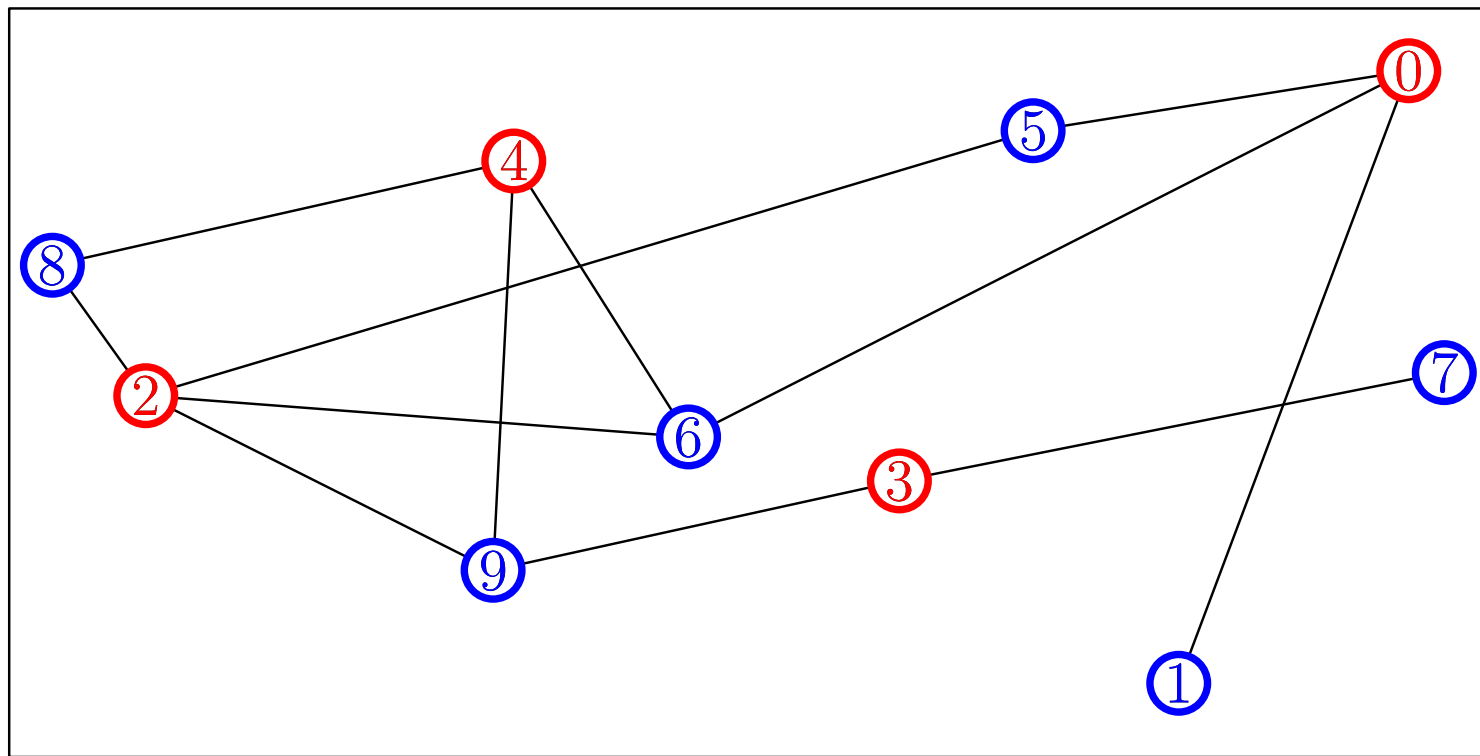# Algorithms and Analysis

## Lesson 20: *Learn to Traverse Graphs*



*Breadth First Search, Depth First Search, Topological Sort*
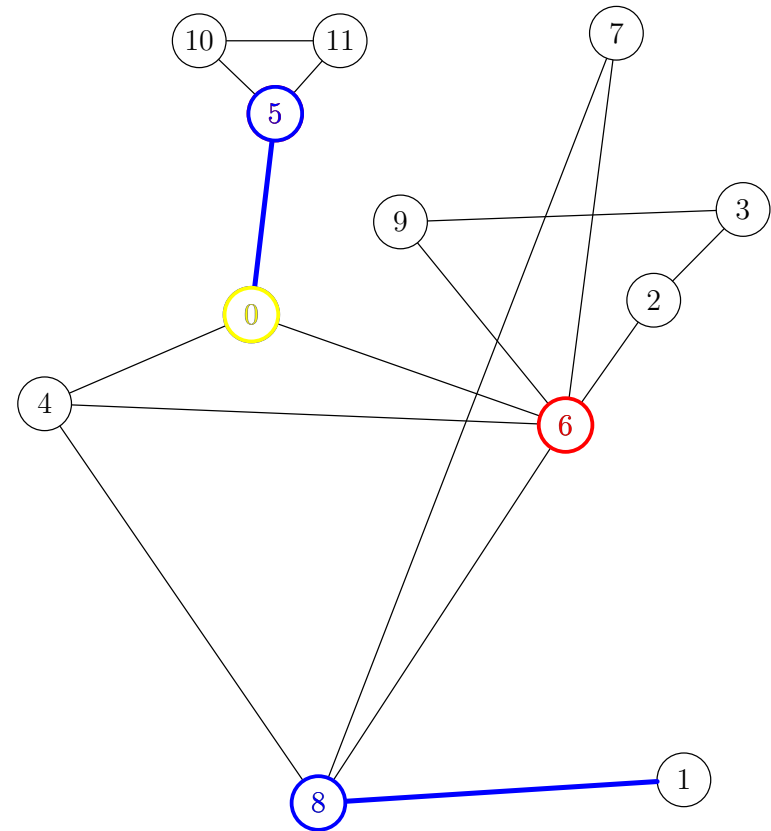
# Outline

1. **Breadth First Search**

   - BFS applications

2. Depth First Search

   - DFS applications

3. Topological Sort

# Basic Graph Algorithms

- Graphs provide an abstraction for a huge number of real world processes: social networks, compute network, road networks, etc.

- Increasing applications focus on very large (sparse) graphs (usually implemented using an adjacency list)

- Require (near) linear time algorithms

- Basic building block are graph traversal algorithms

  ⋆ Breadth First Search
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# Advanced Generic

- To make these algorithm general purpose (generic) we allow ourselves to call arbitrary functions to act on the vertices and edges at different points in the algorithm

- This introduces a new level of generics which makes the algorithms very powerful

- Increases the steepness of the learning curve to use these algorithms

- Once you get familiar with using these algorithms this level of generics starts to pay off

- Libraries which does this include Boost Graph Library and LEDA in C++; JDSL and JGraphT in Java

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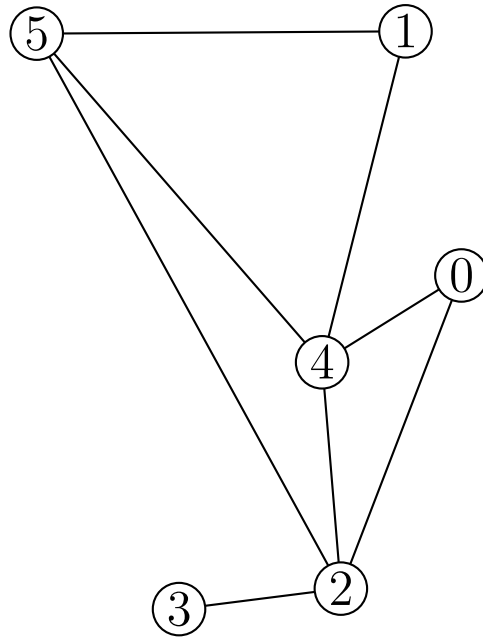
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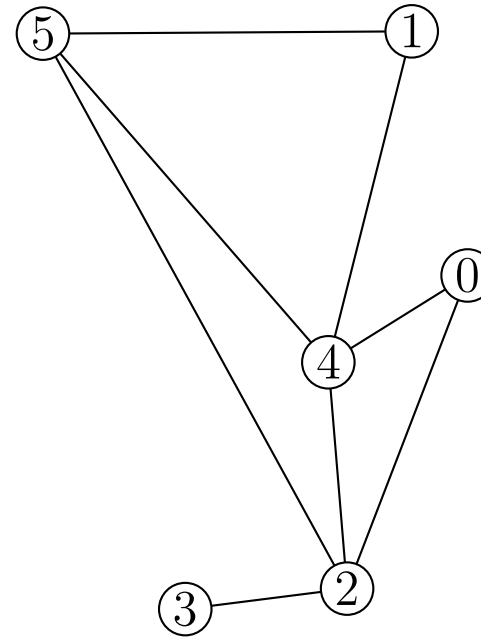
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---

# Graph Traversal

- To traverse a graph we start at a (arbitrary) *root* vertex

- We then follow edges to create a tree



**bfs**

dfs
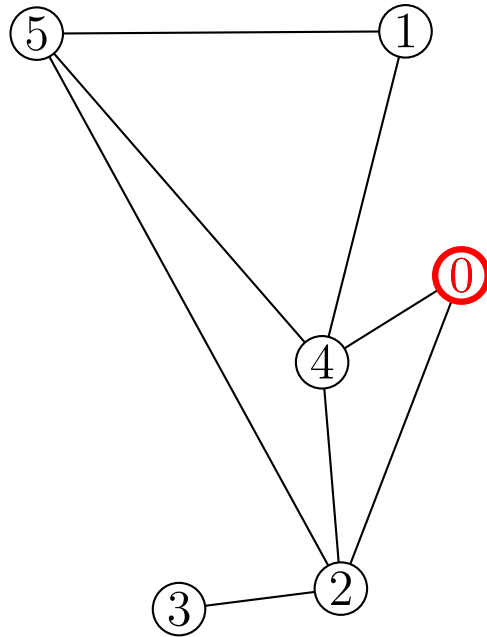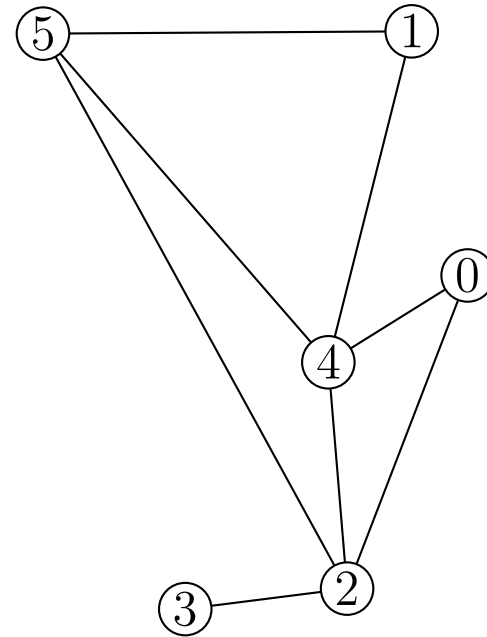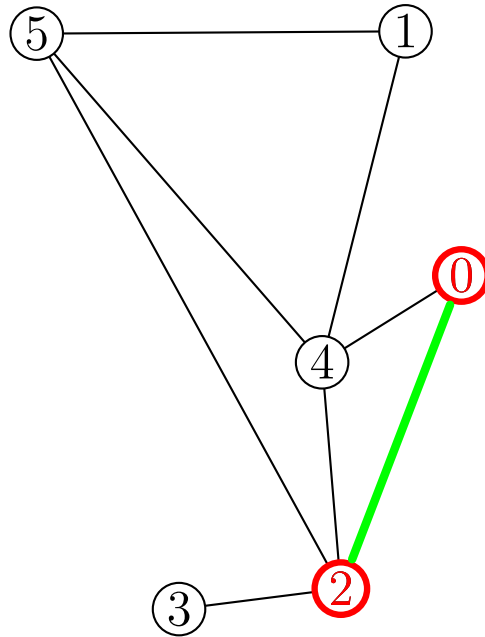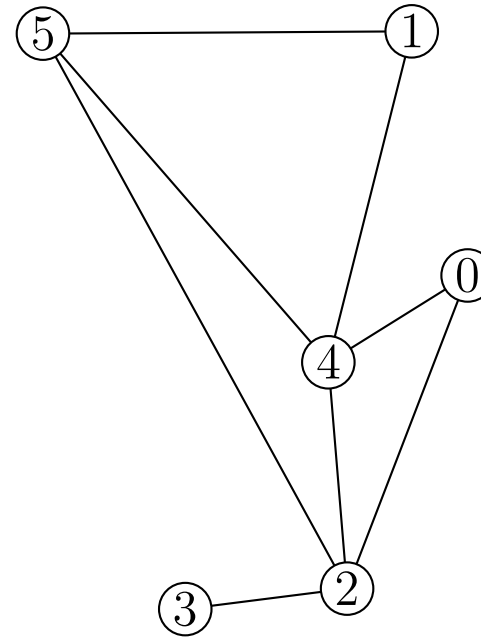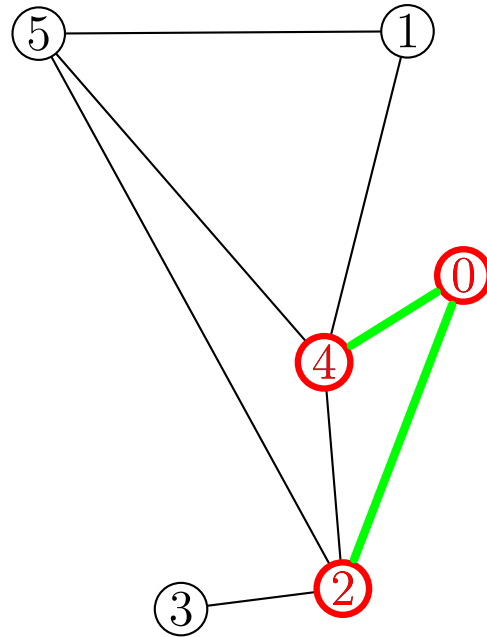
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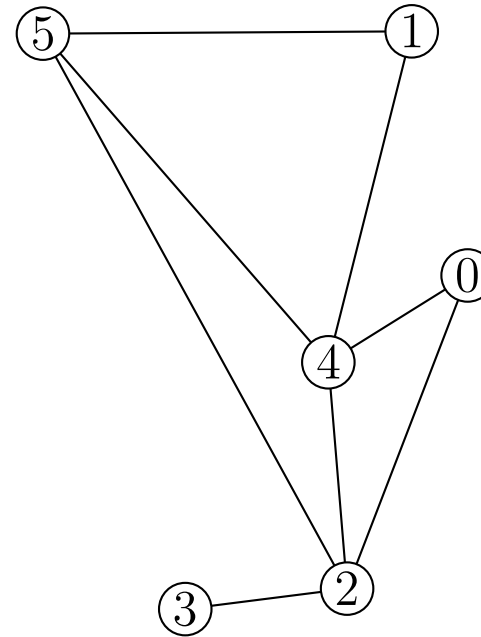
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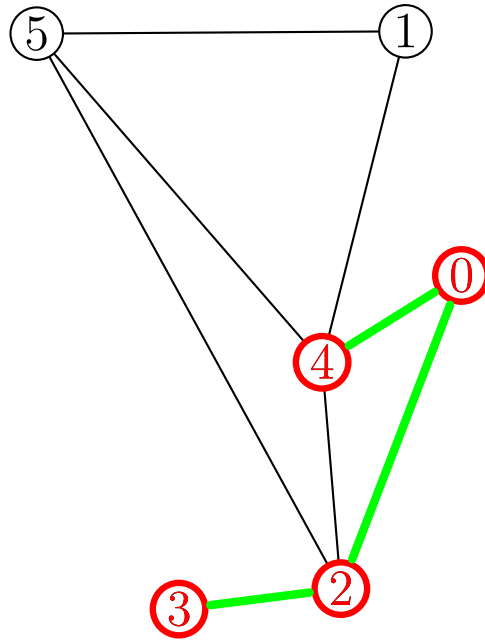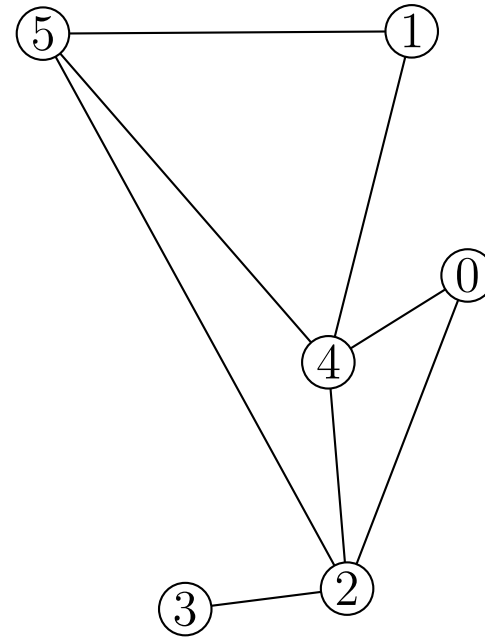**bfs**                                                                dfs

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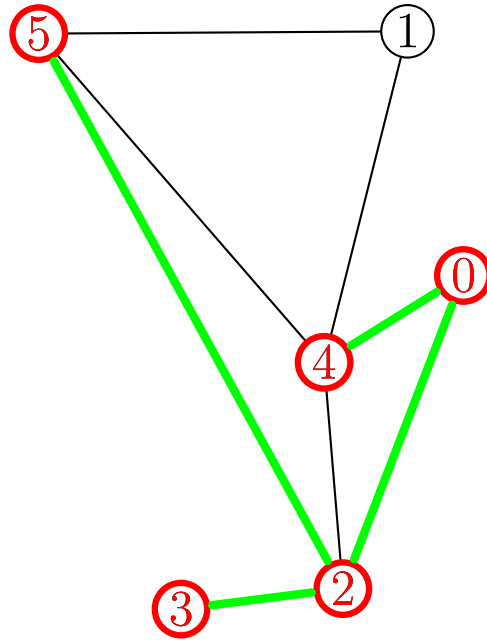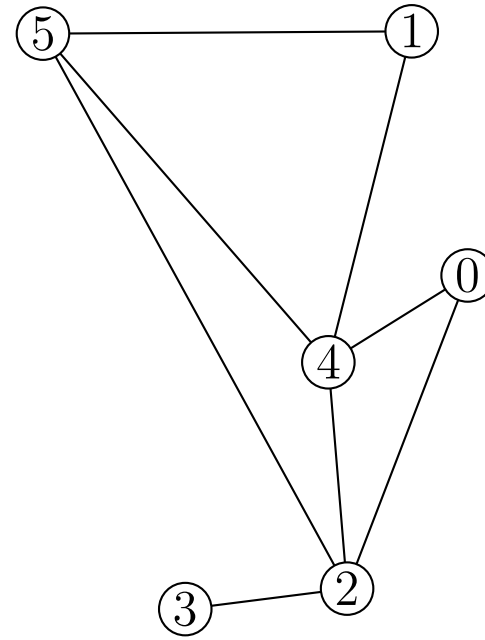**bfs**                                        dfs

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bfs                              **dfs**

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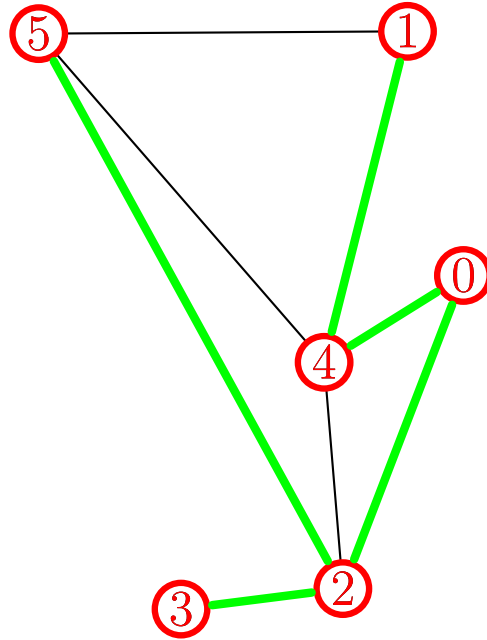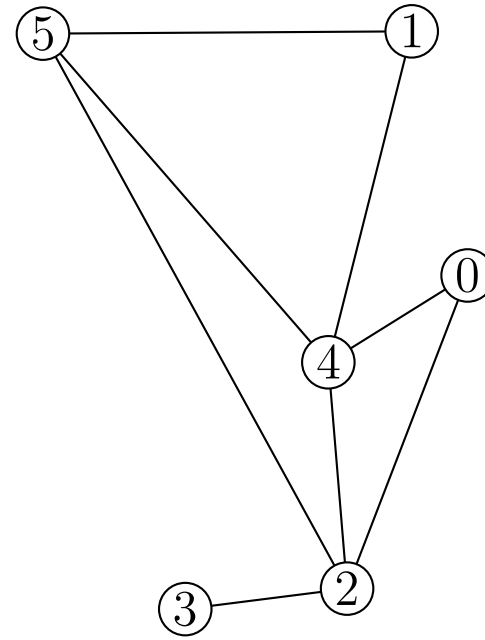


bfs            **dfs**

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bfs                                    **dfs**

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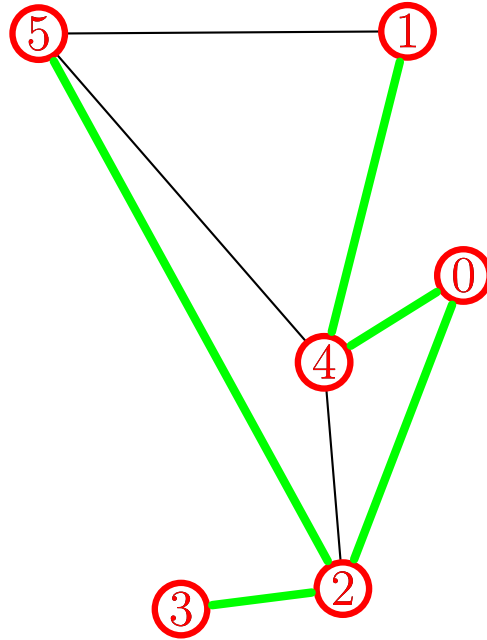
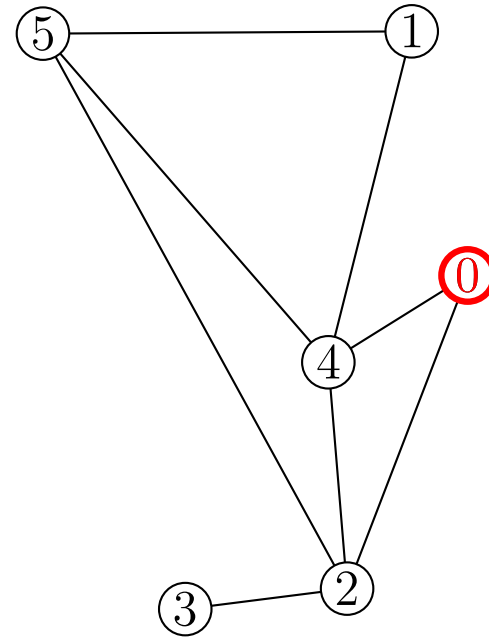- We then follow edges to create a tree



bfs

**dfs**

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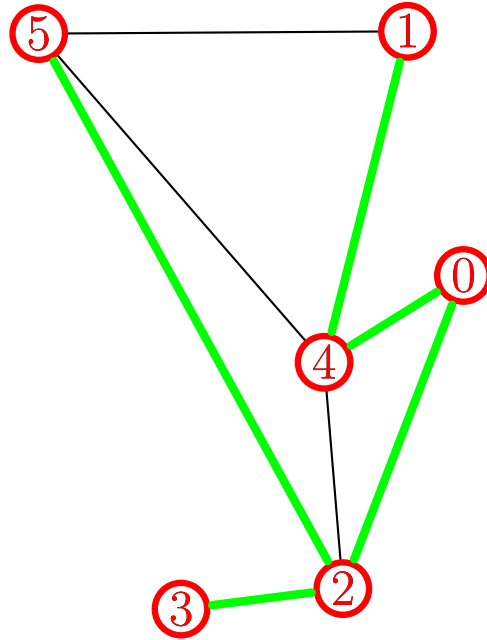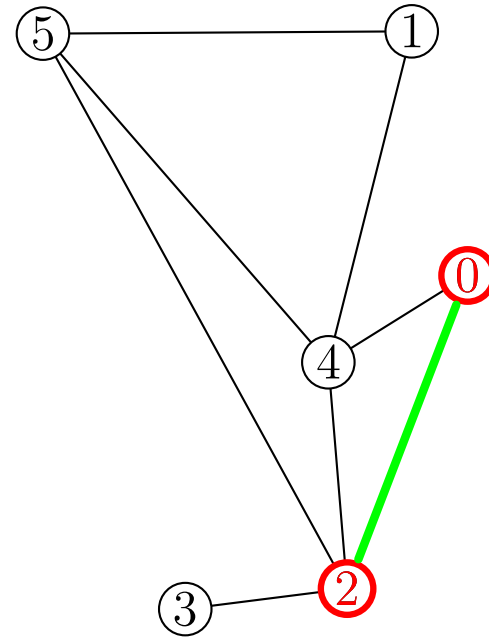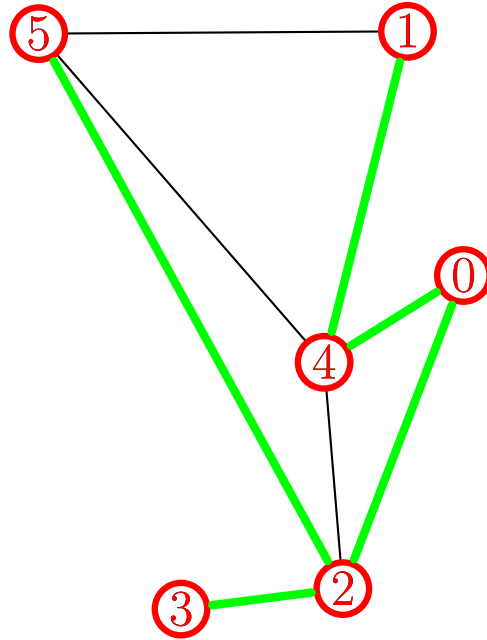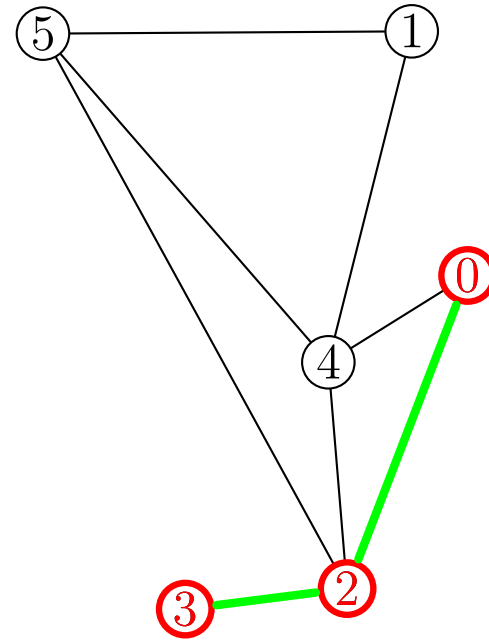- We then follow edges to create a tree



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bfs                    **dfs**

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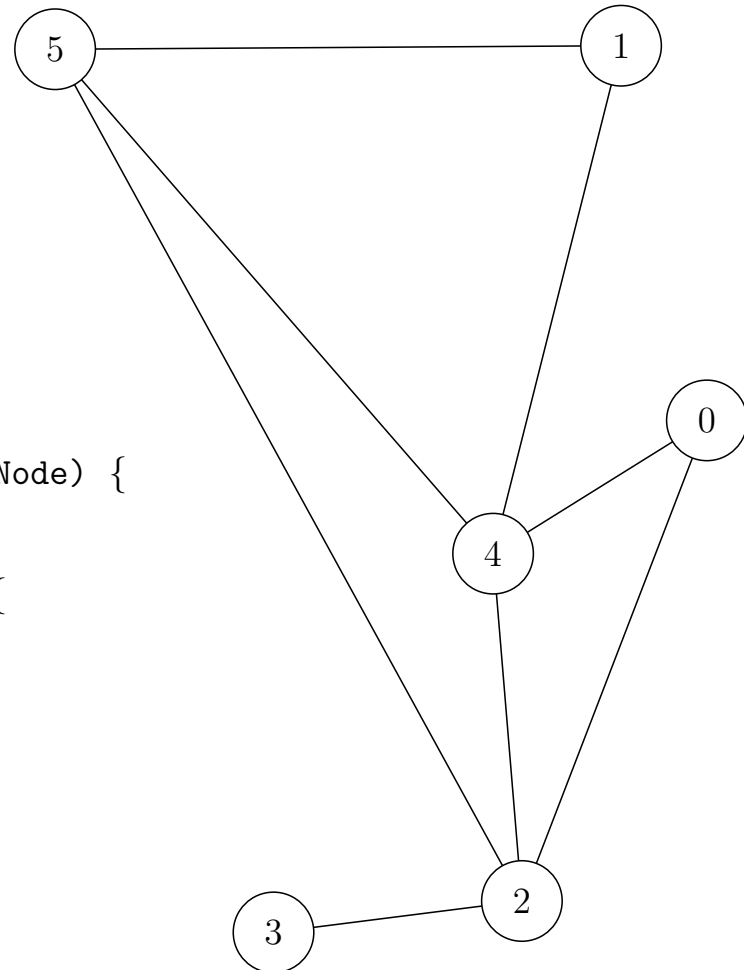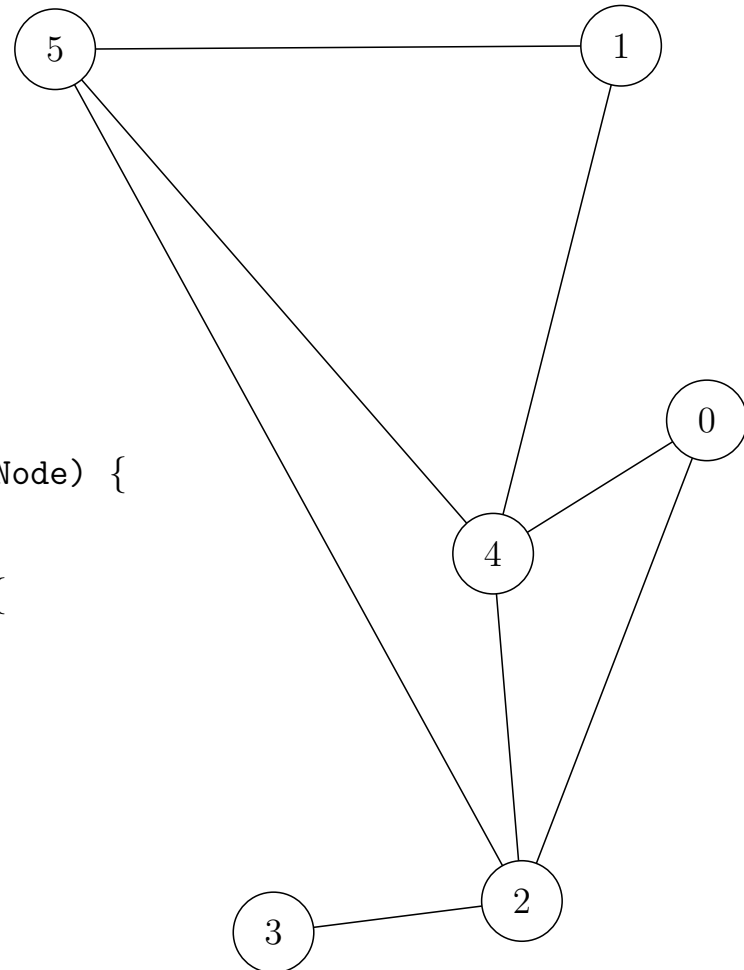
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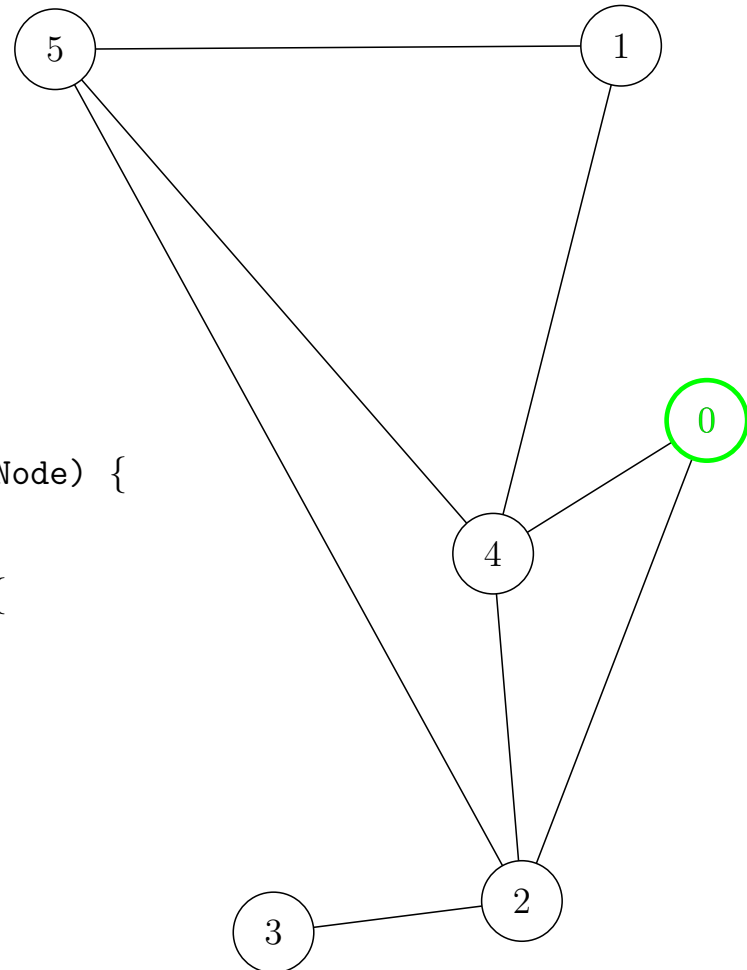
q=

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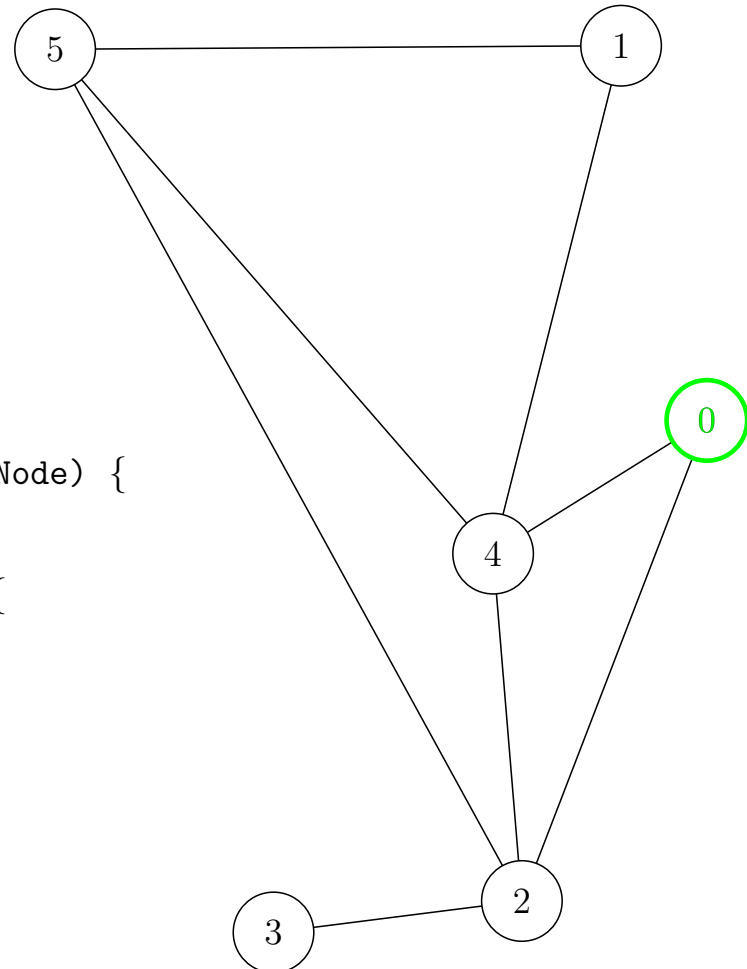q= | 0 | | | | | |

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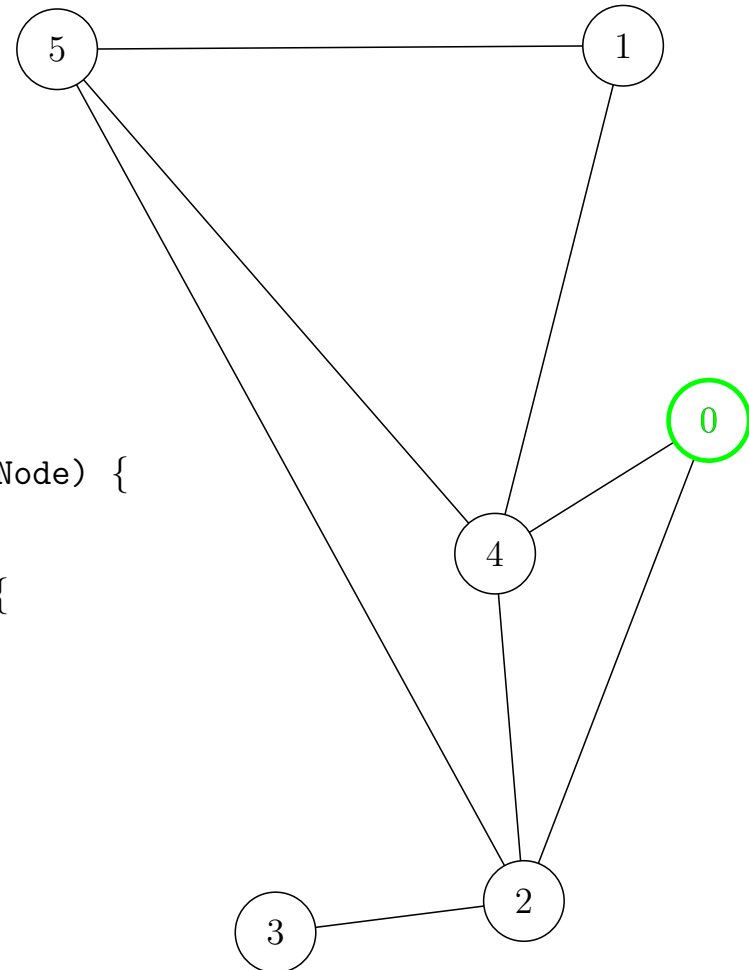currentNode=0

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currentNode=0    neighbour=2

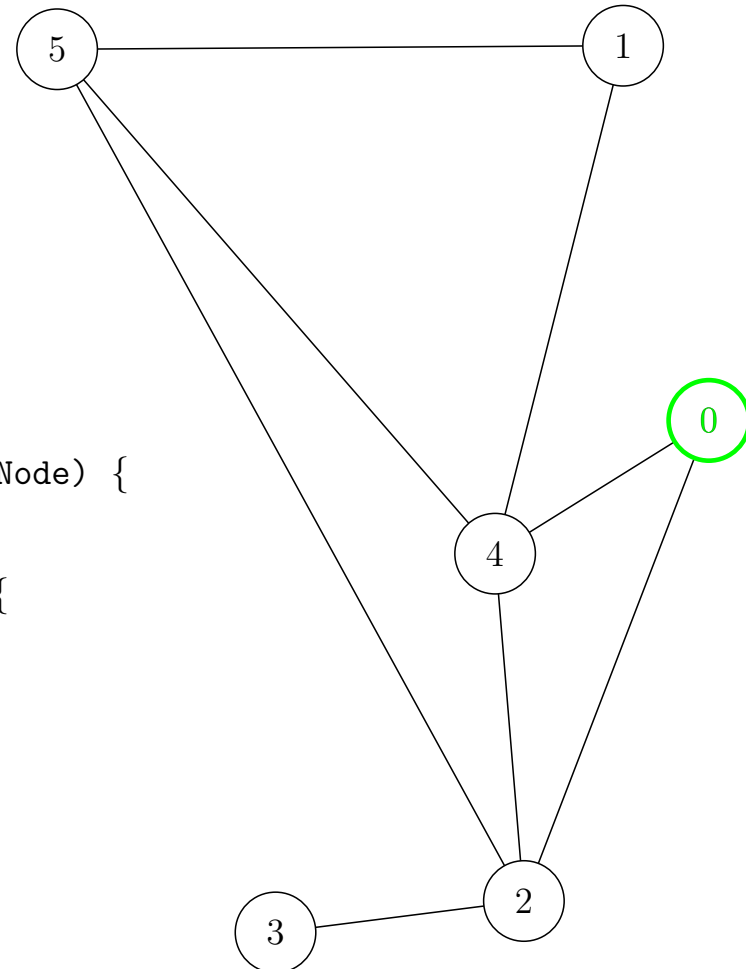q=

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currentNode=0    neighbour=2

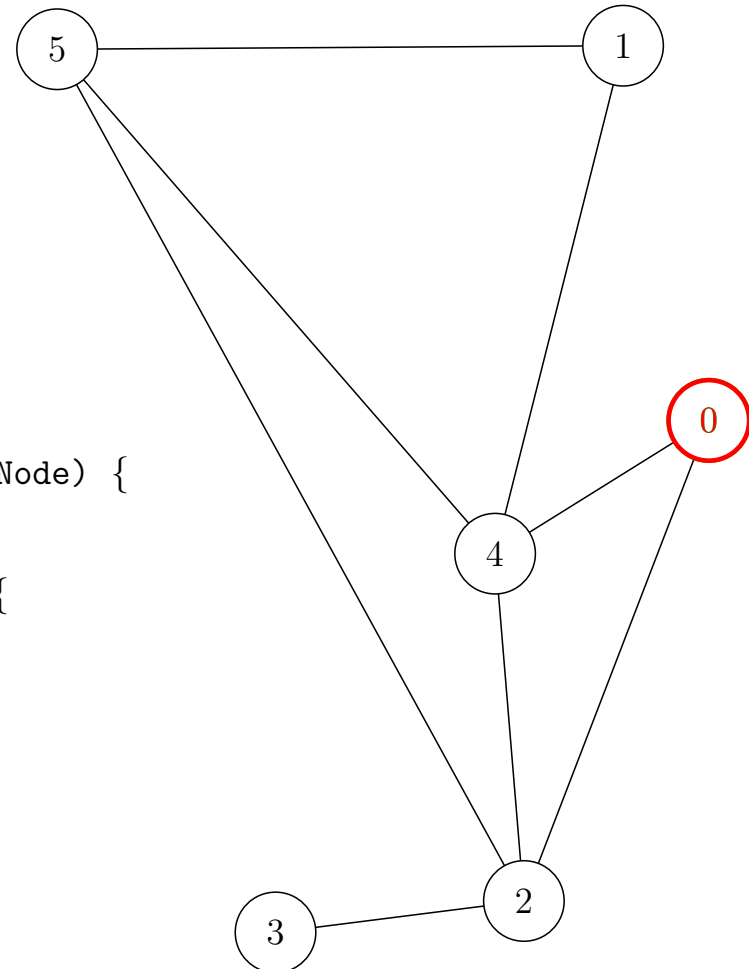q=

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currentNode=0     neighbour=2
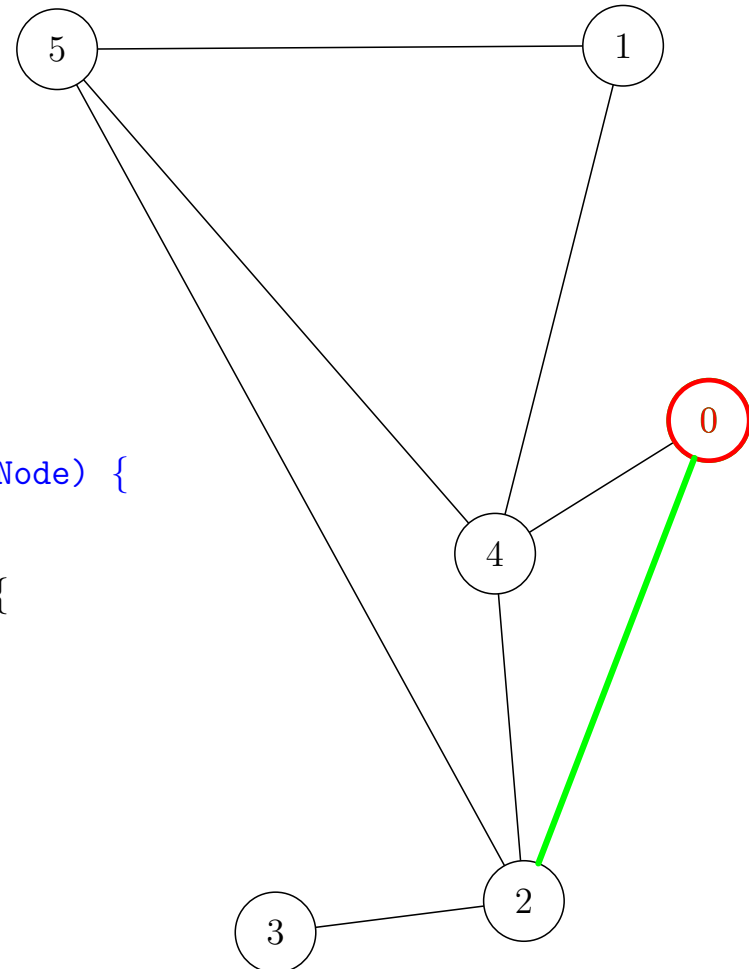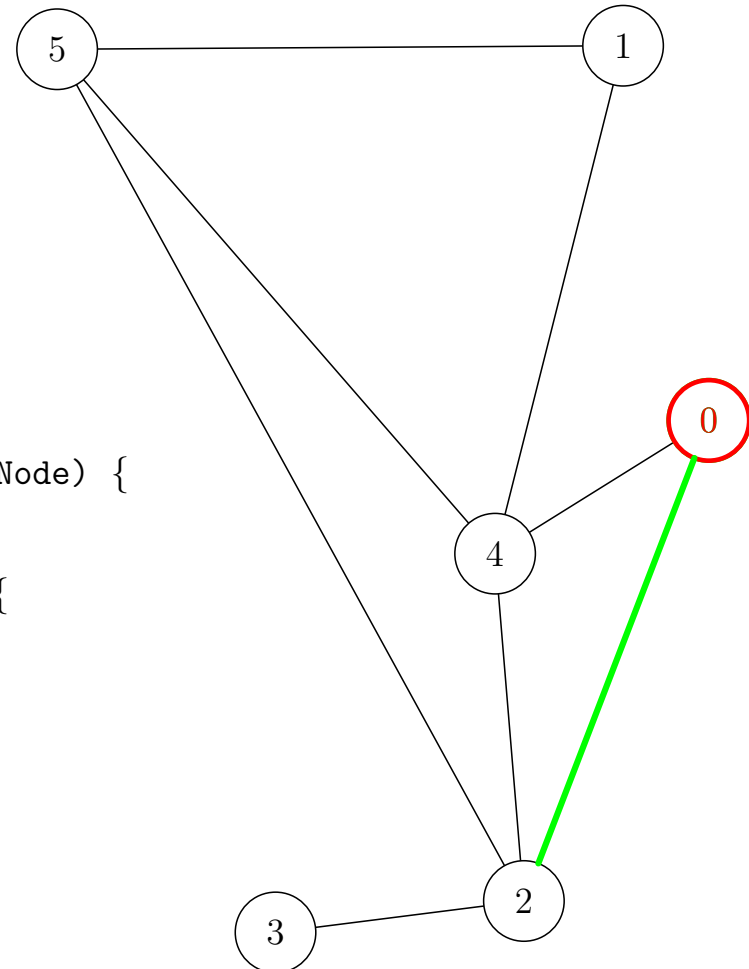
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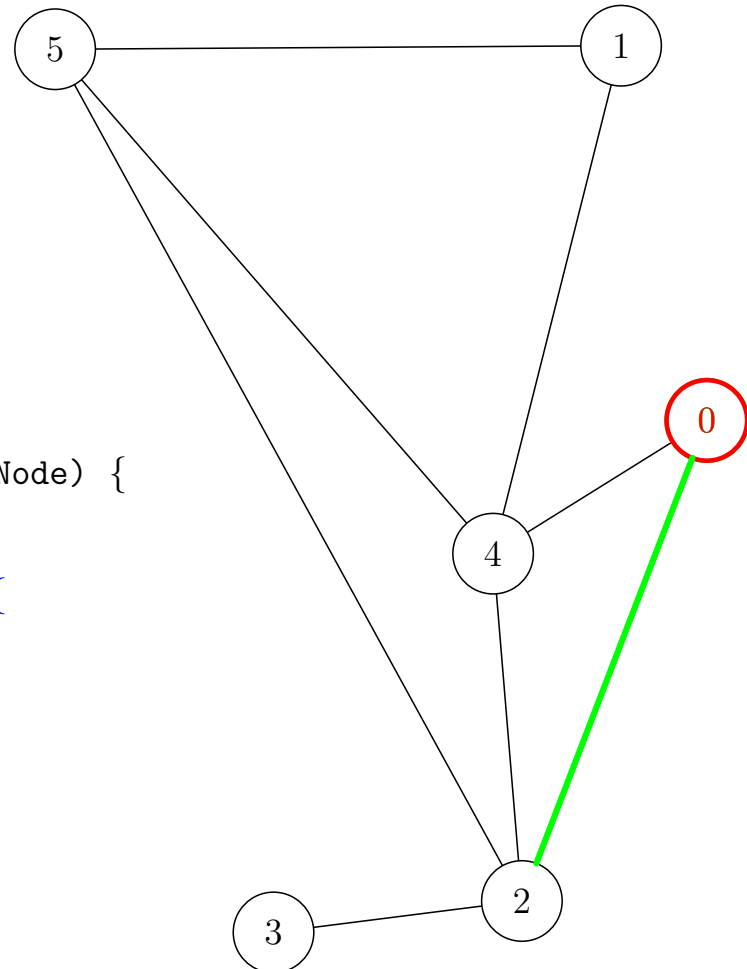| 2 |   |   |   |   |   |
|---|---|---|---|---|---|

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currentNode=0    neighbour=4

q=

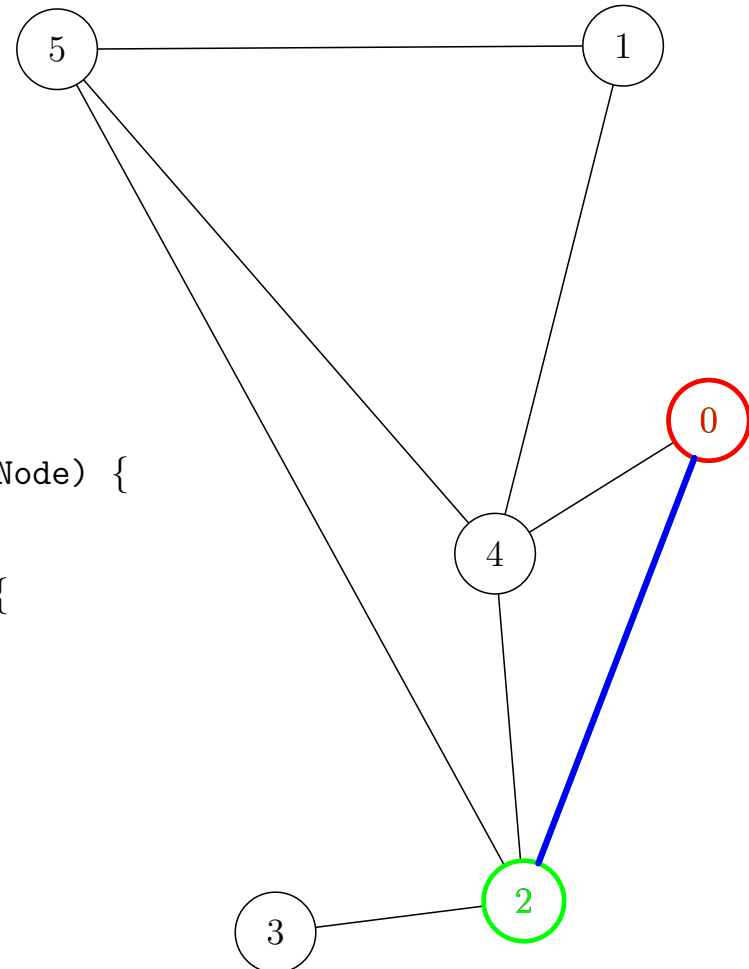| 2 |   |   |   |   |   |
|---|---|---|---|---|---|

