Further Mathematics and Algorithms
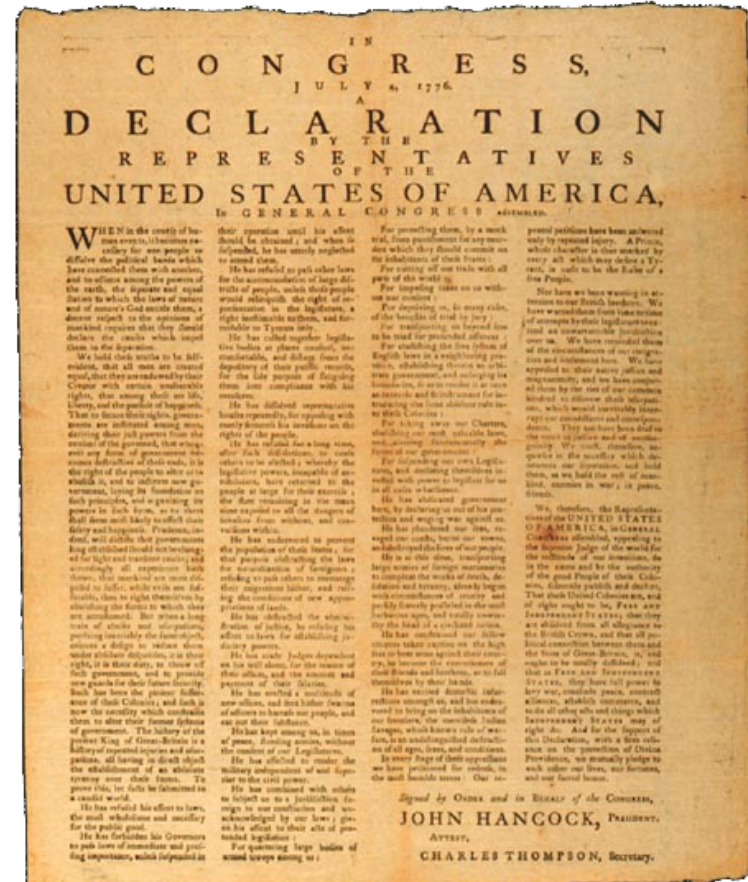
Lesson 3: *Declare your intentions (not your actions)*

*ADTs, stacks, queues, priority queues, sets, maps*

Outline

1. **Abstract Data Types (ADTs)**

2. Stacks

3. Queues and Priority Queues

4. Lists, Sets and Maps

5. Putting it Together

Object Oriented Programming

- OO-programming allows you to build large systems reliably

- In the OO-methodology you separate the interface from the implementation

- The **interface** is the public methods (functions) of a class

- The implementation is hidden (**encapsulated**) and may be changed without affecting how the class is used

- There exist other ways of programming, but C++ is designed to support the OO-methodology—for building systems it is brilliant

