Algorithms and Analysis

Lesson 15: *Sort Wisely*

Merge sort, quick sort and radix sort

Outline

1. **Merge Sort**
2. Quick Sort
3. Radix Sort

Merge Sort

- Merge sort is an example of sort performed in log-linear (i.e. $O(n \log(n))$) time complexity
- It was invented in 1945 by John von Neumann
- It is an example of a divide-and-conquer strategy
  - ⋆ That is, the problem is divided into a number of parts recursively
  - ⋆ The full solution is obtained by recombining the parts

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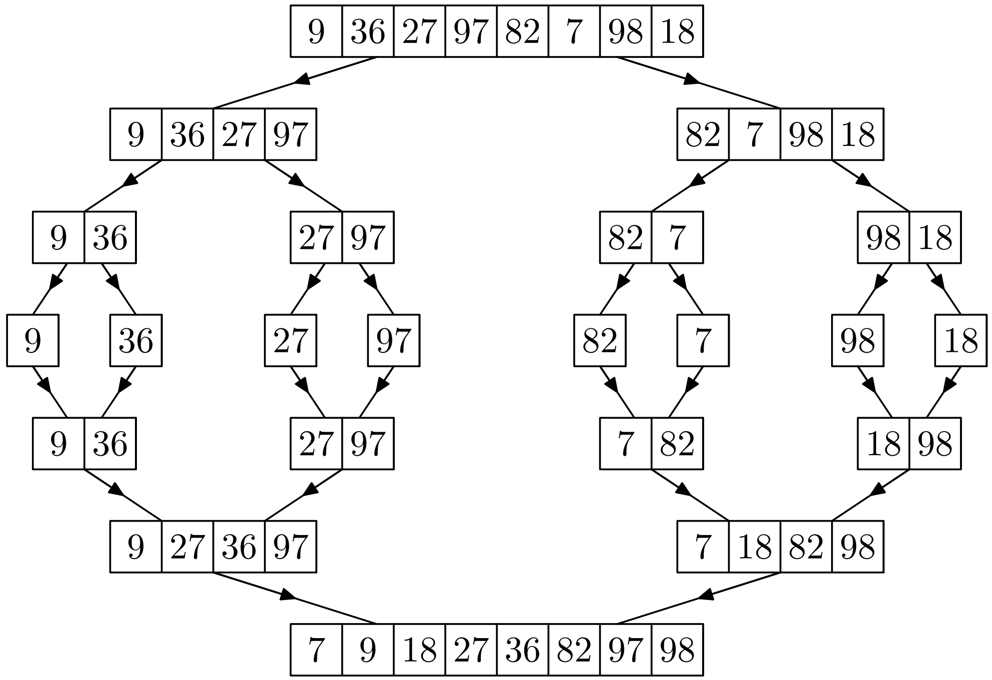
Algorithm

```
MERGESORT(a)
{
  if n > 1
    copy a[1 : ⌊n/2⌋]  to b
    copy a[⌊n/2⌋ + 1 : n]  to c
    MERGESORT(b)
    MERGESORT(c)
    MERGE(b, c, a)
  endif
}
```



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MERGE(b[1 : p], c[1 : q], a[1 : p + q])
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  j←1
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  while i ≤ p  and j ≤ q  do
    if b_i ≤ c_j
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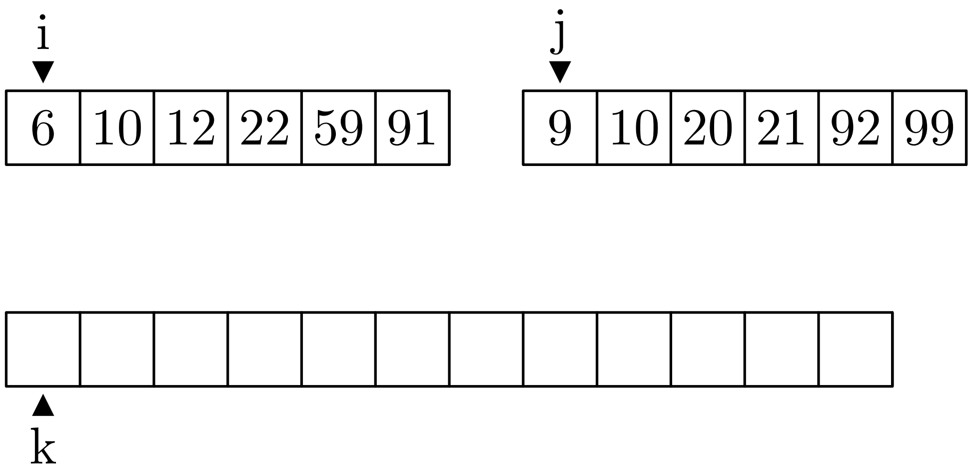

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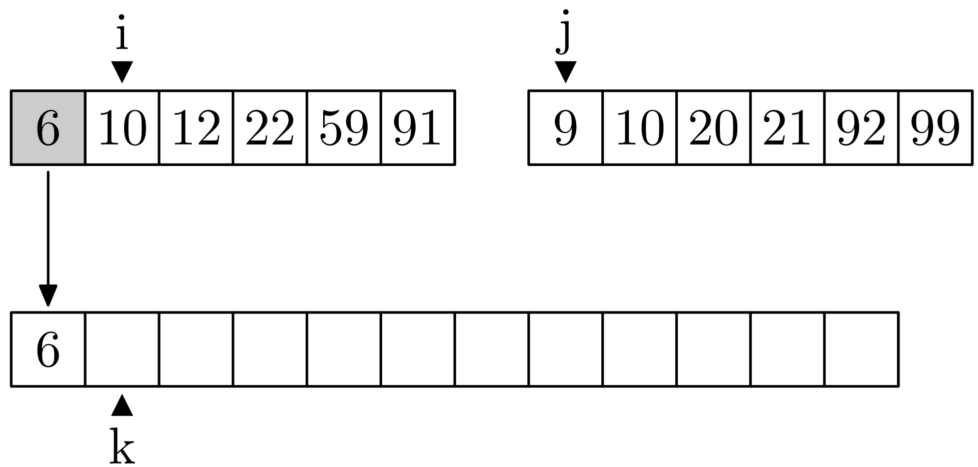

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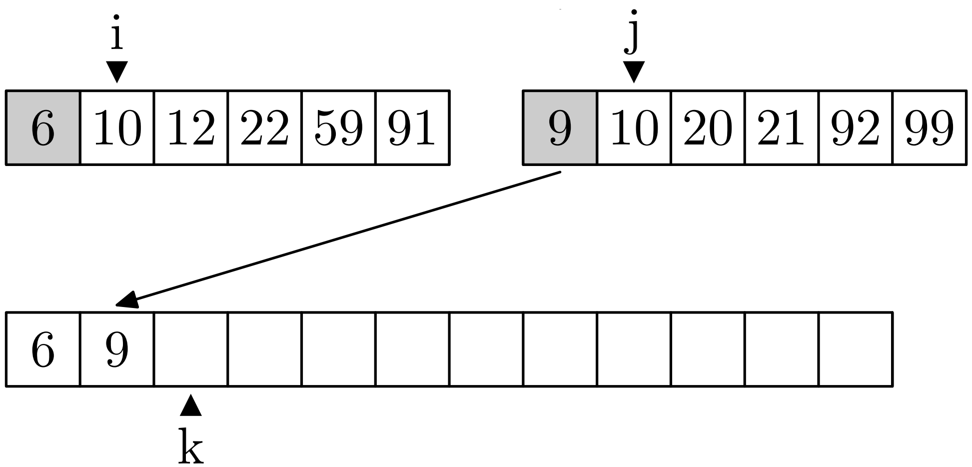

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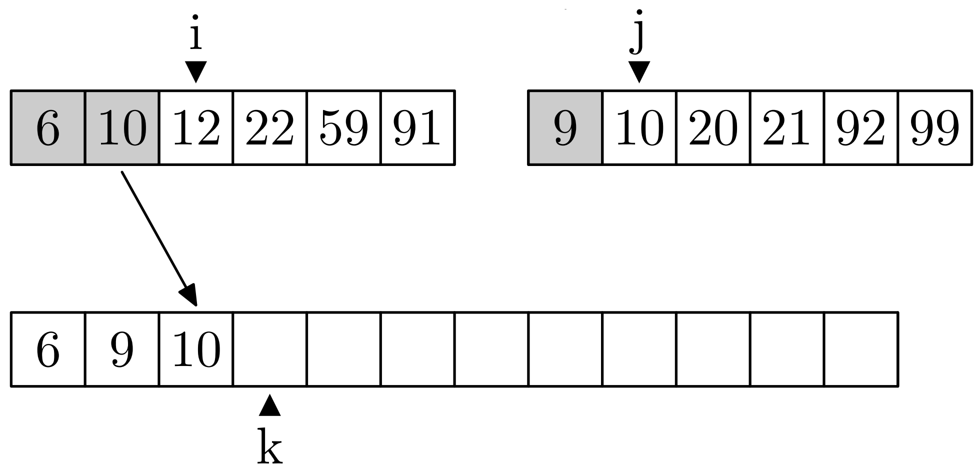

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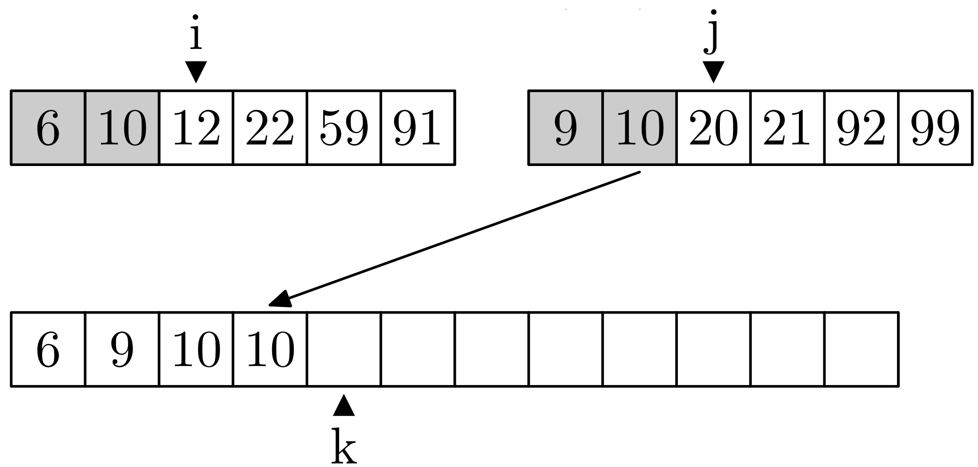

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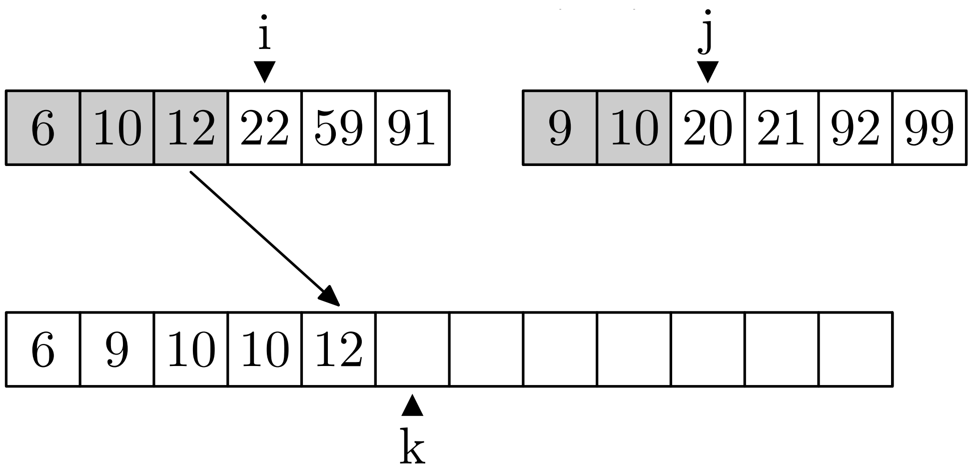

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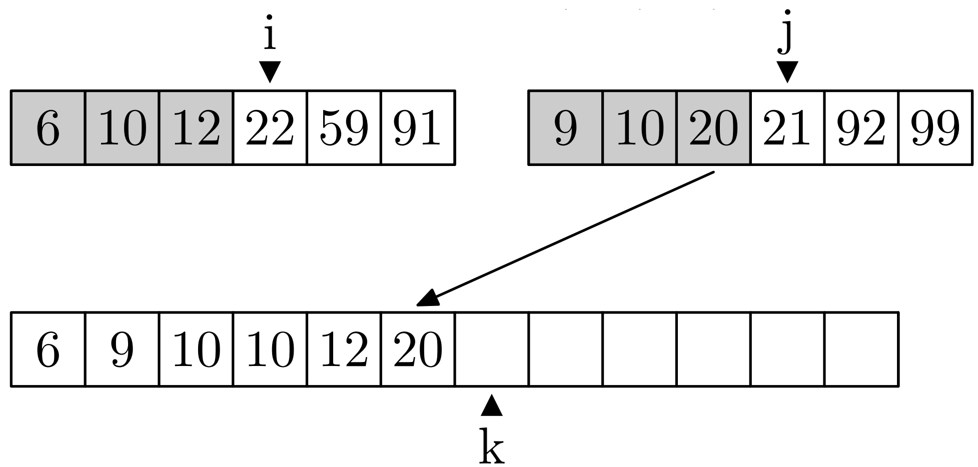

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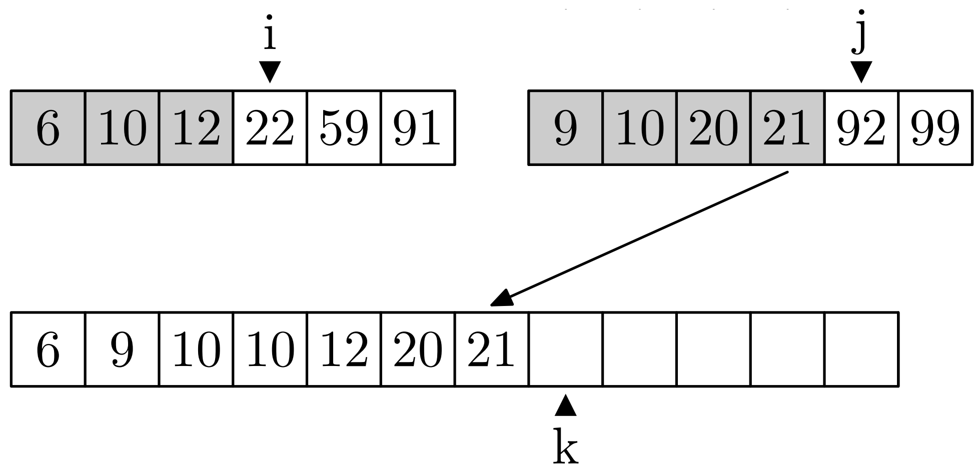

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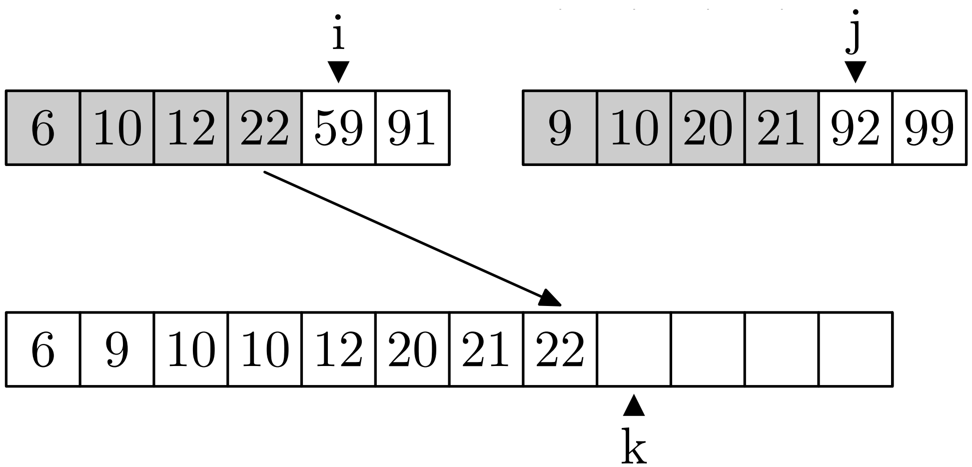

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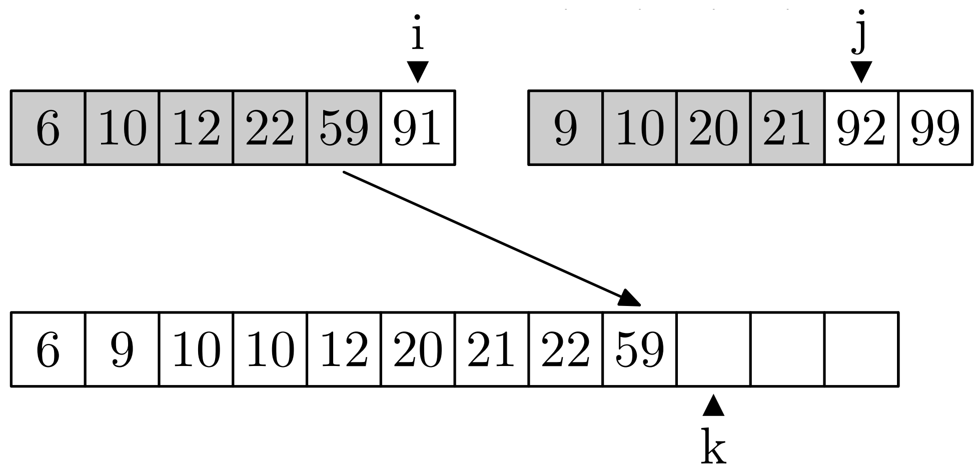

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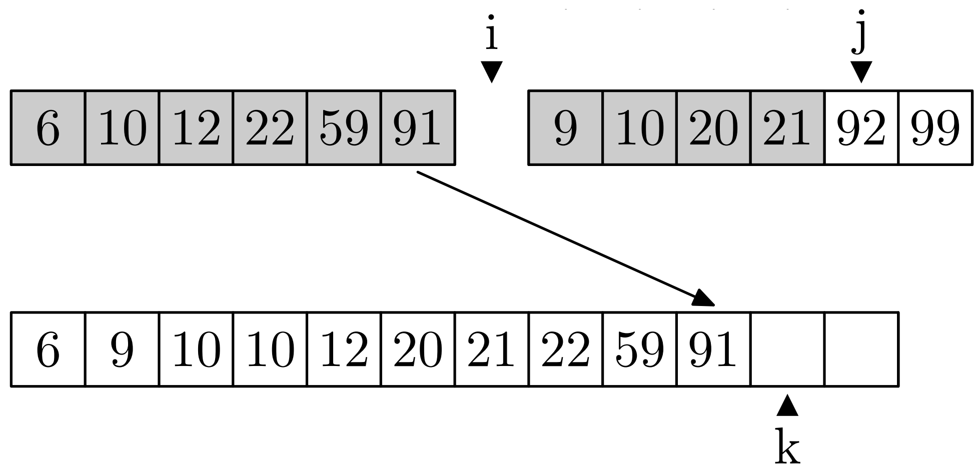

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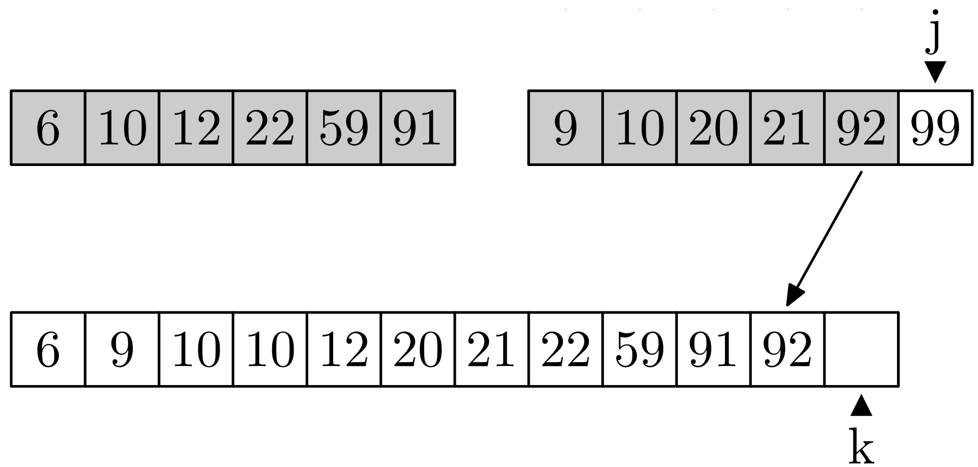

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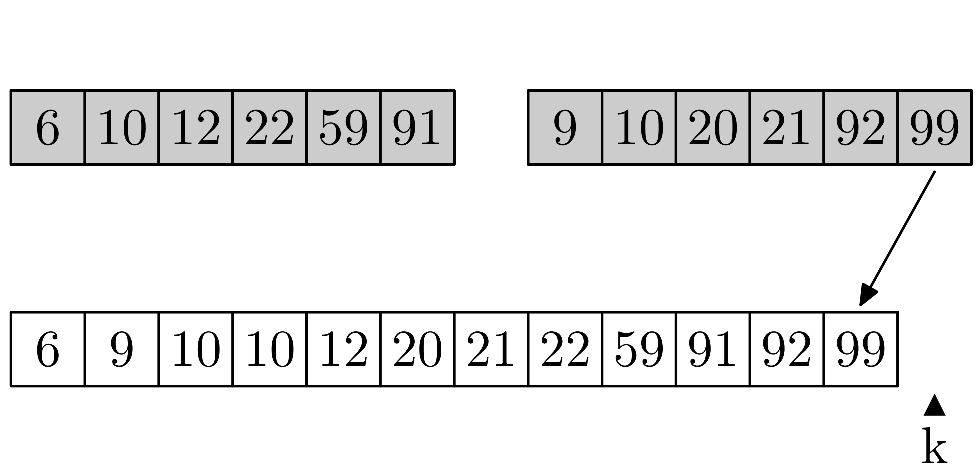

Properties of Merge Sort

- Merge sort is stable provided we merge carefully (i.e. it preserves the order of two entries with the same value)
- Merge sort isn't in-place—we need an array of at most size n to do the merging
- Merging is quick. Given two arrays of size n the most number of comparisons we need to perform is $n-1$

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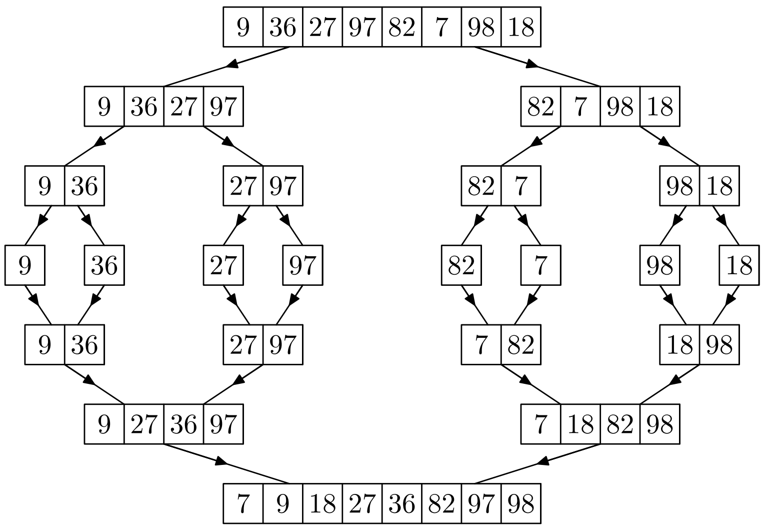

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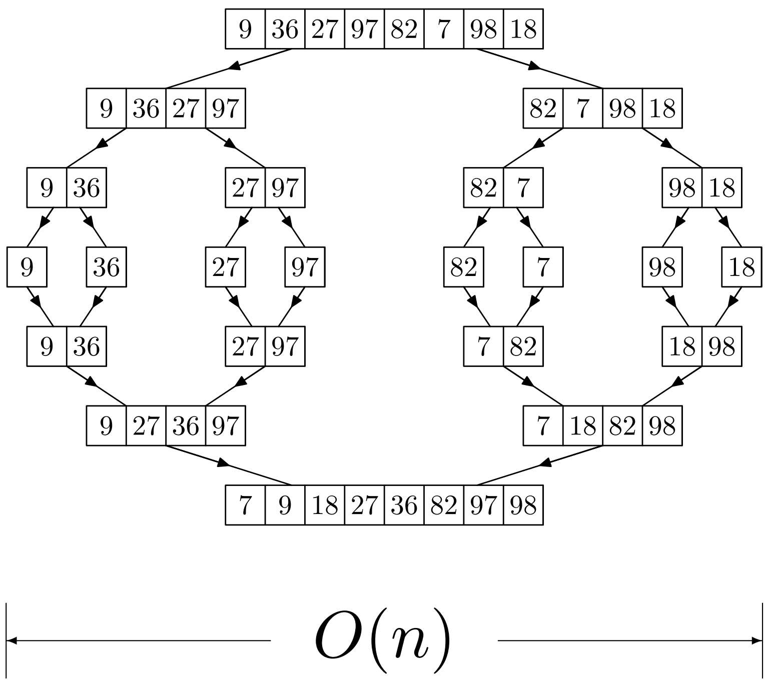

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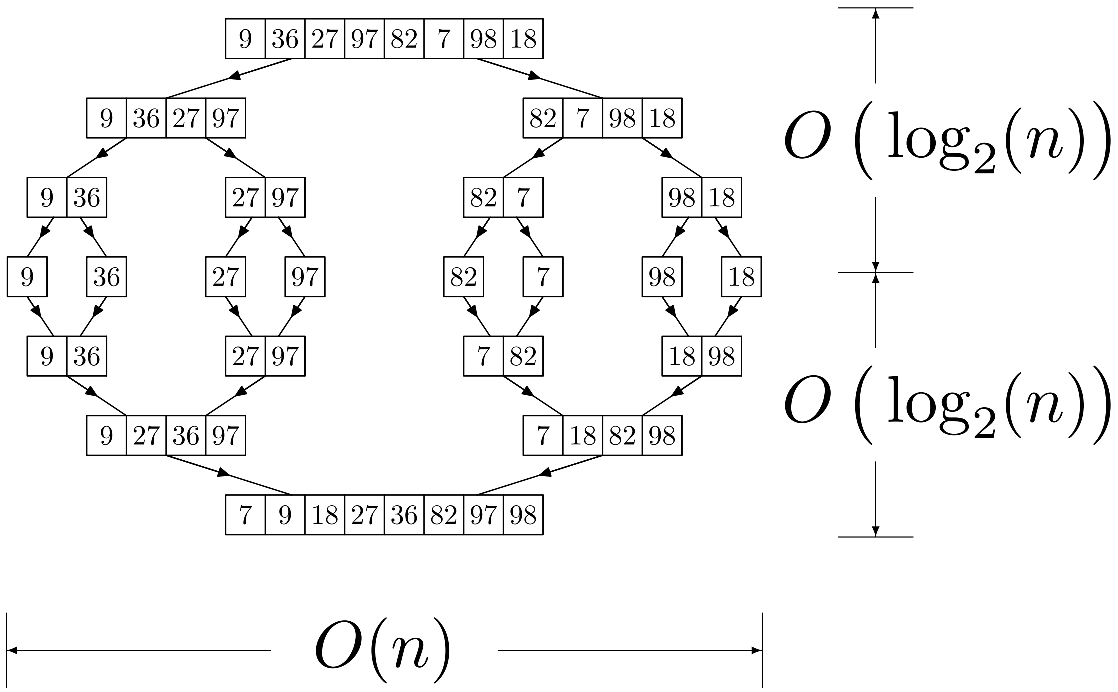

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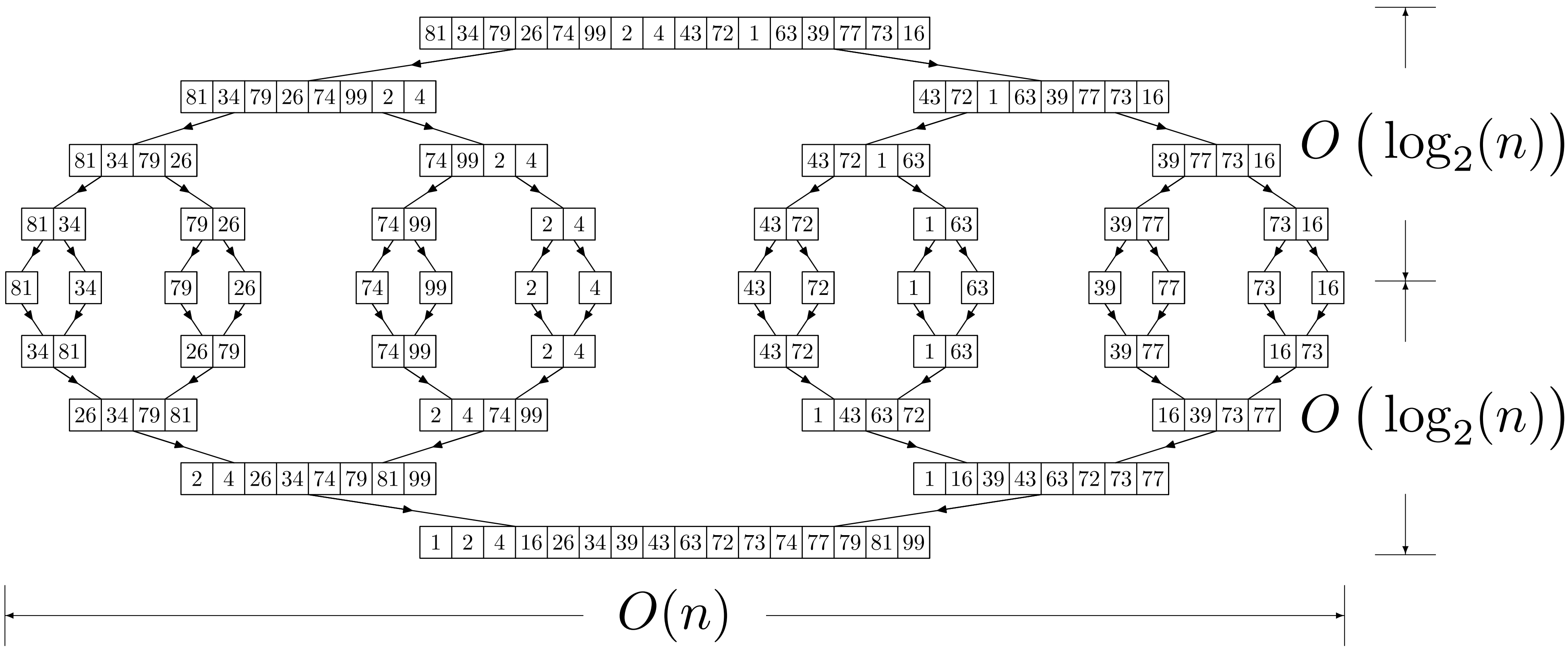

$$n = 16$$

Time Complexity

- We again measure the complexity in the number of comparisons
- From the above argument $C(n) = O(n \times \log_2(n))$
- We can be a bit more formal

$$C(n) = 2C(\lfloor n/2 \rfloor) + C_{\text{merge}}(n) \qquad \text{for } n > 1$$
$$C(0) = 1$$

- But in the worst case $C_{\text{merge}}(n) = n - 1$
- Leads to $C_{\text{worst}}(n) = n \log_2(n) - n + 1$

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General Time Complexity

- In general if we have a recursion formula

$$T(n) = aT(n/b) + f(n)$$

with $a \geq 1$, $b > 1$

- If $f(n) \in \Theta(n^d)$ where $d \geq 0$ then

$$T(n) \in \begin{cases} \Theta\left(n^d\right) & \text{if } a < b^d \\ \Theta\left(n^d \log(n)\right) & \text{if } a = b^d \\ \Theta\left(n^{\log_d(a)}\right) & \text{if } a > b^d \end{cases}$$

- Analogous results hold for the family O and Ω

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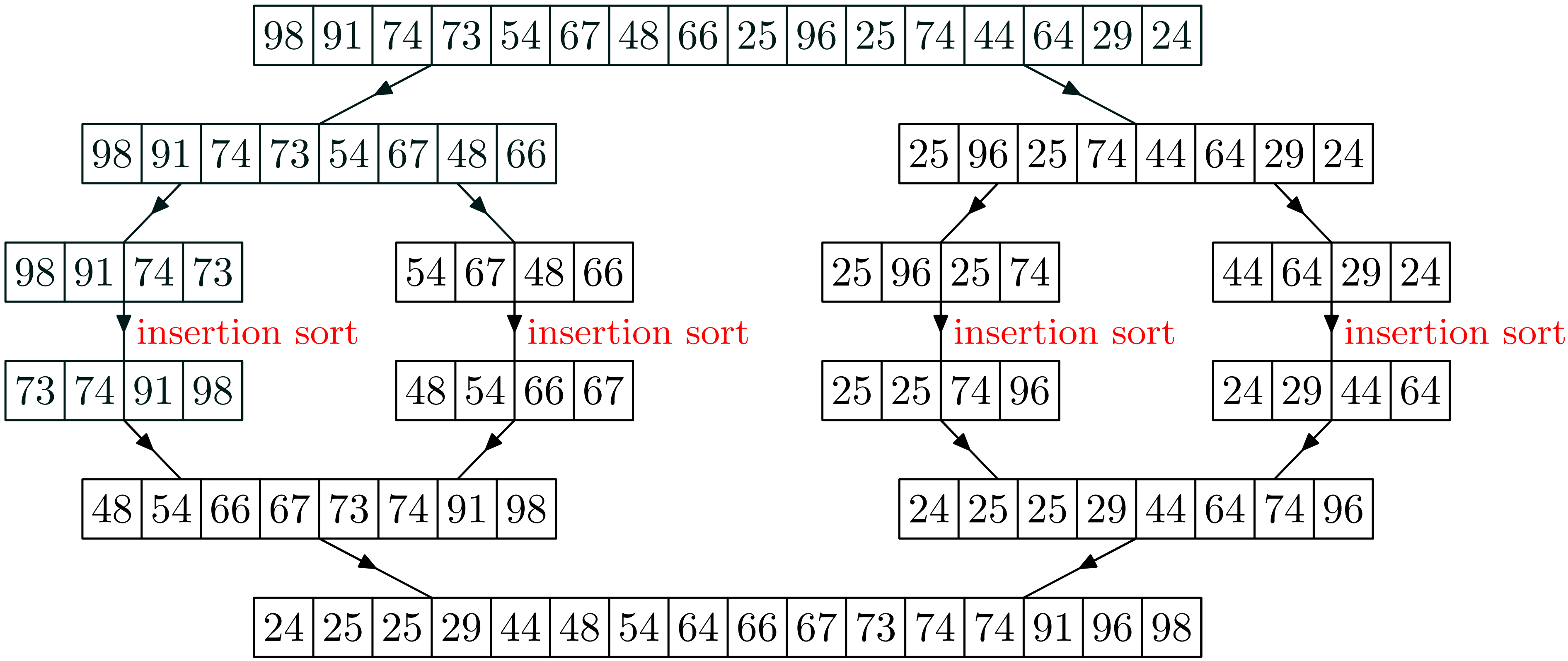
- For very short sequences it is faster to use insertion sort than to pay the overhead of function calls



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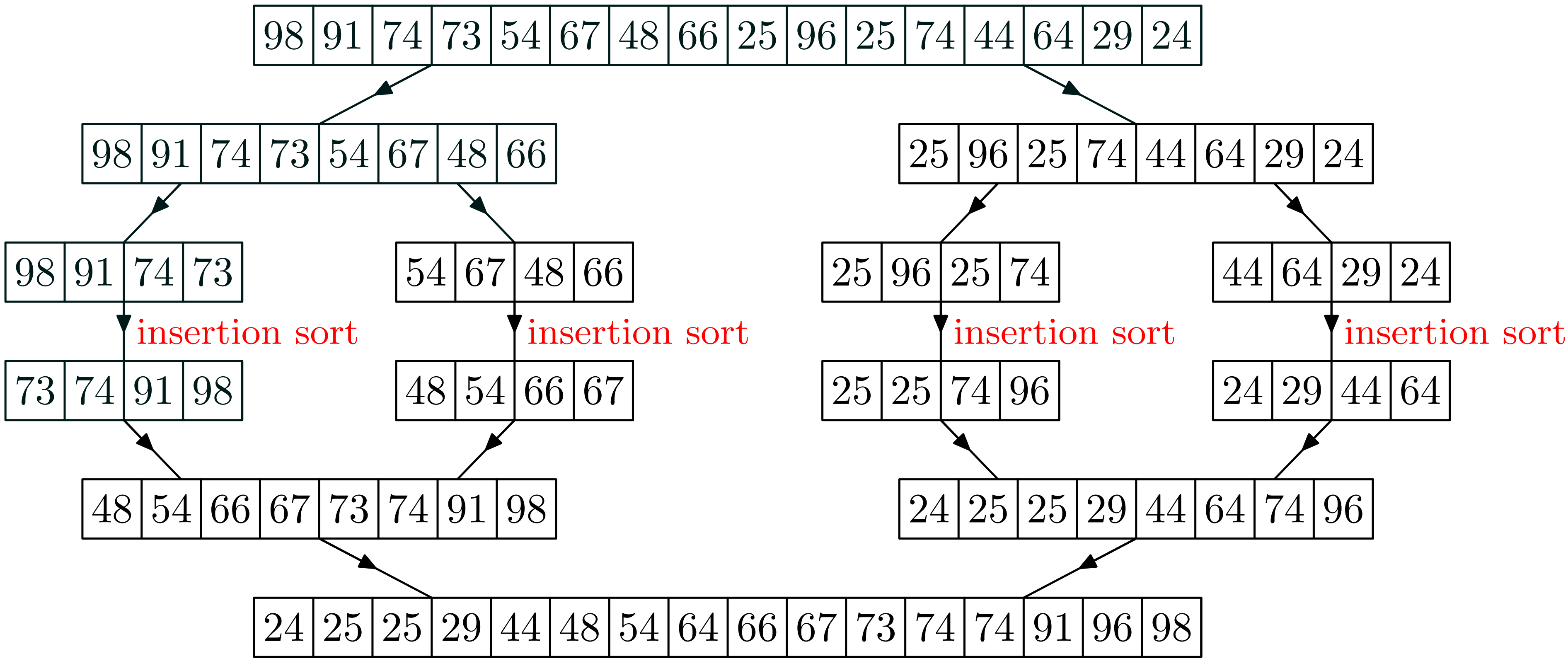

