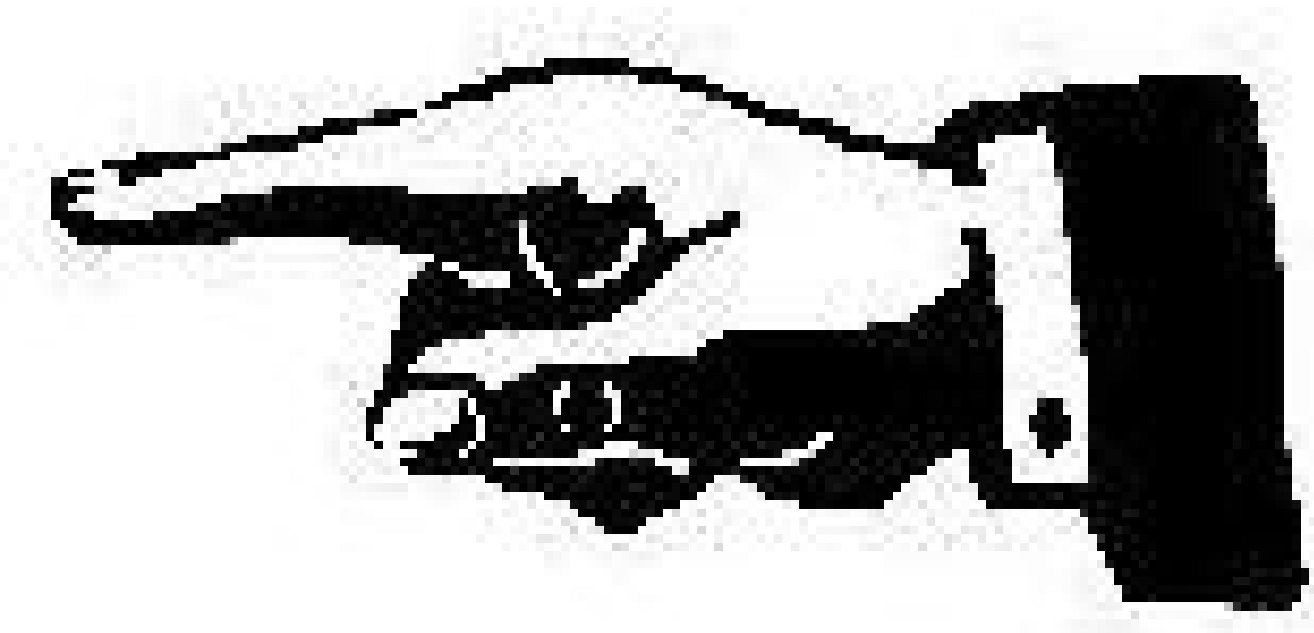
Further Mathematics and Algorithms

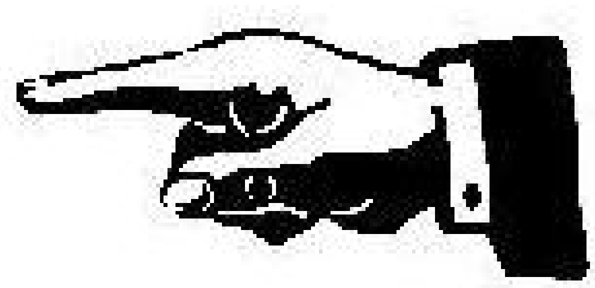
Lesson 8: *Point to where you are going: links*



Linked lists

Outline

1. **References**
2. Singly Linked List
3. Stacks and Queues
4. Doubly Linked List
5. Using Linked Lists
6. Skip Lists



Non-Contiguous Data

- So far we have considered arrays where the data is stored in a contiguous chunk of memory
- This has the great advantage of allowing random access
- It has the disadvantage that it is expensive to add or remove data from the middle of the list or to rearrange the data
- A different approach is to use units of data that point to other units

Non-Contiguous Data Structures

- There are a lot of important data structures using non-contiguous memory
  - ⋆ Binary trees
  - ⋆ Graphs
- In this lecture we consider **linked-lists**
- This is a classic data structure, which is almost entirely useless
- However, it serves as a good introduction to much more useful data structures

Self-Referential Classes

- The building block for a linked list is a node class

```
struct Node<T>
{
  Node(U value, Node<U> *node): value(value), next(node) {}
  T element;
  Node<T> *next;
}
```

