UNIVERSITY OF SOUTHAMPTON COMP1201W1

SEMESTER 2 EXAMINATION 2013/2014

ALGORITHMICS

Duration: 120 mins

You must enter your Student ID and your ISS login ID (as a cross-check) on this page. You must not write your name anywhere on the paper.

Student ID:	
ISS ID:	

Question	Marks
1	
2	
3	
4	
Total	

Answer all parts of the question in section A (25 marks) and TWO *questions from section B (30 marks each).*

This examination is worth 85%. The tutorials were worth 15%.

University approved calculators MAY be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Each answer must be completely contained within the box under the corresponding question. No credit will be given for answers presented elsewhere.

You are advised to write using a soft pencil so that you may readily correct mistakes with an eraser.

You may use a blue book for scratch—it will be discarded without being looked at.

 Number of pages 16

Section A

Question A 1

(a) Write pseudo code for the quick sort algorithm. What is its average and worst case time complexity? *(5 marks)*

Average case complexity:

Worst case complexity:

5

(b) Heaps use a binary tree encoded into an array. Show the binary tree represented by the following array.

3	5	5	13	20	12	29	25	29	29

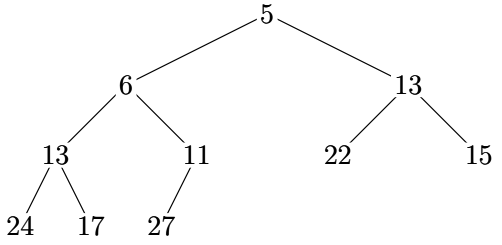
(1 marks)

1

(c) Show what happens to the heap shown on the left when you add 2.

(2 marks)

Before

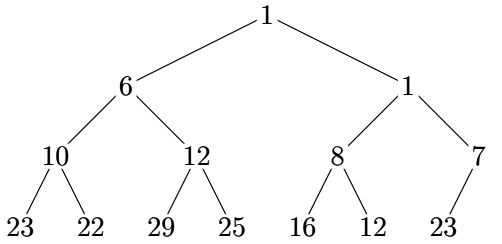


After `add(2)`

2

(d) Show what happens to the heap shown on the left when you remove the minimum entry. *(2 marks)*

Before



After `removeMin()`

2

TURN OVER

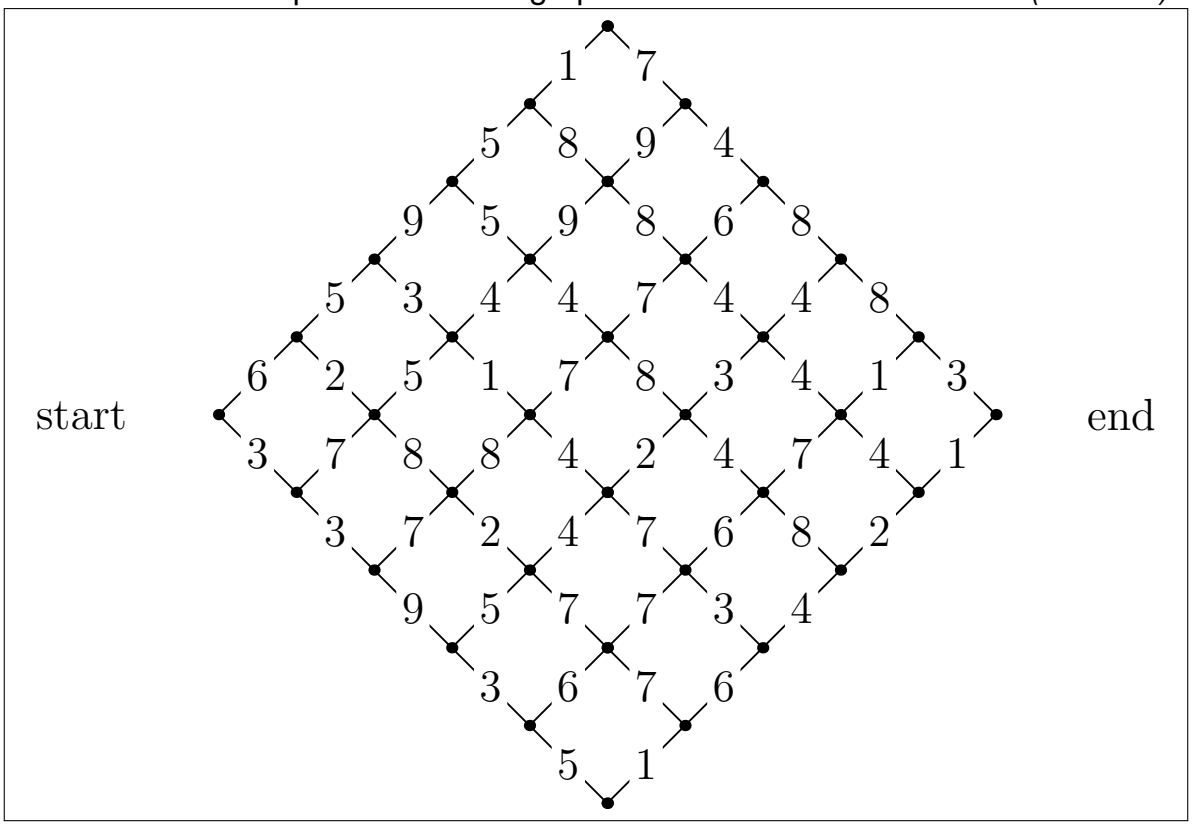
(e) Show how the numbers 12, 3, 7, 19, 15, 52, 46, 23, 22 would be hashed using a hash function $d_2 + 3d_1$ where d_1 is the first (least significant) digit and d_2 the second digit. Show how these would be stored in a hash table using separate chaining. *(10 marks)*