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2. **Quick Sort**
3. Radix Sort

Quicksort

- The most commonly used fast sorting algorithm is **quicksort**
- It was invented by the British computer scientist by C. A. R. Hoare in 1962
- It again uses the divide-and-conquer strategy
- It can be performed in-place, but it is **not** stable
- It works by splitting an array into two depending on whether the elements are less than or greater than a **pivot** value
- This is done recursively until the full array is sorted

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- It can be performed in-place, but it is **not** stable
- It works by splitting an array into two depending on whether the elements are less than or greater than a **pivot** value
- This is done recursively until the full array is sorted

Quicksort

- The most commonly used fast sorting algorithm is **quicksort**
- It was invented by the British computer scientist by C. A. R. Hoare in 1962
- It again uses the divide-and-conquer strategy
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- This is done recursively until the full array is sorted

Partition

- We need to partition the array around the pivot p such that

all elements < p	all elements >= p

```
PARTITION(a, p, left, right)
{
  i ←left
  j ←right
  repeat {
    while a_i < p
       i++
    while a_j ≥ p
       j--
    if i ≥ j
       break
    SWAP(a_i, a_j)
  }
}
```

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pivot $= 52$

52	49	96	29	40	87	73	10	47	60	6	11	33	94	57	85
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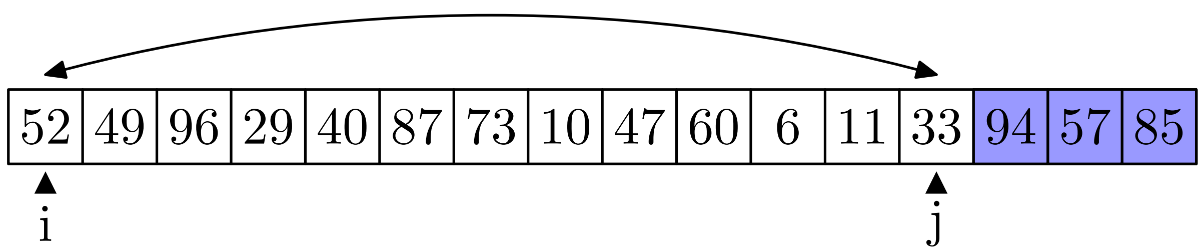

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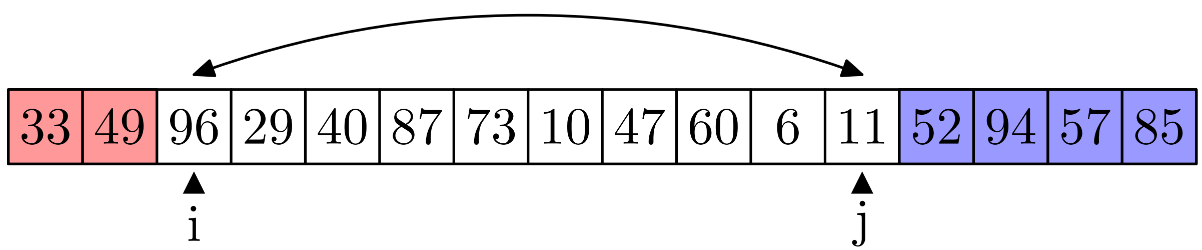

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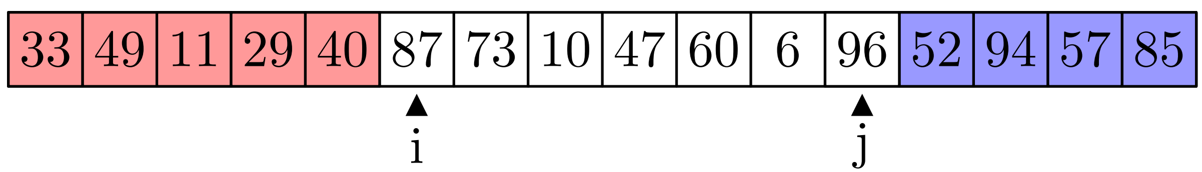

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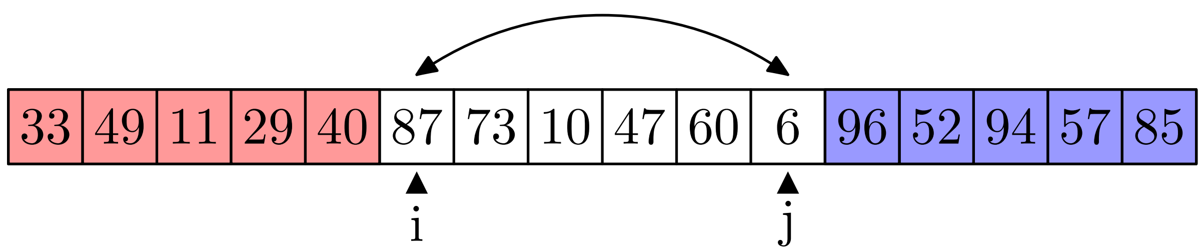

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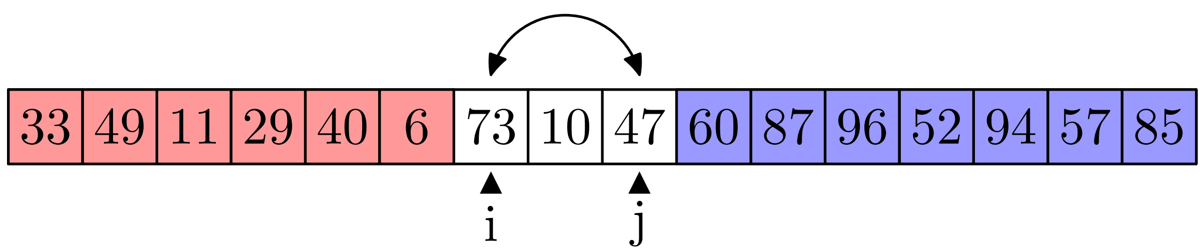

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Optimising Partitioning

- There are different ways of performing the partitioning
- We want to minimise the time taken on the inner loop
- This means we want to perform as few checks as possible
- One method of doing this is to place $sentinels$ at the ends of the array
- We can also reduce work by placing the partition in its correct position

all elements <= p	p	all elements >= p

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Choosing the Pivot

- There are different strategies to choosing the pivot
- Choose the first element in the array
- Choose the median of the first, middle and last element of the array
- This increases the likelihood of the pivot being close to the median of the whole array
- For large arrays (above 40) the median of 3 medians is often used

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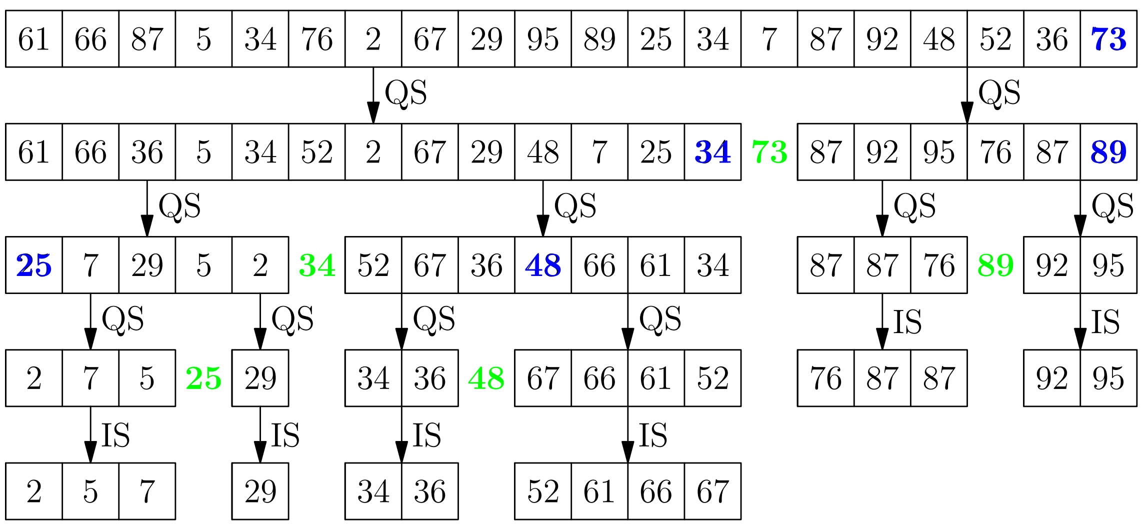
Quicksort

We recursively partition the array until each partition is small enough to sort using insertion sort

```
QuickSort(a, left, right) {
  if (right-left < threshold)
    InsertionSort(a, left, right)
  else
    pivot = ChoosePivot(a, left, right)
    part = Partition(a, pivot, left, right)
    QuickSort(a, left, part-1)
    QuickSort(a, part+1, right)
  endif
}
```



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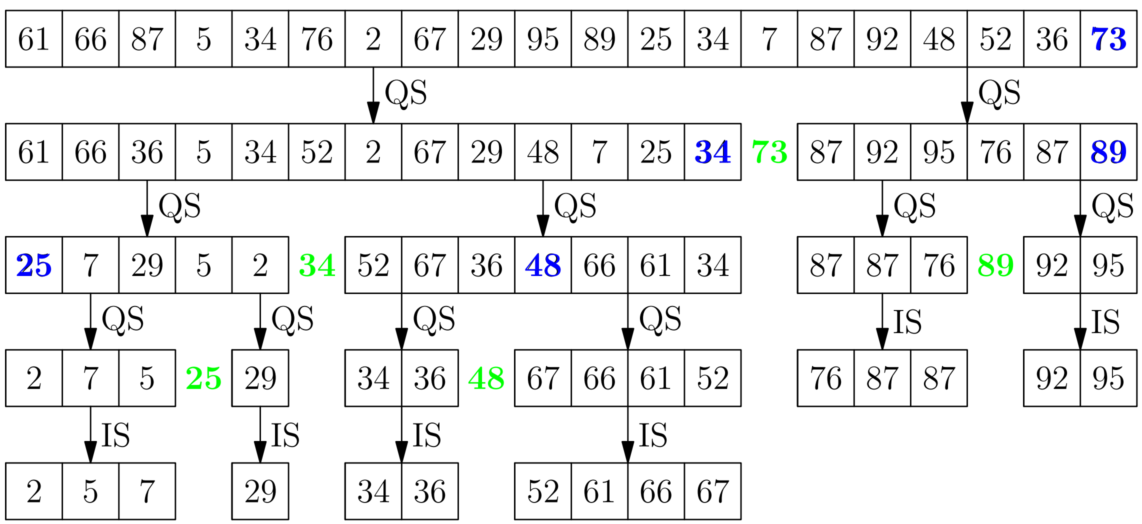

Quicksort

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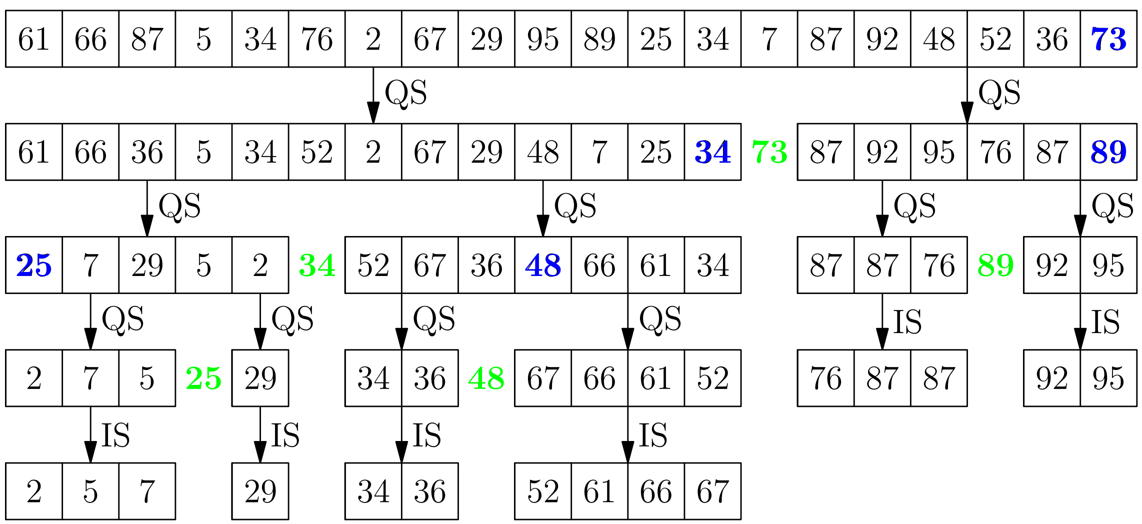
QUICKSORT(a, left, right) {
  if (right-left < threshold)
    INSERTIONSORT(a, left, right)
  else
    pivot = CHOOSEPIVOT(a, left, right)
    part = PARTITION(a, pivot, left, right)
    QUICKSORT(a, left, part-1)
    QUICKSORT(a, part+1, right)
  endif
}
```



Time Complexity

- Partitioning an array of size n takes $\Theta(n)$ operations
- If we split the array in half then number of partitions we need to do is $\lceil \log_2(n) \rceil$
- This is the best case thus quicksort is $\Omega\left(n \log(n)\right)$
- If the pivot is the minimum element of the array then we have to partition $n-1$ times
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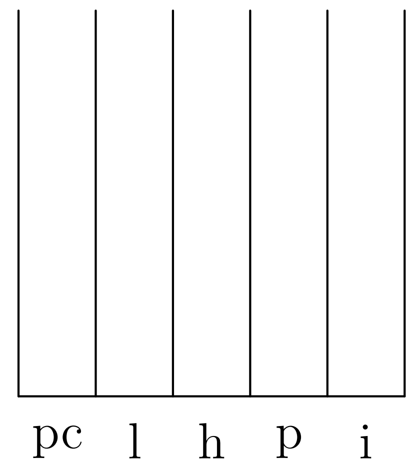
QuickSort

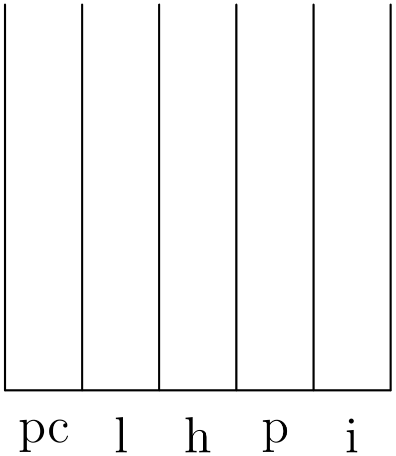
```
quickSort(a, l, h) {
  if(h-l>3) {
    p = choosePivot(a, l, h)
    i = partition(a, p, l, h)
    quickSort(a, l, i-1)
    quickSort(a, i+1, h)
  } else
    insertionSort(a, l, h)
  return
}
```



QuickSort

```
quickSort(a, 0, 19) {
  if(19-0>3) {
    p = choosePivot(a, 0, 19)
    i = partition(a, p, 0, 19)
    quickSort(a, 0, i-1)
    quickSort(a, i+1, 19)
  } else
    insertionSort(a, 0, 19)
  return
}
```

```
PC = 0
l = 0
h = 19
p = #
i = #
```



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
94	48	88	91	63	20	1	97	15	2	79	96	90	25	93	32	4	76	14	1

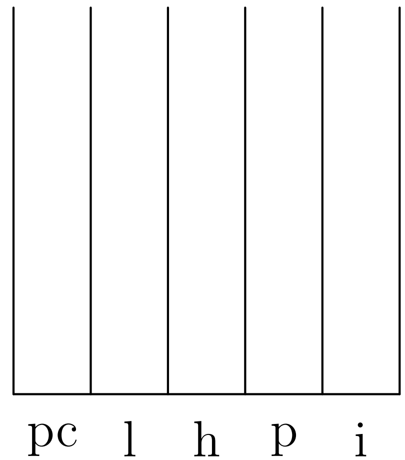
low

high

QuickSort

```
0  quickSort(a, 0, 19) {
1    if(19-0>3) {
2      p = choosePivot(a, 0, 19)
3      i = partition(a, p, 0, 19)
4      quickSort(a, 0, i-1)
5      quickSort(a, i+1, 19)
6    } else
7      insertionSort(a, 0, 19)
8    return
9  }
```

```
PC = 1
l = 0
h = 19
p = #
i = #
```

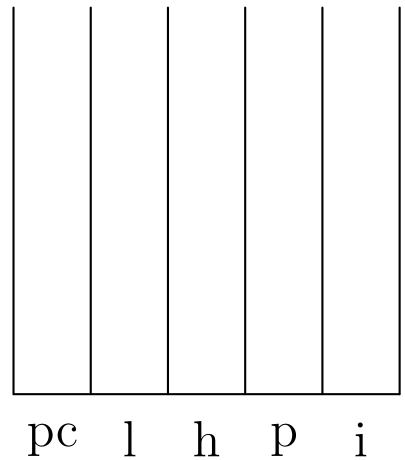


0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
94	48	88	91	63	20	1	97	15	2	79	96	90	25	93	32	4	76	14	1
low																			high

QuickSort

```
0 quickSort(a, 0, 19) {
1   if(19-0>3) {
2     p = choosePivot(a, 0, 19)
3     i = partition(a, p, 0, 19)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 0, 19)
8   return
9 }
```

```
PC = 2
l = 0
h = 19
p = #
i = #
```



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
94	48	88	91	63	20	1	97	15	2	79	96	90	25	93	32	4	76	14	1
low																			high

QuickSort

```
0  quickSort(a, 0, 19) {
1    if(19-0>3) {
2      p = choosePivot(a, 0, 19)
3      i = partition(a, p, 0, 19)
4      quickSort(a, 0, i-1)
5      quickSort(a, i+1, 19)
6    } else
7      insertionSort(a, 0, 19)
8    return
9  }
```

```
PC = 2
l = 0
h = 19
p = #
i = #
```

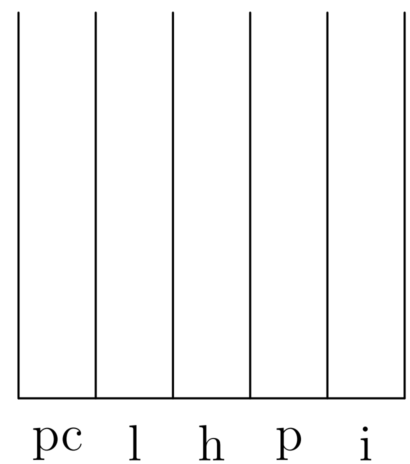
pc	l	h	p	i
2	0	19	#	#

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
94	48	88	91	63	20	1	97	15	2	79	96	90	25	93	32	4	76	14	1
low																			high

QuickSort

```
quickSort(a, 0, 19) {
  if(19-0>3) {
    p = choosePivot(a, 0, 19)
    i = partition(a, 2, 0, 19)
    quickSort(a, 0, i-1)
    quickSort(a, i+1, 19)
  } else
    insertionSort(a, 0, 19)
  return
}
```

```
PC = 3
l = 0
h = 19
p = 2
i = #
```



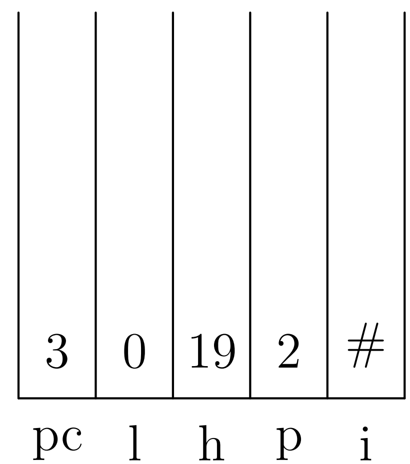
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
94	48	88	91	63	20	1	97	15	2	79	96	90	25	93	32	4	76	14	1

low high

QuickSort

```
0 quickSort(a, 0, 19) {
1   if(19-0>3) {
2     p = choosePivot(a, 0, 19)
3     i = partition(a, 2, 0, 19)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 0, 19)
8   return
9 }
```

```
PC = 3
l = 0
h = 19
p = 2
i = #
```

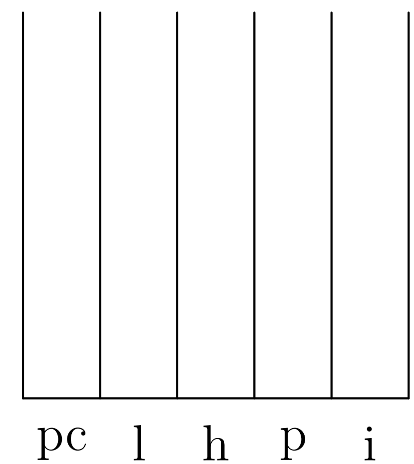


0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	97	15	88	79	96	90	25	93	32	4	76	14	94
low																			high

QuickSort

```
quickSort(a, 0, 19) {
  if(19-0>3) {
    p = choosePivot(a, 0, 19)
    i = partition(a, 2, 0, 19)
    quickSort(a, 0, 2-1)
    quickSort(a, 2+1, 19)
  } else
    insertionSort(a, 0, 19)
  return
}
```

```
PC = 4
l = 0
h = 19
p = 2
i = 2
```



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	97	15	88	79	96	90	25	93	32	4	76	14	94
low																			high

QuickSort

```
quickSort(a, 0, 1) {
  if(1-0>3) {
    p = choosePivot(a, 0, 1)
    i = partition(a, p, 0, 1)
    quickSort(a, 0, i-1)
    quickSort(a, i+1, 1)
  } else
    insertionSort(a, 0, 1)
  return
}
```

```
PC = 0
l = 0
h = 1
p = #
i = #
```

pc	l	h	p	i
4	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	97	15	88	79	96	90	25	93	32	4	76	14	94
low	high																		

QuickSort

```
quickSort(a, 0, 1) {
  if(1-0>3) {
    p = choosePivot(a, 0, 1)
    i = partition(a, p, 0, 1)
    quickSort(a, 0, i-1)
    quickSort(a, i+1, 1)
  } else
    insertionSort(a, 0, 1)
  return
}
```

```
PC = 1
l = 0
h = 1
p = #
i = #
```

pc	l	h	p	i
4	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	97	15	88	79	96	90	25	93	32	4	76	14	94
low	high																		

QuickSort

```
0 quickSort(a, 0, 1) {
1   if(1-0>3) {
2     p = choosePivot(a, 0, 1)
3     i = partition(a, p, 0, 1)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 1)
6   } else
7     insertionSort(a, 0, 1)
8   return
9 }
```

```
PC = 7
l = 0
h = 1
p = #
i = #
```

pc	l	h	p	i
4	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	97	15	88	79	96	90	25	93	32	4	76	14	94
low	high																		

QuickSort

```
quickSort(a, 0, 1) {
  if(1-0>3) {
    p = choosePivot(a, 0, 1)
    i = partition(a, p, 0, 1)
    quickSort(a, 0, i-1)
    quickSort(a, i+1, 1)
  } else
    insertionSort(a, 0, 1)
  return
}
```

```
PC = 7
l = 0
h = 1
p = #
i = #
```

pc	l	h	p	i
7	0	1	#	#
4	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	97	15	88	79	96	90	25	93	32	4	76	14	94
low	high																		

QuickSort

```
0 quickSort(a, 0, 1) {
1   if(1-0>3) {
2     p = choosePivot(a, 0, 1)
3     i = partition(a, p, 0, 1)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 1)
6   } else
7     insertionSort(a, 0, 1)
8   return
9 }
```

```
PC = 8
l = 0
h = 1
p = #
i = #
```

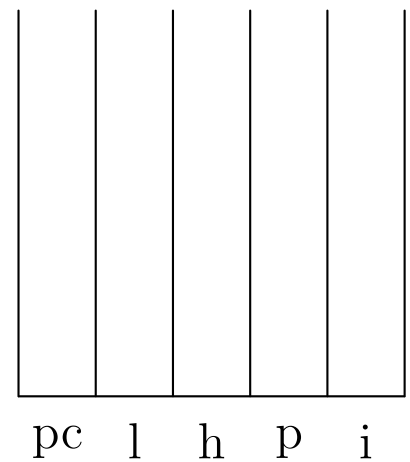
pc	l	h	p	i
4	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	97	15	88	79	96	90	25	93	32	4	76	14	94
low	high																		

QuickSort

```
quickSort(a, 0, 19) {
  if(19-0>3) {
    p = choosePivot(a, 0, 19)
    i = partition(a, 2, 0, 19)
    quickSort(a, 0, 2-1)
    quickSort(a, 2+1, 19)
  } else
    insertionSort(a, 0, 19)
  return
}
```

```
PC = 5
l = 0
h = 19
p = 2
i = 2
```



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	97	15	88	79	96	90	25	93	32	4	76	14	94
low																			high

QuickSort

```
quickSort(a, 3, 19) {
  if(19-3>3) {
    p = choosePivot(a, 3, 19)
    i = partition(a, p, 3, 19)
    quickSort(a, 3, i-1)
    quickSort(a, i+1, 19)
  } else
    insertionSort(a, 3, 19)
  return
}
```

```
PC = 0
l = 3
h = 19
p = #
i = #
```

pc	l	h	p	i
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	97	15	88	79	96	90	25	93	32	4	76	14	94
			low																high

QuickSort

```
quickSort(a, 3, 19) {
  if(19-3>3) {
    p = choosePivot(a, 3, 19)
    i = partition(a, p, 3, 19)
    quickSort(a, 3, i-1)
    quickSort(a, i+1, 19)
  } else
    insertionSort(a, 3, 19)
  return
}
```

```
PC = 1
l = 3
h = 19
p = #
i = #
```

5	0	19	2	2
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	97	15	88	79	96	90	25	93	32	4	76	14	94
			low																high

QuickSort

```
0 quickSort(a, 3, 19) {
1   if(19-3>3) {
2     p = choosePivot(a, 3, 19)
3     i = partition(a, p, 3, 19)
4     quickSort(a, 3, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 3, 19)
8   return
9 }
```

```
PC = 2
l = 3
h = 19
p = #
i = #
```

pc	l	h	p	i
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	97	15	88	79	96	90	25	93	32	4	76	14	94
			low																high

QuickSort

```
quickSort(a, 3, 19) {
  if(19-3>3) {
    p = choosePivot(a, 3, 19)
    i = partition(a, p, 3, 19)
    quickSort(a, 3, i-1)
    quickSort(a, i+1, 19)
  } else
    insertionSort(a, 3, 19)
  return
}
```

```
PC = 2
l = 3
h = 19
p = #
i = #
```

pc	l	h	p	i
2	3	19	#	#
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	97	15	88	79	96	90	25	93	32	4	76	14	94
			low																high

QuickSort

```
quickSort(a, 3, 19) {
  if(19-3>3) {
    p = choosePivot(a, 3, 19)
    i = partition(a, 94, 3, 19)
    quickSort(a, 3, i-1)
    quickSort(a, i+1, 19)
  } else
    insertionSort(a, 3, 19)
  return
}
```

```
PC = 3
l = 3
h = 19
p = 94
i = #
```

pc	l	h	p	i
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	97	15	88	79	96	90	25	93	32	4	76	14	94
			low																high

QuickSort

```
quickSort(a, 3, 19) {
  if(19-3>3) {
    p = choosePivot(a, 3, 19)
    i = partition(a, 94, 3, 19)
    quickSort(a, 3, i-1)
    quickSort(a, i+1, 19)
  } else
    insertionSort(a, 3, 19)
  return
}
```

```
PC = 3
l = 3
h = 19
p = 94
i = #
```

pc	l	h	p	i
3	3	19	94	#
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	14	15	88	79	76	90	25	93	32	4	94	97	96
			low																high

QuickSort

```
0 quickSort(a, 3, 19) {
1   if(19-3>3) {
2     p = choosePivot(a, 3, 19)
3     i = partition(a, 94, 3, 19)
4     quickSort(a, 3, 17-1)
5     quickSort(a, 17+1, 19)
6   } else
7     insertionSort(a, 3, 19)
8   return
9 }
```

```
PC = 4
l = 3
h = 19
p = 94
i = 17
```

pc	l	h	p	i
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	14	15	88	79	76	90	25	93	32	4	94	97	96
			low																high

QuickSort

```
quickSort(a, 3, 16) {
  if(16-3>3) {
    p = choosePivot(a, 3, 16)
    i = partition(a, p, 3, 16)
    quickSort(a, 3, i-1)
    quickSort(a, i+1, 16)
  } else
    insertionSort(a, 3, 16)
  return
}
```

```
PC = 0
l = 3
h = 16
p = #
i = #
```

pc	l	h	p	i
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	14	15	88	79	76	90	25	93	32	4	94	97	96
			low													high			

QuickSort

```
0 quickSort(a, 3, 16) {
1   if(16-3>3) {
2     p = choosePivot(a, 3, 16)
3     i = partition(a, p, 3, 16)
4     quickSort(a, 3, i-1)
5     quickSort(a, i+1, 16)
6   } else
7     insertionSort(a, 3, 16)
8   return
9 }
```

```
PC = 1
l = 3
h = 16
p = #
i = #
```

pc	l	h	p	i
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	14	15	88	79	76	90	25	93	32	4	94	97	96
			low													high			

QuickSort

```
quickSort(a, 3, 16) {
  if(16-3>3) {
    p = choosePivot(a, 3, 16)
    i = partition(a, p, 3, 16)
    quickSort(a, 3, i-1)
    quickSort(a, i+1, 16)
  } else
    insertionSort(a, 3, 16)
  return
}
```

```
PC = 2
l = 3
h = 16
p = #
i = #
```

pc	l	h	p	i
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	14	15	88	79	76	90	25	93	32	4	94	97	96
			low													high			

QuickSort

```
0 quickSort(a, 3, 16) {
1   if(16-3>3) {
2     p = choosePivot(a, 3, 16)
3     i = partition(a, p, 3, 16)
4     quickSort(a, 3, i-1)
5     quickSort(a, i+1, 16)
6   } else
7     insertionSort(a, 3, 16)
8   return
9 }
```

```
PC = 2
l = 3
h = 16
p = #
i = #
```

pc	l	h	p	i
2	3	16	#	#
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	14	15	88	79	76	90	25	93	32	4	94	97	96
			low													high			

QuickSort

```
0  quickSort(a, 3, 16) {
1    if(16-3>3) {
2      p = choosePivot(a, 3, 16)
3      i = partition(a, 88, 3, 16)
4      quickSort(a, 3, i-1)
5      quickSort(a, i+1, 16)
6    } else
7      insertionSort(a, 3, 16)
8    return
9  }
```

```
PC = 3
l = 3
h = 16
p = 88
i = #
```

pc	l	h	p	i
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	91	63	20	48	14	15	88	79	76	90	25	93	32	4	94	97	96
			low													high			

QuickSort

```
quickSort(a, 3, 16) {
  if(16-3>3) {
    p = choosePivot(a, 3, 16)
    i = partition(a, 88, 3, 16)
    quickSort(a, 3, i-1)
    quickSort(a, i+1, 16)
  } else
    insertionSort(a, 3, 16)
  return
}
```

```
PC = 3
l = 3
h = 16
p = 88
i = #
```

pc	l	h	p	i
3	3	16	88	#
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	63	20	48	14	15	32	79	76	25	88	93	90	91	94	97	96
			low													high			

QuickSort

```
quickSort(a, 3, 16) {
  if(16-3>3) {
    p = choosePivot(a, 3, 16)
    i = partition(a, 88, 3, 16)
    quickSort(a, 3, 13-1)
    quickSort(a, 13+1, 16)
  } else
    insertionSort(a, 3, 16)
  return
}
```

```
PC = 4
l = 3
h = 16
p = 88
i = 13
```

pc	l	h	p	i
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	63	20	48	14	15	32	79	76	25	88	93	90	91	94	97	96
			low													high			

QuickSort

```
quickSort(a, 3, 12) {
  if(12-3>3) {
    p = choosePivot(a, 3, 12)
    i = partition(a, p, 3, 12)
    quickSort(a, 3, i-1)
    quickSort(a, i+1, 12)
  } else
    insertionSort(a, 3, 12)
  return
}
```

```
PC = 0
l = 3
h = 12
p = #
i = #
```

pc	l	h	p	i
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	63	20	48	14	15	32	79	76	25	88	93	90	91	94	97	96
			low									high							

QuickSort

```
quickSort(a, 3, 12) {
  if(12-3>3) {
    p = choosePivot(a, 3, 12)
    i = partition(a, p, 3, 12)
    quickSort(a, 3, i-1)
    quickSort(a, i+1, 12)
  } else
    insertionSort(a, 3, 12)
  return
}
```

```
PC = 1
l = 3
h = 12
p = #
i = #
```

pc	l	h	p	i
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	63	20	48	14	15	32	79	76	25	88	93	90	91	94	97	96
			low									high							

QuickSort

```
quickSort(a, 3, 12) {
  if(12-3>3) {
    p = choosePivot(a, 3, 12)
    i = partition(a, p, 3, 12)
    quickSort(a, 3, i-1)
    quickSort(a, i+1, 12)
  } else
    insertionSort(a, 3, 12)
  return
}
```

```
PC = 2
l = 3
h = 12
p = #
i = #
```

pc	l	h	p	i
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	63	20	48	14	15	32	79	76	25	88	93	90	91	94	97	96
			low									high							

QuickSort

```
0 quickSort(a, 3, 12) {
1   if(12-3>3) {
2     p = choosePivot(a, 3, 12)
3     i = partition(a, p, 3, 12)
4     quickSort(a, 3, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 3, 12)
8   return
9 }
```

```
PC = 2
l = 3
h = 12
p = #
i = #
```

pc	l	h	p	i
2	3	12	#	#
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	63	20	48	14	15	32	79	76	25	88	93	90	91	94	97	96
			low									high							

QuickSort

```
quickSort(a, 3, 12) {
  if(12-3>3) {
    p = choosePivot(a, 3, 12)
    i = partition(a, 14, 3, 12)
    quickSort(a, 3, i-1)
    quickSort(a, i+1, 12)
  } else
    insertionSort(a, 3, 12)
  return
}
```

```
PC = 3
l = 3
h = 12
p = 14
i = #
```

pc	l	h	p	i
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	63	20	48	14	15	32	79	76	25	88	93	90	91	94	97	96
			low									high							

QuickSort

```
quickSort(a, 3, 12) {
  if(12-3>3) {
    p = choosePivot(a, 3, 12)
    i = partition(a, 14, 3, 12)
    quickSort(a, 3, i-1)
    quickSort(a, i+1, 12)
  } else
    insertionSort(a, 3, 12)
  return
}
```

```
PC = 3
l = 3
h = 12
p = 14
i = #
```

pc	l	h	p	i
3	3	12	14	#
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	20	48	63	15	32	79	76	25	88	93	90	91	94	97	96
			low									high							

QuickSort

```
0 quickSort(a, 3, 12) {
1   if(12-3>3) {
2     p = choosePivot(a, 3, 12)
3     i = partition(a, 14, 3, 12)
4     quickSort(a, 3, 4-1)
5     quickSort(a, 4+1, 12)
6   } else
7     insertionSort(a, 3, 12)
8   return
9 }
```

```
PC = 4
l = 3
h = 12
p = 14
i = 4
```

pc	l	h	p	i
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	20	48	63	15	32	79	76	25	88	93	90	91	94	97	96
			low									high							

QuickSort

```
0 quickSort(a, 3, 3) {
1   if(3-3>3) {
2     p = choosePivot(a, 3, 3)
3     i = partition(a, p, 3, 3)
4     quickSort(a, 3, i-1)
5     quickSort(a, i+1, 3)
6   } else
7     insertionSort(a, 3, 3)
8   return
9 }
```

```
PC = 0
l = 3
h = 3
p = #
i = #
```

pc	l	h	p	i
4	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	20	48	63	15	32	79	76	25	88	93	90	91	94	97	96
			high low																

QuickSort

```
0 quickSort(a, 3, 3) {
1   if(3-3>3) {
2     p = choosePivot(a, 3, 3)
3     i = partition(a, p, 3, 3)
4     quickSort(a, 3, i-1)
5     quickSort(a, i+1, 3)
6   } else
7     insertionSort(a, 3, 3)
8   return
9 }
```

```
PC = 1
l = 3
h = 3
p = #
i = #
```

pc	l	h	p	i
4	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	20	48	63	15	32	79	76	25	88	93	90	91	94	97	96
			high low i																

QuickSort

```
0  quickSort(a, 3, 3) {
1    if(3-3>3) {
2      p = choosePivot(a, 3, 3)
3      i = partition(a, p, 3, 3)
4      quickSort(a, 3, i-1)
5      quickSort(a, i+1, 3)
6    } else
7      insertionSort(a, 3, 3)
8    return
9  }
```

```
PC = 7
l = 3
h = 3
p = #
i = #
```

pc	l	h	p	i
4	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	20	48	63	15	32	79	76	25	88	93	90	91	94	97	96
			high low																

QuickSort

```
0  quickSort(a, 3, 3) {
1    if(3-3>3) {
2      p = choosePivot(a, 3, 3)
3      i = partition(a, p, 3, 3)
4      quickSort(a, 3, i-1)
5      quickSort(a, i+1, 3)
6    } else
7      insertionSort(a, 3, 3)
8    return
9  }
```

