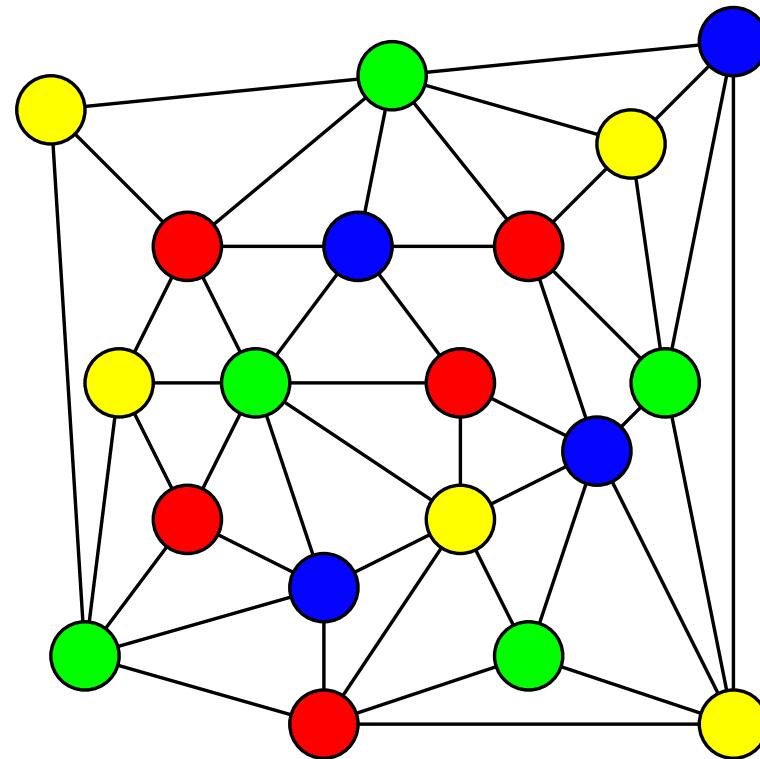


Algorithms and Analysis

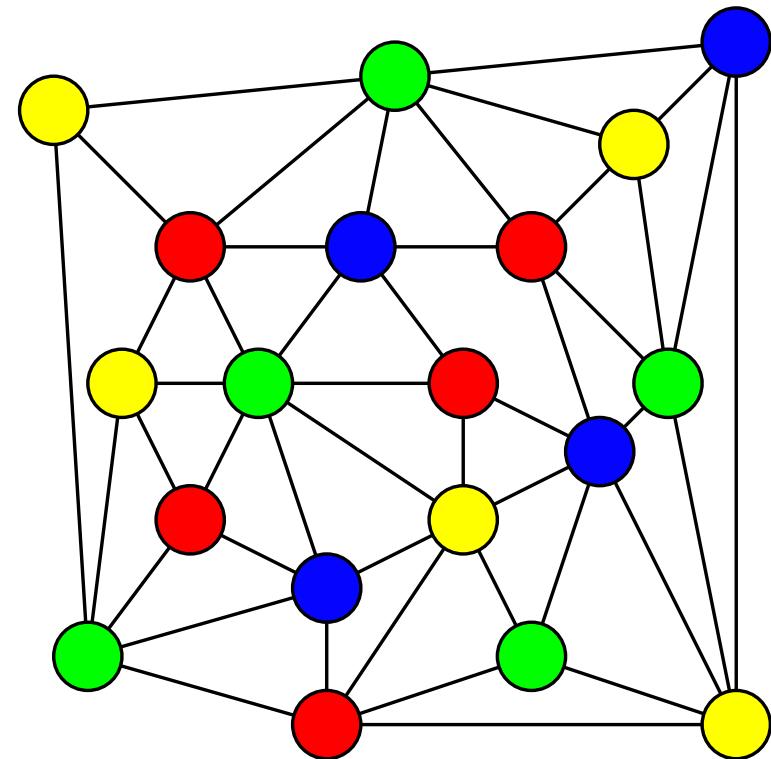
Lesson 16: *Think Graphically*



Graph theory, applications of graphs, graph problems

Outline

1. **Graph Theory**
2. Applications of Graphs
 - Geometric applications
 - Relational applications
3. Implementing Graphs
4. Graph Problems



Motivation

- Many different problems can be described in terms of graphs
- This often reveals the true nature of the problem
- It unifies many apparently different problems
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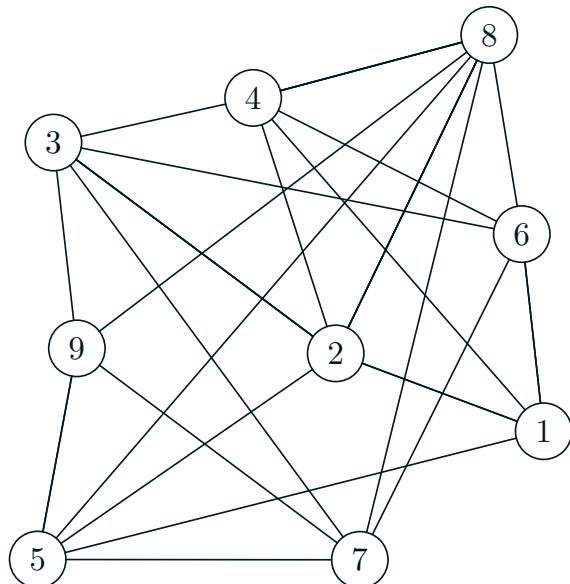
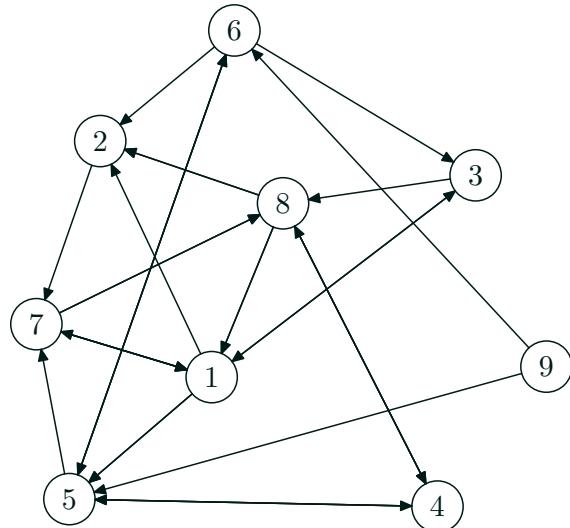
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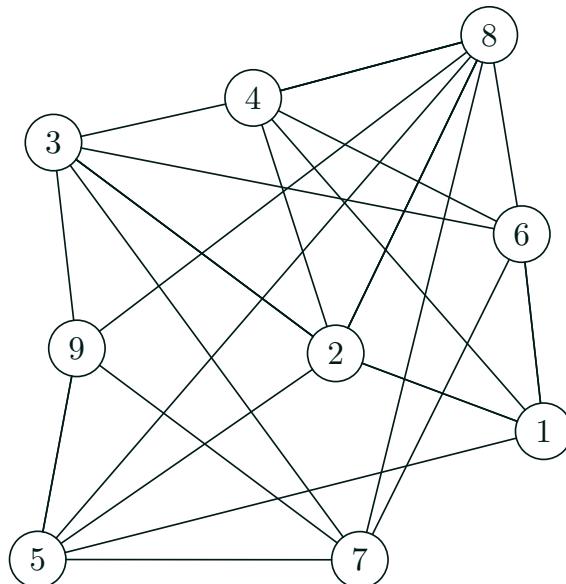
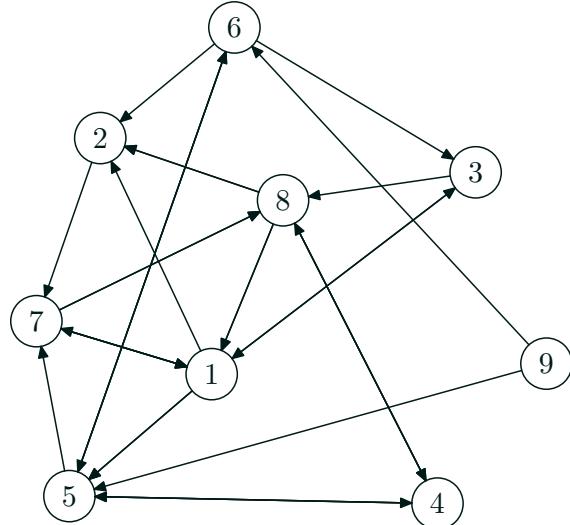
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 - ★ A set of vertices or nodes $\mathcal{V} = \{1, 2, 3 \dots n\}$
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- The edges may be
 - ★ **directed**—sometimes called a **digraph**
 - ★ **undirected**



