

Algorithms and Analysis

Lesson 11: *Use Heaps!*



Heaps, Priority queues, Heap Sort, Other heaps

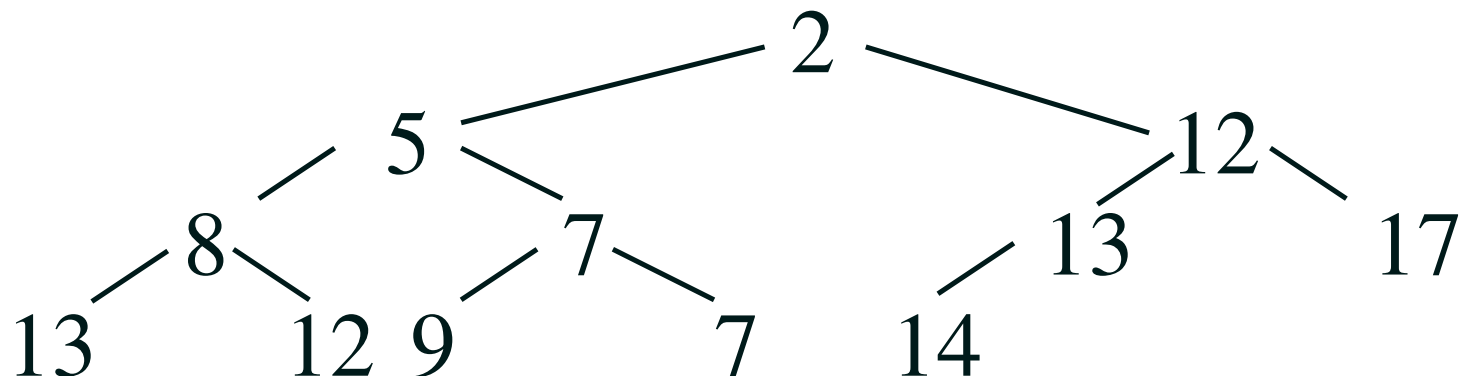
Outline

1. **Heaps**
2. Priority Queues
 - Array Implementation
3. Heap Sort
4. Other Heaps



Heaps

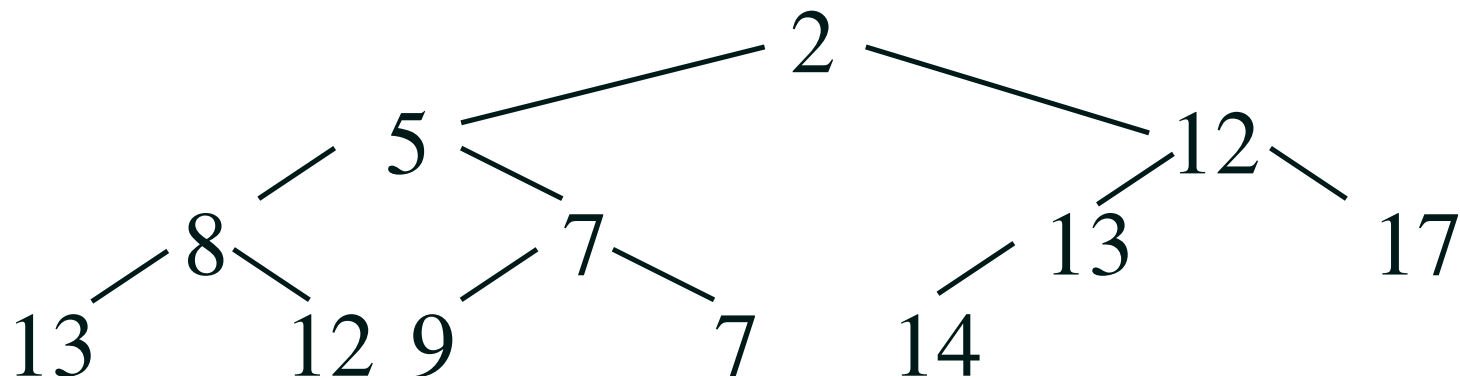
- A (min-)heap is (from one perspective) a binary tree
- It is a binary tree satisfying two constraints
 - ★ It is a **complete** tree
 - ★ Each child has a value 'greater than or equal to' its parent



- **complete** means that every level is fully occupied above the lowest level and the nodes on the lowest level are all to the left

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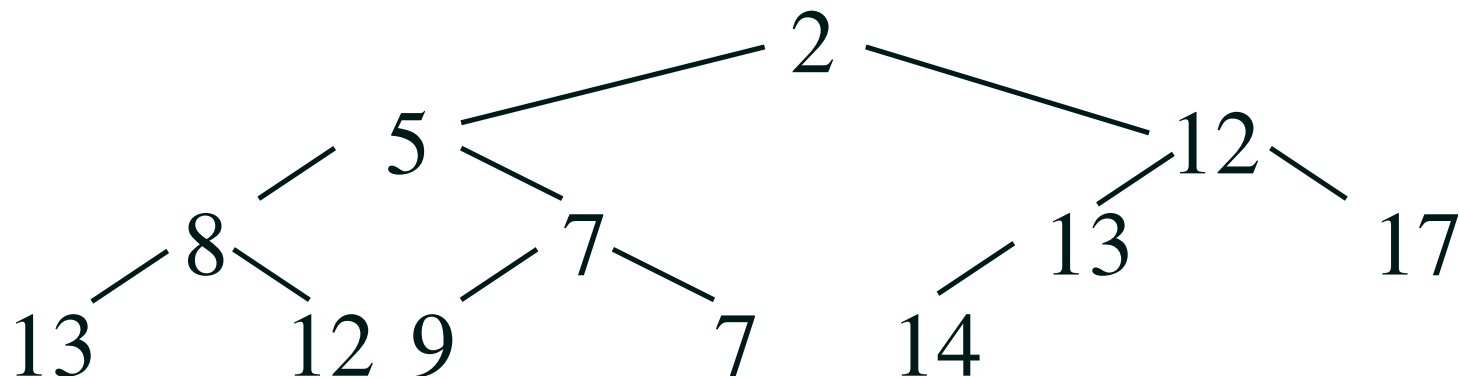
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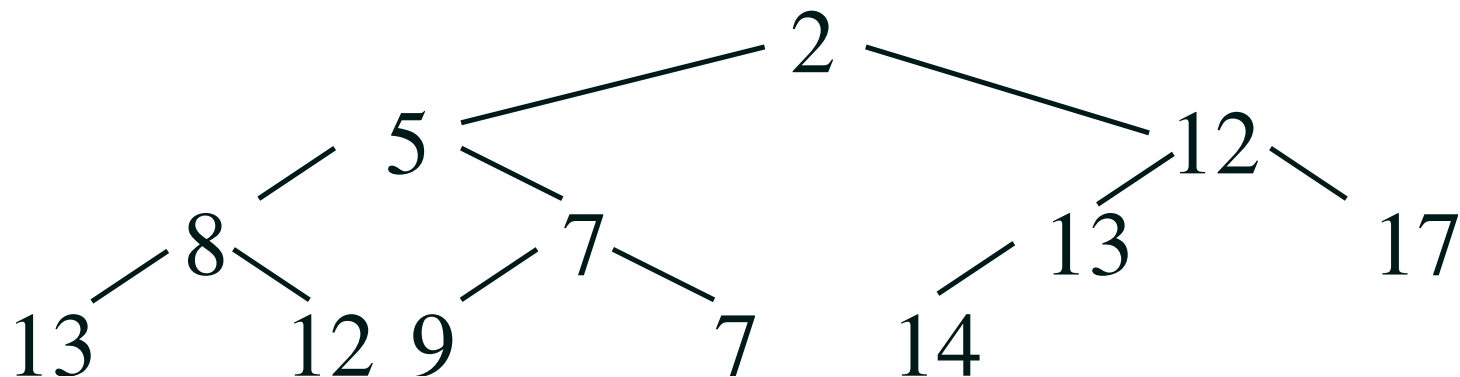
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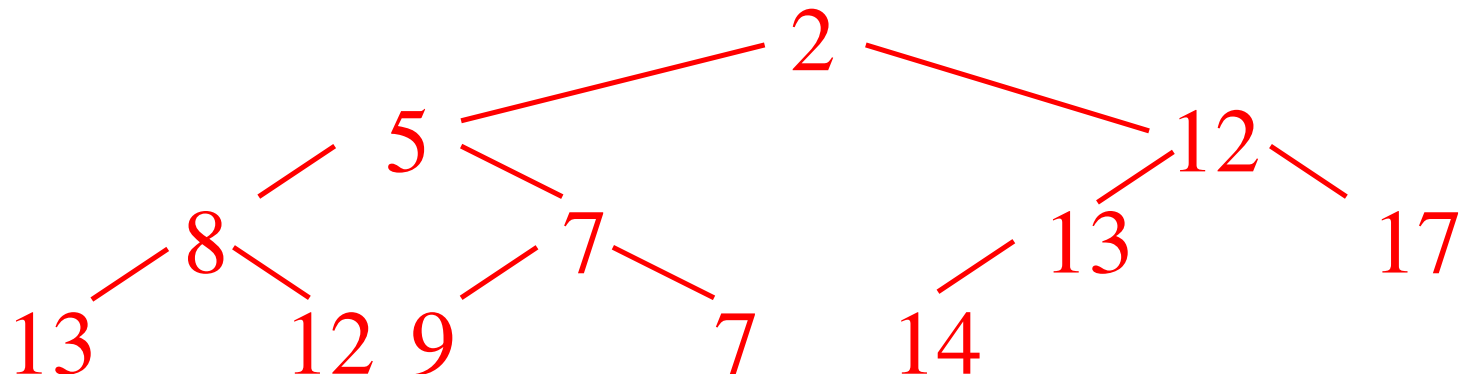
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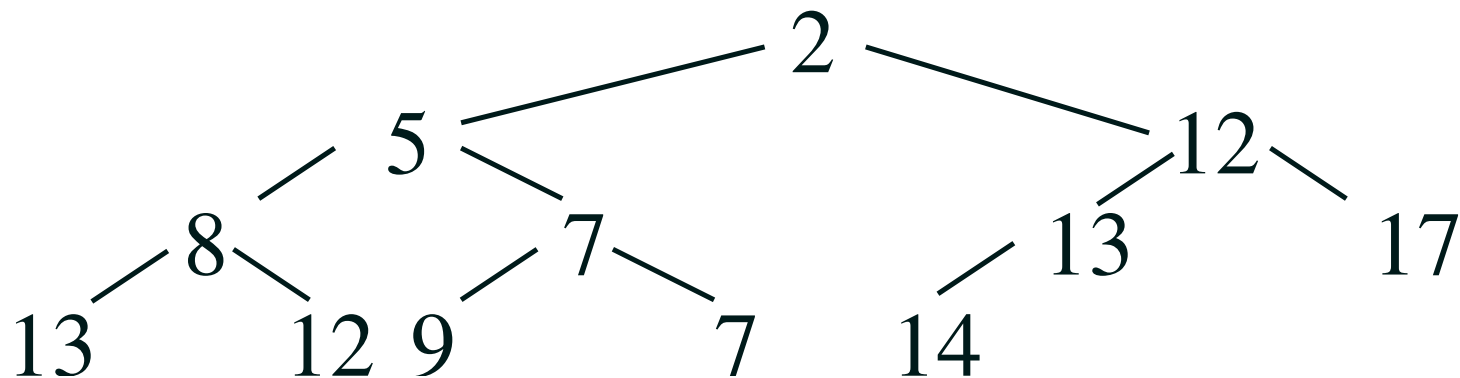
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Priority Queues

- One of the prime uses of heaps is to implement a priority queue
- A Priority Queue is a queue with priorities
- That is, we assign a priority to each element we add
- The head of the queue is the element with highest priority (smallest number)
- Used, for example, in simulating real time events
- Used to implement “greedy algorithms”

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Priority Queue

- A simple Priority Queue might include
 - ★ `unsigned size()` returning the the number of elements
 - ★ `bool empty()` returns true if empty
 - ★ `void push(T element, int priority)` adds an element
 - ★ `T top()` returns head of queue
 - ★ `void pop()` dequeues head of queue

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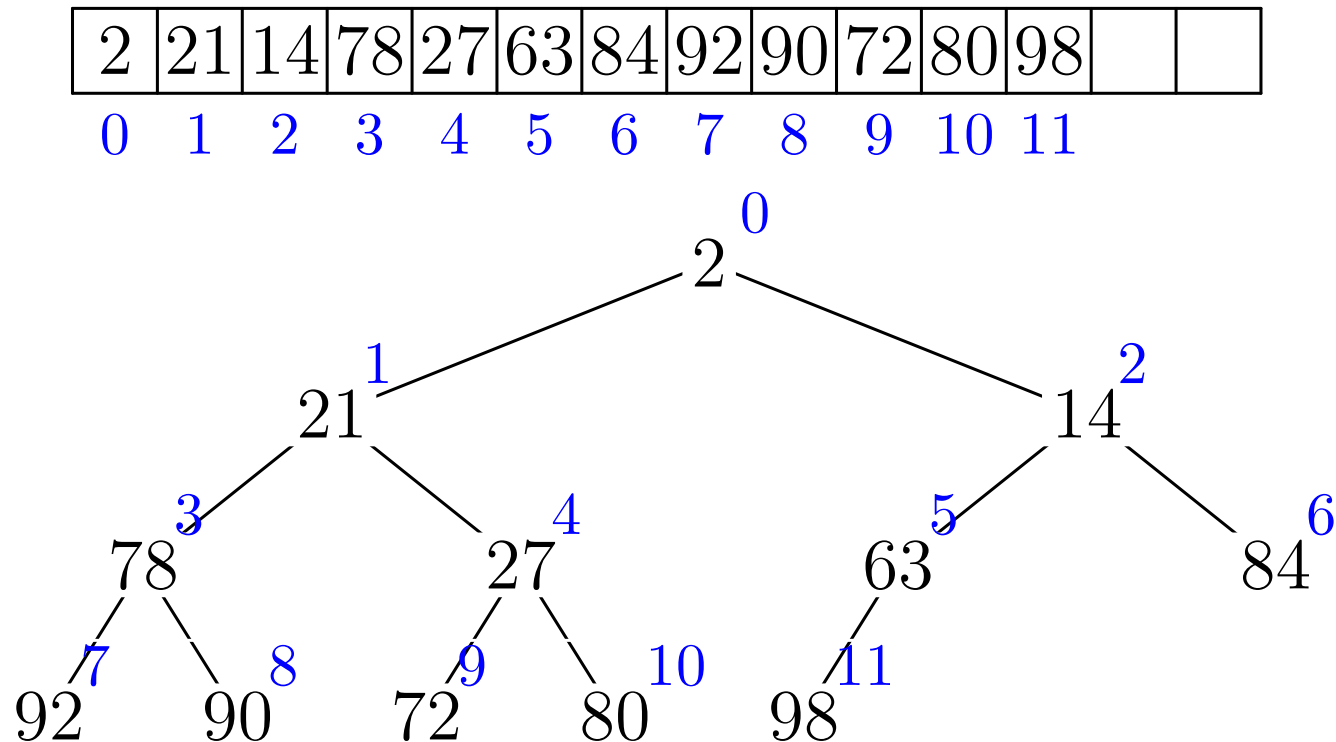
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Array Implementation of Heaps

- Because the tree is complete we can implemented the heap efficiently using an array

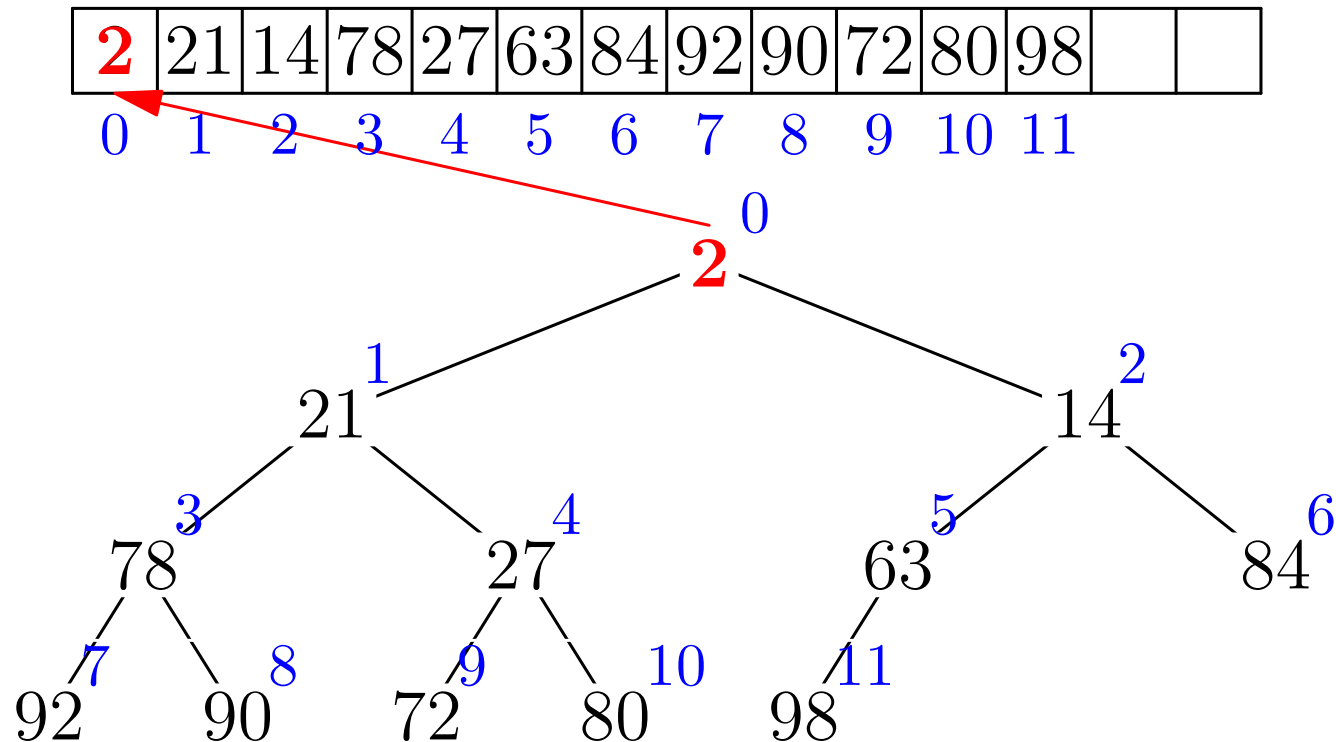
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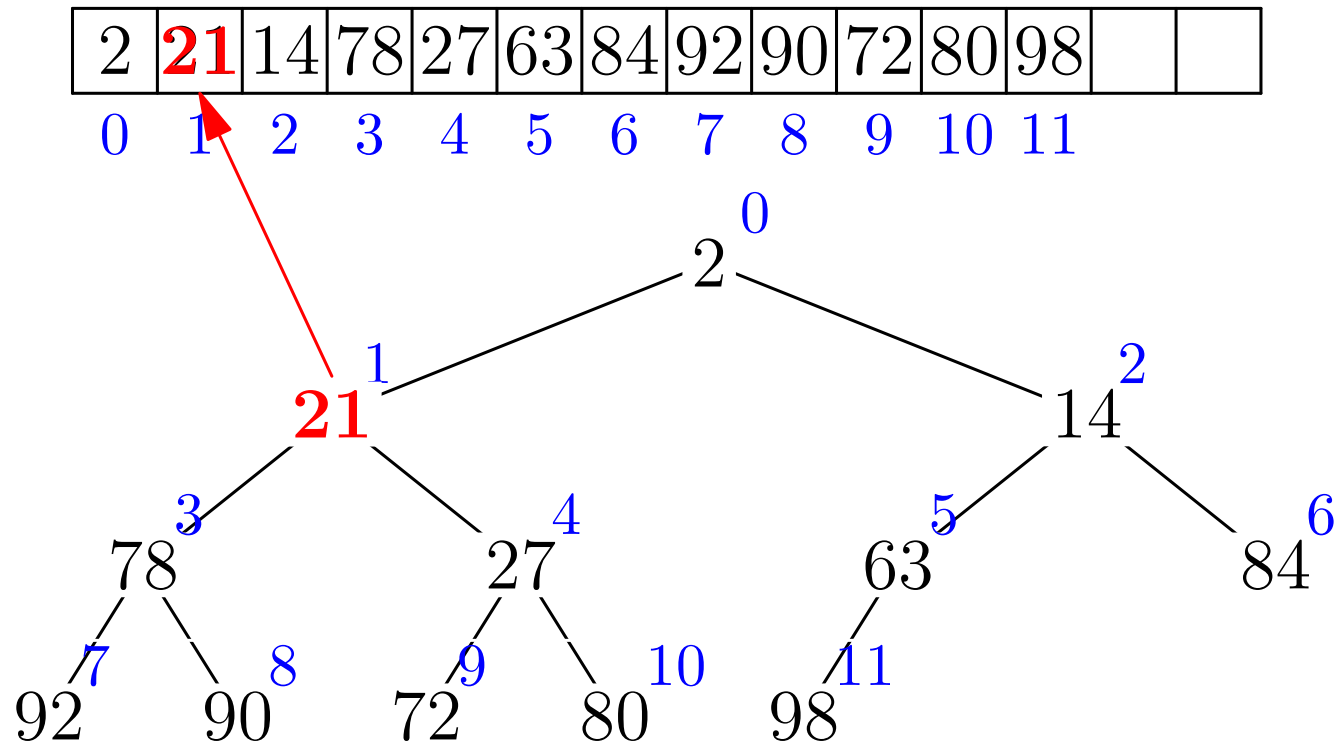
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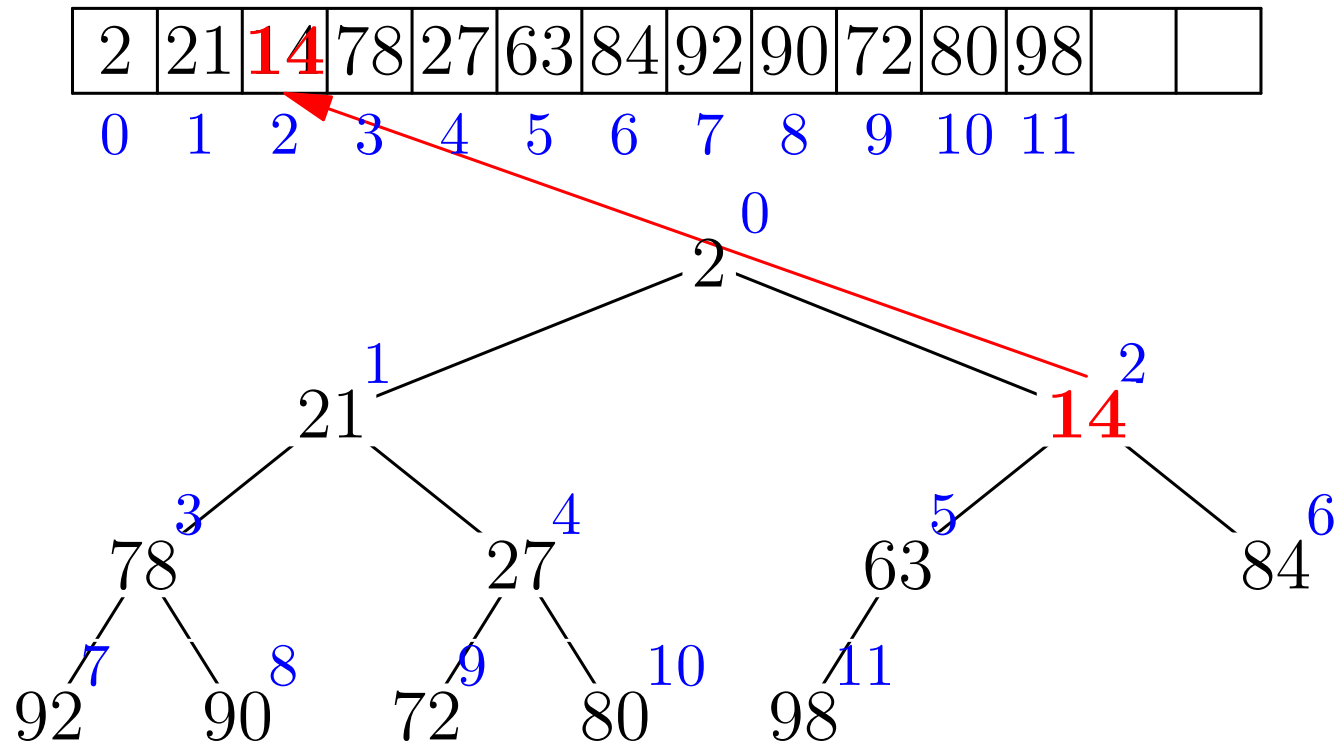
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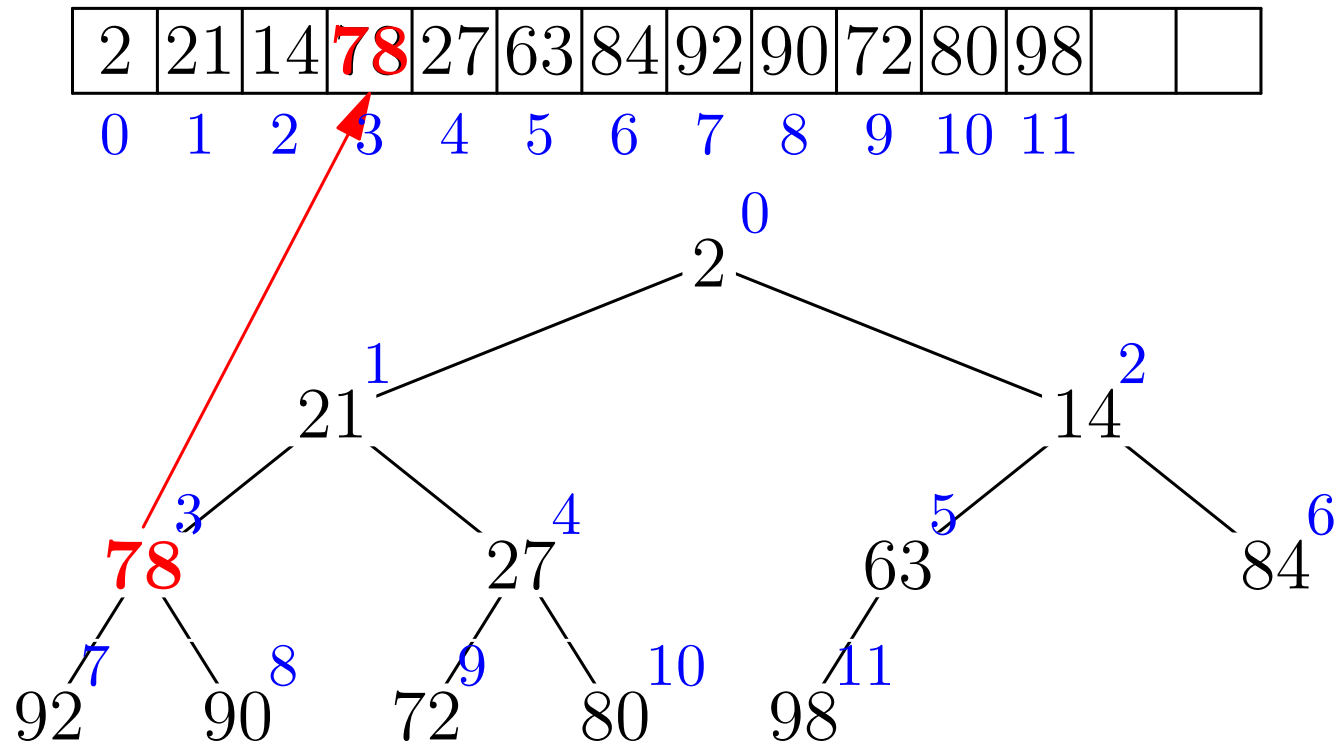
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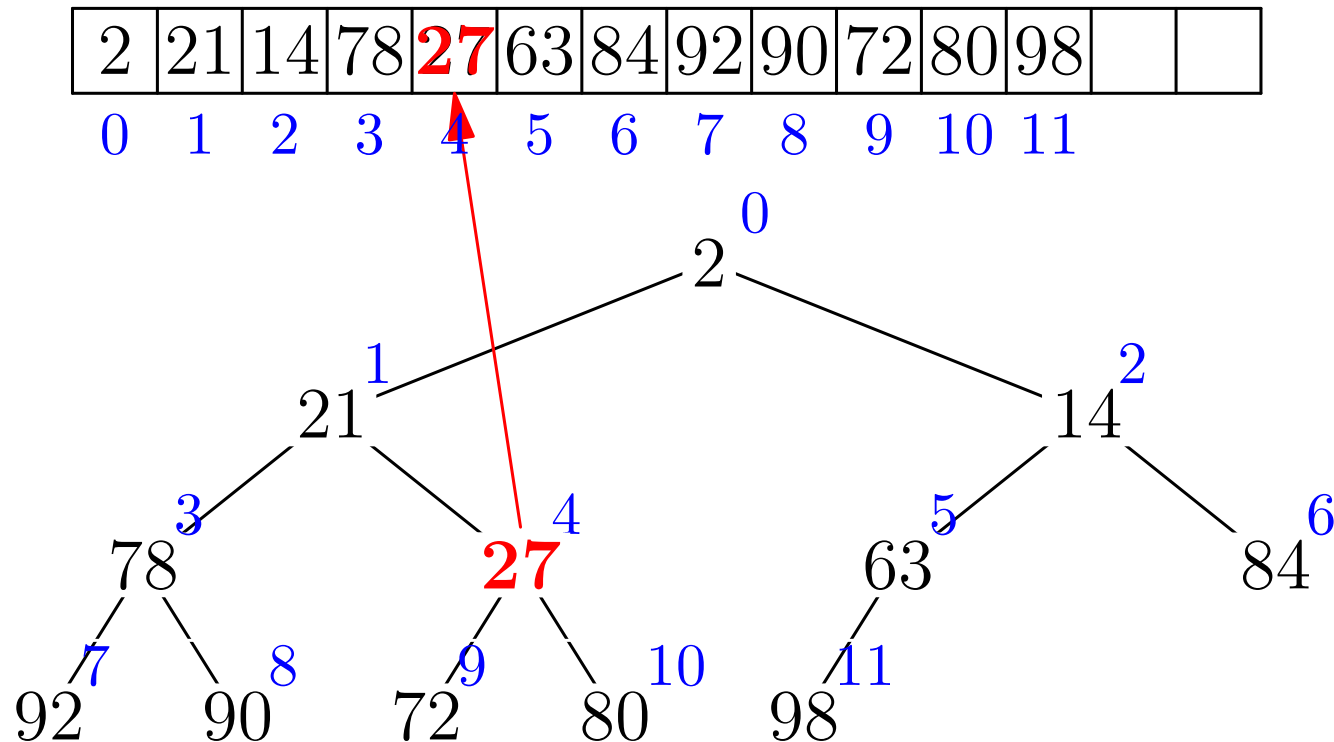
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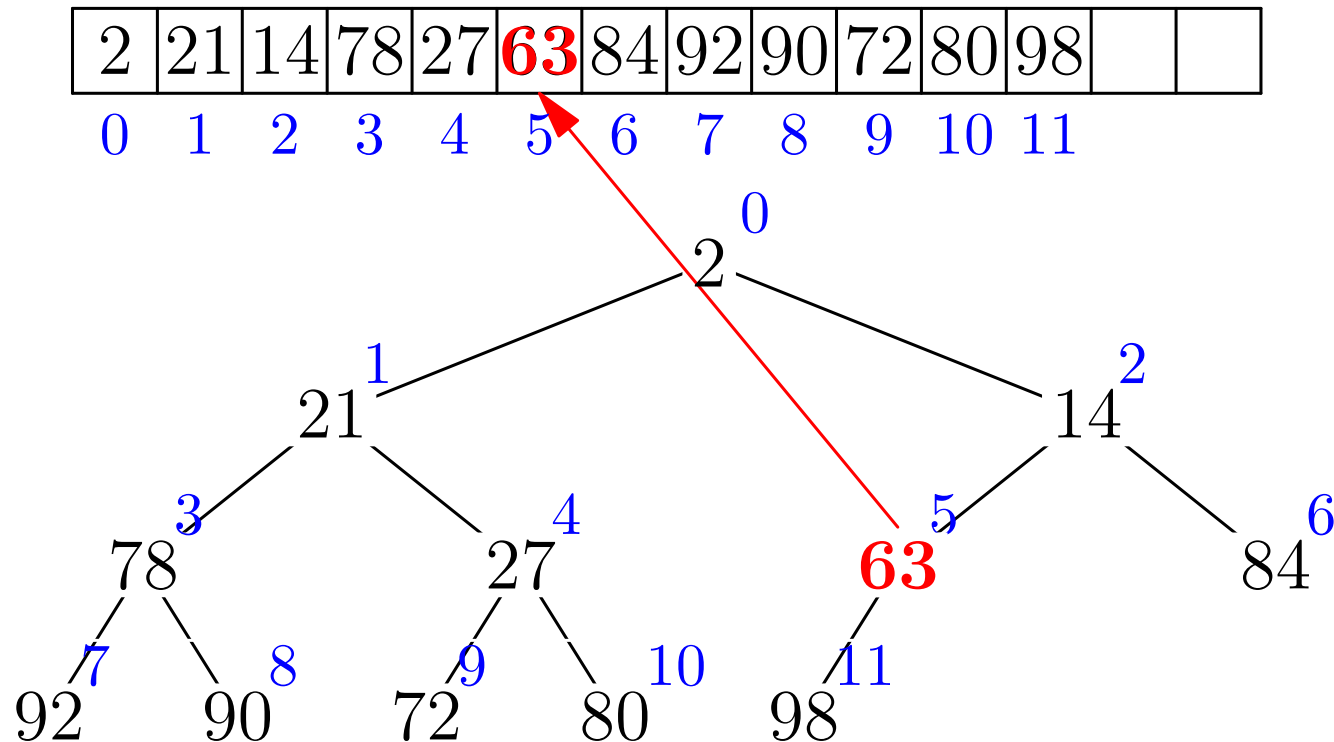
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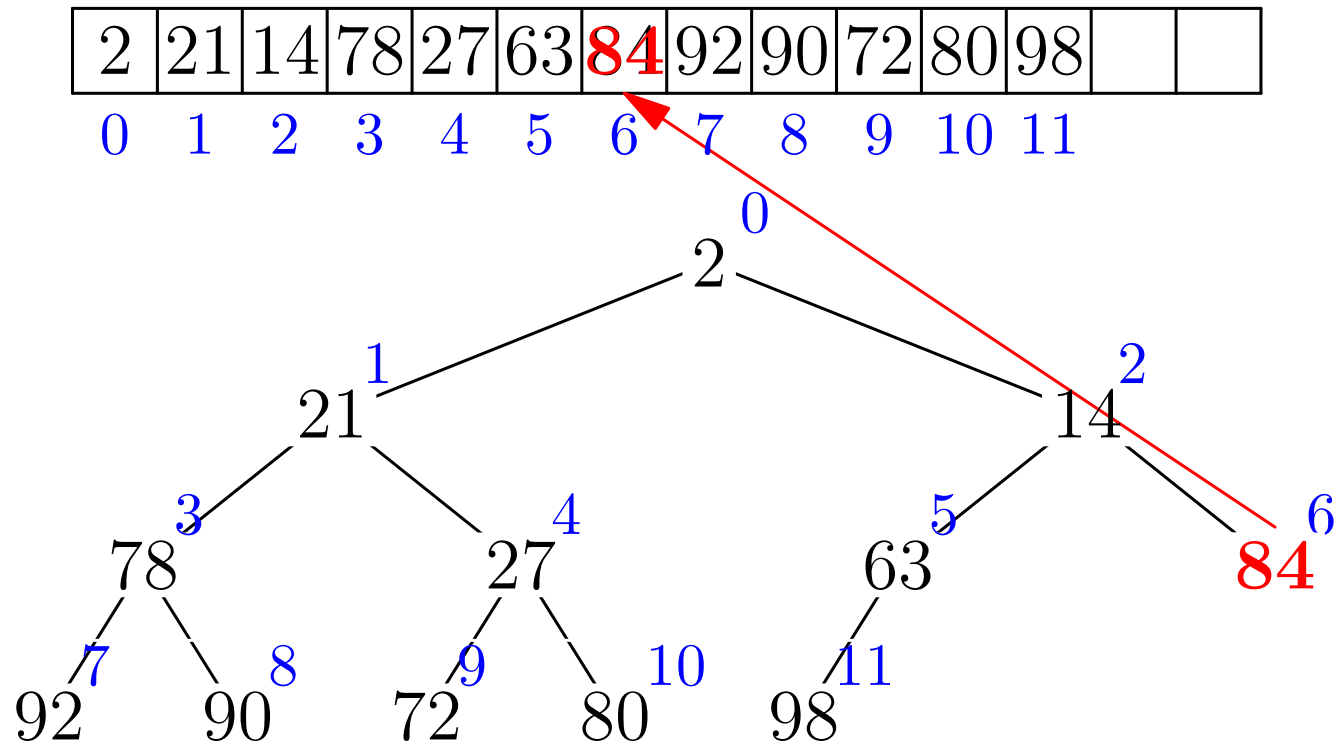
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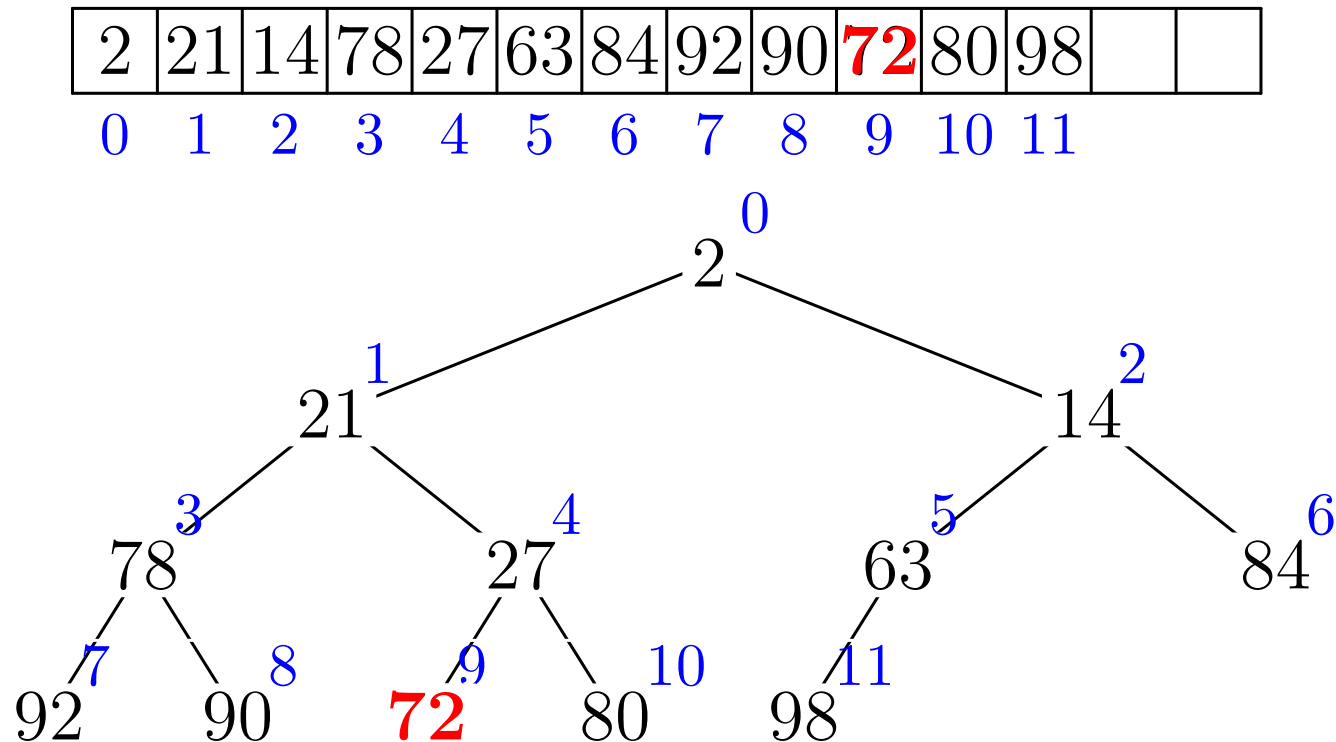
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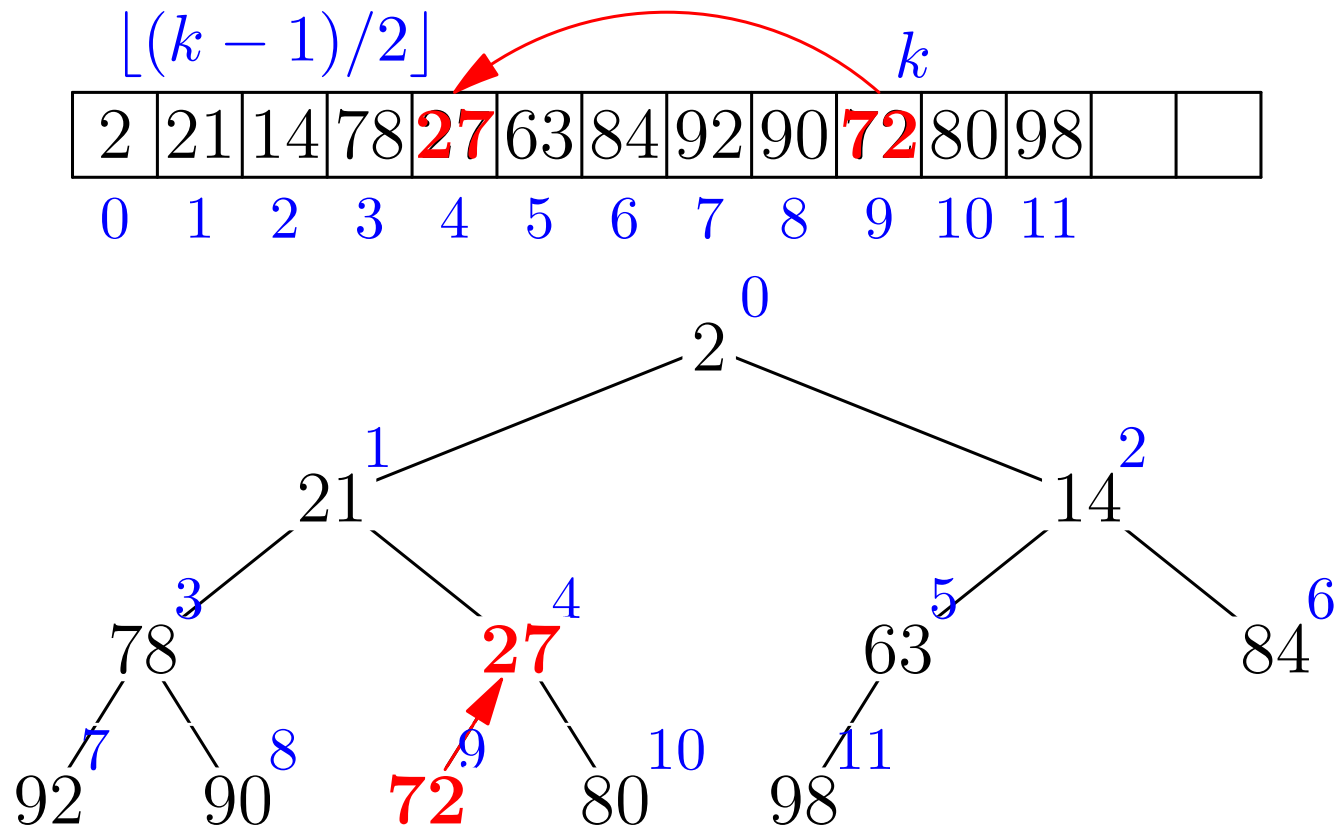
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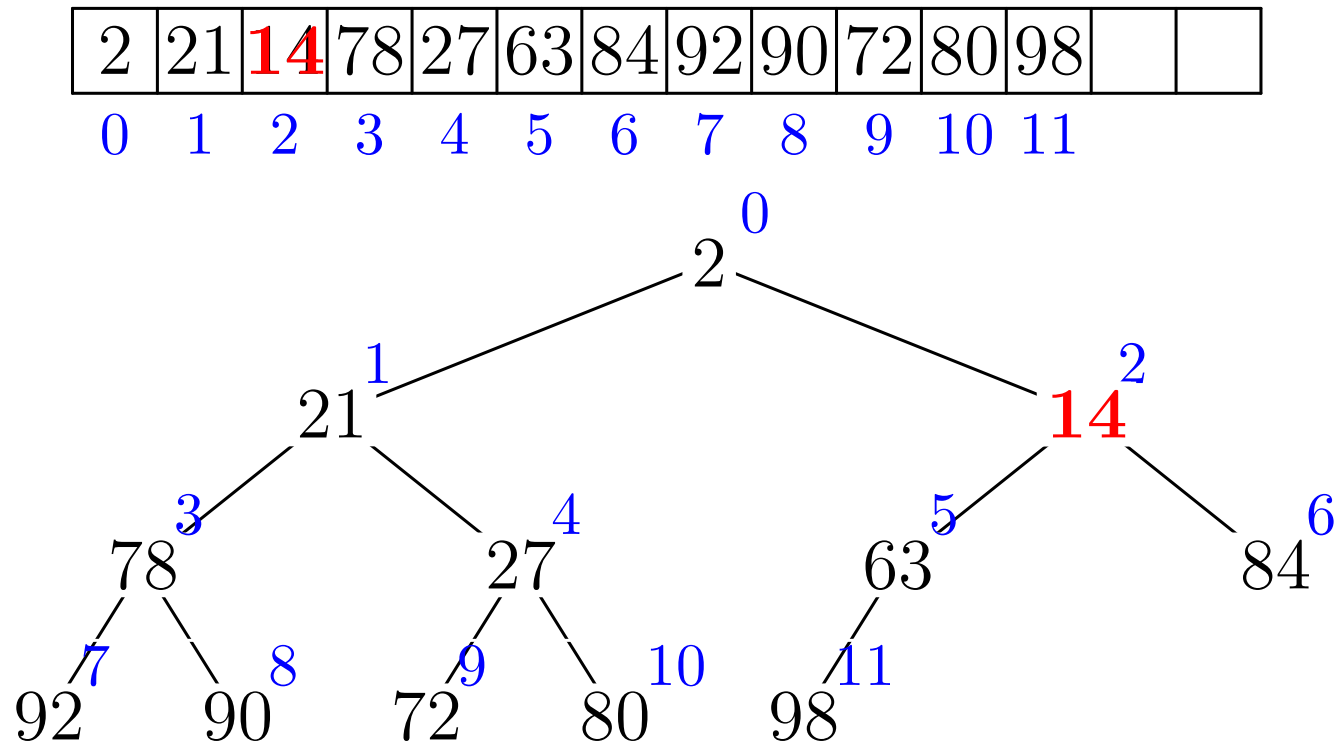
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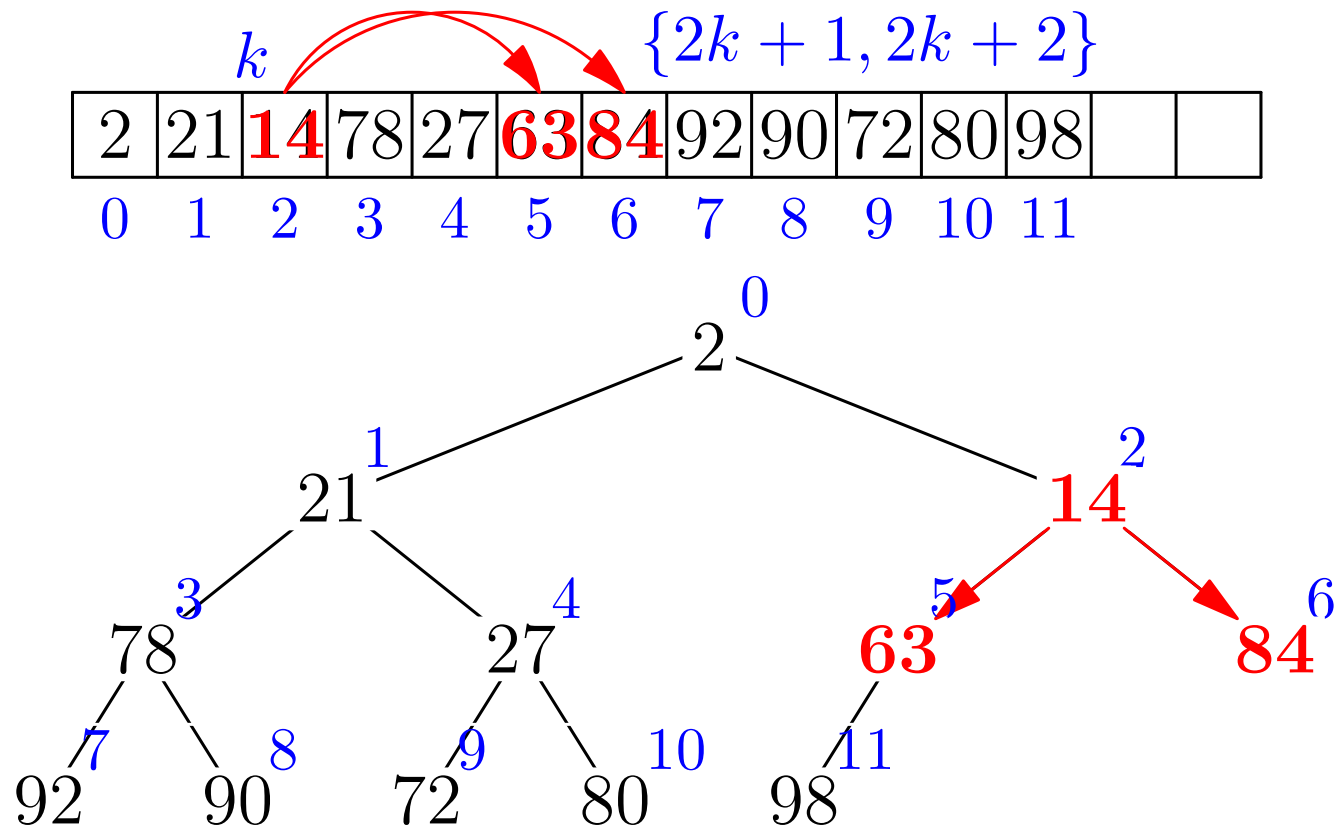
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Code for a Priority Queue