```
PC = 7
l = 3
h = 3
p = #
i = #
```

pc	l	h	p	i
7	3	3	#	#
4	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	20	48	63	15	32	79	76	25	88	93	90	91	94	97	96
			high low																

QuickSort

```
quickSort(a, 3, 3) {
  if(3-3>3) {
    p = choosePivot(a, 3, 3)
    i = partition(a, p, 3, 3)
    quickSort(a, 3, i-1)
    quickSort(a, i+1, 3)
  } else
    insertionSort(a, 3, 3)
  return
}
```

```
PC = 8
l = 3
h = 3
p = #
i = #
```

pc	l	h	p	i
4	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	20	48	63	15	32	79	76	25	88	93	90	91	94	97	96
			low high																

QuickSort

```
quickSort(a, 3, 12) {
  if(12-3>3) {
    p = choosePivot(a, 3, 12)
    i = partition(a, 14, 3, 12)
    quickSort(a, 3, 4-1)
    quickSort(a, 4+1, 12)
  } else
    insertionSort(a, 3, 12)
  return
}
```

```
PC = 5
l = 3
h = 12
p = 14
i = 4
```

pc	l	h	p	i
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	20	48	63	15	32	79	76	25	88	93	90	91	94	97	96
			low									high							

QuickSort

```
quickSort(a, 5, 12) {
  if(12-5>3) {
    p = choosePivot(a, 5, 12)
    i = partition(a, p, 5, 12)
    quickSort(a, 5, i-1)
    quickSort(a, i+1, 12)
  } else
    insertionSort(a, 5, 12)
  return
}
```

```
PC = 0
l = 5
h = 12
p = #
i = #
```

pc	l	h	p	i
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	20	48	63	15	32	79	76	25	88	93	90	91	94	97	96
					low							high							

QuickSort

```
0 quickSort(a, 5, 12) {
1   if(12-5>3) {
2     p = choosePivot(a, 5, 12)
3     i = partition(a, p, 5, 12)
4     quickSort(a, 5, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 5, 12)
8   return
9 }
```

```
PC = 1
l = 5
h = 12
p = #
i = #
```

pc	l	h	p	i
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	20	48	63	15	32	79	76	25	88	93	90	91	94	97	96
					low							high							

QuickSort

```
0  quickSort(a, 5, 12) {
1    if(12-5>3) {
2      p = choosePivot(a, 5, 12)
3      i = partition(a, p, 5, 12)
4      quickSort(a, 5, i-1)
5      quickSort(a, i+1, 12)
6    } else
7      insertionSort(a, 5, 12)
8    return
9  }
```

```
PC = 2
l = 5
h = 12
p = #
i = #
```

pc	l	h	p	i
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	20	48	63	15	32	79	76	25	88	93	90	91	94	97	96
					low							high							

QuickSort

```
quickSort(a, 5, 12) {
  if(12-5>3) {
    p = choosePivot(a, 5, 12)
    i = partition(a, p, 5, 12)
    quickSort(a, 5, i-1)
    quickSort(a, i+1, 12)
  } else
    insertionSort(a, 5, 12)
  return
}
```

```
PC = 2
l = 5
h = 12
p = #
i = #
```

pc	l	h	p	i
2	5	12	#	#
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	20	48	63	15	32	79	76	25	88	93	90	91	94	97	96
					low							high							

QuickSort

```
0 quickSort(a, 5, 12) {
1   if(12-5>3) {
2     p = choosePivot(a, 5, 12)
3     i = partition(a, 20, 5, 12)
4     quickSort(a, 5, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 5, 12)
8   return
9 }
```

```
PC = 3
l = 5
h = 12
p = 20
i = #
```

pc	l	h	p	i
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	20	48	63	15	32	79	76	25	88	93	90	91	94	97	96
					low							high							

QuickSort

```
quickSort(a, 5, 12) {
  if(12-5>3) {
    p = choosePivot(a, 5, 12)
    i = partition(a, 20, 5, 12)
    quickSort(a, 5, i-1)
    quickSort(a, i+1, 12)
  } else
    insertionSort(a, 5, 12)
  return
}
```

```
PC = 3
l = 5
h = 12
p = 20
i = #
```

pc	l	h	p	i
3	5	12	20	#
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	63	48	32	79	76	25	88	93	90	91	94	97	96
					low							high							

QuickSort

```
0  quickSort(a, 5, 12) {
1    if(12-5>3) {
2      p = choosePivot(a, 5, 12)
3      i = partition(a, 20, 5, 12)
4      quickSort(a, 5, 6-1)
5      quickSort(a, 6+1, 12)
6    } else
7      insertionSort(a, 5, 12)
8    return
9  }
```

```
PC = 4
l = 5
h = 12
p = 20
i = 6
```

pc	l	h	p	i
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	63	48	32	79	76	25	88	93	90	91	94	97	96
					low							high							

QuickSort

```
0 quickSort(a, 5, 5) {
1   if(5-5>3) {
2     p = choosePivot(a, 5, 5)
3     i = partition(a, p, 5, 5)
4     quickSort(a, 5, i-1)
5     quickSort(a, i+1, 5)
6   } else
7     insertionSort(a, 5, 5)
8   return
9 }
```

```
PC = 0
l = 5
h = 5
p = #
i = #
```

pc	l	h	p	i
4	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	63	48	32	79	76	25	88	93	90	91	94	97	96
					low high														

QuickSort

```
0 quickSort(a, 5, 5) {
1   if(5-5>3) {
2     p = choosePivot(a, 5, 5)
3     i = partition(a, p, 5, 5)
4     quickSort(a, 5, i-1)
5     quickSort(a, i+1, 5)
6   } else
7     insertionSort(a, 5, 5)
8   return
9 }
```

```
PC = 1
l = 5
h = 5
p = #
i = #
```

pc	l	h	p	i
4	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	63	48	32	79	76	25	88	93	90	91	94	97	96
					low high														

QuickSort

```
quickSort(a, 5, 5) {
  if(5-5>3) {
    p = choosePivot(a, 5, 5)
    i = partition(a, p, 5, 5)
    quickSort(a, 5, i-1)
    quickSort(a, i+1, 5)
  } else
    insertionSort(a, 5, 5)
  return
}
```

```
PC = 7
l = 5
h = 5
p = #
i = #
```

pc	l	h	p	i
4	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	63	48	32	79	76	25	88	93	90	91	94	97	96

low high

QuickSort

```
quickSort(a, 5, 5) {
  if(5-5>3) {
    p = choosePivot(a, 5, 5)
    i = partition(a, p, 5, 5)
    quickSort(a, 5, i-1)
    quickSort(a, i+1, 5)
  } else
    insertionSort(a, 5, 5)
  return
}
```

```
PC = 7
l = 5
h = 5
p = #
i = #
```

pc	l	h	p	i
7	5	5	#	#
4	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	63	48	32	79	76	25	88	93	90	91	94	97	96

high low

QuickSort

```
0 quickSort(a, 5, 5) {
1   if(5-5>3) {
2     p = choosePivot(a, 5, 5)
3     i = partition(a, p, 5, 5)
4     quickSort(a, 5, i-1)
5     quickSort(a, i+1, 5)
6   } else
7     insertionSort(a, 5, 5)
8   return
9 }
```

```
PC = 8
l = 5
h = 5
p = #
i = #
```

pc	l	h	p	i
4	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	63	48	32	79	76	25	88	93	90	91	94	97	96

high low

QuickSort

```
quickSort(a, 5, 12) {
  if(12-5>3) {
    p = choosePivot(a, 5, 12)
    i = partition(a, 20, 5, 12)
    quickSort(a, 5, 6-1)
    quickSort(a, 6+1, 12)
  } else
    insertionSort(a, 5, 12)
  return
}
```

```
PC = 5
l = 5
h = 12
p = 20
i = 6
```

pc	l	h	p	i
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	63	48	32	79	76	25	88	93	90	91	94	97	96
					low							high							

QuickSort

```
quickSort(a, 7, 12) {
  if(12-7>3) {
    p = choosePivot(a, 7, 12)
    i = partition(a, p, 7, 12)
    quickSort(a, 7, i-1)
    quickSort(a, i+1, 12)
  } else
    insertionSort(a, 7, 12)
  return
}
```

```
PC = 0
l = 7
h = 12
p = #
i = #
```

pc	l	h	p	i
5	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	63	48	32	79	76	25	88	93	90	91	94	97	96
							low					high							

QuickSort

```
0 quickSort(a, 7, 12) {
1   if(12-7>3) {
2     p = choosePivot(a, 7, 12)
3     i = partition(a, p, 7, 12)
4     quickSort(a, 7, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 7, 12)
8   return
9 }
```

```
PC = 1
l = 7
h = 12
p = #
i = #
```

pc	l	h	p	i
5	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	63	48	32	79	76	25	88	93	90	91	94	97	96
							low					high							

QuickSort

```
quickSort(a, 7, 12) {
  if(12-7>3) {
    p = choosePivot(a, 7, 12)
    i = partition(a, p, 7, 12)
    quickSort(a, 7, i-1)
    quickSort(a, i+1, 12)
  } else
    insertionSort(a, 7, 12)
  return
}
```

```
PC = 2
l = 7
h = 12
p = #
i = #
```

pc	l	h	p	i
5	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	63	48	32	79	76	25	88	93	90	91	94	97	96
							low					high							

QuickSort

```
quickSort(a, 7, 12) {
  if(12-7>3) {
    p = choosePivot(a, 7, 12)
    i = partition(a, p, 7, 12)
    quickSort(a, 7, i-1)
    quickSort(a, i+1, 12)
  } else
    insertionSort(a, 7, 12)
  return
}
```

```
PC = 2
l = 7
h = 12
p = #
i = #
```

pc	l	h	p	i
2	7	12	#	#
5	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	63	48	32	79	76	25	88	93	90	91	94	97	96
							low					high							

QuickSort

```
0 quickSort(a, 7, 12) {
1   if(12-7>3) {
2     p = choosePivot(a, 7, 12)
3     i = partition(a, 32, 7, 12)
4     quickSort(a, 7, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 7, 12)
8   return
9 }
```

```
PC = 3
l = 7
h = 12
p = 32
i = #
```

pc	l	h	p	i
5	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	63	48	32	79	76	25	88	93	90	91	94	97	96
							low					high							

QuickSort

```
quickSort(a, 7, 12) {
  if(12-7>3) {
    p = choosePivot(a, 7, 12)
    i = partition(a, 32, 7, 12)
    quickSort(a, 7, i-1)
    quickSort(a, i+1, 12)
  } else
    insertionSort(a, 7, 12)
  return
}
```

```
PC = 3
l = 7
h = 12
p = 32
i = #
```

pc	l	h	p	i
3	7	12	32	#
5	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	25	32	48	79	76	63	88	93	90	91	94	97	96
							low					high							

QuickSort

```
0  quickSort(a, 7, 12) {
1    if(12-7>3) {
2      p = choosePivot(a, 7, 12)
3      i = partition(a, 32, 7, 12)
4      quickSort(a, 7, 8-1)
5      quickSort(a, 8+1, 12)
6    } else
7      insertionSort(a, 7, 12)
8    return
9  }
```

```
PC = 4
l = 7
h = 12
p = 32
i = 8
```

pc	l	h	p	i
5	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	25	32	48	79	76	63	88	93	90	91	94	97	96
							low					high							

QuickSort

```
quickSort(a, 7, 7) {
  if(7-7>3) {
    p = choosePivot(a, 7, 7)
    i = partition(a, p, 7, 7)
    quickSort(a, 7, i-1)
    quickSort(a, i+1, 7)
  } else
    insertionSort(a, 7, 7)
  return
}
```

```
PC = 0
l = 7
h = 7
p = #
i = #
```

pc	l	h	p	i
4	7	12	32	8
5	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	25	32	48	79	76	63	88	93	90	91	94	97	96

high
low

QuickSort

```
quickSort(a, 7, 7) {
  if(7-7>3) {
    p = choosePivot(a, 7, 7)
    i = partition(a, p, 7, 7)
    quickSort(a, 7, i-1)
    quickSort(a, i+1, 7)
  } else
    insertionSort(a, 7, 7)
  return
}
```

```
PC = 1
l = 7
h = 7
p = #
i = #
```

pc	l	h	p	i
4	7	12	32	8
5	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	25	32	48	79	76	63	88	93	90	91	94	97	96

QuickSort

```
0 quickSort(a, 7, 7) {
1   if(7-7>3) {
2     p = choosePivot(a, 7, 7)
3     i = partition(a, p, 7, 7)
4     quickSort(a, 7, i-1)
5     quickSort(a, i+1, 7)
6   } else
7     insertionSort(a, 7, 7)
8   return
9 }
```

```
PC = 7
l = 7
h = 7
p = #
i = #
```

pc	l	h	p	i
4	7	12	32	8
5	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	25	32	48	79	76	63	88	93	90	91	94	97	96

high low

QuickSort

```
0 quickSort(a, 7, 7) {
1   if(7-7>3) {
2     p = choosePivot(a, 7, 7)
3     i = partition(a, p, 7, 7)
4     quickSort(a, 7, i-1)
5     quickSort(a, i+1, 7)
6   } else
7     insertionSort(a, 7, 7)
8   return
9 }
```

```
PC = 7
l = 7
h = 7
p = #
i = #
```

pc	l	h	p	i
7	7	7	#	#
4	7	12	32	8
5	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	25	32	48	79	76	63	88	93	90	91	94	97	96

high low

QuickSort

```
0  quickSort(a, 7, 7) {
1    if(7-7>3) {
2      p = choosePivot(a, 7, 7)
3      i = partition(a, p, 7, 7)
4      quickSort(a, 7, i-1)
5      quickSort(a, i+1, 7)
6    } else
7      insertionSort(a, 7, 7)
8    return
9  }
```

```
PC = 8
l = 7
h = 7
p = #
i = #
```

pc	l	h	p	i
4	7	12	32	8
5	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	25	32	48	79	76	63	88	93	90	91	94	97	96
							high low												

QuickSort

```
0 quickSort(a, 7, 12) {
1   if(12-7>3) {
2     p = choosePivot(a, 7, 12)
3     i = partition(a, 32, 7, 12)
4     quickSort(a, 7, 8-1)
5     quickSort(a, 8+1, 12)
6   } else
7     insertionSort(a, 7, 12)
8   return
9 }
```

```
PC = 5
l = 7
h = 12
p = 32
i = 8
```

pc	l	h	p	i
5	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	25	32	48	79	76	63	88	93	90	91	94	97	96
							low					high							

QuickSort

```
quickSort(a, 9, 12) {
  if(12-9>3) {
    p = choosePivot(a, 9, 12)
    i = partition(a, p, 9, 12)
    quickSort(a, 9, i-1)
    quickSort(a, i+1, 12)
  } else
    insertionSort(a, 9, 12)
  return
}
```

```
PC = 0
l = 9
h = 12
p = #
i = #
```

pc	l	h	p	i
5	7	12	32	8
5	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	25	32	48	79	76	63	88	93	90	91	94	97	96
									low			high							

QuickSort

```
quickSort(a, 9, 12) {
  if(12-9>3) {
    p = choosePivot(a, 9, 12)
    i = partition(a, p, 9, 12)
    quickSort(a, 9, i-1)
    quickSort(a, i+1, 12)
  } else
    insertionSort(a, 9, 12)
  return
}
```

```
PC = 1
l = 9
h = 12
p = #
i = #
```

pc	l	h	p	i
5	7	12	32	8
5	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	25	32	48	79	76	63	88	93	90	91	94	97	96
									low			high							

QuickSort

```
0  quickSort(a, 9, 12) {
1    if(12-9>3) {
2      p = choosePivot(a, 9, 12)
3      i = partition(a, p, 9, 12)
4      quickSort(a, 9, i-1)
5      quickSort(a, i+1, 12)
6    } else
7      insertionSort(a, 9, 12)
8    return
9  }
```

```
PC = 7
l  = 9
h  = 12
p  = #
i  = #
```

pc	l	h	p	i
5	7	12	32	8
5	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	25	32	48	79	76	63	88	93	90	91	94	97	96
									low			high							

QuickSort

```
quickSort(a, 9, 12) {
  if(12-9>3) {
    p = choosePivot(a, 9, 12)
    i = partition(a, p, 9, 12)
    quickSort(a, 9, i-1)
    quickSort(a, i+1, 12)
  } else
    insertionSort(a, 9, 12)
  return
}
```

```
PC = 7
l = 9
h = 12
p = #
i = #
```

pc	l	h	p	i
7	9	12	#	#
5	7	12	32	8
5	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	25	32	48	63	76	79	88	93	90	91	94	97	96
									low			high							

QuickSort

```
0  quickSort(a, 9, 12) {
1    if(12-9>3) {
2      p = choosePivot(a, 9, 12)
3      i = partition(a, p, 9, 12)
4      quickSort(a, 9, i-1)
5      quickSort(a, i+1, 12)
6    } else
7      insertionSort(a, 9, 12)
8    return
9  }
```

```
PC = 8
l = 9
h = 12
p = #
i = #
```

pc	l	h	p	i
5	7	12	32	8
5	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	25	32	48	63	76	79	88	93	90	91	94	97	96
									low			high							

QuickSort

```
quickSort(a, 7, 12) {
  if(12-7>3) {
    p = choosePivot(a, 7, 12)
    i = partition(a, 32, 7, 12)
    quickSort(a, 7, 8-1)
    quickSort(a, 8+1, 12)
  } else
    insertionSort(a, 7, 12)
  return
}
```

```
PC = 8
l = 7
h = 12
p = 32
i = 8
```

pc	l	h	p	i
5	5	12	20	6
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	25	32	48	63	76	79	88	93	90	91	94	97	96
							low					high							

QuickSort

```
0  quickSort(a, 5, 12) {
1    if(12-5>3) {
2      p = choosePivot(a, 5, 12)
3      i = partition(a, 20, 5, 12)
4      quickSort(a, 5, 6-1)
5      quickSort(a, 6+1, 12)
6    } else
7      insertionSort(a, 5, 12)
8    return
9  }
```

```
PC = 8
l = 5
h = 12
p = 20
i = 6
```

pc	l	h	p	i
5	3	12	14	4
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	25	32	48	63	76	79	88	93	90	91	94	97	96
					low							high							

QuickSort

```
0 quickSort(a, 3, 12) {
1   if(12-3>3) {
2     p = choosePivot(a, 3, 12)
3     i = partition(a, 14, 3, 12)
4     quickSort(a, 3, 4-1)
5     quickSort(a, 4+1, 12)
6   } else
7     insertionSort(a, 3, 12)
8   return
9 }
```

```
PC = 8
l = 3
h = 12
p = 14
i = 4
```

pc	l	h	p	i
4	3	16	88	13
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	25	32	48	63	76	79	88	93	90	91	94	97	96
			low									high							

QuickSort

```
quickSort(a, 3, 16) {
  if(16-3>3) {
    p = choosePivot(a, 3, 16)
    i = partition(a, 88, 3, 16)
    quickSort(a, 3, 13-1)
    quickSort(a, 13+1, 16)
  } else
    insertionSort(a, 3, 16)
  return
}
```

