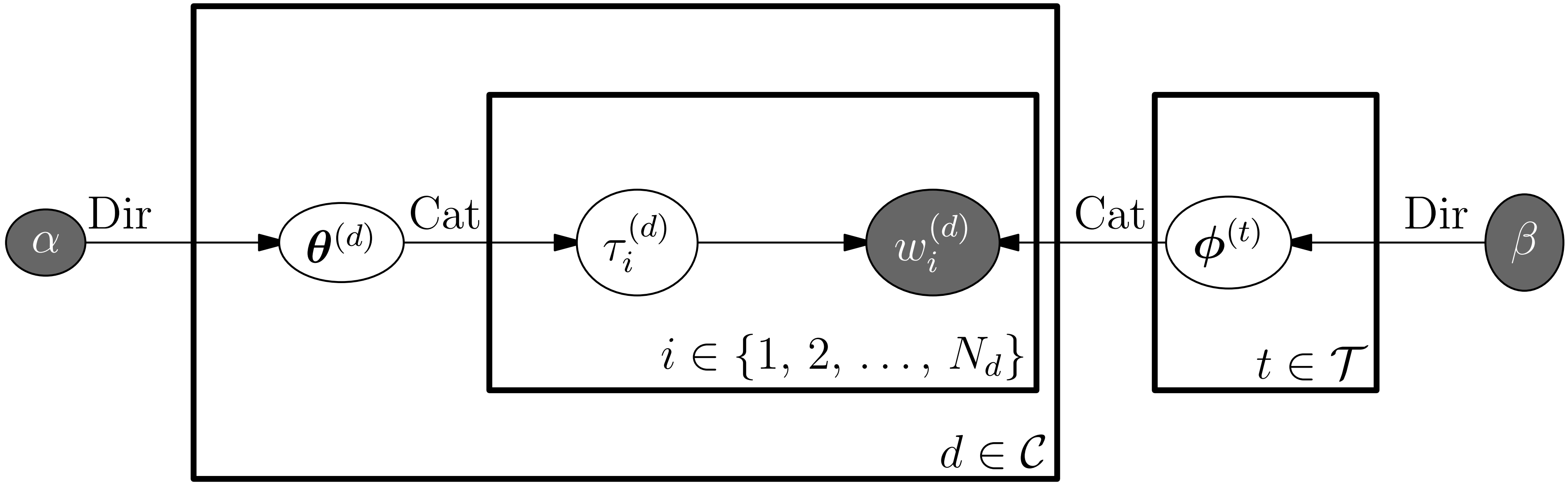
Advanced Machine Learning

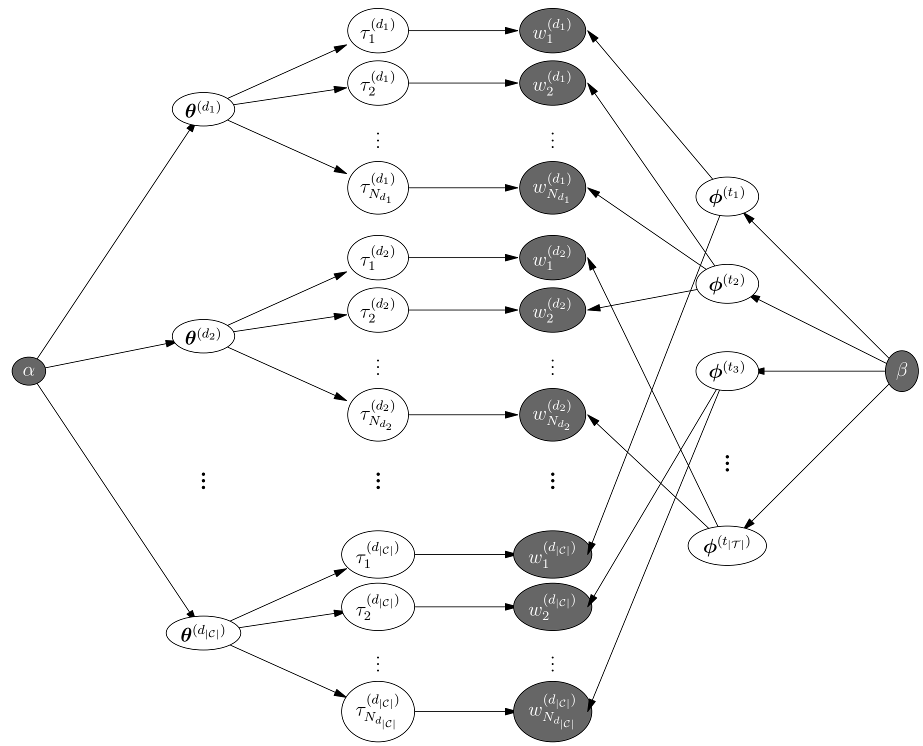
Graphical Models



Conditional Independence, Graphical models, LDA

Outline

1. **Graphical Models**
2. Cakes!
3. Latent Dirichlet Allocation



Graphical Models

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  - ⋆ AI Doctor
  - ⋆ Fault diagnostic system for a computer

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Dependencies Between Variables

- In building a probabilistic model we want to know which random variables depend on each other directly and which don't
- Variables that don't will typically still be correlated
- If two random variables X and Y are correlated then
  - ⋆ X could affect Y
  - ⋆ Y could affect X
  - ⋆ X and Y could not influence each other, but both be affected by another random variable Z

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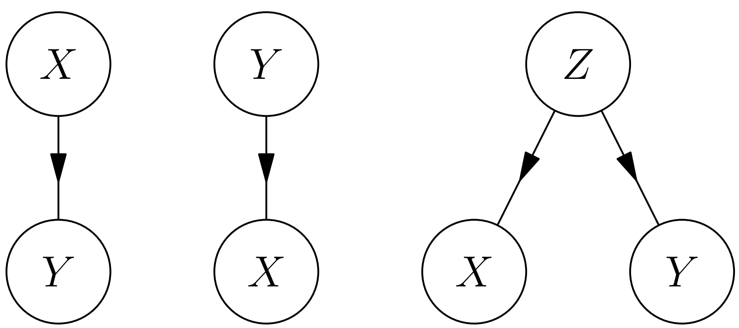
- We could represent the three conditions described above by



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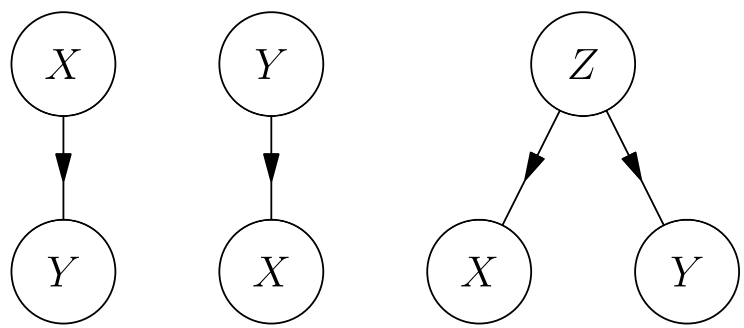

