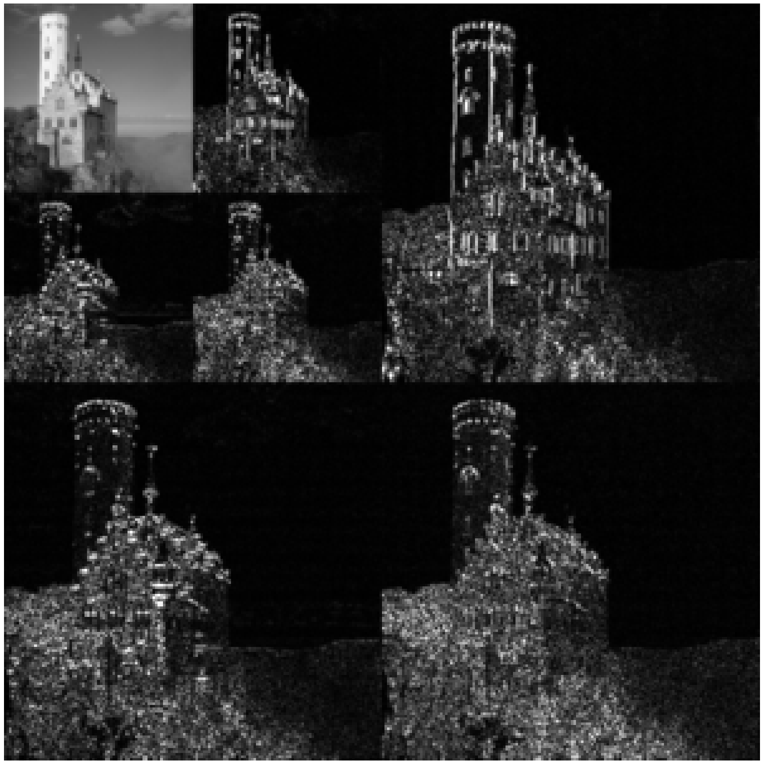
Algorithms and Analysis

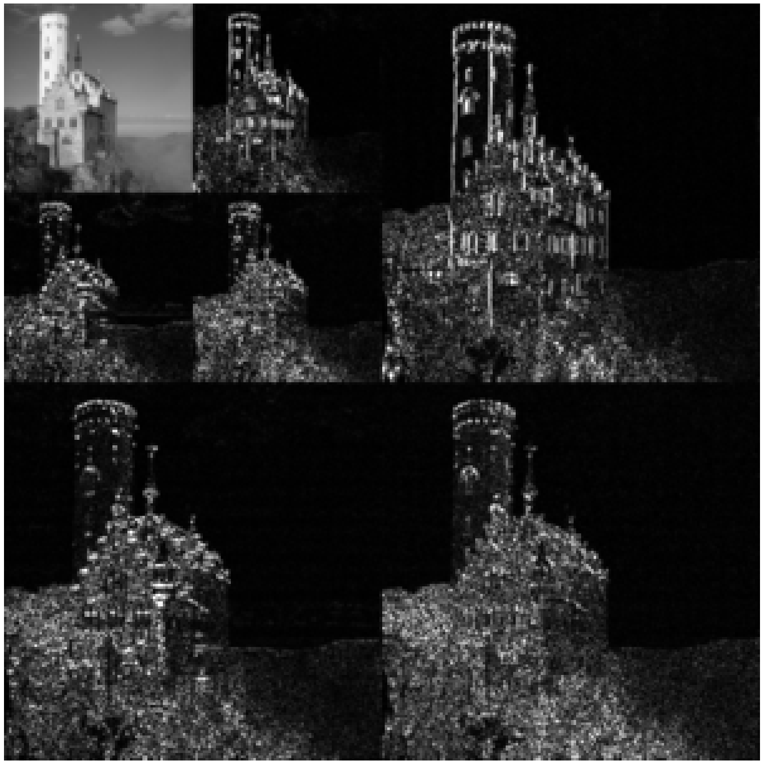
Lesson 24: *Use Smart Encoding!*



File compression, Huffman codes, wavelets

Outline

1. **Huffman codes**
2. Wavelets



File Compression

- File compression comes in two varieties
  - ⋆ Exact compression (e.g. zip used on text files)
  - ⋆ Lossy compression (e.g. jpeg used on pictures—jpeg can also be loss-less or exact)
- Good exact compression (also known as entropy encodings) can give a compression ratio around 25%
- Lossy compression can give a compression ratio from 10-1%
- Important for saving space, but lossy compression can also be used for noise reduction

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- Even used for plagiarism detection!

Entropy Encoding

- Exact encodings use the principle of using short words for frequently occurring sequences (symbols) and longer words for sequences that occur less often

- Claude Shannon showed that for an alphabet of n symbols where the probability of symbol i occurring is p_i no code exists which can transmit information in less than

$$-\sum_{i=1}^{n} p_i \log_2(p_i) \text{ bits}$$

asymptotically this compression can be achieved

- Different encoding schemes differ in the way they identify symbols of the alphabet

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Huffman Coding

- Given a sequence of symbols and their probabilities of occurance, Huffman code provides a way of coding up the information
- It is an example of a **greedy** strategy that happens to be optimal
- Like many greedy strategies it is easily implemented using a priority queue
- It is used in the UNIX compress program and in the exact part of JPEG
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Symbol Frequency

- We start from an alphabet describing the original document
  - ⋆ This might be the set of characters
  - ⋆ For an image it might be the set of pixel values
  - ⋆ It might be pairs of pixel values
- We compute the number of occurrences of each symbol

Symbol	# Occurrences
a	145
b	67
⋮	⋮

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- To save space we want to assign short codes to frequently used symbols
- There is a problem: decoding
- If we assigned a code

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\begin{array}{ccc}
e \to 0 & a \to 1 & r \to 01 \\
o \to 10 & i \to 11 & t \to 000
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etc. we could compress a document very efficiently but we could never decode it uniquely

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- We assign each symbol to a leaf of a binary tree
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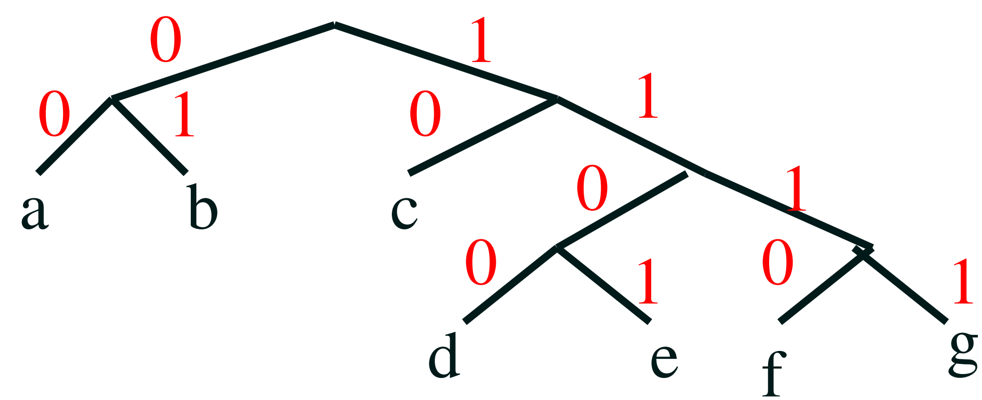

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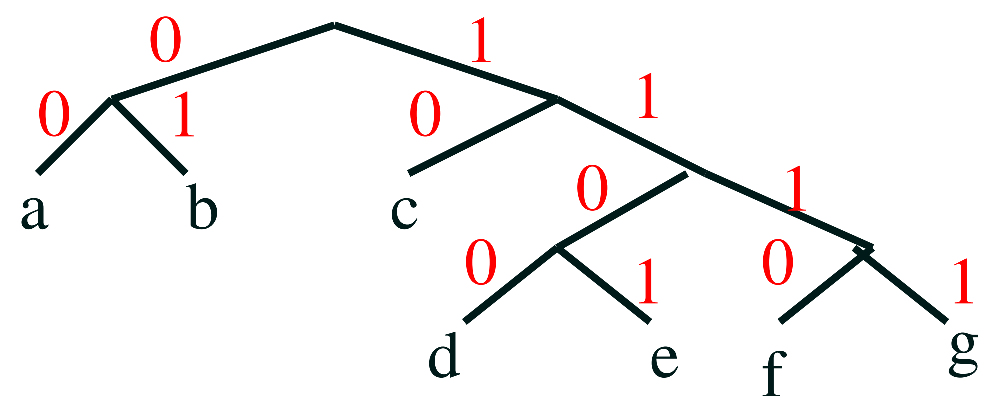

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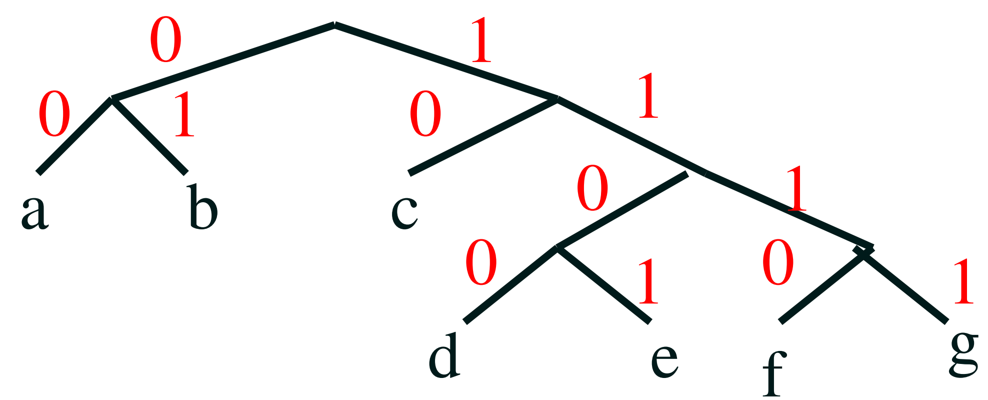

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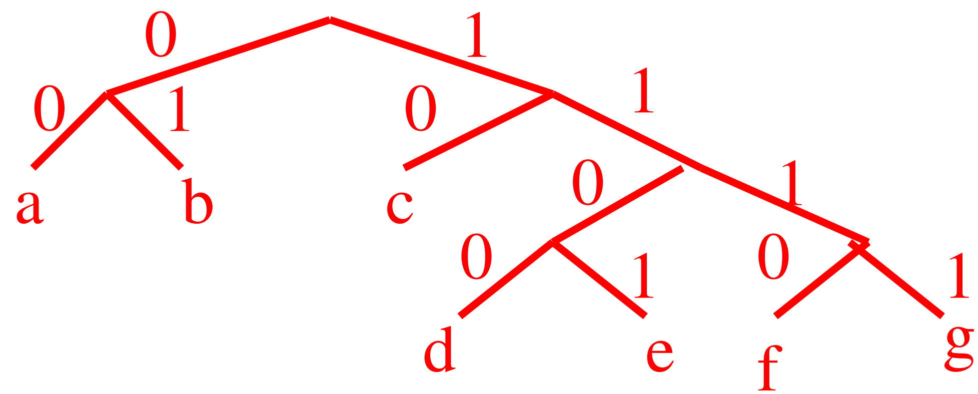

