

Further Mathematics and Algorithms

Lesson 16: *Sort Wisely*



Merge sort, quick sort and radix sort

Outline

1. **Merge Sort**
2. Quick Sort
3. Radix Sort



Merge Sort

- Merge sort is an example of sort performed in log-linear (i.e. $O(n \log(n))$) time complexity
- It was invented in 1945 by John von Neumann
- It is an example of a divide-and-conquer strategy
 - ★ That is, the problem is divided into a number of parts recursively
 - ★ The full solution is obtained by recombining the parts

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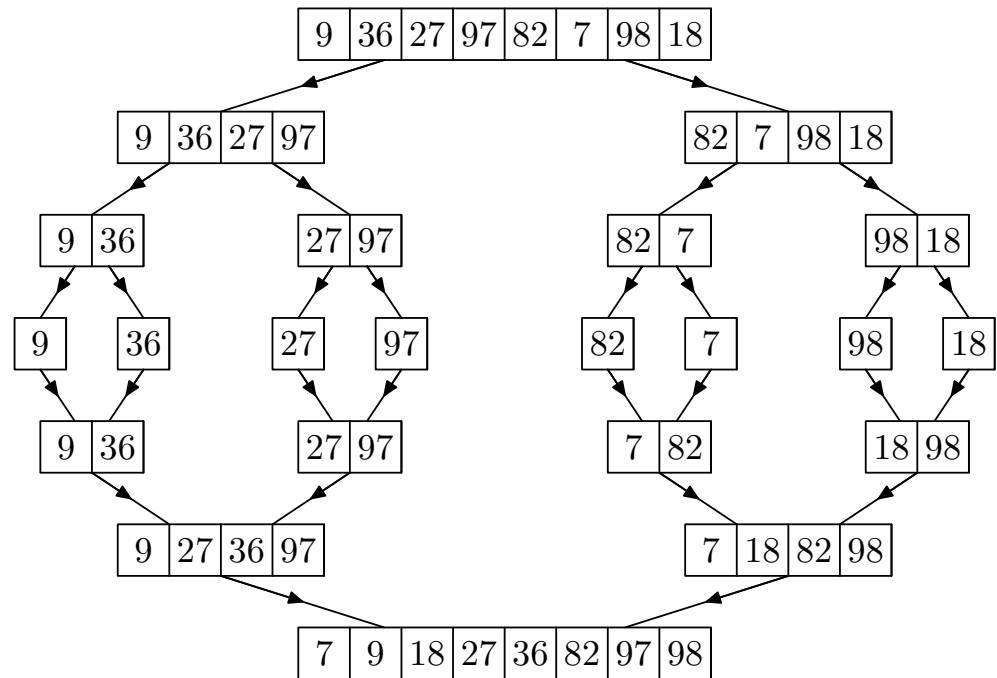
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Algorithm

```

MERGESORT (a)
{
  if  $n > 1$ 
    copy  $a[1 : \lfloor n/2 \rfloor]$  to b
    copy  $a[\lfloor n/2 \rfloor + 1 : n]$  to c
    MERGESORT (b)
    MERGESORT (c)
    MERGE (b, c, a)
  endif
}
    
```

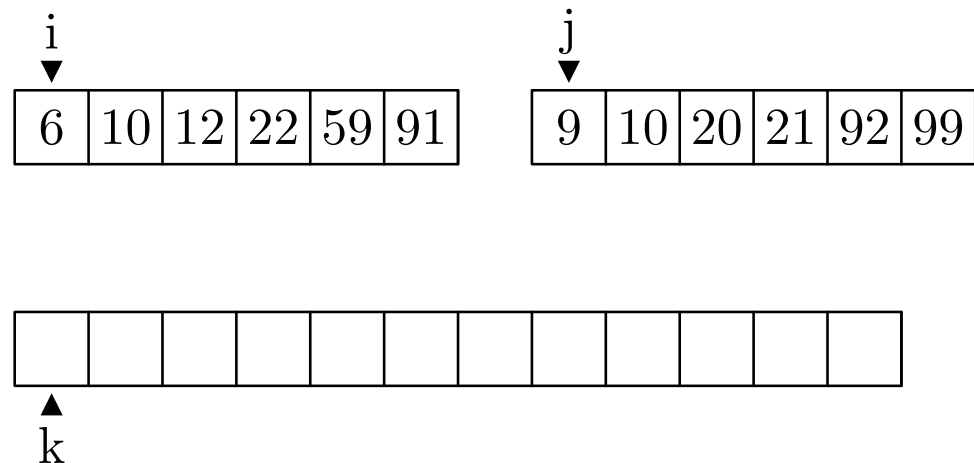


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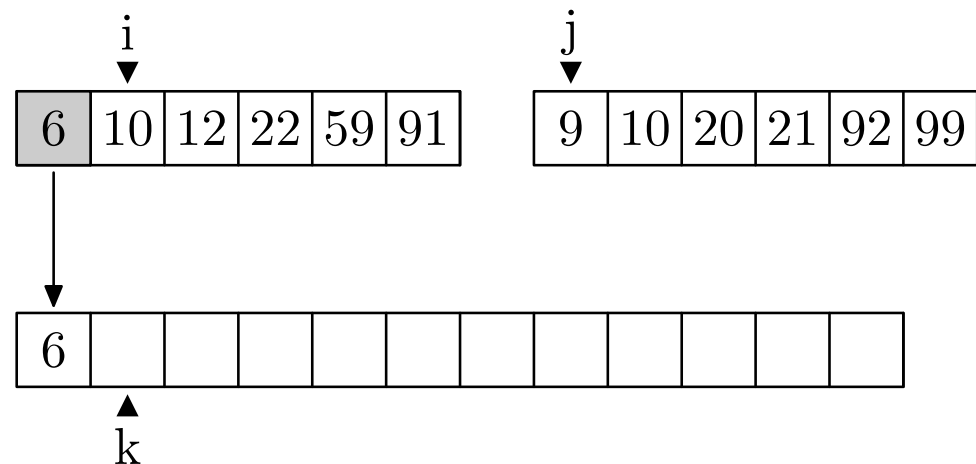
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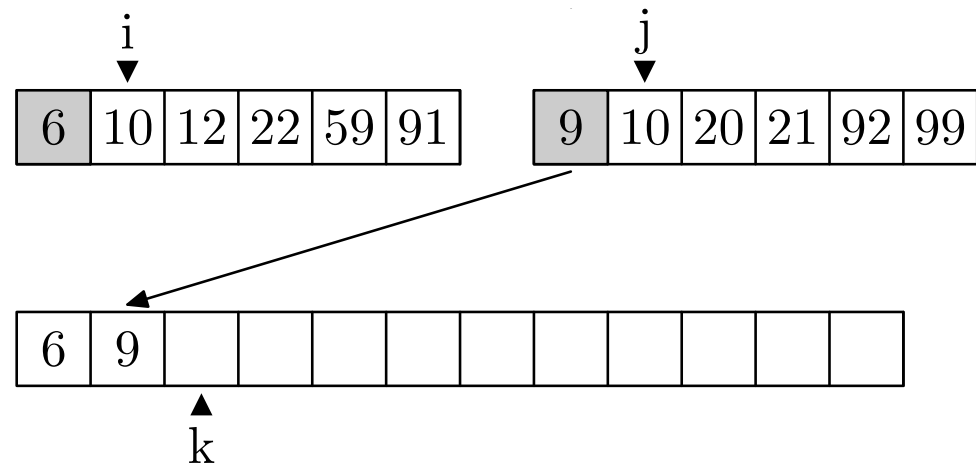
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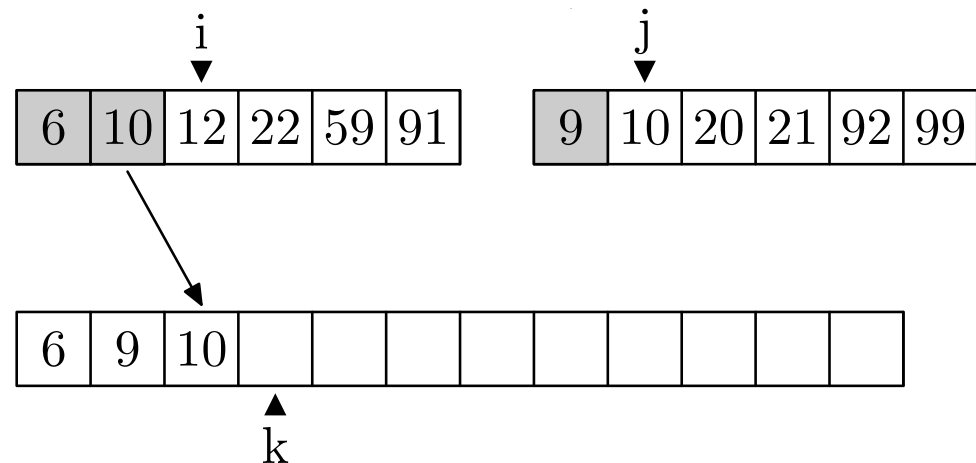
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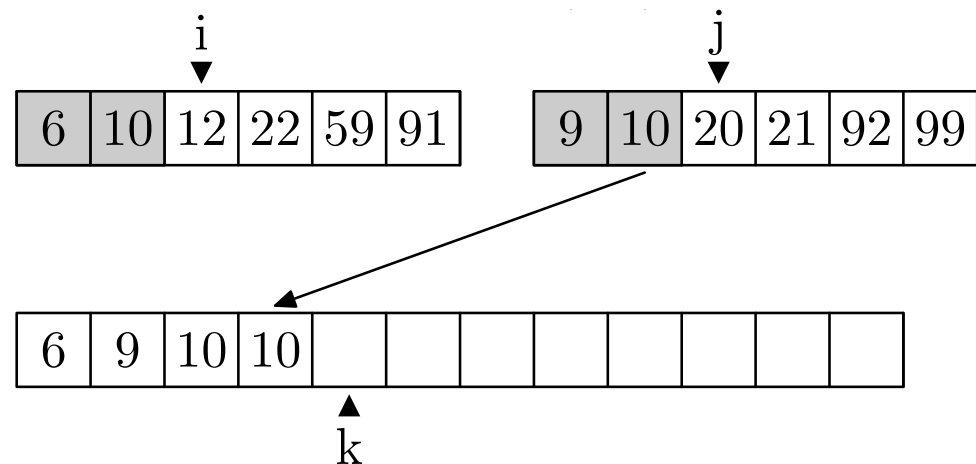
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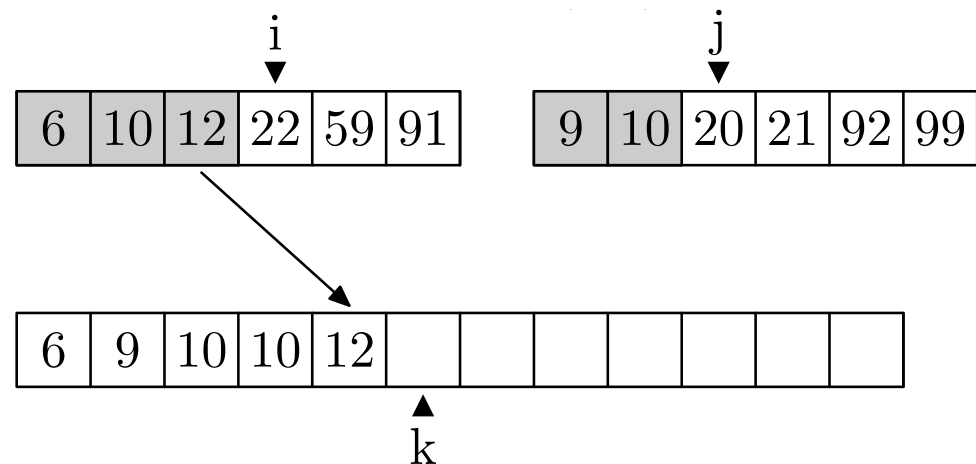
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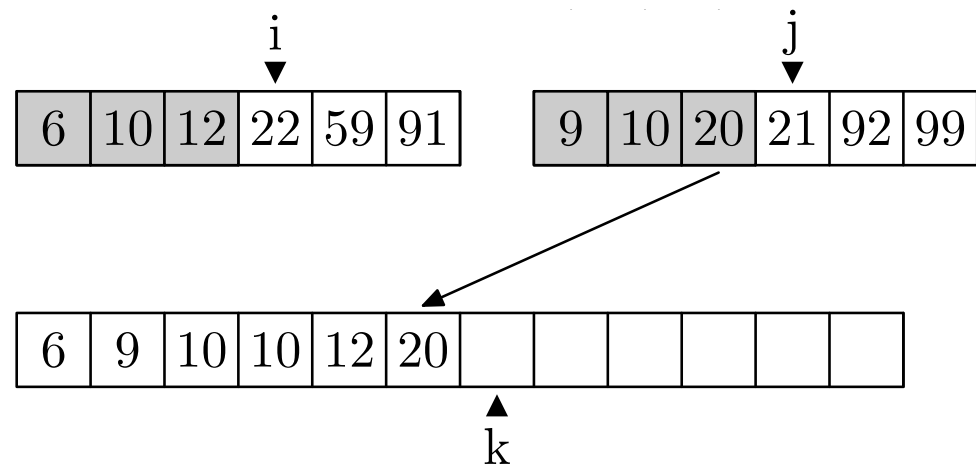
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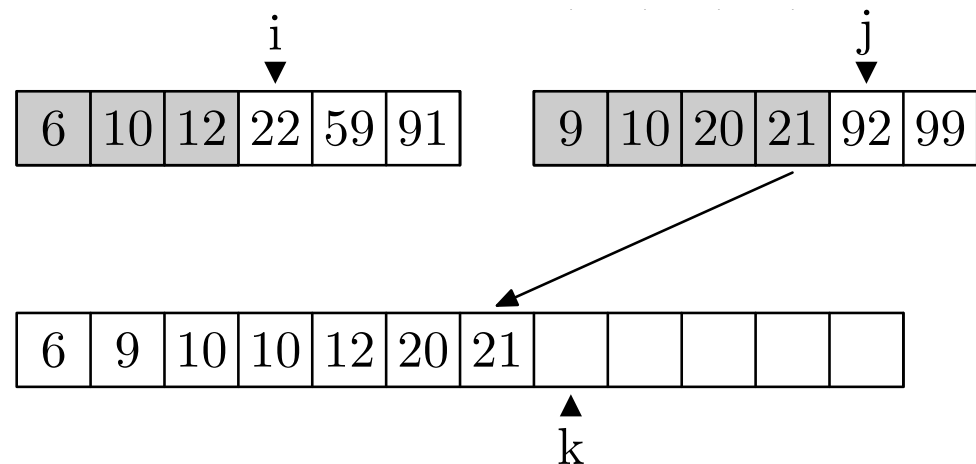
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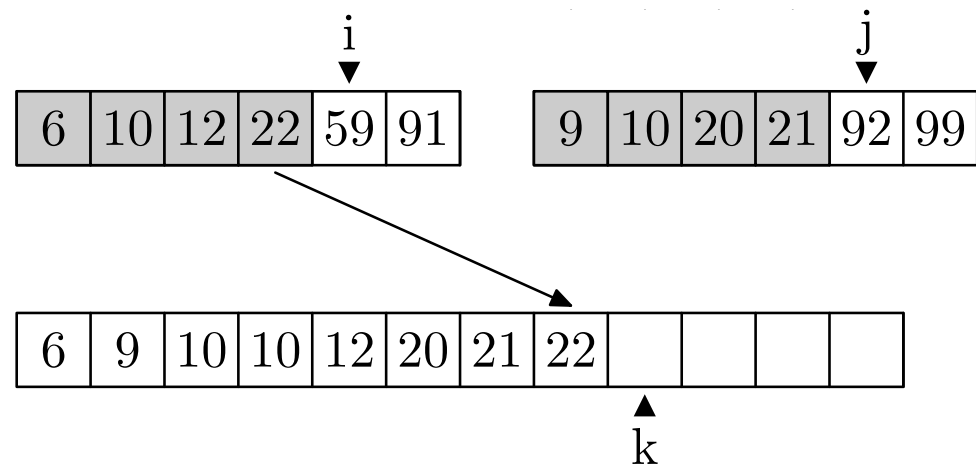


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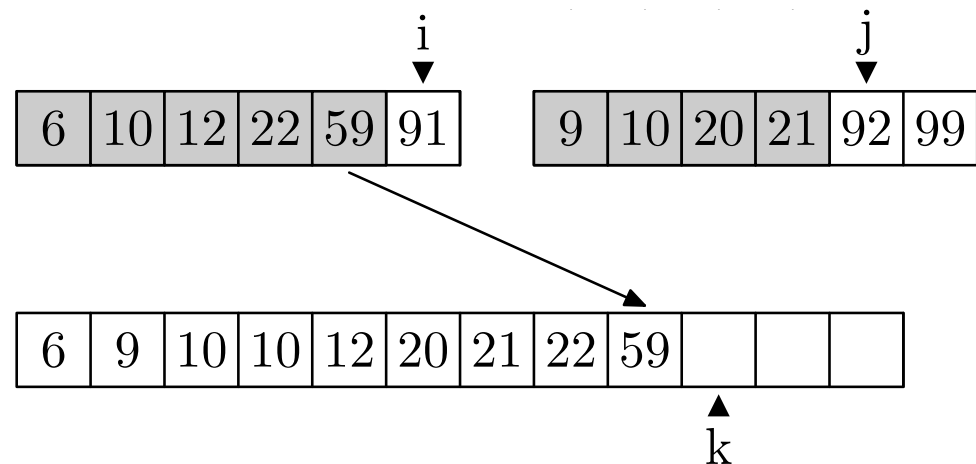
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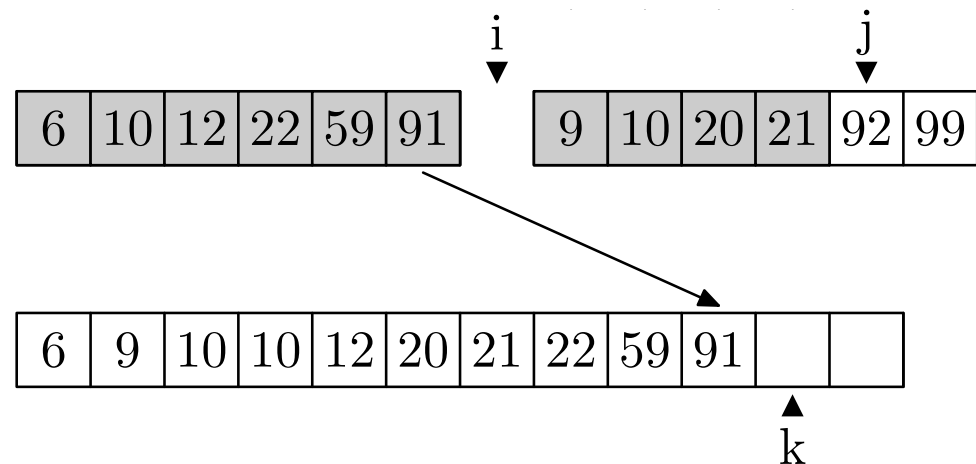
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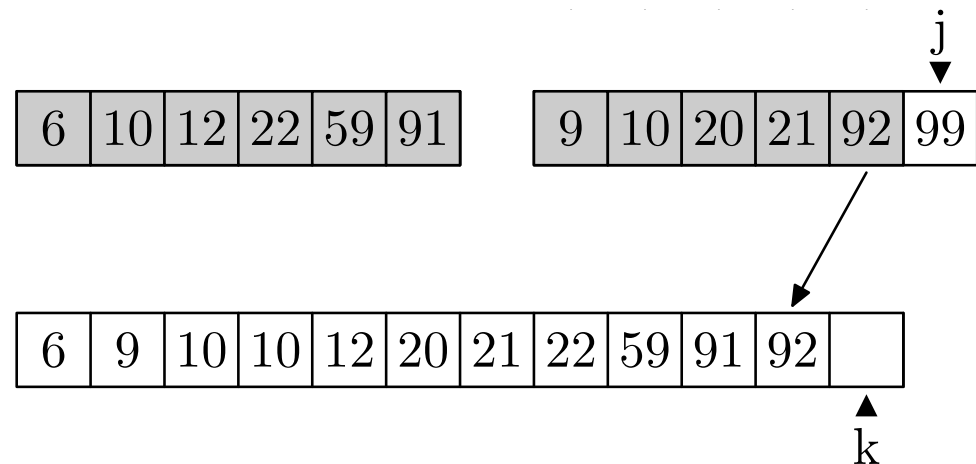
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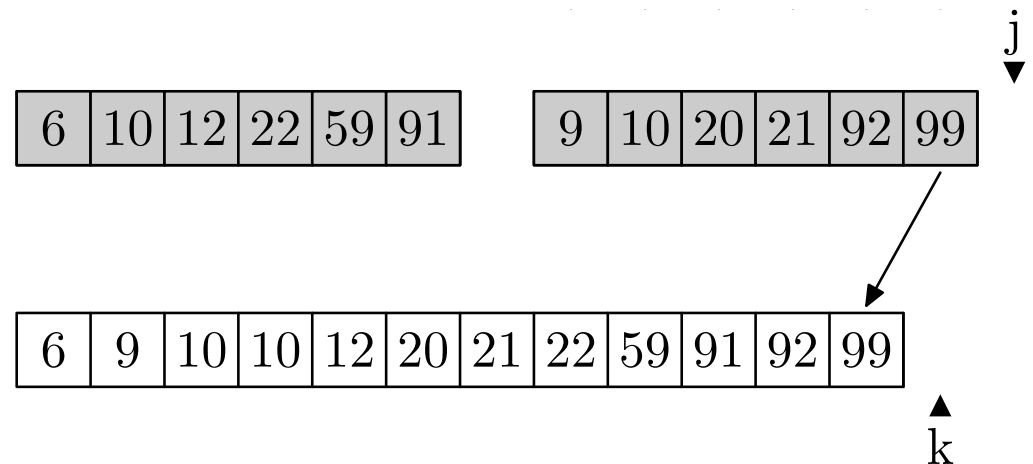


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Properties of Merge Sort

- Merge sort is stable provided we merge carefully (i.e. it preserves the order of two entries with the same value)
- Merge sort isn't in-place—we need an array of at most size n to do the merging
- Merging is quick. Given two arrays of size n the most number of comparisons we need to perform is $n - 1$

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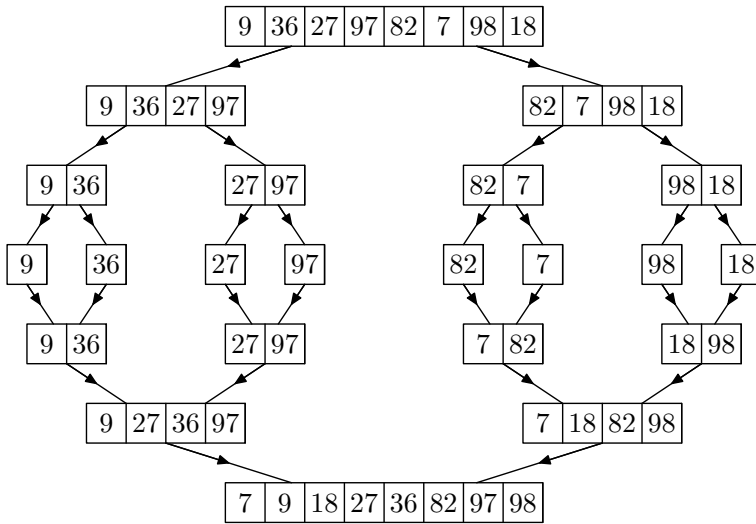
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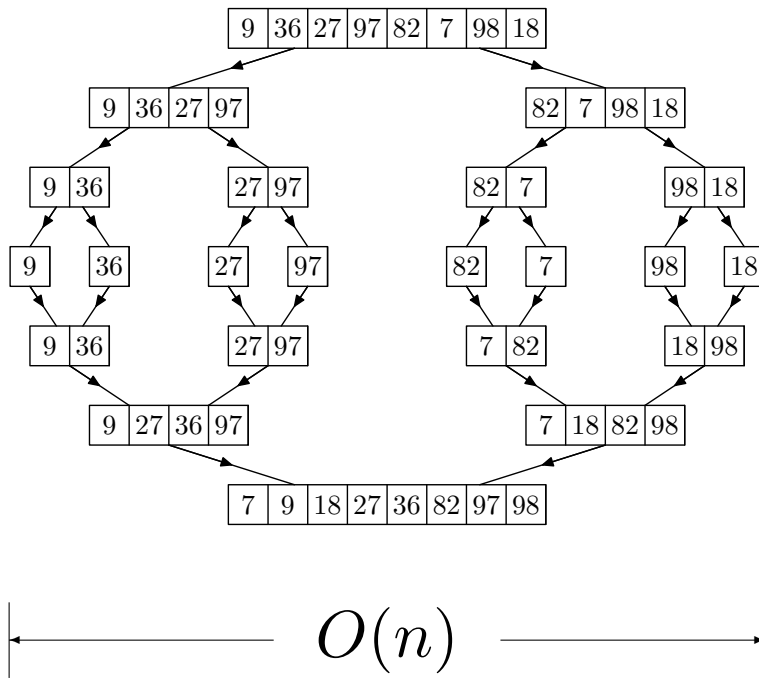
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Time Complexity of Merge Sort



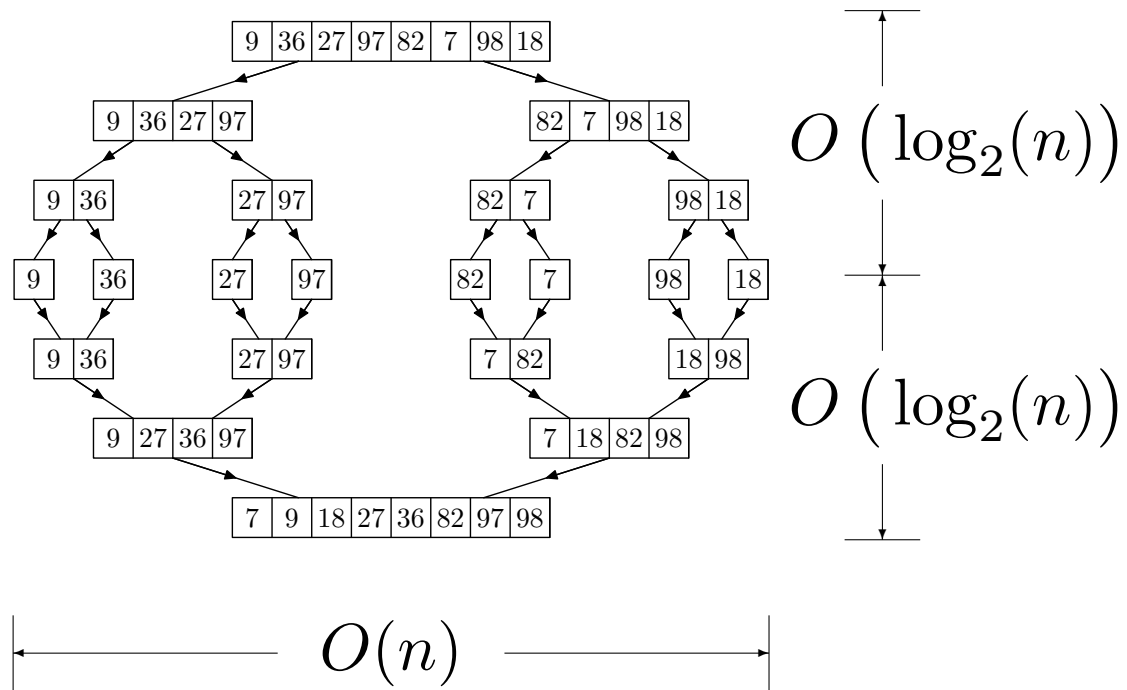
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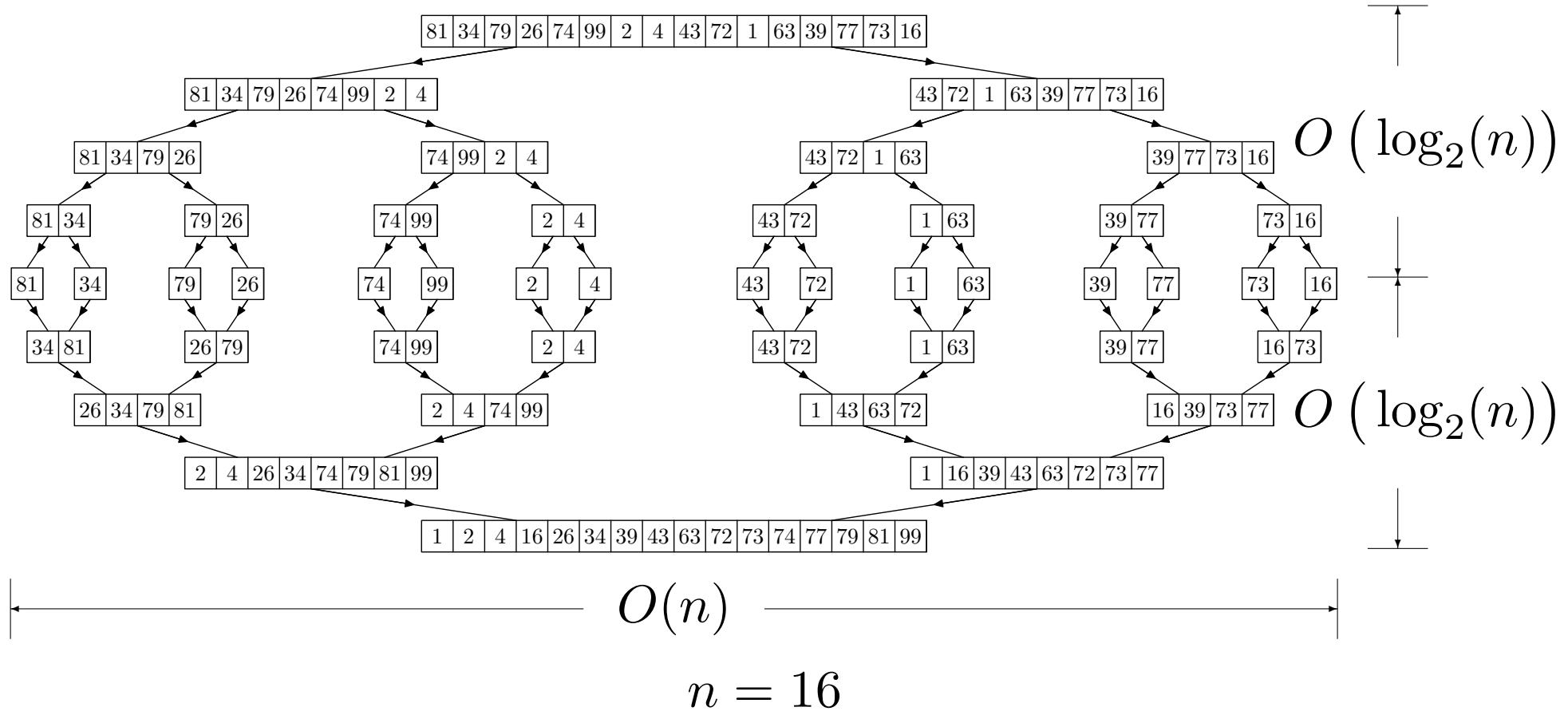
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Time Complexity of Merge Sort



Time Complexity

- We again measure the complexity in the number of comparisons
- From the above argument $C(n) = O(n \times \log_2(n))$
- We can be a bit more formal

$$C(n) = 2C(\lfloor n/2 \rfloor) + C_{\text{merge}}(n) \quad \text{for } n > 1$$

$$C(0) = 1$$

- But in the worst case $C_{\text{merge}}(n) = n - 1$
- Leads to $C_{\text{worst}}(n) = n \log_2(n) - n + 1$

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General Time Complexity

- In general if we have a recursion formula

$$T(n) = aT(n/b) + f(n)$$

with $a \geq 1, b > 1$

- If $f(n) \in \Theta(n^d)$ where $d \geq 0$ then

$$T(n) \in \begin{cases} \Theta(n^d) & \text{if } a < b^d \\ \Theta(n^d \log(n)) & \text{if } a = b^d \\ \Theta(n^{\log_d(a)}) & \text{if } a > b^d \end{cases}$$

- Analogous results hold for the family O and Ω

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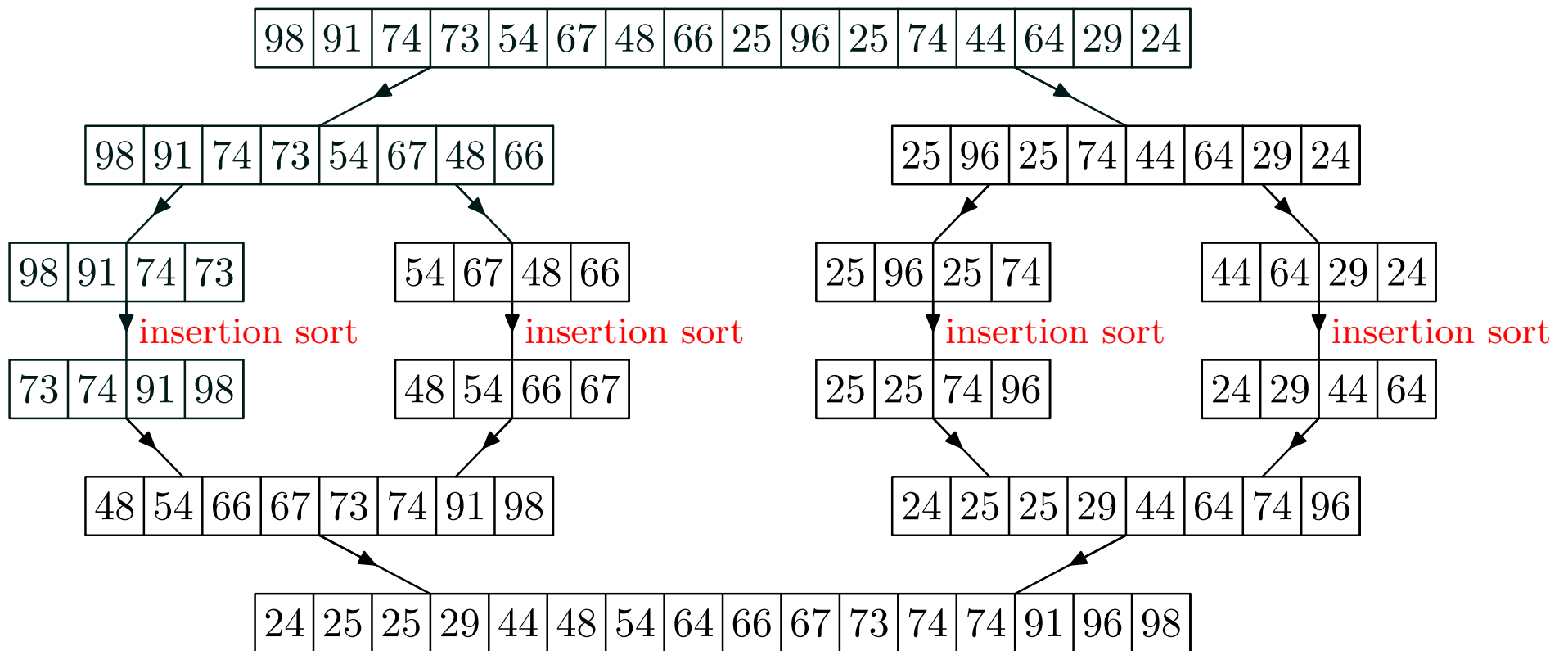
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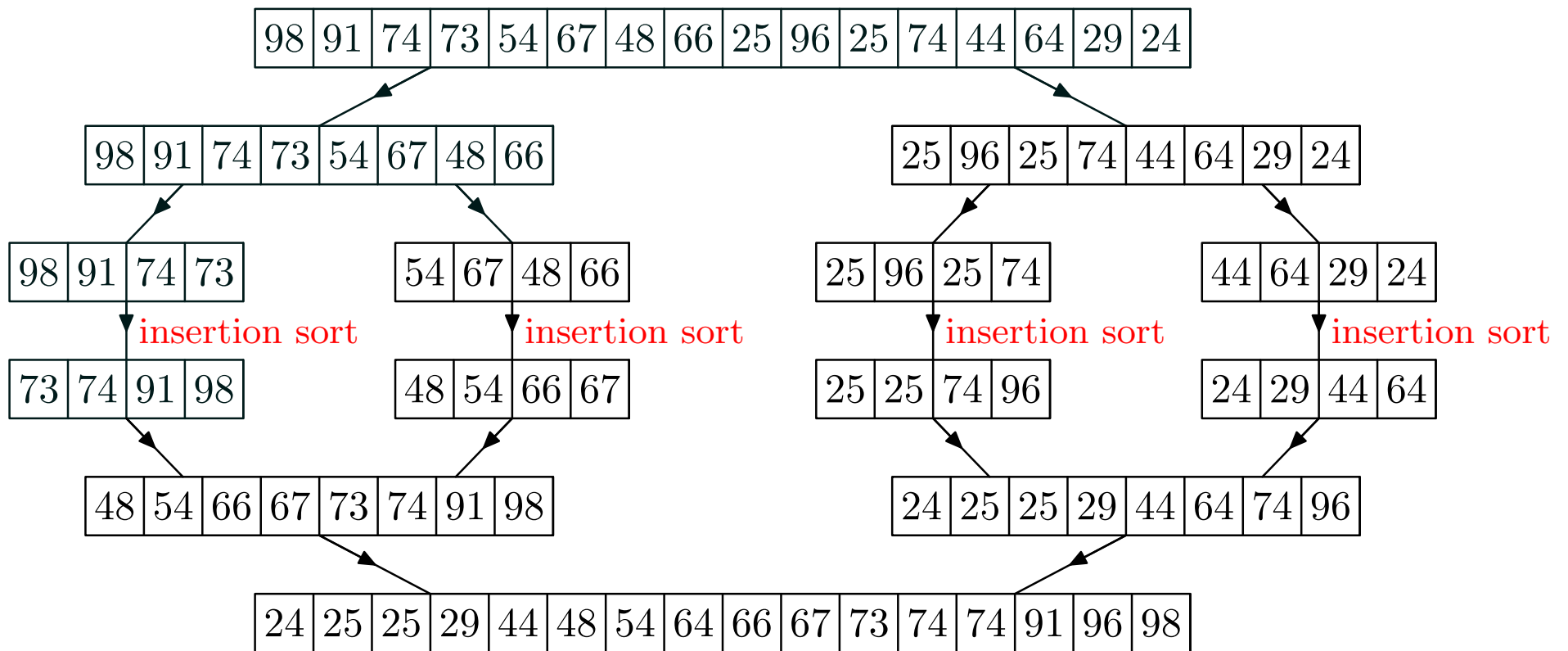
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- For very short sequences it is faster to use insertion sort than to pay the overhead of function calls



