UNIVERSITY OF SOUTHAMPTON AICE1005W1

SEMESTER 2 EXAMINATION 2024/25

ALGORITHMS AND ANALYSIS

Duration 120 mins (2 hours)

This paper is a WRITE-ON examination paper.

You ***must*** write your Student ID on this Page and must not write your name anywhere on the paper.

All answers should be written within the designated boxes in this examination paper and sufficient space is provided for each question.

If, for some reason, space is required to complete or correct an answer to a question, use the "Additional Space" provided on the facing or adjacent page to the question. Clearly indicate which question the answer corresponds to.

No credit will be given for answers presented elsewhere and without clear indication of to what question they correspond. Blue answer books may be used for scratch; they will be discarded without being looked at.

Answer ALL parts of question 1 in part A (worth 40 marks each) and TWO questions in part B worth 30 marks each.

Student ID:

Question	Mark	*Arithmetic checked*	*Double Marked*
A1	/40		
B2	/30		
B3	/30		
B4	/30		
B5	/30		
Total:	/100		

University approved calculators MAY be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

17 page examination paper

 Version 1

Section A

A 1

(a) If a program takes 1s on an input of size $n = 100$, how long will take on an input of size $n = 1000$ if the time complexity is

(i) logarithmic, i.e. $\Theta(\log(n))$

(ii) linear, i.e. $\Theta(n)$

(iii) log-linear, i.e. $\Theta(n \log(n))$

(iv) quadratic, i.e. $\Theta(n^2)$

(v) cubic, i.e. $\Theta(n^3)$

[10 marks]

1

2

3

4

5

$\overline{10}$

(b) Give the average time complexity for checking the time it takes to search for an item in a set implemented as a (1) a binary tree, (2) a hash table and (3) a trie. Discuss any assumptions being made and give the worst case complexity when the assumptions fail. [10 marks]

1

2

3

10

(c) Give the average (amortorised) time complexity for a resizable array (e.g. `vector<T>`) and a doubly linked list (e.g. `list<T>`) for the following operations. [10 marks]

	`vector<T>`	`list<T>`
Adding an element to end of list		
Adding an element to beginning of list		
Accessing the i^{th} element		
Searching for an element		
Adding two lists of size n		

10

(d) Compute a Huffman tree for the following alphabet.

Letter	a	b	c	d	e	f	g
Frequency	120	65	33	54	110	10	83

[7 marks]

$\overline{7}$

(e) How would the word “bagged” be coded in your Huffman tree (show the letter break with a hyphen). [3 marks]

$\overline{3}$

End of question A1

(a) $\overline{10}$ (b) $\overline{10}$ (c) $\overline{10}$ (d) $\overline{7}$ (e) $\overline{3}$ Total $\overline{40}$

Additional space. Do not use unless necessary. Clearly mark corresponding question.

Section B

B 2 The Towers of Hanoi problem is solved by the program

```
hanoi(n, A, B, C)
{
   if (n>0) {
     hanoi(n-1, A, C, B);
     move(A, C);
     hanoi(n-1, B, A, C);
   }
}
```

(a) Let $T(n)$ be the number of times `move` is called to solve the Hanoi problem of size n. Write down a recurrence relation for $T(n)$. [5 marks]

$T(n) =$

5

(b) Write down the boundary condition $T(1)$ and use the recurrence relation to compute $T(2)$, $T(3)$, and $T(4)$

[5 marks]

$T(1) =$

$T(2) =$

$T(3) =$

$T(4) =$

5

(c) Prove by induction that, $f(n) = 2^n - 1$ satisfies the recurrence relation in part (a)
[20 marks]

$\overline{20}$

End of question B2

(a) $\overline{5}$ (b) $\overline{5}$ (c) $\overline{20}$ Total $\overline{30}$

Additional space. Do not use unless necessary. Clearly mark corresponding question.

