

# Further Mathematics and Algorithms

## Lesson 4: *Use Arrays*



*Variable length arrays, implementing stacks*

# Outline

1. **Why Arrays?**
2. Variable Length Arrays
3. Programming Language
4. Implementing Stacks



# Use Arrays

- An array is a contiguous chunk of memory■
- In C we can create arrays using  
`int *array = new int[20]`■
- The array has an access time of  $\Theta(1)$ ■
- The constant factor is small (i.e. access time  $\approx 1$  time step)■
- Arrays provide a very efficient use of memory■
- 95% of the time using arrays is going to give you the best performance, although never use raw arrays!■

# Disadvantages of Arrays

- Arrays have a fixed length■
- Very often we don't know how big an array we want
  - ★ E.g. reading words from a file■
- Adding or deleting elements from the middle of an array is costly■
- Sorted arrays are expensive to maintain■
- Arrays don't know how big they are■—annoying■

# Outline

1. Why Arrays?
2. **Variable Length Arrays**
3. Programming Language
4. Implementing Stacks



# Variable Length Arrays

- We want a variable length array■
- Initially a variable length array would have length zero■
- We should be able to
  - ★ Add an element to an array
  - ★ Access any element in the array
  - ★ Change an element
  - ★ Delete elements
  - ★ Know how many elements we have■

# ADT for a List

- What do we want of a list of `ints`?
  - ★ `void push_back(int value)`
  - ★ random access `array[i]`
  - ★ `int size()`
- It would be useful if it resized
- It would be great to have some algorithms (e.g. sort) that can be run on a list

# Implementation

- How should we implement a list?■
- Use an array, of course!■
- We need to distinguish between
  - ★ the number of elements in the list `size()`
  - ★ the number of elements in the array `capacity()`■
- If the number of elements grows larger than the capacity then we need to increase the capacity■



# Initial Capacity

- We could prevent resizing arrays by using a huge initial capacity■
- However, how big is big enough?■
- What happens when we have an array of arrays?■
- Memory like time is resource we should care about■
- In an analogy with **time complexity** we also care about **space complexity** (i.e. how much memory we need)■
- If we want to store  $n$  elements it is reasonable to expect that we use  $cn$  bits of memory where we want to keep  $c$  small■

# Resizing Memory

- We start with some reasonable capacity■
- We can add elements■ until we reach the capacity■
- A simple method for resizing memory is
  - ★ create a new array with double the capacity of the old array■
  - ★ copy the existing elements from the old array to the new array■

`list.push_back(62)` —→ 

31	35	85	23
----	----	----	----

31	35	85	23	62			
----	----	----	----	----	--	--	--

# Amortised Time Analysis

- How efficient is resizing?■
- Most `push_back(elem)` operations are  $\Theta(1)$ ■
- When we are at full capacity we have to copy all elements■
- Adding to a full array is slow but it is **amortised** by other quick adds■

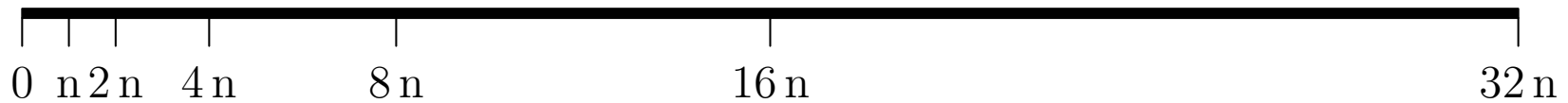
**amortised:** effect of a single operations 'deadened' by other operations■

# Example

- If we have an initial capacity of 10 and add 100 elements then the number of operations needed is
  - ★ adds: 100
  - ★ copies:  $10 + 20 + 40 + 80 = 150$
  - ★ `new int[]`: 4
- 250 adds and copies operations + 4 `new` operations

# General Time Analysis

- If we perform  $N$  adds with an initial capacity of  $n$
- We must perform  $m$  copies where



$$n \times 2^{m-1} < N \leq n \times 2^m \quad \text{i.e.} \quad m = \left\lceil \log_2 \left( \frac{N}{n} \right) \right\rceil$$

- The number of elements copied is

$$n + 2n + 4n + \dots + 2^{m-1}n = n(1 + 2 + \dots + 2^{m-1}) = n(2^m - 1)$$

- Total number of operations is (using  $\lceil \log(a) \rceil < \log(a) + 1$ )

$$N + n(2^m - 1) = N + n2^{\lceil \log_2(\frac{N}{n}) \rceil} - n < N + 2N - n < 3N$$

# Insertion and Deletion

- `vector<T>` is very useful and very fast for lots of things■
- But if you try to insert or delete an element anywhere other than the end then you have to shove all the subsequent elements one space forward■
- This is not the right data structure if you want to keep elements in order■(binary trees will do that for you much more efficiently)■
- Linked lists allow you to splice in a sublist into a list in constant time■although linked lists have a lot of drawbacks■

# Outline

1. Why Arrays?
2. Variable Length Arrays
3. **Programming Language**
4. Implementing Stacks



# Computer Languages

- Different computer languages are designed for different roles and have different advantages and disadvantages■
- **C++** was designed to be fast (as fast as C)■ it pays the price of allowing bugs that hard to detect■
- **Java** was designed to be vary safe (avoiding lots of bugs)■ but is not fast and a bit long winded■
- **Python** was designed so you can rapidly write powerful programmes with a small amount of code■ but it is not fast or safe■



