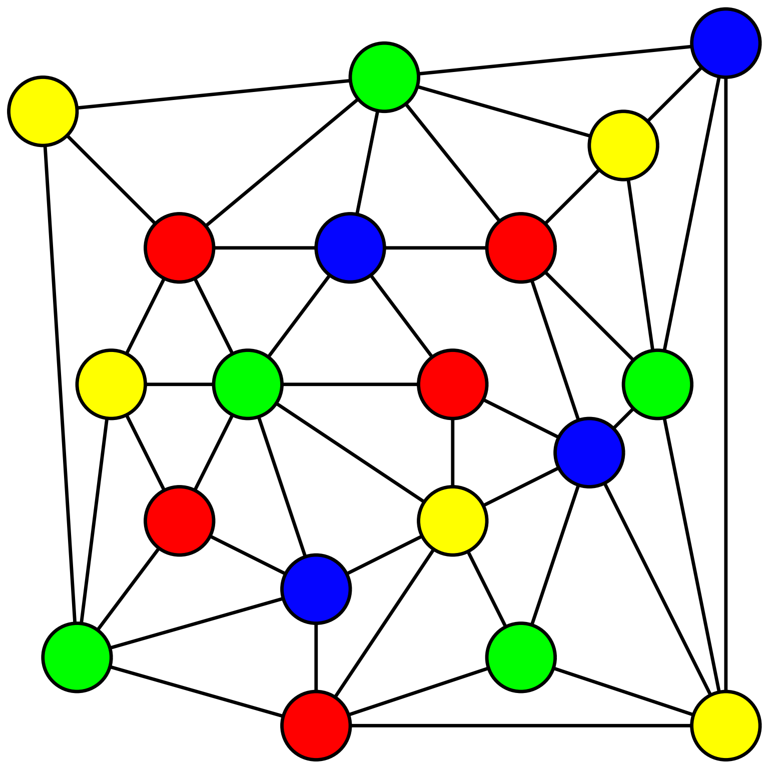
Algorithms and Analysis

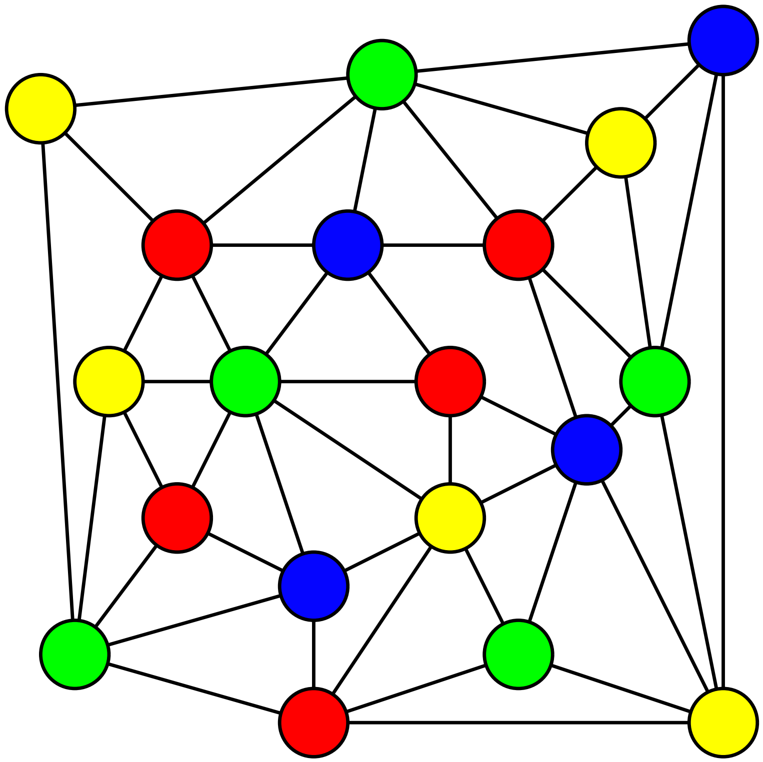
Lesson 19: *Think Graphically*



Graph theory, applications of graphs, graph problems

Outline

1. **Graph Theory**
2. Applications of Graphs
   - Geometric applications
   - Relational applications
3. Implementing Graphs
4. Graph Problems



Motivation

- Many different problems can be described in terms of graphs
- This often reveals the true nature of the problem
- It unifies many apparently different problems
- As much is known about graph problems it often provides a pointer to the solution

Motivation

- Many different problems can be described in terms of graphs
- This often reveals the true nature of the problem
- It unifies many apparently different problems
- As much is known about graph problems it often provides a pointer to the solution

Motivation

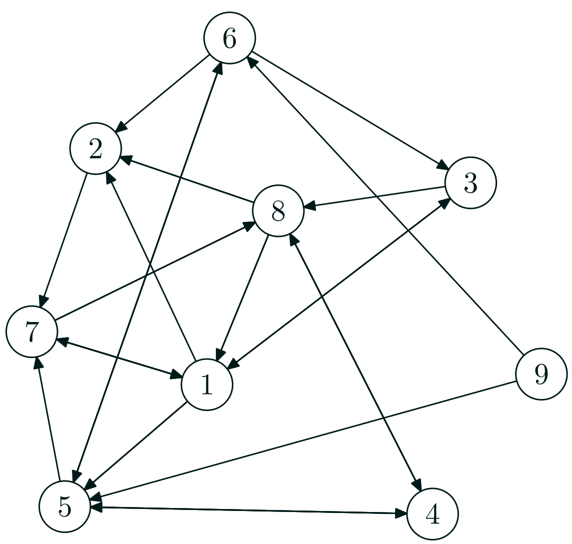
- Many different problems can be described in terms of graphs
- This often reveals the true nature of the problem
- It unifies many apparently different problems
- As much is known about graph problems it often provides a pointer to the solution

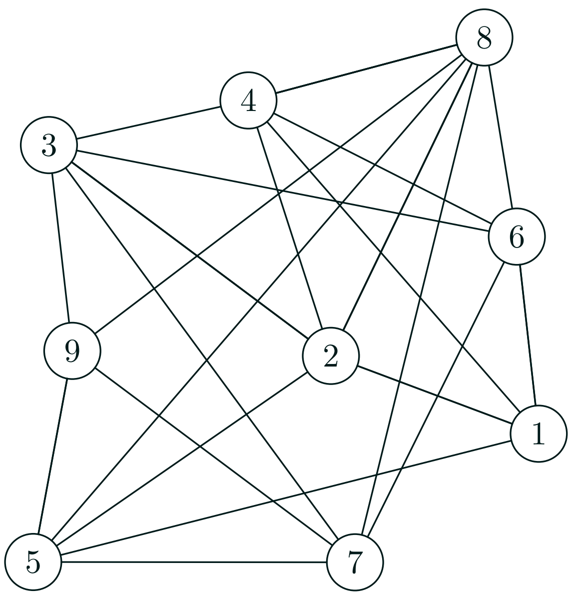
Motivation

- Many different problems can be described in terms of graphs
- This often reveals the true nature of the problem
- It unifies many apparently different problems
- As much is known about graph problems it often provides a pointer to the solution

Definition of a Graph

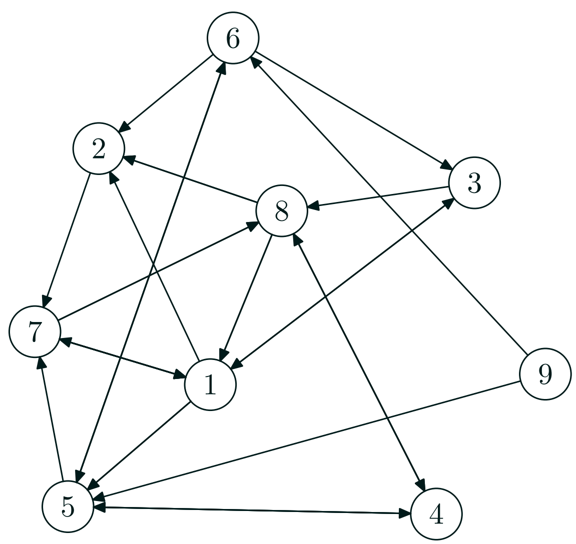
- A graph, G, can be described by
  - ⋆ A set of vertices or nodes $\mathcal{V} = \{1, 2, 3 \ldots n\}$
  - ⋆ A set of edges $\mathcal{E} = \{(i, j) | \text{vertex } i \text{ is connected to vertex } j\}$
- The edges may be
  - ⋆ **directed**—sometimes called a **digraph**
  - ⋆ **undirected**

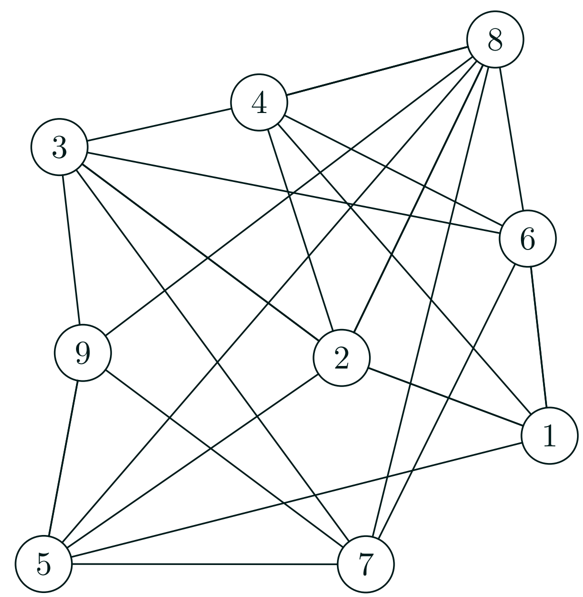




Definition of a Graph

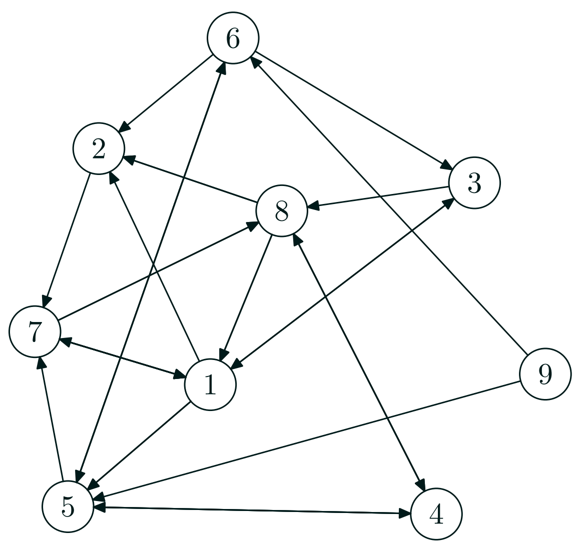
- A graph, G, can be described by
  - ⋆ A set of vertices or nodes $\mathcal{V} = \{1, 2, 3 \ldots n\}$
  - ⋆ A set of edges $\mathcal{E} = \{(i, j) | \text{vertex } i \text{ is connected to vertex } j\}$
- The edges may be
  - ⋆ **directed**—sometimes called a **digraph**
  - ⋆ **undirected**

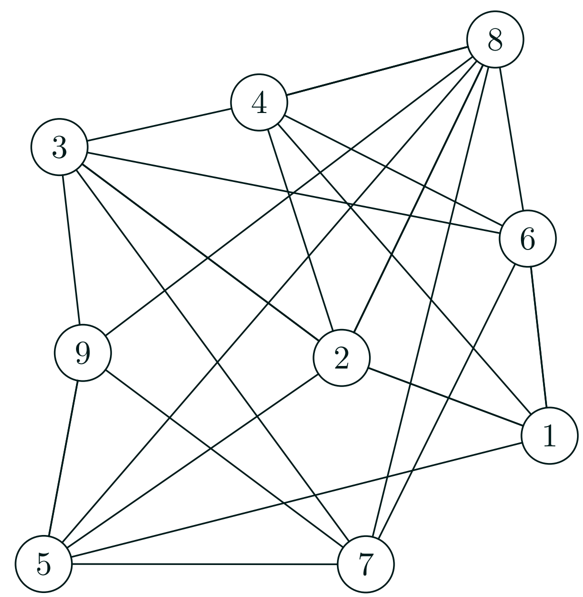




Definition of a Graph

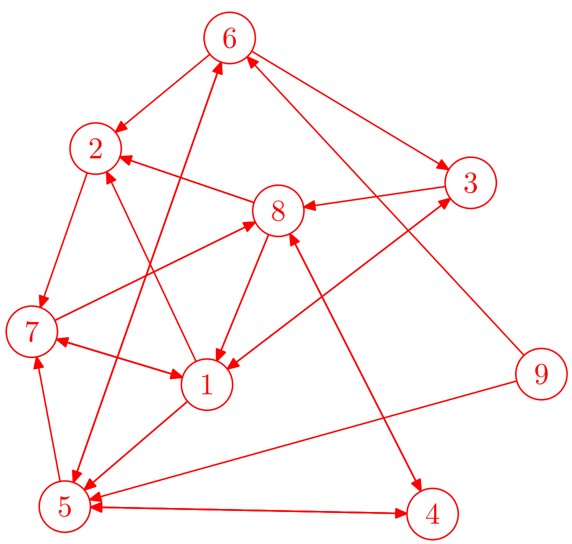
- A graph, G, can be described by
  - ⋆ A set of vertices or nodes $\mathcal{V} = \{1, 2, 3 \dots n\}$
  - ⋆ A set of edges $\mathcal{E} = \{(i,j) | \text{vertex } i \text{ is connected to vertex } j\}$
- The edges may be
  - ⋆ **directed**—sometimes called a **digraph**
  - ⋆ **undirected**

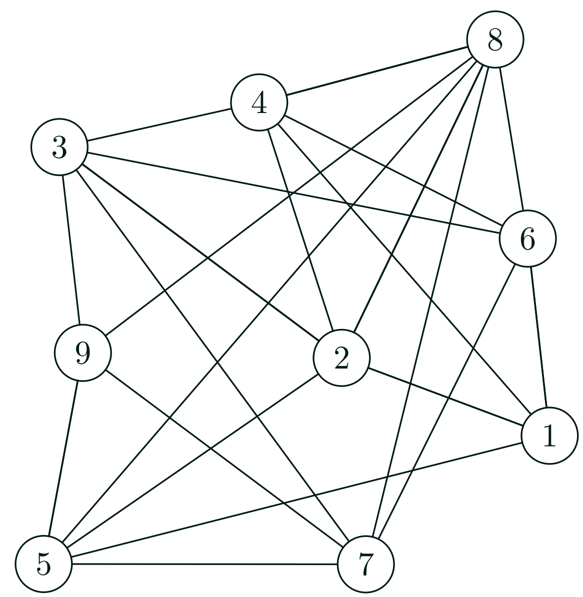




Definition of a Graph

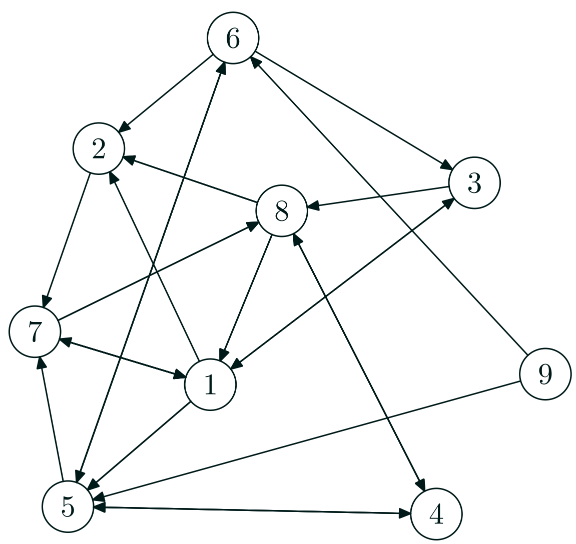
- A graph, G, can be described by
  - ⋆ A set of vertices or nodes $\mathcal{V} = \{1, 2, 3 \ldots n\}$
  - ⋆ A set of edges $\mathcal{E} = \{(i, j) | \text{vertex } i \text{ is connected to vertex } j\}$
- The edges may be
  - ⋆ **directed**—sometimes called a **digraph**
  - ⋆ **undirected**