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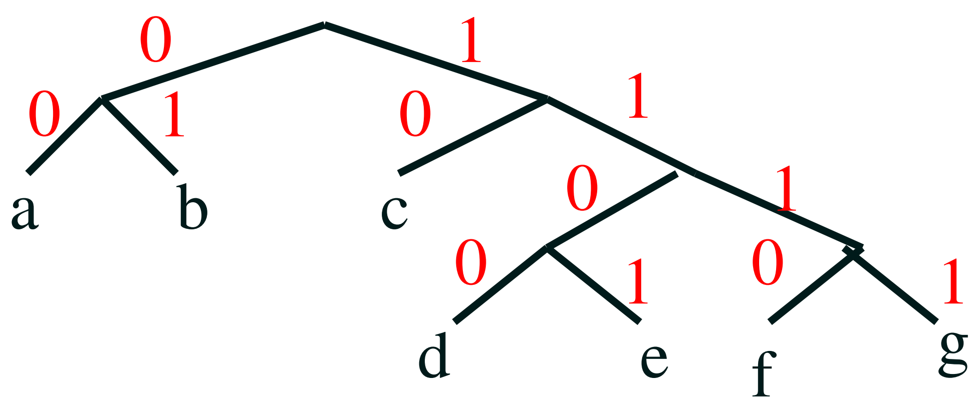

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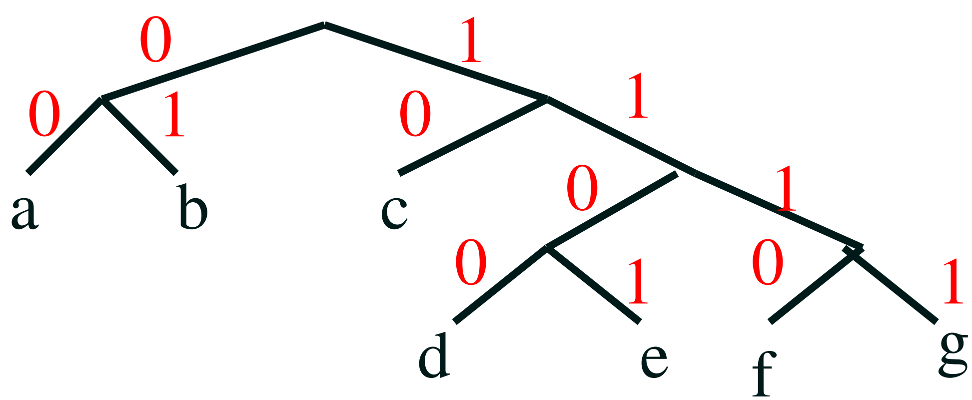

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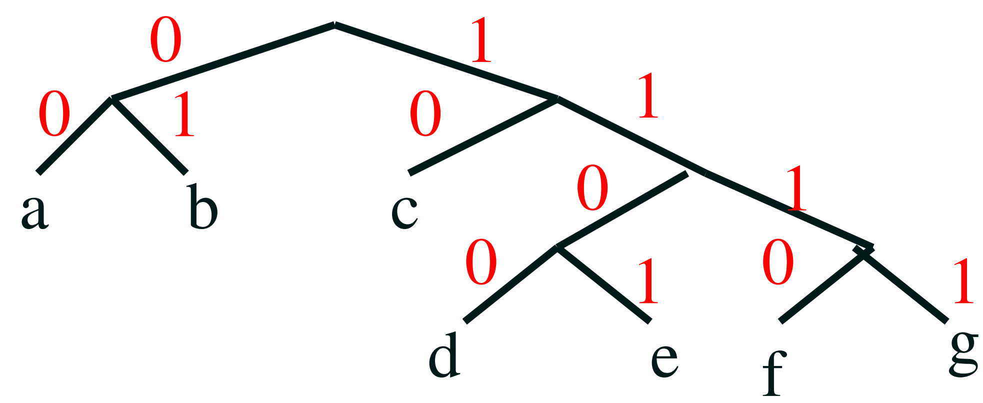

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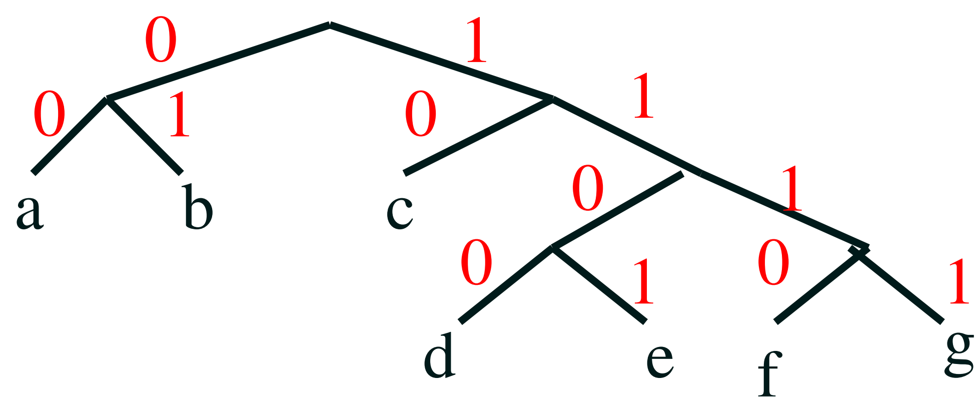

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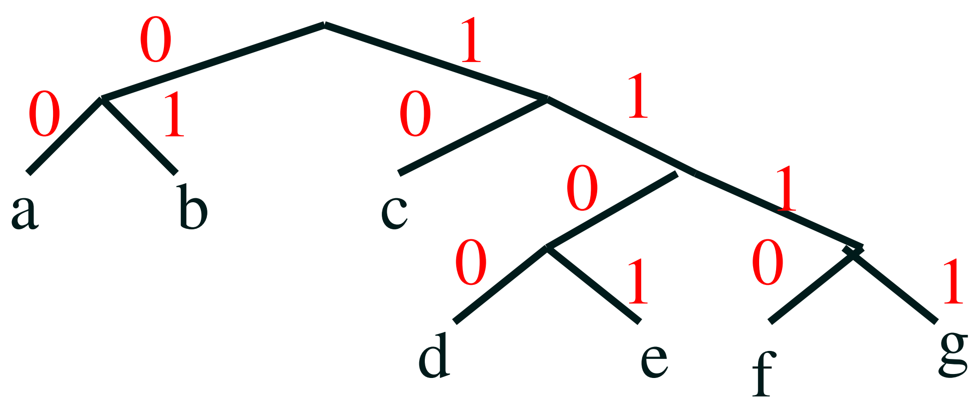

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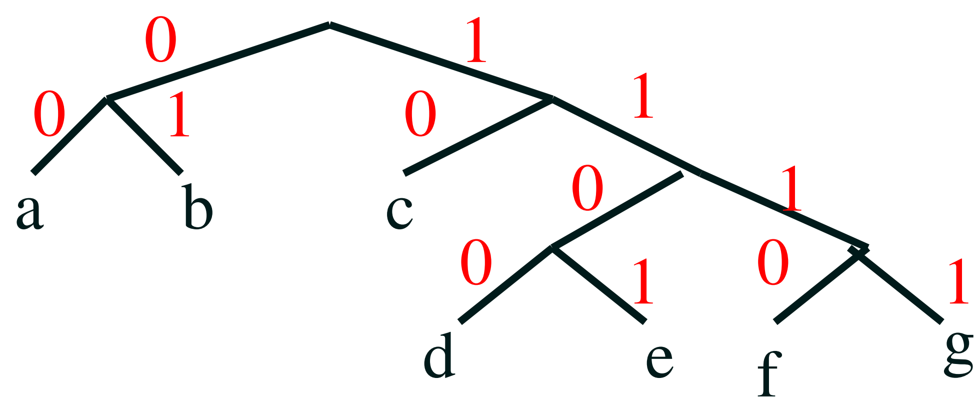

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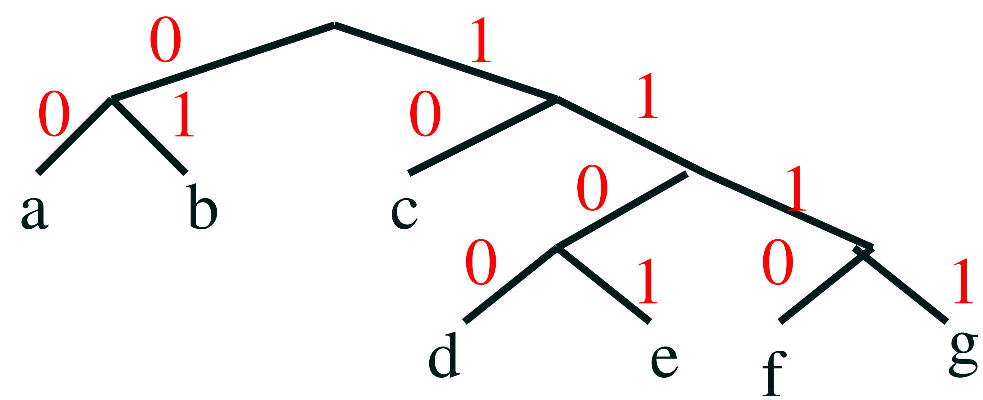

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Generating the Huffman Tree

- We are left with the problem of constructing the Huffman tree such that frequently occurring letters have short codes
- A greedy approach is to iteratively build a tree by
  1. combine the two most infrequent symbols into a subtree
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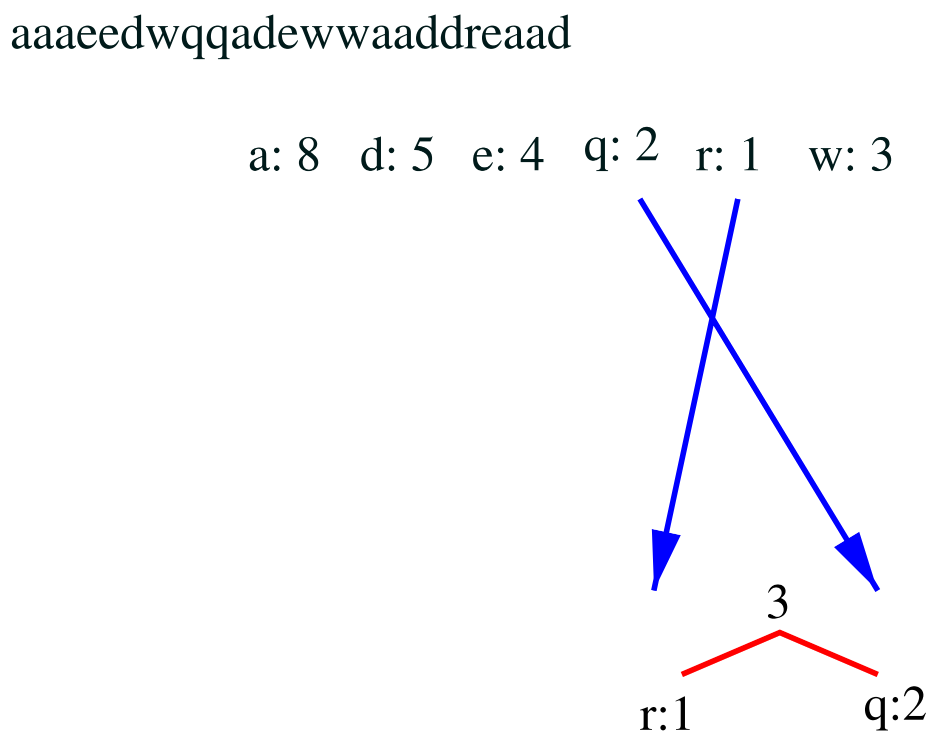