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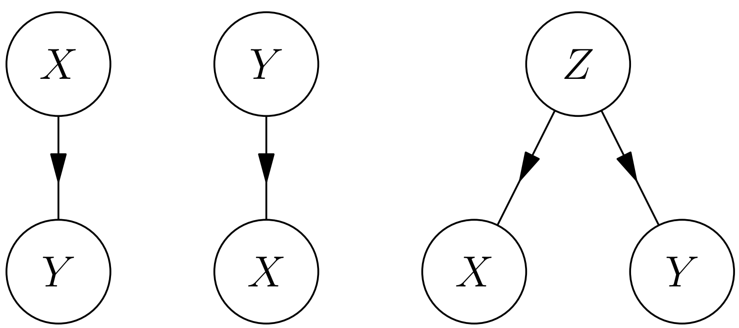

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Statistical Independence

- Two random variables are statistically independent if

$$\mathbb{P}(X,Y) = \mathbb{P}(X)\,\mathbb{P}(Y)$$

- Equally this implies $\mathbb{P}(X|Y) = \mathbb{P}(X)$ and $\mathbb{P}(Y|X) = \mathbb{P}(Y)$
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Conditional Independence

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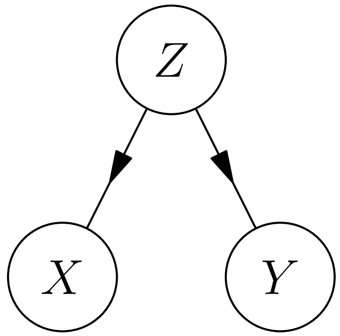

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- But it doesn't imply zero correlation
- Conditional independence reduces computational complexity, e.g.

$$\mathbb{E}[XY] = \sum_{X,Y,Z} X\,Y\,\mathbb{P}(X,Y,Z) = \sum_{Z} P(Z) \left(\sum_{X} X\,P(X|Z) \right) \left(\sum_{Y} Y\,P(Y|Z) \right)$$

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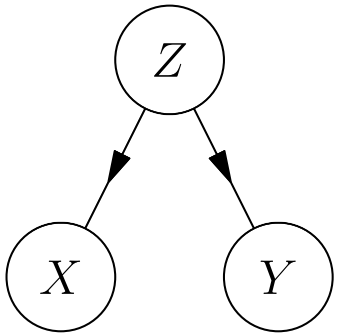

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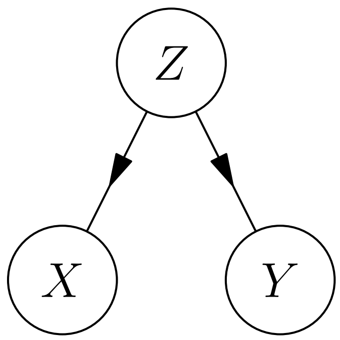

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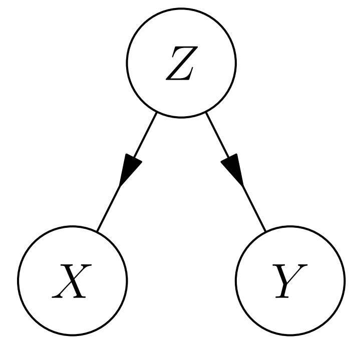

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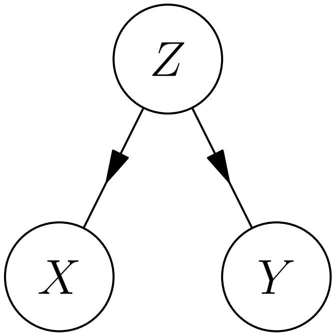

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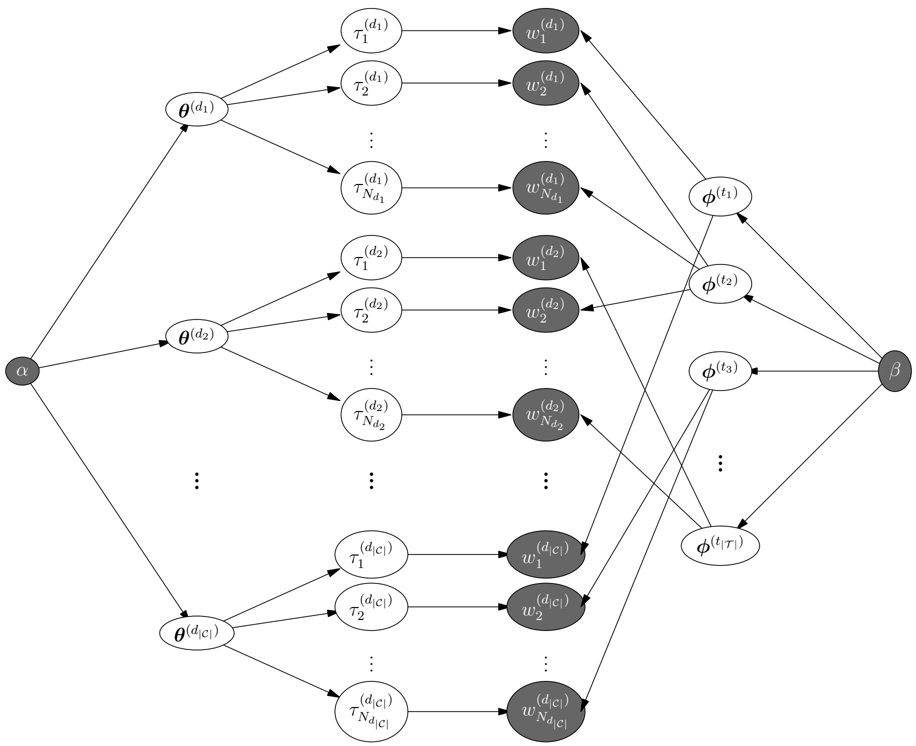

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- It illustrates some simple principles
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- It illustrates some simple principles
- In the subsidiary notes I present a very simple program for computing all the probabilities—I would encourage you to do this as it makes things much clearer

The Cake Scenario

- Abi and Ben both bake cakes and bring them into the coffee room
- Abi will bring in cakes 20% of the time: $\mathbb{P}(A = 1) = 0.2$
- Ben will bring in cakes 10% of the time: $\mathbb{P}(B = 1) = 0.1$
- 90% of the time if either Abi or Ben have put cakes in the coffee room there is some left when I enter
  $\mathbb{P}(C = 1|A = 1, B = 0) = \mathbb{P}(C = 1|A = 0, B = 1) = 0.9$
- If they both make cake then there is always cake left
  $\mathbb{P}(C = 1|A = 1, B = 1) = 1$
- If neither Abi or Ben has made cake there is still a 5% chance someone else has put cake in the coffee room
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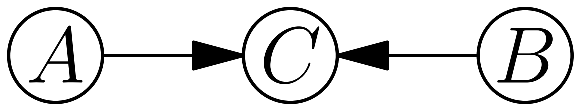
Computing with Probabilities

- Other probabilities I can deduce, e.g.
  $\mathbb{P}(C=0|A,B) = 1 - \mathbb{P}(C=1|A,B)$
- I can depict the causal relationship as



- The quantity that I really want is the joint probability

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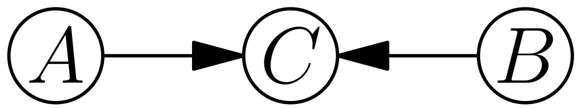

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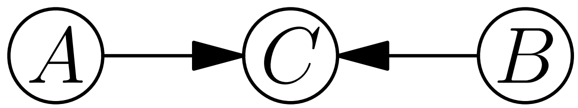

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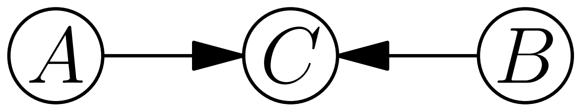

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Computing Expectations

- By using the joint probability and summing over all unknown quantities, we can compute expectations of anything we are interested in

- These sums are often sped up using knowledge of conditional independence

- To compute the probability of and event $\mathcal{E}$ we introduce an indicator function $[\![\mathcal{E}]\!]$ which is equal to 1 if the event happens and 0 otherwise

$$\mathbb{P}(\mathcal{E}) = \mathbb{E}[[\![\mathcal{E}]\!]]$$

- If E is a random variable equal to 1 if event $\mathcal{E}$ happens and 0 otherwise then $E = [\![\mathcal{E}]\!]$

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Are There Any Cakes Left?

