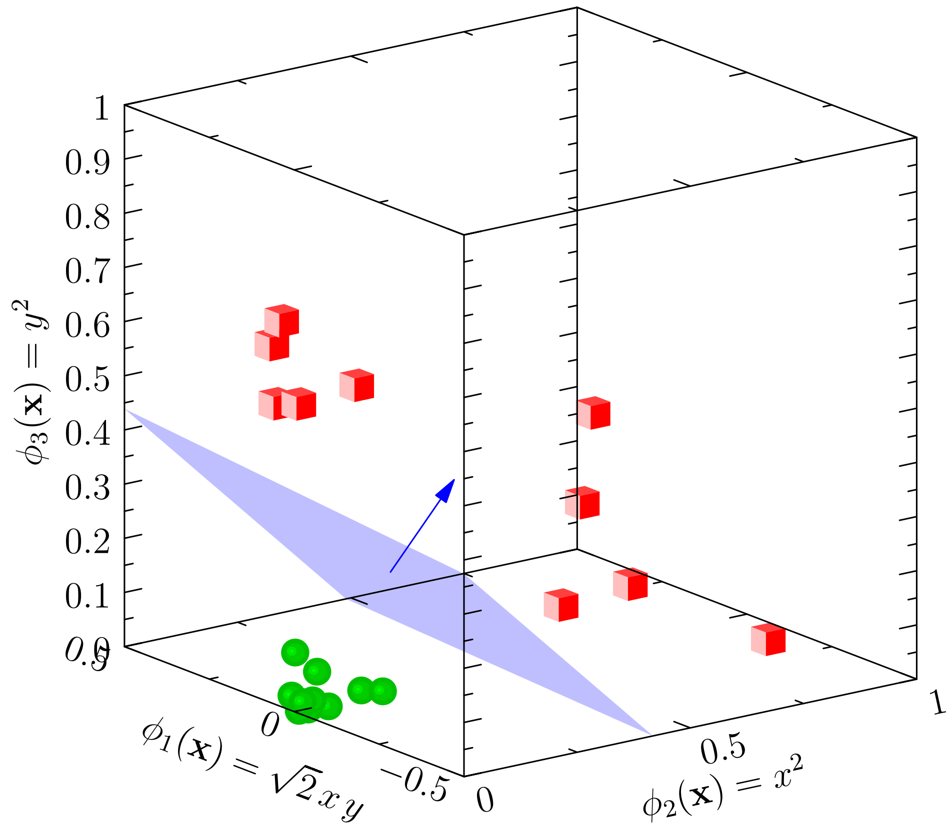
Advanced Machine Learning

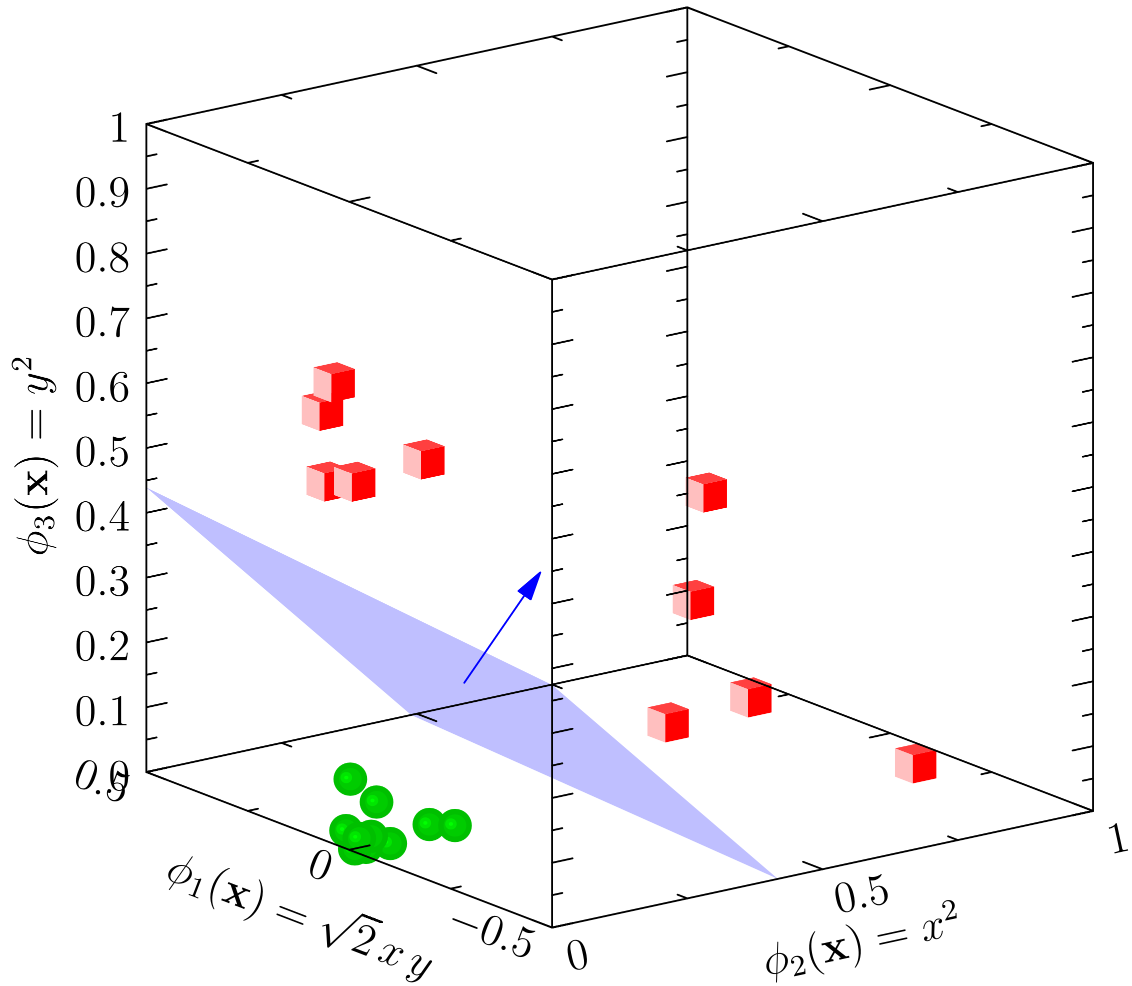
Support Vector Machines



Support Vector Machines, maximum margins

Outline

1. **The Big Picture**
2. Maximum Margins
3. Duality
4. Practice



Support Vector Machines

- Support vector machines, when used right, often have the best generalisation results
- They are typically used on numerical data, but can and have been adapted to text, sequences, etc.
- Although not as trendy as deep learning, they will often be the method of choice on small data sets
- They subtly regularise themselves, choosing a solution that generalises well from a host of different solutions

Support Vector Machines

- Support vector machines, when used right, often have the best generalisation results
- They are typically used on numerical data, but can and have been adapted to text, sequences, etc.
- Although not as trendy as deep learning, they will often be the method of choice on small data sets
- They subtly regularise themselves, choosing a solution that generalises well from a host of different solutions

Support Vector Machines

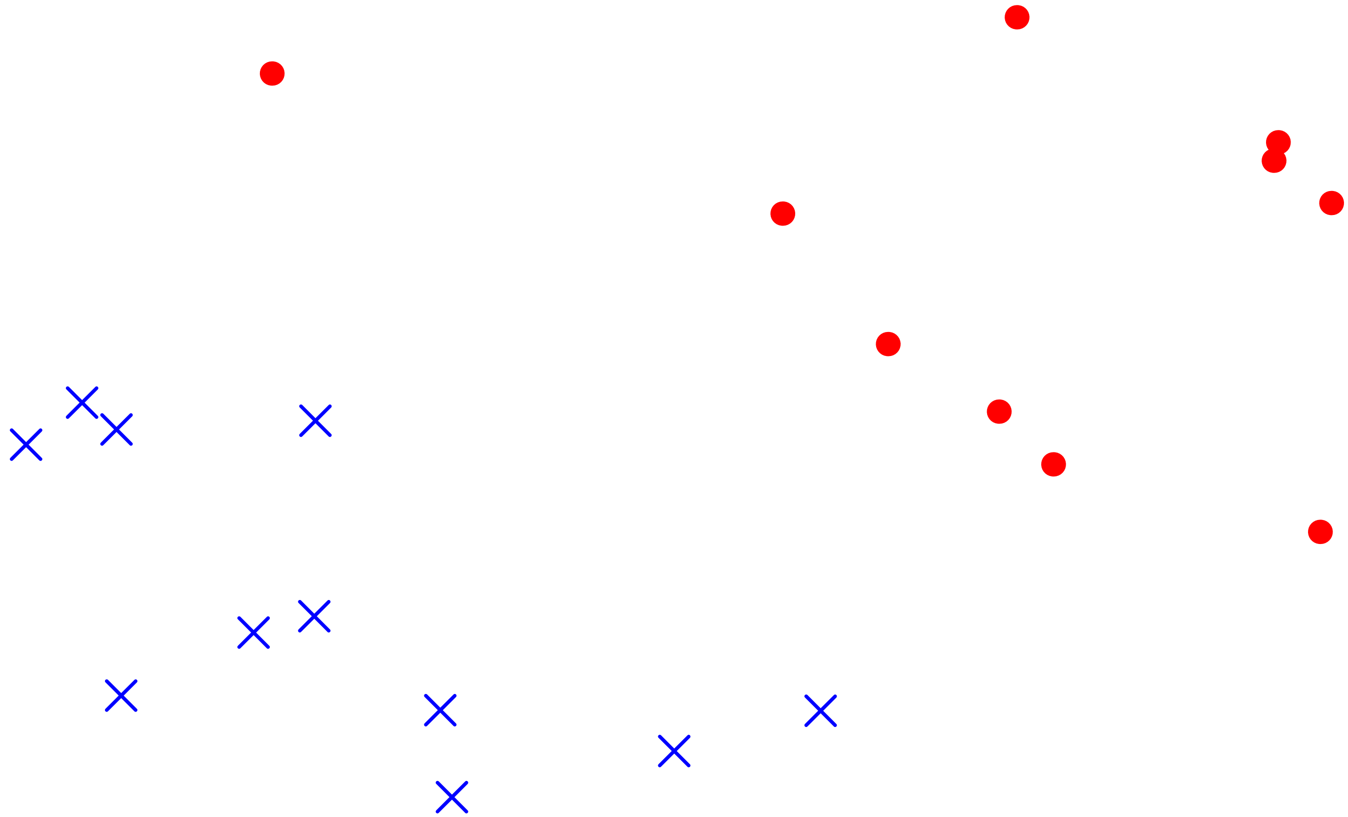
- Support vector machines, when used right, often have the best generalisation results
- They are typically used on numerical data, but can and have been adapted to text, sequences, etc.
- Although not as trendy as deep learning, they will often be the method of choice on small data sets
- They subtly regularise themselves, choosing a solution that generalises well from a host of different solutions

Support Vector Machines

- Support vector machines, when used right, often have the best generalisation results
- They are typically used on numerical data, but can and have been adapted to text, sequences, etc.
- Although not as trendy as deep learning, they will often be the method of choice on small data sets
- They subtly regularise themselves, choosing a solution that generalises well from a host of different solutions

