Algorithms and Analysis

Lesson 17: *Sort Wisely*

Merge sort, quick sort and radix sort

Outline

1. **Merge Sort**
2. Quick Sort
3. Radix Sort

Merge Sort

- Merge sort is an example of sort performed in log-linear (i.e. $O(n\log(n))$) time complexity

- It was invented in 1945 by John von Neumann

- It is an example of a divide-and-conquer strategy
  - ⋆ That is, the problem is divided into a number of parts recursively
  - ⋆ The full solution is obtained by recombining the parts

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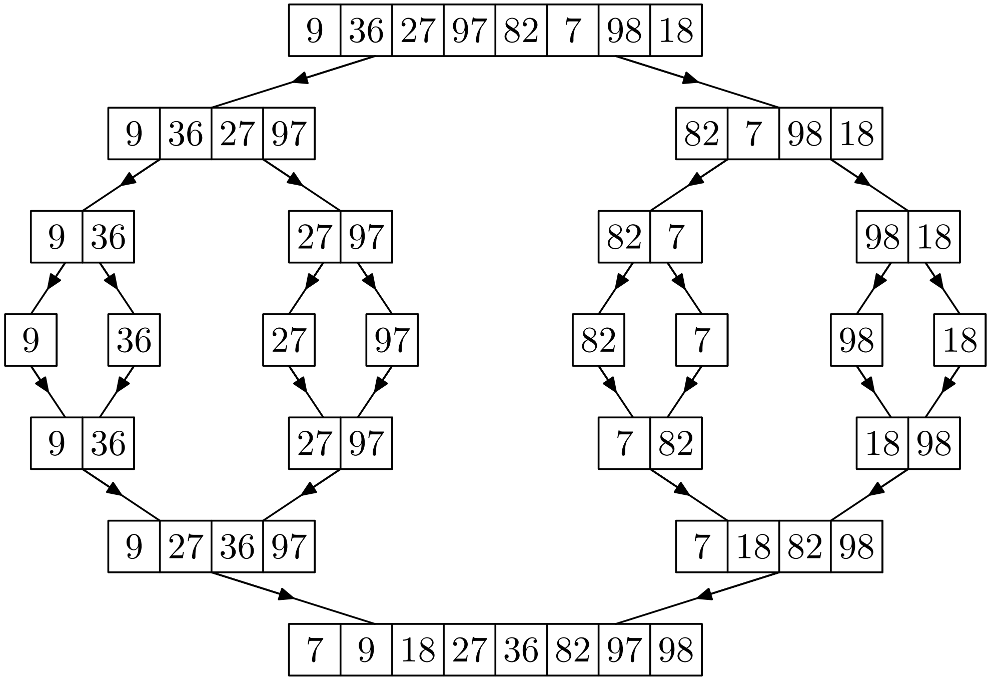
Algorithm

```
MergeSort($\boldsymbol{a}$)
{
  if $n > 1$
    copy $\boldsymbol{a}[1 : \lfloor n/2 \rfloor]$  to $\boldsymbol{b}$
    copy $\boldsymbol{a}[\lfloor n/2 \rfloor + 1 : n]$  to $\boldsymbol{c}$
    MergeSort($\boldsymbol{b}$)
    MergeSort($\boldsymbol{c}$)
    Merge($\boldsymbol{b}, \boldsymbol{c}, \boldsymbol{a}$)
  endif
}
```



Merge

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MERGE($\boldsymbol{b}[1:p]$, $\boldsymbol{c}[1:q]$, $\boldsymbol{a}[1:p+q]$)
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  j←1
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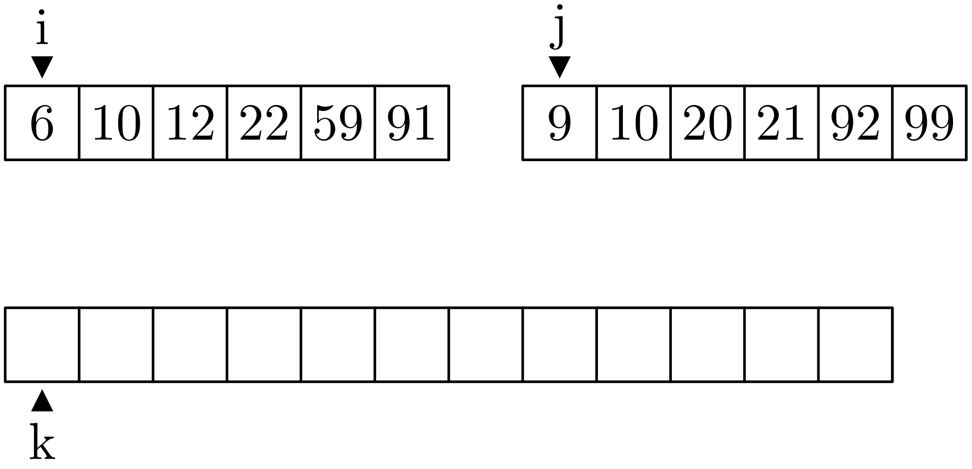

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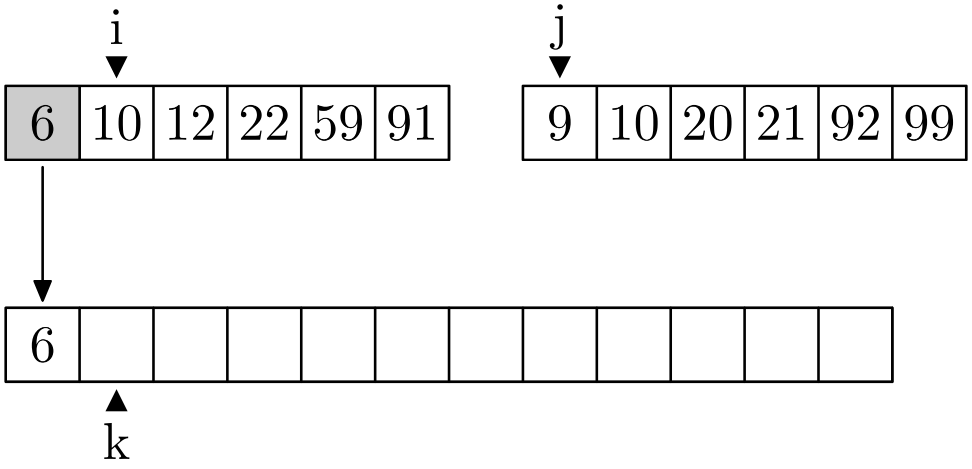

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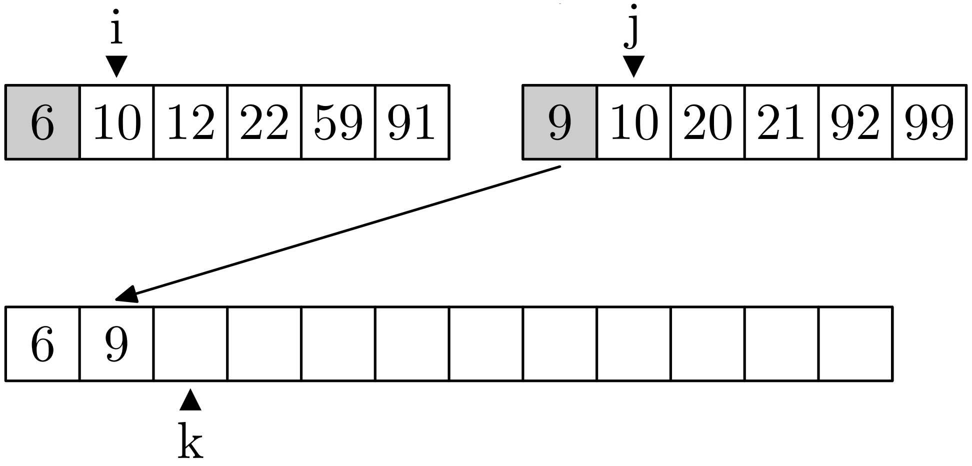

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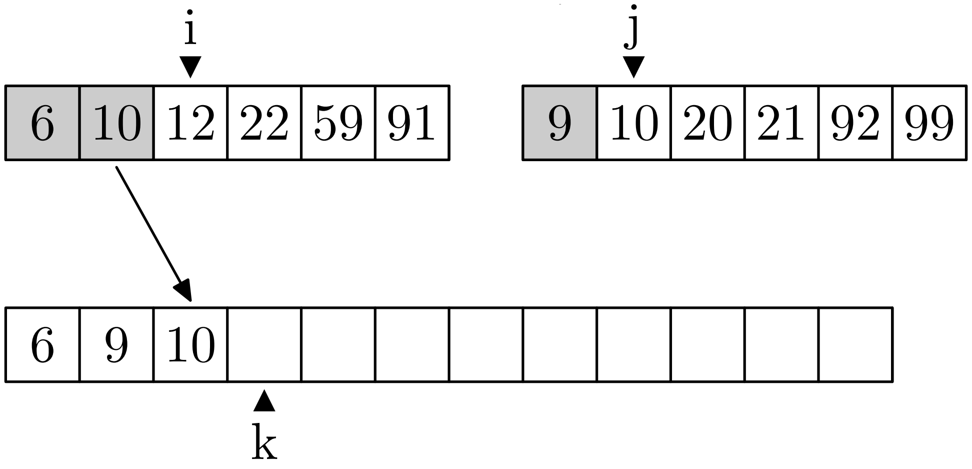

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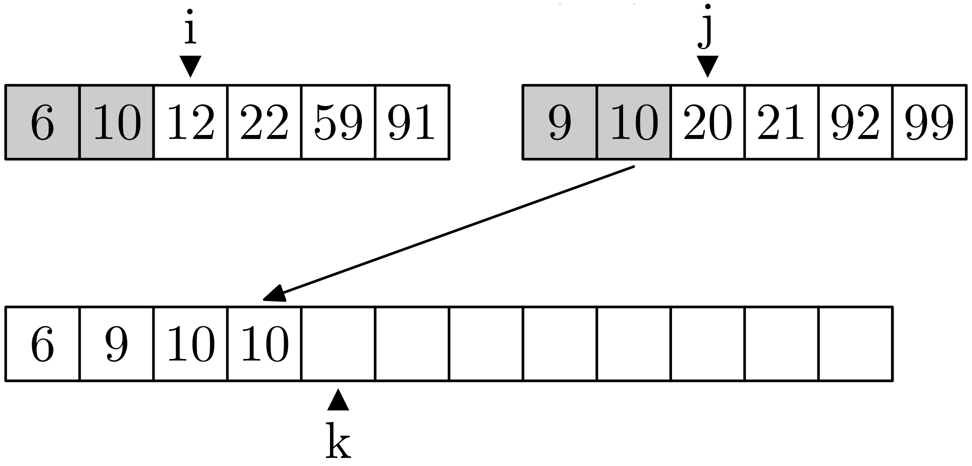

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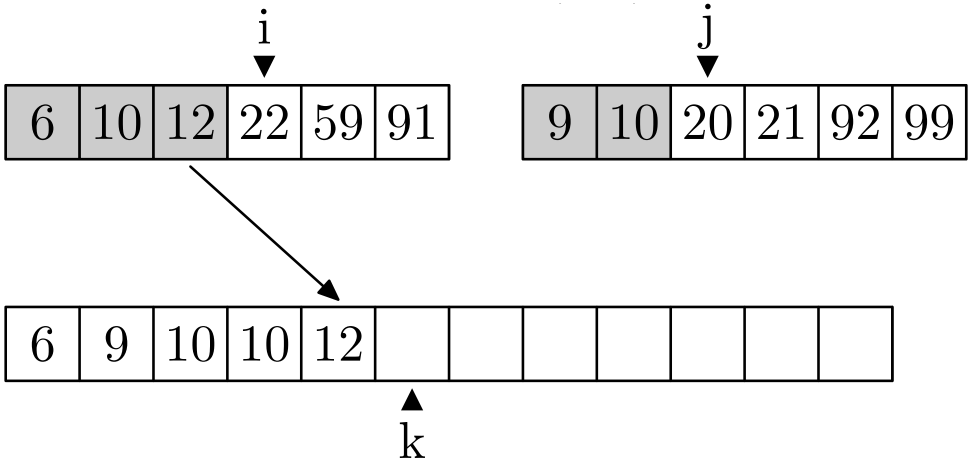

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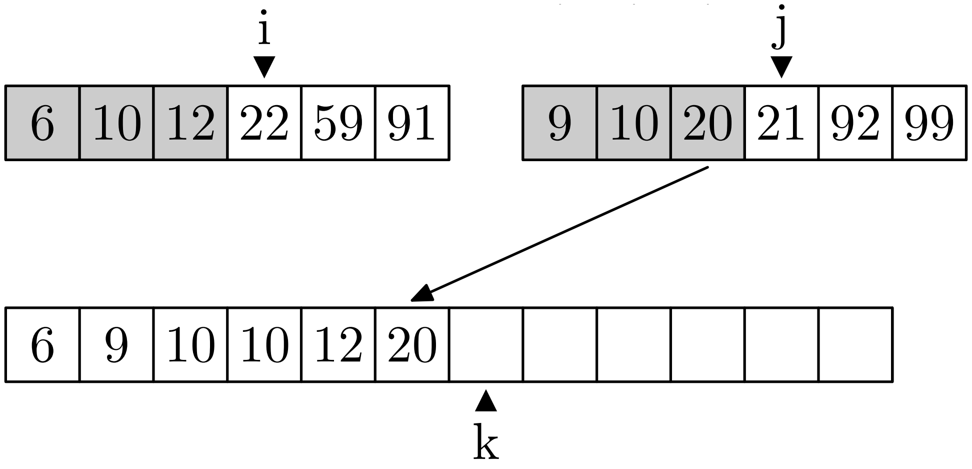

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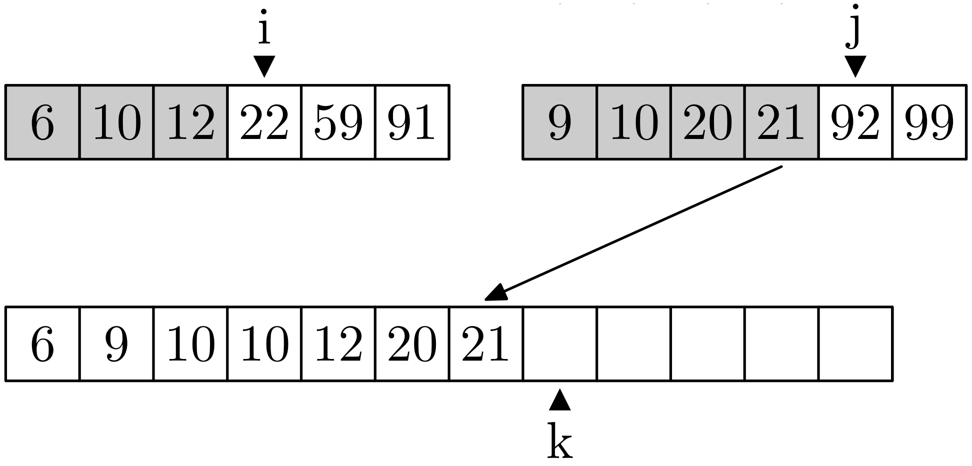

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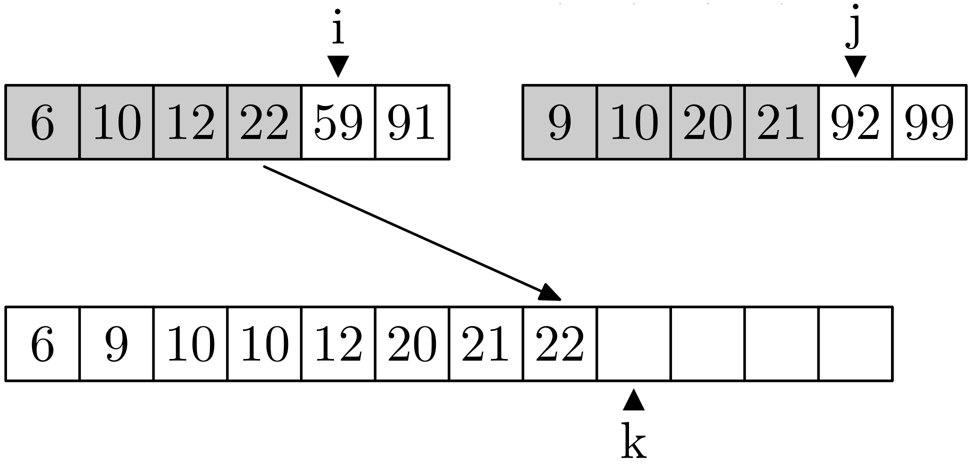

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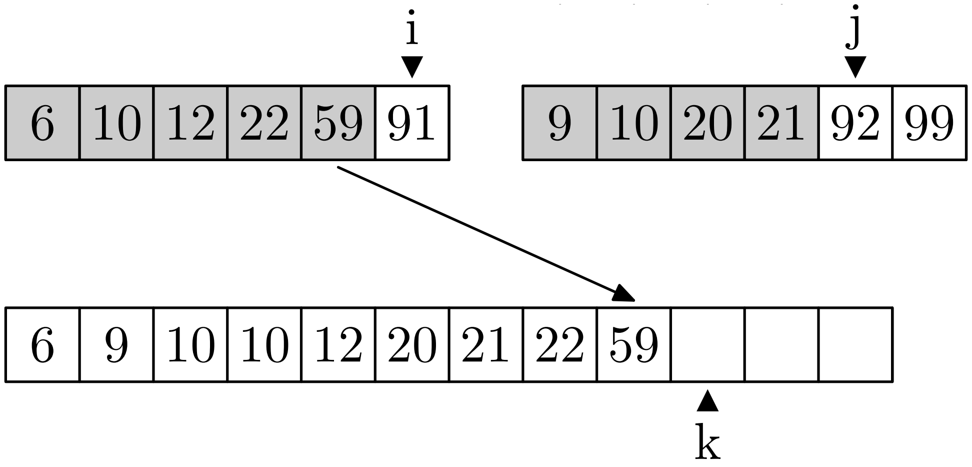

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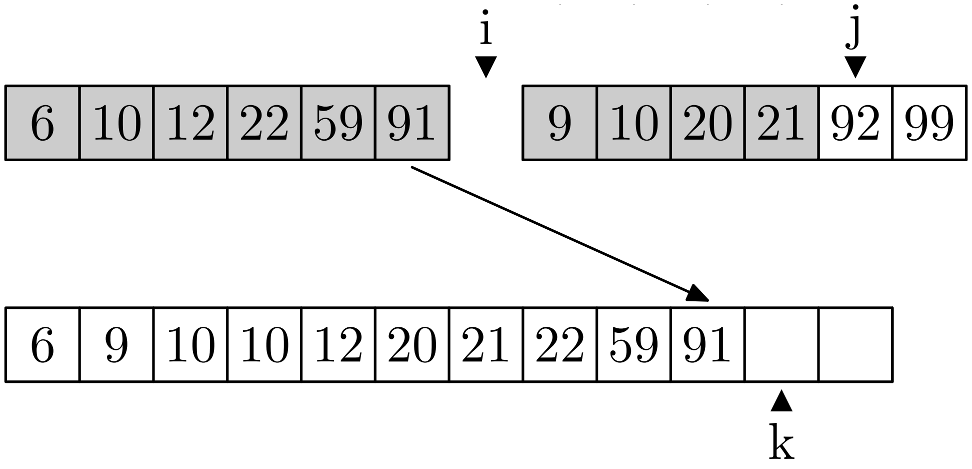

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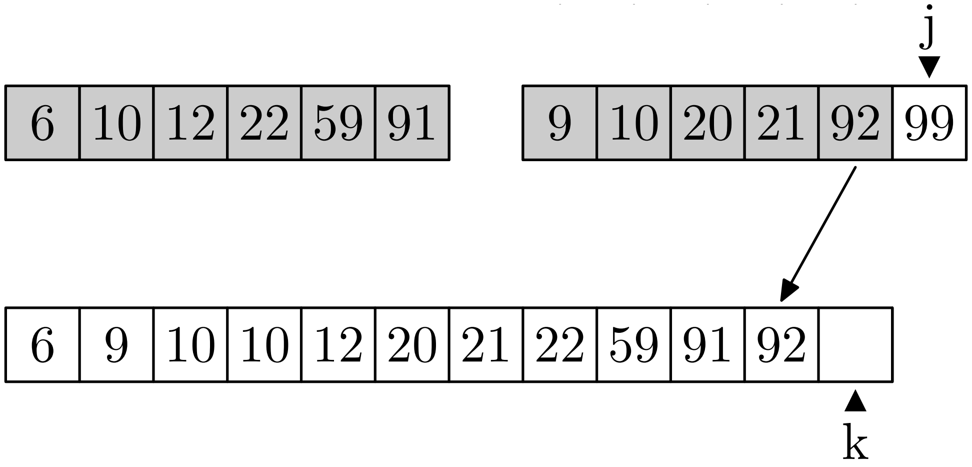

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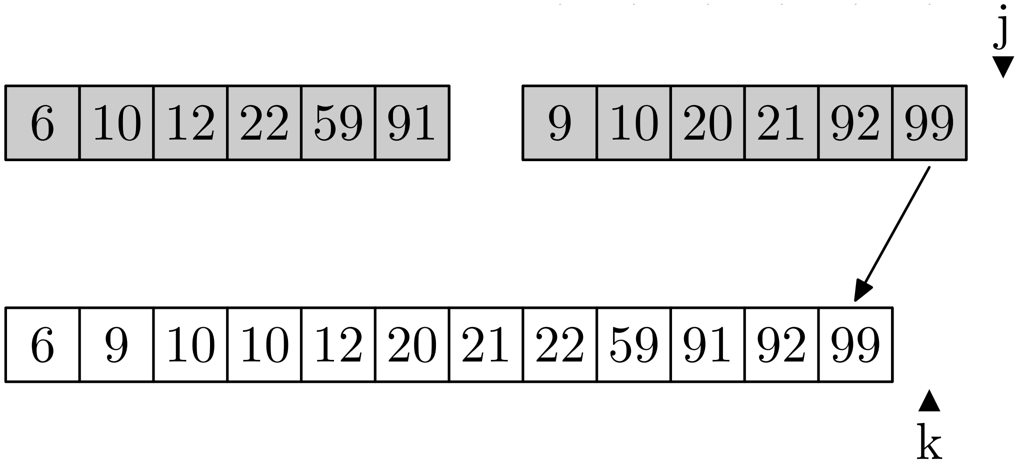

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Properties of Merge Sort

- Merge sort is stable provided we merge carefully (i.e. it preserves the order of two entries with the same value)

- Merge sort isn't in-place—we need an array of at most size n to do the merging

- Merging is quick. Given two arrays of size n the most number of comparisons we need to perform is $n-1$

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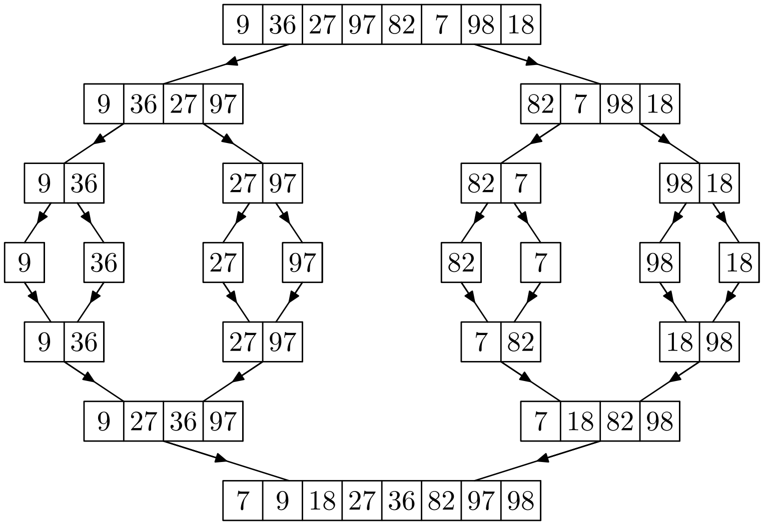
Time Complexity of Merge Sort



9 36 27 97 82 7 98 18

9 36 27 97   82 7 98 18

9 36   27 97   82 7   98 18

9   36   27   97   82   7   98   18

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$$n = 8$$

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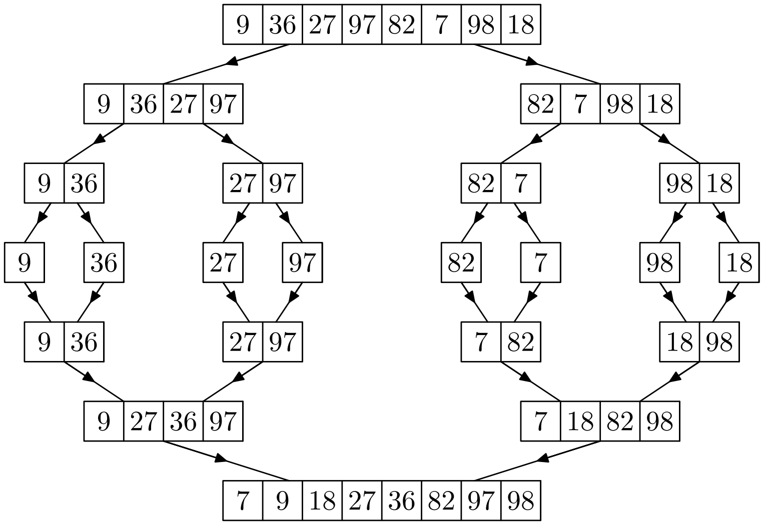

$$\longleftarrow \quad O(n) \quad \longrightarrow$$

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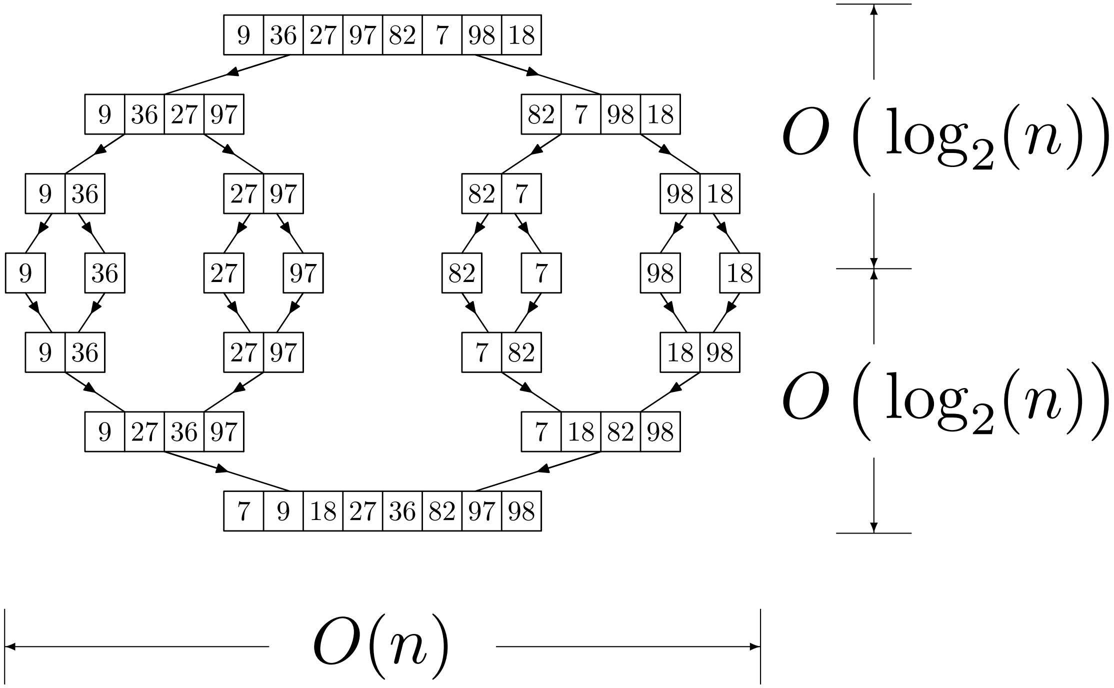

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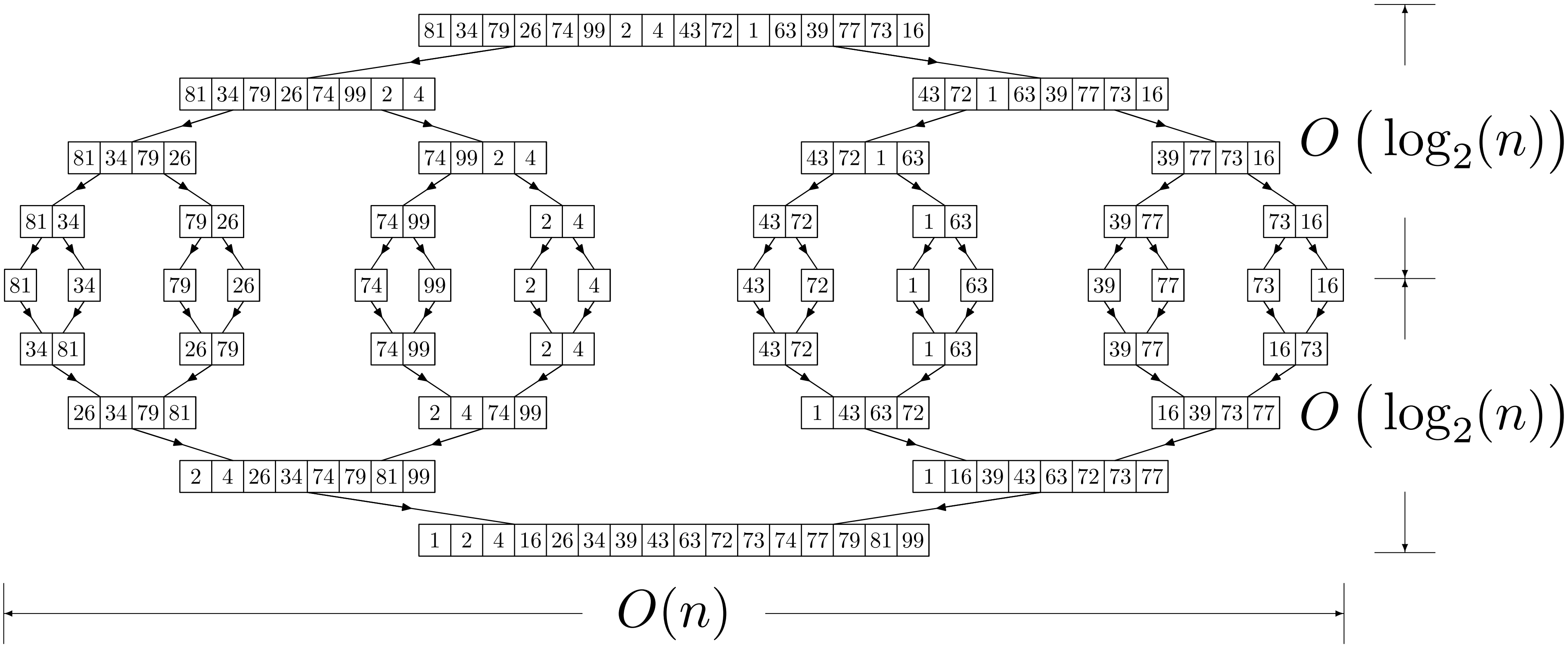

81 34 79 26 74 99 2 4 43 72 1 63 39 77 73 16

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1 63
39 77
73 16

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34
79
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74
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72
1
63
39
77
73
16

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Time Complexity

- We again measure the complexity in the number of comparisons
- From the above argument $C(n) = O(n \times \log_2(n))$
- We can be a bit more formal

$$\begin{aligned} C(n) &= 2C(\lfloor n/2 \rfloor) + C_{\textsf{merge}}(n) && \text{for } n > 1 \\ C(0) &= 1 \end{aligned}$$

- But in the worst case $C_{\textsf{merge}}(n) = n - 1$
- Leads to $C_{\textsf{worst}}(n) = n \log_2(n) - n + 1$

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General Time Complexity

- In general if we have a recursion formula

$$T(n) = aT(n/b) + f(n)$$

with $a \geq 1$, $b > 1$

- If $f(n) \in \Theta(n^d)$ where $d \geq 0$ then

$$T(n) \in \begin{cases} \Theta\left(n^d\right) & \text{if } a < b^d \\ \Theta\left(n^d \log(n)\right) & \text{if } a = b^d \\ \Theta\left(n^{\log_d(a)}\right) & \text{if } a > b^d \end{cases}$$

- Analogous results hold for the family O and Ω

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with $a \geq 1$, $b > 1$

- If $f(n) \in \Theta(n^d)$ where $d \geq 0$ then

$$T(n) \in \begin{cases} \Theta\left(n^d\right) & \text{if } a < b^d \\ \Theta\left(n^d \log(n)\right) & \text{if } a = b^d \\ \Theta\left(n^{\log_d(a)}\right) & \text{if } a > b^d \end{cases}$$

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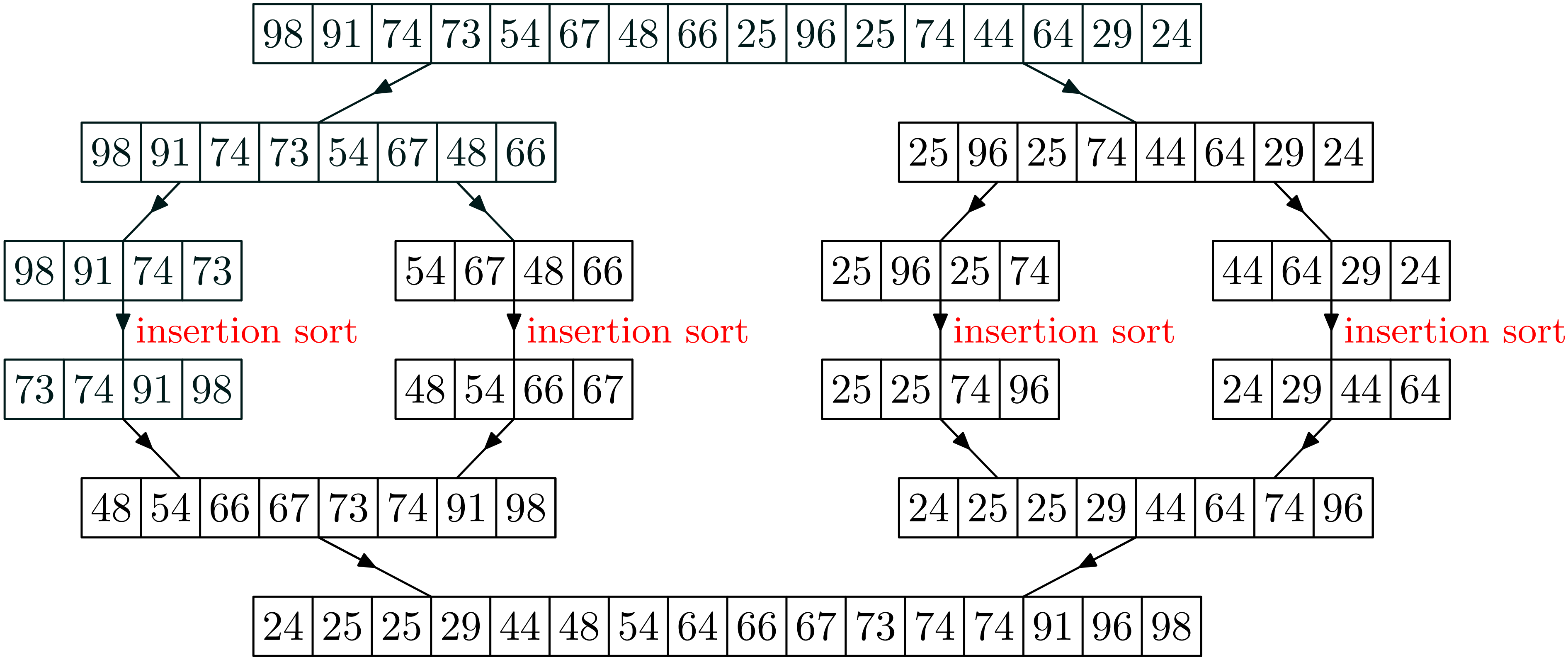
Mixing Sort

- For very short sequences it is faster to use insertion sort than to pay the overhead of function calls



98 91 74 73 54 67 48 66 25 96 25 74 44 64 29 24

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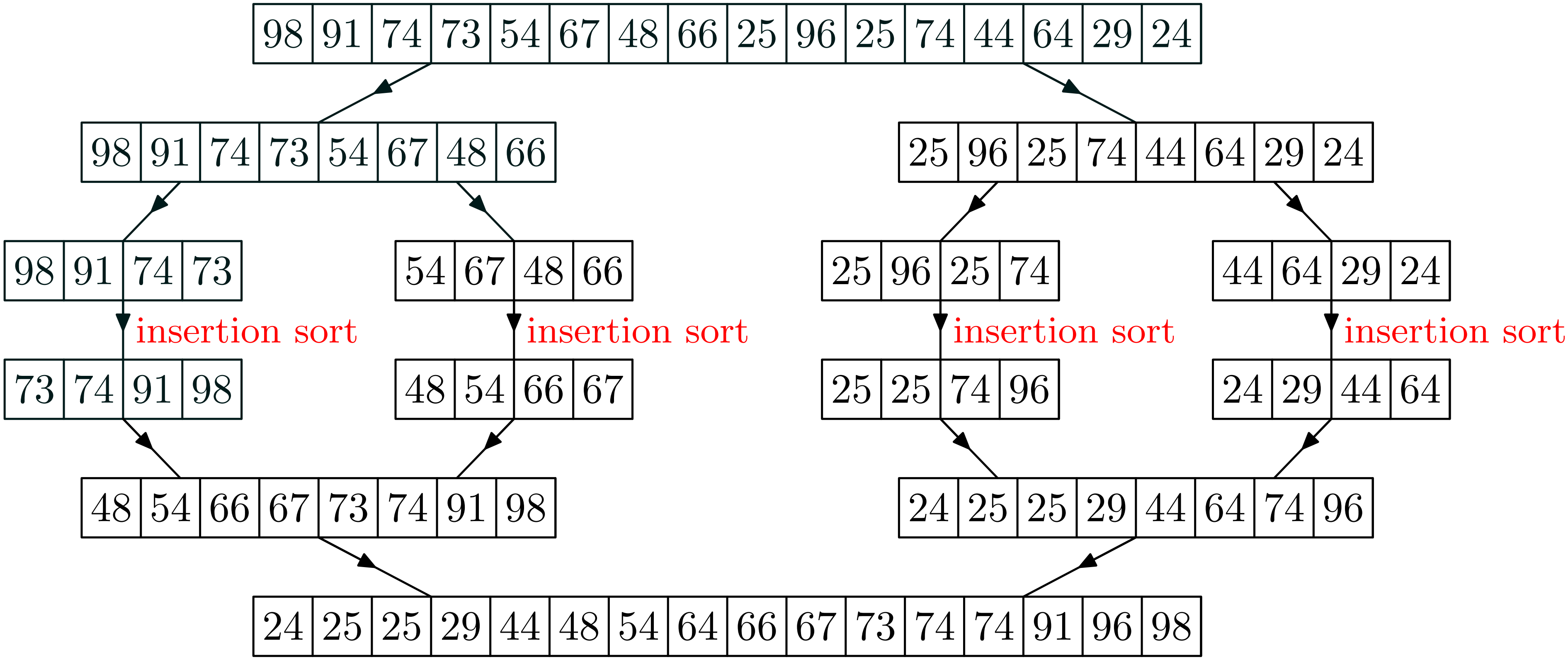

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Outline

1. Merge Sort
2. **Quick Sort**
3. Radix Sort

Quicksort

- The most commonly used fast sorting algorithm is **quicksort**
- It was invented by the British computer scientist by C. A. R. Hoare in 1962
- It again uses the divide-and-conquer strategy
- It can be performed in-place, but it is **not** stable
- It works by splitting an array into two depending on whether the elements are less than or greater than a **pivot** value
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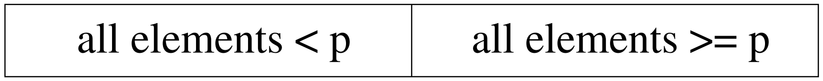

all elements < p

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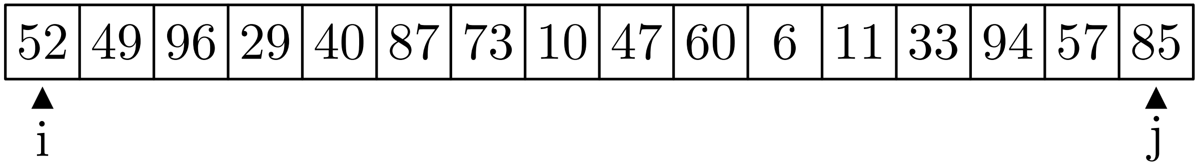

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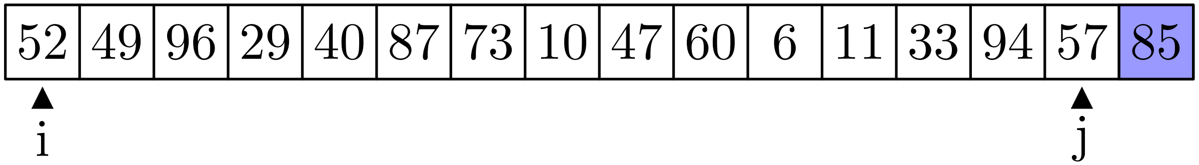

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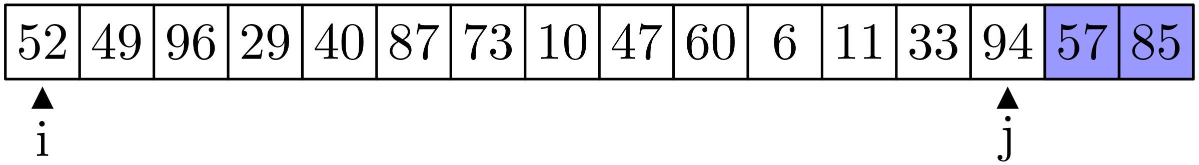

