UNIVERSITY OF SOUTHAMPTON COMP1009W1

SEMESTER 2 EXAMINATION 2005/2006

DATA STRUCTURES AND ALGORITHMS

Duration: 120 mins

Answer THREE *questions*

This examination is worth 85%. The tutorials were worth 15%.

University approved calculators MAY be used.

 Number of pages 5

Question 1

The algorithm for bubble sort to sort an array $\boldsymbol{a} = (a_0, a_1, \ldots a_{n-1})$ is given by

```
BubbleSort(a)
  for i←0 to n-2
    for j←0 to n-2-i
      if a_{j+1} < a_j
        swap a_j and a_{j+1}
      endif
    endfor
  endfor
```

(a) Write a Java method to performing bubble sort on an array of integers.

(7 marks)

(b) What is the time complexity of bubble sort? Explain your answer.

(3 marks)

(c) Draw a decision tree showing how bubble sort would process an array of three elements (a, b, c). *(13 marks)*

(d) Prove a lower bound on the time complexity for any sort algorithm that uses binary decisions? *(10 marks)*

Question 2

(a) Draw the binary search tree which results from inserting the following list of numbers into the tree

$$30, 40, 20, 50, 90, 10, 50, 70, 60, 80.$$

(5 marks)

(b) Explain what happens when you delete 10, 40 and 30?

(10 marks)

(c) Derive the typical and worst case time complexities for insertion. Briefly explain the strategies used to reduce the worst case time complexity. *(10 marks)*

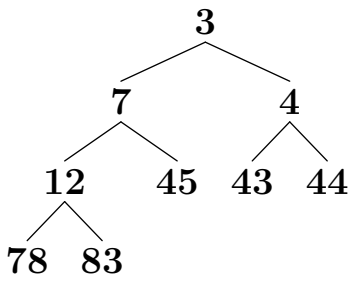
(d) Explain why binary search trees are commonly used to represent sets. What alternative data structure is also used for this purpose? How do these data structures compare? *(8 marks)*

Question 3

(a) What are the properties of a heap? *(5 marks)*

(b) Explain in detail how a heap is stored in memory? *(5 marks)*

(c) Consider the following heap



Explain what happens when we add an new element 5. Draw the new heap. *(5 marks)*

(d) Explain what happens when we remove the minimum element from the heap.

(5 marks)

(e) Explain how a heap is used to perform sort? Is it a stable sort? *(5 marks)*

(f) Prove the time complexity for heap sort. *(4 marks)*

(g) Give two other applications of heaps. *(4 marks)*

Question 4

(a) Assume we are hashing a two digit number d_2d_1 and we have a hash function

$$hash(d_2d_1) = (d_2 + 3d_1) \bmod 10.$$

Calculate the hash addresses for 12, 14, 31, 83, 17, 54, 22.

(3 marks)

(b) Show how the numbers above would be stored in a hash table of size 10 using separate chaining. Calculate the average number of comparisons needed to find an entry. *(7 marks)*

(c) Show how the numbers above would be stored in the same size hash table using linear probing and calculated the average number of comparisons needed to find an entry. *(7 marks)*

(d) Explain the problem specific to linear probing and give an alternative method to implement a hash table using open addressing.

(7 marks)

(e) Explain why deletions cause problems for open hashing. What is the most commonly used fix? *(7 marks)*

(f) What hashing scheme is used by `HashSet<T>` in the Java collection class and why? *(2 marks)*

END OF PAPER