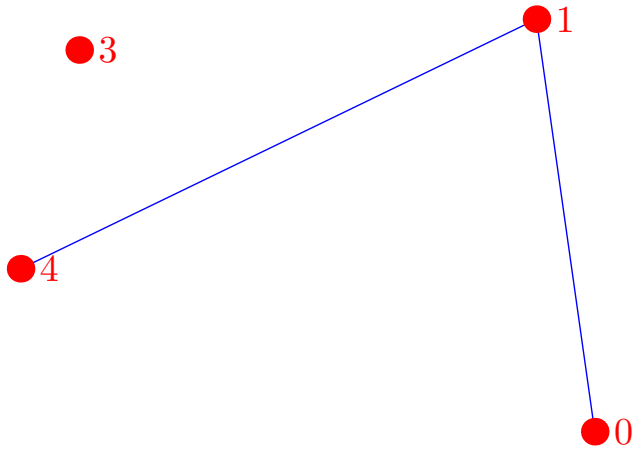
(0,1,3,4,2)

Branch and Bound after Pruning



bound = 302.31

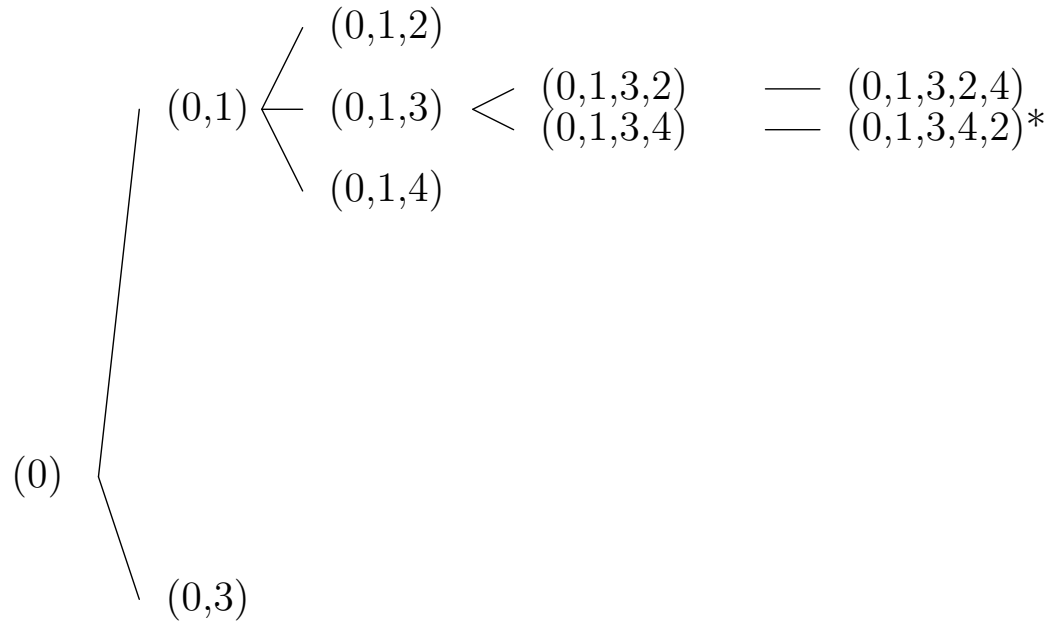
length = 145.41 + 159.95 = 305.37



lower bound exceeded 2

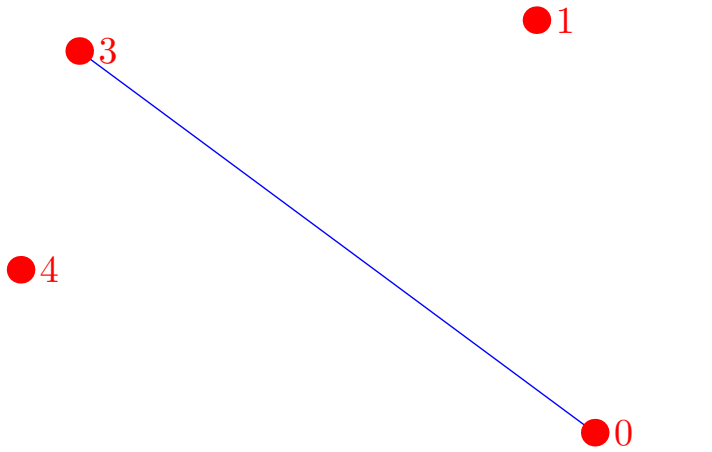
(0,1,4)