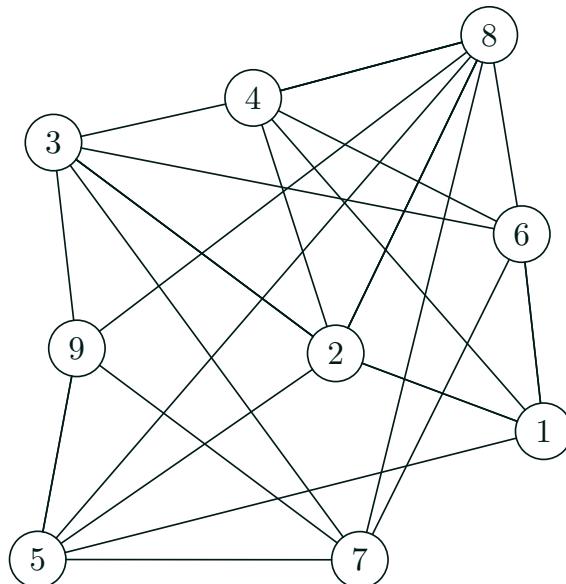
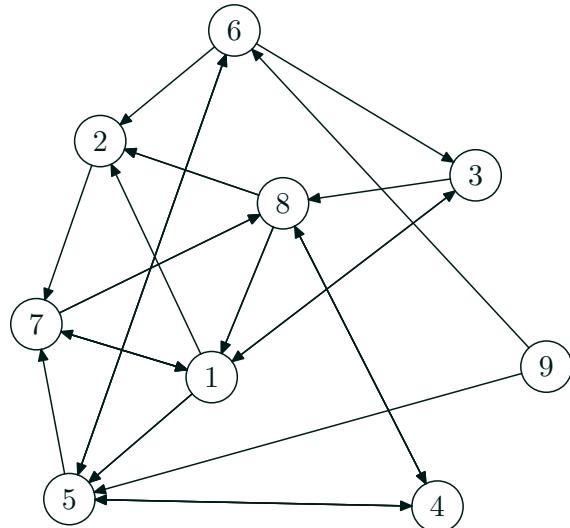
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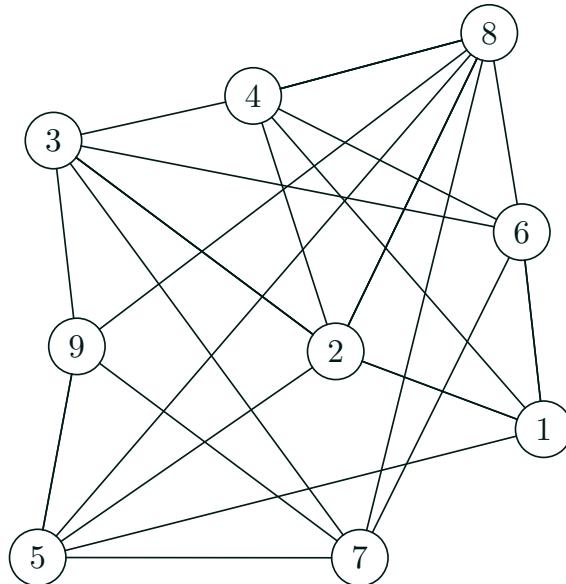
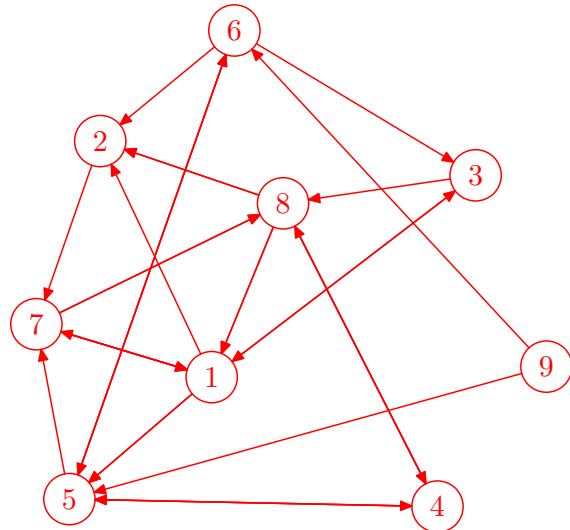
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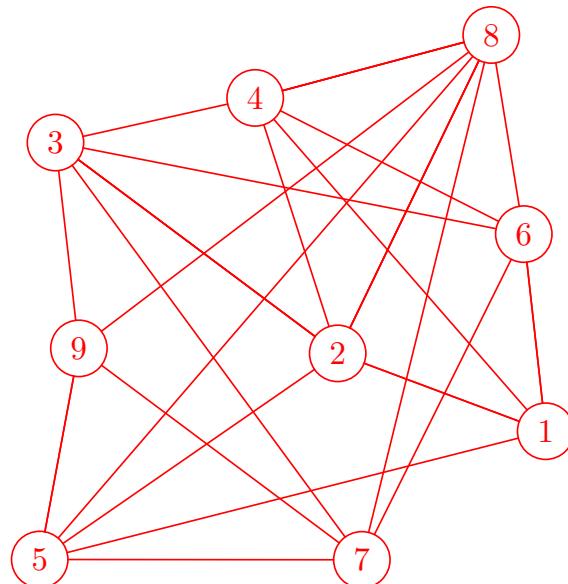
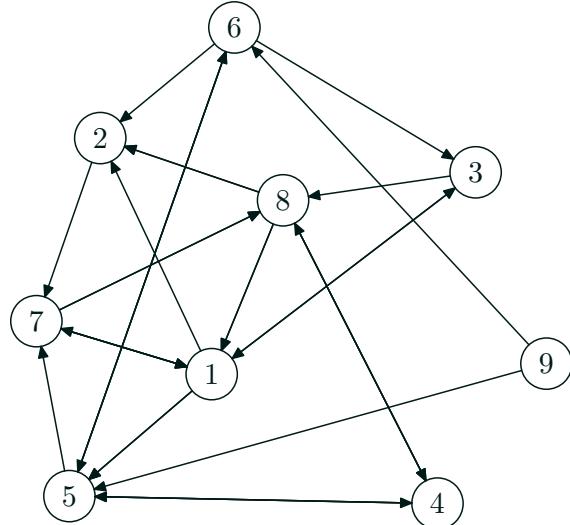
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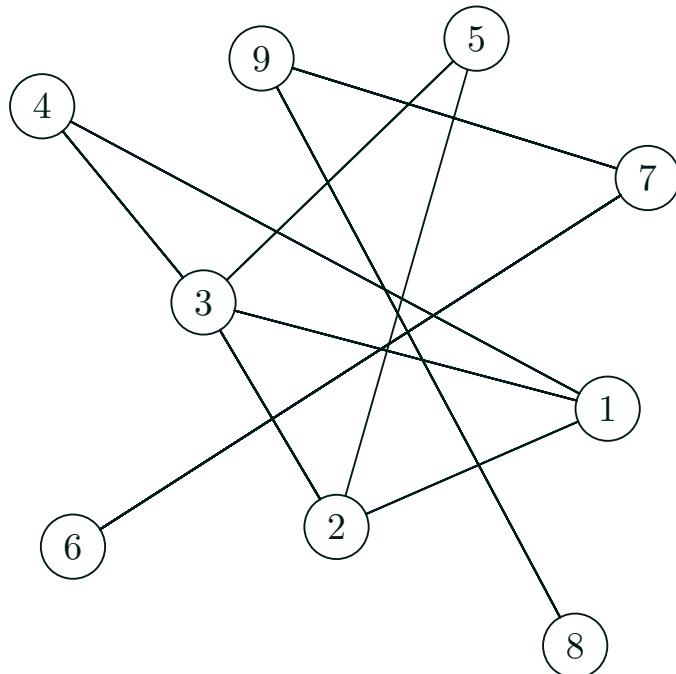
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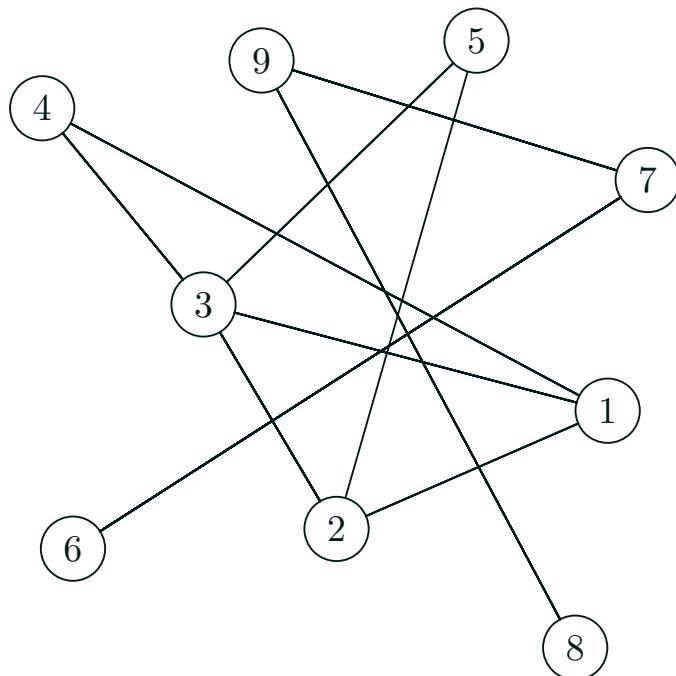
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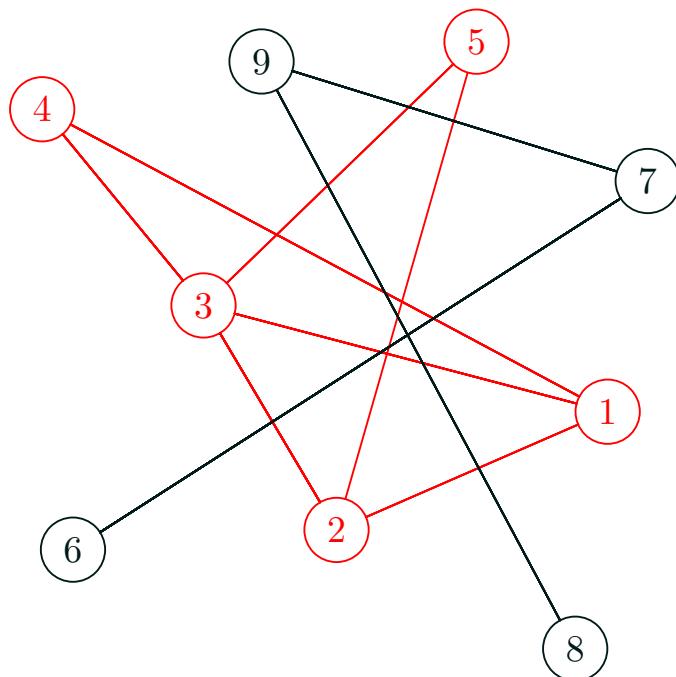
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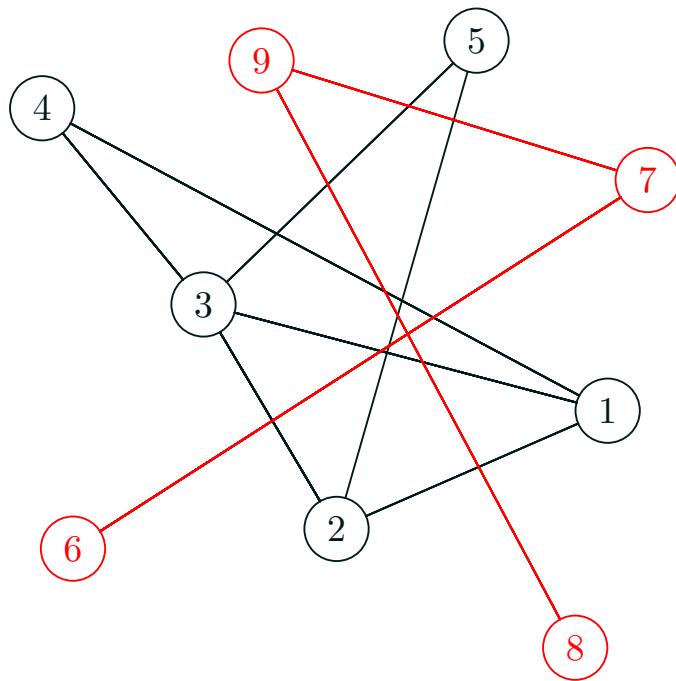
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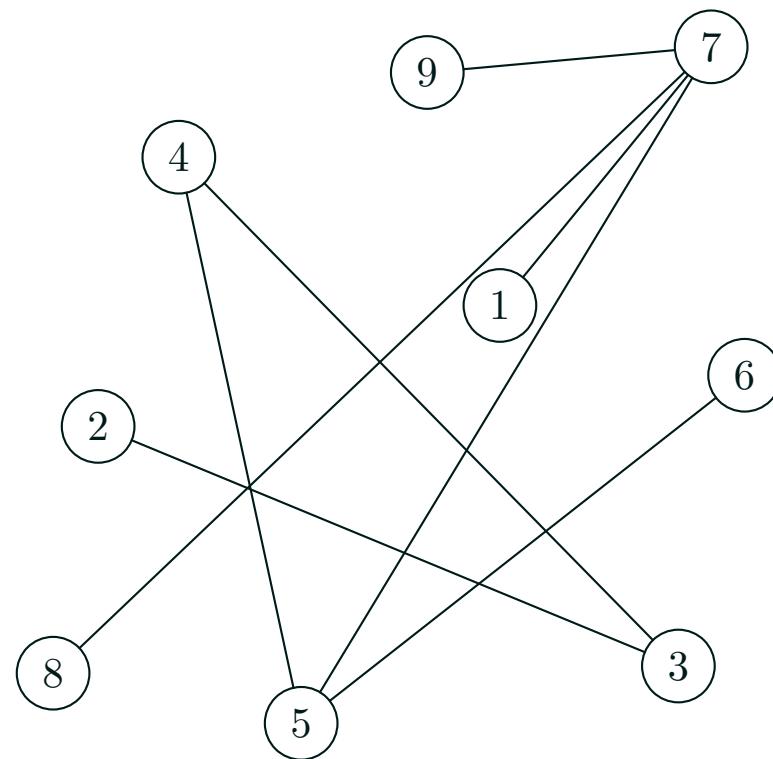
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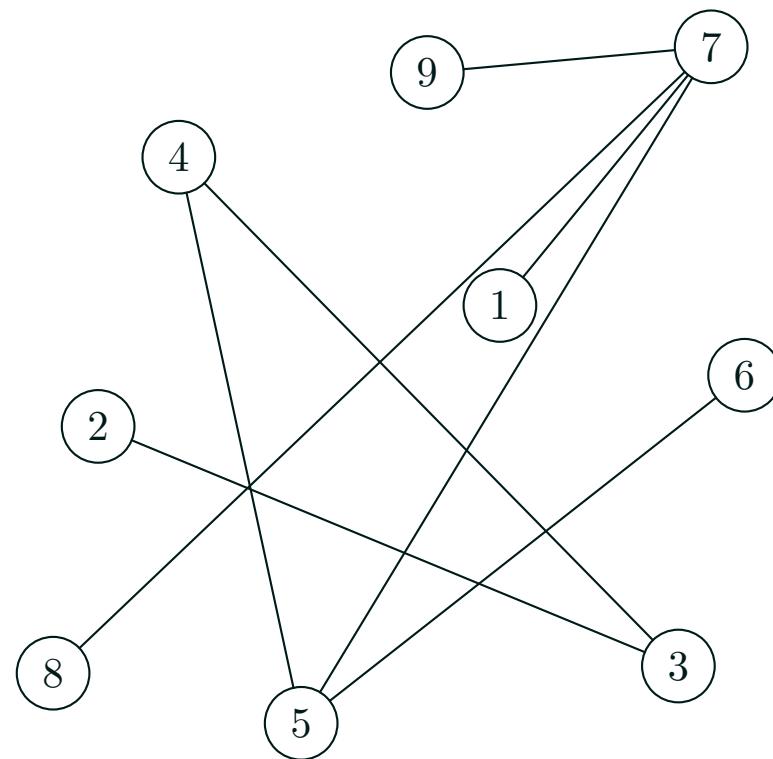
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- A tree will have $n - 1$ edges



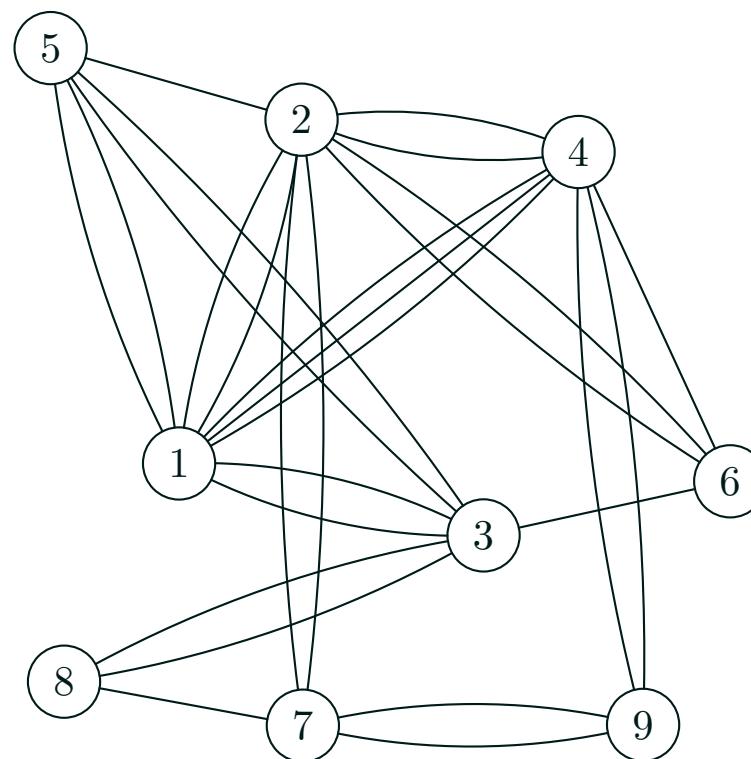
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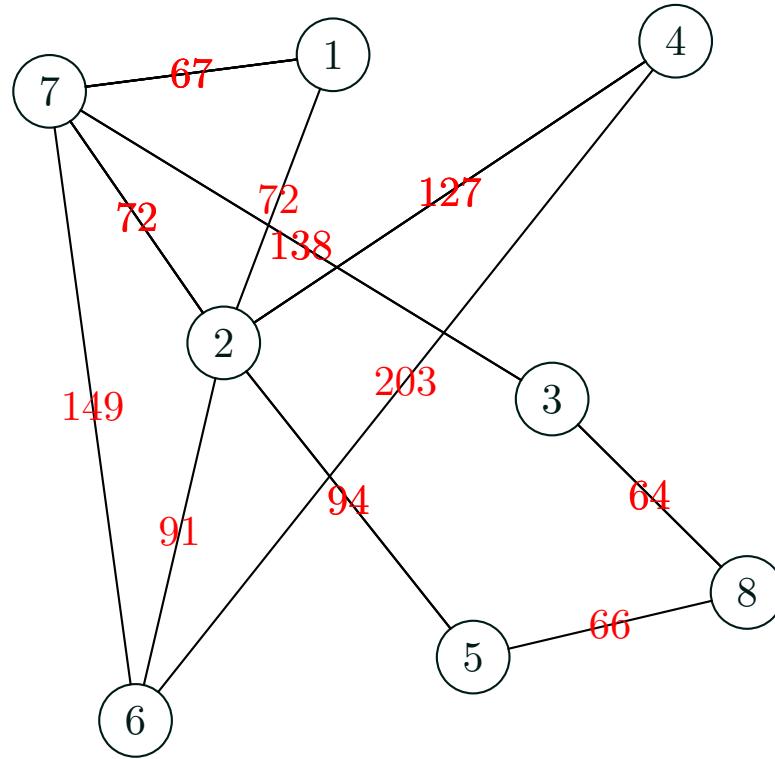
Multigraphs