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```

currentNode=0    neighbour=4

q= | 2 |   |   |   |   |   |
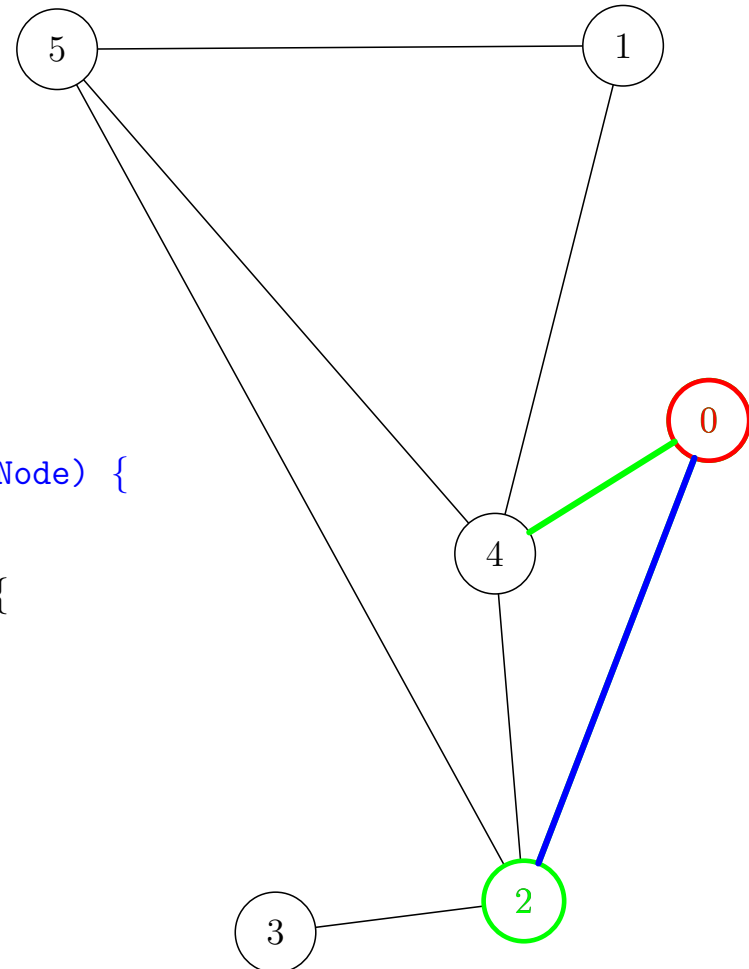
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currentNode=0    neighbour=4

q=

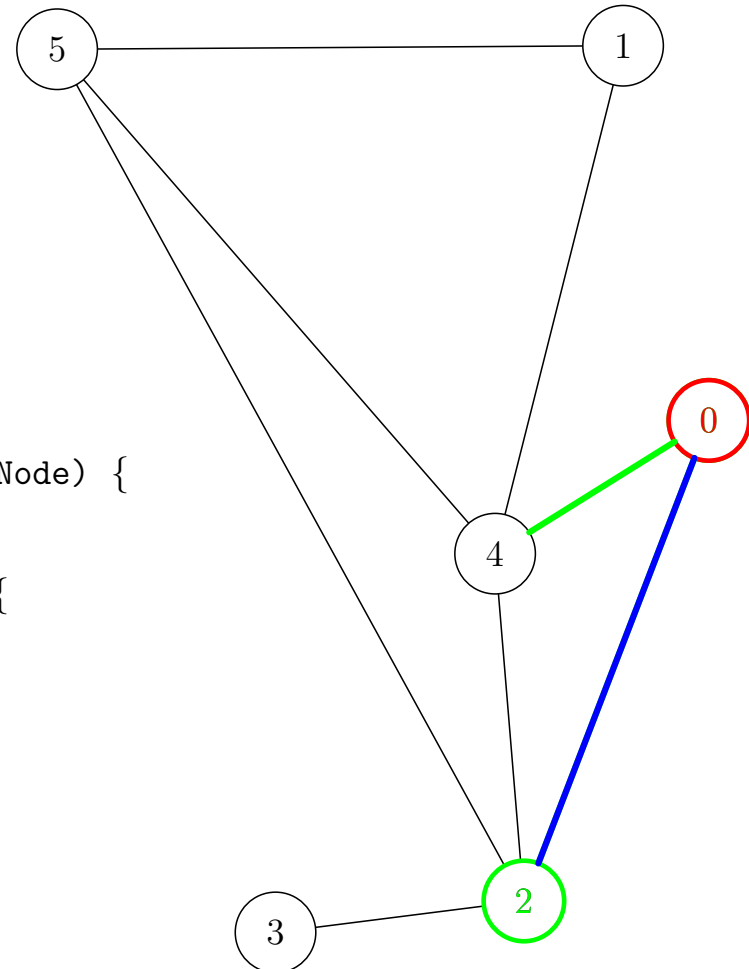| 2 |   |   |   |   |   |
|---|---|---|---|---|---|

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=0    neighbour=4

q=

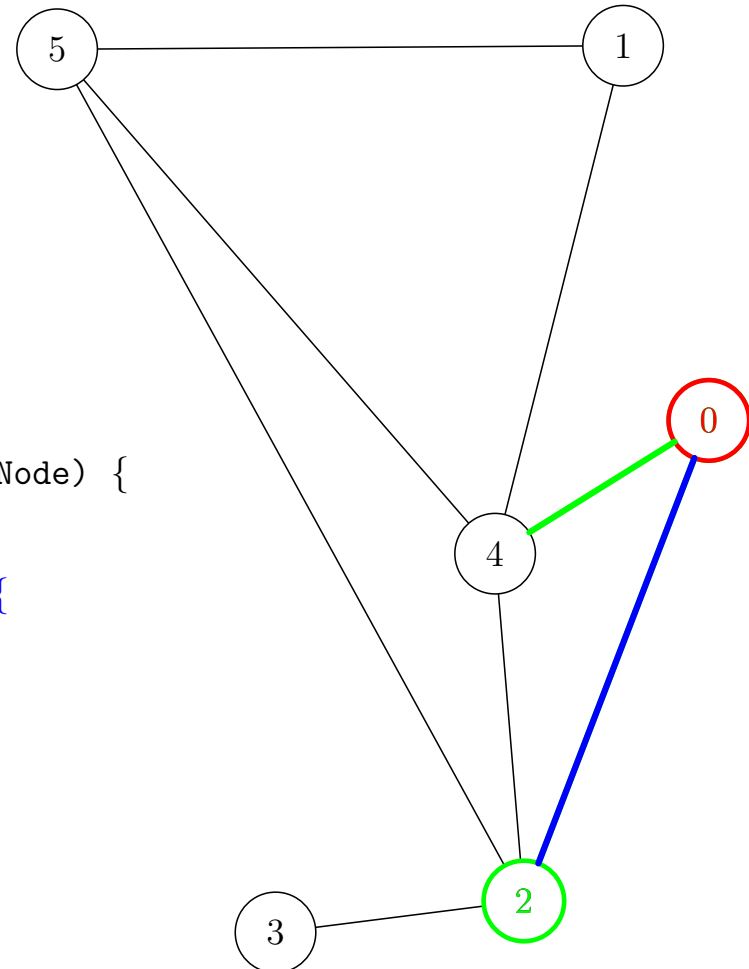| 2 | 4 |  |  |  |  |
|---|---|---|---|---|---|

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=0

q=

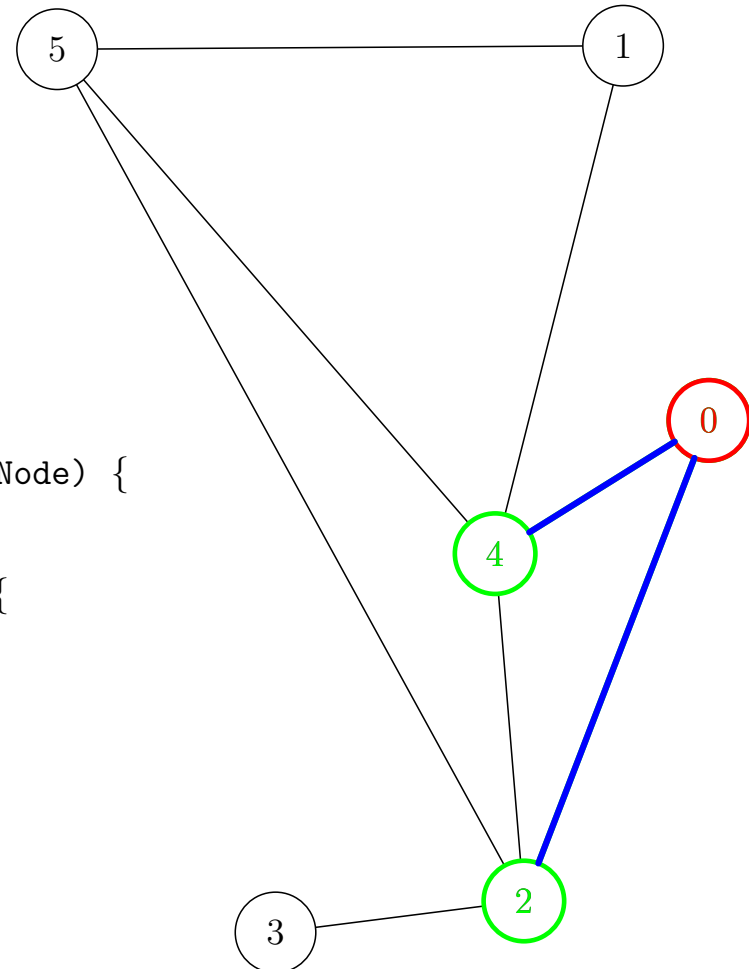| 2 | 4 |   |   |   |   |

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

q=

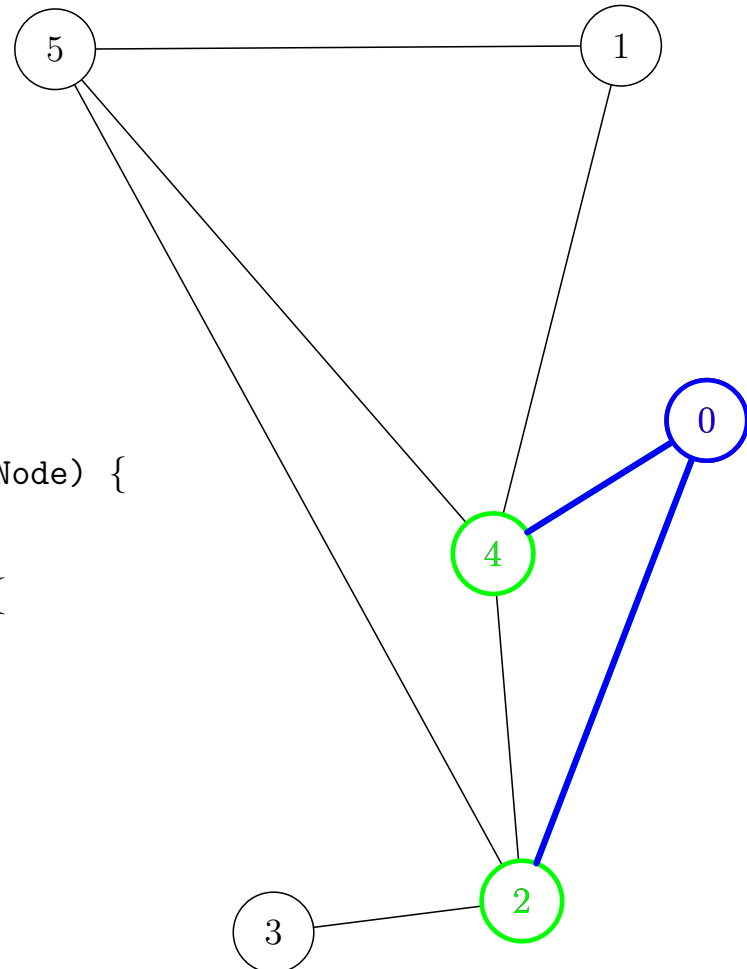| 2 | 4 |  |  |  |  |

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=2

q= 

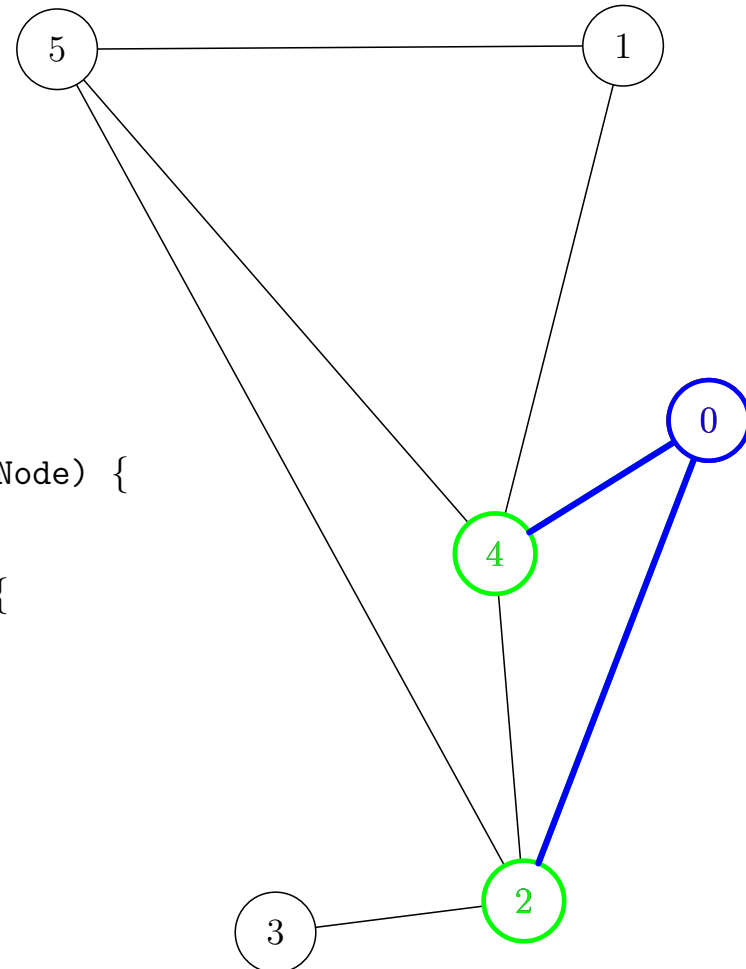| 4 |   |   |   |   |   |
|---|---|---|---|---|---|

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=2    neighbour=3

q=

| 4 |   |   |   |   |   |
|---|---|---|---|---|---|

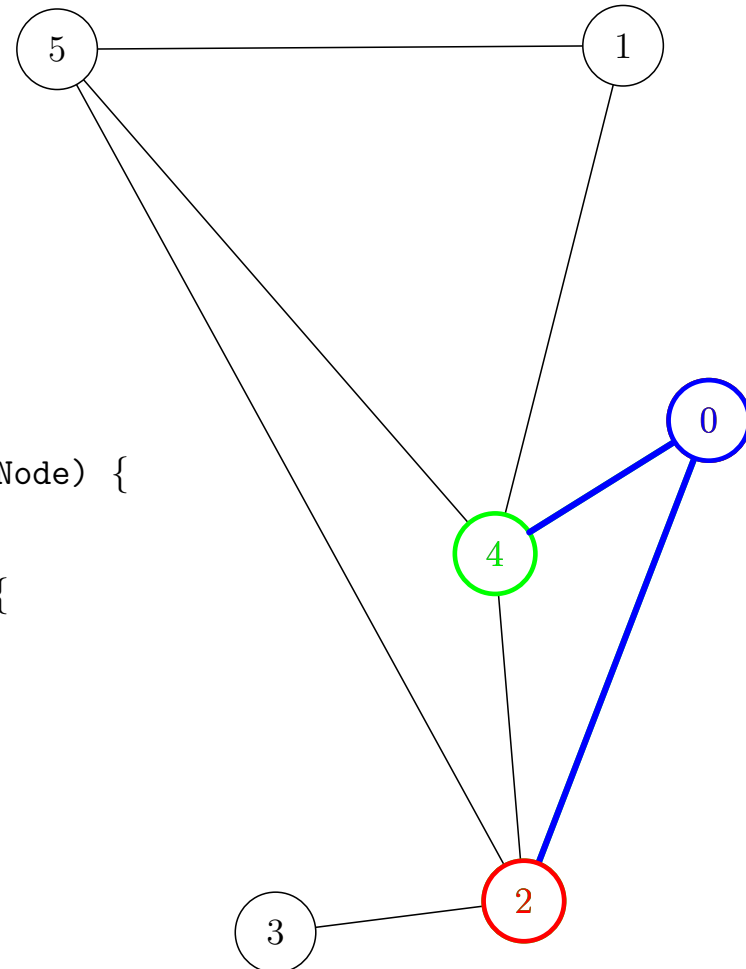# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=2    neighbour=3

q=

| 4 |   |   |   |   |   |
|---|---|---|---|---|---|

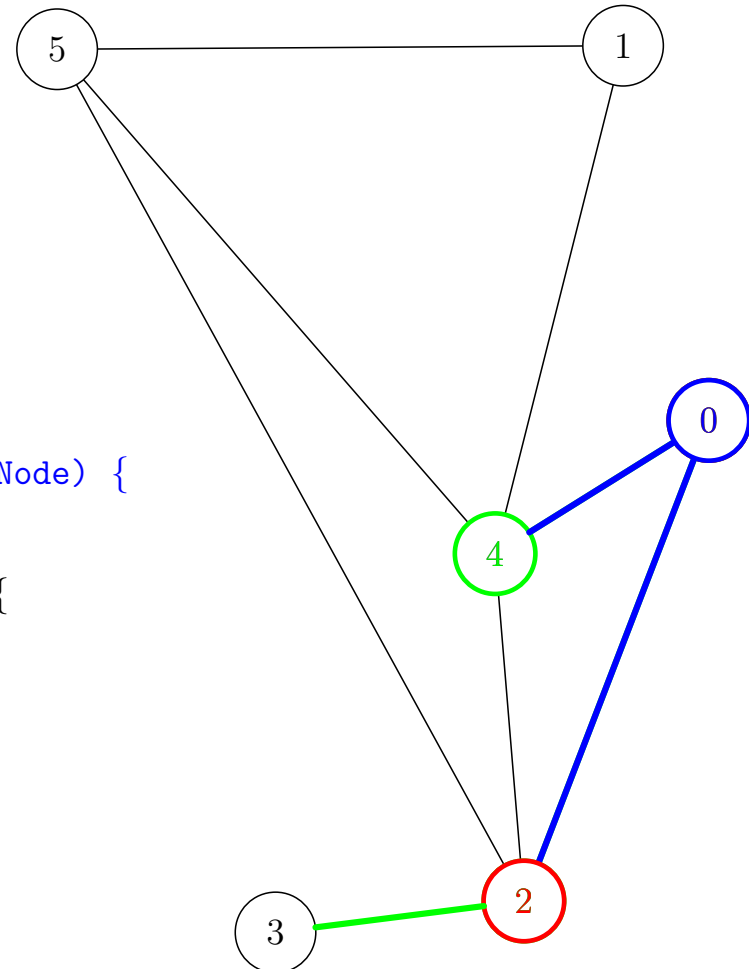# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=2     neighbour=3

q=

| 4 |   |   |   |   |   |
|---|---|---|---|---|---|

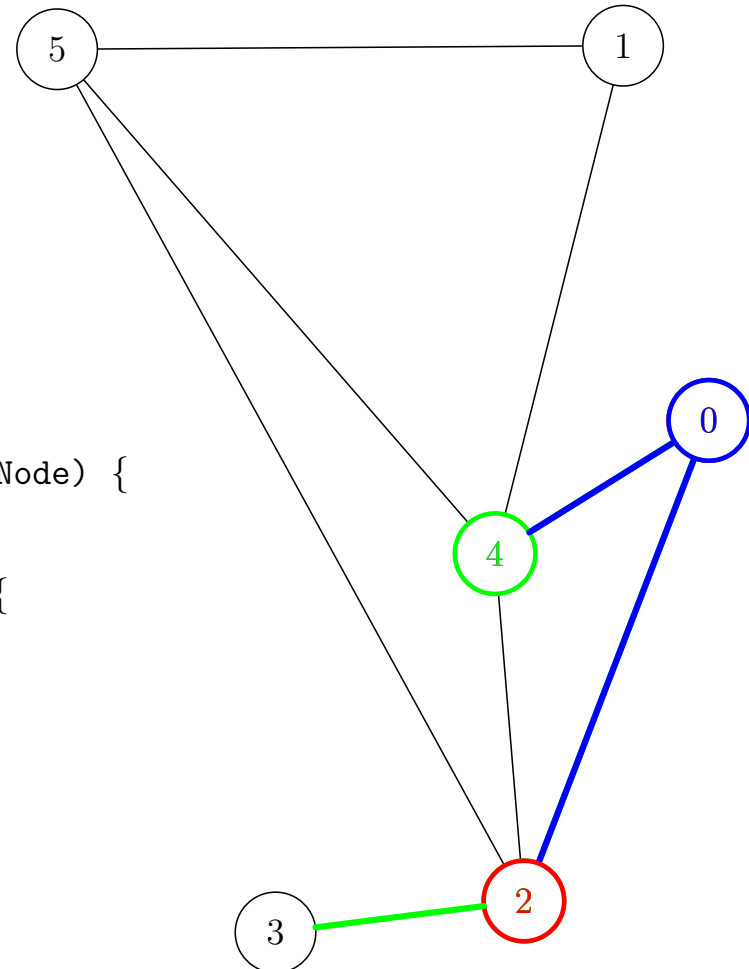# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=2    neighbour=3

q=

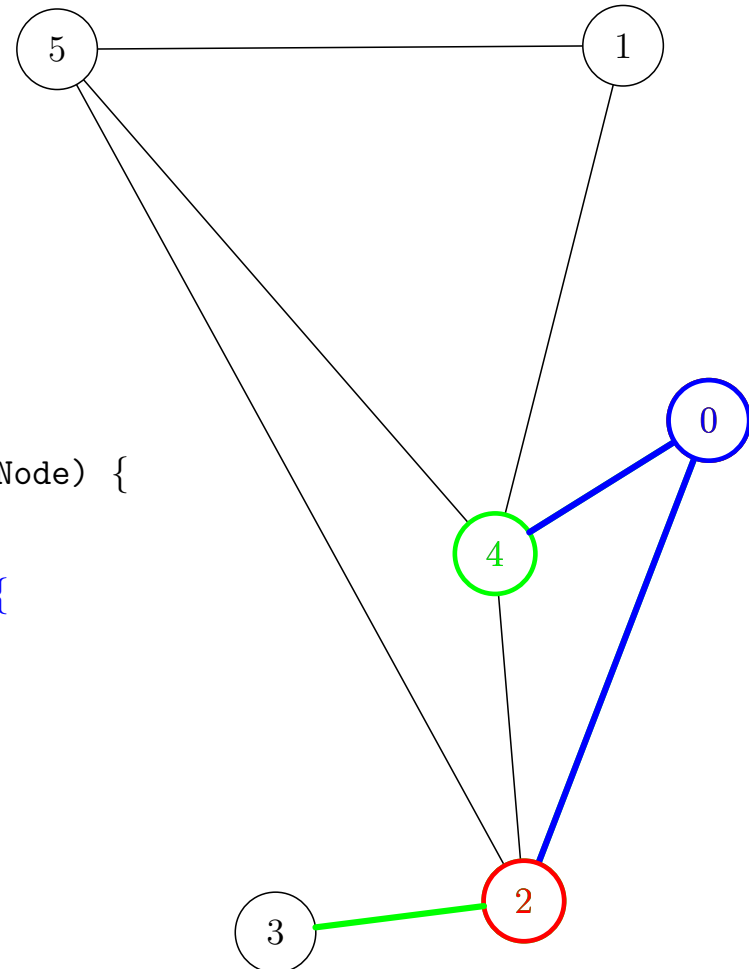| 4 | 3 |   |   |   |   |

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=2    neighbour=4

q=

| 4 | 3 |  |  |  |  |
|---|---|---|---|---|---|

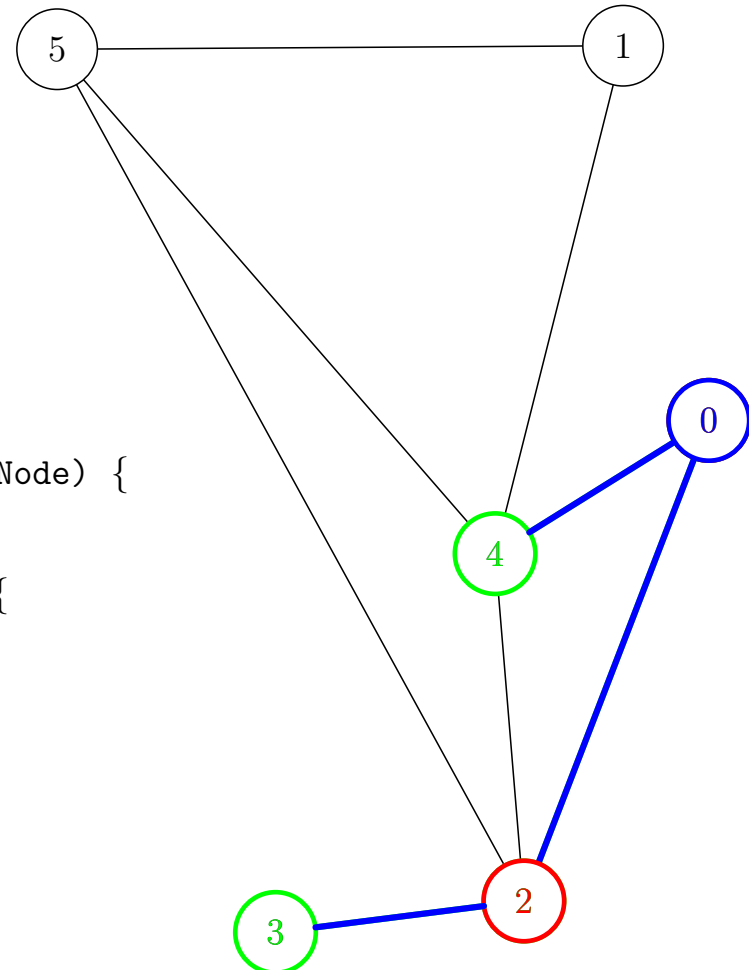# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=2     neighbour=4

q=

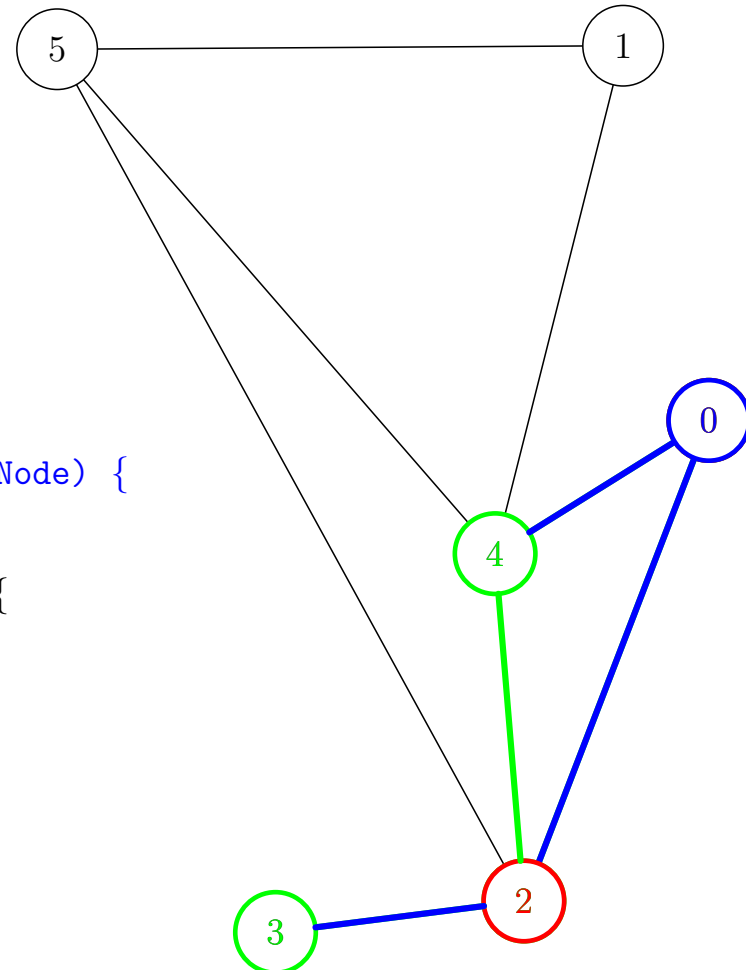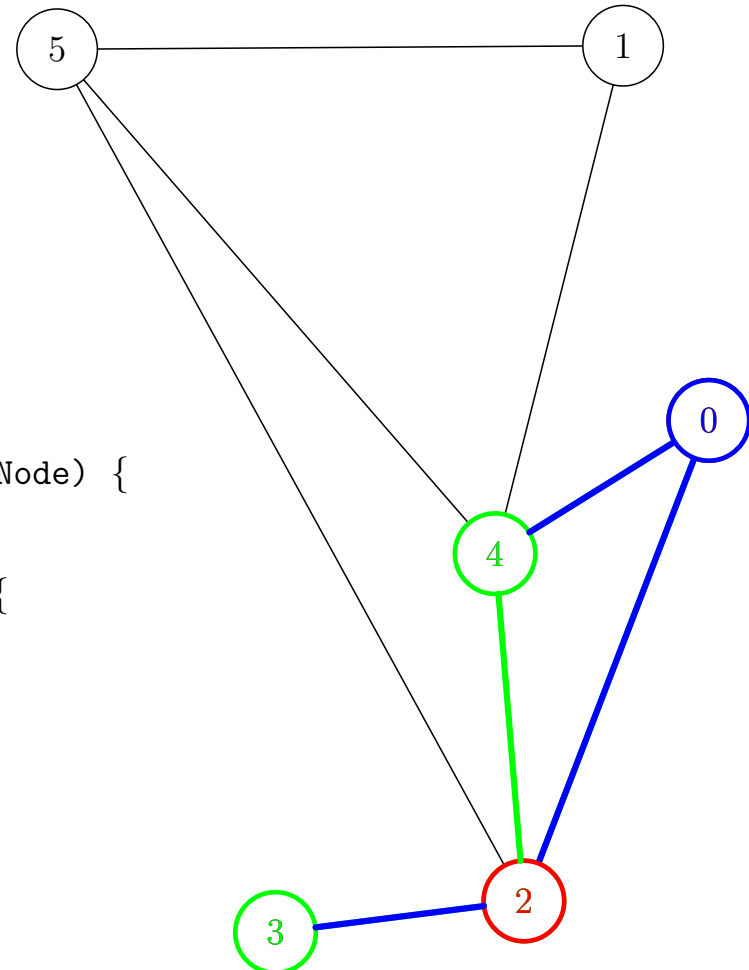| 4 | 3 |   |   |   |   |

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=2    neighbour=4

q=

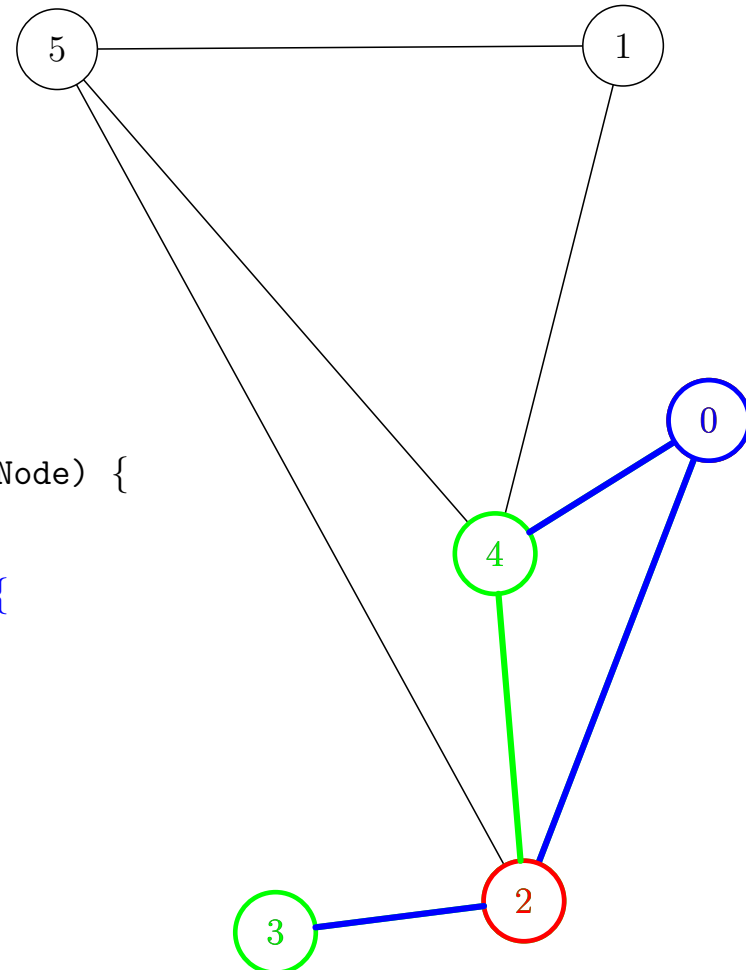| 4 | 3 |  |  |  |  |
|---|---|--|--|--|--|

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=2    neighbour=5

q=

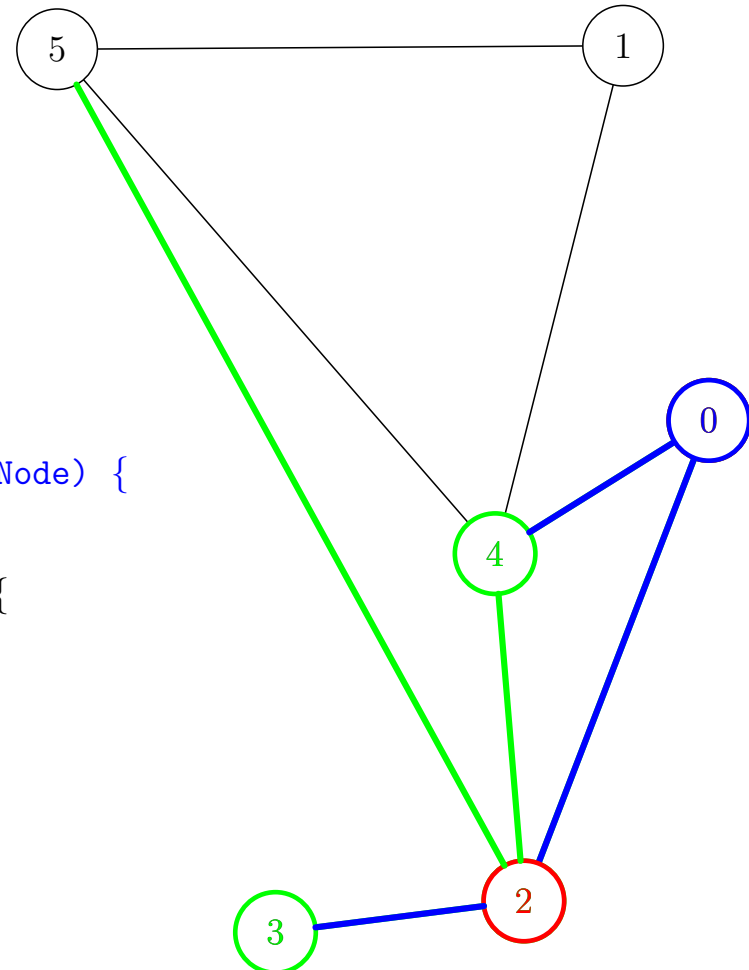| 4 | 3 |  |  |  |  |
|---|---|---|---|---|---|

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=2    neighbour=5

q= | 4 | 3 |   |   |   |   |
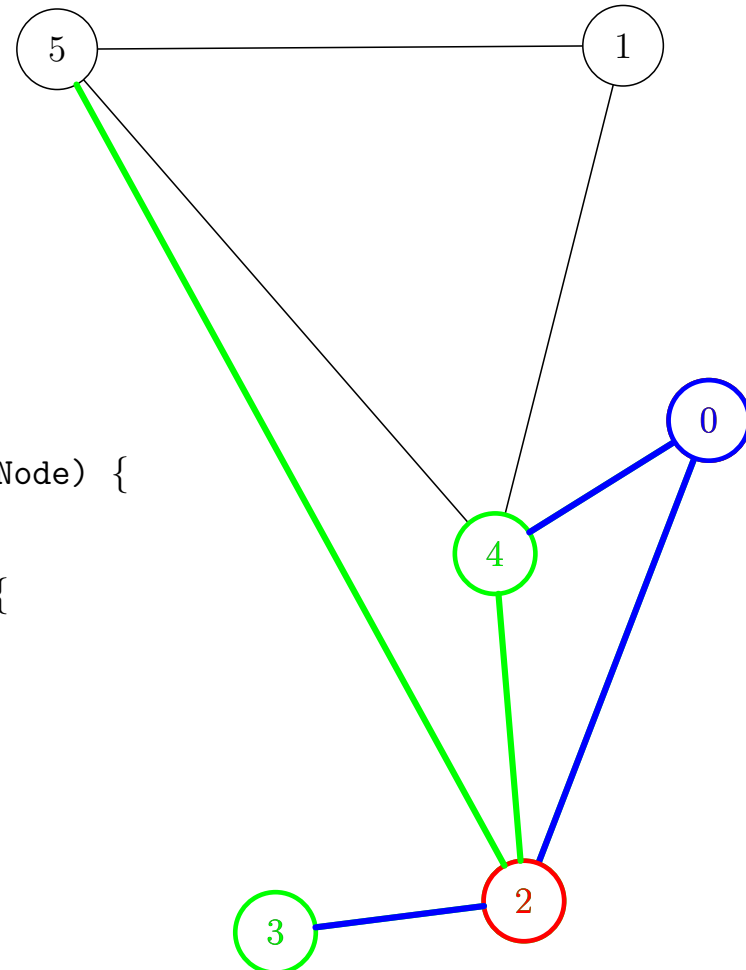
# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=2    neighbour=5

q=

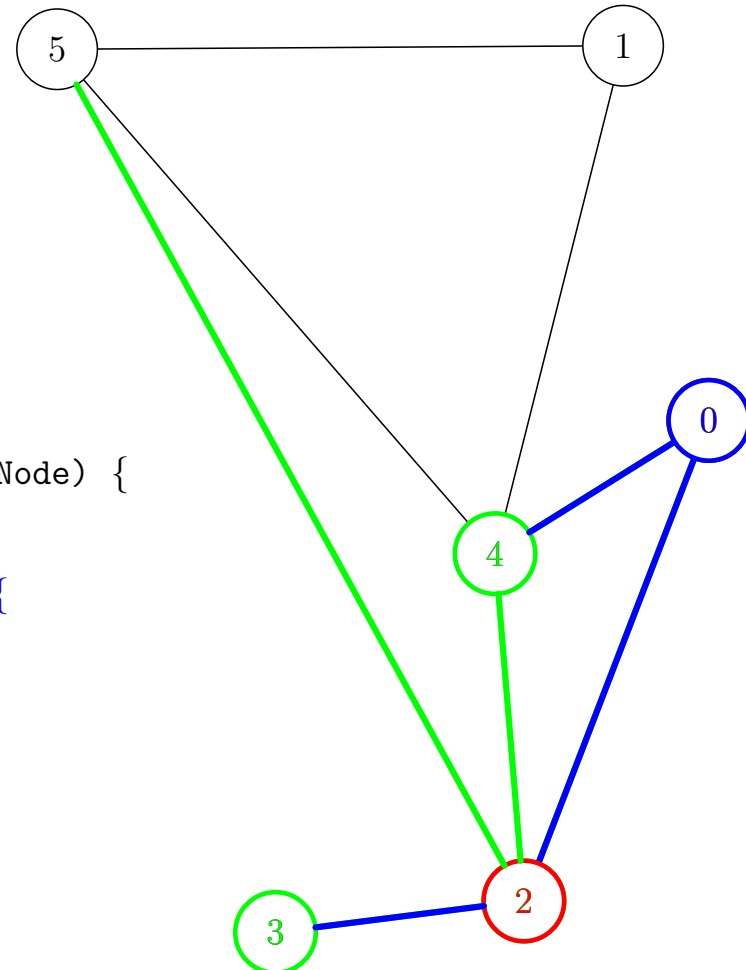| 4 | 3 |   |   |   |   |

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=2    neighbour=5

q=

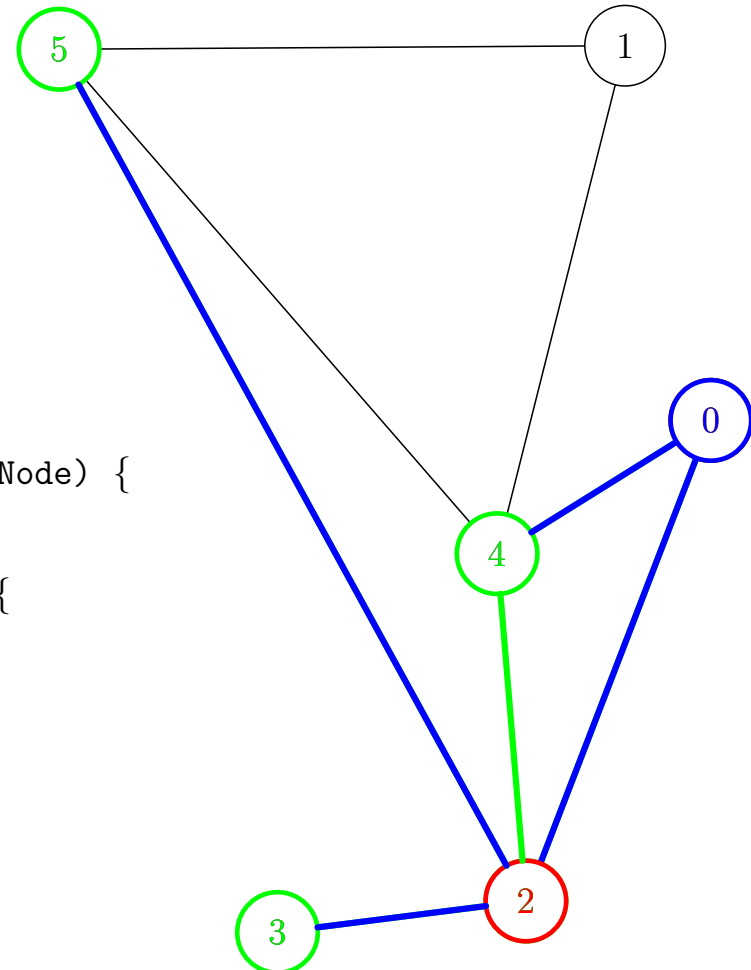| 4 | 3 | 5 |   |   |   |
|---|---|---|---|---|---|

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=2

q=

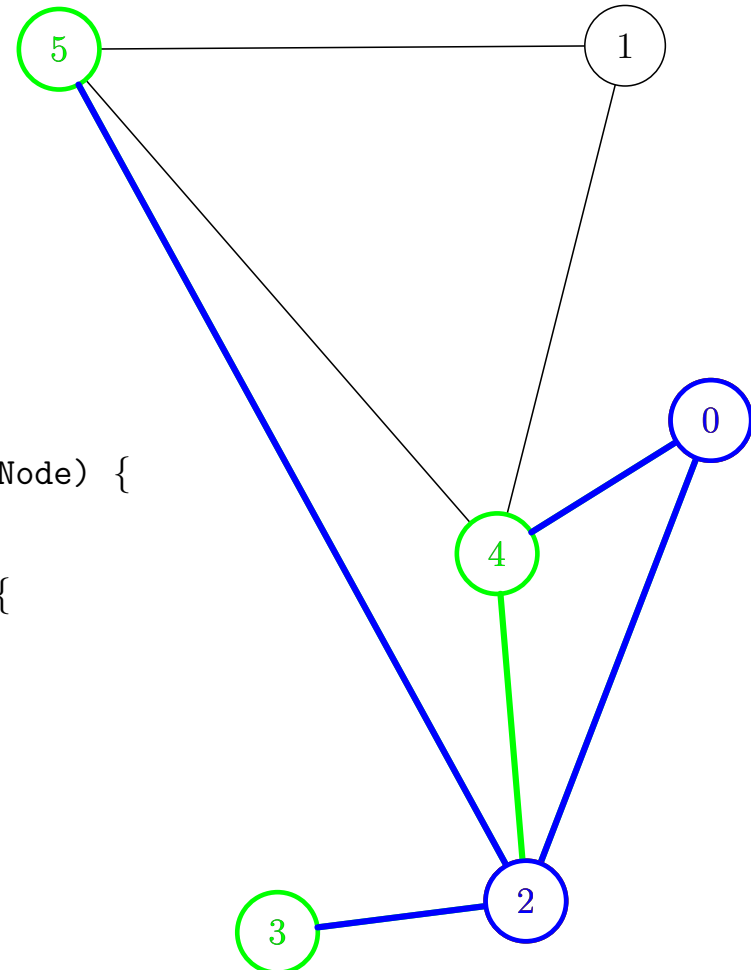| 4 | 3 | 5 |   |   |   |
|---|---|---|---|---|---|

