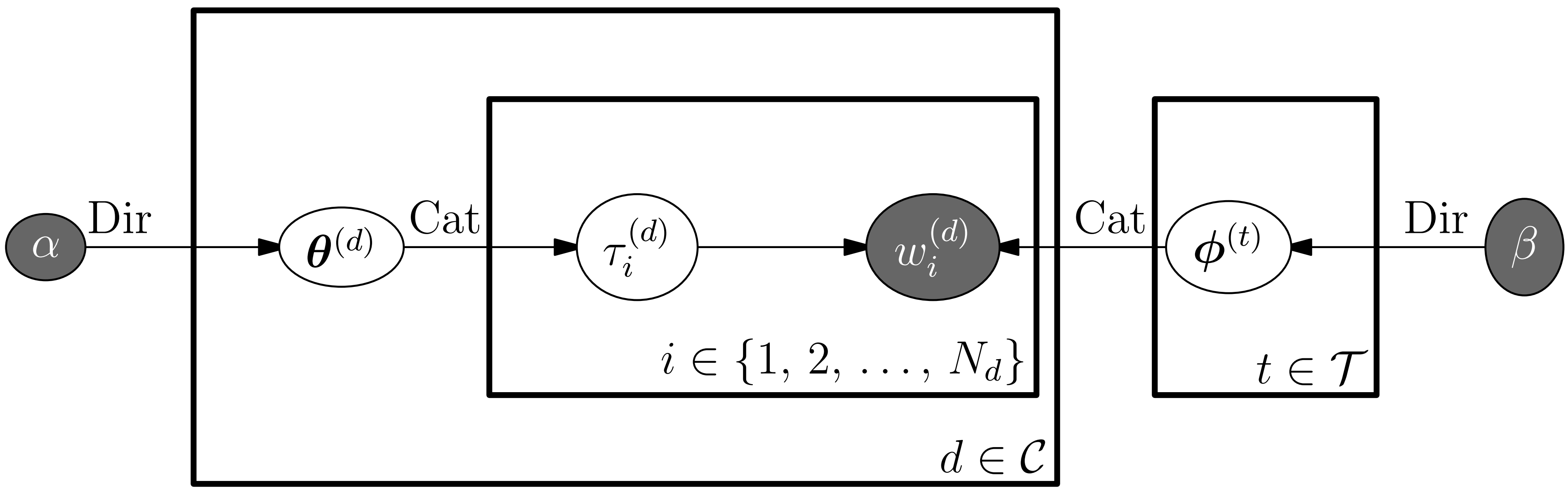
Advanced Machine Learning

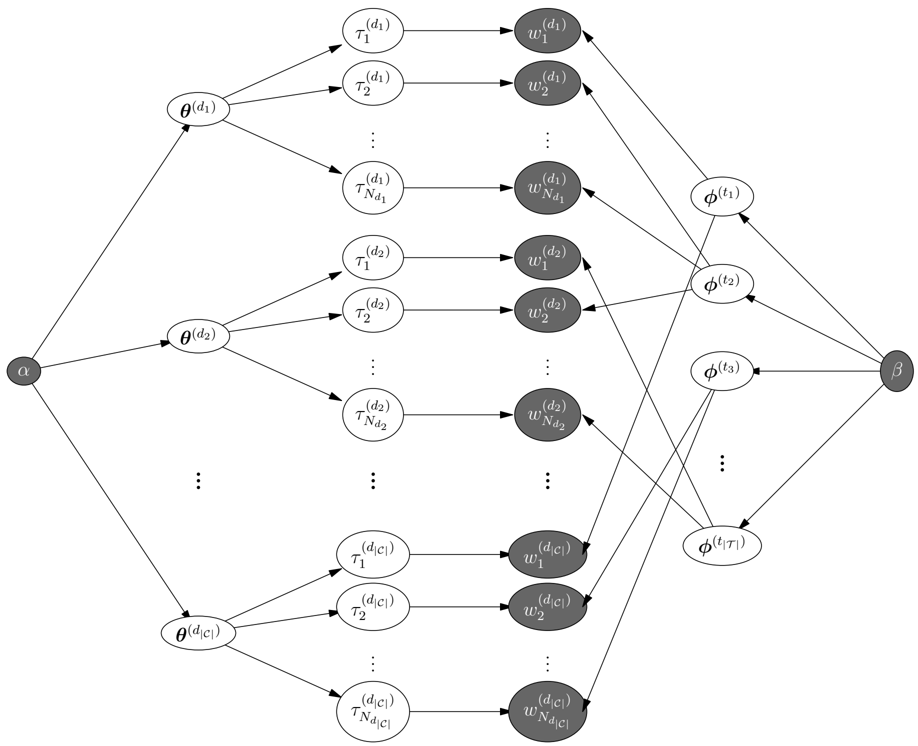
Graphical Models



Conditional Independence, Graphical models, LDA

Outline

1. **Graphical Models**
2. Cakes!
3. Latent Dirichlet Allocation



Graphical Models

- If we want to build large probabilistic inference systems
  - ⋆ AI Doctor
  - ⋆ Fault diagnostic system for a computer

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Dependencies Between Variables

- In building a probabilistic model we want to know which random variables depend on each other directly and which don't
- Variables that don't will typically still be correlated
- If two random variables X and Y are correlated then
  - X could affect Y
  - Y could affect X
