

Algorithms and Analysis

Lesson 9: *Make Friends with Trees*



Binary trees, binary search trees, sets, tree iterators

Outline

1. **Trees**
2. Binary Trees
 - Implementing Binary Trees
3. Binary Search Trees
 - Definition
 - Implementing a Set
4. Tree Iterators



Trees

- Trees are one of the major ways of structuring data
- They are used in a vast number of data structures
 - ★ Binary search trees
 - ★ B-trees
 - ★ splay trees
 - ★ heaps
 - ★ tries
 - ★ suffix trees
- We shall cover most of these

Trees

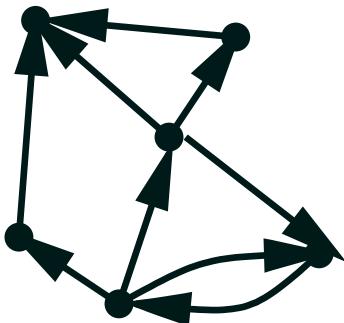
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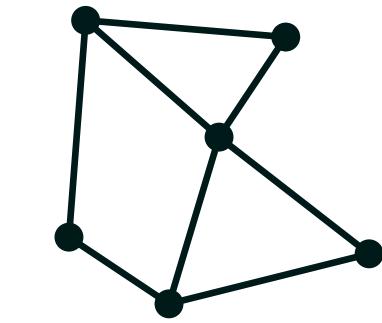
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Defining Trees

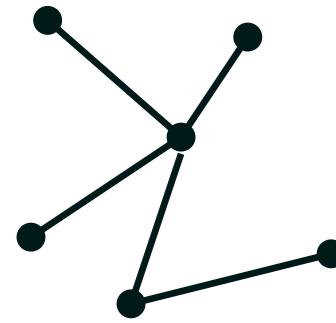
- Mathematically a tree is an **acyclic undirected graph**
 - ★ **graph**: a structure consisting of **nodes** or **vertices** joined by **edges**
 - ★ **undirected**: the edges goes both ways
 - ★ **acyclic**: there are no cycles in the graph



graph



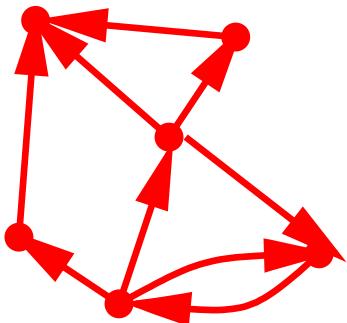
undirected graph



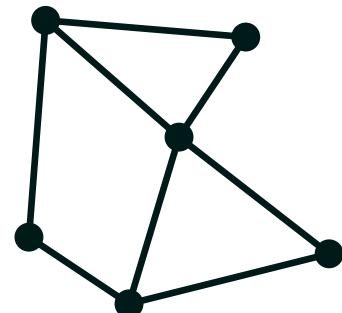
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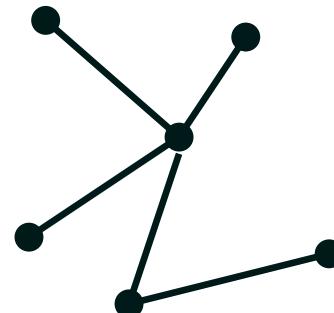
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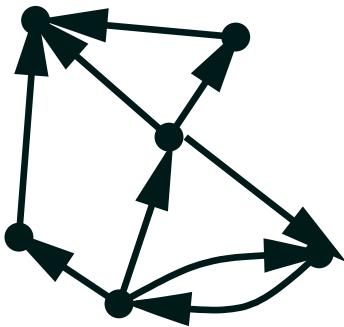
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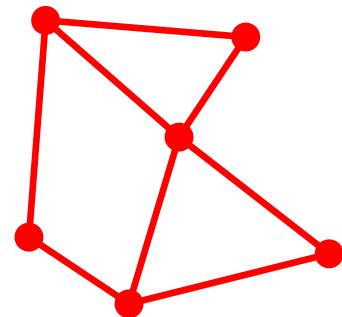
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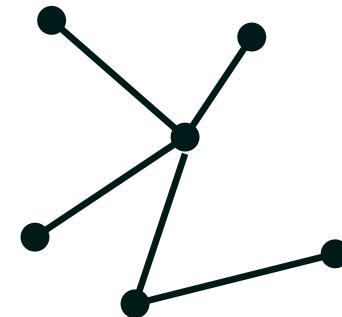
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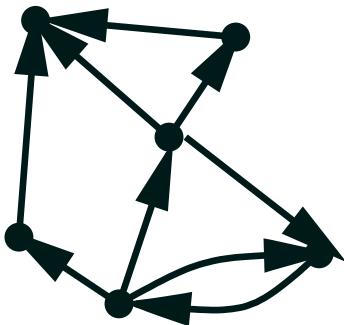
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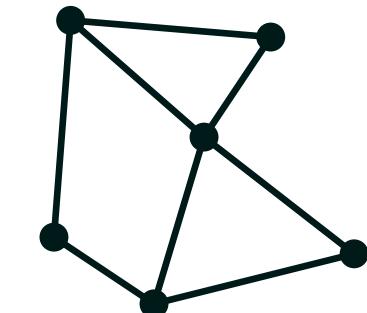
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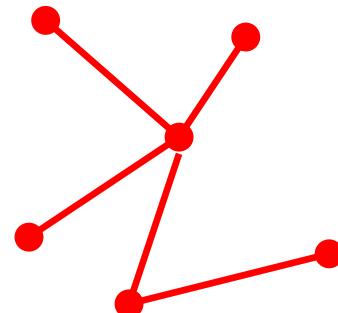
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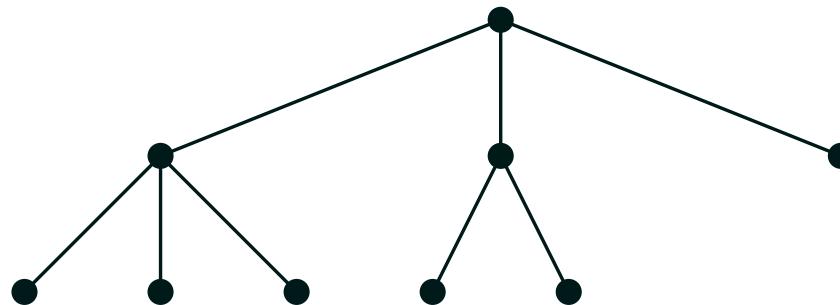
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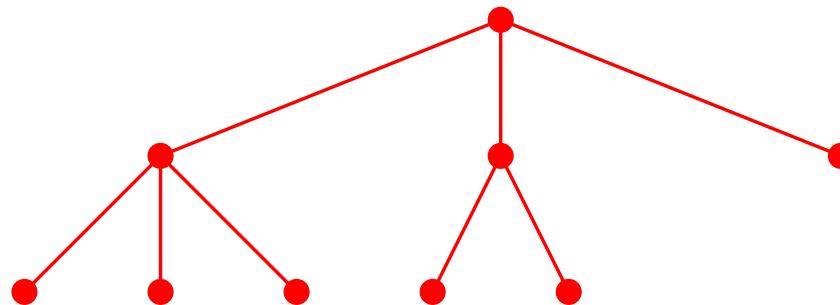
Borrowing from Nature

- We often impose an ordering on the nodes (or a direction on the edges)—known as a rooted tree
- Borrowing from nature, we recognise one node as the **root** node
- Nodes have **children** nodes living beneath them
- Each child has a **parent** node above them except the root
- Nodes with no children are **leaf** nodes



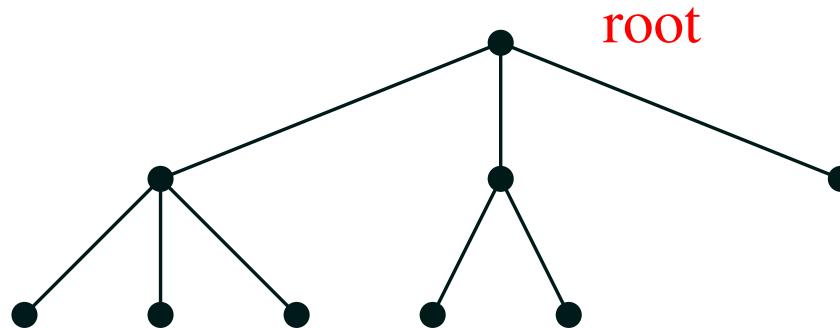
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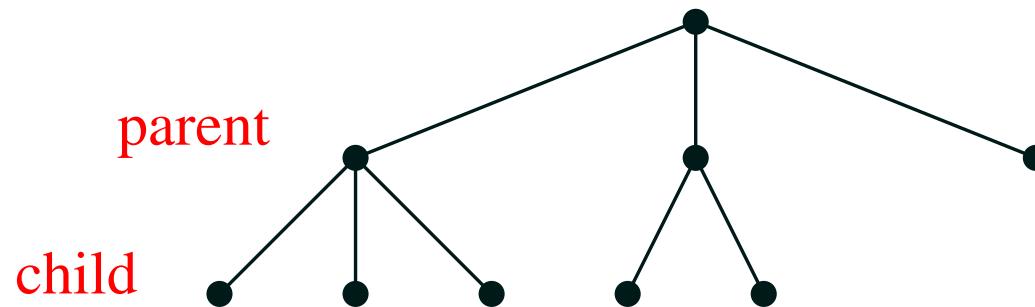
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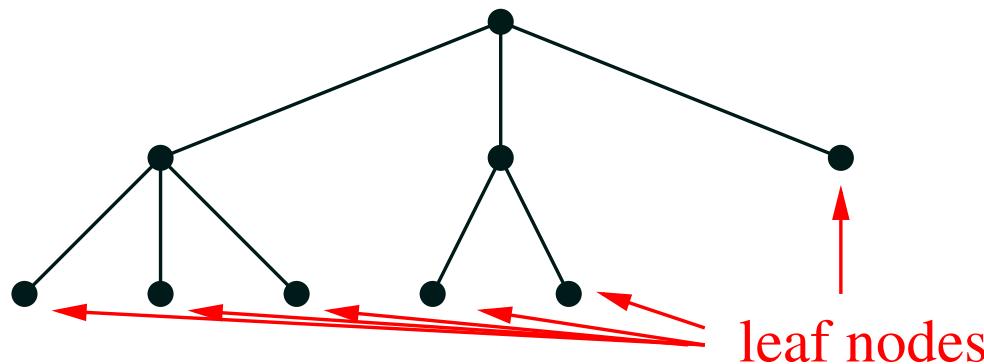
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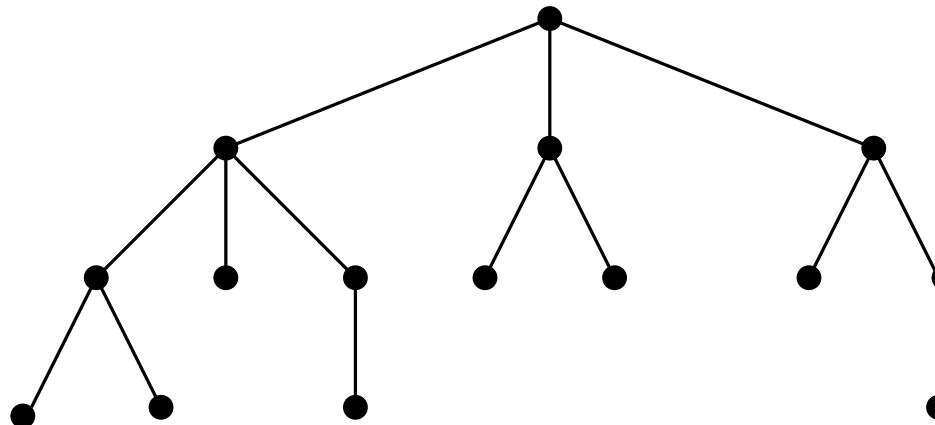


Spot the Error

- One small biological inconsistency

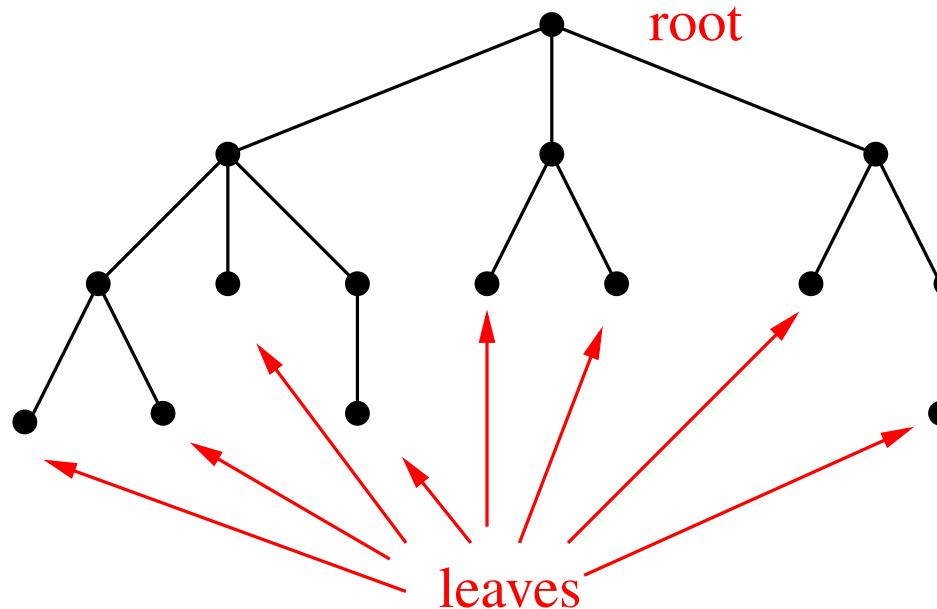
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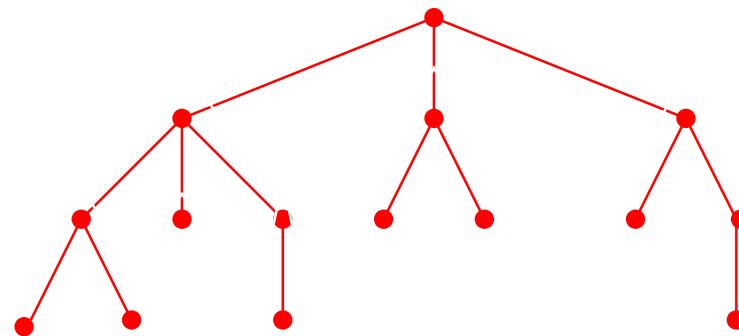
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 - ★ root at the top
 - ★ leaves at the bottom



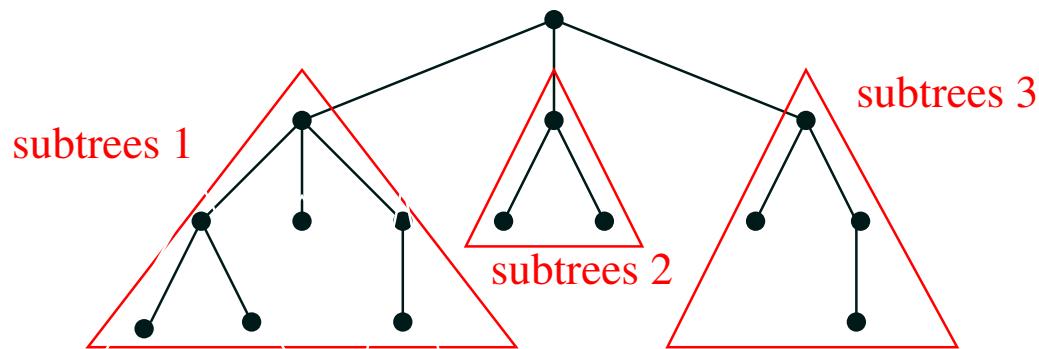
Subtrees

- We can think of the tree made up of **subtrees**



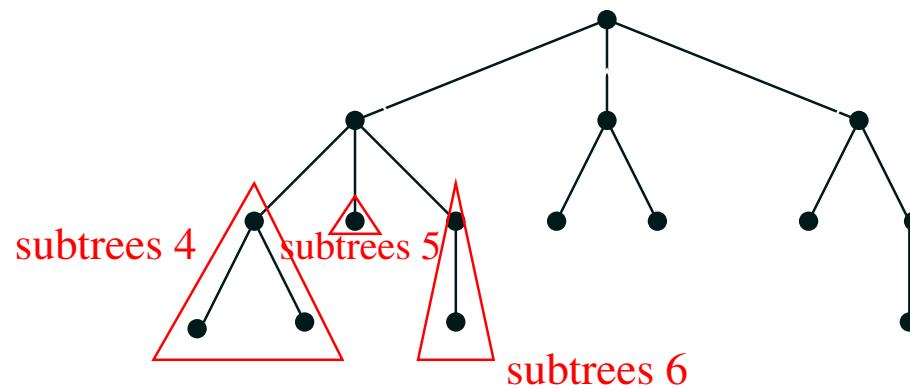
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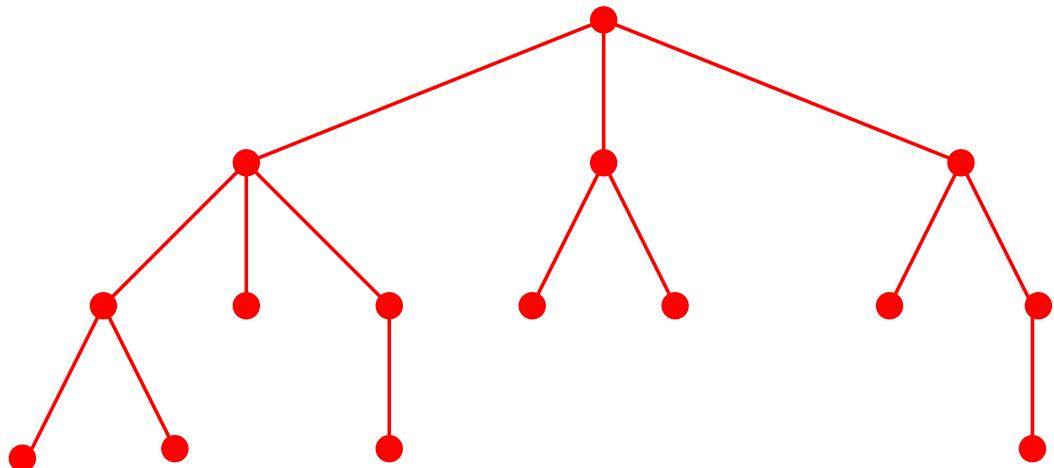
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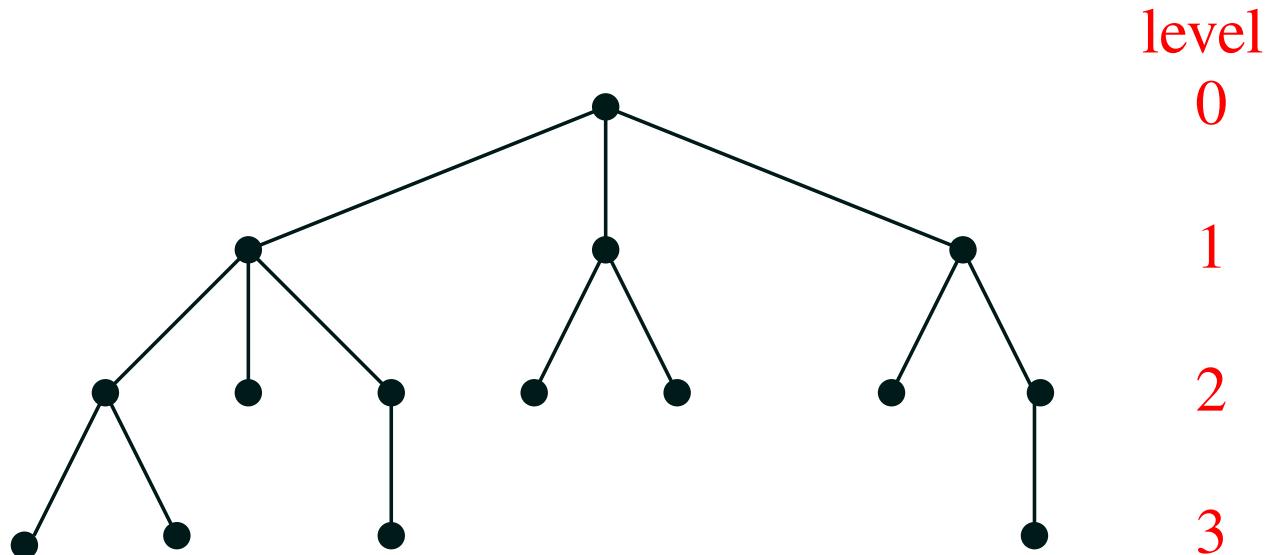
Level of Nodes