- If the collection of edges is a *multiset* then we obtain a **multigraphs** where more than one edge is allowed between pairs of vertices



Weighted Graphs

- If we assign a number to an edge we obtain a **weighted graph**



Networks

- Sometimes we add more information to the graph
- E.g. attributes to the nodes or edges
- Graphs with many attributes are often referred to as **networks**

Networks

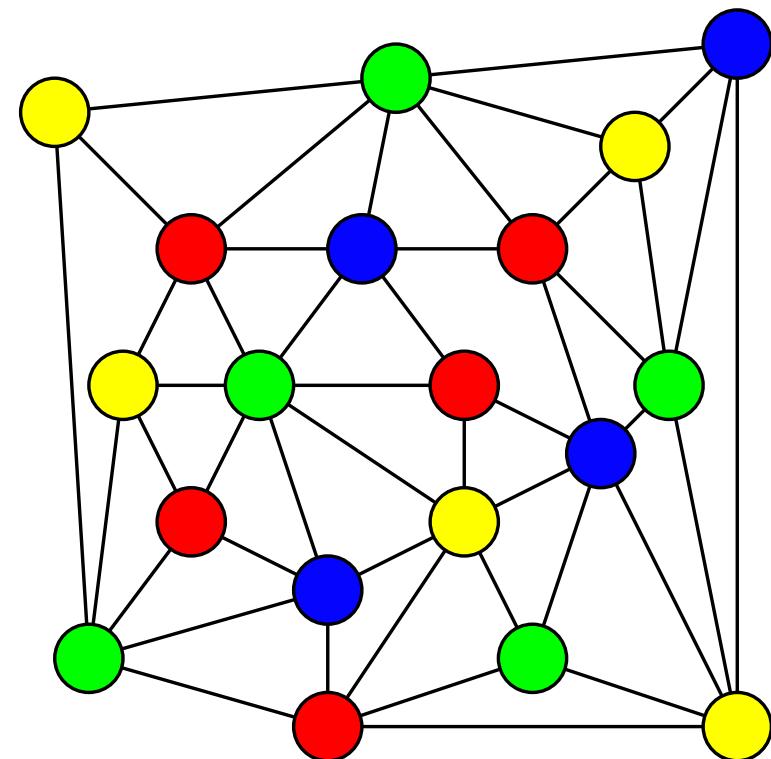
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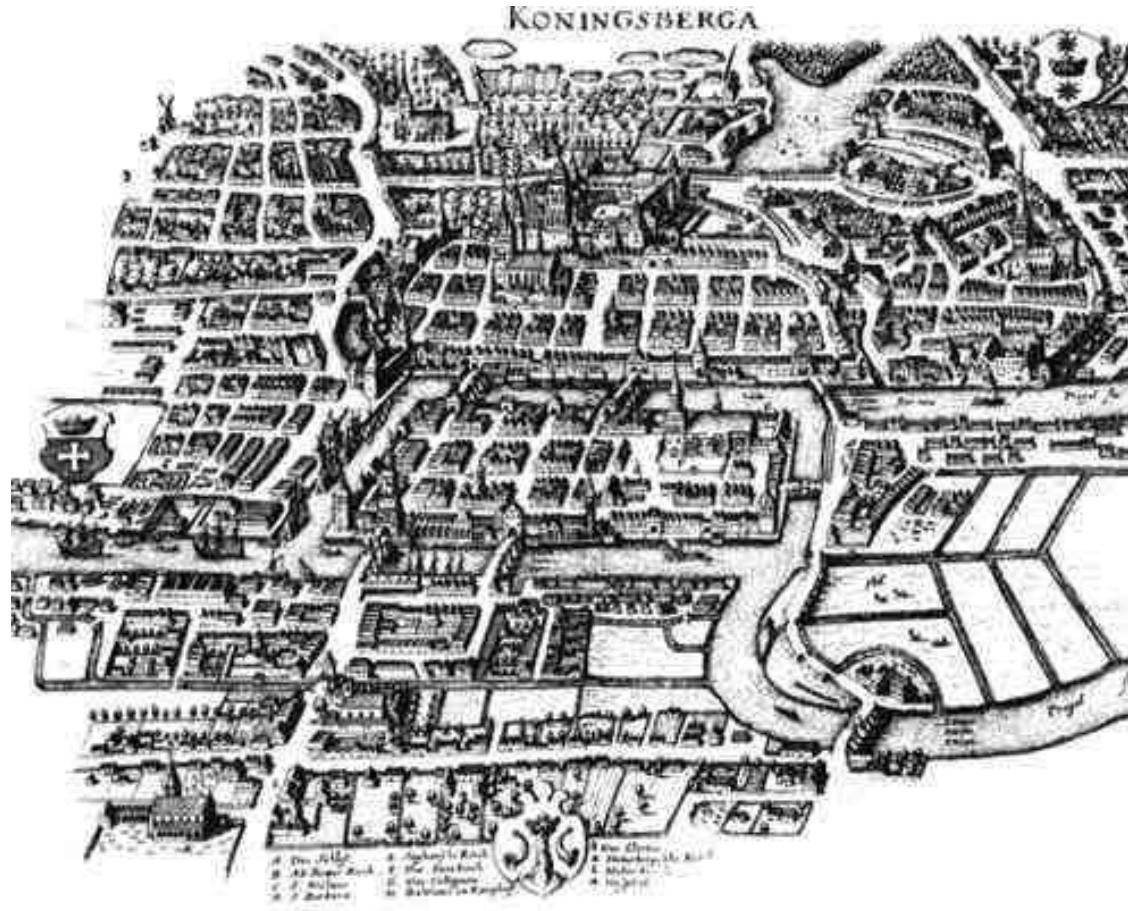
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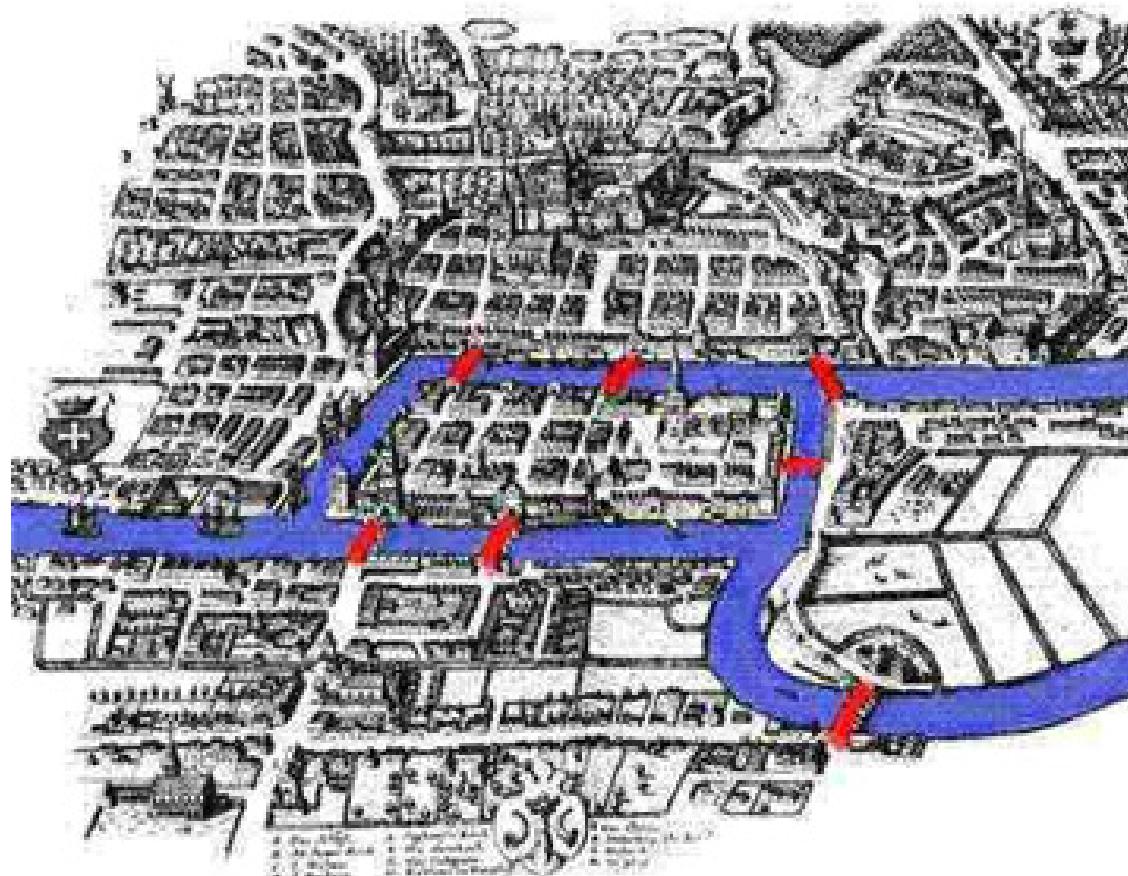
Bridges of Königsberg

Is there a tour around Königsberg going over every bridge once?



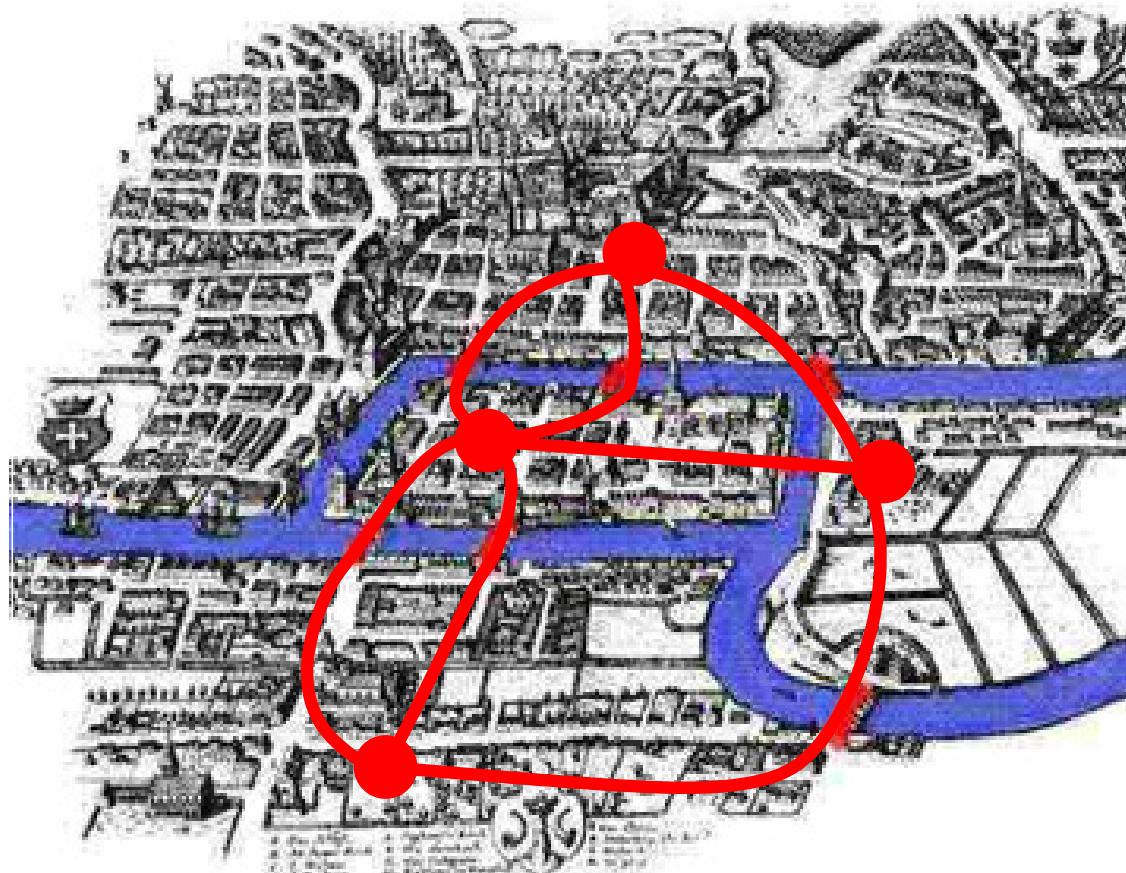
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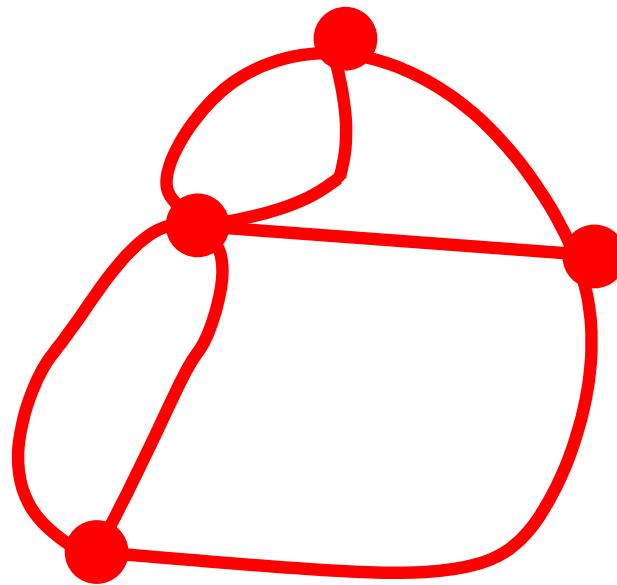
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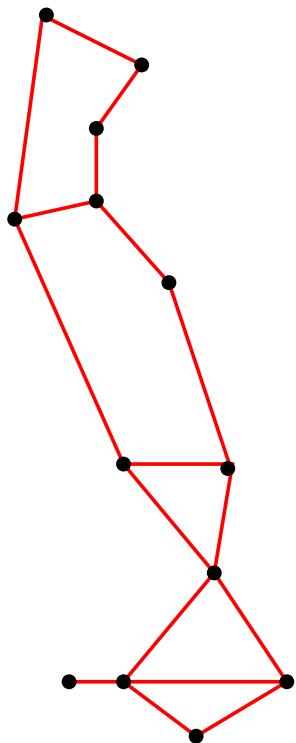
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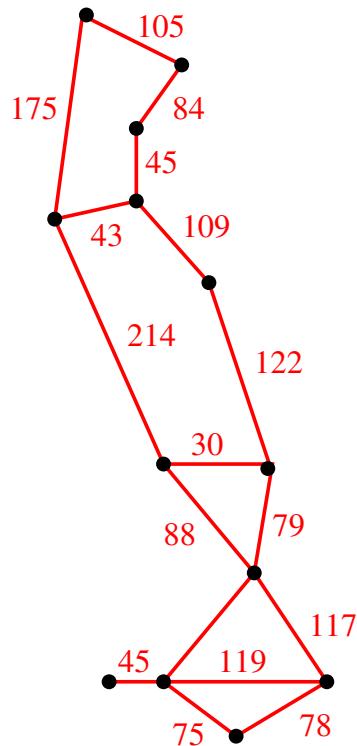
In 1736 Euler published a paper answering this question and founding graph theory