  - X and Y could not influence each other, but both be affected by another random variable Z

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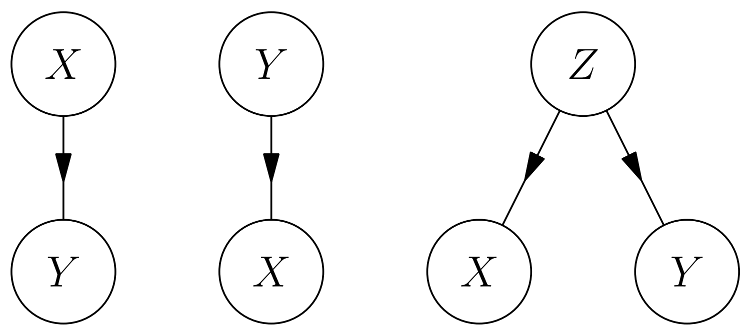
- We could represent the three conditions described above by



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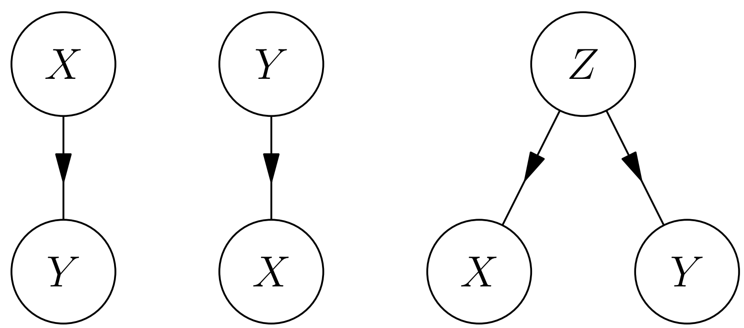

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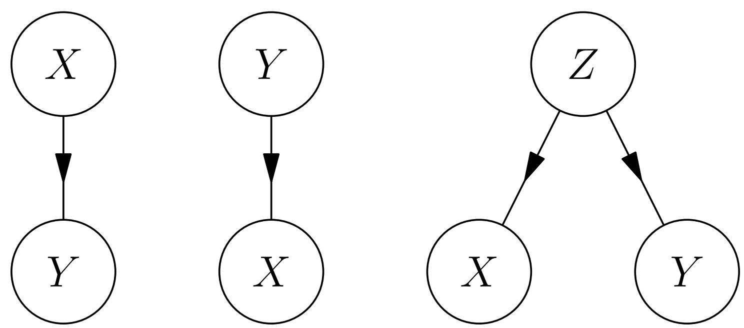

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Statistical Independence

- Two random variables are statistically independent if

$$\mathbb{P}(X,Y) = \mathbb{P}(X)\,\mathbb{P}(Y)$$

- Equally this implies $\mathbb{P}(X|Y) = \mathbb{P}(X)$ and $\mathbb{P}(Y|X) = \mathbb{P}(Y)$
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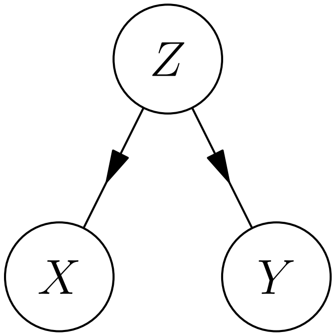

