# Algorithms and Analysis

## Lesson 9: *Make Friends with Trees*


*Binary trees, binary search trees, sets, tree iterators*

# Outline

1. **Trees**
2. Binary Trees
   - Implementing Binary Trees
3. Binary Search Trees
   - Definition
   - Implementing a Set
4. Tree Iterators

# Trees

- Trees are one of the major ways of structuring data
- They are used in a vast number of data structures
  - Binary search trees
  - B-trees
  - splay trees
  - heaps
  - tries
  - suffix trees
- We shall cover most of these

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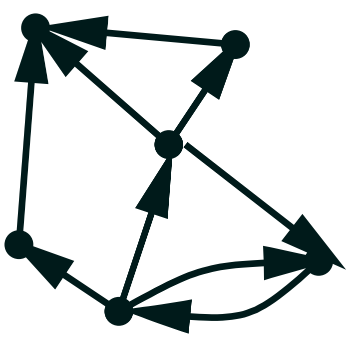
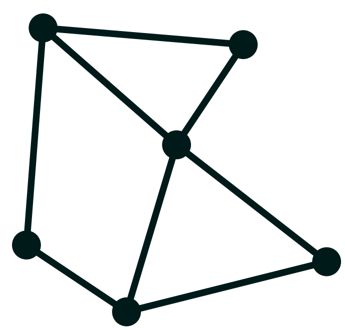
# Defining Trees

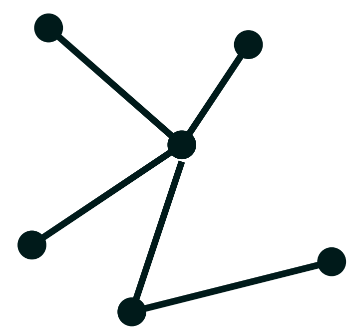
- Mathematically a tree is an **acyclic undirected graph**
  - ⋆ **graph**: a structure consisting of **nodes** or **vertices** joined by **edges**
  - ⋆ **undirected**: the edges goes both ways
  - ⋆ **acyclic**: there are no cycles in the graph



graph



undirected graph



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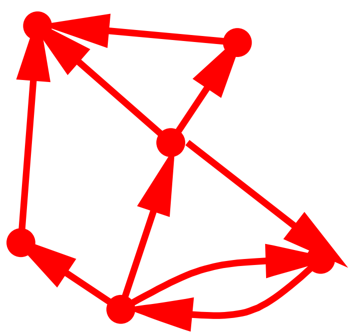

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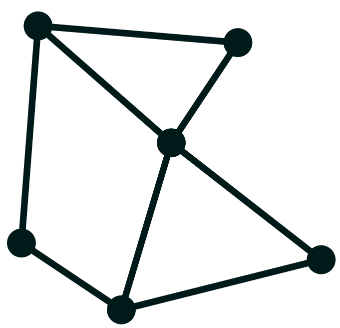

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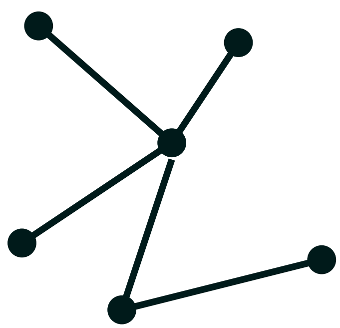

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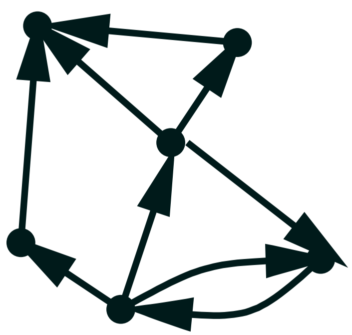

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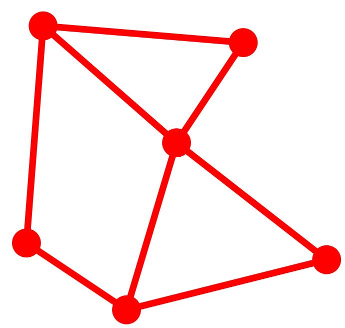

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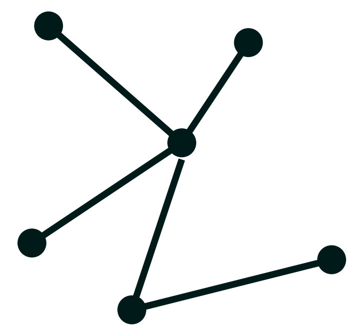

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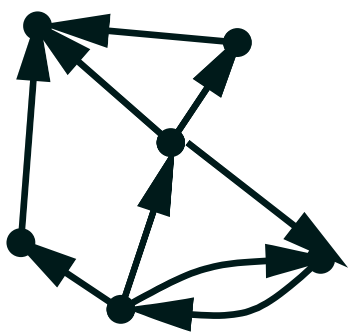

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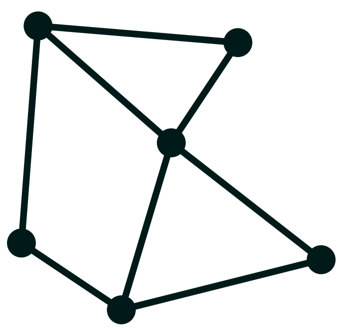

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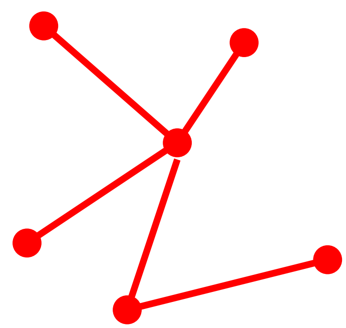

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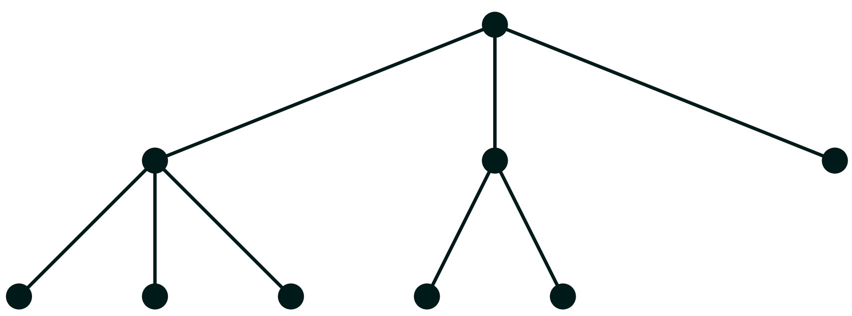
# Borrowing from Nature

- We often impose an ordering on the nodes (or a direction on the edges)—known as a rooted tree
- Borrowing from nature, we recognise one node as the **root** node
- Nodes have **children** nodes living beneath them
- Each child has a **parent** node above them except the root
- Nodes with no children are **leaf** nodes



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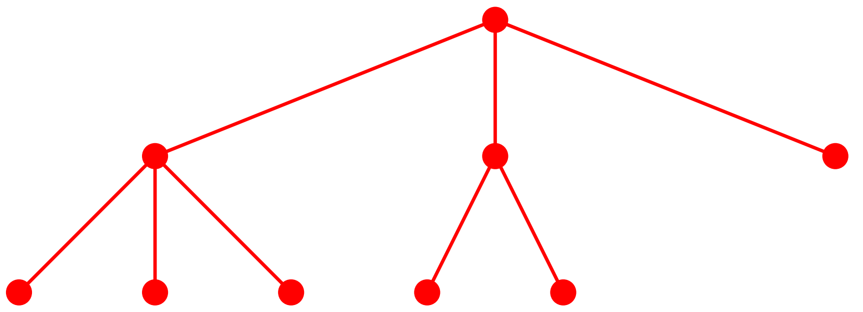

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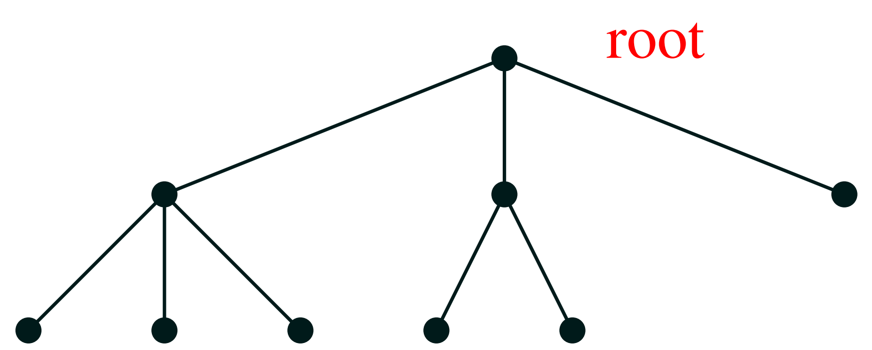

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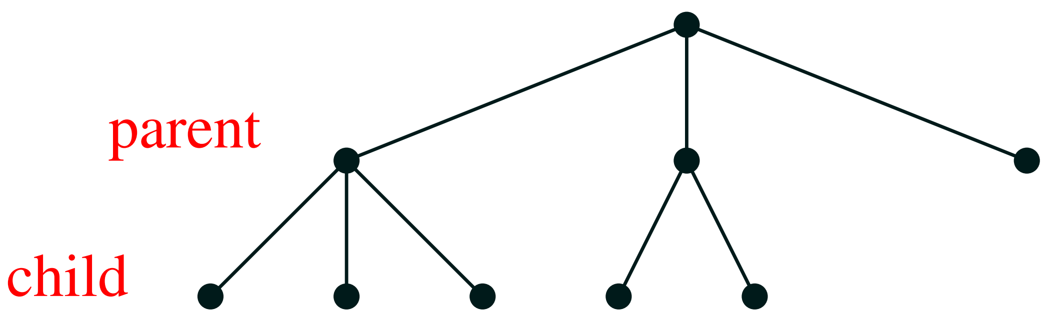

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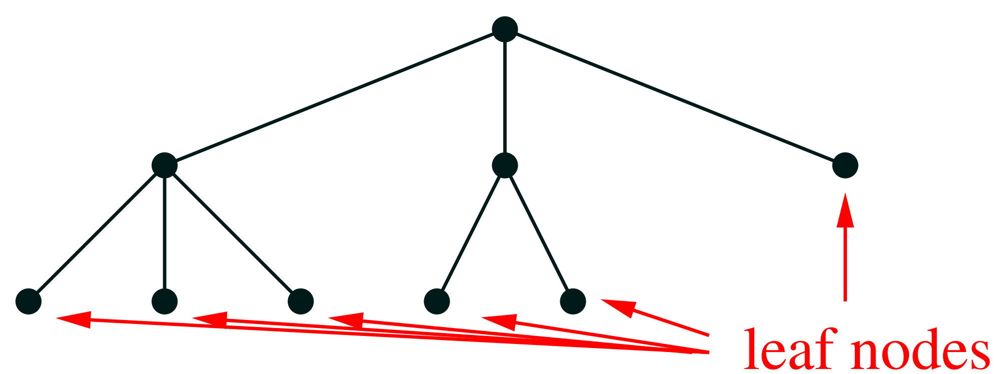

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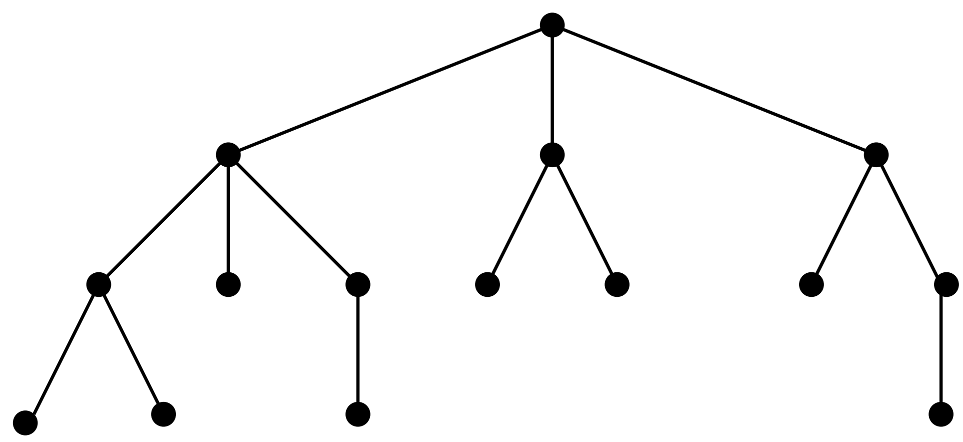

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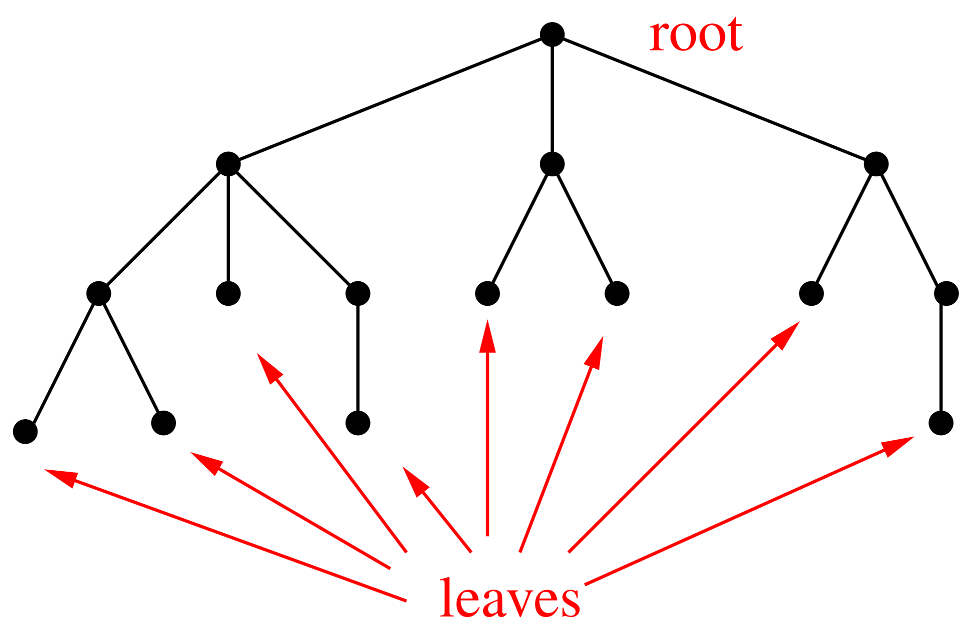
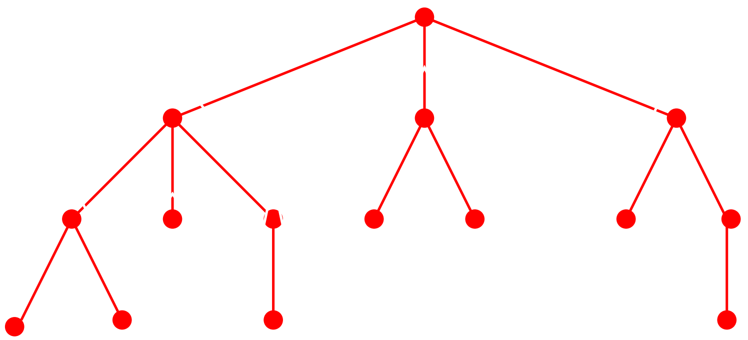
- Yep!, computer scientists draw there trees upside down
  - ⋆ root at the top
  - ⋆ leaves at the bottom



# Subtrees

- We can think of the tree made up of **subtrees**



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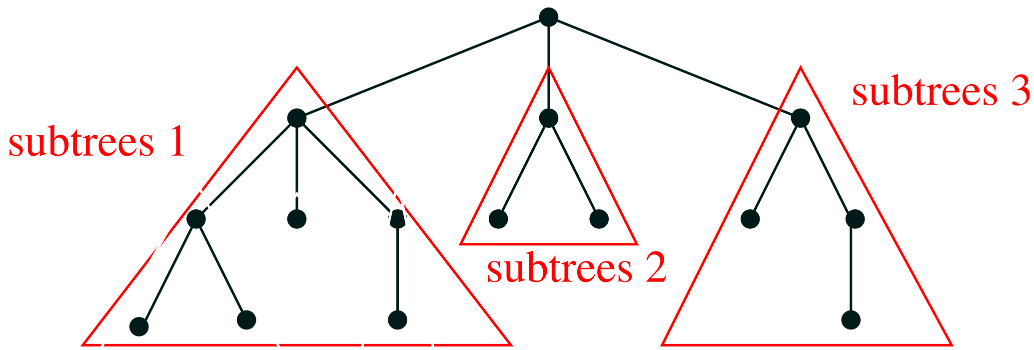

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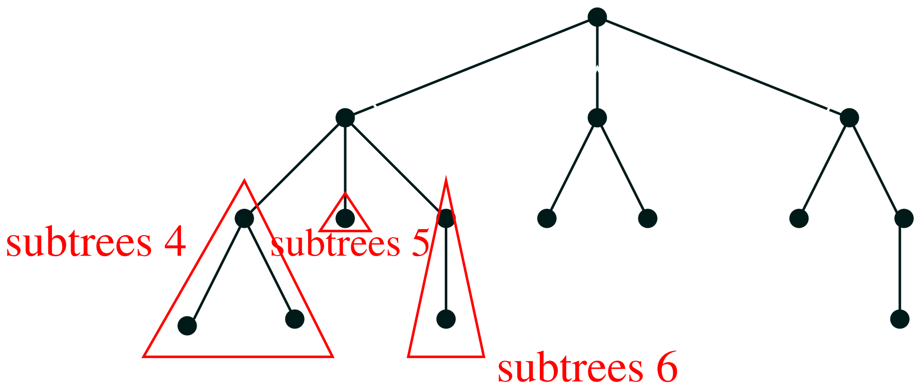
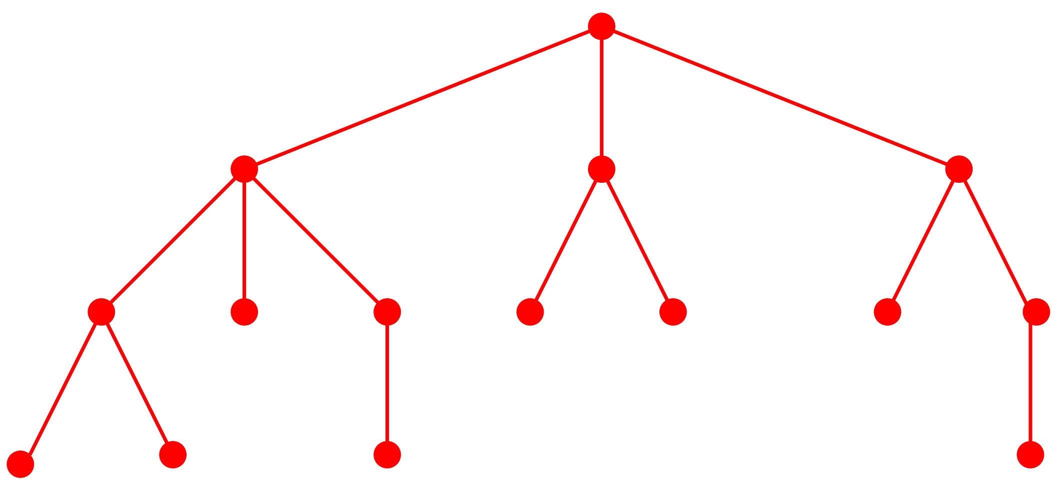