Representing Distances



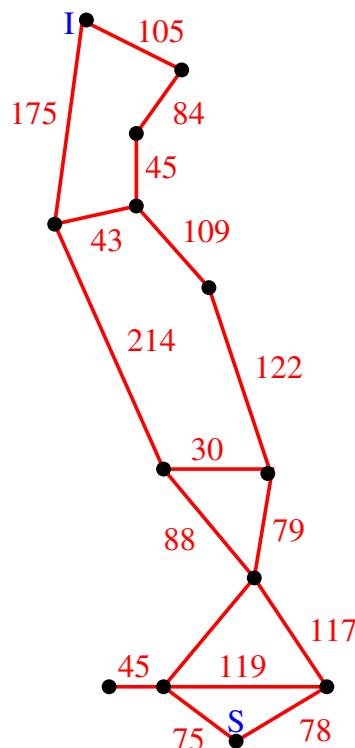
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- With weights representing the distance between nodes
- What is the shortest distance between S and I ?

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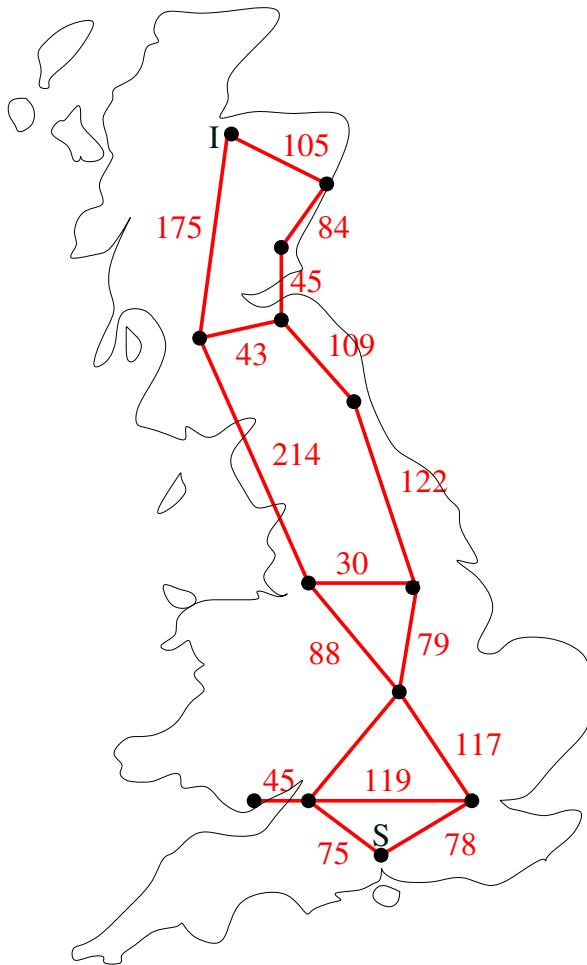
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Other Applications

- We could take the weights to represent the time taken to travel between nodes
- In a computer network the weights might represent the bandwidth
- In a representation of a transport system the weights might represent the carrying capacity of the traffic on a road
- Graphs can be used to represent other kinds of relationships
- E.g. We could create a digraph of links between web pages

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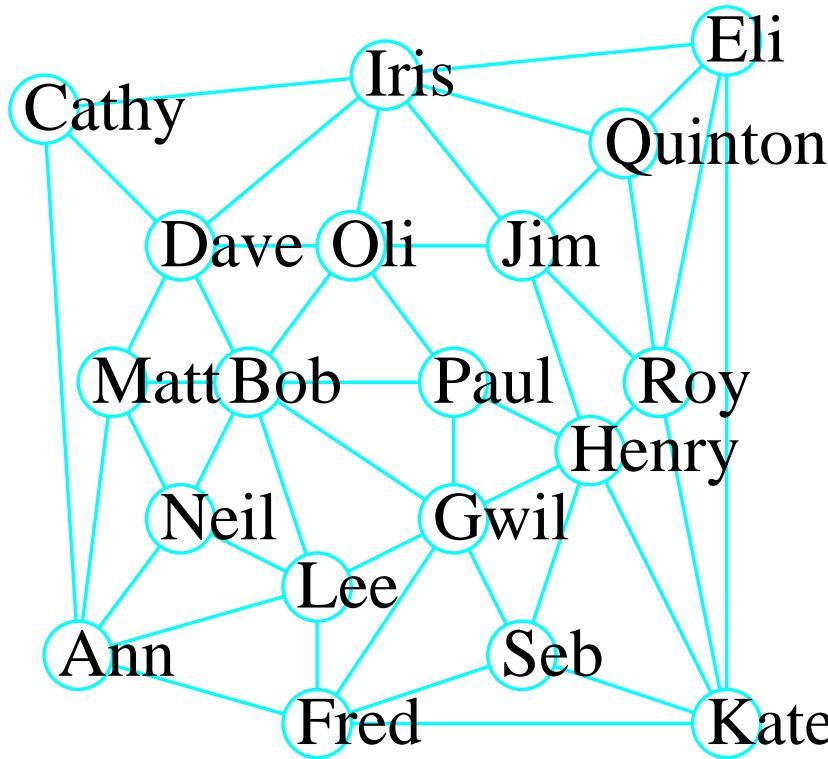
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Christmas Card Problem

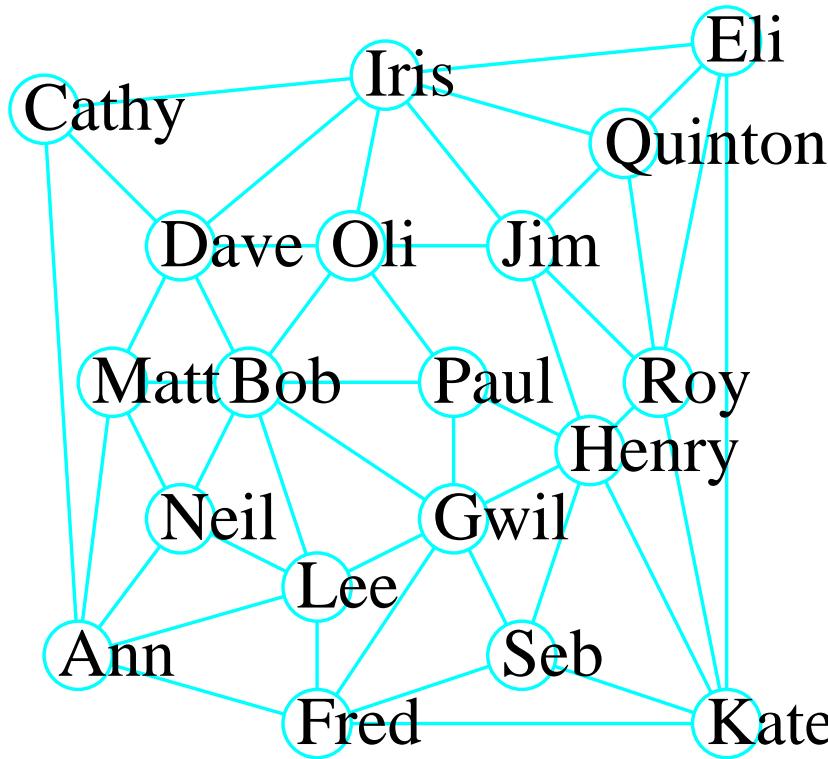
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- Some of my friends know each other



