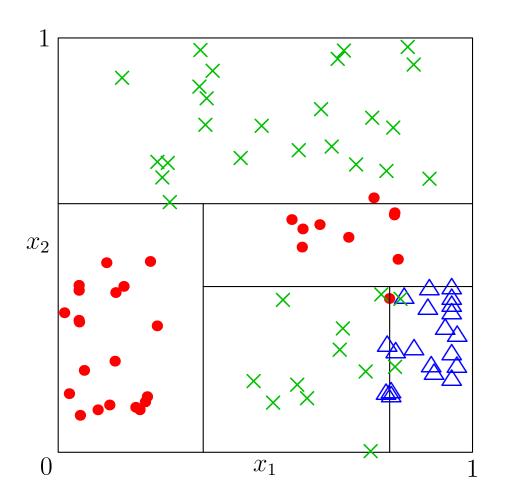
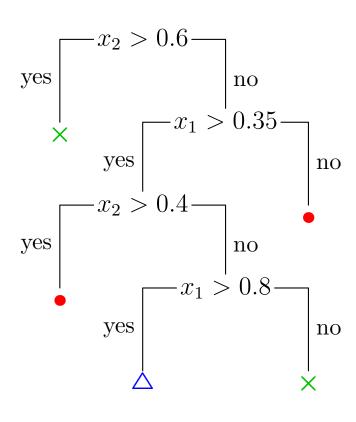
# **Advanced Machine Learning**

## **Boosting**

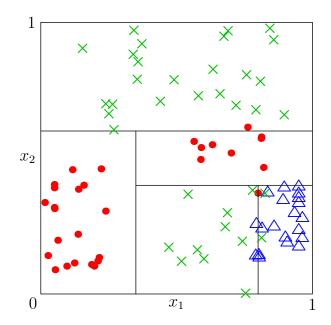


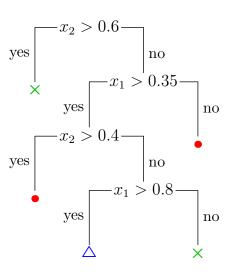


Boosting, AdaBoost, Gradient Boosting

### **Outline**

- 1. Boosting
- 2. AdaBoost
- 3. Gradient Boosting
- 4. Dropout





$$C_n(\boldsymbol{x}) = \sum_{i=1}^n \alpha_i \hat{h}_i(\boldsymbol{x})$$

- Weak learners,  $\hat{h}_i(\boldsymbol{x})$ , are learning machine that do a little better than chance
- ullet The trick is to choose the weights,  $lpha_i$
- Because the weak learners do little better than chance we (miraculously) don't overfit

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- Because the weak learners do little better than chance we (miraculously) don't overfit that much

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- Sometimes we just use one variable (the stump)

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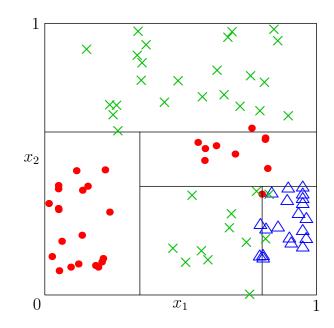
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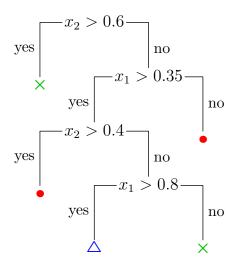
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  - ⋆ adaboost—a classic algorithm for binary classification
  - \* gradient boosting—used for regression, trains a weak learner on the residual errors

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- Suppose we have a binary classification task with data  $\mathcal{D}=\{(\boldsymbol{x}^{\mu},y^{\mu})|\mu=1,2,...,m\} \text{ with } y^{\mu}\in\{-1,1\}$
- ullet Our  $i^{th}$  weak learner provides a prediction  $\hat{h}_i(oldsymbol{x}^\mu) \in \{-1,1\}$
- We ask, can we find a linear combination

$$C_n(\mathbf{x}) = \alpha_1 \hat{h}_1(\mathbf{x}) + \alpha_2 \hat{h}_2(\mathbf{x}) + \dots + \alpha_n \hat{h}_n(\mathbf{x})$$

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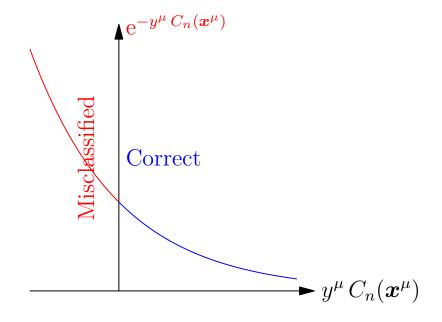
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- So that  $\operatorname{sgn} \big( C_n(\boldsymbol{x}) \big)$  is a strong learner?
- Note we want  $y^{\mu}C_n(\boldsymbol{x}^{\mu})>0$

#### **AdaBoost**

- AdaBoost is a classic solution to this problem
- It assigns an "loss function"

$$L_n = \sum_{\mu=1}^m e^{-y^{\mu} C_n(\boldsymbol{x}^{\mu})}$$

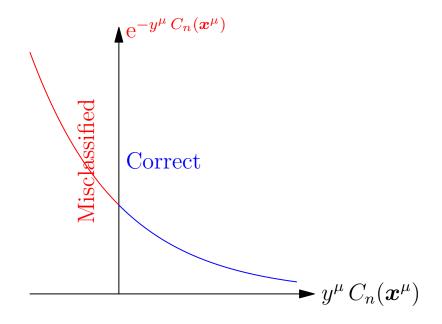


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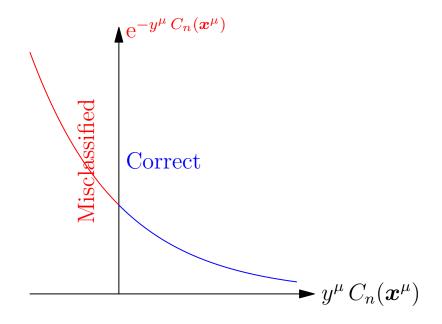