# Level of Nodes

- It is useful to label different levels of the tree
- We take the **level** of a node in a tree as its distance from the root
- We take the **height** of a tree to be the number of levels



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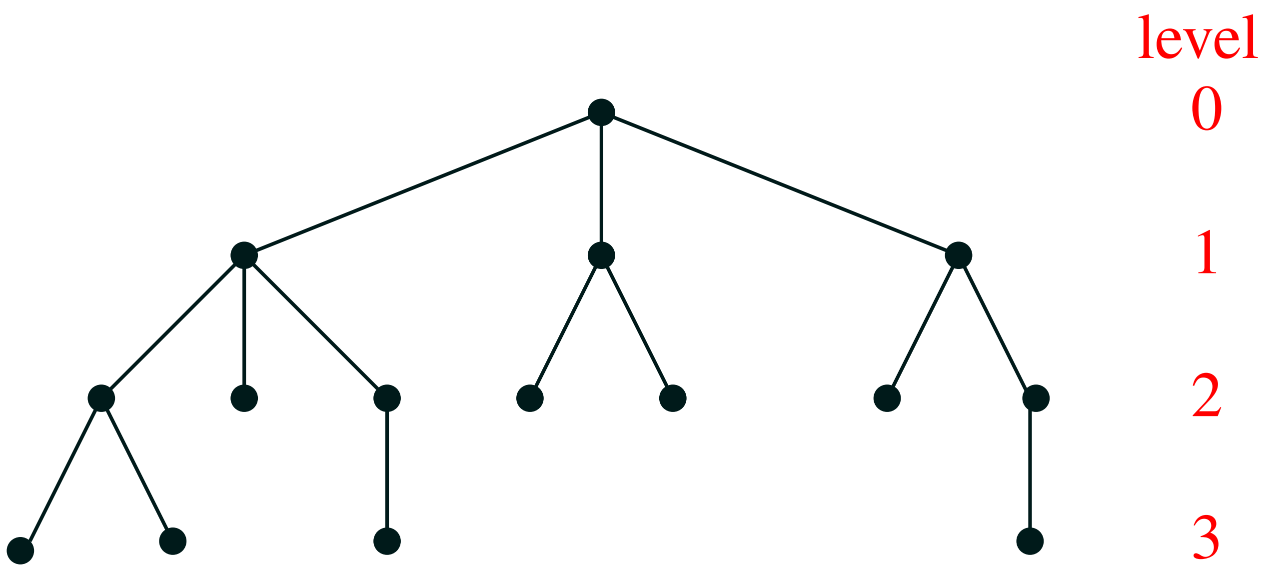

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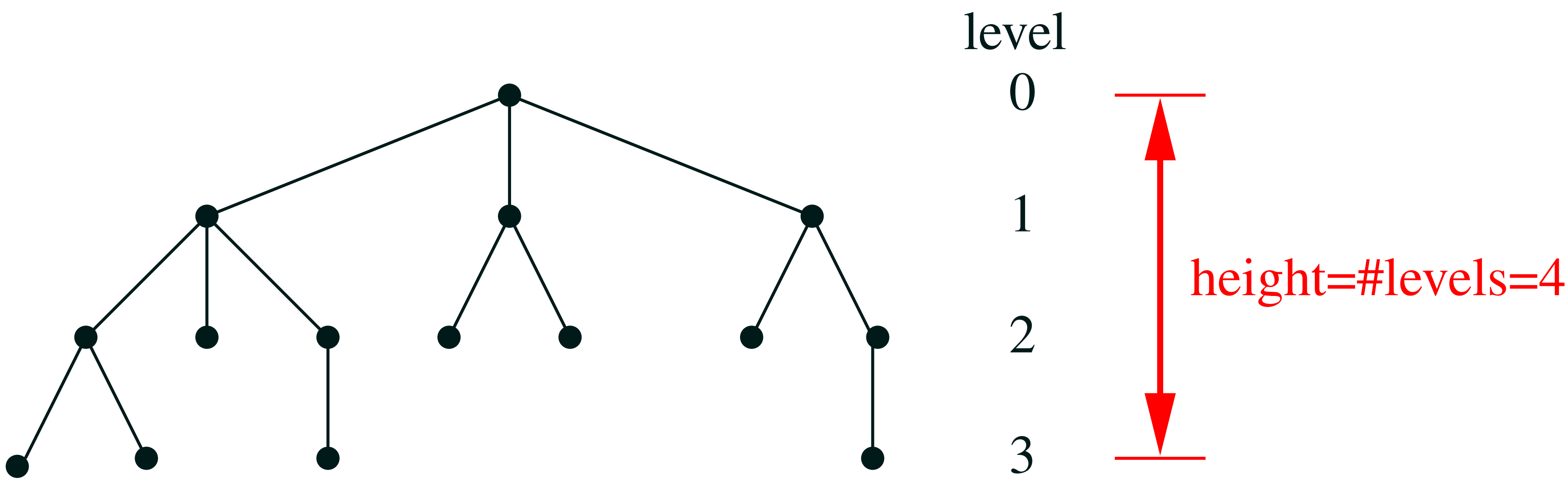


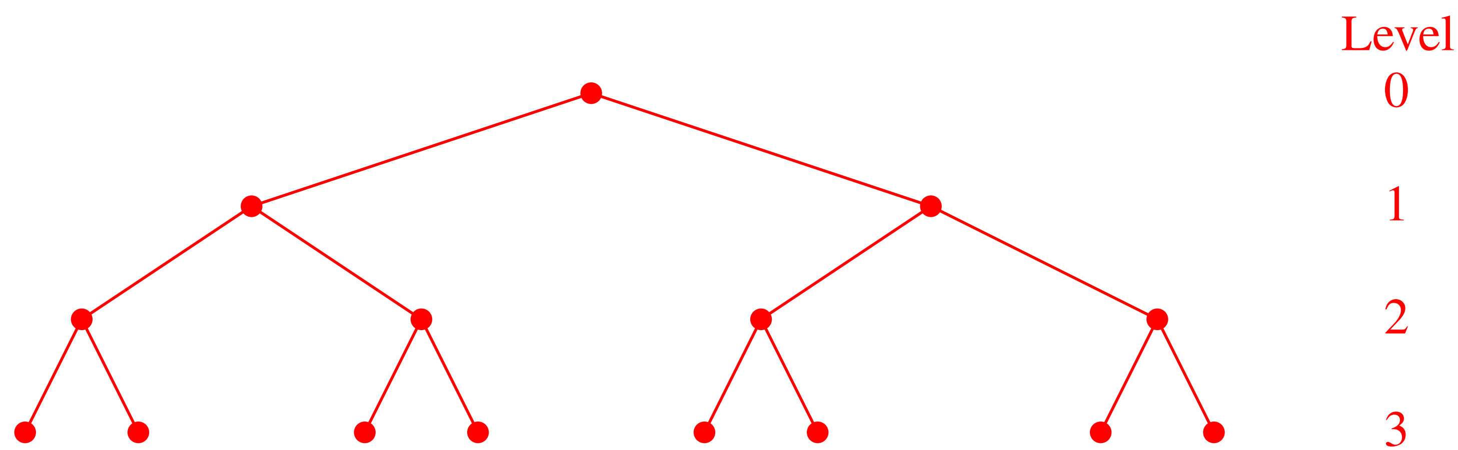
# Outline

1. Trees
2. **Binary Trees**
   - Implementing Binary Trees
3. Binary Search Trees
   - Definition
   - Implementing a Set
4. Tree Iterators

# Binary Trees

- A **binary tree** is a tree where each node can have zero, one or two children
- The total number of possible nodes at level $l$ is $2^l$
- The total number of possible nodes of a tree of height $h$ is

$$1 + 2 + \cdots + 2^{h-1} = 2^h - 1$$



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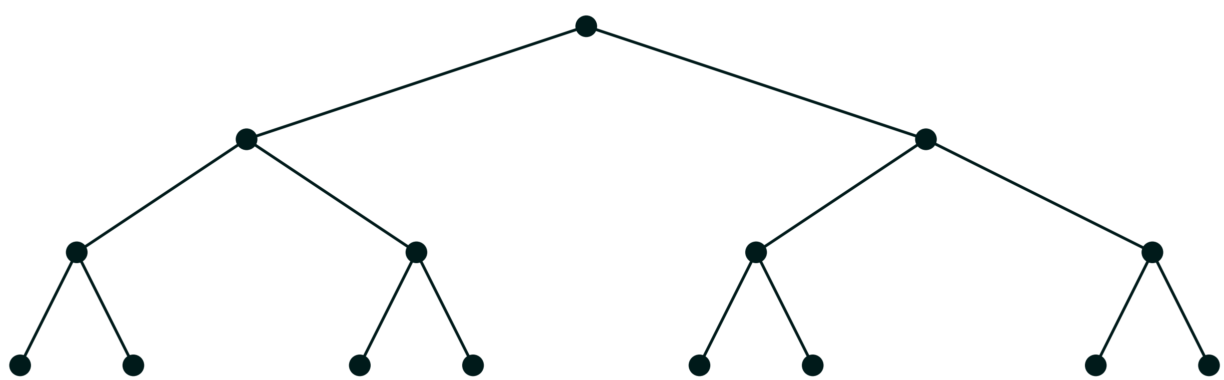

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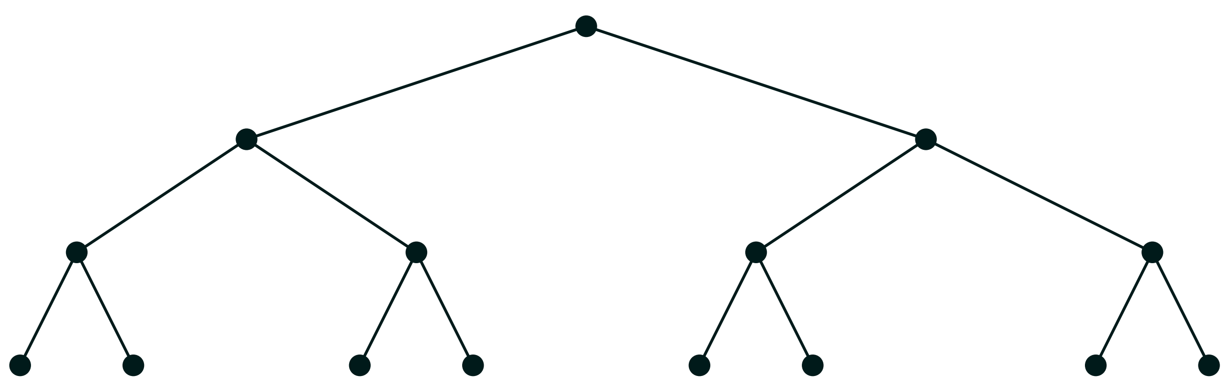

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# Uses of Binary Trees

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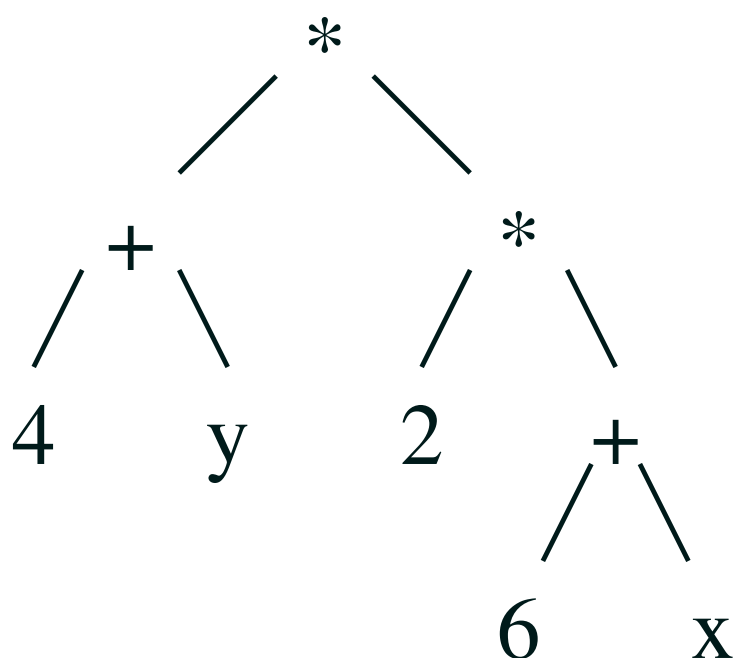
- For example, they are used as **expression trees** to represent formulae

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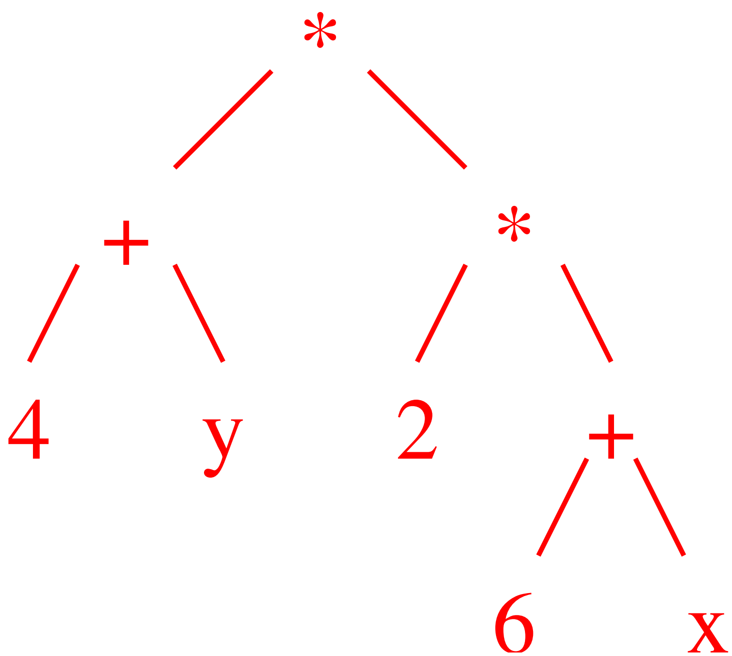


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# Implementation

- We wish to build a generic binary tree class with each node housing an element
- Again we use a `Node<T>` class as the building block for our data structure—in this case a node of the tree
- The `Node<T>` class will contain a pointer to left and right children
- To help navigate the tree each node will contain a pointer to its parent

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template <typename T>
class binary_tree {
private:
  class Node {
  public:
    T element;
    Node* parent;
    Node* left = nullptr;
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    Node(const T& value, Node* parent_node) {
      element = value;
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  unsigned no_elements = 0;
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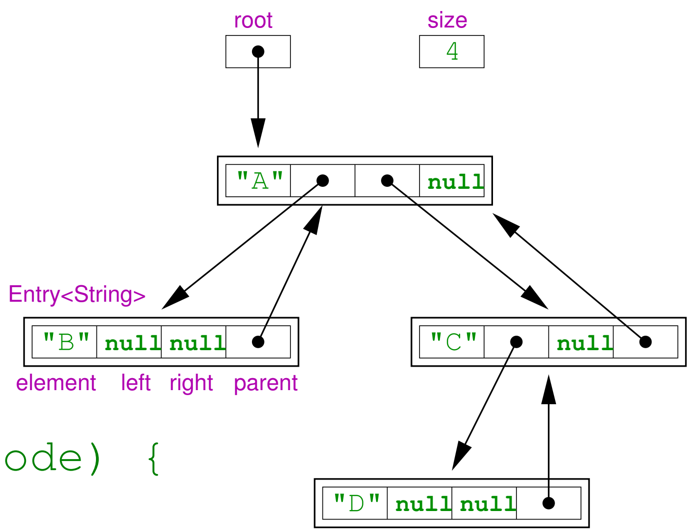

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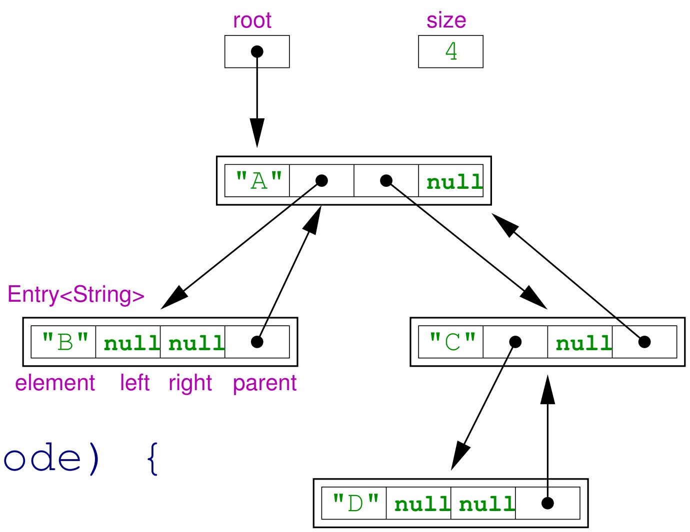