- It is useful to label different levels of the tree
- We take the **level** of a node in a tree as its distance from the root
- We take the **height** of a tree to be the number of levels



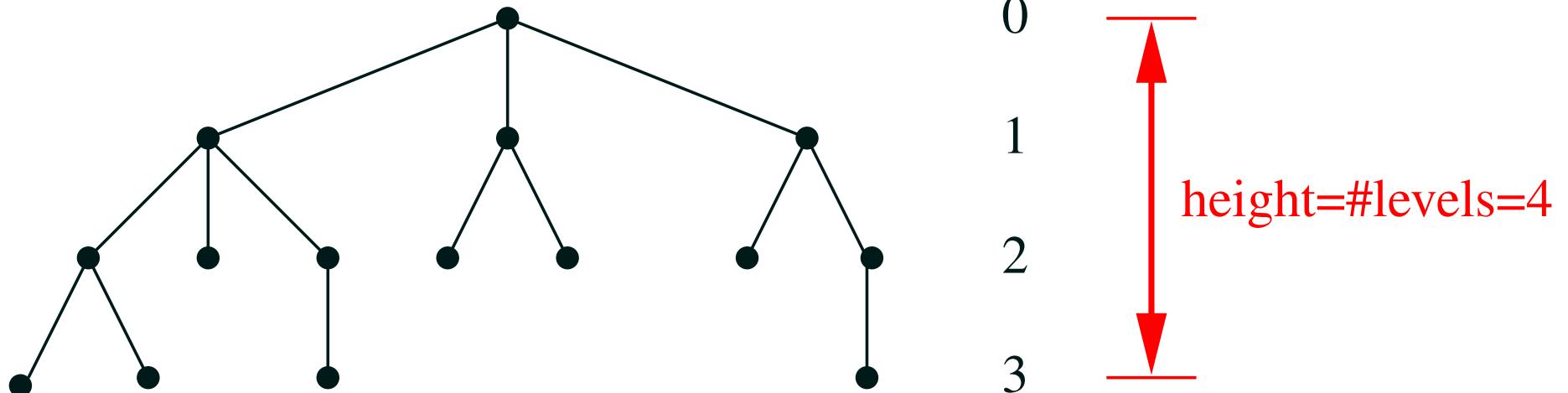
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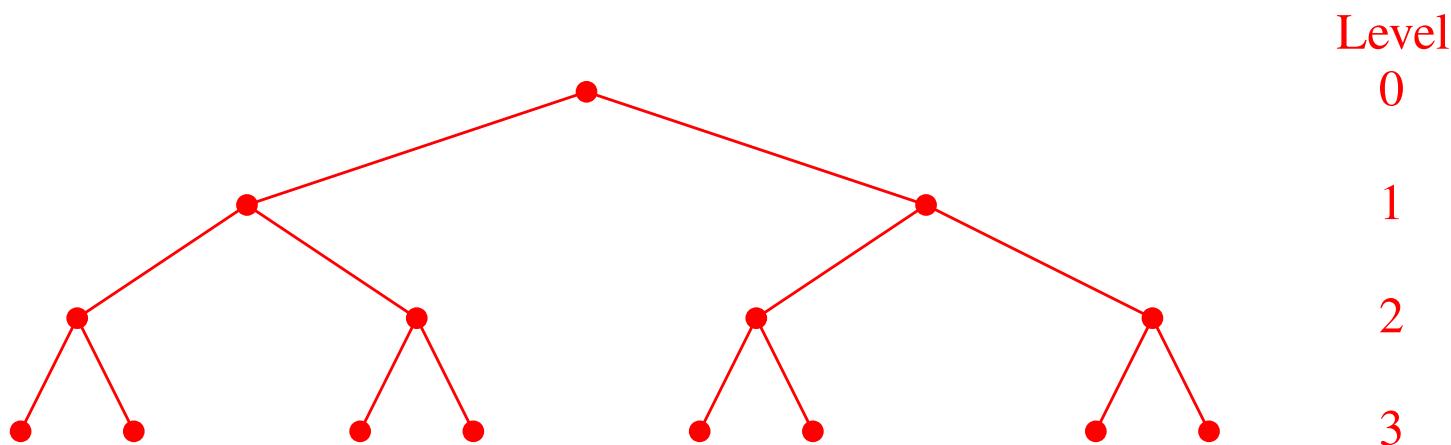
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3. Binary Search Trees
 - Definition
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Binary Trees

- A **binary tree** is a tree where each node can have zero, one or two children
- The total number of possible nodes at level l is 2^l
- The total number of possible nodes of a tree of height h is

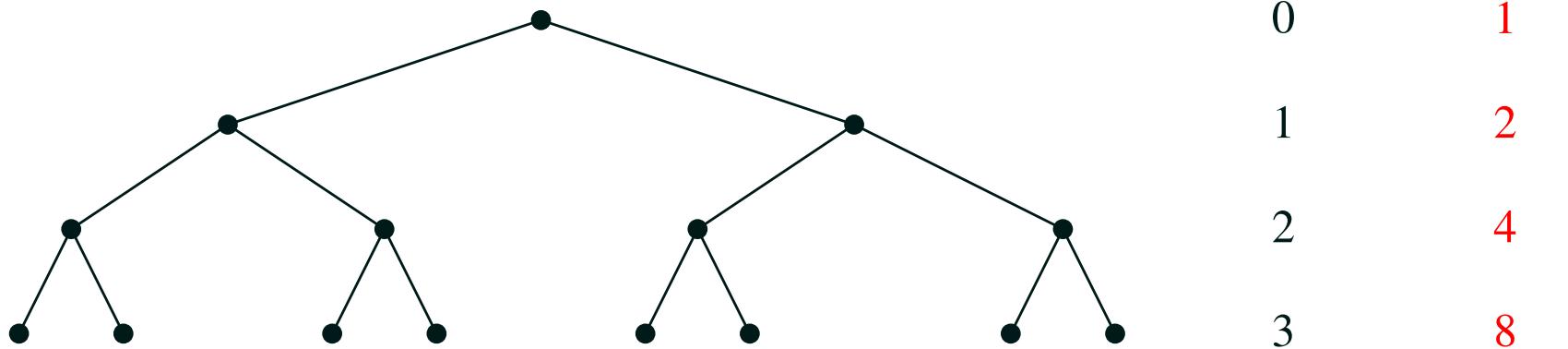
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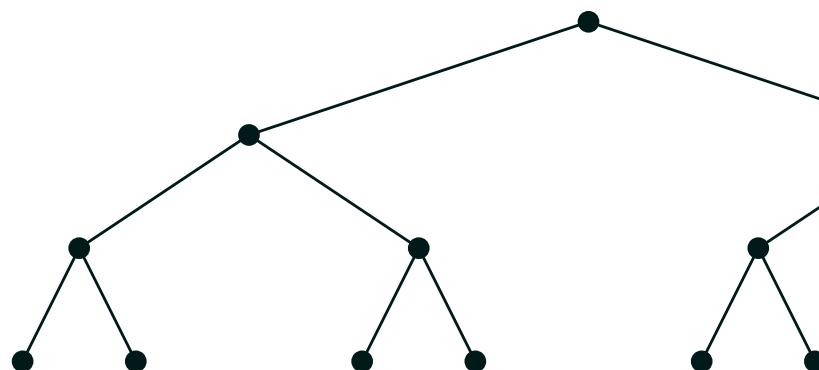
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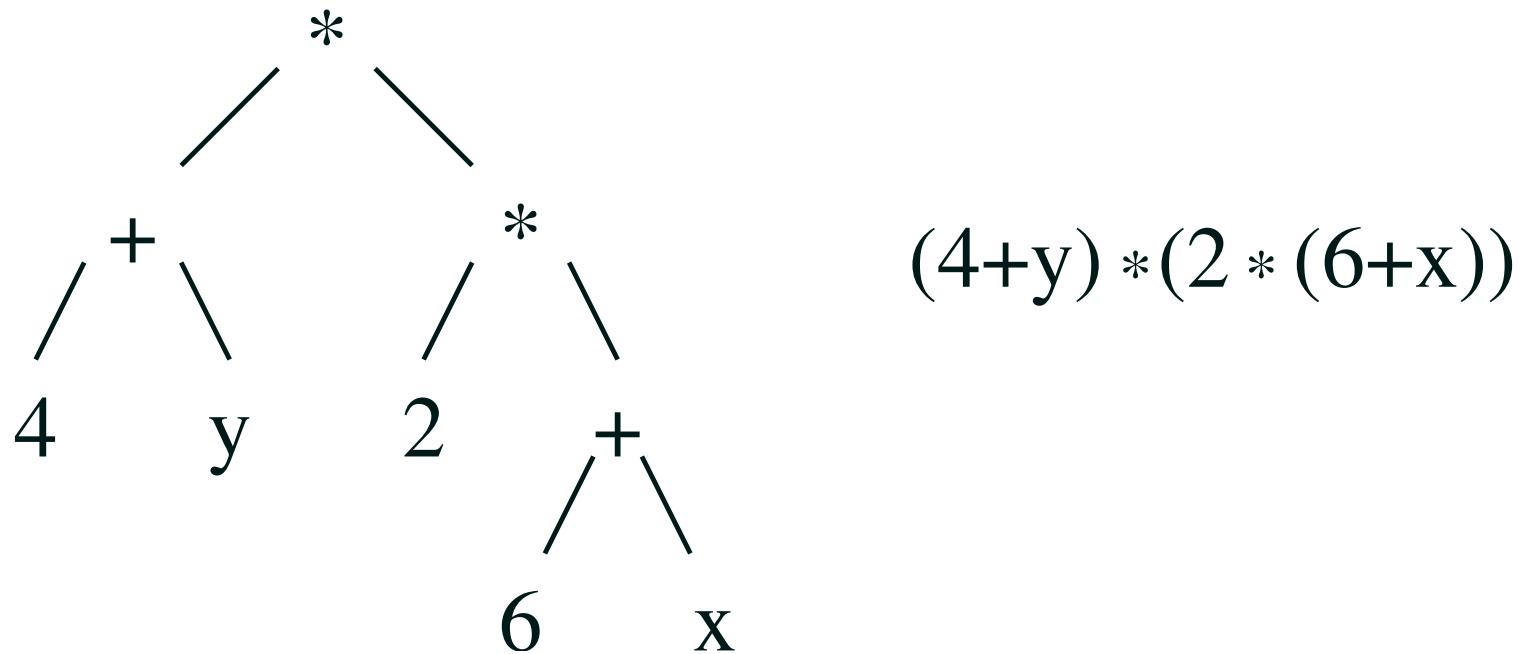
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| Level | # Nodes |
|-------|-----------|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| | <u>15</u> |

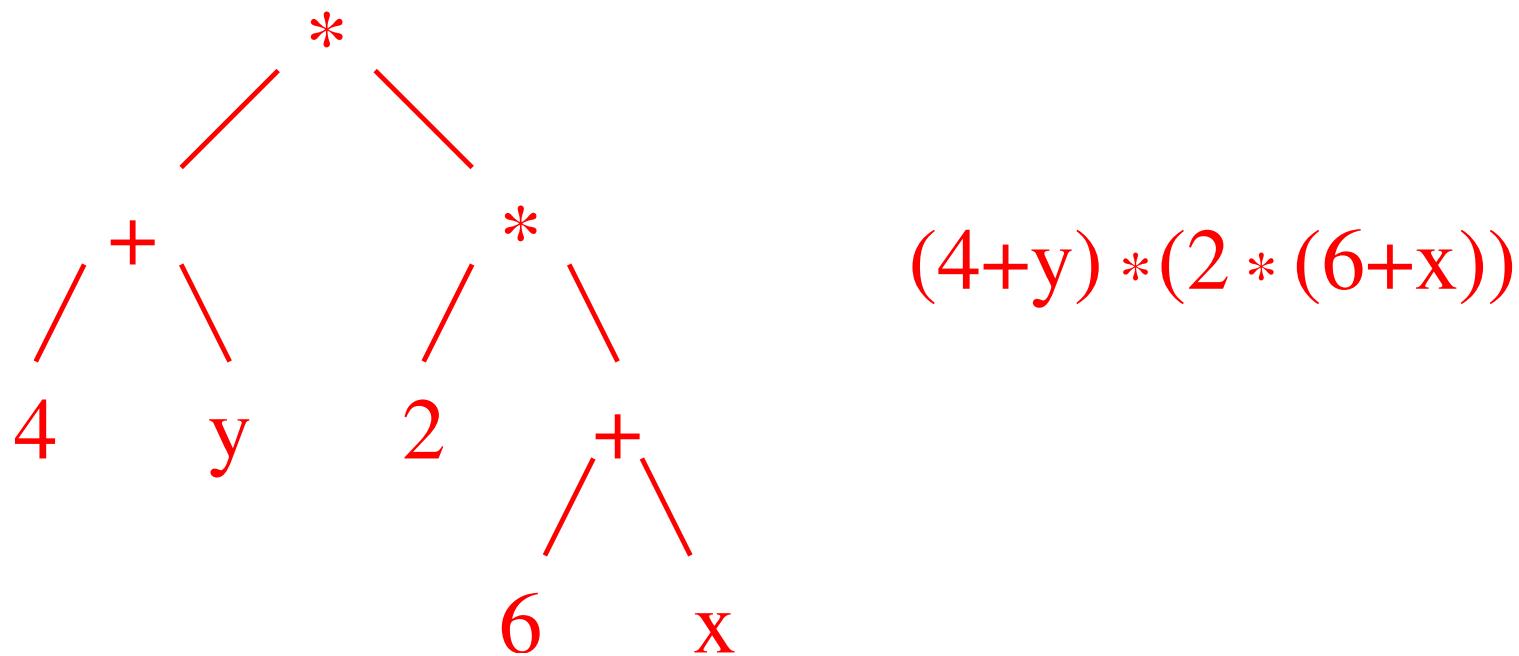
Uses of Binary Trees

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Implementation

- We wish to build a generic binary tree class with each node housing an element
- Again we use a `Node<T>` class as the building block for our data structure—in this case a node of the tree
- The `Node<T>` class will contain a pointer to left and right children
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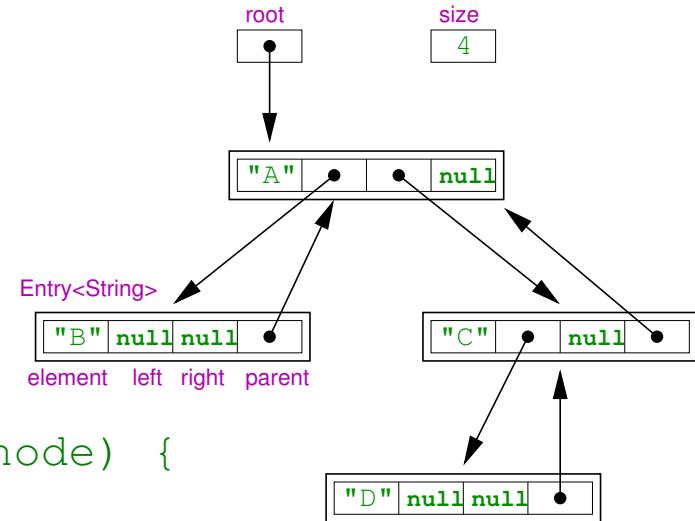
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template <typename T>
class binary_tree {
private:
    class Node {
public:
    T element;
    Node* parent;
    Node* left = nullptr;
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    Node(const T& value, Node* parent_node) {
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unsigned no_elements = 0;
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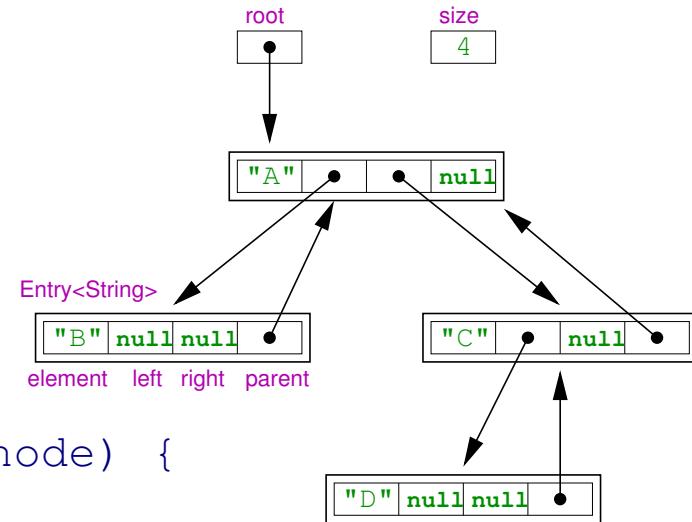
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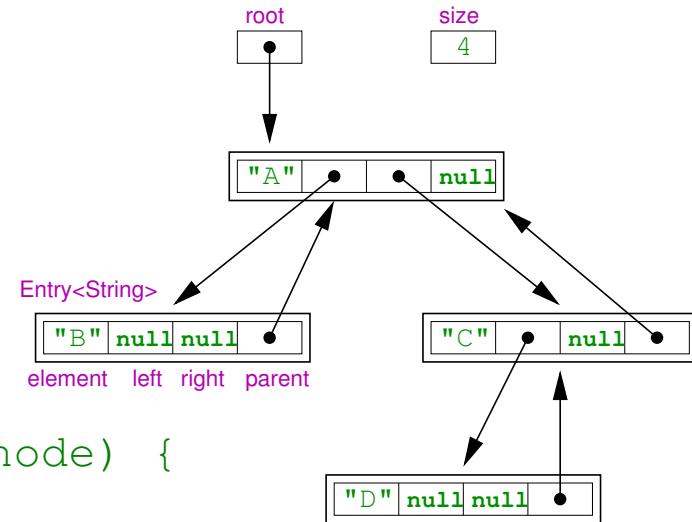
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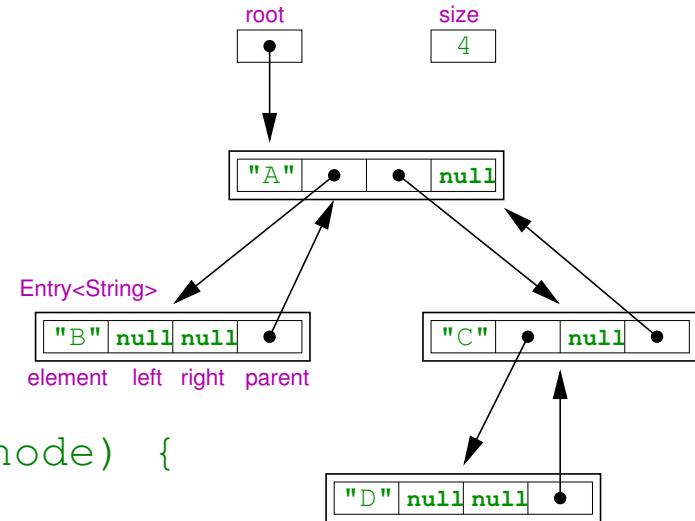
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Binary Search Trees