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```



q=

| 4 | 3 | 5 |  |  |  |
|---|---|---|---|---|---|

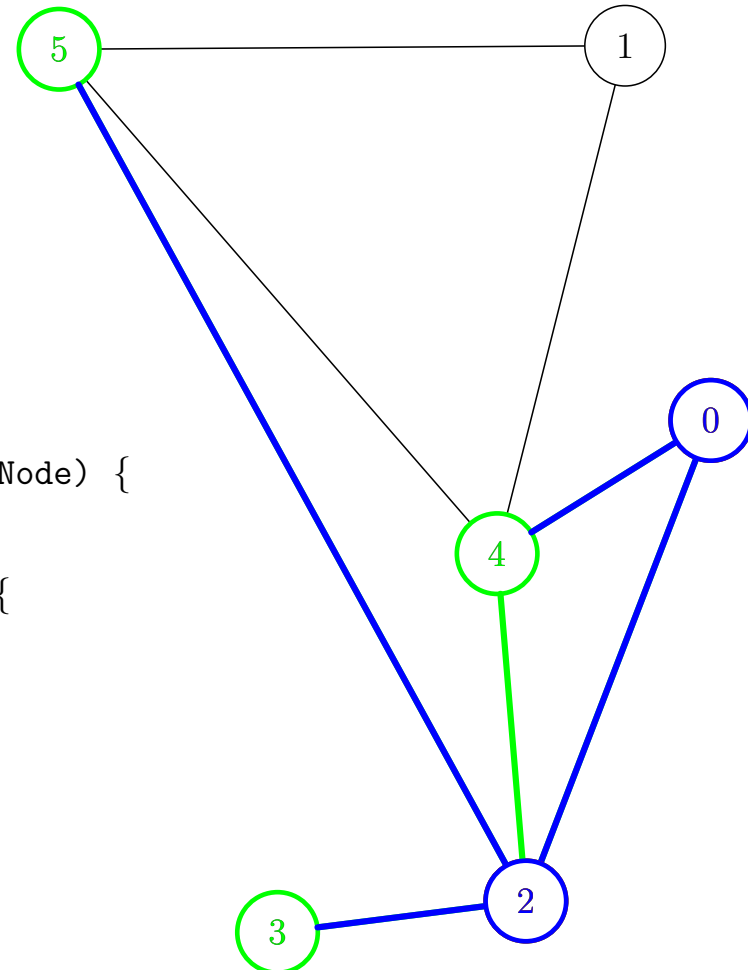# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=4

q=

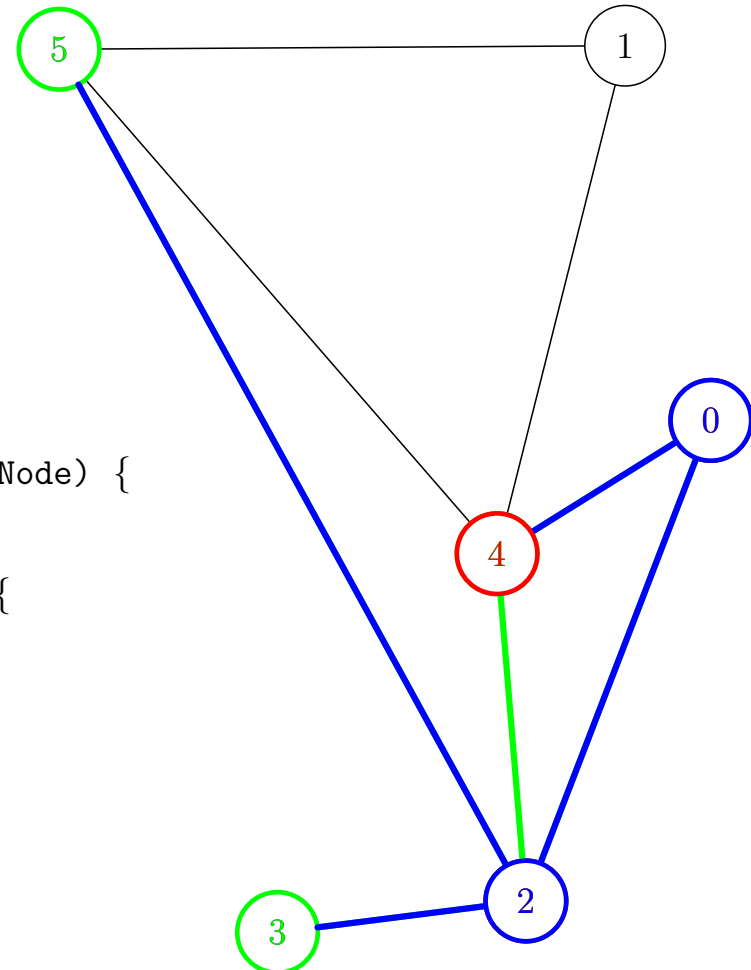| 3 | 5 |   |   |   |   |

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=4    neighbour=1

q=

| 3 | 5 |  |  |  |  |
|---|---|---|---|---|---|

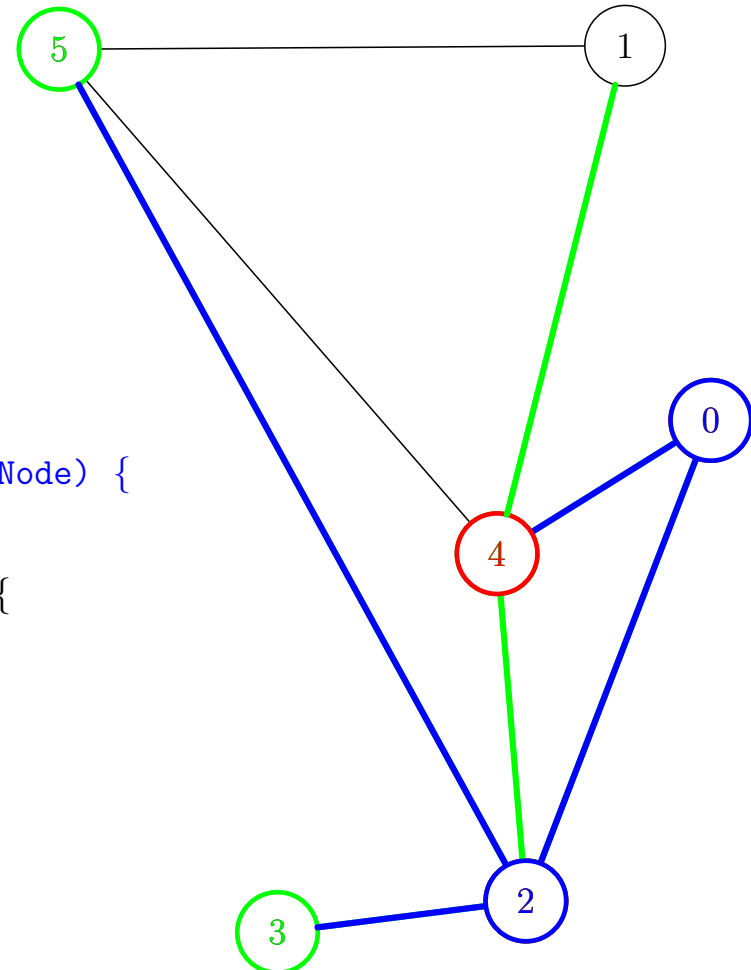# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=4    neighbour=1

q=

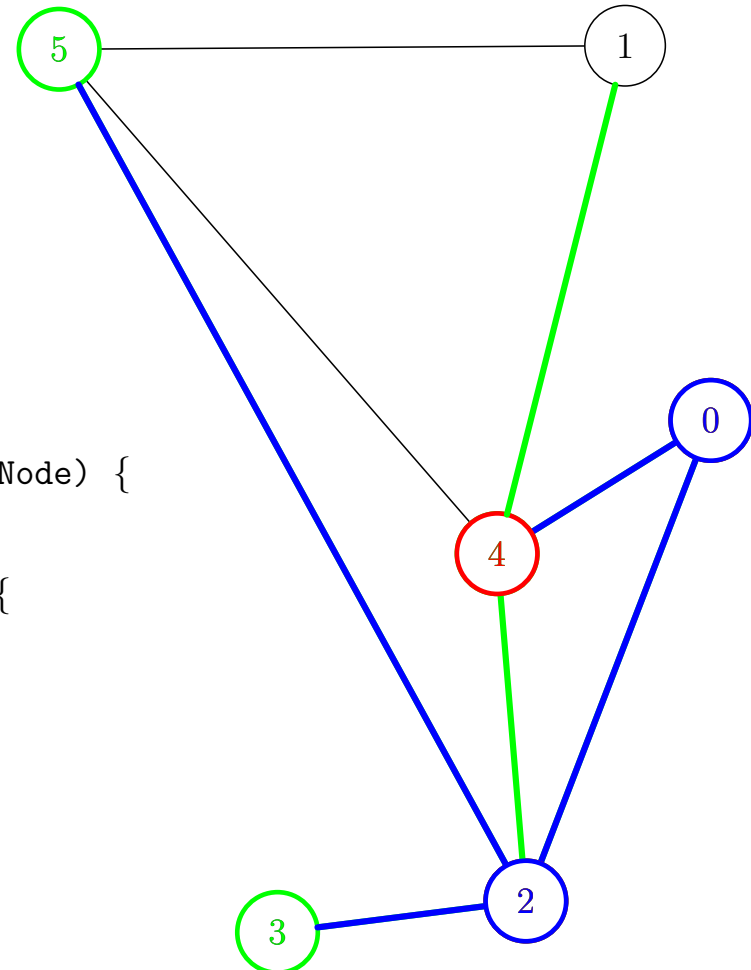| 3 | 5 |   |   |   |   |

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=4    neighbour=1

q=

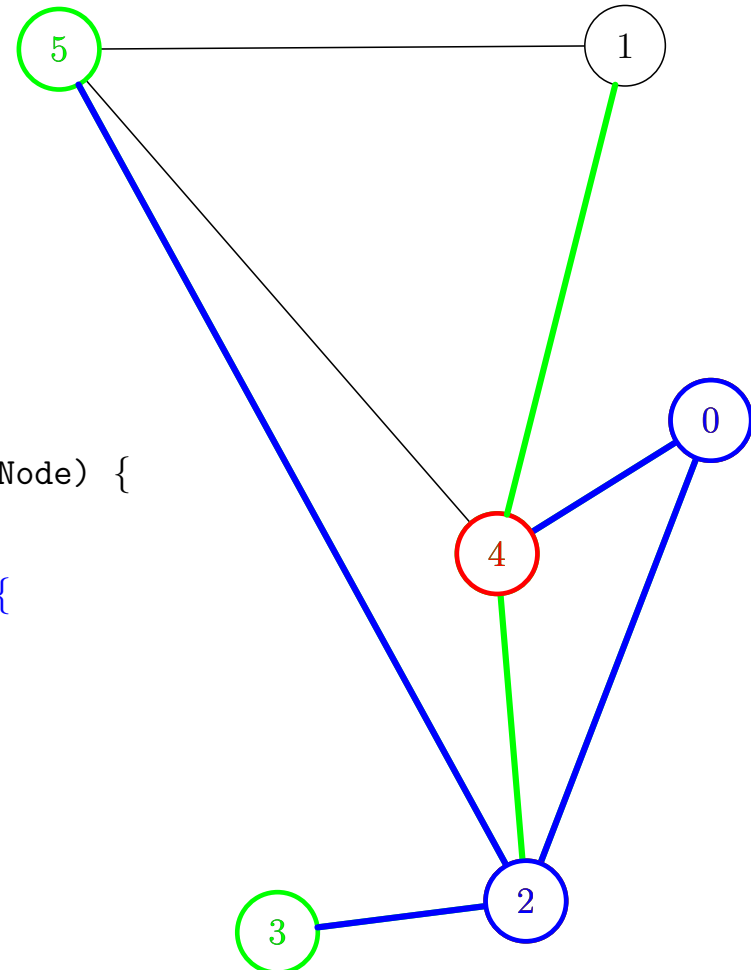| 3 | 5 |   |   |   |   |

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=4     neighbour=1

q= | 3 | 5 | 1 | | | |

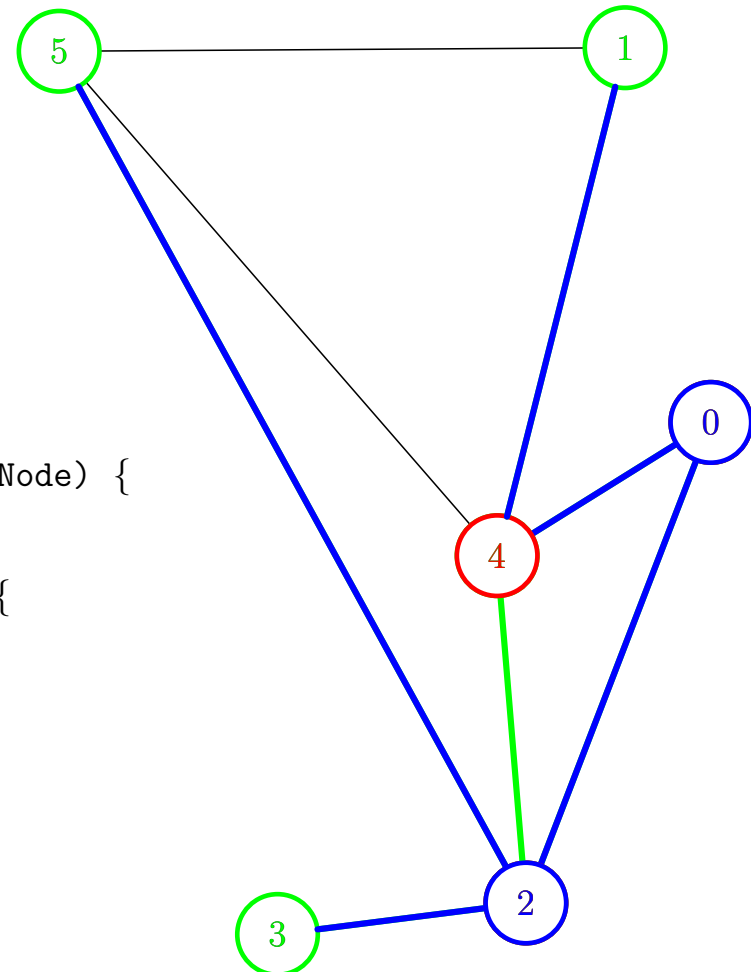# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=4    neighbour=5

q=

| 3 | 5 | 1 |  |  |  |
|---|---|---|---|---|---|

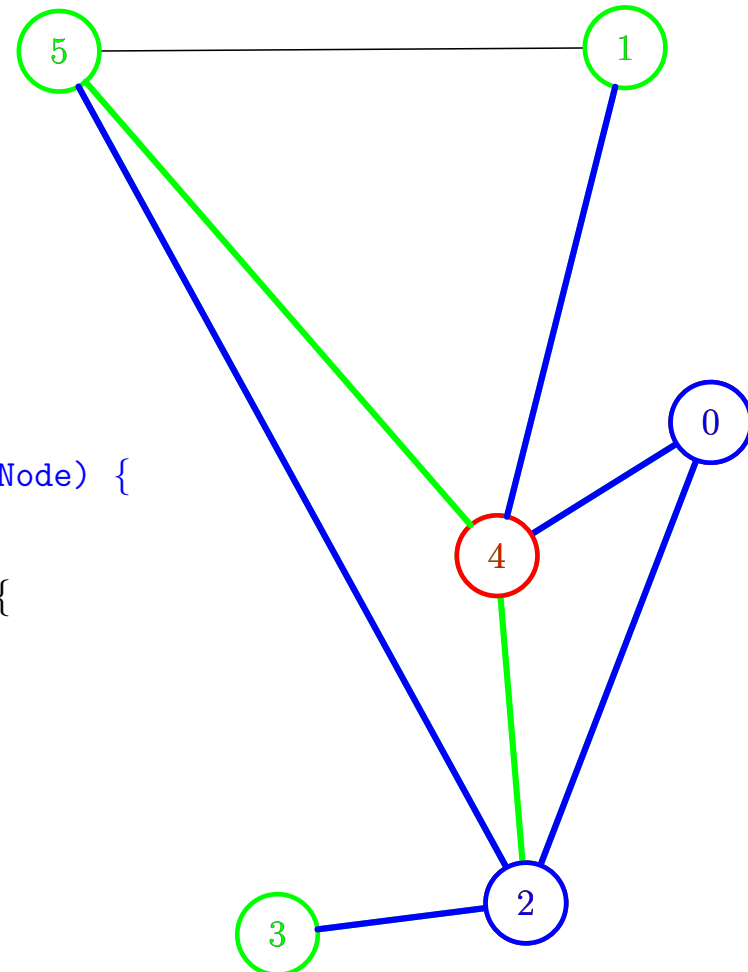# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=4     neighbour=5

q=

| 3 | 5 | 1 |  |  |  |
|---|---|---|---|---|---|

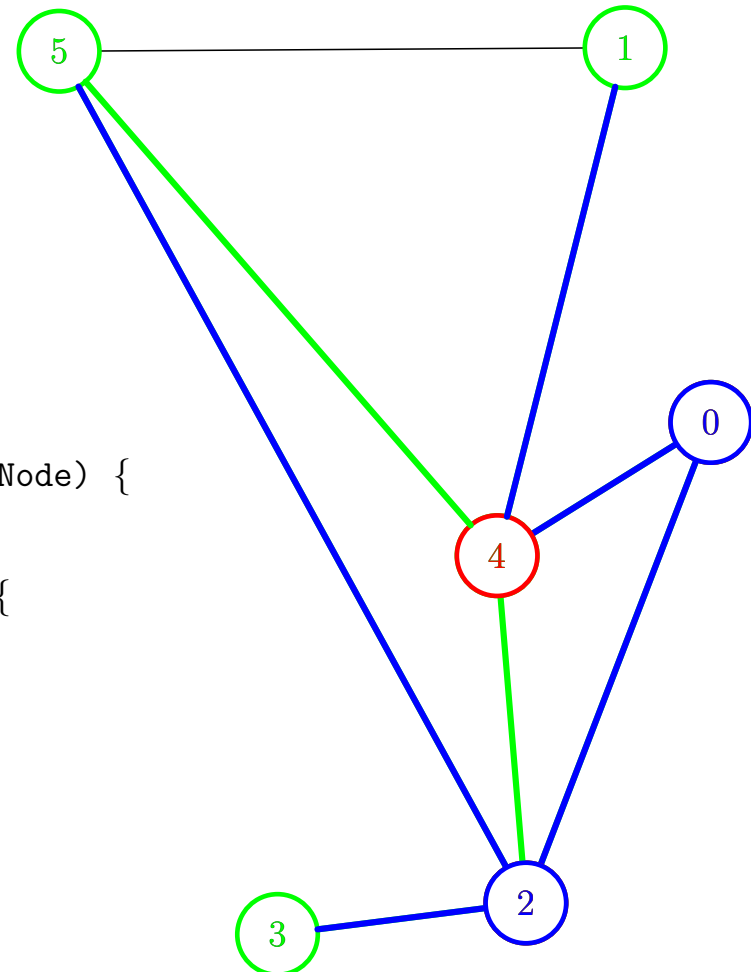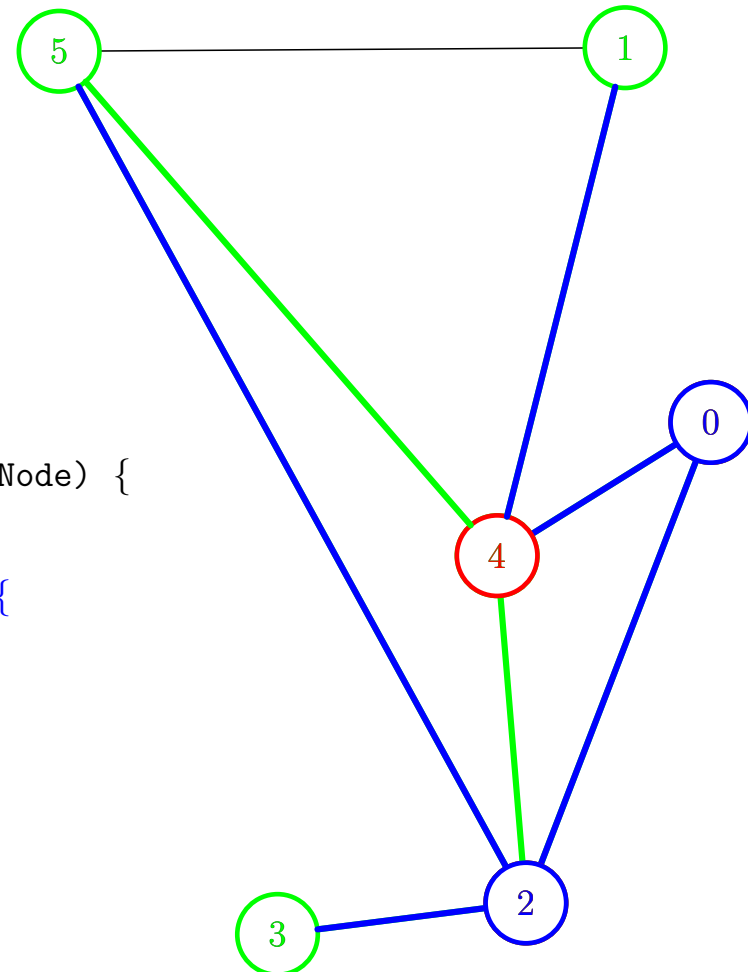# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=4    neighbour=5

q=

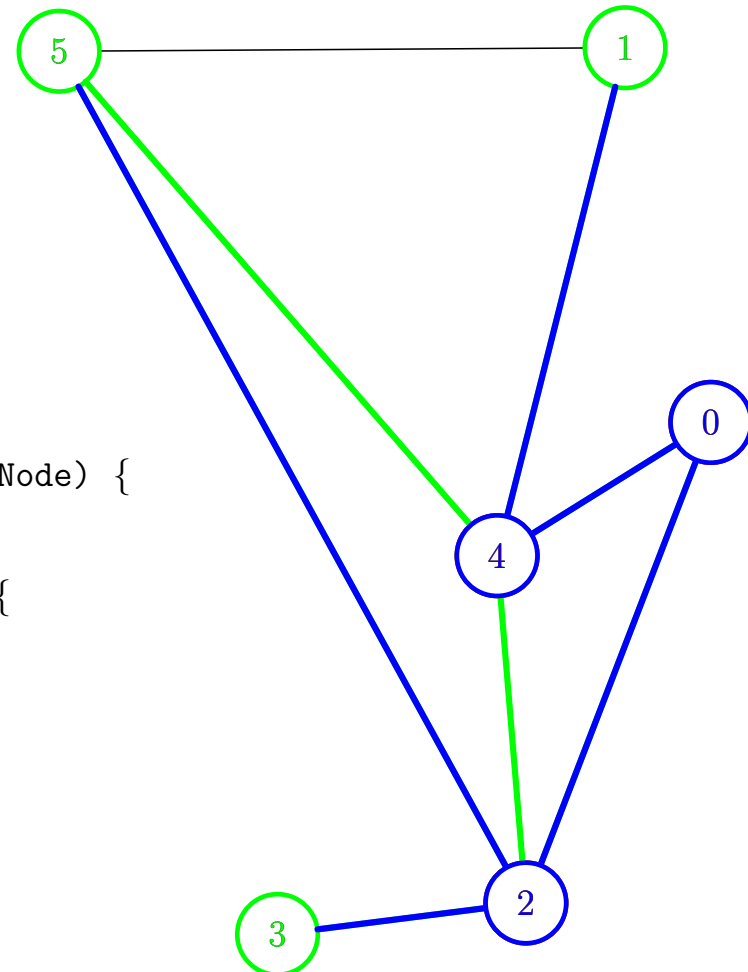| 3 | 5 | 1 |  |  |  |
|---|---|---|---|---|---|

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=4

q=

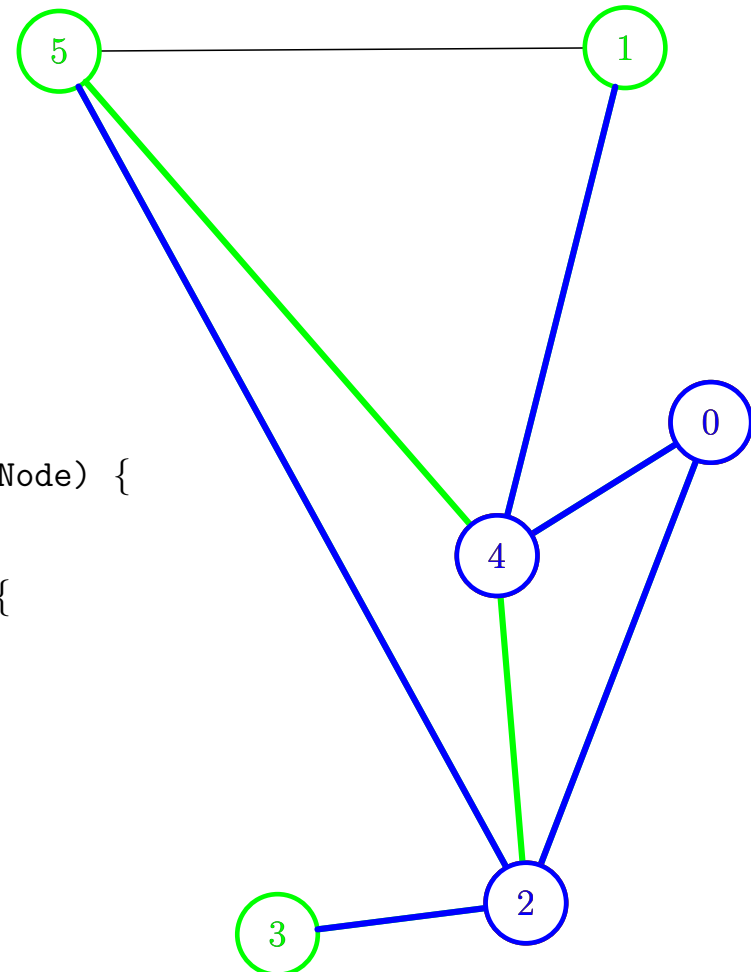| 3 | 5 | 1 | | | |
|---|---|---|---|---|---|

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

q=

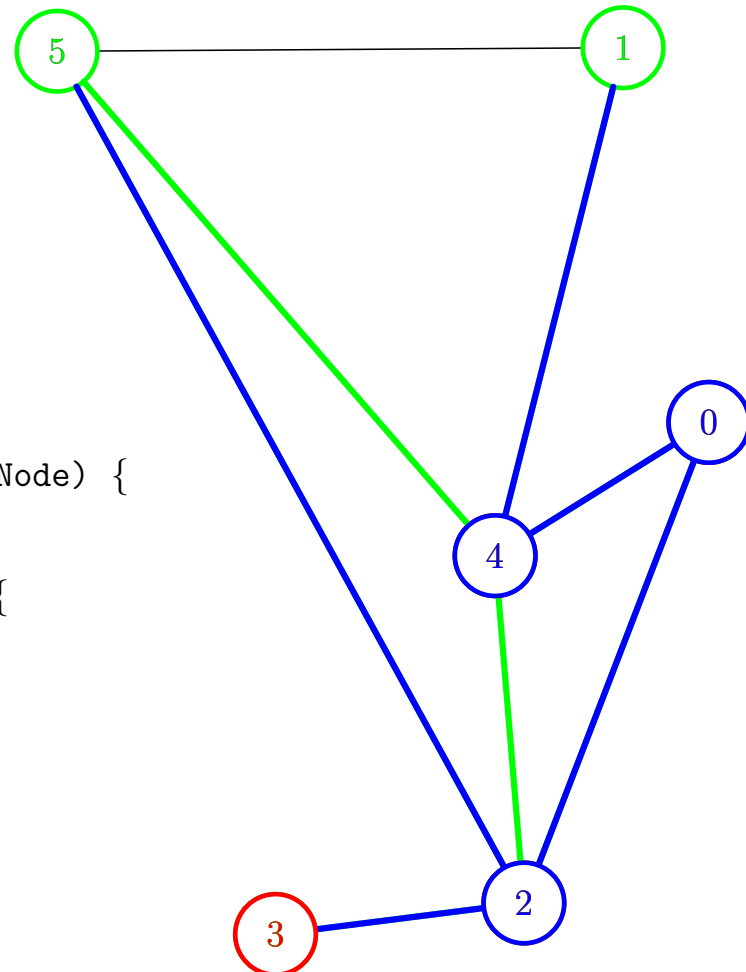| 3 | 5 | 1 | | | |
|---|---|---|---|---|---|

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=3

q=

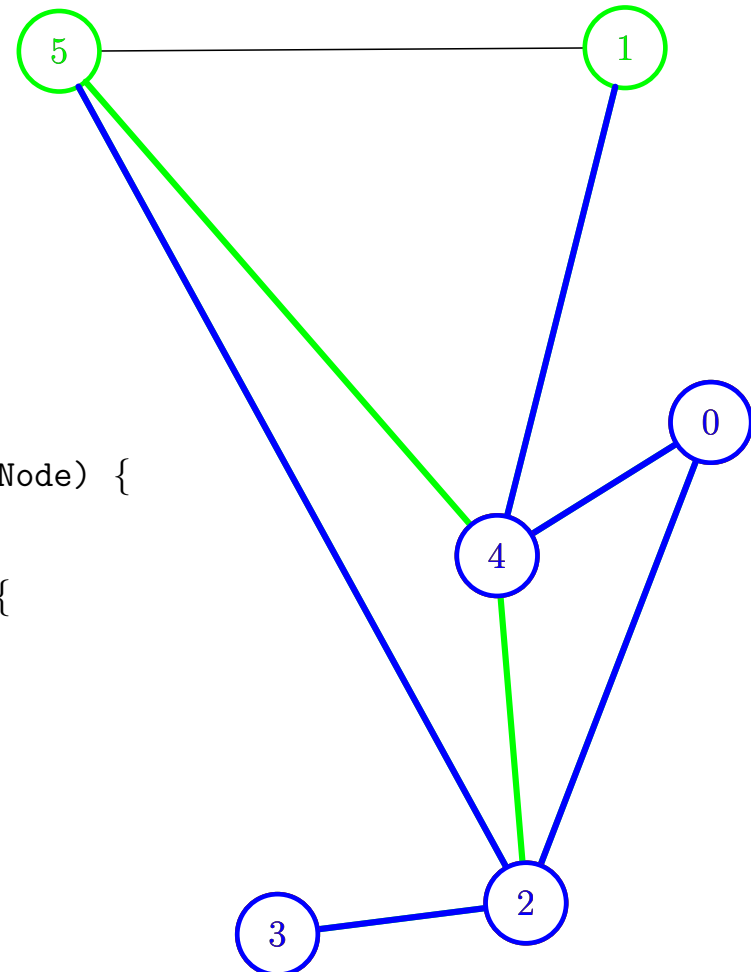| 5 | 1 |  |  |  |  |
|---|---|---|---|---|---|

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=3

q= | 5 | 1 | | | | |
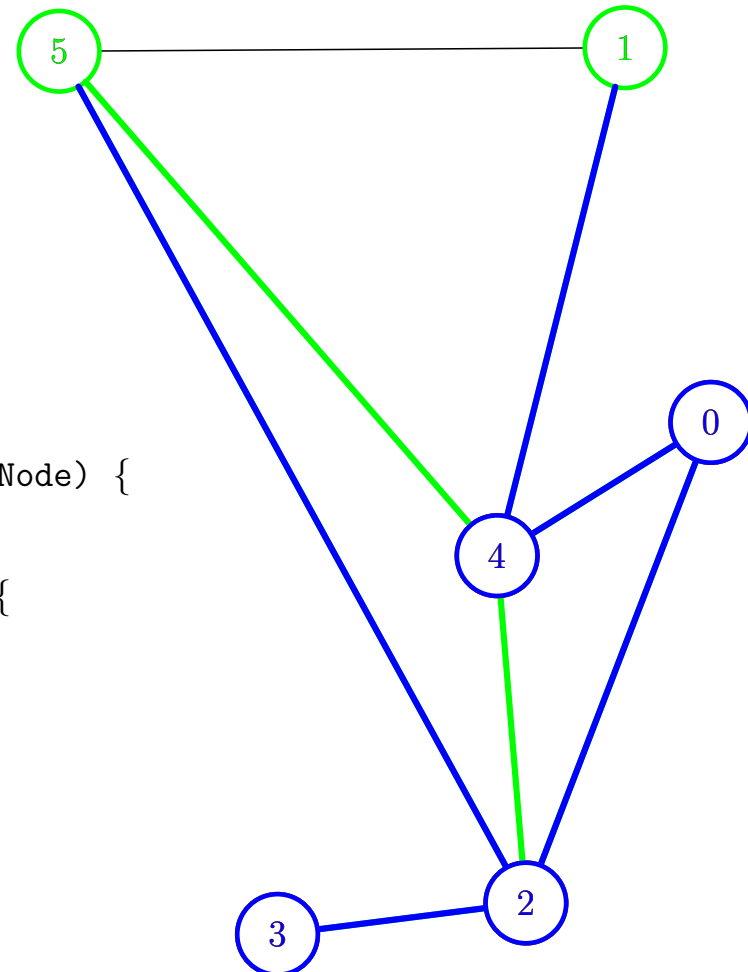
# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

q= | 5 | 1 | | | | |
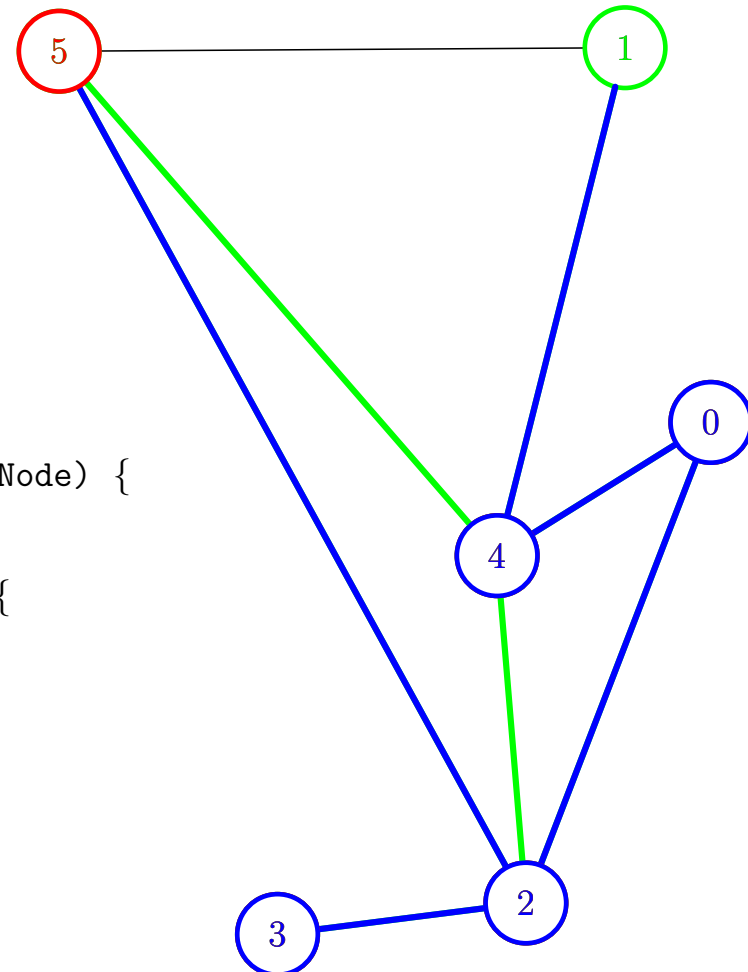
# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=5

q=

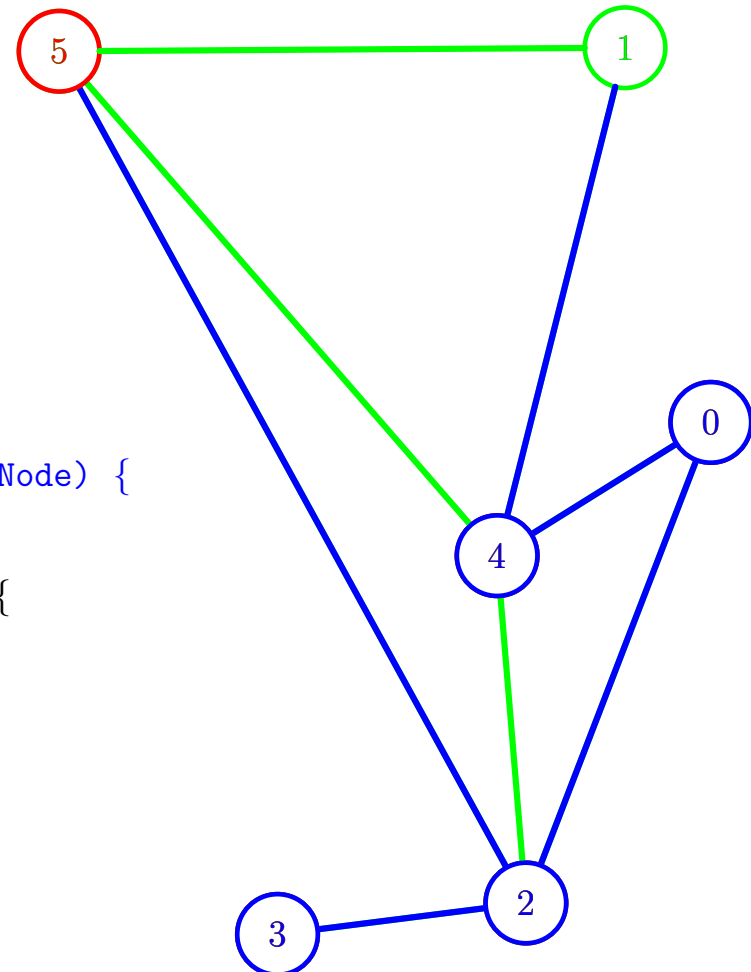| 1 |   |   |   |   |   |
|---|---|---|---|---|---|

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=5    neighbour=1

q=

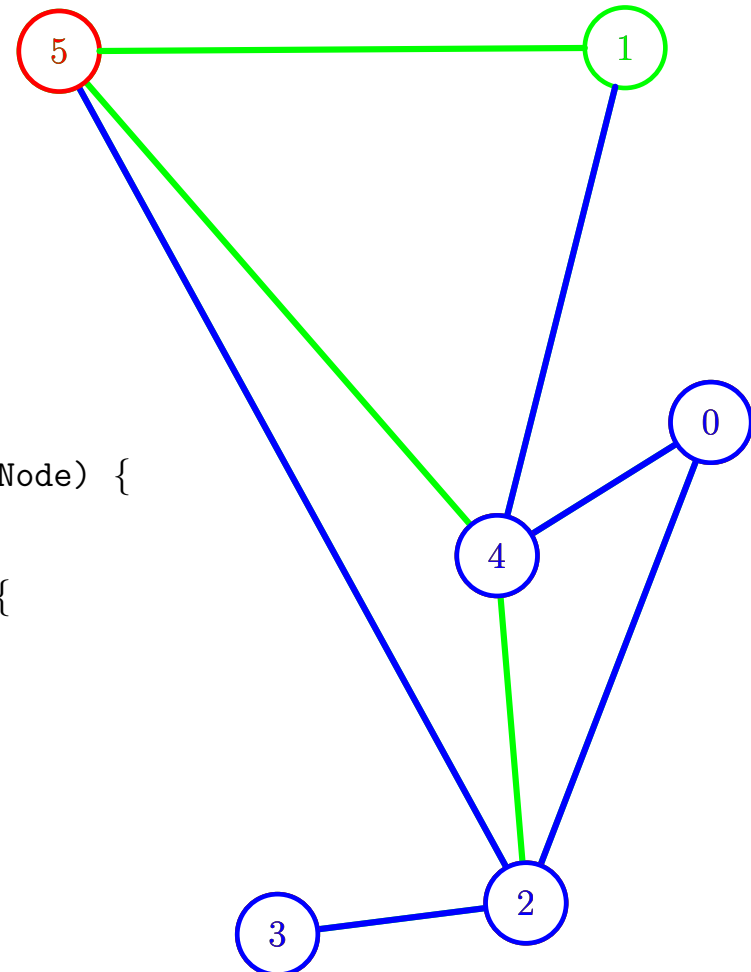| 1 |  |  |  |  |  |
|---|---|---|---|---|---|

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=5    neighbour=1

q= | 1 | | | | | |
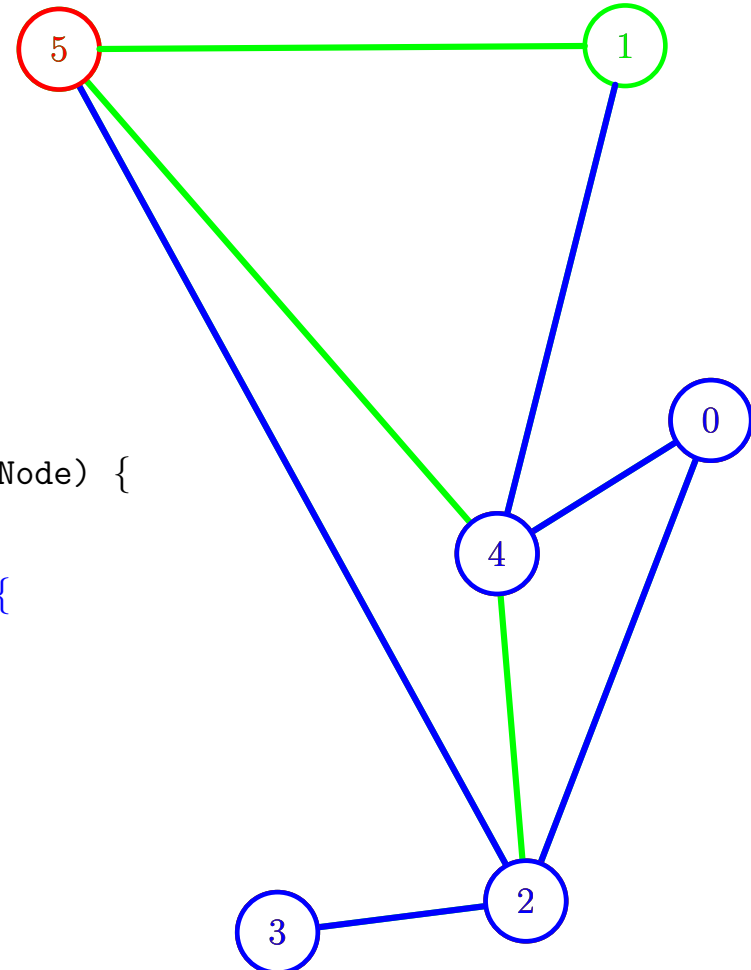
# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
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      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=5      neighbour=1

q= | 1 | | | | | |

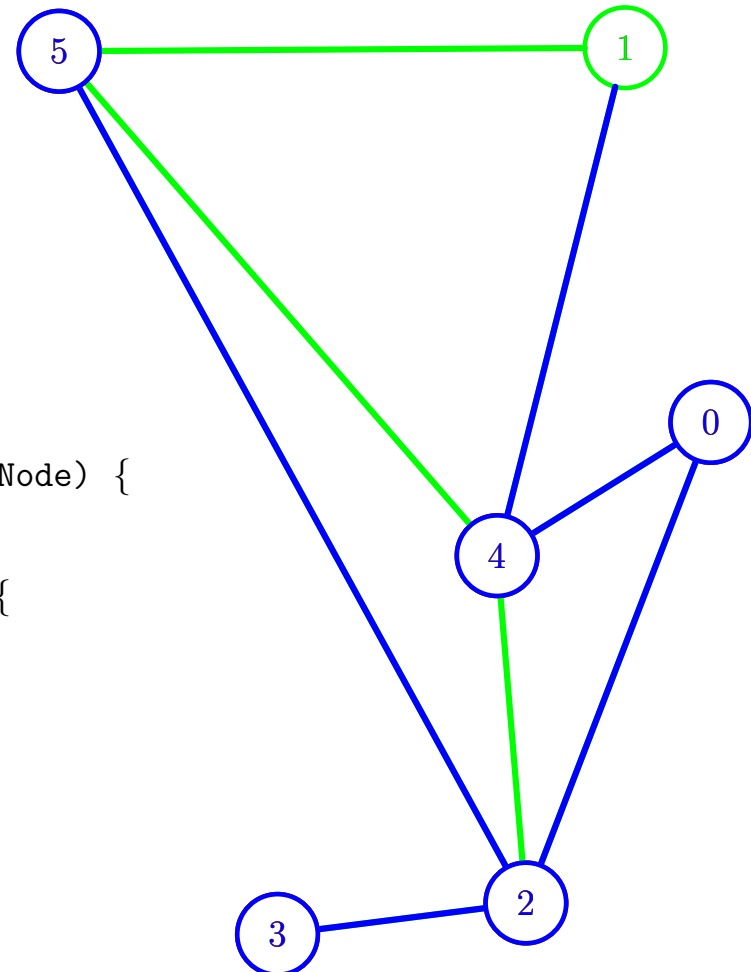# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
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        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=5

q=

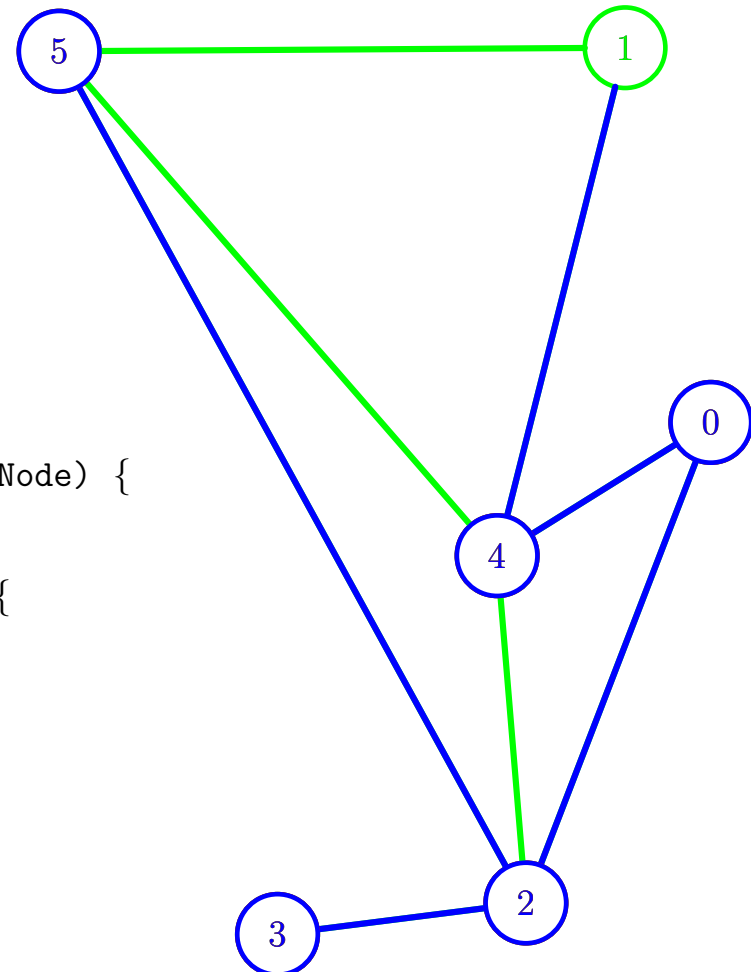| 1 |   |   |   |   |   |
|---|---|---|---|---|---|

# Breadth First Search

```
bfs(graph, node) {
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  List parent(graph.noNodes)
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        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

q=

| 1 |   |   |   |   |   |
|---|---|---|---|---|---|

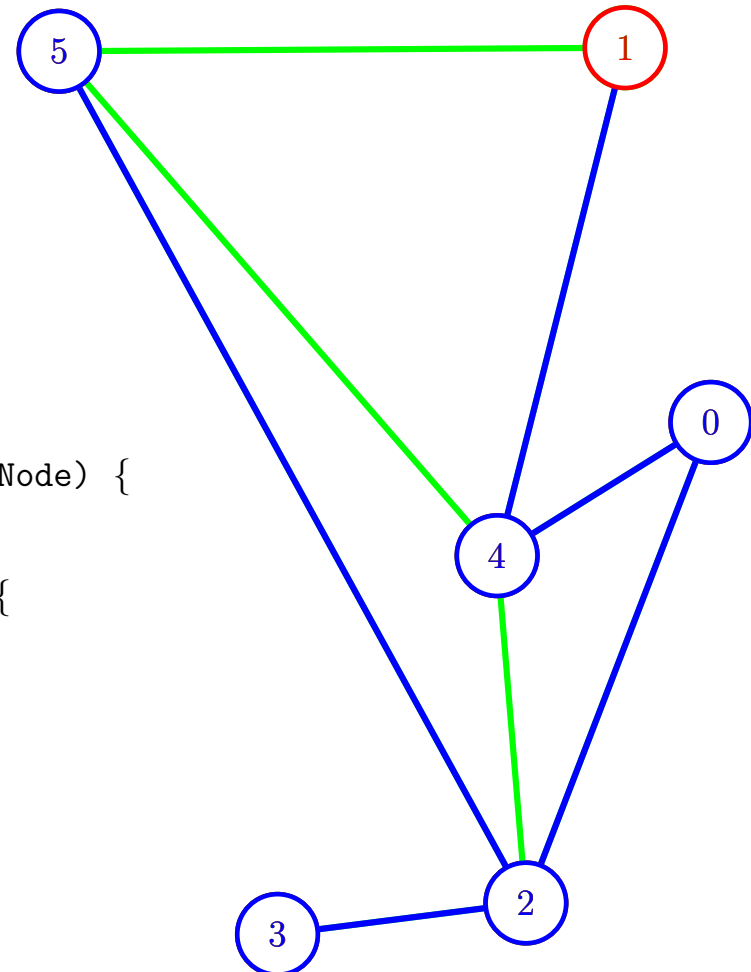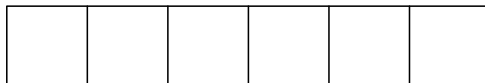# Breadth First Search

```
bfs(graph, node) {
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    foreach neighbour ∈ Neighbourhood(currentNode) {
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      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

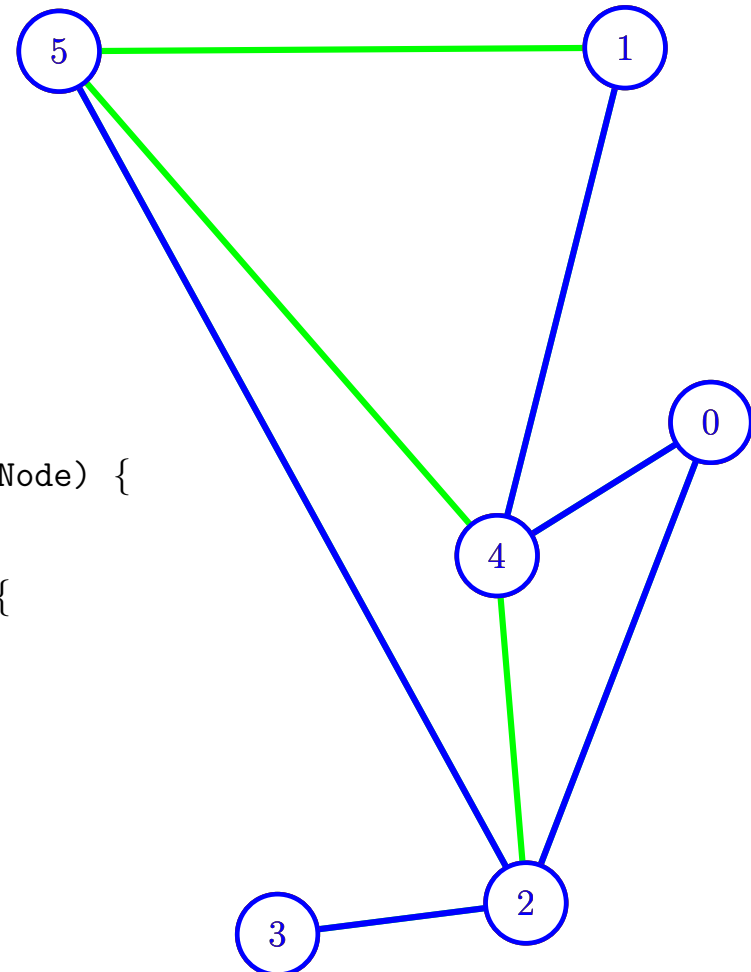currentNode=1
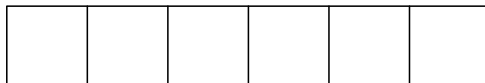
q=

# Breadth First Search

```
bfs(graph, node) {
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  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
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    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```
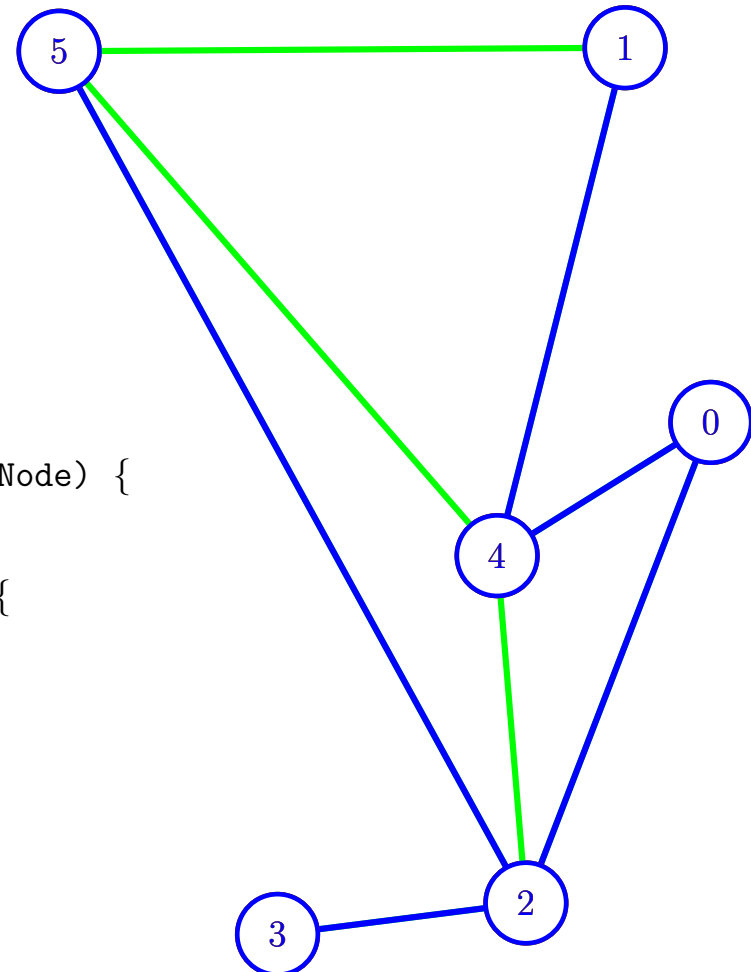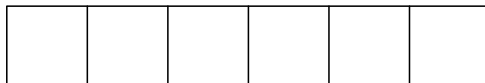
currentNode=1

q=

# Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
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    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

q=

# Applications of Breadth First Search

- Breadth first search can be used to find the shortest path from a source node to a destination node for an **unweighted** graph

  - ⋆ Run `bfs(graph,source)`
  - ⋆ Use parent information to find path from destination back to source

- BFS (as well as DFS) can be used to find connected components

  - ⋆ Use $processVertexEarly$ to mark vertices connected to the current connected component
  - ⋆ Run $bfs$ from all vertices that are not labelled

# Applications of Breadth First Search

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  - ⋆ Use $processVertexEarly$ to mark vertices connected to the current connected component
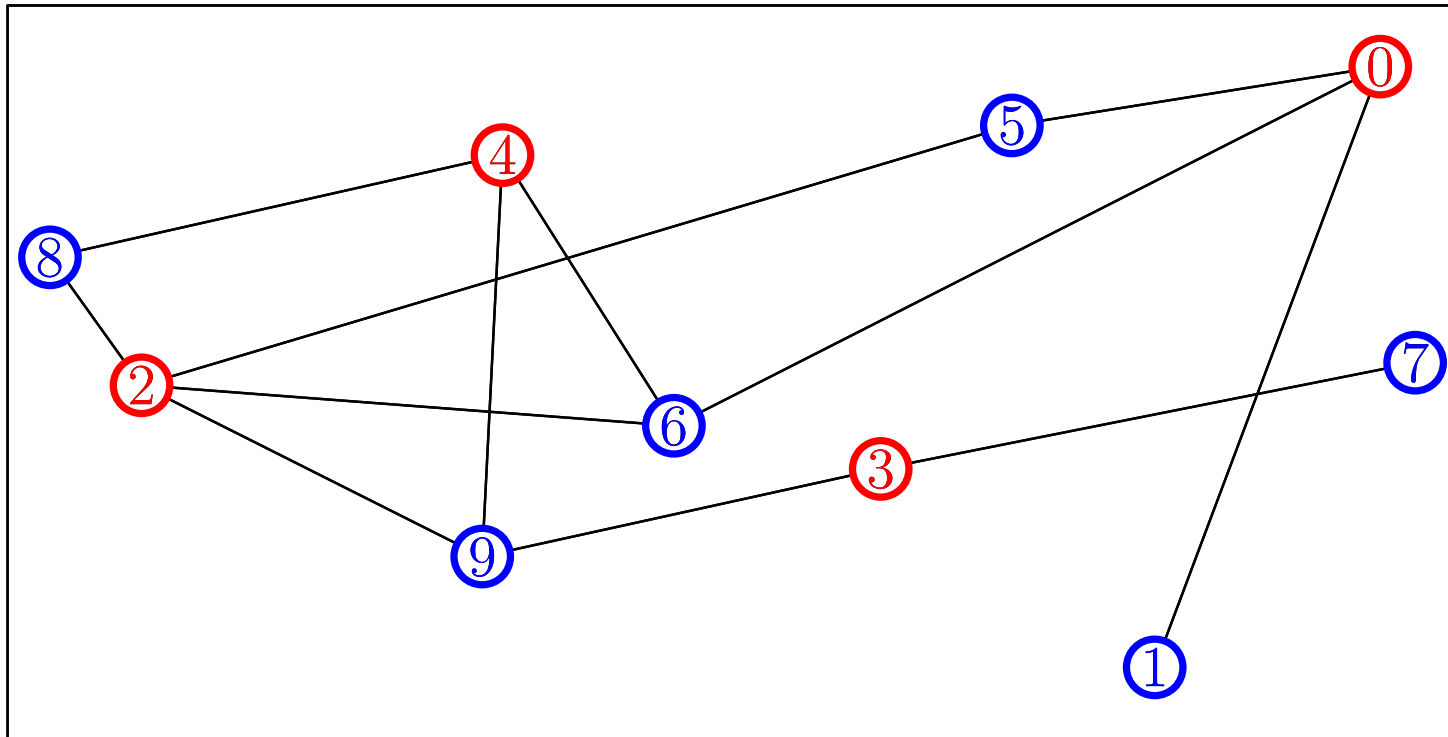  - ⋆ Run $bfs$ from all vertices that are not labelled

# Bipartite Graphs

- Bipartite graphs are graphs where the vertices can be split into two sets so that there are no edges between vertices in the same graph
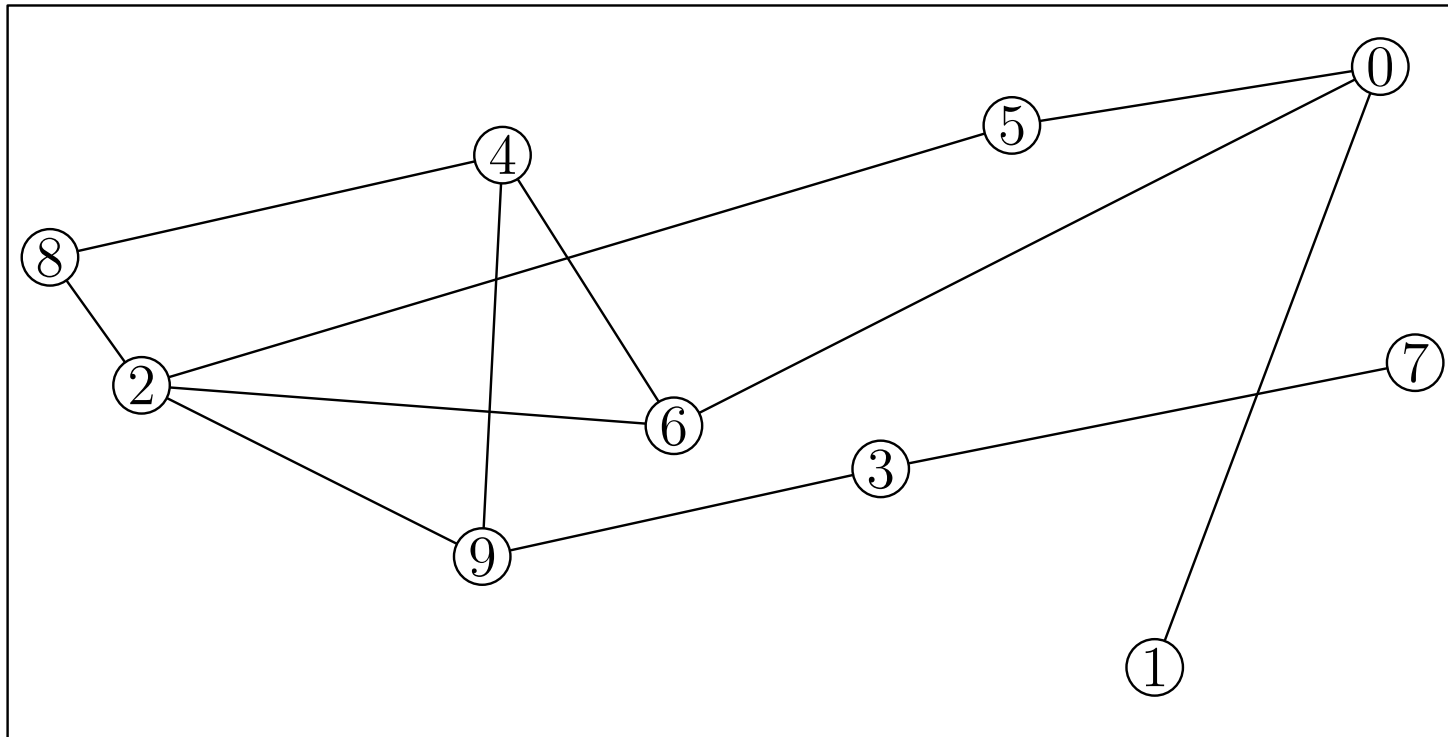
# Bipartite Graphs

- Bipartite graphs are graphs where the vertices can be split into two sets so that there are no edges between vertices in the same graph

# Checking Bipartiteness (Two-colourability)

- Each edge must connect nodes from different sets