```
PC = 5
l = 3
h = 16
p = 88
i = 13
```

pc	l	h	p	i
4	3	19	94	17
5	0	19	2	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	4	14	15	20	25	32	48	63	76	79	88	93	90	91	94	97	96
			low													high			

Sort in Practice

- The STL in C++ offers three sorts
  - ⋆ `sort()` implemented using quicksort
  - ⋆ `stable_sort()` implemented using mergesort
  - ⋆ `partial_sort()` implemented using heapsort
- Java uses
  - ⋆ Quicksort to sort arrays of primitive types
  - ⋆ Mergesort to sort Collections of objects
- Quicksort is typically fastest but has worst case quadratic time complexity

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Selection

- A related problem to sorting is selection
- That is we want to select the k^{th} largest element
- We could do this by first sorting the array
- A full sort is not however necessary—we can use a modified quicksort where we only continue to sort the part of the array we are interested in
- This leads to a $\Theta(n\log(n))$ algorithm which is considerably faster then sorting

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Outline

1. Merge Sort
2. Quick Sort
3. **Radix Sort**

Radix Sort

- Can we get a sort algorithm to run faster than $O\left(n\log(n)\right)$?
- Our proof that this was optimal assumed we were performing binary decisions (is a_i less than a_j?)
- If we don't perform pairwise comparisons then the proof doesn't apply
- Radix sort is the classic example of a sort algorithm that doesn't use pairwise comparisons

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Sorting Into Buckets

- The idea behind radix sort is to sort the elements of an array into some number of buckets
- This is done successively until the whole array is sorted
- Consider sorting integers in decimals (base 10 or radix 10)
- We can successively sort on the digits
- The sort finishes when we have got through all the digits

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- The sort finishes when we have got through all the digits

