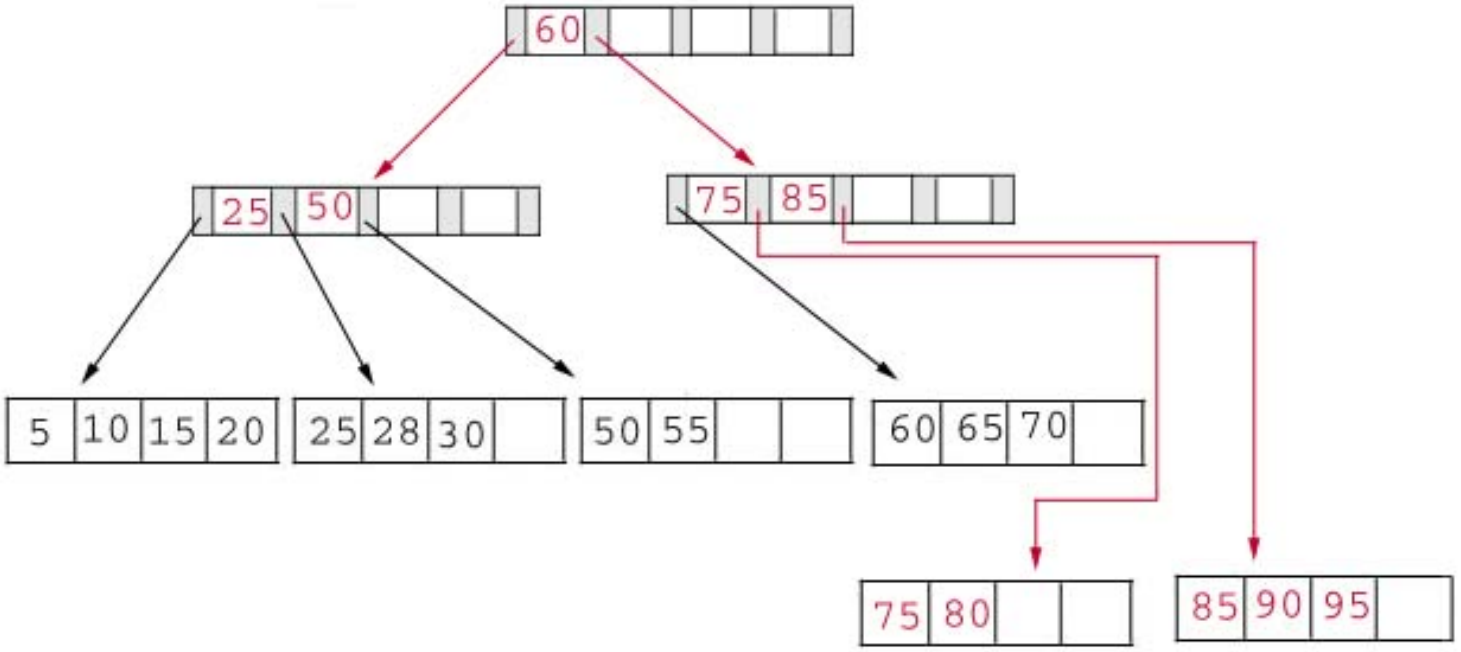
Algorithms and Analysis

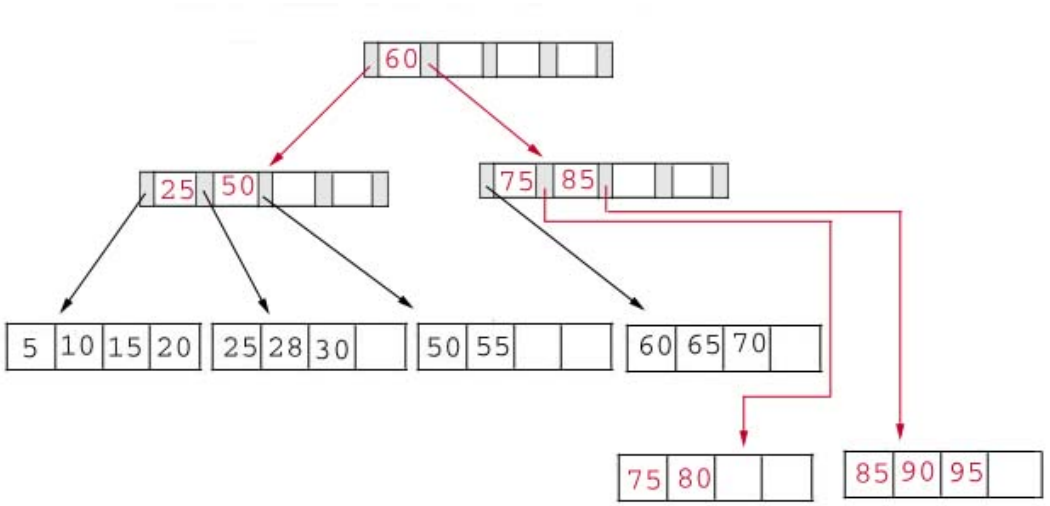
Lesson 9: *Sometimes It Pays Not to Be Binary*



B-Trees, Tries, Suffix Trees

Outline

1. **B-Trees**
2. Tries
3. Suffix Tree



B-Trees

- **B-trees** are balanced trees for fast search, finding successors and predecessors, insert, delete, maximum, minimum, etc.
- Not to be confused with binary trees
- They are designed to keep related data close to each other in (disk) memory to minimise retrieval time
- Important when working with large amount of data that is stored on secondary storage (e.g. disks)
- Used extensively in databases

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When Big-O Doesn't Work

- An underlying assumption of Big-O is that all elementary operations take roughly the same amount of time
- This just isn't true of disk look-up
- The typical time of an elementary operation on a modern processor is 10^{-9} seconds
- But a typical hard disk might do 7 200 revolutions per minute or 120 revolutions per second
- The typical time it takes to locate a record is around 10ms or 10^7 times slower than an elementary operation

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Accessing Data from Disk

- When accessing data from disk minimising the number of disk accesses is critical for good performance
- In database applications we want to store data as large sets
- Storing data in binary trees is disastrous as we typically need around $\log_2(n)$ disk accesses before we locate our data
- It is not unusual in databases for $n = 10\,000\,000$ so that $\log_2(n) \approx 24$
- Using binary trees it would often take several seconds to find a record

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Multiway-Trees

- To remedy this we can use M-way trees so that the access time is

$$\log_M(n) = \frac{\log_2(n)}{\log_2(M)}$$

- In practice we might use $M \approx 200 \approx 2^8$ so we can reduce the depth of the tree by around a factor of 8
- The basic data structures for doing this is the B-tree
- There are many variants of B-tree, all trying to squeeze a bit more performances from the basic structure

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$\mathbf{B^+}$ Tree

- A pretty basic implementation would obey the following rules

1. The data items are stored at leaves
2. The non-leaf nodes store up to M-1 keys to guide the search: key i represents the smallest key in subtree $i + 1$
3. The root is either a leaf or has between 2 and M children
4. All non-leaf nodes except the root have between $\lceil M/2 \rceil$ and M children
5. All leaves are at the same depth and have between $\lceil L/2 \rceil$ and L data entries

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Choosing M and L

- The choice of M and L depends on the block size (the information read in one go from disk)
- It also depends on the type of data that is being stored (integer, reals, strings, etc.)
- M and L might be in the hundreds or thousands
- In the examples below we consider tiny $M = L = 5$ which is unrealistic, but drawable

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- $M = 5,\ L = 5$

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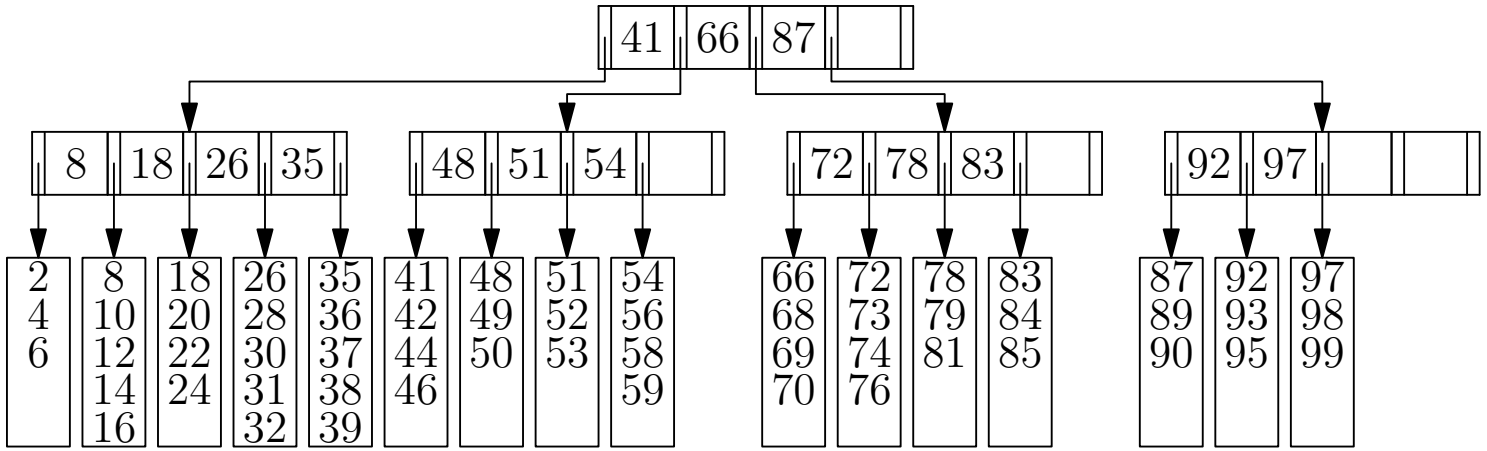
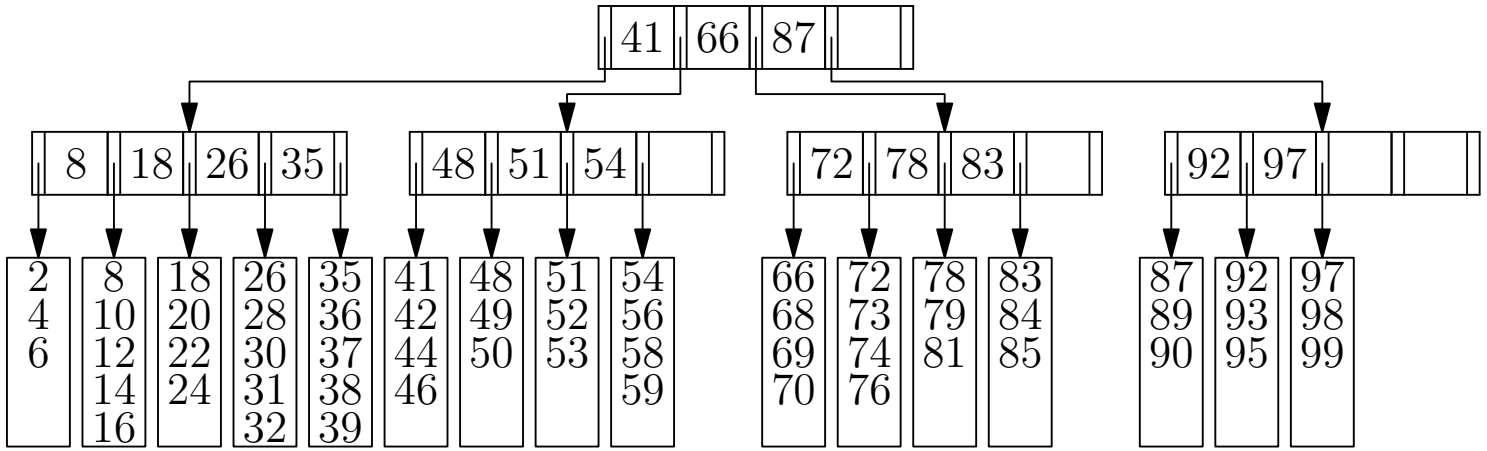