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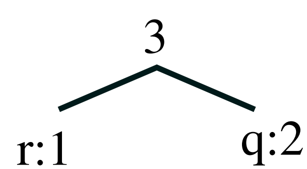

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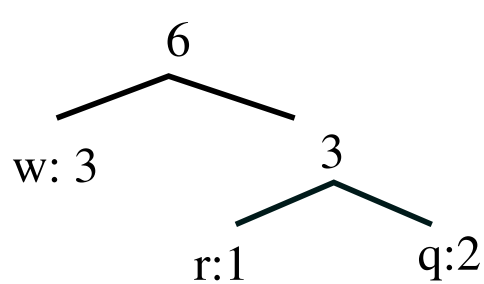

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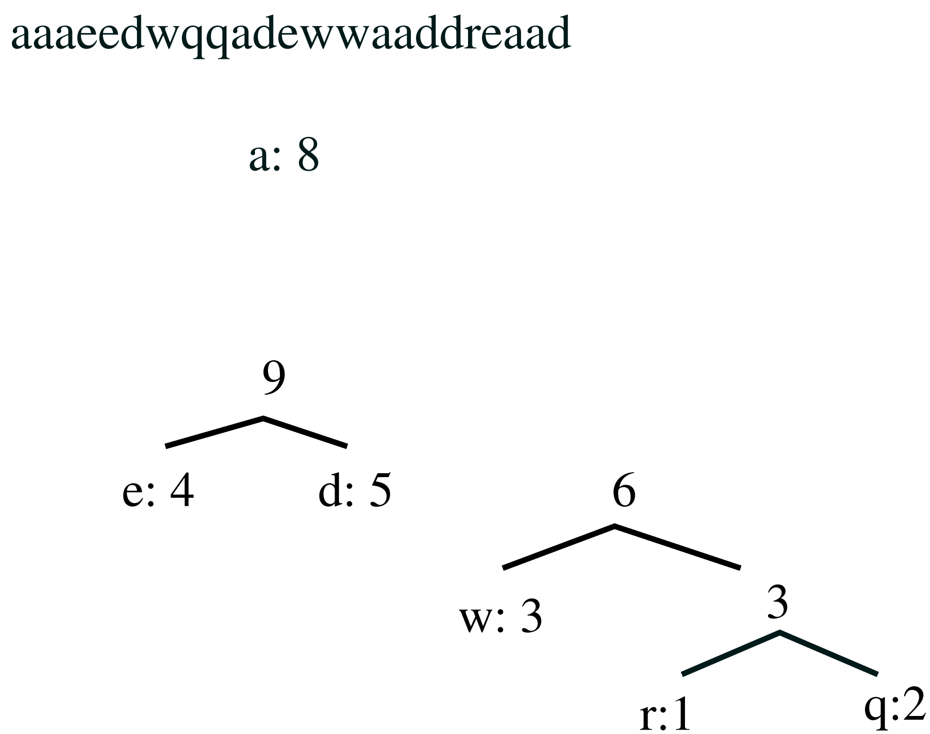

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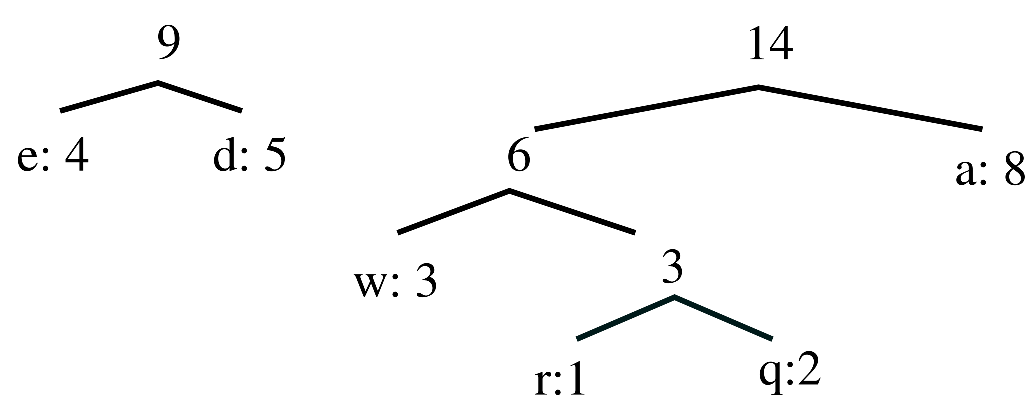

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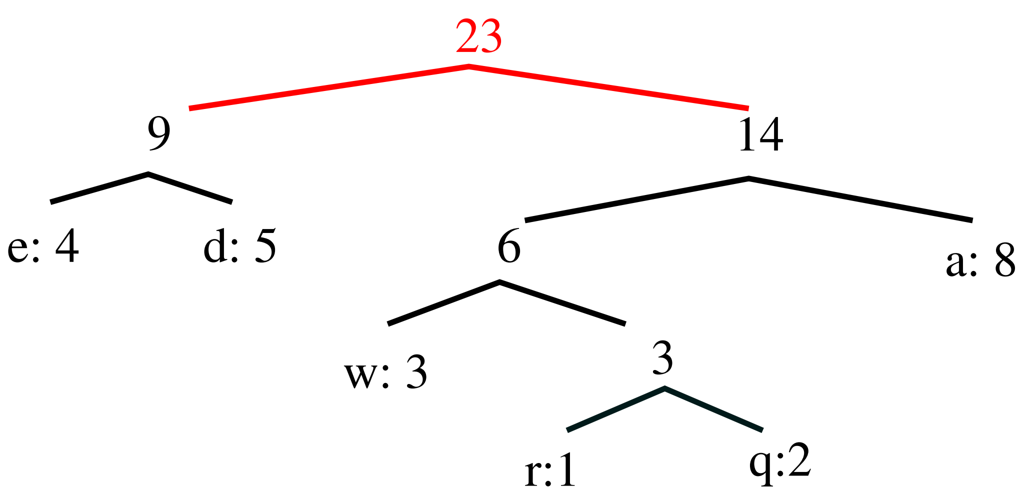

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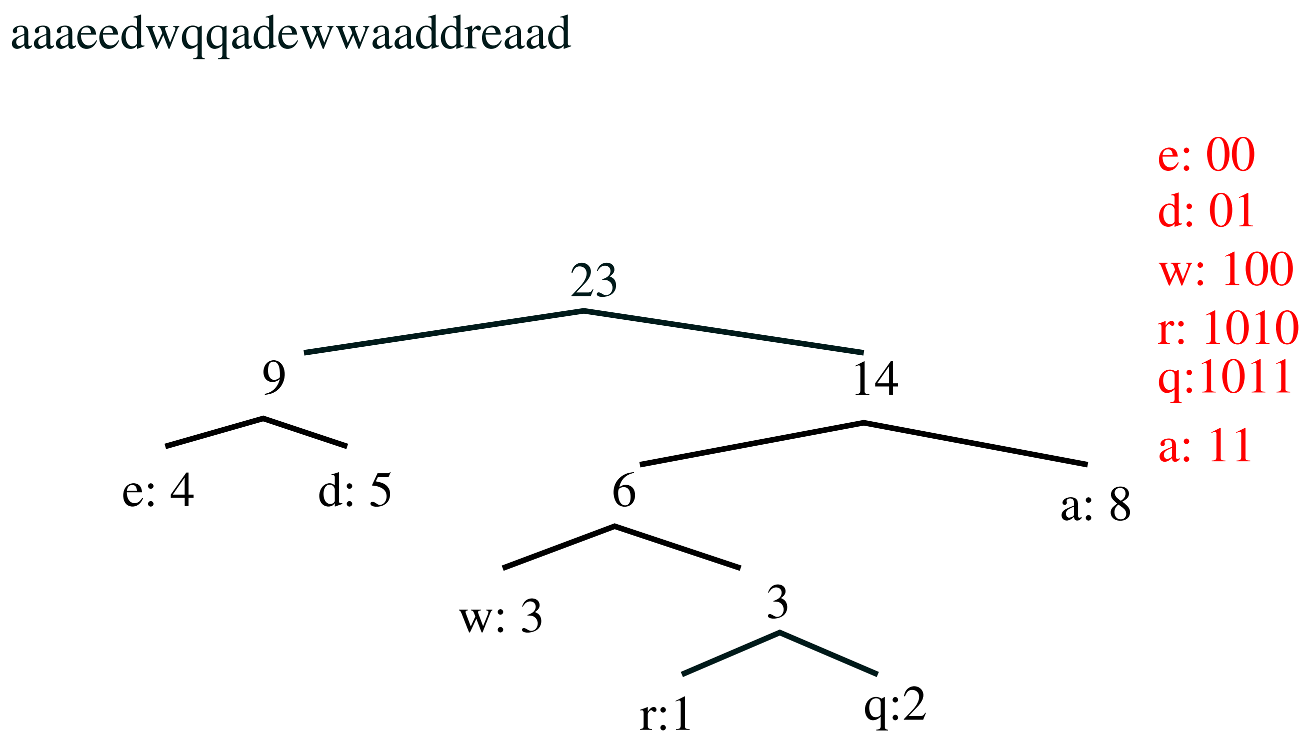

