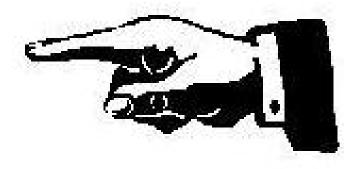
# **Further Mathematics and Algorithms**

# Outline

# Lesson 8: Point to where you are going: links



Linked lists

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# Non-Contiguous Data

- So far we have considered arrays where the data is stored in a contiguous chunk of memory!
- This has the great advantage of allowing random access
- It has the disadvantage that it is expensive to add or remove data from the middle of the list or to rearrange the data
- A different approach is to use units of data that point to other units

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# **Non-Contiguous Data Structures**

- There are a lot of important data structures using non-contiguous memory
  - ★ Binary trees
  - ⋆ Graphs
- In this lecture we consider linked-lists
- This is a classic data structure, which is almost entirely useless
- However, it serves as a good introduction to much more useful data structures

#### **Self-Referential Classes**

The building block for a linked list is a node class

```
struct Node<T>
{
   Node(U value, Node<U> *node): value(value), next(node) {}
   T element;
   Node<T> *next;
}
```

We create new nodes

```
Node<int> *node = new Node<int>(10, pt_to_next)
```

- Note that node is the address of this node!
- I make it a struct as this is a class where I want public access to the element and next
- I can make this class a private class of my linked list

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# Singly Linked List

• We can build a linked list by stringing nodes together



We don't show the "pointer" to element

- A singly linked list has a single "pointer" to the next element
- A doubly linked list has "pointers" to the next and previous element—we will see this later
- We should be able to create a linked list, add elements, remove elements, see if an element exists, etc.

#### **Outline**

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### **Implementation**

- We consider a lightweight implementation
- The class will have a head, a size counter and have a Node as a nested class

```
class MyList {
private:
    template <typename U>
    struct Node{
       Node(U value, Node<U> *node): value(value), next(node) {}
       U value;
       Node<U> *next;
    };
    Node<T> *head;
    unsigned noElements;
```

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# **Simple Methods**

• The constructor is simple (and not strictly necessary)

```
\texttt{MyList():} \ \texttt{n(0),} \ \texttt{head(0)} \ \{\,\}
```

• Other simple methods are

```
unsigned size() const {return noElements;}
bool empty() const {
  return head == 0;
}
```

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### Remove Head of List

```
void remove_head()
{
    Node<T>* dead = head;
    head = head->next
    noElements--;
    delete dead;
}

head

head

remove_head()
```

# **Adding elements**

```
void add(T element)
{

   Node<T> *newNode = new Node<E>();
   newNode.element = element;
   newNode.next = head;
   head = newNode;
   noElements++;
   return true;
}

head

bead

A

newNode

noElements

noElements

noElements
```

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#### Other Methods

- We can easily implement many other methods
  - $\star$  get (int i) —return  $i^{th}$  item in list
  - ★ remove(T obj)-remove obj from list
  - ★ insert(int position, element)
- Note that get (int i) requires moving down the list so is O(n)
   (i.e. not random access)

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noElements |

**Outline** 

Stack

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## **Complexity of Stack**

- All operations of the stack is constant time, i.e. O(1)
- This is the same time complexity as an array implementation
- Memory requirement is approximately  $2 \times n$  reference and n objects—same as worst case for an array!
- However, hidden cost of creating and destroying Node objects
- The array implementation is therefore slightly faster

```
class Stack<E>
{
   private Mylist<T> list = new mylist<T>();

   boolean push(E obj) {list.add(obj);}

   E top() {return list.get_head();} // throw exception

   E pop() {
     T tmp = list.get_head();
     list.remove_head();
     return tmp;
   }

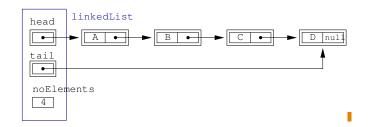
   boolean empty() {return list.empty();}
}
```

• It is easy to implement a stack using a linked list

template <typename T>

# Point to the Back

- ullet To find the end of the queue takes n jumps ullet
- Thus our linked list isn't the right data structure to implement a queuel
- However, we could include a pointer to the end of the queue



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# Implementing a Queue

Outline

- We can then add elements to the tail in constant time
- ullet We can the implement a queue in O(1) time by
  - ★ enqueueing at the back
  - ★ dequeueing at the head
- I leave the implementation details as an exercise for youl
- Note that although adding an element to the tail is constant time, removing an element from the tail is O(n) as we have to find the new tail

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class Node<T>

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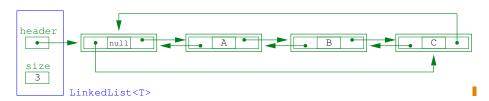
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# **Doubly linked list**

- In a more powerful linked list we would like to navigate the list in either direction
- To achieve this it uses a doubly-linked lists with elements to next and previous

# **Dummy Node**

• List includes a dummy node—this make the implementations slicker



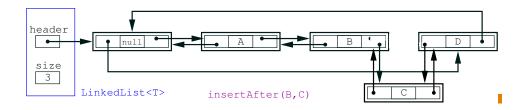
 Symmetric data structure so processing head and tail is equally efficient.

element

# **Time Complexity**

Outline

- add and remove from head and tail O(1)
- find O(n) and slow
- insert and delete O(1) (faster than an array list) once position is found



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**Line Editor** 

#### When To Use Linked Lists

- It is difficult to think of applications where linked lists are the best data structure
- lists—variable length arrays are usually better
- queues—linked list OK, but circular arrays are probably better
- sorted lists—binary trees much better
- linked lists have efficient insertion and deletion but it is difficult to think of an application where this matters

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- One application where efficient insertion and deletion matters is a line editor
- We are usually working at a particular location in the text
- We often want to add or delete whole lines
- Storing the lines as strings in a linked list would allow a fairly efficient implementation

# Outline

Skip Lists

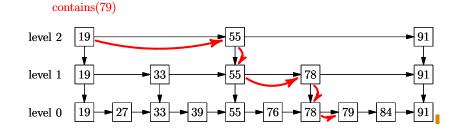
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- **Efficiency of Skip Lists**
- Skip lists provide  $\Theta(\log_2(n))$  search as opposed to  $\Theta(n)$
- They have the similar time complexity to binary trees, although binary trees are slightly faster
- They have one advantage over binary trees—they allow efficient concurrent access
- The standard template library provides a doubly linked list, list<T>| as well as a slingly linked-list slist<T>|

- Linked lists have the disadvantage that to get to anywhere in the list takes on average  $\Theta(n)$  steps
- Even if you kept an ordered list you still need to traverse it
- Skip lists are hierarchies of linked lists which allow binary search



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Lessons

- Node structures that point to other Node structures are used in many important data structures
- Linked lists are the simplest examples of this kind of structure and consequently has a dominant position in most DSA books
- In practice linked lists are seldom the data structure of choice before choosing to use a linked list consider the alternatives
- There are some important uses for linked lists, e.g. skip lists and hash tables (see lecture on hashing)

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