Branch and Bound after Pruning



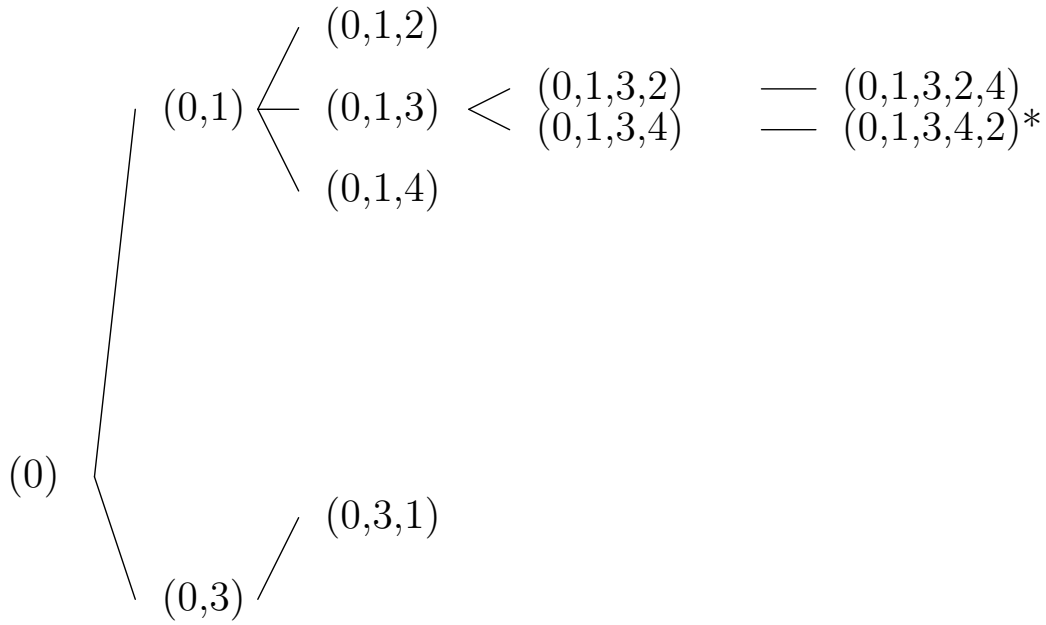
bound = 302.31

length = 94.244 + 193.23 = 287.47



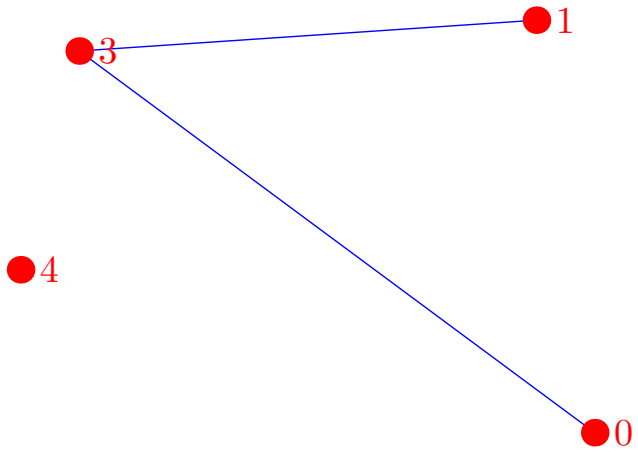
(0,3)

Branch and Bound after Pruning



bound = 302.31

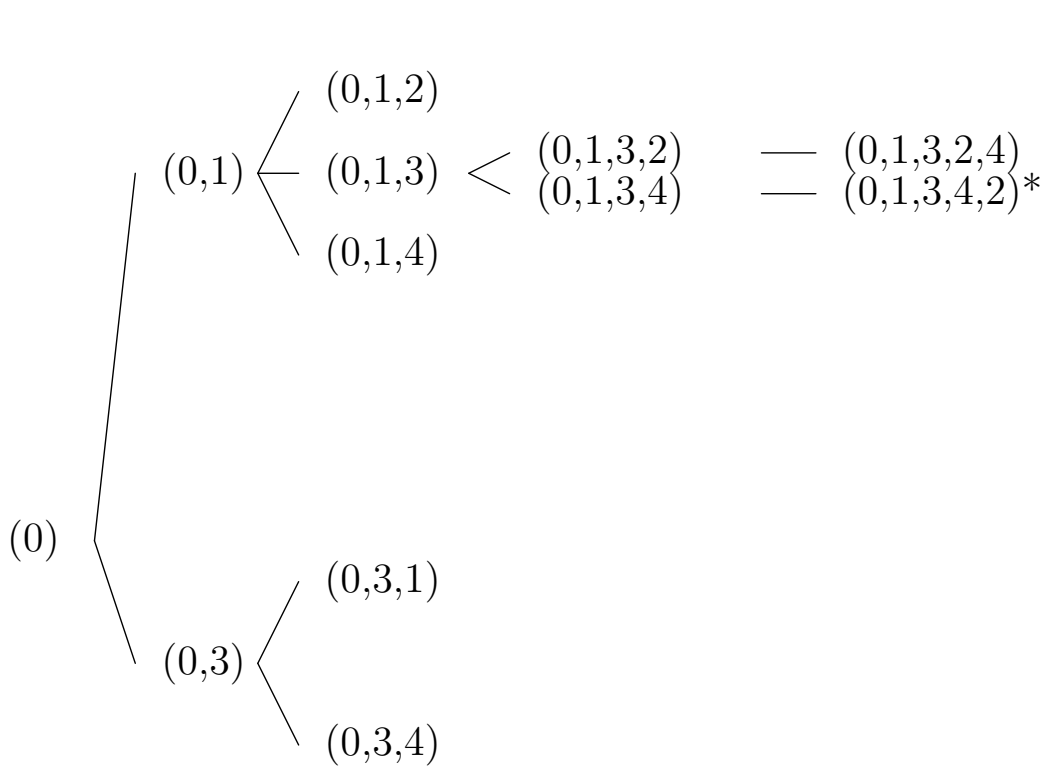
length = 161.58 + 159.95 = 321.53



lower bound exceeded 2

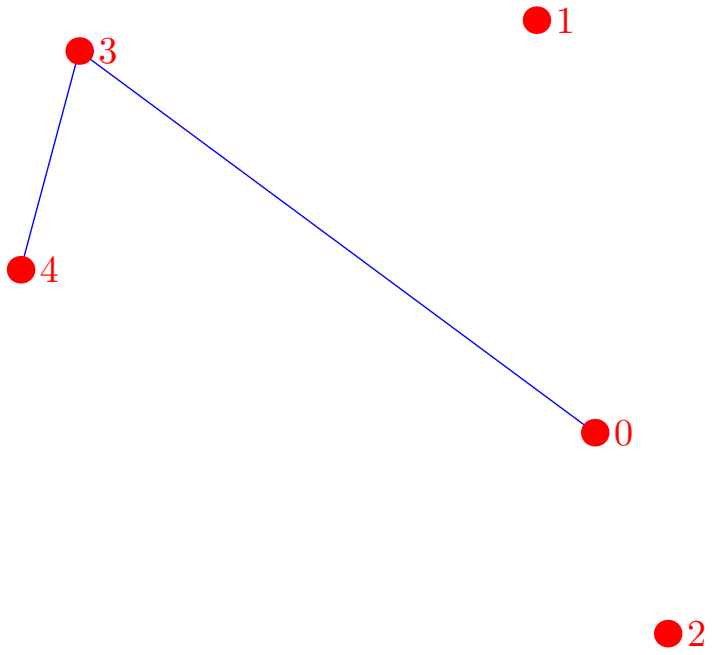
(0,3,1)

