$$|\mathbf{I} + \epsilon\,\mathbf{M}| = \begin{vmatrix} 1 + \epsilon\,M_{11} & \epsilon\,M_{21} & \epsilon\,M_{31} & \epsilon\,M_{41} & \epsilon\,M_{51} \\ \epsilon\,M_{12} & 1 + \epsilon\,M_{22} & \epsilon\,M_{32} & \epsilon\,M_{42} & \epsilon\,M_{52} \\ \epsilon\,M_{13} & \epsilon\,M_{23} & 1 + \epsilon\,M_{33} & \epsilon\,M_{43} & \epsilon\,M_{53} \\ \epsilon\,M_{14} & \epsilon\,M_{24} & \epsilon\,M_{34} & 1 + \epsilon\,M_{44} & \epsilon\,M_{54} \\ \epsilon\,M_{15} & \epsilon\,M_{25} & \epsilon\,M_{35} & \epsilon\,M_{45} & 1 + \epsilon\,M_{55} \end{vmatrix}$$

$$|\mathbf{I} + \epsilon\,\mathbf{M}| = \begin{vmatrix} 1 + \epsilon\,M_{11} & \epsilon\,M_{21} & \epsilon\,M_{31} & \epsilon\,M_{41} & \epsilon\,M_{51} \\ \epsilon\,M_{12} & 1 + \epsilon\,M_{22} & \epsilon\,M_{32} & \epsilon\,M_{42} & \epsilon\,M_{52} \\ \epsilon\,M_{13} & \epsilon\,M_{23} & 1 + \epsilon\,M_{33} & \epsilon\,M_{43} & \epsilon\,M_{53} \\ \epsilon\,M_{14} & \epsilon\,M_{24} & \epsilon\,M_{34} & 1 + \epsilon\,M_{44} & \epsilon\,M_{54} \\ \epsilon\,M_{15} & \epsilon\,M_{25} & \epsilon\,M_{35} & \epsilon\,M_{45} & 1 + \epsilon\,M_{55} \end{vmatrix}$$

$$= (1 + \epsilon\,M_{11})\,C_{11}$$

$$|\mathbf{I} + \epsilon\,\mathbf{M}| = \begin{vmatrix} 1 & M_{11} & \epsilon\,M_{21} & & 1 \\ \epsilon\,M_{12} & 1+ & M_{22} & \epsilon\,M_{32} & \epsilon\,M_{42} & \epsilon\,M_{52} \\ \epsilon\,M_{13} & \epsilon & {}_{3} & 1+\epsilon\,M_{33} & \epsilon\,M_{43} & \epsilon\,M_{53} \\ \epsilon\,M_{14} & \epsilon & {}_{4} & \epsilon\,M_{34} & 1+\epsilon\,M_{44} & \epsilon\,M_{54} \\ \epsilon\,M_{15} & \epsilon & {}_{5} & \epsilon\,M_{35} & \epsilon\,M_{45} & 1+\epsilon\,M_{55} \end{vmatrix}$$

$$= (1 + \epsilon\,M_{11})\,C_{11} - \epsilon\,M_{21}\,C_{21}$$

$$|\mathbf{I} + \epsilon\,\mathbf{M}| = \begin{vmatrix} 1 & & \epsilon\,M_{31} & & 1 \\ \epsilon\,M_{12} & 1 + \epsilon\,M_{22} & \epsilon\, & \epsilon\,M_{42} & \epsilon\,M_{52} \\ \epsilon\,M_{13} & \epsilon\,M_{23} & 1 + \,M_{33} & \epsilon\,M_{43} & \epsilon\,M_{53} \\ \epsilon\,M_{14} & \epsilon\,M_{24} & \epsilon\, & 1 + \epsilon\,M_{44} & \epsilon\,M_{54} \\ \epsilon\,M_{15} & \epsilon\,M_{25} & \epsilon\, & \epsilon\,M_{45} & 1 + \epsilon\,M_{55} \end{vmatrix}$$

$$= (1 + \epsilon\,M_{11})\,C_{11} \; -\epsilon\,M_{21}\,C_{21} +\epsilon\,M_{31}\,C_{31}$$