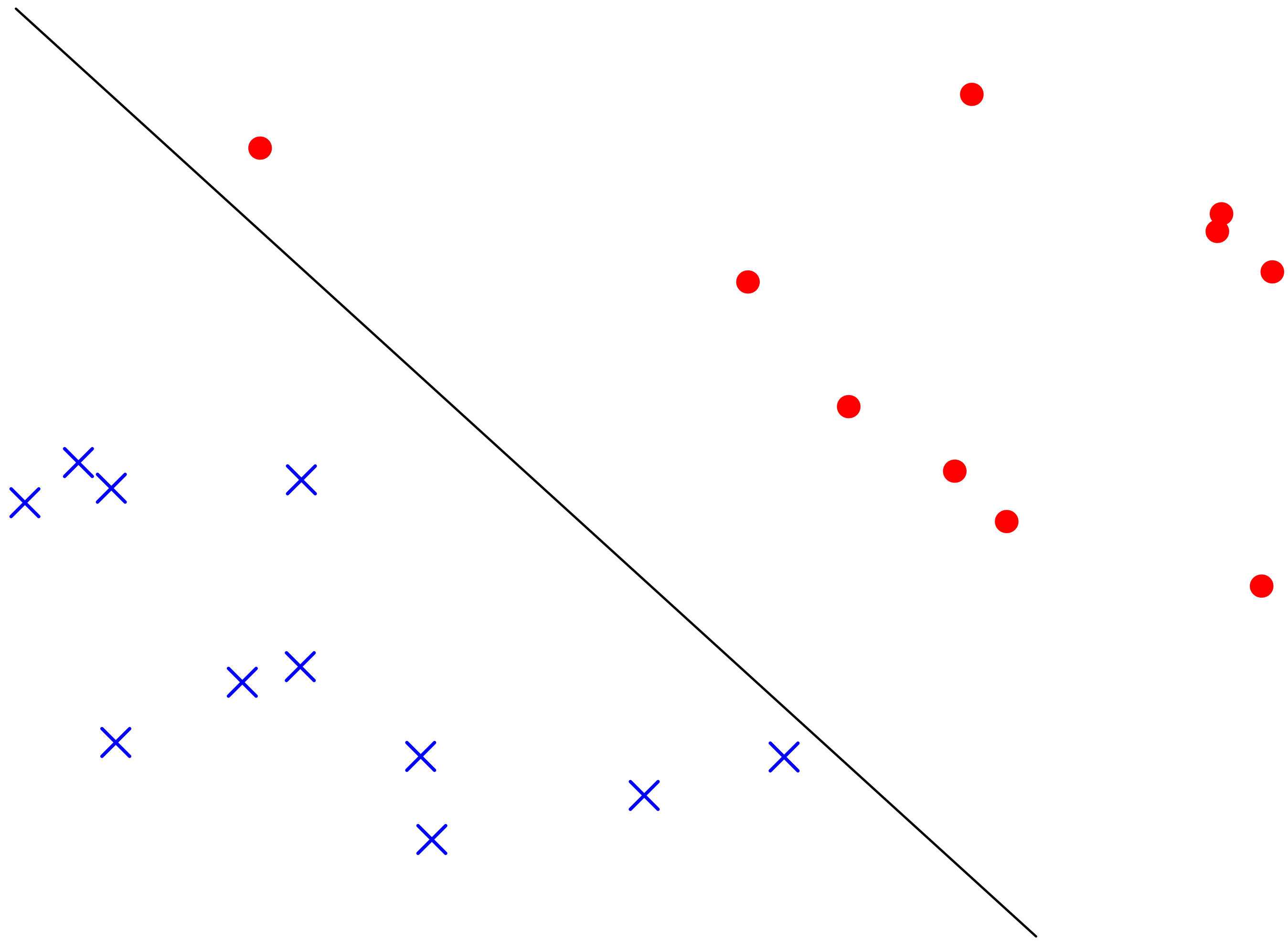
Linear Separation of Data

- SVMs classify linearly separable data



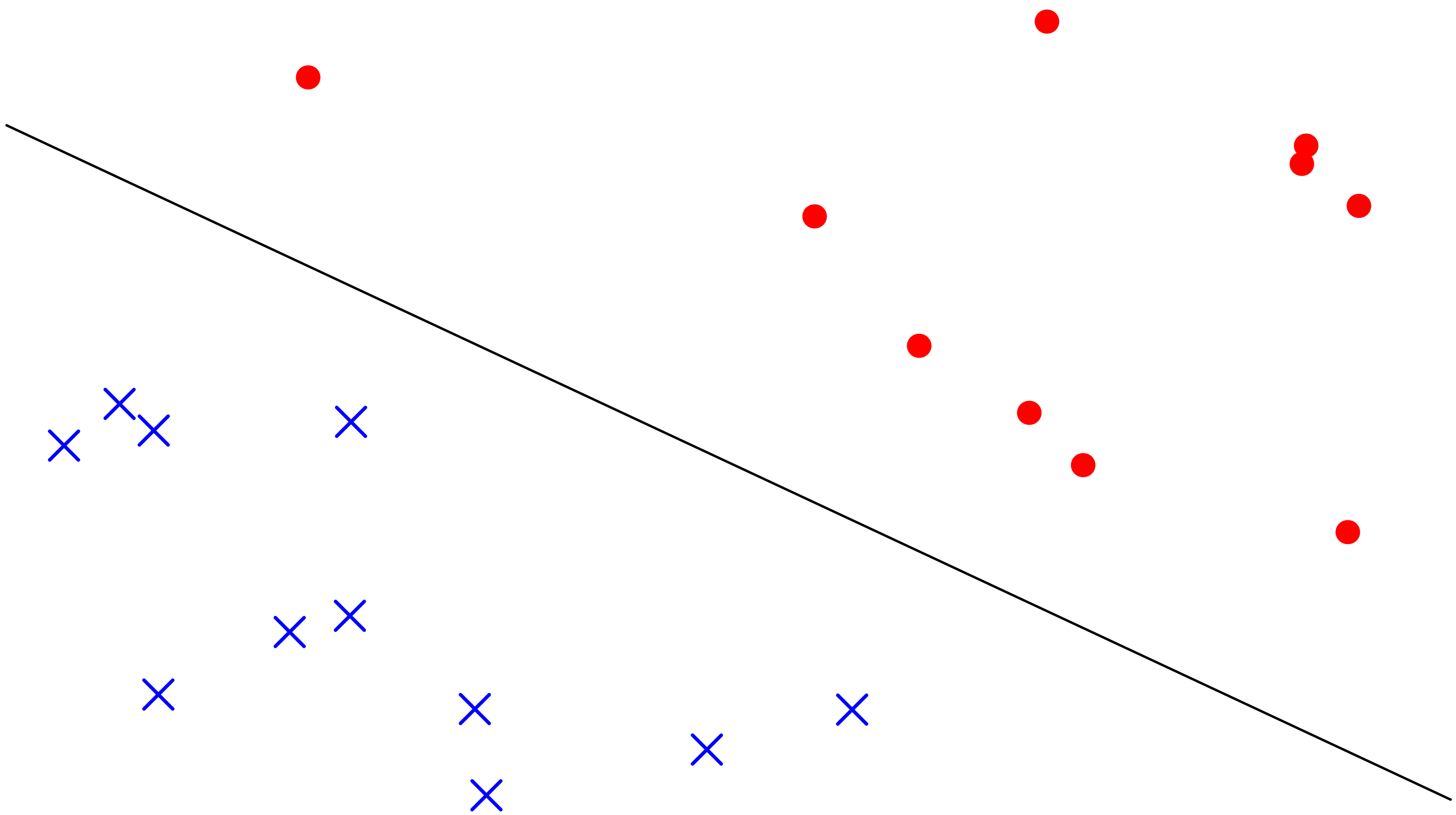
Linear Separation of Data

- SVMs classify linearly separable data



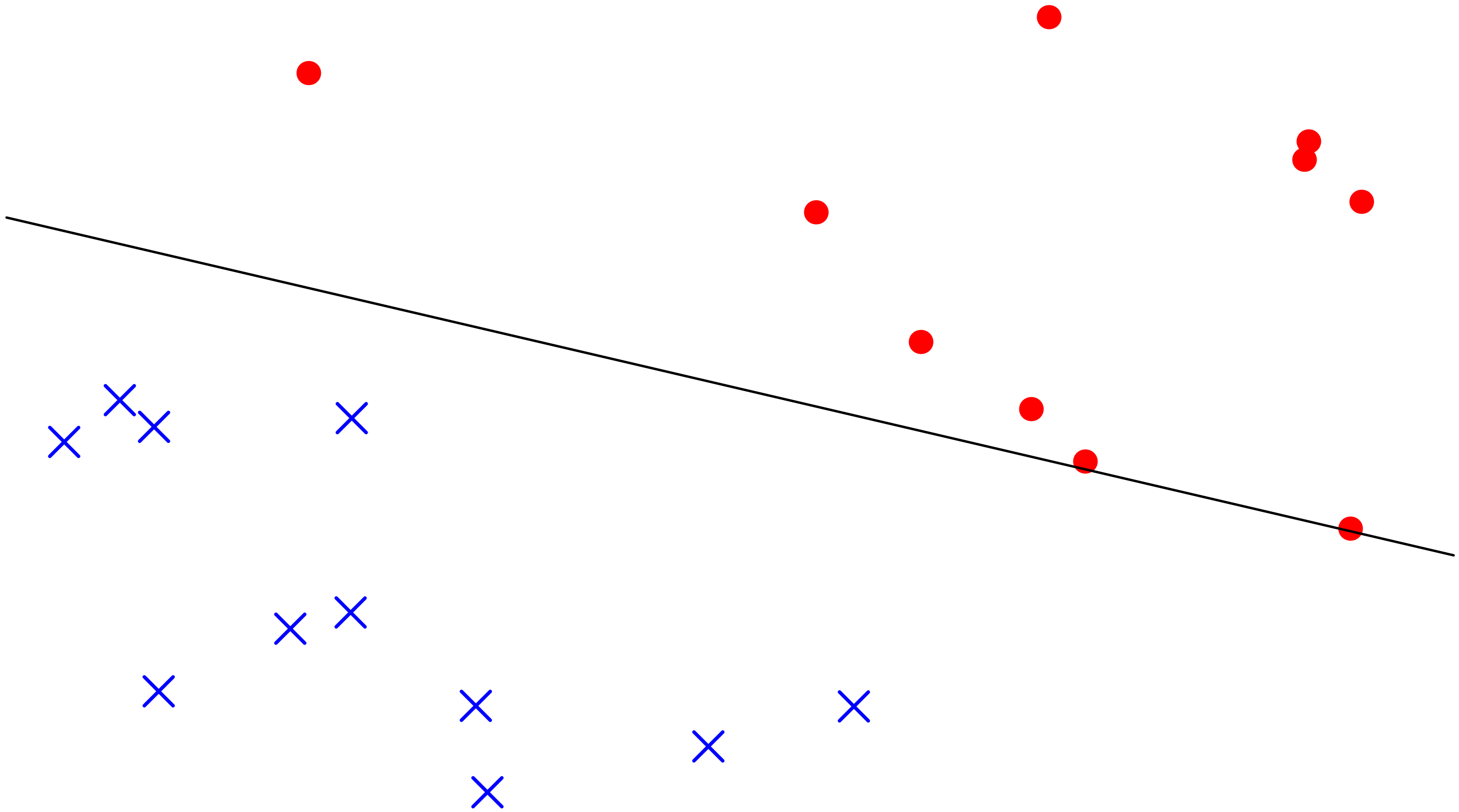
Linear Separation of Data

- SVMs classify linearly separable data



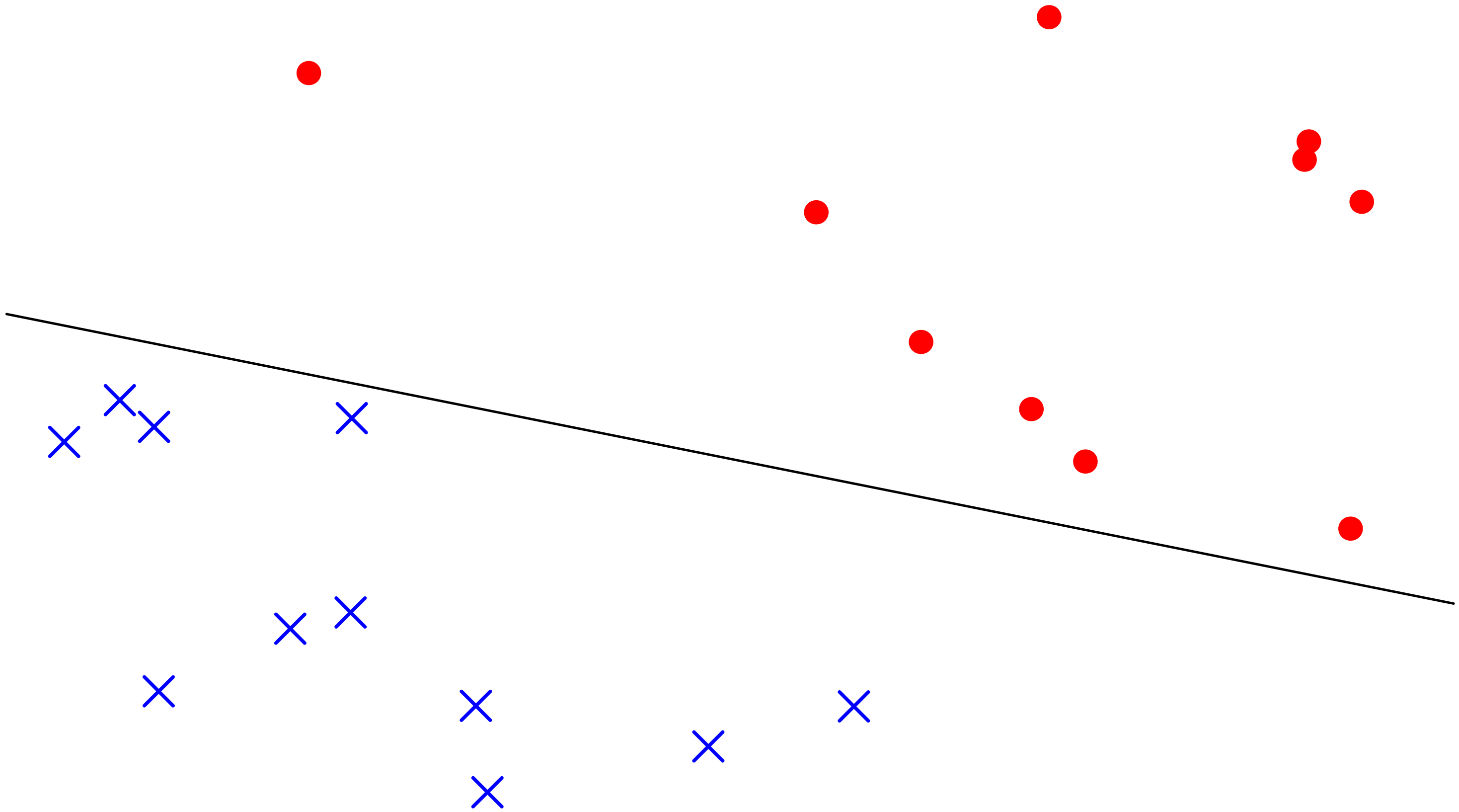
Linear Separation of Data

- SVMs classify linearly separable data



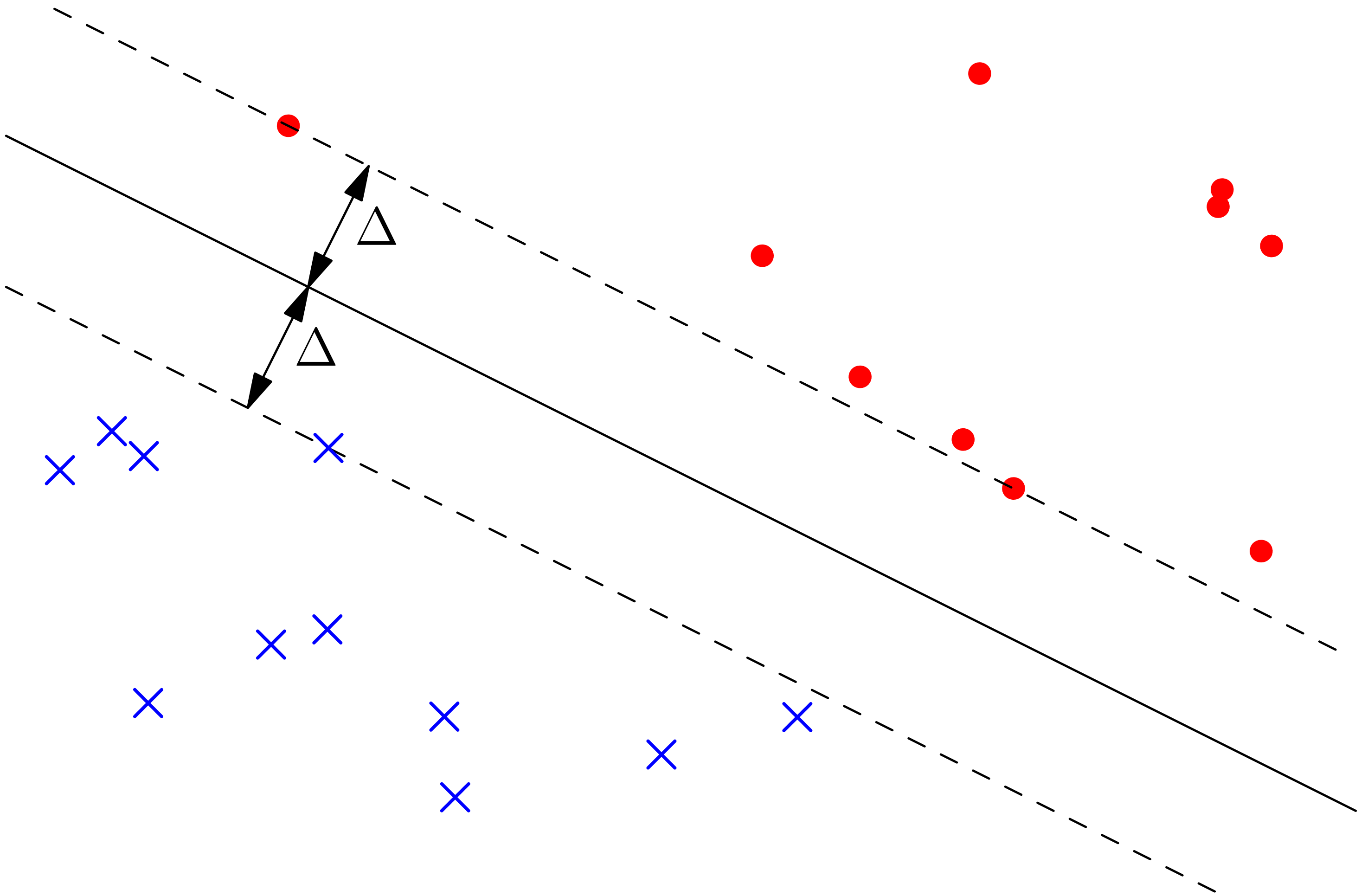
Linear Separation of Data

- SVMs classify linearly separable data



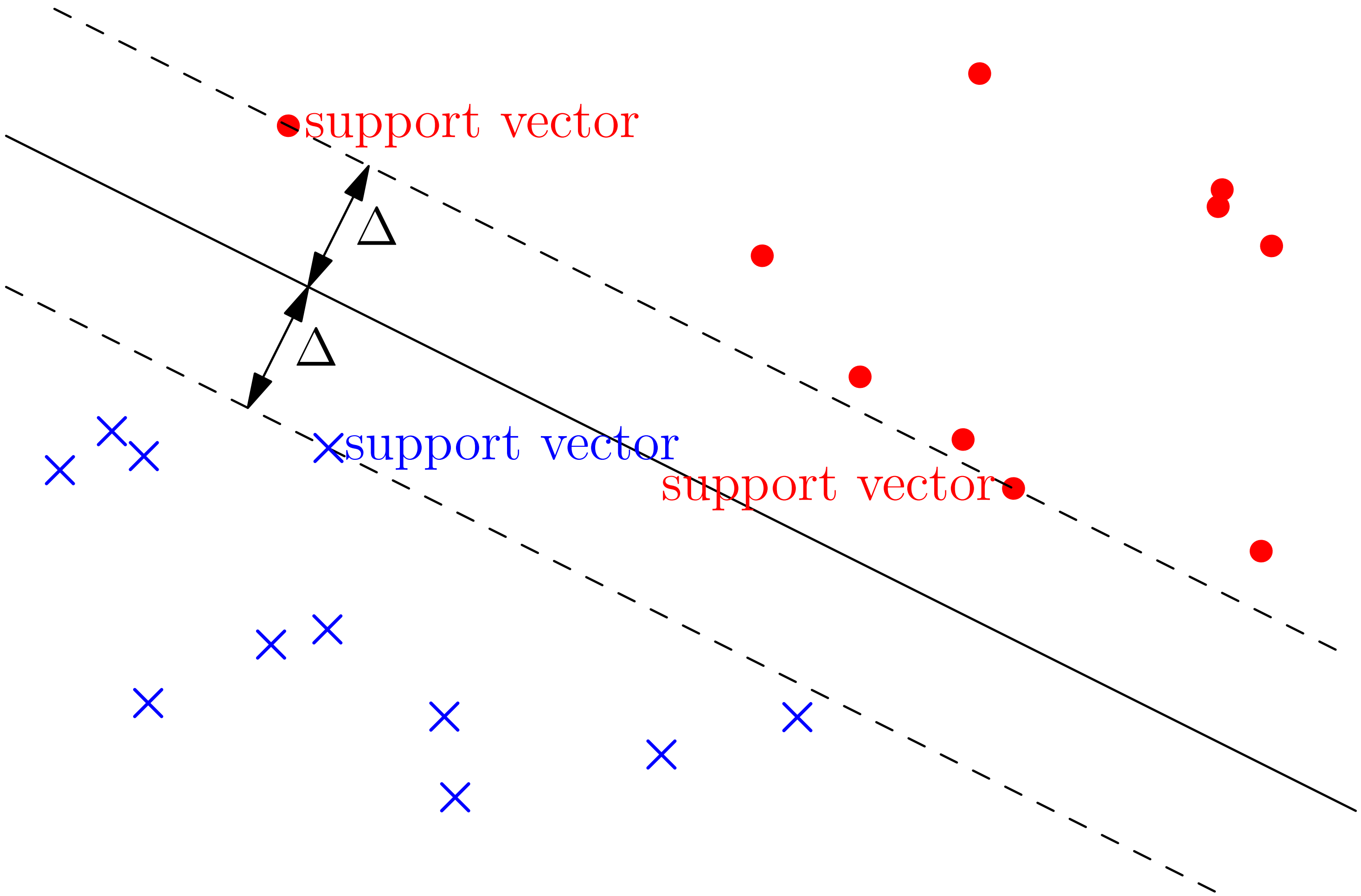
Linear Separation of Data

- SVMs classify linearly separable data



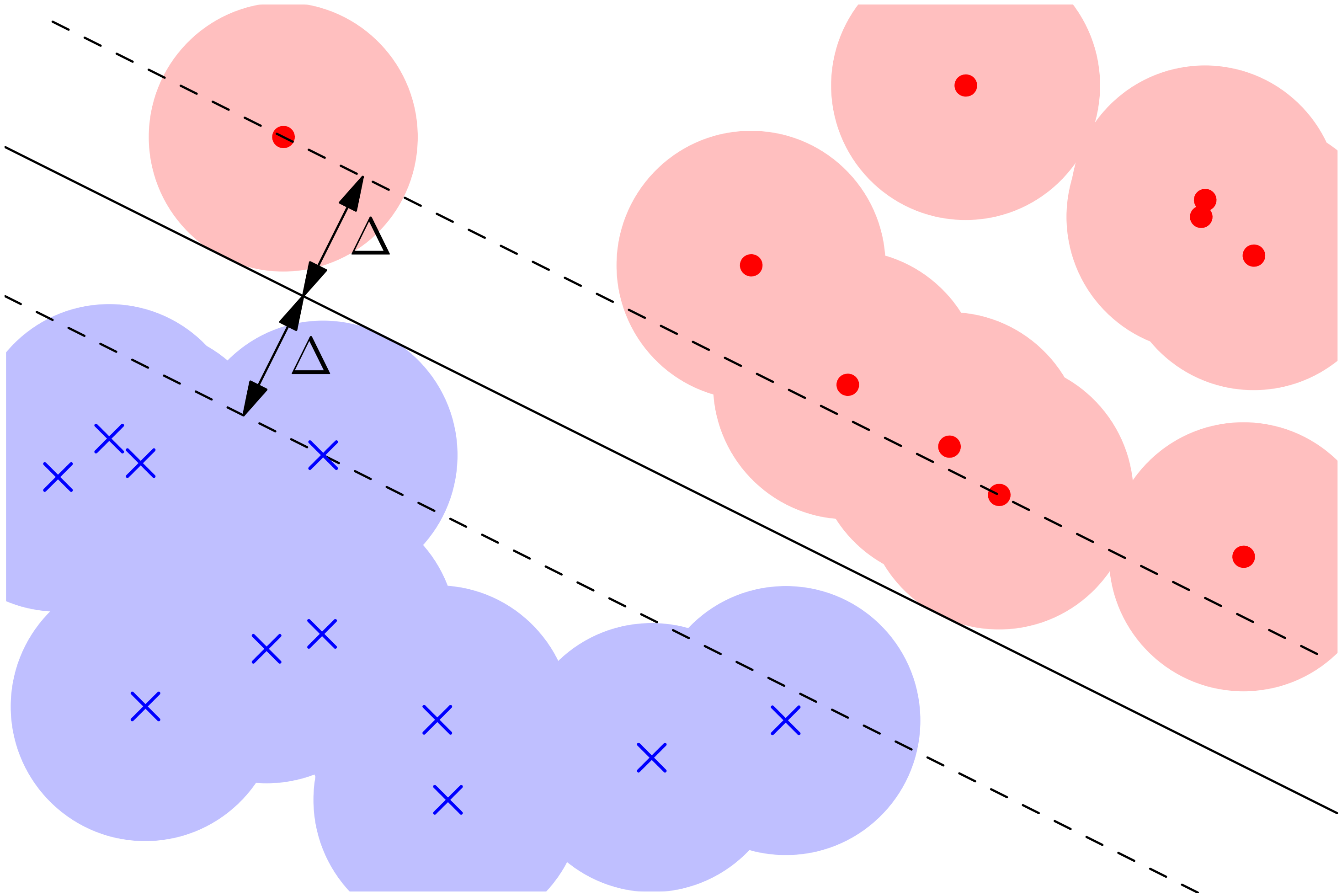
Linear Separation of Data

- SVMs classify linearly separable data



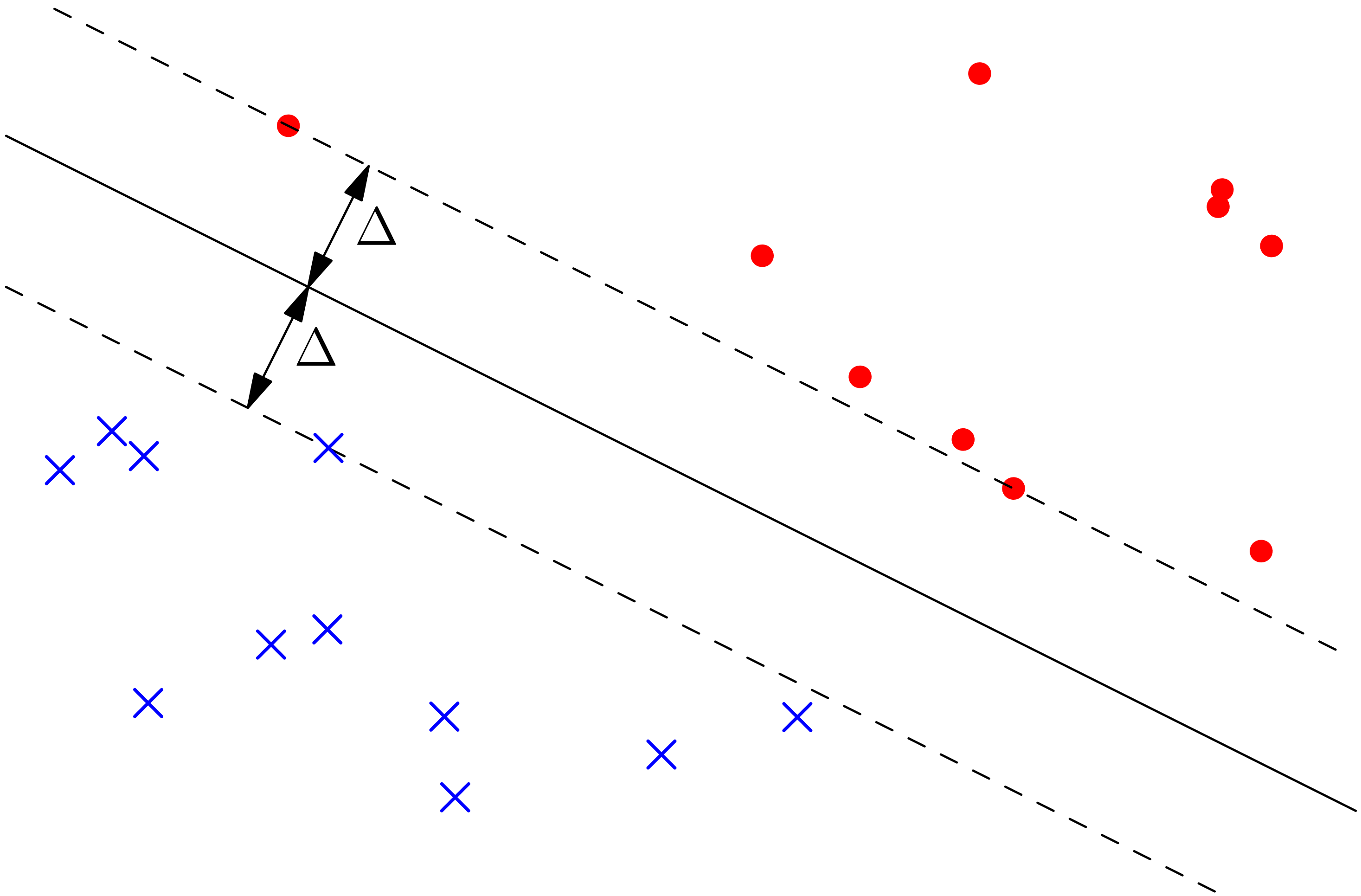
Linear Separation of Data

- SVMs classify linearly separable data



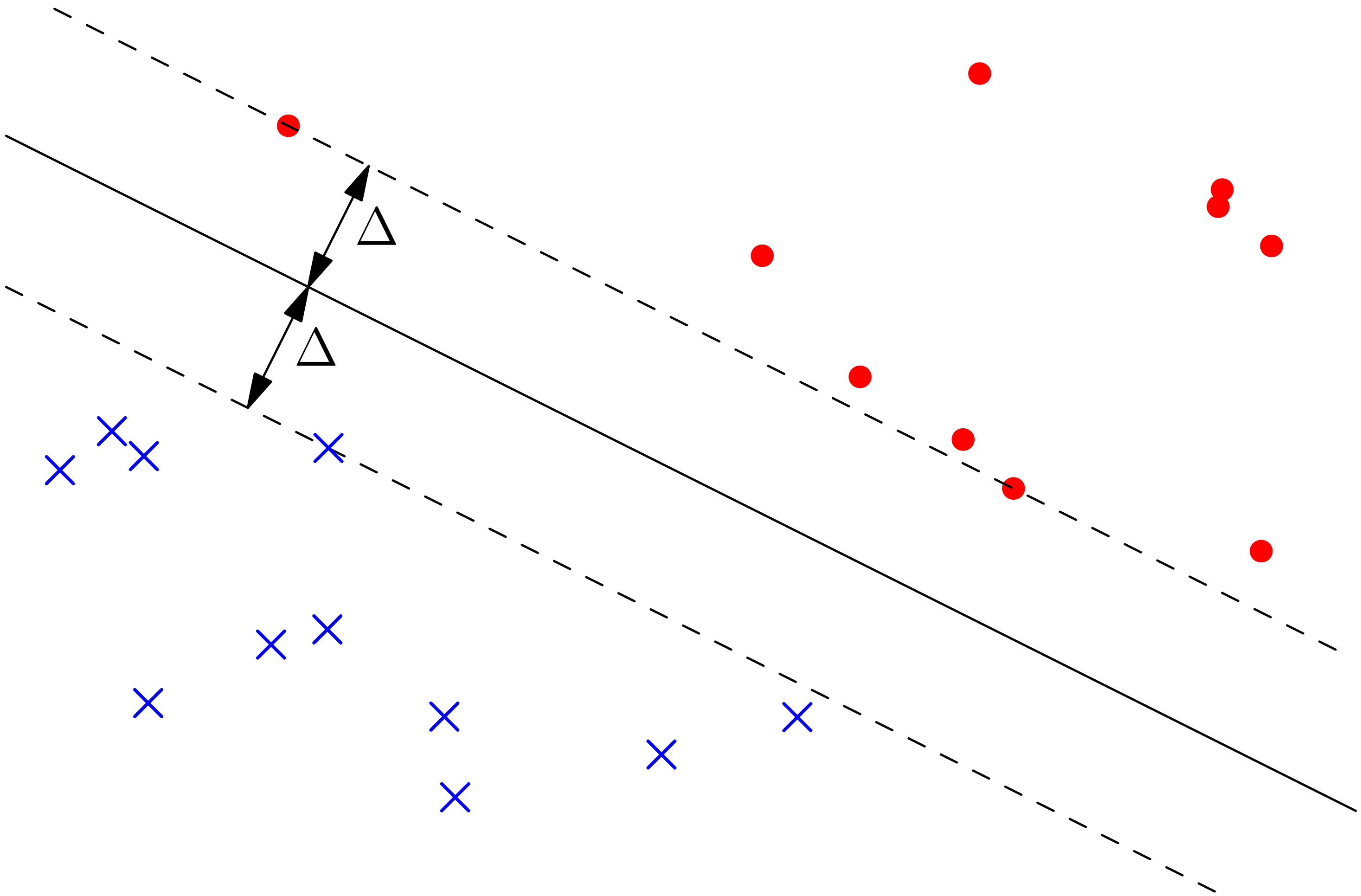
Linear Separation of Data

- SVMs classify linearly separable data



Linear Separation of Data

- SVMs classify linearly separable data



- Finds maximum-margin separating plane

Extended Feature Space

- To increase the likelihood of linear-separability we often use a high-dimensional mapping

$$\boldsymbol{x} = (x_1, x_2, \ldots, x_p)^{\mathsf{T}} \rightarrow \boldsymbol{\phi}(\boldsymbol{x}) = (\phi_1(\boldsymbol{x}), \phi_2(\boldsymbol{x}), \ldots, \phi_r(\boldsymbol{x}))^{\mathsf{T}}$$

$r \gg p$

- Finding the maximum margin hyper-plane is time consuming in "primal" form if r is large

- We can work in the "dual" space of patterns, then we only need to compute inner-products

$$\langle \boldsymbol{\phi}(\boldsymbol{x}_i), \boldsymbol{\phi}(\boldsymbol{x}_j) \rangle = \boldsymbol{\phi}(\boldsymbol{x}_i)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_j)$$

Extended Feature Space

- To increase the likelihood of linear-separability we often use a high-dimensional mapping

$$\boldsymbol{x} = (x_1,\, x_2,\, \ldots,\, x_p)^{\mathsf{T}} \to \boldsymbol{\phi}(\boldsymbol{x}) = (\phi_1(\boldsymbol{x}),\, \phi_2(\boldsymbol{x}),\, \ldots,\, \phi_r(\boldsymbol{x}))^{\mathsf{T}}$$

$r \gg p$

- Finding the maximum margin hyper-plane is time consuming in "primal" form if r is large

- We can work in the "dual" space of patterns, then we only need to compute inner-products

$$\langle \boldsymbol{\phi}(\boldsymbol{x}_i), \boldsymbol{\phi}(\boldsymbol{x}_j) \rangle = \boldsymbol{\phi}(\boldsymbol{x}_i)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_j)$$

Extended Feature Space

- To increase the likelihood of linear-separability we often use a high-dimensional mapping

$$\boldsymbol{x} = (x_1, x_2, \ldots, x_p)^{\mathsf{T}} \rightarrow \boldsymbol{\phi}(\boldsymbol{x}) = (\phi_1(\boldsymbol{x}), \phi_2(\boldsymbol{x}), \ldots, \phi_r(\boldsymbol{x}))^{\mathsf{T}}$$

$r \gg p$

- Finding the maximum margin hyper-plane is time consuming in "primal" form if r is large

- We can work in the "dual" space of patterns, then we only need to compute inner-products

$$\langle \boldsymbol{\phi}(\boldsymbol{x}_i), \boldsymbol{\phi}(\boldsymbol{x}_j) \rangle = \boldsymbol{\phi}(\boldsymbol{x}_i)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_j)$$

Extended Feature Space

- To increase the likelihood of linear-separability we often use a high-dimensional mapping

$$\boldsymbol{x} = (x_1, x_2, \ldots, x_p)^{\mathsf{T}} \to \boldsymbol{\phi}(\boldsymbol{x}) = (\phi_1(\boldsymbol{x}), \phi_2(\boldsymbol{x}), \ldots, \phi_r(\boldsymbol{x}))^{\mathsf{T}}$$

  $r \gg p$

- Finding the maximum margin hyper-plane is time consuming in "primal" form if r is large

- We can work in the "dual" space of patterns, then we only need to compute inner-products

$$\langle \boldsymbol{\phi}(\boldsymbol{x}_i), \boldsymbol{\phi}(\boldsymbol{x}_j) \rangle = \boldsymbol{\phi}(\boldsymbol{x}_i)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_j) = \sum_{k=1}^{r} \phi_k(\boldsymbol{x}_i)\, \phi_k(\boldsymbol{x}_j)$$

Kernel Trick

- If we choose a **positive semi-definite** kernel function $K(\boldsymbol{x}, \boldsymbol{y})$ then there exists functions $\boldsymbol{\phi}(\boldsymbol{x}) = (\phi_k(\boldsymbol{x}) | k = 1,\, 2,\, \ldots,\, r)$, such that

$$K(\boldsymbol{x}_i, \boldsymbol{x}_j) = \boldsymbol{\phi}(\boldsymbol{x}_i)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_j)$$

  (like an eigenvector decomposition of a matrix)

- Never need to compute $\phi_k(\boldsymbol{x}_i)$ explicitly as we only need the inner-product $\boldsymbol{\phi}(\boldsymbol{x}_i)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_j) = K(\boldsymbol{x}_i, \boldsymbol{x}_j)$ to compute maximum margin separating hyper-plane

- Sometimes $\boldsymbol{\phi}(\boldsymbol{x}_i)$ is an infinite dimensional vector so it is good we don't have to compute all the elements!

Kernel Trick

- If we choose a **positive semi-definite** kernel function $K(\boldsymbol{x}, \boldsymbol{y})$ then there exists functions $\boldsymbol{\phi}(\boldsymbol{x}) = (\phi_k(\boldsymbol{x}) | k = 1, 2, \ldots, r)$, such that

$$K(\boldsymbol{x}_i, \boldsymbol{x}_j) = \boldsymbol{\phi}(\boldsymbol{x}_i)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_j)$$

  (like an eigenvector decomposition of a matrix)

- Never need to compute $\phi_k(\boldsymbol{x}_i)$ explicitly as we only need the inner-product $\boldsymbol{\phi}(\boldsymbol{x}_i)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_j) = K(\boldsymbol{x}_i, \boldsymbol{x}_j)$ to compute maximum margin separating hyper-plane

- Sometimes $\boldsymbol{\phi}(\boldsymbol{x}_i)$ is an infinite dimensional vector so it is good we don't have to compute all the elements!

Kernel Trick

- If we choose a **positive semi-definite** kernel function $K(\boldsymbol{x}, \boldsymbol{y})$ then there exists functions $\boldsymbol{\phi}(\boldsymbol{x}) = (\phi_k(\boldsymbol{x}) | k = 1, 2, \ldots, r)$, such that

$$K(\boldsymbol{x}_i, \boldsymbol{x}_j) = \boldsymbol{\phi}(\boldsymbol{x}_i)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_j)$$

  (like an eigenvector decomposition of a matrix)

- Never need to compute $\phi_k(\boldsymbol{x}_i)$ explicitly as we only need the inner-product $\boldsymbol{\phi}(\boldsymbol{x}_i)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_j) = K(\boldsymbol{x}_i, \boldsymbol{x}_j)$ to compute maximum margin separating hyper-plane

- Sometimes $\boldsymbol{\phi}(\boldsymbol{x}_i)$ is an infinite dimensional vector so it is good we don't have to compute all the elements!

Kernel Trick

- If we choose a **positive semi-definite** kernel function $K(\boldsymbol{x}, \boldsymbol{y})$ then there exists functions $\boldsymbol{\phi}(\boldsymbol{x}) = (\phi_k(\boldsymbol{x}) | k = 1, 2, \ldots, r)$, such that

$$K(\boldsymbol{x}_i, \boldsymbol{x}_j) = \boldsymbol{\phi}(\boldsymbol{x}_i)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_j)$$

  (like an eigenvector decomposition of a matrix)

- Never need to compute $\phi_k(\boldsymbol{x}_i)$ explicitly as we only need the inner-product $\boldsymbol{\phi}(\boldsymbol{x}_i)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_j) = K(\boldsymbol{x}_i, \boldsymbol{x}_j)$ to compute maximum margin separating hyper-plane

- Sometimes $\boldsymbol{\phi}(\boldsymbol{x}_i)$ is an infinite dimensional vector so it is good we don't have to compute all the elements!