Branch and Bound after Pruning



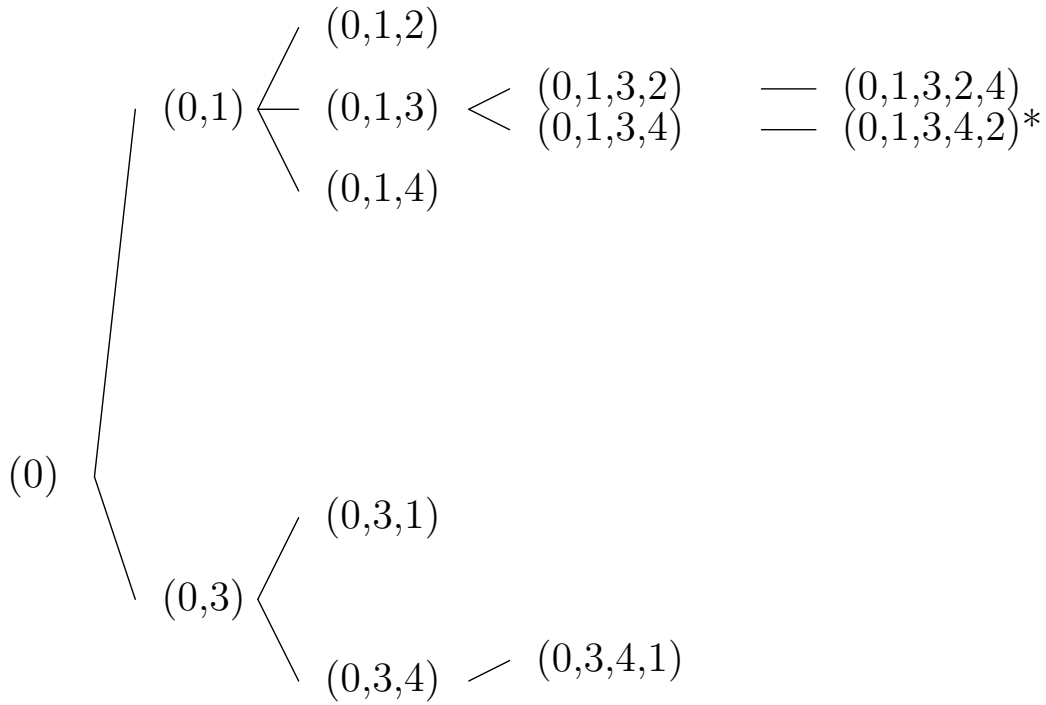
bound = 302.31

length = 127.52 + 159.95 = 287.47



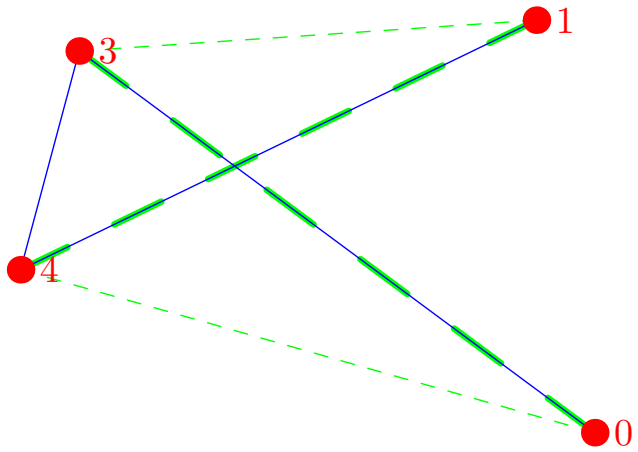
(0,3,4)