- I don't want to send friends that know each other the same card

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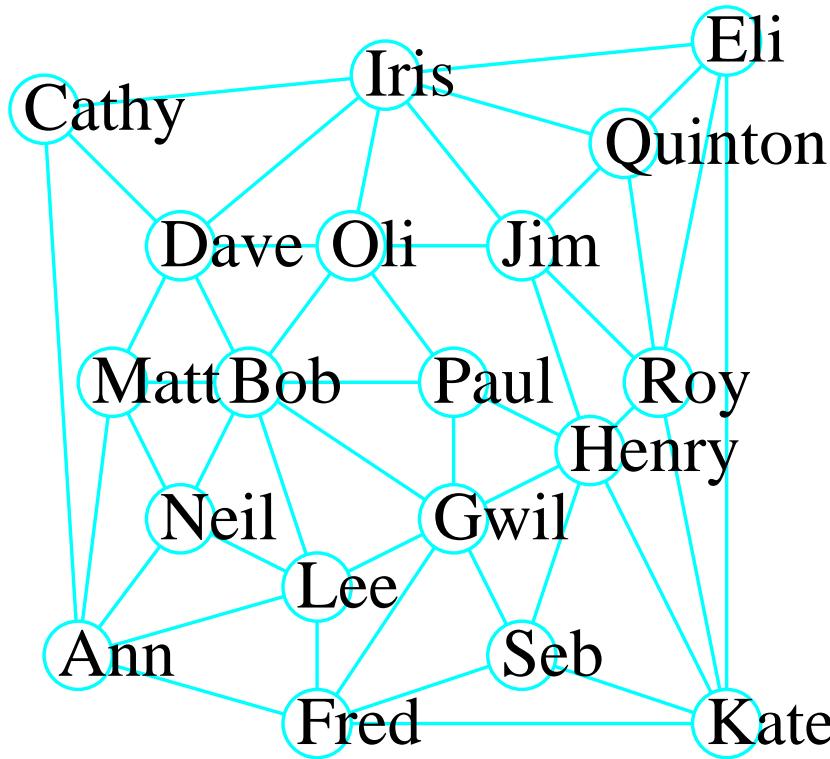
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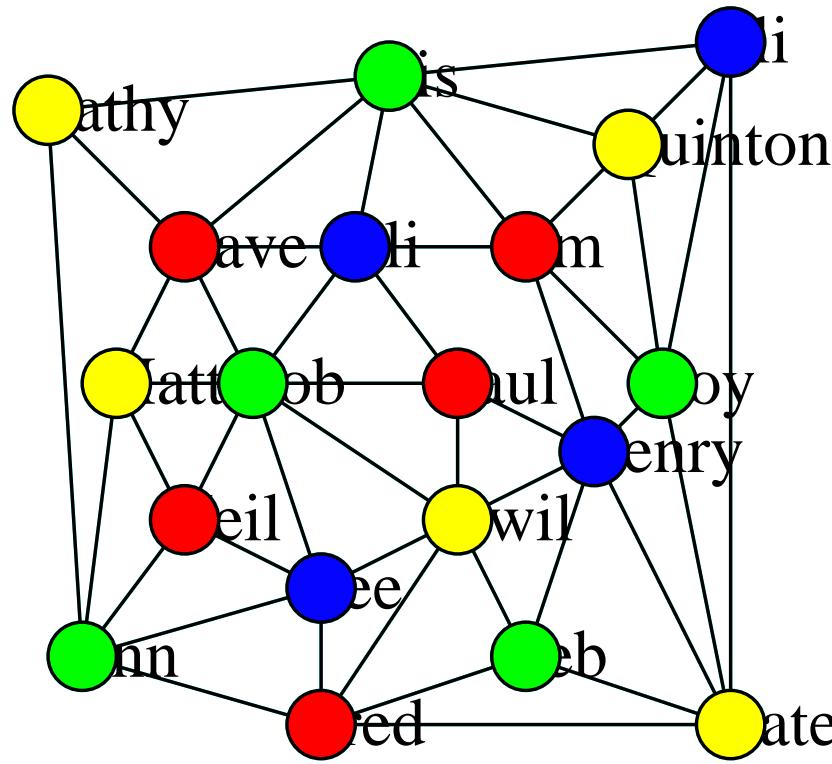
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- A food company used different colour bags for each of its products
- To save money they reduced the stock of bags to 25
- They wanted to know what items to put in what bags so that as few customers as possible would have items with the same colour bags
- This can again be reduced to a graph colouring problem
 - ★ Each node represents an item
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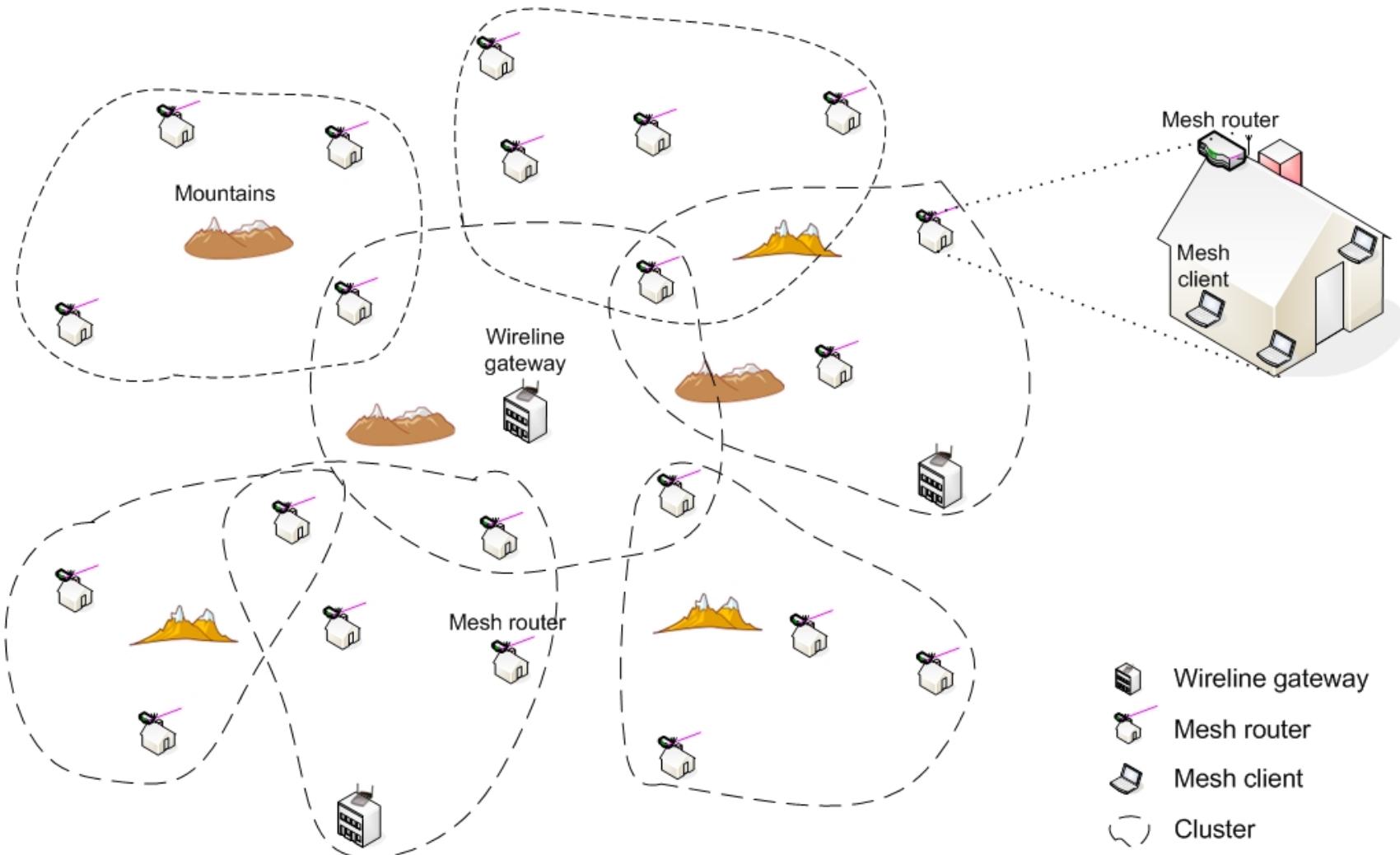
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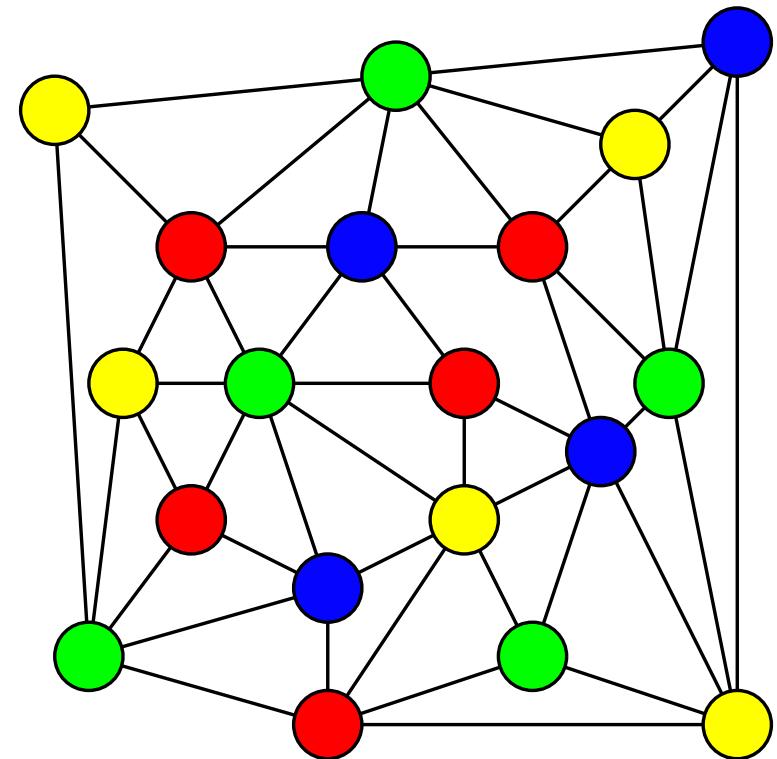
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Frequency Assignment Problem



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- There is no single way to represent graphs
- The best representation depends on the graph
- Some books describe a *Graph ADT*—graphs are too varied for this to be very useful
- An important issue in representing a graph is how to store the edge information

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Adjacency Matrices

- One representation of a graph $G = (\mathcal{V}, \mathcal{E})$ is in term of an $n \times n$ **adjacency matrix \mathbf{A}** with elements

$$A_{ij} = \begin{cases} 1 & \text{if } (i, j) \in \mathcal{E} \\ 0 & \text{if } (i, j) \notin \mathcal{E} \end{cases}$$

where $n = |\mathcal{V}|$

- For undirected graphs \mathbf{A} is a symmetric matrix, i.e. $\mathbf{A} = \mathbf{A}^T$
- For weighted graphs we often store the **connectivity matrix** or **cost-adjacency matrix**, \mathbf{C} , where

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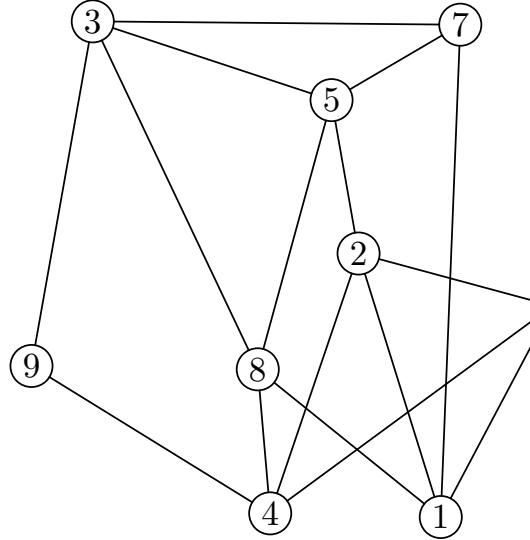
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Representing Undirected Graphs

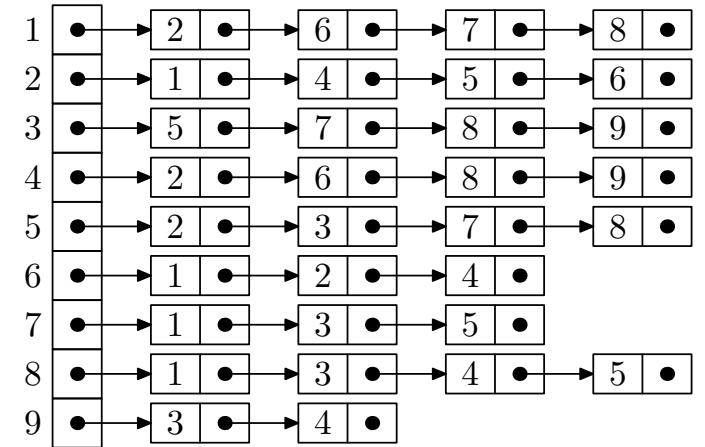


Graph

from

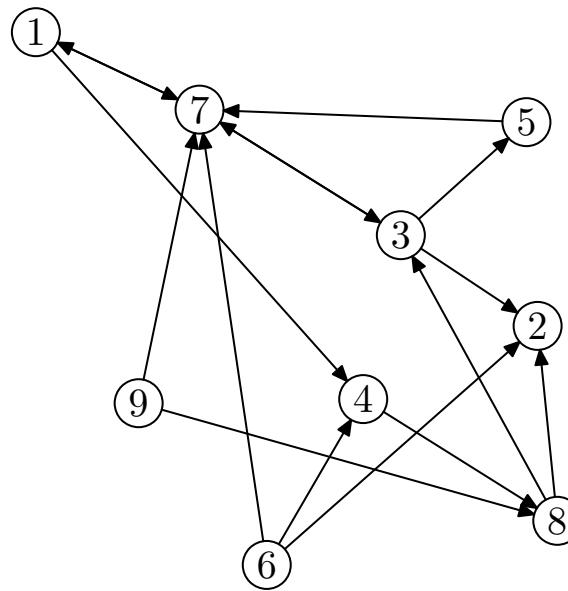
	1	2	3	4	5	6	7	8	9
1	0	1	0	0	0	1	1	1	0
2	1	0	0	1	1	1	0	0	0
3	0	0	0	0	1	0	1	1	1
4	0	1	0	0	0	1	0	1	1
5	0	1	1	0	0	0	1	1	0
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Adjacency Matrix

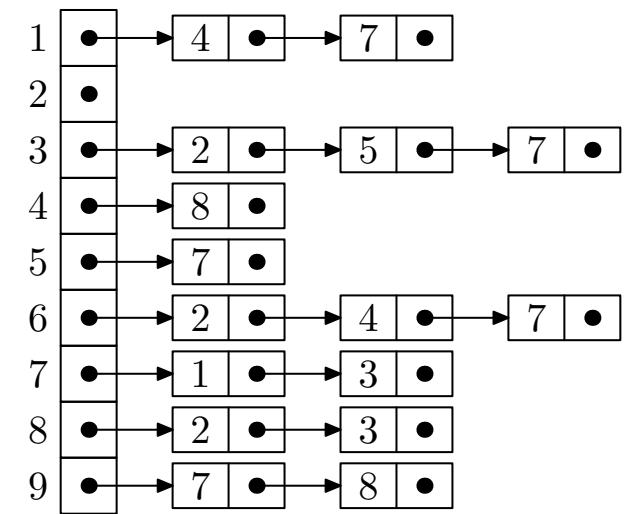


Adjacency List

Representing Digraphs



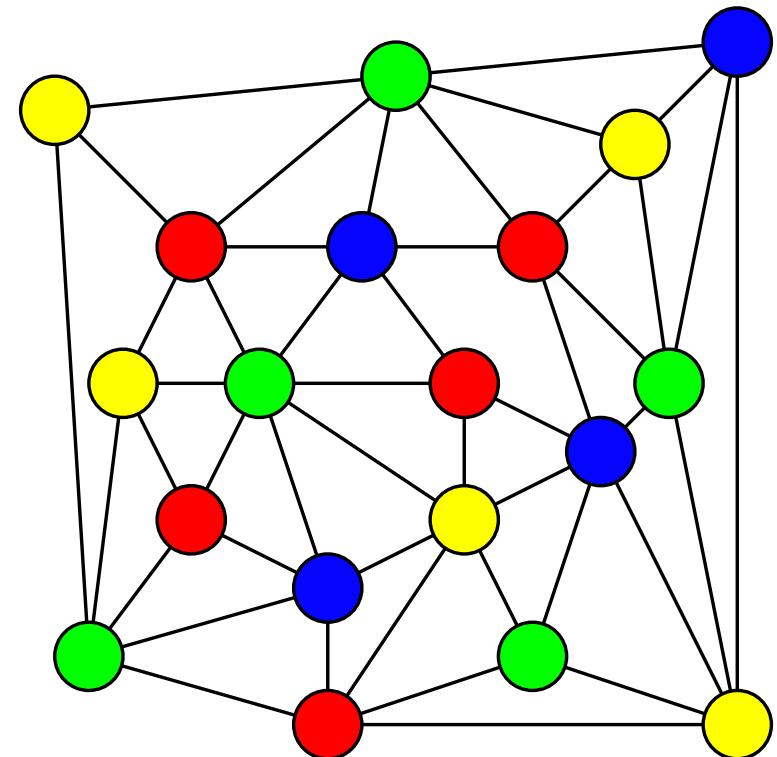
Graph



Adjacency List

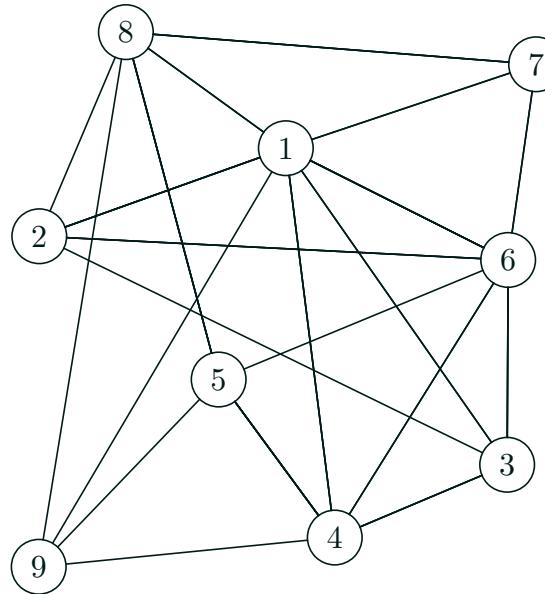
Outline

1. Graph Theory
2. Applications of Graphs
 - Geometric applications
 - Relational applications
3. Implementing Graphs
4. **Graph Problems**