Kernel Functions

- Kernel functions are symmetric functions of two variable
- Strong restriction: *positive semi-definite*
- Examples

$$\text{Quadratic kernel:} \qquad K(\boldsymbol{x}_1, \boldsymbol{x}_2) = \left(\boldsymbol{x}_1^{\mathsf{T}} \boldsymbol{x}_2\right)^2$$

$$\text{Gaussian (RBF) kernel:} \qquad K(\boldsymbol{x}_1, \boldsymbol{x}_2) = \mathrm{e}^{-\gamma \, \|\boldsymbol{x}_1 - \boldsymbol{x}_2\|^2}$$

- Consider the mapping

$$\boldsymbol{x}_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix} \rightarrow \boldsymbol{\phi}(\boldsymbol{x}_i) = \begin{pmatrix} x_i^2 \\ y_i^2 \\ \sqrt{2}\, x_i\, y_i \end{pmatrix}$$

Kernel Functions

- Kernel functions are symmetric functions of two variable
- Strong restriction: *positive semi-definite*
- Examples

$$\text{Quadratic kernel:} \qquad K(\boldsymbol{x}_1, \boldsymbol{x}_2) = \left(\boldsymbol{x}_1^{\mathsf{T}} \boldsymbol{x}_2\right)^2$$

$$\text{Gaussian (RBF) kernel:} \qquad K(\boldsymbol{x}_1, \boldsymbol{x}_2) = \mathrm{e}^{-\gamma \, \|\boldsymbol{x}_1 - \boldsymbol{x}_2\|^2}$$

- Consider the mapping

$$\boldsymbol{x}_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix} \to \boldsymbol{\phi}(\boldsymbol{x}_i) = \begin{pmatrix} x_i^2 \\ y_i^2 \\ \sqrt{2}\, x_i \, y_i \end{pmatrix}$$

Kernel Functions

- Kernel functions are symmetric functions of two variable
- Strong restriction: *positive semi-definite*
- Examples

$$\text{Quadratic kernel:} \qquad K(\boldsymbol{x}_1, \boldsymbol{x}_2) = \left(\boldsymbol{x}_1^{\mathsf{T}} \boldsymbol{x}_2\right)^2$$

$$\text{Gaussian (RBF) kernel:} \qquad K(\boldsymbol{x}_1, \boldsymbol{x}_2) = \mathrm{e}^{-\gamma \, \|\boldsymbol{x}_1 - \boldsymbol{x}_2\|^2}$$

- Consider the mapping

$$\boldsymbol{x}_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix} \rightarrow \boldsymbol{\phi}(\boldsymbol{x}_i) = \begin{pmatrix} x_i^2 \\ y_i^2 \\ \sqrt{2}\, x_i \, y_i \end{pmatrix}$$

Kernel Functions

- Kernel functions are symmetric functions of two variable
- Strong restriction: *positive semi-definite*
- Examples

$$\text{Quadratic kernel:} \qquad K(\boldsymbol{x}_1, \boldsymbol{x}_2) = \left(\boldsymbol{x}_1^\mathsf{T} \boldsymbol{x}_2\right)^2$$

$$\text{Gaussian (RBF) kernel:} \qquad K(\boldsymbol{x}_1, \boldsymbol{x}_2) = \mathrm{e}^{-\gamma \, \|\boldsymbol{x}_1 - \boldsymbol{x}_2\|^2}$$

- Consider the mapping

$$\boldsymbol{x}_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix} \to \boldsymbol{\phi}(\boldsymbol{x}_i) = \begin{pmatrix} x_i^2 \\ y_i^2 \\ \sqrt{2}\, x_i \, y_i \end{pmatrix}$$

Non-linearly Separation of Data

$$K(\boldsymbol{x}_1, \boldsymbol{x}_2) = \begin{pmatrix} x_1^2 & y_1^2 & \sqrt{2}\, x_1\, y_1 \end{pmatrix} \begin{pmatrix} x_2^2 \\ y_2^2 \\ \sqrt{2}\, x_2\, y_2 \end{pmatrix}$$

Non-linearly Separation of Data

$$K(\boldsymbol{x}_1, \boldsymbol{x}_2) = \begin{pmatrix} x_1^2 & y_1^2 & \sqrt{2}\, x_1\, y_1 \end{pmatrix} \begin{pmatrix} x_2^2 \\ y_2^2 \\ \sqrt{2}\, x_2\, y_2 \end{pmatrix} = x_1^2\, x_2^2 + y_1^2\, y_2^2 + 2\, x_1\, y_1\, x_2\, y_2$$

Non-linearly Separation of Data

$$K(\boldsymbol{x}_1, \boldsymbol{x}_2) = \begin{pmatrix} x_1^2 & y_1^2 & \sqrt{2}\, x_1\, y_1 \end{pmatrix} \begin{pmatrix} x_2^2 \\ y_2^2 \\ \sqrt{2}\, x_2\, y_2 \end{pmatrix} = x_1^2\, x_2^2 + y_1^2\, y_2^2 + 2\, x_1\, y_1\, x_2\, y_2$$

$$= (x_1\, x_2 + y_1\, y_2)^2$$

Non-linearly Separation of Data

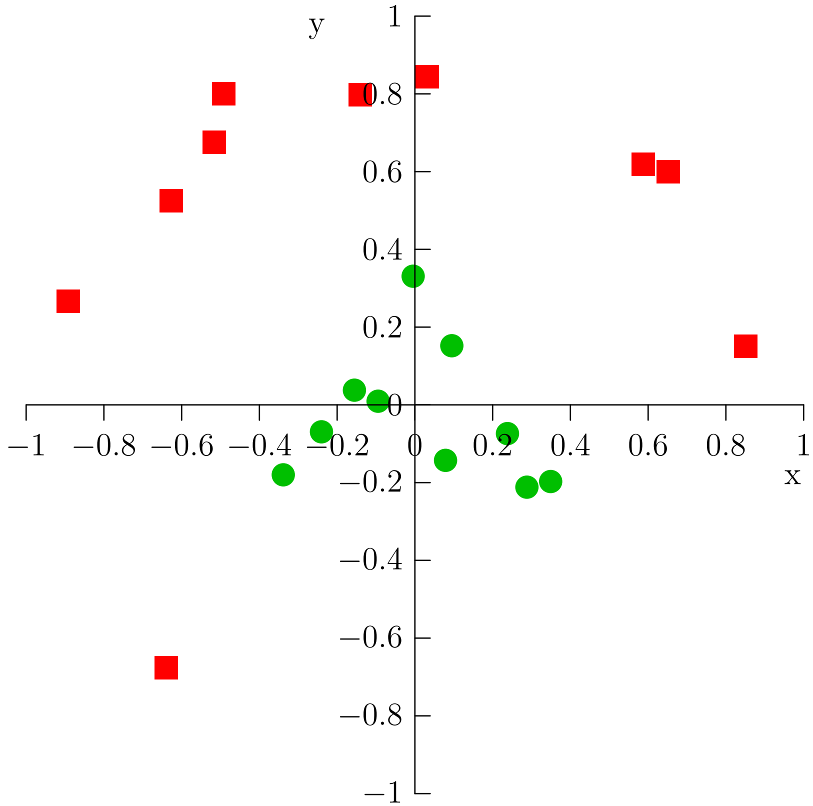
$$K(\boldsymbol{x}_1, \boldsymbol{x}_2) = \begin{pmatrix} x_1^2 & y_1^2 & \sqrt{2}\, x_1\, y_1 \end{pmatrix} \begin{pmatrix} x_2^2 \\ y_2^2 \\ \sqrt{2}\, x_2\, y_2 \end{pmatrix} = x_1^2\, x_2^2 + y_1^2\, y_2^2 + 2\, x_1\, y_1\, x_2\, y_2$$

$$= (x_1\, x_2 + y_1\, y_2)^2 = \left(\boldsymbol{x}_1^T \boldsymbol{x}_2\right)^2$$

Non-linearly Separation of Data

$$K(\boldsymbol{x}_1, \boldsymbol{x}_2) = \begin{pmatrix} x_1^2 & y_1^2 & \sqrt{2}\, x_1\, y_1 \end{pmatrix} \begin{pmatrix} x_2^2 \\ y_2^2 \\ \sqrt{2}\, x_2\, y_2 \end{pmatrix} = x_1^2\, x_2^2 + y_1^2\, y_2^2 + 2\, x_1\, y_1\, x_2\, y_2$$

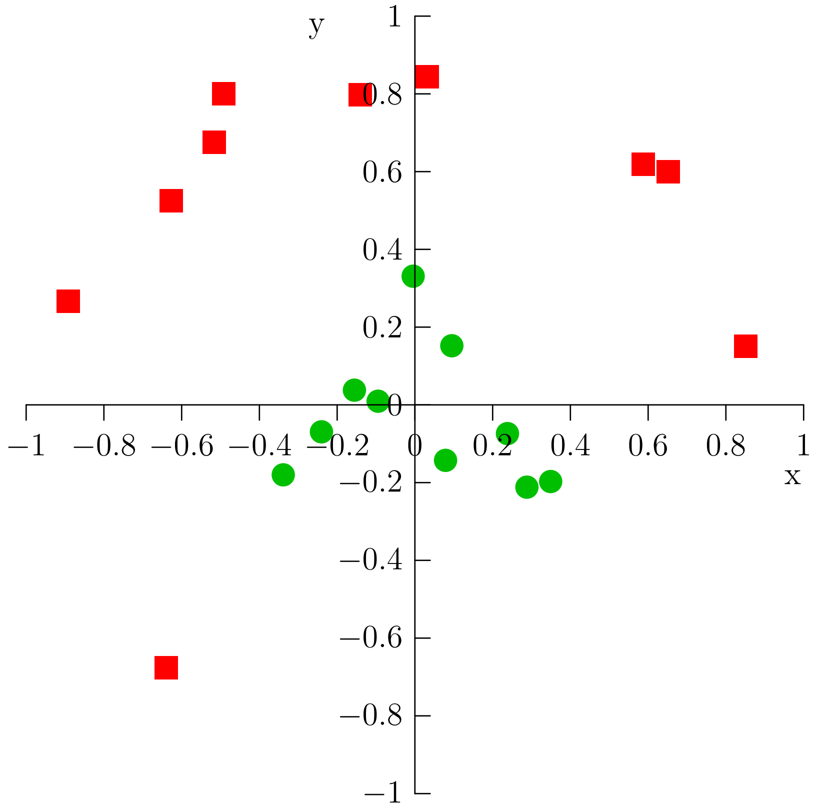
$$= (x_1\, x_2 + y_1\, y_2)^2 = \left(\boldsymbol{x}_1^T \boldsymbol{x}_2\right)^2$$



Non-linearly Separation of Data

$$K(\boldsymbol{x}_1, \boldsymbol{x}_2) = \begin{pmatrix} x_1^2 & y_1^2 & \sqrt{2}\, x_1\, y_1 \end{pmatrix} \begin{pmatrix} x_2^2 \\ y_2^2 \\ \sqrt{2}\, x_2\, y_2 \end{pmatrix} = x_1^2\, x_2^2 + y_1^2\, y_2^2 + 2\, x_1\, y_1\, x_2\, y_2$$

$$= (x_1\, x_2 + y_1\, y_2)^2 = \left(\boldsymbol{x}_1^T \boldsymbol{x}_2\right)^2$$

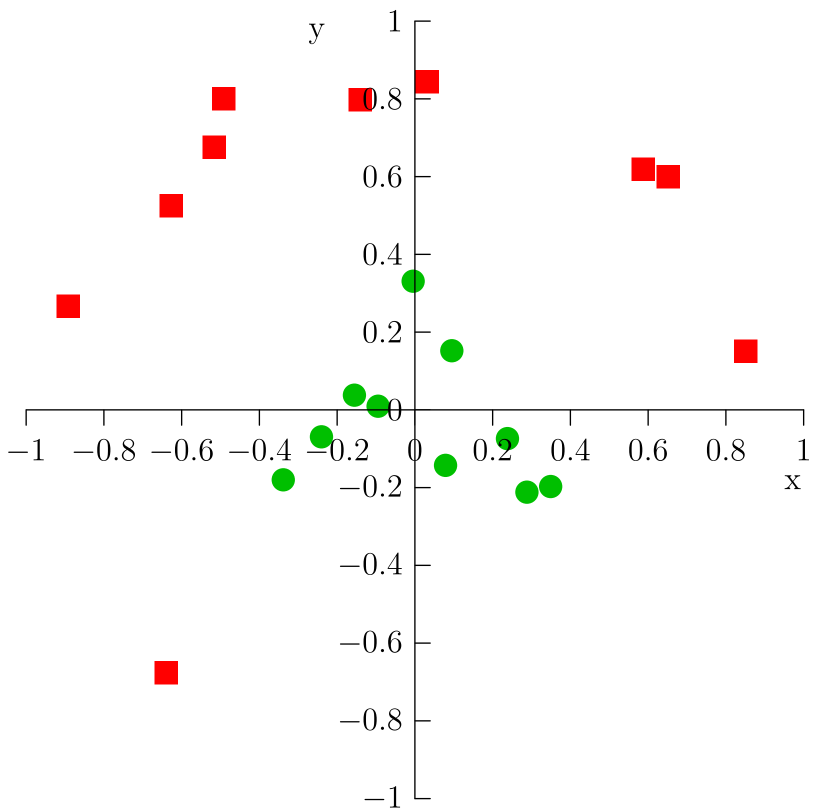


$\xrightarrow{\boldsymbol{x} \to \boldsymbol{\phi}(\boldsymbol{x})}$

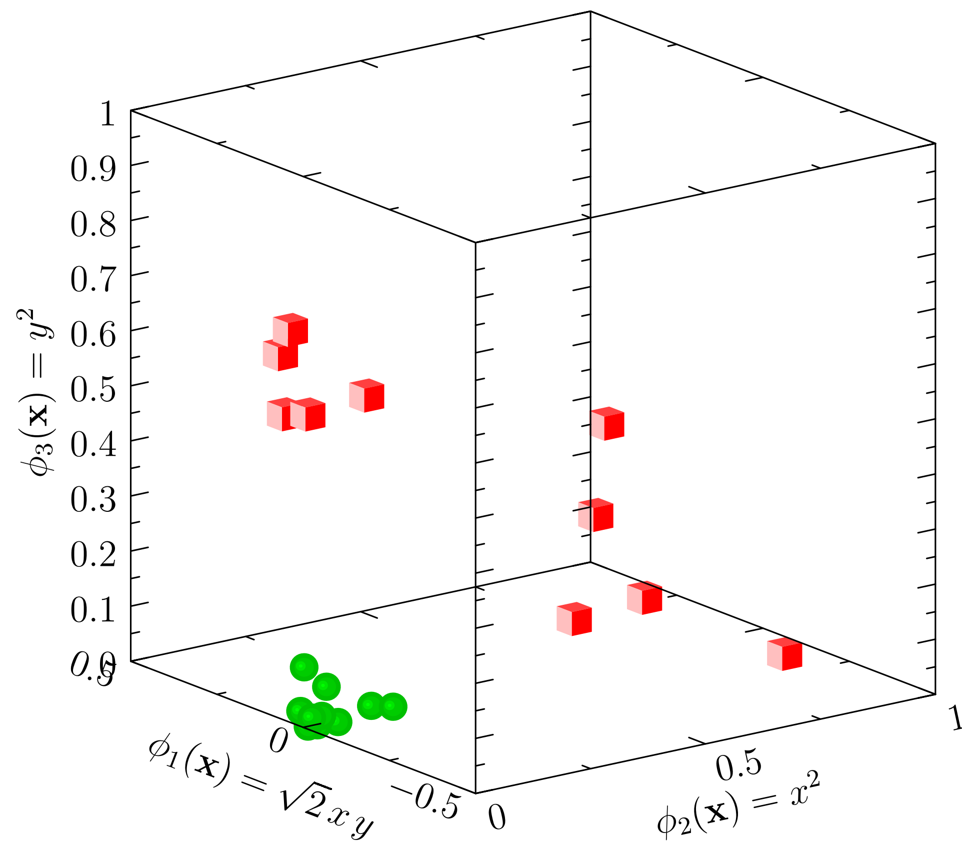
Non-linearly Separation of Data

$$K(\boldsymbol{x}_1, \boldsymbol{x}_2) = \begin{pmatrix} x_1^2 & y_1^2 & \sqrt{2}\, x_1\, y_1 \end{pmatrix} \begin{pmatrix} x_2^2 \\ y_2^2 \\ \sqrt{2}\, x_2\, y_2 \end{pmatrix} = x_1^2\, x_2^2 + y_1^2\, y_2^2 + 2\, x_1\, y_1\, x_2\, y_2$$

$$= (x_1\, x_2 + y_1\, y_2)^2 = \left(\boldsymbol{x}_1^T \boldsymbol{x}_2\right)^2$$



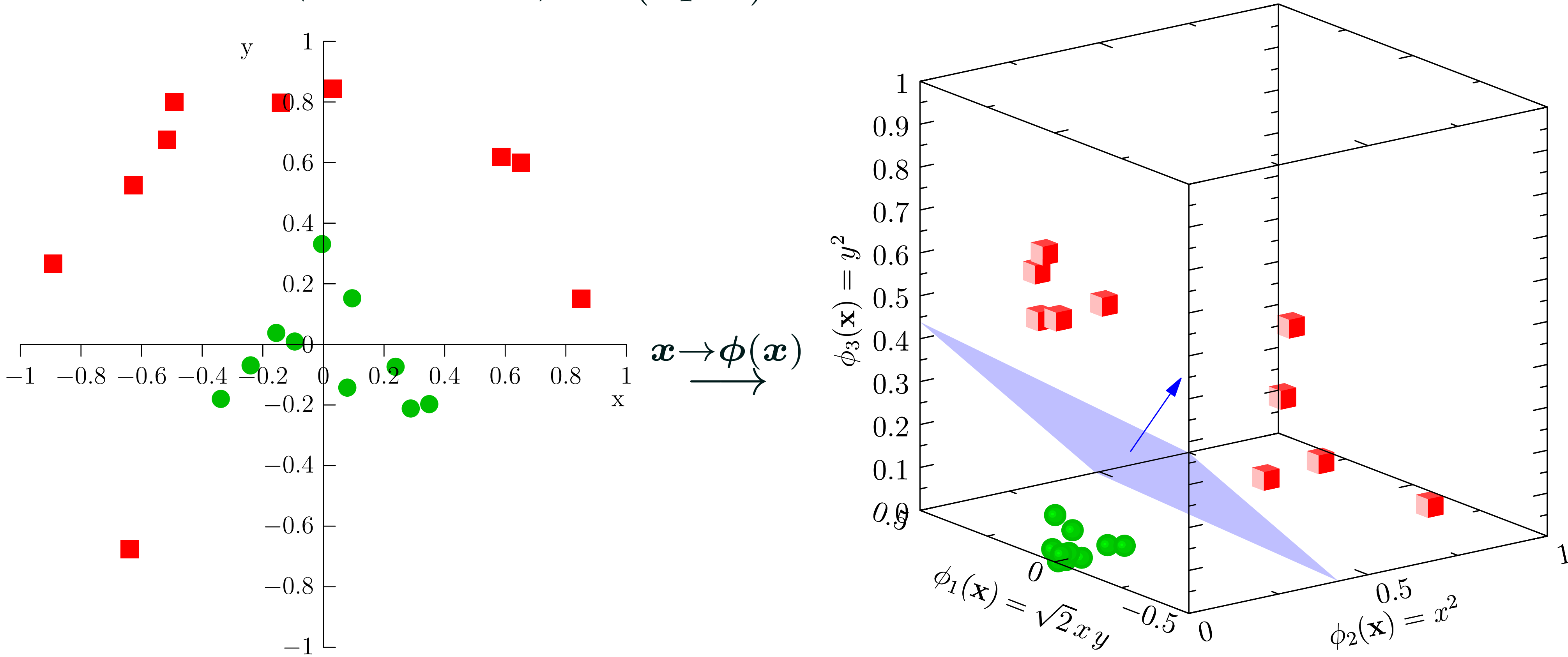
$$\xrightarrow{\boldsymbol{x} \to \boldsymbol{\phi}(\boldsymbol{x})}$$



Non-linearly Separation of Data

$$K(\boldsymbol{x}_1, \boldsymbol{x}_2) = \begin{pmatrix} x_1^2 & y_1^2 & \sqrt{2}\,x_1\,y_1 \end{pmatrix} \begin{pmatrix} x_2^2 \\ y_2^2 \\ \sqrt{2}\,x_2\,y_2 \end{pmatrix} = x_1^2\,x_2^2 + y_1^2\,y_2^2 + 2\,x_1\,y_1\,x_2\,y_2$$

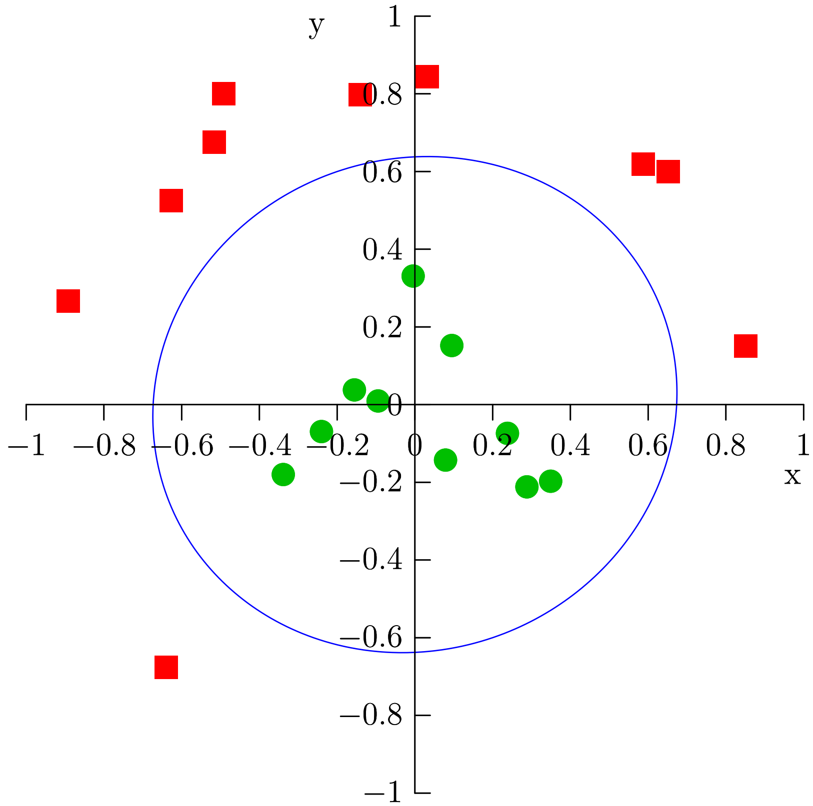
$$= (x_1\,x_2 + y_1\,y_2)^2 = \left(\boldsymbol{x}_1^T \boldsymbol{x}_2\right)^2$$



Non-linearly Separation of Data

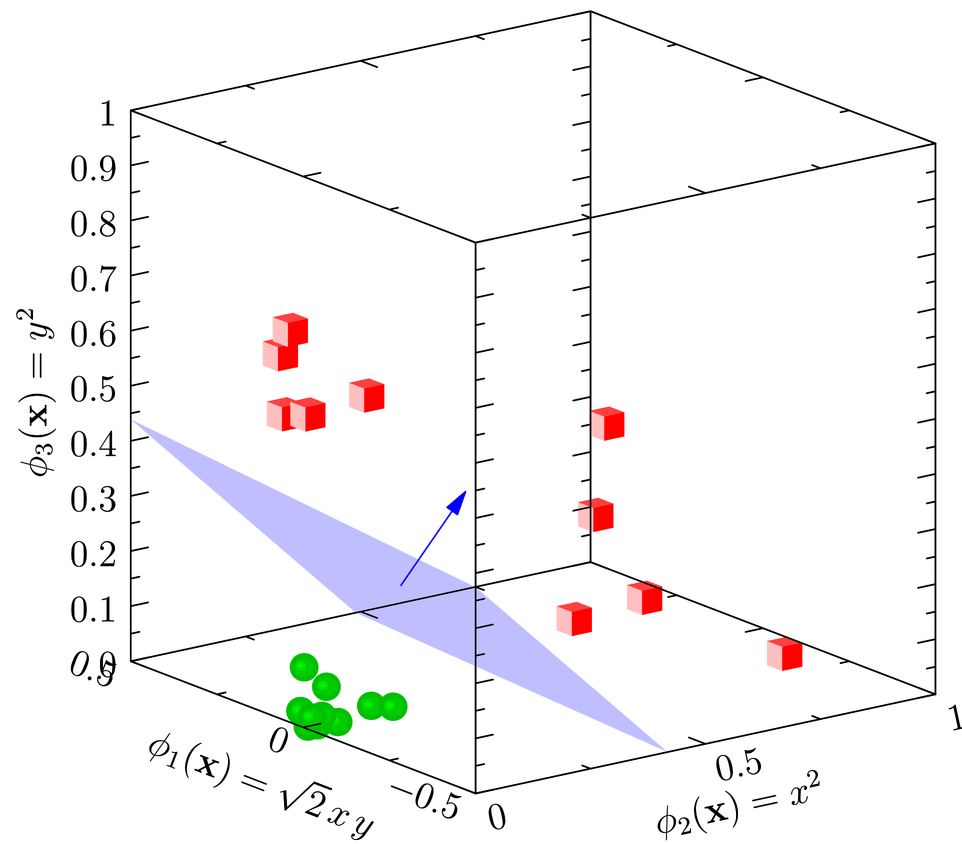
$$K(\boldsymbol{x}_1, \boldsymbol{x}_2) = \begin{pmatrix} x_1^2 & y_1^2 & \sqrt{2}\, x_1\, y_1 \end{pmatrix} \begin{pmatrix} x_2^2 \\ y_2^2 \\ \sqrt{2}\, x_2\, y_2 \end{pmatrix} = x_1^2\, x_2^2 + y_1^2\, y_2^2 + 2\, x_1\, y_1\, x_2\, y_2$$

$$= (x_1\, x_2 + y_1\, y_2)^2 = \left(\boldsymbol{x}_1^T \boldsymbol{x}_2\right)^2$$