Branch and Bound after Pruning



bound = 302.31

length = 211.72 + 61.211 = 272.93

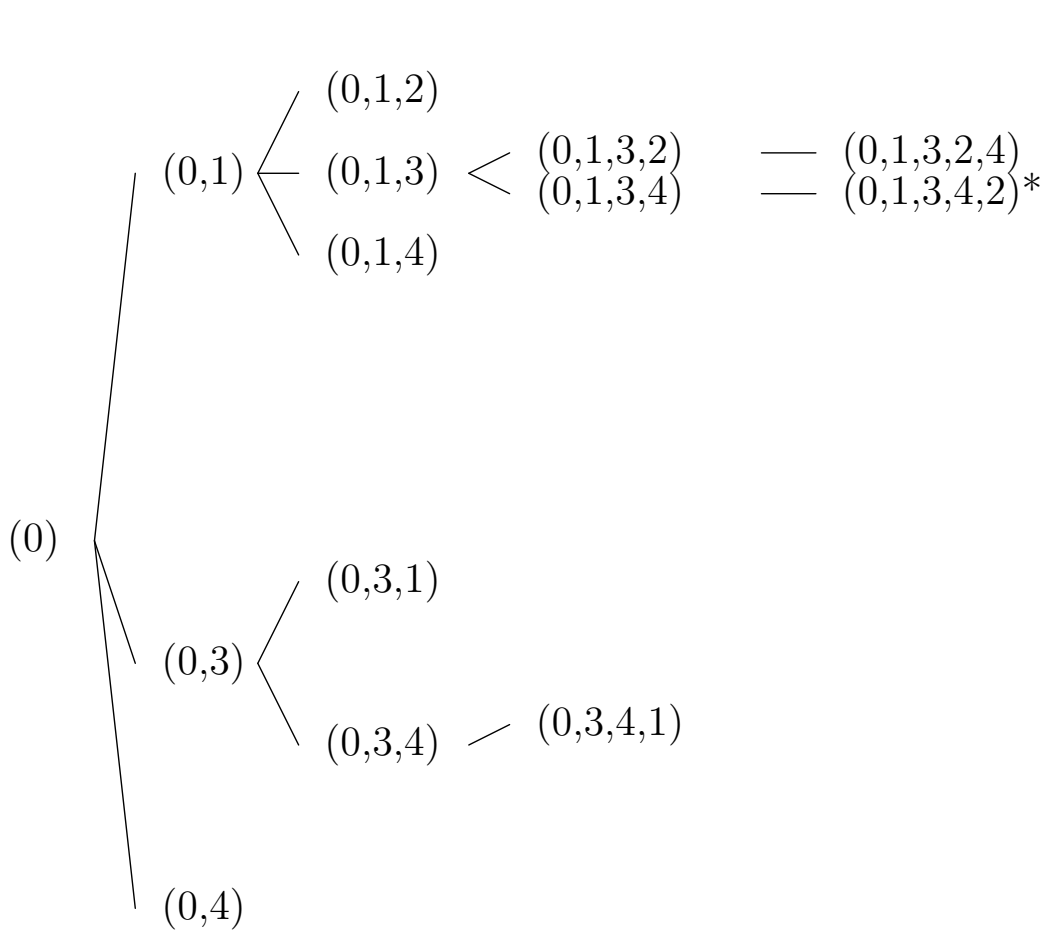


2

Not 2-opt

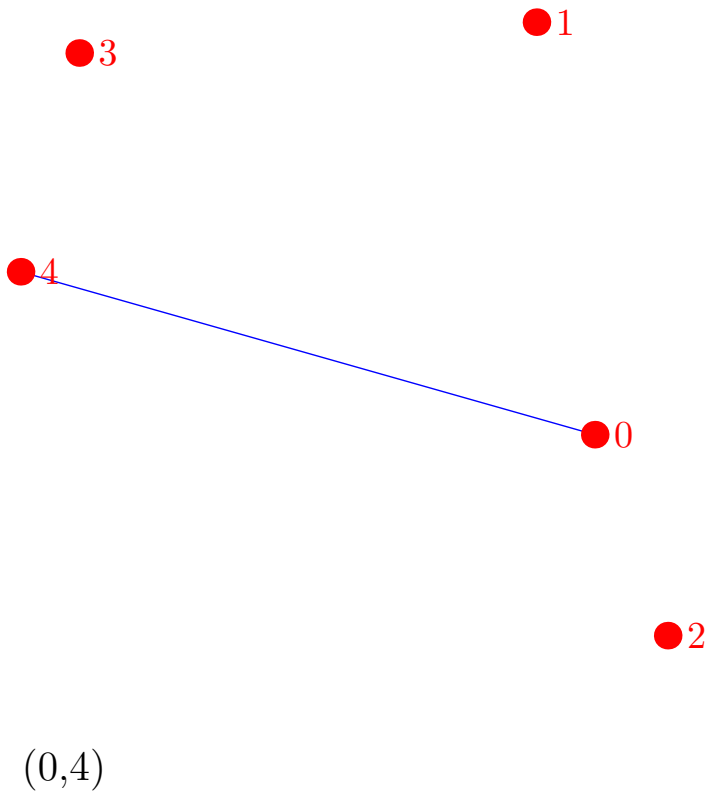
(0,3,4,1)