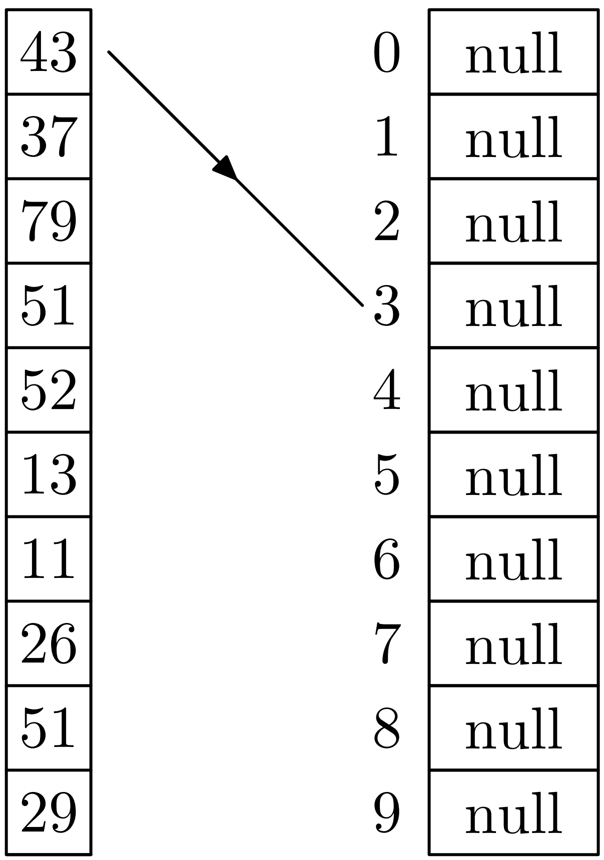
Sorting Into Buckets

- The idea behind radix sort is to sort the elements of an array into some number of buckets
- This is done successively until the whole array is sorted
- Consider sorting integers in decimals (base 10 or radix 10)
- We can successively sort on the digits
- The sort finishes when we have got through all the digits

Radix Sort in Action

43		0	null
37		1	null
79		2	null
51		3	null
52		4	null
13		5	null
11		6	null
26		7	null
51		8	null
29		9	null

Radix Sort in Action



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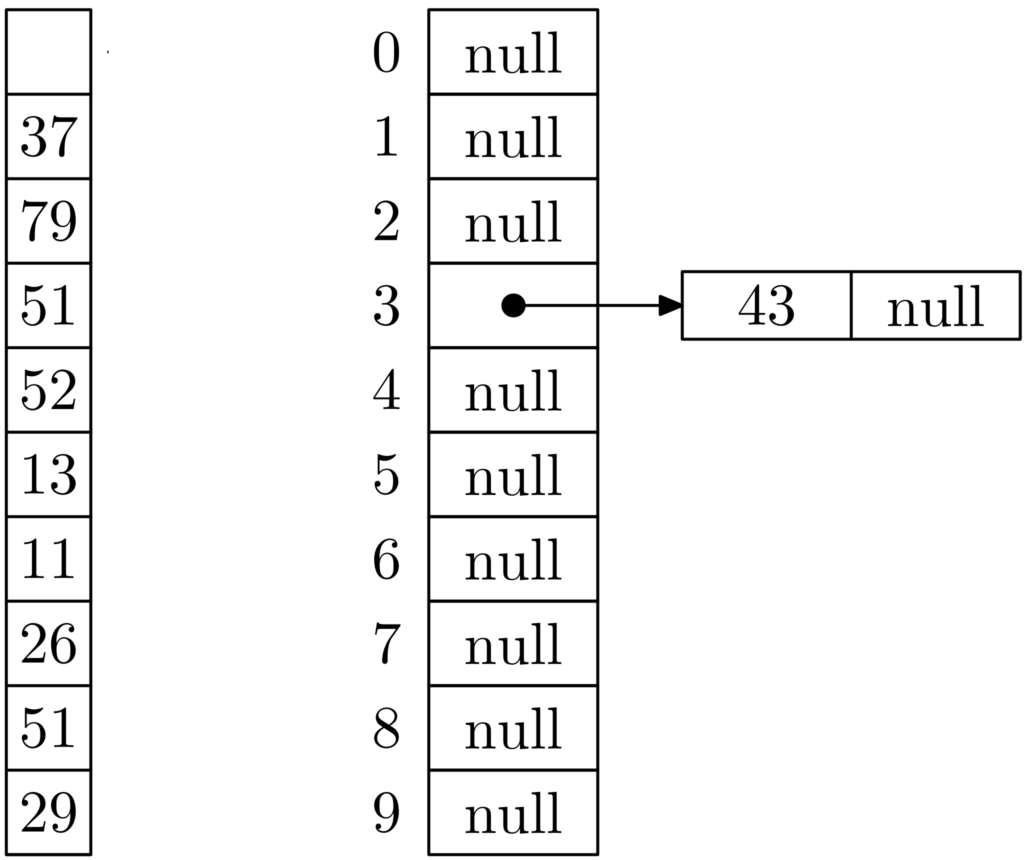

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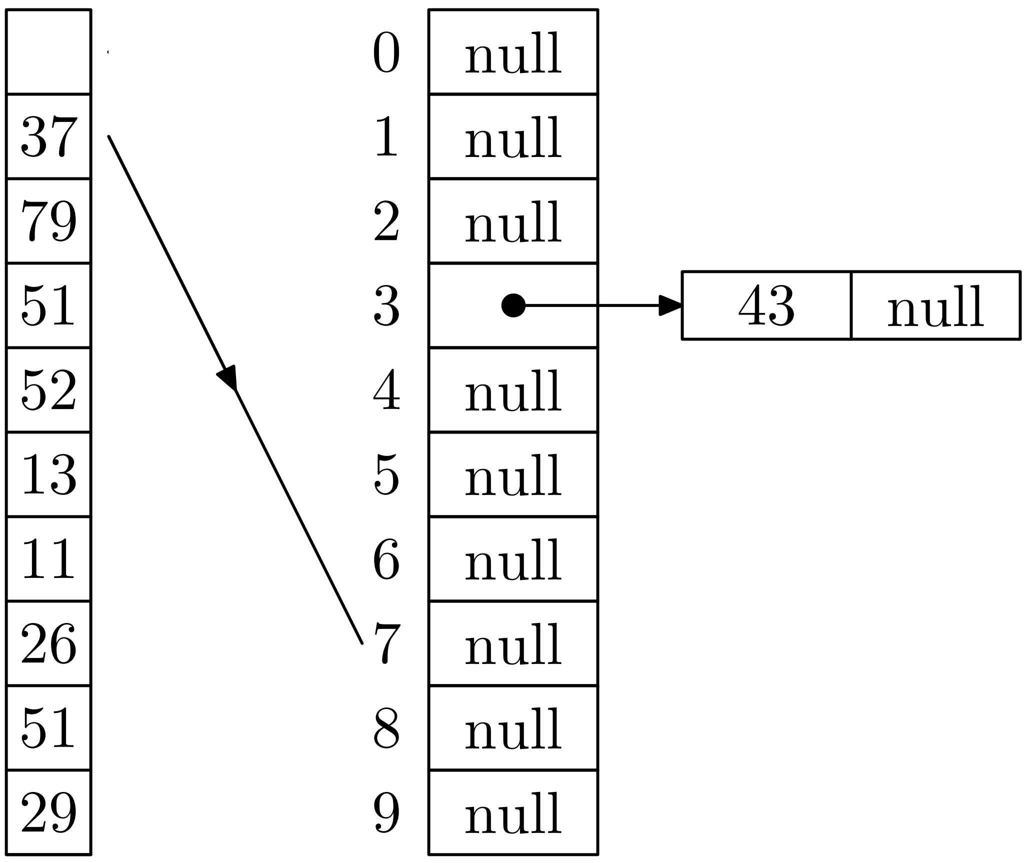
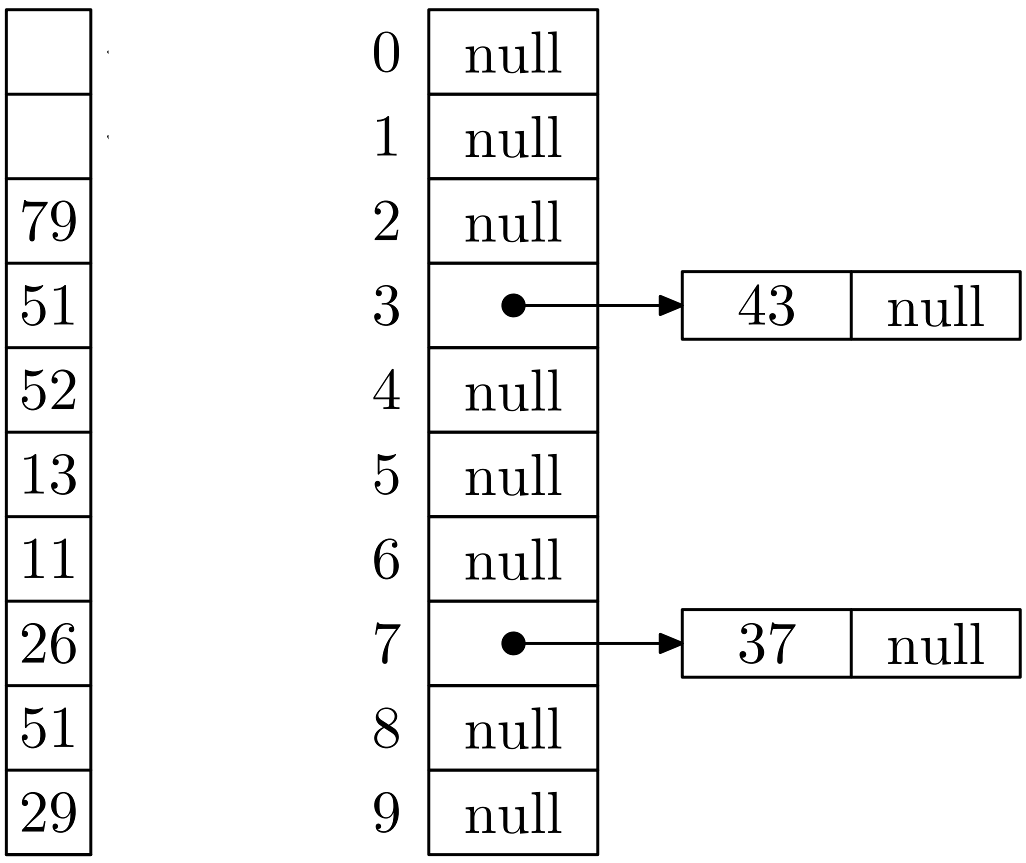


Radix Sort in Action



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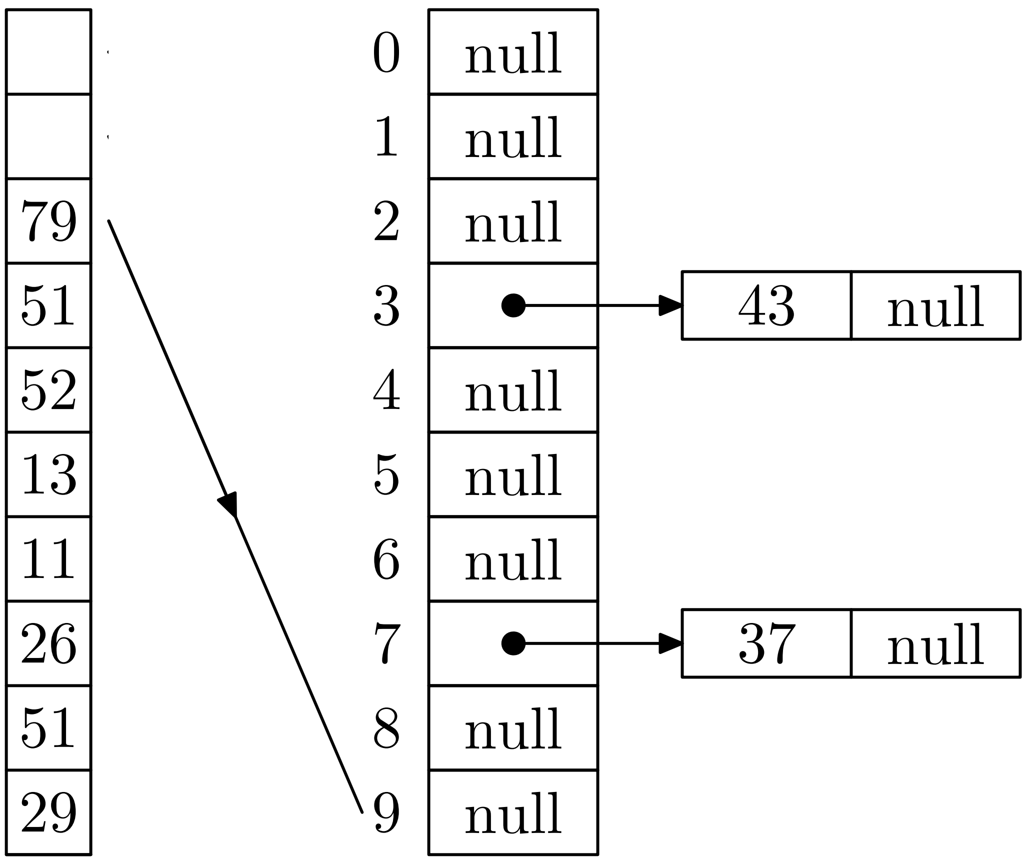

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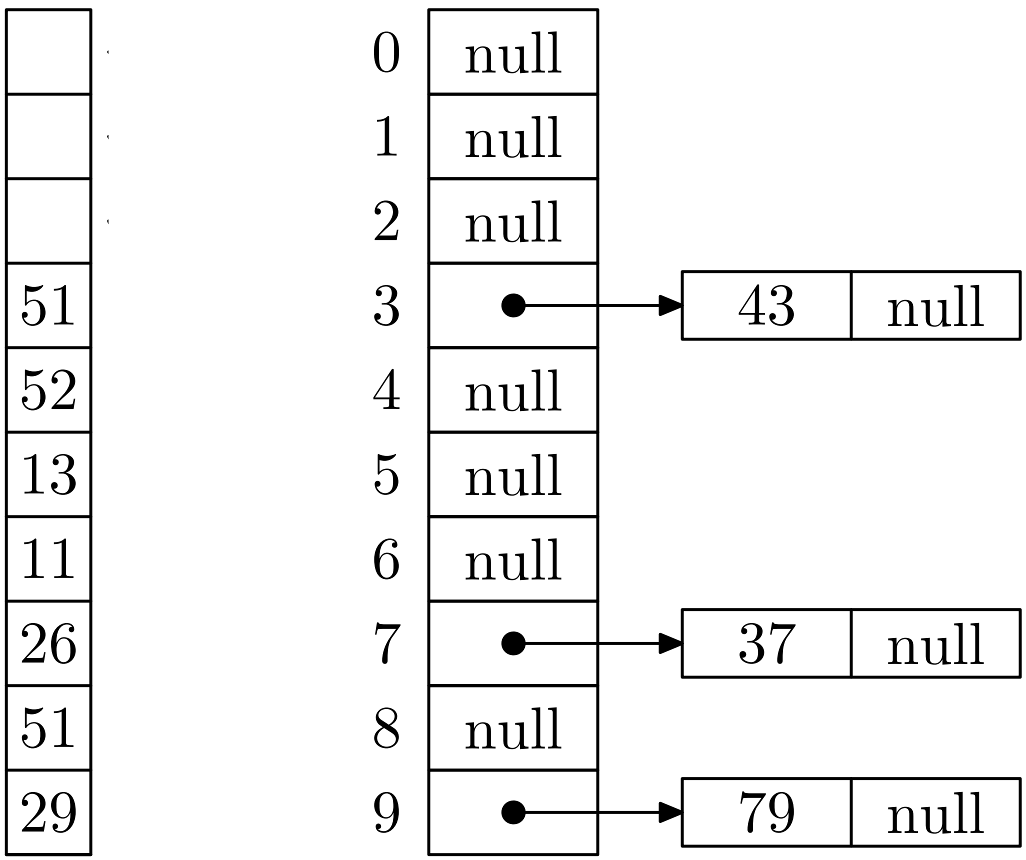

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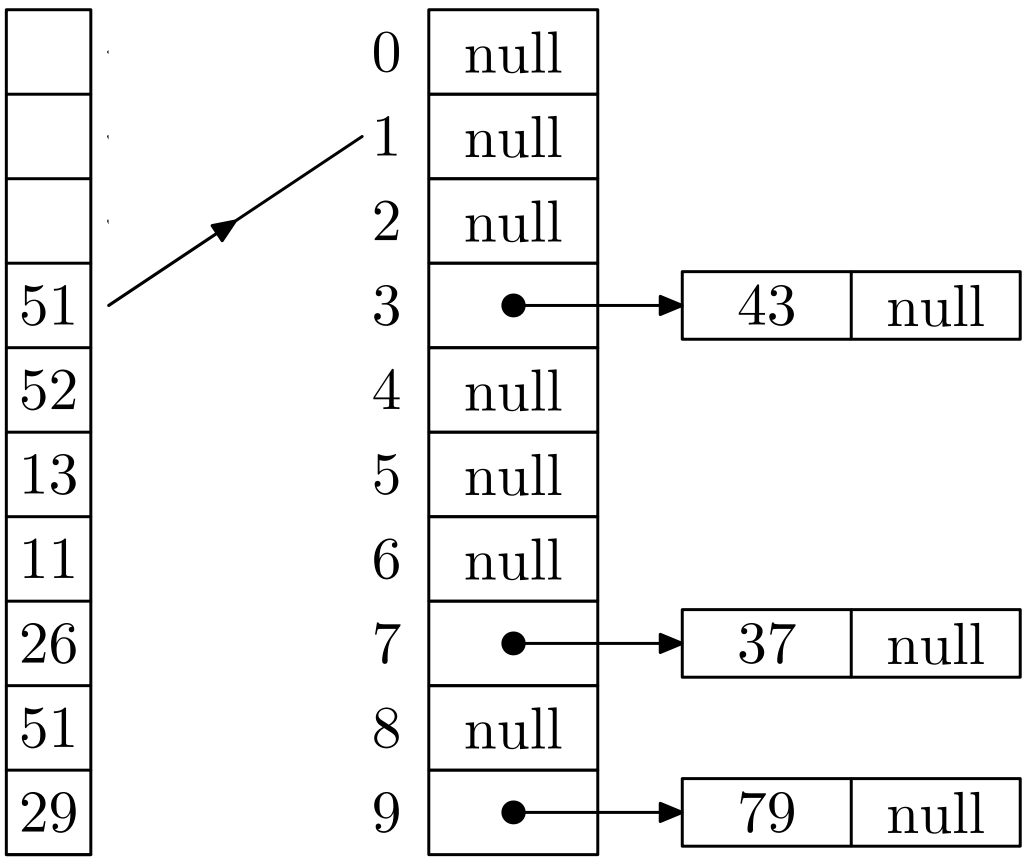

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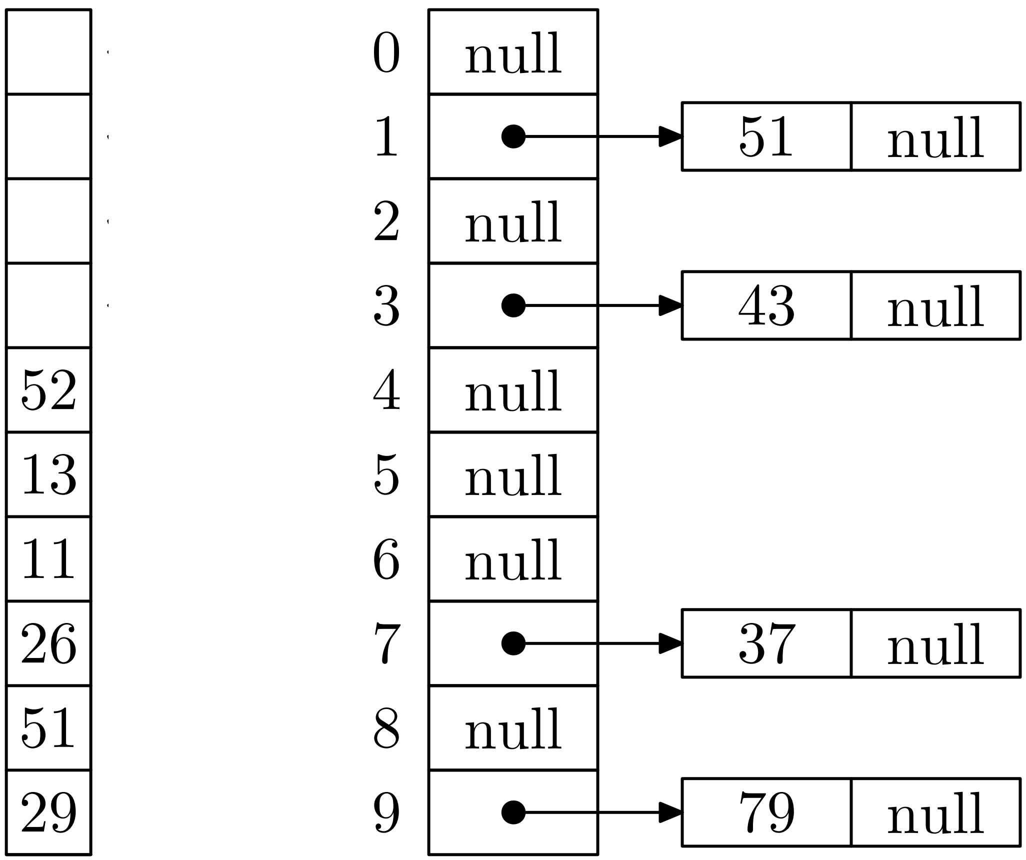

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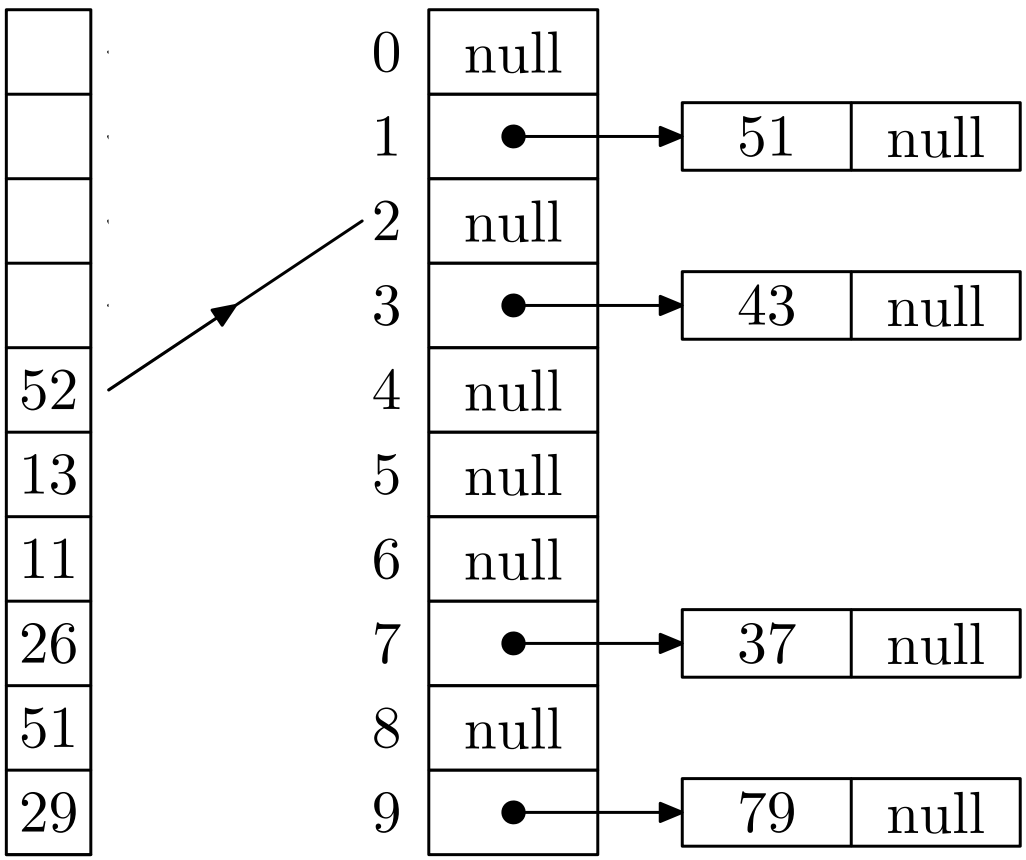

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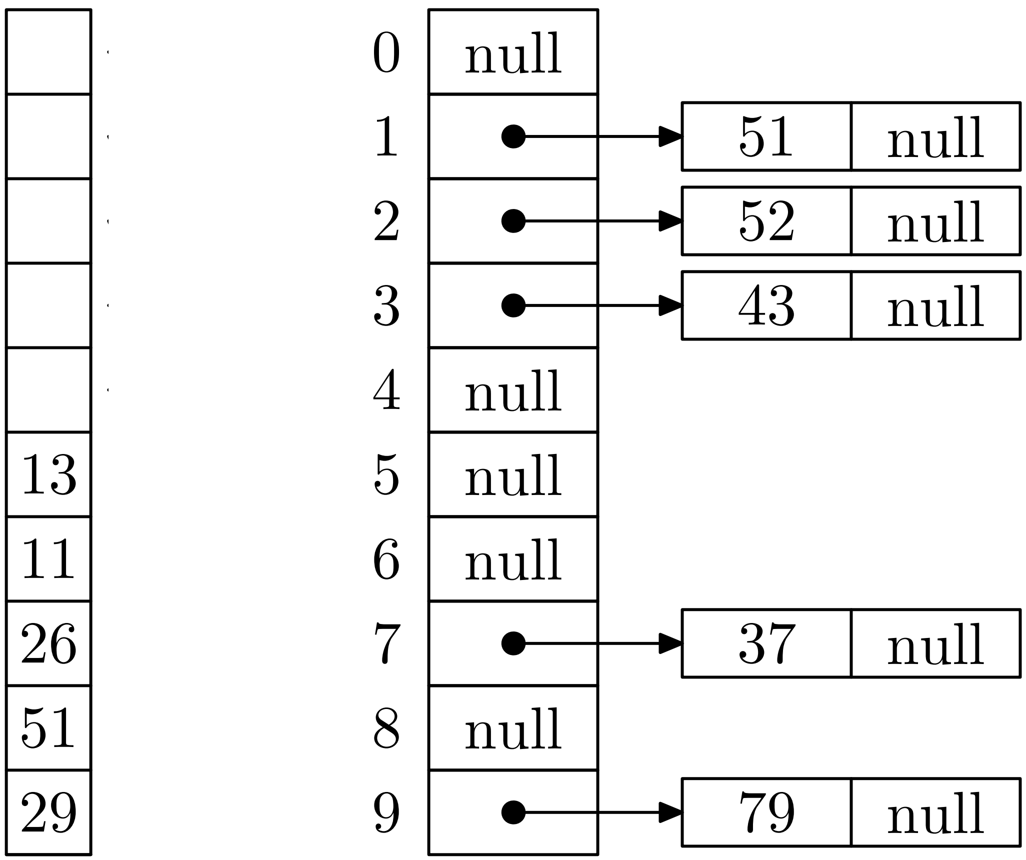

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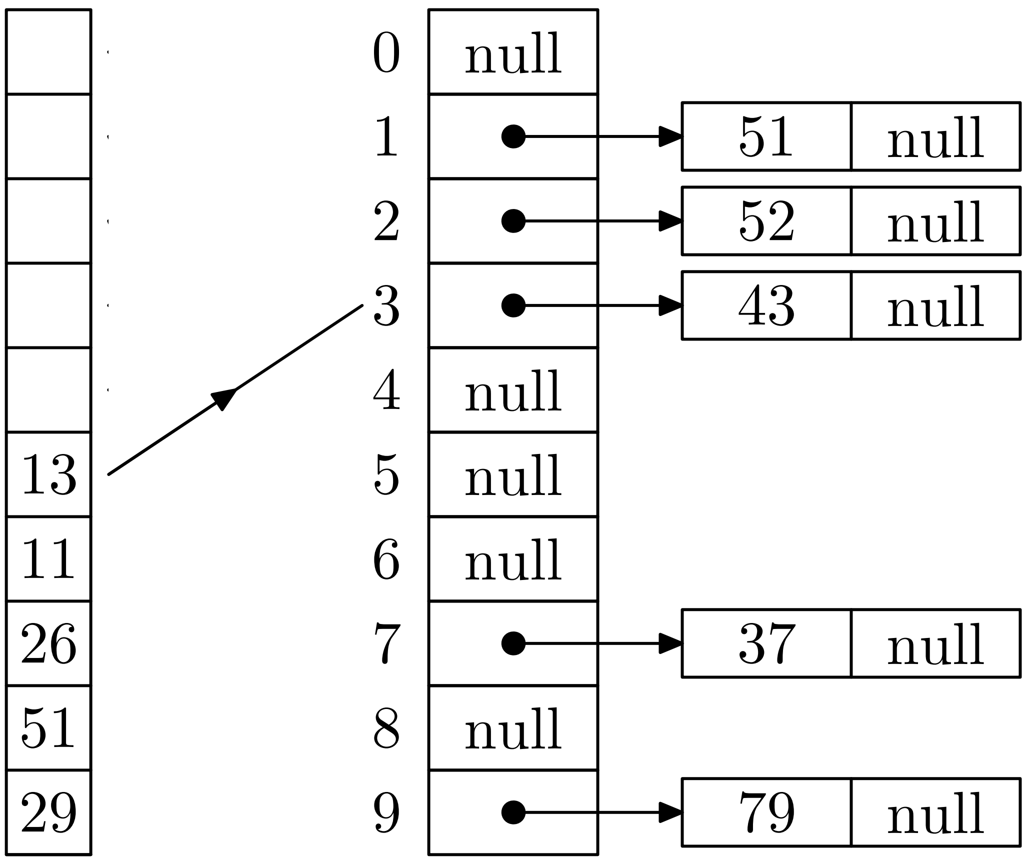

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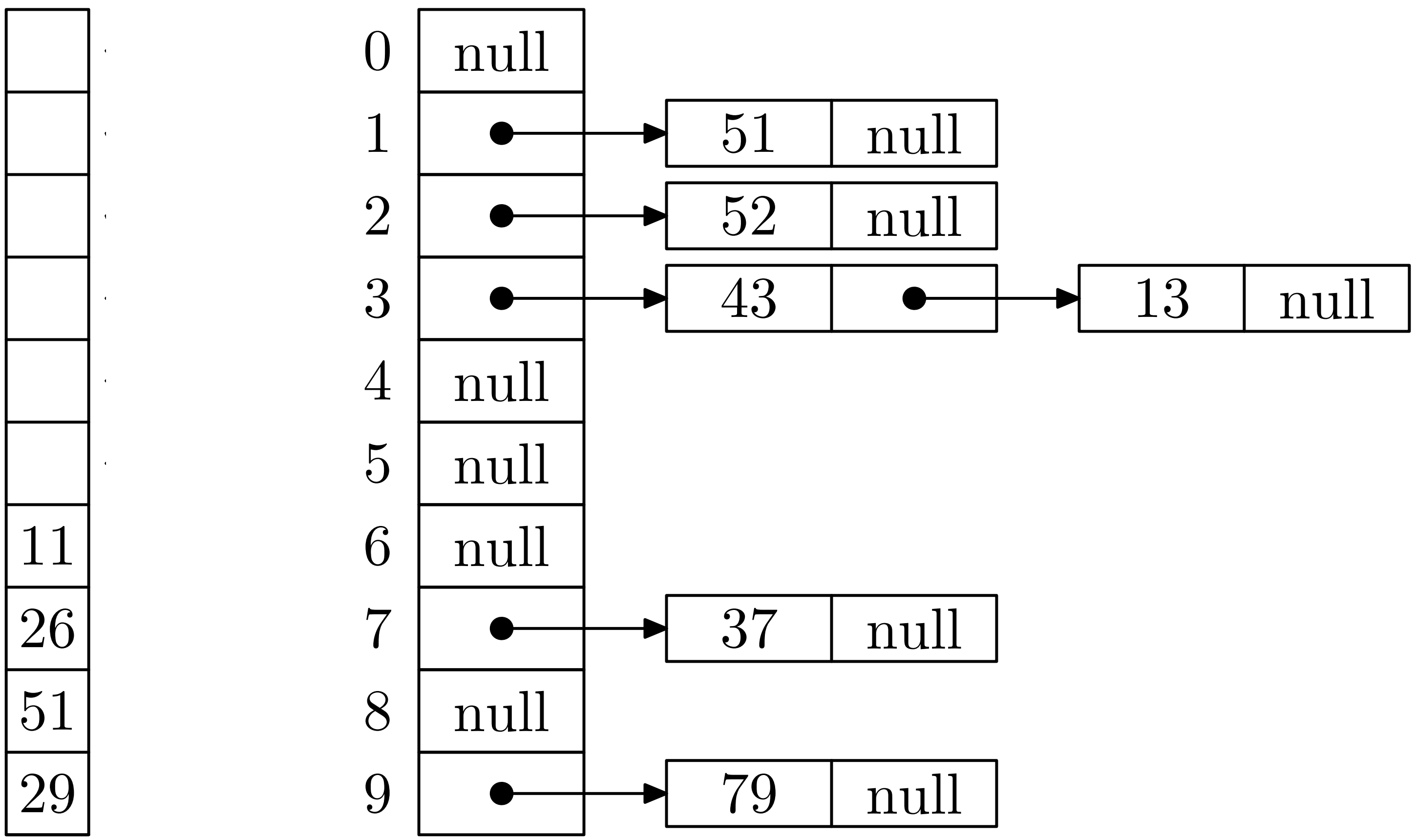

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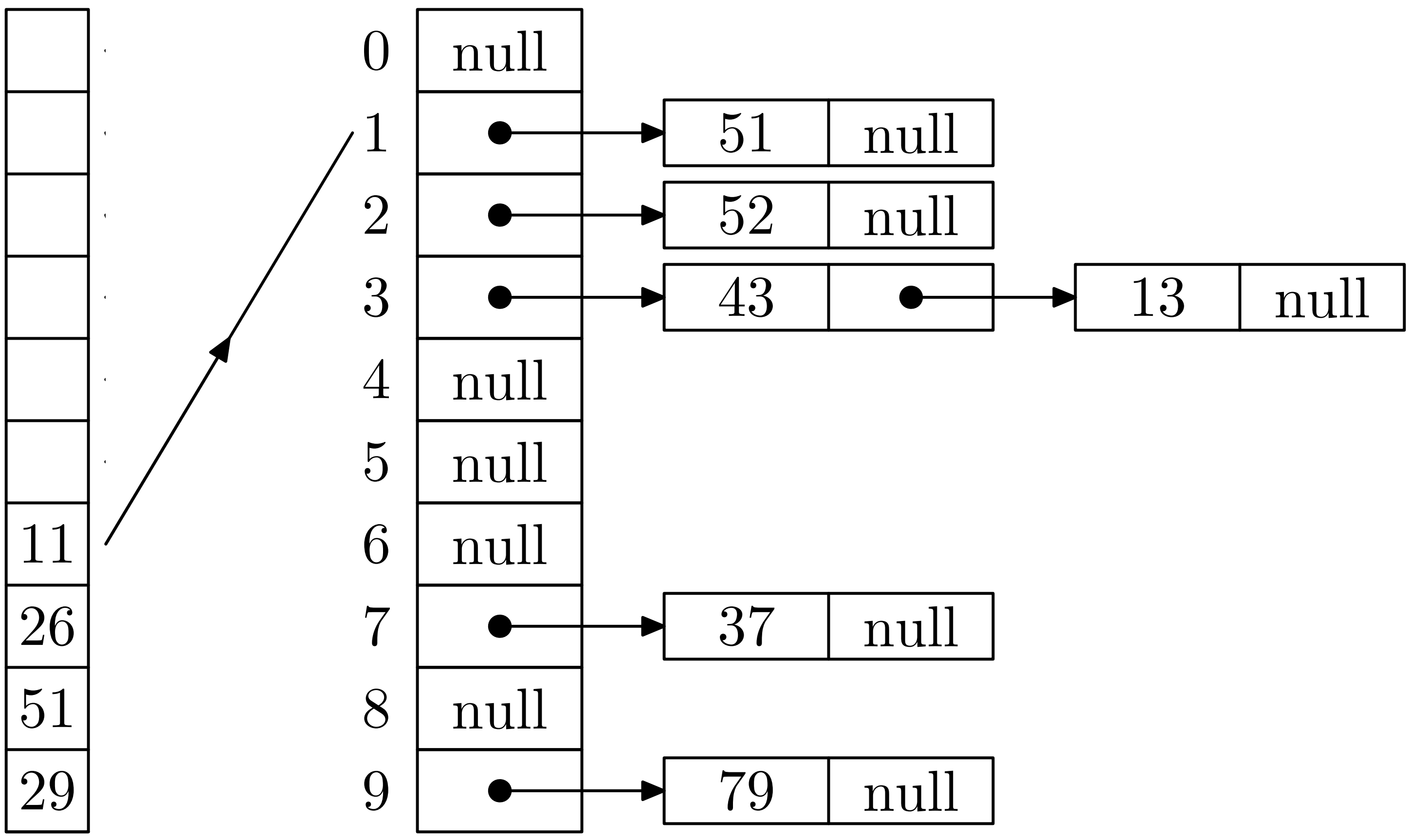

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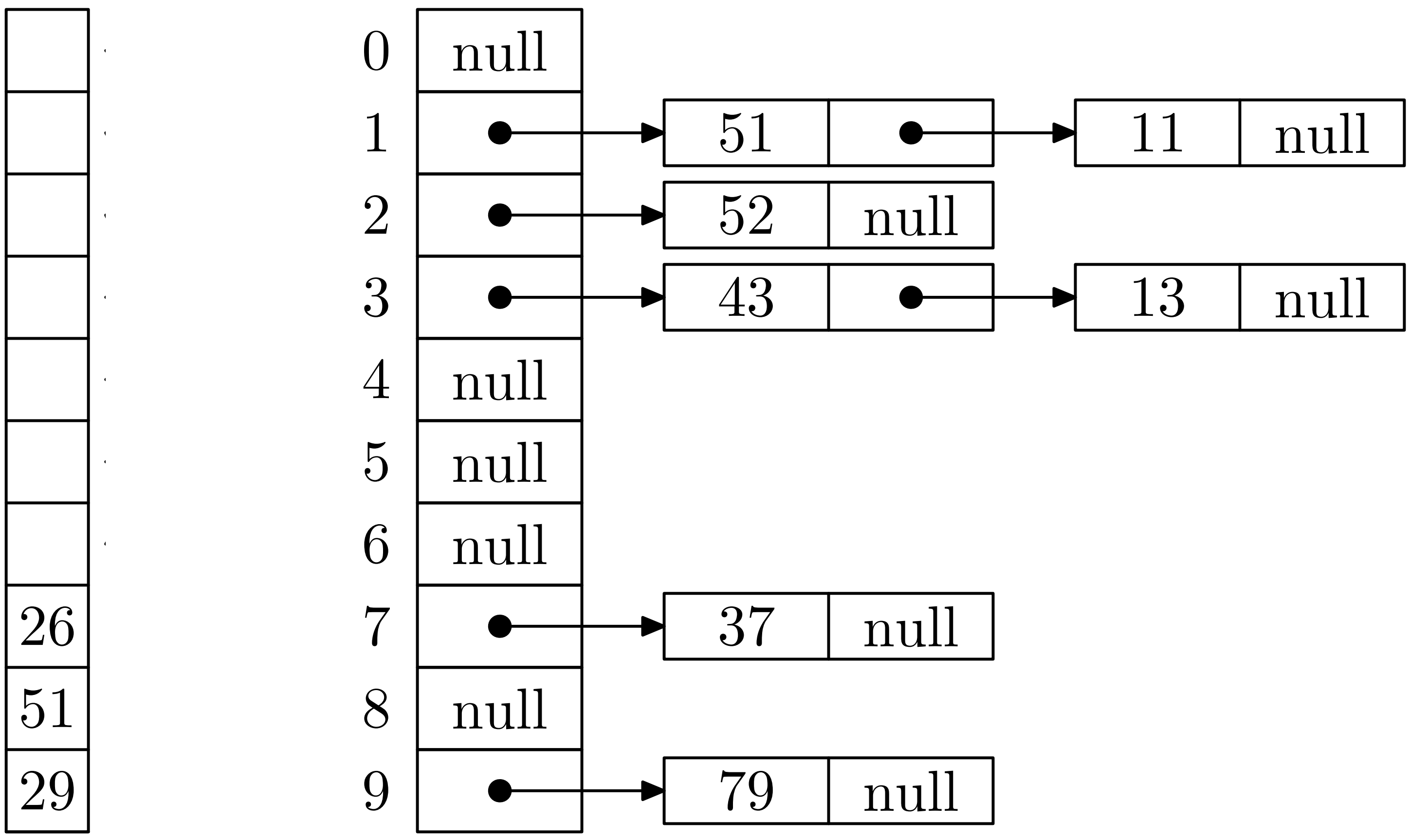

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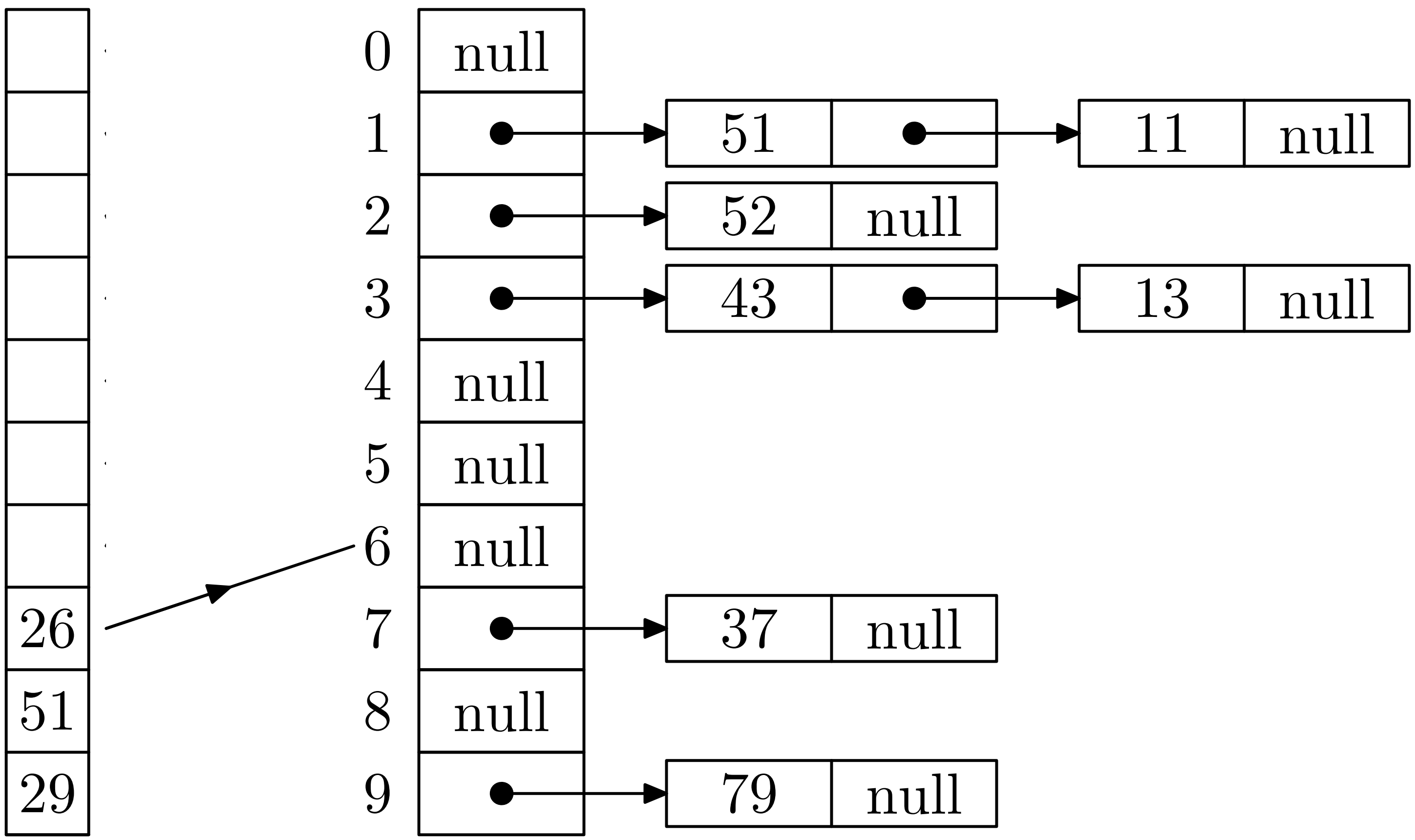

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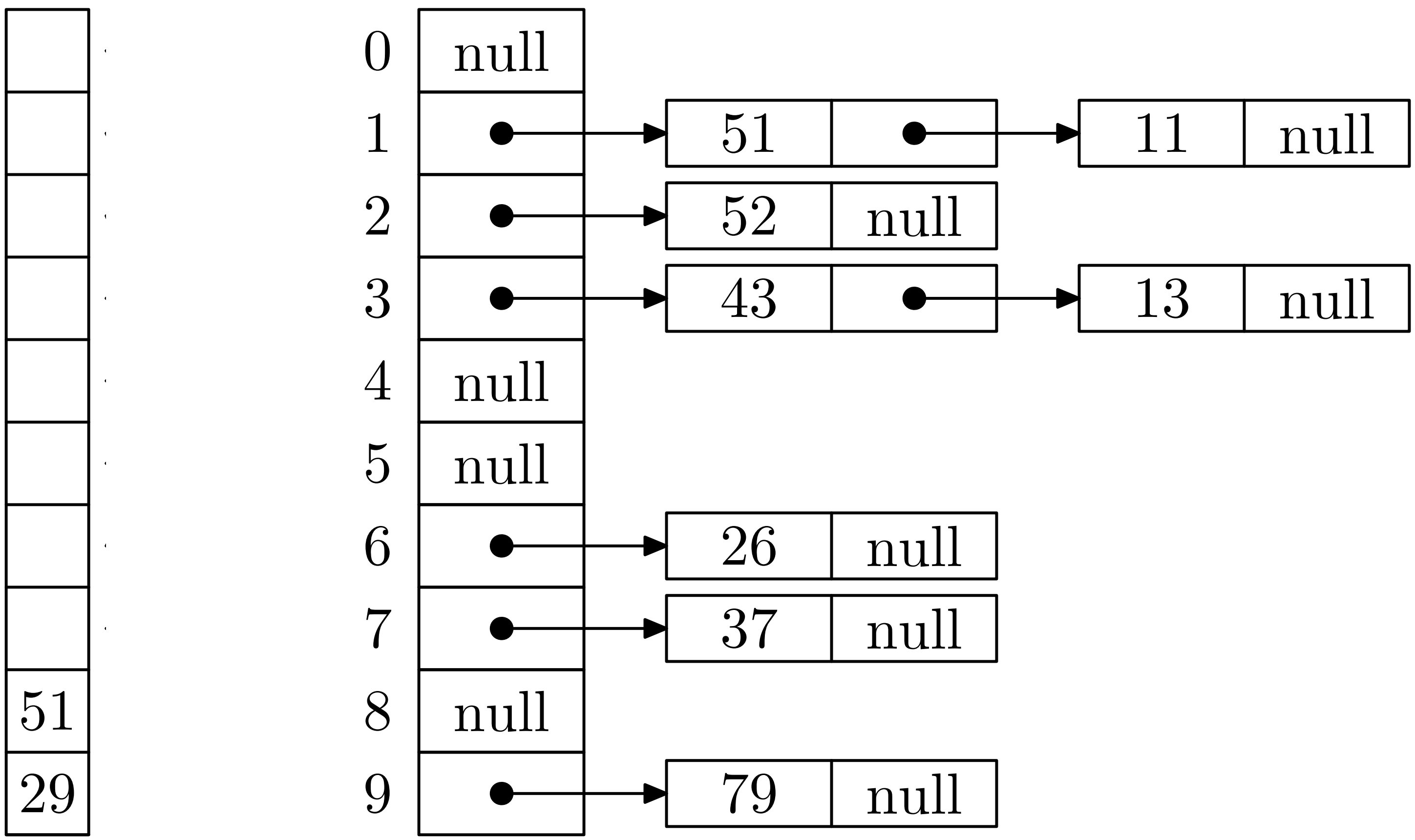