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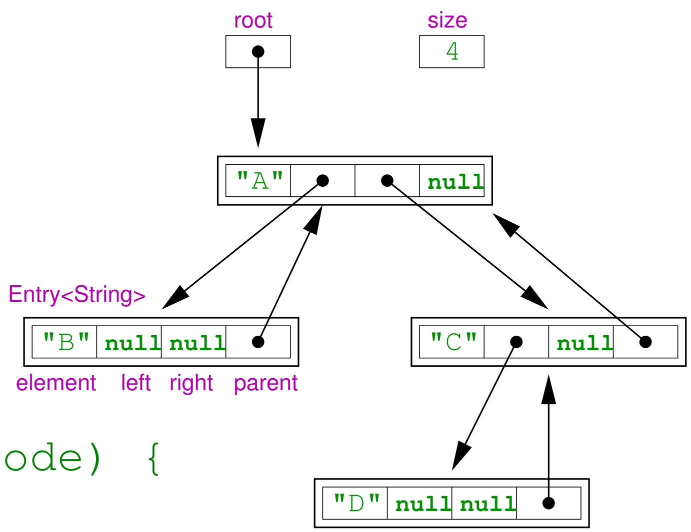

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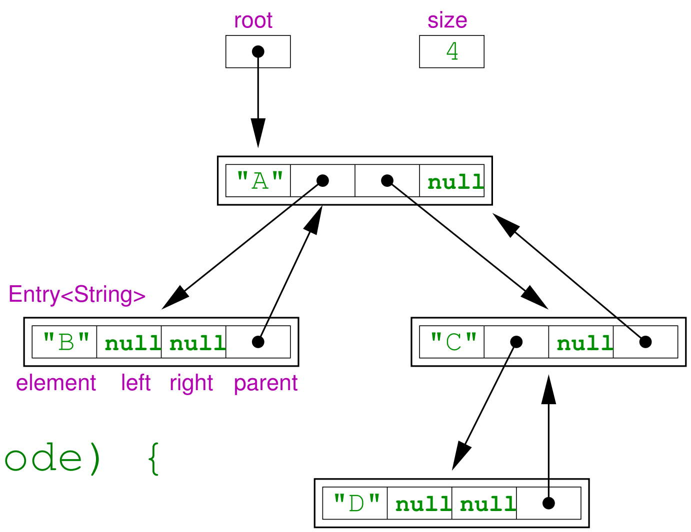

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# Binary Search Trees

- We will concentrate on one of the most important binary trees, namely the **binary search tree**
- The binary search tree keeps the elements ordered
- We can define a binary search tree recursively
  1. Each element in the left subtree is less than the root element
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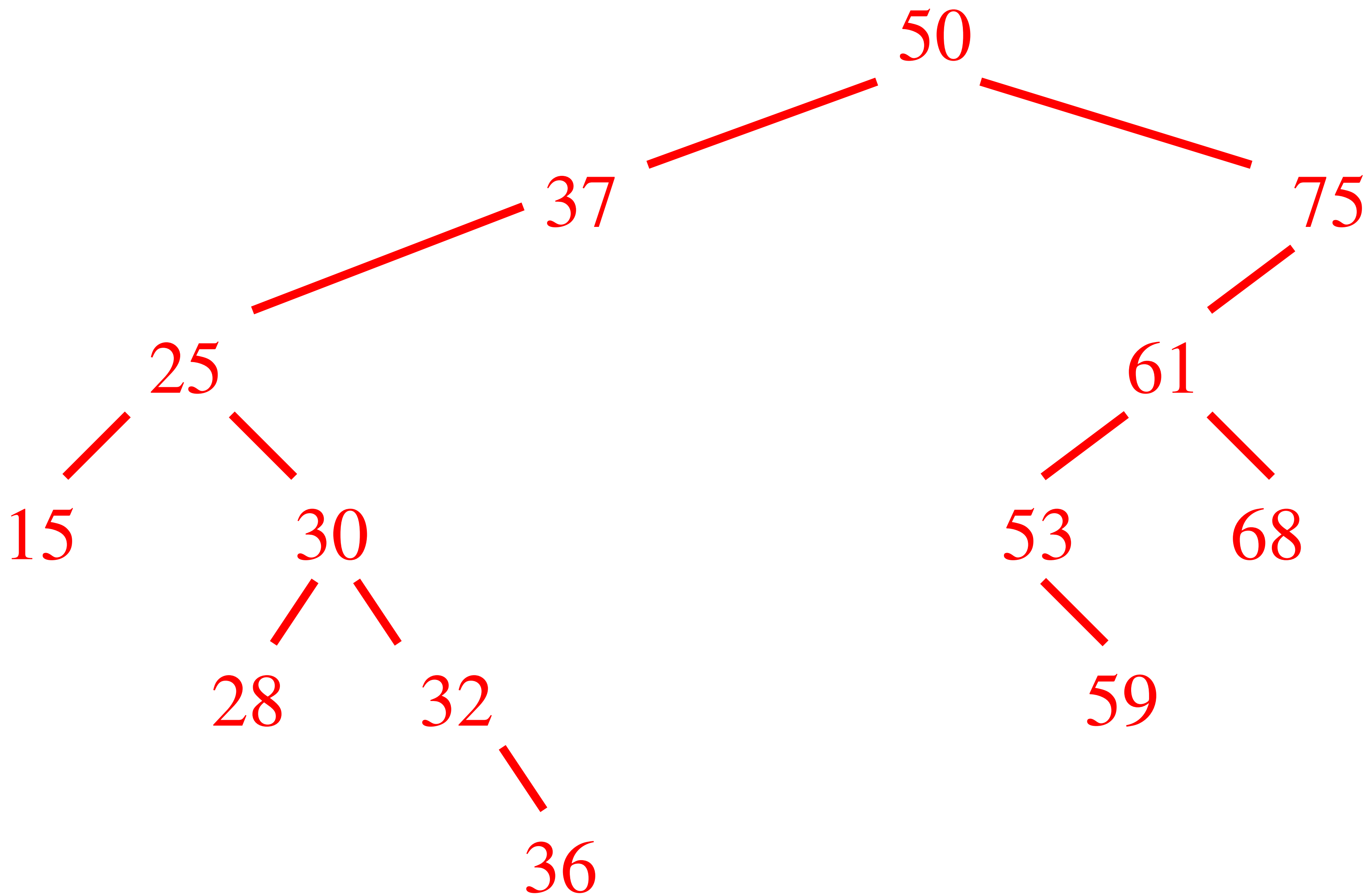

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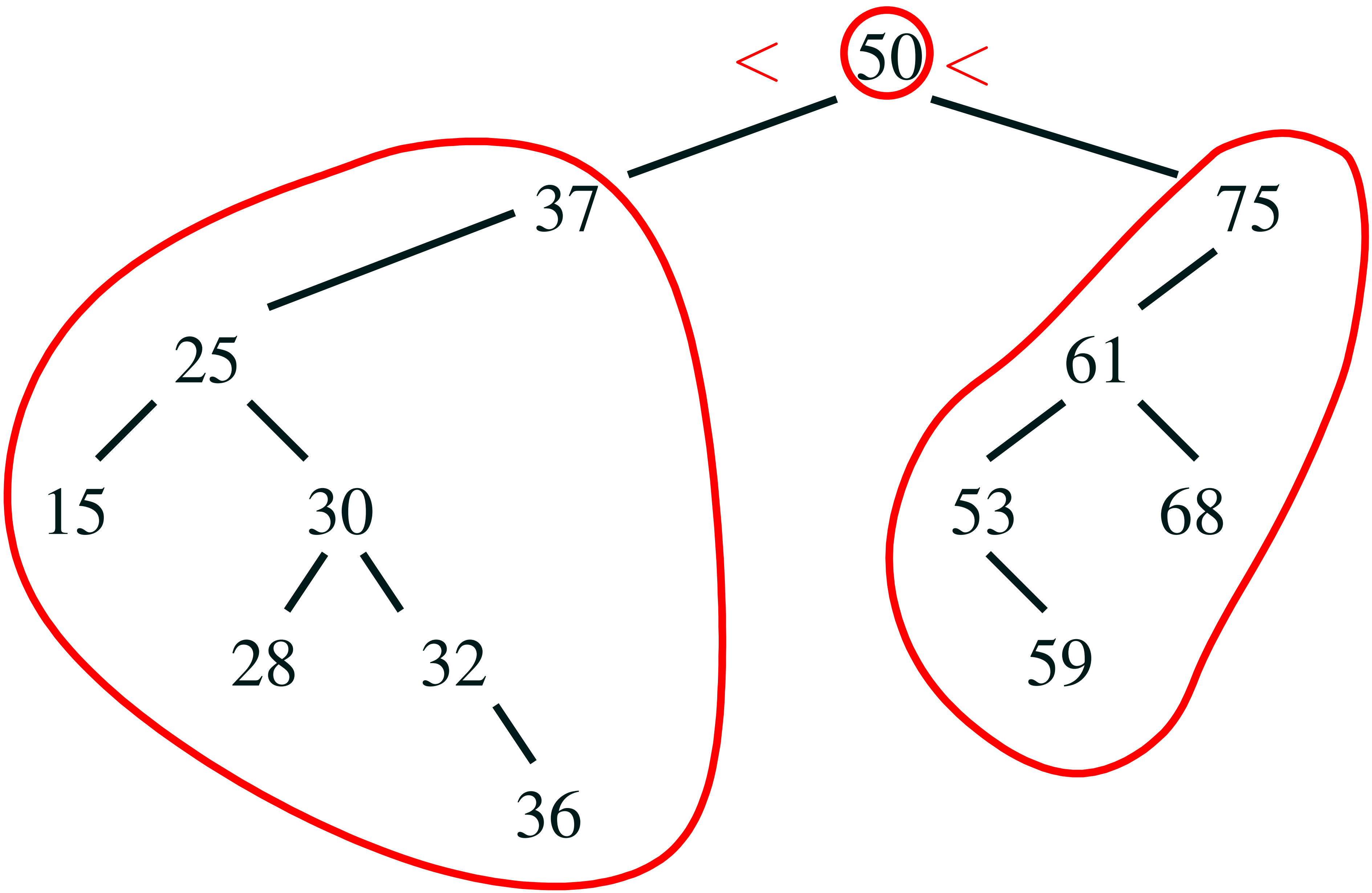

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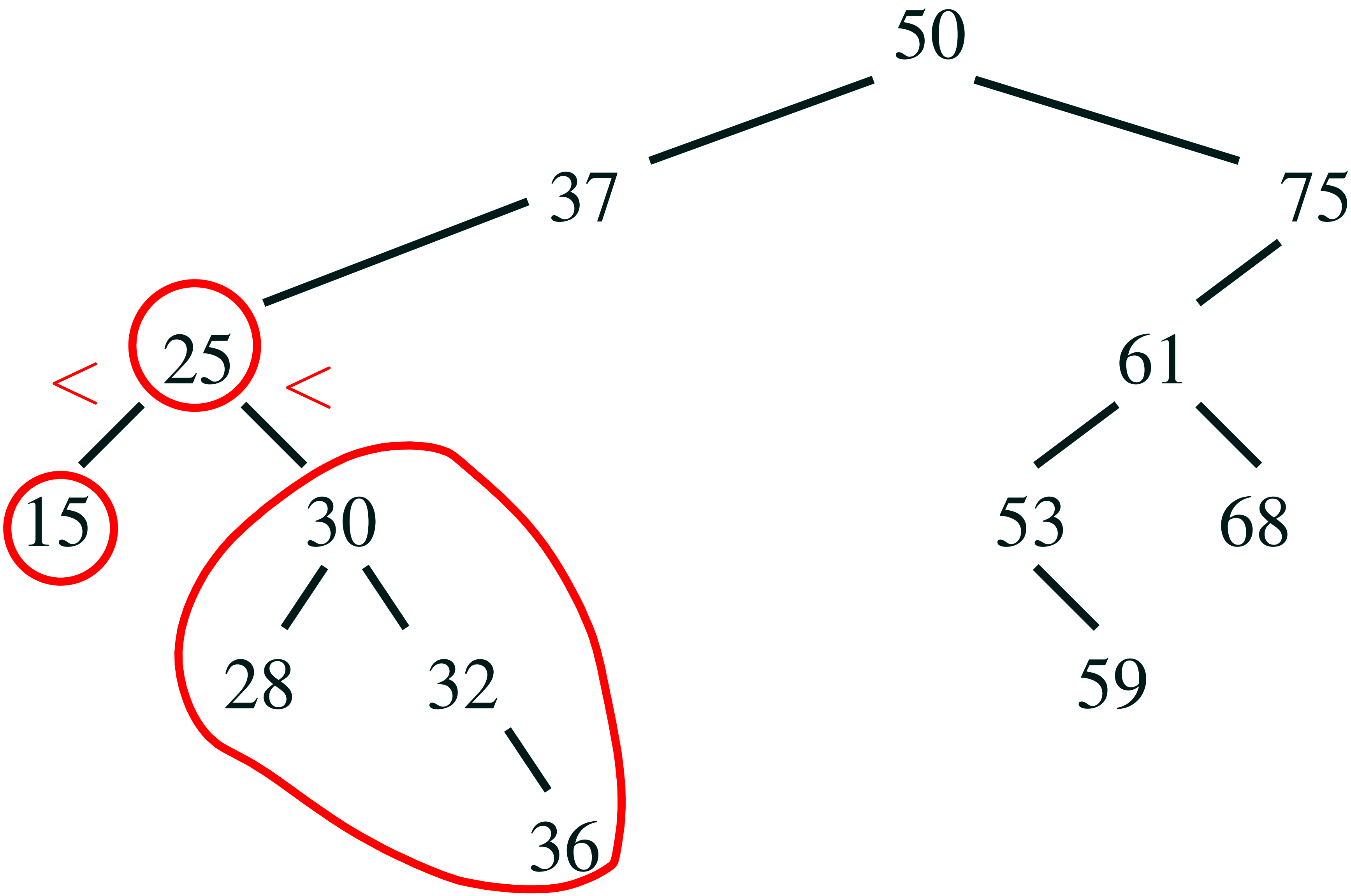

# Searching A Binary Search Tree

- Searching a binary search tree is easy
- Start at the root
- Compare with element
  - ⋆ If less than element go left
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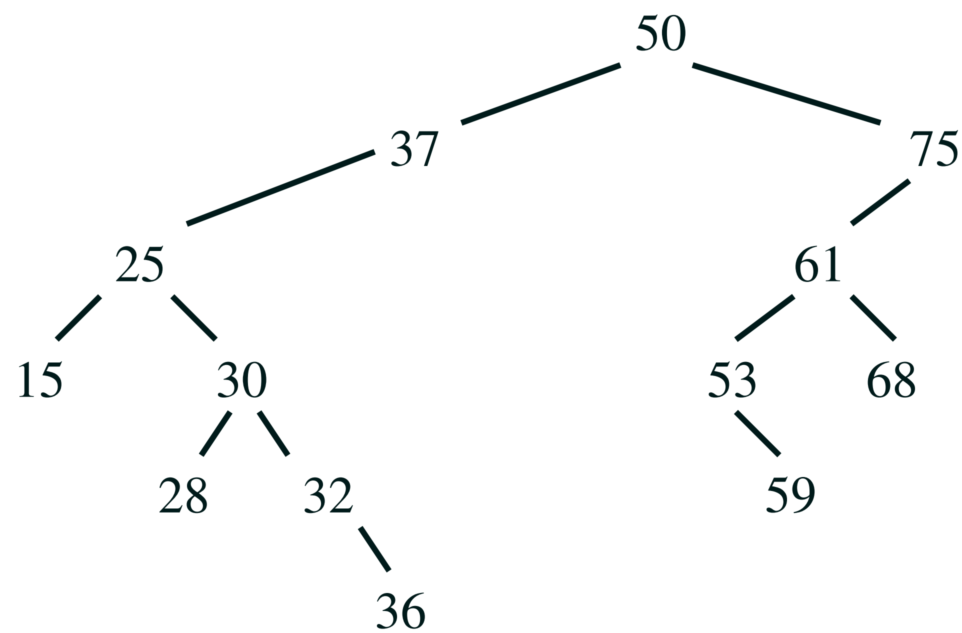

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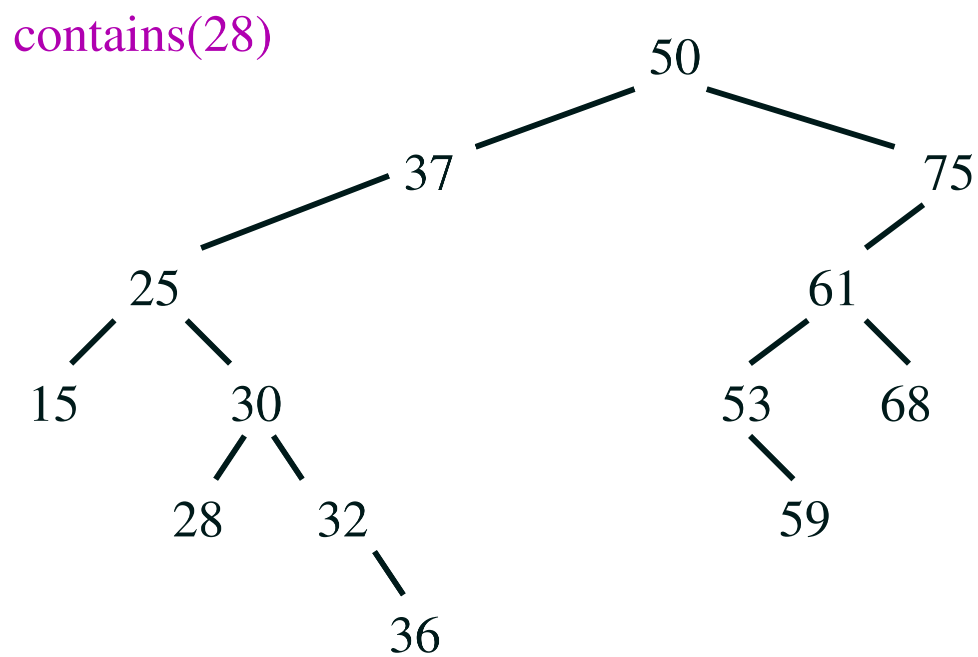