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- The binary search tree keeps the elements ordered
- We can define a binary search tree recursively
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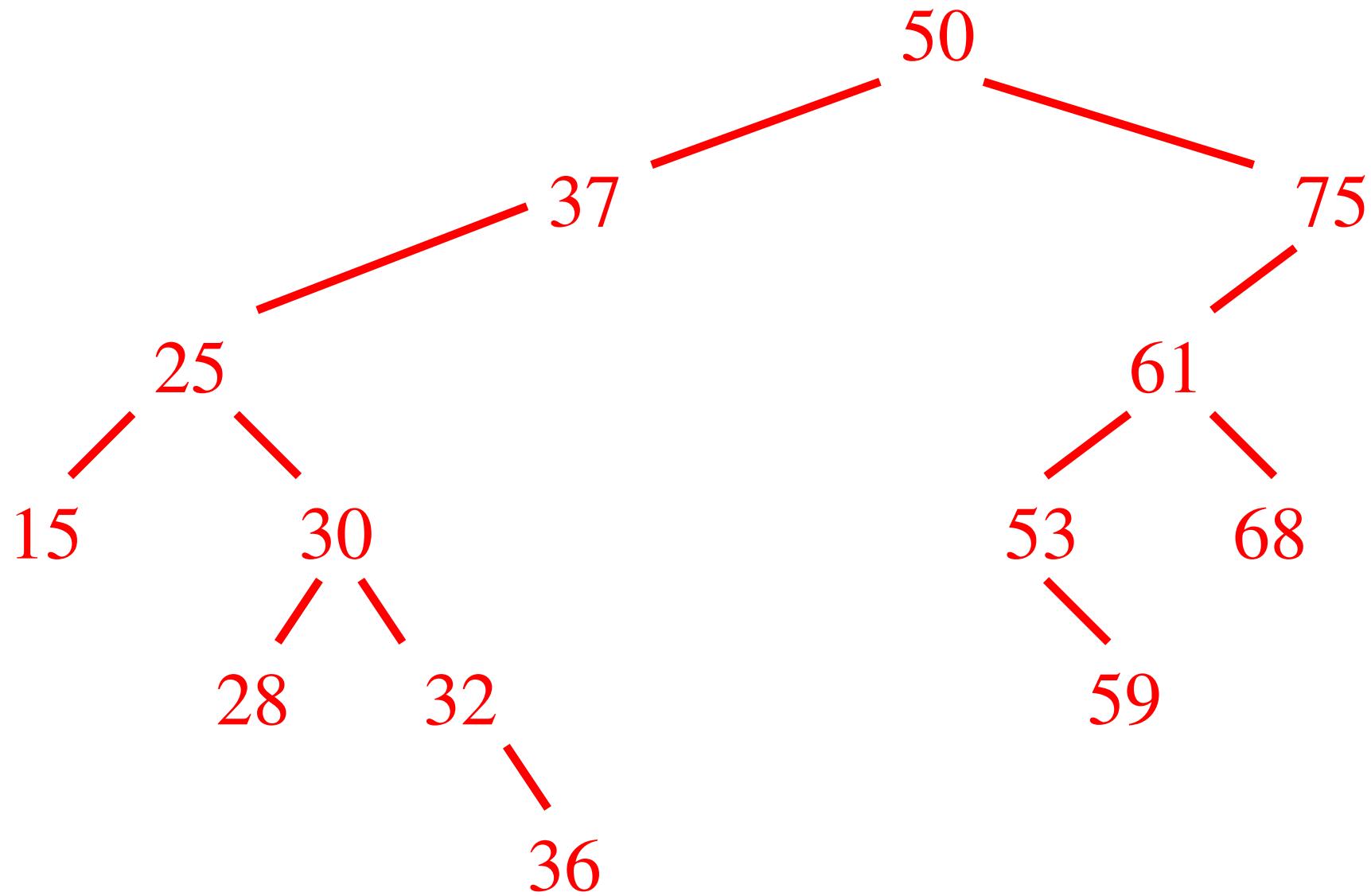
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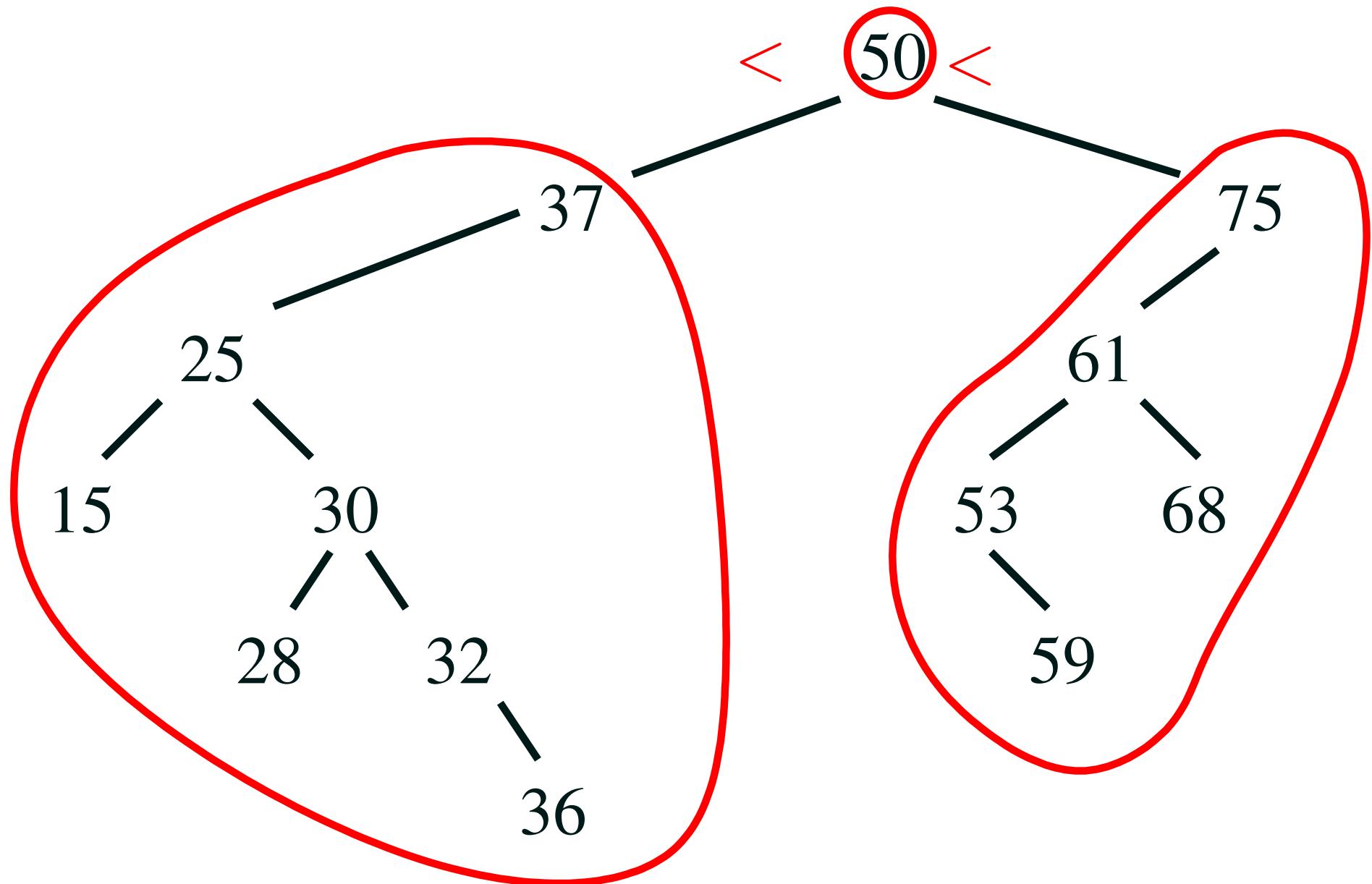
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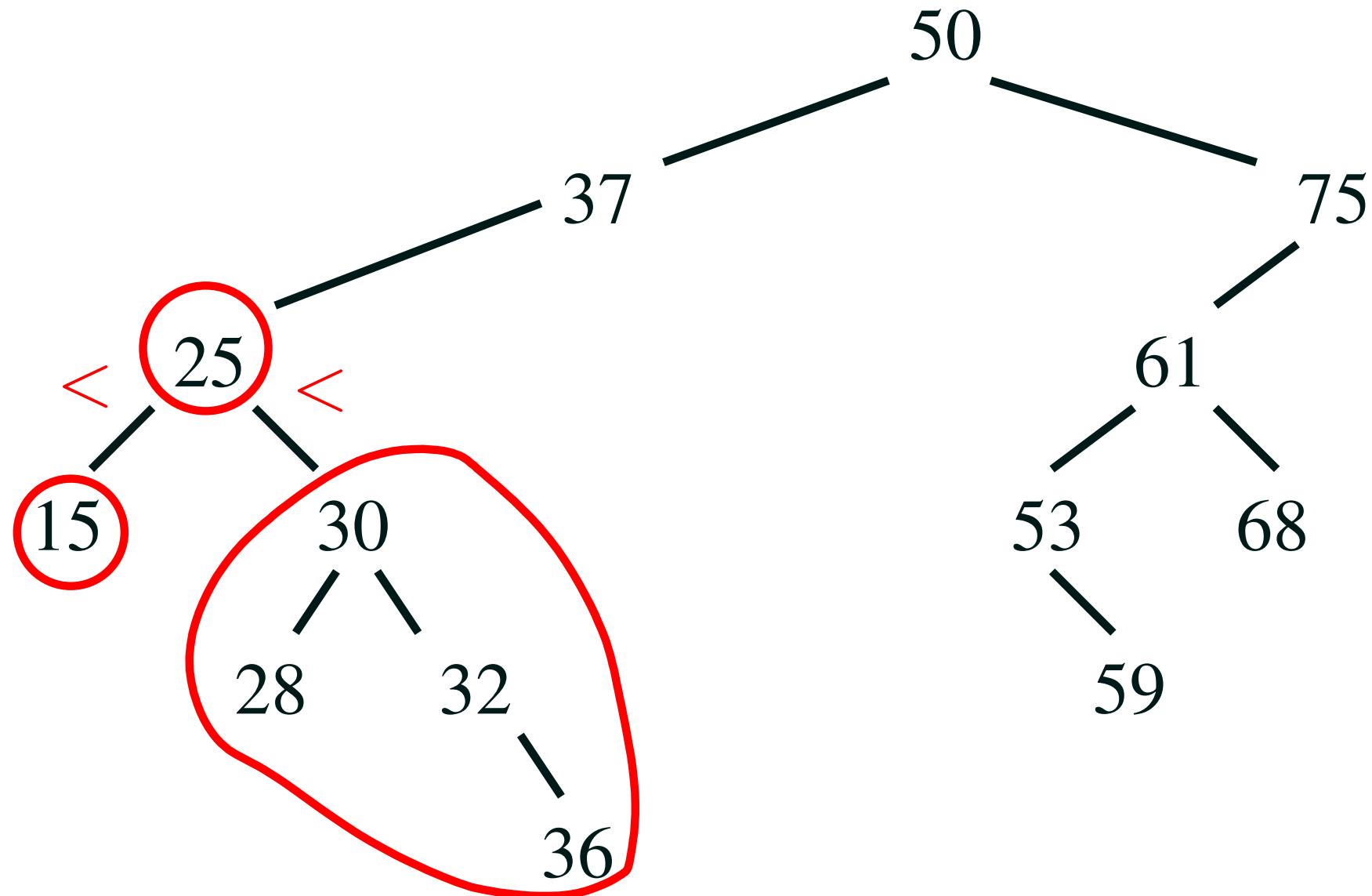
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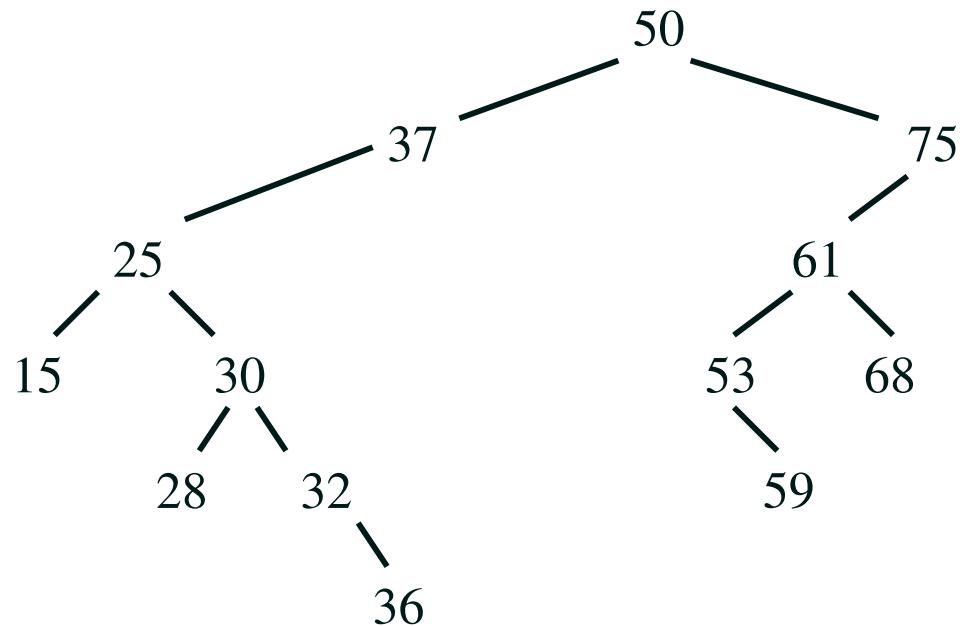


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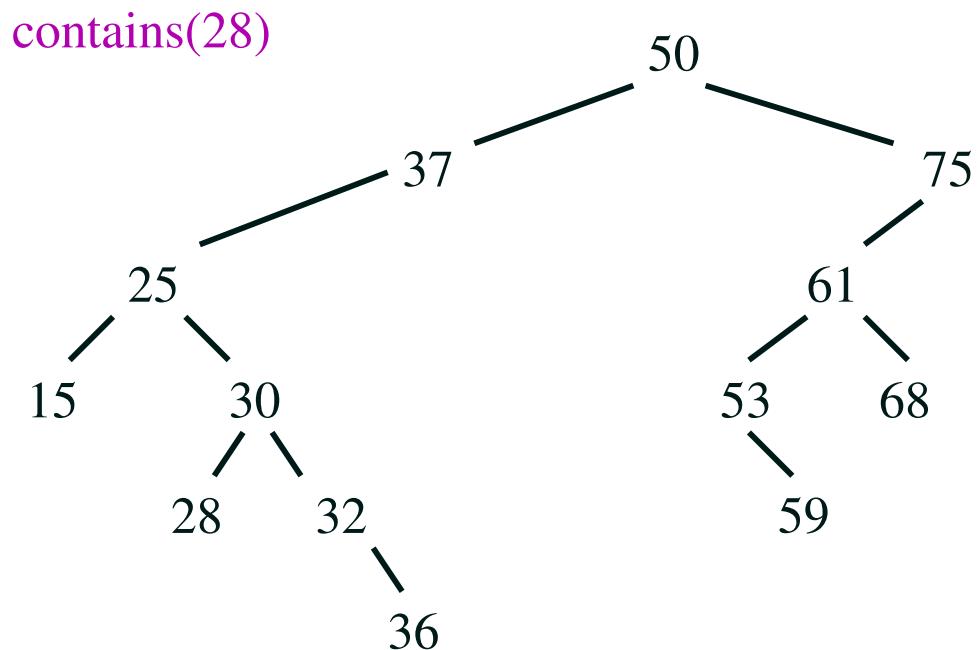
Searching A Binary Search Tree

