

Further Mathematics and Algorithms

Lesson 13: *Make a hash of it*



Hash tables, separate chaining, open addressing, linear/quadratic probing, double hashing

Outline

1. **Why Hash?**
2. Separate Chaining
3. Open Addressing
 - Quadratic Probing
 - Double Hashing
4. Hash Set and Map



Content Addressable Memory

- Suppose we have a list of objects which we want to look up according to its contents
- This is often referred to as **associative memory** structures
- A classical example would be a telephone directory
 - ★ We look up a name
 - ★ We want to know the number
- What data structure should we use?

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Lists and Trees

- To find an entry in a normal list takes $\Theta(n)$ operations
- If we had a sorted list we could use “binary search” to reduce this to $\Theta(\log(n))$
 - ★ We will study binary search later
 - ★ Maintaining an ordered list is costly ($\Theta(n)$ insertions)
- We could use a binary search tree
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Thinking Outside the Box

- As with many data structures thinking about the problem differently can lead to much better solutions
- Let us consider the content we want to search on as a **key**
- For telephone numbers the key would be the name of the person we want to phone
- We could get $O(1)$ search, insertion and deletion if we used the key as an index into a big array
- That is the key is a string of, say, 100 characters so can be represented by an 800 digit binary number
- We could look up the key in a table of 2^{800} items

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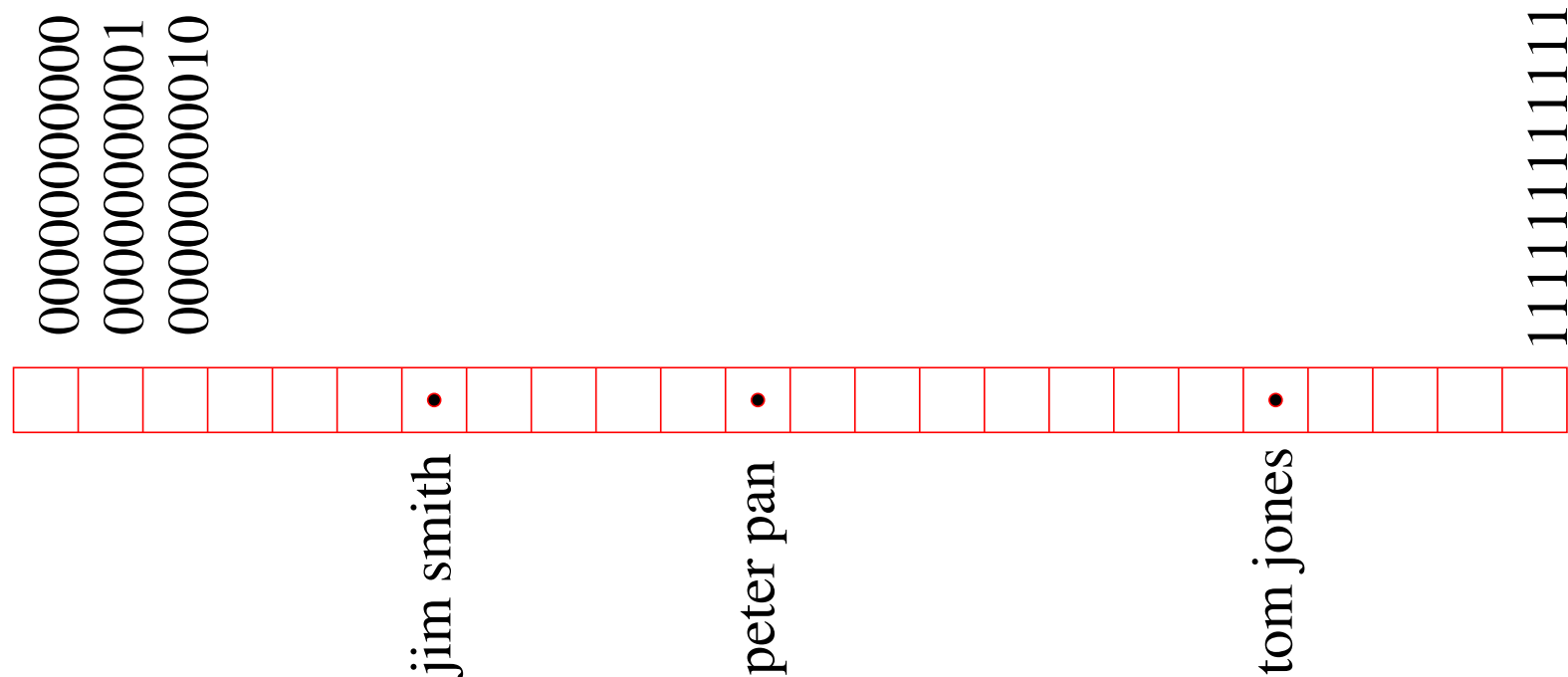
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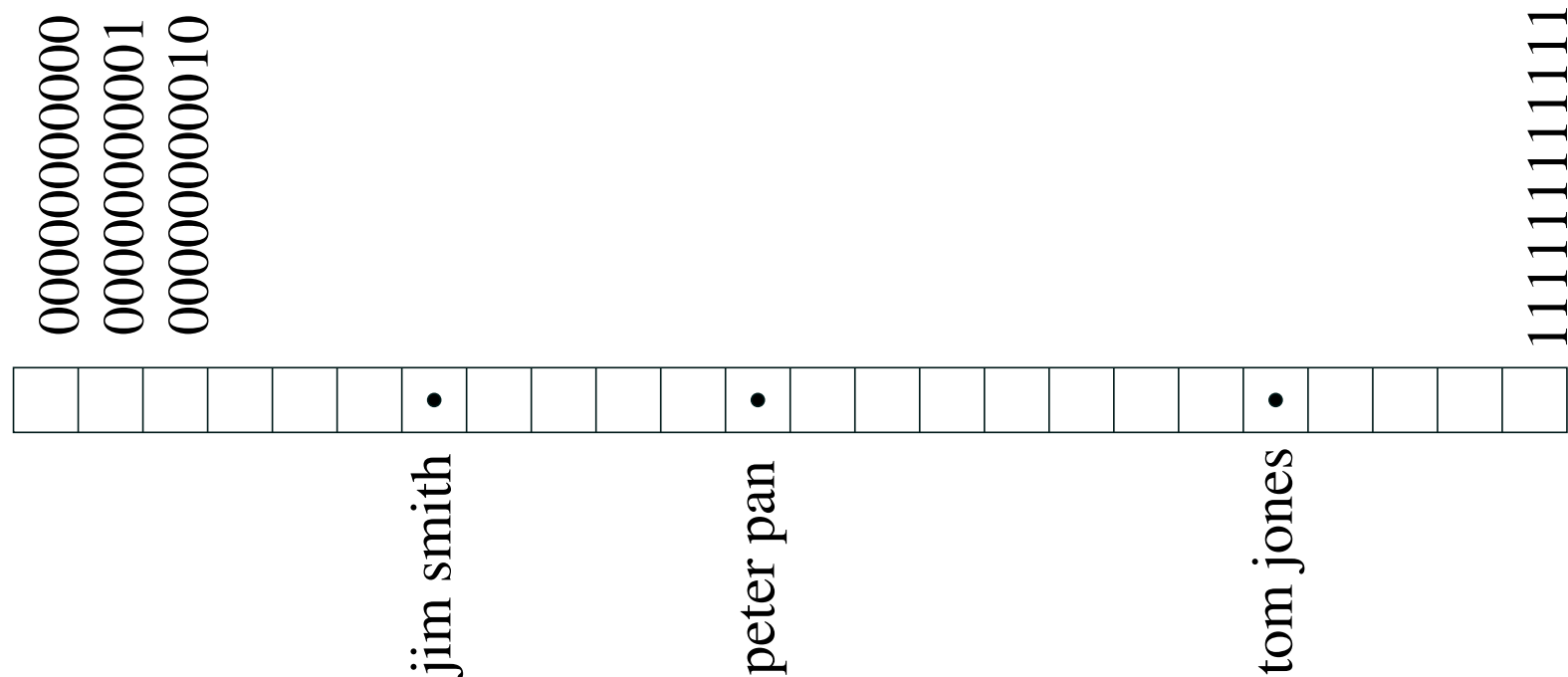
Hashing

- This approach is slightly wasteful of memory
- Almost all memory locations would be empty
- We can save on memory by folding up the table up onto itself



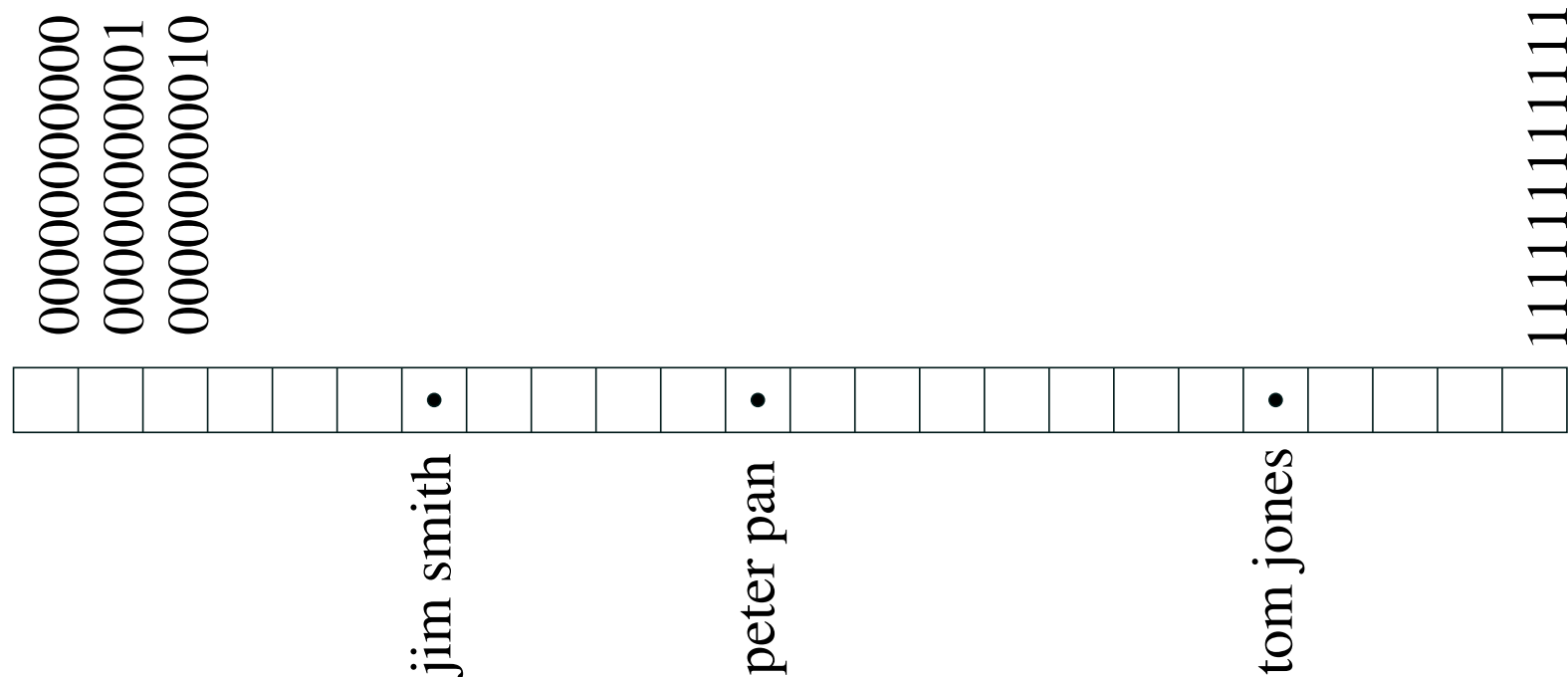
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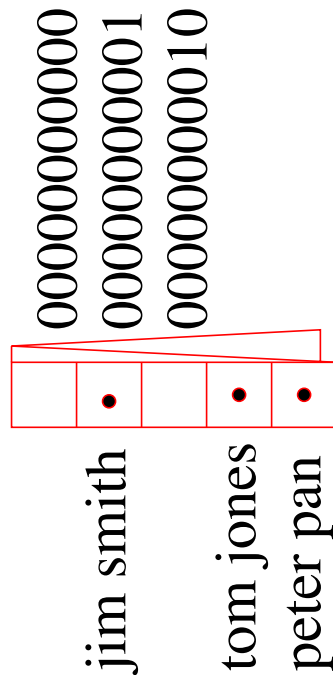
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Hashing Codes

- A **hashing function** `hashCode(x)` takes an object, `x`, and returns a positive integer, the **hash code**
- To turn the hash code into an address take the modulus of the table size

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int index = abs(hashCode(x) % tableSize);
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- If $\text{tableSize} = 2^n$ we can compute this more efficiently using a mask

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Hashing Functions

- Hashing functions take an object and return an integer
- Hashing functions aren't magic
 - ★ They tend to add up integers representing the parts of the object
- We want the integers to be close to random so that similar objects are mapped to different integers
- Sometimes two objects will be mapped to the same address—this is known as a **collision**
- Collision resolution is an important part of hashing