d_2d_1	12	3	7	19	15	52	46	23	22
$d_2 + 3d_1$									
$(d_2 + 3d_1)\%10$									

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

$\overline{10}$

(f) Use the dynamic programming forward algorithm to compute the minimum cost of each path from the left most node to each other node where the cost of moving along an edge is equal to the number shown. An edge can only be traversed from left to right. Use the backwards algorithm to find the minimum cost path across the graph. *(5 marks)*



$\frac{}{5}$

End of question 1

Q1: (a) $\frac{}{5}$ (b) $\frac{}{1}$ (c) $\frac{}{2}$ (d) $\frac{}{2}$ (e) $\frac{}{10}$ (f) $\frac{}{5}$ Total $\frac{}{25}$

TURN OVER

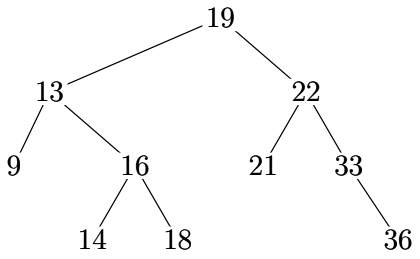
Section B

Question B 2

(a) Draw the binary search tree produced when inserting 64, 48, 35, 50, 26, 77, 43, 94, 0, 61. *(2 marks)*

2

(b) Draw the tree obtained by deleting 22 from the binary search tree shown. *(2 marks)*




```
delete(22)
```

2

(c) Describe the two rules that define an AVL tree *(4 marks)*

1

2

$\frac{}{4}$

(d) Let $m(h)$ be the minimum number of elements in an AVL tree of height h. Write down a recurrence relationship for $m(h)$ *(4 marks)*

$\frac{}{4}$

(e) Write down the boundary condition for $m(1)$ and $m(2)$ *(2 marks)*

$\frac{}{2}$

(f) Using proof by induction show that the minimum number of elements in an AVL tree of height h is greater than or equal to $b(h) = \left(\frac{3}{2}\right)^{h-1}$ *(6 marks)*

6

(g) Use this bound to show that the complexity of insertion and search for an AVL tree is $O(\log(n))$. *(3 marks)*

3

(h) What is the average and worst case time complexity of search in an unbalanced binary search tree and why is balancing a binary search tree regarded as so important? *(3 marks)*

Average case complexity:

Worst case complexity:

$\frac{}{3}$

(i) Describe how a tree map is implemented in the Java collections. *(4 marks)*

$\frac{}{4}$

End of question 2

Q2: (a) $\frac{}{2}$ (b) $\frac{}{2}$ (c) $\frac{}{4}$ (d) $\frac{}{4}$ (e) $\frac{}{2}$ (f) $\frac{}{6}$ (g) $\frac{}{3}$ (h) $\frac{}{3}$ (i) $\frac{}{4}$ Total $\frac{}{30}$

TURN OVER

Question B 3

(a) Describe the difference between depth first search and breadth first search. Give an application of each. *(4 marks)*

DFS Application:

BFS Application:

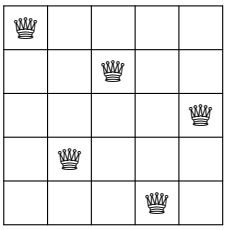
4

(b) What is topological sort? Give an application. *(4 marks)*

Application:

4

(c) The n-queens problem is to put n queens on a chess board such that no queen is in the same row, column or diagonal as any other queen. Either write pseudo code or describe in outline an algorithm to solve the n-queens problem.



(10 marks)

$\overline{10}$

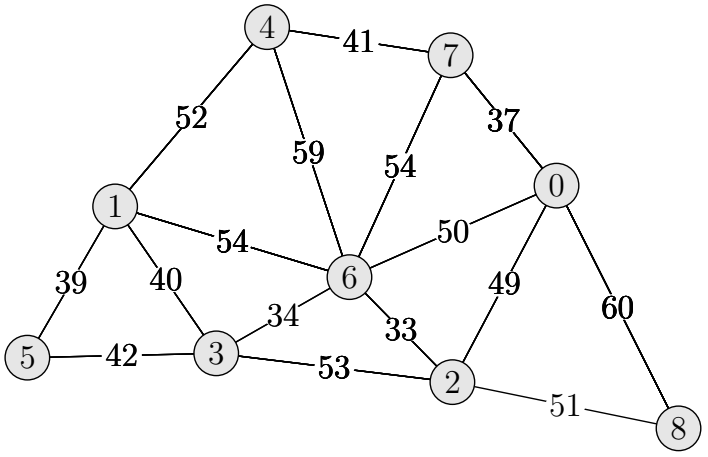
TURN OVER

(d) Describe Kruskal's algorithm for constructing a minimum spanning tree. What data structures are need to implement the algorithm? *(6 marks)*