- Searching a binary search tree is easy
 - Start at the root
 - Compare with element
 - ★ If less than element go left
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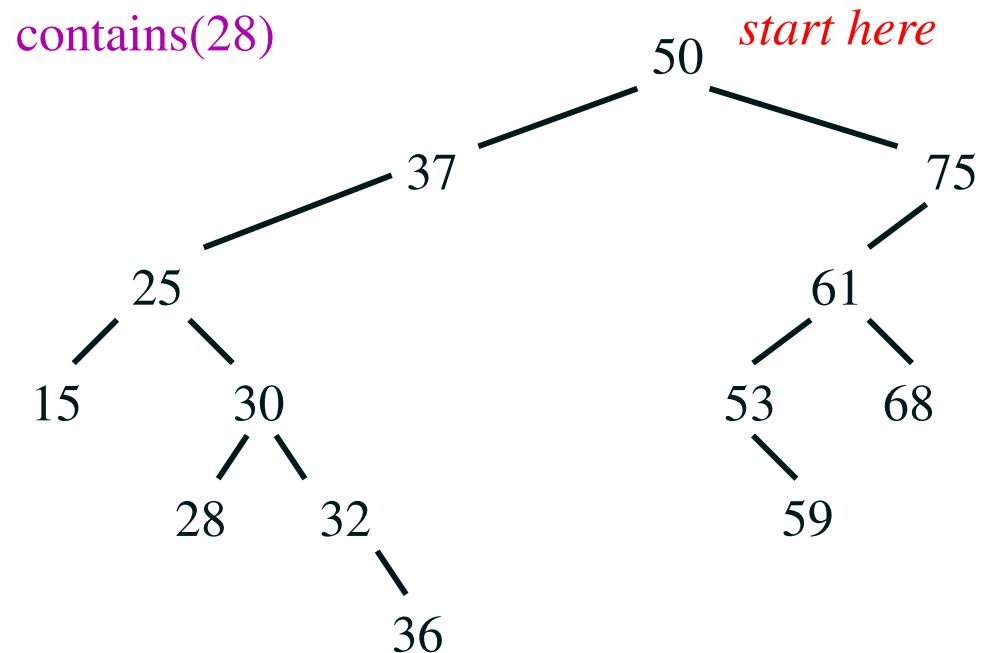
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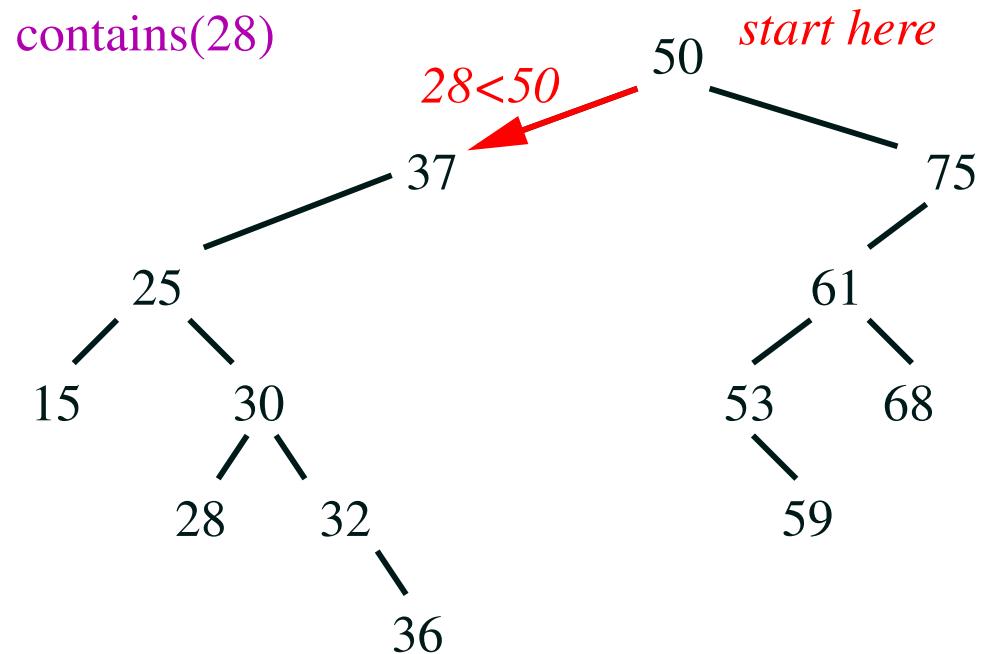
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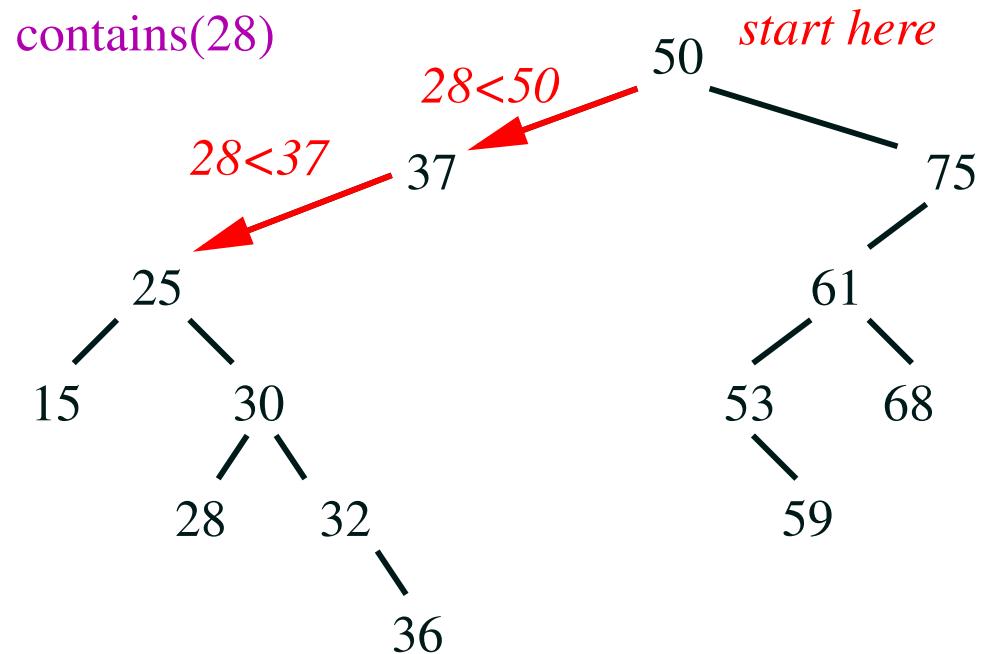
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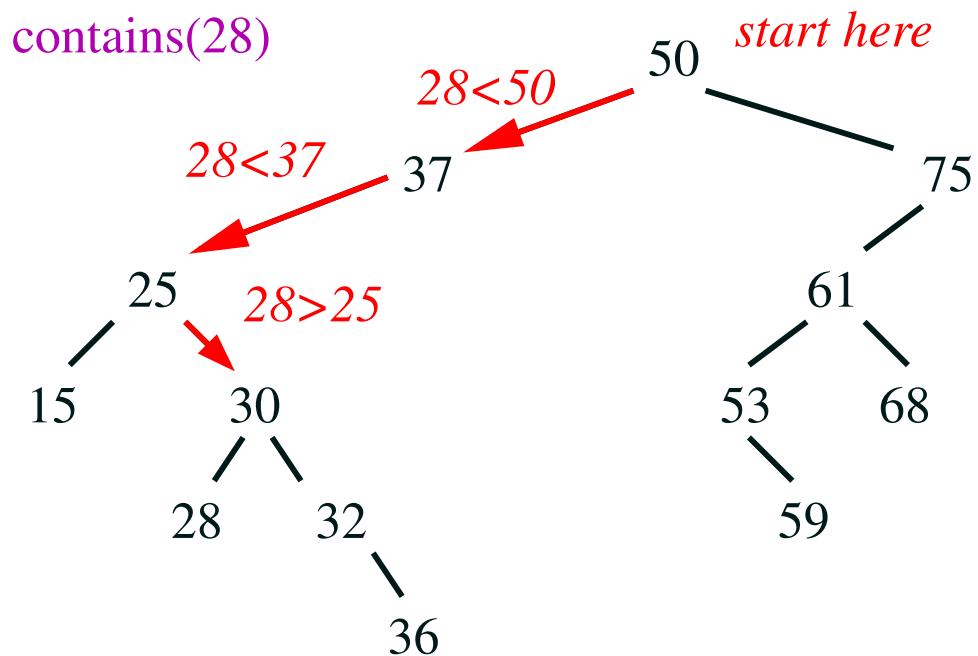
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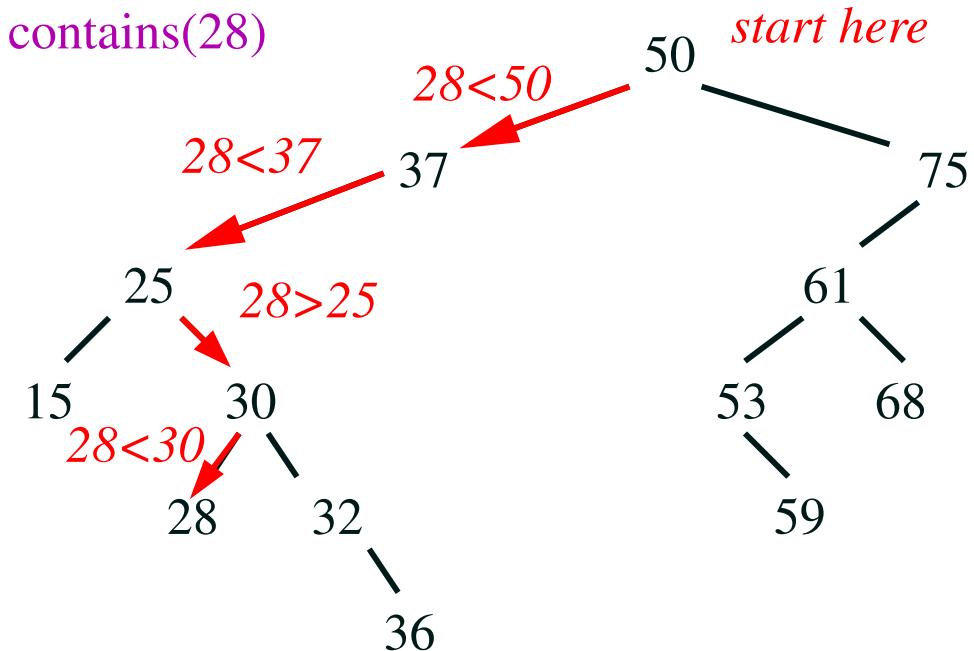
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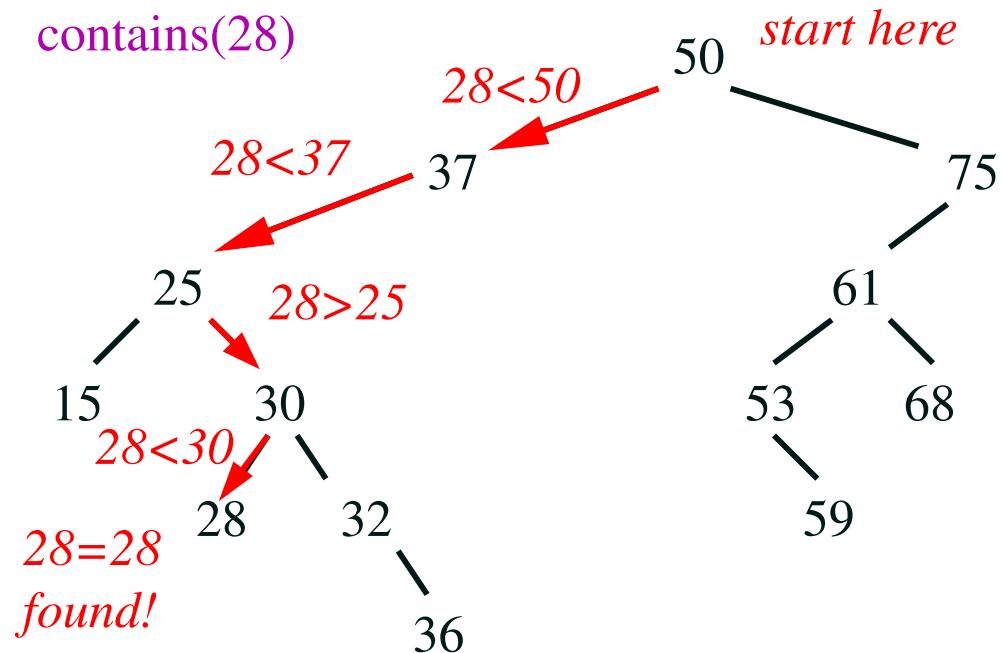
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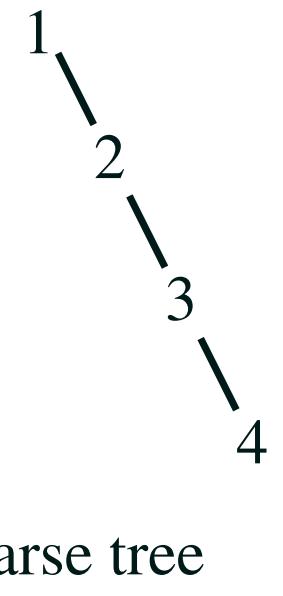
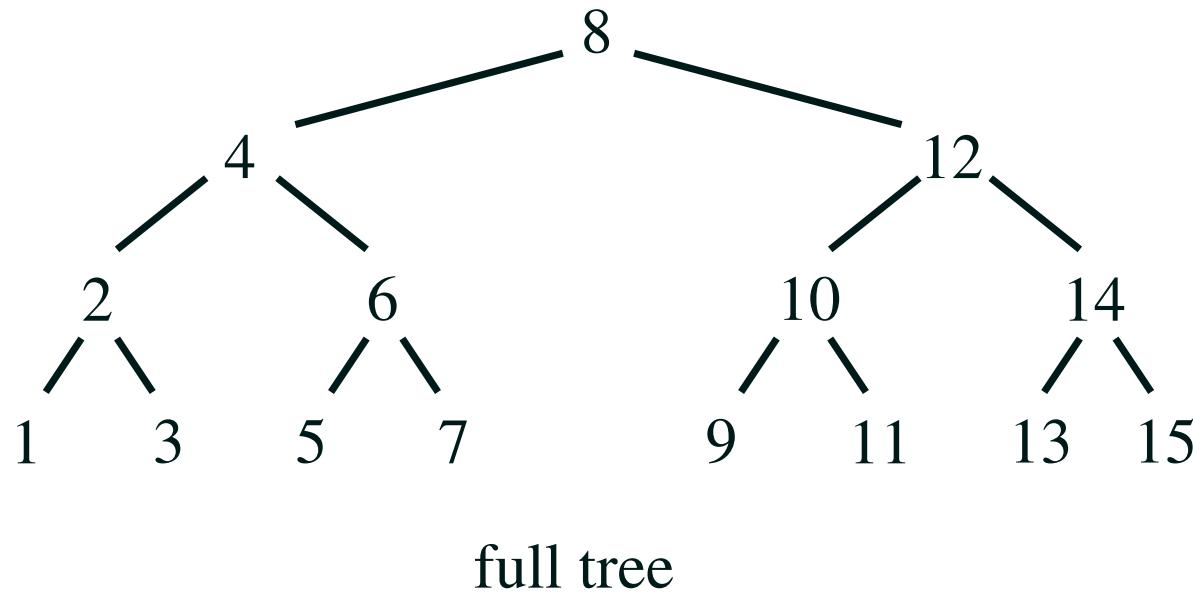
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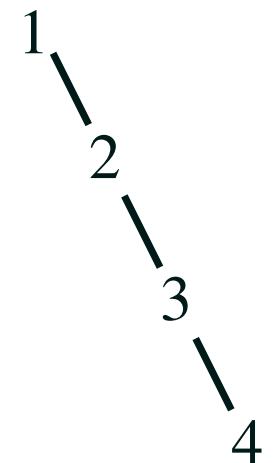
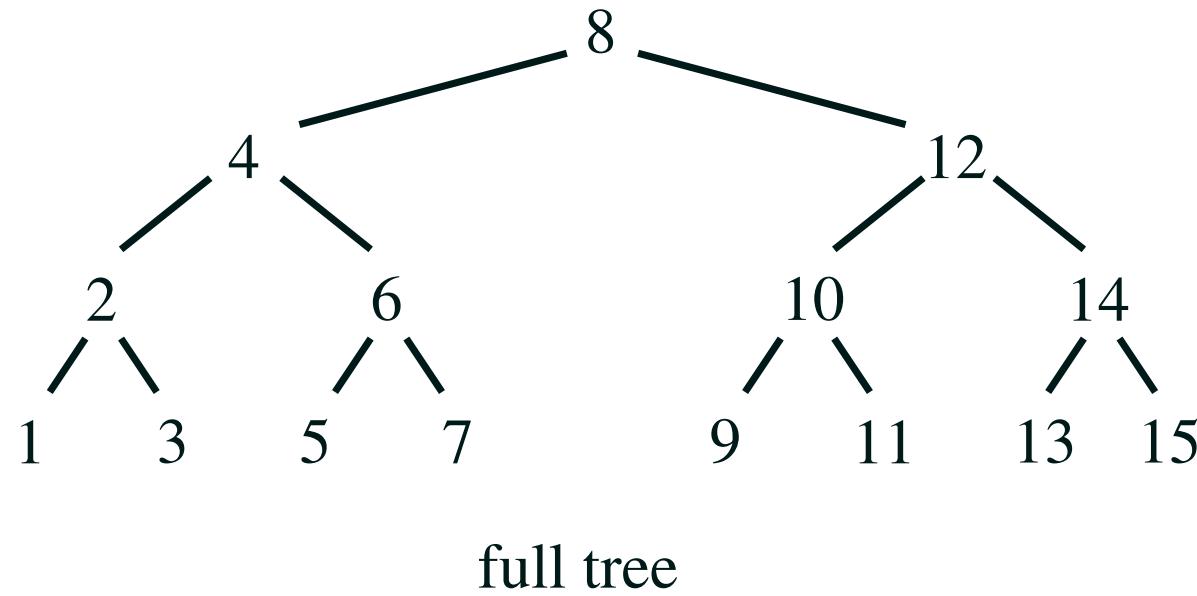
Speed of Search

- The number of comparisons necessary to find an element in a binary tree depends on the level of the node in the tree
- The worst case number of comparisons is therefore the height of the tree
- This depends on the density of the tree



Speed of Search

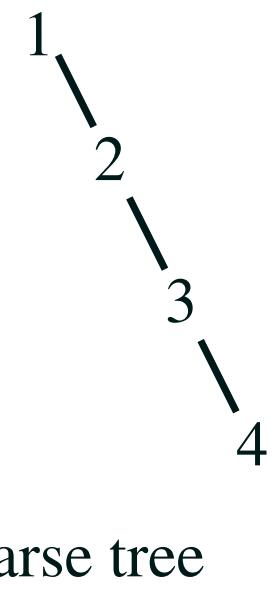
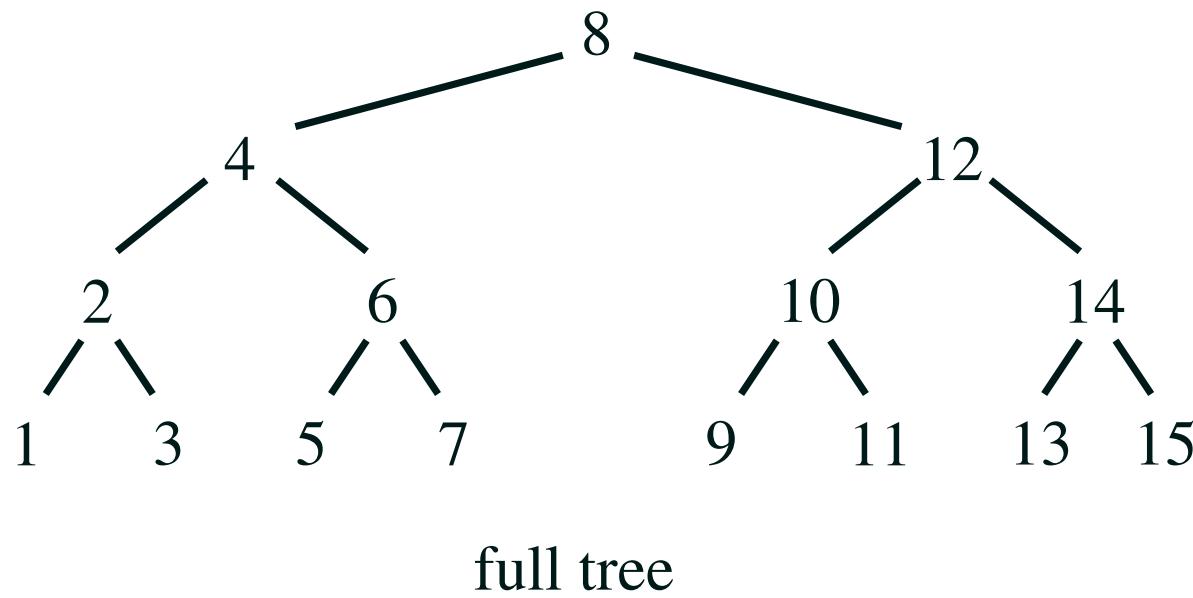
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sparse tree