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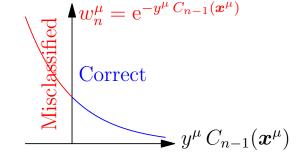
## **Choosing a Weak Classifier**

To minimise the loss

$$L_n(\alpha_n) = e^{-\alpha_n} \sum_{\mu=1}^m w_n^{\mu} + (e^{\alpha_n} - e^{-\alpha_n}) \sum_{\mu: y^{\mu} \neq \hat{h}_n(\mathbf{x}^{\mu})} w_n^{\mu}$$

We choose the weak learner with the lowest value of

$$\sum_{\mu:y^{\mu}\neq\hat{h}_n(\boldsymbol{x}^{\mu})} w_n^{\mu} = \sum_{\mu:y^{\mu}\neq\hat{h}_n(\boldsymbol{x}^{\mu})} e^{-y^{\mu}C_{n-1}(\boldsymbol{x}^{\mu})}$$



That is, it misclassifies only where the other learners classify well

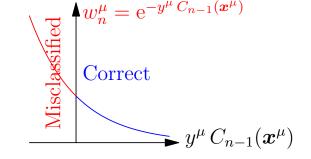
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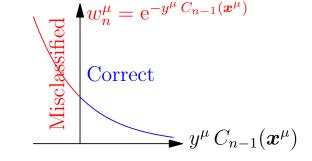
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## **Choosing Weights**

• We now choose the weight  $\alpha_n$  to minimise the loss  $L_n(\alpha_n)$ 

$$\frac{\partial L_n(\alpha_n)}{\partial \alpha_n} = e^{\alpha_n} \sum_{\mu: y^{\mu} \neq \hat{h}_n(\mathbf{x}^{\mu})} w_n^{\mu} - e^{-\alpha_n} \sum_{\mu: y^{\mu} = \hat{h}_n(\mathbf{x}^{\mu})} w_n^{\mu} = 0$$

That is

$$e^{2\alpha_n} = \frac{\sum_{\mu:y^{\mu} = \hat{h}_n(\boldsymbol{x}^{\mu})} \sum_{\mu:y^{\mu} \neq \hat{h}_n(\boldsymbol{x}^{\mu})} \qquad \text{or} \qquad \alpha_n = \frac{1}{2} \log \left( \frac{\sum_{\mu:y^{\mu} = \hat{h}_n(\boldsymbol{x}^{\mu})} \sum_{\mu:y^{\mu} \neq \hat{h}_n(\boldsymbol{x}^{\mu})} \sum_{\mu:y^{\mu} \neq \hat{h}_n(\boldsymbol{x}^{\mu})} \sum_{\mu:y^{\mu} \neq \hat{h}_n(\boldsymbol{x}^{\mu})} \right)$$

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# **Algorithm**

- 1. Start with a set of weak learners  $\mathcal{W}$
- 2. Associate a weight,  $w_n^\mu$ , with every data point  $({m x}^\mu,y^\mu)$ ,  $\mu=1,2,\ldots,m$
- 3. Initially  $w_1^{\mu} = 1$
- 4. Choose the weak learning,  $\hat{h}_n(x) \in \mathcal{W}$ , that minimises  $\sum_{\mu:y^\mu 
  eq \hat{h}_n(x^\mu)} w_n^\mu$
- 5. Update predictor  $C_n(\boldsymbol{x}) = C_{n-1}(\boldsymbol{x}) + \alpha_n \hat{h}_n(\boldsymbol{x})$  where

$$\alpha_n = \frac{1}{2} \log \left( \frac{\sum\limits_{\mu:y^{\mu} = \hat{h}_n(\boldsymbol{x}^{\mu})} w_n^{\mu}}{\sum\limits_{\mu:y^{\mu} \neq \hat{h}_n(\boldsymbol{x}^{\mu})} w_n^{\mu}} \right)$$

- 6. Update  $w_{n+1}^{\mu} = w_n^{\mu} e^{-y^{\mu} \alpha_n \hat{h}_n(\boldsymbol{x}^{\mu})}$
- 7. Go to 4