- We create new nodes
  `Node<int> *node = new Node<int>(10, pt_to_next)`

- Note that `node` is the address of this node

- I make it a `struct` as this is a class where I want public access to the element and next

- I can make this class a private class of my linked list

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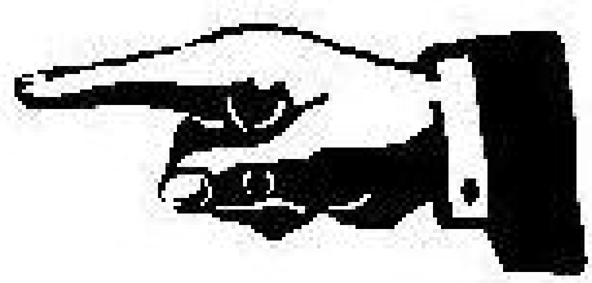
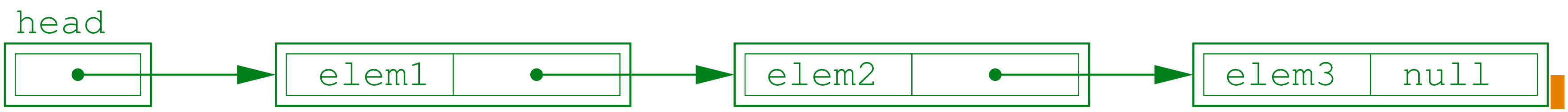


Singly Linked List

- We can build a linked list by stringing nodes together



We don't show the "pointer" to `element`

- A singly linked list has a single "pointer" to the next element
- A doubly linked list has "pointers" to the next and previous element—we will see this later
- We should be able to create a linked list, add elements, remove elements, see if an element exists, etc.

Implementation

- We consider a lightweight implementation
- The class will have a head, a size counter and have a `Node` as a nested class

```
class MyList {
private:
  template <typename U>
  struct Node{
    Node(U value, Node<U> *node): value(value), next(node) {}
    U value;
    Node<U> *next;
  };
  Node<T> *head;
  unsigned noElements;
```

Simple Methods

- The constructor is simple (and not strictly necessary)

```
MyList(): n(0), head(0) {}
```

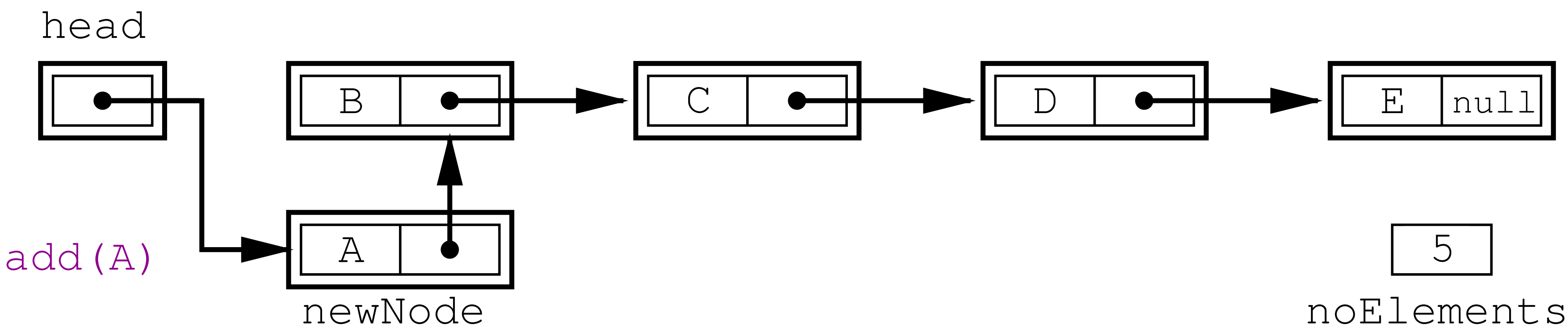
- Other simple methods are

```
unsigned size() const {return noElements;}

bool empty() const {
  return head == 0;
}
```

