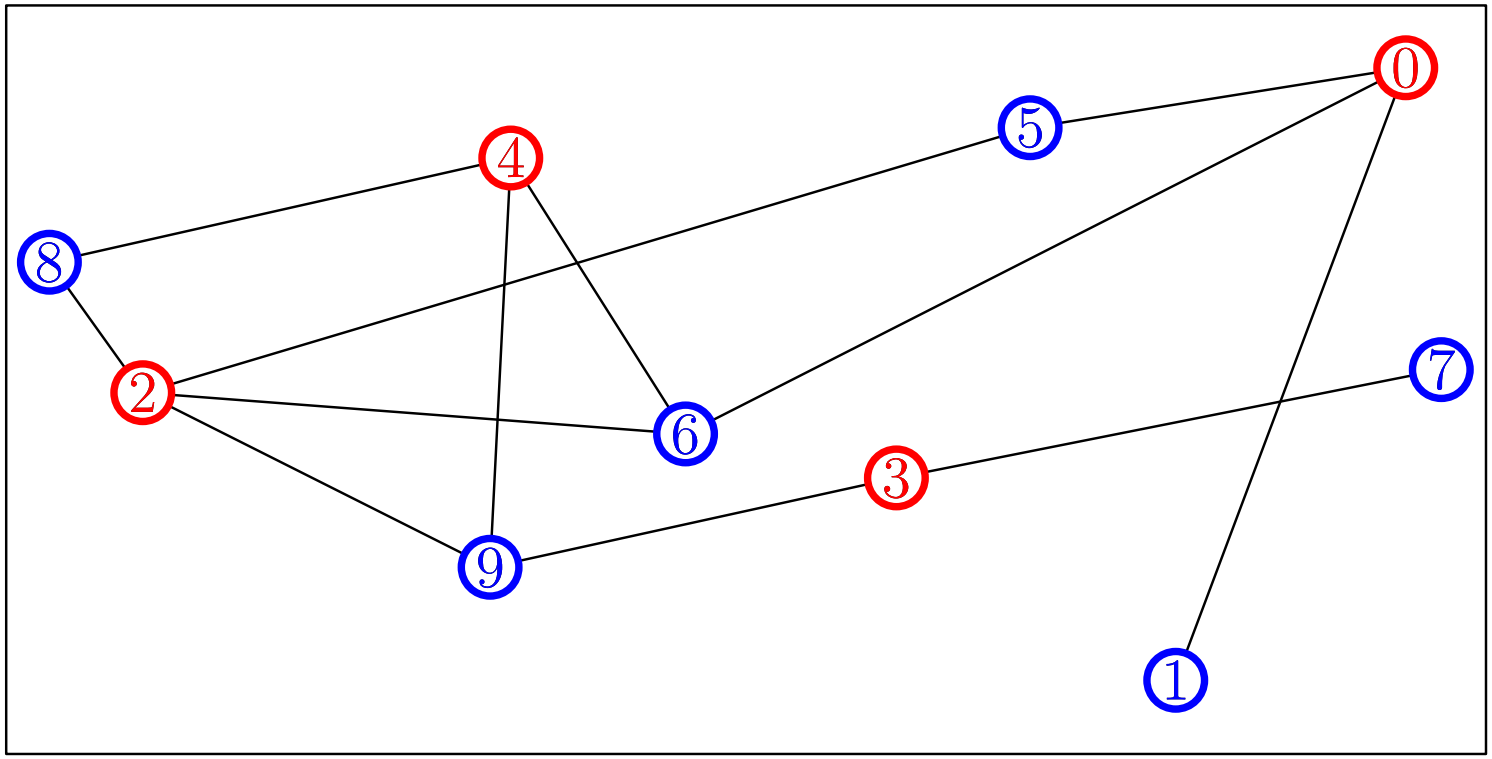
Algorithms and Analysis

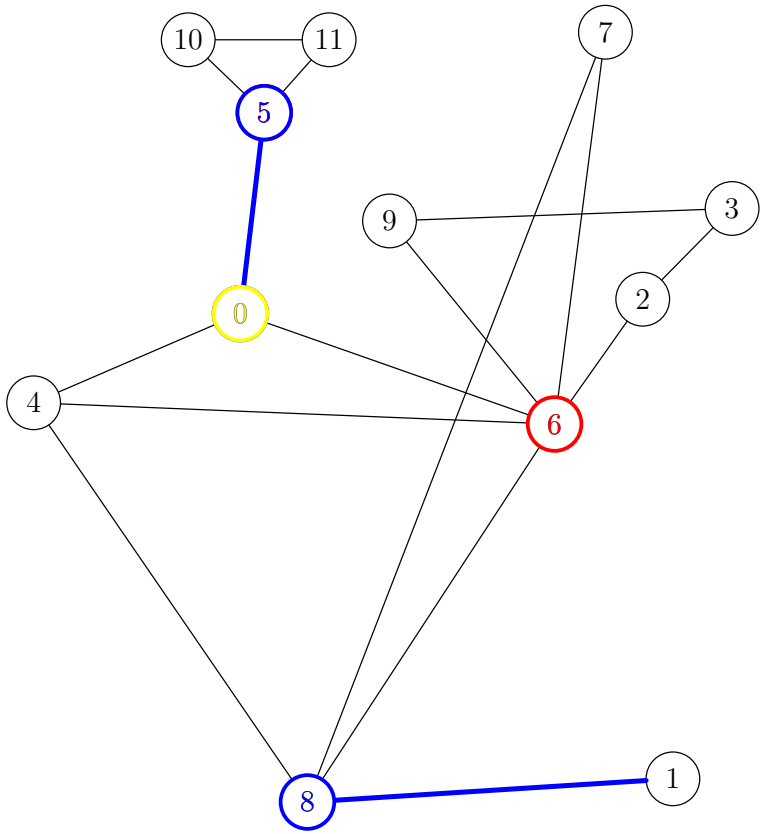
Lesson 20: *Learn to Traverse Graphs*



Breadth First Search, Depth First Search, Topological Sort

Outline

1. **Breadth First Search**
   - BFS applications
2. Depth First Search
   - DFS applications
3. Topological Sort



Basic Graph Algorithms

- Graphs provide an abstraction for a huge number of real world processes: social networks, compute network, road networks, etc.
- Increasing applications focus on very large (sparse) graphs (usually implemented using an adjacency list)
- Require (near) linear time algorithms
- Basic building block are graph traversal algorithms
  - ⋆ Breadth First Search
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Advanced Generic

- To make these algorithm general purpose (generic) we allow ourselves to call arbitrary functions to act on the vertices and edges at different points in the algorithm
- This introduces a new level of generics which makes the algorithms very powerful
- Increases the steepness of the learning curve to use these algorithms
- Once you get familiar with using these algorithms this level of generics starts to pay off
- Libraries which does this include Boost Graph Library and LEDA in C++; JDSL and JGraphT in Java

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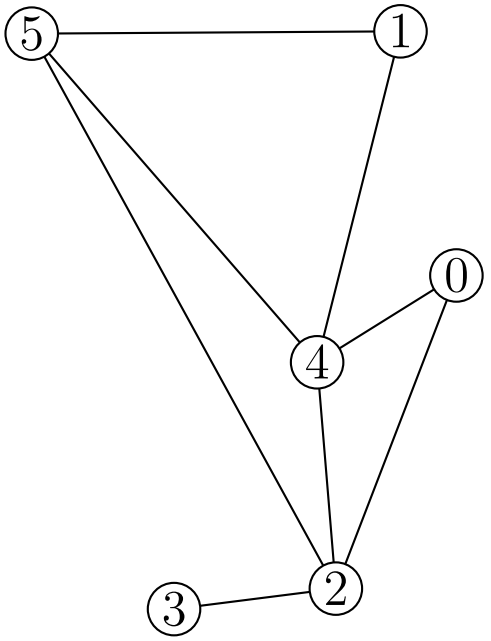
Graph Traversal

- To traverse a graph we start at a (arbitrary) $root$ vertex
- We then follow edges to create a tree



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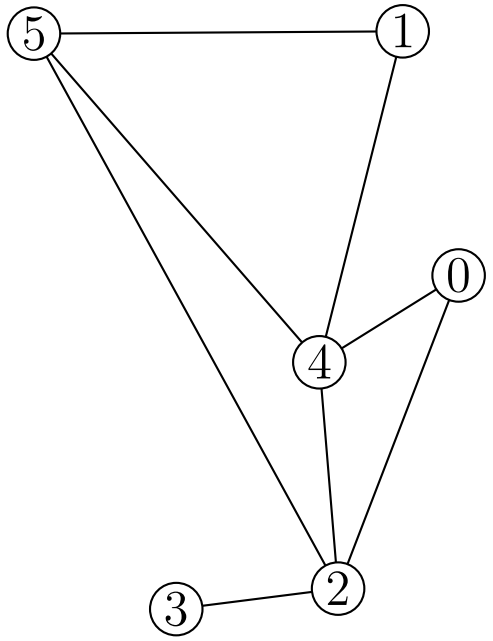

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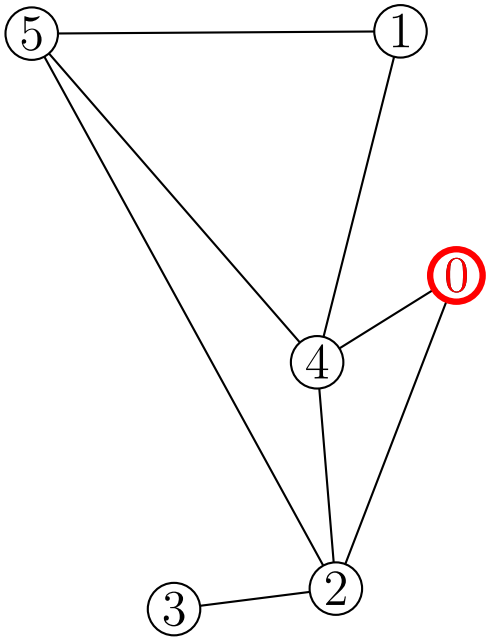

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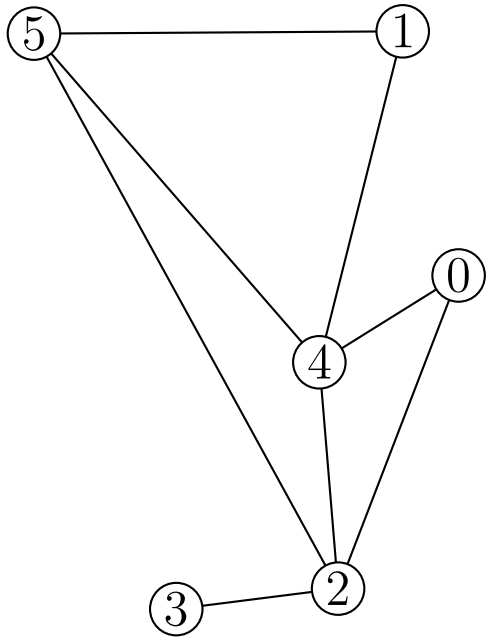

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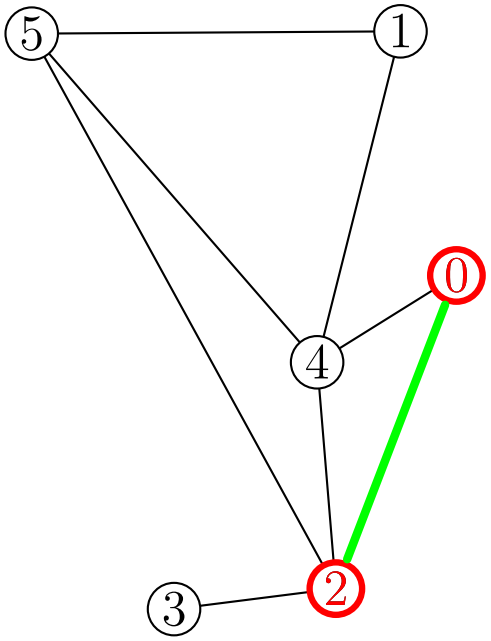

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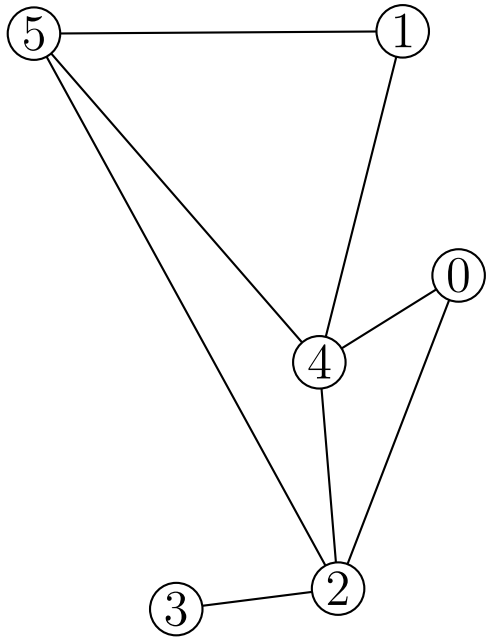

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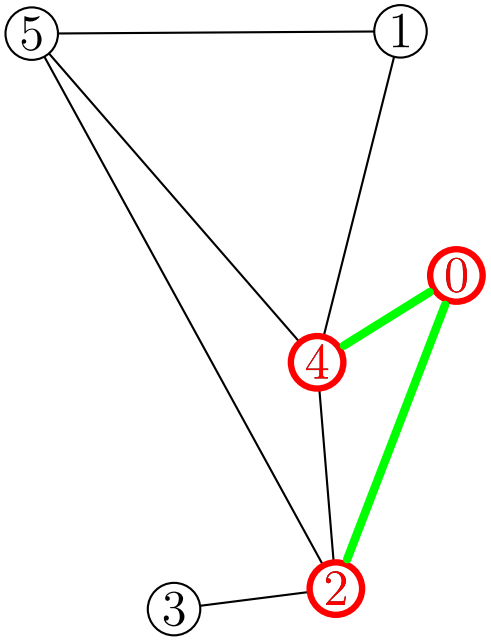

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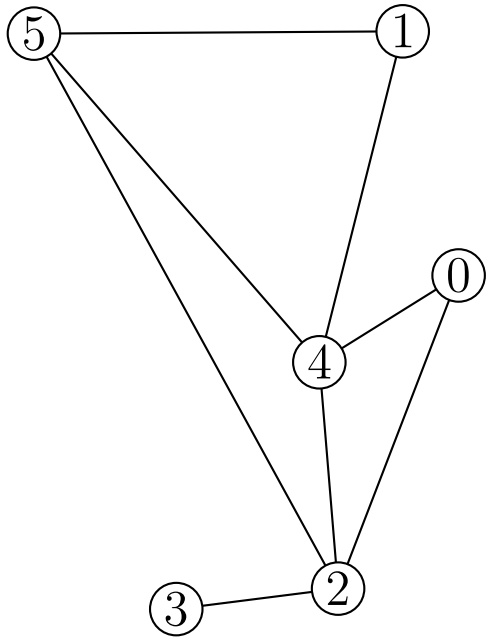

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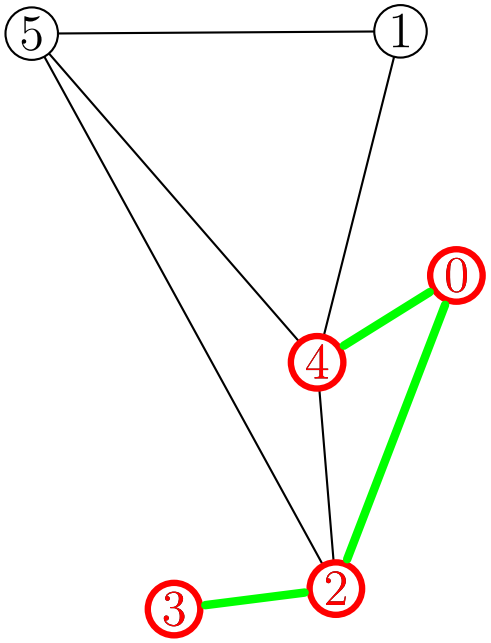

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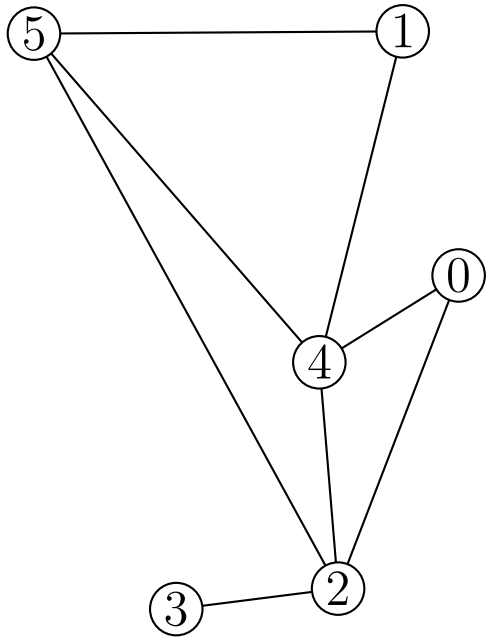

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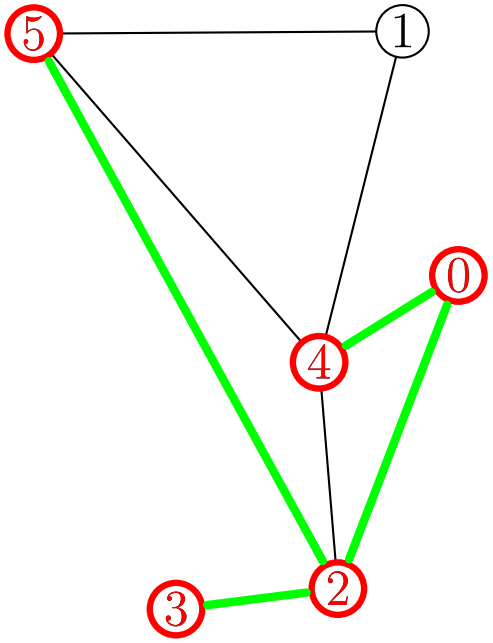

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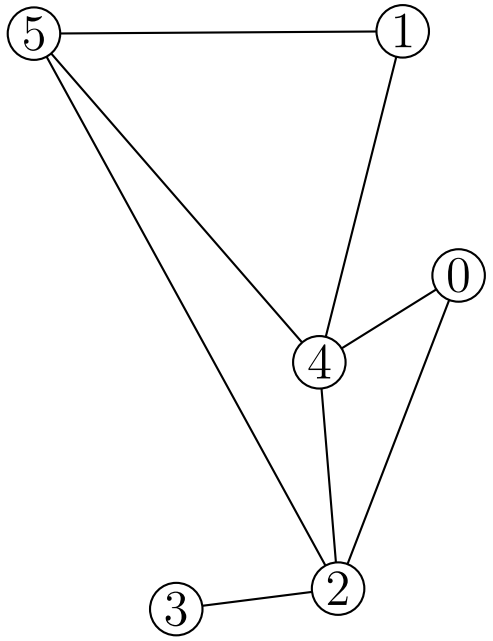

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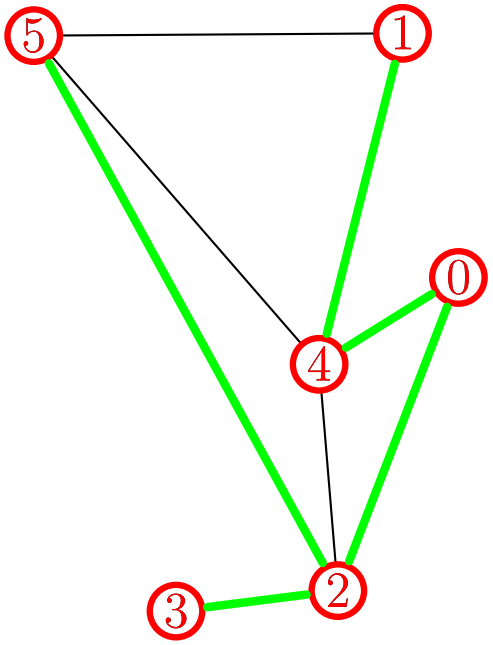

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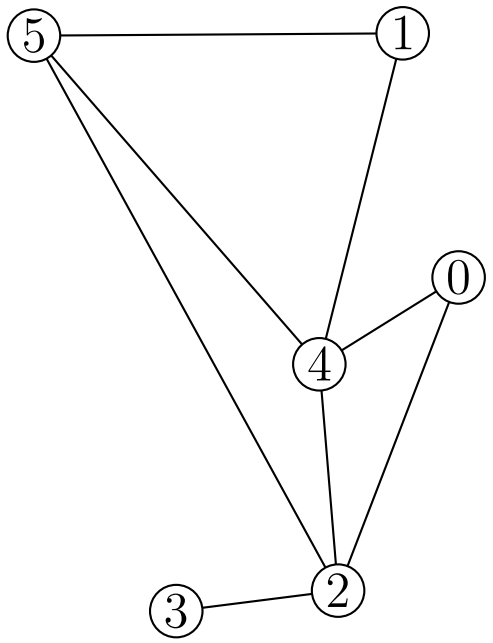

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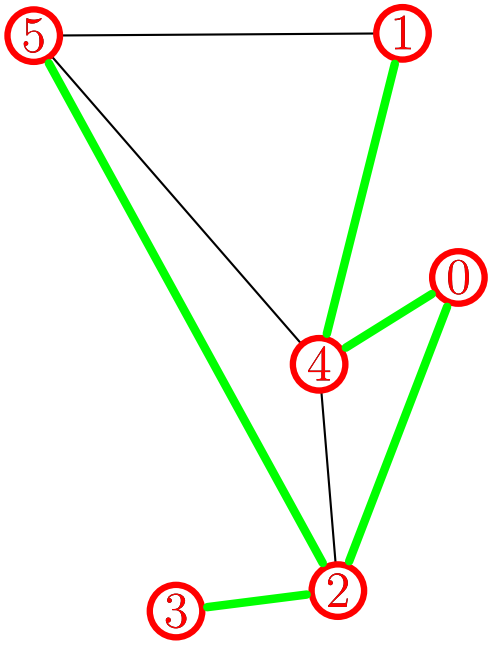

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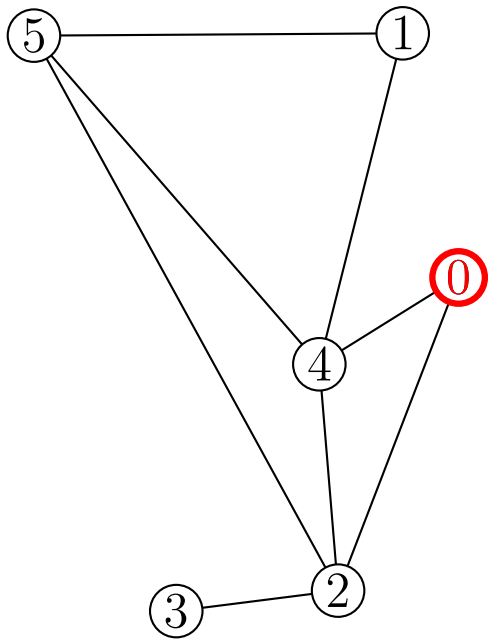

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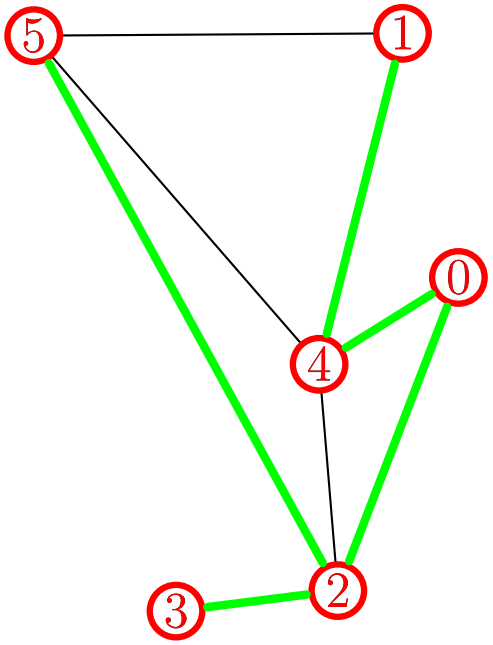

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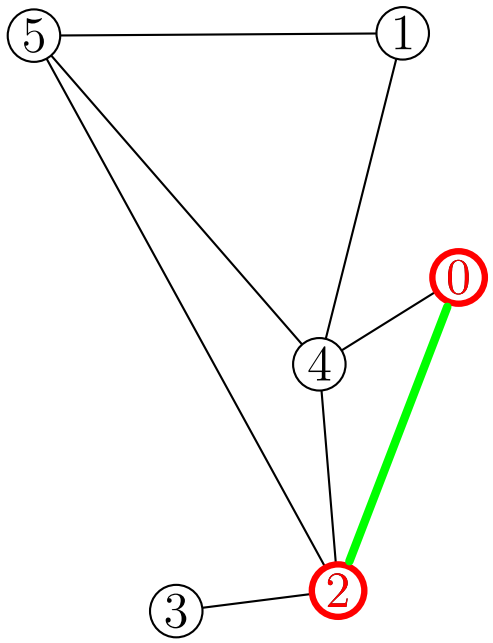

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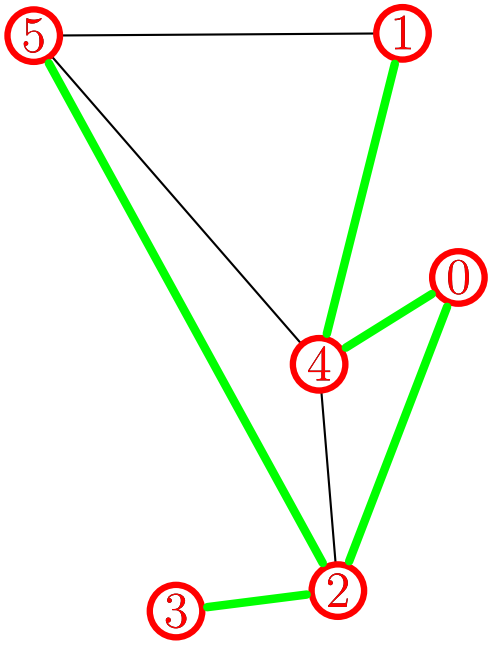

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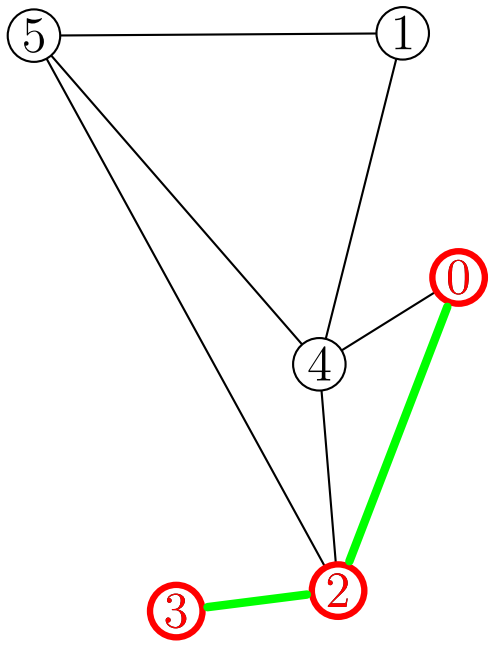

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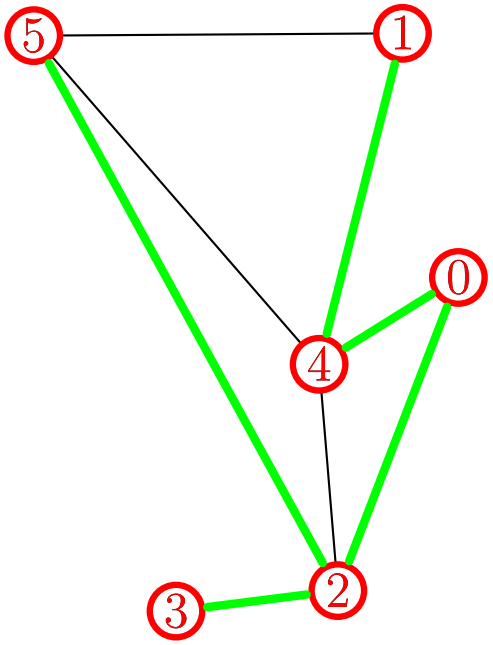

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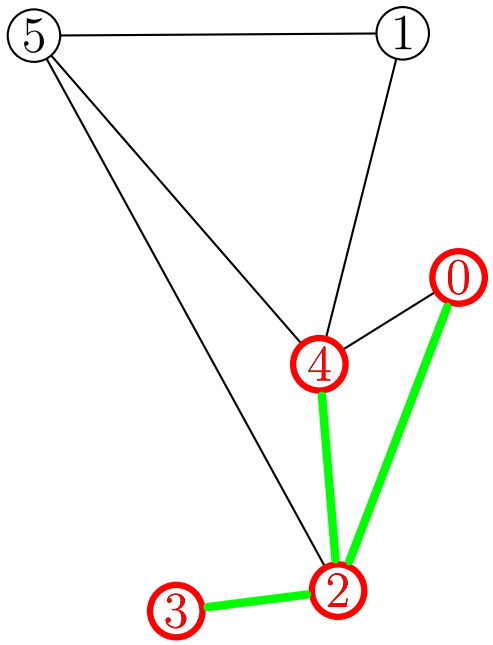

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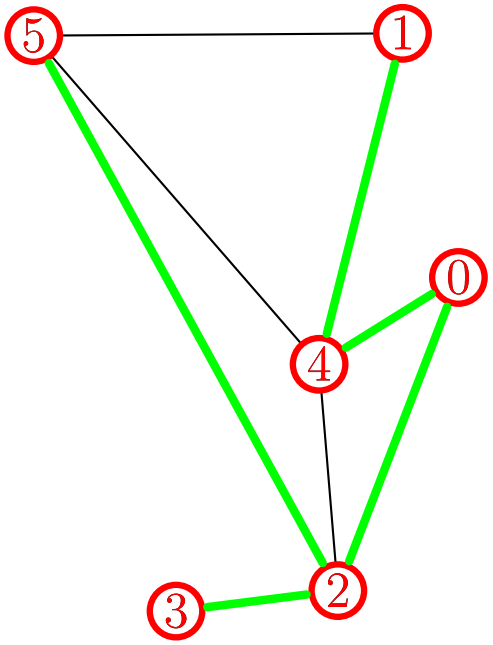

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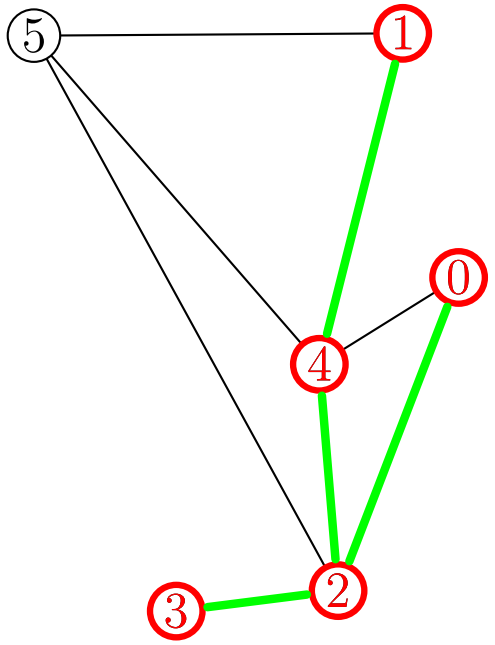

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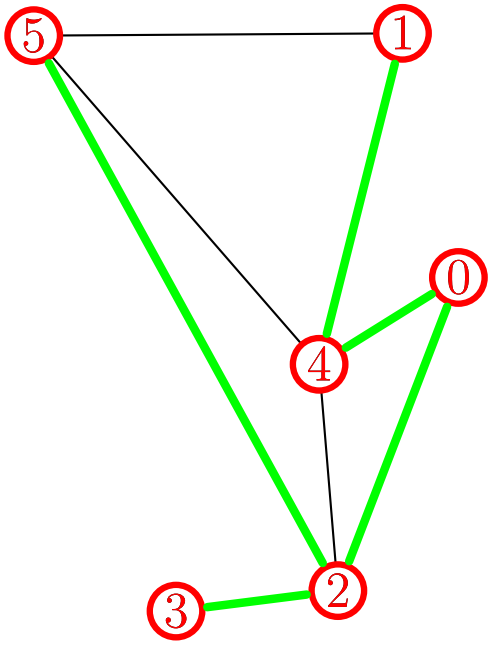

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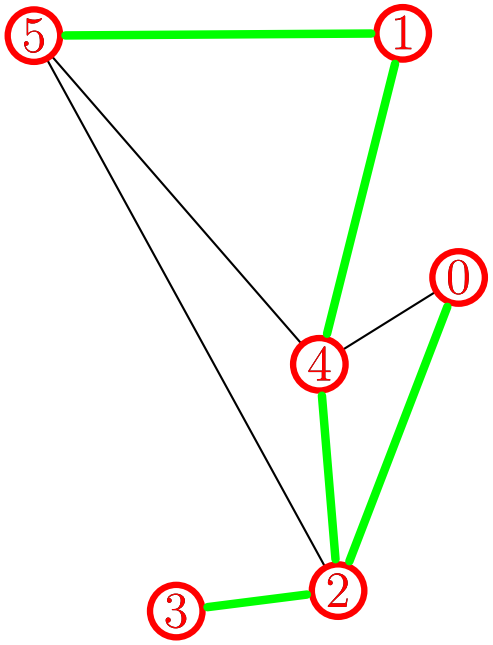

dfs

Breadth First Search

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  List parent(graph.noNodes)
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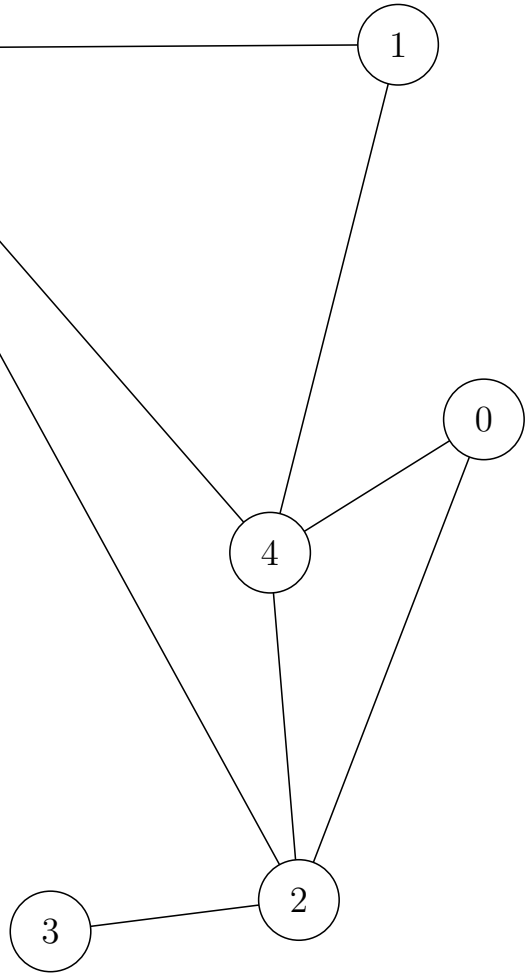

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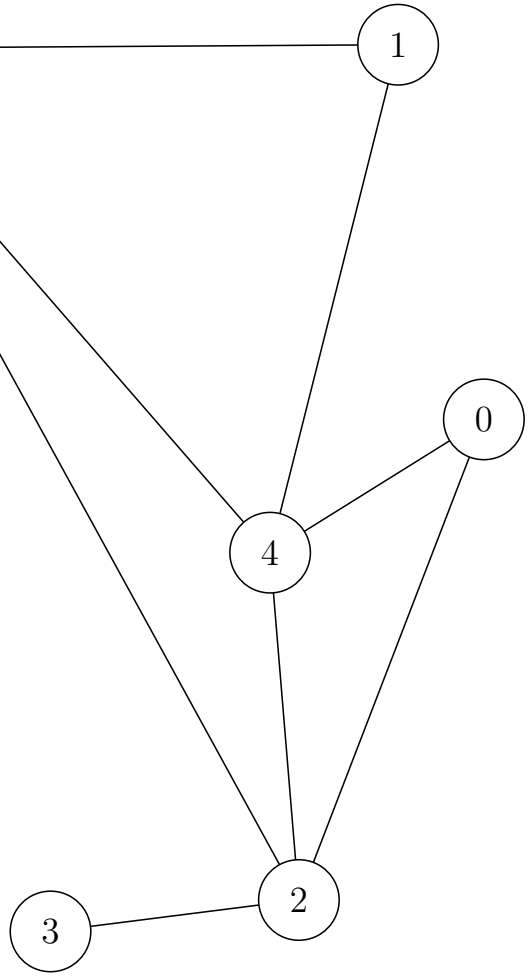

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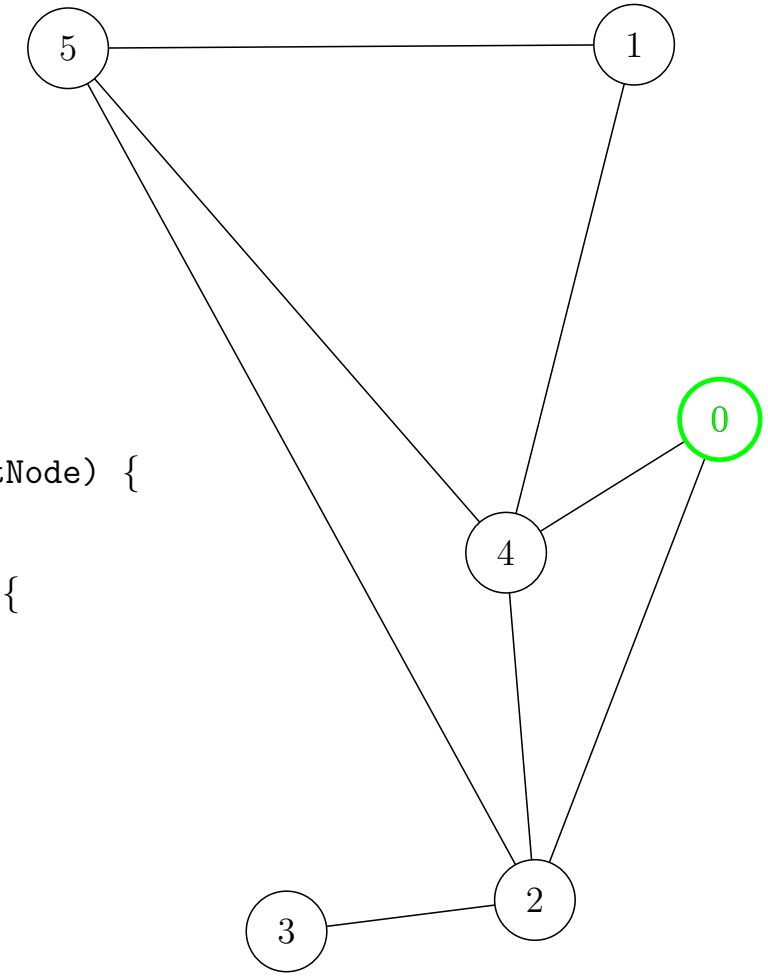

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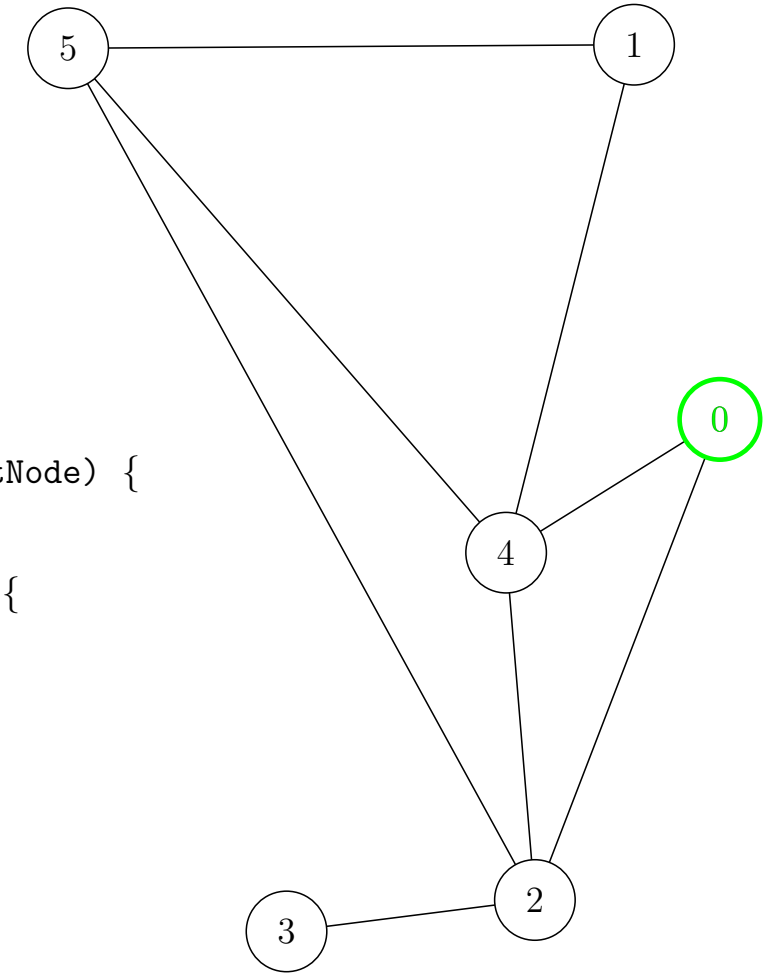
q= 



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    processVertexLlate(currentNode)
  }
}
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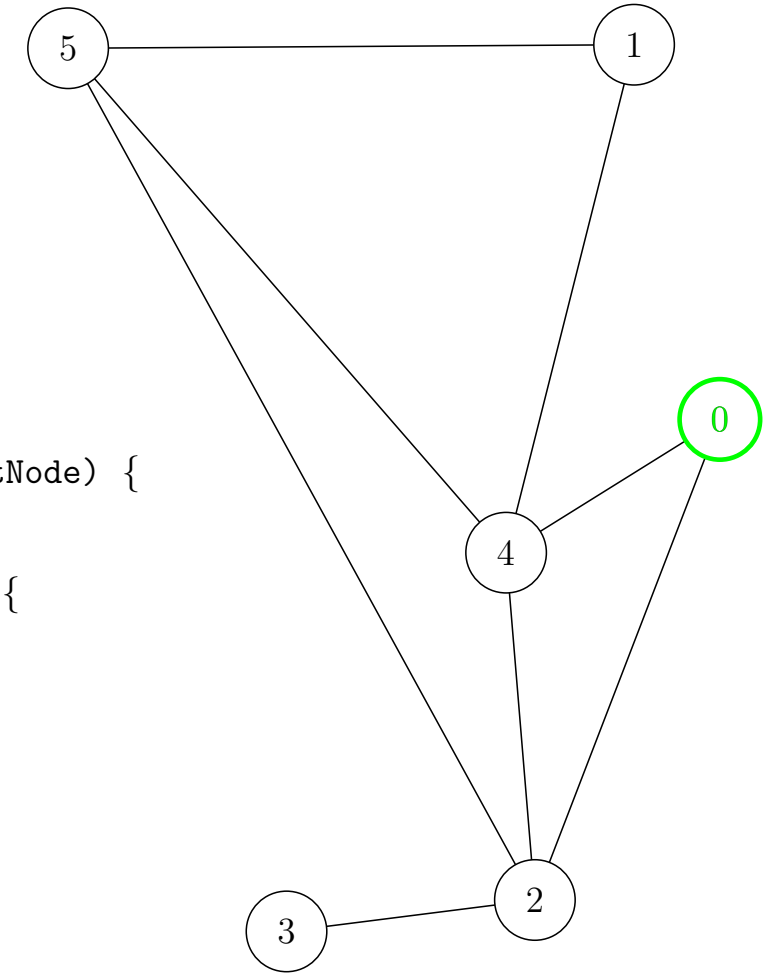
q=

0					



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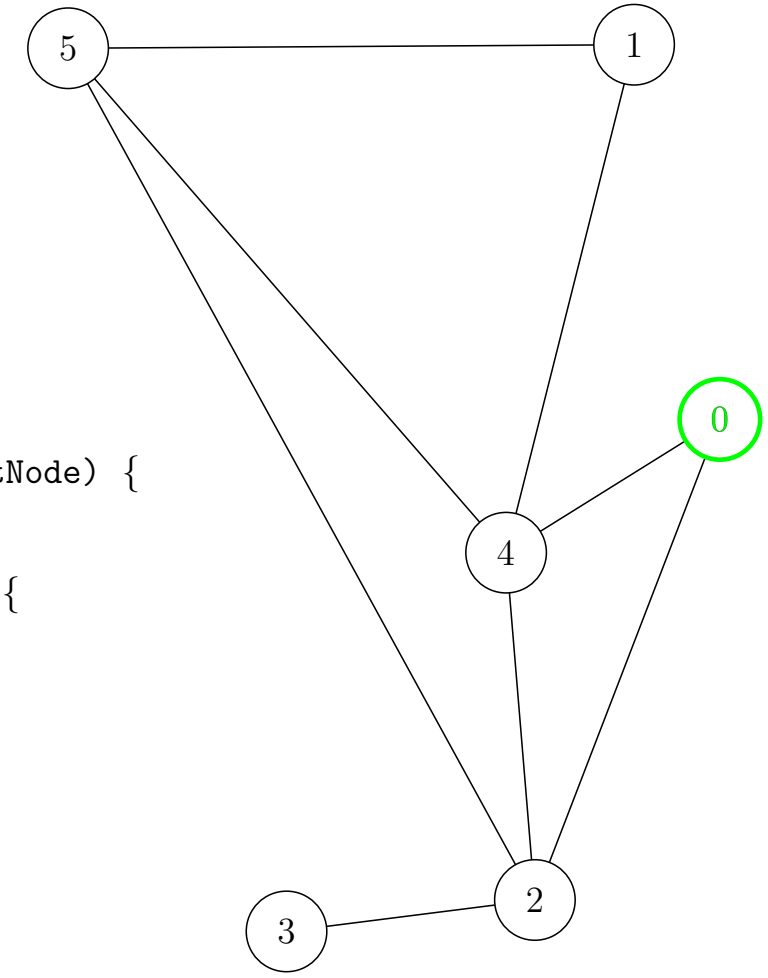