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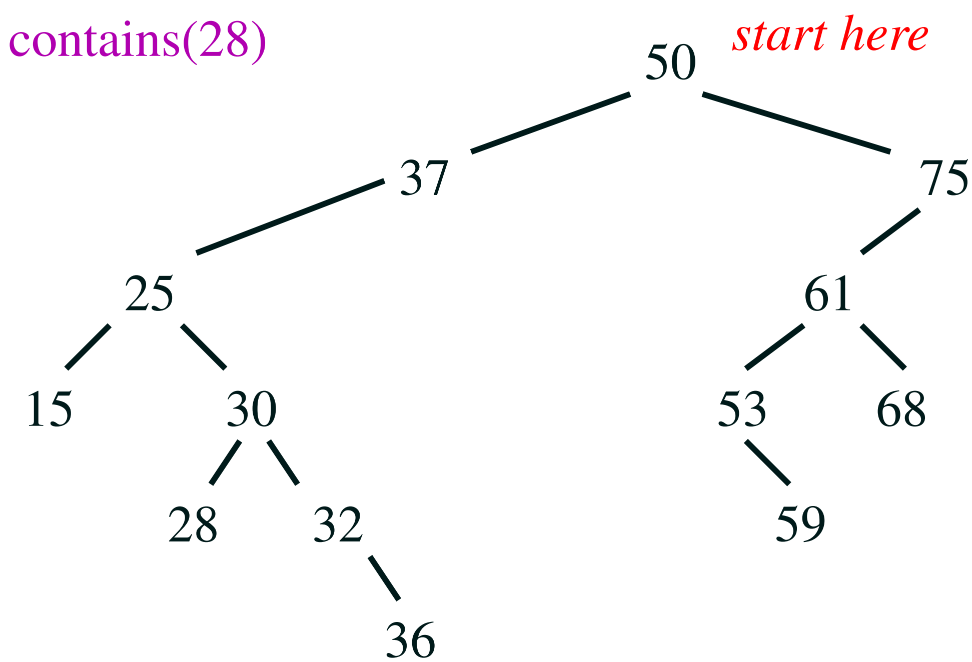

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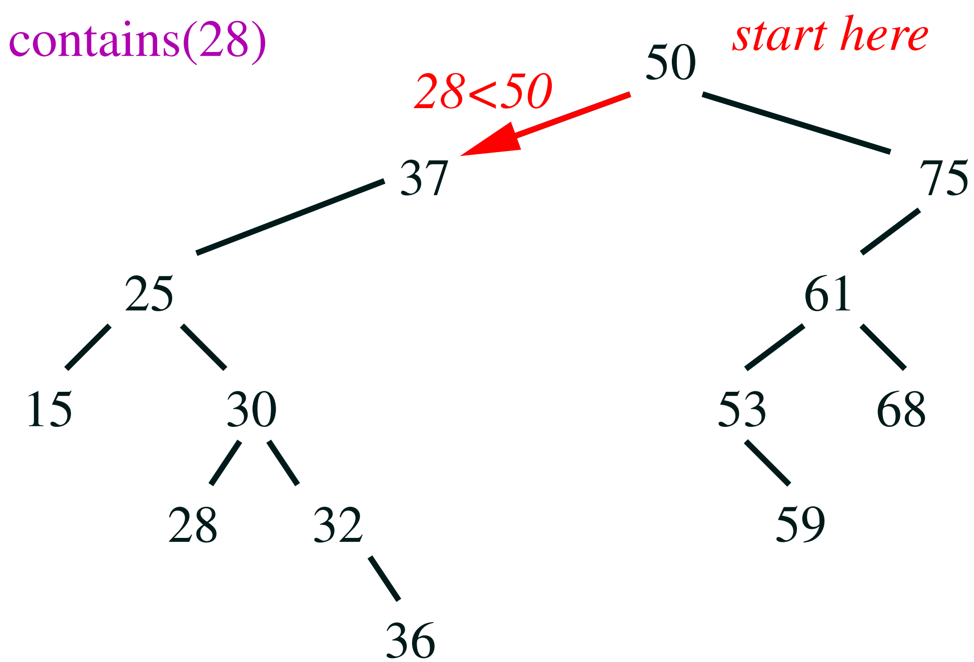

# Searching A Binary Search Tree

- Searching a binary search tree is easy
- Start at the root
- Compare with element
  - ⋆ If less than element go left
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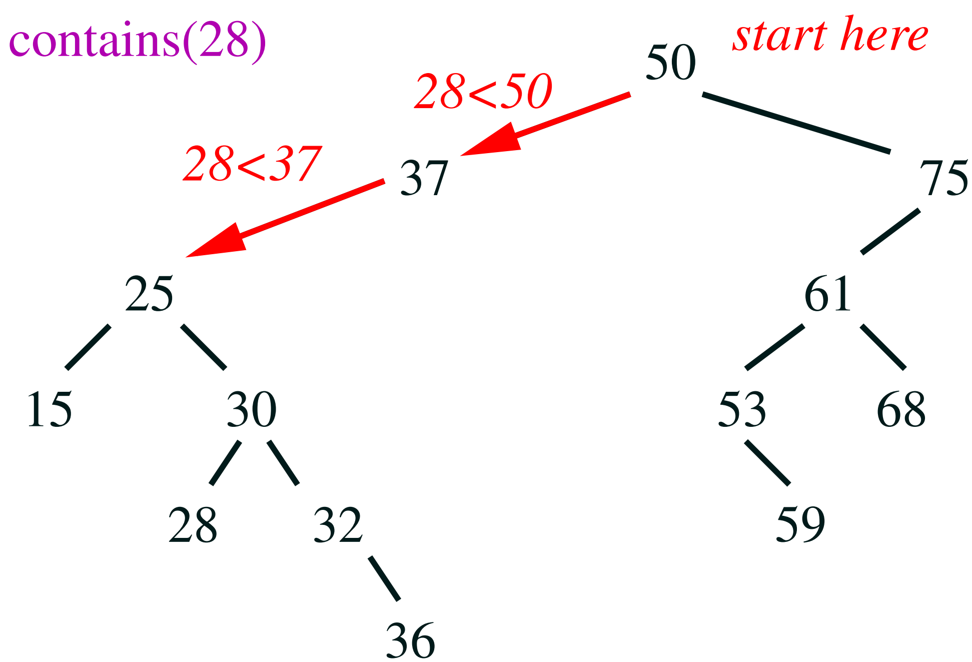

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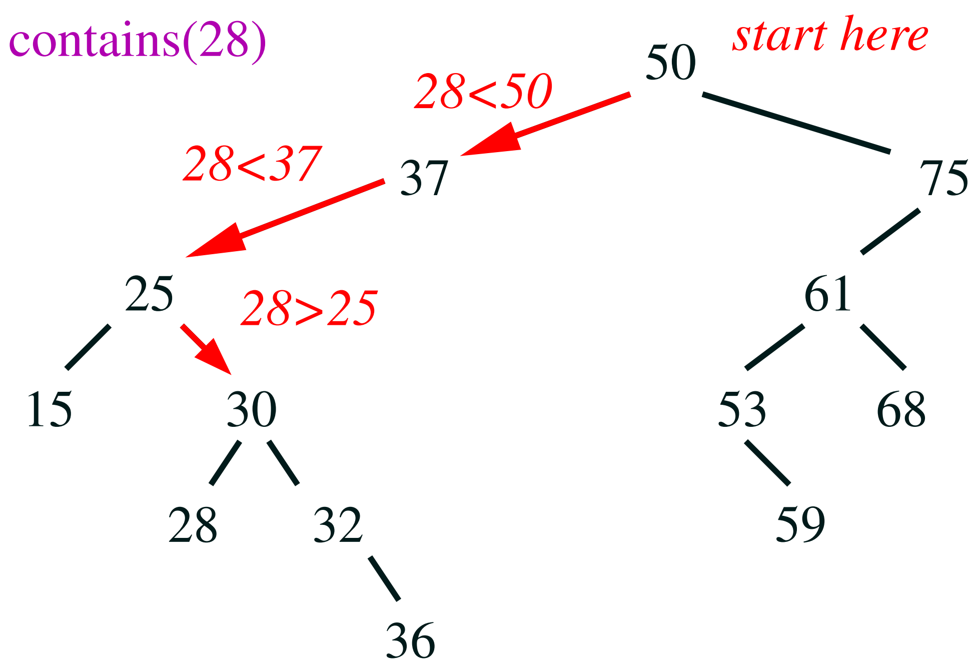

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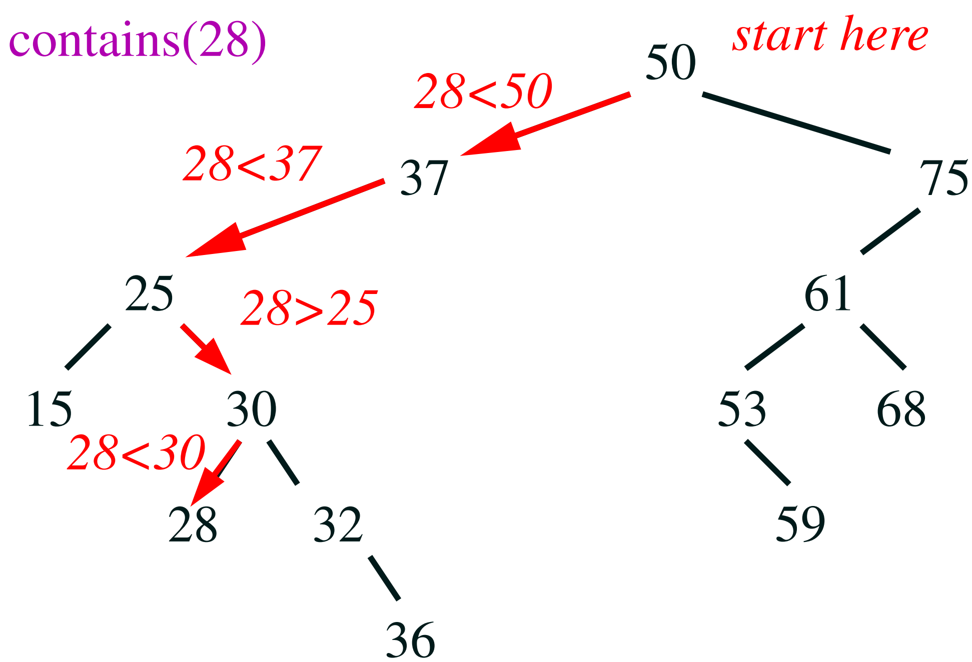

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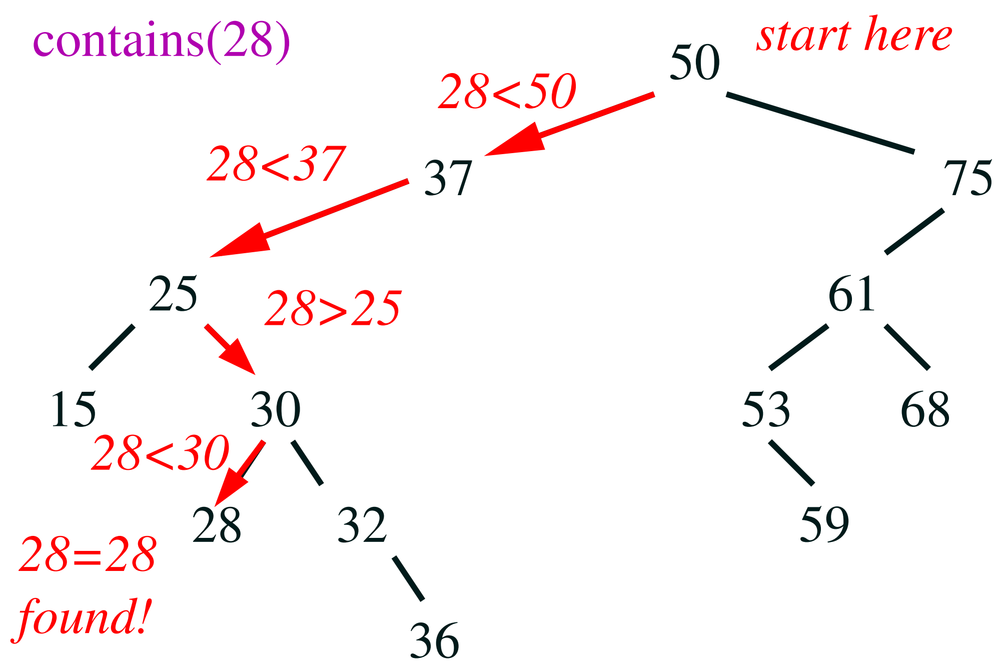
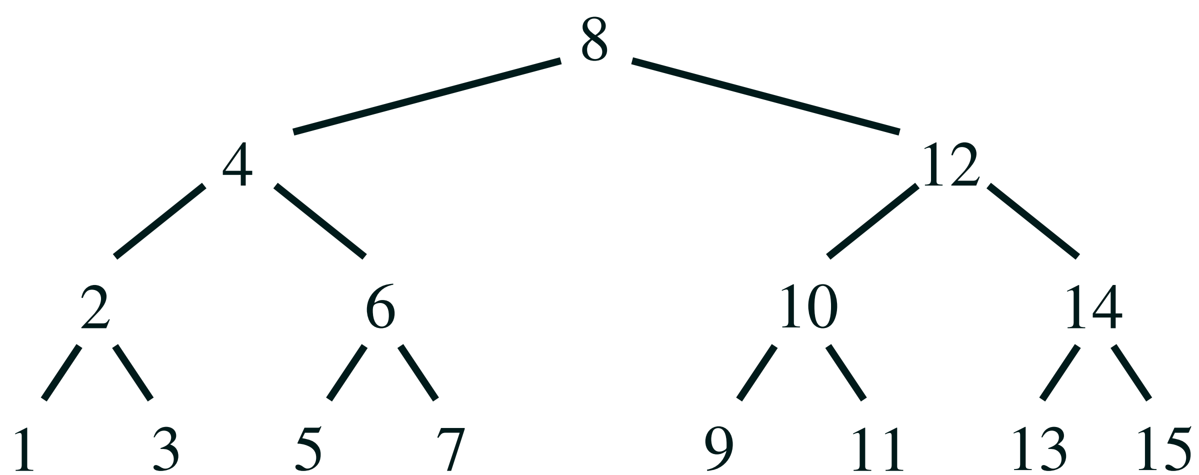
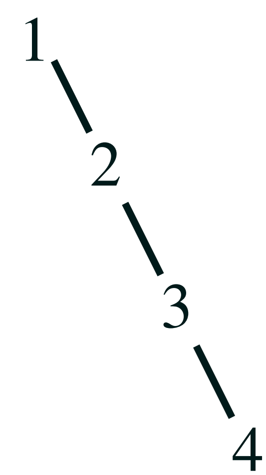


# Speed of Search

- The number of comparisons necessary to find an element in a binary tree depends on the level of the node in the tree
- The worst case number of comparisons is therefore the height of the tree
- This depends on the density of the tree




full tree

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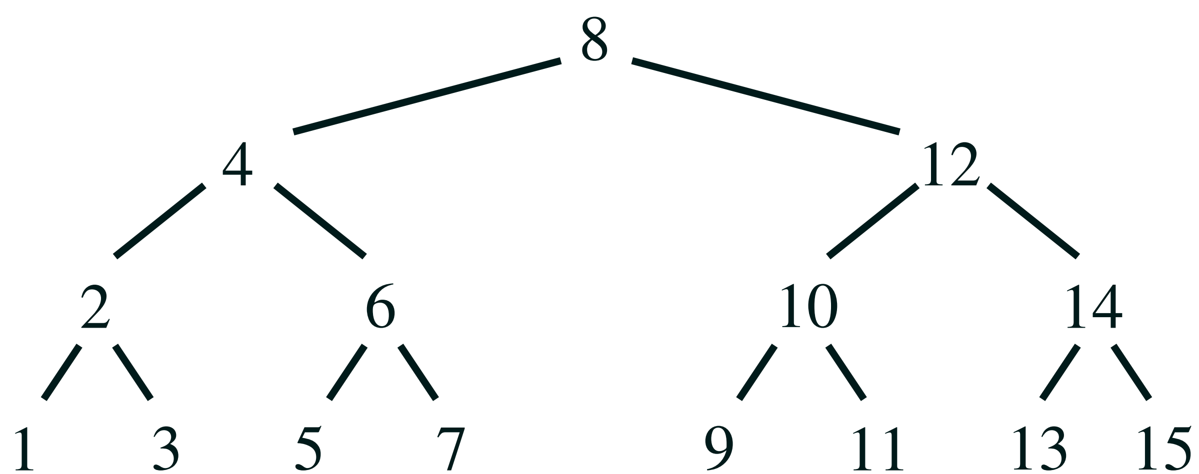


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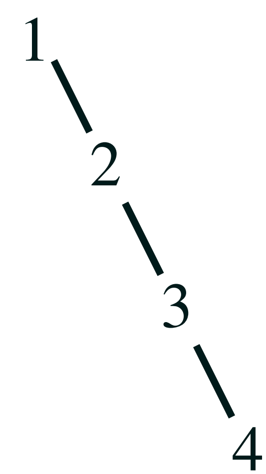


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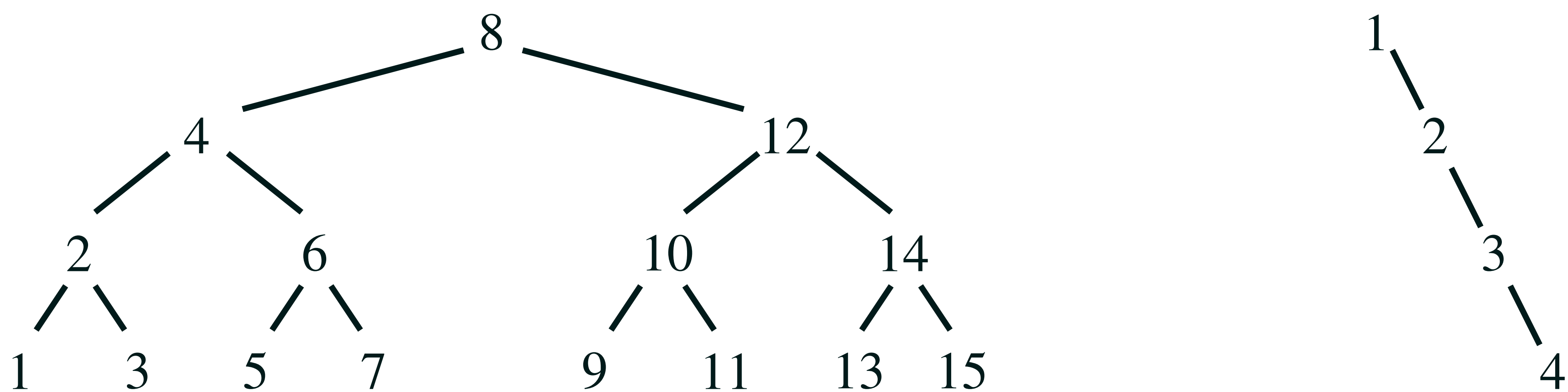