Object-Oriented Classes
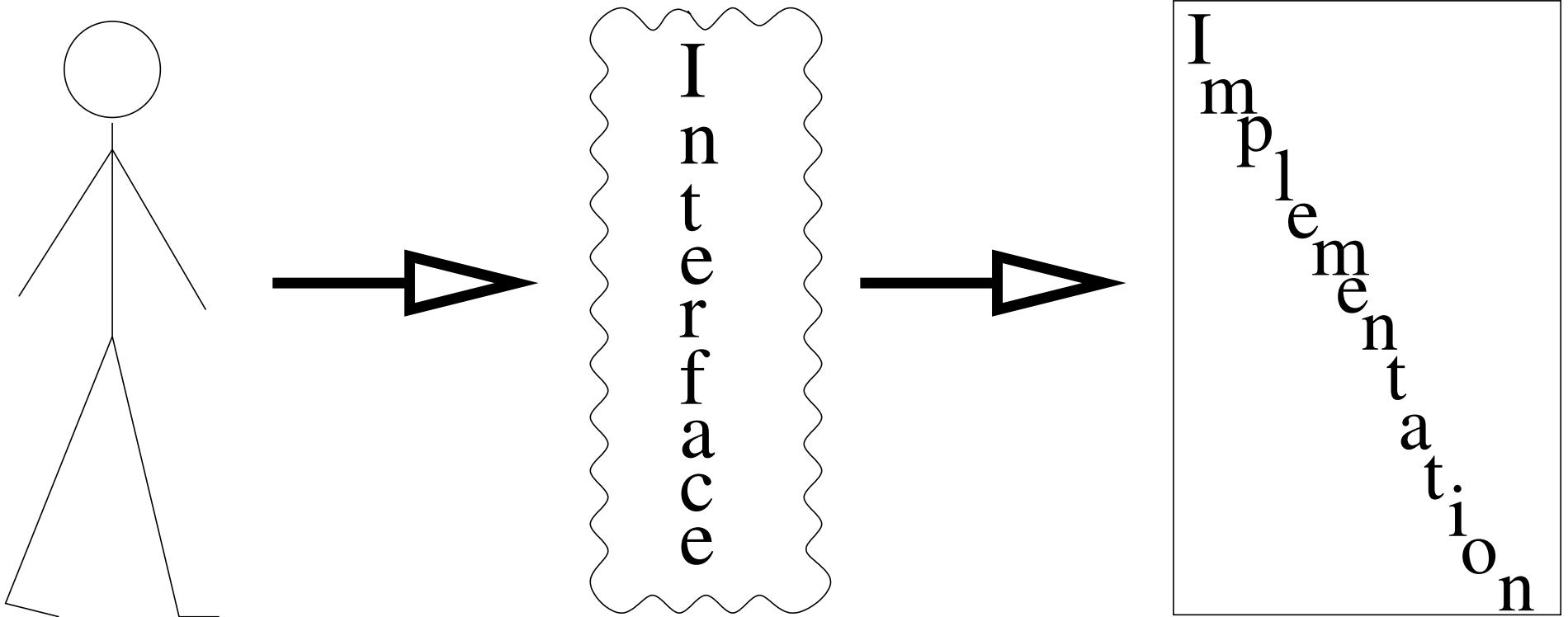
Abstract Data Types

- With data structures there are some traditional interfaces called **Abstract Data Types** or ADTs

- These are implementation free data structures

- They are mathematical abstractions of the data structure

- Their purpose is to allow you to declare you intentions

- You are entering into an agreement that you only intend to use the underlying data structure in the way specified by the interface