Data Structures:

6

(e) In the graph below draw the minimum spanning tree (by highlighting the edges) and write down the order in which the edges would be found by Kruskal's algorithm. *(6 marks)*



6

1. **2.**

3. **4.**

5. **6.**

7. **8.**

End of question 3

Q3: (a) $\frac{}{4}$ (b) $\frac{}{4}$ (c) $\frac{}{10}$ (d) $\frac{}{6}$ (e) $\frac{}{6}$ Total $\frac{}{30}$

Question B 4

(a) Describe the local (neighbourhood) search strategy. What is its main drawback? *(4 marks)*

Drawback:

4

(b) Describe a strategy to overcome this. *(4 marks)*

4

(c) How efficient would you expect heuristic search to be on a problem like TSP or graph colouring? *(2 marks)*

2

(d) Describe the three conditions required of a linear programing problem. *(6 marks)*

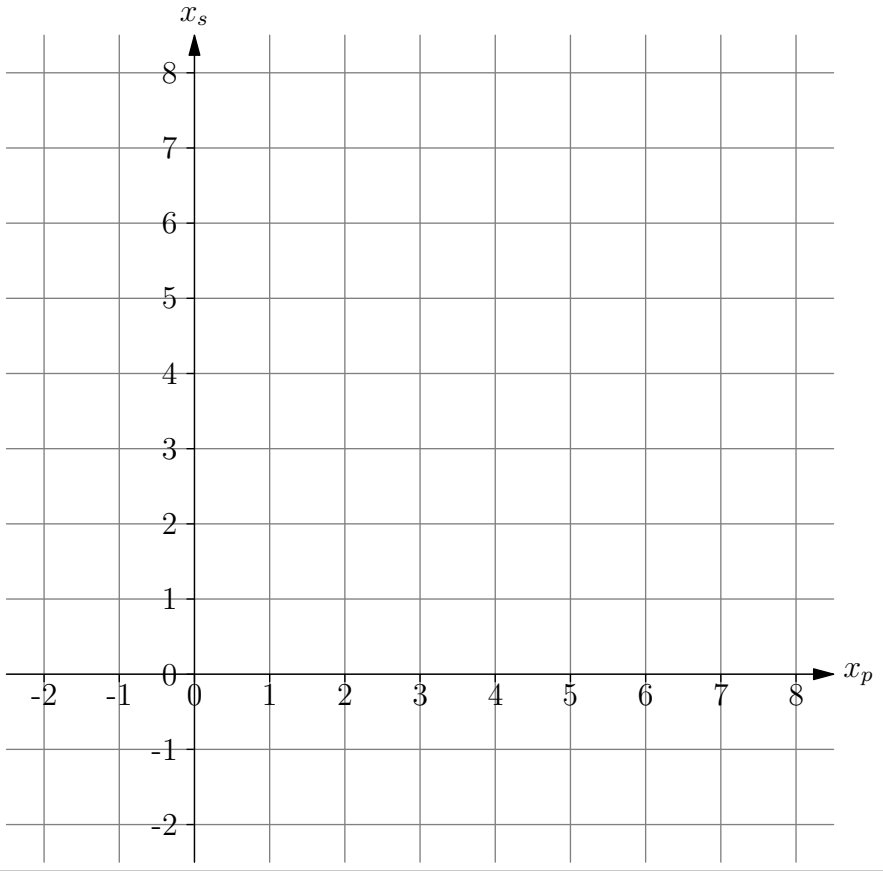
1

2

3

6

(e) Consider the problem of deciding the cheapest balance between potatoes and soybean. Let x_p be the amount of potatoes and x_s the amount of soybeans. The cost of soybean is taken to be twice the cost of potatoes. We require the vitamin C content from the two ingredients should be at least 12 units where the vitamin content of 1 unit of potatoes is 2 and for soybean 3. We also require the vitamin B6 content of the two ingredients should be at least 8 units where one unit of potatoes supplies 4 while soybean provides 1 unit. Write down the linear programming problem and draw the direction that minimises the cost and the constraints (shade the infeasible region). Mark the optimal solution. *(10 marks)*



$\overline{10}$

(f) Describe in brief outline the simplex algorithm. *(4 marks)*

$\frac{}{4}$

End of question 4

Q4: (a) $\frac{}{4}$	(b) $\frac{}{4}$	(c) $\frac{}{2}$	(d) $\frac{}{6}$	(e) $\frac{}{10}$	(f) $\frac{}{4}$	Total $\frac{}{30}$

END OF PAPER