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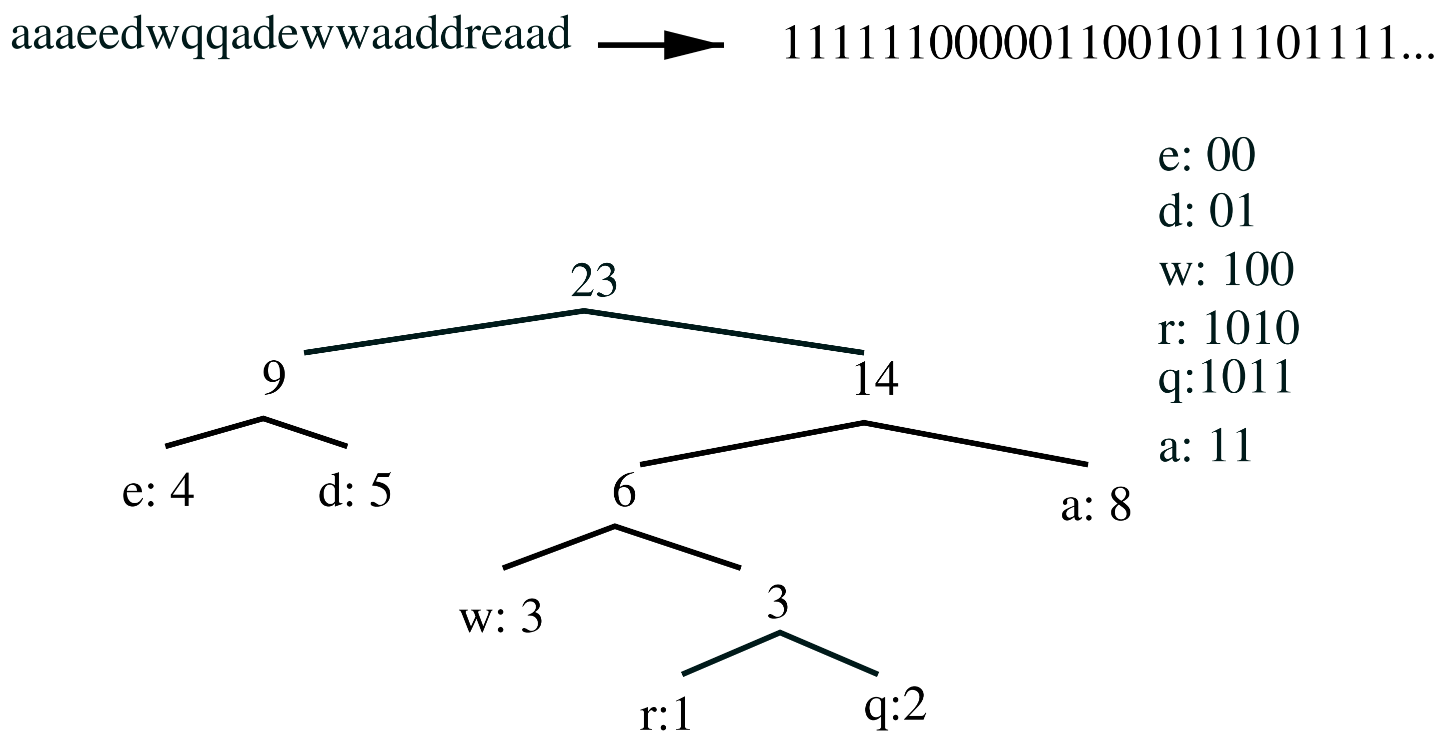
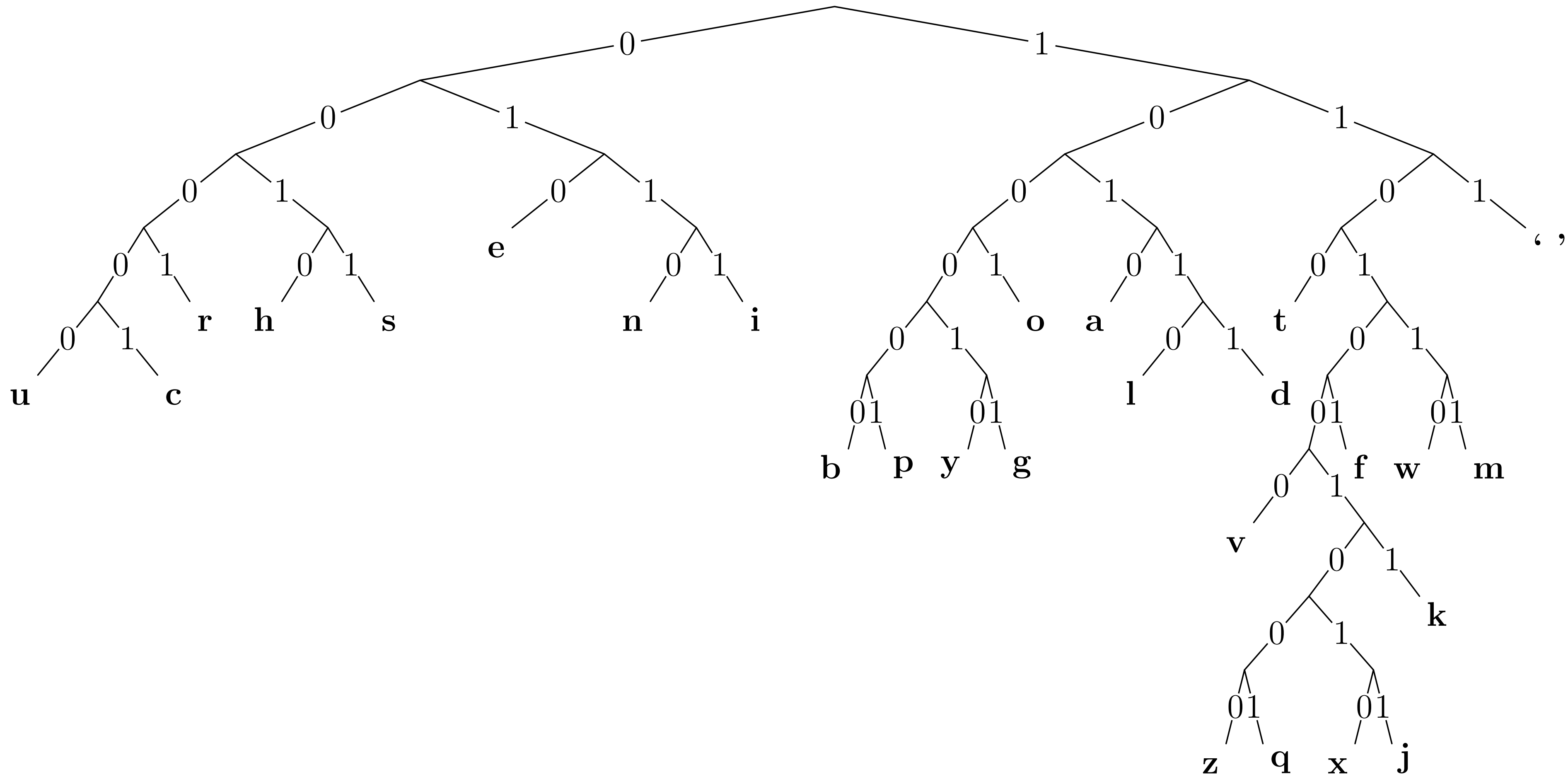


English Letters



the quick brown fox jumps over the lazy dog — 344 bits

11000010010111110100100100000011100001110100111111000000001100111011001101111101
01100111010010101111101001011000001101111000010011111100111010000100001111110000
100101111011010101101001000100010111101111001100011 — 211 bits

Implementing Huffman Encoding

- To implement Huffman encoding you need
  1. A class to build Huffman trees by combining subtrees
  2. A way to find the least frequently used symbols or symbol combinations
- Priority queues are ideal for this application
- They allow you to find the least frequently used symbols (`removeMin`) and to add new symbols (`add`)
- To decode you follow the Huffman tree

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- Huffman encoding is an example of a **Greedy solution pattern**
- That is we look for local optimality (i.e. we combine the two least frequently used symbols)
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Advanced Techniques

- Huffman code is optimal given the frequency of symbols
- However, there is considerable art in identifying which 'symbols' to use
- Advanced compression algorithms (LZ78, LZW Lempel-Ziv-Welch) build dictionaries of sequences seen in the files—they tend to be rather specialised
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File Compression and Plagiarism Detection

- One way of spotting plagiarism is to compare the compressed lengths of two files and the length of the compressed file when the two files are concatenated first
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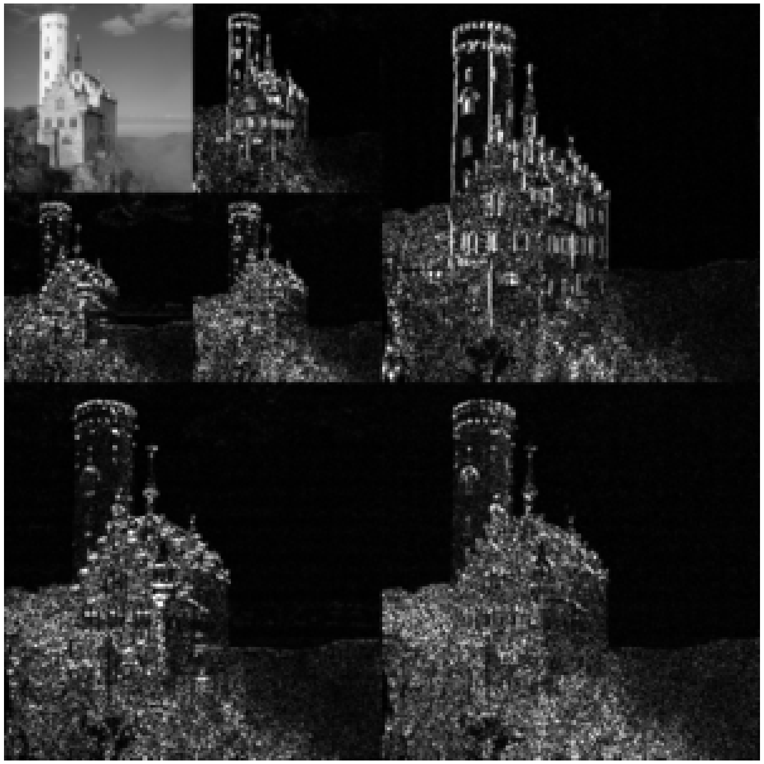
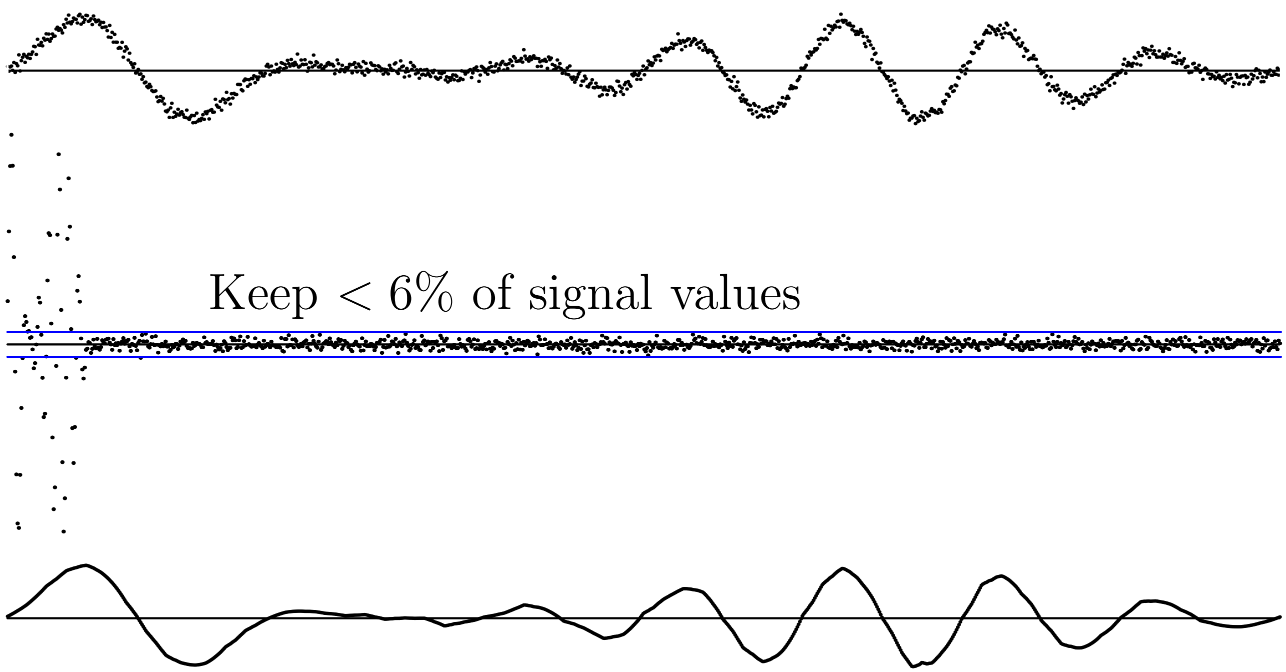
Outline