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Hashing Strings

- A strings might be hashed using a function

```
unsigned long long hash(string const& s) {  
    unsigned long long results = 12345;  
  
    for (auto ch = s.begin(); ch != s.end(); ++ch) {  
        results = 127*results + static_cast<unsigned char>(*ch);  
    }  
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- The numbers 12345 and 127 is to try to prevent clashes

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- The numbers 12345 and 127 is to try to prevent clashes—there are lots of alternatives
- What we want is that strings that might be similar receive very different hash codes

DIY

- The `unordered_set<T, Hash<T> >` allows you to define your own hash function
- By default this is set to `std::hash<T>(T)`
- Not all classes have hash function defined so you will need to do this
- Care is needed to make you hash function produce near random hash codes

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Collision Resolution

- Collisions are inevitable and must be dealt with
- There are two commonly used strategies
 - ★ Separate chaining—make a hash table of lists
 - ★ Open addressing—find a new position in the hash table
- Collisions add computational cost
- They occur when the hash table becomes full
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Resizing a Hash Table

- Resizing a hash table is easy
 - ★ Create a new hash table of, say, twice the size
 - ★ Iterate through the old hash table adding each element to the new hash table
- Note that you have to recompute all the hash codes
- Resizing a hash table has a modest amortised cost, but can give you a very hiccupy performance
- The size of a hash table is a classic example of a memory-space versus execution time trade off—using bigger (sparser) hash tables speeds up performance

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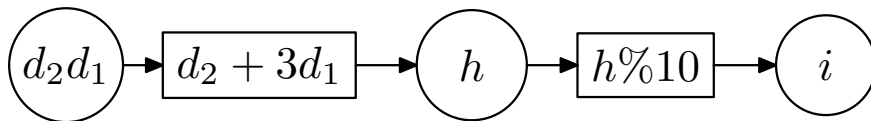
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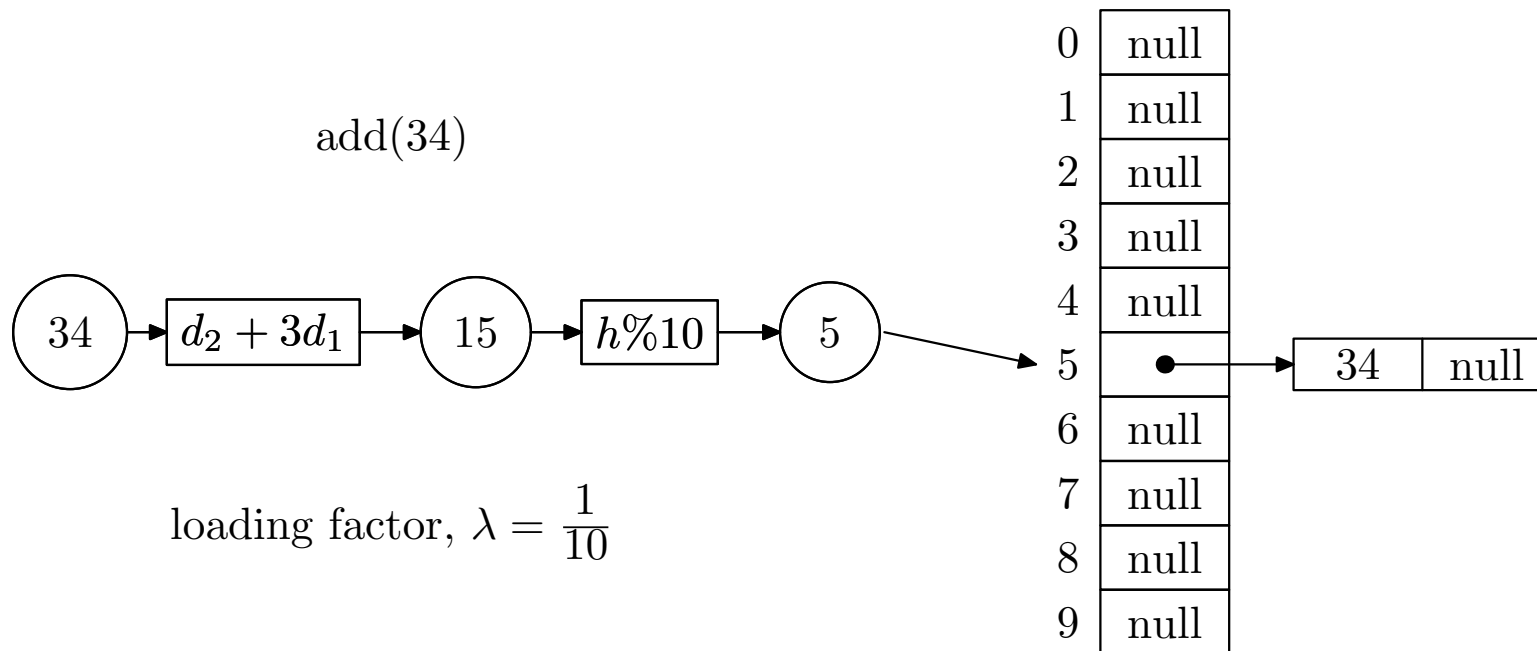


loading factor, $\lambda = \frac{0}{10}$

0	null
1	null
2	null
3	null
4	null
5	null
6	null
7	null
8	null
9	null

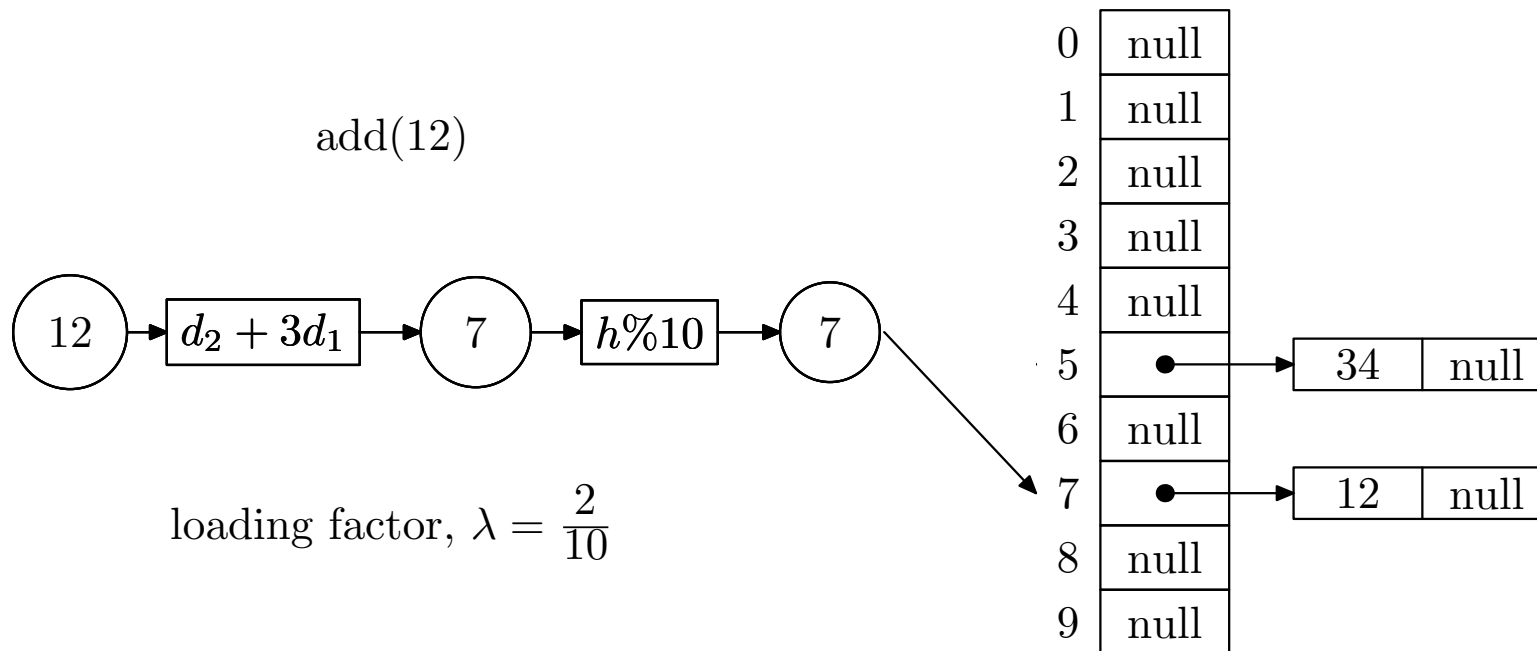
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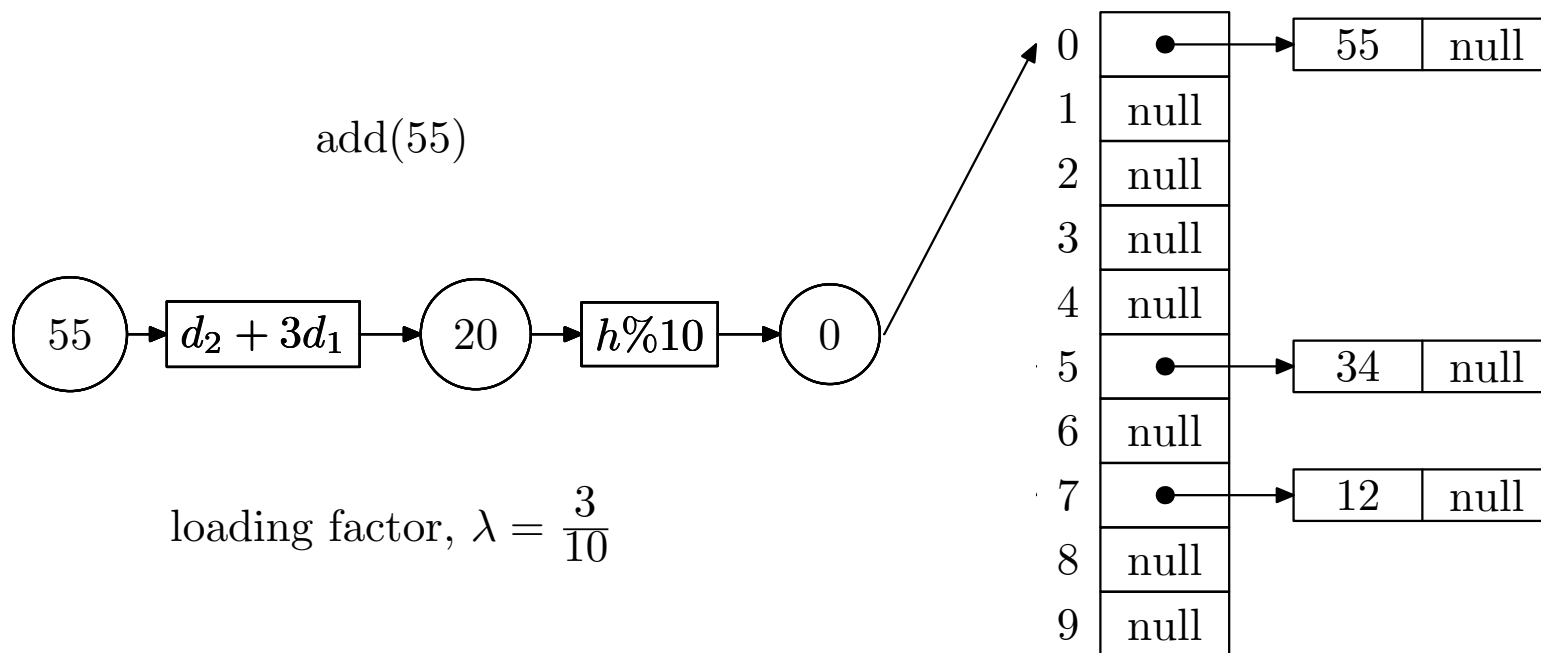
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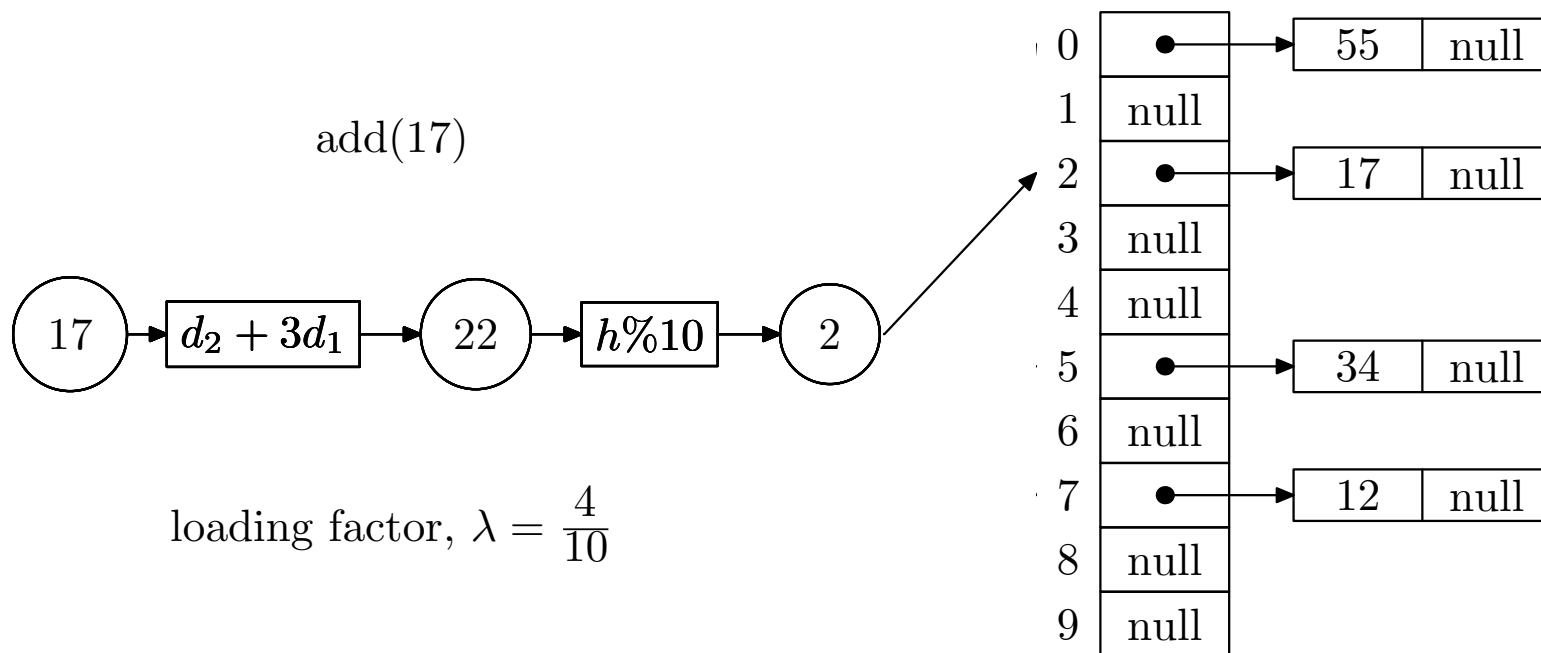
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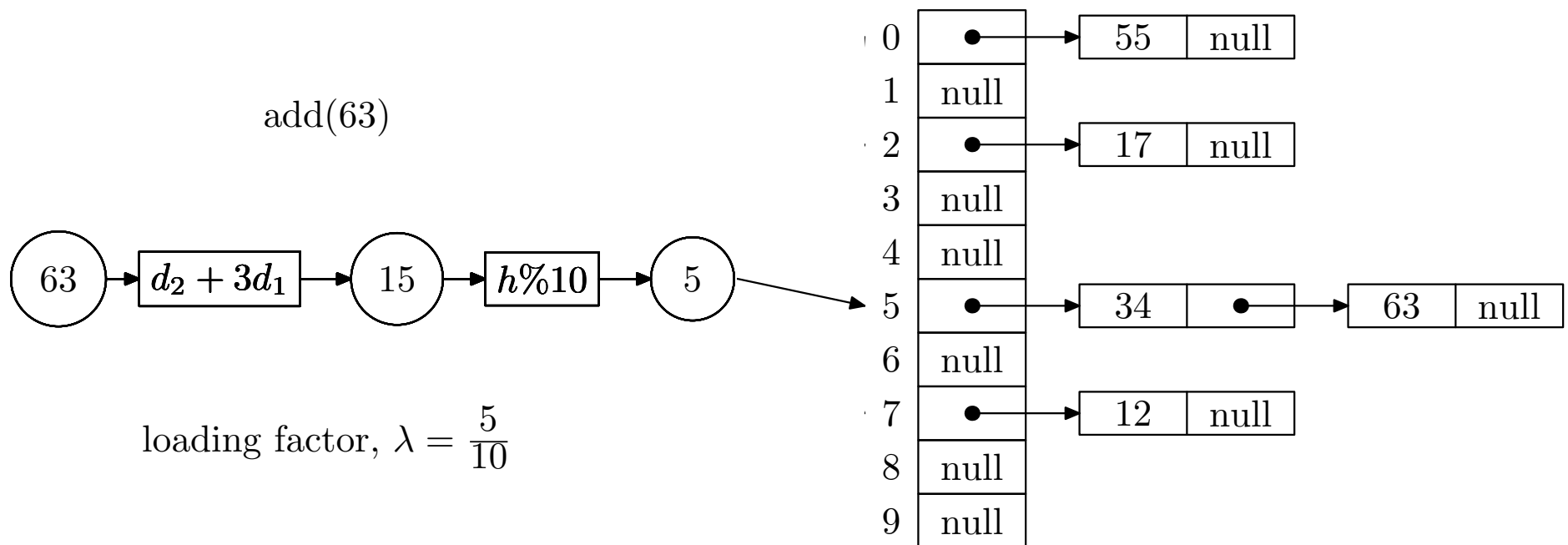
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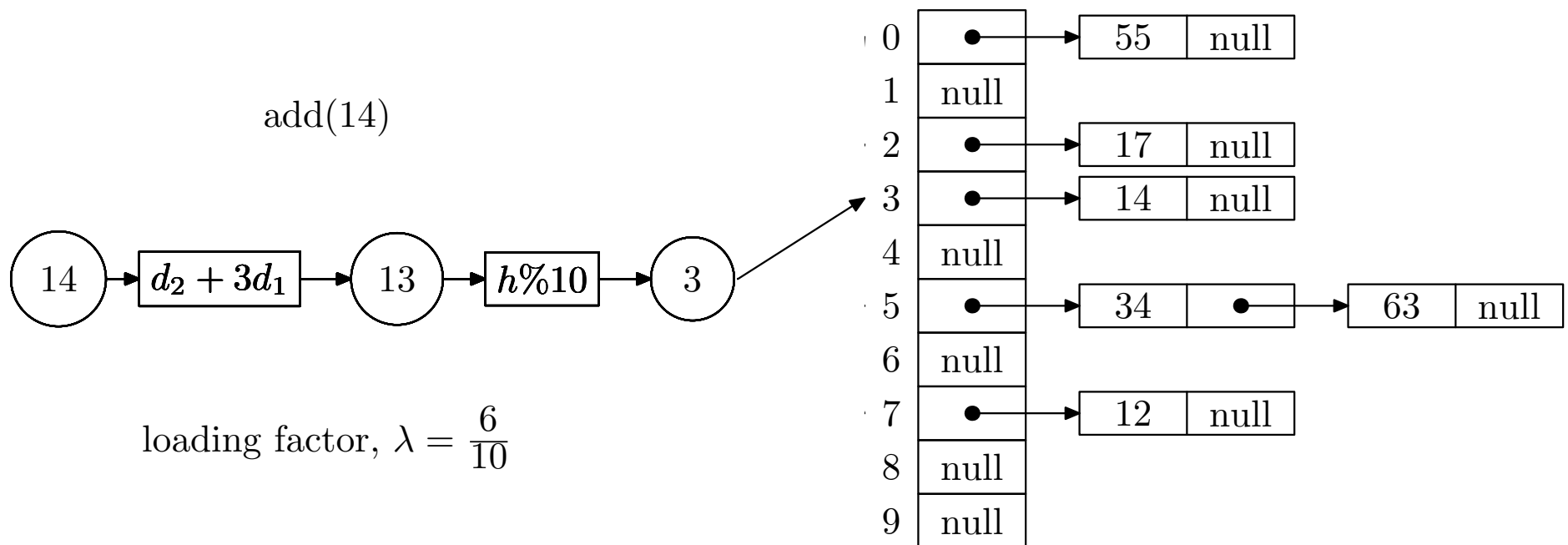
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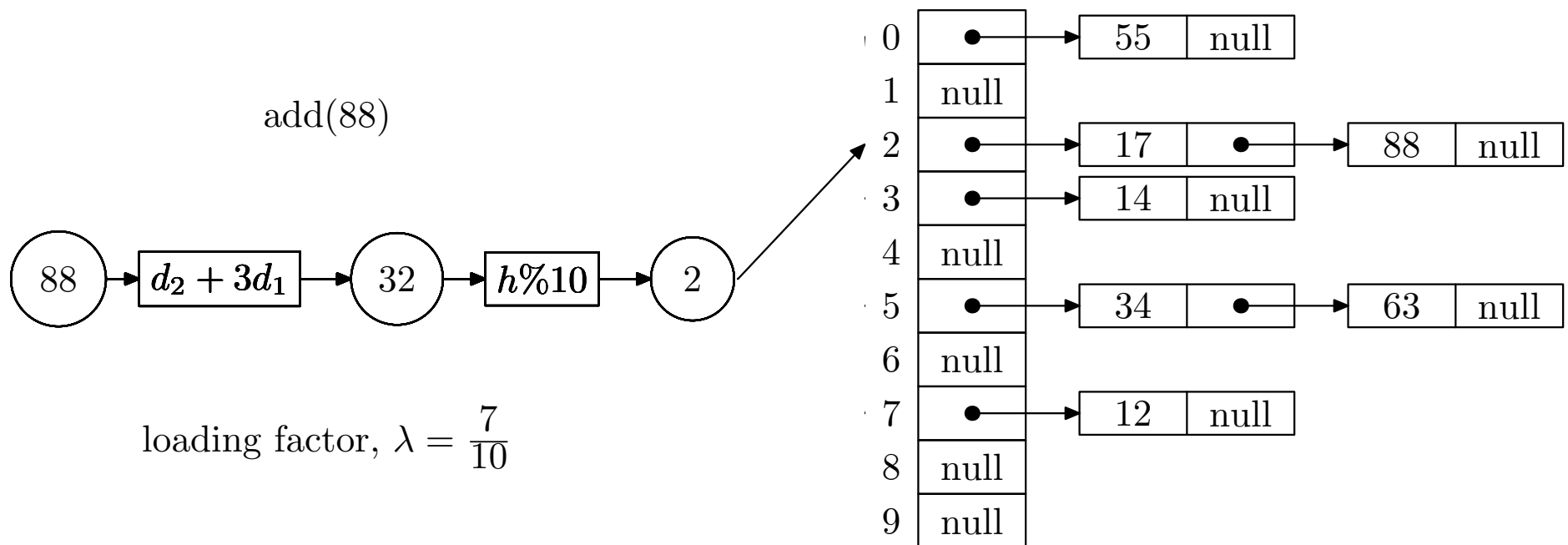
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Search

- To find an entry in a hash table we again use the hash function on a key to find the table entry and then we search the list
- The time complexity depends on where objects are hashed
- If the objects are evenly dispersed in the table, search (and insertion) is $\Omega(1)$