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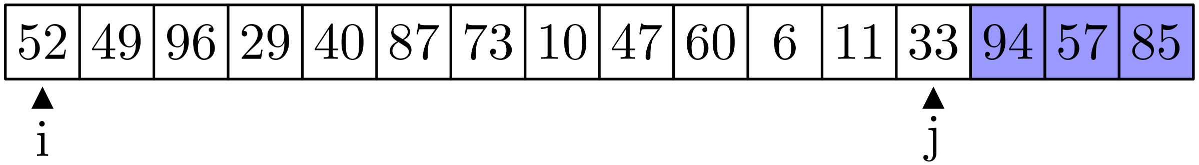

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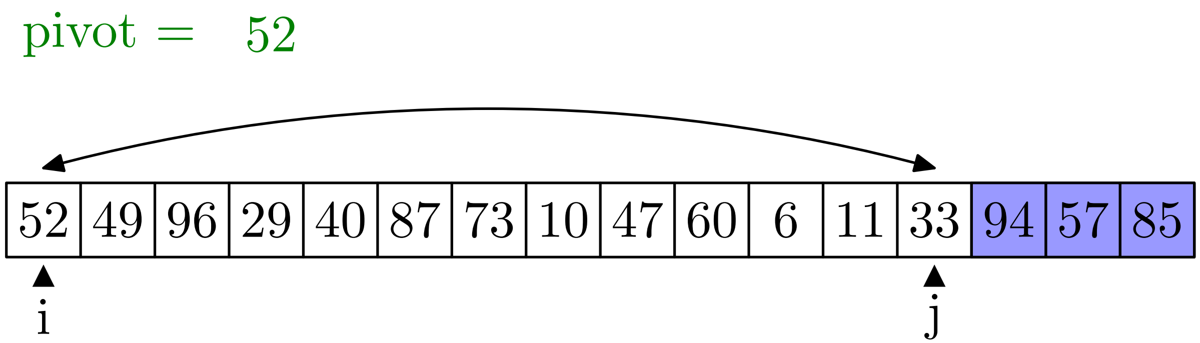

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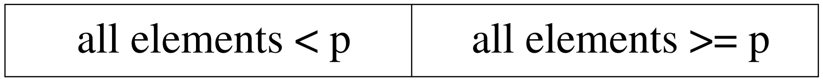

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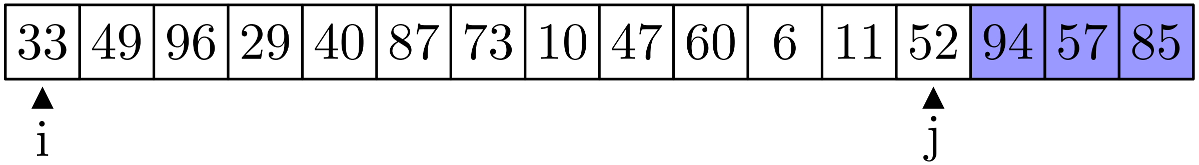

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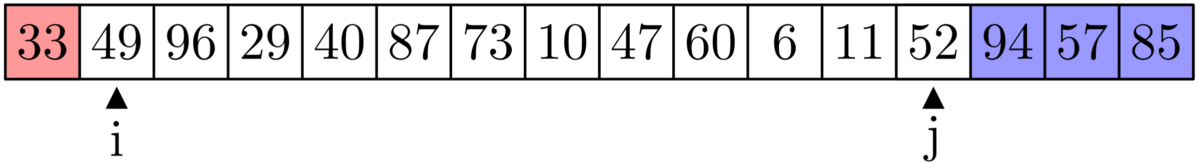

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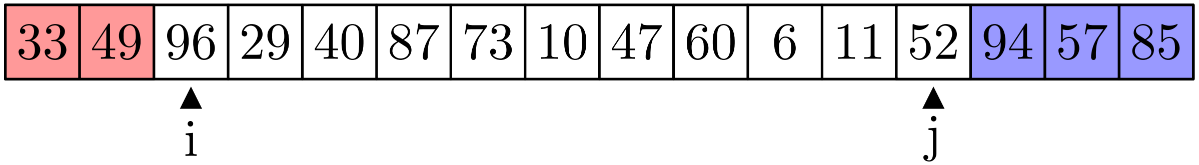

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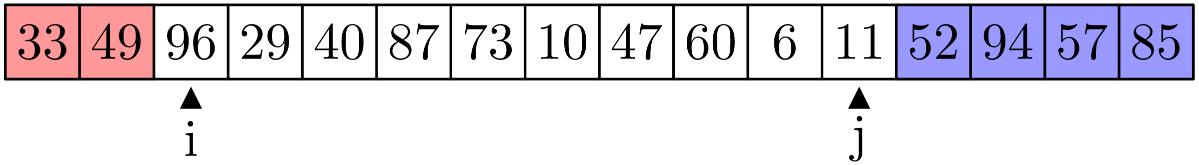

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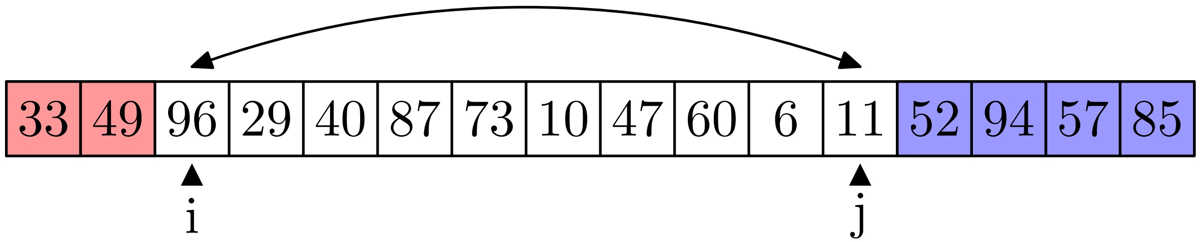

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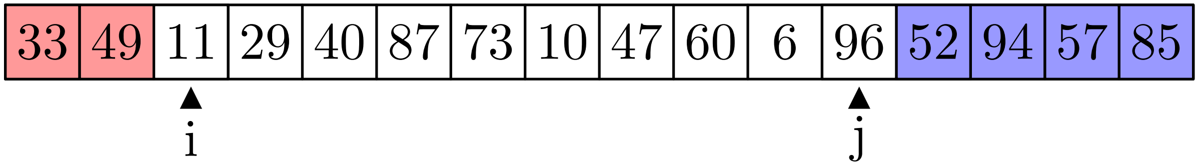

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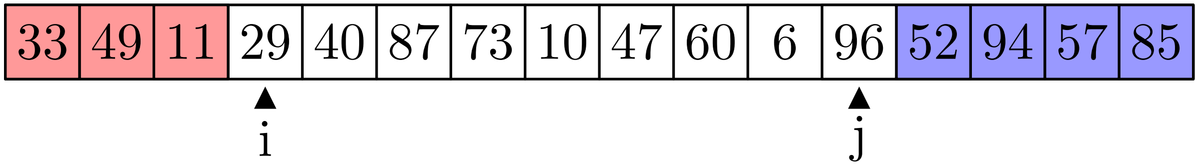

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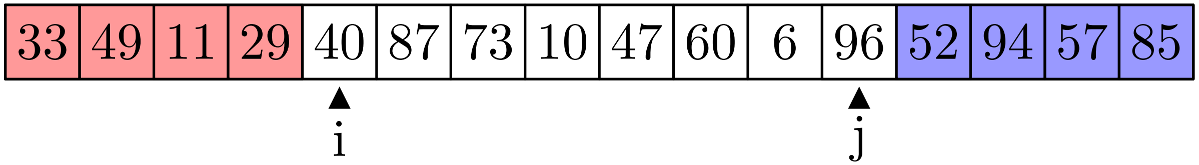

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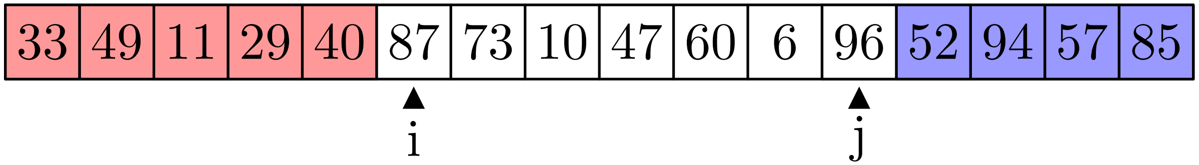

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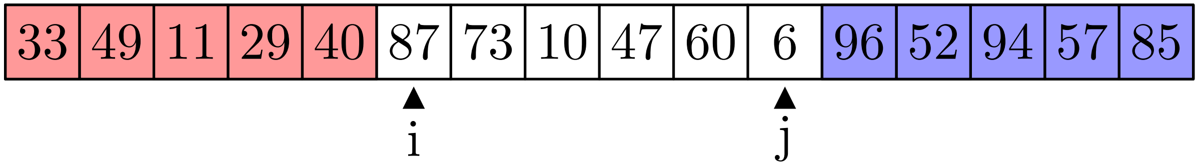

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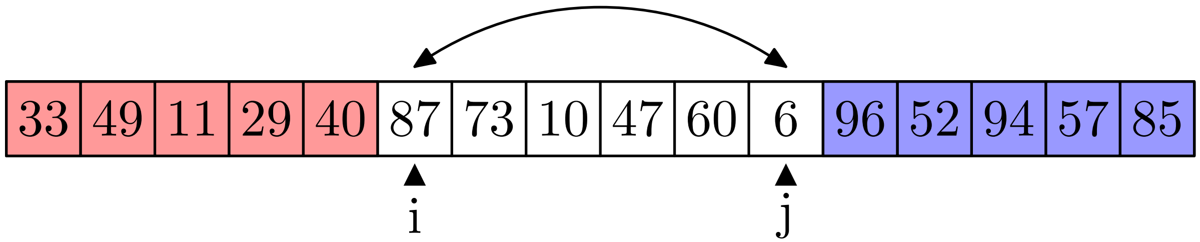

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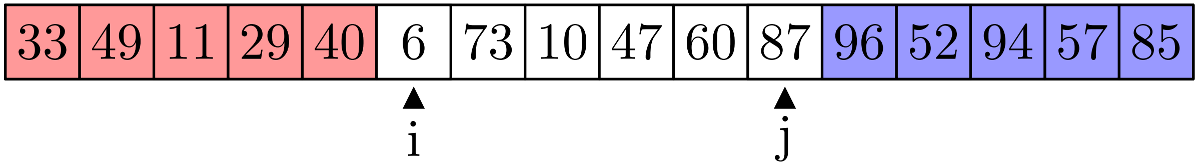

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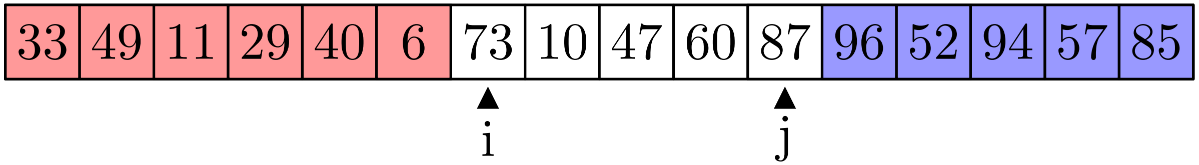

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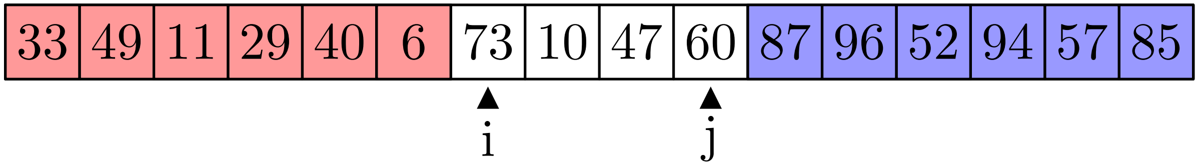

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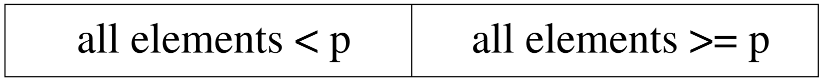

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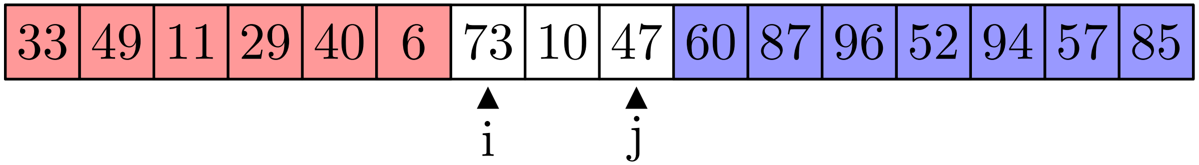

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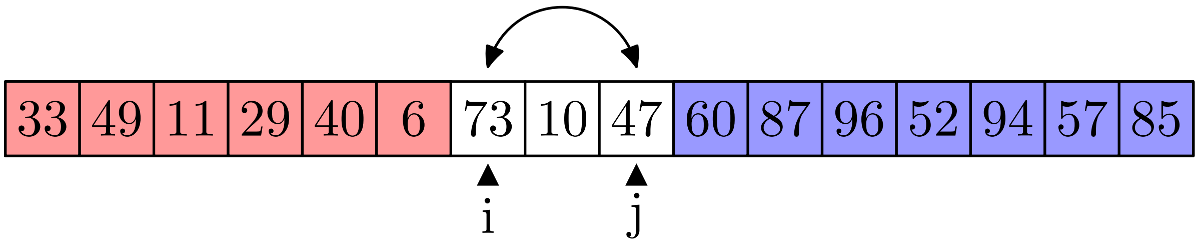

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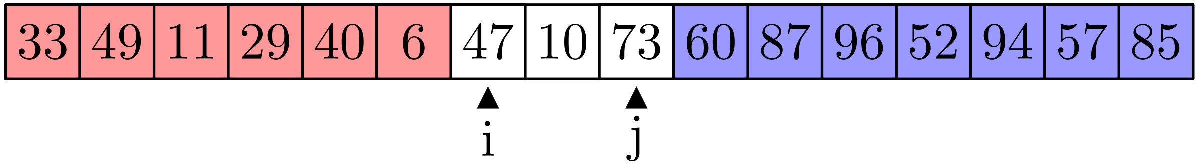


33	49	11	29	40	6	47	10	73	60	87	96	52	94	57	85

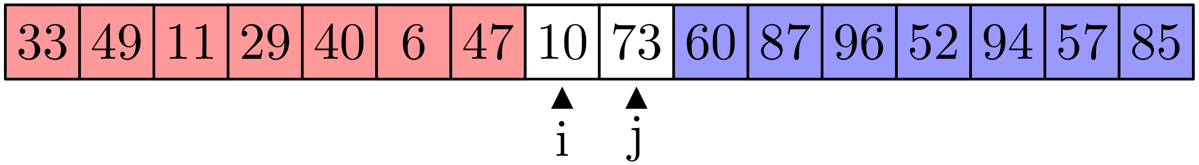
Partition

- We need to partition the array around the pivot p such that

all elements < p	all elements >= p

```
PARTITION(a, p, left, right)
{
  i ← left
  j ← right
  repeat {
    while $a_i < p$
      i++
    while $a_j \geq p$
      j--
    if i $\geq$ j
      break
    SWAP($a_i$, $a_j$)
  }
}
```

pivot $=$ 52



33	49	11	29	40	6	47	10	73	60	87	96	52	94	57	85

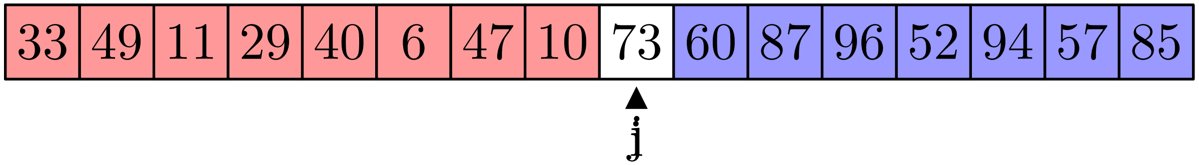
Partition

- We need to partition the array around the pivot p such that

all elements < p	all elements >= p

```
PARTITION(a, p, left, right)
{
  i ←left
  j ←right
  repeat {
    while $a_i < p$
      i++
    while $a_j \ge p$
      j--
    if i ≥ j
      break
    SWAP($a_i$,$a_j$)
  }
}
```

pivot = 52



33	49	11	29	40	6	47	10	73	60	87	96	52	94	57	85

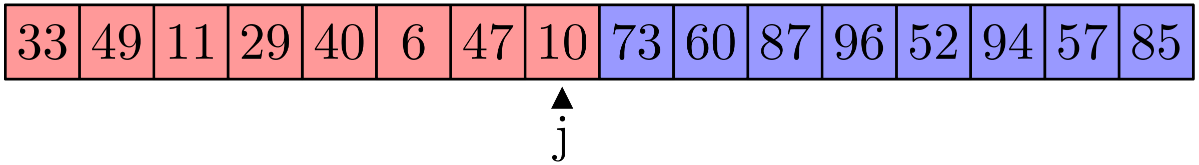
Partition

- We need to partition the array around the pivot p such that

all elements < p	all elements >= p

```
PARTITION(a, p, left, right)
{
  i ← left
  j ← right
  repeat {
    while $a_i < p$
      i++
    while $a_j \geq p$
      j--
    if i $\geq$ j
      break
    SWAP($a_i$, $a_j$)
  }
}
```

pivot = 52



33	49	11	29	40	6	47	10	73	60	87	96	52	94	57	85

Optimising Partitioning

- There are different ways of performing the partitioning
- We want to minimise the time taken on the inner loop
- This means we want to perform as few checks as possible
- One method of doing this is to place *sentinels* at the ends of the array
- We can also reduce work by placing the partition in its correct position

all elements <= p	p	all elements >= p

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all elements <= p	p	all elements >= p

Choosing the Pivot

- There are different strategies to choosing the pivot
- Choose the first element in the array
- Choose the median of the first, middle and last element of the array
- This increases the likelihood of the pivot being close to the median of the whole array
- For large arrays (above 40) the median of 3 medians is often used

Choosing the Pivot

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Choosing the Pivot

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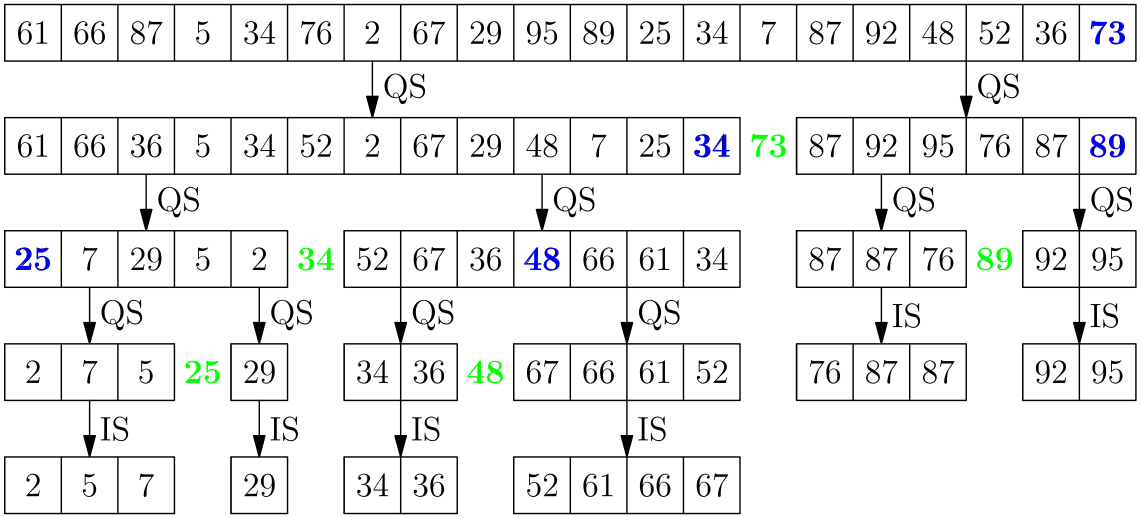
- For large arrays (above 40) the median of 3 medians is often used

Quicksort

We recursively partition the array until each partition is small enough to sort using insertion sort

```
QuickSort(a, left, right) {
  if (right-left < threshold)
    InsertionSort(a, left, right)
  else
    pivot = ChoosePivot(a, left, right)
    part = Partition(a, pivot, left, right)
    QuickSort(a, left, part-1)
    QuickSort(a, part+1, right)
  endif
}
```



61 66 87 5 34 76 2 67 29 95 89 25 34 7 87 92 48 52 36 **73**

QS QS

61 66 36 5 34 52 2 67 29 48 7 25 **34** **73** 87 92 95 76 87 **89**

QS QS QS QS

25 7 29 5 2 **34** 52 67 36 **48** 66 61 34 87 87 76 **89** 92 95

QS QS QS QS IS IS

2 7 5 **25** 29 34 36 **48** 67 66 61 52 76 87 87 92 95

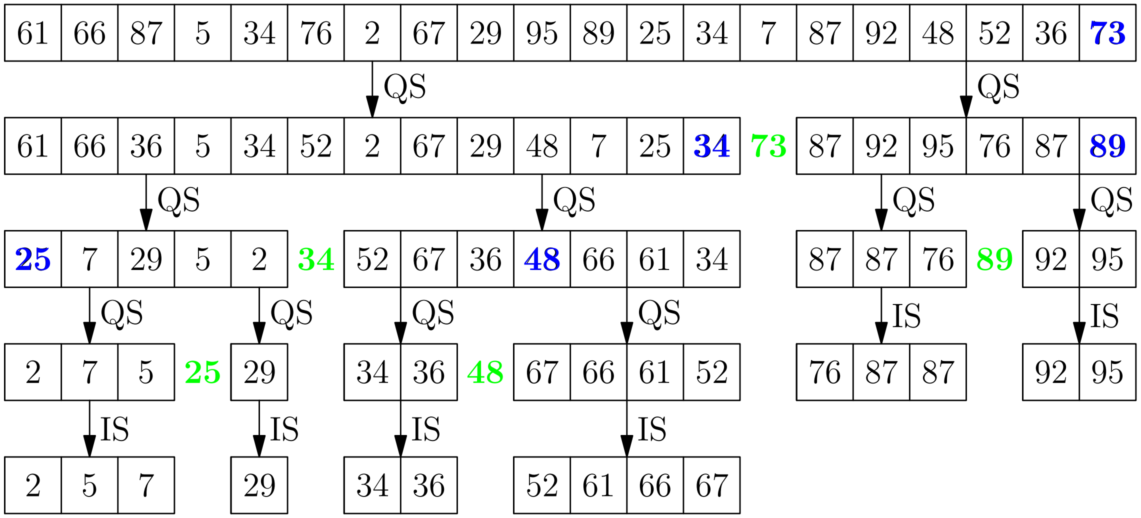
IS IS IS IS

2 5 7 29 34 36 52 61 66 67

Quicksort

We recursively partition the array until each partition is small enough to sort using insertion sort

```
QuickSort(a, left, right) {
  if (right-left < threshold)
    InsertionSort(a, left, right)
  else
    pivot = ChoosePivot(a, left, right)
    part = Partition(a, pivot, left, right)
    QuickSort(a, left, part-1)
    QuickSort(a, part+1, right)
  endif
}
```



61 66 87 5 34 76 2 67 29 95 89 25 34 7 87 92 48 52 36 **73**

QS QS

61 66 36 5 34 52 2 67 29 48 7 25 **34** **73** 87 92 95 76 87 **89**

QS QS QS QS

25 7 29 5 2 **34** 52 67 36 **48** 66 61 34 87 87 76 **89** 92 95

QS QS QS QS IS IS

2 7 5 **25** 29 34 36 **48** 67 66 61 52 76 87 87 92 95

IS IS IS IS

2 5 7 29 34 36 52 61 66 67

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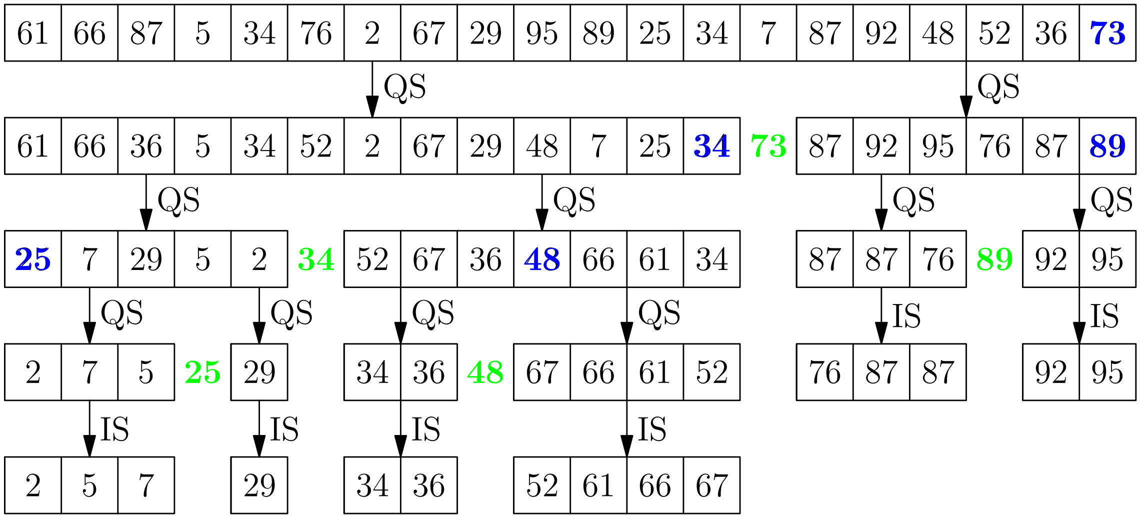
    part = Partition(a, pivot, left, right)
    QuickSort(a, left, part-1)
    QuickSort(a, part+1, right)
  endif
}
```



61 66 87 5 34 76 2 67 29 95 89 25 34 7 87 92 48 52 36 **73**

QS QS

61 66 36 5 34 52 2 67 29 48 7 25 **34** **73** 87 92 95 76 87 **89**

QS QS QS QS

25 7 29 5 2 **34** 52 67 36 **48** 66 61 34 87 87 76 **89** 92 95

QS QS QS QS IS IS

2 7 5 **25** 29 34 36 **48** 67 66 61 52 76 87 87 92 95

IS IS IS IS

2 5 7 29 34 36 52 61 66 67

Time Complexity

- Partitioning an array of size n takes $\Theta(n)$ operations
- If we split the array in half then number of partitions we need to do is $\lceil \log_2(n) \rceil$
- This is the best case thus quicksort is $\Omega\left(n \log(n)\right)$
- If the pivot is the minimum element of the array then we have to partition $n-1$ times
- This is the worst case so quicksort is $O\left(n^2\right)$
- This worst case will happen if the array is already sorted and we choose the pivot to be the first element in the array!

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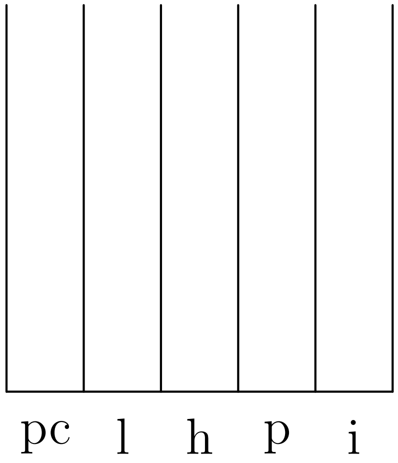
- This worst case will happen if the array is already sorted and we choose the pivot to be the first element in the array!

QuickSort

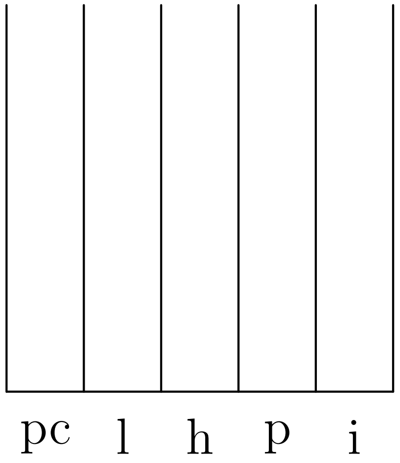
```
0 quickSort(a, l, h) {
1   if(h-l>3) {
2     p = choosePivot(a, l, h)
3     i = partition(a, p, l, h)
4     quickSort(a, l, i-1)
5     quickSort(a, i+1, h)
6   } else
7     insertionSort(a, l, h)
8   return
9 }
```



QuickSort

```
0 quickSort(a, 0, 19) {
1   if(19-0>3) {
2     p = choosePivot(a, 0, 19)
3     i = partition(a, p, 0, 19)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 0, 19)
8   return
9 }
```

```
PC = 0
l = 0
h = 19
p = #
i = #
```

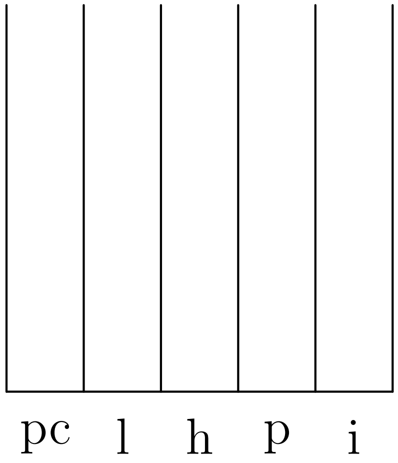


0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	87	5	34	76	2	67	29	95	89	25	34	7	87	92	48	52	36	73
low																			high

QuickSort

```
0 quickSort(a, 0, 19) {
1   if(19-0>3) {
2     p = choosePivot(a, 0, 19)
3     i = partition(a, p, 0, 19)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 0, 19)
8   return
9 }
```

```
PC = 1
l = 0
h = 19
p = #
i = #
```

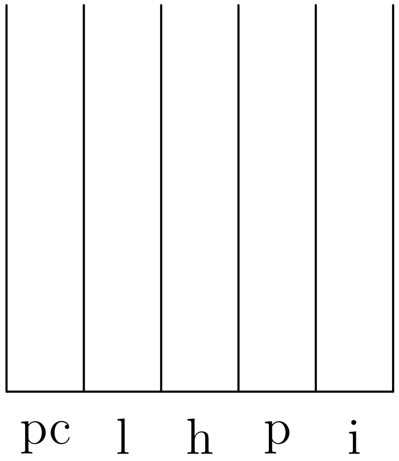


0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	87	5	34	76	2	67	29	95	89	25	34	7	87	92	48	52	36	73
low																			high

QuickSort

```
0 quickSort(a, 0, 19) {
1   if(19-0>3) {
2     p = choosePivot(a, 0, 19)
3     i = partition(a, p, 0, 19)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 0, 19)
8   return
9 }
```

```
PC = 2
l = 0
h = 19
p = #
i = #
```



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	87	5	34	76	2	67	29	95	89	25	34	7	87	92	48	52	36	73
low																			high

QuickSort

```
0  quickSort(a, 0, 19) {
1    if(19-0>3) {
2      p = choosePivot(a, 0, 19)
3      i = partition(a, p, 0, 19)
4      quickSort(a, 0, i-1)
5      quickSort(a, i+1, 19)
6    } else
7      insertionSort(a, 0, 19)
8    return
9  }
```

```
PC = 2
l = 0
h = 19
p = #
i = #
```

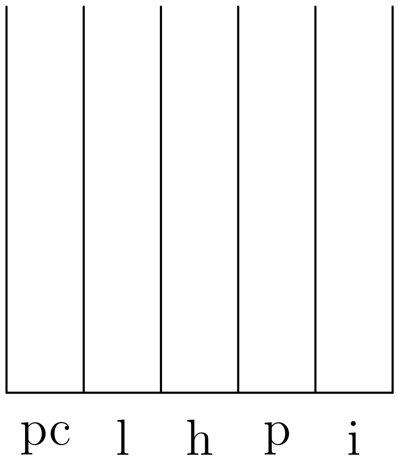
2	0	19	#	#
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	87	5	34	76	2	67	29	95	89	25	34	7	87	92	48	52	36	73
low																			high

QuickSort

```
PC = 3
l = 0
h = 19
p = 73
i = #
```

```
0 quickSort(a, 0, 19) {
1   if(19-0>3) {
2     p = choosePivot(a, 0, 19)
3     i = partition(a, 73, 0, 19)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 0, 19)
8   return
9 }
```



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	87	5	34	76	2	67	29	95	89	25	34	7	87	92	48	52	36	73
low																			high

QuickSort

```
PC = 3
l  = 0
h  = 19
p  = 73
i  = #
```

```
0  quickSort(a, 0, 19) {
1    if(19-0>3) {
2      p = choosePivot(a, 0, 19)
3      i = partition(a, 73, 0, 19)
4      quickSort(a, 0, i-1)
5      quickSort(a, i+1, 19)
6    } else
7      insertionSort(a, 0, 19)
8    return
9  }
```

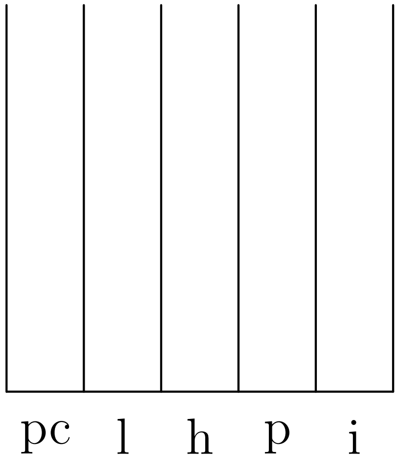
3	0	19	73	#
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	36	5	34	52	2	67	29	48	7	25	34	73	87	92	95	76	87	89
low																			high

QuickSort

```
0  quickSort(a, 0, 19) {
1    if(19-0>3) {
2      p = choosePivot(a, 0, 19)
3      i = partition(a, 73, 0, 19)
4      quickSort(a, 0, 13-1)
5      quickSort(a, 13+1, 19)
6    } else
7      insertionSort(a, 0, 19)
8    return
9  }
```

```
PC = 4
l = 0
h = 19
p = 73
i = 13
```



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	36	5	34	52	2	67	29	48	7	25	34	73	87	92	95	76	87	89
low																			high

QuickSort

```
0 quickSort(a, 0, 12) {
1   if(12-0>3) {
2     p = choosePivot(a, 0, 12)
3     i = partition(a, p, 0, 12)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 0, 12)
8   return
9 }
```

```
PC = 0
l  = 0
h  = 12
p  = #
i  = #
```

pc	l	h	p	i
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	36	5	34	52	2	67	29	48	7	25	34	73	87	92	95	76	87	89
low												high							

QuickSort

```
0 quickSort(a, 0, 12) {
1   if(12-0>3) {
2     p = choosePivot(a, 0, 12)
3     i = partition(a, p, 0, 12)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 0, 12)
8   return
9 }
```

```
PC = 1
l = 0
h = 12
p = #
i = #
```

pc	l	h	p	i
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	36	5	34	52	2	67	29	48	7	25	34	73	87	92	95	76	87	89
low												high							

QuickSort

```
PC = 2
l = 0
h = 12
p = #
i = #
```

```
0 quickSort(a, 0, 12) {
1   if(12-0>3) {
2     p = choosePivot(a, 0, 12)
3     i = partition(a, p, 0, 12)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 0, 12)
8   return
9 }
```

pc	l	h	p	i
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	36	5	34	52	2	67	29	48	7	25	34	73	87	92	95	76	87	89
low												high							

QuickSort

```
PC = 2
l = 0
h = 12
p = #
i = #
```

```
0 quickSort(a, 0, 12) {
1   if(12-0>3) {
2     p = choosePivot(a, 0, 12)
3     i = partition(a, p, 0, 12)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 0, 12)
8   return
9 }
```

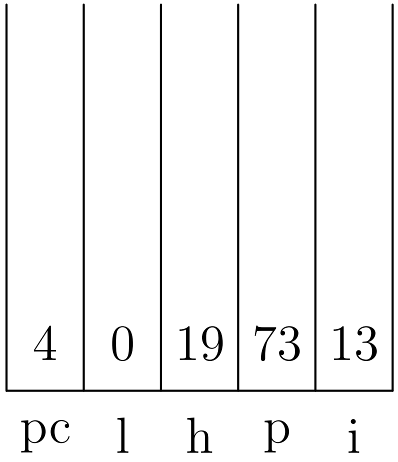
pc	l	h	p	i
2	0	12	#	#
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	36	5	34	52	2	67	29	48	7	25	34	73	87	92	95	76	87	89
low												high							

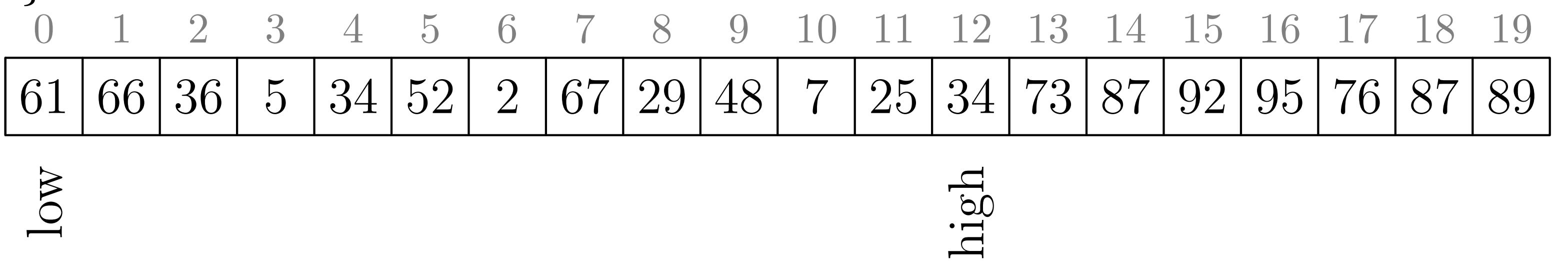
QuickSort

```
0  quickSort(a, 0, 12) {
1    if(12-0>3) {
2      p = choosePivot(a, 0, 12)
3      i = partition(a, 34, 0, 12)
4      quickSort(a, 0, i-1)
5      quickSort(a, i+1, 12)
6    } else
7      insertionSort(a, 0, 12)
8    return
9  }
```

```
PC = 3
l = 0
h = 12
p = 34
i = #
```



pc	l	h	p	i
4	0	19	73	13



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
61	66	36	5	34	52	2	67	29	48	7	25	34	73	87	92	95	76	87	89
low												high							

QuickSort

```
0  quickSort(a, 0, 12) {
1    if(12-0>3) {
2      p = choosePivot(a, 0, 12)
3      i = partition(a, 34, 0, 12)
4      quickSort(a, 0, i-1)
5      quickSort(a, i+1, 12)
6    } else
7      insertionSort(a, 0, 12)
8    return
9  }
```

```
PC = 3
l = 0
h = 12
p = 34
i = #
```

pc	l	h	p	i
3	0	12	34	#
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
25	7	29	5	2	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low												high							

QuickSort

```
0  quickSort(a, 0, 12) {
1    if(12-0>3) {
2      p = choosePivot(a, 0, 12)
3      i = partition(a, 34, 0, 12)
4      quickSort(a, 0, 5-1)
5      quickSort(a, 5+1, 12)
6    } else
7      insertionSort(a, 0, 12)
8    return
9  }
```

```
PC = 4
l = 0
h = 12
p = 34
i = 5
```

4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
25	7	29	5	2	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low												high							

QuickSort

```
0 quickSort(a, 0, 4) {
1   if(4-0>3) {
2     p = choosePivot(a, 0, 4)
3     i = partition(a, p, 0, 4)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 4)
6   } else
7     insertionSort(a, 0, 4)
8   return
9 }
```

```
PC = 0
l = 0
h = 4
p = #
i = #
```

pc	l	h	p	i
4	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
25	7	29	5	2	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low				high															

QuickSort

```
0 quickSort(a, 0, 4) {
1   if(4-0>3) {
2     p = choosePivot(a, 0, 4)
3     i = partition(a, p, 0, 4)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 4)
6   } else
7     insertionSort(a, 0, 4)
8   return
9 }
```

```
PC = 1
l = 0
h = 4
p = #
i = #
```

pc	l	h	p	i
4	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
25	7	29	5	2	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low				high															

QuickSort

```
0  quickSort(a, 0, 4) {
1    if(4-0>3) {
2      p = choosePivot(a, 0, 4)
3      i = partition(a, p, 0, 4)
4      quickSort(a, 0, i-1)
5      quickSort(a, i+1, 4)
6    } else
7      insertionSort(a, 0, 4)
8    return
9  }
```

```
PC = 2
l = 0
h = 4
p = #
i = #
```

pc	l	h	p	i
4	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
25	7	29	5	2	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low				high															

QuickSort

```
0 quickSort(a, 0, 4) {
1   if(4-0>3) {
2     p = choosePivot(a, 0, 4)
3     i = partition(a, p, 0, 4)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 4)
6   } else
7     insertionSort(a, 0, 4)
8   return
9 }
```

```
PC = 2
l = 0
h = 4
p = #
i = #
```

pc	l	h	p	i
2	0	4	#	#
4	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
25	7	29	5	2	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low				high															

QuickSort

```
0  quickSort(a, 0, 4) {
1    if(4-0>3) {
2      p = choosePivot(a, 0, 4)
3      i = partition(a, 25, 0, 4)
4      quickSort(a, 0, i-1)
5      quickSort(a, i+1, 4)
6    } else
7      insertionSort(a, 0, 4)
8    return
9  }
```

```
PC = 3
l = 0
h = 4
p = 25
i = #
```

pc	l	h	p	i
4	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
25	7	29	5	2	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low				high															

QuickSort

```
0 quickSort(a, 0, 4) {
1   if(4-0>3) {
2     p = choosePivot(a, 0, 4)
3     i = partition(a, 25, 0, 4)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 4)
6   } else
7     insertionSort(a, 0, 4)
8   return
9 }
```

```
PC = 3
l = 0
h = 4
p = 25
i = #
```

pc	l	h	p	i
3	0	4	25	#
4	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	7	5	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low				high															

QuickSort

