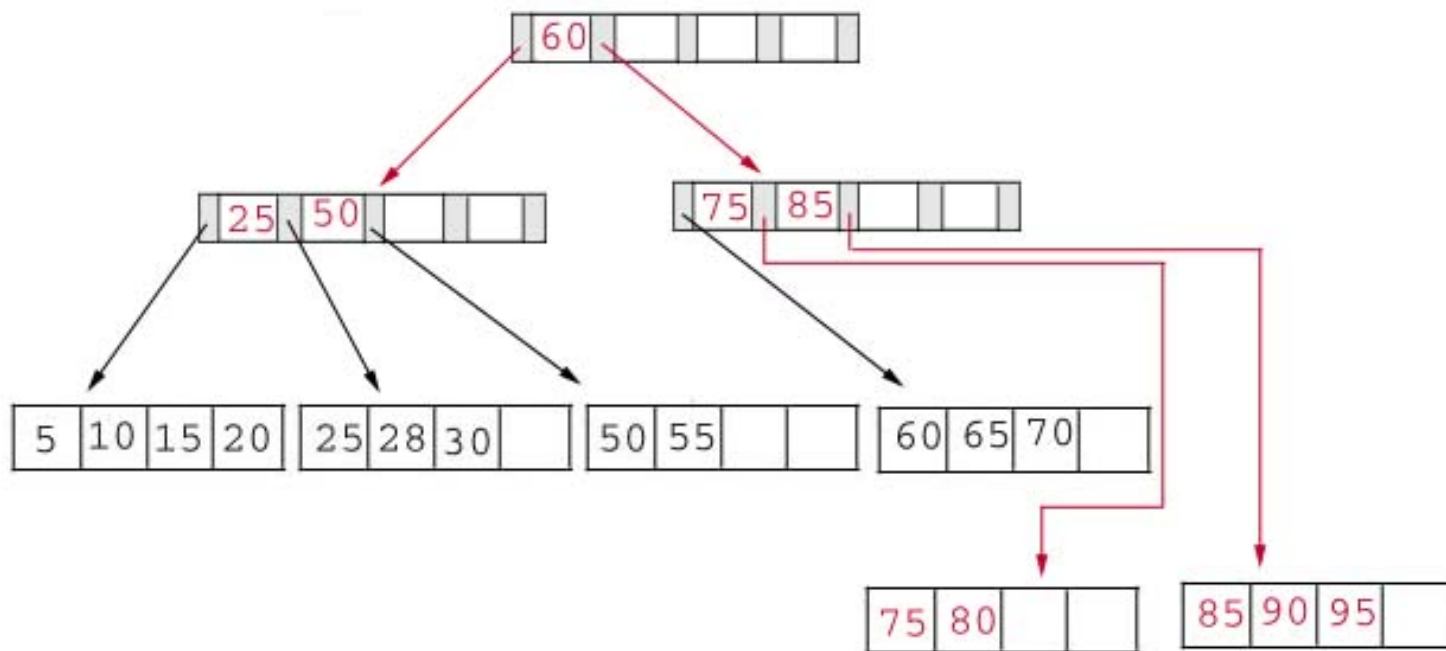


# Algorithms and Analysis

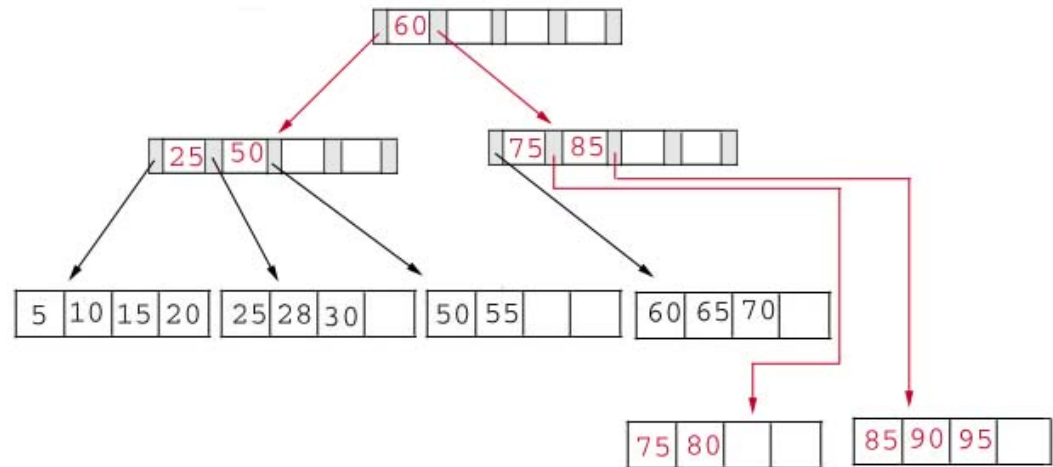
## Lesson 12: *Sometimes It Pays Not to Be Binary*



*B-Trees, Tries, Suffix Trees*

# Outline

1. **B-Trees**
2. Tries
3. Suffix Tree



# B-Trees

- **B-trees** are balanced trees for fast search, finding successors and predecessors, insert, delete, maximum, minimum, etc.
- Not to be confused with binary trees
- They are designed to keep related data close to each other in (disk) memory to minimise retrieval time
- Important when working with large amount of data that is stored on secondary storage (e.g. disks)
- Used extensively in databases

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# When Big-O Doesn't Work

- An underlying assumption of Big-O is that all elementary operations take roughly the same amount of time
- This just isn't true of disk look-up
- The typical time of an elementary operation on a modern processor is  $10^{-9}$  seconds
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# Accessing Data from Disk

- When accessing data from disk minimising the number of disk accesses is critical for good performance
- In database applications we want to store data as large sets
- Storing data in binary trees is disastrous as we typically need around  $\log_2(n)$  disk accesses before we locate our data
- It is not unusual in databases for  $n = 10\,000\,000$  so that  $\log_2(n) \approx 24$
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# Multiway-Trees

- To remedy this we can use M-way trees so that the access time is

$$\log_M(n) = \frac{\log_2(n)}{\log_2(M)}$$

- In practice we might use  $M \approx 200 \approx 2^8$  so we can reduce the depth of the tree by around a factor of 8
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- There are many variants of B-tree, all trying to squeeze a bit more performances from the basic structure

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# B<sup>+</sup> Tree

- A pretty basic implementation would obey the following rules
  1. The data items are stored at leaves
  2. The non-leaf nodes store up to  $M-1$  keys to guide the search:  
key  $i$  represents the smallest key in subtree  $i + 1$
  3. The root is either a leaf or has between 2 and  $M$  children
  4. All non-leaf nodes except the root have between  $\lceil M/2 \rceil$  and  $M$  children
  5. All leaves are at the same depth and have between  $\lceil L/2 \rceil$  and  $L$  data entries

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# Choosing $M$ and $L$

- The choice of  $M$  and  $L$  depends on the block size (the information read in one go from disk)
- It also depends on the type of data that is being stored (integer, reals, strings, etc.)
- $M$  and  $L$  might be in the hundreds or thousands
- In the examples below we consider tiny  $M = L = 5$  which is unrealistic, but drawable

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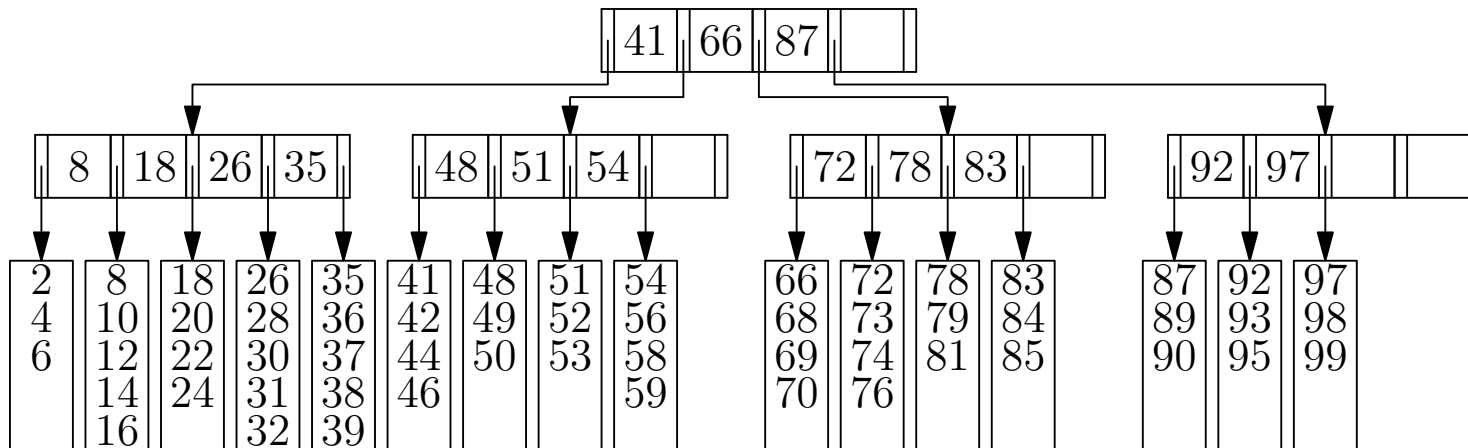
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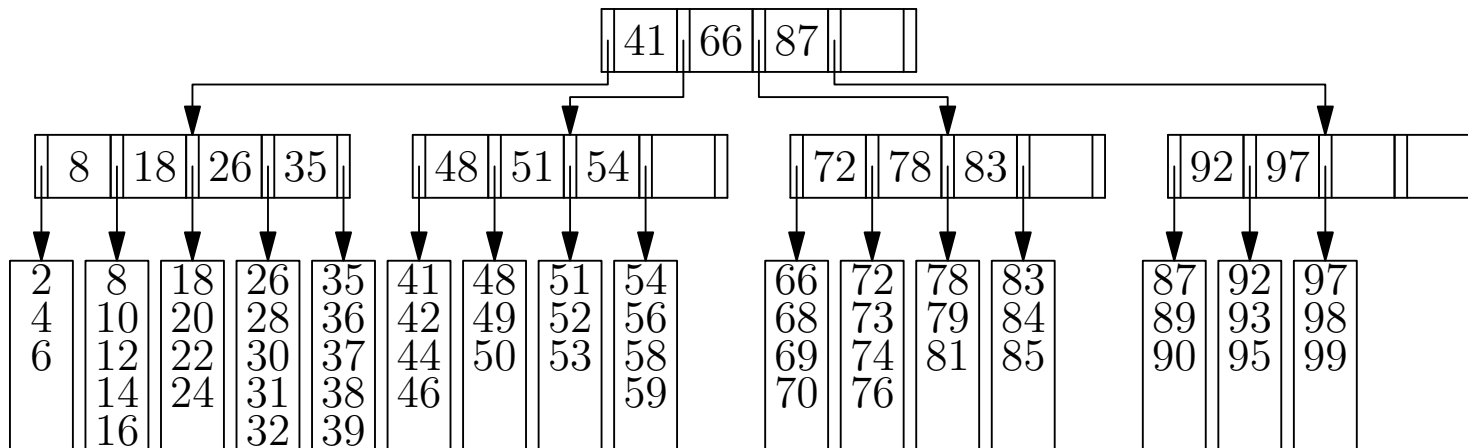