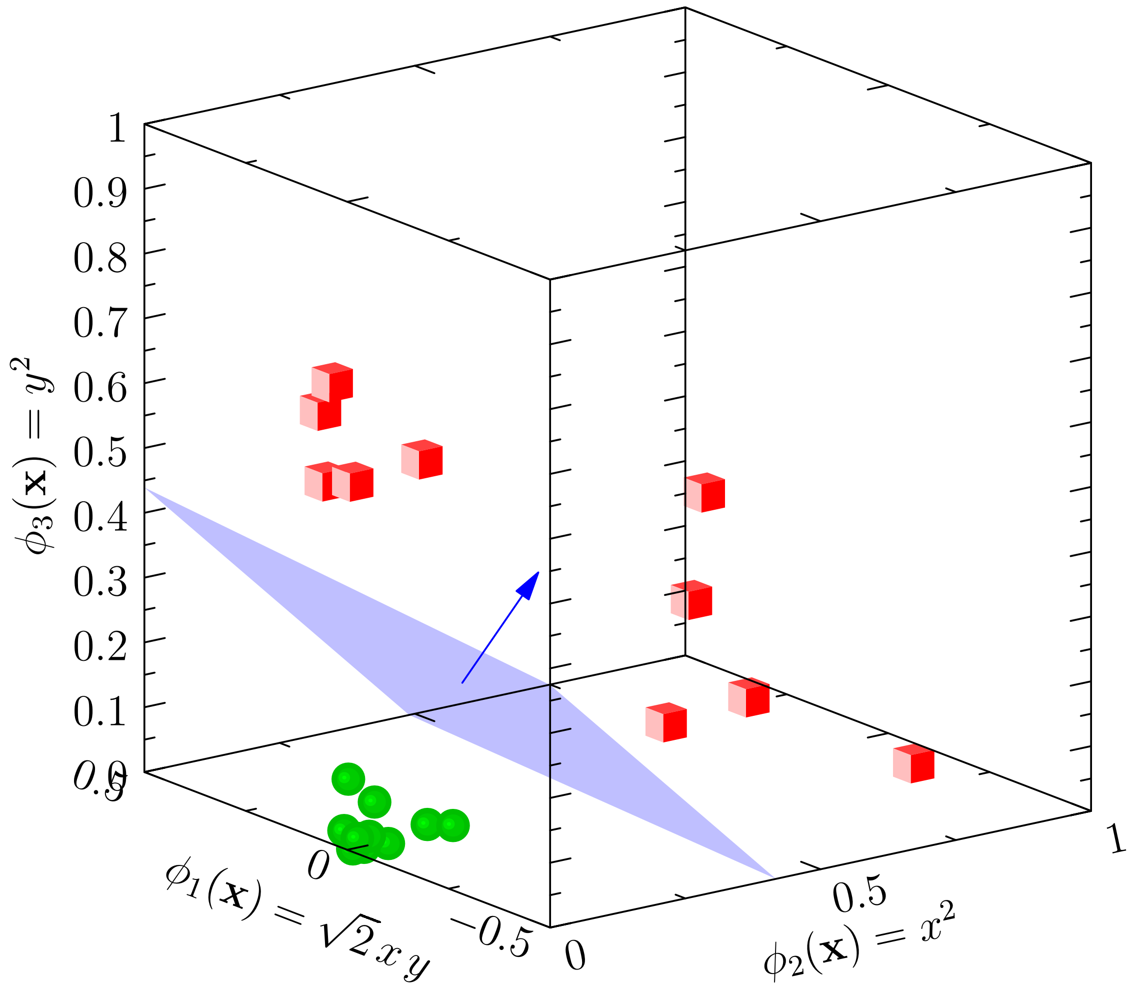
$\boldsymbol{x} \to \boldsymbol{\phi}(\boldsymbol{x})$

$\longrightarrow$



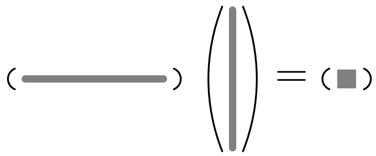
Outline

1. The Big Picture
2. **Maximum Margins**
3. Duality
4. Practice



Inner Product

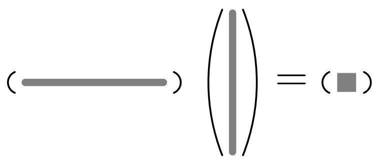
- Recall the inner or dot product



$$\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \boldsymbol{x} \cdot \boldsymbol{y}$$

Inner Product

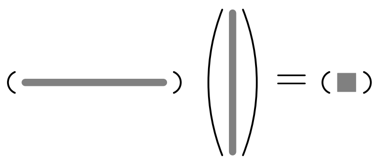
- Recall the inner or dot product



$$\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \boldsymbol{x} \cdot \boldsymbol{y} = \boldsymbol{x}^{\mathsf{T}} \boldsymbol{y}$$

Inner Product

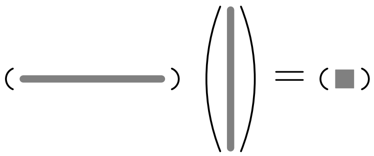
- Recall the inner or dot product



$$\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \boldsymbol{x} \cdot \boldsymbol{y} = \boldsymbol{x}^{\mathsf{T}} \boldsymbol{y} = \sum_{i=1}^{n} x_i \, y_i$$

Inner Product

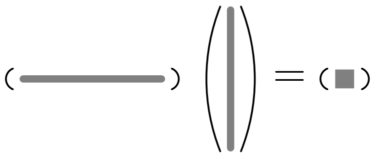
- Recall the inner or dot product



$$\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \boldsymbol{x} \cdot \boldsymbol{y} = \boldsymbol{x}^{\mathsf{T}} \boldsymbol{y} = \sum_{i=1}^{n} x_i \, y_i = \|\boldsymbol{x}\| \, \|\boldsymbol{y}\| \, \cos(\theta)$$

Inner Product

- Recall the inner or dot product

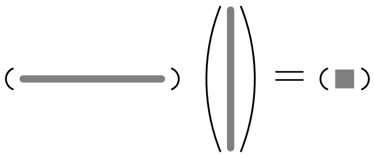


$$\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \boldsymbol{x} \cdot \boldsymbol{y} = \boldsymbol{x}^\mathsf{T} \boldsymbol{y} = \sum_{i=1}^{n} x_i \, y_i = \|\boldsymbol{x}\| \, \|\boldsymbol{y}\| \, \cos(\theta)$$

- If $\|\boldsymbol{w}\| = 1$ then $\boldsymbol{x}^\mathsf{T} \boldsymbol{w} = \|\boldsymbol{x}\| \, \cos(\theta)$

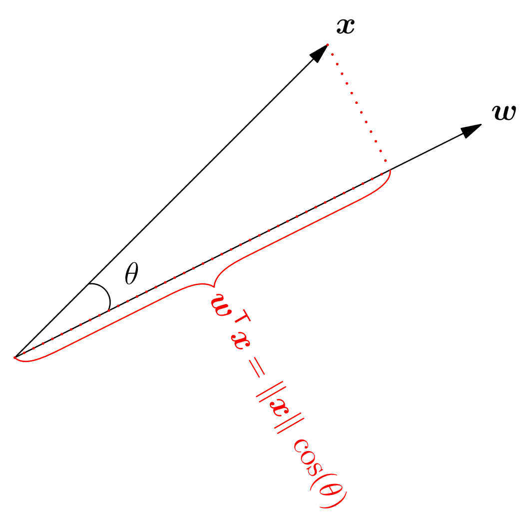
Inner Product

- Recall the inner or dot product



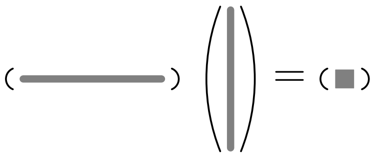
$$\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \boldsymbol{x} \cdot \boldsymbol{y} = \boldsymbol{x}^{\mathsf{T}} \boldsymbol{y} = \sum_{i=1}^{n} x_i \, y_i = \|\boldsymbol{x}\| \, \|\boldsymbol{y}\| \, \cos(\theta)$$

- If $\|\boldsymbol{w}\| = 1$ then $\boldsymbol{x}^{\mathsf{T}} \boldsymbol{w} = \|\boldsymbol{x}\| \, \cos(\theta)$



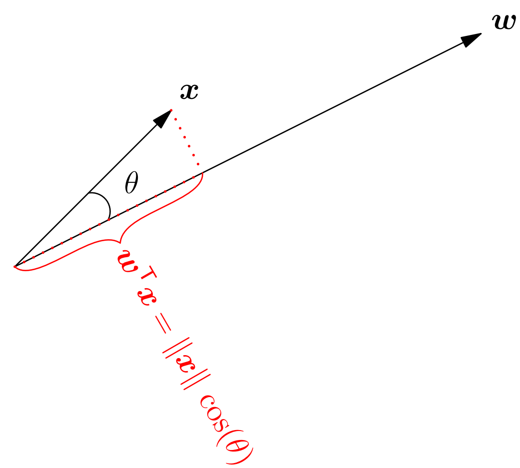
Inner Product

- Recall the inner or dot product



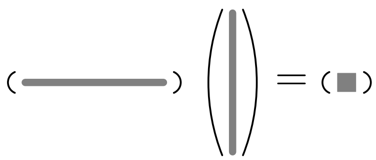
$$\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \boldsymbol{x} \cdot \boldsymbol{y} = \boldsymbol{x}^{\mathsf{T}} \boldsymbol{y} = \sum_{i=1}^{n} x_i \, y_i = \|\boldsymbol{x}\| \, \|\boldsymbol{y}\| \, \cos(\theta)$$

- If $\|\boldsymbol{w}\| = 1$ then $\boldsymbol{x}^{\mathsf{T}} \boldsymbol{w} = \|\boldsymbol{x}\| \, \cos(\theta)$



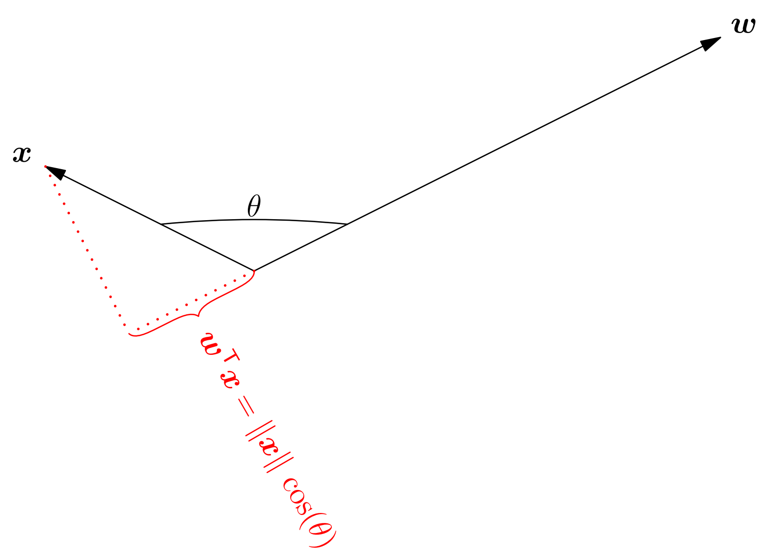
Inner Product

- Recall the inner or dot product



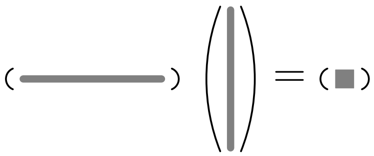
$$\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \boldsymbol{x} \cdot \boldsymbol{y} = \boldsymbol{x}^{\mathsf{T}} \boldsymbol{y} = \sum_{i=1}^{n} x_i \, y_i = \|\boldsymbol{x}\| \, \|\boldsymbol{y}\| \, \cos(\theta)$$

- If $\|\boldsymbol{w}\| = 1$ then $\boldsymbol{x}^{\mathsf{T}} \boldsymbol{w} = \|\boldsymbol{x}\| \, \cos(\theta)$



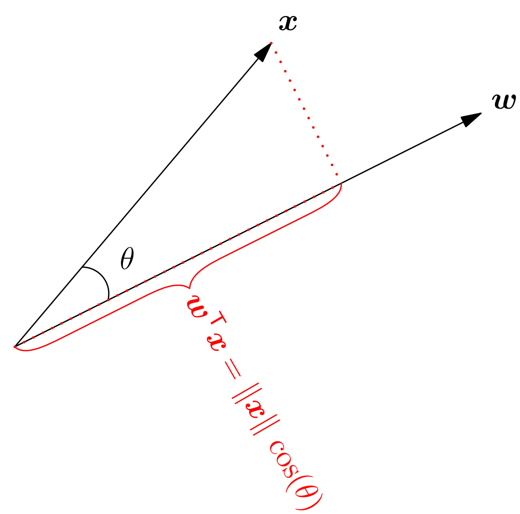
Inner Product

- Recall the inner or dot product



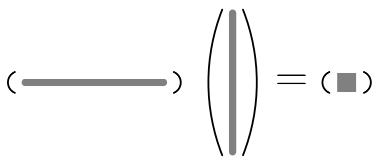
$$\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \boldsymbol{x} \cdot \boldsymbol{y} = \boldsymbol{x}^{\mathsf{T}} \boldsymbol{y} = \sum_{i=1}^{n} x_i \, y_i = \|\boldsymbol{x}\| \, \|\boldsymbol{y}\| \, \cos(\theta)$$

- If $\|\boldsymbol{w}\| = 1$ then $\boldsymbol{x}^{\mathsf{T}} \boldsymbol{w} = \|\boldsymbol{x}\| \, \cos(\theta)$



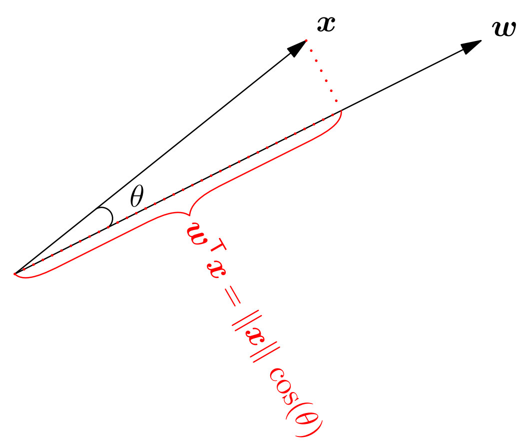
Inner Product

- Recall the inner or dot product



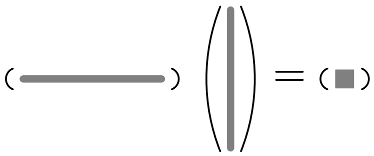
$$\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \boldsymbol{x} \cdot \boldsymbol{y} = \boldsymbol{x}^{\mathsf{T}} \boldsymbol{y} = \sum_{i=1}^{n} x_i \, y_i = \|\boldsymbol{x}\| \, \|\boldsymbol{y}\| \, \cos(\theta)$$

- If $\|\boldsymbol{w}\| = 1$ then $\boldsymbol{x}^{\mathsf{T}} \boldsymbol{w} = \|\boldsymbol{x}\| \, \cos(\theta)$



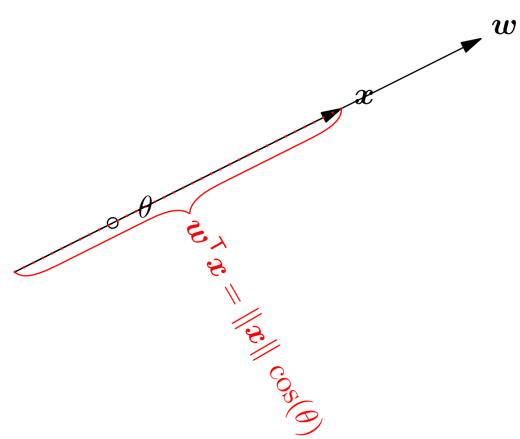
Inner Product

- Recall the inner or dot product



$$\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \boldsymbol{x} \cdot \boldsymbol{y} = \boldsymbol{x}^{\mathsf{T}} \boldsymbol{y} = \sum_{i=1}^{n} x_i \, y_i = \|\boldsymbol{x}\| \, \|\boldsymbol{y}\| \, \cos(\theta)$$

- If $\|\boldsymbol{w}\| = 1$ then $\boldsymbol{x}^{\mathsf{T}} \boldsymbol{w} = \|\boldsymbol{x}\| \, \cos(\theta)$

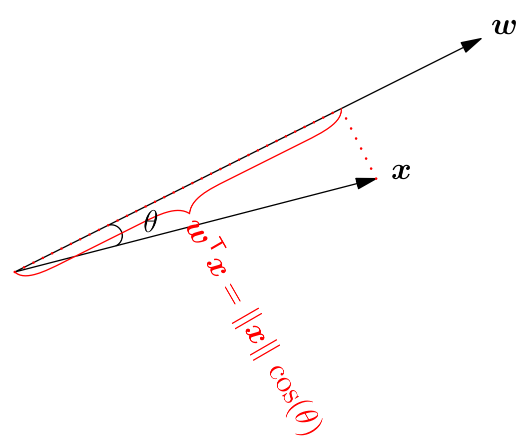


Inner Product

- Recall the inner or dot product

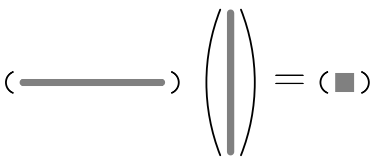
$$\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \boldsymbol{x} \cdot \boldsymbol{y} = \boldsymbol{x}^\mathsf{T} \boldsymbol{y} = \sum_{i=1}^{n} x_i \, y_i = \|\boldsymbol{x}\| \, \|\boldsymbol{y}\| \, \cos(\theta)$$

- If $\|\boldsymbol{w}\| = 1$ then $\boldsymbol{x}^\mathsf{T} \boldsymbol{w} = \|\boldsymbol{x}\| \, \cos(\theta)$



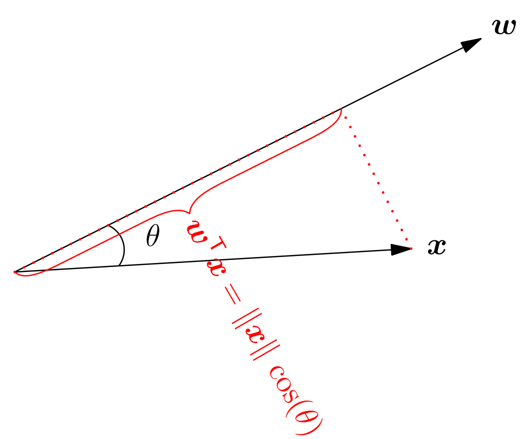
Inner Product

- Recall the inner or dot product



$$\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \boldsymbol{x} \cdot \boldsymbol{y} = \boldsymbol{x}^{\mathsf{T}} \boldsymbol{y} = \sum_{i=1}^{n} x_i \, y_i = \|\boldsymbol{x}\| \, \|\boldsymbol{y}\| \, \cos(\theta)$$

- If $\|\boldsymbol{w}\| = 1$ then $\boldsymbol{x}^{\mathsf{T}} \boldsymbol{w} = \|\boldsymbol{x}\| \, \cos(\theta)$



Maximise Margin

- Consider a linearly separable set of data
  - $\mathcal{D} = \{(\boldsymbol{x}_k,\, y_k)\}_{k=1}^{m}$
  - $y_k \in \{-1, 1\}$
- Our task is to find a separating plane defined by the orthogonal vector $\boldsymbol{w}$ and a threshold b such that

$$y_k \left(\frac{\boldsymbol{w}^{\mathsf{T}} \boldsymbol{x}_k}{\|\boldsymbol{w}\|} - b \right) \geq \Delta$$

where Δ is the margin

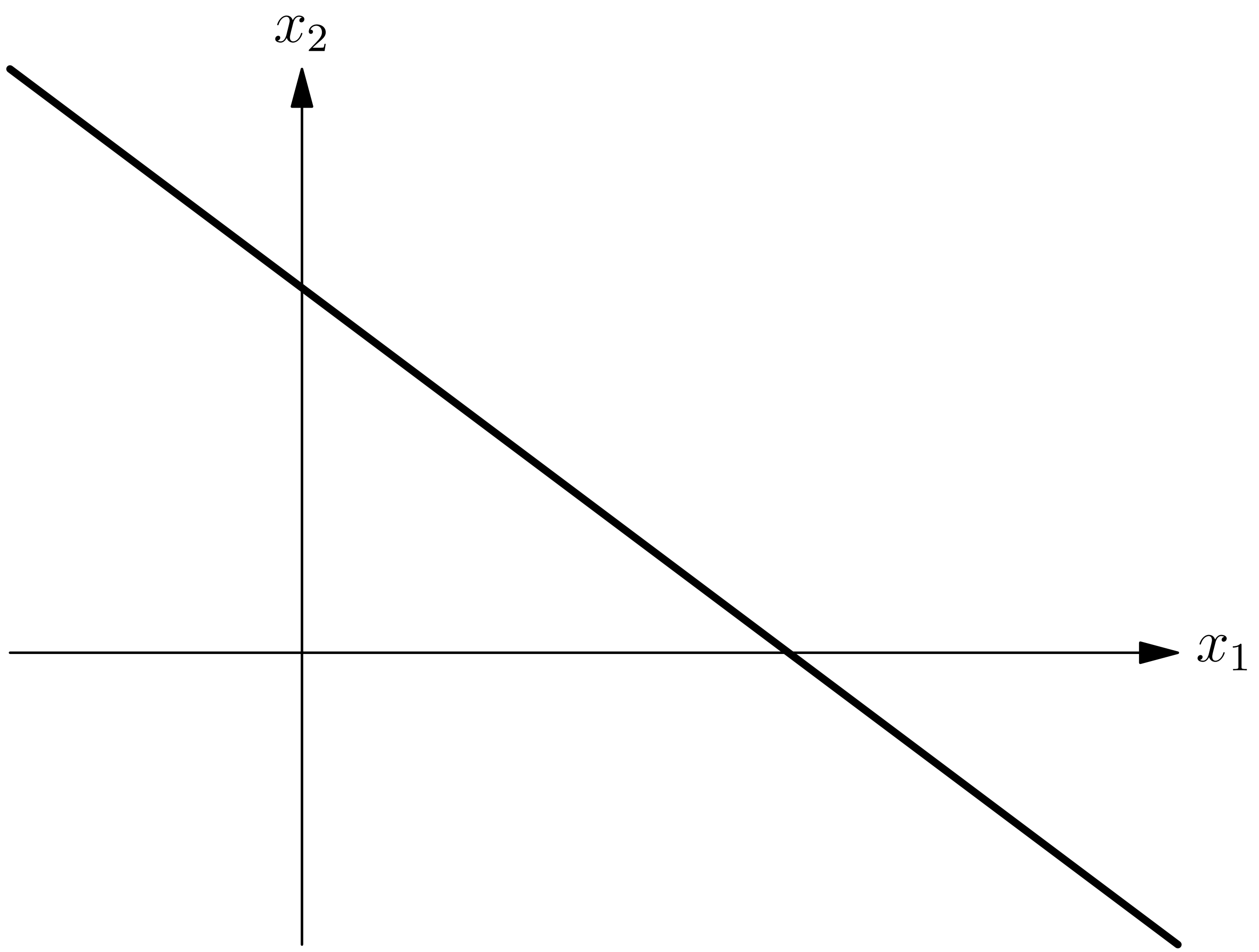
Maximise Margin

- Consider a linearly separable set of data
  - ⋆ $\mathcal{D} = \{(\boldsymbol{x}_k,\, y_k)\}_{k=1}^{m}$
  - ⋆ $y_k \in \{-1, 1\}$
- Our task is to find a separating plane defined by the orthogonal vector $\boldsymbol{w}$ and a threshold b such that

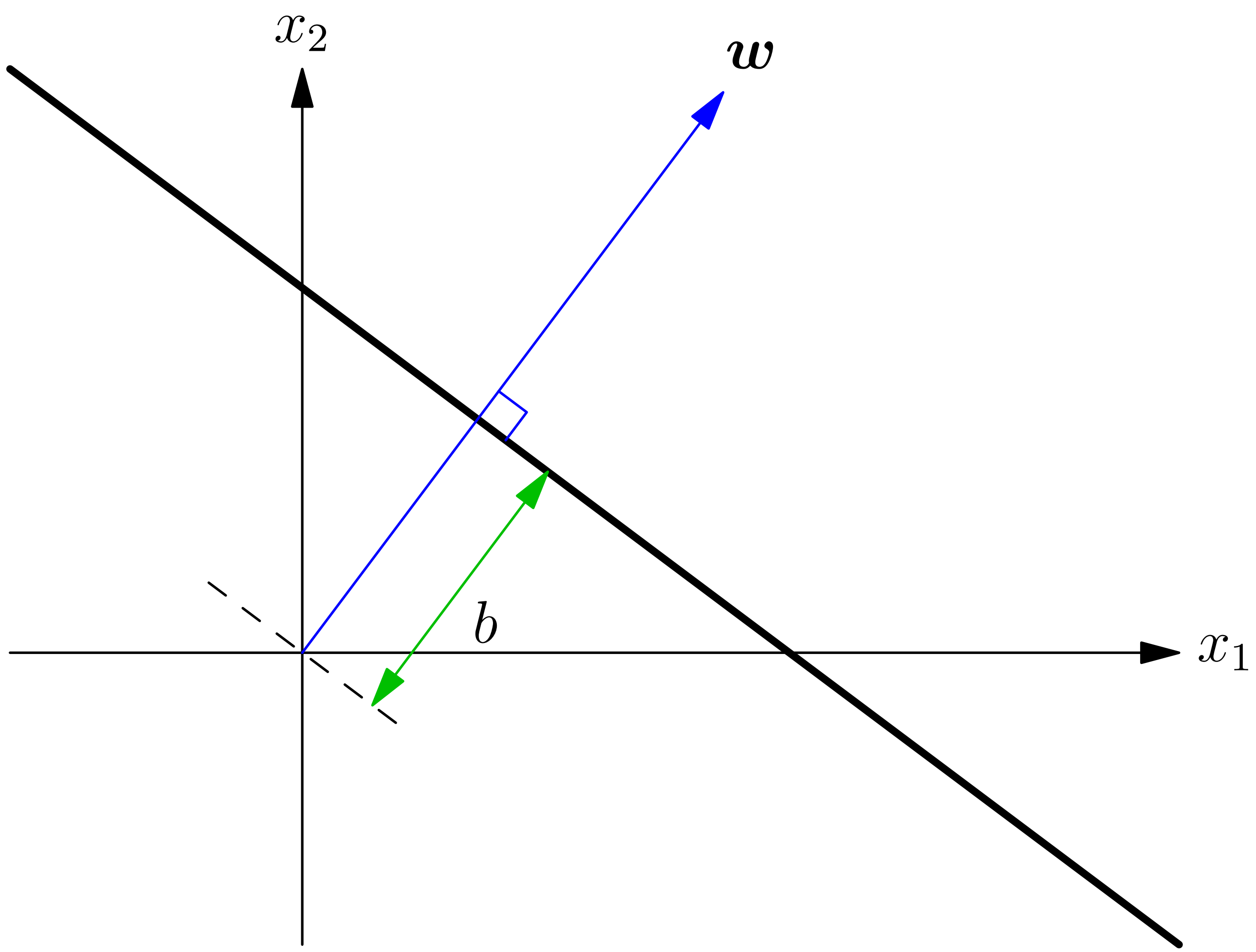
$$y_k \left(\frac{\boldsymbol{w}^\mathsf{T} \boldsymbol{x}_k}{\|\boldsymbol{w}\|} - b \right) \geq \Delta$$