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- But it doesn't imply zero correlation
- Conditional independence reduces computational complexity, e.g.

$$\mathbb{E}[XY] = \sum_{X,Y,Z} XY\,\mathbb{P}(X,Y,Z) = \sum_{Z} P(Z) \left(\sum_{X} X P(X|Z) \right) \left(\sum_{Y} Y P(Y|Z) \right)$$

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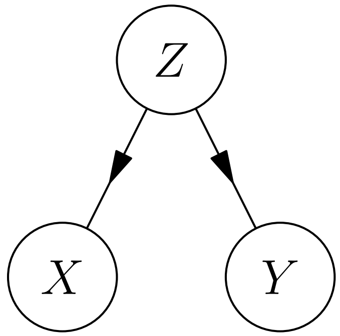

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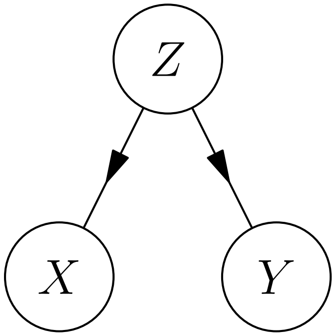

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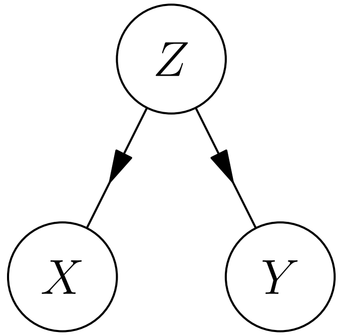

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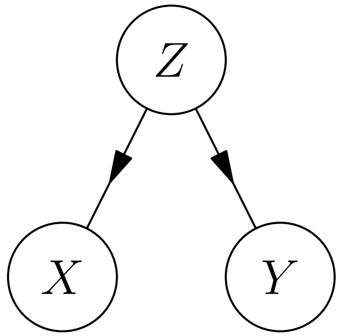

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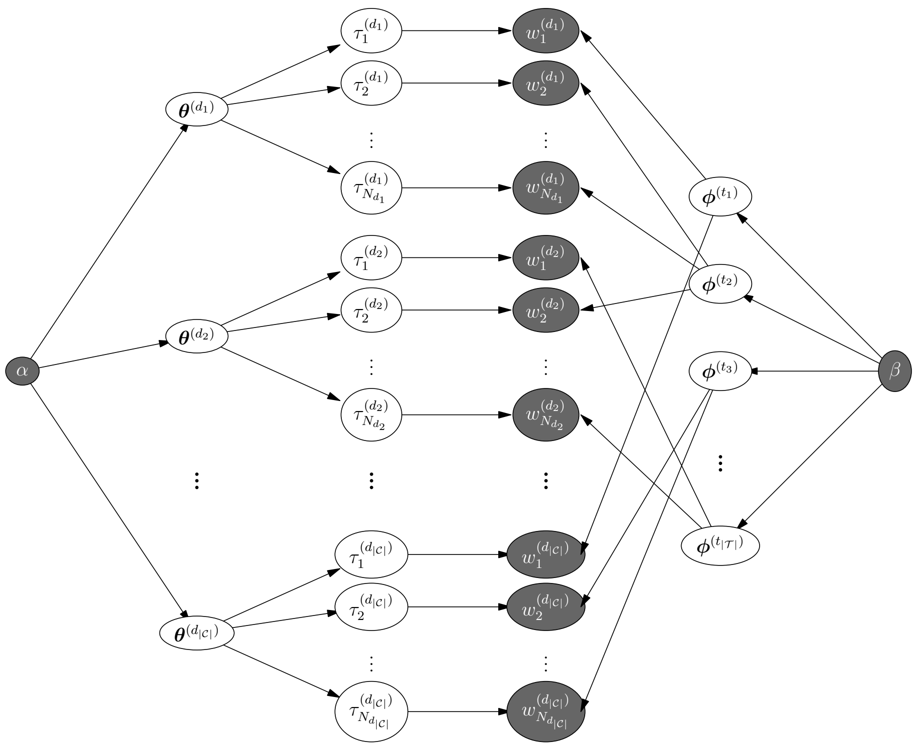

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- It illustrates some simple principles
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- It illustrates some simple principles
- In the subsidiary notes I present a very simple program for computing all the probabilities—I would encourage you to do this as it makes things much clearer