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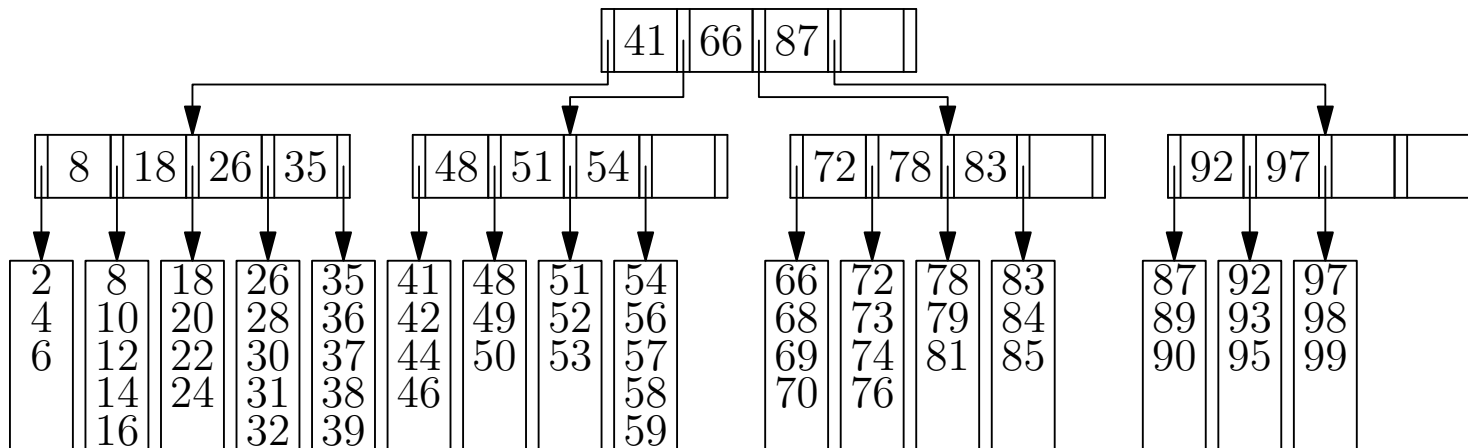
- $M = 5, L = 5$

insert(57)



# B-Tree Example

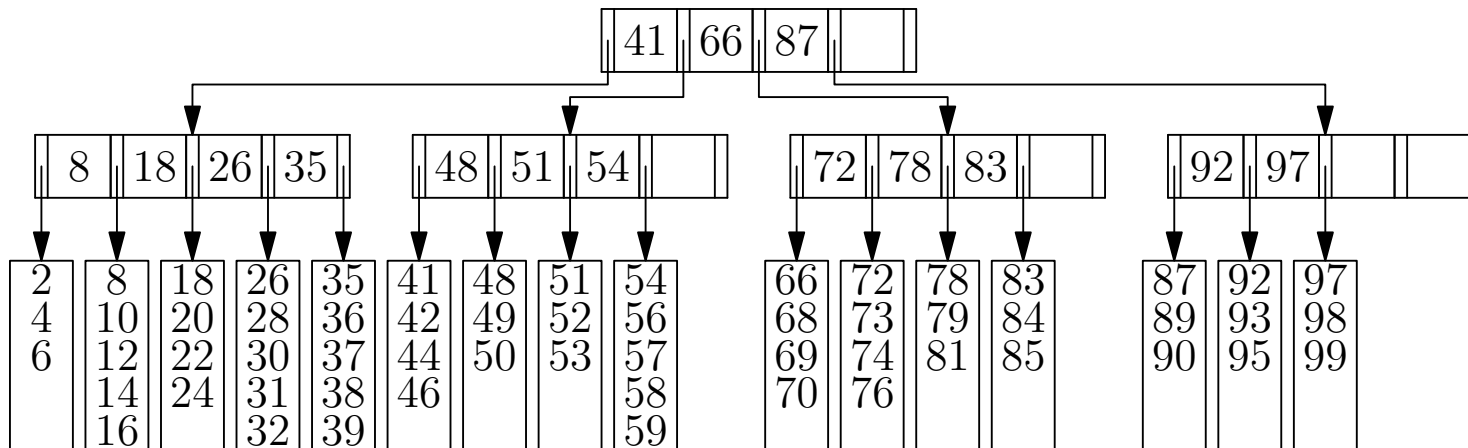
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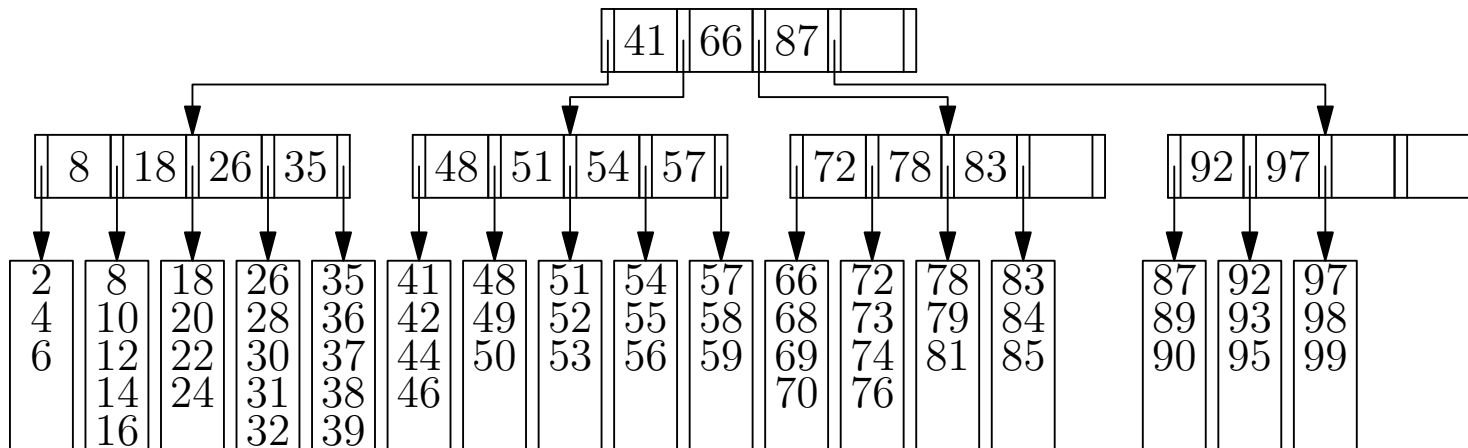
- $M = 5, L = 5$

insert(55)