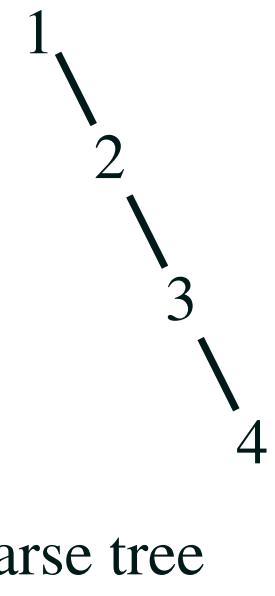
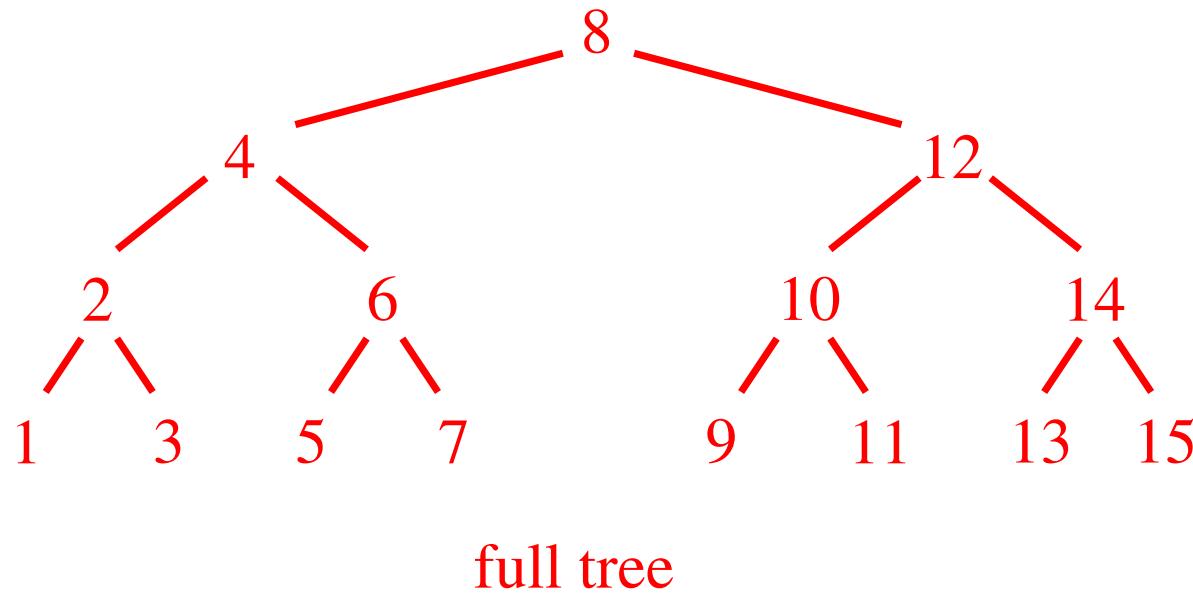
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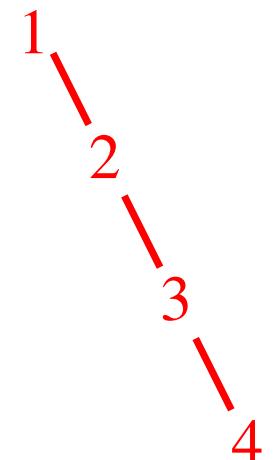
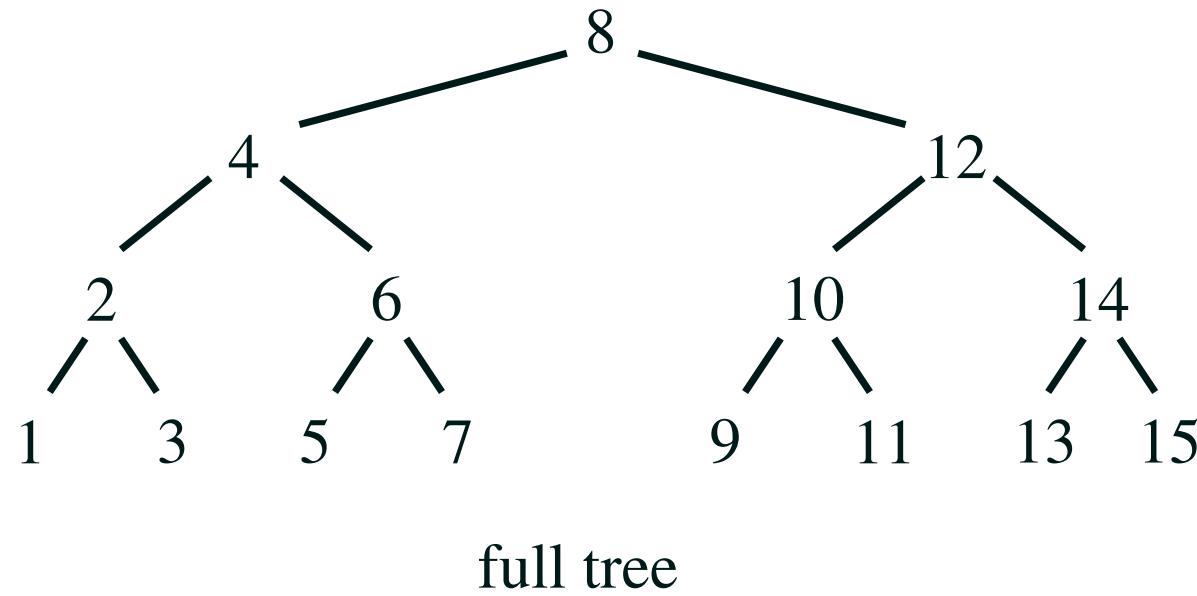
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Implementing a Set

- A set is a fundamental **abstract data type**
- It is a collection of things with no repetition and no order
- Ironically because order doesn't matter we can order the elements

$$\{1, 3, 5, 5, 3, 4\} = \{5, 3, 4, 1\} = \{1, 3, 4, 5\}$$

- This allows rapid search—a feature we care about
- Binary trees are one of the efficient ways of implementing a set

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Fitting In

- The standard template library provides a class `std::set<T>`
- This contains many functions like
 - ★ Constructors
 - ★ `size()`
 - ★ `insert(T o)`
 - ★ `find(T o)`
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 - ★ `begin()` and `end()`

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Comparable

- To sort any objects they must be comparable
- In the STL the set implementation has a second template parameter: `std::set<T, Compare = less<T>>`
- by default this is defined to be `less<T>` (which is a function already defined for most common types) which you can define
- If you have a set of complex objects you will have to define Compare

```
bool MyCompare(MyObject left, MyObject right) {  
    return something  
}
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```
mySet = set<MyObject, MyCompare>;
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Find an Element

- One of the core operations of a binary tree is to find a node

```
iterator find(const T& element) {  
    Node* current = root;  
    while (current!=nullptr) {  
        if (current->element == element) {  
            return iterator(current);  
        }  
        if (element < current->element) {  
            current = current->left;  
        } else {  
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    }  
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}
```

Add an Element

```
pair<iterator, bool> insert(const T& element) {
    if (no_elements==0) {
        root = new Node(element, nullptr);
        ++no_elements;
    }
    return pair<iterator, bool>(iterator(root), true);
}

Node* parent = nullptr;
Node* current = root;
while (current != nullptr) {
    if (current->element == element) {
        return pair<iterator, bool>(iterator(nullptr), false);
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    parent = current;
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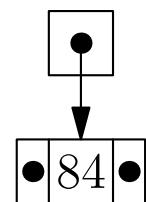
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current = new Node(element, parent);
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    parent->left = current;
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}
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```

Tree in Action

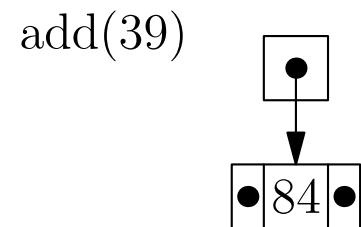
add(84)



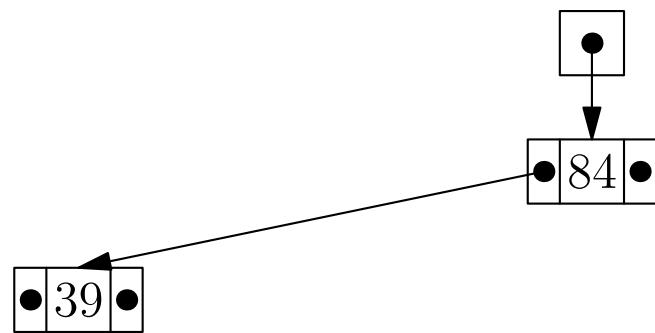
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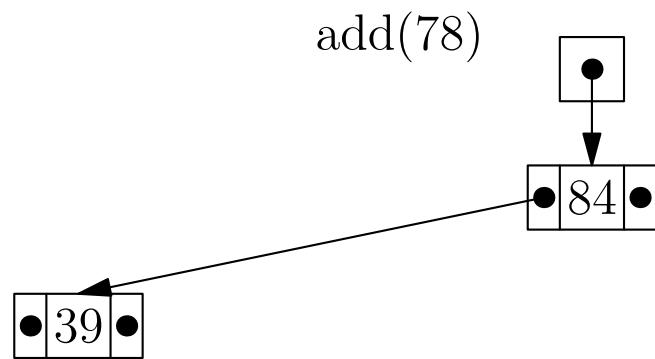
Tree in Action



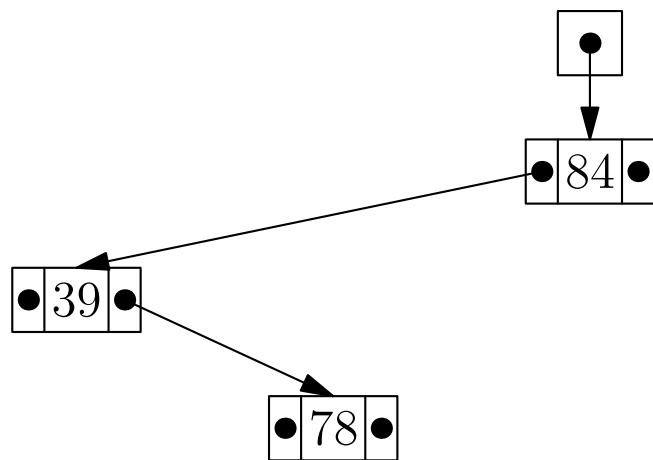
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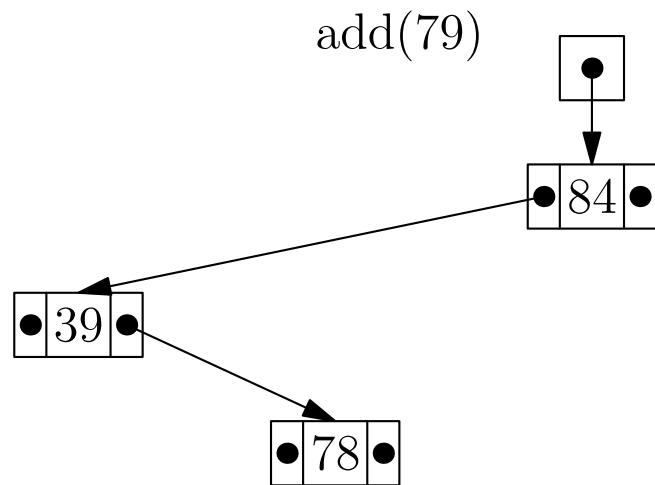
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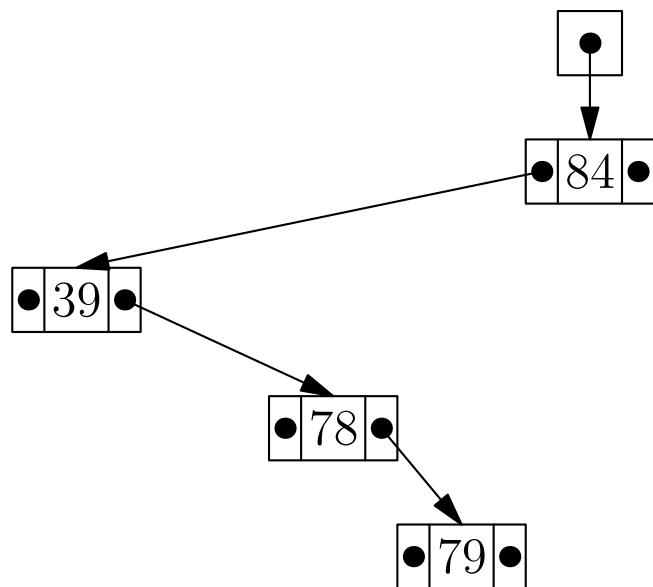
Tree in Action