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Quicksort

- The most commonly used fast sorting algorithm is **quicksort**
- It was invented by the British computer scientist by C. A. R. Hoare in 1962
- It again uses the divide-and-conquer strategy
- It can be performed in-place, but it is **not** stable
- It works by splitting an array into two depending on whether the elements are less than or greater than a **pivot** value
- This is done recursively until the full array is sorted

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- It works by splitting an array into two depending on whether the elements are less than or greater than a **pivot** value
- This is done recursively until the full array is sorted

Quicksort

- The most commonly used fast sorting algorithm is **quicksort**
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Partition

- We need to partition the array around the pivot p such that

all elements $< p$	all elements $\geq p$
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PARTITION( $a$ ,  $p$ , left, right)
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pivot = 52

52	49	96	29	40	87	73	10	47	60	6	11	33	94	57	85
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pivot = 52

52	49	96	29	40	87	73	10	47	60	6	11	33	94	57	85
----	----	----	----	----	----	----	----	----	----	---	----	----	----	----	----

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↑
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↑
 j

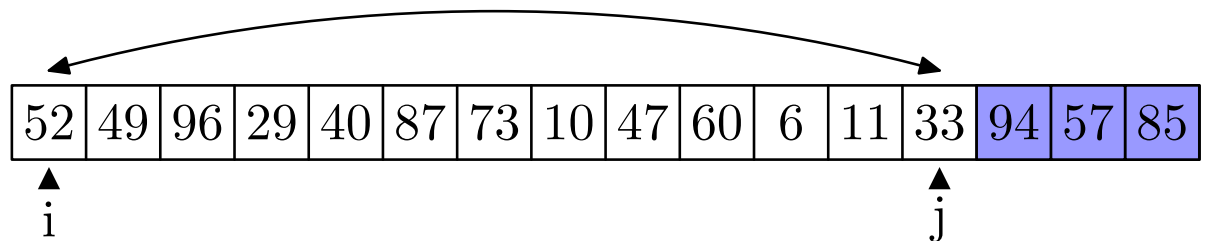
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↑
 i

↑
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Partition

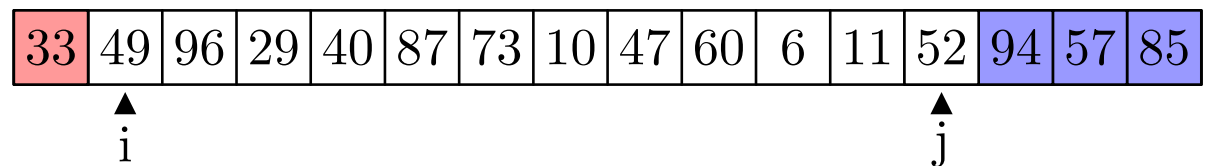
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PARTITION(*a*, p, left, right)

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```
i ← left
```

```
j ← right
```

```
repeat {
```

```
while  $a_i < p$ 
```

 $i++$

```
while  $a_j \geq p$ 
```

j--

```
if i ≥ j
```

break

$$\text{SWAP}(a_i, a_j)$$

}

}

pivot = 52

33	49	96	29	40	87	73	10	47	60	6	11	52	94	57	85
		▲ i											▲ j		

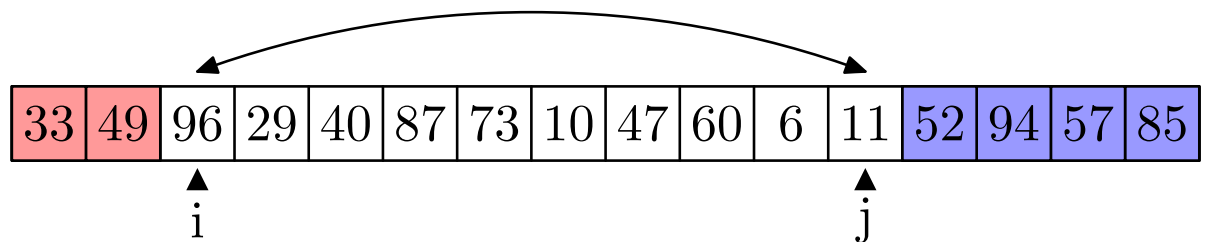
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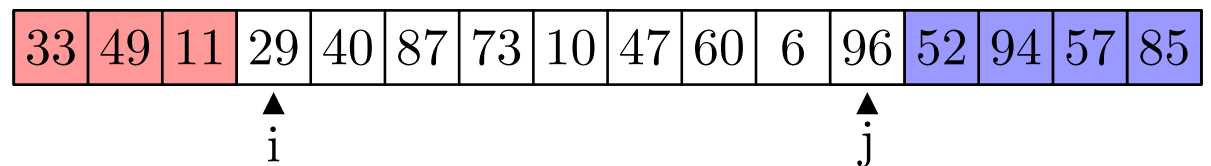
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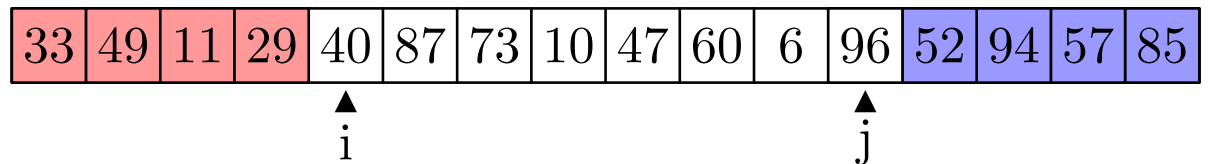
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Partition

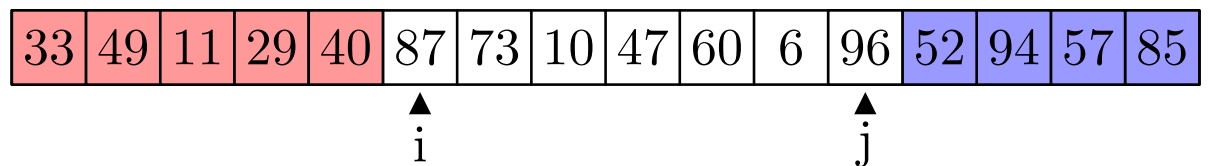
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▲
 i

▲
 j

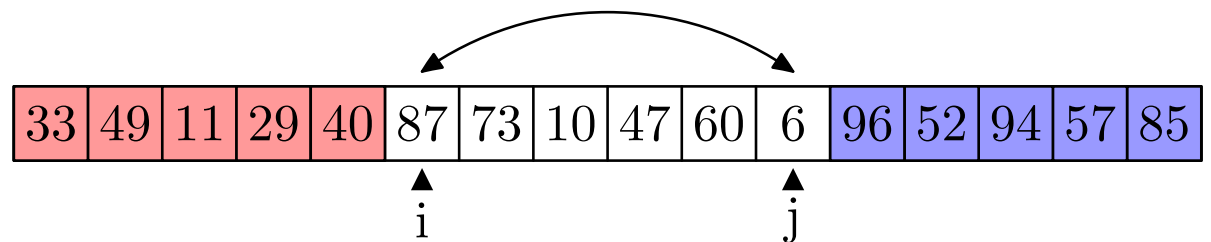
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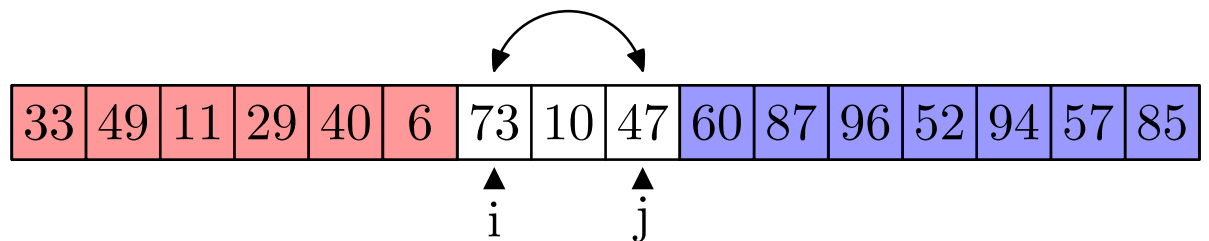
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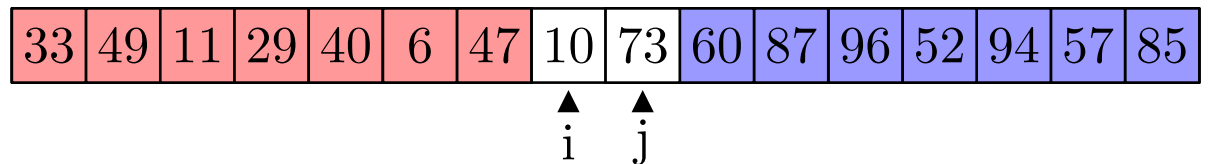
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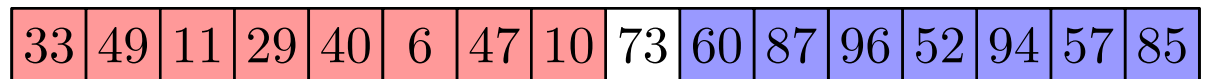
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Partition

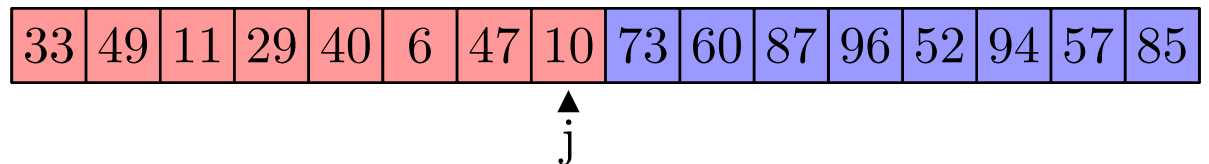
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  repeat {  
    while  $a_i < p$   
       $i++$   
    while  $a_j \geq p$   
       $j--$   
    if  $i \geq j$   
      break  
    SWAP( $a_i, a_j$ )  
  }  
}
```

pivot = 52



Optimising Partitioning

- There are different ways of performing the partitioning
- We want to minimise the time taken on the inner loop
- This means we want to perform as few checks as possible
- One method of doing this is to place *sentinels* at the ends of the array
- We can also reduce work by placing the partition in its correct position

all elements $\leq p$	p	all elements $\geq p$
-----------------------	-----	-----------------------

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all elements $\leq p$	p	all elements $\geq p$
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all elements $\leq p$	p	all elements $\geq p$
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all elements $\leq p$	p	all elements $\geq p$
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all elements $\leq p$	p	all elements $\geq p$
-----------------------	-----	-----------------------

Choosing the Pivot

- There are different strategies to choosing the pivot
- Choose the first element in the array
- Choose the median of the first, middle and last element of the array
- This increases the likelihood of the pivot being close to the median of the whole array
- For large arrays (above 40) the median of 3 medians is often used

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- Choose the first element in the array
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Choosing the Pivot

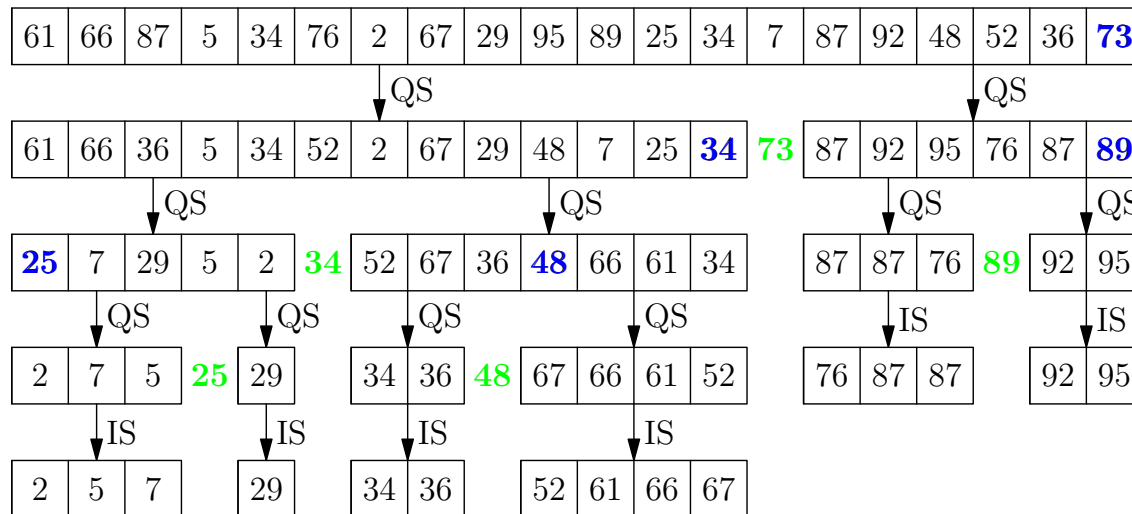
- There are different strategies to choosing the pivot
- Choose the first element in the array
- Choose the median of the first, middle and last element of the array
- This increases the likelihood of the pivot being close to the median of the whole array
- For large arrays (above 40) the median of 3 medians is often used

Quicksort

We recursively partition the array until each partition is small enough to sort using insertion sort

```

QUICKSORT(a, left, right) {
    if (right-left < threshold)
        INSERTIONSORT(a, left, right)
    else
        pivot = CHOOSEPIVOT(a, left, right)
        part = PARTITION(a, pivot, left, right)
        QUICKSORT(a, left, part-1)
        QUICKSORT(a, part+1, right)
    endif
}
    
```

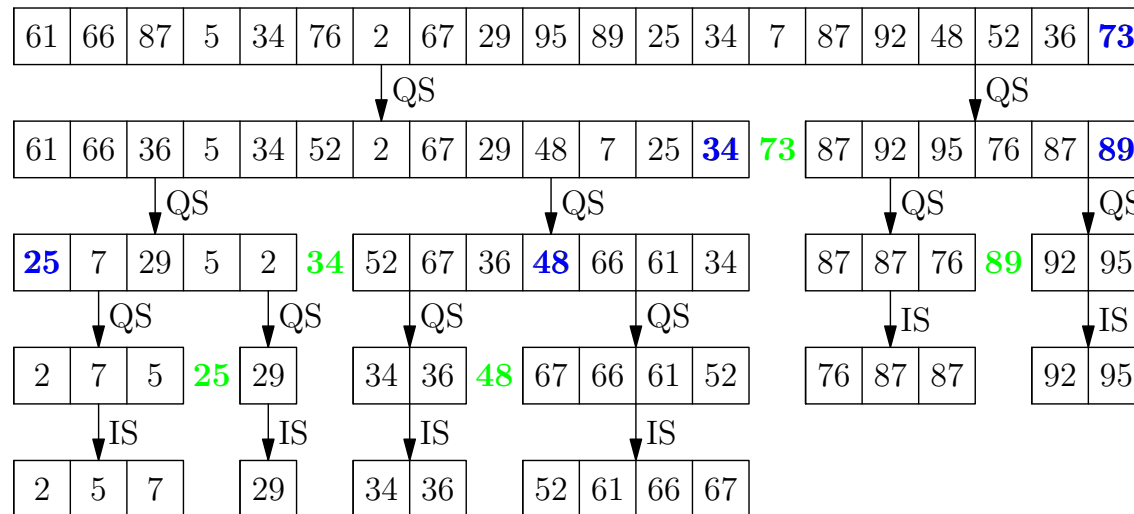


Quicksort

We recursively partition the array until each partition is small enough to sort using insertion sort

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    if (right-left < threshold)
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    else
        pivot = CHOOSEPIVOT(a, left, right)
        part = PARTITION(a, pivot, left, right)
        QUICKSORT(a, left, part-1)
        QUICKSORT(a, part+1, right)
    endif
}
    
```



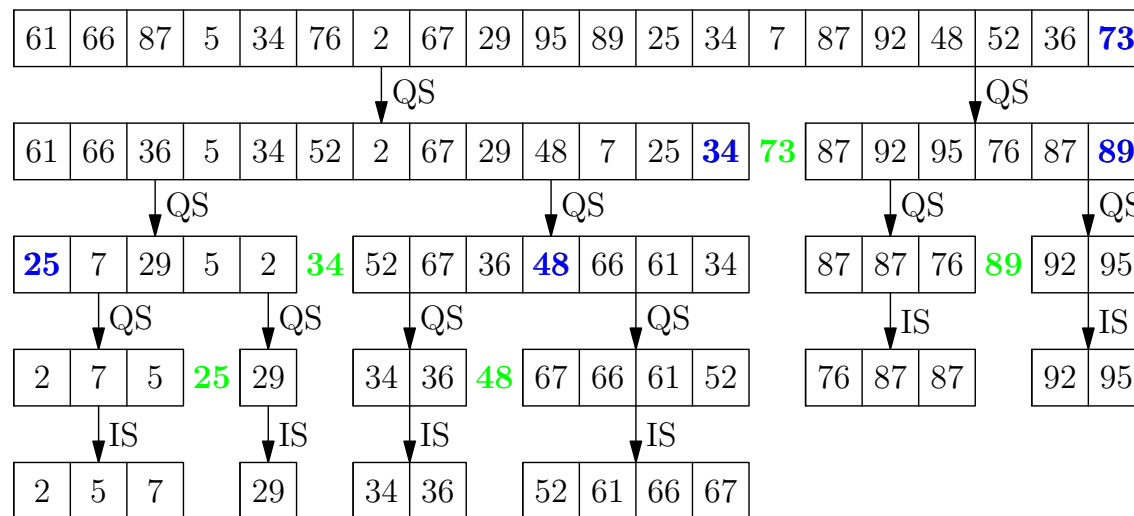
Quicksort

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```

QUICKSORT(a, left, right) {
    if (right-left < threshold)
        INSERTIONSORT(a, left, right)
    else
        pivot = CHOOSEPIVOT(a, left, right)
        part = PARTITION(a, pivot, left, right)
        QUICKSORT(a, left, part-1)
        QUICKSORT(a, part+1, right)
    endif
}

```



Time Complexity

- Partitioning an array of size n takes $\Theta(n)$ operations
- If we split the array in half then number of partitions we need to do is $\lceil \log_2(n) \rceil$
- This is the best case thus quicksort is $\Omega(n \log(n))$
- If the pivot is the minimum element of the array then we have to partition $n - 1$ times
- This is the worst case so quicksort is $O(n^2)$
- This worst case will happen if the array is already sorted and we choose the pivot to be the first element in the array!

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Time Complexity

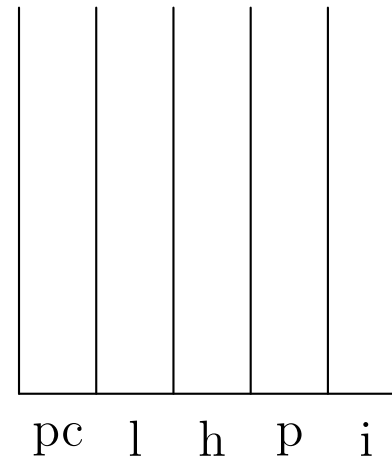
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Time Complexity

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- This worst case will happen if the array is already sorted and we choose the pivot to be the first element in the array!

QuickSort

```
0 quickSort(a, l, h) {  
1   if(h-l>3) {  
2       p = choosePivot(a, l, h)  
3       i = partition(a, p, l, h)  
4       quickSort(a, l, i-1)  
5       quickSort(a, i+1, h)  
6   } else  
7       insertionSort(a, l, h)  
8   return  
9 }
```



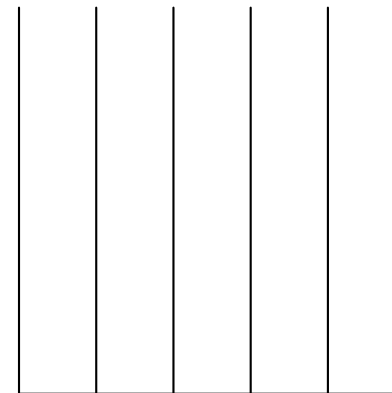
QuickSort

```

0 quickSort(a, 0, 19) {
1   if(19-0>3) {
2     p = choosePivot(a, 0, 19)
3     i = partition(a, p, 0, 19)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 0, 19)
8   return
9 }

```

PC	=	0
l	=	0
h	=	19
p	=	#
i	=	#



pc l h p i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	87	5	34	76	2	67	29	95	89	25	34	7	87	92	48	52	36	73

low

high

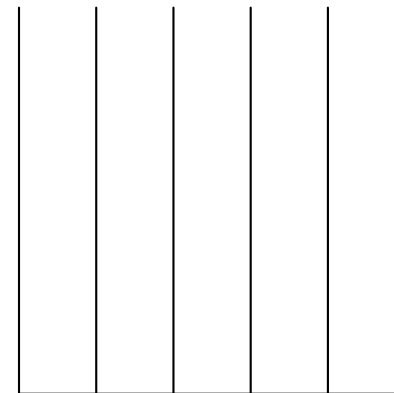
QuickSort

```

0 quickSort(a, 0, 19) {
1   if(19-0>3) {
2       p = choosePivot(a, 0, 19)
3       i = partition(a, p, 0, 19)
4       quickSort(a, 0, i-1)
5       quickSort(a, i+1, 19)
6   } else
7       insertionSort(a, 0, 19)
8   return
9 }

```

PC	=	1
l	=	0
h	=	19
p	=	#
i	=	#



pc l h p i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	87	5	34	76	2	67	29	95	89	25	34	7	87	92	48	52	36	73

low

high

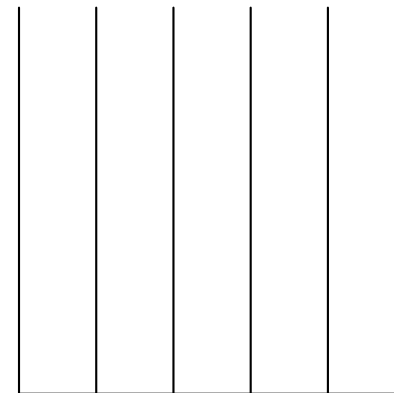
QuickSort

```

0 quickSort(a, 0, 19) {
1   if(19-0>3) {
2       p = choosePivot(a, 0, 19)
3       i = partition(a, p, 0, 19)
4       quickSort(a, 0, i-1)
5       quickSort(a, i+1, 19)
6   } else
7       insertionSort(a, 0, 19)
8   return
9 }

```

PC	=	2
l	=	0
h	=	19
p	=	#
i	=	#



pc l h p i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	87	5	34	76	2	67	29	95	89	25	34	7	87	92	48	52	36	73

low

high

QuickSort

```

0 quickSort(a, 0, 19) {
1   if(19-0>3) {
2     p = choosePivot(a, 0, 19)
3     i = partition(a, p, 0, 19)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 0, 19)
8   return
9 }

```

PC	=	2
l	=	0
h	=	19
p	=	#
i	=	#

2	0	19	#	#
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	87	5	34	76	2	67	29	95	89	25	34	7	87	92	48	52	36	73
low																			high

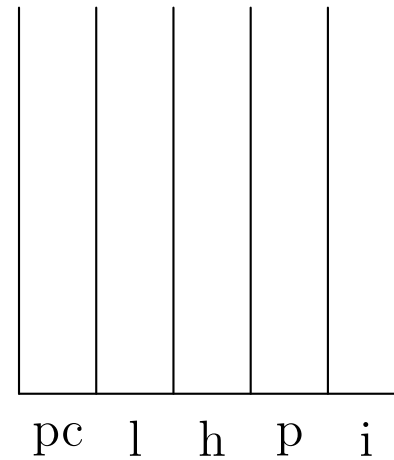
QuickSort

```

0 quickSort(a, 0, 19) {
1   if(19-0>3) {
2     p = choosePivot(a, 0, 19)
3     i = partition(a, 73, 0, 19)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 0, 19)
8   return
9 }

```

PC	=	3
l	=	0
h	=	19
p	=	73
i	=	#



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	87	5	34	76	2	67	29	95	89	25	34	7	87	92	48	52	36	73

low high

QuickSort

```

0 quickSort(a, 0, 19) {
1   if(19-0>3) {
2     p = choosePivot(a, 0, 19)
3     i = partition(a, 73, 0, 19)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 0, 19)
8   return
9 }

```

PC	=	3
l	=	0
h	=	19
p	=	73
i	=	#

3	0	19	73	#
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	36	5	34	52	2	67	29	48	7	25	34	73	87	92	95	76	87	89
low																			high

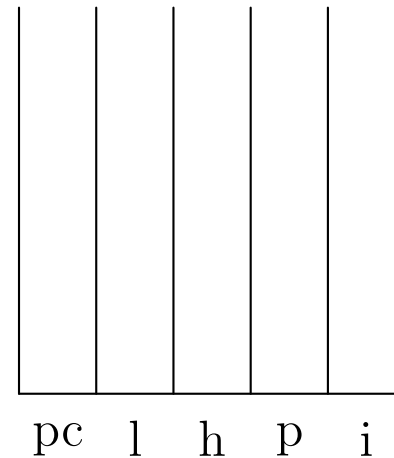
QuickSort

```

0 quickSort(a, 0, 19) {
1   if(19-0>3) {
2     p = choosePivot(a, 0, 19)
3     i = partition(a, 73, 0, 19)
4     quickSort(a, 0, 13-1)
5     quickSort(a, 13+1, 19)
6   } else
7     insertionSort(a, 0, 19)
8   return
9 }

```

PC	=	4
l	=	0
h	=	19
p	=	73
i	=	13



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	36	5	34	52	2	67	29	48	7	25	34	73	87	92	95	76	87	89

low high

QuickSort

```

0 quickSort(a, 0, 12) {
1   if(12-0>3) {
2     p = choosePivot(a, 0, 12)
3     i = partition(a, p, 0, 12)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 0, 12)
8   return
9 }

```

PC	=	0
l	=	0
h	=	12
p	=	#
i	=	#

4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	36	5	34	52	2	67	29	48	7	25	34	73	87	92	95	76	87	89
low										high									

QuickSort

```

0 quickSort(a, 0, 12) {
1   if(12-0>3) {
2       p = choosePivot(a, 0, 12)
3       i = partition(a, p, 0, 12)
4       quickSort(a, 0, i-1)
5       quickSort(a, i+1, 12)
6   } else
7       insertionSort(a, 0, 12)
8   return
9 }

```

PC = 1
l = 0
h = 12
p = #
i = #

4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	36	5	34	52	2	67	29	48	7	25	34	73	87	92	95	76	87	89
low																			high

QuickSort

```

0 quickSort(a, 0, 12) {
1   if(12-0>3) {
2     p = choosePivot(a, 0, 12)
3     i = partition(a, p, 0, 12)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 0, 12)
8   return
9 }

```

PC = 2
l = 0
h = 12
p = #
i = #

4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	36	5	34	52	2	67	29	48	7	25	34	73	87	92	95	76	87	89
low										high									

QuickSort

```

0 quickSort(a, 0, 12) {
1   if(12-0>3) {
2     p = choosePivot(a, 0, 12)
3     i = partition(a, p, 0, 12)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 0, 12)
8   return
9 }

```

PC	=	2
l	=	0
h	=	12
p	=	#
i	=	#

2	0	12	#	#
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	36	5	34	52	2	67	29	48	7	25	34	73	87	92	95	76	87	89
low												high							

QuickSort

```

0 quickSort(a, 0, 12) {
1   if(12-0>3) {
2     p = choosePivot(a, 0, 12)
3     i = partition(a, 34, 0, 12)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 0, 12)
8   return
9 }

```

PC	=	3
l	=	0
h	=	12
p	=	34
i	=	#

4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	36	5	34	52	2	67	29	48	7	25	34	73	87	92	95	76	87	89
low										high									

QuickSort

```

0 quickSort(a, 0, 12) {
1   if(12-0>3) {
2     p = choosePivot(a, 0, 12)
3     i = partition(a, 34, 0, 12)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 0, 12)
8   return
9 }

```

PC	=	3
l	=	0
h	=	12
p	=	34
i	=	#

3	0	12	34	#
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19		
25	7	29	5	2	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89		
					low											high					

QuickSort

```

0 quickSort(a, 0, 12) {
1   if(12-0>3) {
2     p = choosePivot(a, 0, 12)
3     i = partition(a, 34, 0, 12)
4     quickSort(a, 0, 5-1)
5     quickSort(a, 5+1, 12)
6   } else
7     insertionSort(a, 0, 12)
8   return
9 }

```

PC	=	4
l	=	0
h	=	12
p	=	34
i	=	5

4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
25	7	29	5	2	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low												high							

QuickSort

```

0 quickSort(a, 0, 4) {
1   if(4-0>3) {
2     p = choosePivot(a, 0, 4)
3     i = partition(a, p, 0, 4)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 4)
6   } else
7     insertionSort(a, 0, 4)
8   return
9 }

```

PC = 0
l = 0
h = 4
p = #
i = #

4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
25	7	29	5	2	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low				high															

QuickSort

```

0 quickSort(a, 0, 4) {
1   if(4-0>3) {
2       p = choosePivot(a, 0, 4)
3       i = partition(a, p, 0, 4)
4       quickSort(a, 0, i-1)
5       quickSort(a, i+1, 4)
6   } else
7       insertionSort(a, 0, 4)
8   return
9 }

```

PC = 1
l = 0
h = 4
p = #
i = #

4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
25	7	29	5	2	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low				high															

QuickSort

```

0 quickSort(a, 0, 4) {
1   if(4-0>3) {
2     p = choosePivot(a, 0, 4)
3     i = partition(a, p, 0, 4)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 4)
6   } else
7     insertionSort(a, 0, 4)
8   return
9 }

```

PC = 2
l = 0
h = 4
p = #
i = #

4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
25	7	29	5	2	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low				high															

QuickSort

```

0 quickSort(a, 0, 4) {
1   if(4-0>3) {
2     p = choosePivot(a, 0, 4)
3     i = partition(a, p, 0, 4)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 4)
6   } else
7     insertionSort(a, 0, 4)
8   return
9 }

```

PC	=	2
l	=	0
h	=	4
p	=	#
i	=	#

2	0	4	#	#
4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
25	7	29	5	2	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low				high															

QuickSort

```

0 quickSort(a, 0, 4) {
1   if(4-0>3) {
2     p = choosePivot(a, 0, 4)
3     i = partition(a, 25, 0, 4)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 4)
6   } else
7     insertionSort(a, 0, 4)
8   return
9 }

```

PC	=	3
l	=	0
h	=	4
p	=	25
i	=	#

4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
25	7	29	5	2	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low				high															

QuickSort

```

0 quickSort(a, 0, 4) {
1   if(4-0>3) {
2     p = choosePivot(a, 0, 4)
3     i = partition(a, 25, 0, 4)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 4)
6   } else
7     insertionSort(a, 0, 4)
8   return
9 }

```

PC	=	3
l	=	0
h	=	4
p	=	25
i	=	#

3	0	4	25	#
4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	7	5	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low			high																

QuickSort

```

0 quickSort(a, 0, 4) {
1   if(4-0>3) {
2     p = choosePivot(a, 0, 4)
3     i = partition(a, 25, 0, 4)
4     quickSort(a, 0, 3-1)
5     quickSort(a, 3+1, 4)
6   } else
7     insertionSort(a, 0, 4)
8   return
9 }

```

PC	=	4
l	=	0
h	=	4
p	=	25
i	=	3

4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	7	5	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low				high															

QuickSort

```

0 quickSort(a, 0, 2) {
1   if(2-0>3) {
2     p = choosePivot(a, 0, 2)
3     i = partition(a, p, 0, 2)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 2)
6   } else
7     insertionSort(a, 0, 2)
8   return
9 }

```

PC	=	0
l	=	0
h	=	2
p	=	#
i	=	#

4	0	4	25	3
4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	7	5	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low		high																	

QuickSort

```

0 quickSort(a, 0, 2) {
1   if(2-0>3) {
2     p = choosePivot(a, 0, 2)
3     i = partition(a, p, 0, 2)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 2)
6   } else
7     insertionSort(a, 0, 2)
8   return
9 }

```

PC	=	1
l	=	0
h	=	2
p	=	#
i	=	#

4	0	4	25	3
4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	7	5	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low		high																	

QuickSort

```

0 quickSort(a, 0, 2) {
1   if(2-0>3) {
2     p = choosePivot(a, 0, 2)
3     i = partition(a, p, 0, 2)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 2)
6   } else
7     insertionSort(a, 0, 2)
8   return
9 }

```

PC = 7
l = 0
h = 2
p = #
i = #

4	0	4	25	3
4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	7	5	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low		high																	

QuickSort

```

0 quickSort(a, 0, 2) {
1   if(2-0>3) {
2     p = choosePivot(a, 0, 2)
3     i = partition(a, p, 0, 2)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 2)
6   } else
7     insertionSort(a, 0, 2)
8   return
9 }

```

PC	=	7
l	=	0
h	=	2
p	=	#
i	=	#

7	0	2	#	#
4	0	4	25	3
4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low			high																

QuickSort

```

0 quickSort(a, 0, 2) {
1   if(2-0>3) {
2     p = choosePivot(a, 0, 2)
3     i = partition(a, p, 0, 2)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 2)
6   } else
7     insertionSort(a, 0, 2)
8   return
9 }

```

PC	=	8
l	=	0
h	=	2
p	=	#
i	=	#

4	0	4	25	3
4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low		high																	

QuickSort

```

0 quickSort(a, 0, 4) {
1   if(4-0>3) {
2     p = choosePivot(a, 0, 4)
3     i = partition(a, 25, 0, 4)
4     quickSort(a, 0, 3-1)
5     quickSort(a, 3+1, 4)
6   } else
7     insertionSort(a, 0, 4)
8   return
9 }

```

PC	=	5
l	=	0
h	=	4
p	=	25
i	=	3

4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low				high															

QuickSort

```

0 quickSort(a, 4, 4) {
1   if(4-4>3) {
2     p = choosePivot(a, 4, 4)
3     i = partition(a, p, 4, 4)
4     quickSort(a, 4, i-1)
5     quickSort(a, i+1, 4)
6   } else
7     insertionSort(a, 4, 4)
8   return
9 }

```

PC = 0
l = 4
h = 4
p = #
i = #

5	0	4	25	3
4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89

high

QuickSort

```

0 quickSort(a, 4, 4) {
1   if(4-4>3) {
2     p = choosePivot(a, 4, 4)
3     i = partition(a, p, 4, 4)
4     quickSort(a, 4, i-1)
5     quickSort(a, i+1, 4)
6   } else
7     insertionSort(a, 4, 4)
8   return
9 }

```

PC = 1
l = 4
h = 4
p = #
i = #

5	0	4	25	3
4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89

high

QuickSort

```

0 quickSort(a, 4, 4) {
1   if(4-4>3) {
2     p = choosePivot(a, 4, 4)
3     i = partition(a, p, 4, 4)
4     quickSort(a, 4, i-1)
5     quickSort(a, i+1, 4)
6   } else
7     insertionSort(a, 4, 4)
8   return
9 }

```

PC = 7
l = 4
h = 4
p = #
i = #

5	0	4	25	3
4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89

high

QuickSort

```

0 quickSort(a, 4, 4) {
1   if(4-4>3) {
2     p = choosePivot(a, 4, 4)
3     i = partition(a, p, 4, 4)
4     quickSort(a, 4, i-1)
5     quickSort(a, i+1, 4)
6   } else
7     insertionSort(a, 4, 4)
8   return
9 }

```

PC = 7
l = 4
h = 4
p = #
i = #

7	4	4	#	#
5	0	4	25	3
4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89

high

QuickSort

```

0 quickSort(a, 4, 4) {
1   if(4-4>3) {
2     p = choosePivot(a, 4, 4)
3     i = partition(a, p, 4, 4)
4     quickSort(a, 4, i-1)
5     quickSort(a, i+1, 4)
6   } else
7     insertionSort(a, 4, 4)
8   return
9 }