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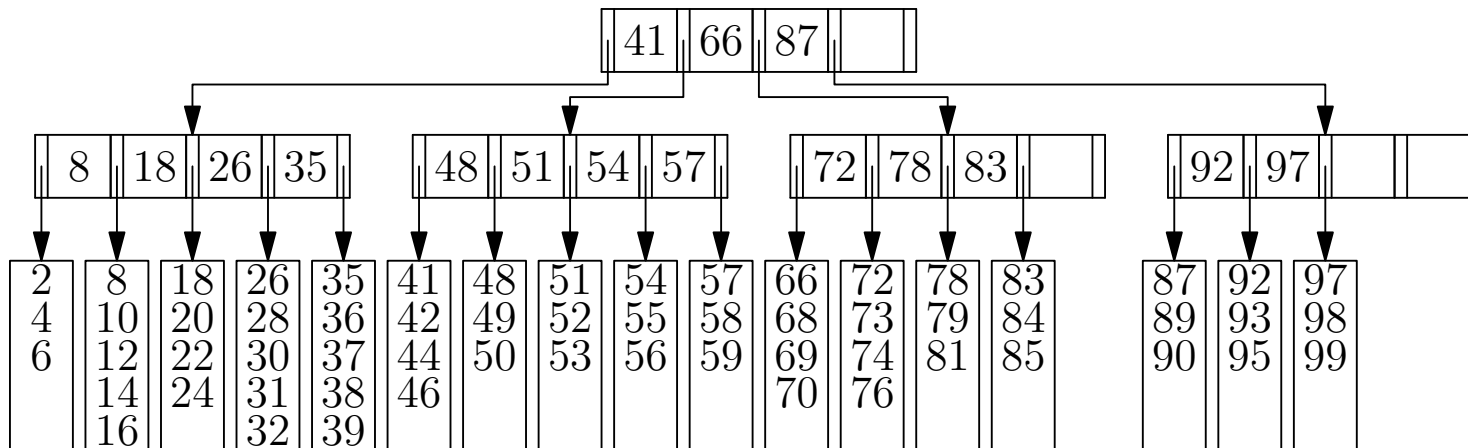
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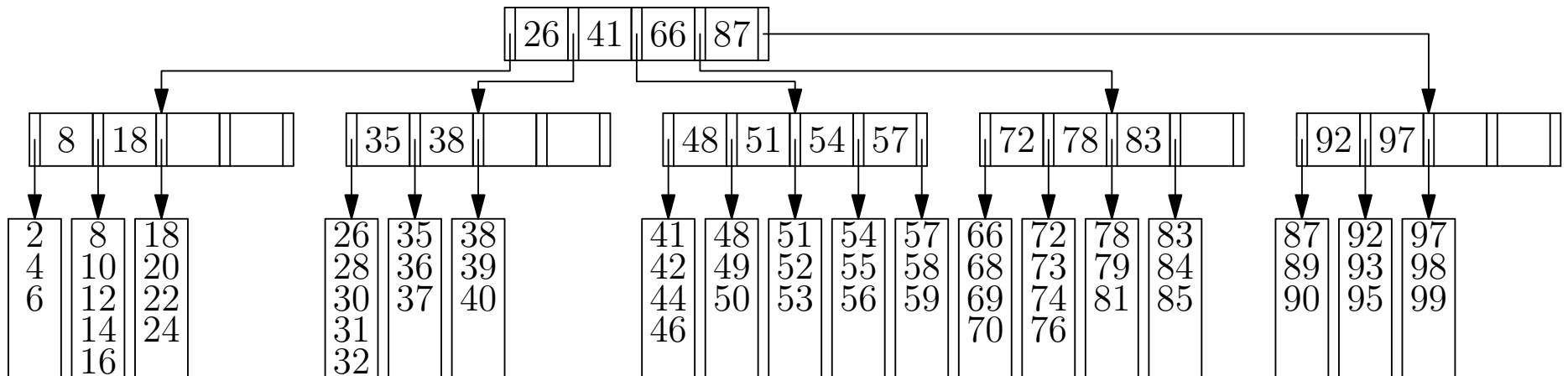
- $M = 5, L = 5$

insert(40)



# B-Tree Example

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# Other Changes

- If the root is full then it can be split into two and a new root created
- B-trees also have to allow the removal of records without losing its structure
- There are a number of variant strategies (e.g. neighbouring nodes can adopt a child if the current node cannot expand any more)
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# B-Tree Summary

- B-trees are an important data structure for databases where reducing the number of disk searches is vital
- They tend to be much more complex than the other data structures we have seen
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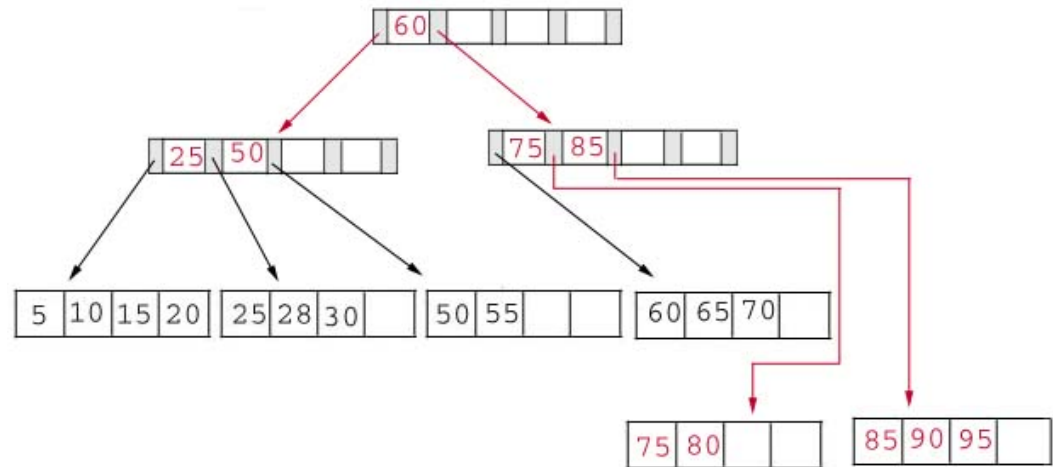
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# Outline

1. B-Trees
2. **Tries**
3. Suffix Tree



# Tries

- A **Trie** (pron. 'try') or **digital tree** is a multiway tree often used for storing large sets of words
- They are trees with a possible branch for every letter of an alphabet
- Their names comes from the word *retrieval*
- Tries usually compactify the edges in the tree
- All words end with a special letter “\$”



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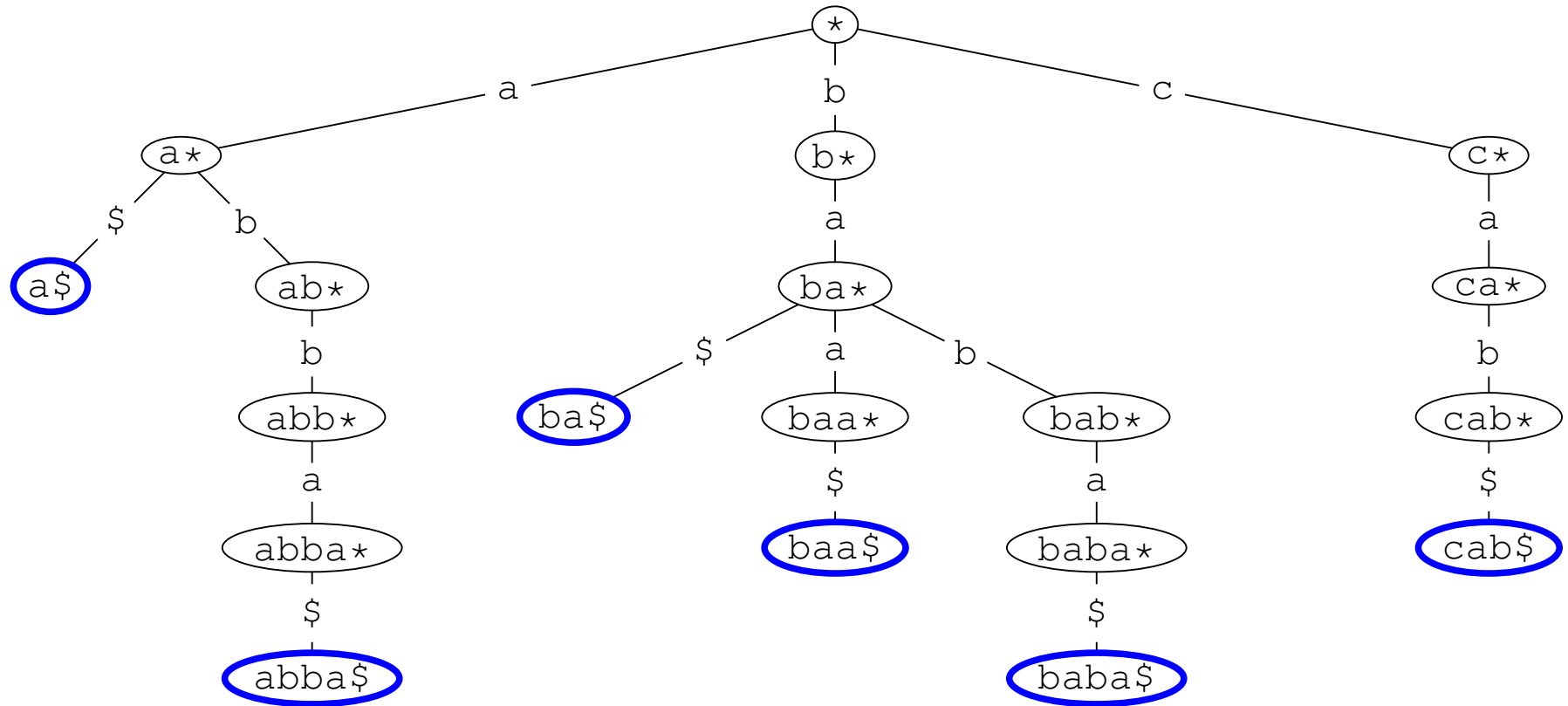
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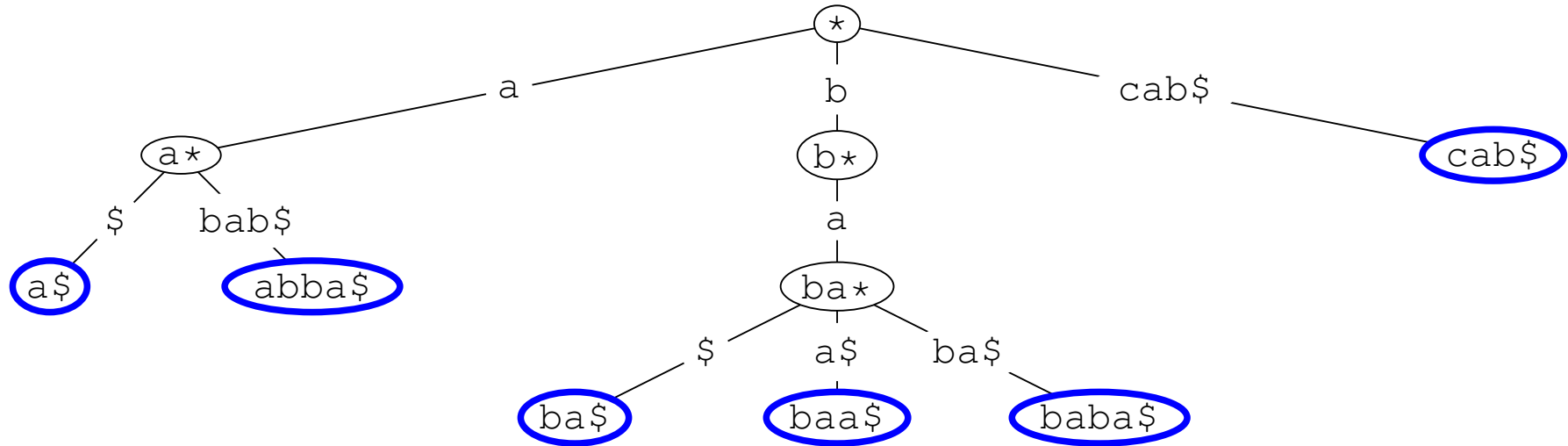
# Trie