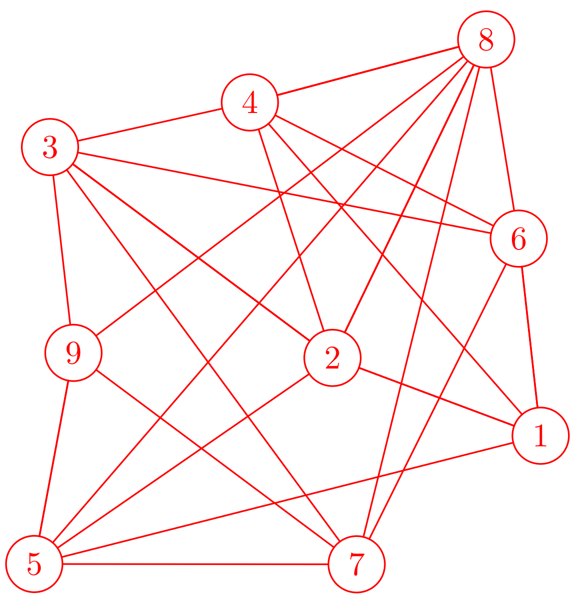




Definition of a Graph

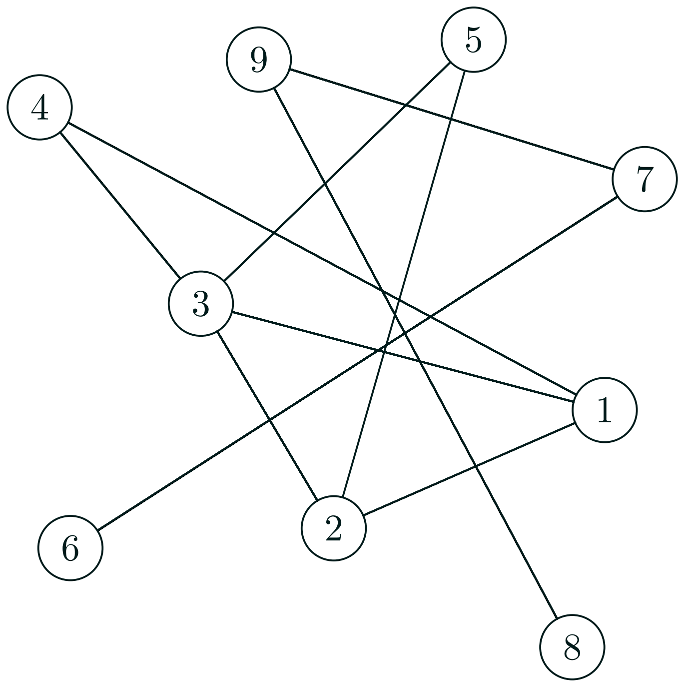
- A graph, G, can be described by
  - ⋆ A set of vertices or nodes $\mathcal{V} = \{1, 2, 3 \ldots n\}$
  - ⋆ A set of edges $\mathcal{E} = \{(i, j) | \text{vertex } i \text{ is connected to vertex } j\}$
- The edges may be
  - ⋆ **directed**—sometimes called a **digraph**
  - ⋆ **undirected**





Connected and Unconnected Graphs

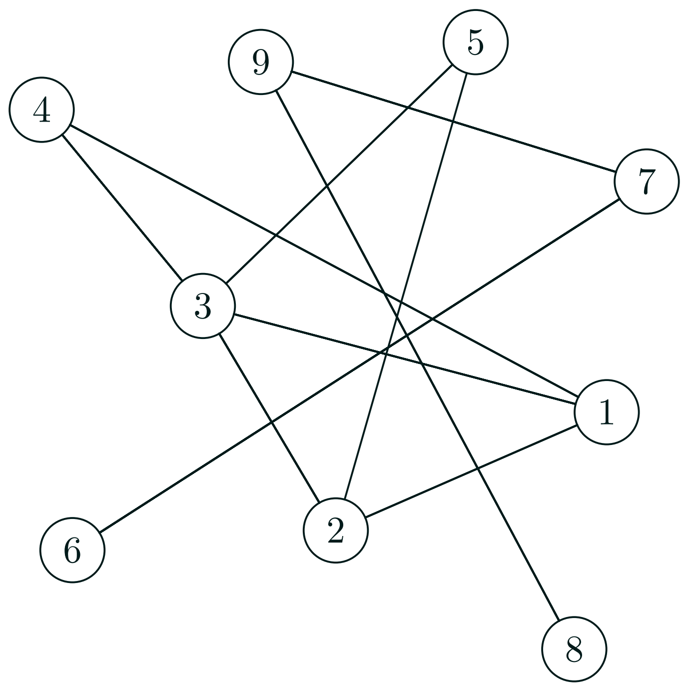
- A graph is **connected** if you can get from one node to any other along a series of edges
- Otherwise it is **disconnected**



Connected and Unconnected Graphs

- A graph is **connected** if you can get from one node to any other along a series of edges

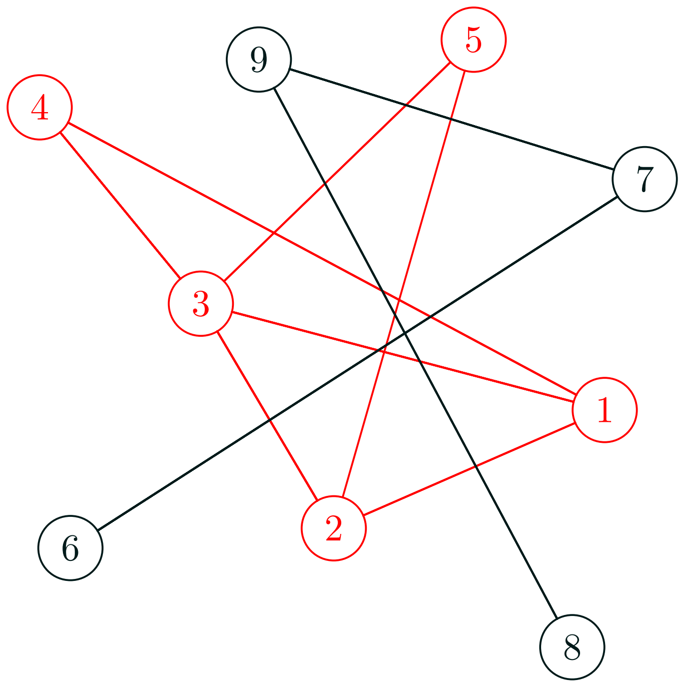
- Otherwise it is **disconnected**



Connected and Unconnected Graphs

- A graph is **connected** if you can get from one node to any other along a series of edges

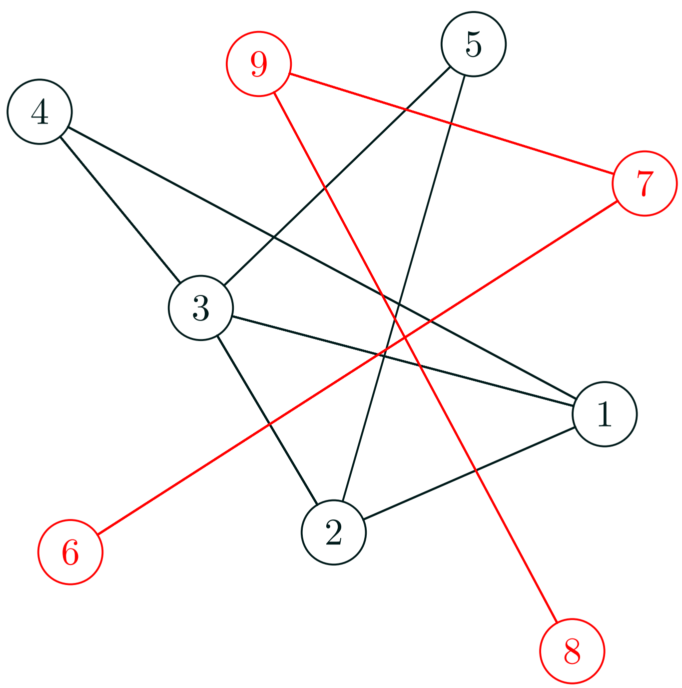
- Otherwise it is **disconnected**



Connected and Unconnected Graphs

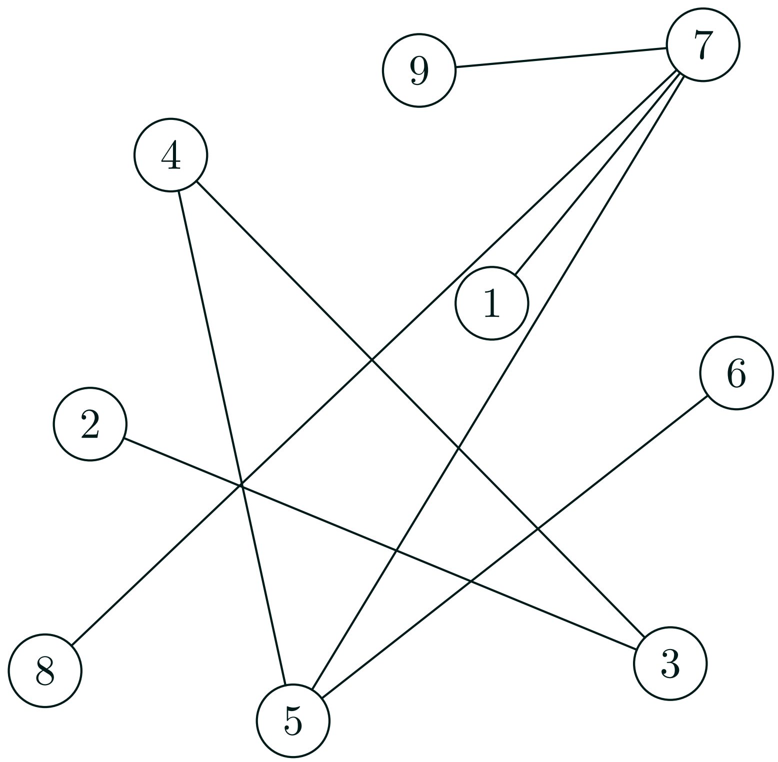
- A graph is **connected** if you can get from one node to any other along a series of edges

- Otherwise it is **disconnected**



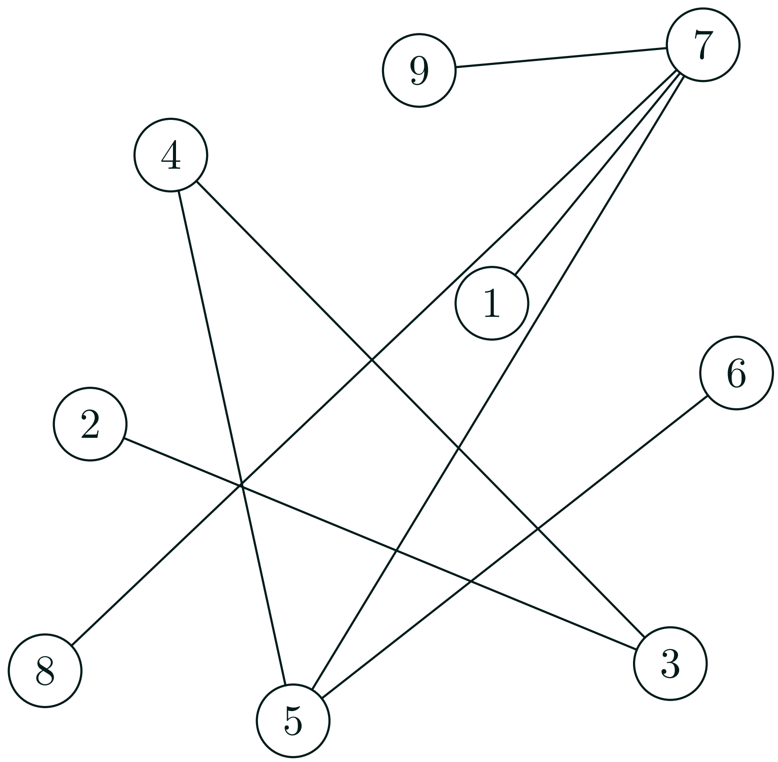
Trees

- A tree is a connected graphs with no cycles
- A tree will have $n-1$ edges



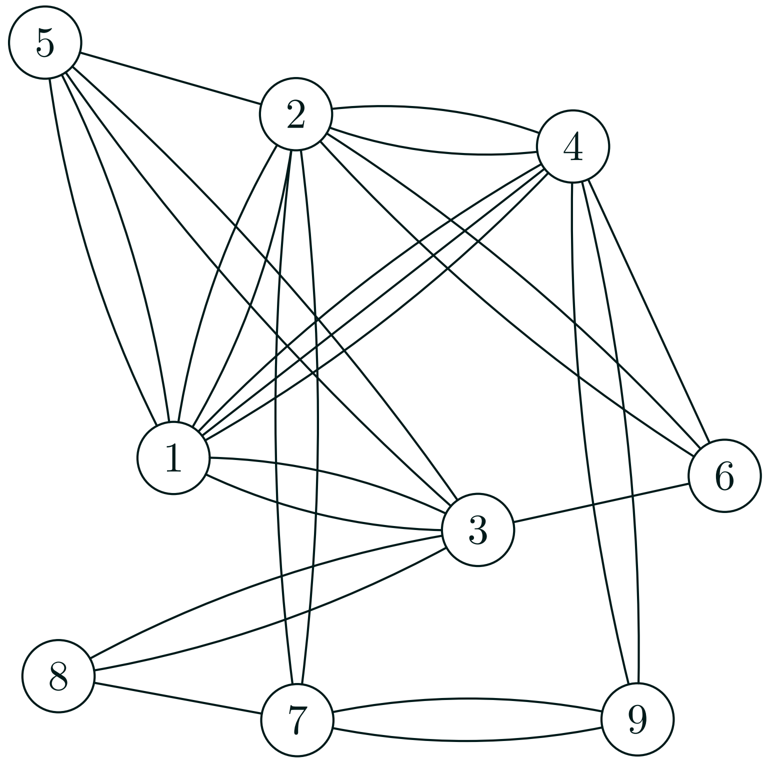
Trees

- A tree is a connected graphs with no cycles
- A tree will have $n - 1$ edges



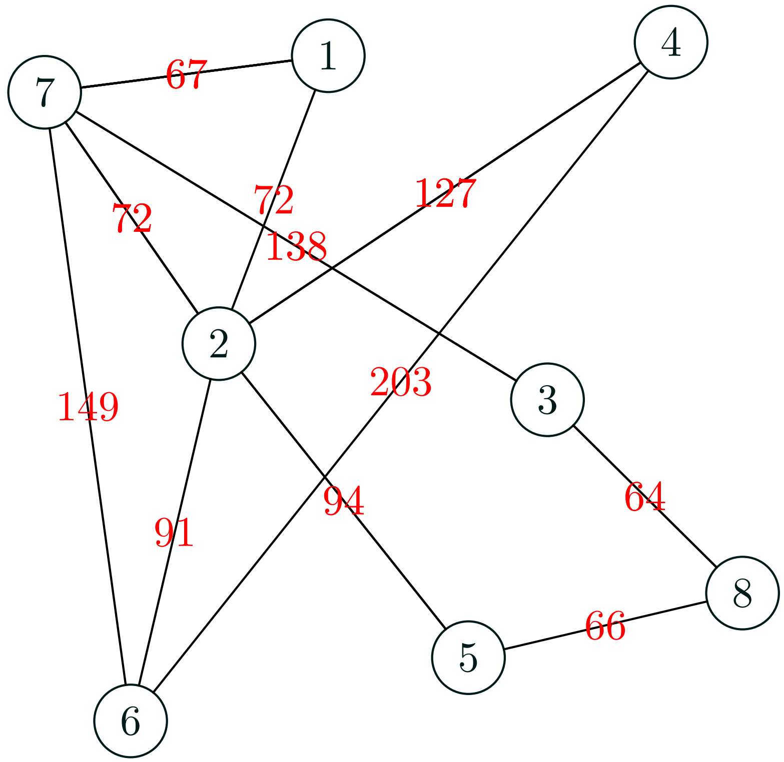
Multigraphs

- If the collection of edges is a *multiset* then we obtain a **multigraphs** where more than one edge is allowed between pairs of vertices



Weighted Graphs

- If we assign a number to an edge we obtain a **weighted graph**



Networks

- Sometimes we add more information to the graph
- E.g. attributes to the nodes or edges
- Graphs with many attributes are often referred to as **networks**

Networks

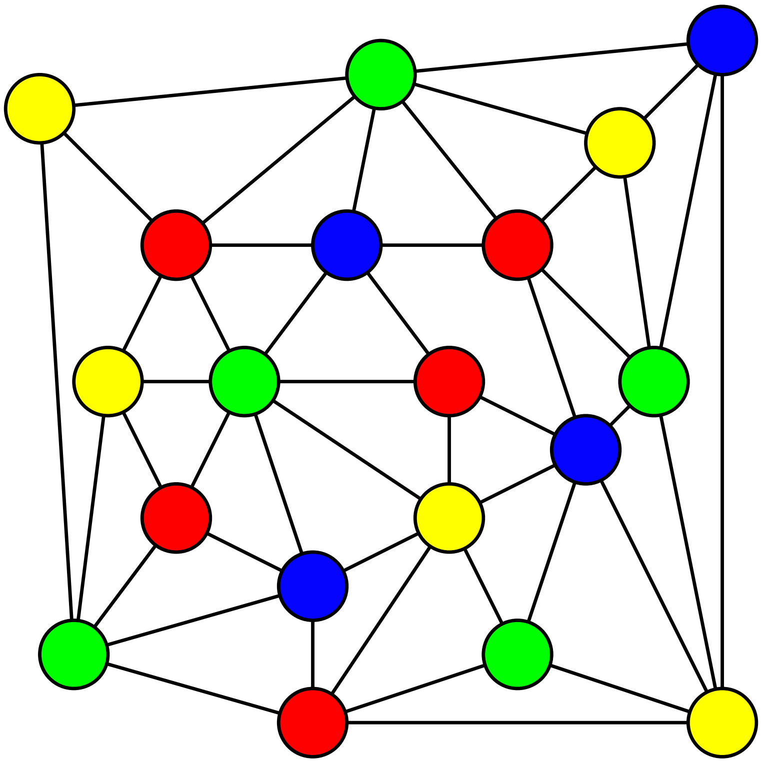
- Sometimes we add more information to the graph
- E.g. attributes to the nodes or edges
- Graphs with many attributes are often referred to as **networks**