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    }
    processVertexLate(currentNode)
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}
```

currentNode=0

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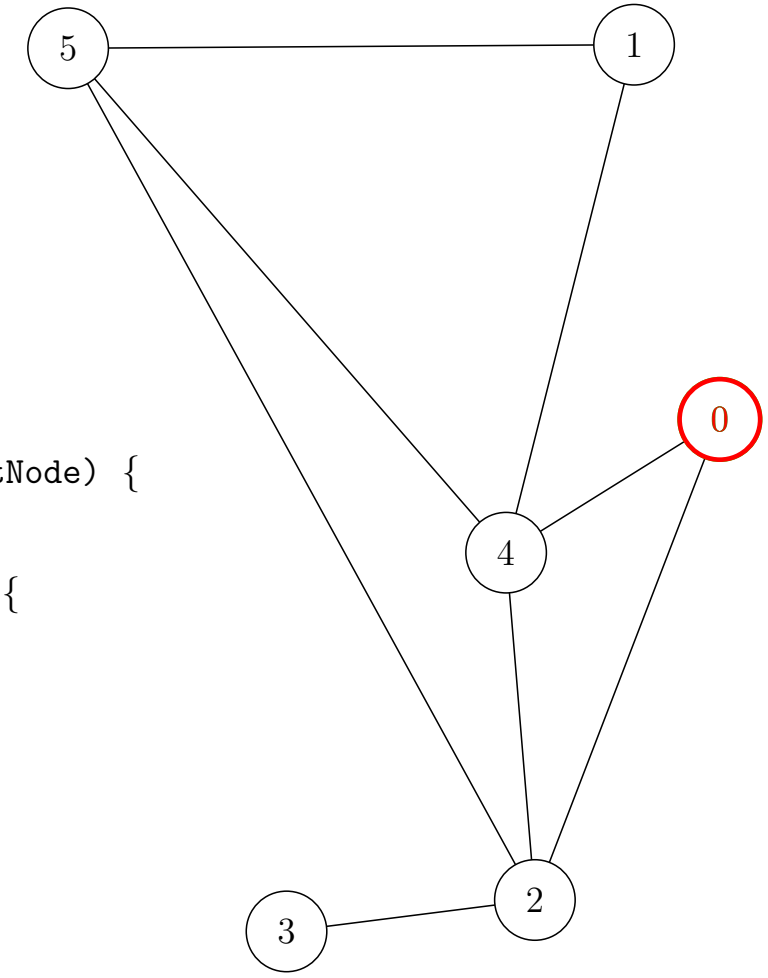

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        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```
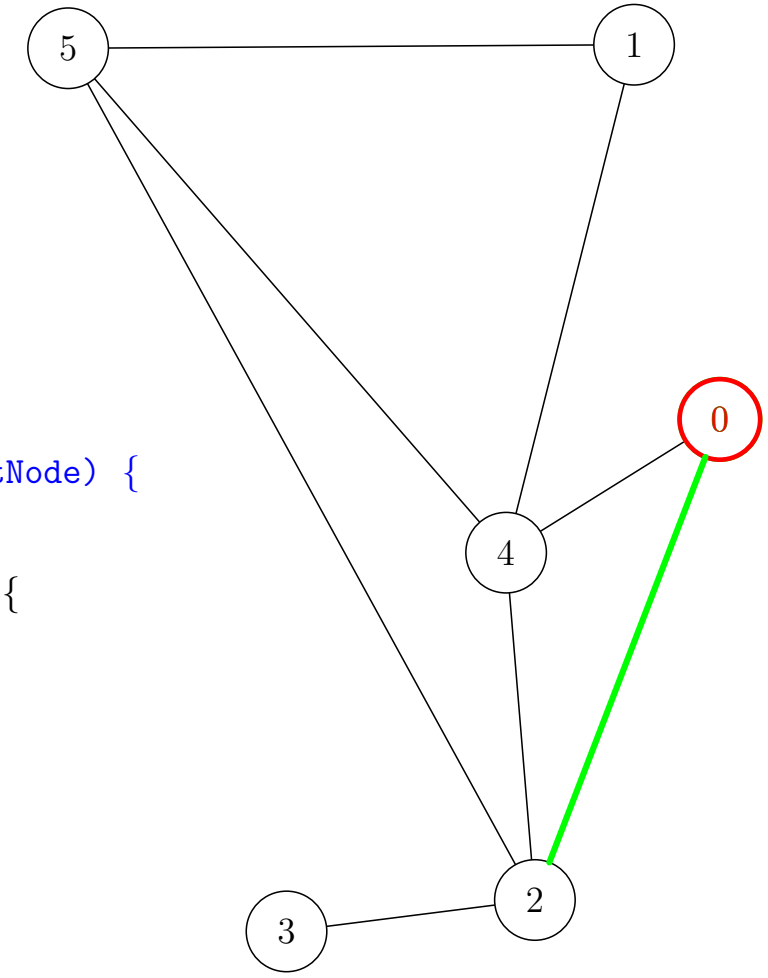

currentNode=0 neighbour=2

q= 

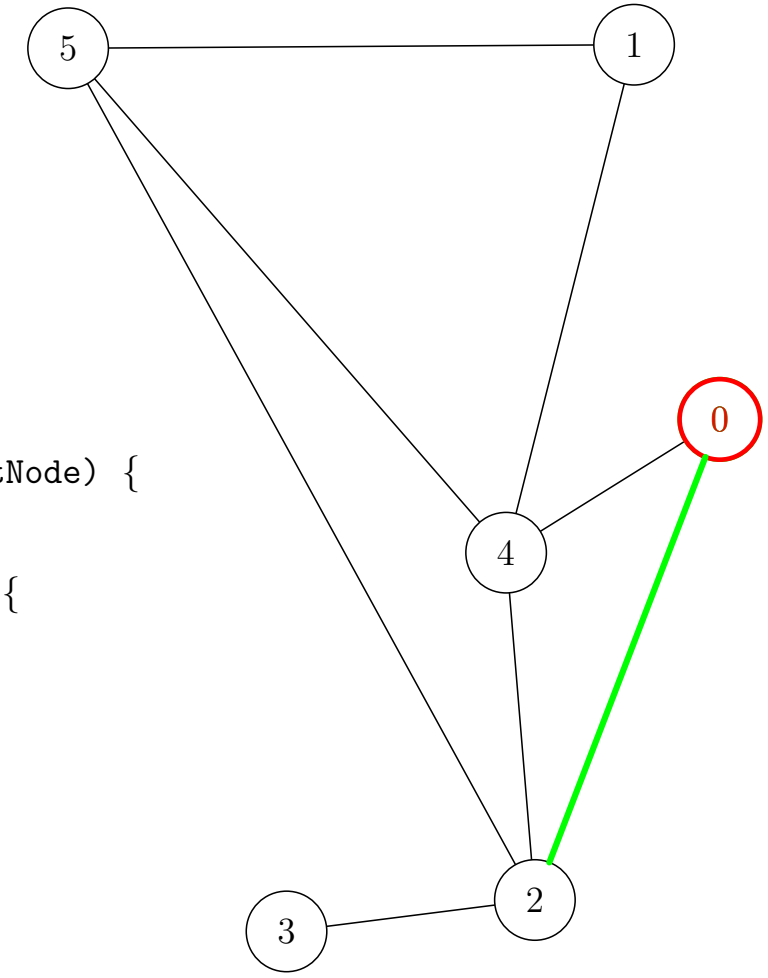


Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=0 neighbour=2

q= 



Breadth First Search

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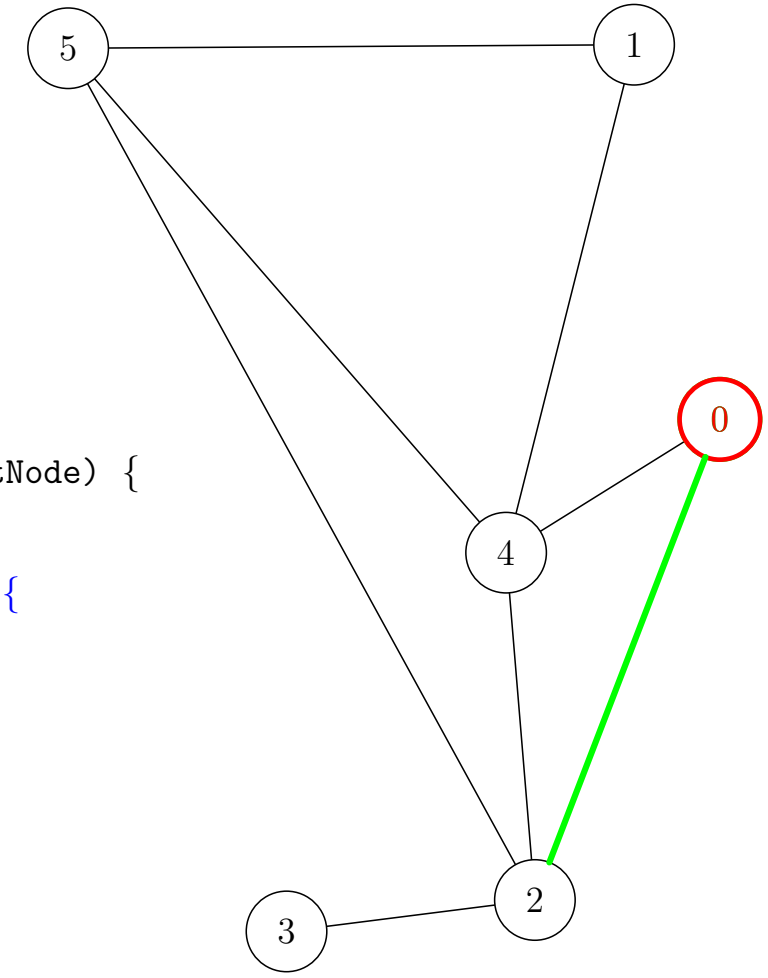
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=0 neighbour=2

q= 



Breadth First Search

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      }
    }
    processVertexLate(currentNode)
  }
}
```
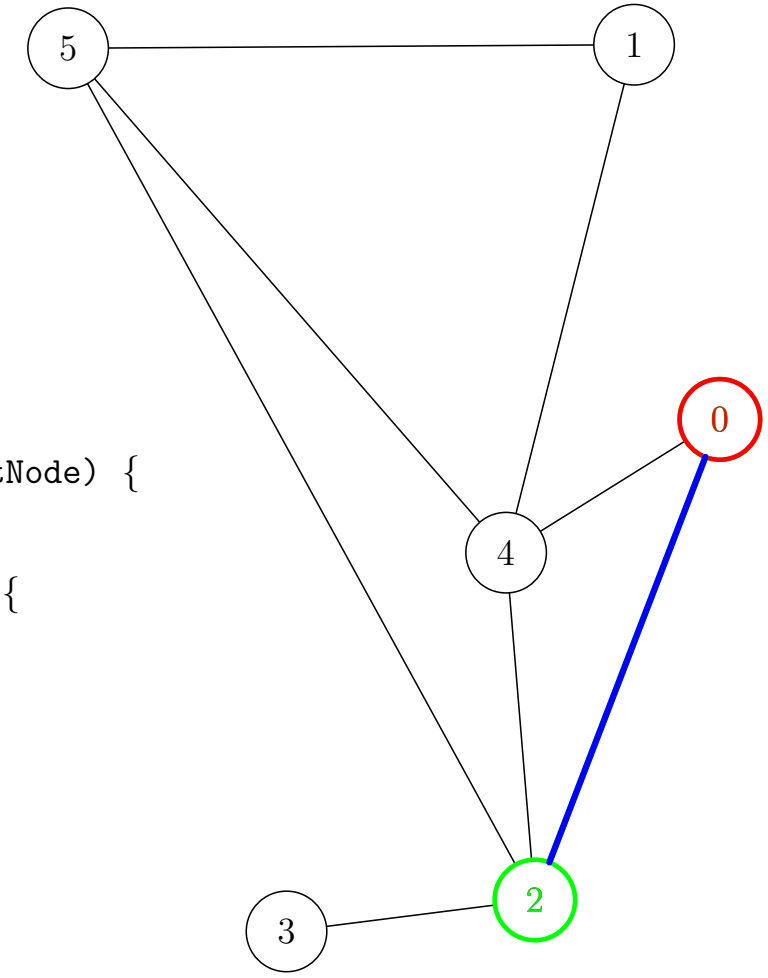

currentNode=0 neighbour=2

q=

2					



Breadth First Search

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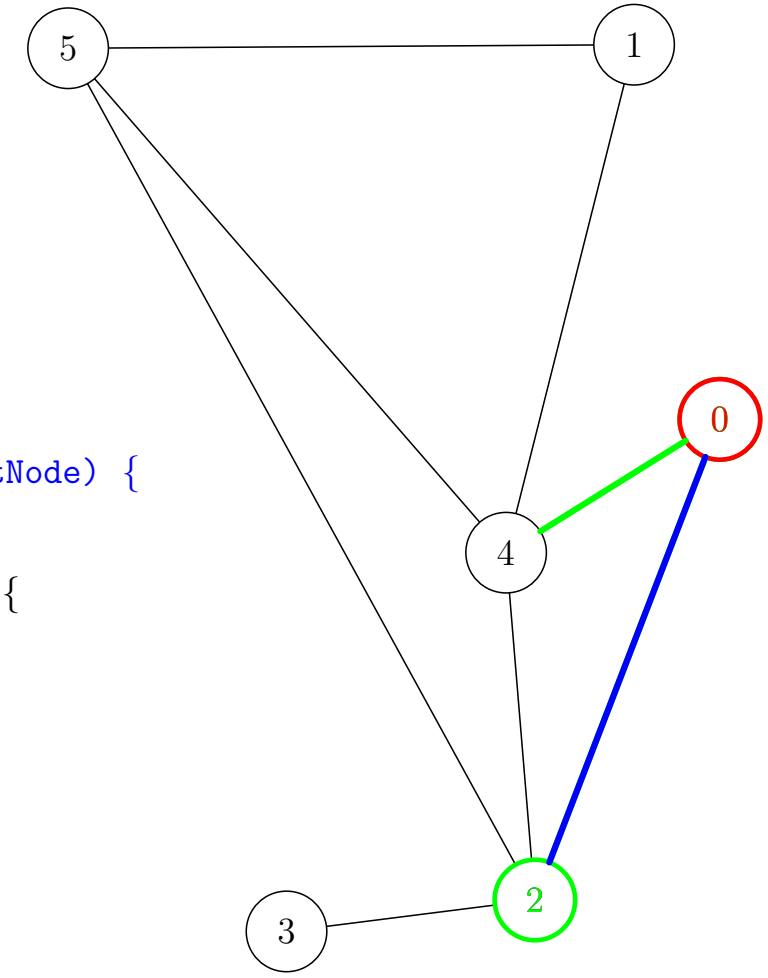
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=0 neighbour=4

q=

2					



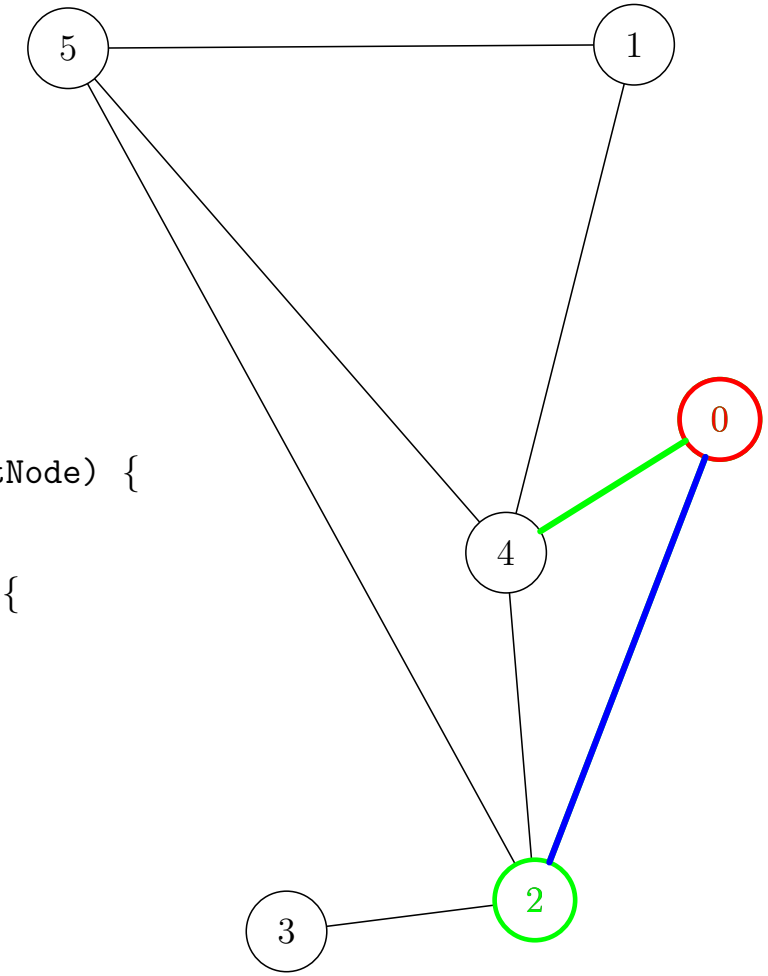
Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=0 neighbour=4

q=

2					



Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
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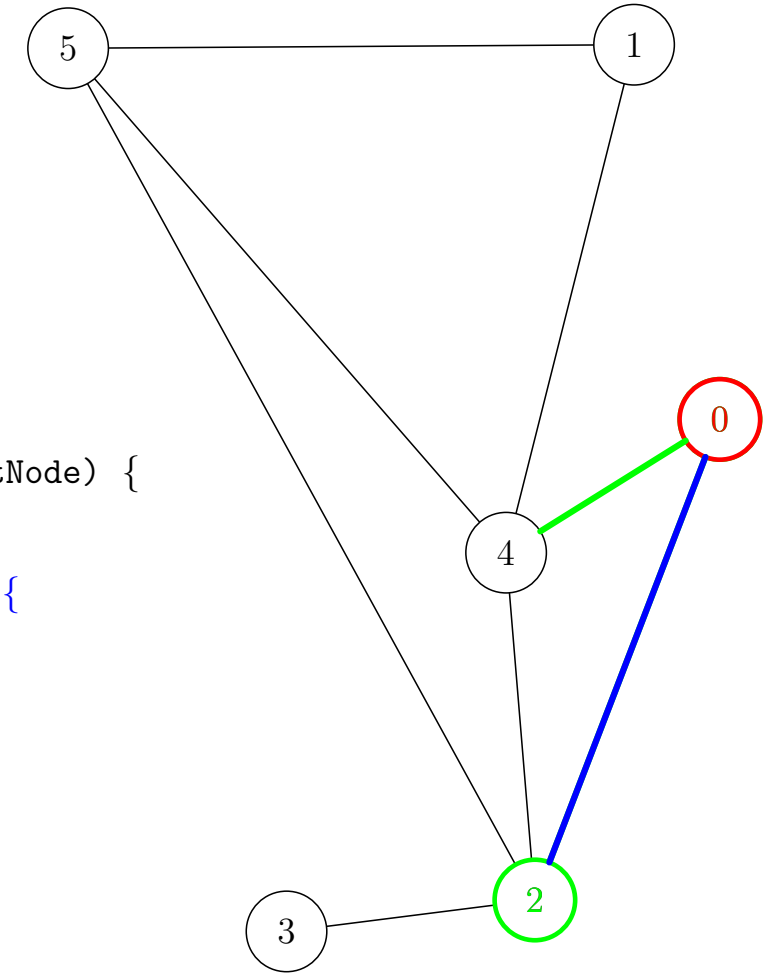
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=0 neighbour=4

q=

2					



Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
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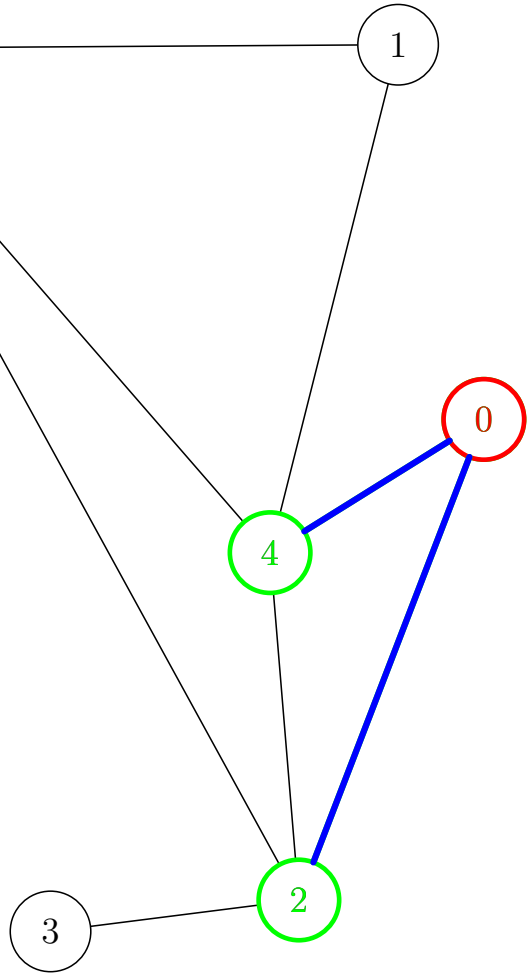
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=0 neighbour=4

q=

2	4				



Breadth First Search

```
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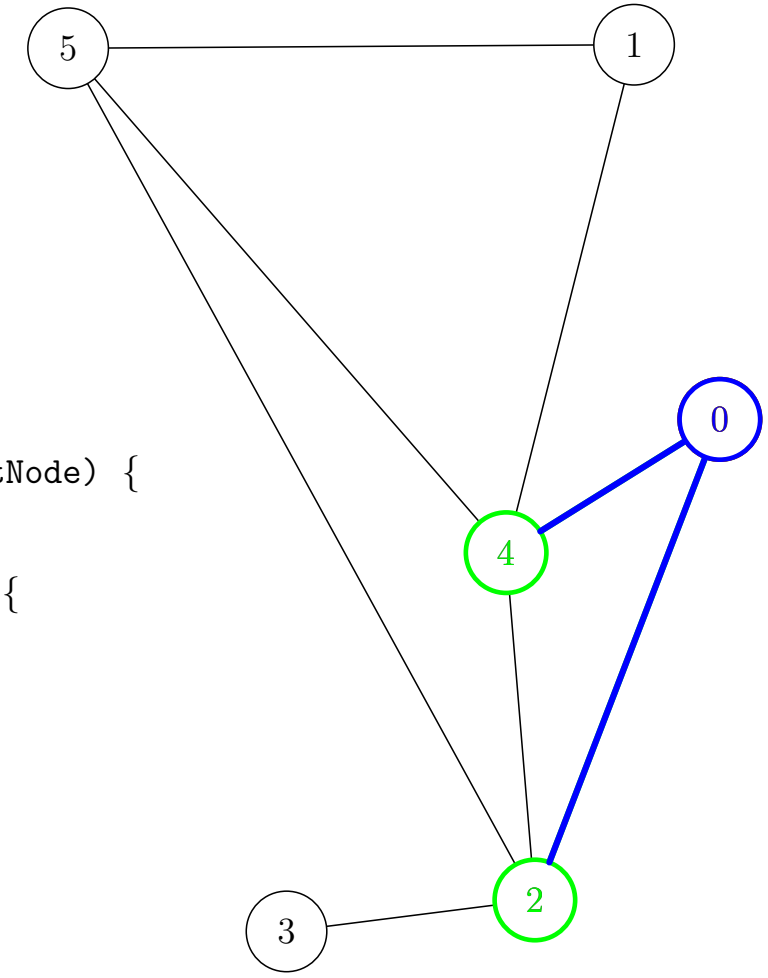
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=0

q=

2	4				



Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
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  q.enqueue(node)
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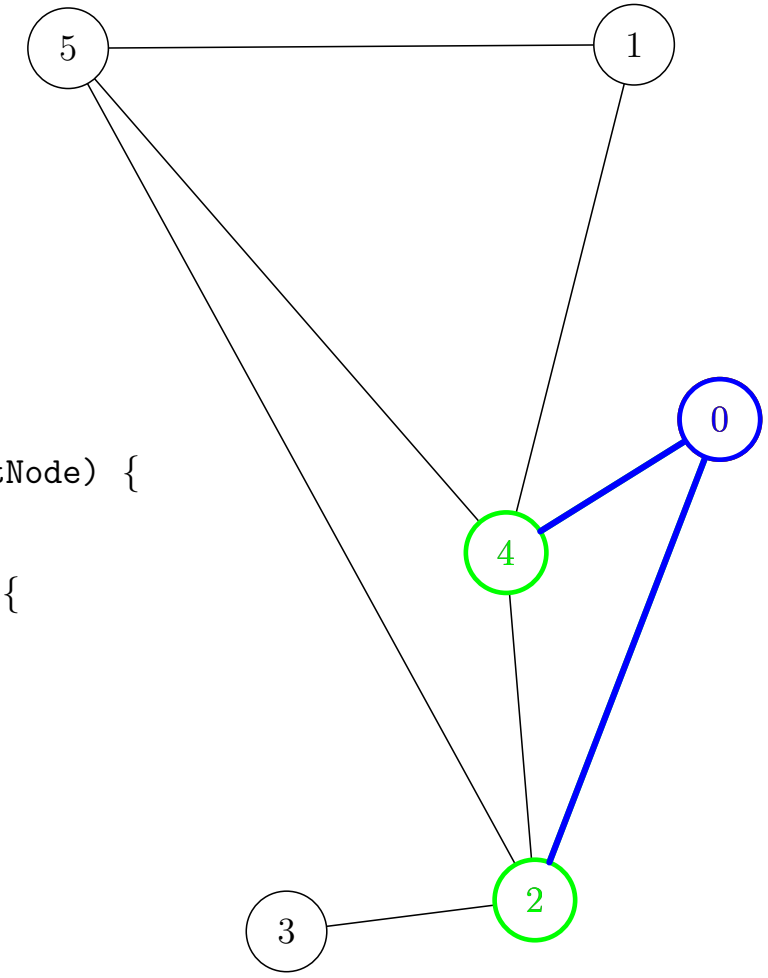
      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLlate(currentNode)
  }
}
```

q=

2	4				



Breadth First Search

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      if (state[neighbour] ≠ "processed")
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```
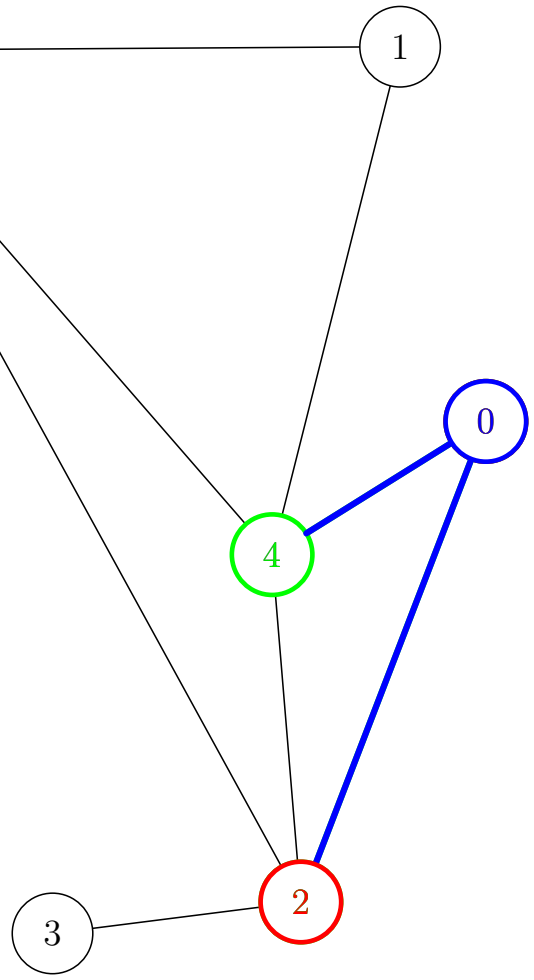

currentNode=2

q=

4					



Breadth First Search

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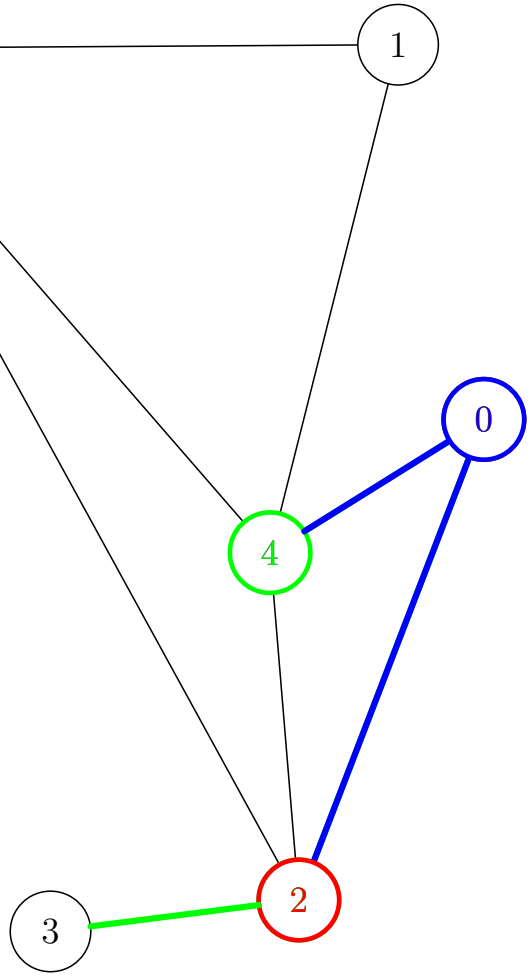
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=2 neighbour=3

q= | 4 | | | | | |



Breadth First Search

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        q.enqueue(neighbour)
      }
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    processVertexLate(currentNode)
  }
}
```
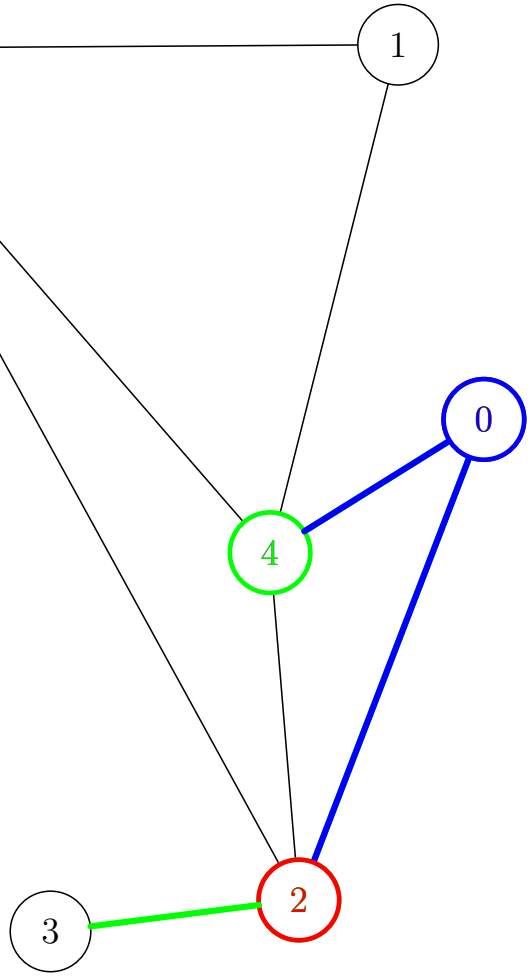

currentNode=2 neighbour=3

q= | 4 | | | | | |



Breadth First Search

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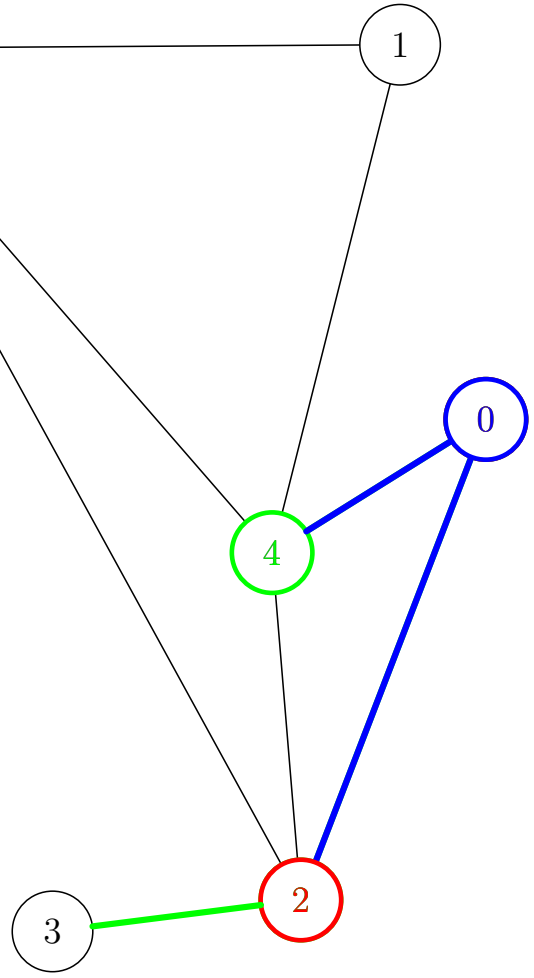
  }
}
```

currentNode=2 neighbour=3

q=

4					



Breadth First Search

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```
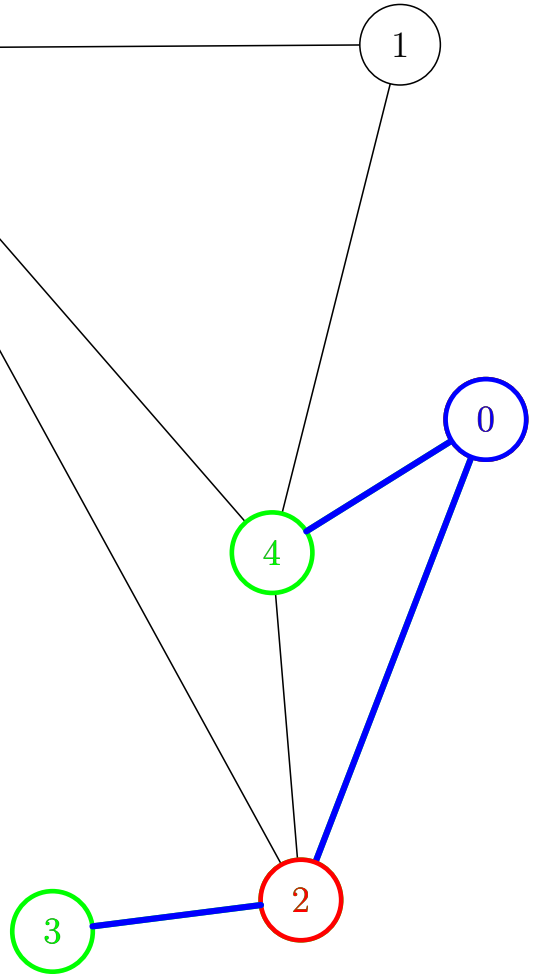

currentNode=2 neighbour=3

q=

4	3				



Breadth First Search

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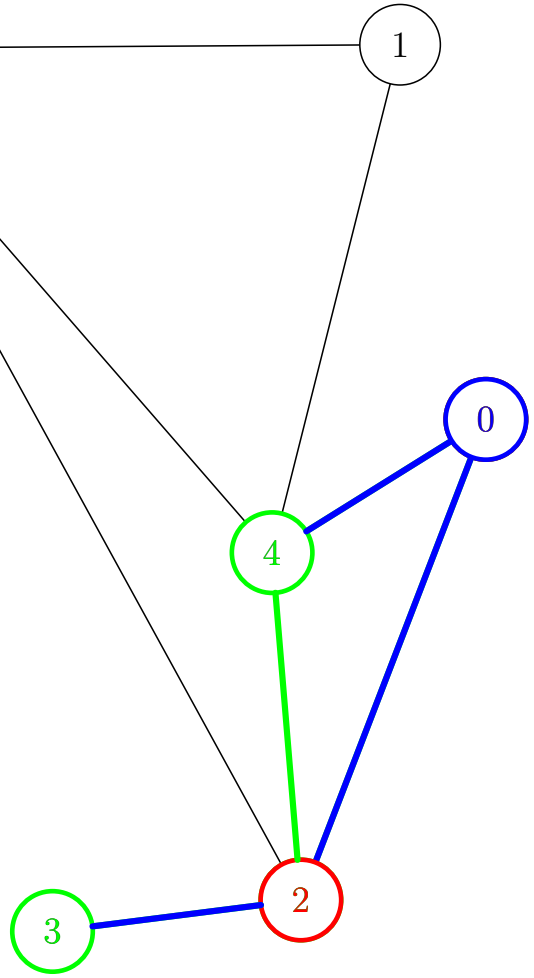
```

currentNode=2 neighbour=4

q=

4	3				



Breadth First Search

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  }
}
```
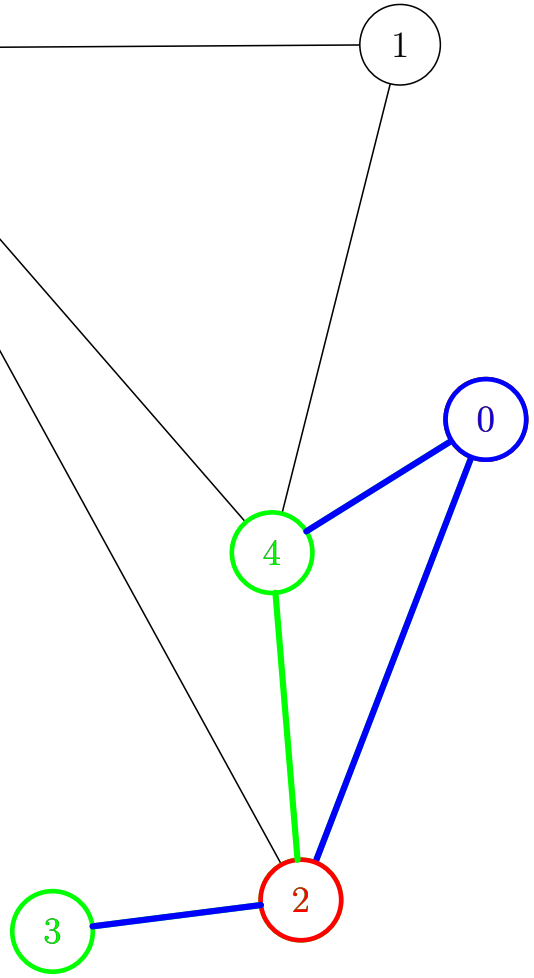

currentNode=2 neighbour=4

q=

4	3				



Breadth First Search

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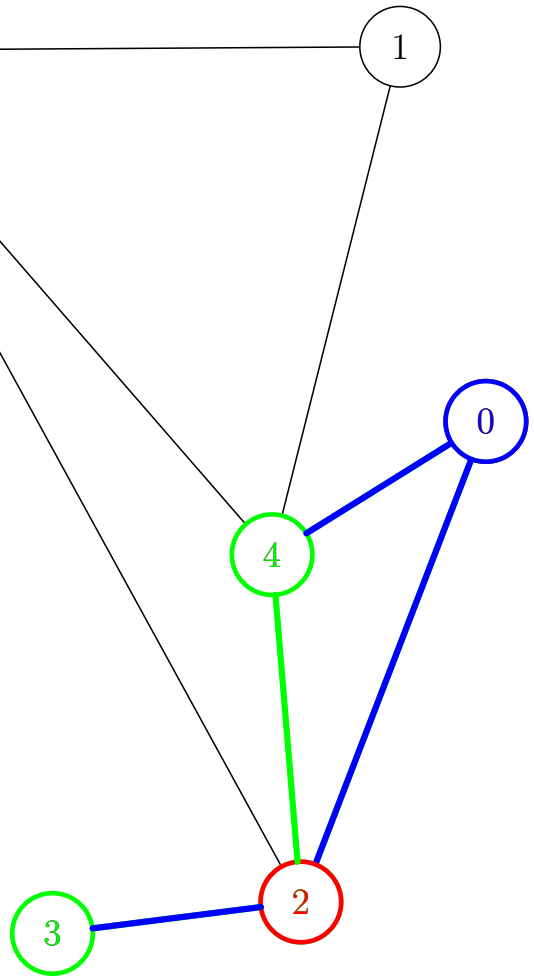
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=2 neighbour=4

q=

4	3				



Breadth First Search

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      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```
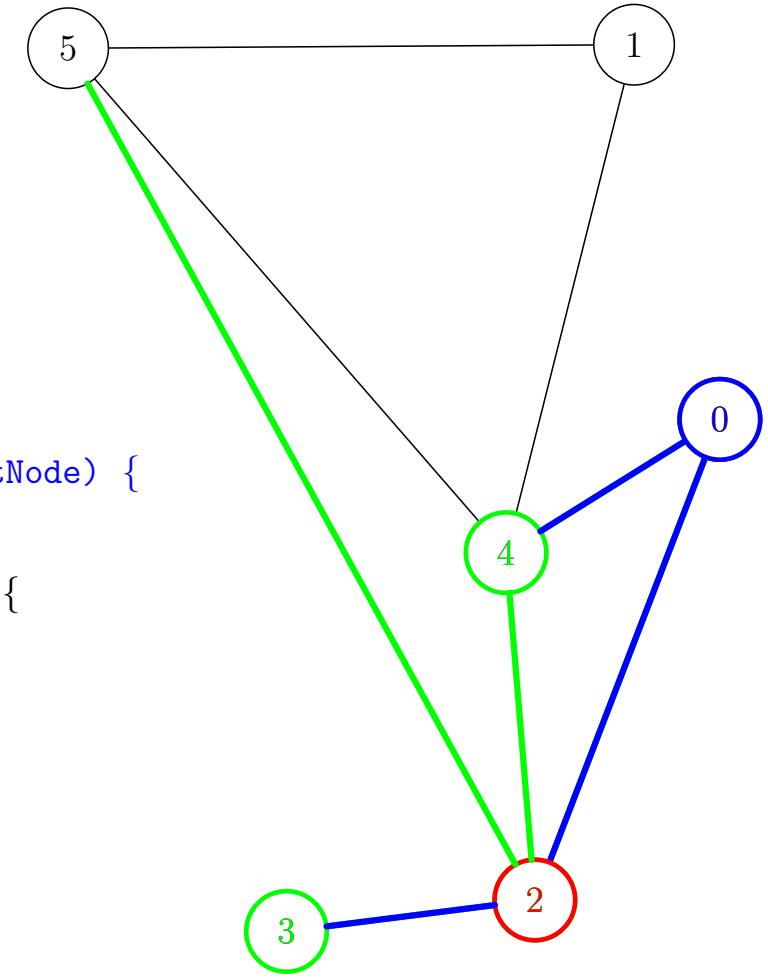

currentNode=2 neighbour=5

q=

4	3				



Breadth First Search

```
bfs(graph, node) {
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  List parent(graph.noNodes)
  state[node] ← "discovered"
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        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```
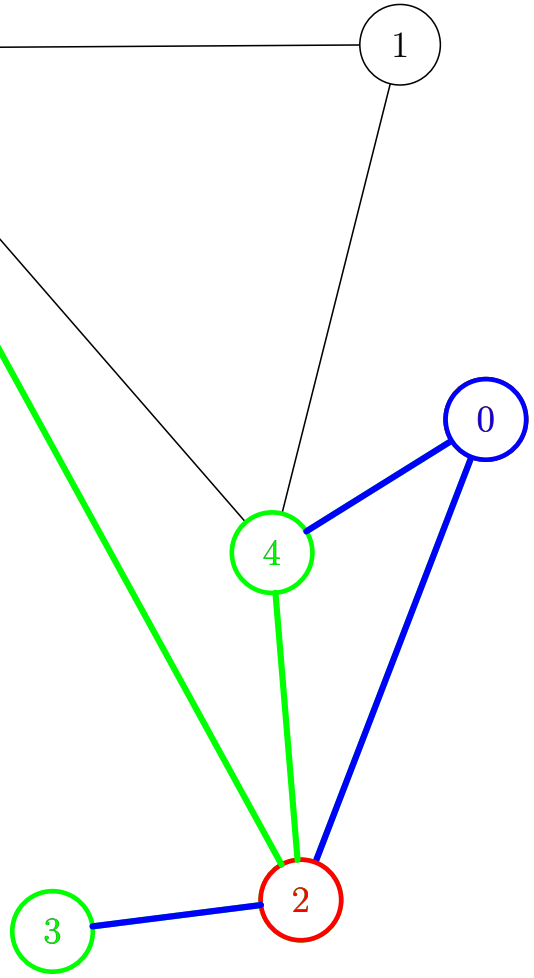

currentNode=2 neighbour=5

q=

4	3				



Breadth First Search

```
bfs(graph, node) {
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        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```
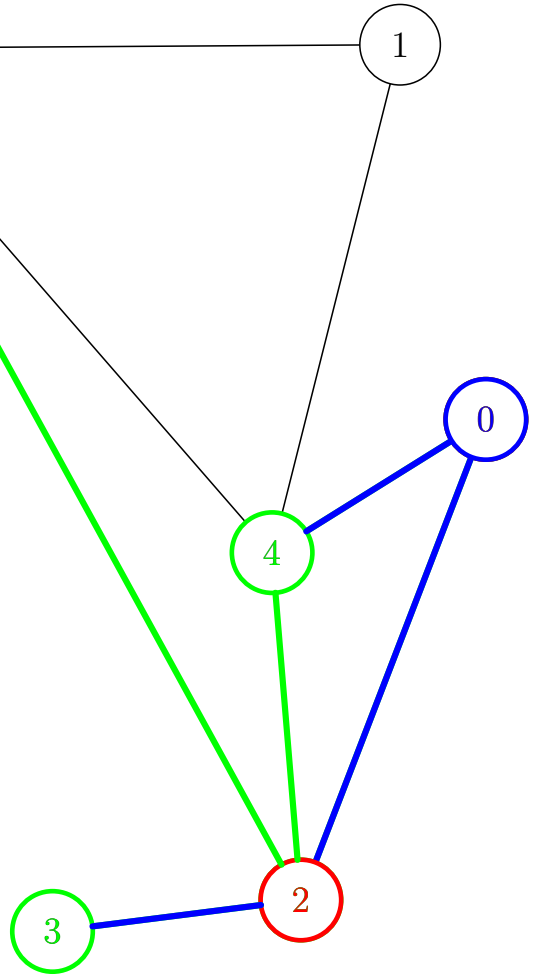

currentNode=2 neighbour=5

q=

4	3				



Breadth First Search

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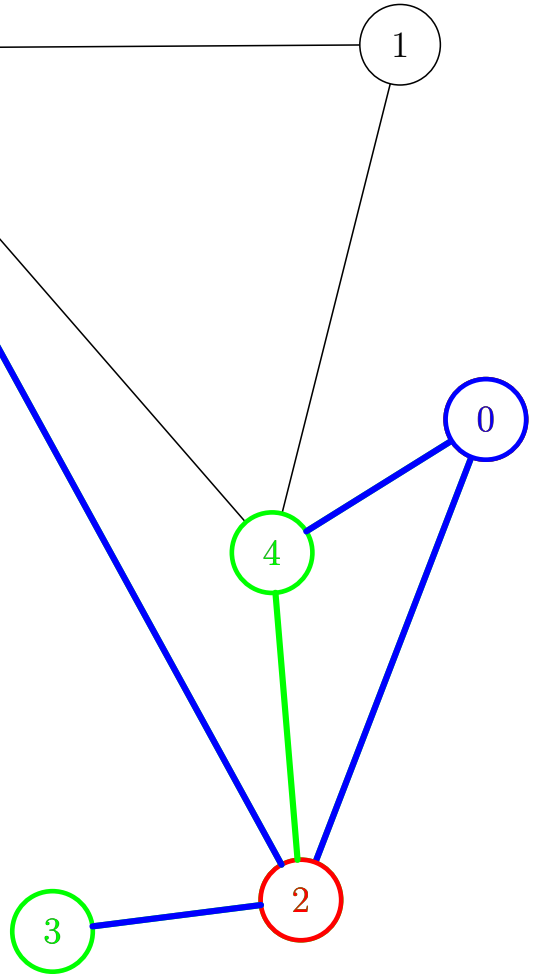
        processEdge(currentNode, neighbour)
      if (state[neighbour] = "undiscovered") {
        state[neighbour] ← "discovered"
        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=2 neighbour=5

q=

4	3	5			



Breadth First Search

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        parent[neighbour] ← currentNode
        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```
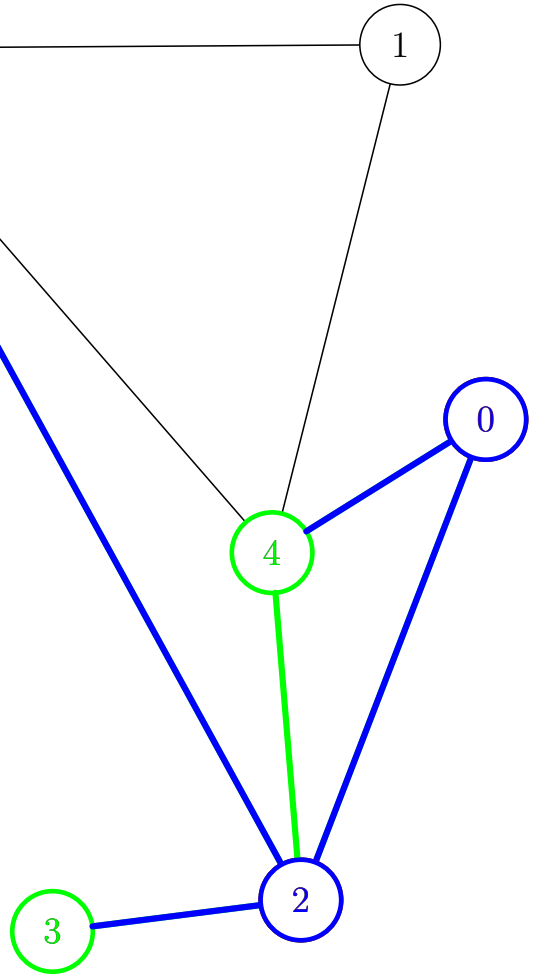

currentNode=2

q=

4	3	5			



Breadth First Search

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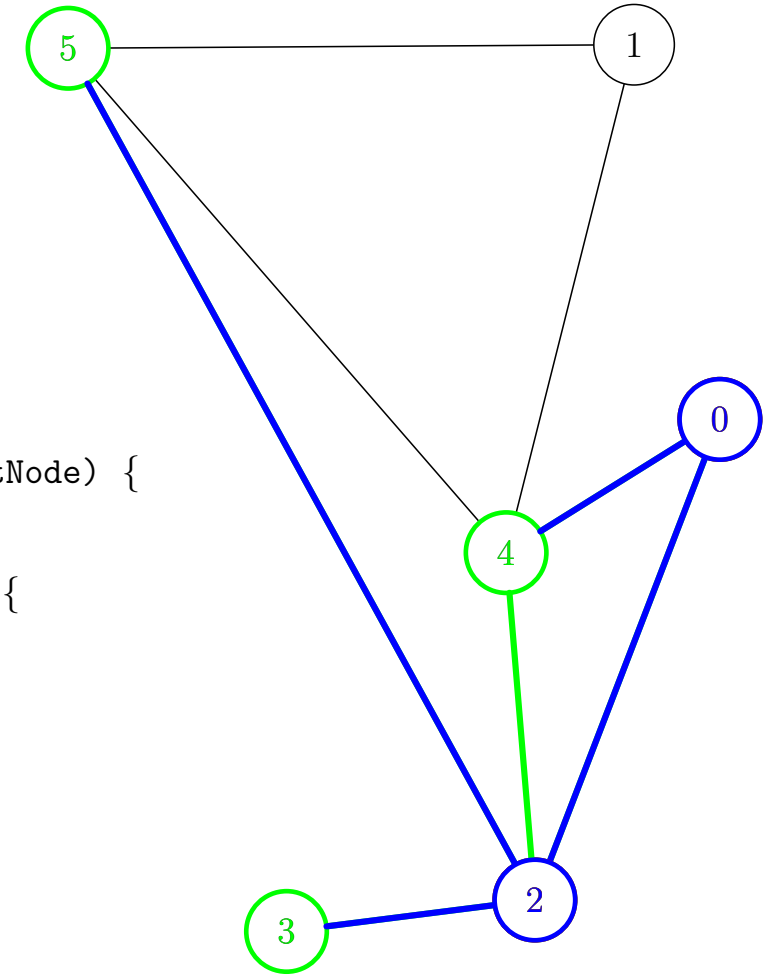
        q.enqueue(neighbour)
      }
    }
    processVertexLlate(currentNode)
  }
}
```

q=

4	3	5			



Breadth First Search

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        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```
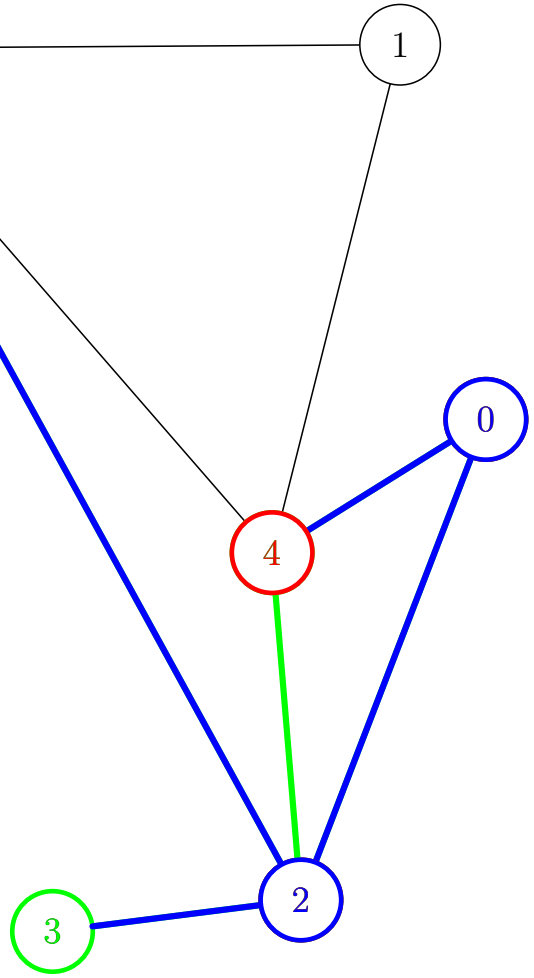

currentNode=4

q=

3	5				



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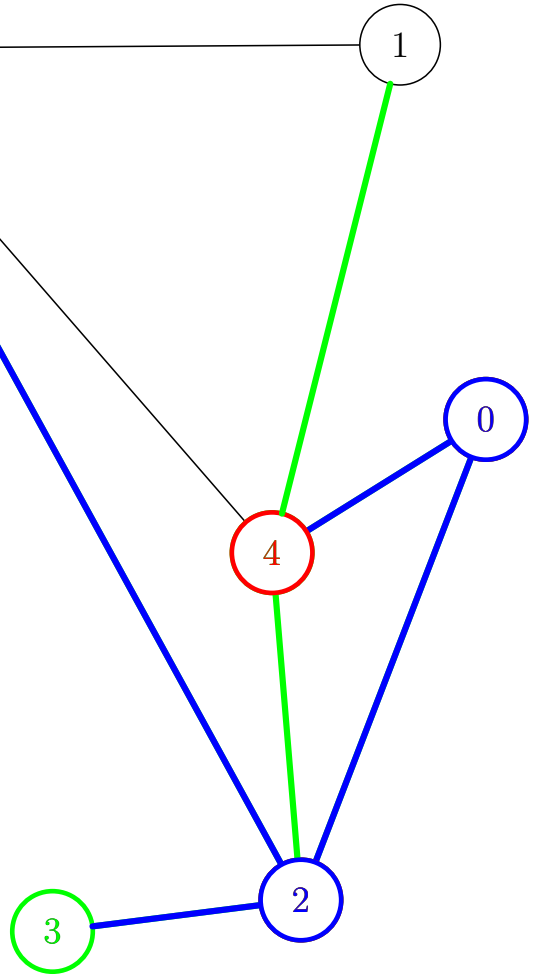
      }
    }
    processVertexLate(currentNode)
  }
}
```

currentNode=4 neighbour=1

q=

3	5				



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```
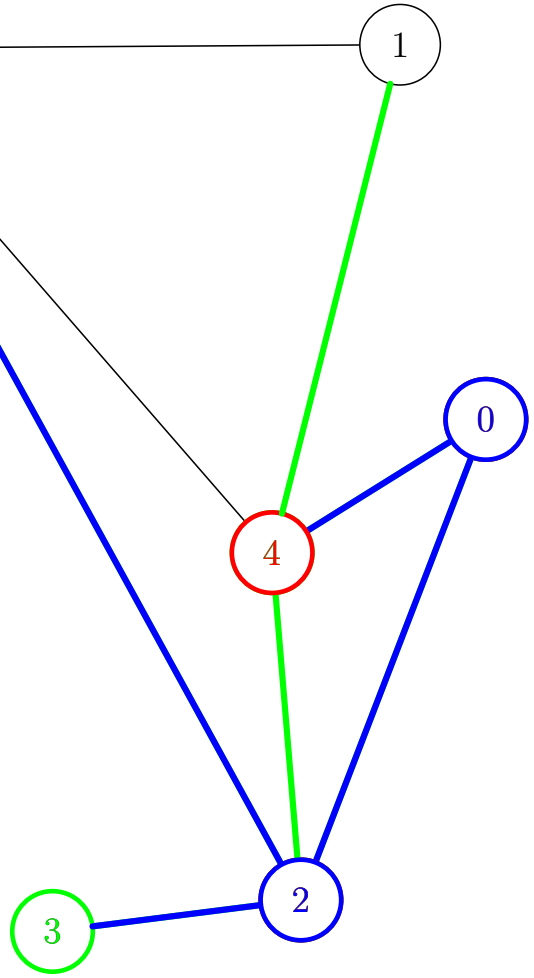

currentNode=4 neighbour=1

q=

3	5				



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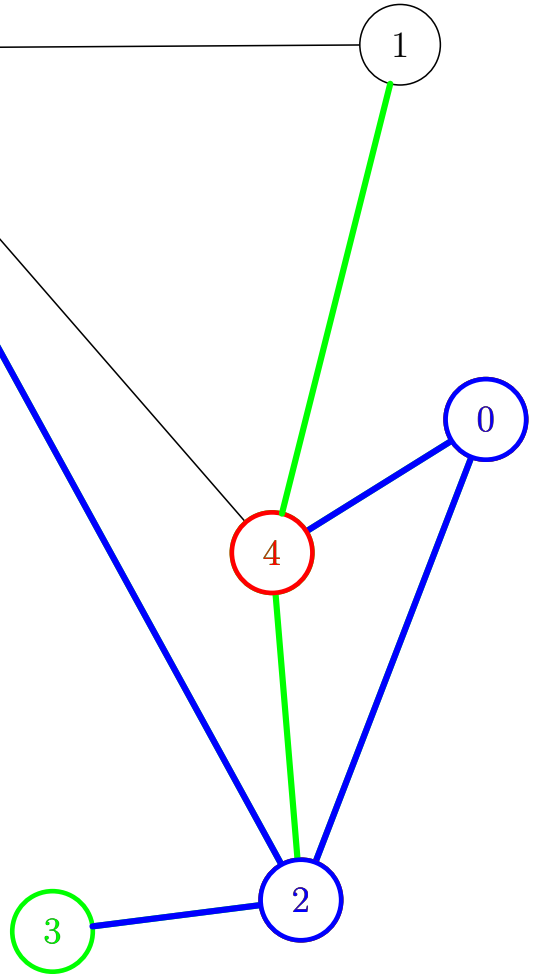
currentNode=4 neighbour=1

q=

3	5				



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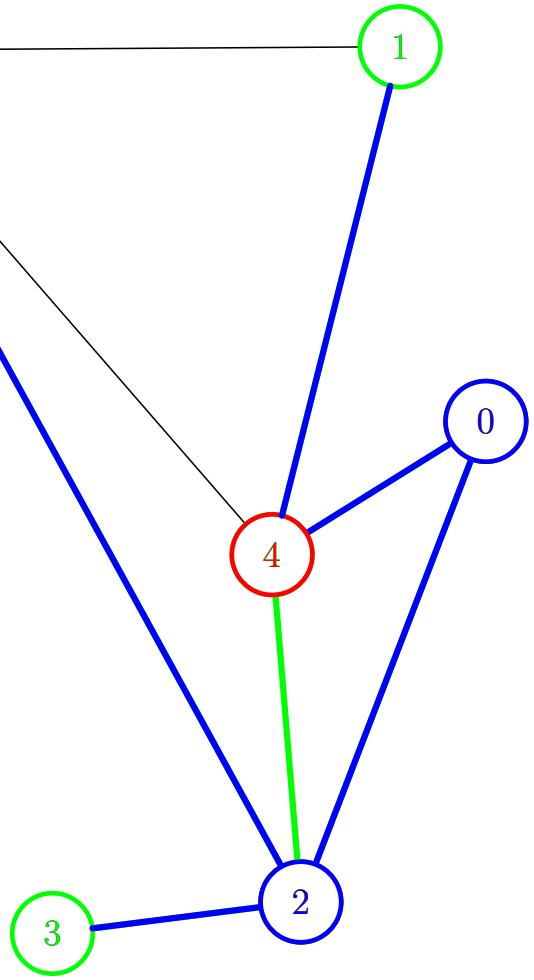
currentNode=4 neighbour=1