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#include <vector>
using namespace std;

template <typename T, typename P>
class heapPQ {
private:
    vector<pair<T, P> > array;
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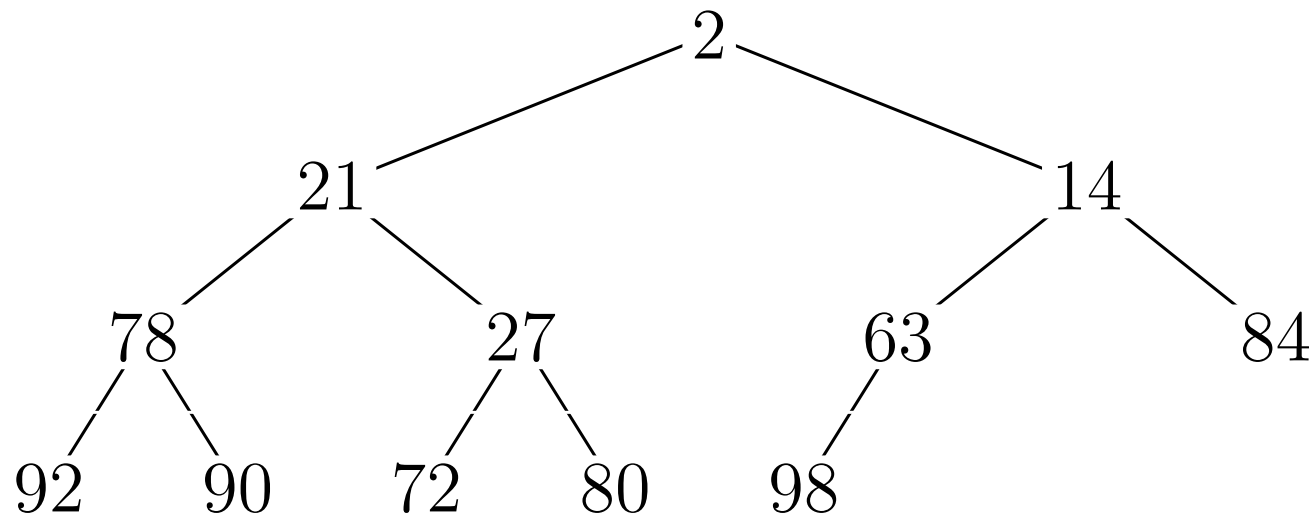
    unsigned size() {return array.size();}

    bool empty() {return array.empty();}

    const T& top() {return array[0].first;}
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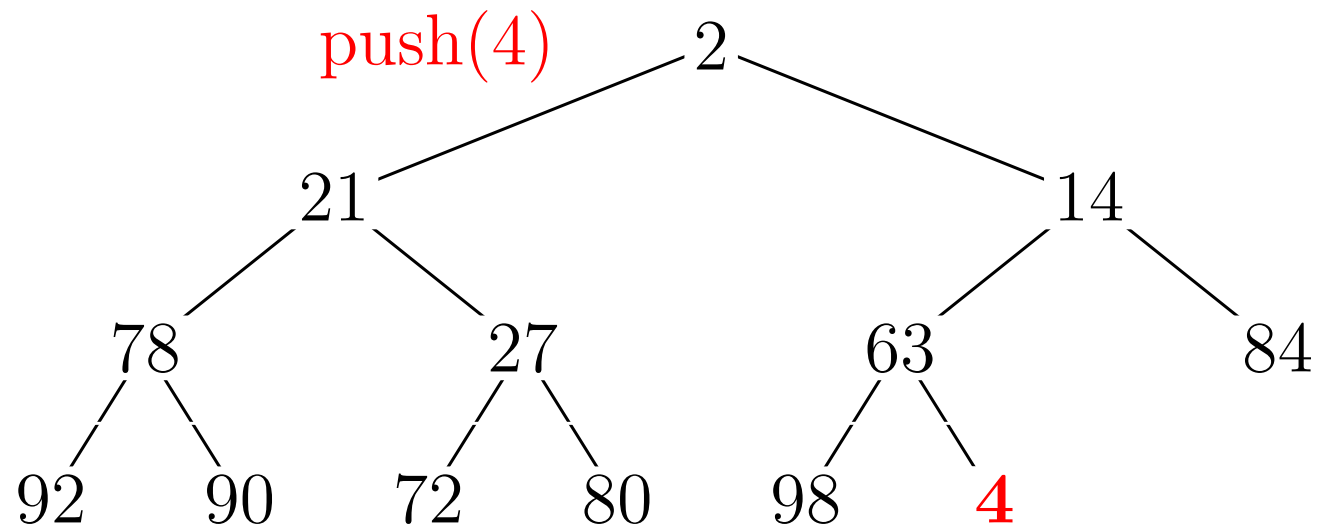
Adding an Element

2	21	14	78	27	63	84	92	90	72	80	98		
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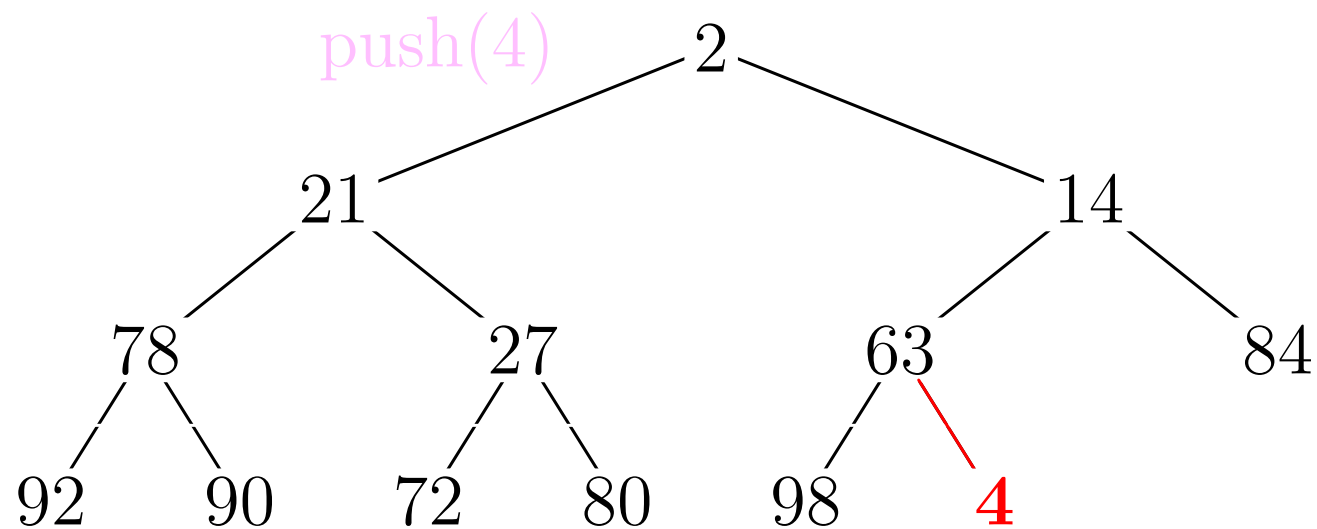
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2	21	14	78	27	63	84	92	90	72	80	98	4	
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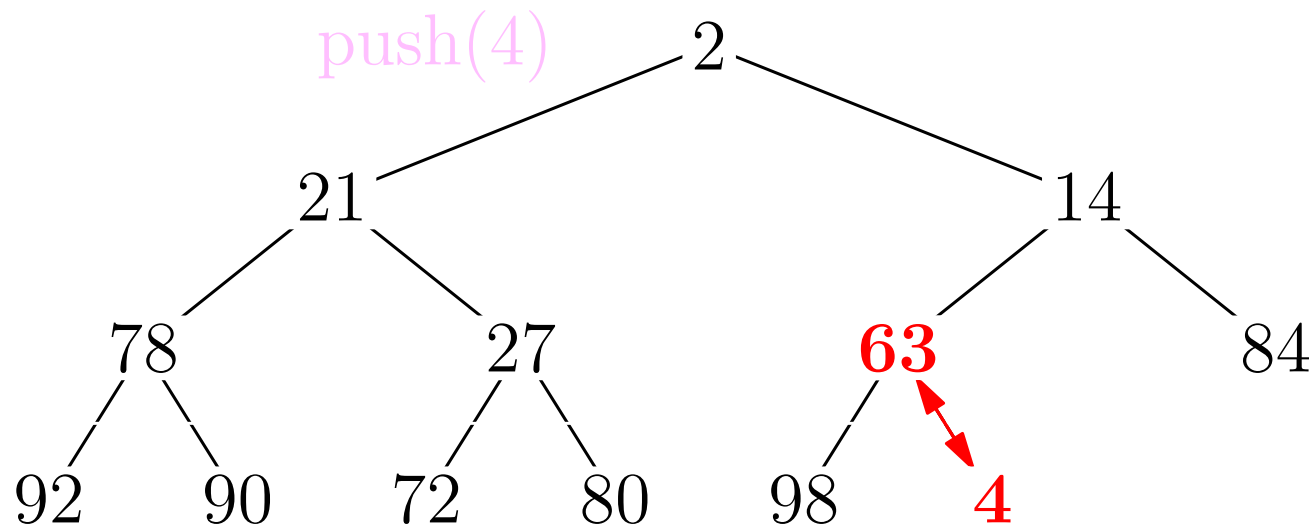
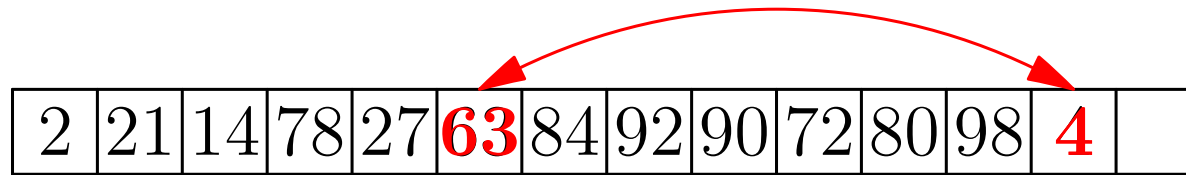


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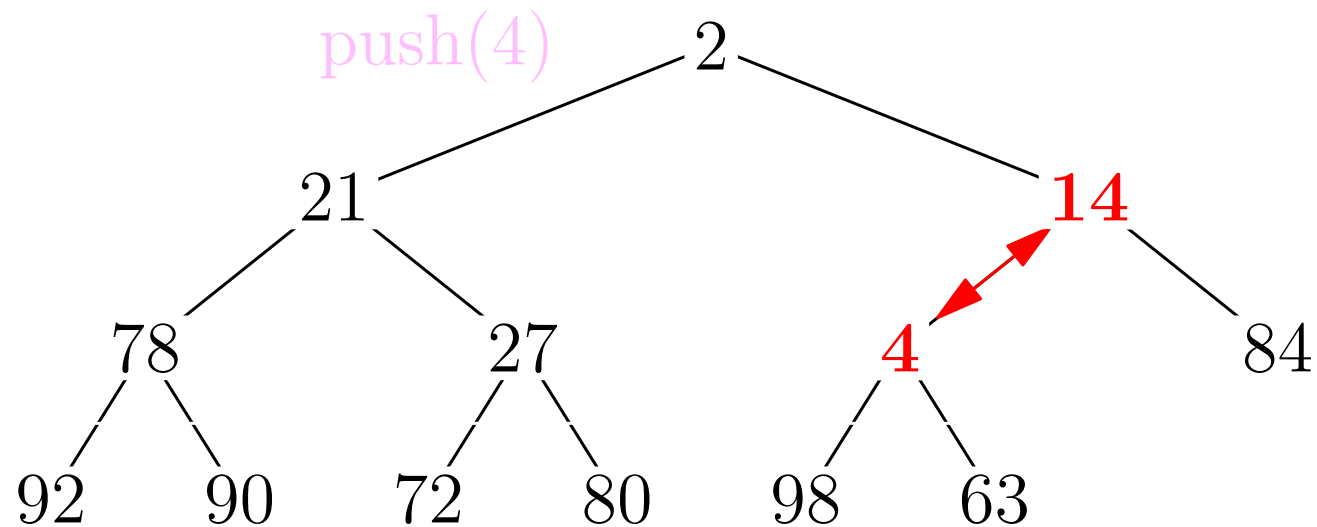
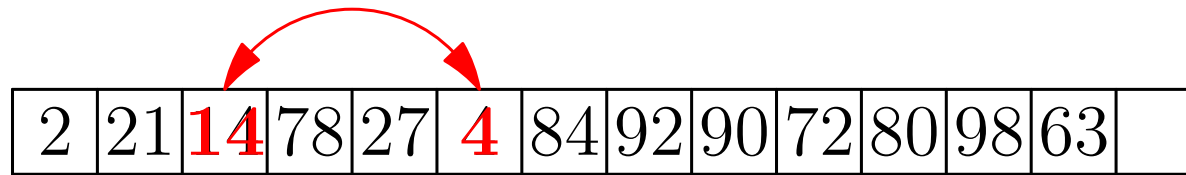
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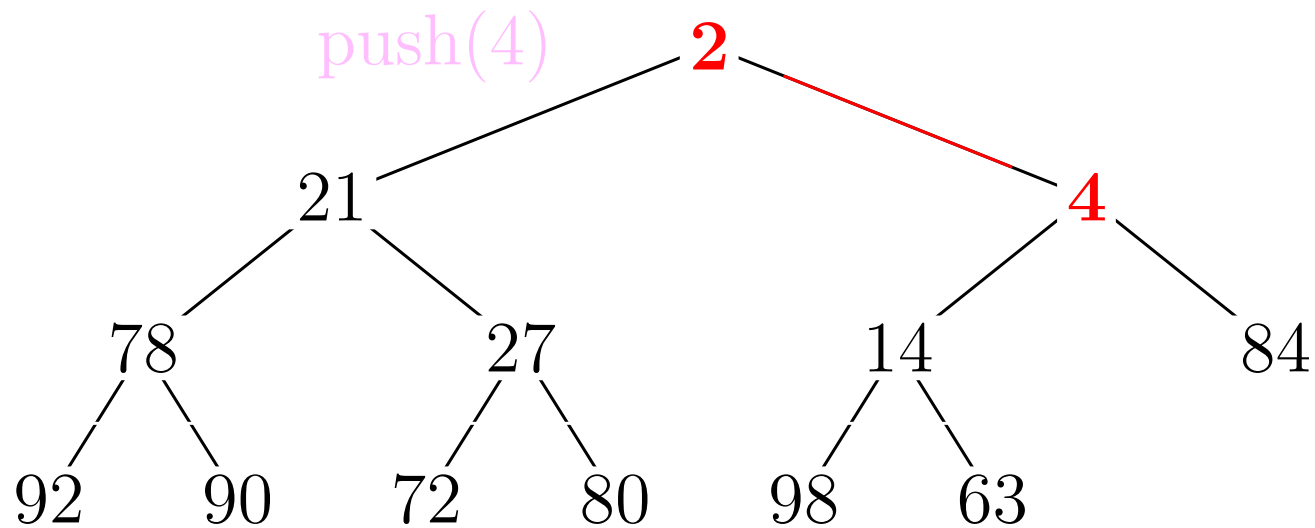
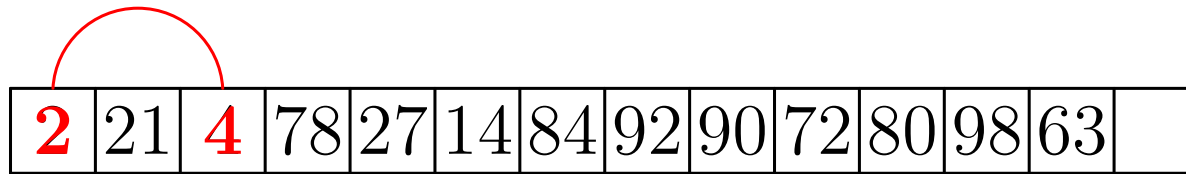
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    pair<T,P> tmp(value, priority);
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    /* Percolate Up */
    while(child!=0) {
        unsigned parent = (child-1)>>1; // floor((child-1)/2)
        if (array[parent].second < array[child].second)
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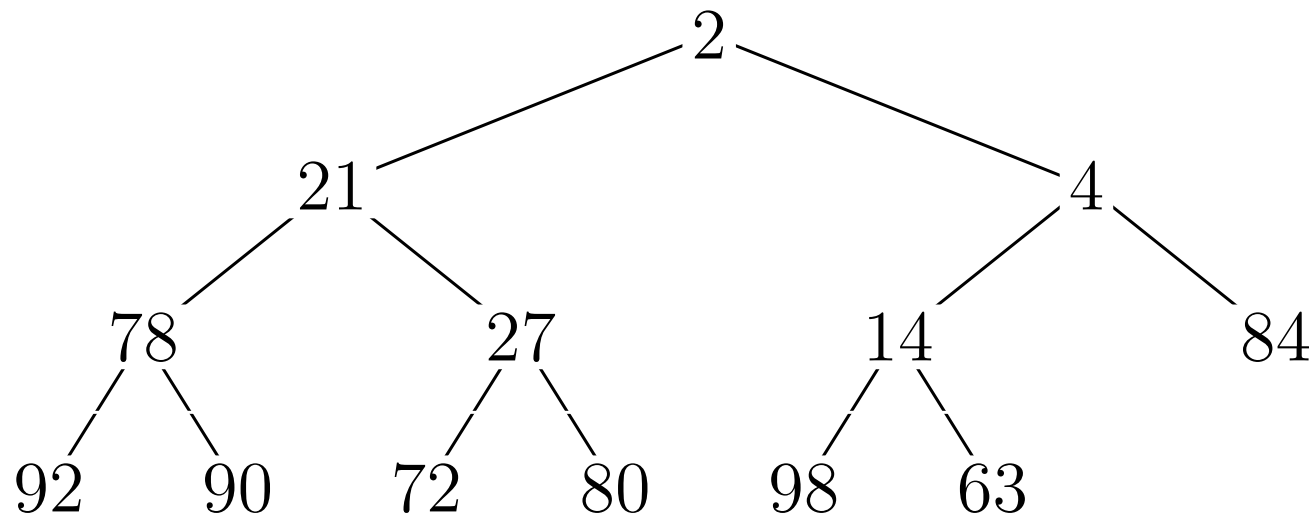
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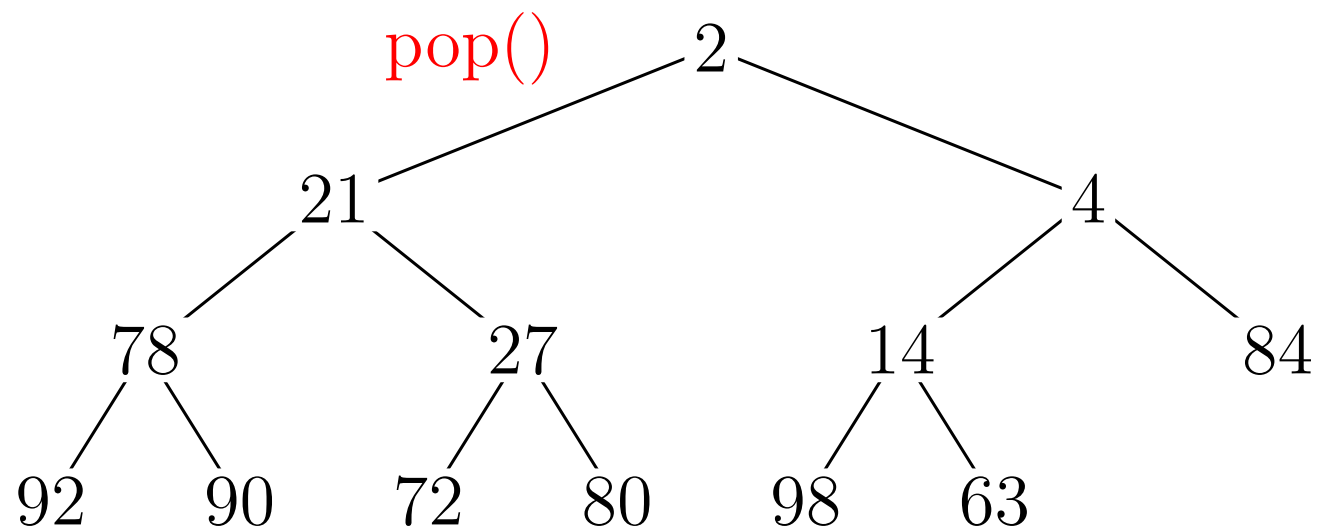
Popping the Top