The Cake Scenario

- Abi and Ben both bake cakes and bring them into the coffee room
- Abi will bring in cakes 20% of the time: $\mathbb{P}(A = 1) = 0.2$
- Ben will bring in cakes 10% of the time: $\mathbb{P}(B = 1) = 0.1$
- 90% of the time if either Abi or Ben have put cakes in the coffee room there is some left when I enter
  $\mathbb{P}(C = 1|A = 1, B = 0) = \mathbb{P}(C = 1|A = 0, B = 1) = 0.9$
- If they both make cake then there is always cake left
  $\mathbb{P}(C = 1|A = 1, B = 1) = 1$
- If neither Abi or Ben has made cake there is still a 5% chance someone else has put cake in the coffee room
  $\mathbb{P}(C = 1|A = 0, B = 0) = 0.05$

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Computing with Probabilities

- Other probabilities I can deduce, e.g.
  $\mathbb{P}(C=0|A,B) = 1 - \mathbb{P}(C=1|A,B)$
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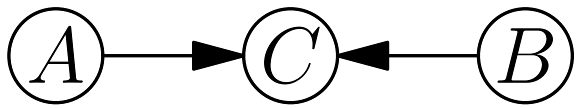

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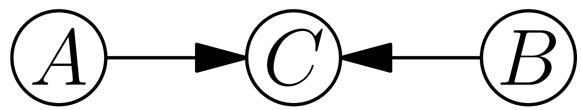

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Computing Expectations

- By using the joint probability and summing over all unknown quantities, we can compute expectations of anything we are interested in

- These sums are often sped up using knowledge of conditional independence

- To compute the probability of and event $\mathcal{E}$ we introduce an indicator function $[\![\mathcal{E}]\!]$ which is equal to 1 if the event happens and 0 otherwise
$$\mathbb{P}(\mathcal{E}) = \mathbb{E}[[\![\mathcal{E}]\!]]$$

- If E is a random variable equal to 1 if event $\mathcal{E}$ happens and 0 otherwise then $E = [\![\mathcal{E}]\!]$

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Are There Any Cakes Left?

- We can use our model to compute the probabilities of there being cakes in the coffee room

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\mathbb{P}(C=1) &= \sum_{A,B,C \in \{0,1\}} [\![C=1]\!]\, \mathbb{P}(A,B,C) \\
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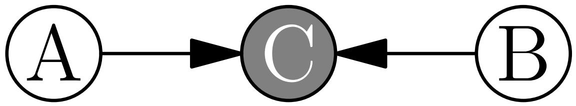
- In graphical models observed random variables are shaded



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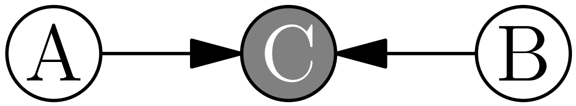

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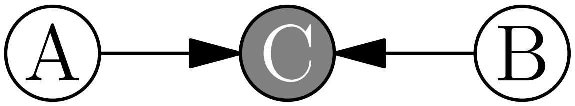

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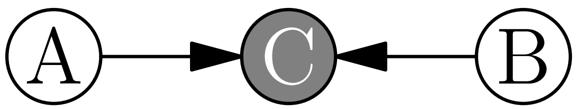

- The probabilities conditioned on C is given by

$$\mathbb{P}(A,B|C) = \frac{\mathbb{P}(A,B,C)}{\mathbb{P}(C)}$$

where

$$\mathbb{P}(C) = \sum_{A,B \in \{0,1\}} \mathbb{P}(A,B,C)$$

Who Made Those Cakes?

- If we observe there are cakes

$$\mathbb{P}(A, B | C = 1) = \mathbb{P}(A, B, C = 1) / \mathbb{P}(C = 1)$$

- A straightforward if tedious calculation shows

$$\mathbb{P}(A = 1 | C = 1) = 0.628, \quad \mathbb{P}(B = 1 | C = 1) = 0.317$$

$$\mathbb{P}(A = 1, B = 1 | C = 1) = 0.069$$

- Note $\mathbb{P}(A = 1, B = 1 | C = 1) \neq \mathbb{P}(A = 1 | C = 1)\,\mathbb{P}(B = 1 | C = 1)$