```
isBipartite(graph) {
    colour = List(graph.noNodes(), "white");
    bipartite = true;

    foreach node in graph {
        if (colour[node] == "white") {
            colour[node] = "red";
            bfs(graph, node);
        }
    }
    return bipartite;
}

processEdge(node1, node2) {
    if (colour[node1] == colour[node2])
        bipartite = false;
    colour[node2] = (colour[node1]=="red")? "blue":"red";
}
```

# Checking Bipartiteness (Two-colourability)

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```
isBipartite(graph) {
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processEdge(node1, node2) {
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        bipartite = false;
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}
```

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            bfs(graph, node);
        }
    }
    return bipartite;
}

processEdge(node1, node2) {
    if (colour[node1] == colour[node2])
        bipartite = false;
    colour[node2] = (colour[node1]=="red")? "blue":"red";
}
```
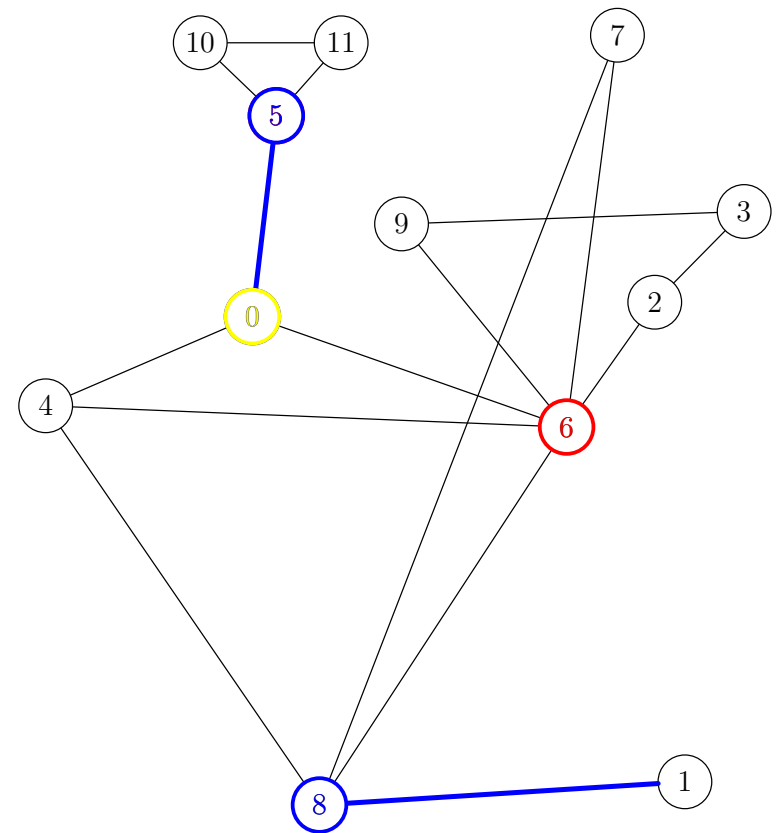
# Checking Bipartiteness (Two-colourability)

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        if (colour[node] == "white") {
            colour[node] = "red";
            bfs(graph, node);
        }
    }
    return bipartite;
}

processEdge(node1, node2) {
    if (colour[node1] == colour[node2])
        bipartite = false;
    colour[node2] = (colour[node1]=="red")? "blue":"red";
}
```

# Outline

1. Breadth First Search

   • BFS applications

2. **Depth First Search**

   • DFS applications

3. Topological Sort

# Depth First Search

- <span style="color:red">Depth first search is essentially like breadth first search except we replace the queue by a stack</span>

- In practice it is often implemented using recursion rather than a stack

- It proves useful to keep a record of the traversal **time** for each vertex

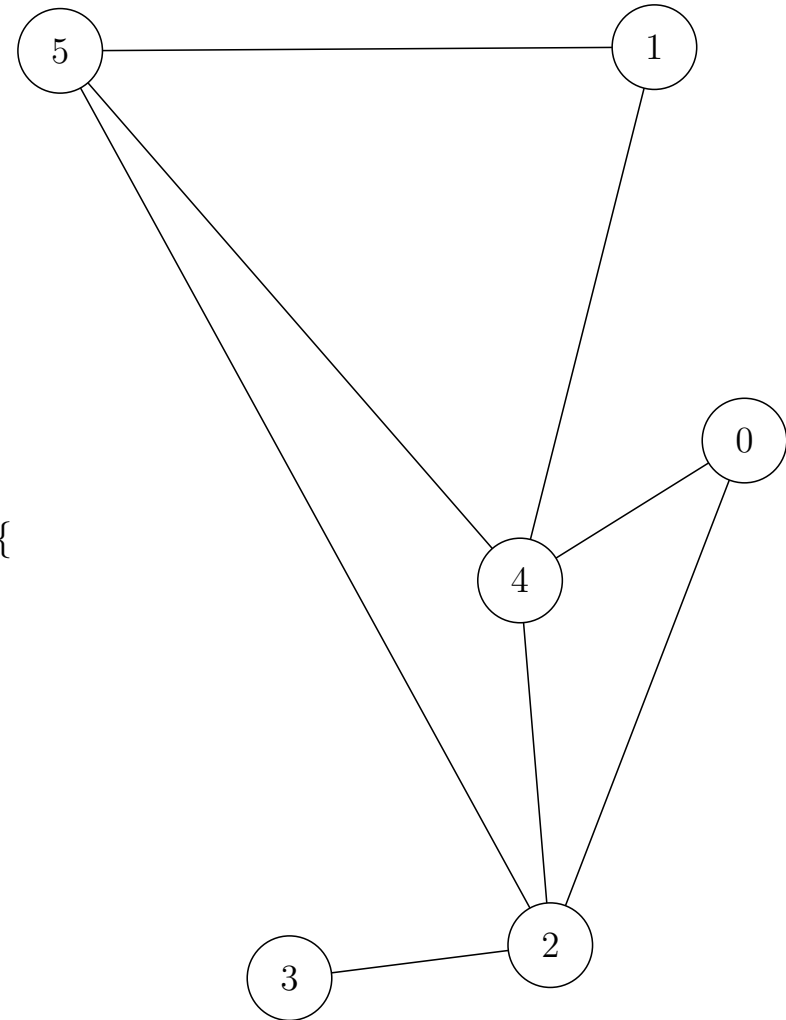  ⋆ the clock ticks each time a vertex is entered or exited

# Depth First Search

- Depth first search is essentially like breadth first search except we replace the queue by a stack

- In practice it is often implemented using recursion rather than a stack

- It proves useful to keep a record of the traversal **time** for each vertex

  ⋆ the clock ticks each time a vertex is entered or exited

# Depth First Search

- Depth first search is essentially like breadth first search except we replace the queue by a stack

- In practice it is often implemented using recursion rather than a stack

- It proves useful to keep a record of the traversal **time** for each vertex

  ★ the clock ticks each time a vertex is entered or exited

# Depth First Search

```
dfs(graph, node) {
    state ← Array[n, "undiscovered"]
    finished ← false
    dfs_recur(graph, node)
}

dfs_recur(graph, node) {
    if (finished) return
    state[node] ← "discovered"
    time ← time + 1
    processVertexEarly(node)
    foreach neighbour ∈ Neighbourhood(node) {
        if (state[neighbour] = "undiscovered") {
            parent[neighbour] ← node
            processEdge(node, neighbour)
            dfs_recur(graph, neighbour)
        } else if (state[neighbour] ≠ "processed") {
            processEdge(node, neighbour)
        }
        if (finished) return
    }
    processVertexLate(currentNode)
    state[currentNode] ← "processed"
    time ← time + 1
}

    node=0      neighbour=nil   time=0
```
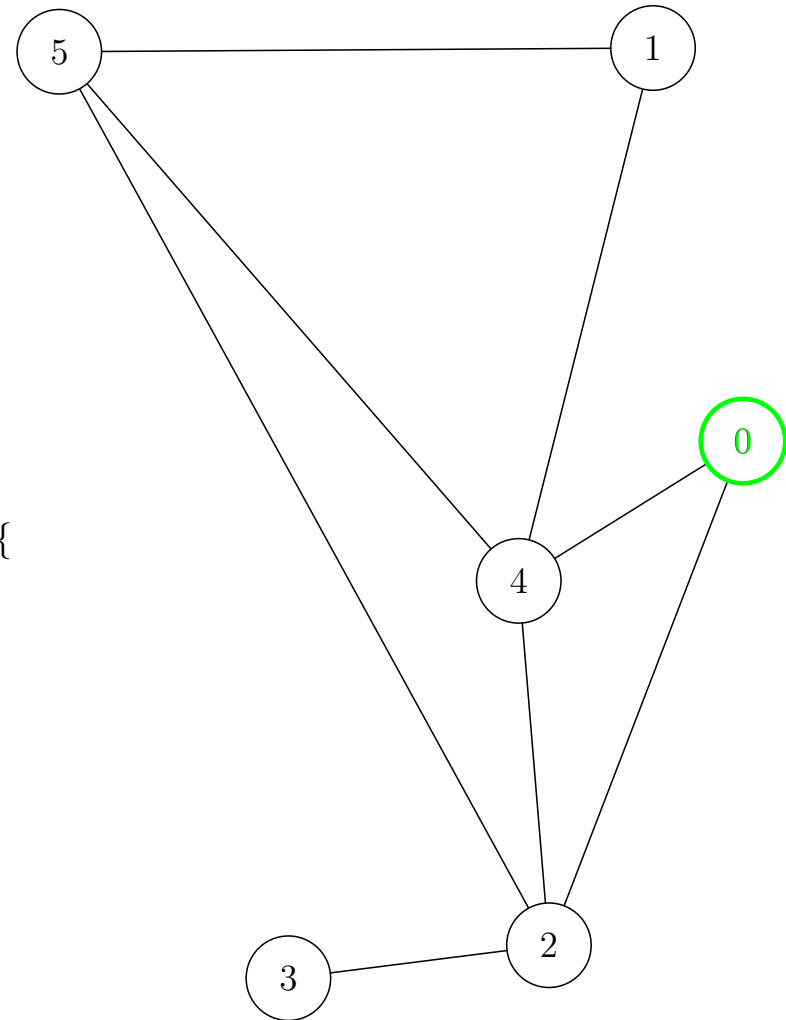
# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

    node=0      neighbour=nil   time=1
```
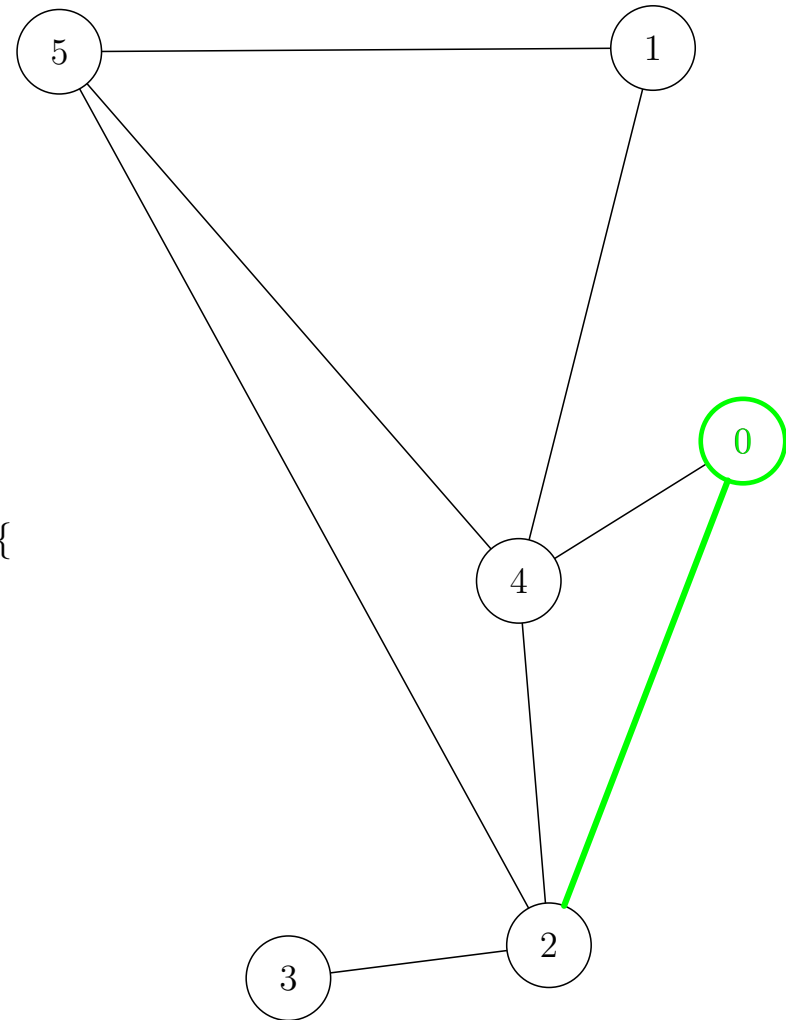
# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

    node=0      neighbour=2      time=1
```
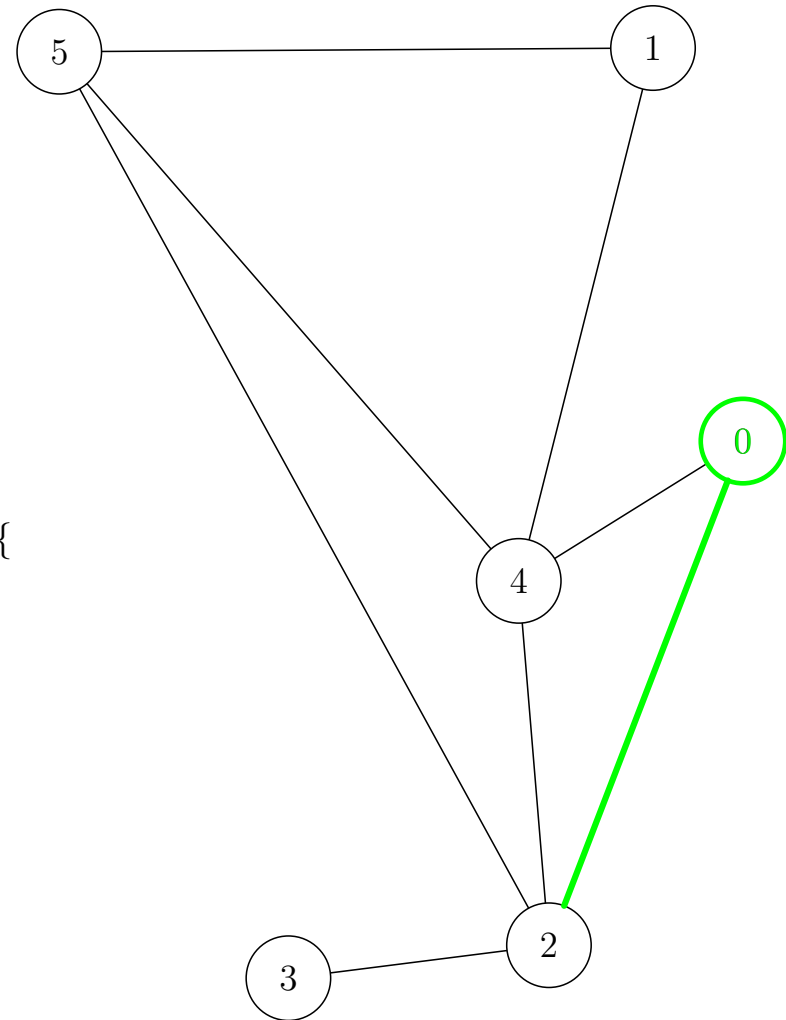
# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
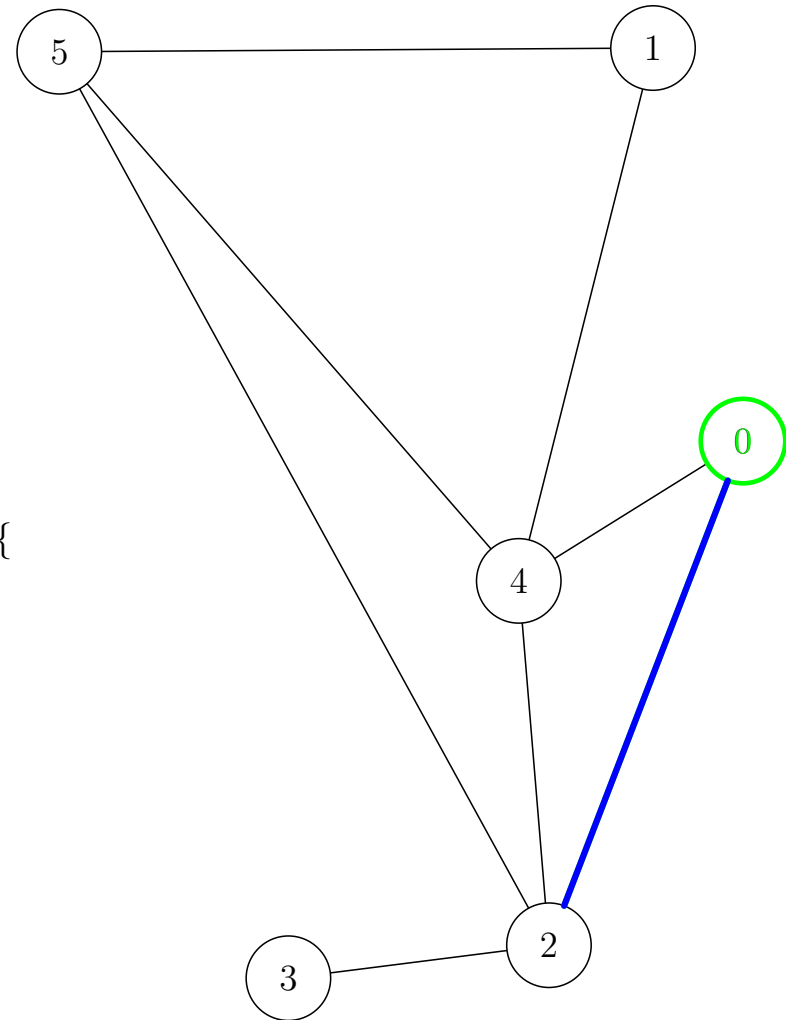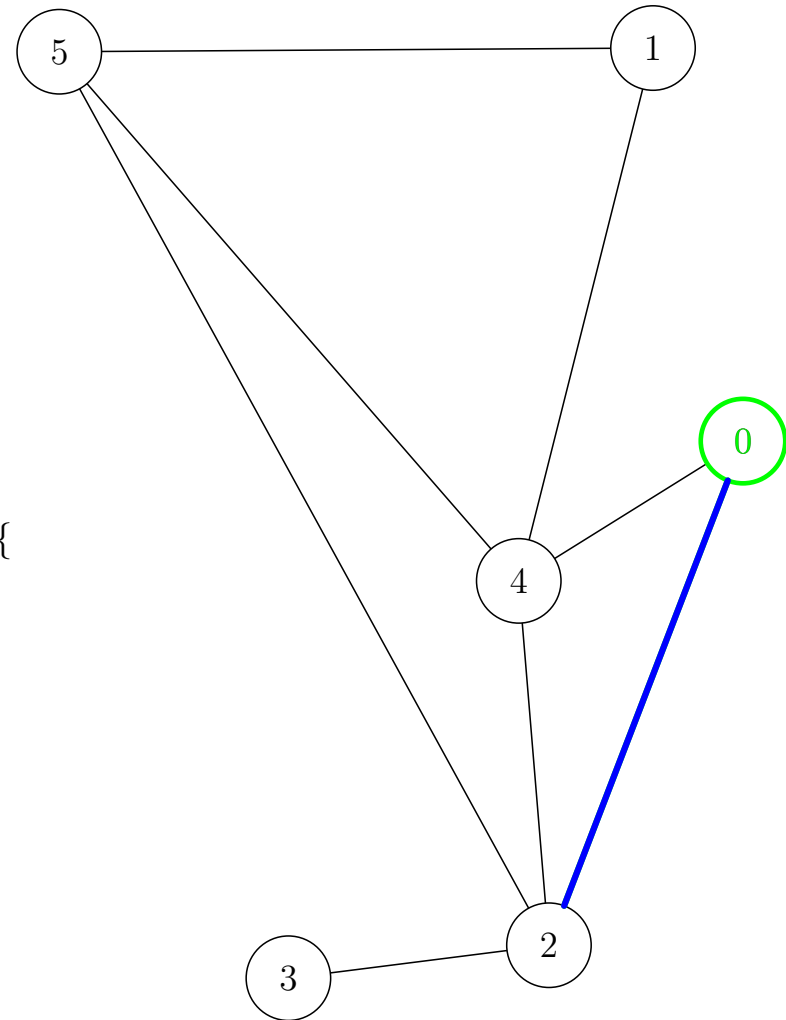
node=0    neighbour=2    time=1

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

    node=0      neighbour=2      time=1
```

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

    node=0      neighbour=2      time=1
```
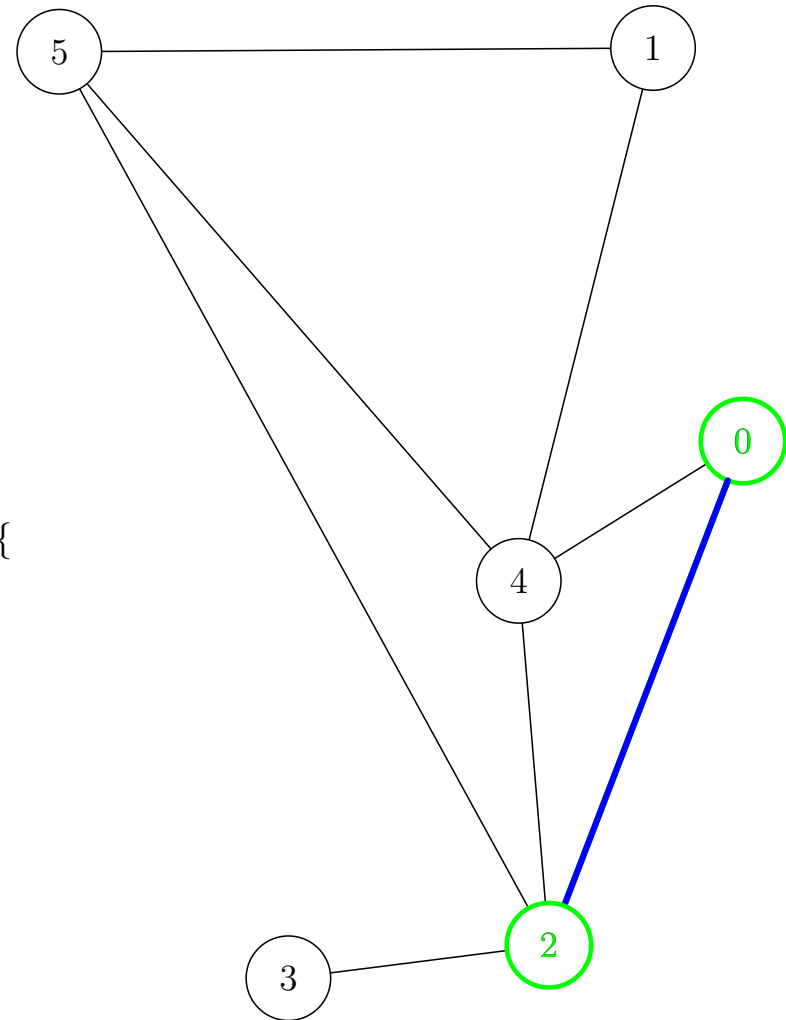
# Depth First Search

```
dfs(graph, node) {
    state ← Array[n, "undiscovered"]
    finished ← false
    dfs_recur(graph, node)
}

dfs_recur(graph, node) {
    if (finished) return
    state[node] ← "discovered"
    time ← time + 1
    processVertexEarly(node)
    foreach neighbour ∈ Neighbourhood(node) {
        if (state[neighbour] = "undiscovered") {
            parent[neighbour] ← node
            processEdge(node, neighbour)
            dfs_recur(graph, neighbour)
        } else if (state[neighbour] ≠ "processed") {
            processEdge(node, neighbour)
        }
        if (finished) return
    }
    processVertexLate(currentNode)
    state[currentNode] ← "processed"
    time ← time + 1
}

        node=2      neighbour=nil   time=2
```
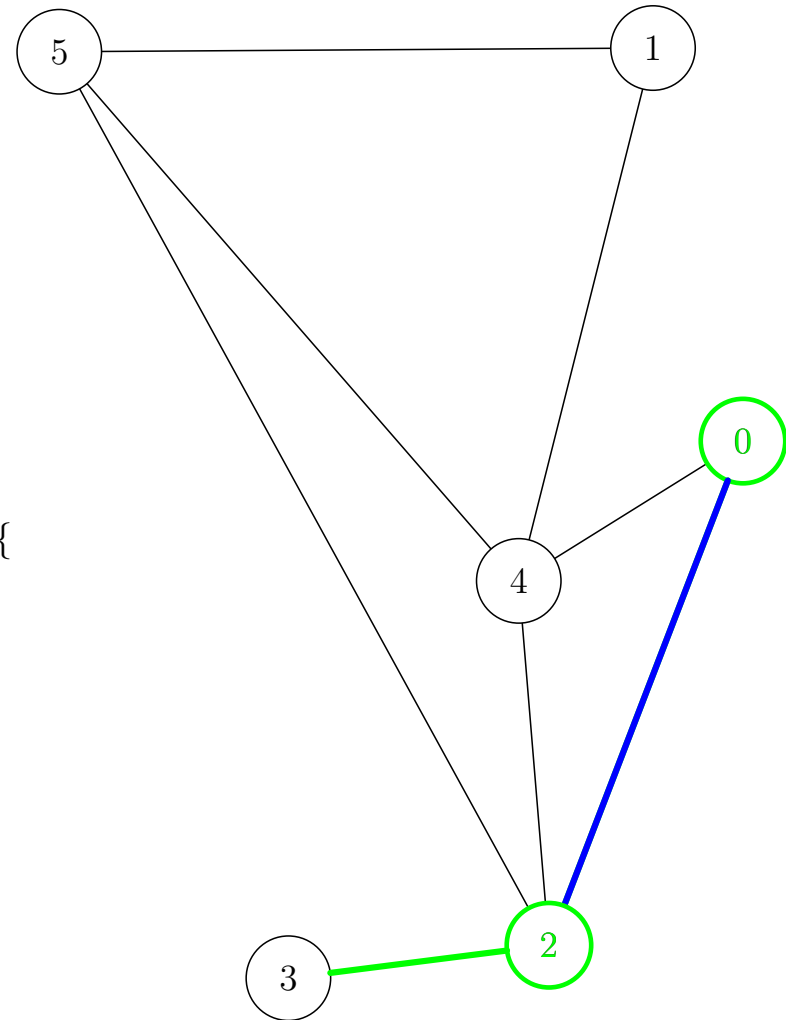
# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

      node=2     neighbour=3     time=2
```
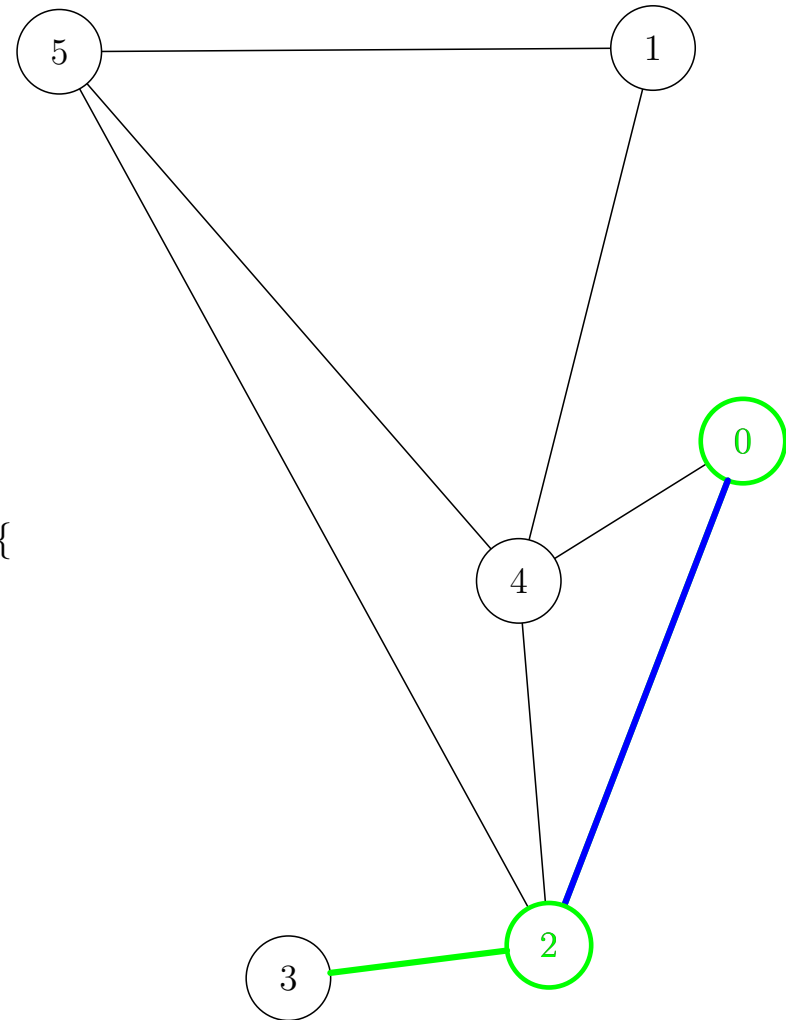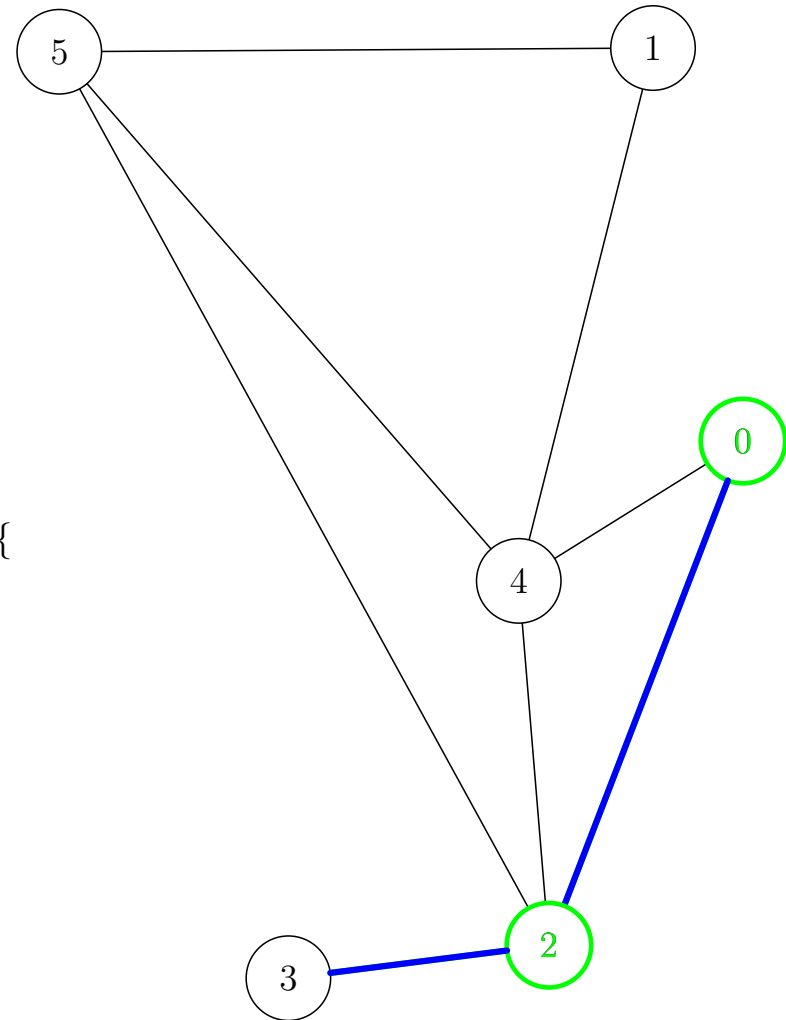
# Depth First Search

```
dfs(graph, node) {
    state ← Array[n, "undiscovered"]
    finished ← false
    dfs_recur(graph, node)
}

dfs_recur(graph, node) {
    if (finished) return
    state[node] ← "discovered"
    time ← time + 1
    processVertexEarly(node)
    foreach neighbour ∈ Neighbourhood(node) {
        if (state[neighbour] = "undiscovered") {
            parent[neighbour] ← node
            processEdge(node, neighbour)
            dfs_recur(graph, neighbour)
        } else if (state[neighbour] ≠ "processed") {
            processEdge(node, neighbour)
        }
        if (finished) return
    }
    processVertexLate(currentNode)
    state[currentNode] ← "processed"
    time ← time + 1
}

        node=2      neighbour=3      time=2
```
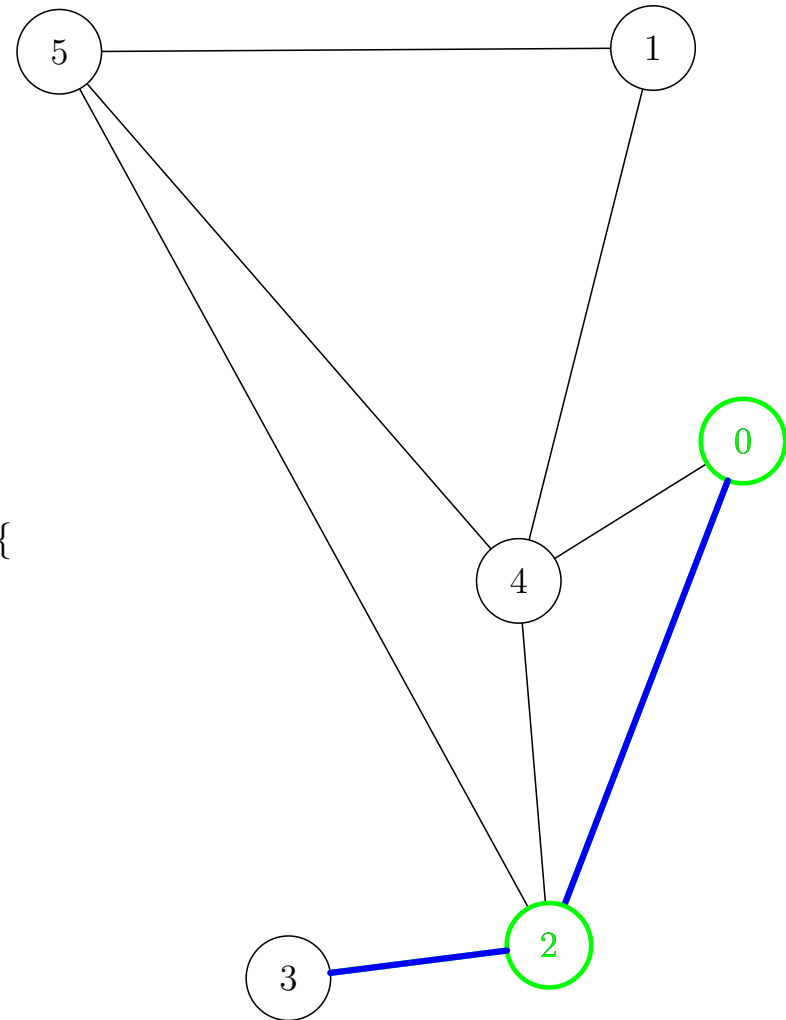
# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

      node=2      neighbour=3      time=2
```
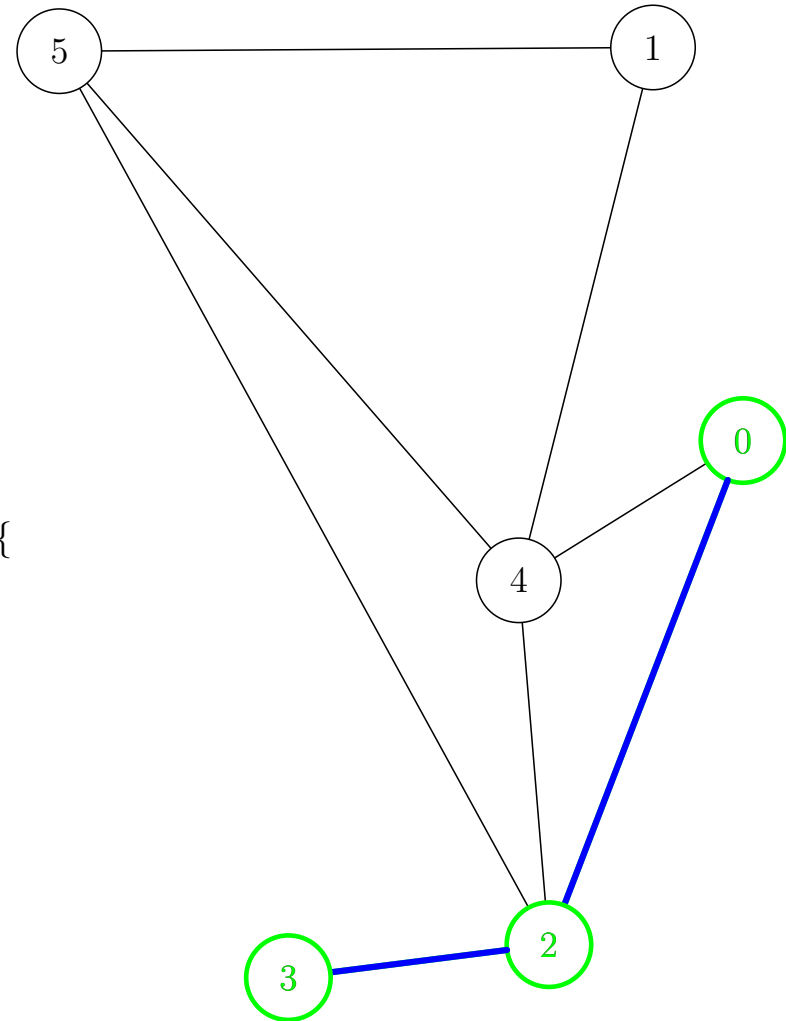
# Depth First Search

```
dfs(graph, node) {
    state ← Array[n, "undiscovered"]
    finished ← false
    dfs_recur(graph, node)
}

dfs_recur(graph, node) {
    if (finished) return
    state[node] ← "discovered"
    time ← time + 1
    processVertexEarly(node)
    foreach neighbour ∈ Neighbourhood(node) {
        if (state[neighbour] = "undiscovered") {
            parent[neighbour] ← node
            processEdge(node, neighbour)
            dfs_recur(graph, neighbour)
        } else if (state[neighbour] ≠ "processed") {
            processEdge(node, neighbour)
        }
        if (finished) return
    }
    processVertexLate(currentNode)
    state[currentNode] ← "processed"
    time ← time + 1
}

        node=2      neighbour=3      time=2
```
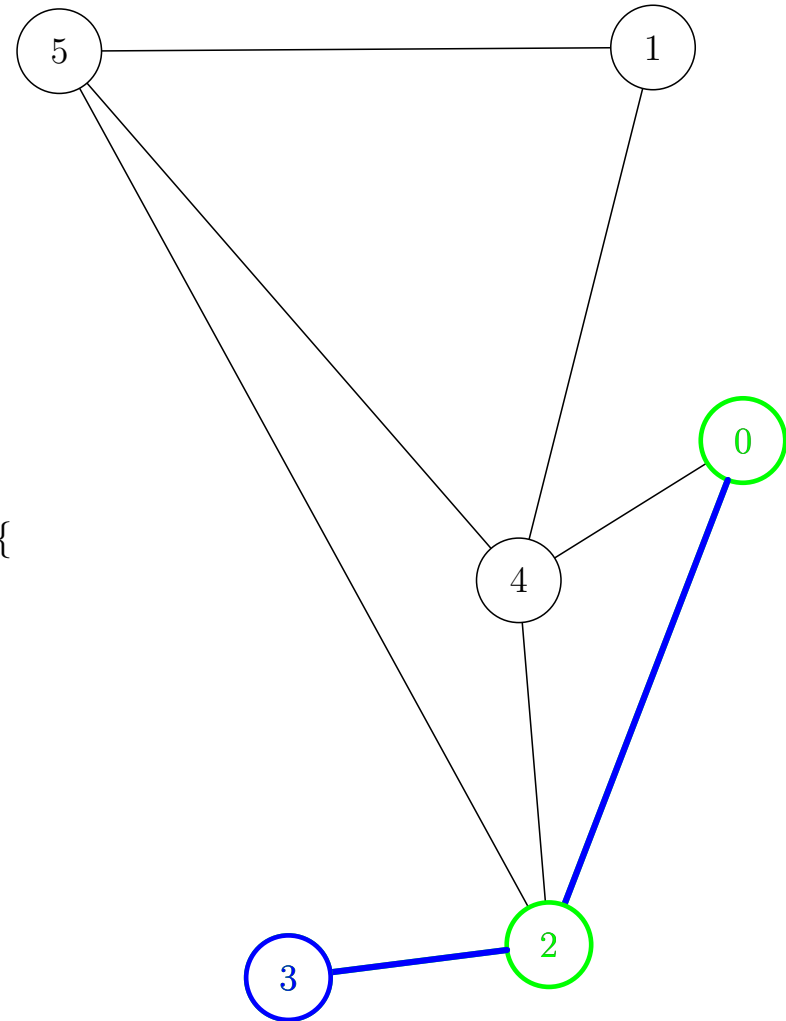
# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

      node=3     neighbour=nil  time=3
```
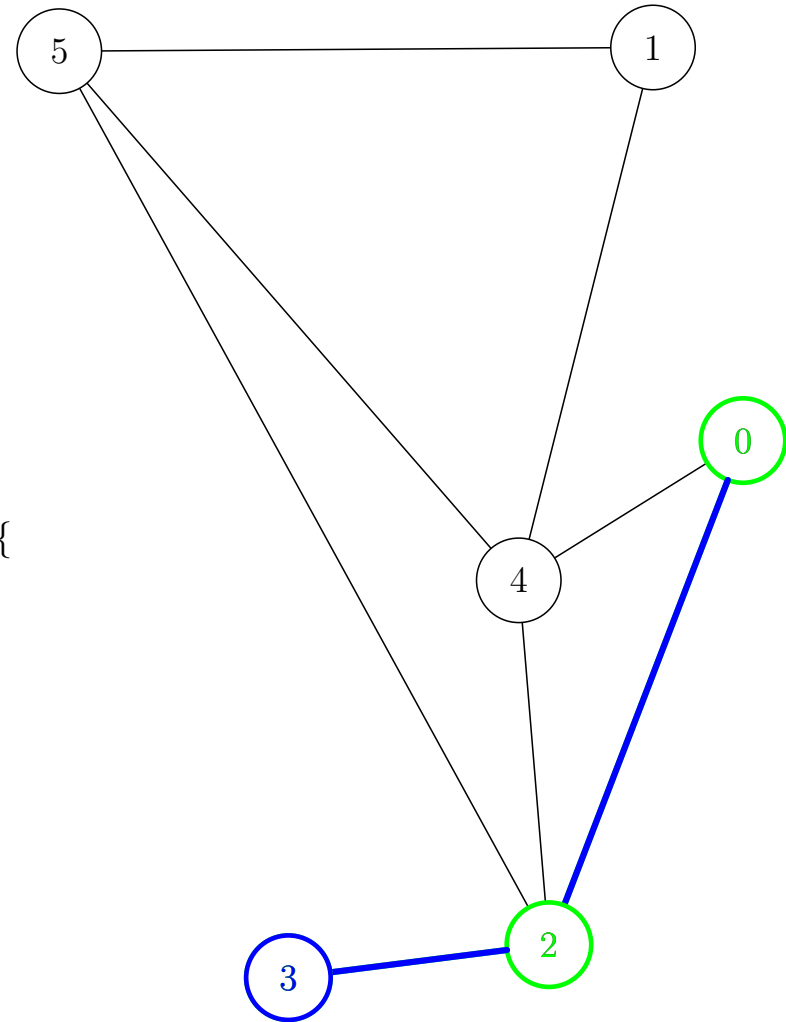
# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
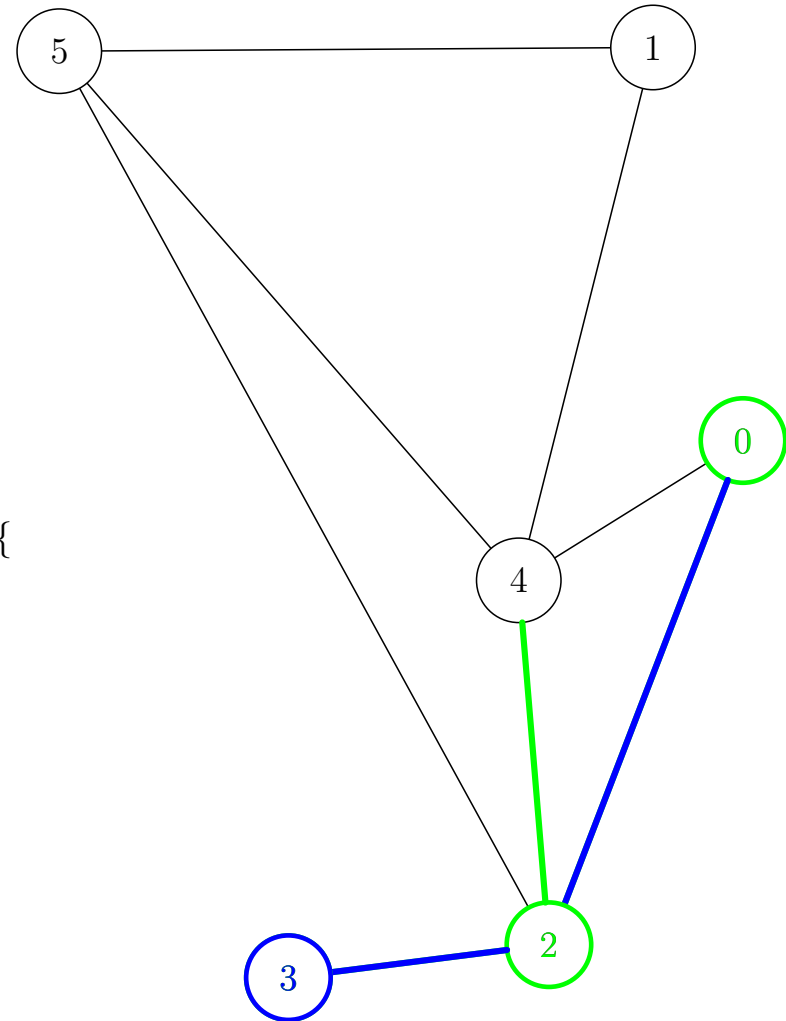
node=3     neighbour=nil    time=4

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
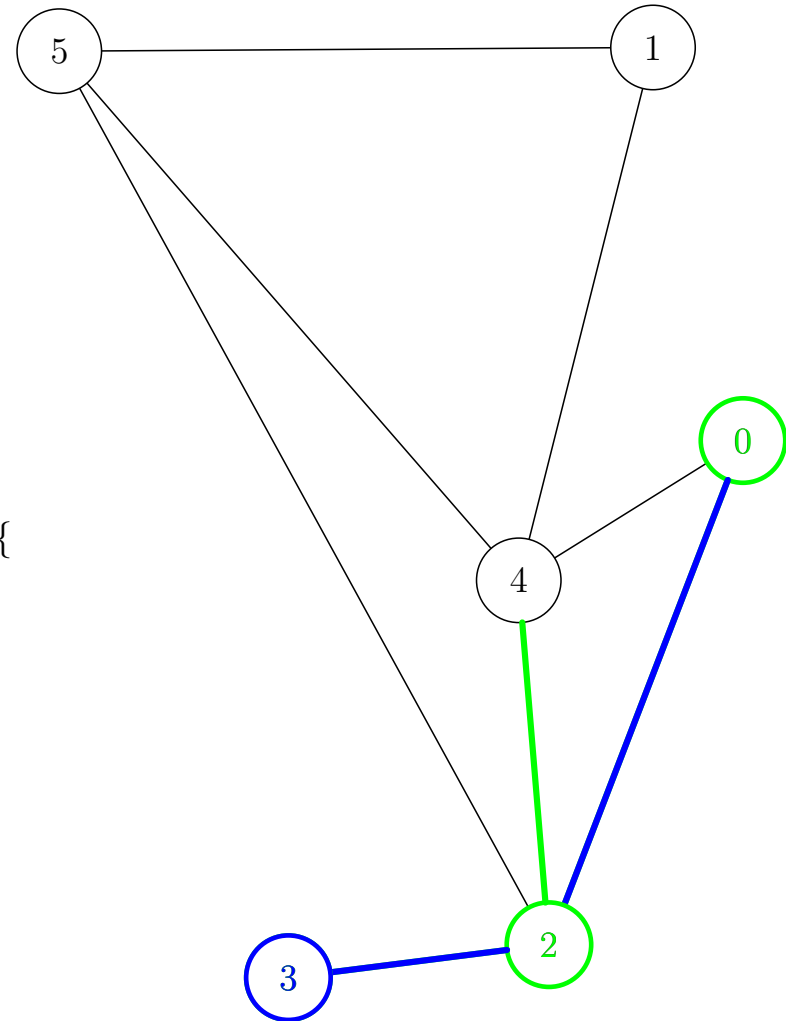
node=2        neighbour=3        time=4

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
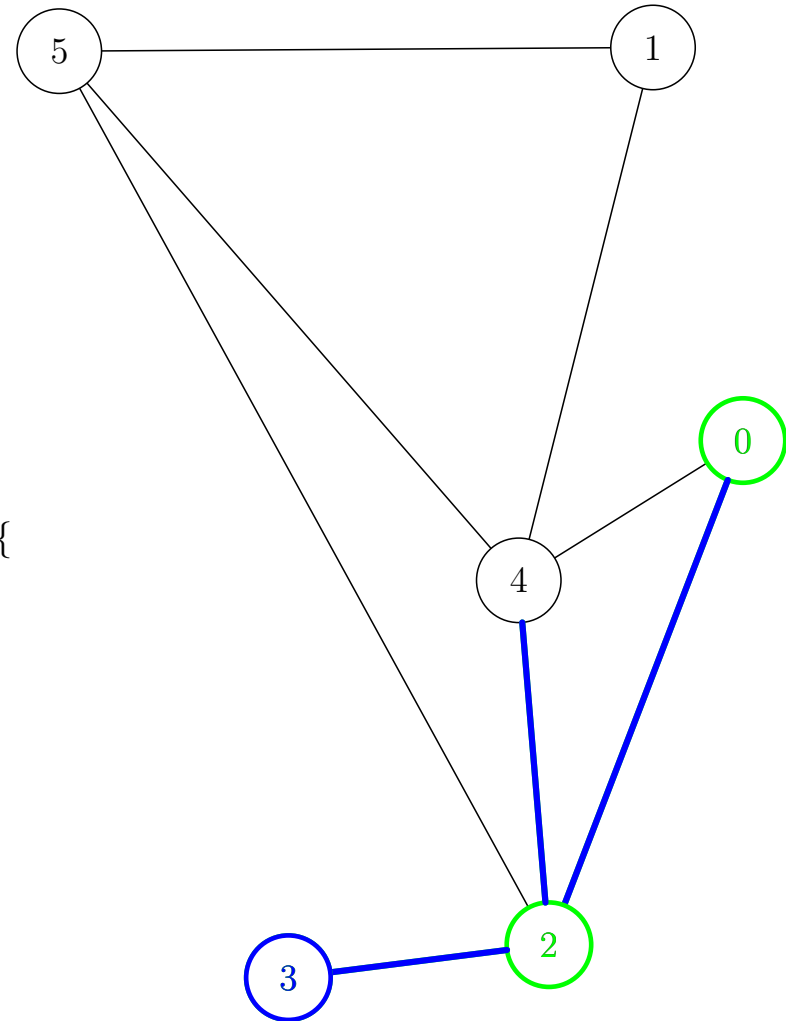
node=2     neighbour=4     time=4

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

    node=2      neighbour=4      time=4
```
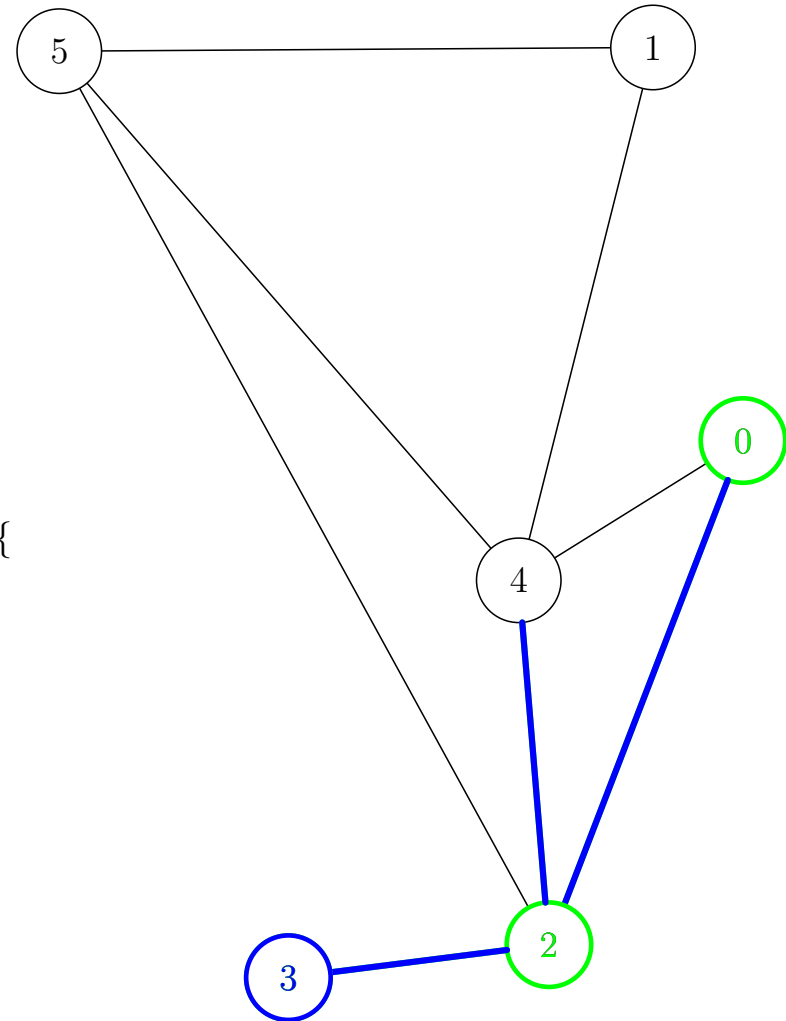
# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
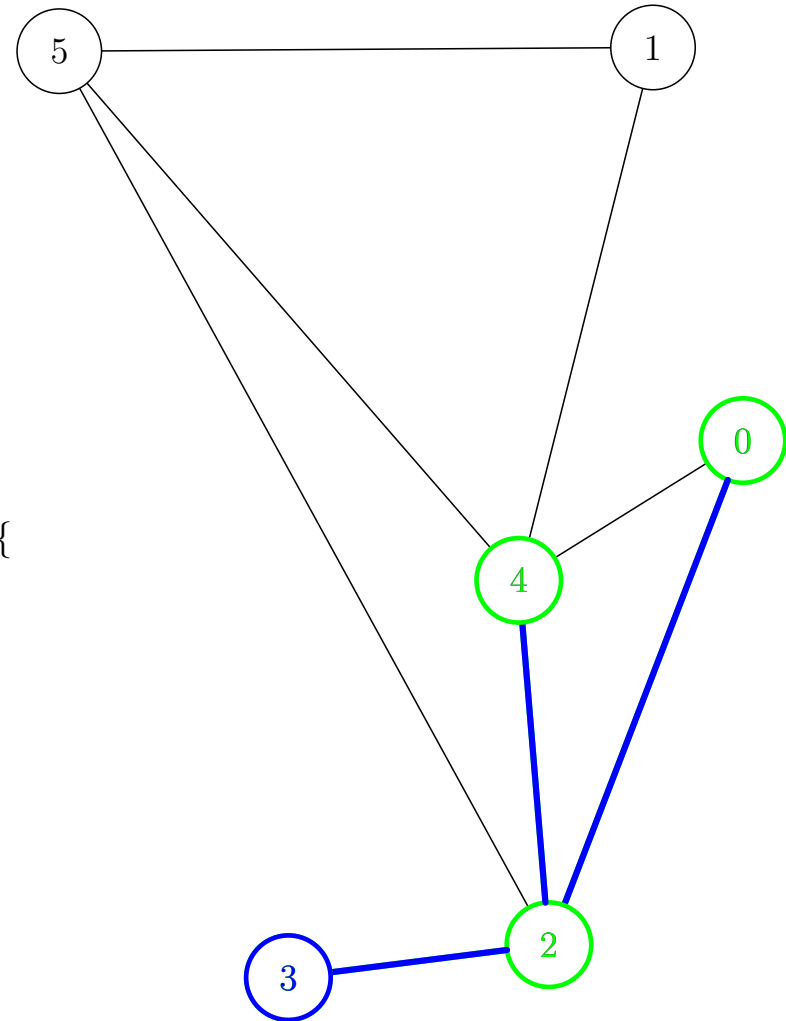
node=2        neighbour=4        time=4