Branch and Bound after Pruning

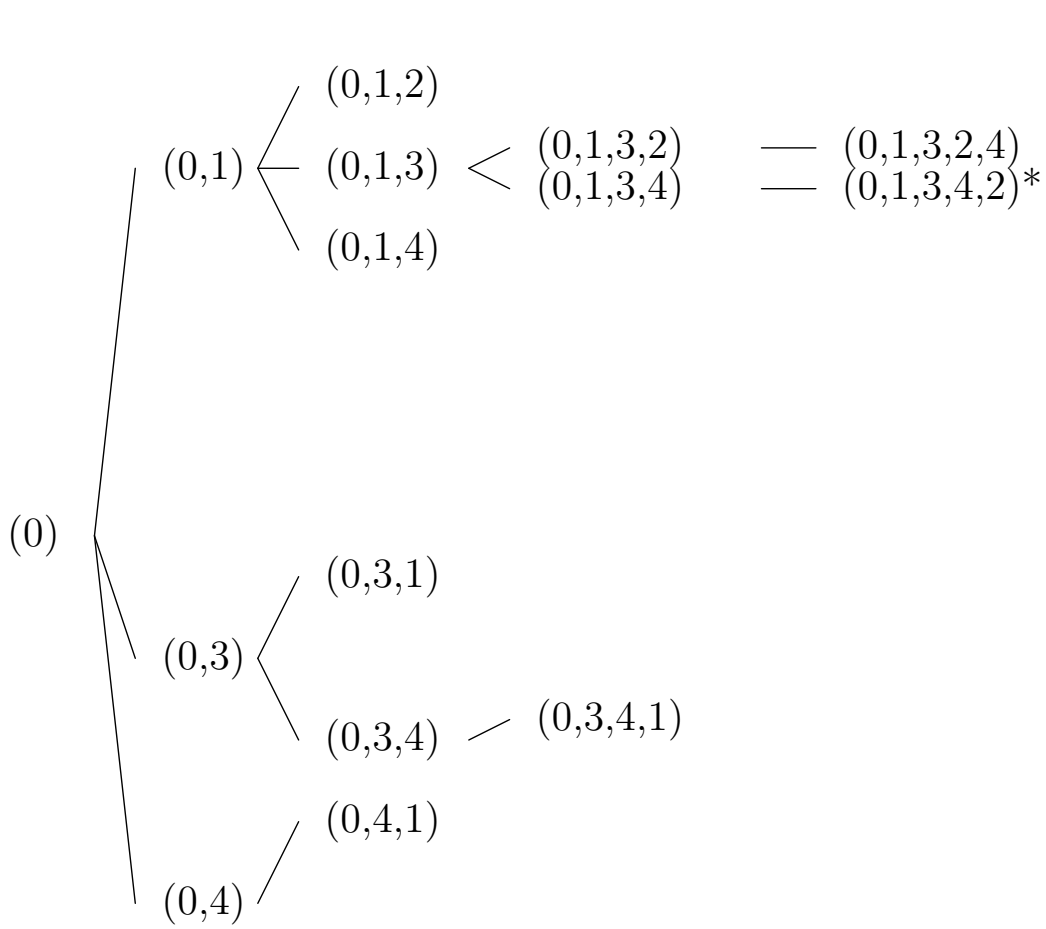


bound = 302.31

length = 87.692 + 193.23 = 280.92

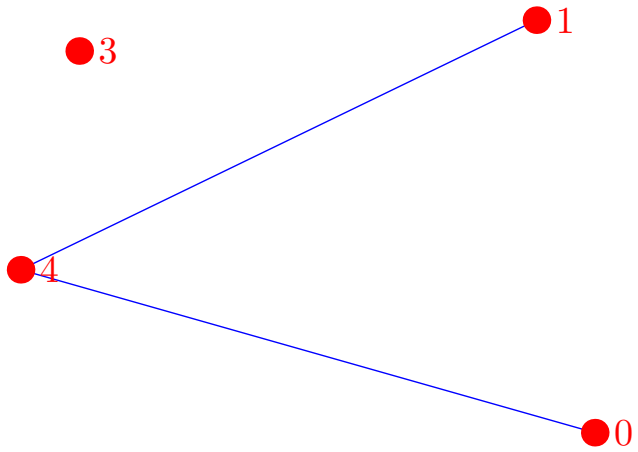


Branch and Bound after Pruning



bound = 302.31

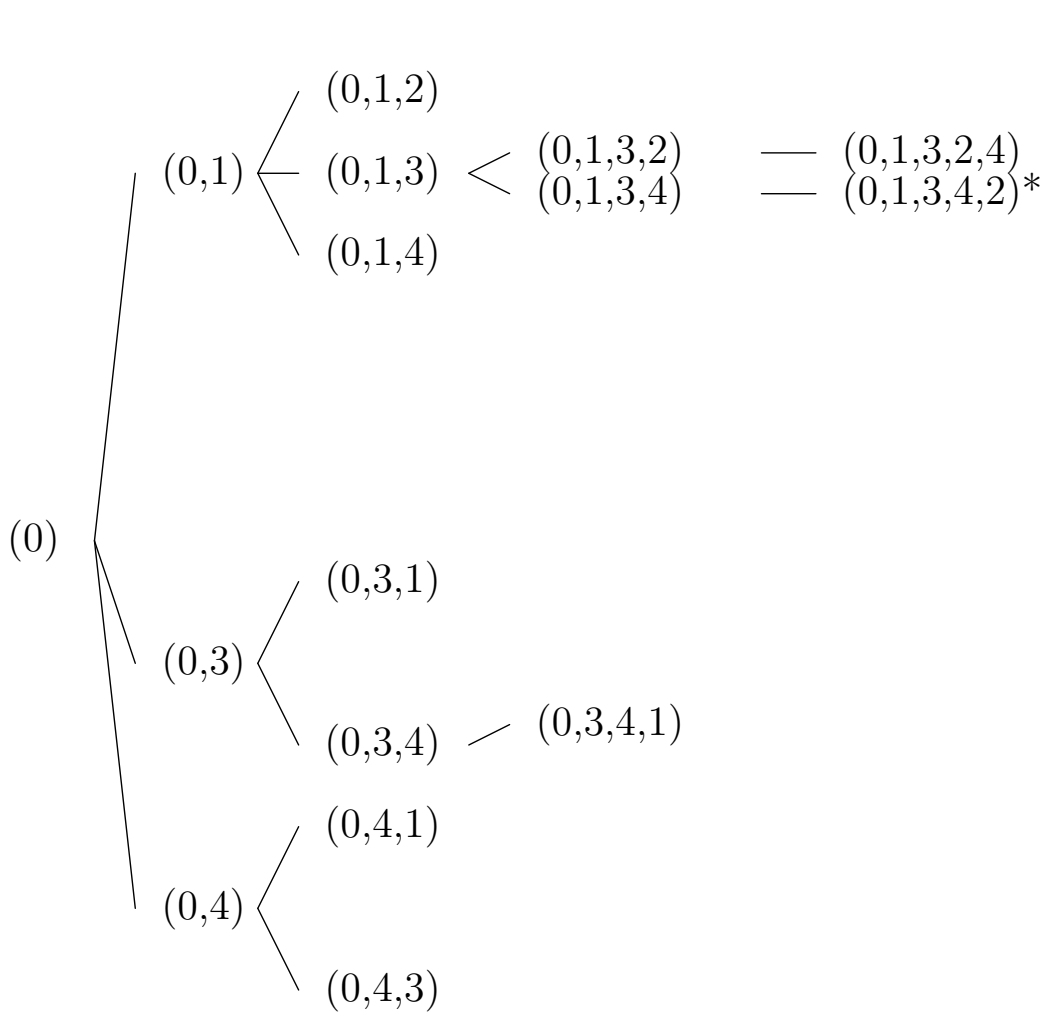
length = 171.9 + 159.95 = 331.85



lower bound exceeded 2

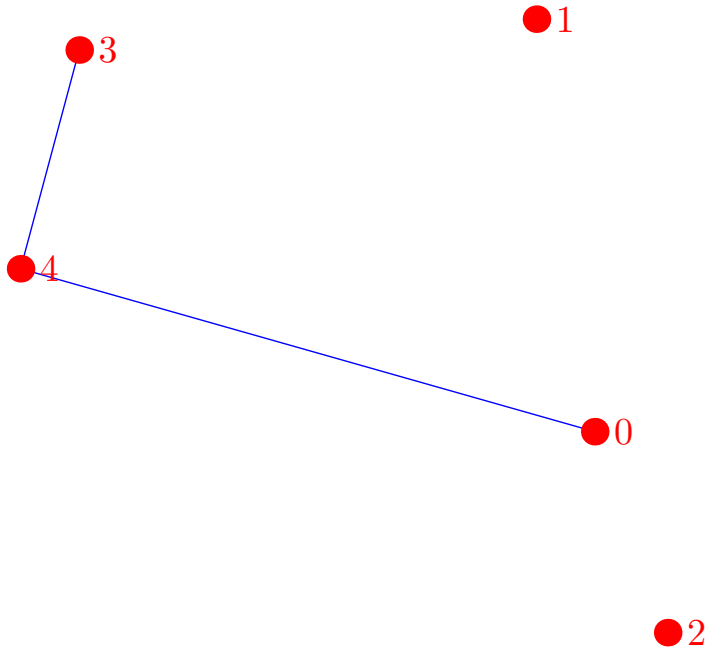
(0,4,1)

