Name:	Student ID:

BIAS VARIANCE PROBLEM SHEET

When modelling systems with uncertainty it is convenient to define *random variables*. The are numbers that we associate with the outcome of some stochastic event. We associate a probability (or probability density) with the set out outcomes such that the random variables take a particular value. We often write random variables using capital letters (e.g. X) while the actual values the X takes we write with small letters x. Thus $\mathbb{P}(X = x)$ is the probability that the random value, X, takes value, x. We write non-random variables (scalars) with a small letter (e.g. c). Note that for most continuous random variables $\mathbb{P}(X = x) = 0$ so instead we define the probability density

$$f_X(x) = \lim_{\delta x \to 0} \frac{\mathbb{P}(x \le X \le x + \delta x)}{\delta x}.$$

The expectation (or average value) of some function, g, of X is written as

$$\mathbb{E}_X\left[g(X)\right] = \begin{cases} \sum_{x \in \mathcal{X}} \mathbb{P}(X = x)\, g(x) \\ \int f_X(x) g(x) \mathrm{d}x \end{cases}$$

depending on whether X is a continuous or discrete random variable. Note $\mathcal{X}$ is the possible values that the random variable can take. When it is clear what random variables we are taking expectation with respect to then we will often write $\mathbb{E}\left[\cdot\right]$ for $\mathbb{E}_X\left[\cdot\right]$.

1

(a) Let X be outcome of an honest dice ($\mathcal{X} = \{1,2,3,4,5,6\}$). What is

(i) $\mathbb{E}\left[X\right]$

(ii) $\mathbb{E}\left[2X\right]$

(iii) $\mathbb{E}\left[X^2\right]$

[3 marks]

i

ii

iii

$\overline{3}$

(b) Let X be a random variable as before and Y be a random variable for a second independent dice. What is

(i) $\mathbb{E}_X\left[X+Y\right]$

(ii) $\mathbb{E}_{X,Y}\left[X+Y\right]$

(iii) $\mathbb{E}_X\left[XY\right]$

(iv) $\mathbb{E}_{X,Y}\left[XY\right]$

[4 marks]

i ______

ii ______

iii ______

iv ______

$\overline{}$
4

(c) Let X be the random variable as before and E be a random variable equal to 0 if X is odd and 1 if X is even. Note that E is not independent of X. What is

(i) $\mathbb{P}(E=1)(=\mathbb{E}_X\left[E\right])$

(ii) $\mathbb{E}_X\left[X+E\right]$

(iii) $\mathbb{E}_X\left[XE\right]$

[3 marks]

i ______

ii ______

iii ______

$\overline{}$
3

End of question 1

(a) $\frac{}{3}$ (b) $\frac{}{4}$ (c) $\frac{}{3}$ Total $\frac{}{10}$