```
0  quickSort(a, 0, 4) {
1    if(4-0>3) {
2      p = choosePivot(a, 0, 4)
3      i = partition(a, 25, 0, 4)
4      quickSort(a, 0, 3-1)
5      quickSort(a, 3+1, 4)
6    } else
7      insertionSort(a, 0, 4)
8    return
9  }
```

```
PC = 4
l = 0
h = 4
p = 25
i = 3
```

pc	l	h	p	i
4	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	7	5	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low				high															

QuickSort

```
0 quickSort(a, 0, 2) {
1   if(2-0>3) {
2     p = choosePivot(a, 0, 2)
3     i = partition(a, p, 0, 2)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 2)
6   } else
7     insertionSort(a, 0, 2)
8   return
9 }
```

```
PC = 0
l = 0
h = 2
p = #
i = #
```

pc	l	h	p	i
4	0	4	25	3
4	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	7	5	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low		high																	

QuickSort

```
0 quickSort(a, 0, 2) {
1   if(2-0>3) {
2     p = choosePivot(a, 0, 2)
3     i = partition(a, p, 0, 2)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 2)
6   } else
7     insertionSort(a, 0, 2)
8   return
9 }
```

```
PC = 1
l = 0
h = 2
p = #
i = #
```

pc	l	h	p	i
4	0	4	25	3
4	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	7	5	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low		high																	

QuickSort

```
0  quickSort(a, 0, 2) {
1    if(2-0>3) {
2      p = choosePivot(a, 0, 2)
3      i = partition(a, p, 0, 2)
4      quickSort(a, 0, i-1)
5      quickSort(a, i+1, 2)
6    } else
7      insertionSort(a, 0, 2)
8    return
9  }
```

```
PC = 7
l = 0
h = 2
p = #
i = #
```

pc	l	h	p	i
4	0	4	25	3
4	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	7	5	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low		high																	

QuickSort

```
0 quickSort(a, 0, 2) {
1   if(2-0>3) {
2     p = choosePivot(a, 0, 2)
3     i = partition(a, p, 0, 2)
4     quickSort(a, 0, i-1)
5     quickSort(a, i+1, 2)
6   } else
7     insertionSort(a, 0, 2)
8   return
9 }
```

```
PC = 7
l = 0
h = 2
p = #
i = #
```

pc	l	h	p	i
7	0	2	#	#
4	0	4	25	3
4	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low		high																	

QuickSort

```
0  quickSort(a, 0, 2) {
1    if(2-0>3) {
2      p = choosePivot(a, 0, 2)
3      i = partition(a, p, 0, 2)
4      quickSort(a, 0, i-1)
5      quickSort(a, i+1, 2)
6    } else
7      insertionSort(a, 0, 2)
8    return
9  }
```

```
PC = 8
l = 0
h = 2
p = #
i = #
```

pc	l	h	p	i
4	0	4	25	3
4	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low		high																	

QuickSort

```
0 quickSort(a, 0, 4) {
1   if(4-0>3) {
2     p = choosePivot(a, 0, 4)
3     i = partition(a, 25, 0, 4)
4     quickSort(a, 0, 3-1)
5     quickSort(a, 3+1, 4)
6   } else
7     insertionSort(a, 0, 4)
8   return
9 }
```

```
PC = 5
l = 0
h = 4
p = 25
i = 3
```

4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low				high															

QuickSort

```
0 quickSort(a, 4, 4) {
1   if(4-4>3) {
2     p = choosePivot(a, 4, 4)
3     i = partition(a, p, 4, 4)
4     quickSort(a, 4, i-1)
5     quickSort(a, i+1, 4)
6   } else
7     insertionSort(a, 4, 4)
8   return
9 }
```

```
PC = 0
l = 4
h = 4
p = #
i = #
```

pc	l	h	p	i
5	0	4	25	3
4	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89

high
low

QuickSort

```
0 quickSort(a, 4, 4) {
1   if(4-4>3) {
2     p = choosePivot(a, 4, 4)
3     i = partition(a, p, 4, 4)
4     quickSort(a, 4, i-1)
5     quickSort(a, i+1, 4)
6   } else
7     insertionSort(a, 4, 4)
8   return
9 }
```

```
PC = 1
l  = 4
h  = 4
p  = #
i  = #
```

pc	l	h	p	i
5	0	4	25	3
4	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89

low
high

QuickSort

```
0  quickSort(a, 4, 4) {
1    if(4-4>3) {
2      p = choosePivot(a, 4, 4)
3      i = partition(a, p, 4, 4)
4      quickSort(a, 4, i-1)
5      quickSort(a, i+1, 4)
6    } else
7      insertionSort(a, 4, 4)
8    return
9  }
```

```
PC = 7
l  = 4
h  = 4
p  = #
i  = #
```

pc	l	h	p	i
5	0	4	25	3
4	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
				low high															

QuickSort

```
0  quickSort(a, 4, 4) {
1    if(4-4>3) {
2      p = choosePivot(a, 4, 4)
3      i = partition(a, p, 4, 4)
4      quickSort(a, 4, i-1)
5      quickSort(a, i+1, 4)
6    } else
7      insertionSort(a, 4, 4)
8    return
9  }
```

```
PC = 7
l = 4
h = 4
p = #
i = #
```

pc	l	h	p	i
7	4	4	#	#
5	0	4	25	3
4	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
				high low															

QuickSort

```
PC = 8
l = 4
h = 4
p = #
i = #
```

```
0 quickSort(a, 4, 4) {
1   if(4-4>3) {
2     p = choosePivot(a, 4, 4)
3     i = partition(a, p, 4, 4)
4     quickSort(a, 4, i-1)
5     quickSort(a, i+1, 4)
6   } else
7     insertionSort(a, 4, 4)
8   return
9 }
```

pc	l	h	p	i
5	0	4	25	3
4	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
				low high															

QuickSort

```
0 quickSort(a, 0, 4) {
1   if(4-0>3) {
2     p = choosePivot(a, 0, 4)
3     i = partition(a, 25, 0, 4)
4     quickSort(a, 0, 3-1)
5     quickSort(a, 3+1, 4)
6   } else
7     insertionSort(a, 0, 4)
8   return
9 }
```

```
PC = 8
l = 0
h = 4
p = 25
i = 3
```

4	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low				high															

QuickSort

```
0 quickSort(a, 0, 12) {
1   if(12-0>3) {
2     p = choosePivot(a, 0, 12)
3     i = partition(a, 34, 0, 12)
4     quickSort(a, 0, 5-1)
5     quickSort(a, 5+1, 12)
6   } else
7     insertionSort(a, 0, 12)
8   return
9 }
```

```
PC = 5
l = 0
h = 12
p = 34
i = 5
```

4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
low												high							

QuickSort

```
0 quickSort(a, 6, 12) {
1   if(12-6>3) {
2     p = choosePivot(a, 6, 12)
3     i = partition(a, p, 6, 12)
4     quickSort(a, 6, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 6, 12)
8   return
9 }
```

```
PC = 0
l  = 6
h  = 12
p  = #
i  = #
```

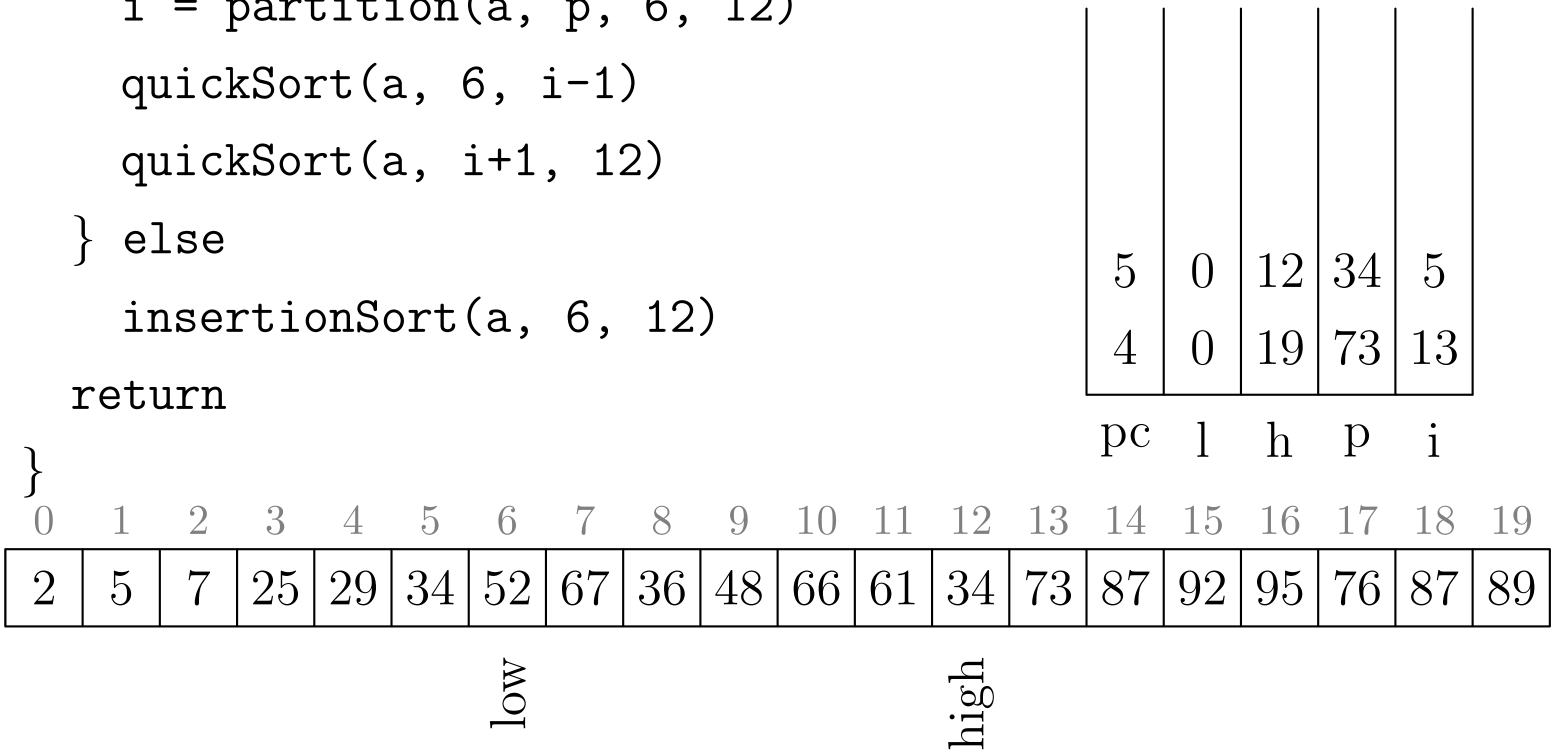
pc	l	h	p	i
5	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
						low						high							

QuickSort

```
0 quickSort(a, 6, 12) {
1   if(12-6>3) {
2     p = choosePivot(a, 6, 12)
3     i = partition(a, p, 6, 12)
4     quickSort(a, 6, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 6, 12)
8   return
9 }
```

```
PC = 1
l = 6
h = 12
p = #
i = #
```



pc	l	h	p	i
5	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
						low						high							

QuickSort

```
0  quickSort(a, 6, 12) {
1    if(12-6>3) {
2      p = choosePivot(a, 6, 12)
3      i = partition(a, p, 6, 12)
4      quickSort(a, 6, i-1)
5      quickSort(a, i+1, 12)
6    } else
7      insertionSort(a, 6, 12)
8    return
9  }
```

```
PC = 2
l = 6
h = 12
p = #
i = #
```

pc	l	h	p	i
5	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
						low						high							

QuickSort

```
0  quickSort(a, 6, 12) {
1    if(12-6>3) {
2      p = choosePivot(a, 6, 12)
3      i = partition(a, p, 6, 12)
4      quickSort(a, 6, i-1)
5      quickSort(a, i+1, 12)
6    } else
7      insertionSort(a, 6, 12)
8    return
9  }
```

```
PC = 2
l = 6
h = 12
p = #
i = #
```

pc	l	h	p	i
2	6	12	#	#
5	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
						low						high							

QuickSort

```
0 quickSort(a, 6, 12) {
1   if(12-6>3) {
2     p = choosePivot(a, 6, 12)
3     i = partition(a, 48, 6, 12)
4     quickSort(a, 6, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 6, 12)
8   return
9 }
```

```
PC = 3
l = 6
h = 12
p = 48
i = #
```

pc	l	h	p	i
5	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	52	67	36	48	66	61	34	73	87	92	95	76	87	89
						low						high							

QuickSort

```
0 quickSort(a, 6, 12) {
1   if(12-6>3) {
2     p = choosePivot(a, 6, 12)
3     i = partition(a, 48, 6, 12)
4     quickSort(a, 6, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 6, 12)
8   return
9 }
```

```
PC = 3
l = 6
h = 12
p = 48
i = #
```

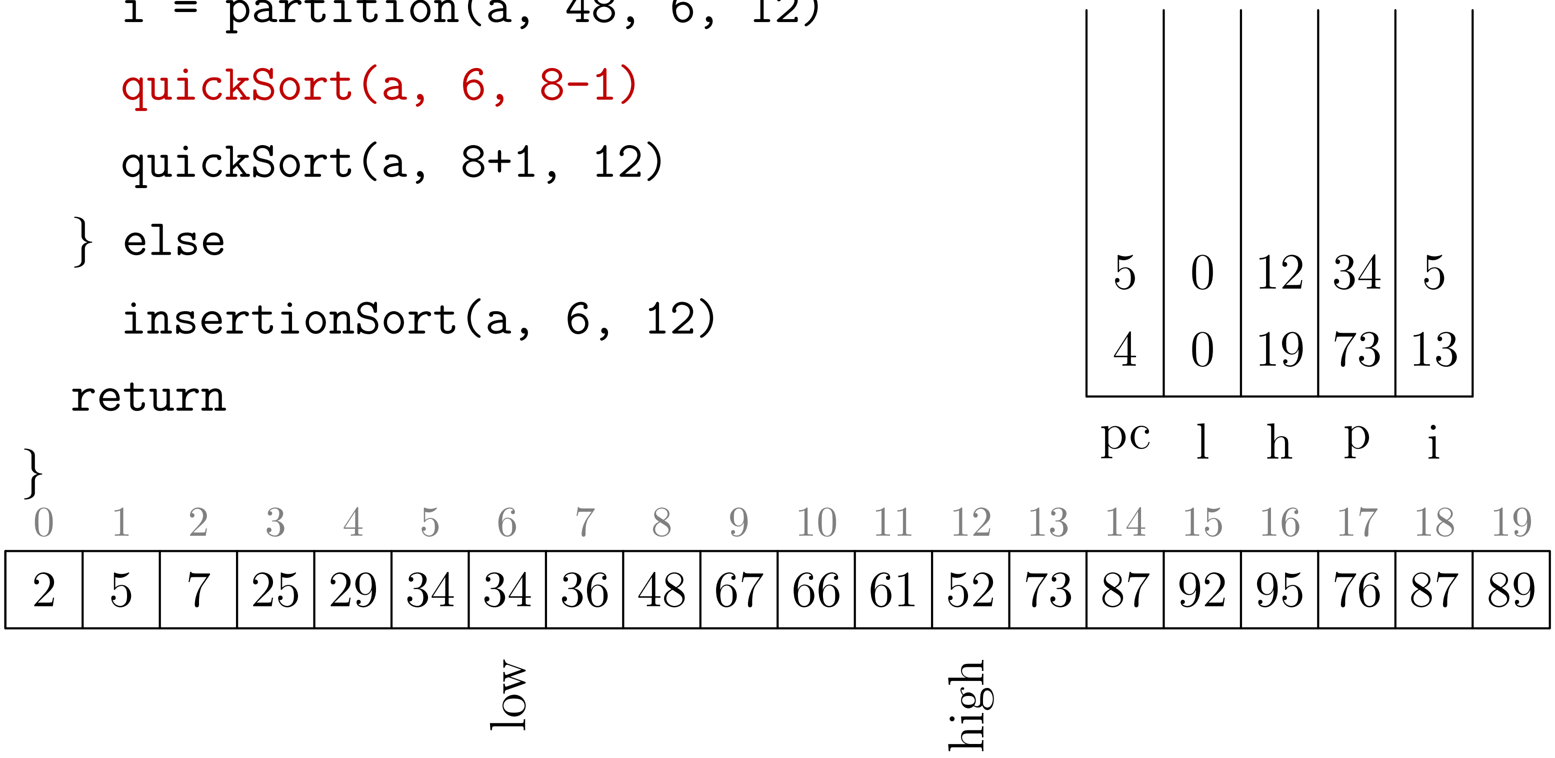
pc	l	h	p	i
3	6	12	48	#
5	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89
						low						high							

QuickSort

```
0 quickSort(a, 6, 12) {
1   if(12-6>3) {
2     p = choosePivot(a, 6, 12)
3     i = partition(a, 48, 6, 12)
4     quickSort(a, 6, 8-1)
5     quickSort(a, 8+1, 12)
6   } else
7     insertionSort(a, 6, 12)
8   return
9 }
```

```
PC = 4
l = 6
h = 12
p = 48
i = 8
```



pc	l	h	p	i
5	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89

QuickSort

```
0 quickSort(a, 6, 7) {
1   if(7-6>3) {
2     p = choosePivot(a, 6, 7)
3     i = partition(a, p, 6, 7)
4     quickSort(a, 6, i-1)
5     quickSort(a, i+1, 7)
6   } else
7     insertionSort(a, 6, 7)
8   return
9 }
```

```
PC = 0
l = 6
h = 7
p = #
i = #
```

pc	l	h	p	i
4	6	12	48	8
5	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89
						low	high												

QuickSort

```
0  quickSort(a, 6, 7) {
1    if(7-6>3) {
2      p = choosePivot(a, 6, 7)
3      i = partition(a, p, 6, 7)
4      quickSort(a, 6, i-1)
5      quickSort(a, i+1, 7)
6    } else
7      insertionSort(a, 6, 7)
8    return
9  }
```

```
PC = 1
l = 6
h = 7
p = #
i = #
```

pc	l	h	p	i
4	6	12	48	8
5	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89
						low	high												

QuickSort

```
0 quickSort(a, 6, 7) {
1   if(7-6>3) {
2     p = choosePivot(a, 6, 7)
3     i = partition(a, p, 6, 7)
4     quickSort(a, 6, i-1)
5     quickSort(a, i+1, 7)
6   } else
7     insertionSort(a, 6, 7)
8   return
9 }
```

```
PC = 7
l = 6
h = 7
p = #
i = #
```

pc	l	h	p	i
4	6	12	48	8
5	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89
						low	high												

QuickSort

```
0 quickSort(a, 6, 7) {
1   if(7-6>3) {
2     p = choosePivot(a, 6, 7)
3     i = partition(a, p, 6, 7)
4     quickSort(a, 6, i-1)
5     quickSort(a, i+1, 7)
6   } else
7     insertionSort(a, 6, 7)
8   return
9 }
```

```
PC = 7
l = 6
h = 7
p = #
i = #
```

7	6	7	#	#
4	6	12	48	8
5	0	12	34	5
4	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89
						low	high												

QuickSort

```
0 quickSort(a, 6, 7) {
1   if(7-6>3) {
2     p = choosePivot(a, 6, 7)
3     i = partition(a, p, 6, 7)
4     quickSort(a, 6, i-1)
5     quickSort(a, i+1, 7)
6   } else
7     insertionSort(a, 6, 7)
8   return
9 }
```

```
PC = 8
l = 6
h = 7
p = #
i = #
```

pc	l	h	p	i
4	6	12	48	8
5	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89
						low	high												

QuickSort

```
PC = 5
l = 6
h = 12
p = 48
i = 8
```

```
0 quickSort(a, 6, 12) {
1   if(12-6>3) {
2     p = choosePivot(a, 6, 12)
3     i = partition(a, 48, 6, 12)
4     quickSort(a, 6, 8-1)
5     quickSort(a, 8+1, 12)
6   } else
7     insertionSort(a, 6, 12)
8   return
9 }
```

pc	l	h	p	i
5	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89
						low						high							

QuickSort

```
0  quickSort(a, 9, 12) {
1    if(12-9>3) {
2      p = choosePivot(a, 9, 12)
3      i = partition(a, p, 9, 12)
4      quickSort(a, 9, i-1)
5      quickSort(a, i+1, 12)
6    } else
7      insertionSort(a, 9, 12)
8    return
9  }
```

```
PC = 0
l  = 9
h  = 12
p  = #
i  = #
```

pc	l	h	p	i
5	6	12	48	8
5	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89
									low			high							

QuickSort

```
0 quickSort(a, 9, 12) {
1   if(12-9>3) {
2     p = choosePivot(a, 9, 12)
3     i = partition(a, p, 9, 12)
4     quickSort(a, 9, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 9, 12)
8   return
9 }
```

```
PC = 1
l = 9
h = 12
p = #
i = #
```

pc	l	h	p	i
5	6	12	48	8
5	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89
									low			high							

QuickSort

```
0  quickSort(a, 9, 12) {
1    if(12-9>3) {
2      p = choosePivot(a, 9, 12)
3      i = partition(a, p, 9, 12)
4      quickSort(a, 9, i-1)
5      quickSort(a, i+1, 12)
6    } else
7      insertionSort(a, 9, 12)
8    return
9  }
```

```
PC = 7
l = 9
h = 12
p = #
i = #
```

pc	l	h	p	i
5	6	12	48	8
5	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	67	66	61	52	73	87	92	95	76	87	89
									low			high							

QuickSort

```
0 quickSort(a, 9, 12) {
1   if(12-9>3) {
2     p = choosePivot(a, 9, 12)
3     i = partition(a, p, 9, 12)
4     quickSort(a, 9, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 9, 12)
8   return
9 }
```

```
PC = 7
l = 9
h = 12
p = #
i = #
```

pc	l	h	p	i
7	9	12	#	#
5	6	12	48	8
5	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89
									low			high							

QuickSort

```
PC = 8
l = 9
h = 12
p = #
i = #
```

```
0 quickSort(a, 9, 12) {
1   if(12-9>3) {
2     p = choosePivot(a, 9, 12)
3     i = partition(a, p, 9, 12)
4     quickSort(a, 9, i-1)
5     quickSort(a, i+1, 12)
6   } else
7     insertionSort(a, 9, 12)
8   return
9 }
```

pc	l	h	p	i
5	6	12	48	8
5	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89
									low			high							

QuickSort

```
0 quickSort(a, 6, 12) {
1   if(12-6>3) {
2     p = choosePivot(a, 6, 12)
3     i = partition(a, 48, 6, 12)
4     quickSort(a, 6, 8-1)
5     quickSort(a, 8+1, 12)
6   } else
7     insertionSort(a, 6, 12)
8   return
9 }
```

```
PC = 8
l = 6
h = 12
p = 48
i = 8
```

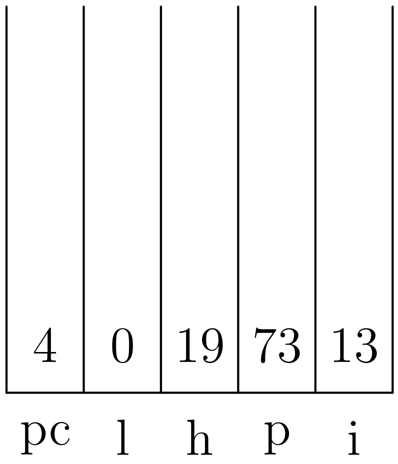
pc	l	h	p	i
5	0	12	34	5
4	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89
						low						high							

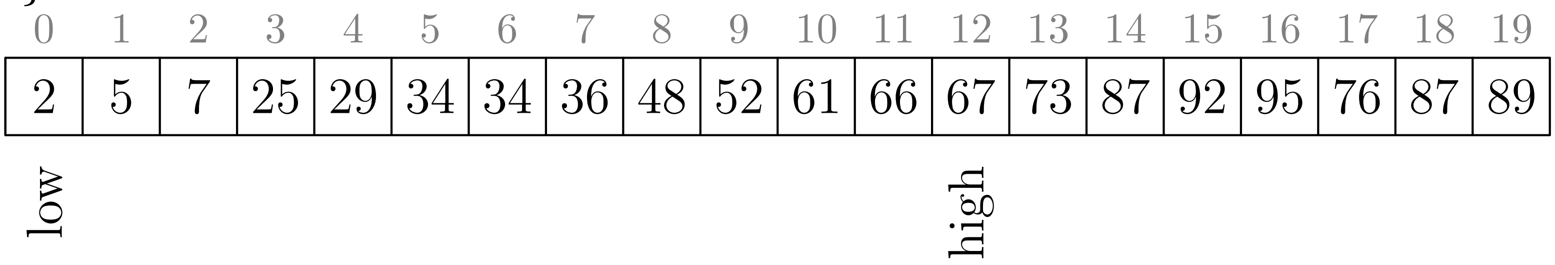
QuickSort

```
0 quickSort(a, 0, 12) {
1   if(12-0>3) {
2     p = choosePivot(a, 0, 12)
3     i = partition(a, 34, 0, 12)
4     quickSort(a, 0, 5-1)
5     quickSort(a, 5+1, 12)
6   } else
7     insertionSort(a, 0, 12)
8   return
9 }
```

```
PC = 8
l = 0
h = 12
p = 34
i = 5
```



pc	l	h	p	i
4	0	19	73	13

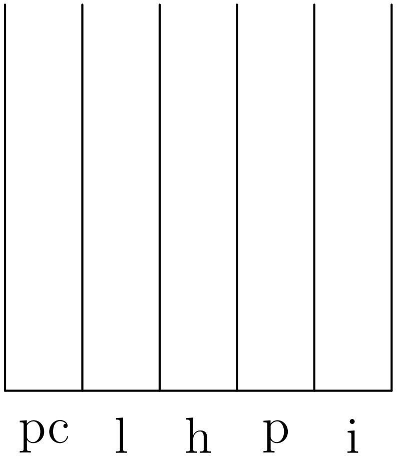


0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89

QuickSort

```
0  quickSort(a, 0, 19) {
1    if(19-0>3) {
2      p = choosePivot(a, 0, 19)
3      i = partition(a, 73, 0, 19)
4      quickSort(a, 0, 13-1)
5      quickSort(a, 13+1, 19)
6    } else
7      insertionSort(a, 0, 19)
8    return
9  }
```

```
PC = 5
l = 0
h = 19
p = 73
i = 13
```

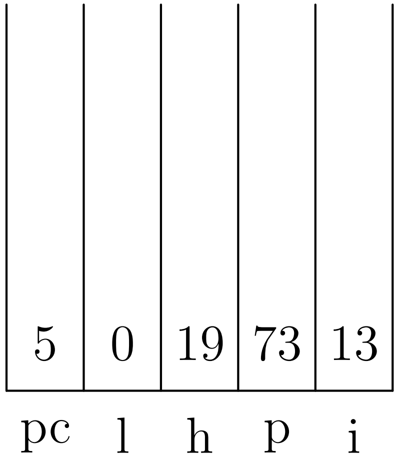


0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89
low																			high

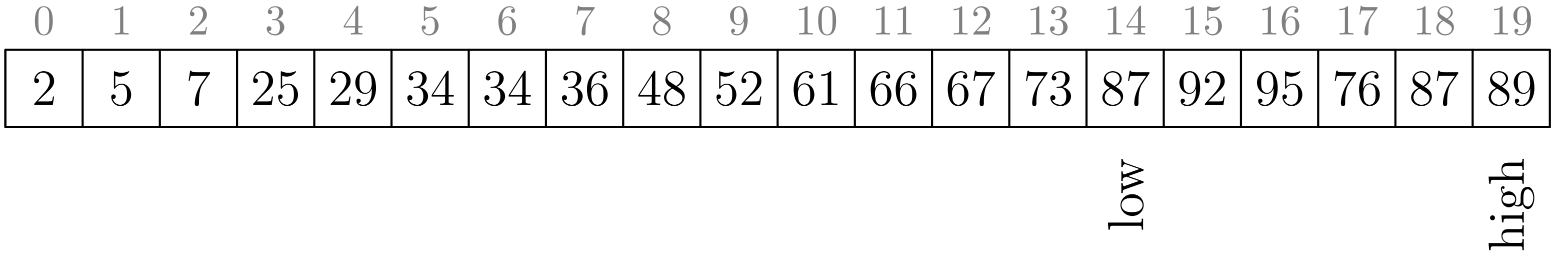
QuickSort

```
0 quickSort(a, 14, 19) {
1   if(19-14>3) {
2     p = choosePivot(a, 14, 19)
3     i = partition(a, p, 14, 19)
4     quickSort(a, 14, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 14, 19)
8   return
9 }
```

```
PC = 0
l = 14
h = 19
p = #
i = #
```



pc	l	h	p	i
5	0	19	73	13



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89
														low					high

QuickSort

```
0  quickSort(a, 14, 19) {
1    if(19-14>3) {
2      p = choosePivot(a, 14, 19)
3      i = partition(a, p, 14, 19)
4      quickSort(a, 14, i-1)
5      quickSort(a, i+1, 19)
6    } else
7      insertionSort(a, 14, 19)
8    return
9  }
```

```
PC = 1
l = 14
h = 19
p = #
i = #
```

5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89
														low					high

QuickSort

```
0  quickSort(a, 14, 19) {
1    if(19-14>3) {
2      p = choosePivot(a, 14, 19)
3      i = partition(a, p, 14, 19)
4      quickSort(a, 14, i-1)
5      quickSort(a, i+1, 19)
6    } else
7      insertionSort(a, 14, 19)
8    return
9  }
```

```
PC = 2
l = 14
h = 19
p = #
i = #
```

5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89
														low					high

QuickSort

```
0 quickSort(a, 14, 19) {
1   if(19-14>3) {
2     p = choosePivot(a, 14, 19)
3     i = partition(a, p, 14, 19)
4     quickSort(a, 14, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 14, 19)
8   return
9 }
```

```
PC = 2
l = 14
h = 19
p = #
i = #
```

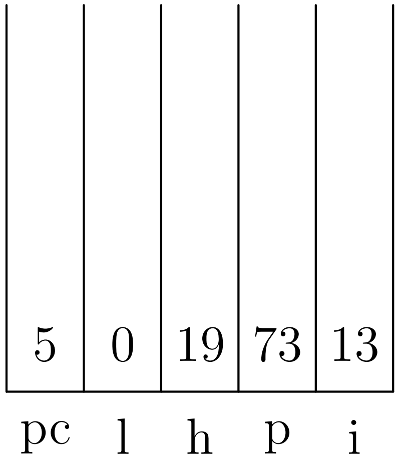
pc	l	h	p	i
2	14	19	#	#
5	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89
														low					high

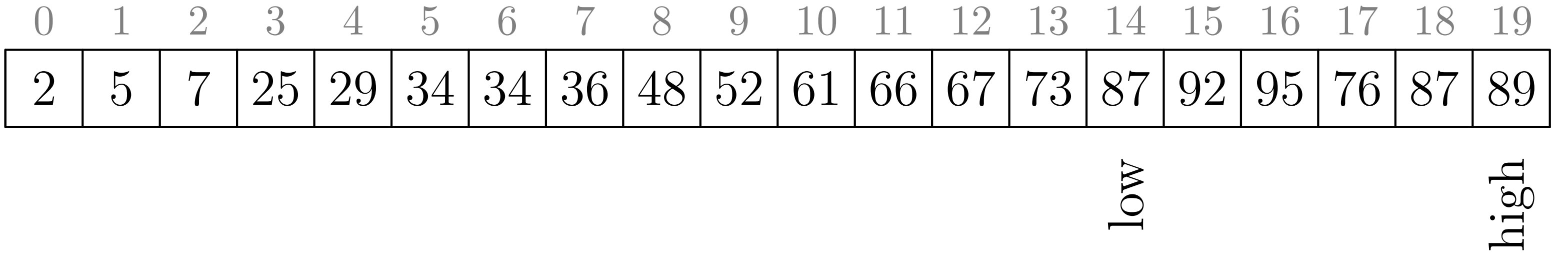
QuickSort

```
0 quickSort(a, 14, 19) {
1   if(19-14>3) {
2     p = choosePivot(a, 14, 19)
3     i = partition(a, 89, 14, 19)
4     quickSort(a, 14, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 14, 19)
8   return
9 }
```

```
PC = 3
l = 14
h = 19
p = 89
i = #
```



pc	l	h	p	i
5	0	19	73	13



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	92	95	76	87	89
														low					high

QuickSort

```
0 quickSort(a, 14, 19) {
1   if(19-14>3) {
2     p = choosePivot(a, 14, 19)
3     i = partition(a, 89, 14, 19)
4     quickSort(a, 14, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 14, 19)
8   return
9 }
```

```
PC = 3
l = 14
h = 19
p = 89
i = #
```

pc	l	h	p	i
3	14	19	89	#
5	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	87	76	89	92	95
														low					high

QuickSort

```
0 quickSort(a, 14, 19) {
1   if(19-14>3) {
2     p = choosePivot(a, 14, 19)
3     i = partition(a, 89, 14, 19)
4     quickSort(a, 14, 17-1)
5     quickSort(a, 17+1, 19)
6   } else
7     insertionSort(a, 14, 19)
8   return
9 }
```

```
PC = 4
l  = 14
h  = 19
p  = 89
i  = 17
```

pc	l	h	p	i
5	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	87	76	89	92	95
														low					high

QuickSort

```
0 quickSort(a, 14, 16) {
1   if(16-14>3) {
2     p = choosePivot(a, 14, 16)
3     i = partition(a, p, 14, 16)
4     quickSort(a, 14, i-1)
5     quickSort(a, i+1, 16)
6   } else
7     insertionSort(a, 14, 16)
8   return
9 }
```

```
PC = 0
l  = 14
h  = 16
p  = #
i  = #
```

pc	l	h	p	i
4	14	19	89	17
5	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	87	76	89	92	95
														low		high			

QuickSort

```
0 quickSort(a, 14, 16) {
1   if(16-14>3) {
2     p = choosePivot(a, 14, 16)
3     i = partition(a, p, 14, 16)
4     quickSort(a, 14, i-1)
5     quickSort(a, i+1, 16)
6   } else
7     insertionSort(a, 14, 16)
8   return
9 }
```

```
PC = 1
l  = 14
h  = 16
p  = #
i  = #
```

pc	l	h	p	i
4	14	19	89	17
5	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	87	76	89	92	95
														low		high			

QuickSort

```
0 quickSort(a, 14, 16) {
1   if(16-14>3) {
2     p = choosePivot(a, 14, 16)
3     i = partition(a, p, 14, 16)
4     quickSort(a, 14, i-1)
5     quickSort(a, i+1, 16)
6   } else
7     insertionSort(a, 14, 16)
8   return
9 }
```

```
PC = 7
l = 14
h = 16
p = #
i = #
```

pc	l	h	p	i
4	14	19	89	17
5	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	87	87	76	89	92	95
														low		high			

QuickSort

```
0 quickSort(a, 14, 16) {
1   if(16-14>3) {
2     p = choosePivot(a, 14, 16)
3     i = partition(a, p, 14, 16)
4     quickSort(a, 14, i-1)
5     quickSort(a, i+1, 16)
6   } else
7     insertionSort(a, 14, 16)
8   return
9 }
```

```
PC = 7
l = 14
h = 16
p = #
i = #
```

pc	l	h	p	i
7	14	16	#	#
4	14	19	89	17
5	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95
														low		high			

QuickSort

```
0  quickSort(a, 14, 16) {
1    if(16-14>3) {
2      p = choosePivot(a, 14, 16)
3      i = partition(a, p, 14, 16)
4      quickSort(a, 14, i-1)
5      quickSort(a, i+1, 16)
6    } else
7      insertionSort(a, 14, 16)
8    return
9  }
```

```
PC = 8
l  = 14
h  = 16
p  = #
i  = #
```

pc	l	h	p	i
4	14	19	89	17
5	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95
														low		high			

QuickSort

```
0  quickSort(a, 14, 19) {
1    if(19-14>3) {
2      p = choosePivot(a, 14, 19)
3      i = partition(a, 89, 14, 19)
4      quickSort(a, 14, 17-1)
5      quickSort(a, 17+1, 19)
6    } else
7      insertionSort(a, 14, 19)
8    return
9  }
```

```
PC = 5
l = 14
h = 19
p = 89
i = 17
```

pc	l	h	p	i
5	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95
														low					high

QuickSort

```
0 quickSort(a, 18, 19) {
1   if(19-18>3) {
2     p = choosePivot(a, 18, 19)
3     i = partition(a, p, 18, 19)
4     quickSort(a, 18, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 18, 19)
8   return
9 }
```

```
PC = 0
l  = 18
h  = 19
p  = #
i  = #
```

pc	l	h	p	i
5	14	19	89	17
5	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95
																		low	high

QuickSort

```
0 quickSort(a, 18, 19) {
1   if(19-18>3) {
2     p = choosePivot(a, 18, 19)
3     i = partition(a, p, 18, 19)
4     quickSort(a, 18, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 18, 19)
8   return
9 }
```

```
PC = 1
l = 18
h = 19
p = #
i = #
```

pc	l	h	p	i
5	14	19	89	17
5	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95
																		low	high

QuickSort

```
0 quickSort(a, 18, 19) {
1   if(19-18>3) {
2     p = choosePivot(a, 18, 19)
3     i = partition(a, p, 18, 19)
4     quickSort(a, 18, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 18, 19)
8   return
9 }
```

```
PC = 7
l = 18
h = 19
p = #
i = #
```

pc	l	h	p	i
5	14	19	89	17
5	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95
																		low	high

QuickSort

```
0 quickSort(a, 18, 19) {
1   if(19-18>3) {
2     p = choosePivot(a, 18, 19)
3     i = partition(a, p, 18, 19)
4     quickSort(a, 18, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 18, 19)
8   return
9 }
```

```
PC = 7
l = 18
h = 19
p = #
i = #
```

pc	l	h	p	i
7	18	19	#	#
5	14	19	89	17
5	0	19	73	13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95
																		low	high

QuickSort

```
0 quickSort(a, 18, 19) {
1   if(19-18>3) {
2     p = choosePivot(a, 18, 19)
3     i = partition(a, p, 18, 19)
4     quickSort(a, 18, i-1)
5     quickSort(a, i+1, 19)
6   } else
7     insertionSort(a, 18, 19)
8   return
9 }
```

```
PC = 8
l  = 18
h  = 19
p  = #
i  = #
```

5	14	19	89	17
5	0	19	73	13
pc	l	h	p	i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95
																		low	high

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```
0 quickSort(a, 14, 19) {
1   if(19-14>3) {
2     p = choosePivot(a, 14, 19)
3     i = partition(a, 89, 14, 19)
4     quickSort(a, 14, 17-1)
5     quickSort(a, 17+1, 19)
6   } else
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```

```
PC = 8
l  = 14
h  = 19
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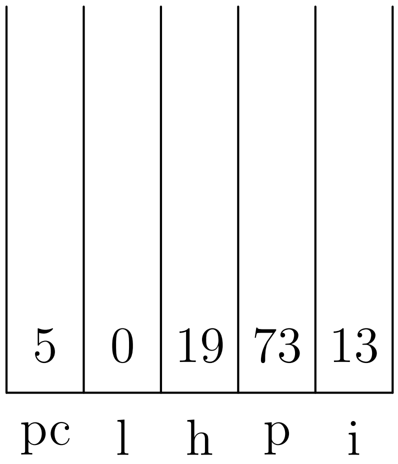
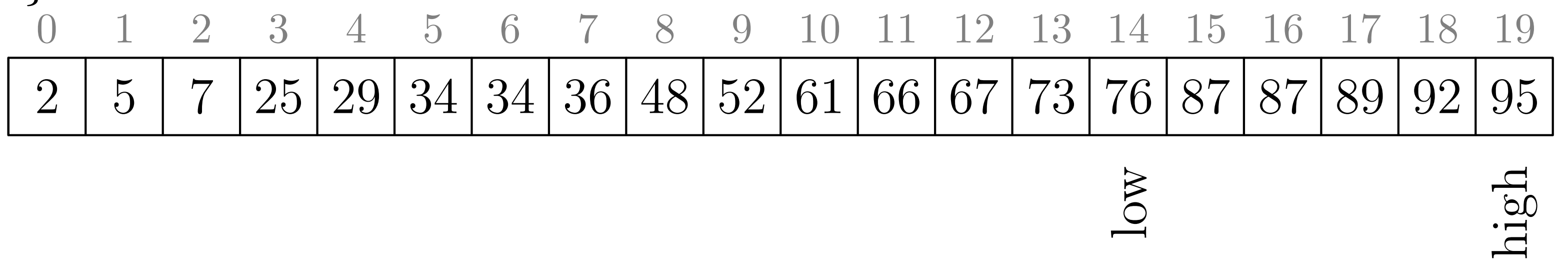
i  = 17
```



pc	l	h	p	i
5	0	19	73	13

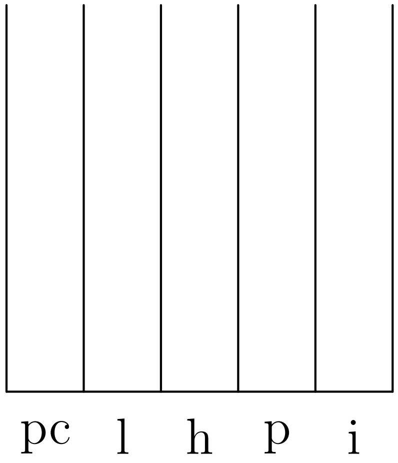


0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95
														low					high

QuickSort

```
0 quickSort(a, 0, 19) {
1   if(19-0>3) {
2     p = choosePivot(a, 0, 19)
3     i = partition(a, 73, 0, 19)
4     quickSort(a, 0, 13-1)
5     quickSort(a, 13+1, 19)
6   } else
7     insertionSort(a, 0, 19)
8   return
9 }
```