# Problems with C++

- Amongst a number of issues that make C++ dangerous are
  - ★ Memory management
  - ★ Writing to parts of memory that you should not
  - ★ Multiple inheritance, although you seldom need to do this
- However, by using existing data structures (STL) and following established programming patterns these don't have to be an issue

# Memory Management

- Most programming languages have two types of memory

**The Stack:** is the area of memory controlled by compiler for local variables, function calls, etc.■

**The Heap:** is area that the programmer (you) can request, which is nice■

- In C++ you are given the **right** to ask for memory

```
int *storage = new int[n];
```

- You have **responsibility** to free the memory

```
delete[] storage;
```

# Trouble with Memory Management

- If you don't release memory acquired with `new` using `delete` you cause a **memory leak**■
- Often memory leaks are no concern, but in large programs memory leaks will rapidly exhaust the computer's memory, slowing down the code and eventually leading to the programme crashing■
- To release a block of memory we can use:  
**delete** [] `storage`;■
- Now `storage` is a **dangling pointer** and must not be used as it is no longer valid■
- If we accidentally delete the storage twice we get an *undefined behaviour*■ but often the programme will crash■

# Resource Acquisition is Initialisation (RAII)

- Java and Python use garbage collectors which automatically checks whether memory can be accessed and if not it is removed■
- In C++ this is your responsibility■
- But there is a standard **programming pattern** to elevate the problem known as **Resource Acquisition is Initialisation (RAII)**■

*Wrap all resources in classes. Request the resources in the constructor and release the resource in the destructor■*

- When the object goes out of scope (you leave a `for` loop, function call, etc.) the destructor is called and the resource is safely released■

# Writing over Memory

- In C++ the following will compile and run

```
int *array = new int[4];  
int *a = new int[2];  
double *darray = new double[4];  
array[4] = 4;
```

- However `array[4]` has not been assigned (unlike `array[0]`, `array[1]`, `array[2]` and `array[3]`)
- The memory on the heap corresponding to the address of `array[4]` might have been assigned to `a[0]` in which case you may inadvertently have set `a[0]` to 4 leading to the program not doing what you want
- It might be that you have put an `int` into `darray[0]` which will then crash the system when you read `darray[0]`

# Guarding Against Mistakes

- These are really hard problems to debug because where the program goes wrong or crashes can be very far from the assignment that caused the error■
- Java takes the approach that it always tests whether you are writing in valid memory■
- By default C++ doesn't even for data structures■—making this check slows down random access■
- Checks can also make pipeline optimisations harder to make■
- The onus is on the user to use the memory correctly■

# Follow Programming Idioms

- Using common data structures and following common idioms will prevent most errors

```
int n = 5;  
vector<int> array(n);
```

```
for(int i=0; i<array.size(); ++i) {  
    array[i] = i;  
}
```

```
for(auto pt=array.begin(); pt != array.end(); ++pt){  
    *pt *= 2  
}
```

```
for(int& element: array) {  
    element += 2;  
}
```



# Outline

1. Why Arrays?
2. Variable Length Arrays
3. Programming Language
4. **Implementing Stacks**





# Stacks

- Lets look at implementing a stack■
- Remember a stack has methods
  - ★ `push (Object )`
  - ★ `pop ()`
  - ★ `top ()`
  - ★ `empty ()`■

# Implementation of Stack

```
template <typename T>
class MyStack
{
private:
    std::vector<T> stack;

public:
    void push(const T& obj) {stack.push_back(obj);}

    T top() const {return stack.back();}

    T pop() {
        T tmp = stack.back();
        stack.pop_back();
        return tmp;
    }

    T empty() {return stack.size()==0;}
};
```

# Notes on Implementation

- I don't need to write a constructor as C++ generates a default constructor that will initialise the stack correctly■
- I don't need to write a desctuctor because by default the destructor for `vector<T>` will be called which releases memory■
- I've written the `pop` command, that I like, but if I run

```
stack<Widget> widget_stack;  
Widget w;  
widget_stack.push(w);  
Widget w1(widget.pop());
```

if the last command throws an exception then the last term on the stack is lost for ever■

# Why not use a vector

- Surely it is mad to use `MyStack<T>` as I could just use the more powerful `vector<T>`■
- I can make `MyStack<T>` as efficient as `vector<T>` by inlining function calls■
- But why would I want to lose functional?■
- By using `MyStack<T>` I am **declaring my intention** of using this data structure as a stack■
- I'm not going to do something weird like modify an element inside the stack■
- My code becomes self-explanatory■—I don't need to write comments as it is clear what I am doing■

# Using MyStack

- Implementing a stack using a dynamically re-sizable array is trivial■
- Stacks have many applications■
- E.g. suppose we want to write a program to reverse the order of strings in a file■

**you can't swallow a cage can't you**

you  
can't  
swallow  
a  
cage  
can  
you

■

# Reversing Strings in File

```
#include <stack>
#include <iostream>
#include <fstream>
#include <iterator>
#include <algorithm>
using namespace std;

int main(int argc, char *argv[]) {
    ifstream in(argv[1]);

    stack<string> stack;

    string word;
    while (in >> word)
        stack.push(word);

    while(!stack.empty()) {
        cout << stack.top() << ' ';
        stack.pop();
    }
}
```

# Lessons

- Arrays are very efficient both in space (memory) and access time■
- Resizing an array is not that costly■
- insertion and deletion are expensive,  $O(n)$ ■
- Arrays are often the simplest way to implement many other data structures, e.g. stacks■
- Use (dynamically re-sizable) arrays frequently!■