- If the objects are hashed to the same entry in the hash table then search is $O(n)$
- Provided you have a good hashing function and the hash table isn't too full you can expect $\Theta(1)$ average case performance

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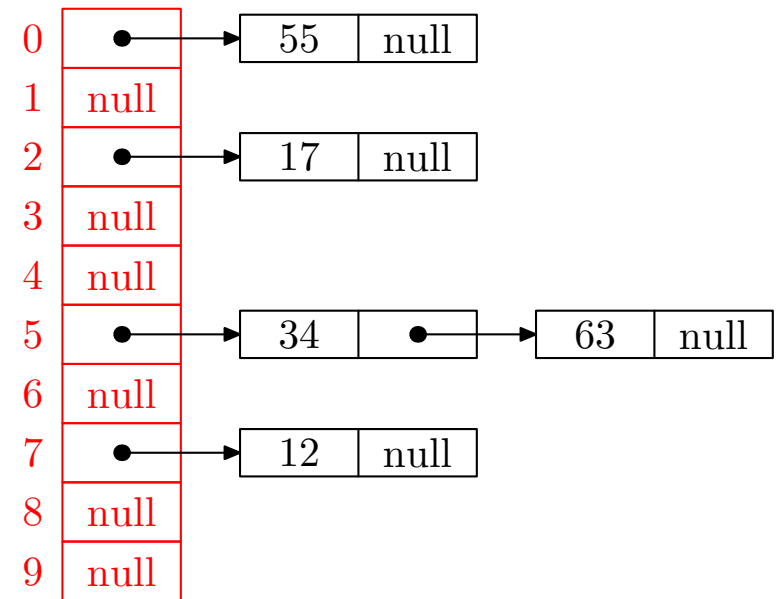
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Iterating Over a Hash Table

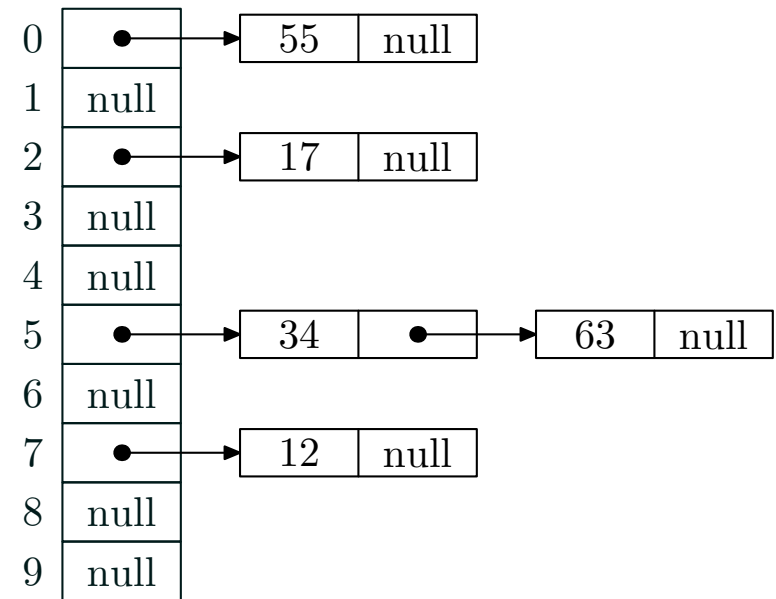
- To iterate over a hash table we
 - ★ Iterate through the array
 - ★ At each element we iterate through the linked list
- The order of the elements appears random
- This becomes more efficient as the table becomes fuller



55, 17, 34, 63, 12

Iterating Over a Hash Table

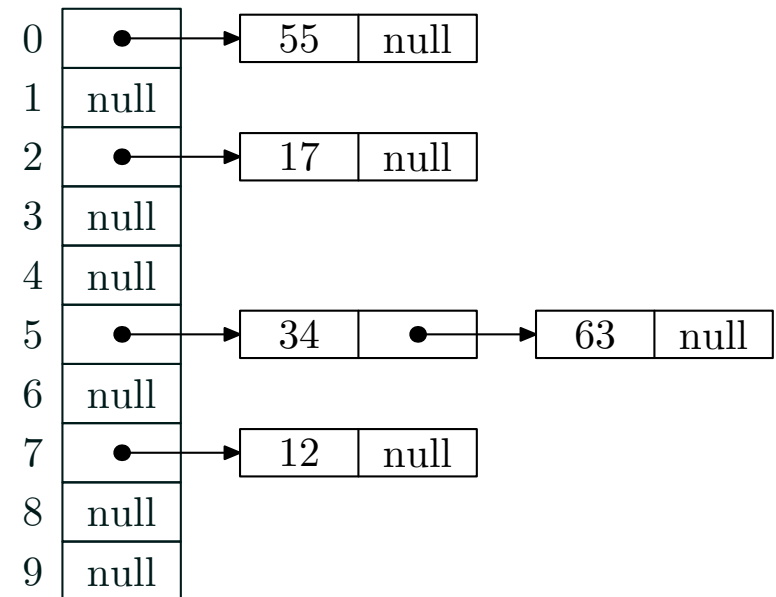
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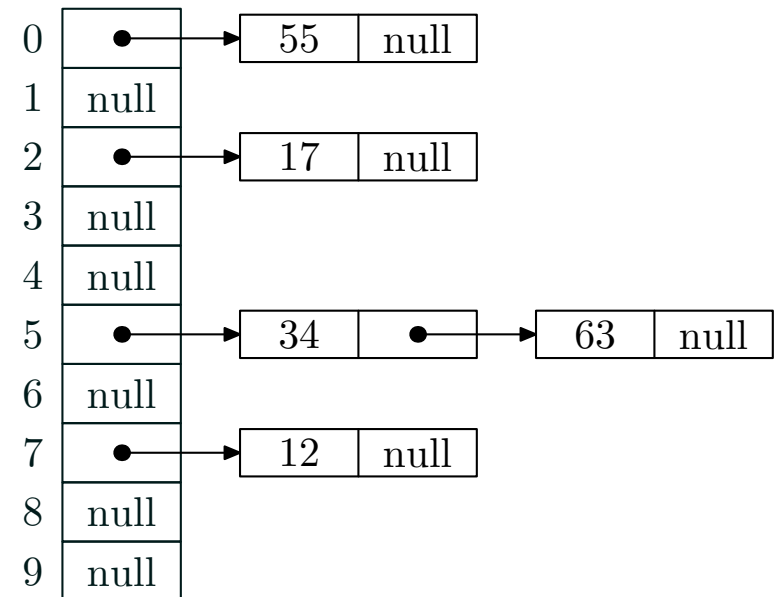
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55, 17, 34, 63, 12

Outline

1. Why Hash?
2. Separate Chaining
3. **Open Addressing**
 - Quadratic Probing
 - Double Hashing
4. Hash Set and Map



Open Addressing

- In open addressing we have a single table of objects (without a linked-list)
- In the case of a collision a new location in the table is found
- The simplest mechanism is known as **linear probing** where we move the entry to the next available location

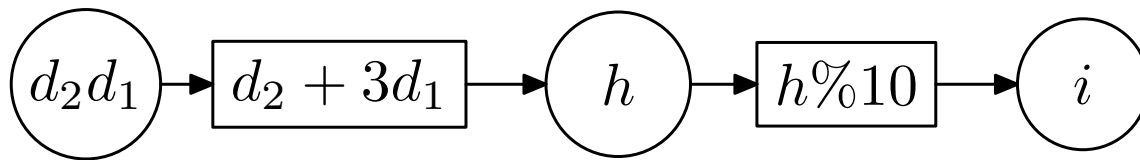
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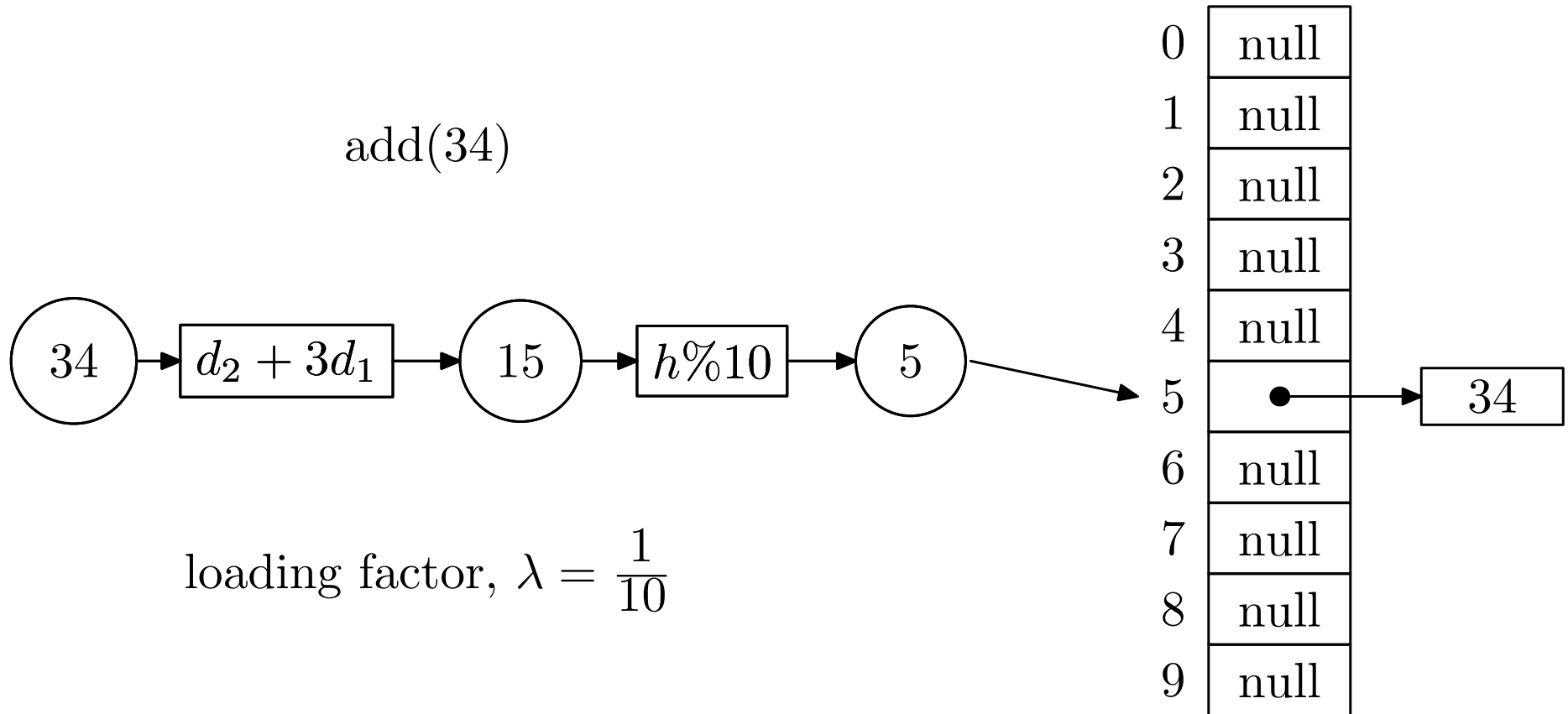
Linear Probing



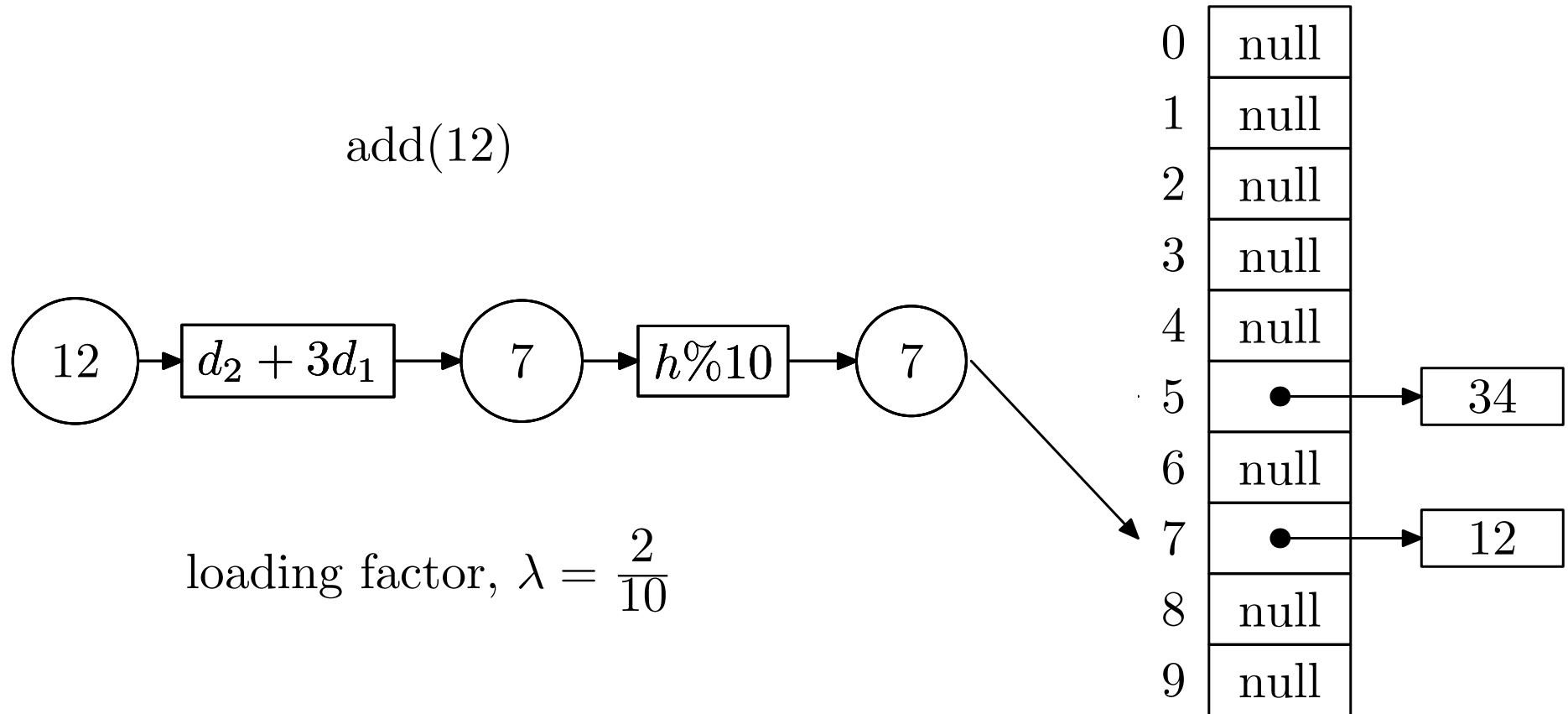
loading factor, $\lambda = \frac{0}{10}$

0	null