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Elaborate Cakes

- We can elaborate on our cake model
- We suppose that Dave likes cakes so if there is a cake in the coffee room there is a 80% chance that I will see him eating a cake: $\mathbb{P}(D=1|C=1)=0.8$
- Even if there are no cakes in the coffee room there is a 10% chance that Dave has bought his own cake: $\mathbb{P}(D=1|C=0)=0.1$
- Eli also likes cakes: there is a 60% chance that I will see her eating cakes if there are cakes in the coffee room: $\mathbb{P}(E=1|C=1)=0.6$
- But she never buys herself cakes $\mathbb{P}(E=1|C=0)=0$

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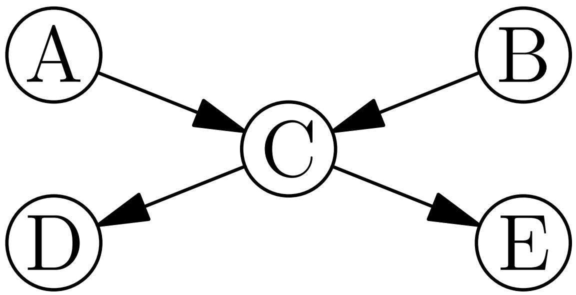

- This allows us to break down the joint probability

$$\begin{aligned}\mathbb{P}(A,B,C,D,E) &= \mathbb{P}(C,D,E|A,B)\,\mathbb{P}(B)\,\mathbb{P}(A)\\ &= \mathbb{P}(D|C)\,\mathbb{P}(E|C)\,\mathbb{P}(C|A,B)\,\mathbb{P}(B)\,\mathbb{P}(A)\end{aligned}$$

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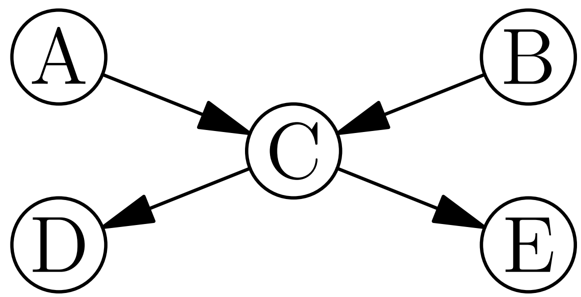

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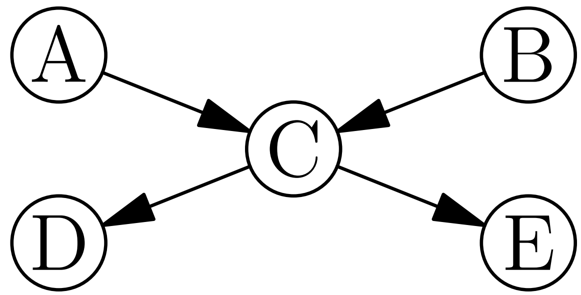

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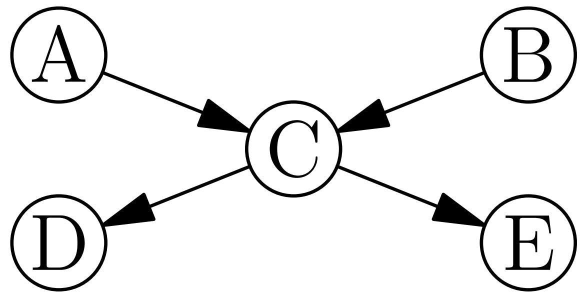

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Dependencies

- If we don't observe cakes then the probability of Dave and Eli eating cake are not independent

$$\mathbb{P}(D = 1) = 0.303, \qquad \mathbb{P}(E = 1) = 0.174$$

$$\mathbb{P}(D = 1, E = 1) = 0.1392$$

- This changes if we know there are cakes in the coffee room

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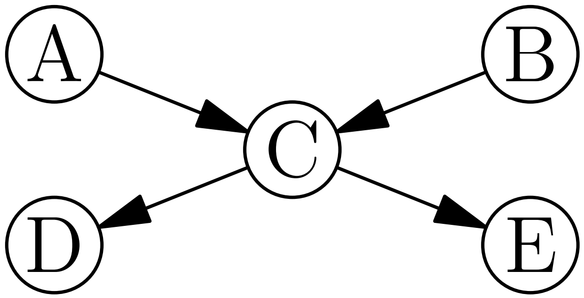