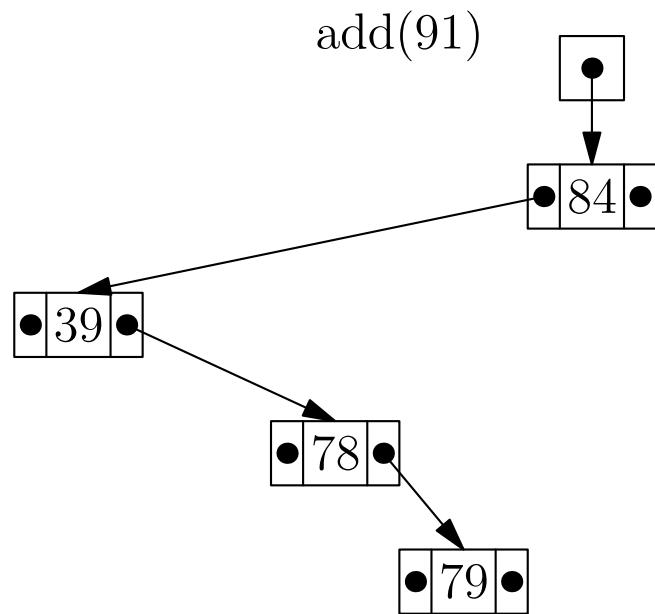
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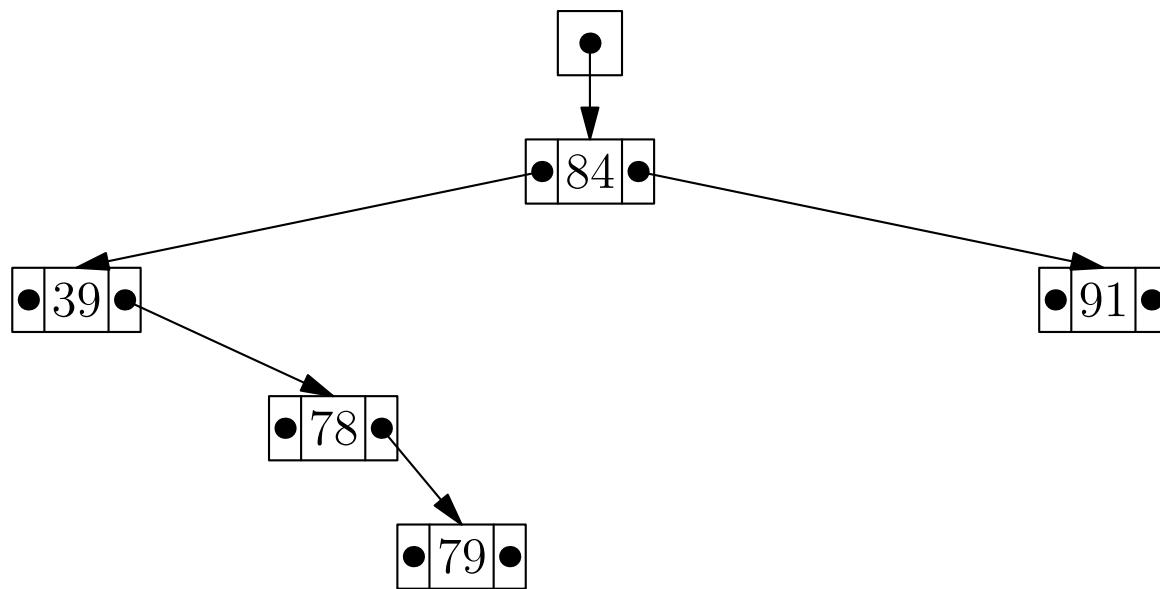
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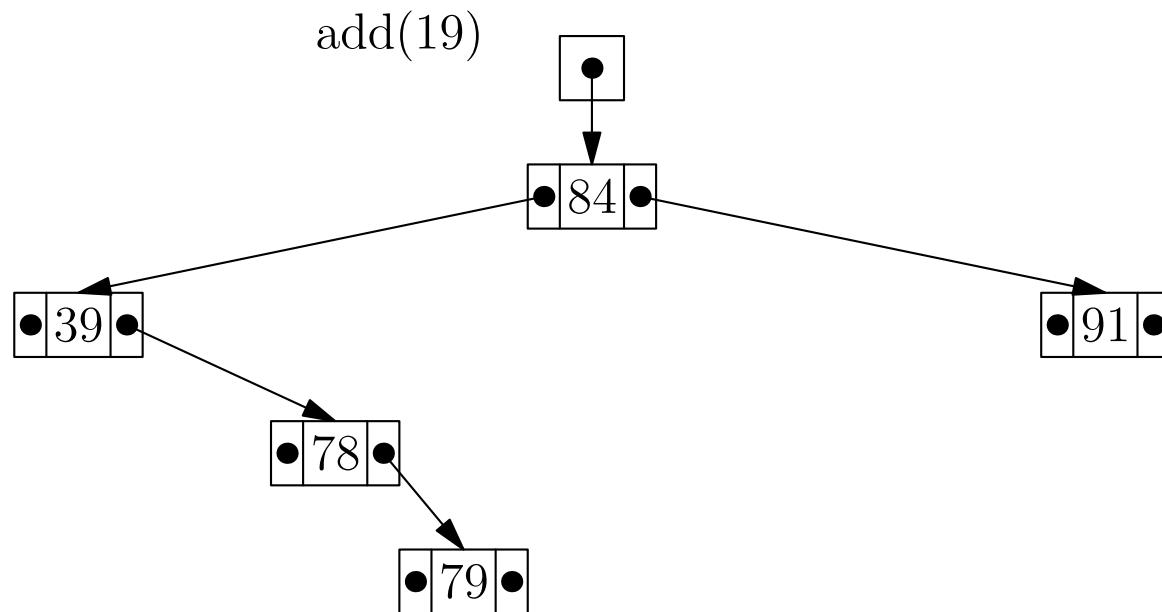
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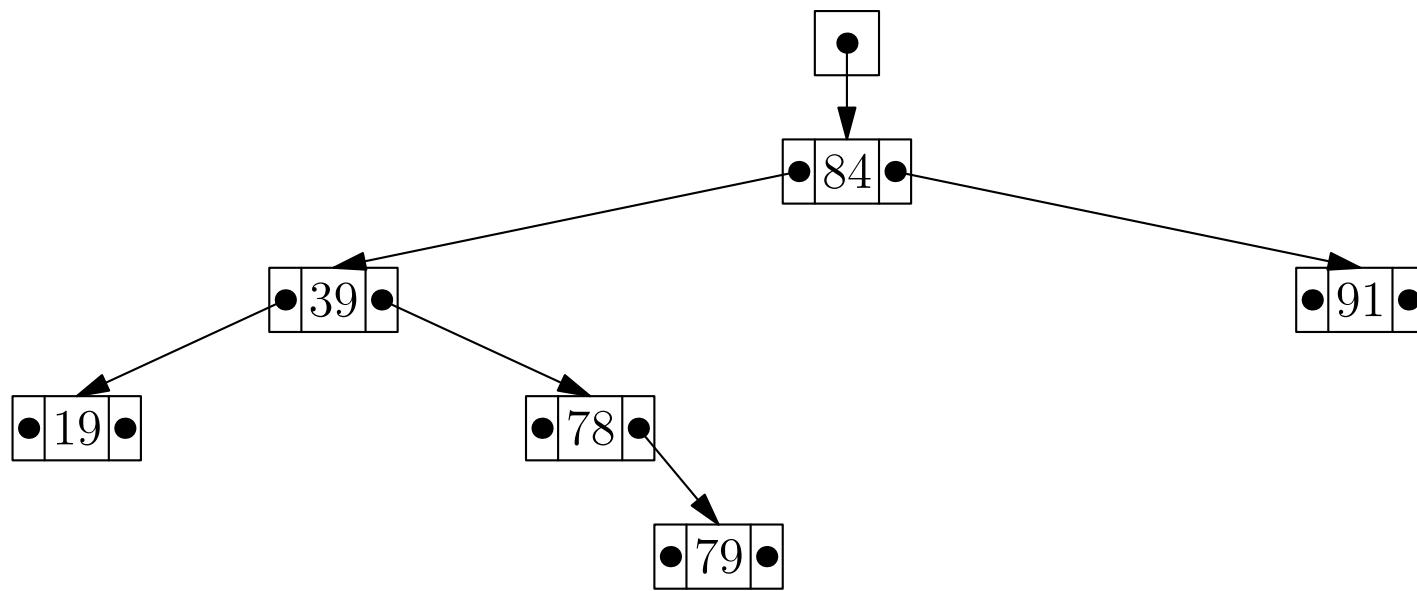
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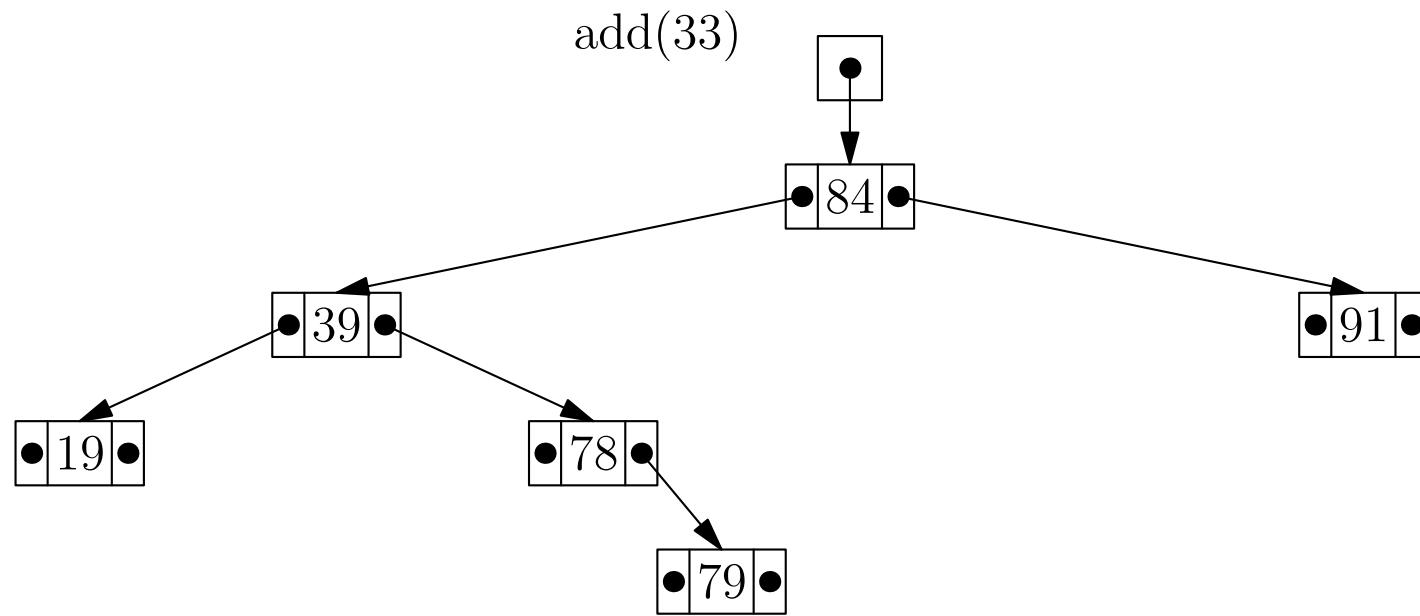
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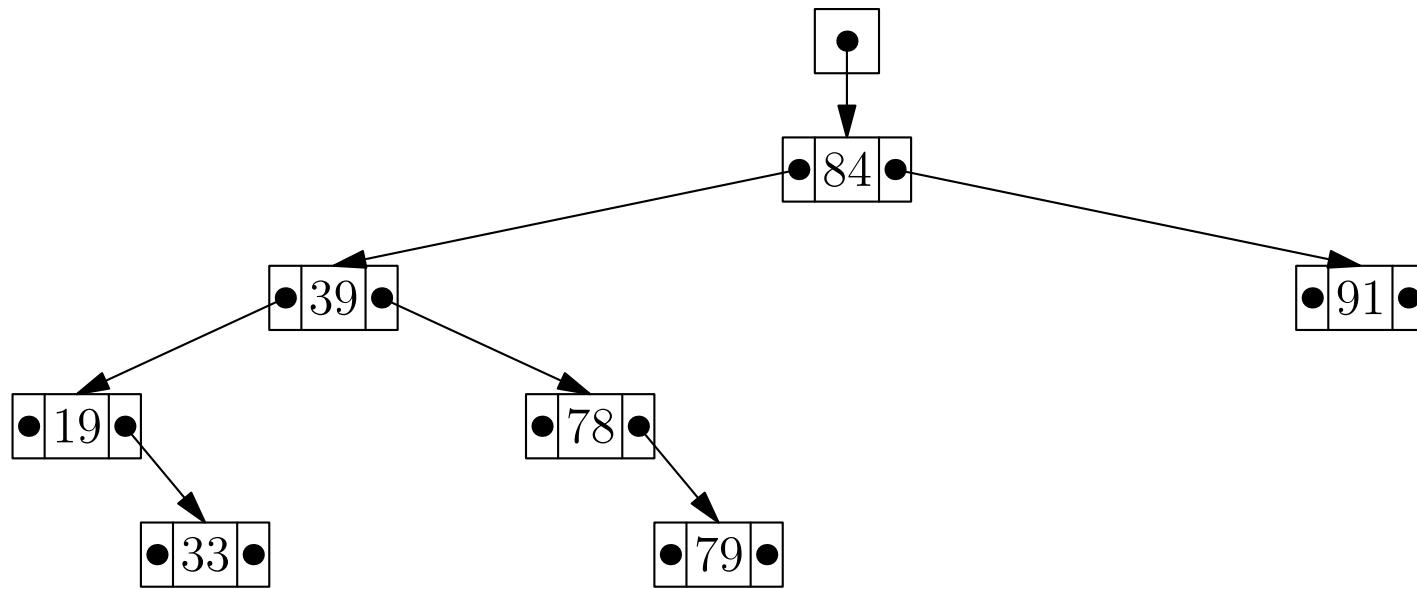
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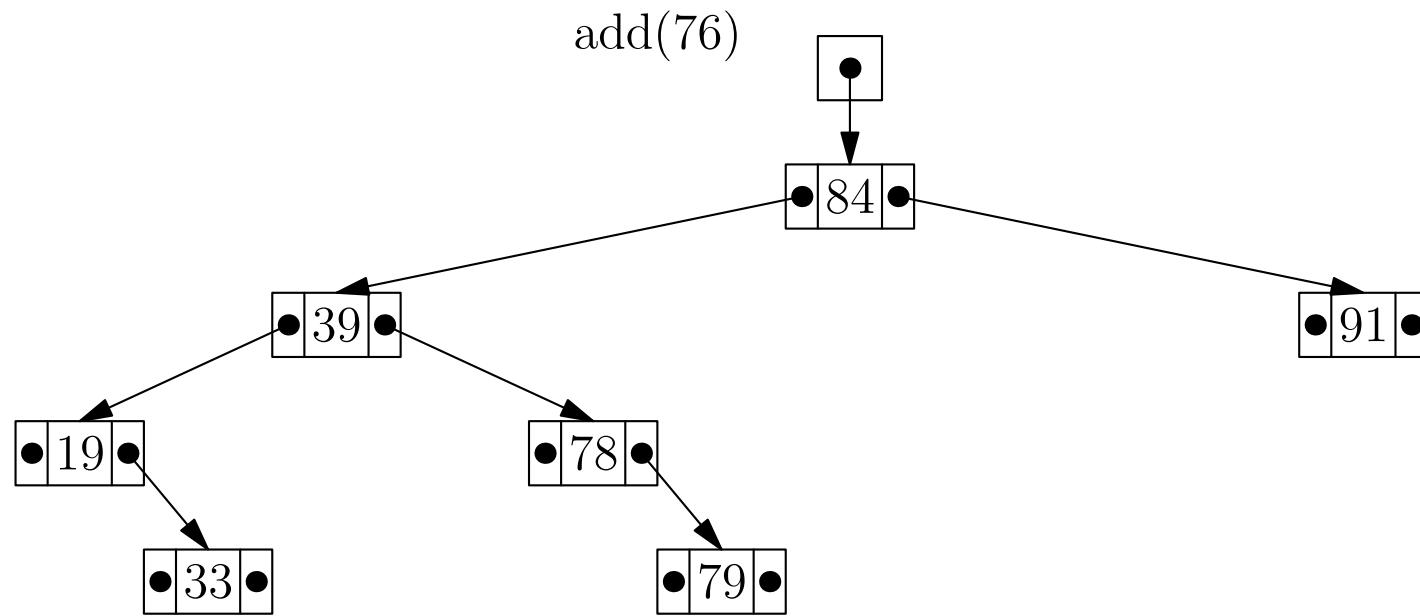
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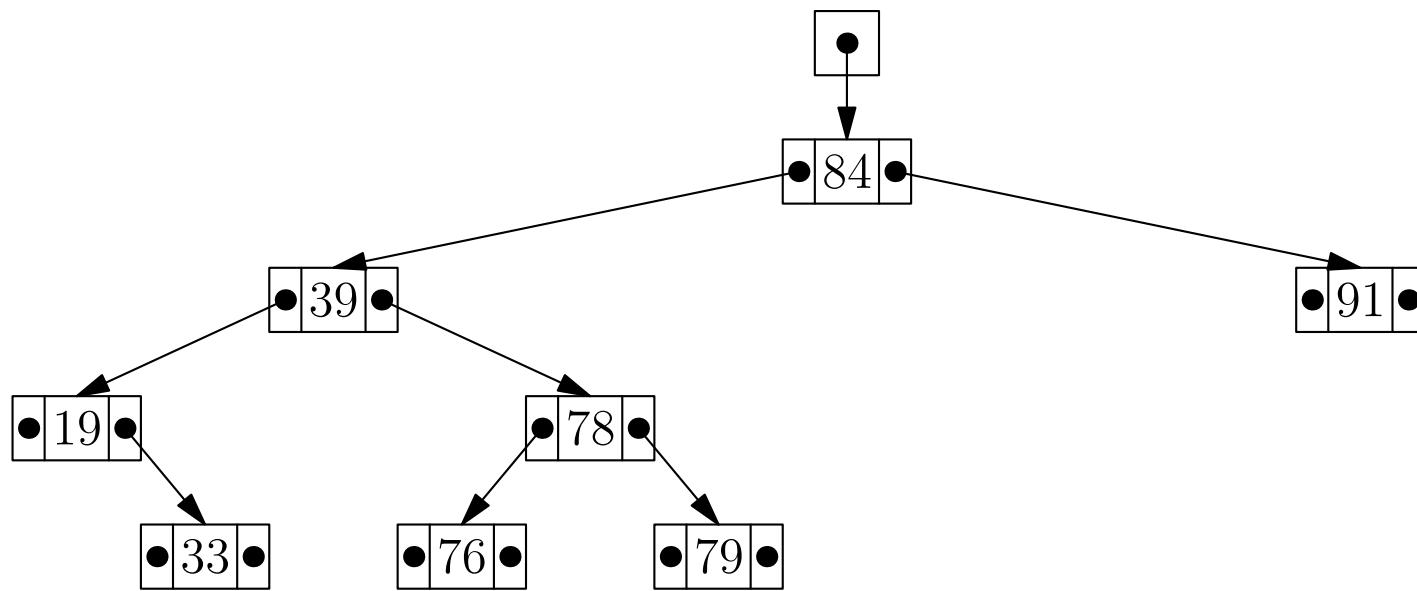
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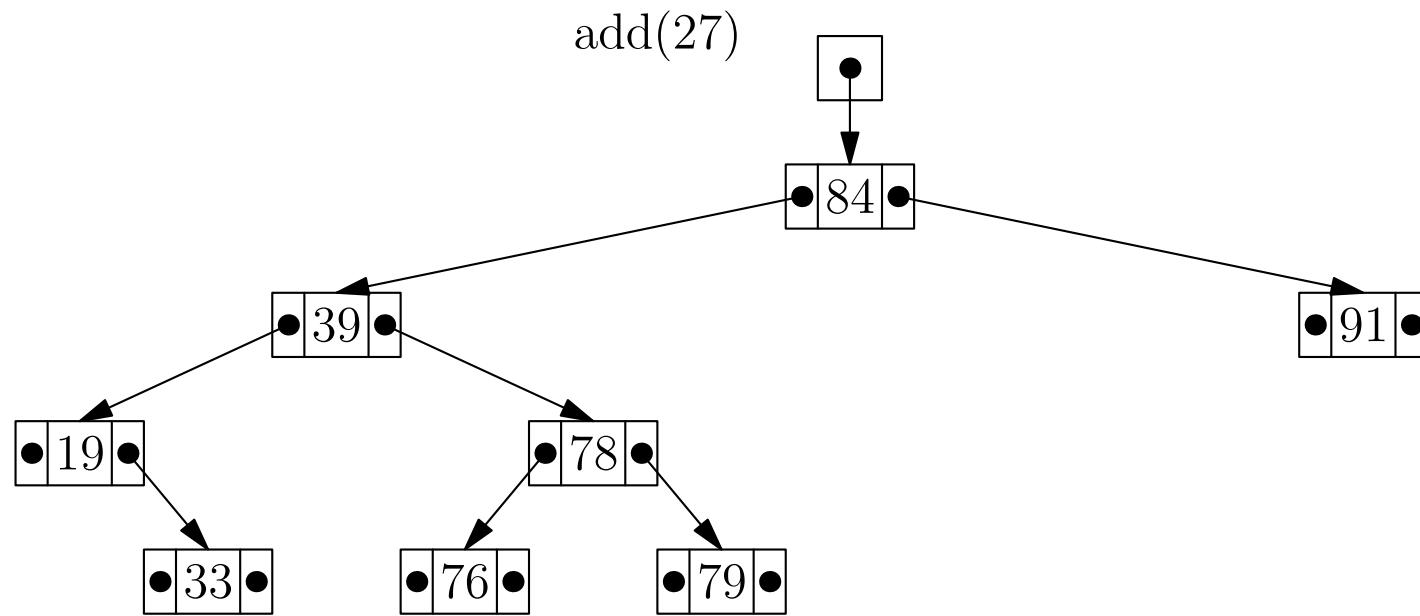
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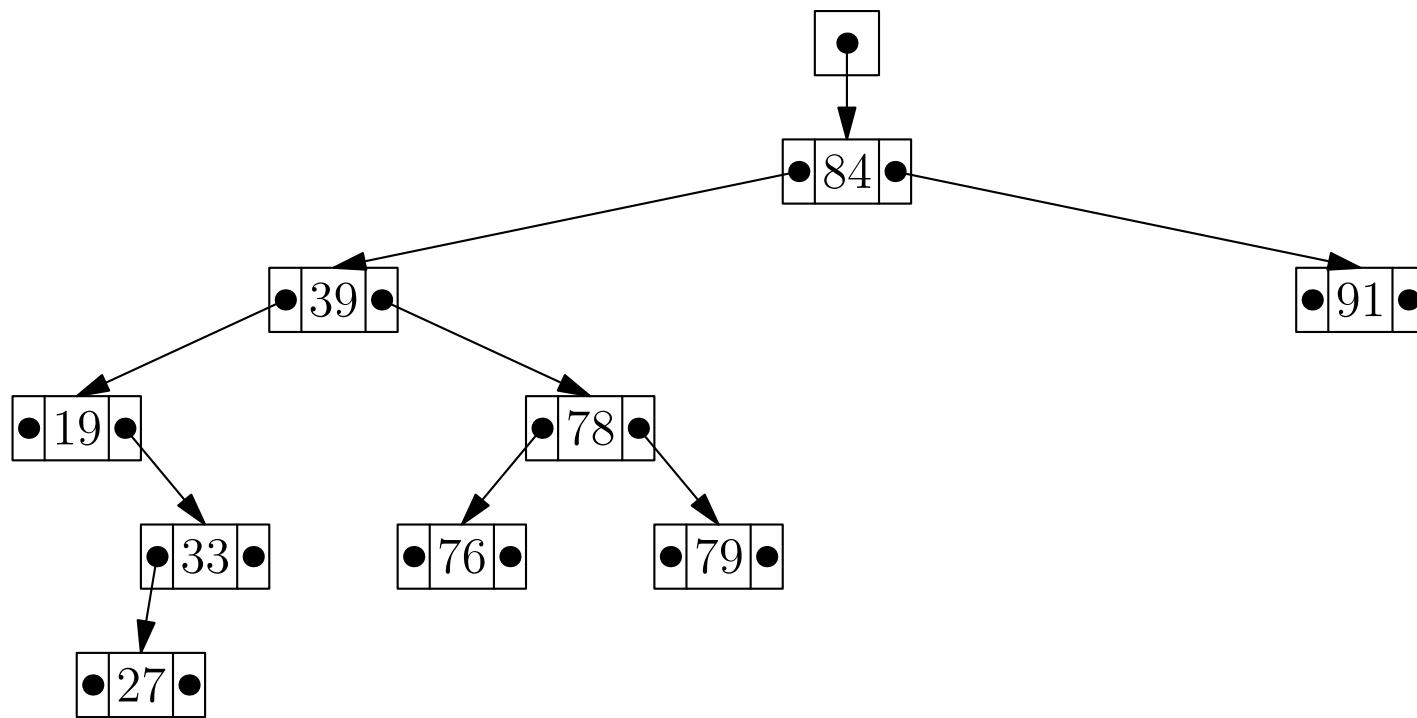
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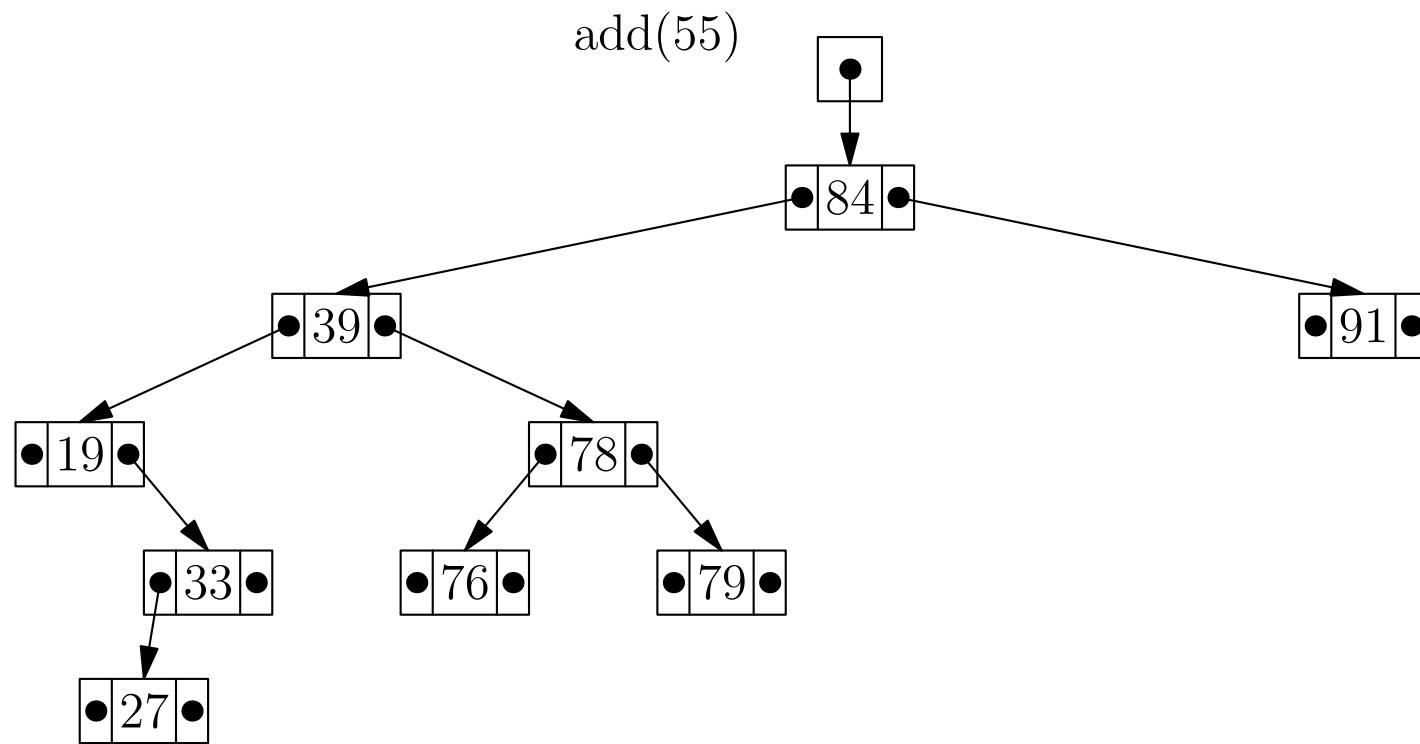
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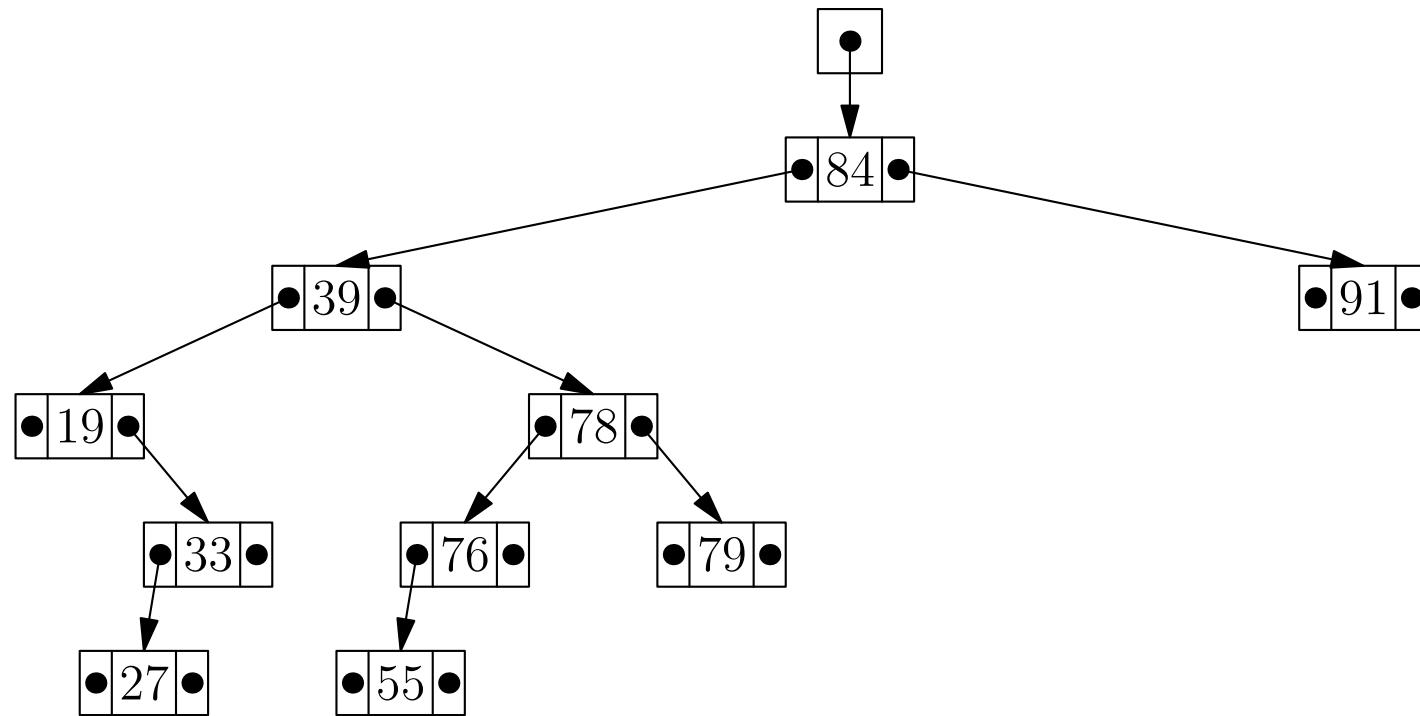
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Shape of Tree