- We can use our model to compute the probabilities of there being cakes in the coffee room

$$\mathbb{P}(C=1) = \sum_{A,B,C \in \{0,1\}} [\![C=1]\!]\,\mathbb{P}(A,B,C)$$

$$= \sum_{A,B \in \{0,1\}} \mathbb{P}(C=1|A,B)\,\mathbb{P}(A)\,\mathbb{P}(B) = 0.303$$

- The probability that Abi baked a cake is just 0.2 and for Ben its 0.1 (which is what we assume at the start)

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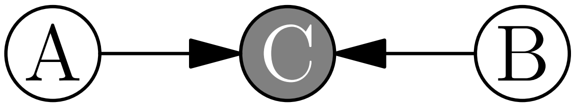
- In graphical models observed random variables are shaded



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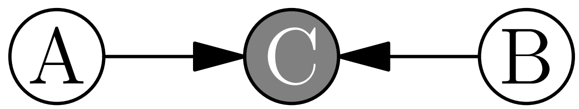

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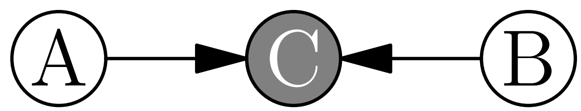

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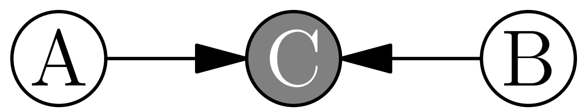

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$$\mathbb{P}(A=1|C=1) = 0.630, \quad \mathbb{P}(B=1|C=1) = 0.317$$
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Elaborate Cakes

- We can elaborate on our cake model
- We suppose that Dave likes cakes so if there is a cake in the coffee room there is a 80% chance that I will see him eating a cake: $\mathbb{P}(D = 1|C = 1) = 0.8$
- Even if there are no cakes in the coffee room there is a 10% chance that Dave has bought his own cake: $\mathbb{P}(D = 1|C = 0) = 0.1$
- Eli also likes cakes: there is a 60% chance that I will see her eating cakes if there are cakes in the coffee room: $\mathbb{P}(E = 1|C = 1) = 0.6$
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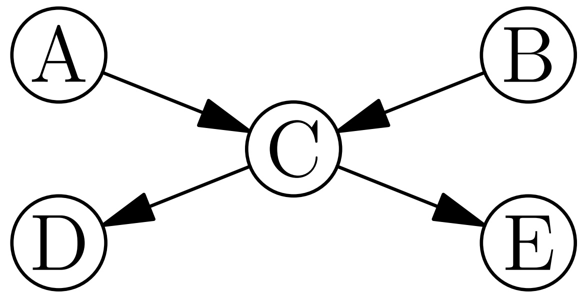

- This allows us to break down the joint probability

$$\begin{aligned}\mathbb{P}(A,B,C,D,E) &= \mathbb{P}(C,D,E|A,B)\,\mathbb{P}(B)\,\mathbb{P}(A)\\ &= \mathbb{P}(D|C)\,\mathbb{P}(E|C)\,\mathbb{P}(C|A,B)\,\mathbb{P}(B)\,\mathbb{P}(A)\end{aligned}$$

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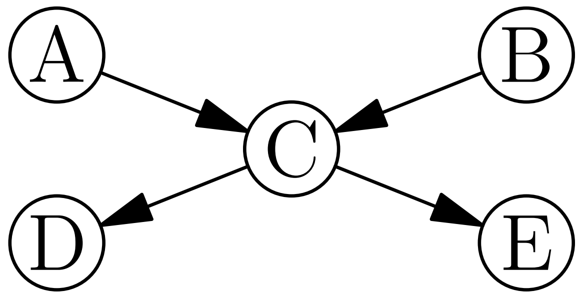

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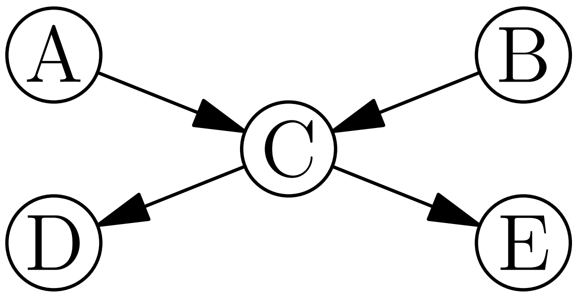

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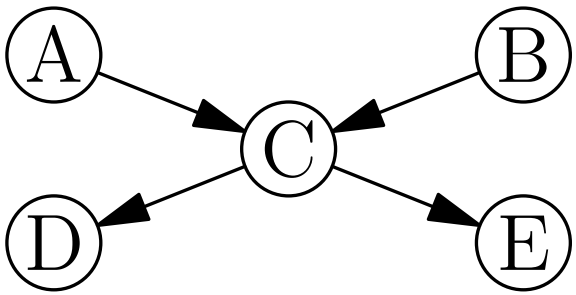

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Dependencies

- If we don't observe cakes then the probability of Dave and Eli eating cake are not independent

$$\mathbb{P}(D=1) = 0.3121, \qquad \mathbb{P}(E=1) = 0.1818$$
$$\mathbb{P}(D=1, E=1) = 0.14544$$

- This changes if we know there are cakes in the coffee room

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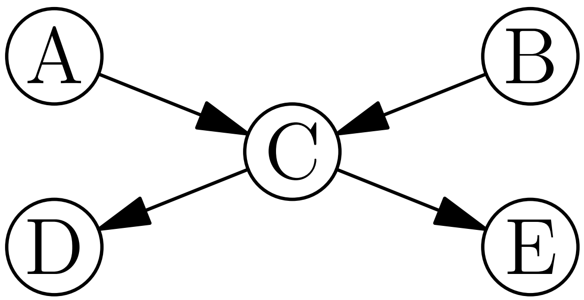

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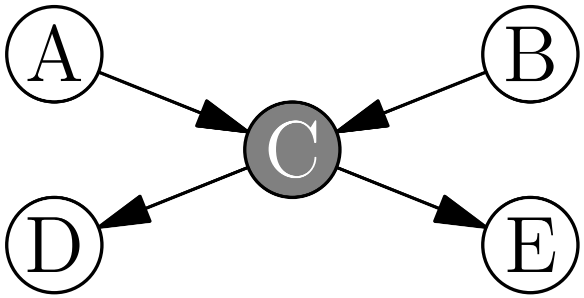