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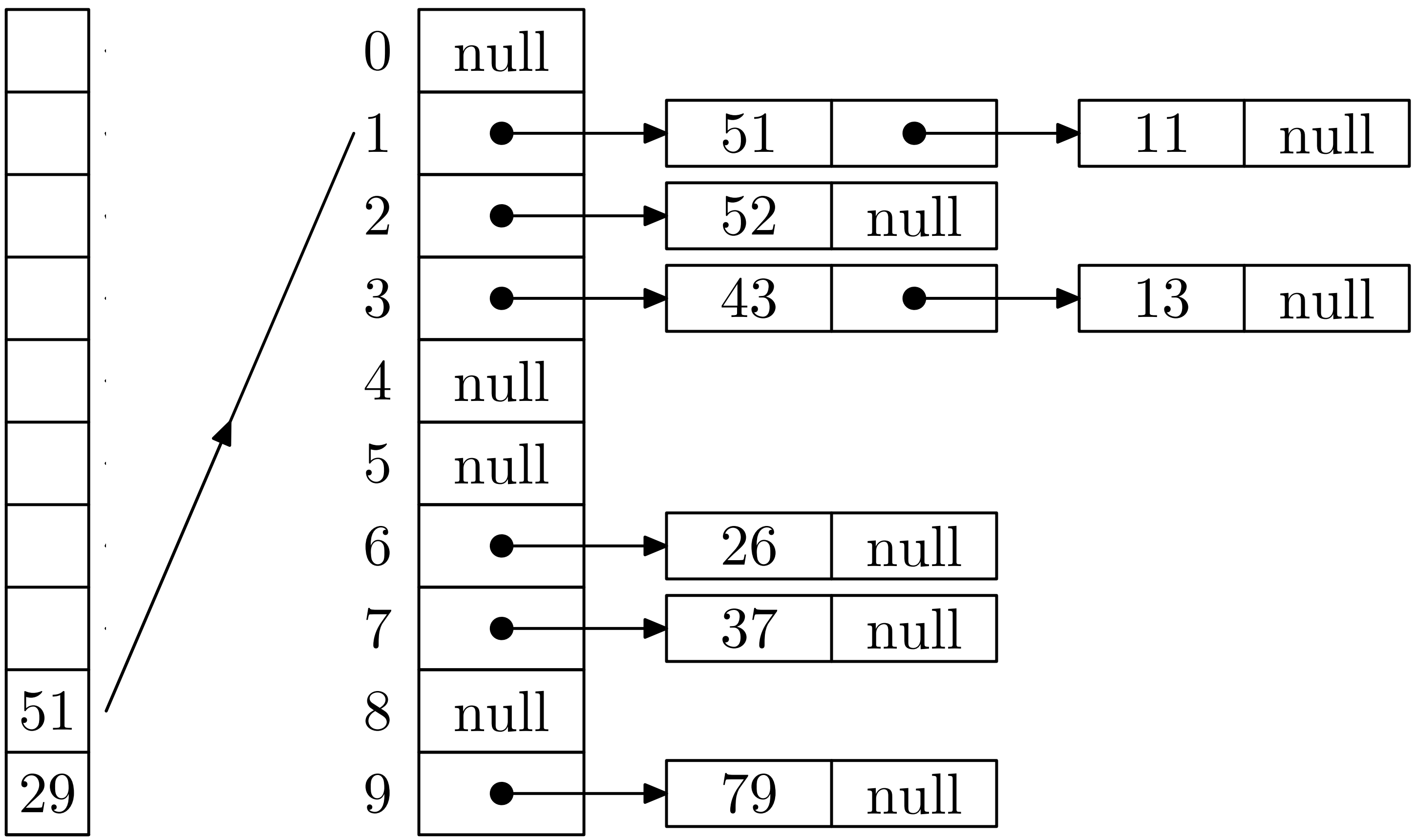

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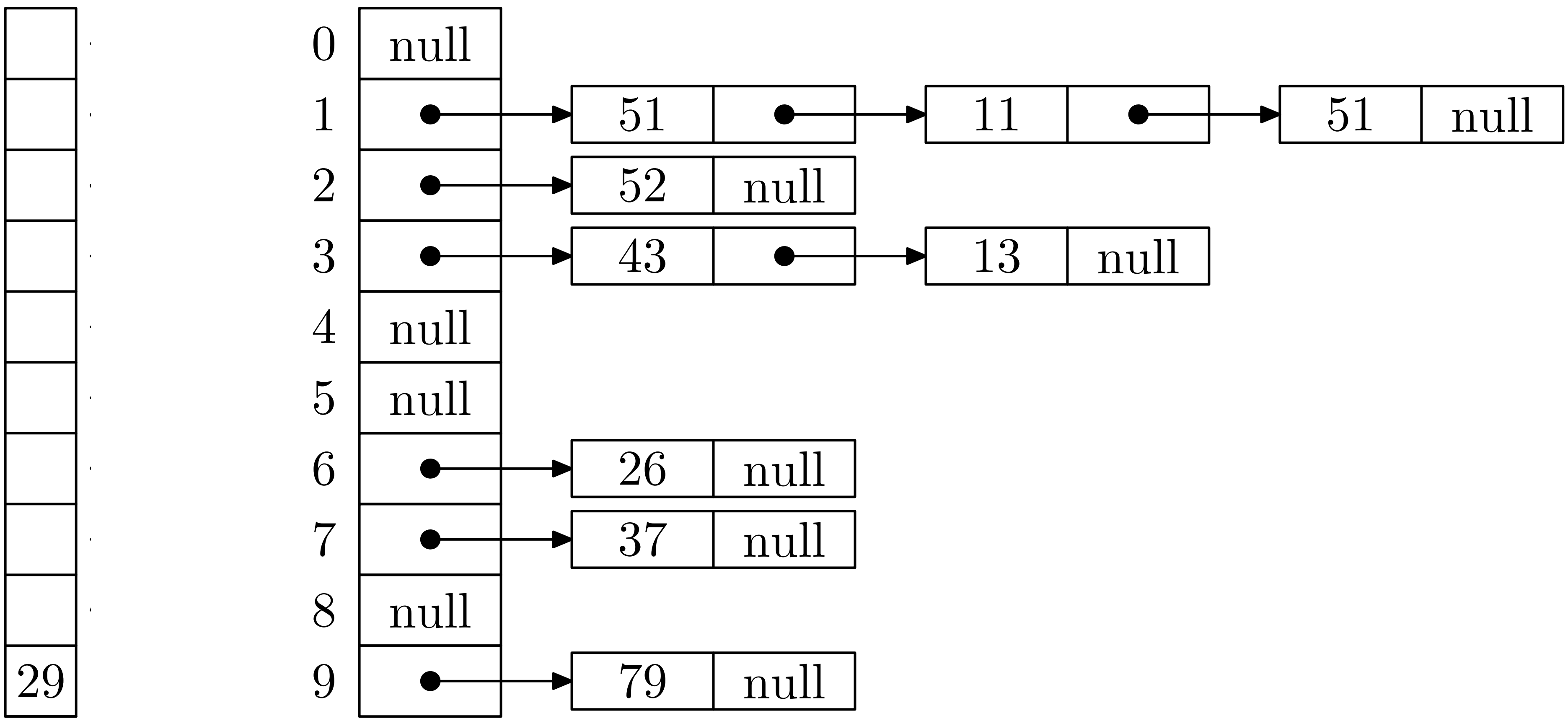

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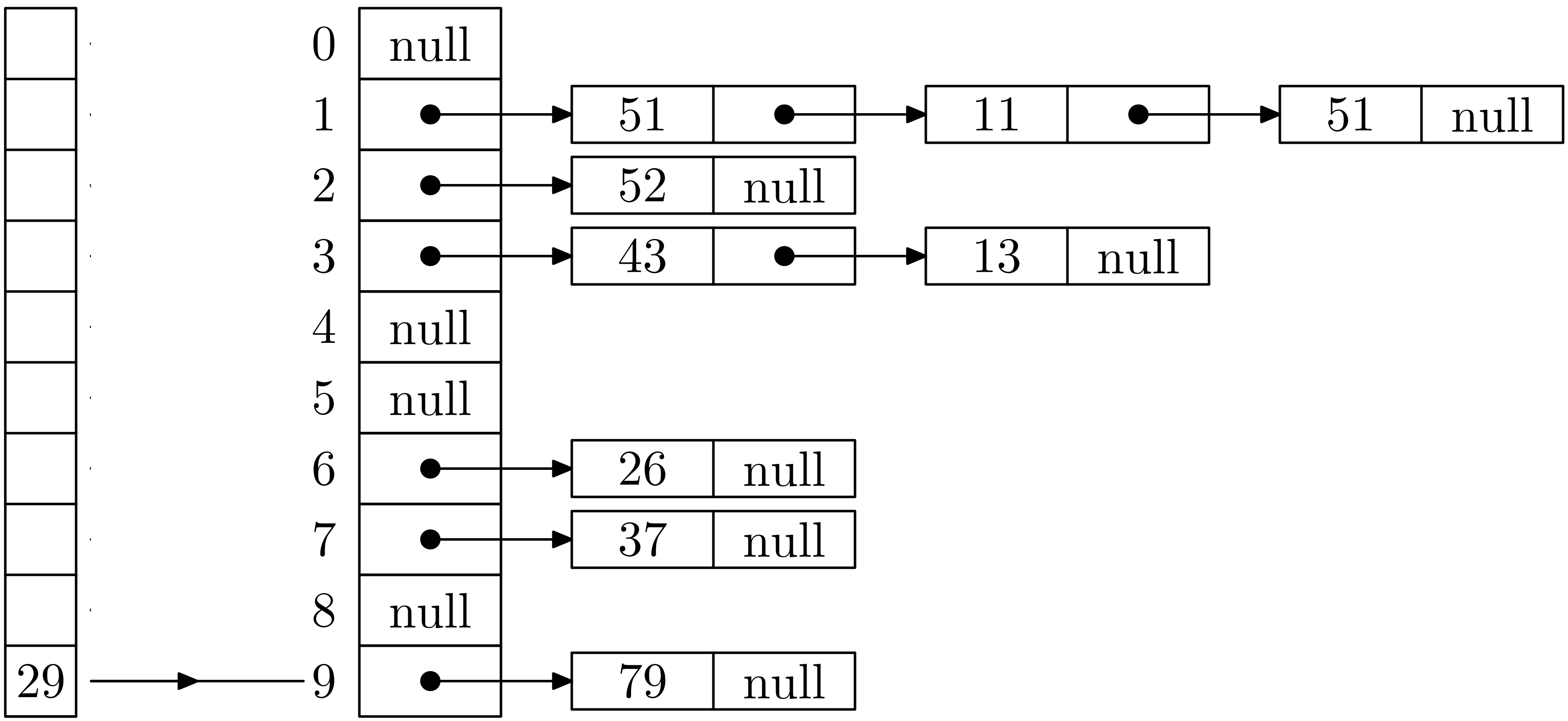

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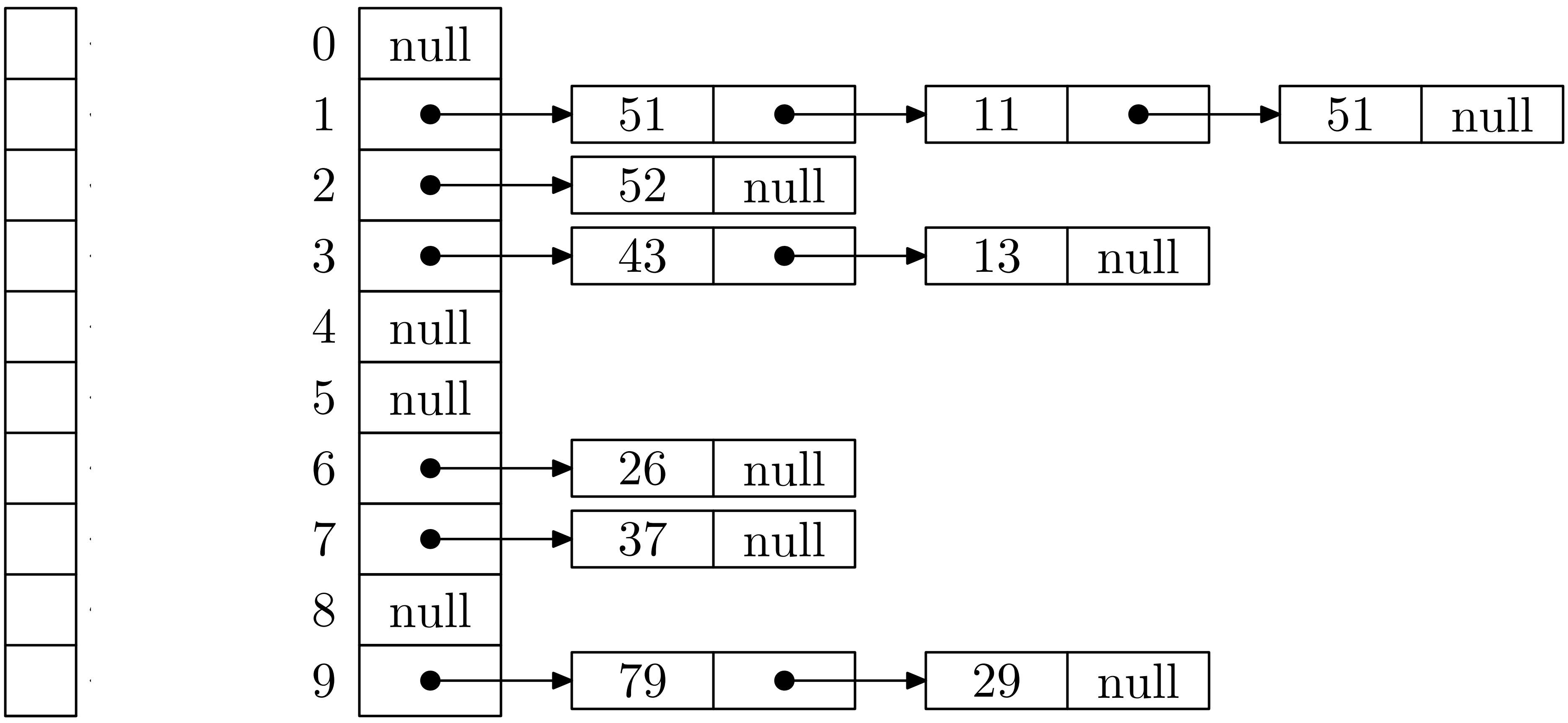

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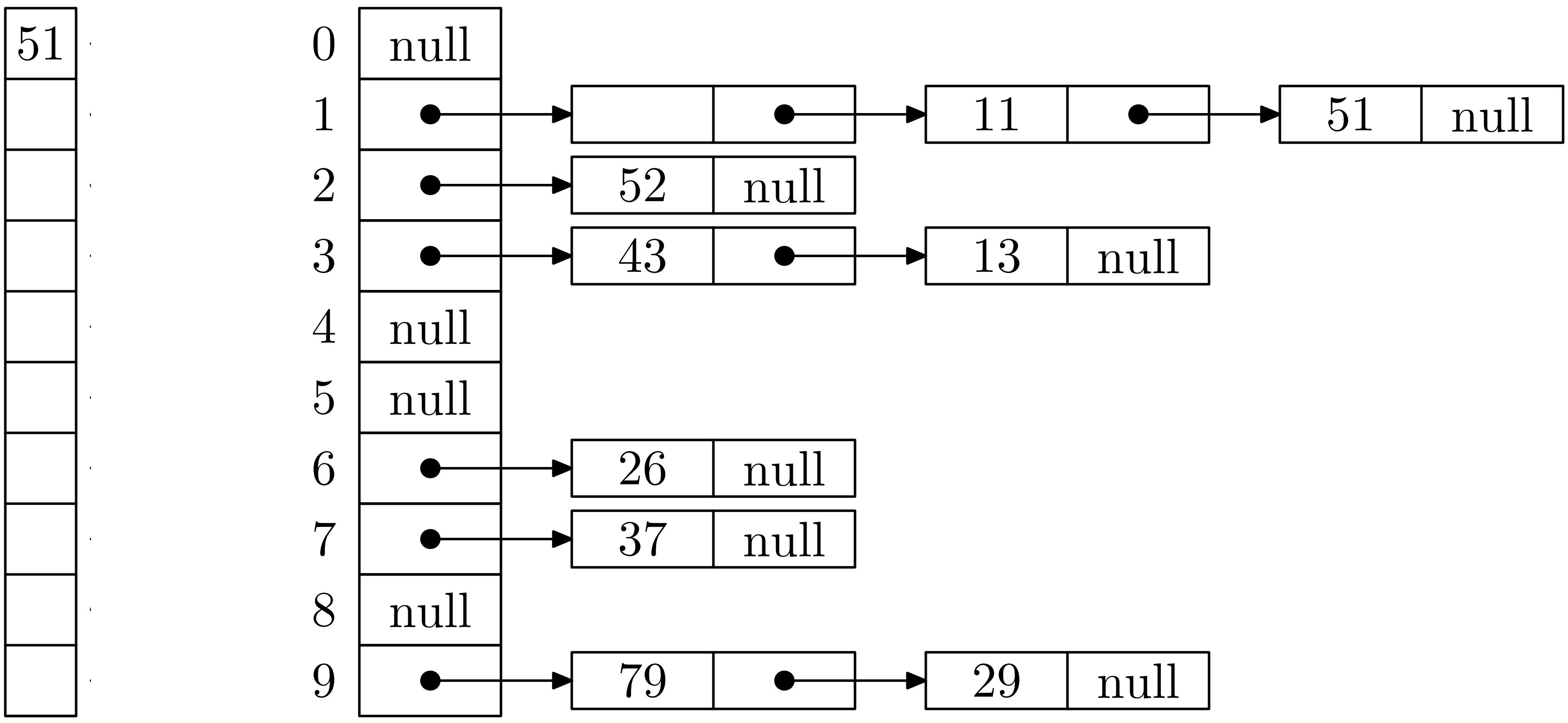

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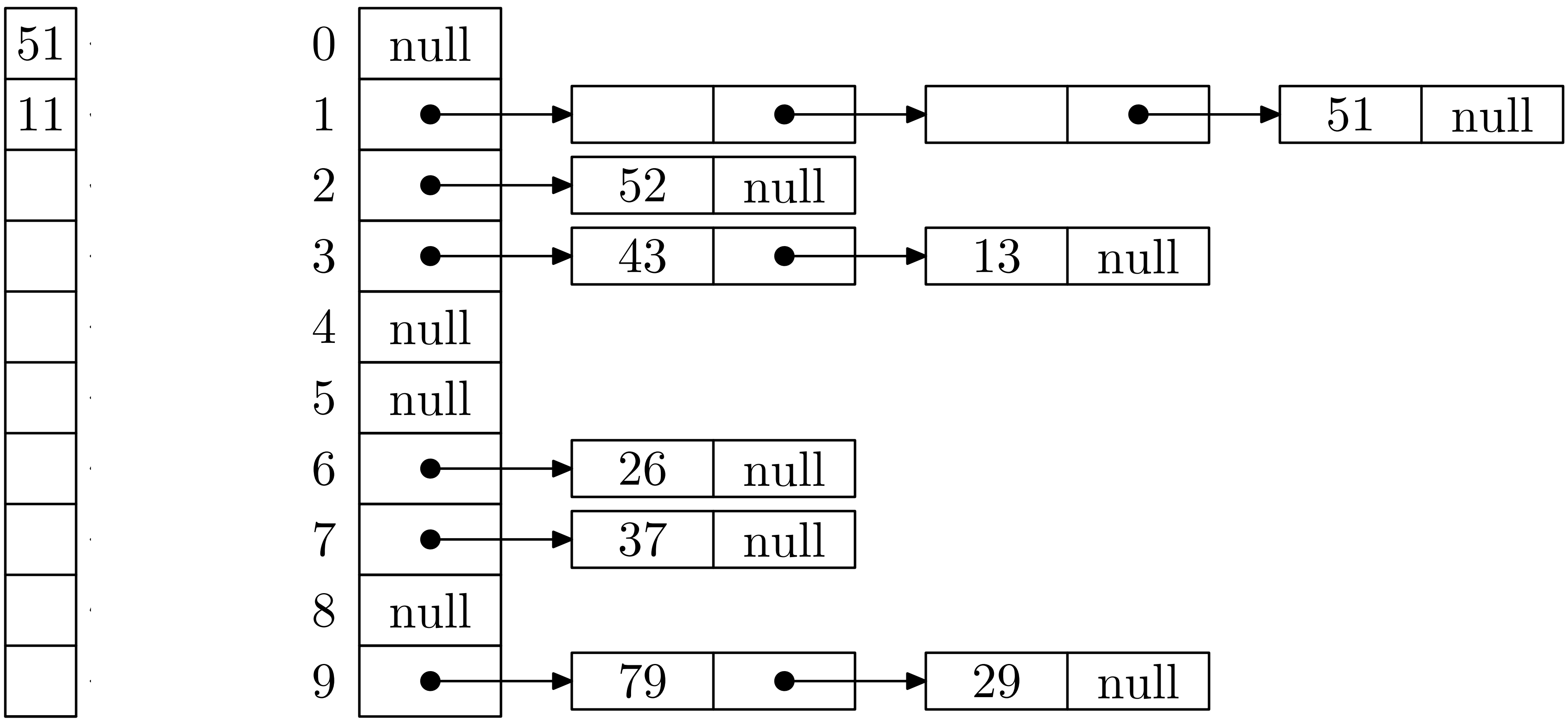

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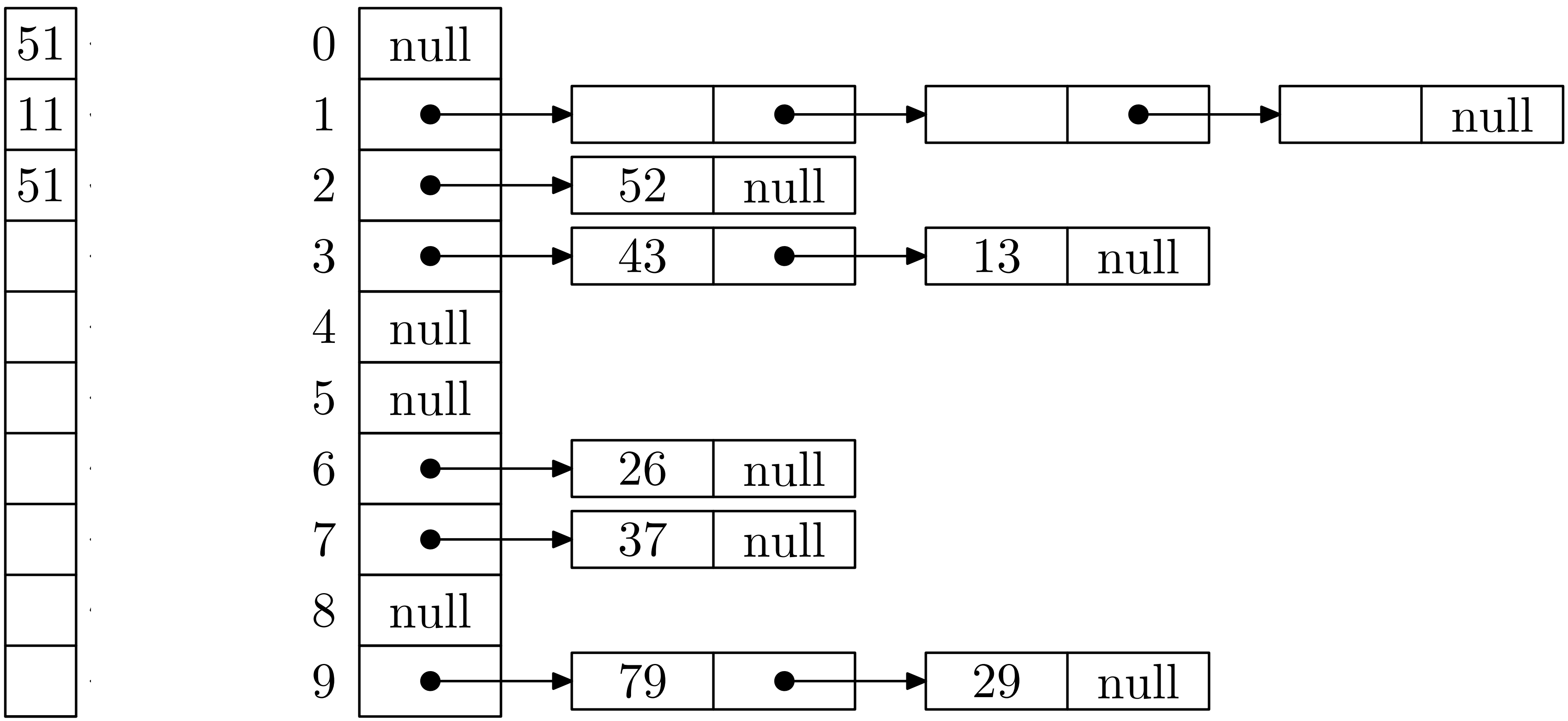

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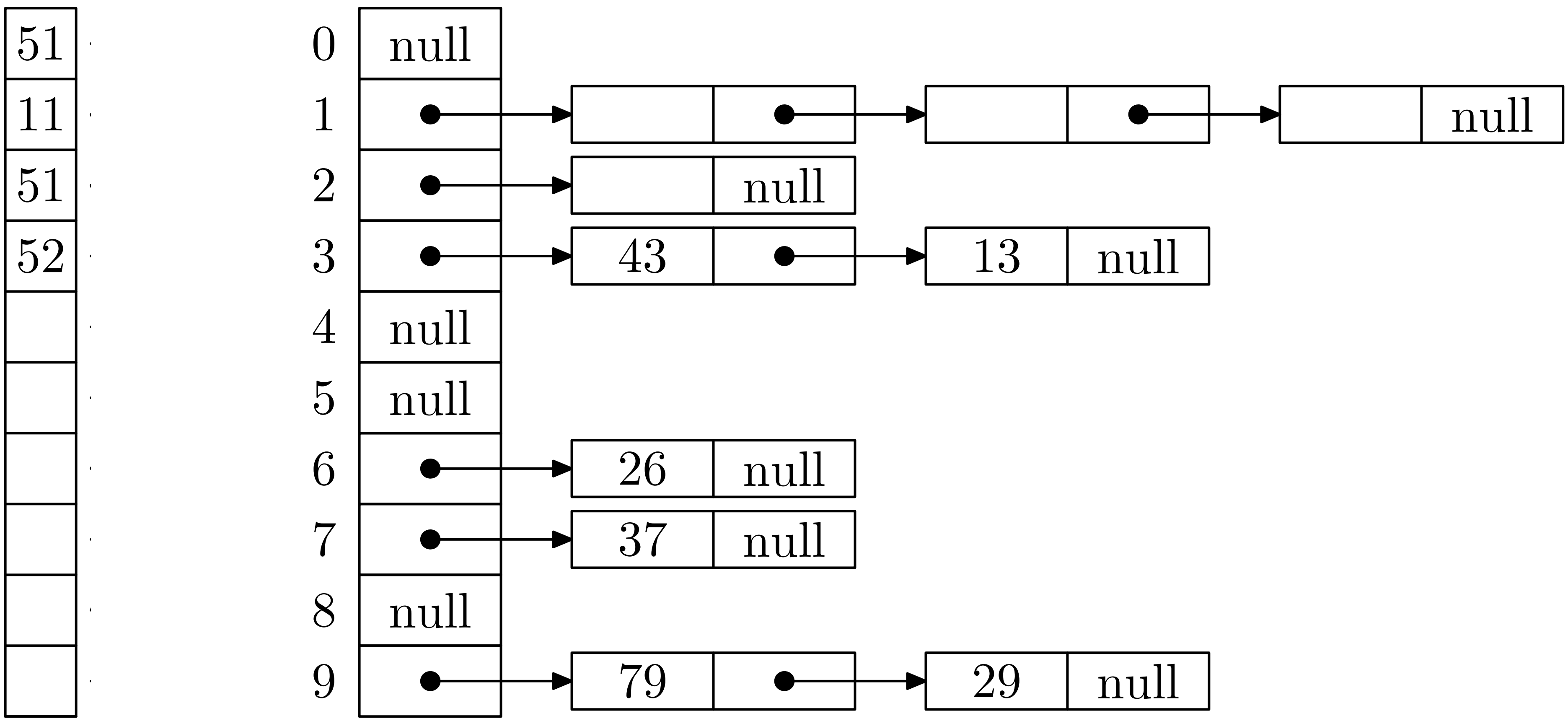

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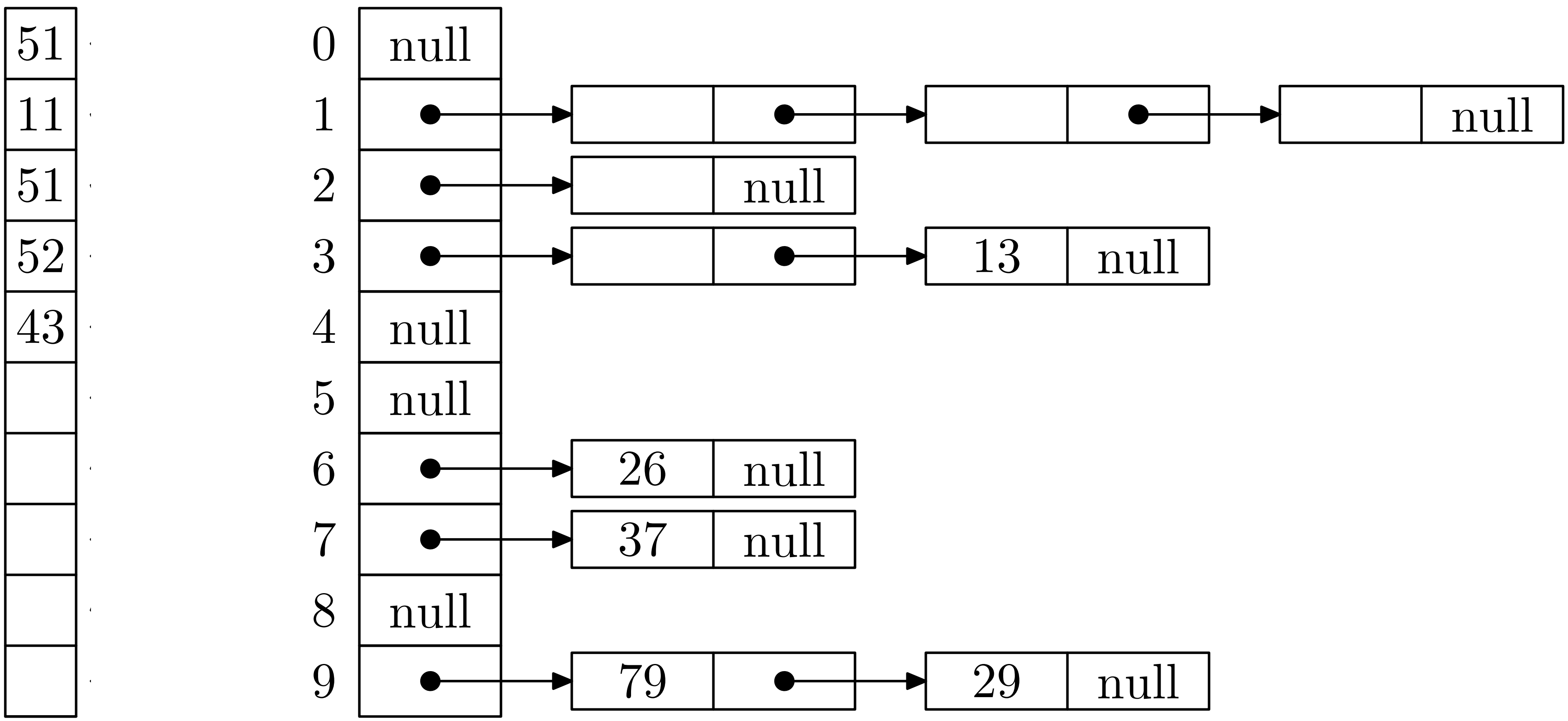

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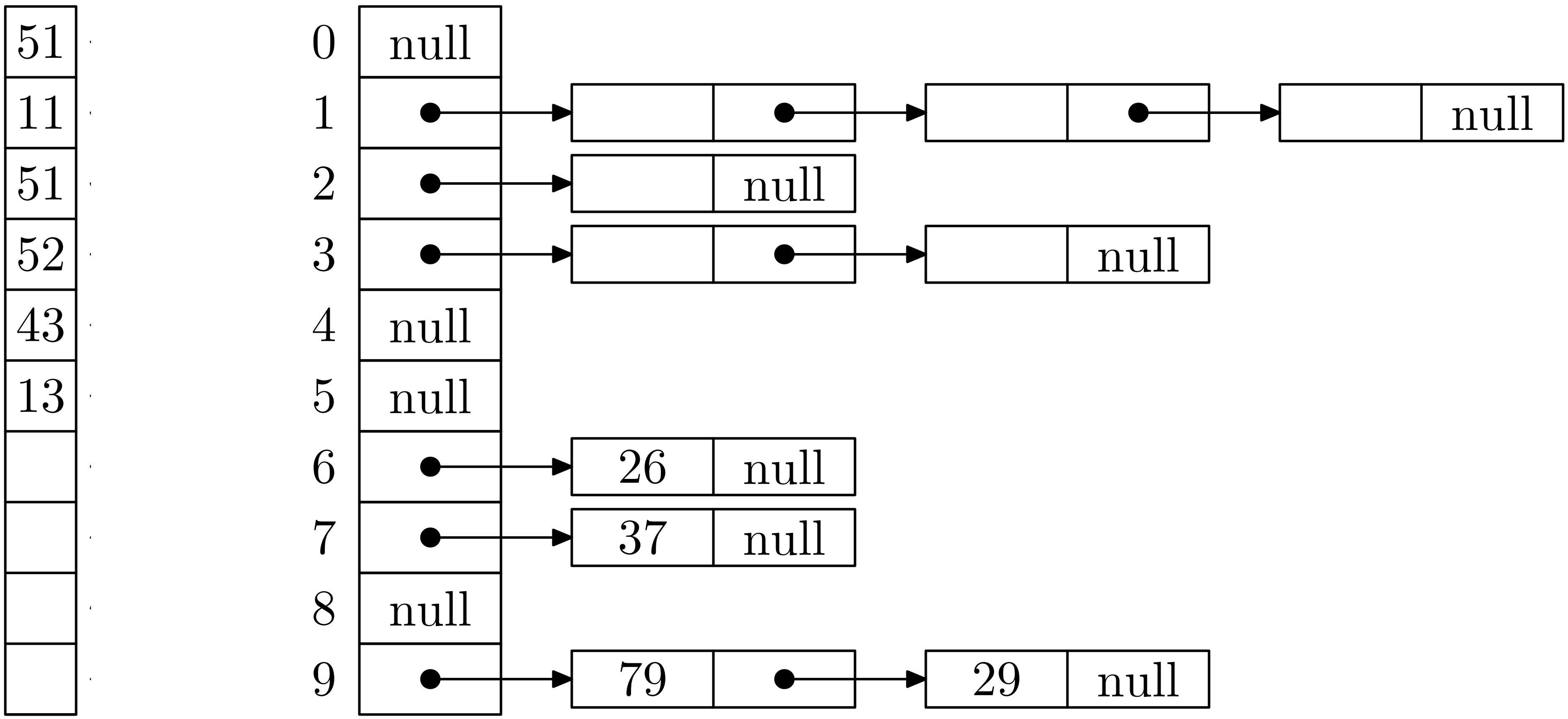

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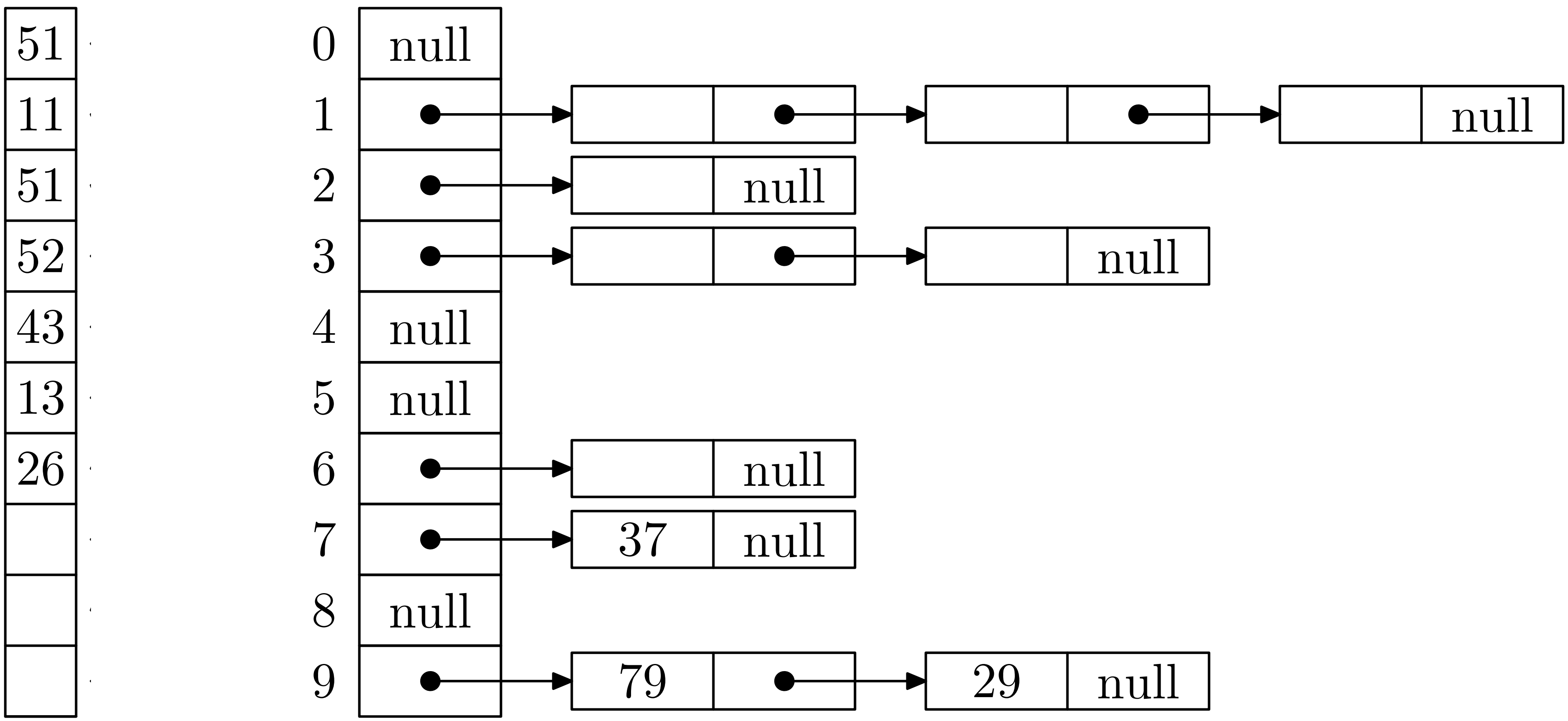

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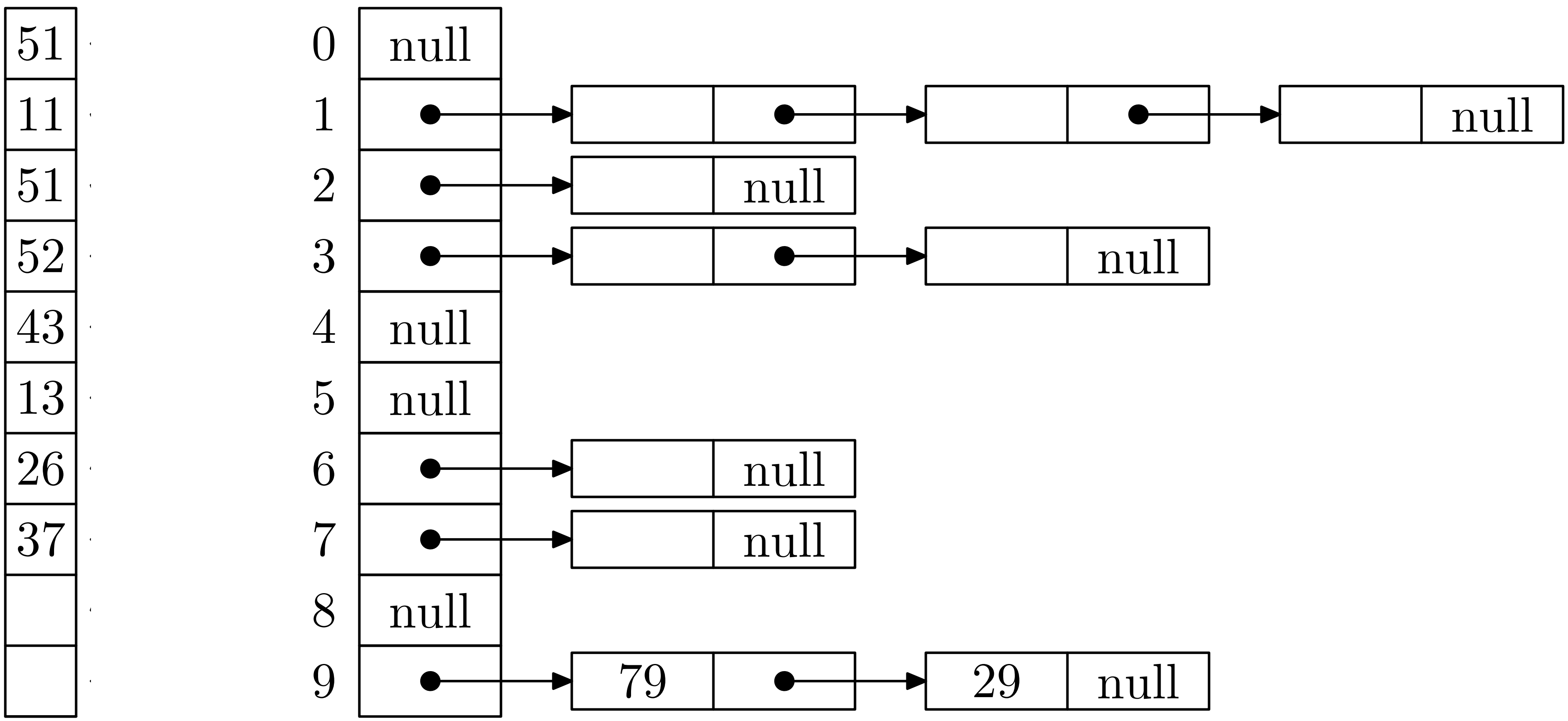

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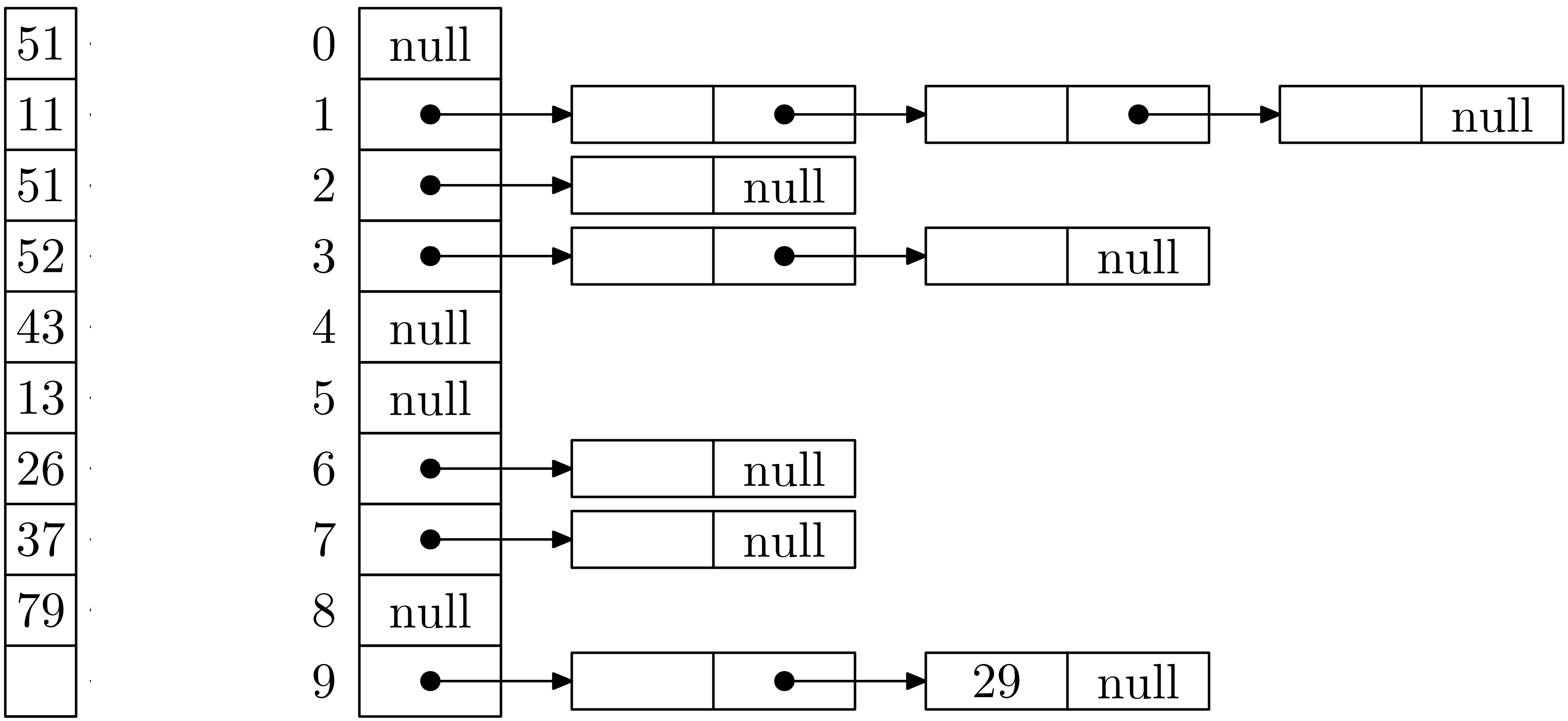

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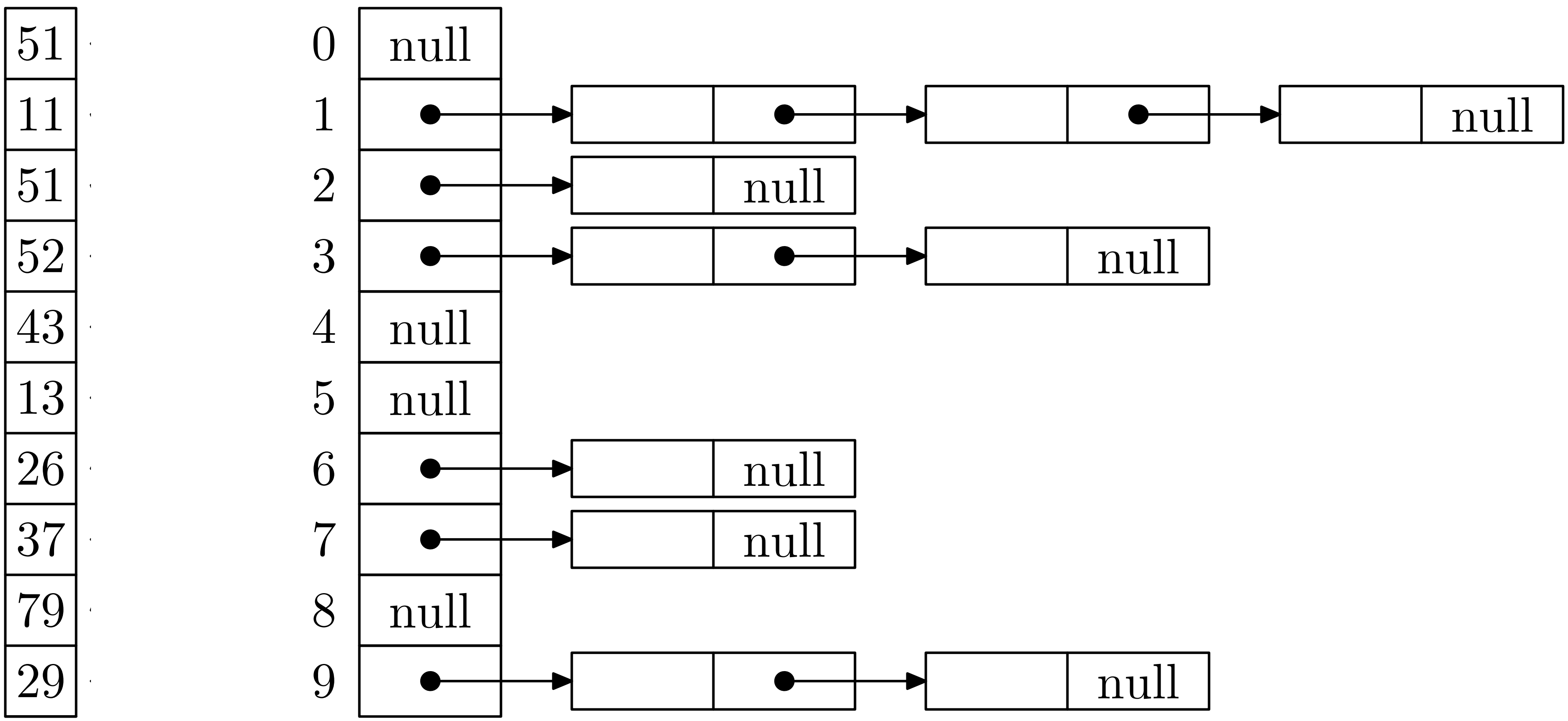

Radix Sort in Action

51		0	null
11		1	null
51		2	null
52		3	null
43		4	null
13		5	null
26		6	null
37		7	null
79		8	null
29		9	null

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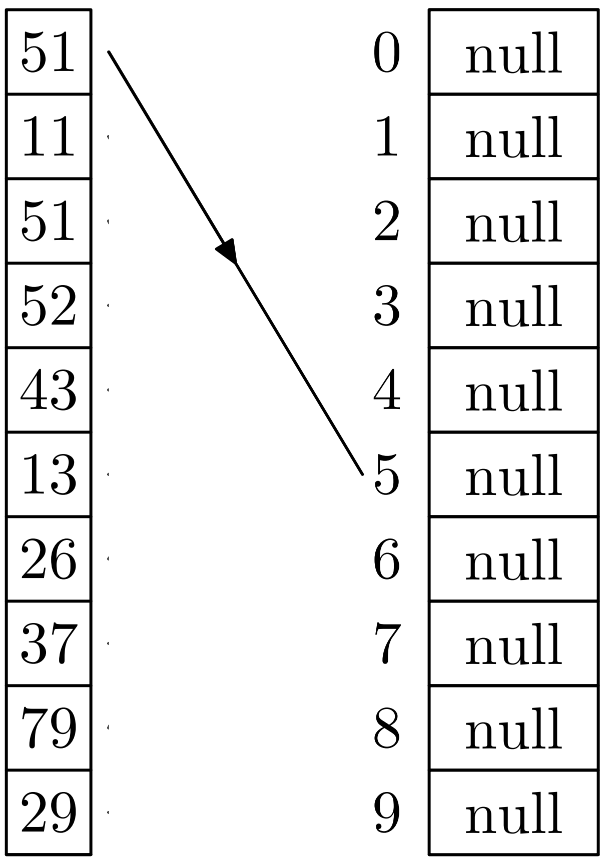

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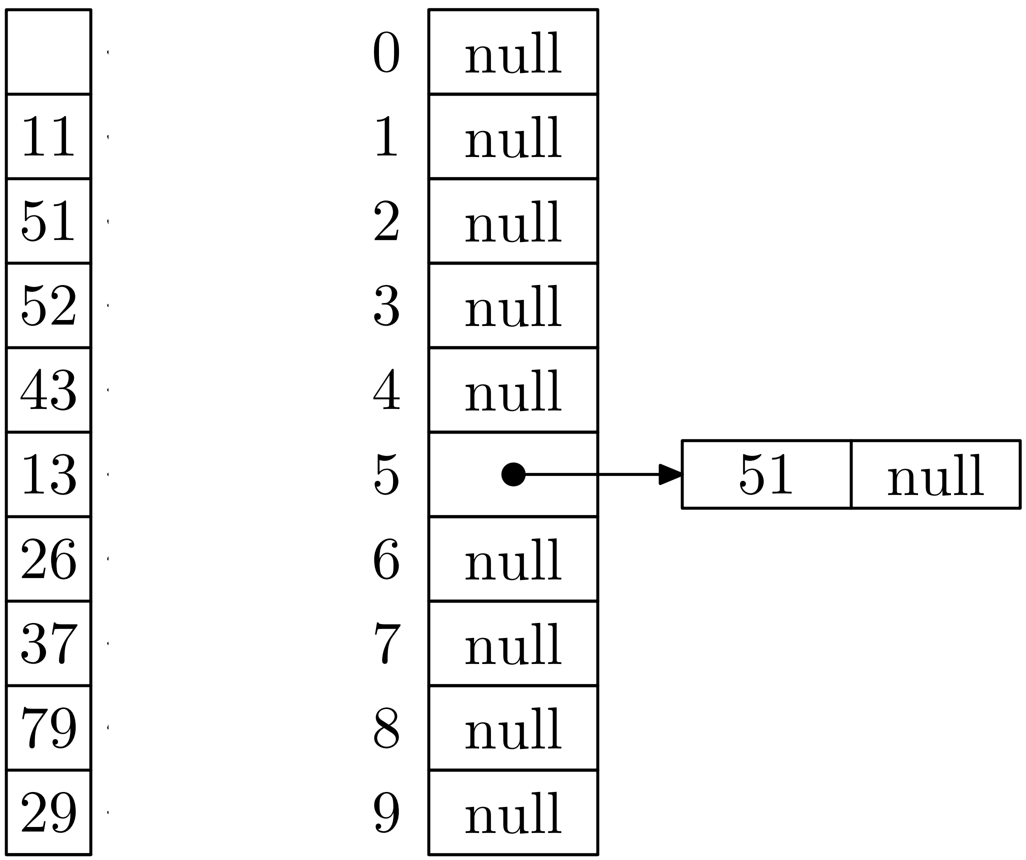

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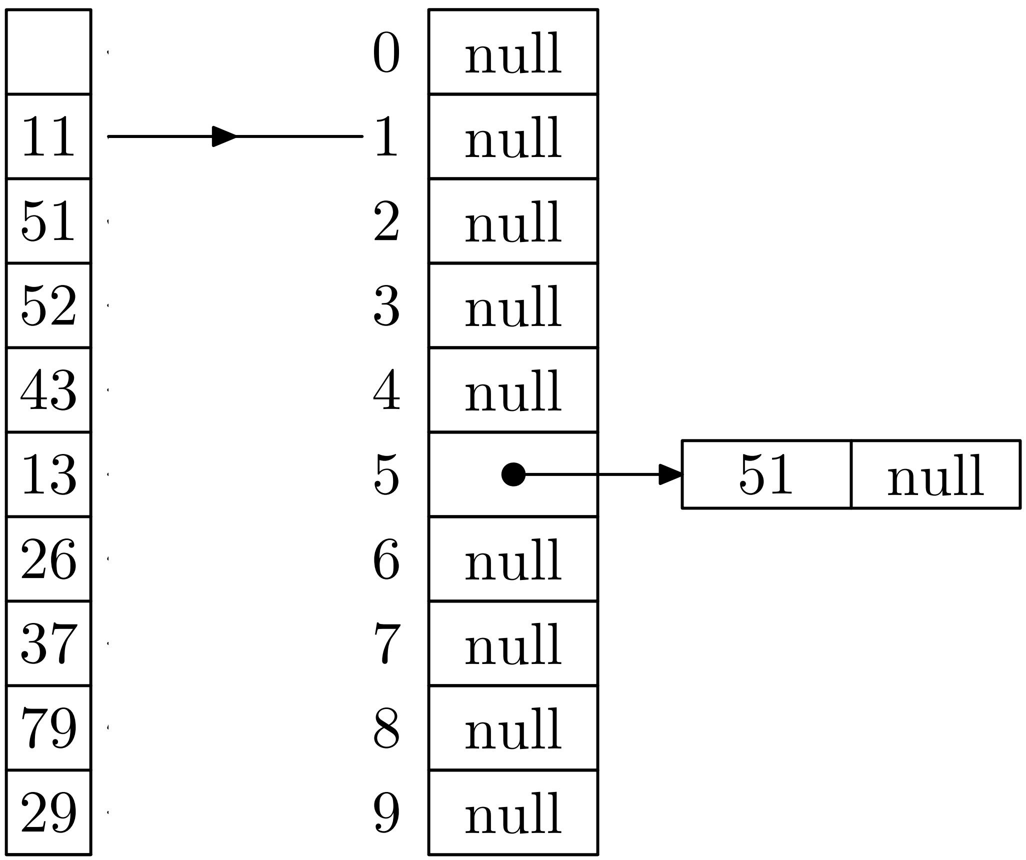

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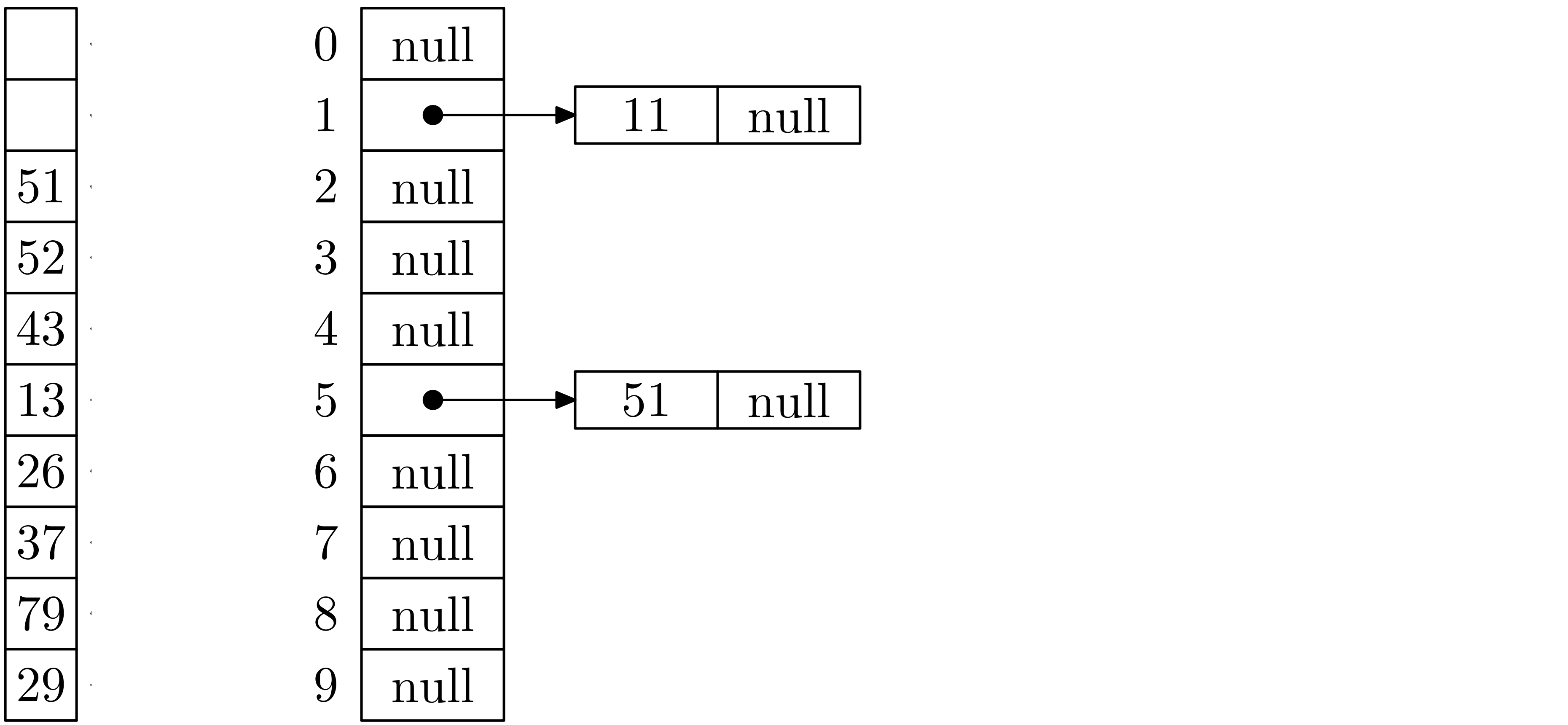

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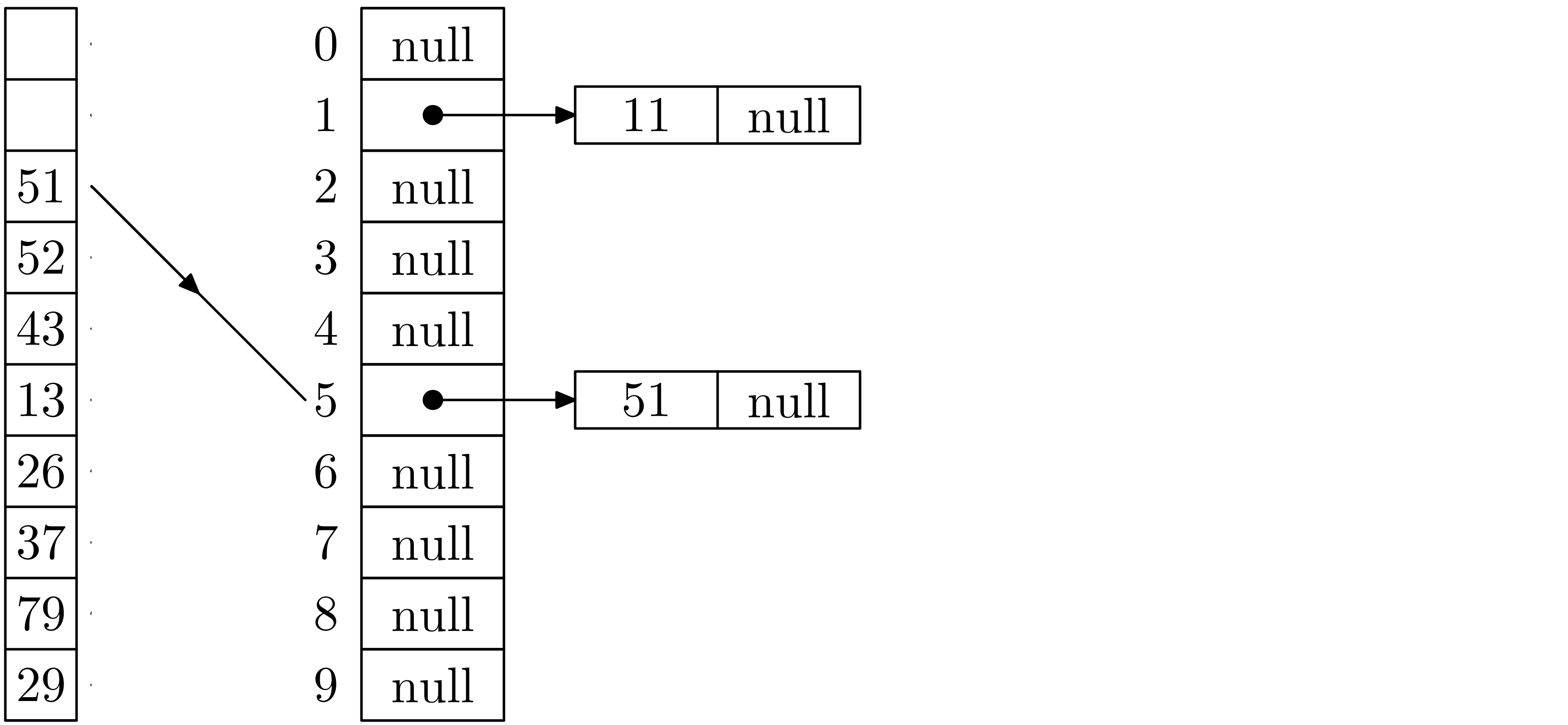

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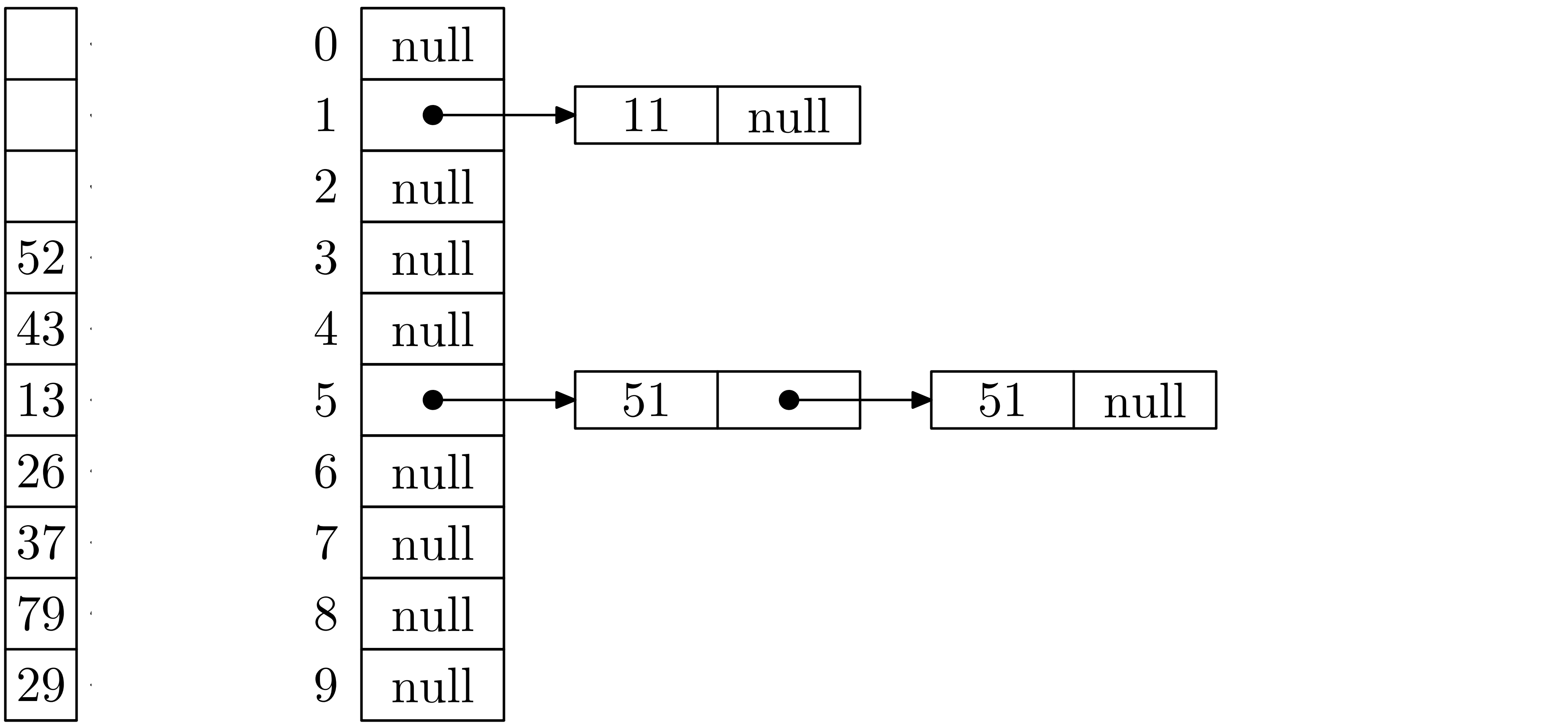