{a\$, abba\$, ba\$, baa\$, baba\$, cab\$}



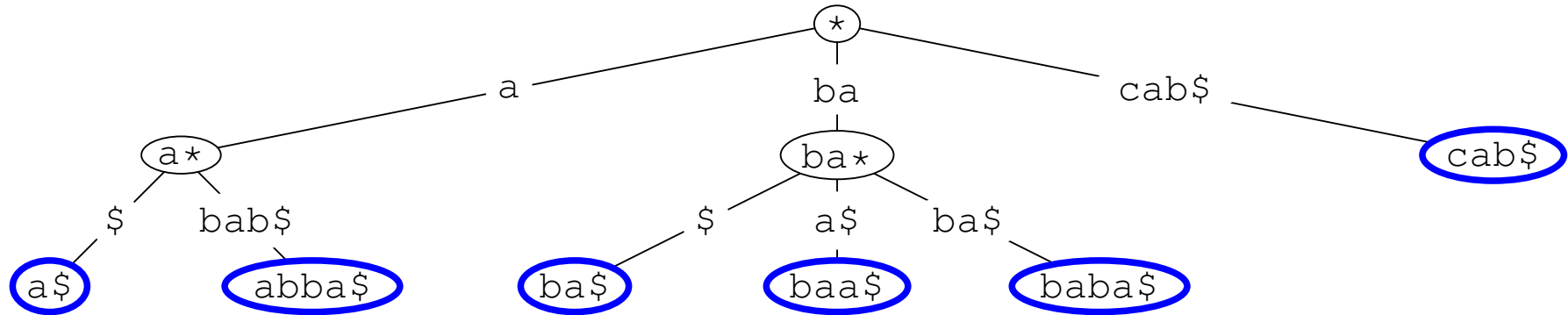
# Trie

{a, abba, ba, baa, baba, cab}



# Trie

{a, abba, ba, baa, baba, cab}



# Uses of Tries

- Tries are yet another way of implementing sets
- They provide quick insertion, deletion and find
- Typically considerably quicker than binary trees and hash tables
- They are particularly good for spell checkers, completion algorithms, longest-prefix matching, hyphenation
- Each search finds the longest match between the words in the set and the query



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# Trie for 31 Most Common English Words

\$
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z

# Trie for 31 Most Common English Words

add("THAT")

\$
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z

# Trie for 31 Most Common English Words

add("THAT")

	0
\$	
A	
B	
C	
D	
E	
F	
G	
H	
I	
J	
K	
L	
M	
N	
O	
P	
Q	
R	
S	
T	THAT\$
U	
V	
W	
X	
Y	
Z	

# Trie for 31 Most Common English Words

add("IN")

	0
\$	
A	
B	
C	
D	
E	
F	
G	
H	
I	
J	
K	
L	
M	
N	
O	
P	
Q	
R	
S	
T	THAT\$
U	
V	
W	
X	
Y	
Z	



# Trie for 31 Most Common English Words

add("IN")

	0
\$	
A	
B	
C	
D	
E	
F	
G	
H	
I	IN\$
J	
K	
L	
M	
N	
O	
P	
Q	
R	
S	
T	THAT\$
U	
V	
W	
X	
Y	
Z	

# Trie for 31 Most Common English Words

add("IT")

	0
\$	
A	
B	
C	
D	
E	
F	
G	
H	
I	IN\$
J	
K	
L	
M	
N	
O	
P	
Q	
R	
S	
T	THAT\$
U	
V	
W	
X	
Y	
Z	

# Trie for 31 Most Common English Words

add("IT")

	0	1
\$		
A		
B		
C		
D		
E		
F		
G		
H		
I	I* (1)	
J		
K		
L		
M		
N		IN\$
O		
P		
Q		
R		
S		
T	THAT\$	IT\$
U		
V		
W		
X		
Y		
Z		

# Trie for 31 Most Common English Words

add("I")

	0	1
\$		
A		
B		
C		
D		
E		
F		
G		
H		
I	I* (1)	
J		
K		
L		
M		
N		IN\$
O		
P		
Q		
R		
S		
T	THAT\$	IT\$
U		
V		
W		
X		
Y		
Z		

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add("I")

	0	1
\$		I\$
A		
B		
C		
D		
E		
F		
G		
H		
I	I* (1)	
J		
K		
L		
M		
N		IN\$
O		
P		
Q		
R		
S		
T	THAT\$	IT\$
U		
V		
W		
X		
Y		
Z		

# Trie for 31 Most Common English Words

add("HAD")

	0	1
\$		I\$
A		
B		
C		
D		
E		
F		
G		
H		
I	I* (1)	
J		
K		
L		
M		
N		IN\$
O		
P		
Q		
R		
S		
T	THAT\$	IT\$
U		
V		
W		
X		
Y		
Z		

# Trie for 31 Most Common English Words

add("HAD")

	0	1
\$		I\$
A		
B		
C		
D		
E		
F		
G		
H	HAD\$	
I	I* (1)	
J		
K		
L		
M		
N		IN\$
O		
P		
Q		
R		
S		
T	THAT\$	IT\$
U		
V		
W		
X		
Y		
Z		

# Trie for 31 Most Common English Words

add("NOT")

	0	1
\$		I\$
A		
B		
C		
D		
E		
F		
G		
H	HAD\$	
I	I* (1)	
J		
K		
L		
M		
N		IN\$
O		
P		
Q		
R		
S		
T	THAT\$	IT\$
U		
V		
W		
X		
Y		
Z		



# Trie for 31 Most Common English Words

add("NOT")

	0	1
\$		I\$
A		
B		
C		
D		
E		
F		
G		
H	HAD\$	
I	I* (1)	
J		
K		
L		
M		
N	NOT\$	IN\$
O		
P		
Q		
R		
S		
T	THAT\$	IT\$
U		
V		
W		
X		
Y		
Z		

# Trie for 31 Most Common English Words

add("WITH")

	0	1
\$		I\$
A		
B		
C		
D		
E		
F		
G		
H	HAD\$	
I	I* (1)	
J		
K		
L		
M		
N	NOT\$	IN\$
O		
P		
Q		
R		
S		
T	THAT\$	IT\$
U		
V		
W		
X		
Y		
Z		

# Trie for 31 Most Common English Words

add("WITH")

	0	1
\$		I\$
A		
B		
C		
D		
E		
F		
G		
H	HAD\$	
I	I* (1)	
J		
K		
L		
M		
N	NOT\$	IN\$
O		
P		
Q		
R		
S		
T	THAT\$	IT\$
U		
V		
W	WITH\$	
X		
Y		
Z		

# Trie for 31 Most Common English Words

add("HIS")

	0	1
\$		I\$
A		
B		
C		
D		
E		
F		
G		
H	HAD\$	
I	I* (1)	
J		
K		
L		
M		
N	NOT\$	IN\$
O		
P		
Q		
R		
S		
T	THAT\$	IT\$
U		
V		
W	WITH\$	
X		
Y		
Z		

# Trie for 31 Most Common English Words

add("HIS")

	0	1	2
\$		I\$	
A			HAD\$
B			
C			
D			
E			
F			
G			
H	H* (2)		
I	I* (1)		HIS\$
J			
K			
L			
M			
N	NOT\$	IN\$	
O			
P			
Q			
R			
S			
T	THAT\$	IT\$	
U			
V			
W	WITH\$		
X			
Y			
Z			

# Trie for 31 Most Common English Words

add("HAVE")

	0	1	2
\$		I\$	
A			HAD\$
B			
C			
D			
E			
F			
G			
H	H* (2)		
I	I* (1)		HIS\$
J			
K			
L			
M			
N	NOT\$	IN\$	
O			
P			
Q			
R			
S			
T	THAT\$	IT\$	
U			
V			
W	WITH\$		
X			
Y			
Z			

# Trie for 31 Most Common English Words

add("HAVE")

	0	1	2	3
\$		I\$		
A			HA* (3)	
B				
C				
D				HAD\$
E				
F				
G				
H	H* (2)			
I	I* (1)		HIS\$	
J				
K				
L				
M				
N	NOT\$	IN\$		
O				
P				
Q				
R				
S				
T	THAT\$	IT\$		
U				
V				HAVE\$
W	WITH\$			
X				
Y				
Z				

# Trie for 31 Most Common English Words

add("A")

	0	1	2	3
\$		I\$		
A			HA* (3)	
B				
C				
D				HAD\$
E				
F				
G				
H	H* (2)			
I	I* (1)		HIS\$	
J				
K				
L				
M				
N	NOT\$	IN\$		
O				
P				
Q				
R				
S				
T	THAT\$	IT\$		
U				
V				HAVE\$
W	WITH\$			
X				
Y				
Z				



# Trie for 31 Most Common English Words

add("A")

	0	1	2	3
\$		I\$		
A	A\$		HA* (3)	
B				
C				
D				HAD\$
E				
F				
G				
H	H* (2)			
I	I* (1)		HIS\$	
J				
K				
L				
M				
N	NOT\$	IN\$		
O				
P				
Q				
R				
S				
T	THAT\$	IT\$		
U				
V				HAVE\$
W	WITH\$			
X				
Y				
Z				