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
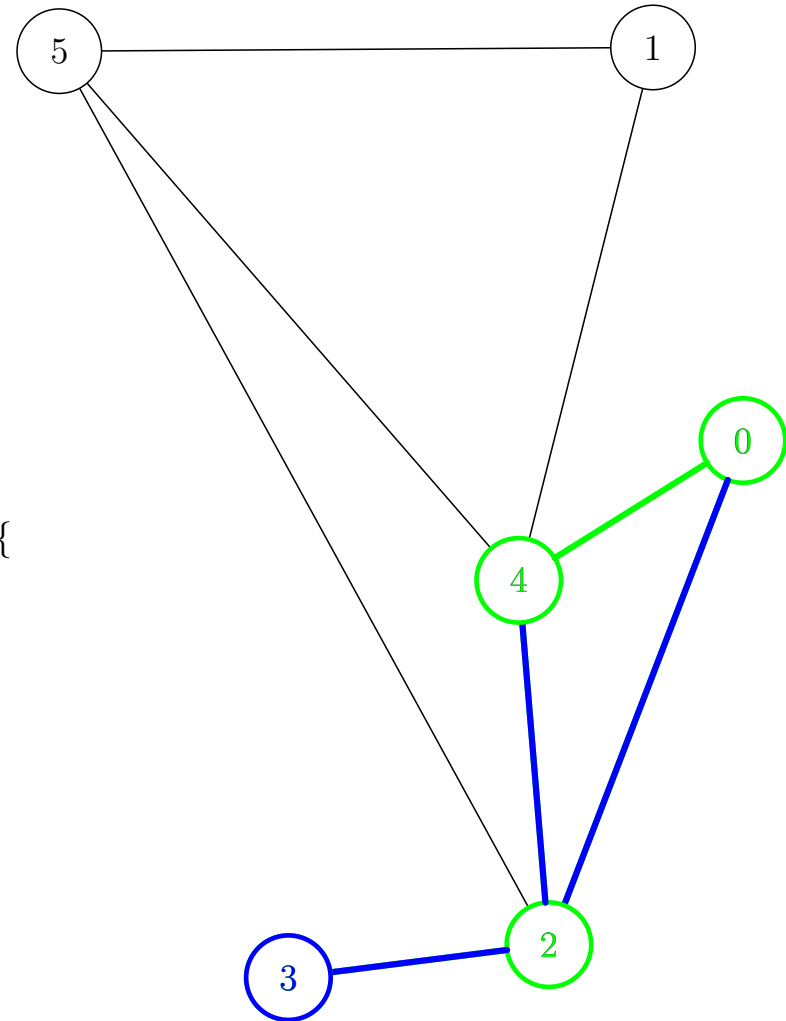
node=2        neighbour=4        time=4

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
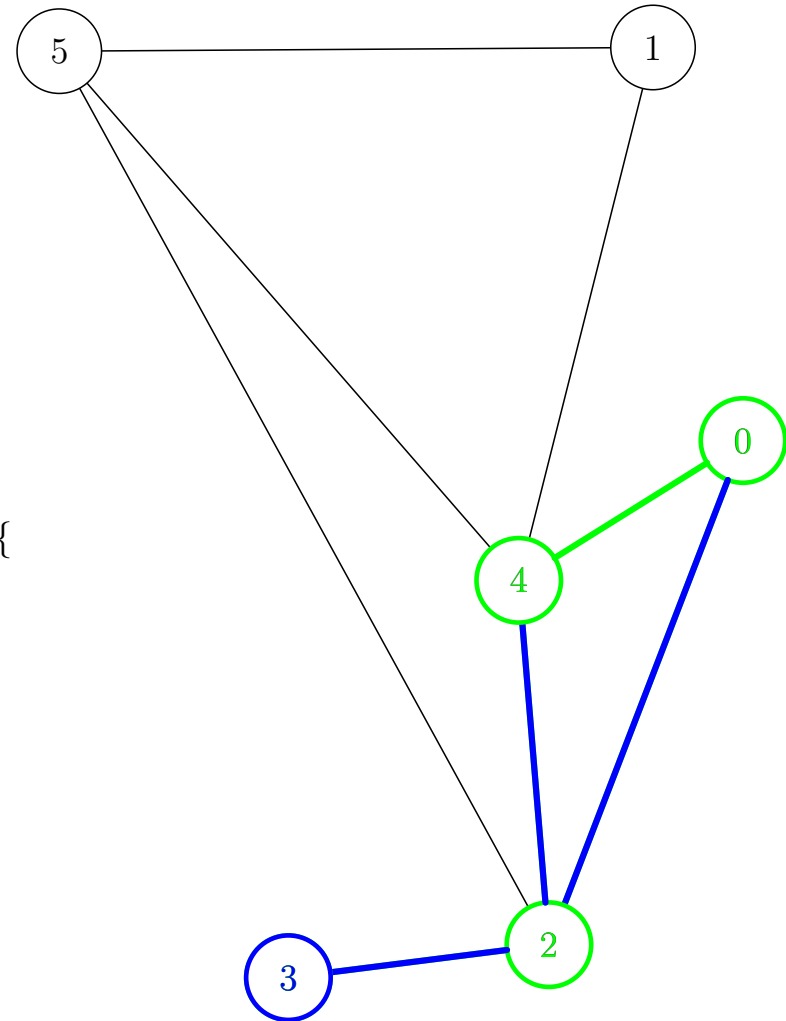
node=4    neighbour=nil   time=5

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

        node=4       neighbour=nil   time=5
```
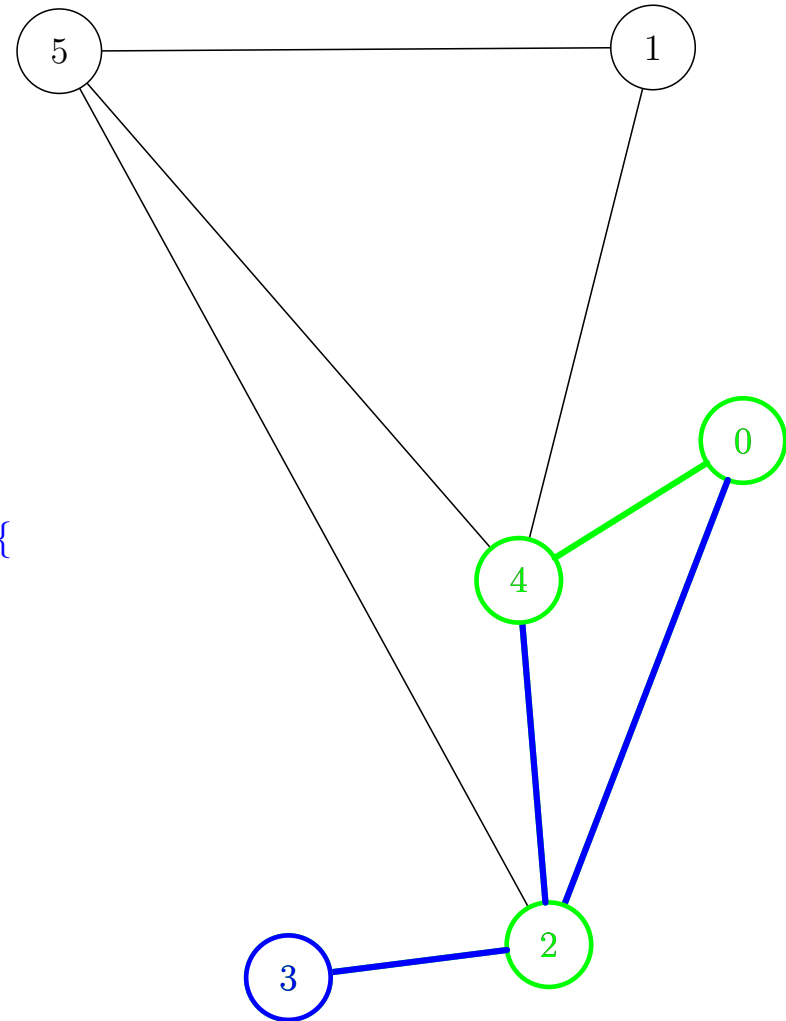
# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

      node=4      neighbour=nil   time=5
```
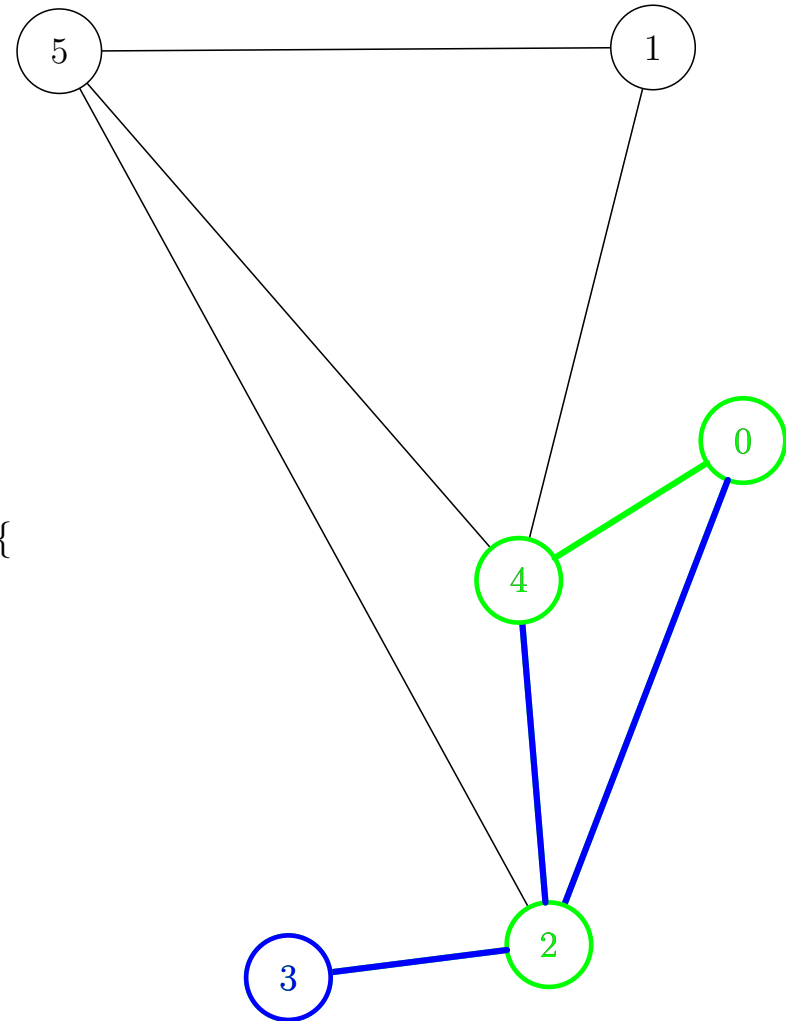
# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

    node=4      neighbour=nil   time=5
```
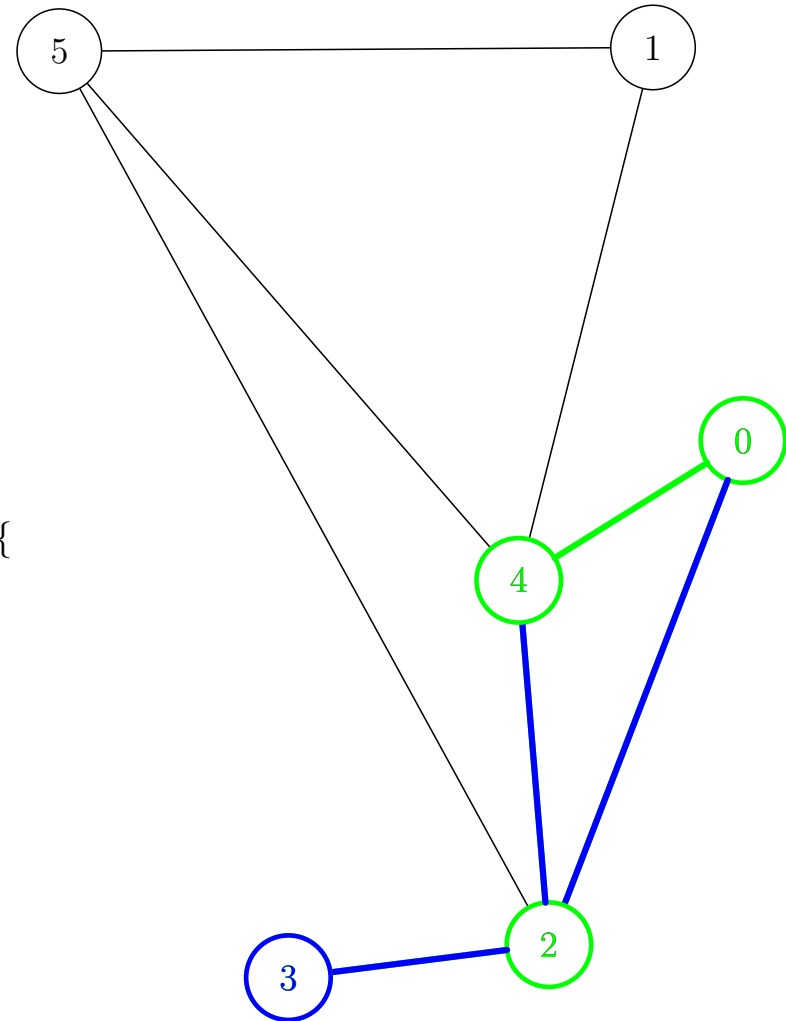
# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
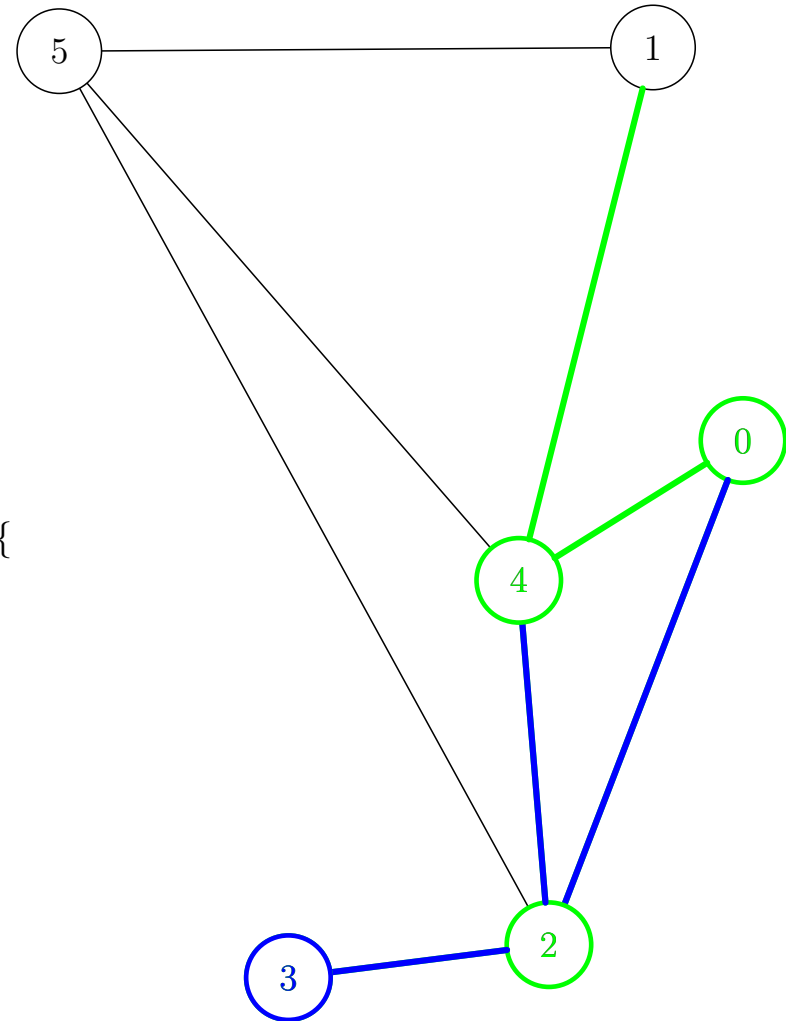
node=4      neighbour=nil   time=5

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

      node=4      neighbour=nil   time=5
```
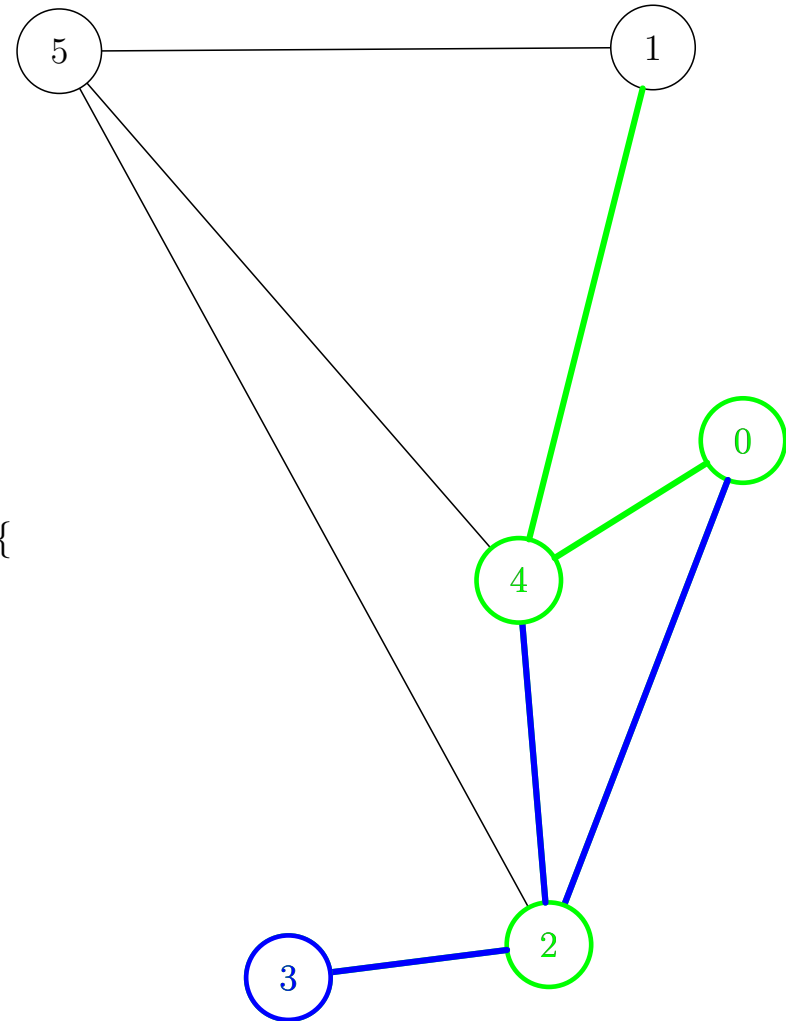
# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
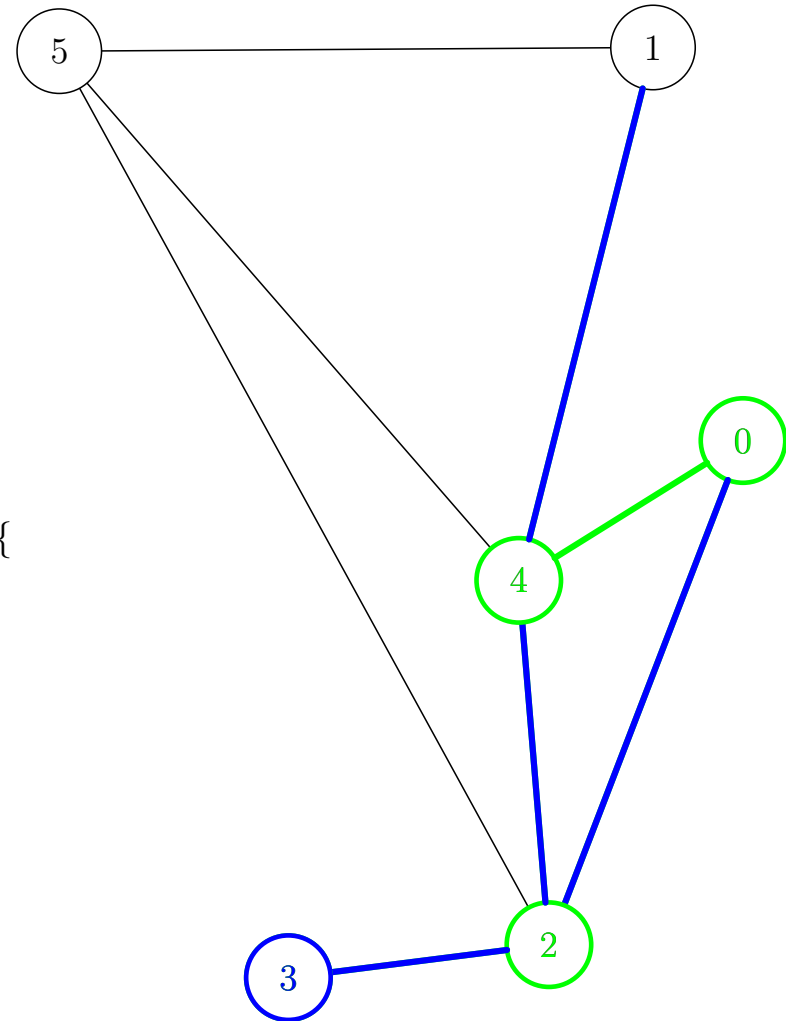
node=4        neighbour=1        time=5

# Depth First Search

```
dfs(graph, node) {
    state ← Array[n, "undiscovered"]
    finished ← false
    dfs_recur(graph, node)
}

dfs_recur(graph, node) {
    if (finished) return
    state[node] ← "discovered"
    time ← time + 1
    processVertexEarly(node)
    foreach neighbour ∈ Neighbourhood(node) {
        if (state[neighbour] = "undiscovered") {
            parent[neighbour] ← node
            processEdge(node, neighbour)
            dfs_recur(graph, neighbour)
        } else if (state[neighbour] ≠ "processed") {
            processEdge(node, neighbour)
        }
        if (finished) return
    }
    processVertexLate(currentNode)
    state[currentNode] ← "processed"
    time ← time + 1
}
```
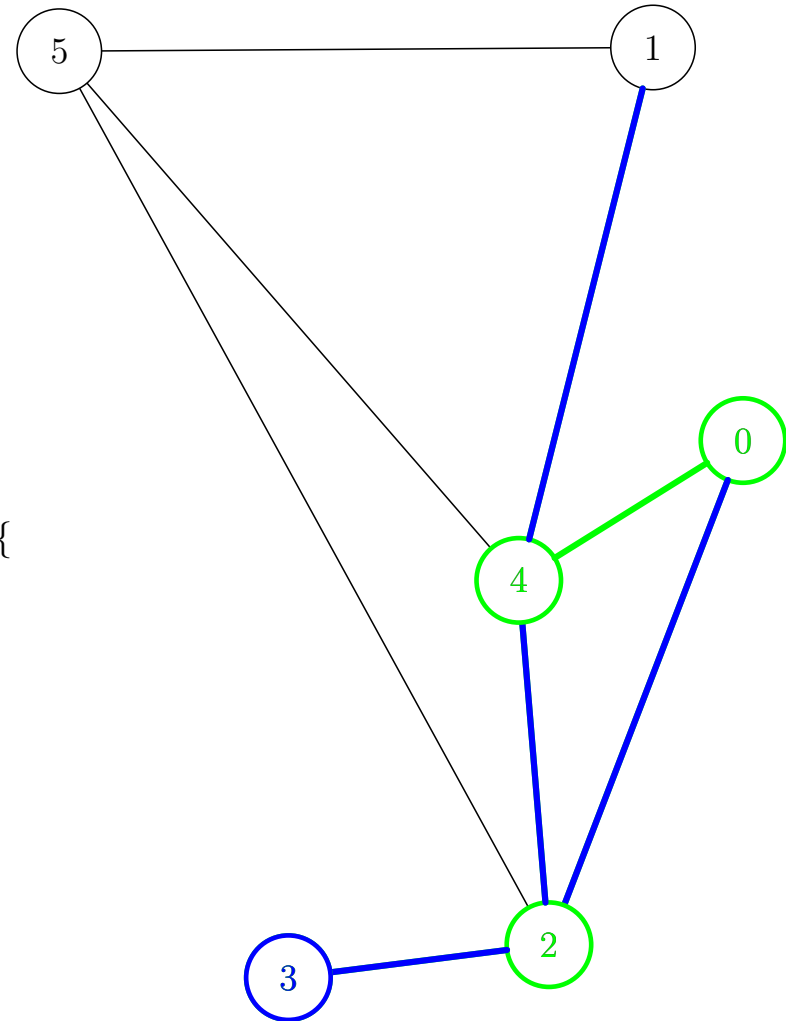
node=4        neighbour=1        time=5

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
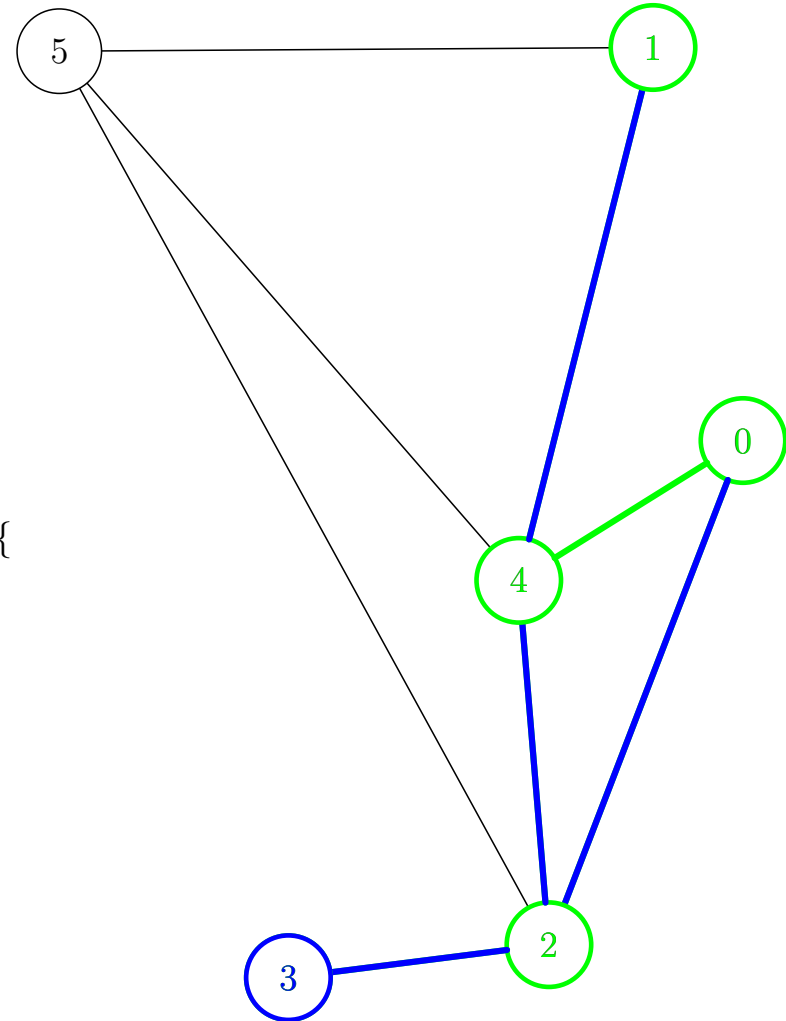
node=4        neighbour=1        time=5

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
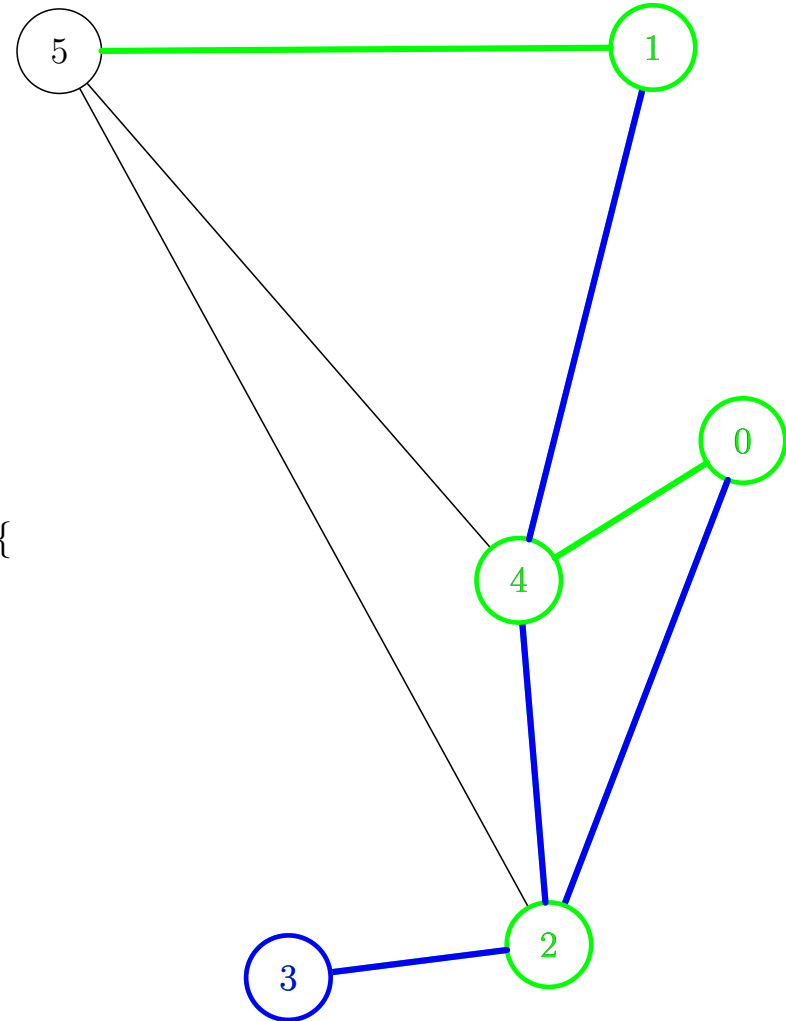
node=4     neighbour=1     time=5

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

    node=1      neighbour=nil   time=6
```
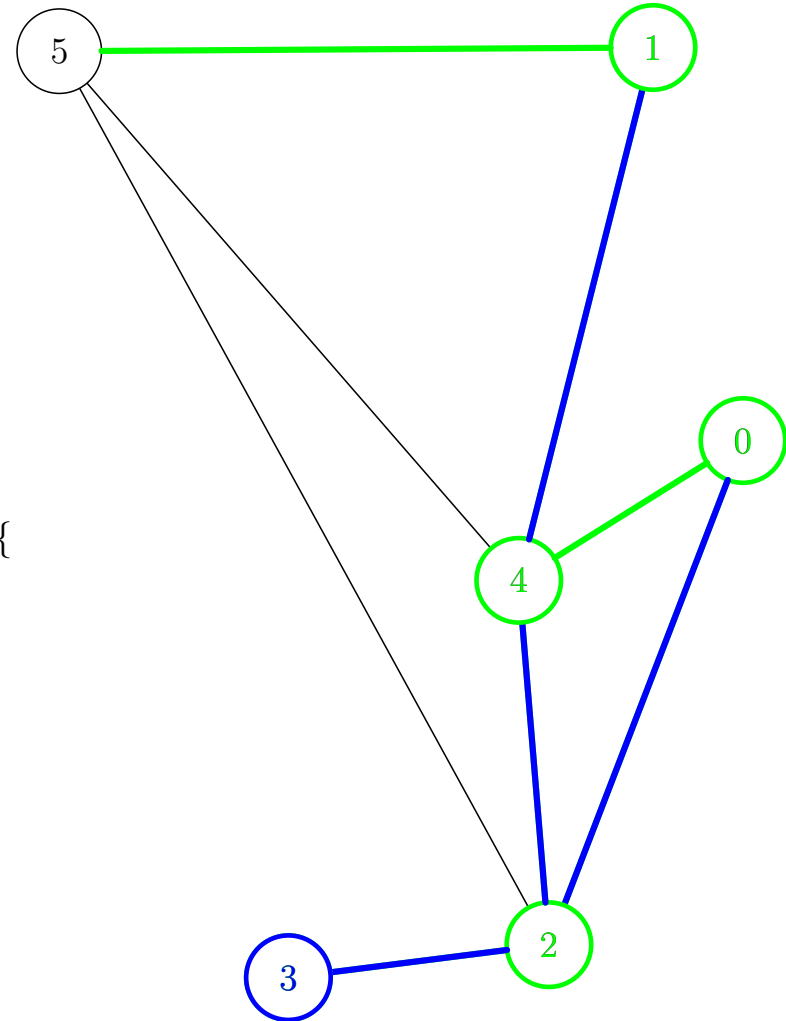
# Depth First Search

```
dfs(graph, node) {
   state ← Array[n, "undiscovered"]
   finished ← false
   dfs_recur(graph, node)
}

dfs_recur(graph, node) {
   if (finished) return
   state[node] ← "discovered"
   time ← time + 1
   processVertexEarly(node)
   foreach neighbour ∈ Neighbourhood(node) {
      if (state[neighbour] = "undiscovered") {
         parent[neighbour] ← node
         processEdge(node, neighbour)
         dfs_recur(graph, neighbour)
      } else if (state[neighbour] ≠ "processed") {
         processEdge(node, neighbour)
      }
      if (finished) return
   }
   processVertexLate(currentNode)
   state[currentNode] ← "processed"
   time ← time + 1
}
```
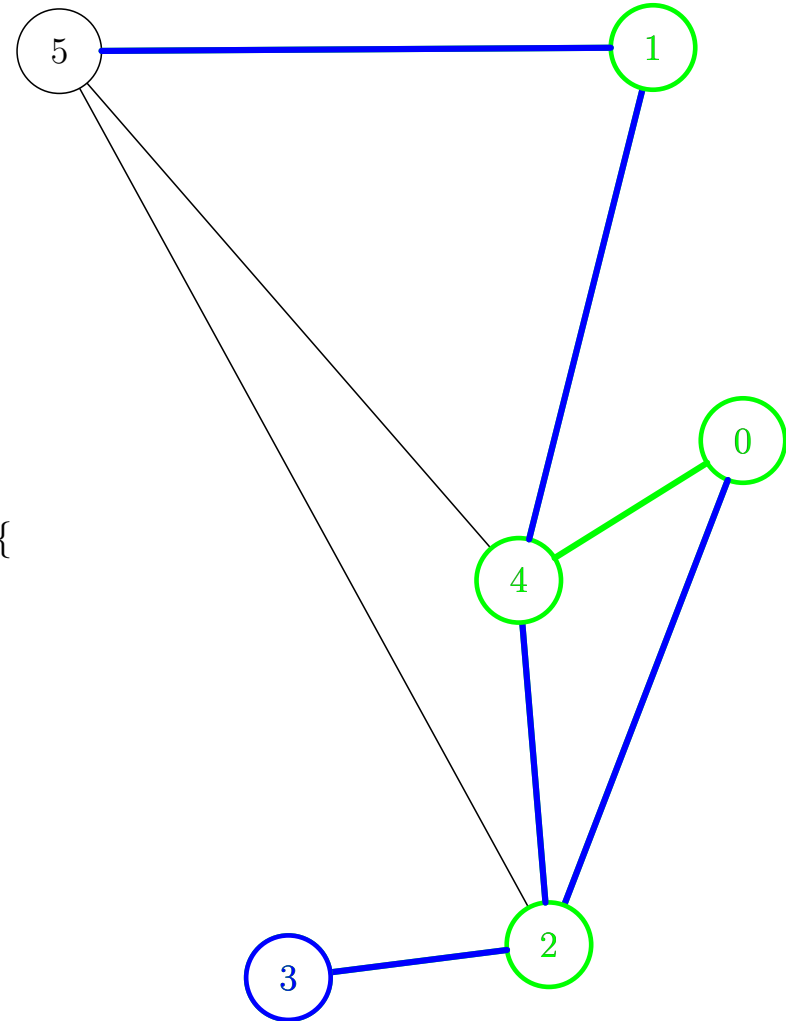
node=1        neighbour=5        time=6



---

# Depth First Search

```
dfs(graph, node) {
    state ← Array[n, "undiscovered"]
    finished ← false
    dfs_recur(graph, node)
}

dfs_recur(graph, node) {
    if (finished) return
    state[node] ← "discovered"
    time ← time + 1
    processVertexEarly(node)
    foreach neighbour ∈ Neighbourhood(node) {
        if (state[neighbour] = "undiscovered") {
            parent[neighbour] ← node
            processEdge(node, neighbour)
            dfs_recur(graph, neighbour)
        } else if (state[neighbour] ≠ "processed") {
            processEdge(node, neighbour)
        }
        if (finished) return
    }
    processVertexLate(currentNode)
    state[currentNode] ← "processed"
    time ← time + 1
}
```
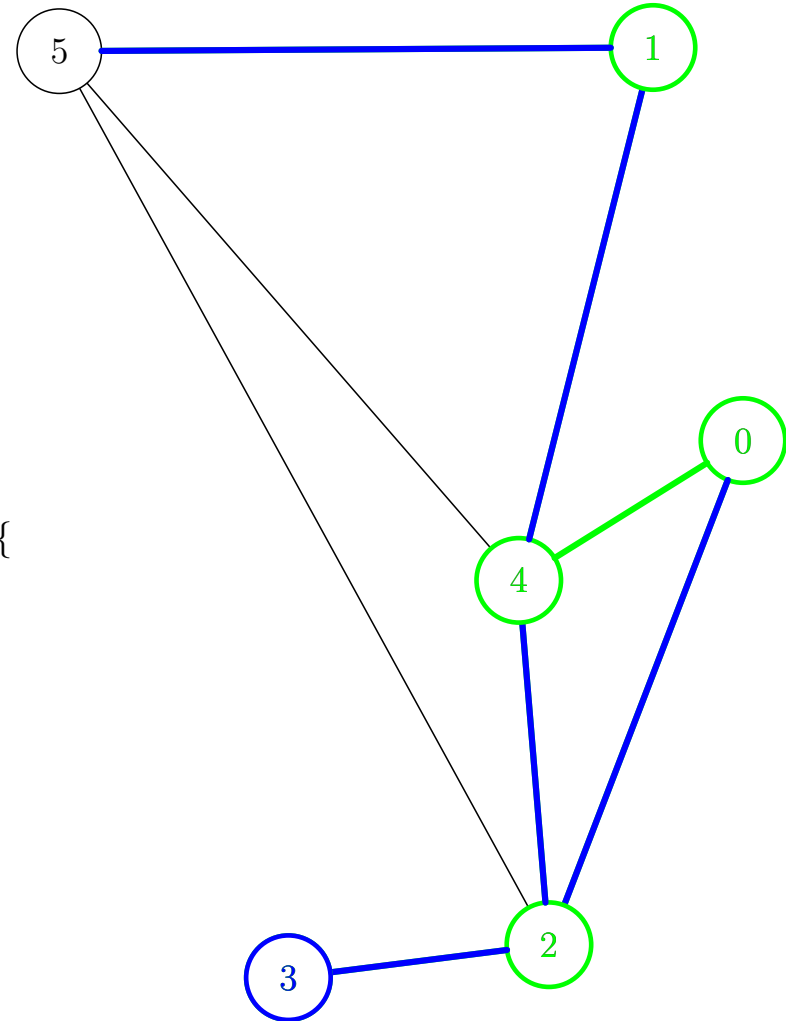
node=1        neighbour=5        time=6

# Depth First Search

```
dfs(graph, node) {
    state ← Array[n, "undiscovered"]
    finished ← false
    dfs_recur(graph, node)
}

dfs_recur(graph, node) {
    if (finished) return
    state[node] ← "discovered"
    time ← time + 1
    processVertexEarly(node)
    foreach neighbour ∈ Neighbourhood(node) {
        if (state[neighbour] = "undiscovered") {
            parent[neighbour] ← node
            processEdge(node, neighbour)
            dfs_recur(graph, neighbour)
        } else if (state[neighbour] ≠ "processed") {
            processEdge(node, neighbour)
        }
        if (finished) return
    }
    processVertexLate(currentNode)
    state[currentNode] ← "processed"
    time ← time + 1
}
```
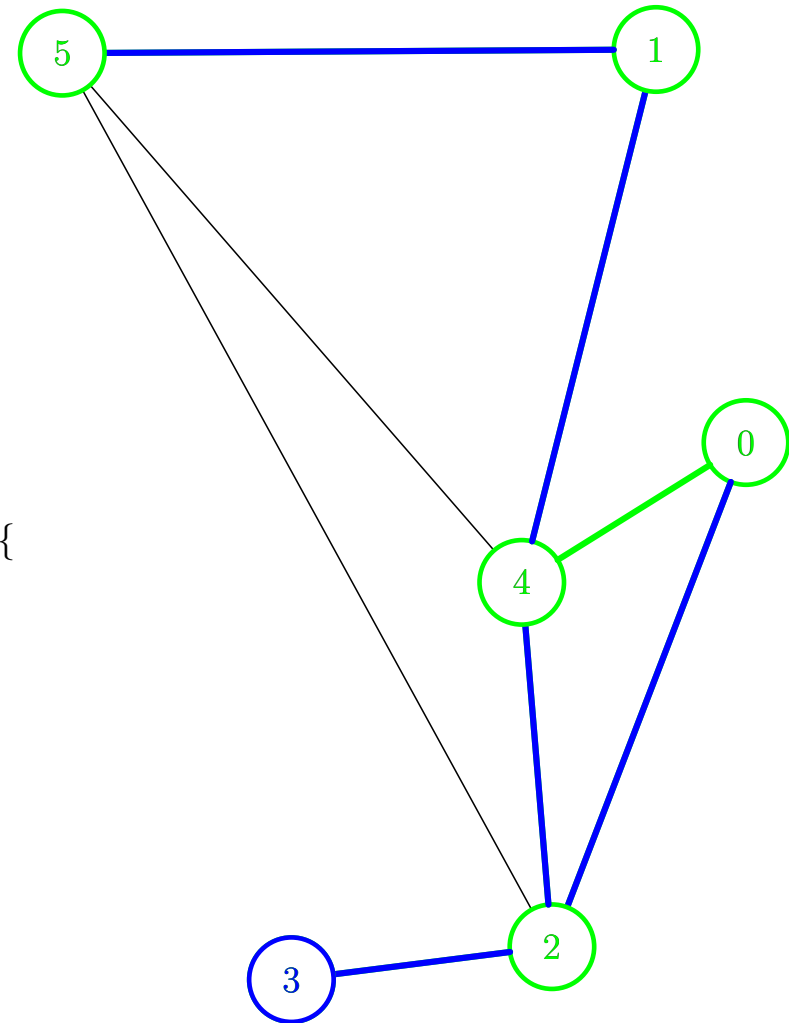
node=1        neighbour=5        time=6

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
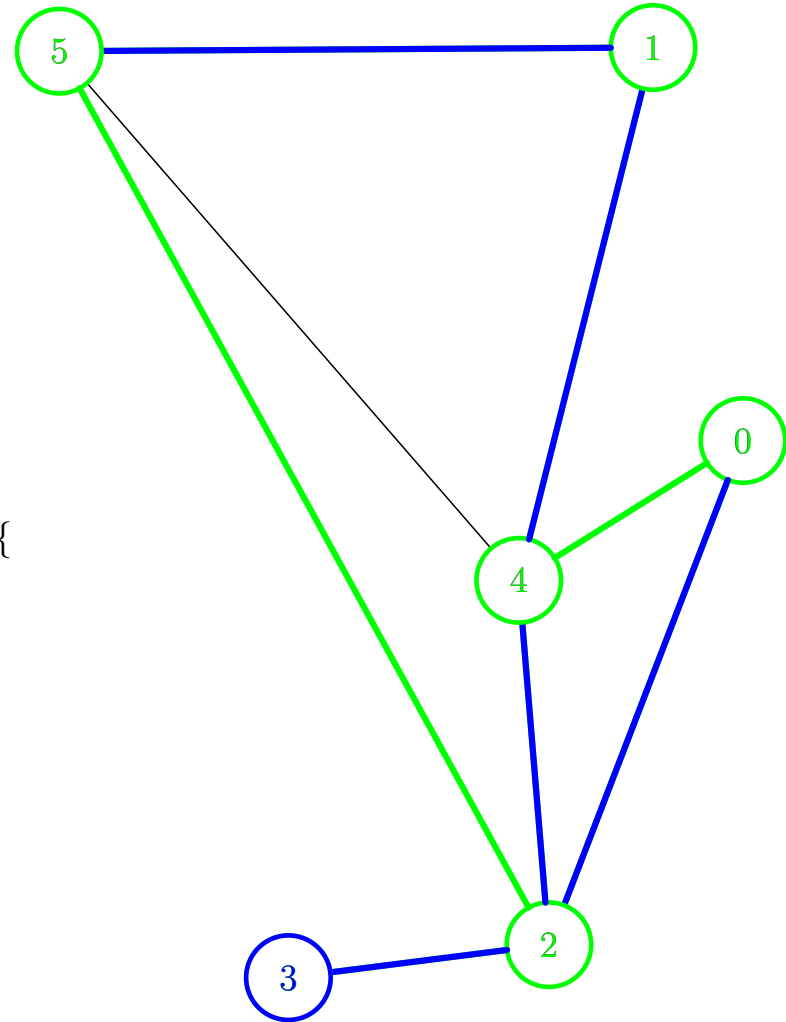
node=1        neighbour=5        time=6

# Depth First Search