Hamilton Cycle

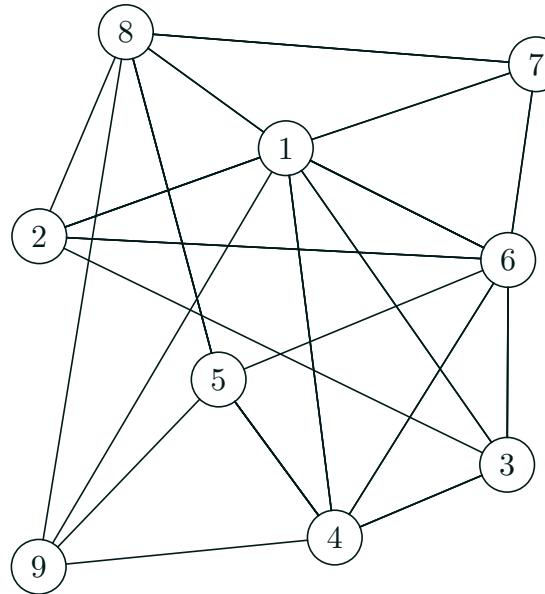
- The Euler path problem is to find a path through a multigraph that passes through every edge once—easy to solve
- The Hamilton cycle problem is to find a cycle that goes through each vertex exactly once



- There is no known efficient algorithm to solve this

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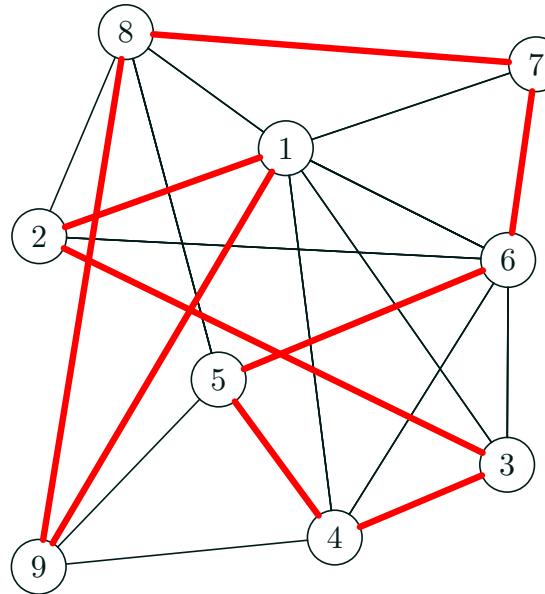
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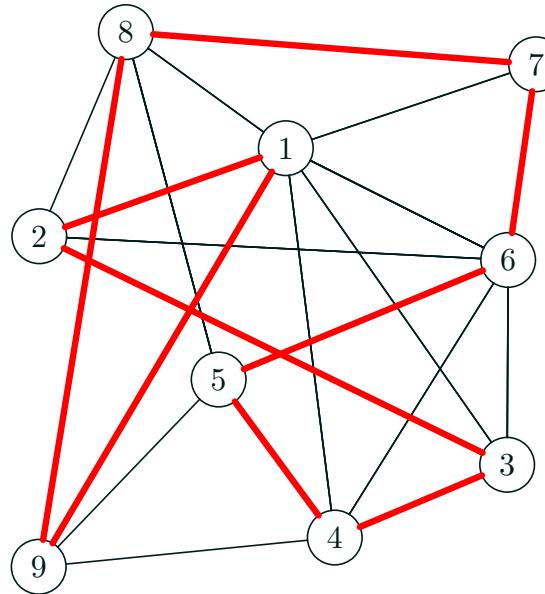
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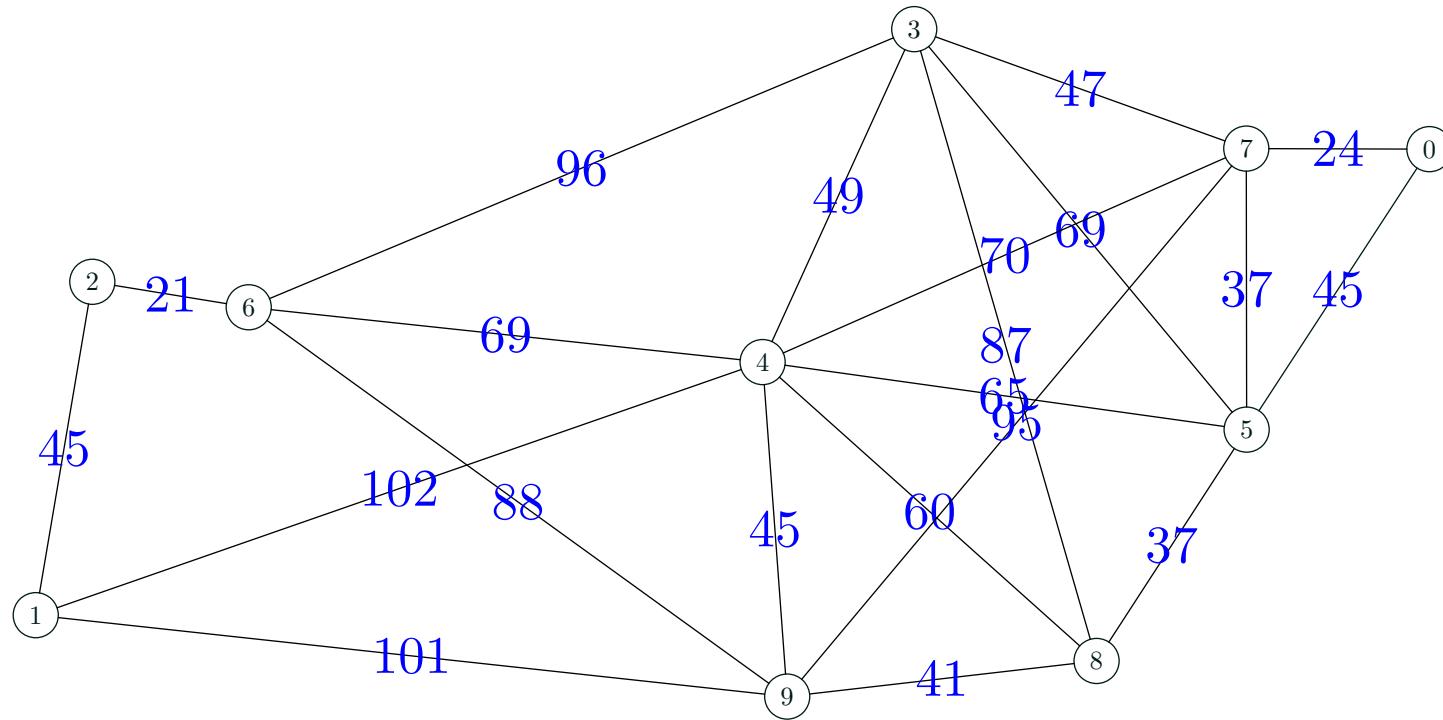
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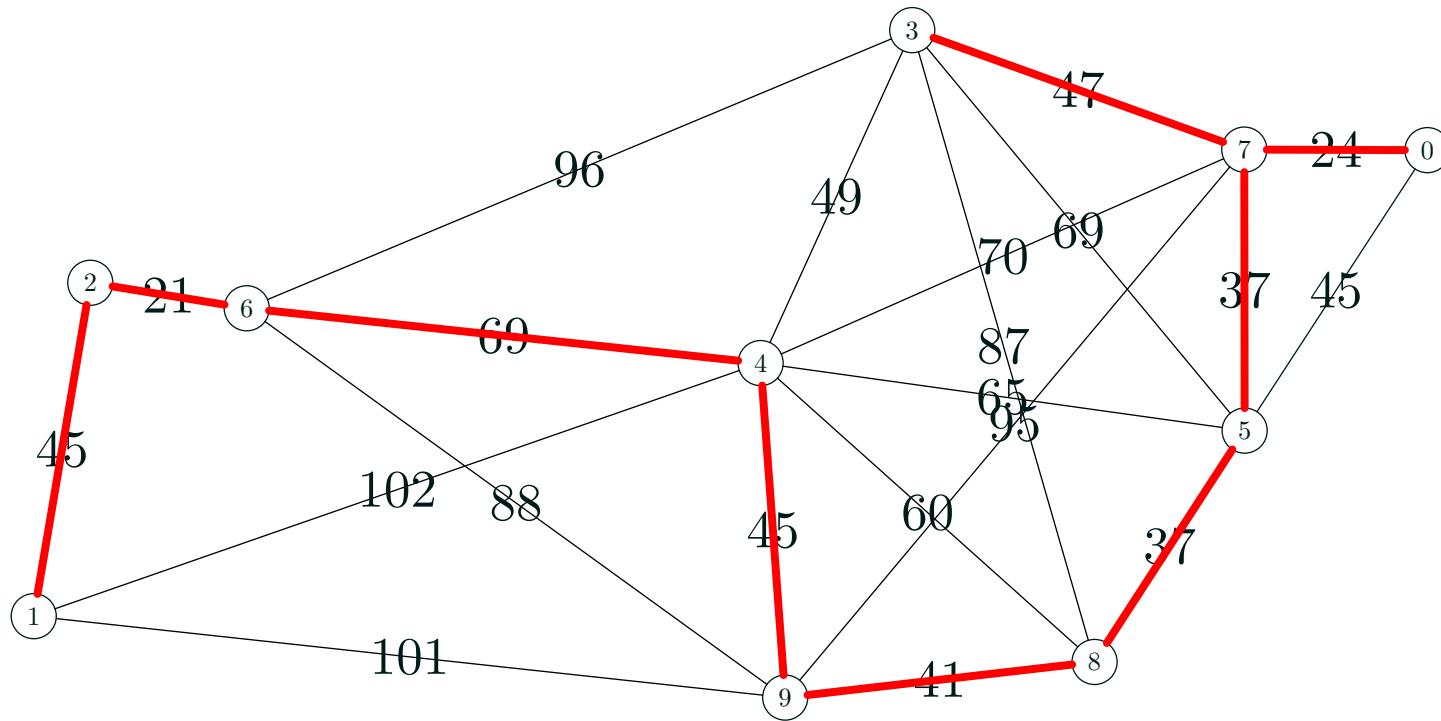
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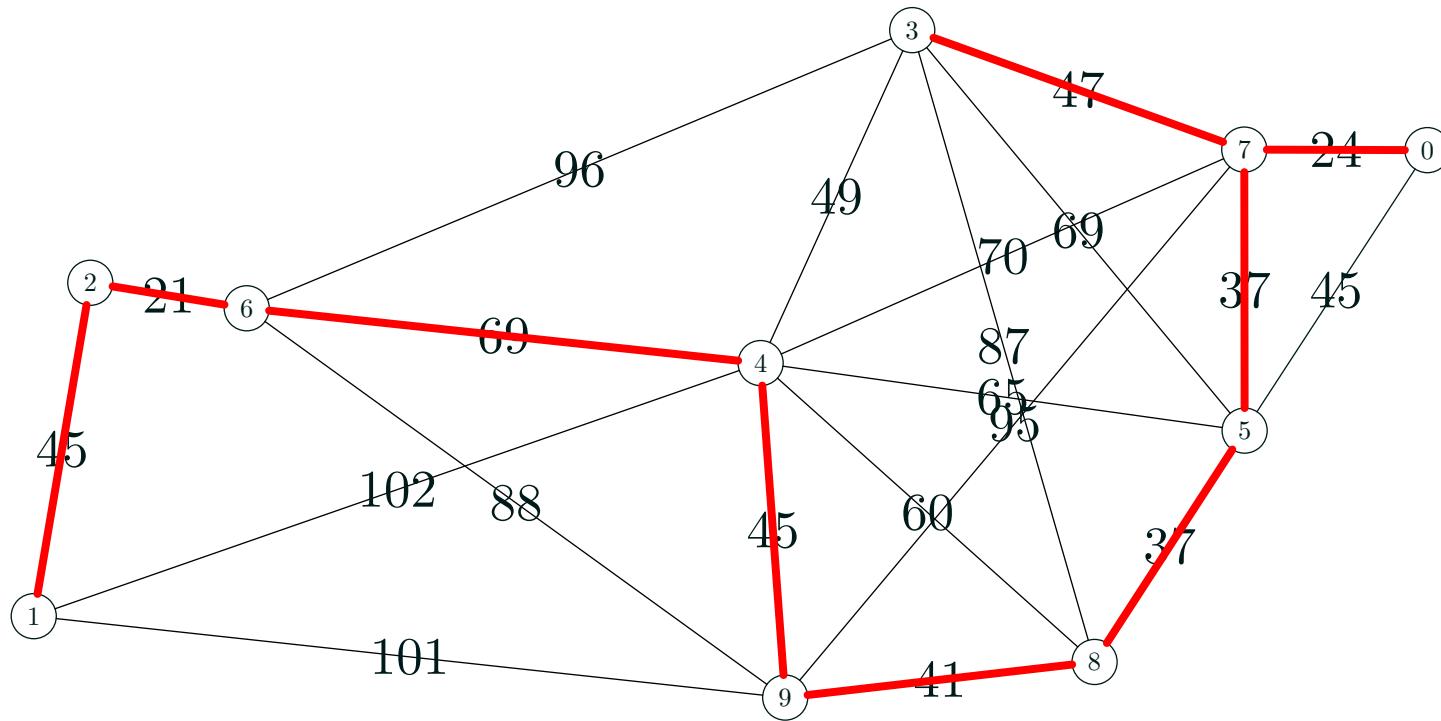
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Graph Partitioning