# Trie for 31 Most Common English Words

add("WHICH")

	0	1	2	3
\$		I\$		
A	A\$		HA* (3)	
B				
C				
D				HAD\$
E				
F				
G				
H	H* (2)			
I	I* (1)		HIS\$	
J				
K				
L				
M				
N	NOT\$	IN\$		
O				
P				
Q				
R				
S				
T	THAT\$	IT\$		
U				
V				HAVE\$
W	WITH\$			
X				
Y				
Z				

# Trie for 31 Most Common English Words

add("WHICH")

	0	1	2	3	4
\$		I\$			
A	A\$		HA* (3)		
B					
C					
D				HAD\$	
E					
F					
G					
H	H* (2)				WHICH\$
I	I* (1)		HIS\$		WITH\$
J					
K					
L					
M					
N	NOT\$	IN\$			
O					
P					
Q					
R					
S					
T	THAT\$	IT\$			
U					
V				HAVE\$	
W	W* (4)				
X					
Y					
Z					

# Trie for 31 Most Common English Words

add("HER")

	0	1	2	3	4
\$		I\$			
A	A\$		HA* (3)		
B					
C					
D				HAD\$	
E					
F					
G					
H	H* (2)				WHICH\$
I	I* (1)		HIS\$		WITH\$
J					
K					
L					
M					
N	NOT\$	IN\$			
O					
P					
Q					
R					
S					
T	THAT\$	IT\$			
U					
V				HAVE\$	
W	W* (4)				
X					
Y					
Z					

# Trie for 31 Most Common English Words

add("HER")

	0	1	2	3	4
\$		I\$			
A	A\$		HA* (3)		
B					
C					
D				HAD\$	
E			HER\$		
F					
G					
H	H* (2)				WHICH\$
I	I* (1)		HIS\$		WITH\$
J					
K					
L					
M					
N	NOT\$	IN\$			
O					
P					
Q					
R					
S					
T	THAT\$	IT\$			
U					
V				HAVE\$	
W	W* (4)				
X					
Y					
Z					

# Trie for 31 Most Common English Words

add("AT")

	0	1	2	3	4
\$		I\$			
A	A\$		HA* (3)		
B					
C					
D				HAD\$	
E			HER\$		
F					
G					
H	H* (2)				WHICH\$
I	I* (1)		HIS\$		WITH\$
J					
K					
L					
M					
N	NOT\$	IN\$			
O					
P					
Q					
R					
S					
T	THAT\$	IT\$			
U					
V				HAVE\$	
W	W* (4)				
X					
Y					
Z					

# Trie for 31 Most Common English Words

add("AT")

	0	1	2	3	4	5
\$		I\$				A\$
A	A* (5)		HA* (3)			
B						
C						
D				HAD\$		
E			HER\$			
F						
G						
H	H* (2)				WHICH\$	
I	I* (1)		HIS\$		WITH\$	
J						
K						
L						
M						
N	NOT\$	IN\$				
O						
P						
Q						
R						
S						
T	THAT\$	IT\$				AT\$
U						
V				HAVE\$		
W	W* (4)					
X						
Y						
Z						

# Trie for 31 Most Common English Words

add("IS")

	0	1	2	3	4	5
\$		I\$				A\$
A	A* (5)		HA* (3)			
B						
C						
D				HAD\$		
E			HER\$			
F						
G						
H	H* (2)				WHICH\$	
I	I* (1)		HIS\$		WITH\$	
J						
K						
L						
M						
N	NOT\$	IN\$				
O						
P						
Q						
R						
S						
T	THAT\$	IT\$				AT\$
U						
V				HAVE\$		
W	W* (4)					
X						
Y						
Z						



# Trie for 31 Most Common English Words

add("IS")

	0	1	2	3	4	5
\$		I\$				A\$
A	A* (5)		HA* (3)			
B						
C						
D				HAD\$		
E			HER\$			
F						
G						
H	H* (2)				WHICH\$	
I	I* (1)		HIS\$		WITH\$	
J						
K						
L						
M						
N	NOT\$	IN\$				
O						
P						
Q						
R						
S		IS\$				
T	THAT\$	IT\$				AT\$
U						
V				HAVE\$		
W	W* (4)					
X						
Y						
Z						

# Trie for 31 Most Common English Words

add("AND")

	0	1	2	3	4	5
\$		I\$				A\$
A	A* (5)		HA* (3)			
B						
C						
D				HAD\$		
E			HER\$			
F						
G						
H	H* (2)				WHICH\$	
I	I* (1)		HIS\$		WITH\$	
J						
K						
L						
M						
N	NOT\$	IN\$				
O						
P						
Q						
R						
S		IS\$				
T	THAT\$	IT\$				AT\$
U						
V				HAVE\$		
W	W* (4)					
X						
Y						
Z						

# Trie for 31 Most Common English Words

add("AND")

	0	1	2	3	4	5
\$		I\$				A\$
A	A* (5)		HA* (3)			
B						
C						
D				HAD\$		
E			HER\$			
F						
G						
H	H* (2)				WHICH\$	
I	I* (1)		HIS\$		WITH\$	
J						
K						
L						
M						
N	NOT\$	IN\$				AND\$
O						
P						
Q						
R						
S		IS\$				
T	THAT\$	IT\$				AT\$
U						
V				HAVE\$		
W	W* (4)					
X						
Y						
Z						

# Trie for 31 Most Common English Words

add("BUT")

	0	1	2	3	4	5
\$		I\$				A\$
A	A* (5)		HA* (3)			
B						
C						
D				HAD\$		
E			HER\$			
F						
G						
H	H* (2)				WHICH\$	
I	I* (1)		HIS\$		WITH\$	
J						
K						
L						
M						
N	NOT\$	IN\$				AND\$
O						
P						
Q						
R						
S		IS\$				
T	THAT\$	IT\$				AT\$
U						
V				HAVE\$		
W	W* (4)					
X						
Y						
Z						

# Trie for 31 Most Common English Words

add("BUT")

	0	1	2	3	4	5
\$		I\$				A\$
A	A* (5)		HA* (3)			
B	BUT\$					
C						
D				HAD\$		
E			HER\$			
F						
G						
H	H* (2)				WHICH\$	
I	I* (1)		HIS\$		WITH\$	
J						
K						
L						
M						
N	NOT\$	IN\$				AND\$
O						
P						
Q						
R						
S		IS\$				
T	THAT\$	IT\$				AT\$
U						
V				HAVE\$		
W	W* (4)					
X						
Y						
Z						

# Trie for 31 Most Common English Words

add("THE")

	0	1	2	3	4	5
\$		I\$				A\$
A	A* (5)		HA* (3)			
B	BUT\$					
C						
D				HAD\$		
E			HER\$			
F						
G						
H	H* (2)				WHICH\$	
I	I* (1)		HIS\$		WITH\$	
J						
K						
L						
M						
N	NOT\$	IN\$				AND\$
O						
P						
Q						
R						
S		IS\$				
T	THAT\$	IT\$				AT\$
U						
V				HAVE\$		
W	W* (4)					
X						
Y						
Z						

# Trie for 31 Most Common English Words

add("THE")

	0	1	2	3	4	5	6
\$		I\$				A\$	
A	A* (5)		HA* (3)				THAT\$
B	BUT\$						
C							
D				HAD\$			
E			HER\$				THE\$
F							
G							
H	H* (2)				WHICH\$		
I	I* (1)		HIS\$		WITH\$		
J							
K							
L							
M							
N	NOT\$	IN\$				AND\$	
O							
P							
Q							
R							
S		IS\$					
T	TH* (6)	IT\$				AT\$	
U							
V				HAVE\$			
W	W* (4)						
X							
Y							
Z							

# Trie for 31 Most Common English Words

add("FOR")

	0	1	2	3	4	5	6
\$		I\$				A\$	
A	A* (5)		HA* (3)				THAT\$
B	BUT\$						
C							
D				HAD\$			
E			HER\$				THE\$
F							
G							
H	H* (2)				WHICH\$		
I	I* (1)		HIS\$		WITH\$		
J							
K							
L							
M							
N	NOT\$	IN\$				AND\$	
O							
P							
Q							
R							
S		IS\$					
T	TH* (6)	IT\$				AT\$	
U							
V				HAVE\$			
W	W* (4)						
X							
Y							
Z							



# Trie for 31 Most Common English Words

add("FOR")

	0	1	2	3	4	5	6
\$		I\$				A\$	
A	A* (5)		HA* (3)				THAT\$
B	BUT\$						
C							
D				HAD\$			
E			HER\$				THE\$
F	FOR\$						
G							
H	H* (2)				WHICH\$		
I	I* (1)		HIS\$		WITH\$		
J							
K							
L							
M							
N	NOT\$	IN\$				AND\$	
O							
P							
Q							
R							
S		IS\$					
T	TH* (6)	IT\$				AT\$	
U							
V				HAVE\$			
W	W* (4)						
X							
Y							
Z							

# Trie for 31 Most Common English Words

add("ON")

	0	1	2	3	4	5	6
\$		I\$				A\$	
A	A* (5)		HA* (3)				THAT\$
B	BUT\$						
C							
D				HAD\$			
E			HER\$				THE\$
F	FOR\$						
G							
H	H* (2)				WHICH\$		
I	I* (1)		HIS\$		WITH\$		
J							
K							
L							
M							
N	NOT\$	IN\$				AND\$	
O							
P							
Q							
R							
S		IS\$					
T	TH* (6)	IT\$				AT\$	
U							
V				HAVE\$			
W	W* (4)						
X							
Y							
Z							

# Trie for 31 Most Common English Words

add("ON")