```

PC = 8
l = 4
h = 4
p = #
i = #

5	0	4	25	3
4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89

high

QuickSort

```

0 quickSort(a, 0, 4) {
1   if(4-0>3) {
2     p = choosePivot(a, 0, 4)
3     i = partition(a, 25, 0, 4)
4     quickSort(a, 0, 3-1)
5     quickSort(a, 3+1, 4)
6   } else
7     insertionSort(a, 0, 4)
8   return
9 }

```

PC	=	8
l	=	0
h	=	4
p	=	25
i	=	3

4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low				high															

QuickSort

```

0 quickSort(a, 0, 12) {
1   if(12-0>3) {
2     p = choosePivot(a, 0, 12)
3     i = partition(a, 34, 0, 12)
4     quickSort(a, 0, 5-1)
5     quickSort(a, 5+1, 12)
6   } else
7     insertionSort(a, 0, 12)
8   return
9 }

```

PC = 5
l = 0
h = 12
p = 34
i = 5

4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low												high							

QuickSort

```

0 quickSort(a, 6, 12) {
1   if(12-6>3) {
2     p = choosePivot(a, 6, 12)
3     i = partition(a, p, 6, 12)
4     quickSort(a, 6, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 6, 12)
8   return
9 }

```

PC	=	0
l	=	6
h	=	12
p	=	#
i	=	#

5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89

low

high

QuickSort

```

0 quickSort(a, 6, 12) {
1   if(12-6>3) {
2     p = choosePivot(a, 6, 12)
3     i = partition(a, p, 6, 12)
4     quickSort(a, 6, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 6, 12)
8   return
9 }

```

PC	=	1
l	=	6
h	=	12
p	=	#
i	=	#

5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89

low

high

QuickSort

```

0 quickSort(a, 6, 12) {
1   if(12-6>3) {
2     p = choosePivot(a, 6, 12)
3     i = partition(a, p, 6, 12)
4     quickSort(a, 6, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 6, 12)
8   return
9 }

```

PC	=	2
l	=	6
h	=	12
p	=	#
i	=	#

5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89

low

high

QuickSort

```

0 quickSort(a, 6, 12) {
1   if(12-6>3) {
2     p = choosePivot(a, 6, 12)
3     i = partition(a, p, 6, 12)
4     quickSort(a, 6, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 6, 12)
8   return
9 }

```

PC	=	2
l	=	6
h	=	12
p	=	#
i	=	#

2	6	12	#	#
5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
						low													
												high							

QuickSort

```

0 quickSort(a, 6, 12) {
1   if(12-6>3) {
2     p = choosePivot(a, 6, 12)
3     i = partition(a, 48, 6, 12)
4     quickSort(a, 6, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 6, 12)
8   return
9 }

```

PC = 3
l = 6
h = 12
p = 48
i = #

5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89

low

high

QuickSort

```

0 quickSort(a, 6, 12) {
1   if(12-6>3) {
2     p = choosePivot(a, 6, 12)
3     i = partition(a, 48, 6, 12)
4     quickSort(a, 6, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 6, 12)
8   return
9 }

```

PC	=	3
l	=	6
h	=	12
p	=	48
i	=	#

3	6	12	48	#
5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89

low

high

QuickSort

```

0 quickSort(a, 6, 12) {
1   if(12-6>3) {
2     p = choosePivot(a, 6, 12)
3     i = partition(a, 48, 6, 12)
4     quickSort(a, 6, 8-1)
5     quickSort(a, 8+1, 12)
6   } else
7     insertionSort(a, 6, 12)
8   return
9 }

```

PC	=	4
l	=	6
h	=	12
p	=	48
i	=	8

5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89

low

high

QuickSort

```

0 quickSort(a, 6, 7) {
1   if(7-6>3) {
2     p = choosePivot(a, 6, 7)
3     i = partition(a, p, 6, 7)
4     quickSort(a, 6, i-1)
5     quickSort(a, i+1, 7)
6   } else
7     insertionSort(a, 6, 7)
8   return
9 }

```

PC = 0
l = 6
h = 7
p = #
i = #

4	6	12	48	8
5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89

low
high

QuickSort

```

0 quickSort(a, 6, 7) {
1   if(7-6>3) {
2     p = choosePivot(a, 6, 7)
3     i = partition(a, p, 6, 7)
4     quickSort(a, 6, i-1)
5     quickSort(a, i+1, 7)
6   } else
7     insertionSort(a, 6, 7)
8   return
9 }

```

PC = 1
l = 6
h = 7
p = #
i = #

4	6	12	48	8
5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89

low
high

QuickSort

```

0 quickSort(a, 6, 7) {
1   if(7-6>3) {
2     p = choosePivot(a, 6, 7)
3     i = partition(a, p, 6, 7)
4     quickSort(a, 6, i-1)
5     quickSort(a, i+1, 7)
6   } else
7     insertionSort(a, 6, 7)
8   return
9 }

```

PC = 7
l = 6
h = 7
p = #
i = #

4	6	12	48	8
5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89

low
high

QuickSort

```

0 quickSort(a, 6, 7) {
1   if(7-6>3) {
2     p = choosePivot(a, 6, 7)
3     i = partition(a, p, 6, 7)
4     quickSort(a, 6, i-1)
5     quickSort(a, i+1, 7)
6   } else
7     insertionSort(a, 6, 7)
8   return
9 }

```

PC	=	7
l	=	6
h	=	7
p	=	#
i	=	#

7	6	7	#	#
4	6	12	48	8
5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89

low

high

QuickSort

```

0 quickSort(a, 6, 7) {
1   if(7-6>3) {
2     p = choosePivot(a, 6, 7)
3     i = partition(a, p, 6, 7)
4     quickSort(a, 6, i-1)
5     quickSort(a, i+1, 7)
6   } else
7     insertionSort(a, 6, 7)
8   return
9 }

```

PC	=	8
l	=	6
h	=	7
p	=	#
i	=	#

4	6	12	48	8
5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89

low
high

QuickSort

```

0 quickSort(a, 6, 12) {
1   if(12-6>3) {
2     p = choosePivot(a, 6, 12)
3     i = partition(a, 48, 6, 12)
4     quickSort(a, 6, 8-1)
5     quickSort(a, 8+1, 12)
6   } else
7     insertionSort(a, 6, 12)
8   return
9 }

```

PC	=	5
l	=	6
h	=	12
p	=	48
i	=	8

5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89

low

high

QuickSort

```

0 quickSort(a, 9, 12) {
1   if(12-9>3) {
2     p = choosePivot(a, 9, 12)
3     i = partition(a, p, 9, 12)
4     quickSort(a, 9, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 9, 12)
8   return
9 }

```

PC	=	0
l	=	9
h	=	12
p	=	#
i	=	#

5	6	12	48	8
5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89

low

high

QuickSort

```

0 quickSort(a, 9, 12) {
1   if(12-9>3) {
2       p = choosePivot(a, 9, 12)
3       i = partition(a, p, 9, 12)
4       quickSort(a, 9, i-1)
5       quickSort(a, i+1, 12)
6   } else
7       insertionSort(a, 9, 12)
8   return
9 }

```

PC	=	1
l	=	9
h	=	12
p	=	#
i	=	#

5	6	12	48	8
5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89

low

high

QuickSort

```

0 quickSort(a, 9, 12) {
1   if(12-9>3) {
2     p = choosePivot(a, 9, 12)
3     i = partition(a, p, 9, 12)
4     quickSort(a, 9, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 9, 12)
8   return
9 }

```

PC = 7
l = 9
h = 12
p = #
i = #

5	6	12	48	8
5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89

low

high

QuickSort

```

0 quickSort(a, 9, 12) {
1   if(12-9>3) {
2     p = choosePivot(a, 9, 12)
3     i = partition(a, p, 9, 12)
4     quickSort(a, 9, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 9, 12)
8   return
9 }

```

PC = 7
l = 9
h = 12
p = #
i = #

7	9	12	#	#
5	6	12	48	8
5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89

low

high

QuickSort

```

0 quickSort(a, 9, 12) {
1   if(12-9>3) {
2     p = choosePivot(a, 9, 12)
3     i = partition(a, p, 9, 12)
4     quickSort(a, 9, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 9, 12)
8   return
9 }

```

PC	=	8
l	=	9
h	=	12
p	=	#
i	=	#

5	6	12	48	8
5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89

low

high

QuickSort

```

0 quickSort(a, 6, 12) {
1   if(12-6>3) {
2     p = choosePivot(a, 6, 12)
3     i = partition(a, 48, 6, 12)
4     quickSort(a, 6, 8-1)
5     quickSort(a, 8+1, 12)
6   } else
7     insertionSort(a, 6, 12)
8   return
9 }

```

PC	=	8
l	=	6
h	=	12
p	=	48
i	=	8

5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89

low

high

QuickSort

```

0 quickSort(a, 0, 12) {
1   if(12-0>3) {
2     p = choosePivot(a, 0, 12)
3     i = partition(a, 34, 0, 12)
4     quickSort(a, 0, 5-1)
5     quickSort(a, 5+1, 12)
6   } else
7     insertionSort(a, 0, 12)
8   return
9 }

```

PC	=	8
l	=	0
h	=	12
p	=	34
i	=	5

4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89
low										high									

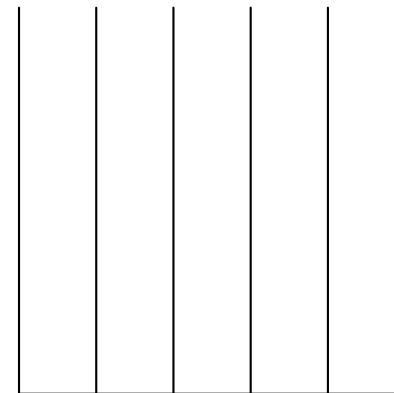
QuickSort

```

0 quickSort(a, 0, 19) {
1   if(19-0>3) {
2     p = choosePivot(a, 0, 19)
3     i = partition(a, 73, 0, 19)
4     quickSort(a, 0, 13-1)
5     quickSort(a, 13+1, 19)
6   } else
7     insertionSort(a, 0, 19)
8   return
9 }

```

PC	=	5
l	=	0
h	=	19
p	=	73
i	=	13



pc l h p i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89

low

high

QuickSort

```

0 quickSort(a, 14, 19) {
1   if(19-14>3) {
2     p = choosePivot(a, 14, 19)
3     i = partition(a, p, 14, 19)
4     quickSort(a, 14, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 14, 19)
8   return
9 }

```

PC	=	0
l	=	14
h	=	19
p	=	#
i	=	#

5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89
														low					high

QuickSort

```

0 quickSort(a, 14, 19) {
1   if(19-14>3) {
2     p = choosePivot(a, 14, 19)
3     i = partition(a, p, 14, 19)
4     quickSort(a, 14, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 14, 19)
8   return
9 }

```

PC	=	1
l	=	14
h	=	19
p	=	#
i	=	#

5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89
														low					high

QuickSort

```

0 quickSort(a, 14, 19) {
1   if(19-14>3) {
2     p = choosePivot(a, 14, 19)
3     i = partition(a, p, 14, 19)
4     quickSort(a, 14, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 14, 19)
8   return
9 }

```

PC	=	2
l	=	14
h	=	19
p	=	#
i	=	#

5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19		
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89		
														low							high

QuickSort

```

0 quickSort(a, 14, 19) {
1   if(19-14>3) {
2     p = choosePivot(a, 14, 19)
3     i = partition(a, p, 14, 19)
4     quickSort(a, 14, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 14, 19)
8   return
9 }

```

PC	=	2
l	=	14
h	=	19
p	=	#
i	=	#

2	14	19	#	#
5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89	
														low						high

QuickSort

```

0 quickSort(a, 14, 19) {
1   if(19-14>3) {
2     p = choosePivot(a, 14, 19)
3     i = partition(a, 89, 14, 19)
4     quickSort(a, 14, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 14, 19)
8   return
9 }

```

PC	=	3
l	=	14
h	=	19
p	=	89
i	=	#

5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89
low														high					

QuickSort

```

0 quickSort(a, 14, 19) {
1   if(19-14>3) {
2     p = choosePivot(a, 14, 19)
3     i = partition(a, 89, 14, 19)
4     quickSort(a, 14, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 14, 19)
8   return
9 }

```

PC	=	3
l	=	14
h	=	19
p	=	89
i	=	#

3	14	19	89	#
5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	87	76	89	92	95	
														low						high

QuickSort

```

0 quickSort(a, 14, 19) {
1   if(19-14>3) {
2     p = choosePivot(a, 14, 19)
3     i = partition(a, 89, 14, 19)
4     quickSort(a, 14, 17-1)
5     quickSort(a, 17+1, 19)
6   } else
7     insertionSort(a, 14, 19)
8   return
9 }

```

PC	=	4
l	=	14
h	=	19
p	=	89
i	=	17

5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	87	76	89	92	95
low														high					

QuickSort

```

0 quickSort(a, 14, 16) {
1   if(16-14>3) {
2     p = choosePivot(a, 14, 16)
3     i = partition(a, p, 14, 16)
4     quickSort(a, 14, i-1)
5     quickSort(a, i+1, 16)
6   } else
7     insertionSort(a, 14, 16)
8   return
9 }

```

PC = 0
l = 14
h = 16
p = #
i = #

4	14	19	89	17
5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	87	76	89	92	95
low														high					

QuickSort

```

0 quickSort(a, 14, 16) {
1   if(16-14>3) {
2     p = choosePivot(a, 14, 16)
3     i = partition(a, p, 14, 16)
4     quickSort(a, 14, i-1)
5     quickSort(a, i+1, 16)
6   } else
7     insertionSort(a, 14, 16)
8   return
9 }

```

PC	=	1
l	=	14
h	=	16
p	=	#
i	=	#

4	14	19	89	17
5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	87	76	89	92	95

low high

QuickSort

```

0 quickSort(a, 14, 16) {
1   if(16-14>3) {
2     p = choosePivot(a, 14, 16)
3     i = partition(a, p, 14, 16)
4     quickSort(a, 14, i-1)
5     quickSort(a, i+1, 16)
6   } else
7     insertionSort(a, 14, 16)
8   return
9 }

```

PC = 7
l = 14
h = 16
p = #
i = #

4	14	19	89	17
5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	87	76	89	92	95

low high

QuickSort

```

0 quickSort(a, 14, 16) {
1   if(16-14>3) {
2     p = choosePivot(a, 14, 16)
3     i = partition(a, p, 14, 16)
4     quickSort(a, 14, i-1)
5     quickSort(a, i+1, 16)
6   } else
7     insertionSort(a, 14, 16)
8   return
9 }

```

PC = 7
l = 14
h = 16
p = #
i = #

7	14	16	#	#
4	14	19	89	17
5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95
low														high					

QuickSort

```

0 quickSort(a, 14, 16) {
1   if(16-14>3) {
2     p = choosePivot(a, 14, 16)
3     i = partition(a, p, 14, 16)
4     quickSort(a, 14, i-1)
5     quickSort(a, i+1, 16)
6   } else
7     insertionSort(a, 14, 16)
8   return
9 }

```

PC	=	8
l	=	14
h	=	16
p	=	#
i	=	#

4	14	19	89	17
5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95

low high

QuickSort

```

0 quickSort(a, 14, 19) {
1   if(19-14>3) {
2     p = choosePivot(a, 14, 19)
3     i = partition(a, 89, 14, 19)
4     quickSort(a, 14, 17-1)
5     quickSort(a, 17+1, 19)
6   } else
7     insertionSort(a, 14, 19)
8   return
9 }

```

PC	=	5
l	=	14
h	=	19
p	=	89
i	=	17

5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19		
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95		
														low							high

QuickSort

```

0 quickSort(a, 18, 19) {
1   if(19-18>3) {
2     p = choosePivot(a, 18, 19)
3     i = partition(a, p, 18, 19)
4     quickSort(a, 18, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 18, 19)
8   return
9 }

```

PC	=	0
l	=	18
h	=	19
p	=	#
i	=	#

5	14	19	89	17
5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95

low
high

QuickSort

```

0 quickSort(a, 18, 19) {
1   if(19-18>3) {
2     p = choosePivot(a, 18, 19)
3     i = partition(a, p, 18, 19)
4     quickSort(a, 18, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 18, 19)
8   return
9 }

```

PC	=	1
l	=	18
h	=	19
p	=	#
i	=	#

5	14	19	89	17
5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95

low
high

QuickSort

```

0 quickSort(a, 18, 19) {
1   if(19-18>3) {
2     p = choosePivot(a, 18, 19)
3     i = partition(a, p, 18, 19)
4     quickSort(a, 18, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 18, 19)
8   return
9 }

```

PC = 7
l = 18
h = 19
p = #
i = #

5	14	19	89	17
5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95

low
high

QuickSort

```

0 quickSort(a, 18, 19) {
1   if(19-18>3) {
2     p = choosePivot(a, 18, 19)
3     i = partition(a, p, 18, 19)
4     quickSort(a, 18, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 18, 19)
8   return
9 }

```

PC	=	7
l	=	18
h	=	19
p	=	#
i	=	#

7	18	19	#	#
5	14	19	89	17
5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95

low
high

QuickSort

```

0 quickSort(a, 18, 19) {
1   if(19-18>3) {
2     p = choosePivot(a, 18, 19)
3     i = partition(a, p, 18, 19)
4     quickSort(a, 18, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 18, 19)
8   return
9 }

```

PC	=	8
l	=	18
h	=	19
p	=	#
i	=	#

5	14	19	89	17
5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95

low
high

QuickSort

```

0 quickSort(a, 14, 19) {
1   if(19-14>3) {
2     p = choosePivot(a, 14, 19)
3     i = partition(a, 89, 14, 19)
4     quickSort(a, 14, 17-1)
5     quickSort(a, 17+1, 19)
6   } else
7     insertionSort(a, 14, 19)
8   return
9 }

```

PC = 8
l = 14
h = 19
p = 89
i = 17

5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95
low														high					

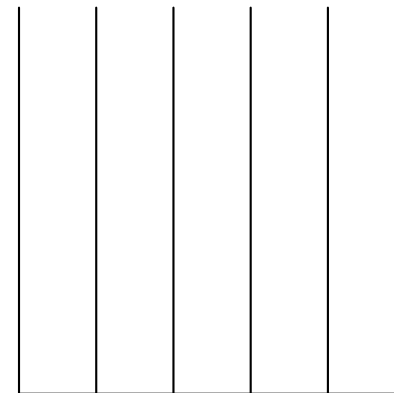
QuickSort

```

0 quickSort(a, 0, 19) {
1   if(19-0>3) {
2     p = choosePivot(a, 0, 19)
3     i = partition(a, 73, 0, 19)
4     quickSort(a, 0, 13-1)
5     quickSort(a, 13+1, 19)
6   } else
7     insertionSort(a, 0, 19)
8   return
9 }

```

PC	=	8
l	=	0
h	=	19
p	=	73
i	=	13



pc l h p i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95

low

high

Sort in Practice

- The STL in C++ offers three sorts
 - ★ `sort()` implemented using quicksort
 - ★ `stable_sort()` implemented using mergesort
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Selection

- A related problem to sorting is selection
- That is we want to select the k^{th} largest element
- We could do this by first sorting the array
- A full sort is not however necessary—we can use a modified quicksort where we only continue to sort the part of the array we are interested in
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Outline

1. Merge Sort
2. Quick Sort
3. **Radix Sort**



Radix Sort

- Can we get a sort algorithm to run faster than $O(n \log(n))$?
- Our proof that this was optimal assumed we were performing binary decisions (is a_i less than a_j ?)
- If we don't perform pairwise comparisons then the proof doesn't apply
- Radix sort is the classic example of a sort algorithm that doesn't use pairwise comparisons

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Sorting Into Buckets

- The idea behind radix sort is to sort the elements of an array into some number of buckets
- This is done successively until the whole array is sorted
- Consider sorting integers in decimals (base 10 or radix 10)
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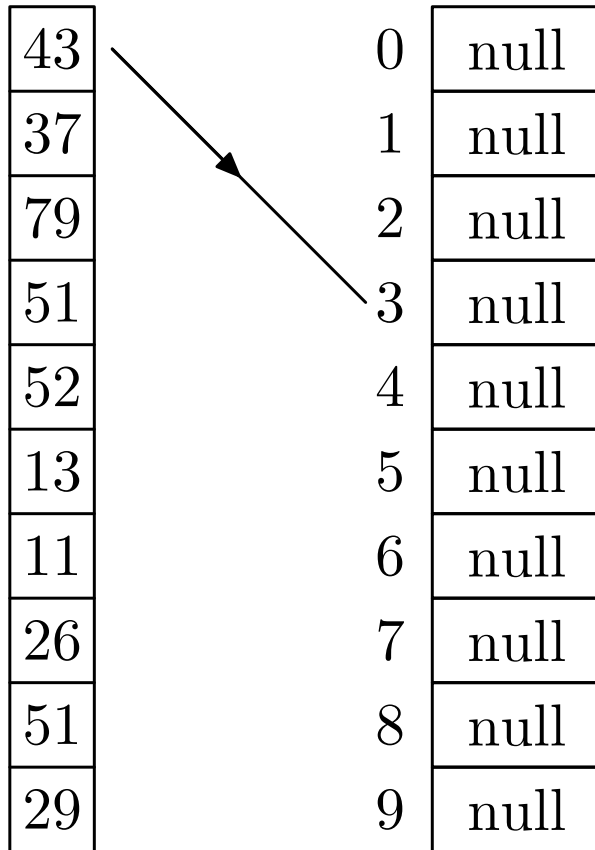
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Radix Sort in Action