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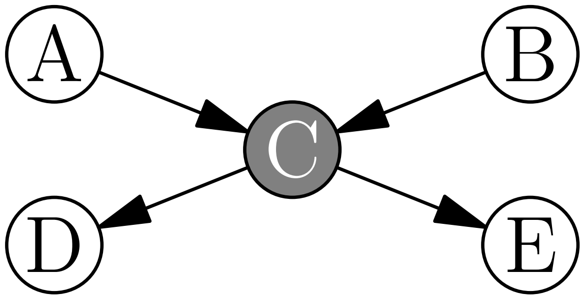

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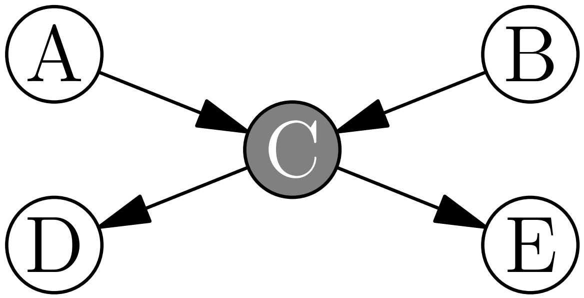

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- There are rules to deduce the conditional independence from a graphical model given which variables have been observed—but these are details that you can look up if needed

Graphical Model Frameworks

- There are sophisticated frameworks for computing probabilities in Bayesian Belief Networks efficiently
- If our graph is a tree then we can evaluate probabilities efficiently
- When there are loops (so that a random variable both influences and is influenced by another random variables) then exact evaluation of expectations requires exhaustive summing over variables
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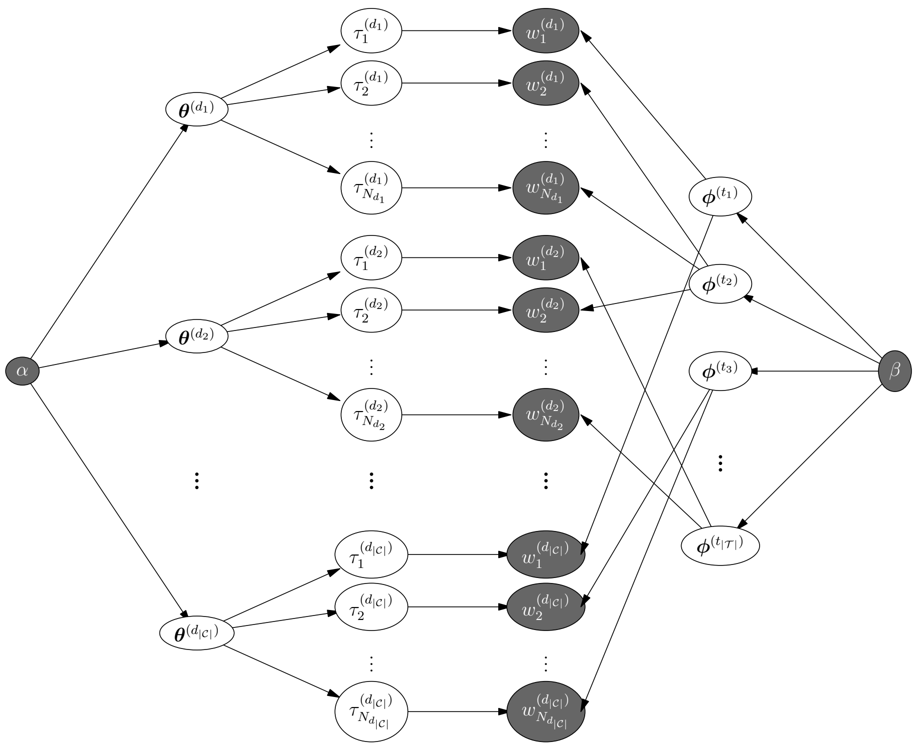
Outline

1. Graphical Models
2. Cakes!
3. **Latent Dirichlet Allocation**



Model for Documents

- We consider a model for the words in a set of documents (we ignore word order)
- We consider a corpus $\mathcal{C} = \{d_i | i = 1, 2, \dots |\mathcal{C}|\}$
- With documents consisting of words