	0	1	2	3	4	5	6
\$		I\$				A\$	
A	A* (5)		HA* (3)				THAT\$
B	BUT\$						
C							
D				HAD\$			
E			HER\$				THE\$
F	FOR\$						
G							
H	H* (2)				WHICH\$		
I	I* (1)		HIS\$		WITH\$		
J							
K							
L							
M							
N	NOT\$	IN\$				AND\$	
O	ON\$						
P							
Q							
R							
S		IS\$					
T	TH* (6)	IT\$				AT\$	
U							
V				HAVE\$			
W	W* (4)						
X							
Y							
Z							

# Trie for 31 Most Common English Words

add("HE")

	0	1	2	3	4	5	6
\$		I\$				A\$	
A	A* (5)		HA* (3)				THAT\$
B	BUT\$						
C							
D				HAD\$			
E			HER\$				THE\$
F	FOR\$						
G							
H	H* (2)				WHICH\$		
I	I* (1)		HIS\$		WITH\$		
J							
K							
L							
M							
N	NOT\$	IN\$				AND\$	
O	ON\$						
P							
Q							
R							
S		IS\$					
T	TH* (6)	IT\$				AT\$	
U							
V				HAVE\$			
W	W* (4)						
X							
Y							
Z							

# Trie for 31 Most Common English Words

add("HE")

	0	1	2	3	4	5	6	7
\$		I\$				A\$		HE\$
A	A* (5)		HA* (3)				THAT\$	
B	BUT\$							
C								
D				HAD\$				
E			HE* (7)				THE\$	
F	FOR\$							
G								
H	H* (2)				WHICH\$			
I	I* (1)		HIS\$		WITH\$			
J								
K								
L								
M								
N	NOT\$	IN\$				AND\$		
O	ON\$							
P								
Q								
R								HER\$
S		IS\$						
T	TH* (6)	IT\$				AT\$		
U								
V				HAVE\$				
W	W* (4)							
X								
Y								
Z								

# Trie for 31 Most Common English Words

add("WAS")

	0	1	2	3	4	5	6	7
\$		I\$				A\$		HE\$
A	A* (5)		HA* (3)				THAT\$	
B	BUT\$							
C								
D				HAD\$				
E			HE* (7)				THE\$	
F	FOR\$							
G								
H	H* (2)				WHICH\$			
I	I* (1)		HIS\$		WITH\$			
J								
K								
L								
M								
N	NOT\$	IN\$				AND\$		
O	ON\$							
P								
Q								
R								HER\$
S		IS\$						
T	TH* (6)	IT\$				AT\$		
U								
V				HAVE\$				
W	W* (4)							
X								
Y								
Z								

# Trie for 31 Most Common English Words

add("WAS")

	0	1	2	3	4	5	6	7
\$		I\$				A\$		HE\$
A	A* (5)		HA* (3)		WAS\$		THAT\$	
B	BUT\$							
C								
D				HAD\$				
E			HE* (7)				THE\$	
F	FOR\$							
G								
H	H* (2)				WHICH\$			
I	I* (1)		HIS\$		WITH\$			
J								
K								
L								
M								
N	NOT\$	IN\$				AND\$		
O	ON\$							
P								
Q								
R								HER\$
S		IS\$						
T	TH* (6)	IT\$				AT\$		
U								
V				HAVE\$				
W	W* (4)							
X								
Y								
Z								

# Trie for 31 Most Common English Words

add("ARE")

	0	1	2	3	4	5	6	7
\$		I\$				A\$		HE\$
A	A* (5)		HA* (3)		WAS\$		THAT\$	
B	BUT\$							
C								
D				HAD\$				
E			HE* (7)				THE\$	
F	FOR\$							
G								
H	H* (2)				WHICH\$			
I	I* (1)		HIS\$		WITH\$			
J								
K								
L								
M								
N	NOT\$	IN\$				AND\$		
O	ON\$							
P								
Q								
R								HER\$
S		IS\$						
T	TH* (6)	IT\$				AT\$		
U								
V				HAVE\$				
W	W* (4)							
X								
Y								
Z								



# Trie for 31 Most Common English Words

add("ARE")

	0	1	2	3	4	5	6	7
\$		I\$				A\$		HE\$
A	A* (5)		HA* (3)		WAS\$		THAT\$	
B	BUT\$							
C								
D				HAD\$				
E			HE* (7)				THE\$	
F	FOR\$							
G								
H	H* (2)				WHICH\$			
I	I* (1)		HIS\$		WITH\$			
J								
K								
L								
M								
N	NOT\$	IN\$				AND\$		
O	ON\$							
P								
Q								
R						ARE\$		HER\$
S		IS\$						
T	TH* (6)	IT\$				AT\$		
U								
V				HAVE\$				
W	W* (4)							
X								
Y								
Z								

# Trie for 31 Most Common English Words

add("BY")

	0	1	2	3	4	5	6	7
\$		I\$				A\$		HE\$
A	A* (5)		HA* (3)		WAS\$		THAT\$	
B	BUT\$							
C								
D				HAD\$				
E			HE* (7)				THE\$	
F	FOR\$							
G								
H	H* (2)				WHICH\$			
I	I* (1)		HIS\$		WITH\$			
J								
K								
L								
M								
N	NOT\$	IN\$				AND\$		
O	ON\$							
P								
Q								
R						ARE\$		HER\$
S		IS\$						
T	TH* (6)	IT\$				AT\$		
U								
V				HAVE\$				
W	W* (4)							
X								
Y								
Z								

# Trie for 31 Most Common English Words

add("BY")

	0	1	2	3	4	5	6	7	8
\$		I\$				A\$		HE\$	
A	A* (5)		HA* (3)		WAS\$		THAT\$		
B	B* (8)								
C									
D				HAD\$					
E			HE* (7)				THE\$		
F	FOR\$								
G									
H	H* (2)				WHICH\$				
I	I* (1)		HIS\$		WITH\$				
J									
K									
L									
M									
N	NOT\$	IN\$				AND\$			
O	ON\$								
P									
Q									
R						ARE\$		HER\$	
S		IS\$							
T	TH* (6)	IT\$				AT\$			
U									BUT\$
V				HAVE\$					
W	W* (4)								
X									
Y									BY\$
Z									

# Trie for 31 Most Common English Words

add("OR")

	0	1	2	3	4	5	6	7	8
\$		I\$				A\$		HE\$	
A	A* (5)		HA* (3)		WAS\$		THAT\$		
B	B* (8)								
C									
D				HAD\$					
E			HE* (7)				THE\$		
F	FOR\$								
G									
H	H* (2)				WHICH\$				
I	I* (1)		HIS\$		WITH\$				
J									
K									
L									
M									
N	NOT\$	IN\$				AND\$			
O	ON\$								
P									
Q									
R						ARE\$		HER\$	
S		IS\$							
T	TH* (6)	IT\$				AT\$			
U									BUT\$
V				HAVE\$					
W	W* (4)								
X									
Y									BY\$
Z									

# Trie for 31 Most Common English Words

add("OR")

	0	1	2	3	4	5	6	7	8	9
\$		I\$				A\$		HE\$		
A	A* (5)		HA* (3)		WAS\$		THAT\$			
B	B* (8)									
C										
D				HAD\$						
E			HE* (7)				THE\$			
F	FOR\$									
G										
H	H* (2)				WHICH\$					
I	I* (1)		HIS\$		WITH\$					
J										
K										
L										
M										
N	NOT\$	IN\$				AND\$				ON\$
O	O* (9)									
P										
Q										
R						ARE\$		HER\$		OR\$
S		IS\$								
T	TH* (6)	IT\$				AT\$				
U									BUT\$	
V				HAVE\$						
W	W* (4)									
X										
Y									BY\$	
Z										

# Trie for 31 Most Common English Words

add("AS")

	0	1	2	3	4	5	6	7	8	9
\$		I\$				A\$		HE\$		
A	A* (5)		HA* (3)		WAS\$		THAT\$			
B	B* (8)									
C										
D				HAD\$						
E			HE* (7)				THE\$			
F	FOR\$									
G										
H	H* (2)				WHICH\$					
I	I* (1)		HIS\$		WITH\$					
J										
K										
L										
M										
N	NOT\$	IN\$				AND\$				ON\$
O	O* (9)									
P										
Q										
R						ARE\$		HER\$		OR\$
S		IS\$								
T	TH* (6)	IT\$				AT\$				
U									BUT\$	
V				HAVE\$						
W	W* (4)									
X										
Y									BY\$	
Z										

# Trie for 31 Most Common English Words

add("AS")

	0	1	2	3	4	5	6	7	8	9
\$		I\$				A\$		HE\$		
A	A* (5)		HA* (3)		WAS\$		THAT\$			
B	B* (8)									
C										
D				HAD\$						
E			HE* (7)				THE\$			
F	FOR\$									
G										
H	H* (2)				WHICH\$					
I	I* (1)		HIS\$		WITH\$					
J										
K										
L										
M										
N	NOT\$	IN\$				AND\$				ON\$
O	O* (9)									
P										
Q										
R						ARE\$		HER\$		OR\$
S		IS\$				AS\$				
T	TH* (6)	IT\$				AT\$				
U									BUT\$	
V				HAVE\$						
W	W* (4)									
X										
Y									BY\$	
Z										

# Trie for 31 Most Common English Words

add("THIS")

	0	1	2	3	4	5	6	7	8	9
\$		I\$				A\$		HE\$		
A	A* (5)		HA* (3)		WAS\$		THAT\$			
B	B* (8)									
C										
D				HAD\$						
E			HE* (7)				THE\$			
F	FOR\$									
G										
H	H* (2)				WHICH\$					
I	I* (1)		HIS\$		WITH\$					
J										
K										
L										
M										
N	NOT\$	IN\$				AND\$				ON\$
O	O* (9)									
P										
Q										
R						ARE\$		HER\$		OR\$
S		IS\$				AS\$				
T	TH* (6)	IT\$				AT\$				
U									BUT\$	
V				HAVE\$						
W	W* (4)									
X										
Y									BY\$	
Z										