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4	null
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6	null
7	null
8	null
9	null

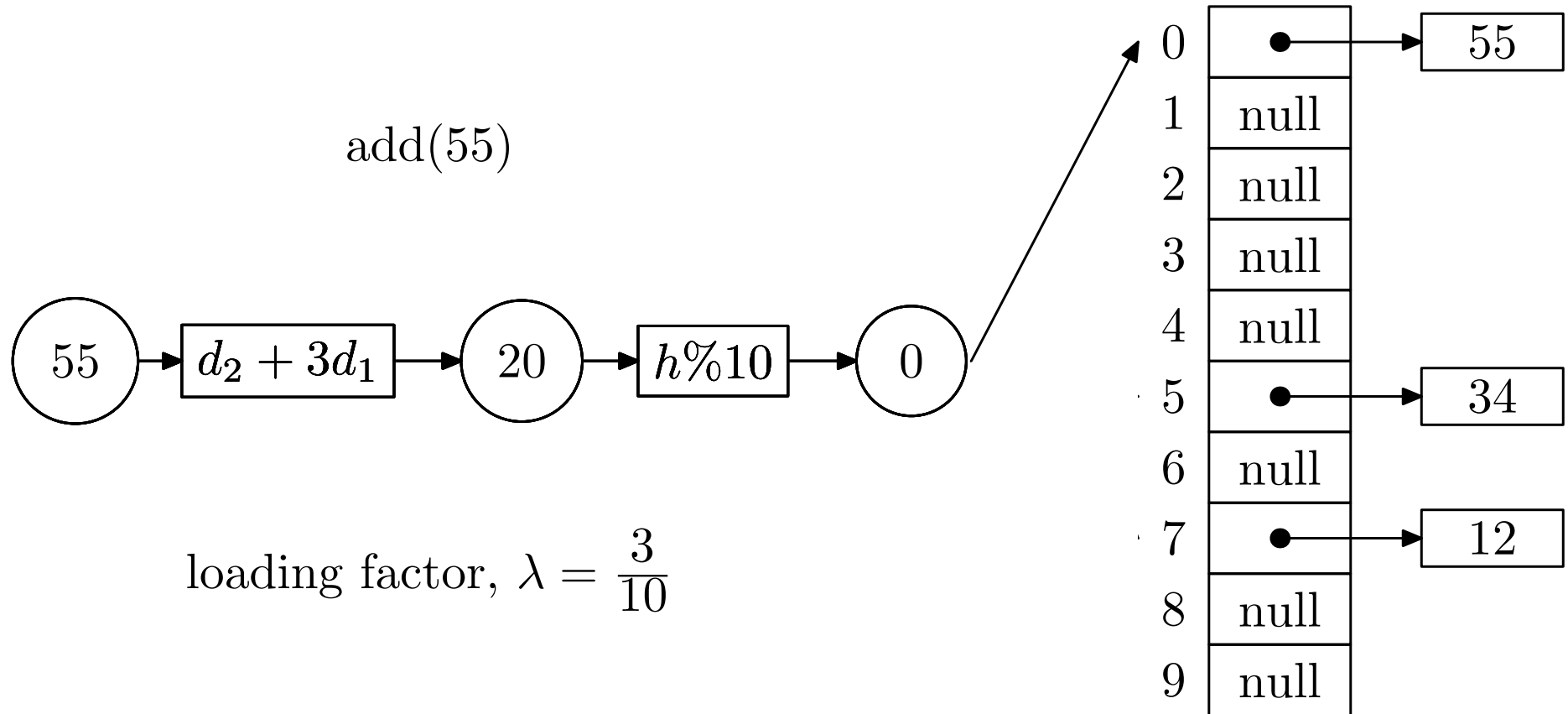
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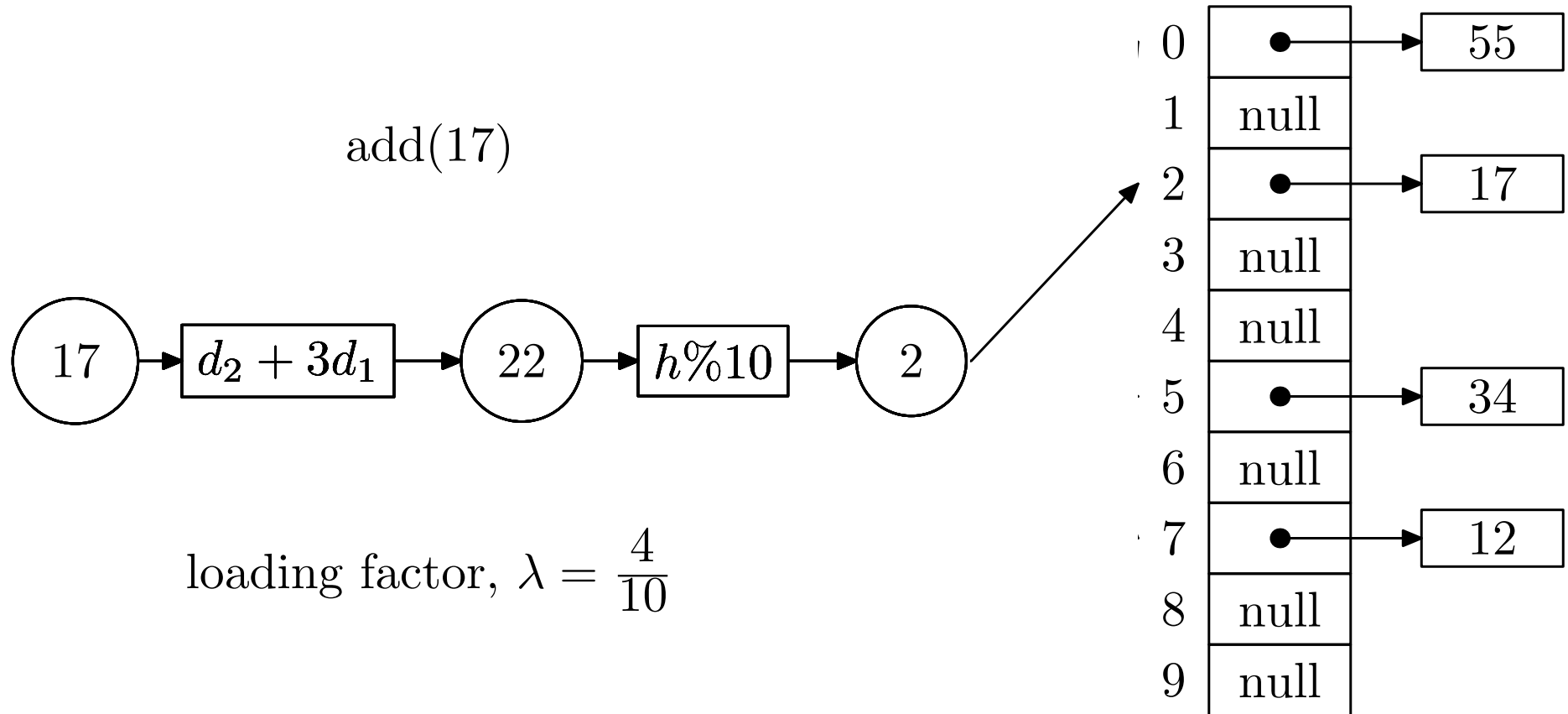
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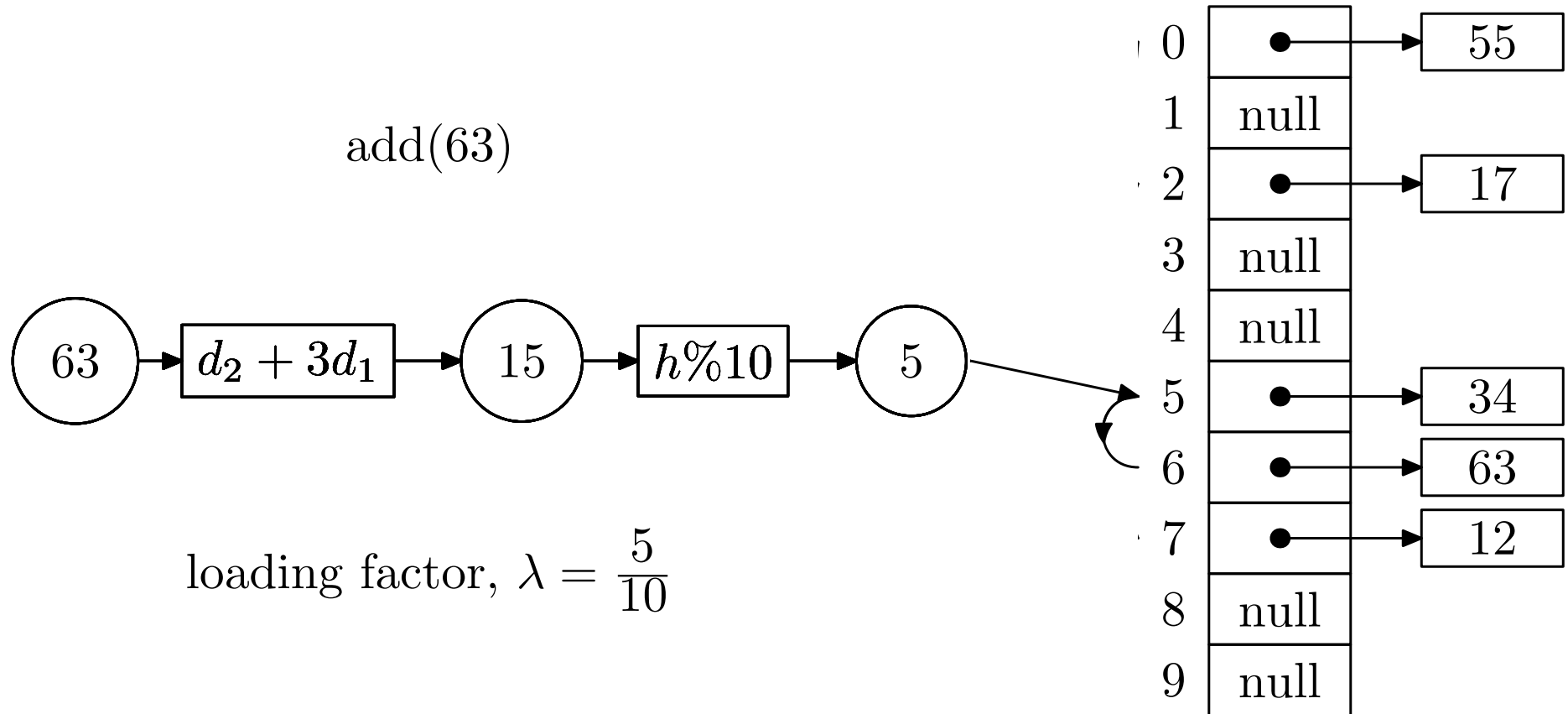
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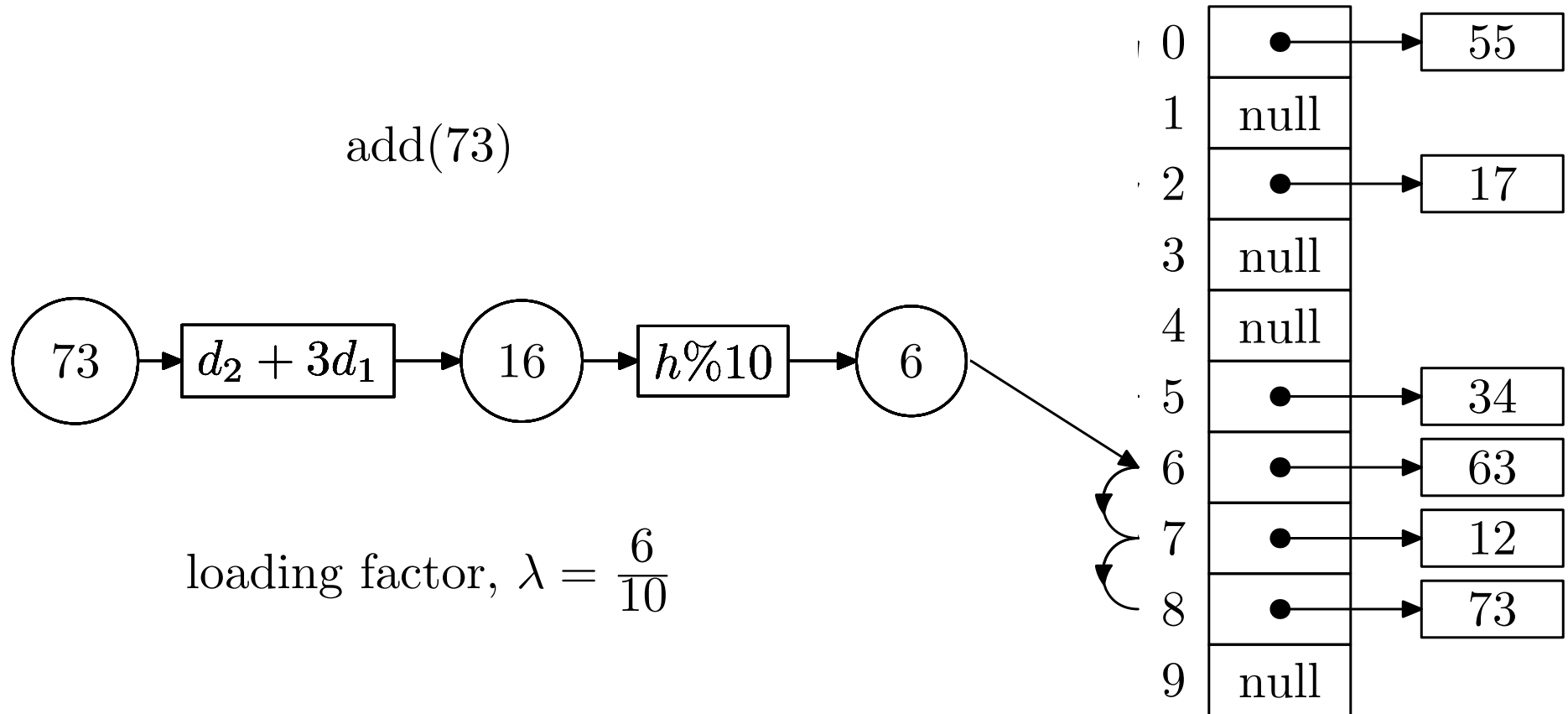
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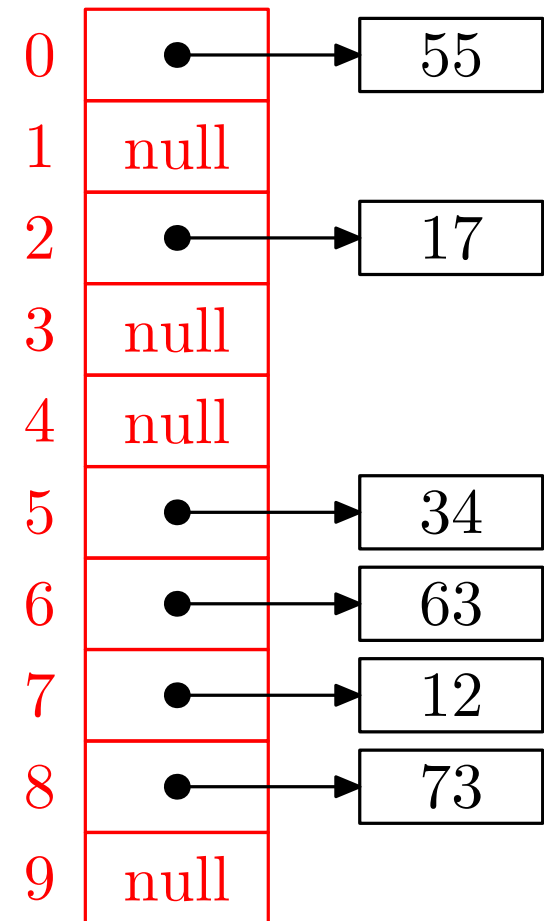


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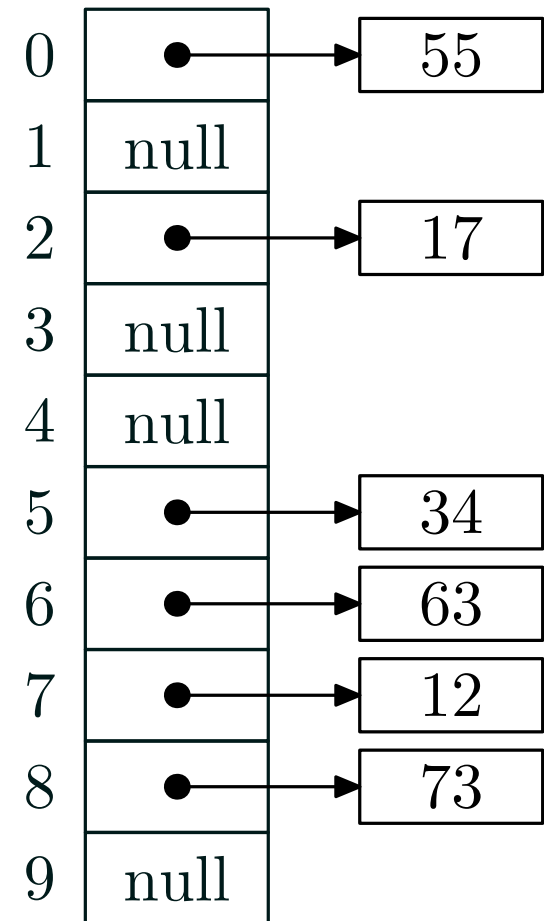
Linear Probing Pile Up

- The entries will tend to pile up or cluster—this is sometimes referred to as **primary clustering**
- Clusters become worse as the number of entries grow
- Clusters will increase the number of probes needed to find an insert location
- The proportion of full entries in the table is known as the **loading factor**



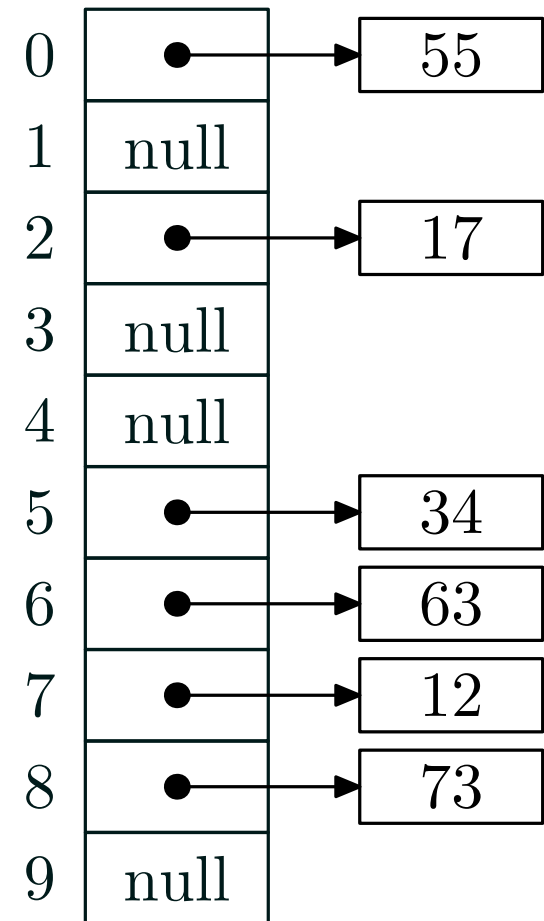
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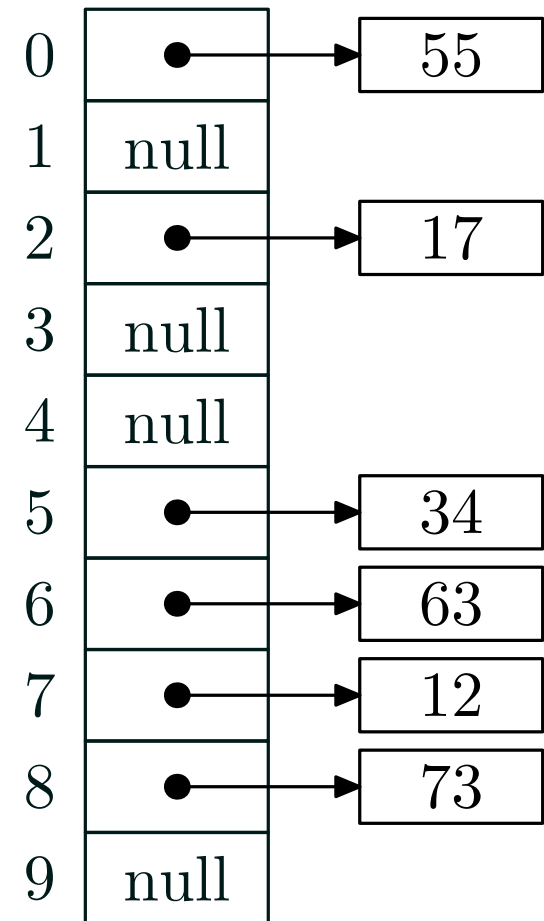
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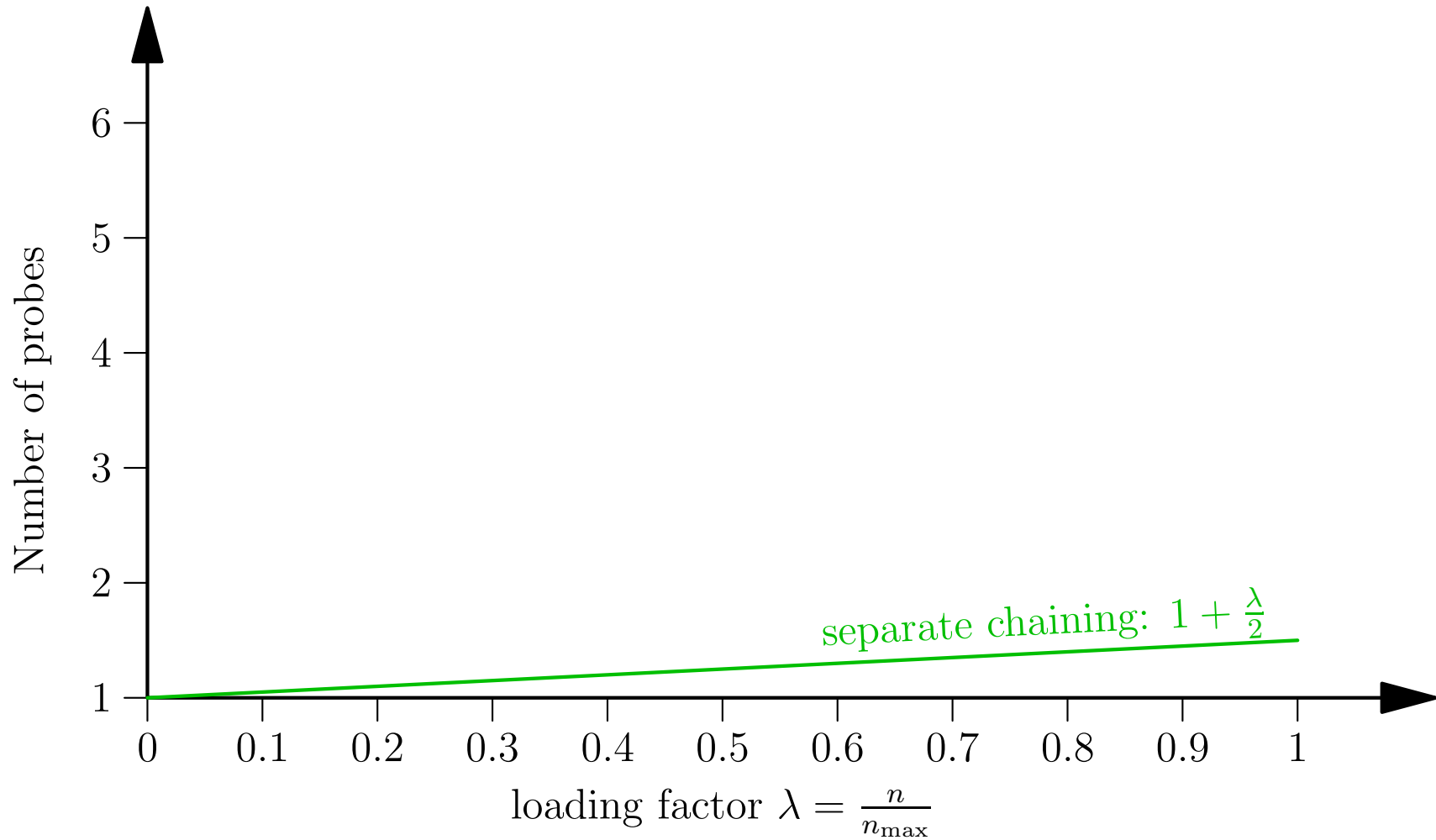


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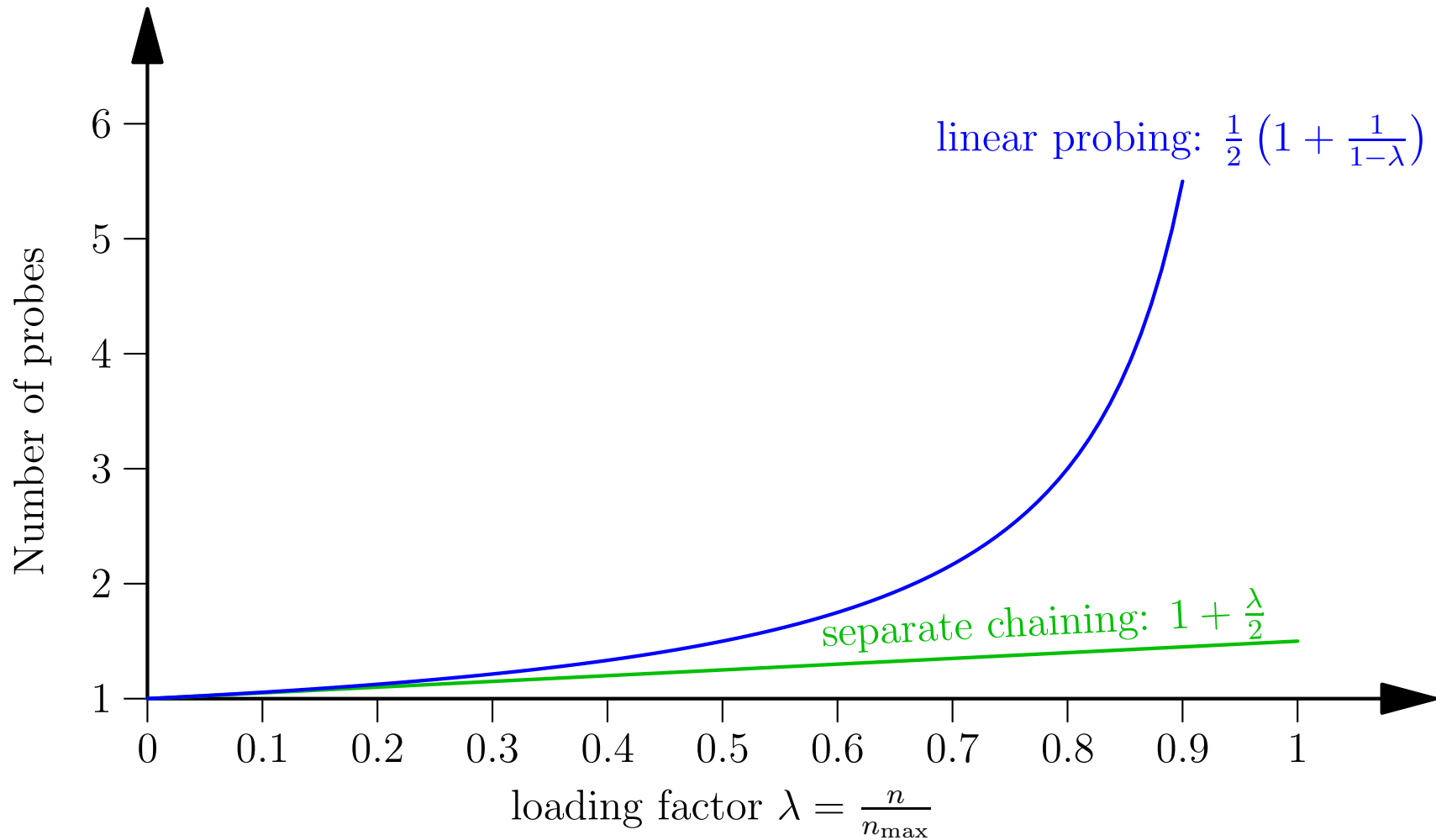
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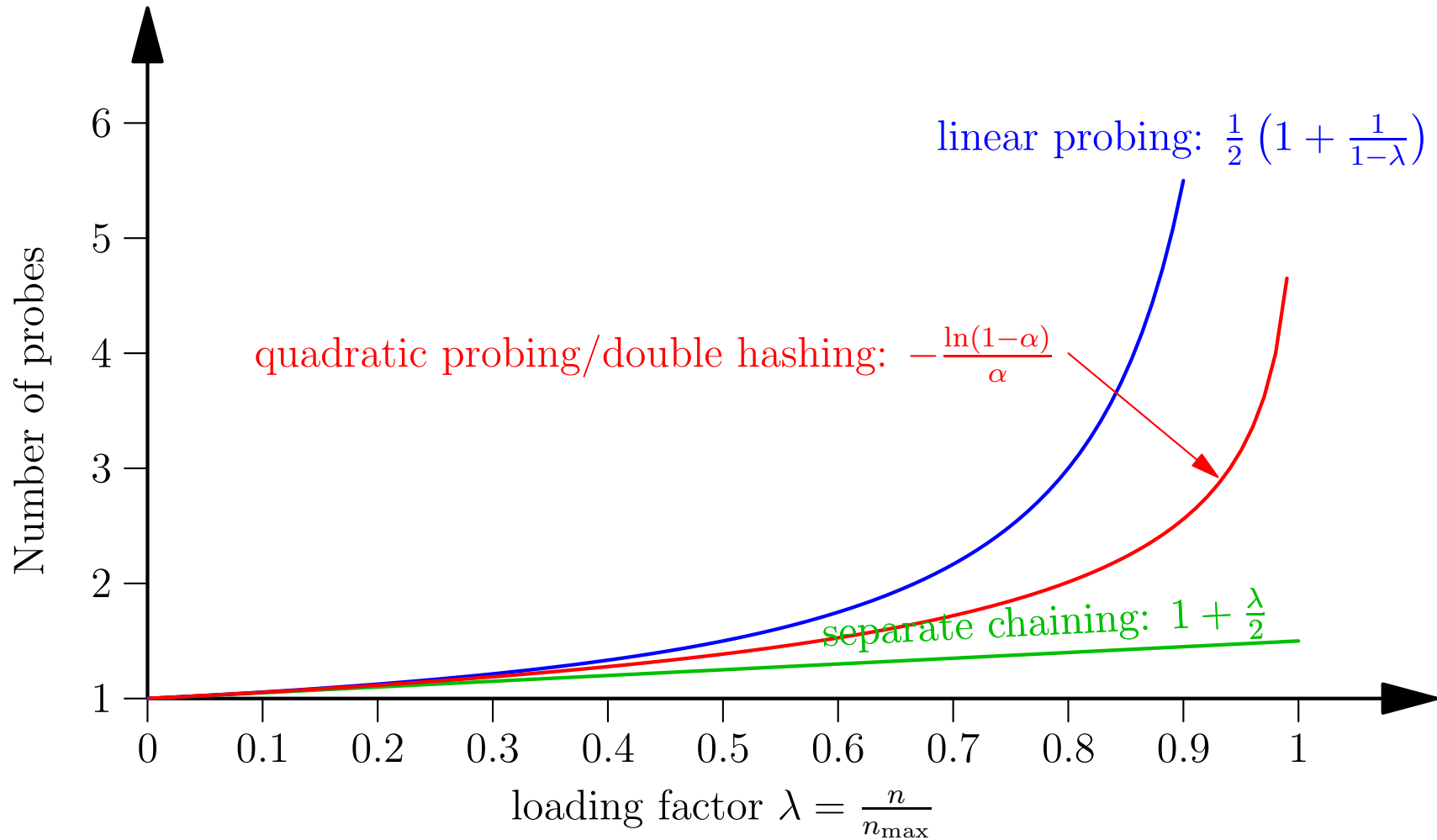
Reducing Number of Probes



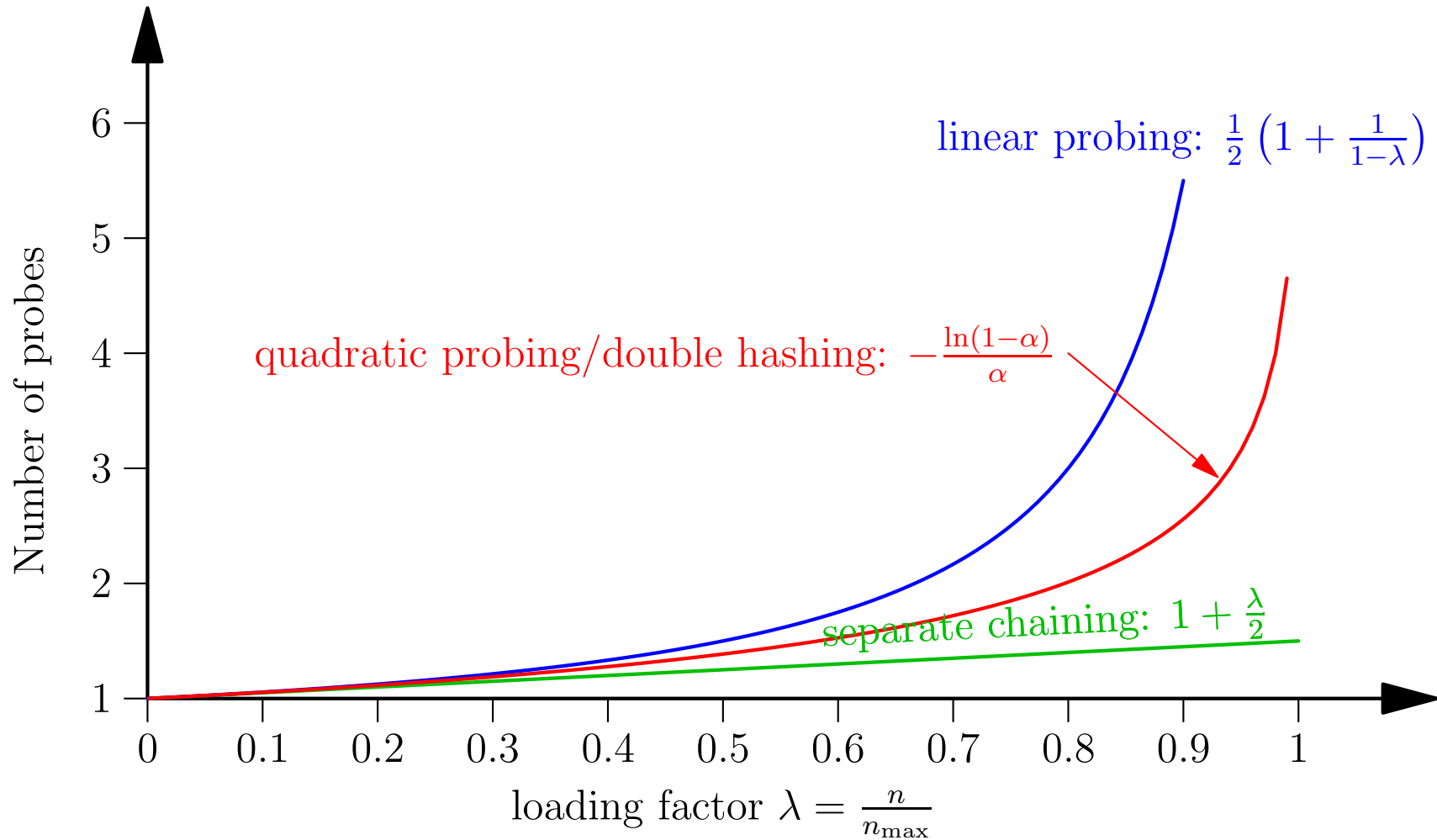