- The simplest version of this problem is to cut a graph into two equal halves so that you minimise the number of edges you cut
- If the edges are weighted then you want to minimise the sum of edges that are cut
- If the vertices are weighted you want to balance the sum of vertex weights in the two partitions
- An example of this problem is in dividing up a problem to run on a parallel computer
 - ★ Nodes are subtasks (weights on nodes are run times)
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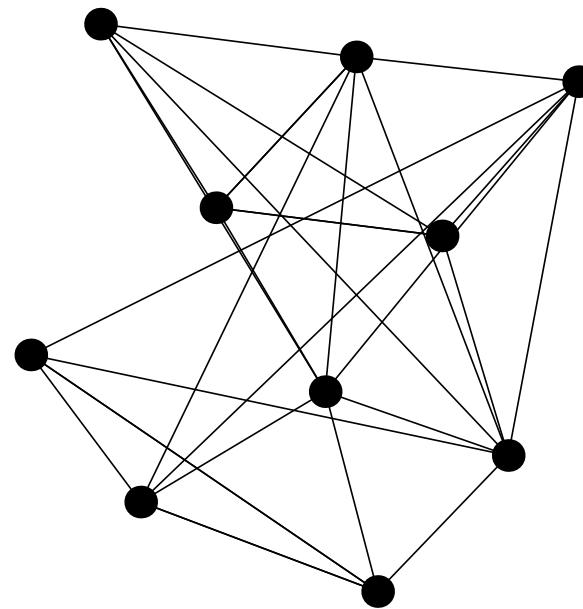
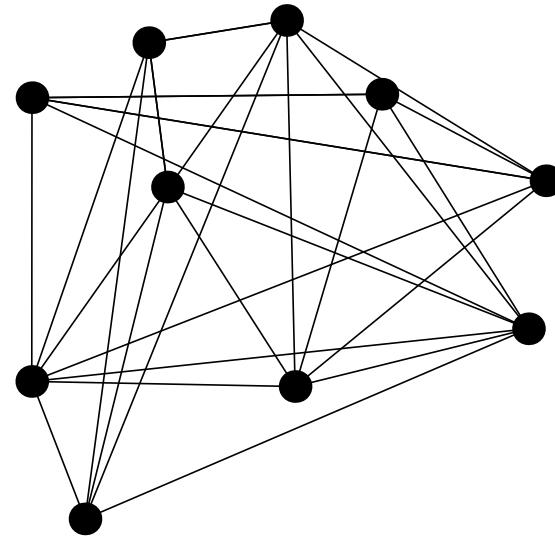
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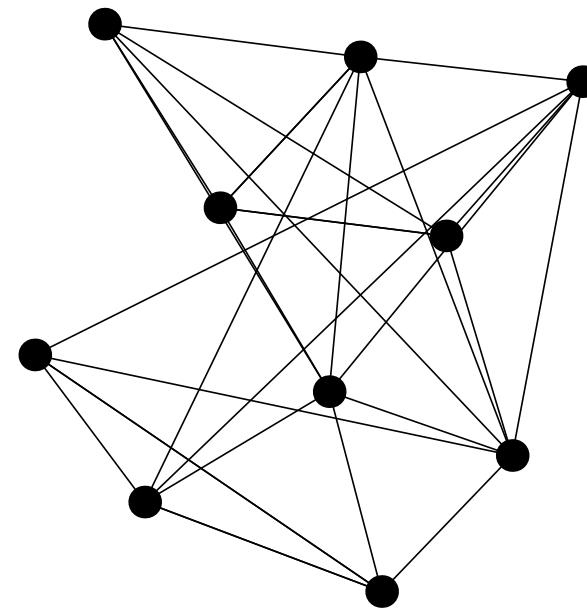
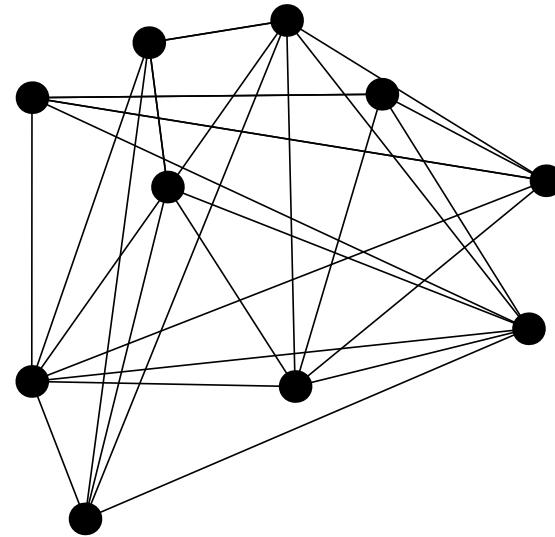
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- Theoretically it is interesting because it is not NP-complete

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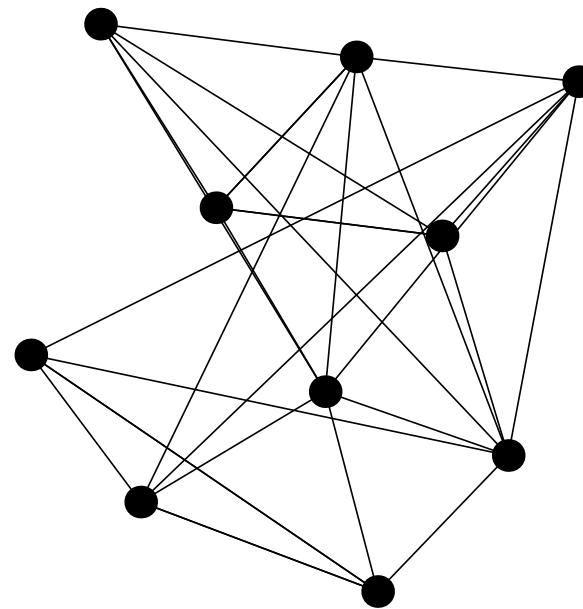
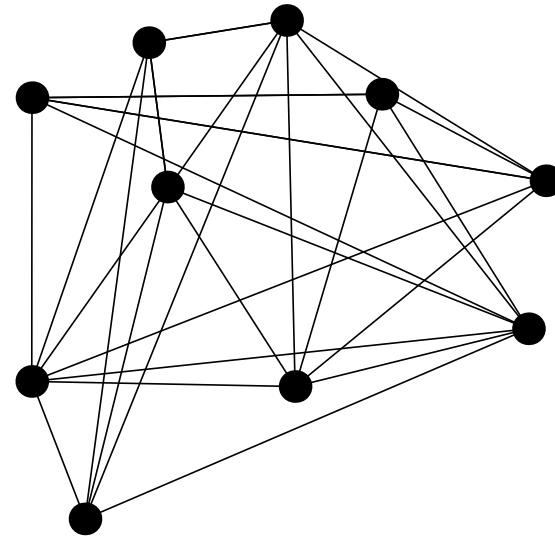
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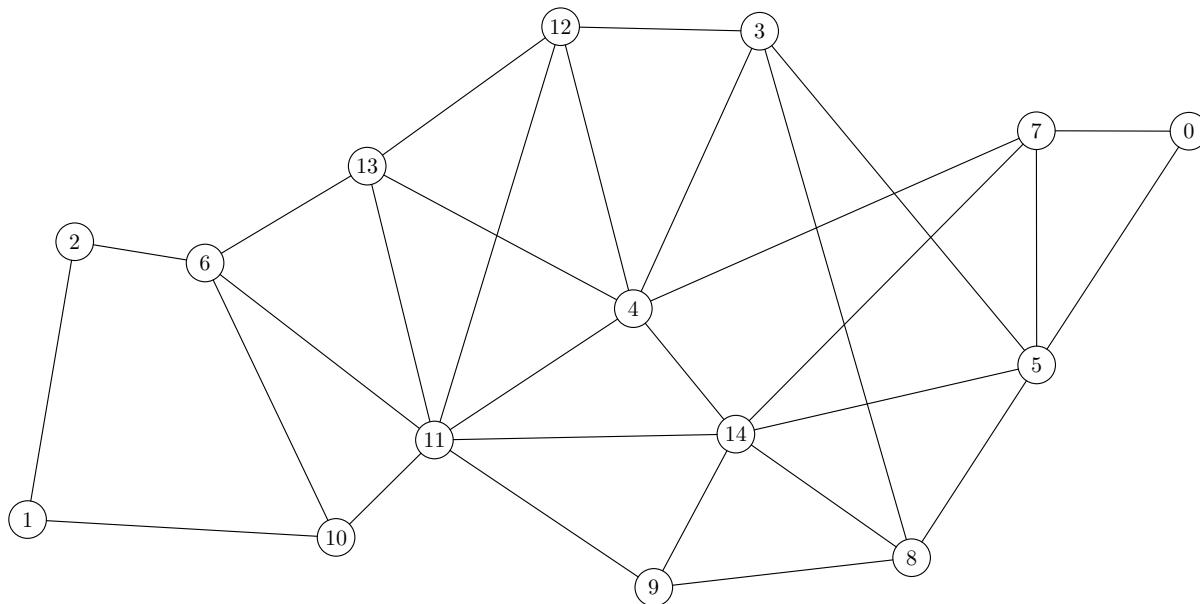
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Vertex Cover