B 3

(a) Briefly describe Prim's algorithm. [10 marks]

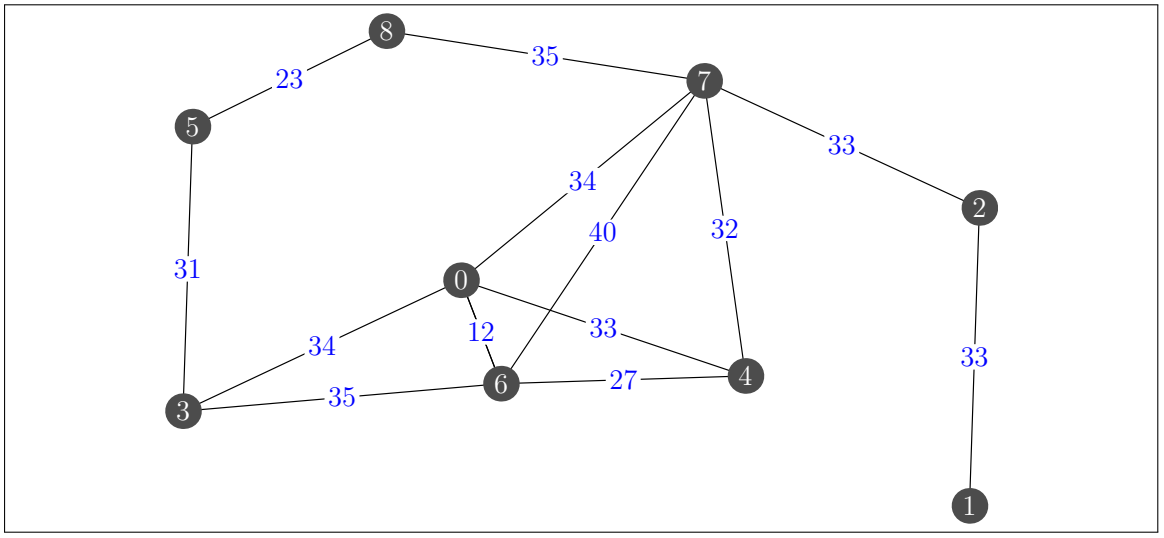
10

(b) What is the time complexity of Prim's algorithm for constructing a minimum spanning tree.

[5 marks]

5

(c) In the graph below draw the minimum spanning tree (by highlighting the edges) and write down the order in which the edges would be found by Prim's algorithm. [15 marks]



1.	**2.**
3.	**4.**
5.	**6.**
7.	**8.**

$\frac{}{15}$

End of question B3

(a) $\frac{}{10}$ (b) $\frac{}{5}$ (c) $\frac{}{15}$ Total $\frac{}{30}$

Additional space. Do not use unless necessary. Clearly mark corresponding question.