  where Δ is the margin

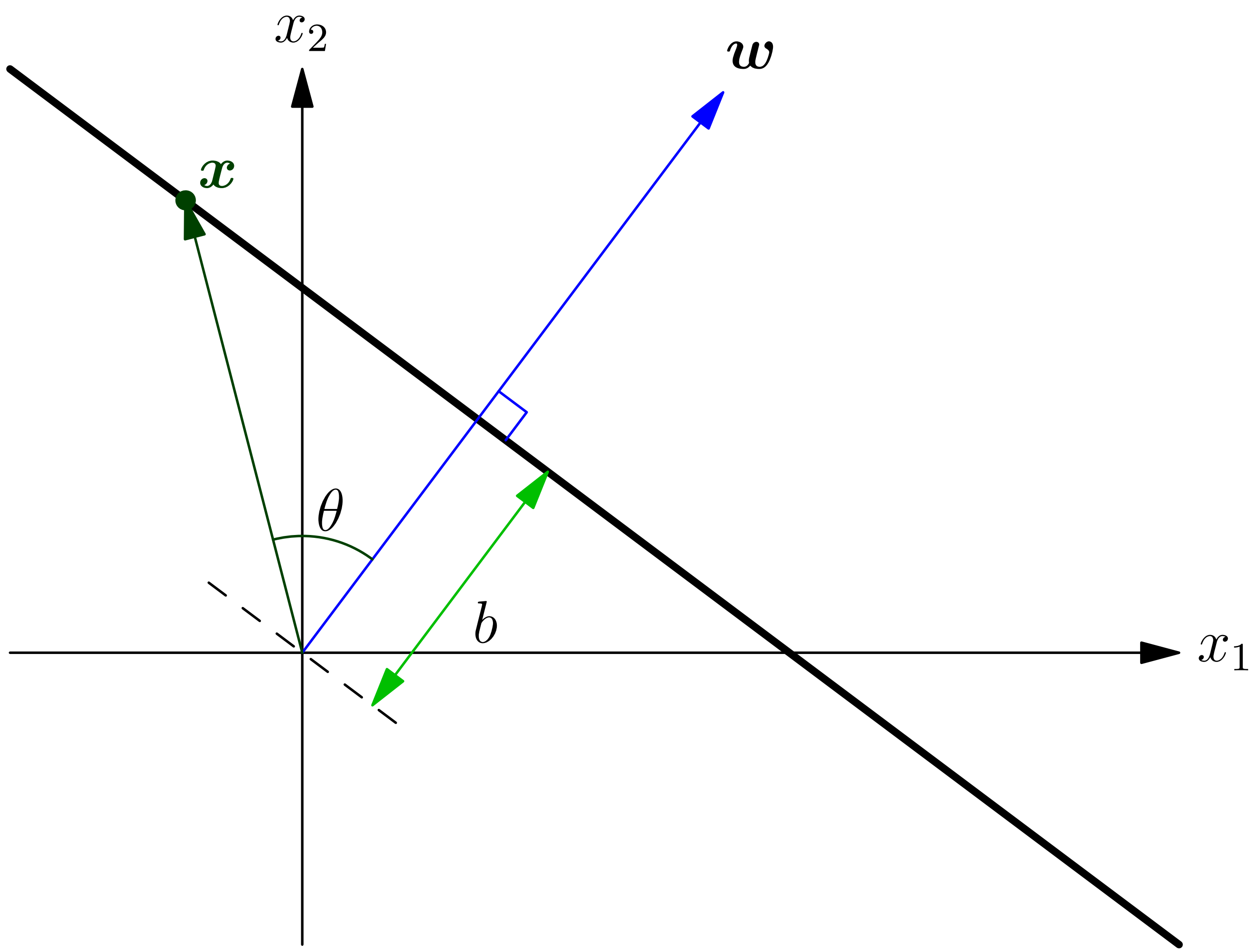
Distance to hyperplanes



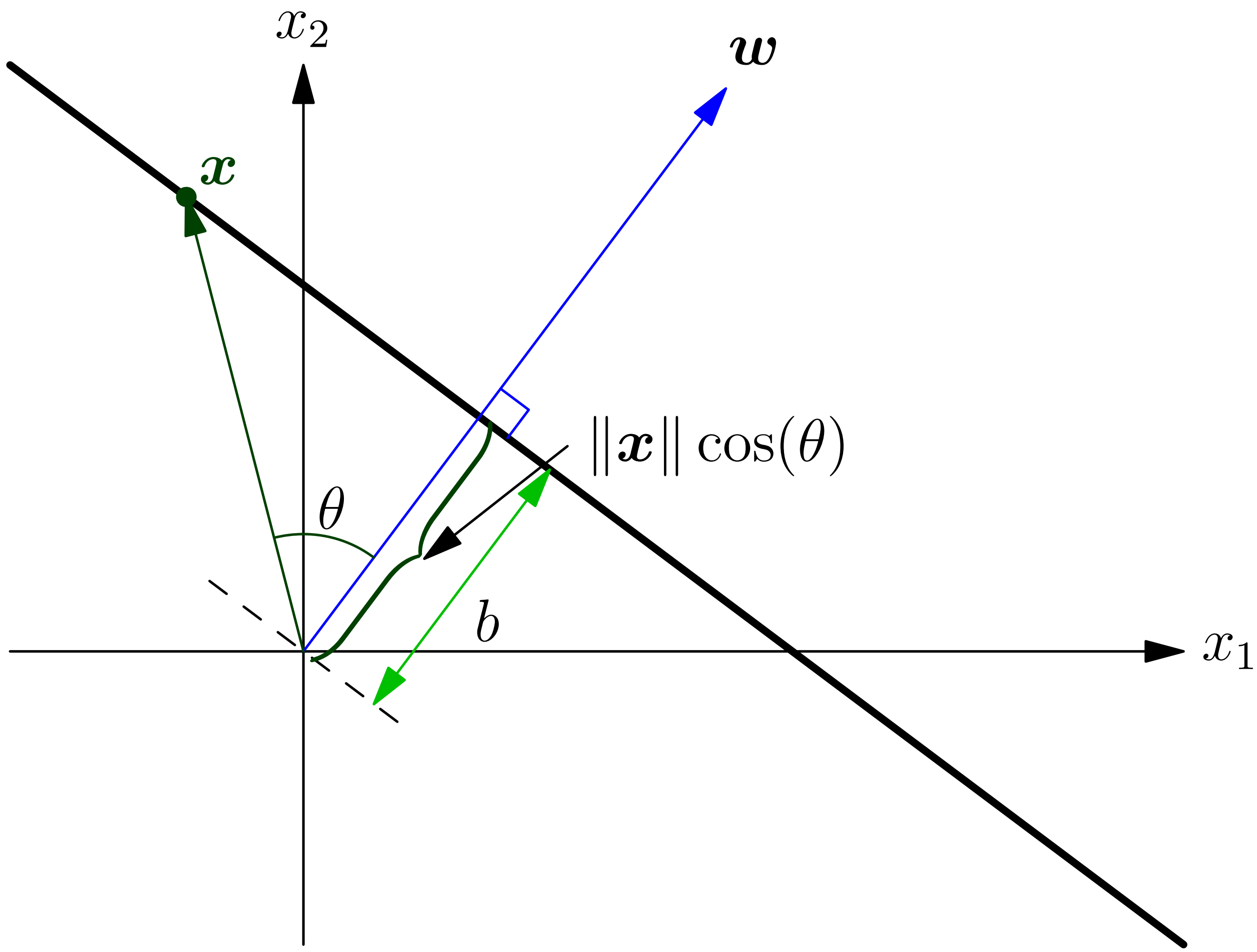
Distance to hyperplanes



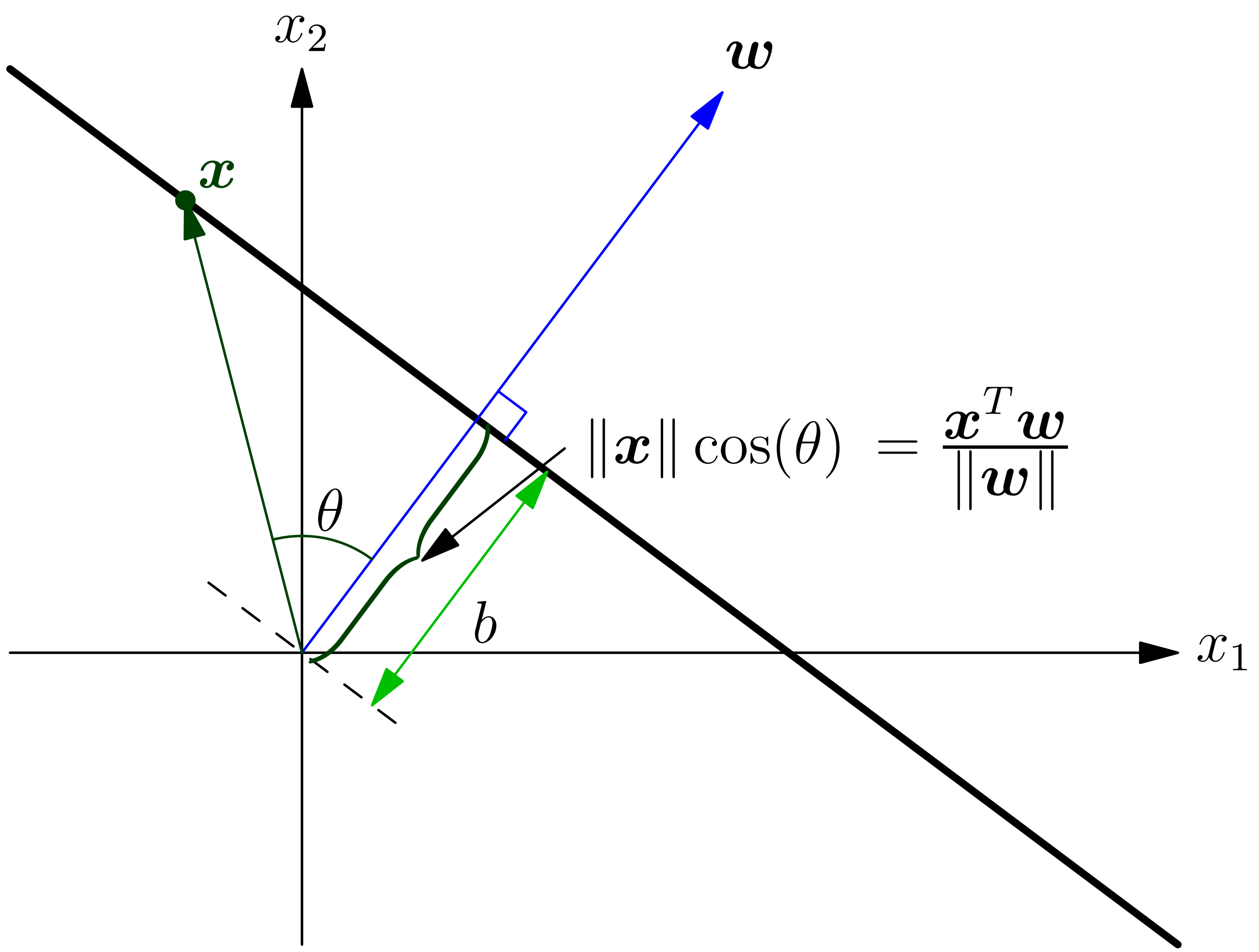
Distance to hyperplanes



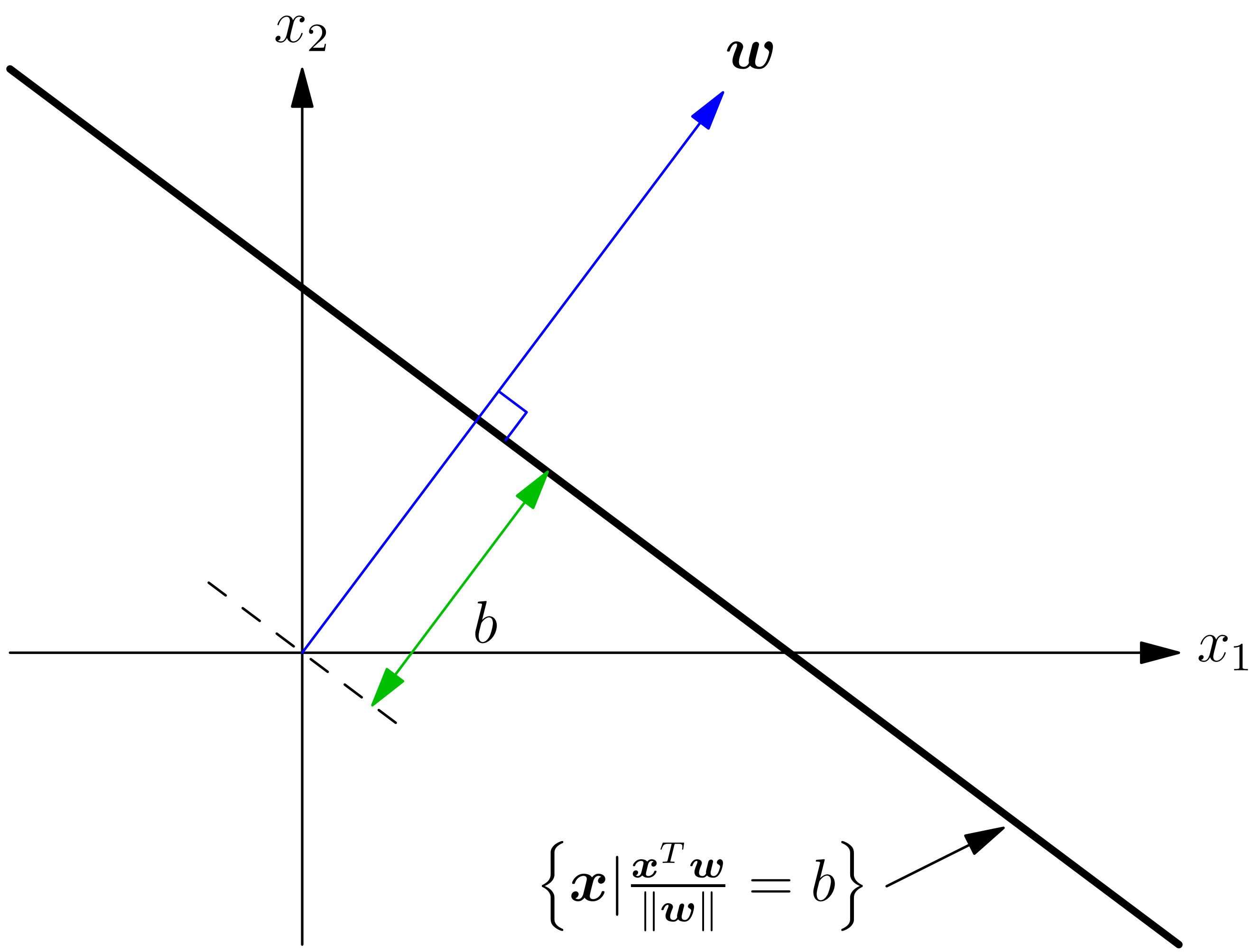
Distance to hyperplanes



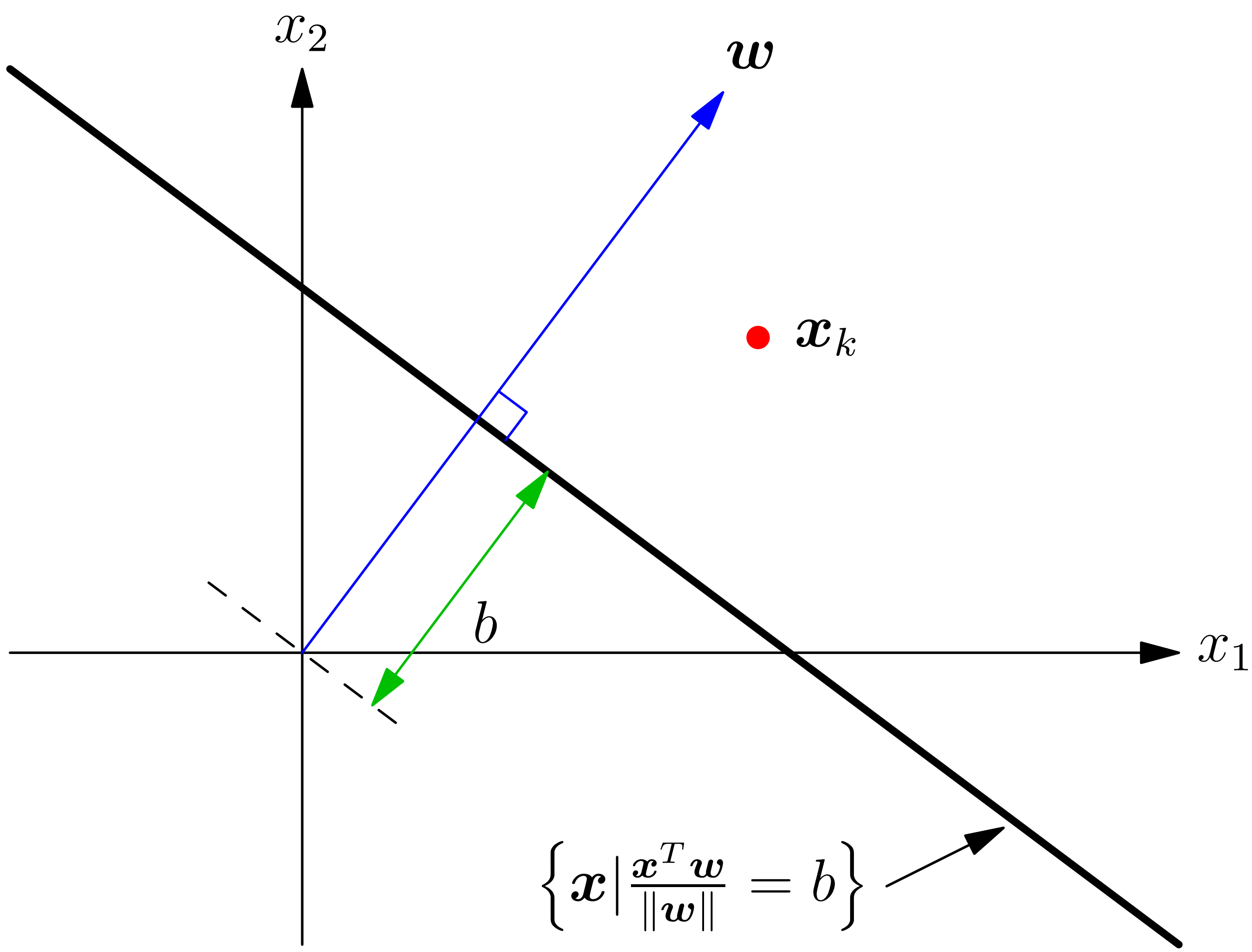
Distance to hyperplanes



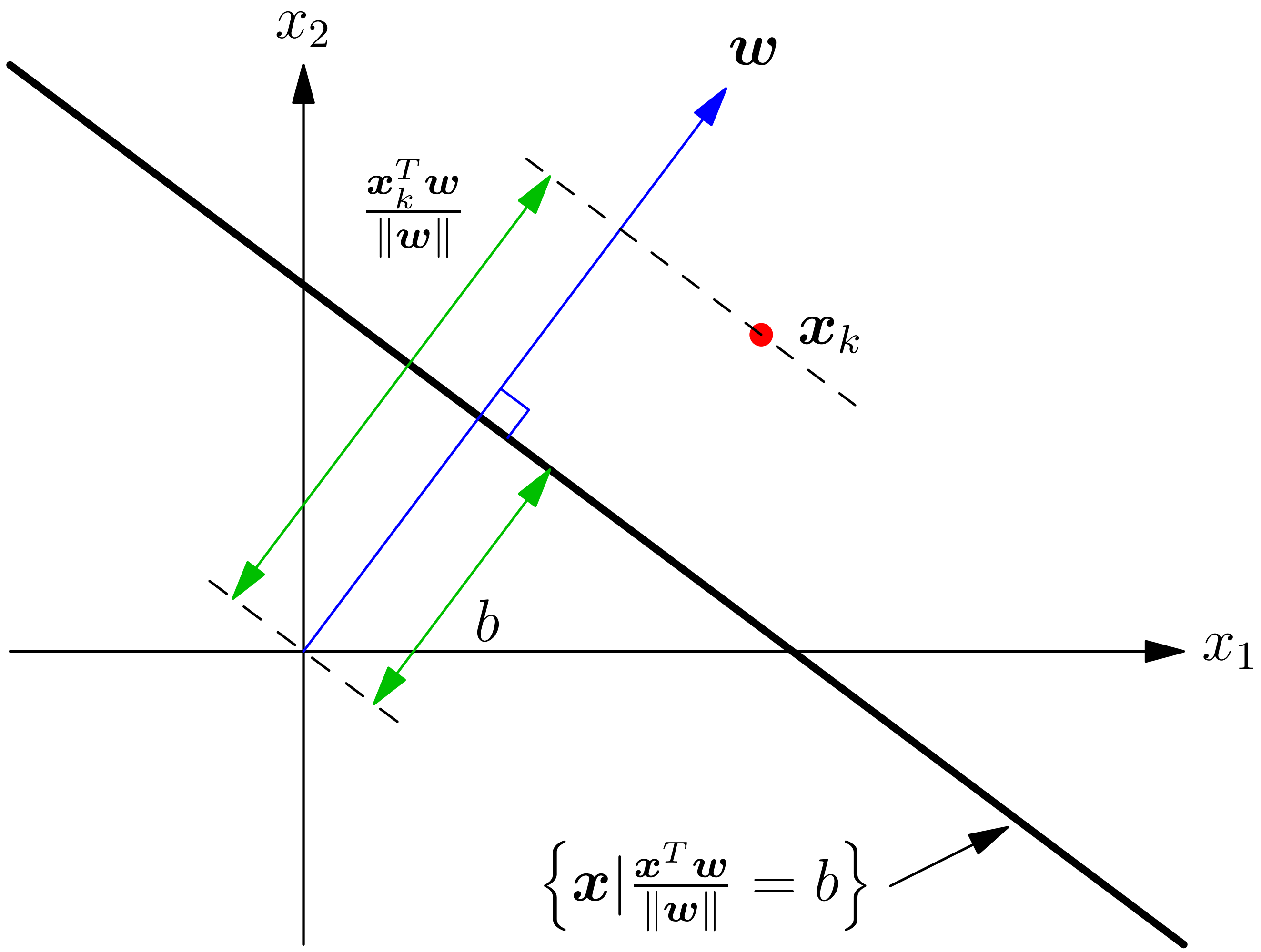
Distance to hyperplanes



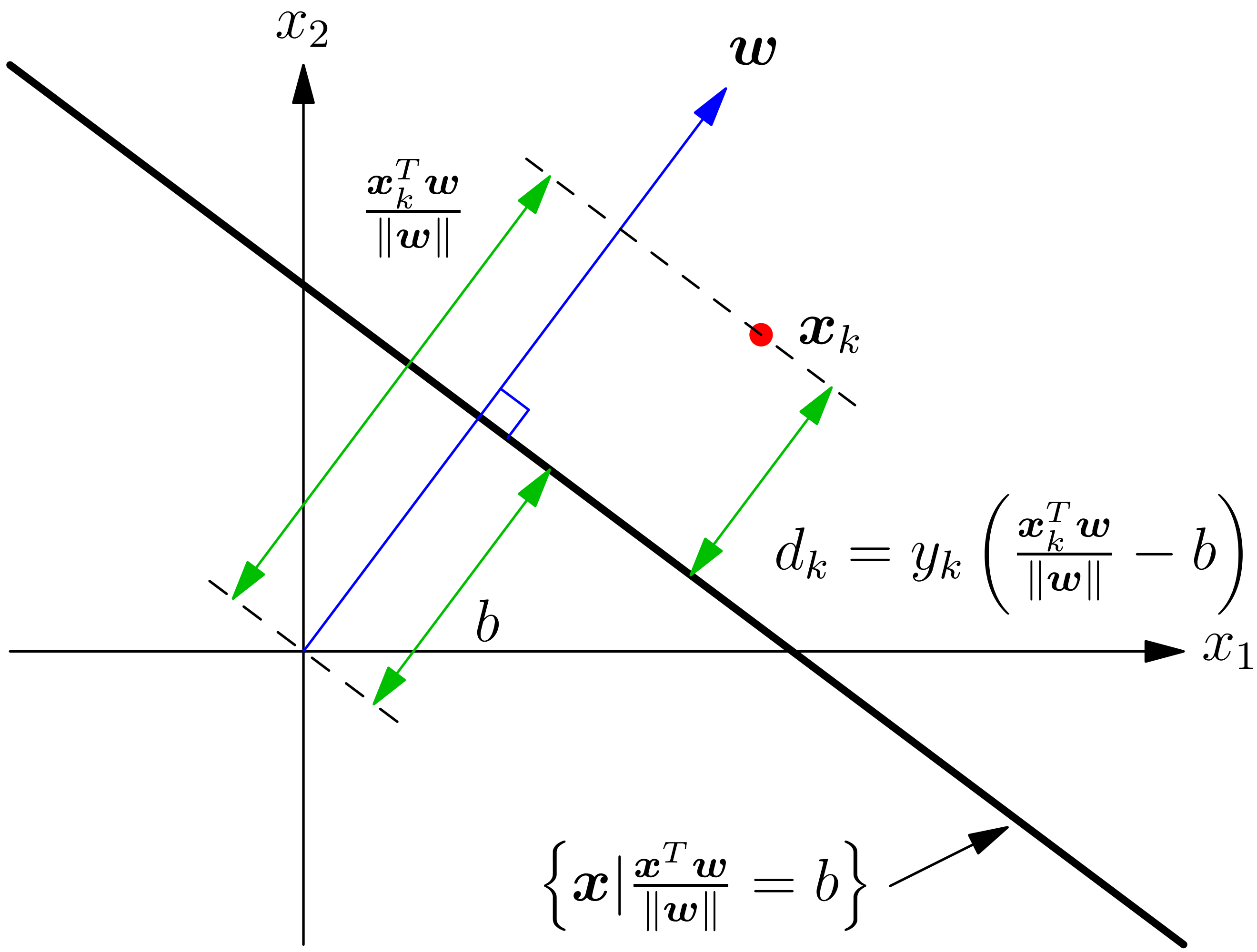
Distance to hyperplanes



Distance to hyperplanes



Distance to hyperplanes



Constrained Optimisation

- Wish to find $\boldsymbol{w}$ and b to maximise Δ subject to constraints

$$y_k \left(\frac{\boldsymbol{w}^\mathsf{T} \boldsymbol{x}_k}{\|\boldsymbol{w}\|} - b \right) \geq \Delta \quad \text{for all } k = 1, 2, \ldots, m$$

- If we divide through by Δ

$$y_k \left(\frac{\boldsymbol{w}^\mathsf{T} \boldsymbol{x}_k}{\Delta \|\boldsymbol{w}\|} - \frac{b}{\Delta} \right) \geq 1 \quad \text{for all } k = 1, 2, \ldots, m$$

- Define $\hat{\boldsymbol{w}} = \boldsymbol{w}/(\Delta\|\boldsymbol{w}\|)$ and $\hat{b} = b/\Delta$

$$y_k \left(\hat{\boldsymbol{w}}^\mathsf{T} \boldsymbol{x}_k - \hat{b} \right) \geq 1$$

Constrained Optimisation

- Wish to find $\boldsymbol{w}$ and b to maximise Δ subject to constraints

$$y_k \left(\frac{\boldsymbol{w}^\mathsf{T} \boldsymbol{x}_k}{\|\boldsymbol{w}\|} - b \right) \geq \Delta \quad \text{for all } k = 1, 2, \ldots, m$$

- If we divide through by Δ

$$y_k \left(\frac{\boldsymbol{w}^\mathsf{T} \boldsymbol{x}_k}{\Delta \|\boldsymbol{w}\|} - \frac{b}{\Delta} \right) \geq 1 \quad \text{for all } k = 1, 2, \ldots, m$$