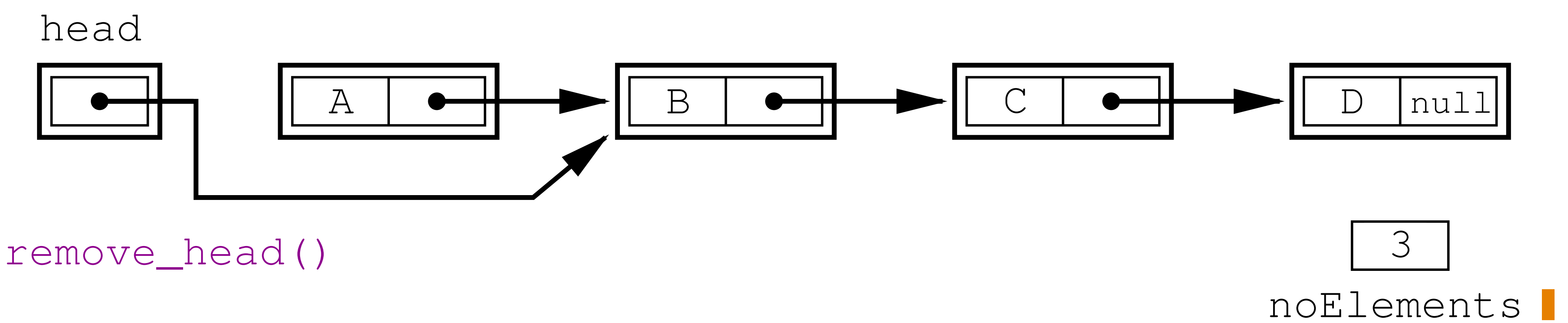
Adding elements

```
void add(T element)
{
    Node<T> *newNode = new Node<E>();
    newNode.element = element;
    newNode.next = head;
    head = newNode;
    noElements++;
    return true;
}
```



Remove Head of List

```
void remove_head()
{
    Node<T>* dead = head;
    head = head->next
    noElements--;
    delete dead;
}
```



Other Methods

- We can easily implement many other methods
  - `get(int i)`—return i^{th} item in list
  - `remove(T obj)`-remove `obj` from list
  - `insert(int position, element)`
- Note that `get(int i)` requires moving down the list so is $O(n)$ (i.e. not random access)

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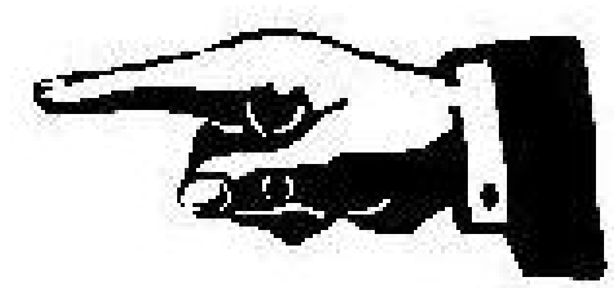


Stack

- It is easy to implement a stack using a linked list

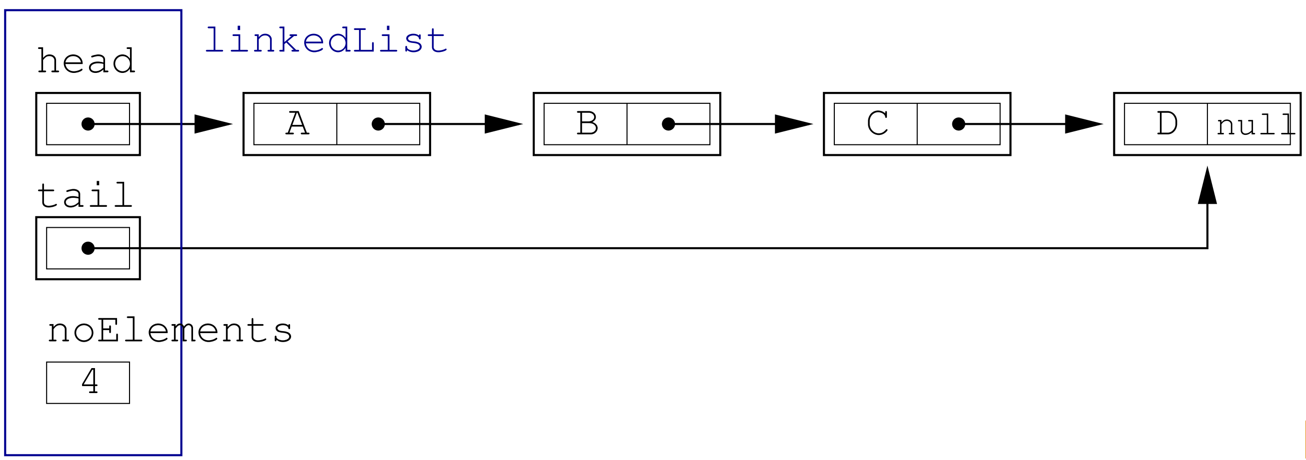
```
template <typename T>
class Stack<E>
{
  private Mylist<T> list = new mylist<T>();

  boolean push(E obj) {list.add(obj);}

  E top() {return list.get_head();} // throw exception

  E pop() {
    T tmp = list.get_head();
    list.remove_head();
    return tmp;
  }

  boolean empty() {return list.empty();}
}
```