- The structure of the tree depends on the order in which we add elements to it
- Suppose we add

*To be, or not to be: that is the question:
Whether 'tis nobler in the mind to suffer
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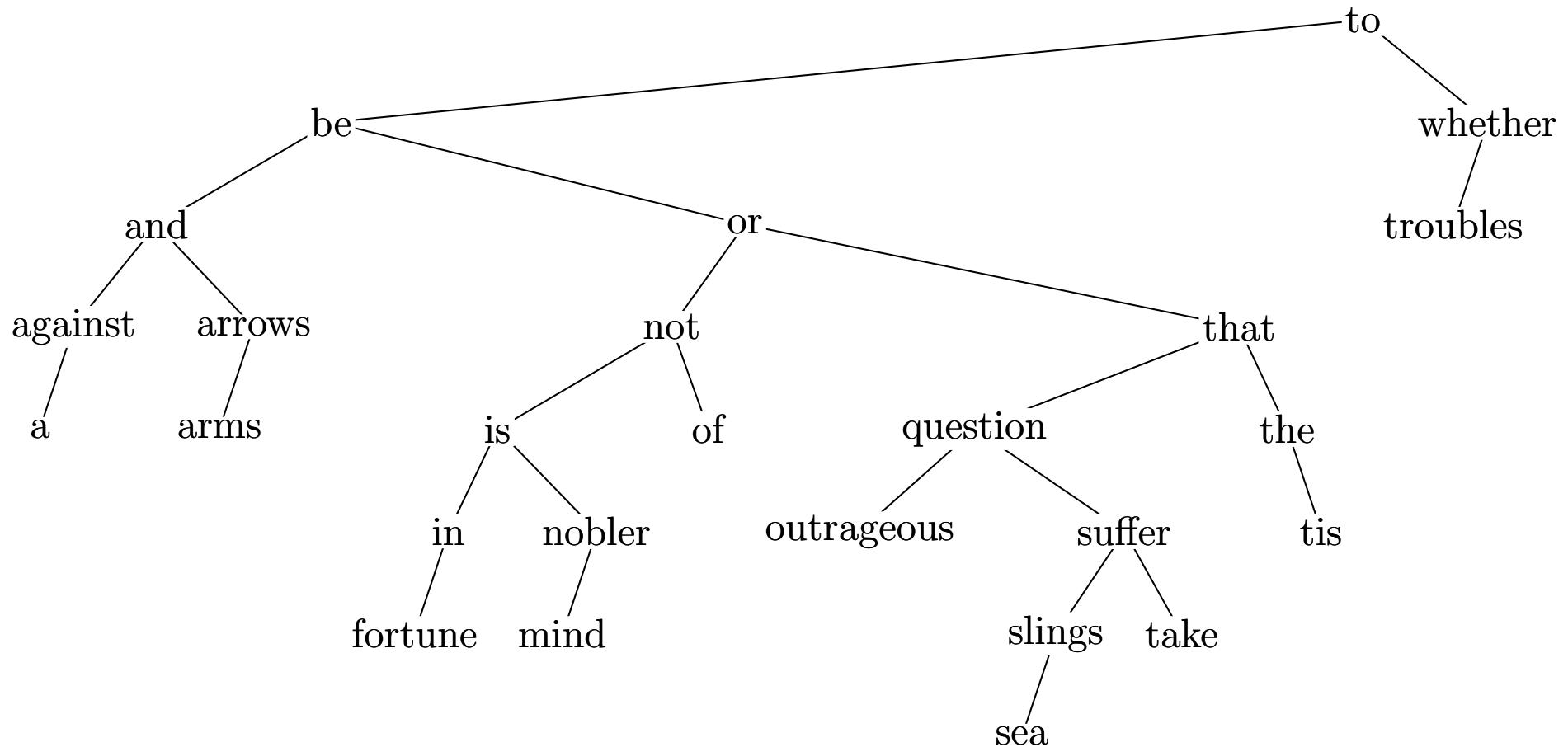
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Hamlet



Outline

1. Trees
2. Binary Trees
 - Implementing Binary Trees
3. Binary Search Trees
 - Definition
 - Implementing a Set
4. Tree Iterators



Tree Iterators

- As with most container classes it is very useful to define iterators
- begin() should return a “pointer” to the start of the tree
- end() provides a “pointer” past the end
- operator*() returns the element
- operator++() increments the “pointer”
- operator!=(lhs, rhs) is used to compare iterators

```
set<int> mySet;  
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for(auto pt=mySet.begin(), pt!=mySet.end(), ++pt) {  
    cout << *pt;  
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C++ Code

```
class binary_tree {  
  
private:  
    class iterator {  
private:  
    Node* current;  
  
public:  
    iterator(Node* node) {current=node;}  
    T operator*() const {return current->element;}  
    iterator operator++()  
    {  
        current = successor(current);  
        return *this;  
    }  
    bool operator!=(const iterator& other)  
    {  
        return current!=other.current;  
    }  
};  
  
iterator begin() {...}  
iterator end() {return iterator(0)}  
};
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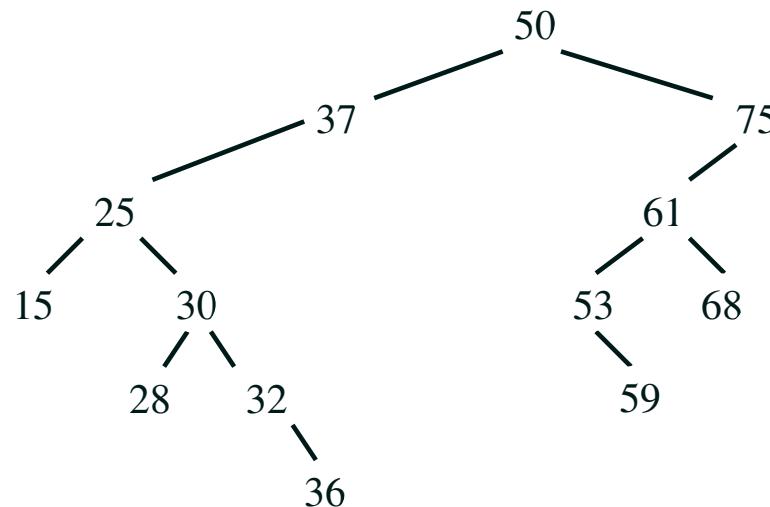
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    iterator operator++()  
    {  
        current = successor(current);  
        return *this;  
    }  
    bool operator==(const iterator& other)  
    {  
        return current==other.current;  
    }  
};  
  