B 4

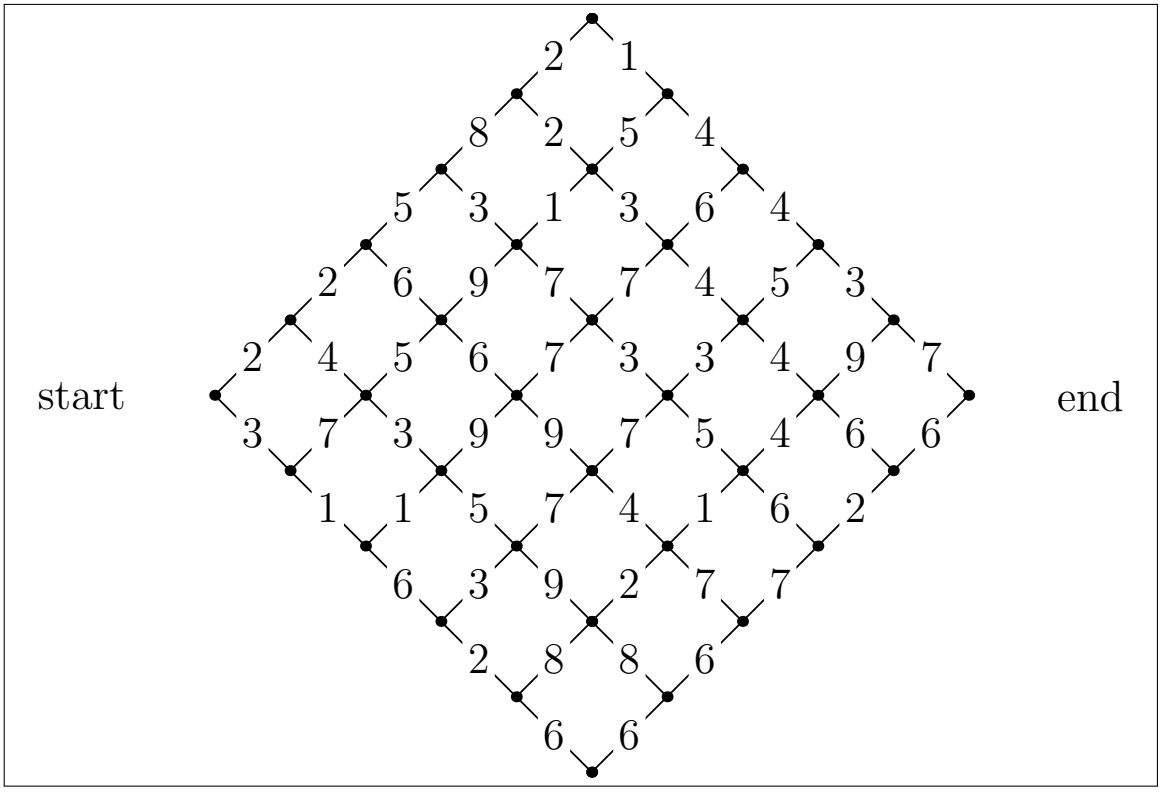
(a) Briefly describe dynamic programming (both forward and backward algorithms). [10 marks]

10

(b) Give an example of where dynamic programming is used. [5 marks]

5

(c) Use the dynamic programming forward algorithm to compute the minimum cost of each path from the left most node to each other node, where the cost of moving along an edge is equal to the number shown. An edge can only be traversed from left to right. Use the backwards algorithm to find the minimum cost path across the graph. [15 marks]