Complexity of Stack

- All operations of the stack is constant time, i.e. $O(1)$
- This is the same time complexity as an array implementation
- Memory requirement is approximately $2 \times n$ reference and n objects—same as worst case for an array
- However, hidden cost of creating and destroying `Node` objects
- The array implementation is therefore slightly faster

Point to the Back

- To find the end of the queue takes n jumps
- Thus our linked list isn't the right data structure to implement a queue
- However, we could include a pointer to the end of the queue



Implementing a Queue

- We can then add elements to the tail in constant time
- We can the implement a queue in $O(1)$ time by
  - ⋆ `enqueue`ing at the back
  - ⋆ `dequeue`ing at the head
- I leave the implementation details as an exercise for you
- Note that although adding an element to the tail is constant time, removing an element from the tail is $O(n)$ as we have to find the new tail

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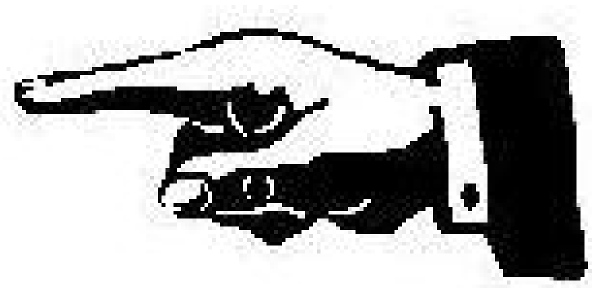
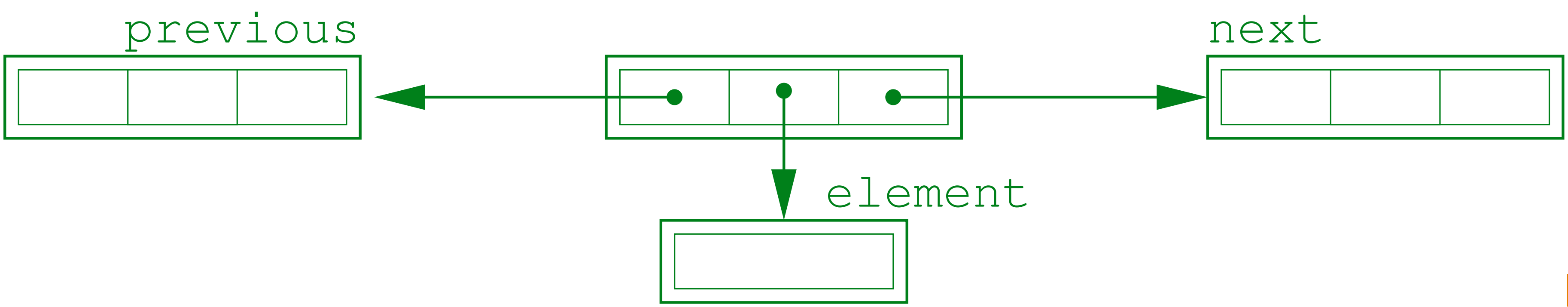