```
PC = 8
l = 0
h = 19
p = 73
i = 13
```



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	7	25	29	34	34	36	48	52	61	66	67	73	76	87	87	89	92	95
low																			high

Sort in Practice

- The STL in C++ offers three sorts
  - ⋆ `sort()` implemented using quicksort
  - ⋆ `stable_sort()` implemented using mergesort
  - ⋆ `partial_sort()` implemented using heapsort
- Java uses
  - ⋆ Quicksort to sort arrays of primitive types
  - ⋆ Mergesort to sort Collections of objects
- Quicksort is typically fastest but has worst case quadratic time complexity

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Selection

- A related problem to sorting is selection
- That is we want to select the k^{th} largest element
- We could do this by first sorting the array
- A full sort is not however necessary—we can use a modified quicksort where we only continue to sort the part of the array we are interested in
- This leads to a $\Theta(n \log(n))$ algorithm which is considerably faster then sorting

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Outline

1. Merge Sort
2. Quick Sort
3. **Radix Sort**

Radix Sort

- Can we get a sort algorithm to run faster than $O\left(n\log(n)\right)$?
- Our proof that this was optimal assumed we were performing binary decisions (is a_i less than a_j?)
- If we don't perform pairwise comparisons then the proof doesn't apply
- Radix sort is the classic example of a sort algorithm that doesn't use pairwise comparisons

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Sorting Into Buckets

- The idea behind radix sort is to sort the elements of an array into some number of buckets
- This is done successively until the whole array is sorted
- Consider sorting integers in decimals (base 10 or radix 10)
- We can successively sort on the digits
- The sort finishes when we have got through all the digits

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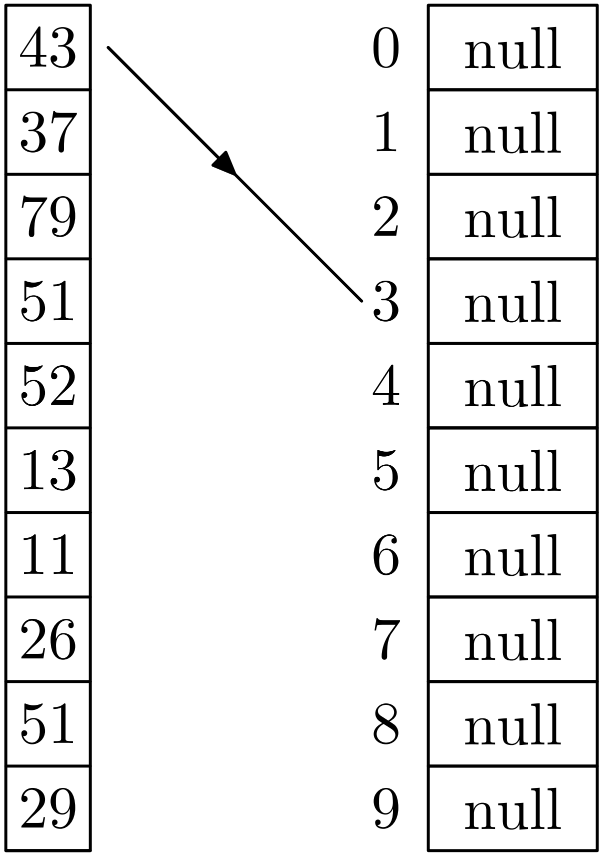