ADTs
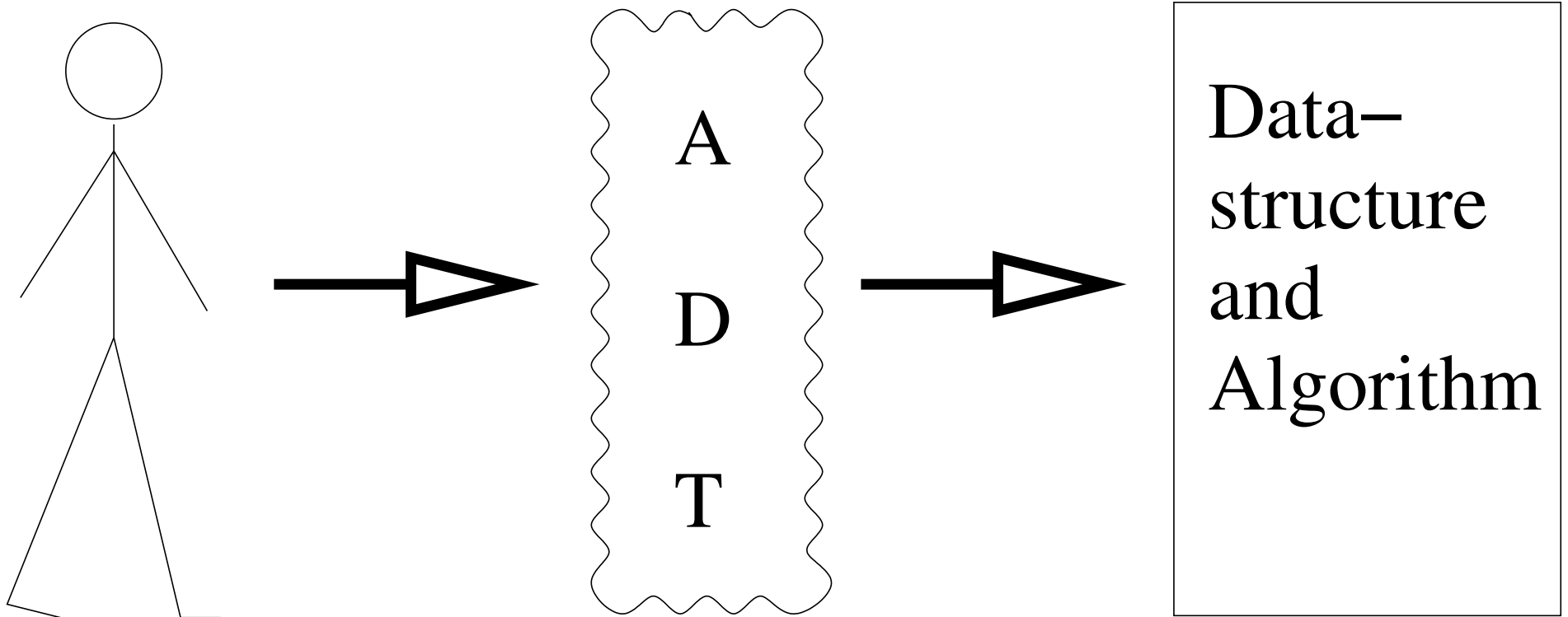
Say it with an ADT

- Common ADTs include stacks, queues, priority queues, sets, multisets and maps

- There are many possible implementations of these ADTs (some far from obvious)

- Each ADT has a limited set of methods associated with it

- They are an abstraction away from the implementation

- By declaring your intentions you are making your code easier to understand and maintain

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Stacks

- Last In First Out (LIFO) memory

- Standard functions

  - ⋆ push(item)
  - ⋆ T top()
  - ⋆ T pop() except in C++ pop() doesn't return the top of the stack
  - ⋆ boolean empty()

- Implemented using an array (or a linked-list)

pop()

push(11) void

| 11 |
| 3 |
| 16 |
| 5 |
| 4 |
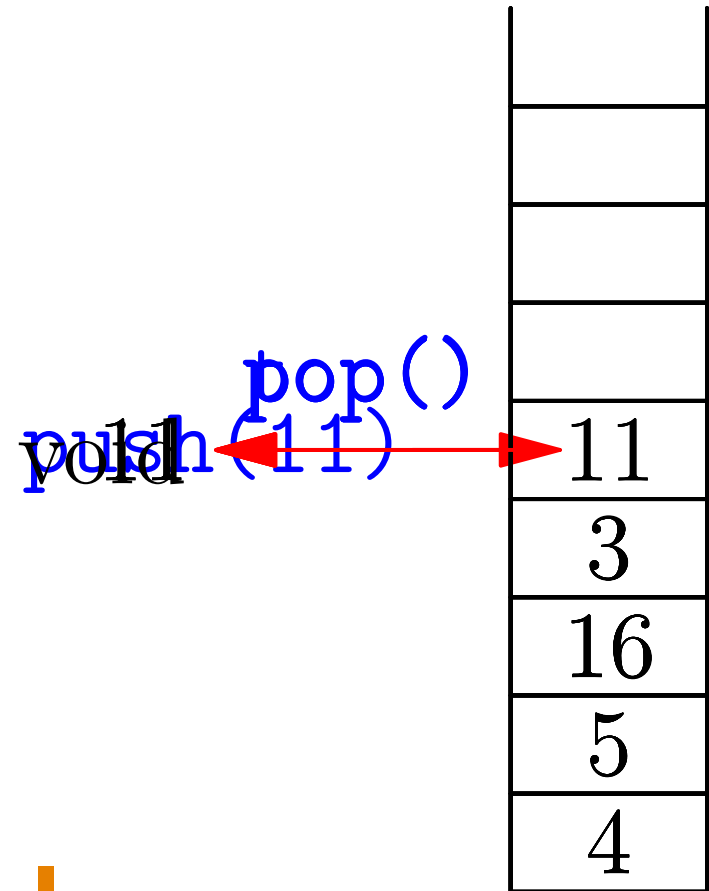
Why Use a Stack?