```
dfs(graph, node) {
    state ← Array[n, "undiscovered"]
    finished ← false
    dfs_recur(graph, node)
}

dfs_recur(graph, node) {
    if (finished) return
    state[node] ← "discovered"
    time ← time + 1
    processVertexEarly(node)
    foreach neighbour ∈ Neighbourhood(node) {
        if (state[neighbour] = "undiscovered") {
            parent[neighbour] ← node
            processEdge(node, neighbour)
            dfs_recur(graph, neighbour)
        } else if (state[neighbour] ≠ "processed") {
            processEdge(node, neighbour)
        }
        if (finished) return
    }
    processVertexLate(currentNode)
    state[currentNode] ← "processed"
    time ← time + 1
}

        node=5      neighbour=nil   time=7
```
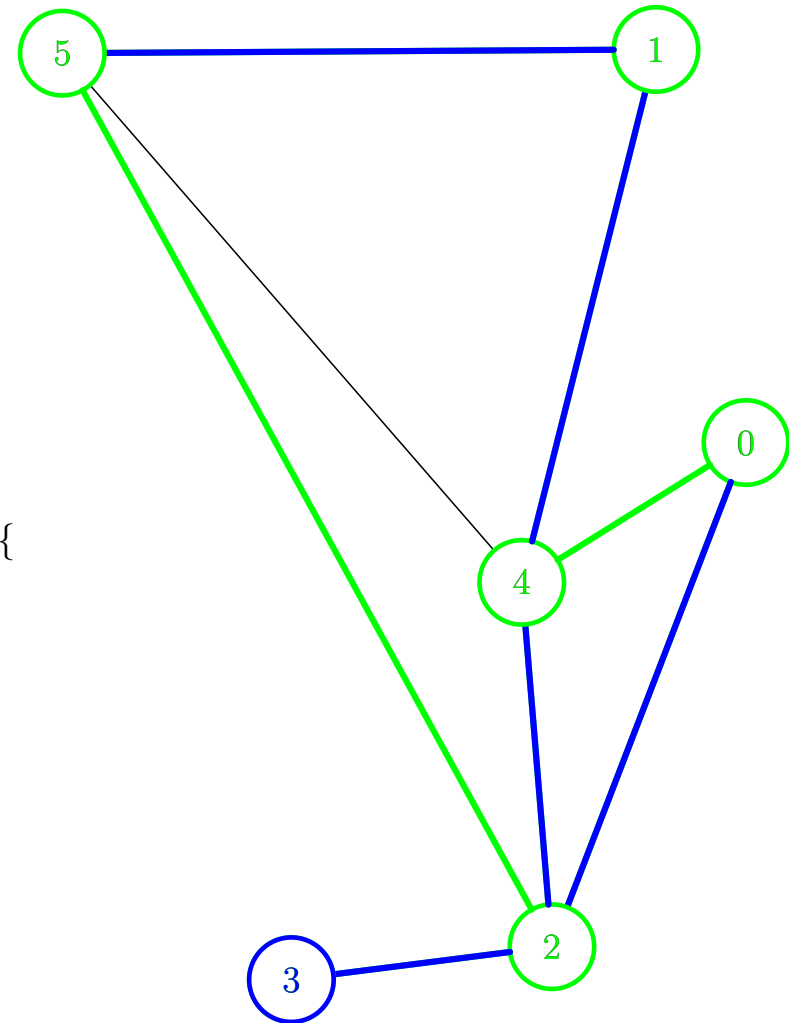
# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

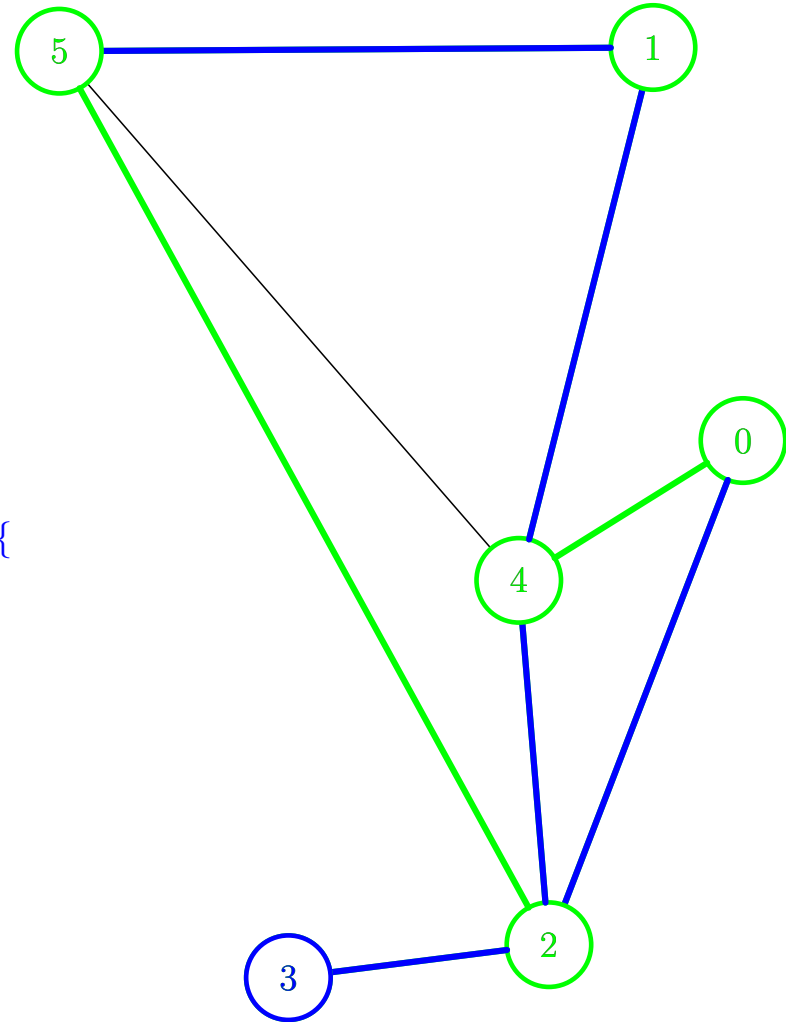node=5      neighbour=2      time=7

# Depth First Search

```
dfs(graph, node) {
   state ← Array[n, "undiscovered"]
   finished ← false
   dfs_recur(graph, node)
}

dfs_recur(graph, node) {
   if (finished) return
   state[node] ← "discovered"
   time ← time + 1
   processVertexEarly(node)
   foreach neighbour ∈ Neighbourhood(node) {
      if (state[neighbour] = "undiscovered") {
         parent[neighbour] ← node
         processEdge(node, neighbour)
         dfs_recur(graph, neighbour)
      } else if (state[neighbour] ≠ "processed") {
         processEdge(node, neighbour)
      }
      if (finished) return
   }
   processVertexLate(currentNode)
   state[currentNode] ← "processed"
   time ← time + 1
}
```
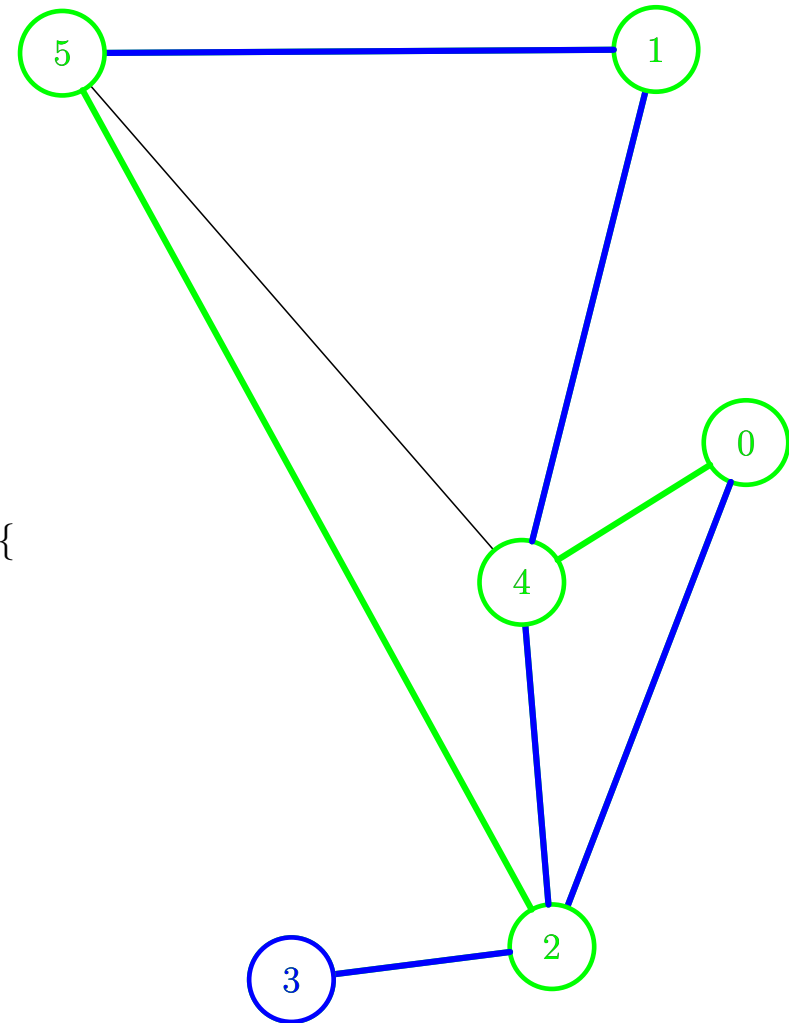
node=5        neighbour=2        time=7

# Depth First Search

```
dfs(graph, node) {
    state ← Array[n, "undiscovered"]
    finished ← false
    dfs_recur(graph, node)
}

dfs_recur(graph, node) {
    if (finished) return
    state[node] ← "discovered"
    time ← time + 1
    processVertexEarly(node)
    foreach neighbour ∈ Neighbourhood(node) {
        if (state[neighbour] = "undiscovered") {
            parent[neighbour] ← node
            processEdge(node, neighbour)
            dfs_recur(graph, neighbour)
        } else if (state[neighbour] ≠ "processed") {
            processEdge(node, neighbour)
        }
        if (finished) return
    }
    processVertexLate(currentNode)
    state[currentNode] ← "processed"
    time ← time + 1
}
```
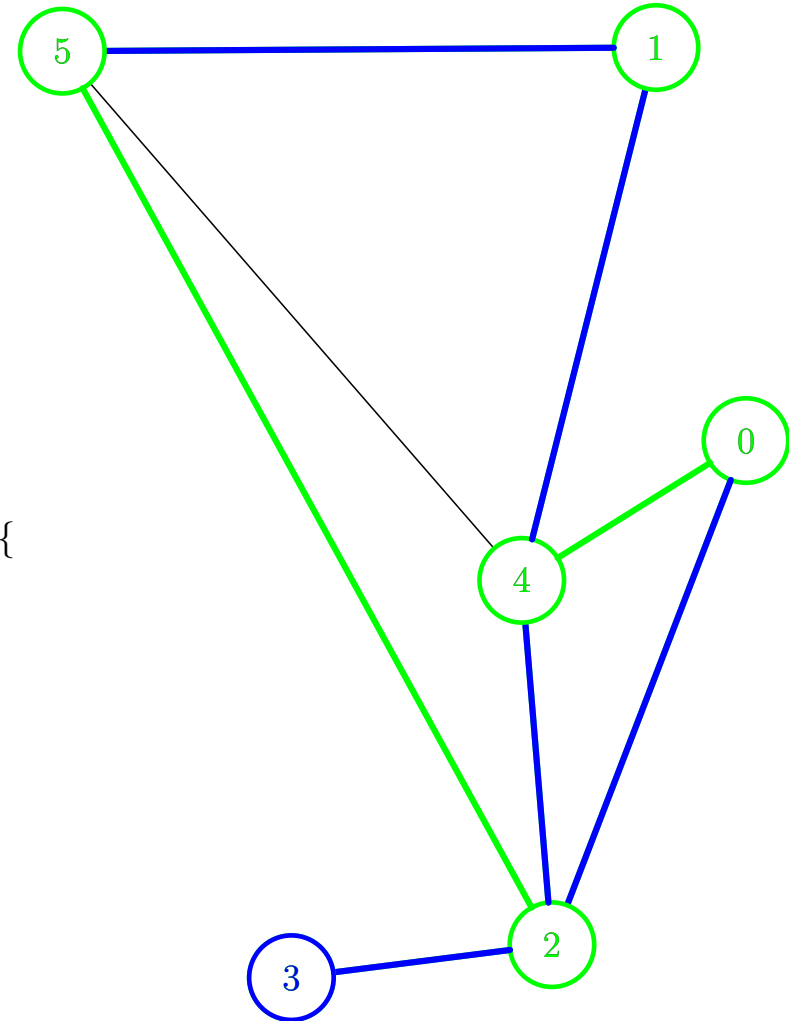
node=5        neighbour=2        time=7

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
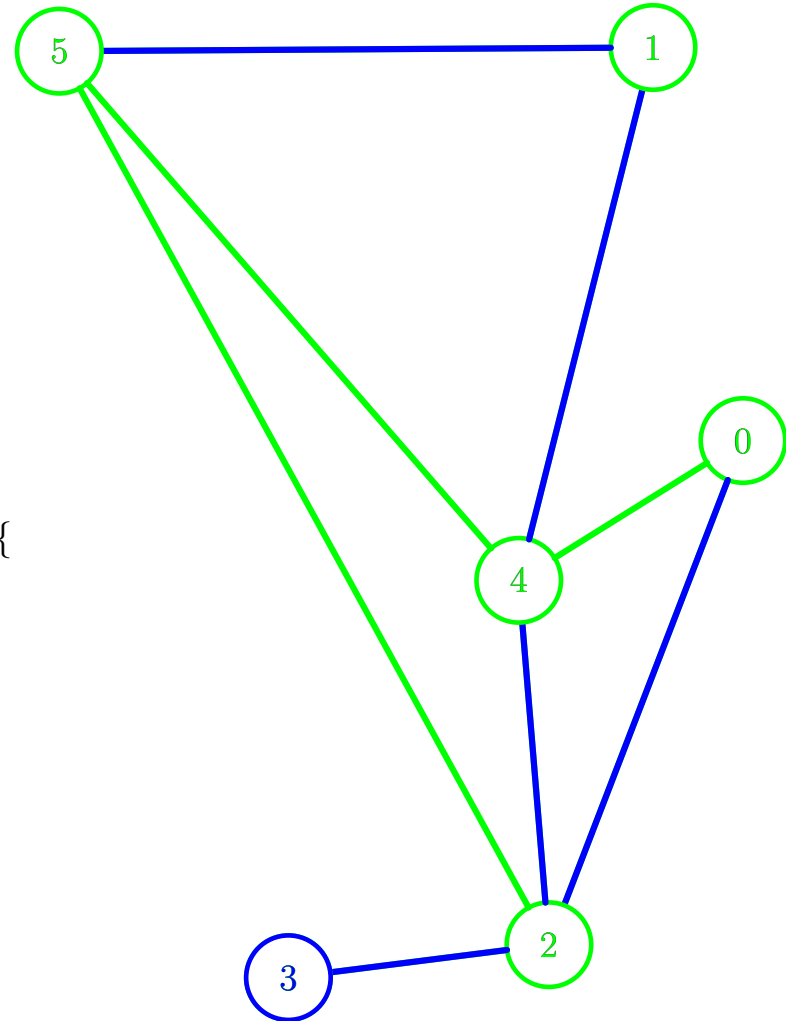
node=5      neighbour=2      time=7

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

     node=5      neighbour=2     time=7
```
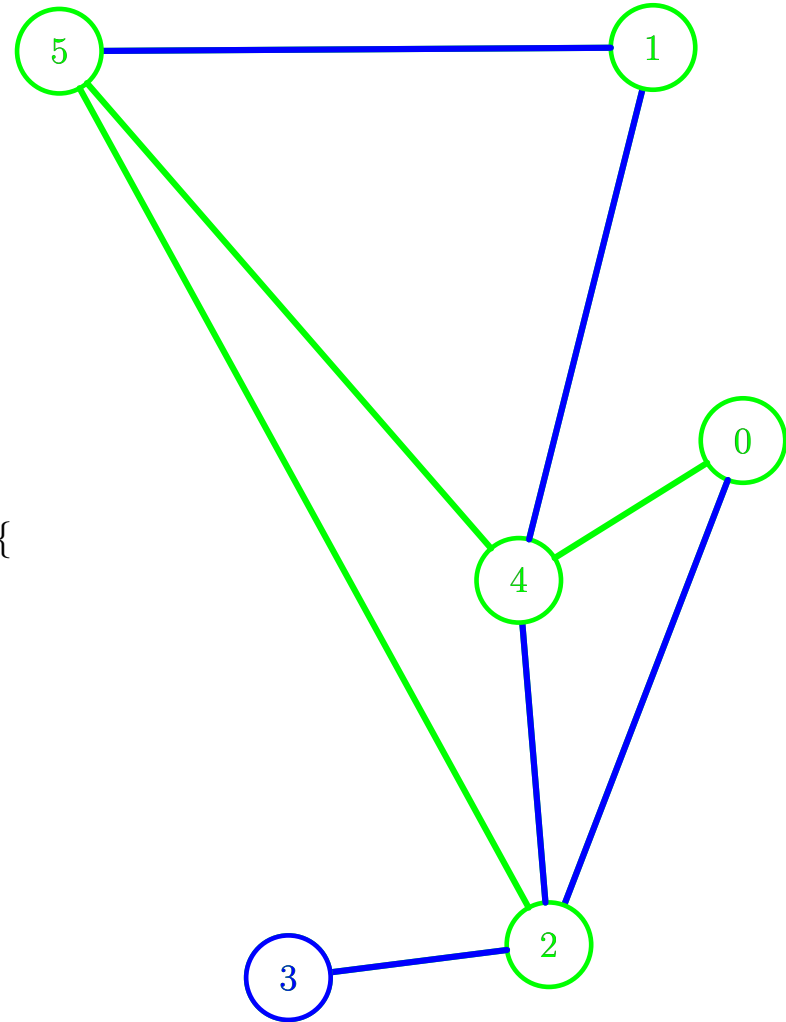
# Depth First Search

```
dfs(graph, node) {
   state ← Array[n, "undiscovered"]
   finished ← false
   dfs_recur(graph, node)
}

dfs_recur(graph, node) {
   if (finished) return
   state[node] ← "discovered"
   time ← time + 1
   processVertexEarly(node)
   foreach neighbour ∈ Neighbourhood(node) {
      if (state[neighbour] = "undiscovered") {
         parent[neighbour] ← node
         processEdge(node, neighbour)
         dfs_recur(graph, neighbour)
      } else if (state[neighbour] ≠ "processed") {
         processEdge(node, neighbour)
      }
      if (finished) return
   }
   processVertexLate(currentNode)
   state[currentNode] ← "processed"
   time ← time + 1
}
```
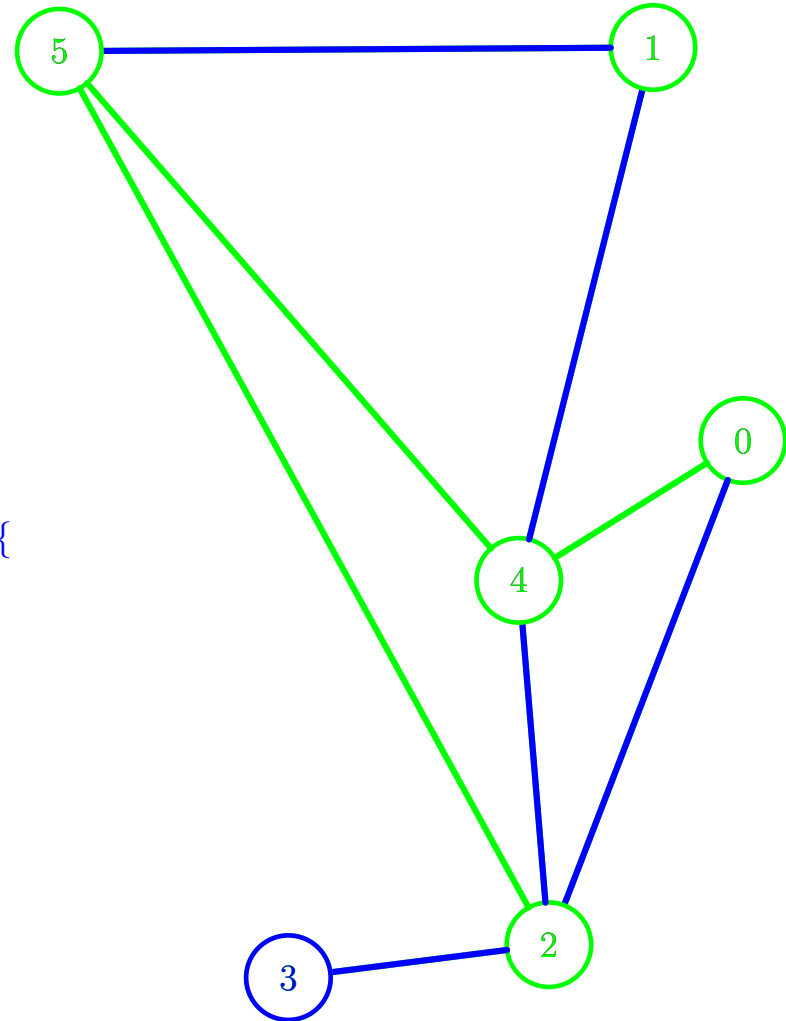
node=5        neighbour=4        time=7

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
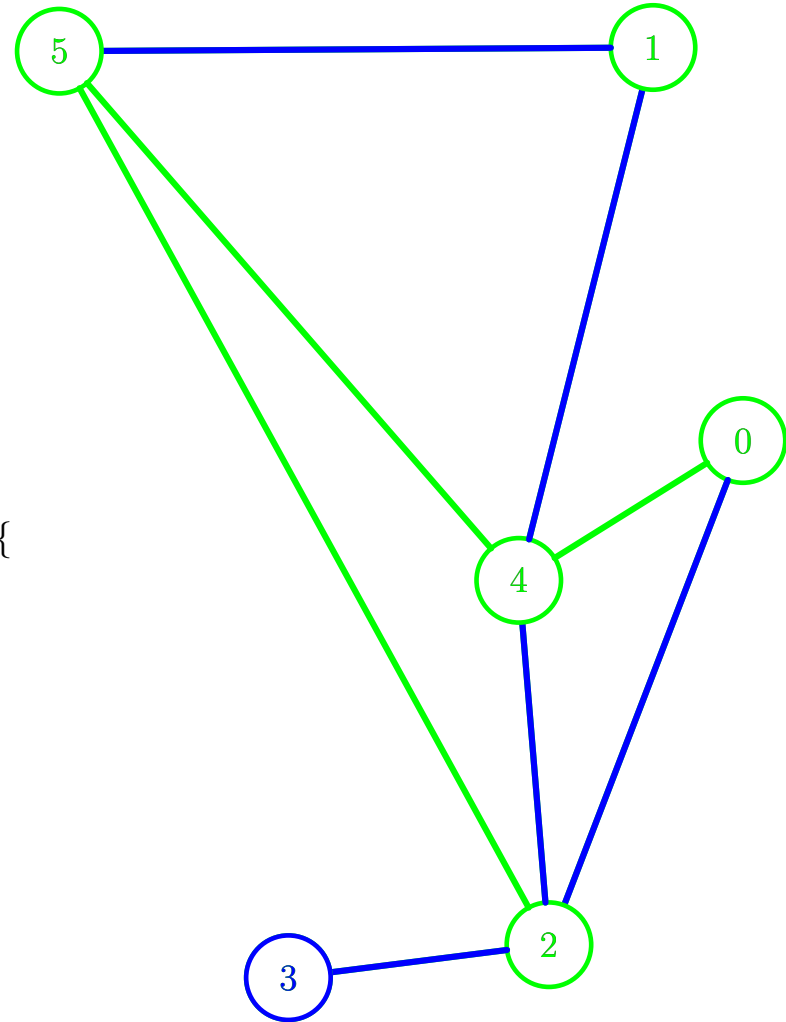
node=5      neighbour=4      time=7

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
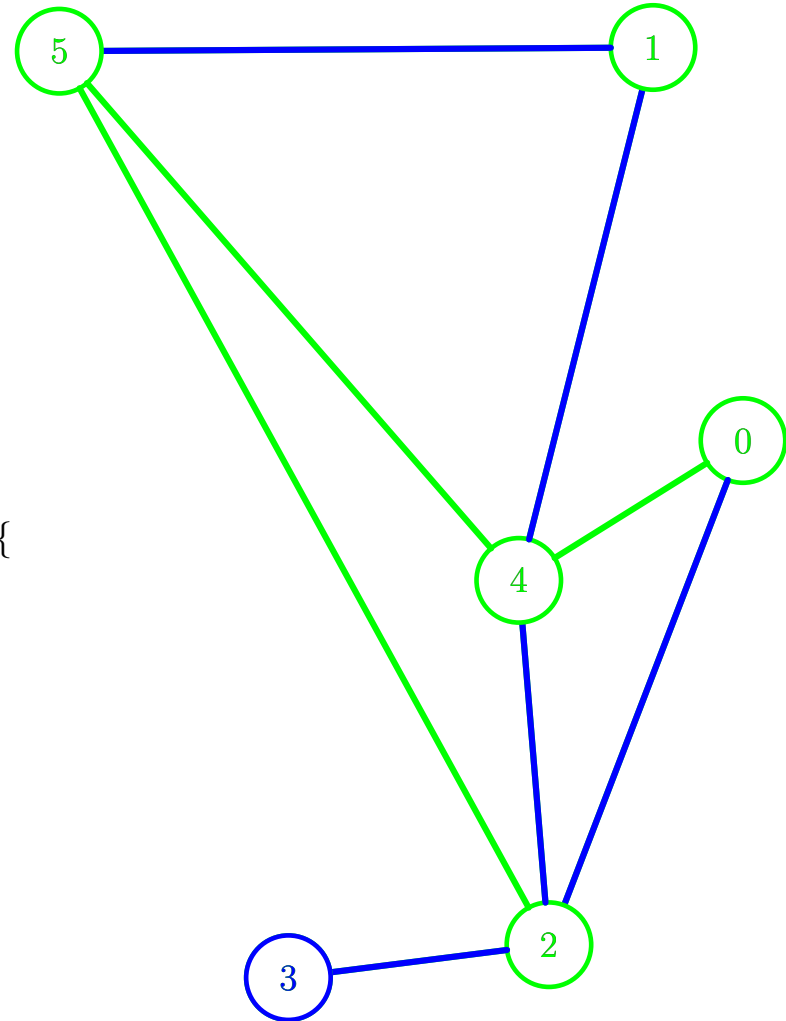
node=5        neighbour=4        time=7

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
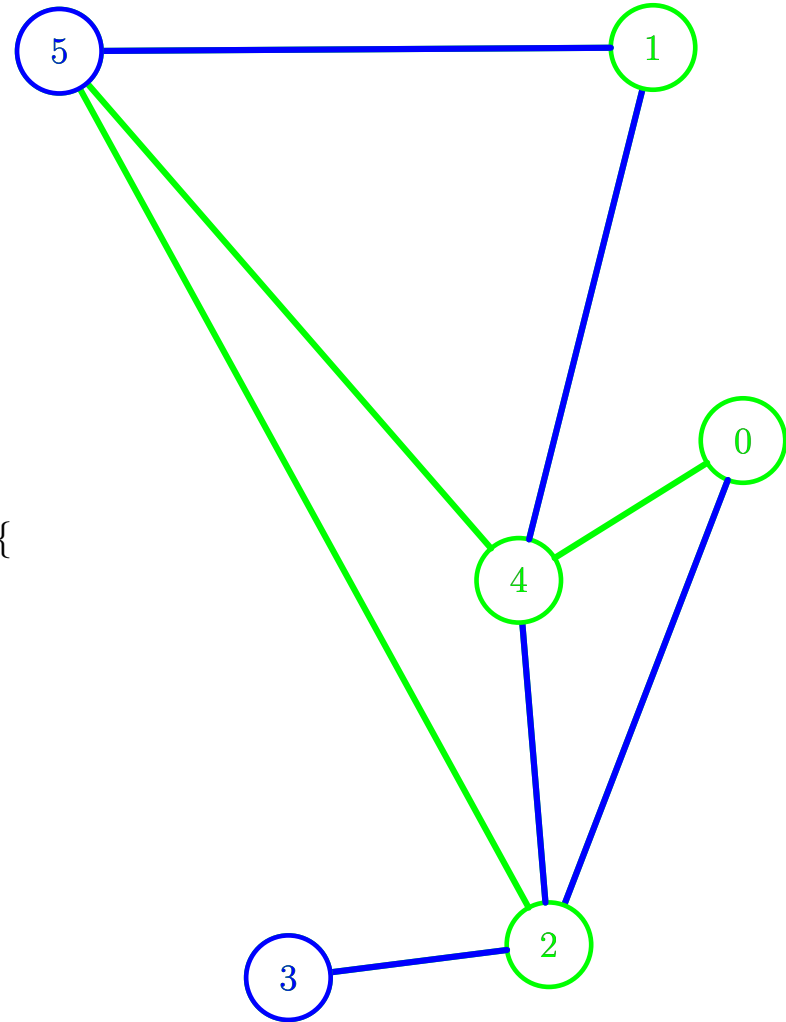
node=5        neighbour=4        time=7

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
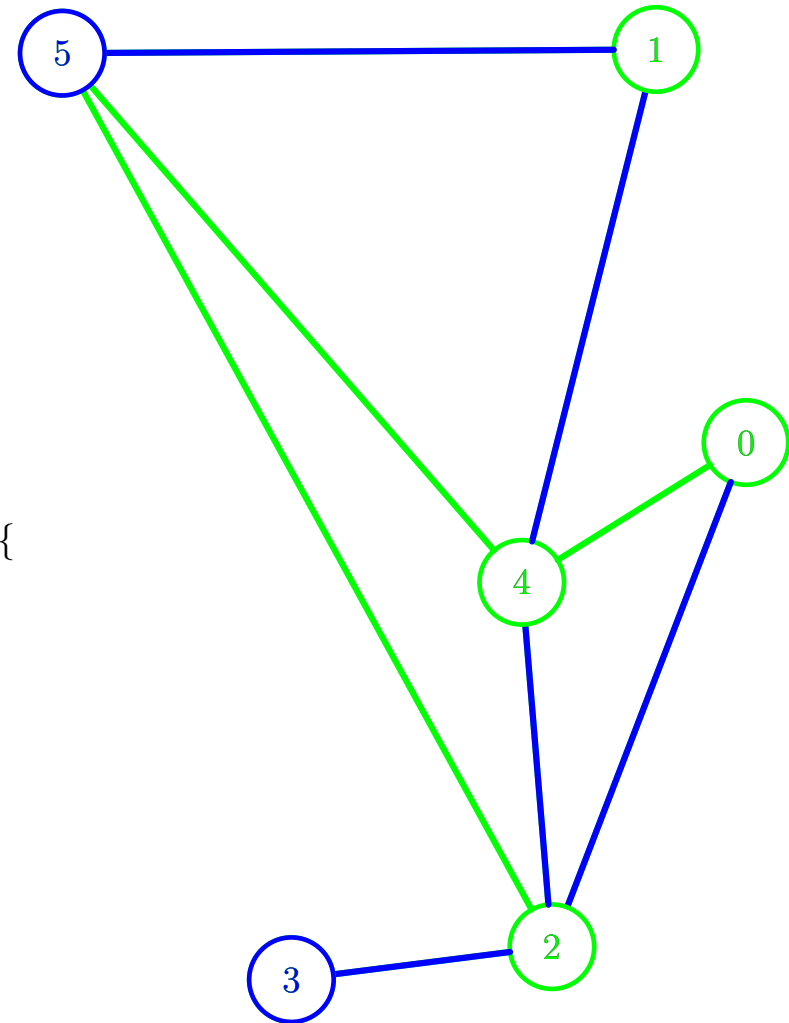
node=5        neighbour=4        time=7

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
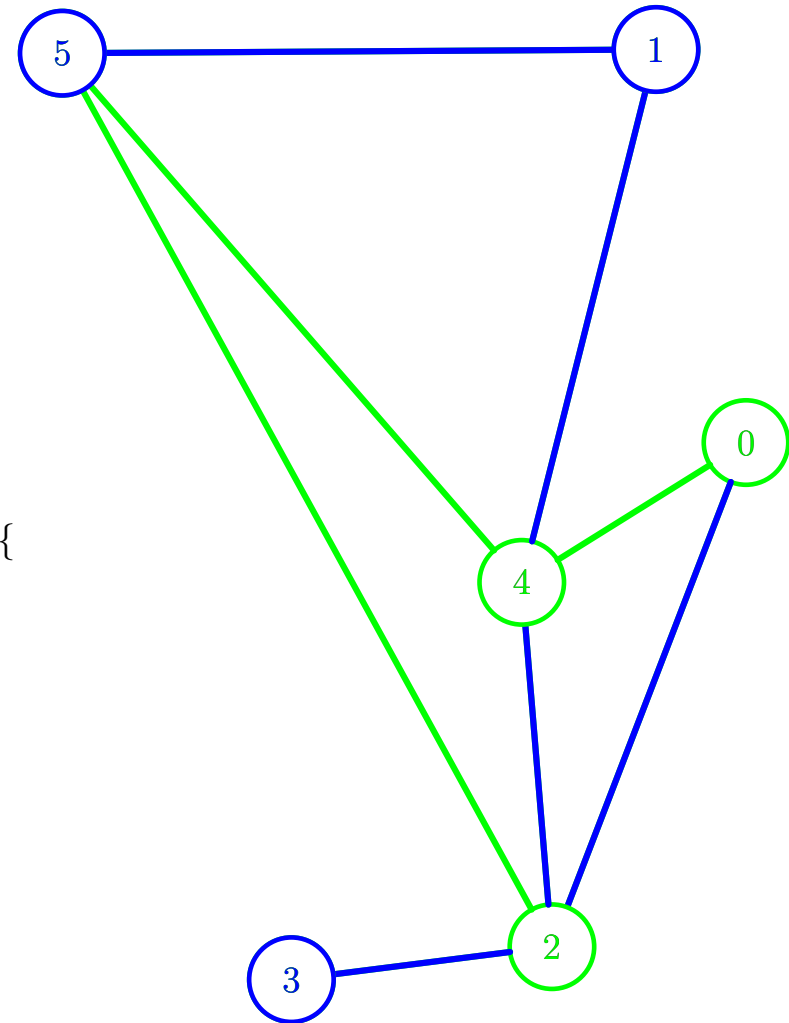
node=5    neighbour=nil    time=8

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

    node=1      neighbour=5      time=8
```
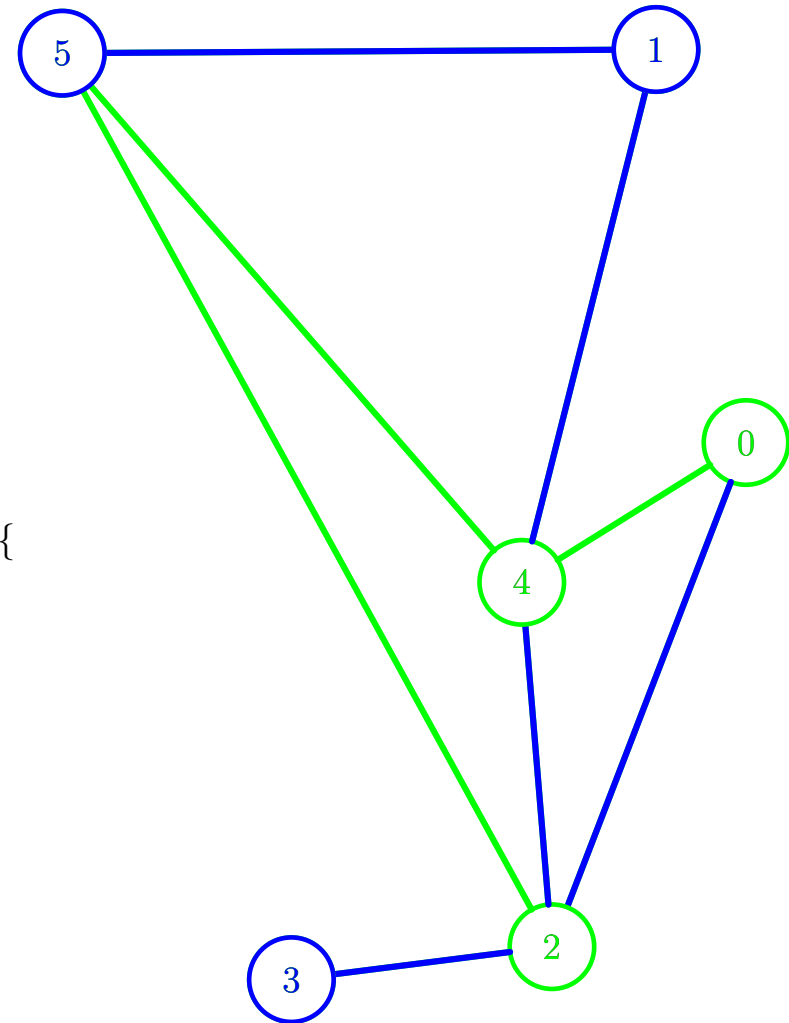
# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

    node=1     neighbour=nil  time=9
```
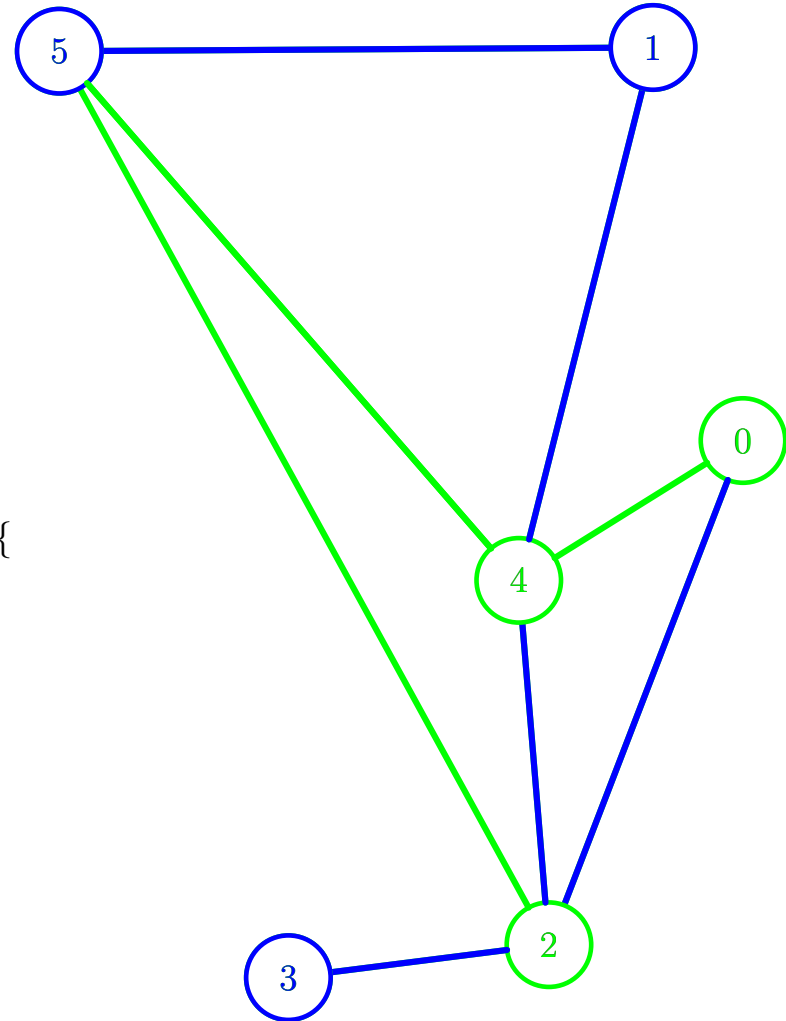
# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
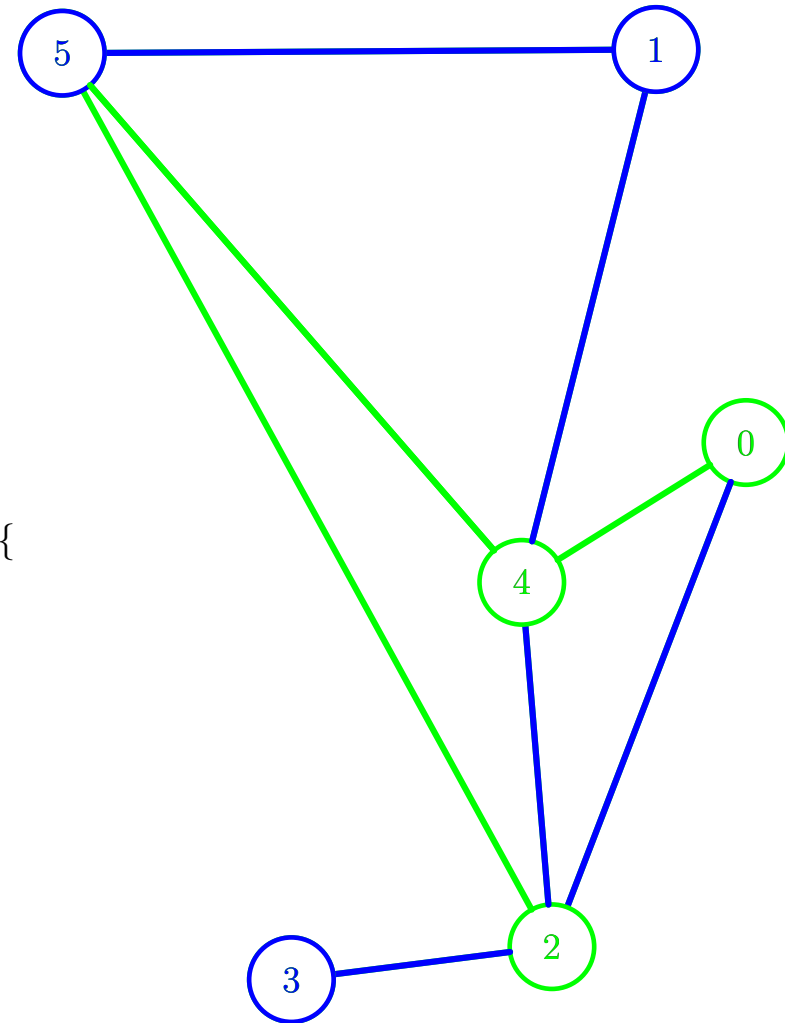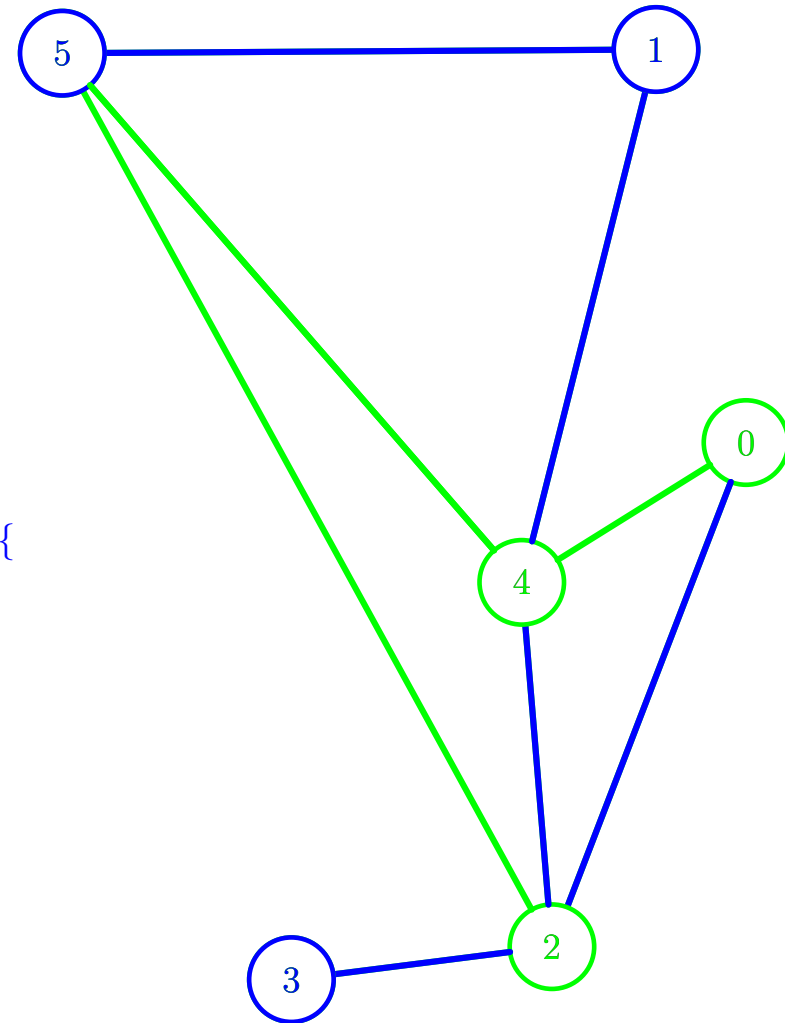
node=4        neighbour=1        time=9

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

    node=4      neighbour=5      time=9
```

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
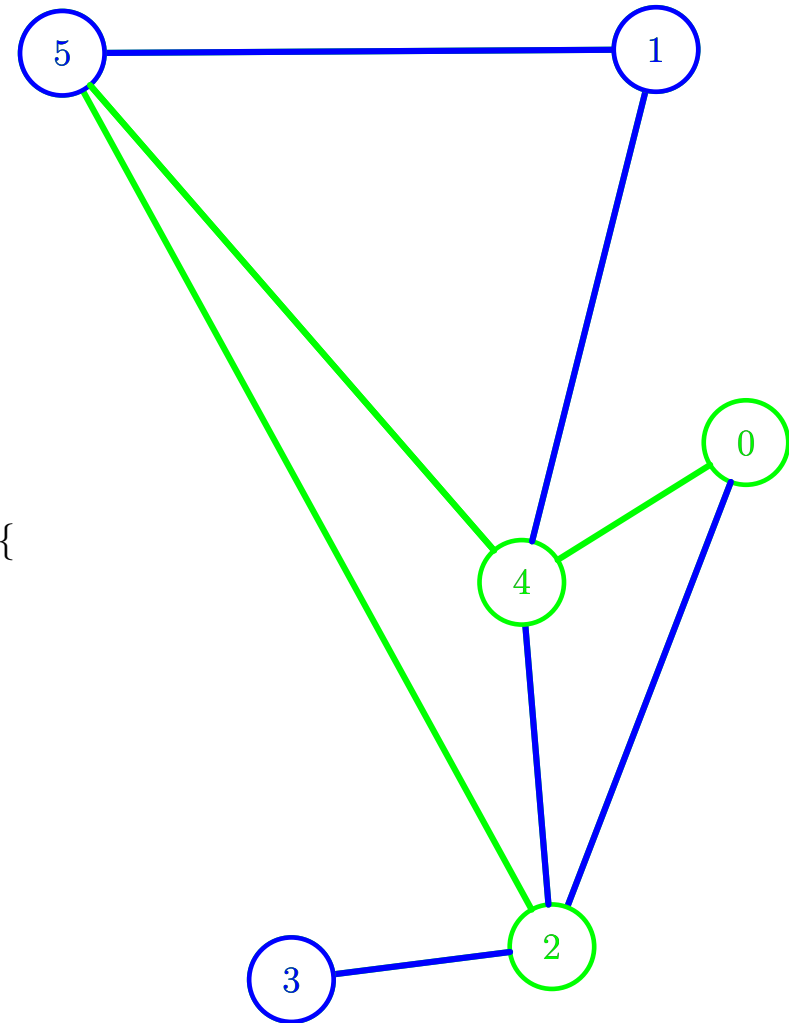
node=4      neighbour=5      time=9

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
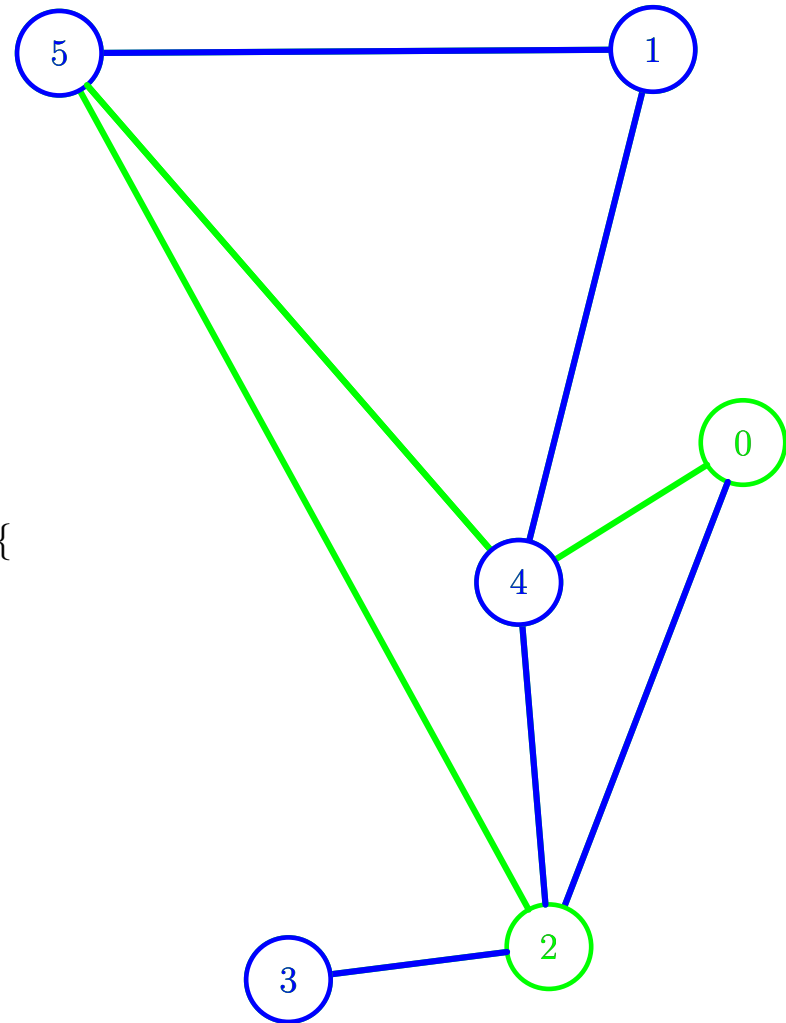
node=4        neighbour=5        time=9

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
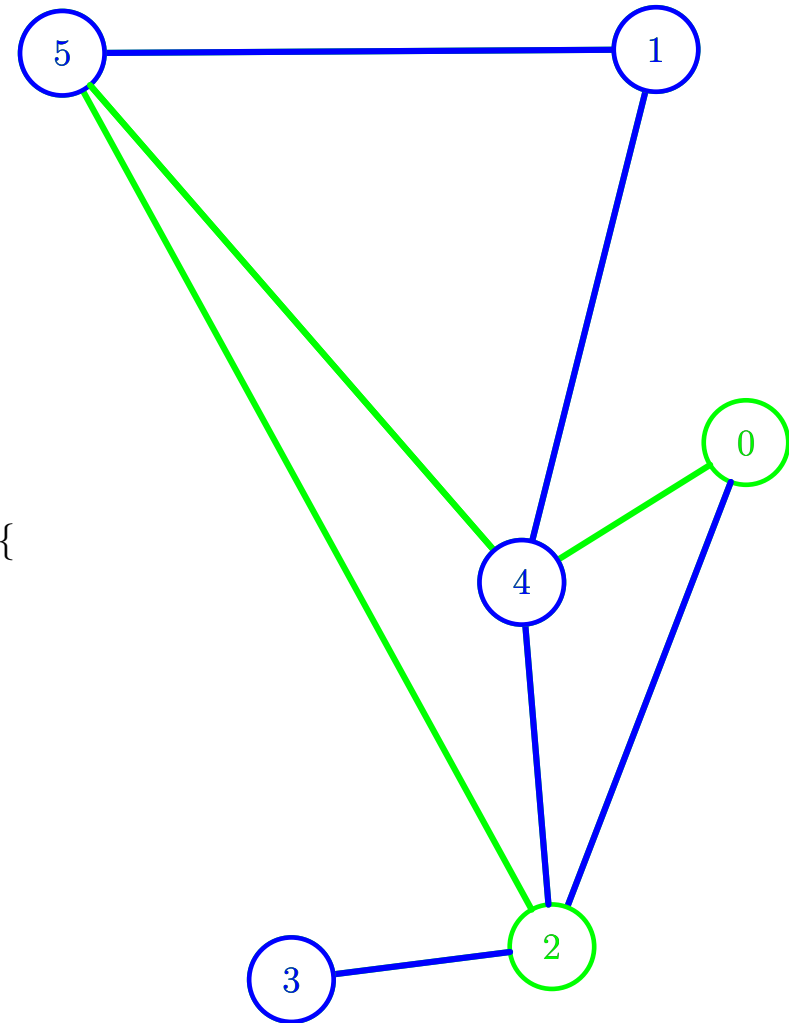
node=4        neighbour=5        time=9

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
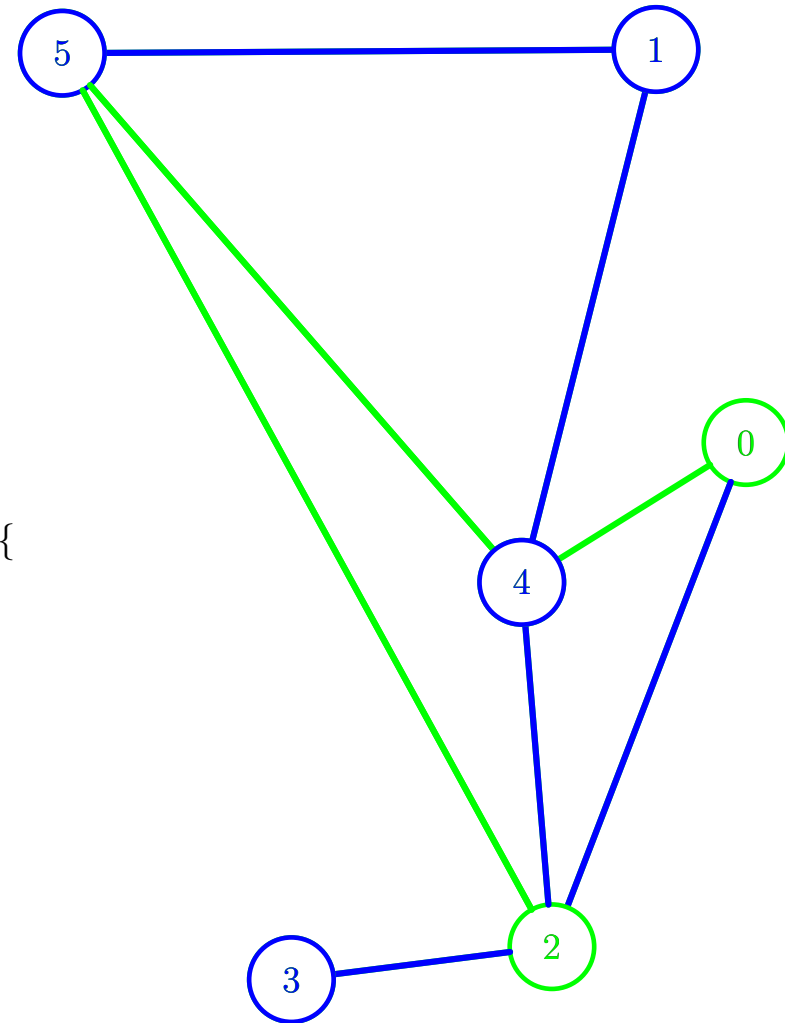
node=4      neighbour=nil   time=10

# Depth First Search