# Trie for 31 Most Common English Words

add("THIS")

	0	1	2	3	4	5	6	7	8	9
\$		I\$				A\$		HE\$		
A	A* (5)		HA* (3)		WAS\$		THAT\$			
B	B* (8)									
C										
D				HAD\$						
E			HE* (7)				THE\$			
F	FOR\$									
G										
H	H* (2)				WHICH\$					
I	I* (1)		HIS\$		WITH\$		THIS\$			
J										
K										
L										
M										
N	NOT\$	IN\$				AND\$				ON\$
O	O* (9)									
P										
Q										
R						ARE\$		HER\$		OR\$
S		IS\$				AS\$				
T	TH* (6)	IT\$				AT\$				
U									BUT\$	
V				HAVE\$						
W	W* (4)									
X										
Y									BY\$	
Z										

# Trie for 31 Most Common English Words

add("YOU")

	0	1	2	3	4	5	6	7	8	9
\$		I\$				A\$		HE\$		
A	A* (5)		HA* (3)		WAS\$		THAT\$			
B	B* (8)									
C										
D				HAD\$						
E			HE* (7)				THE\$			
F	FOR\$									
G										
H	H* (2)				WHICH\$					
I	I* (1)		HIS\$		WITH\$		THIS\$			
J										
K										
L										
M										
N	NOT\$	IN\$				AND\$				ON\$
O	O* (9)									
P										
Q										
R						ARE\$		HER\$		OR\$
S		IS\$				AS\$				
T	TH* (6)	IT\$				AT\$				
U									BUT\$	
V				HAVE\$						
W	W* (4)									
X										
Y									BY\$	
Z										

# Trie for 31 Most Common English Words

add("YOU")

	0	1	2	3	4	5	6	7	8	9
\$		I\$				A\$		HE\$		
A	A* (5)		HA* (3)		WAS\$		THAT\$			
B	B* (8)									
C										
D				HAD\$						
E			HE* (7)				THE\$			
F	FOR\$									
G										
H	H* (2)				WHICH\$					
I	I* (1)		HIS\$		WITH\$		THIS\$			
J										
K										
L										
M										
N	NOT\$	IN\$				AND\$				ON\$
O	O* (9)									
P										
Q										
R						ARE\$		HER\$		OR\$
S		IS\$				AS\$				
T	TH* (6)	IT\$				AT\$				
U									BUT\$	
V				HAVE\$						
W	W* (4)									
X										
Y	YOU\$								BY\$	
Z										

# Trie for 31 Most Common English Words

add("BE")

	0	1	2	3	4	5	6	7	8	9
\$		I\$				A\$		HE\$		
A	A* (5)		HA* (3)		WAS\$		THAT\$			
B	B* (8)									
C										
D				HAD\$						
E			HE* (7)				THE\$			
F	FOR\$									
G										
H	H* (2)				WHICH\$					
I	I* (1)		HIS\$		WITH\$		THIS\$			
J										
K										
L										
M										
N	NOT\$	IN\$				AND\$				ON\$
O	O* (9)									
P										
Q										
R						ARE\$		HER\$		OR\$
S		IS\$				AS\$				
T	TH* (6)	IT\$				AT\$				
U									BUT\$	
V				HAVE\$						
W	W* (4)									
X										
Y	YOU\$								BY\$	
Z										

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add("BE")

	0	1	2	3	4	5	6	7	8	9
\$		I\$				A\$		HE\$		
A	A* (5)		HA* (3)		WAS\$		THAT\$			
B	B* (8)									
C										
D				HAD\$						
E			HE* (7)				THE\$		BE\$	
F	FOR\$									
G										
H	H* (2)				WHICH\$					
I	I* (1)		HIS\$		WITH\$		THIS\$			
J										
K										
L										
M										
N	NOT\$	IN\$				AND\$				ON\$
O	O* (9)									
P										
Q										
R						ARE\$		HER\$		OR\$
S		IS\$				AS\$				
T	TH* (6)	IT\$				AT\$				
U									BUT\$	
V				HAVE\$						
W	W* (4)									
X										
Y	YOU\$								BY\$	
Z										

# Trie for 31 Most Common English Words

add("FROM")

	0	1	2	3	4	5	6	7	8	9
\$		I\$				A\$		HE\$		
A	A* (5)		HA* (3)		WAS\$		THAT\$			
B	B* (8)									
C										
D				HAD\$						
E			HE* (7)				THE\$		BE\$	
F	FOR\$									
G										
H	H* (2)				WHICH\$					
I	I* (1)		HIS\$		WITH\$		THIS\$			
J										
K										
L										
M										
N	NOT\$	IN\$				AND\$				ON\$
O	O* (9)									
P										
Q										
R						ARE\$		HER\$		OR\$
S		IS\$				AS\$				
T	TH* (6)	IT\$				AT\$				
U									BUT\$	
V				HAVE\$						
W	W* (4)									
X										
Y	YOU\$								BY\$	
Z										

# Trie for 31 Most Common English Words

add("FROM")

	0	1	2	3	4	5	6	7	8	9	10
\$		I\$				A\$		HE\$			
A	A* (5)		HA* (3)		WAS\$		THAT\$				
B	B* (8)										
C											
D				HAD\$							
E			HE* (7)				THE\$		BE\$		
F	F* (10)										
G											
H	H* (2)				WHICH\$						
I	I* (1)		HIS\$		WITH\$		THIS\$				
J											
K											
L											
M											
N	NOT\$	IN\$				AND\$				ON\$	
O	O* (9)										FOR\$
P											
Q											
R						ARE\$		HER\$		OR\$	FROM\$
S		IS\$				AS\$					
T	TH* (6)	IT\$				AT\$					
U									BUT\$		
V				HAVE\$							
W	W* (4)										
X											
Y	YOU\$								BY\$		
Z											

# Trie for 31 Most Common English Words

add("OF")

	0	1	2	3	4	5	6	7	8	9	10
\$		I\$				A\$		HE\$			
A	A* (5)		HA* (3)		WAS\$		THAT\$				
B	B* (8)										
C											
D				HAD\$							
E			HE* (7)				THE\$		BE\$		
F	F* (10)										
G											
H	H* (2)				WHICH\$						
I	I* (1)		HIS\$		WITH\$		THIS\$				
J											
K											
L											
M											
N	NOT\$	IN\$				AND\$				ON\$	
O	O* (9)										FOR\$
P											
Q											
R						ARE\$		HER\$		OR\$	FROM\$
S		IS\$				AS\$					
T	TH* (6)	IT\$				AT\$					
U									BUT\$		
V				HAVE\$							
W	W* (4)										
X											
Y	YOU\$								BY\$		
Z											



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add("OF")

	0	1	2	3	4	5	6	7	8	9	10
\$		I\$				A\$		HE\$			
A	A* (5)		HA* (3)		WAS\$		THAT\$				
B	B* (8)										
C											
D				HAD\$							
E			HE* (7)				THE\$		BE\$		
F	F* (10)									OF\$	
G											
H	H* (2)				WHICH\$						
I	I* (1)		HIS\$		WITH\$		THIS\$				
J											
K											
L											
M											
N	NOT\$	IN\$				AND\$				ON\$	
O	O* (9)										FOR\$
P											
Q											
R						ARE\$		HER\$		OR\$	FROM\$
S		IS\$				AS\$					
T	TH* (6)	IT\$				AT\$					
U									BUT\$		
V				HAVE\$							
W	W* (4)										
X											
Y	YOU\$								BY\$		
Z											

# Disadvantage of Tries

- Table-based tries typically waste large amounts of memory
- Often table-based tries are used for the first few layers, while lower levels use a less memory intensive data structure
- These days memory is less of a problem so table-based tries are acceptable for some applications
- There are many implementations of tries each suited to a particular task

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# Binary Tries

- One extreme (though not uncommon) solution to address memory issues is to build a bit-level trie so the data-structure is a binary tree
- It differs from a binary tree in that the decisions to go left or right depends on the current bit
- Although you lose the advantage of a multiway tree (of reducing the depth) it does find the longest match and it speeds up finds which fail

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# Why Tries?