- Define $\hat{\boldsymbol{w}} = \boldsymbol{w}/(\Delta\|\boldsymbol{w}\|)$ and $\hat{b} = b/\Delta$

$$y_k \left(\hat{\boldsymbol{w}}^\mathsf{T} \boldsymbol{x}_k - \hat{b} \right) \geq 1$$

Constrained Optimisation

- Wish to find $\boldsymbol{w}$ and b to maximise Δ subject to constraints

$$y_k \left(\frac{\boldsymbol{w}^\mathsf{T} \boldsymbol{x}_k}{\|\boldsymbol{w}\|} - b \right) \geq \Delta \quad \text{for all } k = 1, 2, \ldots, m$$

- If we divide through by Δ

$$y_k \left(\frac{\boldsymbol{w}^\mathsf{T} \boldsymbol{x}_k}{\Delta \|\boldsymbol{w}\|} - \frac{b}{\Delta} \right) \geq 1 \quad \text{for all } k = 1, 2, \ldots, m$$

- Define $\hat{\boldsymbol{w}} = \boldsymbol{w}/(\Delta\|\boldsymbol{w}\|)$ and $\hat{b} = b/\Delta$

$$y_k \left(\hat{\boldsymbol{w}}^\mathsf{T} \boldsymbol{x}_k - \hat{b} \right) \geq 1$$

Quadratic Programming Problem

- Note that as $\hat{\boldsymbol{w}} = \boldsymbol{w}/(\Delta\|\boldsymbol{w}\|)$

$$\|\hat{\boldsymbol{w}}\| = \left\| \frac{\boldsymbol{w}}{\Delta \, \|\boldsymbol{w}\|} \right\| = \frac{1}{\Delta \, \|\boldsymbol{w}\|} \, \|\boldsymbol{w}\| = \frac{1}{\Delta}$$

- Minimising $\|\hat{\boldsymbol{w}}\|^2$ is equivalent to maximising the margin Δ

- Can write the optimisation problem as a *quadratic programming problem*

$$\min_{\hat{\boldsymbol{w}}, \hat{b}} \frac{\|\hat{\boldsymbol{w}}\|^2}{2} \quad \text{subject to } y_k \left(\hat{\boldsymbol{w}}^\mathsf{T} \boldsymbol{x}_k - \hat{b} \right) \geq 1 \text{ for all } k = 1, 2, \ldots, m$$

Quadratic Programming Problem

- Note that as $\hat{\boldsymbol{w}} = \boldsymbol{w}/(\Delta\|\boldsymbol{w}\|)$

$$\|\hat{\boldsymbol{w}}\| = \left\|\frac{\boldsymbol{w}}{\Delta\,\|\boldsymbol{w}\|}\right\| = \frac{1}{\Delta\,\|\boldsymbol{w}\|}\,\|\boldsymbol{w}\| = \frac{1}{\Delta}$$

- Minimising $\|\hat{\boldsymbol{w}}\|^2$ is equivalent to maximising the margin Δ

- Can write the optimisation problem as a *quadratic programming problem*

$$\min_{\hat{\boldsymbol{w}},\,\hat{b}} \frac{\|\hat{\boldsymbol{w}}\|^2}{2} \quad \text{subject to } y_k\left(\hat{\boldsymbol{w}}^{\mathsf{T}}\boldsymbol{x}_k - \hat{b}\right) \geq 1 \text{ for all } k = 1,\, 2,\, \ldots,\, m$$

Quadratic Programming Problem

- Note that as $\hat{\boldsymbol{w}} = \boldsymbol{w}/(\Delta\|\boldsymbol{w}\|)$

$$\|\hat{\boldsymbol{w}}\| = \left\| \frac{\boldsymbol{w}}{\Delta \, \|\boldsymbol{w}\|} \right\| = \frac{1}{\Delta \, \|\boldsymbol{w}\|} \, \|\boldsymbol{w}\| = \frac{1}{\Delta}$$

- Minimising $\|\hat{\boldsymbol{w}}\|^2$ is equivalent to maximising the margin Δ

- Can write the optimisation problem as a *quadratic programming problem*

$$\min_{\hat{\boldsymbol{w}},\, \hat{b}} \frac{\|\hat{\boldsymbol{w}}\|^2}{2} \quad \text{subject to } y_k \left(\hat{\boldsymbol{w}}^{\mathsf{T}} \boldsymbol{x}_k - \hat{b} \right) \geq 1 \text{ for all } k = 1,\, 2,\, \ldots,\, m$$

Quadratic Programming Problem

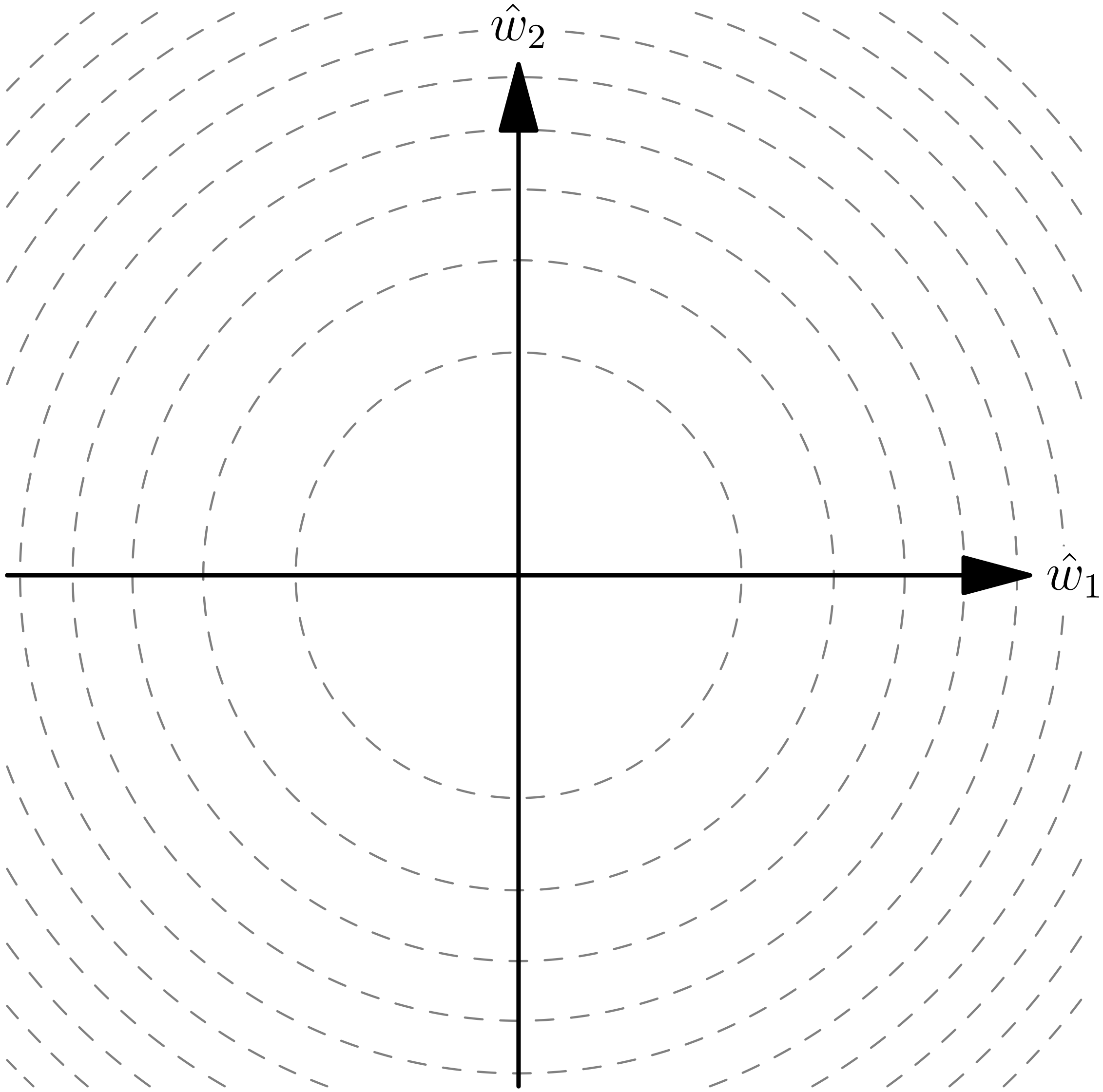
- Note that as $\hat{\boldsymbol{w}} = \boldsymbol{w}/(\Delta\|\boldsymbol{w}\|)$

$$\|\hat{\boldsymbol{w}}\| = \left\| \frac{\boldsymbol{w}}{\Delta\,\|\boldsymbol{w}\|} \right\| = \frac{1}{\Delta\,\|\boldsymbol{w}\|}\,\|\boldsymbol{w}\| = \frac{1}{\Delta}$$

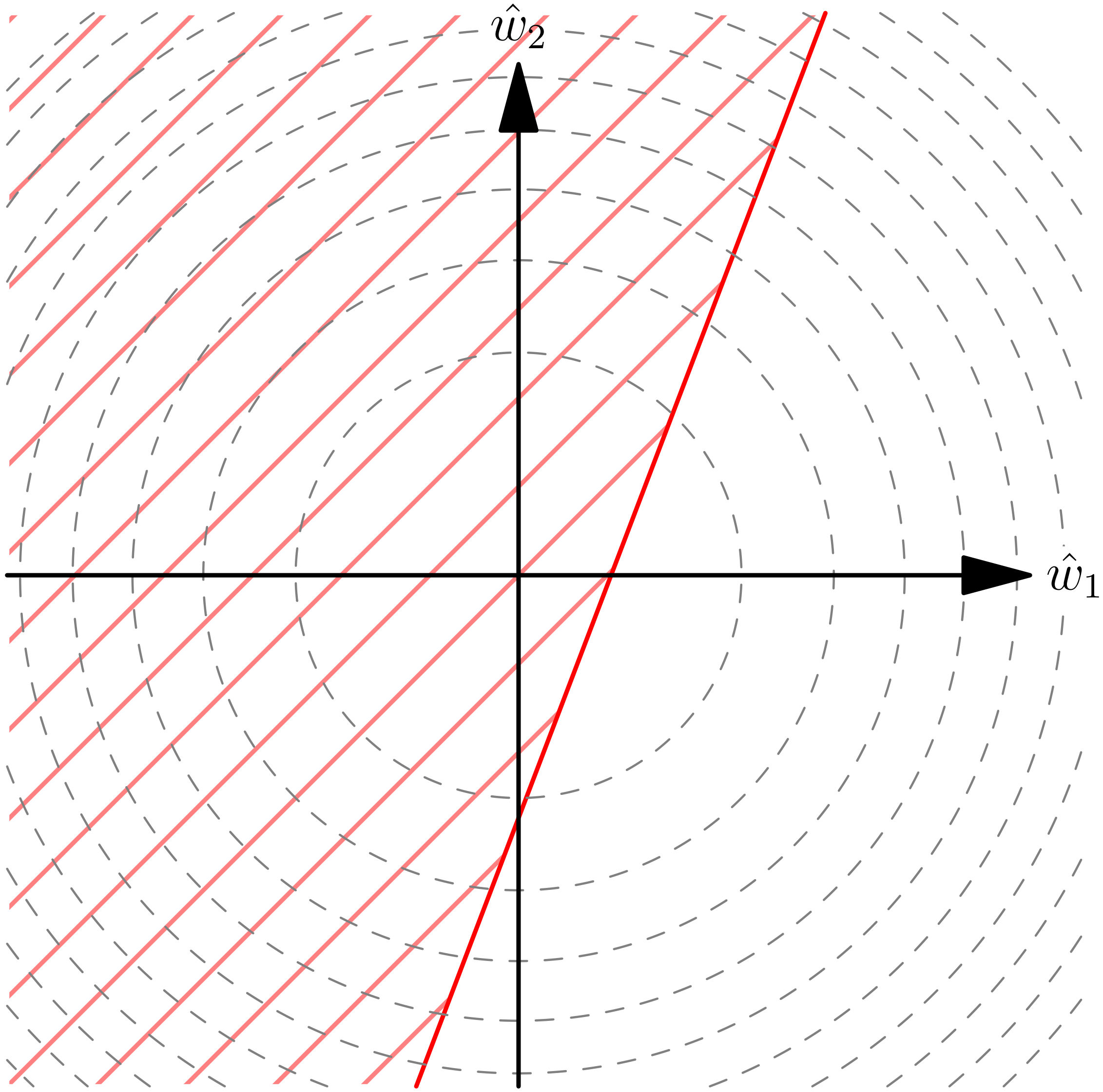
- Minimising $\|\hat{\boldsymbol{w}}\|^2$ is equivalent to maximising the margin Δ
- Can write the optimisation problem as a *quadratic programming problem*

$$\min_{\hat{\boldsymbol{w}},\,\hat{b}} \frac{\|\hat{\boldsymbol{w}}\|^2}{2} \quad \text{subject to } y_k \left(\hat{\boldsymbol{w}}^\mathsf{T} \boldsymbol{x}_k - \hat{b} \right) \geq 1 \text{ for all } k = 1, 2, \ldots, m$$

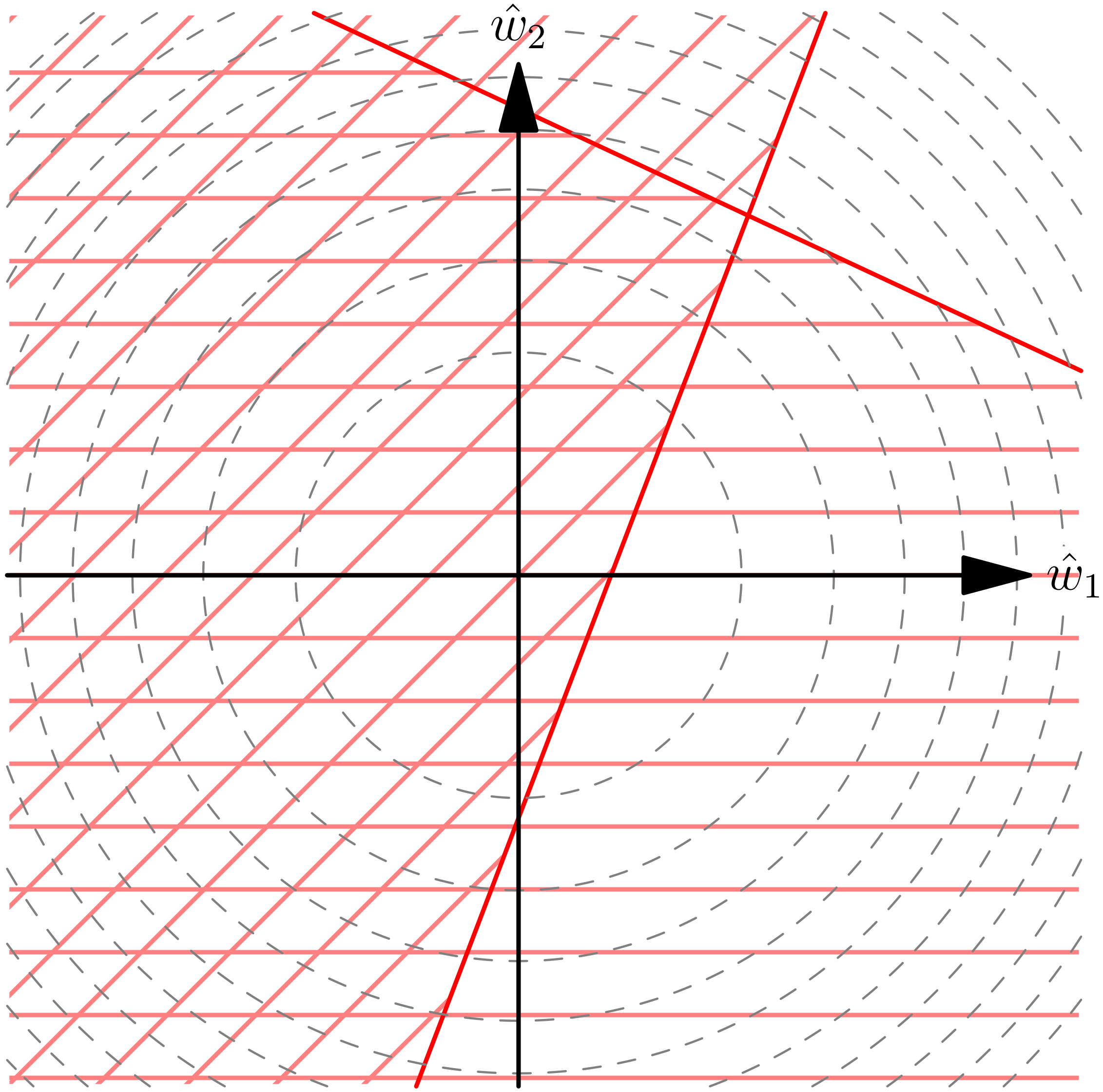
Quadratic Programming in SVMs



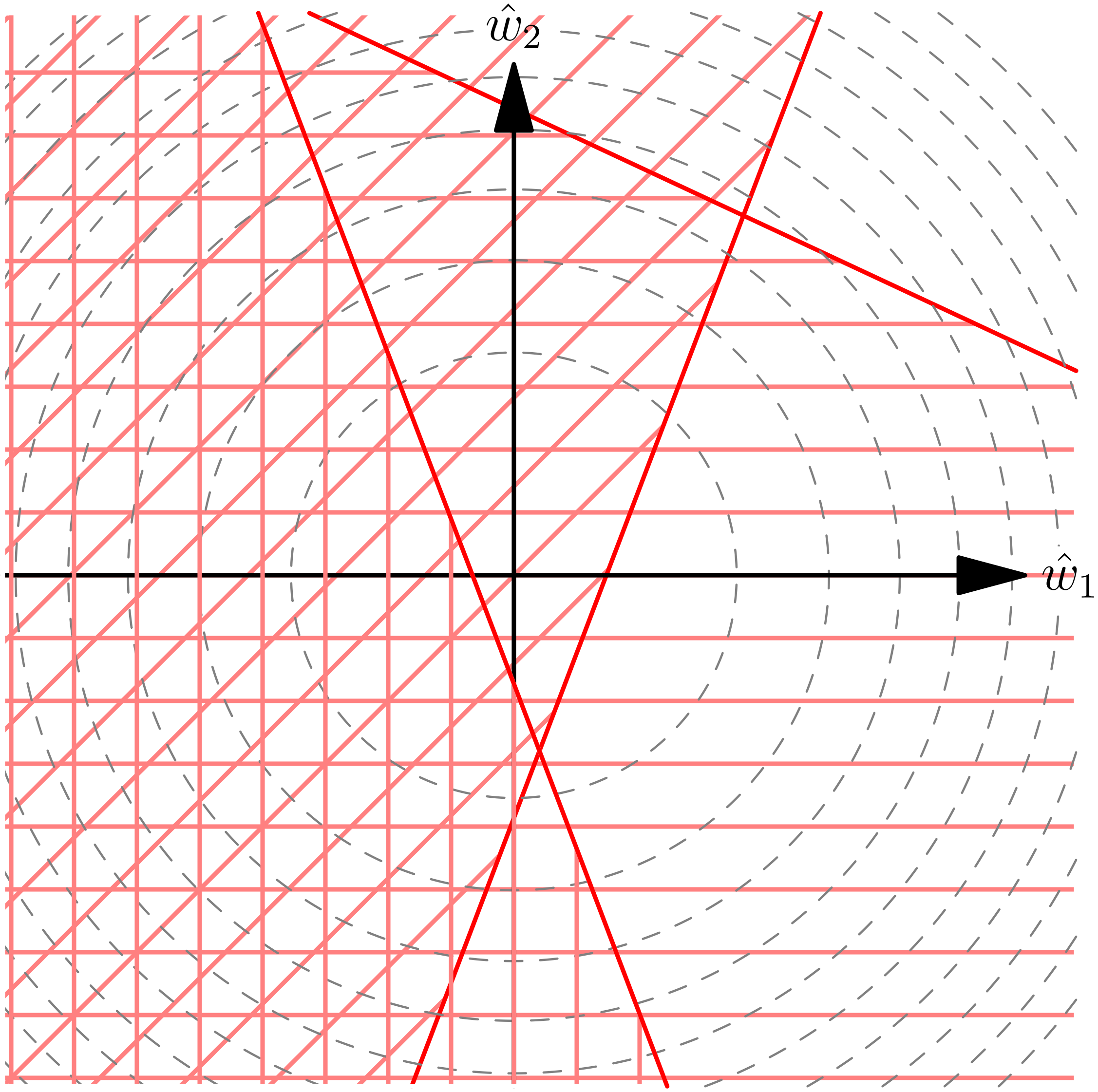
Quadratic Programming in SVMs



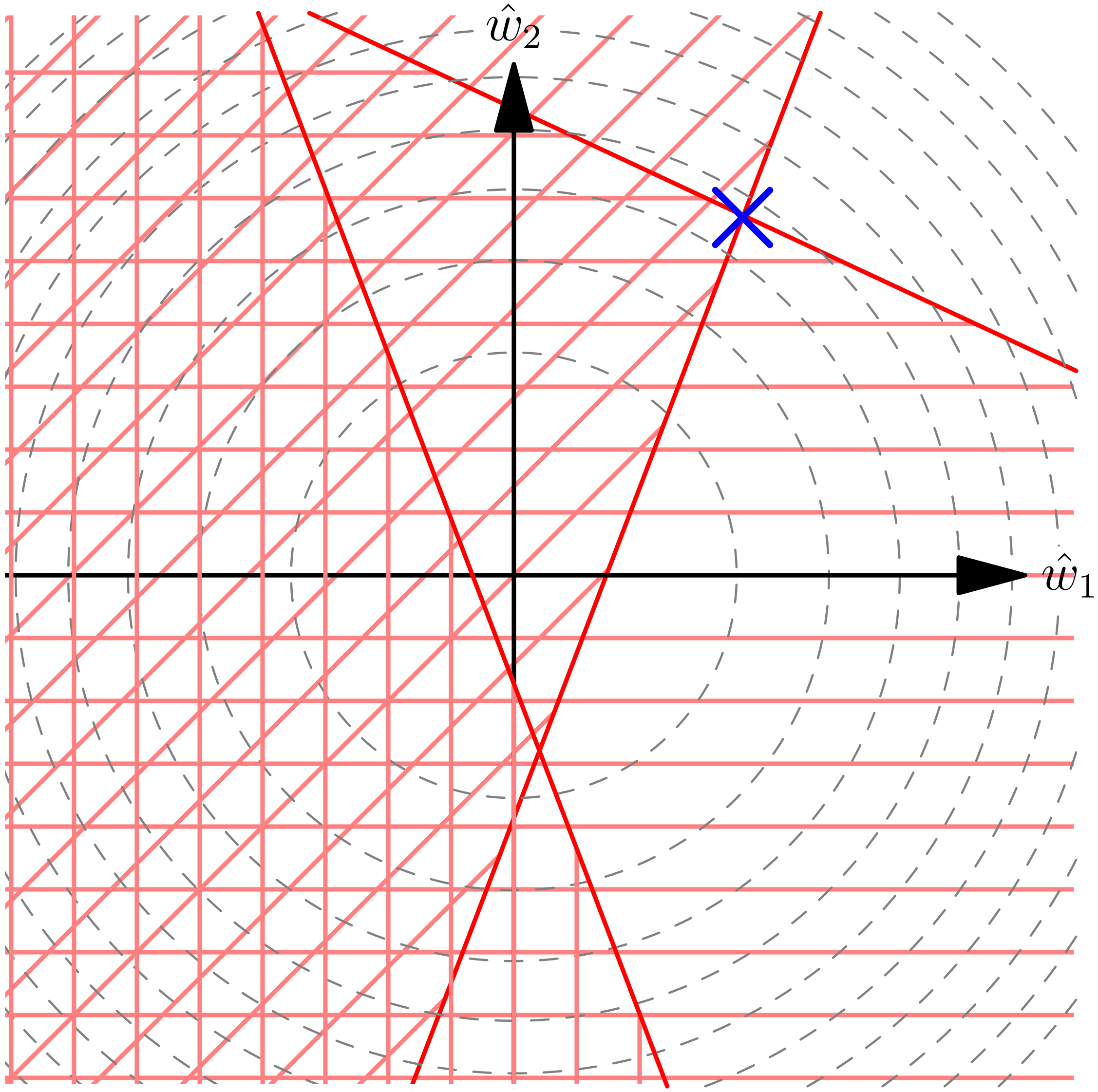
Quadratic Programming in SVMs



Quadratic Programming in SVMs



Quadratic Programming in SVMs



Quadratic Programming

- We have a quadratic programming problem for the weights $\hat{\boldsymbol{w}} = (\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_p)$ and bias $\hat{b}$ and m constraints

- This is a classic but fiddly optimisation problems

- It can be solved in $O(p^3)$ time (it involves inverting matrices)

- We will see that there is an equivalent dual problem which allows us to use the kernel trick with time complexity $O(m^3)$

Quadratic Programming

- We have a quadratic programming problem for the weights $\hat{\boldsymbol{w}} = (\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_p)$ and bias $\hat{b}$ and m constraints
- This is a classic but fiddly optimisation problems
- It can be solved in $O(p^3)$ time (it involves inverting matrices)
- We will see that there is an equivalent dual problem which allows us to use the kernel trick with time complexity $O(m^3)$

Quadratic Programming

- We have a quadratic programming problem for the weights $\hat{\boldsymbol{w}} = (\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_p)$ and bias $\hat{b}$ and m constraints
- This is a classic but fiddly optimisation problems
- It can be solved in $O(p^3)$ time (it involves inverting matrices)
- We will see that there is an equivalent dual problem which allows us to use the kernel trick with time complexity $O(m^3)$

Quadratic Programming

- We have a quadratic programming problem for the weights $\hat{\boldsymbol{w}} = (\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_p)$ and bias $\hat{b}$ and m constraints
- This is a classic but fiddly optimisation problems
- It can be solved in $O(p^3)$ time (it involves inverting matrices) (phew it is not NP-complete!)
- We will see that there is an equivalent dual problem which allows us to use the kernel trick with time complexity $O(m^3)$

Quadratic Programming

- We have a quadratic programming problem for the weights $\hat{\boldsymbol{w}} = (\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_p)$ and bias $\hat{b}$ and m constraints

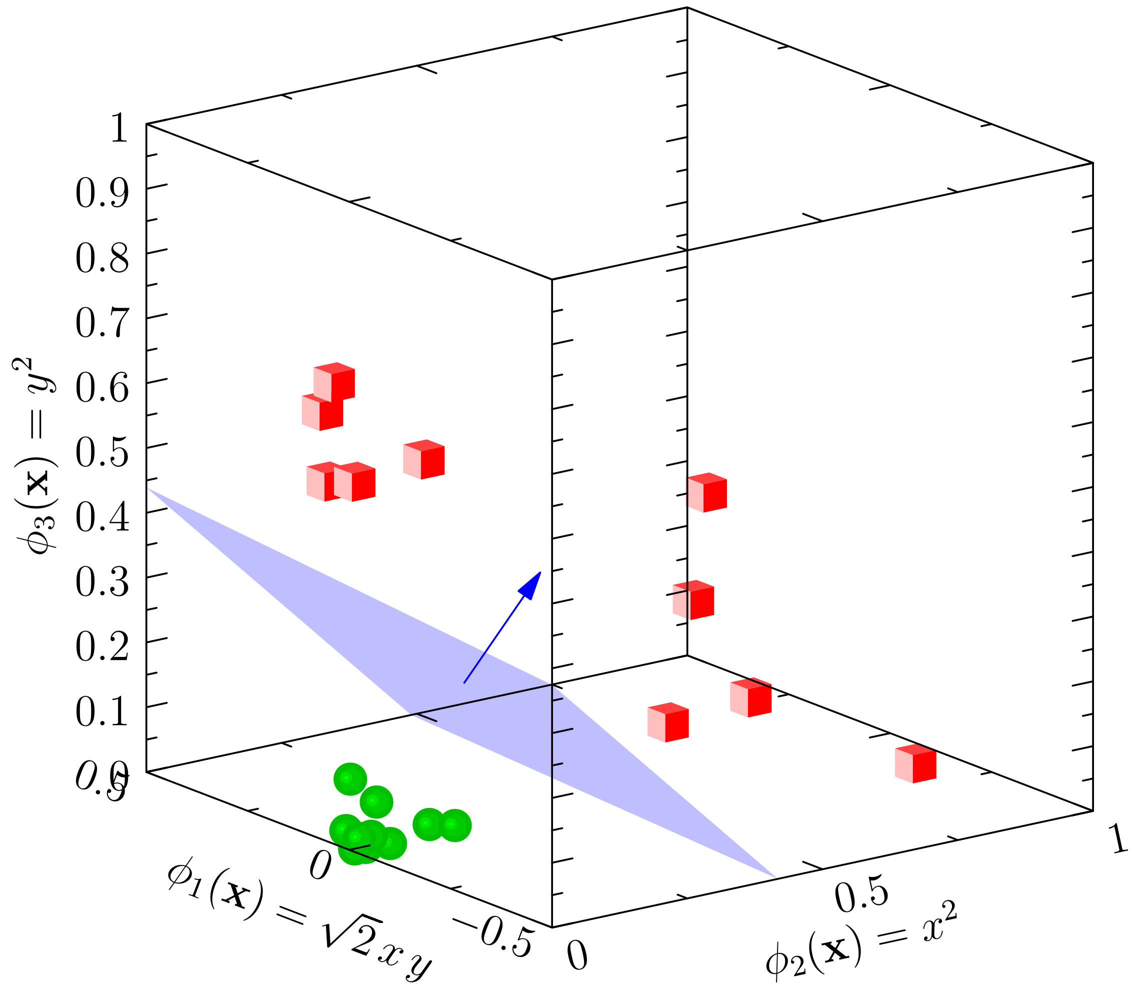
- This is a classic but fiddly optimisation problems

- It can be solved in $O(p^3)$ time (it involves inverting matrices) (phew it is not NP-complete!)