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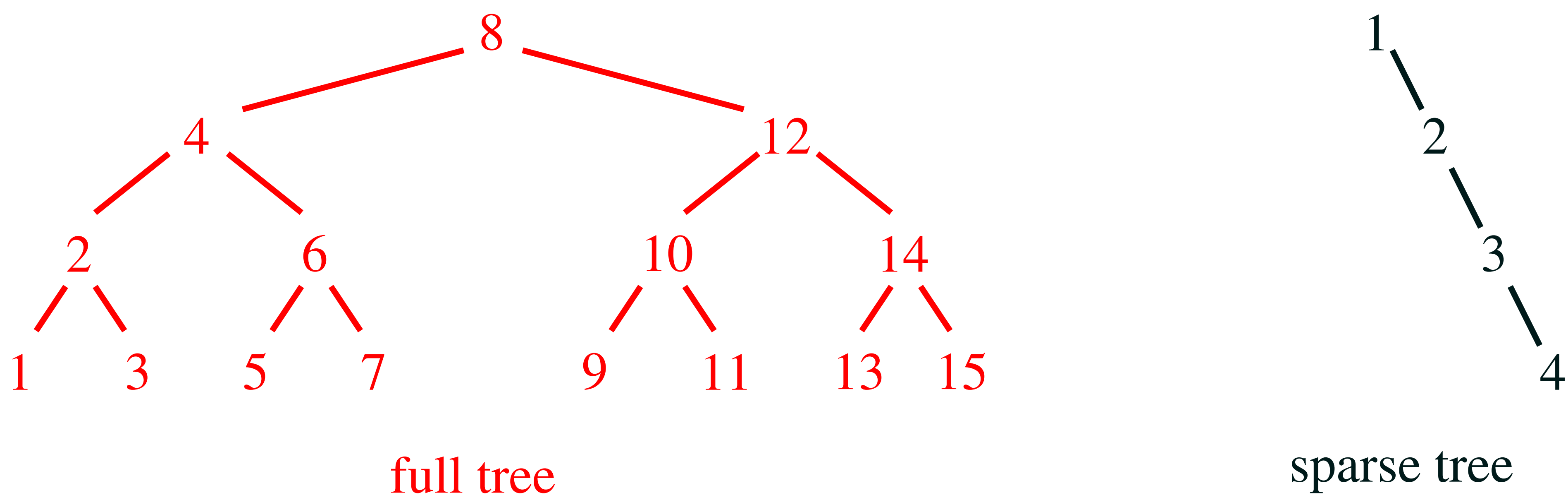

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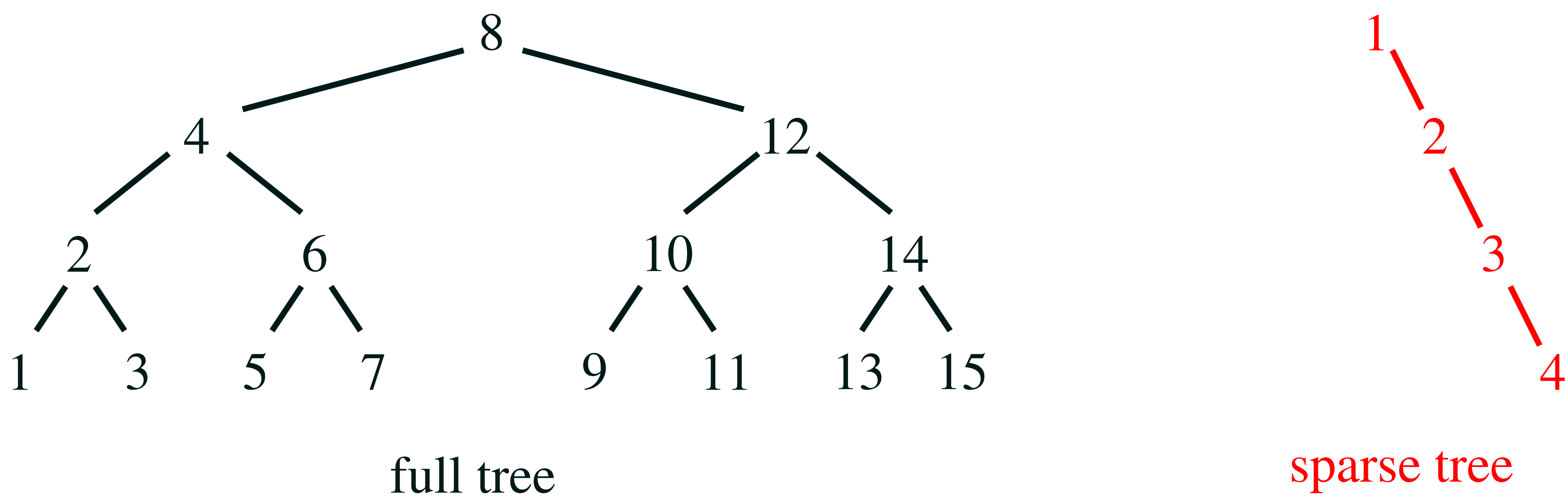

# Implementing a Set

- A set is a fundamental **abstract data type**
- It is a collection of things with no repetition and no order
- Ironically because order doesn't matter we can order the elements

$$\{1, 3, 5, 5, 3, 4\} = \{5, 3, 4, 1\} = \{1, 3, 4, 5\}$$

- This allows rapid search—a feature we care about
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# Fitting In

- The standard template library provides a class `std:set<T>`
- This contains many functions like
  - ⋆ Constructors
  - ⋆ `size()`
  - ⋆ `insert(T o)`
  - ⋆ `find(T o)`
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# Comparable

- To sort any objects they must be comparable
- In the STL the set implementation has a second template parameter: `std::set<T, Compare = less<T>>`
- by default this is defined to be `less<T>` (which is a function already defined for most common types) which you can define
- If you have a set of complex objects you will have to define `Compare`

```
bool MyCompare(MyObject left, MyObject right) {
  return something
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mySet = set<MyObject, MyCompare>;
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# Find an Element

- One of the core operations of a binary tree is to find a node

```
iterator find(const T& element) {
  Node* current = root;
  while (current!=nullptr) {
    if (current->element == element) {
      return iterator(current);
    }
    if (element < current->element) {
      current = current->left;
    } else {
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# Add an Element

```
pair<iterator, bool> insert(const T& element) {
  if (no_elements==0) {
    root = new Node(element, nullptr);
    ++no_elements;
    return pair<iterator, bool>(iterator(root), true);
  }
  Node* parent = nullptr;
  Node* current = root;
  while(current != nullptr) {
    if (current->element == element) {
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    parent = current;
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  current = new Node(element, parent);
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    parent->right = current;
  }
  ++no_elements;
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}
```
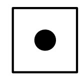

# Tree in Action

add(84)



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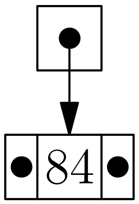
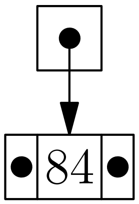


# Tree in Action

add(39)



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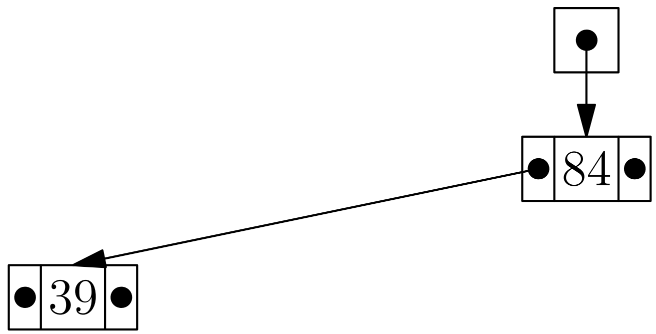

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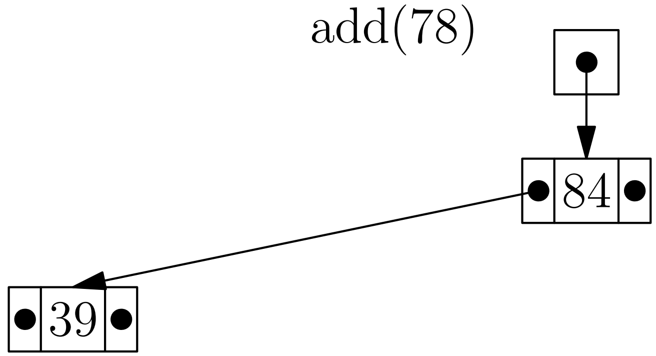

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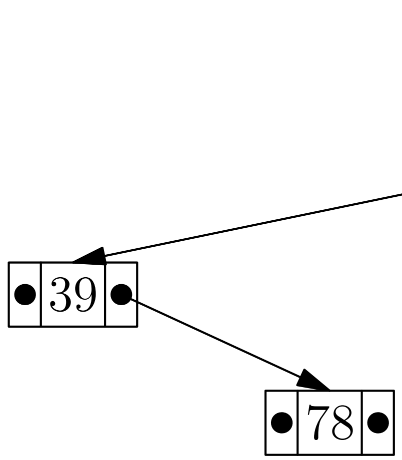

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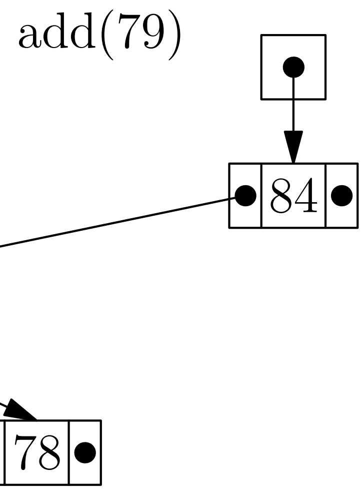

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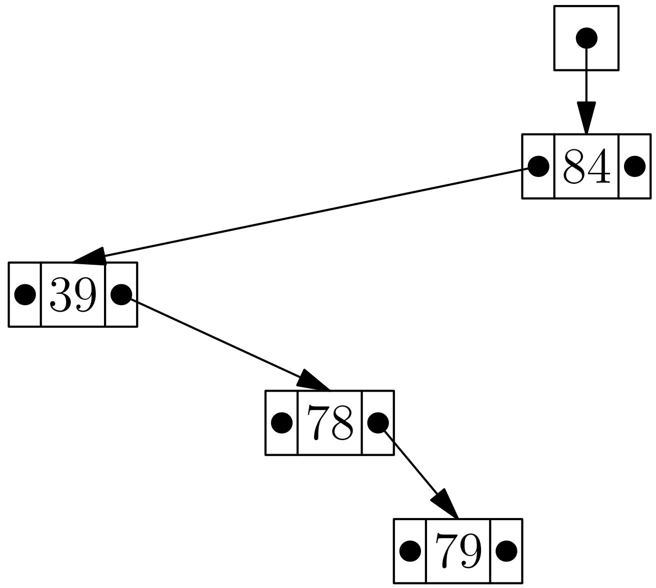

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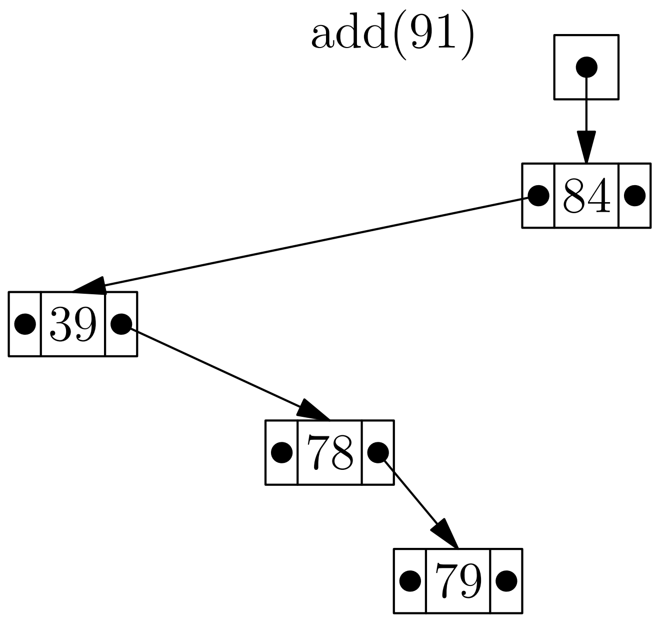

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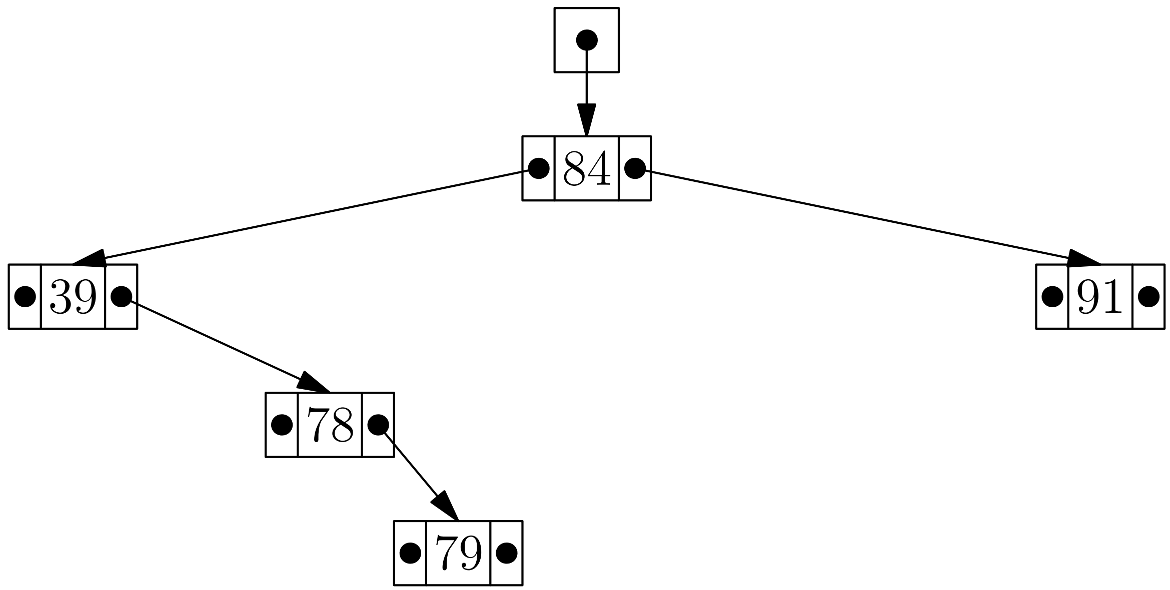
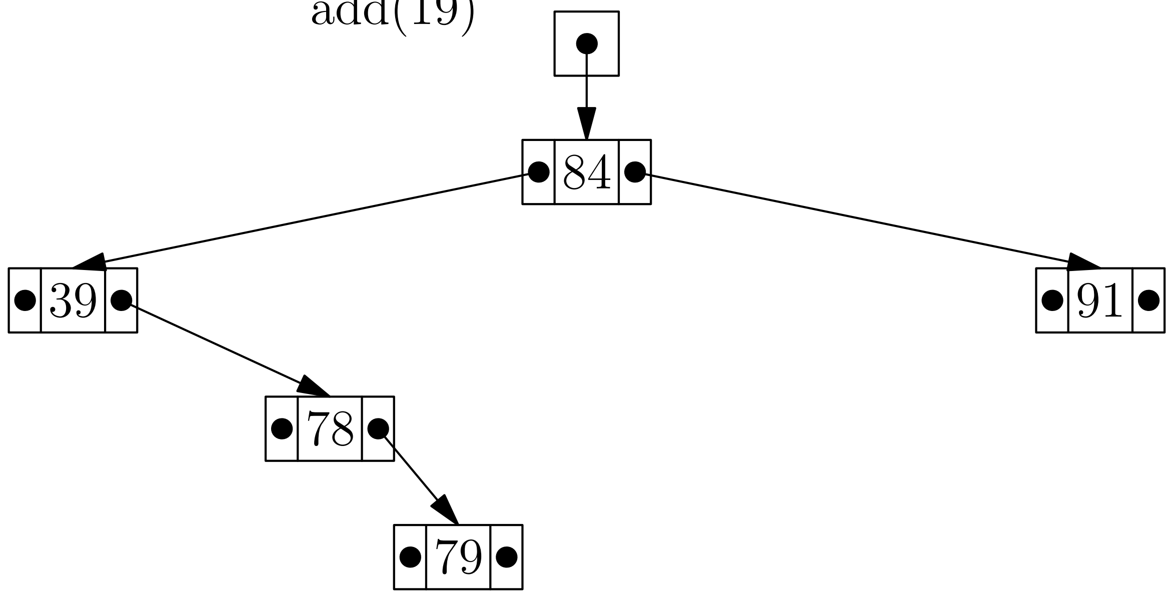


# Tree in Action

add(19)



