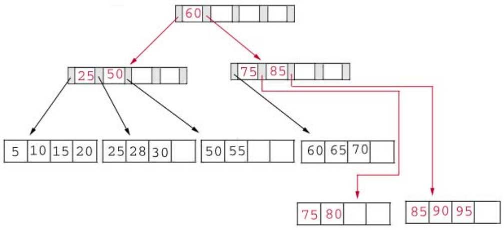
Further Mathematics and Algorithms

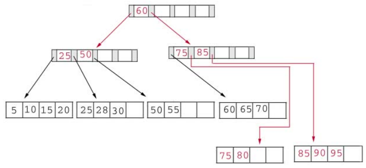
Lesson 12: *Sometimes It Pays Not to Be Binary*



B-Trees, Tries, Suffix Trees

Outline

1. **B-Trees**
2. Tries
3. Suffix Tree



B-Trees

- **B-trees** are balanced trees for fast search, finding successors and predecessors, insert, delete, maximum, minimum, etc.
- Not to be confused with binary trees
- They are designed to keep related data close to each other in (disk) memory to minimise retrieval time
- Important when working with large amount of data that is stored on secondary storage (e.g. disks)
- Used extensively in databases

When Big-O Doesn't Work

- An underlying assumption of Big-O is that all elementary operations take roughly the same amount of time
- This just isn't true of disk look-up
- The typical time of an elementary operation on a modern processor is 10^{-9} seconds
- But a typical hard disk might do 7 200 revolutions per minute or 120 revolutions per second
- The typical time it takes to locate a record is around 10ms or 10^7 times slower than an elementary operation

Accessing Data from Disk

- When accessing data from disk minimising the number of disk accesses is critical for good performance
- In database applications we want to store data as large sets
- Storing data in binary trees is disastrous as we typically need around $\log_2(n)$ disk accesses before we locate our data
- It is not unusual in databases for $n = 10\,000\,000$ so that $\log_2(n) \approx 24$
- Using binary trees it would often take several seconds to find a record

Multiway-Trees

- To remedy this we can use M-way trees so that the access time is

$$\log_M(n) = \frac{\log_2(n)}{\log_2(M)}$$

- In practice we might use $M \approx 200 \approx 2^8$ so we can reduce the depth of the tree by around a factor of 8
- The basic data structures for doing this is the B-tree
- There are many variants of B-tree, all trying to squeeze a bit more performances from the basic structure

B^+ Tree

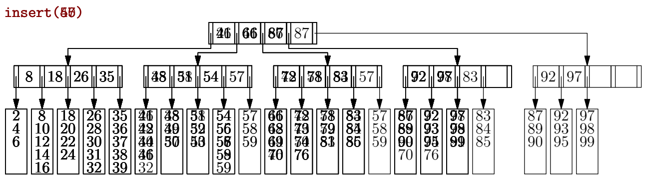
- A pretty basic implementation would obey the following rules
  1. The data items are stored at leaves
  2. The non-leaf nodes store up to M-1 keys to guide the search: key i represents the smallest key in subtree $i+1$
  3. The root is either a leaf or has between 2 and M children
  4. All non-leaf nodes except the root have between $\lceil M/2 \rceil$ and M children
  5. All leaves are at the same depth and have between $\lceil L/2 \rceil$ and L data entries

Choosing M and L

- The choice of M and L depends on the block size (the information read in one go from disk)
- It also depends on the type of data that is being stored (integer, reals, strings, etc.)
- M and L might be in the hundreds or thousands
- In the examples below we consider tiny $M = L = 5$ which is unrealistic, but drawable

B-Tree Example

- $M = 5,\ L = 5$



Other Changes

- If the root is full then it can be split into two and a new root created
- B-trees also have to allow the removal of records without losing its structure
- There are a number of variant strategies (e.g. neighbouring nodes can adopt a child if the current node cannot expand any more)
- The actual implementation of B-trees is tricky because there are many special cases

B-Tree Summary

- B-trees are an important data structure for databases where reducing the number of disk searches is vital
- They tend to be much more complex than the other data structures we have seen
- The problem of disk access can be improved by replacing disk memory with solid-state drives (still slow compared to memory)
- For massive databases new data structures have been developed to allow faster (although less flexible) information access (e.g. NOSQL, MongoDB, Neo4j)

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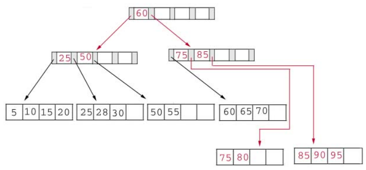
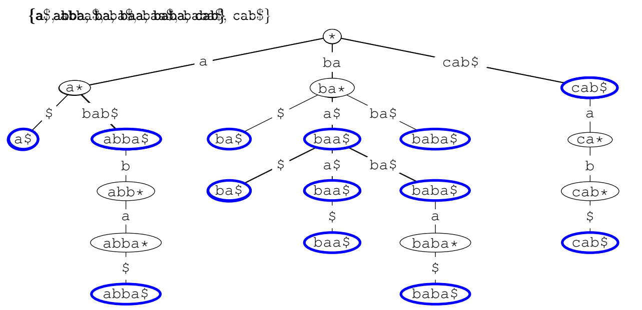