q=

3	5	1			



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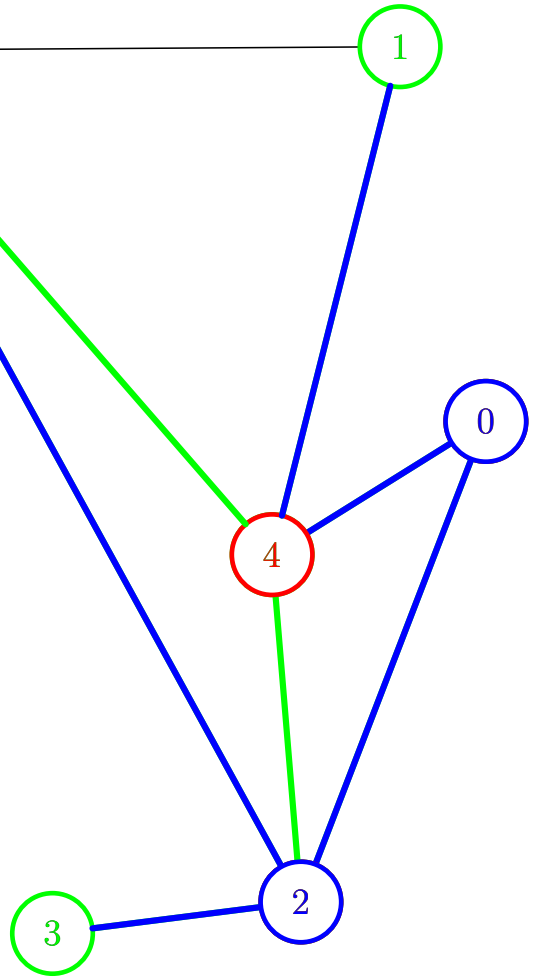
currentNode=4 neighbour=5

q=

3	5	1			



Breadth First Search

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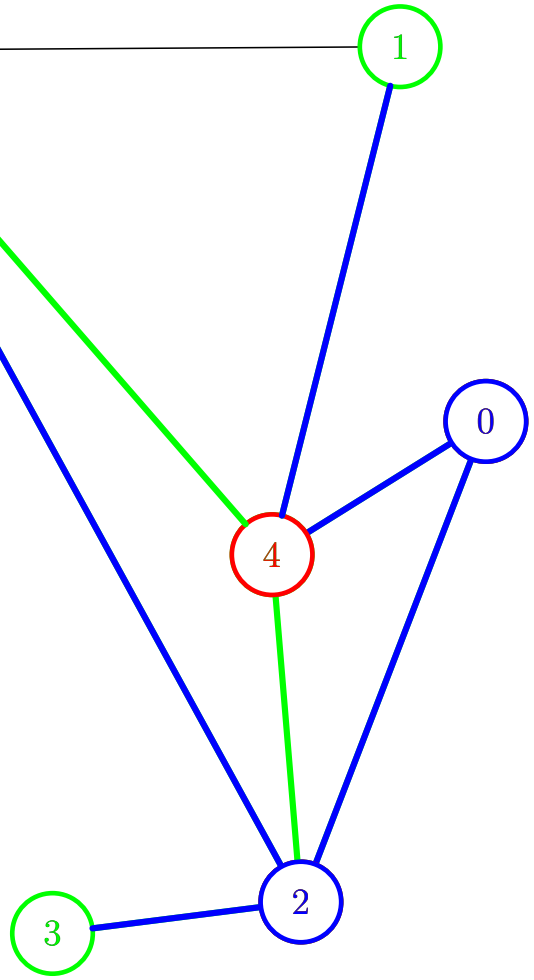
currentNode=4 neighbour=5

q=

3	5	1			



Breadth First Search

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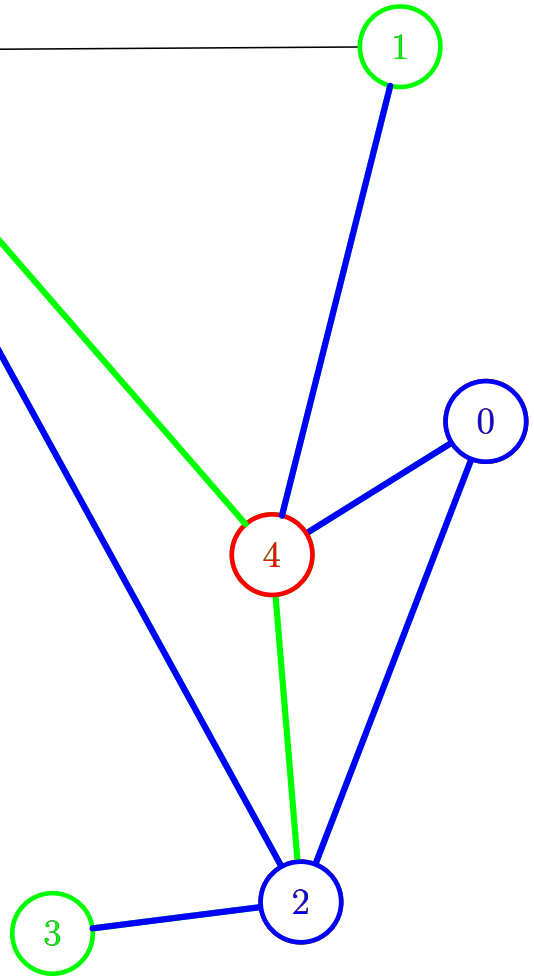
currentNode=4 neighbour=5

q=

3	5	1			



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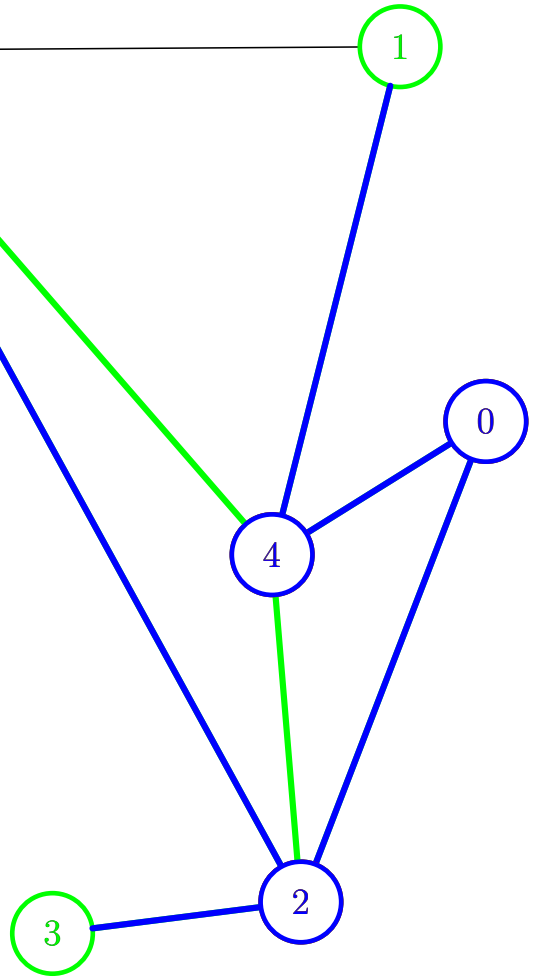
currentNode=4

q=

3	5	1			



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      }
    }
    processVertexLlate(currentNode)
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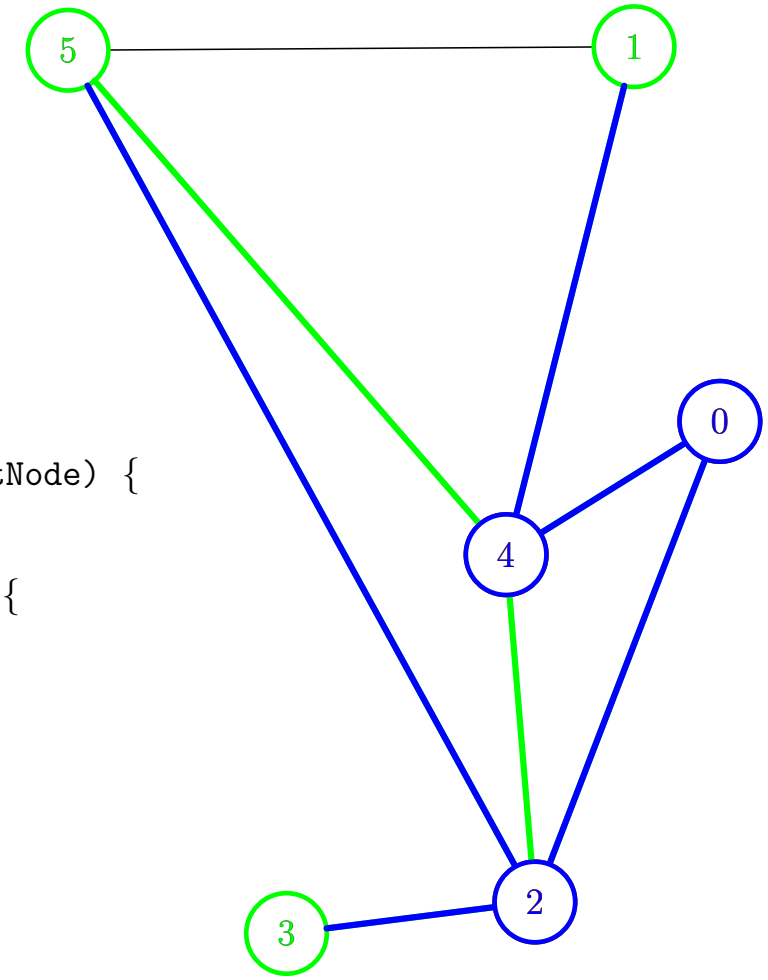
}
```

q=

3	5	1			



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        q.enqueue(neighbour)
      }
    }
    processVertexLate(currentNode)
  }
}
```
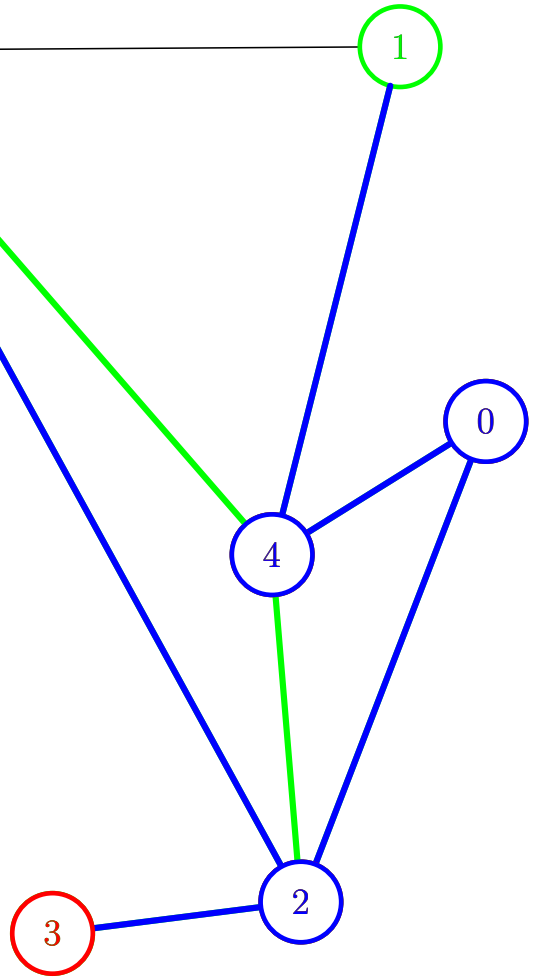

currentNode=3

q=

5	1				



Breadth First Search

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    }
    processVertexLate(currentNode)
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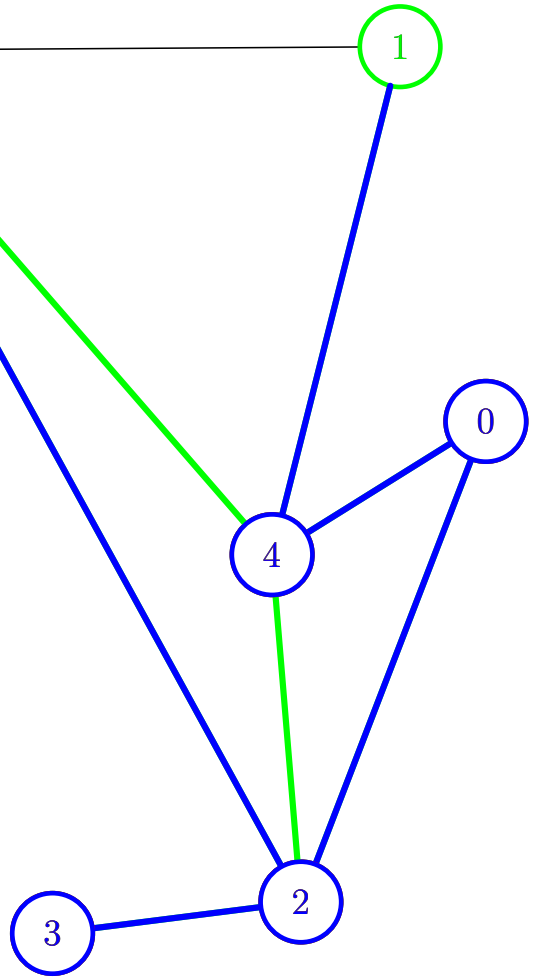
}
```

currentNode=3

q=

5	1				



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```
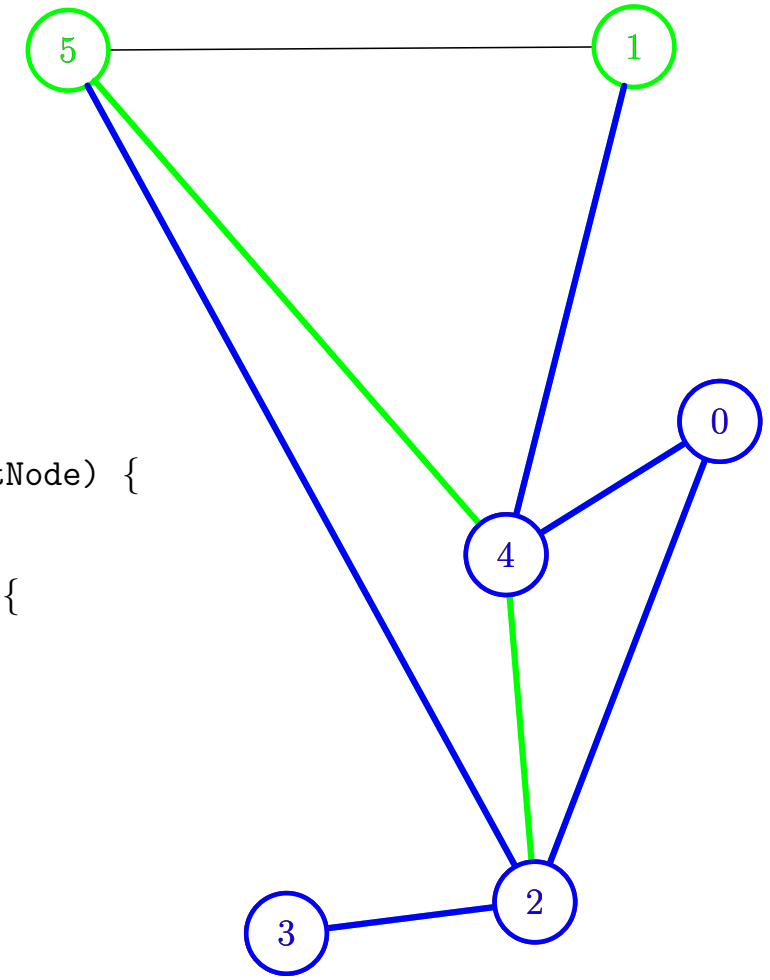

q=

5	1				



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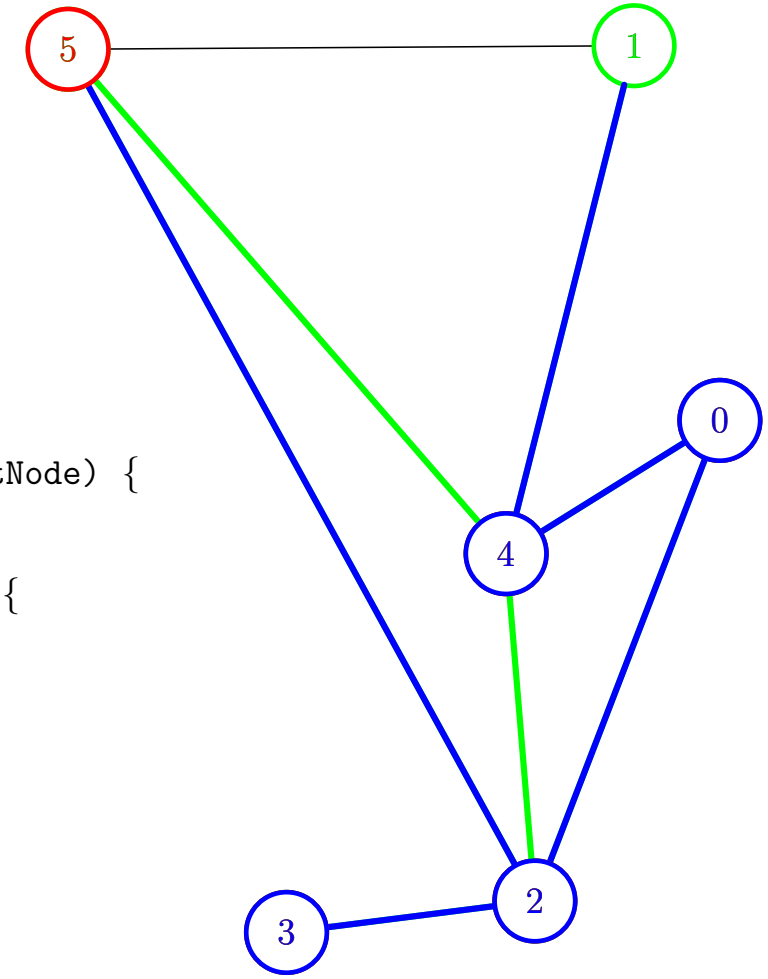
```

currentNode=5

q=

1					



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```
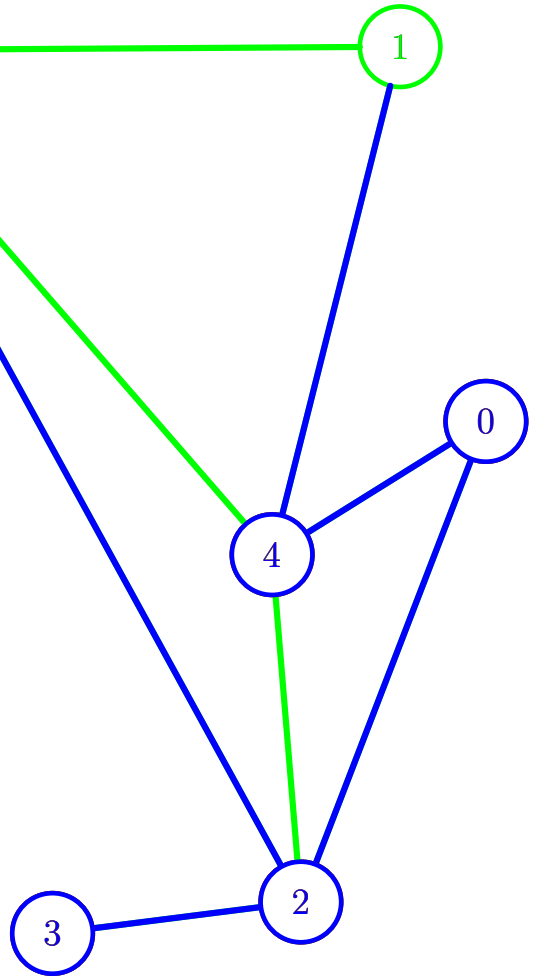

currentNode=5 neighbour=1

q=

1					



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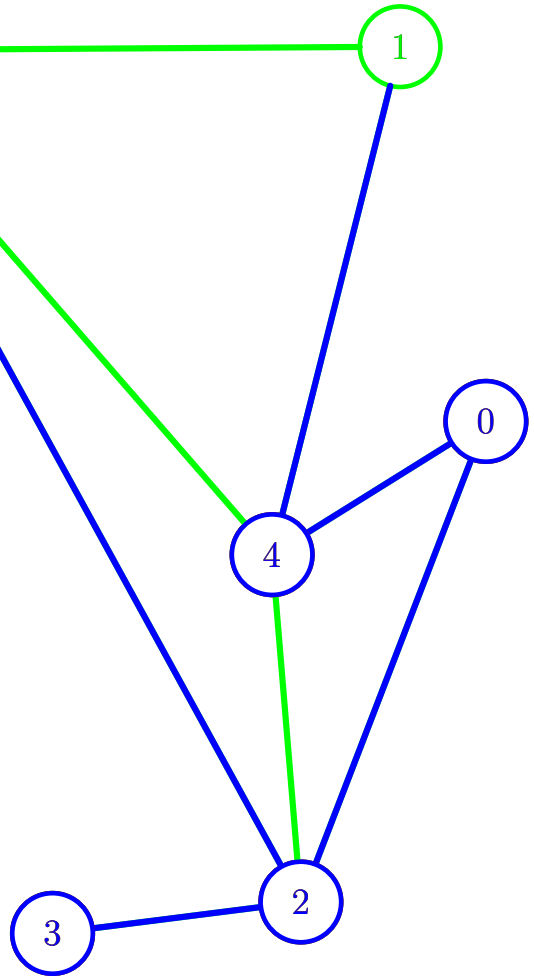
currentNode=5 neighbour=1

q=

1					



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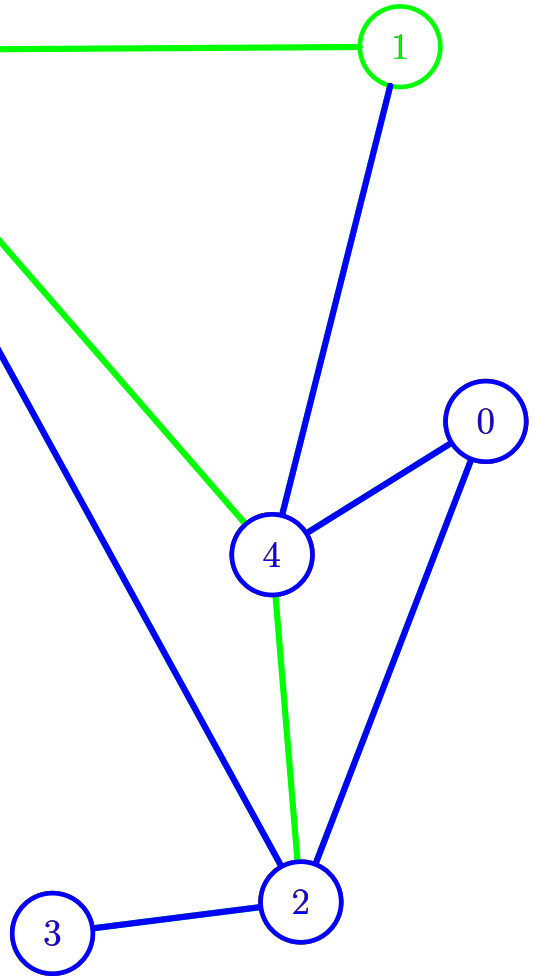
q=

1					



Breadth First Search

```
bfs(graph, node) {
  List state(graph.noNodes, "undiscovered")
  List parent(graph.noNodes)
  state[node] ← "discovered"
  parent[node] ← nil
  Queue q
  q.enqueue(node)
  while (!q.isEmpty()) {
    currentNode ← q.dequeue()
    processVertexEarly(currentNode)
    state[currentNode] ← "processed"
    foreach neighbour ∈ Neighbourhood(currentNode) {
      if (state[neighbour] ≠ "processed")
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        q.enqueue(neighbour)
      }
    }
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```
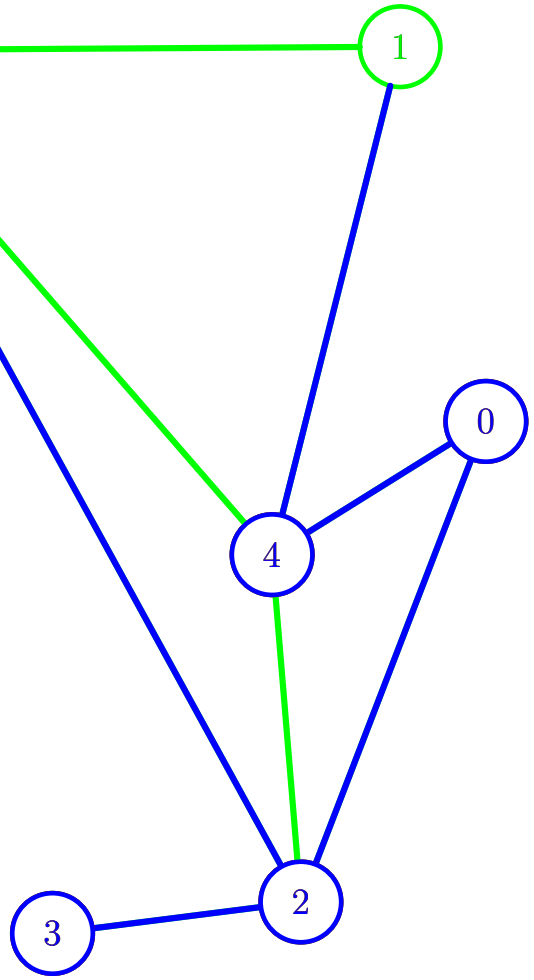

currentNode=5

q=

1					



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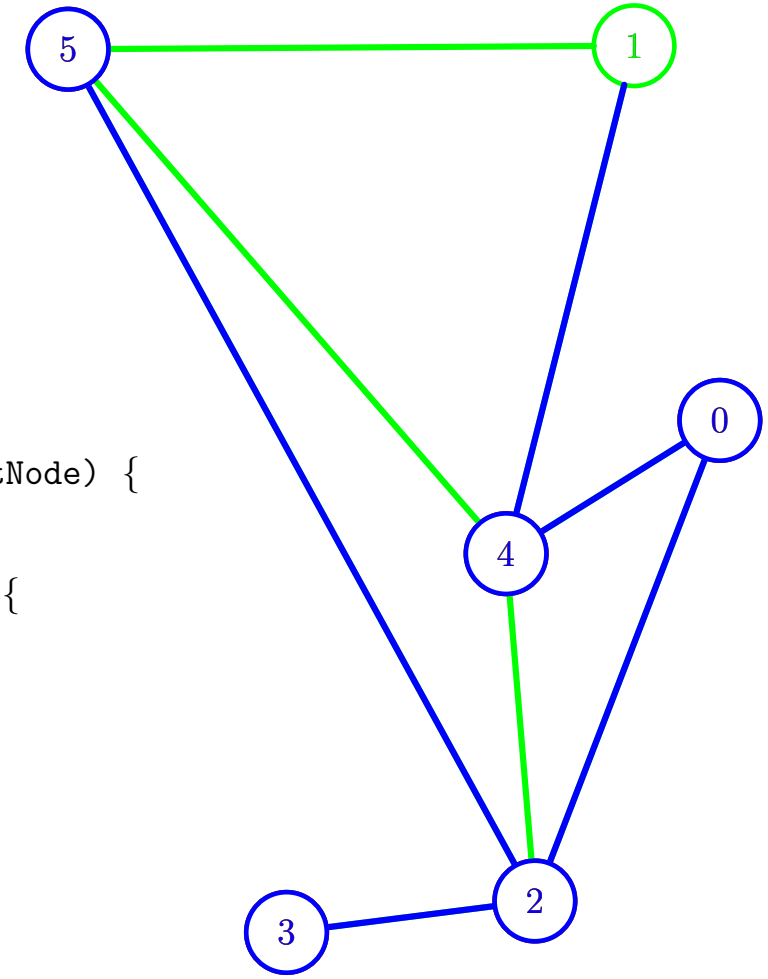

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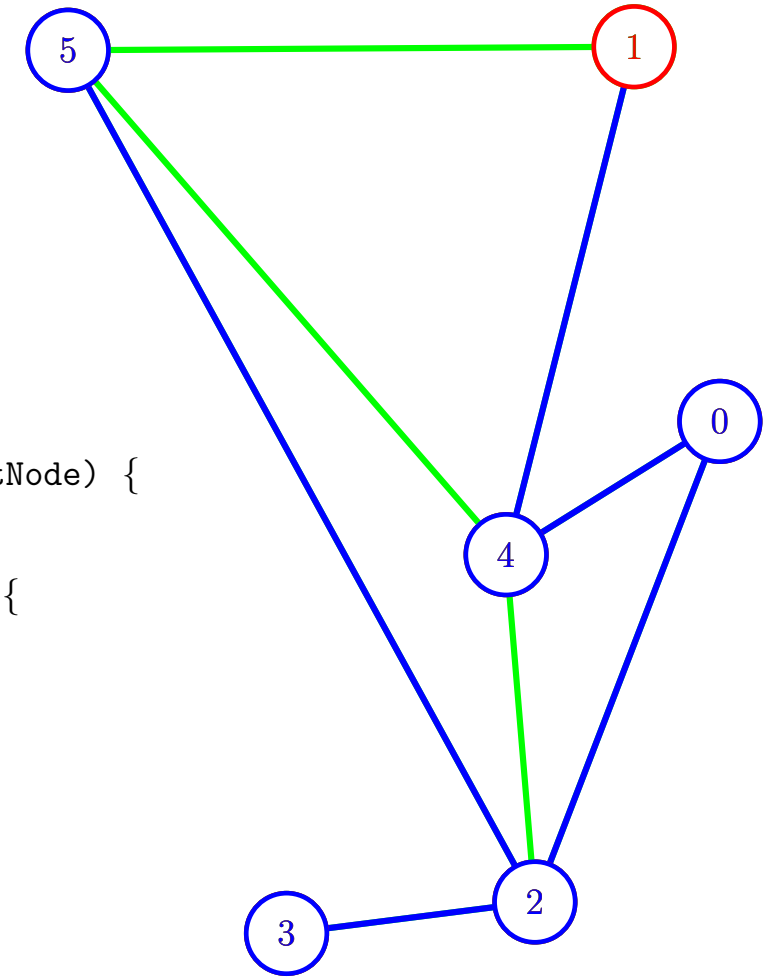
q= | | | | | | |



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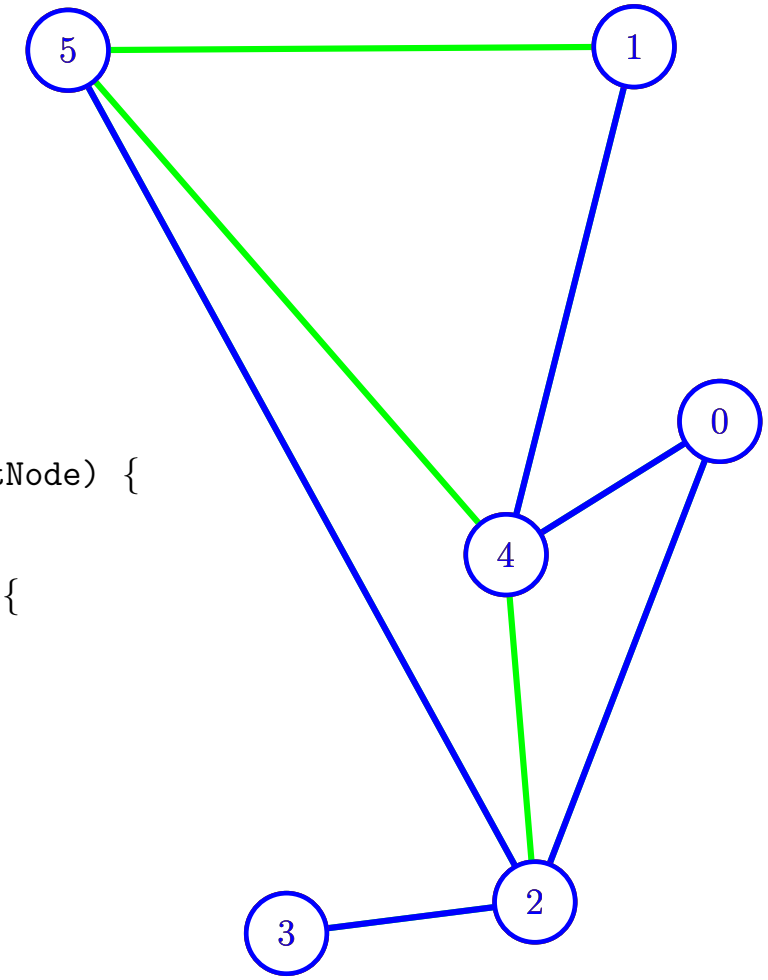

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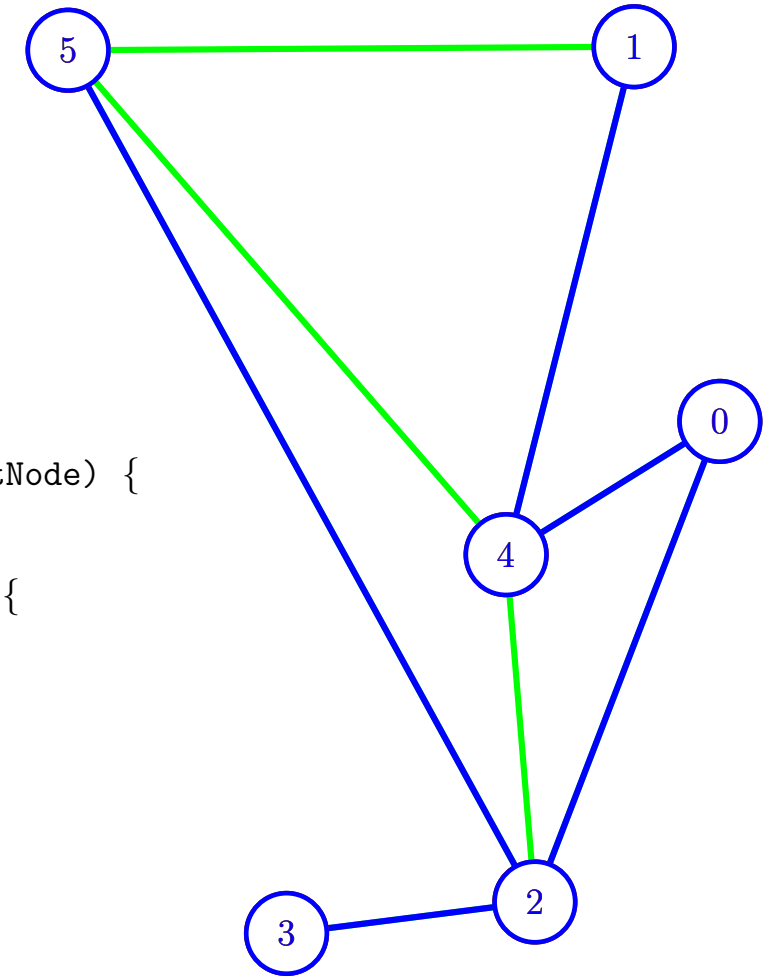

Applications of Breadth First Search

- Breadth first search can be used to find the shortest path from a source node to a destination node for an **unweighted** graph
  - ⋆ Run `bfs(graph,source)`
  - ⋆ Use parent information to find path from destination back to source
- BFS (as well as DFS) can be used to find connected components
  - ⋆ Use $processVertexEarly$ to mark vertices connected to the current connected component
  - ⋆ Run bfs from all vertices that are not labelled

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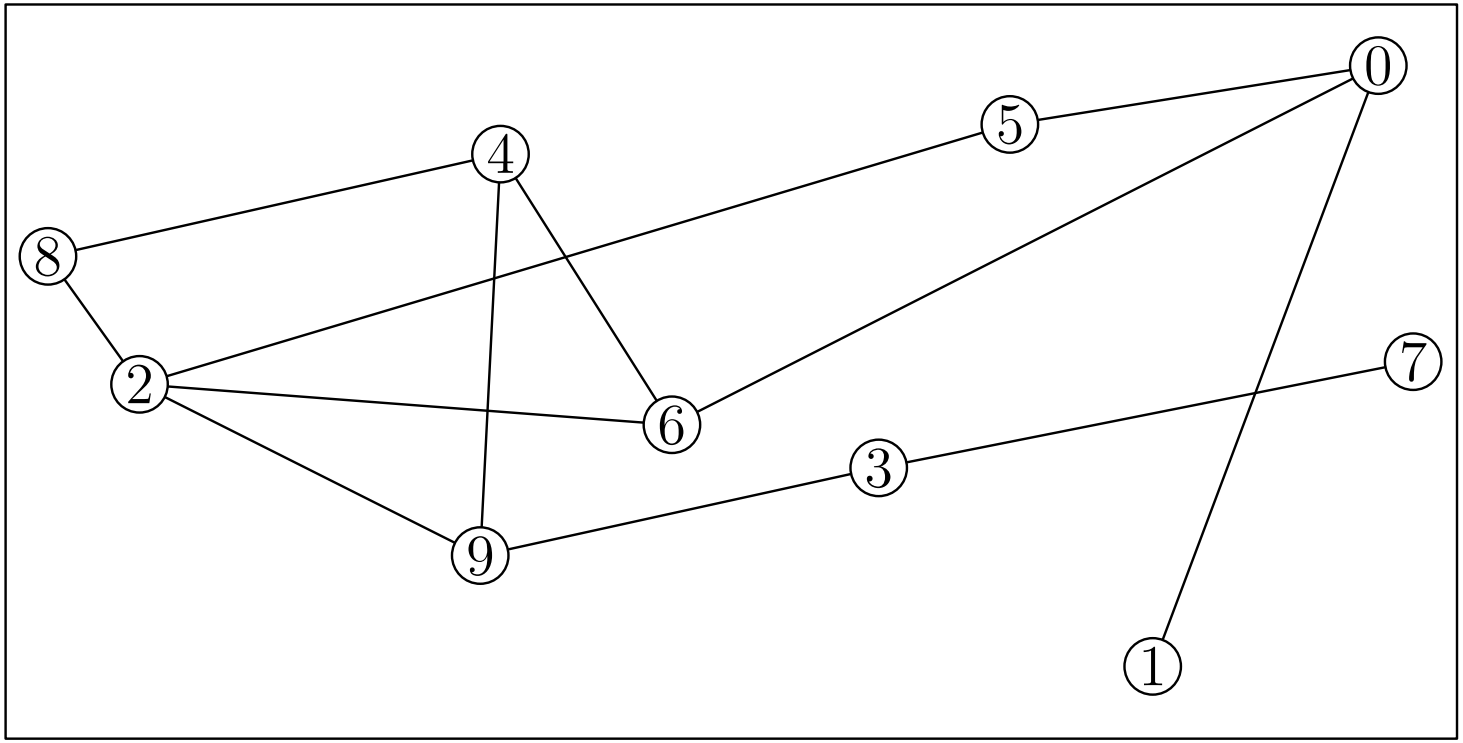
Bipartite Graphs

- Bipartite graphs are graphs where the vertices can be split into two sets so that there are no edges between vertices in the same graph



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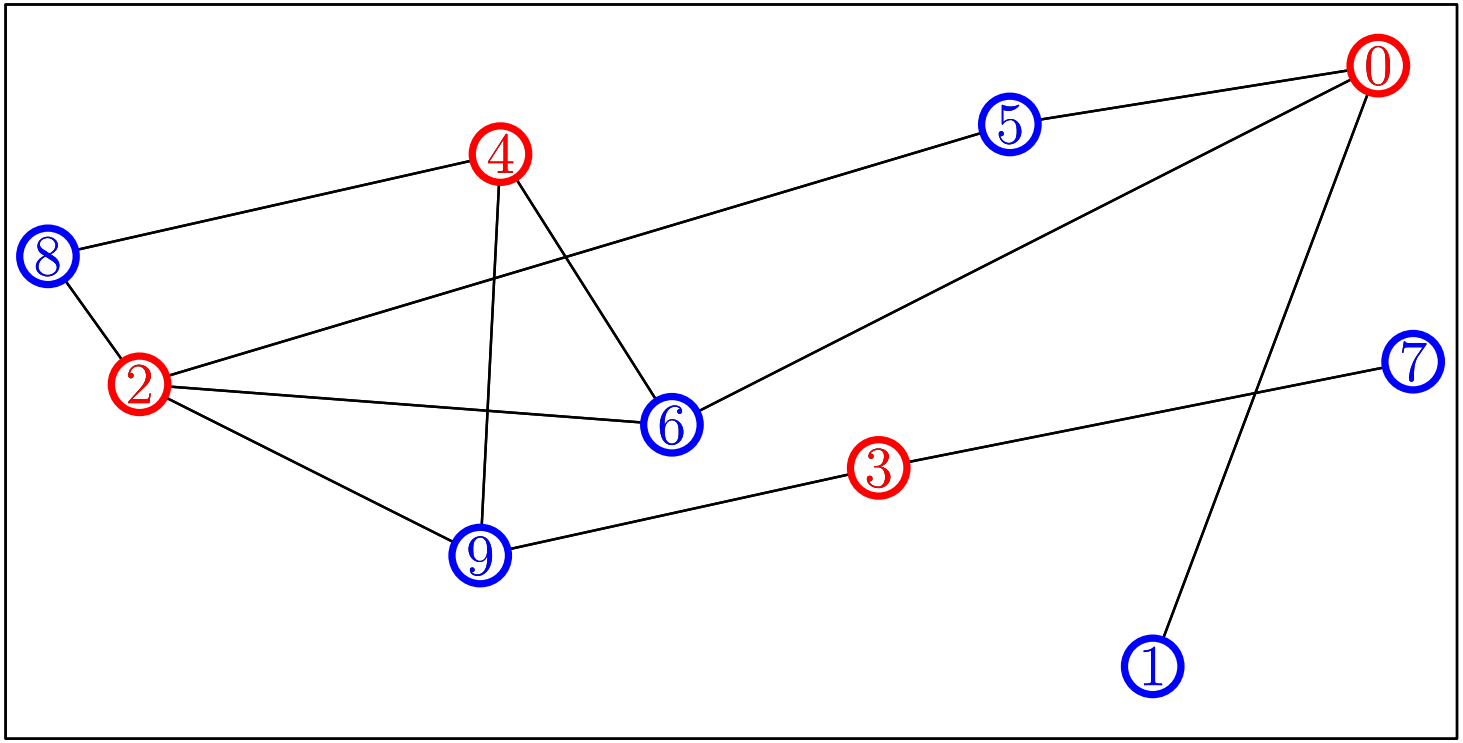

Checking Bipartiteness (Two-colourability)

- Each edge must connect nodes from different sets

```
isBipartite(graph) {
   colour = List(graph.noNodes(), "white");
   bipartite = true;

   foreach node in graph {
      if (colour[node] == "white") {
         colour[node] = "red";
         bfs(graph, node);
      }
   }
   return bipartite;
}

processEdge(node1, node2) {
   if (colour[node1] == colour[node2])
       bipartite = false;
   colour[node2] = (colour[node1]=="red")? "blue":"red";
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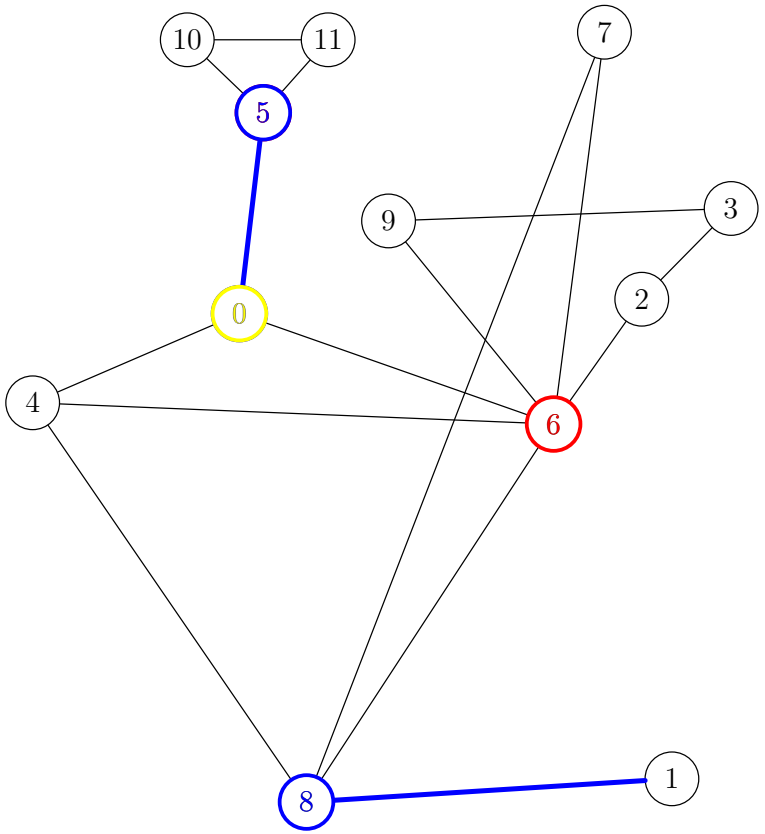
Outline

1. Breadth First Search
   - BFS applications
2. **Depth First Search**
   - DFS applications
3. Topological Sort



Depth First Search

- Depth first search is essentially like breadth first search except we replace the queue by a stack
- In practice it is often implemented using recursion rather than a stack
- It proves useful to keep a record of the traversal **time** for each vertex
  - ⋆ the clock ticks each time a vertex is entered or exited

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  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
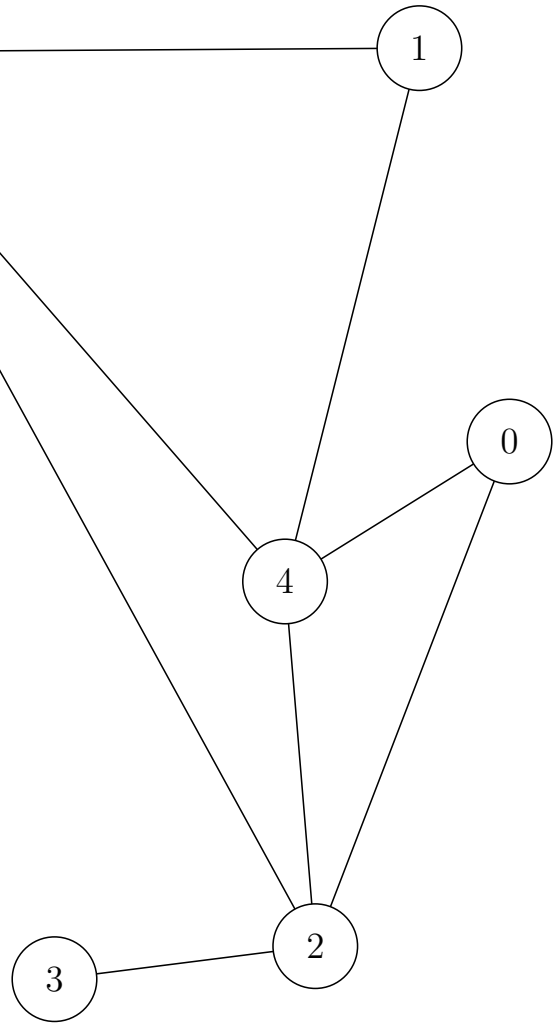

node=0 neighbour=nil time=0



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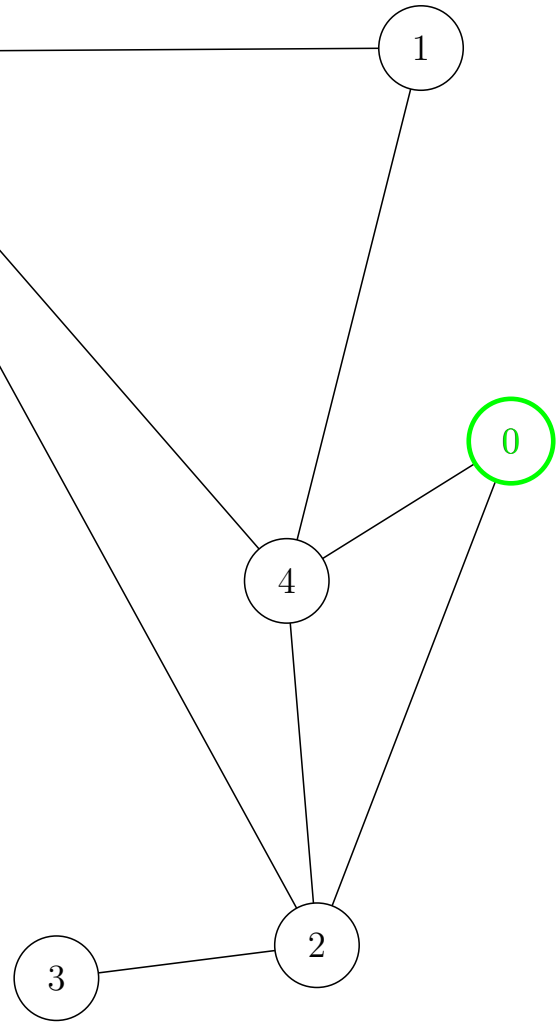