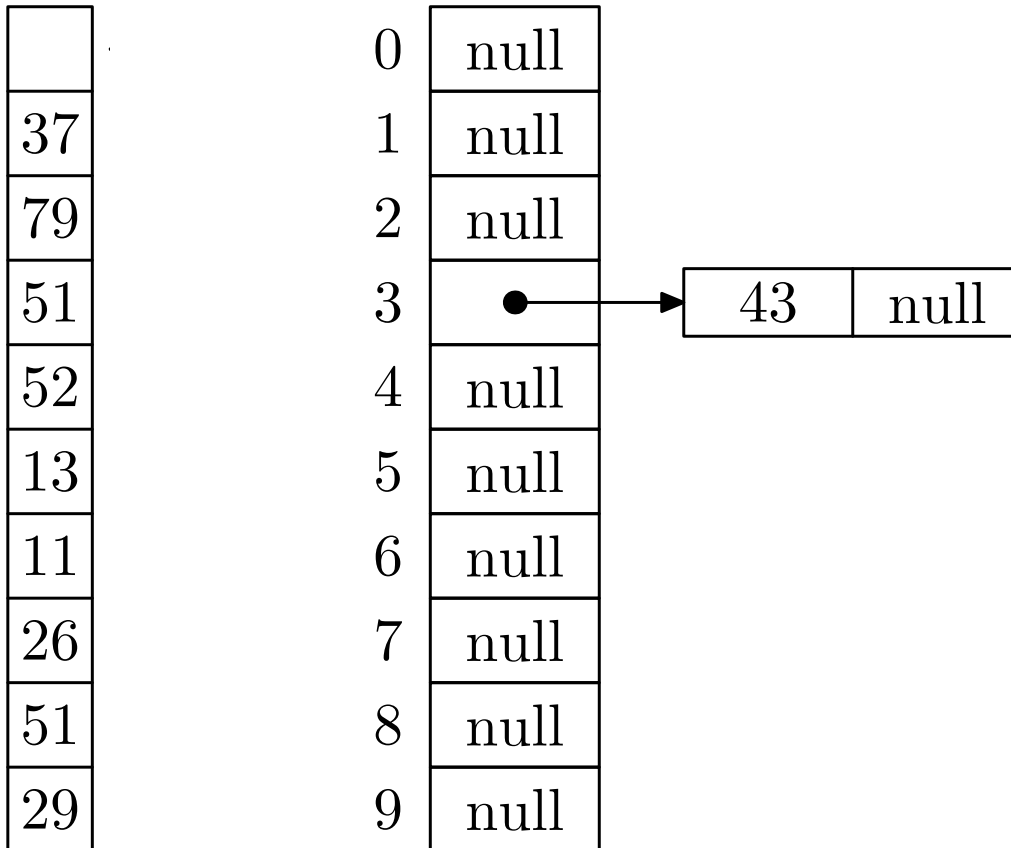
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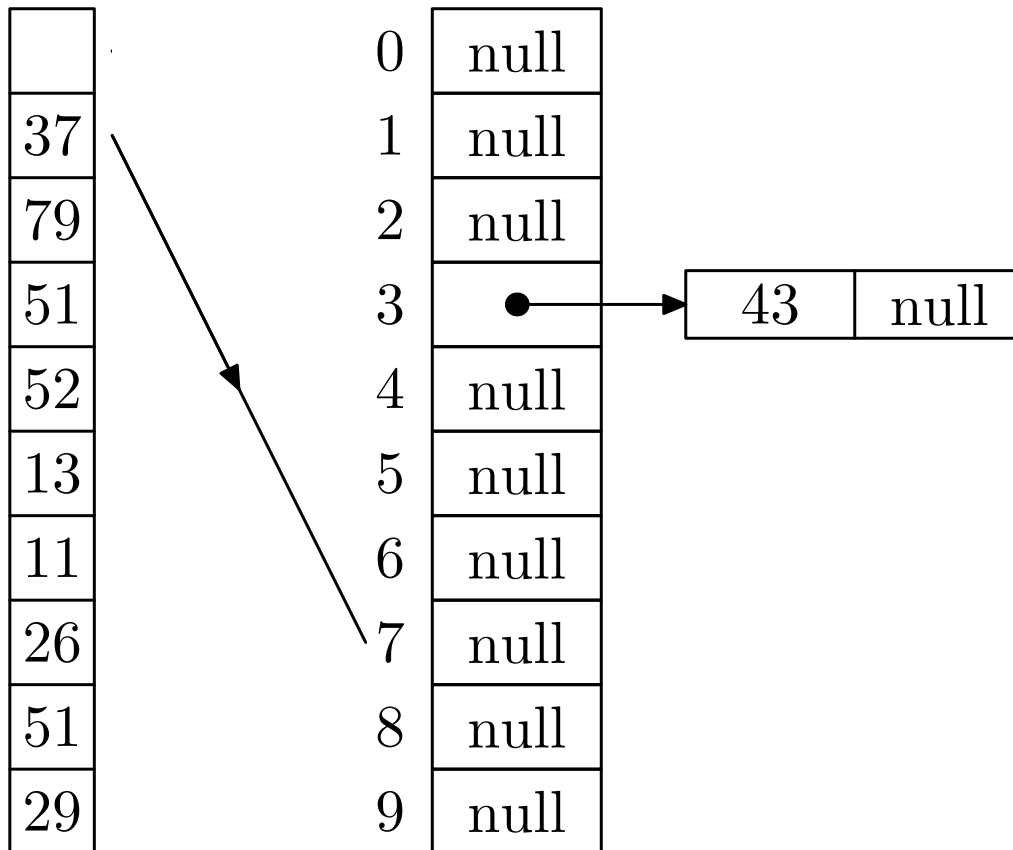
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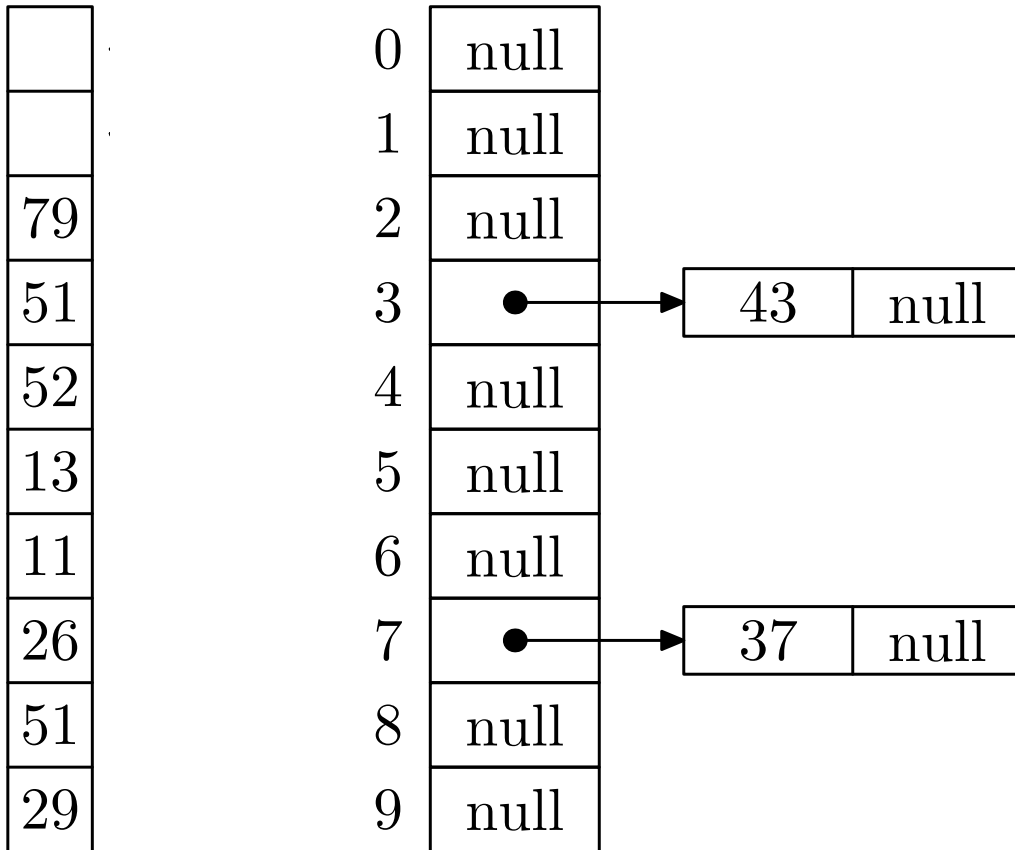
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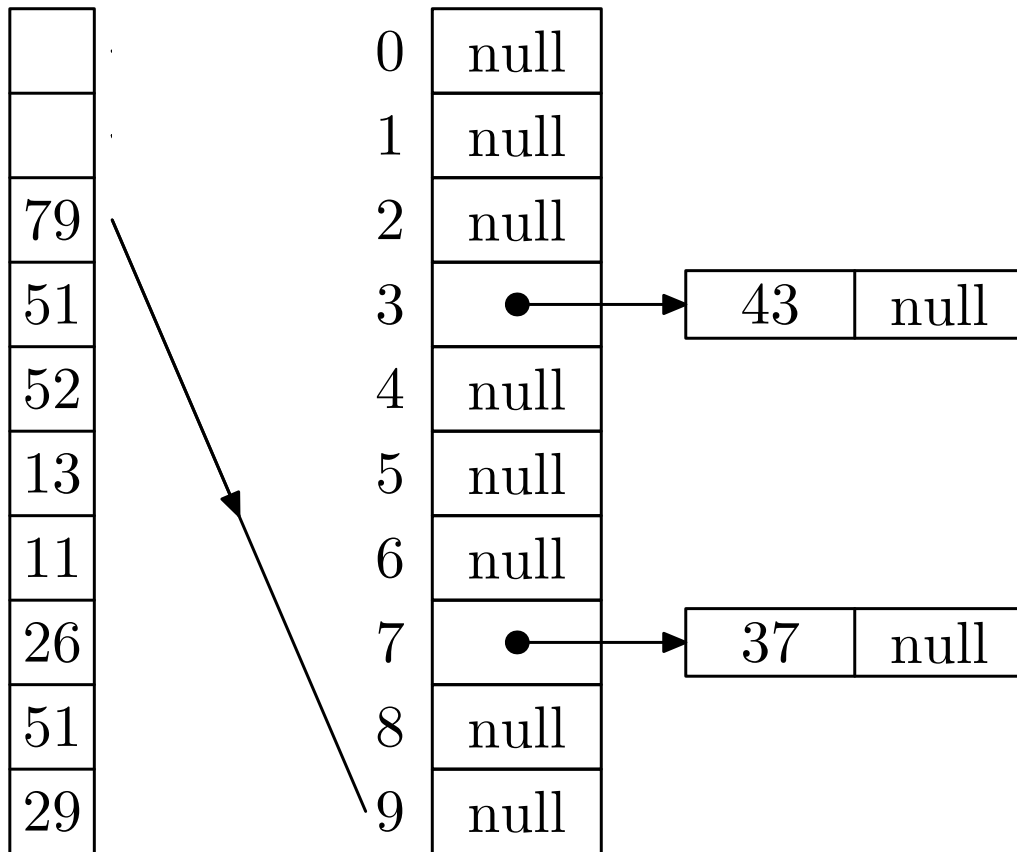
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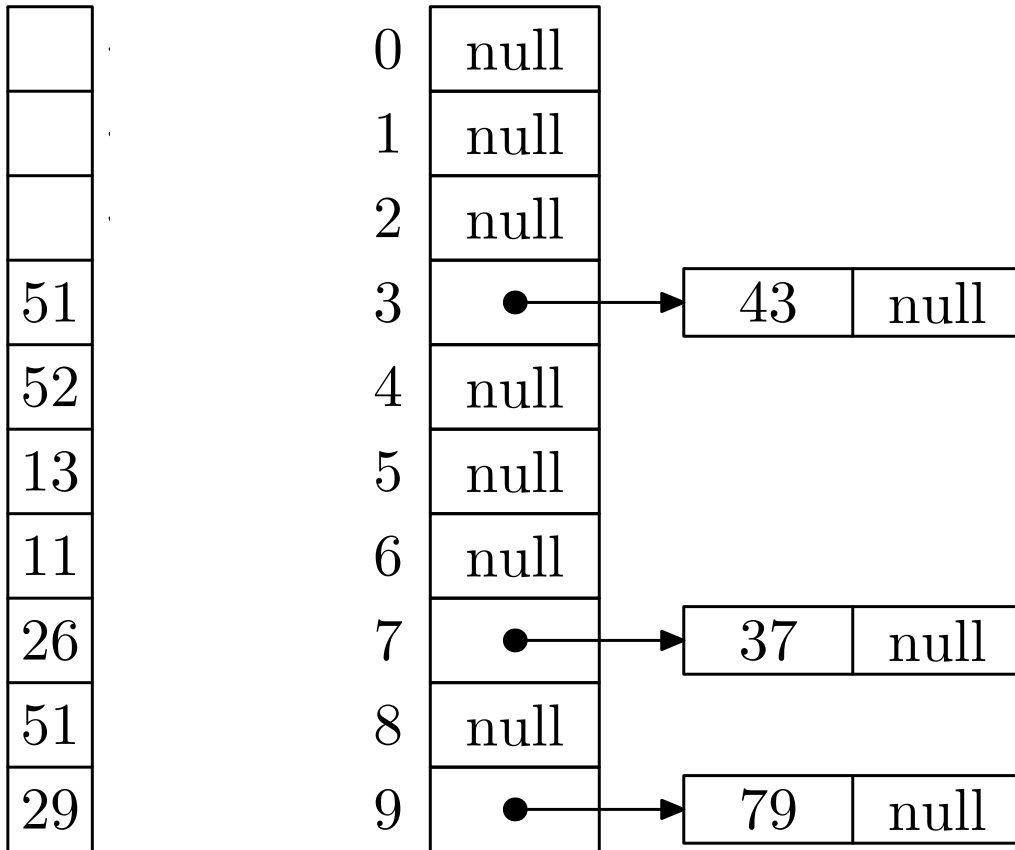
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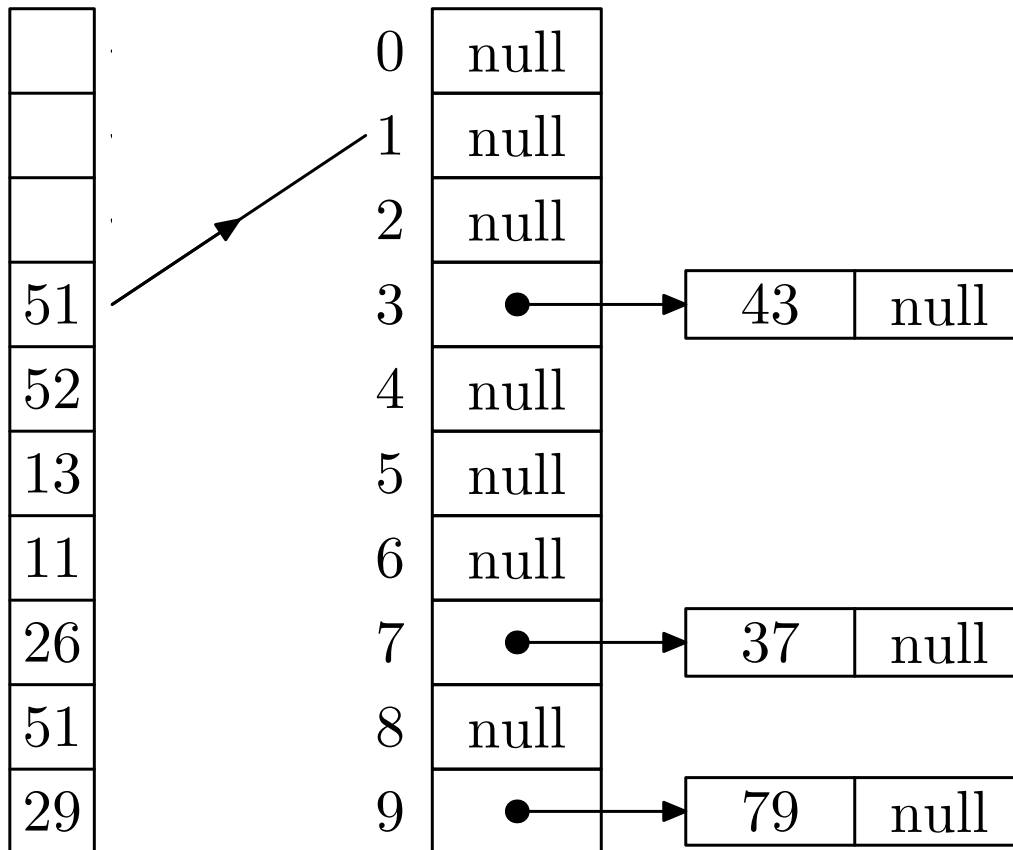
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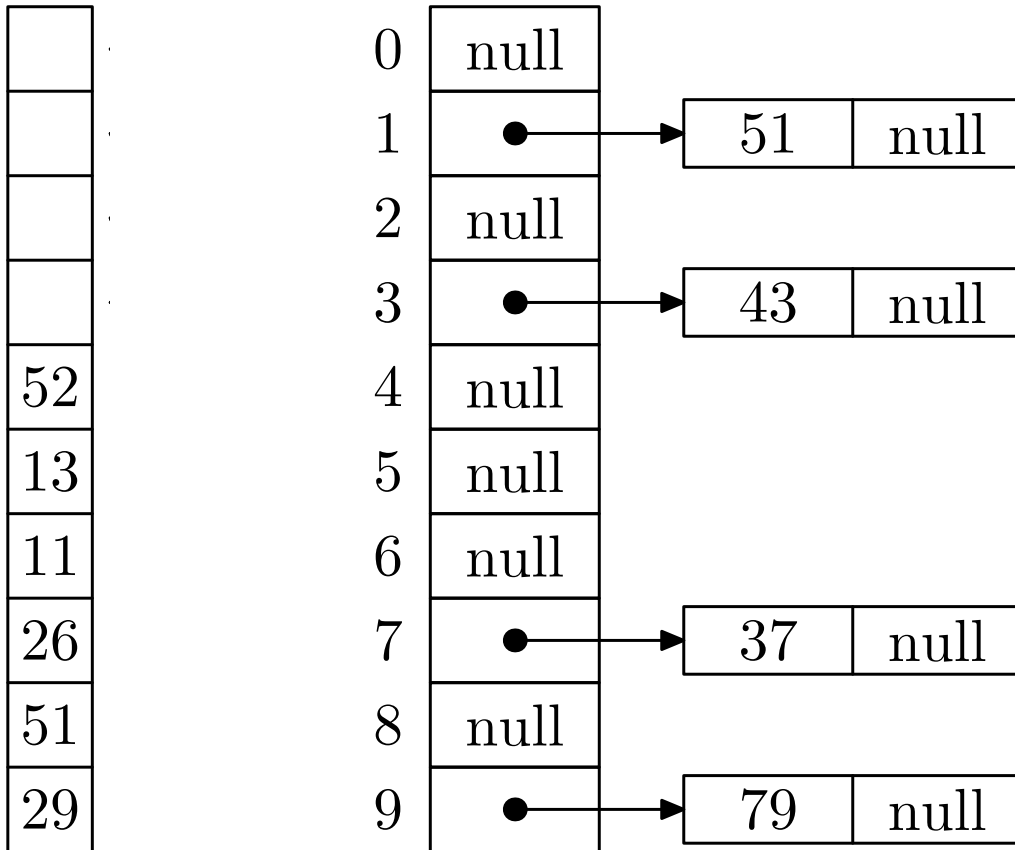
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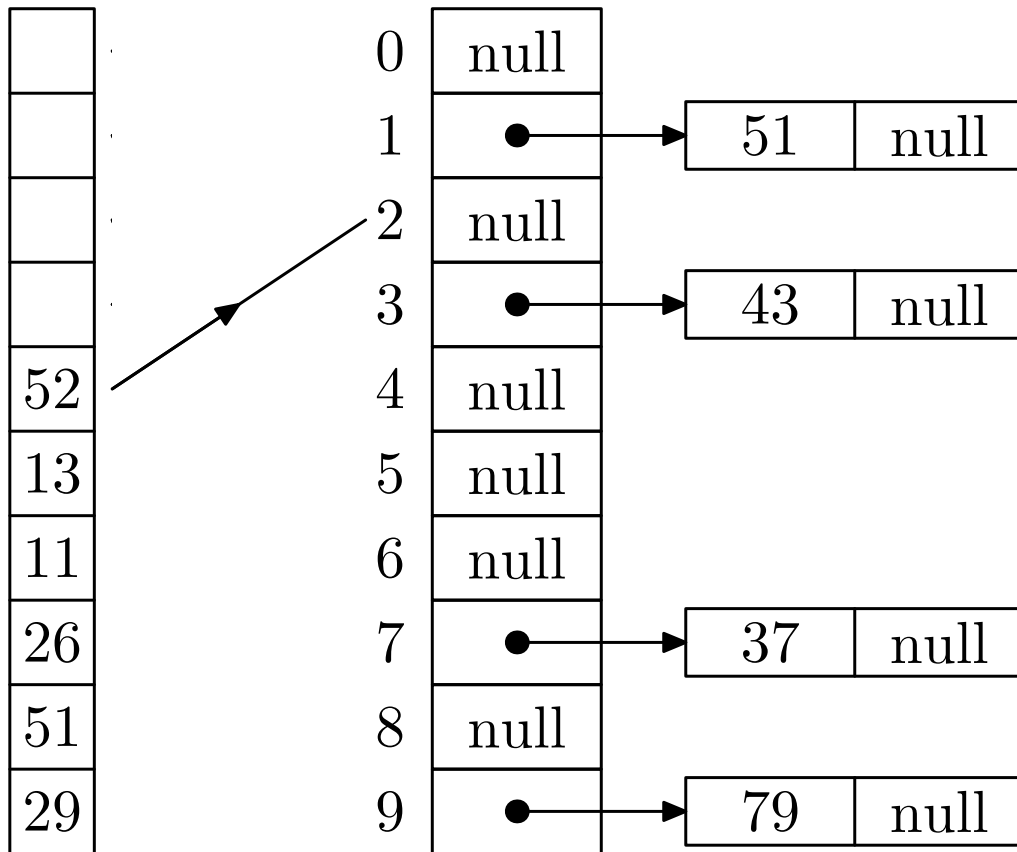
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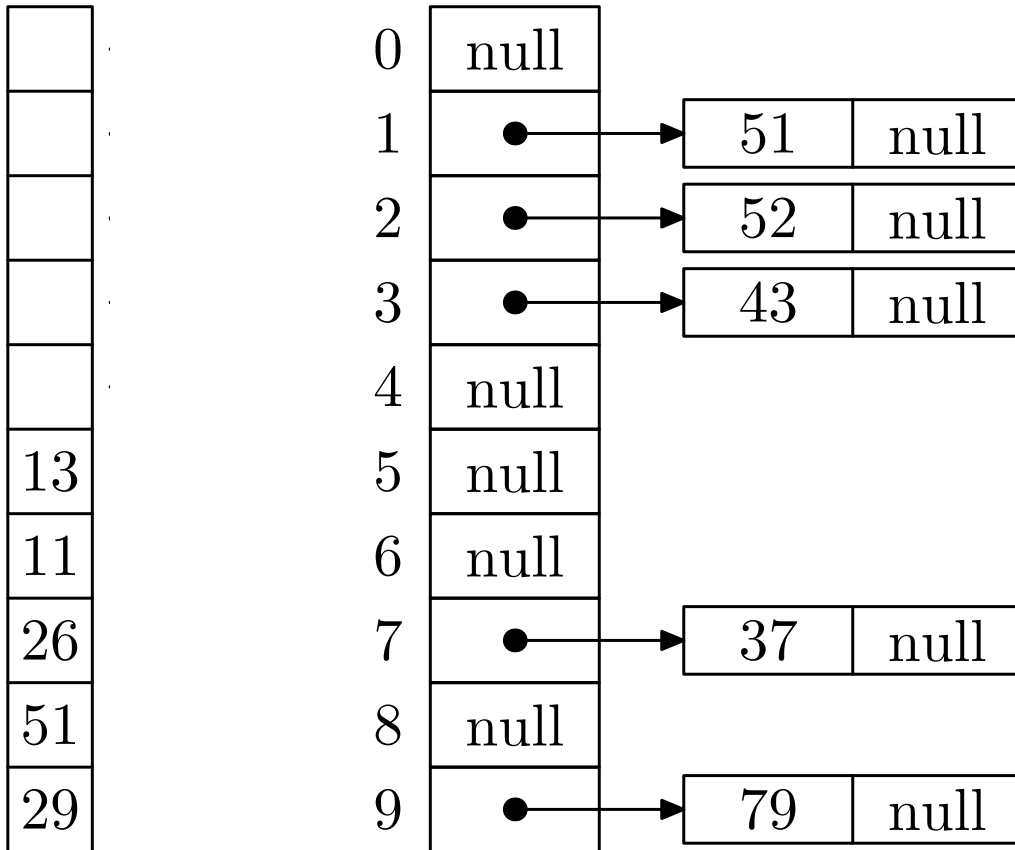
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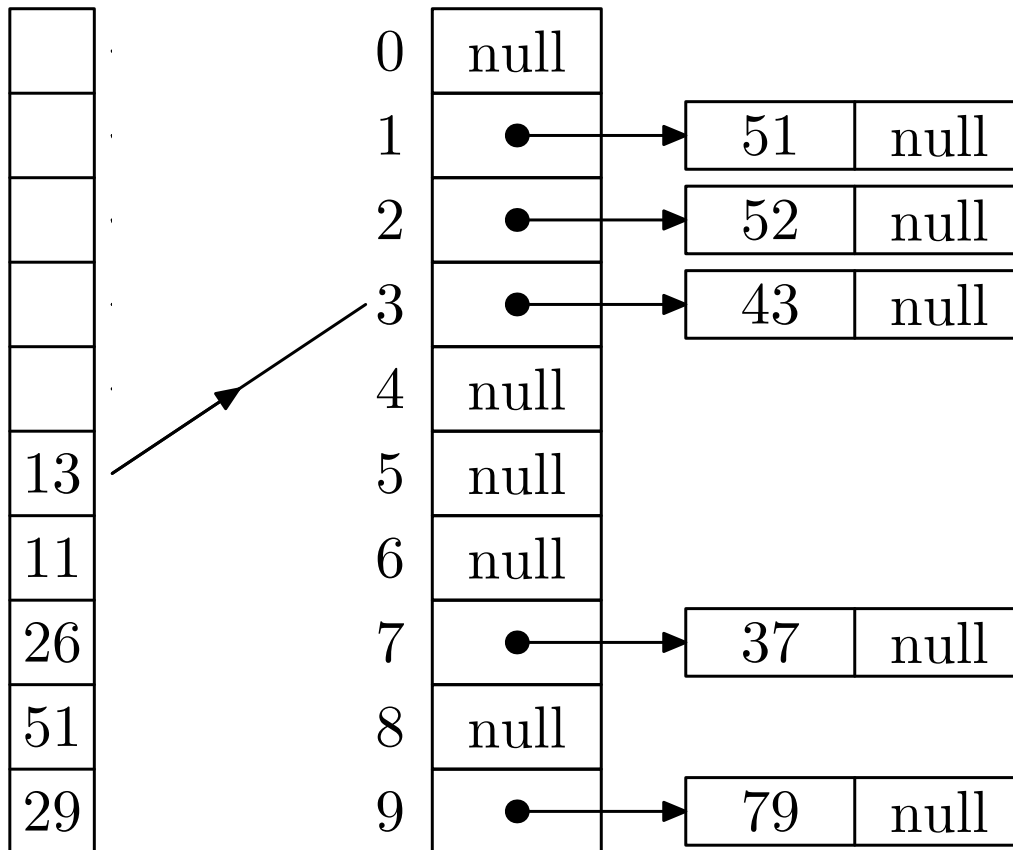
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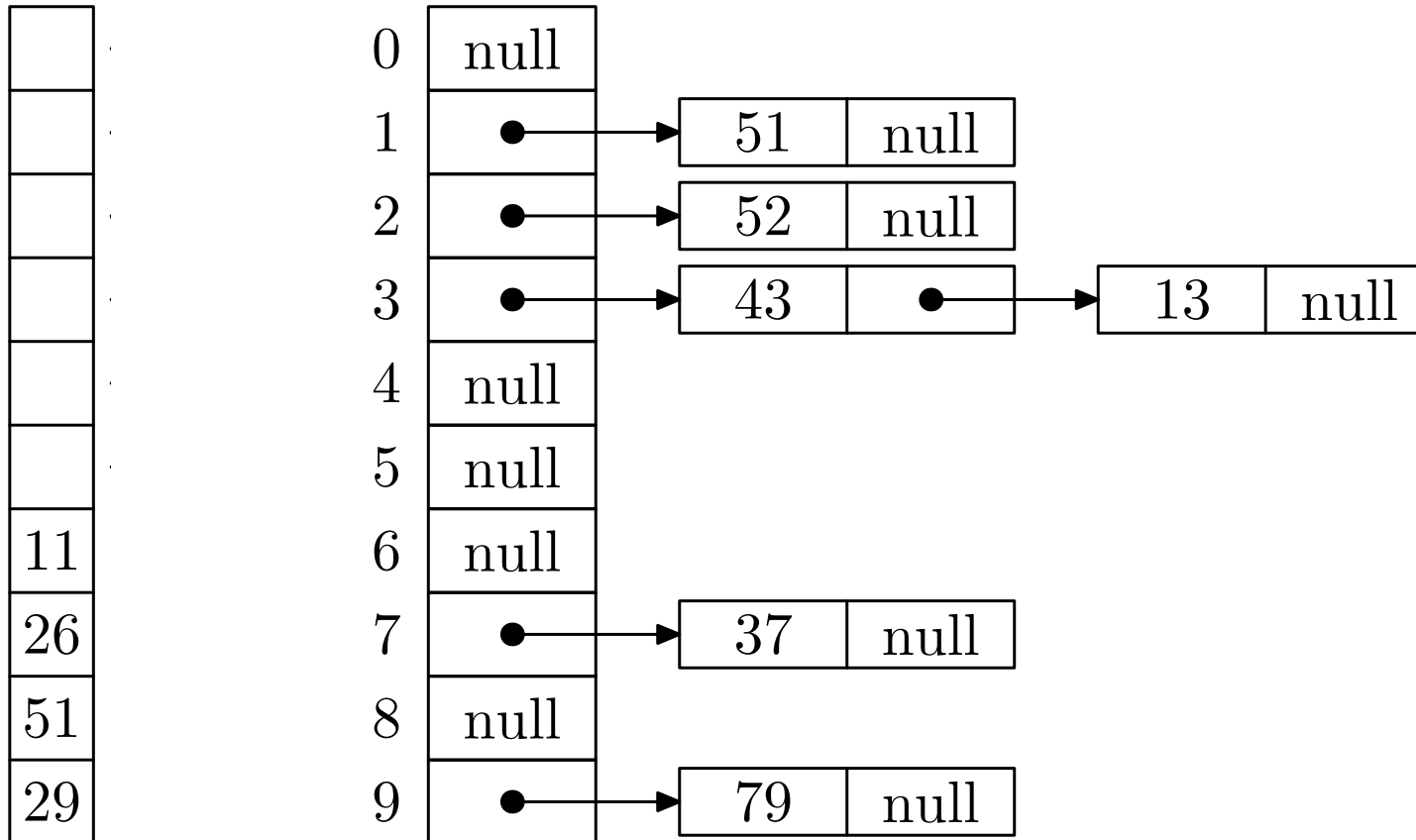
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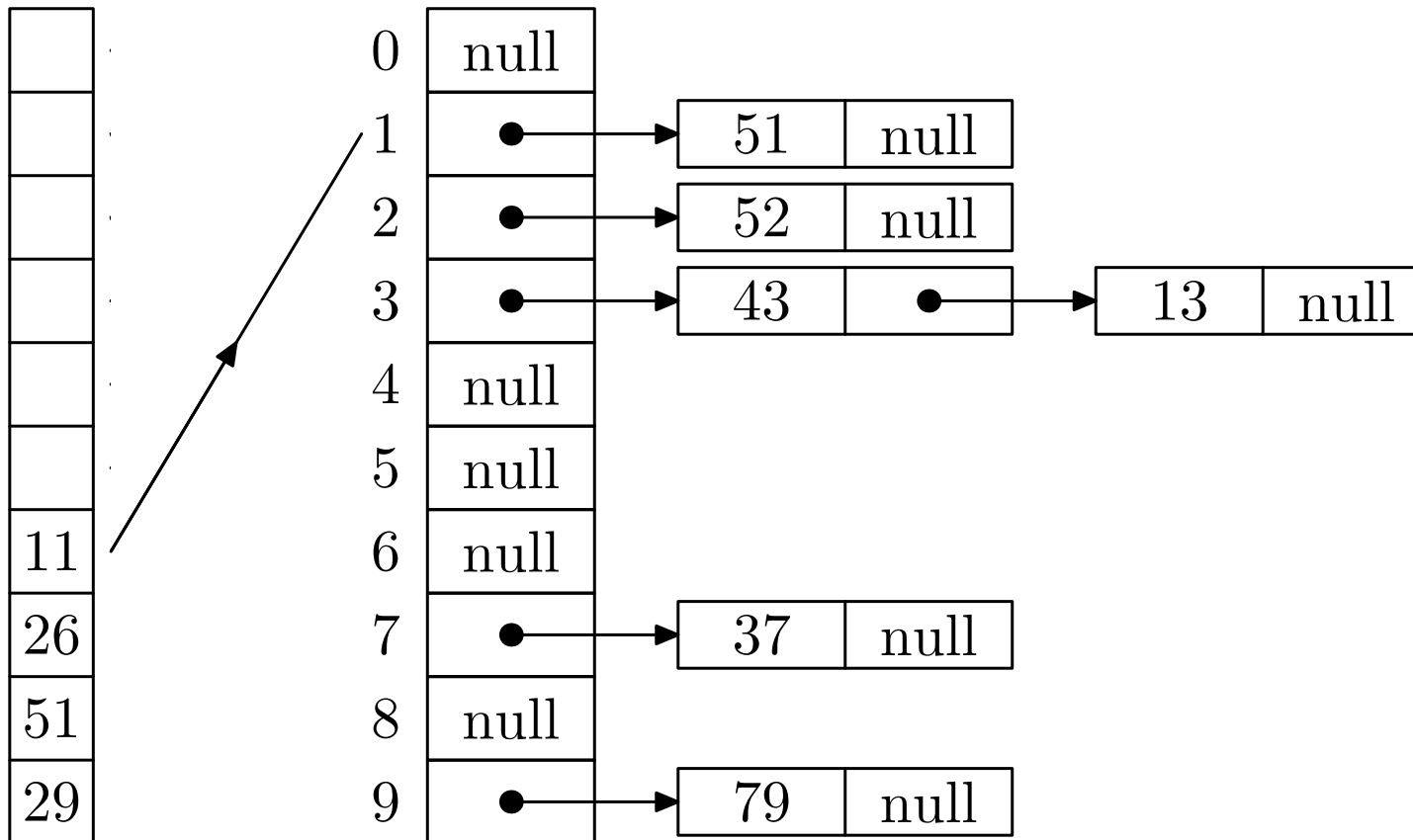
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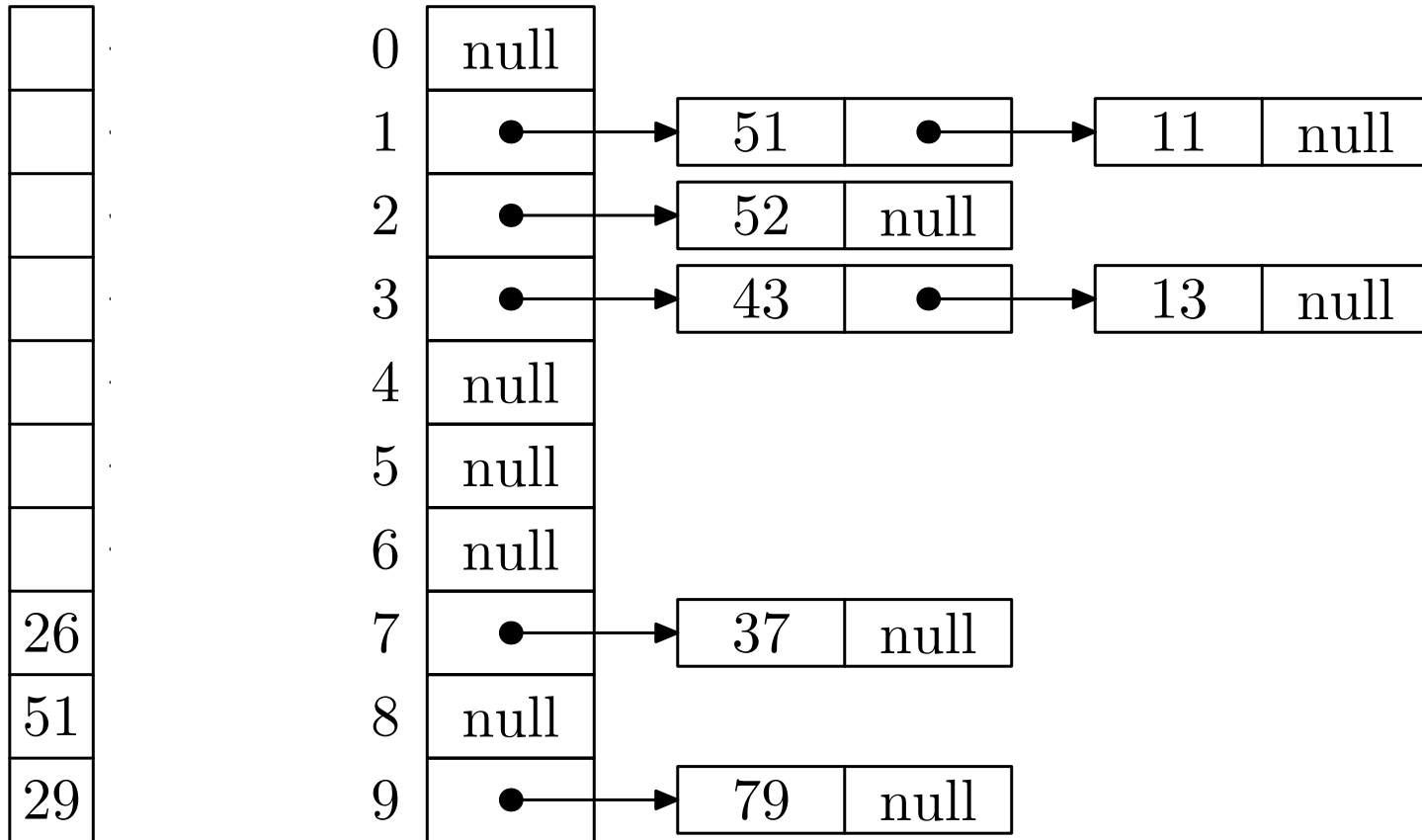
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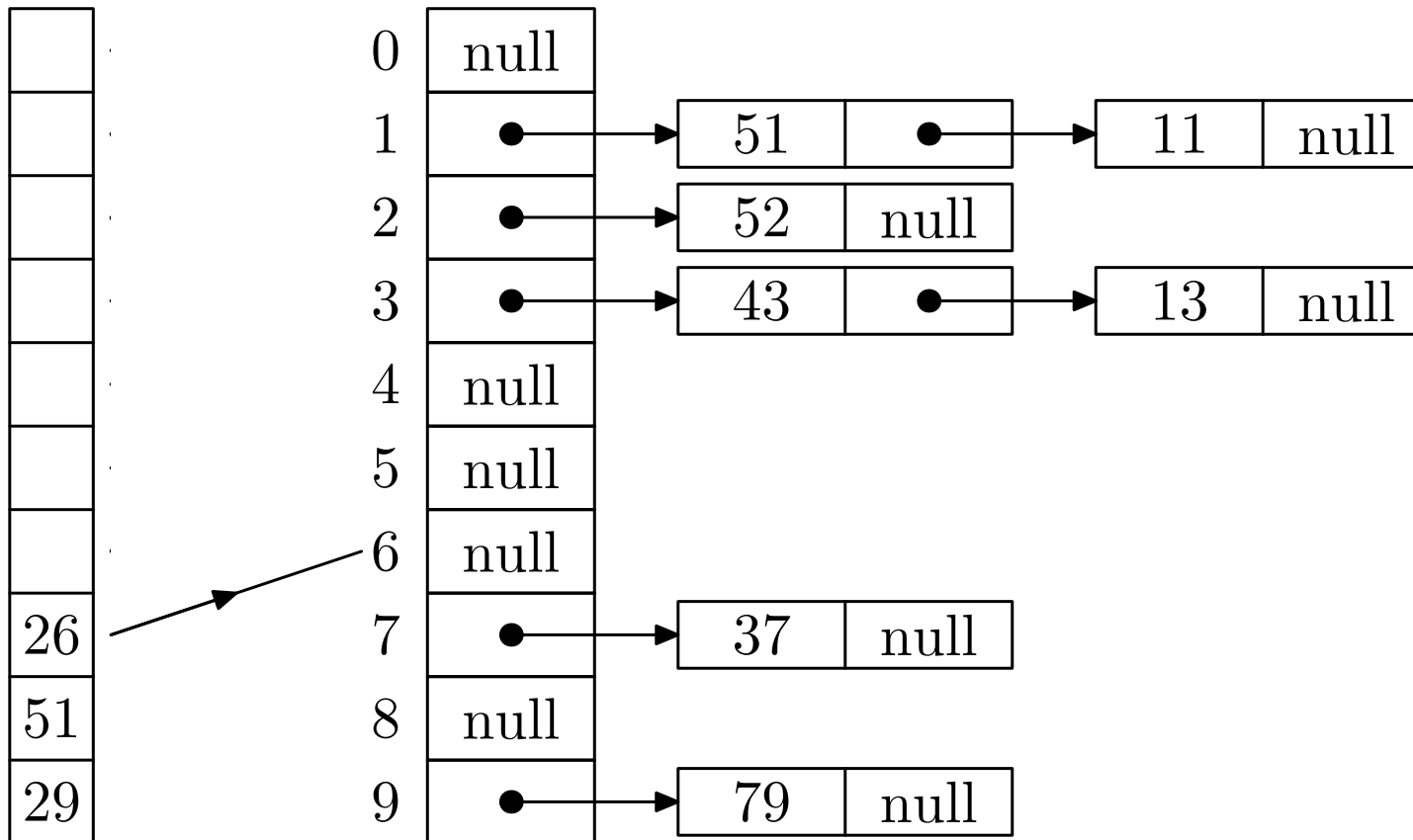
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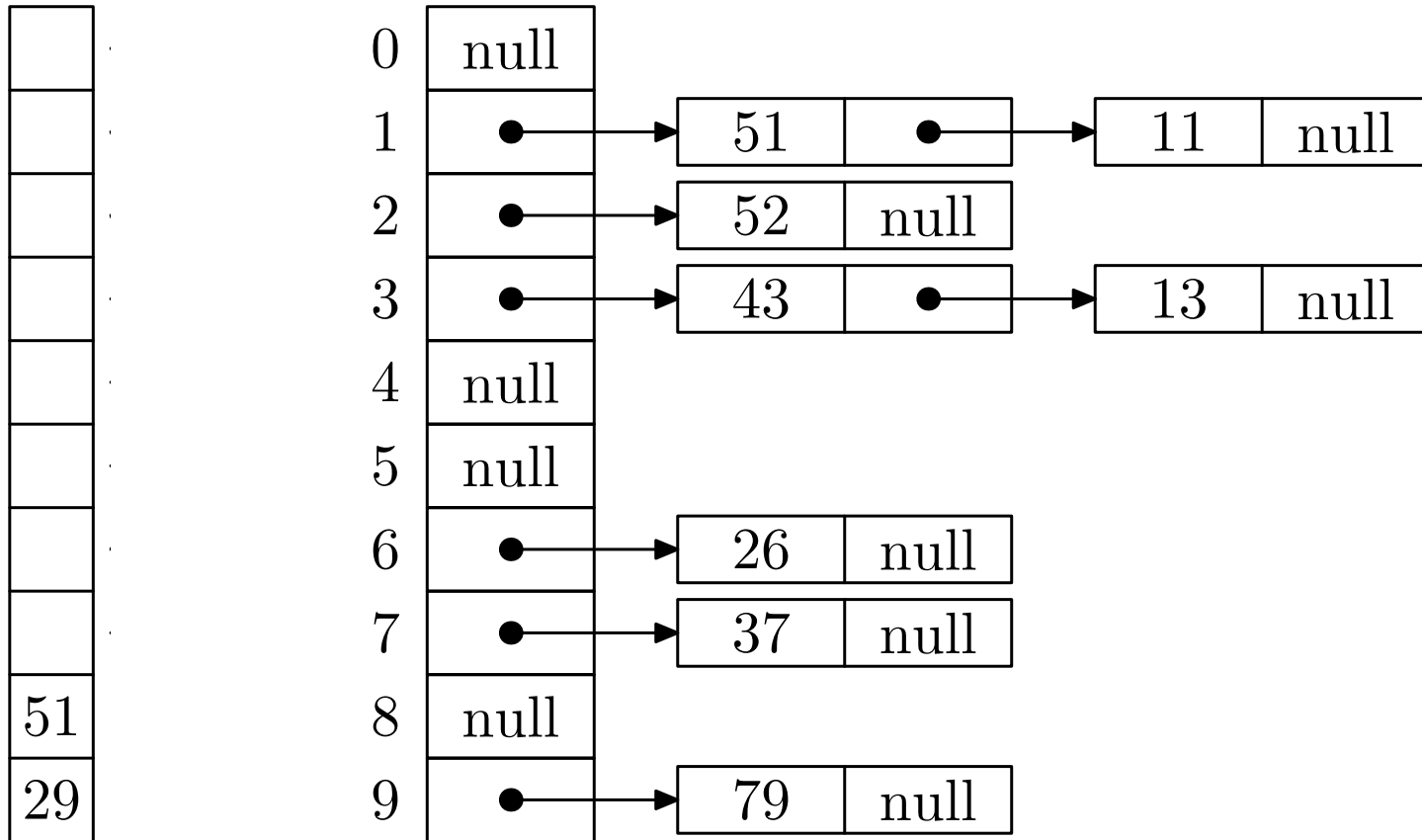