Tries

- A **Trie** (pron. 'try') or **digital tree** is a multiway tree often used for storing large sets of words
- They are trees with a possible branch for every letter of an alphabet
- Their names comes from the word *re**trie**val*
- Tries usually compactify the edges in the tree
- All words end with a special letter "\$"

Trie



Uses of Tries

- Tries are yet another way of implementing sets
- They provide quick insertion, deletion and find
- Typically considerably quicker than binary trees and hash tables
- They are particularly good for spell checkers, completion algorithms, longest-prefix matching, hyphenation
- Each search finds the longest match between the words in the set and the query

Trie for 31 Most Common English Words

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
A	A*(5)		HA*(8)		WAS$		THAT$				
B	BUT(8)										
C											
D				HAD$							
E			HER(7)				THE$		BE$		
F	FOR(3)									OF$	
G											
H	HA(2)				WHICH$						
I	IN(1)		HIS$		WITH$		THIS$				
J											
K											
L											
M											
N	NOT$	IN$				AND$				ON$	
O	ON(9)										FOR$
P											
Q											
R						ARE$		HER$		OR$	FROM$
S		IS$				AS$					
T	THAT(6)	IT$				AT$					
U									BUT$		
V				HAVE$							
W	WITH$										
X											
Y	YOU$								BY$		
Z											

Disadvantage of Tries

- Table-based tries typically waste large amounts of memory
- Often table-based tries are used for the first few layers, while lower levels use a less memory intensive data structure
- These days memory is less of a problem so table-based tries are acceptable for some applications
- There are many implementations of tries each suited to a particular task

Binary Tries

- One extreme (though not uncommon) solution to address memory issues is to build a bit-level trie so the data-structure is a binary tree
- It differs from a binary tree in that the decisions to go left or right depends on the current bit
- Although you lose the advantage of a multiway tree (of reducing the depth) it does find the longest match and it speeds up finds which fail

Why Tries?

- Tries are a classic example of a trade-off between memory and computational complexity
- Tries are slightly specialist and tend to get used in very particular applications
  - Finding longest matches
  - Completion, spell checking, etc.
- A basic trie is not too complicated, however, . . .
- There are many implementation which try to overcome the difficulty of wasting too much memory

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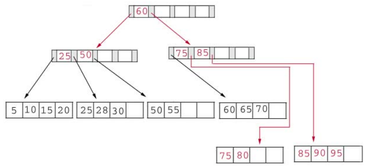
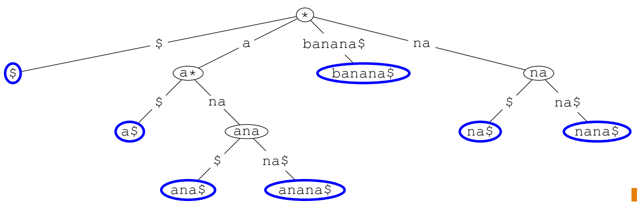


Suffix Tree

- Suffix tree is a trie of all suffixes of a string
- E.g. banana

`{$, a$, na$, ana$, nana$, anana$, banana$}`



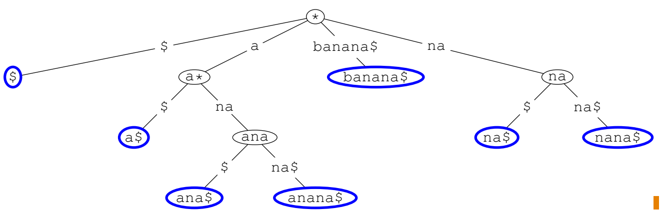
Importance of Suffix Tree

- The first linear-time algorithm for computing suffix trees was proposed by Peter Weiner in 1973, a more space efficient algorithm was proposed by Edward M. McCreight in 1976
- Esko Ukkonen in 1995 proposed a variant of McCreight's algorithm, but in a way that was much easier to understand
- It really only got implemented after this
- They are very important for string-based algorithms
- The classic application is in finding a match for a query string, Q, in a text, T

String Matching

- To find a match of a query string, Q, in a text, T, we can first construct the suffix tree of the string T we then simple look up the query, Q, using the trie

`{$, a$, na$, ana$, nana$, anana$, banana$}`



Complexity of Suffix Trees

- Using a regular trie for a suffix tree would typically use far too much memory to be useful
- However, by using pointers to the original text it is possible to build a suffix tree using $O(n)$ memory where n is the length of the text
- Furthermore (and rather incredibly) there is a linear time ($O(n)$) algorithm to construct the trie
- The algorithm is not however trivial to understand

Uses of Suffix Trees

- Suffix trees are efficient whenever it is likely that you will do multiple searches
- Exact word matching is in itself a very important application
- Suffix trees in combination with dynamic programming (which we will eventually get to) can be used to do inexact matching (finding the match with the smallest edit distance)
- Suffix trees get used in bioinformatics, advanced machine learning algorithms, . . .

Lessons

- Multiway trees can considerable speed up search over binary trees
- They are very important in some specialised applications (e.g. databases, spell-checking, completion, suffix trees)
- They are not as general purpose as binary search trees and are more complicated to implement
- But they can give the best performance—sometimes performance matters enough to make it worthwhile implementing multiway trees