```
dfs(graph, node) {
   state ← Array[n, "undiscovered"]
   finished ← false
   dfs_recur(graph, node)
}

dfs_recur(graph, node) {
   if (finished) return
   state[node] ← "discovered"
   time ← time + 1
   processVertexEarly(node)
   foreach neighbour ∈ Neighbourhood(node) {
      if (state[neighbour] = "undiscovered") {
         parent[neighbour] ← node
         processEdge(node, neighbour)
         dfs_recur(graph, neighbour)
      } else if (state[neighbour] ≠ "processed") {
         processEdge(node, neighbour)
      }
      if (finished) return
   }
   processVertexLate(currentNode)
   state[currentNode] ← "processed"
   time ← time + 1
}
```
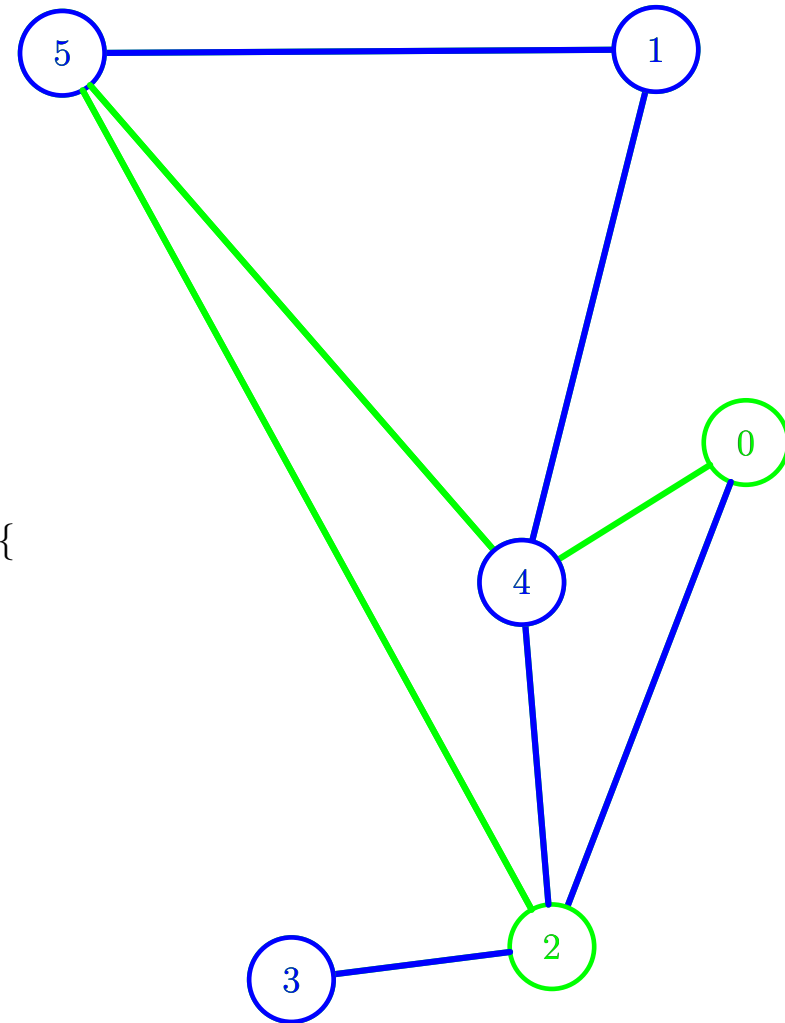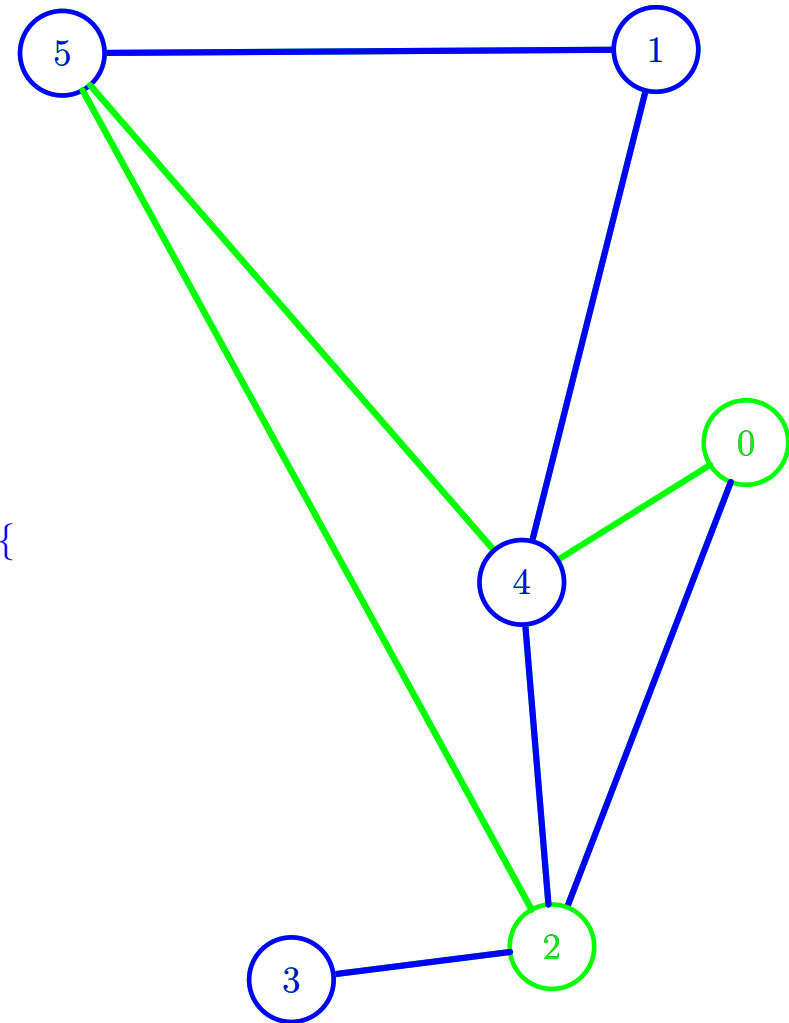
node=2      neighbour=4      time=10

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

    node=2      neighbour=5      time=10
```

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
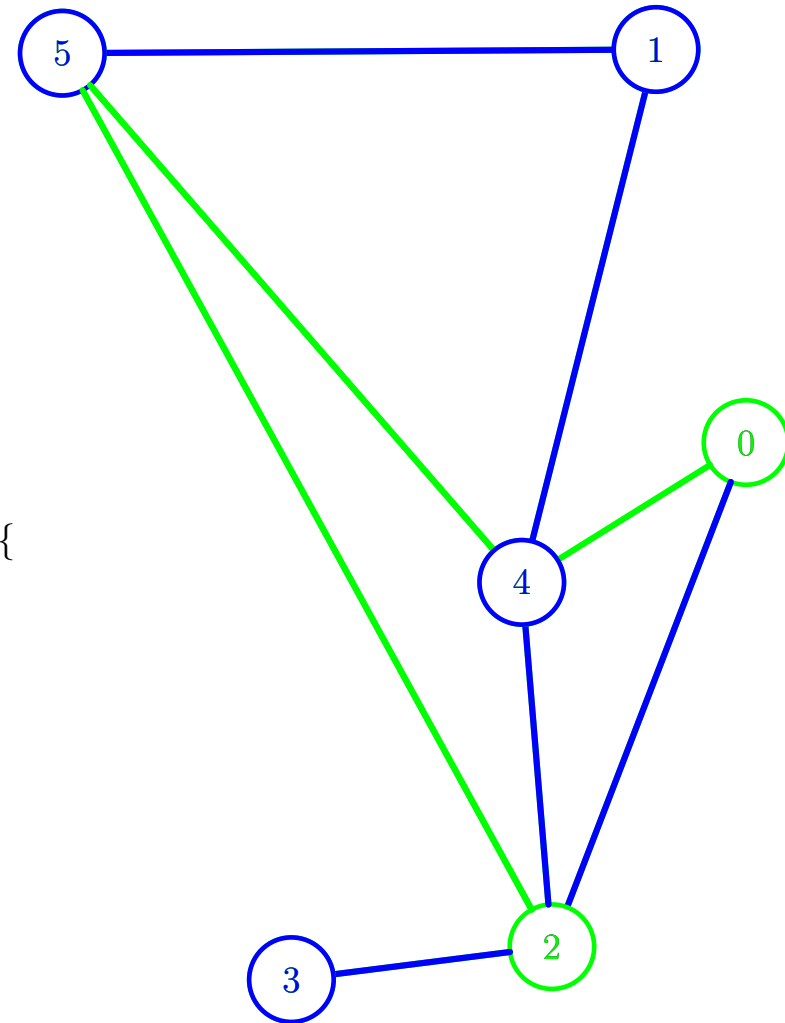
node=2        neighbour=5        time=10

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
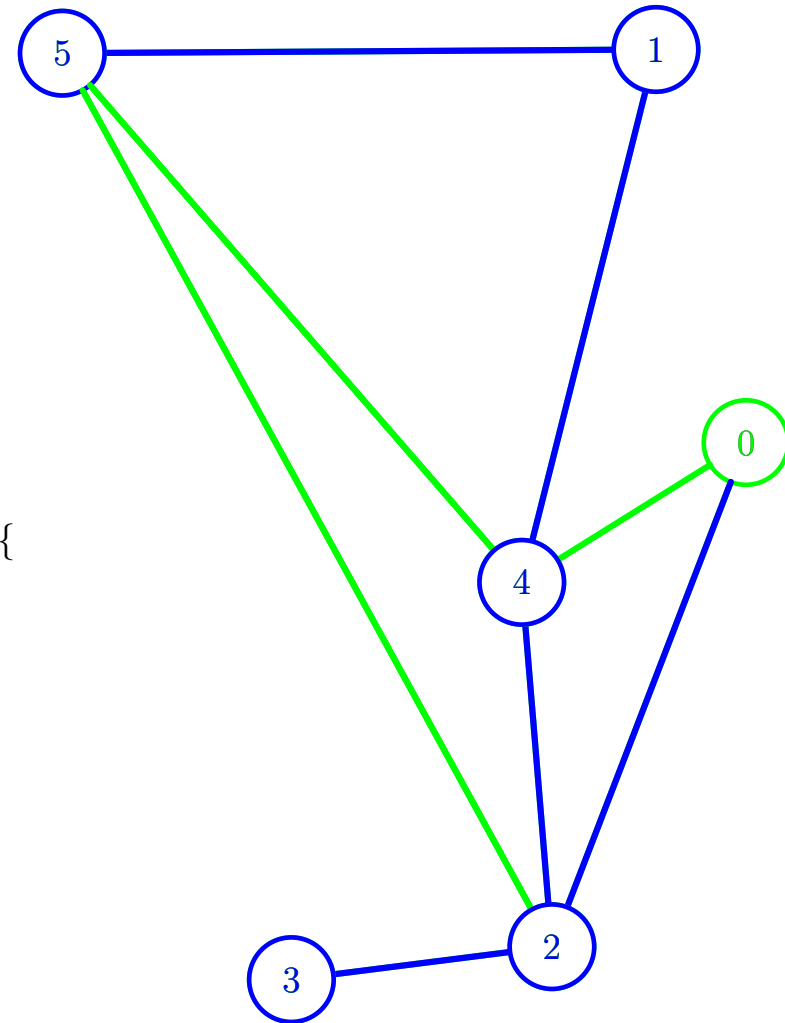
node=2        neighbour=5        time=10

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
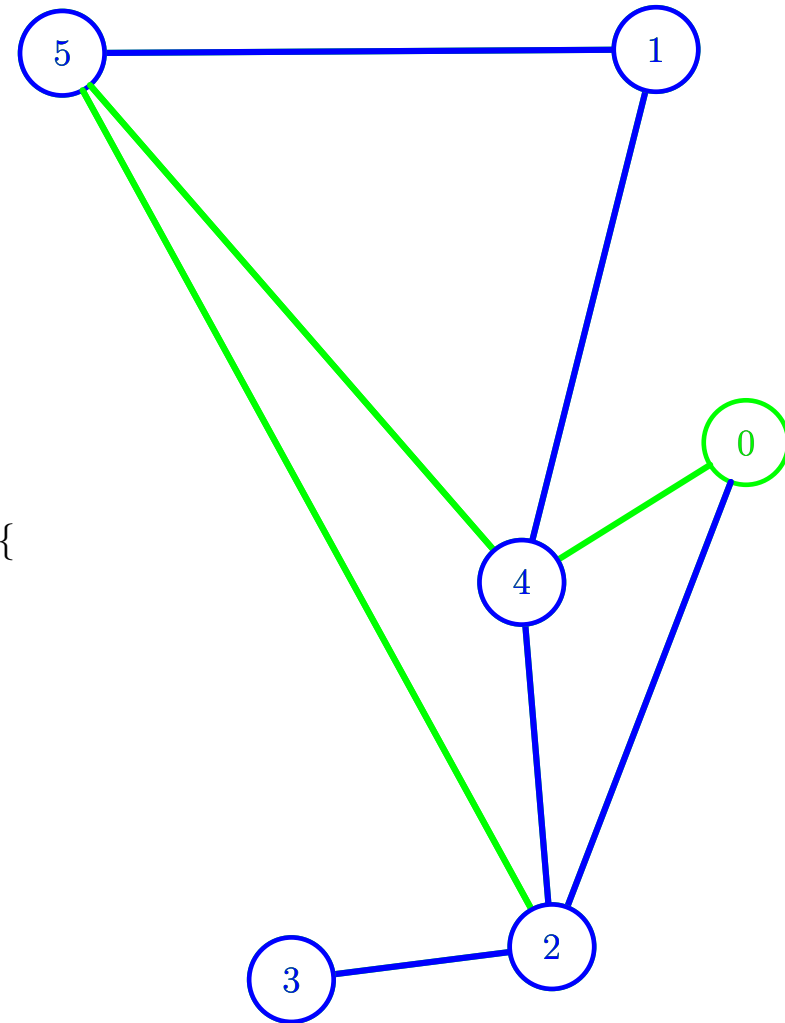
node=2        neighbour=5        time=10

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

    node=2      neighbour=nil   time=11
```
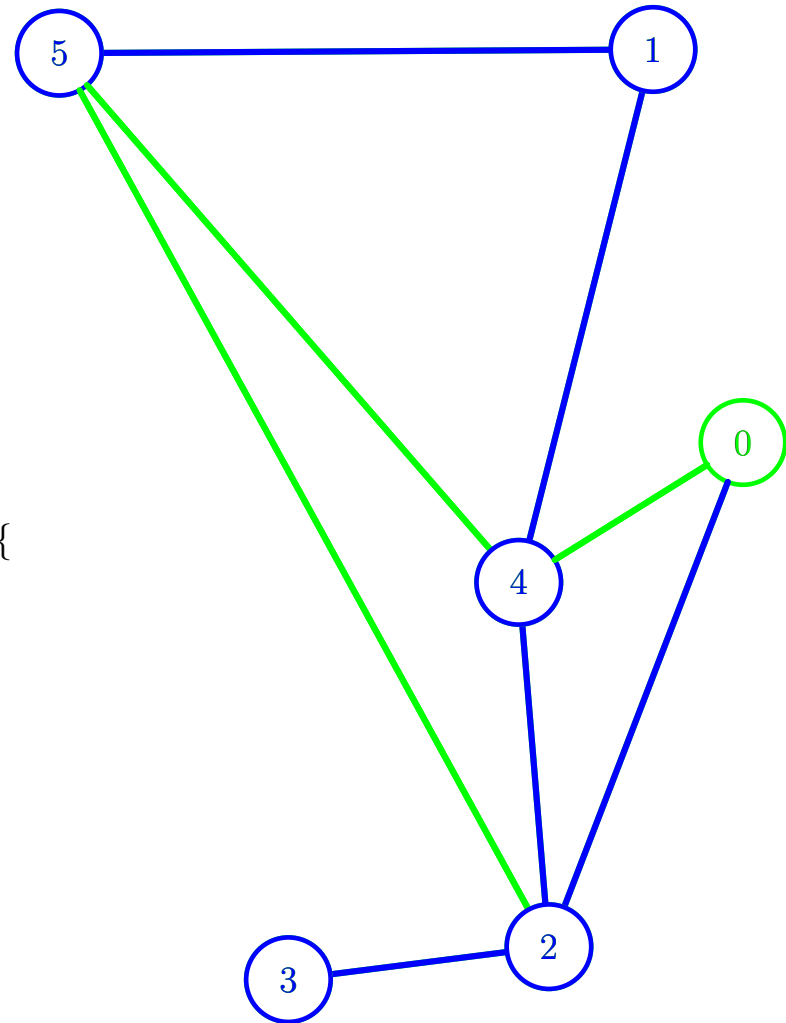
# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
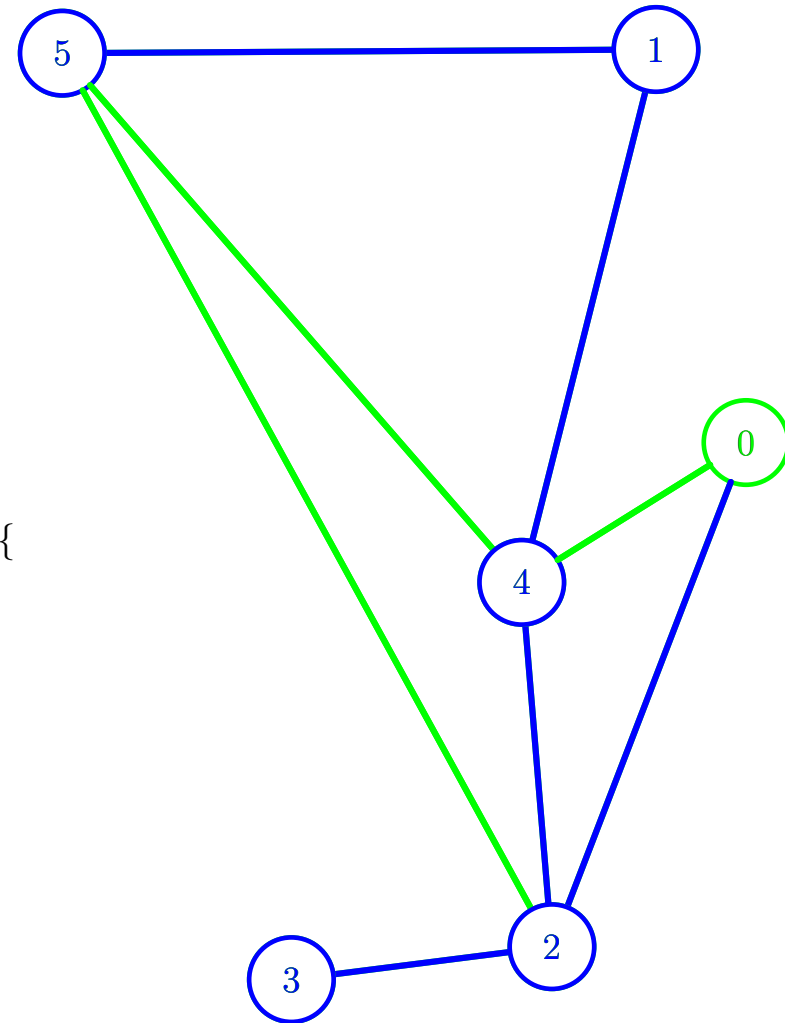
node=0      neighbour=2      time=11

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
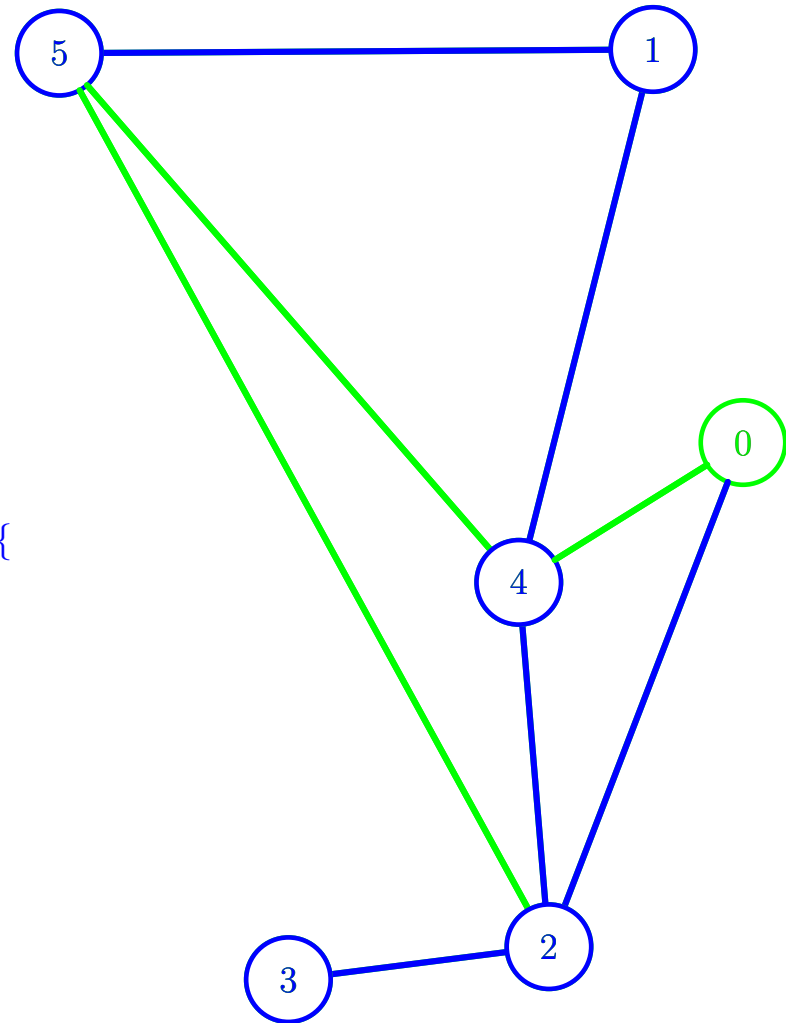
node=0     neighbour=4     time=11

# Depth First Search

```
dfs(graph, node) {
   state ← Array[n, "undiscovered"]
   finished ← false
   dfs_recur(graph, node)
}

dfs_recur(graph, node) {
   if (finished) return
   state[node] ← "discovered"
   time ← time + 1
   processVertexEarly(node)
   foreach neighbour ∈ Neighbourhood(node) {
      if (state[neighbour] = "undiscovered") {
         parent[neighbour] ← node
         processEdge(node, neighbour)
         dfs_recur(graph, neighbour)
      } else if (state[neighbour] ≠ "processed") {
         processEdge(node, neighbour)
      }
      if (finished) return
   }
   processVertexLate(currentNode)
   state[currentNode] ← "processed"
   time ← time + 1
}
```
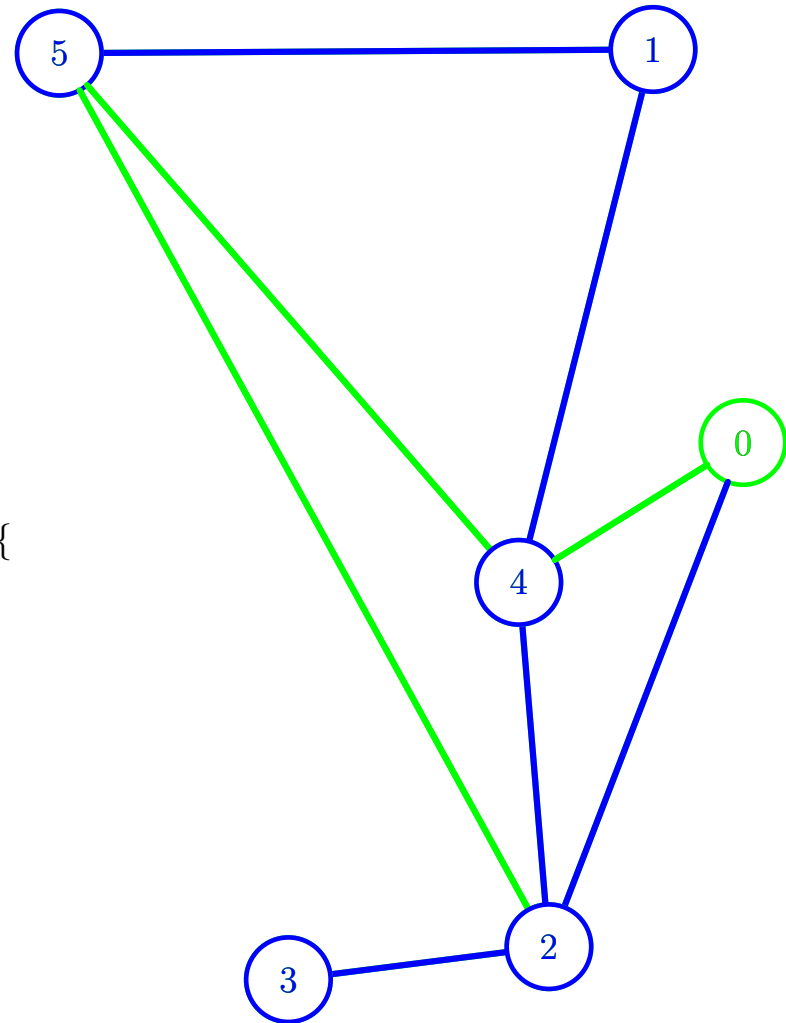
node=0        neighbour=4        time=11

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
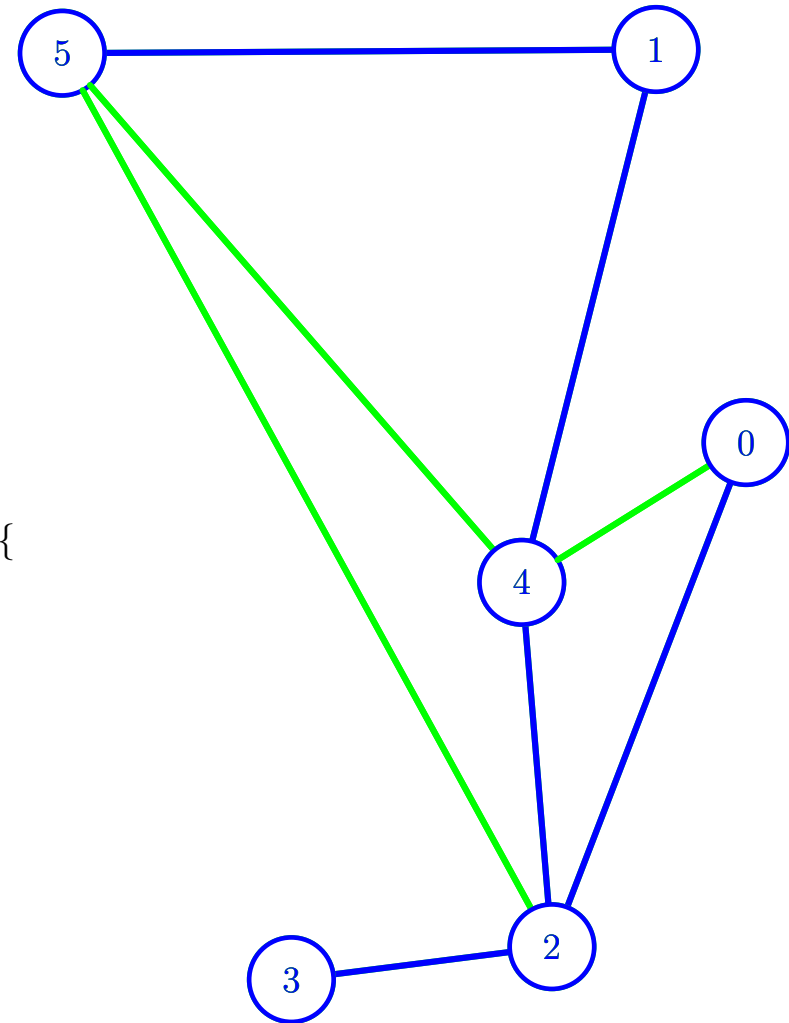
node=0      neighbour=4      time=11

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

node=0        neighbour=4        time=11

# Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}

        node=0      neighbour=nil   time=12
```

# Applications of DFS

- Depth first search has many applications

- Suppose we want to check if the graph is a tree (i.e. has no cycles)

- The only edges that are allowed are parent edges

```
processEdges(node1, node2) {
    if (parent[node1] ≠ node2) {
        isTree ←false
        finish ←true
    }
}
```

(note that we set `finish` to stop DFS prematurely)

# Applications of DFS

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- Suppose we want to check if the graph is a tree (i.e. has no cycles)

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processEdges(node1, node2) {
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}
```

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---

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```
processEdges(node1, node2) {
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        isTree ←false
        finish ←true
    }
}
```

(note that we set `finish` to stop DFS prematurely)

# Biconnected Graphs and Articulation Vertices

- If we removed a vertex (and all its edges) from a graph would the graph become disconnected?

- Such nodes are called **articulation vertices**

- Graphs with no articulation vertices are said to be **biconnected**

- In many applications articulation vertices are important, e.g. they represent single point of failures in communication networks

- A brute force method for identifying articulation vertices is to remove each and check for connectivity—this would take $O(n(m+n))$ time
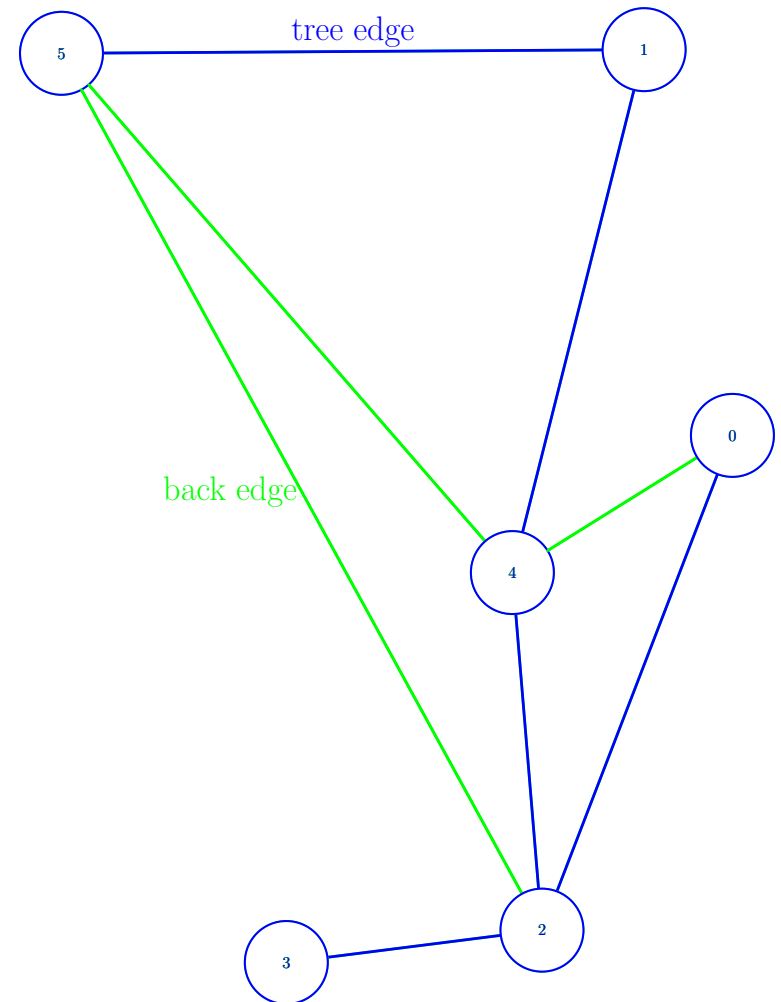
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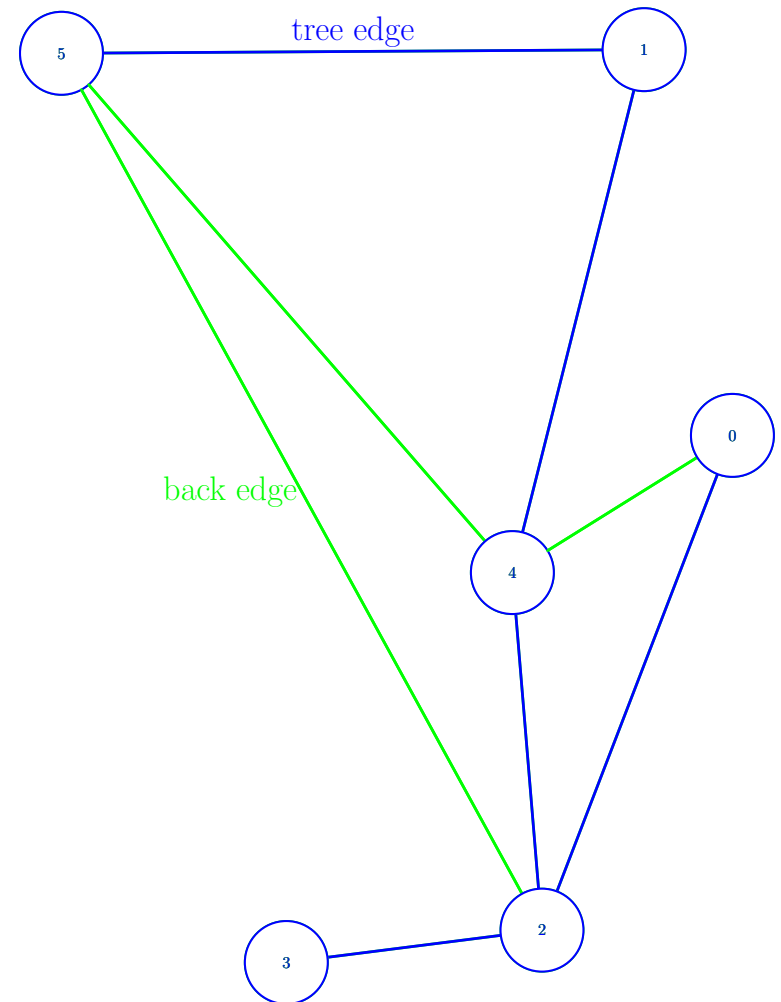
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- If we removed a vertex (and all its edges) from a graph would the graph become disconnected?

- Such nodes are called **articulation vertices**

- Graphs with no articulation vertices are said to be **biconnected**

- In many applications articulation vertices are important, e.g. they represent single point of failures in communication networks

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# Single Pass Algorithm

- In DFS we divide the edges into tree edges that define the search tree (edges between nodes and parents) and back edges which take us back to vertices we have already seen

- Without the back edges we have a tree where all non-leaf nodes are articulation nodes

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# Reachable Ancestors

- Need to check if there exist back edges to nodes that have been visited earlier

- We maintain an array noting the reachable ancestors of all nodes

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- In the `processEdge` method, if the edge is a

  **back edge** we update the reachable ancestor (we check the $entryTime$ to see if the edge leads to a vertex which was discovered earlier)

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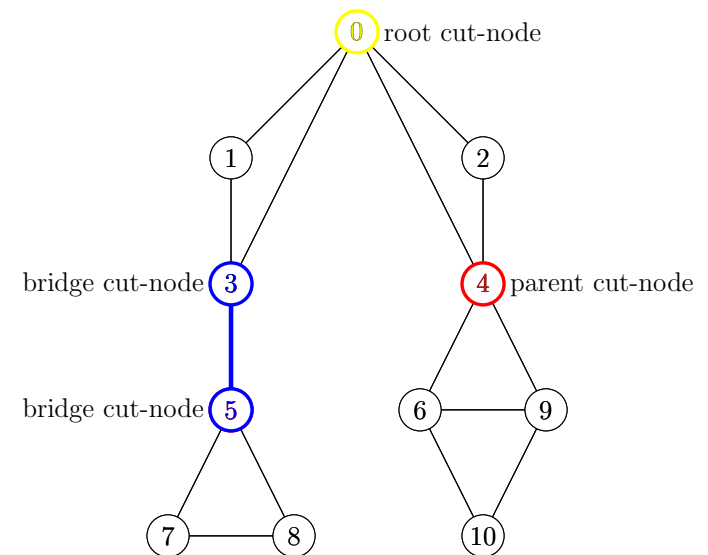
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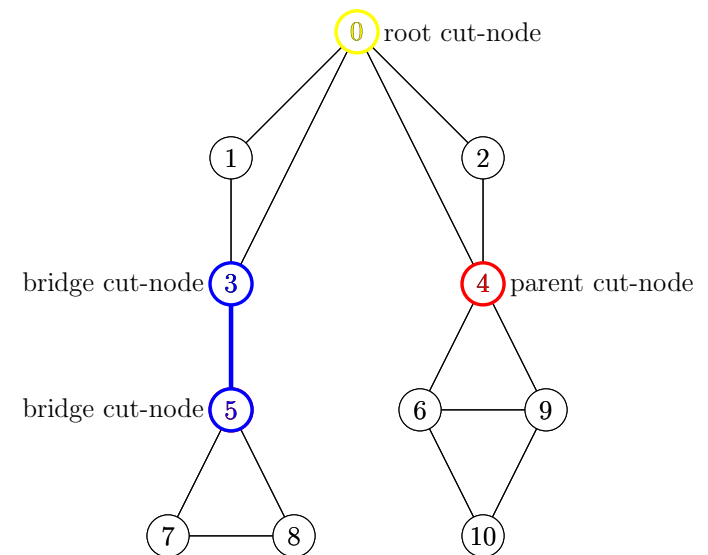# Types of Articulated Vertices

- Key is to recognise that articulated vertices only occur in three version

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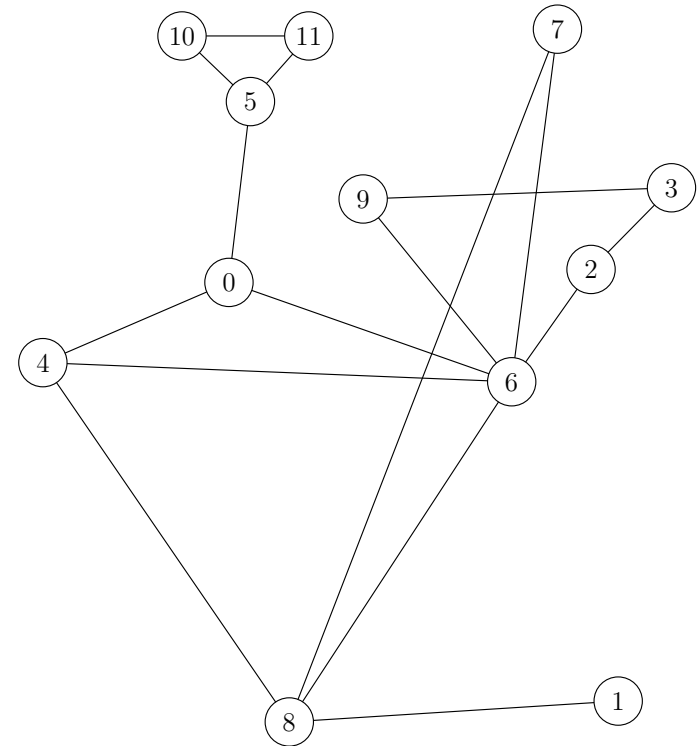
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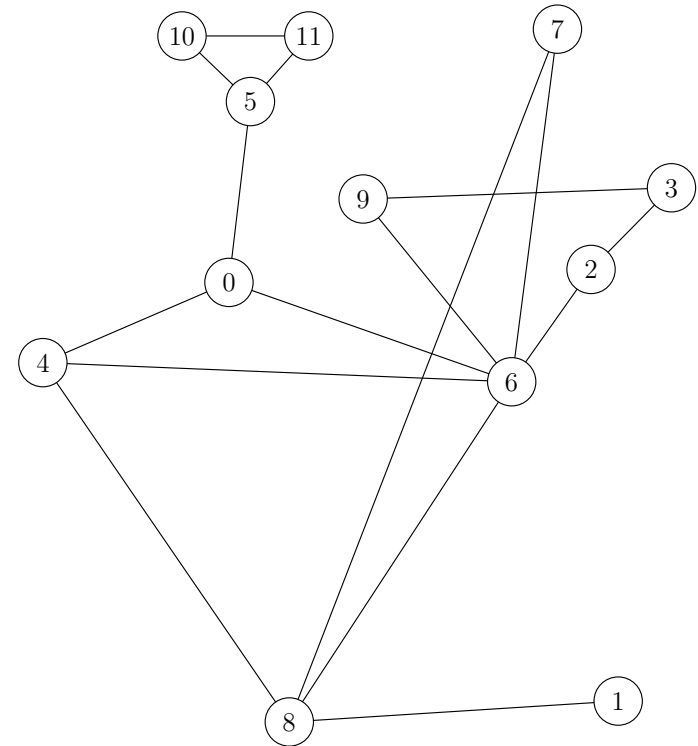- These are determined in `processVertexLate` method.

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- Algorithmic details are not too important

- One pass $(O(n + m))$ algorithm

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# Outline

1. Breadth First Search

   • BFS applications

2. Depth First Search

   • DFS applications

3. **Topological Sort**
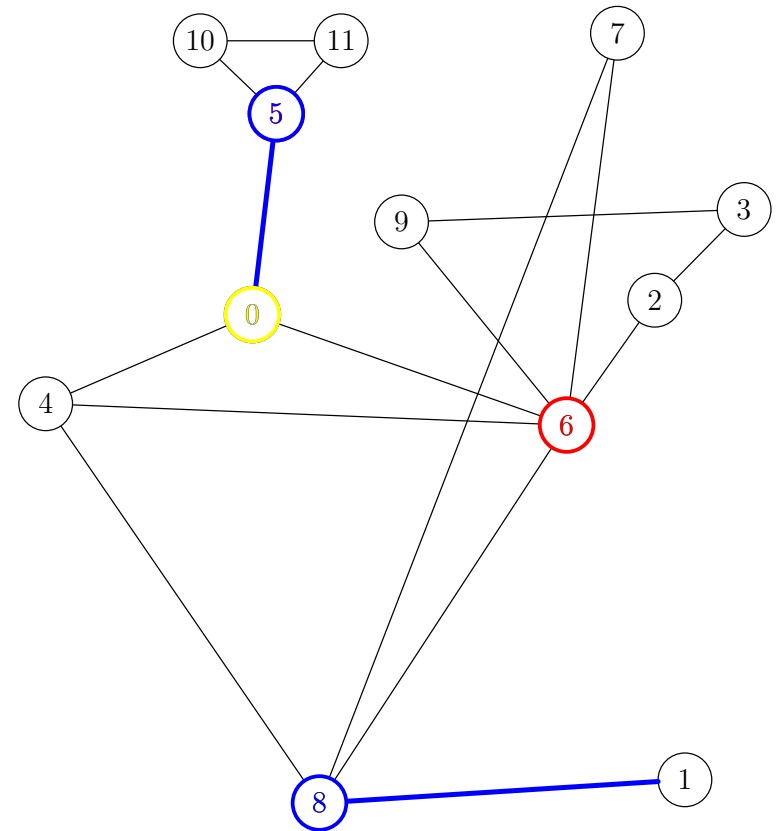
# DAGs

- Directed acyclic graphs or DAGs are directed graphs without cycles

- They are often used to represent complex processes

  ⋆ Vertices are processes
  ⋆ Directed edge $(i, j)$ indicates process $i$ needs to occur before process $j$
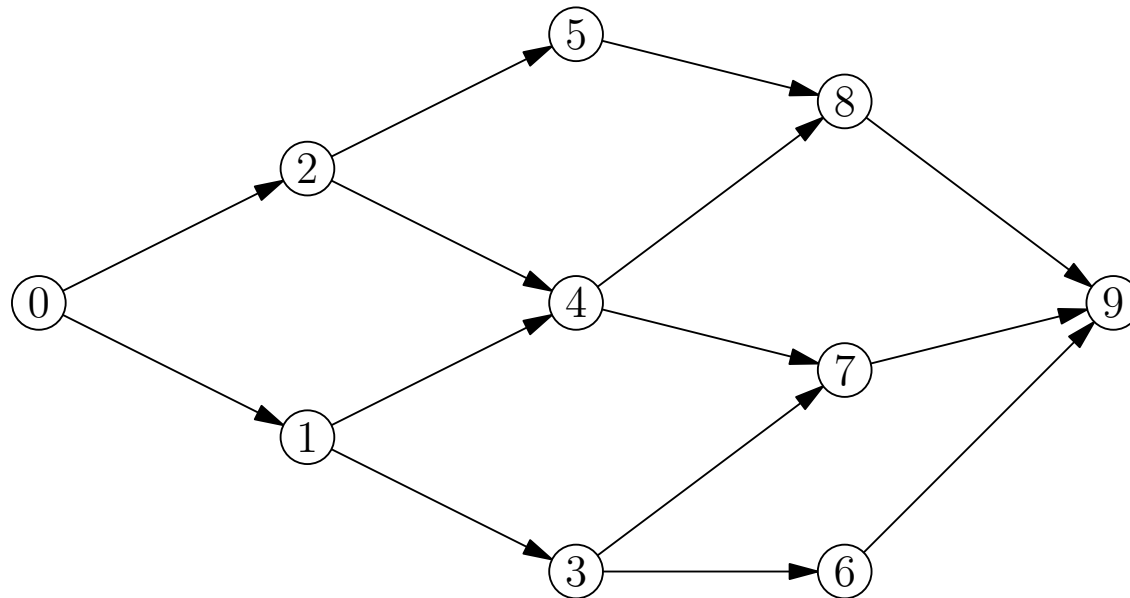
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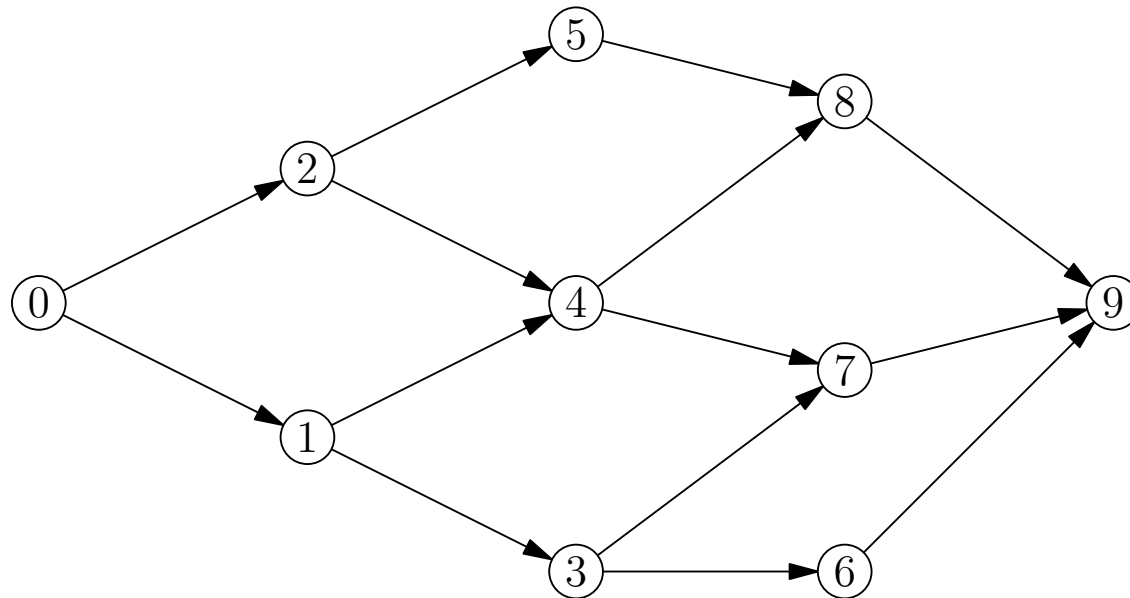
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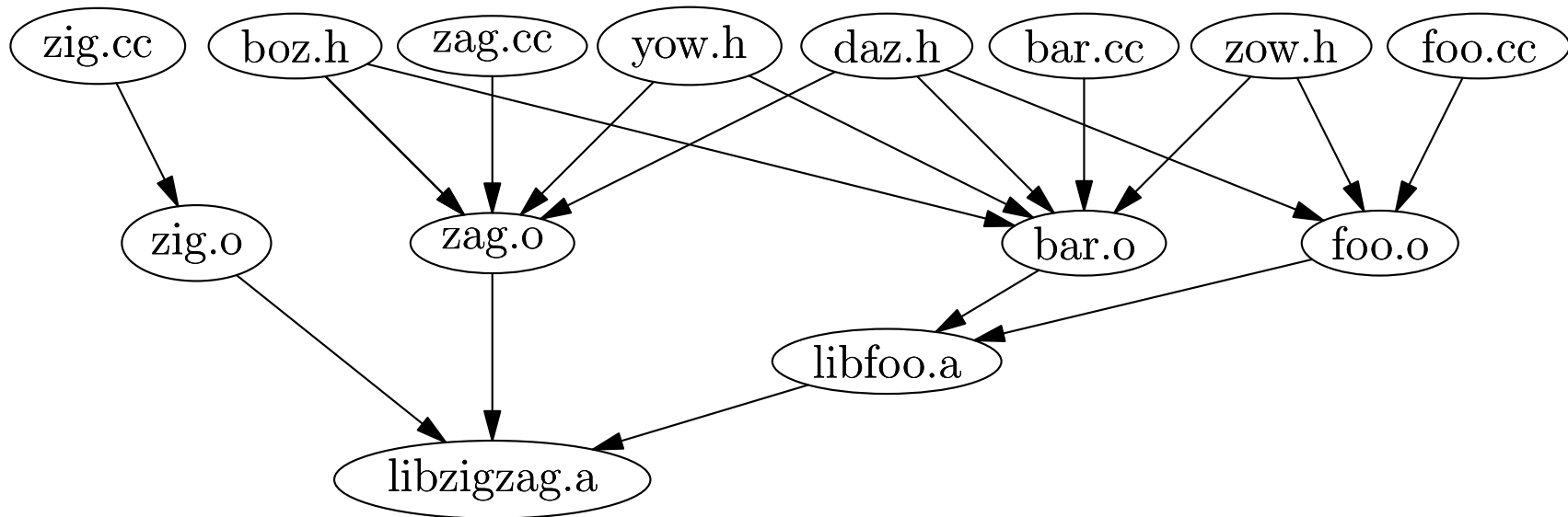
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- The same problem occurs in compiling classes

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- If the graphs were not acyclic it would be impossible process them!