B-Tree Example

- $M = 5,\ L = 5$

`insert(57)`



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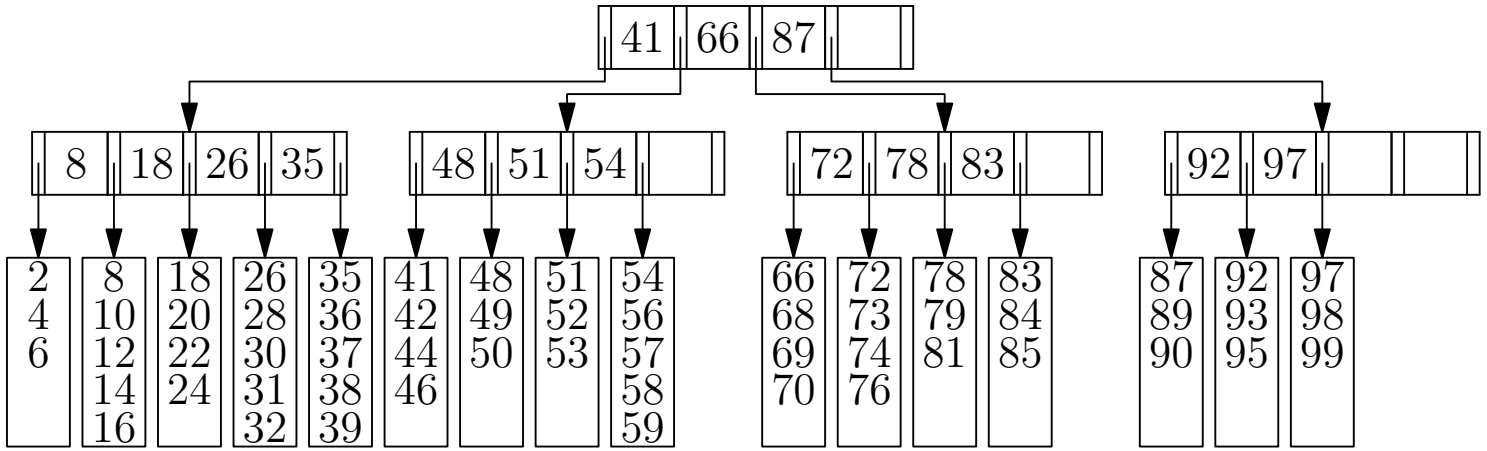

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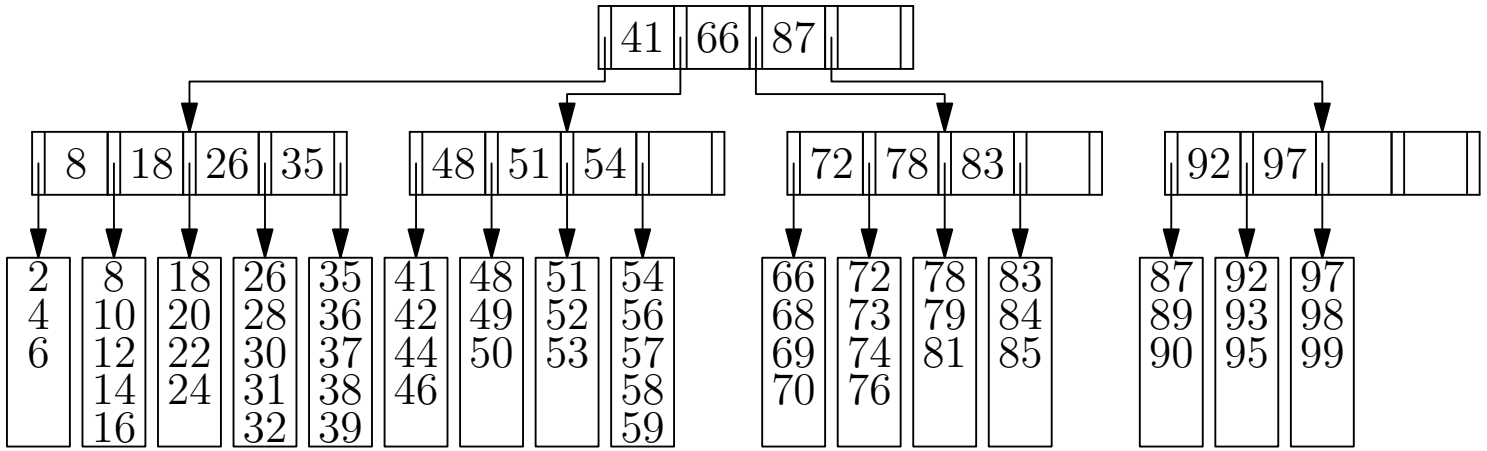
`insert(55)`



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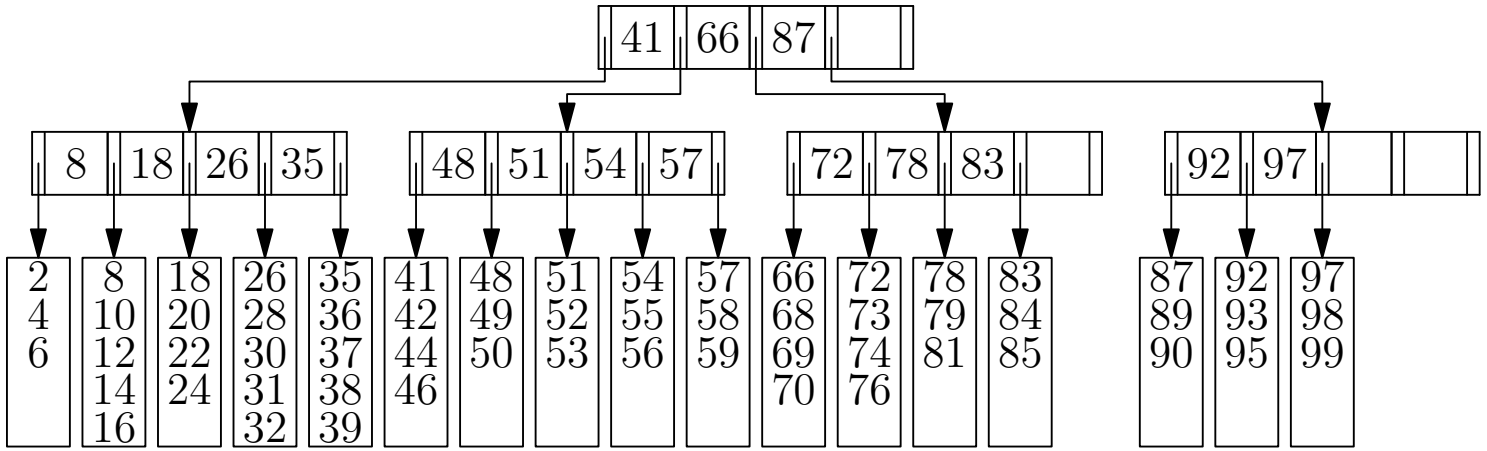