- We will see that there is an equivalent dual problem which allows us to use the kernel trick with time complexity $O(m^3)$

Outline

1. The Big Picture
2. Maximum Margins
3. **Duality**
4. Practice



Extended Feature Space

- We can generalise the SVM if we map all our features vectors to an extended feature space

$$\boldsymbol{x} \to \boldsymbol{\phi}(\boldsymbol{x})$$

- The components of $\boldsymbol{\phi}(\boldsymbol{x})$ will typically be (non-linear) functions of $\boldsymbol{x}$ (e.g. $\phi_1(\boldsymbol{x}) = x_1^2$, $\phi_1(\boldsymbol{x}) = x_2^2$, $\phi_1(\boldsymbol{x}) = \sqrt{2}\, x_1\, x_2$)

- We are free to choose whatever mappings we like

- There may be many more components of $\boldsymbol{\phi}(\boldsymbol{x})$ than of $\boldsymbol{x}$

- But in the extended feature space (involving $\boldsymbol{\phi}(\boldsymbol{x}) = \big(\phi_1(\boldsymbol{x}), \phi_2(\boldsymbol{x}), \ldots, \phi_r(\boldsymbol{x})\big)$) the time complexity is $O(r^3)$

Extended Feature Space

- We can generalise the SVM if we map all our features vectors to an extended feature space

$$\boldsymbol{x} \to \boldsymbol{\phi}(\boldsymbol{x})$$

- The components of $\boldsymbol{\phi}(\boldsymbol{x})$ will typically be (non-linear) functions of $\boldsymbol{x}$ (e.g. $\phi_1(\boldsymbol{x}) = x_1^2$, $\phi_1(\boldsymbol{x}) = x_2^2$, $\phi_1(\boldsymbol{x}) = \sqrt{2}\, x_1\, x_2$)
- We are free to choose whatever mappings we like
- There may be many more components of $\boldsymbol{\phi}(\boldsymbol{x})$ than of $\boldsymbol{x}$
- But in the extended feature space (involving $\boldsymbol{\phi}(\boldsymbol{x}) = \big(\phi_1(\boldsymbol{x}), \phi_2(\boldsymbol{x}), \ldots, \phi_r(\boldsymbol{x})\big)$) the time complexity is $O(r^3)$

Extended Feature Space

- We can generalise the SVM if we map all our features vectors to an extended feature space

$$\boldsymbol{x} \to \boldsymbol{\phi}(\boldsymbol{x})$$

- The components of $\boldsymbol{\phi}(\boldsymbol{x})$ will typically be (non-linear) functions of $\boldsymbol{x}$ (e.g. $\phi_1(\boldsymbol{x}) = x_1^2$, $\phi_1(\boldsymbol{x}) = x_2^2$, $\phi_1(\boldsymbol{x}) = \sqrt{2}\, x_1\, x_2$)

- We are free to choose whatever mappings we like

- There may be many more components of $\boldsymbol{\phi}(\boldsymbol{x})$ than of $\boldsymbol{x}$

- But in the extended feature space (involving $\boldsymbol{\phi}(\boldsymbol{x}) = \big(\phi_1(\boldsymbol{x}), \phi_2(\boldsymbol{x}), \ldots, \phi_r(\boldsymbol{x})\big)$) the time complexity is $O(r^3)$

Extended Feature Space

- We can generalise the SVM if we map all our features vectors to an extended feature space

$$\boldsymbol{x} \to \boldsymbol{\phi}(\boldsymbol{x})$$

- The components of $\boldsymbol{\phi}(\boldsymbol{x})$ will typically be (non-linear) functions of $\boldsymbol{x}$ (e.g. $\phi_1(\boldsymbol{x}) = x_1^2$, $\phi_1(\boldsymbol{x}) = x_2^2$, $\phi_1(\boldsymbol{x}) = \sqrt{2}\, x_1\, x_2$)
- We are free to choose whatever mappings we like
- There may be many more components of $\boldsymbol{\phi}(\boldsymbol{x})$ than of $\boldsymbol{x}$
- But in the extended feature space (involving $\boldsymbol{\phi}(\boldsymbol{x}) = \big(\phi_1(\boldsymbol{x}), \phi_2(\boldsymbol{x}), \ldots, \phi_r(\boldsymbol{x})\big)$) the time complexity is $O(r^3)$

Extended Feature Space

- We can generalise the SVM if we map all our features vectors to an extended feature space

$$\boldsymbol{x} \to \boldsymbol{\phi}(\boldsymbol{x})$$

- The components of $\boldsymbol{\phi}(\boldsymbol{x})$ will typically be (non-linear) functions of $\boldsymbol{x}$ (e.g. $\phi_1(\boldsymbol{x}) = x_1^2$, $\phi_1(\boldsymbol{x}) = x_2^2$, $\phi_1(\boldsymbol{x}) = \sqrt{2}\, x_1\, x_2$)

- We are free to choose whatever mappings we like

- There may be many more components of $\boldsymbol{\phi}(\boldsymbol{x})$ than of $\boldsymbol{x}$ making it easier to find a linear separation of the two classes

- But in the extended feature space (involving $\boldsymbol{\phi}(\boldsymbol{x}) = \big(\phi_1(\boldsymbol{x}), \phi_2(\boldsymbol{x}), \ldots, \phi_r(\boldsymbol{x})\big)$) the time complexity is $O(r^3)$

Extended Feature Space

- We can generalise the SVM if we map all our features vectors to an extended feature space

$$\boldsymbol{x} \to \boldsymbol{\phi}(\boldsymbol{x})$$

- The components of $\boldsymbol{\phi}(\boldsymbol{x})$ will typically be (non-linear) functions of $\boldsymbol{x}$ (e.g. $\phi_1(\boldsymbol{x}) = x_1^2$, $\phi_1(\boldsymbol{x}) = x_2^2$, $\phi_1(\boldsymbol{x}) = \sqrt{2}\, x_1\, x_2$)

- We are free to choose whatever mappings we like

- There may be many more components of $\boldsymbol{\phi}(\boldsymbol{x})$ than of $\boldsymbol{x}$ making it easier to find a linear separation of the two classes

- But in the extended feature space (involving $\boldsymbol{\phi}(\boldsymbol{x}) = \big(\phi_1(\boldsymbol{x}), \phi_2(\boldsymbol{x}), \ldots, \phi_r(\boldsymbol{x})\big)$) the time complexity is $O(r^3)$

Lagrangian

- In the extended feature space we can find a separating plane (given by $\boldsymbol{w}$ and b) with maximum margine by solving the problem

$$\min_{\boldsymbol{w},\, b} \frac{\|\boldsymbol{w}\|^2}{2} \quad \text{subject to } y_k \left(\boldsymbol{w}^\mathsf{T}\boldsymbol{\phi}(\boldsymbol{x}_k) - b\right) \geq 1 \text{ for all } k = 1,\, 2,\, \ldots,\, m$$

- We can write this as a Lagrange problem

$$\min_{\boldsymbol{w},\, b} \max_{\boldsymbol{\alpha} \geq \mathbf{0}} \mathcal{L}(\boldsymbol{w}, b, \boldsymbol{\alpha})$$

where

$$\mathcal{L}(\boldsymbol{w}, b, \boldsymbol{\alpha}) = \frac{\|\boldsymbol{w}\|^2}{2} - \sum_{k=1}^{m} \alpha_k \left(y_k \left(\boldsymbol{w}^\mathsf{T}\boldsymbol{\phi}(\boldsymbol{x}_k) - b\right) - 1\right)$$

subject to $\alpha_k \geq 0$

Lagrangian

- In the extended feature space we can find a separating plane (given by $\boldsymbol{w}$ and b) with maximum margine by solving the problem

$$\min_{\boldsymbol{w},\, b} \frac{\|\boldsymbol{w}\|^2}{2} \quad \text{subject to } y_k \left(\boldsymbol{w}^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_k) - b\right) \geq 1 \text{ for all } k = 1,\, 2,\, \ldots,\, m$$

- We can write this as a Lagrange problem

$$\min_{\boldsymbol{w},\, b} \max_{\boldsymbol{\alpha} \geq \boldsymbol{0}} \mathcal{L}(\boldsymbol{w}, b, \boldsymbol{\alpha})$$

where

$$\mathcal{L}(\boldsymbol{w}, b, \boldsymbol{\alpha}) = \frac{\|\boldsymbol{w}\|^2}{2} - \sum_{k=1}^{m} \alpha_k \left(y_k \left(\boldsymbol{w}^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_k) - b\right) - 1\right)$$

subject to $\alpha_k \geq 0$

Obtaining the Dual Form of the Problem

- Differentiating the Lagrangian with respect to $\boldsymbol{w}$

$$\mathcal{L}(\boldsymbol{w}, b, \boldsymbol{\alpha}) = \frac{1}{2}\|\boldsymbol{w}\|^2 - \sum_{k=1}^{m} \alpha_k \left(y_k \left(\boldsymbol{w}^\mathsf{T} \boldsymbol{\phi}(\boldsymbol{x}_k) - b \right) - 1 \right)$$

- $\boldsymbol{\nabla}_{\boldsymbol{w}} \mathcal{L} = \boldsymbol{w} - \sum_{k=1}^{m} \alpha_k \, y_k \, \boldsymbol{\phi}(\boldsymbol{x}_k) = 0$ implies that $\boldsymbol{w}^* = \sum_{k=1}^{m} \alpha_k \, y_k \, \boldsymbol{\phi}(\boldsymbol{x}_k)$

- $\frac{\partial \mathcal{L}}{\partial b} = \sum_{k=1}^{m} \alpha_k \, y_k = 0$ implies $\sum_{k=1}^{m} \alpha_k \, y_k = 0$

- Substituting back into the Lagrangian

$$\max_{\boldsymbol{\alpha} \geq \boldsymbol{0}} \sum_{k=1}^{m} \alpha_k - \frac{1}{2} \sum_{k,l=1}^{m} \alpha_k \, \alpha_l \, y_k \, y_l \, \boldsymbol{\phi}(\boldsymbol{x}_k)^\mathsf{T} \boldsymbol{\phi}(\boldsymbol{x}_l)$$

Obtaining the Dual Form of the Problem

- Differentiating the Lagrangian with respect to $\boldsymbol{w}$

$$\mathcal{L}(\boldsymbol{w}, b, \boldsymbol{\alpha}) = \frac{1}{2}\|\boldsymbol{w}\|^2 - \sum_{k=1}^{m} \alpha_k \left(y_k \left(\boldsymbol{w}^\mathsf{T} \boldsymbol{\phi}(\boldsymbol{x}_k) - b\right) - 1\right)$$

- $\boldsymbol{\nabla}_{\boldsymbol{w}} \mathcal{L} = \boldsymbol{w} - \sum_{k=1}^{m} \alpha_k \, y_k \, \boldsymbol{\phi}(\boldsymbol{x}_k) = 0$ implies that
  $\boldsymbol{w}^* = \sum_{k=1}^{m} \alpha_k \, y_k \, \boldsymbol{\phi}(\boldsymbol{x}_k)$

- $\frac{\partial \mathcal{L}}{\partial b} = \sum_{k=1}^{m} \alpha_k \, y_k = 0$ implies $\sum_{k=1}^{m} \alpha_k \, y_k = 0$

- Substituting back into the Lagrangian

$$\max_{\boldsymbol{\alpha} \geq \mathbf{0}} \sum_{k=1}^{m} \alpha_k - \frac{1}{2} \sum_{k,l=1}^{m} \alpha_k \, \alpha_l \, y_k \, y_l \, \boldsymbol{\phi}(\boldsymbol{x}_k)^\mathsf{T} \, \boldsymbol{\phi}(\boldsymbol{x}_l)$$

Obtaining the Dual Form of the Problem

- Differentiating the Lagrangian with respect to $\boldsymbol{w}$

$$\mathcal{L}(\boldsymbol{w}, b, \boldsymbol{\alpha}) = \frac{1}{2}\|\boldsymbol{w}\|^2 - \sum_{k=1}^{m} \alpha_k \left(y_k \left(\boldsymbol{w}^\mathsf{T}\boldsymbol{\phi}(\boldsymbol{x}_k) - b\right) - 1\right)$$

- $\boldsymbol{\nabla}_{\boldsymbol{w}}\mathcal{L} = \boldsymbol{w} - \sum_{k=1}^{m} \alpha_k\, y_k\, \boldsymbol{\phi}(\boldsymbol{x}_k) = 0$ implies that $\boldsymbol{w}^* = \sum_{k=1}^{m} \alpha_k\, y_k\, \boldsymbol{\phi}(\boldsymbol{x}_k)$

- $\frac{\partial\, \mathcal{L}}{\partial\, b} = \sum_{k=1}^{m} \alpha_k\, y_k = 0$ implies $\sum_{k=1}^{m} \alpha_k\, y_k = 0$

- Substituting back into the Lagrangian

$$\max_{\boldsymbol{\alpha} \geq \boldsymbol{0}} \sum_{k=1}^{m} \alpha_k - \frac{1}{2} \sum_{k,l=1}^{m} \alpha_k\, \alpha_l\, y_k\, y_l\, \boldsymbol{\phi}(\boldsymbol{x}_k)^\mathsf{T}\, \boldsymbol{\phi}(\boldsymbol{x}_l)$$

Obtaining the Dual Form of the Problem

- Differentiating the Lagrangian with respect to $\boldsymbol{w}$

$$\mathcal{L}(\boldsymbol{w}, b, \boldsymbol{\alpha}) = \frac{1}{2}\|\boldsymbol{w}\|^2 - \sum_{k=1}^{m} \alpha_k \left(y_k \left(\boldsymbol{w}^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_k) - b \right) - 1 \right)$$

- $\boldsymbol{\nabla}_{\boldsymbol{w}} \mathcal{L} = \boldsymbol{w} - \sum_{k=1}^{m} \alpha_k \, y_k \, \boldsymbol{\phi}(\boldsymbol{x}_k) = 0$ implies that $\boldsymbol{w}^* = \sum_{k=1}^{m} \alpha_k \, y_k \, \boldsymbol{\phi}(\boldsymbol{x}_k)$

- $\frac{\partial \mathcal{L}}{\partial b} = \sum_{k=1}^{m} \alpha_k \, y_k = 0$ implies $\sum_{k=1}^{m} \alpha_k \, y_k = 0$

- Substituting back into the Lagrangian

$$\max_{\boldsymbol{\alpha} \geq \boldsymbol{0}} \sum_{k=1}^{m} \alpha_k - \frac{1}{2} \sum_{k,l=1}^{m} \alpha_k \, \alpha_l \, y_k \, y_l \, \boldsymbol{\phi}(\boldsymbol{x}_k)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_l)$$

The Dual Problem

- The dual problem is now to find α_k's that maximise

$$\mathcal{L}(\boldsymbol{\alpha}) = \sum_{k=1}^{m} \alpha_k - \frac{1}{2} \sum_{k,l=1}^{m} \alpha_k \, \alpha_l \, y_k \, y_l \, \boldsymbol{\phi}(\boldsymbol{x}_k)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_l)$$

- subject to constraints

$$\sum_{k=1}^{m} \alpha_k \, y_k = 0 \qquad \qquad \forall k = 1, 2, \ldots, m \quad \alpha_k \geq 0$$

- The Hessian of $\mathcal{L}(\boldsymbol{\alpha})$ is a matrix of the form $-\mathbf{X}^{\mathsf{T}} \mathbf{X}$ where $X_{ik} = y_k \, \phi_i(\boldsymbol{x}_k)$

The Dual Problem

- The dual problem is now to find α_k's that maximise

$$\mathcal{L}(\boldsymbol{\alpha}) = \sum_{k=1}^{m} \alpha_k - \frac{1}{2} \sum_{k,l=1}^{m} \alpha_k \, \alpha_l \, y_k \, y_l \, \boldsymbol{\phi}(\boldsymbol{x}_k)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_l)$$

- subject to constraints

$$\sum_{k=1}^{m} \alpha_k \, y_k = 0 \qquad \forall k = 1, 2, \ldots, m \quad \alpha_k \geq 0$$

- The Hessian of $\mathcal{L}(\boldsymbol{\alpha})$ is a matrix of the form $-\mathbf{X}^{\mathsf{T}} \mathbf{X}$ where $X_{ik} = y_k \, \phi_i(\boldsymbol{x}_k)$

The Dual Problem

- The dual problem is now to find α_k's that maximise

$$\mathcal{L}(\boldsymbol{\alpha}) = \sum_{k=1}^{m} \alpha_k - \frac{1}{2} \sum_{k,l=1}^{m} \alpha_k \, \alpha_l \, y_k \, y_l \, \boldsymbol{\phi}(\boldsymbol{x}_k)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_l)$$

- subject to constraints

$$\sum_{k=1}^{m} \alpha_k \, y_k = 0 \qquad\qquad \forall k = 1, 2, \ldots, m \quad \alpha_k \geq 0$$

- The Hessian of $\mathcal{L}(\boldsymbol{\alpha})$ is a matrix of the form $-\mathbf{X}^{\mathsf{T}} \mathbf{X}$ where $X_{ik} = y_k \, \phi_i(\boldsymbol{x}_k)$

The Dual Problem

- The dual problem is now to find α_k's that maximise

$$\mathcal{L}(\boldsymbol{\alpha}) = \sum_{k=1}^{m} \alpha_k - \frac{1}{2} \sum_{k,l=1}^{m} \alpha_k \, \alpha_l \, y_k \, y_l \, \boldsymbol{\phi}(\boldsymbol{x}_k)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_l)$$

- subject to constraints

$$\sum_{k=1}^{m} \alpha_k \, y_k = 0 \qquad\qquad \forall k = 1, 2, \ldots, m \quad \alpha_k \geq 0$$

- The Hessian of $\mathcal{L}(\boldsymbol{\alpha})$ is a matrix of the form $-\mathbf{X}^{\mathsf{T}} \mathbf{X}$ where $X_{ik} = y_k \, \phi_i(\boldsymbol{x}_k)$ (note this is negative semi-definite so there is a unique maximum)

Kernel Trick

- We will show in the next lecture that if $K(\boldsymbol{x}, \boldsymbol{y})$ is a positive semi-definite function then it can always be written as

$$K(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{\phi}(\boldsymbol{x})^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{y})$$

- As $\boldsymbol{\phi}(\boldsymbol{x}_k)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_l)$ appears in the dual problem we can express the dual problem as finding α_k's that maximise

$$\mathcal{L}(\boldsymbol{\alpha}) = \sum_{k=1}^{m} \alpha_k - \frac{1}{2} \sum_{k,l=1}^{m} \alpha_k \, \alpha_l \, y_k \, y_l \, K(\boldsymbol{x}_k, \boldsymbol{x}_l)$$

- We therefore never have to compute $\boldsymbol{\phi}(\boldsymbol{x})$

Kernel Trick

- We will show in the next lecture that if $K(\boldsymbol{x}, \boldsymbol{y})$ is a positive semi-definite function then it can always be written as

$$K(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{\phi}(\boldsymbol{x})^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{y})$$

- As $\boldsymbol{\phi}(\boldsymbol{x}_k)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_l)$ appears in the dual problem we can express the dual problem as finding α_k's that maximise

$$\mathcal{L}(\boldsymbol{\alpha}) = \sum_{k=1}^{m} \alpha_k - \frac{1}{2} \sum_{k,l=1}^{m} \alpha_k \, \alpha_l \, y_k \, y_l \, K(\boldsymbol{x}_k, \boldsymbol{x}_l)$$

- We therefore never have to compute $\boldsymbol{\phi}(\boldsymbol{x})$

Kernel Trick

- We will show in the next lecture that if $K(\boldsymbol{x}, \boldsymbol{y})$ is a positive semi-definite function then it can always be written as

$$K(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{\phi}(\boldsymbol{x})^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{y})$$

- As $\boldsymbol{\phi}(\boldsymbol{x}_k)^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_l)$ appears in the dual problem we can express the dual problem as finding α_k's that maximise

$$\mathcal{L}(\boldsymbol{\alpha}) = \sum_{k=1}^{m} \alpha_k - \frac{1}{2} \sum_{k,l=1}^{m} \alpha_k \, \alpha_l \, y_k \, y_l \, K(\boldsymbol{x}_k, \boldsymbol{x}_l)$$

- We therefore never have to compute $\boldsymbol{\phi}(\boldsymbol{x})$

Sequential Minimal Optimisation

- One of the most efficient techniques for training SVMs is *Sequential Minimal Optimisation* or SMO
- This takes two Lagrange multipliers α_i and α_j and adjusts them to maximise the dual objective function
- This is very quick as it can be done in closed form
- Note that because $\sum_{k=1}^{m} y_k \alpha_k = 0$ we have to change at least two variables at the same time
- A heuristic is used to choose the best pair of α's to optimise
- Run until close to the optimum

Sequential Minimal Optimisation

- One of the most efficient techniques for training SVMs is *Sequential Minimal Optimisation* or SMO
- This takes two Lagrange multipliers α_i and α_j and adjusts them to maximise the dual objective function
- This is very quick as it can be done in closed form
- Note that because $\sum_{k=1}^{m} y_k \, \alpha_k = 0$ we have to change at least two variables at the same time
- A heuristic is used to choose the best pair of α's to optimise
- Run until close to the optimum

Sequential Minimal Optimisation

- One of the most efficient techniques for training SVMs is *Sequential Minimal Optimisation* or SMO
- This takes two Lagrange multipliers α_i and α_j and adjusts them to maximise the dual objective function
- This is very quick as it can be done in closed form
- Note that because $\sum_{k=1}^{m} y_k \, \alpha_k = 0$ we have to change at least two variables at the same time
- A heuristic is used to choose the best pair of α's to optimise
- Run until close to the optimum

Sequential Minimal Optimisation

- One of the most efficient techniques for training SVMs is *Sequential Minimal Optimisation* or SMO
- This takes two Lagrange multipliers α_i and α_j and adjusts them to maximise the dual objective function
- This is very quick as it can be done in closed form
- Note that because $\sum_{k=1}^{m} y_k \alpha_k = 0$ we have to change at least two variables at the same time
- A heuristic is used to choose the best pair of α's to optimise
- Run until close to the optimum

Sequential Minimal Optimisation

- One of the most efficient techniques for training SVMs is *Sequential Minimal Optimisation* or SMO

- This takes two Lagrange multipliers α_i and α_j and adjusts them to maximise the dual objective function

- This is very quick as it can be done in closed form

- Note that because $\sum_{k=1}^{m} y_k\,\alpha_k = 0$ we have to change at least two variables at the same time

- A heuristic is used to choose the best pair of α's to optimise

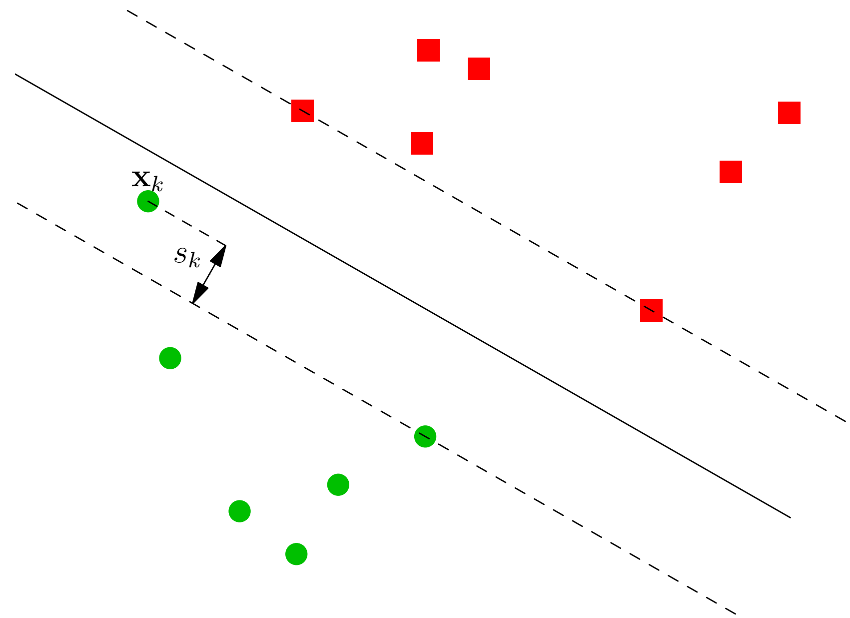
- Run until close to the optimum

Sequential Minimal Optimisation

- One of the most efficient techniques for training SVMs is *Sequential Minimal Optimisation* or SMO
- This takes two Lagrange multipliers α_i and α_j and adjusts them to maximise the dual objective function
- This is very quick as it can be done in closed form
- Note that because $\sum_{k=1}^{m} y_k \alpha_k = 0$ we have to change at least two variables at the same time
- A heuristic is used to choose the best pair of α's to optimise
- Run until close to the optimum