Branch and Bound after Pruning



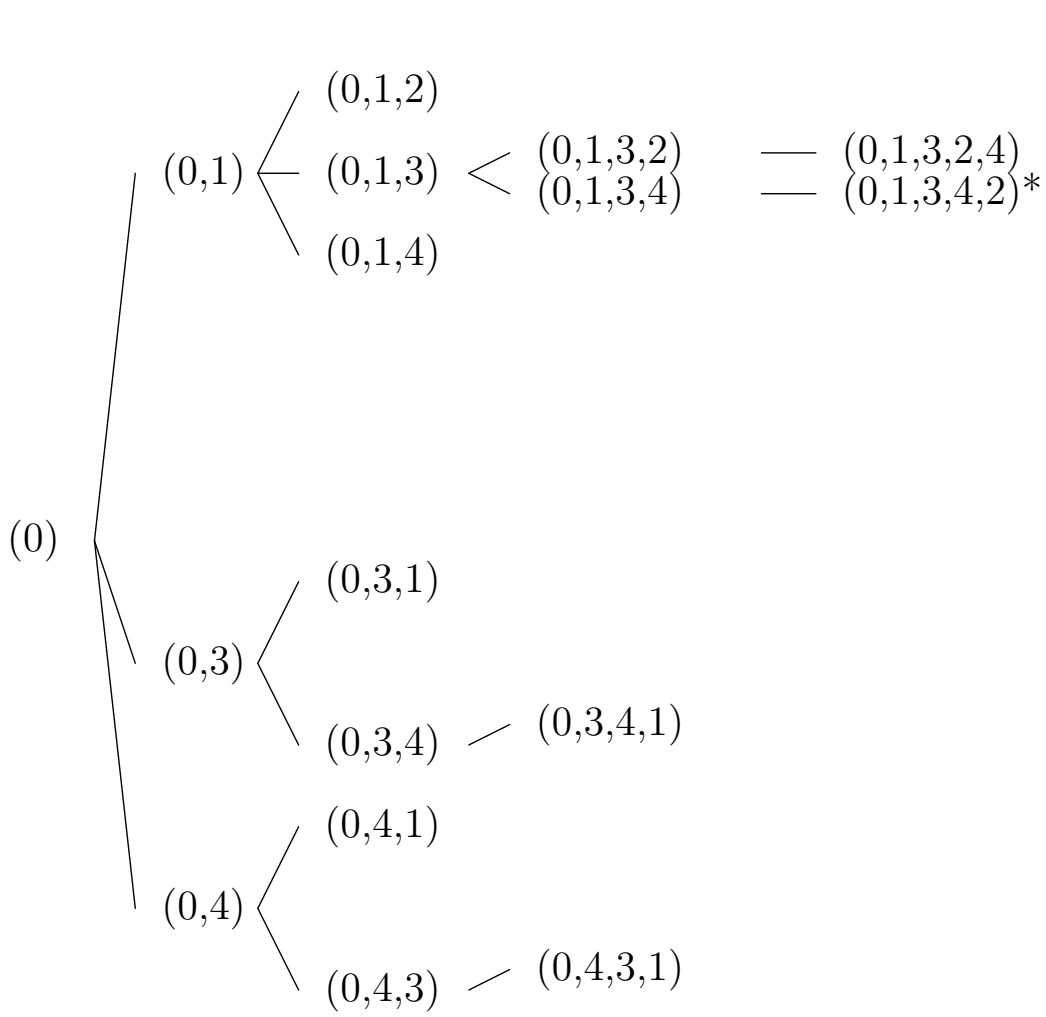
bound = 302.31

length = 120.97 + 159.95 = 280.92



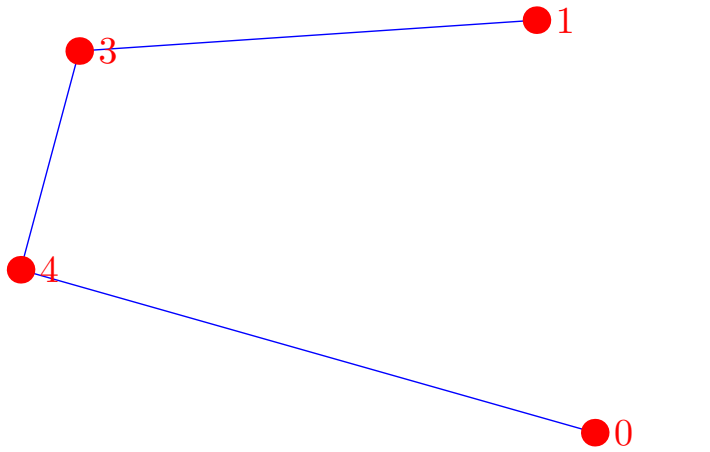
(0,4,3)