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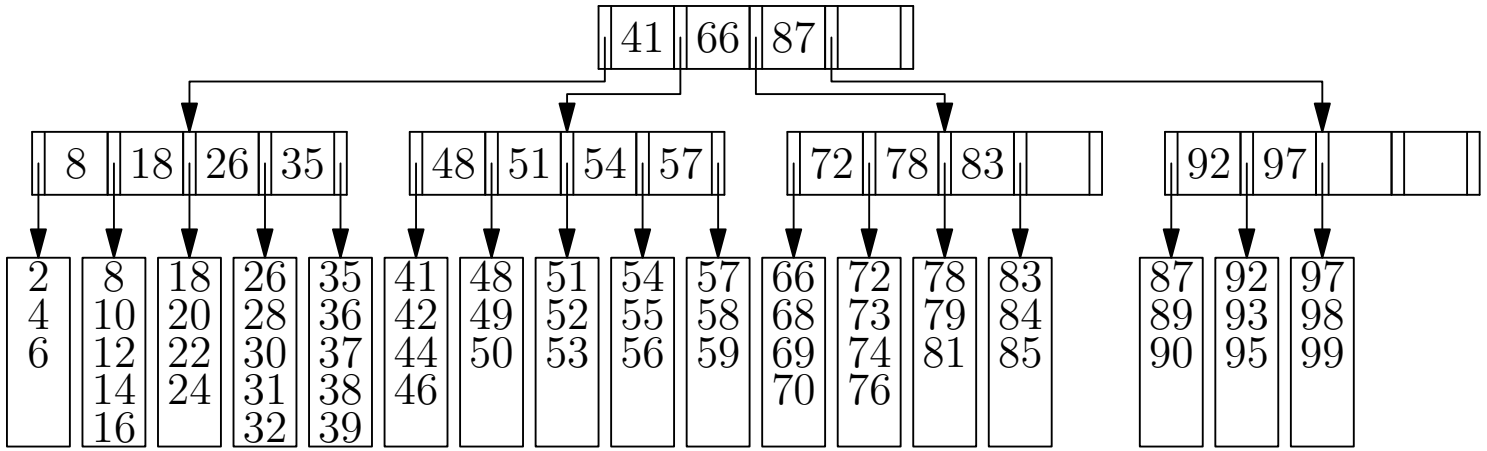
- $M = 5,\ L = 5$

`insert(40)`



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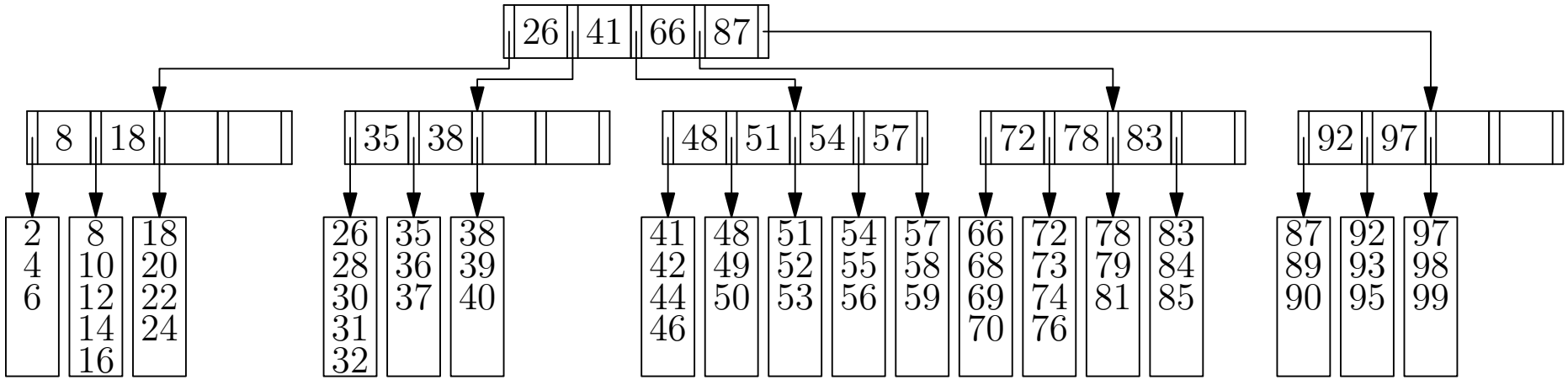

Other Changes

- If the root is full then it can be split into two and a new root created
- B-trees also have to allow the removal of records without losing its structure
- There are a number of variant strategies (e.g. neighbouring nodes can adopt a child if the current node cannot expand any more)
- The actual implementation of B-trees is tricky because there are many special cases

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B-Tree Summary

- B-trees are an important data structure for databases where reducing the number of disk searches is vital
- They tend to be much more complex than the other data structures we have seen
- The problem of disk access can be improved by replacing disk memory with solid-state drives (still slow compared to memory)
- For massive databases new data structures have been developed to allow faster (although less flexible) information access (e.g. NOSQL, MongoDB, Neo4j)

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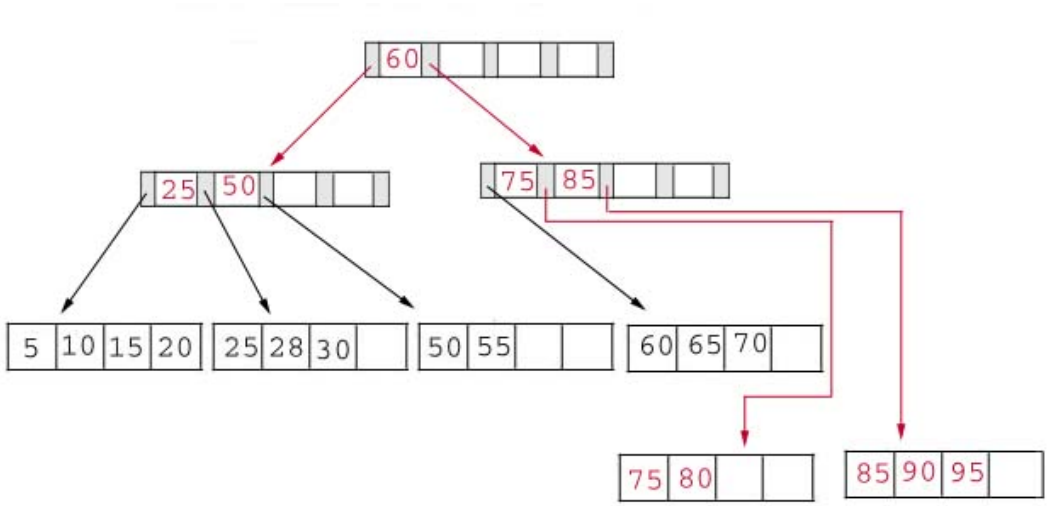

Tries

- A **Trie** (pron. 'try') or **digital tree** is a multiway tree often used for storing large sets of words
- They are trees with a possible branch for every letter of an alphabet
- Their names comes from the word $re\boldsymbol{trie}val$
- Tries usually compactify the edges in the tree
- All words end with a special letter "$"

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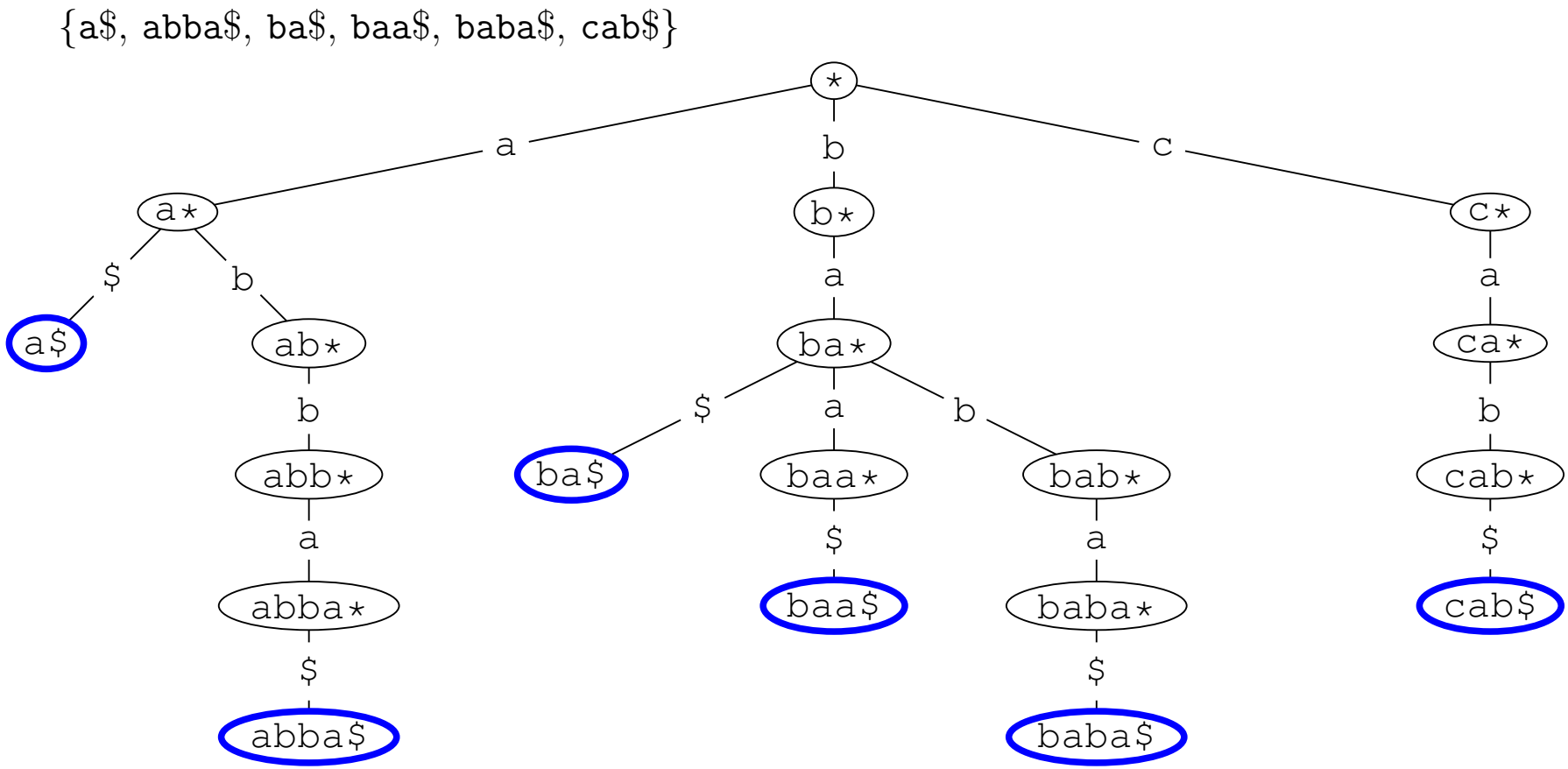
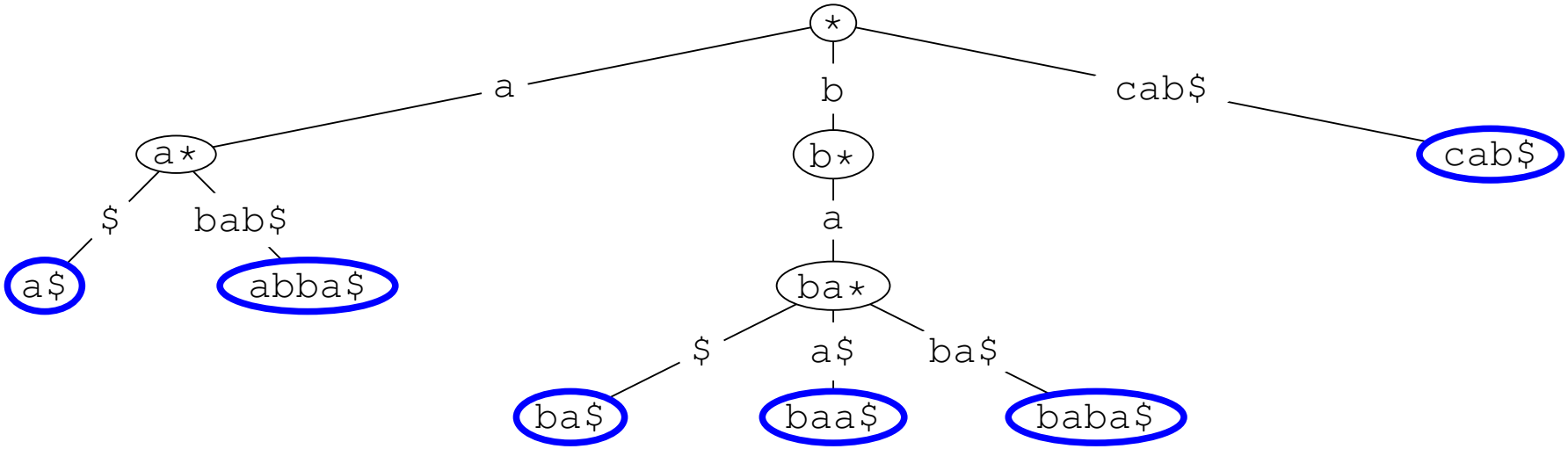