Soft Margins

- We can relax the margin constraints by introducing *slack variables*, $s_k \geq 0$



$$y_k(\boldsymbol{x}_k^{\mathsf{T}}\boldsymbol{w} - b) \geq 1 - s_k$$

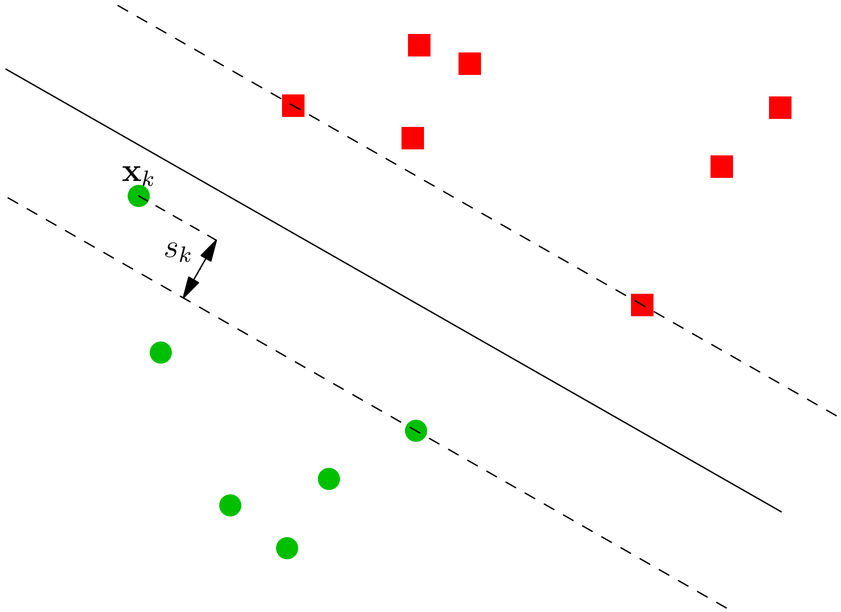
- Minimise $\frac{\|\boldsymbol{w}\|^2}{2} + C\sum_{k=1}^{n} s_k$

- Large C punishes slack variables

Soft Margins

- We can relax the margin constraints by introducing *slack variables*, $s_k \geq 0$

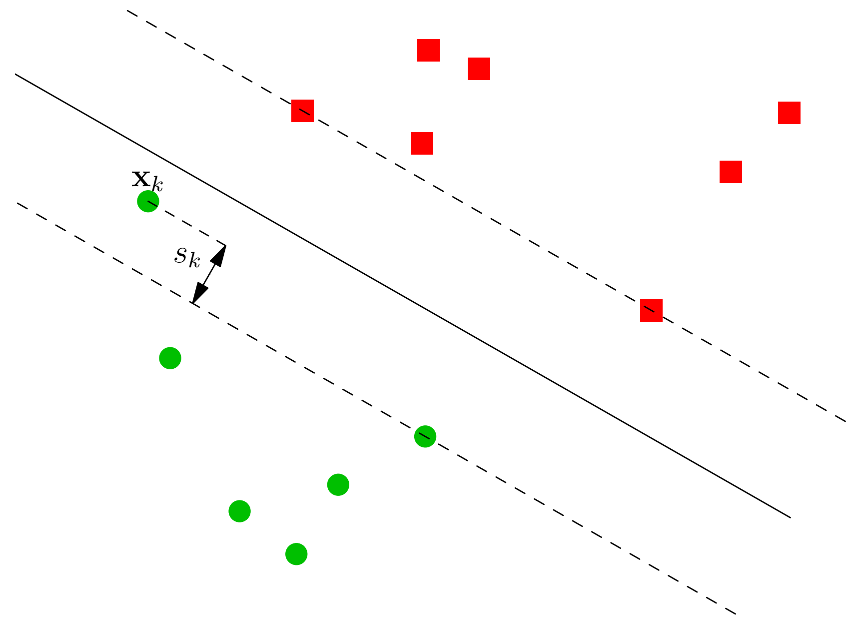
$$y_k(\boldsymbol{x}_k^{\mathsf{T}}\boldsymbol{w} - b) \geq 1 - s_k$$



- Minimise $\frac{\|\boldsymbol{w}\|^2}{2} + C\sum_{k=1}^{n} s_k$
- Large C punishes slack variables

Soft Margins

- We can relax the margin constraints by introducing *slack variables*, $s_k \geq 0$



$$y_k(\boldsymbol{x}_k^{\mathsf{T}} \boldsymbol{w} - b) \geq 1 - s_k$$

- Minimise $\frac{\|\boldsymbol{w}\|^2}{2} + C \sum_{k=1}^{n} s_k$

- Large C punishes slack variables

Dual Problem with Slack Variables

- The Lagrangian with slack variables is

$$\mathcal{L} = \frac{1}{2}\|\boldsymbol{w}\|^2 + C\sum_{k=1}^{m} s_k - \sum_{k=1}^{m} \alpha_k \left(y_k \left(\boldsymbol{w}^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_k) - b \right) - 1 + s_k \right) - \sum_{k=1}^{m} \beta_k \, s_k$$

  where β_k are Lagrange multipliers that ensure $s_k \geq 0$ (note that $\beta_k \geq 0$—this is the KKT condition)

- Now minimising with respect to s_i

$$\frac{\partial \mathcal{L}}{\partial s_i} = C - \alpha_i - \beta_i = 0$$

- Or $\alpha_i = C - \beta_i$. Since $\beta_i \geq 0$ the constraint is $\alpha_i \leq C$

Dual Problem with Slack Variables

- The Lagrangian with slack variables is

$$\mathcal{L} = \frac{1}{2}\|\boldsymbol{w}\|^2 + C\sum_{k=1}^{m} s_k - \sum_{k=1}^{m} \alpha_k \left(y_k \left(\boldsymbol{w}^{\mathsf{T}} \boldsymbol{\phi}(\boldsymbol{x}_k) - b \right) - 1 + s_k \right) - \sum_{k=1}^{m} \beta_k \, s_k$$

  where β_k are Lagrange multipliers that ensure $s_k \geq 0$ (note that $\beta_k \geq 0$—this is the KKT condition)

- Now minimising with respect to s_i

$$\frac{\partial \mathcal{L}}{\partial s_i} = C - \alpha_i - \beta_i = 0$$

- Or $\alpha_i = C - \beta_i$. Since $\beta_i \geq 0$ the constraint is $\alpha_i \leq C$

Dual Problem with Slack Variables

- The Lagrangian with slack variables is

$$\mathcal{L} = \frac{1}{2}\|\boldsymbol{w}\|^2 + C\sum_{k=1}^{m} s_k - \sum_{k=1}^{m} \alpha_k \left(y_k \left(\boldsymbol{w}^{\mathsf{T}}\boldsymbol{\phi}(\boldsymbol{x}_k) - b\right) - 1 + s_k\right) - \sum_{k=1}^{m} \beta_k\, s_k$$

  where β_k are Lagrange multipliers that ensure $s_k \geq 0$ (note that $\beta_k \geq 0$—this is the KKT condition)

- Now minimising with respect to s_i

$$\frac{\partial\,\mathcal{L}}{\partial\, s_i} = C - \alpha_i - \beta_i = 0$$

- Or $\alpha_i = C - \beta_i$. Since $\beta_i \geq 0$ the constraint is $\alpha_i \leq C$

Dual Problem with Slack Variables

- The Lagrangian with slack variables is

$$\mathcal{L} = \frac{1}{2}\|\boldsymbol{w}\|^2 + C\sum_{k=1}^{m} s_k - \sum_{k=1}^{m} \alpha_k \left(y_k \left(\boldsymbol{w}^\mathsf{T}\boldsymbol{\phi}(\boldsymbol{x}_k) - b\right) - 1 + s_k\right) - \sum_{k=1}^{m} \beta_k\, s_k$$

  where β_k are Lagrange multipliers that ensure $s_k \geq 0$ (note that $\beta_k \geq 0$—this is the KKT condition)

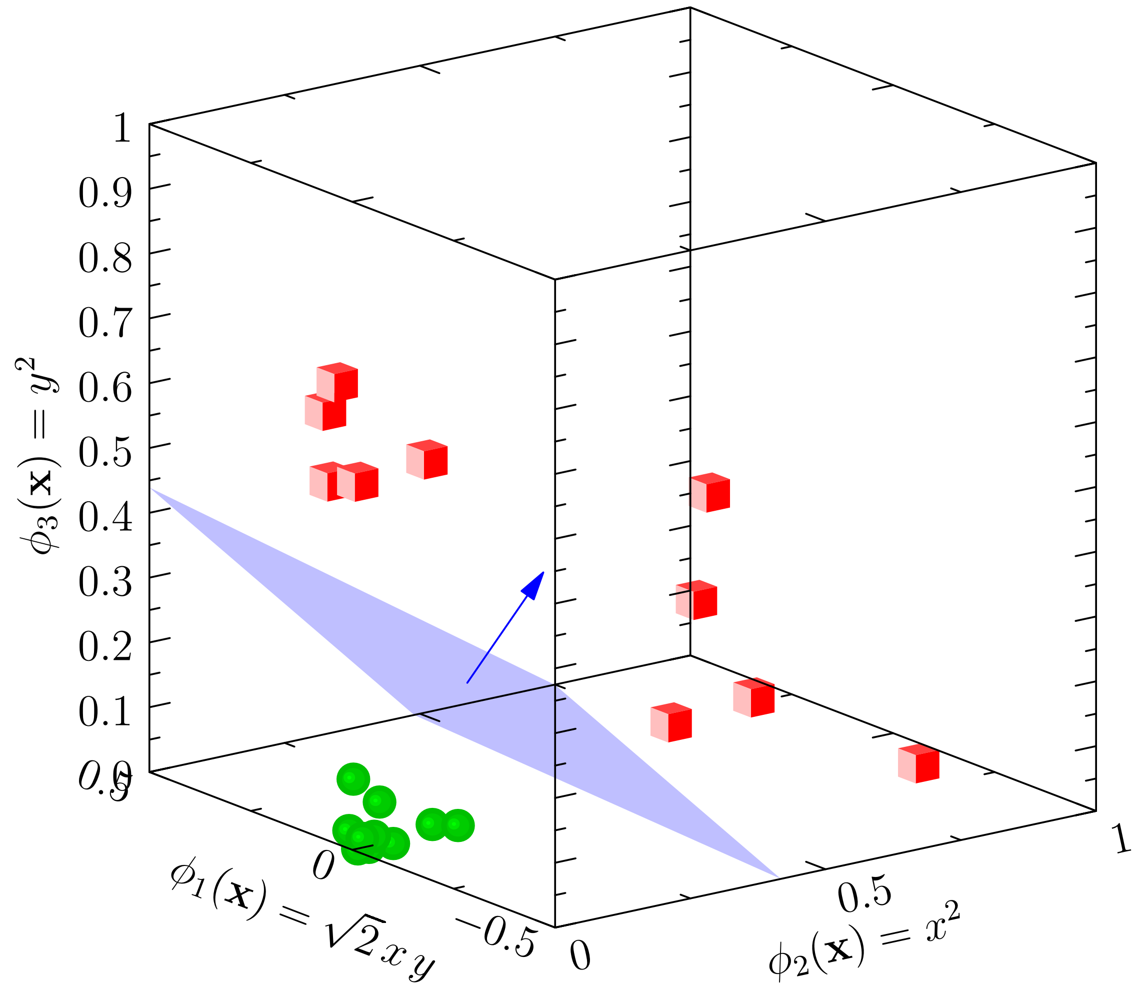
- Now minimising with respect to s_i

$$\frac{\partial\,\mathcal{L}}{\partial\, s_i} = C - \alpha_i - \beta_i = 0$$

- Or $\alpha_i = C - \beta_i$. Since $\beta_i \geq 0$ the constraint is $\alpha_i \leq C$ (recall also $\alpha_i \geq 0$)

Outline

1. The Big Picture
2. Maximum Margins
3. Duality
4. **Practice**



Getting SVMs to Work Well

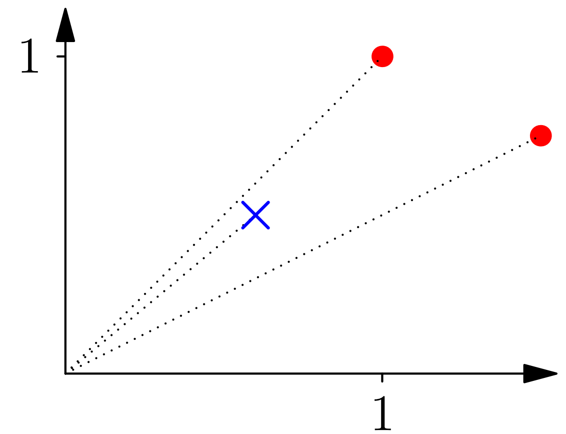
- SVMs rely on distances between data points
- These will change relative to each other if we rescale some features but not other—giving different maximum-margin hyper-planes

Getting SVMs to Work Well

- SVMs rely on distances between data points
- These will change relative to each other if we rescale some features but not other—giving different maximum-margin hyper-planes

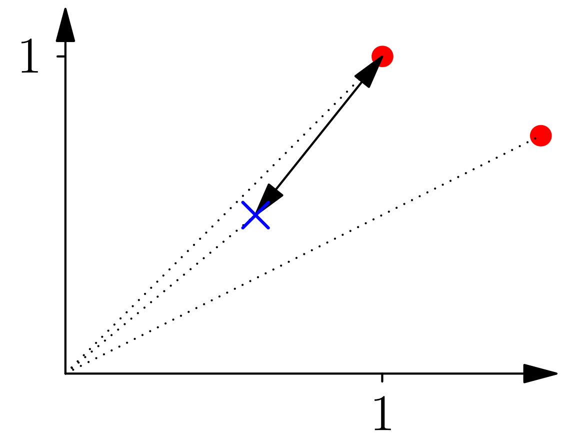
Getting SVMs to Work Well

- SVMs rely on distances between data points
- These will change relative to each other if we rescale some features but not other—giving different maximum-margin hyper-planes



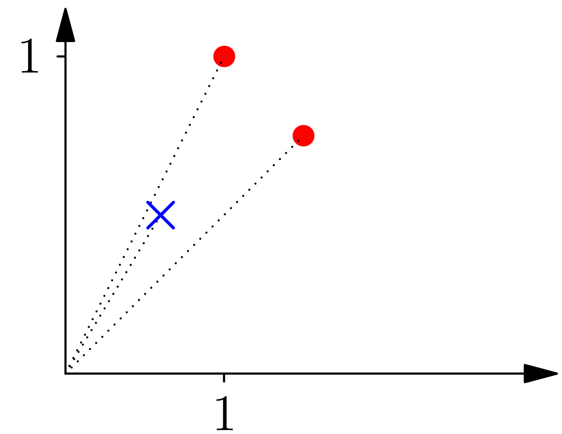
Getting SVMs to Work Well

- SVMs rely on distances between data points
- These will change relative to each other if we rescale some features but not other—giving different maximum-margin hyper-planes



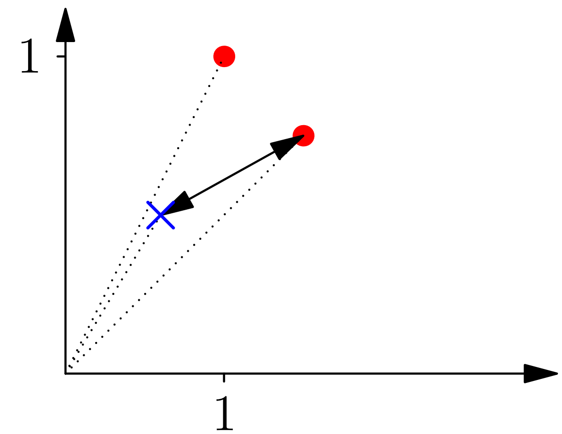
Getting SVMs to Work Well

- SVMs rely on distances between data points
- These will change relative to each other if we rescale some features but not other—giving different maximum-margin hyper-planes



Getting SVMs to Work Well

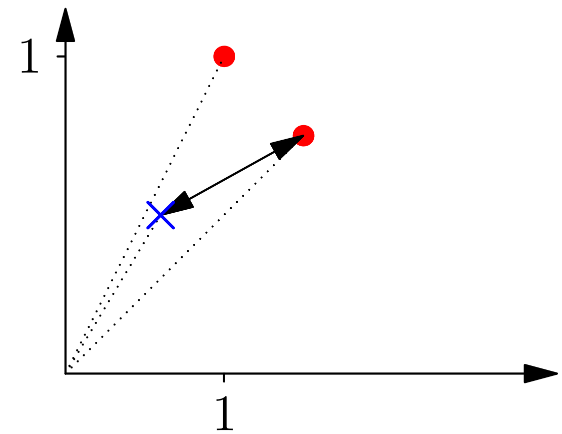
- SVMs rely on distances between data points
- These will change relative to each other if we rescale some features but not other—giving different maximum-margin hyper-planes



Getting SVMs to Work Well

- SVMs rely on distances between data points

- These will change relative to each other if we rescale some features but not other—giving different maximum-margin hyper-planes



- If we don't know what features are important (most often the case), then it is worth scaling each feature (for example, so their range is between 0 and 1 or their variance is 1)

Optimising C

- Recall that we can introduce soft-margins using slack variables where we minimise $\frac{\|\hat{\boldsymbol{w}}\|^2}{2} + C \sum_{k=1}^{m} s_k$ subject to constraints
- In practice it can make a huge difference to the performance if we change C
- Optimal C values changes by many orders of magnitude e.g. 2^{-5}–2^{15}
- Typically optimised by a grid search (start from 2^{-5} say and double until you reach 2^{15})
- Measure performance on a validation set

Optimising C

- Recall that we can introduce soft-margins using slack variables where we minimise $\frac{\|\hat{\boldsymbol{w}}\|^2}{2} + C\sum_{k=1}^{m} s_k$ subject to constraints
- In practice it can make a huge difference to the performance if we change C
- Optimal C values changes by many orders of magnitude e.g. 2^{-5}–2^{15}
- Typically optimised by a grid search (start from 2^{-5} say and double until you reach 2^{15})
- Measure performance on a validation set

Optimising C

- Recall that we can introduce soft-margins using slack variables where we minimise $\frac{\|\hat{\boldsymbol{w}}\|^2}{2} + C\sum_{k=1}^{m} s_k$ subject to constraints

- In practice it can make a huge difference to the performance if we change C

- Optimal C values changes by many orders of magnitude e.g. 2^{-5}–2^{15}

- Typically optimised by a grid search (start from 2^{-5} say and double until you reach 2^{15})

- Measure performance on a validation set

Optimising C

- Recall that we can introduce soft-margins using slack variables where we minimise $\frac{\|\hat{\boldsymbol{w}}\|^2}{2} + C \sum_{k=1}^{m} s_k$ subject to constraints
- In practice it can make a huge difference to the performance if we change C
- Optimal C values changes by many orders of magnitude e.g. 2^{-5}–2^{15}
- Typically optimised by a grid search (start from 2^{-5} say and double until you reach 2^{15})
- Measure performance on a validation set

Optimising C

- Recall that we can introduce soft-margins using slack variables where we minimise $\frac{\|\hat{\boldsymbol{w}}\|^2}{2} + C \sum_{k=1}^{m} s_k$ subject to constraints

- In practice it can make a huge difference to the performance if we change C

- Optimal C values changes by many orders of magnitude e.g. 2^{-5}–2^{15}

- Typically optimised by a grid search (start from 2^{-5} say and double until you reach 2^{15})

- Measure performance on a validation set

Choosing the Right Kernel Function

- There are kernels design for particular data types (e.g. string kernels for text or biological sequences)

- For numerical data people tend to look at using no kernel (linear SVM), a radial basis function (Gaussian) kernel or polynomial kernels

- Kernel's often come with parameters, e.g. the popular radial basis function kernel

$$K(\boldsymbol{x}, \boldsymbol{y}) = \mathrm{e}^{-\gamma \, \|\boldsymbol{x}-\boldsymbol{y}\|^2}$$

- Optimal γ values range over 2^{-15}–2^3

Choosing the Right Kernel Function

- There are kernels design for particular data types (e.g. string kernels for text or biological sequences)

- For numerical data people tend to look at using no kernel (linear SVM), a radial basis function (Gaussian) kernel or polynomial kernels

- Kernel's often come with parameters, e.g. the popular radial basis function kernel

$$K(\boldsymbol{x}, \boldsymbol{y}) = \mathrm{e}^{-\gamma \, \|\boldsymbol{x}-\boldsymbol{y}\|^2}$$

- Optimal γ values range over 2^{-15}–2^{3}

Choosing the Right Kernel Function

- There are kernels design for particular data types (e.g. string kernels for text or biological sequences)
- For numerical data people tend to look at using no kernel (linear SVM), a radial basis function (Gaussian) kernel or polynomial kernels
- Kernel's often come with parameters, e.g. the popular radial basis function kernel

$$K(\boldsymbol{x}, \boldsymbol{y}) = \mathrm{e}^{-\gamma \, \|\boldsymbol{x} - \boldsymbol{y}\|^2}$$

- Optimal γ values range over 2^{-15}–2^{3}

Choosing the Right Kernel Function

- There are kernels design for particular data types (e.g. string kernels for text or biological sequences)
- For numerical data people tend to look at using no kernel (linear SVM), a radial basis function (Gaussian) kernel or polynomial kernels
- Kernel's often come with parameters, e.g. the popular radial basis function kernel

$$K(\boldsymbol{x}, \boldsymbol{y}) = \mathrm{e}^{-\gamma \, \|\boldsymbol{x}-\boldsymbol{y}\|^2}$$

- Optimal γ values range over 2^{-15}–2^{3}

Multi-Class Problems

- By construction SVMs separate only two classes
- If we have a multi-class problem we have to use multiple SVMs
- There are two major ways practitioners do this

  One-versus-all: for each class, train a separate classify to determine that class versus all others

  All-pairs: train a classify for all pairs of classes
- In both cases choose the class which the classifier is most certain about

Multi-Class Problems

- By construction SVMs separate only two classes
- If we have a multi-class problem we have to use multiple SVMs
- There are two major ways practitioners do this

  One-versus-all: for each class, train a separate classify to determine that class versus all others

  All-pairs: train a classify for all pairs of classes

- In both cases choose the class which the classifier is most certain about

Multi-Class Problems

- By construction SVMs separate only two classes
- If we have a multi-class problem we have to use multiple SVMs
- There are two major ways practitioners do this

  One-versus-all: for each class, train a separate classify to determine that class versus all others

  All-pairs: train a classify for all pairs of classes
- In both cases choose the class which the classifier is most certain about

Multi-Class Problems

- By construction SVMs separate only two classes
- If we have a multi-class problem we have to use multiple SVMs
- There are two major ways practitioners do this

  One-versus-all: for each class, train a separate classify to determine that class versus all others

  All-pairs: train a classify for all pairs of classes

- In both cases choose the class which the classifier is most certain about

Multi-Class Problems

- By construction SVMs separate only two classes
- If we have a multi-class problem we have to use multiple SVMs
- There are two major ways practitioners do this

  One-versus-all: for each class, train a separate classify to determine that class versus all others

  All-pairs: train a classify for all pairs of classes
- In both cases choose the class which the classifier is most certain about

SVM Libraries

- Although SVMs have unique solutions, they require very well written optimisers
- If you have a large data set they can be very slow
- There are good libraries out there, svmlib, svm-lite, etc.
- These will often automate normalisation of data and grid search for parameters

SVM Libraries

- Although SVMs have unique solutions, they require very well written optimisers
- If you have a large data set they can be very slow
- There are good libraries out there, svmlib, svm-lite, etc.
- These will often automate normalisation of data and grid search for parameters

SVM Libraries

- Although SVMs have unique solutions, they require very well written optimisers
- If you have a large data set they can be very slow
- There are good libraries out there, svmlib, svm-lite, etc.
- These will often automate normalisation of data and grid search for parameters

SVM Libraries

- Although SVMs have unique solutions, they require very well written optimisers
- If you have a large data set they can be very slow
- There are good libraries out there, svmlib, svm-lite, etc.
- These will often automate normalisation of data and grid search for parameters

Conclusions

- We've seen how SVMs work
- We've learnt how to use them
- We've seen that we can find the maximum margin hyper-plane by solving a quadratic programming problem (with a unique solution)
- This is a convex optimisation problem with a unique optimum
- The **dual problem** of an SVM is particularly simple, especially if we use a positive semi-definite kernel

Conclusions

- We've seen how SVMs work
- We've learnt how to use them
- We've seen that we can find the maximum margin hyper-plane by solving a quadratic programming problem (with a unique solution)
- This is a convex optimisation problem with a unique optimum
- The **dual problem** of an SVM is particularly simple, especially if we use a positive semi-definite kernel

Conclusions

- We've seen how SVMs work
- We've learnt how to use them
- We've seen that we can find the maximum margin hyper-plane by solving a quadratic programming problem (with a unique solution)
- This is a convex optimisation problem with a unique optimum
- The **dual problem** of an SVM is particularly simple, especially if we use a positive semi-definite kernel

Conclusions

- We've seen how SVMs work
- We've learnt how to use them
- We've seen that we can find the maximum margin hyper-plane by solving a quadratic programming problem (with a unique solution)
- This is a convex optimisation problem with a unique optimum
- The **dual problem** of an SVM is particularly simple, especially if we use a positive semi-definite kernel

Conclusions

- We've seen how SVMs work
- We've learnt how to use them
- We've seen that we can find the maximum margin hyper-plane by solving a quadratic programming problem (with a unique solution)
- This is a convex optimisation problem with a unique optimum
- The **dual problem** of an SVM is particularly simple, especially if we use a positive semi-definite kernel

Conclusions

- We've seen how SVMs work
- We've learnt how to use them
- We've seen that we can find the maximum margin hyper-plane by solving a quadratic programming problem (with a unique solution)
- This is a convex optimisation problem with a unique optimum
- The **dual problem** of an SVM is particularly simple, especially if we use a positive semi-definite kernel (we explore these in the next lecture)