Branch and Bound after Pruning



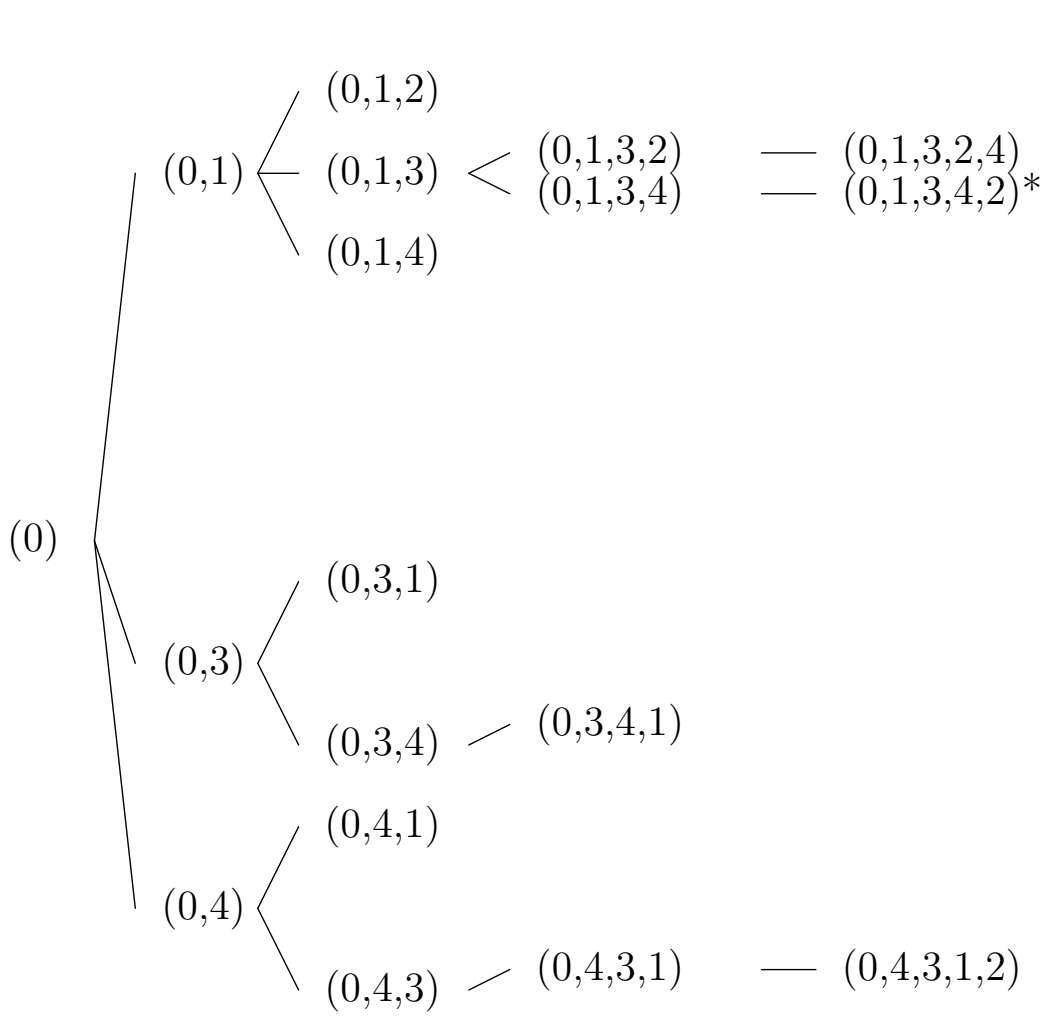
bound = 302.31

length = 188.3 + 61.211 = 249.51



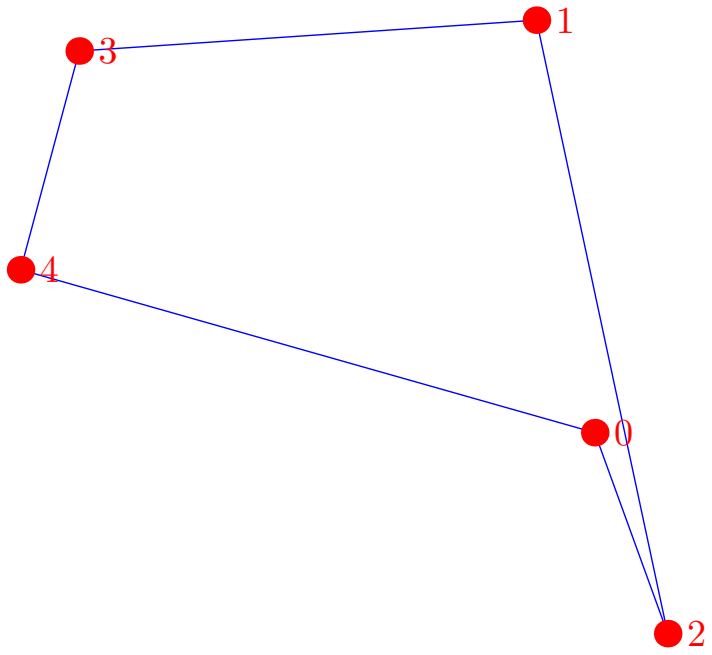
(0,4,3,1)

Branch and Bound after Pruning



bound = 302.31

length = 311.88 + 0 = 311.88



(0,4,3,1,2)

Applications of Branch and Bound

- Branch and bound works for many optimisation problems
- It's drawback is that you often end up still searching an exponentially large search space even though it might be massively faster than exhaustive enumeration
- To make it work well requires considerable work
- This is not an instantaneous algorithm, you may be waiting hours before you find a solution
- For really large problems branch and bound might be too slow

Applications of Branch and Bound

- Branch and bound works for many optimisation problems
- It's drawback is that you often end up still searching an exponentially large search space even though it might be massively faster than exhaustive enumeration
- To make it work well requires considerable work
- This is not an instantaneous algorithm, you may be waiting hours before you find a solution
- For really large problems branch and bound might be too slow

Applications of Branch and Bound

- Branch and bound works for many optimisation problems
- It's drawback is that you often end up still searching an exponentially large search space even though it might be massively faster than exhaustive enumeration
- To make it work well requires considerable work
- This is not an instantaneous algorithm, you may be waiting hours before you find a solution
- For really large problems branch and bound might be too slow

Applications of Branch and Bound

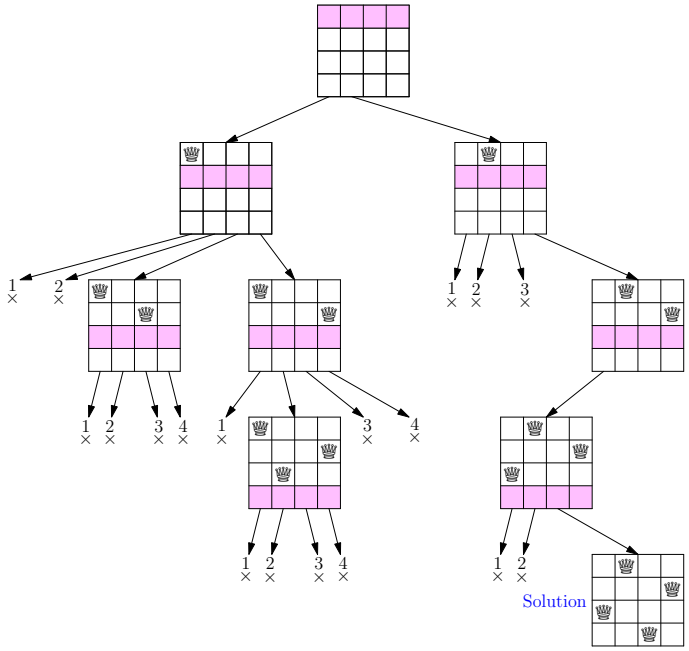
- Branch and bound works for many optimisation problems
- It's drawback is that you often end up still searching an exponentially large search space even though it might be massively faster than exhaustive enumeration
- To make it work well requires considerable work
- This is not an instantaneous algorithm, you may be waiting hours before you find a solution
- For really large problems branch and bound might be too slow

Applications of Branch and Bound

- Branch and bound works for many optimisation problems
- It's drawback is that you often end up still searching an exponentially large search space even though it might be massively faster than exhaustive enumeration
- To make it work well requires considerable work
- This is not an instantaneous algorithm, you may be waiting hours before you find a solution
- For really large problems branch and bound might be too slow

Outline

1. Search Trees
2. Backtracking
3. Branch and Bound
4. **Search in AI**



Other Search Strategies

- Search is a big topic in AI
- The algorithms used depends on the information available
- A classic search scenario is when there is "heuristic" information which provides a hint as to where an optimal solution lies
- Algorithms such as A^* exist which will finds the best route given an (admissible) heuristic as efficiently as possible
- You should learn about this next year in AI

Other Search Strategies

- Search is a big topic in AI
- The algorithms used depends on the information available
- A classic search scenario is when there is "heuristic" information which provides a hint as to where an optimal solution lies
- Algorithms such as A^* exist which will finds the best route given an (admissible) heuristic as efficiently as possible
- You should learn about this next year in AI

Other Search Strategies

- Search is a big topic in AI
- The algorithms used depends on the information available
- A classic search scenario is when there is "heuristic" information which provides a hint as to where an optimal solution lies
- Algorithms such as A^* exist which will finds the best route given an (admissible) heuristic as efficiently as possible
- You should learn about this next year in AI

Other Search Strategies

- Search is a big topic in AI
- The algorithms used depends on the information available
- A classic search scenario is when there is "heuristic" information which provides a hint as to where an optimal solution lies
- Algorithms such as A^* exist which will finds the best route given an (admissible) heuristic as efficiently as possible
- You should learn about this next year in AI

Other Search Strategies

- Search is a big topic in AI
- The algorithms used depends on the information available
- A classic search scenario is when there is "heuristic" information which provides a hint as to where an optimal solution lies
- Algorithms such as A^* exist which will finds the best route given an (admissible) heuristic as efficiently as possible
- You should learn about this next year in AI

Planning and Game Paying

- Search is also used to find the best action to take in planning problems and game playing (e.g. computer chess)
- Again it is useful to think in terms of a search tree
- Searching all paths on the search tree is usually infeasible
- Look for ways of pruning the search tree to focus on good moves
- Strategies include $minimax$ and $alpha\text{-}beta\ pruning$