{a$, abba$, ba$, baa$, baba$, cab$}
*
a
b
c
a*
b*
c*
$
b
a
a
ba*
ca*
a$
ab*
$
a
b
b
abb*
ba$
baa*
bab*
cab*
a
$
a
b
abba*
baa$
baba*
cab$
$
$
abba$
baba$

Trie

{a, abba, ba, baa, baba, cab}



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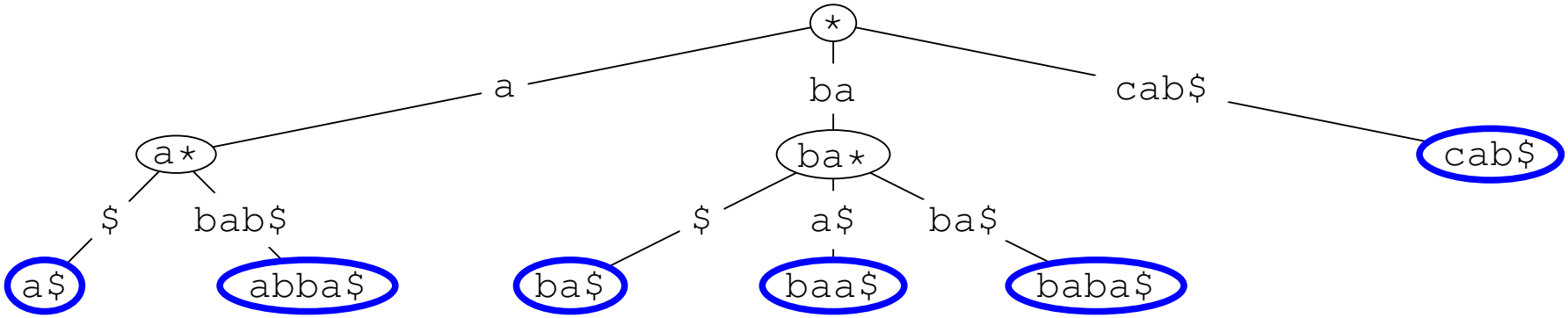

Uses of Tries

- Tries are yet another way of implementing sets
- They provide quick insertion, deletion and find
- Typically considerably quicker than binary trees and hash tables
- They are particularly good for spell checkers, completion algorithms, longest-prefix matching, hyphenation
- Each search finds the longest match between the words in the set and the query

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Trie for 31 Most Common English Words

$
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z

Trie for 31 Most Common English Words

```
add("THAT")
```

$
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z

Trie for 31 Most Common English Words

```
add("THAT")
```

	0
$	
A	
B	
C	
D	
E	
F	
G	
H	
I	
J	
K	
L	
M	
N	
O	
P	
Q	
R	
S	
T	THAT$
U	
V	
W	
X	
Y	
Z	

Trie for 31 Most Common English Words

add("IN")

	0
$	
A	
B	
C	
D	
E	
F	
G	
H	
I	
J	
K	
L	
M	
N	
O	
P	
Q	
R	
S	
T	THAT$
U	
V	
W	
X	
Y	
Z	

Trie for 31 Most Common English Words

```
add("IN")
```

	0
$	
A	
B	
C	
D	
E	
F	
G	
H	
I	IN$
J	
K	
L	
M	
N	
O	
P	
Q	
R	
S	
T	THAT$
U	
V	
W	
X	
Y	
Z	

Trie for 31 Most Common English Words

```
add("IT")
```

	0
$	
A	
B	
C	
D	
E	
F	
G	
H	
I	IN$
J	
K	
L	
M	
N	
O	
P	
Q	
R	
S	
T	THAT$
U	
V	
W	
X	
Y	
Z	

Trie for 31 Most Common English Words

```
add("IT")
```

	0	1
$		
A		
B		
C		
D		
E		
F		
G		
H		
I	I* (1)	
J		
K		
L		
M		
N		IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W		
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("I")
```

	0	1
$		
A		
B		
C		
D		
E		
F		
G		
H		
I	I* (1)	
J		
K		
L		
M		
N		IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W		
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("I")
```

	0	1
$		I$
A		
B		
C		
D		
E		
F		
G		
H		
I	I* (1)	
J		
K		
L		
M		
N		IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W		
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("HAD")
```

	0	1
$		I$
A		
B		
C		
D		
E		
F		
G		
H		
I	I* (1)	
J		
K		
L		
M		
N		IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W		
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("HAD")
```

	0	1
$		I$
A		
B		
C		
D		
E		
F		
G		
H	HAD$	
I	I* (1)	
J		
K		
L		
M		
N		IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W		
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("NOT")
```

	0	1
$		I$
A		
B		
C		
D		
E		
F		
G		
H	HAD$	
I	I* (1)	
J		
K		
L		
M		
N		IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W		
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("NOT")
```

	0	1
$		I$
A		
B		
C		
D		
E		
F		
G		
H	HAD$	
I	I* (1)	
J		
K		
L		
M		
N	NOT$	IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W		
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("WITH")
```

	0	1
$		I$
A		
B		
C		
D		
E		
F		
G		
H	HAD$	
I	I* (1)	
J		
K		
L		
M		
N	NOT$	IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W		
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("WITH")
```

	0	1
$		I$
A		
B		
C		
D		
E		
F		
G		
H	HAD$	
I	I* (1)	
J		
K		
L		
M		
N	NOT$	IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W	WITH$	
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("HIS")
```

	0	1
$		I$
A		
B		
C		
D		
E		
F		
G		
H	HAD$	
I	I* (1)	
J		
K		
L		
M		
N	NOT$	IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W	WITH$	
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("HIS")
```

	0	1	2
$		I$	
A			HAD$
B			
C			
D			
E			
F			
G			
H	H* (2)		
I	I* (1)		HIS$
J			
K			
L			
M			
N	NOT$	IN$	
O			
P			
Q			
R			
S			
T	THAT$	IT$	
U			
V			
W	WITH$		
X			
Y			
Z			

Trie for 31 Most Common English Words

add("HAVE")

	0	1	2
$		I$	
A			HAD$
B			
C			
D			
E			
F			
G			
H	H* (2)		
I	I* (1)		HIS$
J			
K			
L			
M			
N	NOT$	IN$	
O			
P			
Q			
R			
S			
T	THAT$	IT$	
U			
V			
W	WITH$		
X			
Y			
Z			

Trie for 31 Most Common English Words

```
add("HAVE")
```

	0	1	2	3
$		I$		
A			HA* (3)	
B				
C				
D				HAD$
E				
F				
G				
H	H* (2)			
I	I* (1)		HIS$	
J				
K				
L				
M				
N	NOT$	IN$		
O				
P				
Q				
R				
S				
T	THAT$	IT$		
U				
V				HAVE$
W	WITH$			
X				
Y				
Z				

Trie for 31 Most Common English Words

```
add("A")
```

	0	1	2	3
$		I$		
A			HA* (3)	
B				
C				
D				HAD$
E				
F				
G				
H	H* (2)			
I	I* (1)		HIS$	
J				
K				
L				
M				
N	NOT$	IN$		
O				
P				
Q				
R				
S				
T	THAT$	IT$		
U				
V				HAVE$
W	WITH$			
X				
Y				
Z				

Trie for 31 Most Common English Words

```
add("A")
```

	0	1	2	3
$		I$		
A	A$		HA* (3)	
B				
C				
D				HAD$
E				
F				
G				
H	H* (2)			
I	I* (1)		HIS$	
J				
K				
L				
M				
N	NOT$	IN$		
O				
P				
Q				
R				
S				
T	THAT$	IT$		
U				
V				HAVE$
W	WITH$			
X				
Y				
Z				