Networks

- Sometimes we add more information to the graph
- E.g. attributes to the nodes or edges
- Graphs with many attributes are often referred to as **networks**

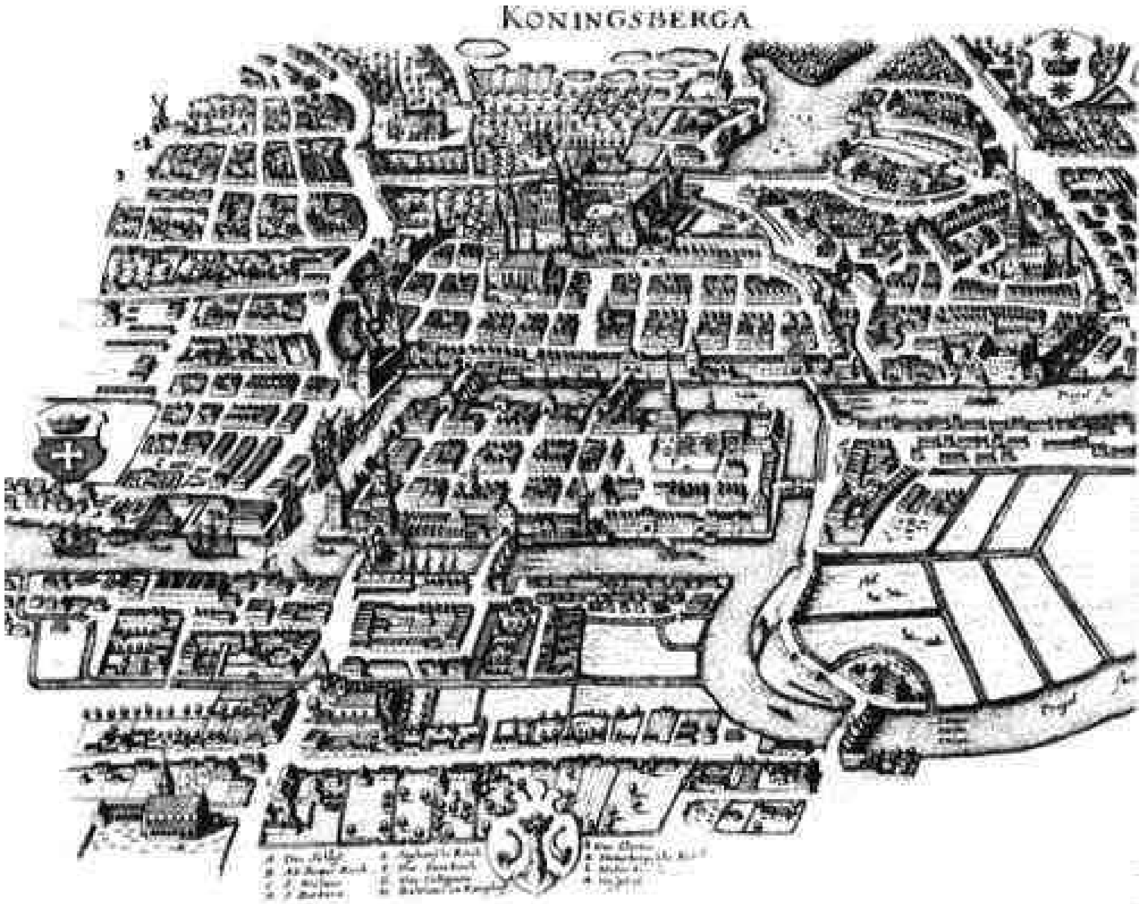
Outline

1. Graph Theory
2. **Applications of Graphs**
   - Geometric applications
   - Relational applications
3. Implementing Graphs
4. Graph Problems



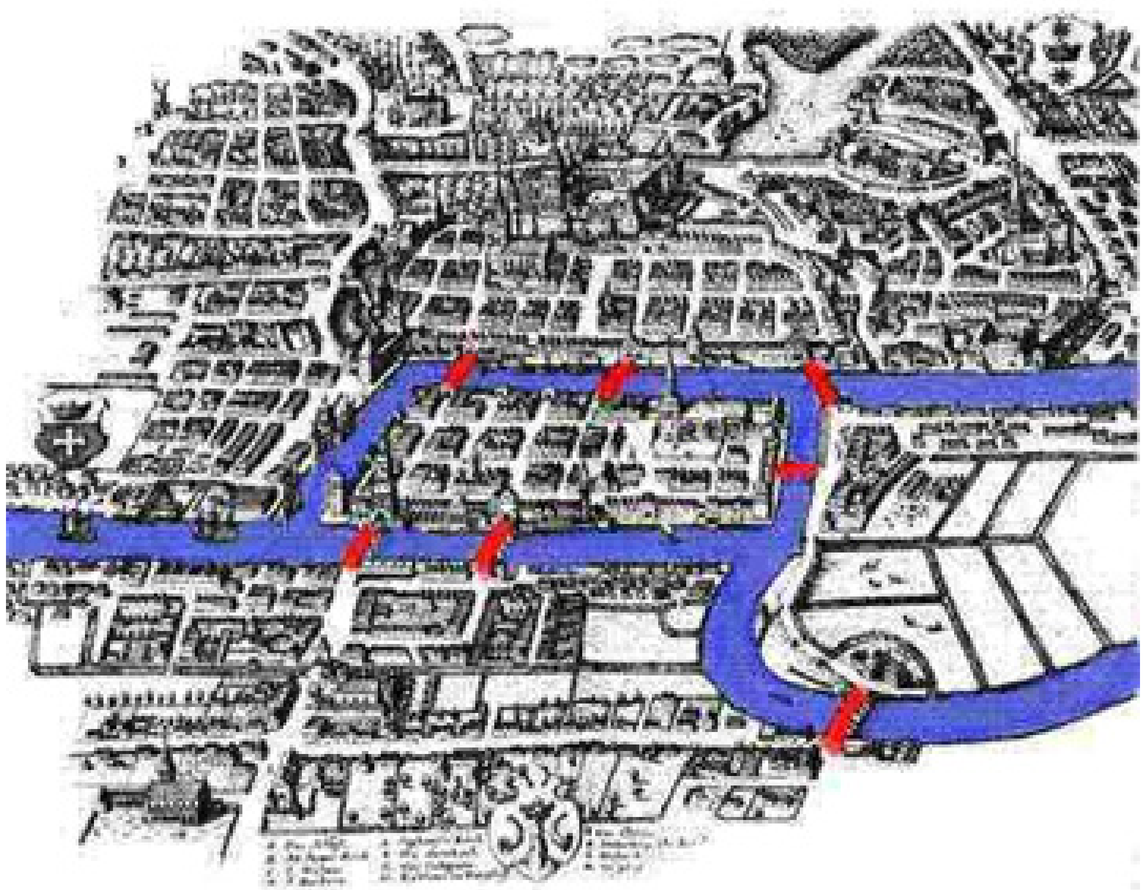
Bridges of Königsberg

Is there a tour around Königsberg going over every bridge once?



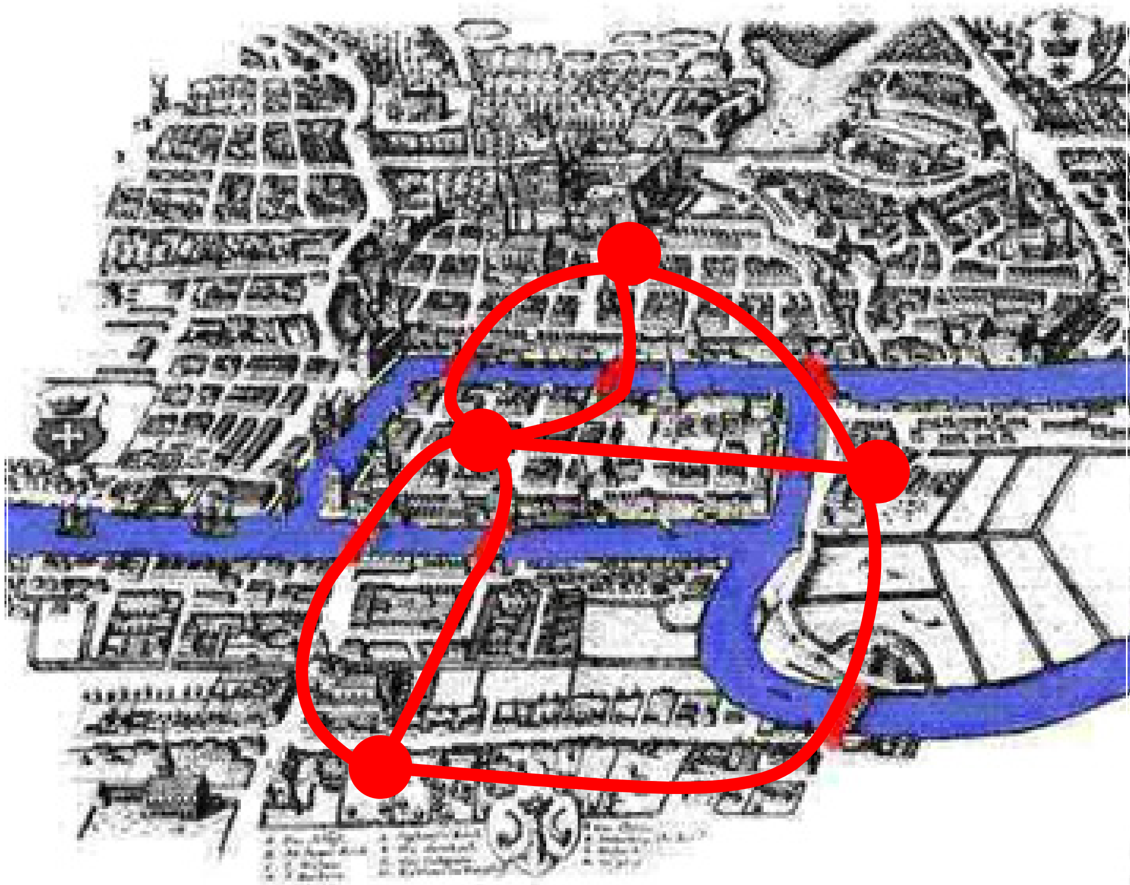
Bridges of Königsberg

Is there a tour around Königsberg going over every bridge once?



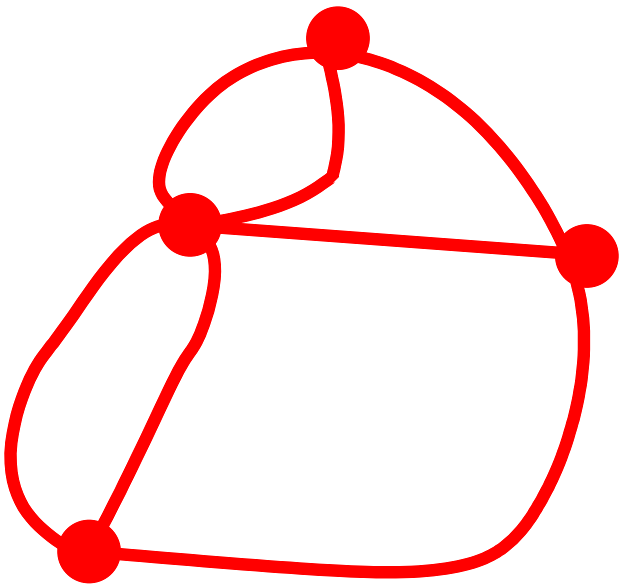
Bridges of Königsberg

Is there a tour around Königsberg going over every bridge once?



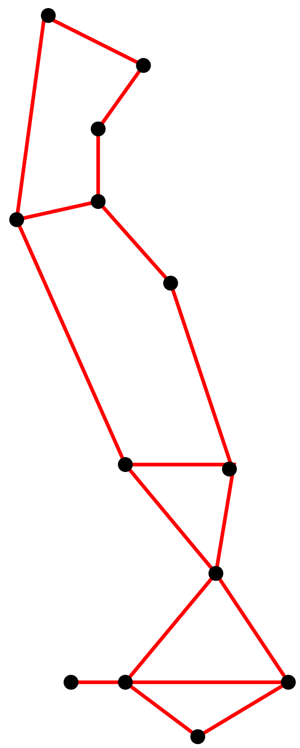
Bridges of Königsberg

Is there a tour around Königsberg going over every bridge once?



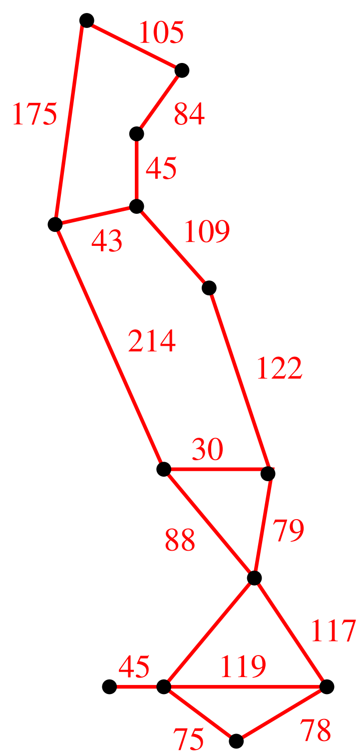
In 1736 Euler published a paper answering this question and founding graph theory

Representing Distances



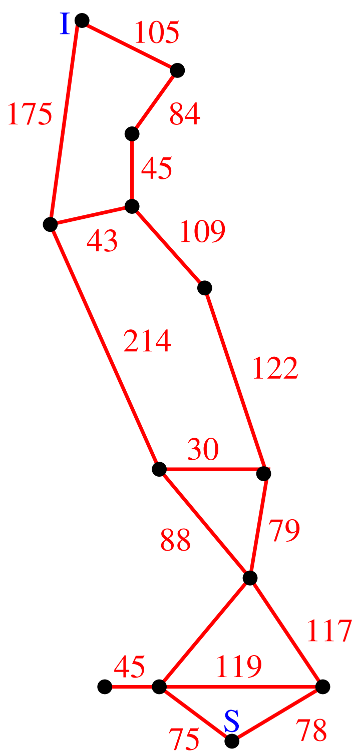
- Consider some graph
- With weights representing the distance between nodes
- What is the shortest distance between S and I?

Representing Distances



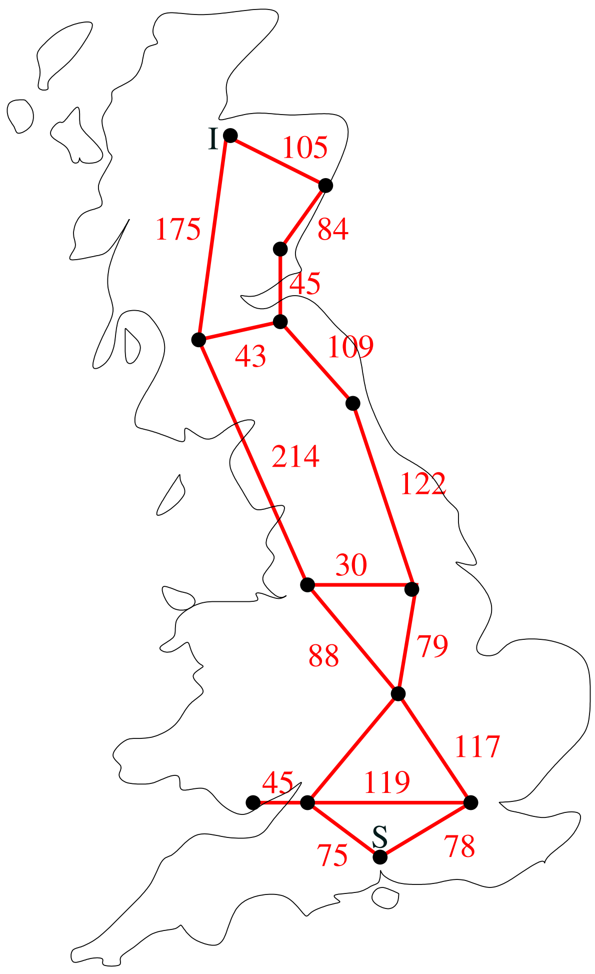
- Consider some graph
- With weights representing the distance between nodes
- What is the shortest distance between S and I?