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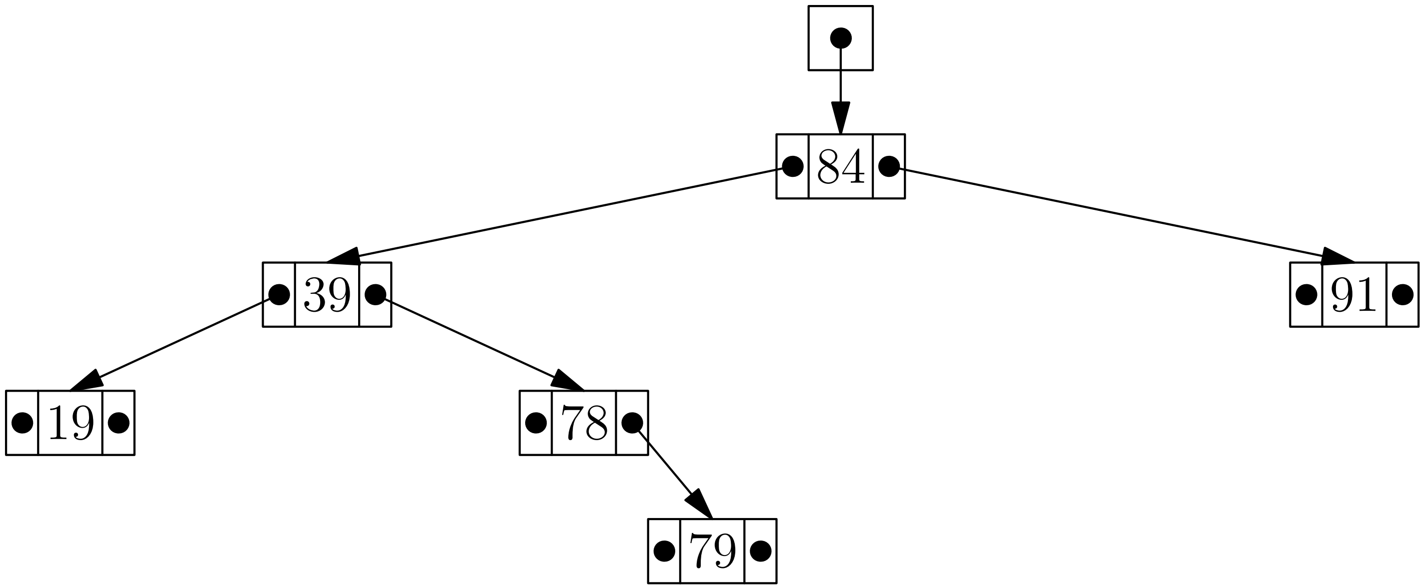

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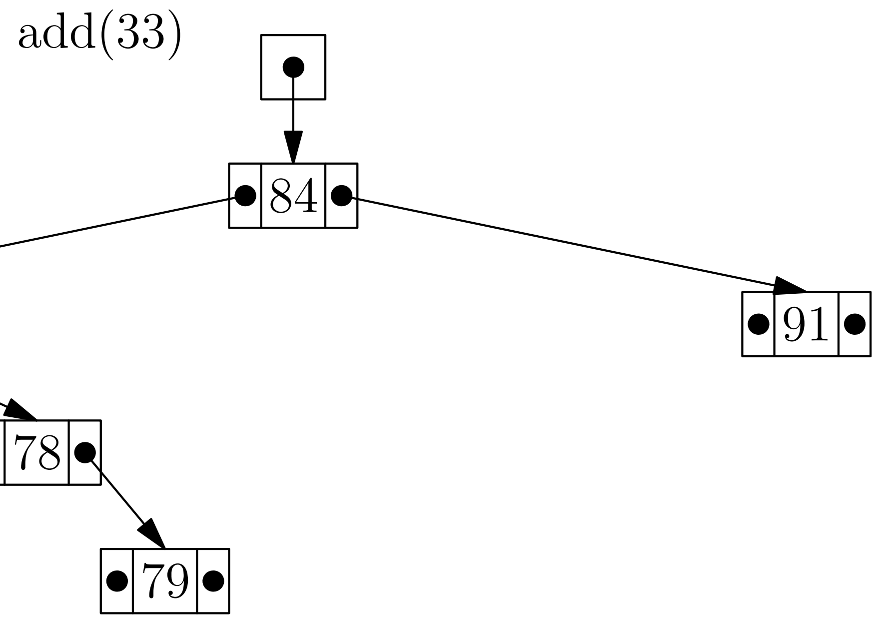

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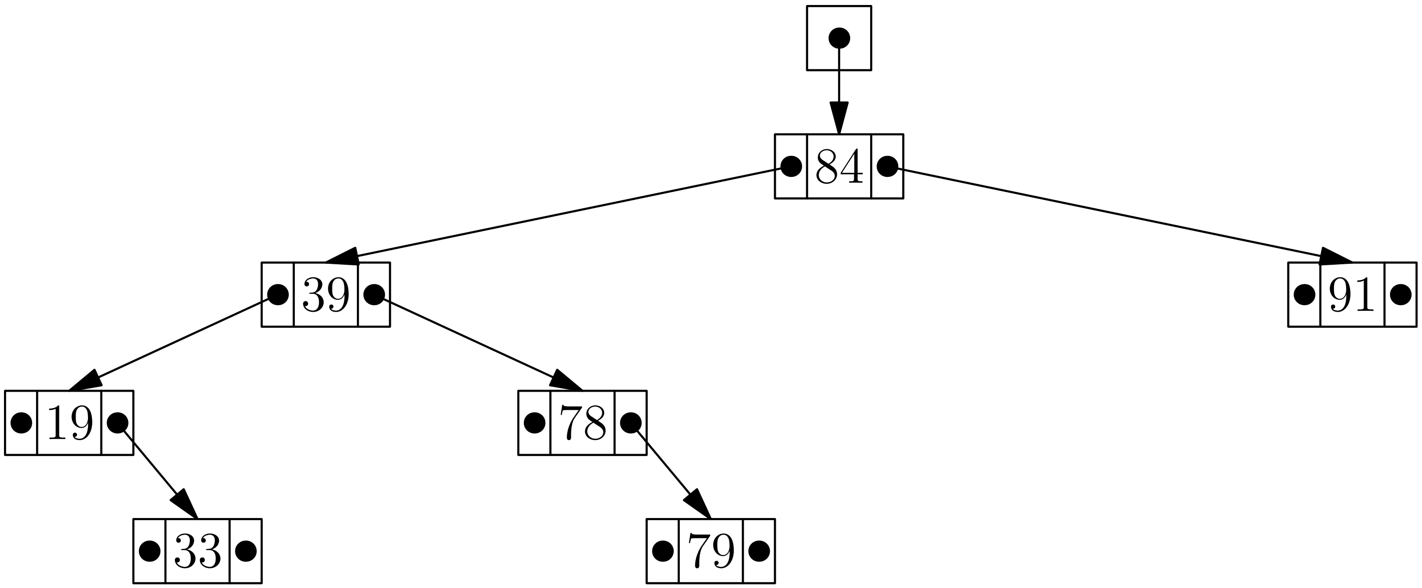

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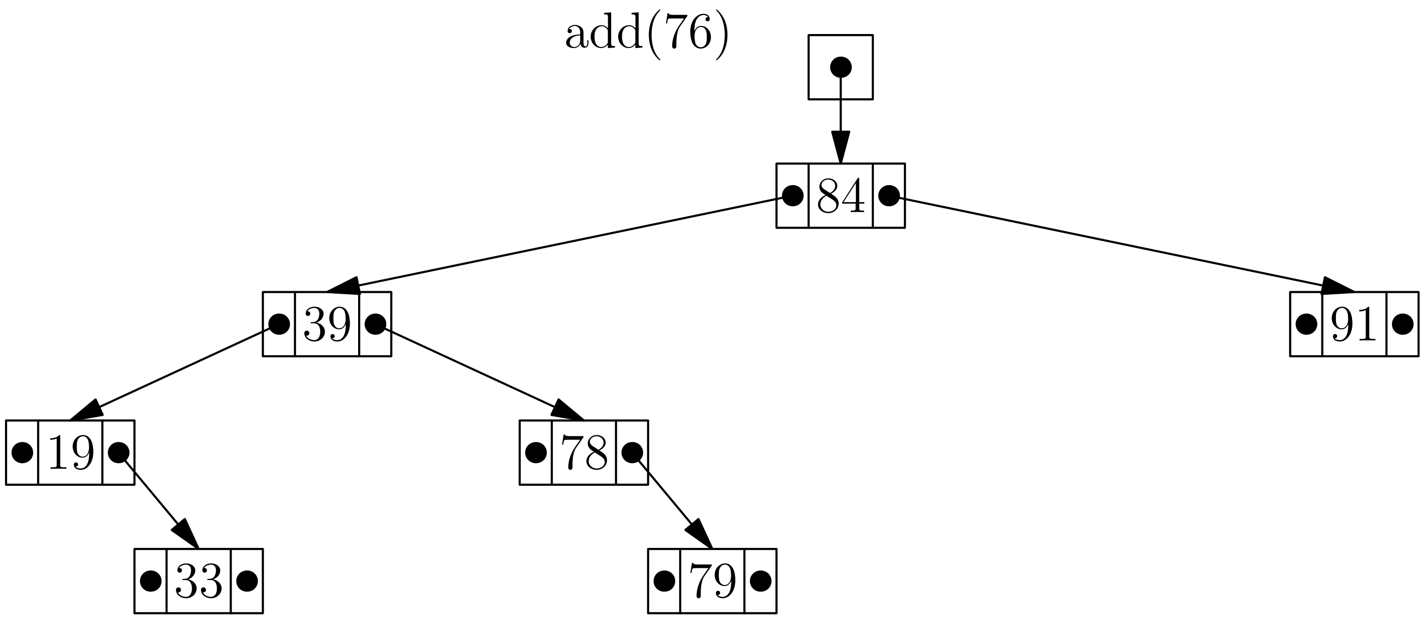

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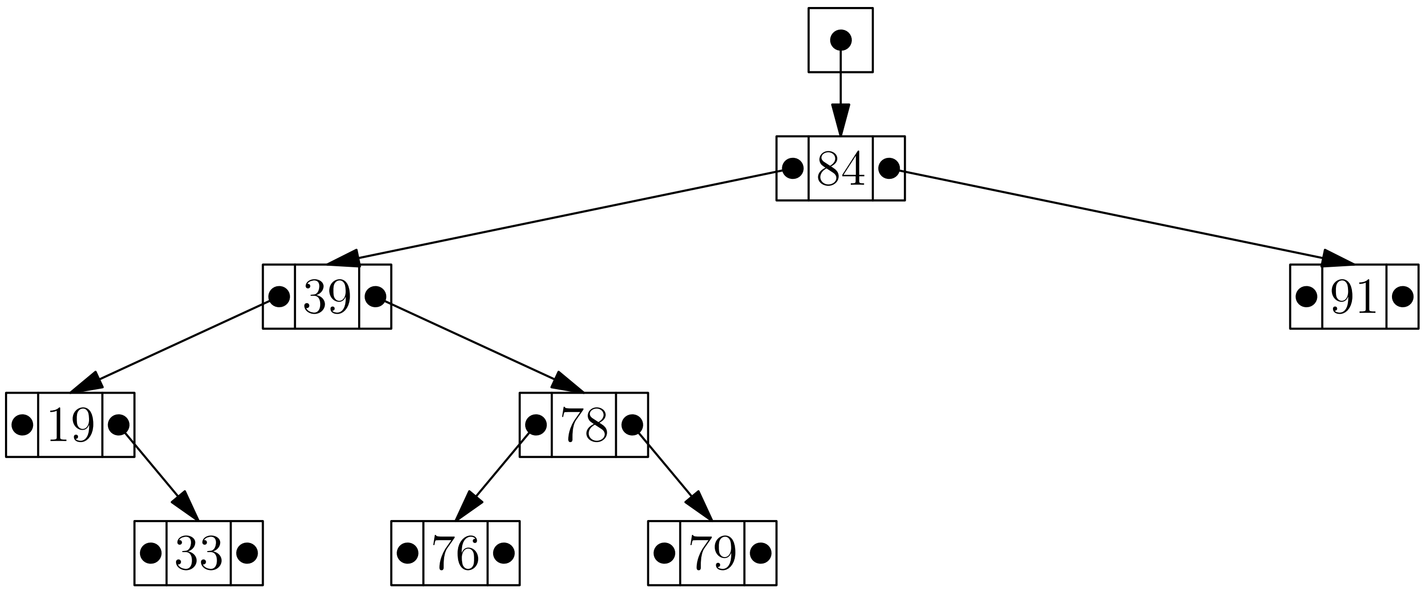

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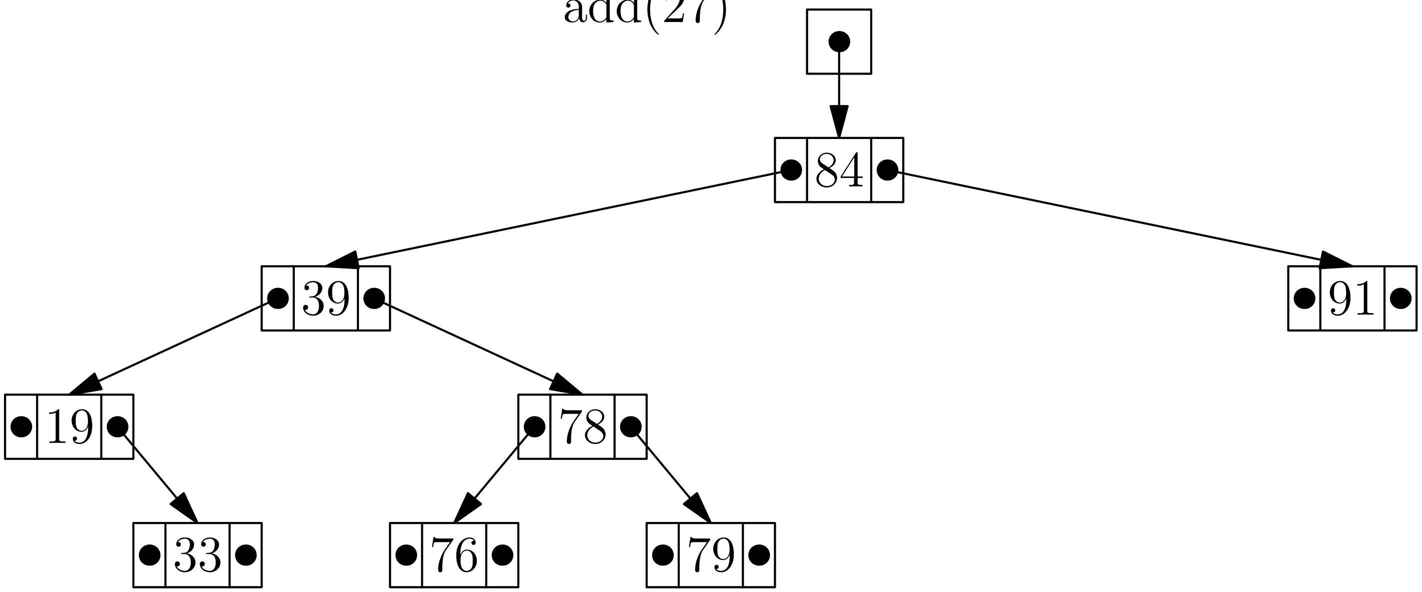
add(27)



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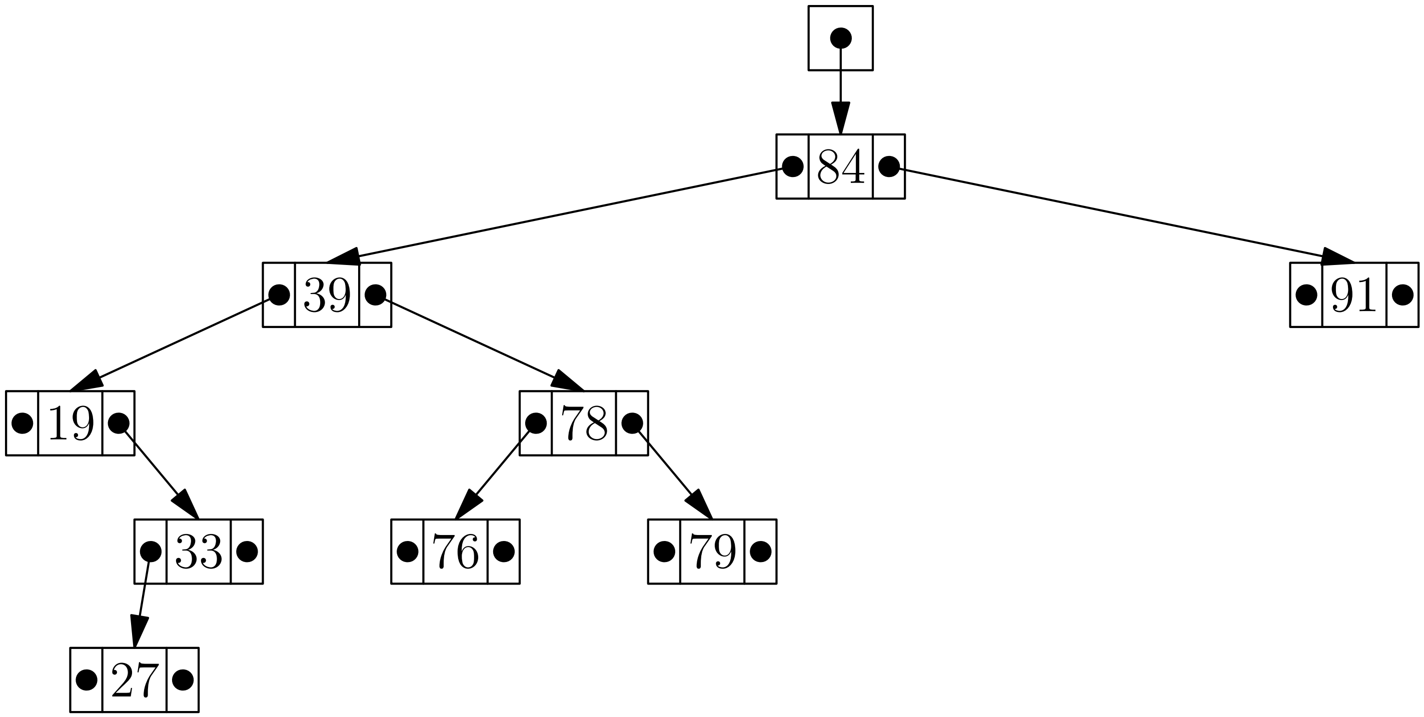