$$d = \left(w_1^{(d)}, w_2^{(d)}, \dots, w_{N_d}^{(d)} \right)$$

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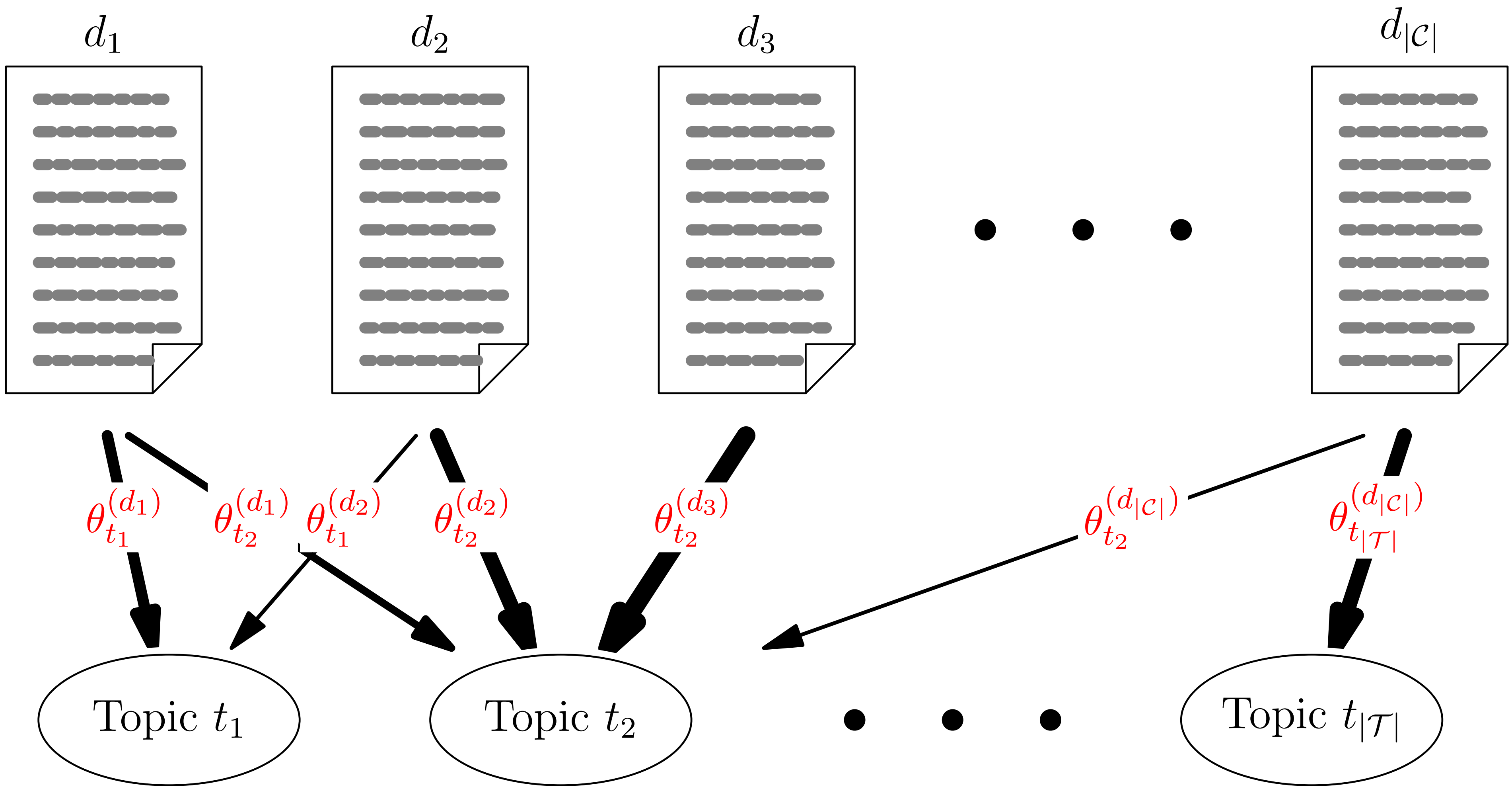
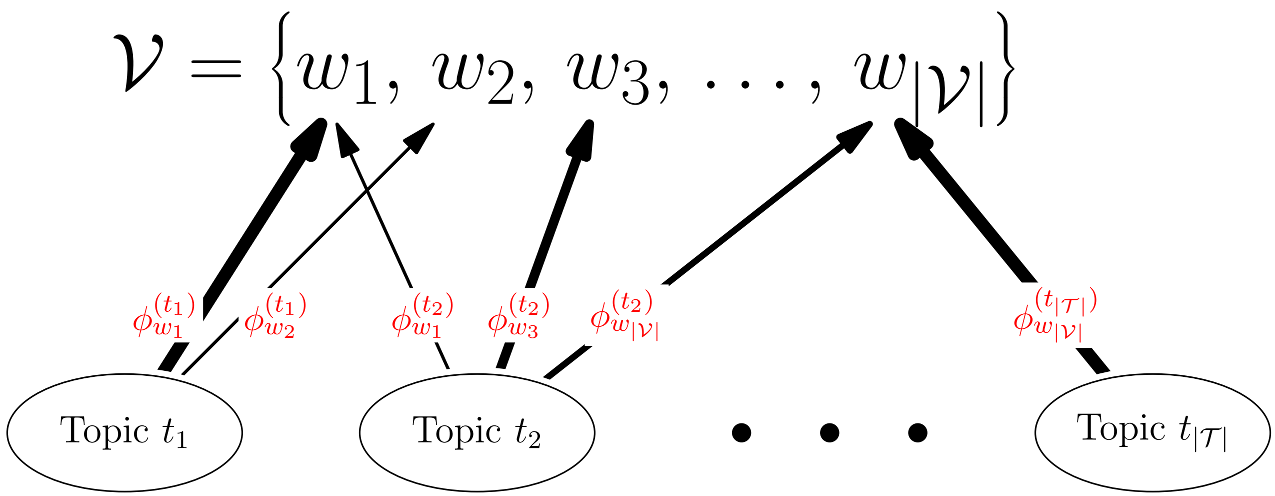


$$\boldsymbol{\theta}^{(d)} = (\theta_t^{(d)} | t \in \mathcal{T}) \qquad \theta_t^{(d)} \geq 0 \qquad \sum_{t=1}^{|\mathcal{T}|} \theta_t^{(d)} = 1$$

Words and Topic

- We associate a probability $\phi_w^{(t)}$ that a word, w, is related to a topic t



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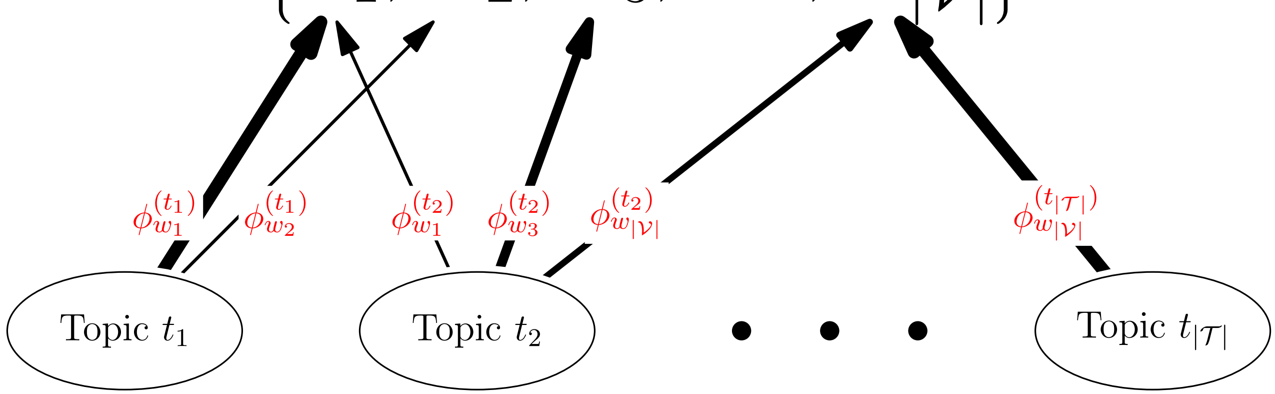

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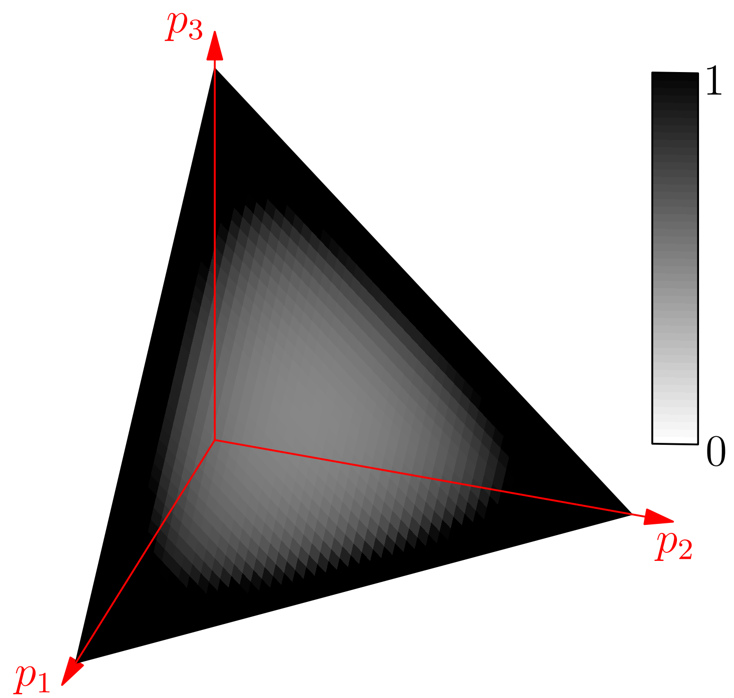
Dirichlet Allocation

- Most documents are predominantly about a few topics and most topic have a small number of words associated to them

- We can generate sparse vectors $\boldsymbol{\theta}^{(d)}$ and $\boldsymbol{\phi}^{(t)}$ from a Dirichlet distribution with small parameters $\boldsymbol{\alpha}$

$$\mathrm{Dir}(\boldsymbol{p}|\boldsymbol{\alpha}) = \Gamma\left(\sum_i \alpha_i\right) \prod_{i=1}^{n} \frac{p_i^{\alpha_i - 1}}{\Gamma(\alpha_i)}$$