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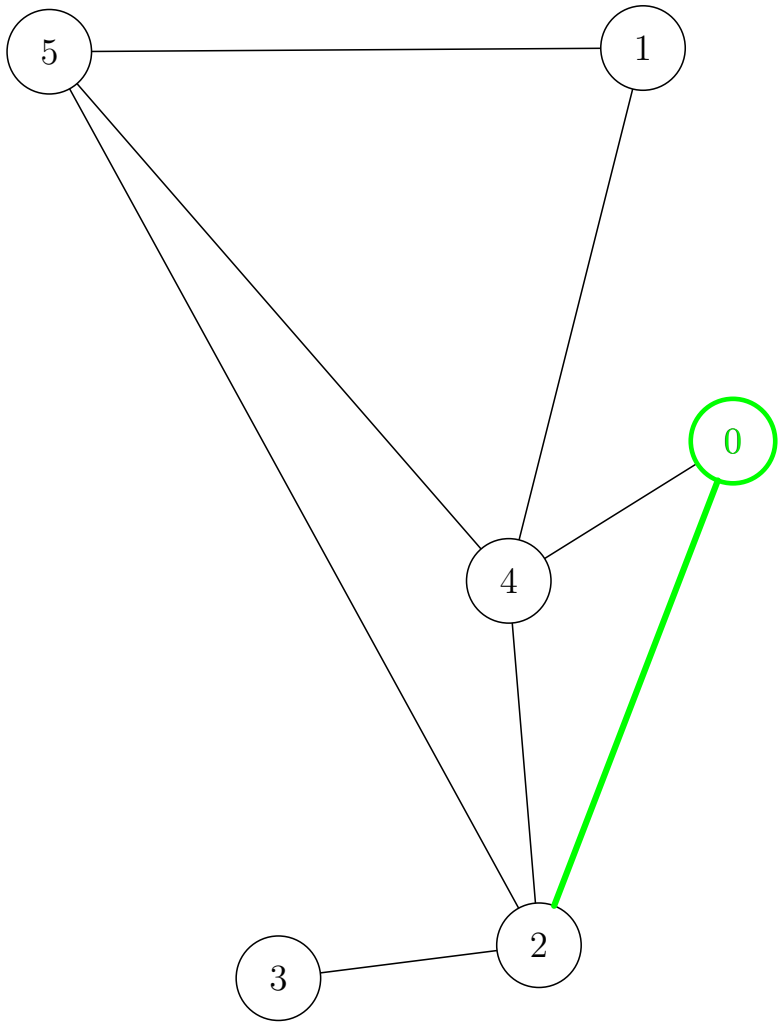
```

node=0 neighbour=2 time=1



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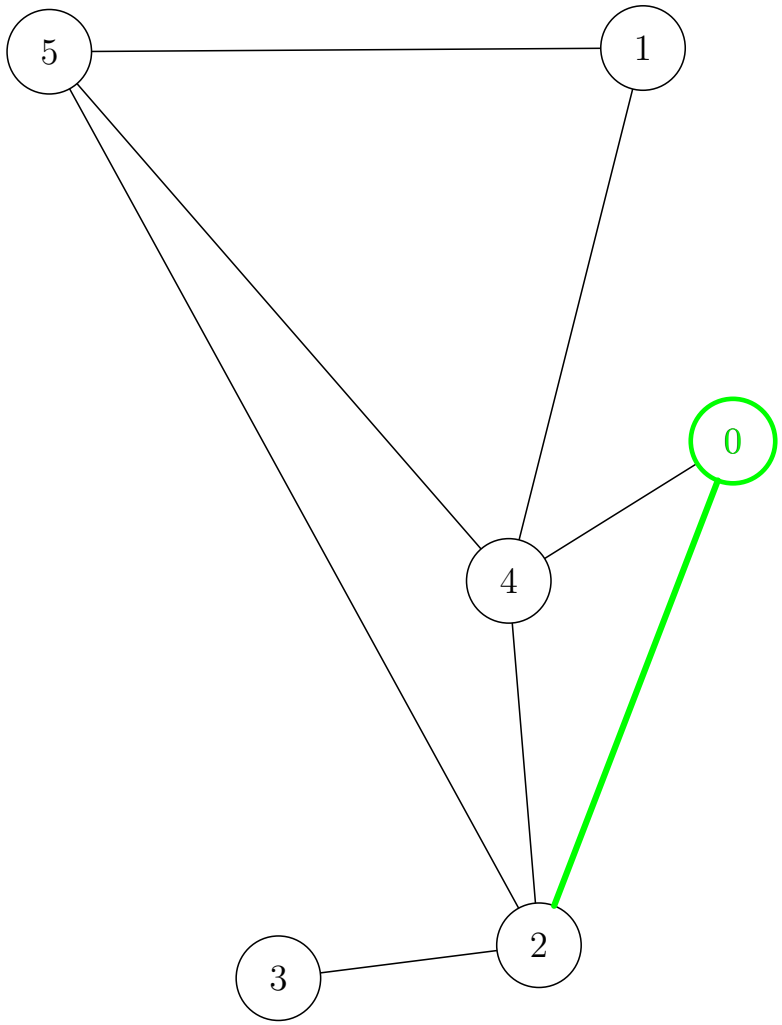

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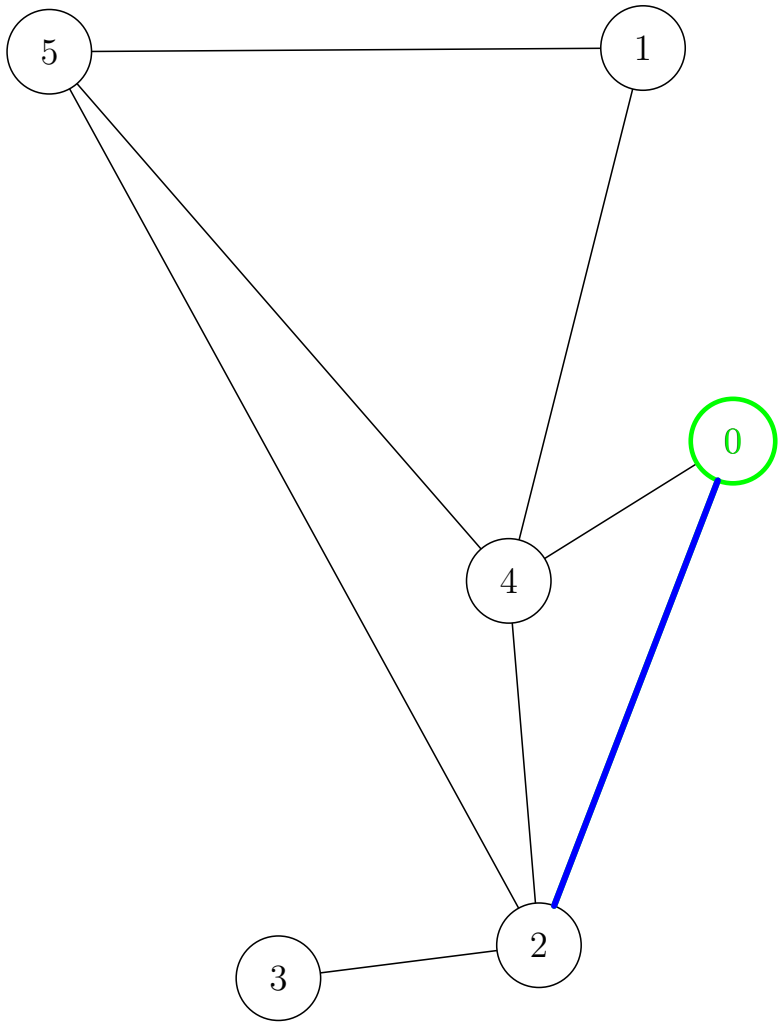

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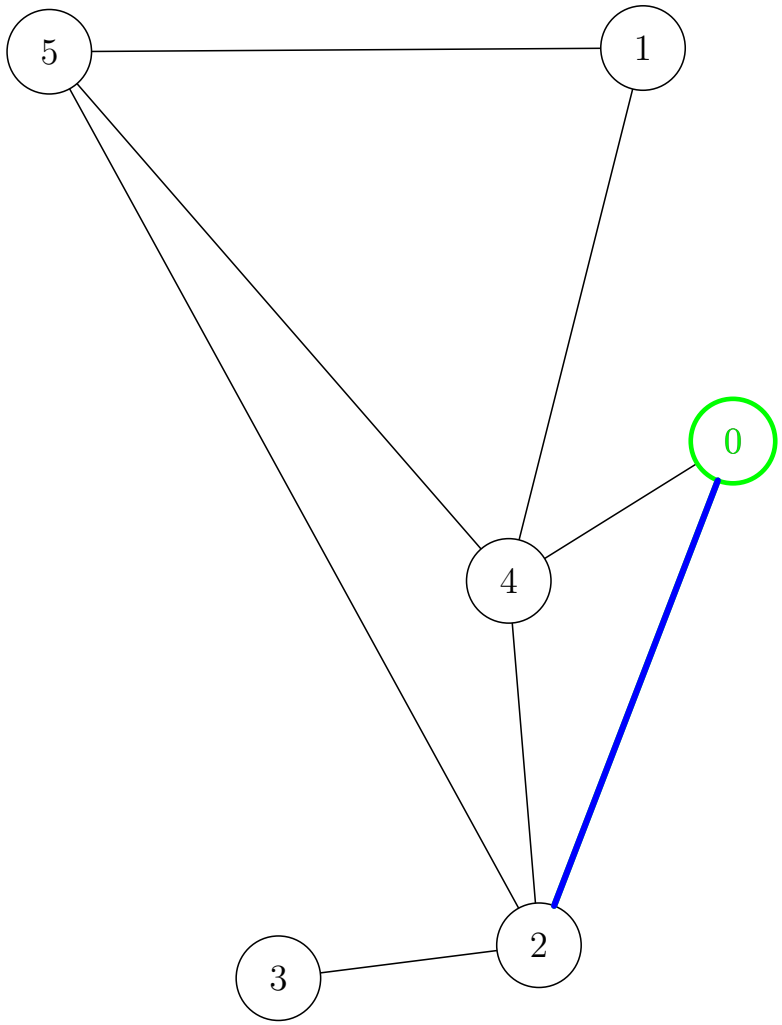

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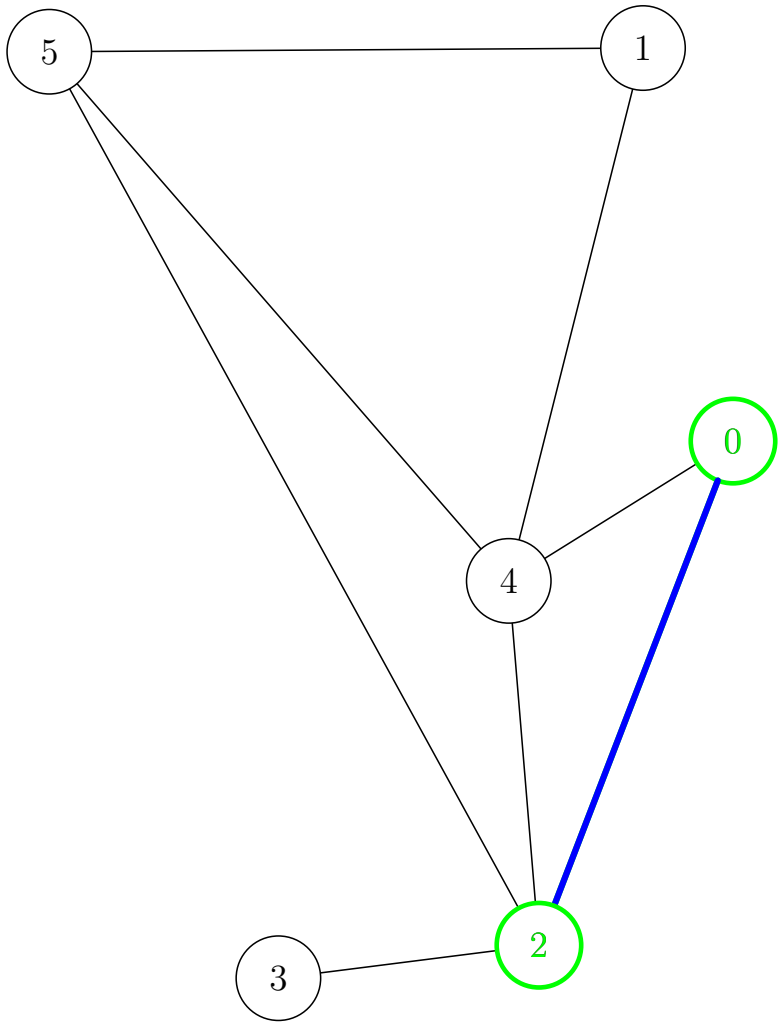

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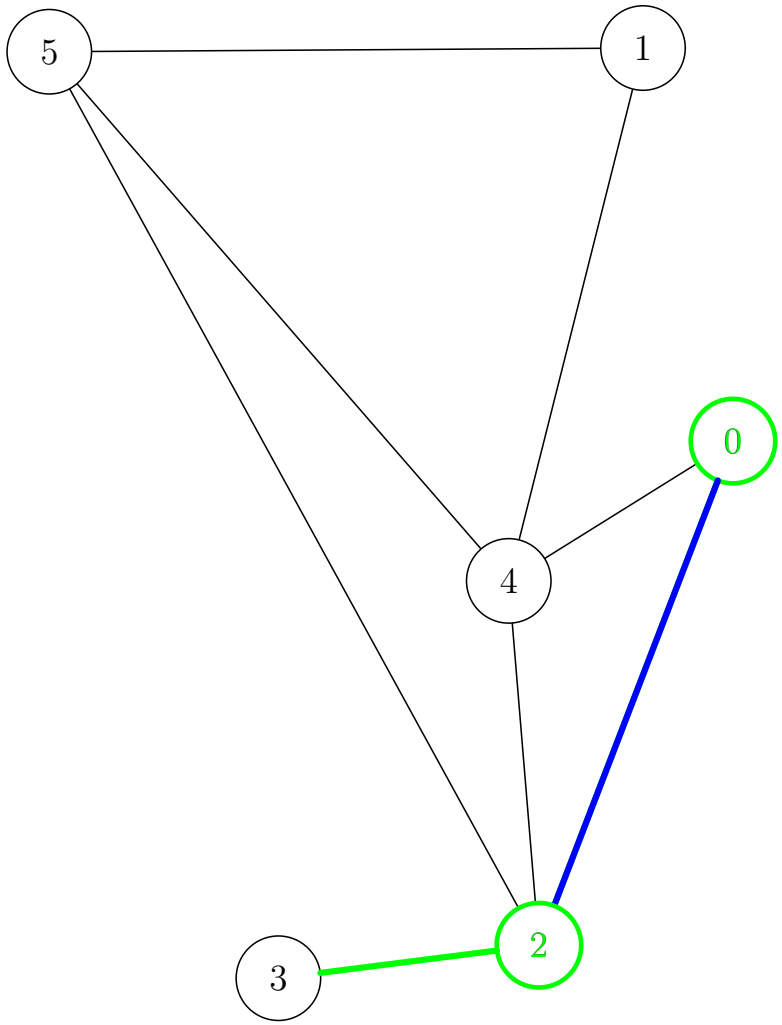

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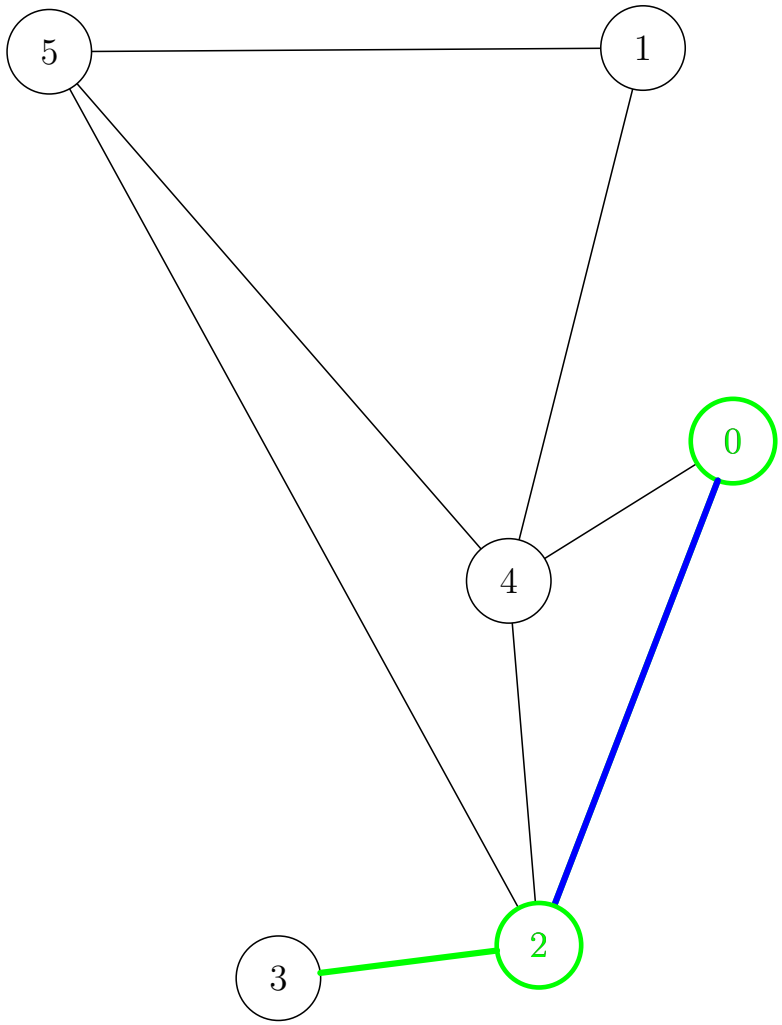

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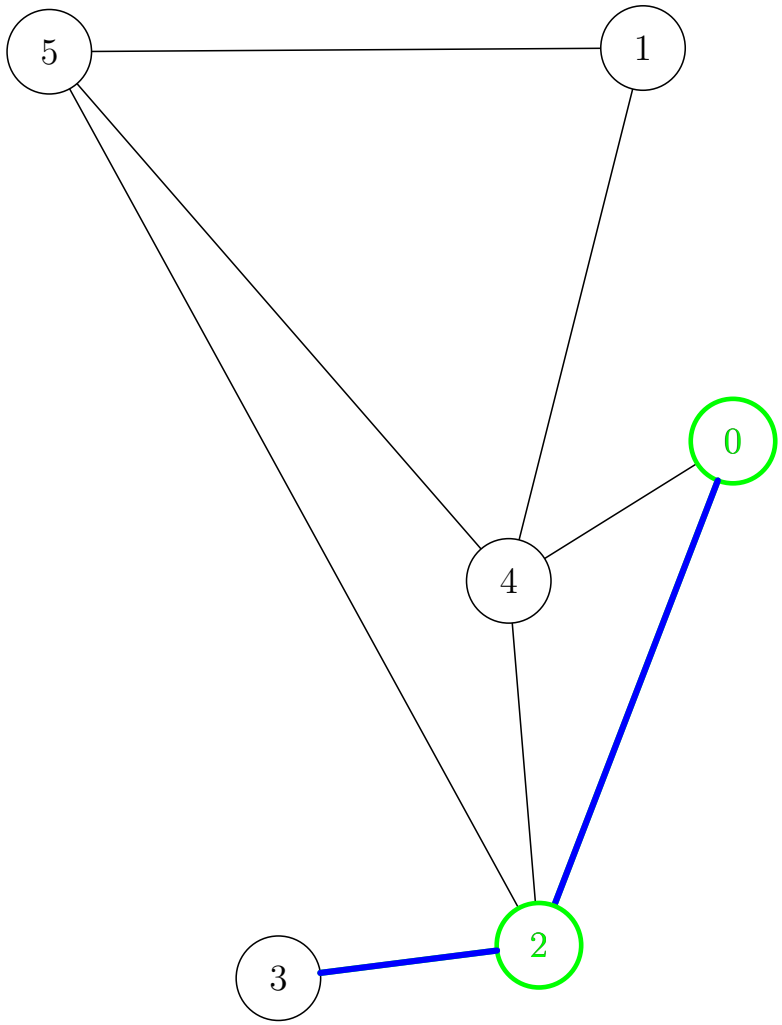

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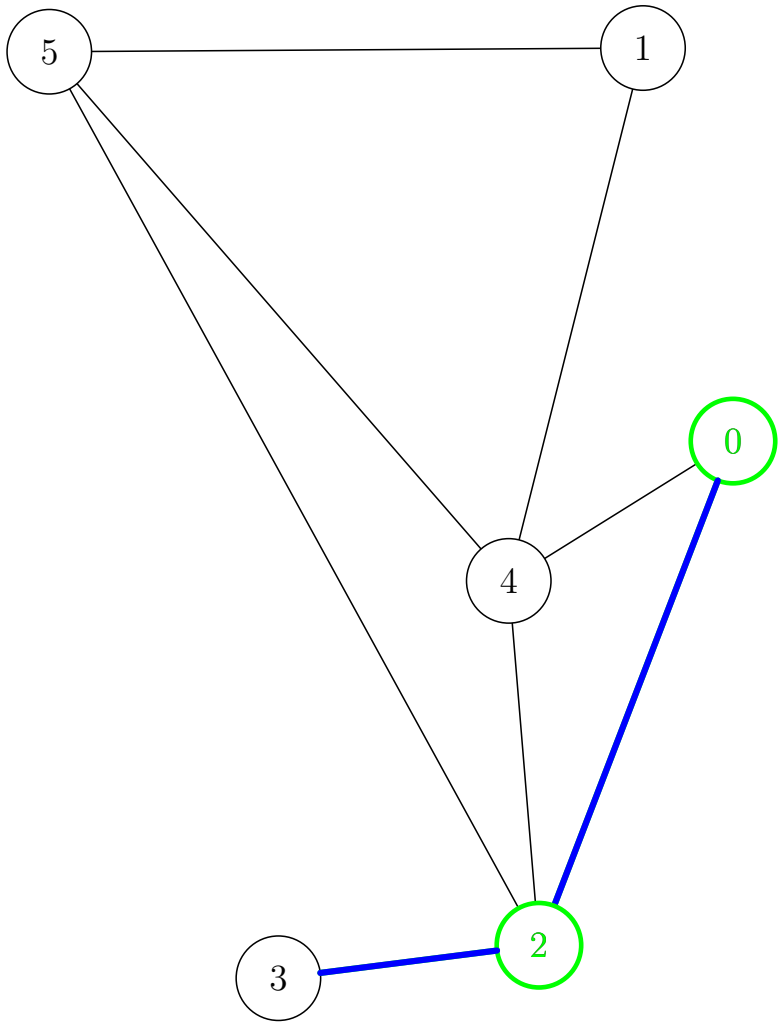
node=2 neighbour=3 time=2

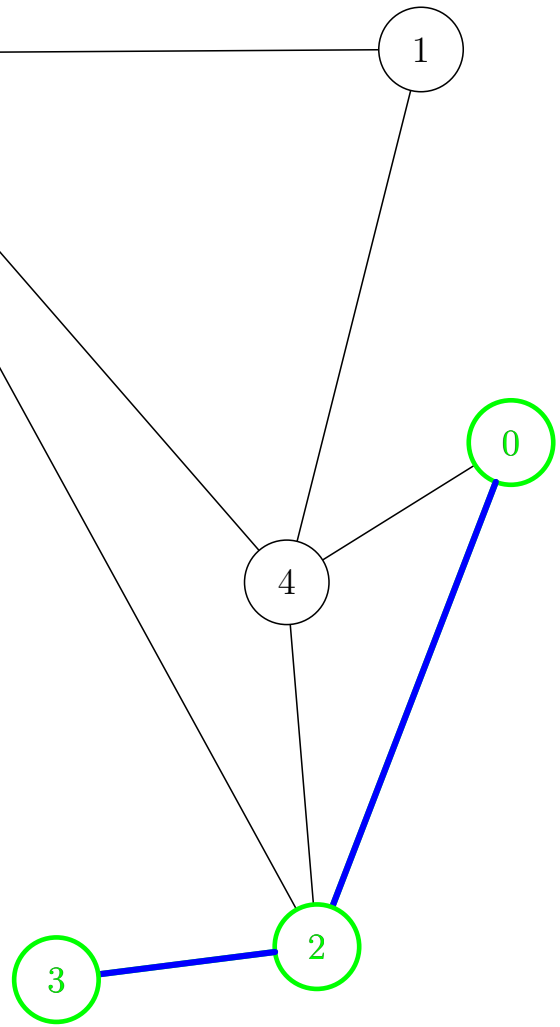


Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

node=3 neighbour=nil time=3



Depth First Search

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  state ← Array[n, "undiscovered"]
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      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
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    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
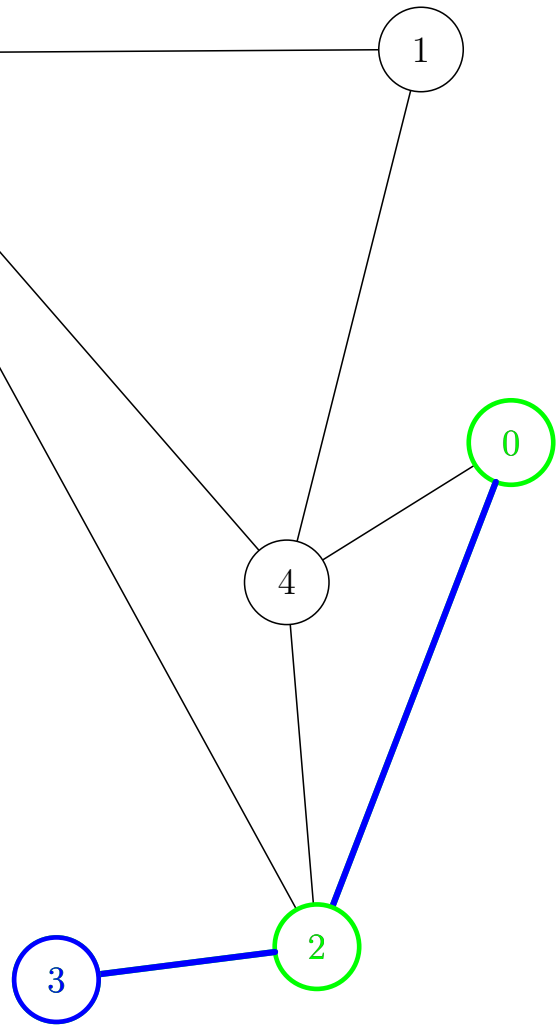

node=3 neighbour=nil time=4



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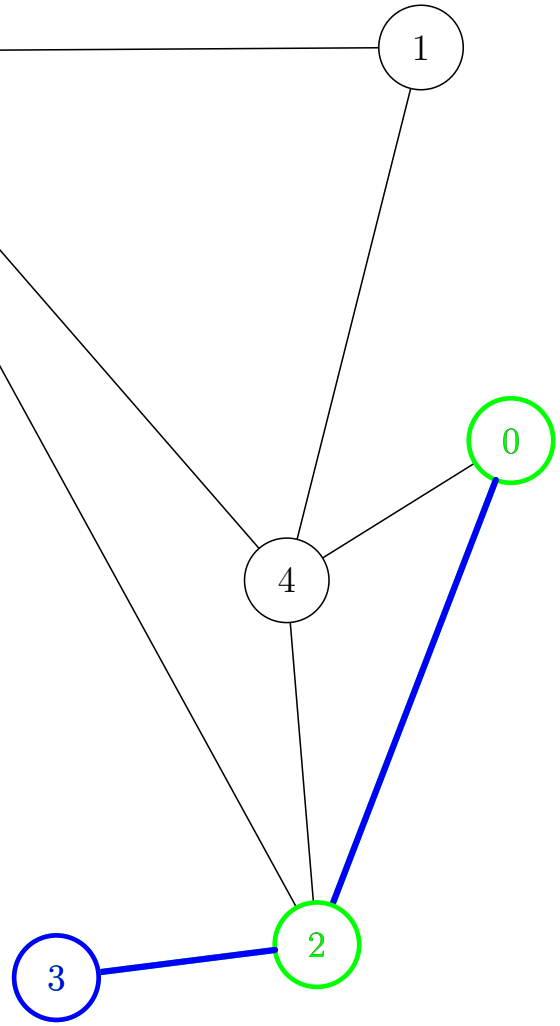
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

node=2 neighbour=3 time=4



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  time ← time + 1
}
```
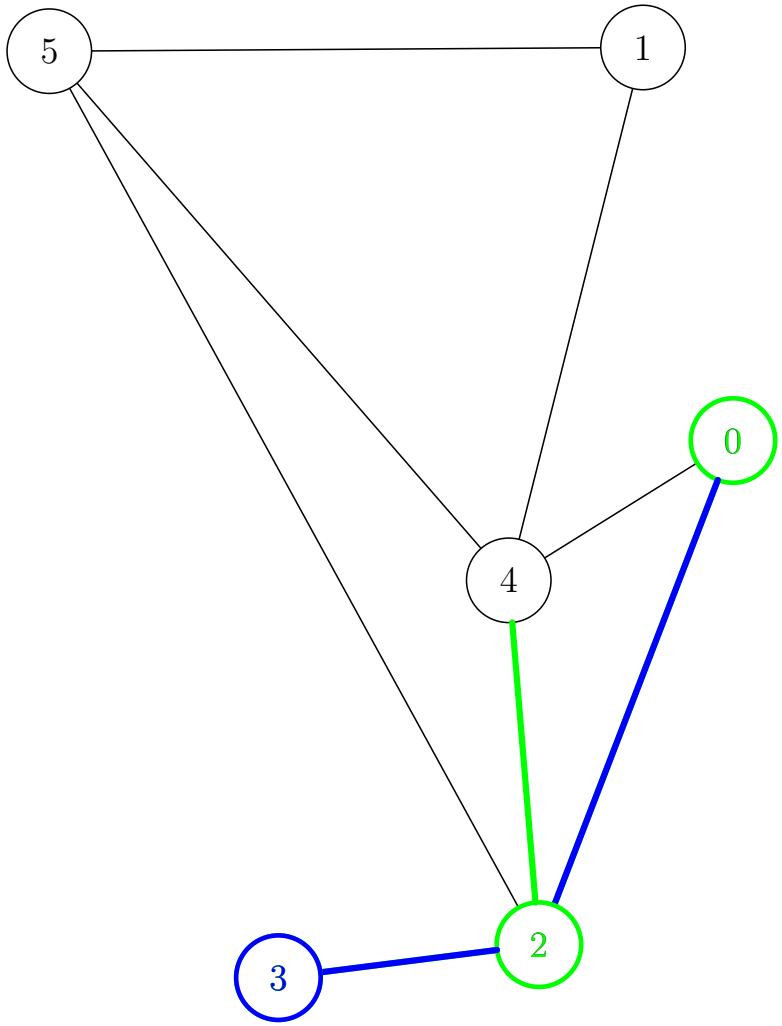

node=2 neighbour=4 time=4



Depth First Search

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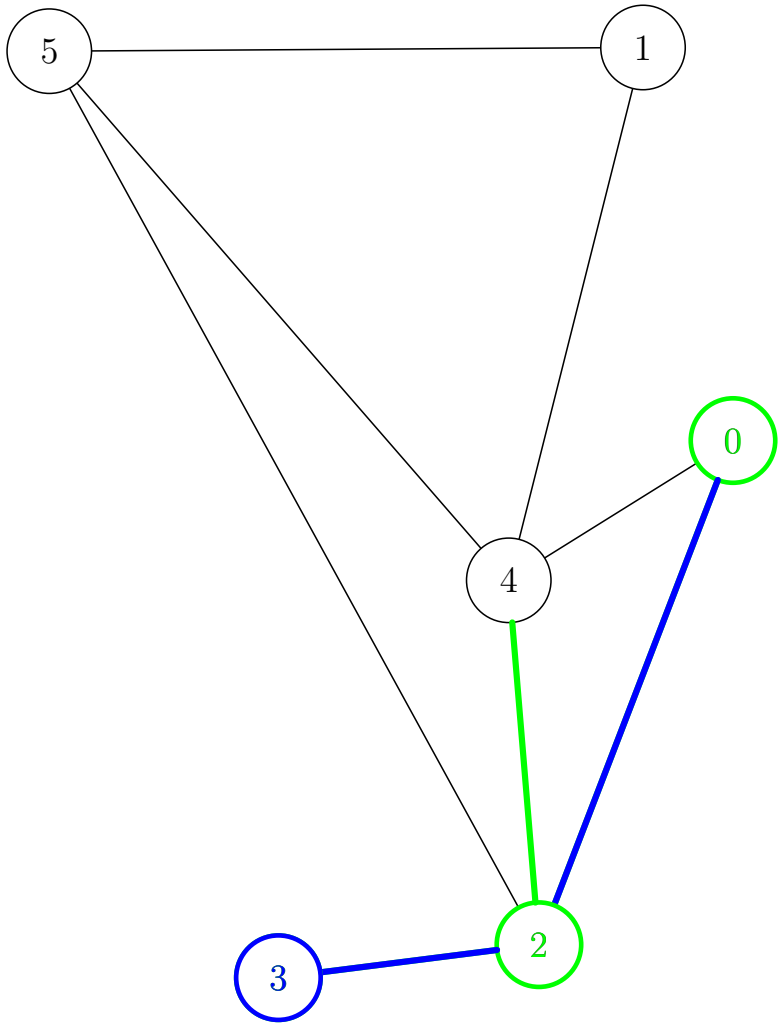
  time ← time + 1
}
```

node=2 neighbour=4 time=4



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      processEdge(node, neighbour)
    }
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  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
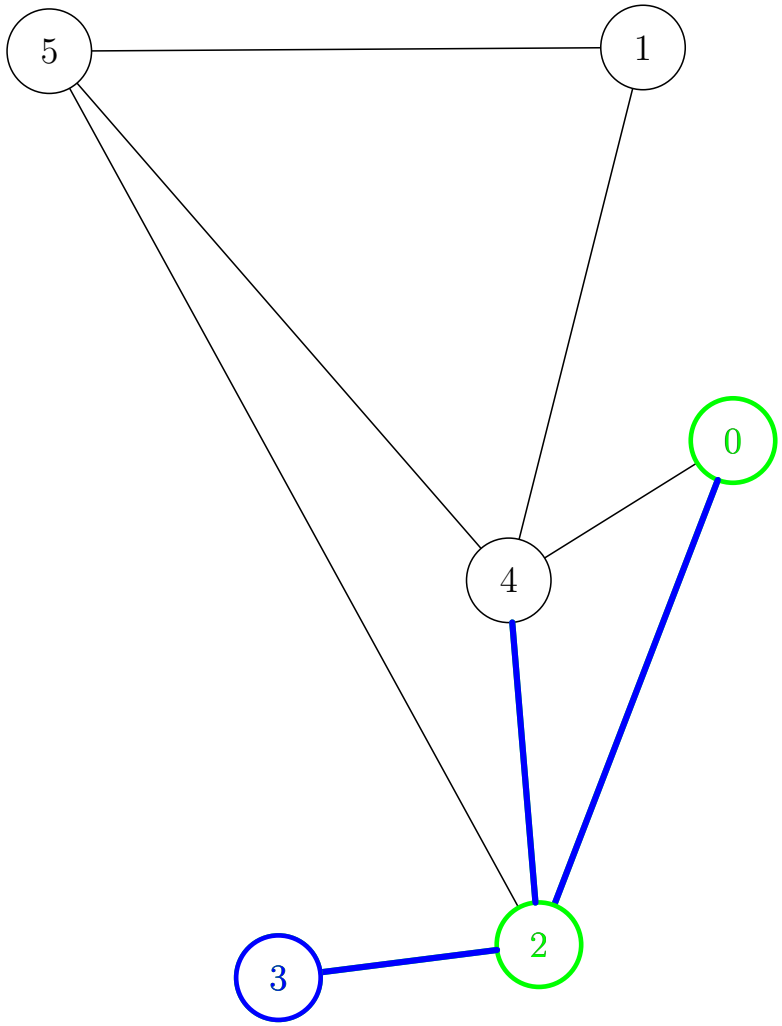

node=2 neighbour=4 time=4



Depth First Search

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    }
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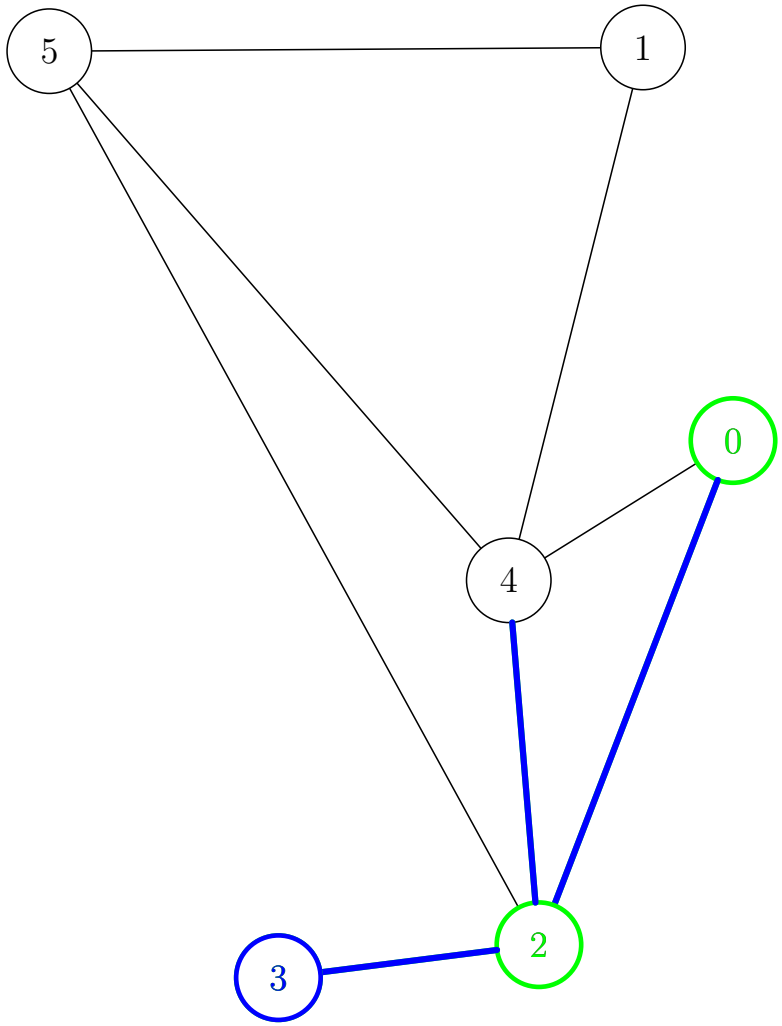
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

node=2 neighbour=4 time=4



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      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
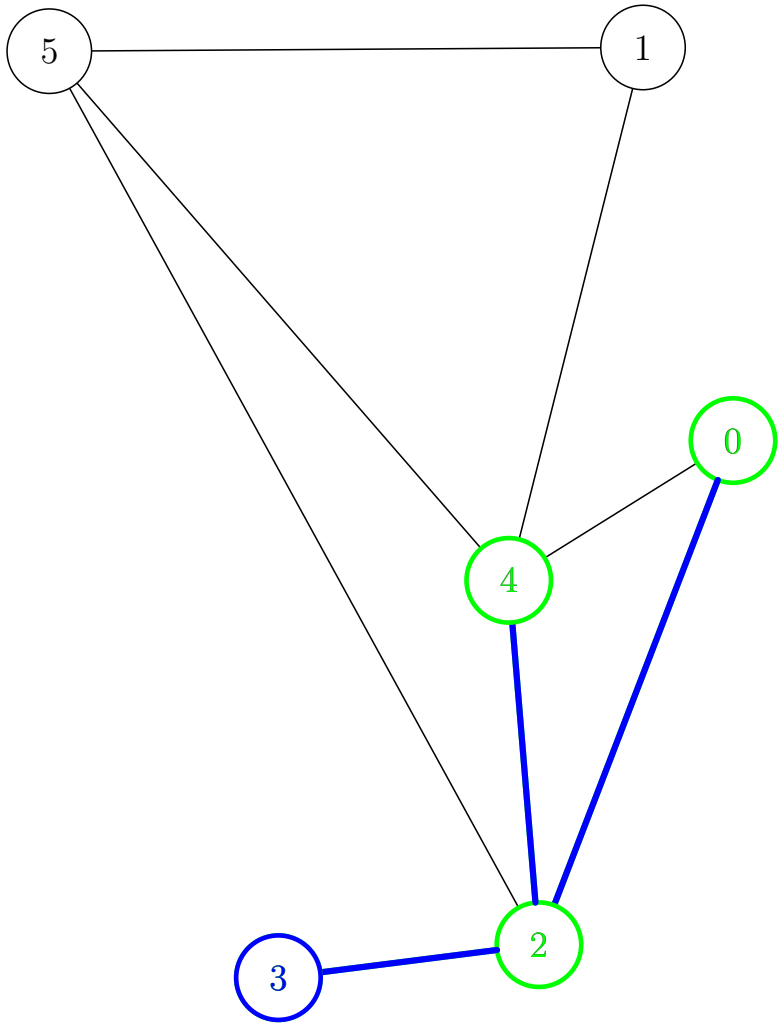

node=4 neighbour=nil time=5



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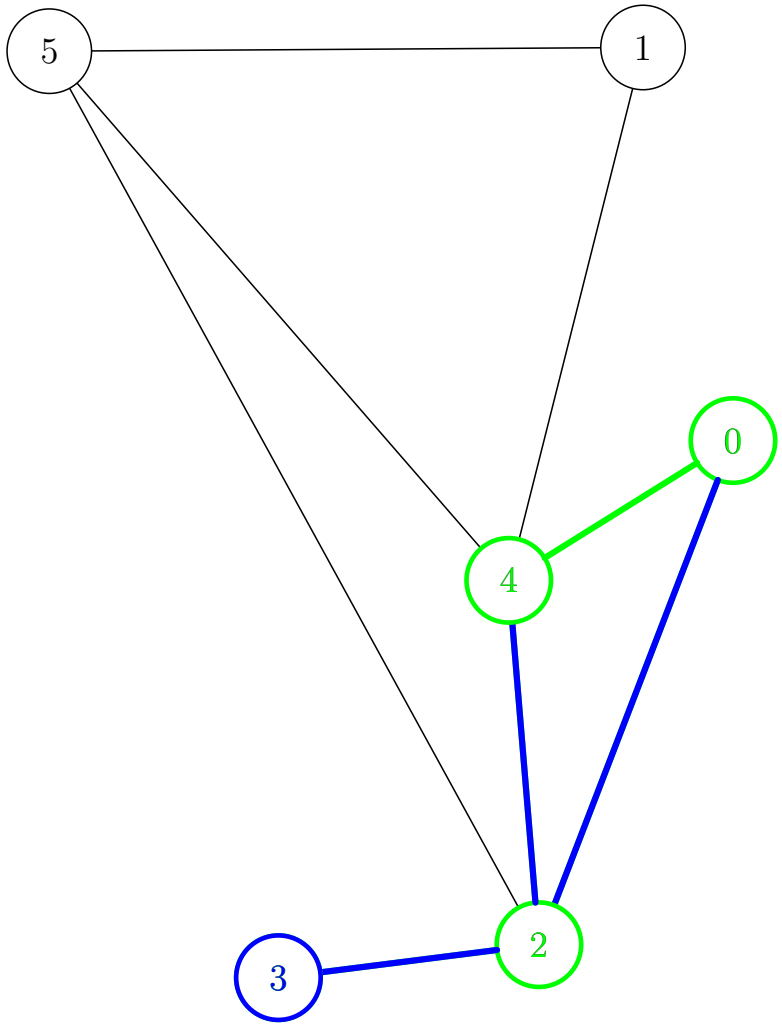
node=4 neighbour=nil time=5



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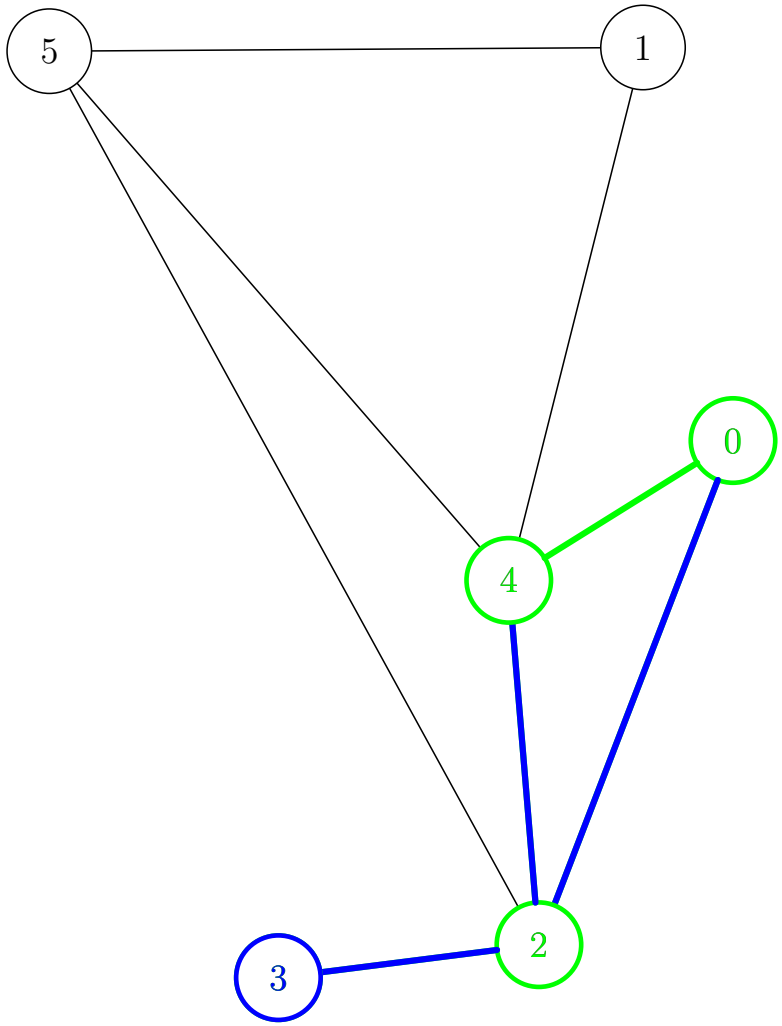
node=4 neighbour=nil time=5



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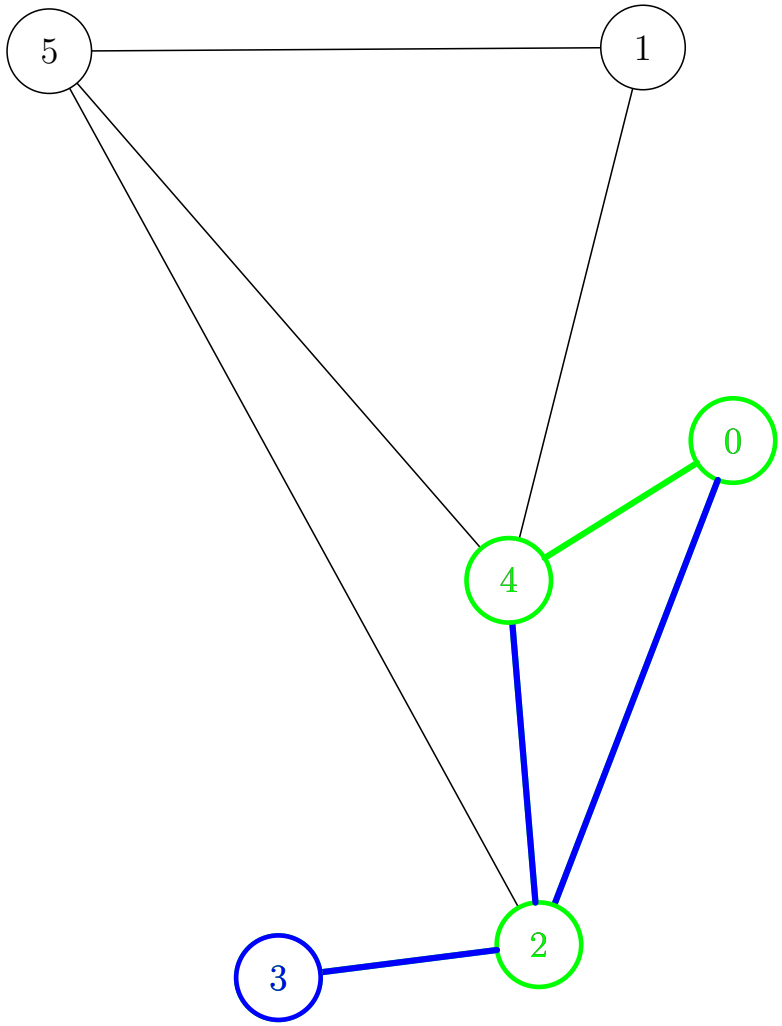
node=4 neighbour=nil time=5



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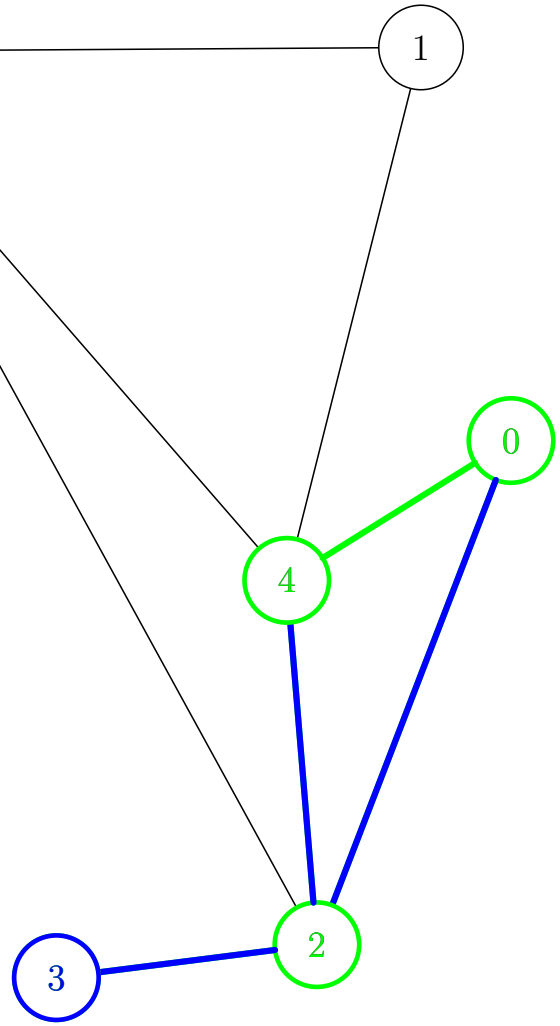
}
```

node=4 neighbour=nil time=5



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  time ← time + 1
}
```
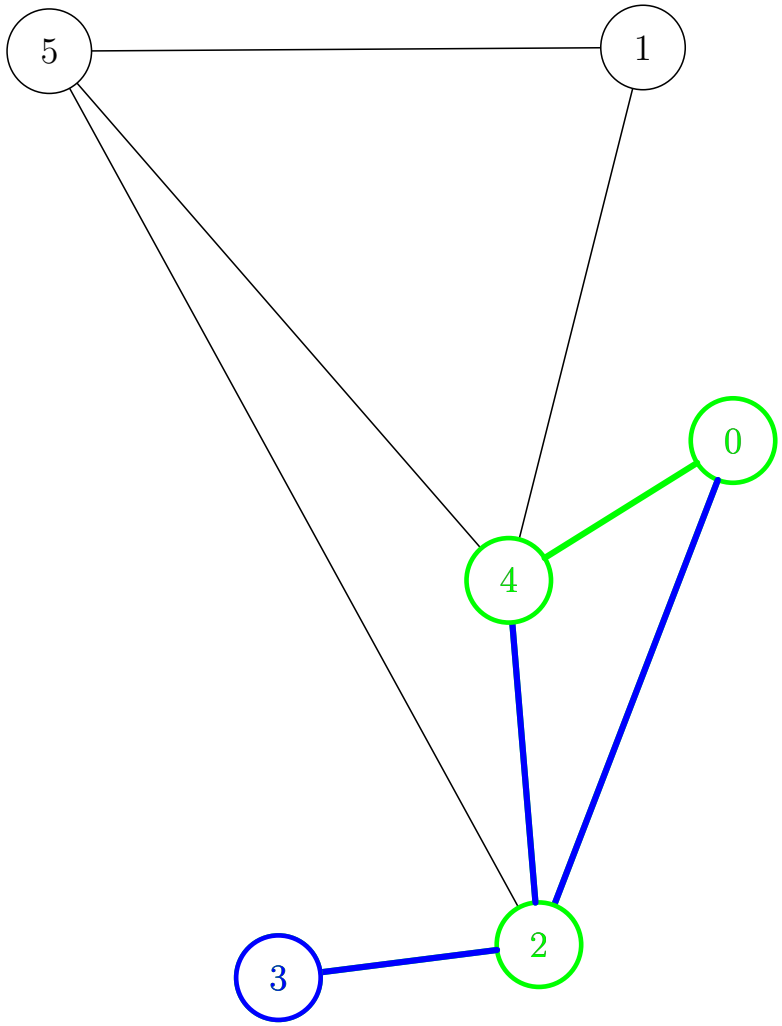

node=4 neighbour=nil time=5



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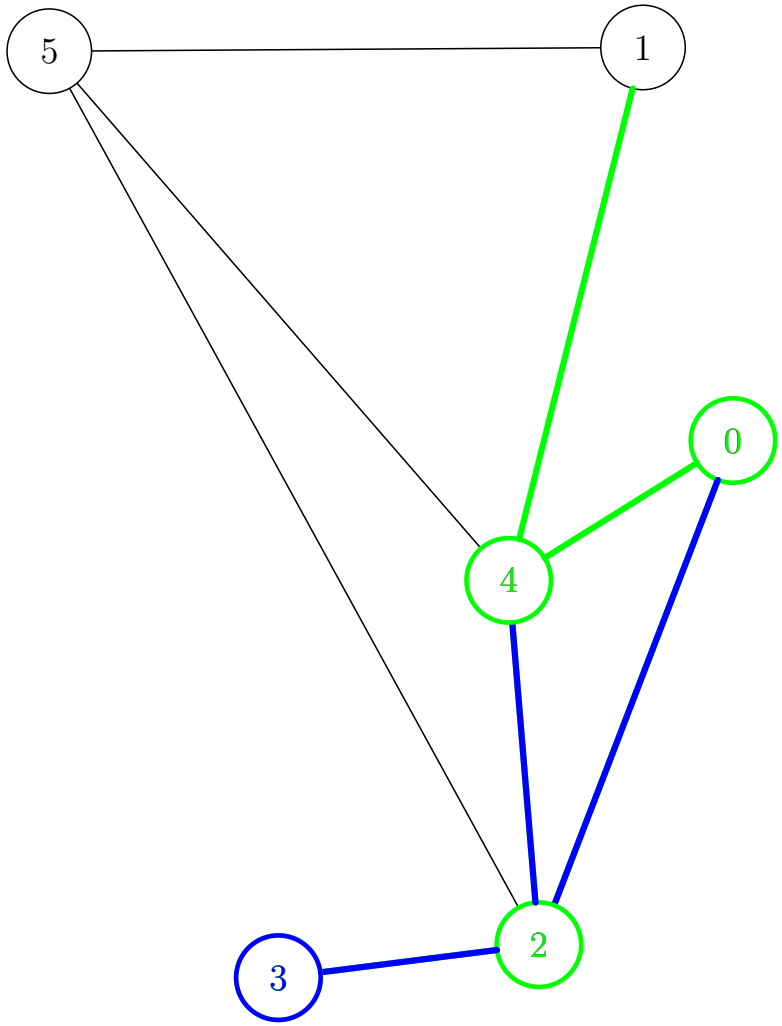
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

node=4 neighbour=1 time=5



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  time ← time + 1
}
```
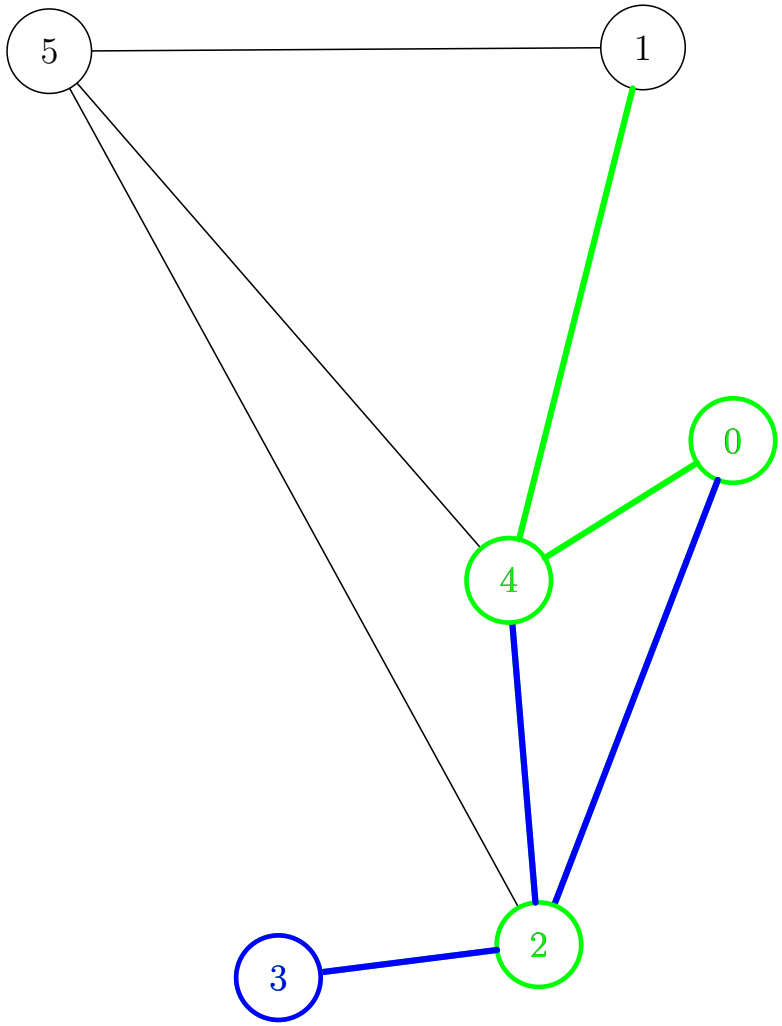