Trie for 31 Most Common English Words

```
add("WHICH")
```

	0	1	2	3
$		I$		
A	A$		HA* (3)	
B				
C				
D				HAD$
E				
F				
G				
H	H* (2)			
I	I* (1)		HIS$	
J				
K				
L				
M				
N	NOT$	IN$		
O				
P				
Q				
R				
S				
T	THAT$	IT$		
U				
V				HAVE$
W	WITH$			
X				
Y				
Z				

Trie for 31 Most Common English Words

```
add("WHICH")
```

	0	1	2	3	4
$		I$			
A	A$		HA* (3)		
B					
C					
D				HAD$	
E					
F					
G					
H	H* (2)				WHICH$
I	I* (1)		HIS$		WITH$
J					
K					
L					
M					
N	NOT$	IN$			
O					
P					
Q					
R					
S					
T	THAT$	IT$			
U					
V				HAVE$	
W	W* (4)				
X					
Y					
Z					

Trie for 31 Most Common English Words

```
add("HER")
```

	0	1	2	3	4
$		I$			
A	A$		HA* (3)		
B					
C					
D				HAD$	
E					
F					
G					
H	H* (2)				WHICH$
I	I* (1)		HIS$		WITH$
J					
K					
L					
M					
N	NOT$	IN$			
O					
P					
Q					
R					
S					
T	THAT$	IT$			
U					
V				HAVE$	
W	W* (4)				
X					
Y					
Z					

Trie for 31 Most Common English Words

```
add("HER")
```

	0	1	2	3	4
$		I$			
A	A$		HA* (3)		
B					
C					
D				HAD$	
E			HER$		
F					
G					
H	H* (2)				WHICH$
I	I* (1)		HIS$		WITH$
J					
K					
L					
M					
N	NOT$	IN$			
O					
P					
Q					
R					
S					
T	THAT$	IT$			
U					
V				HAVE$	
W	W* (4)				
X					
Y					
Z					

Trie for 31 Most Common English Words

```
add("AT")
```

	0	1	2	3	4
$		I$			
A	A$		HA* (3)		
B					
C					
D				HAD$	
E			HER$		
F					
G					
H	H* (2)				WHICH$
I	I* (1)		HIS$		WITH$
J					
K					
L					
M					
N	NOT$	IN$			
O					
P					
Q					
R					
S					
T	THAT$	IT$			
U					
V				HAVE$	
W	W* (4)				
X					
Y					
Z					

Trie for 31 Most Common English Words

`add("AT")`

	0	1	2	3	4	5
$		I$				A$
A	A* (5)		HA* (3)			
B						
C						
D				HAD$		
E			HER$			
F						
G						
H	H* (2)				WHICH$	
I	I* (1)		HIS$		WITH$	
J						
K						
L						
M						
N	NOT$	IN$				
O						
P						
Q						
R						
S						
T	THAT$	IT$				AT$
U						
V				HAVE$		
W	W* (4)					
X						
Y						
Z						

Trie for 31 Most Common English Words

```
add("IS")
```

	0	1	2	3	4	5
$		I$				A$
A	A* (5)		HA* (3)			
B						
C						
D				HAD$		
E			HER$			
F						
G						
H	H* (2)				WHICH$	
I	I* (1)		HIS$		WITH$	
J						
K						
L						
M						
N	NOT$	IN$				
O						
P						
Q						
R						
S						
T	THAT$	IT$				AT$
U						
V				HAVE$		
W	W* (4)					
X						
Y						
Z						

Trie for 31 Most Common English Words

`add("IS")`

	0	1	2	3	4	5
$		I$				A$
A	A* (5)		HA* (3)			
B						
C						
D				HAD$		
E			HER$			
F						
G						
H	H* (2)				WHICH$	
I	I* (1)		HIS$		WITH$	
J						
K						
L						
M						
N	NOT$	IN$				
O						
P						
Q						
R						
S		IS$				
T	THAT$	IT$				AT$
U						
V				HAVE$		
W	W* (4)					
X						
Y						
Z						

Trie for 31 Most Common English Words

`add("AND")`

	0	1	2	3	4	5
$		I$				A$
A	A* (5)		HA* (3)			
B						
C						
D				HAD$		
E			HER$			
F						
G						
H	H* (2)				WHICH$	
I	I* (1)		HIS$		WITH$	
J						
K						
L						
M						
N	NOT$	IN$				
O						
P						
Q						
R						
S		IS$				
T	THAT$	IT$				AT$
U						
V				HAVE$		
W	W* (4)					
X						
Y						
Z						

Trie for 31 Most Common English Words

`add("AND")`

	0	1	2	3	4	5
$		I$				A$
A	A* (5)		HA* (3)			
B						
C						
D				HAD$		
E			HER$			
F						
G						
H	H* (2)				WHICH$	
I	I* (1)		HIS$		WITH$	
J						
K						
L						
M						
N	NOT$	IN$				AND$
O						
P						
Q						
R						
S		IS$				
T	THAT$	IT$				AT$
U						
V				HAVE$		
W	W* (4)					
X						
Y						
Z						

Trie for 31 Most Common English Words

`add("BUT")`

	0	1	2	3	4	5
$		I$				A$
A	A* (5)		HA* (3)			
B						
C						
D				HAD$		
E			HER$			
F						
G						
H	H* (2)				WHICH$	
I	I* (1)		HIS$		WITH$	
J						
K						
L						
M						
N	NOT$	IN$				AND$
O						
P						
Q						
R						
S		IS$				
T	THAT$	IT$				AT$
U						
V				HAVE$		
W	W* (4)					
X						
Y						
Z						

Trie for 31 Most Common English Words

`add("BUT")`

	0	1	2	3	4	5
$		I$				A$
A	A* (5)		HA* (3)			
B	BUT$					
C						
D				HAD$		
E			HER$			
F						
G						
H	H* (2)				WHICH$	
I	I* (1)		HIS$		WITH$	
J						
K						
L						
M						
N	NOT$	IN$				AND$
O						
P						
Q						
R						
S		IS$				
T	THAT$	IT$				AT$
U						
V				HAVE$		
W	W* (4)					
X						
Y						
Z						

Trie for 31 Most Common English Words

```
add("THE")
```

	0	1	2	3	4	5
$		I$				A$
A	A* (5)		HA* (3)			
B	BUT$					
C						
D				HAD$		
E			HER$			
F						
G						
H	H* (2)				WHICH$	
I	I* (1)		HIS$		WITH$	
J						
K						
L						
M						
N	NOT$	IN$				AND$
O						
P						
Q						
R						
S		IS$				
T	THAT$	IT$				AT$
U						
V				HAVE$		
W	W* (4)					
X						
Y						
Z						

Trie for 31 Most Common English Words

`add("THE")`

	0	1	2	3	4	5	6
$		I$				A$	
A	A* (5)		HA* (3)				THAT$
B	BUT$						
C							
D				HAD$			
E			HER$				THE$
F							
G							
H	H* (2)				WHICH$		
I	I* (1)		HIS$		WITH$		
J							
K							
L							
M							
N	NOT$	IN$				AND$	
O							
P							
Q							
R							
S		IS$					
T	TH* (6)	IT$				AT$	
U							
V				HAVE$			
W	W* (4)						
X							
Y							
Z							

Trie for 31 Most Common English Words

`add("FOR")`

	0	1	2	3	4	5	6
$		I$				A$	
A	A* (5)		HA* (3)				THAT$
B	BUT$						
C							
D				HAD$			
E			HER$				THE$
F							
G							
H	H* (2)				WHICH$		
I	I* (1)		HIS$		WITH$		
J							
K							
L							
M							
N	NOT$	IN$				AND$	
O							
P							
Q							
R							
S		IS$					
T	TH* (6)	IT$				AT$	
U							
V				HAVE$			
W	W* (4)						
X							
Y							
Z							

Trie for 31 Most Common English Words

`add("FOR")`

	0	1	2	3	4	5	6
$		I$				A$	
A	A* (5)		HA* (3)				THAT$
B	BUT$						
C							
D				HAD$			
E			HER$				THE$
F	FOR$						
G							
H	H* (2)				WHICH$		
I	I* (1)		HIS$		WITH$		
J							
K							
L							
M							
N	NOT$	IN$				AND$	
O							
P							
Q							
R							
S		IS$					
T	TH* (6)	IT$				AT$	
U							
V				HAVE$			
W	W* (4)						
X							
Y							
Z							

Trie for 31 Most Common English Words

`add("ON")`

	0	1	2	3	4	5	6
$		I$				A$	
A	A* (5)		HA* (3)				THAT$
B	BUT$						
C							
D				HAD$			
E			HER$				THE$
F	FOR$						
G							
H	H* (2)				WHICH$		
I	I* (1)		HIS$		WITH$		
J							
K							
L							
M							
N	NOT$	IN$				AND$	
O							
P							
Q							
R							
S		IS$					
T	TH* (6)	IT$				AT$	
U							
V				HAVE$			
W	W* (4)						
X							
Y							
Z							

Trie for 31 Most Common English Words

add("ON")

	0	1	2	3	4	5	6
$		I$				A$	
A	A* (5)		HA* (3)				THAT$
B	BUT$						
C							
D				HAD$			
E			HER$				THE$
F	FOR$						
G							
H	H* (2)				WHICH$		
I	I* (1)		HIS$		WITH$		
J							
K							
L							
M							
N	NOT$	IN$				AND$	
O	ON$						
P							
Q							
R							
S		IS$					
T	TH* (6)	IT$				AT$	
U							
V				HAVE$			
W	W* (4)						
X							
Y							
Z							