Doubly linked list

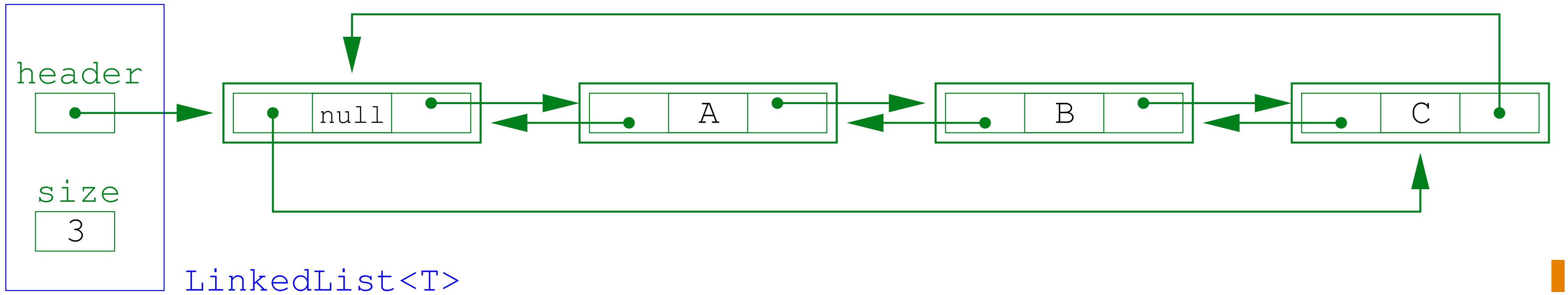
- In a more powerful linked list we would like to navigate the list in either direction
- To achieve this it uses a doubly-linked lists with elements to next and previous

```
class Node<T>
{
  T element;
  Node<T> *next;
  Node<T> *previous;
}
```



Dummy Node

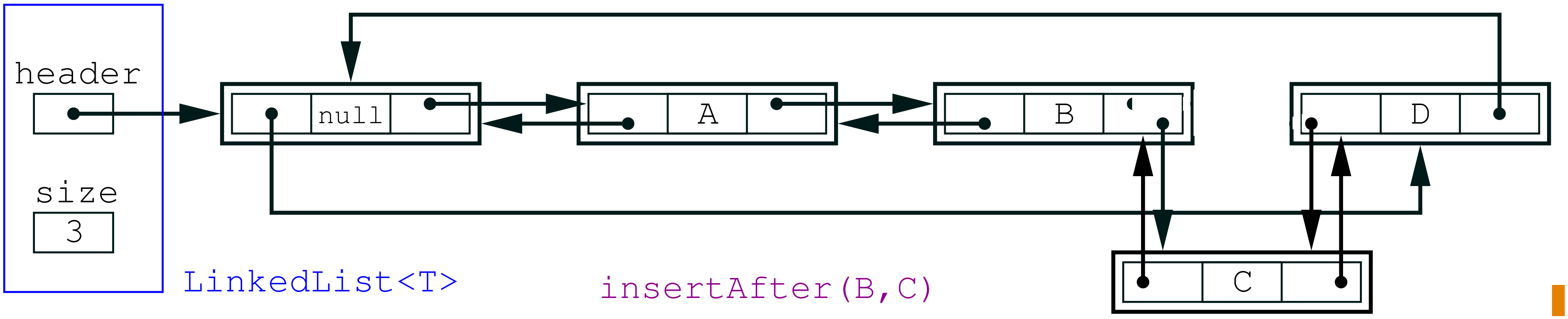
- List includes a dummy node—this make the implementations slicker



- Symmetric data structure so processing head and tail is equally efficient

Time Complexity

- `add` and `remove` from head and tail $O(1)$
- `find` $O(n)$ and slow
- `insert` and `delete` $O(1)$ (faster than an array list) once position is found



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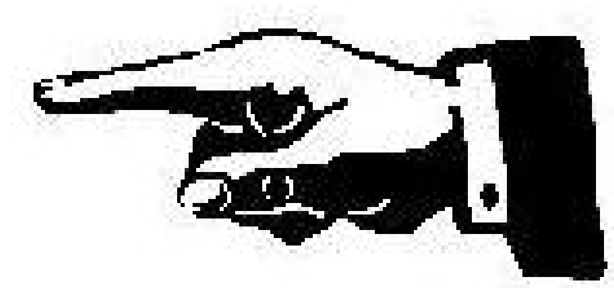

When To Use Linked Lists

- It is difficult to think of applications where linked lists are the best data structure
- lists—variable length arrays are usually better
- queues—linked list OK, but circular arrays are probably better
- sorted lists—binary trees much better
- linked lists have efficient insertion and deletion but it is difficult to think of an application where this matters