2	21	4	78	27	14	84	92	90	72	80	98	63	
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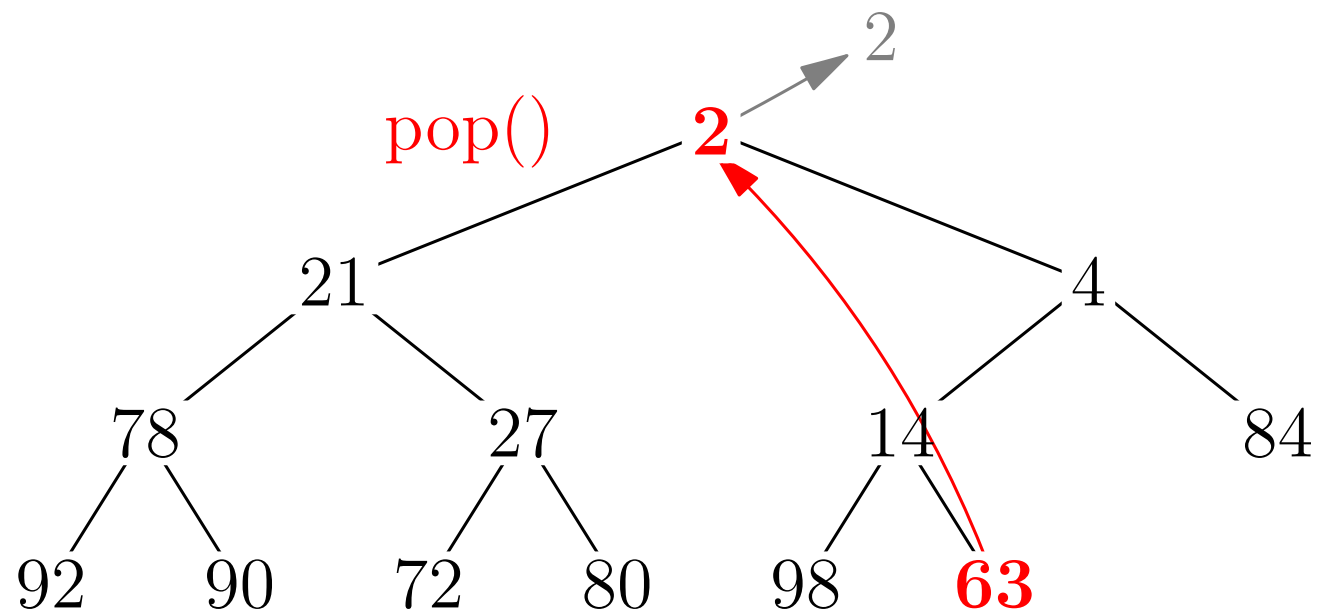
Popping the Top

2	21	4	78	27	14	84	92	90	72	80	98	63	
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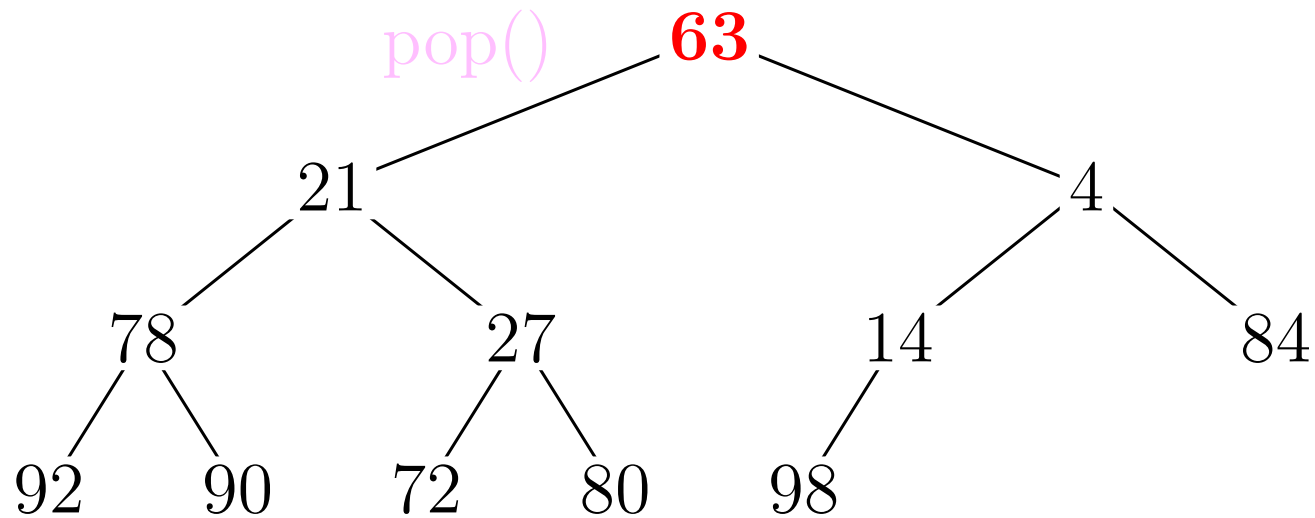
Popping the Top

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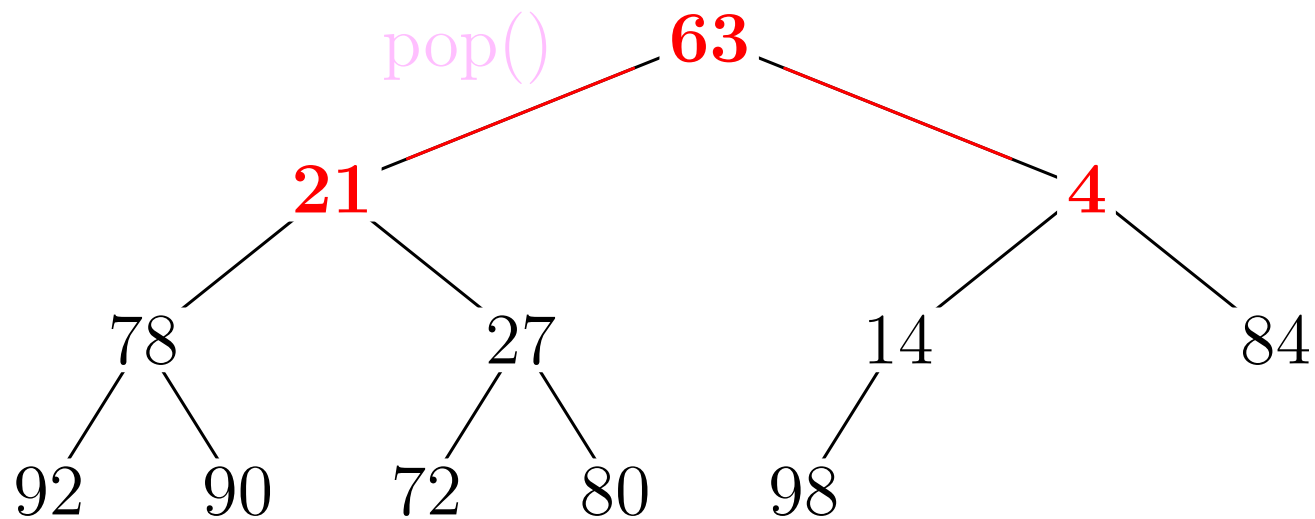
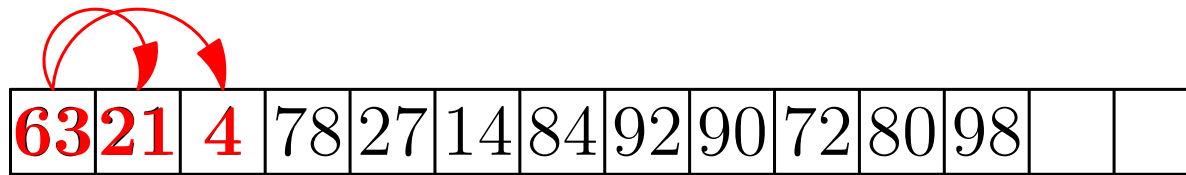


Popping the Top

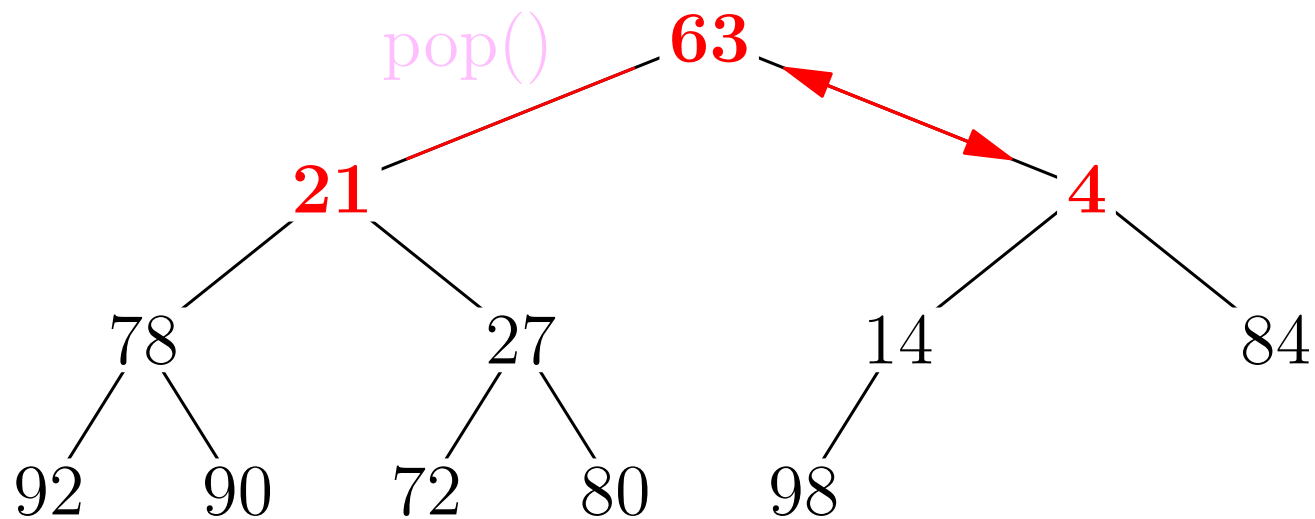
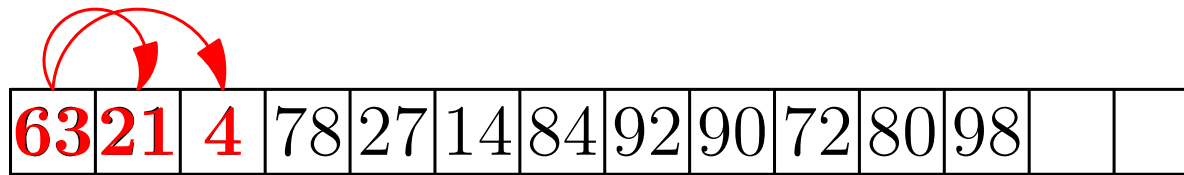
63	21	4	78	27	14	84	92	90	72	80	98		
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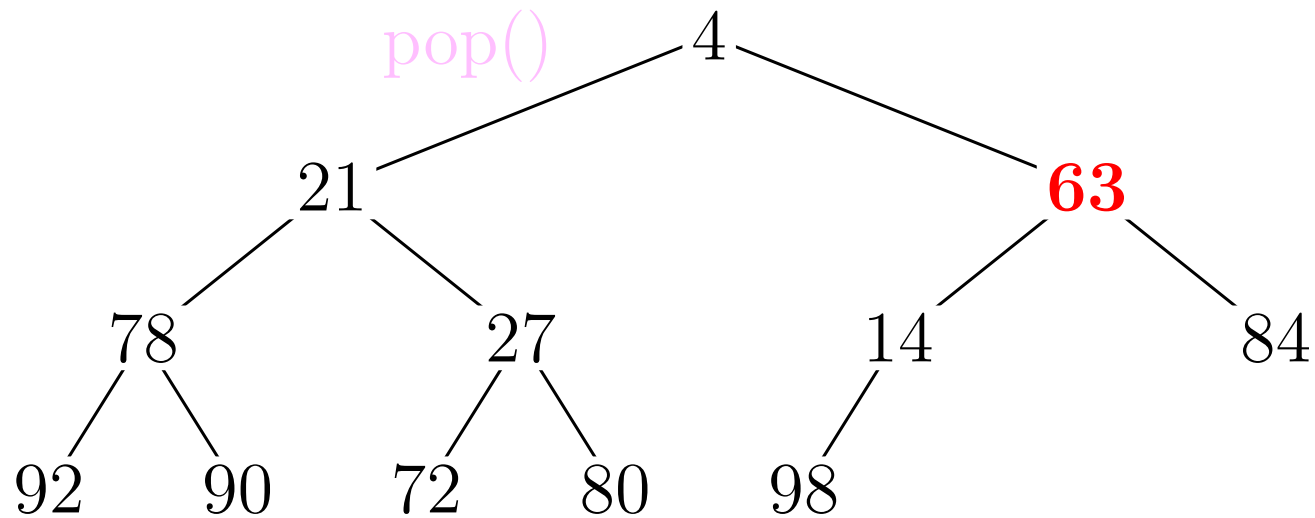


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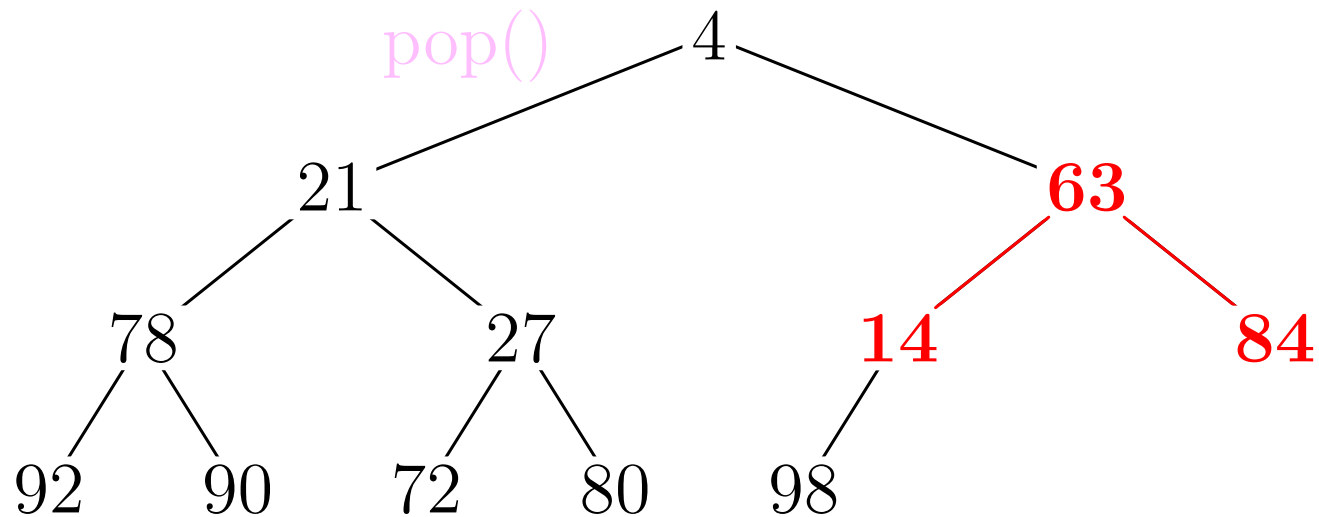
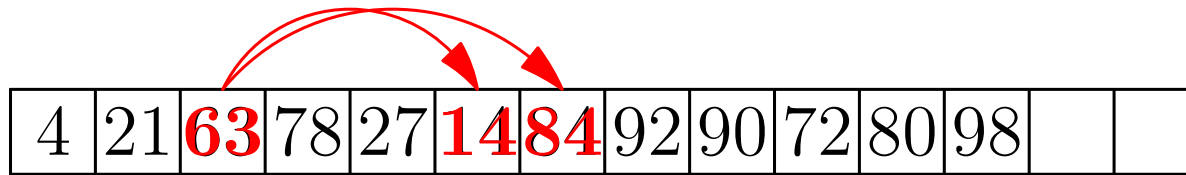


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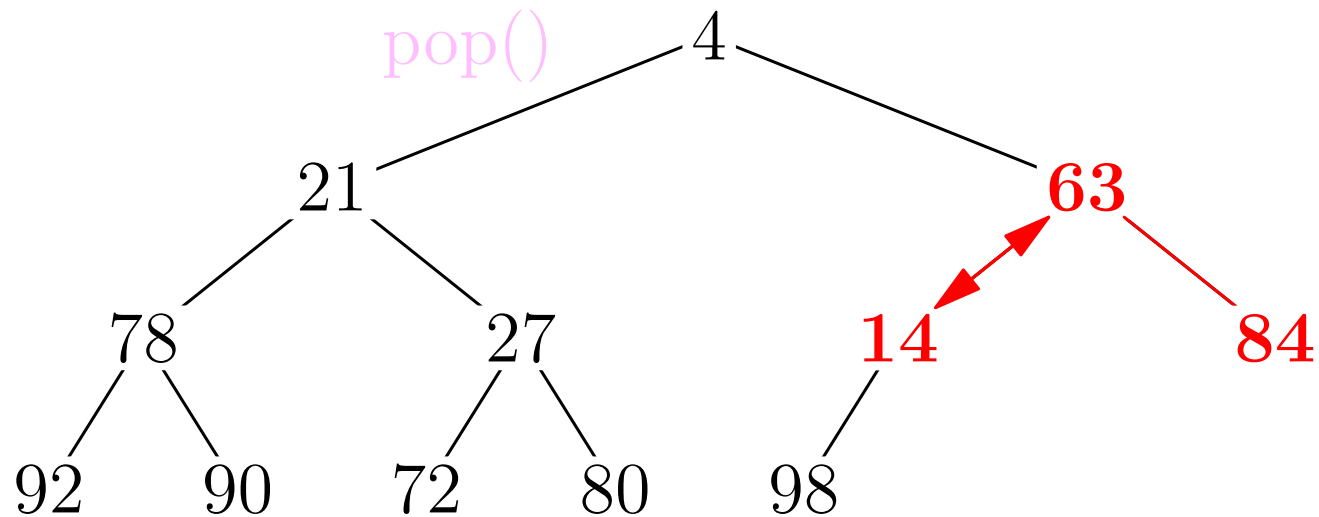
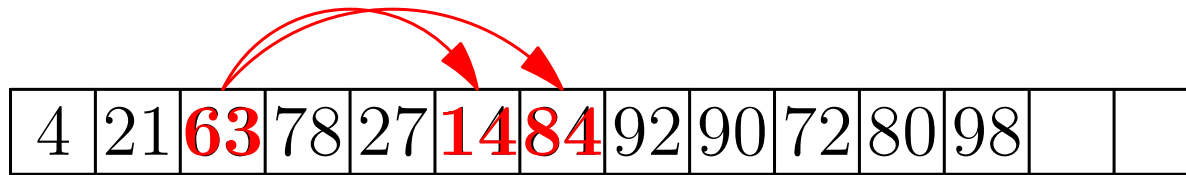
4	21	63	78	27	14	84	92	90	72	80	98		
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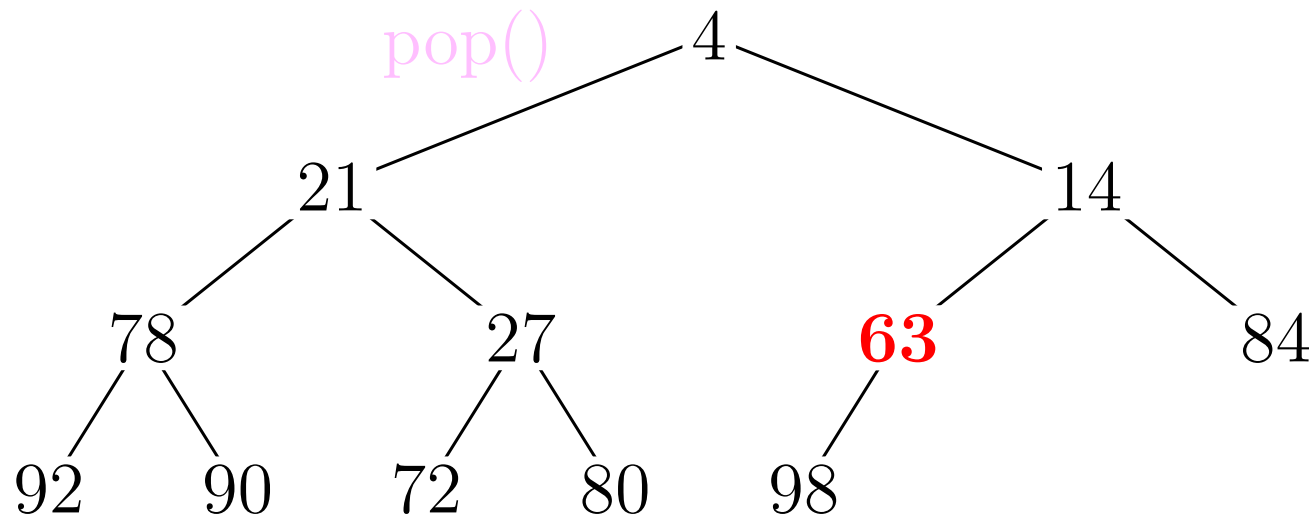


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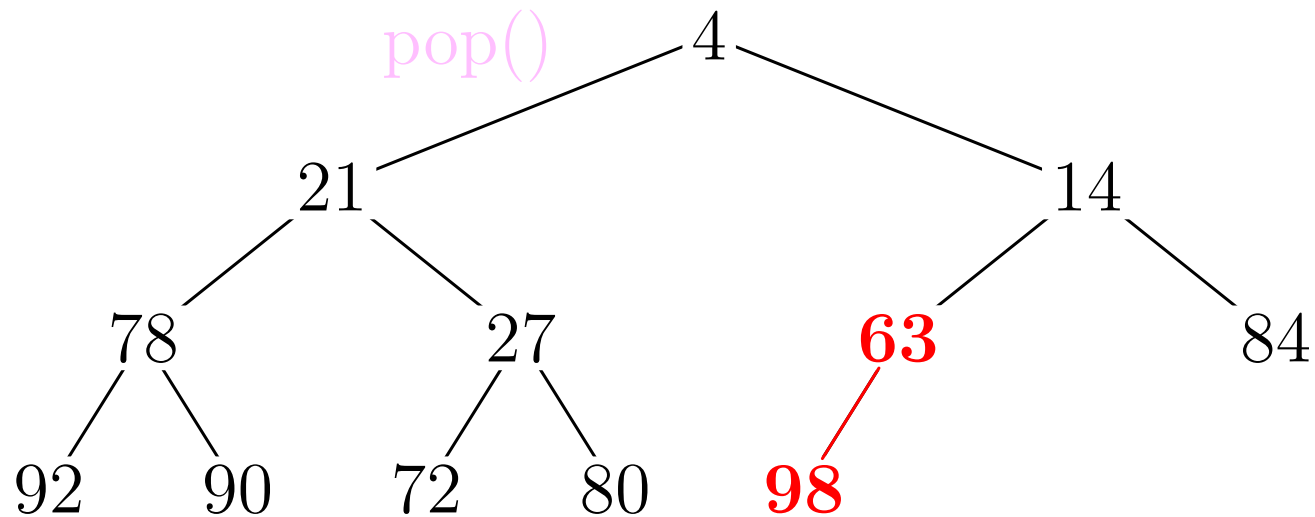
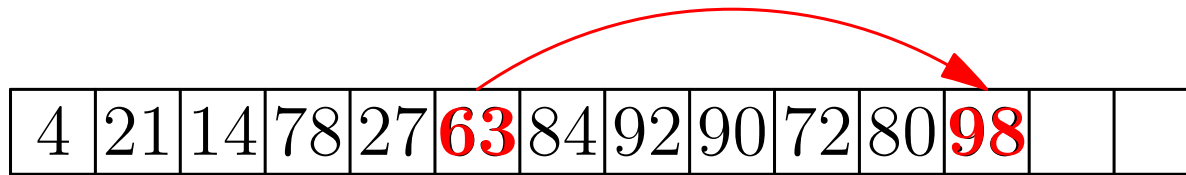


Popping the Top

4	21	14	78	27	63	84	92	90	72	80	98		
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Popping the Top



Popping the Top

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void pop() {  
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    pair<T, P> tmp = array.back();  
    array[0] = tmp;  
    array.pop_back();  
}
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    /* Percolate down */  
    while(child < size()) {
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Heaps in Action

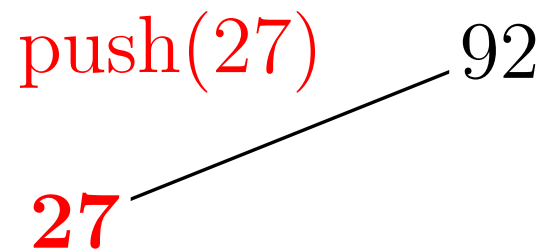
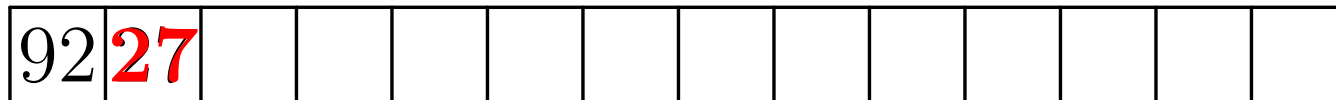


Heaps in Action

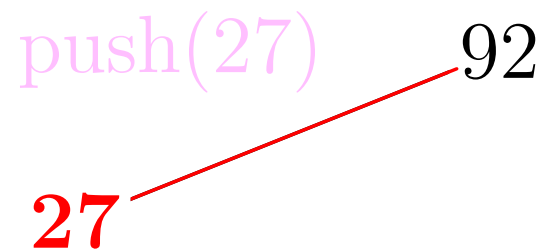


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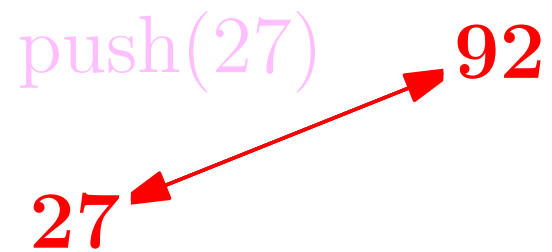
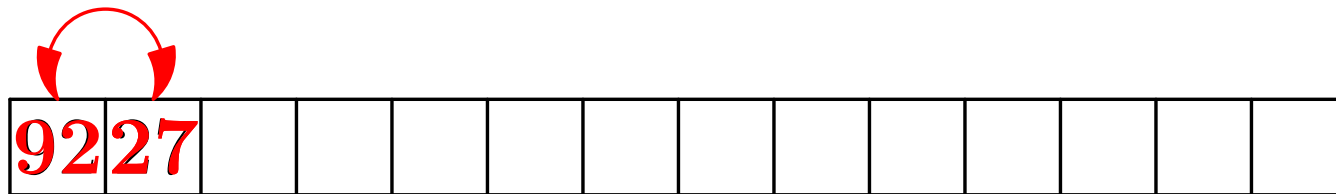
Heaps in Action