Radix Sort in Action

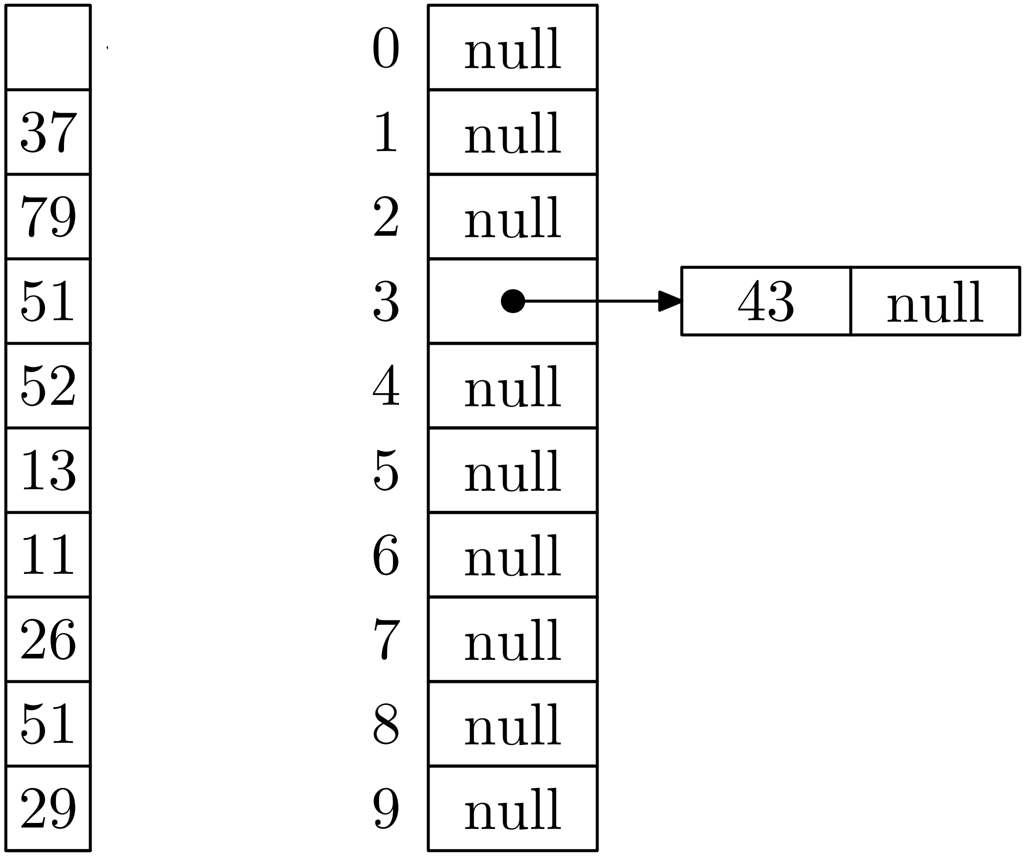
43	0	null
37	1	null
79	2	null
51	3	null
52	4	null
13	5	null
11	6	null
26	7	null
51	8	null
29	9	null

Radix Sort in Action



43	0	null
37	1	null
79	2	null
51	3	null
52	4	null
13	5	null
11	6	null
26	7	null
51	8	null
29	9	null

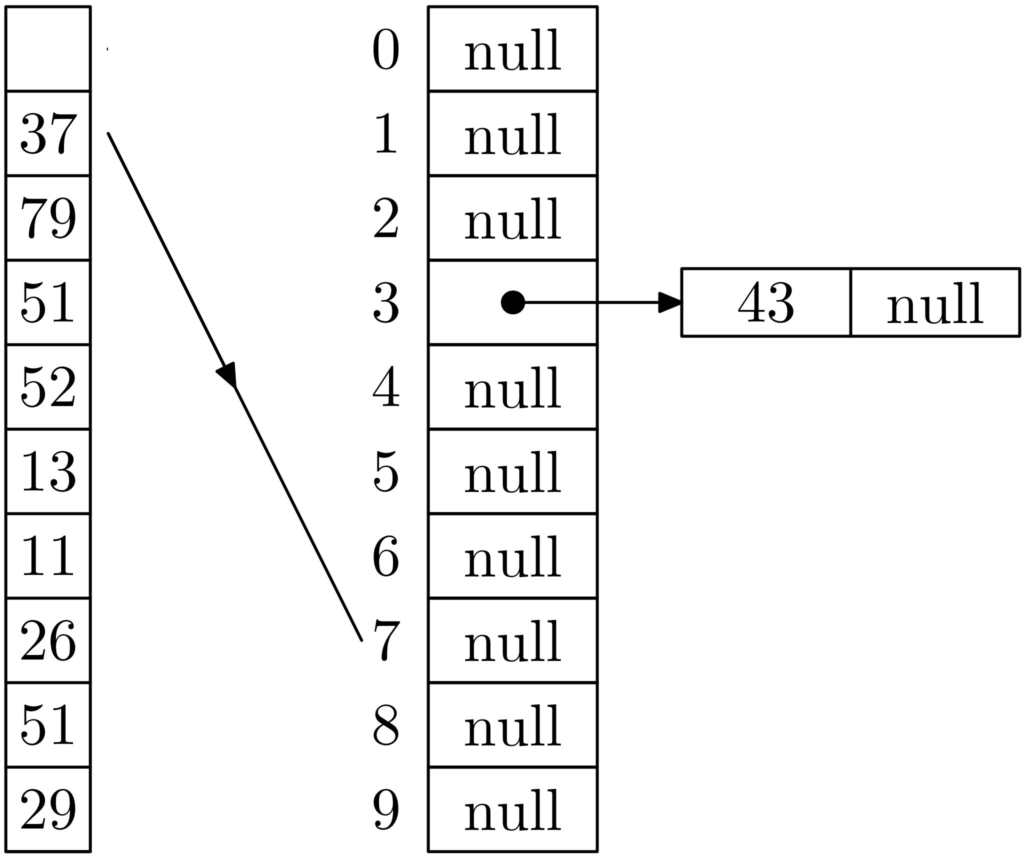
Radix Sort in Action



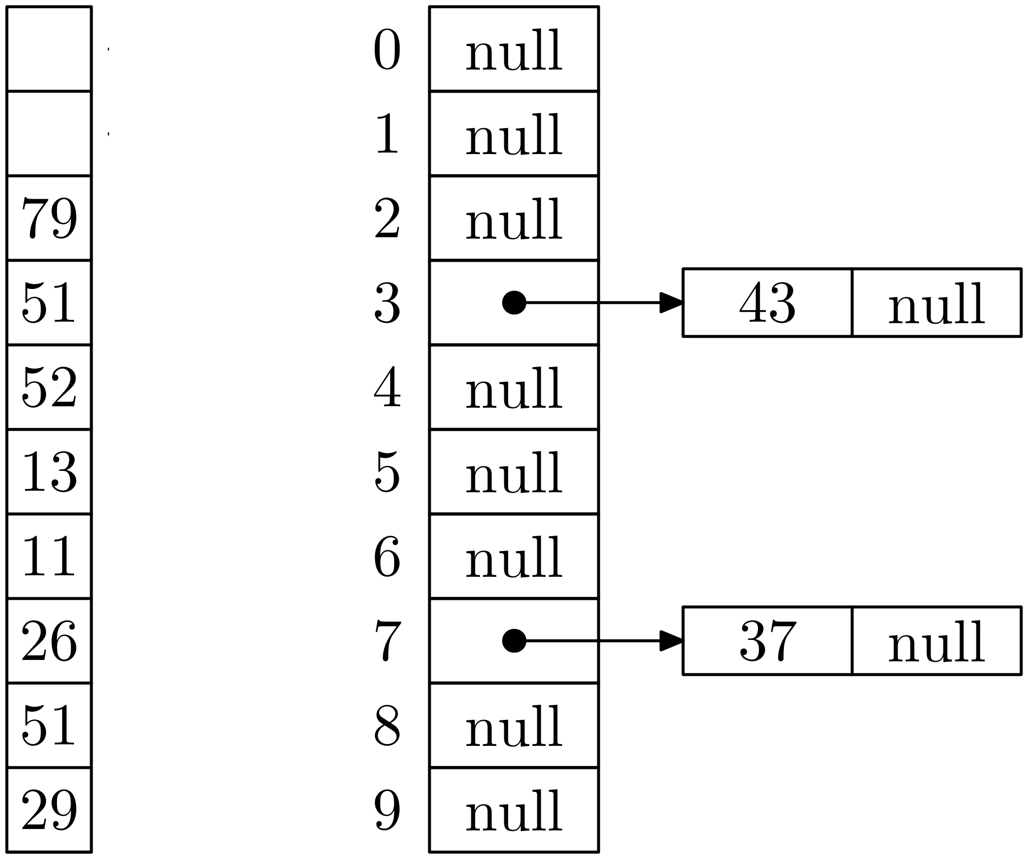
37
79
51
52
13
11
26
51
29

0 null
1 null
2 null
3 → 43 null
4 null
5 null
6 null
7 null
8 null
9 null

Radix Sort in Action

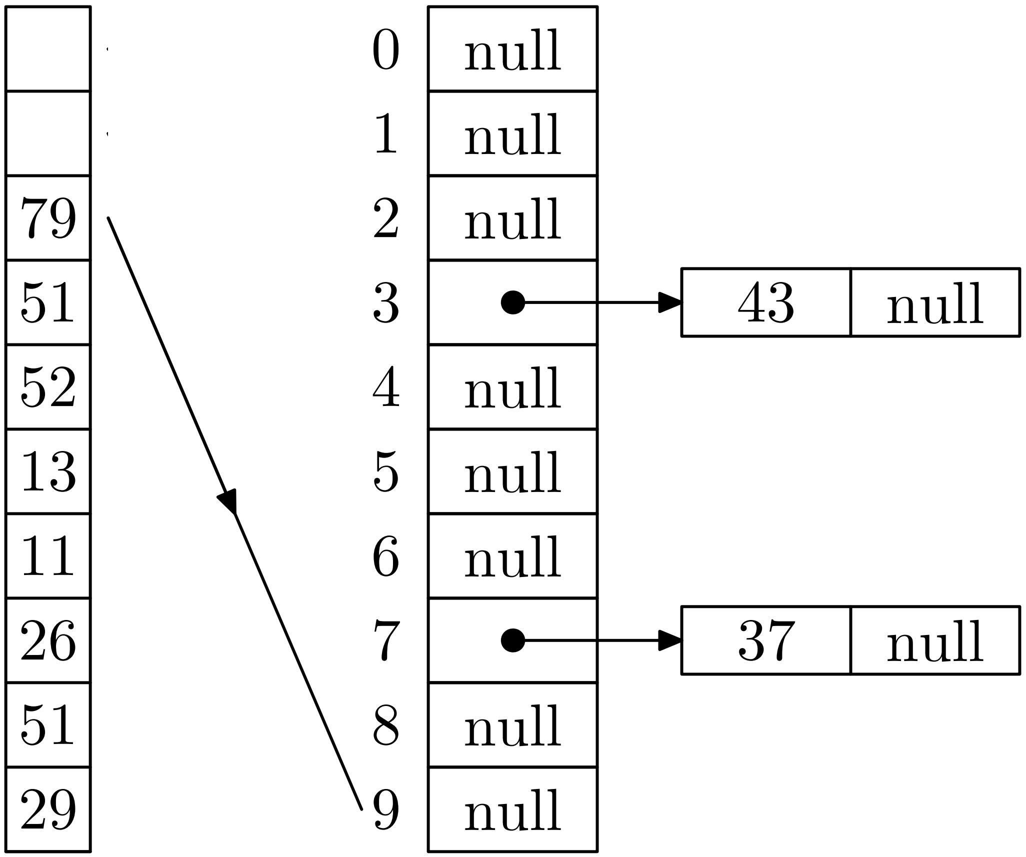


Radix Sort in Action

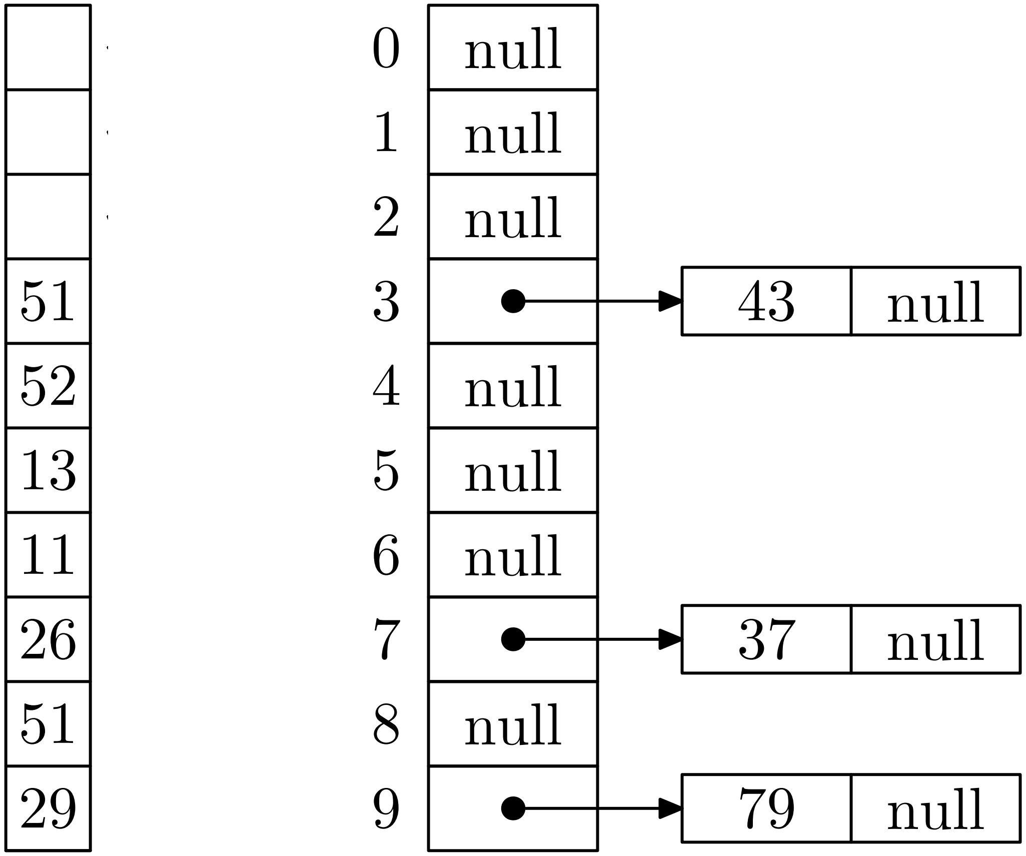


	0	null		
	1	null		
79	2	null		
51	3	●	43	null
52	4	null		
13	5	null		
11	6	null		
26	7	●	37	null
51	8	null		
29	9	null		

Radix Sort in Action



Radix Sort in Action



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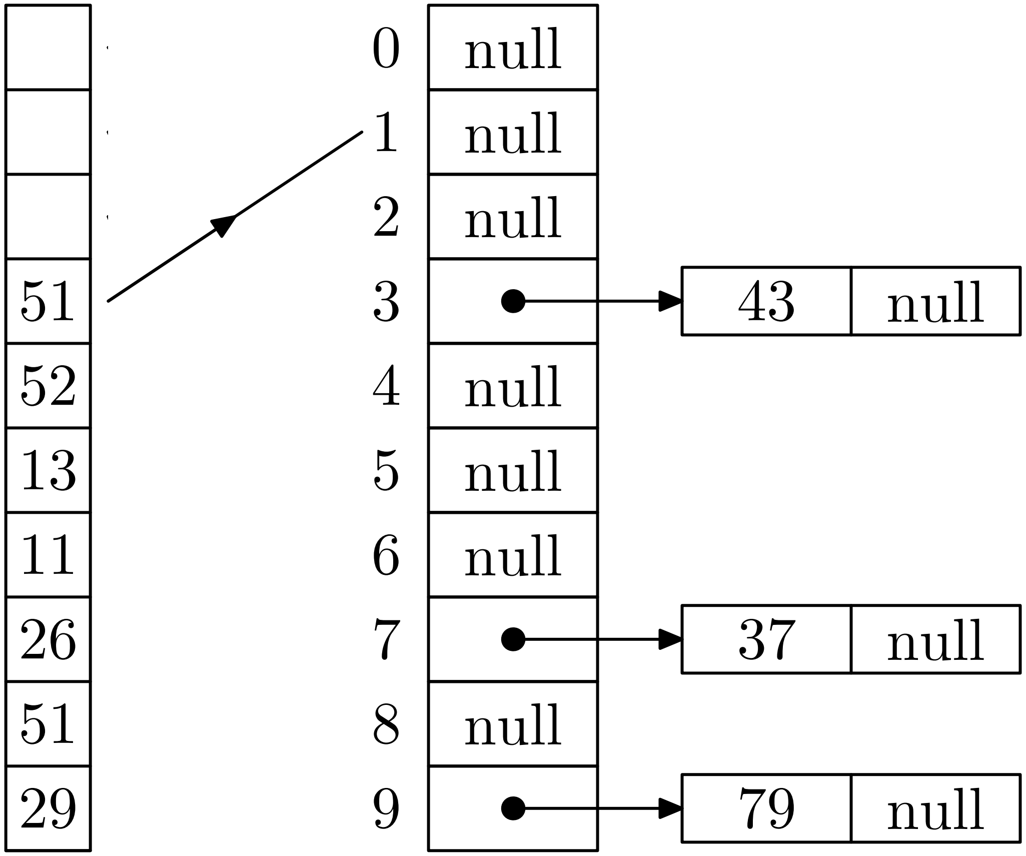
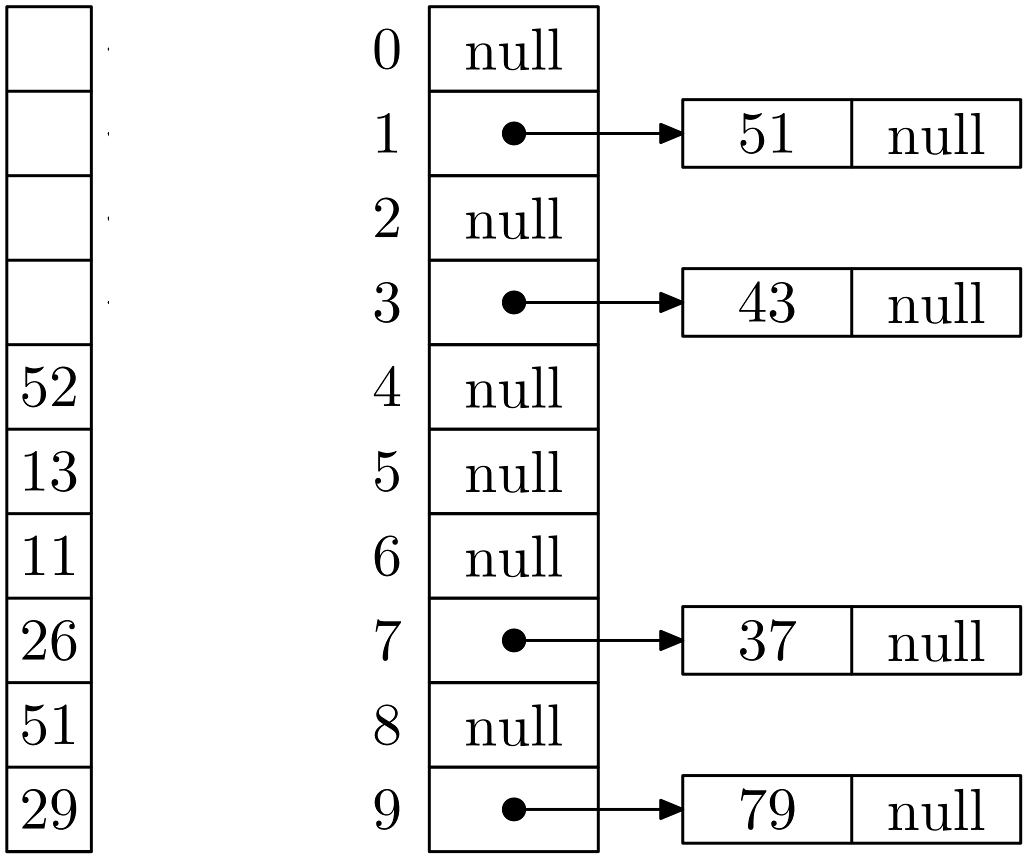
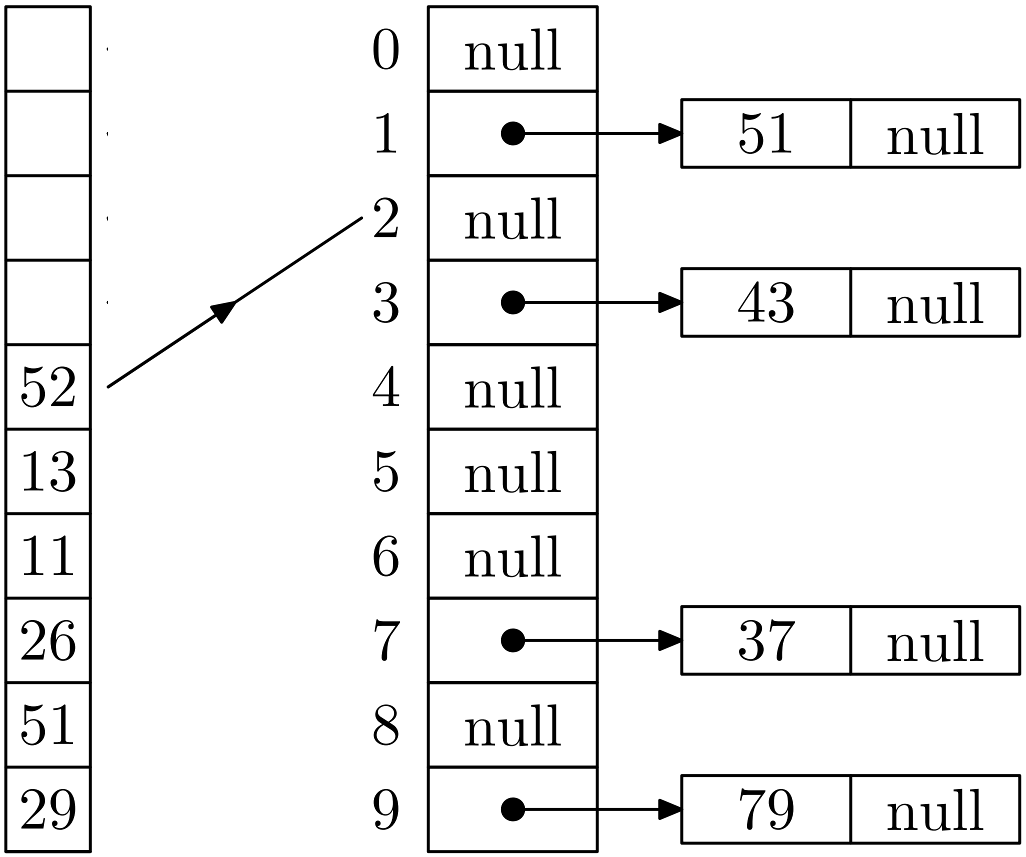


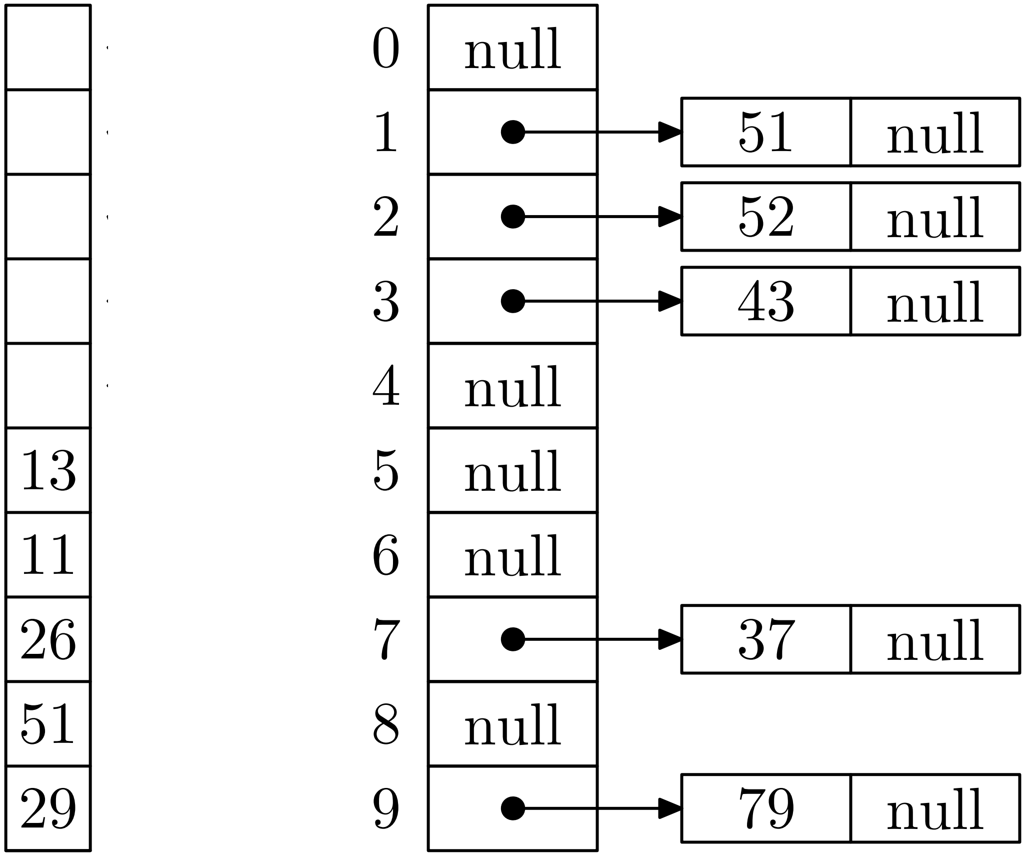
Radix Sort in Action



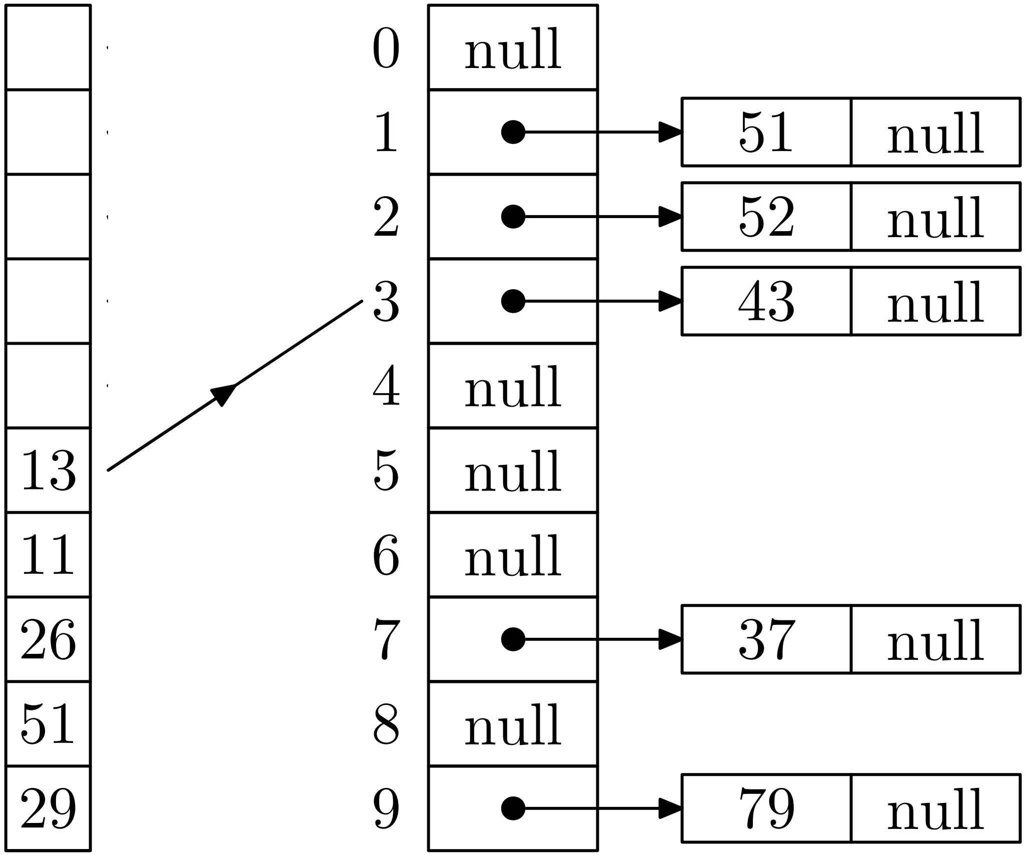
Radix Sort in Action



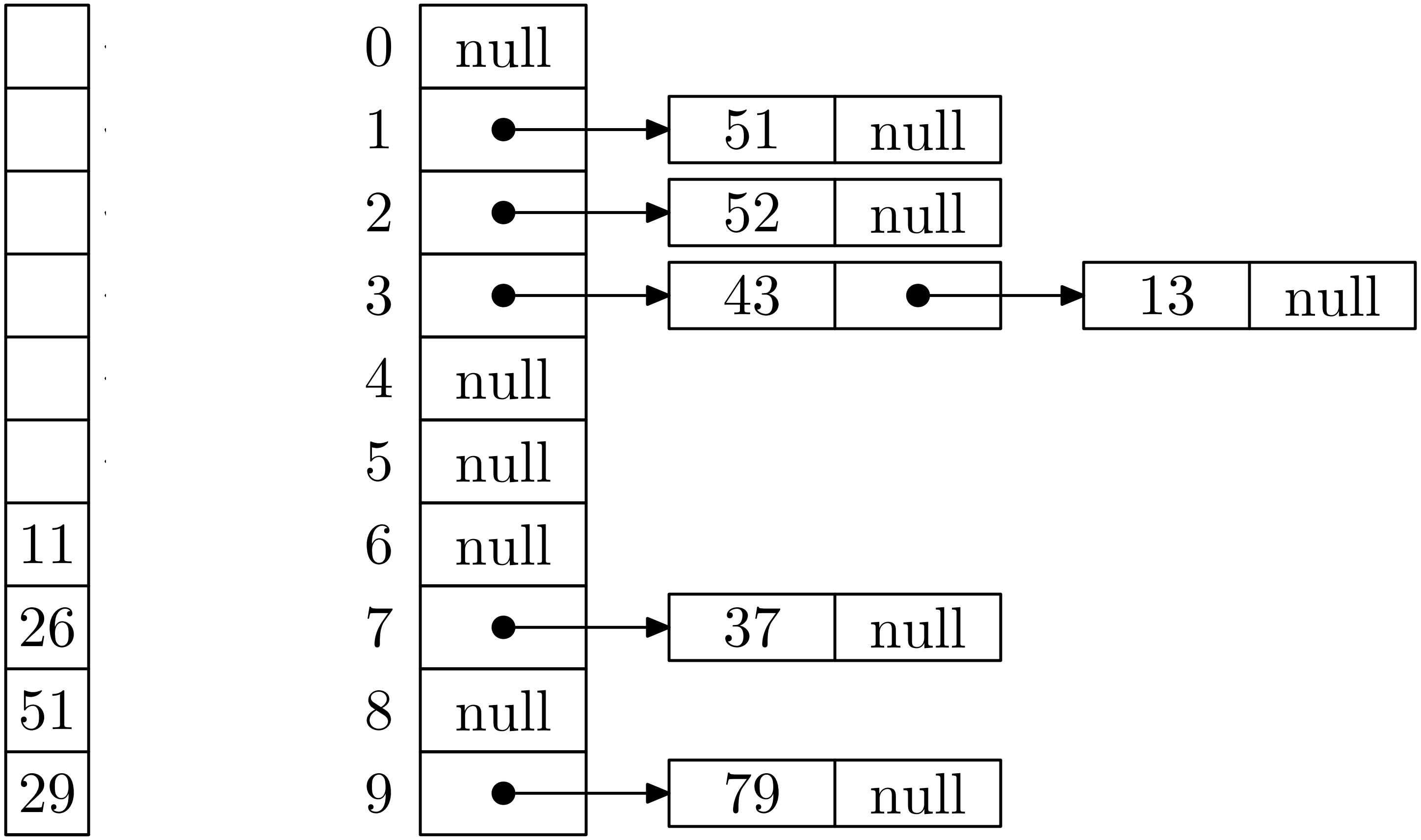
Radix Sort in Action



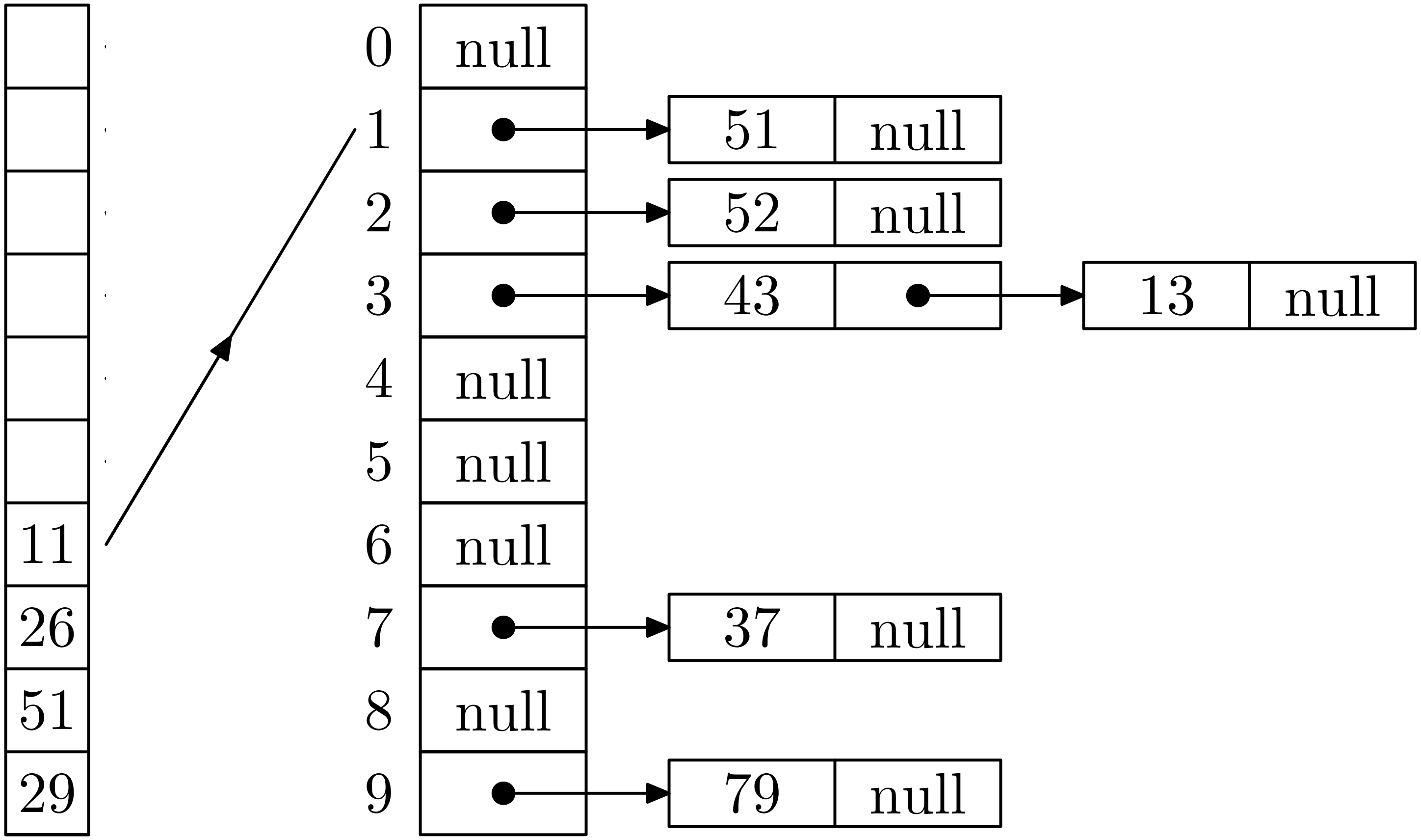
Radix Sort in Action



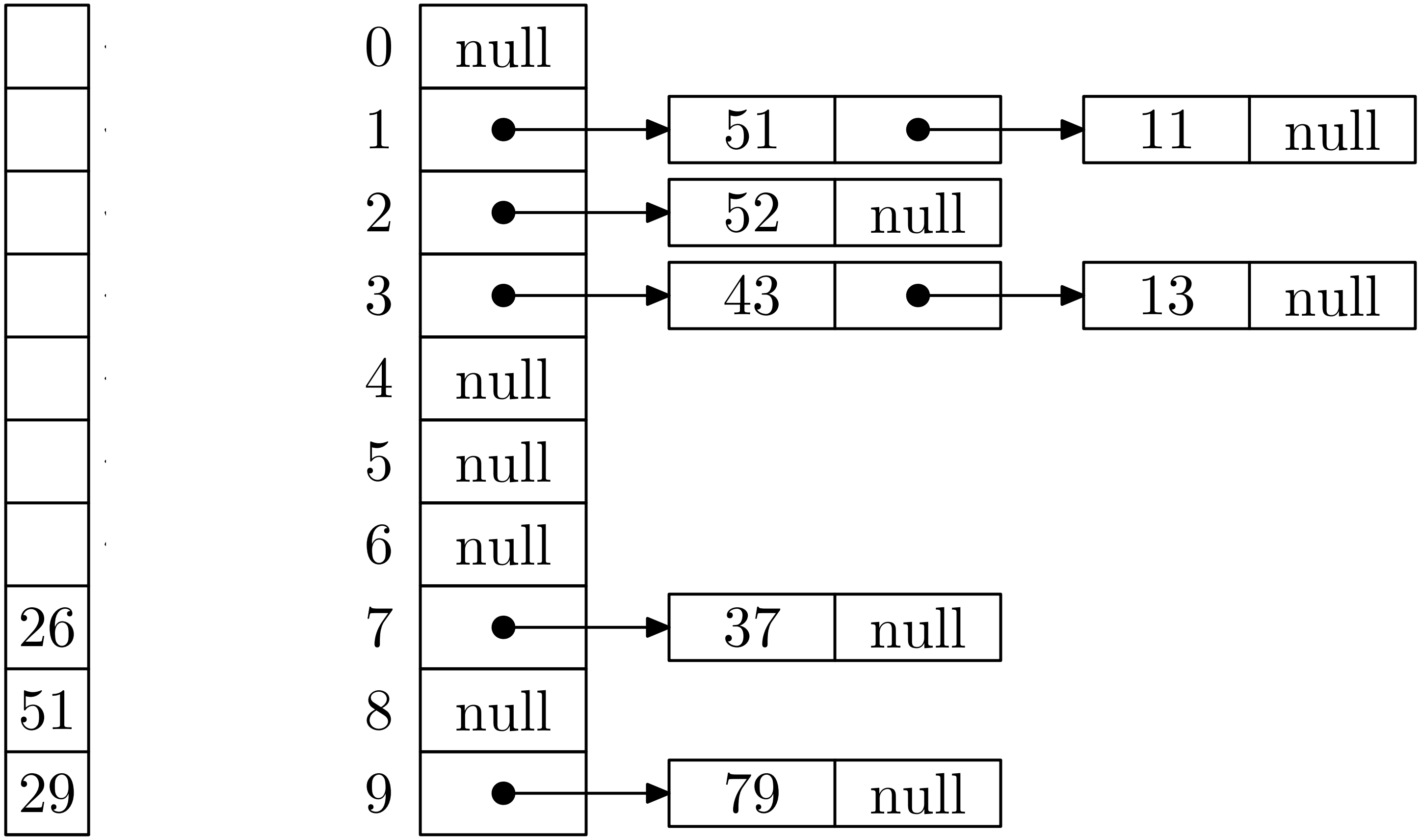
Radix Sort in Action



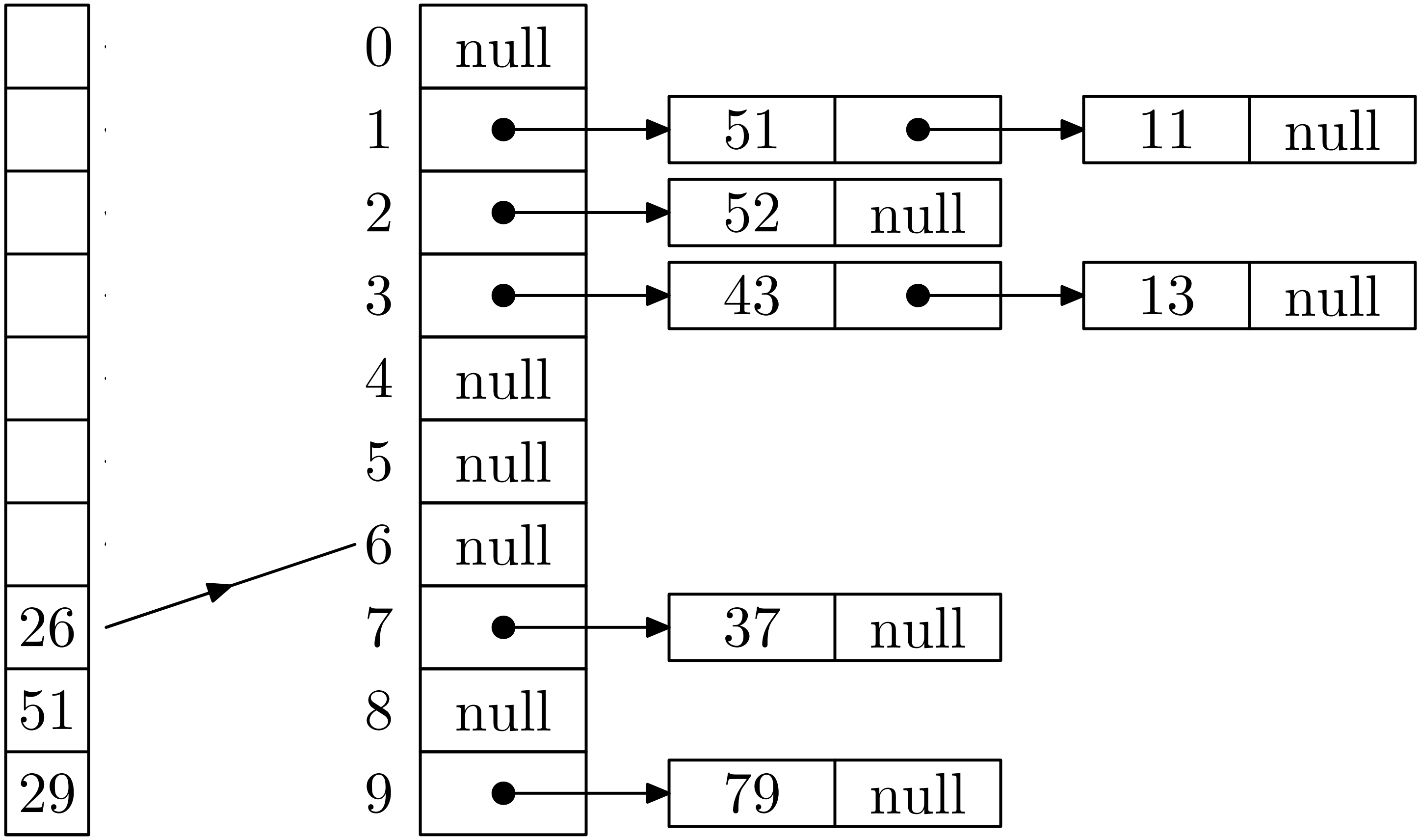
Radix Sort in Action



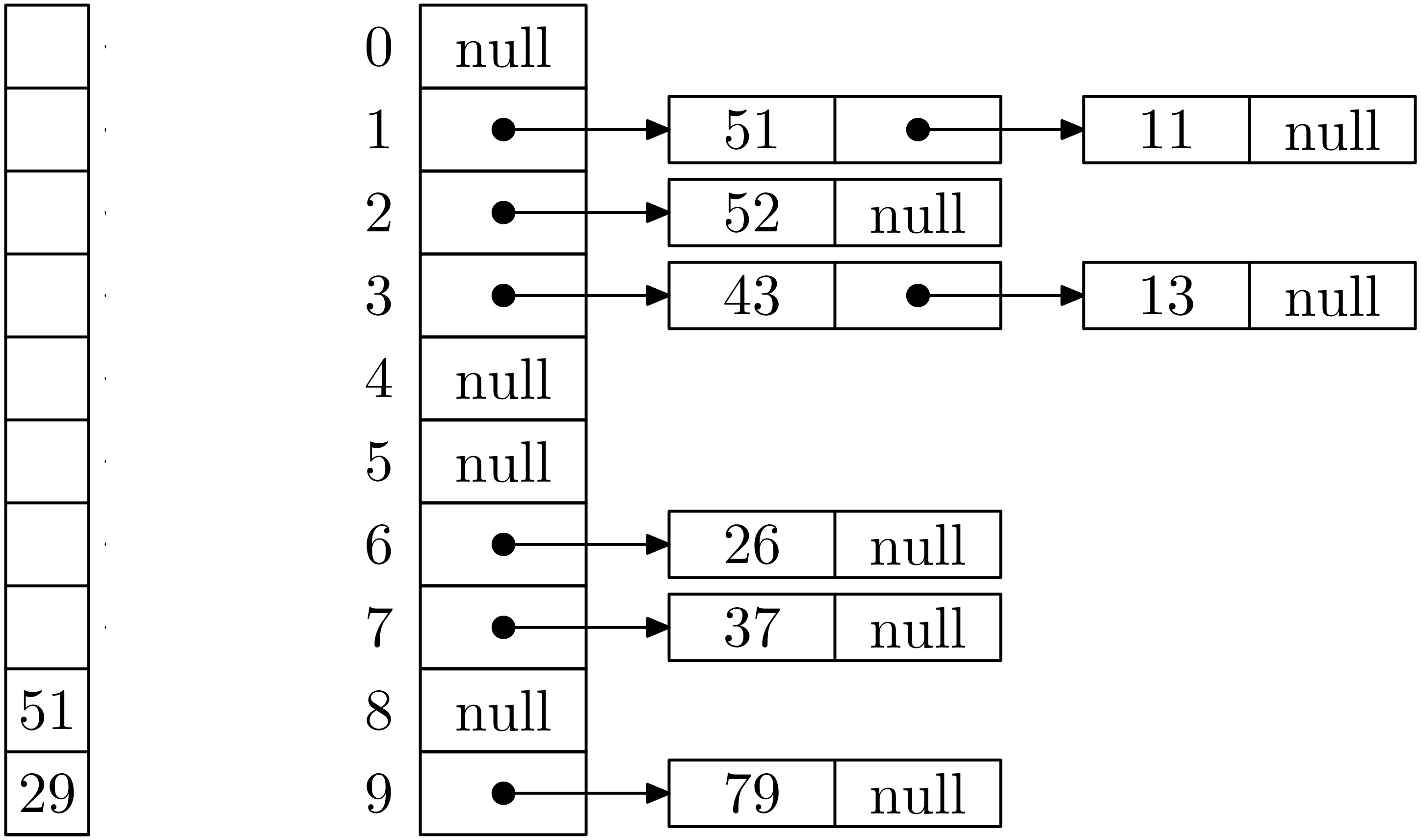
Radix Sort in Action



Radix Sort in Action



Radix Sort in Action



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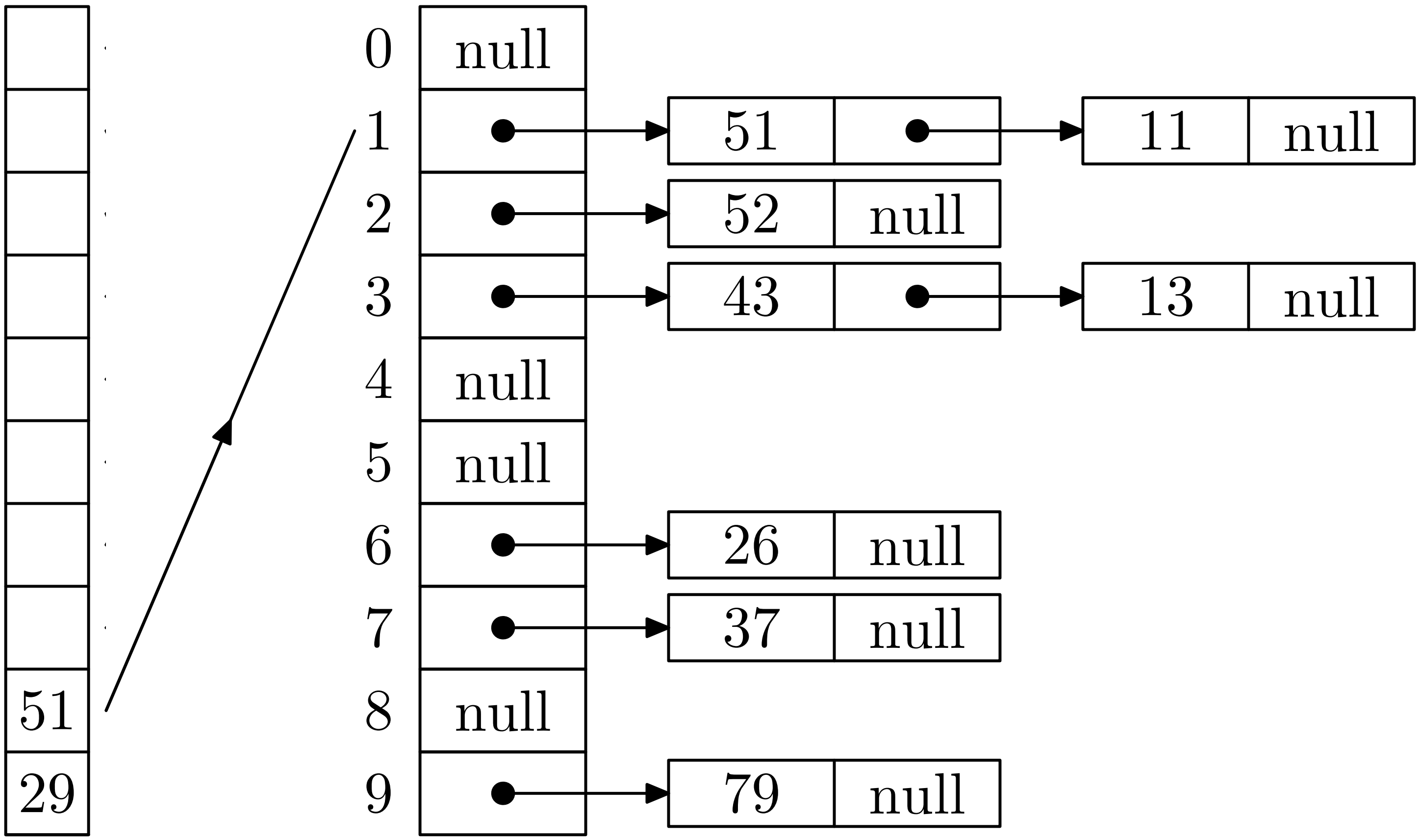
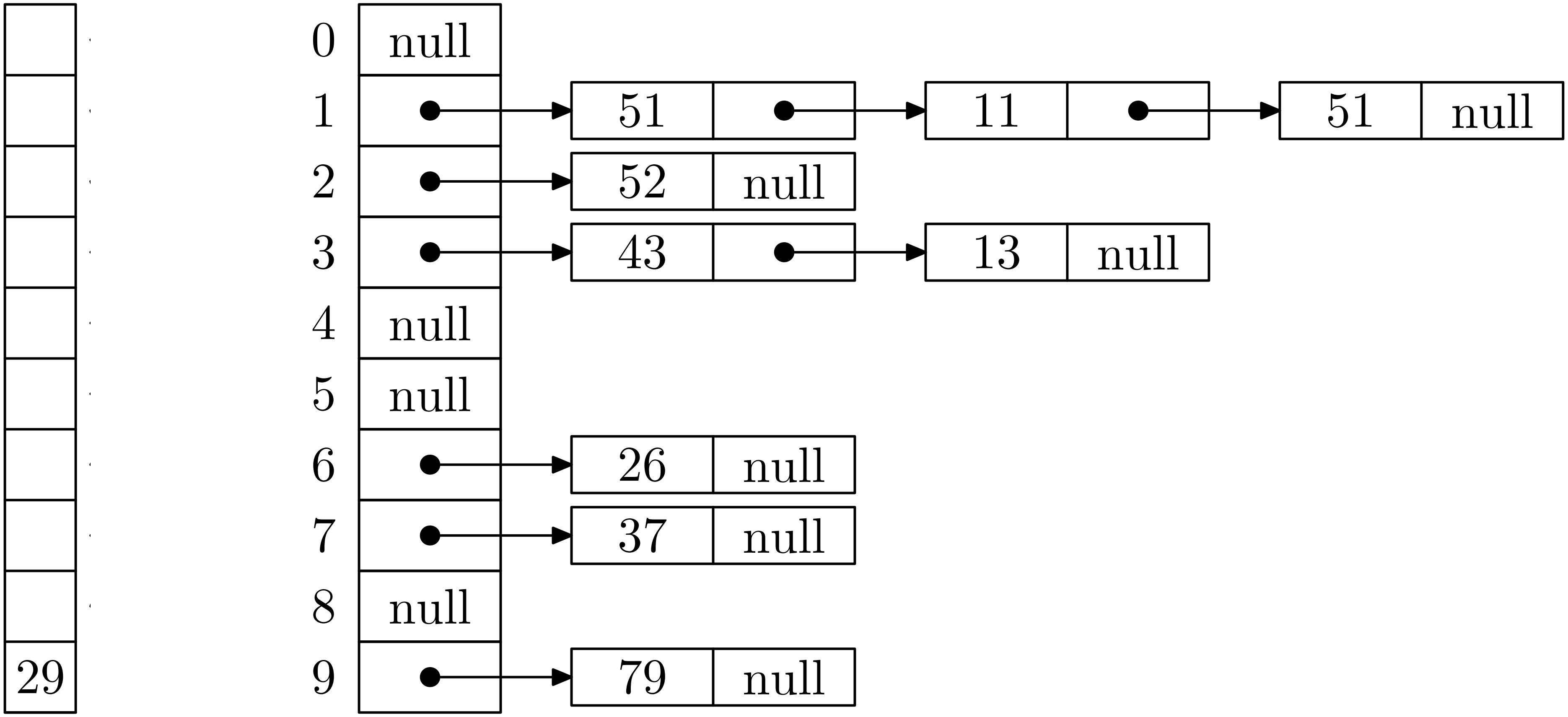


Radix Sort in Action



29

0 null

1 → 51 → 11 → 51 null

2 → 52 null

3 → 43 → 13 null

4 null

5 null

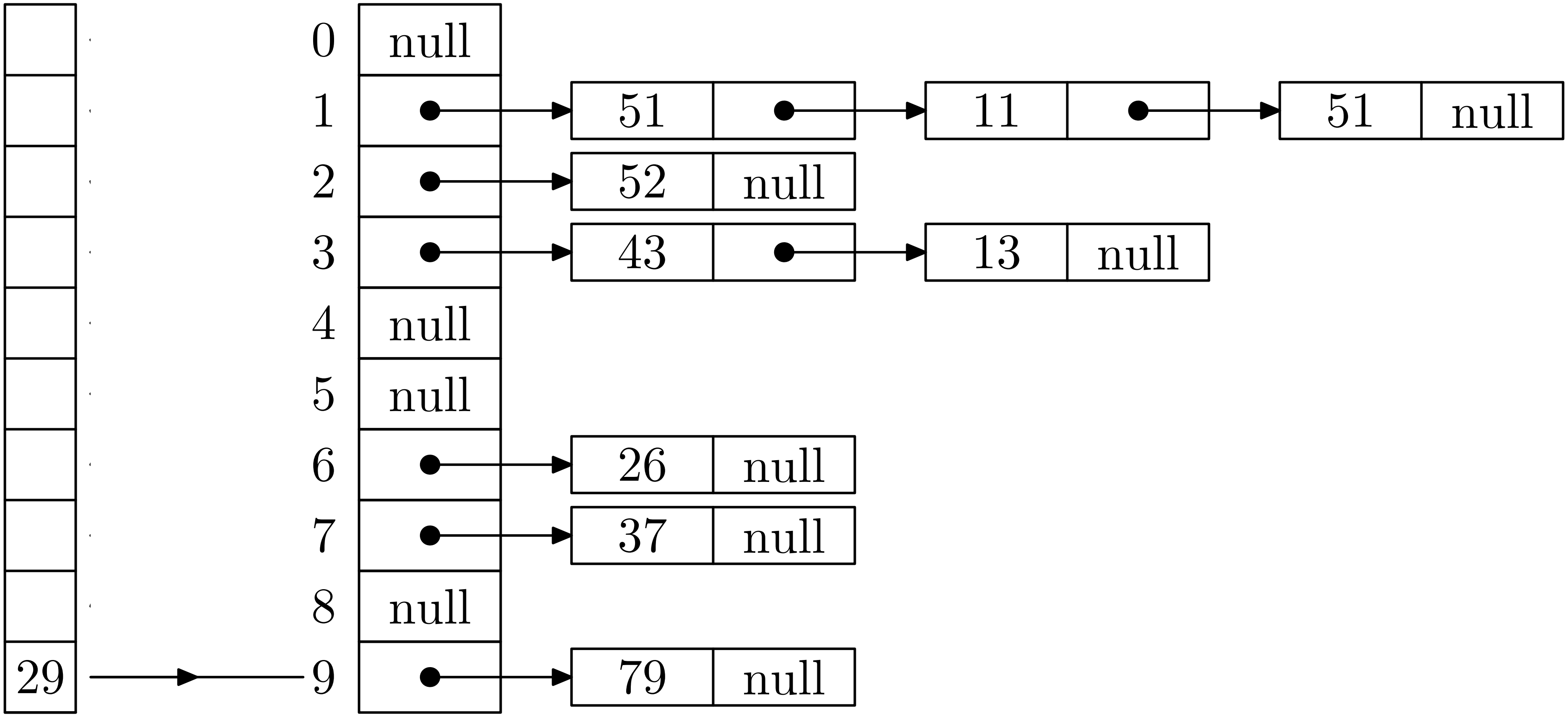
6 → 26 null

7 → 37 null

8 null

9 → 79 null

Radix Sort in Action