iterator begin() {...}  
iterator end() {return iterator(0)}  
};
```

Successor

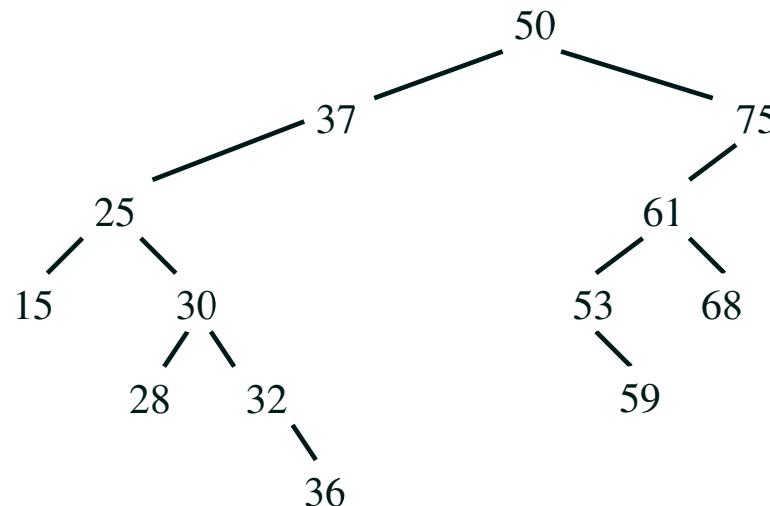
- To find the successor we first start in the left most branch
- We follow two rules
 1. **If** right child exist **then** move right once and then move as far left as possible
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Successor

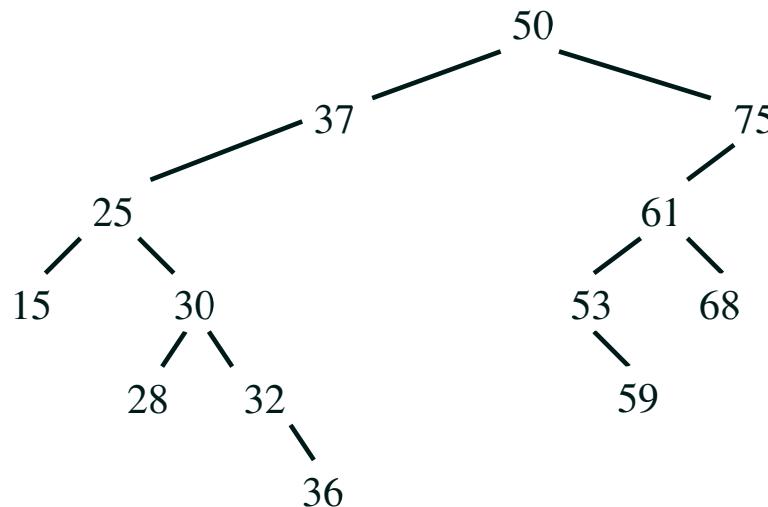
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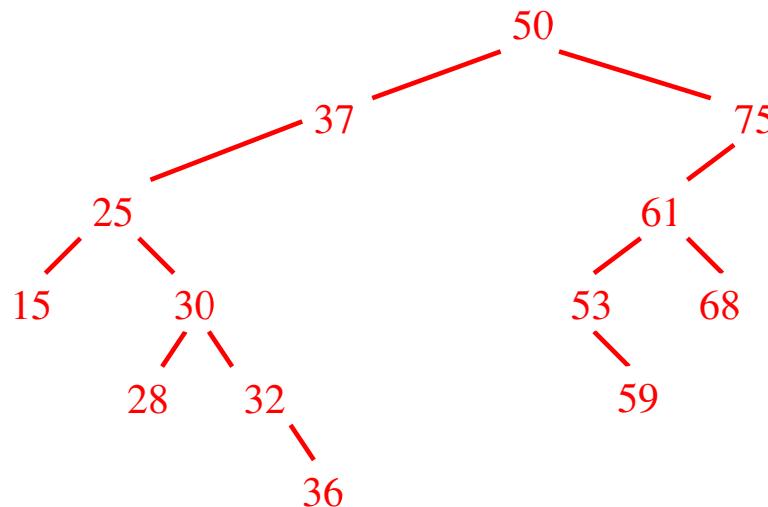
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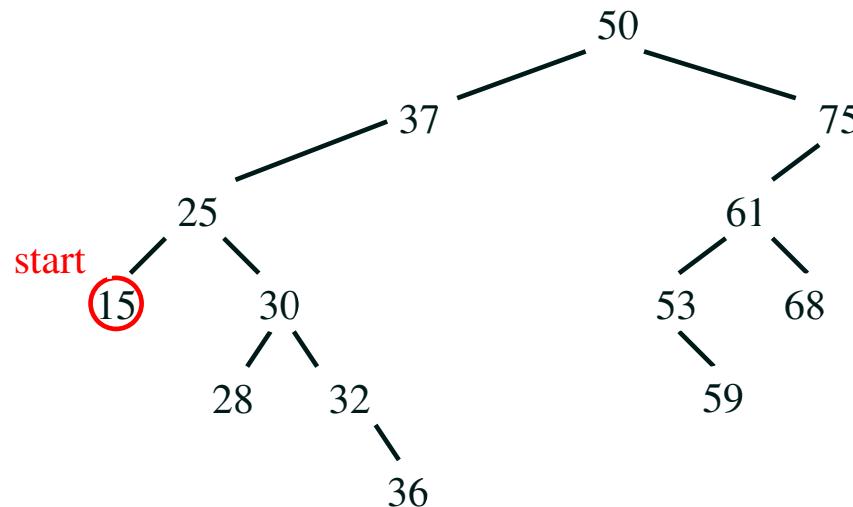
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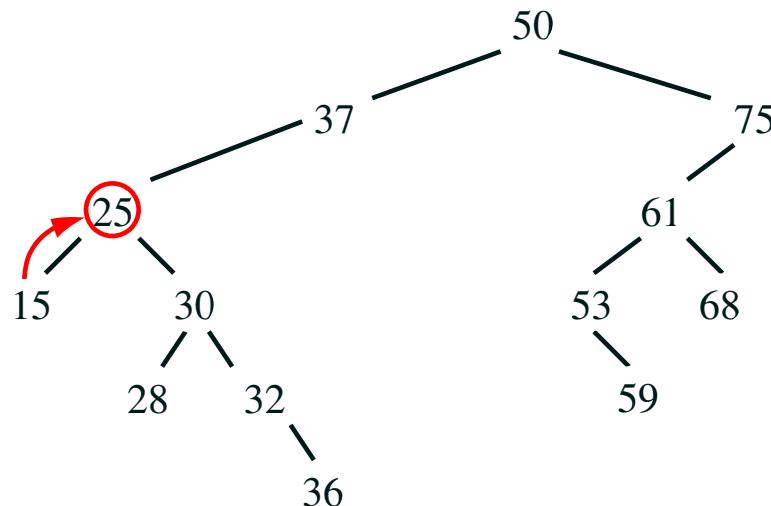
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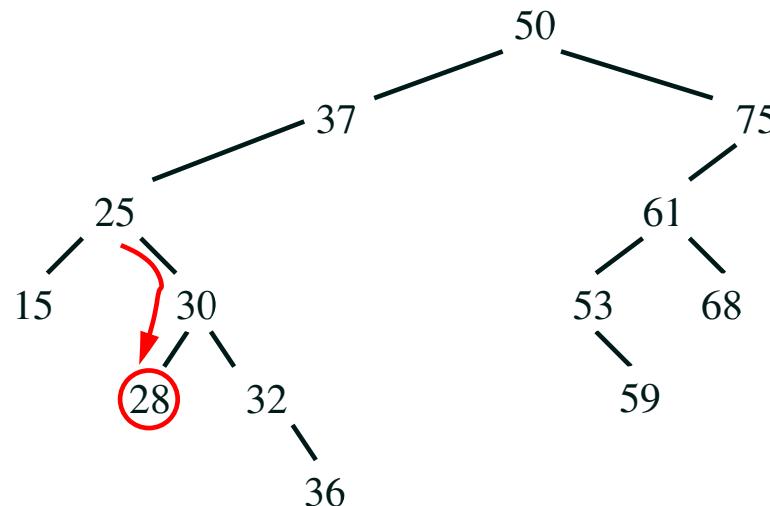
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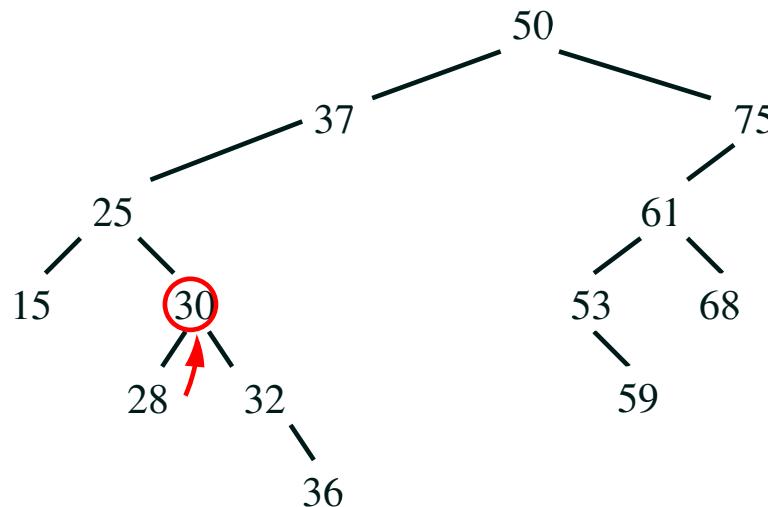
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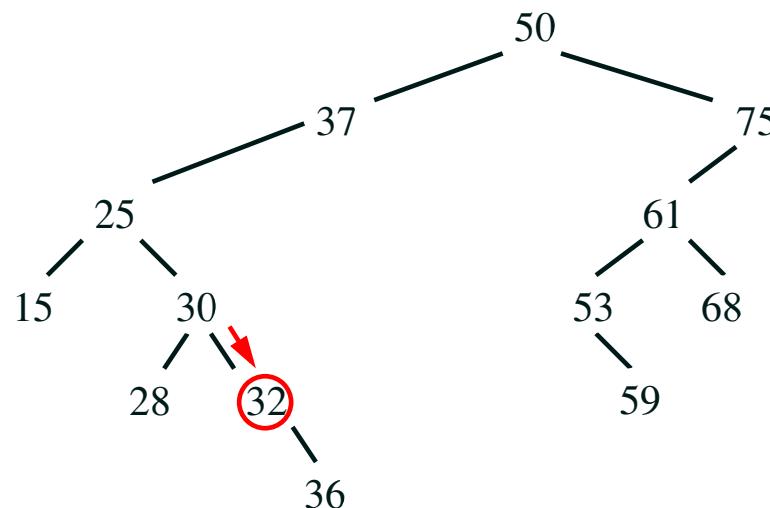
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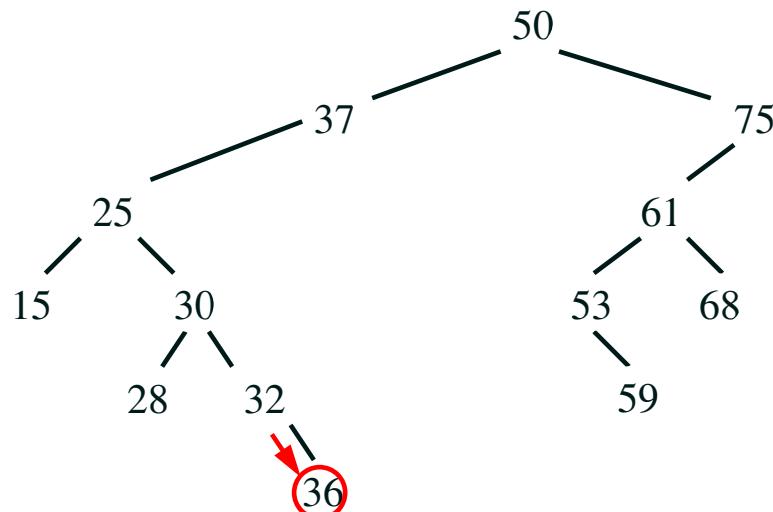
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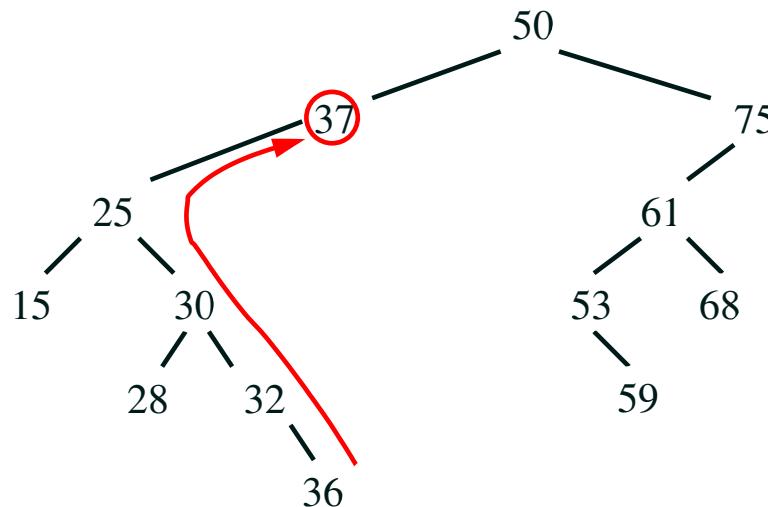
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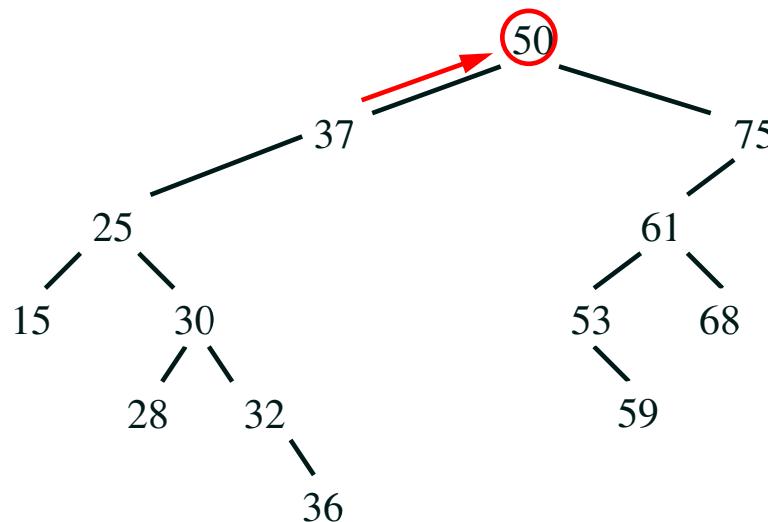
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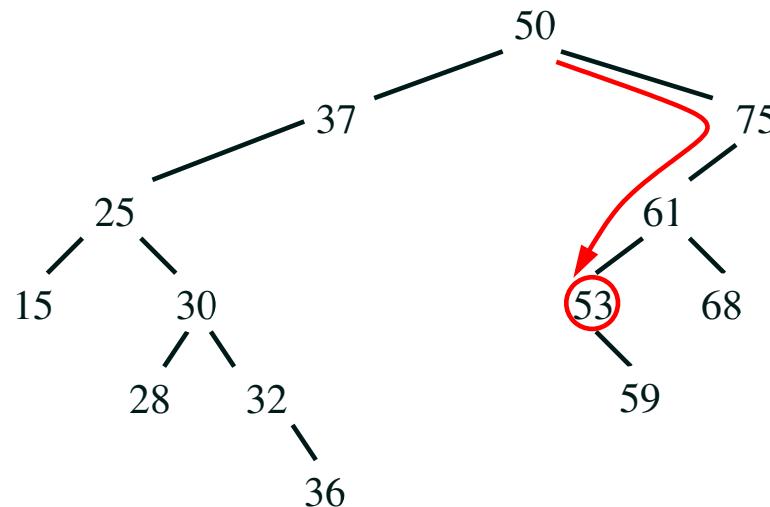
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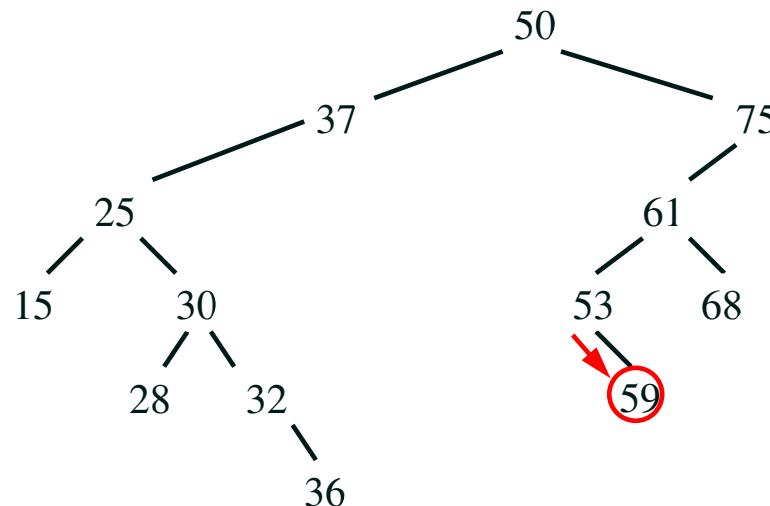
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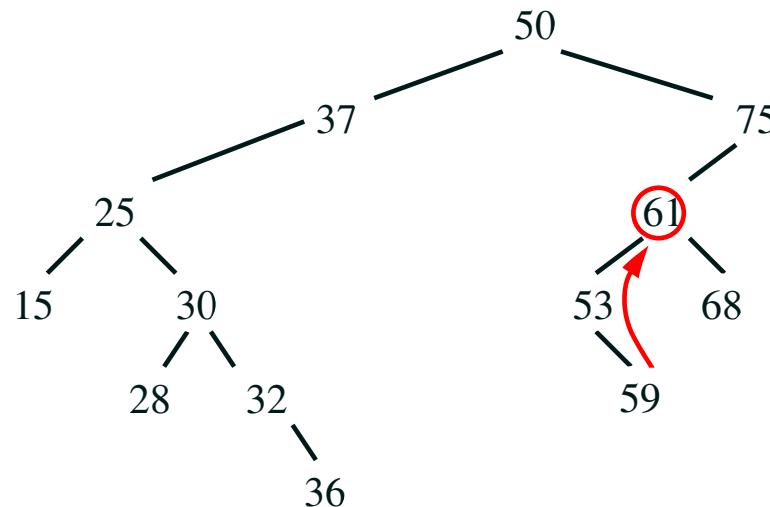
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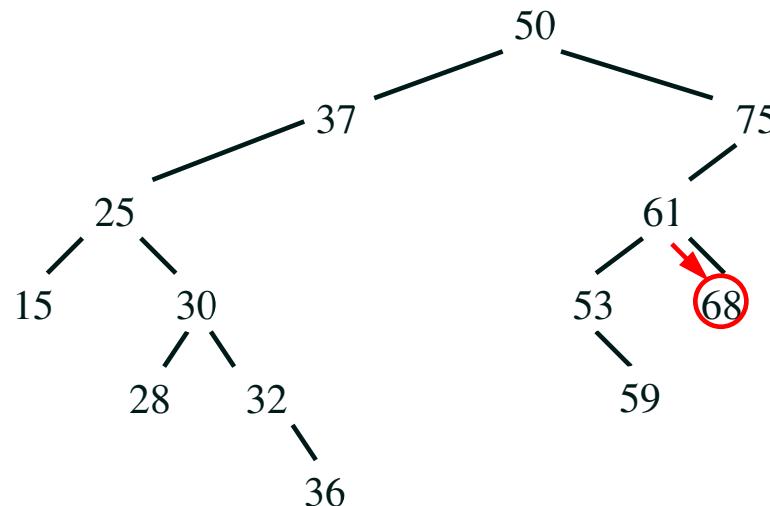
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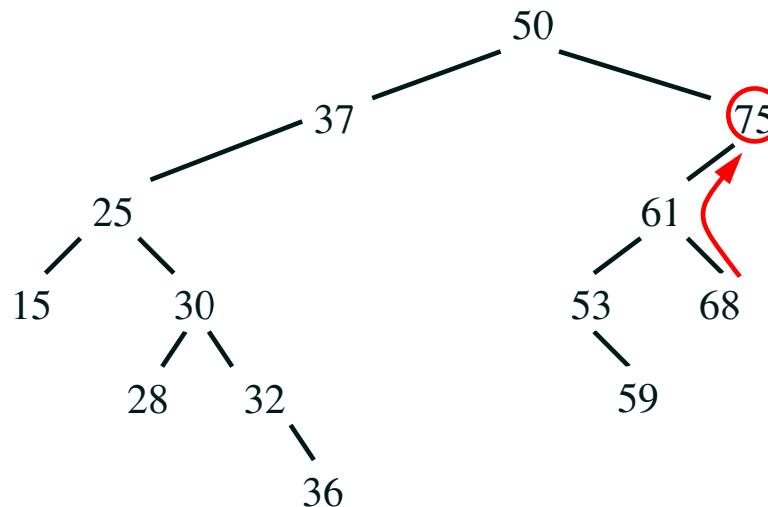
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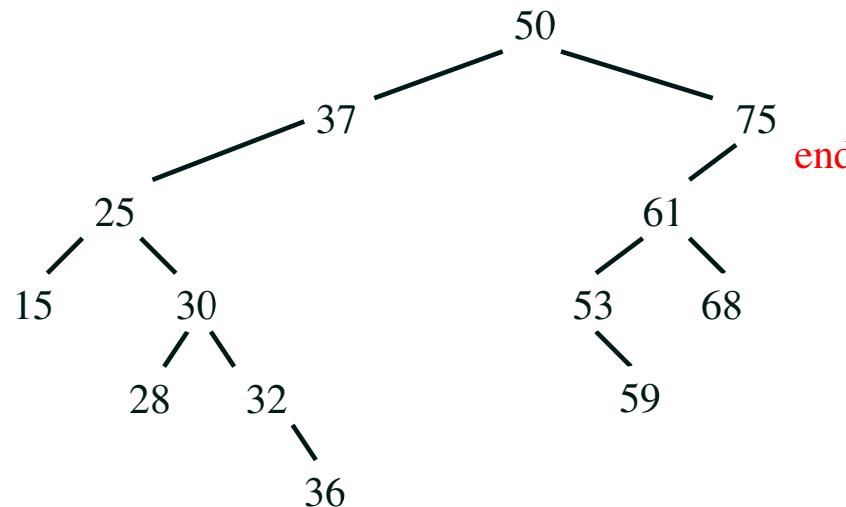
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Lessons

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- Conceptually they are quite simple
- However, there are a lot of details that need to be understood
- Coding even simple trees needs great care
- As we will see things get more complicated

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