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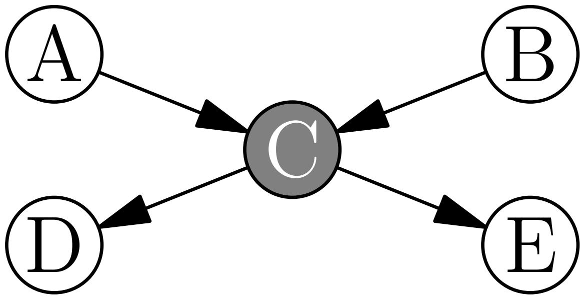

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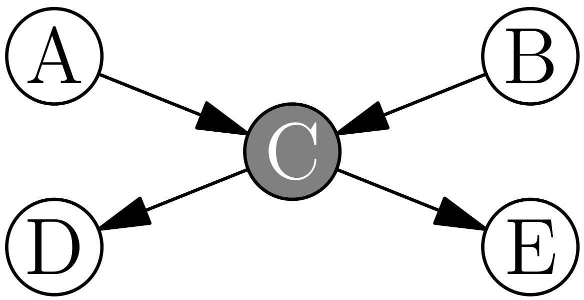

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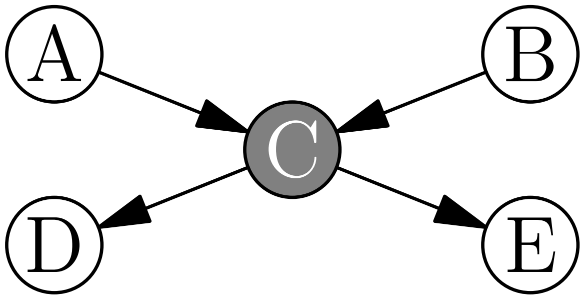

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- There are rules to deduce the conditional independence from a graphical model given which variables have been observed—but these are details that you can look up if needed

Graphical Model Frameworks

- There are sophisticated frameworks for computing probabilities in Bayesian Belief Networks efficiently
- If our graph is a tree then we can evaluate probabilities efficiently
- When there are loops (so that a random variable both influences and is influenced by another random variables) then exact evaluation of expectations requires exhaustive summing over variables
- There are various message passing algorithms designed to obtain approximations of expectations

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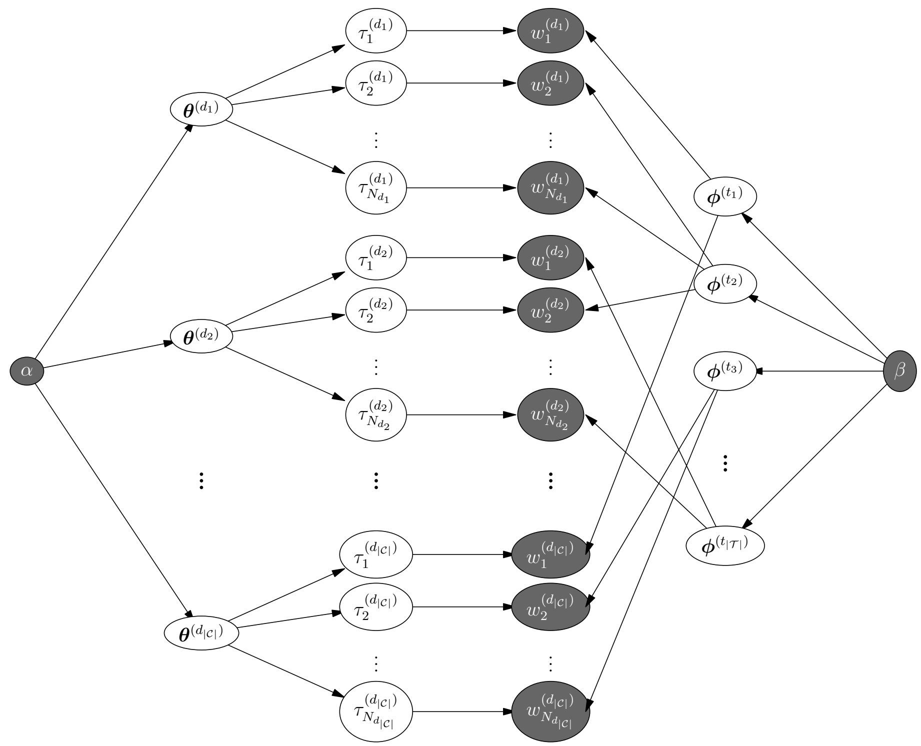
Outline