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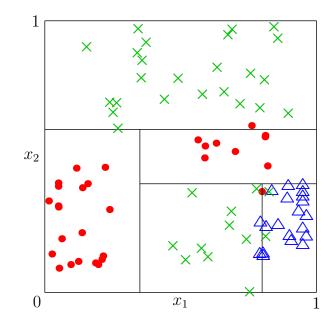
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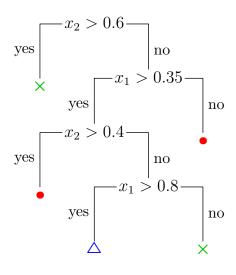
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- 3. **Gradient Boosting**
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- At each step  $\hat{h}_n(x)$  is trained to predict the **residual error**,  $\Delta_{n-1} = y C_{n-1}(x)$ , (i.e. the target minus the current prediction)
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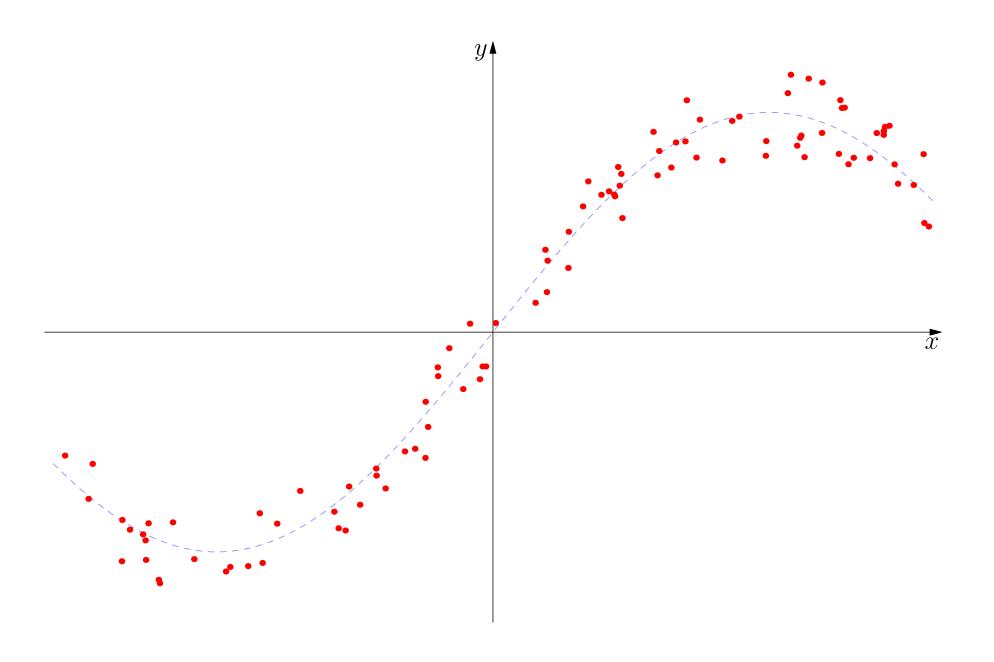
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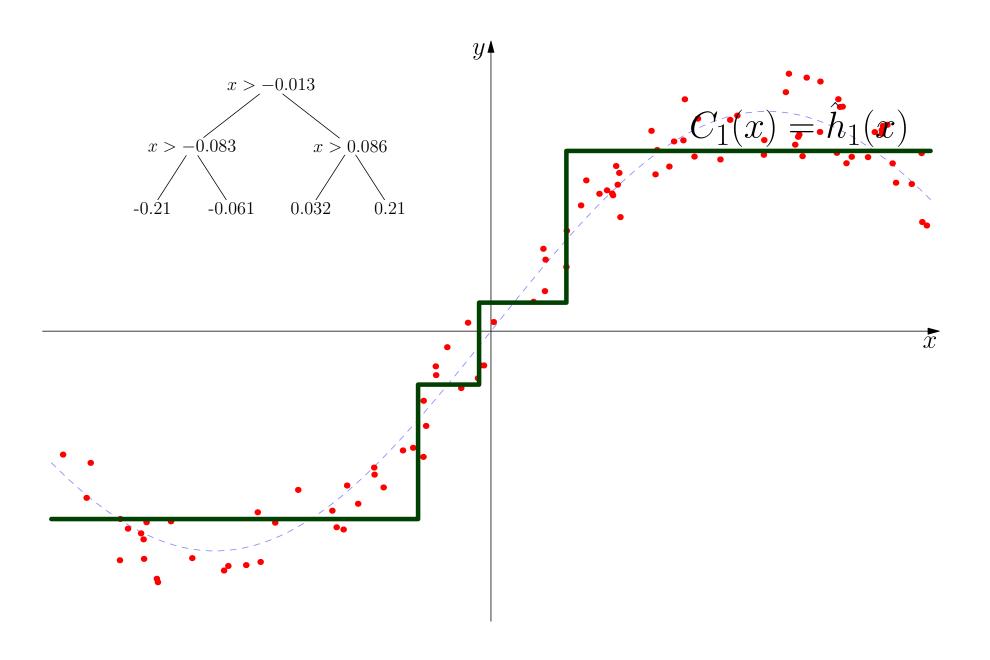
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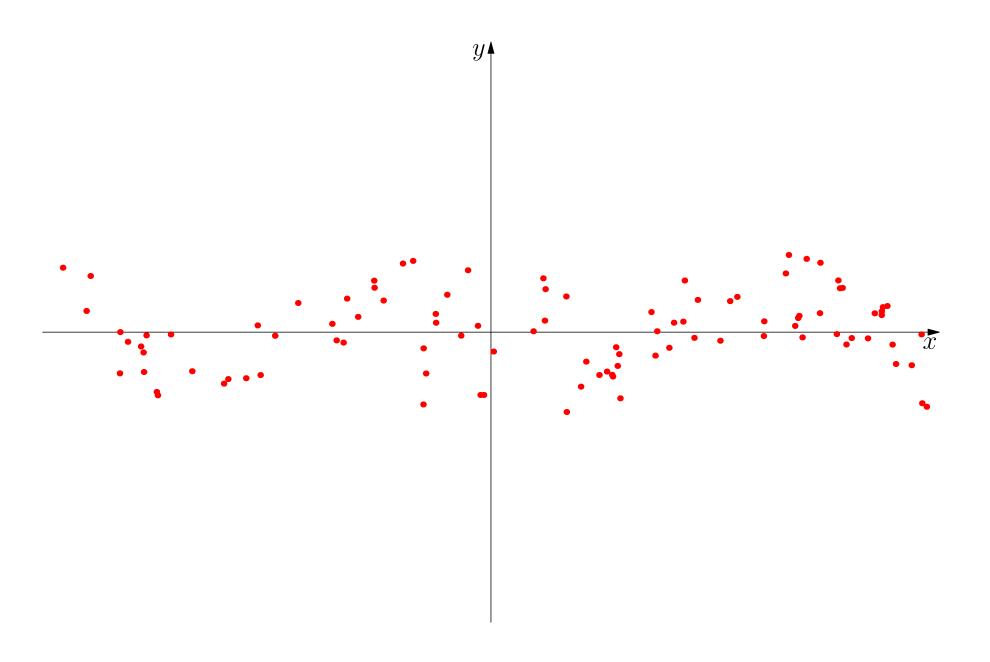
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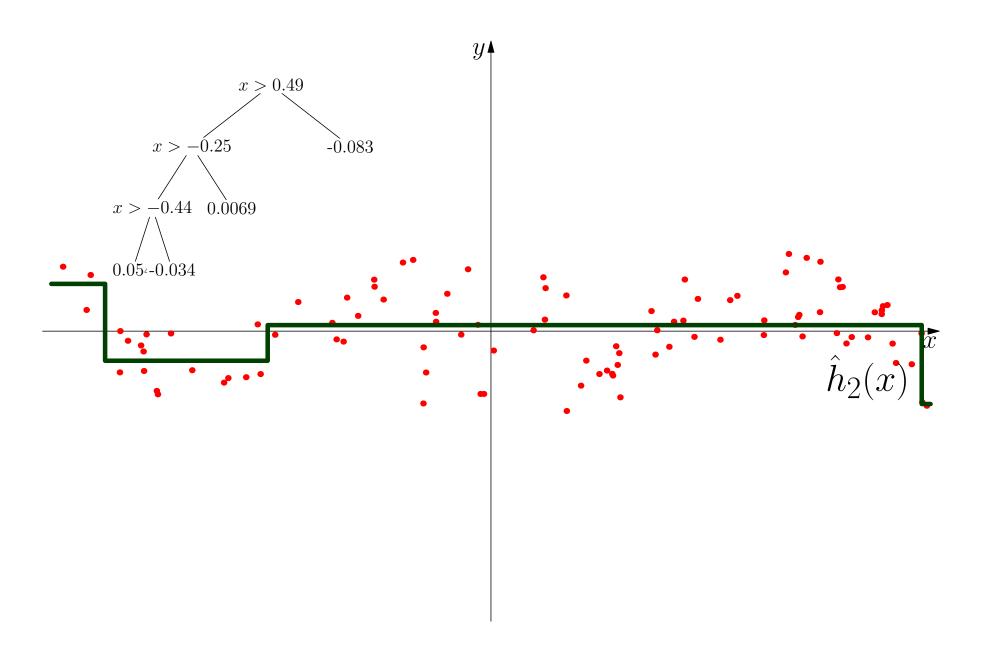
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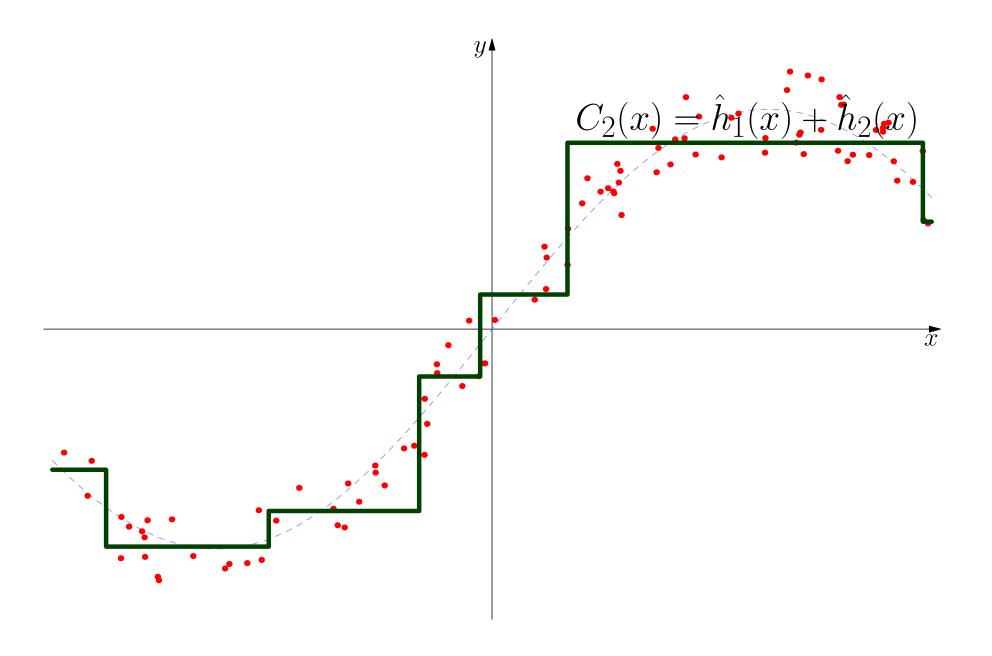
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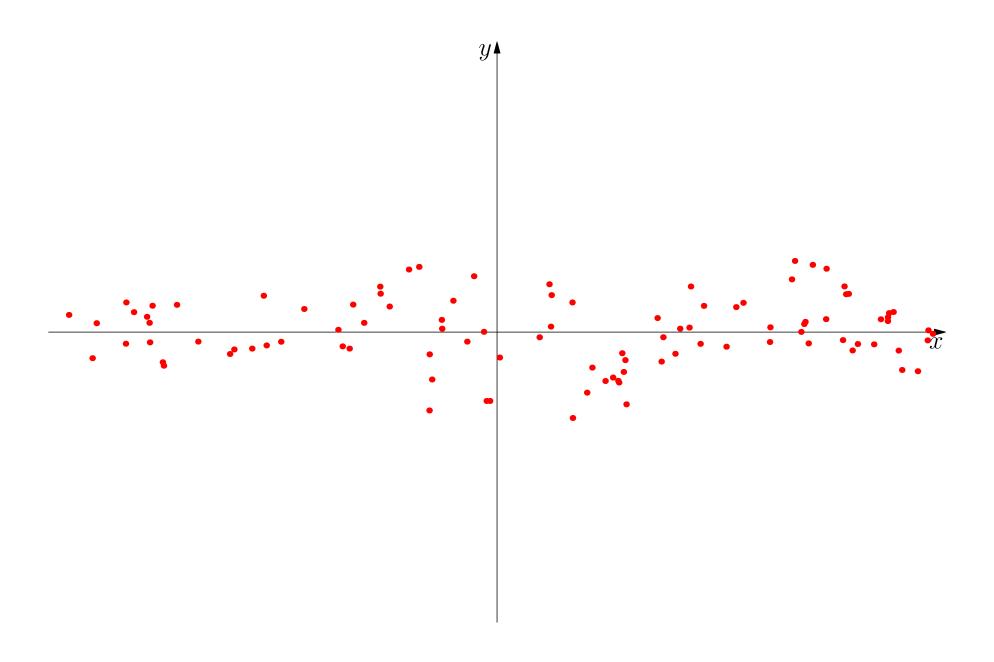