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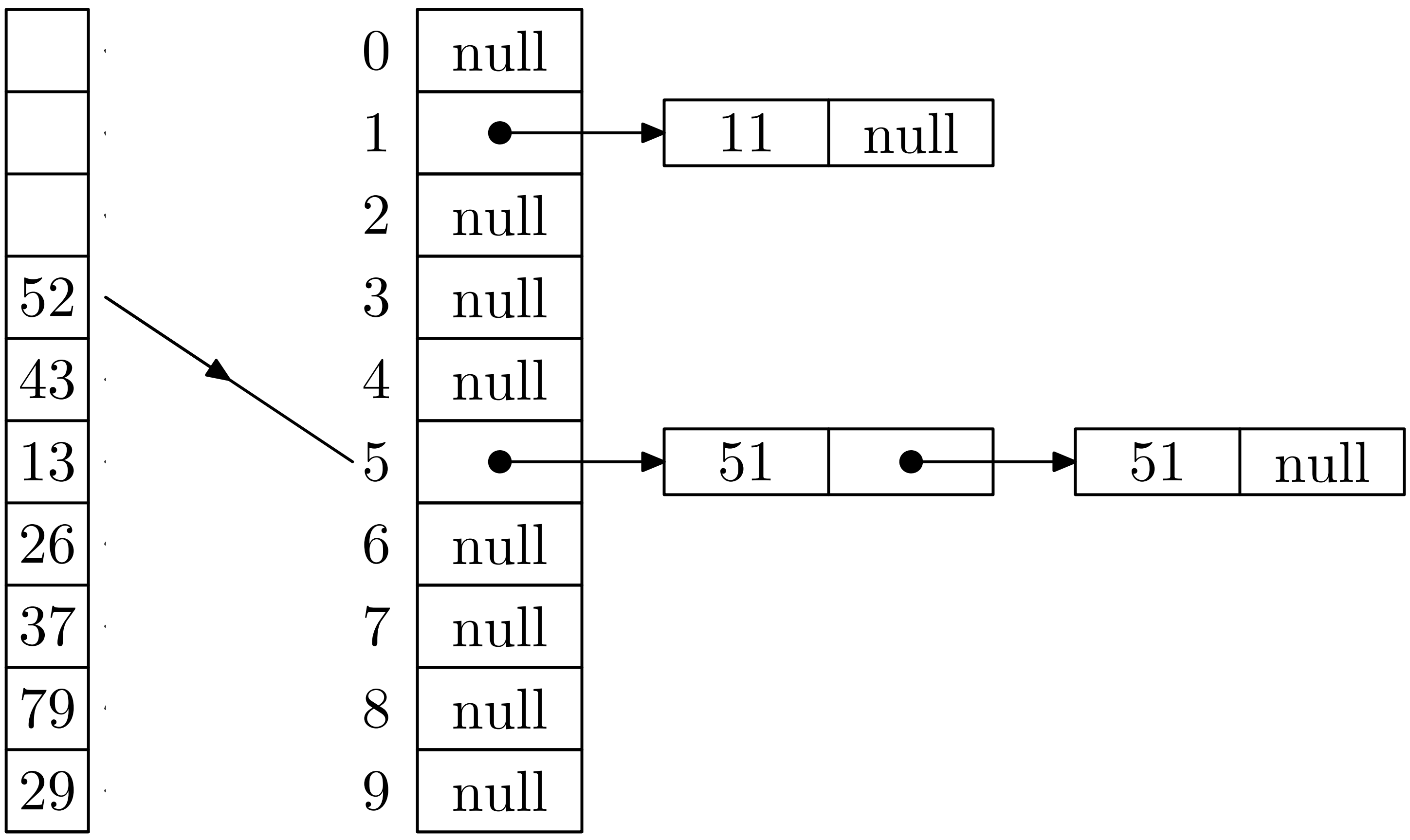

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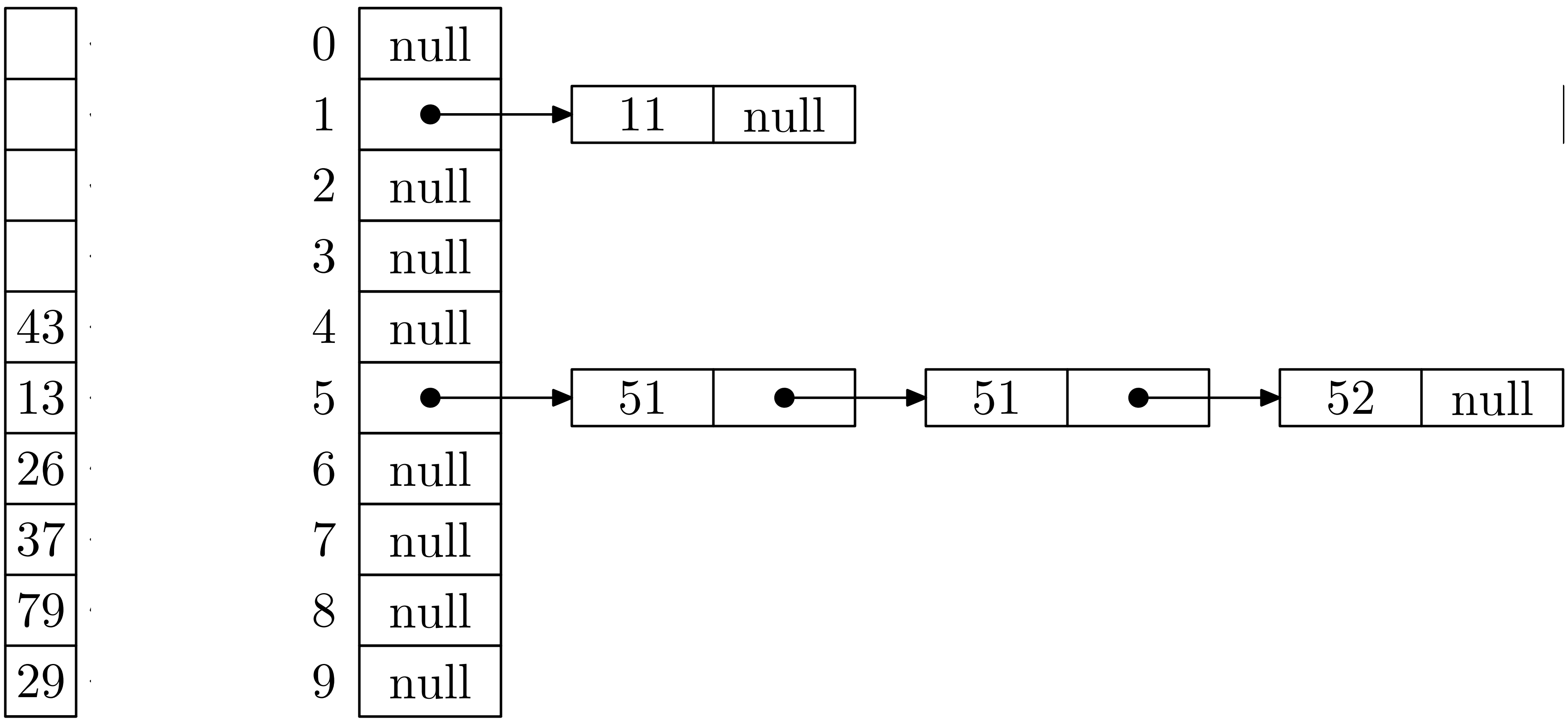

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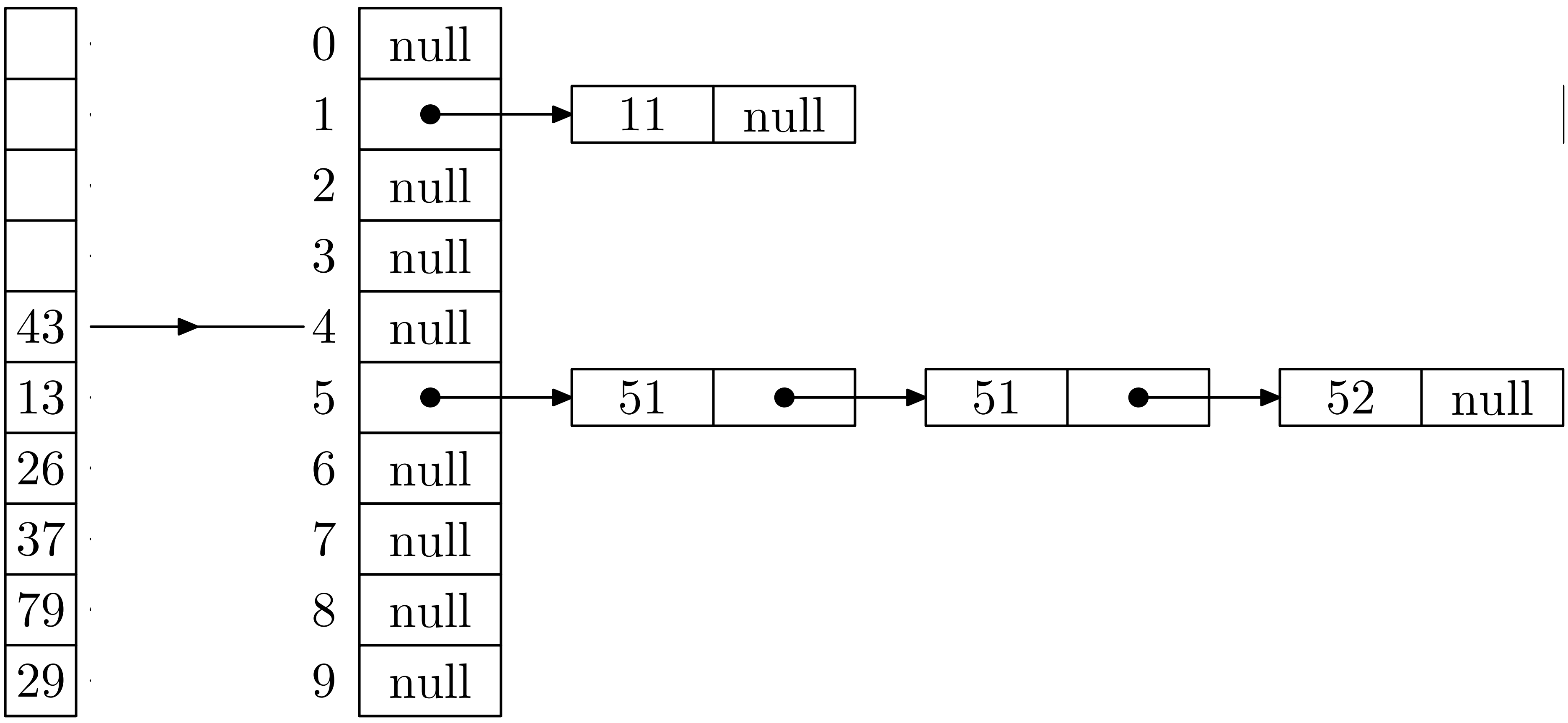

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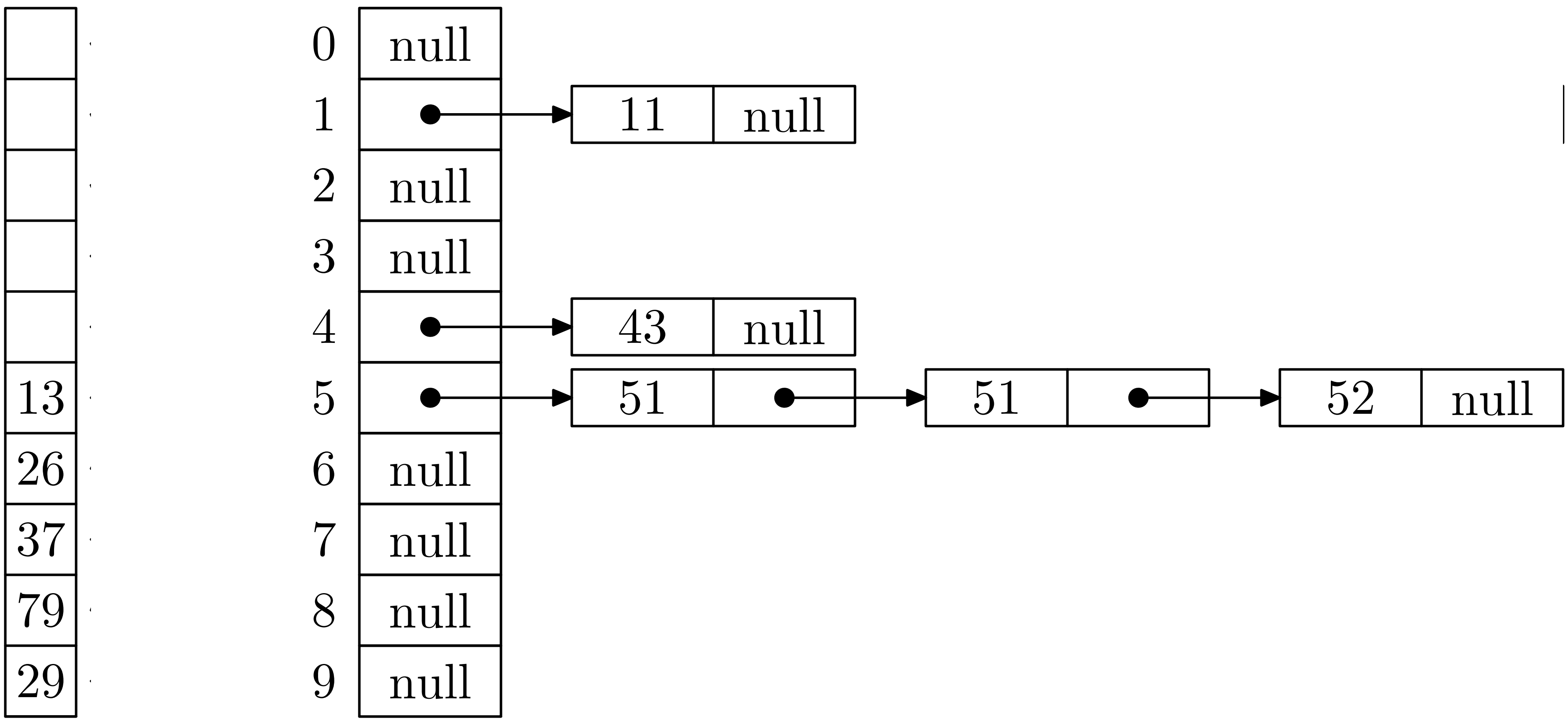

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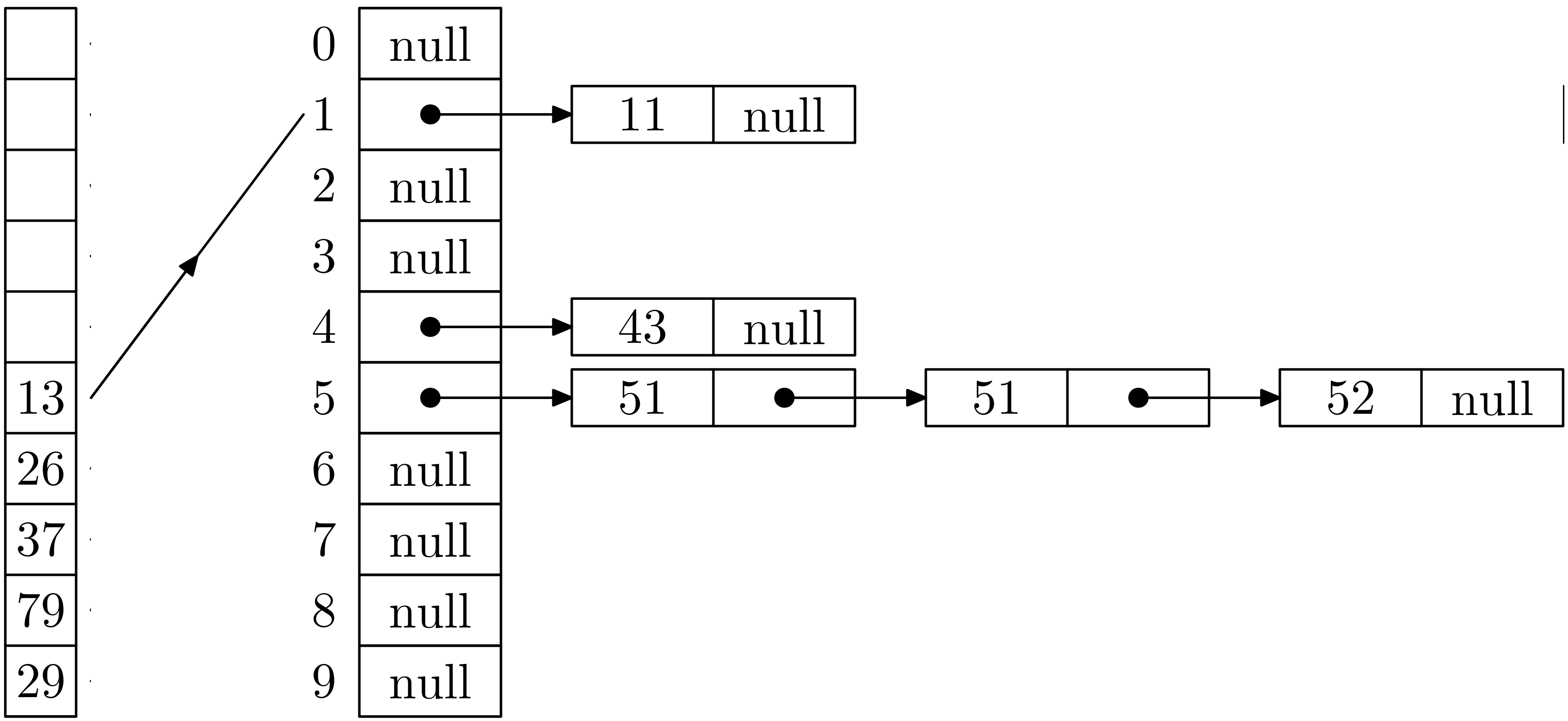

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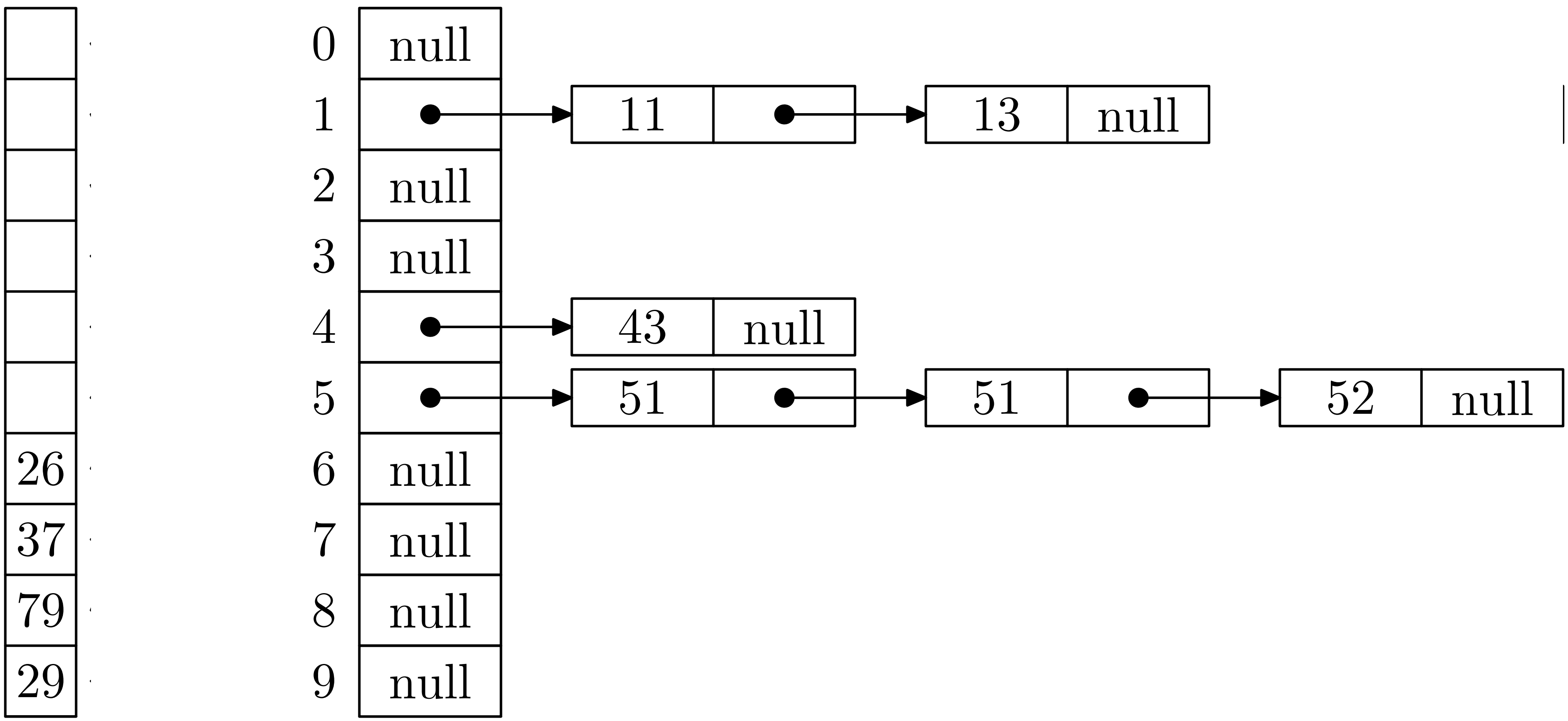

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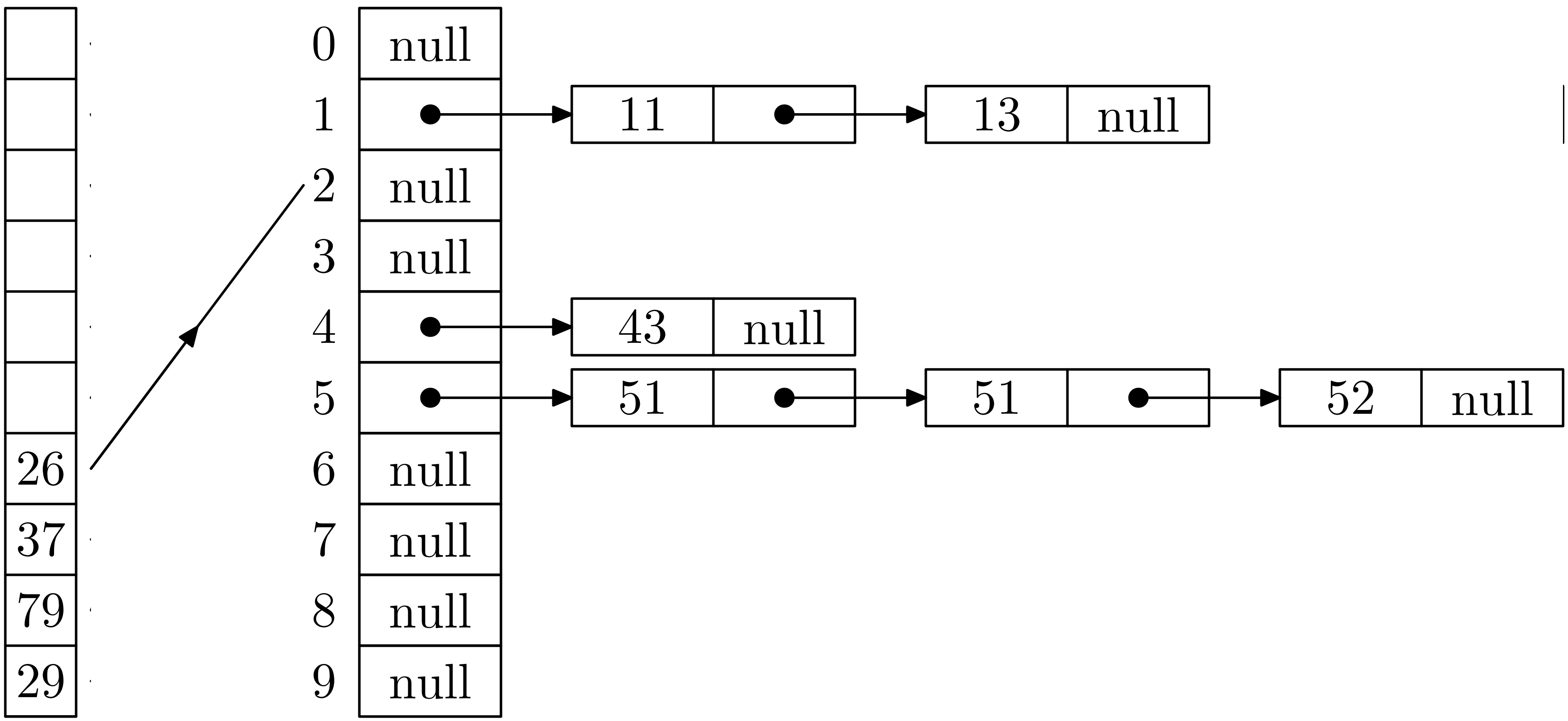

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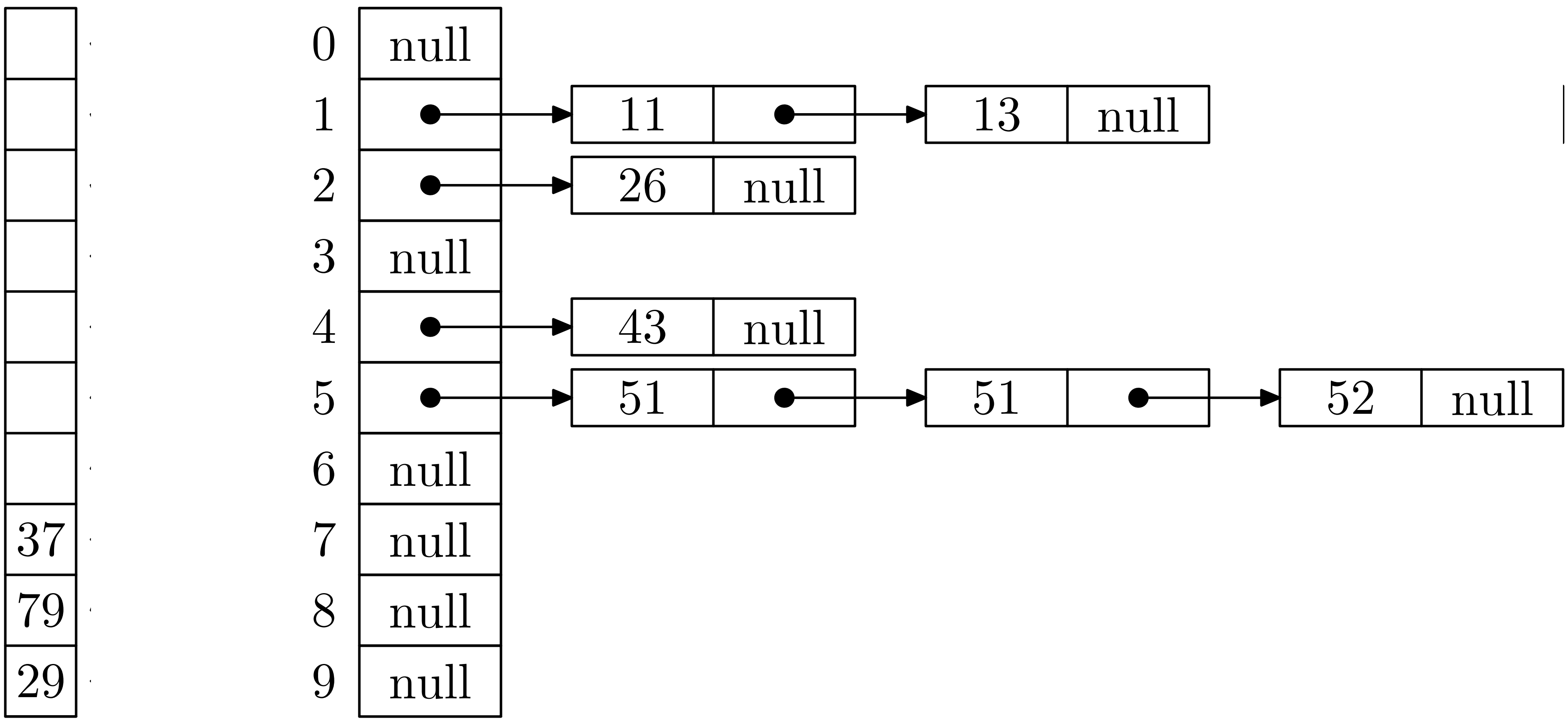

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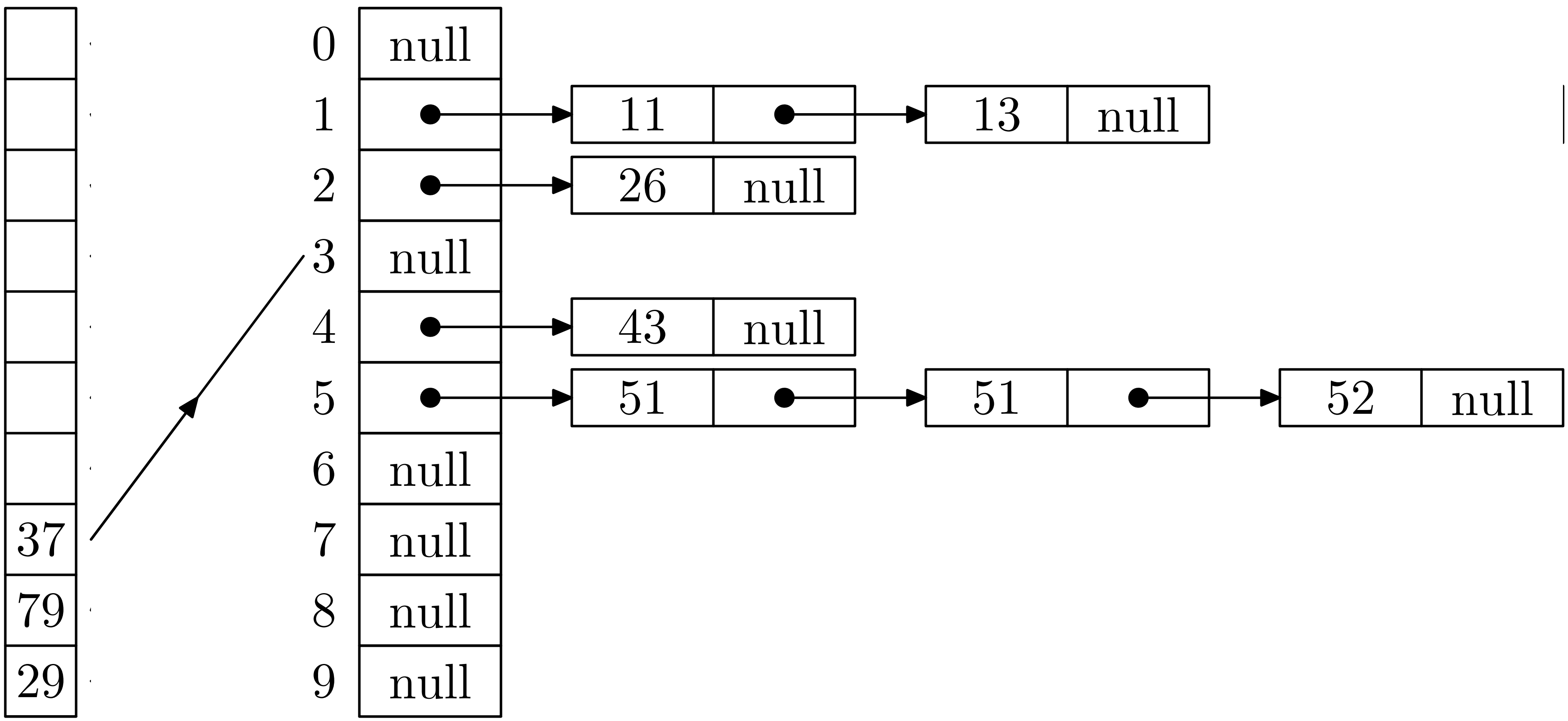

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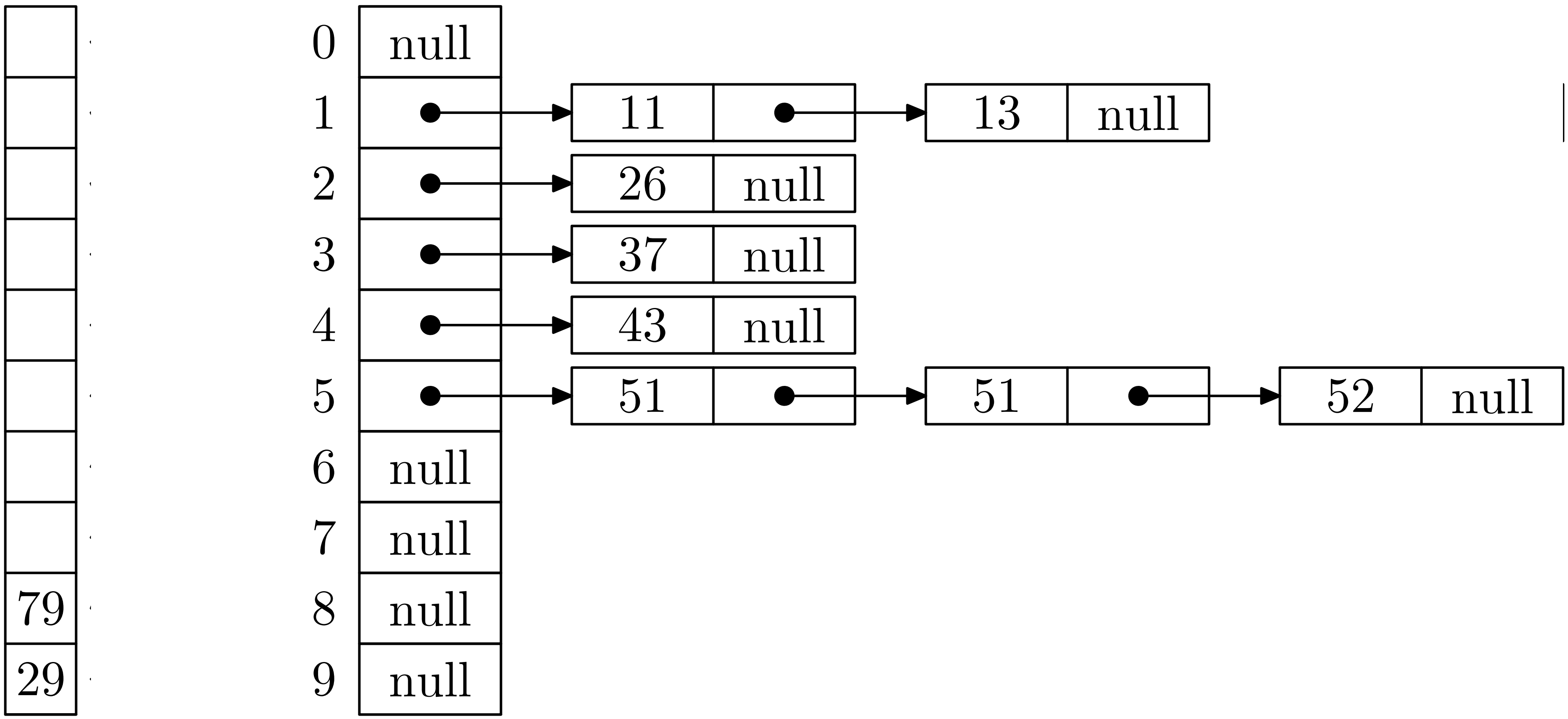

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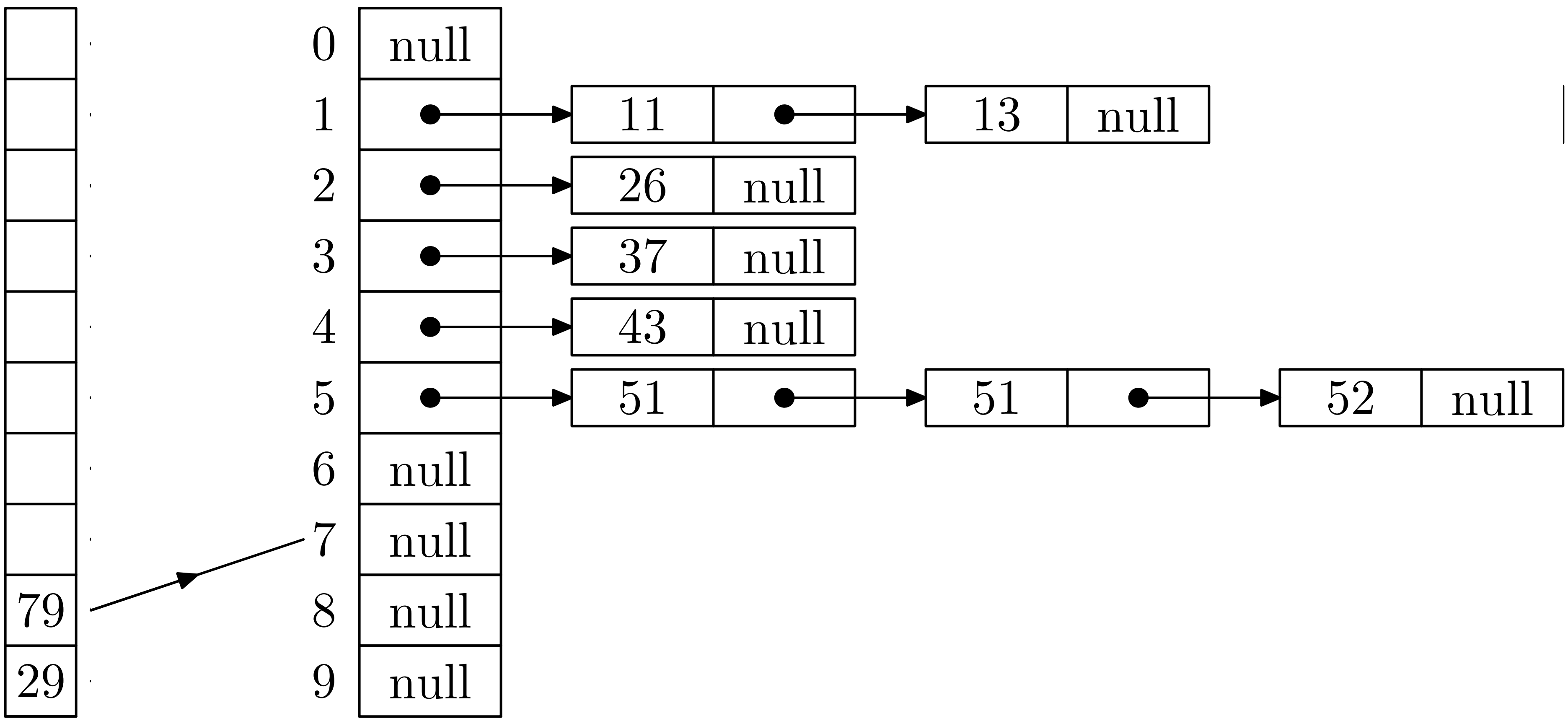

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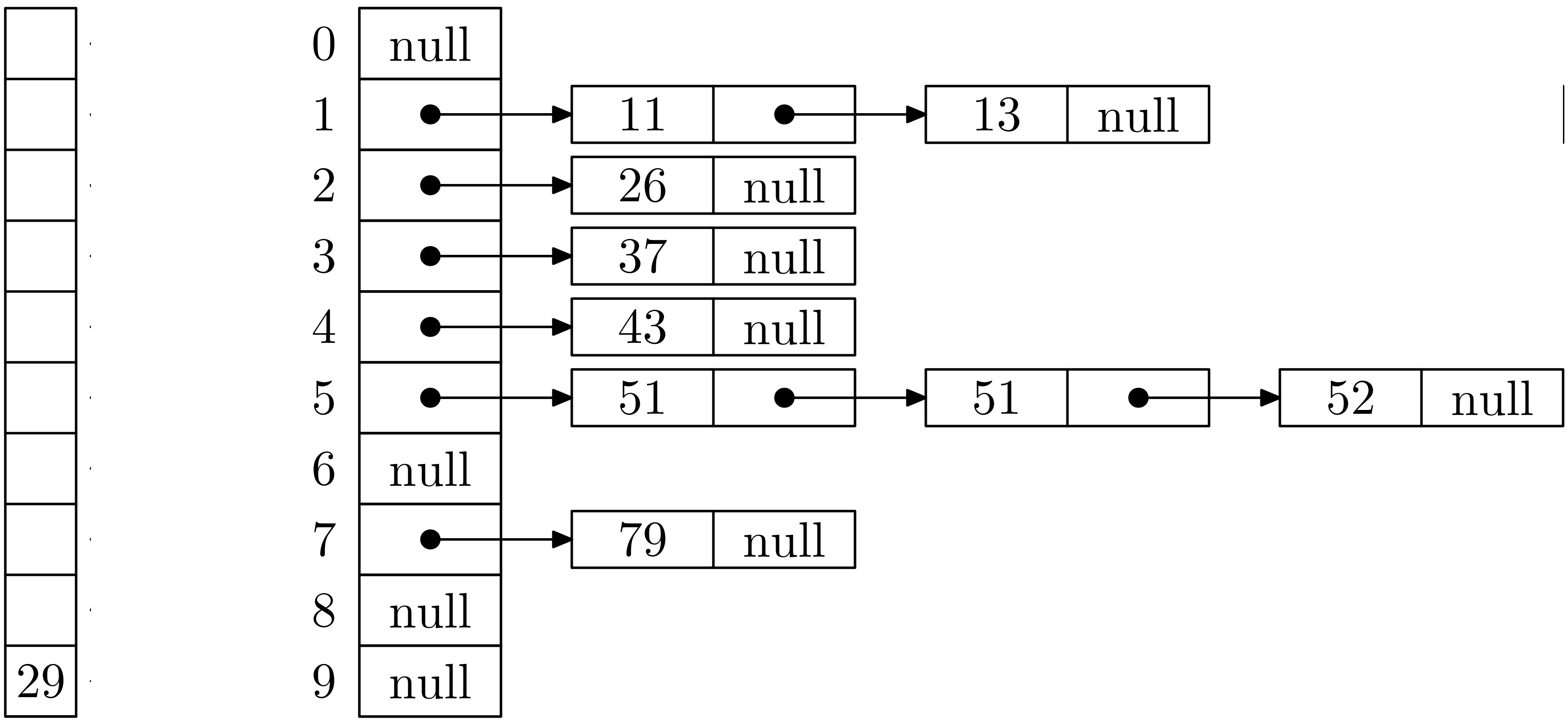

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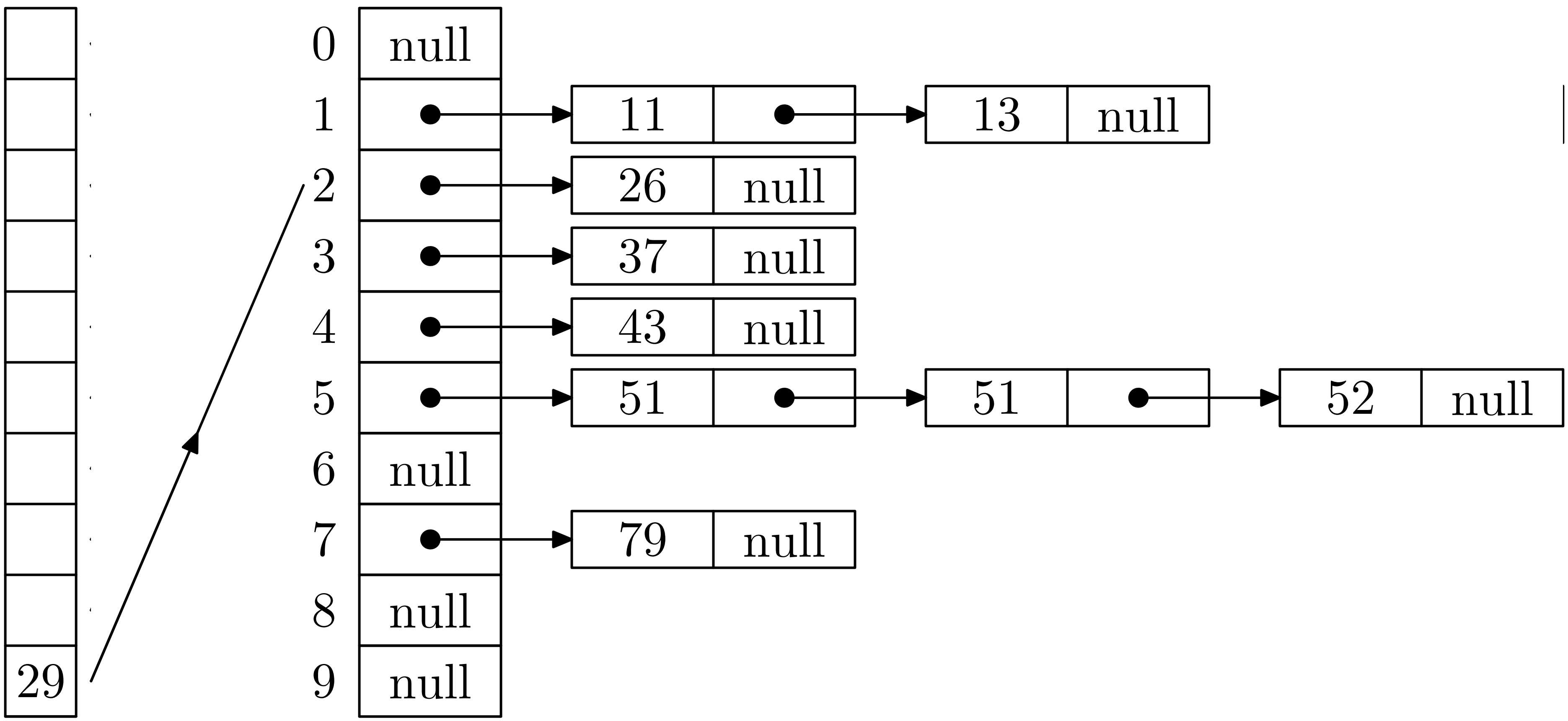

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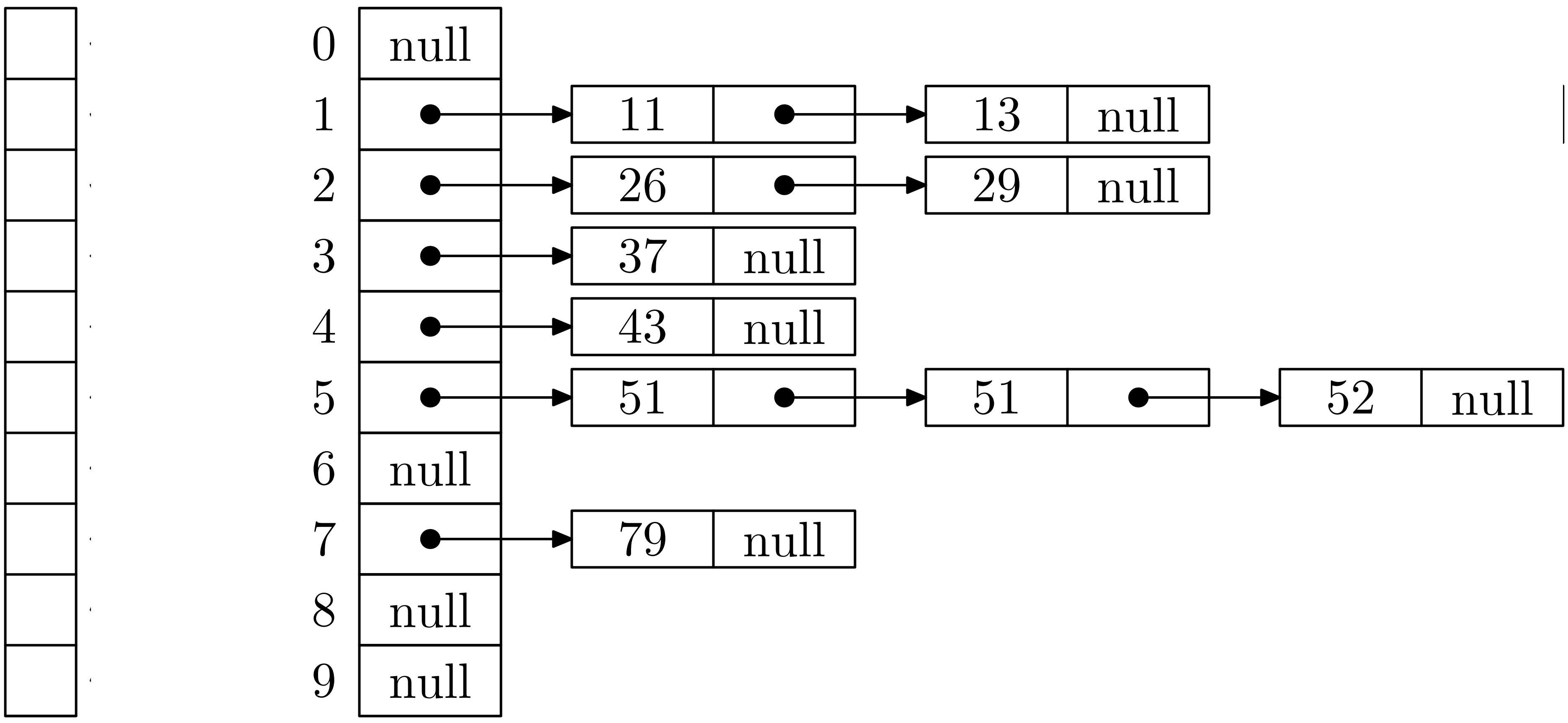

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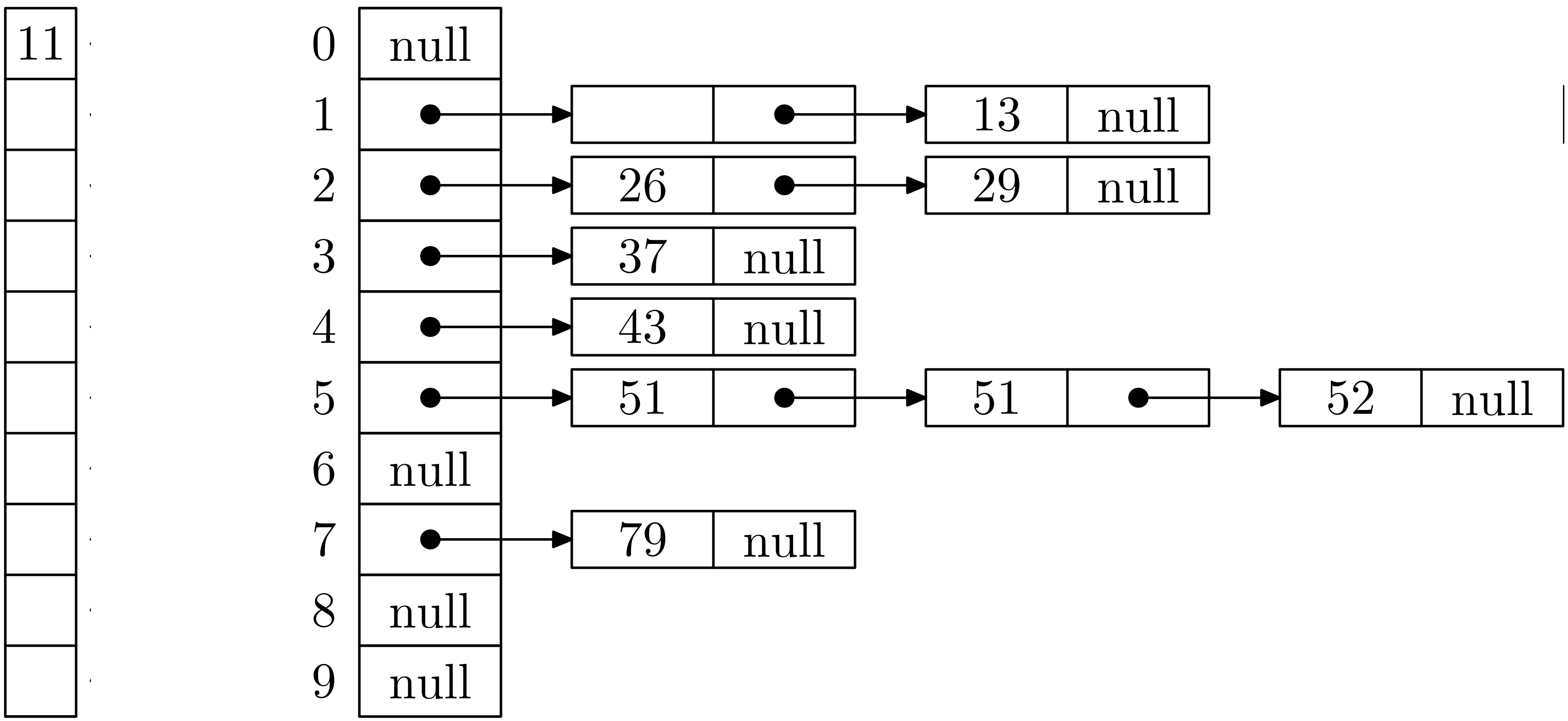

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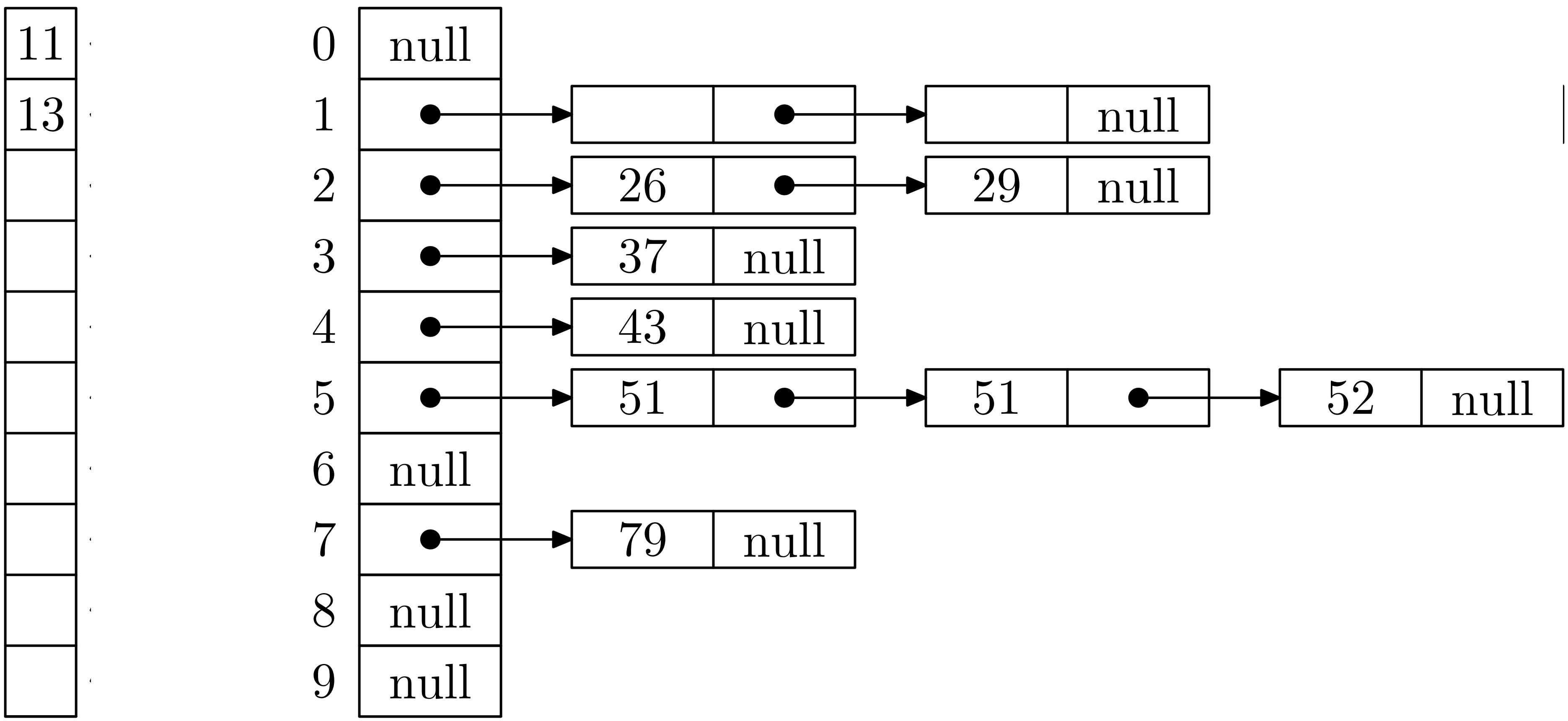

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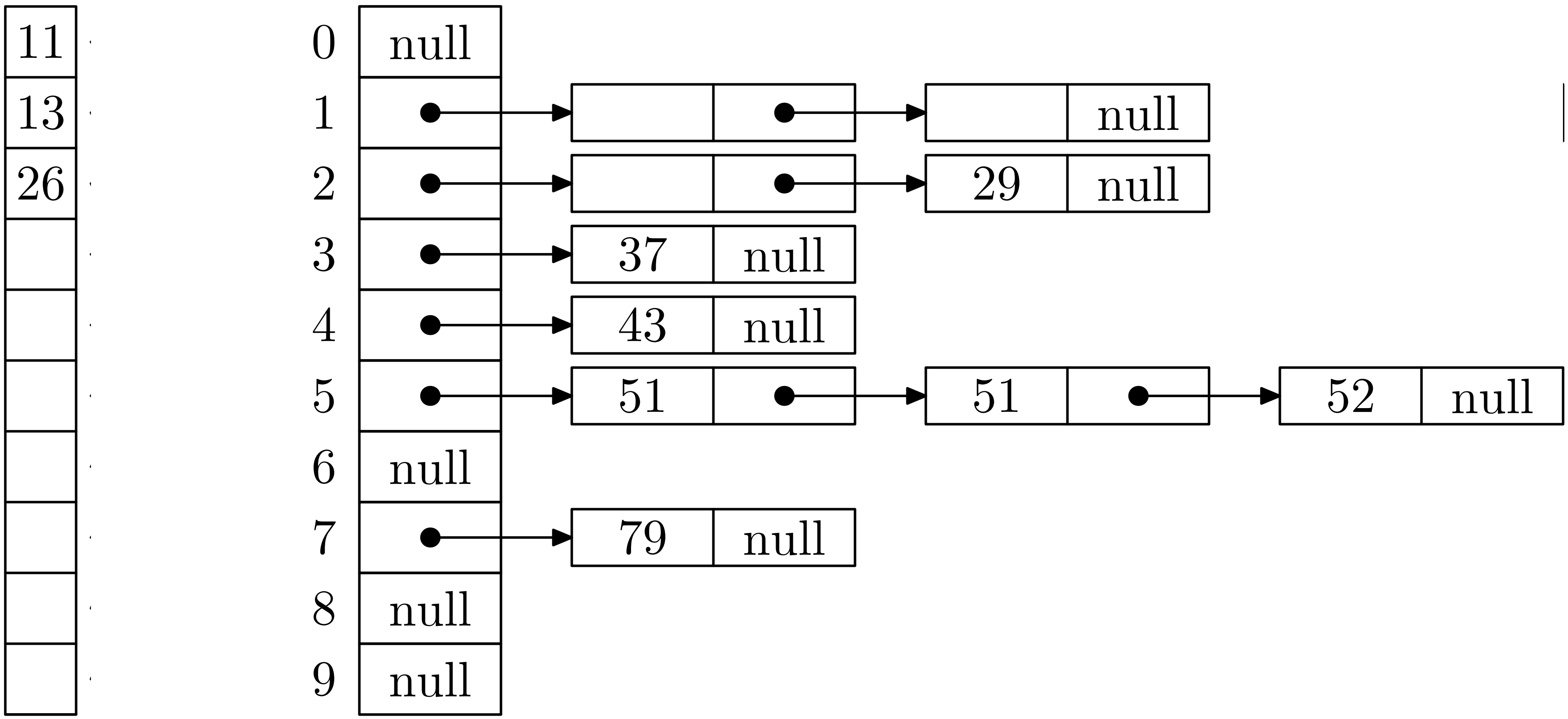

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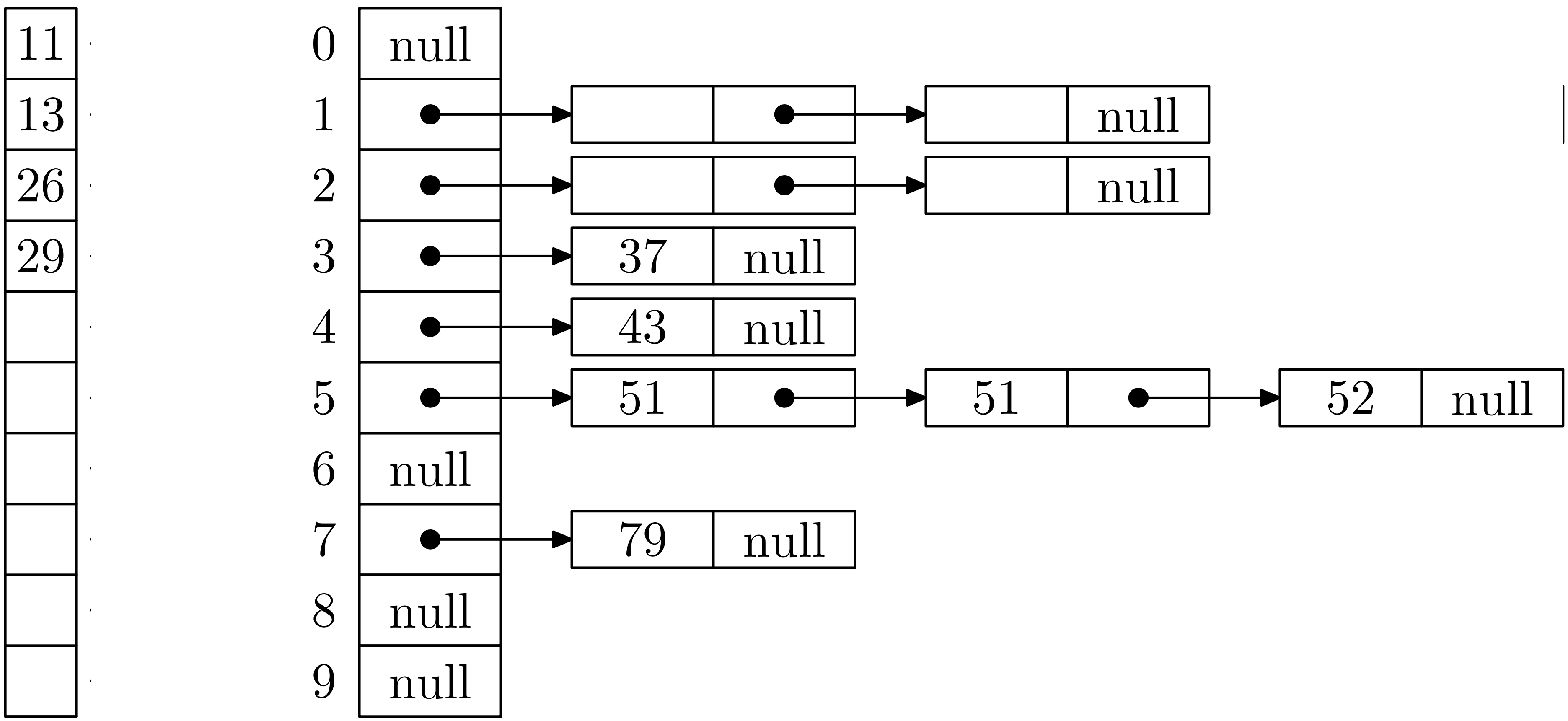

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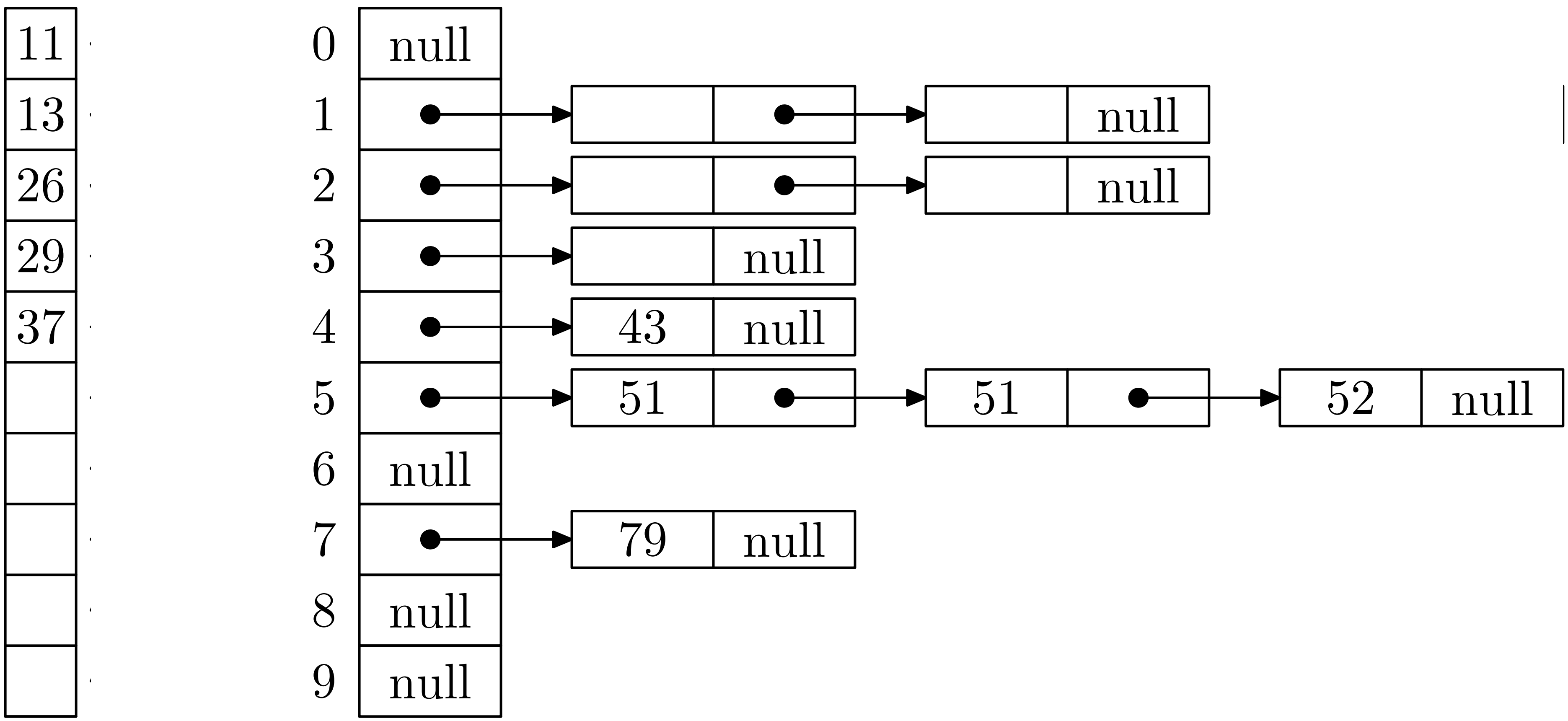

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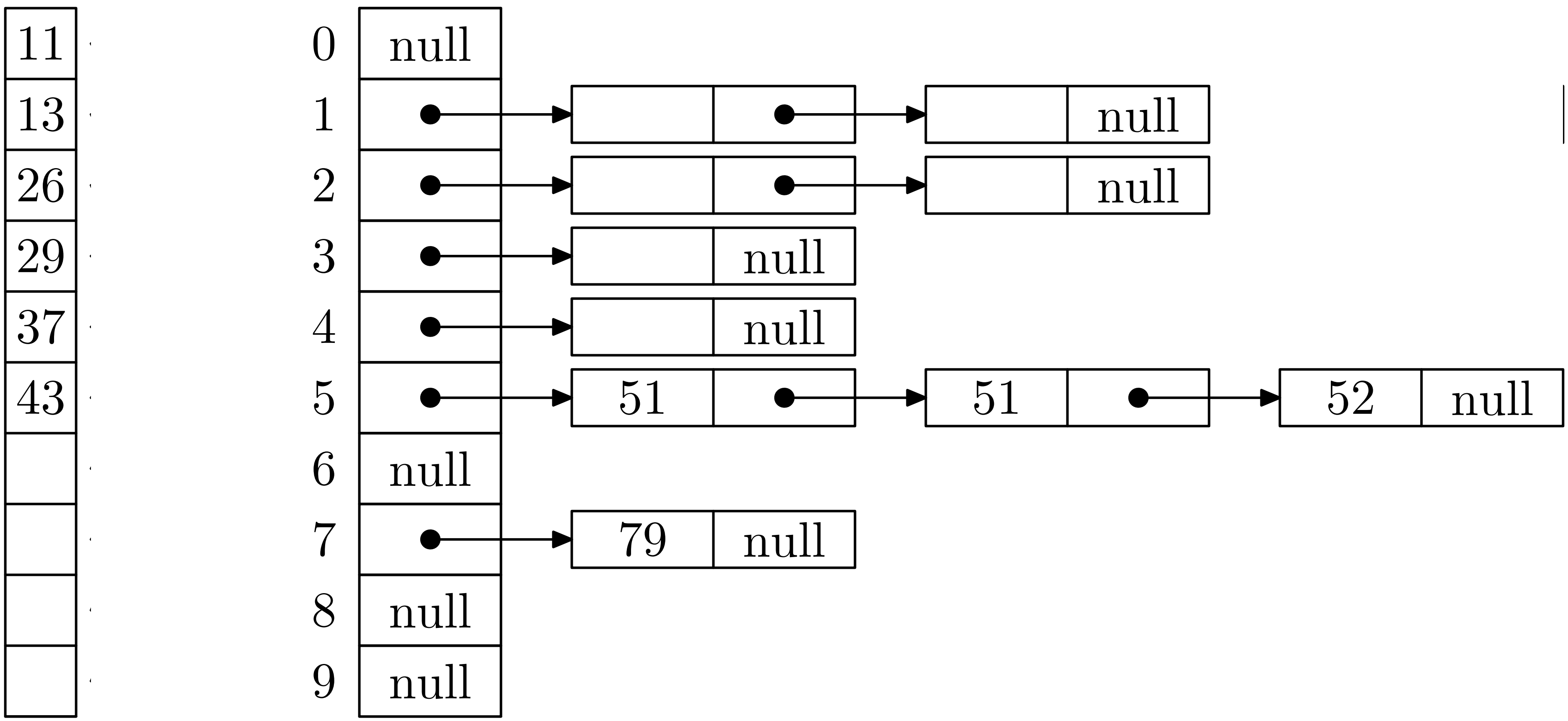

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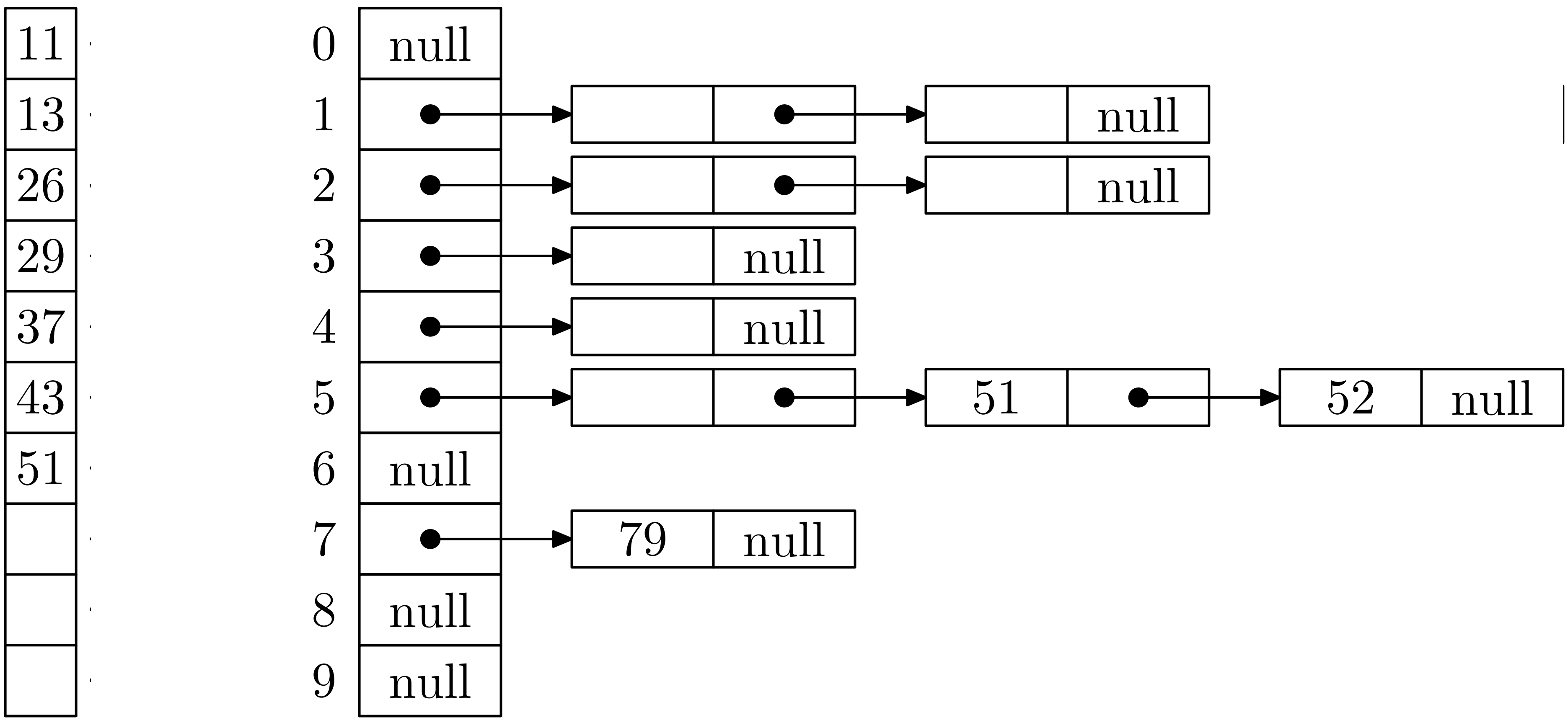

