Further Mathematics and Algorithms

Lesson 8: Point to where you are going: links



Linked lists

Outline

- 1. References
- 2. Singly Linked List
- 3. Stacks and Queues
- 4. Doubly Linked List
- 5. Using Linked Lists
- 6. Skip Lists



- So far we have considered arrays where the data is stored in a contiguous chunk of memory
- This has the great advantage of allowing random access
- It has the disadvantage that it is expensive to add or remove data from the middle of the list or to rearrange the data
- A different approach is to use units of data that point to other units

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- There are a lot of important data structures using non-contiguous memory
 - ★ Binary trees
 - ★ Graphs
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- This is a classic data structure
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The building block for a linked list is a node class

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struct Node<T>
{
   Node(U value, Node<U> *node): value(value), next(node) {}
   T element;
   Node<T> *next;
}
```

```
Node<int> *node = new Node<int>(10, pt_to_next)
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- Note that node is the address of this node
- I make it a struct as this is a class where I want public access to the element and next
- I can make this class a private class of my linked list

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We can build a linked list by stringing nodes together



- A singly linked list has a single "pointer" to the next element
- A doubly linked list has "pointers" to the next and previous element—we will see this later
- We should be able to create a linked list, add elements, remove elements, see if an element exists, etc.

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Implementation

- We consider a lightweight implementation
- The class will have a head, a size counter and have a Node as a nested class

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class MyList {
private:
   template <typename U>
   struct Node{
     Node(U value, Node<U> *node): value(value), next(node) {}
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   Node<T> *head;
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Simple Methods

The constructor is simple (and not strictly necessary)

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unsigned size() const {return noElements;}
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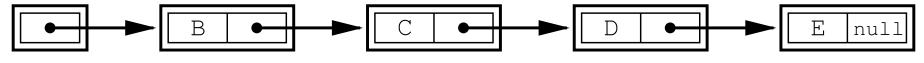
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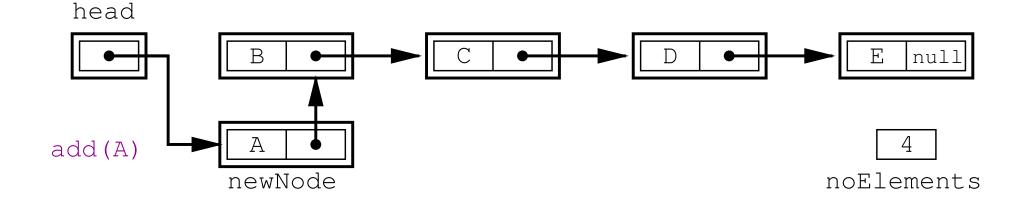
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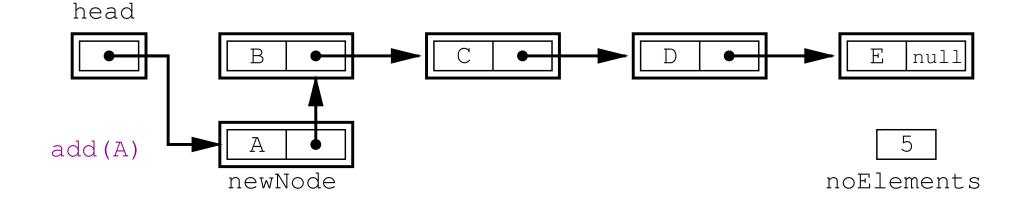
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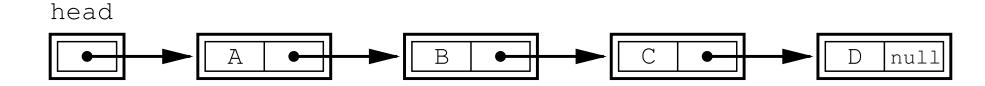
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Remove Head of List