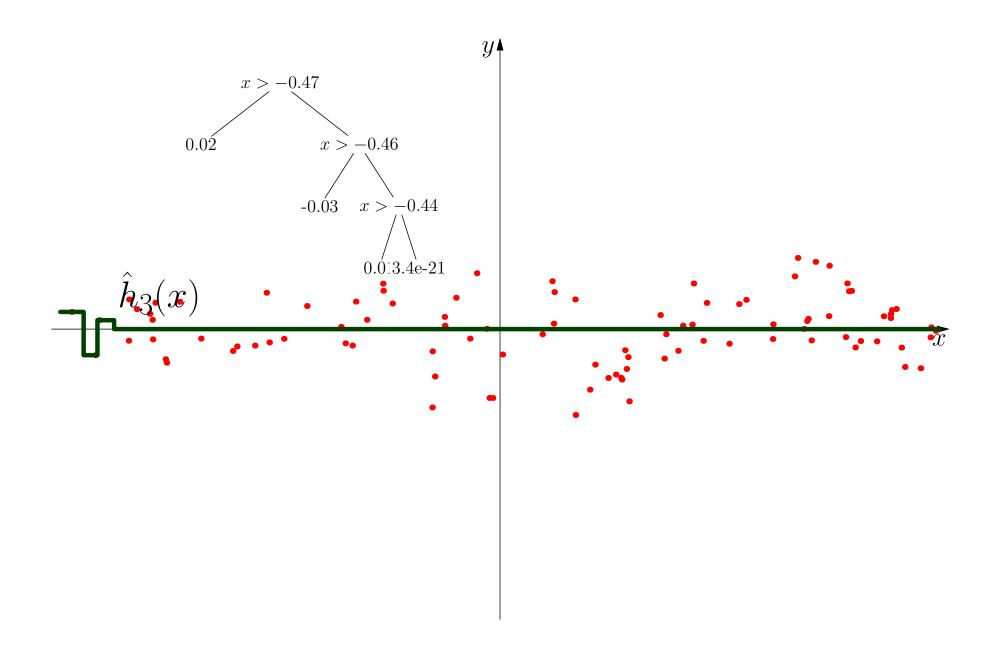


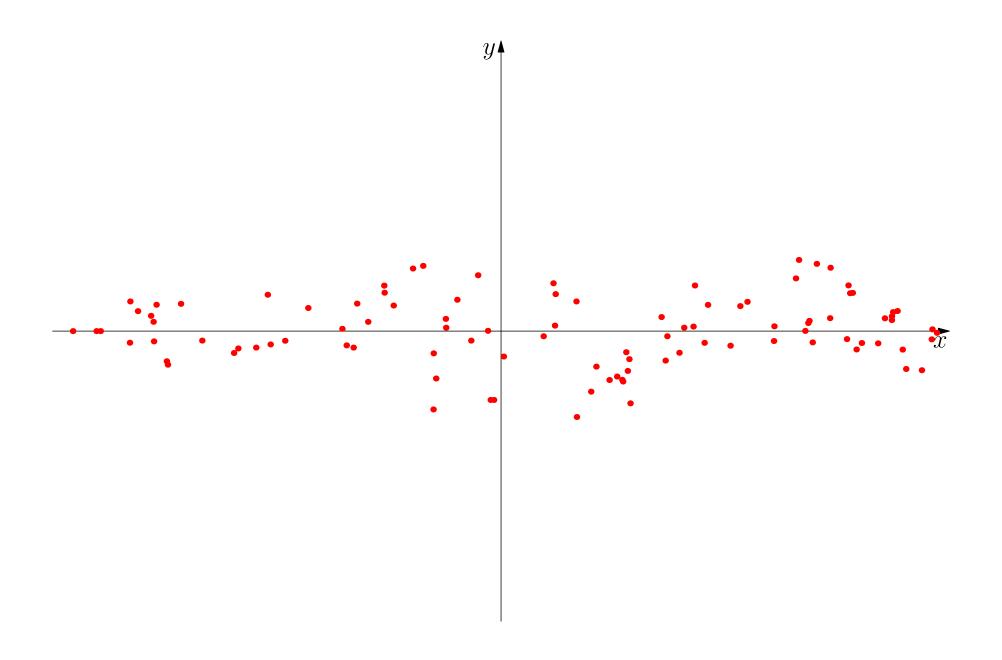


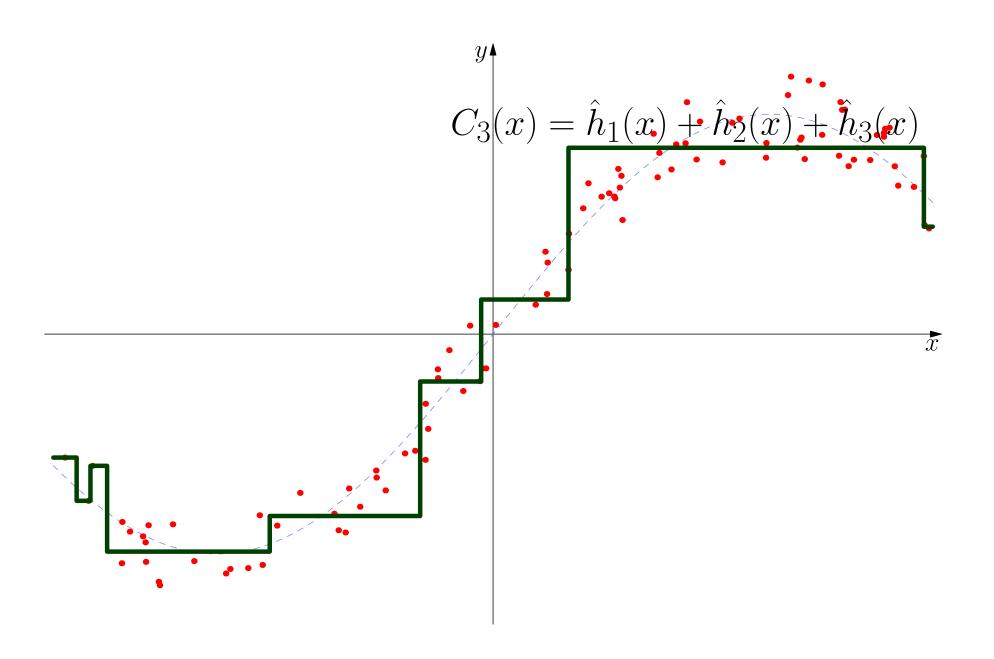


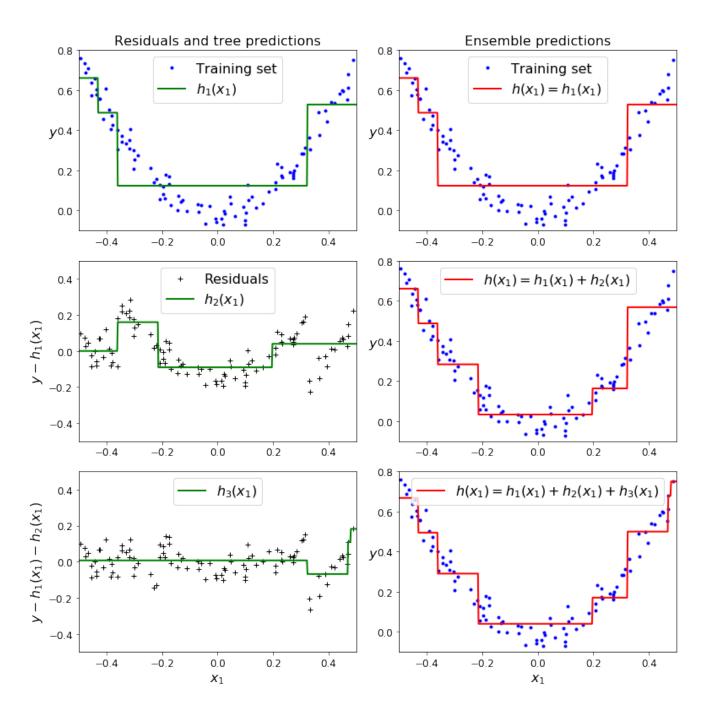






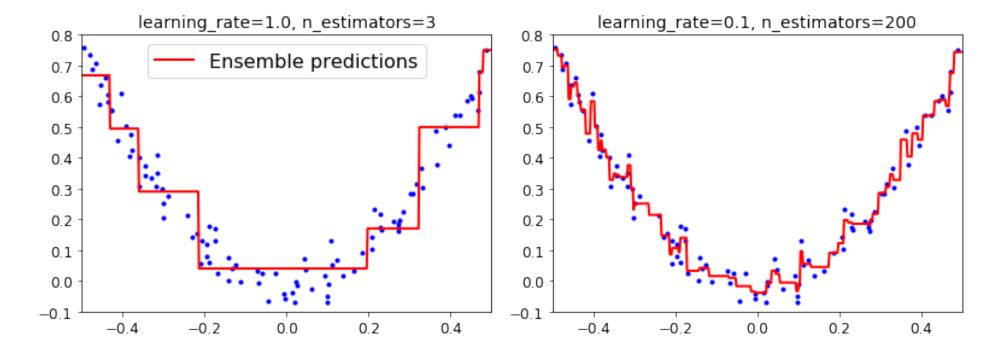