$\frac{}{15}$

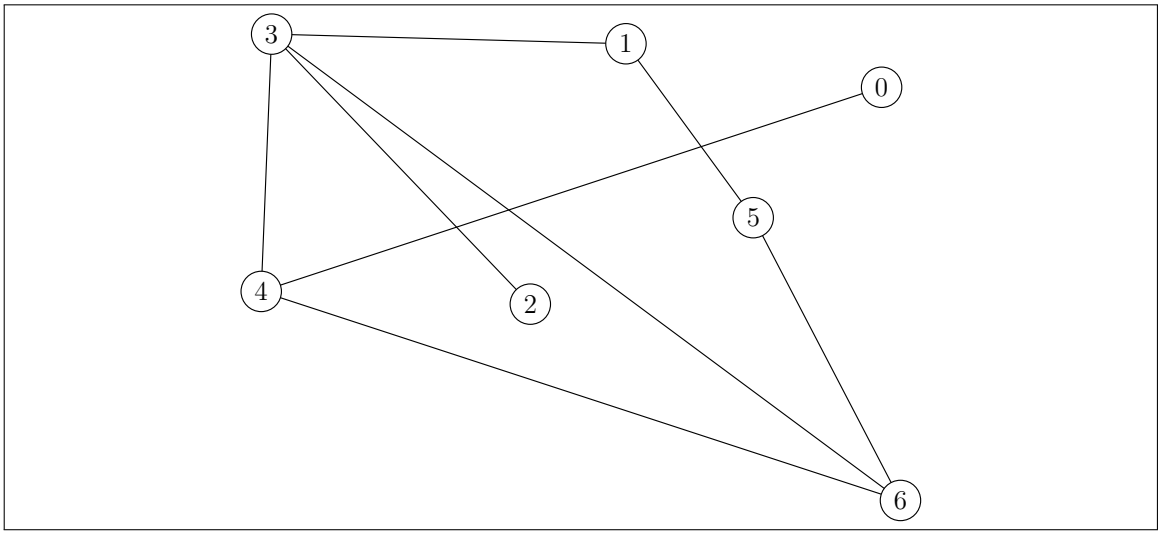
End of question B4

(a) $\frac{}{10}$	(b) $\frac{}{5}$	(c) $\frac{}{15}$	Total $\frac{}{30}$

Additional space. Do not use unless necessary. Clearly mark corresponding question.

B 5

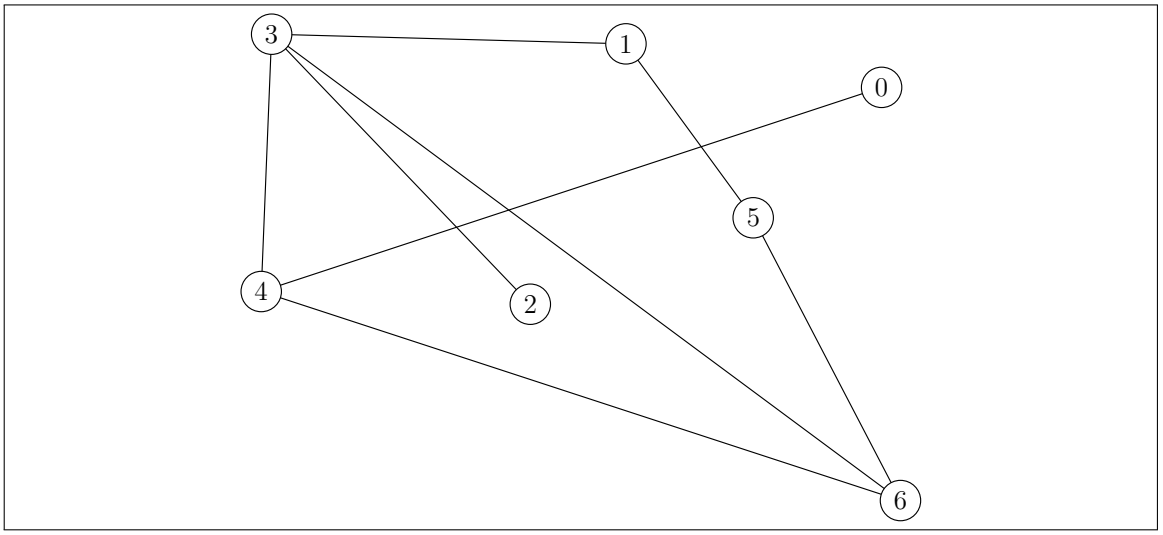
(a) Draw the edges on the graph used to find the vertices using **breadth first search** starting from vertex 0 where the lower numbered vertices are searched first. Write the order in which the vertices are discovered. [10 marks]



$\overline{10}$

bfs order = _____

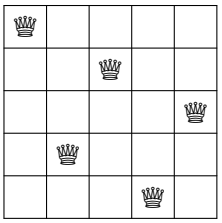
(b) Draw the edges on the graph used to find the vertices using **depth first search** starting from vertex 0 where the lower numbered vertices are searched first. Write the order in which the vertices are discovered. [10 marks]



$\overline{10}$

dfs order = _____

(c) The n-queens problem is to put n queens on a chess board such that no queen is in the same row, column or diagonal as any other queen. Either write pseudo code or describe in outline an algorithm to solve the n-queens problem.



[10 marks]

$\overline{10}$

End of question B5

(a) $\overline{10}$ (b) $\overline{10}$ (c) $\overline{10}$ Total $\overline{30}$

Additional space. Do not use unless necessary. Clearly mark corresponding question.