Line Editor

- One application where efficient insertion and deletion matters is a line editor
- We are usually working at a particular location in the text
- We often want to add or delete whole lines
- Storing the lines as strings in a linked list would allow a fairly efficient implementation

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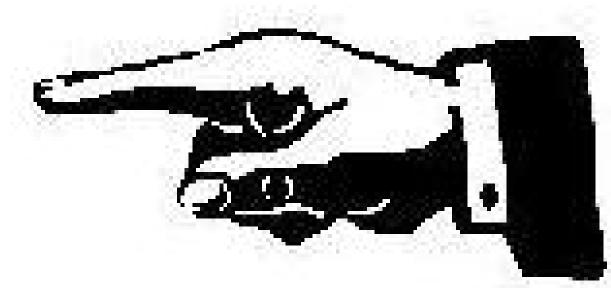
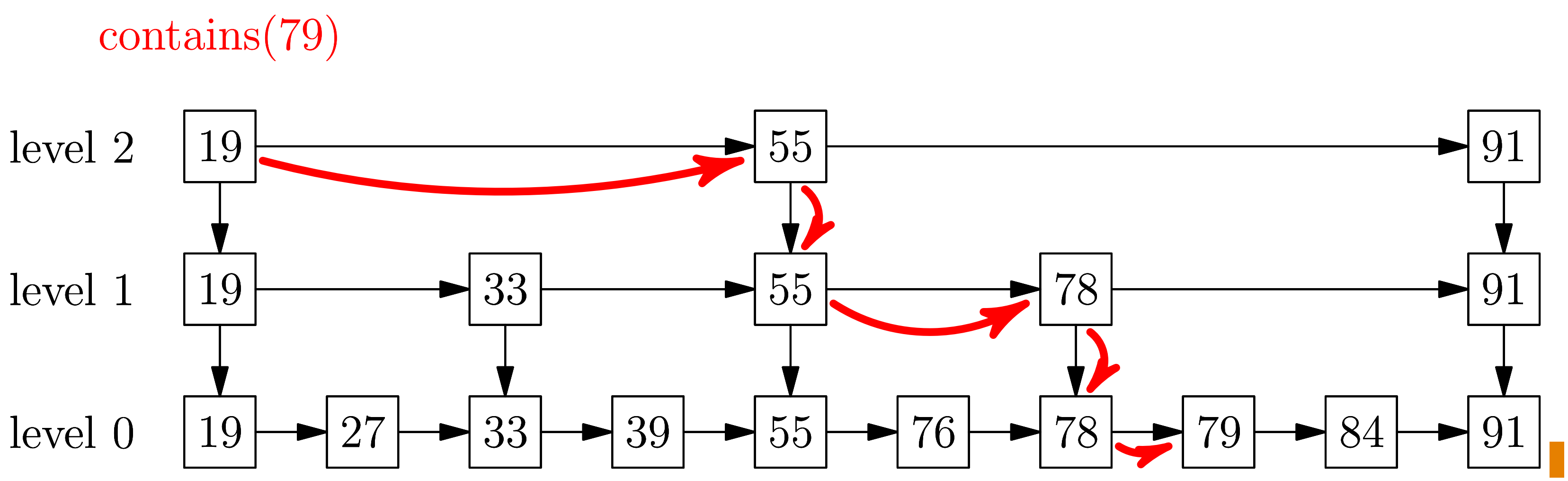


Skip Lists

- Linked lists have the disadvantage that to get to anywhere in the list takes on average $\Theta(n)$ steps
- Even if you kept an ordered list you still need to traverse it
- Skip lists are hierarchies of linked lists which allow binary search



Efficiency of Skip Lists

- Skip lists provide $\Theta(\log_2(n))$ search as opposed to $\Theta(n)$
- They have the similar time complexity to binary trees, although binary trees are slightly faster
- They have one advantage over binary trees—they allow efficient concurrent access
- The standard template library provides a doubly linked list, `list<T>`, as well as a slingly linked-list `slist<T>`

Lessons

- Node structures that point to other Node structures are used in many important data structures
- Linked lists are the simplest examples of this kind of structure and consequently has a dominant position in most DSA books
- In practice linked lists are seldom the data structure of choice—before choosing to use a linked list consider the alternatives
- There are some important uses for linked lists, e.g. skip lists and hash tables (see lecture on hashing)