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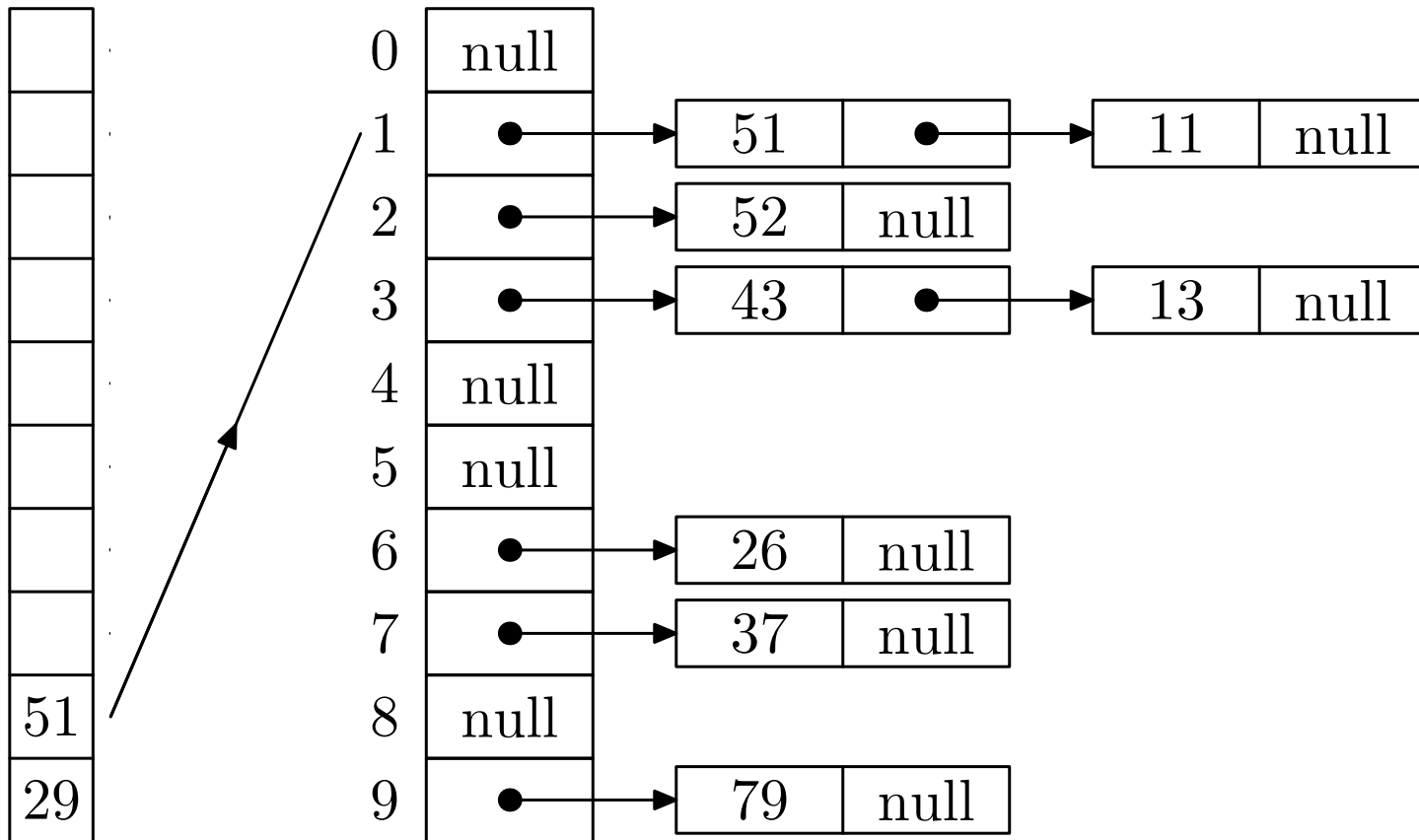
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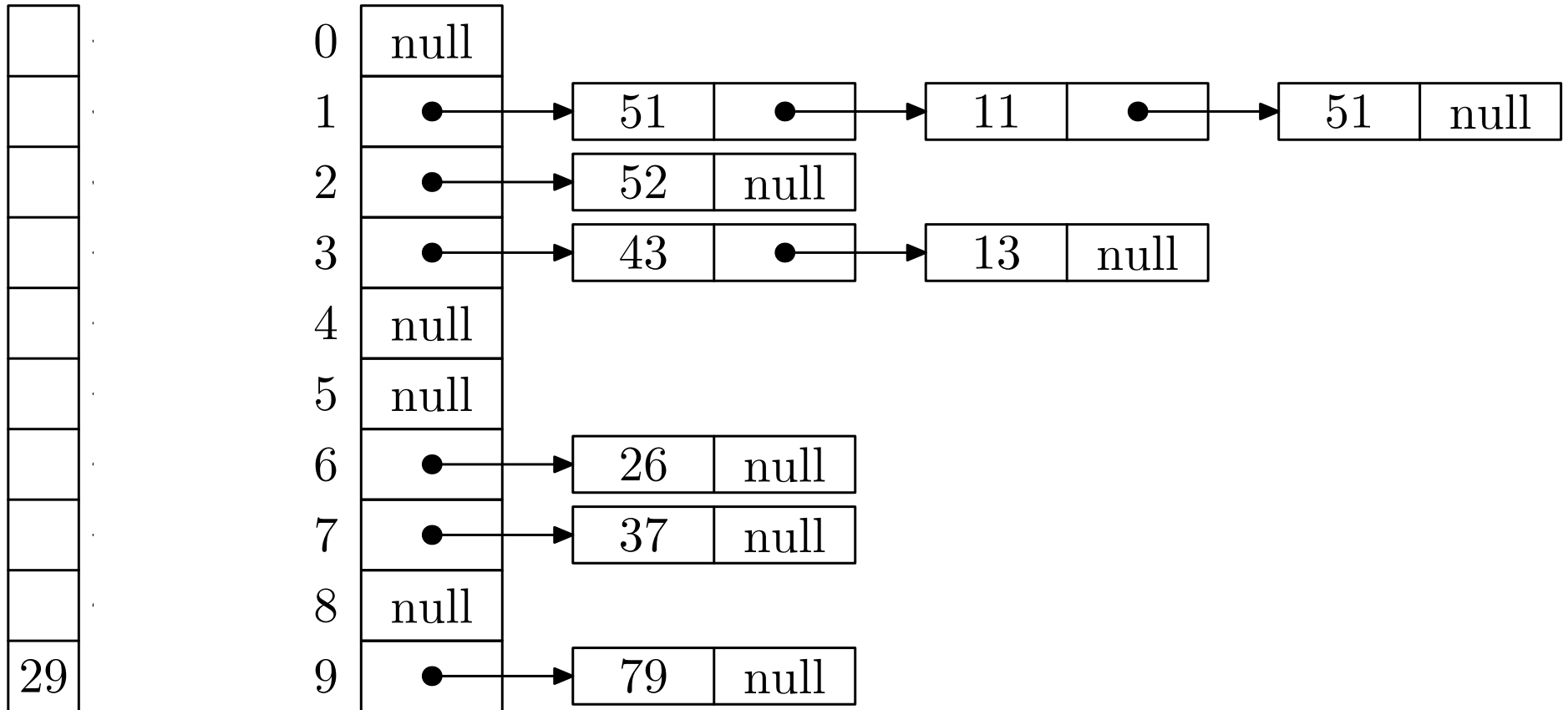
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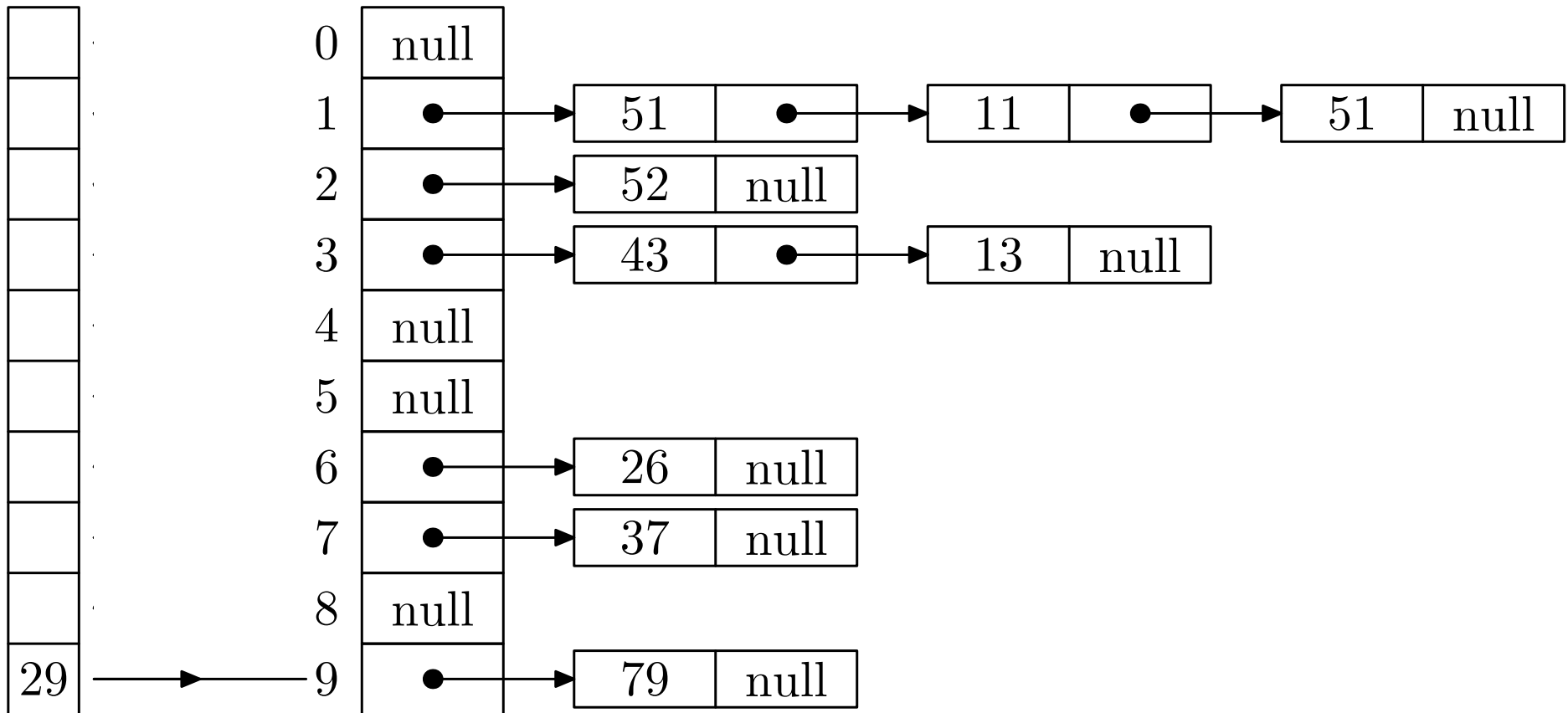
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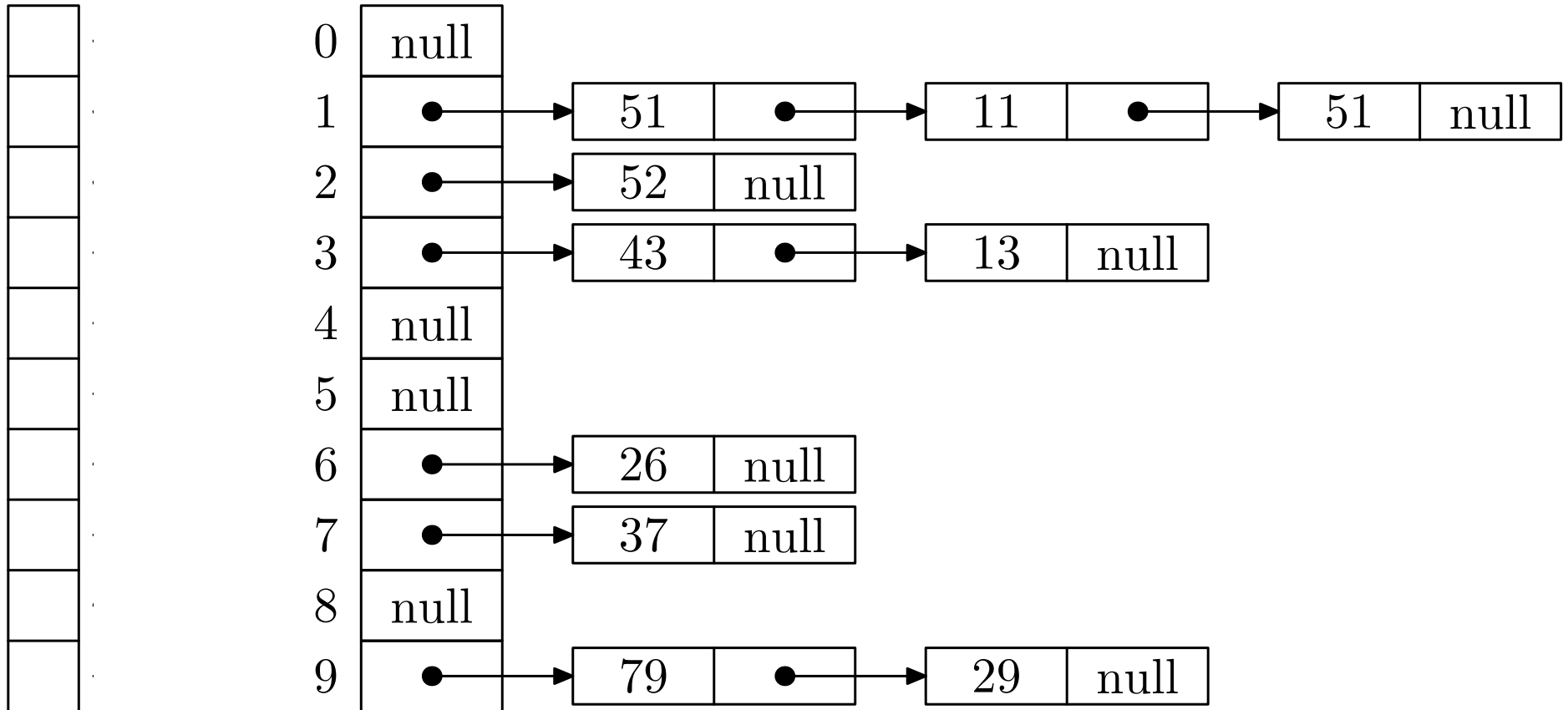
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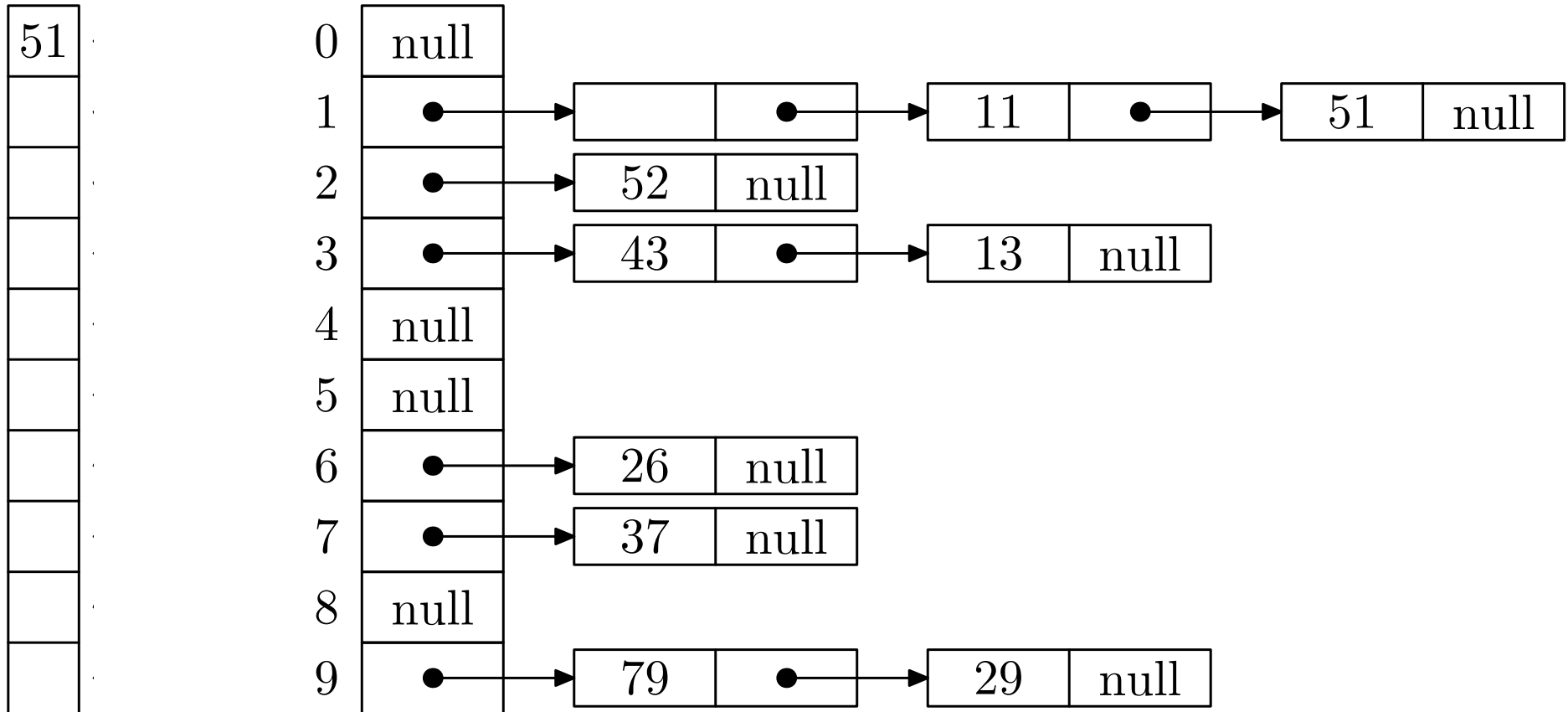
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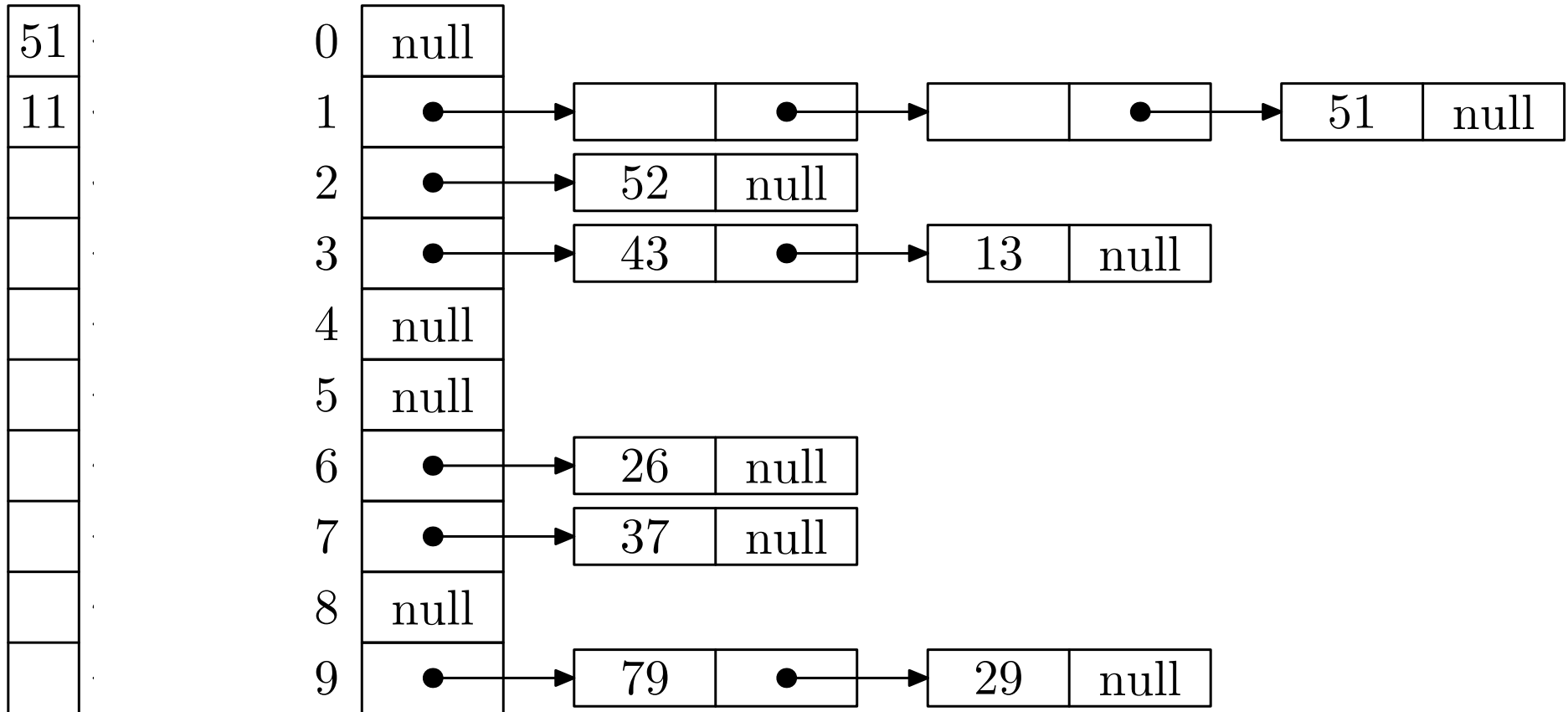
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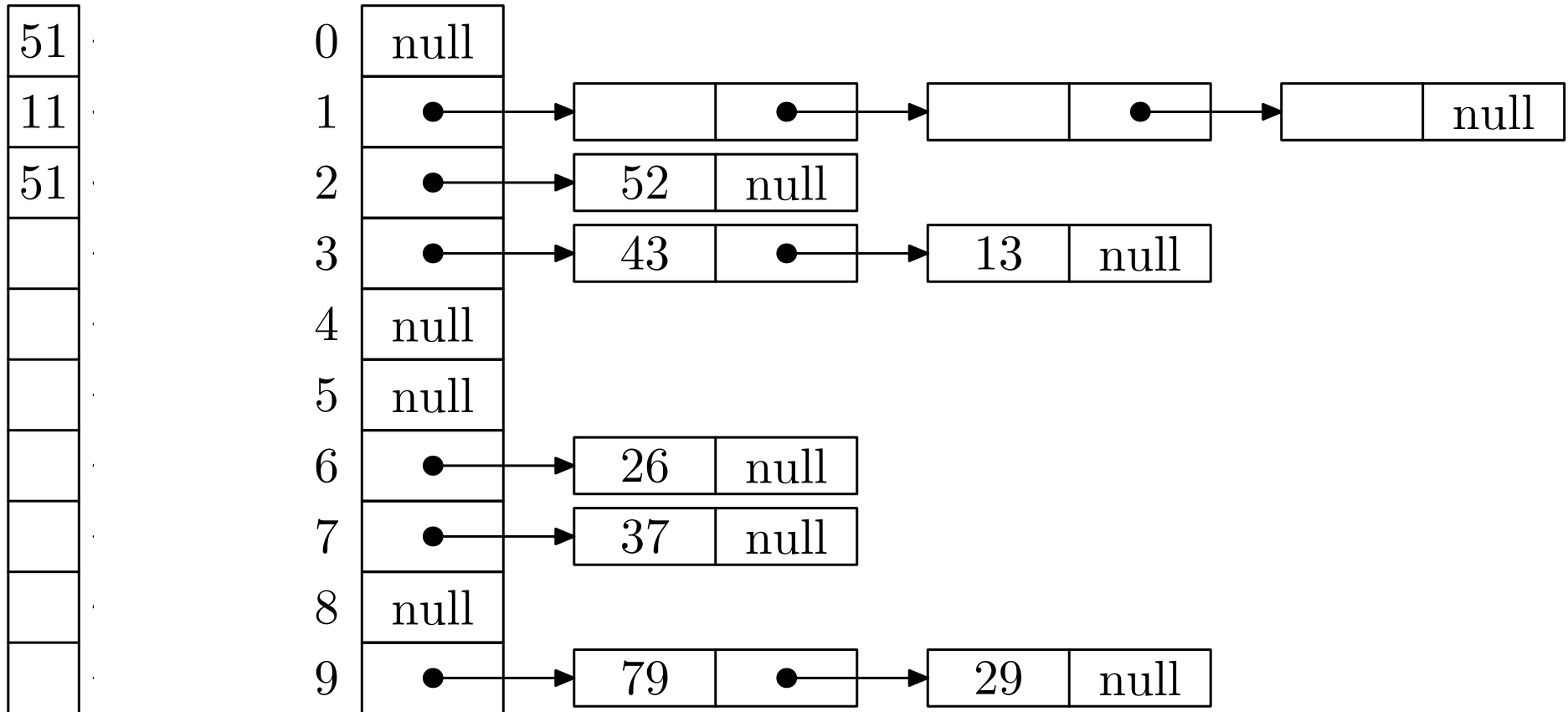
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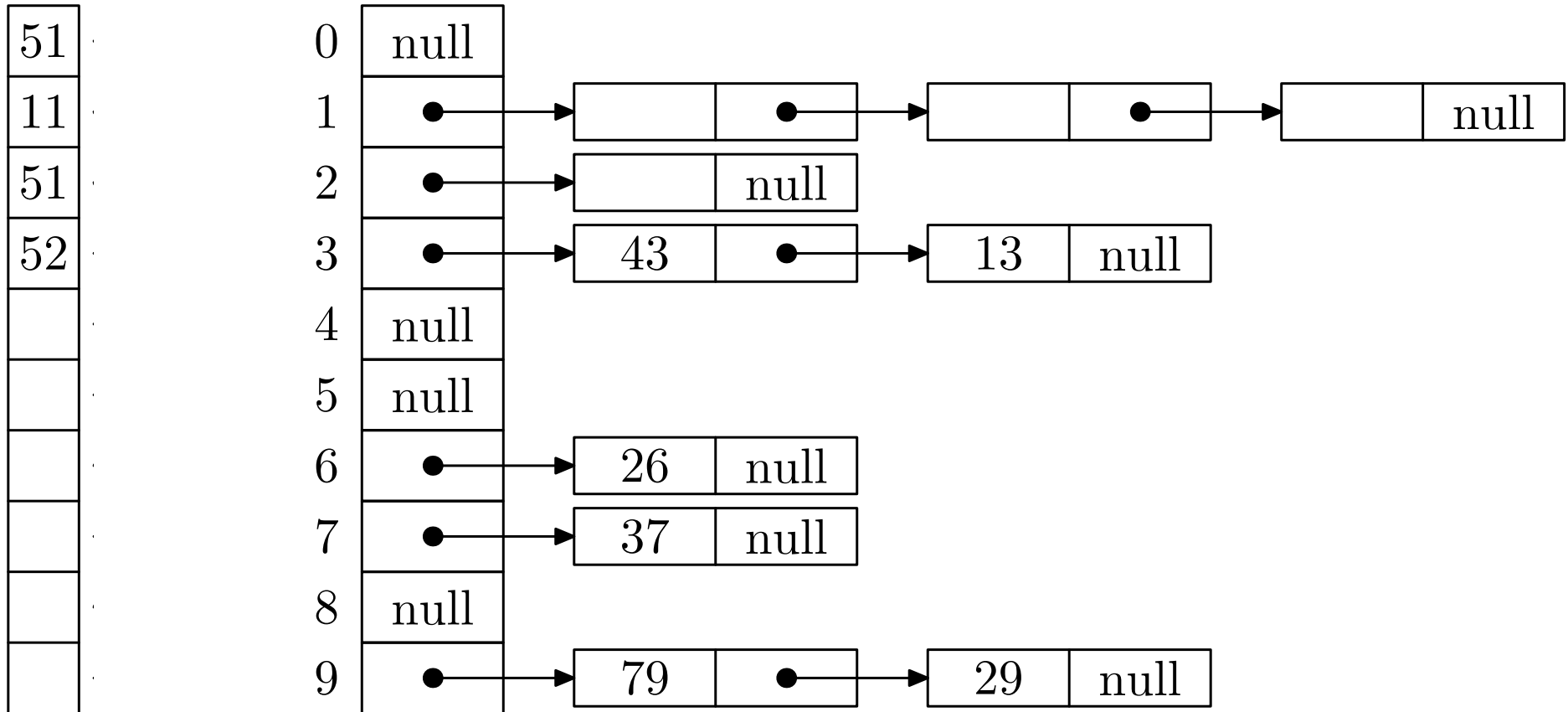
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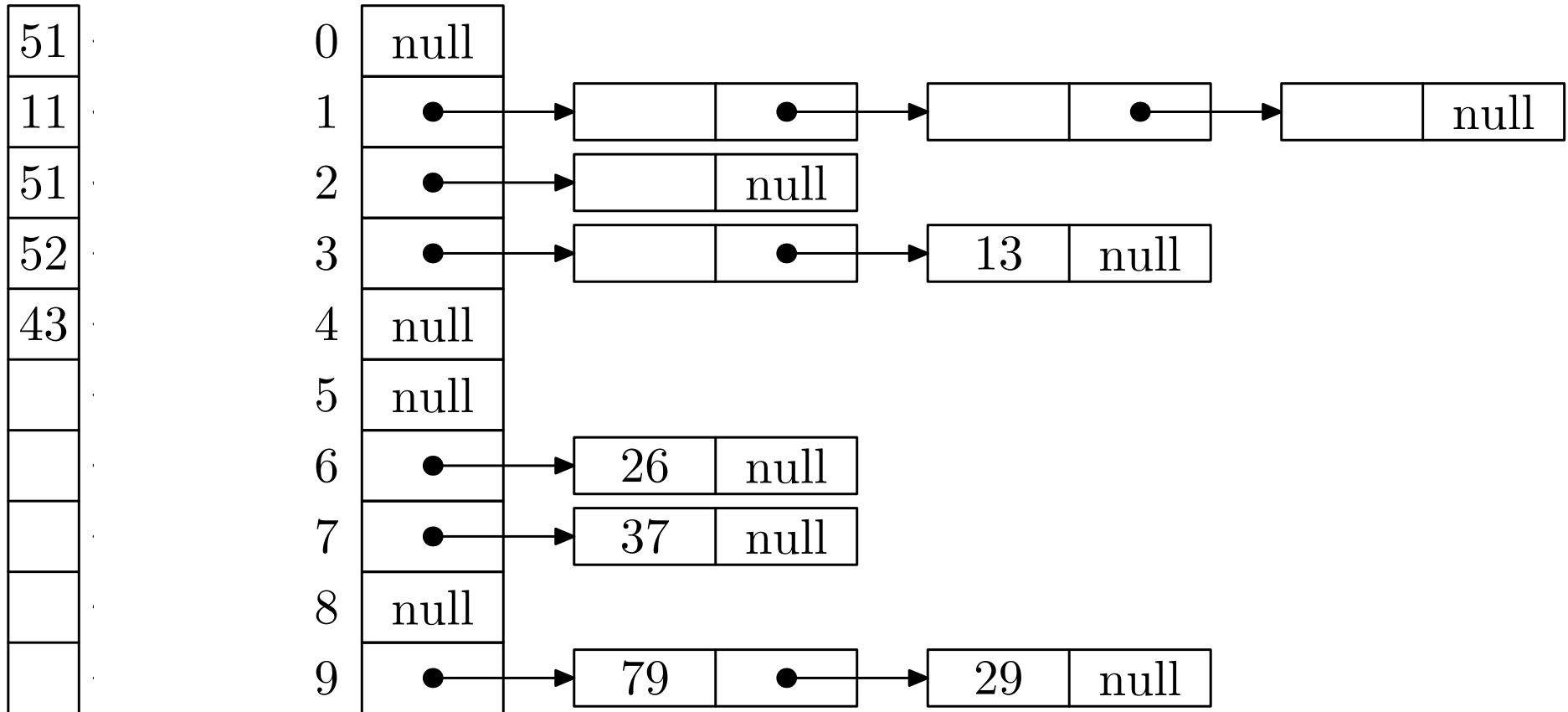
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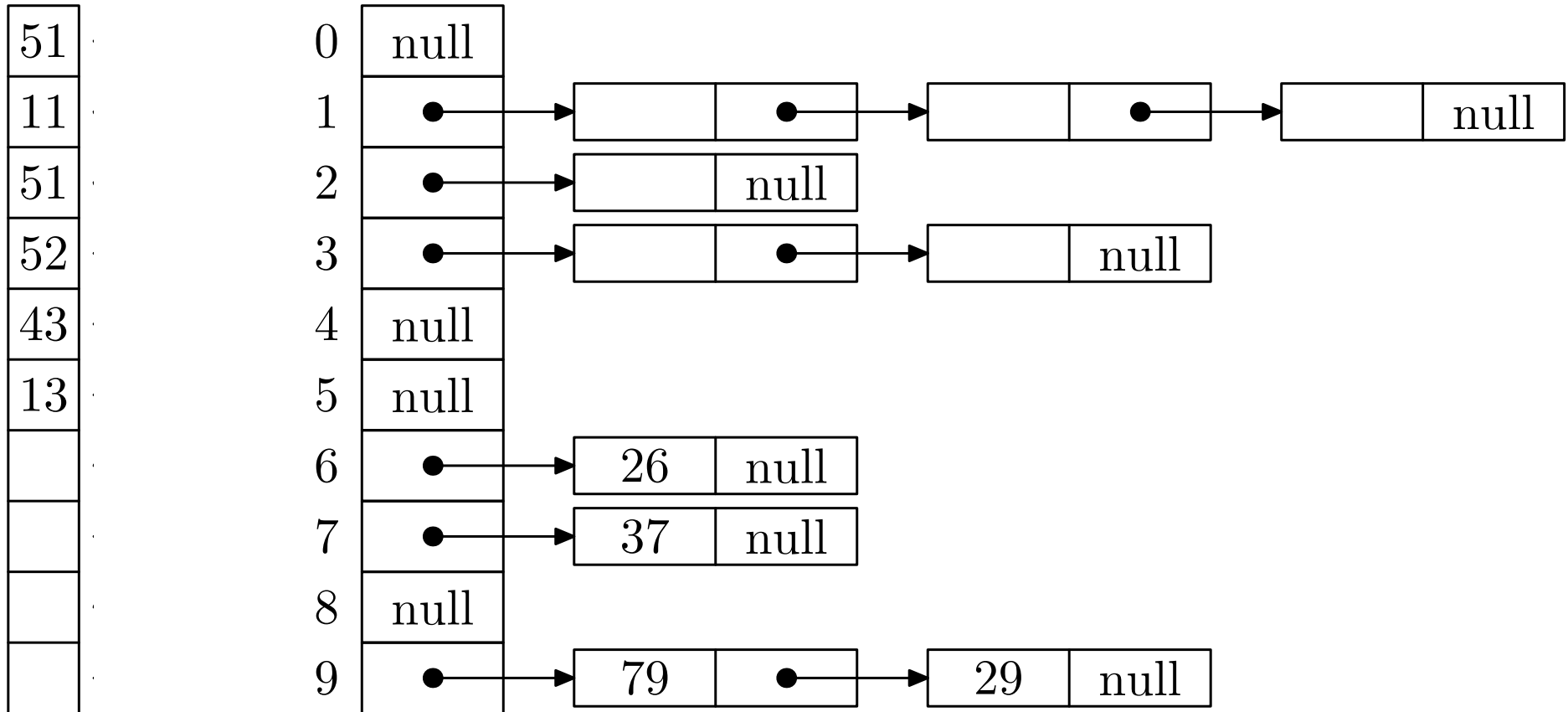
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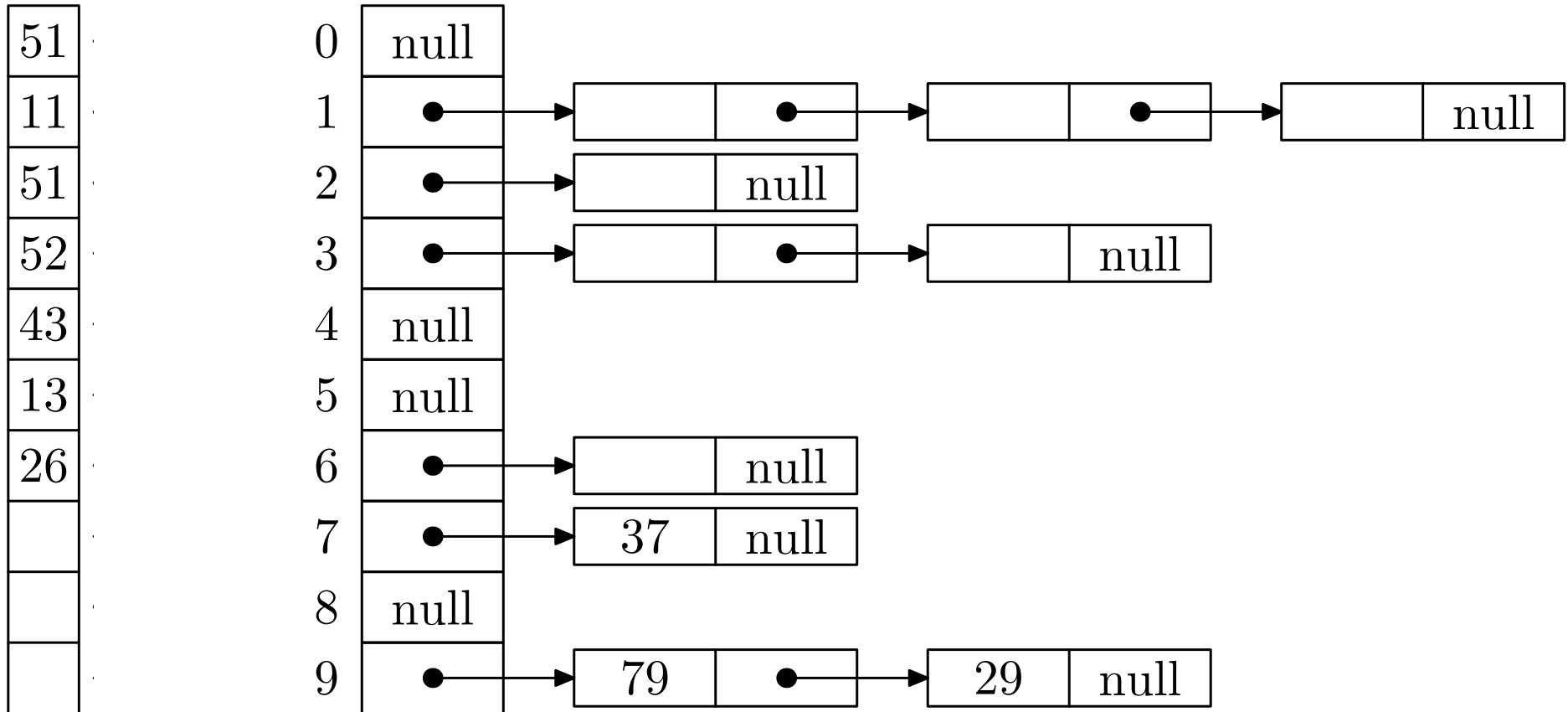
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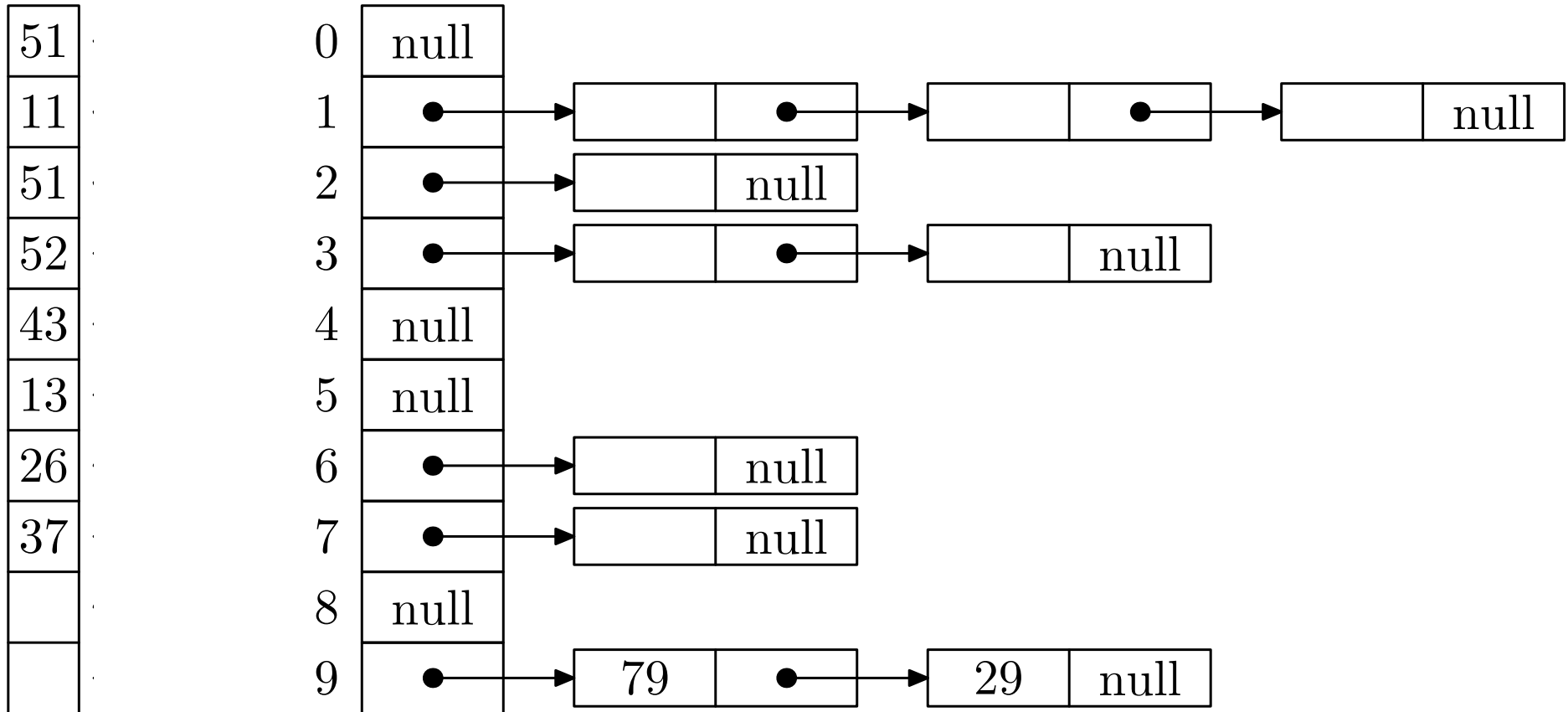
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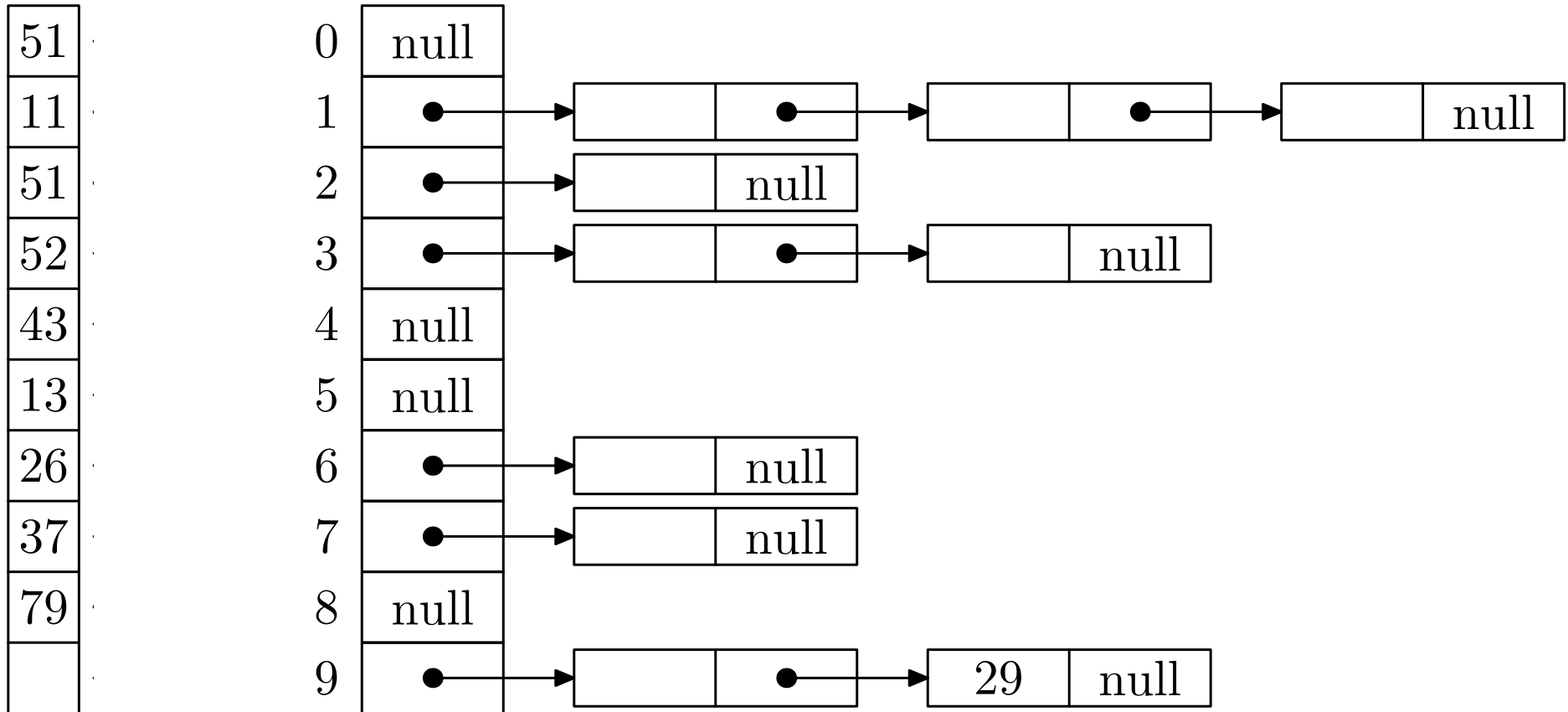
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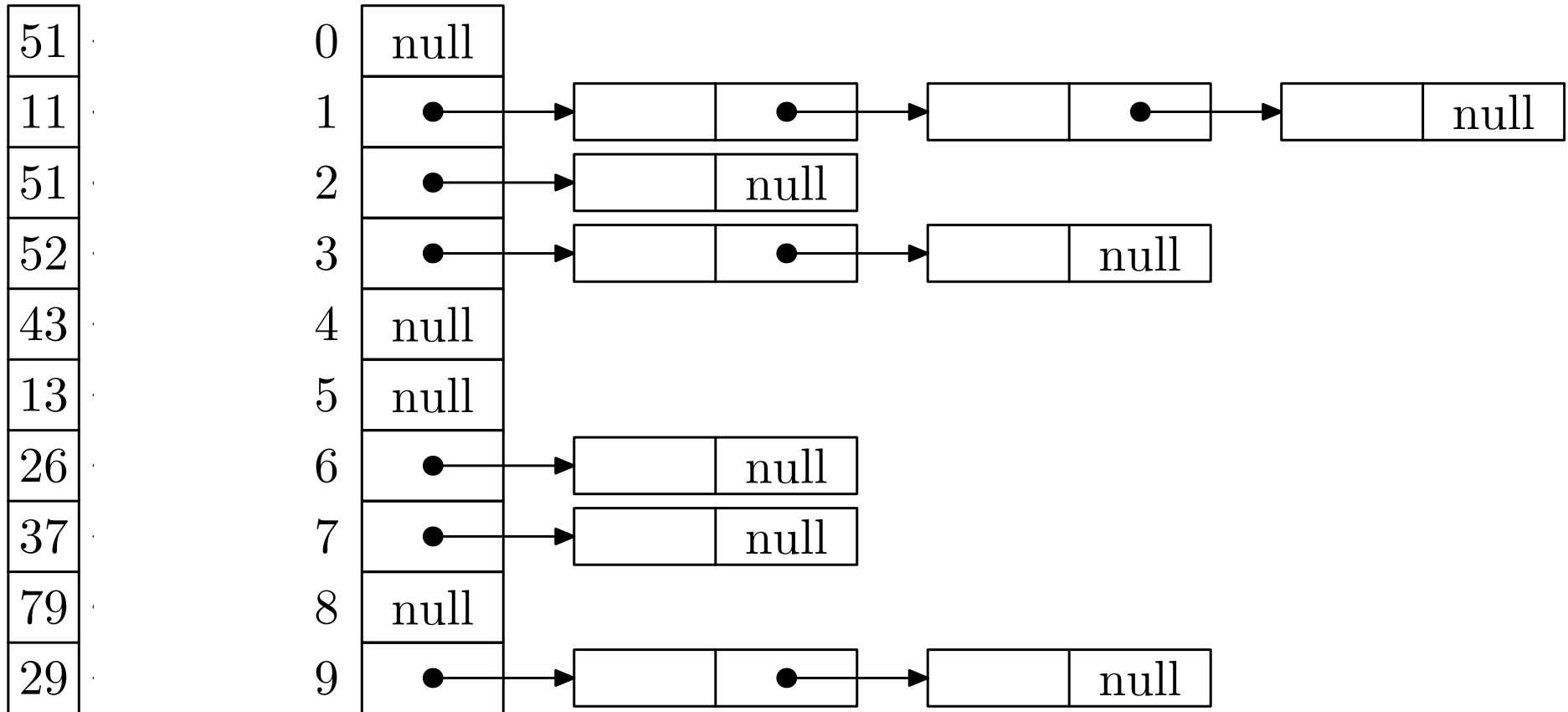
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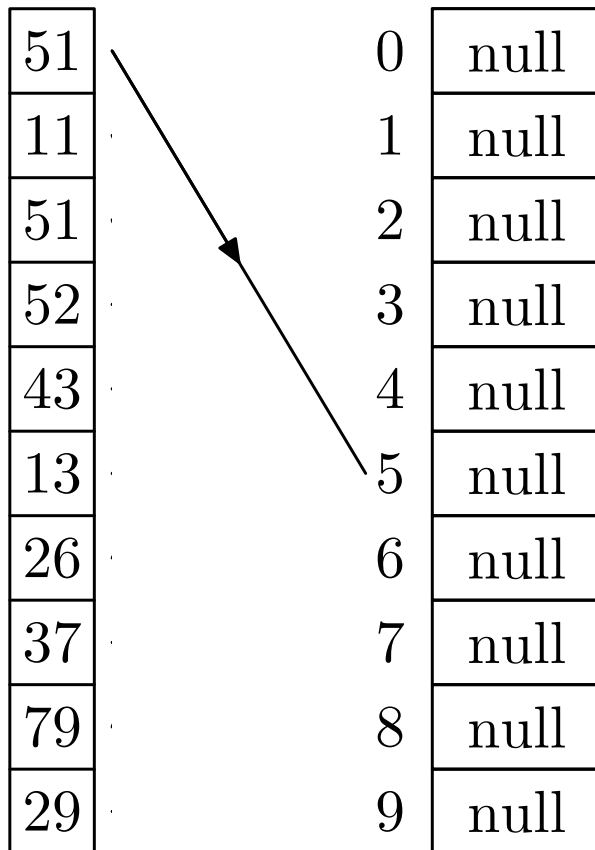