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    delete dead;
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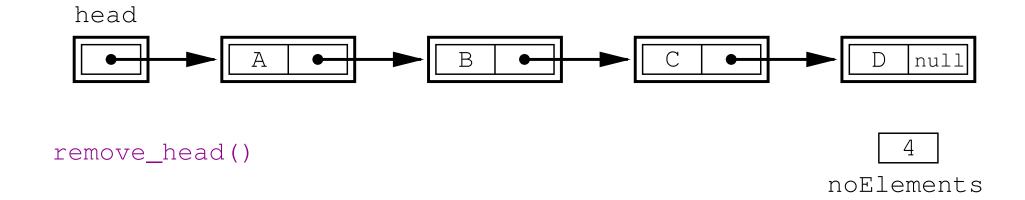
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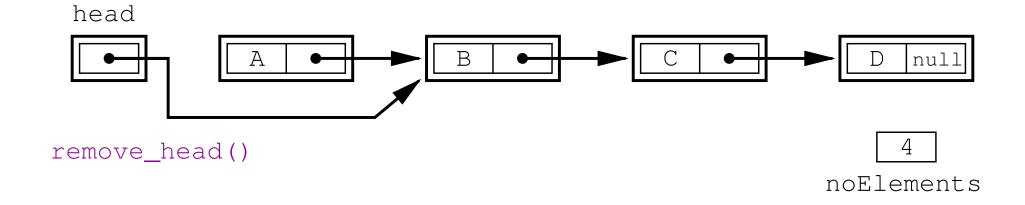
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noElements

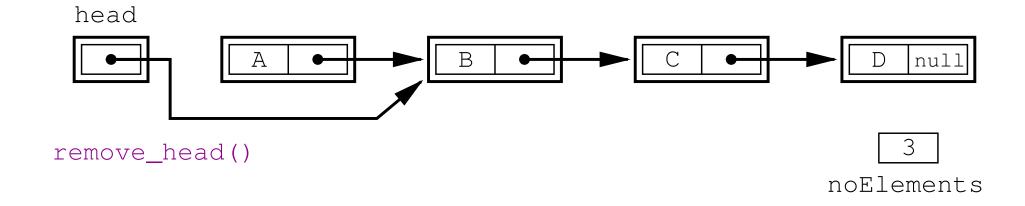
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We can easily implement many other methods

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\star get (int i) — return i^{th} item in list
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- ★ remove(T obj)-remove obj from list
- ★ insert(int position, element)
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Stack

It is easy to implement a stack using a linked list

```
template <typename T>
class Stack<E>
 private Mylist<T> list = new mylist<T>();
 boolean push(E obj) {list.add(obj);}
  E top() {return list.get_head();} // throw exception
 E pop() {
    T tmp = list.get_head();
    list.remove_head();
    return tmp;
 boolean empty() {return list.empty();}
```

- All operations of the stack is constant time, i.e. O(1)
- This is the same time complexity as an array implementation
- Memory requirement is approximately $2 \times n$ reference and n objects—same as worst case for an array
- However, hidden cost of creating and destroying Node objects
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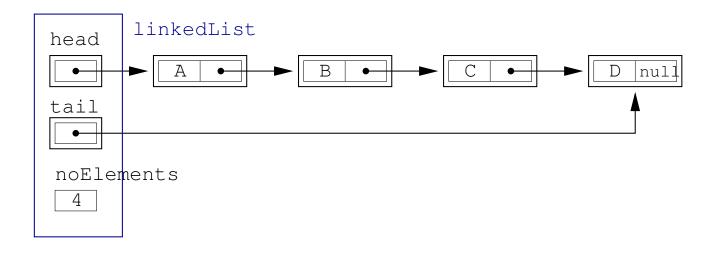
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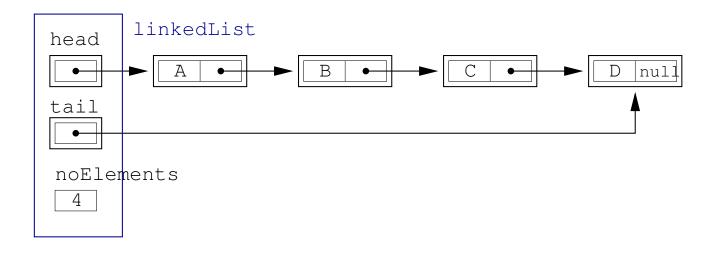
Point to the Back