- Stacks reduces the access to memory—no longer random access

- Seems counter intuitive to reduce what you can do

- Gives you a very simple interface

- Prevents another programmer from using memory in a way that will break existing code

- Sufficient for large number of algorithms
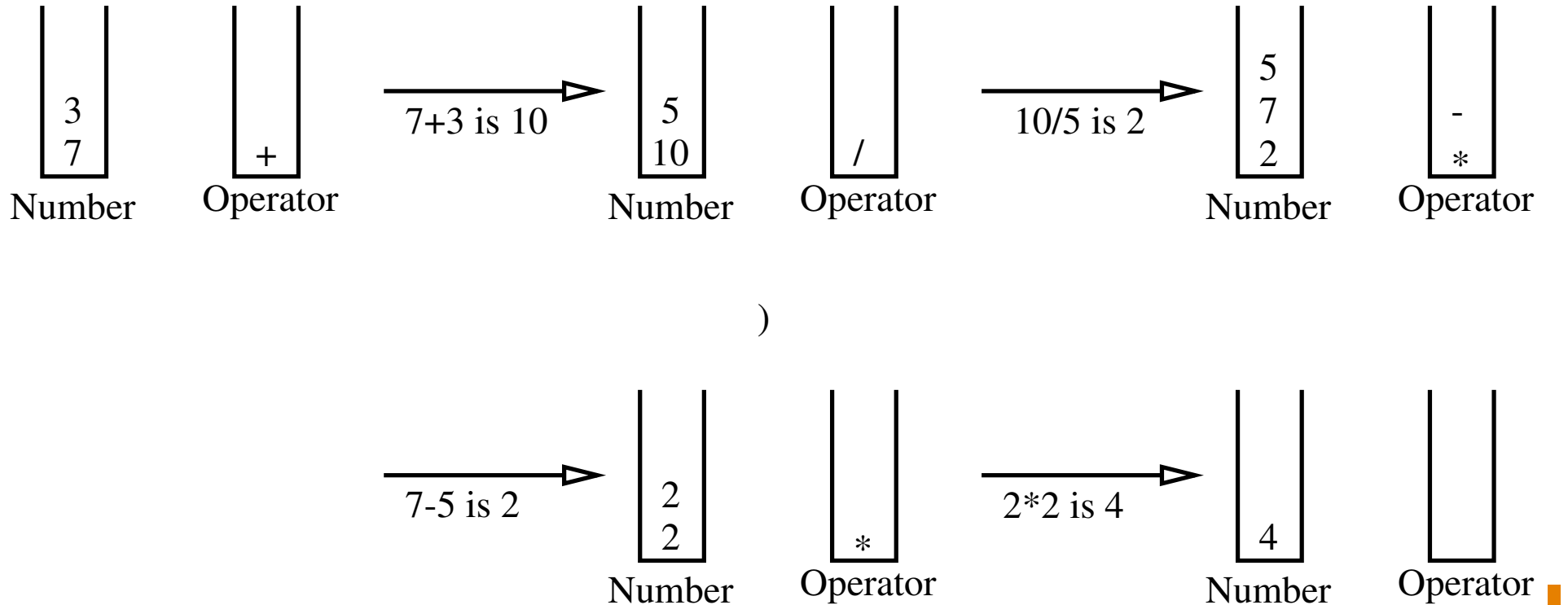

Uses of Stacks

- Reversing an array▮

- Parsing expression for compilers▮

  ⋆ balancing parentheses▮
  ⋆ matching XML tags▮
  ⋆ evaluating arithmetic expression▮

- Clustering algorithm▮

Evaluating Arithmetic Expressions

(((7+3)/5)*(7-5))   X/5)*(7-5))   X*(7-5))

```
|   |      |   |
| 3 |      |   |        7+3 is 10
| 7 |      | + |      ───────────▶
└───┘      └───┘
Number    Operator
```

```
|   |      |   |
| 5 |      |   |        10/5 is 2
| 10|      | / |      ───────────▶
└───┘      └───┘
Number    Operator
```

```
| 5 |      |   |
| 7 |      | - |
| 2 |      | * |
└───┘      └───┘
Number    Operator
```