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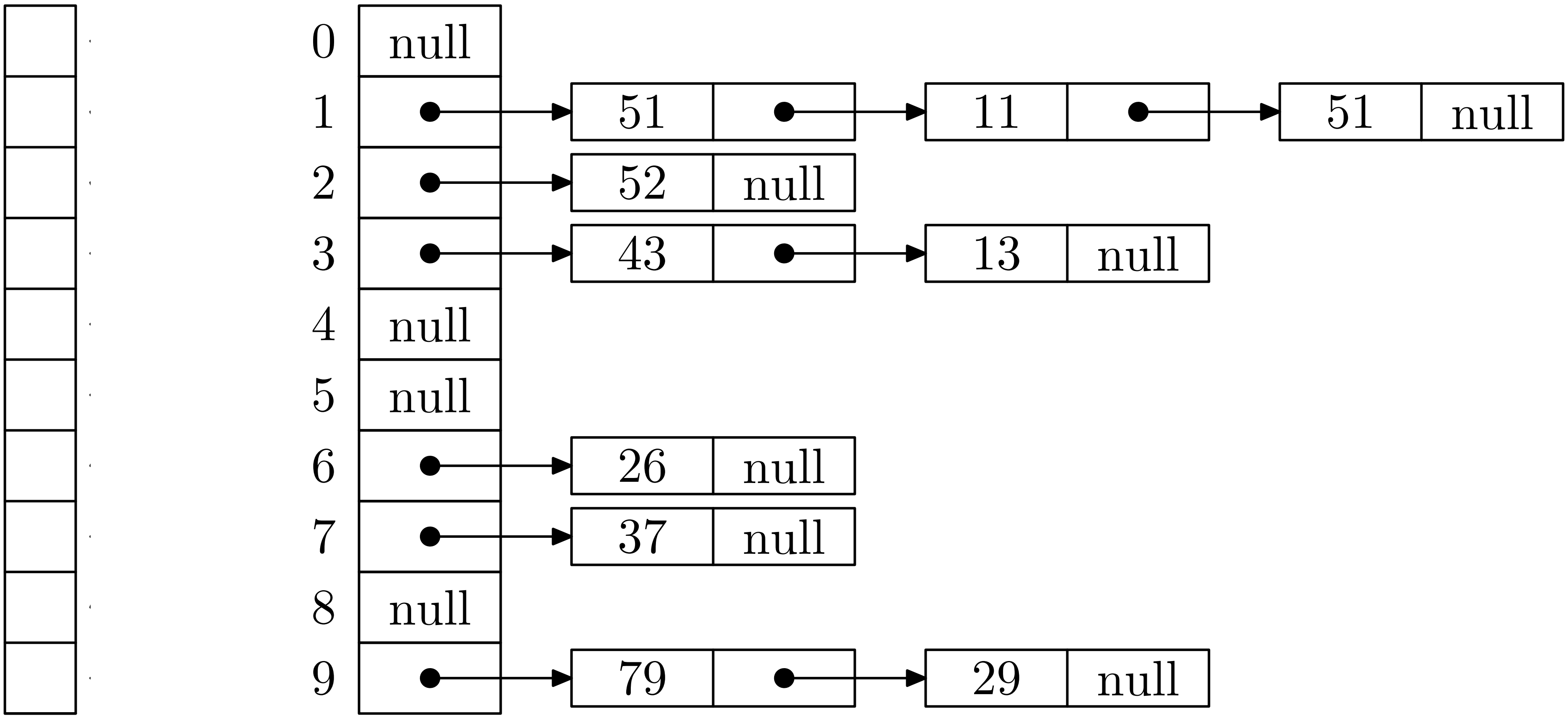

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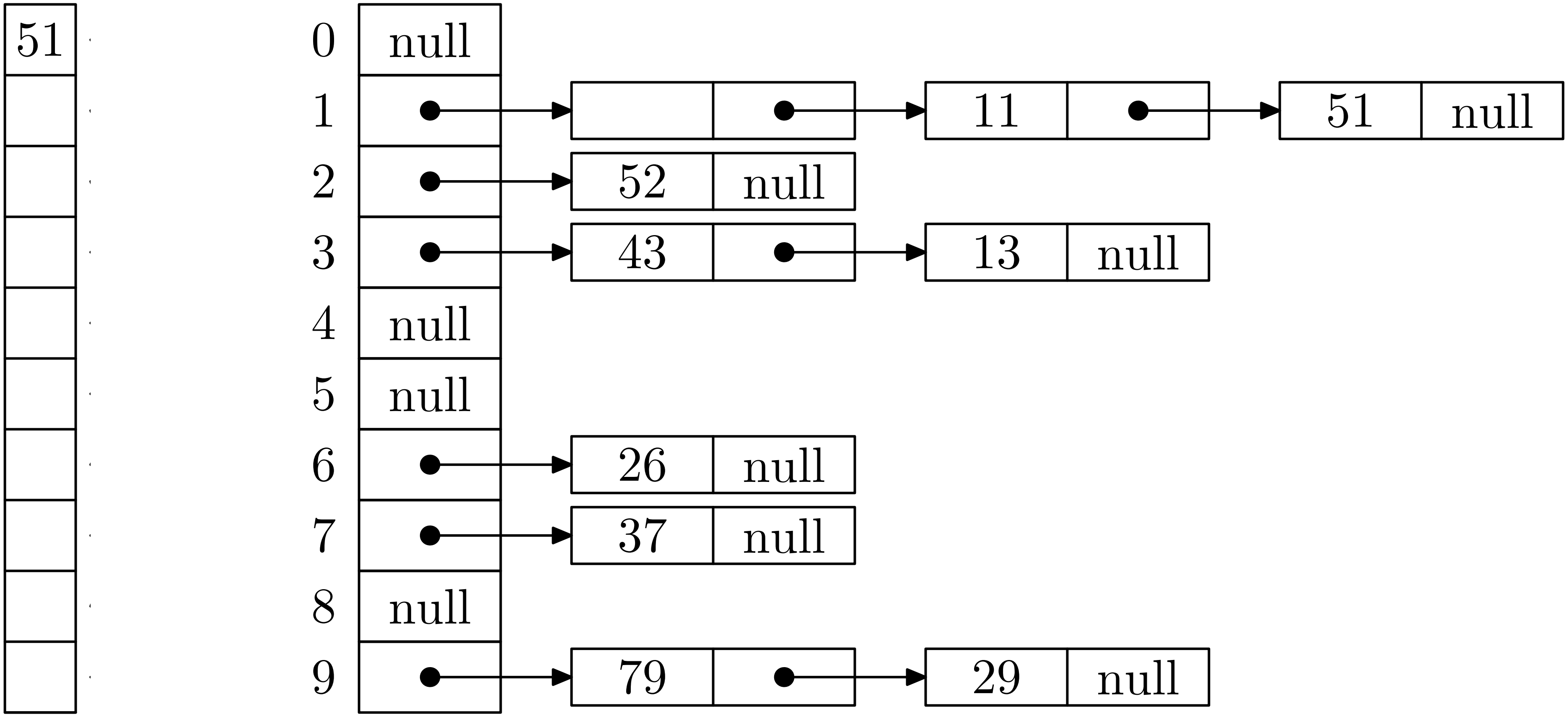

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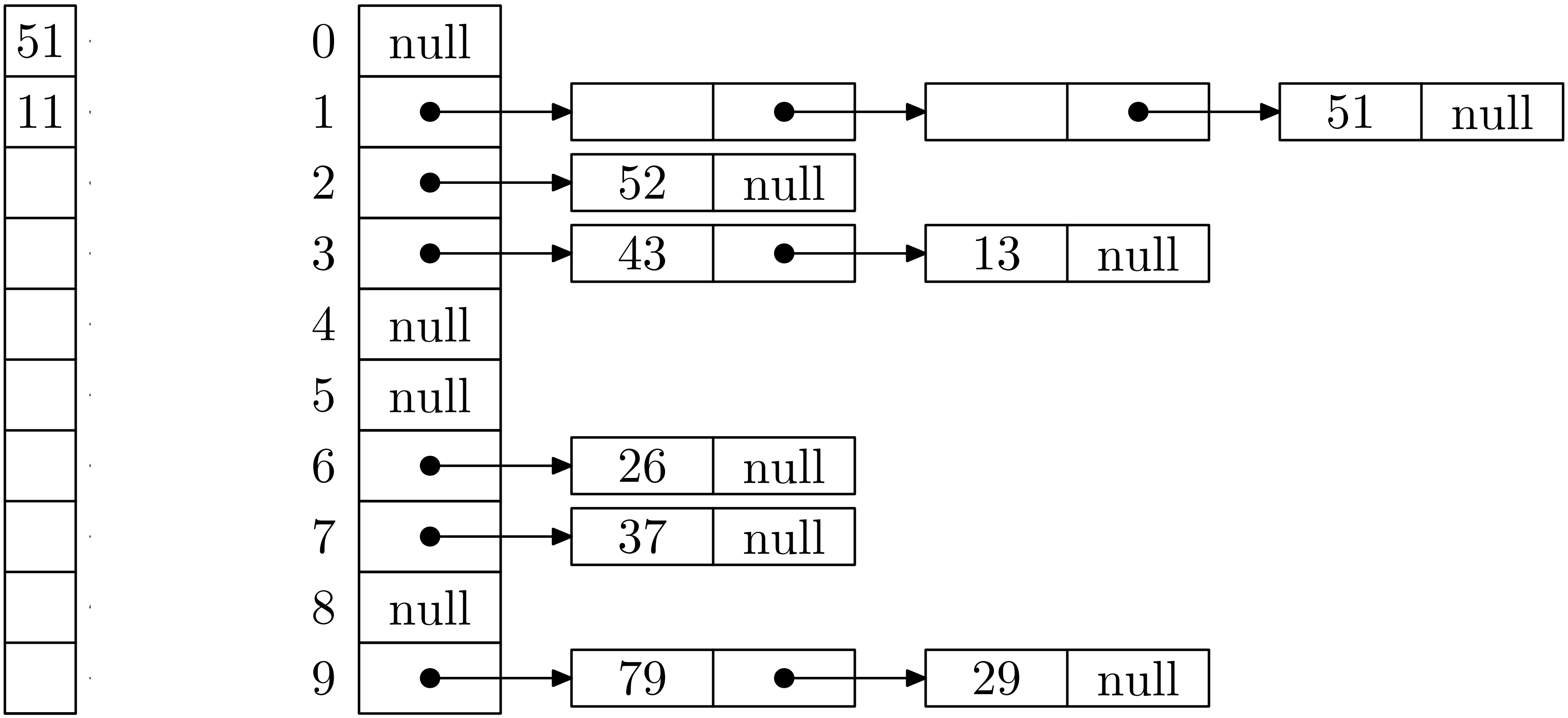
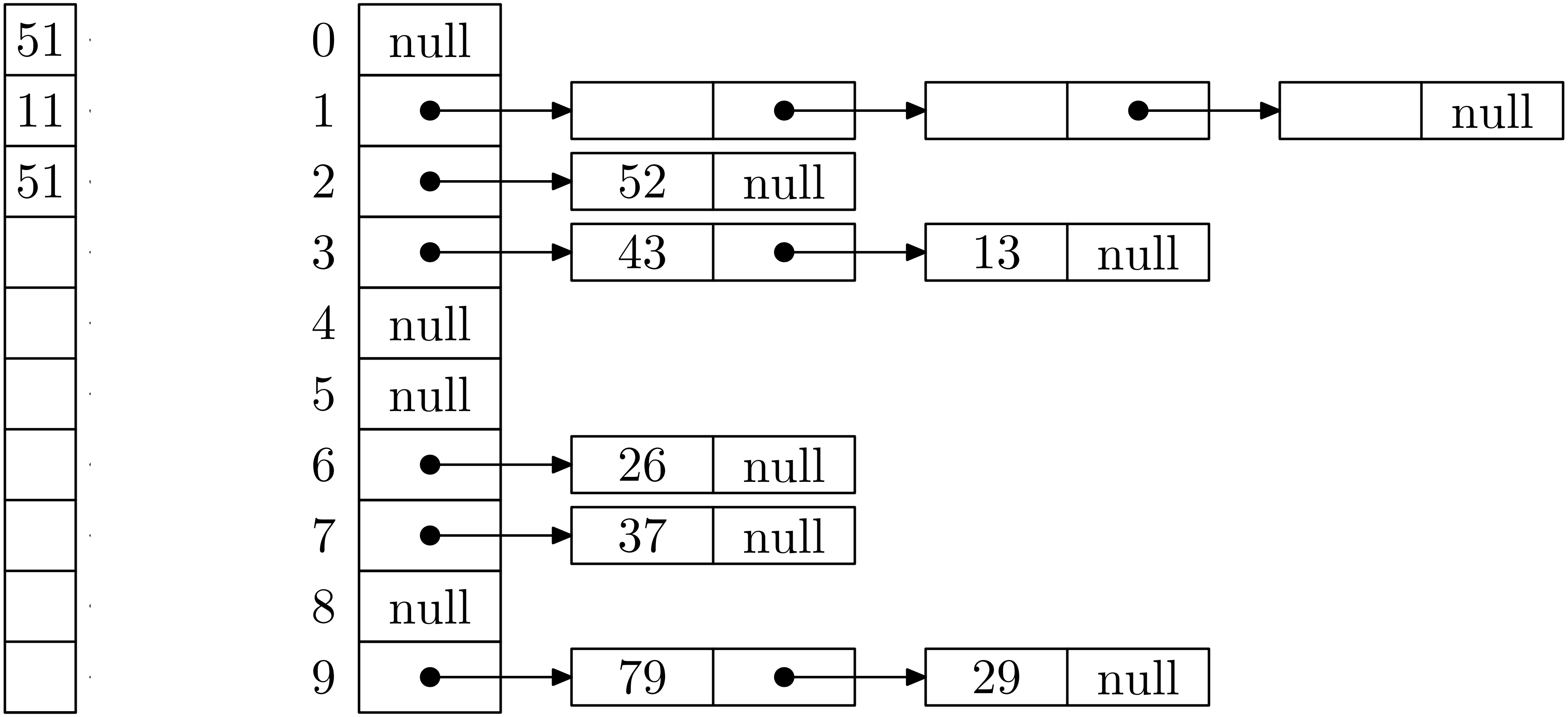


Radix Sort in Action



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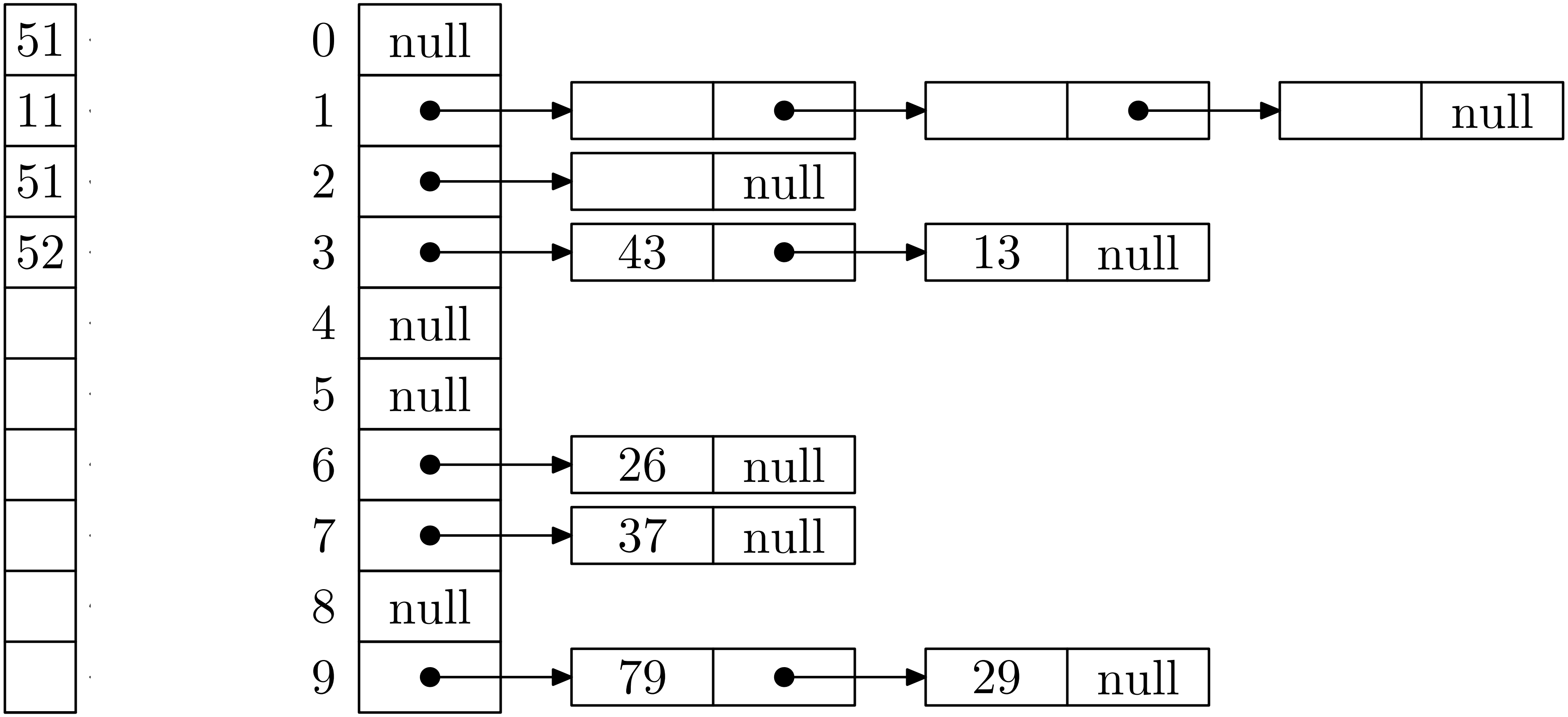
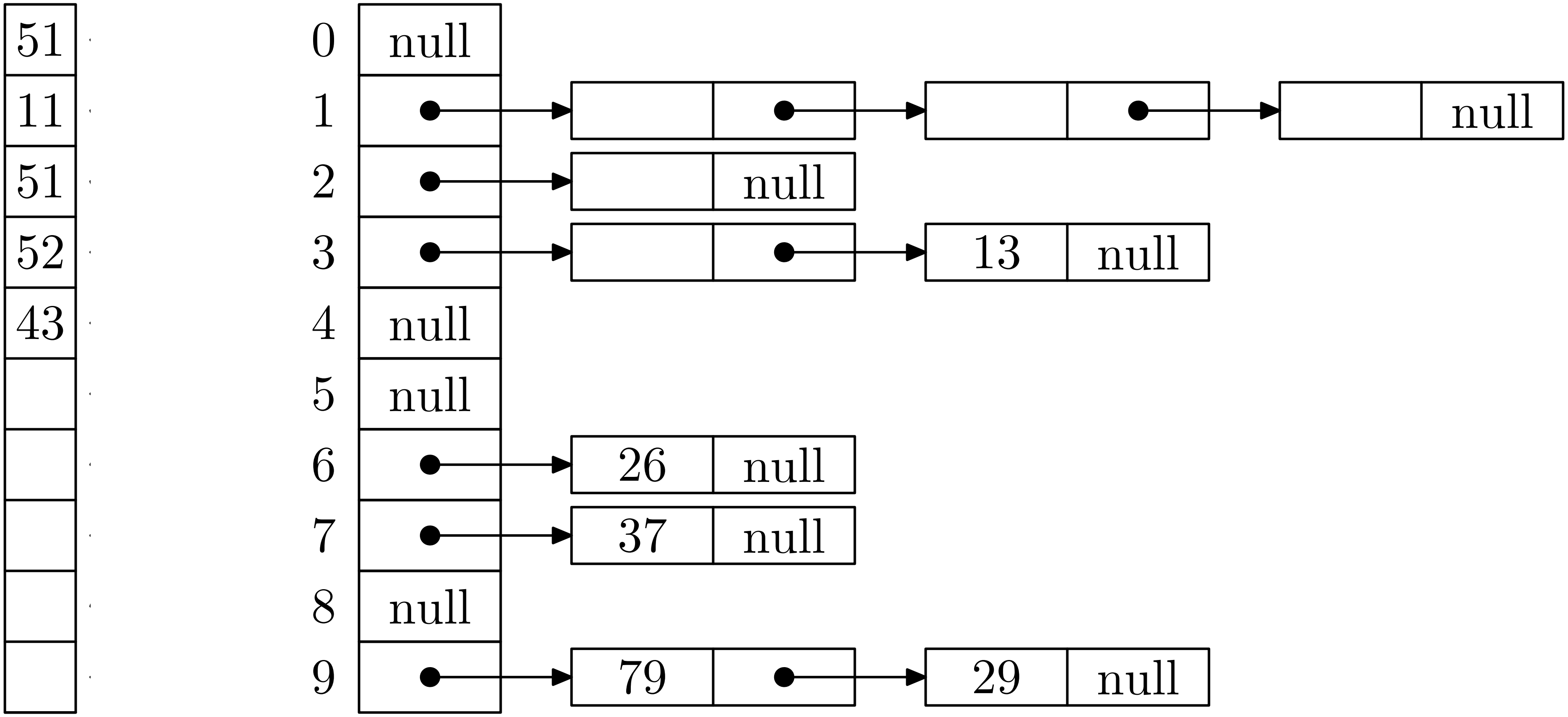


Radix Sort in Action



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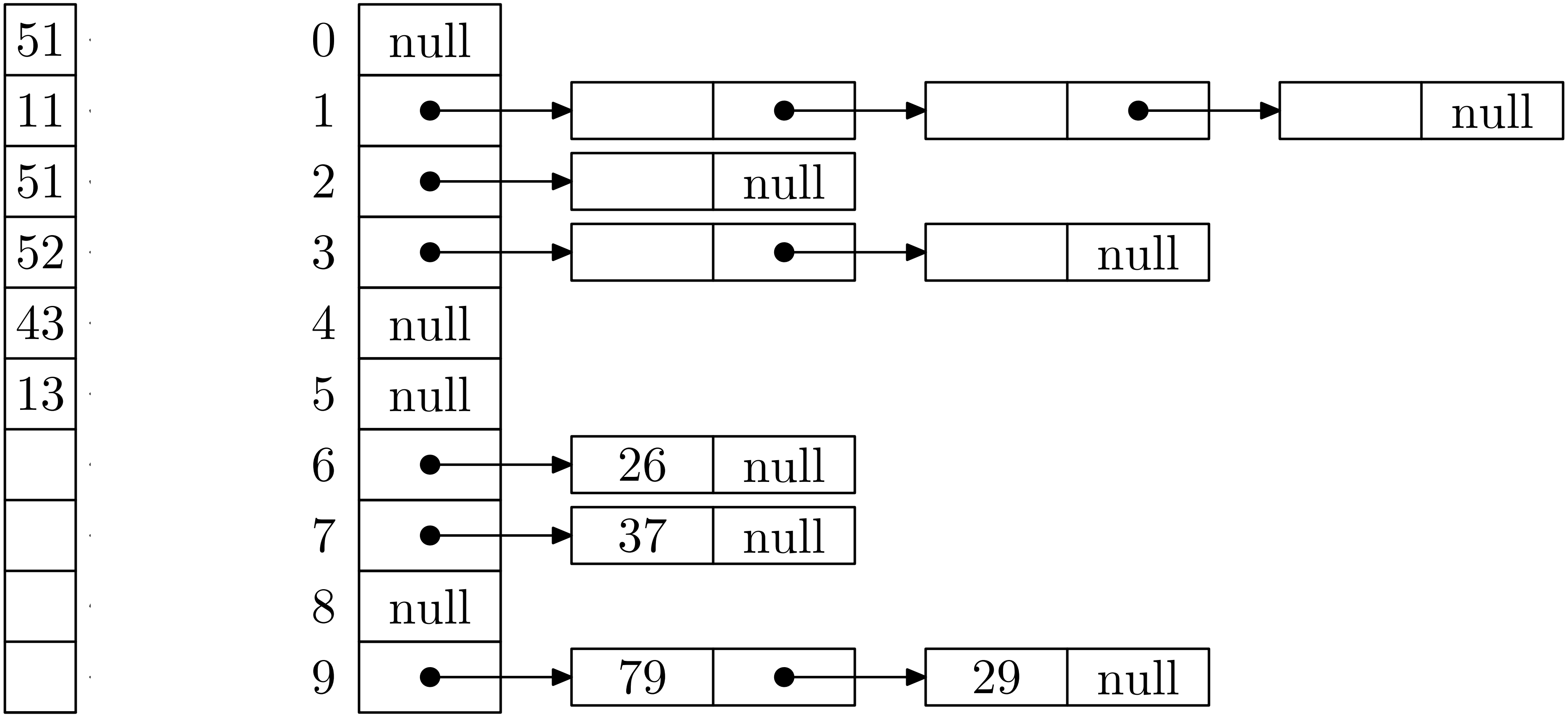

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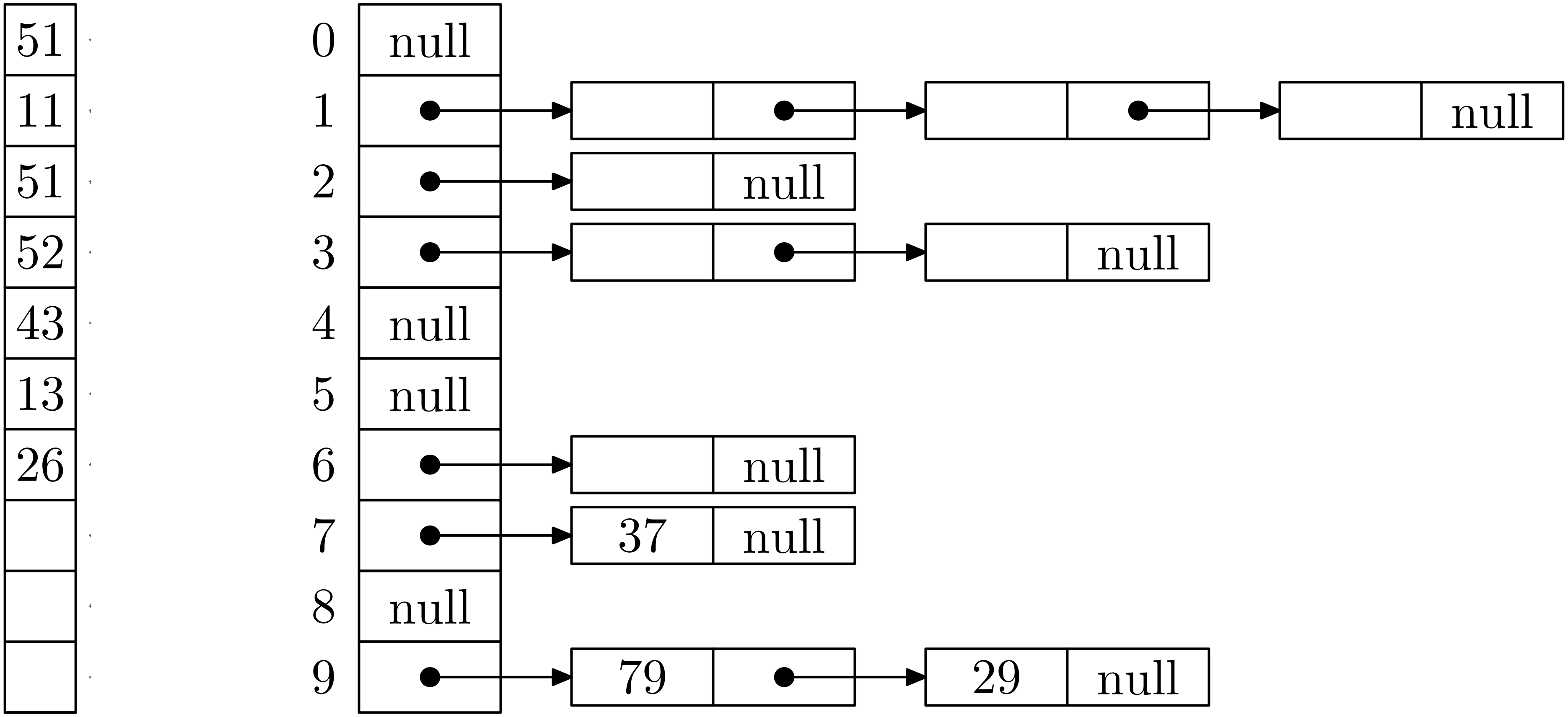

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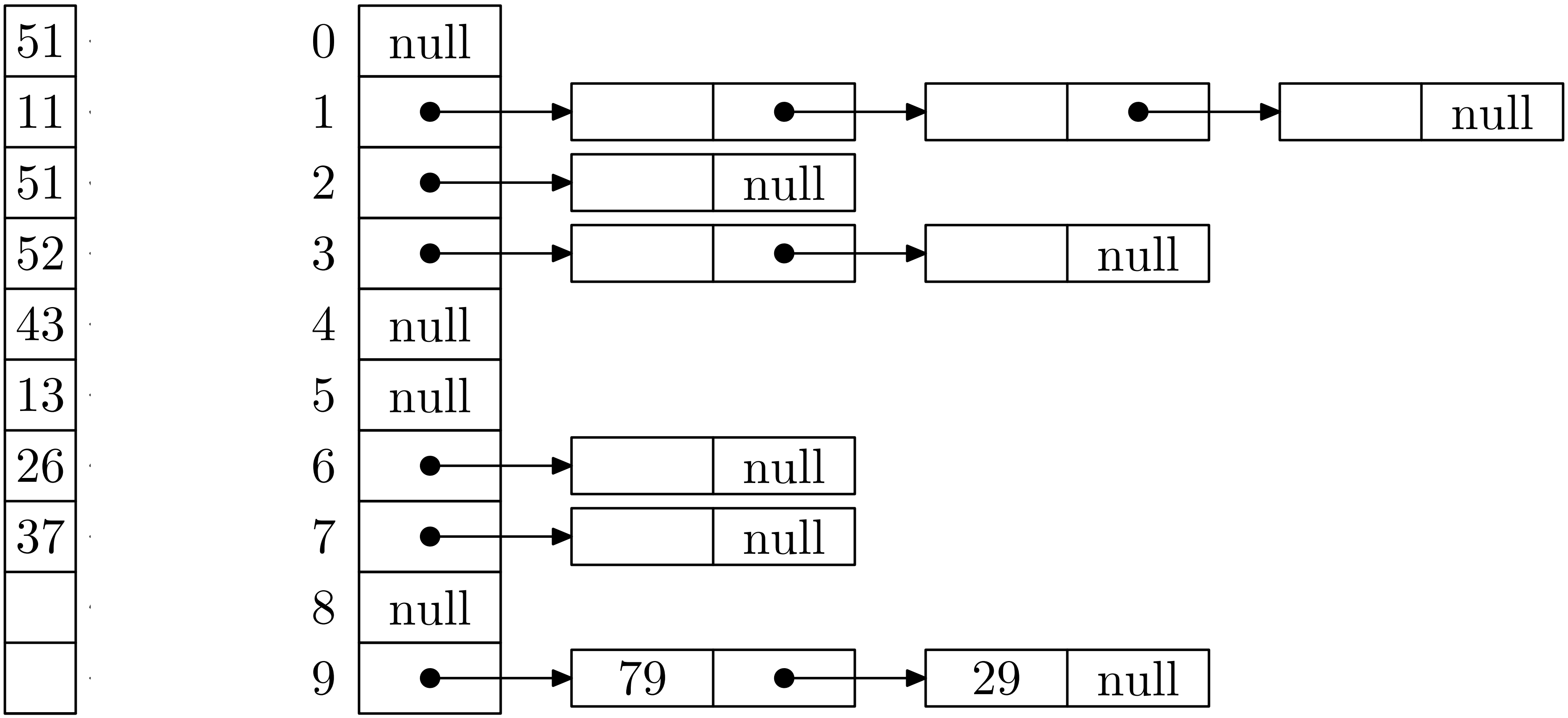
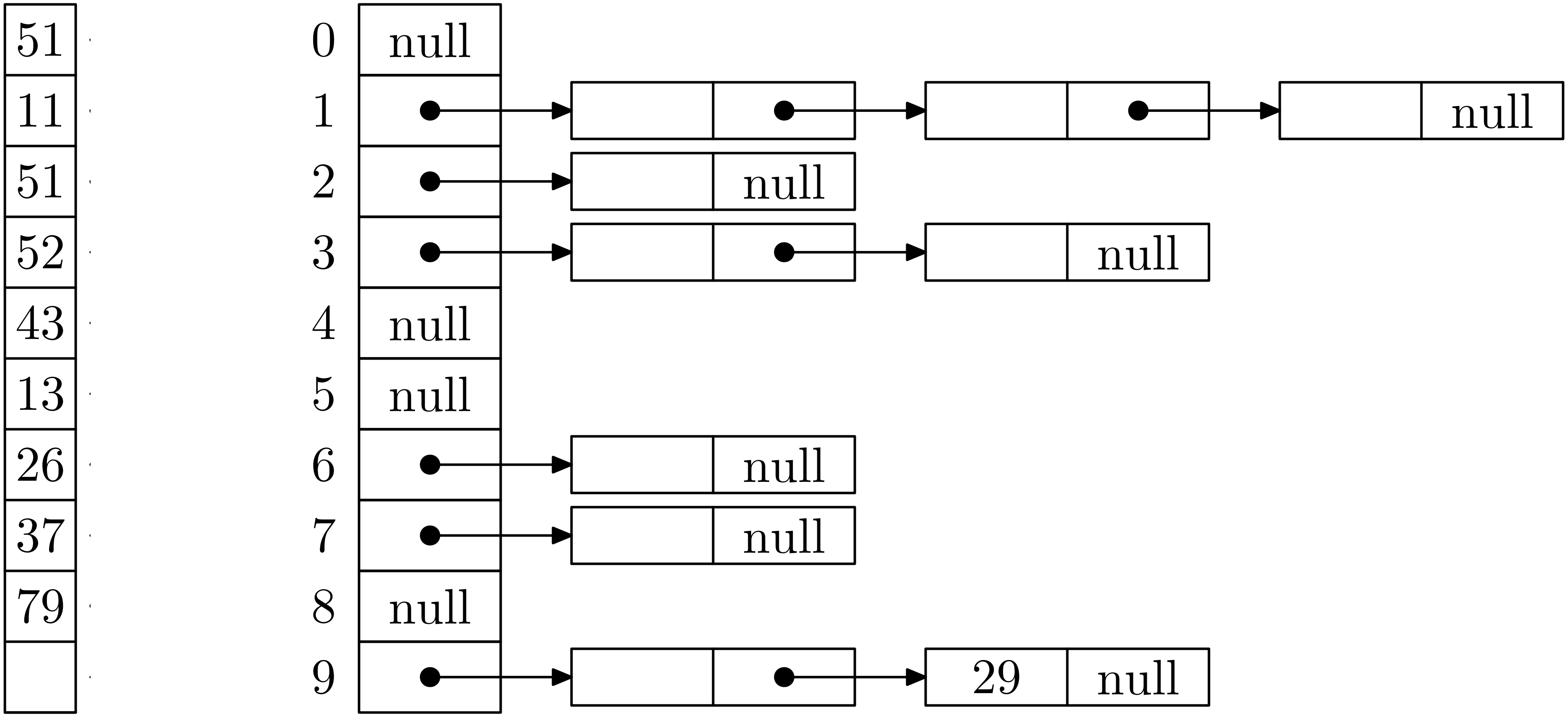


51
11
51
52
43
13
26
37

0 null
1
2 null
3 null
4 null
5 null
6 null
7 null
8 null
9 79 29 null

Radix Sort in Action



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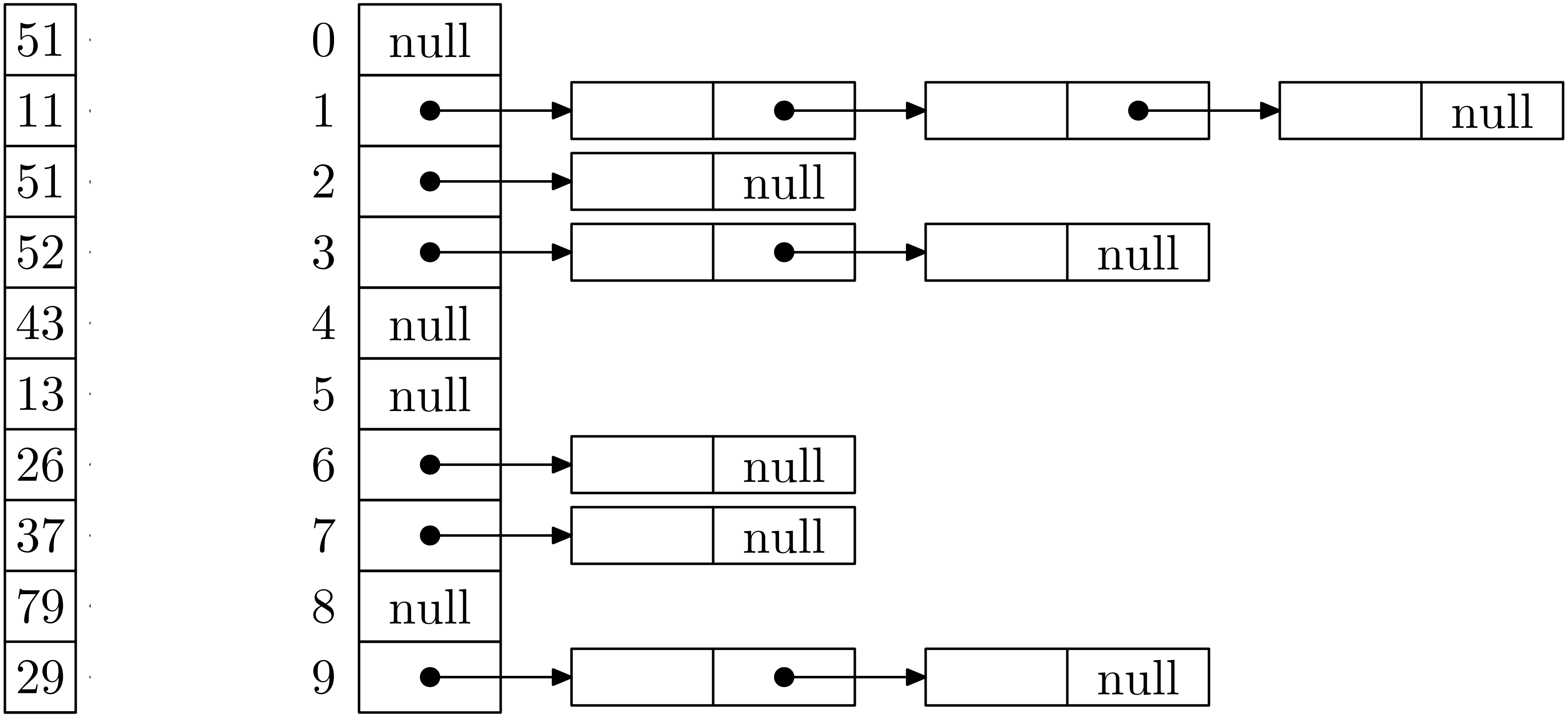

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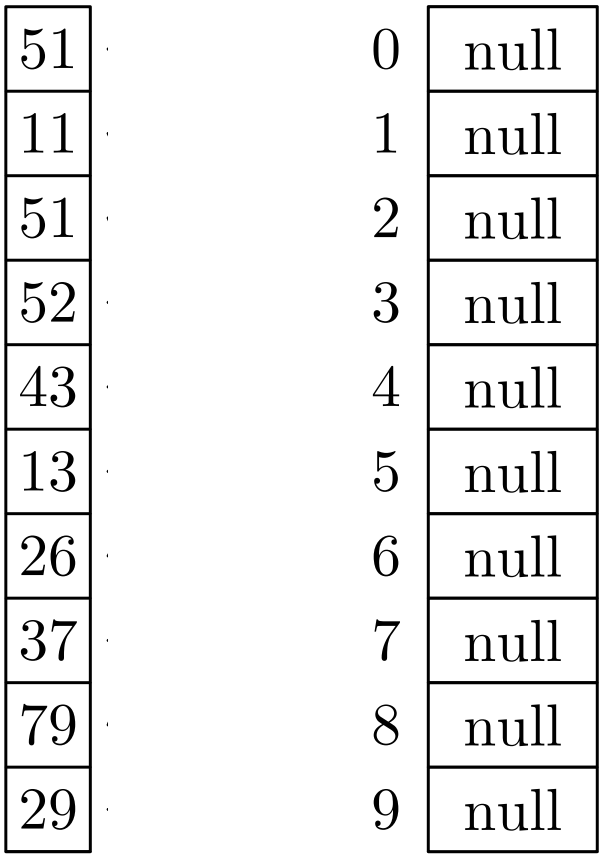

51
11
51
52
43
13
26
37
79
29

0	null
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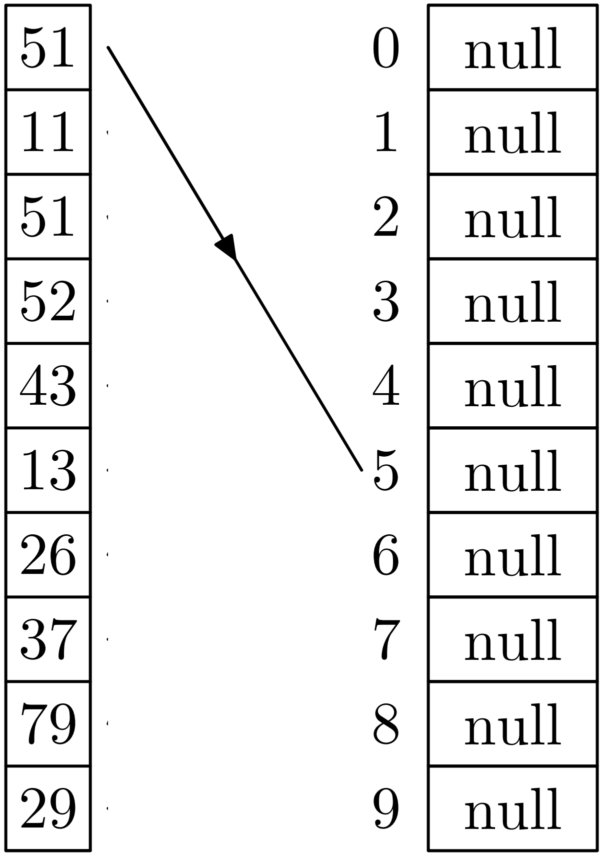
2	null
3	null
4	null
5	null
6	null
7	null
8	null
9	null

Radix Sort in Action



51
11
51
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43
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79
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0	null
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3	null
4	null
5	null
6	null
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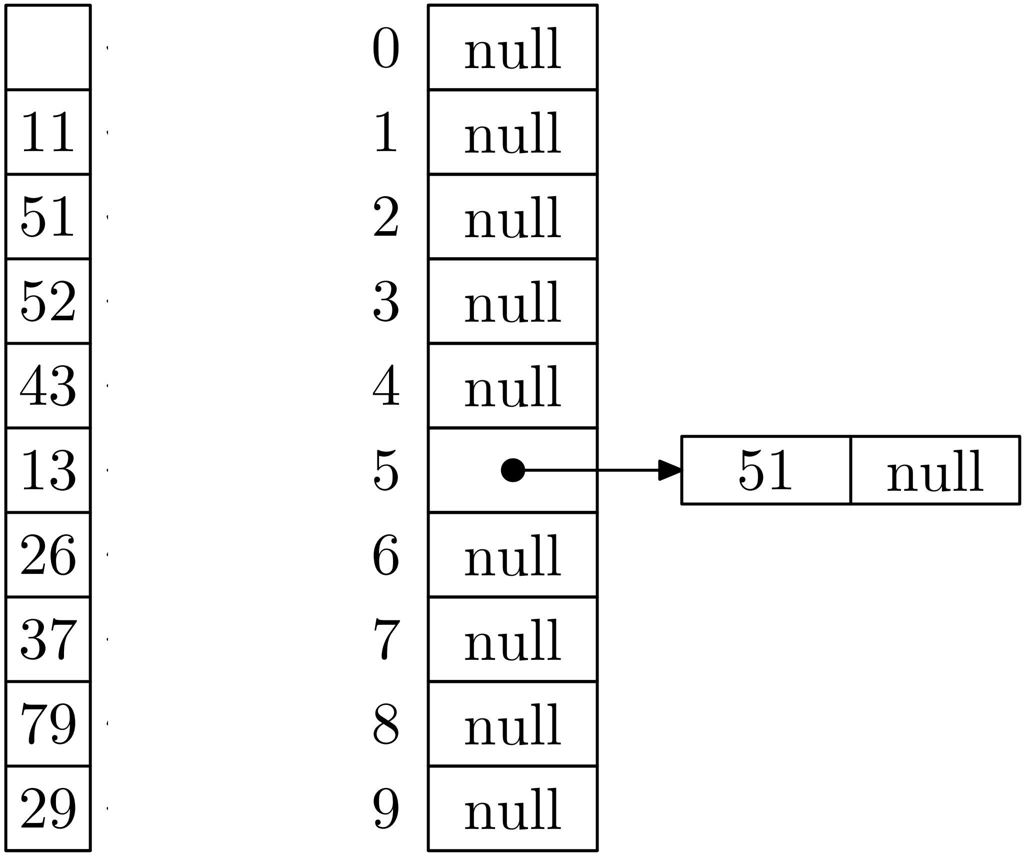

11
51
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43
13
26
37
79
29

0 null
1 null
2 null
3 null
4 null
5 → 51 null
6 null
7 null
8 null
9 null

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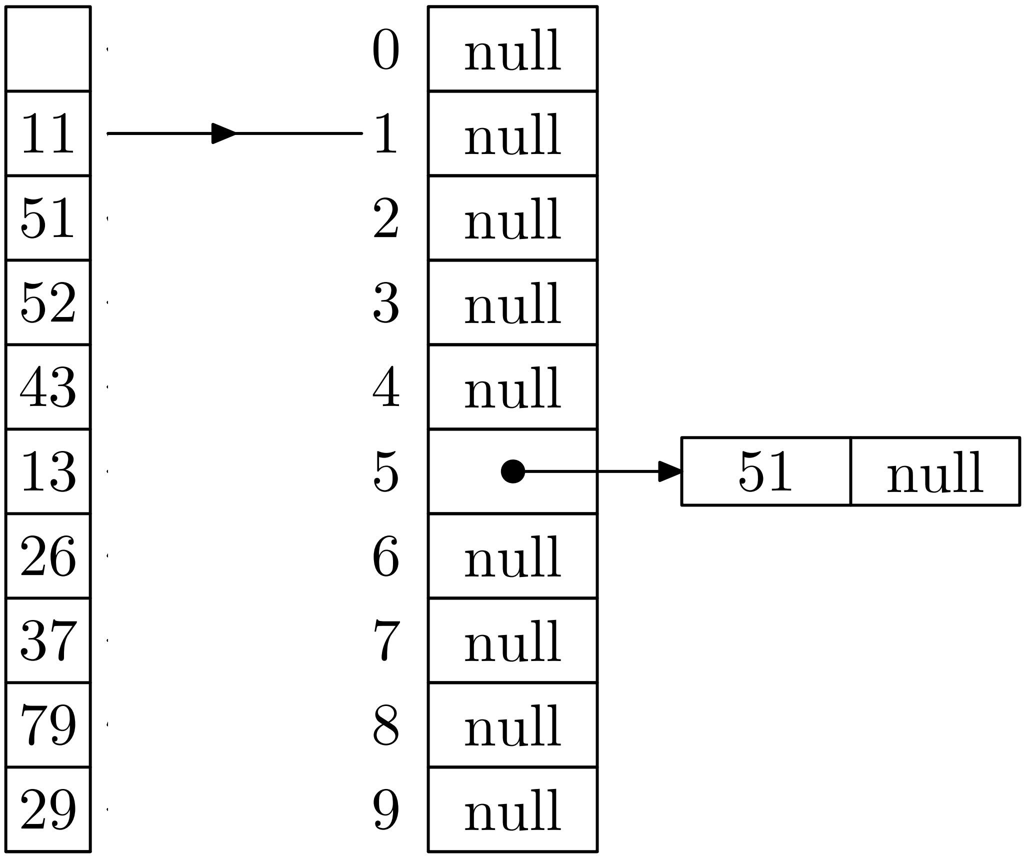

