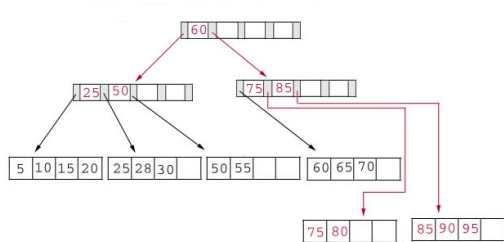


## Lesson 9: Sometimes It Pays Not to Be Binary



B-Trees, Tries, Suffix Trees

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### B-Trees

- **B-trees** are balanced trees for fast search, finding successors and predecessors, insert, delete, maximum, minimum, etc.■
- Not to be confused with binary trees■
- They are designed to keep related data close to each other in (disk) memory to minimise retrieval time■
- Important when working with large amount of data that is stored on secondary storage (e.g. disks)■
- Used extensively in databases■

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### Accessing Data from Disk

- When accessing data from disk minimising the number of disk accesses is critical for good performance■
- In database applications we want to store data as large sets■
- Storing data in binary trees is disastrous as we typically need around  $\log_2(n)$  disk accesses before we locate our data■
- It is not unusual in databases for  $n = 10\,000\,000$  so that  $\log_2(n) \approx 24$ ■
- Using binary trees it would often take several seconds to find a record■

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### B<sup>+</sup> Tree

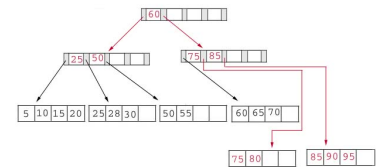
- A pretty basic implementation would obey the following rules
  1. The data items are stored at leaves■
  2. The non-leaf nodes store up to  $M-1$  keys to guide the search: key  $i$  represents the smallest key in subtree  $i + 1$ ■
  3. The root is either a leaf or has between 2 and  $M$  children■
  4. All non-leaf nodes except the root have between  $\lceil M/2 \rceil$  and  $M$  children■
  5. All leaves are at the same depth and have between  $\lceil L/2 \rceil$  and  $L$  data entries■

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1. **B-Trees**
2. **Tries**
3. **Suffix Tree**



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### When Big-O Doesn't Work

- An underlying assumption of Big-O is that all elementary operations take roughly the same amount of time■
- This just isn't true of disk look-up■
- The typical time of an elementary operation on a modern processor is  $10^{-9}$  seconds■
- But a typical hard disk might do 7200 revolutions per minute or 120 revolutions per second■
- The typical time it takes to locate a record is around 10ms or  $10^7$  times slower than an elementary operation■

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### Multiway-Trees

- To remedy this we can use  $M$ -way trees so that the access time is

$$\log_M(n) = \frac{\log_2(n)}{\log_2(M)}$$

- In practice we might use  $M \approx 200 \approx 2^8$  so we can reduce the depth of the tree by around a factor of 8■
- The basic data structures for doing this is the B-tree■
- There are many variants of B-tree, all trying to squeeze a bit more performances from the basic structure■

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### Choosing $M$ and $L$

- The choice of  $M$  and  $L$  depends on the block size (the information read in one go from disk)■
- It also depends on the type of data that is being stored (integer, reals, strings, etc.)■
- $M$  and  $L$  might be in the hundreds or thousands■
- In the examples below we consider tiny  $M = L = 5$  which is unrealistic, but drawable■

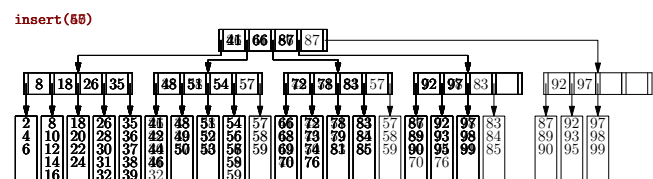
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## B-Tree Example

- $M = 5, L = 5$



## Disadvantage of Tries

- Table-based tries typically waste large amounts of memory
- Often table-based tries are used for the first few layers, while lower levels use a less memory intensive data structure
- These days memory is less of a problem so table-based tries are acceptable for some applications
- There are many implementations of tries each suited to a particular task

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## Why Tries?