Trie for 31 Most Common English Words

`add("HE")`

	0	1	2	3	4	5	6
$		I$				A$	
A	A* (5)		HA* (3)				THAT$
B	BUT$						
C							
D				HAD$			
E			HER$				THE$
F	FOR$						
G							
H	H* (2)				WHICH$		
I	I* (1)		HIS$		WITH$		
J							
K							
L							
M							
N	NOT$	IN$				AND$	
O	ON$						
P							
Q							
R							
S		IS$					
T	TH* (6)	IT$				AT$	
U							
V				HAVE$			
W	W* (4)						
X							
Y							
Z							

Trie for 31 Most Common English Words

add("HE")

	0	1	2	3	4	5	6	7
$		I$				A$		HE$
A	A* (5)		HA* (3)				THAT$	
B	BUT$							
C								
D				HAD$				
E			HE* (7)				THE$	
F	FOR$							
G								
H	H* (2)				WHICH$			
I	I* (1)		HIS$		WITH$			
J								
K								
L								
M								
N	NOT$	IN$				AND$		
O	ON$							
P								
Q								
R								HER$
S		IS$						
T	TH* (6)	IT$				AT$		
U								
V				HAVE$				
W	W* (4)							
X								
Y								
Z								

Trie for 31 Most Common English Words

add("WAS")

	0	1	2	3	4	5	6	7
$		I$				A$		HE$
A	A* (5)		HA* (3)				THAT$	
B	BUT$							
C								
D				HAD$				
E			HE* (7)				THE$	
F	FOR$							
G								
H	H* (2)				WHICH$			
I	I* (1)		HIS$		WITH$			
J								
K								
L								
M								
N	NOT$	IN$				AND$		
O	ON$							
P								
Q								
R								HER$
S		IS$						
T	TH* (6)	IT$				AT$		
U								
V				HAVE$				
W	W* (4)							
X								
Y								
Z								

Trie for 31 Most Common English Words

add("WAS")

	0	1	2	3	4	5	6	7
$		I$				A$		HE$
A	A* (5)		HA* (3)		WAS$		THAT$	
B	BUT$							
C								
D				HAD$				
E			HE* (7)				THE$	
F	FOR$							
G								
H	H* (2)				WHICH$			
I	I* (1)		HIS$		WITH$			
J								
K								
L								
M								
N	NOT$	IN$				AND$		
O	ON$							
P								
Q								
R								HER$
S		IS$						
T	TH* (6)	IT$				AT$		
U								
V				HAVE$				
W	W* (4)							
X								
Y								
Z								

Trie for 31 Most Common English Words

add("ARE")

	0	1	2	3	4	5	6	7
$		I$				A$		HE$
A	A* (5)		HA* (3)		WAS$		THAT$	
B	BUT$							
C								
D				HAD$				
E			HE* (7)				THE$	
F	FOR$							
G								
H	H* (2)				WHICH$			
I	I* (1)		HIS$		WITH$			
J								
K								
L								
M								
N	NOT$	IN$				AND$		
O	ON$							
P								
Q								
R								HER$
S		IS$						
T	TH* (6)	IT$				AT$		
U								
V				HAVE$				
W	W* (4)							
X								
Y								
Z								

Trie for 31 Most Common English Words

add("ARE")

	0	1	2	3	4	5	6	7
$		I$				A$		HE$
A	A* (5)		HA* (3)		WAS$		THAT$	
B	BUT$							
C								
D				HAD$				
E			HE* (7)				THE$	
F	FOR$							
G								
H	H* (2)				WHICH$			
I	I* (1)		HIS$		WITH$			
J								
K								
L								
M								
N	NOT$	IN$				AND$		
O	ON$							
P								
Q								
R						ARE$		HER$
S		IS$						
T	TH* (6)	IT$				AT$		
U								
V				HAVE$				
W	W* (4)							
X								
Y								
Z								

Trie for 31 Most Common English Words

`add("BY")`

	0	1	2	3	4	5	6	7
$		I$				A$		HE$
A	A* (5)		HA* (3)		WAS$		THAT$	
B	BUT$							
C								
D				HAD$				
E			HE* (7)				THE$	
F	FOR$							
G								
H	H* (2)				WHICH$			
I	I* (1)		HIS$		WITH$			
J								
K								
L								
M								
N	NOT$	IN$				AND$		
O	ON$							
P								
Q								
R						ARE$		HER$
S		IS$						
T	TH* (6)	IT$				AT$		
U								
V				HAVE$				
W	W* (4)							
X								
Y								
Z								

Trie for 31 Most Common English Words

`add("BY")`

	0	1	2	3	4	5	6	7	8
$		I$				A$		HE$	
A	A* (5)		HA* (3)		WAS$		THAT$		
B	B* (8)								
C									
D				HAD$					
E			HE* (7)				THE$		
F	FOR$								
G									
H	H* (2)				WHICH$				
I	I* (1)		HIS$		WITH$				
J									
K									
L									
M									
N	NOT$	IN$				AND$			
O	ON$								
P									
Q									
R						ARE$		HER$	
S		IS$							
T	TH* (6)	IT$				AT$			
U									BUT$
V				HAVE$					
W	W* (4)								
X									
Y									BY$
Z									

Trie for 31 Most Common English Words

add("OR")

	0	1	2	3	4	5	6	7	8
$		I$				A$		HE$	
A	A* (5)		HA* (3)		WAS$		THAT$		
B	B* (8)								
C									
D				HAD$					
E			HE* (7)				THE$		
F	FOR$								
G									
H	H* (2)				WHICH$				
I	I* (1)		HIS$		WITH$				
J									
K									
L									
M									
N	NOT$	IN$				AND$			
O	ON$								
P									
Q									
R						ARE$		HER$	
S		IS$							
T	TH* (6)	IT$				AT$			
U									BUT$
V				HAVE$					
W	W* (4)								
X									
Y									BY$
Z									

Trie for 31 Most Common English Words

add("OR")

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$			
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$					
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$								
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y									BY$	
Z										

Trie for 31 Most Common English Words

`add("AS")`

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$			
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$					
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$								
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y									BY$	
Z										

Trie for 31 Most Common English Words

`add("AS")`

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$			
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$					
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$				AS$				
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y									BY$	
Z										

Trie for 31 Most Common English Words

add("THIS")

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$			
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$					
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$				AS$				
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y									BY$	
Z										

Trie for 31 Most Common English Words

add("THIS")

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$			
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$		THIS$			
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$				AS$				
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y									BY$	
Z										

Trie for 31 Most Common English Words

add("YOU")

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$			
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$		THIS$			
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$				AS$				
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y									BY$	
Z										

Trie for 31 Most Common English Words

add("YOU")

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$			
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$		THIS$			
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$				AS$				
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y	YOU$								BY$	
Z										

Trie for 31 Most Common English Words

`add("BE")`

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$			
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$		THIS$			
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$				AS$				
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y	YOU$								BY$	
Z										

Trie for 31 Most Common English Words

`add("BE")`

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$		BE$	
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$		THIS$			
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$				AS$				
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y	YOU$								BY$	
Z										

Trie for 31 Most Common English Words

add("FROM")

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$		BE$	
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$		THIS$			
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$				AS$				
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y	YOU$								BY$	
Z										

Trie for 31 Most Common English Words

`add("FROM")`

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
A	A* (5)		HA* (3)		WAS$		THAT$				
B	B* (8)										
C											
D				HAD$							
E			HE* (7)				THE$		BE$		
F	F* (10)										
G											
H	H* (2)				WHICH$						
I	I* (1)		HIS$		WITH$		THIS$				
J											
K											
L											
M											
N	NOT$	IN$				AND$				ON$	
O	O* (9)										FOR$
P											
Q											
R						ARE$		HER$		OR$	FROM$
S		IS$				AS$					
T	TH* (6)	IT$				AT$					
U									BUT$		
V				HAVE$							
W	W* (4)										
X											
Y	YOU$								BY$		
Z											

Trie for 31 Most Common English Words

add("OF")

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
A	A* (5)		HA* (3)		WAS$		THAT$				
B	B* (8)										
C											
D				HAD$							
E			HE* (7)				THE$		BE$		
F	F* (10)										
G											
H	H* (2)				WHICH$						
I	I* (1)		HIS$		WITH$		THIS$				
J											
K											
L											
M											
N	NOT$	IN$				AND$				ON$	
O	O* (9)										FOR$
P											
Q											
R						ARE$		HER$		OR$	FROM$
S		IS$				AS$					
T	TH* (6)	IT$				AT$					
U									BUT$		
V				HAVE$							
W	W* (4)										
X											
Y	YOU$								BY$		
Z											

Trie for 31 Most Common English Words

add("OF")

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
A	A* (5)		HA* (3)		WAS$		THAT$				
B	B* (8)										
C											
D				HAD$							
E			HE* (7)				THE$		BE$		
F	F* (10)									OF$	
G											
H	H* (2)				WHICH$						
I	I* (1)		HIS$		WITH$		THIS$				
J											
K											
L											
M											
N	NOT$	IN$				AND$				ON$	
O	O* (9)										FOR$
P											
Q											
R						ARE$		HER$		OR$	FROM$
S		IS$				AS$					
T	TH* (6)	IT$				AT$					
U									BUT$		
V				HAVE$							
W	W* (4)										
X											
Y	YOU$								BY$		
Z											

Trie for 31 Most Common English Words

`search("YOU")`

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
A	A* (5)		HA* (3)		WAS$		THAT$				
B	B* (8)										
C											
D				HAD$							
E			HE* (7)				THE$		BE$		
F	F* (10)									OF$	
G											
H	H* (2)				WHICH$						
I	I* (1)		HIS$		WITH$		THIS$				
J											
K											
L											
M											
N	NOT$	IN$				AND$				ON$	
O	O* (9)										FOR$
P											
Q											
R						ARE$		HER$		OR$	FROM$
S		IS$				AS$					
T	TH* (6)	IT$				AT$					
U									BUT$		
V				HAVE$							
W	W* (4)										
X											
Y	YOU$								BY$		
Z											