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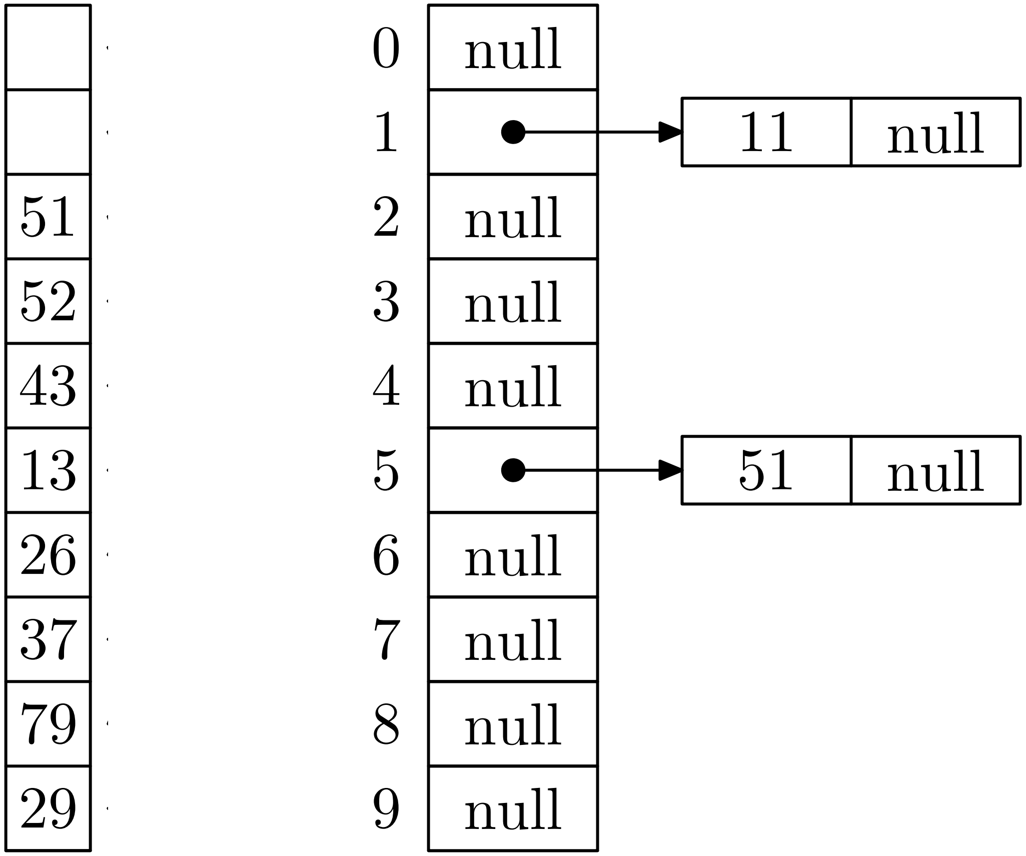

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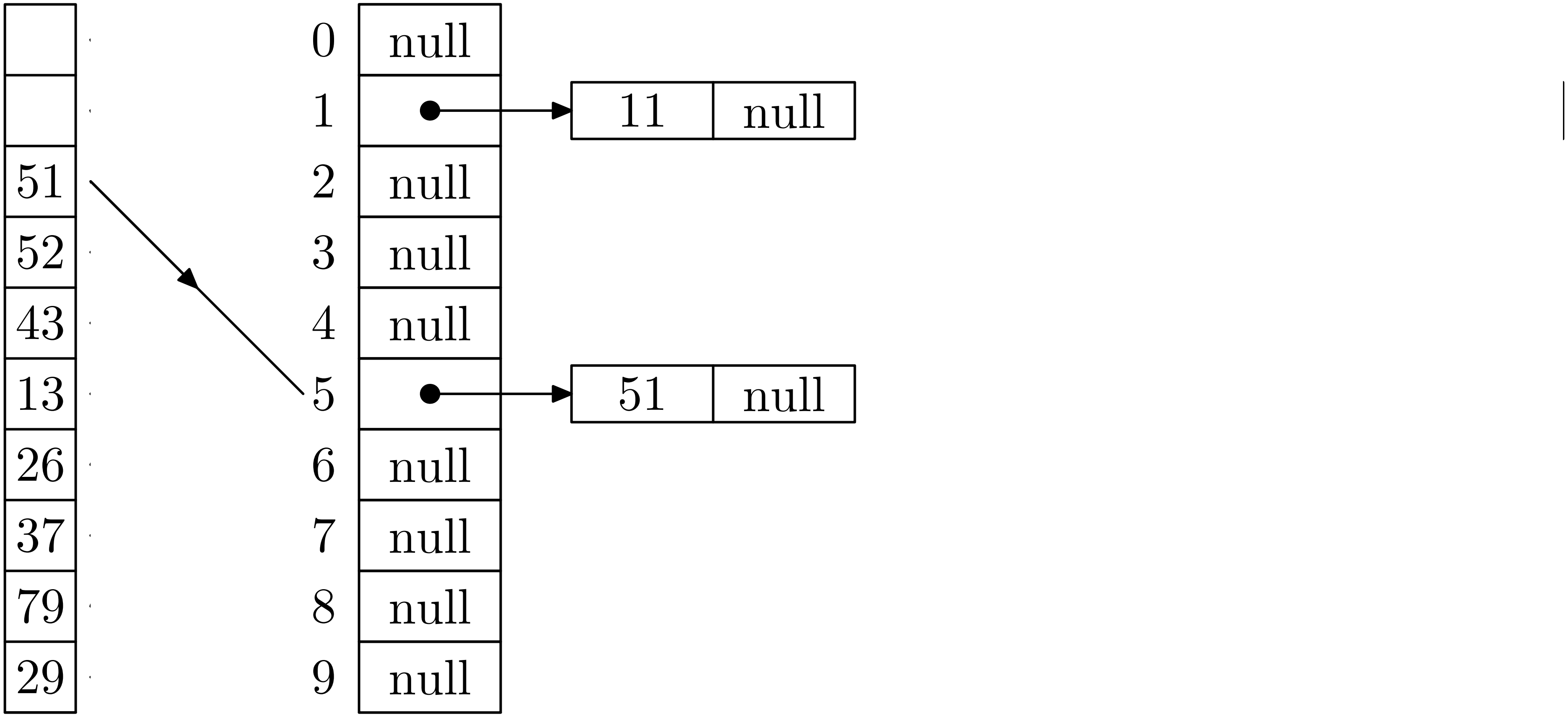

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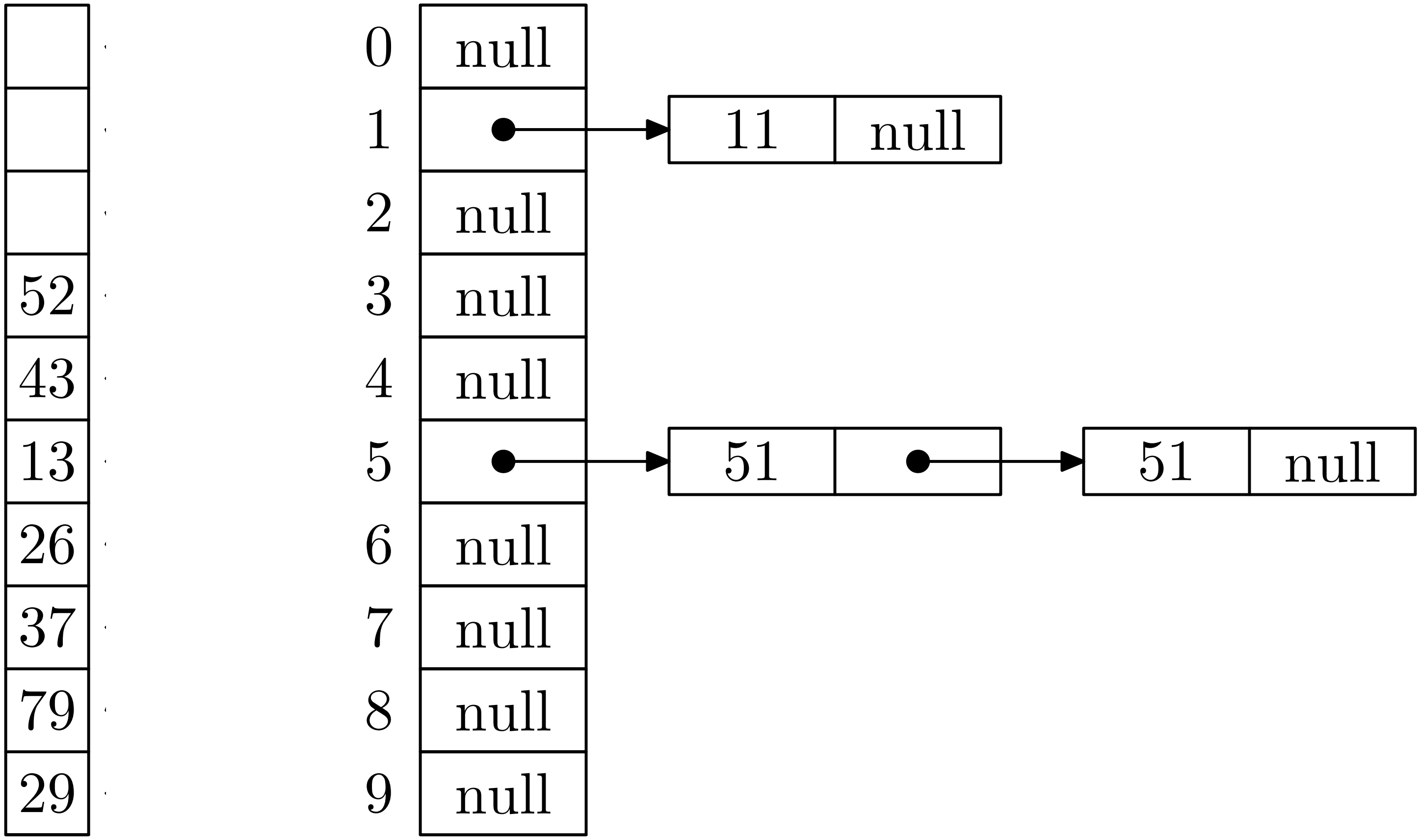

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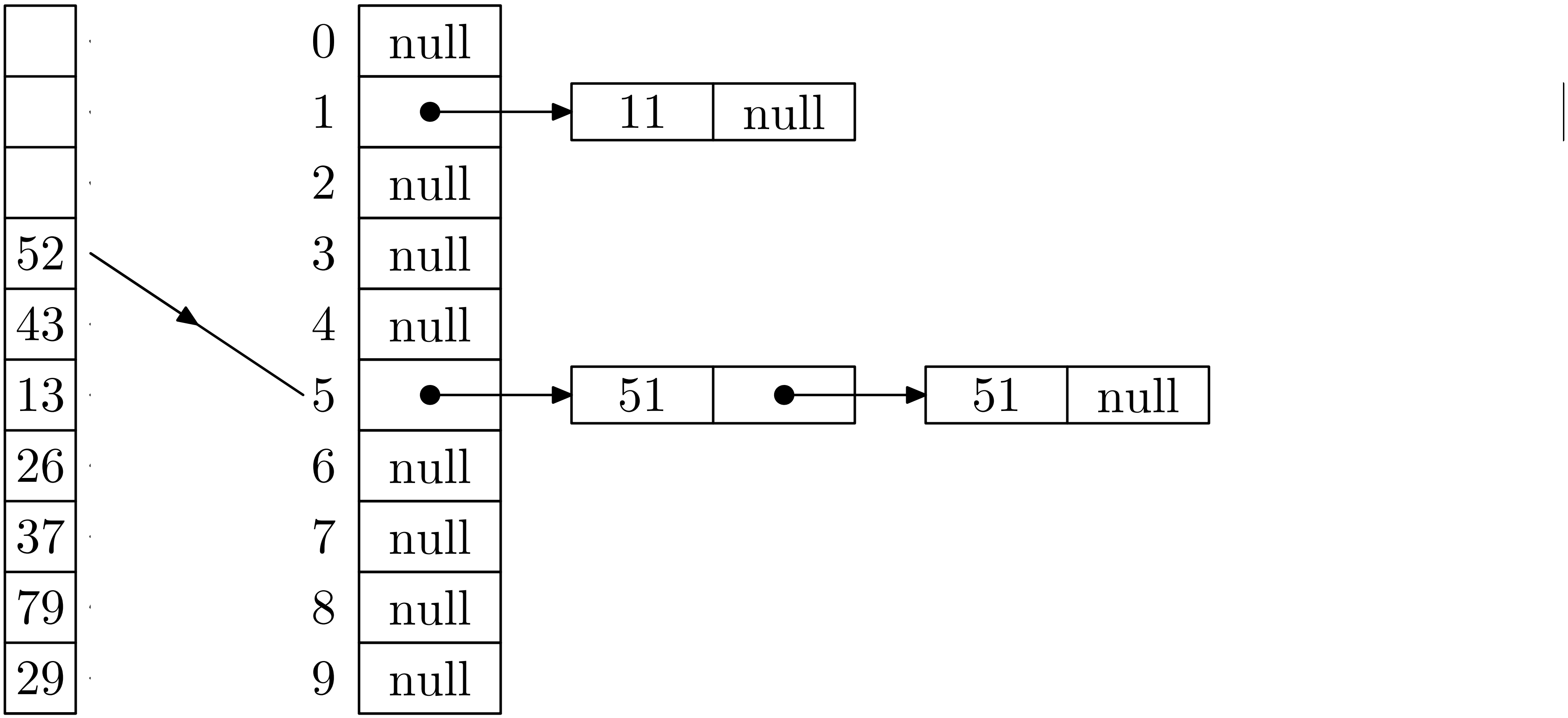

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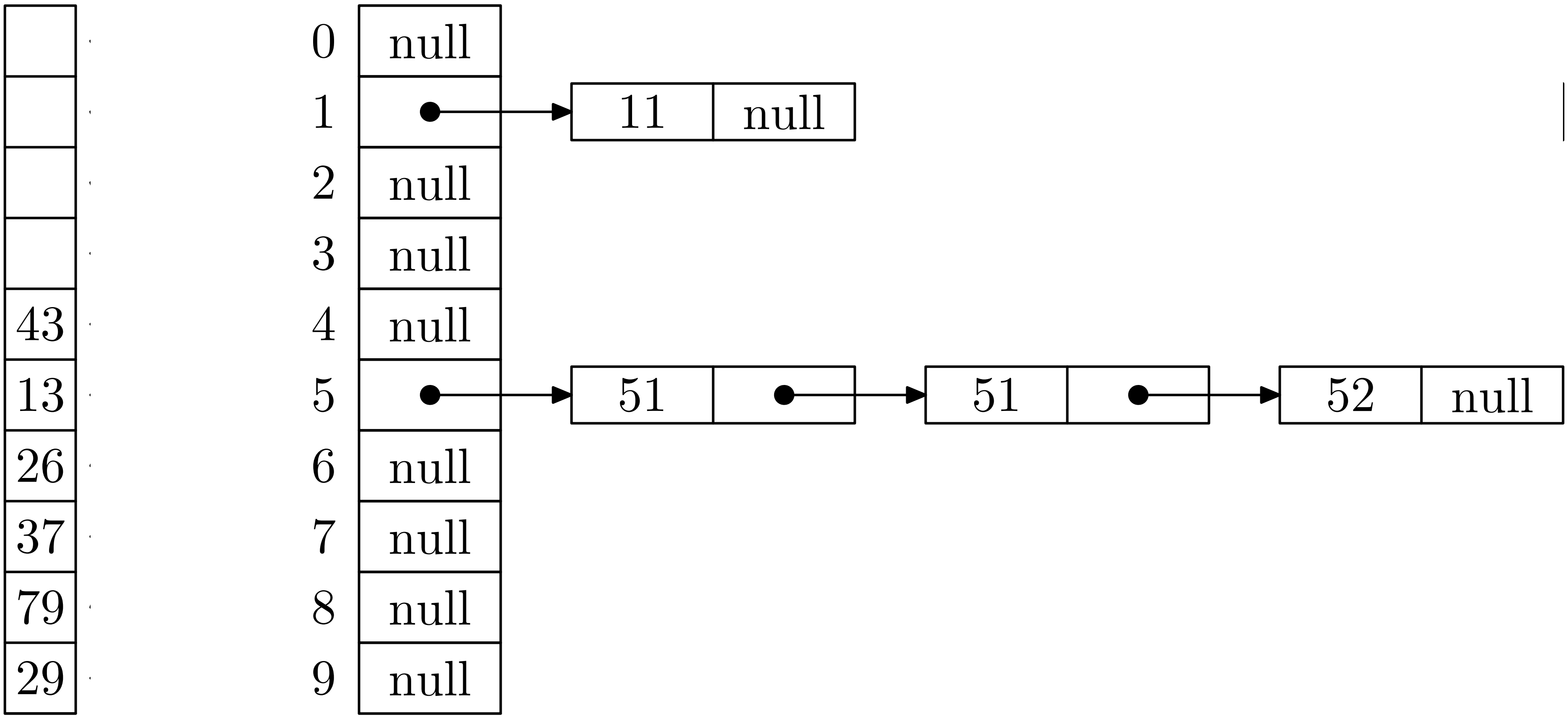

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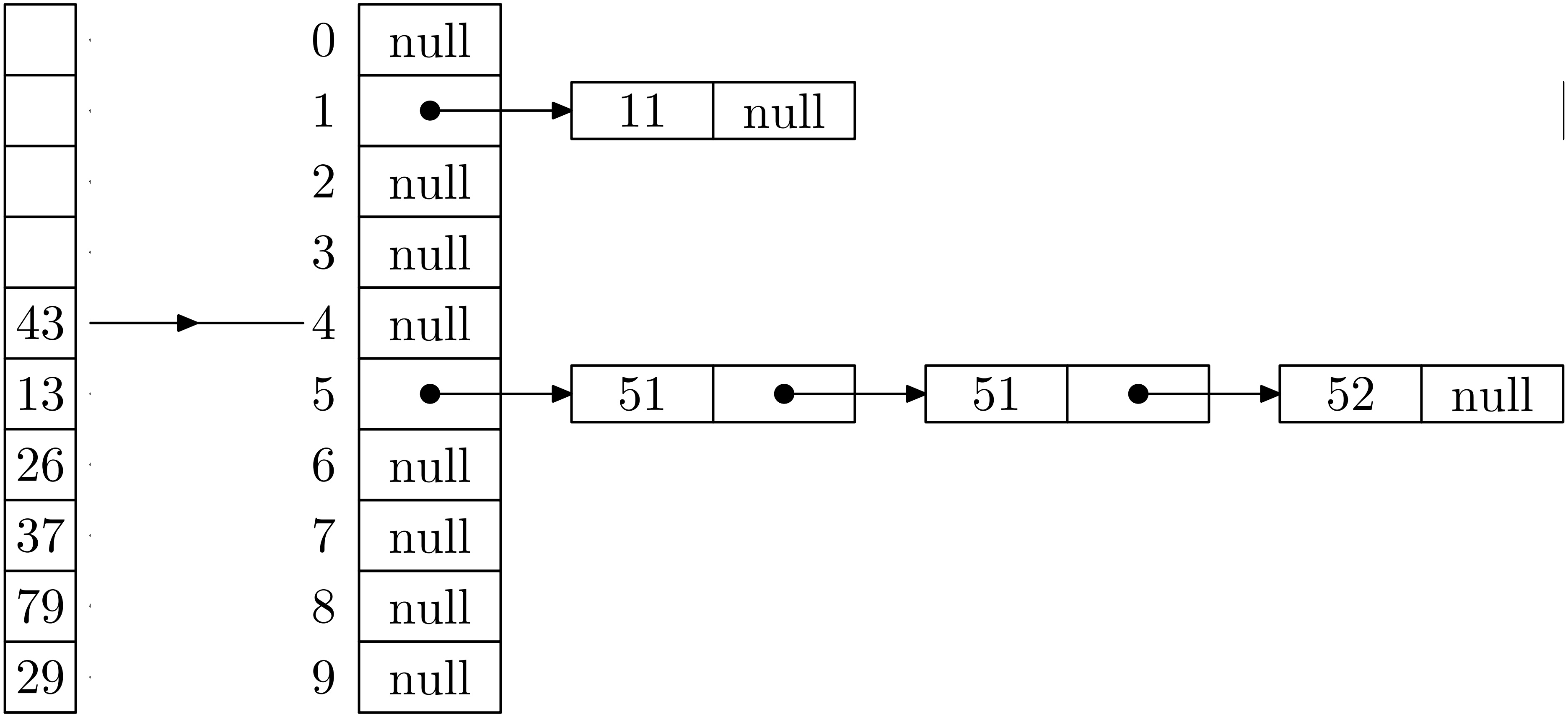

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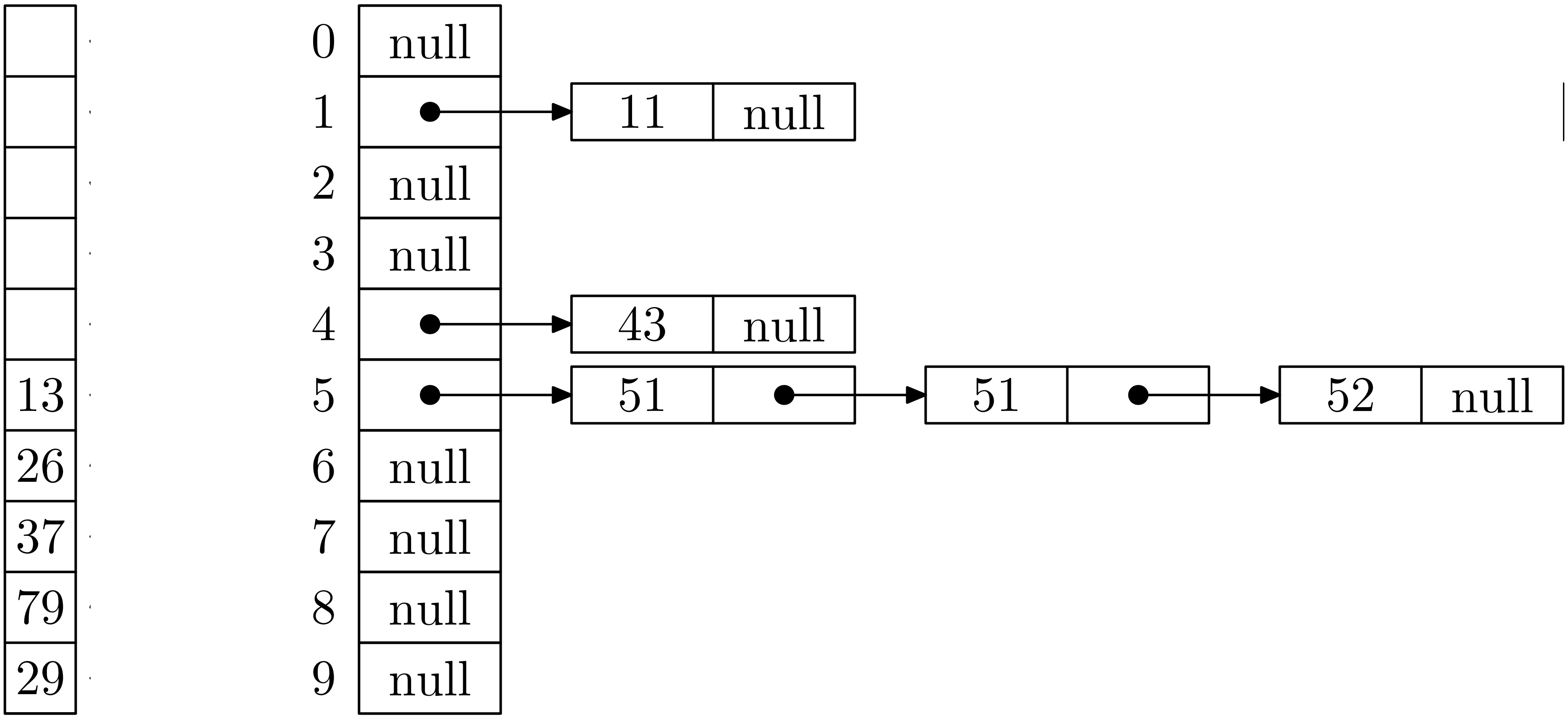

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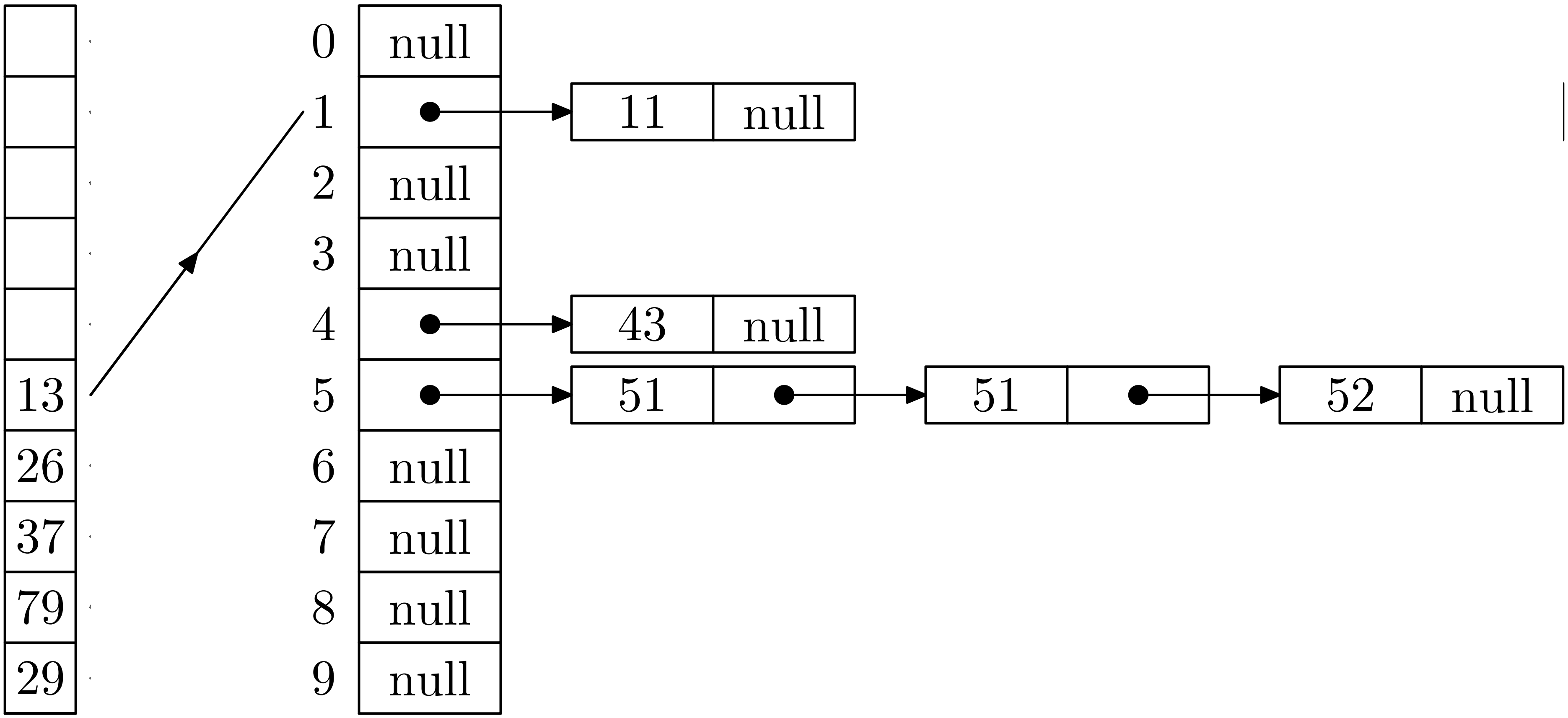

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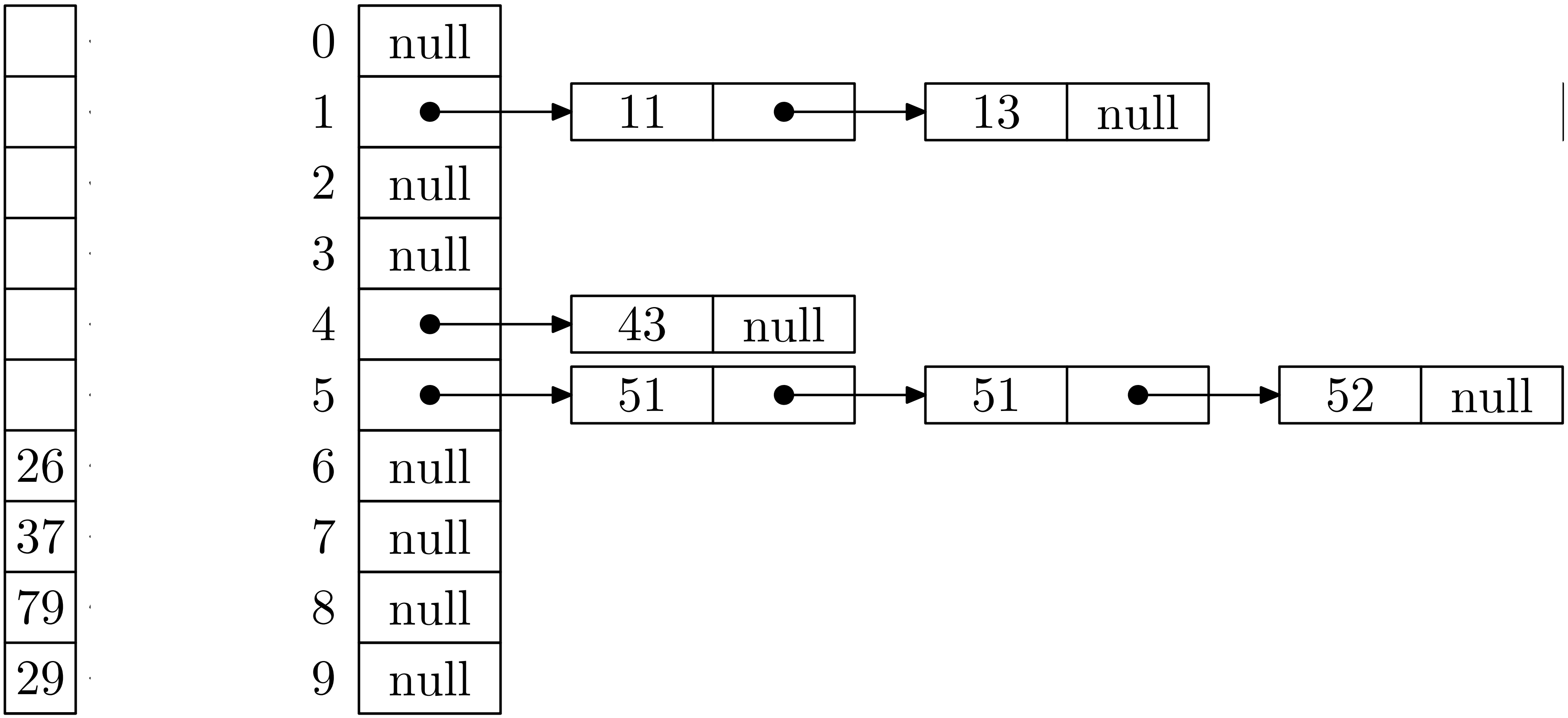

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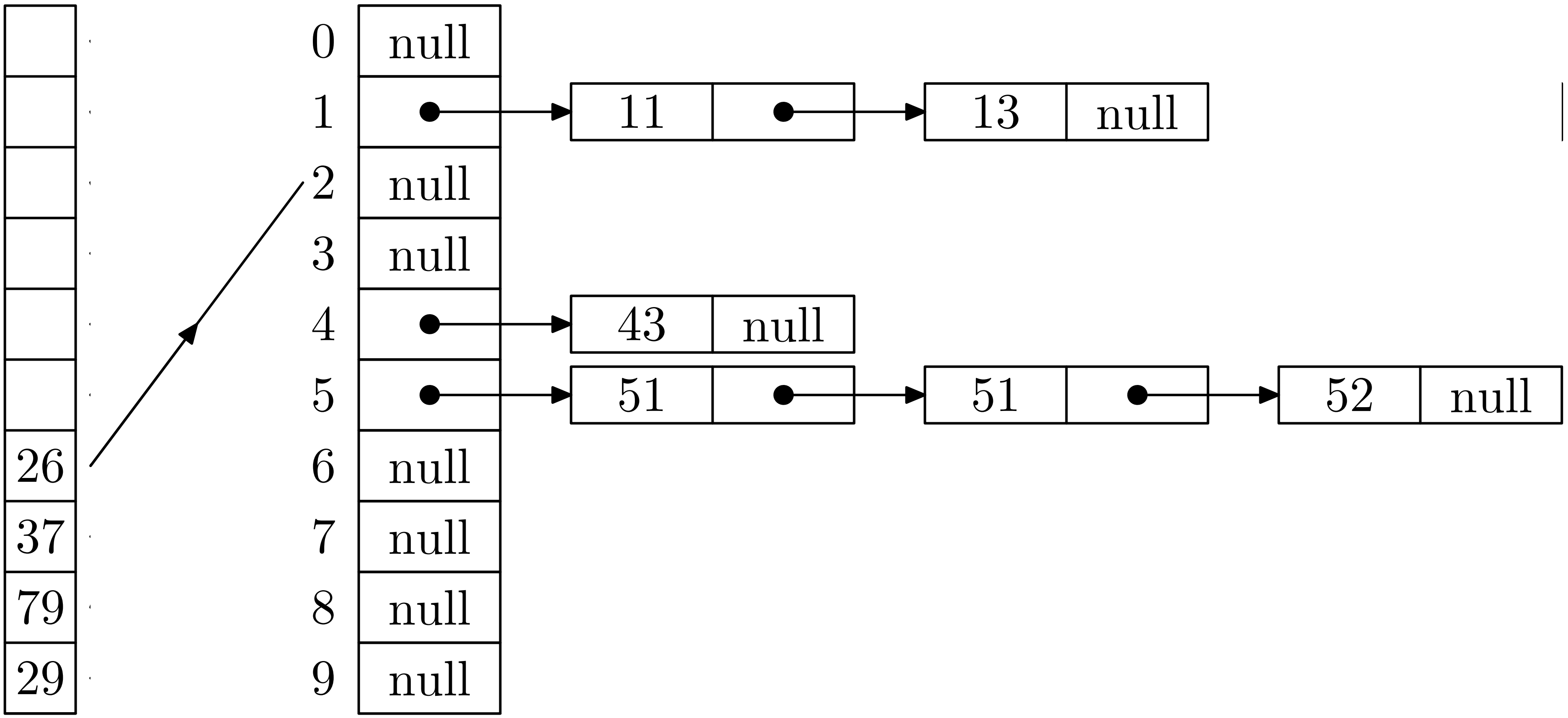

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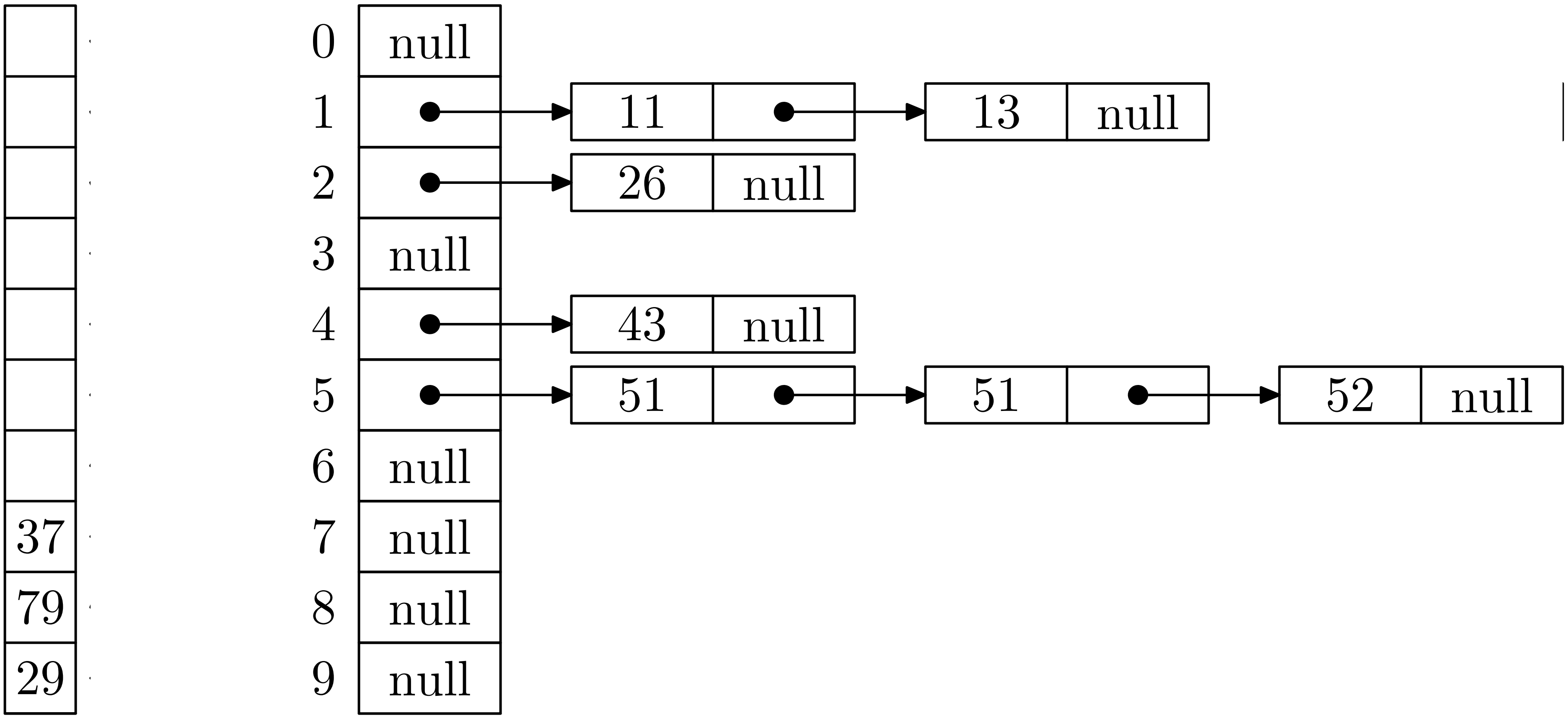

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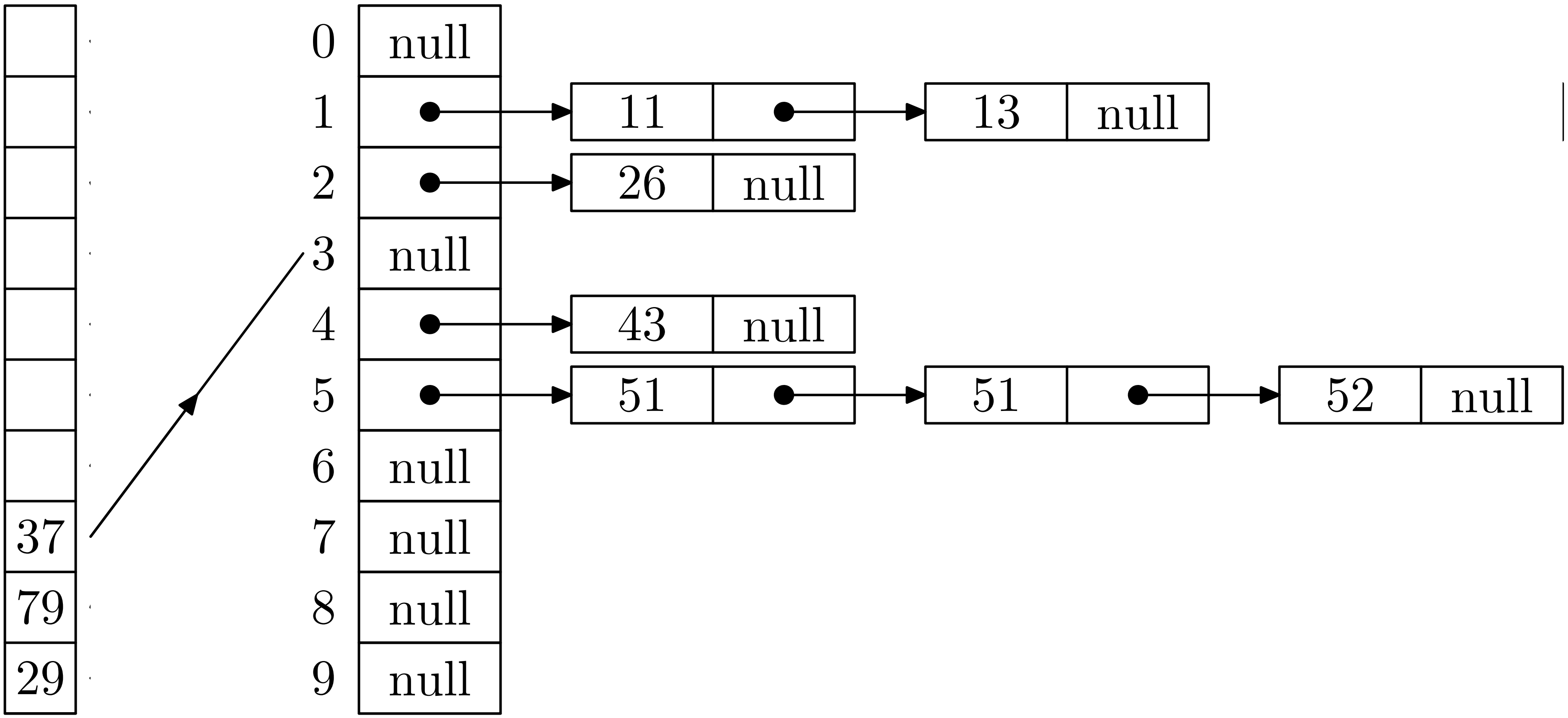

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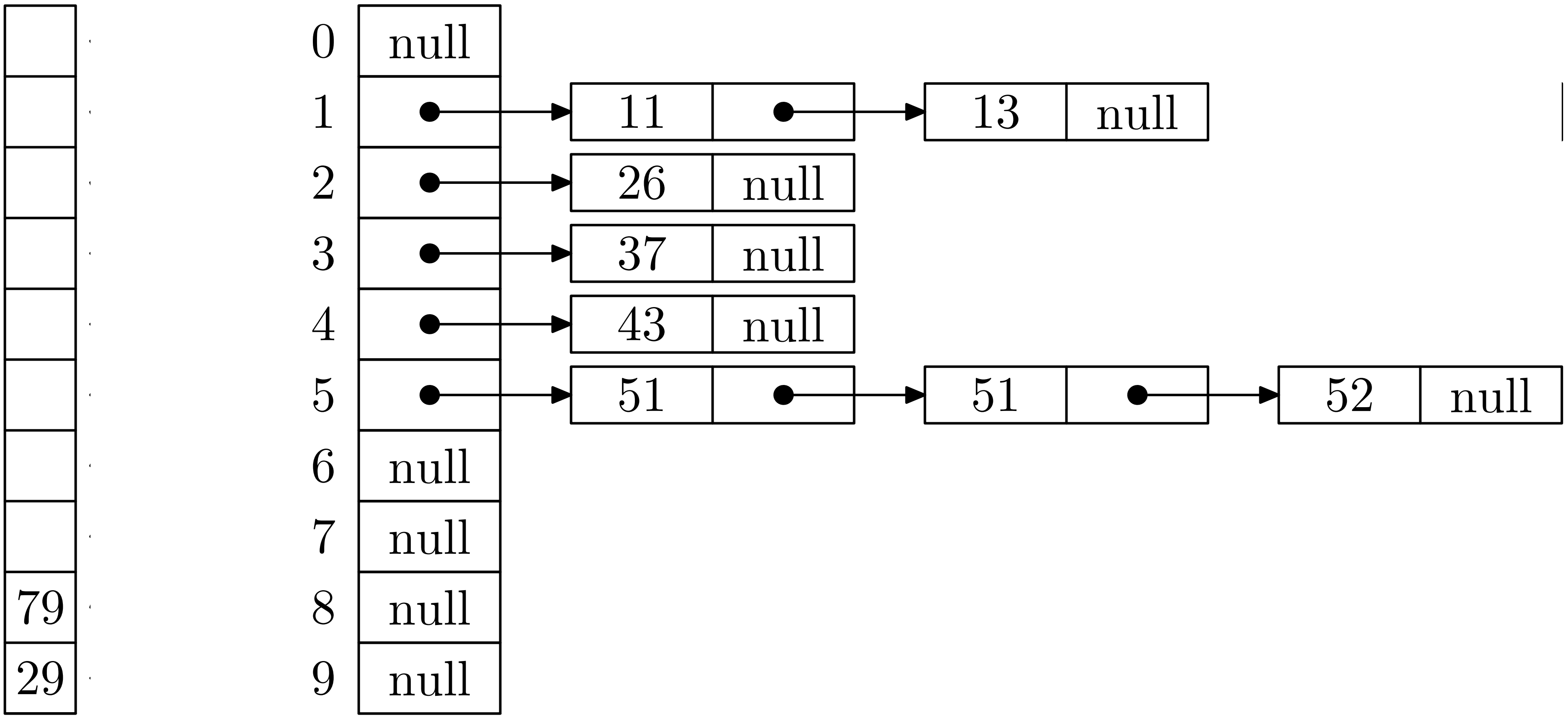

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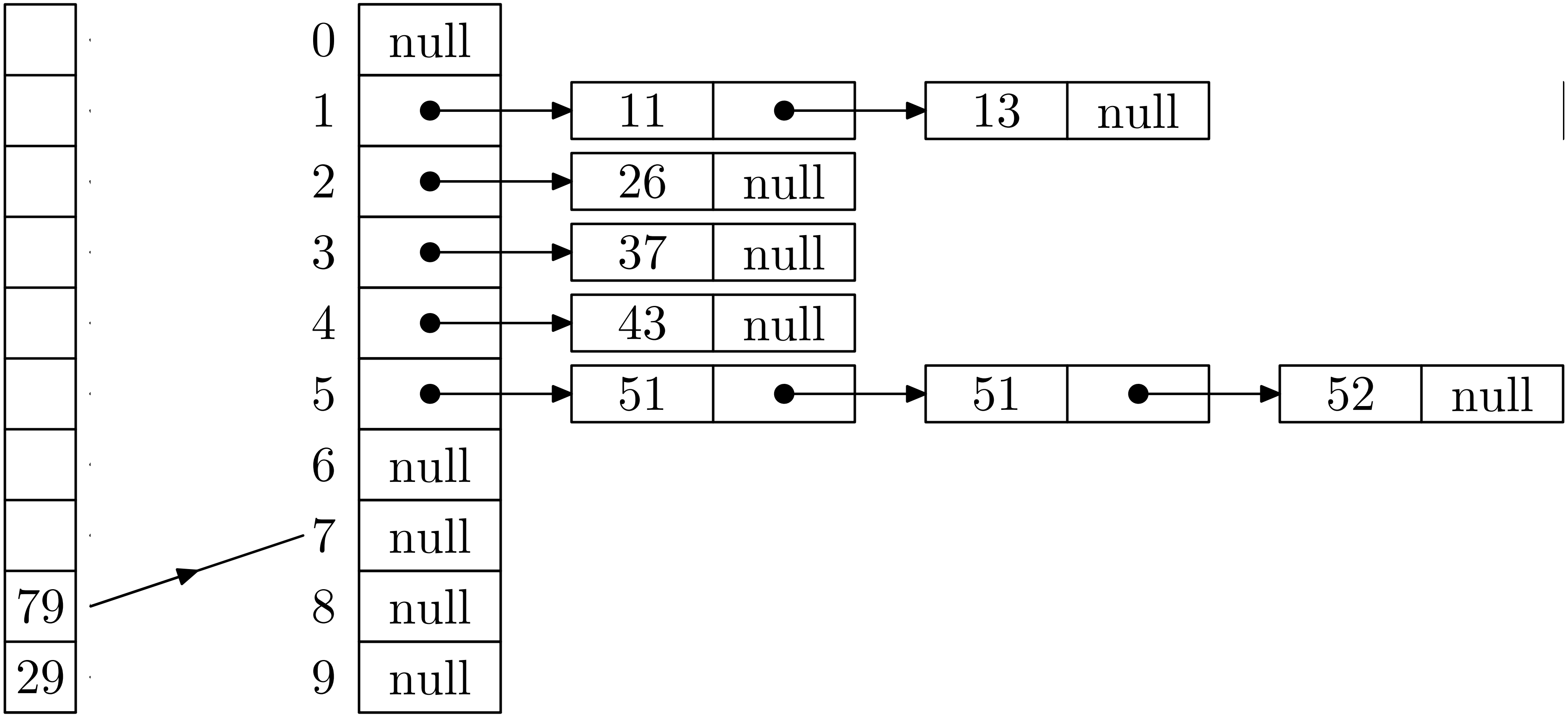

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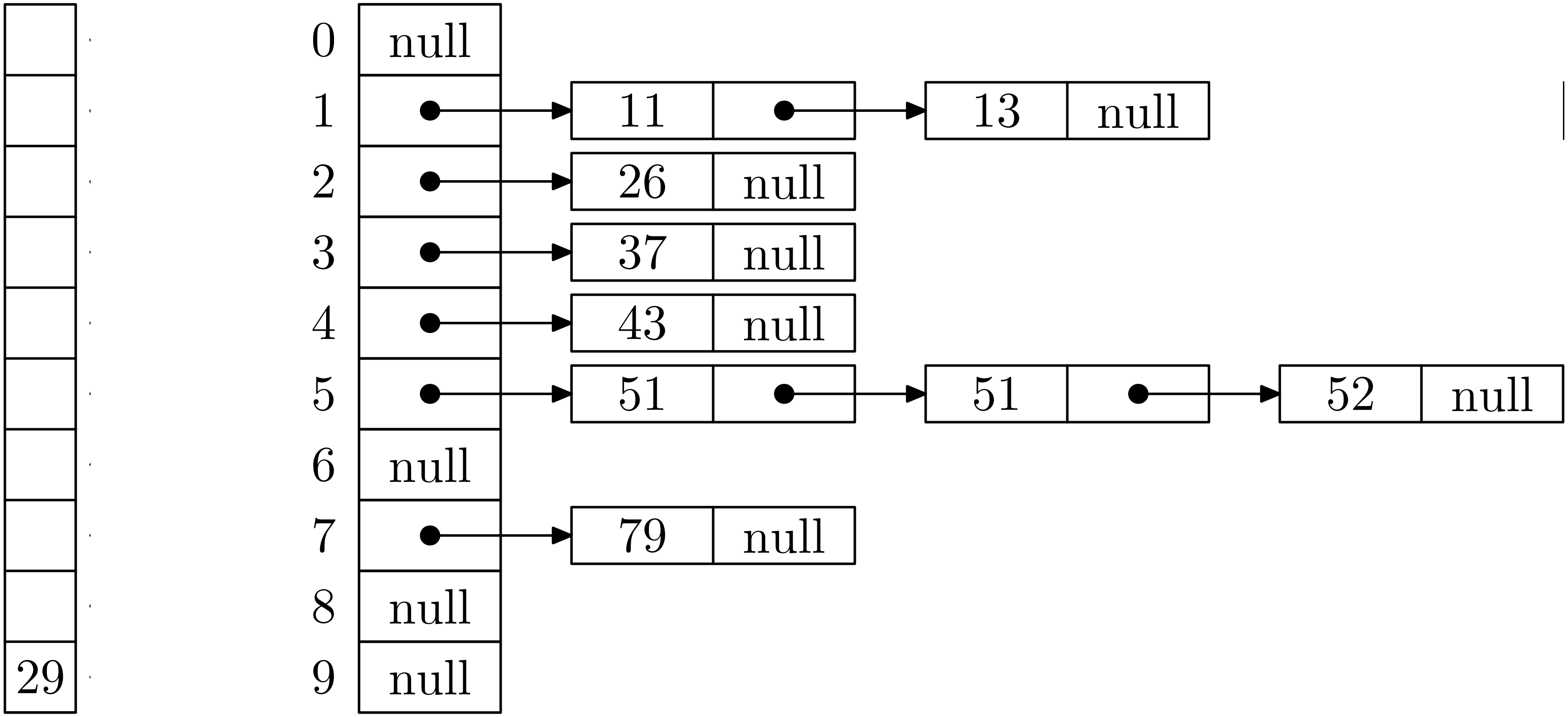

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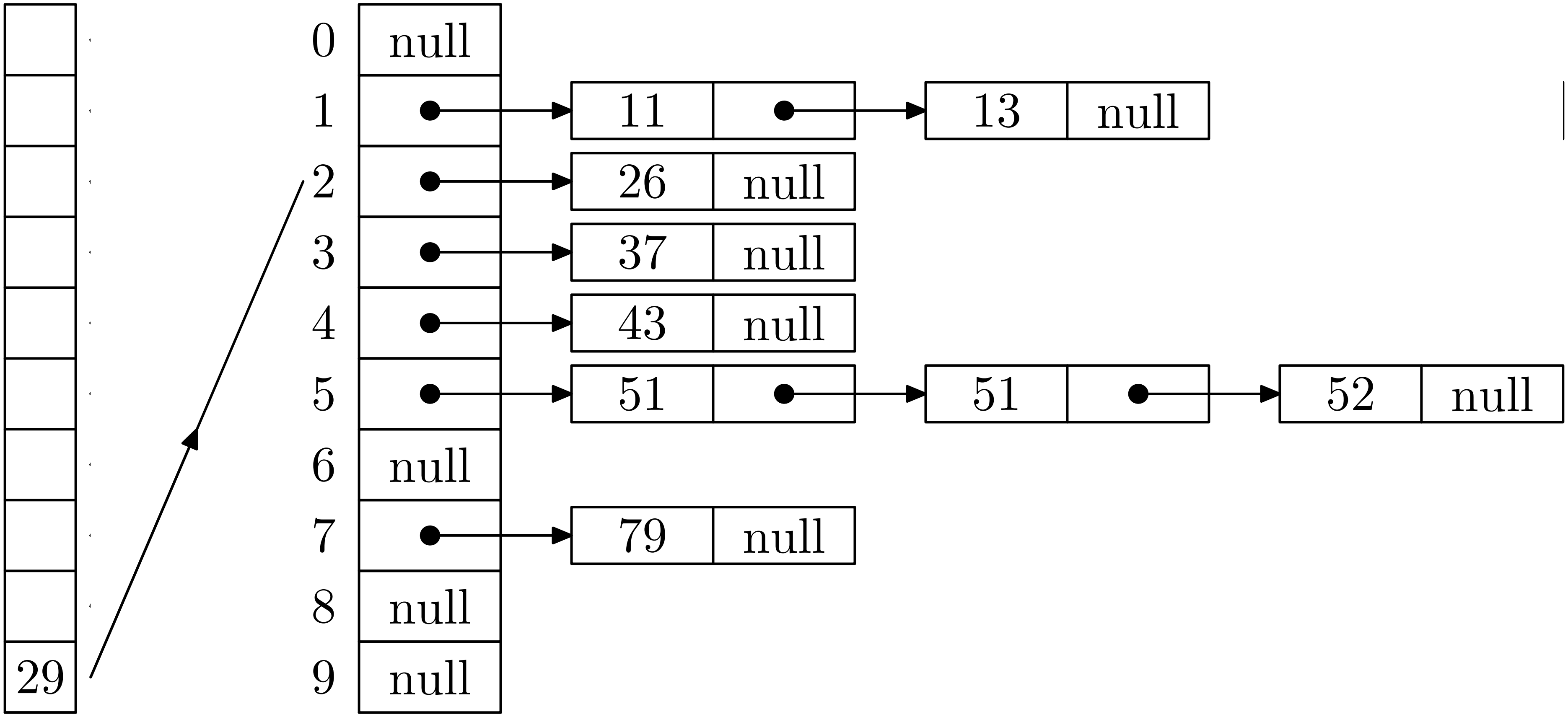

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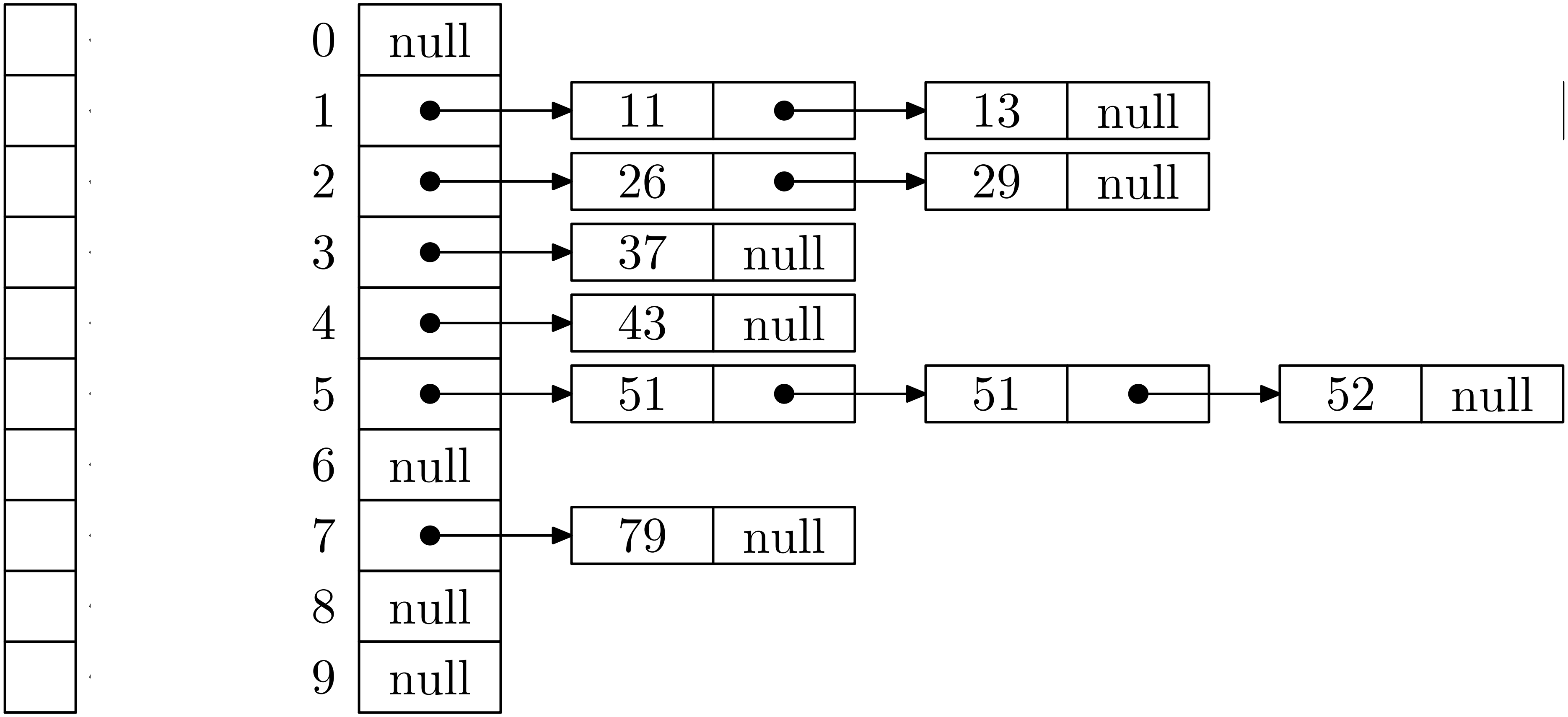

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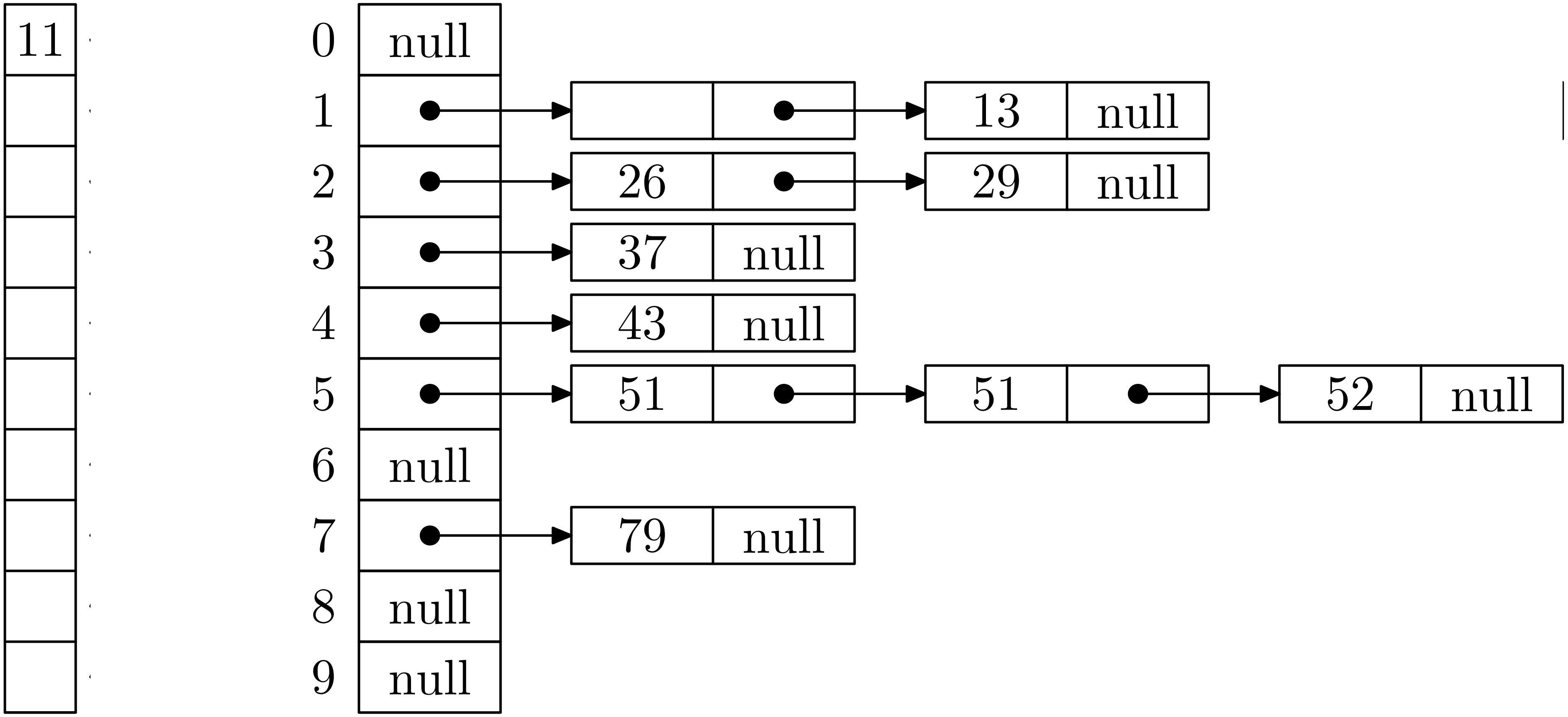

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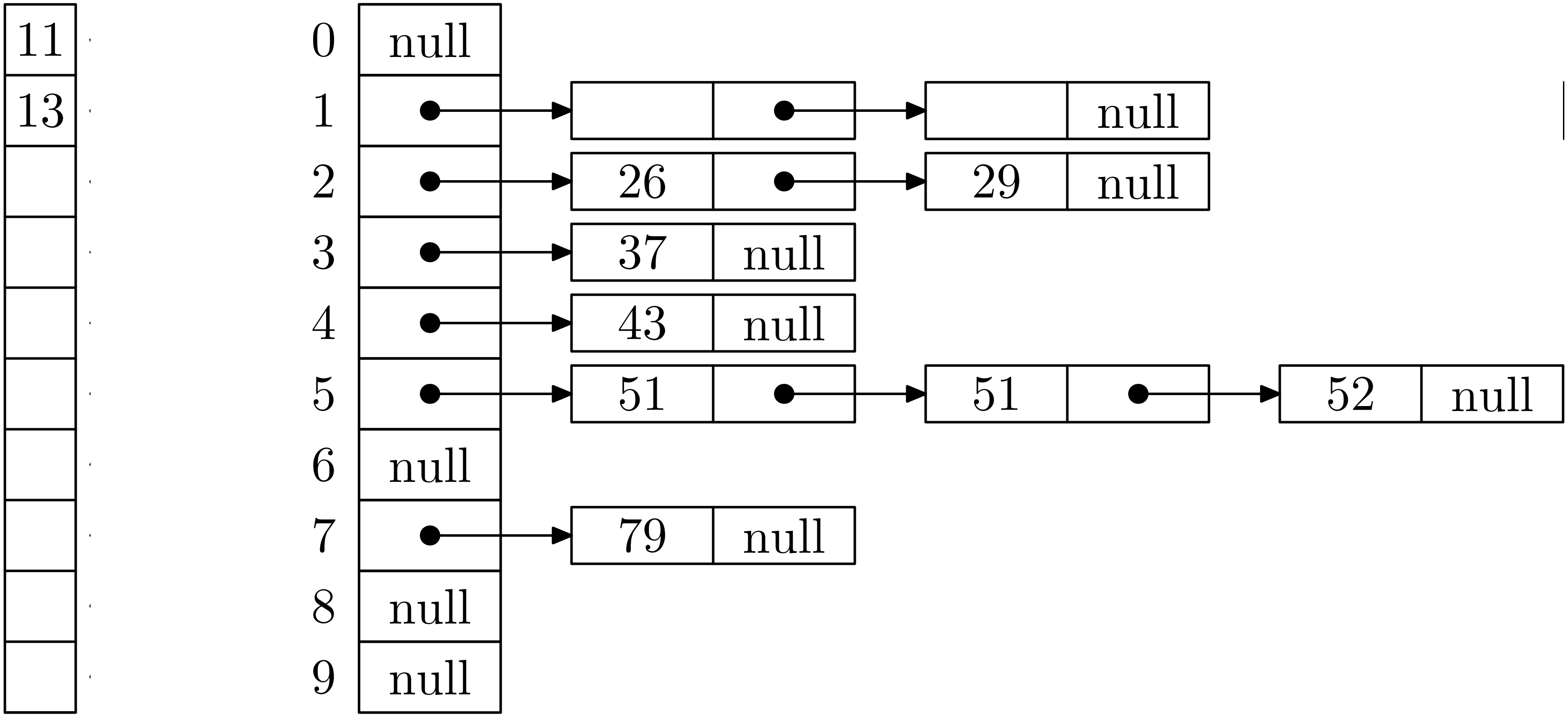

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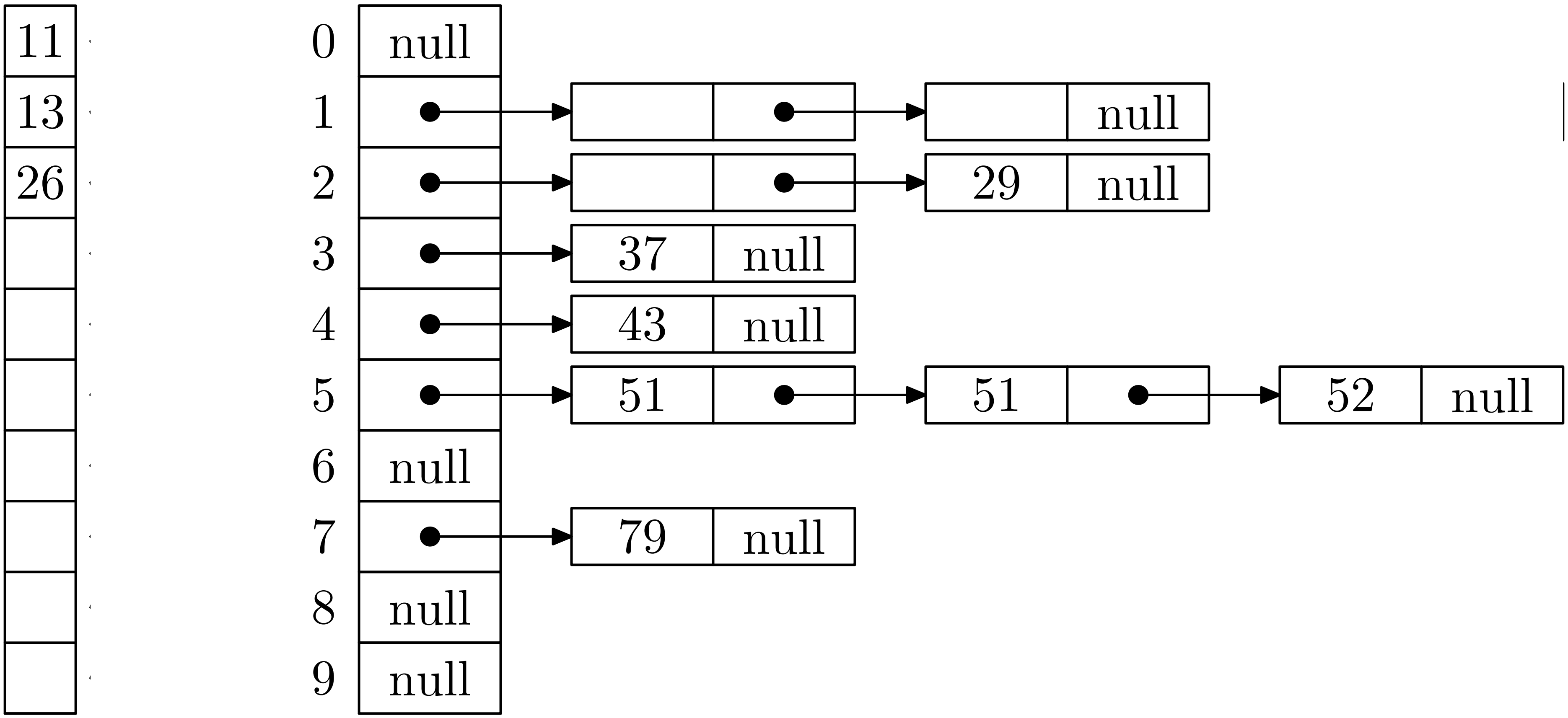