Planning and Game Paying

- Search is also used to find the best action to take in planning problems and game playing (e.g. computer chess)
- Again it is useful to think in terms of a search tree
- Searching all paths on the search tree is usually infeasible
- Look for ways of pruning the search tree to focus on good moves
- Strategies include $minimax$ and $alpha\text{-}beta\ pruning$

Planning and Game Paying

- Search is also used to find the best action to take in planning problems and game playing (e.g. computer chess)
- Again it is useful to think in terms of a search tree
- Searching all paths on the search tree is usually infeasible
- Look for ways of pruning the search tree to focus on good moves
- Strategies include $minimax$ and $alpha\text{-}beta\ pruning$

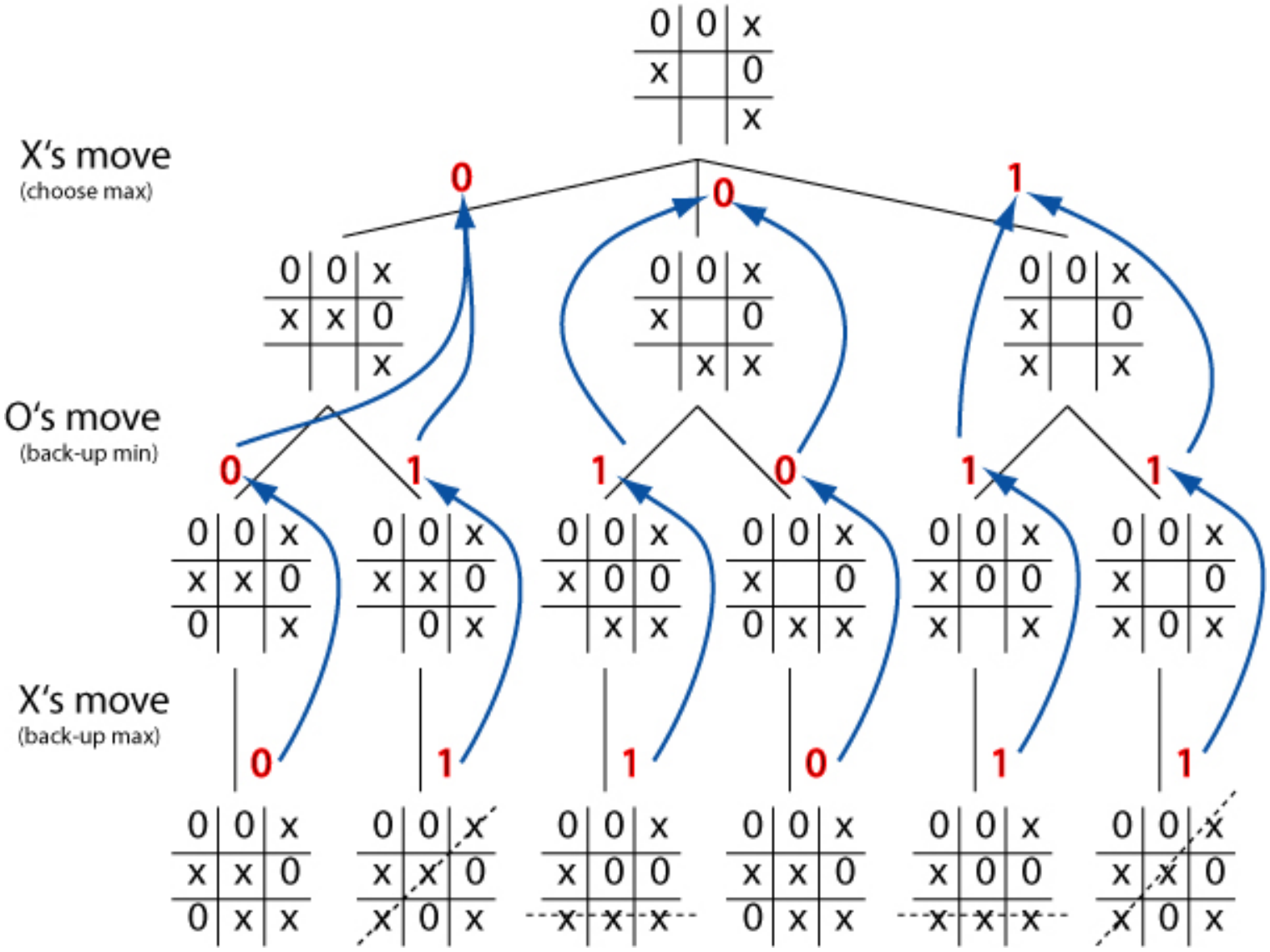
Planning and Game Paying

- Search is also used to find the best action to take in planning problems and game playing (e.g. computer chess)
- Again it is useful to think in terms of a search tree
- Searching all paths on the search tree is usually infeasible
- Look for ways of pruning the search tree to focus on good moves
- Strategies include $minimax$ and $alpha\text{-}beta\ pruning$

Planning and Game Paying

- Search is also used to find the best action to take in planning problems and game playing (e.g. computer chess)
- Again it is useful to think in terms of a search tree
- Searching all paths on the search tree is usually infeasible
- Look for ways of pruning the search tree to focus on good moves
- Strategies include $minimax$ and $alpha\text{-}beta\ pruning$

Minimax with Alpha-Beta Prunning



Lessons

- Search has many applications
- It is helpful to consider the search space as a tree whose branch corresponds to possible actions
- Backtracking is useful in search trees with constraints
- For optimisation problems branch and bound uses backtracking and costs of partial solutions as constraints
- Widely applicable, but can take too long

Lessons

- Search has many applications
- It is helpful to consider the search space as a tree whose branch corresponds to possible actions
- Backtracking is useful in search trees with constraints
- For optimisation problems branch and bound uses backtracking and costs of partial solutions as constraints
- Widely applicable, but can take too long

Lessons

- Search has many applications
- It is helpful to consider the search space as a tree whose branch corresponds to possible actions
- Backtracking is useful in search trees with constraints
- For optimisation problems branch and bound uses backtracking and costs of partial solutions as constraints
- Widely applicable, but can take too long

Lessons

- Search has many applications
- It is helpful to consider the search space as a tree whose branch corresponds to possible actions
- Backtracking is useful in search trees with constraints
- For optimisation problems branch and bound uses backtracking and costs of partial solutions as constraints
- Widely applicable, but can take too long

Lessons

- Search has many applications
- It is helpful to consider the search space as a tree whose branch corresponds to possible actions
- Backtracking is useful in search trees with constraints
- For optimisation problems branch and bound uses backtracking and costs of partial solutions as constraints
- Widely applicable, but can take too long