# Topological Sort

- Given a DAG a topological sort outputs an ordered list of vertices which respects the ordering imposed by the edges

- That is, for each edge $(i, j)$, vertex $i$ will occur before vertex $j$

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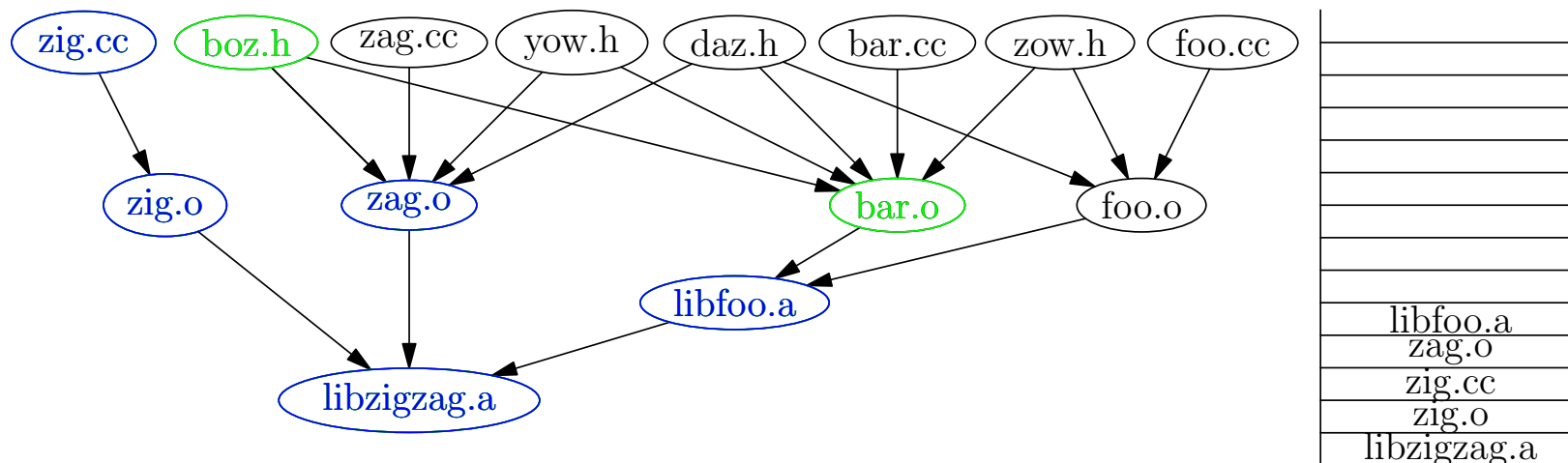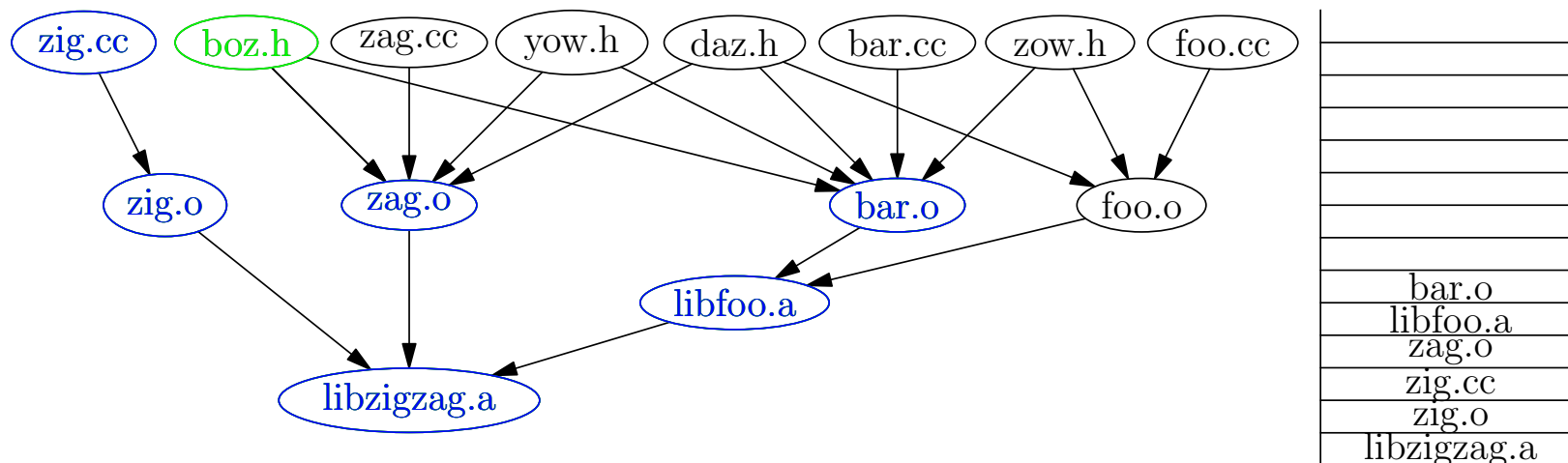- A topological sort is generated by a reversed order list of how DFS processes nodes
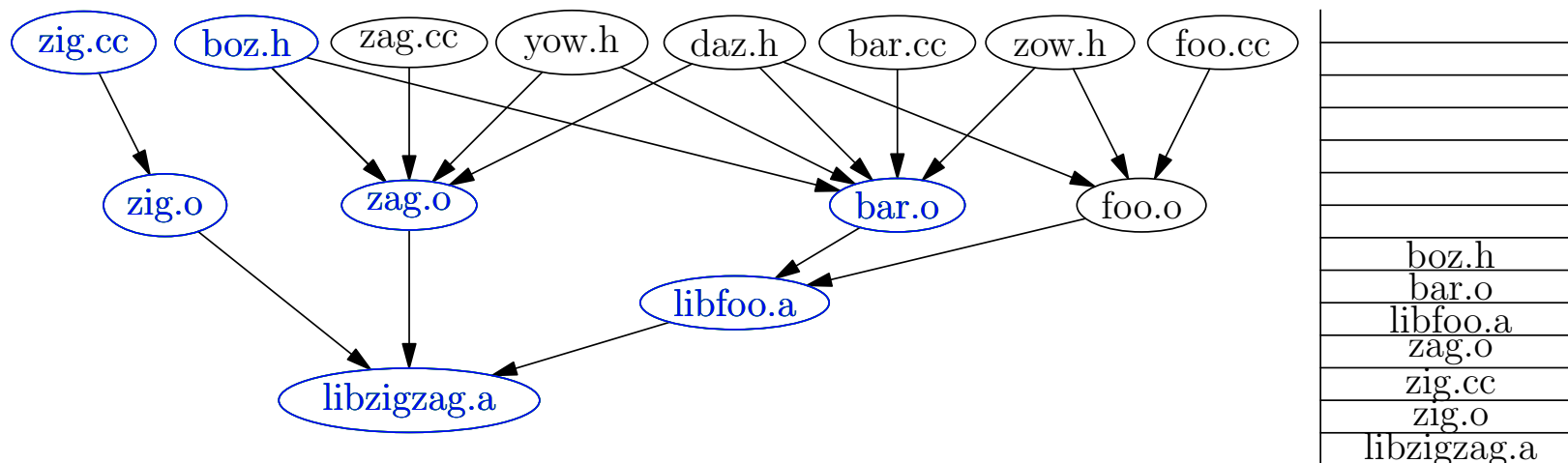
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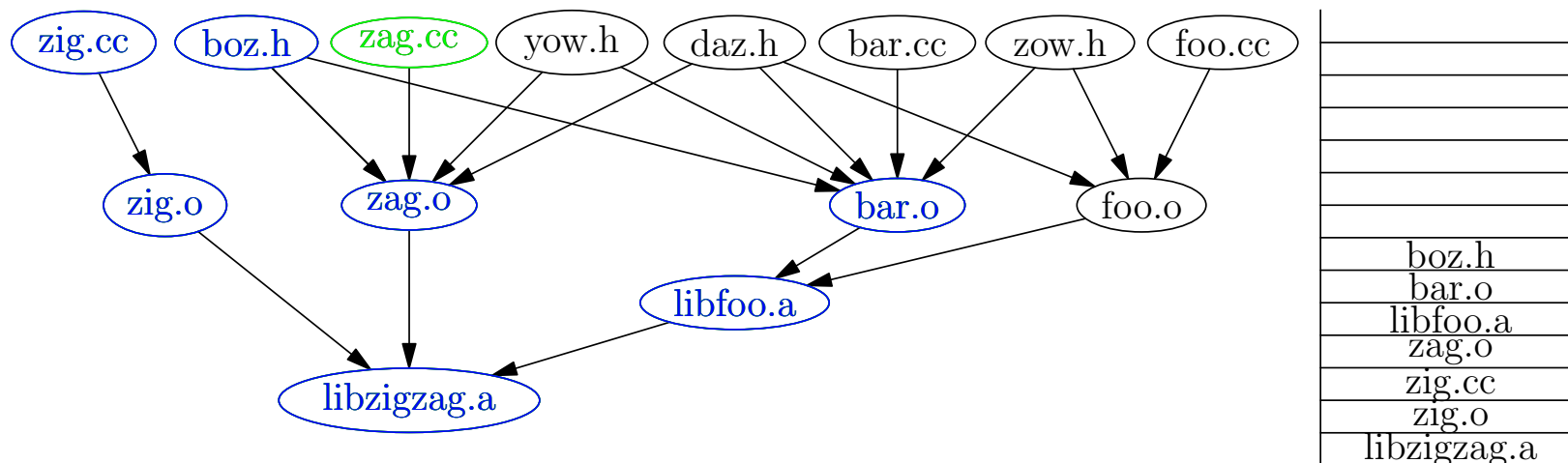
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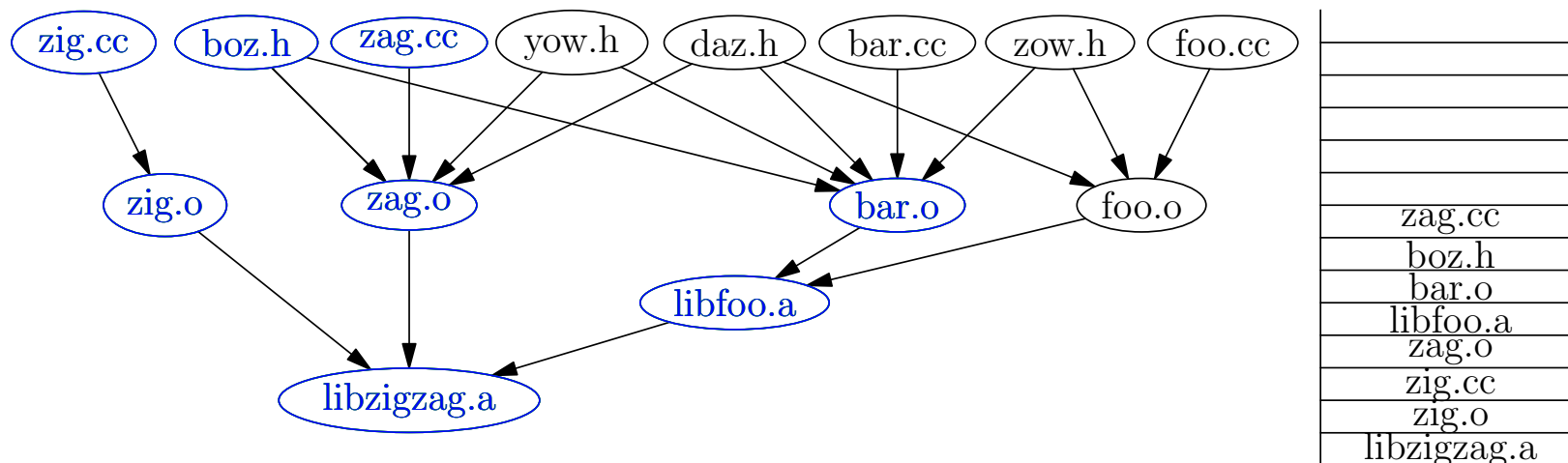
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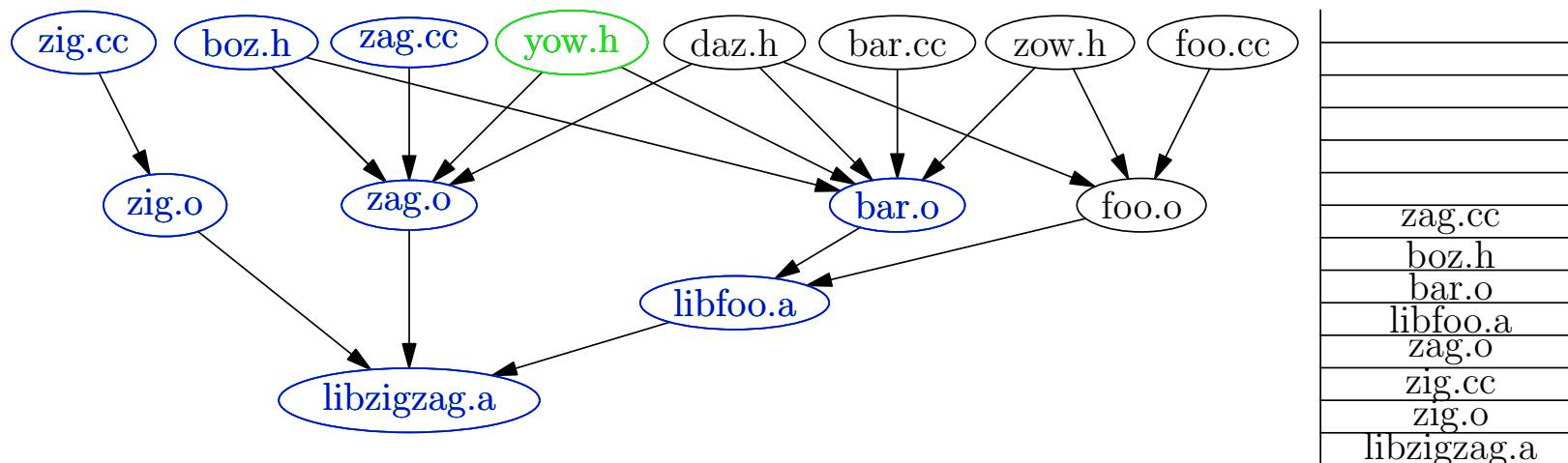
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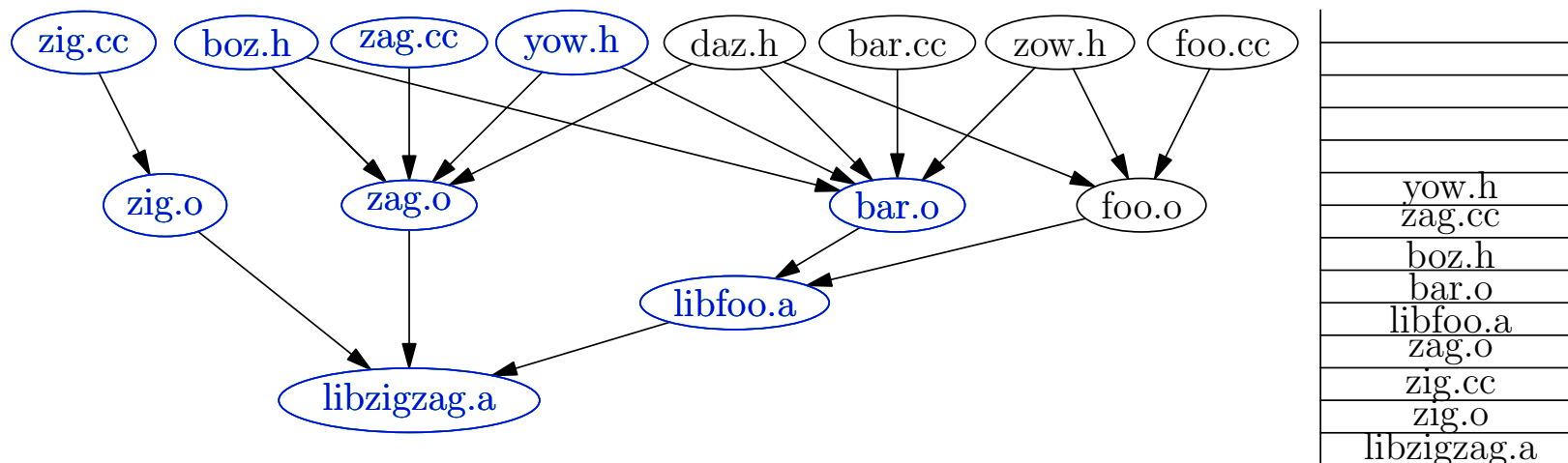
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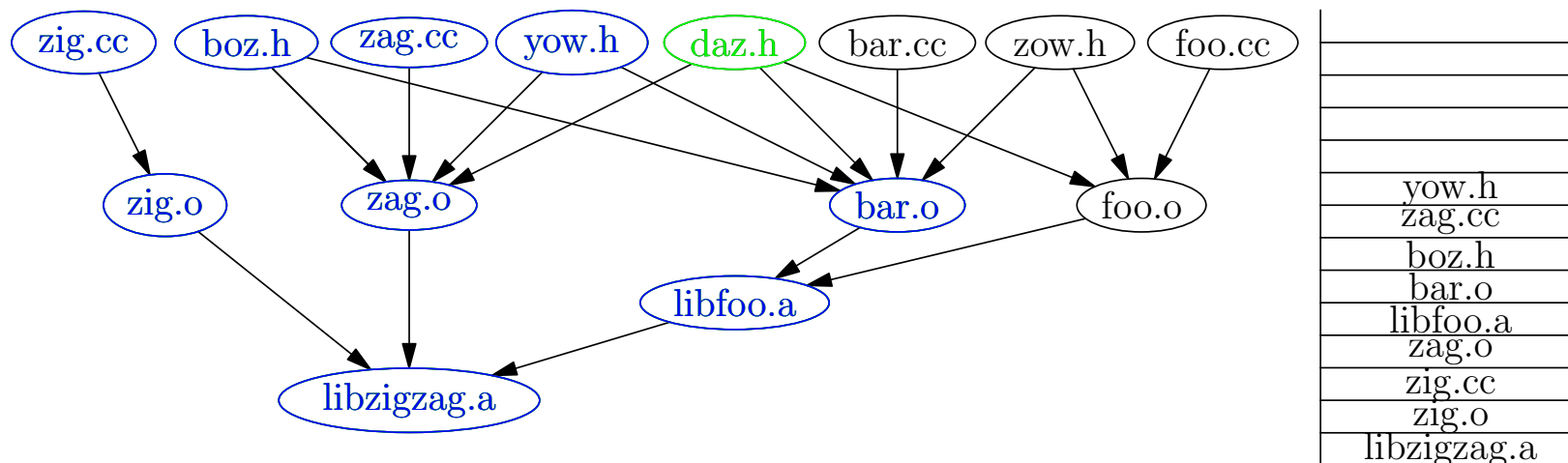
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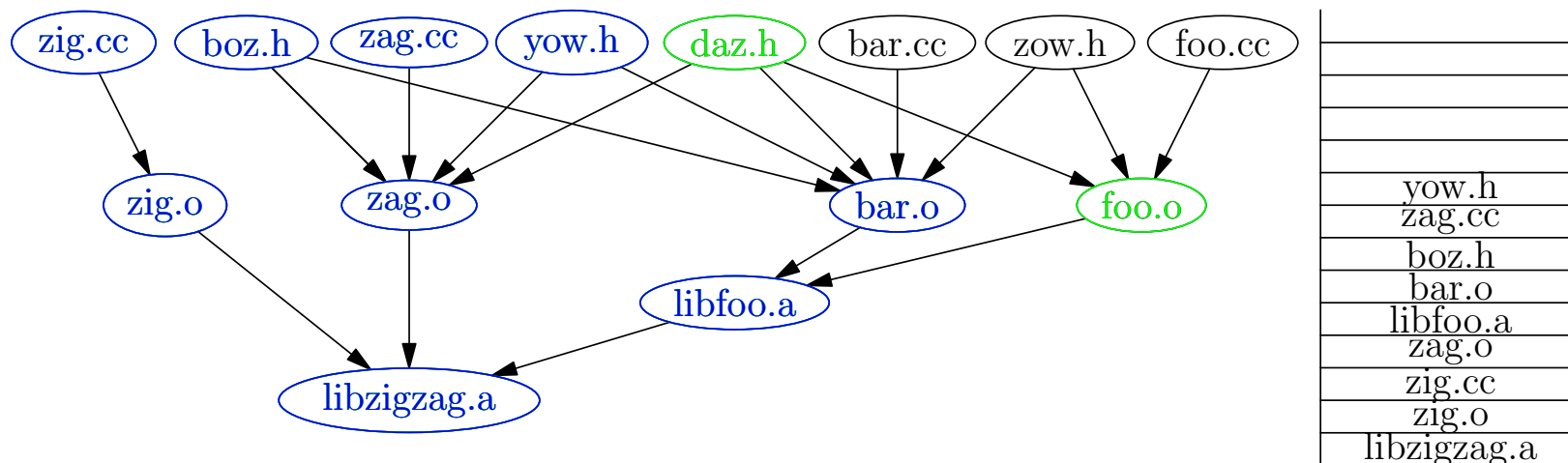
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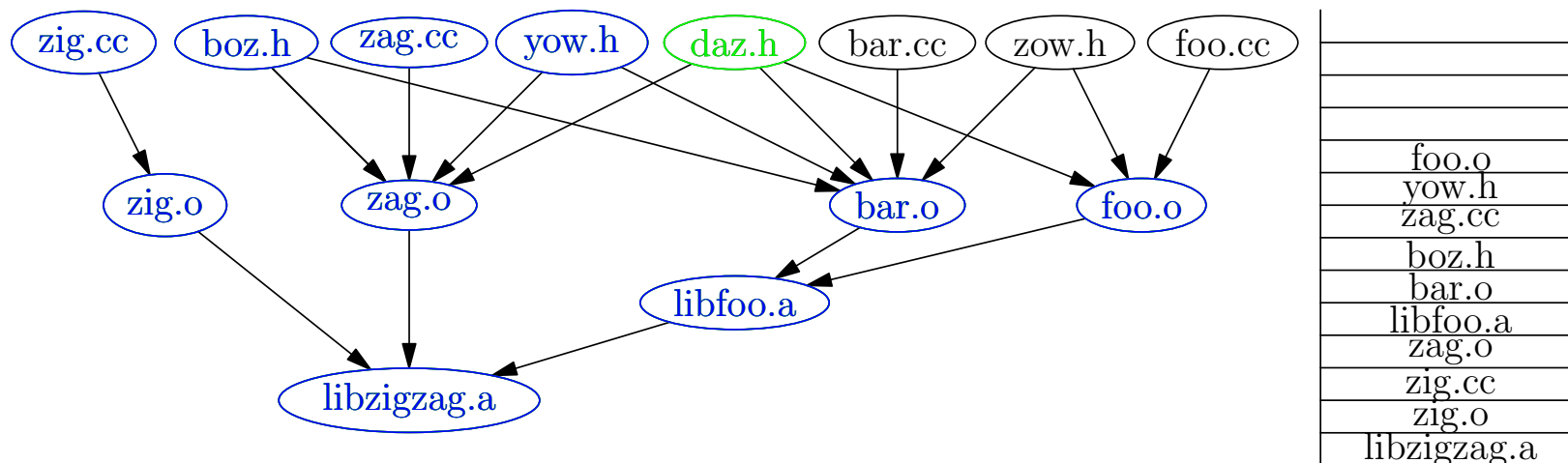
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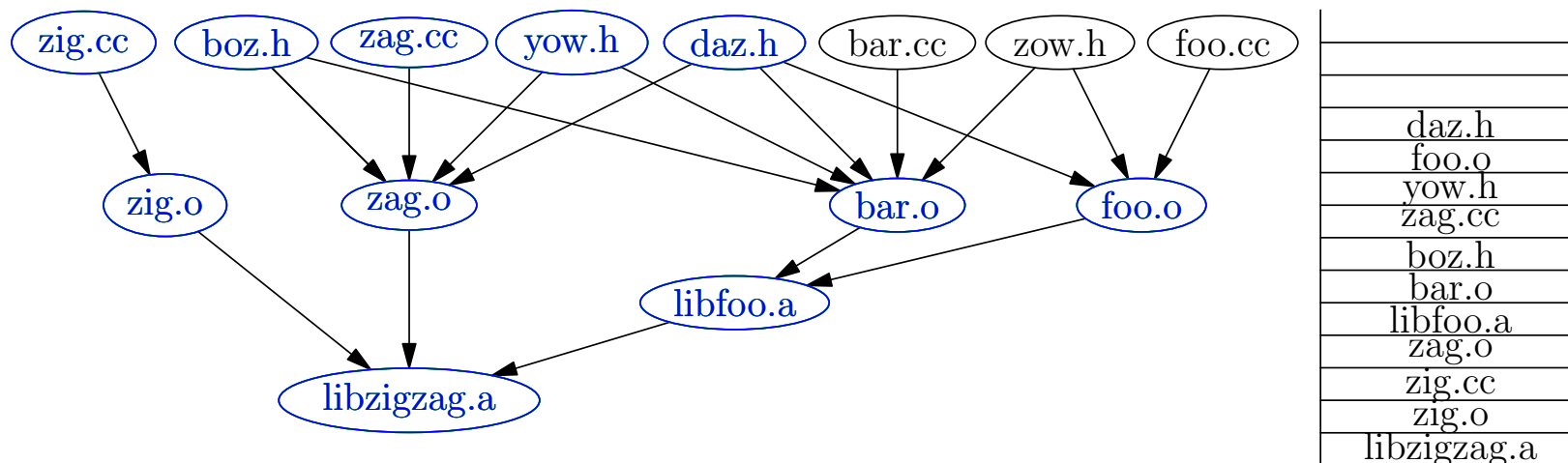
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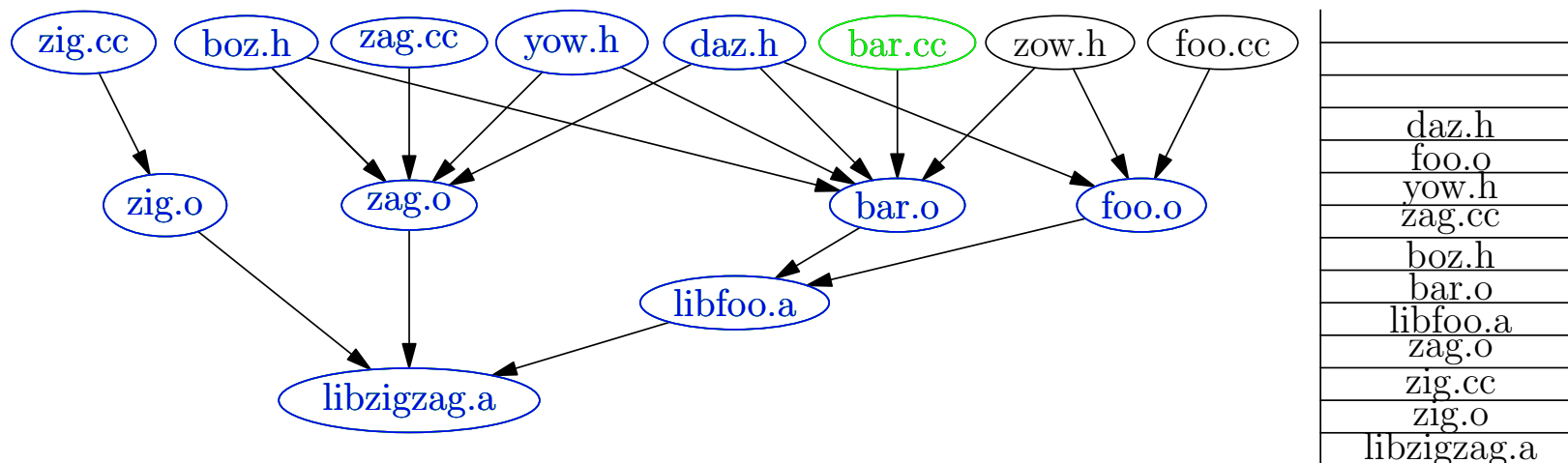
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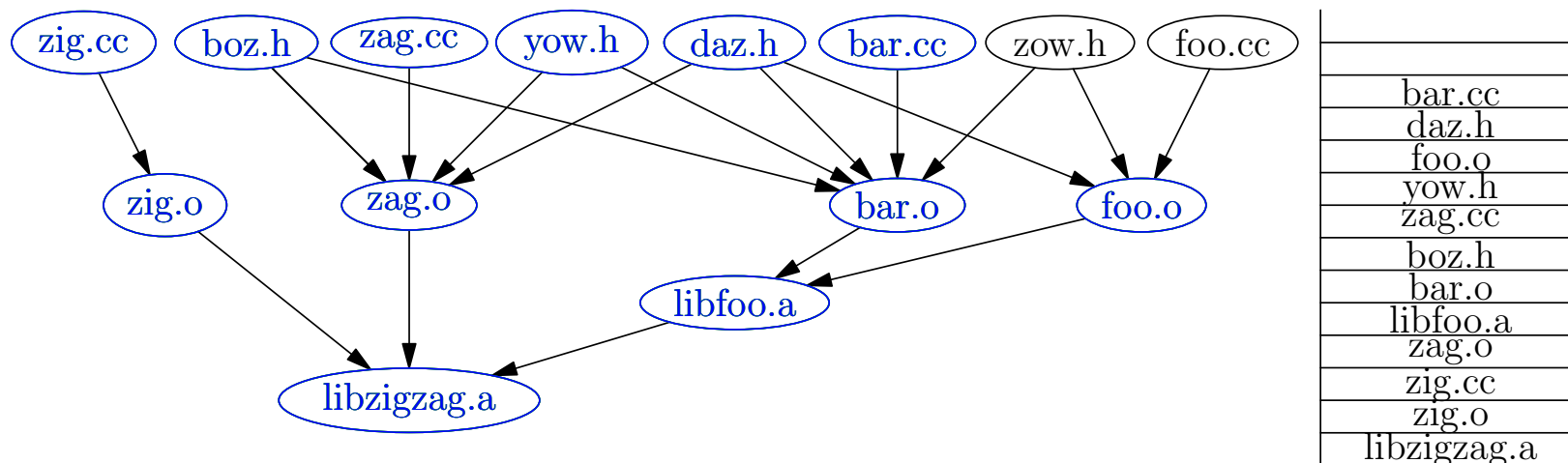
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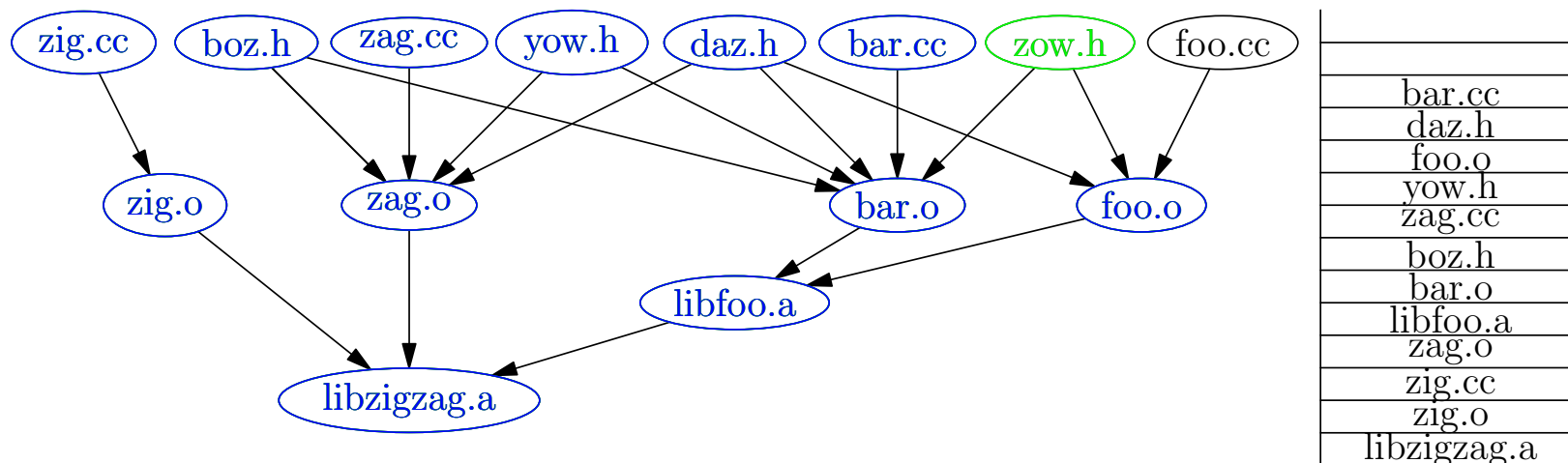
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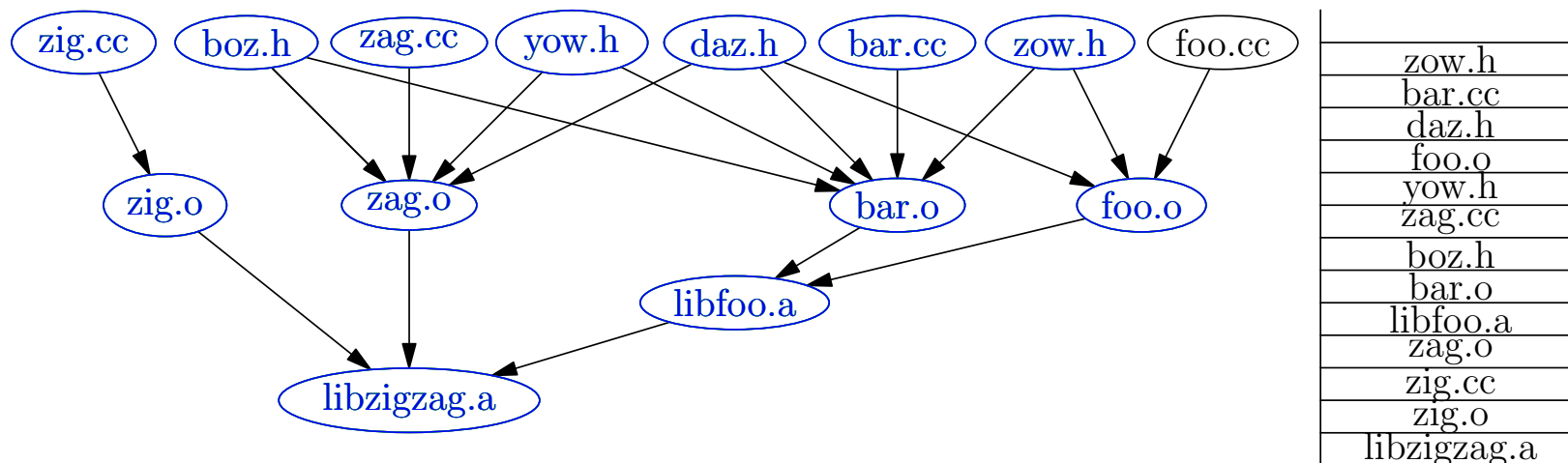
Algorithms and Analysis

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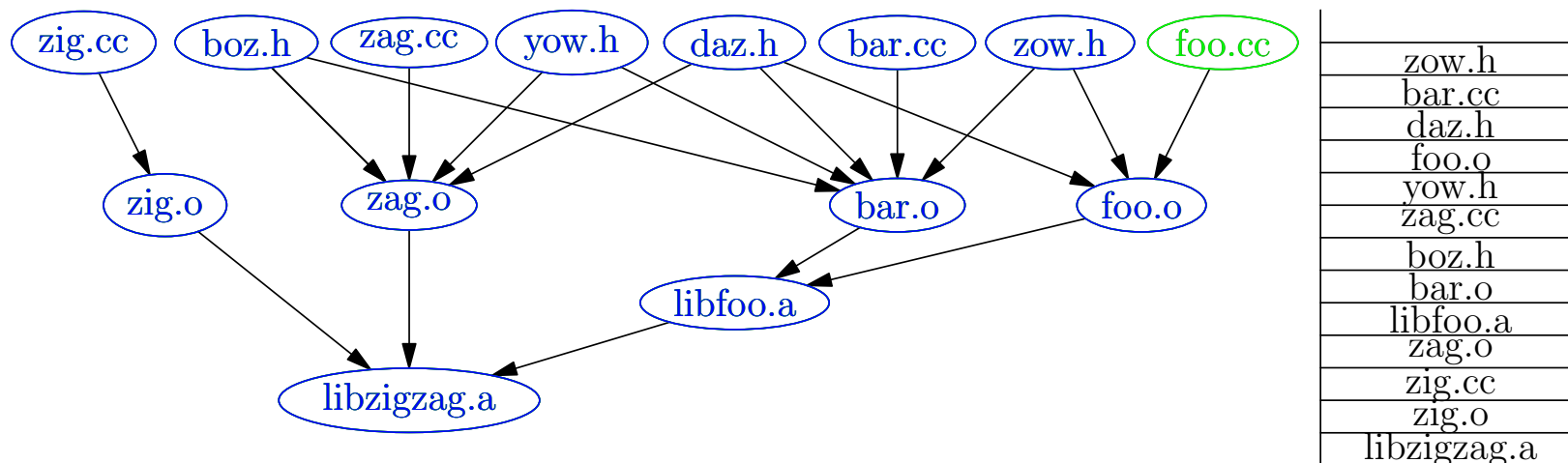
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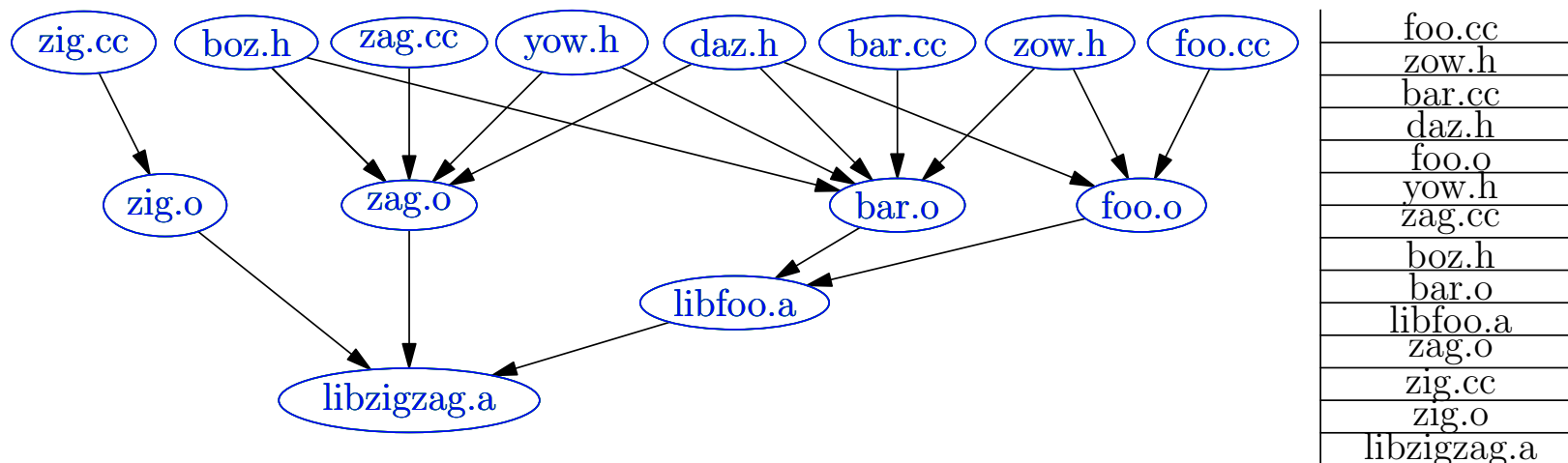
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# DFS on Digraphs

DFS for undirected graphs

```
dfs(graph, node) {
  if ("finished") return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] ≠ "discovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if ("finished") return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

# DFS on Digraphs

**DFS for directed graphs**

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  state[currentNode] ← "processed"
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}
```

# Implementing Topological Sort

• Given our DFS programme we define

```
topologicalSort(graph) {
    Stack stack
    for node ∈ graph.vertexSet()
        if (¬discovered[node])
            dfs(graph, node)

    List topSortList
    while (¬stack.isEmpty())
        topSortList.add(stack.pop())

    return topSortList
}
```

# Enhance DFS

- Requires us to define a couple of helper function

```
processVertexLast(node) {
  stack.push(node)
}

processEdge(currentNode, neighbour) {
  if (state[neighbour] == "processed") {
    print "error: graph not a DAG"
    finished = true
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# Implementation Issues

- Most awkward part of the implementation is that the `topologicalSort` algorithm needs access to dfs structures (`discovered[]`)

- `processVertexLast(node)` needs access to the stack

- Need to be able to redefine `processVertexFirst`, `processEdge` and `processVertexLast`

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    - ⋆ Java: JDSL, JGraphT
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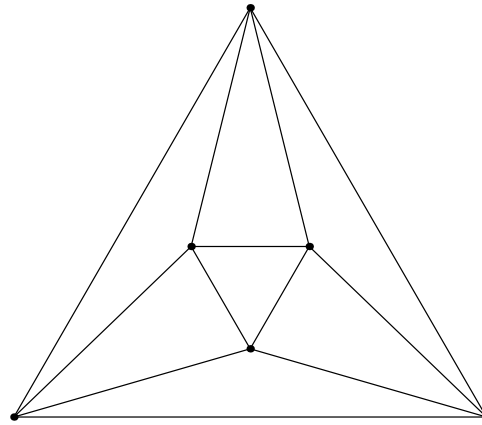    - ⋆ C++: Boost Graph Library, LEDA

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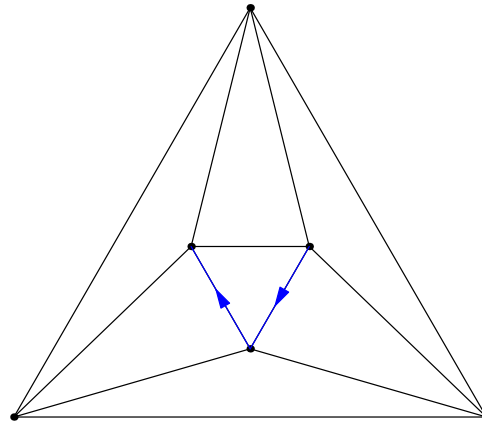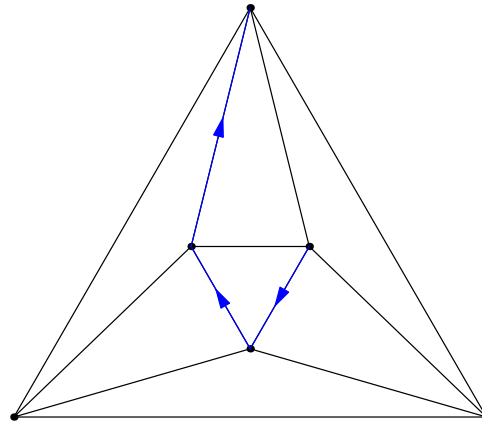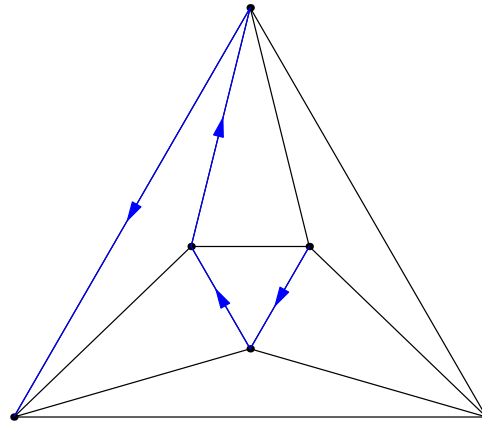
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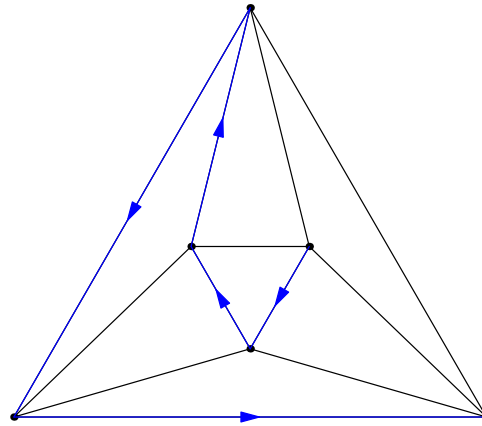
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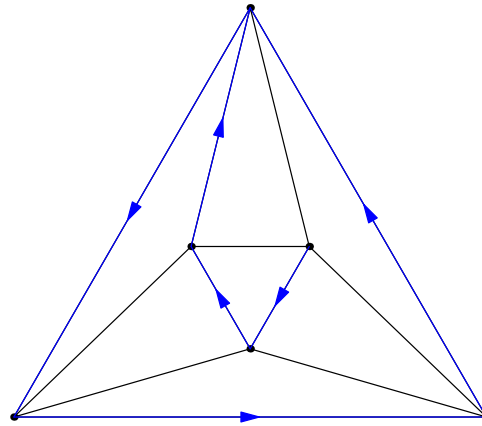- DFS is used for many other classic problems

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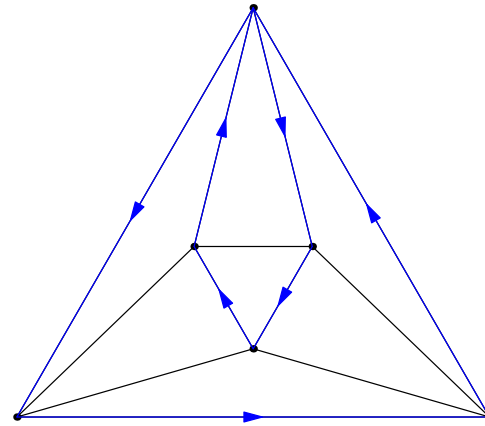
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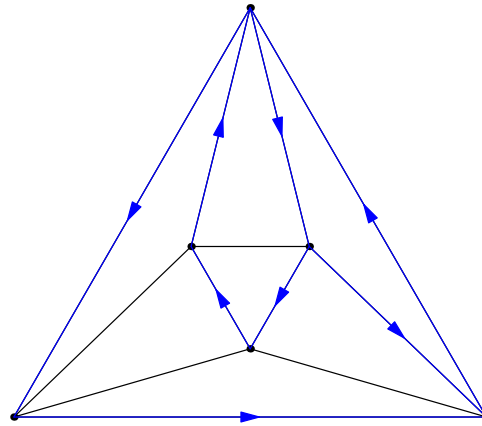- DFS is used for many other classic problems

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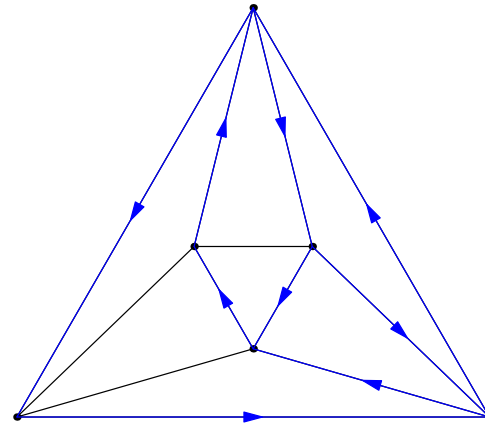- DFS is used for many other classic problems

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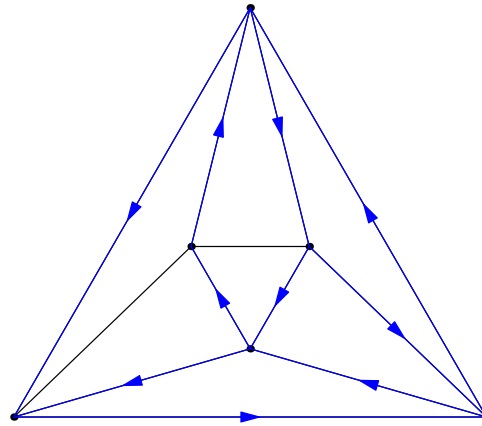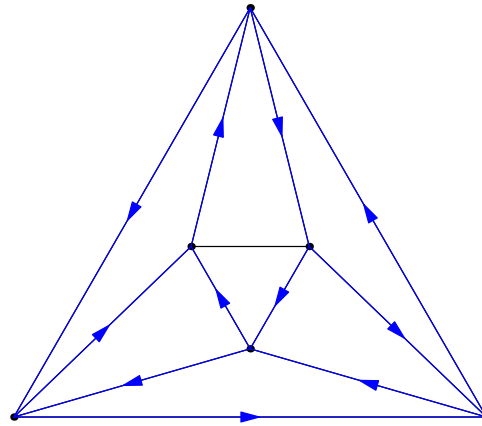
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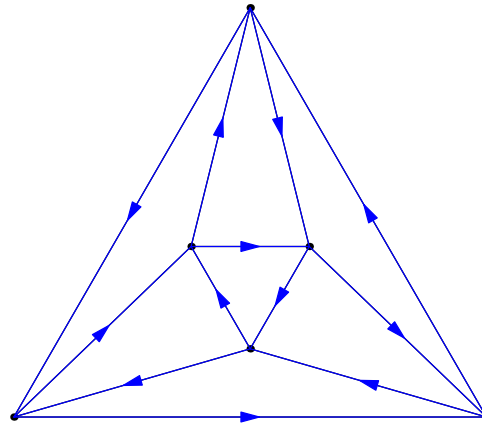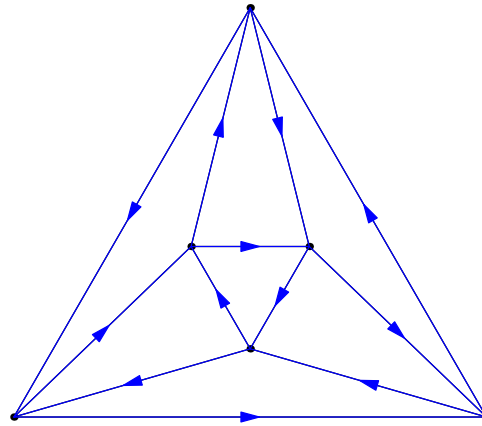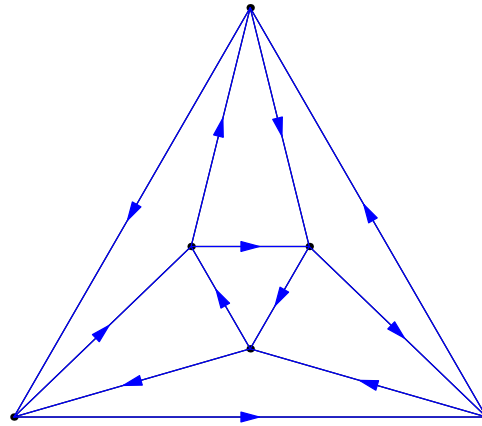
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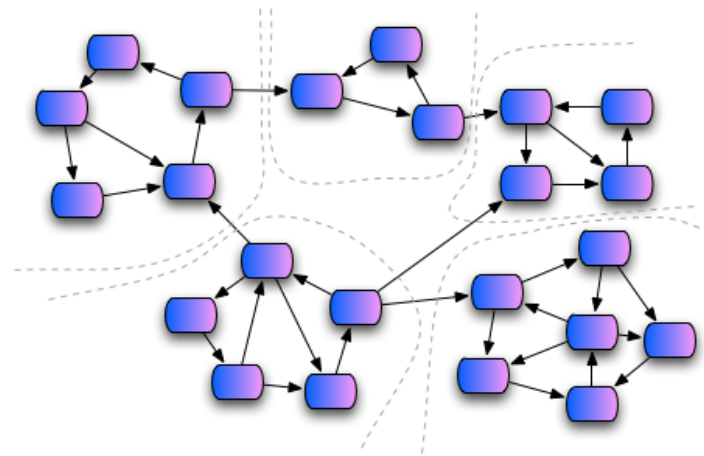
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# Lessons

- <span style="color:red">Breadth first and depth first search are different methods for traversing graphs</span>

- They are used as part of many specific algorithms for discovering graph properties

- Breadth first search is particularly important for finding shortest paths in unweighted graphs

- Depth first search is used in a whole host of applications (finding articulation points, Euler cycles, strongly connected components)

- One of the most used application is in topological sort (finding an ordering of processes represented by a DAG)

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