node=4 neighbour=1 time=5



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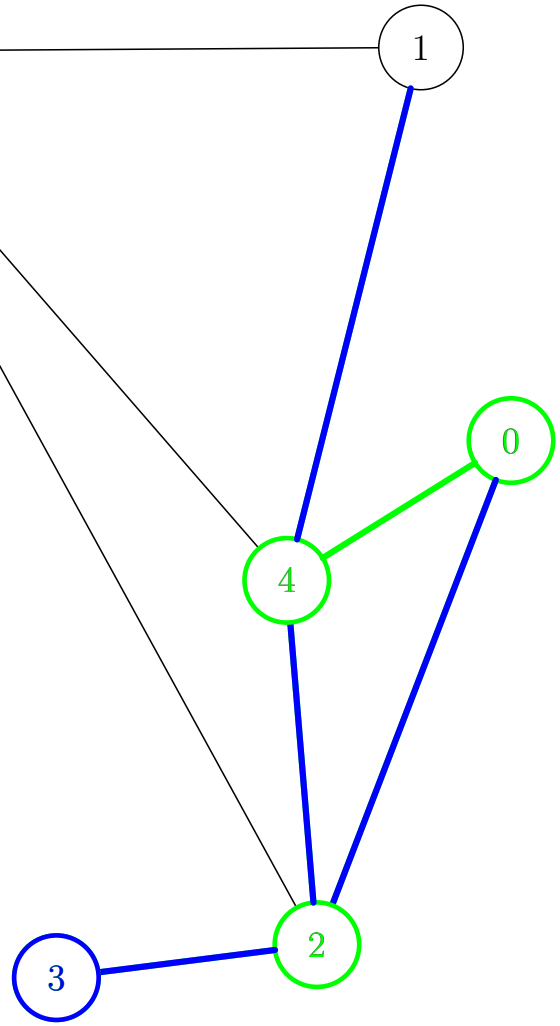
  time ← time + 1
}
```

node=4 neighbour=1 time=5



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  state[currentNode] ← "processed"
  time ← time + 1
}
```
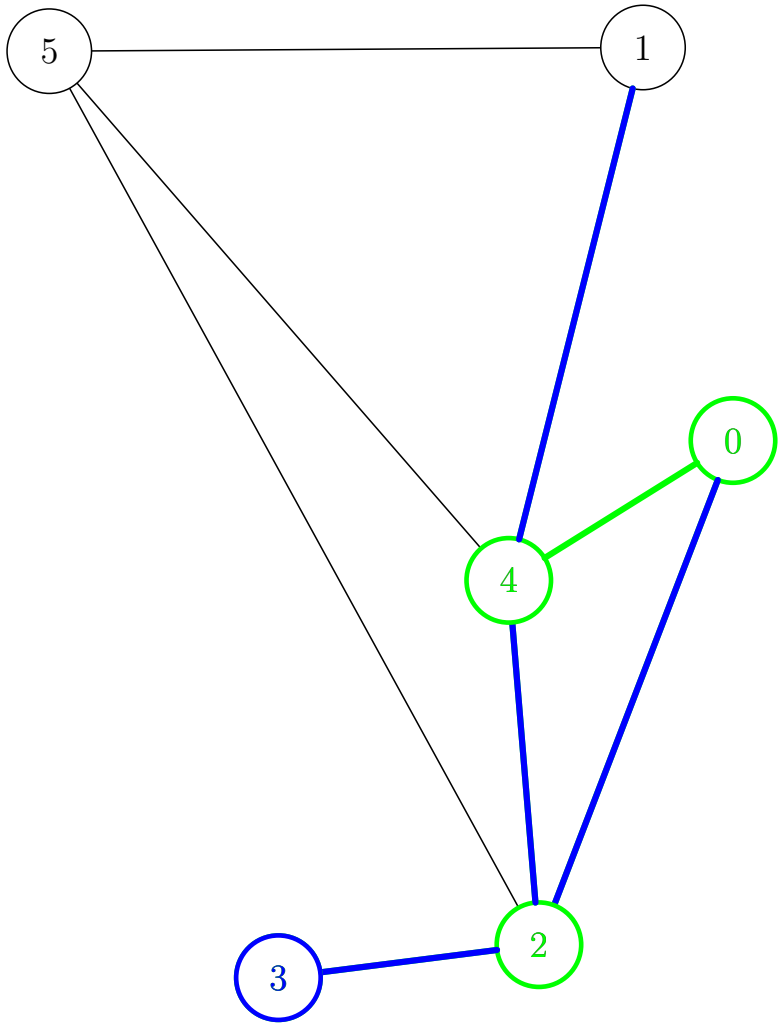

node=4 neighbour=1 time=5



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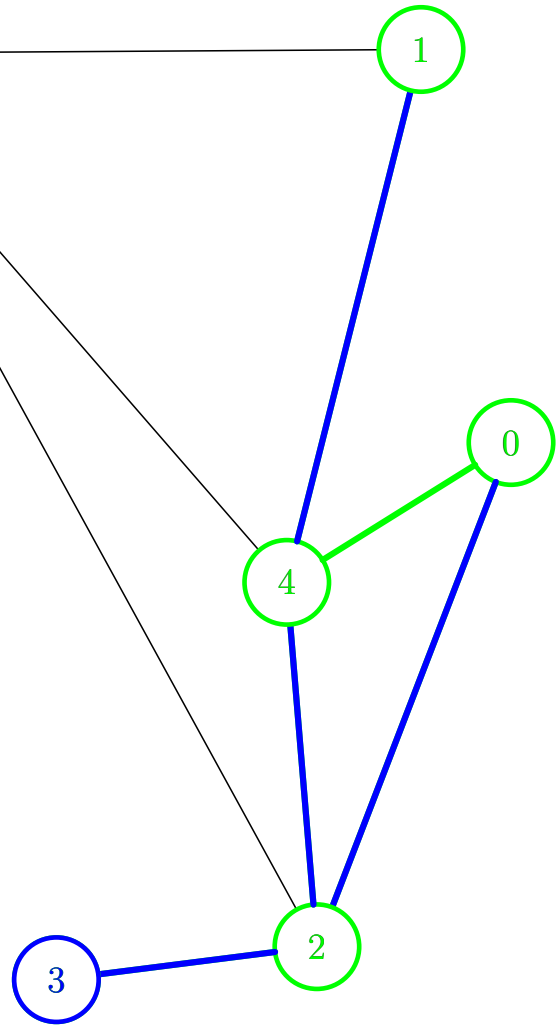
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

node=1 neighbour=nil time=6



Depth First Search

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    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
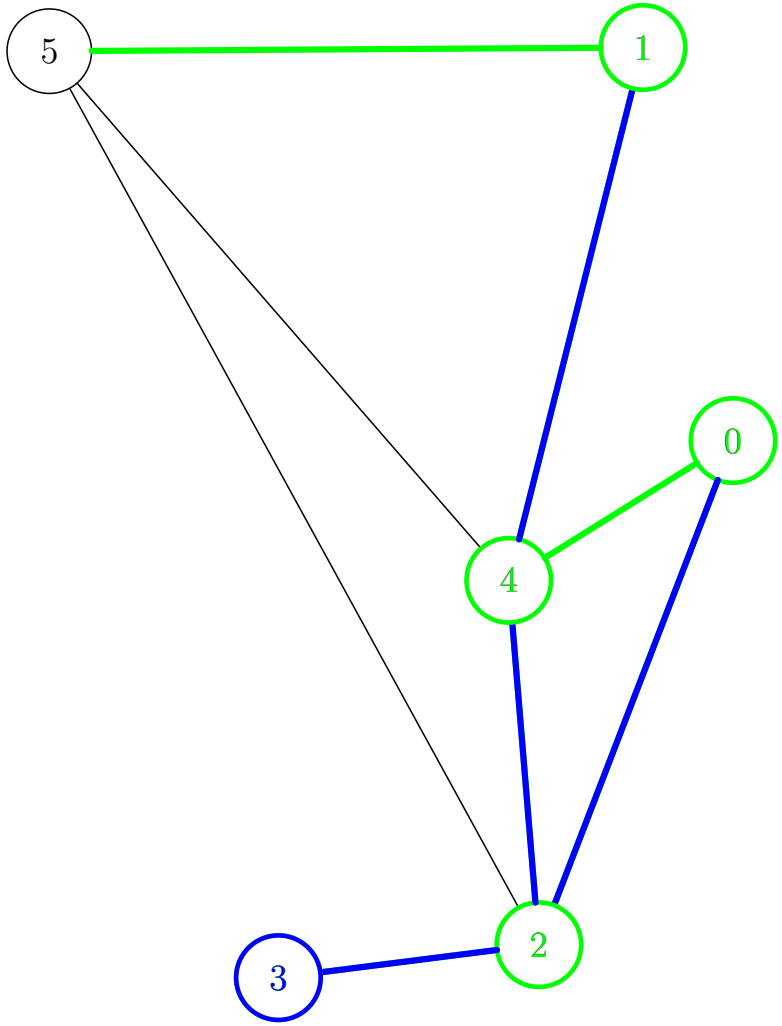

node=1 neighbour=5 time=6



Depth First Search

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  processVertexLate(currentNode)
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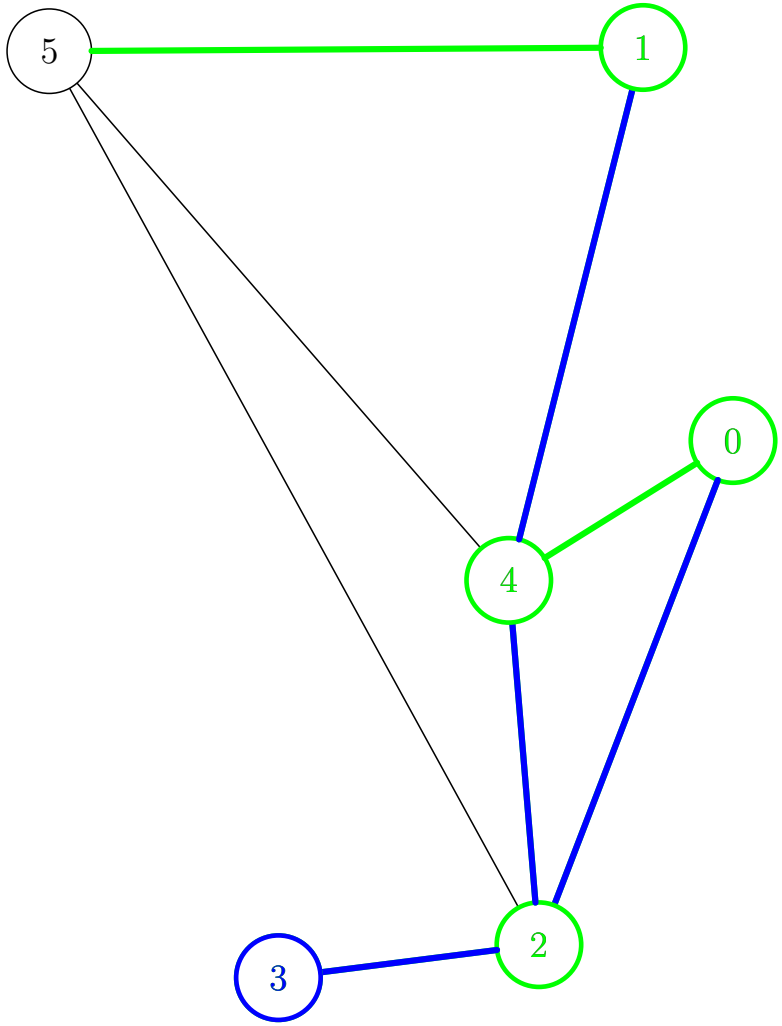
  time ← time + 1
}
```

node=1 neighbour=5 time=6



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}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
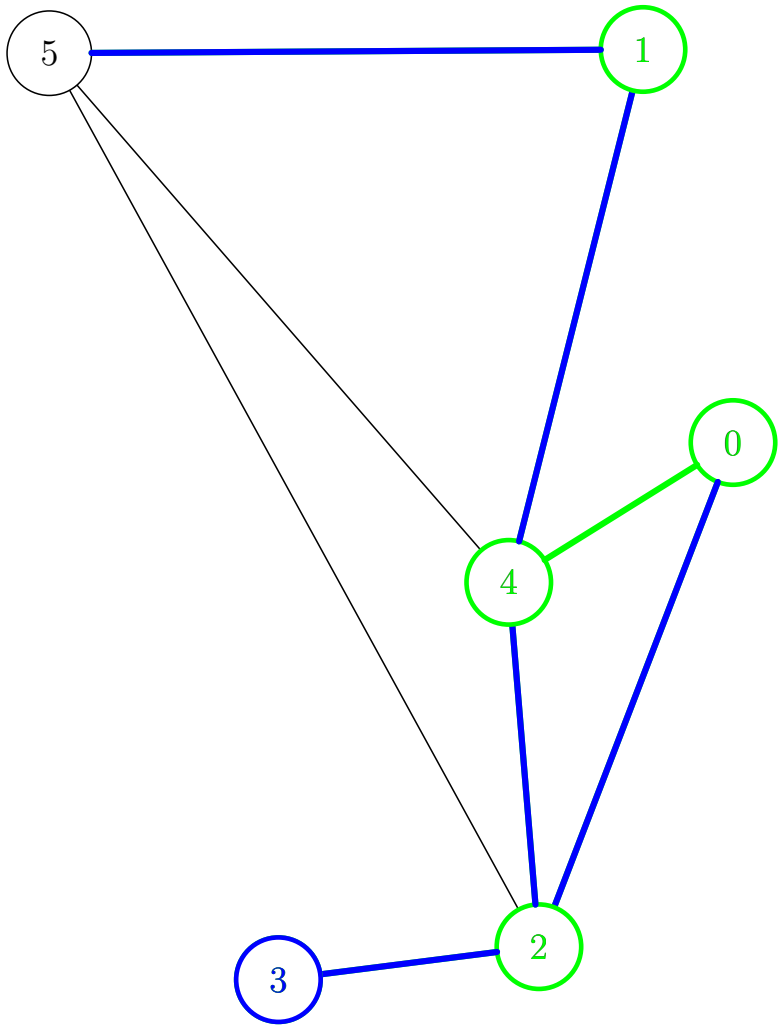
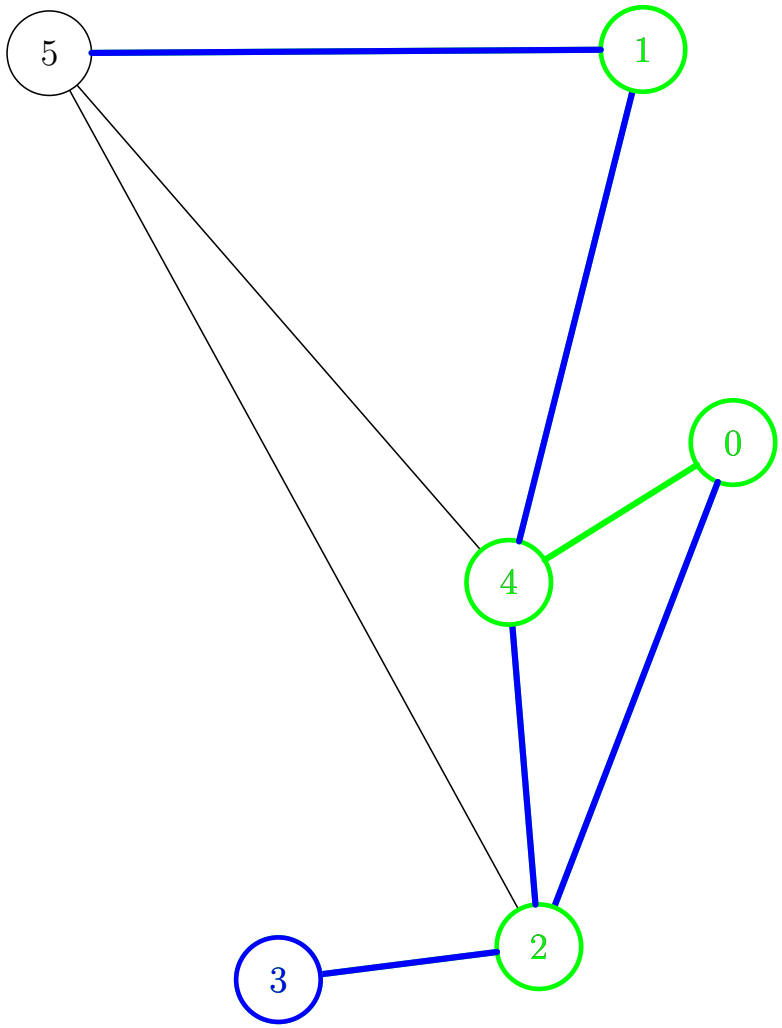

node=1 neighbour=5 time=6



Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

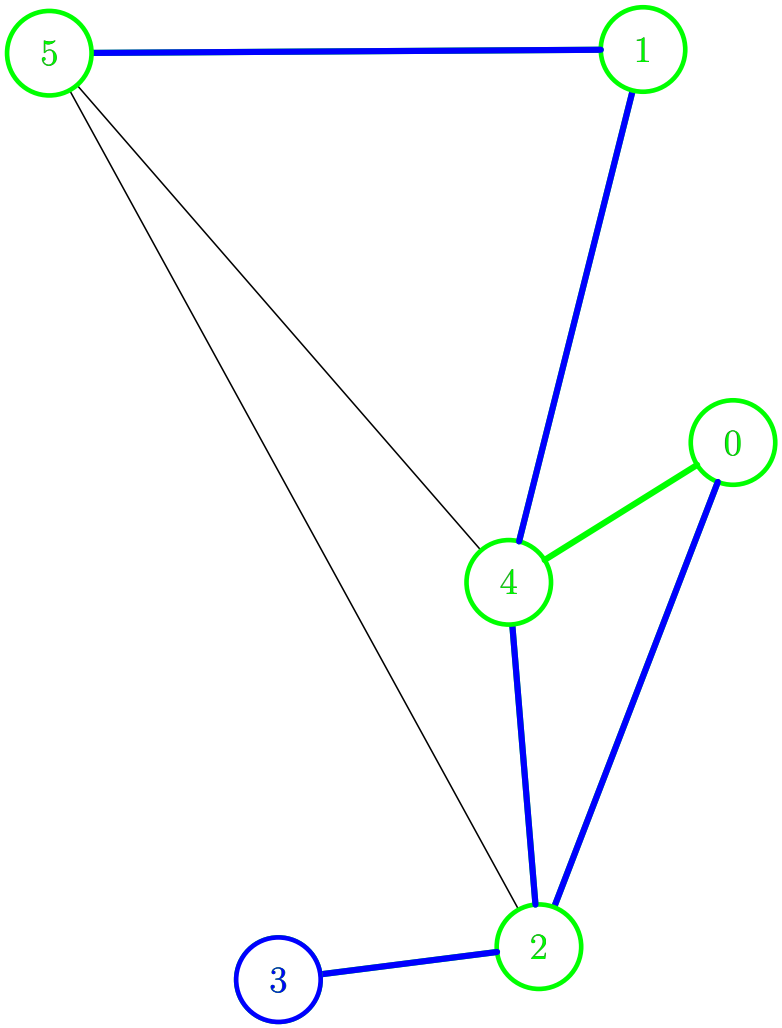
node=1 neighbour=5 time=6



Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

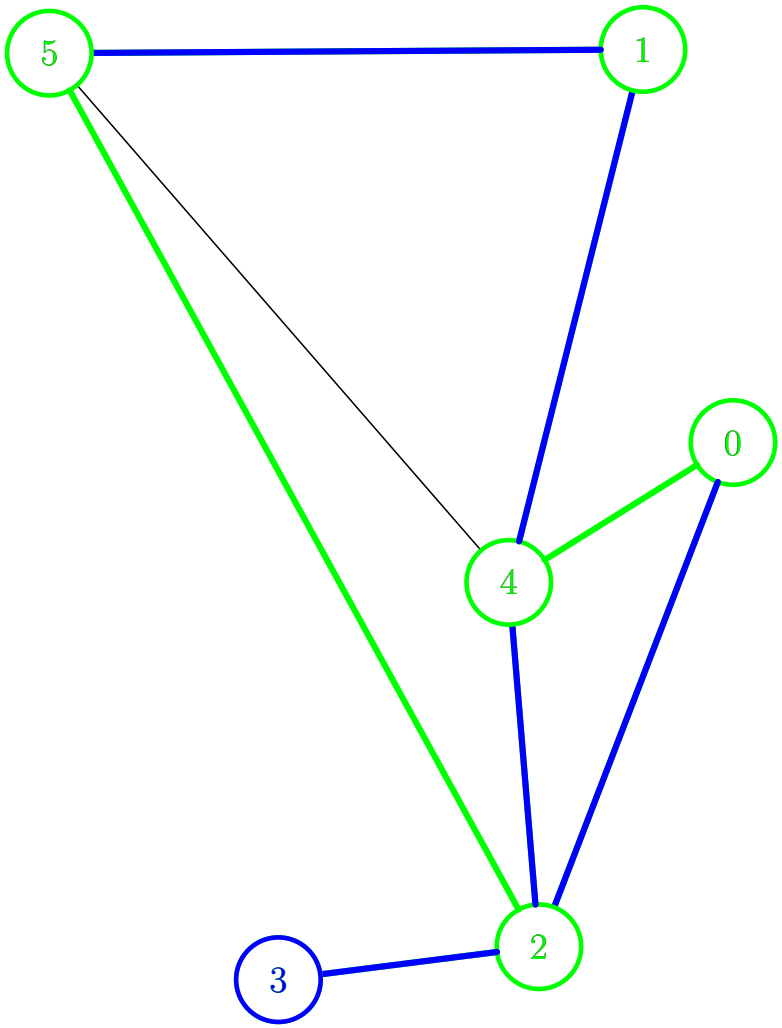
node=5 neighbour=nil time=7



Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

node=5 neighbour=2 time=7



Depth First Search

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dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
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  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
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      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
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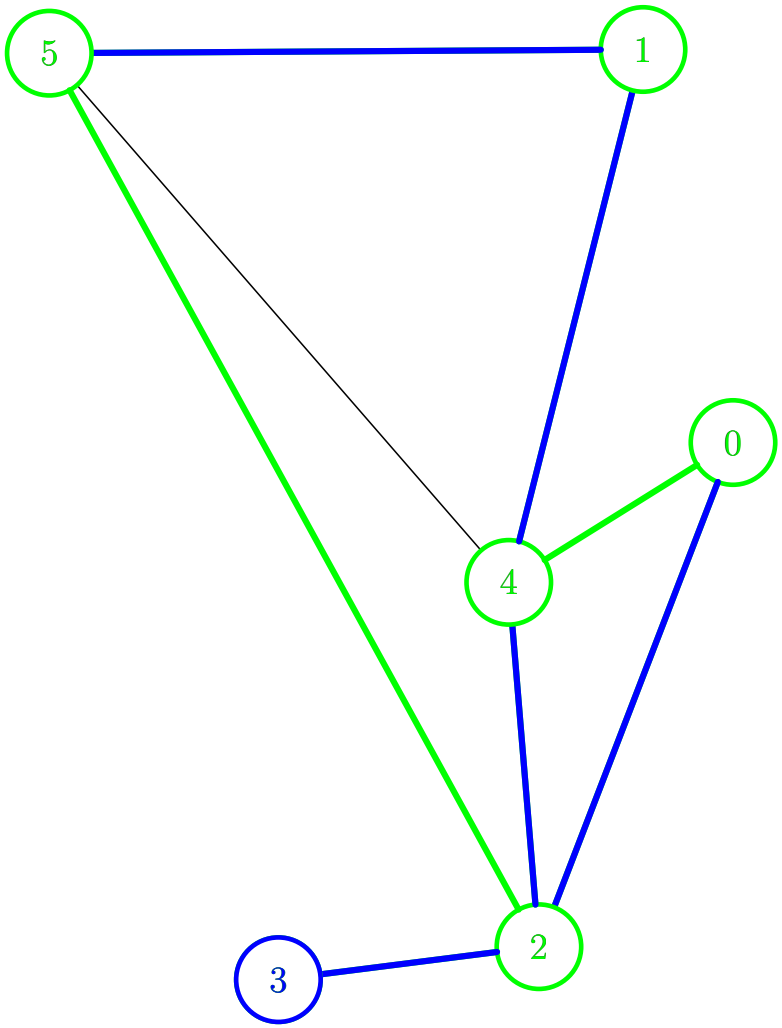
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

node=5 neighbour=2 time=7



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  time ← time + 1
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```
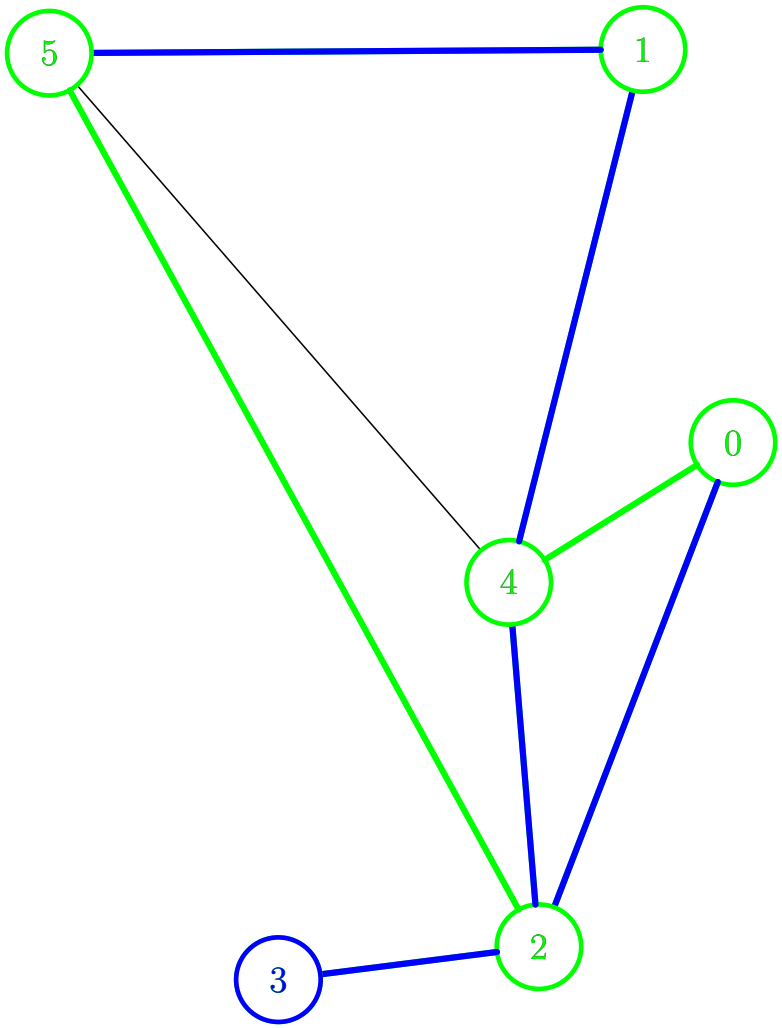

node=5 neighbour=2 time=7



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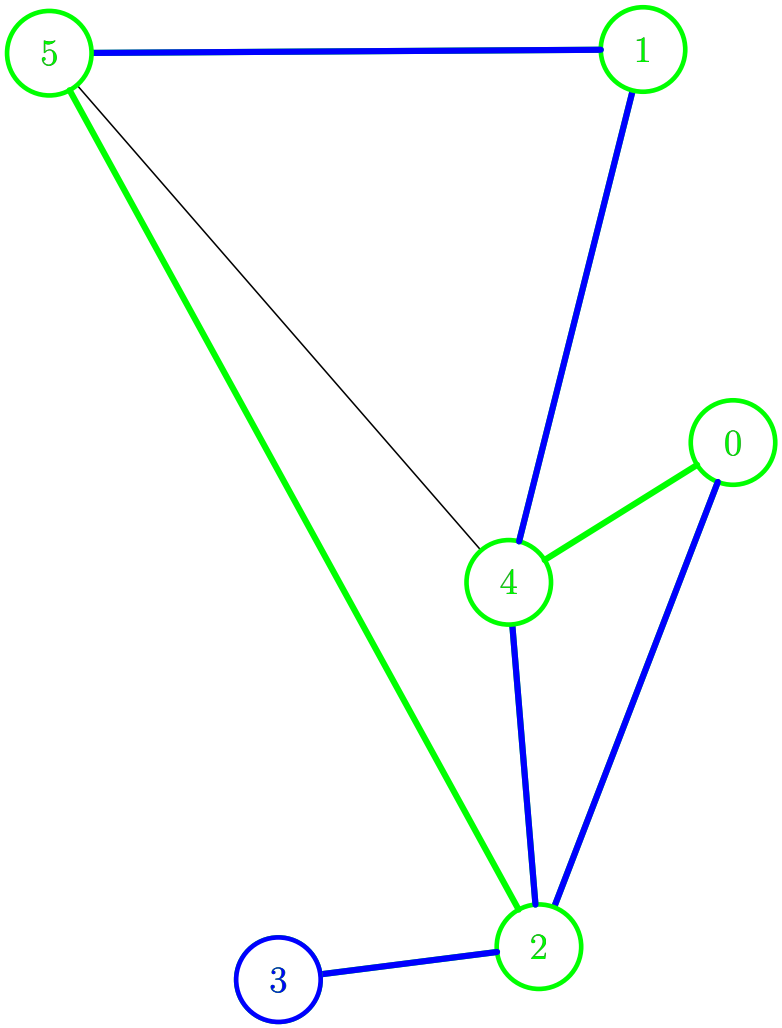
node=5 neighbour=2 time=7



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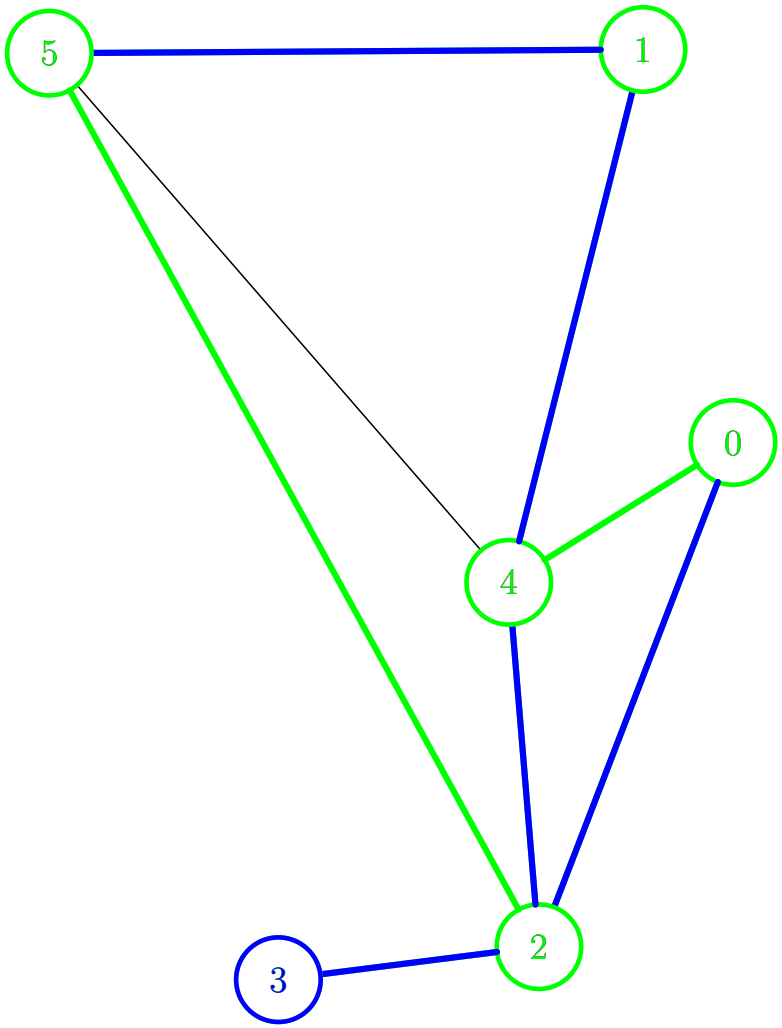
node=5 neighbour=2 time=7



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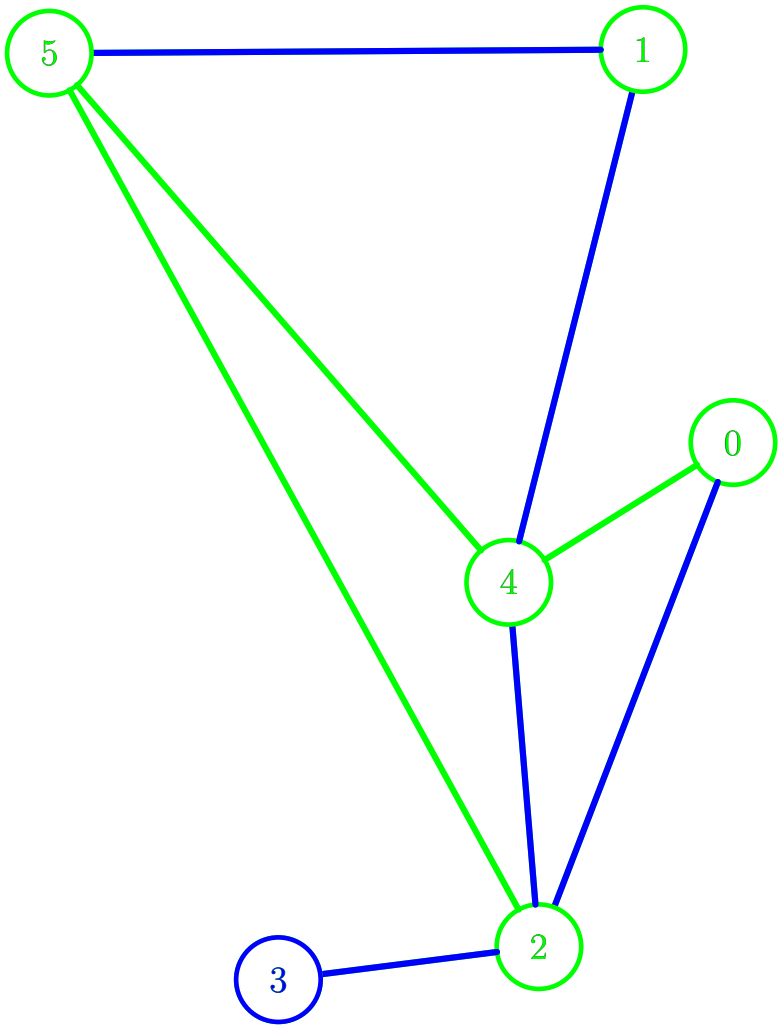
  state[currentNode] ← "processed"
  time ← time + 1
}
```

node=5 neighbour=4 time=7



Depth First Search

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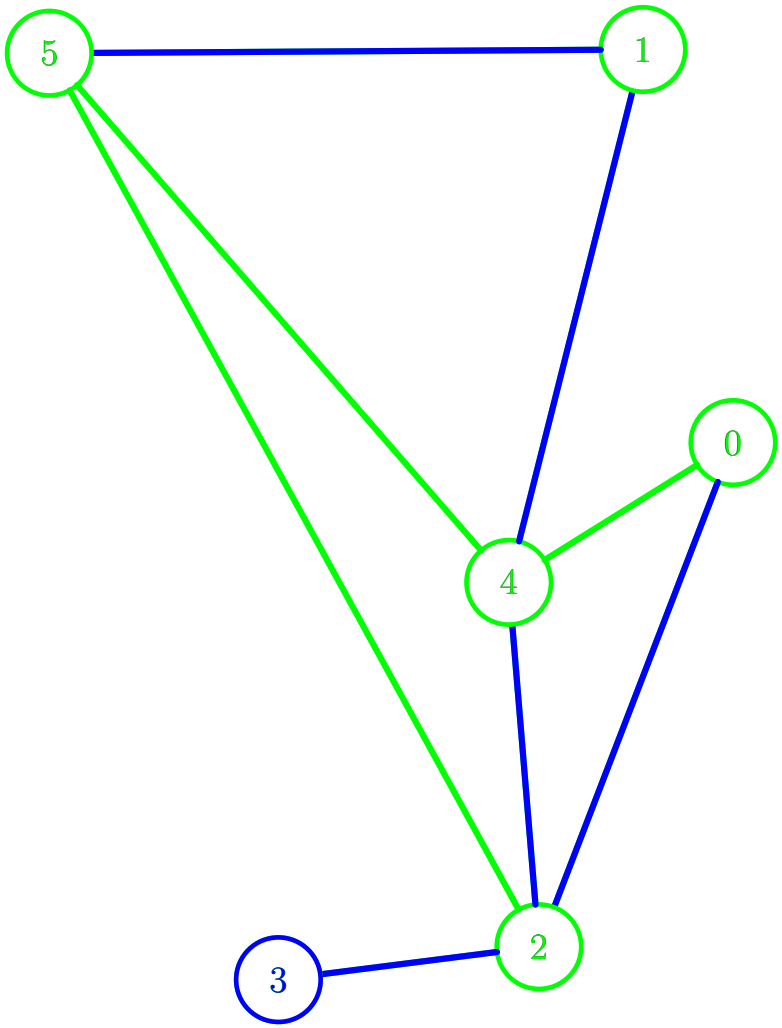
node=5 neighbour=4 time=7



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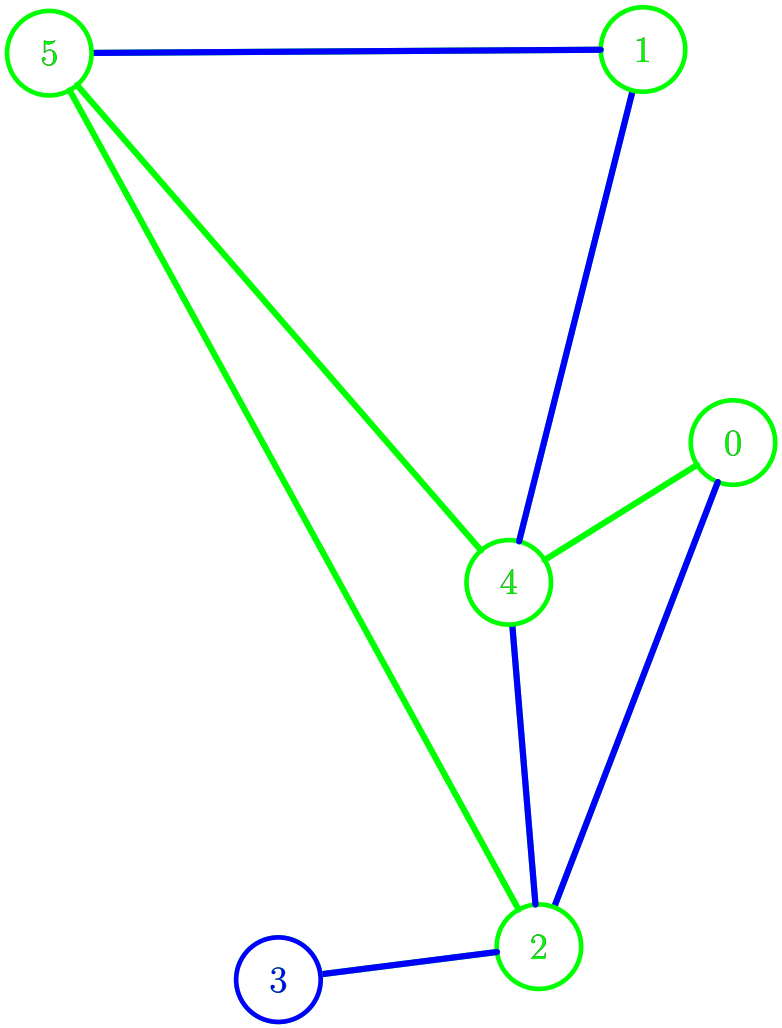
node=5 neighbour=4 time=7



Depth First Search

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```
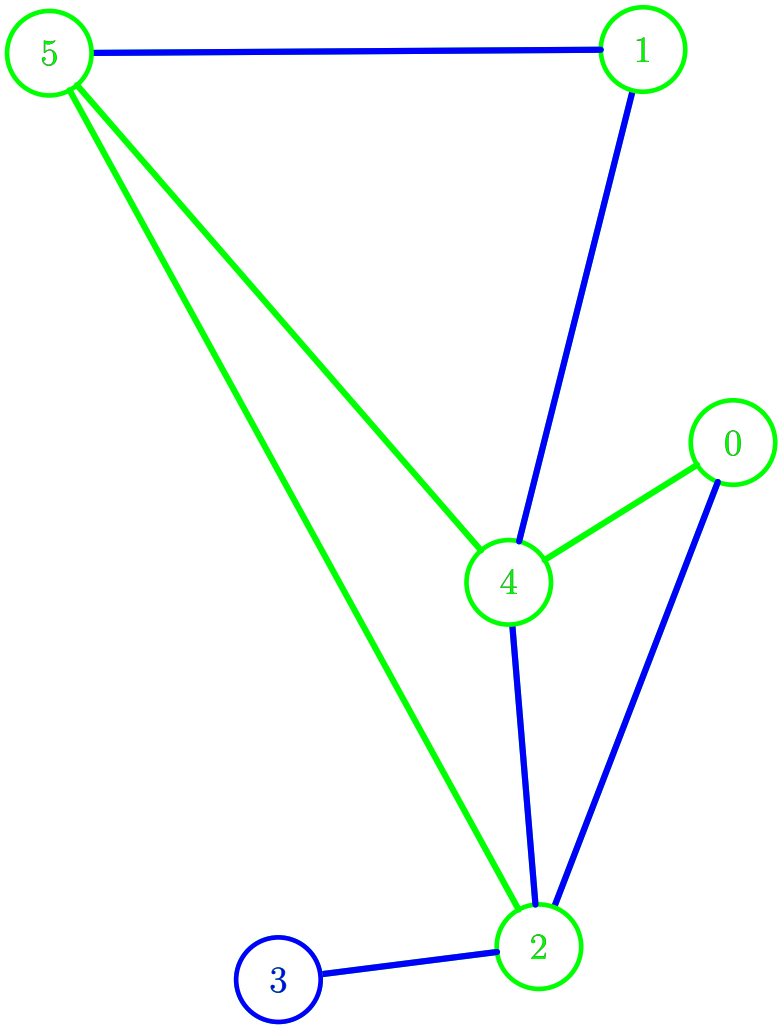

node=5 neighbour=4 time=7



Depth First Search

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  finished ← false
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      processEdge(node, neighbour)
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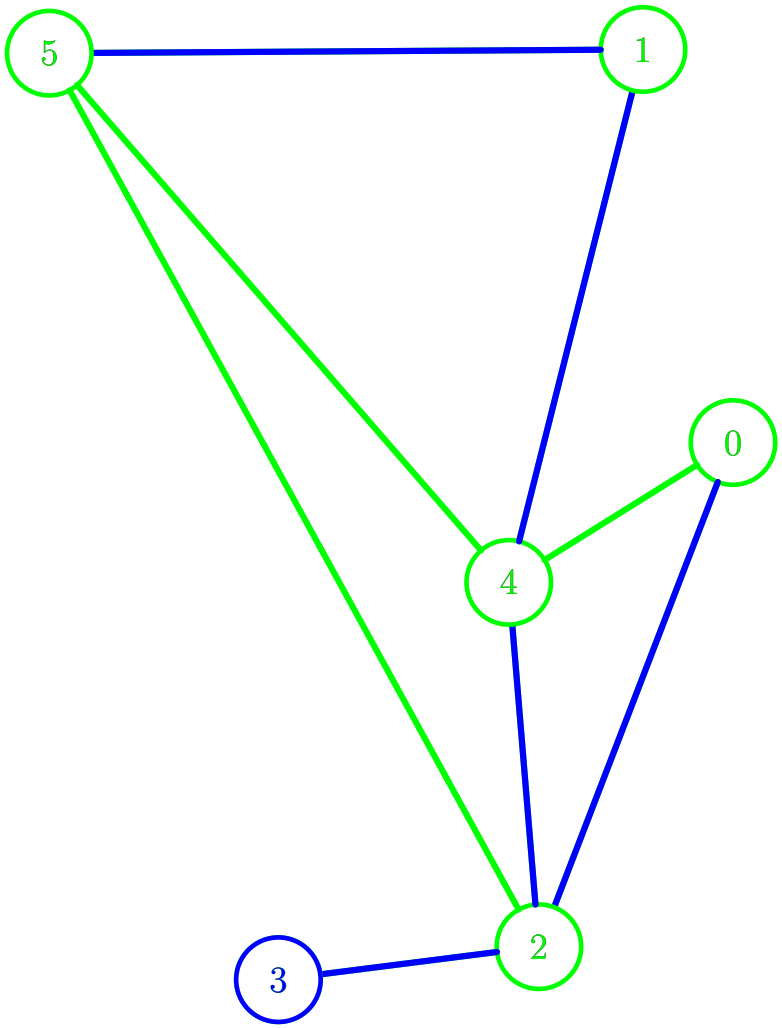
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

node=5 neighbour=4 time=7



Depth First Search

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  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
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dfs_recur(graph, node) {
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  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
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      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
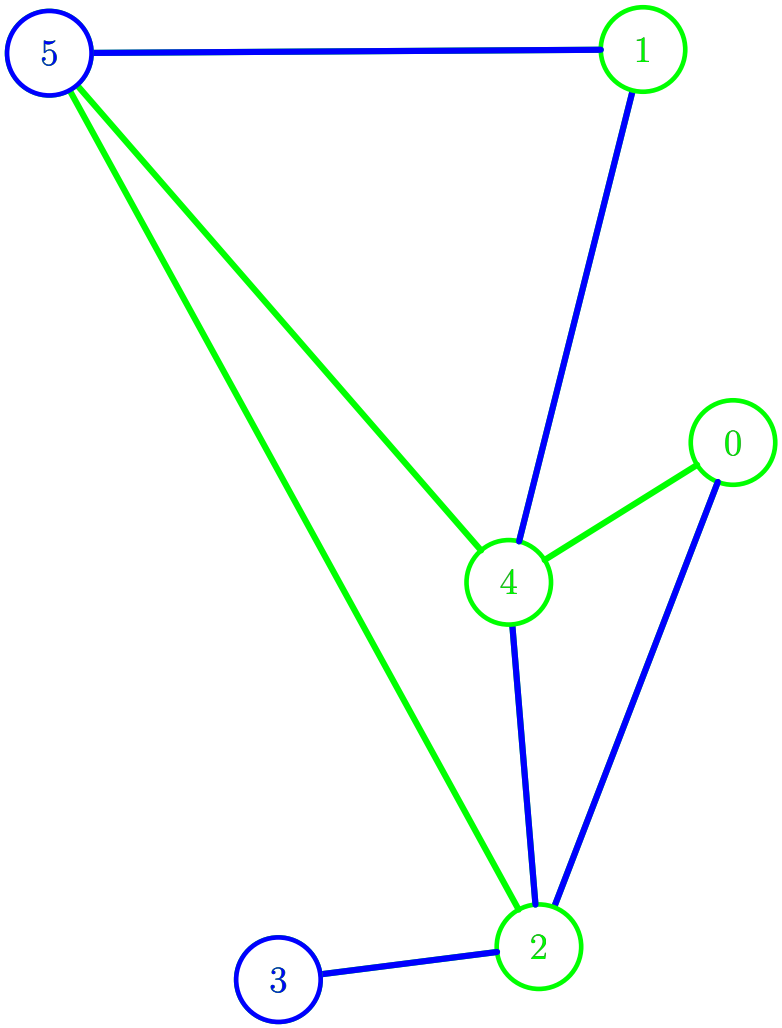
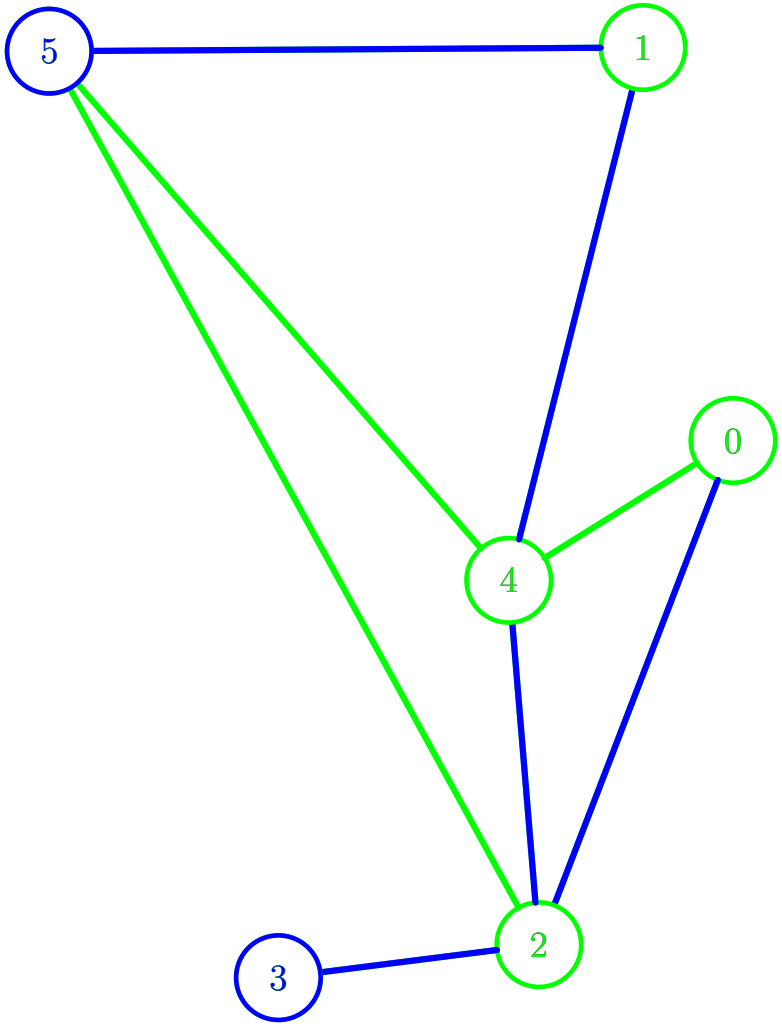