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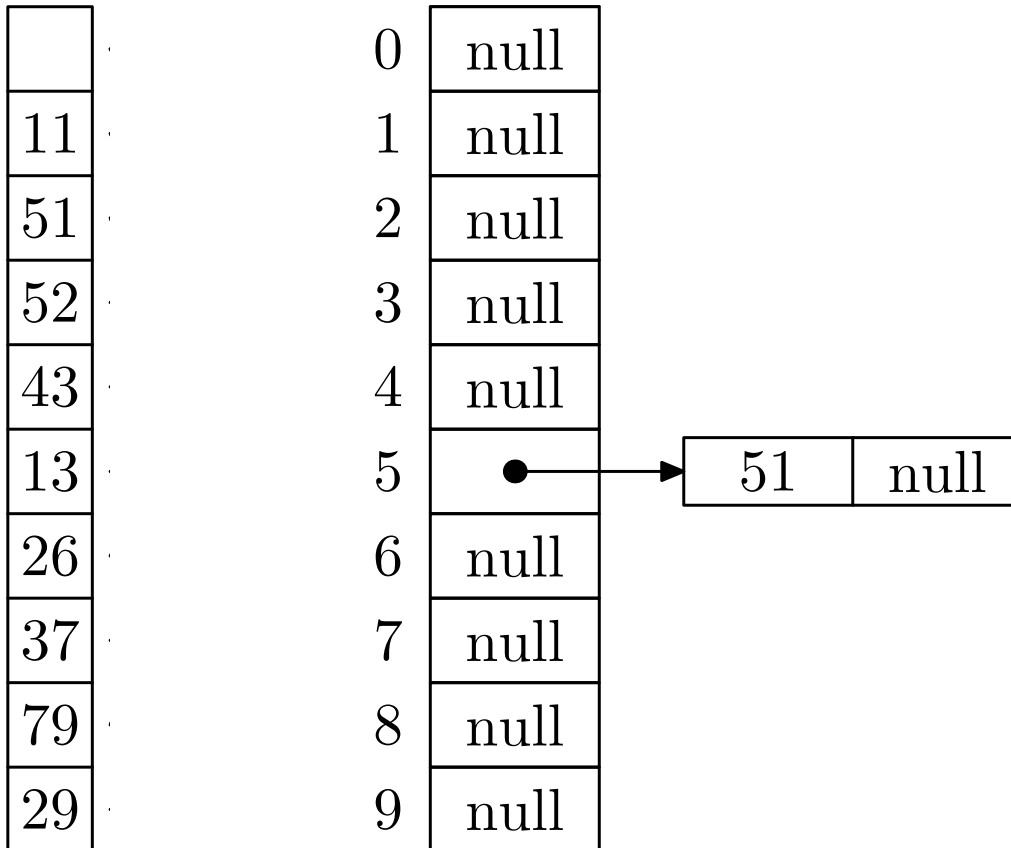
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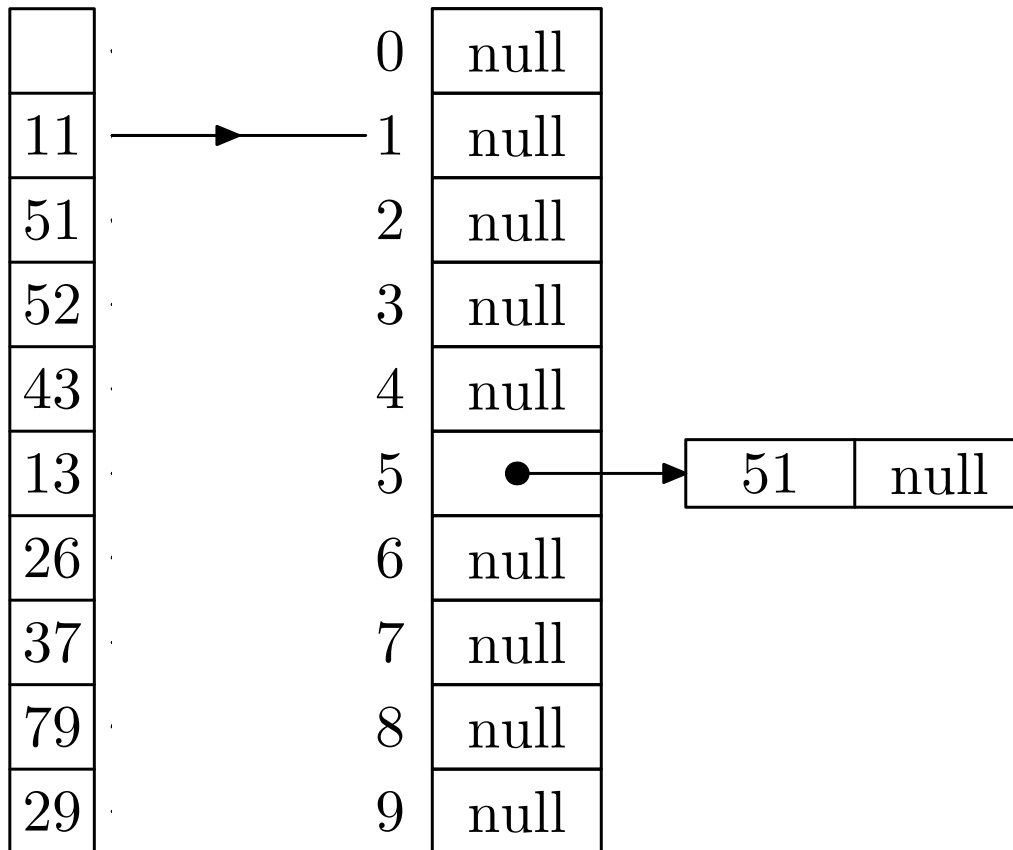
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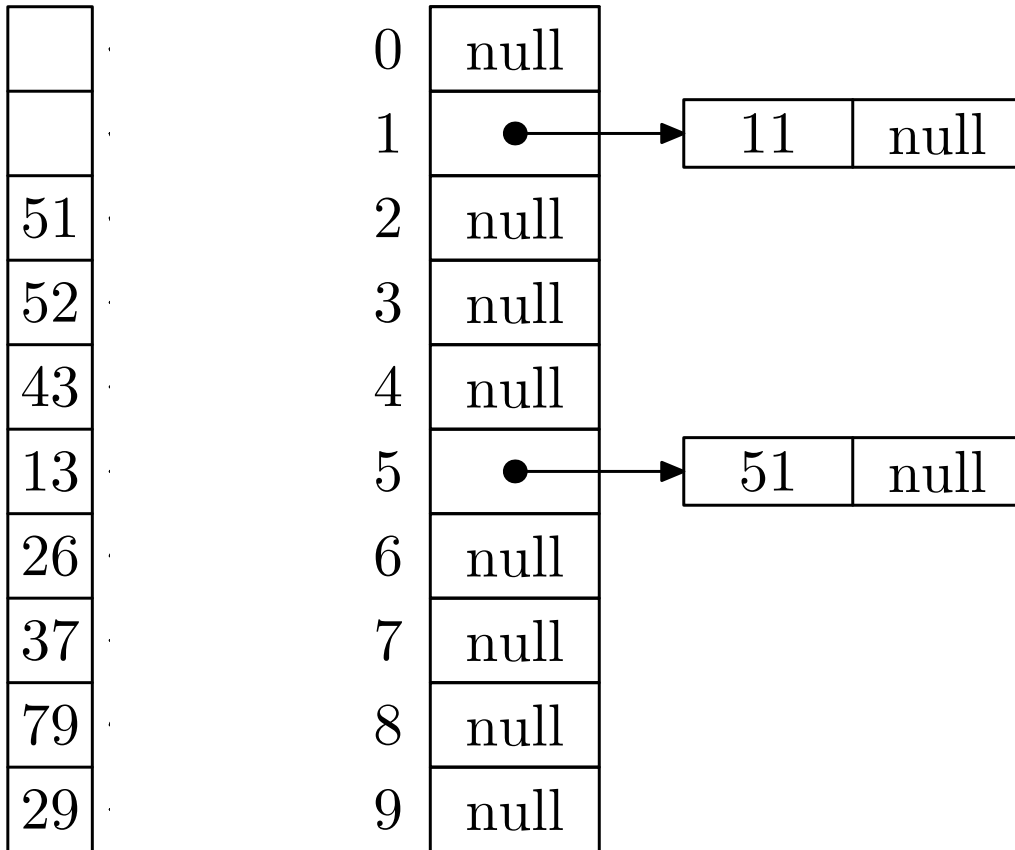
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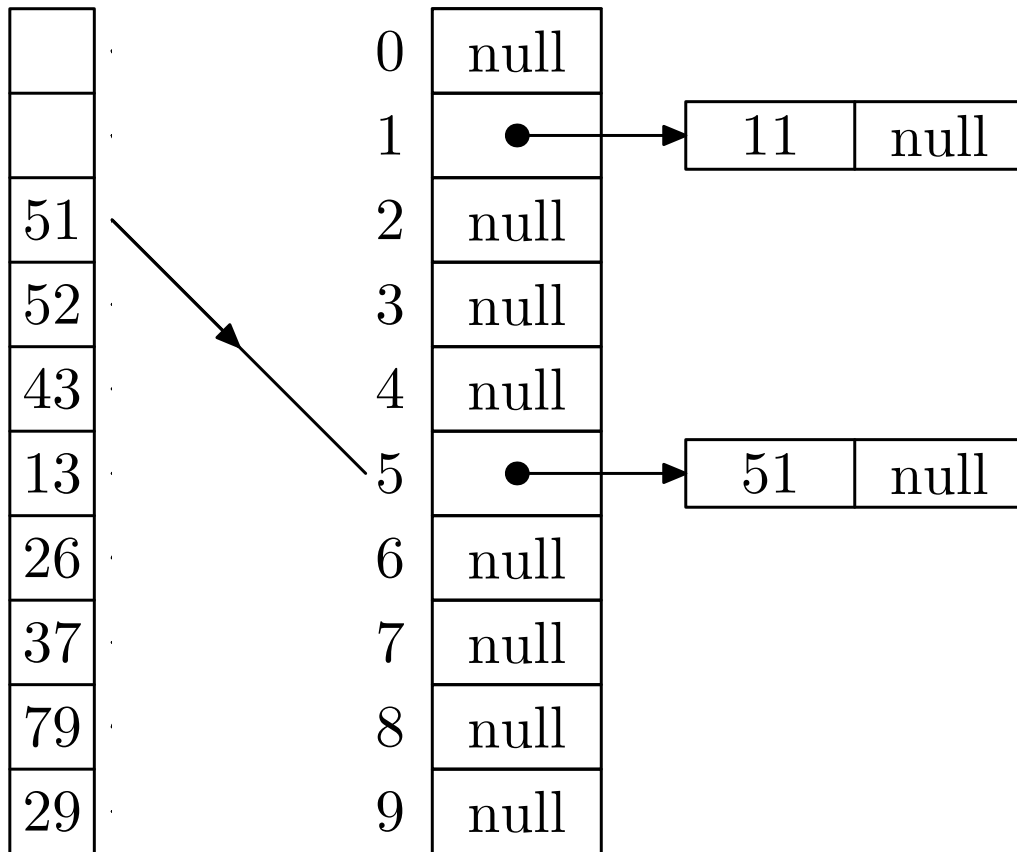
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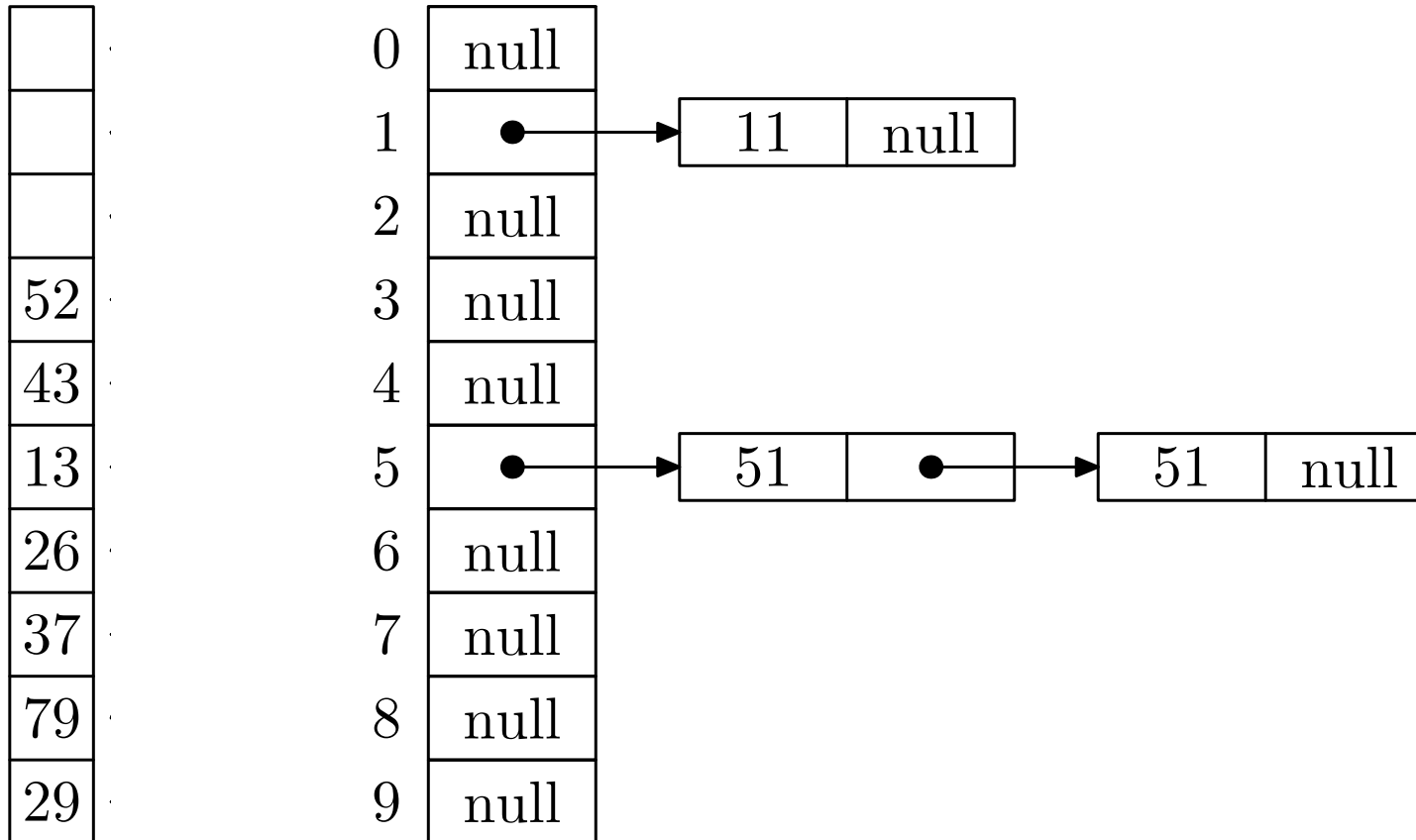
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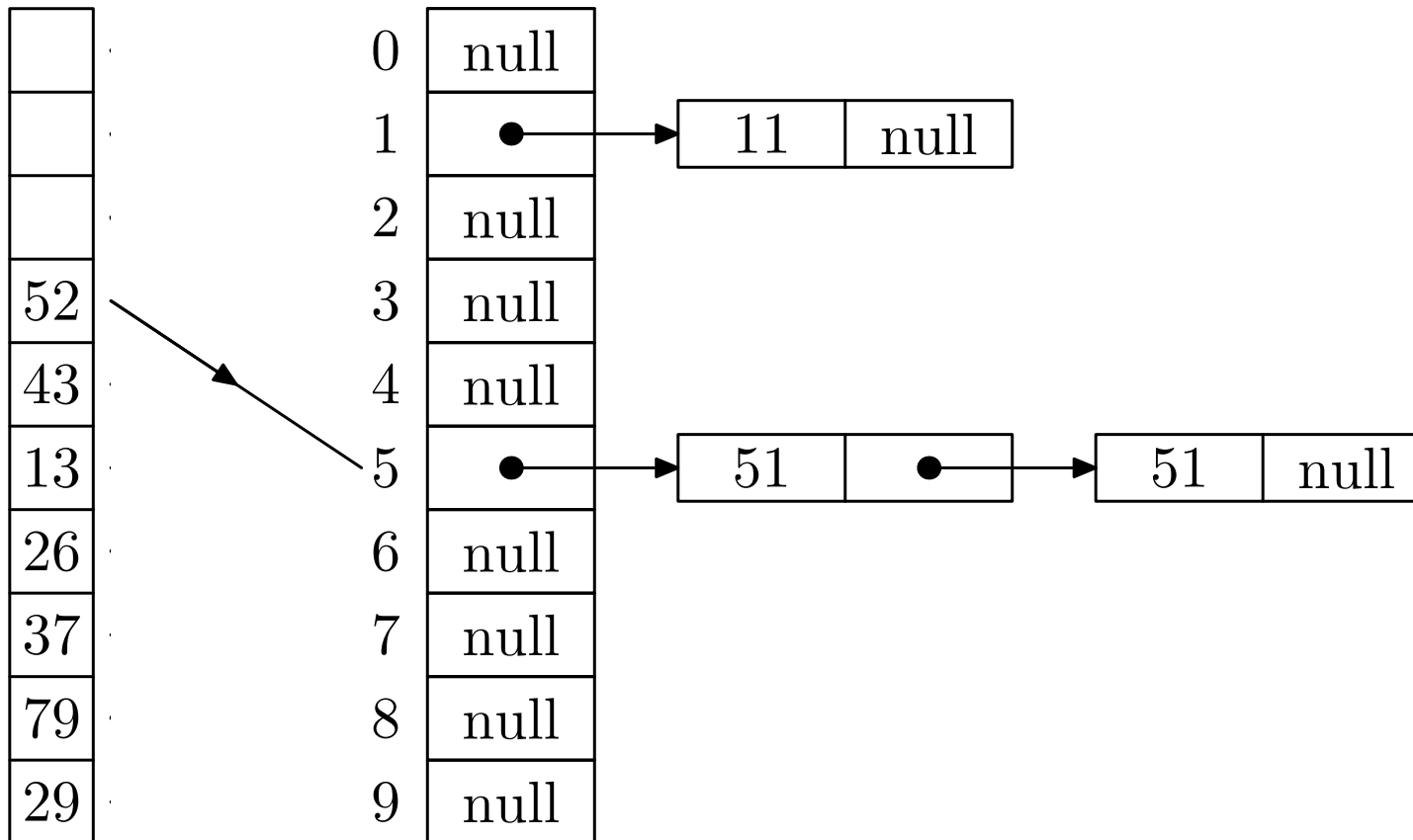
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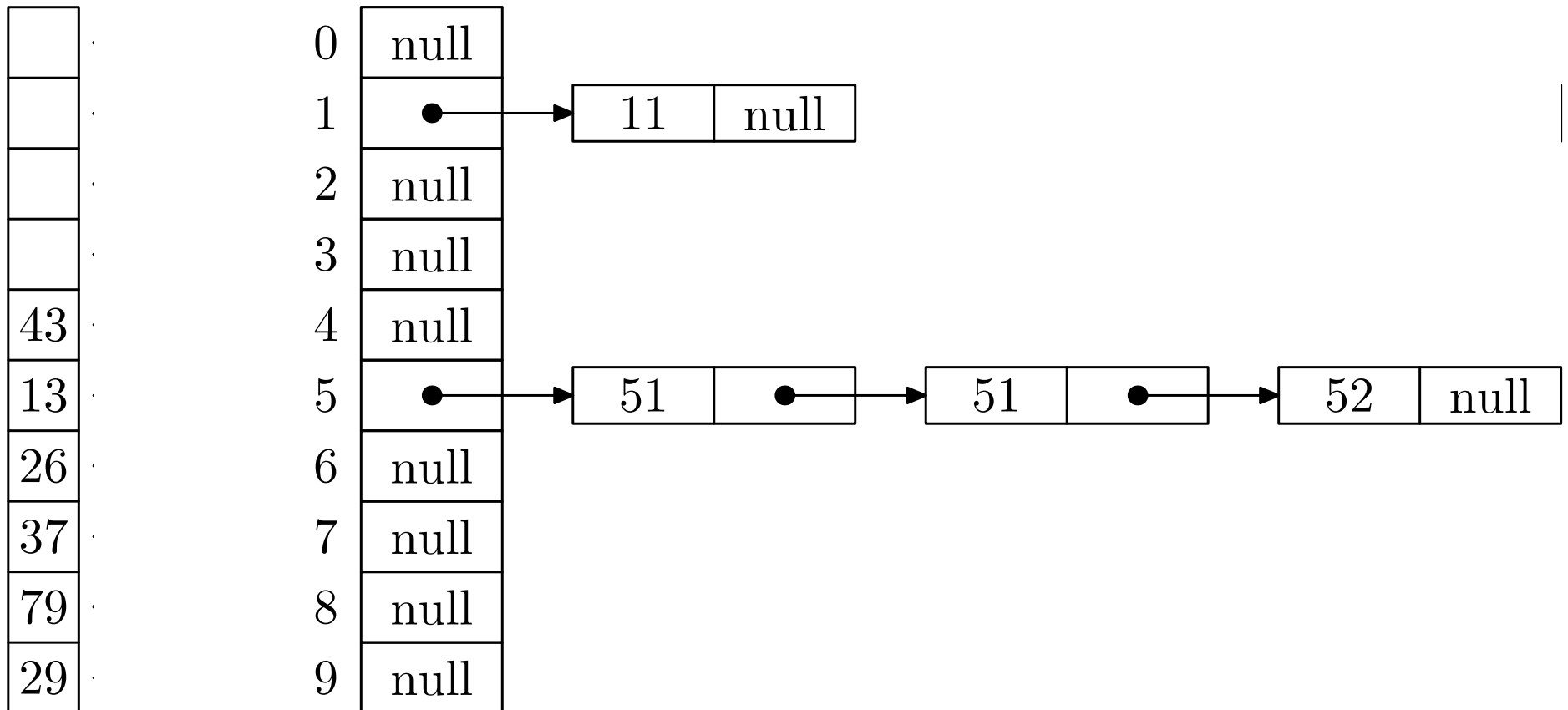
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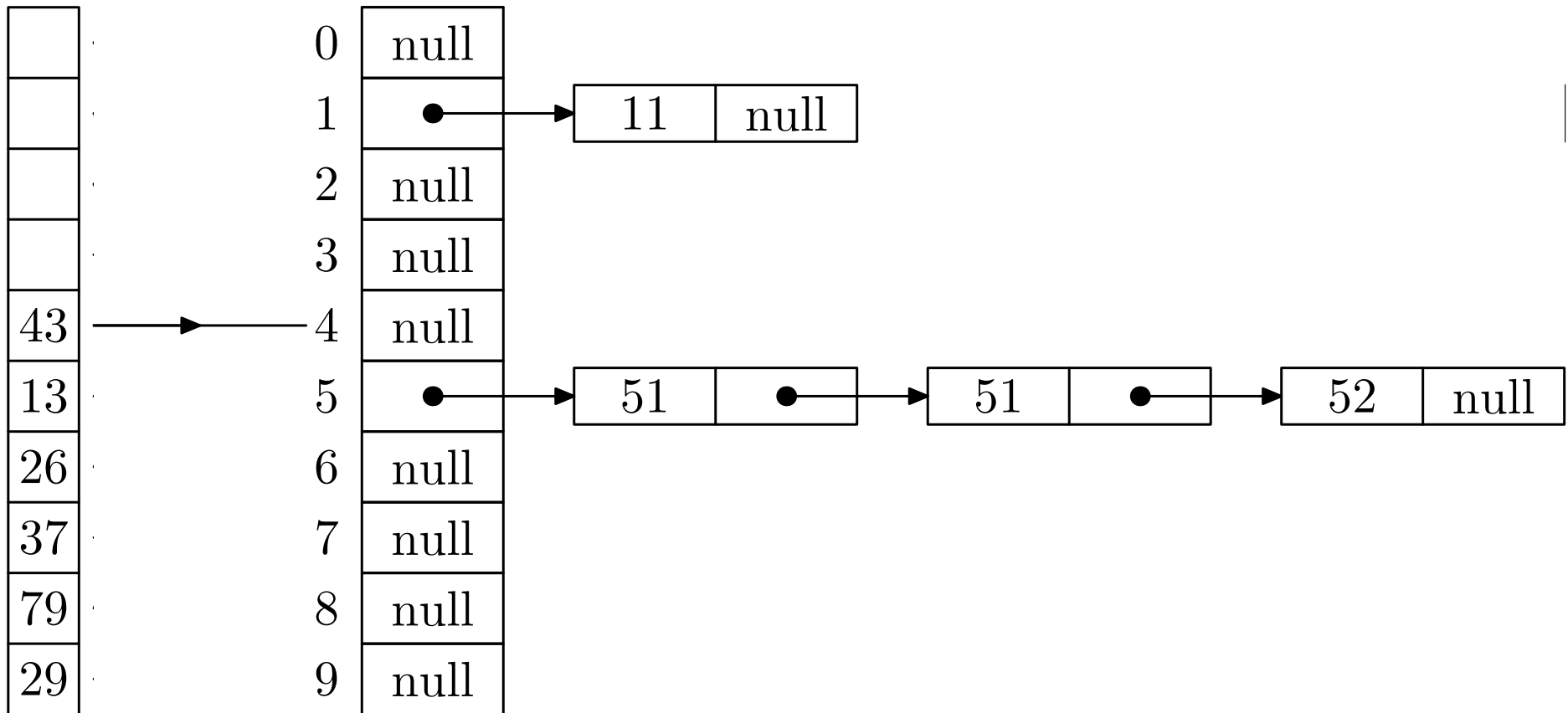
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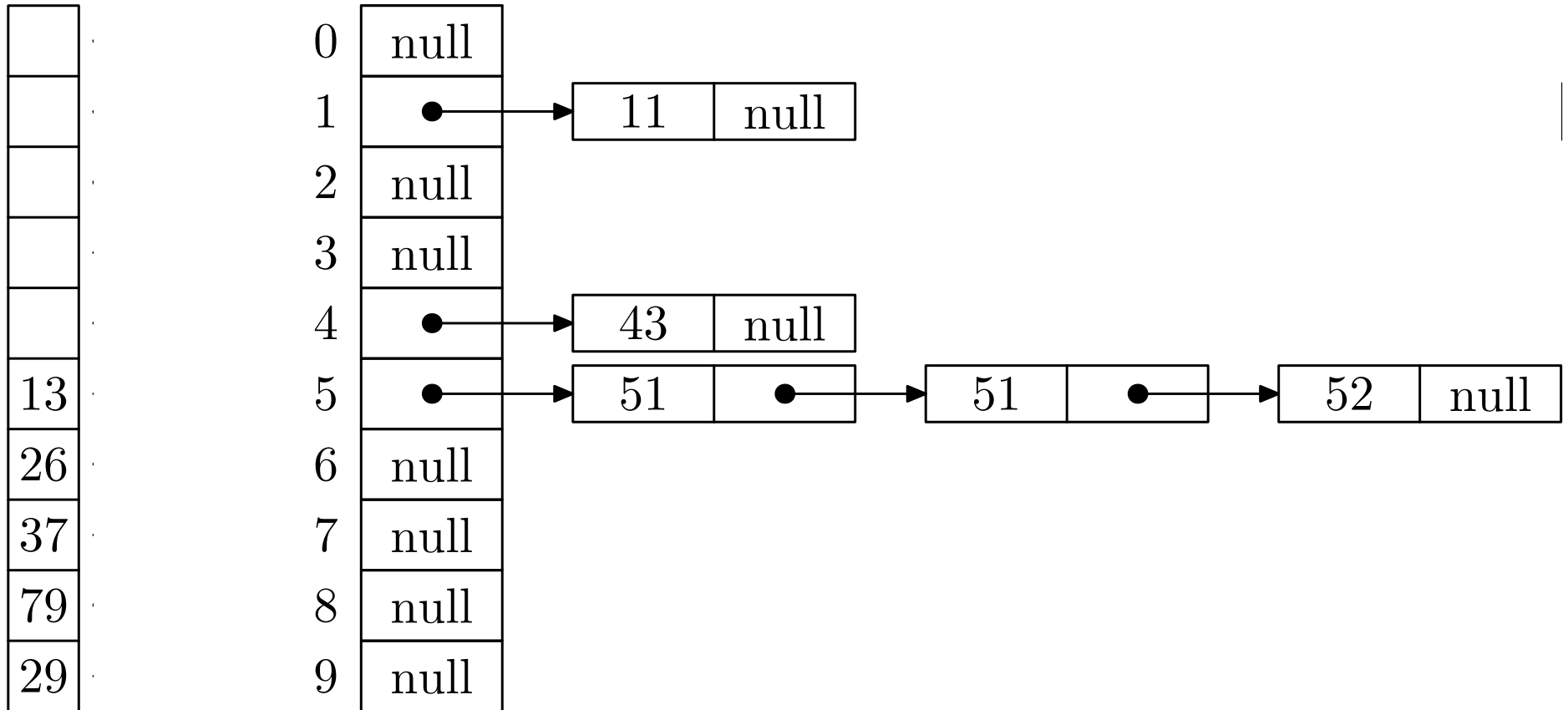
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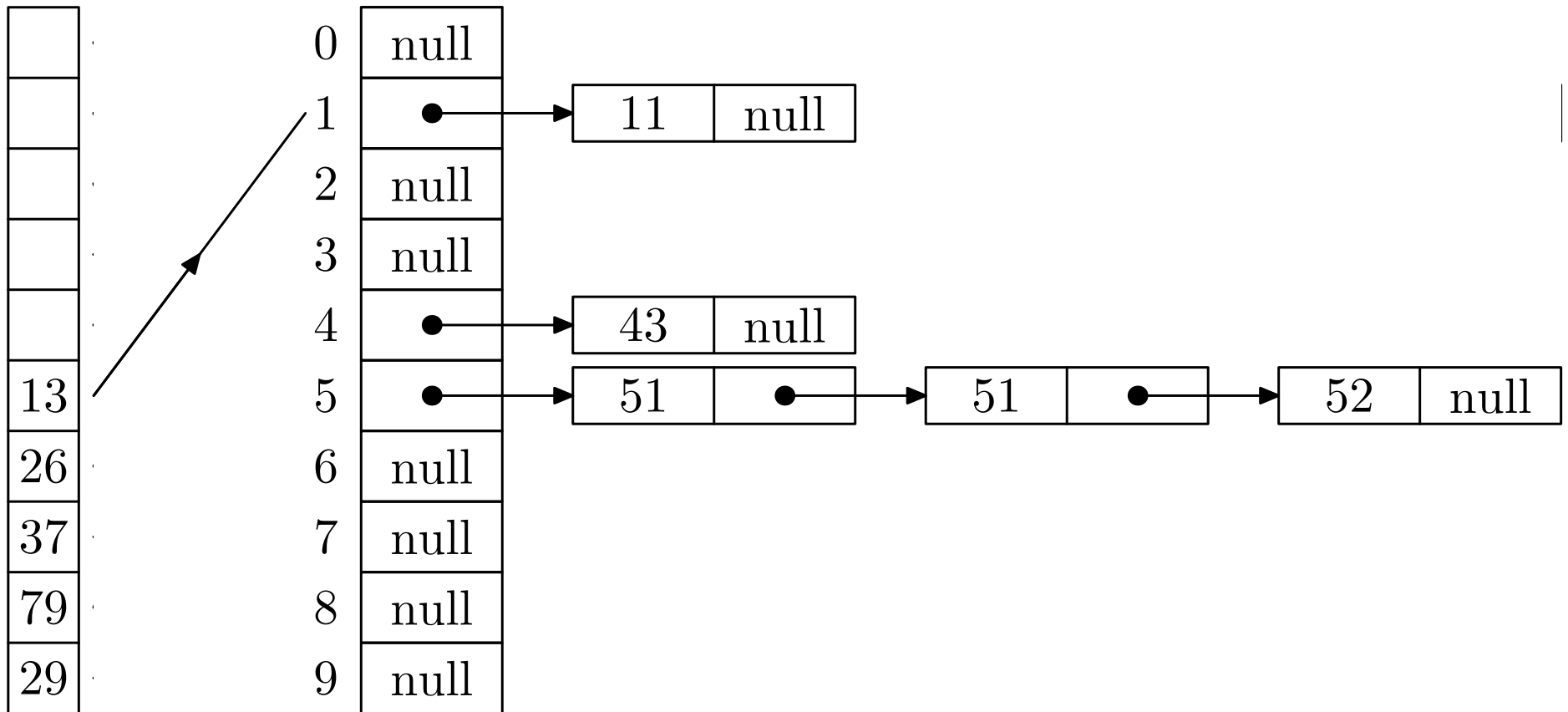
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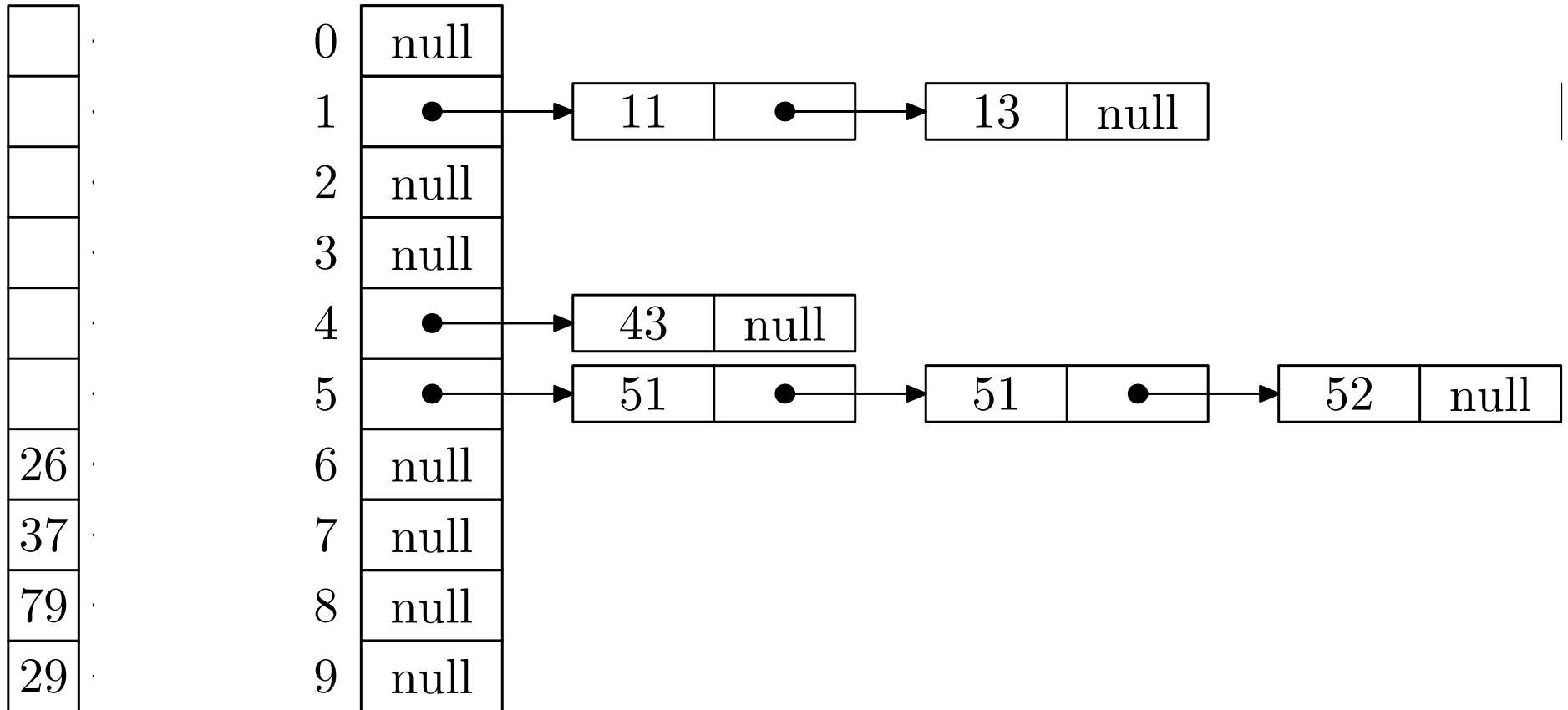
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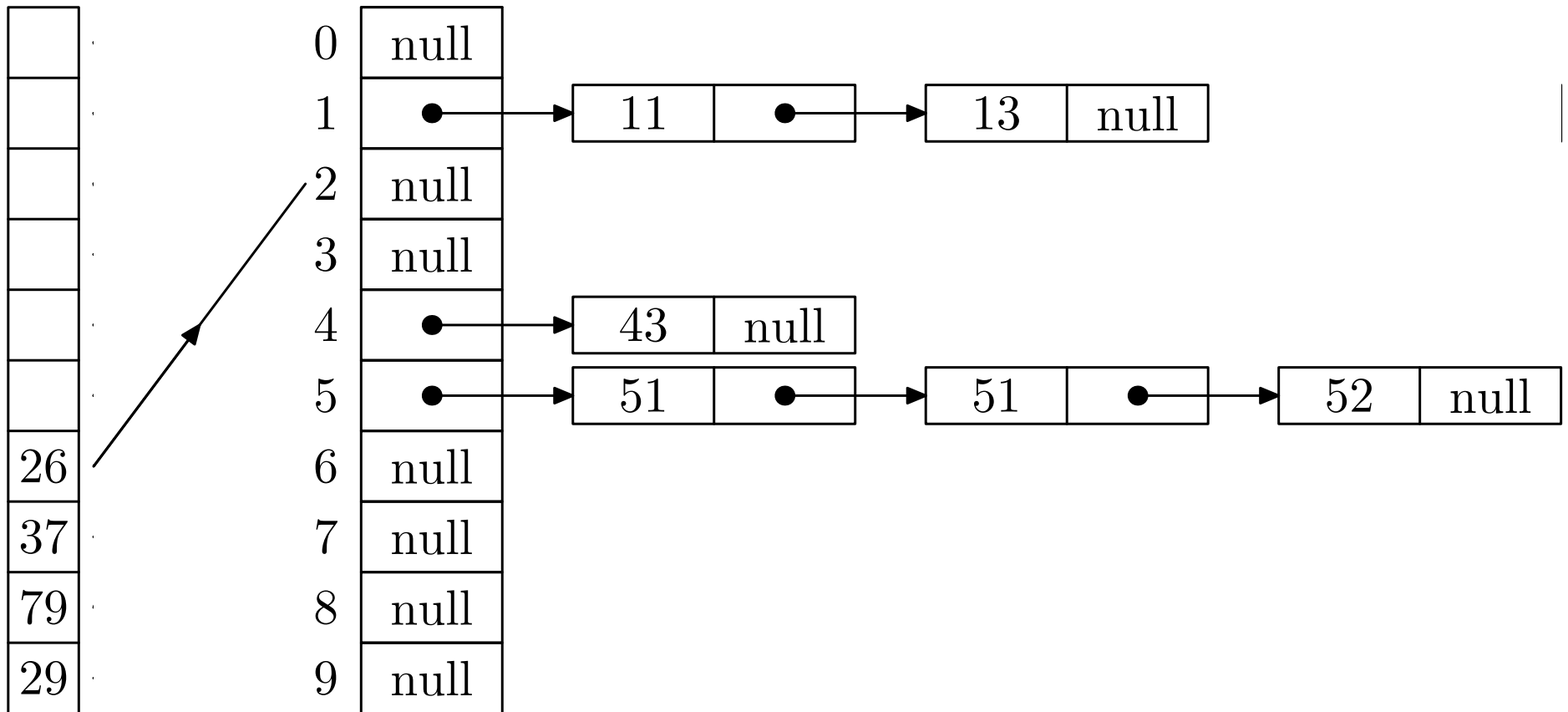
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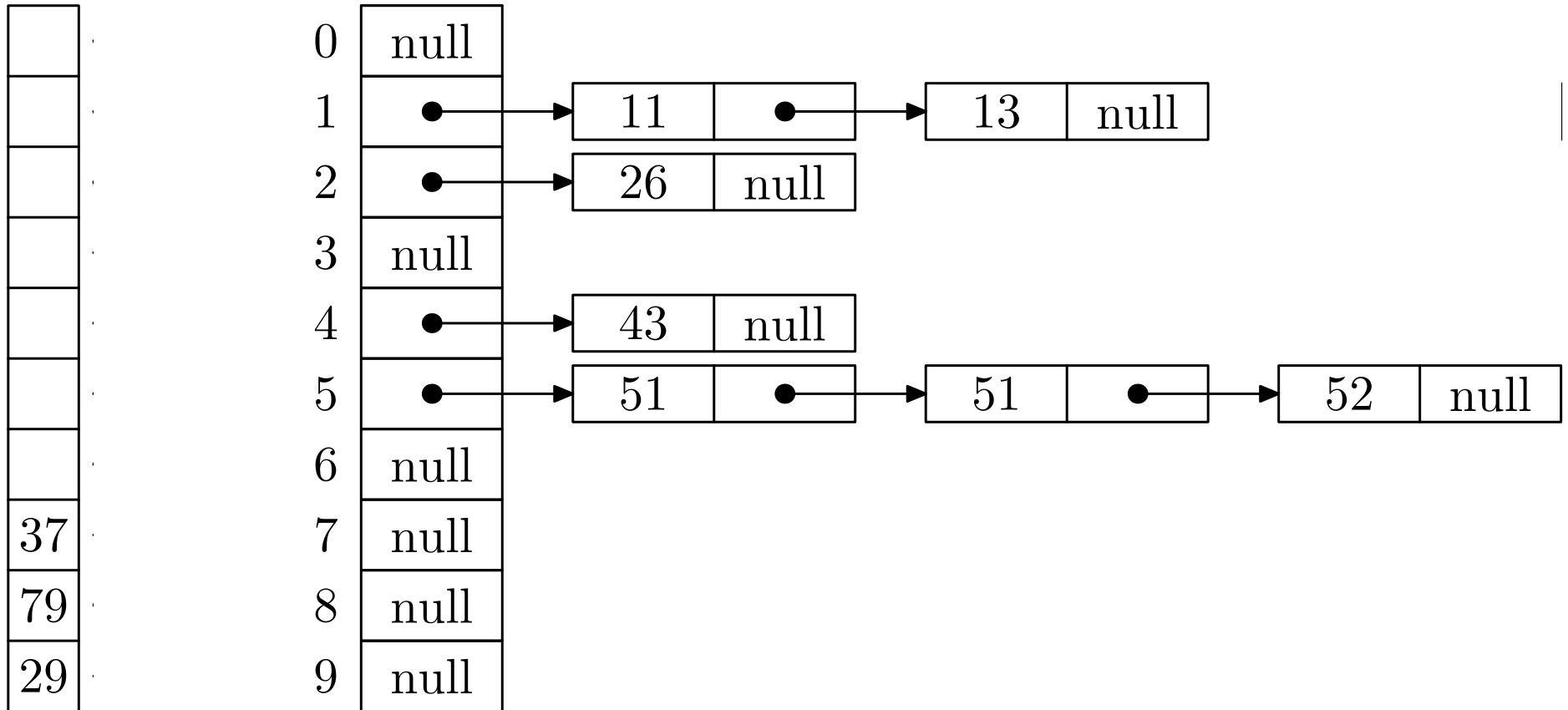
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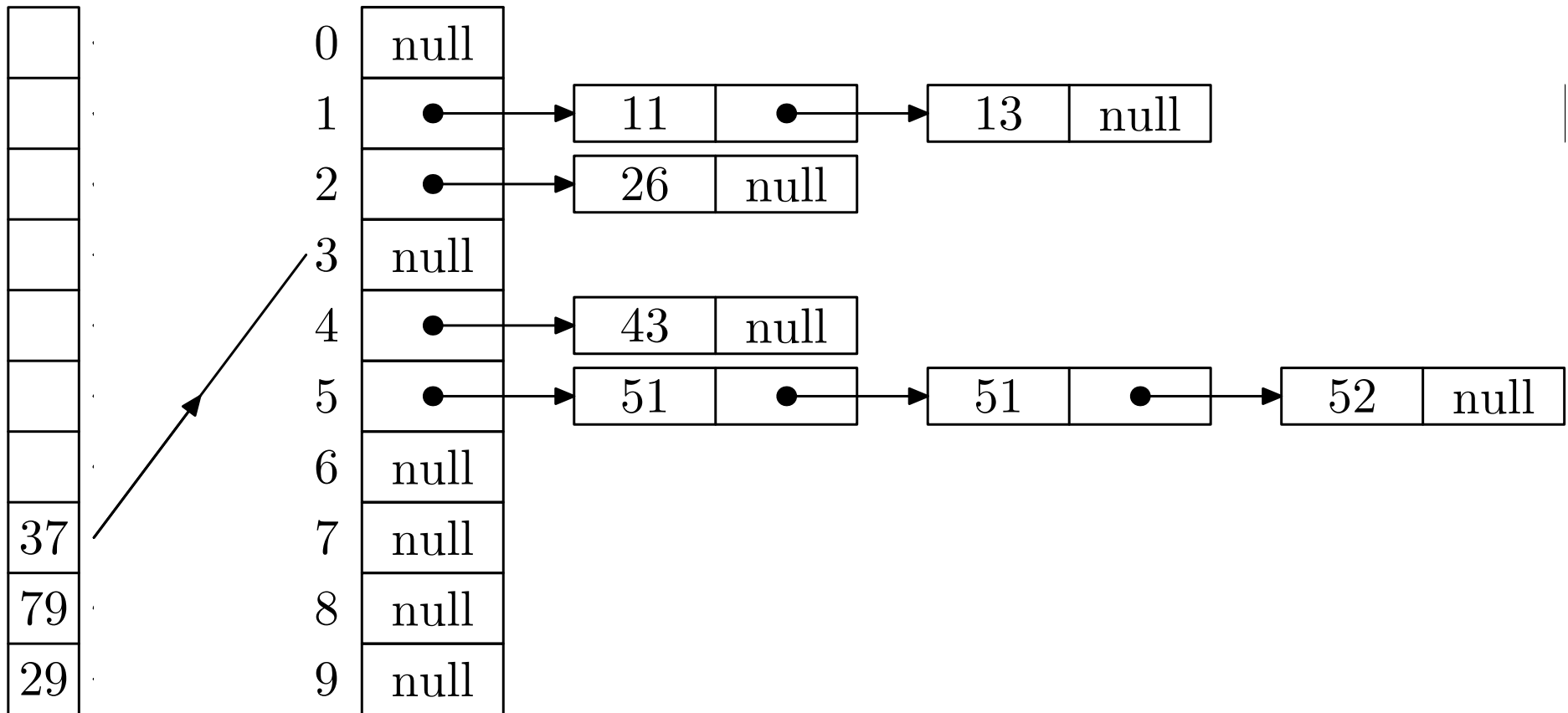
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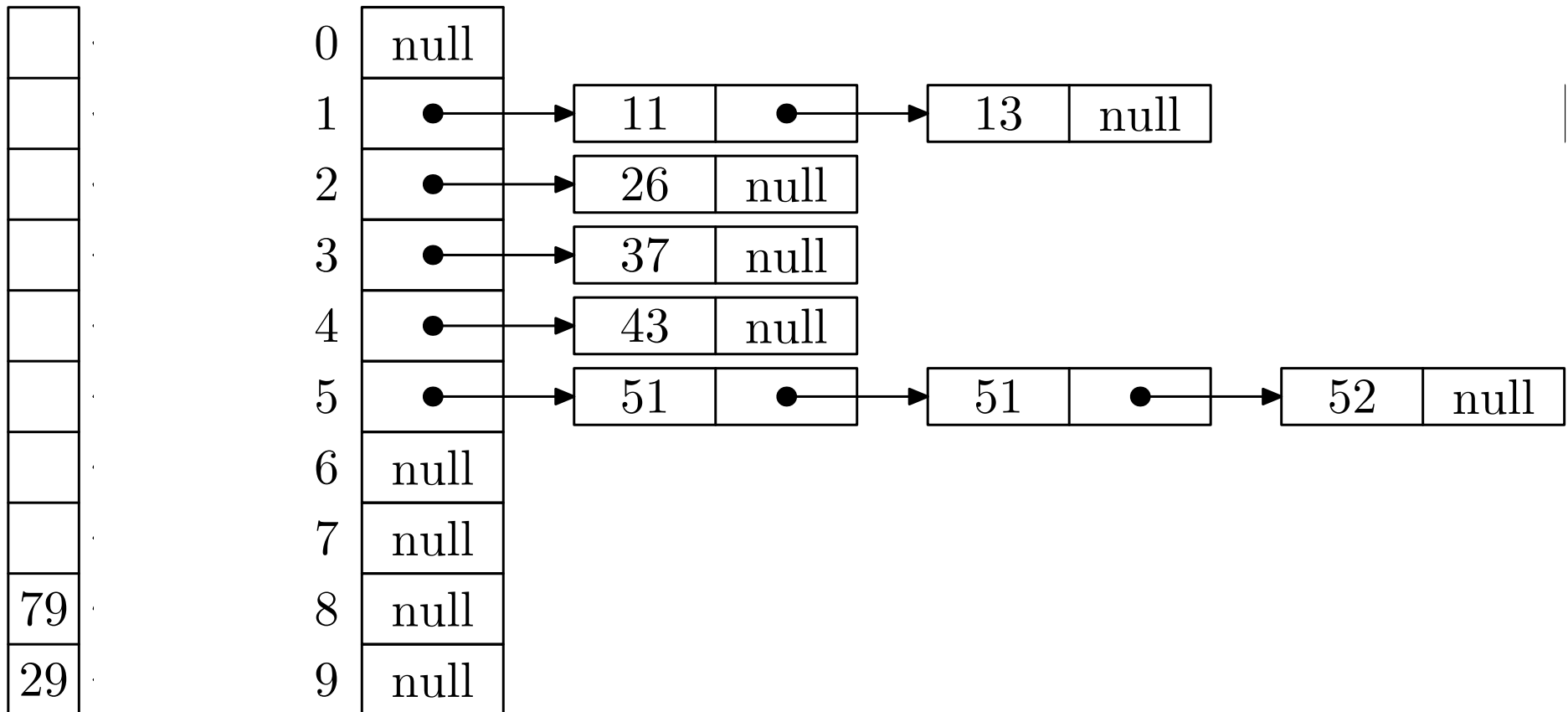