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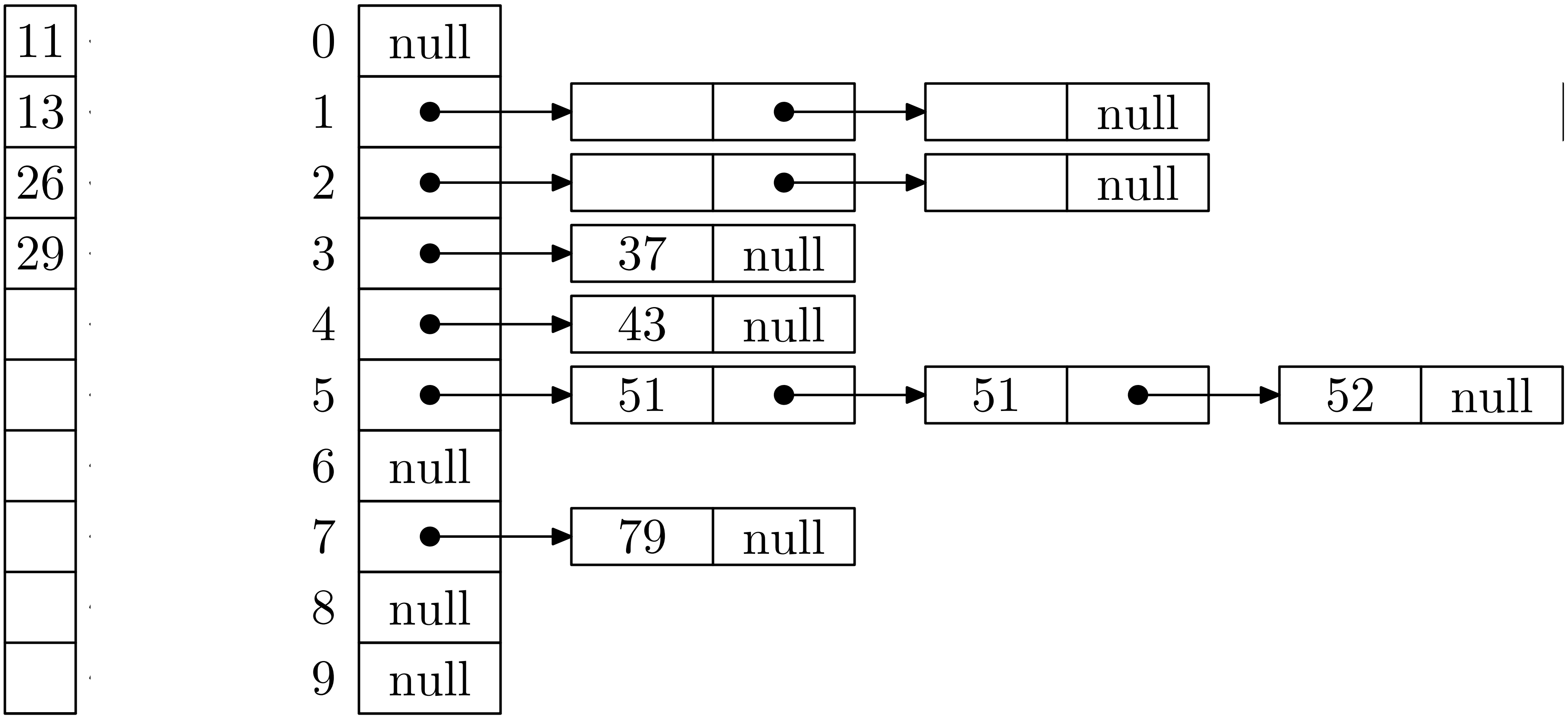

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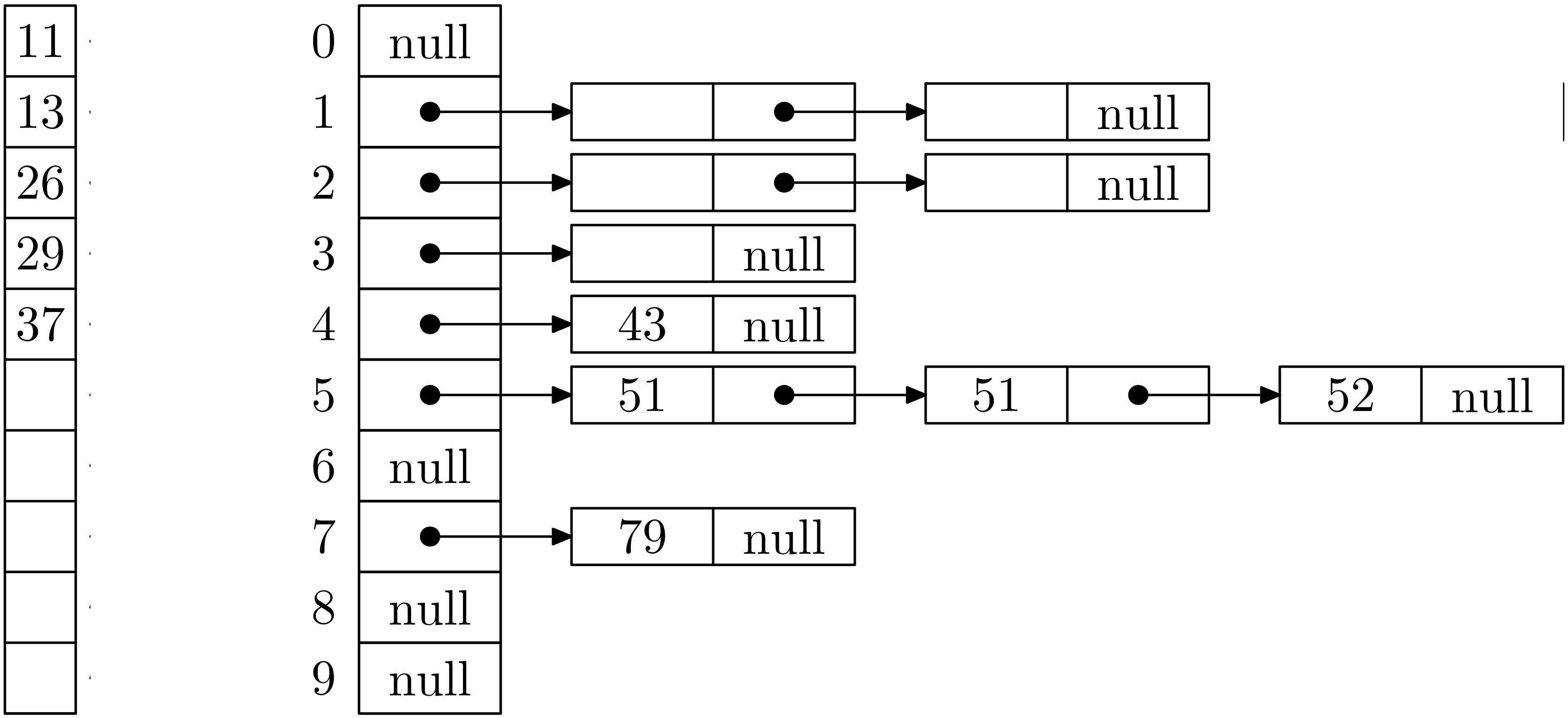

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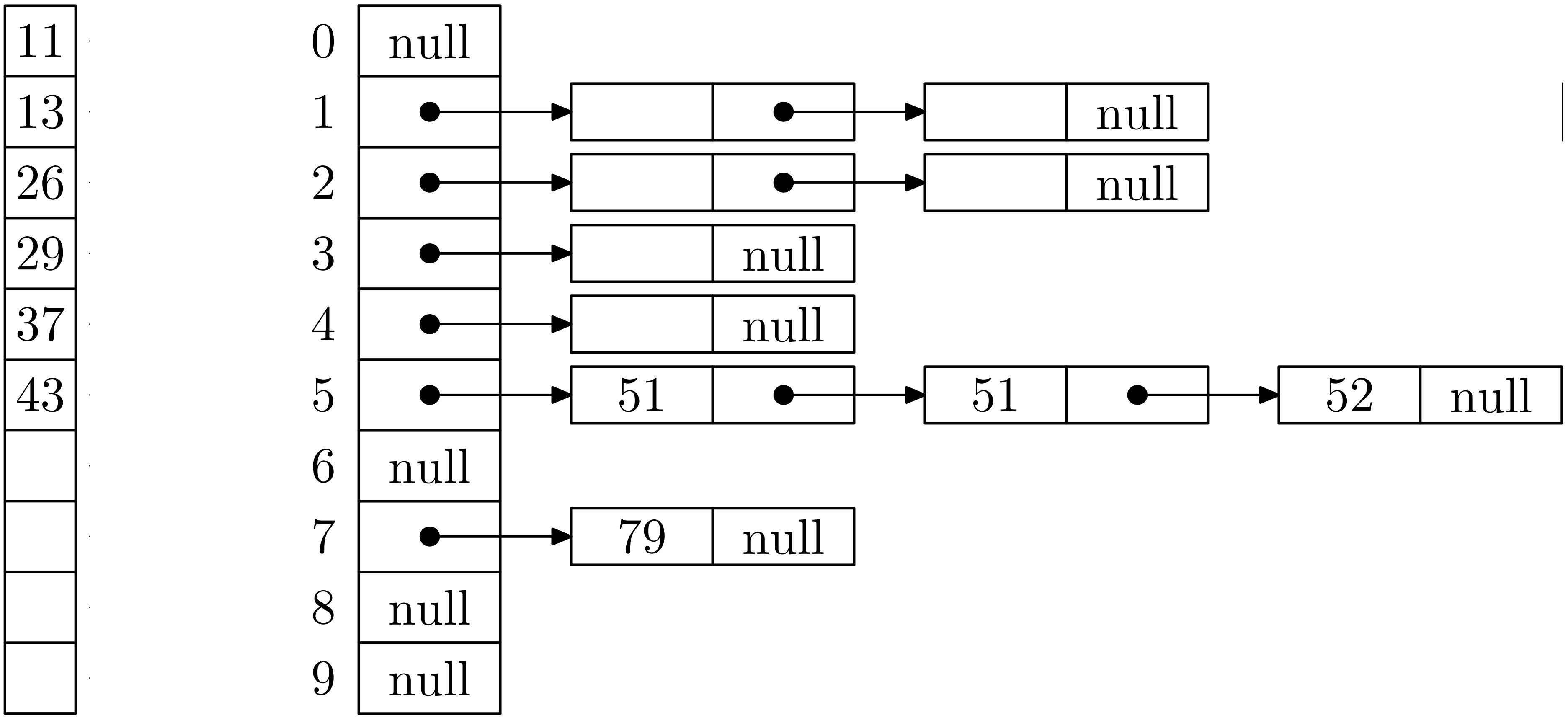

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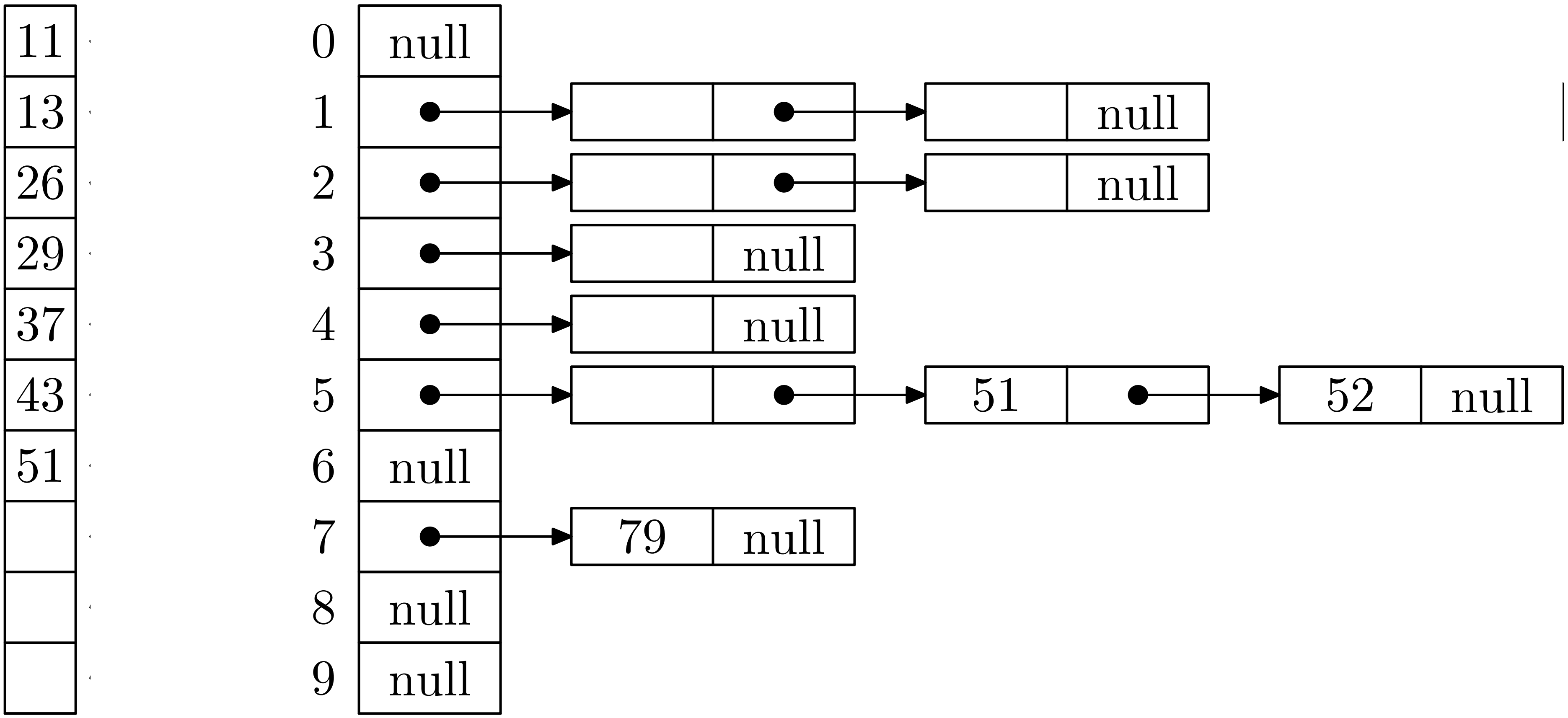

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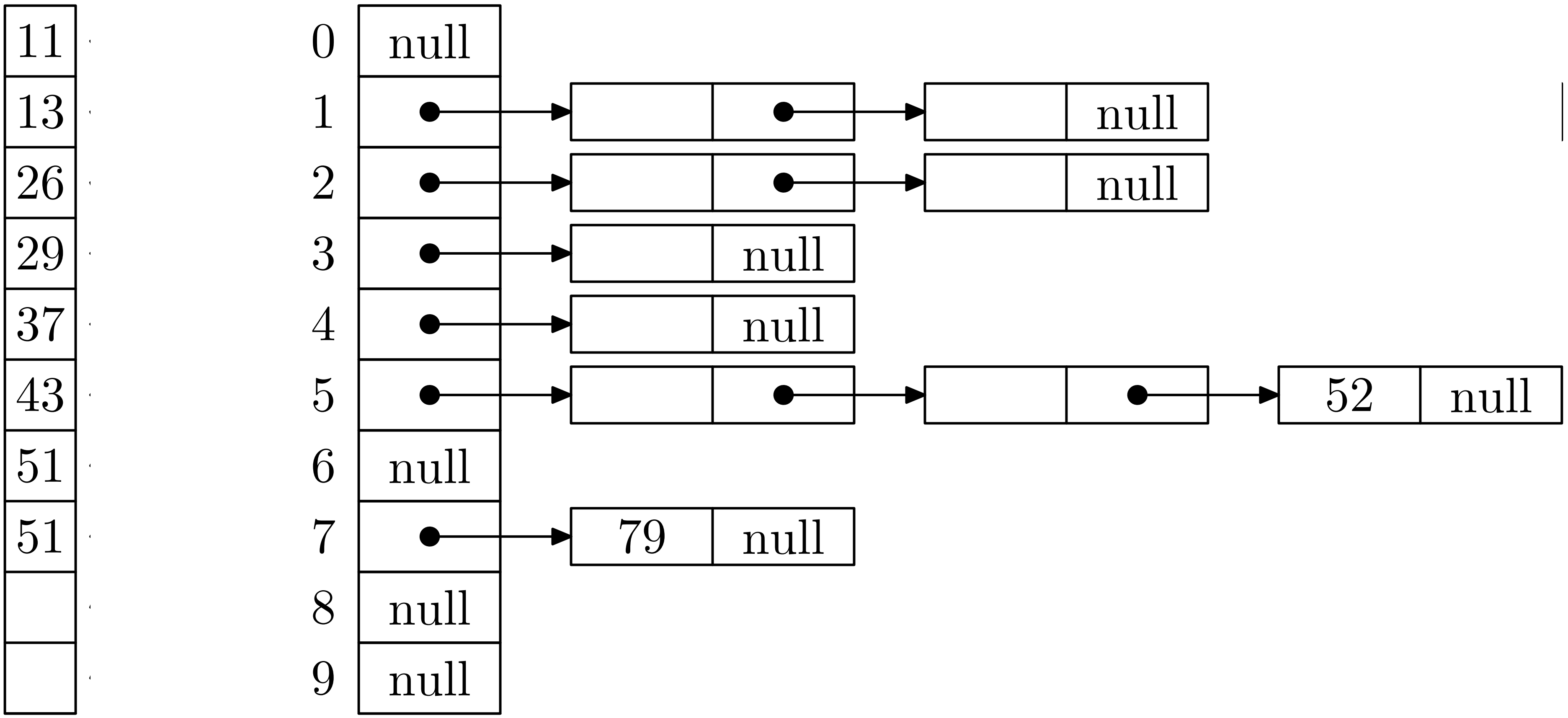

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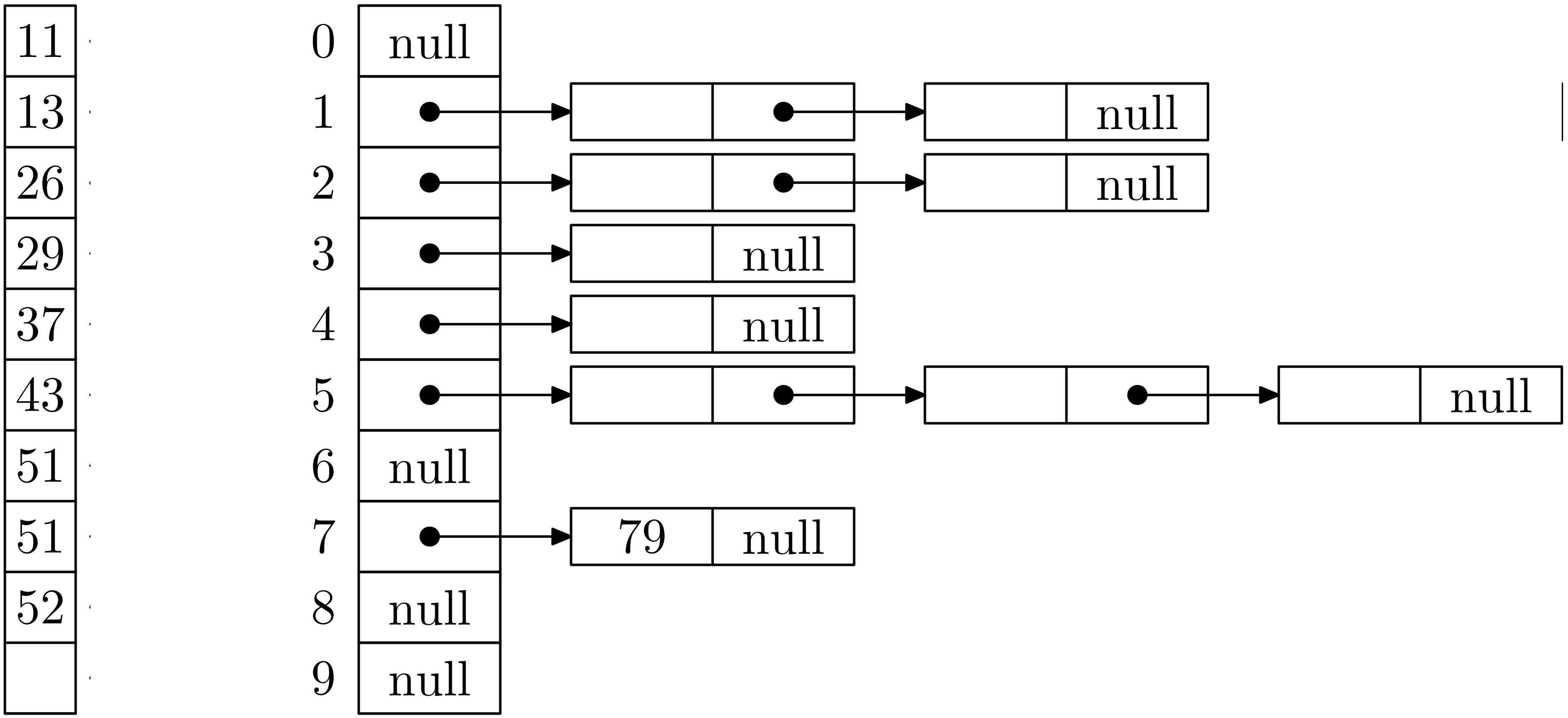

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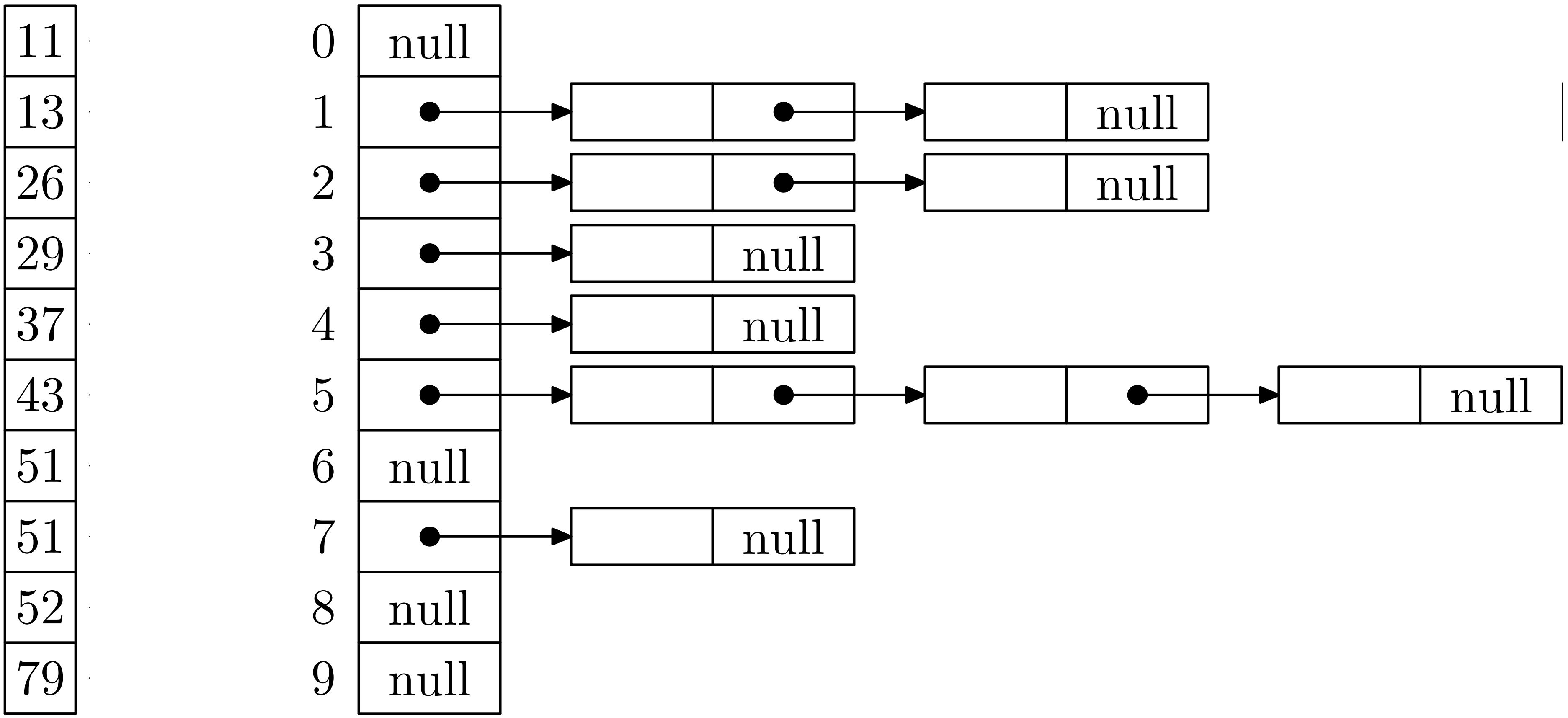

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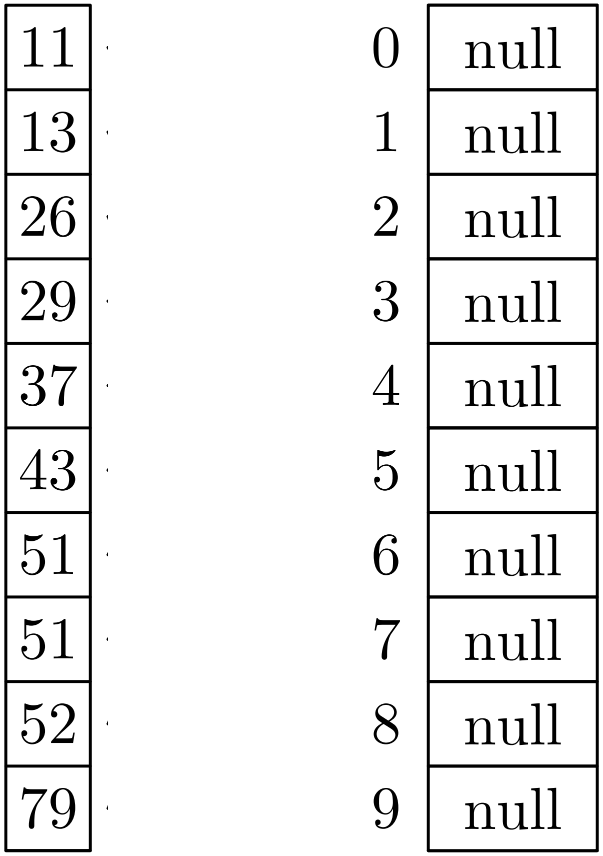

11
13
26
29
37
43
51
51
52
79

0 null

1 null

2 null

3 null

4 null

5 null

6 null

7 null

8 null

9 null

Time Complexity of Radix Sort

- We need not use base 10 we could use base r (the radix)
- If the maximum number to be sorted is N then the number of iterations of radix sort is $\log_r(N)$
- Each sort involves n operations
- Thus the total number of operations is $O\left(n\lceil\log_r(N)\rceil\right)$
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- We then sort the buckets on less significant figures

- Quicksort is a bucket sort with two buckets, but where we choose a pivot to determine which bucket to use

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- In practice, radix sort or bucket sort are rarely used
- The overhead of maintaining the buckets make them less efficient than they might appear
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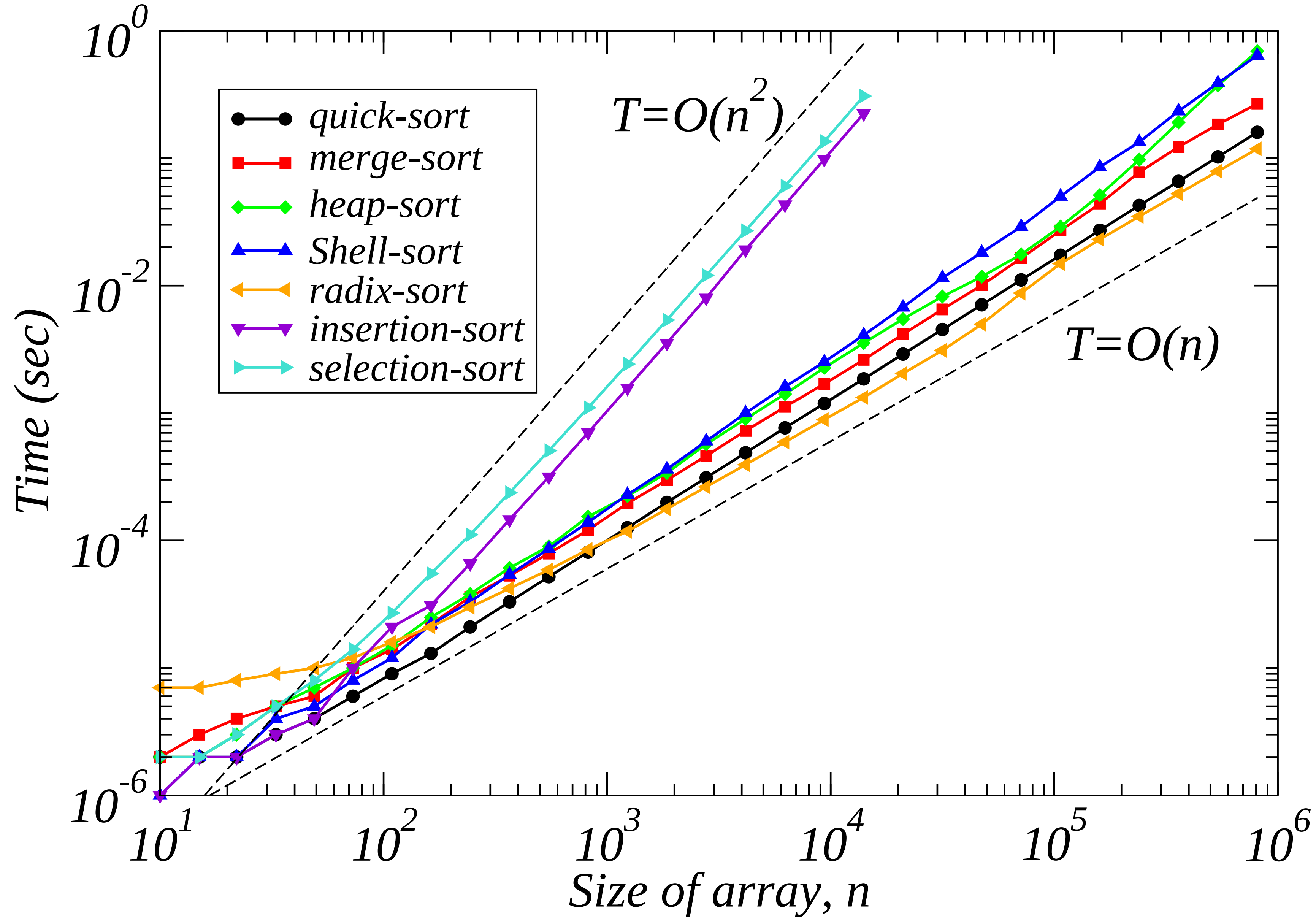
Comparison of Sort Algorithms



Lessons

- Sort is important—it is one of the commonest high level operations
- Merge sort and quick sort are the most commonly used sort
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