1. Huffman codes
2. **Wavelets**



Signals and Energies

- We consider compressing a signal $\boldsymbol{x} = (x_0, x_1, \ldots, x_{n-1})$



- We can define the “energy” as the squared deviations

$$E = \sum_{i=1}^{n} x_i^2$$

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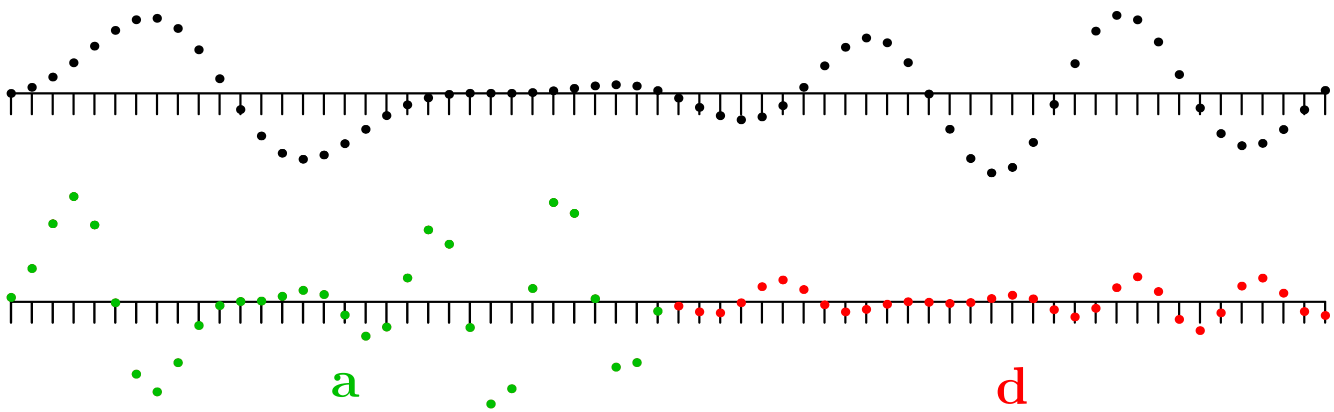
Wavelets

- With wavelets we try to re-represent the signal so as to squeeze as much energy as possible into fewer bits

- The easiest way to do this is with Haar wavelets

$$a_i = \frac{x_{2i} + x_{2i+1}}{\sqrt{2}} \qquad d_i = \frac{x_{2i} - x_{2i+1}}{\sqrt{2}}$$

- Define new signal $(a_0, a_1, a_2, \ldots, a_{n/2-1}, d_0, d_1, \ldots, d_{n/2-1})$



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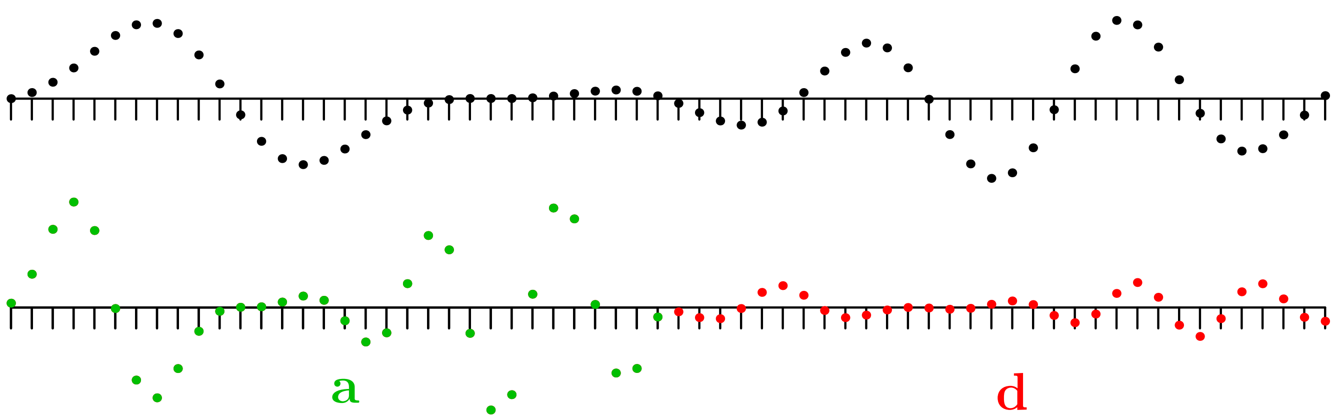

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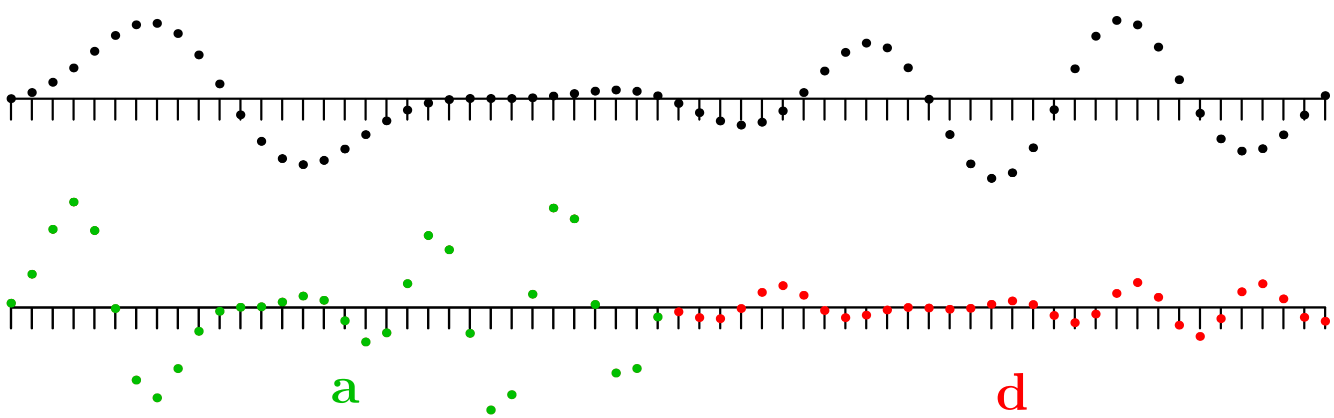

Carrier and Difference Signals

- The terms $a_i = (x_{2i} + x_{2i+1})/\sqrt{2}$ takes the “average” of the signal, but compresses it in half the space

- The terms $d_i = (x_{2i} - x_{2i+1})/\sqrt{2}$ takes the difference and is small if the signal does not change much

- The energy is conserved since

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- The wavelet transform can be easily reversed

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- Can compute transform using vectors (wavelets)

$$a_i = \boldsymbol{V_i} \cdot \boldsymbol{x} \qquad d_i = \boldsymbol{W_i} \cdot \boldsymbol{x}$$

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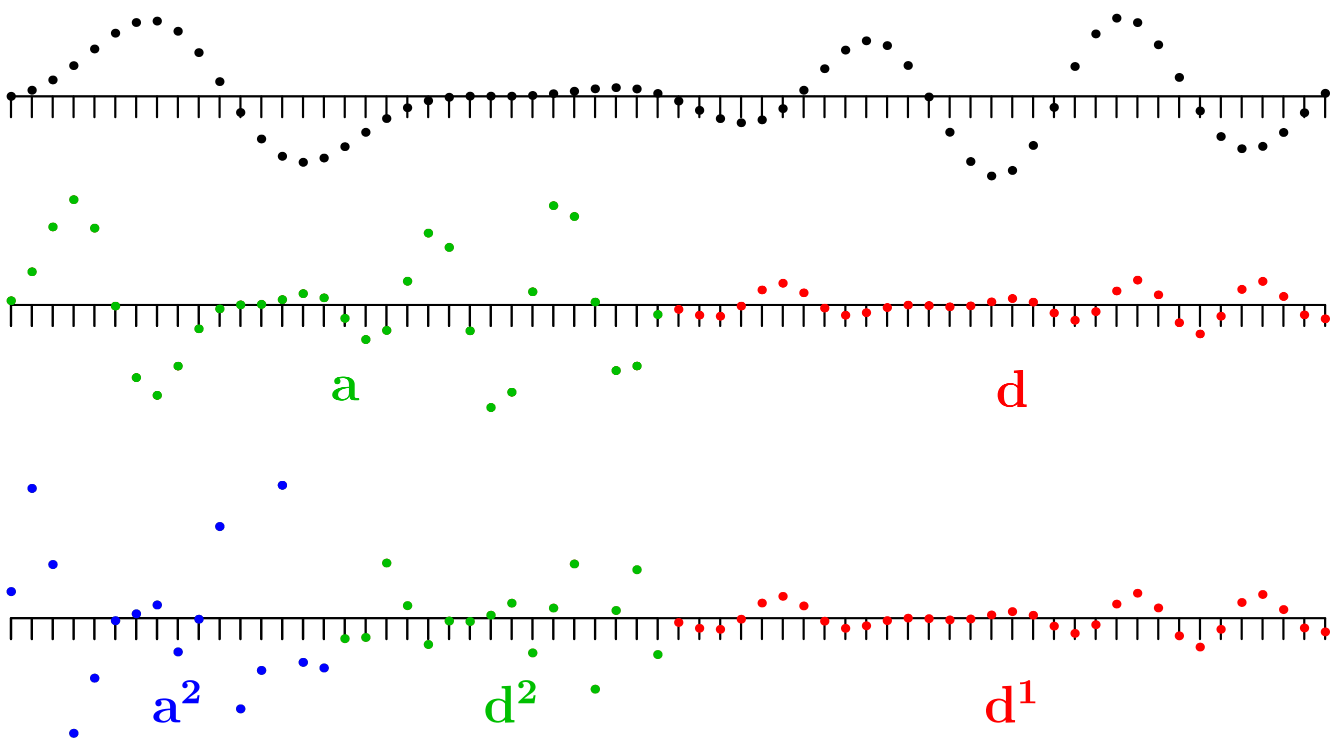