)

```
                     |   |      |   |
      7-5 is 2       | 2 |      |   |        2*2 is 4       |   |      |   |
   ───────────▶      | 2 |      | * |      ───────────▶     | 4 |      |   |
                     └───┘      └───┘                       └───┘      └───┘
                     Number    Operator                    Number    Operator
```

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Queues

- First-in-first-out (FIFO) memory model

- `enqueue(elem)`

- `peek()`

- `dequeue()`

- C++ has a double ended queue (`deque`) with `push_front()`, `push_back()`, etc.

|	F	E	D	C	B	

Uses of Queues

- Queues are heavily used in multi-threaded applications (e.g. operating systems)

- Multi-threaded applications need to minimise waiting and ensure the integrity of the data structure (for instance when an exception is thrown)

- Because of this they are more complicated than most data structures

- They can be implemented using linked-lists or circular arrays

Priority Queues

- Queue with priorities

- `insert(elem,priority)` (in C++ `push()`)

- `findMin()` (in C++ `top()`)

- `deleteMin()` (in C++ `pop()`)

| B | 5 | D | 4 | A | 3 |

$\xrightarrow{\text{deleteMin()}}$ C
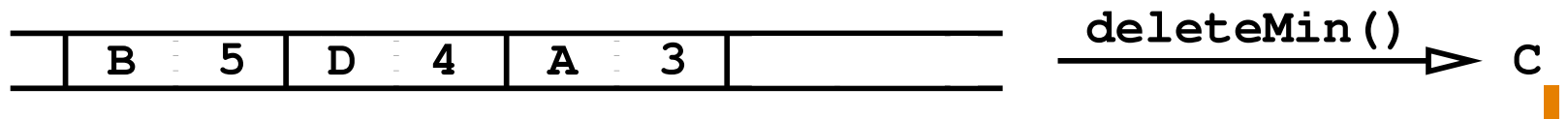
Uses of Priority Queues

- Queues with priorities (e.g. which threads should run)

- Real time simulation

- Often used in "greedy algorithms"

  ⋆ Huffman encoding
  ⋆ Prim's minimum spanning tree algorithm

Implementation of Priority Queue

- Could be implemented using a binary tree or linked list

- Most efficient implementation uses a heap

- A heap is a binary tree implemented using an array

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Lists

- In C++ the standard list is known as `vector<T>`

- That is, it is a collection where the order in which you put items into the list counts

- You can have repetitions of elements

- It has random access, e.g. `v[i]`

- You can `push_back(i)`, `insert`, `erase`, etc.

- C++ has a linked list class `List<T>`

Sets

- Models mathematical sets

- Container with no ordering or repetitions

- Methods include `insert`, `find`, `size`, `erase`

- Provides fast search (`find`)

- This is the class to use when you have to rapidly find whether an object is in the set or not—don't use a list like `vector<T>`!

Iterators

- Wish to act on all members of the set

- Performed using an `iterator`

- Iterators are used by many collections

- In C++ iterators follow the pointer convention

```
set<string> words;

words.insert("hello");
words.insert("world");

for(auto iter = words.begin(); iter != words.end(); ++iter) {
  cout << *iter << endl;
}
```

Implementation of Sets

- Sets are very important and there are many implementations depending on their usage

- Two common implementations of sets are

    ★ hash tables: `unordered_set<T>`
    ★ binary trees: `set<T>`

- Which is most efficient depends on the application

- Binary trees allow you to iterate in order (iterating over a hash table will give you outputs in random order)