- $\sum_i p_i = 1$



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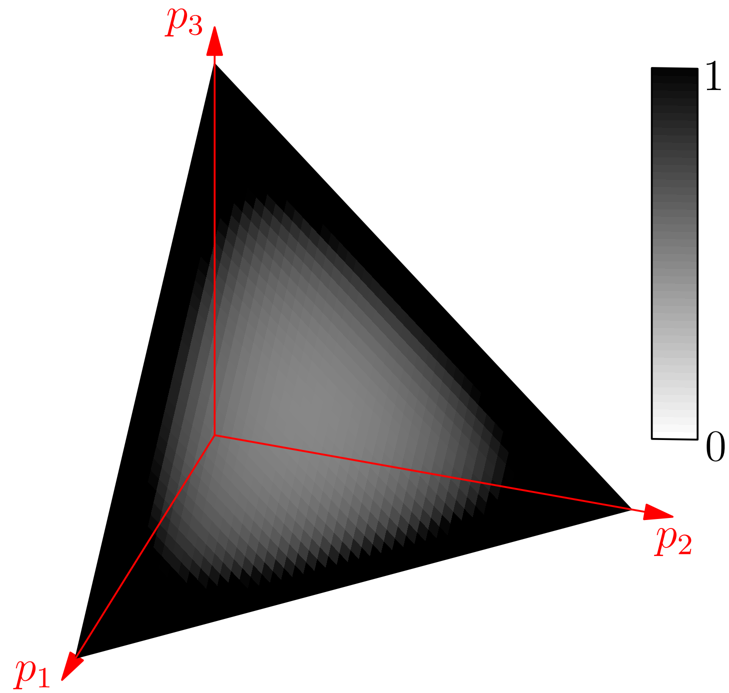

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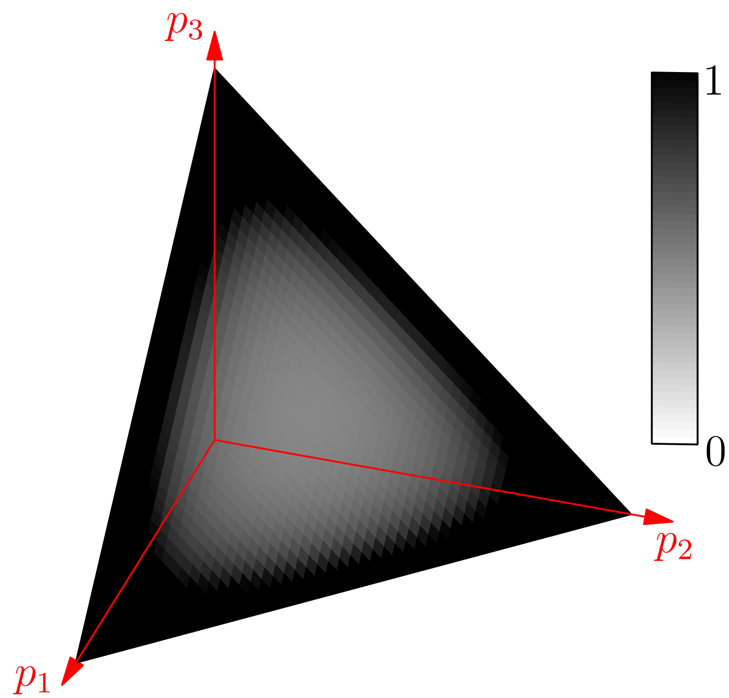

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- To generate a document we choose a topic for each word and a word for each topic

$$\begin{aligned} \forall d \in \mathcal{C} \quad & \boldsymbol{\theta}^{(d)} \sim \mathrm{Dir}(\alpha \mathbf{1}) \\ \forall t \in \mathcal{T} \quad & \boldsymbol{\phi}^{(t)} \sim \mathrm{Dir}(\beta \mathbf{1}) \\ \forall d \in \mathcal{C} \ \wedge \ \forall i \in \{1, 2, \ldots, N_d\} \quad & \tau_i^{(d)} \sim \mathrm{Cat}(\boldsymbol{\theta}^{(d)}), \ w_i^{(d)} \sim \mathrm{Cat}(\boldsymbol{\phi}^{(\tau_i^{(d)})}) \end{aligned}$$

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- Where $\mathrm{Cat}(i|\boldsymbol{p}) = p_i$ is the categorical distribution (we choose one of a number of options)

- This model is known as **Latent Dirichlet Allocation**

Generating Document

- To generate a document we choose a topic for each word and a word for each topic

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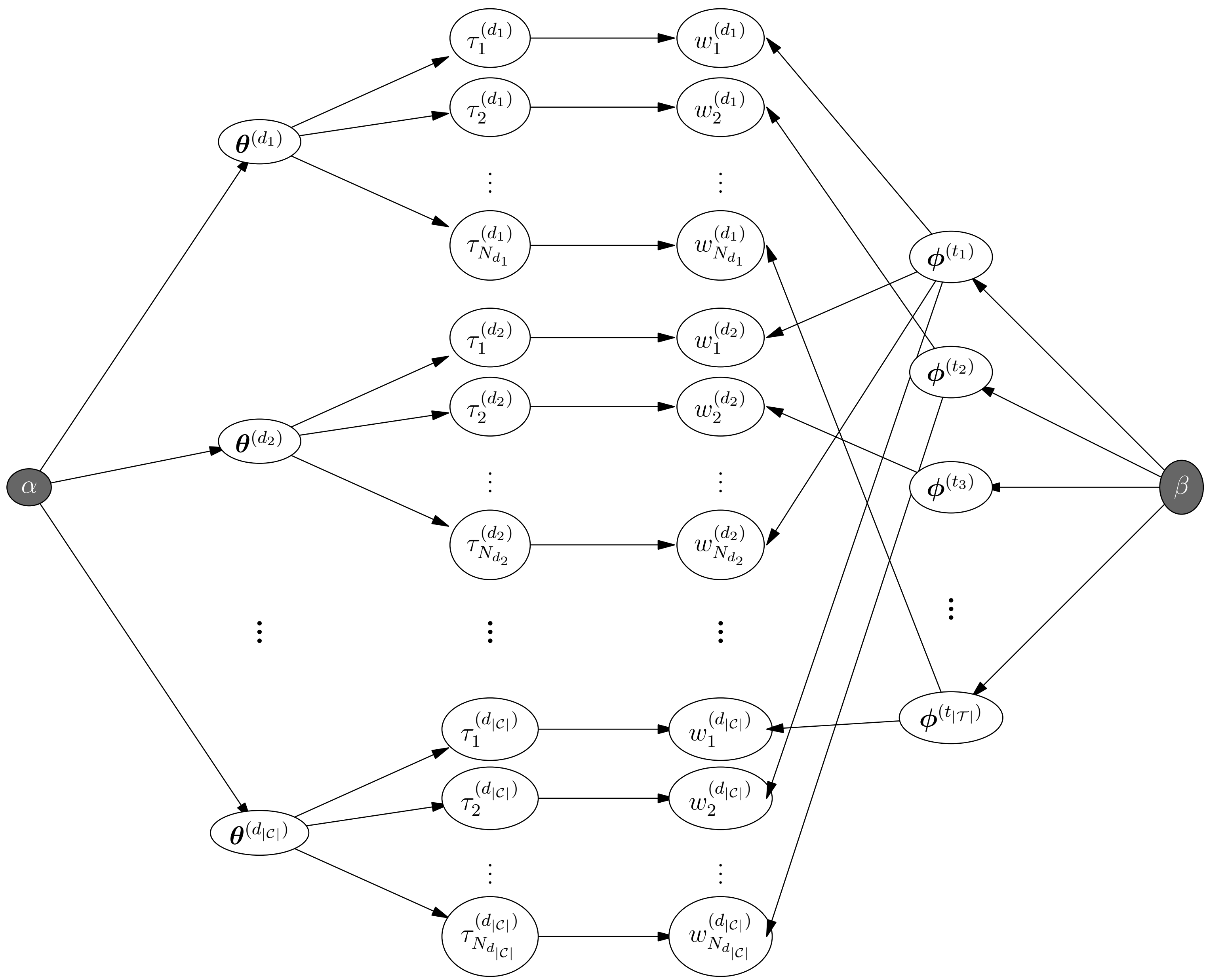