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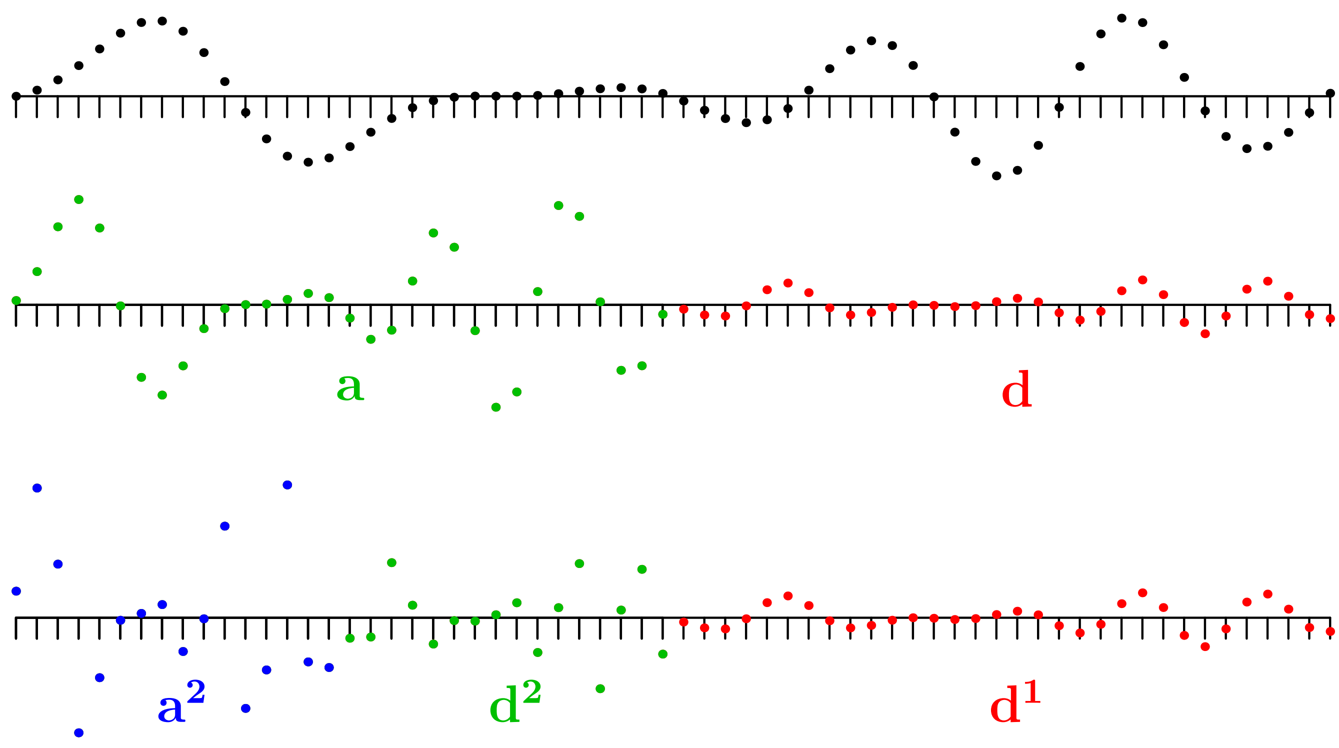

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- Ingrid Daubechies suggested a host of wavelets which do better than Haar for smooth signals

- The simplest is Daub4 defined by

$$a_i = c_0 x_{2i} + c_1 x_{2i+1} + c_2 x_{2i+2} + c_3 x_{2i+3}$$
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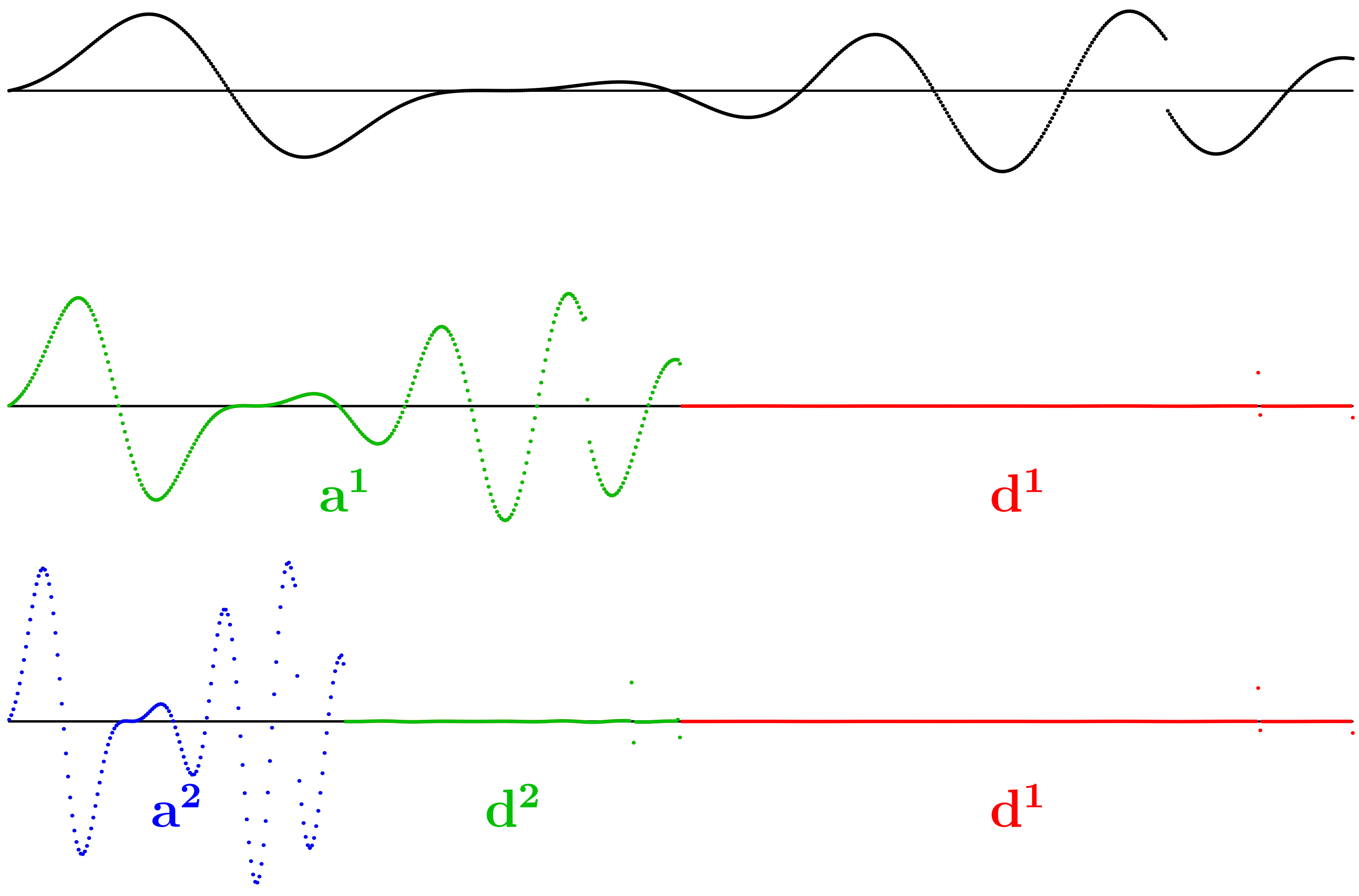

Signal Compression

- To compress the signal we can set all components of the transformed signal whose magnitude lies below a threshold to 0
- We transmit the non-zero magnitude together with a binary mask showing the position of the non-zero magnitude
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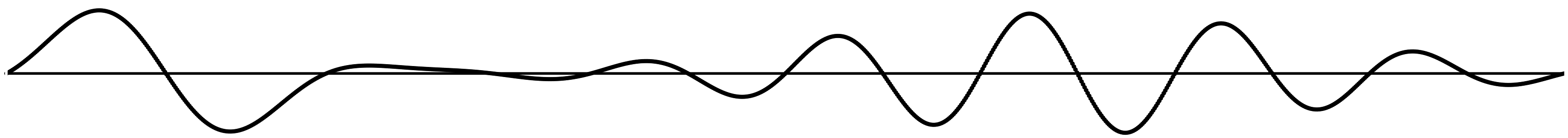

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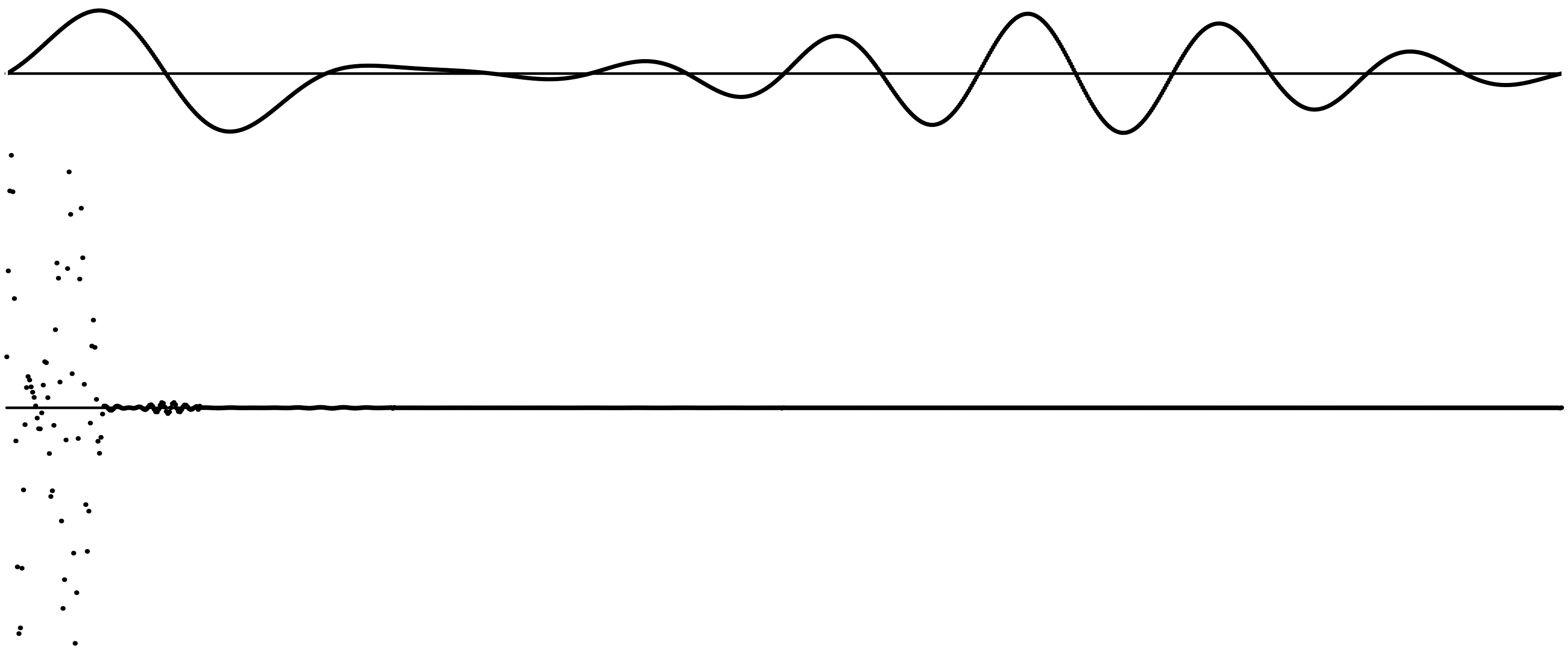