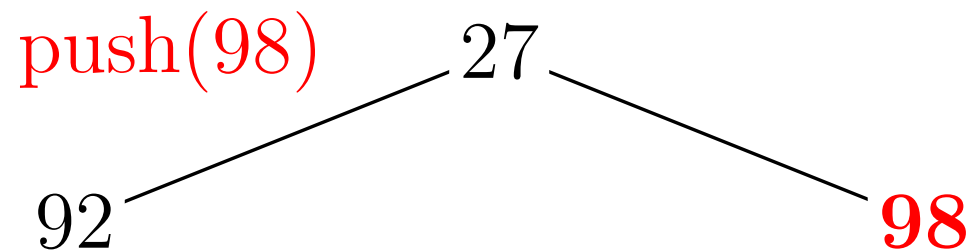
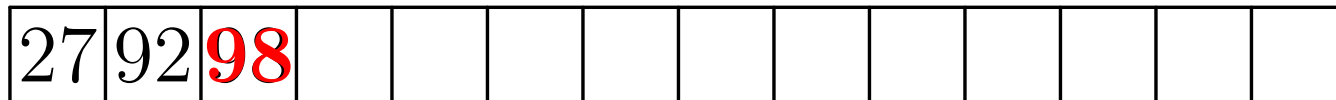
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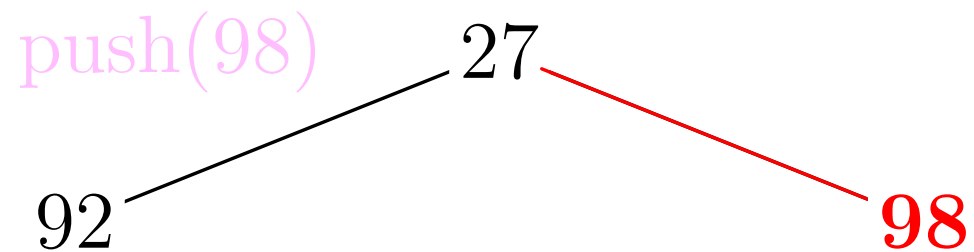
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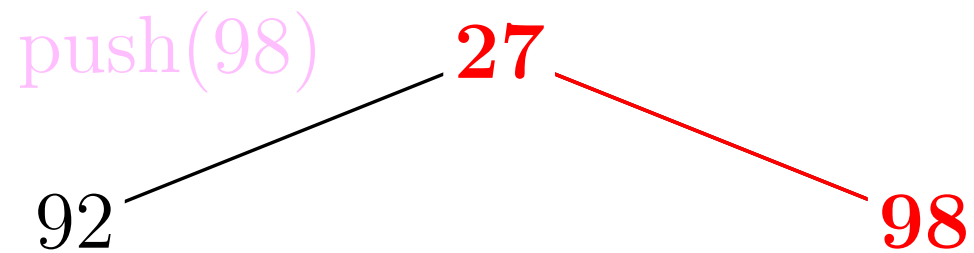
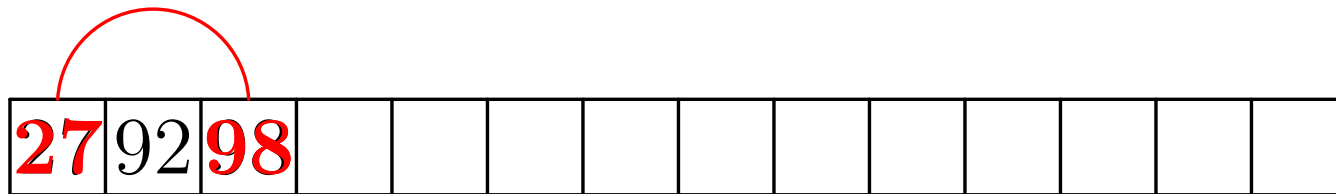
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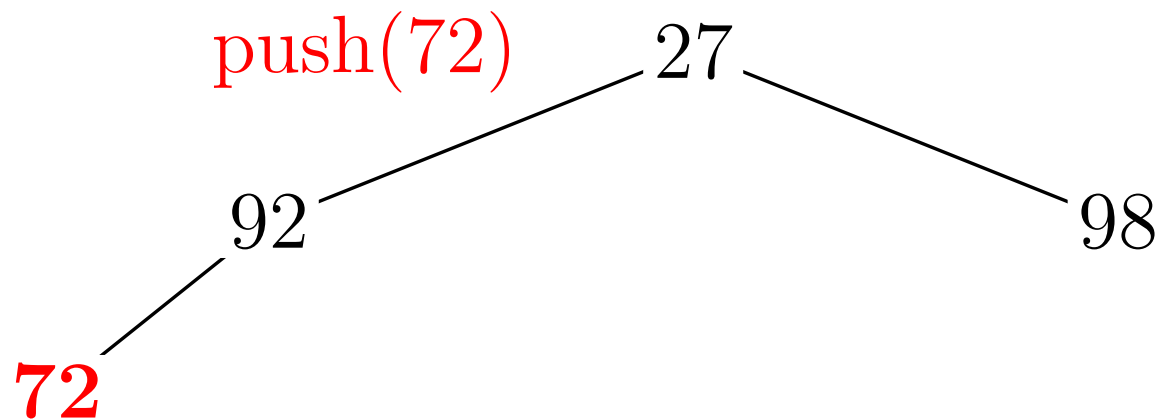
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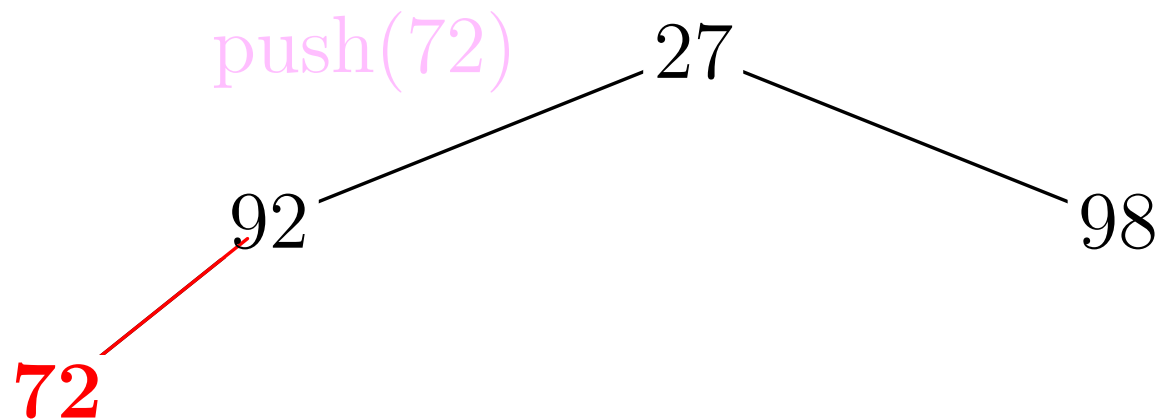
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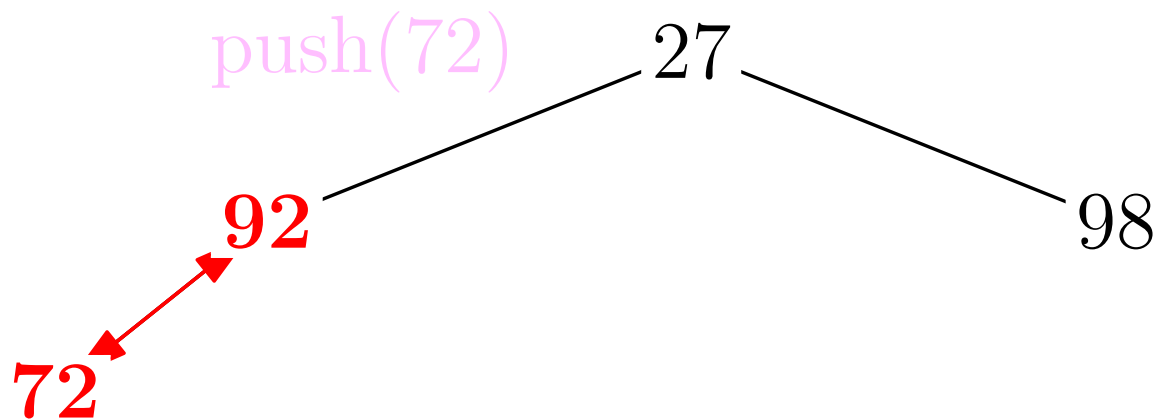
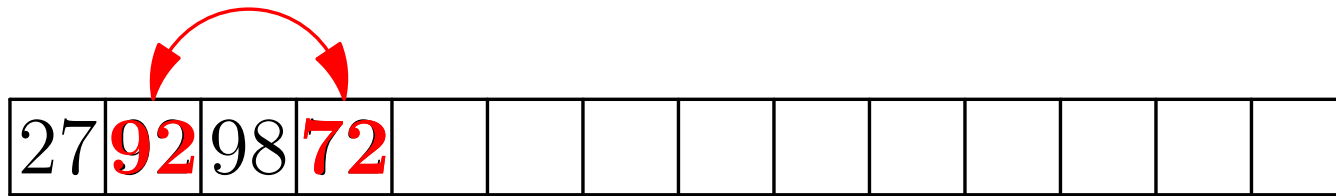
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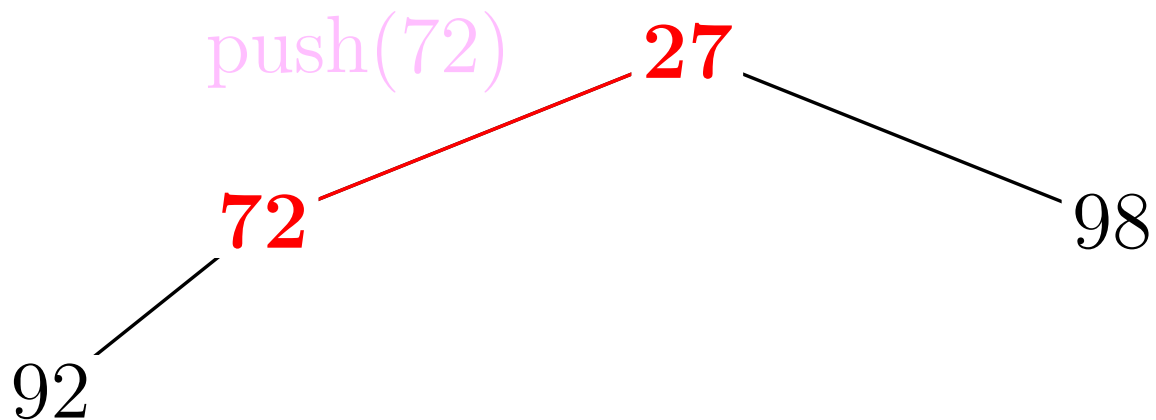
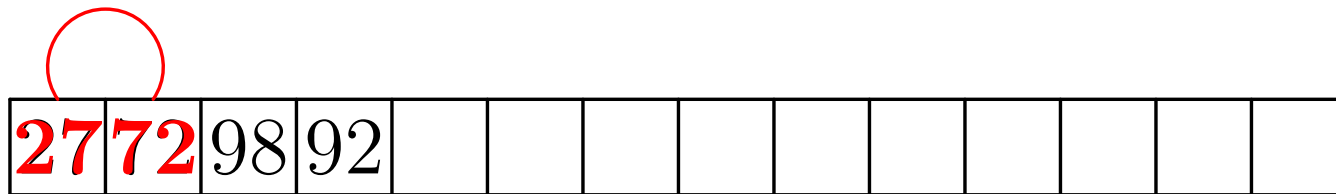
Heaps in Action



Heaps in Action

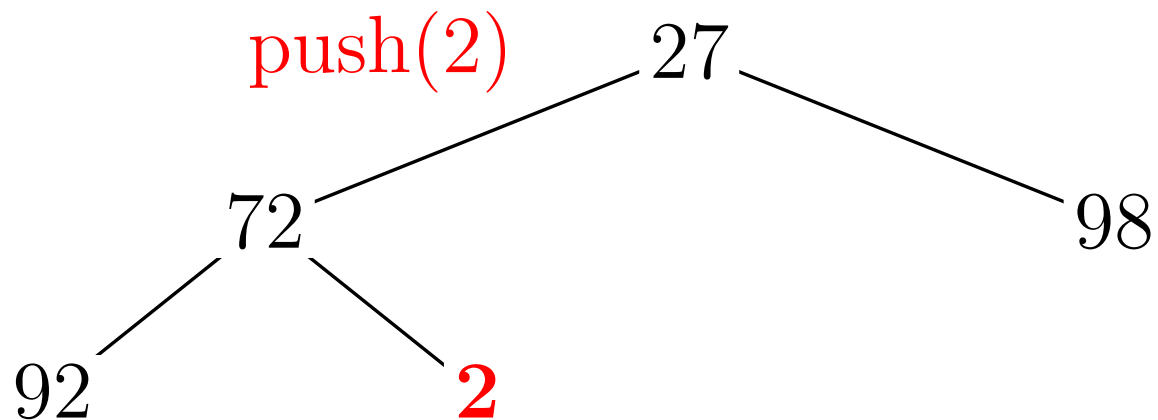


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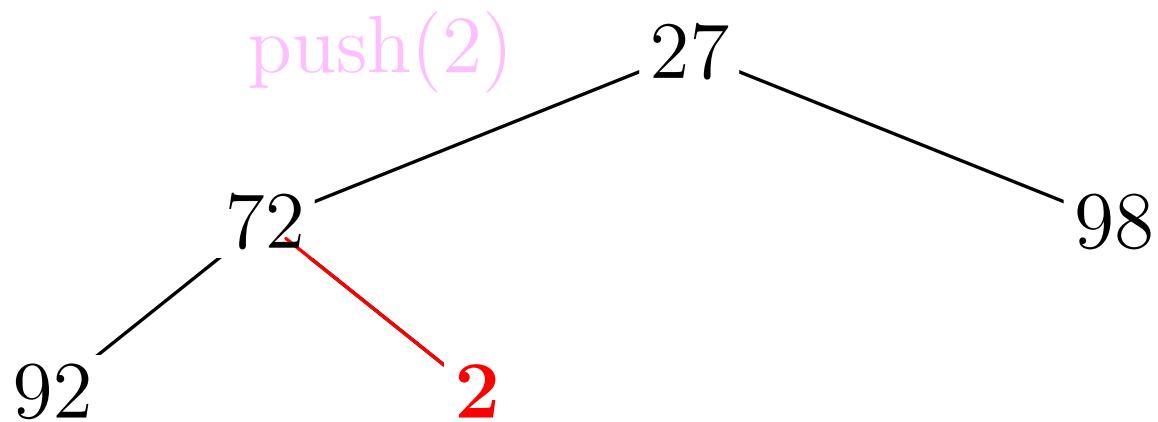


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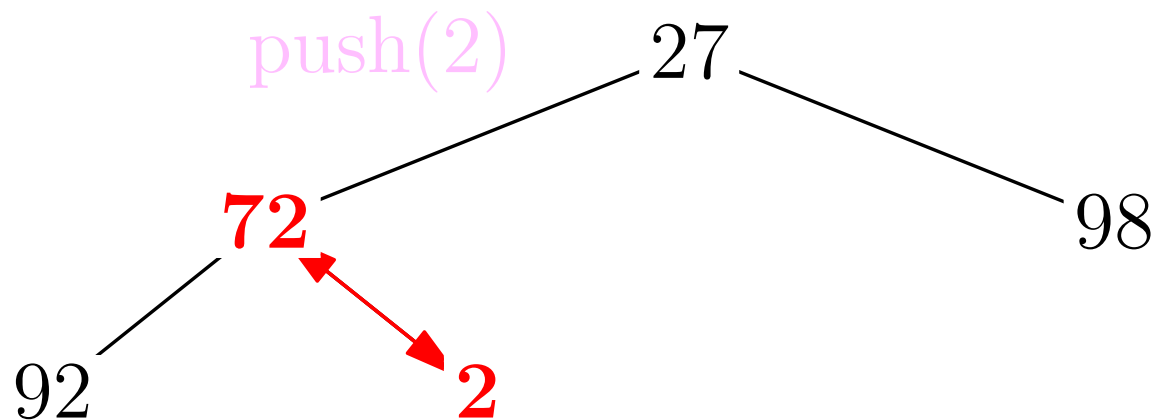
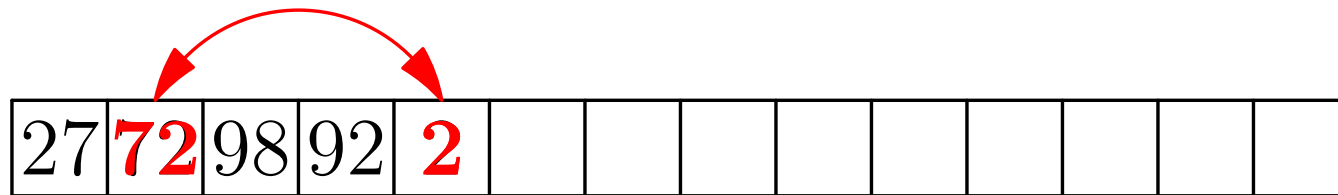
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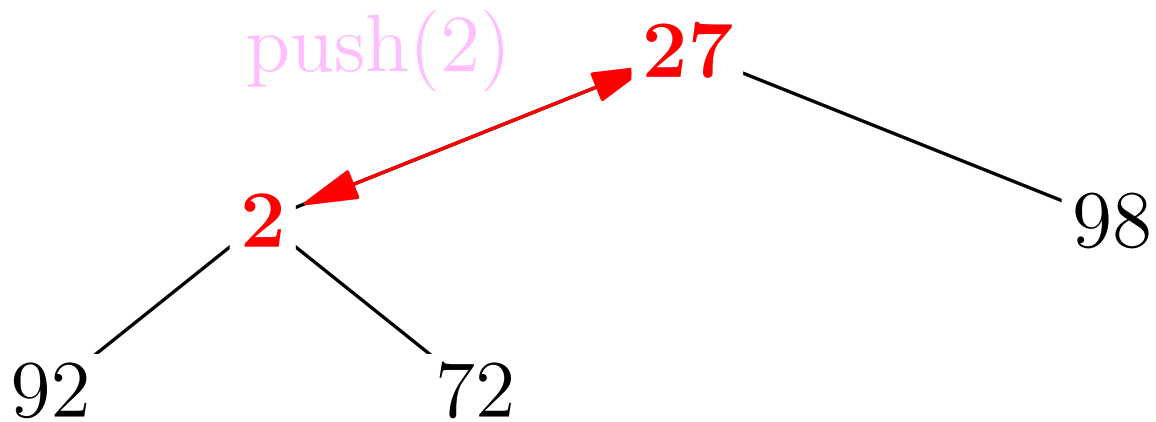
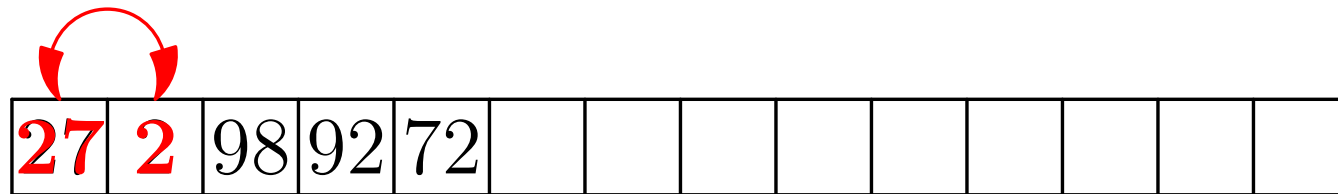
Heaps in Action



Heaps in Action

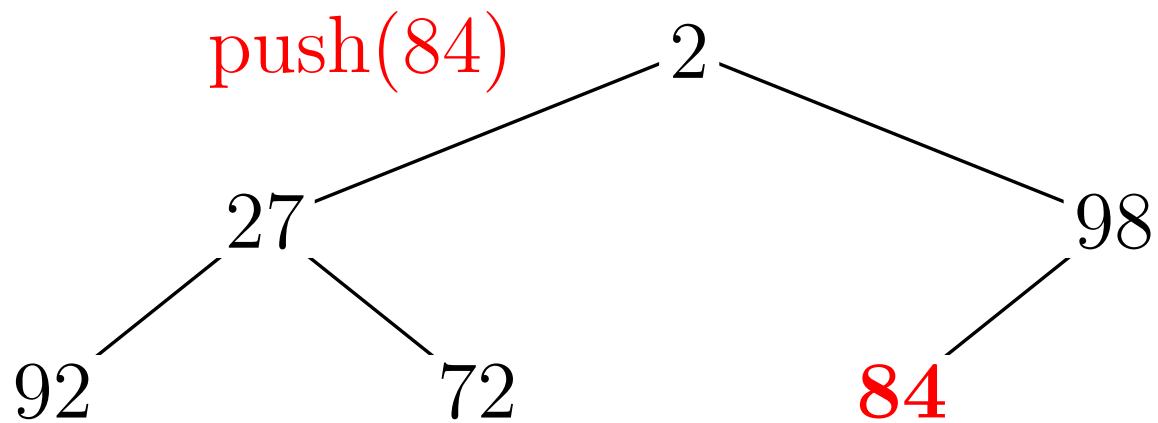


Heaps in Action



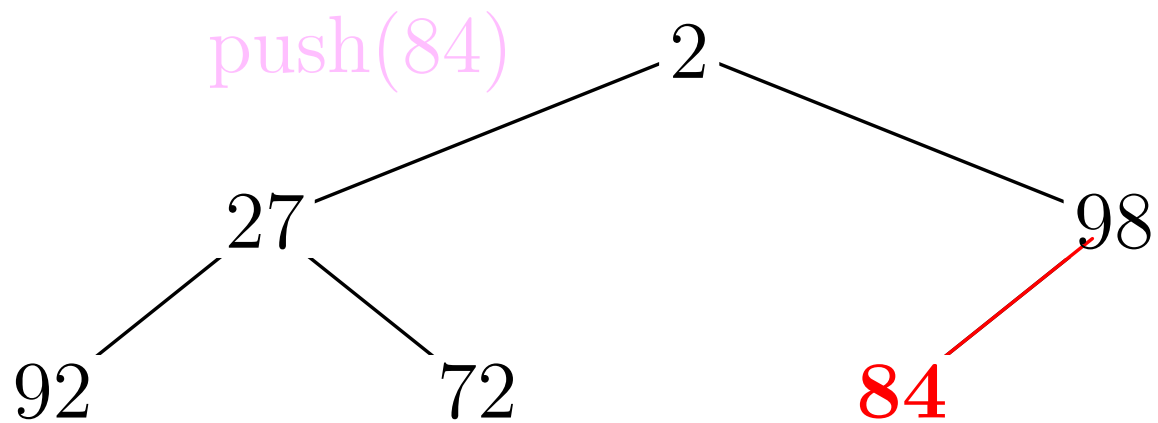
Heaps in Action

2	27	98	92	72	84								
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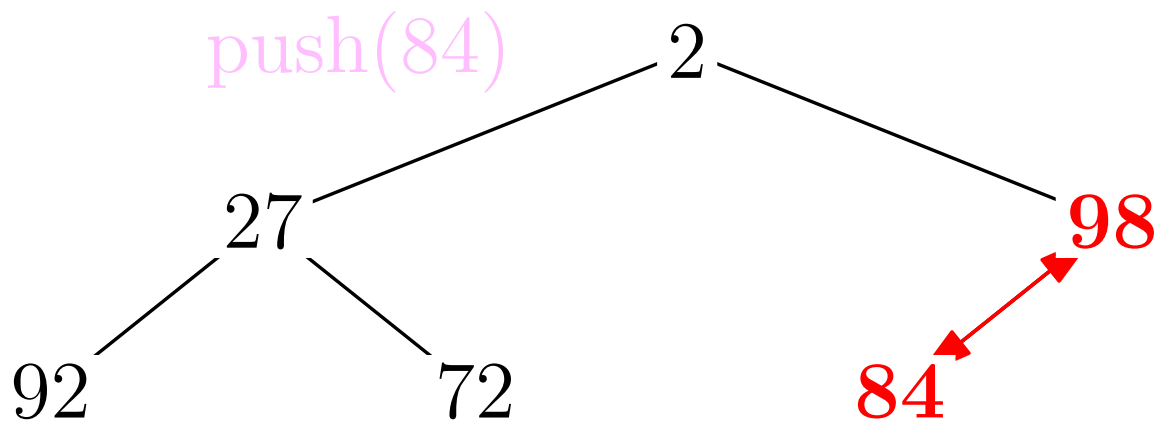
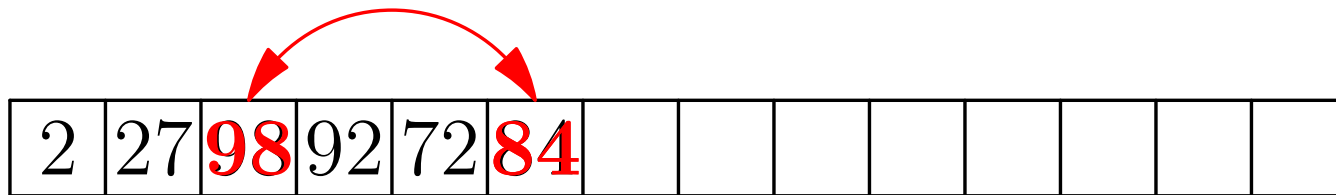


Heaps in Action

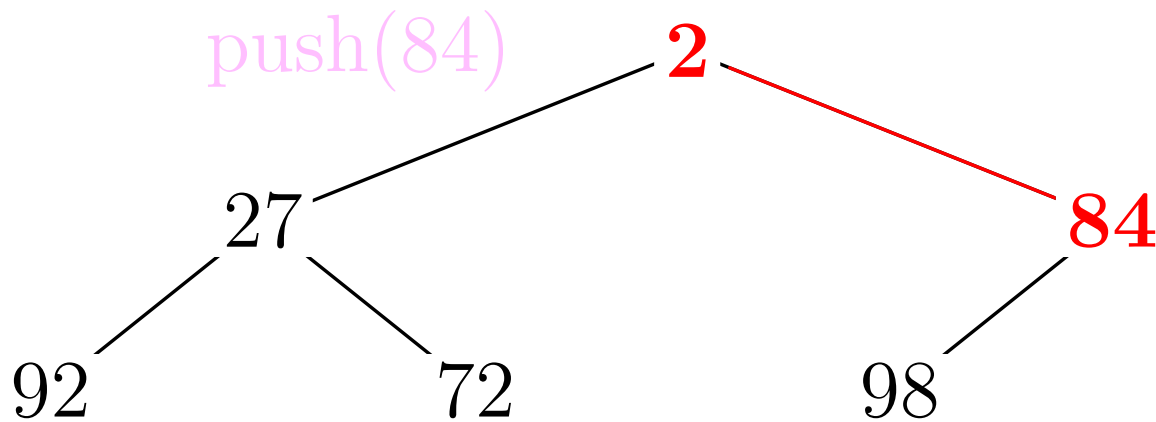
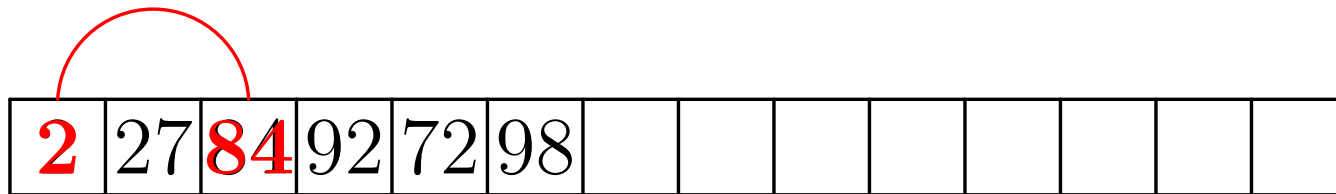
2	27	98	92	72	84								
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Heaps in Action

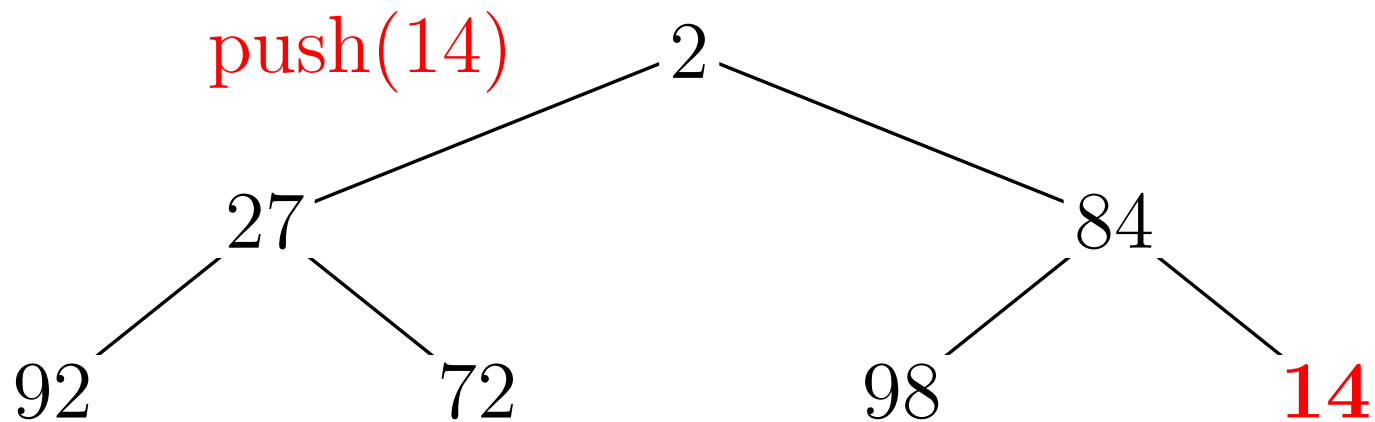


Heaps in Action



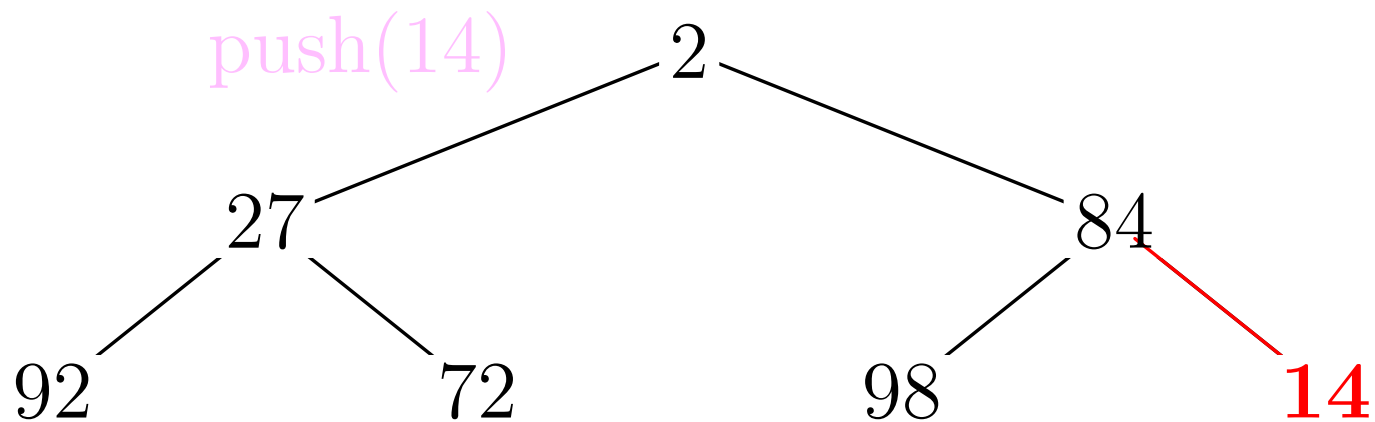
Heaps in Action

2	27	84	92	72	98	14							
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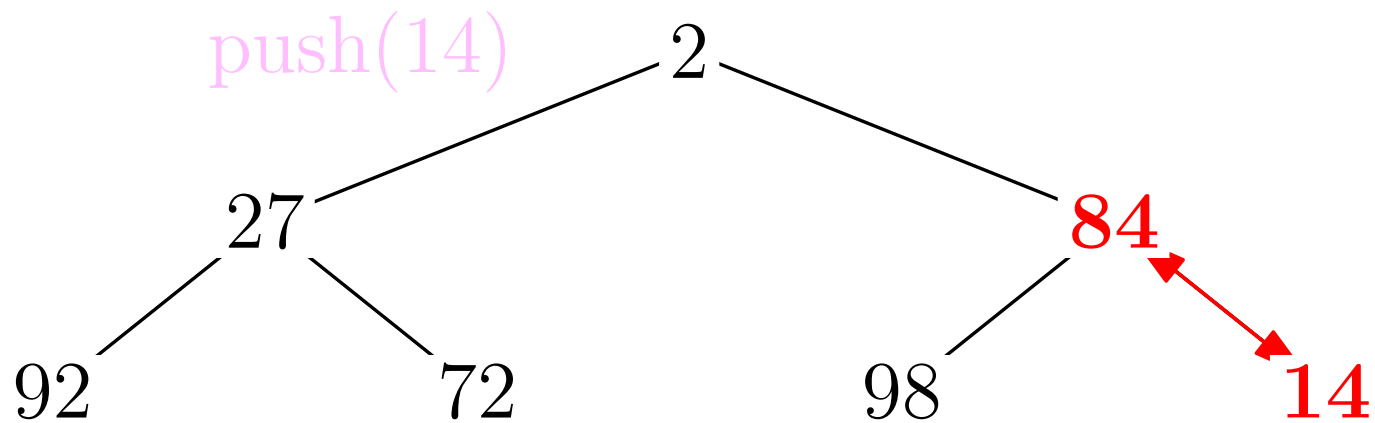
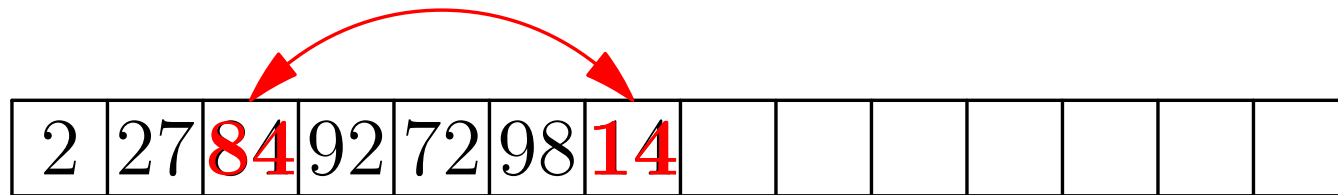


Heaps in Action

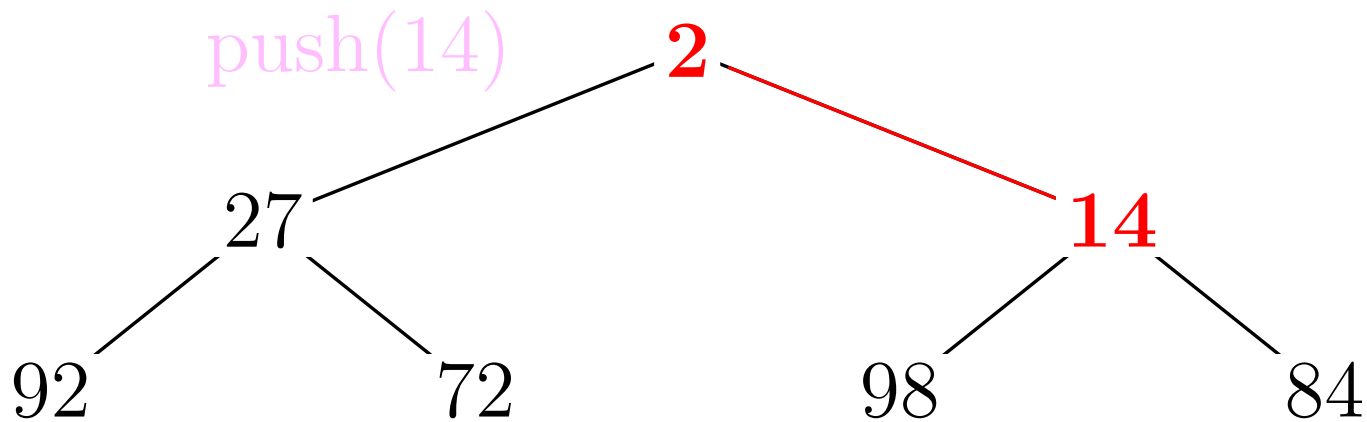
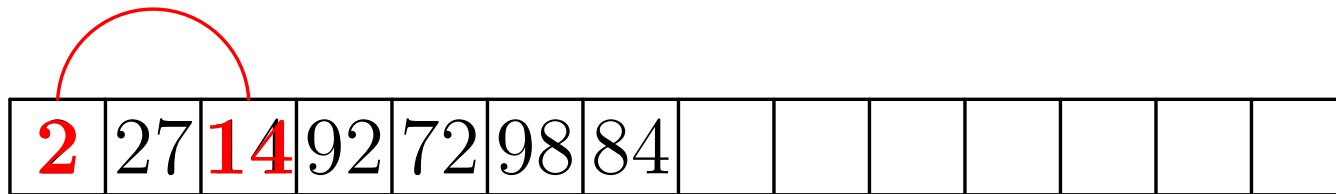
2	27	84	92	72	98	14							
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Heaps in Action