1. Graphical Models
2. Cakes!
3. **Latent Dirichlet Allocation**



Model for Documents

- We consider a model for the words in a set of documents (we ignore word order)
- We consider a corpus $\mathcal{C} = \{d_i | i = 1, 2, \ldots |\mathcal{C}|\}$
- With documents consisting of words

$$d = \left(w_1^{(d)}, w_2^{(d)}, \ldots, w_{N_d}^{(d)}\right)$$

- We assume that there is a set of topics $\mathcal{T} = \{t_1, t_2, \ldots, t_{|\mathcal{T}|}\}$
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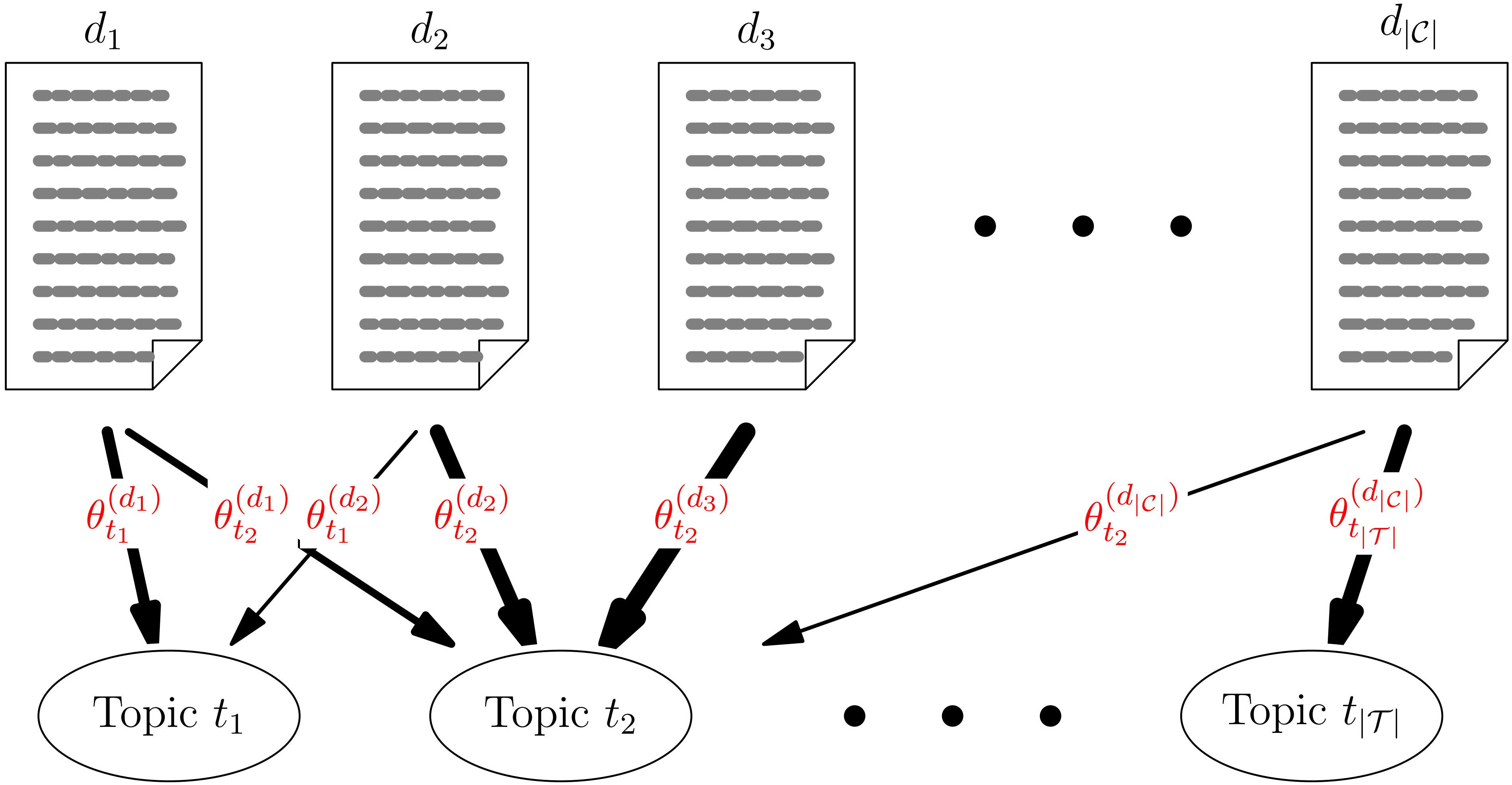
Documents and Topic