$$|\mathbf{I} + \epsilon\,\mathbf{M}| = \begin{vmatrix} 1 + \epsilon\,M_{11} & \epsilon\,M_{21} & \epsilon\,M_{31} & \epsilon\,M_{41} & \epsilon\,M_{51} \\ \epsilon\,M_{12} & 1 + \epsilon\,M_{22} & \epsilon\,M_{32} & \epsilon\,M_{42} & \epsilon\,M_{52} \\ \epsilon\,M_{13} & \epsilon\,M_{23} & 1 + \epsilon\,M_{33} & \epsilon\,M_{43} & \epsilon\,M_{53} \\ \epsilon\,M_{14} & \epsilon\,M_{24} & \epsilon\,M_{34} & 1 + \epsilon\,M_{44} & \epsilon\,M_{54} \\ \epsilon\,M_{15} & \epsilon\,M_{25} & \epsilon\,M_{35} & \epsilon\,M_{45} & 1 + \epsilon\,M_{55} \end{vmatrix}$$

$$= (1 + \epsilon\,M_{11})\,C_{11} \; - \epsilon\,M_{21}\,C_{21} \; + \epsilon\,M_{31}\,C_{31} \; - \epsilon\,M_{41}\,C_{41}$$

$$|\mathbf{I} + \epsilon\,\mathbf{M}| = \begin{vmatrix} 1 & & & & \epsilon\,M_{51} \\ \epsilon\,M_{12} & 1+\epsilon\,M_{22} & \epsilon\,M_{32} & \epsilon\,M_{42} & \\ \epsilon\,M_{13} & \epsilon\,M_{23} & 1+\epsilon\,M_{33} & \epsilon\,M_{43} & \\ \epsilon\,M_{14} & \epsilon\,M_{24} & \epsilon\,M_{34} & 1+\epsilon\,M_{44} & \\ \epsilon\,M_{15} & \epsilon\,M_{25} & \epsilon\,M_{35} & \epsilon\,M_{45} & 1+\epsilon\,M_{55} \end{vmatrix}$$

$$= (1+\epsilon\,M_{11})\,C_{11} \;-\epsilon\,M_{21}\,C_{21} \;+\epsilon\,M_{31}\,C_{31} \;-\epsilon\,M_{41}\,C_{41} \;+\epsilon\,M_{51}\,C_{51}$$

$$|\mathbf{I} + \epsilon\,\mathbf{M}| = \begin{vmatrix} 1 + \epsilon\,M_{11} & \epsilon\,M_{21} & \epsilon\,M_{31} & \epsilon\,M_{41} & \epsilon\,M_{51} \\ \epsilon\,M_{12} & 1 + \epsilon\,M_{22} & \epsilon\,M_{32} & \epsilon\,M_{42} & \epsilon\,M_{52} \\ \epsilon\,M_{13} & \epsilon\,M_{23} & 1 + \epsilon\,M_{33} & \epsilon\,M_{43} & \epsilon\,M_{53} \\ \epsilon\,M_{14} & \epsilon\,M_{24} & \epsilon\,M_{34} & 1 + \epsilon\,M_{44} & \epsilon\,M_{54} \\ \epsilon\,M_{15} & \epsilon\,M_{25} & \epsilon\,M_{35} & \epsilon\,M_{45} & 1 + \epsilon\,M_{55} \end{vmatrix}$$

$$= (1 + \epsilon\,M_{11})\,C_{11} + O(\epsilon^2)$$

$$|\mathbf{I} + \epsilon\,\mathbf{M}| = \begin{vmatrix} 1 + \epsilon\,M_{11} & \epsilon & & & \\ \epsilon & 1 + \epsilon\,M_{22} & \epsilon\,M_{32} & \epsilon\,M_{42} & \epsilon\,M_{52} \\ \epsilon & \epsilon\,M_{23} & 1 + \epsilon\,M_{33} & \epsilon\,M_{43} & \epsilon\,M_{53} \\ \epsilon & \epsilon\,M_{24} & \epsilon\,M_{34} & 1 + \epsilon\,M_{44} & \epsilon\,M_{54} \\ \epsilon & \epsilon\,M_{25} & \epsilon\,M_{35} & \epsilon\,M_{45} & 1 + \epsilon\,M_{55} \end{vmatrix}$$