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- Thus our linked list isn't the right data structure to implement a queue
- However, we could include a pointer to the end of the queue



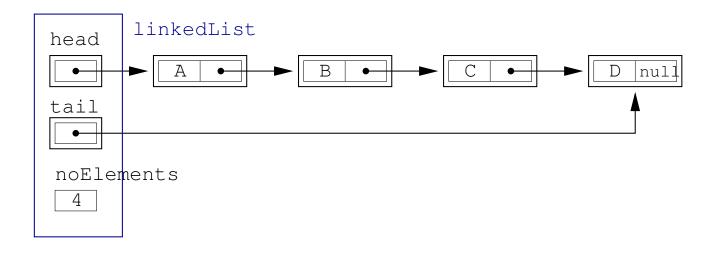
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- We can the implement a queue in O(1) time by
 - ★ enqueueing at the back
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- I leave the implementation details as an exercise for you
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- To achieve this it uses a doubly-linked lists with elements to next and previous

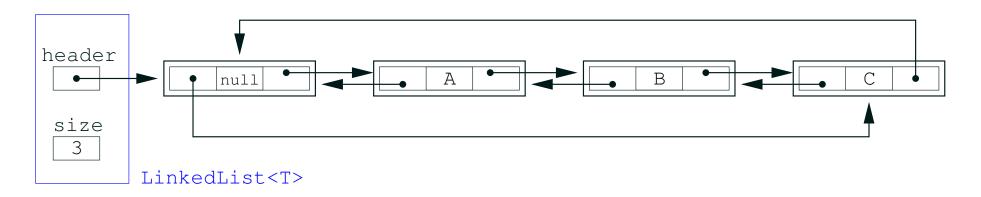
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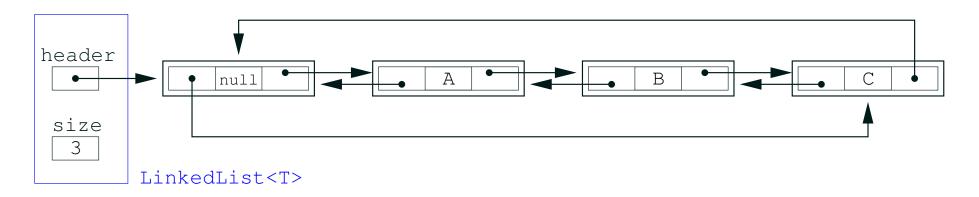
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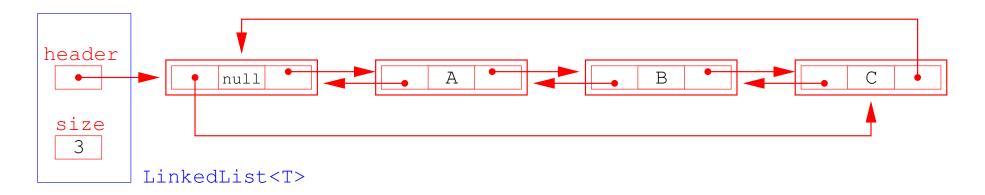
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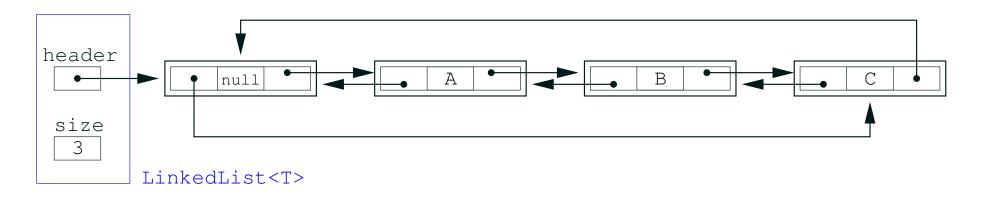
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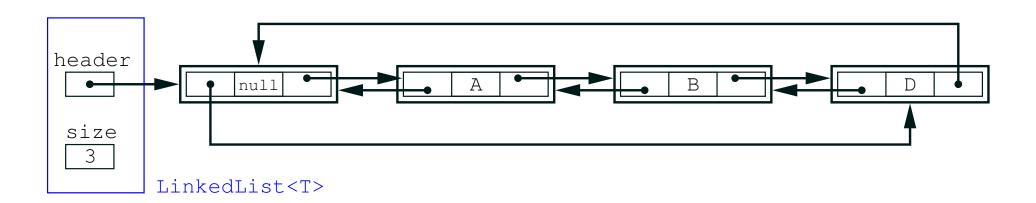