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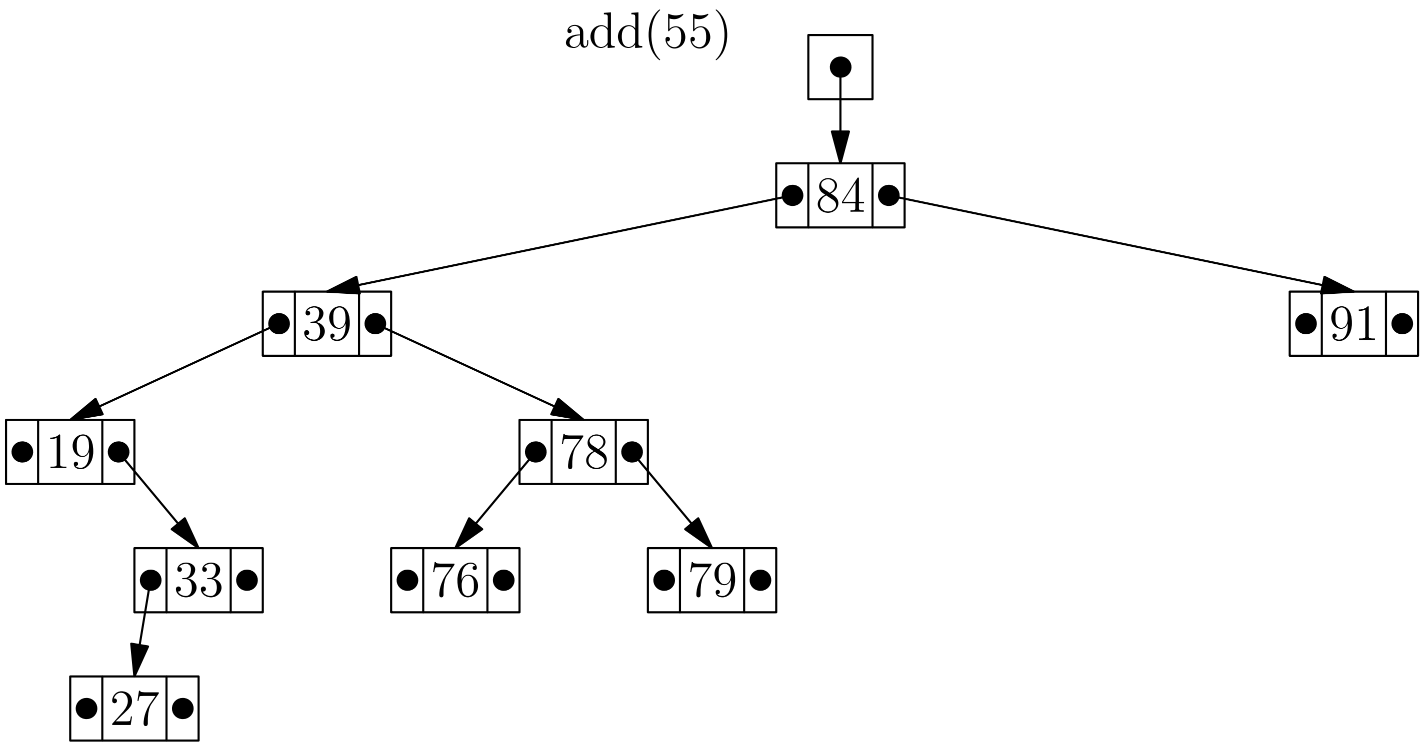

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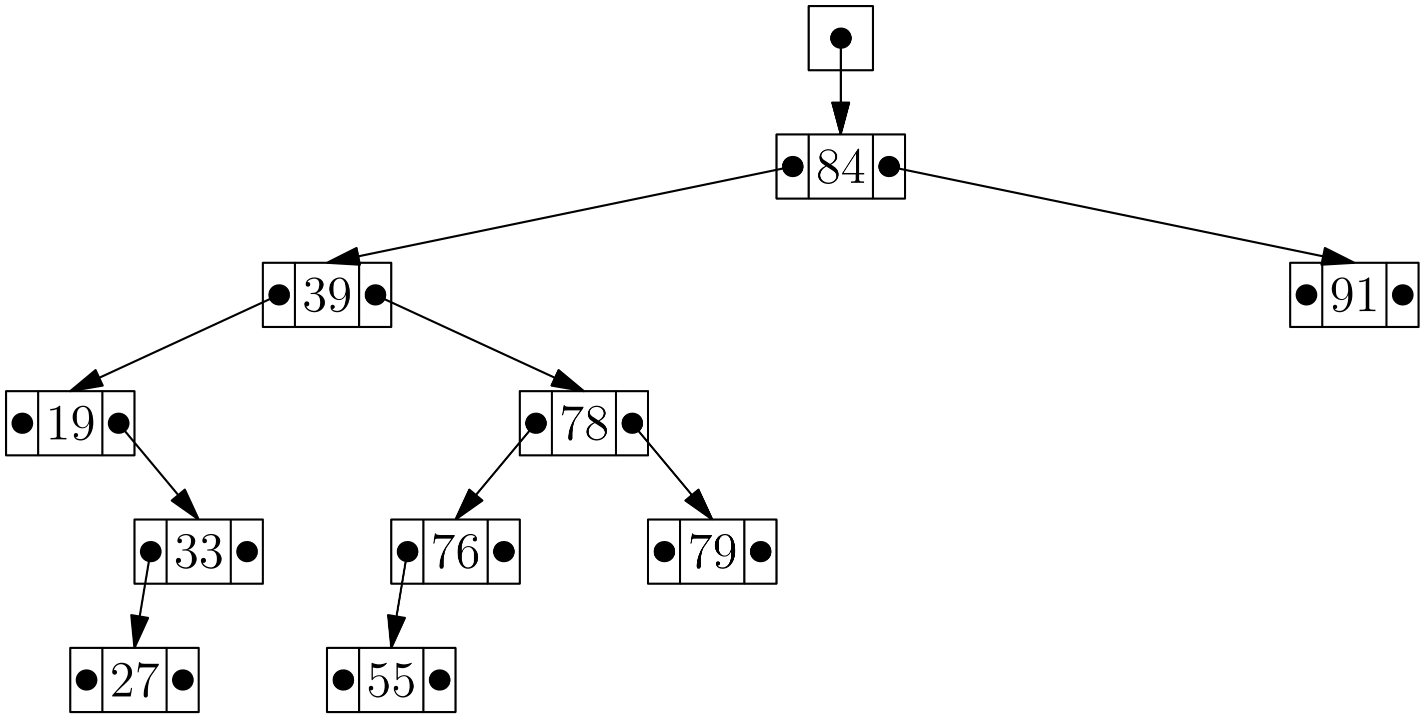

# Shape of Tree

- The structure of the tree depends on the order in which we add elements to it

- Suppose we add

  *To be, or not to be: that is the question:*
  *Whether 'tis nobler in the mind to suffer*
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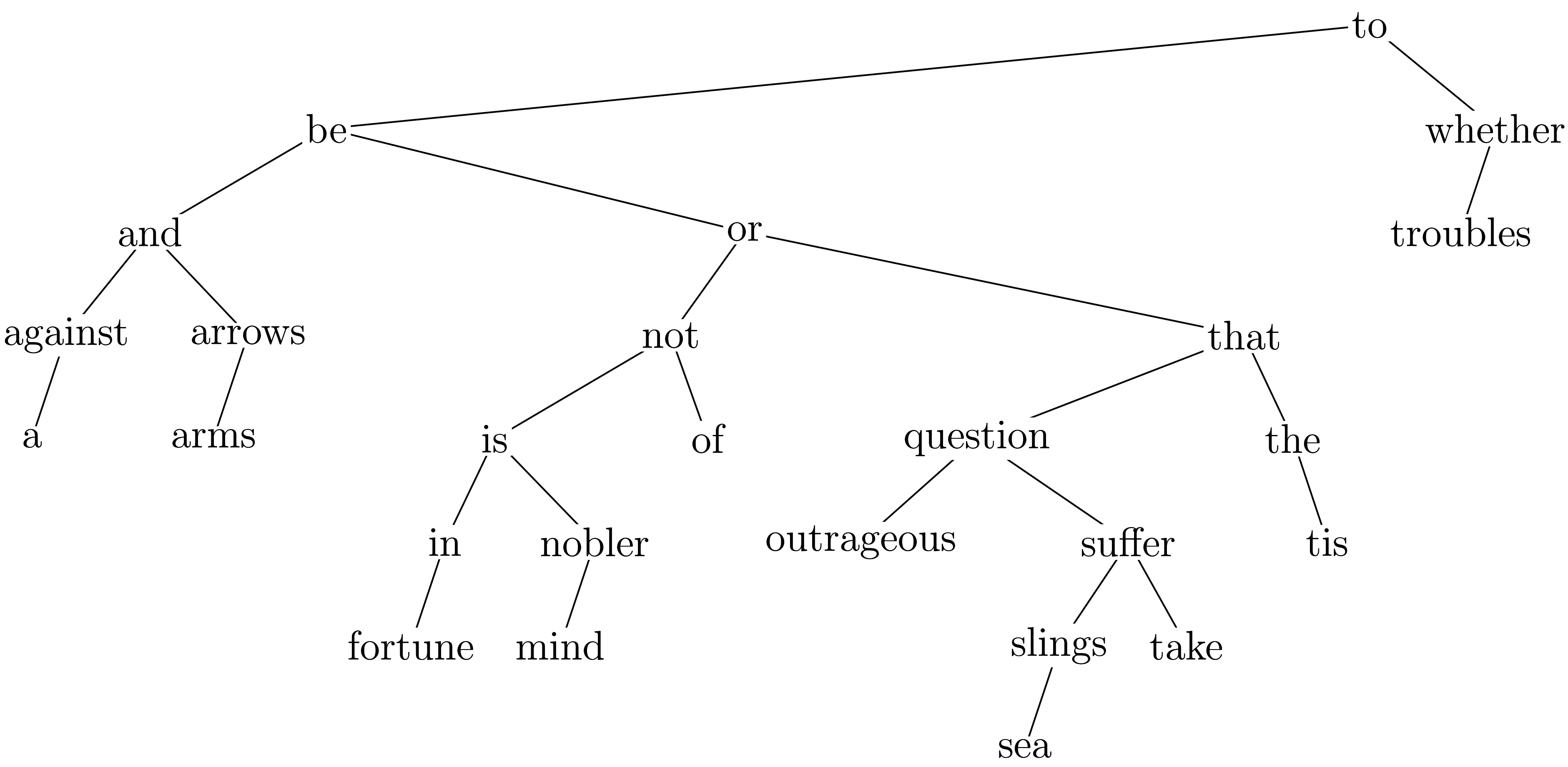
# Hamlet


to
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question
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in
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outrageous
suffer
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# Outline

1. Trees
2. Binary Trees
   - Implementing Binary Trees
3. Binary Search Trees
   - Definition
   - Implementing a Set
4. **Tree Iterators**

# Tree Iterators

- As with most container classes it is very useful to define iterators
- `begin()` should return a “pointer” to the start of the tree
- `end()` provides a “pointer” past the end
- `operator*()` returns the element
- `opeator++()` increments the “pointer”
- `operator!=(lhs, rhs)` is used to compare iterators

```
set<int> mySet;
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for(auto pt=mySet.begin(), pt!=mySet.end(), ++pt) {
  cout << *pt;
}
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- `operator!=(lhs, rhs)` is used to compare iterators

```
set<int> mySet;
...
for(auto pt=mySet.begin(), pt!=mySet.end(), ++pt) {
  cout << *pt;
}
```

# Tree Iterators

- As with most container classes it is very useful to define iterators
- `begin()` should return a "pointer" to the start of the tree
- `end()` provides a "pointer" past the end
- `operator*()` returns the element
- `opeator++()` increments the "pointer"
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set<int> mySet;
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for(auto pt=mySet.begin(), pt!=mySet.end(), ++pt) {
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# C++ Code

```
class binary_tree {

private:
  class iterator {
  private:
    Node* current;

  public:
    iterator(Node* node) {current=node;}
    T operator*() const {return current->element;}
    iterator operator++() {
      current = successor(current);
      return *this;
    }
    bool operator!=(const iterator& other) {
      return current!=other.current;
    }
  };

  iterator begin() {...}
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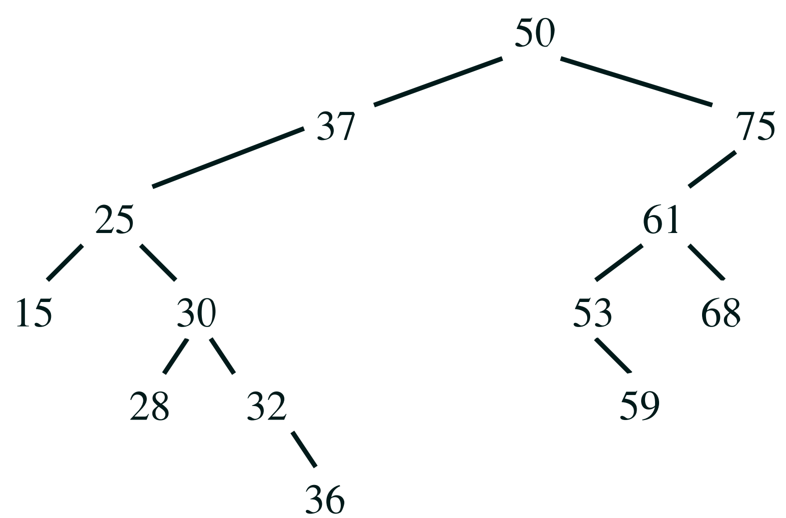
# Successor

- To find the successor we first start in the left most branch
- We follow two rules

1. **If** right child exist **then** move right once and then move as far left as possible
2. **else** go $up$ to the left as far as possible and then move up right

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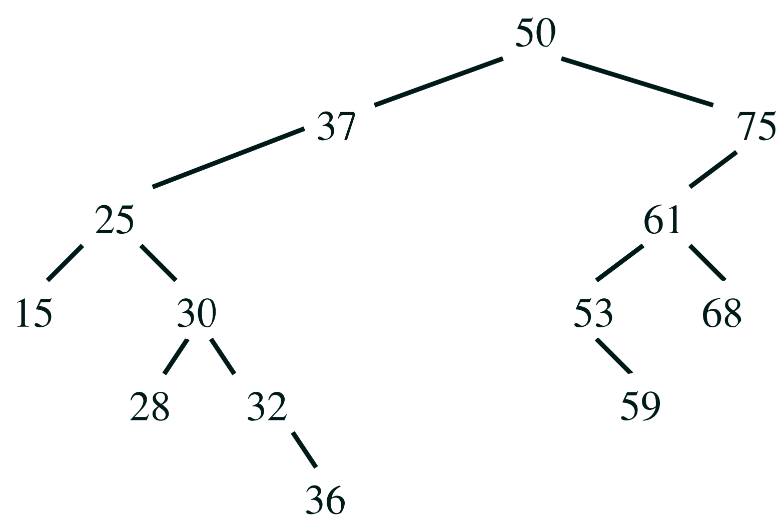


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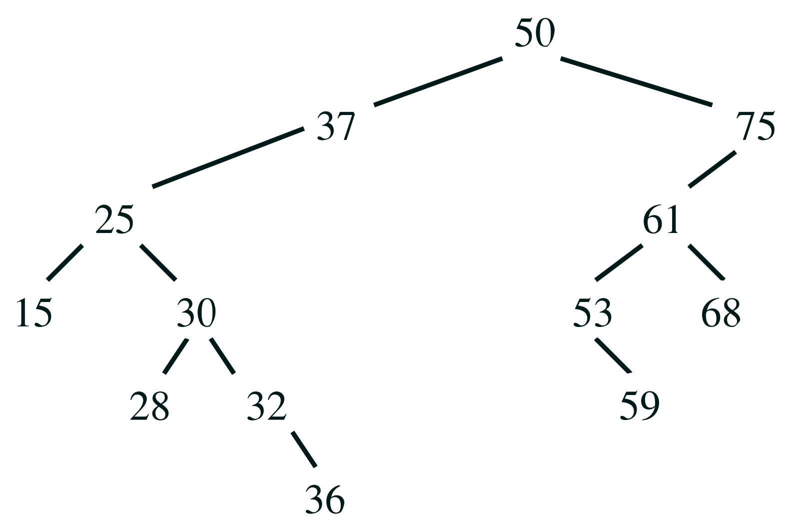


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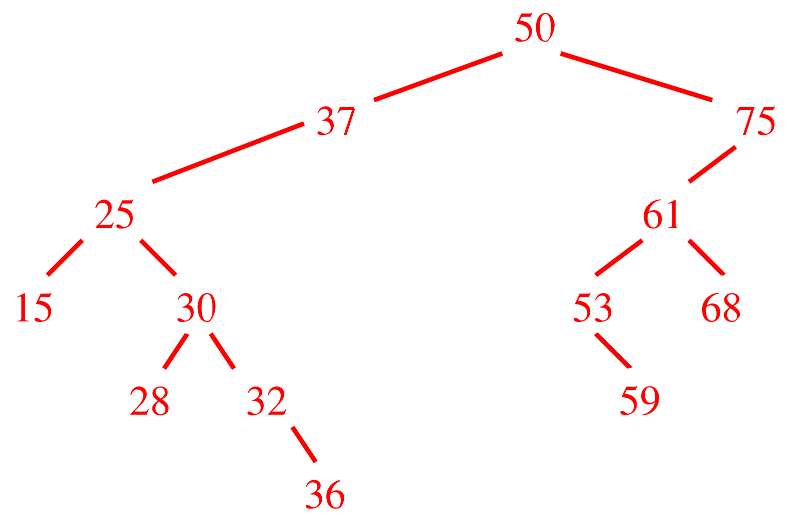


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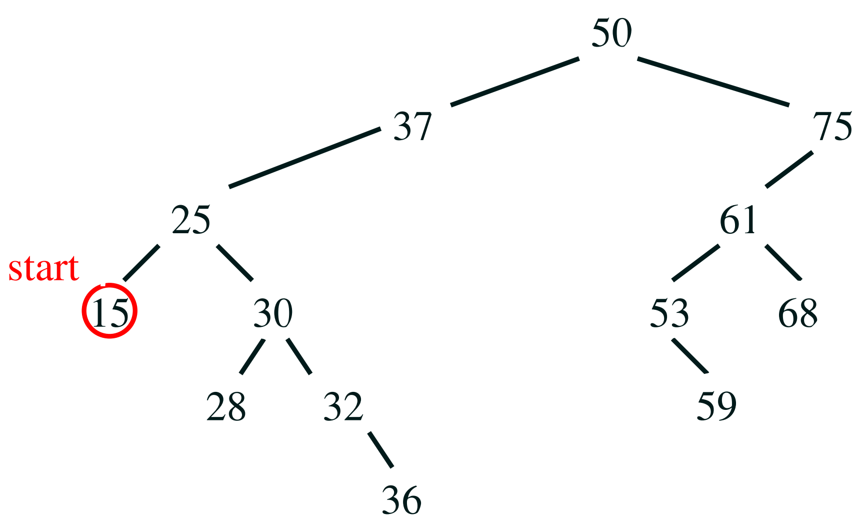