- Tries are a classic example of a trade-off between memory and computational complexity
- Tries are slightly specialist and tend to get used in very particular applications
  - ★ Finding longest matches
  - ★ Completion, spell checking, etc.
- A basic trie is not too complicated, however, . . .
- There are many implementation which try to overcome the difficulty of wasting too much memory

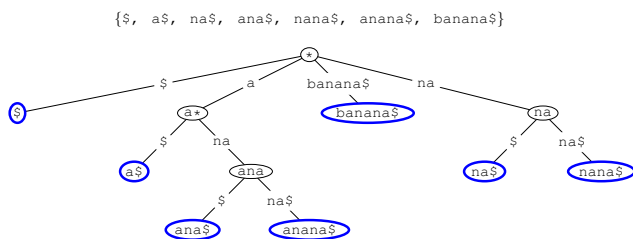
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## Suffix Tree

- Suffix tree is a trie of all suffixes of a string
- E.g. banana



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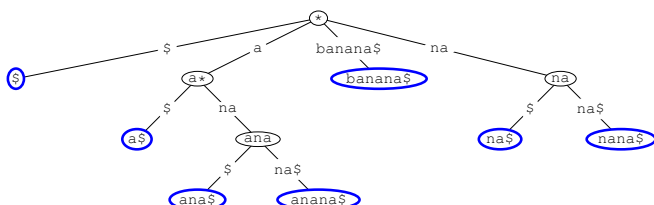
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## String Matching

- To find a match of a query string,  $Q$ , in a text,  $T$ , we can first construct the suffix tree of the string  $T$  we then simply look up the query,  $Q$ , using the trie

{\$, a\$, na\$, ana\$, nana\$, anana\$, banana\$}



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## Binary Tries

- One extreme (though not uncommon) solution to address memory issues is to build a bit-level trie so the data-structure is a binary tree
- It differs from a binary tree in that the decisions to go left or right depends on the current bit
- Although you lose the advantage of a multiway tree (of reducing the depth) it does find the longest match and it speeds up finds which fail

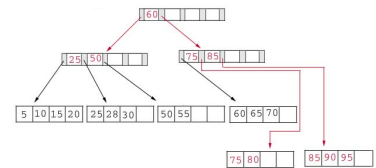
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## Outline

1. B-Trees
2. Tries
3. Suffix Tree



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## Importance of Suffix Tree

- The first linear-time algorithm for computing suffix trees was proposed by Peter Weiner in 1973, a more space efficient algorithm was proposed by Edward M. McCreight in 1976
- Esko Ukkonen in 1995 proposed a variant of McCreight's algorithm, but in a way that was much easier to understand
- It really only got implemented after this
- They are very important for string-based algorithms
- The classic application is in finding a match for a query string,  $Q$ , in a text,  $T$

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## Complexity of Suffix Trees

- Using a regular trie for a suffix tree would typically use far too much memory to be useful
- However, by using pointers to the original text it is possible to build a suffix tree using  $O(n)$  memory where  $n$  is the length of the text
- Furthermore (and rather incredibly) there is a linear time ( $O(n)$ ) algorithm to construct the trie
- The algorithm is not however trivial to understand

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- Suffix trees are efficient whenever it is likely that you will do multiple searches
- Exact word matching is in itself a very important application
- Suffix trees in combination with dynamic programming (which we will eventually get to) can be used to do inexact matching (finding the match with the smallest edit distance)
- Suffix trees get used in bioinformatics, advanced machine learning algorithms, . . .

- Multiway trees can considerably speed up search over binary trees
- They are very important in some specialised applications (e.g. databases, spell-checking, completion, suffix trees)
- They are not as general purpose as binary search trees and are more complicated to implement
- But they can give the best performance—sometimes performance matters enough to make it worthwhile implementing multiway trees