Trie for 31 Most Common English Words

```
search("YOU")
```

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
A	A* (5)		HA* (3)		WAS$		THAT$				
B	B* (8)										
C											
D				HAD$							
E			HE* (7)				THE$		BE$		
F	F* (10)									OF$	
G											
H	H* (2)				WHICH$						
I	I* (1)		HIS$		WITH$		THIS$				
J											
K											
L											
M											
N	NOT$	IN$				AND$				ON$	
O	O* (9)										FOR$
P											
Q											
R						ARE$		HER$		OR$	FROM$
S		IS$				AS$					
T	TH* (6)	IT$				AT$					
U									BUT$		
V				HAVE$							
W	W* (4)										
X											
Y	YOU$								BY$		
Z											

Trie for 31 Most Common English Words

```
search("YOU")  True
```

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
A	A* (5)		HA* (3)		WAS$		THAT$				
B	B* (8)										
C											
D				HAD$							
E			HE* (7)				THE$		BE$		
F	F* (10)									OF$	
G											
H	H* (2)				WHICH$						
I	I* (1)		HIS$		WITH$		THIS$				
J											
K											
L											
M											
N	NOT$	IN$				AND$				ON$	
O	O* (9)										FOR$
P											
Q											
R						ARE$		HER$		OR$	FROM$
S		IS$				AS$					
T	TH* (6)	IT$				AT$					
U									BUT$		
V				HAVE$							
W	W* (4)										
X											
Y	YOU$								BY$		
Z											

Trie for 31 Most Common English Words

`search("THE")`

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
A	A* (5)		HA* (3)		WAS$		THAT$				
B	B* (8)										
C											
D				HAD$							
E			HE* (7)				THE$		BE$		
F	F* (10)									OF$	
G											
H	H* (2)				WHICH$						
I	I* (1)		HIS$		WITH$		THIS$				
J											
K											
L											
M											
N	NOT$	IN$				AND$				ON$	
O	O* (9)										FOR$
P											
Q											
R						ARE$		HER$		OR$	FROM$
S		IS$				AS$					
T	TH* (6)	IT$				AT$					
U									BUT$		
V				HAVE$							
W	W* (4)										
X											
Y	YOU$								BY$		
Z											

Trie for 31 Most Common English Words

`search("THE")`

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
A	A* (5)		HA* (3)		WAS$		THAT$				
B	B* (8)										
C											
D				HAD$							
E			HE* (7)				THE$		BE$		
F	F* (10)									OF$	
G											
H	H* (2)				WHICH$						
I	I* (1)		HIS$		WITH$		THIS$				
J											
K											
L											
M											
N	NOT$	IN$				AND$				ON$	
O	O* (9)										FOR$
P											
Q											
R						ARE$		HER$		OR$	FROM$
S		IS$				AS$					
T	TH* (6)	IT$				AT$					
U									BUT$		
V				HAVE$							
W	W* (4)										
X											
Y	YOU$								BY$		
Z											

Trie for 31 Most Common English Words

`search("THE")`

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
A	A* (5)		HA* (3)		WAS$		THAT$				
B	B* (8)										
C											
D				HAD$							
E			HE* (7)				THE$		BE$		
F	F* (10)									OF$	
G											
H	H* (2)				WHICH$						
I	I* (1)		HIS$		WITH$		THIS$				
J											
K											
L											
M											
N	NOT$	IN$				AND$				ON$	
O	O* (9)										FOR$
P											
Q											
R						ARE$		HER$		OR$	FROM$
S		IS$				AS$					
T	TH* (6)	IT$				AT$					
U									BUT$		
V				HAVE$							
W	W* (4)										
X											
Y	YOU$								BY$		
Z											

Trie for 31 Most Common English Words

```
search("THE")  True
```

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
A	A* (5)		HA* (3)		WAS$		THAT$				
B	B* (8)										
C											
D				HAD$							
E			HE* (7)				THE$		BE$		
F	F* (10)									OF$	
G											
H	H* (2)				WHICH$						
I	I* (1)		HIS$		WITH$		THIS$				
J											
K											
L											
M											
N	NOT$	IN$				AND$				ON$	
O	O* (9)										FOR$
P											
Q											
R						ARE$		HER$		OR$	FROM$
S		IS$				AS$					
T	TH* (6)	IT$				AT$					
U									BUT$		
V				HAVE$							
W	W* (4)										
X											
Y	YOU$								BY$		
Z											

Trie for 31 Most Common English Words

`search("NO")`

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
A	A* (5)		HA* (3)		WAS$		THAT$				
B	B* (8)										
C											
D				HAD$							
E			HE* (7)				THE$		BE$		
F	F* (10)									OF$	
G											
H	H* (2)				WHICH$						
I	I* (1)		HIS$		WITH$		THIS$				
J											
K											
L											
M											
N	NOT$	IN$				AND$				ON$	
O	O* (9)										FOR$
P											
Q											
R						ARE$		HER$		OR$	FROM$
S		IS$				AS$					
T	TH* (6)	IT$				AT$					
U									BUT$		
V				HAVE$							
W	W* (4)										
X											
Y	YOU$								BY$		
Z											

Trie for 31 Most Common English Words

`search("NO")`

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
A	A* (5)		HA* (3)		WAS$		THAT$				
B	B* (8)										
C											
D				HAD$							
E			HE* (7)				THE$		BE$		
F	F* (10)									OF$	
G											
H	H* (2)				WHICH$						
I	I* (1)		HIS$		WITH$		THIS$				
J											
K											
L											
M											
N	NOT$	IN$				AND$				ON$	
O	O* (9)										FOR$
P											
Q											
R						ARE$		HER$		OR$	FROM$
S		IS$				AS$					
T	TH* (6)	IT$				AT$					
U									BUT$		
V				HAVE$							
W	W* (4)										
X											
Y	YOU$								BY$		
Z											

Trie for 31 Most Common English Words

```
search("NO")  False
```

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
A	A* (5)		HA* (3)		WAS$		THAT$				
B	B* (8)										
C											
D				HAD$							
E			HE* (7)				THE$		BE$		
F	F* (10)									OF$	
G											
H	H* (2)				WHICH$						
I	I* (1)		HIS$		WITH$		THIS$				
J											
K											
L											
M											
N	NOT$	IN$				AND$				ON$	
O	O* (9)										FOR$
P											
Q											
R						ARE$		HER$		OR$	FROM$
S		IS$				AS$					
T	TH* (6)	IT$				AT$					
U									BUT$		
V				HAVE$							
W	W* (4)										
X											
Y	YOU$								BY$		
Z											

Trie for 31 Most Common English Words

`search("THEM")`

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
A	A* (5)		HA* (3)		WAS$		THAT$				
B	B* (8)										
C											
D				HAD$							
E			HE* (7)				THE$		BE$		
F	F* (10)									OF$	
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H	H* (2)				WHICH$						
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Trie for 31 Most Common English Words

```
search("THEM") False
```

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
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`search("HERE")`

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
A	A* (5)		HA* (3)		WAS$		THAT$				
B	B* (8)										
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E			HE* (7)				THE$		BE$		
F	F* (10)									OF$	
G											
H	H* (2)				WHICH$						
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J											
K											
L											
M											
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O	O* (9)										FOR$
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Trie for 31 Most Common English Words

```
search("HERE") False
```

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Disadvantage of Tries

- Table-based tries typically waste large amounts of memory
- Often table-based tries are used for the first few layers, while lower levels use a less memory intensive data structure
- These days memory is less of a problem so table-based tries are acceptable for some applications
- There are many implementations of tries each suited to a particular task

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Binary Tries

- One extreme (though not uncommon) solution to address memory issues is to build a bit-level trie so the data-structure is a binary tree
- It differs from a binary tree in that the decisions to go left or right depends on the current bit
- Although you lose the advantage of a multiway tree (of reducing the depth) it does find the longest match and it speeds up finds which fail

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Why Tries?

- Tries are a classic example of a trade-off between memory and computational complexity
- Tries are slightly specialist and tend to get used in very particular applications
  - ⋆ Finding longest matches
  - ⋆ Completion, spell checking, etc.
- A basic trie is not too complicated, however, . . .
- There are many implementation which try to overcome the difficulty of wasting too much memory