- add and remove from head and tail O(1)
- find O(n) and slow
- insert and delete O(1) (faster than an array list) once position is found

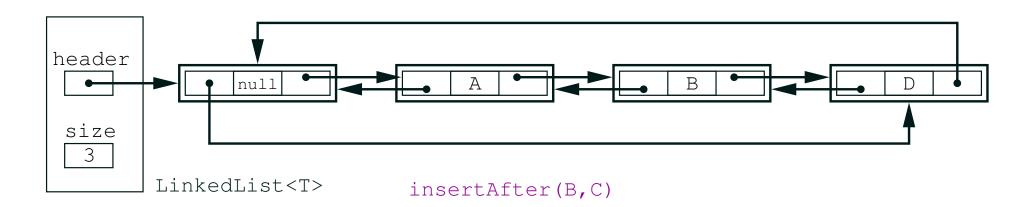
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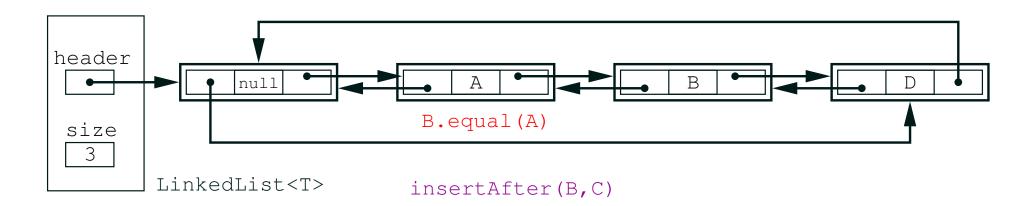
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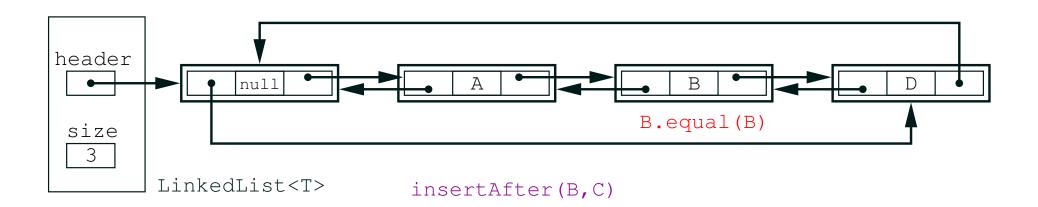
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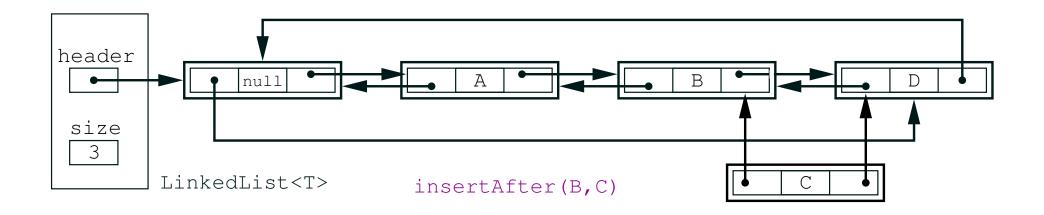
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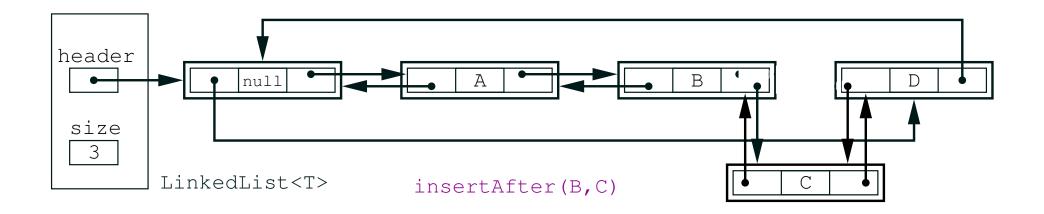
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- queues—linked list OK, but circular arrays are probably better
- sorted lists—binary trees much better
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- We are usually working at a particular location in the text
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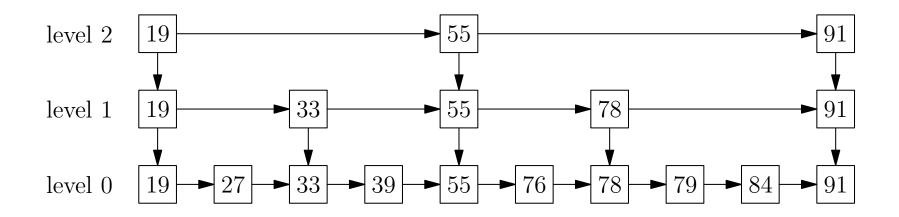
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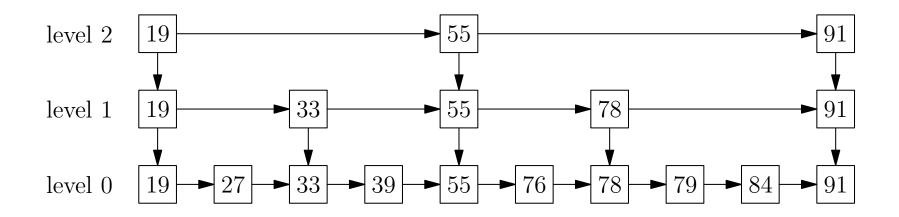
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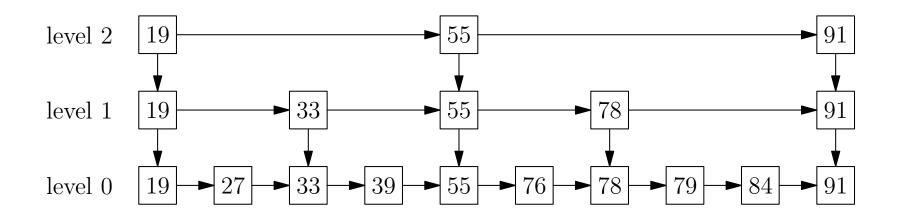