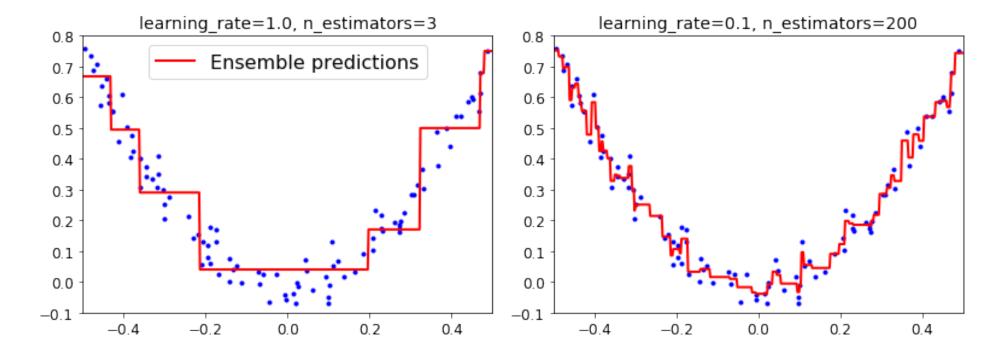
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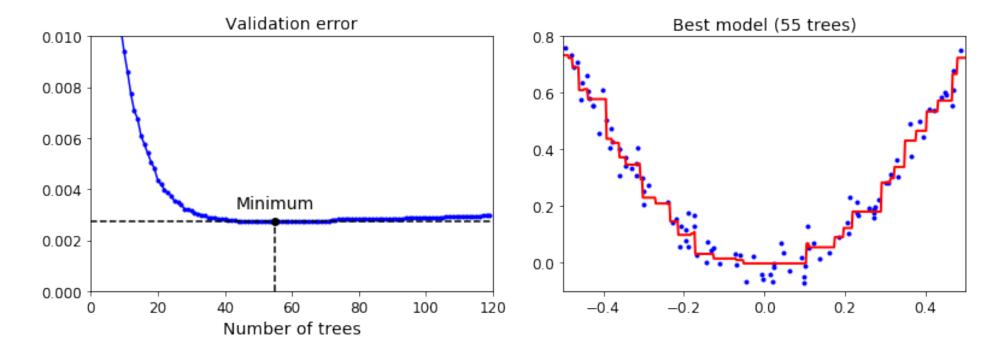
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But we will over-fit eventually