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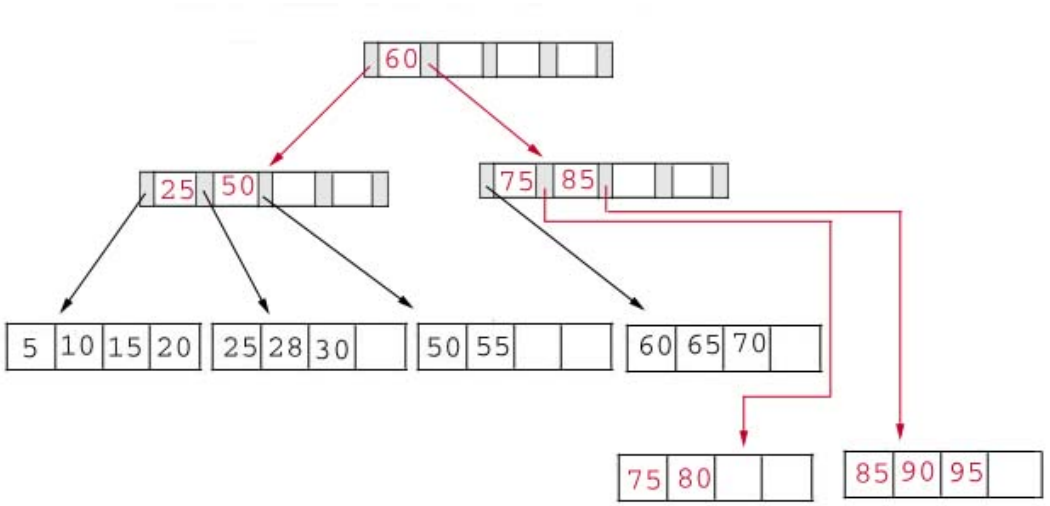
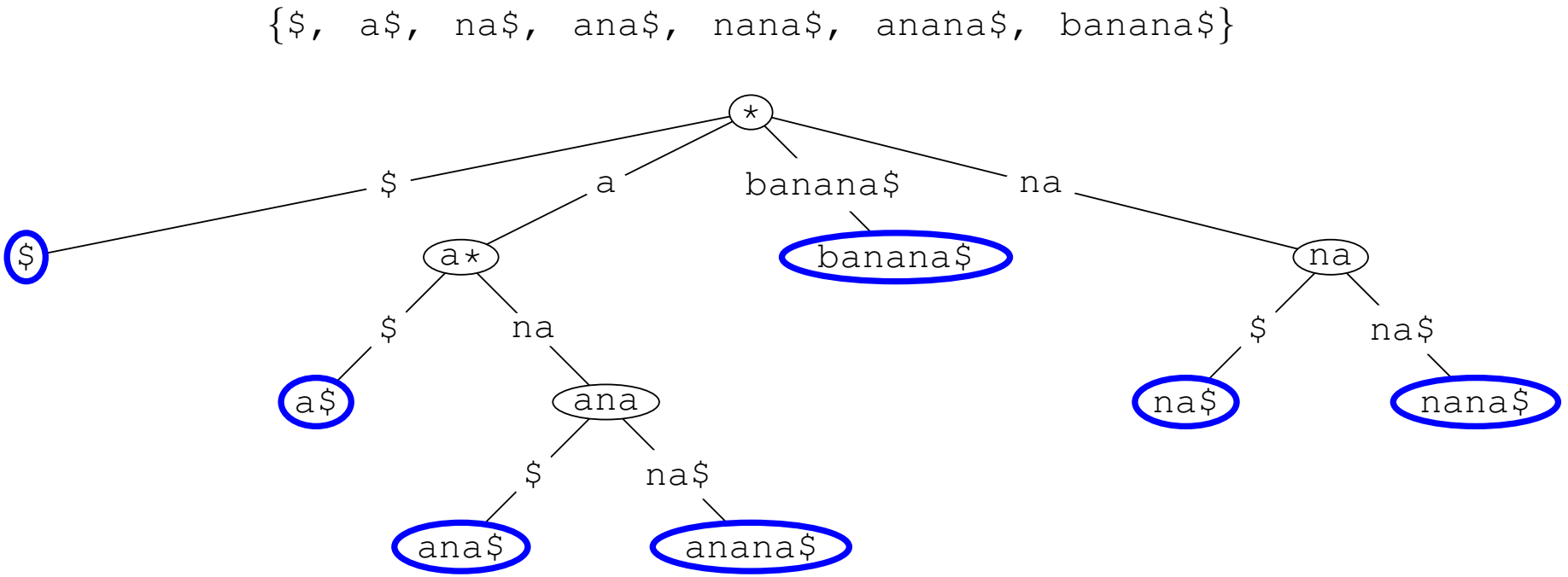
Outline

1. B-Trees
2. Tries
3. **Suffix Tree**



Suffix Tree

- Suffix tree is a trie of all suffixes of a string
- E.g. banana



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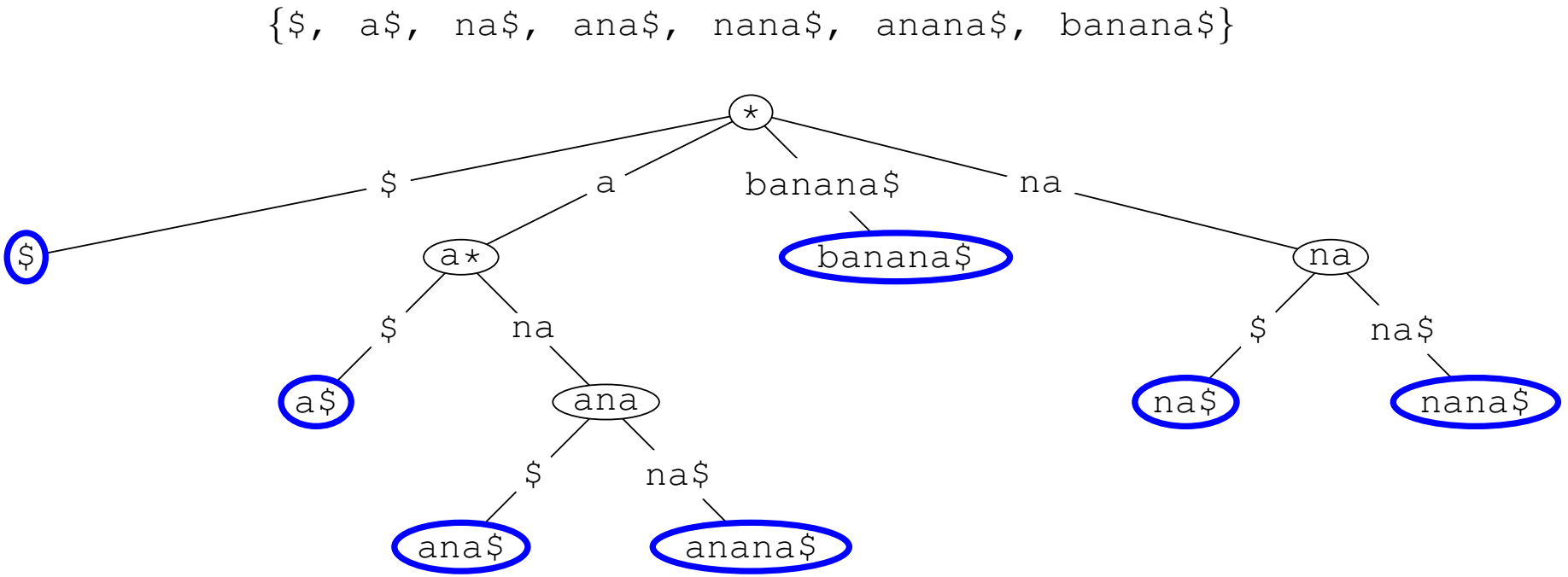

Importance of Suffix Tree

- The first linear-time algorithm for computing suffix trees was proposed by Peter Weiner in 1973, a more space efficient algorithm was proposed by Edward M. McCreight in 1976
- Esko Ukkonen in 1995 proposed a variant of McCreight's algorithm, but in a way that was much easier to understand
- It really only got implemented after this
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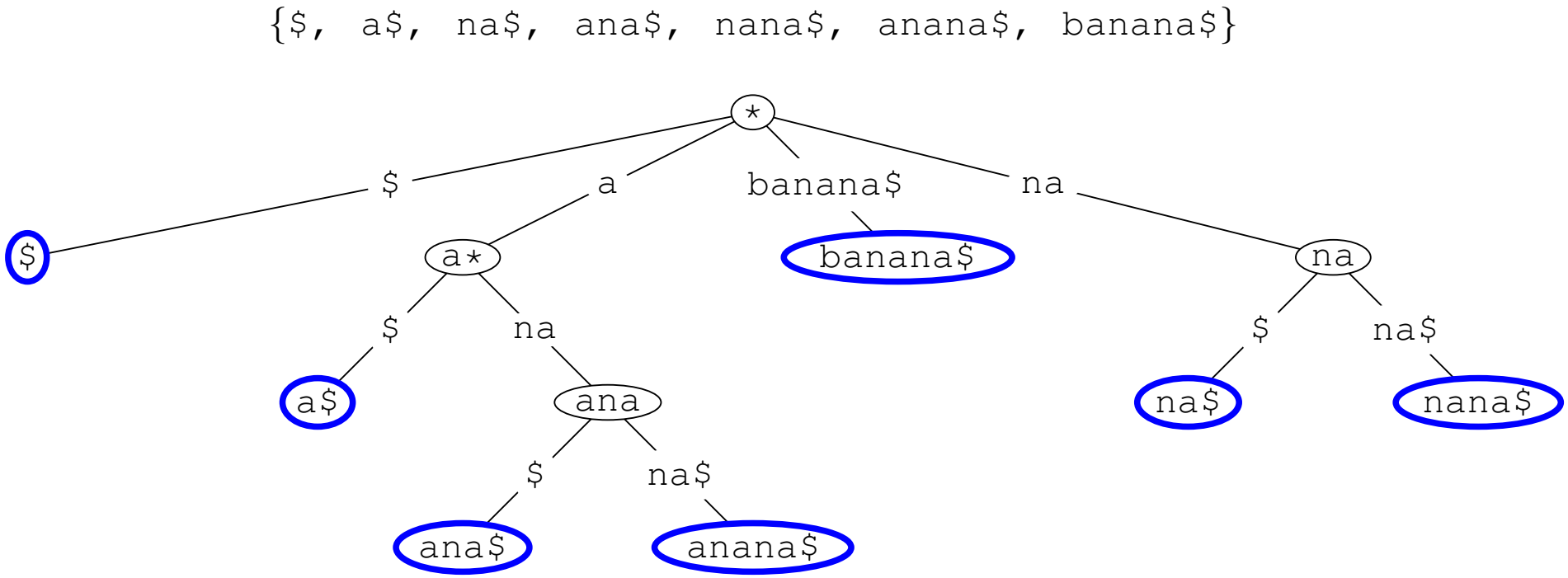
String Matching

- To find a match of a query string, Q, in a text, T, we can first construct the suffix tree of the string T we then simple look up the query, Q, using the trie



Complexity of Suffix Trees

- Using a regular trie for a suffix tree would typically use far too much memory to be useful
- However, by using pointers to the original text it is possible to build a suffix tree using $O(n)$ memory where n is the length of the text
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Uses of Suffix Trees

- Suffix trees are efficient whenever it is likely that you will do multiple searches
- Exact word matching is in itself a very important application
- Suffix trees in combination with dynamic programming (which we will eventually get to) can be used to do inexact matching (finding the match with the smallest edit distance)
- Suffix trees get used in bioinformatics, advanced machine learning algorithms, . . .

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Lessons

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- They are very important in some specialised applications (e.g. databases, spell-checking, completion, suffix trees)
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