- Linked lists have the disadvantage that to get to anywhere in the list takes on average $\Theta(n)$ steps
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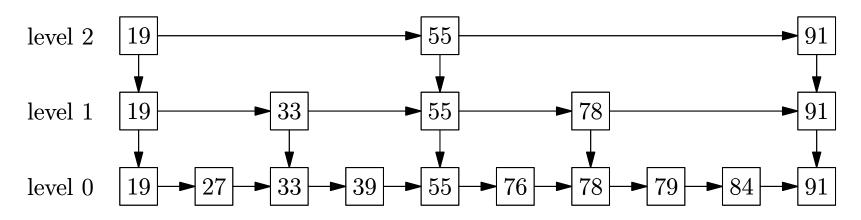


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contains(79)



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→ 39 |

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contains(79) level 2 19 55 91 level 1 19 78 91

level 0

→ 55 |

- Skip lists provide $\Theta(\log_2(n))$ search as opposed to $\Theta(n)$
- They have the similar time complexity to binary trees, although binary trees are slightly faster
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- Linked lists are the simplest examples of this kind of structure and consequently has a dominant position in most DSA books
- In practice linked lists are seldom the data structure of choice—before choosing to use a linked list consider the alternatives
- There are some important uses for linked lists, e.g. skip lists and hash tables (see lecture on hashing)

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