Representing Distances



- Consider some graph
- With weights representing the distance between nodes
- What is the shortest distance between S and I?

Representing Distances



- Consider some graph
- With weights representing the distance between nodes
- What is the shortest distance between S and I?

Other Applications

- We could take the weights to represent the time taken to travel between nodes
- In a computer network the weights might represent the bandwidth
- In a representation of a transport system the weights might represent the carrying capacity of the traffic on a road
- Graphs can be used to represent other kinds of relationships
- E.g. We could create a digraph of links between web pages

Other Applications

- We could take the weights to represent the time taken to travel between nodes
- In a computer network the weights might represent the bandwidth
- In a representation of a transport system the weights might represent the carrying capacity of the traffic on a road
- Graphs can be used to represent other kinds of relationships
- E.g. We could create a digraph of links between web pages

Other Applications

- We could take the weights to represent the time taken to travel between nodes
- In a computer network the weights might represent the bandwidth
- In a representation of a transport system the weights might represent the carrying capacity of the traffic on a road
- Graphs can be used to represent other kinds of relationships
- E.g. We could create a digraph of links between web pages

Other Applications

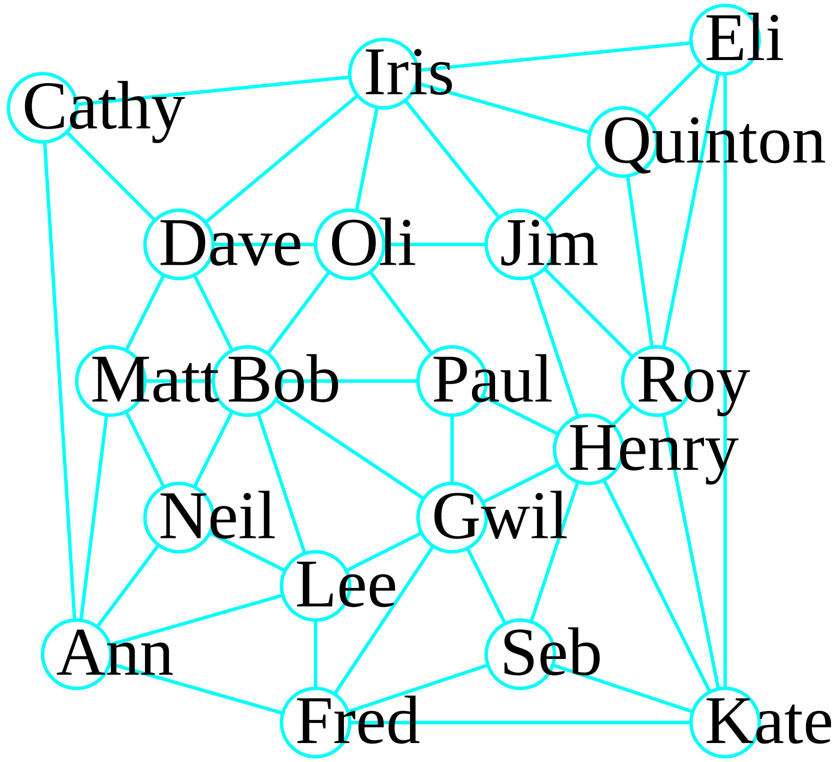
- We could take the weights to represent the time taken to travel between nodes
- In a computer network the weights might represent the bandwidth
- In a representation of a transport system the weights might represent the carrying capacity of the traffic on a road
- Graphs can be used to represent other kinds of relationships
- E.g. We could create a digraph of links between web pages

Other Applications

- We could take the weights to represent the time taken to travel between nodes
- In a computer network the weights might represent the bandwidth
- In a representation of a transport system the weights might represent the carrying capacity of the traffic on a road
- Graphs can be used to represent other kinds of relationships
- E.g. We could create a digraph of links between web pages

Christmas Card Problem

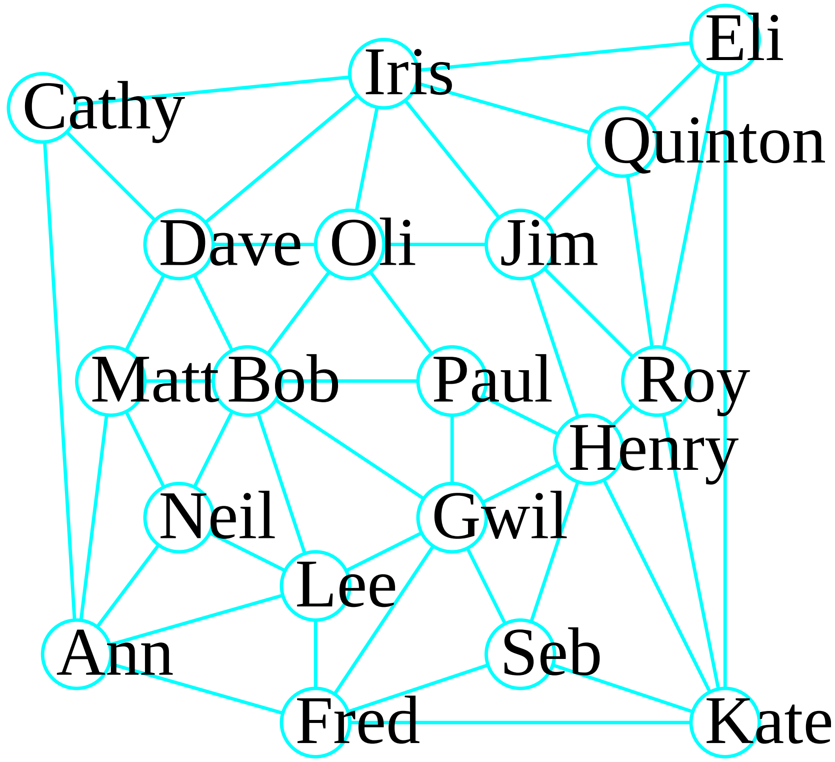
- I have four types of Christmas cards
- Some of my friends know each other



- I don't want to send friends that know each other the same card

Christmas Card Problem

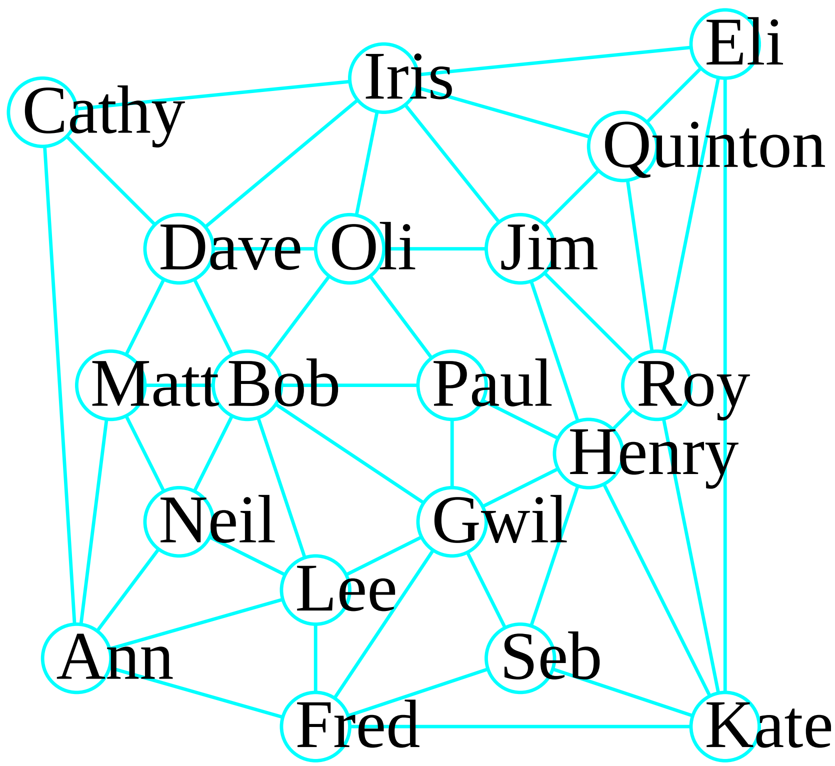
- I have four types of Christmas cards
- Some of my friends know each other



- I don't want to send friends that know each other the same card

Christmas Card Problem

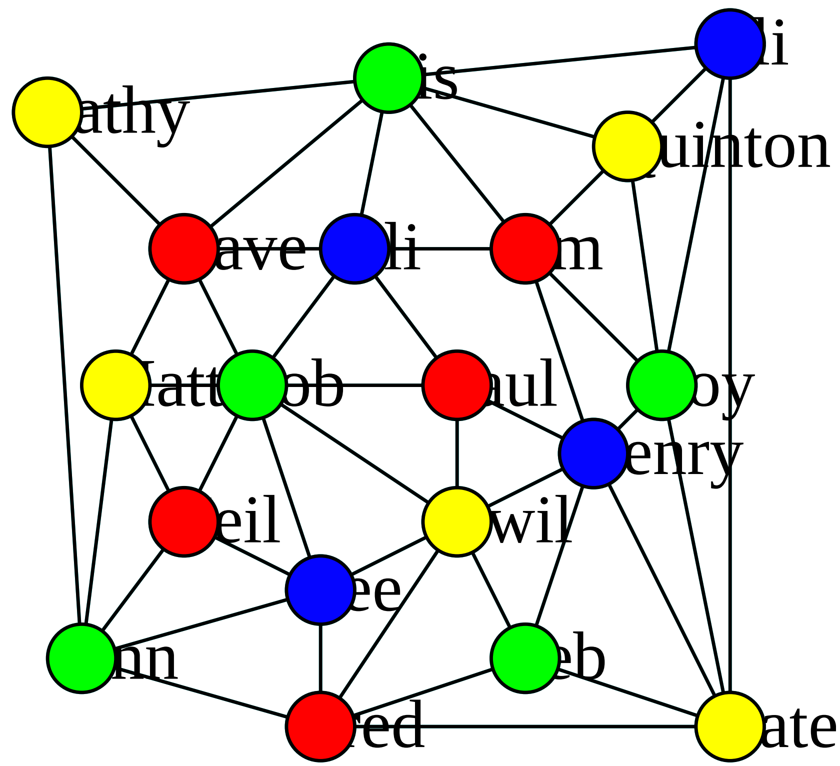
- I have four types of Christmas cards
- Some of my friends know each other



- I don't want to send friends that know each other the same card

Christmas Card Problem

- I have four types of Christmas cards
- Some of my friends know each other



- I don't want to send friends that know each other the same card

A Real World Problem

- A food company used different colour bags for each of it products
- To save money they reduced the stock of bags to 25
- They wanted to know what items to put in what bags so that as few customers as possible would have items with the same colour bags
- This can again be reduced to a graph colouring problem
  - ⋆ Each node represents an item
  - ⋆ The edges were weighted by the number of customers that took both items
  - ⋆ The aim was to colour the nodes with 25 colours to minimise the weights where the edges shared the same colour

A Real World Problem

- A food company used different colour bags for each of it products
- To save money they reduced the stock of bags to 25
- They wanted to know what items to put in what bags so that as few customers as possible would have items with the same colour bags
- This can again be reduced to a graph colouring problem
  - ⋆ Each node represents an item
  - ⋆ The edges were weighted by the number of customers that took both items
  - ⋆ The aim was to colour the nodes with 25 colours to minimise the weights where the edges shared the same colour

A Real World Problem

- A food company used different colour bags for each of it products
- To save money they reduced the stock of bags to 25
- They wanted to know what items to put in what bags so that as few customers as possible would have items with the same colour bags
- This can again be reduced to a graph colouring problem
  - ⋆ Each node represents an item
  - ⋆ The edges were weighted by the number of customers that took both items
  - ⋆ The aim was to colour the nodes with 25 colours to minimise the weights where the edges shared the same colour

A Real World Problem

- A food company used different colour bags for each of it products
- To save money they reduced the stock of bags to 25
- They wanted to know what items to put in what bags so that as few customers as possible would have items with the same colour bags
- This can again be reduced to a graph colouring problem
  - ⋆ Each node represents an item
  - ⋆ The edges were weighted by the number of customers that took both items
  - ⋆ The aim was to colour the nodes with 25 colours to minimise the weights where the edges shared the same colour

A Real World Problem

- A food company used different colour bags for each of it products
- To save money they reduced the stock of bags to 25
- They wanted to know what items to put in what bags so that as few customers as possible would have items with the same colour bags
- This can again be reduced to a graph colouring problem
  - ⋆ Each node represents an item
  - ⋆ The edges were weighted by the number of customers that took both items
  - ⋆ The aim was to colour the nodes with 25 colours to minimise the weights where the edges shared the same colour

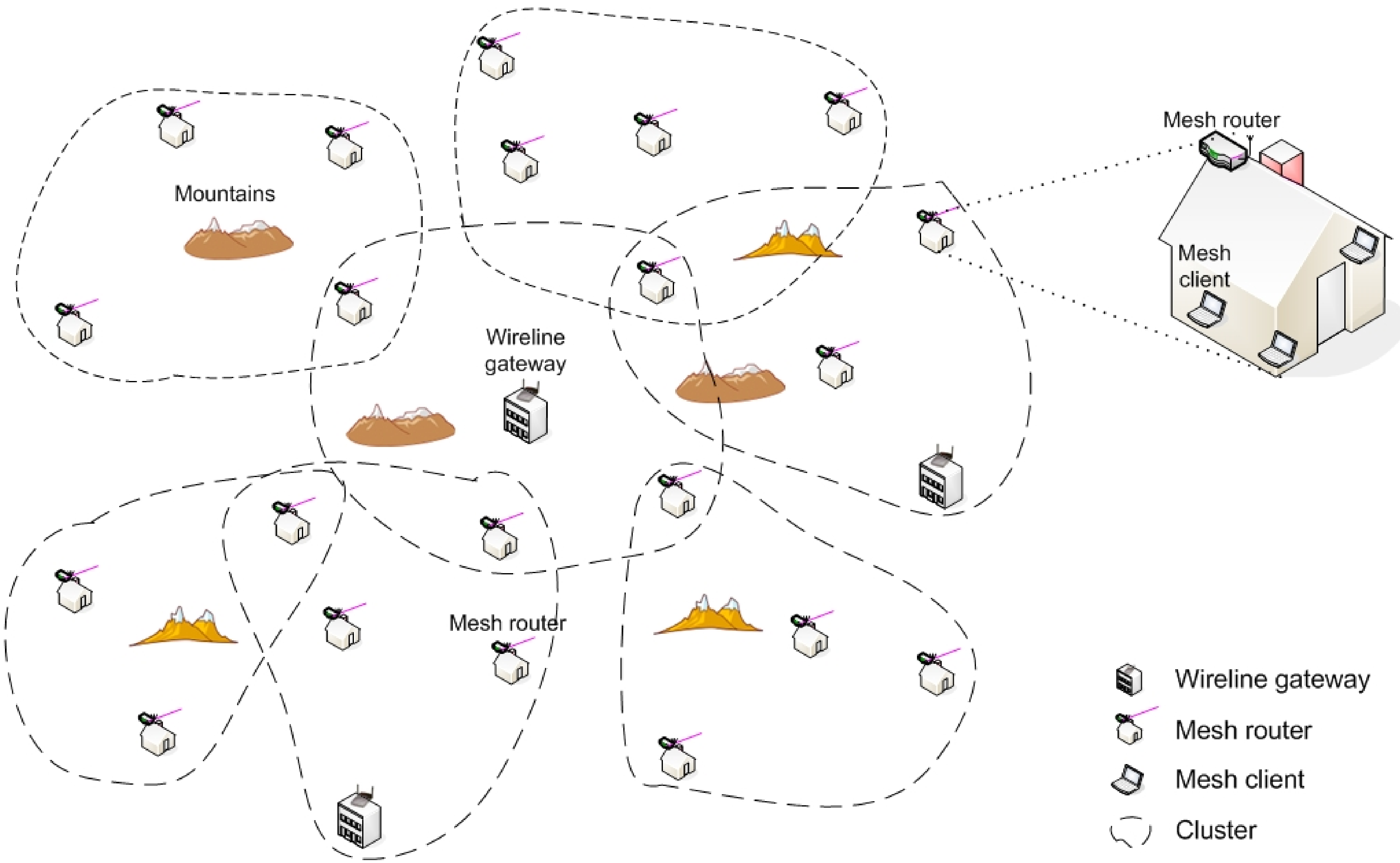
A Real World Problem

- A food company used different colour bags for each of it products
- To save money they reduced the stock of bags to 25
- They wanted to know what items to put in what bags so that as few customers as possible would have items with the same colour bags
- This can again be reduced to a graph colouring problem
  - ⋆ Each node represents an item
  - ⋆ The edges were weighted by the number of customers that took both items
  - ⋆ The aim was to colour the nodes with 25 colours to minimise the weights where the edges shared the same colour