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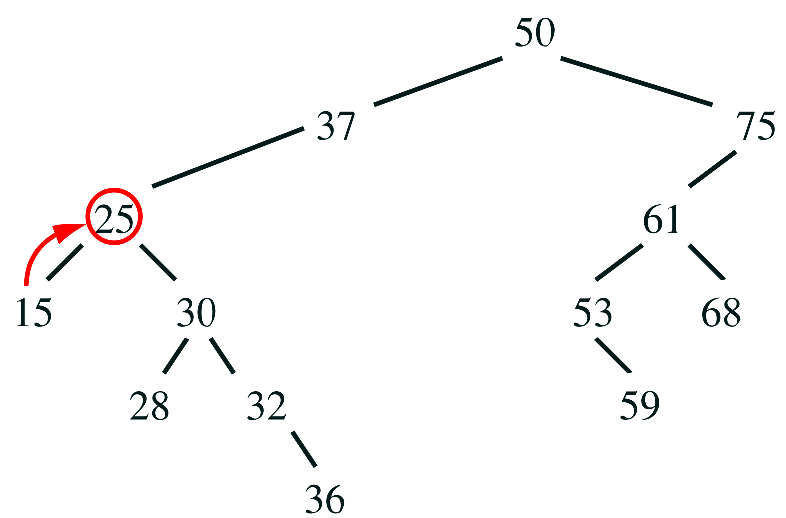


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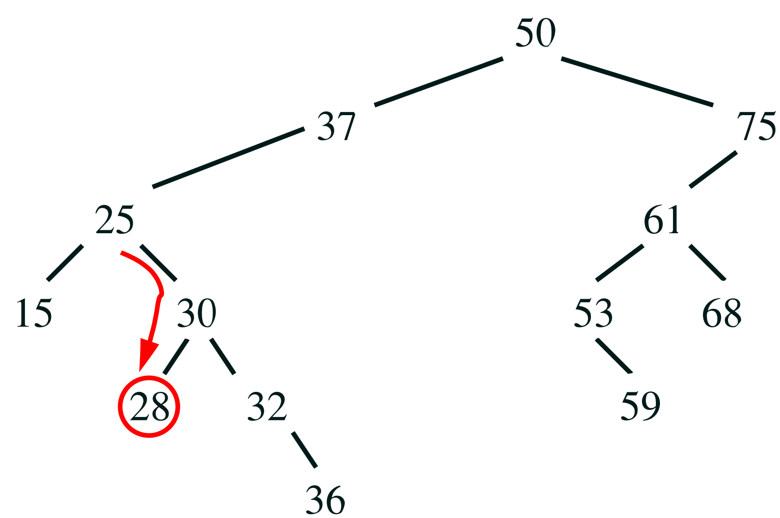


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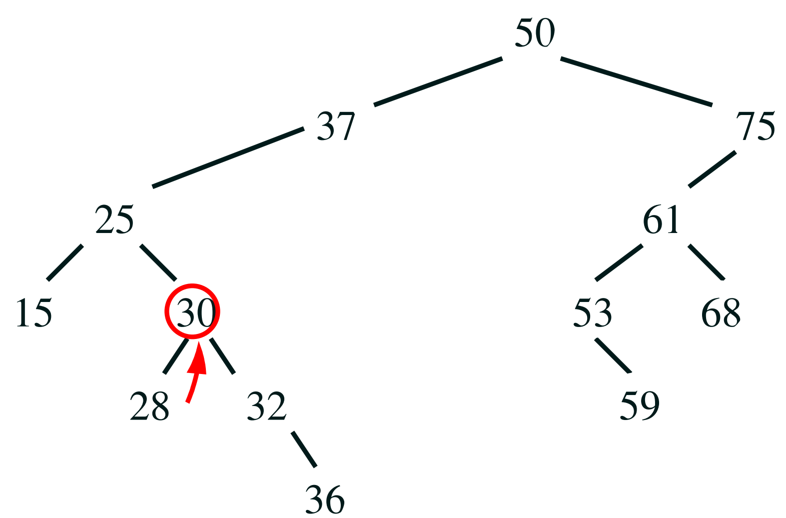


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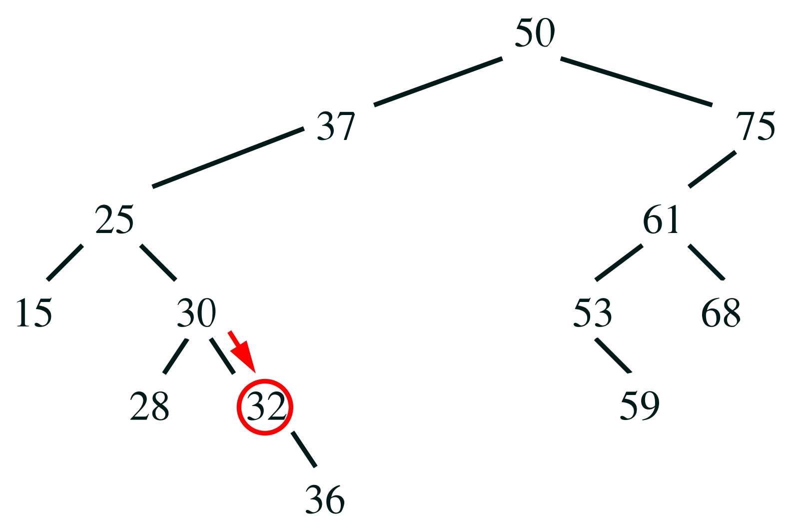


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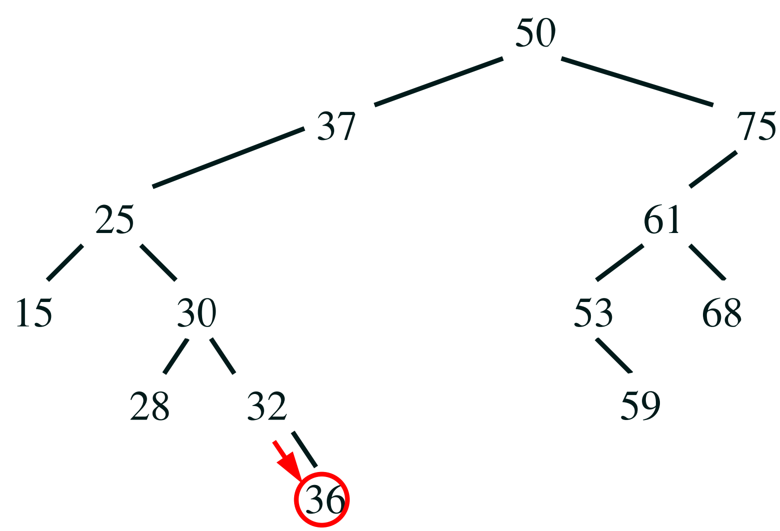


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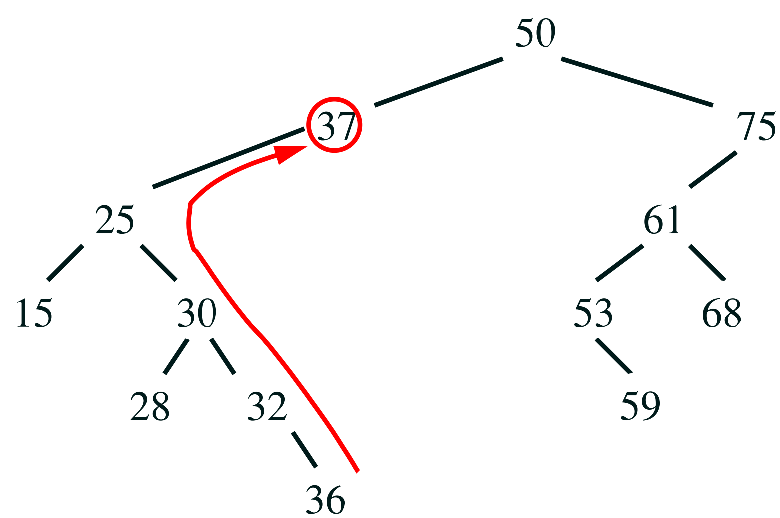


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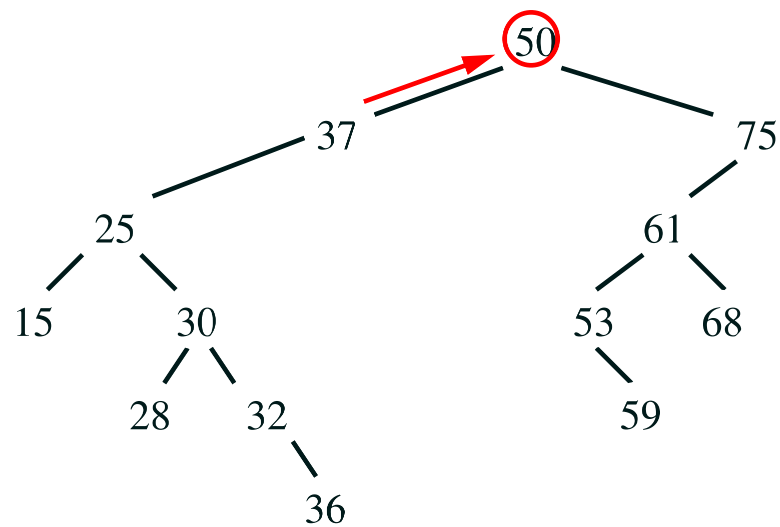


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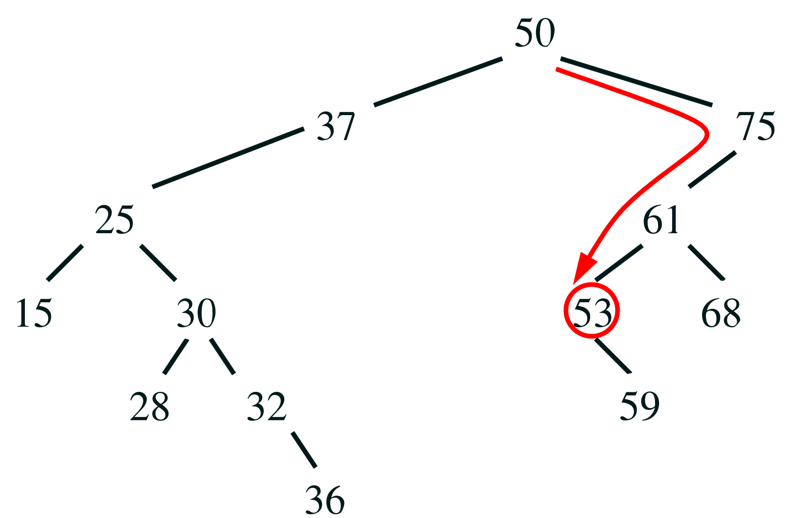


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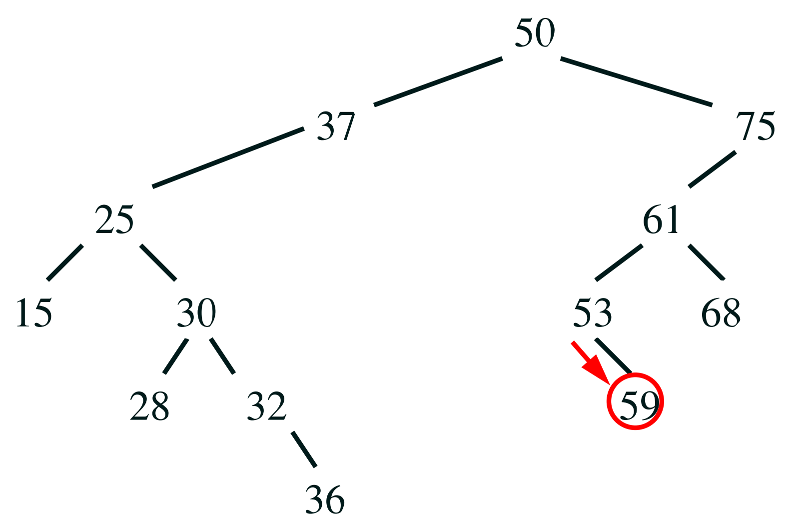


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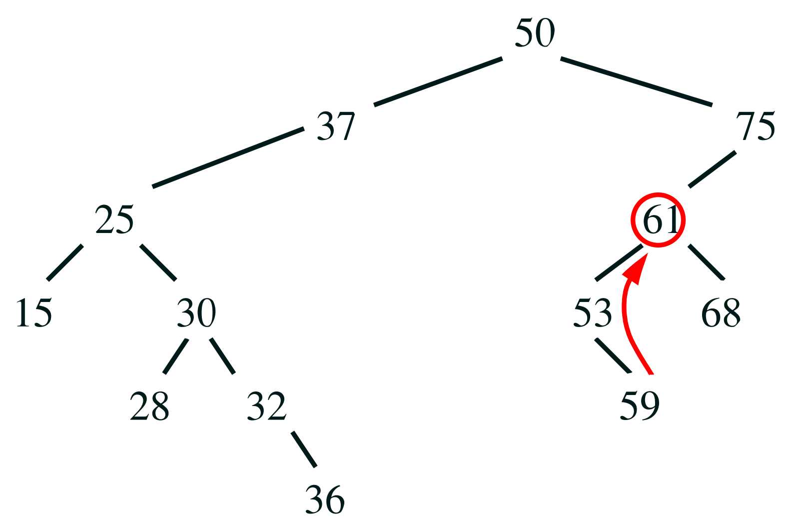


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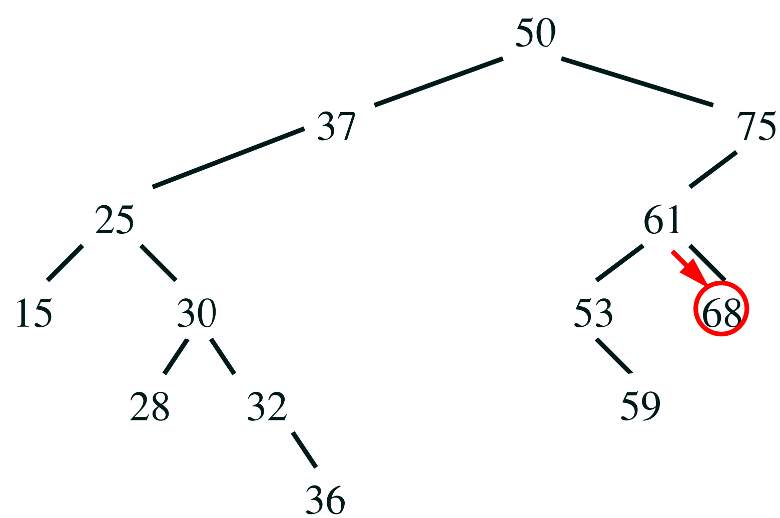


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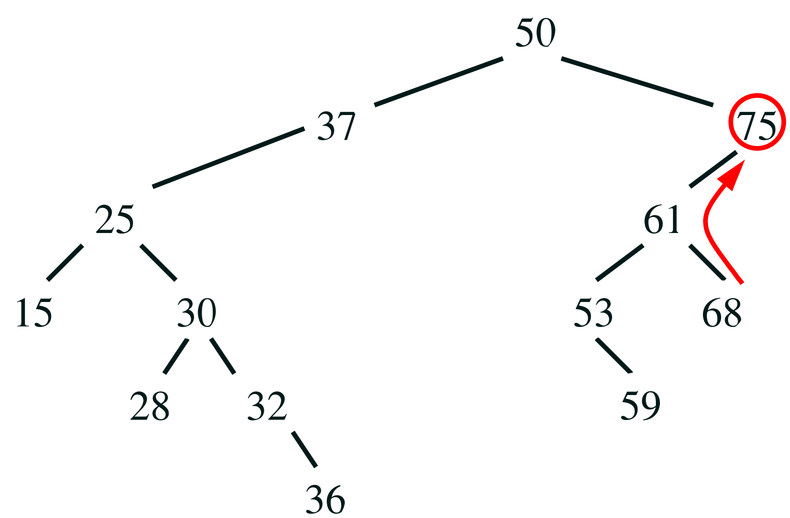


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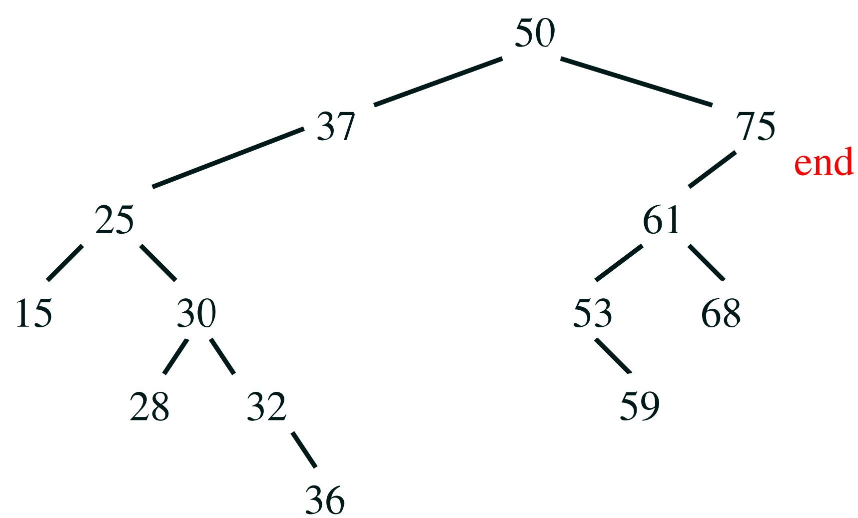


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# Lessons

- Trees and particularly binary trees are one of the most important tools of a computer scientist
- Conceptually they are quite simple
- However, there are a lot of details that need to be understood
- Coding even simple trees needs great care
- As we will see things get more complicated

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