node=5 neighbour=nil time=8



Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

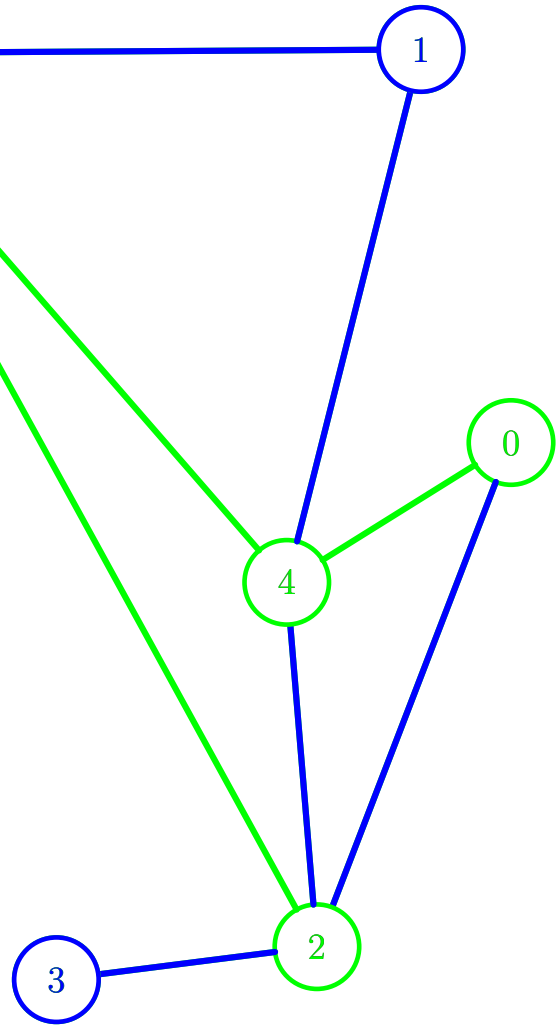
node=1 neighbour=5 time=8



Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

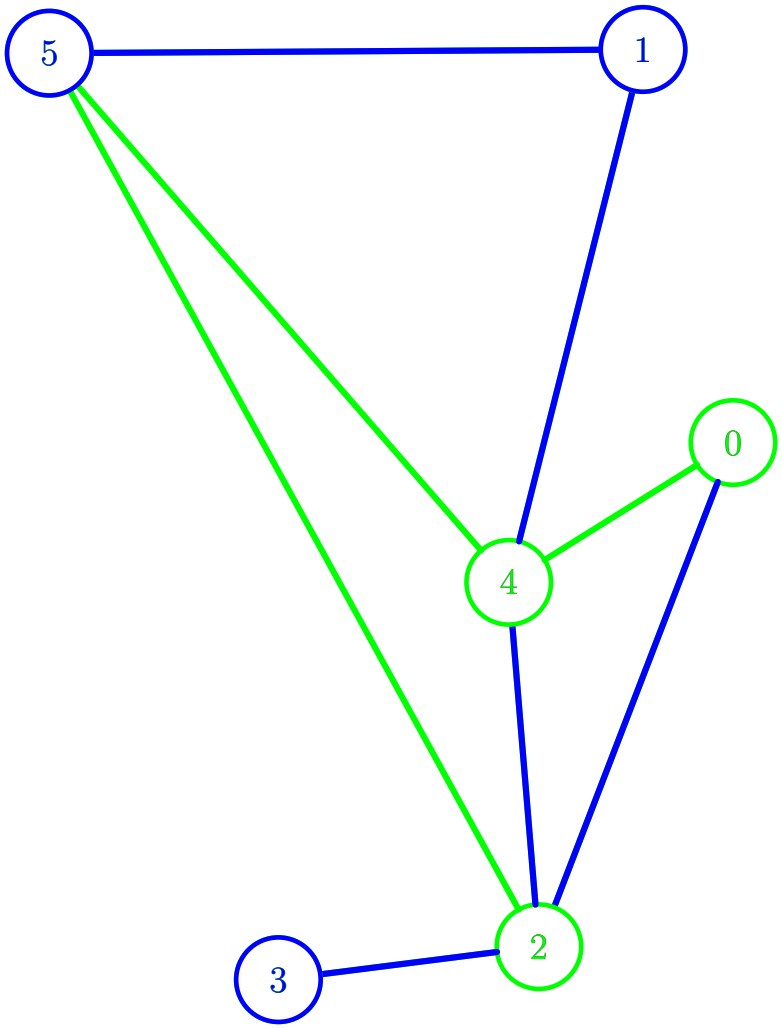
node=1 neighbour=nil time=9



Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

node=4 neighbour=1 time=9



Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
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      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
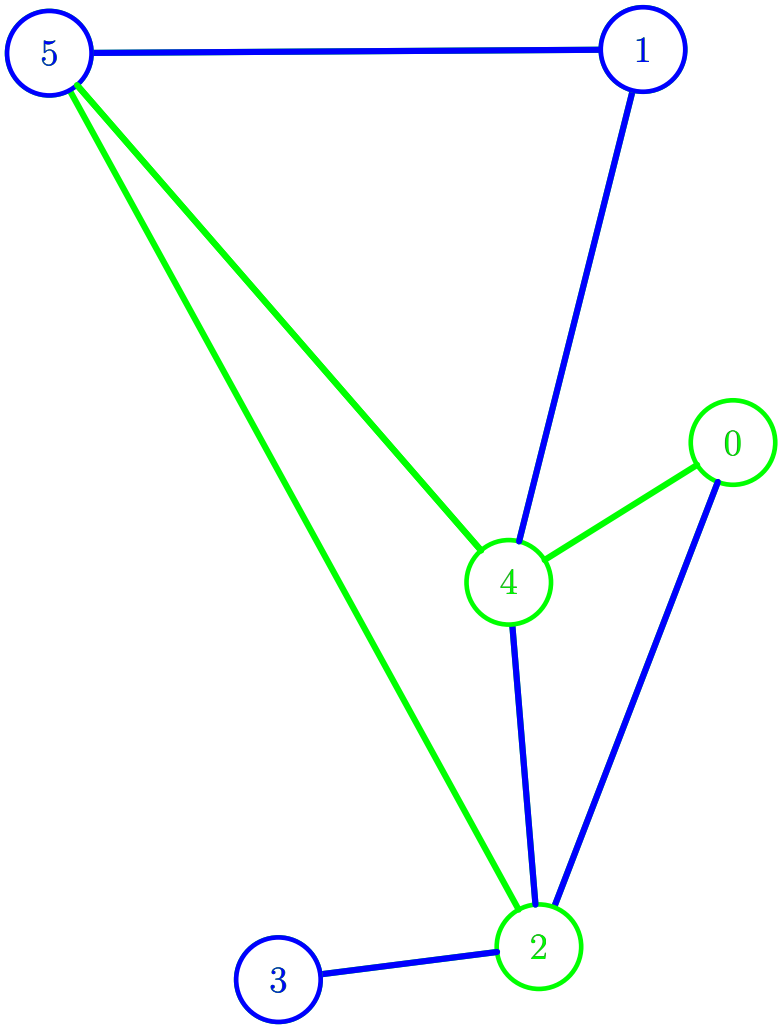

node=4 neighbour=5 time=9



Depth First Search

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dfs(graph, node) {
  state ← Array[n, "undiscovered"]
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  time ← time + 1
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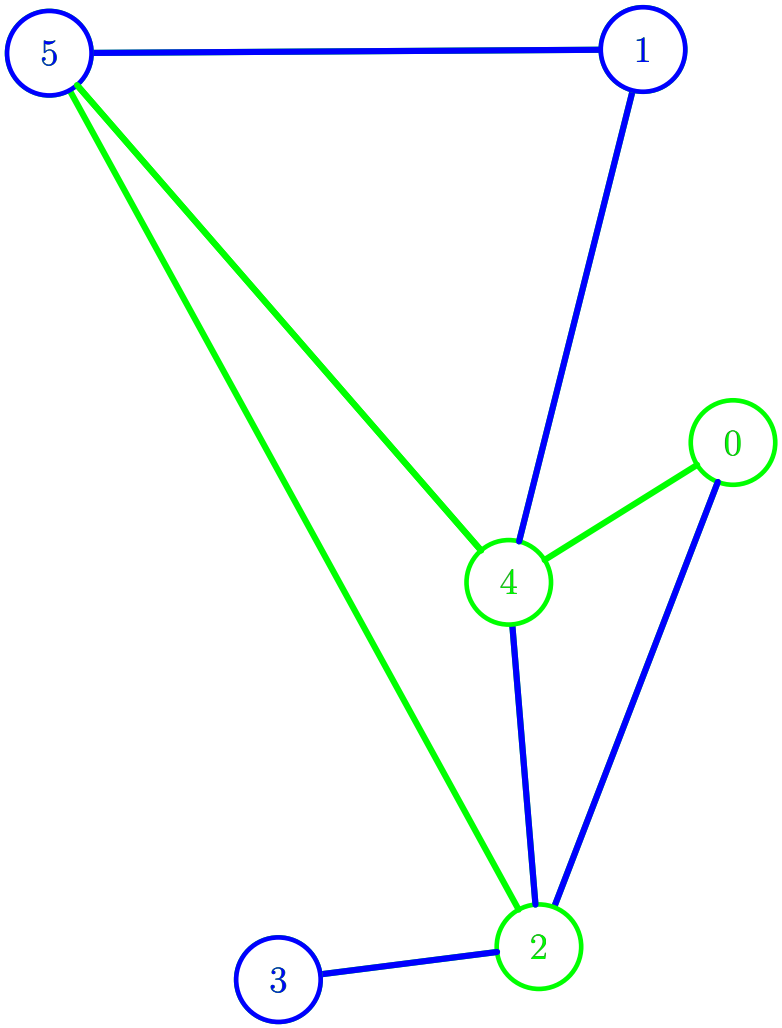
```

node=4 neighbour=5 time=9



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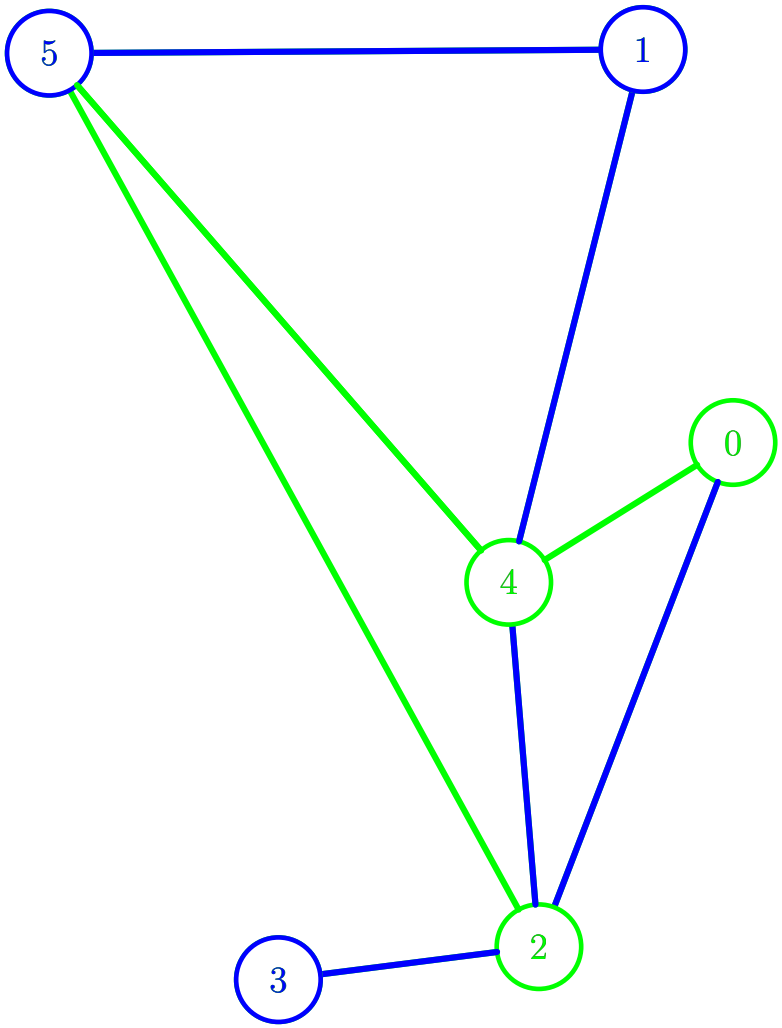
node=4 neighbour=5 time=9



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  time ← time + 1
}
```
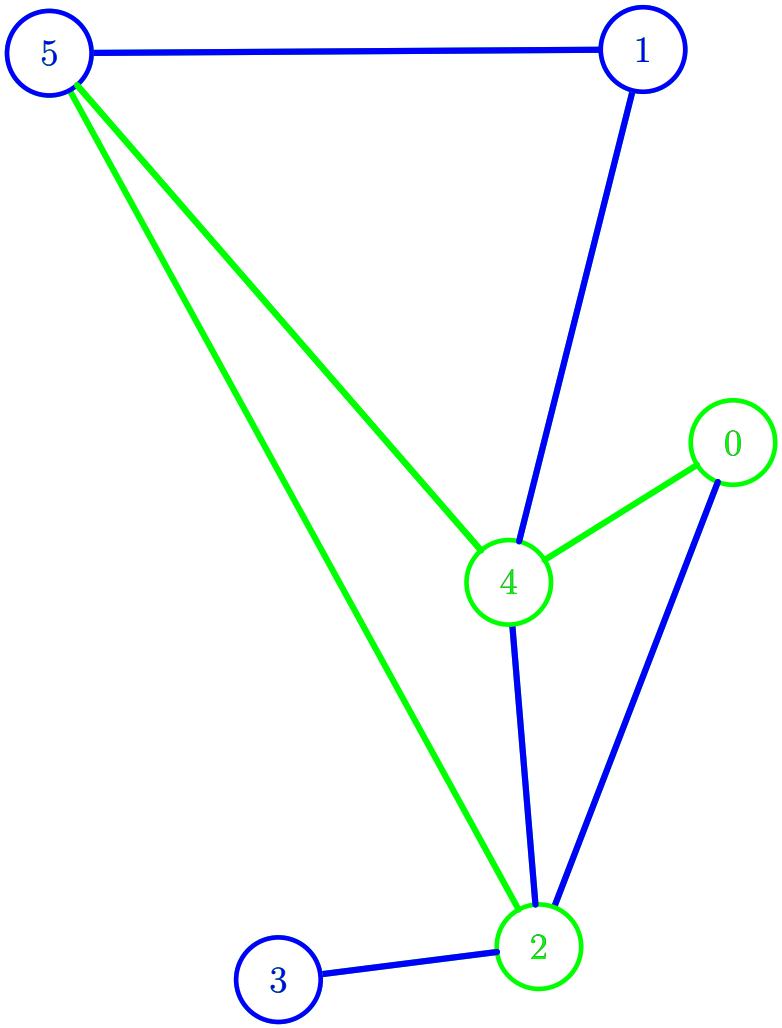
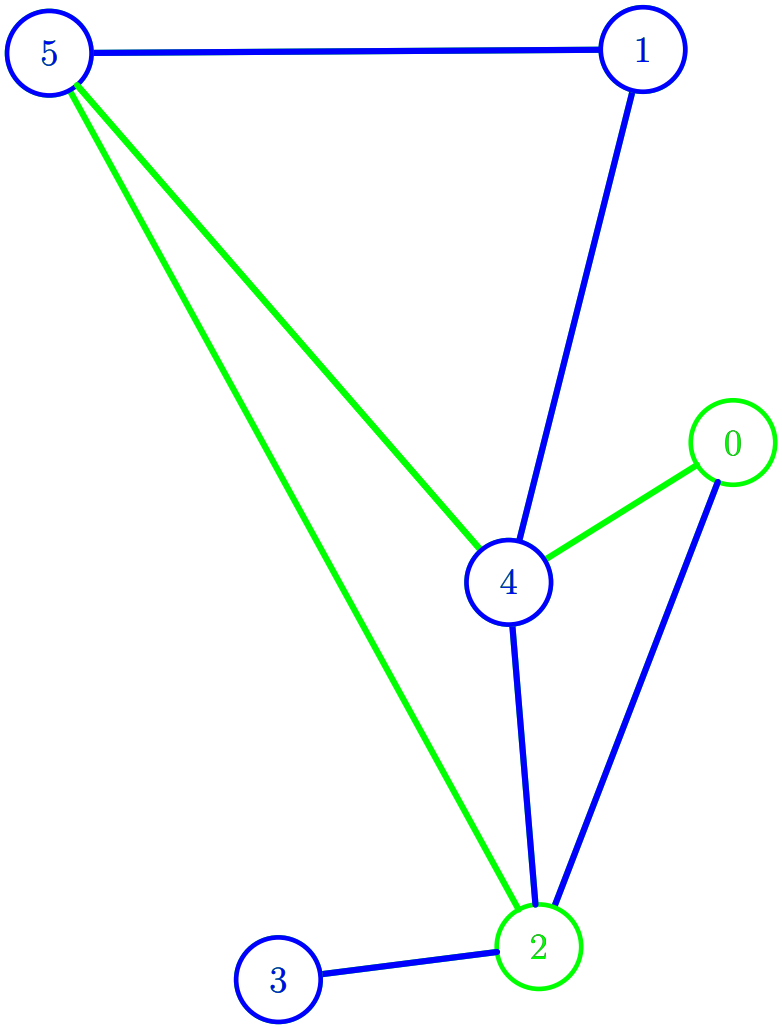

node=4 neighbour=5 time=9



Depth First Search

```
dfs(graph, node) {
  state ← Array[n, "undiscovered"]
  finished ← false
  dfs_recur(graph, node)
}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] = "undiscovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

node=4 neighbour=nil time=10



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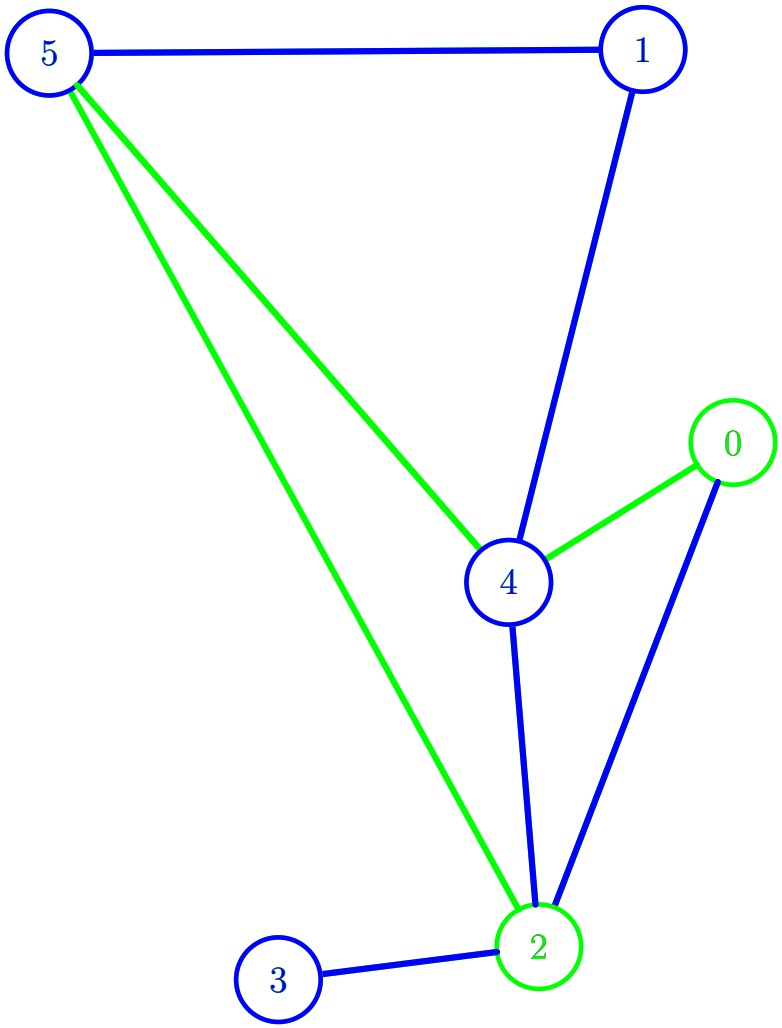
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

node=2 neighbour=4 time=10



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  state[currentNode] ← "processed"
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}
```
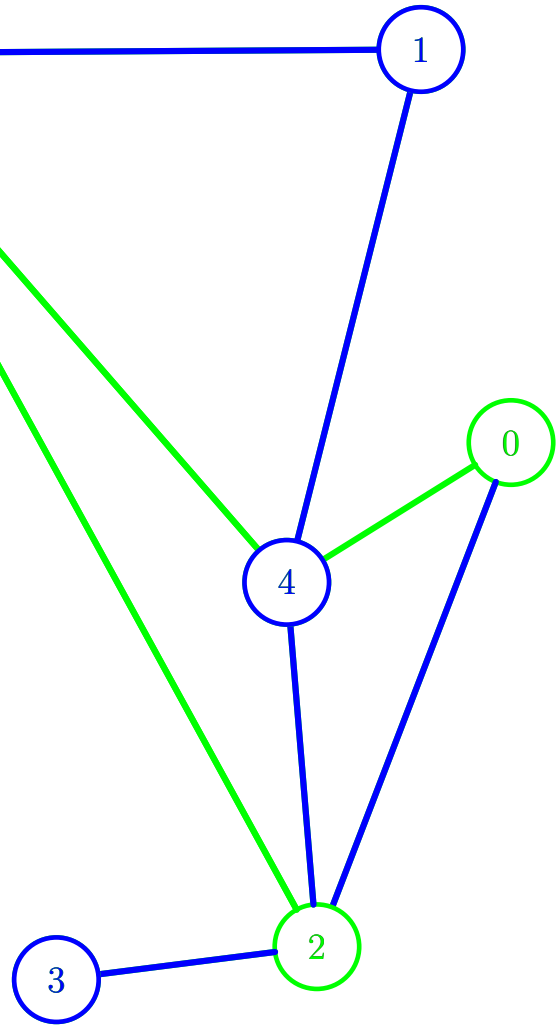

node=2 neighbour=5 time=10



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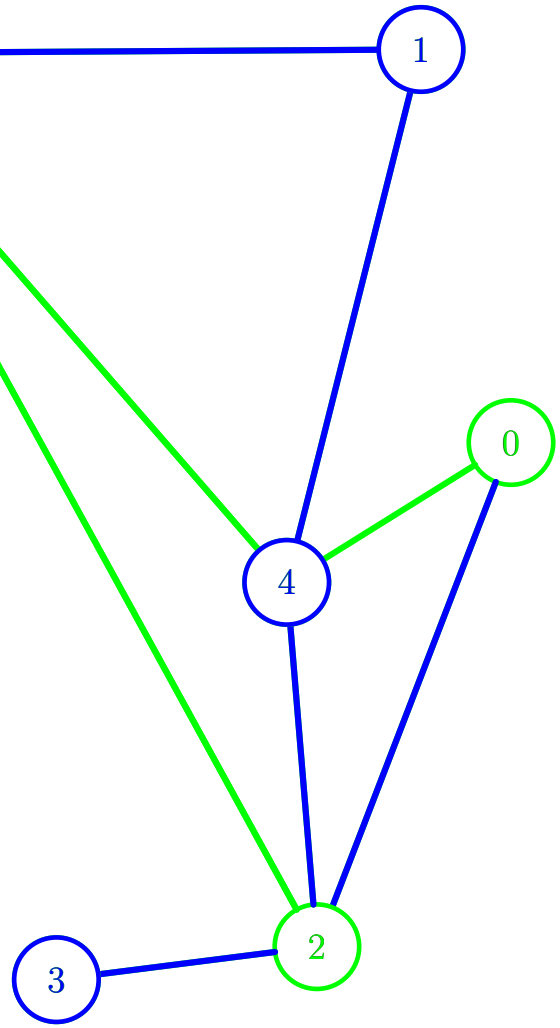
  time ← time + 1
}
```

node=2 neighbour=5 time=10



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  state[node] ← "discovered"
  time ← time + 1
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  foreach neighbour ∈ Neighbourhood(node) {
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      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
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      processEdge(node, neighbour)
    }
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  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
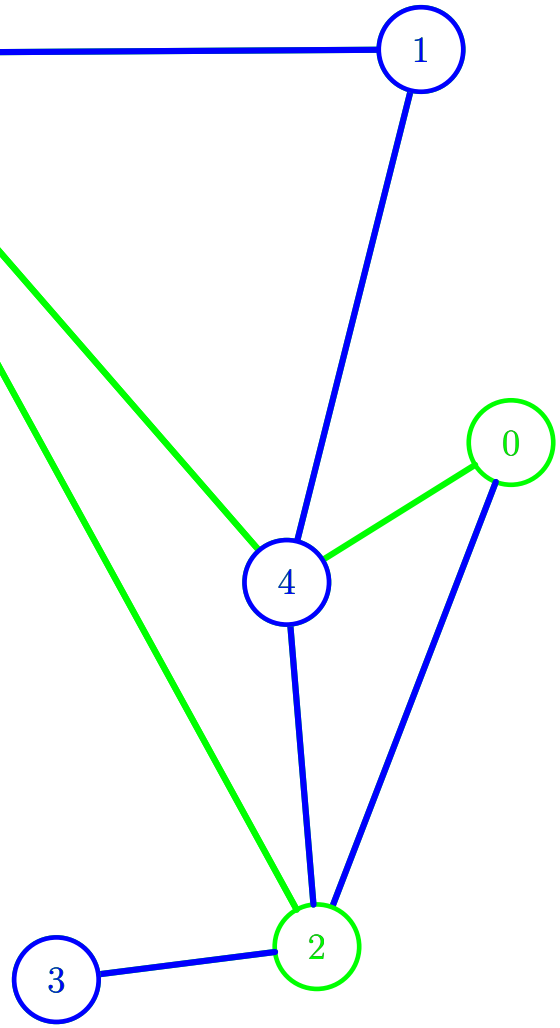

node=2 neighbour=5 time=10



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```

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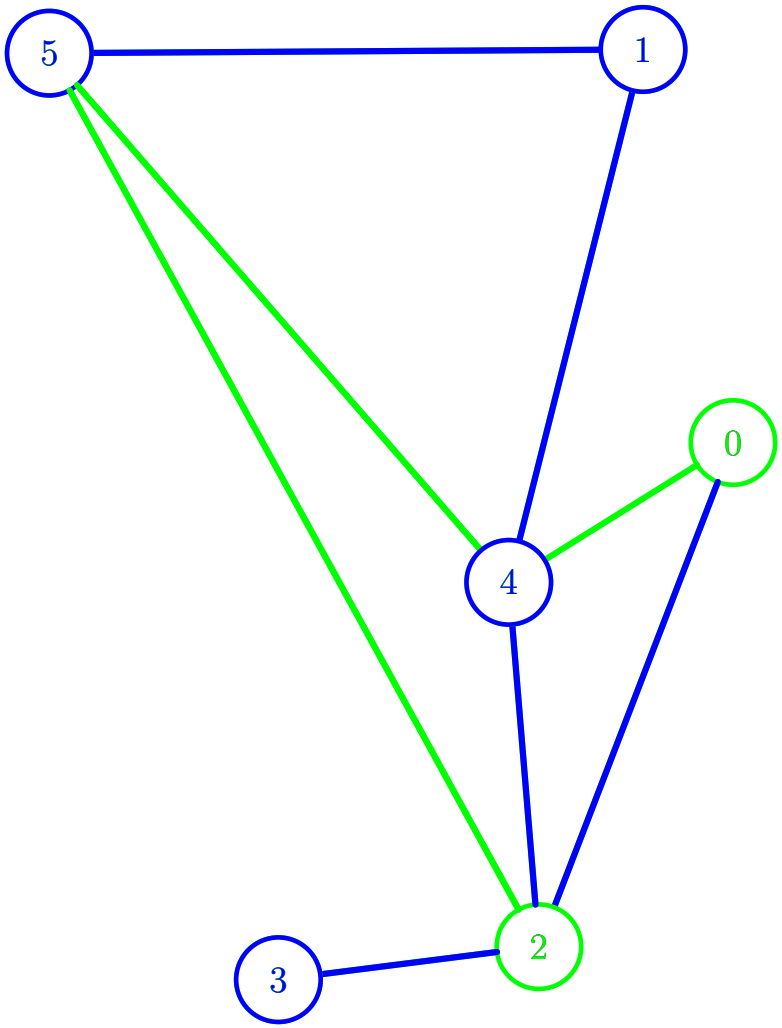

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}

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  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
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      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
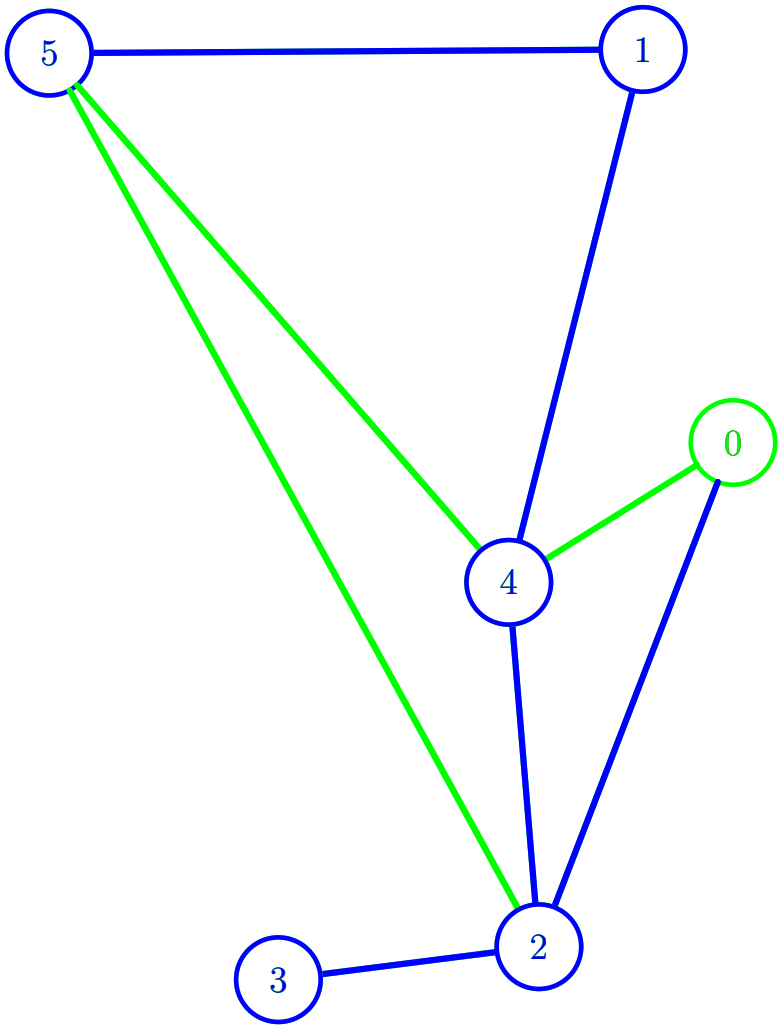

node=2 neighbour=nil time=11



Depth First Search

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  finished ← false
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}

dfs_recur(graph, node) {
  if (finished) return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
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      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
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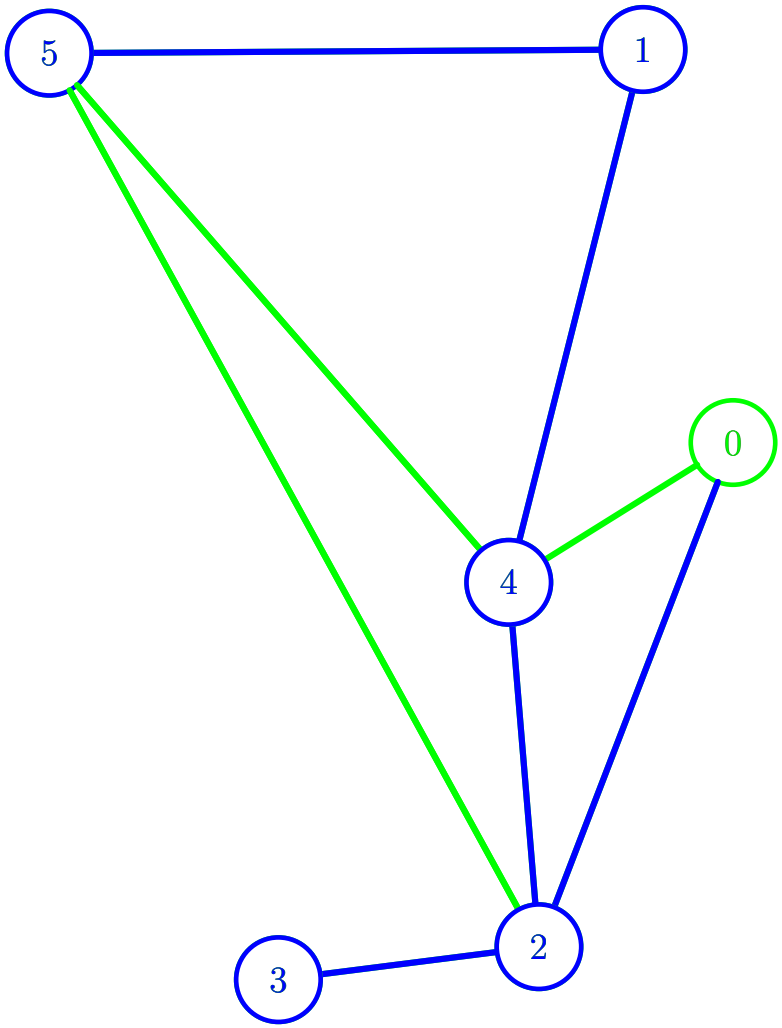
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

node=0 neighbour=2 time=11



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  time ← time + 1
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  foreach neighbour ∈ Neighbourhood(node) {
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      processEdge(node, neighbour)
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    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
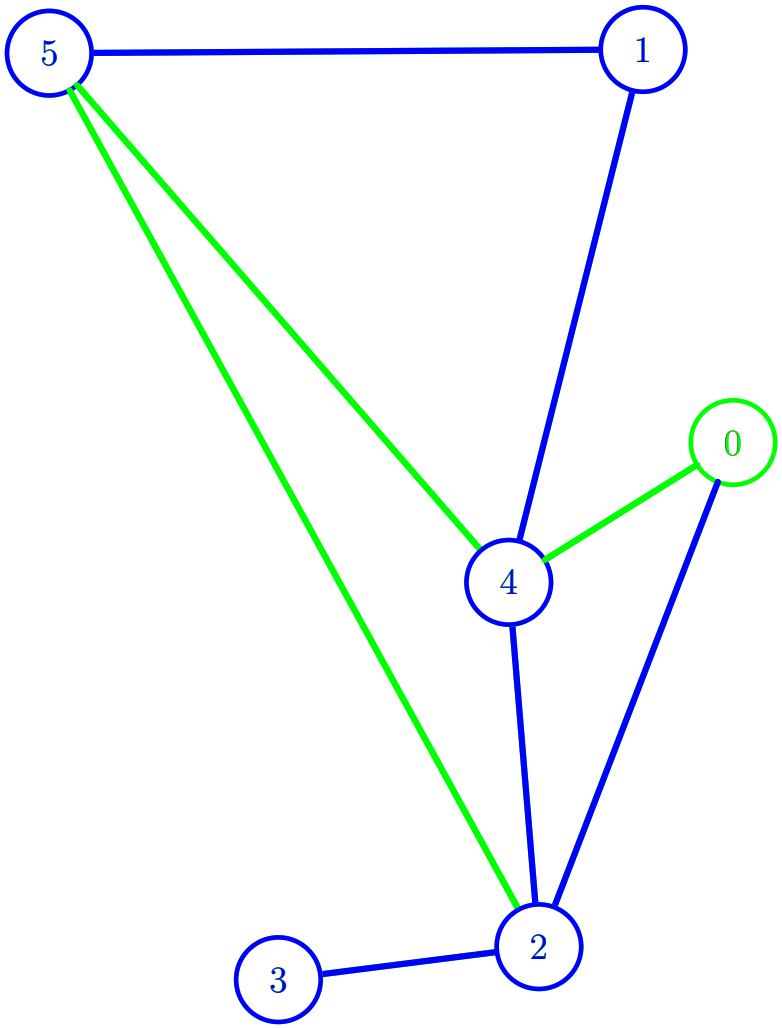

node=0 neighbour=4 time=11



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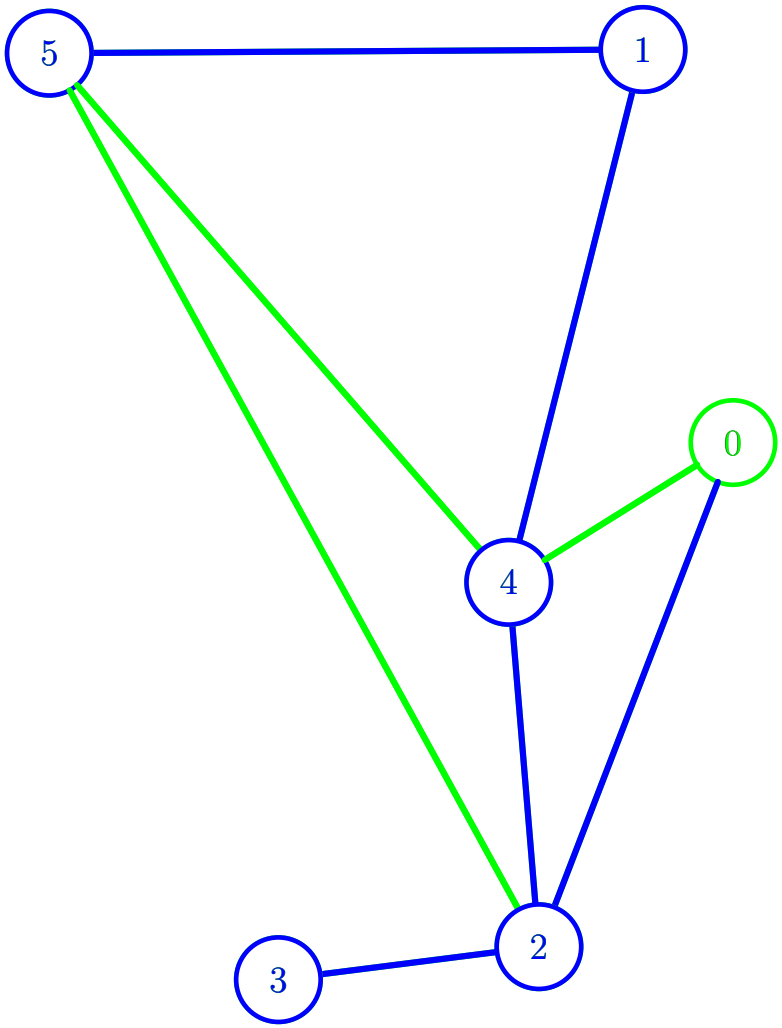
}
```

node=0 neighbour=4 time=11



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  foreach neighbour ∈ Neighbourhood(node) {
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  state[currentNode] ← "processed"
  time ← time + 1
}
```
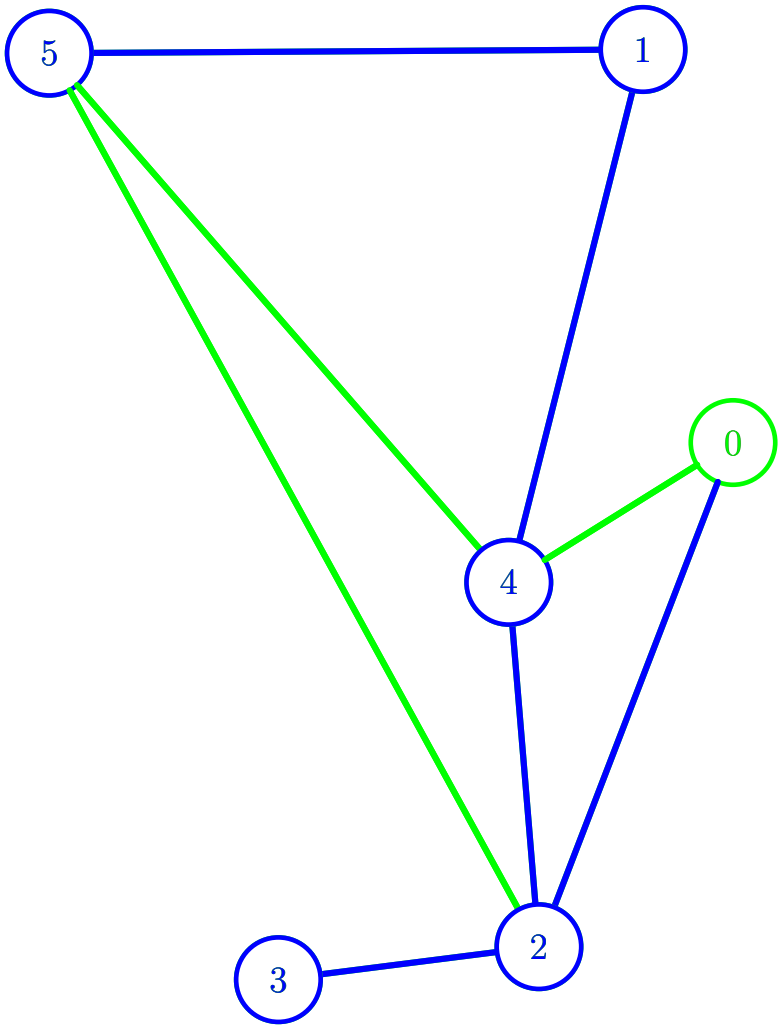

node=0 neighbour=4 time=11



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  foreach neighbour ∈ Neighbourhood(node) {
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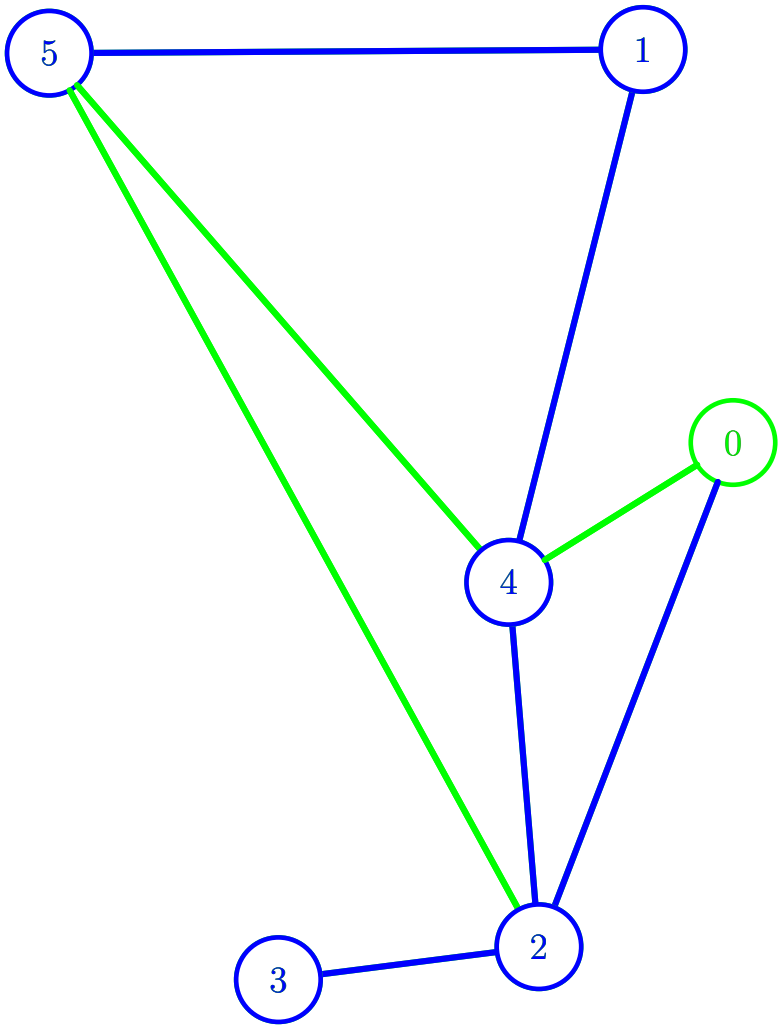
}
```

node=0 neighbour=4 time=11



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      processEdge(node, neighbour)
      dfs_recur(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if (finished) return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```
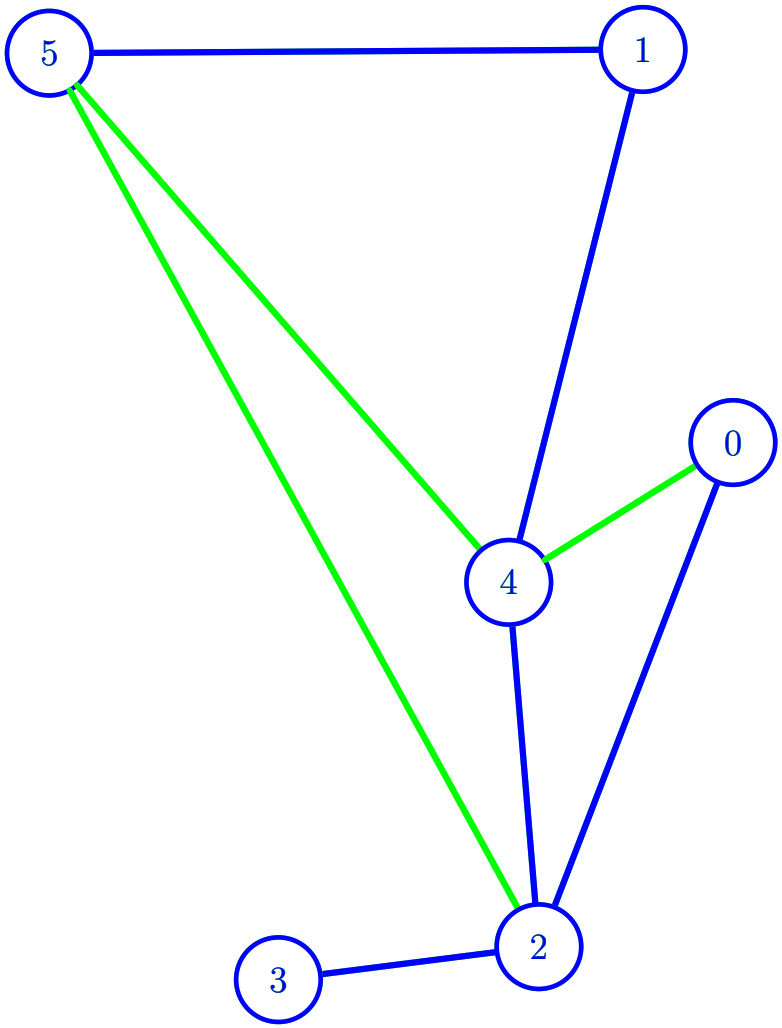