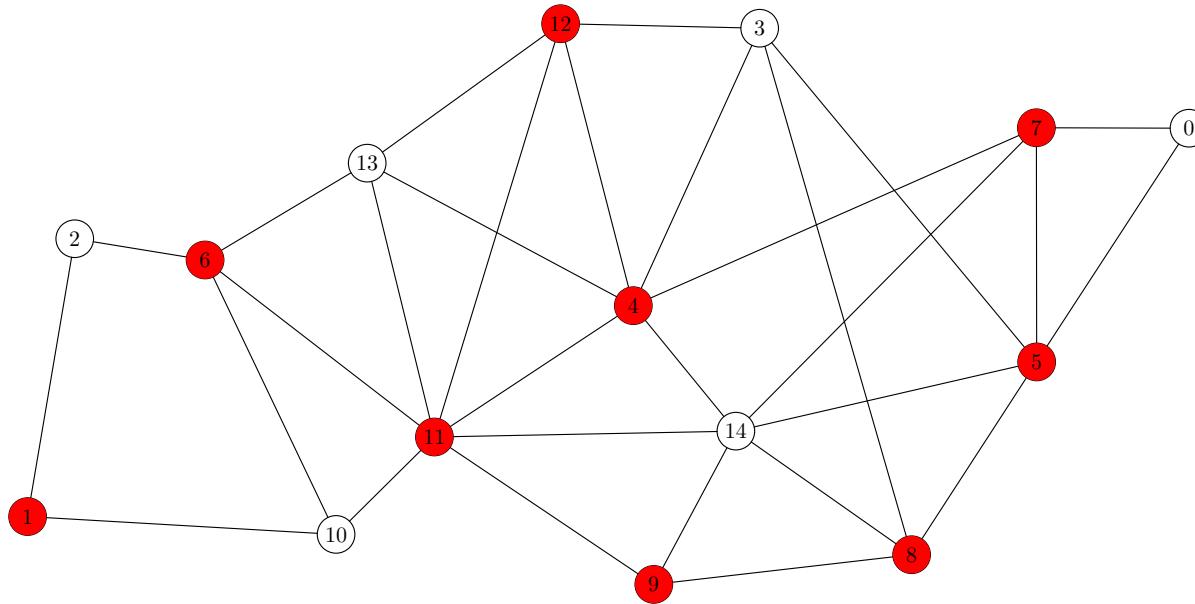
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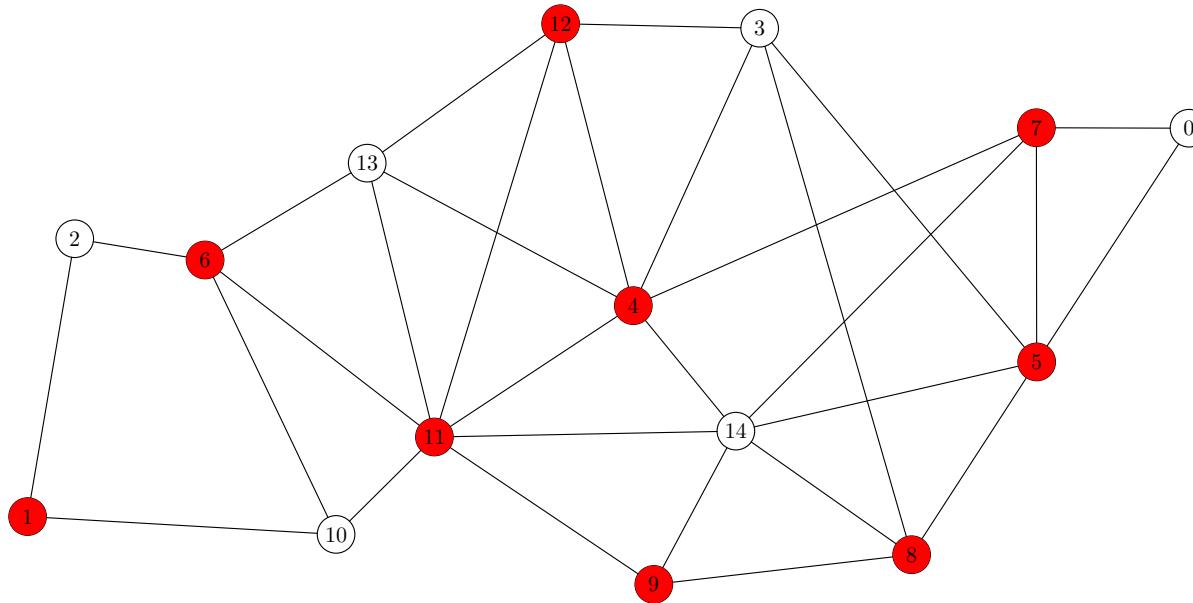
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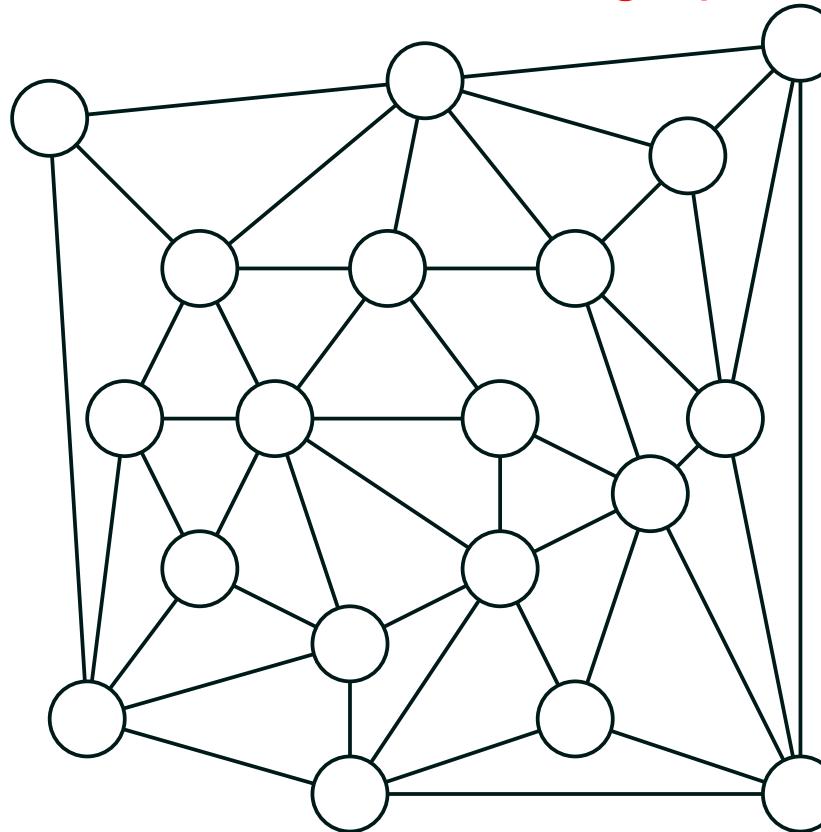
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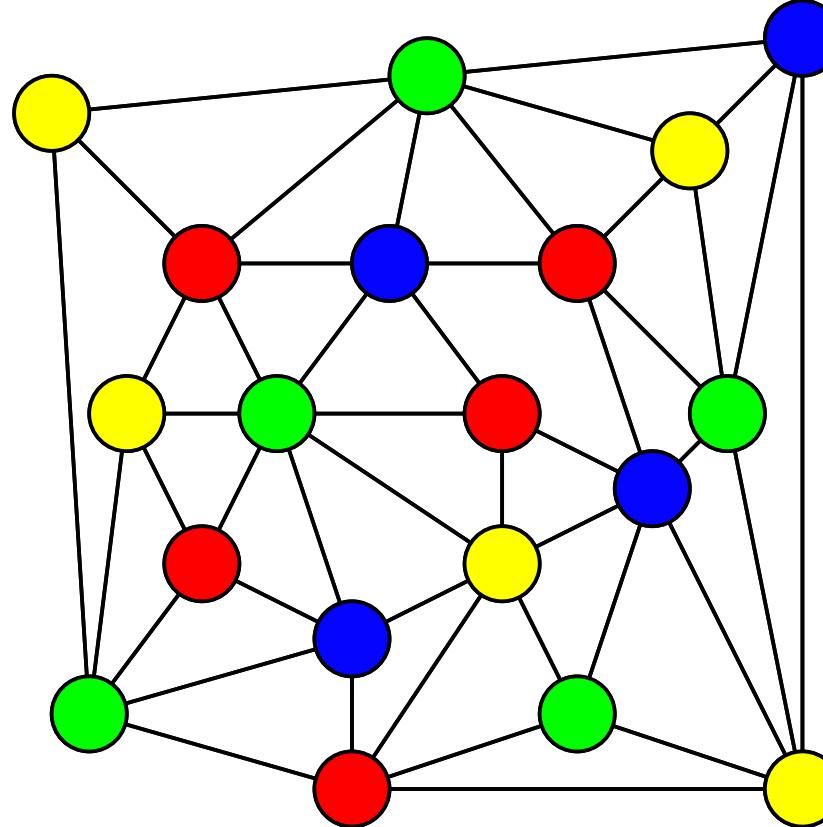
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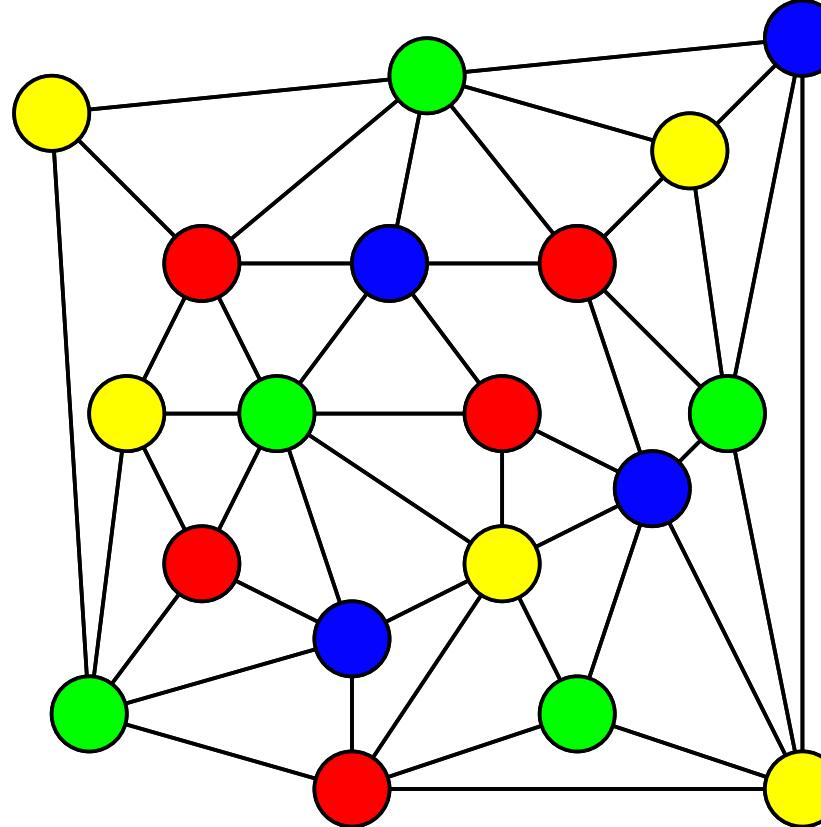
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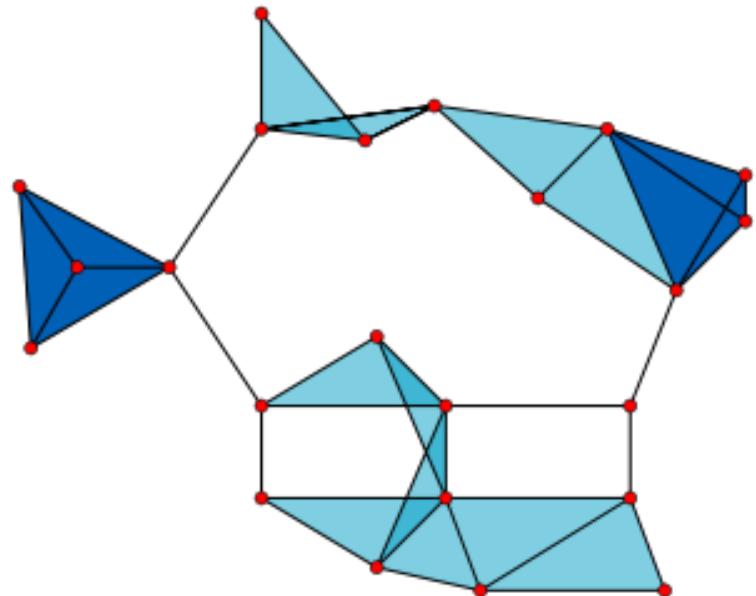
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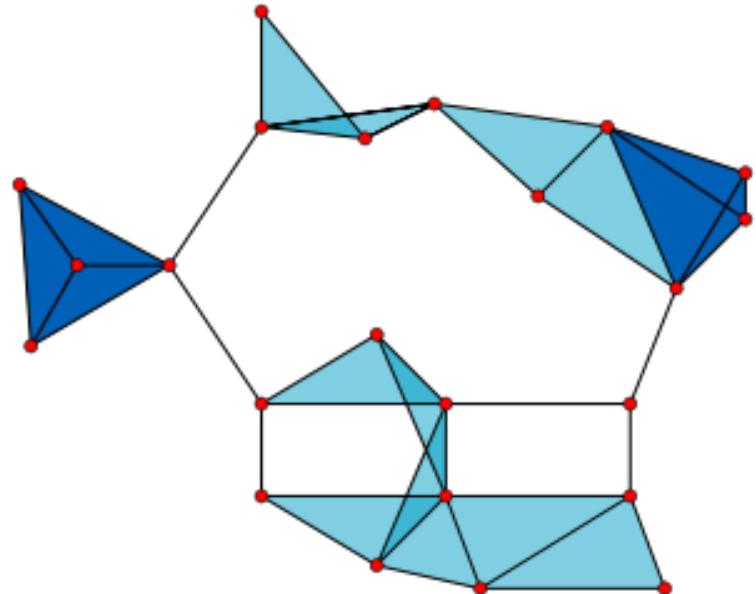
Other Graph Problems

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 - ★ Max-clique (hard)
 - ★ Maximal independent set (hard)
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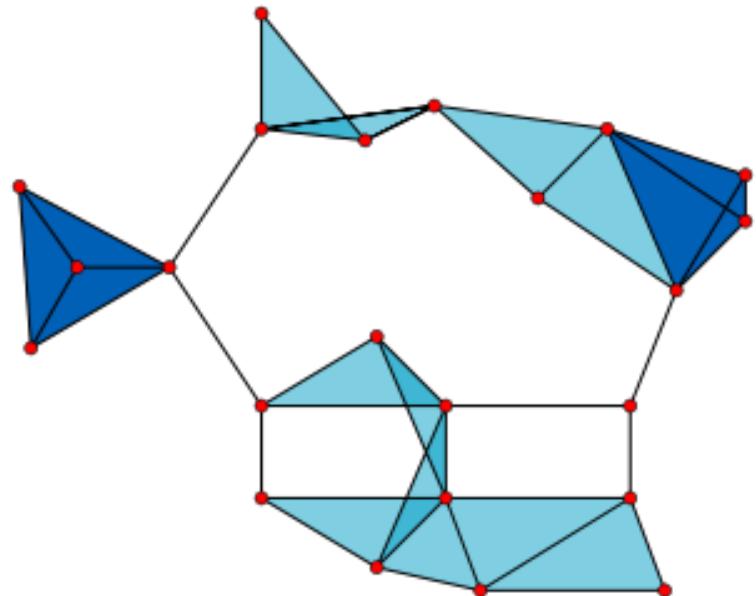
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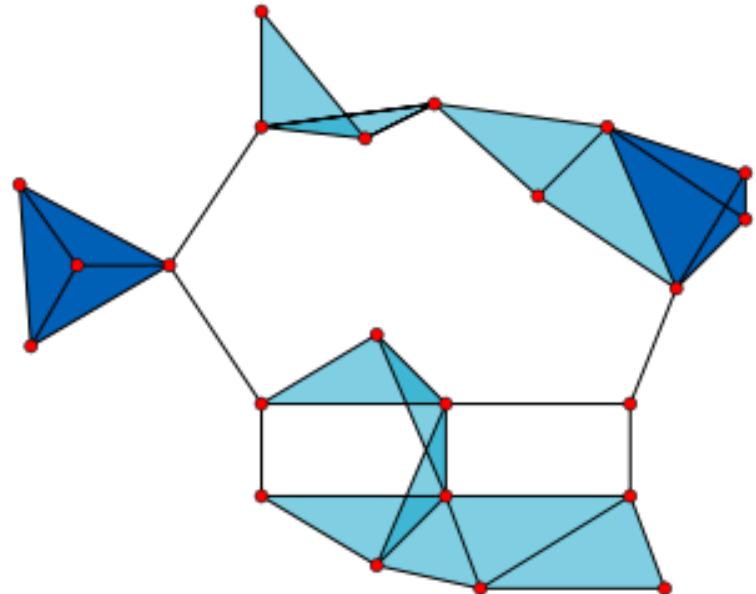
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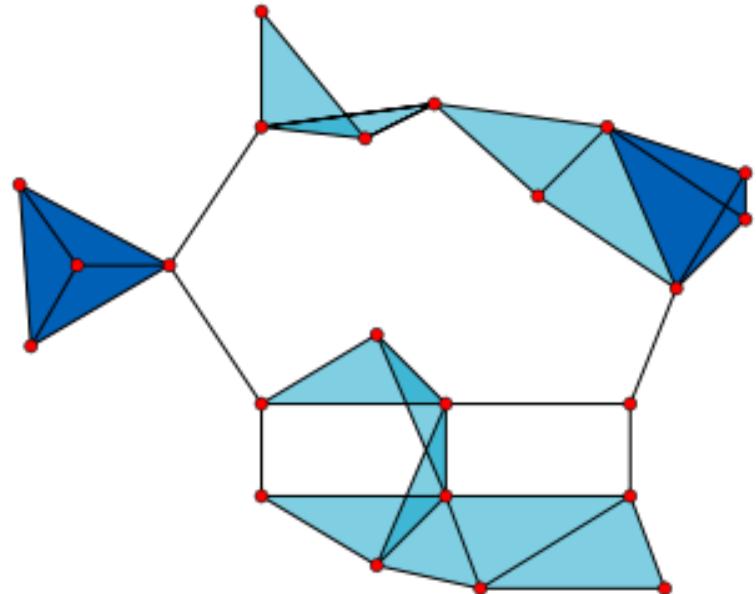
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Lessons

- Graphs are an important method for abstracting problems
- They appear in a huge number of disparate fields
- There are many problems for which efficient algorithms are known
- There are many problems which are believed to be hard—i.e. there aren't any efficient algorithms
- Even for hard problems there are good algorithms for finding approximated solutions

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