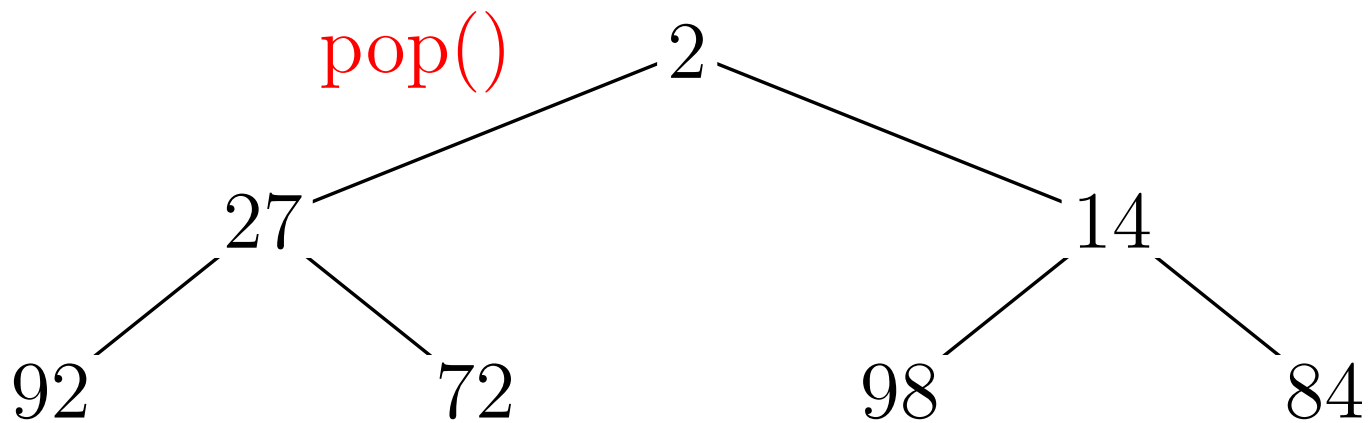


Heaps in Action

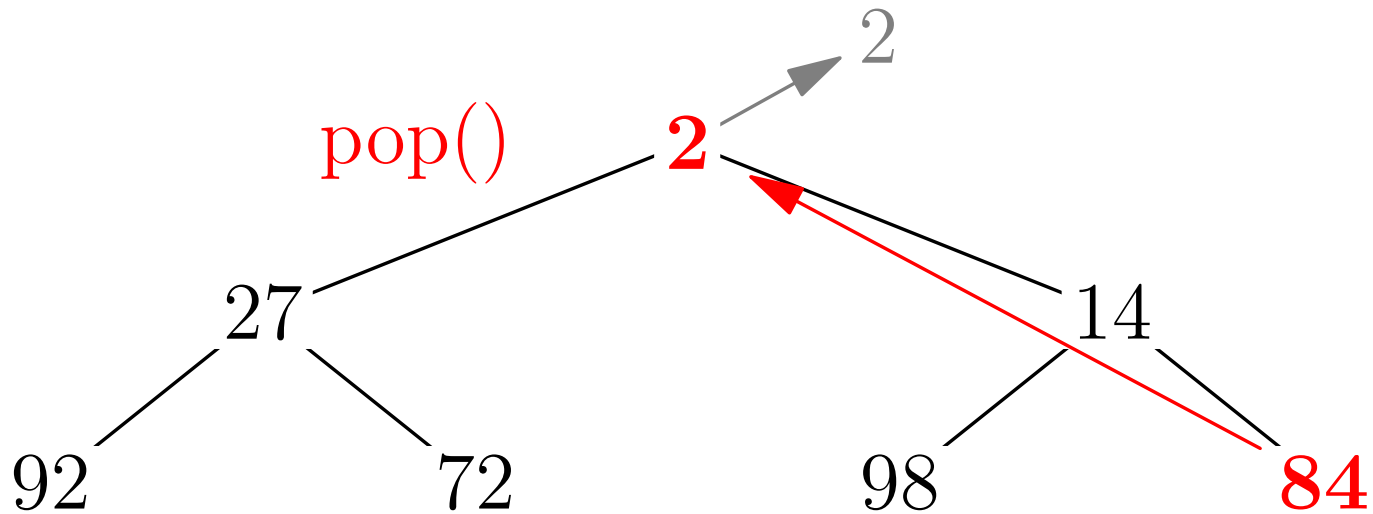
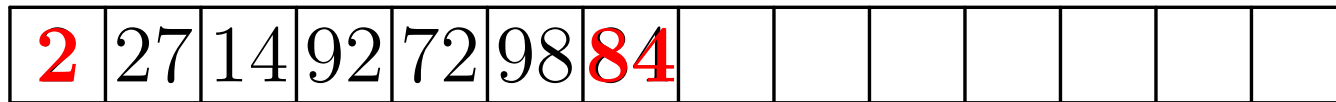


Heaps in Action

2	27	14	92	72	98	84							
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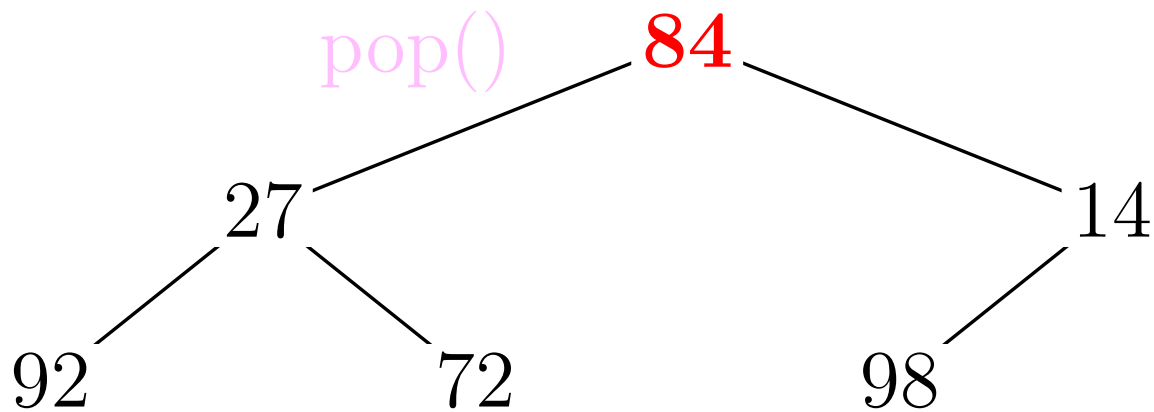


Heaps in Action



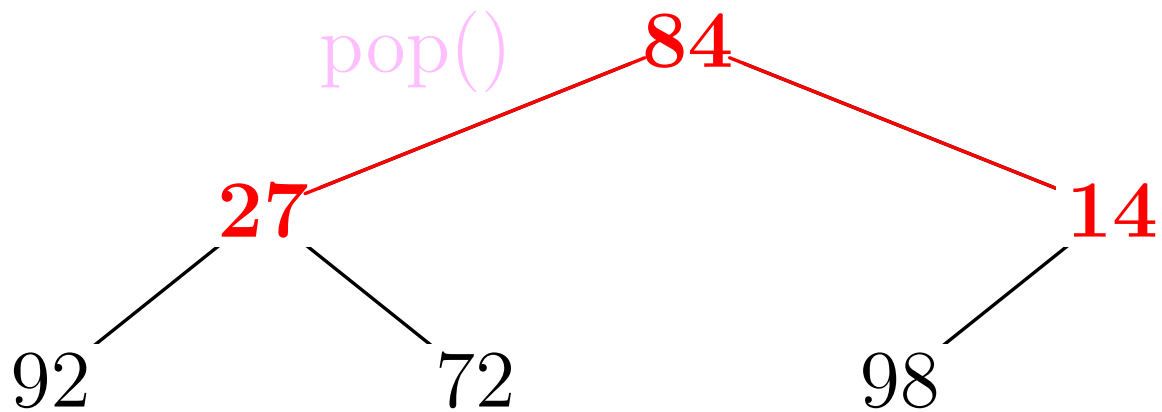
Heaps in Action

84	27	14	92	72	98								
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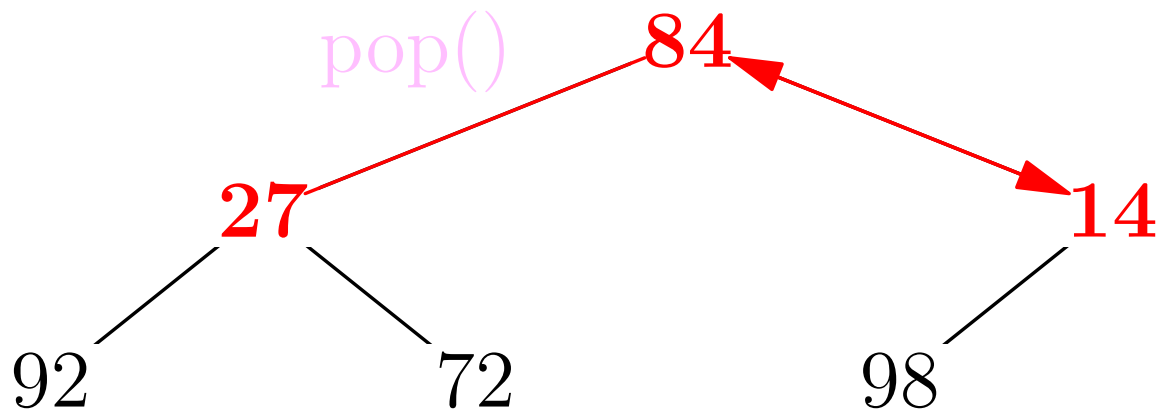
Heaps in Action

84	27	14	92	72	98								
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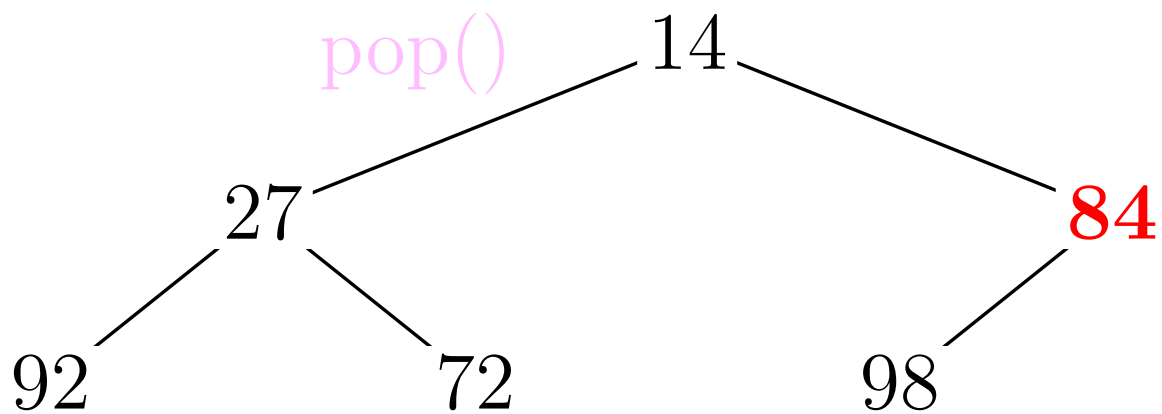
Heaps in Action

84	27	14	92	72	98								
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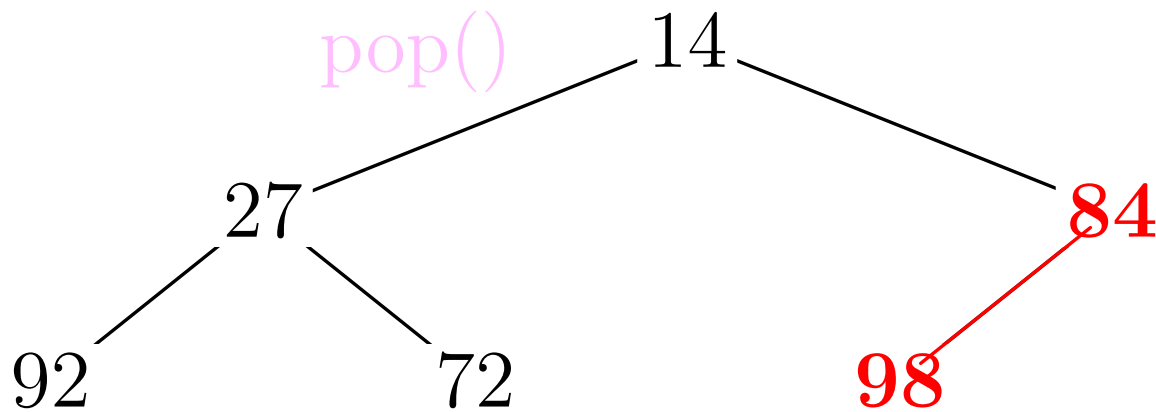
Heaps in Action

14	27	84	92	72	98								
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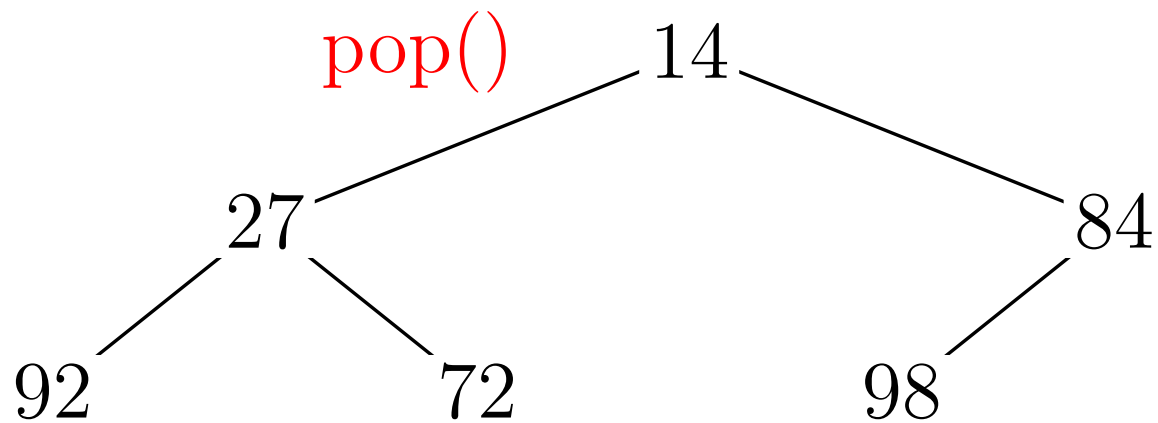
Heaps in Action

14	27	84	92	72	98								
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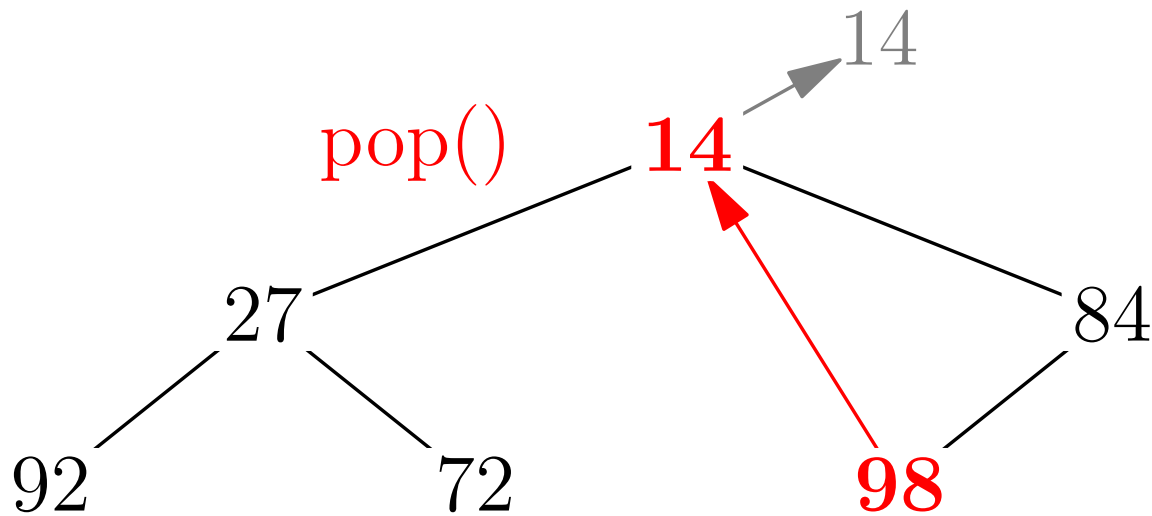


Heaps in Action

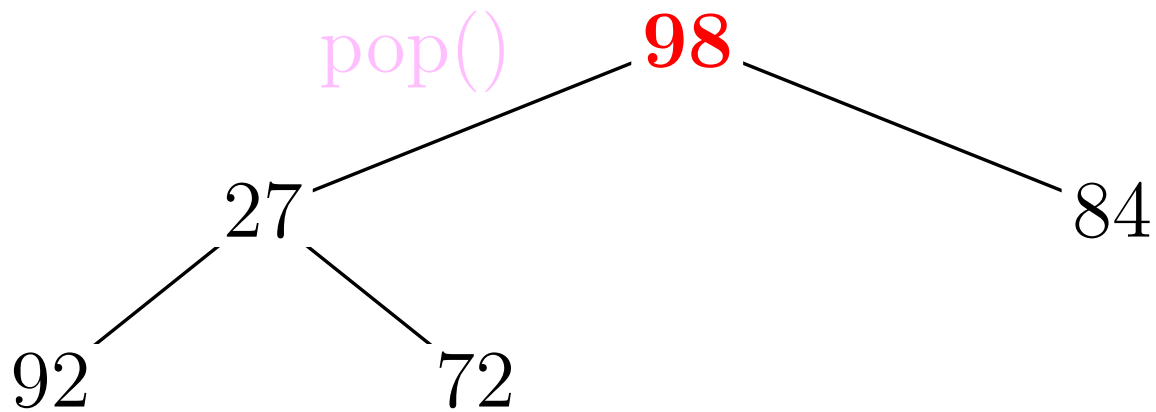
14	27	84	92	72	98								
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Heaps in Action

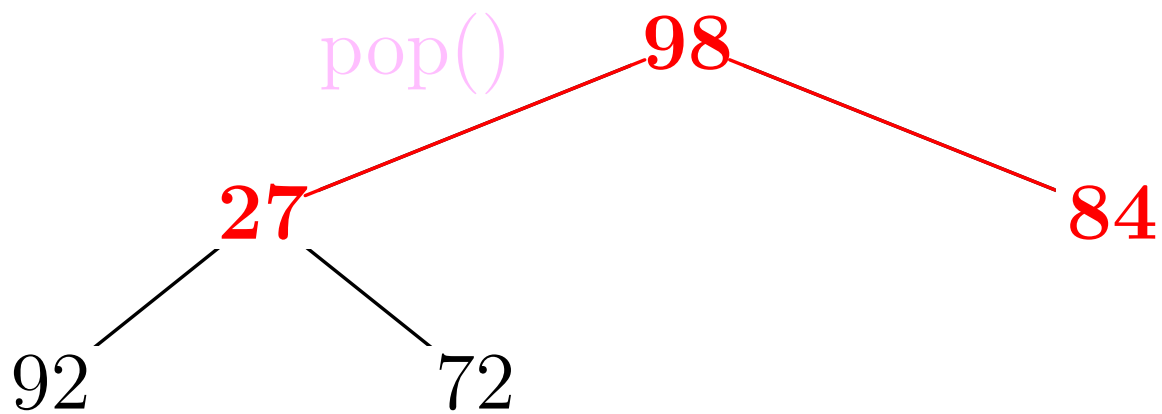


Heaps in Action



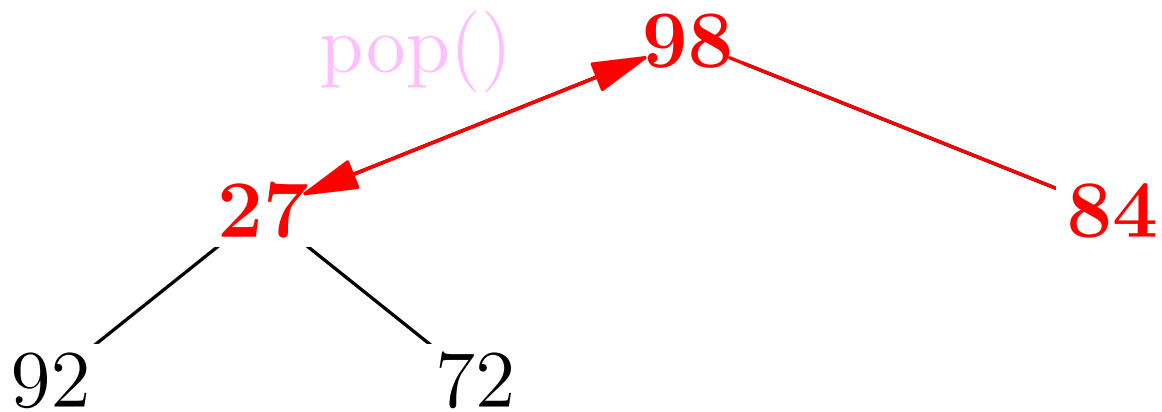
Heaps in Action

98	27	84	92	72									
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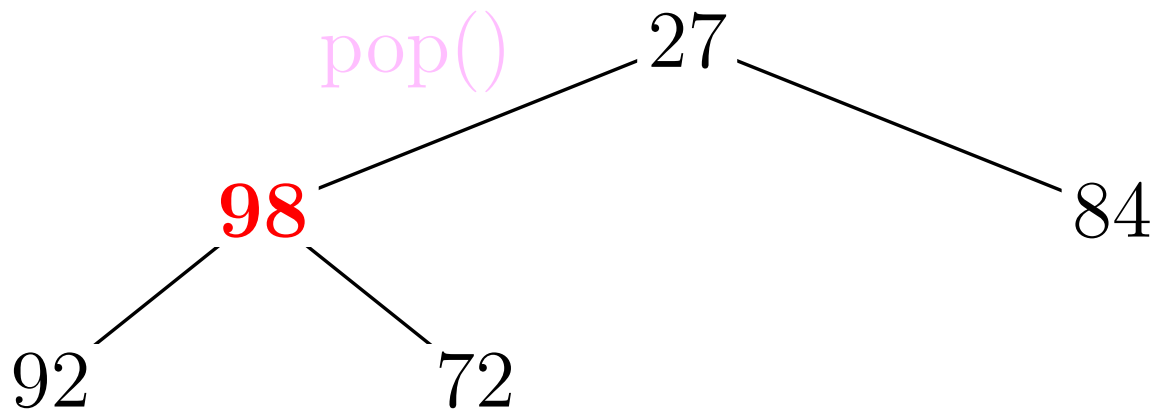


Heaps in Action

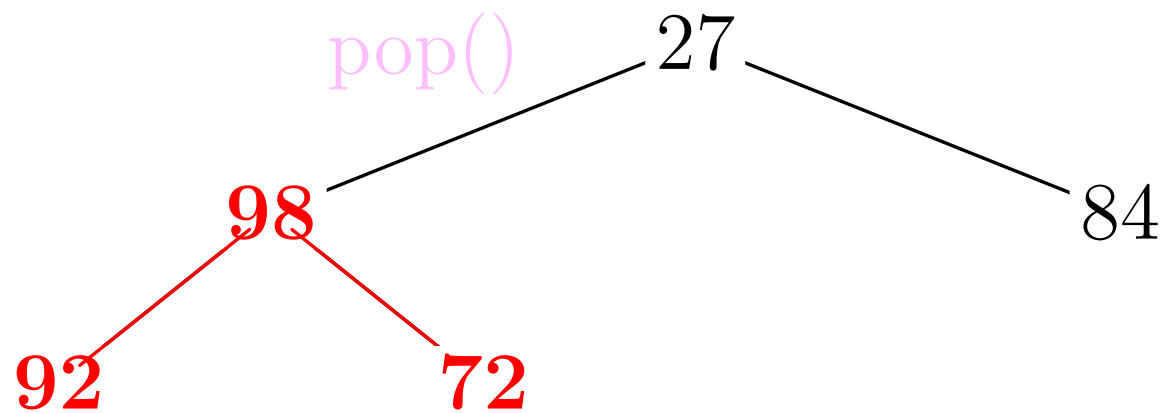
98	27	84	92	72									
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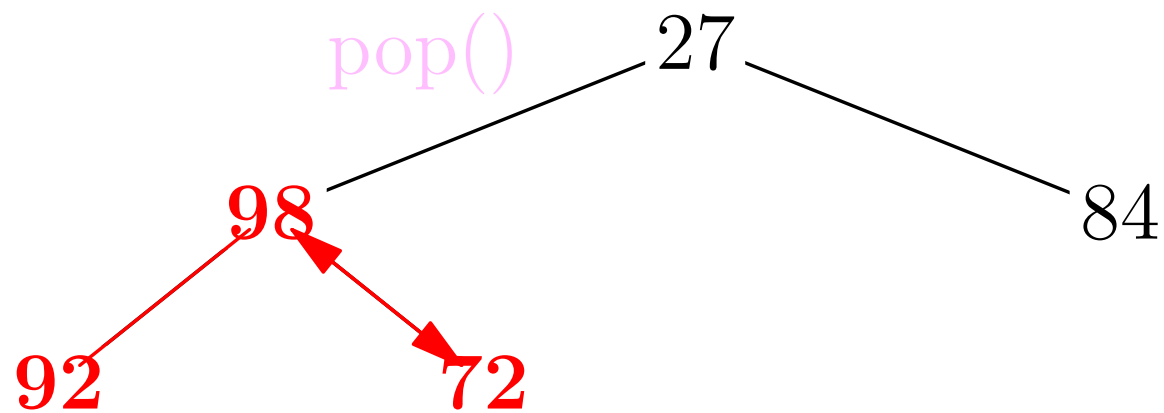
Heaps in Action



Heaps in Action

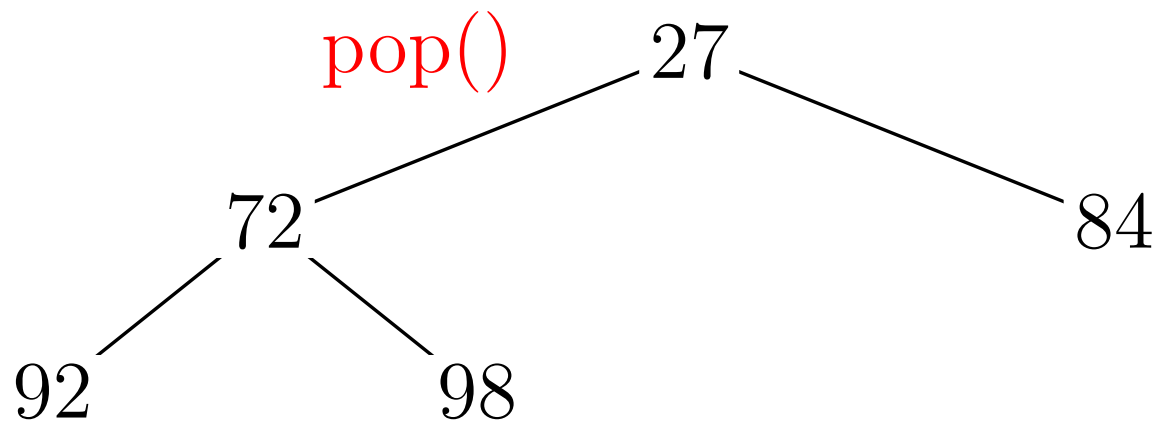


Heaps in Action

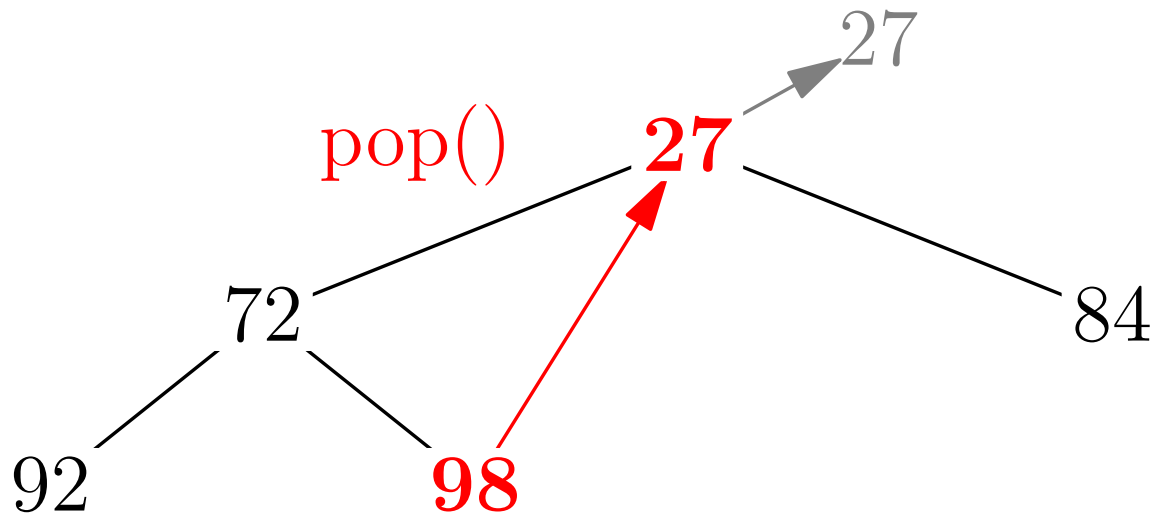


Heaps in Action

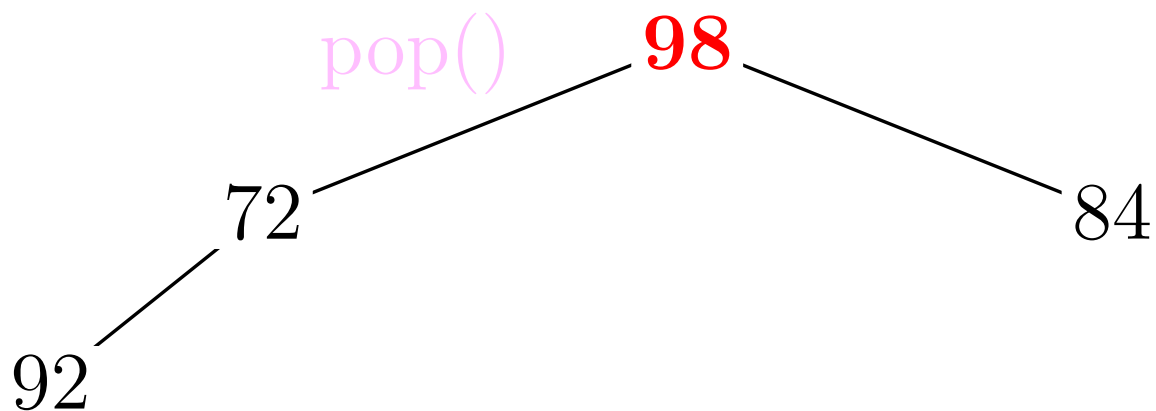
27	72	84	92	98									
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Heaps in Action

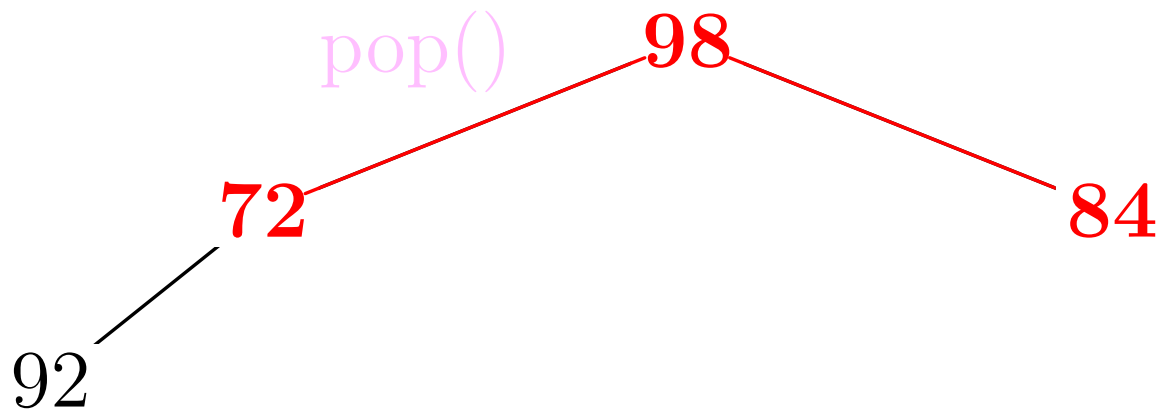


Heaps in Action



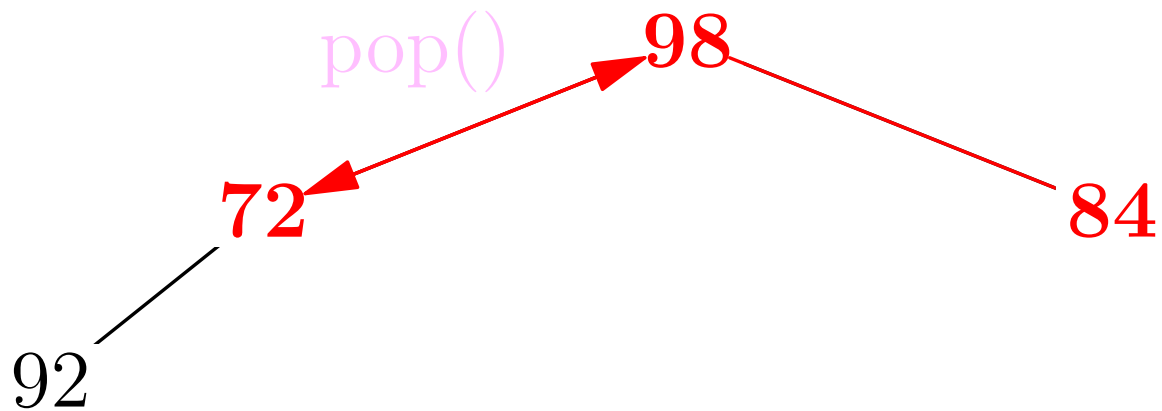
Heaps in Action

98	72	84	92										
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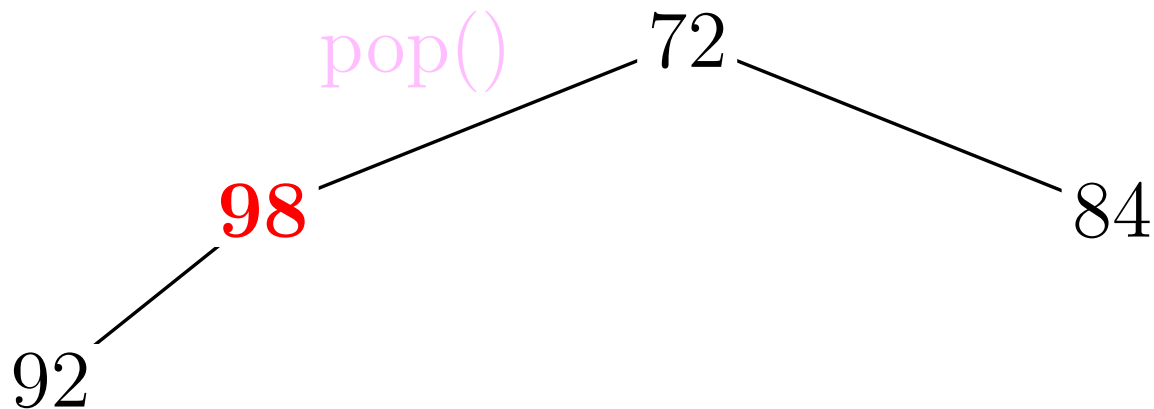