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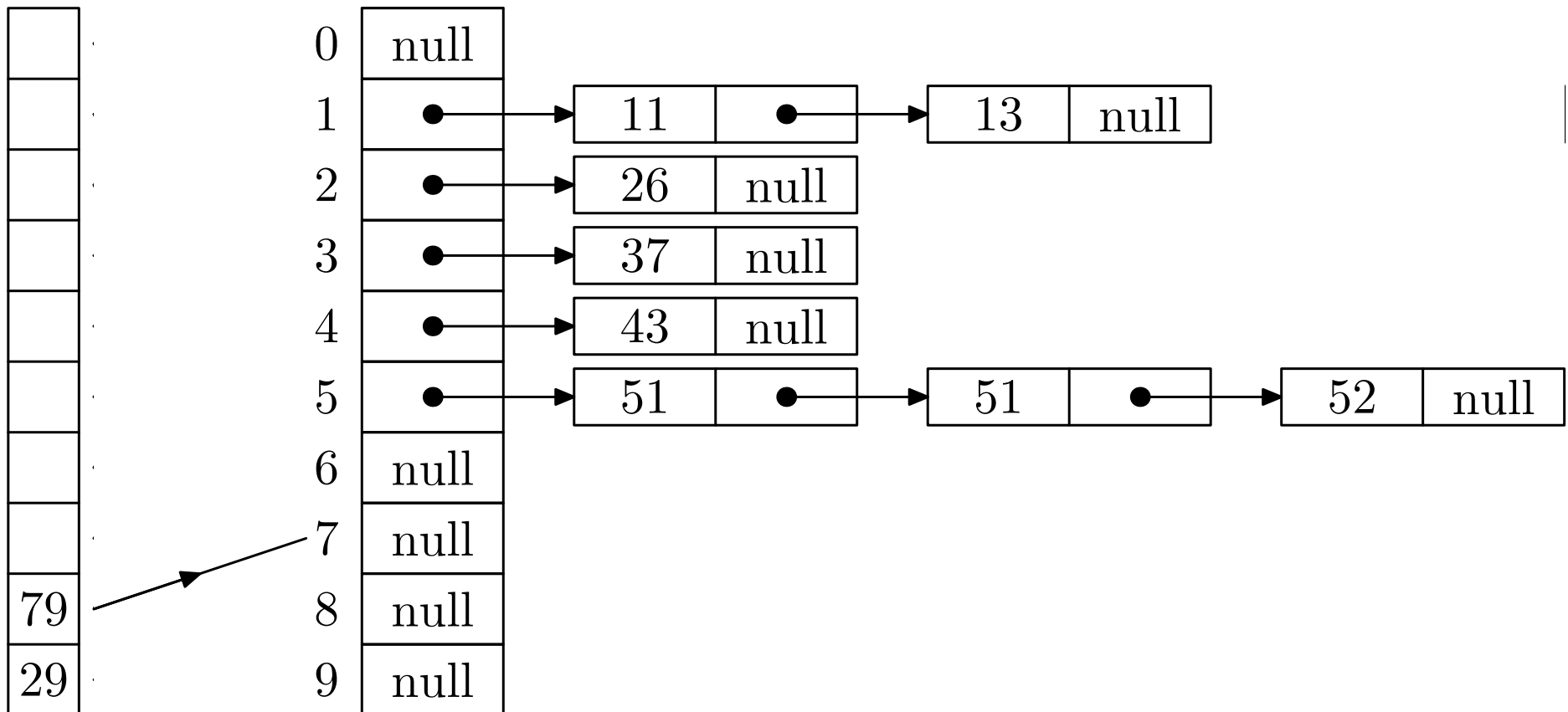
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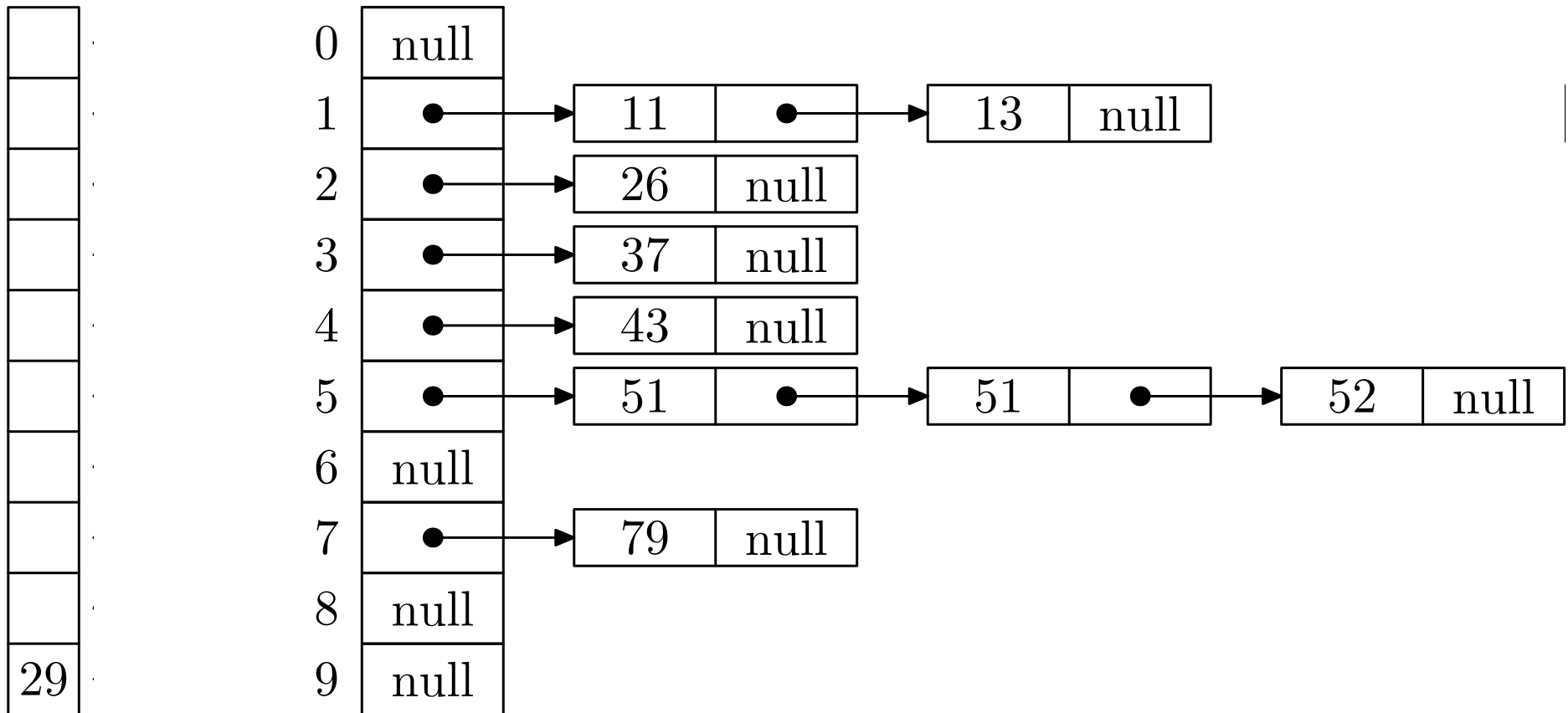
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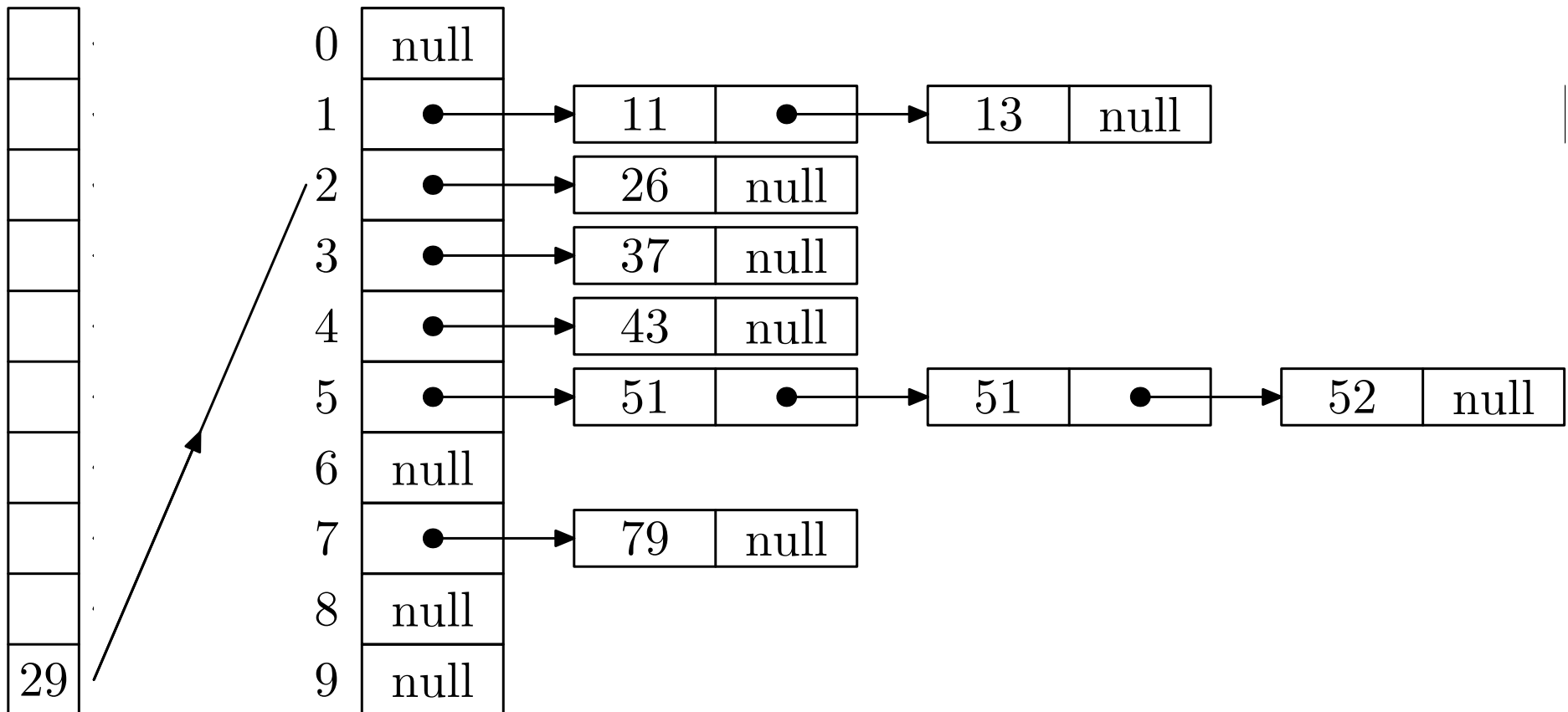
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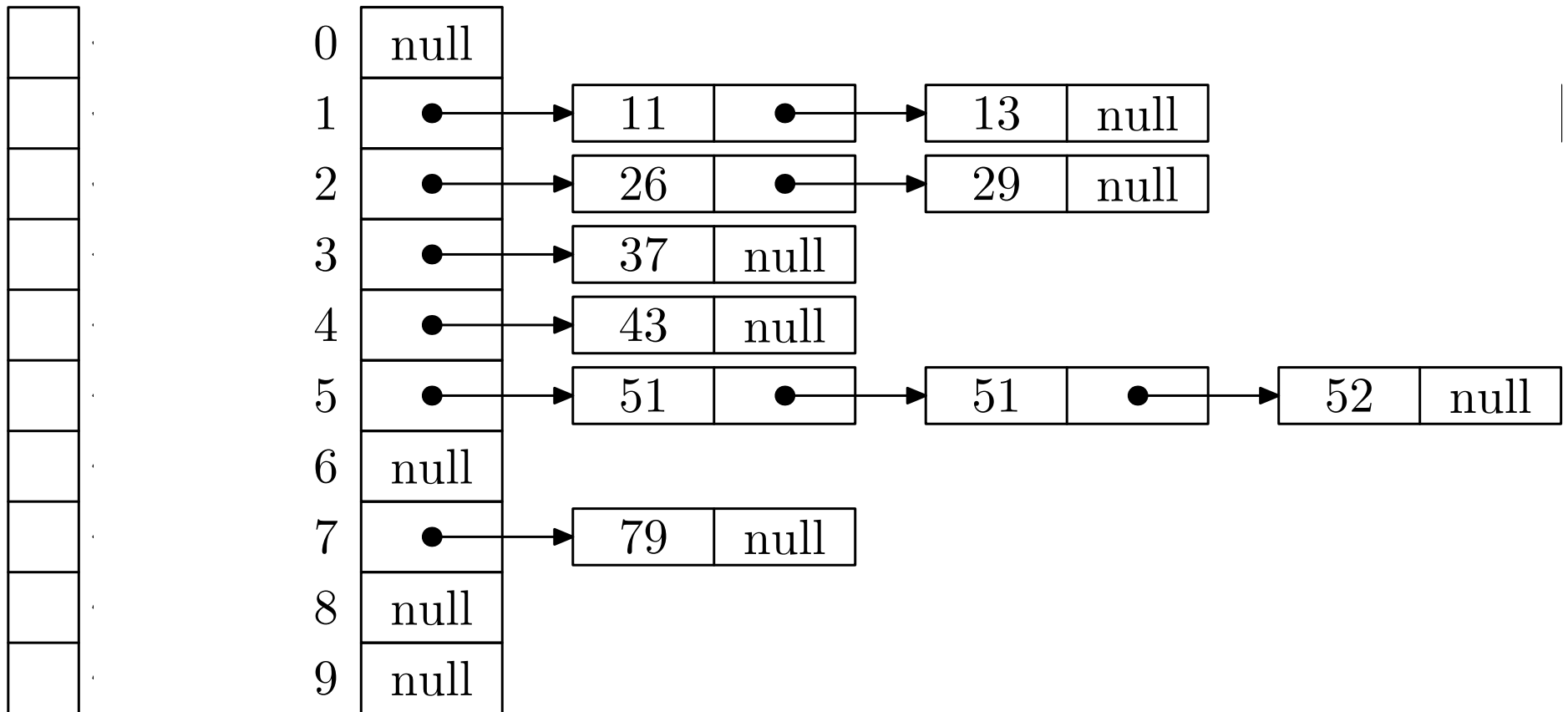
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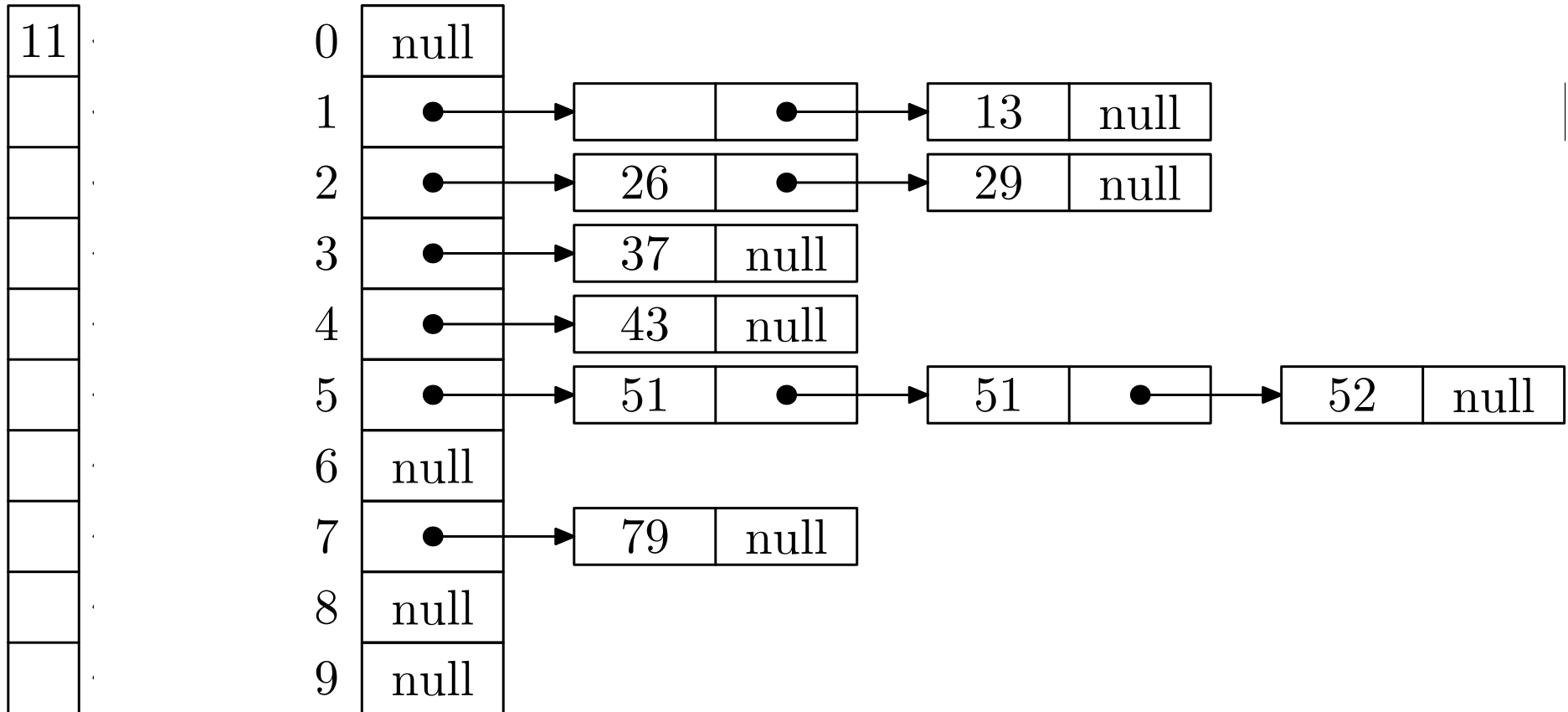
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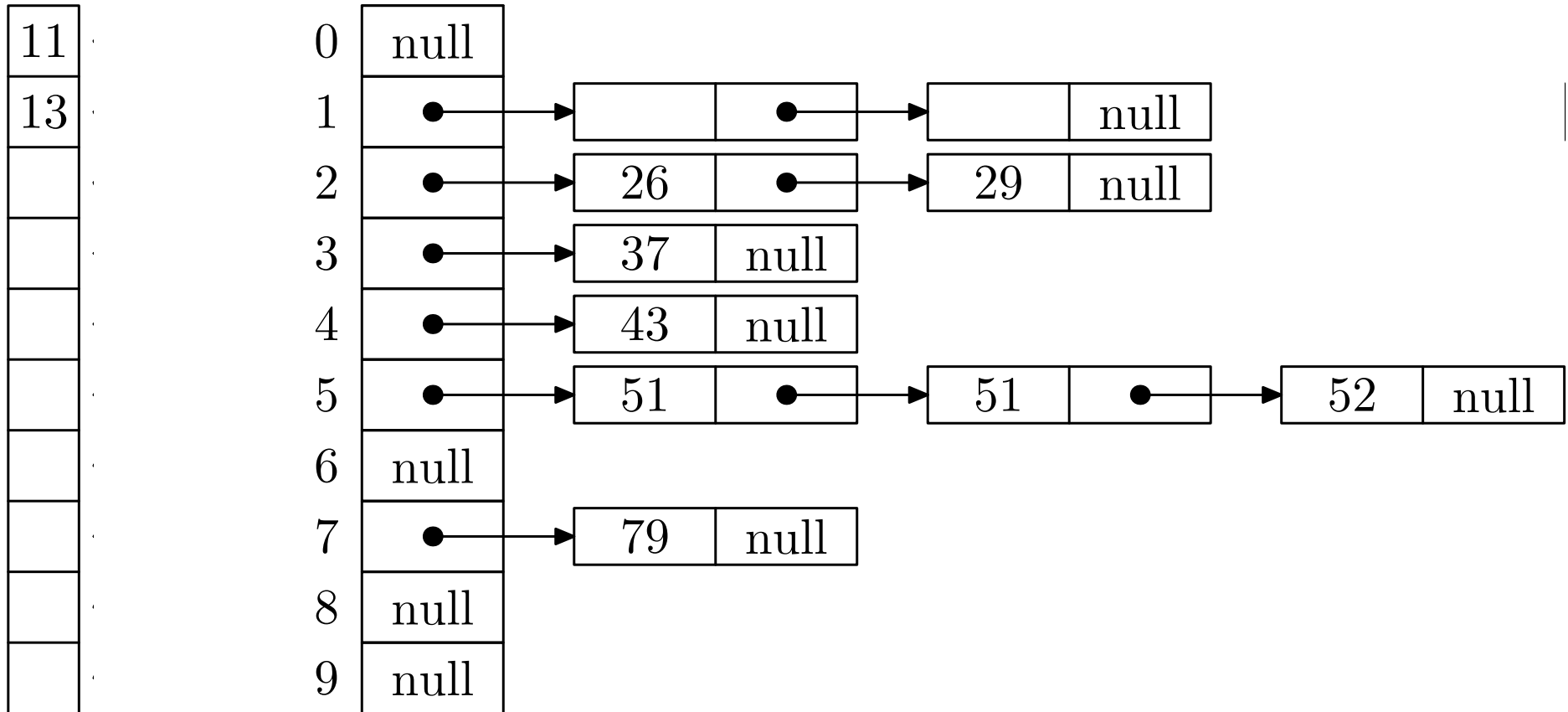
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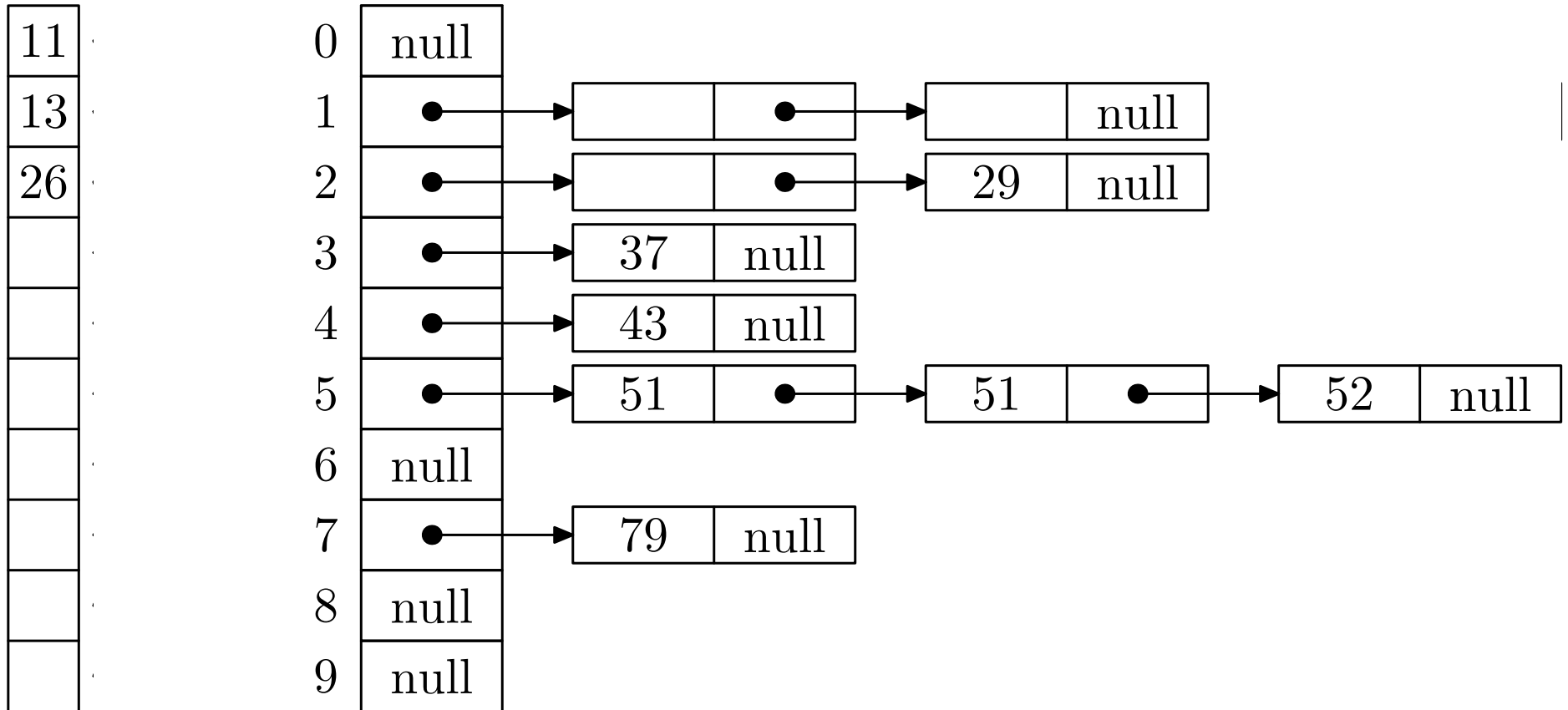
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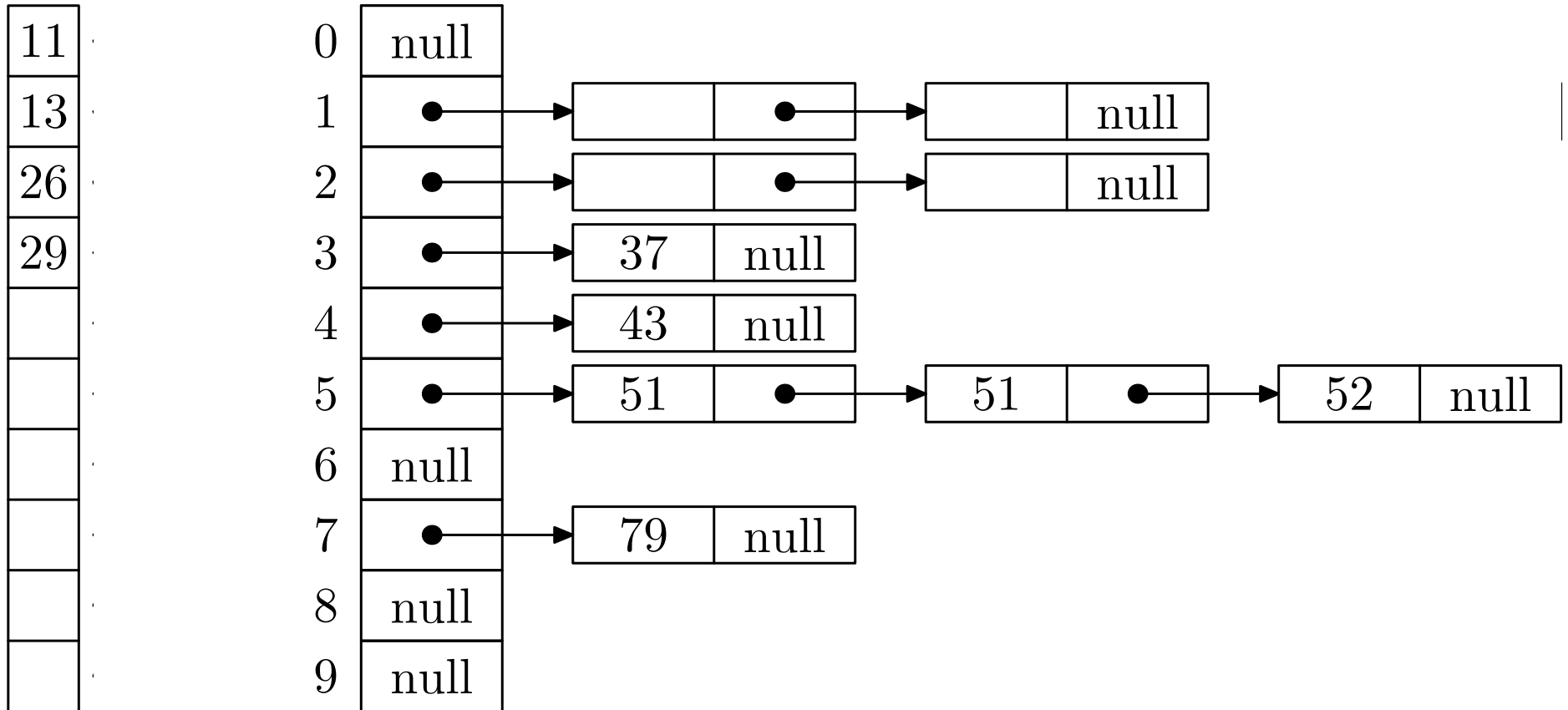
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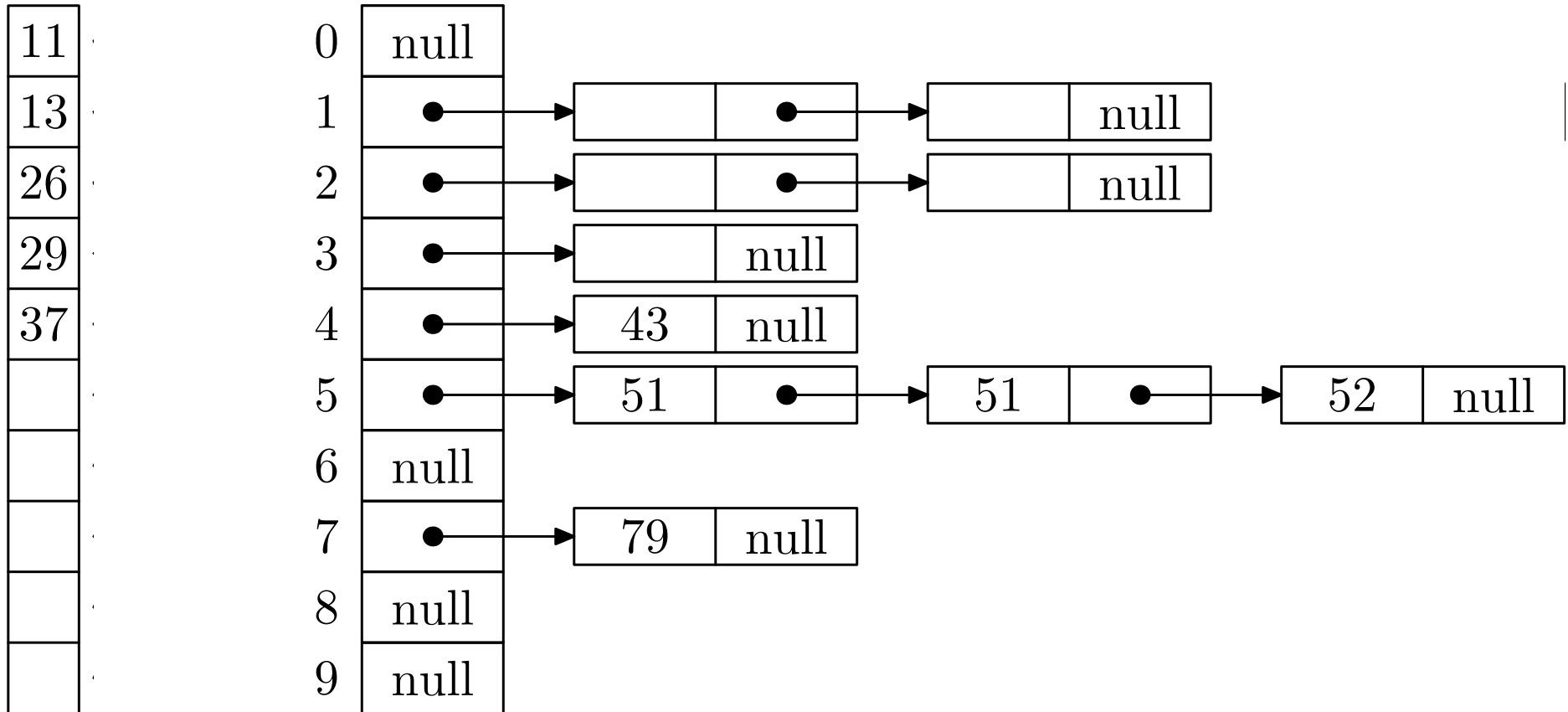
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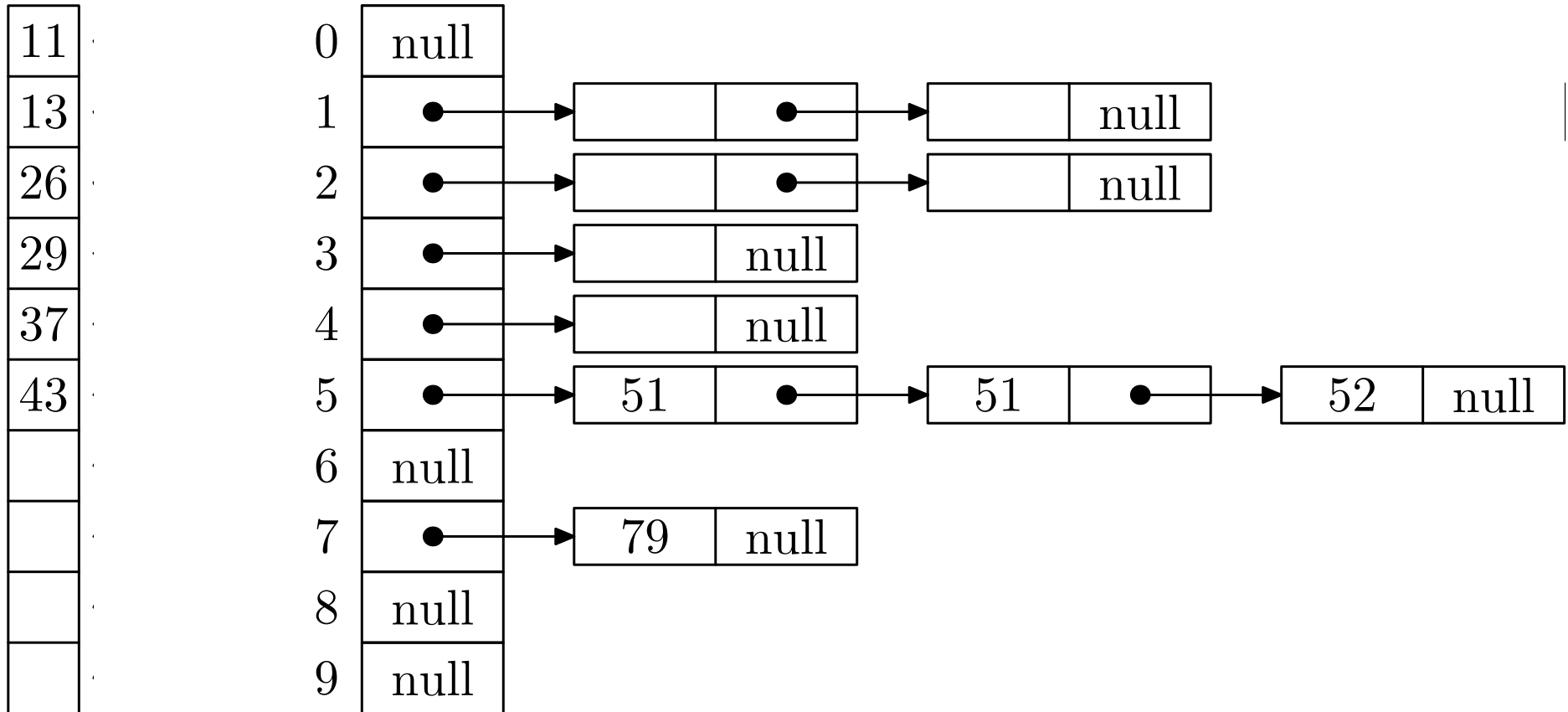
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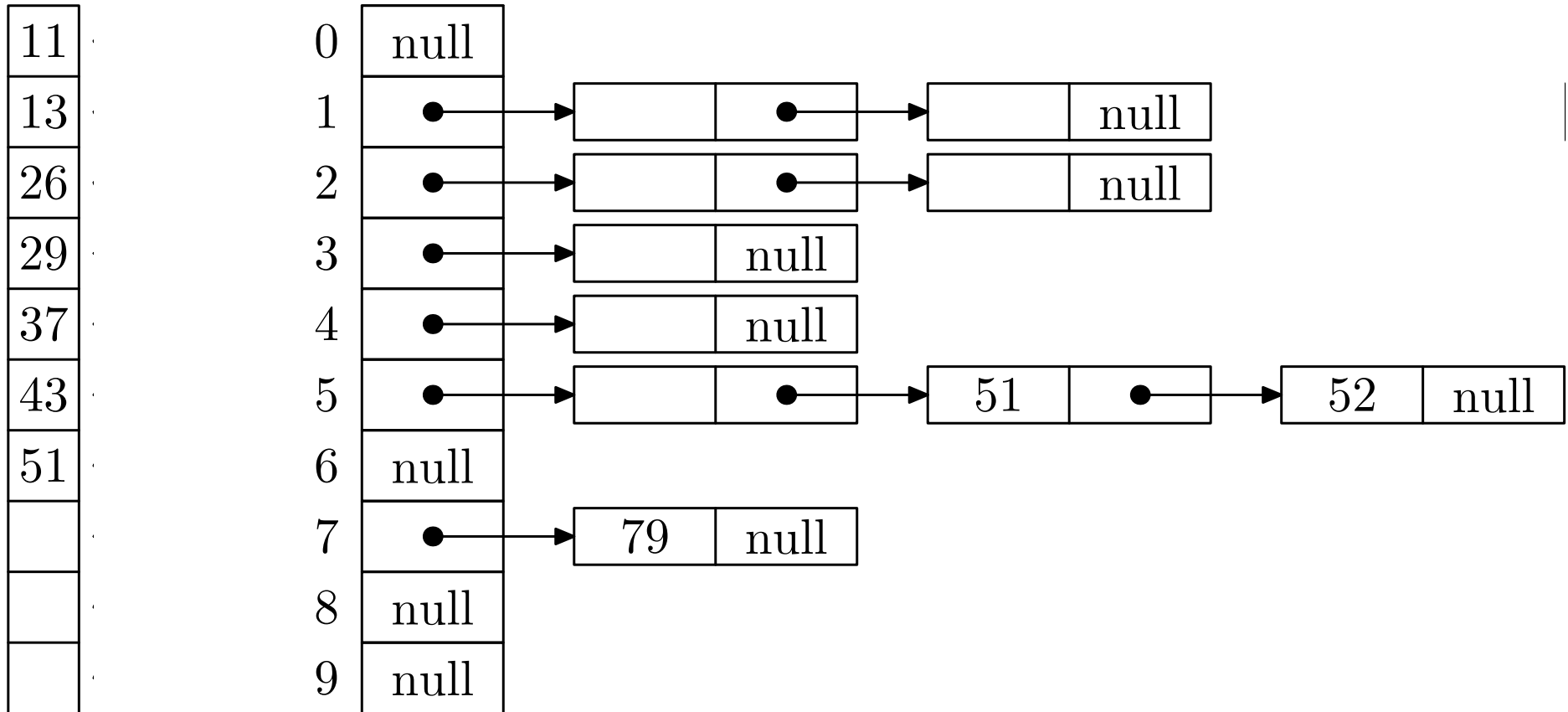
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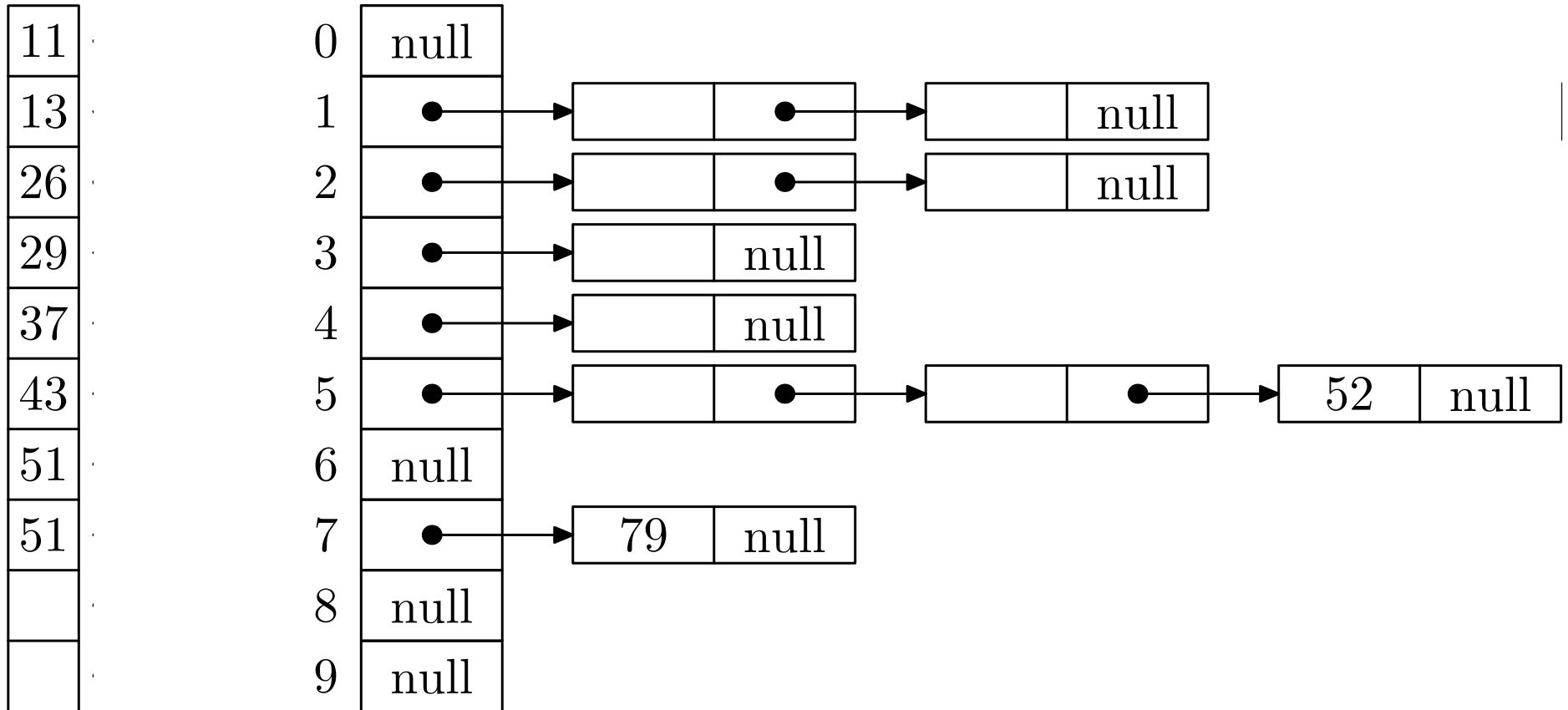
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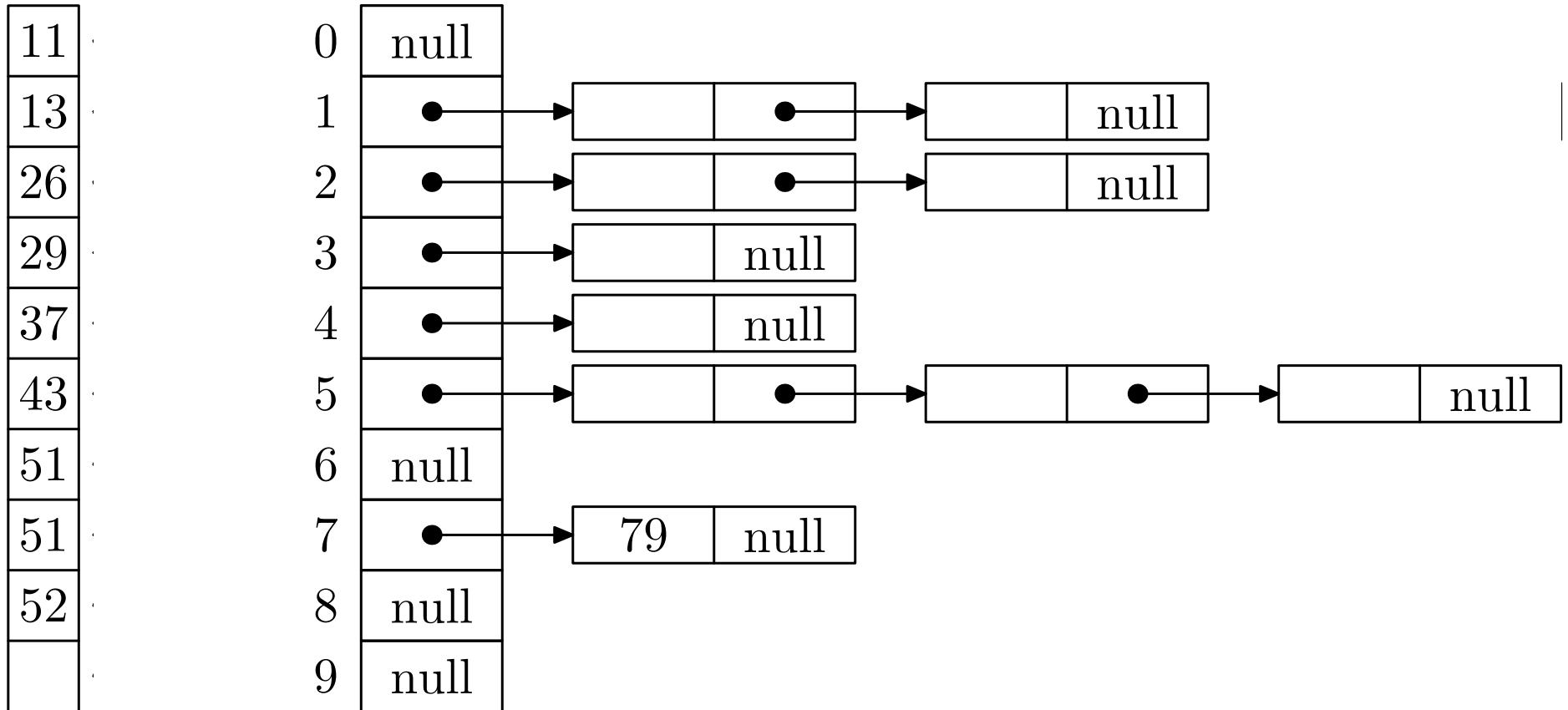
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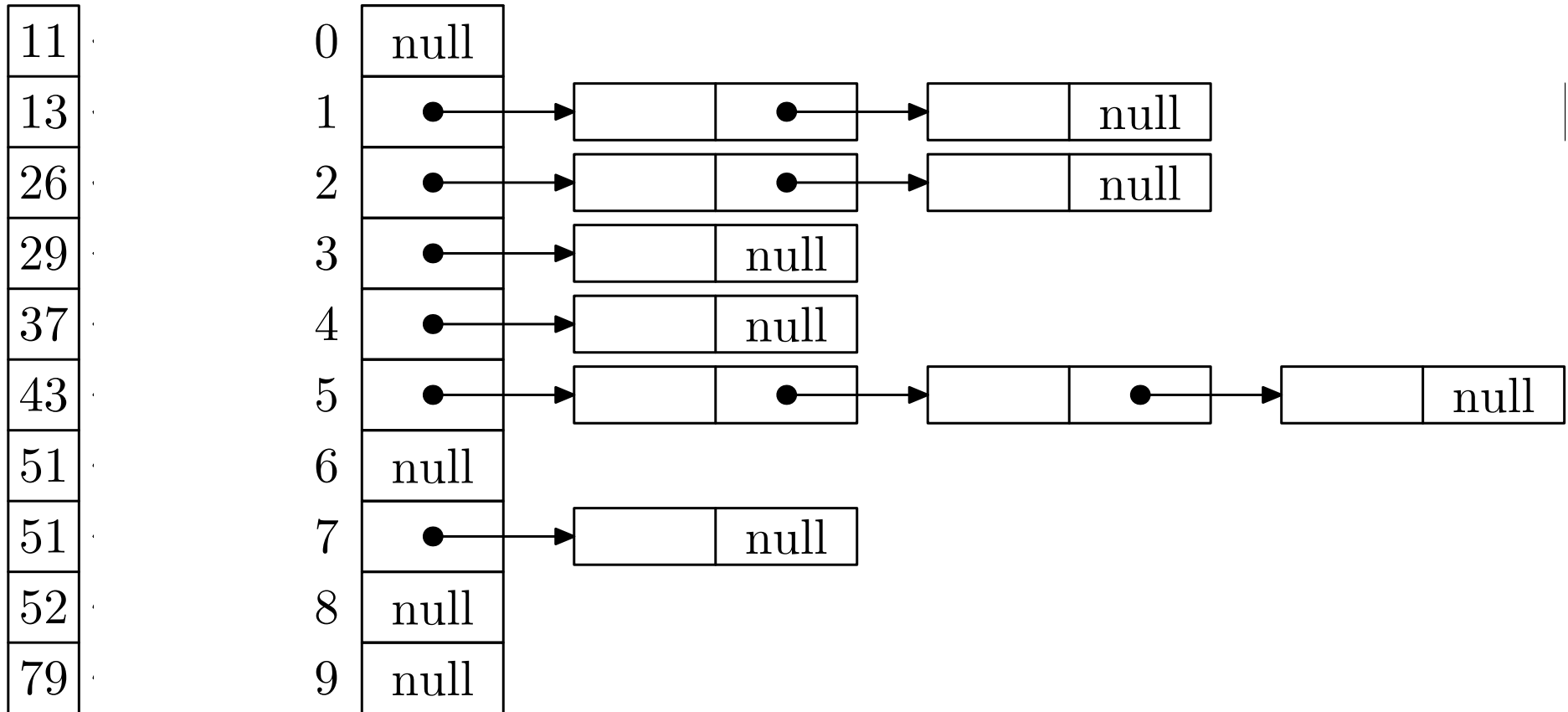
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Radix Sort in Action