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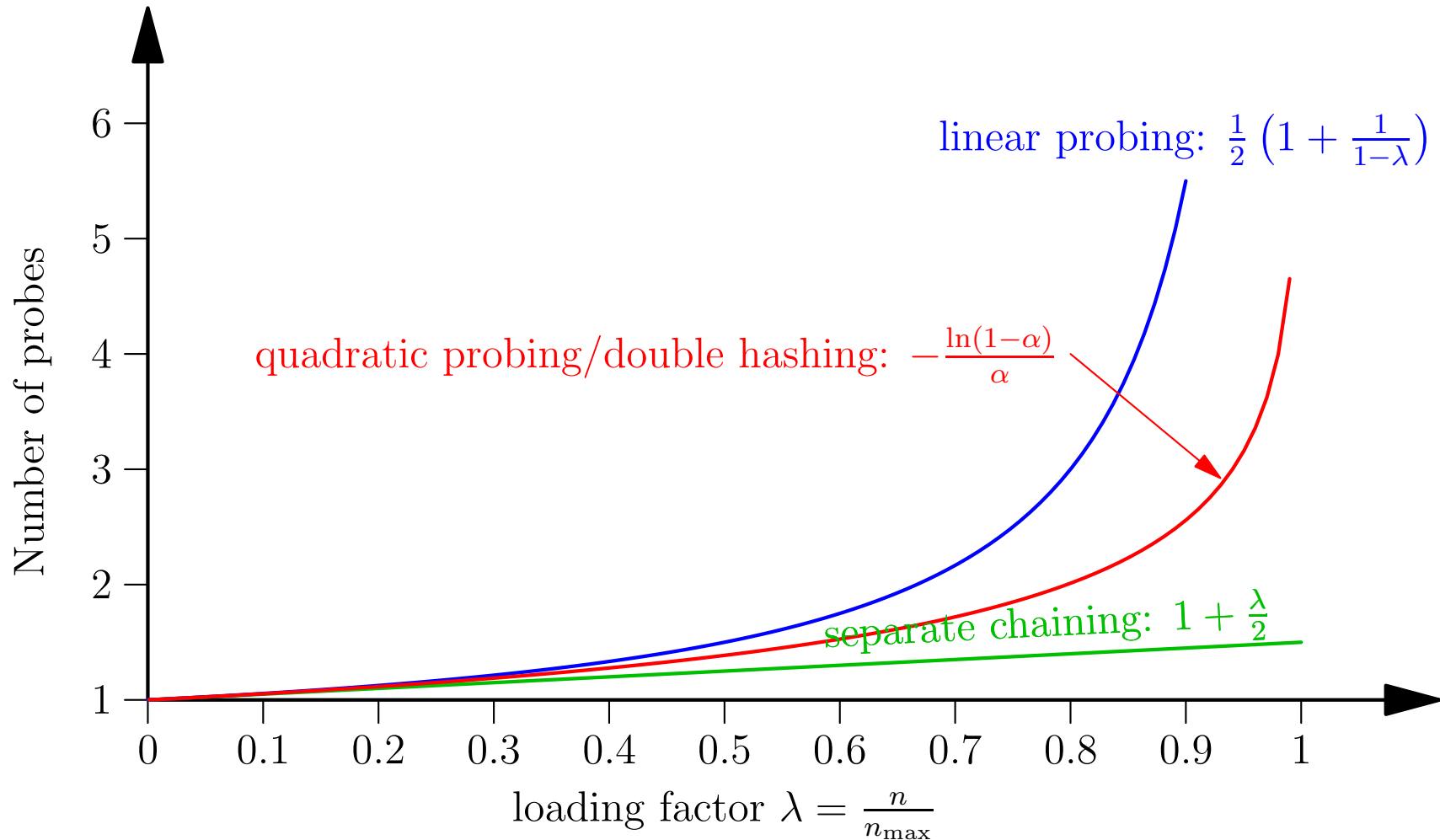


Reducing Number of Probes



- To avoid clustering we can use **quadratic probing**

Reducing Number of Probes



- To avoid clustering we can use **quadratic probing** or **double hashing**

Quadratic Probing

- In quadratic probing we try the locations $h(x) + d_i$ where $h(x)$ is the original hash code and $d_i = i^2$
- That is we takes steps 1, 4, 9, 16, . . .
- Quadratic probing prevents primary clustering so dramatically decreases the number of probes needed to find a free location when the table is reasonably full
- One problem is that if we are unlucky we might not be able to add an element to the hash table even if the table isn't full
- However, if the size of the table is prime then quadratic probing will always find a free position provided it is not more than half full

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Double Hashing

- An alternative strategy is to known as double hashing where the locations tried are $h(x) + d_i$ where $d_i = i \times h_2(x)$
- $h_2(x)$ is a second hash function that depends on the key
- A good choice is $h_2(x) = R - (x \bmod R)$ where R is a prime smaller than the table size
- It is important that $h_2(x)$ is not a divisor of the table size
 - ★ Either make sure the table size is prime or
 - ★ Set the step size to 1 if $h_2(x)$ is a divisor of the table size

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Problems with Remove

- For all open addressing hash systems removing an entry is a problem