Heaps in Action

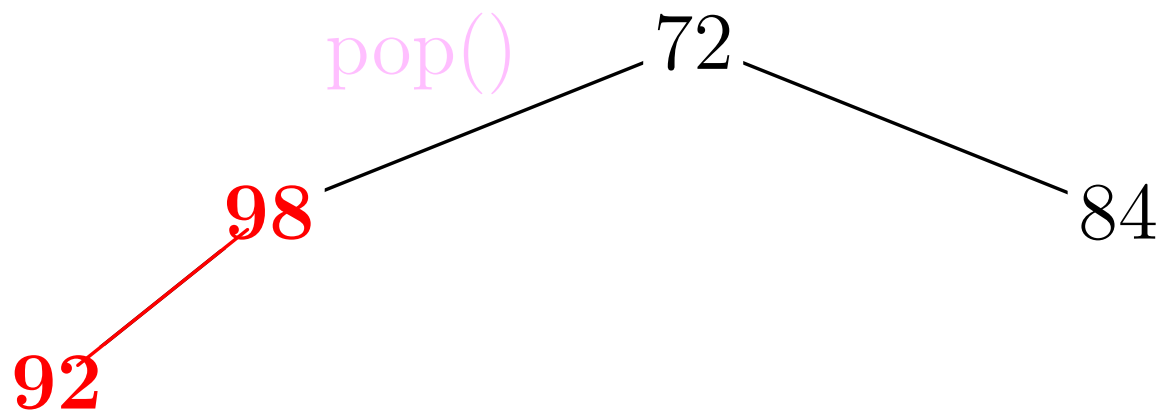
98	72	84	92										
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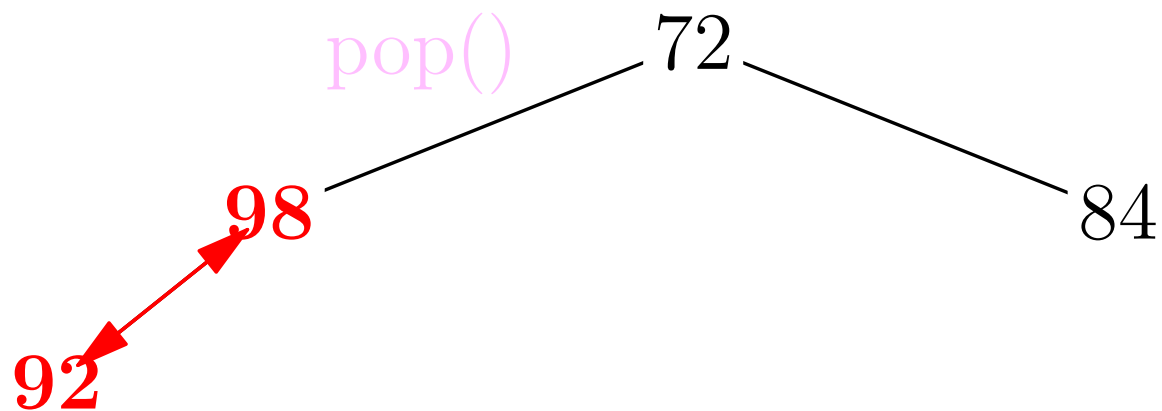
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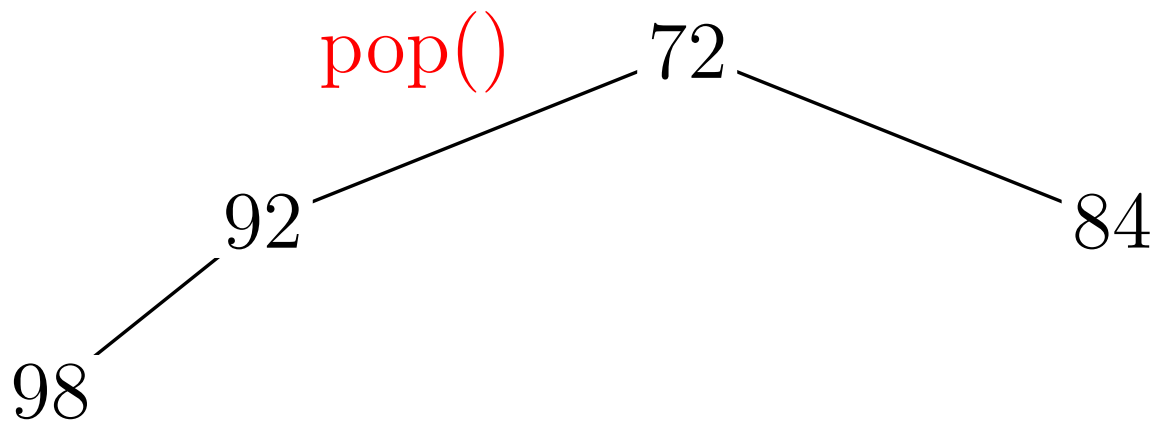
Heaps in Action



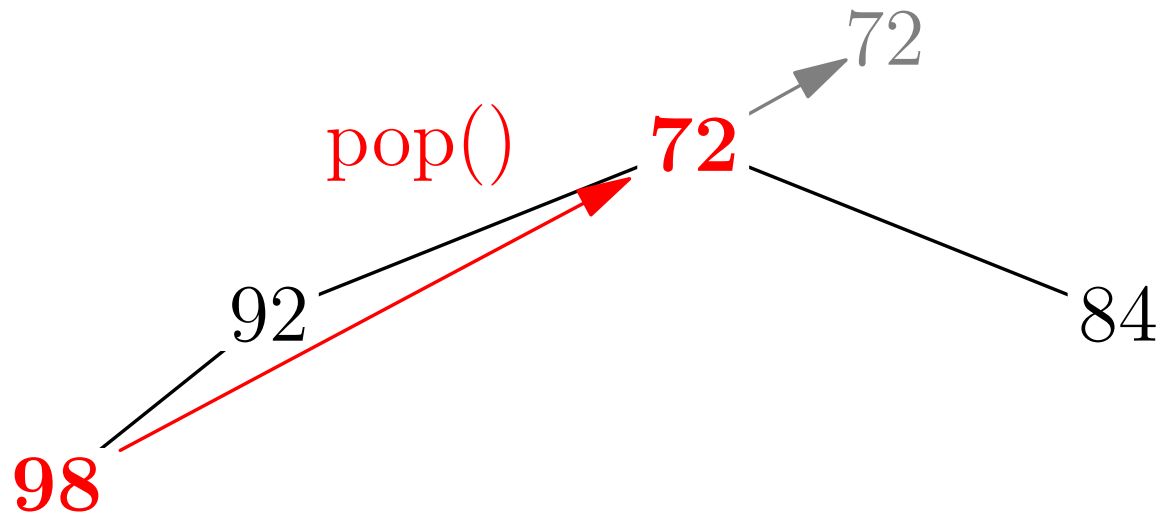
Heaps in Action



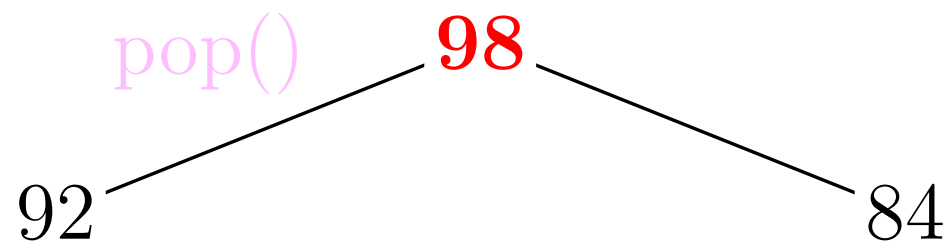
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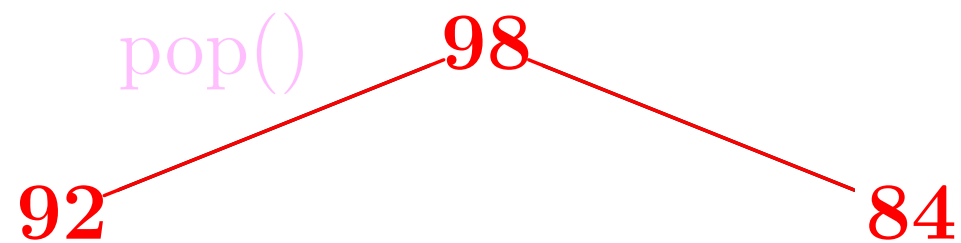
Heaps in Action



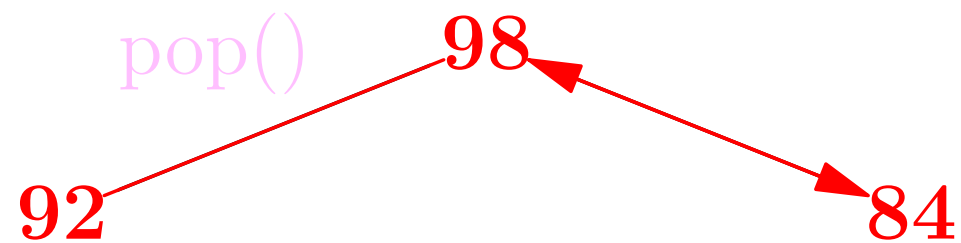
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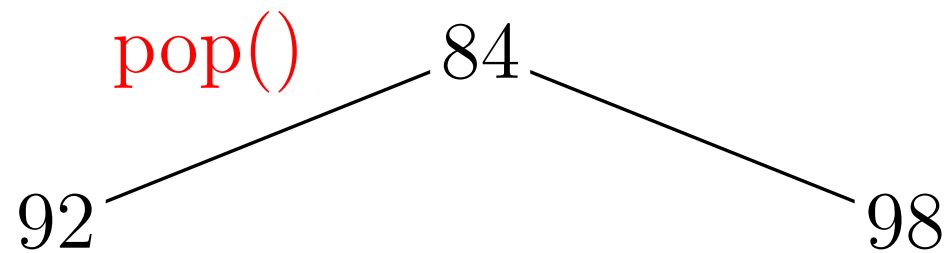
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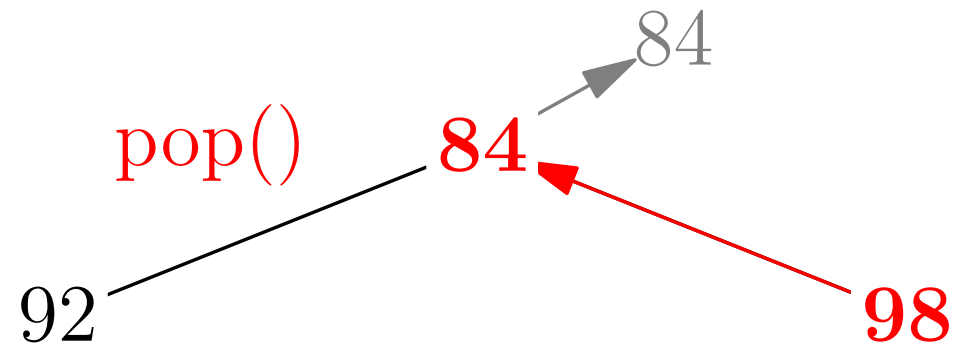
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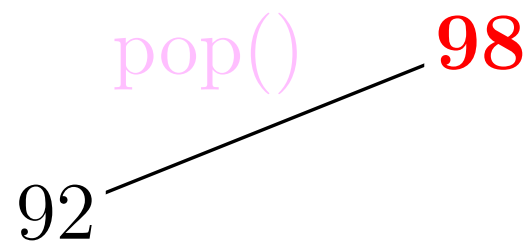
Heaps in Action



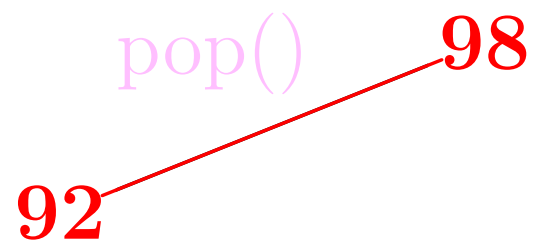
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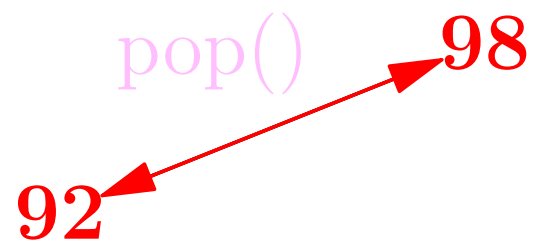
Heaps in Action



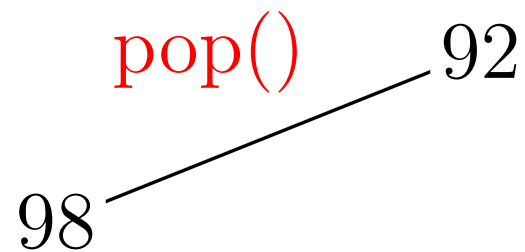
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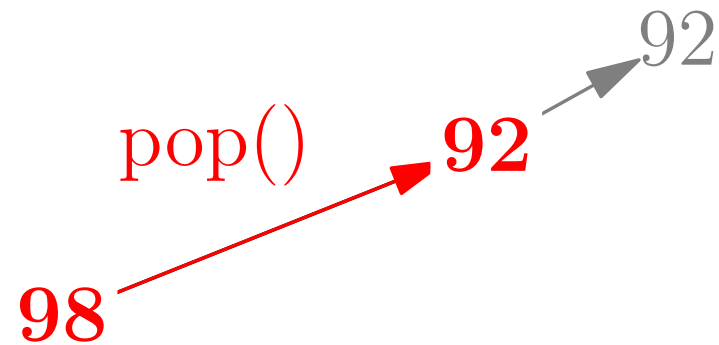
Heaps in Action



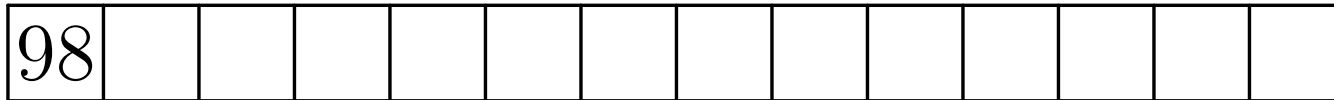
Heaps in Action



Heaps in Action



Heaps in Action



pop() 98

Heaps in Action



pop() 98

Time Complexity of Heaps

- The two important operation are `add` and `removeMin`
- These both either percolating an element up the tree or percolating an element down the tree
- The number of operations depends on the depth of the tree which is $\Theta(\log(n))$
- Thus `add` and `removeMin` are $O(\log(n))$
- Except `add` could also require resizing the array, but the amortised cost of this is low

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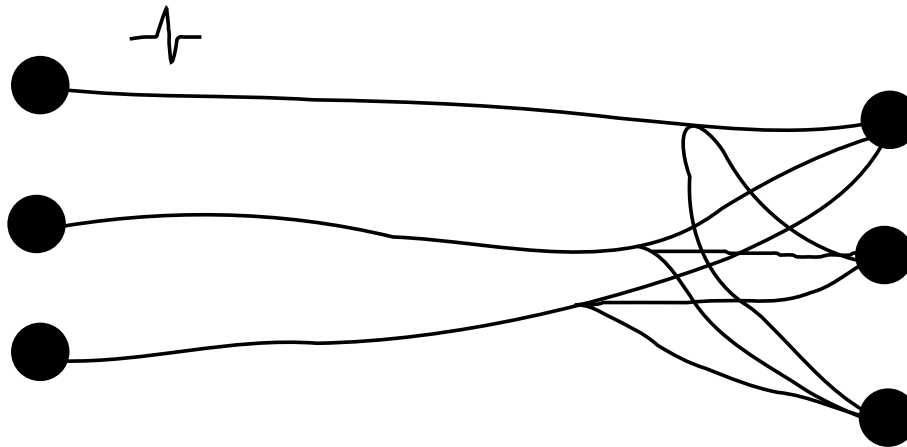
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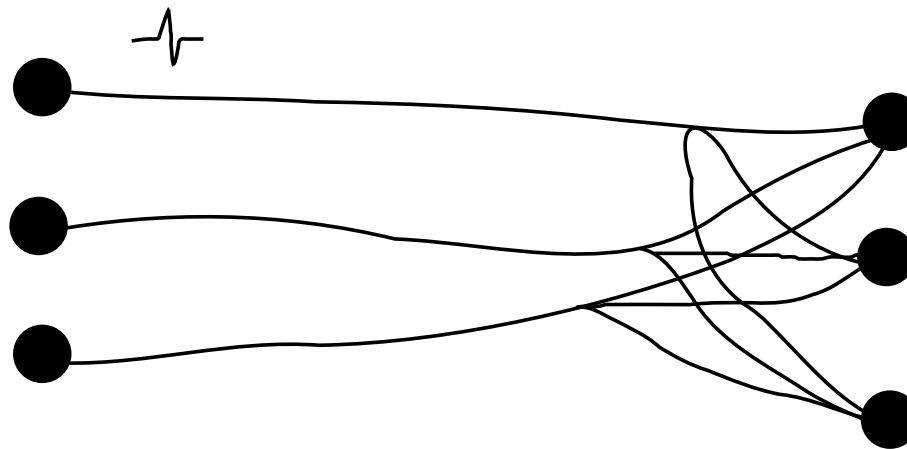
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- Using a priority queue meant we knew when the next event would happen
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Outline