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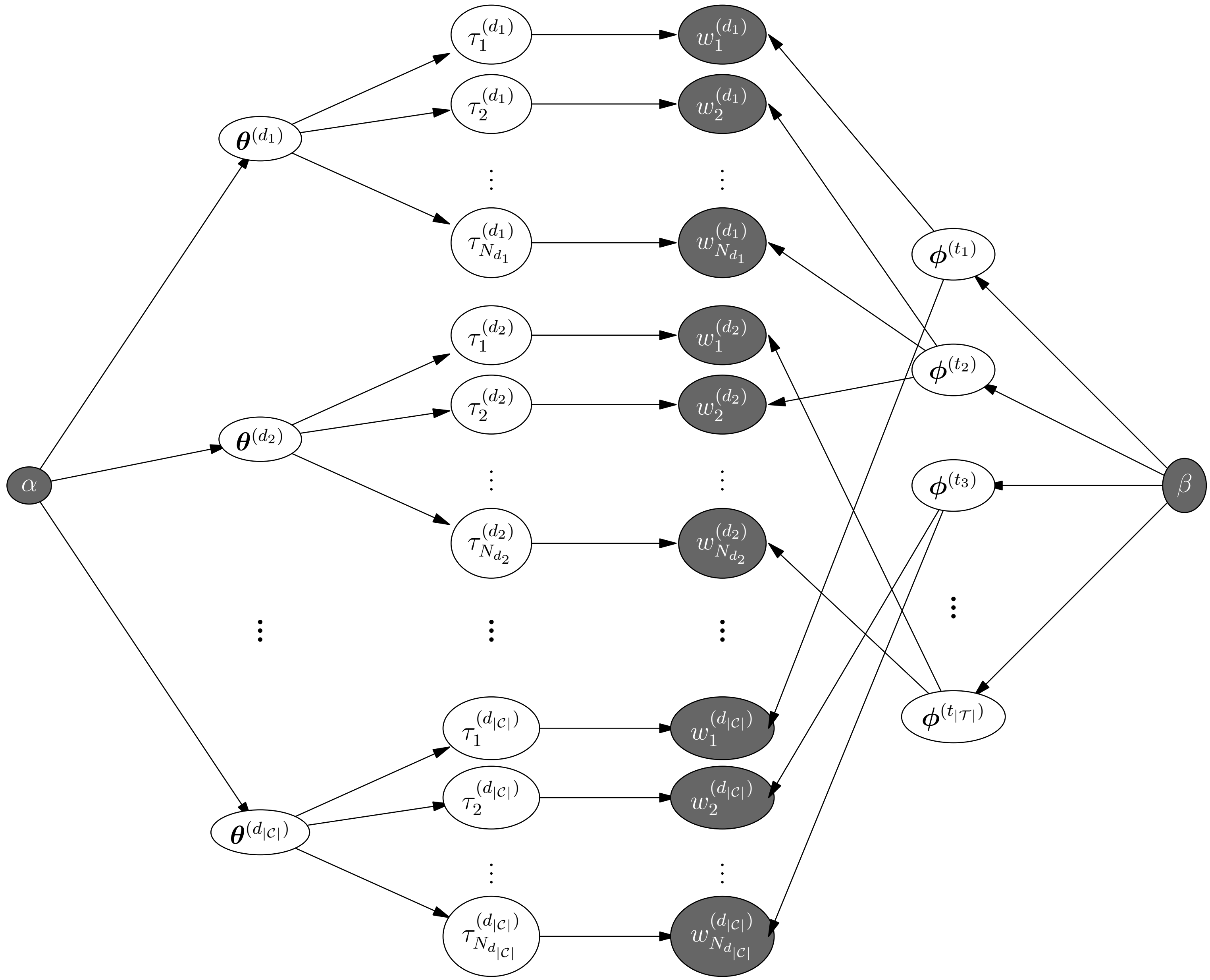
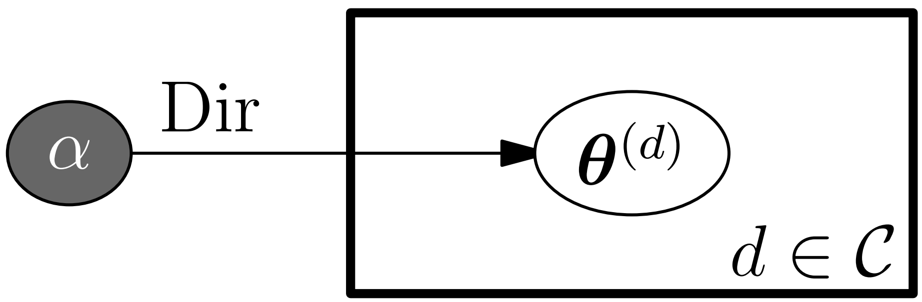


Plate Diagrams

- Drawing every random variable is tedious (and not really possible)
- A short-hand is to draw a box (plate) meaning repeat



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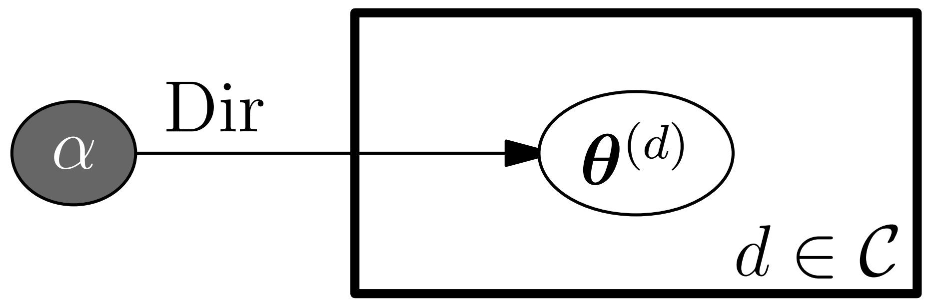