- Remember our strategy to find an input x is
 1. Compute the array index based on the hash code of x
 2. If the array location is empty then the search fails
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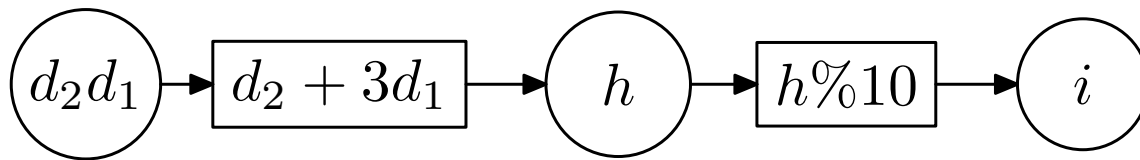
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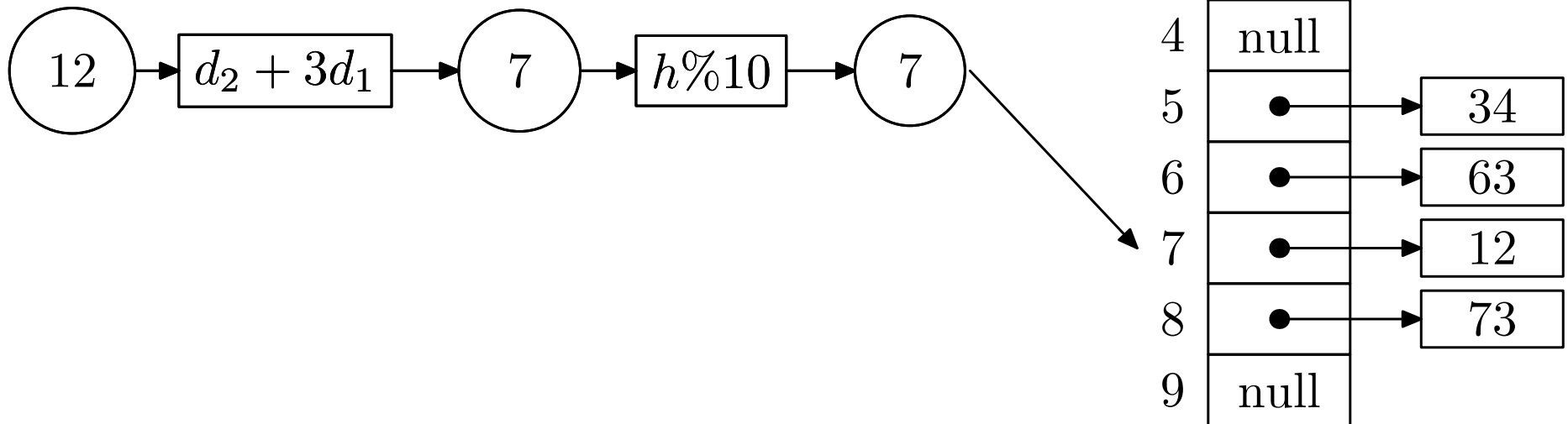
Linear Probing Example



0	●	→	55
1	null		
2	●	→	17
3	null		
4	null		
5	●	→	34
6	●	→	63
7	●	→	12
8	●	→	73
9	null		

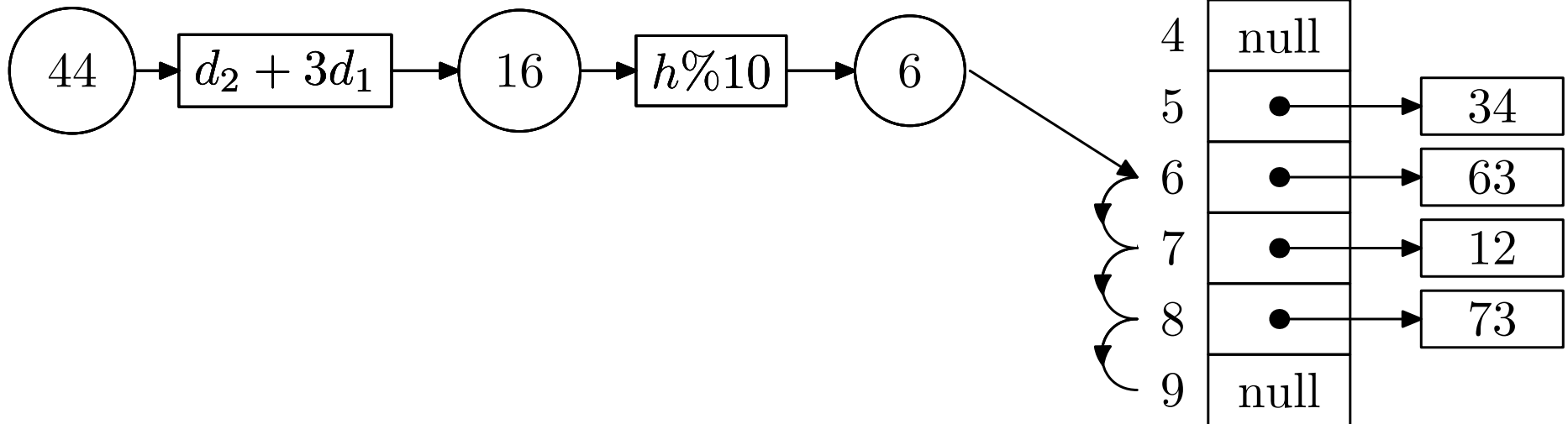
Linear Probing Example

$\text{find}(12) \rightarrow \text{true}$



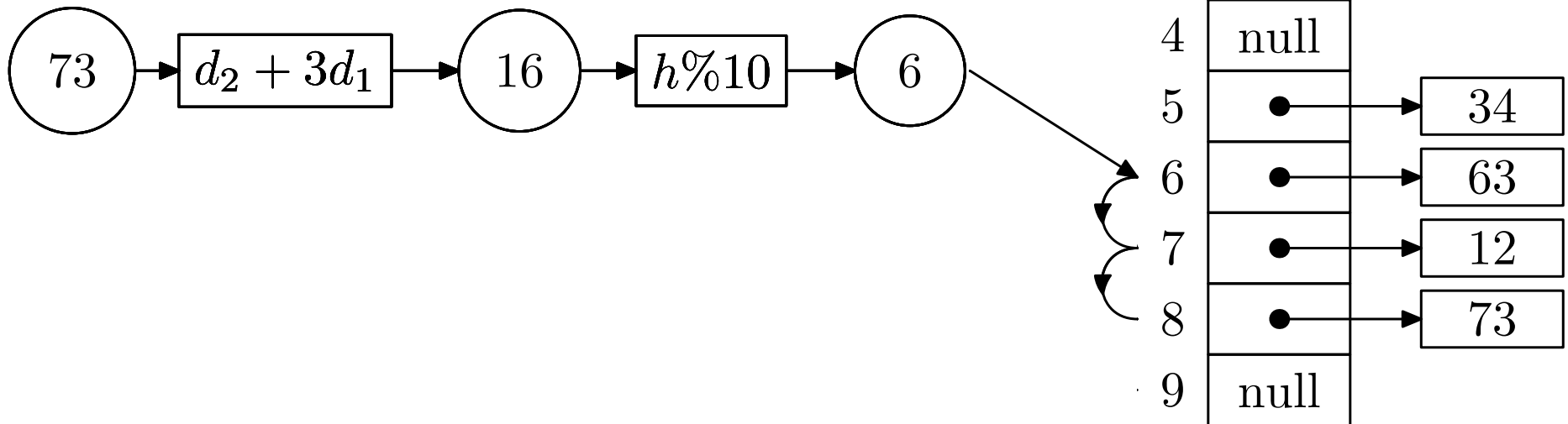
Linear Probing Example

find(44) → fail



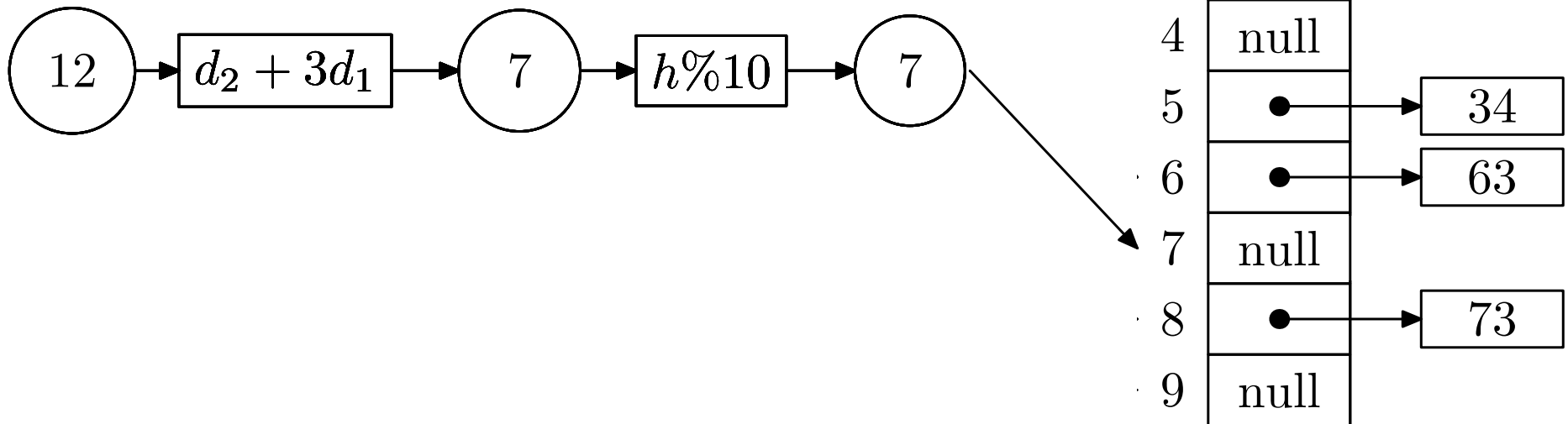
Linear Probing Example

find(73) → true



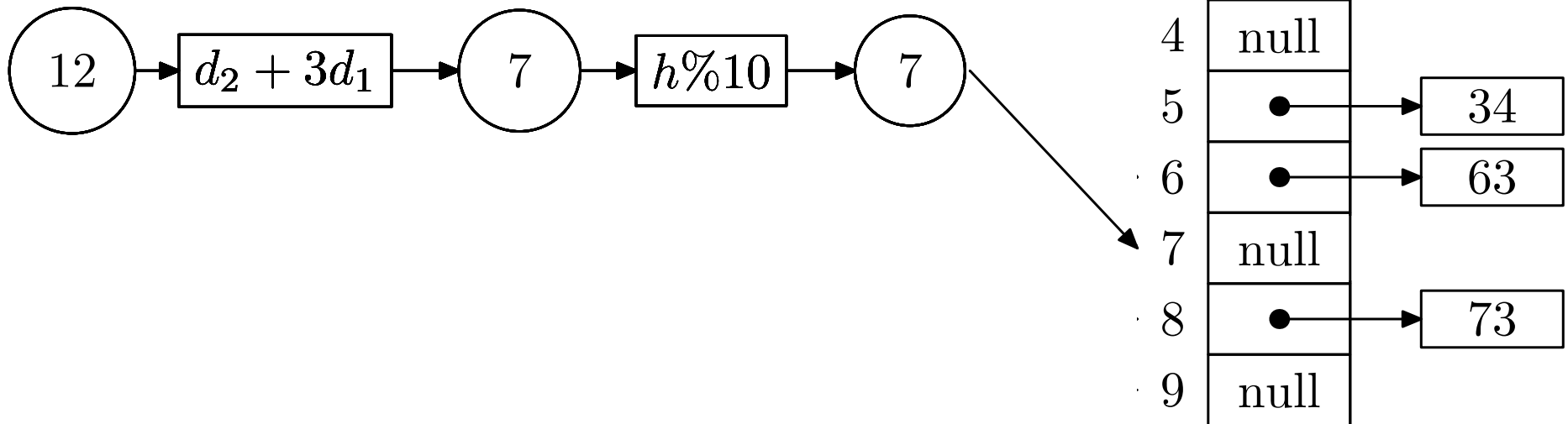
Linear Probing Example

$\text{delete}(12) \rightarrow \text{true}$



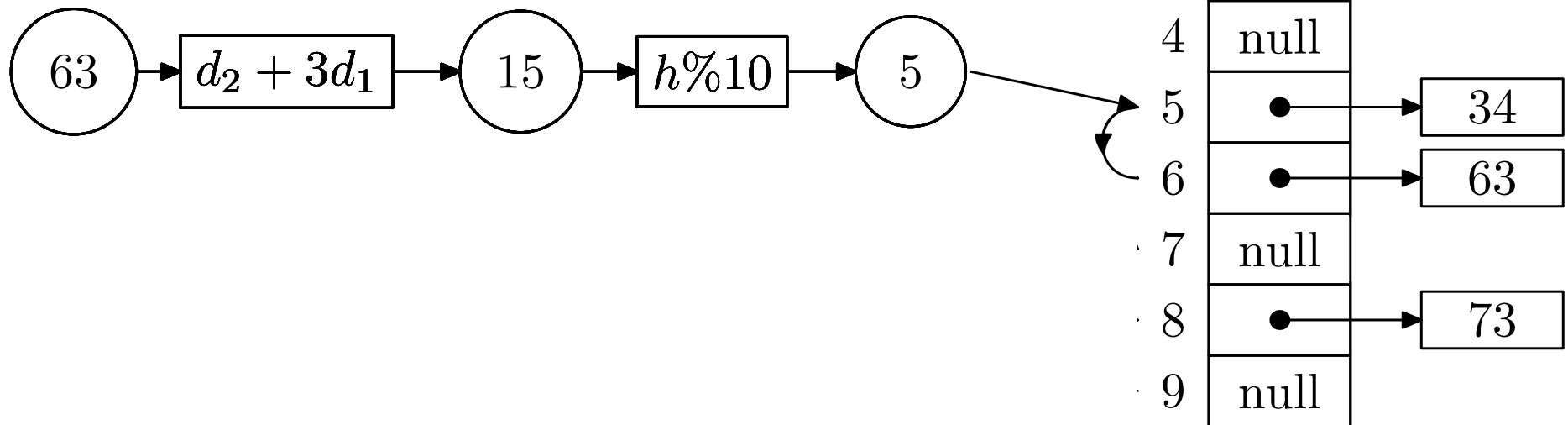
Linear Probing Example

find(12) → fail



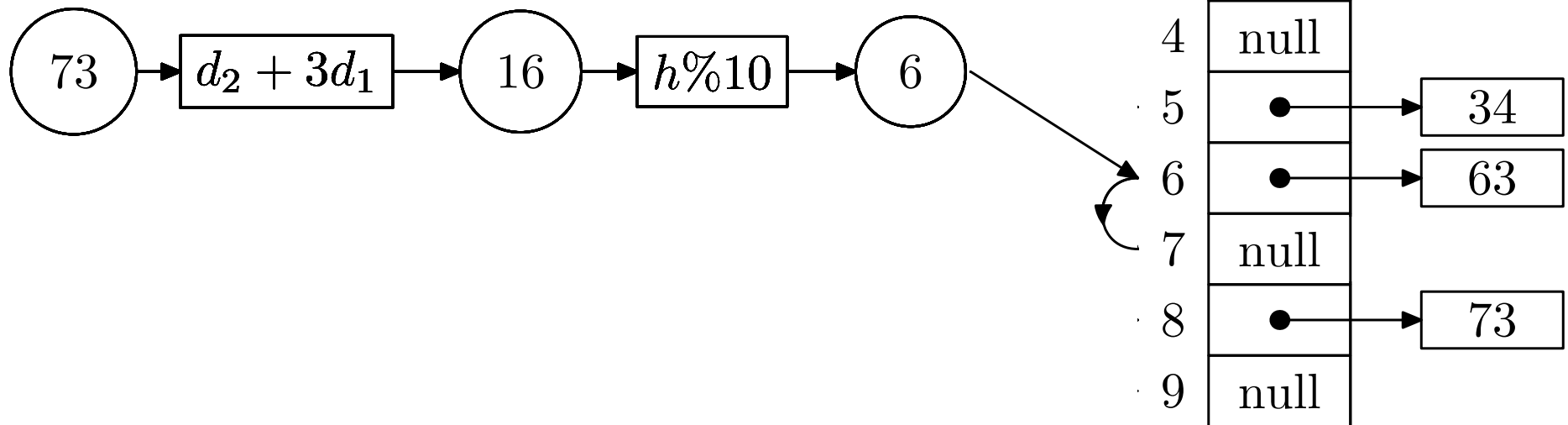
Linear Probing Example

$\text{find}(63) \rightarrow \text{true}$



Linear Probing Example

find(73) → fail



Lazy Remove

- One easy fix is to mark the deleted table with a special entry
- A find method would consider this entry as full
- An iterator would ignore this entry
- An insert operator could insert a new entry in these special locations

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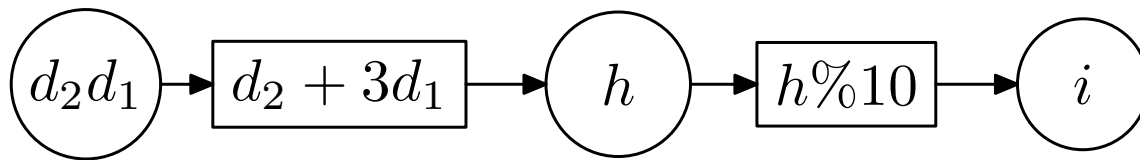
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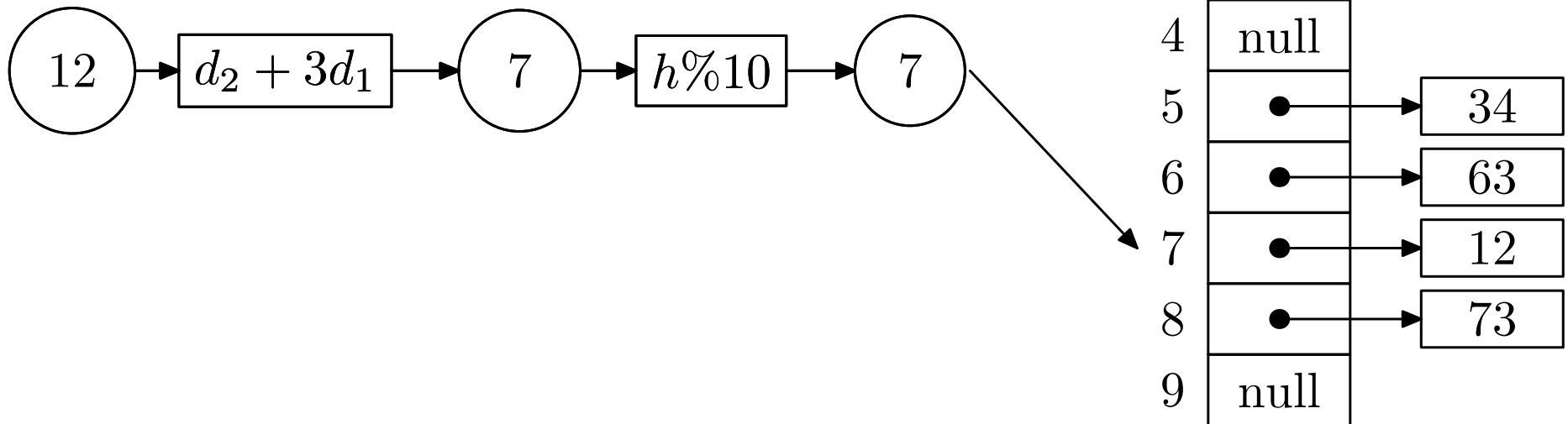