1. Heaps
2. Priority Queues
 - Array Implementation
3. **Heap Sort**
4. Other Heaps



Heap Sort

- A priority queue suggests a very simple way of performing sort
- We simply add elements to a heap and then take them off again

```
template <typename T>
void sort(vector<T> aList)
{
    HeapPQ<T> aHeap = new HeapPQ<T>(aList.size());
    for (T element: aList)
        aHeap.push(element, element);

    aList.clear();
    while(aHeap.size() > 0) {
        aList.push_back(aHeap.top());
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- Note that this is not an in-place sort algorithm (i.e. it uses lots of memory)

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Example of Heap Sort

92	27	98	72	2	84	14	90	78	21	80	63
----	----	----	----	---	----	----	----	----	----	----	----

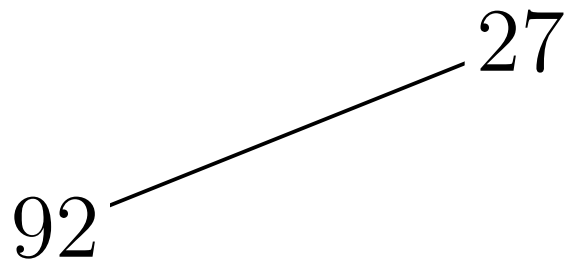
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92	27	98	72	2	84	14	90	78	21	80	63
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92

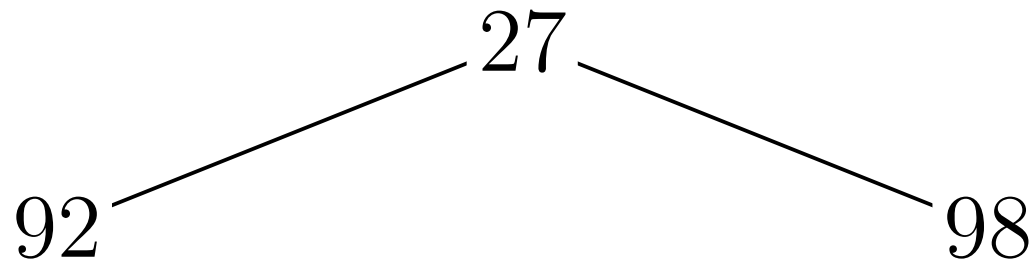
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92	27	98	72	2	84	14	90	78	21	80	63
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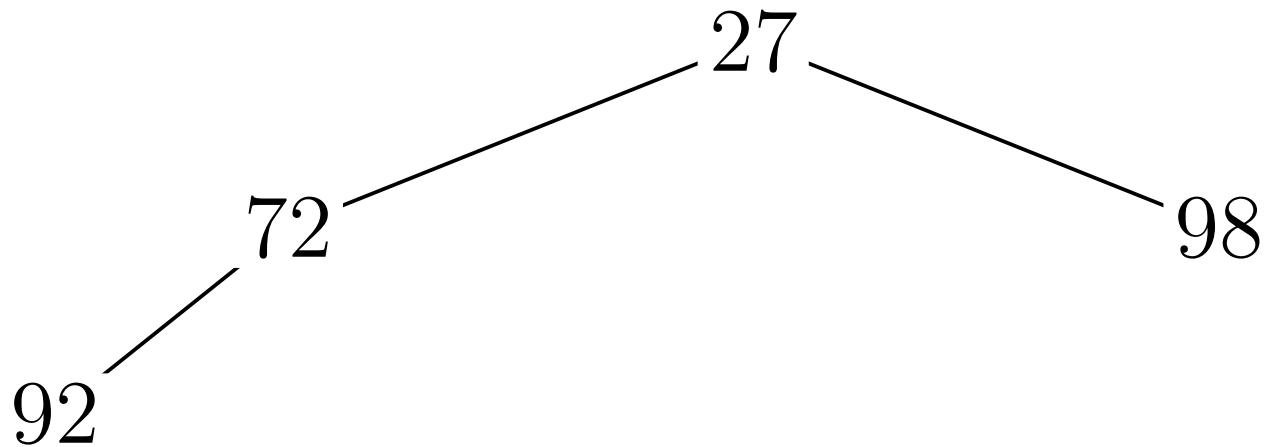
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92	27	98	72	2	84	14	90	78	21	80	63
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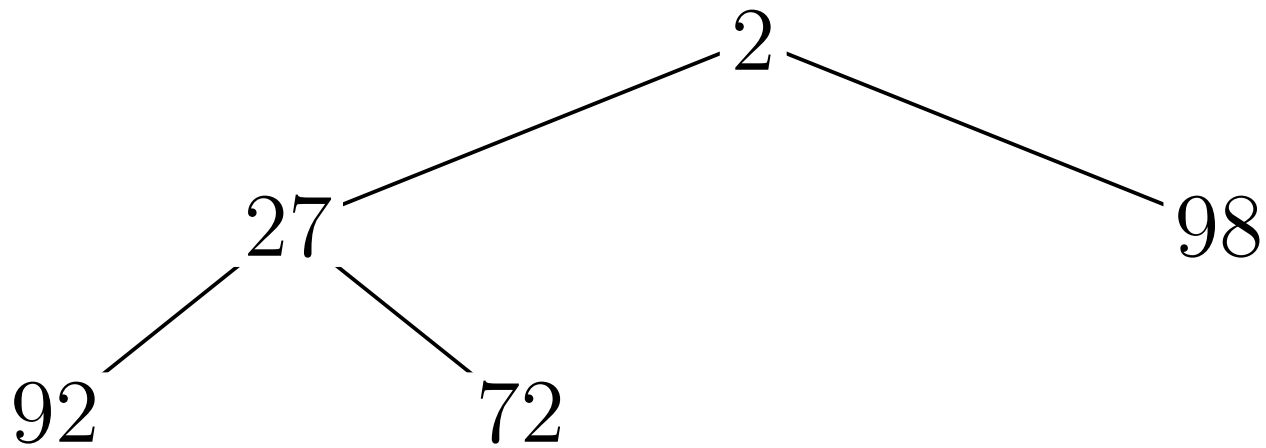
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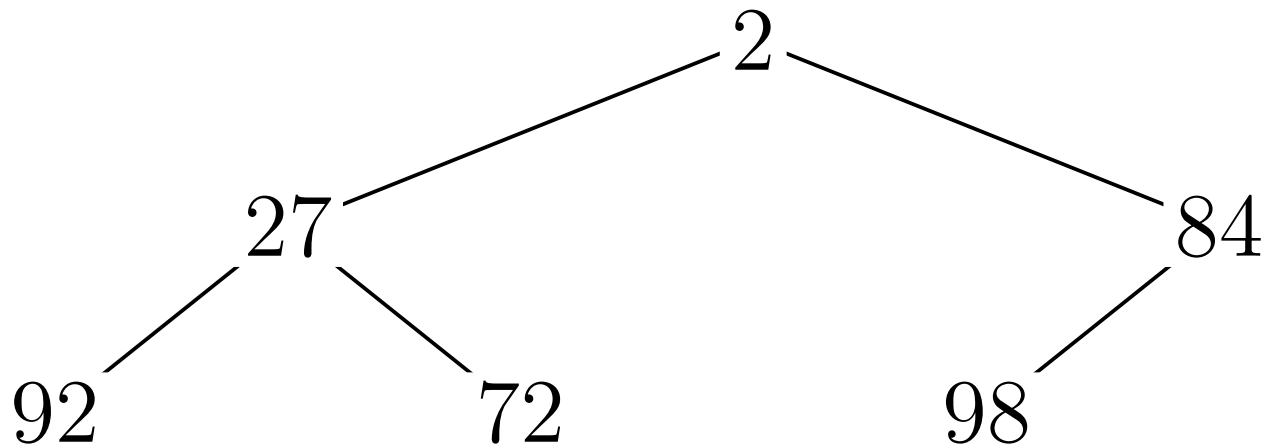
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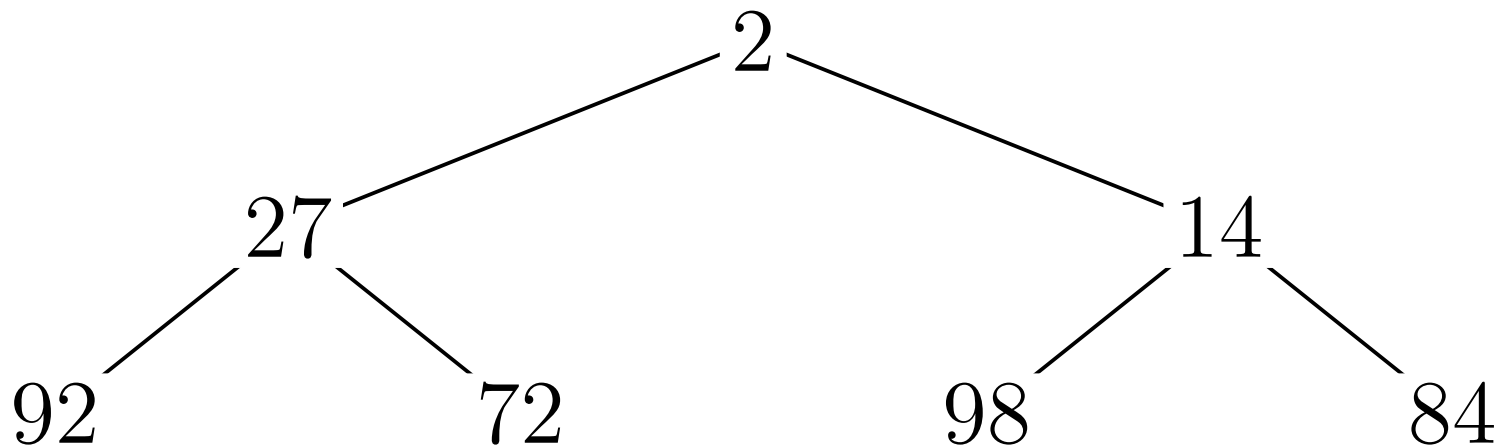
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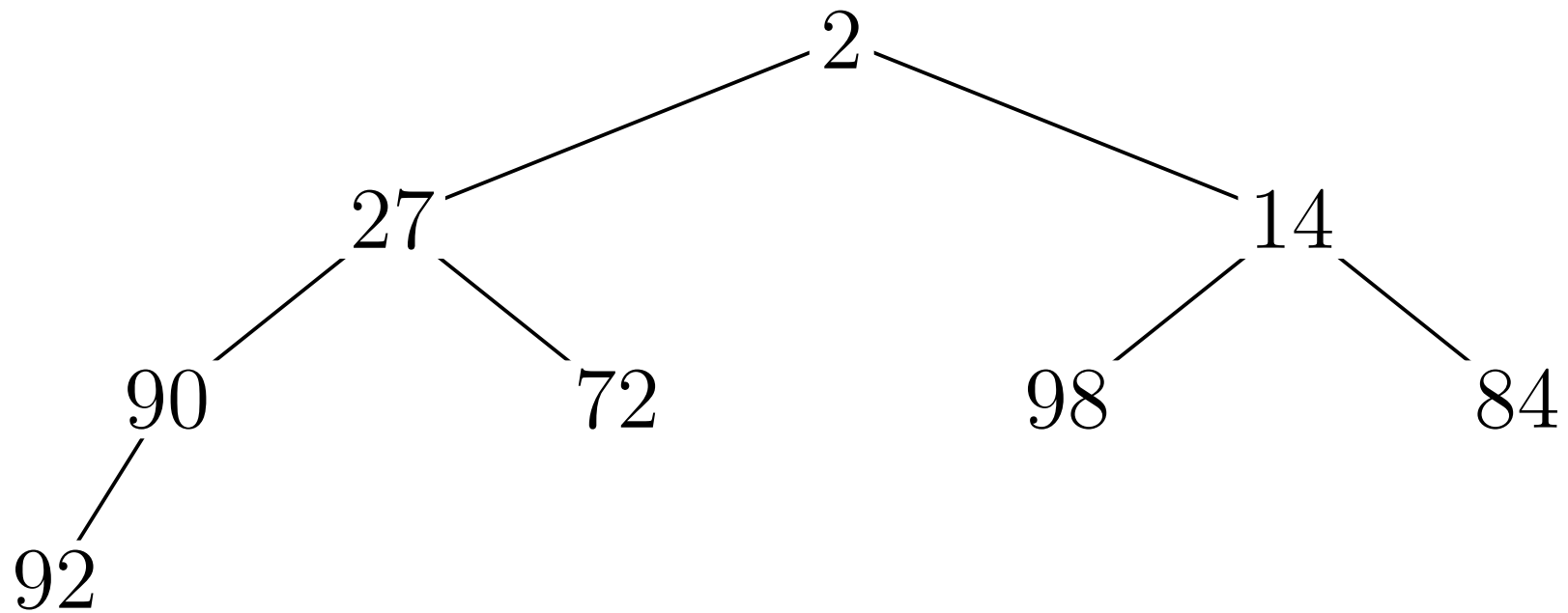
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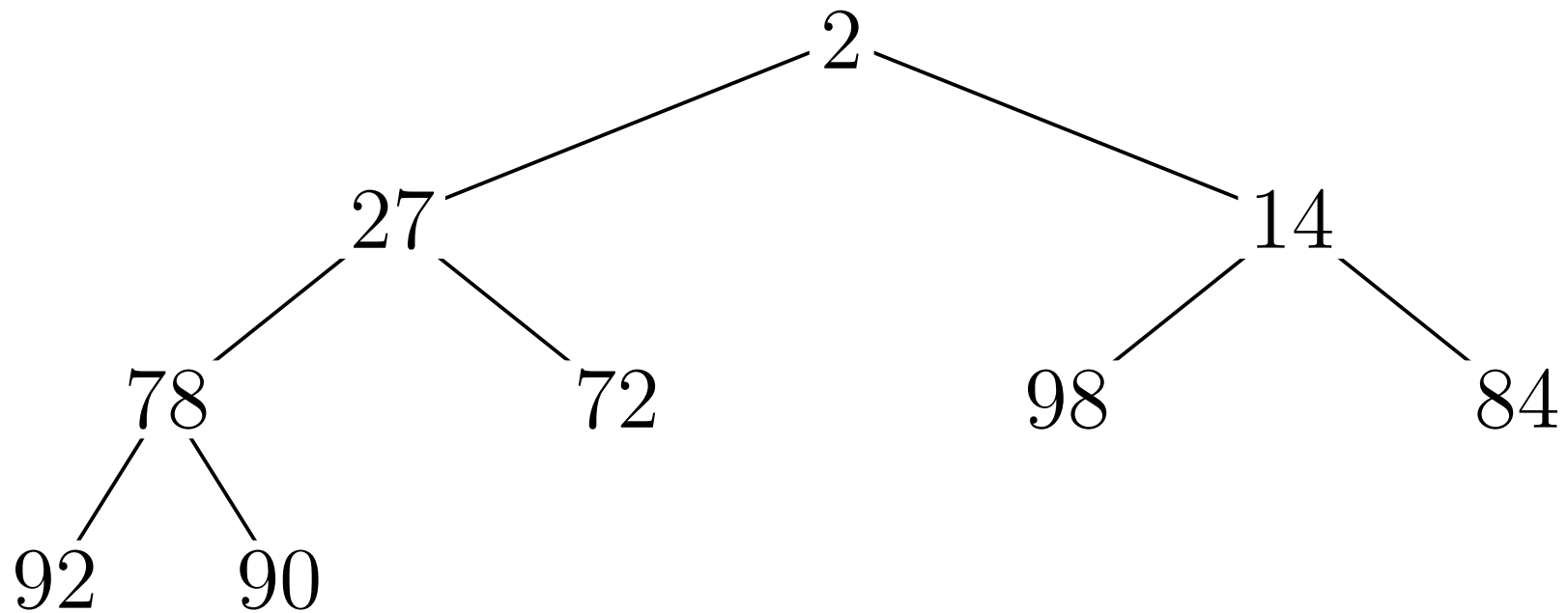
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92	27	98	72	2	84	14	90	78	21	80	63
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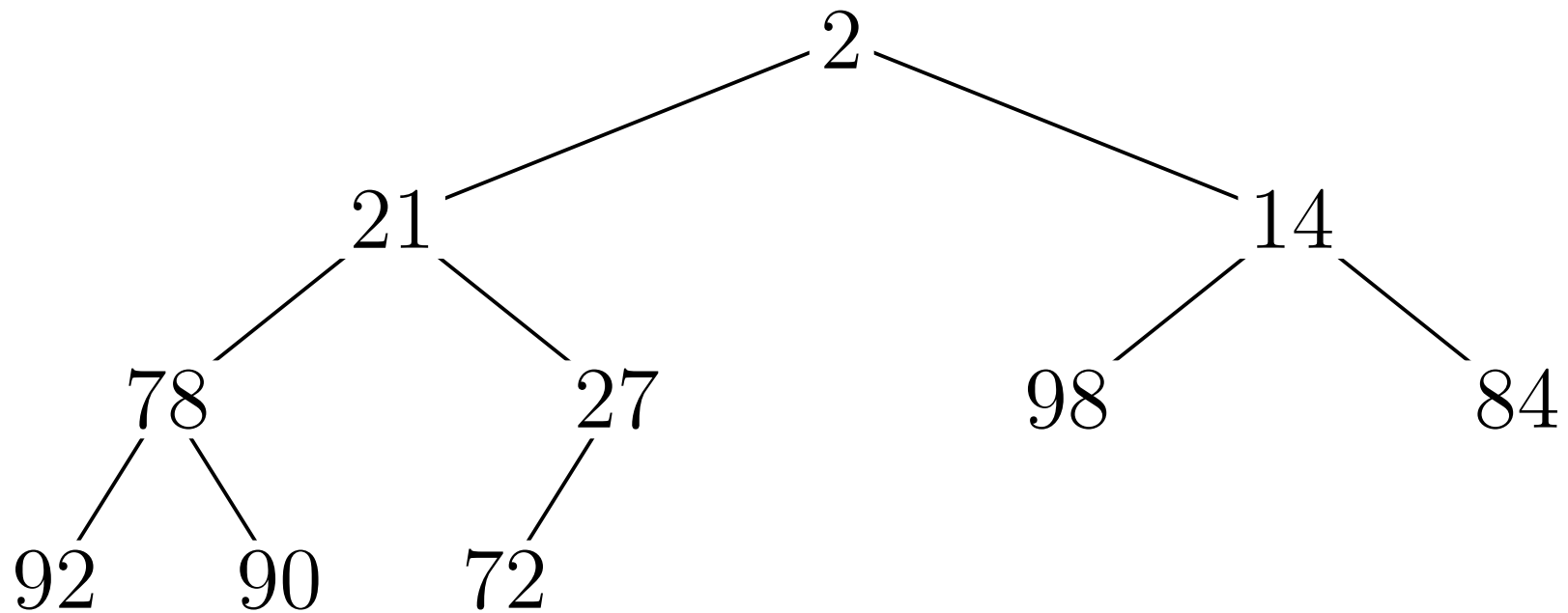
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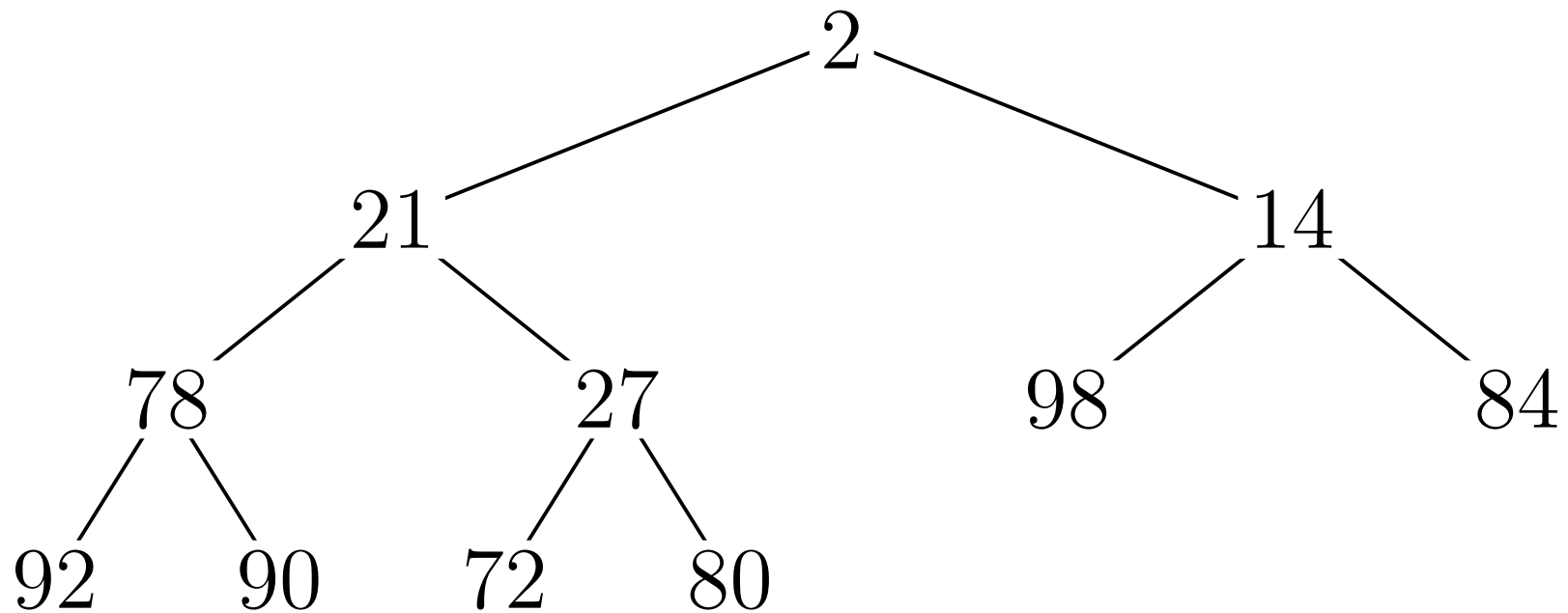
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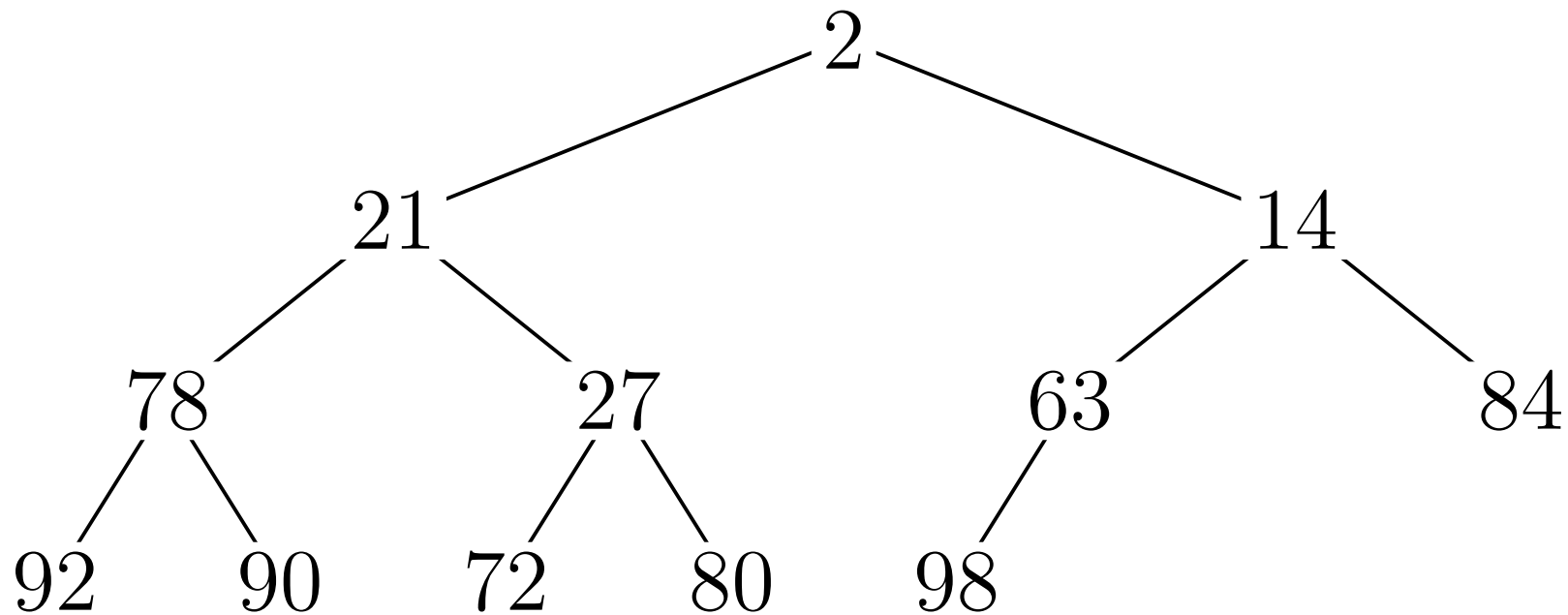
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92	27	98	72	2	84	14	90	78	21	80	63
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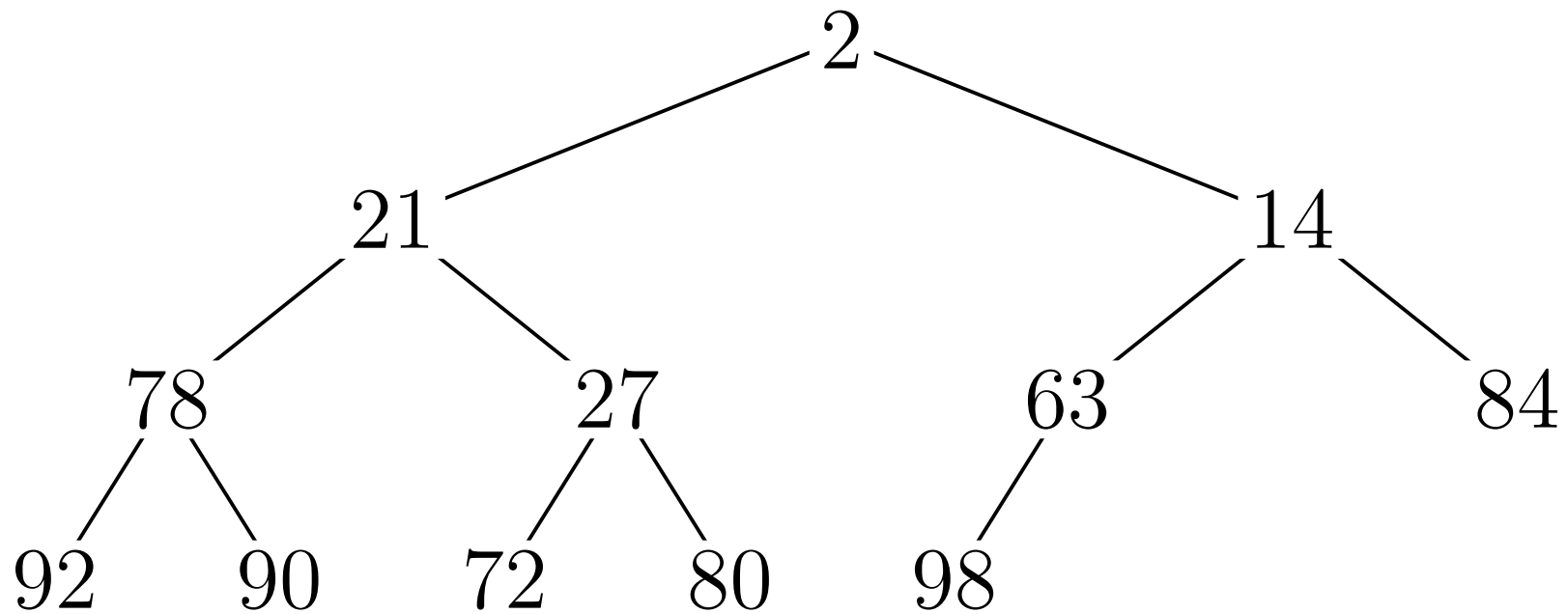


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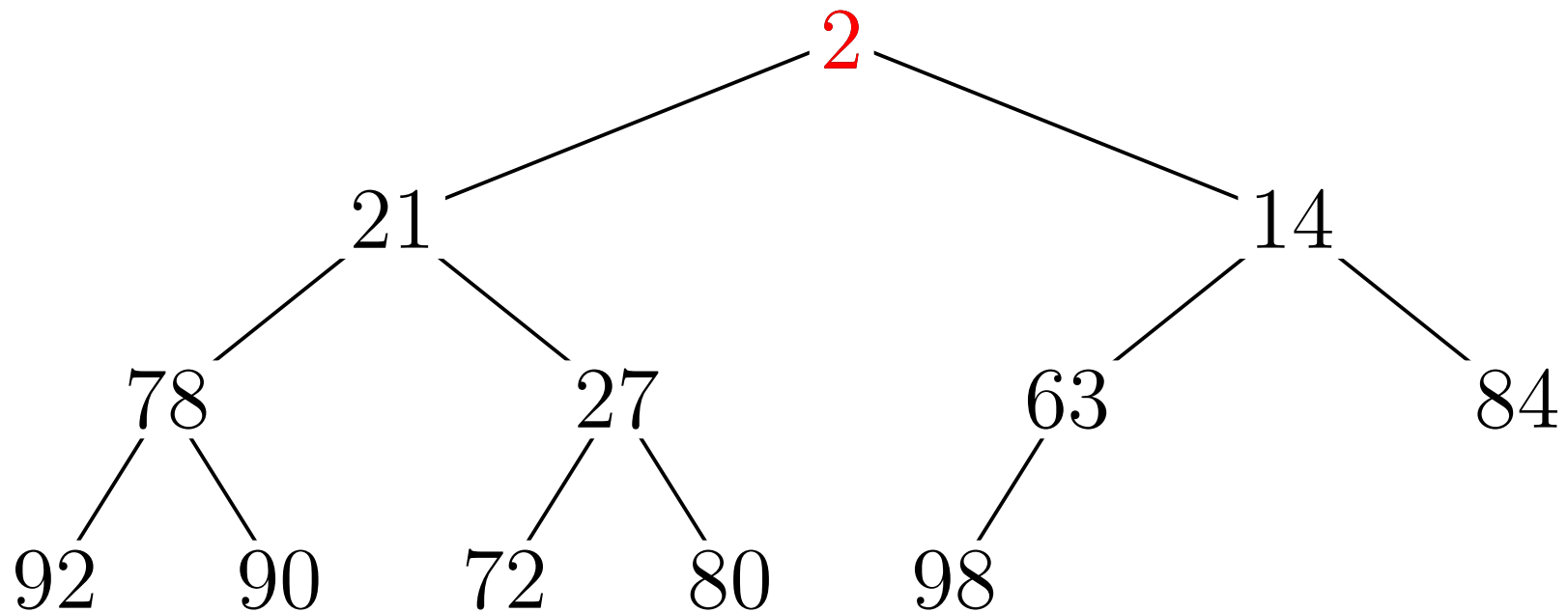
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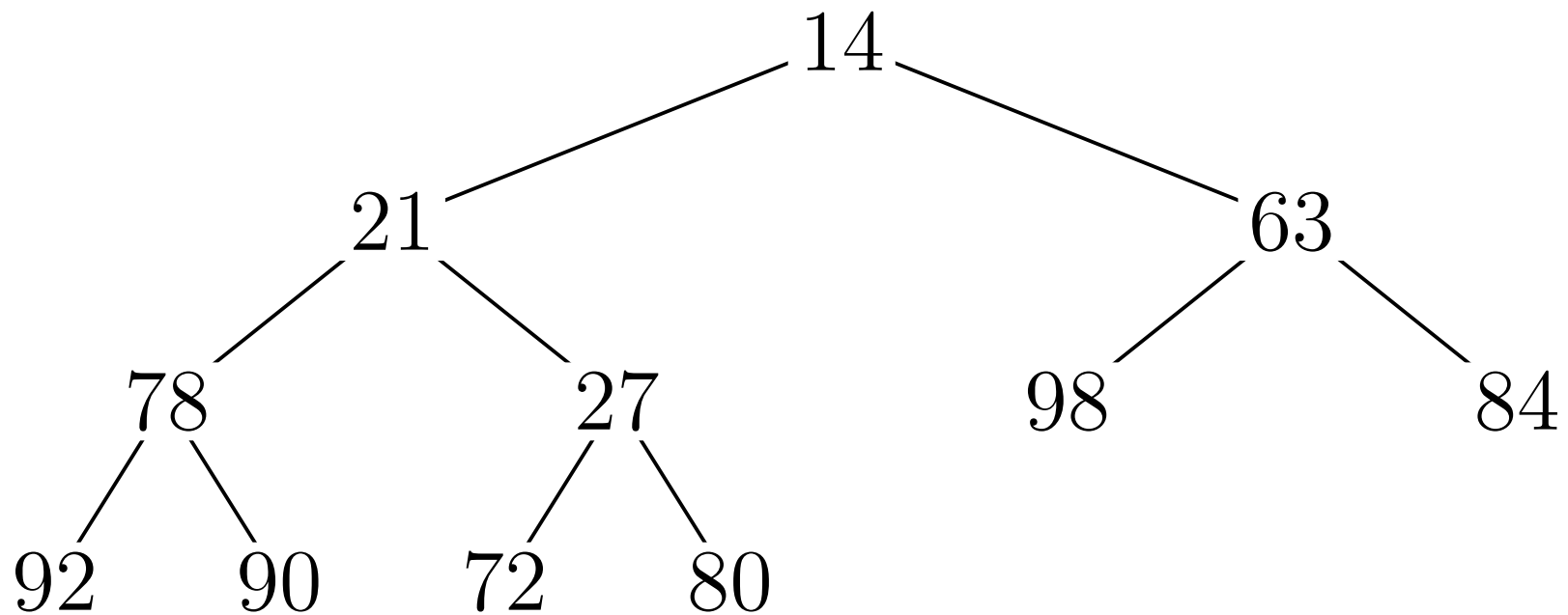
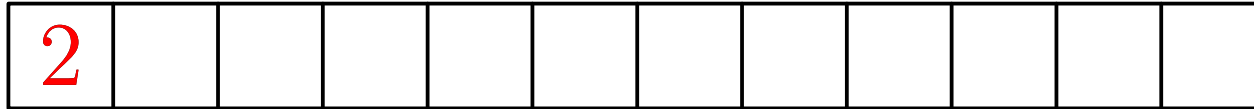
Example of Heap Sort



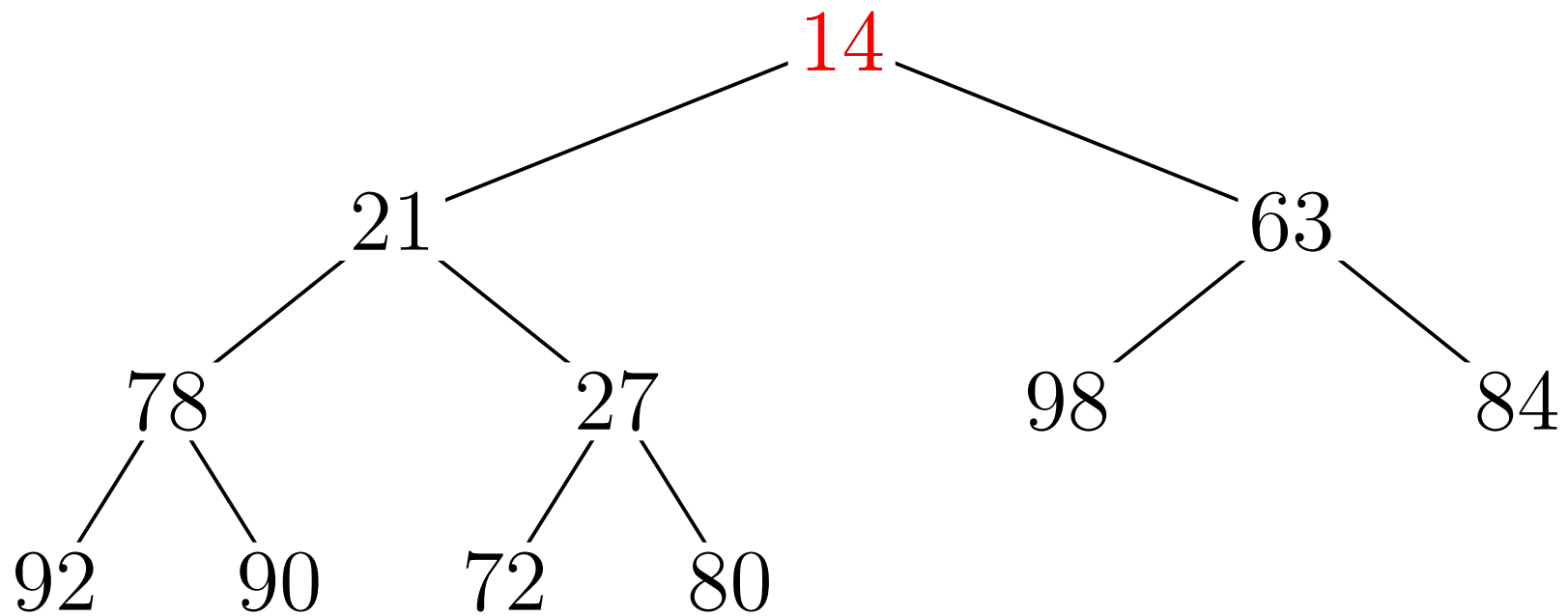
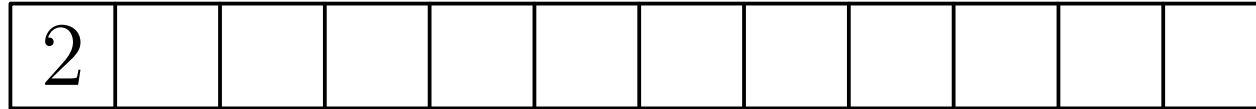
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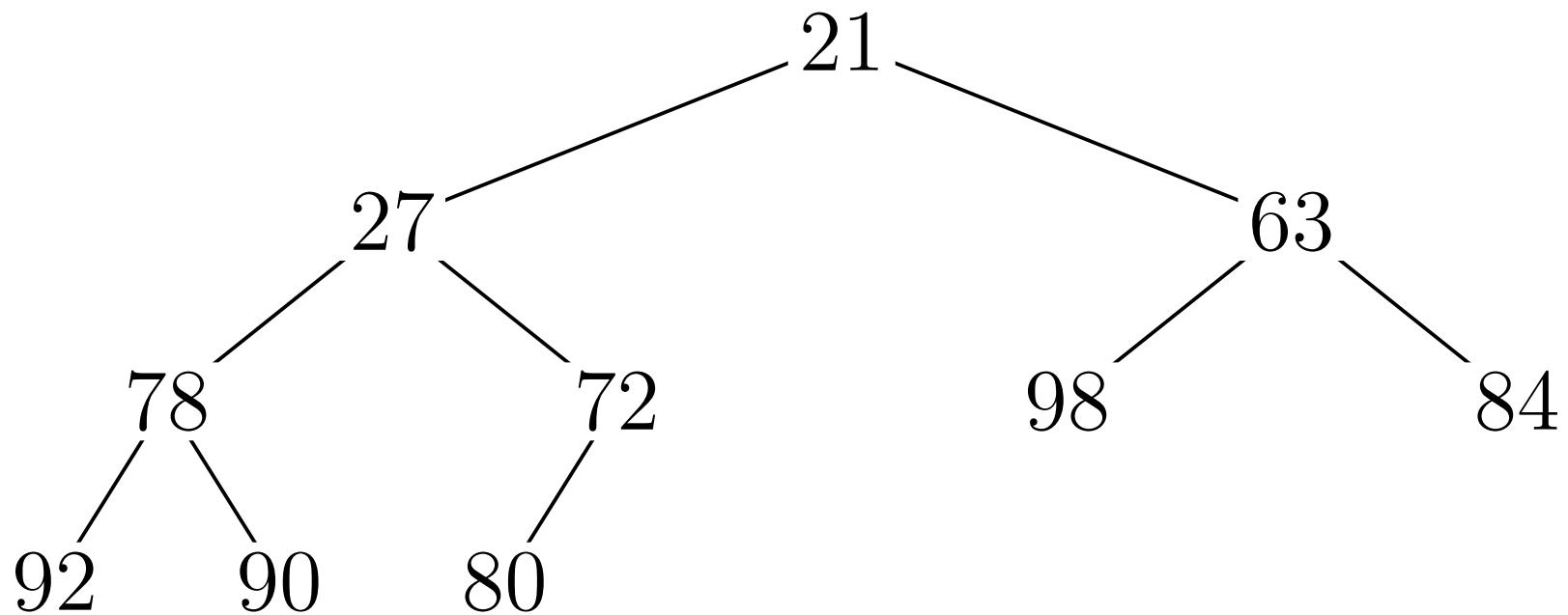


Example of Heap Sort



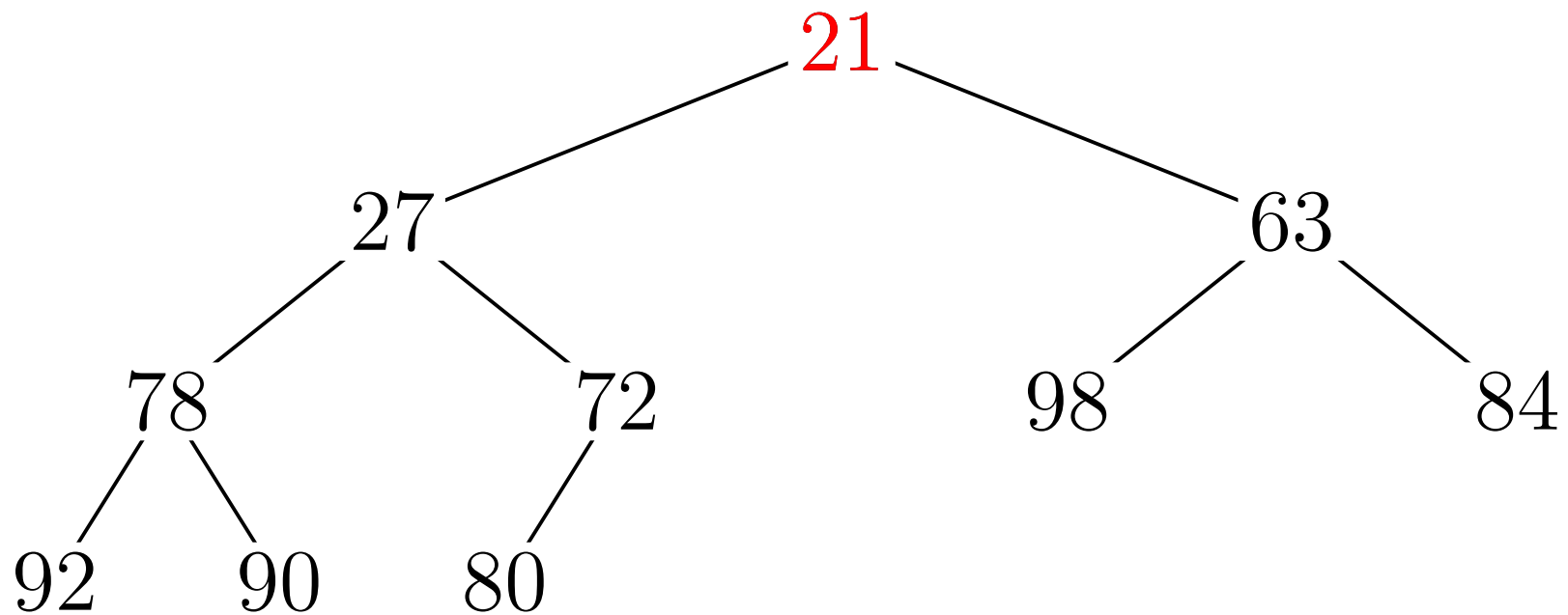
Example of Heap Sort

2	14										
---	----	--	--	--	--	--	--	--	--	--	--



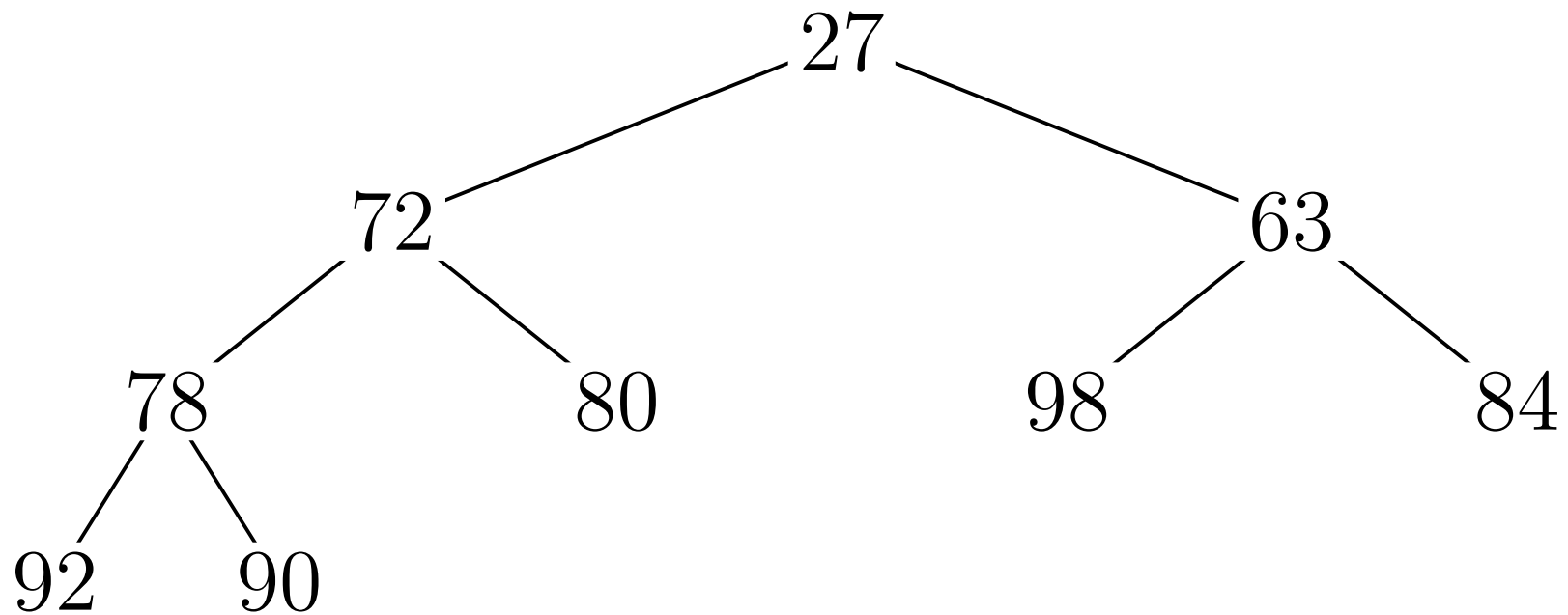
Example of Heap Sort

2	14										
---	----	--	--	--	--	--	--	--	--	--	--



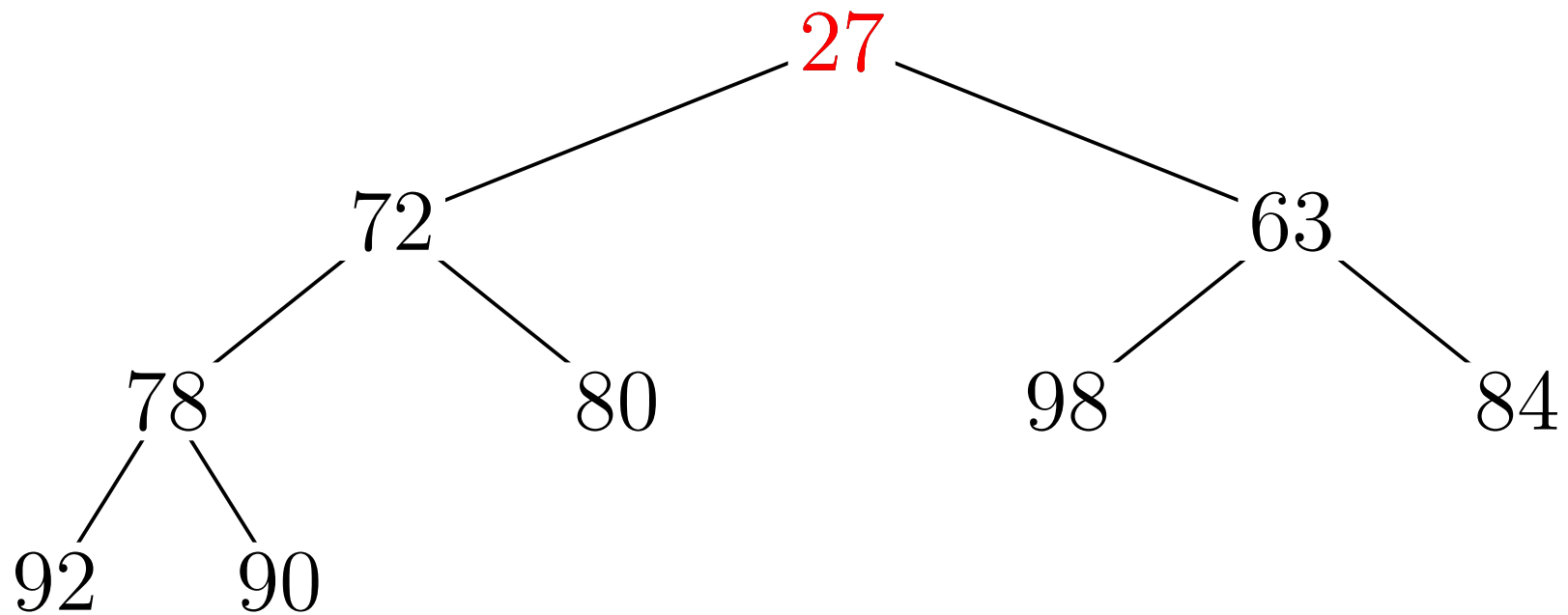
Example of Heap Sort

2	14	21									
---	----	----	--	--	--	--	--	--	--	--	--



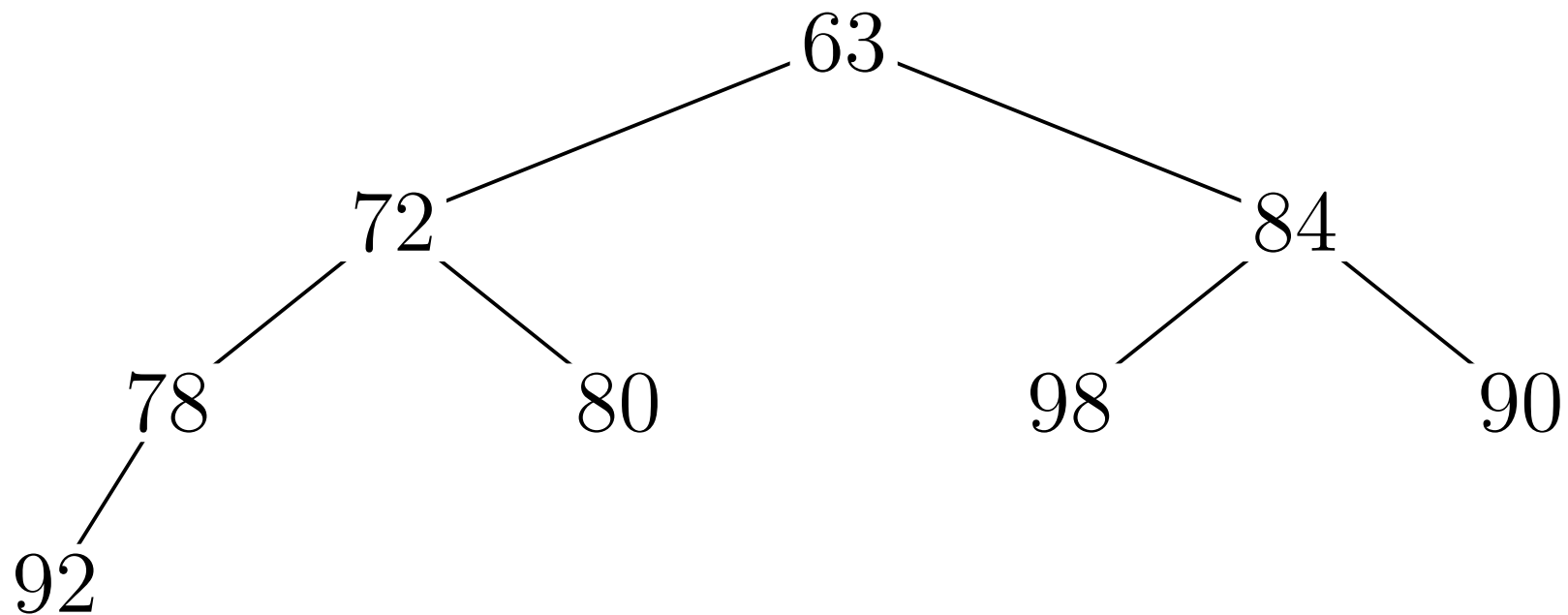
Example of Heap Sort

2	14	21									
---	----	----	--	--	--	--	--	--	--	--	--



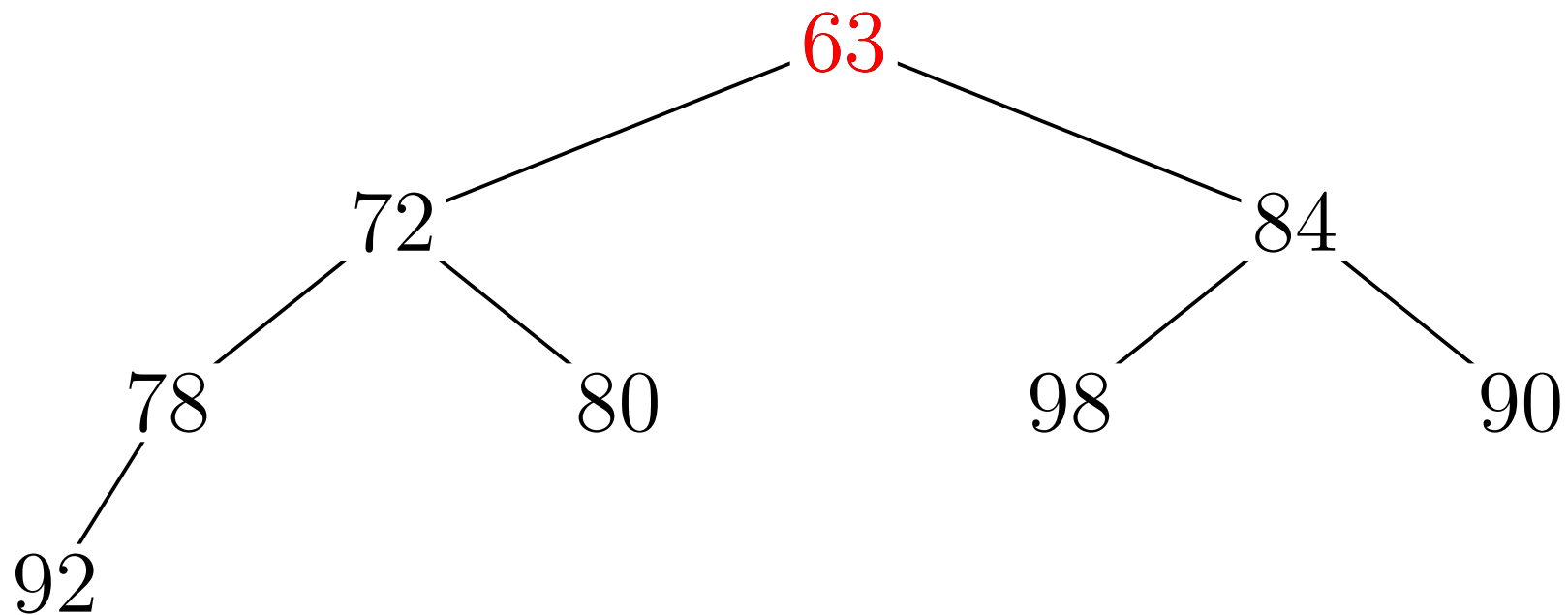
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2	14	21	27								
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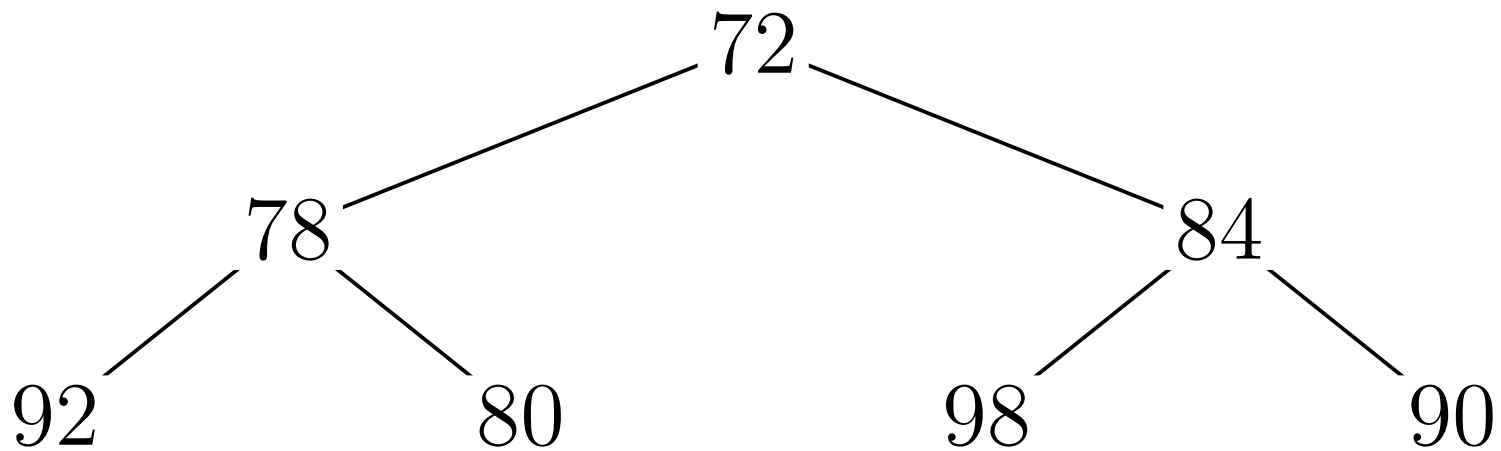
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2	14	21	27								
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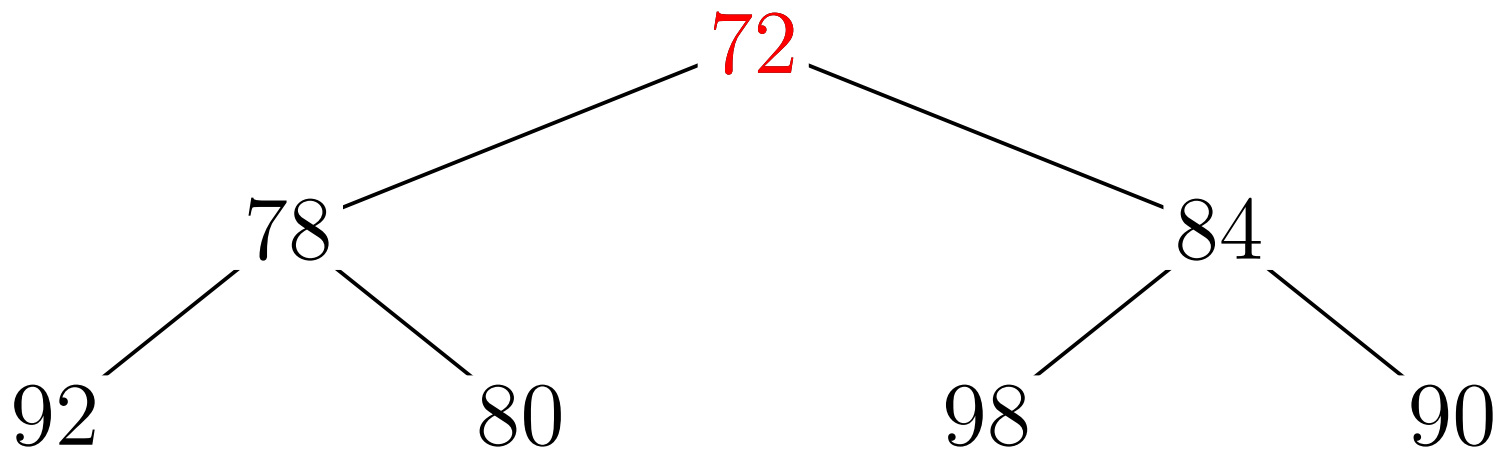
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2	14	21	27	63							