## **Early Stopping**

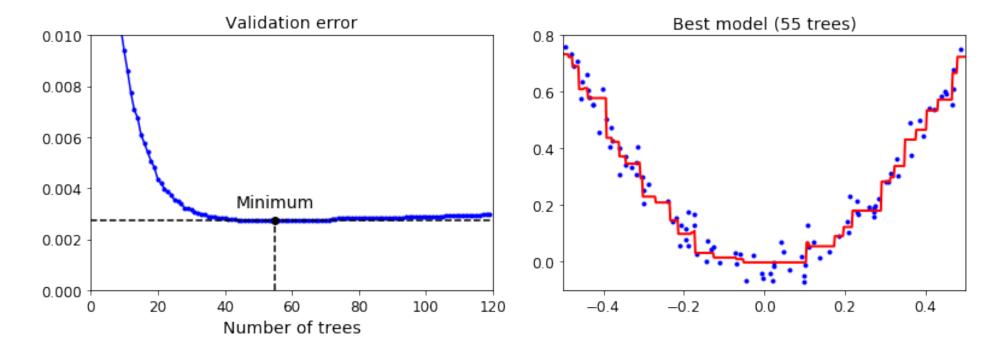
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 Use cross-validation against a validation set to decide when to stop

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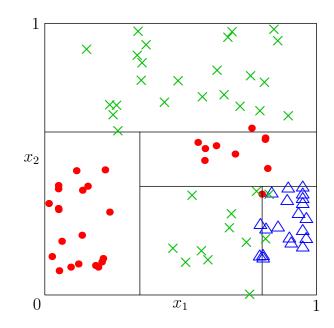
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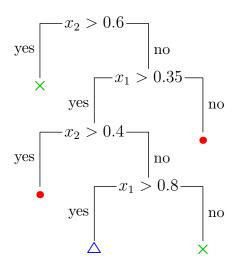
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- The machines should have roughly the same performance
- Of course, this comes at the price of having to train multiple machines
- One can try to train a machine to decide how to combine different machines (stacking)
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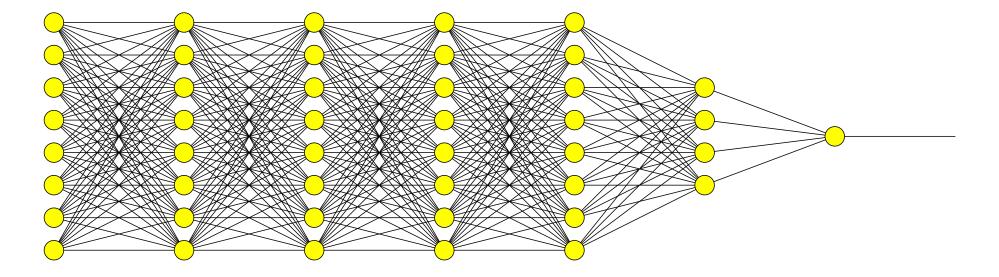
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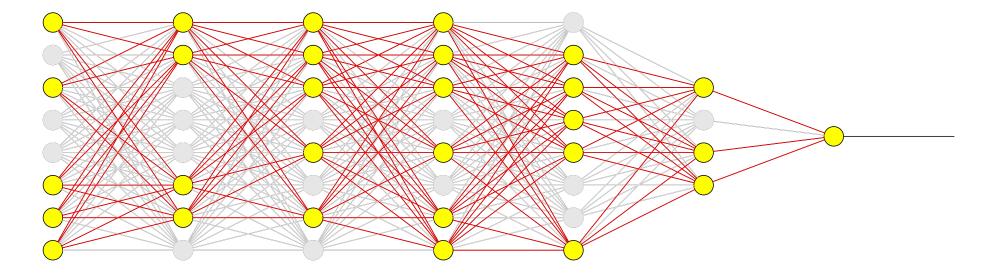
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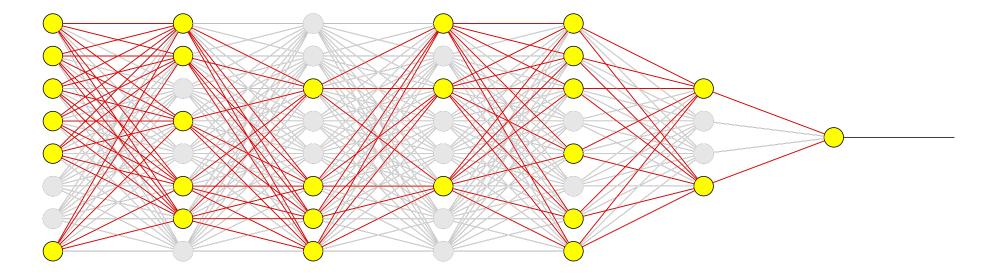
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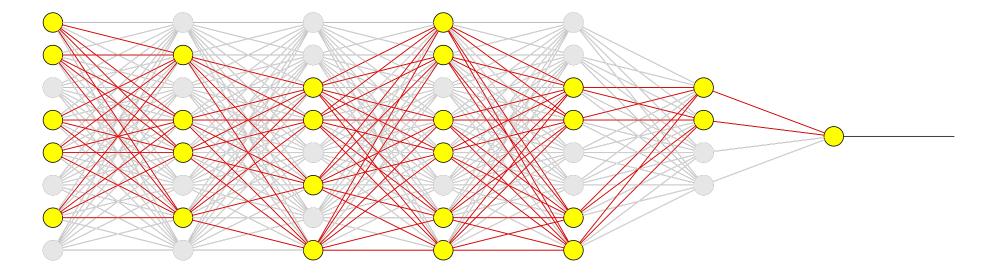
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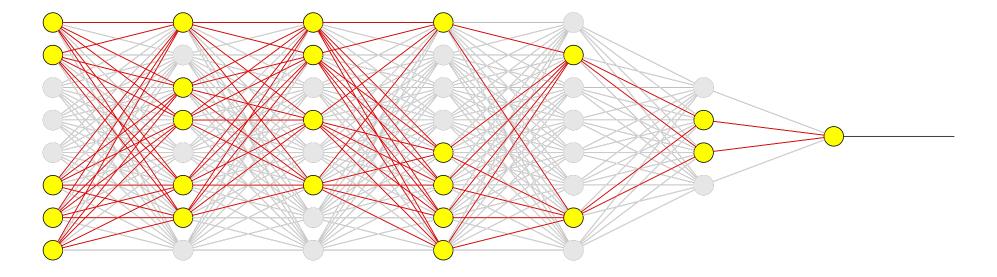