node=0 neighbour=nil time=12



Applications of DFS

- Depth first search has many applications
- Suppose we want to check if the graph is a tree (i.e. has no cycles)
- The only edges that are allowed are parent edges

```
processEdges(node1, node2) {
  if (parent[node1] ≠ node2) {
    isTree ←false
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(note that we set `finish` to stop DFS prematurely)

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Biconnected Graphs and Articulation Vertices

- If we removed a vertex (and all its edges) from a graph would the graph become disconnected?
- Such nodes are called **articulation vertices**
- Graphs with no articulation vertices are said to be **biconnected**
- In many applications articulation vertices are important, e.g. they represent single point of failures in communication networks
- A brute force method for identifying articulation vertices is to remove each and check for connectivity—this would take $O(n(m+n))$ time

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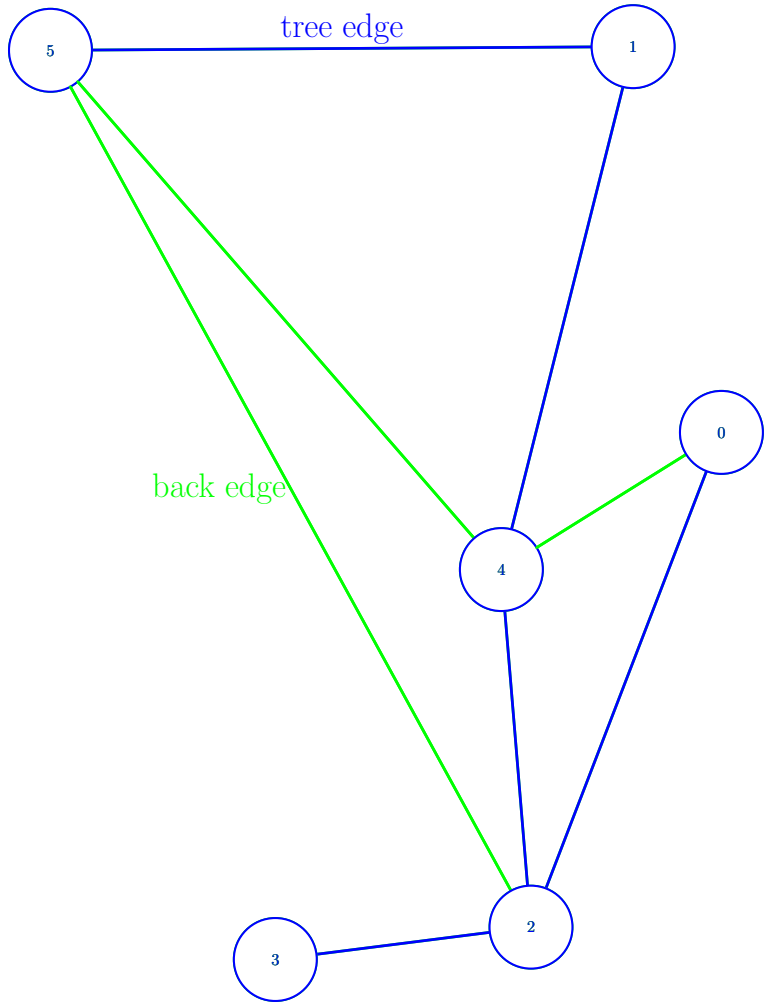
- A brute force method for identifying articulation vertices is to remove each and check for connectivity—this would take $O(n(m+n))$ time. Can we do this any faster?

Single Pass Algorithm

- In DFS we divide the edges into tree edges that define the search tree (edges between nodes and parents) and back edges which take us back to vertices we have already seen
- Without the back edges we have a tree where all non-leaf nodes are articulation nodes
- The back edges secure the edges to the rest of the tree



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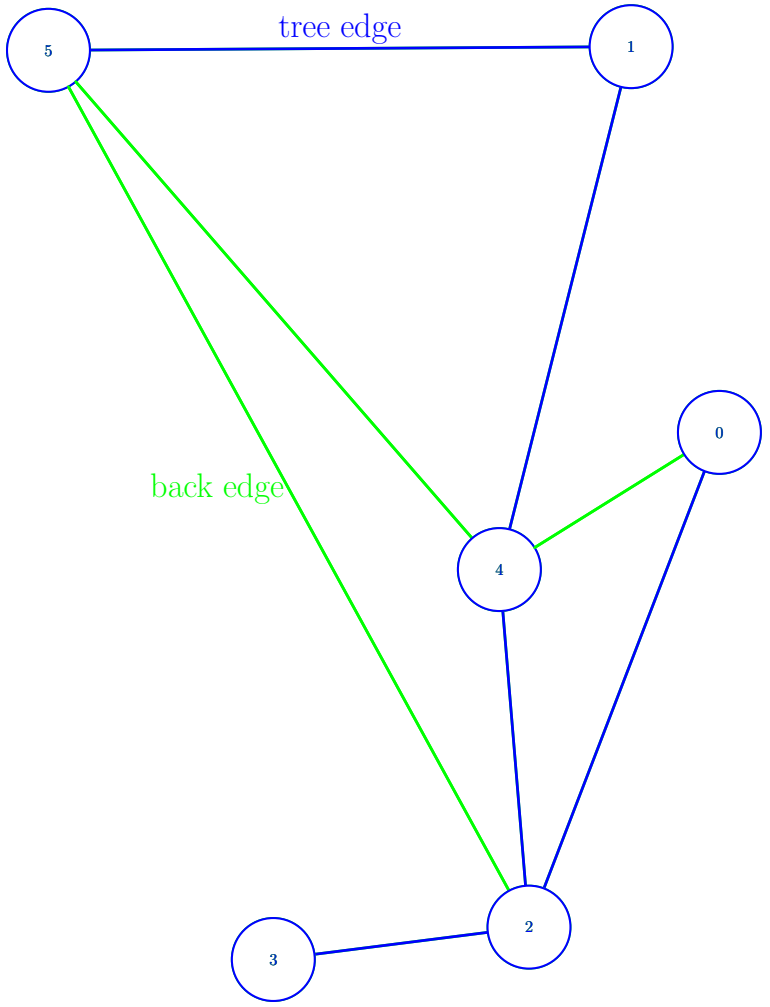

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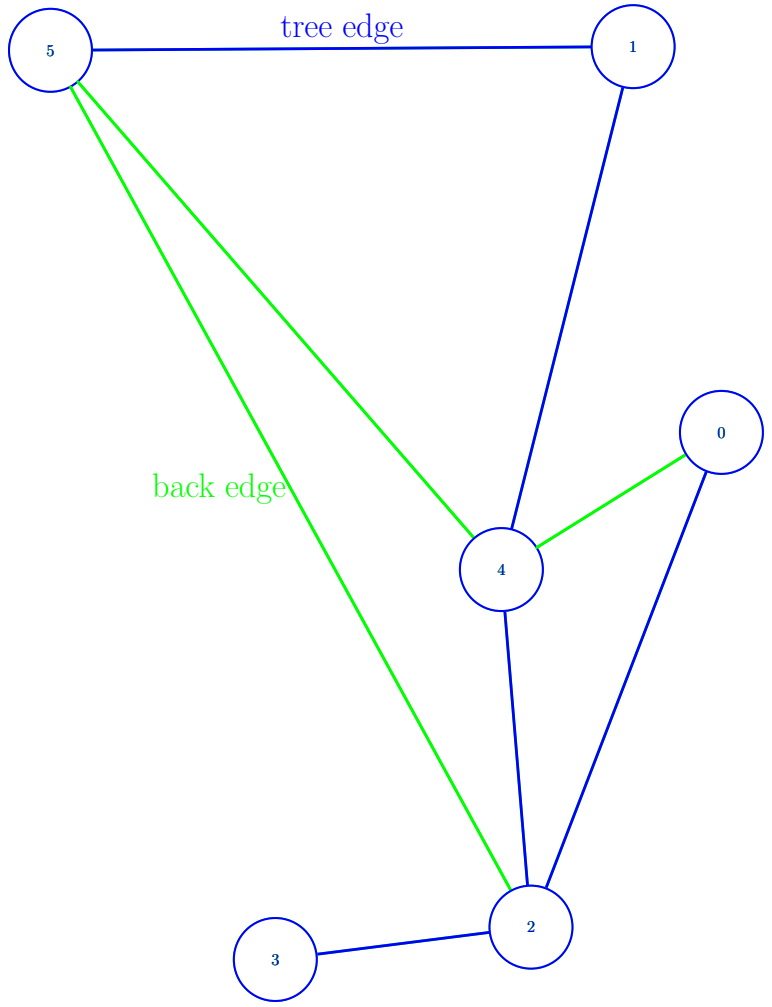

Reachable Ancestors

- Need to check if there exist back edges to nodes that have been visited earlier
- We maintain an array noting the reachable ancestors of all nodes
- This is initialised in the `processVertexEarly` method to the node itself
- In the `processEdge` method, if the edge is a

  back edge we update the reachable ancestor (we check the $entryTime$ to see if the edge leads to a vertex which was discovered earlier)

  tree edge we maintain a count of the number of tree edges connected to the vertex (used to determine if we are at a leaf node)

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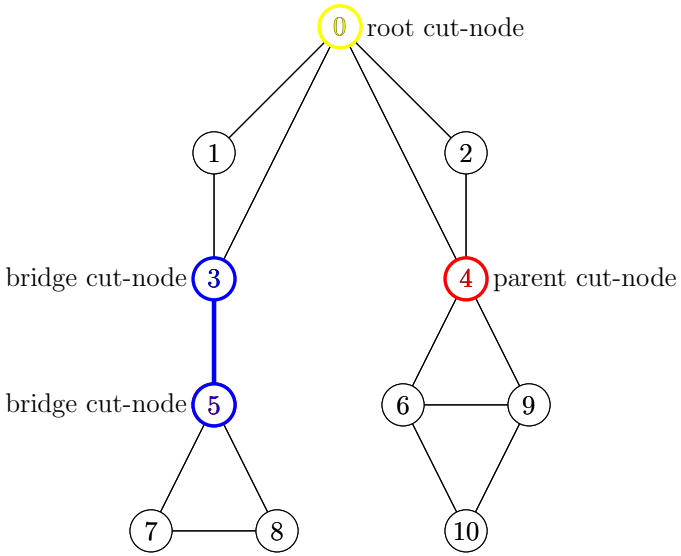
Types of Articulated Vertices

- Key is to recognise that articulated vertices only occur in three version

  Root cut-nodes Occur when the root has more than one child

  Bridge cut-nodes Occurs when the earliest reachable vertex (not including the tree edge to the parent) is the vertex itself. The parent will be an articulation node as will be the node itself if it is not a leaf node

  Parent cut-nodes If the earliest reachable vertex is it parent then the parent is an articulation node



- These are determined in `processVertexLate` method.

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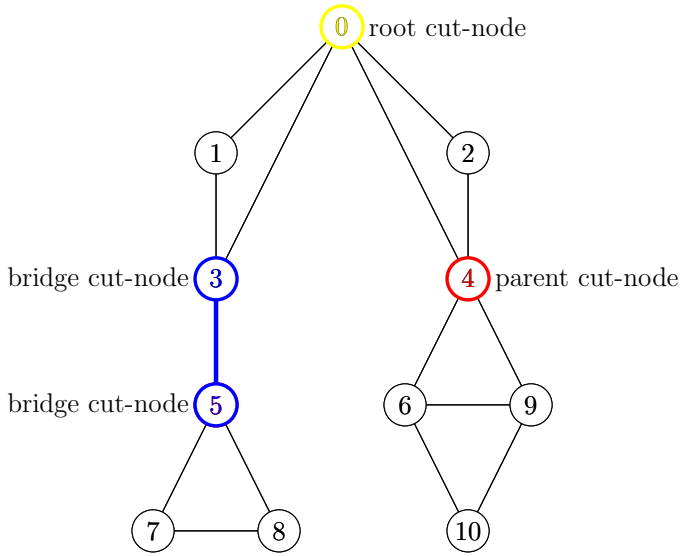

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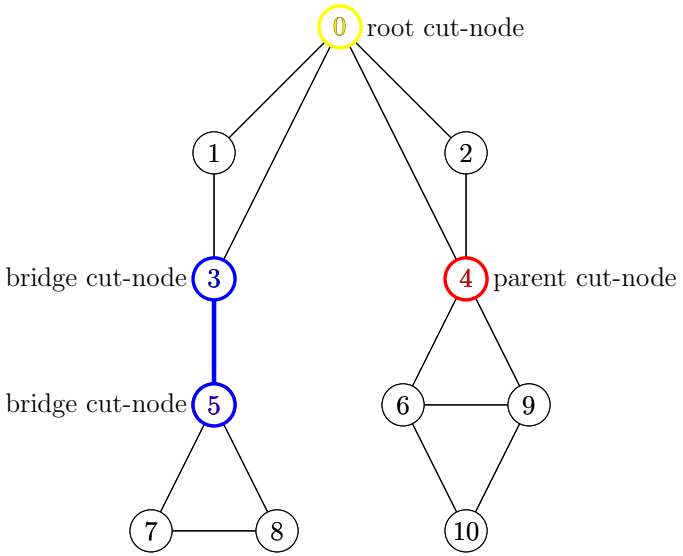

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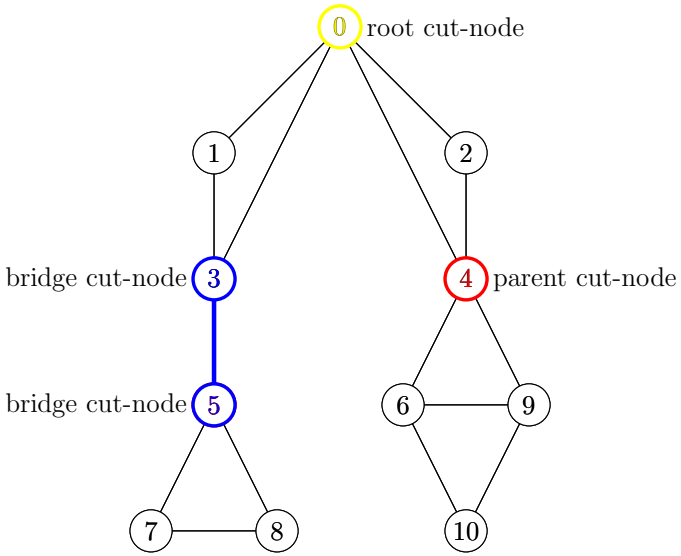

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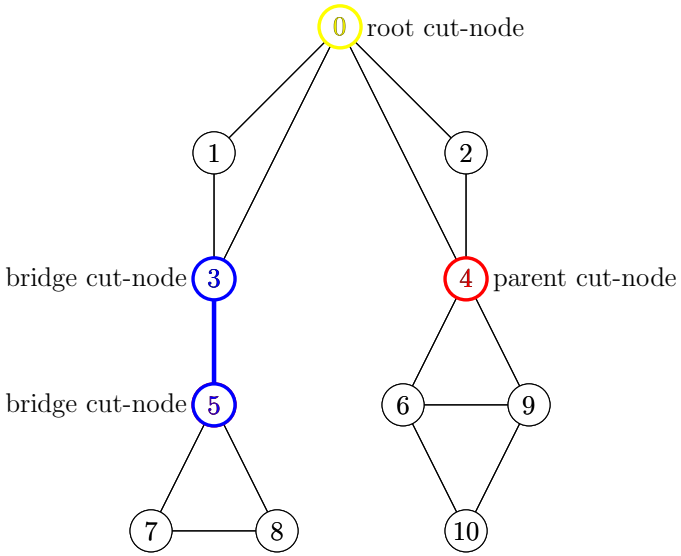

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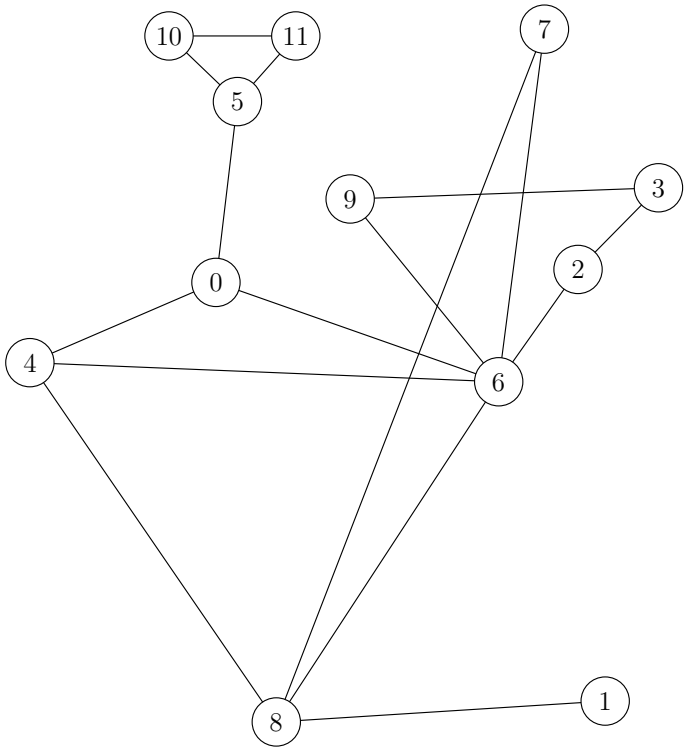

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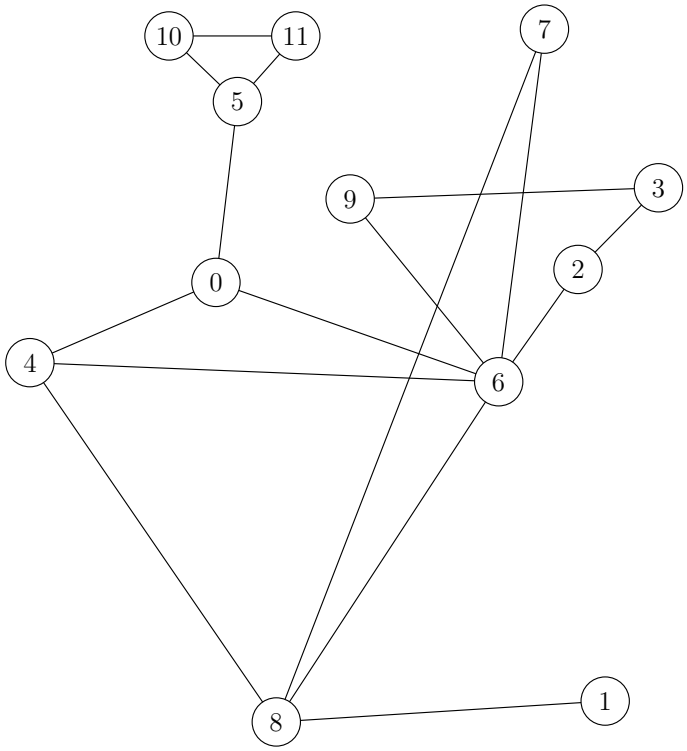

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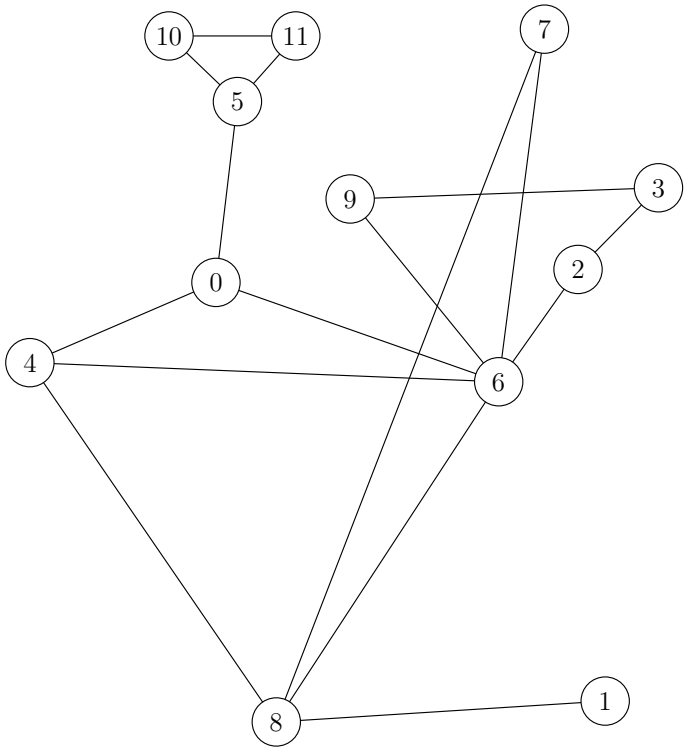

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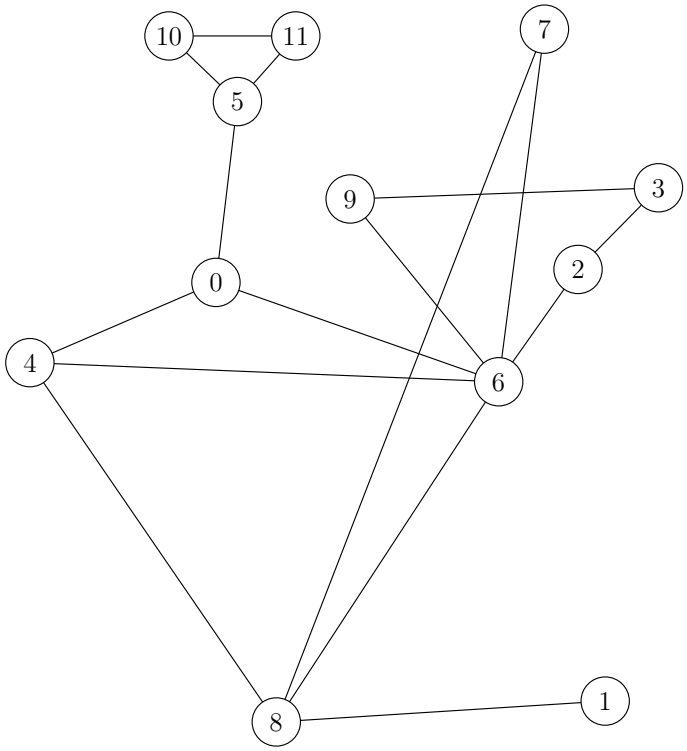

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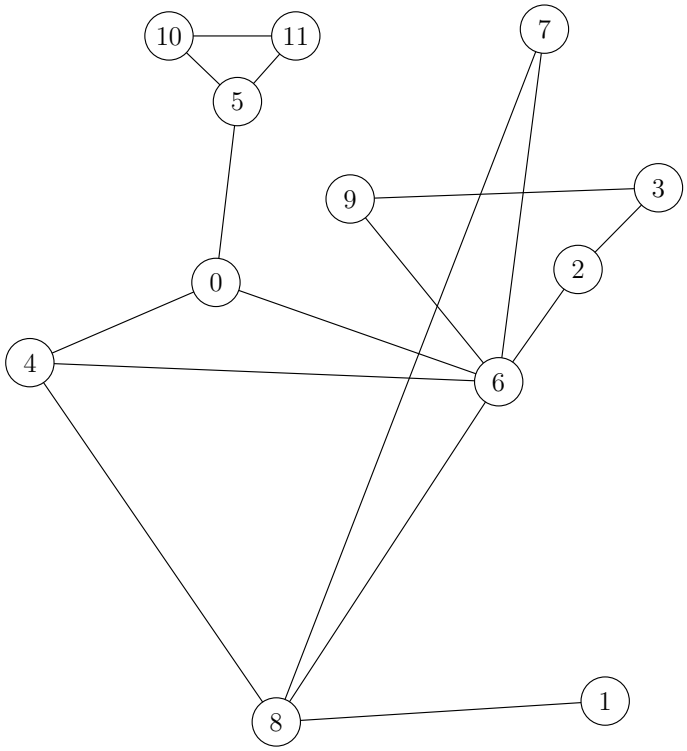

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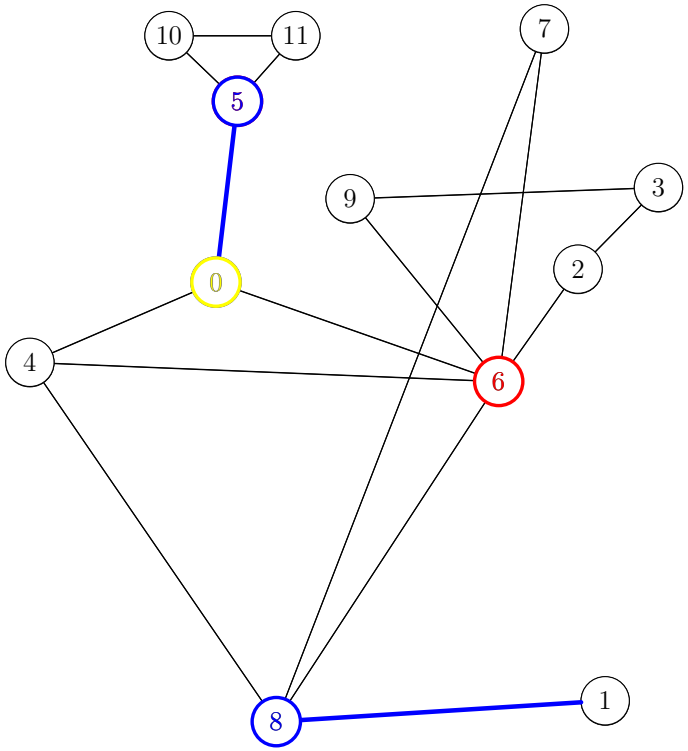
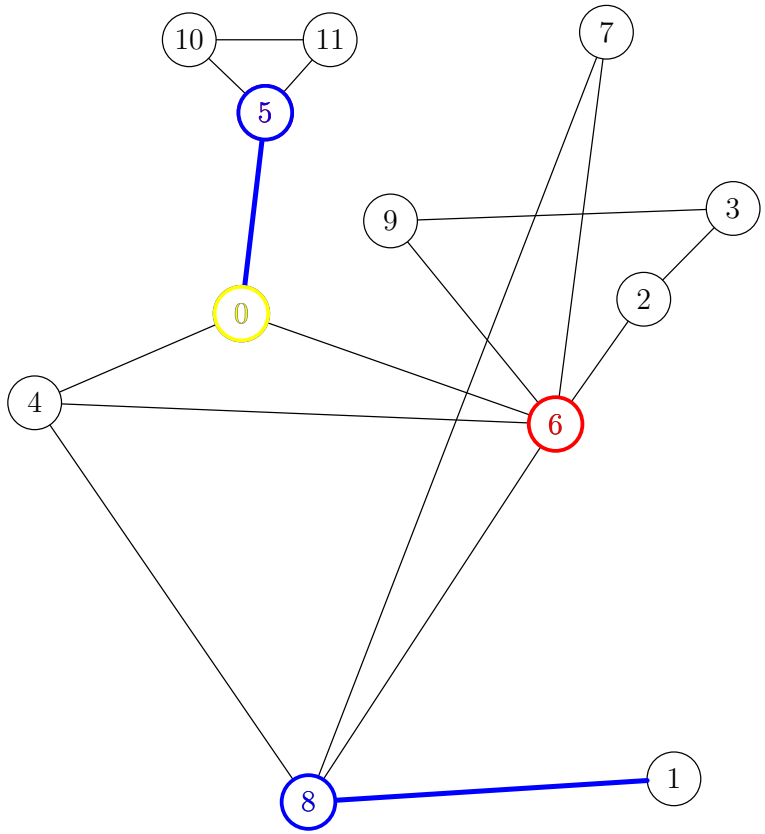


Outline

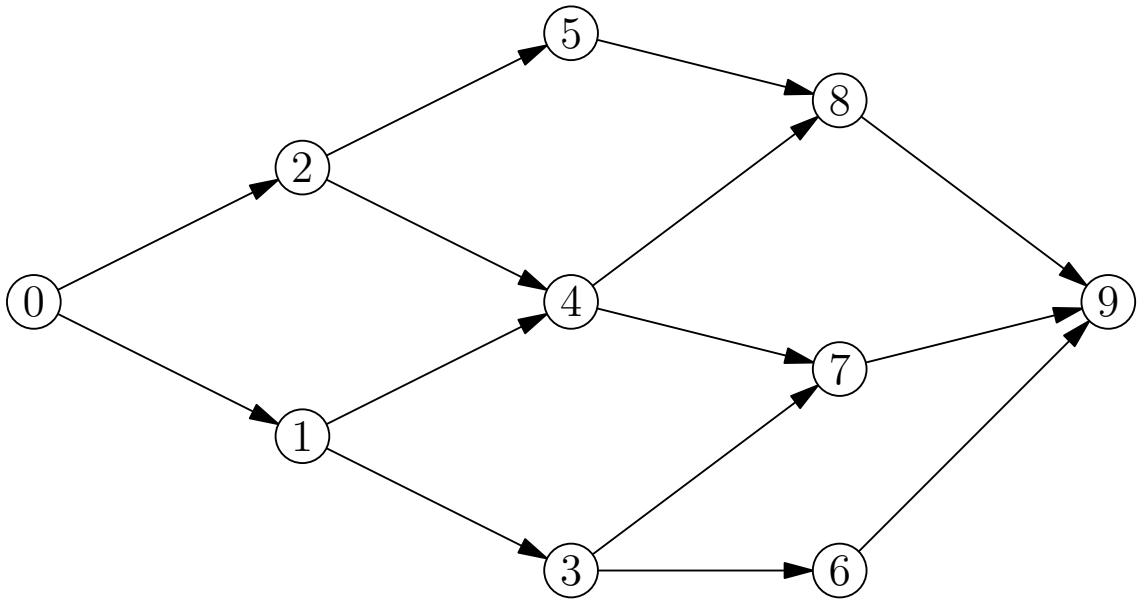
1. Breadth First Search
   - BFS applications
2. Depth First Search
   - DFS applications
3. **Topological Sort**



DAGs

- Directed acyclic graphs or DAGs are directed graphs without cycles

- They are often used to represent complex processes
  - ⋆ Vertices are processes
  - ⋆ Directed edge (i, j) indicates process i needs to occur before process j



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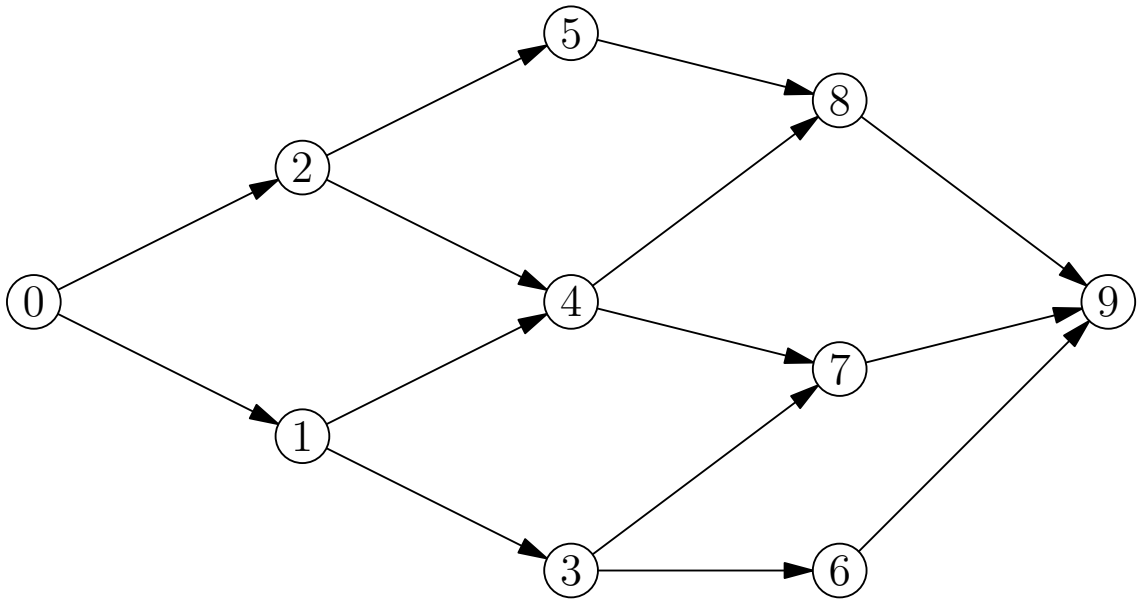

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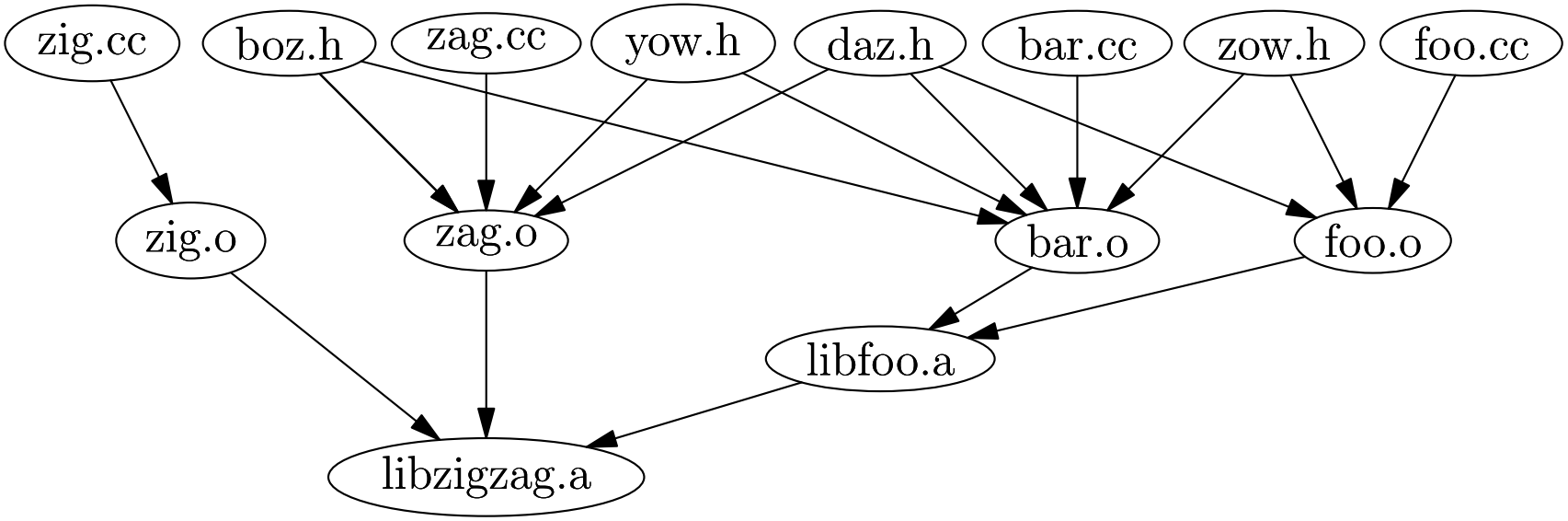

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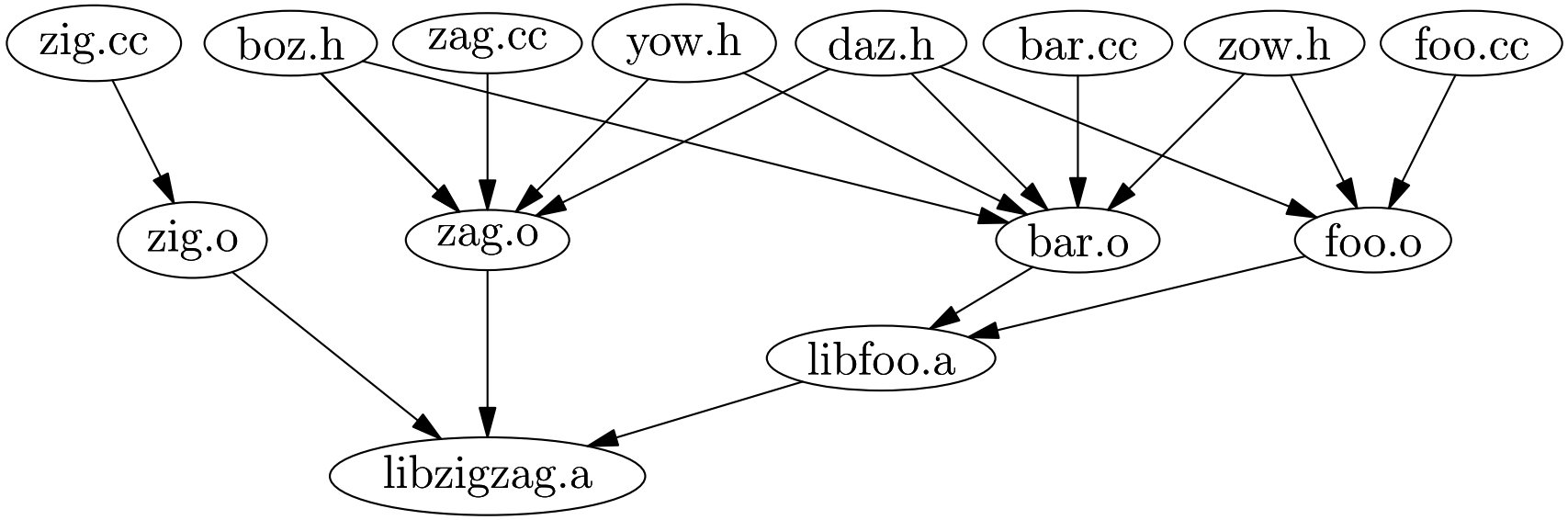

Other Applications

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- If the graphs were not acyclic it would be impossible process them!

Topological Sort

- Given a DAG a topological sort outputs an ordered list of vertices which respects the ordering imposed by the edges
- That is, for each edge (i, j), vertex i will occur before vertex j
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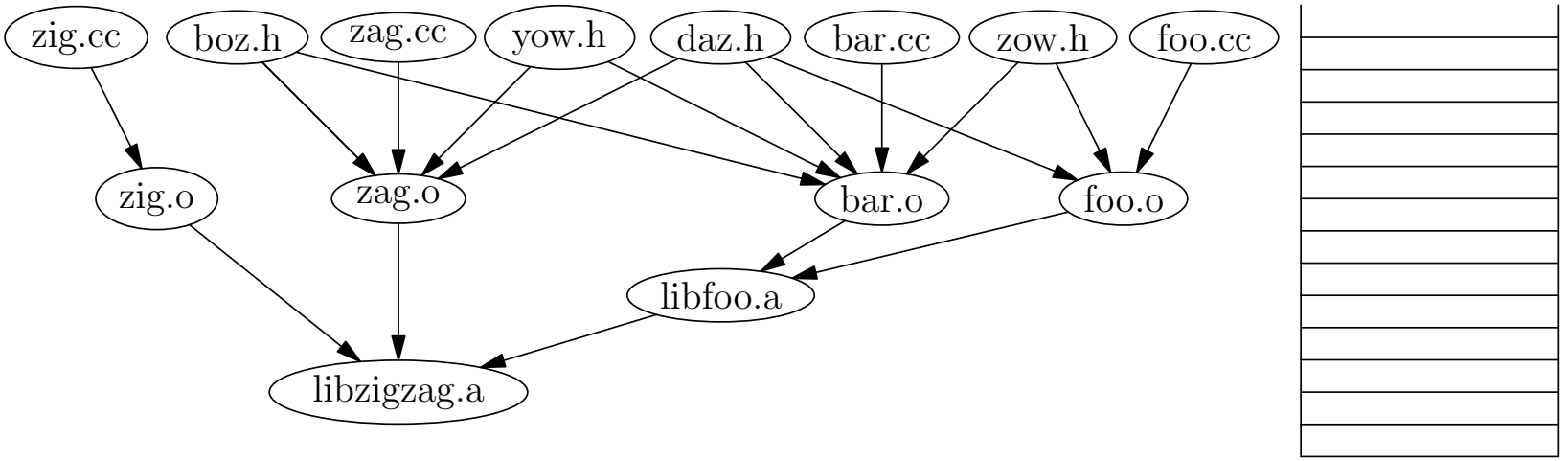

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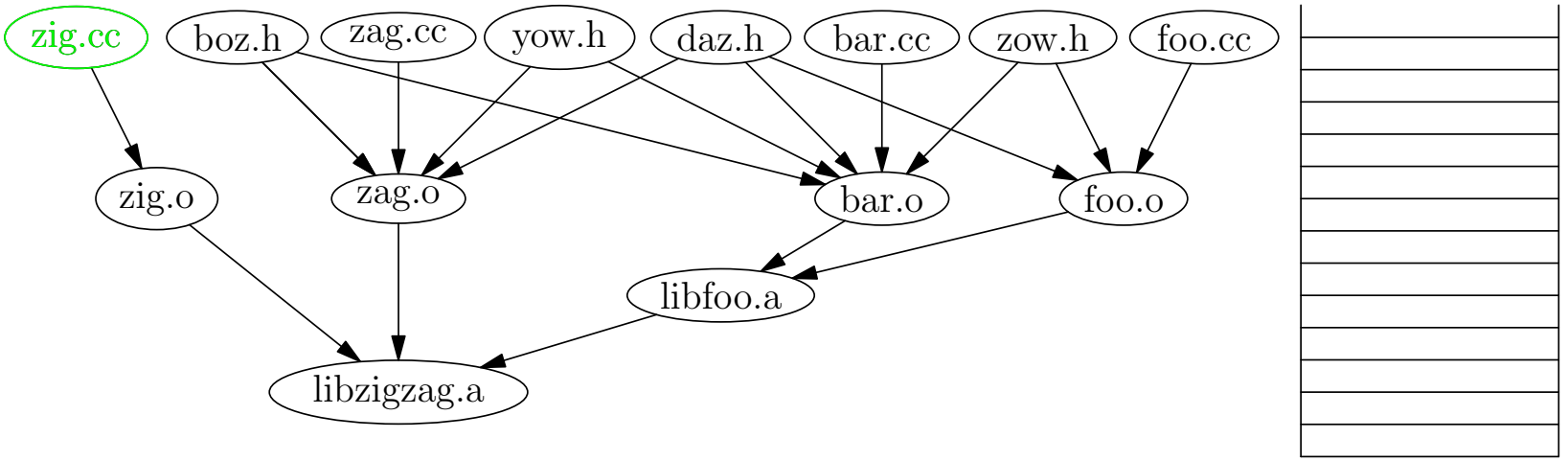

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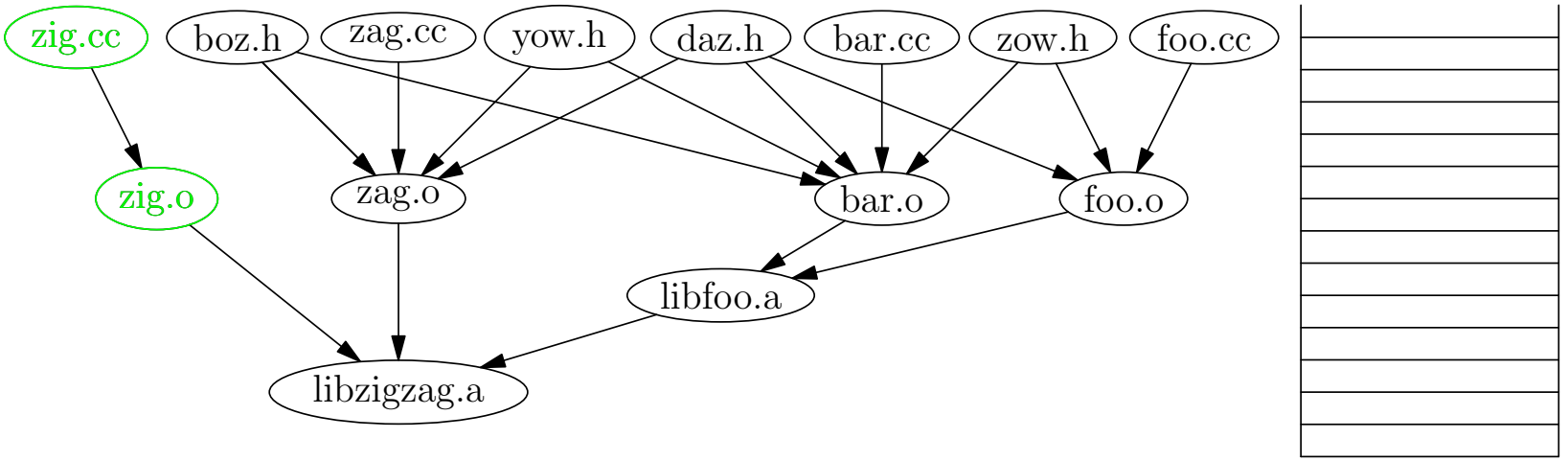

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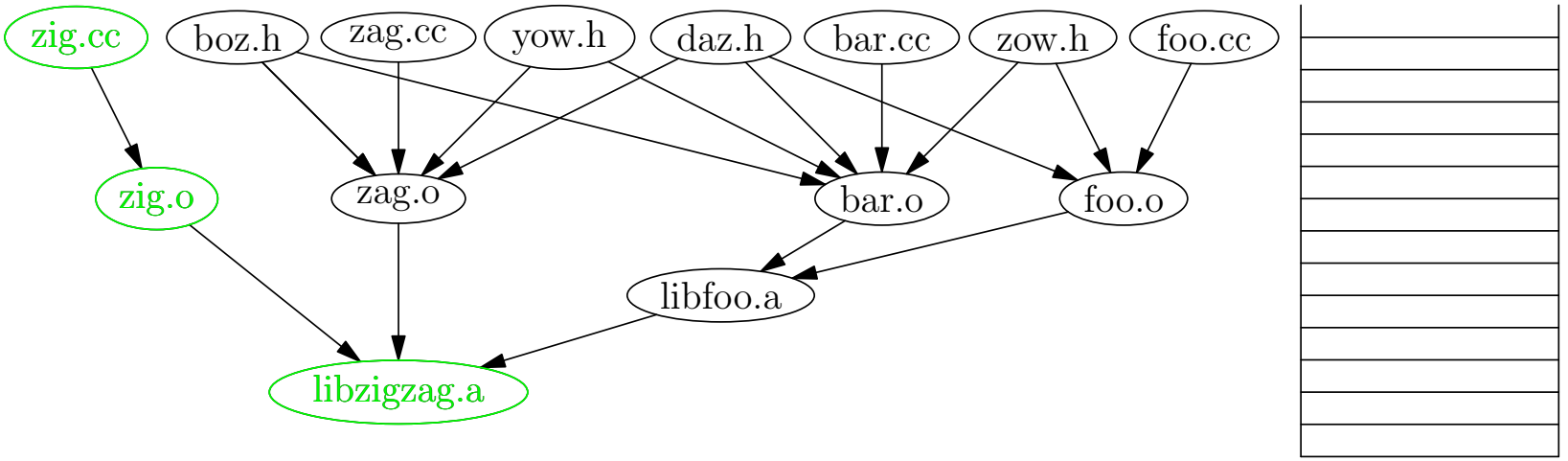

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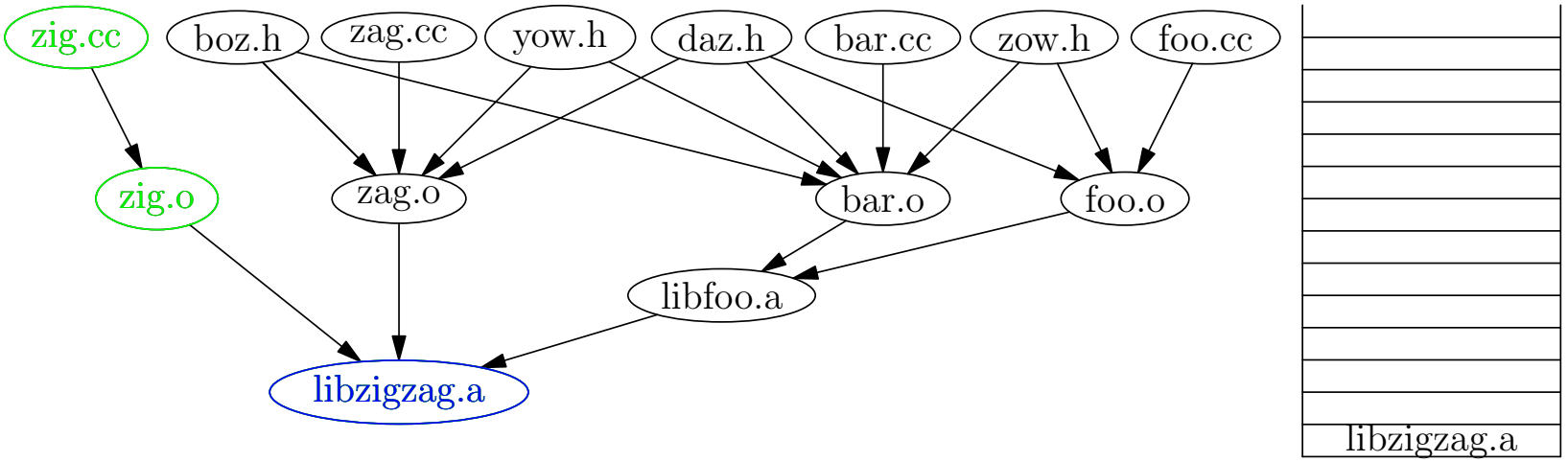

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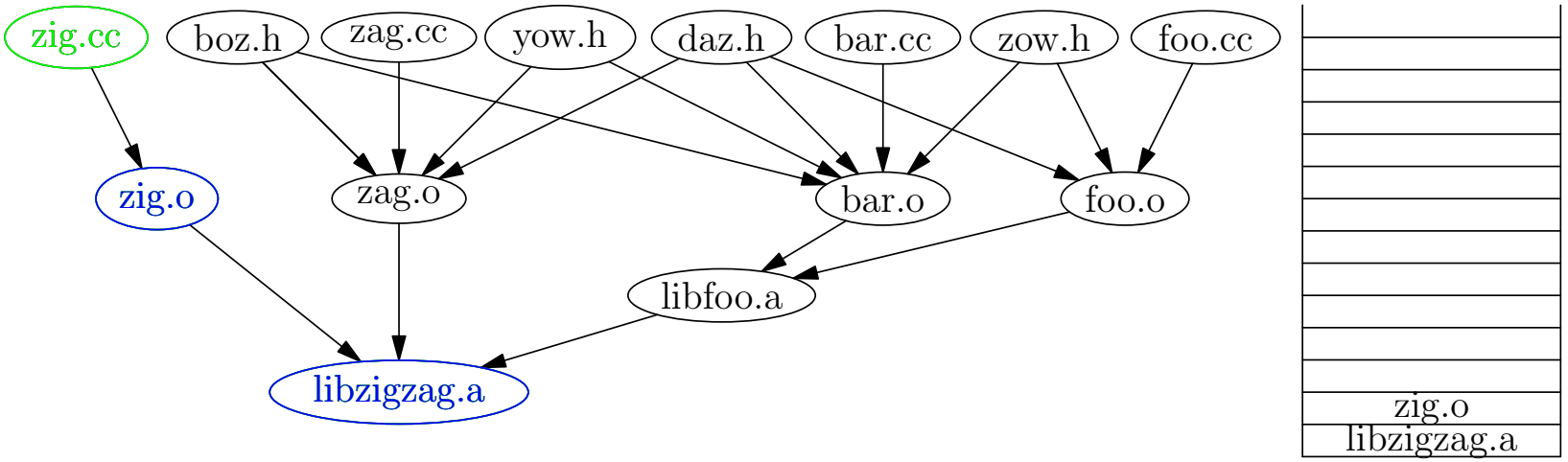

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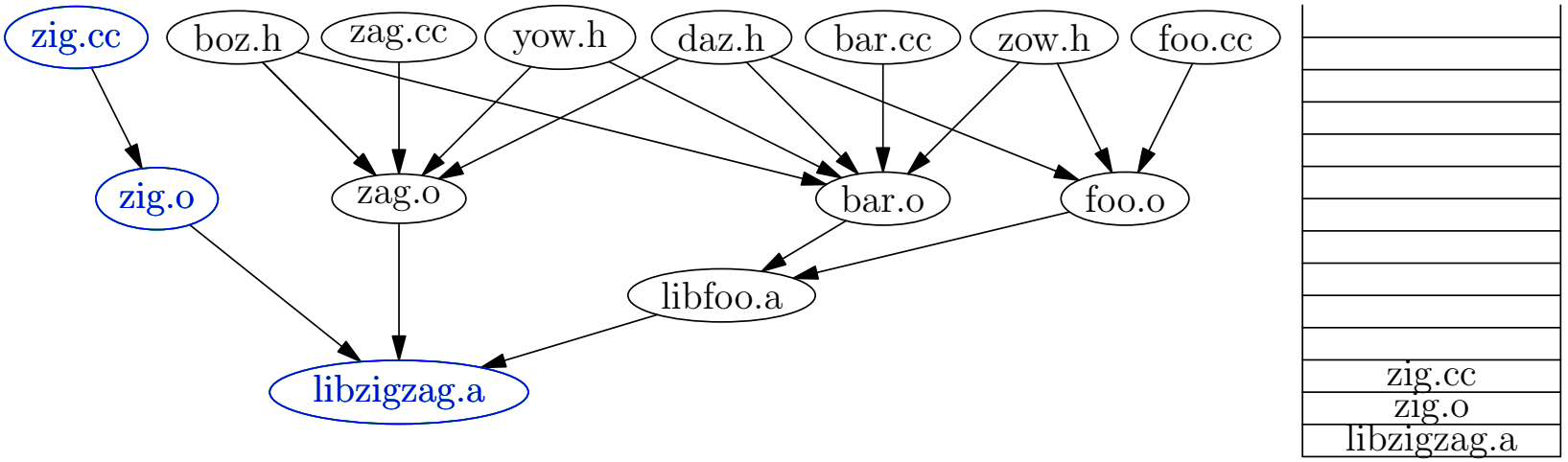

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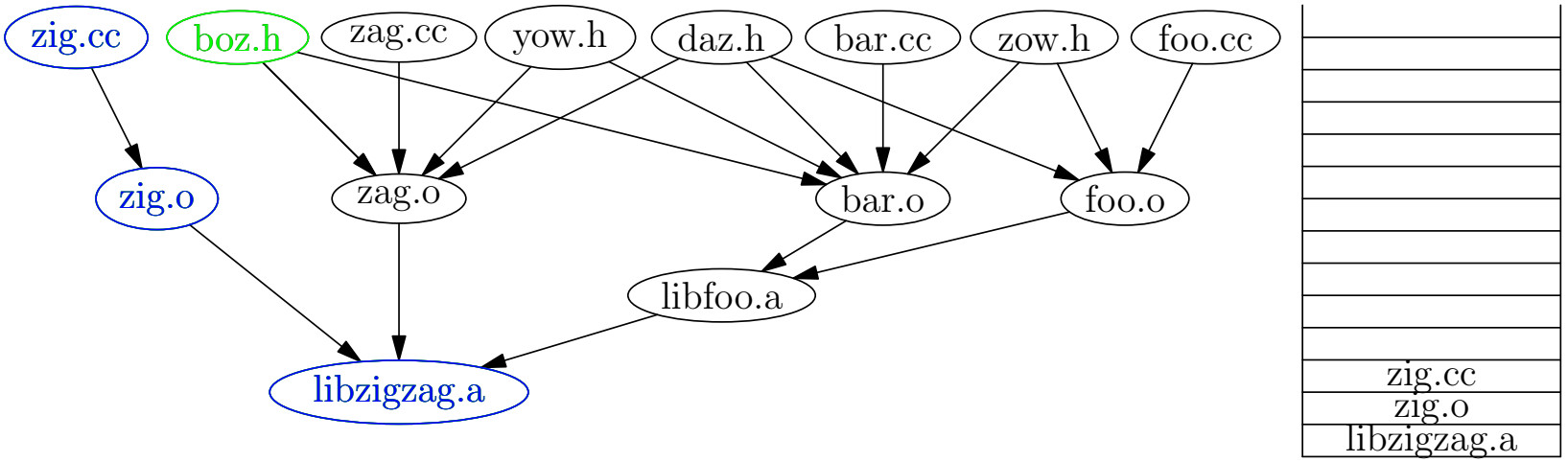

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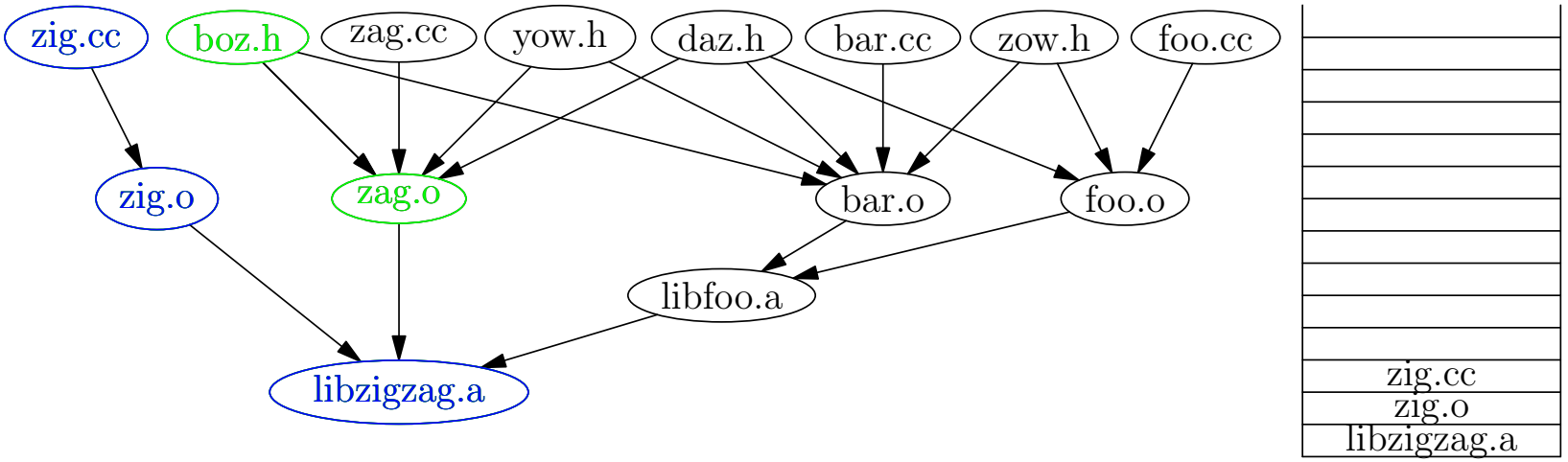

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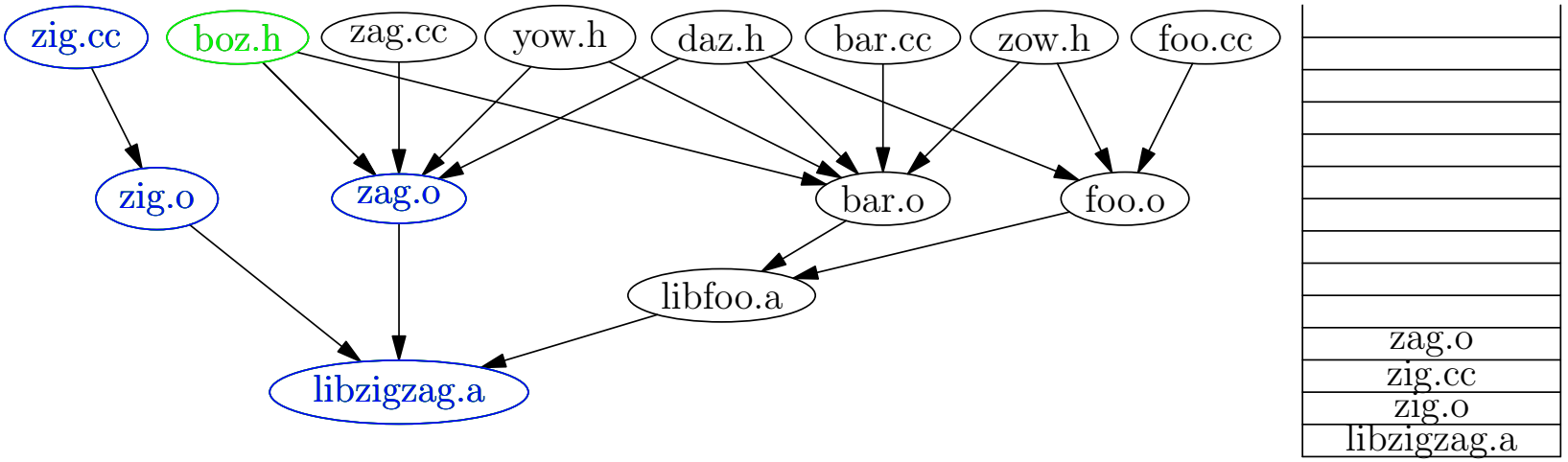

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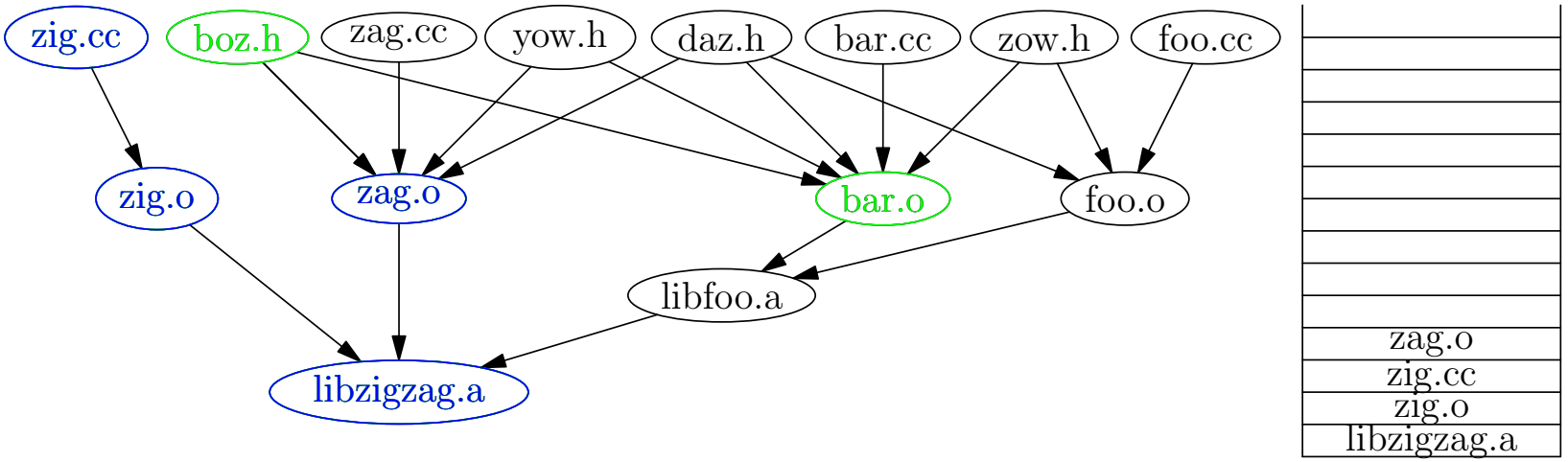

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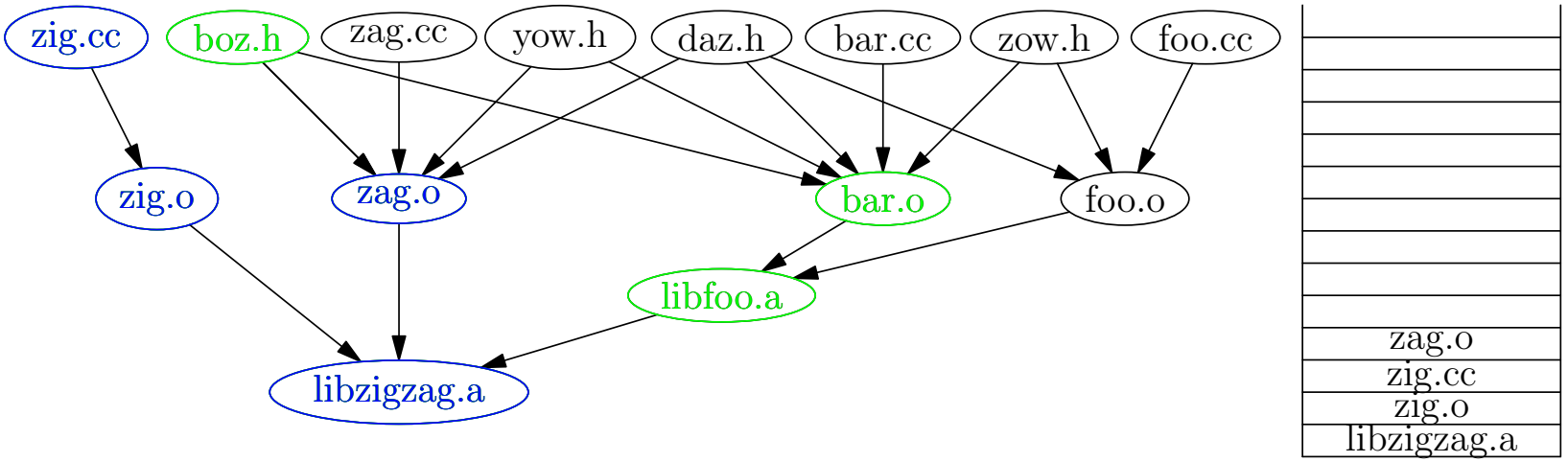

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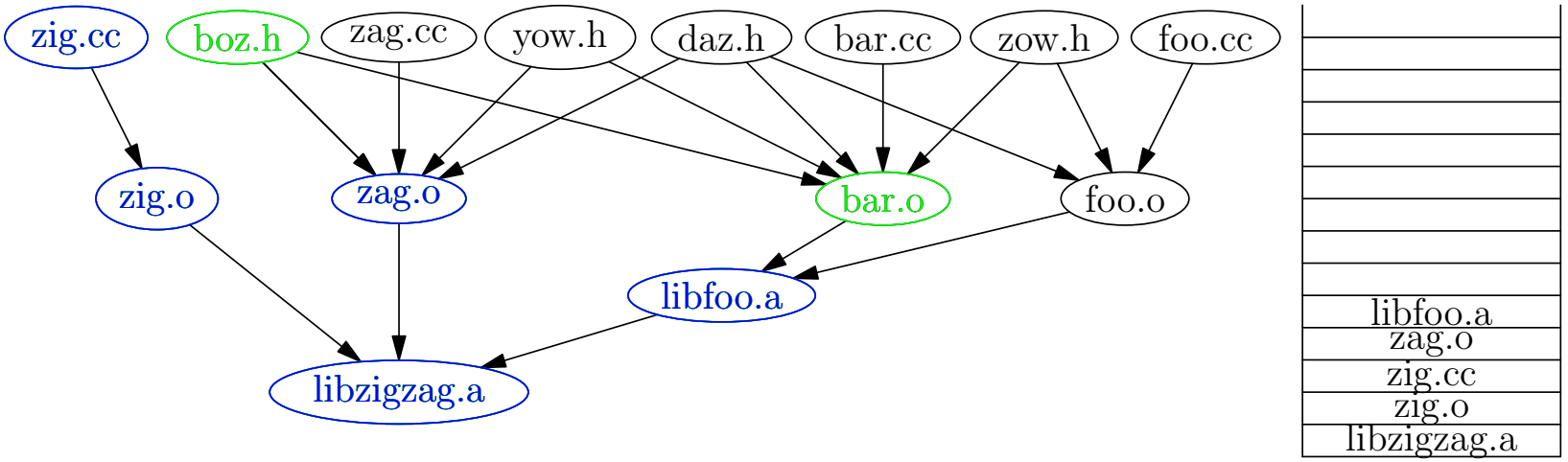

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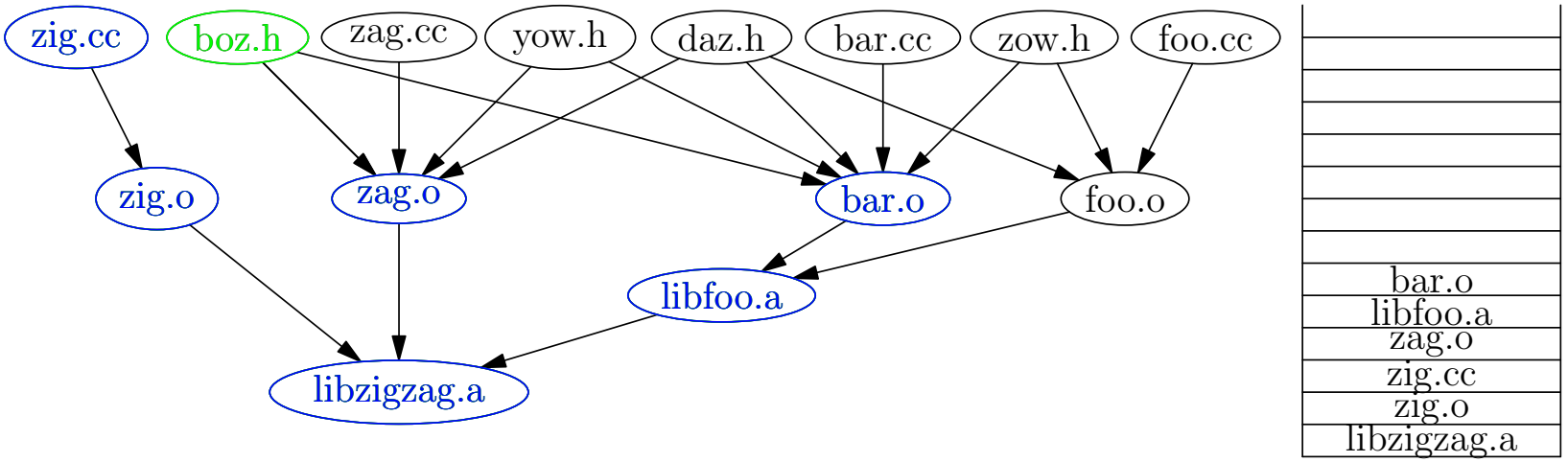

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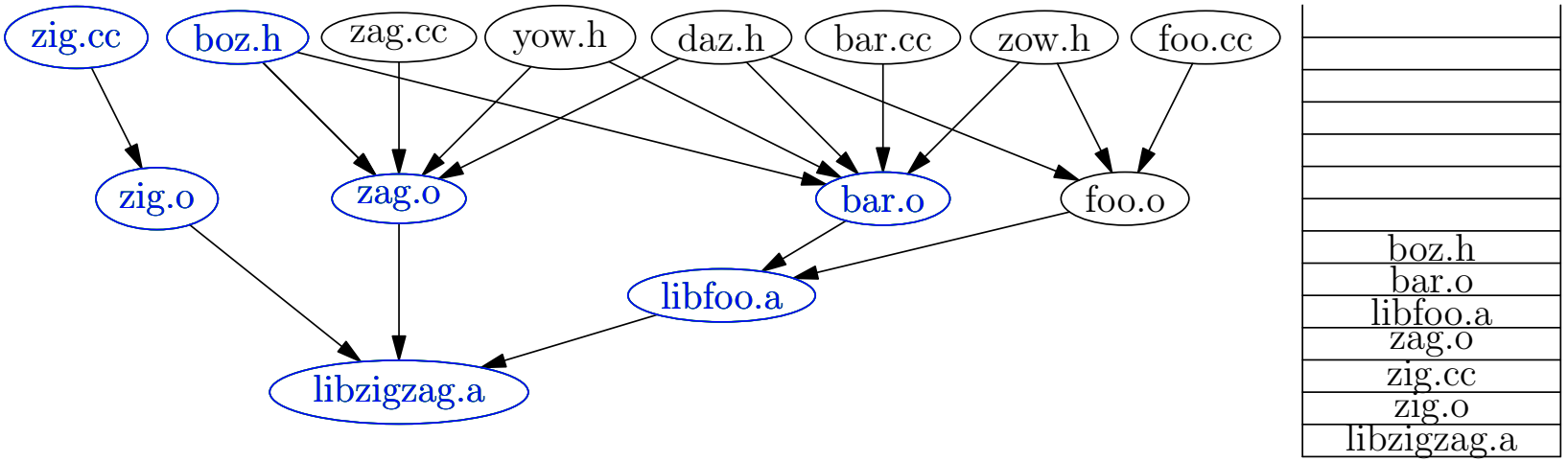

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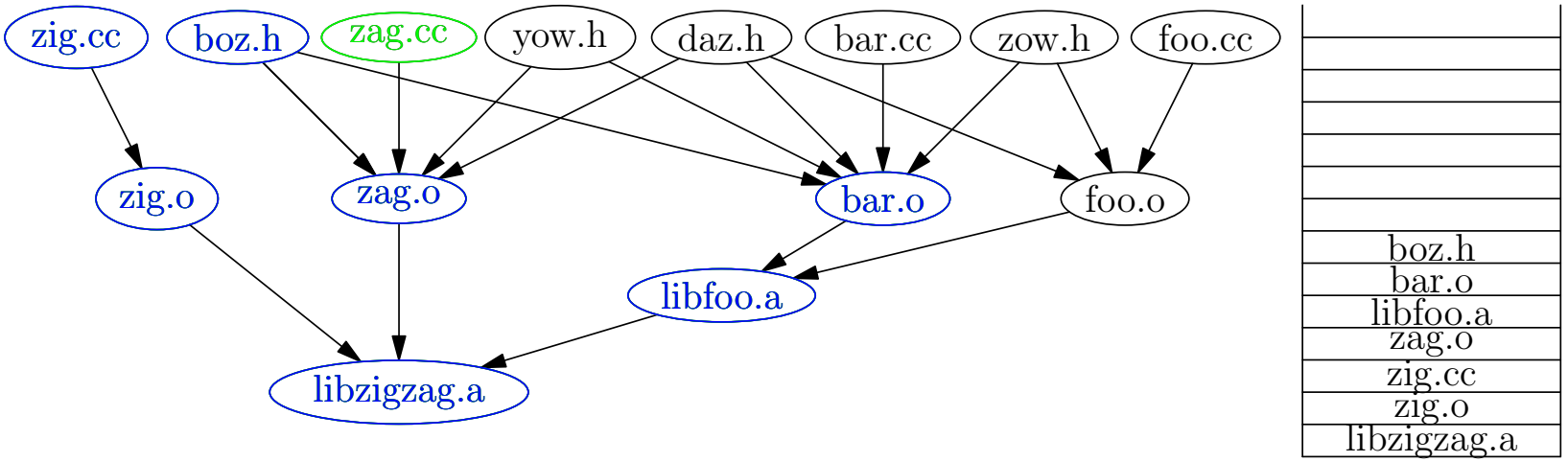

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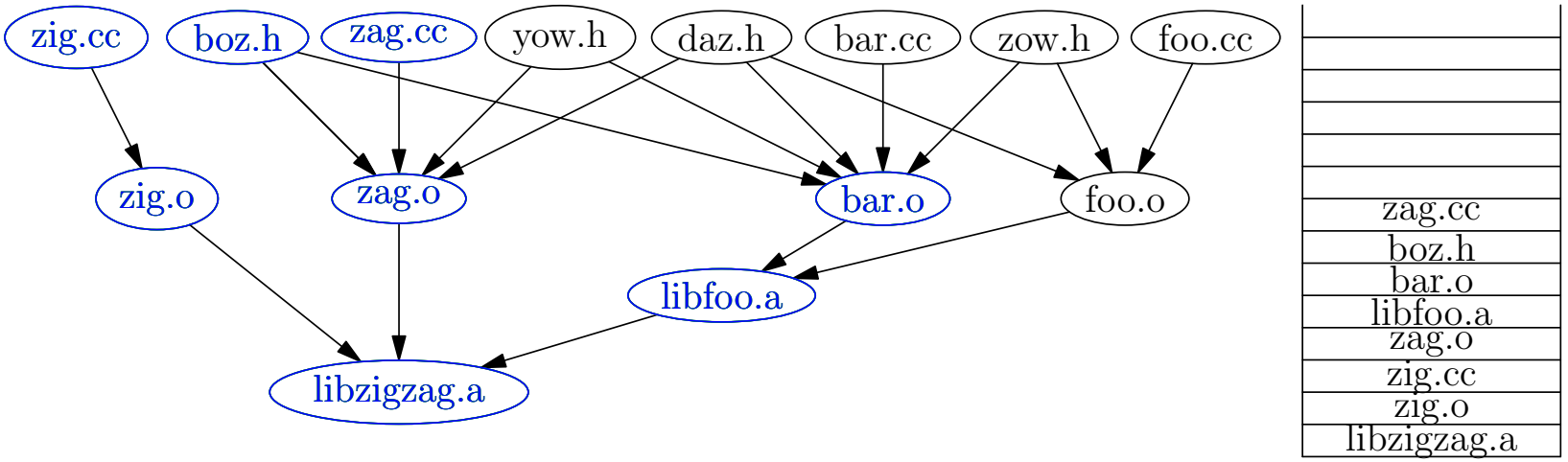

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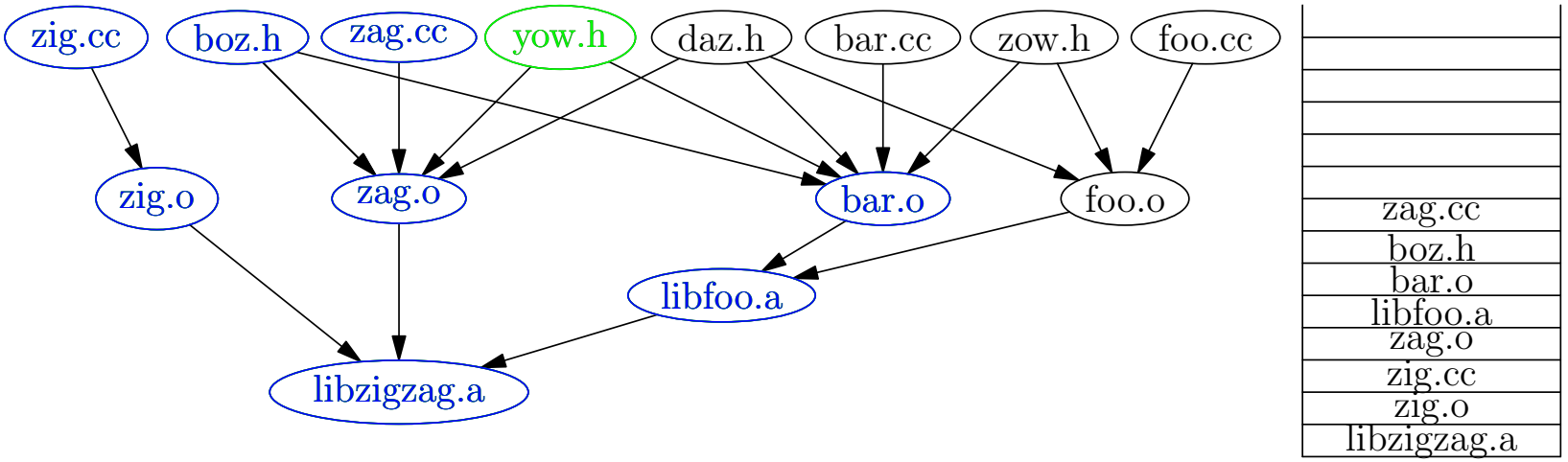

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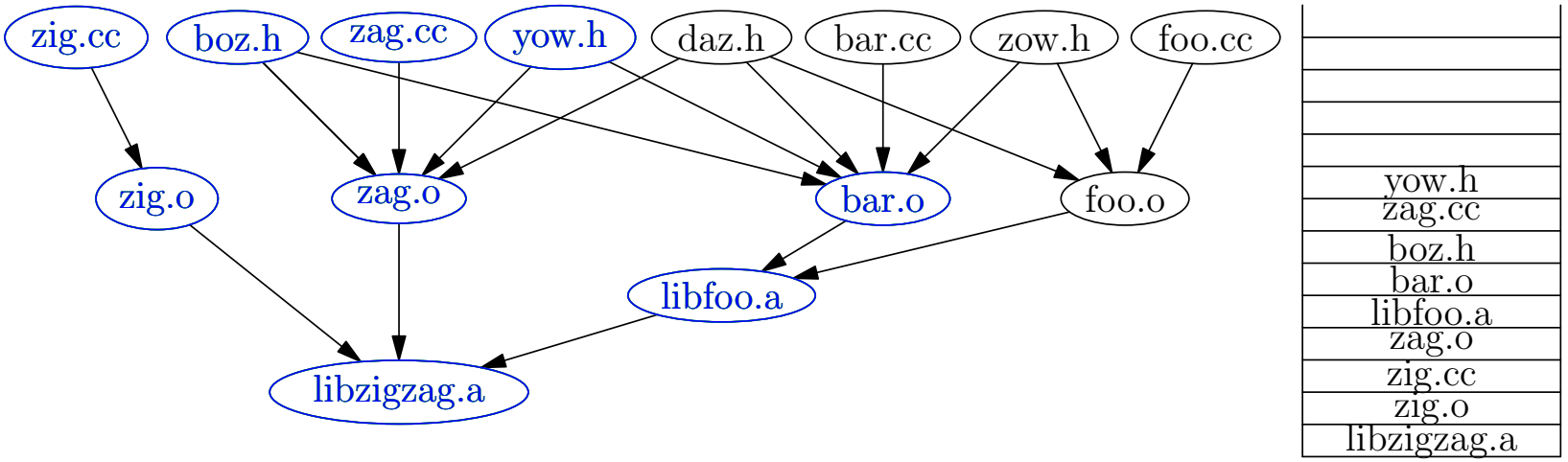

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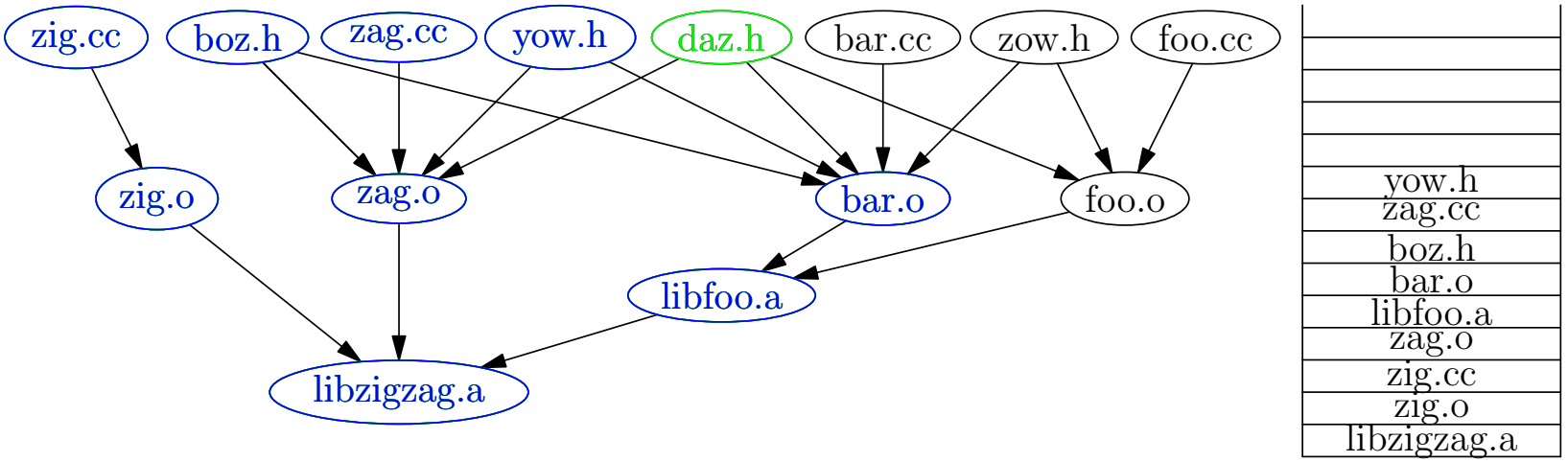

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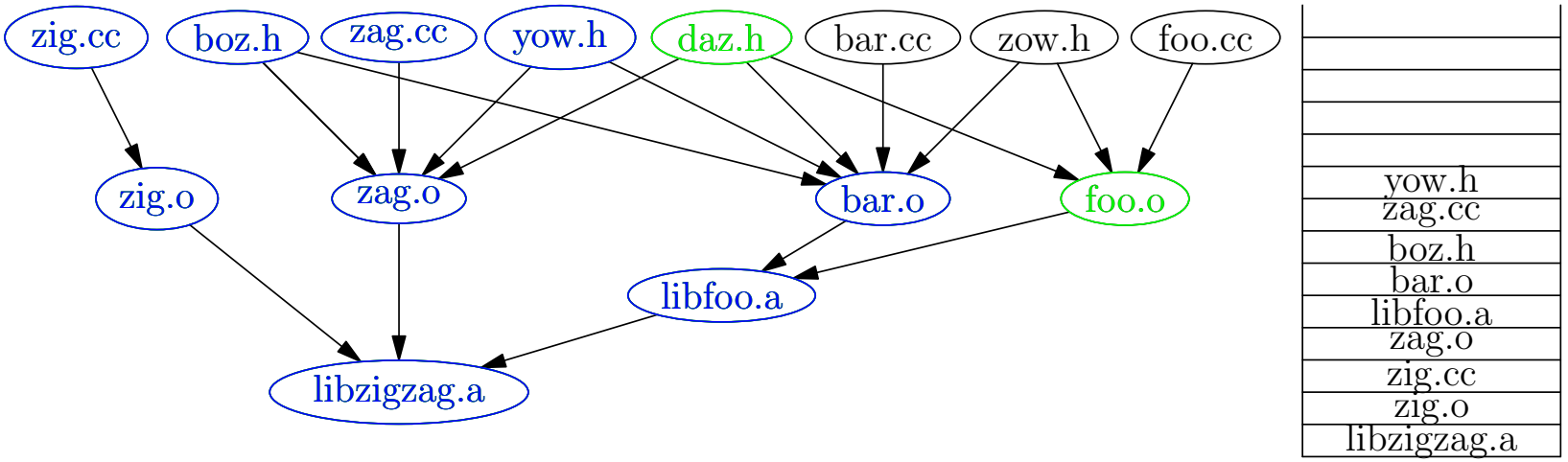

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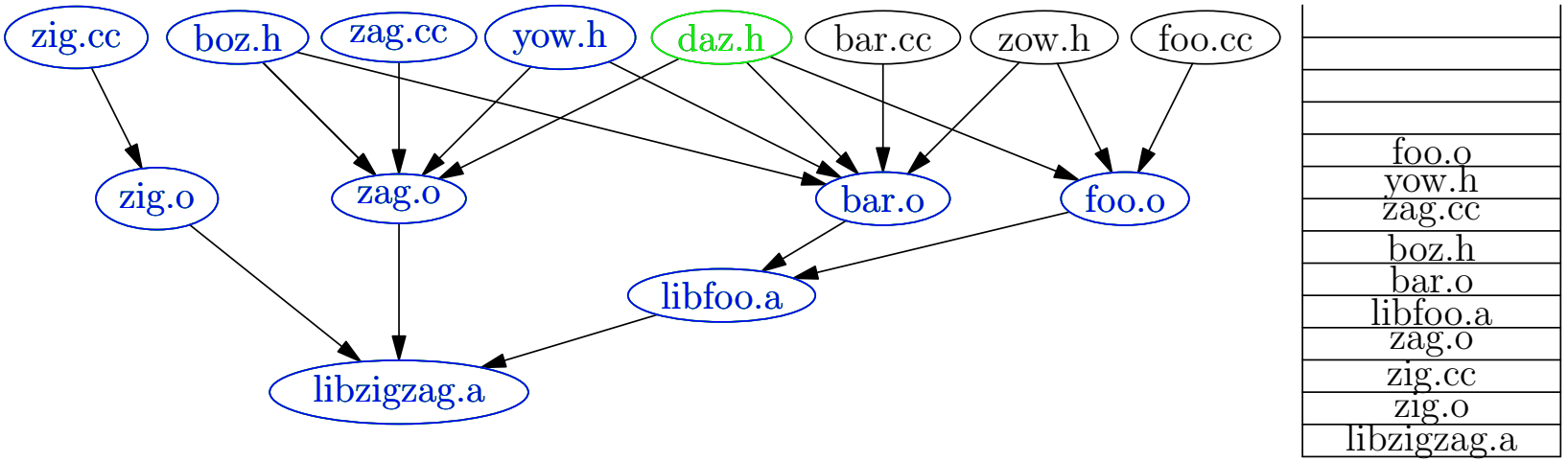

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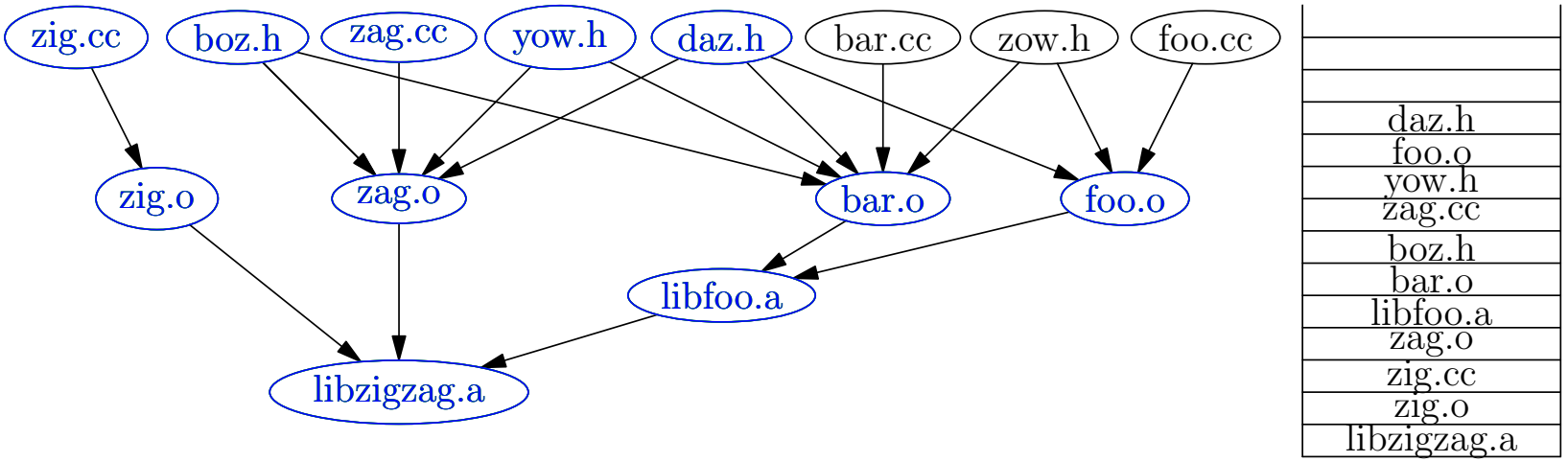

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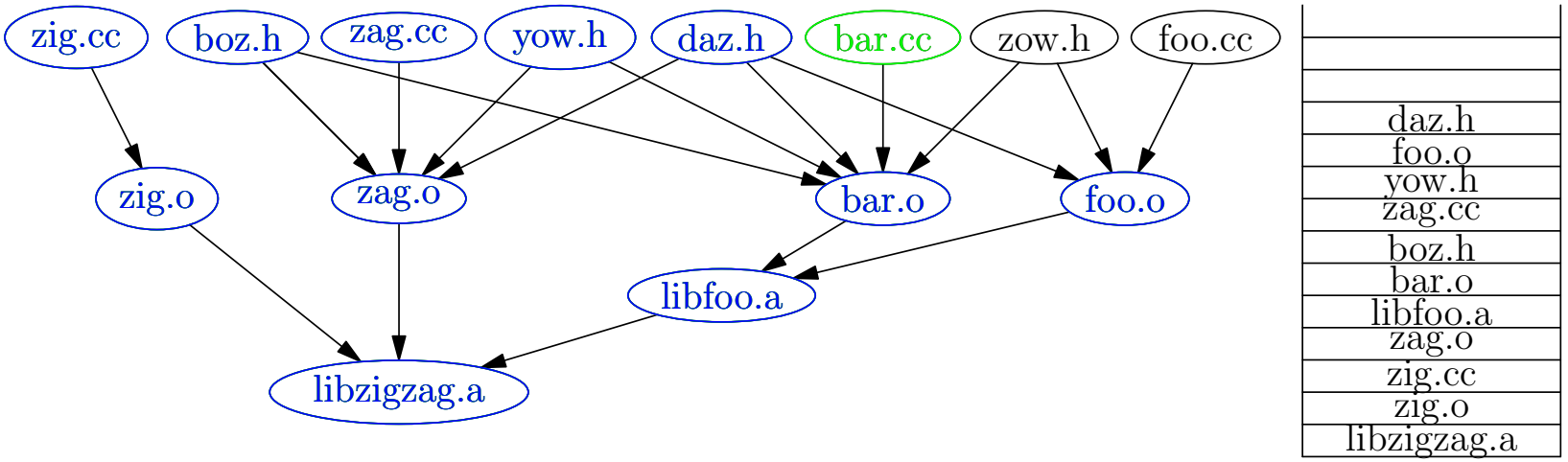

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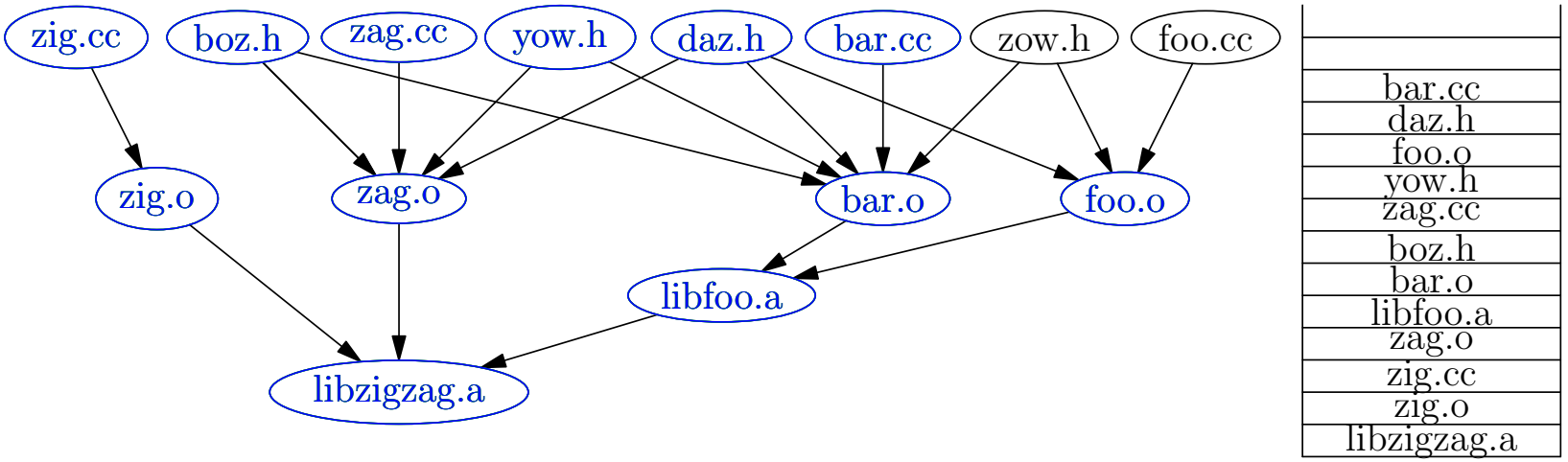

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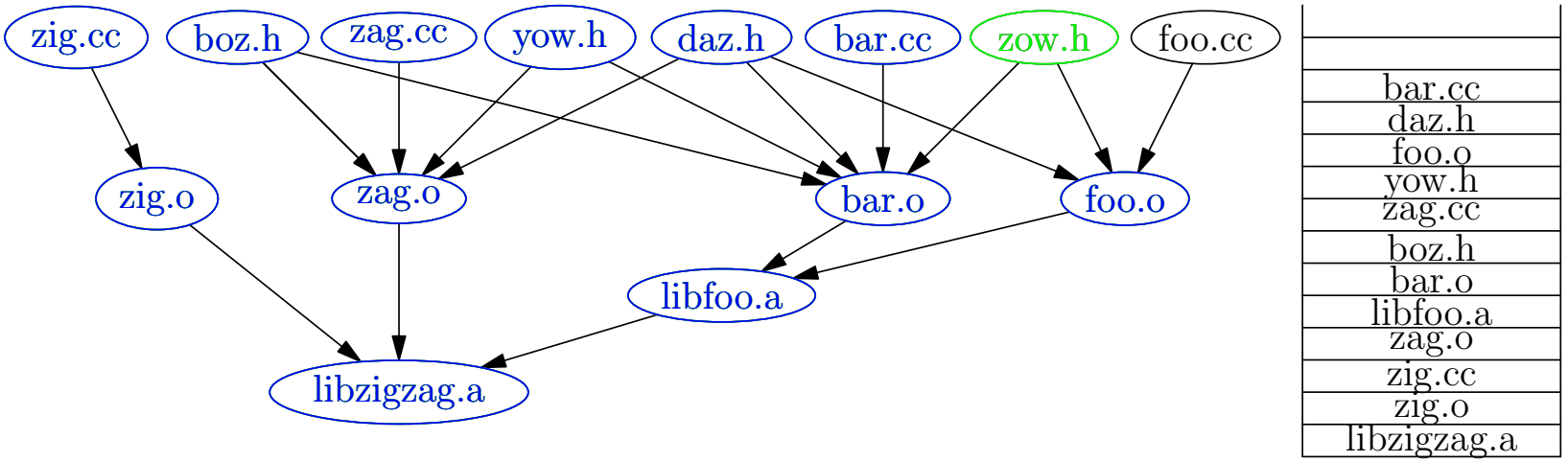

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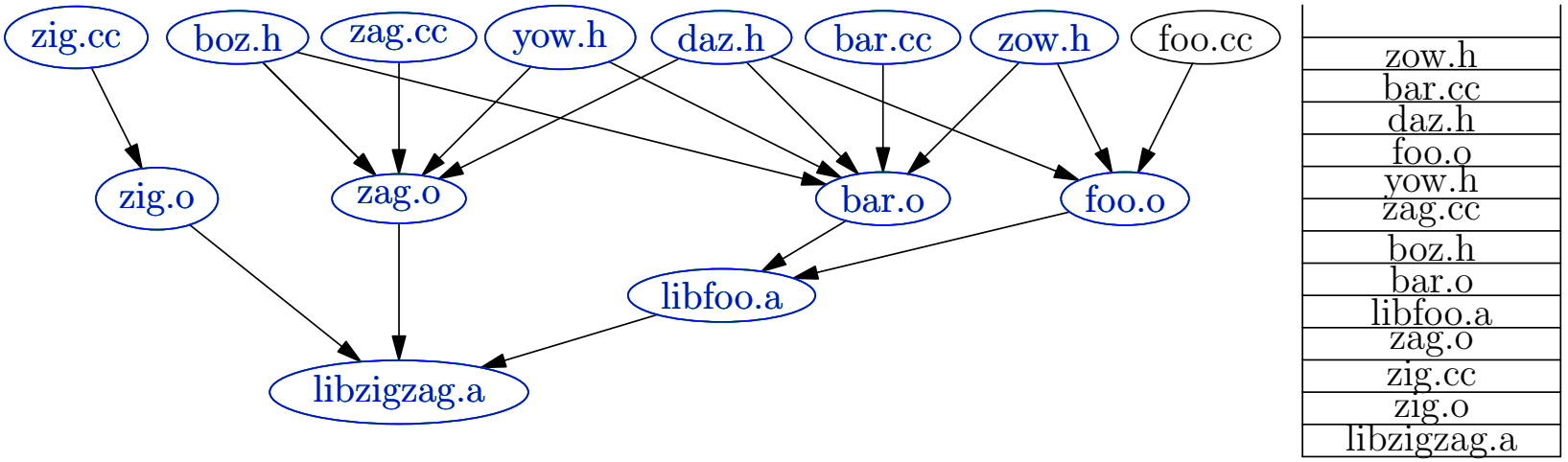

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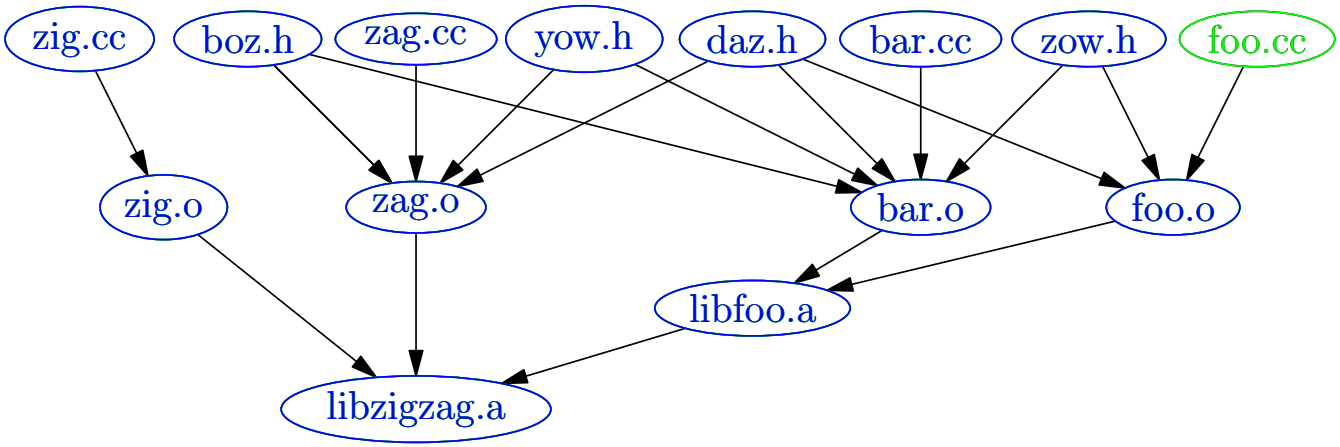

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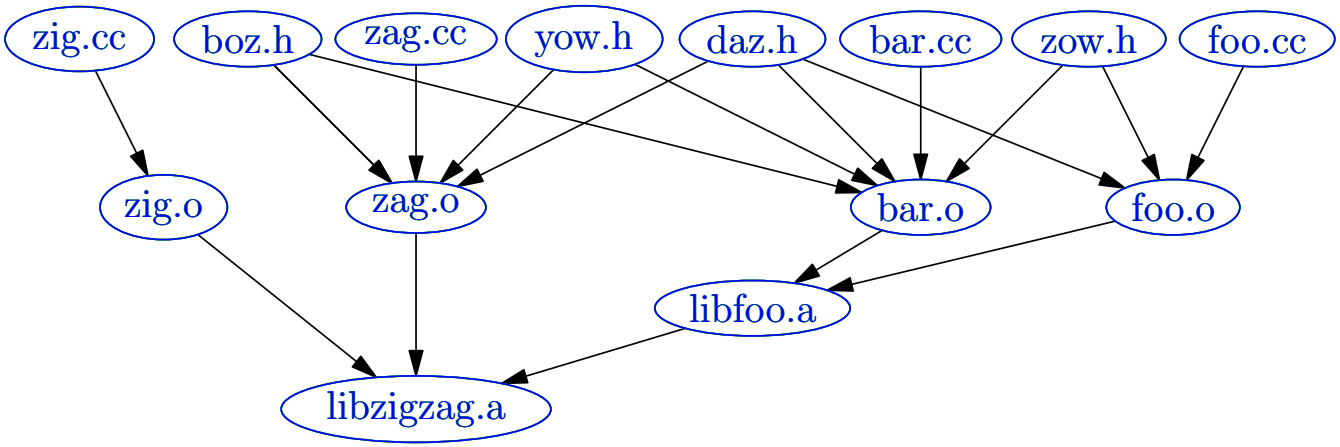

foo.cc
zow.h
bar.cc
daz.h
foo.o
yow.h
zag.cc
boz.h
bar.o
libfoo.a
zag.o
zig.cc
zig.o
libzigzag.a

DFS on Digraphs

DFS for undirected graphs