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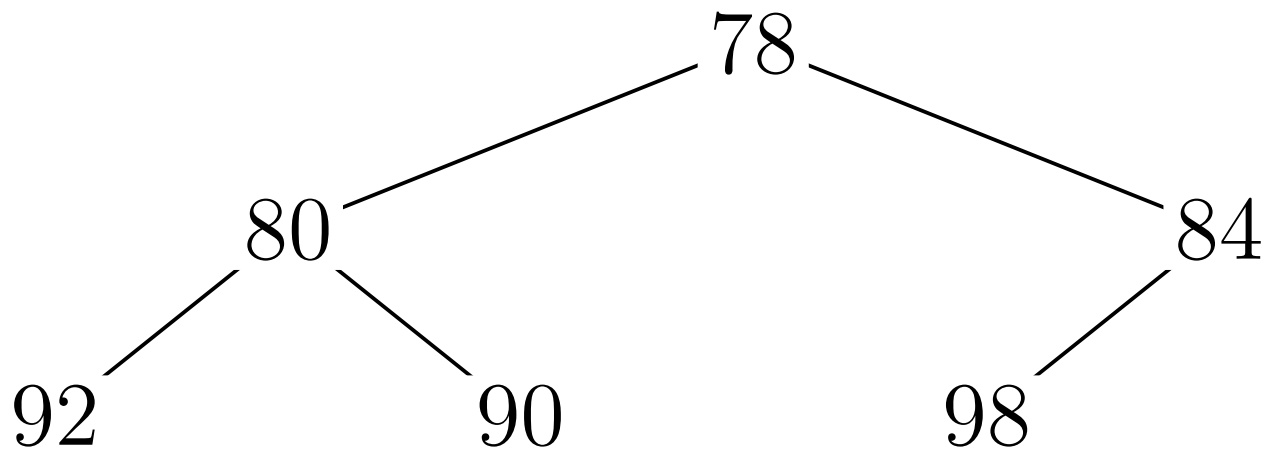
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2	14	21	27	63							
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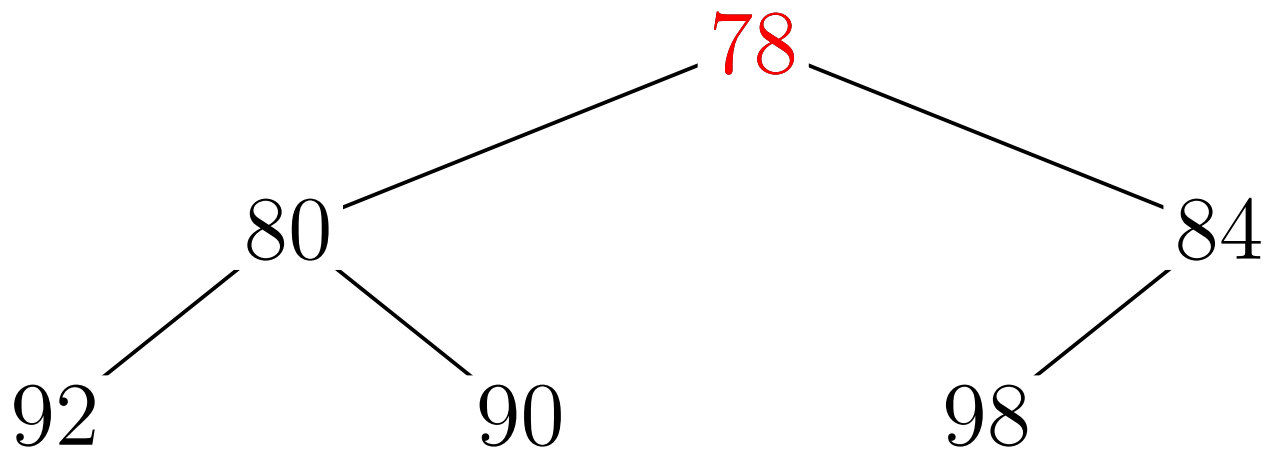
Example of Heap Sort

2	14	21	27	63	72						
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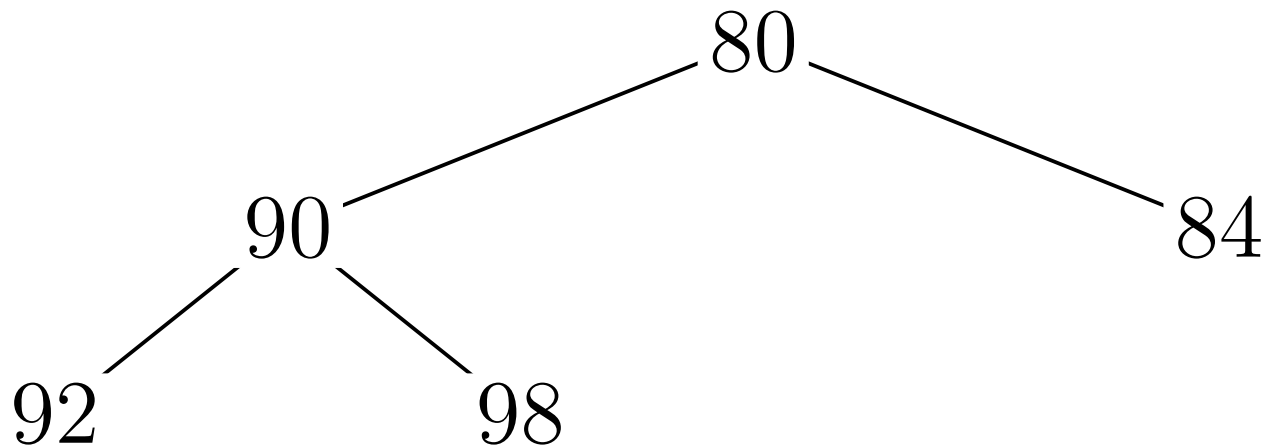
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2	14	21	27	63	72						
---	----	----	----	----	----	--	--	--	--	--	--



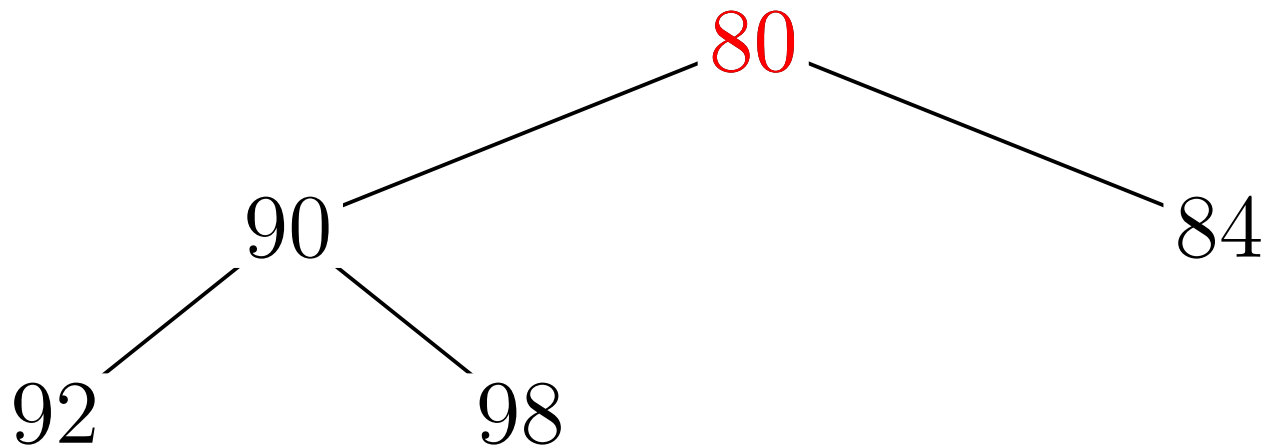
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2	14	21	27	63	72	78					
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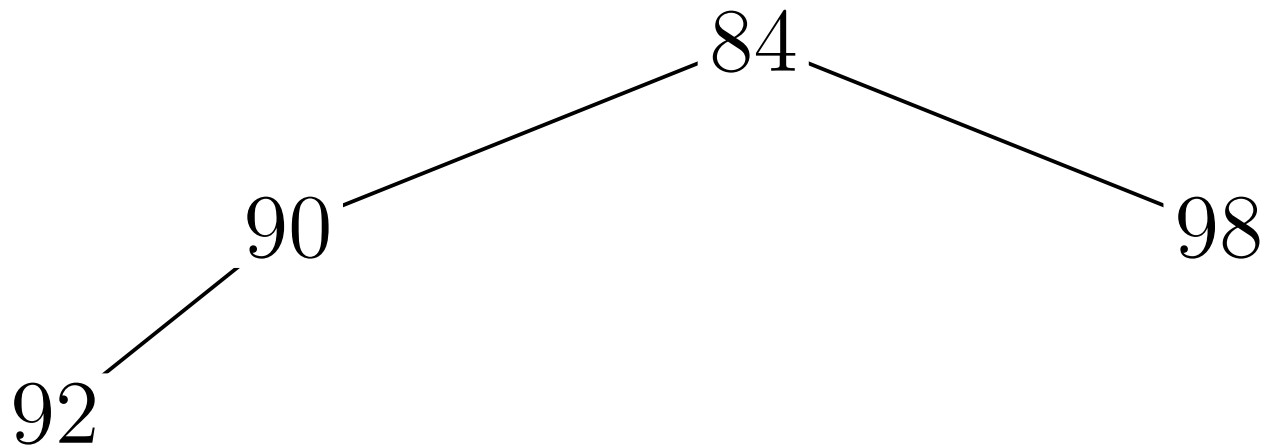
Example of Heap Sort

2	14	21	27	63	72	78					
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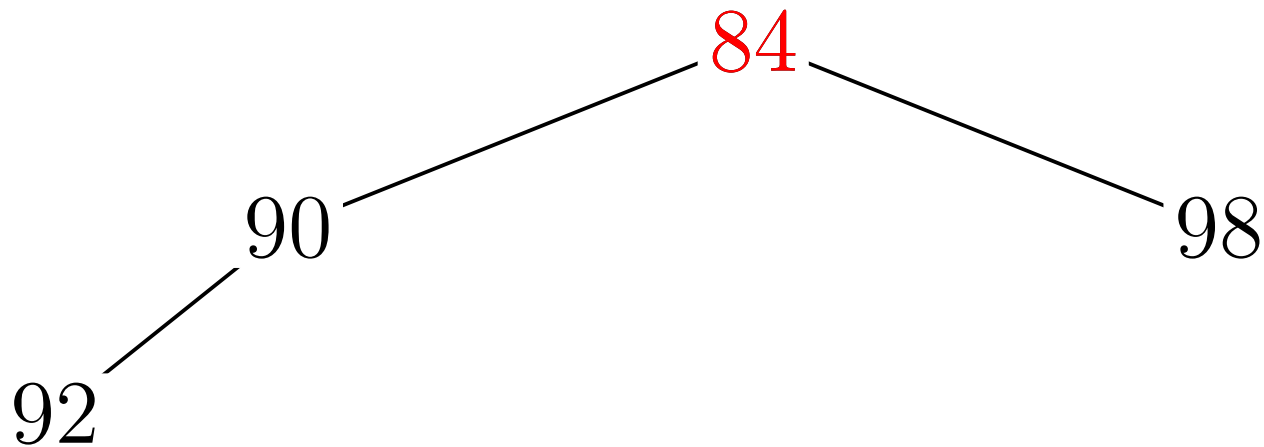
Example of Heap Sort

2	14	21	27	63	72	78	80				
---	----	----	----	----	----	----	----	--	--	--	--



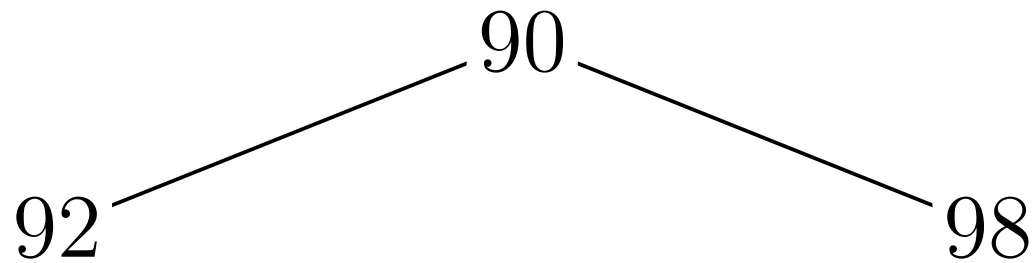
Example of Heap Sort

2	14	21	27	63	72	78	80				
---	----	----	----	----	----	----	----	--	--	--	--



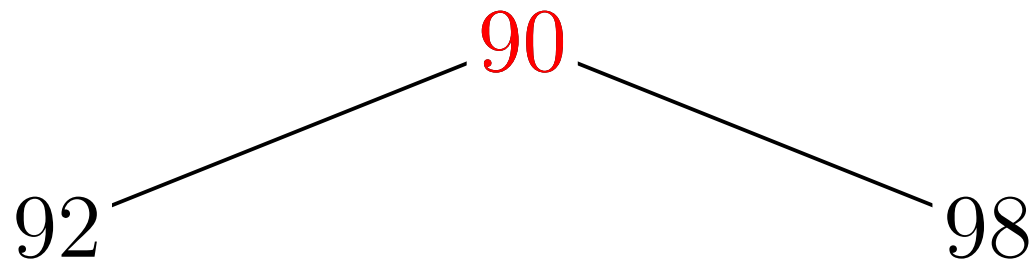
Example of Heap Sort

2	14	21	27	63	72	78	80	84			
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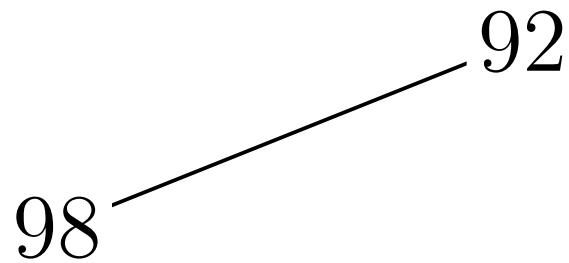
Example of Heap Sort

2	14	21	27	63	72	78	80	84			
---	----	----	----	----	----	----	----	----	--	--	--



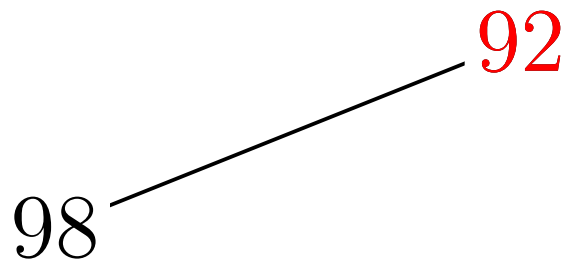
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2	14	21	27	63	72	78	80	84	90		
---	----	----	----	----	----	----	----	----	----	--	--



Example of Heap Sort

2	14	21	27	63	72	78	80	84	90		
---	----	----	----	----	----	----	----	----	----	--	--



Example of Heap Sort

2	14	21	27	63	72	78	80	84	90	92	
---	----	----	----	----	----	----	----	----	----	----	--

98

Example of Heap Sort

2	14	21	27	63	72	78	80	84	90	92	
---	----	----	----	----	----	----	----	----	----	----	--

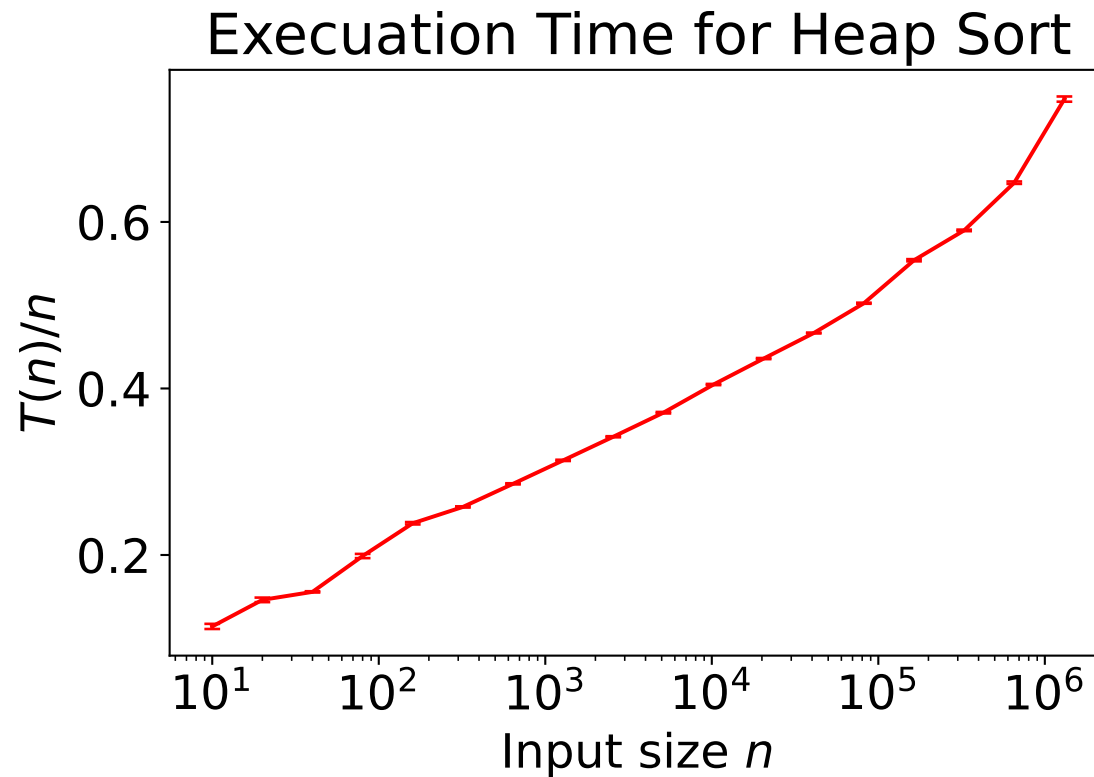
98

Example of Heap Sort

2	14	21	27	63	72	78	80	84	90	92	98
---	----	----	----	----	----	----	----	----	----	----	----

Complexity of Heap Sort

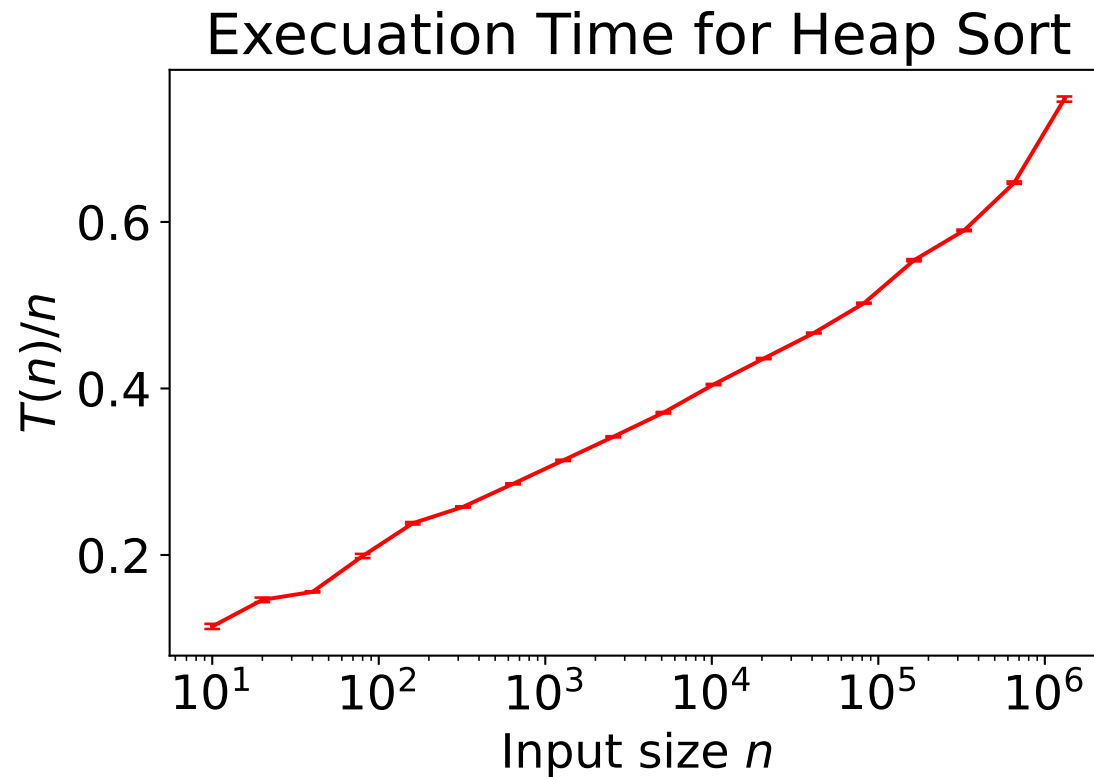
- As we have to add n elements and then remove n elements the time complexity is log-linear, i.e. $O(n \log(n))$



- This is actually a very efficient algorithm

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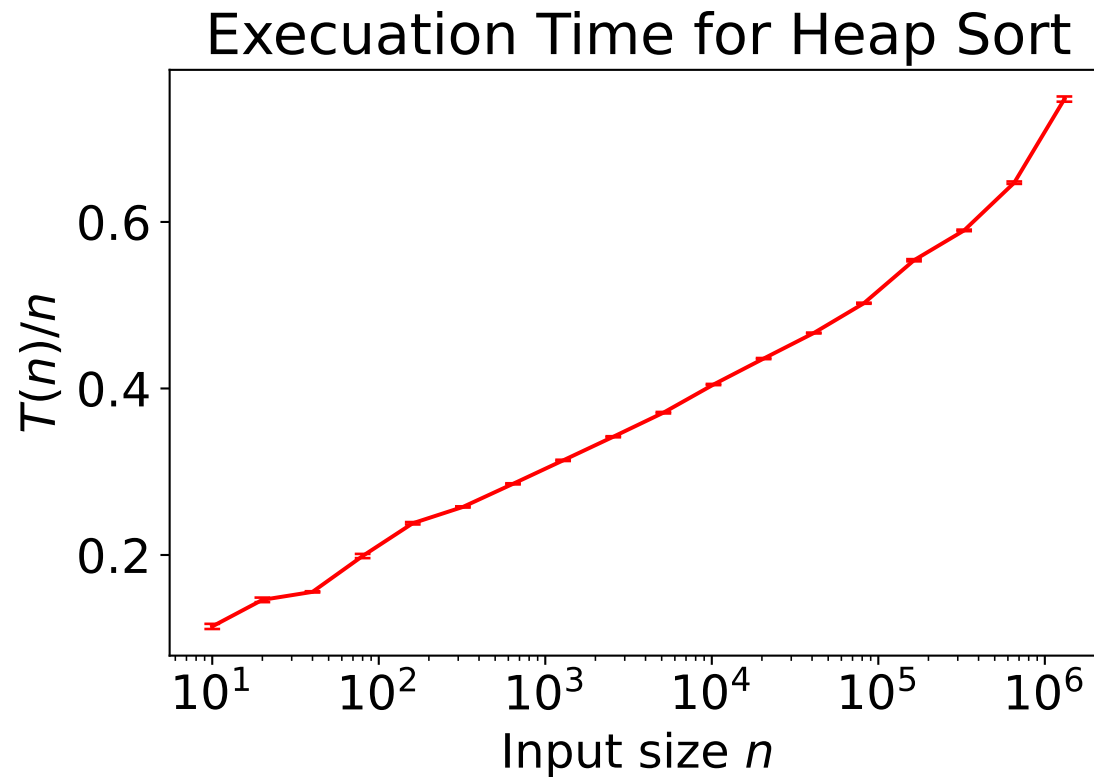
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1. Heaps
2. Priority Queues
 - Array Implementation
3. Heap Sort
4. **Other Heaps**



Other Heaps

- Binary Heaps are so useful that other types of heaps have been developed
- The simplest enhancement is to combine a binary heap with a map which maintains a pointer to each element
- The map has to be updated every time elements are moved in the heap (fortunately only $O(\log(n))$ elements are move each time the heap is updated)
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Merging Heaps

- One common demand on a heaps is to merge two heaps
- Unfortunately binary heaps are not efficiently merged
- There are a variety of different heaps (leftist heaps, skew heaps, binomial queues, . . .) designed to be merged
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Other Operations

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- Heaps are a powerful data structure—they are particularly useful for implementing priority queues
- Heaps are binary trees that can be implemented as arrays
- Priority queue have many (often surprising uses)
 - ★ They are used when you need a queue with priorities, e.g. in operating systems
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 - ★ One important application is in real time simulations
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