- `multiset<T>` are sets with repetition

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Maps

- A map provides a content addressable memory for pairs $keyword: data$∎

- It provides fast access to the data through the keyword∎

- Implement as tree or hash table∎

- Multimaps allows different data to be stored with the same keyword∎

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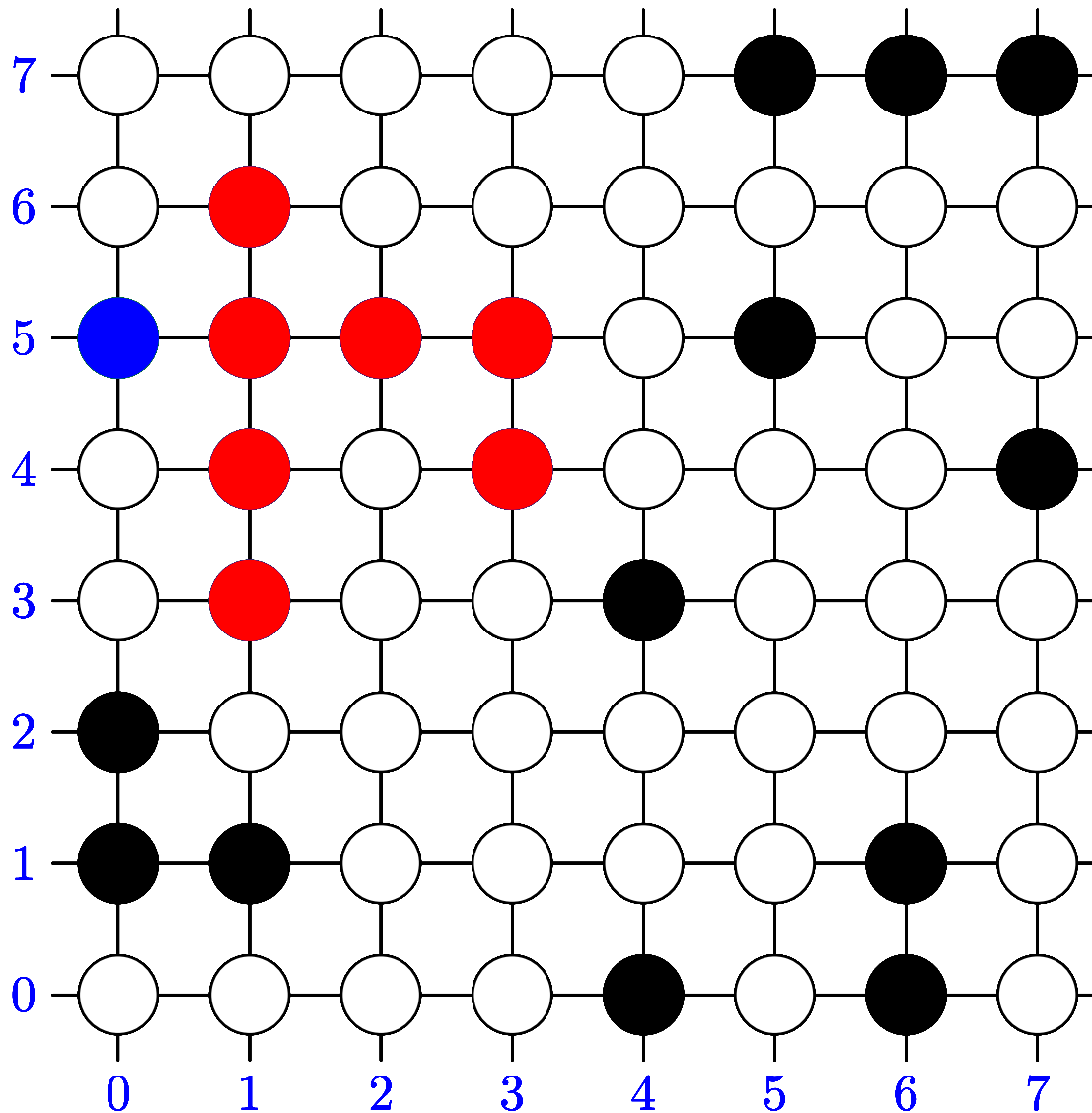
5. **Putting it Together**