$$= (1 + \epsilon\,M_{11})\,C_{11} + O(\epsilon^2)$$

$$|\mathbf{I} + \epsilon\,\mathbf{M}| = \begin{vmatrix} \boxed{1+\epsilon\,M_{11}} & \epsilon\,\blacksquare_{1} & & & \\[4pt] \epsilon\,\blacksquare_{2} & \boxed{1+\epsilon\,M_{22}} & \epsilon\,\blacksquare_{2} & & \\[4pt] \epsilon\,\blacksquare_{3} & \epsilon\,\blacksquare_{3} & 1+\epsilon\,M_{33} & \epsilon\,M_{43} & \epsilon\,M_{53} \\[4pt] \epsilon\,\blacksquare_{4} & \epsilon\,\blacksquare_{4} & \epsilon\,M_{34} & 1+\epsilon\,M_{44} & \epsilon\,M_{54} \\[4pt] \epsilon\,\blacksquare_{5} & \epsilon\,\blacksquare_{5} & \epsilon\,M_{35} & \epsilon\,M_{45} & 1+\epsilon\,M_{55} \end{vmatrix}$$

$$= (1+\epsilon\,M_{11})\,(1+\epsilon\,M_{22})\,C'_{22} + O(\epsilon^2)$$

$$|\mathbf{I} + \epsilon\,\mathbf{M}| = \begin{vmatrix} 1 + \epsilon\,M_{11} & \epsilon\,M_{21} & \epsilon\,M_{31} & \epsilon\,M_{41} & \epsilon\,M_{51} \\ \epsilon\,M_{12} & 1 + \epsilon\,M_{22} & \epsilon\,M_{32} & \epsilon\,M_{42} & \epsilon\,M_{52} \\ \epsilon\,M_{13} & \epsilon\,M_{23} & 1 + \epsilon\,M_{33} & \epsilon\,M_{43} & \epsilon\,M_{53} \\ \epsilon\,M_{14} & \epsilon\,M_{24} & \epsilon\,M_{34} & 1 + \epsilon\,M_{44} & \epsilon\,M_{54} \\ \epsilon\,M_{15} & \epsilon\,M_{25} & \epsilon\,M_{35} & \epsilon\,M_{45} & 1 + \epsilon\,M_{55} \end{vmatrix}$$

$$= \prod_i (1 + \epsilon\,M_{ii}) + O(\epsilon^2)$$

$$|\mathbf{I} + \epsilon\,\mathbf{M}| = \begin{vmatrix} 1 + \epsilon\,M_{11} & \epsilon\,M_{21} & \epsilon\,M_{31} & \epsilon\,M_{41} & \epsilon\,M_{51} \\ \epsilon\,M_{12} & 1 + \epsilon\,M_{22} & \epsilon\,M_{32} & \epsilon\,M_{42} & \epsilon\,M_{52} \\ \epsilon\,M_{13} & \epsilon\,M_{23} & 1 + \epsilon\,M_{33} & \epsilon\,M_{43} & \epsilon\,M_{53} \\ \epsilon\,M_{14} & \epsilon\,M_{24} & \epsilon\,M_{34} & 1 + \epsilon\,M_{44} & \epsilon\,M_{54} \\ \epsilon\,M_{15} & \epsilon\,M_{25} & \epsilon\,M_{35} & \epsilon\,M_{45} & 1 + \epsilon\,M_{55} \end{vmatrix}$$

$$= (1 + \epsilon \sum_i M_{ii}) + O(\epsilon^2)$$

$$|\mathbf{I} + \epsilon\,\mathbf{M}| = \begin{vmatrix} 1 + \epsilon\,M_{11} & \epsilon\,M_{21} & \epsilon\,M_{31} & \epsilon\,M_{41} & \epsilon\,M_{51} \\ \epsilon\,M_{12} & 1 + \epsilon\,M_{22} & \epsilon\,M_{32} & \epsilon\,M_{42} & \epsilon\,M_{52} \\ \epsilon\,M_{13} & \epsilon\,M_{23} & 1 + \epsilon\,M_{33} & \epsilon\,M_{43} & \epsilon\,M_{53} \\ \epsilon\,M_{14} & \epsilon\,M_{24} & \epsilon\,M_{34} & 1 + \epsilon\,M_{44} & \epsilon\,M_{54} \\ \epsilon\,M_{15} & \epsilon\,M_{25} & \epsilon\,M_{35} & \epsilon\,M_{45} & 1 + \epsilon\,M_{55} \end{vmatrix}$$

$$= (1 + \epsilon\,\text{tr}\,\mathbf{M}) + O(\epsilon^2)$$