Lazy Remove in Action



0	●	→	55
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3	null		
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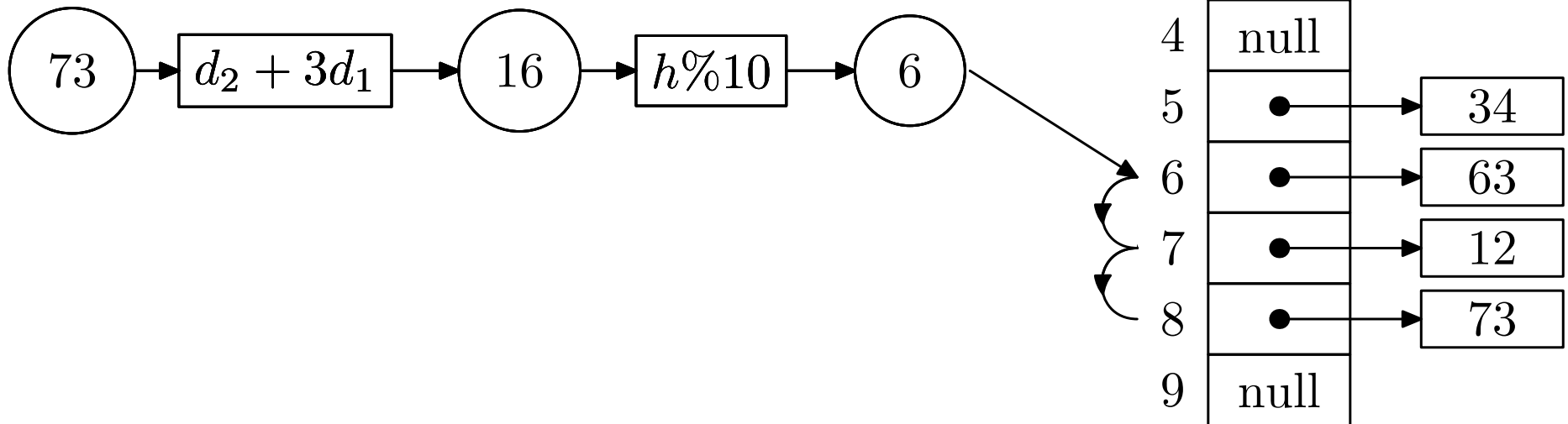
Lazy Remove in Action

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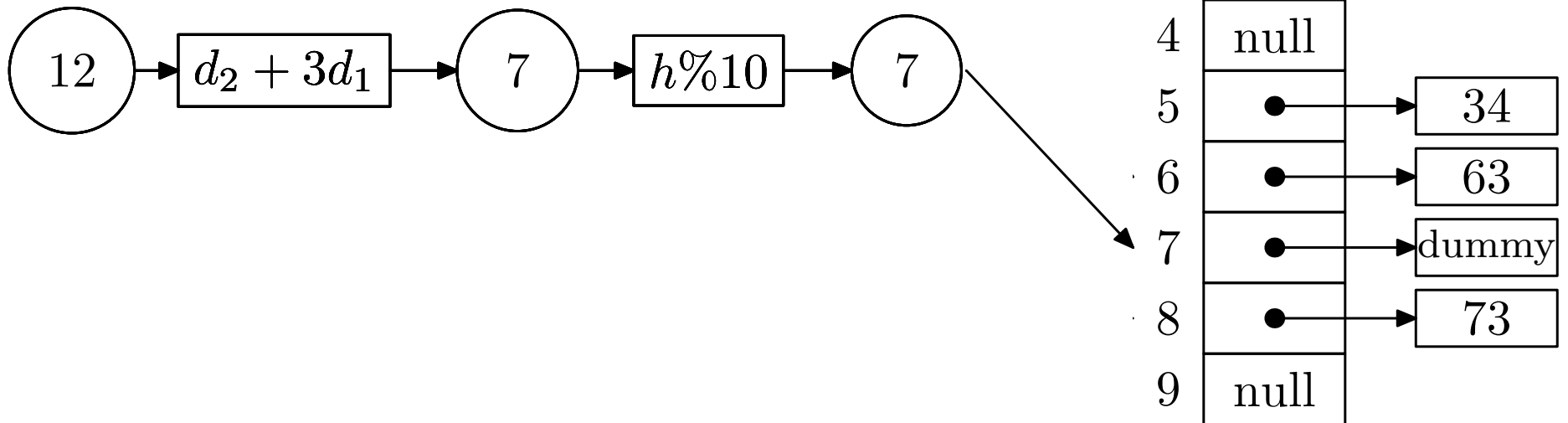
Lazy Remove in Action

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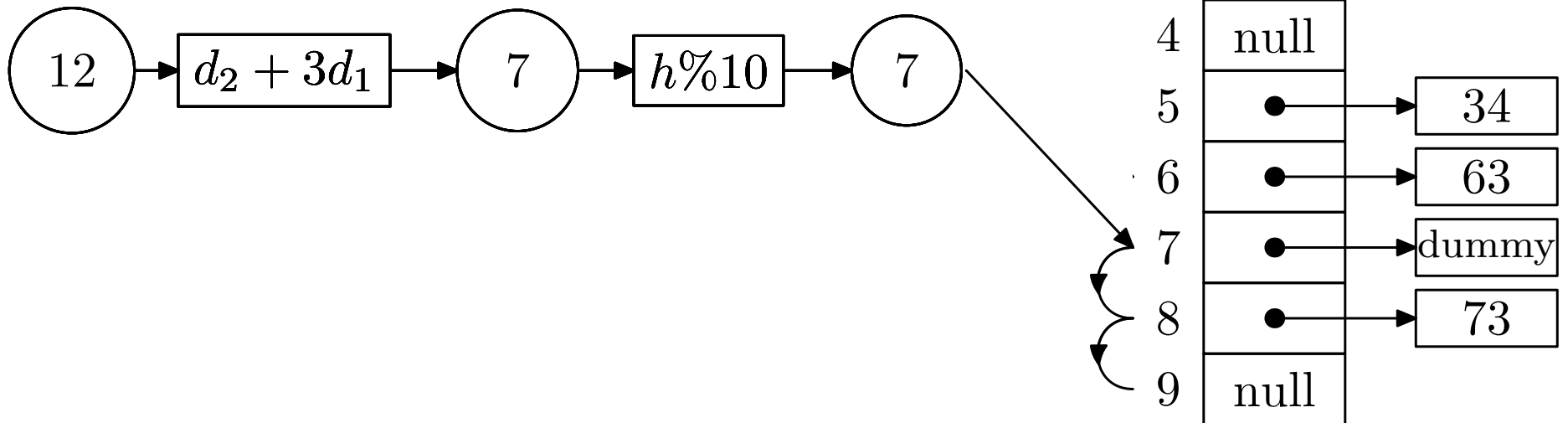
Lazy Remove in Action

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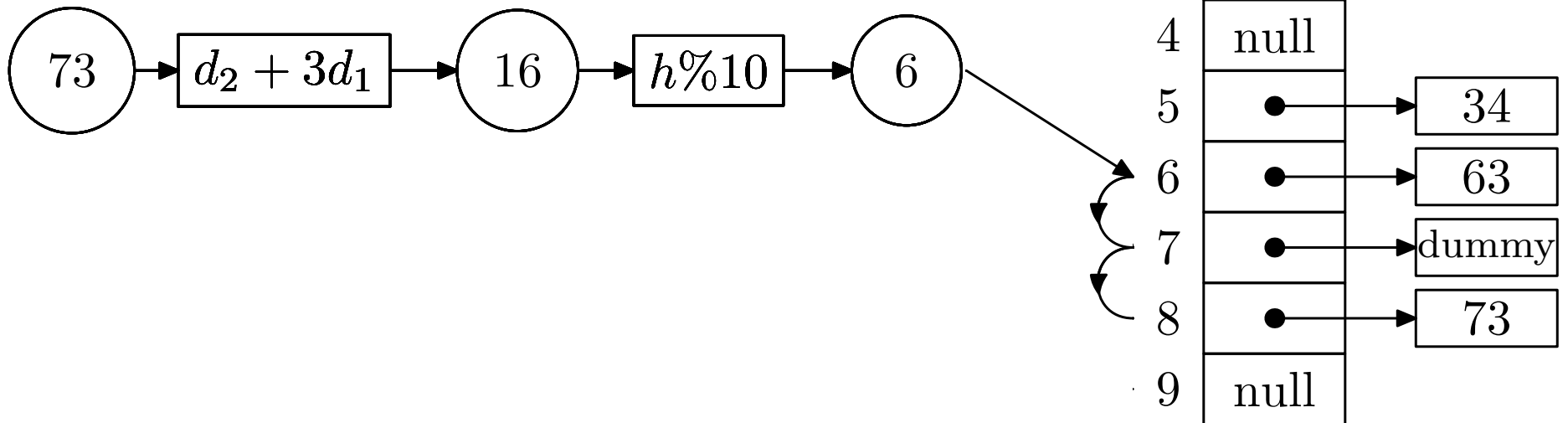
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find(12) → fail



Lazy Remove in Action

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What Strategy to Use?

- Most libraries including the STL (and the Java Collection class) use separate chaining
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Hash Sets and Maps

- C++ also provides an `unordered_map<Key, V>` class
- It's performance is asymptotically superior to `map`, $O(1)$ rather than $O(\log(n))$
- Hash functions can take time to compute (it is often $O(\log(n))$) so `unordered_sets` might not be faster than `sets`
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Applications

- Hash tables are used everywhere
- E.g. most databases use hash tables to speed up search
- In many document applications hash tables will be being generated in the background
- Content addressability is ubiquitous to many application where hash tables are used as standard

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