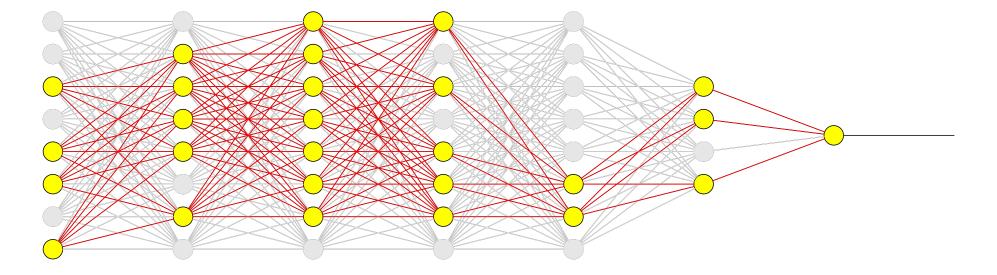


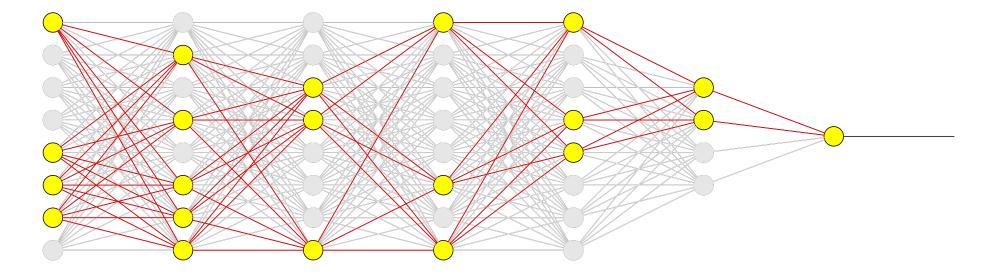


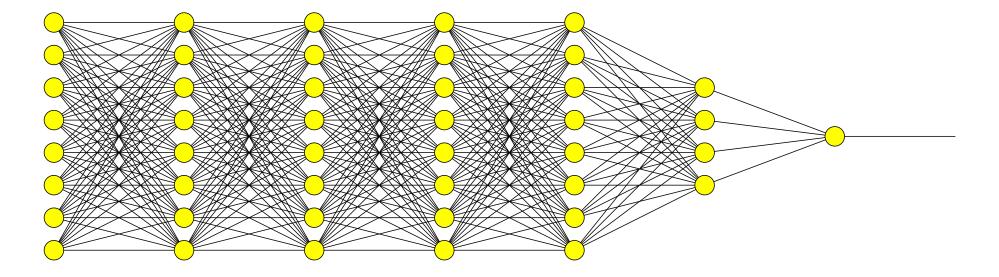




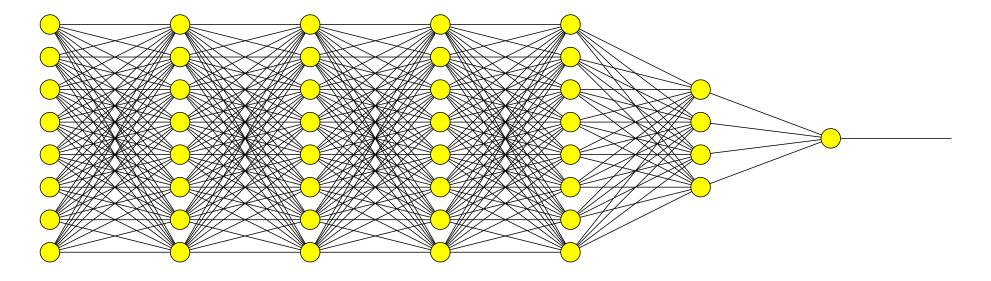








For deep learning we can control for over-fitting using dropout



• This can be seen as ensembling lots of much simpler machines

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- Tend to work best with very simple models (true of random forest and boosting)
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