Radix Sort in Action

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Time Complexity of Radix Sort

- We need not use base 10 we could use base r (the radix)
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- The overhead of maintaining the buckets make them less efficient than they might appear
- Radix sort is harder to generalise to other data types than comparison based sorts
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- Having said that there are some very neat implementations of radix sort

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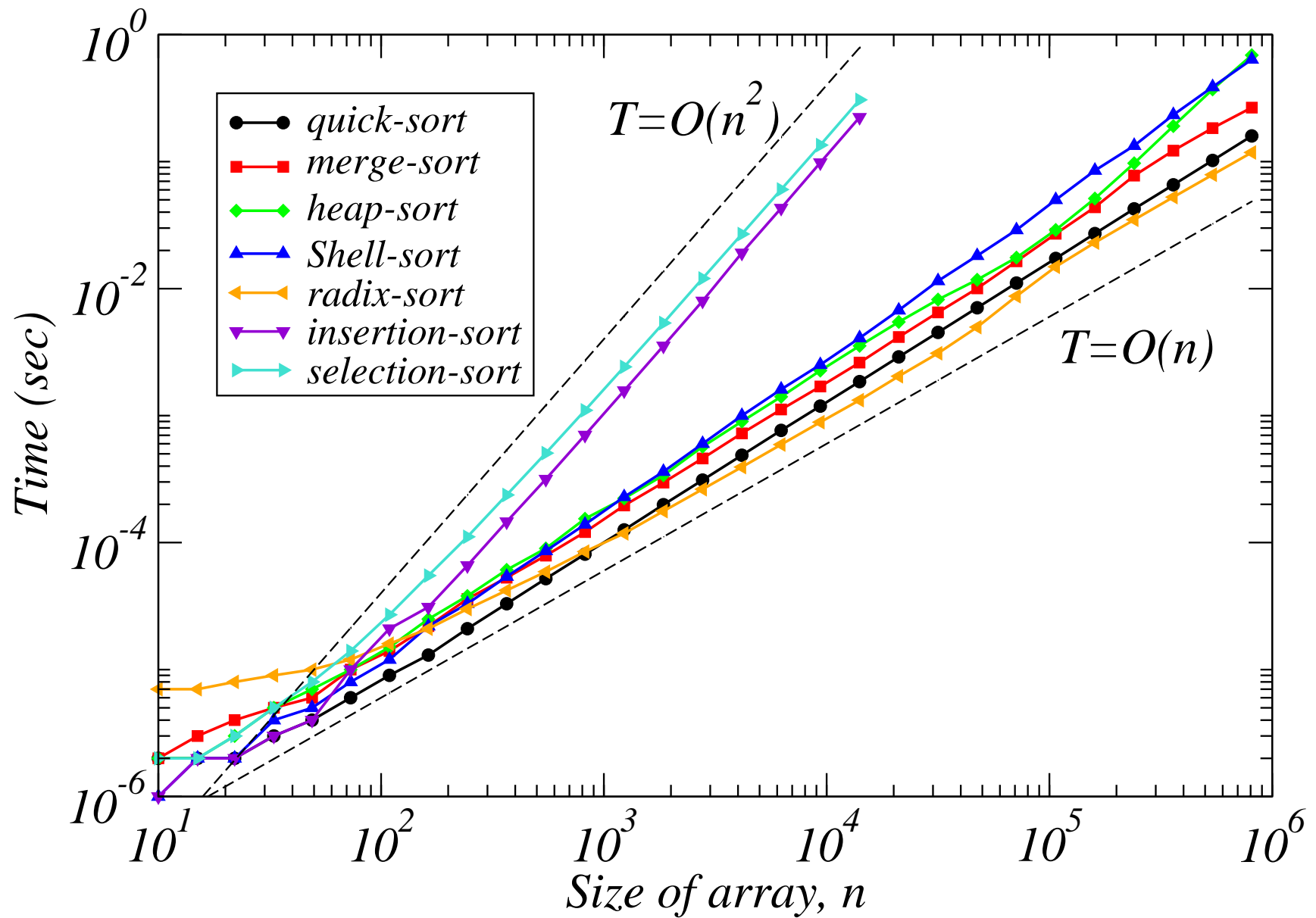
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Comparison of Sort Algorithms



Lessons

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- Merge sort and quick sort are the most commonly used sort
- There are sorts that have a better time complexity than quicksort
- In practice it is difficult to beat quicksort

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