A Real World Problem

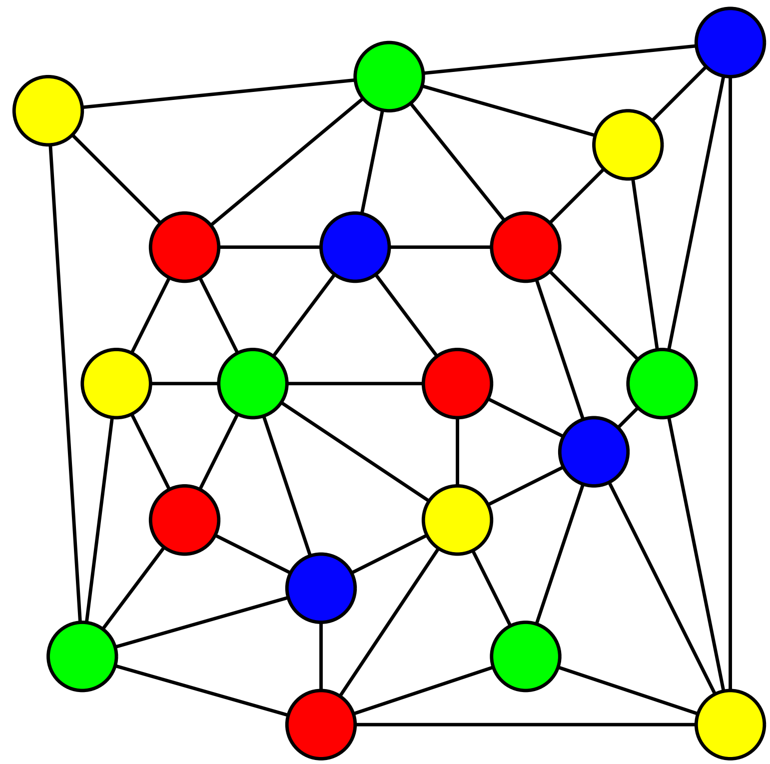
- A food company used different colour bags for each of it products
- To save money they reduced the stock of bags to 25
- They wanted to know what items to put in what bags so that as few customers as possible would have items with the same colour bags
- This can again be reduced to a graph colouring problem
  - ⋆ Each node represents an item
  - ⋆ The edges were weighted by the number of customers that took both items
  - ⋆ The aim was to colour the nodes with 25 colours to minimise the weights where the edges shared the same colour

Frequency Assignment Problem



Outline

1. Graph Theory
2. Applications of Graphs
   - Geometric applications
   - Relational applications
3. **Implementing Graphs**
4. Graph Problems



Representations

- There is no single way to represent graphs
- The best representation depends on the graph
- Some books describe a $Graph\ ADT$—graphs are too varied for this to be very useful
- An important issue in representing a graph is how to store the edge information

Representations

- There is no single way to represent graphs
- The best representation depends on the graph
- Some books describe a $Graph\ ADT$—graphs are too varied for this to be very useful
- An important issue in representing a graph is how to store the edge information

Representations

- There is no single way to represent graphs
- The best representation depends on the graph
- Some books describe a $Graph\ ADT$—graphs are too varied for this to be very useful
- An important issue in representing a graph is how to store the edge information

Representations

- There is no single way to represent graphs
- The best representation depends on the graph
- Some books describe a $Graph\ ADT$—graphs are too varied for this to be very useful
- An important issue in representing a graph is how to store the edge information

Adjacency Matrices

- One representation of a graph $G = (\mathcal{V}, \mathcal{E})$ is in term of an $n \times n$ **adjacency matrix** **A** with elements

$$A_{ij} = \begin{cases} 1 & \text{if } (i,j) \in \mathcal{E} \\ 0 & \text{if } (i,j) \notin \mathcal{E} \end{cases}$$

  where $n = |\mathcal{V}|$

- For undirected graphs **A** is a symmetric matrix, i.e. $\mathbf{A} = \mathbf{A}^{\mathsf{T}}$

- For weighted graphs we often store the **connectivity matrix** or **cost-adjacency matrix**, **C**, where

$$C_{ij} = \begin{cases} w_{ij} & \text{if } (i,j) \in \mathcal{E} \\ 0 & \text{if } (i,j) \notin \mathcal{E} \end{cases}$$

Adjacency Matrices

- One representation of a graph $G = (\mathcal{V}, \mathcal{E})$ is in term of an $n \times n$ **adjacency matrix A** with elements

$$A_{ij} = \begin{cases} 1 & \text{if } (i,j) \in \mathcal{E} \\ 0 & \text{if } (i,j) \notin \mathcal{E} \end{cases}$$

  where $n = |\mathcal{V}|$

- For undirected graphs **A** is a symmetric matrix, i.e. $\mathbf{A} = \mathbf{A}^{\mathsf{T}}$

- For weighted graphs we often store the **connectivity matrix** or **cost-adjacency matrix**, **C**, where

$$C_{ij} = \begin{cases} w_{ij} & \text{if } (i,j) \in \mathcal{E} \\ 0 & \text{if } (i,j) \notin \mathcal{E} \end{cases}$$

Adjacency Matrices

- One representation of a graph $G = (\mathcal{V}, \mathcal{E})$ is in term of an $n \times n$ **adjacency matrix A** with elements

$$A_{ij} = \begin{cases} 1 & \text{if } (i,j) \in \mathcal{E} \\ 0 & \text{if } (i,j) \notin \mathcal{E} \end{cases}$$

  where $n = |\mathcal{V}|$

- For undirected graphs **A** is a symmetric matrix, i.e. $\mathbf{A} = \mathbf{A}^\mathsf{T}$

- For weighted graphs we often store the **connectivity matrix** or **cost-adjacency matrix**, **C**, where

$$C_{ij} = \begin{cases} w_{ij} & \text{if } (i,j) \in \mathcal{E} \\ 0 & \text{if } (i,j) \notin \mathcal{E} \end{cases}$$

Adjacency Lists

- For **dense** graphs where the number of edges is $\Theta(n^2)$ the adjacency matrix is often a useful representation
- But in **sparse** graphs where the number of edges is $\Theta(n)$ the adjacency matrix has a very large number of zeros
- A more efficient representation is in terms of the adjacency list where the set of outgoing edges is stored for each node
- In some applications it is useful to store both the adjacency matrix and the adjacency list

Adjacency Lists

- For **dense** graphs where the number of edges is $\Theta(n^2)$ the adjacency matrix is often a useful representation
- But in **sparse** graphs where the number of edges is $\Theta(n)$ the adjacency matrix has a very large number of zeros
- A more efficient representation is in terms of the adjacency list where the set of outgoing edges is stored for each node
- In some applications it is useful to store both the adjacency matrix and the adjacency list

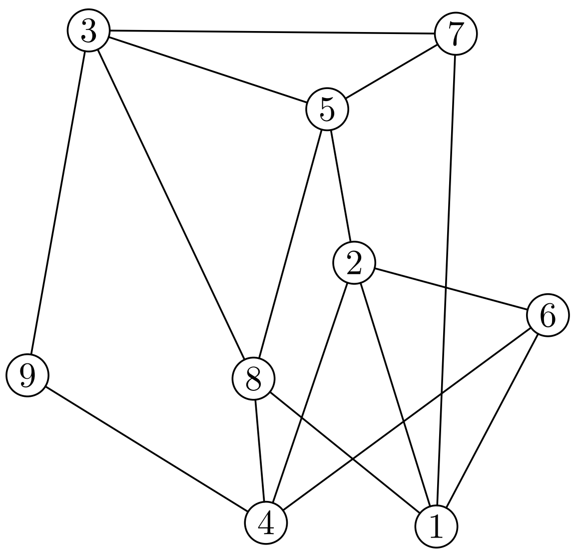
Adjacency Lists

- For **dense** graphs where the number of edges is $\Theta(n^2)$ the adjacency matrix is often a useful representation
- But in **sparse** graphs where the number of edges is $\Theta(n)$ the adjacency matrix has a very large number of zeros
- A more efficient representation is in terms of the adjacency list where the set of outgoing edges is stored for each node
- In some applications it is useful to store both the adjacency matrix and the adjacency list

Adjacency Lists

- For **dense** graphs where the number of edges is $\Theta(n^2)$ the adjacency matrix is often a useful representation
- But in **sparse** graphs where the number of edges is $\Theta(n)$ the adjacency matrix has a very large number of zeros
- A more efficient representation is in terms of the adjacency list where the set of outgoing edges is stored for each node
- In some applications it is useful to store both the adjacency matrix and the adjacency list

Representing Undirected Graphs

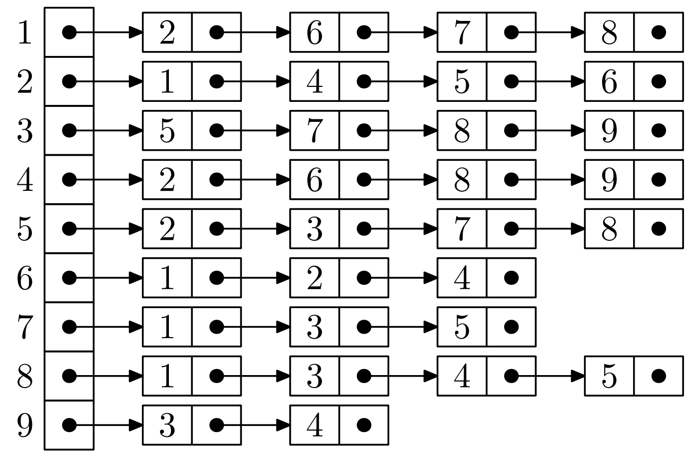


Graph

from

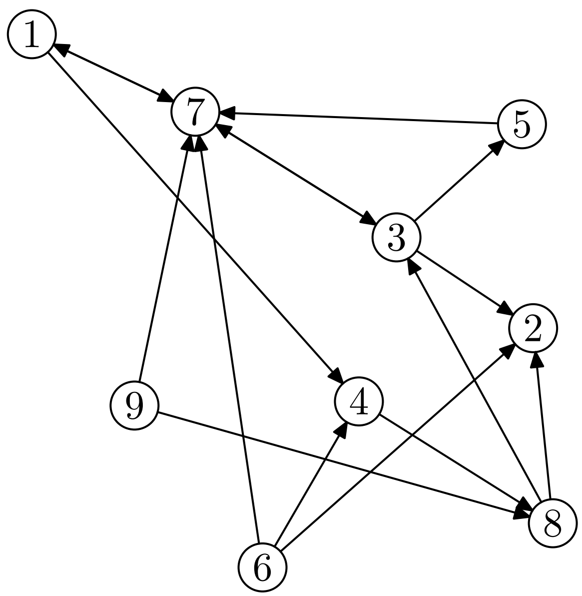
		1	2	3	4	5	6	7	8	9
	1	0	1	0	0	0	1	1	1	0
	2	1	0	0	1	1	1	0	0	0
	3	0	0	0	0	1	0	1	1	1
	4	0	1	0	0	0	1	0	1	1
to	5	0	1	1	0	0	0	1	1	0
	6	1	1	0	1	0	0	0	0	0
	7	1	0	1	0	1	0	0	0	0
	8	1	0	1	1	1	0	0	0	0
	9	0	0	1	1	0	0	0	0	0

Adjacency Matrix



Adjacency List

Representing Digraphs

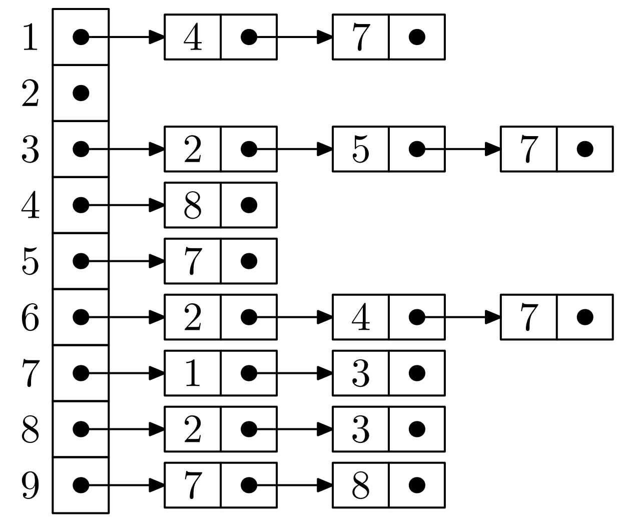


Graph

from

to	1	2	3	4	5	6	7	8	9
1	0	0	0	0	0	0	1	0	0
2	0	0	1	0	0	1	0	1	0
3	0	0	0	0	0	0	1	1	0
4	1	0	0	0	0	1	0	0	0
5	0	0	1	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0
7	1	0	1	0	1	1	0	0	1
8	0	0	0	1	0	0	0	0	1
9	0	0	0	0	0	0	0	0	0

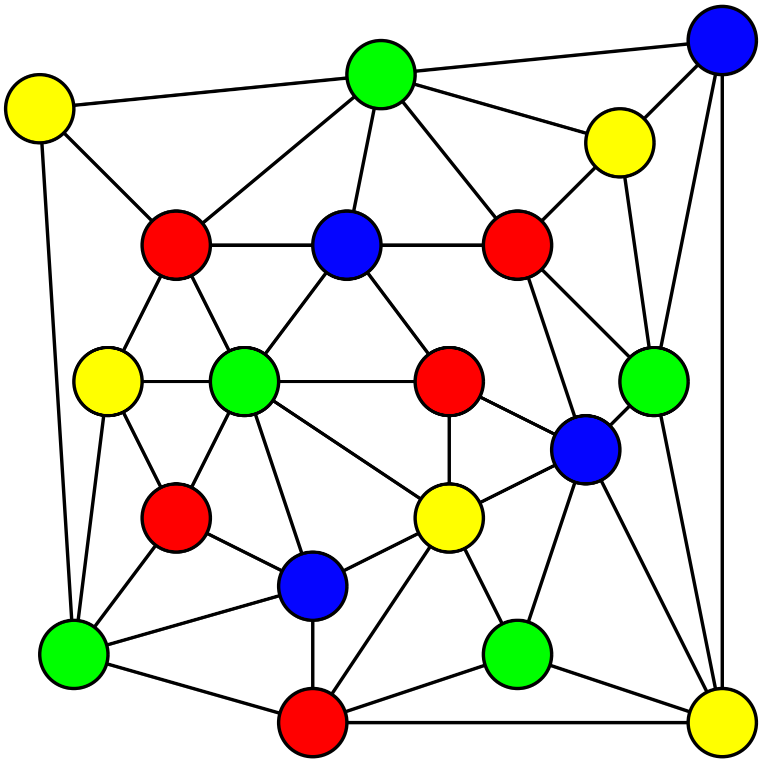
Adjacency Matrix



Adjacency List

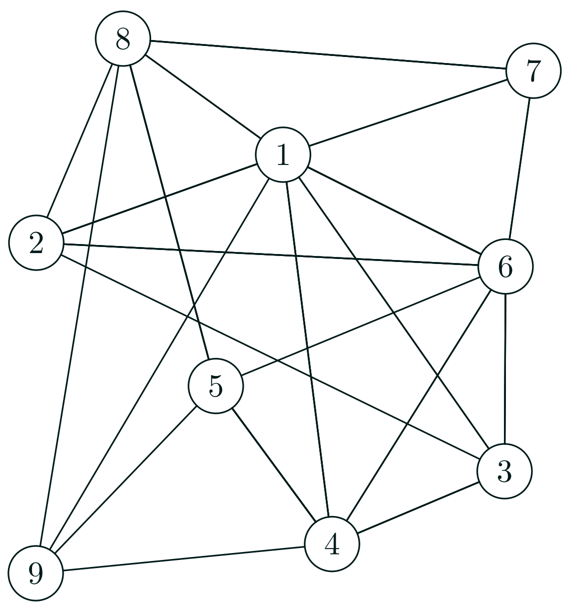
Outline

1. Graph Theory
2. Applications of Graphs
   - Geometric applications
   - Relational applications
3. Implementing Graphs
4. **Graph Problems**



Hamilton Cycle

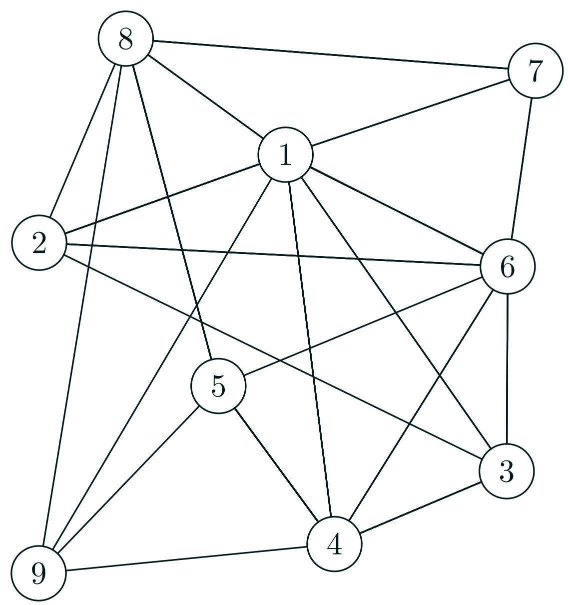
- The Euler path problem is to find a path through a multigraph that passes through every edge once—easy to solve
- The Hamilton cycle problem is to find a cycle that goes through each vertex exactly once