Connected Nodes
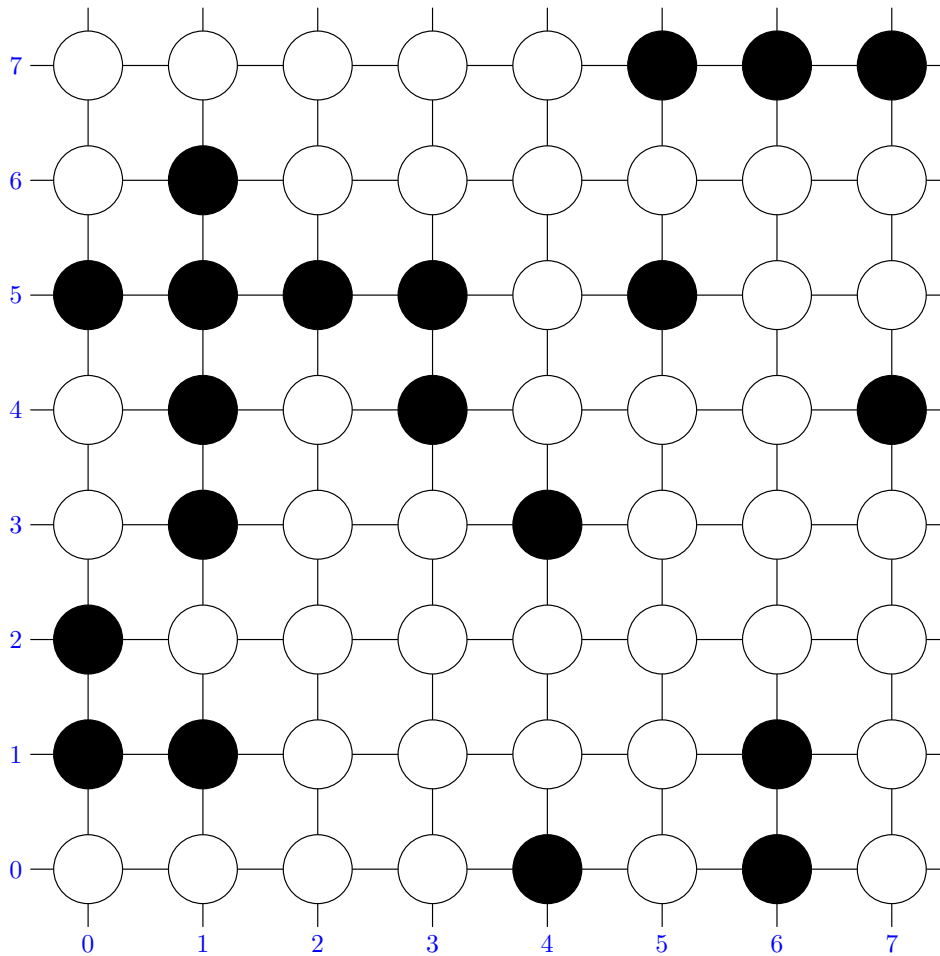
- A frequent problem is to find clusters of connected cells▮

- Applications in computer vision, computer go, graph connectedness, . . . ▮

Connected Nodes

nextNode = (2,5)

uncheckedNodes =

$(1,6)$
$(3,4)$
$(0,5)$

clusterNodes =
$\{(2,5), (1,5), (3,5),$
$(3,4), (0,5), (1,4),$
$(1,6), (1,3) \}$

Connected Node Algorithm

```
set<Node> findCluster(Node startNode, Graph graph)
{
  stack<Node> uncheckedNodes = new Stack<Node>();
  set<Node> clusterNodes = new HashSet<Node>();

  uncheckedNodes.push(startNode);
  clusterNodes.add(startNode);

  while (!uncheckedNodes.empty()) {
    Node next = uncheckedNodes.top();    uncheckedNodes.pop();
    vector<Node> neighbours = graph.getNeighbours(next);
    for (Node neigh: neighbours) {
      if (graph.isOccupied(neigh) && !clusterNodes.contains(neigh) ) {
        uncheckedNodes.push(neigh);
        clusterNodes.insert(neigh);
      }
    }
  }

  return clusterNodes;
}
```

Lessons

- Abstract Data Types (ADT) are interfaces to data

- Their purpose is to allow the programmer to declare their intentions

- They often have different implementations with different properties

- The most efficient implementation is not always obvious—we will see many of these implementations as we go through this course

- You need to know the common ADTs (e.g. Stack, Queue, List, Set, Map) and how and when to use them