- Tries are a classic example of a trade-off between memory and computational complexity
- Tries are slightly specialist and tend to get used in very particular applications
  - ★ Finding longest matches
  - ★ Completion, spell checking, etc.
- A basic trie is not too complicated, however, . . .
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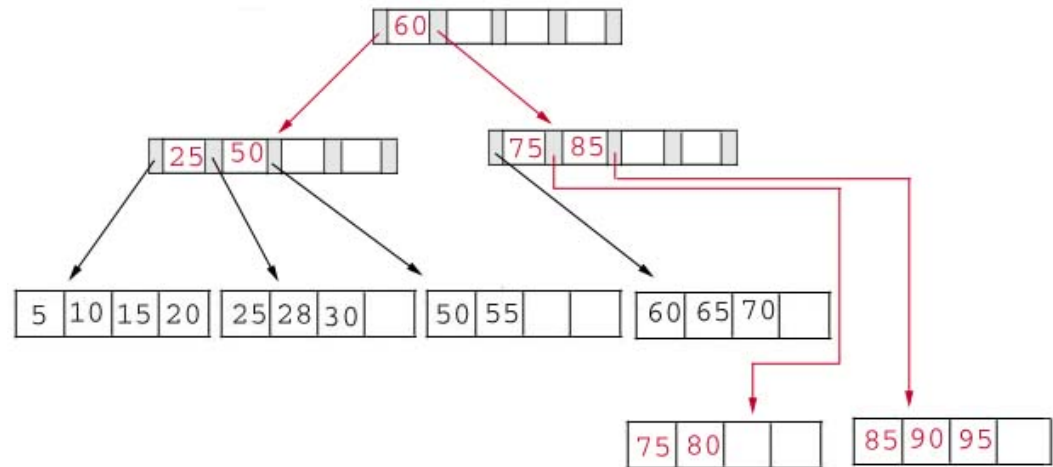
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# Outline

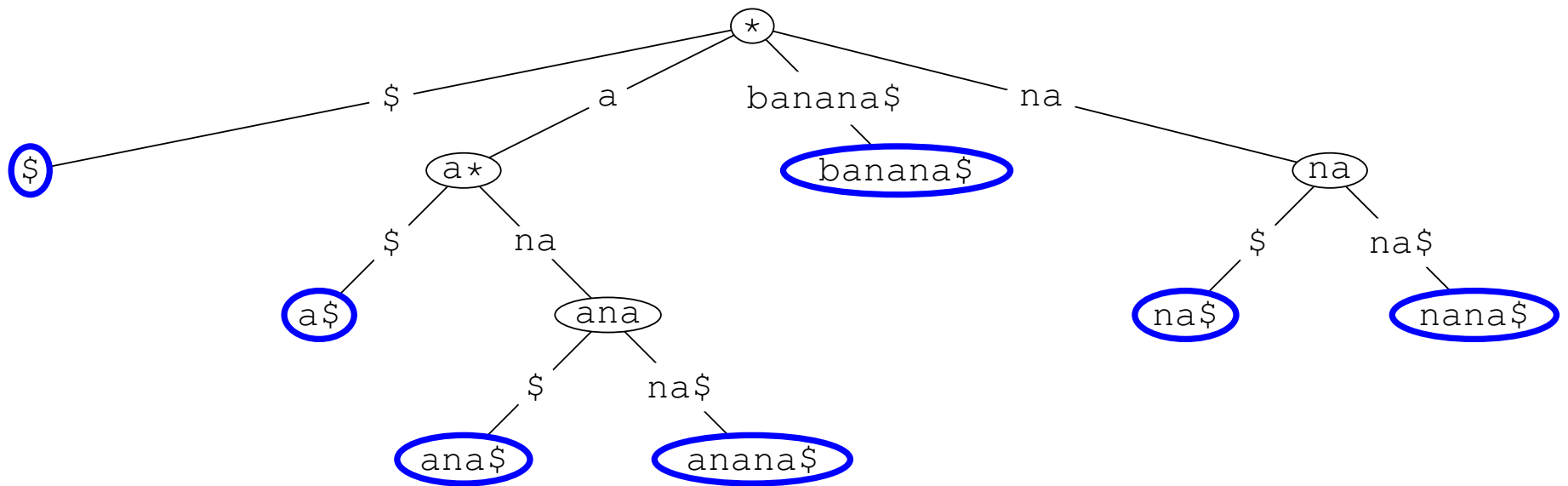
1. B-Trees
2. Tries
3. **Suffix Tree**



# Suffix Tree

- Suffix tree is a trie of all suffixes of a string
- E.g. banana

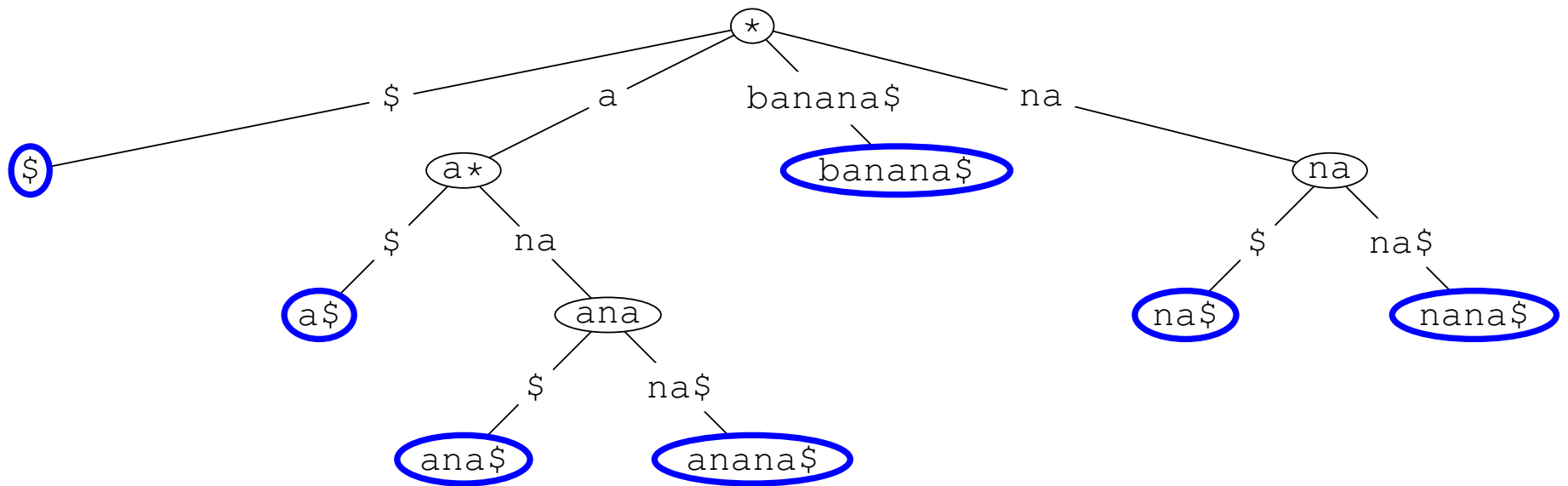
{\$, a\$, na\$, ana\$, nana\$, anana\$, banana\$}



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# Importance of Suffix Tree

- The first linear-time algorithm for computing suffix trees was proposed by Peter Weiner in 1973, a more space efficient algorithm was proposed by Edward M. McCreight in 1976
- Esko Ukkonen in 1995 proposed a variant of McCreight's algorithm, but in a way that was much easier to understand
- It really only got implemented after this
- They are very important for string-based algorithms
- The classic application is in finding a match for a query string,  $Q$ , in a text,  $T$



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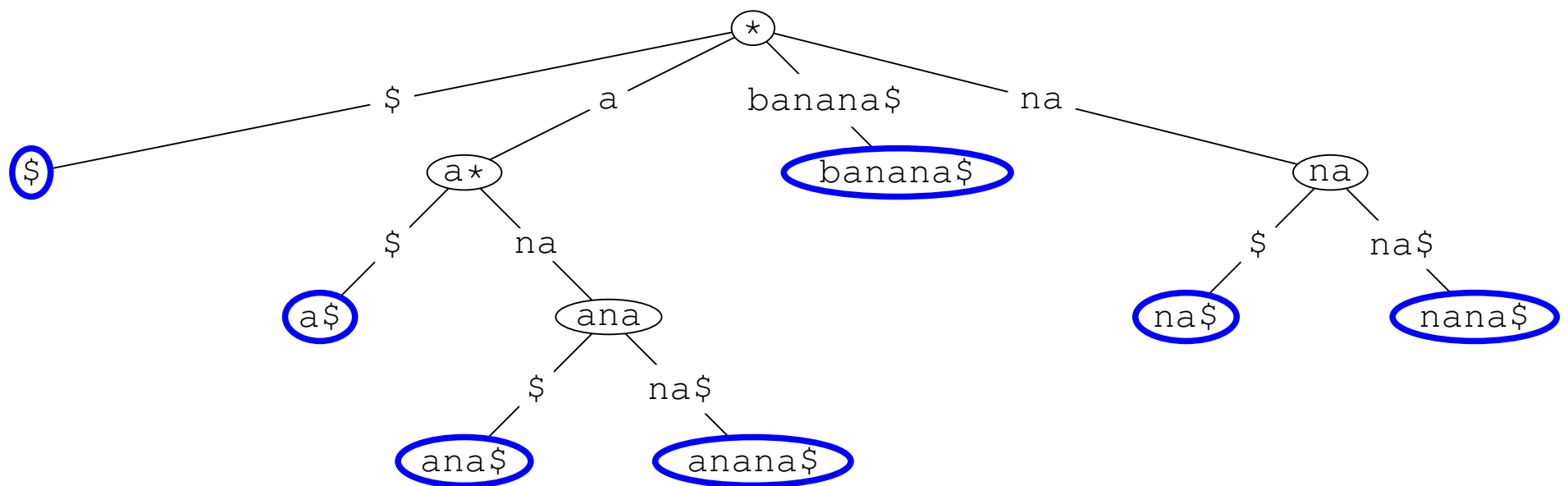
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# String Matching

- To find a match of a query string,  $Q$ , in a text,  $T$ , we can first construct the suffix tree of the string  $T$  we then simply look up the query,  $Q$ , using the trie

$\{\$, a\$, na\$, ana\$, nana\$, anana\$, banana\$\}$



# Complexity of Suffix Trees

- Using a regular trie for a suffix tree would typically use far too much memory to be useful
- However, by using pointers to the original text it is possible to build a suffix tree using  $O(n)$  memory where  $n$  is the length of the text
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# Uses of Suffix Trees

- Suffix trees are efficient whenever it is likely that you will do multiple searches
- Exact word matching is in itself a very important application
- Suffix trees in combination with dynamic programming (which we will eventually get to) can be used to do inexact matching (finding the match with the smallest edit distance)
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# Lessons

- Multiway trees can considerably speed up search over binary trees
- They are very important in some specialised applications (e.g. databases, spell-checking, completion, suffix trees)
- They are not as general purpose as binary search trees and are more complicated to implement
- But they can give the best performance

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- But they can give the best performance—sometimes performance matters enough to make it worthwhile implementing multiway trees