- There is no known efficient algorithm to solve this

Hamilton Cycle

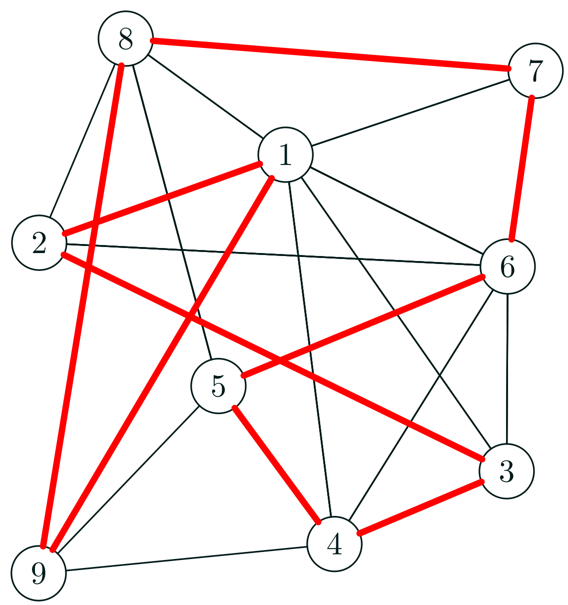
- The Euler path problem is to find a path through a multigraph that passes through every edge once—easy to solve
- The Hamilton cycle problem is to find a cycle that goes through each vertex exactly once



- There is no known efficient algorithm to solve this

Hamilton Cycle

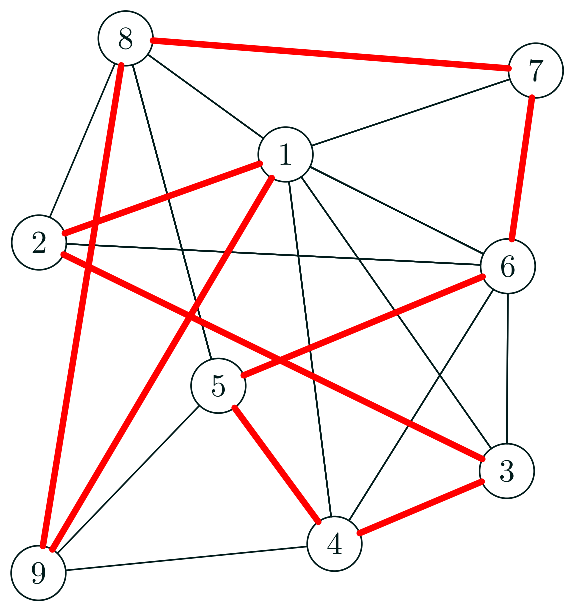
- The Euler path problem is to find a path through a multigraph that passes through every edge once—easy to solve
- The Hamilton cycle problem is to find a cycle that goes through each vertex exactly once



- There is no known efficient algorithm to solve this

Hamilton Cycle

- The Euler path problem is to find a path through a multigraph that passes through every edge once—easy to solve
- The Hamilton cycle problem is to find a cycle that goes through each vertex exactly once



- There is no known efficient algorithm to solve this

Shortest Path and TSP

- The shortest path problem is to find a path between two nodes
- There is an efficient algorithm—see next lecture
- In the travelling salesperson problem the task is to find the shortest tour (Hamilton cycle)—we usually assume there is an edge between every pair of nodes
- There is no know efficient algorithm to solve all TSPs

Shortest Path and TSP

- The shortest path problem is to find a path between two nodes
- There is an efficient algorithm—see next lecture
- In the travelling salesperson problem the task is to find the shortest tour (Hamilton cycle)—we usually assume there is an edge between every pair of nodes
- There is no know efficient algorithm to solve all TSPs

Shortest Path and TSP

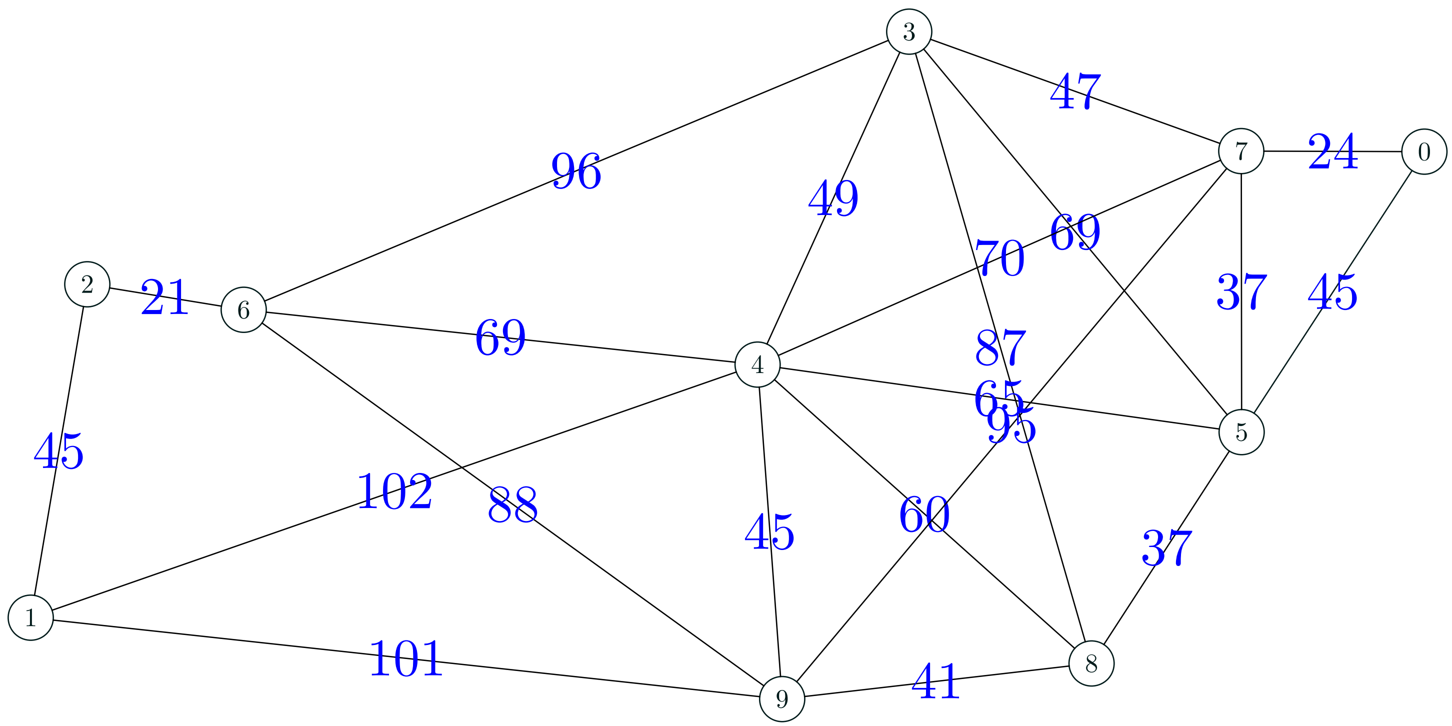
- The shortest path problem is to find a path between two nodes
- There is an efficient algorithm—see next lecture
- In the travelling salesperson problem the task is to find the shortest tour (Hamilton cycle)—we usually assume there is an edge between every pair of nodes
- There is no know efficient algorithm to solve all TSPs

Shortest Path and TSP

- The shortest path problem is to find a path between two nodes
- There is an efficient algorithm—see next lecture
- In the travelling salesperson problem the task is to find the shortest tour (Hamilton cycle)—we usually assume there is an edge between every pair of nodes
- There is no know efficient algorithm to solve all TSPs

Minimum Spanning Tree

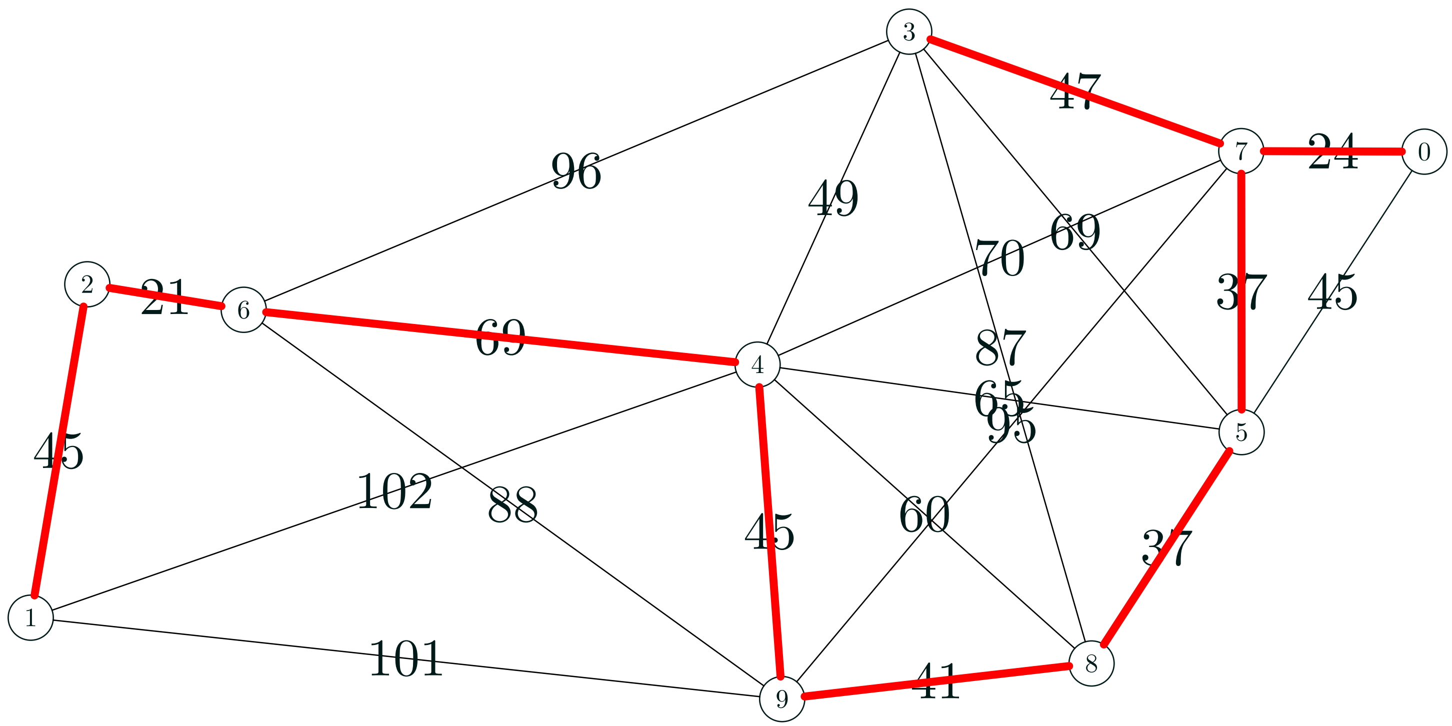
- Suppose we want to construct pylons connecting a number of cities using the least amount of cable



- We will study an efficient algorithm to solve this in the next but one lecture

Minimum Spanning Tree

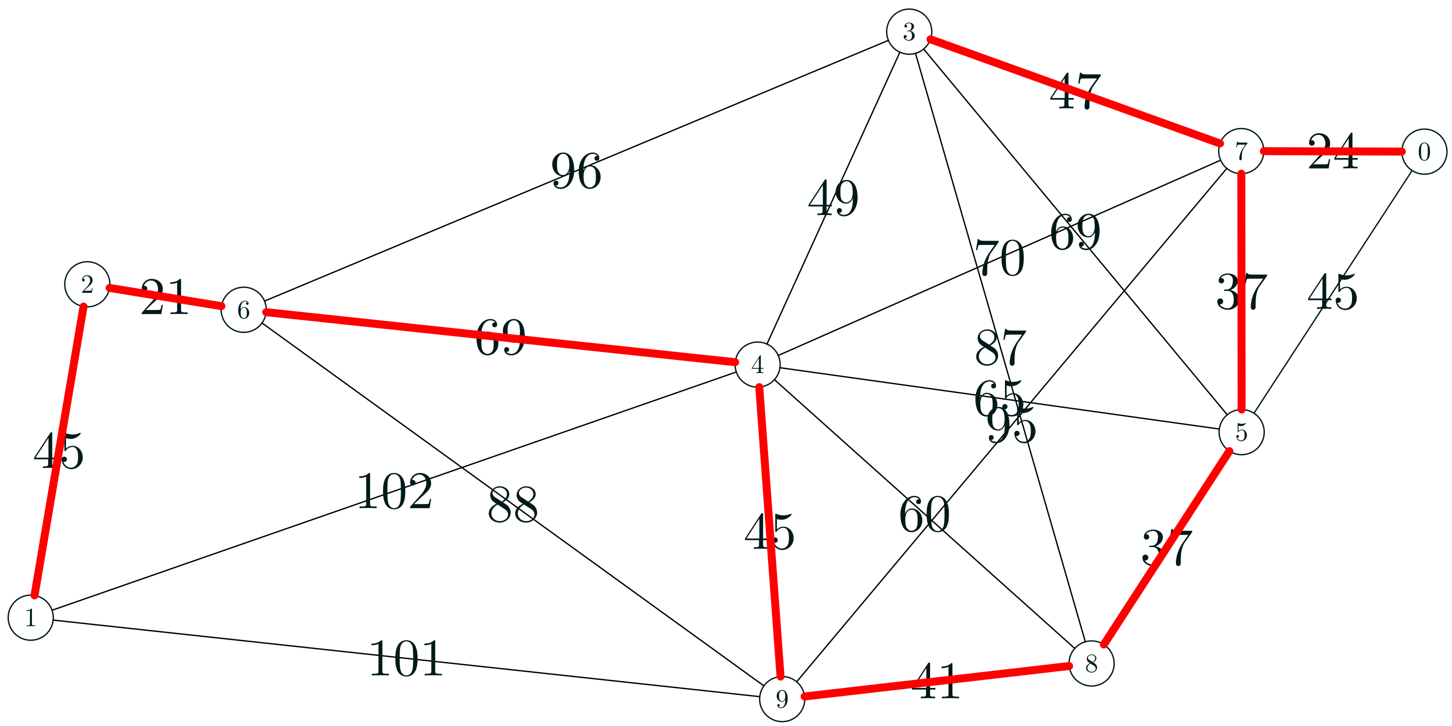
- Suppose we want to construct pylons connecting a number of cities using the least amount of cable



- We will study an efficient algorithm to solve this in the next but one lecture

Minimum Spanning Tree

- Suppose we want to construct pylons connecting a number of cities using the least amount of cable



- We will study an efficient algorithm to solve this in the next but one lecture

Graph Partitioning

- The simplest version of this problem is to cut a graph into two equal halves so that you minimise the number of edges you cut
- If the edges are weighted then you want to minimise the sum of edges that are cut
- If the vertices are weighted you want to balance the sum of vertex weights in the two partitions
- An example of this problem is in dividing up a problem to run on a parallel computer
  - ⋆ Nodes are subtasks (weights on nodes are run times)
  - ⋆ Edge weights indicate communication cost
- There is no known efficient algorithm to solve this

Graph Partitioning

- The simplest version of this problem is to cut a graph into two equal halves so that you minimise the number of edges you cut
- If the edges are weighted then you want to minimise the sum of edges that are cut
- If the vertices are weighted you want to balance the sum of vertex weights in the two partitions
- An example of this problem is in dividing up a problem to run on a parallel computer
  - ⋆ Nodes are subtasks (weights on nodes are run times)
  - ⋆ Edge weights indicate communication cost
- There is no known efficient algorithm to solve this

Graph Partitioning

- The simplest version of this problem is to cut a graph into two equal halves so that you minimise the number of edges you cut
- If the edges are weighted then you want to minimise the sum of edges that are cut
- If the vertices are weighted you want to balance the sum of vertex weights in the two partitions
- An example of this problem is in dividing up a problem to run on a parallel computer
  - ⋆ Nodes are subtasks (weights on nodes are run times)
  - ⋆ Edge weights indicate communication cost
- There is no known efficient algorithm to solve this

Graph Partitioning

- The simplest version of this problem is to cut a graph into two equal halves so that you minimise the number of edges you cut
- If the edges are weighted then you want to minimise the sum of edges that are cut
- If the vertices are weighted you want to balance the sum of vertex weights in the two partitions
- An example of this problem is in dividing up a problem to run on a parallel computer
  - ⋆ Nodes are subtasks (weights on nodes are run times)
  - ⋆ Edge weights indicate communication cost
- There is no known efficient algorithm to solve this