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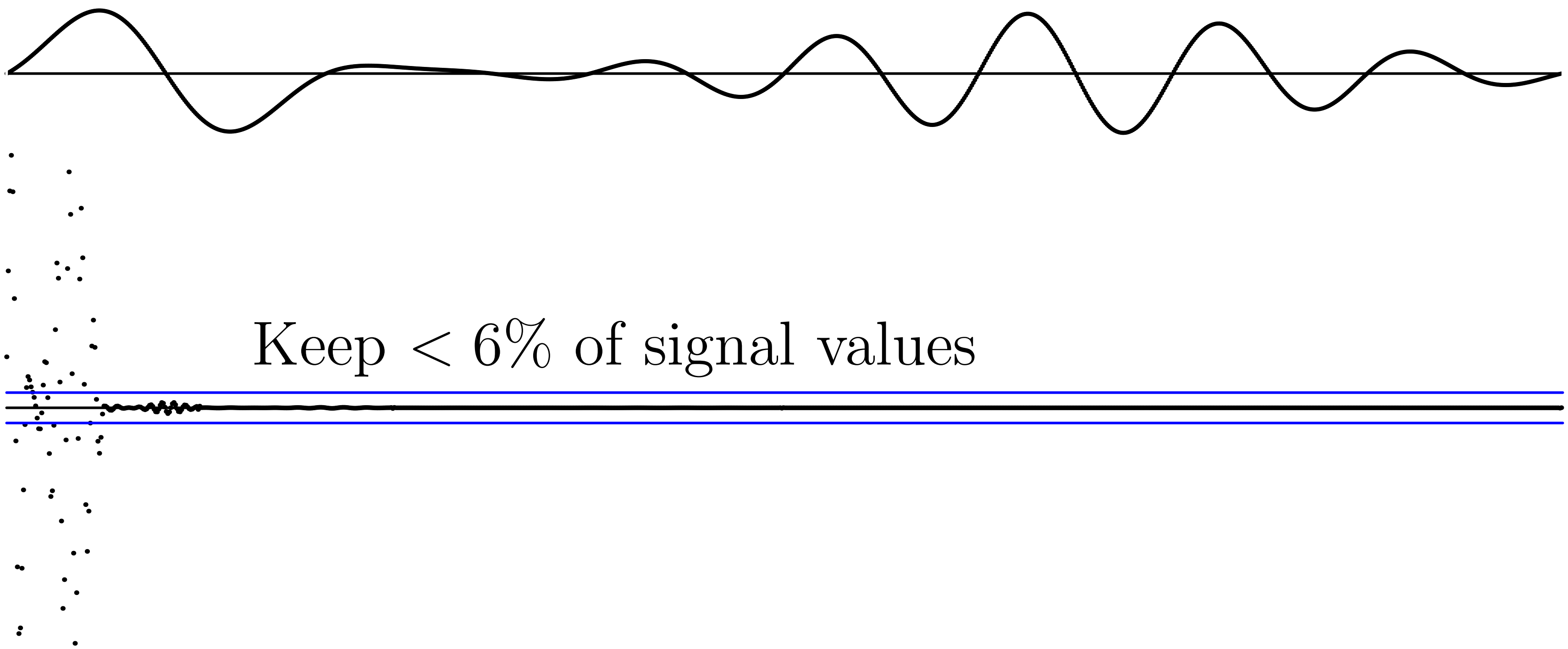

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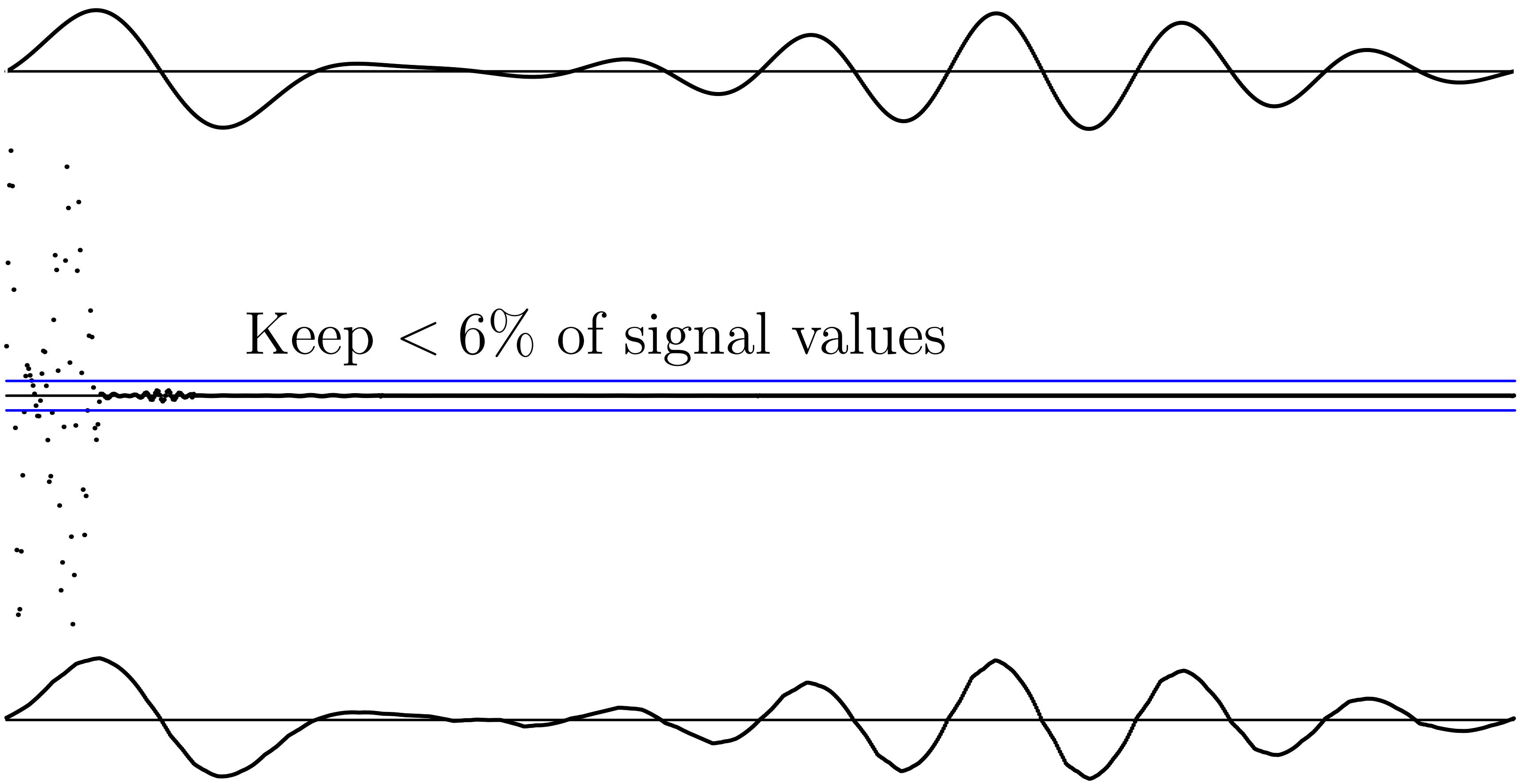

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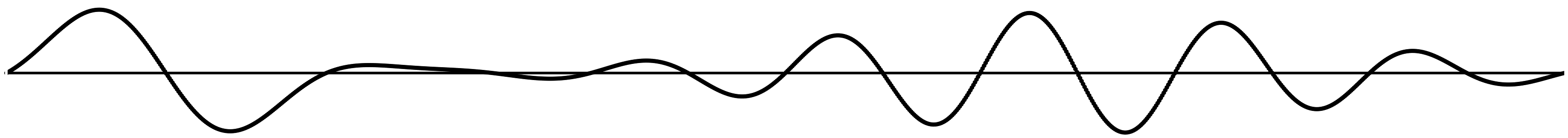

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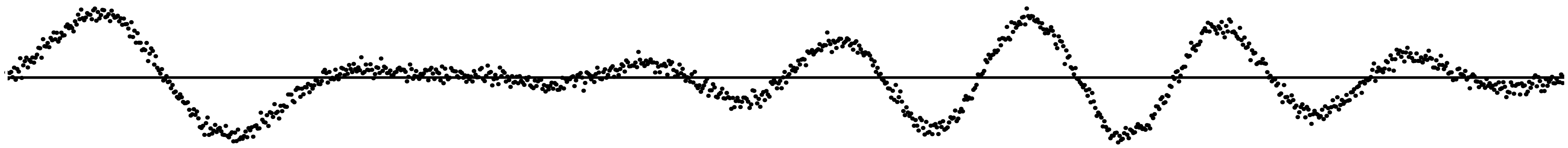

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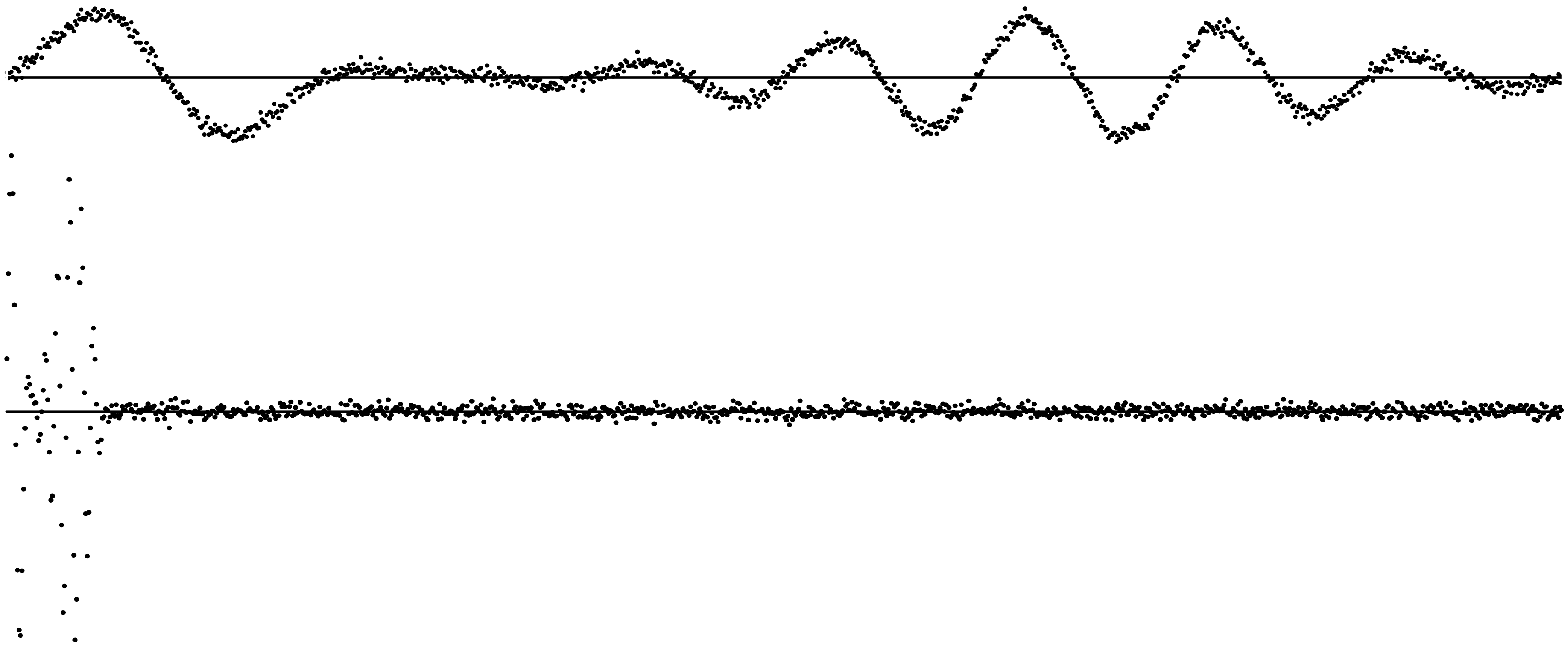