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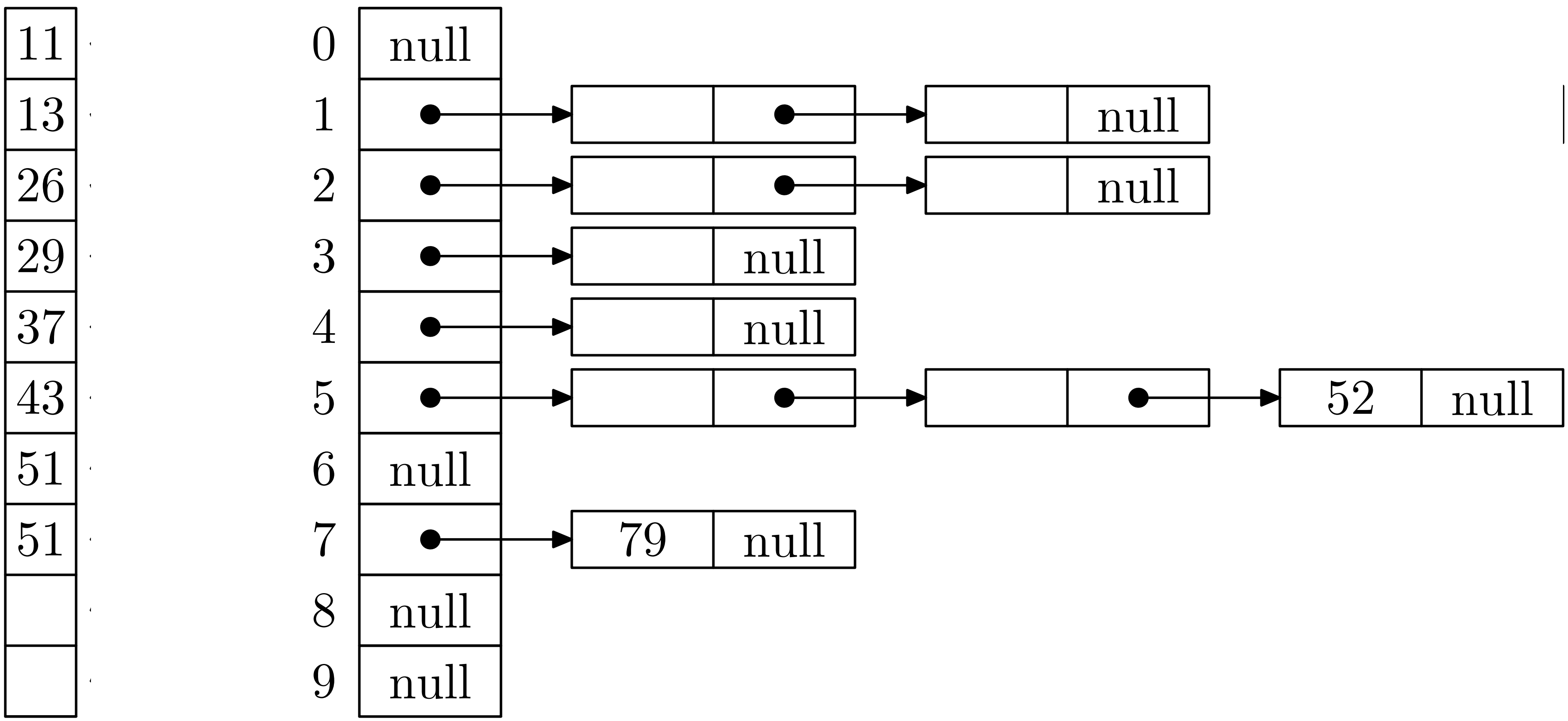

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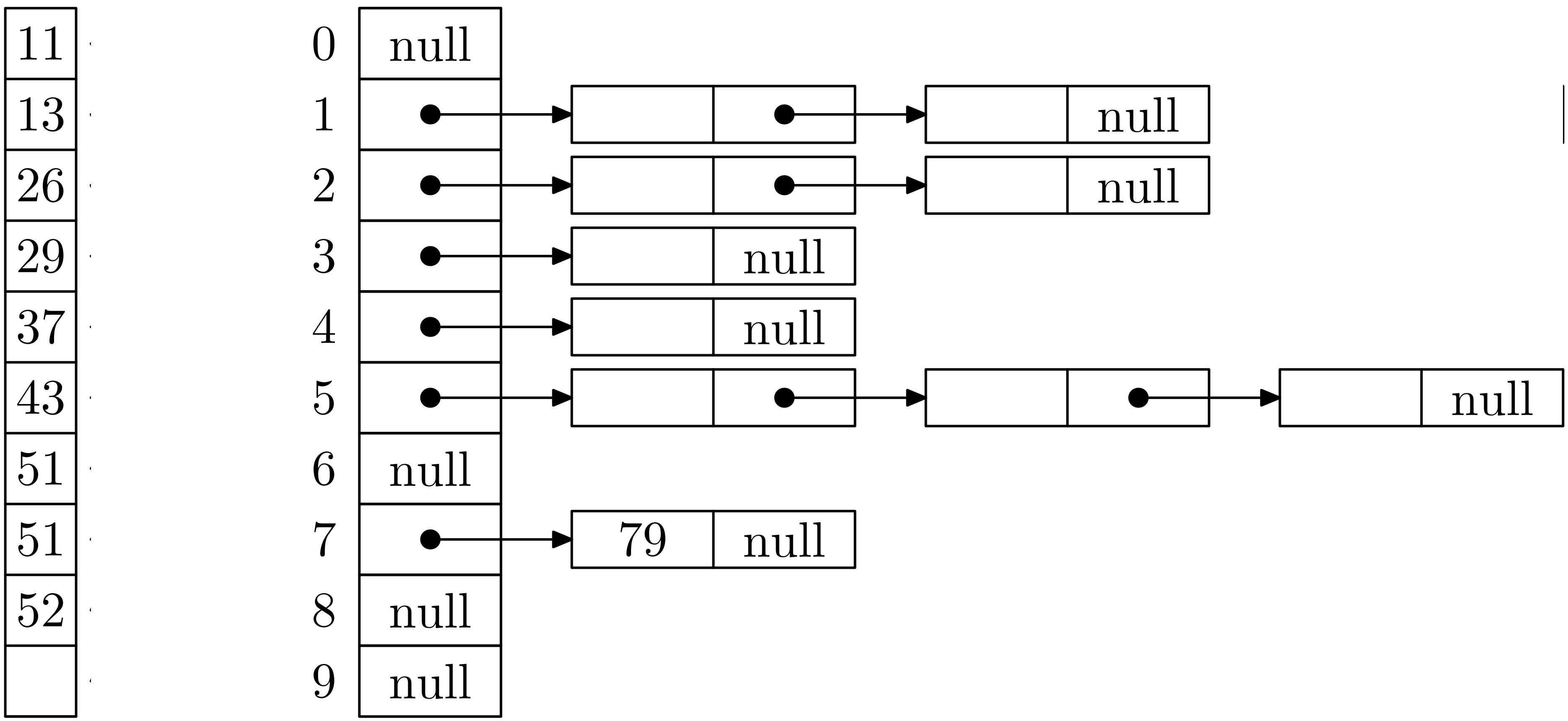

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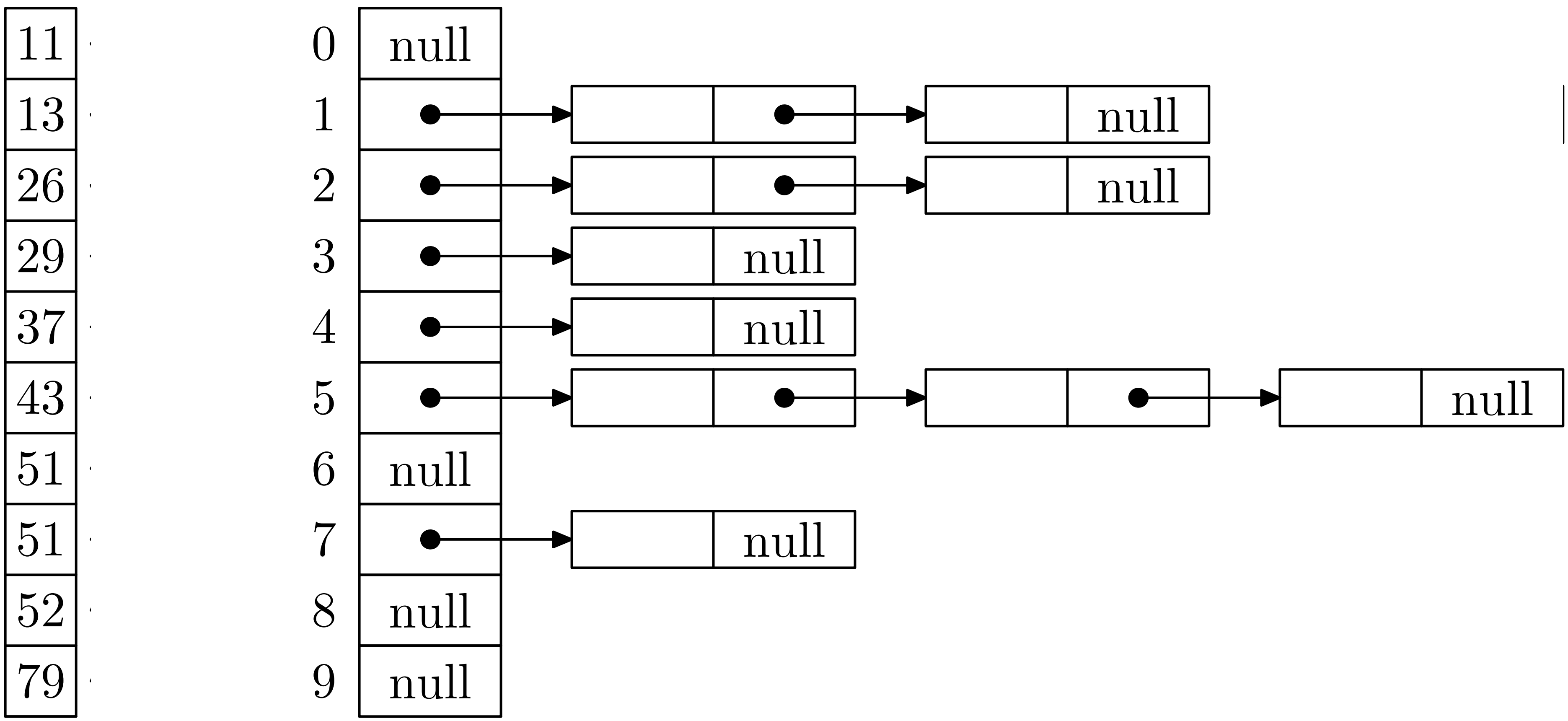

Radix Sort in Action

11
13
26
29
37
43
51
51
52
79

0	null
1	null
2	null
3	null
4	null
5	null
6	null
7	null
8	null
9	null

Time Complexity of Radix Sort

- We need not use base 10 we could use base r (the radix)
- If the maximum number to be sorted is N then the number of iterations of radix sort is $\log_r(N)$
- Each sort involves n operations
- Thus the total number of operations is $O\left(n\lceil\log_r(N)\rceil\right)$
- Since N does not depend on n we can write this as $O\left(n\right)$

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Bucket Sort

- A closely related sort is bucket sort where we divide up the inputs into buckets based on the most significant figure
- We then sort the buckets on less significant figures
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Minimum Time for Sort

- Can we do better?
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- If there is an element that isn't examined then we don't know where to put it
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Practical Sort

- In practice, radix sort or bucket sort are rarely used
- The overhead of maintaining the buckets make them less efficient than they might appear
- Radix sort is harder to generalise to other data types than comparison based sorts
- In practice quick sort and merge sort are usually preferred
- Having said that there are some very neat implementations of radix sort

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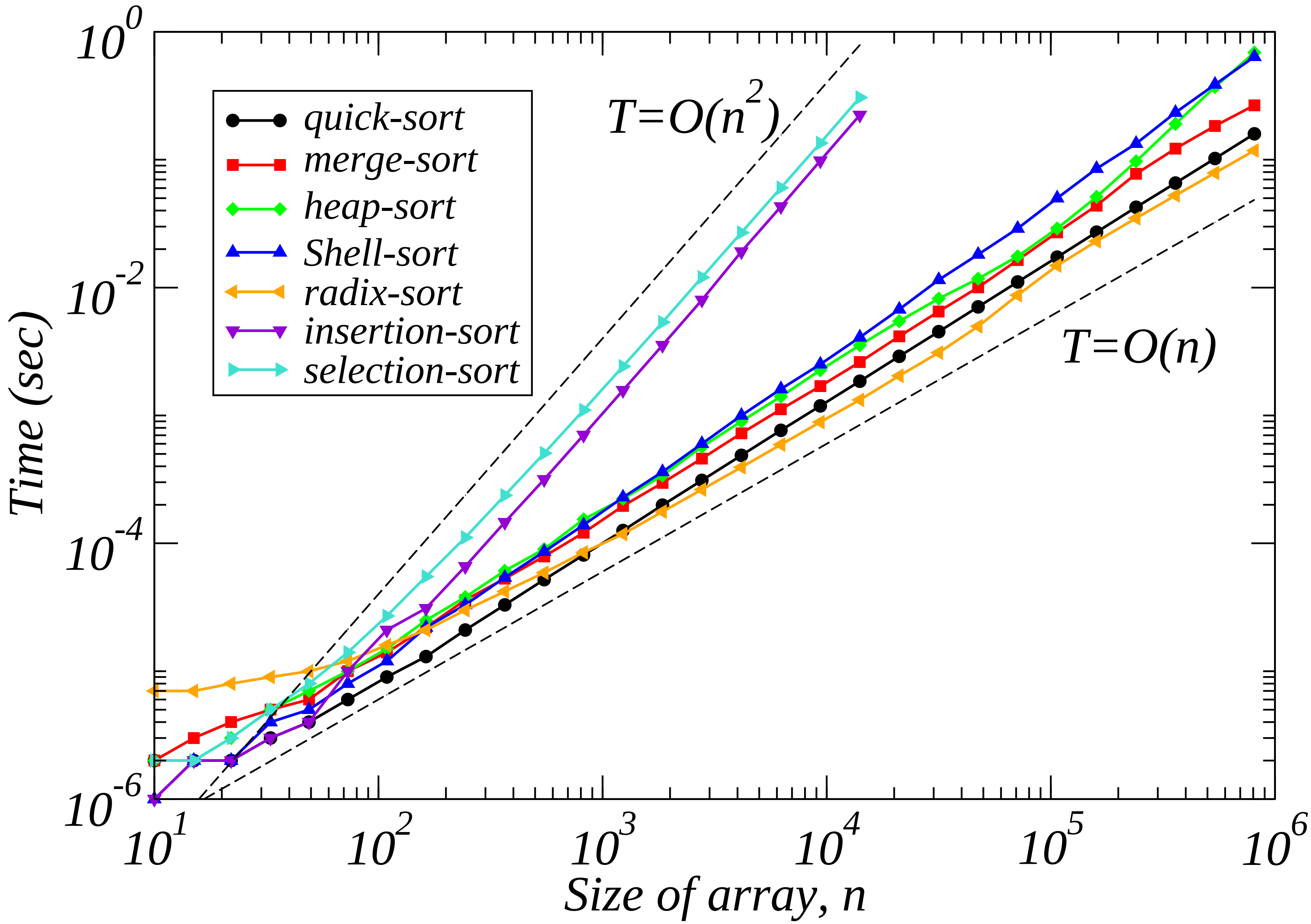
Comparison of Sort Algorithms



Lessons

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- There are sorts that have a better time complexity that quicksort
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