Graph Partitioning

- The simplest version of this problem is to cut a graph into two equal halves so that you minimise the number of edges you cut
- If the edges are weighted then you want to minimise the sum of edges that are cut
- If the vertices are weighted you want to balance the sum of vertex weights in the two partitions
- An example of this problem is in dividing up a problem to run on a parallel computer
  - ⋆ Nodes are subtasks (weights on nodes are run times)
  - ⋆ Edge weights indicate communication cost
- There is no known efficient algorithm to solve this

Graph Partitioning

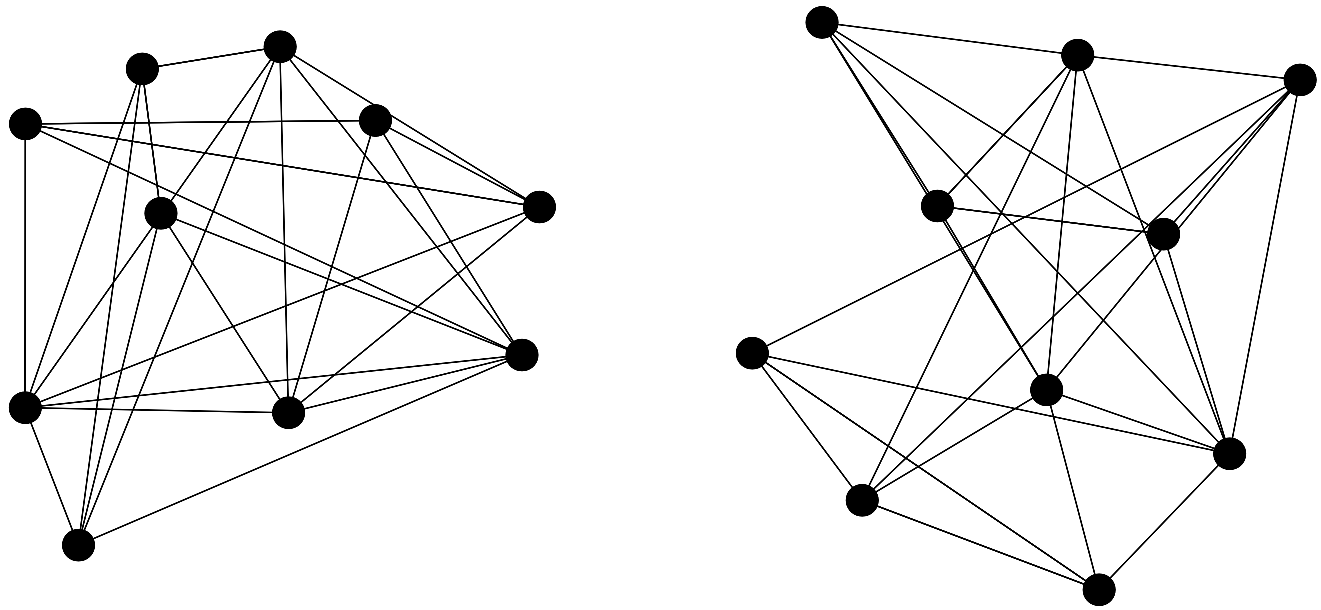
- The simplest version of this problem is to cut a graph into two equal halves so that you minimise the number of edges you cut
- If the edges are weighted then you want to minimise the sum of edges that are cut
- If the vertices are weighted you want to balance the sum of vertex weights in the two partitions
- An example of this problem is in dividing up a problem to run on a parallel computer
  - ⋆ Nodes are subtasks (weights on nodes are run times)
  - ⋆ Edge weights indicate communication cost
- There is no known efficient algorithm to solve this

Graph Partitioning

- The simplest version of this problem is to cut a graph into two equal halves so that you minimise the number of edges you cut
- If the edges are weighted then you want to minimise the sum of edges that are cut
- If the vertices are weighted you want to balance the sum of vertex weights in the two partitions
- An example of this problem is in dividing up a problem to run on a parallel computer
  - ⋆ Nodes are subtasks (weights on nodes are run times)
  - ⋆ Edge weights indicate communication cost
- There is no known efficient algorithm to solve this

Graph Isomorphism

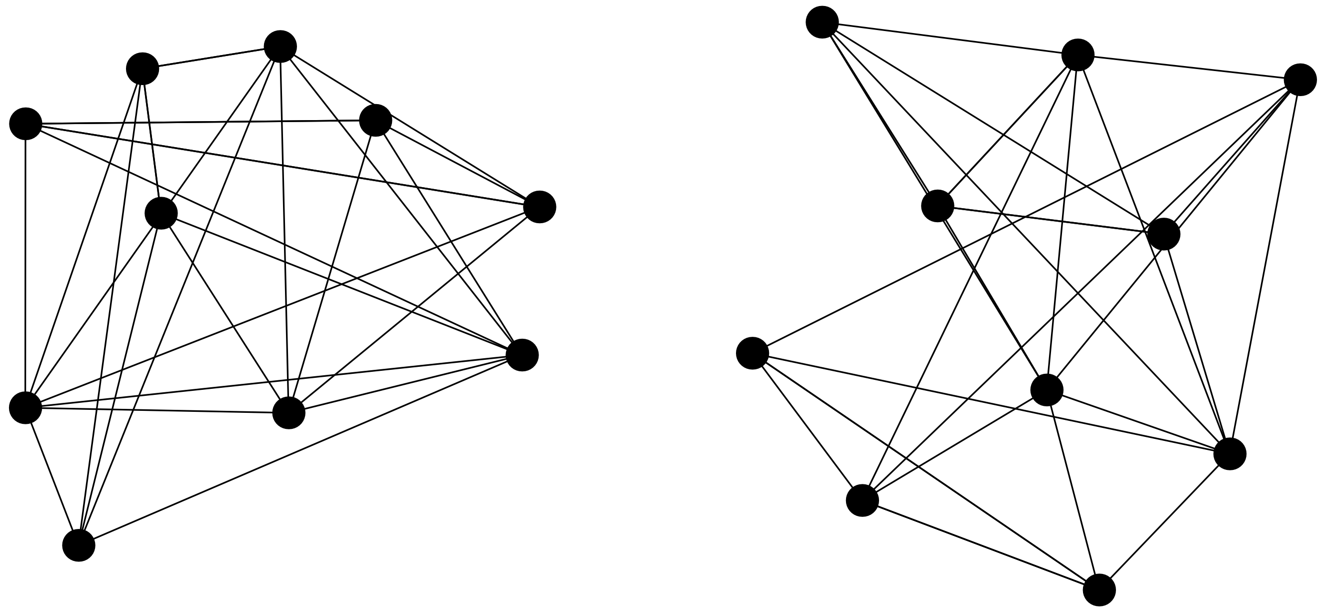
- Do two graphs have the same structure?



- There is no known efficient algorithm to solve this problem
- Theoretically it is interesting because it is not NP-complete

Graph Isomorphism

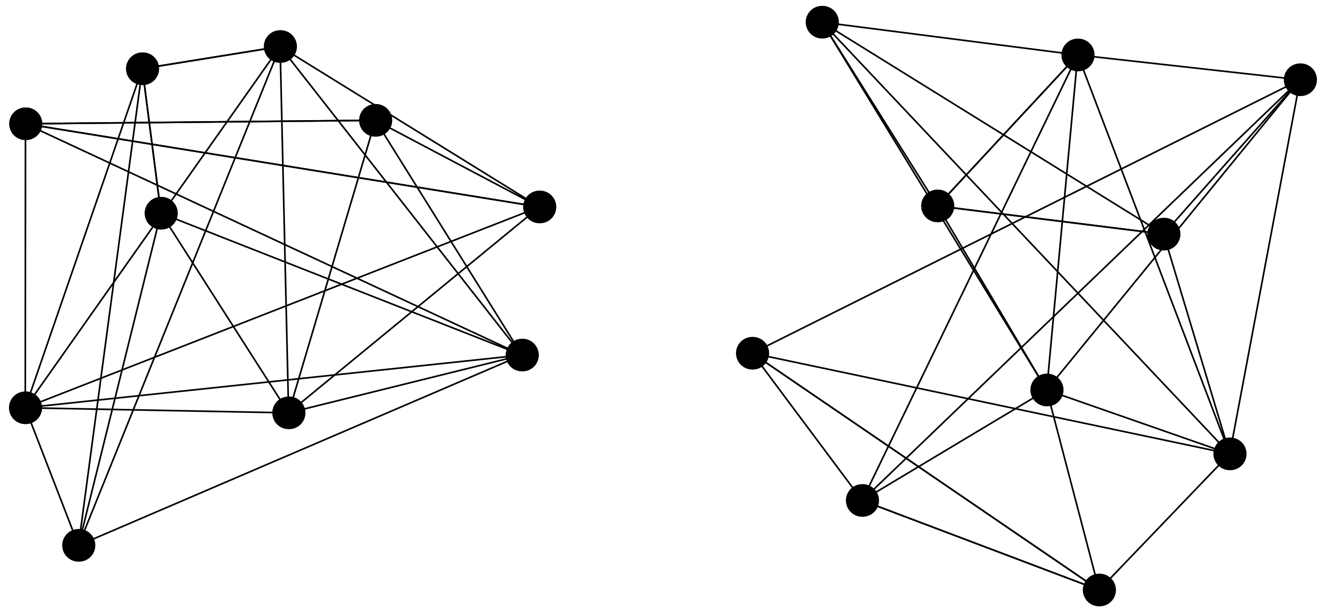
- Do two graphs have the same structure?



- There is no known efficient algorithm to solve this problem
- Theoretically it is interesting because it is not NP-complete

Graph Isomorphism

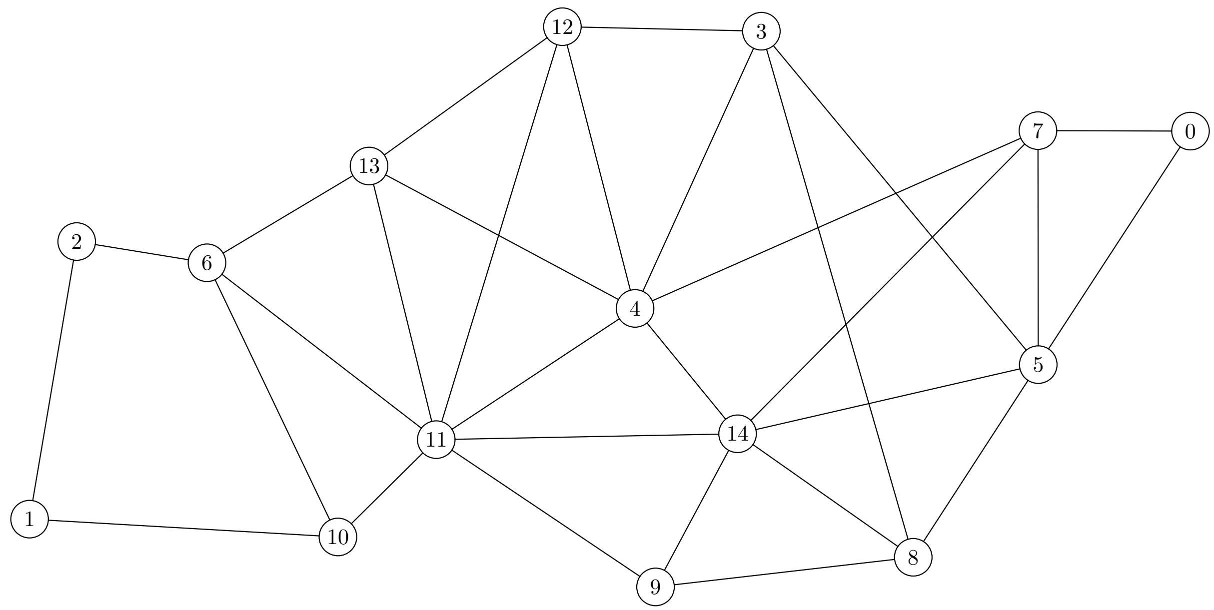
- Do two graphs have the same structure?



- There is no known efficient algorithm to solve this problem
- Theoretically it is interesting because it is not NP-complete

Vertex Cover

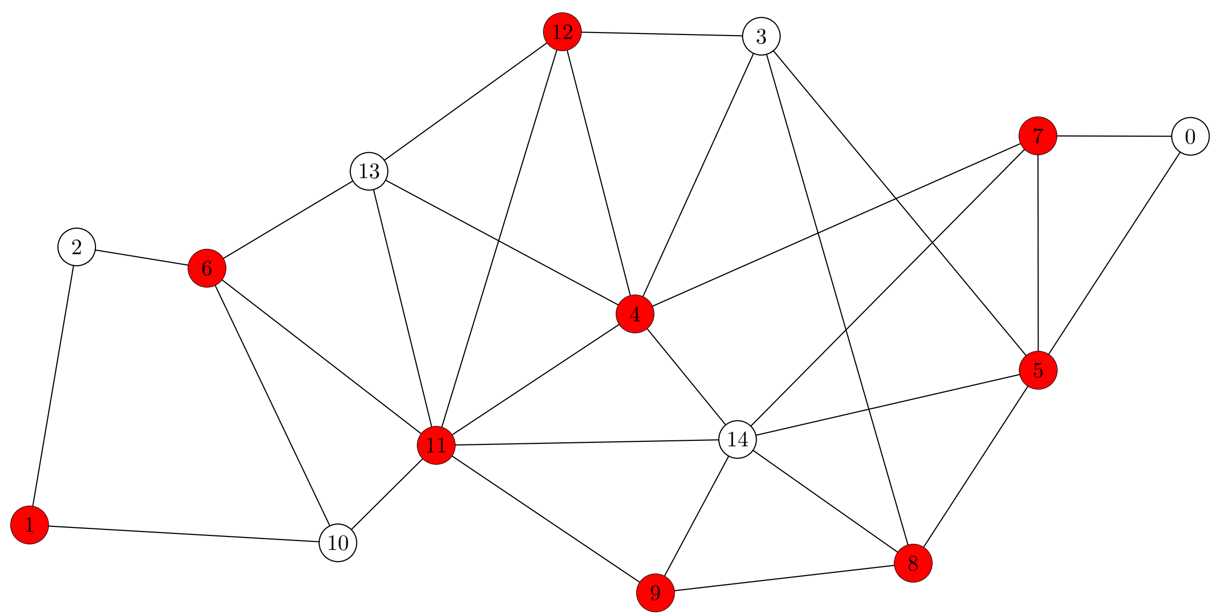
- How many guards do you need to cover all the corridors in a museum



- There is no known efficient algorithm to solve this

Vertex Cover

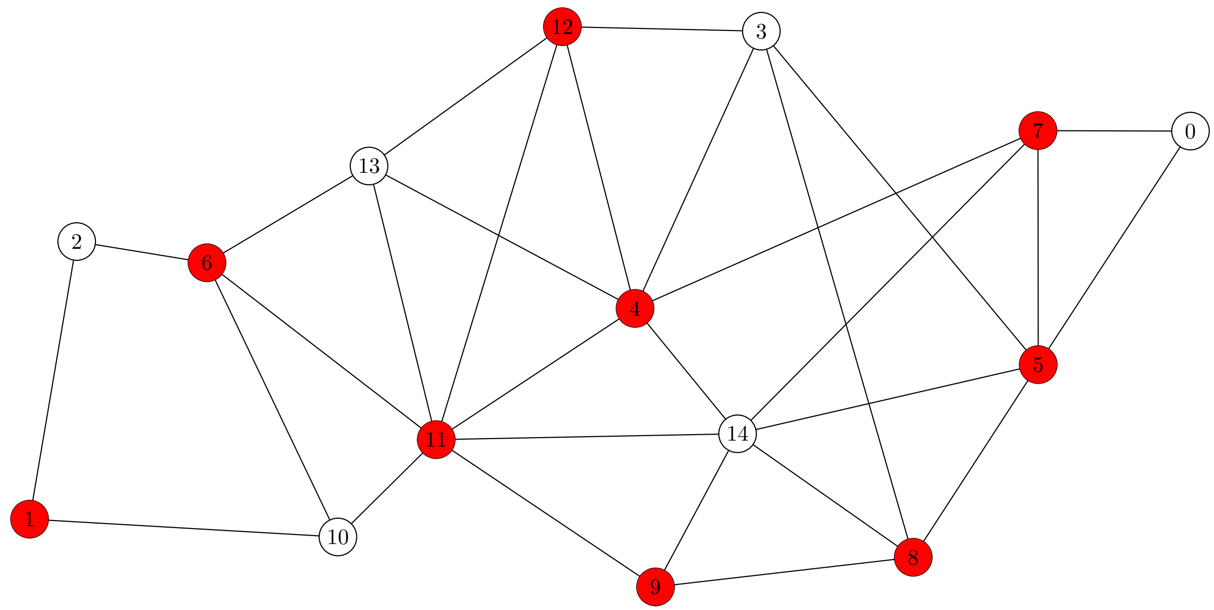
- How many guards do you need to cover all the corridors in a museum