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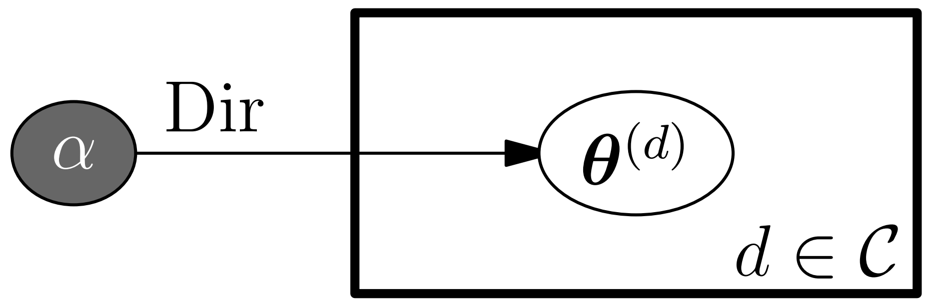

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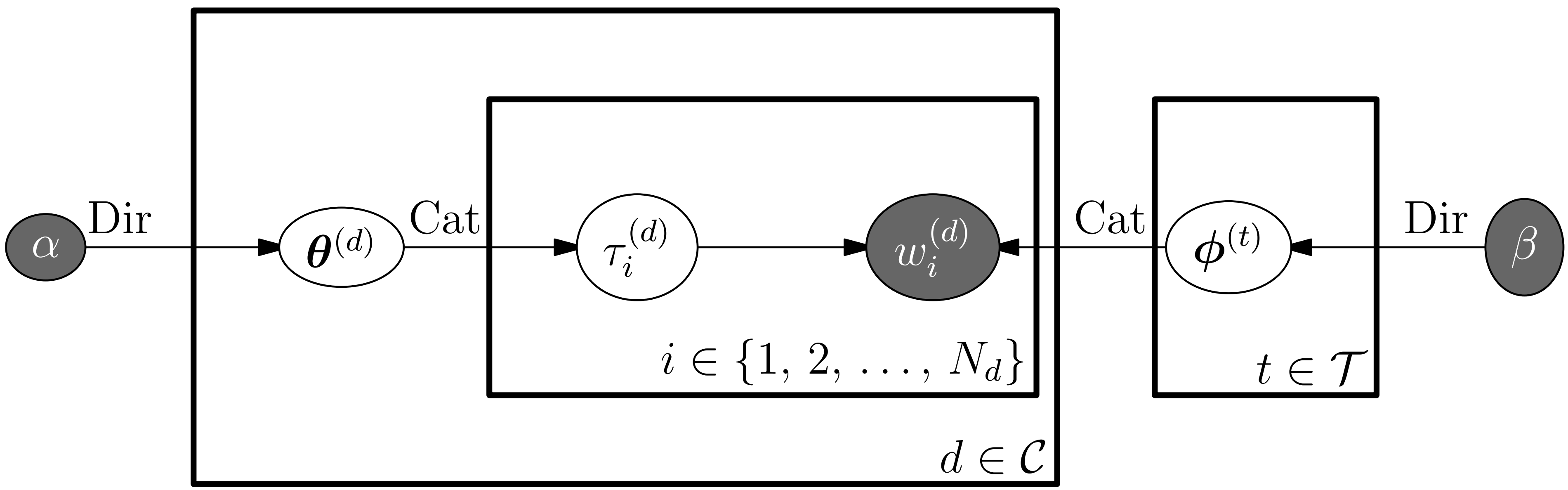

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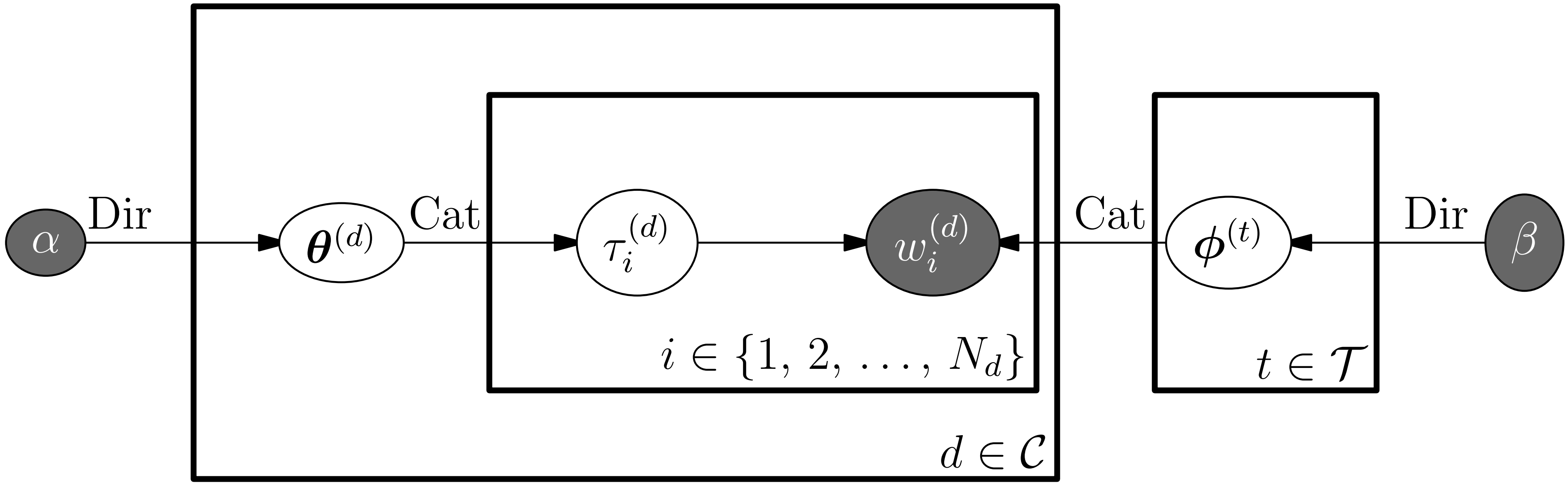

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Probabilistic Model

- The graphical Model is shorthand for the variables

$$\boldsymbol{W} = (\boldsymbol{w}^{(d)} | d \in \mathcal{C}) \quad \text{with} \quad \boldsymbol{w}^{(d)} = (w_1^{(d)}, w_2^{(d)}, \dots, w_{N_d}^{(d)}), \quad \text{and} \quad w_i^{(d)} \in \mathcal{V}$$

$$\boldsymbol{T} = (\tau_i^{(d)} | d \in \mathcal{C} \wedge i \in \{1, 2, \dots, N_d\}) \quad \text{with} \quad \tau_i^{(d)} \in \mathcal{T}$$

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- Distributed according to

$$\mathbb{P}\left(\boldsymbol{W}, \boldsymbol{T}, \boldsymbol{\Theta}, \boldsymbol{\Phi} \middle| \alpha, \beta\right) = \left(\prod_{t \in \mathcal{T}} \mathrm{Dir}\left(\boldsymbol{\phi}^{(t)} \middle| \beta \mathbf{1}\right)\right)$$

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Finding Topics

- We are given the set of words $\boldsymbol{W}$ and don't really care about τ_i^d the topic associated with word i in document d
- But we are interested in the words associated with each topic $\boldsymbol{\phi}^{(t_i)}$
- And the topics associated with each document $\boldsymbol{\theta}^{(d)}$
- To compute them we need to sample the probability distribution
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Summary

- Building probabilistic models is an intricate process
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- This allows us to break down the joint probability of all the variables into conditional probabilities
- This is useful for building the model, but also can speed up evaluating expectations
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