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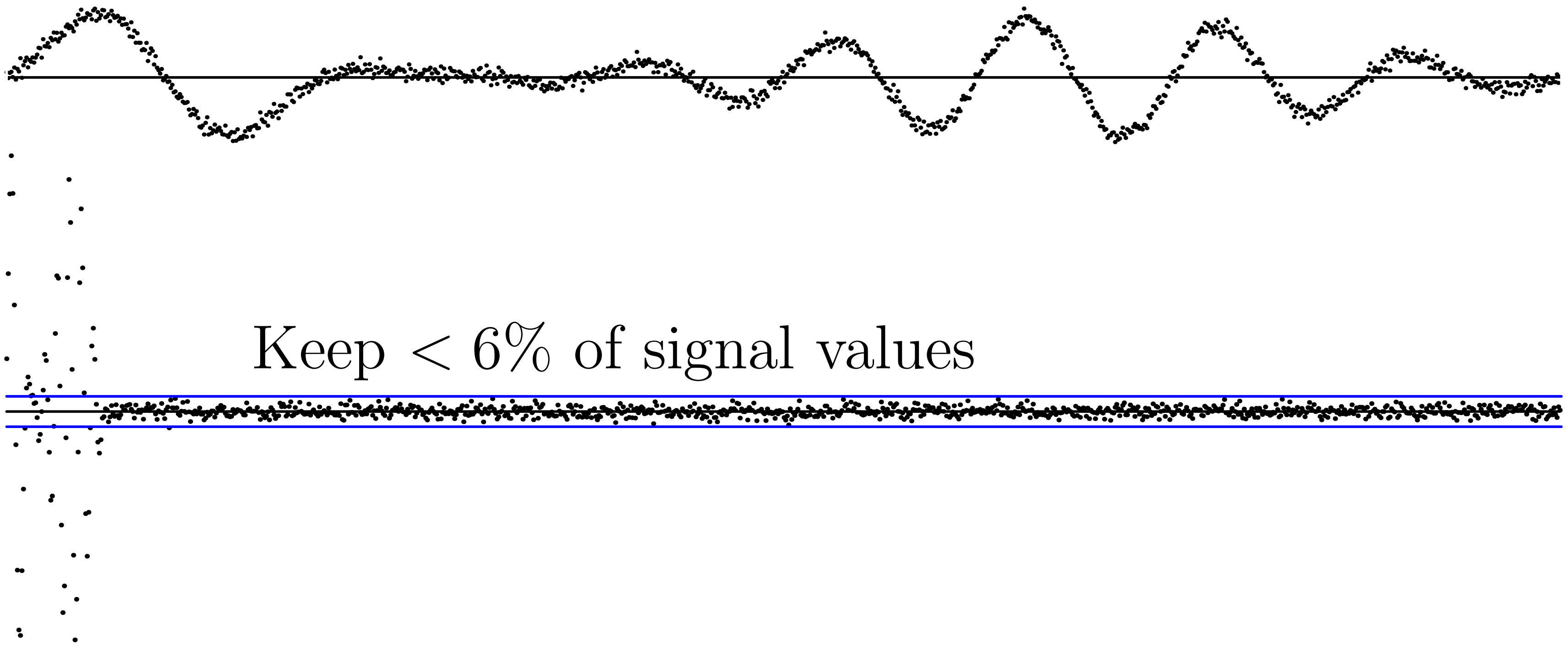

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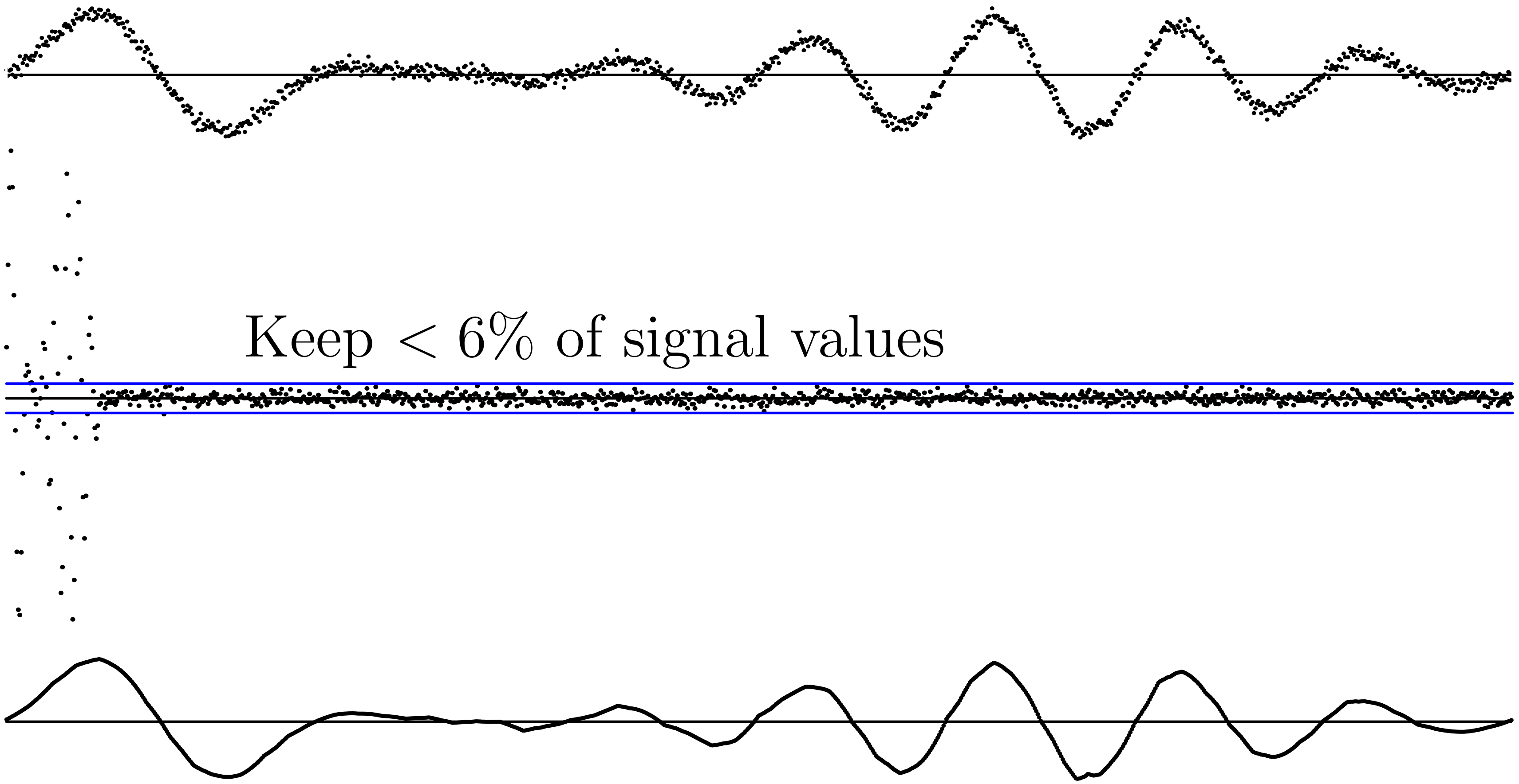

Other Wavelets

- Can use high-order wavelets which captures more energy in the carrier signal, e.g. Daub10 or Daub20
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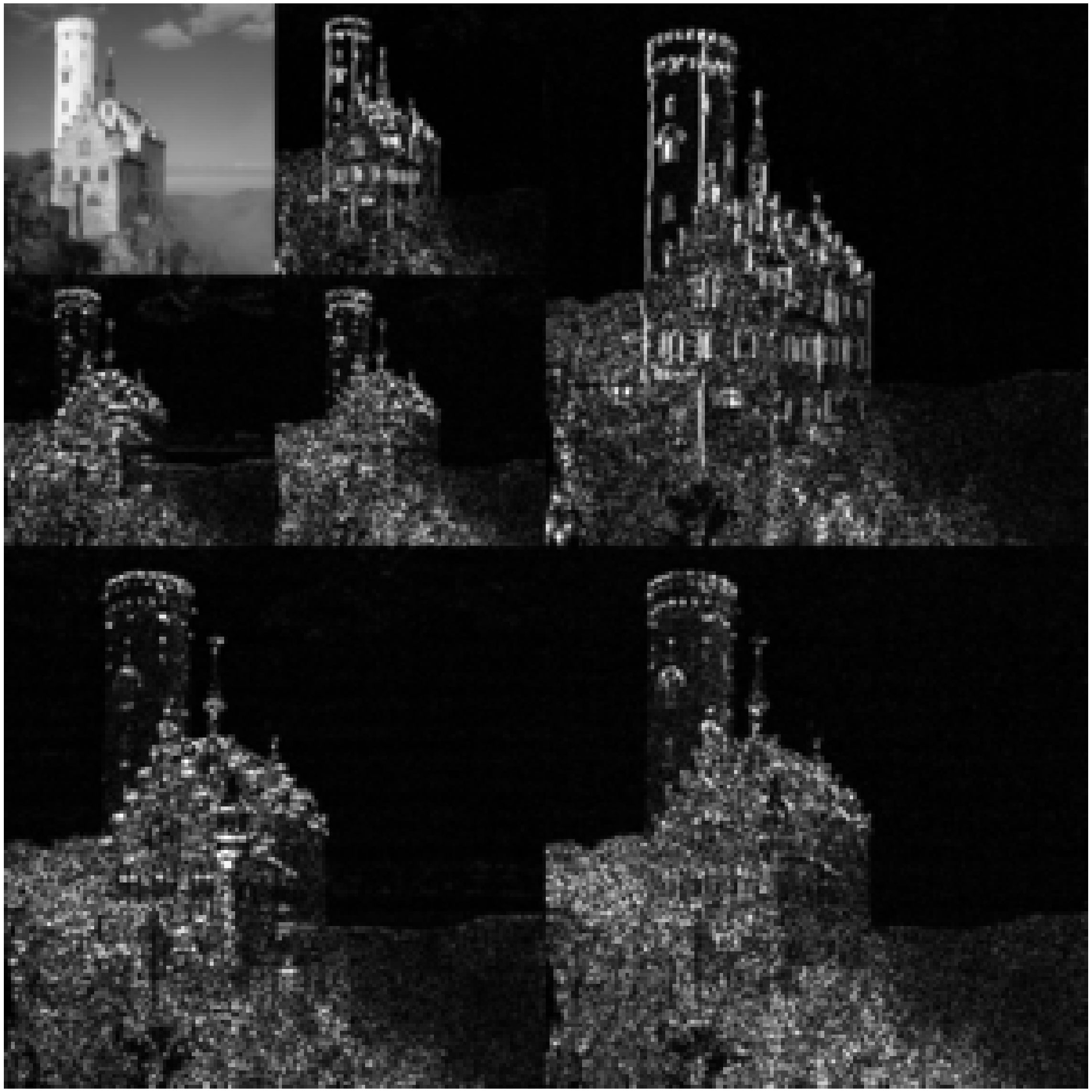

Summary

- File compression is an important task in its own right
- Files may either be compressed losslessly or lossily
- Lossy compression is typically much more efficient (e.g. an order of magnitude smaller)
- Huffman encoding often lies at the lowest level in many compression algorithms
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