- There is no known efficient algorithm to solve this

Vertex Cover

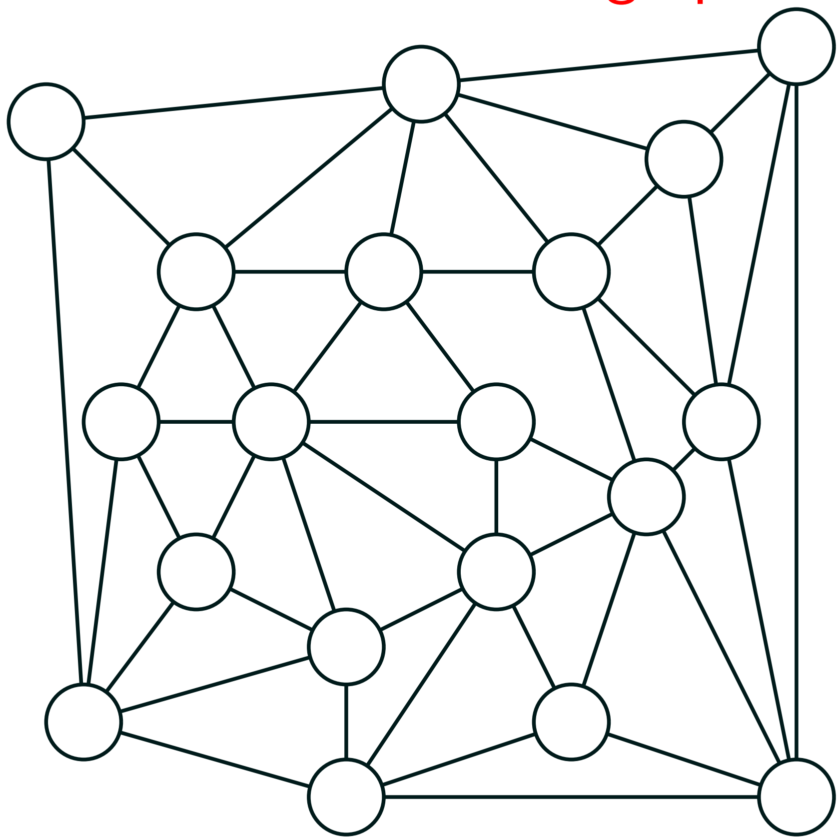
- How many guards do you need to cover all the corridors in a museum



- There is no known efficient algorithm to solve this

Graph Colouring

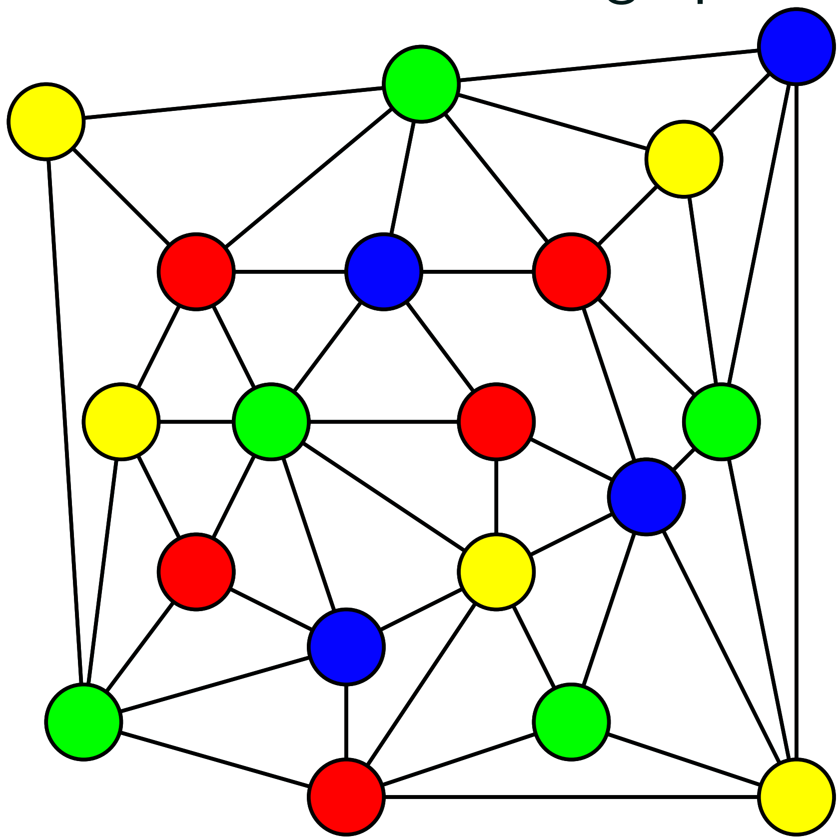
- How many colours do I need to colour a graph with no conflicts



- There is no known efficient algorithm to solve this

Graph Colouring

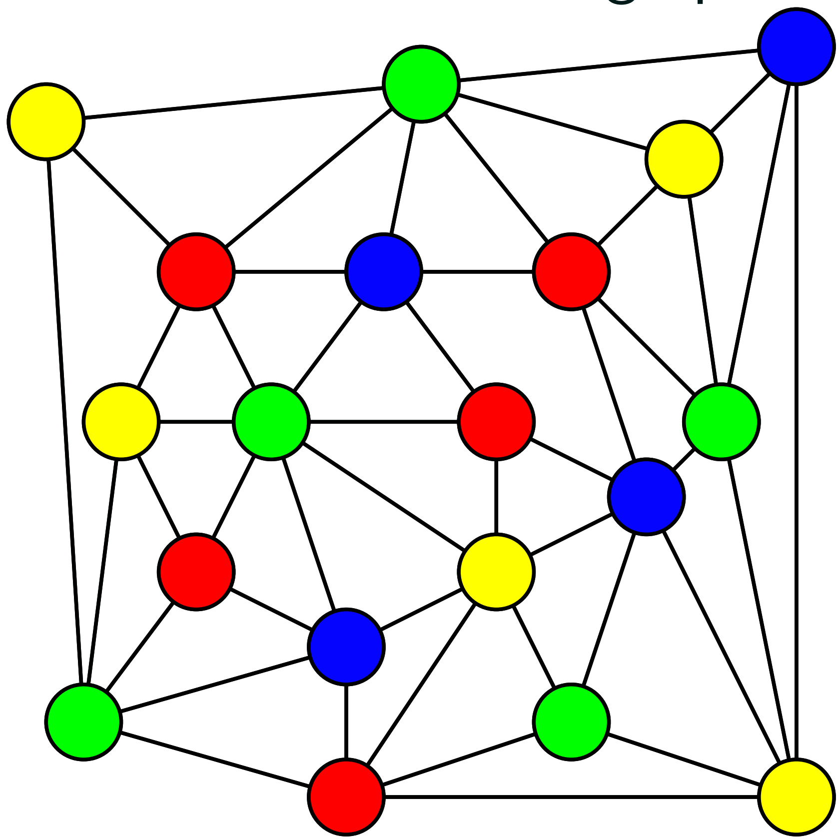
- How many colours do I need to colour a graph with no conflicts



- There is no known efficient algorithm to solve this

Graph Colouring

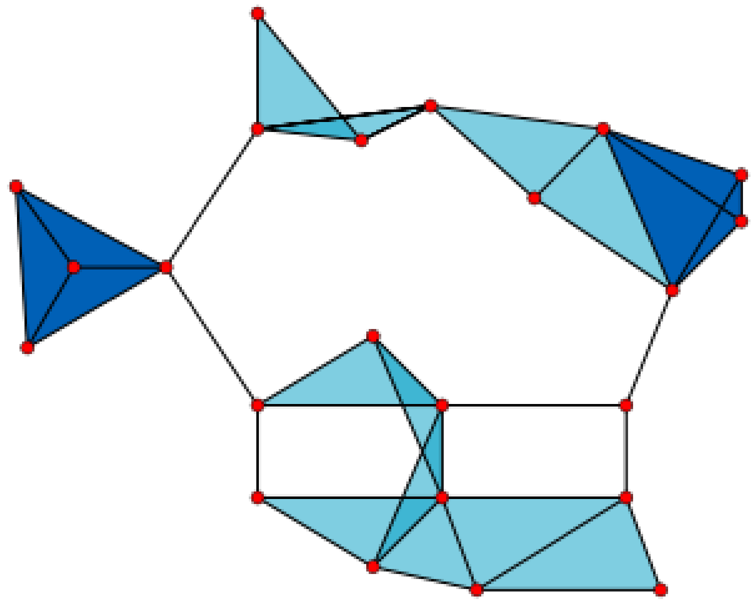
- How many colours do I need to colour a graph with no conflicts



- There is no known efficient algorithm to solve this

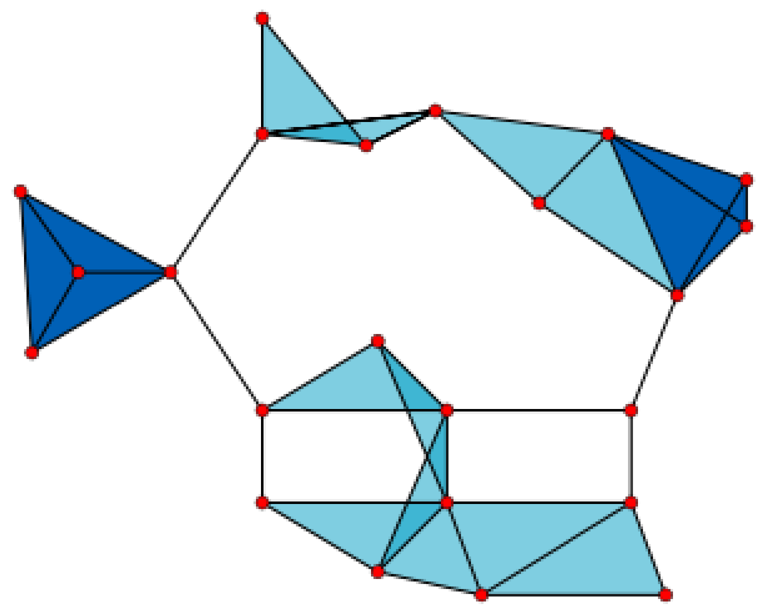
Other Graph Problems

- These are only a sample of the many famous graph problems
- Others include
  - ⋆ Max-clique (hard)
  - ⋆ Maximal independent set (hard)
  - ⋆ Maximal flow problem (easy)
  - ⋆ Max-cut (hard)



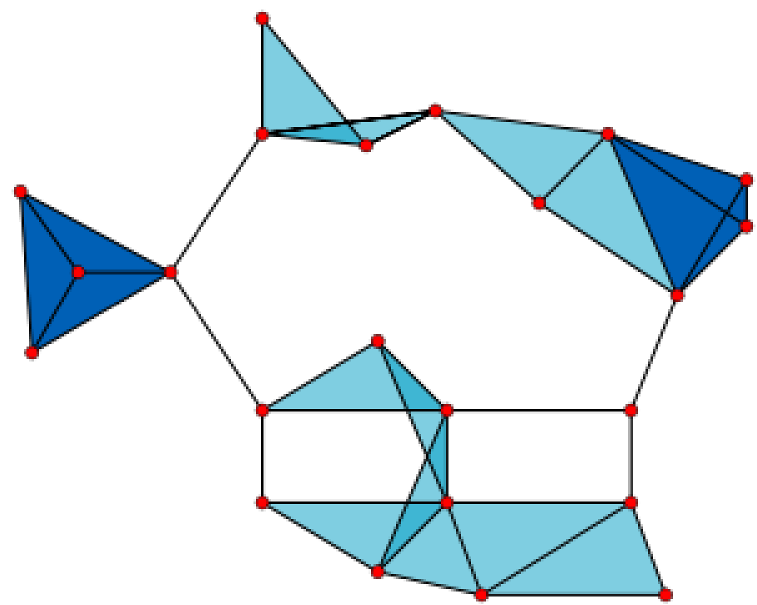
Other Graph Problems

- These are only a sample of the many famous graph problems
- Others include
  - ⋆ Max-clique (hard)
  - ⋆ Maximal independent set (hard)
  - ⋆ Maximal flow problem (easy)
  - ⋆ Max-cut (hard)



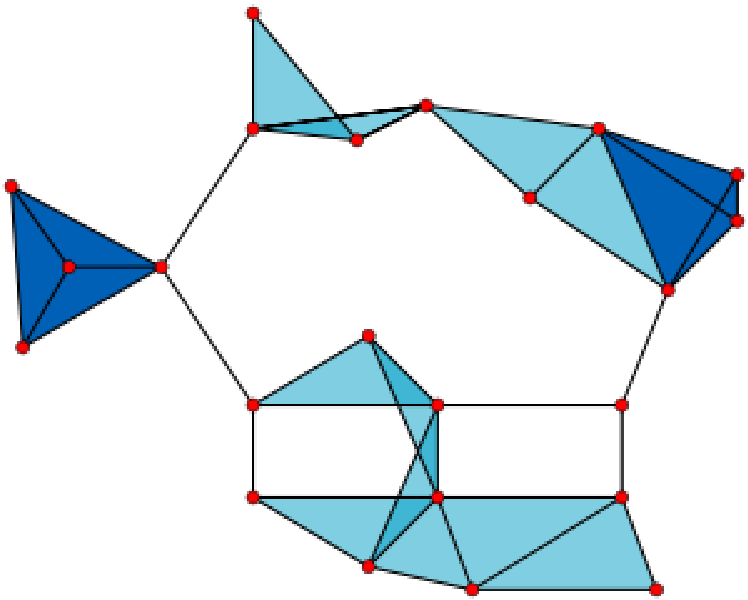
Other Graph Problems

- These are only a sample of the many famous graph problems
- Others include
  - ⋆ Max-clique (hard)
  - ⋆ Maximal independent set (hard)
  - ⋆ Maximal flow problem (easy)
  - ⋆ Max-cut (hard)



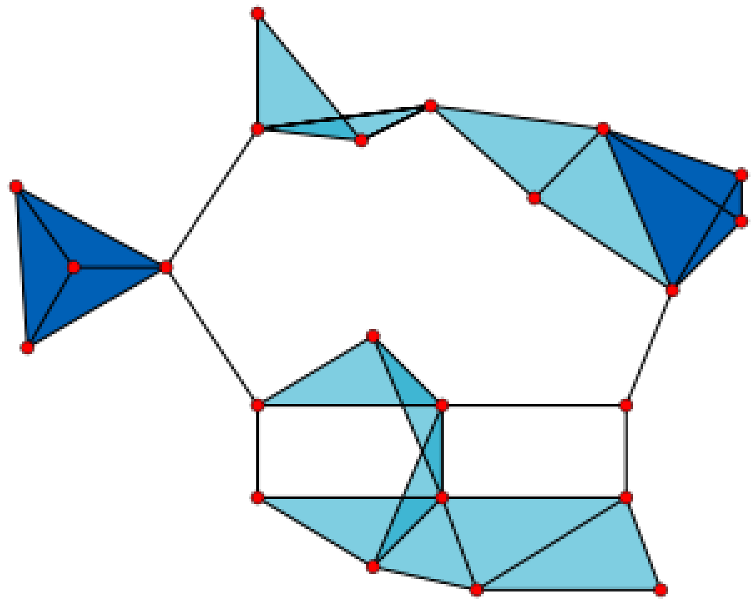
Other Graph Problems

- These are only a sample of the many famous graph problems
- Others include
  - ⋆ Max-clique (hard)
  - ⋆ Maximal independent set (hard)
  - ⋆ Maximal flow problem (easy)
  - ⋆ Max-cut (hard)



Other Graph Problems

- These are only a sample of the many famous graph problems
- Others include
  - ⋆ Max-clique (hard)
  - ⋆ Maximal independent set (hard)
  - ⋆ Maximal flow problem (easy)
  - ⋆ Max-cut (hard)



Lessons

- Graphs are an important method for abstracting problems
- They appear in a huge number of disparate fields
- There are many problems for which efficient algorithms are known
- There are many problems which are believed to be hard—i.e. there aren't any efficient algorithms
- Even for hard problems there are good algorithms for finding approximated solutions

Lessons

- Graphs are an important method for abstracting problems
- They appear in a huge number of disparate fields
- There are many problems for which efficient algorithms are known
- There are many problems which are believed to be hard—i.e. there aren't any efficient algorithms
- Even for hard problems there are good algorithms for finding approximated solutions

Lessons

- Graphs are an important method for abstracting problems
- They appear in a huge number of disparate fields
- There are many problems for which efficient algorithms are known
- There are many problems which are believed to be hard—i.e. there aren't any efficient algorithms
- Even for hard problems there are good algorithms for finding approximated solutions

Lessons

- Graphs are an important method for abstracting problems
- They appear in a huge number of disparate fields
- There are many problems for which efficient algorithms are known
- There are many problems which are believed to be hard—i.e. there aren't any efficient algorithms
- Even for hard problems there are good algorithms for finding approximated solutions

Lessons

- Graphs are an important method for abstracting problems
- They appear in a huge number of disparate fields
- There are many problems for which efficient algorithms are known
- There are many problems which are believed to be hard—i.e. there aren't any efficient algorithms
- Even for hard problems there are good algorithms for finding approximated solutions