```
dfs(graph, node) {
  if ("finished") return
  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] ≠ "discovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs(graph, neighbour)
    } else if (state[neighbour] ≠ "processed") {
      processEdge(node, neighbour)
    }
    if ("finished") return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
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}
```

DFS on Digraphs

DFS for directed graphs

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  state[node] ← "discovered"
  time ← time + 1
  processVertexEarly(node)
  foreach neighbour ∈ Neighbourhood(node) {
    if (state[neighbour] ≠ "discovered") {
      parent[neighbour] ← node
      processEdge(node, neighbour)
      dfs(graph, neighbour)
    } else if (state[neighbour] ≠ "processed" ∨ graph is directed) {
      processEdge(node, neighbour)
    }
    if ("finished") return
  }
  processVertexLate(currentNode)
  state[currentNode] ← "processed"
  time ← time + 1
}
```

Implementing Topological Sort

- Given our DFS programme we define

```
topologicalSort(graph) {
  Stack stack
  for node ∈ graph.vertexSet()
    if (¬discovered[node])
      dfs(graph, node)

  List topSortList
  while (¬stack.isEmpty())
    topSortList.add(stack.pop())

  return topSortList
}
```

Enhance DFS

- Requires us to define a couple of helper function

```
processVertexLast(node) {
   stack.push(node)
}

processEdge(currentNode, neighbour) {
   if (state[neighbour] == "processed") {
      print "error: graph not a DAG"
      finished = true
   }
}
```

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Implementation Issues

- Most awkward part of the implementation is that the `topologicalSort` algorithm needs access to dfs structures (`discovered[]`)
- `processVertexLast(node)` needs access to the stack
- Need to be able to redefine `processVertexFirst`, `processEdge` and `processVertexLast`
- Different languages and libraries cope with this differently
  - ⋆ Java: JDSL, JGraphT
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Other Applications

- DFS is used for many other classic problems
- Euler Cycles
- Strongly Connected Components

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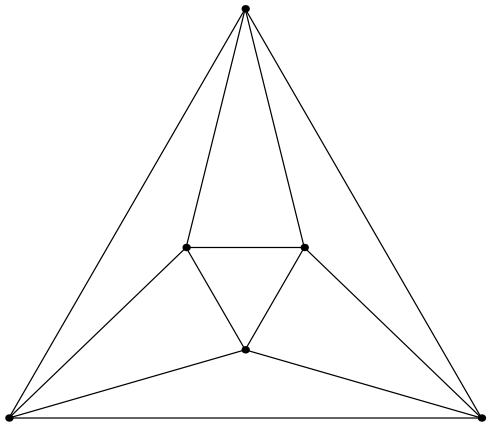

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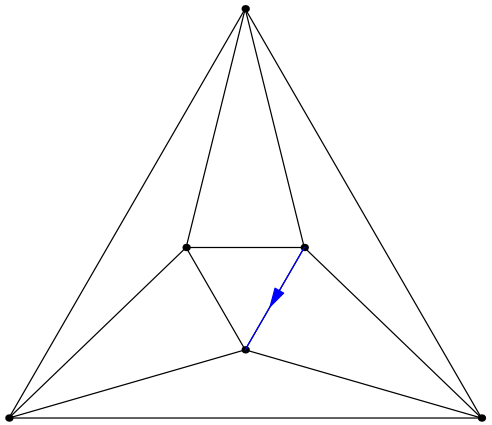

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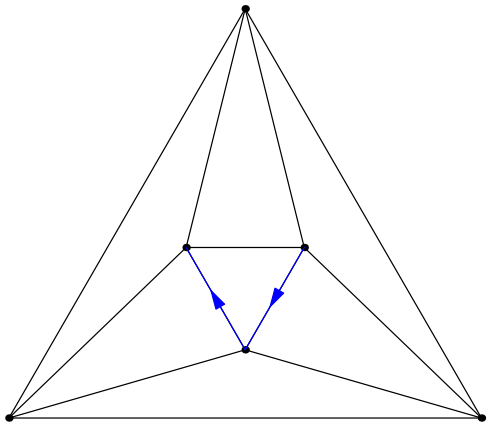

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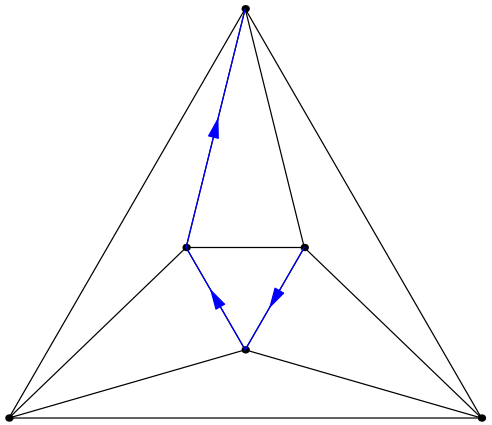

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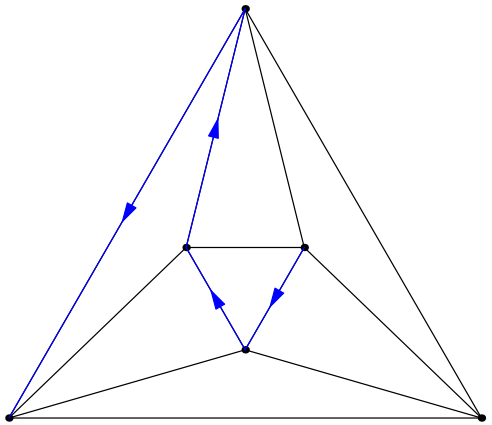

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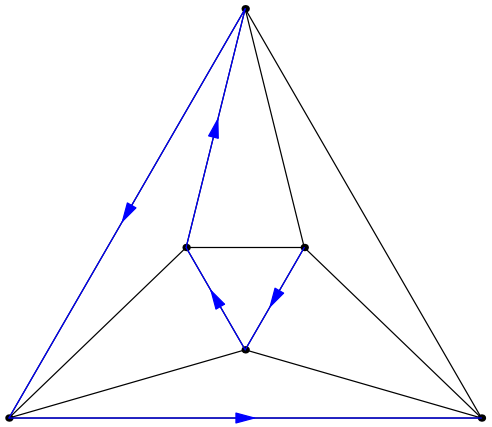

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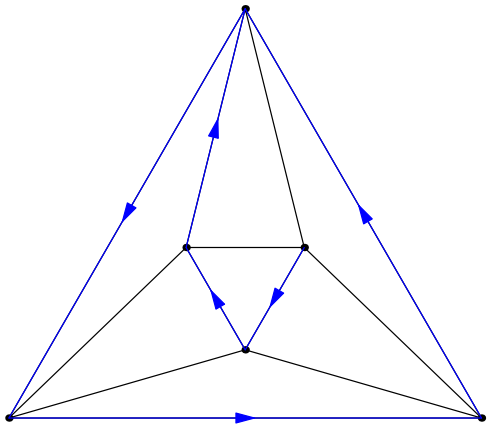

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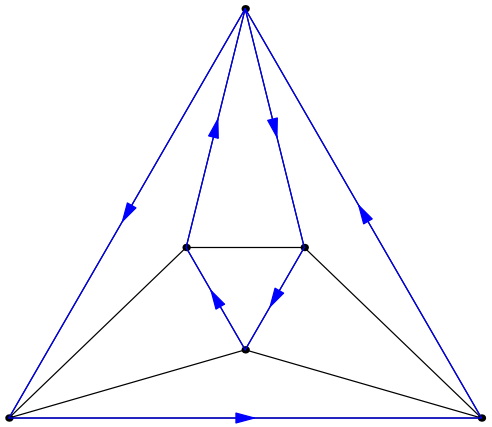

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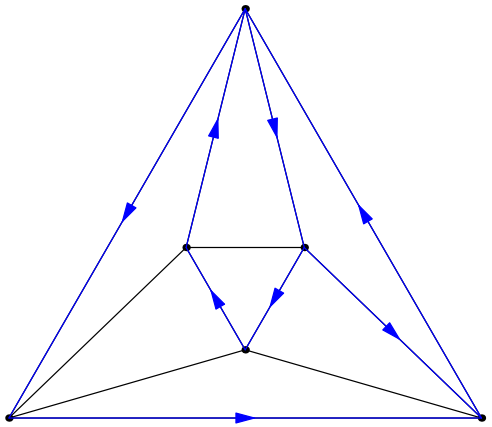

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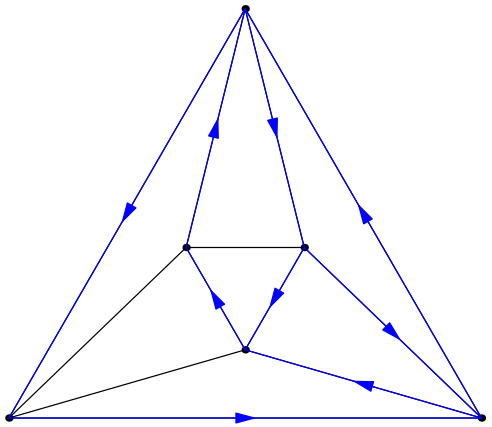

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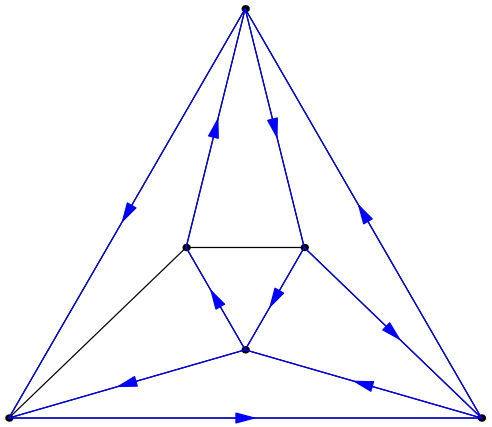

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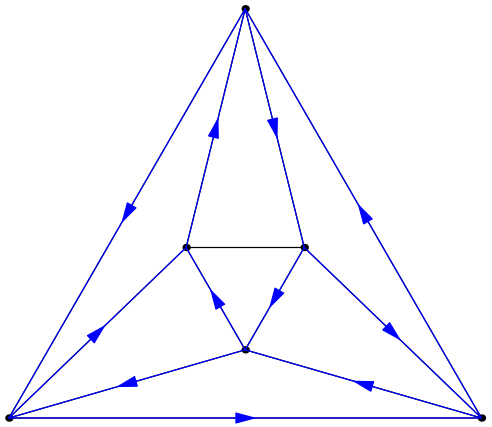

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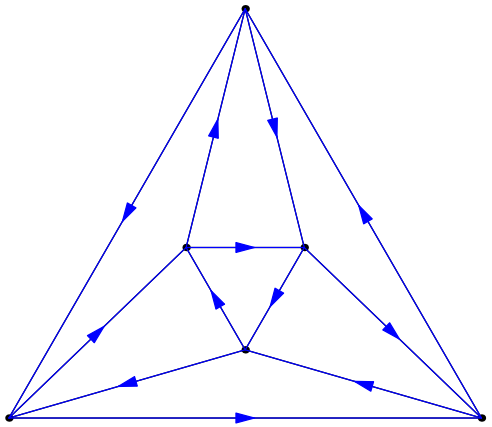

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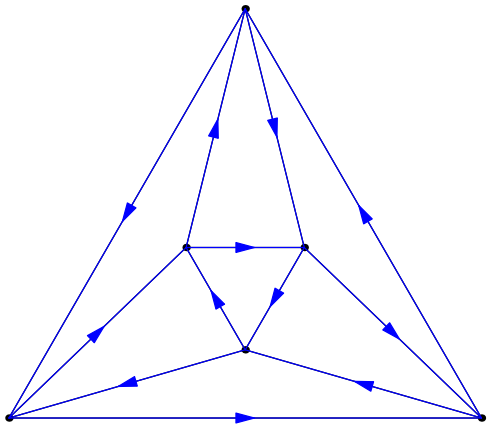

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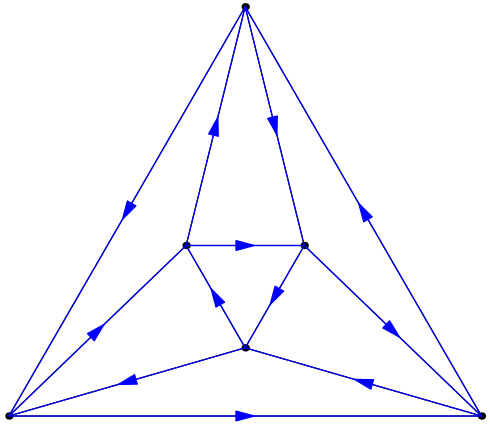

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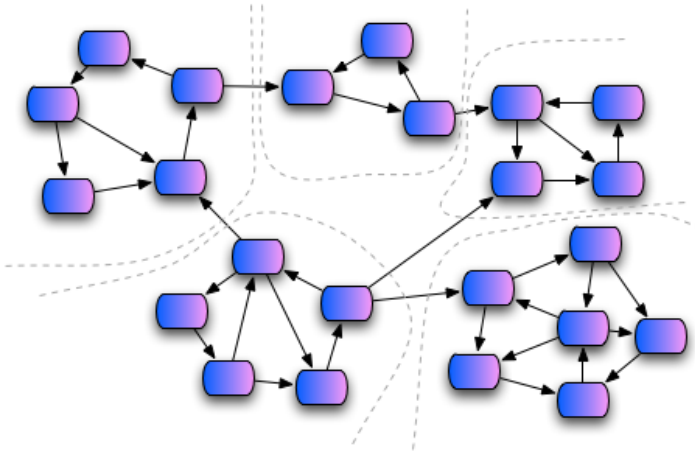

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- They are used as part of many specific algorithms for discovering graph properties
- Breadth first search is particularly important for finding shortest paths in unweighted graphs
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