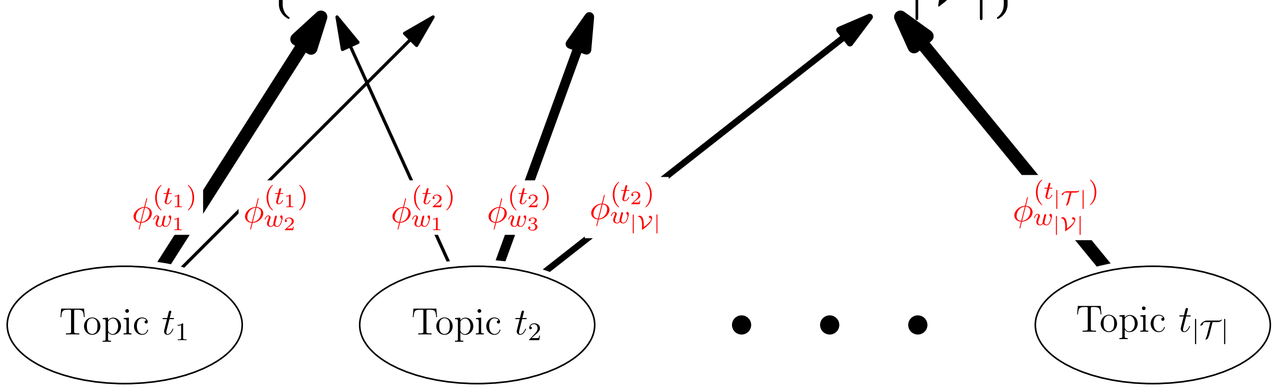


$$\boldsymbol{\theta}^{(d)} = (\theta_t^{(d)} | t \in \mathcal{T}) \qquad \theta_t^{(d)} \geq 0 \qquad \sum_{t=1}^{|\mathcal{T}|} \theta_t^{(d)} = 1$$

Words and Topic

- We associate a probability $\phi_w^{(t)}$ that a word, w, is related to a topic t

$$\mathcal{V} = \left\{w_1,\, w_2,\, w_3,\, \ldots,\, w_{|\mathcal{V}|}\right\}$$



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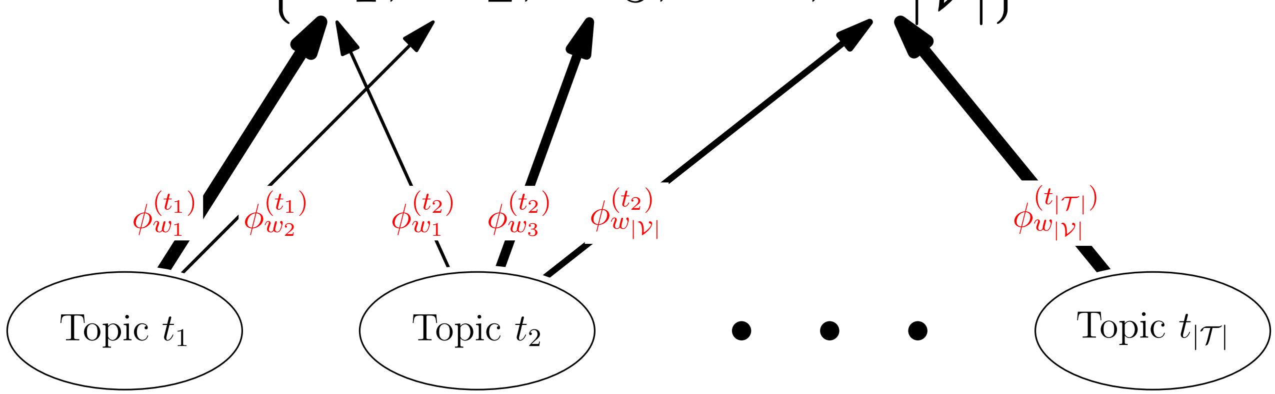


$$\boldsymbol{\phi}^{(t)} = \left(\phi_w^{(t)} \middle| w \in \mathcal{V} \right)$$

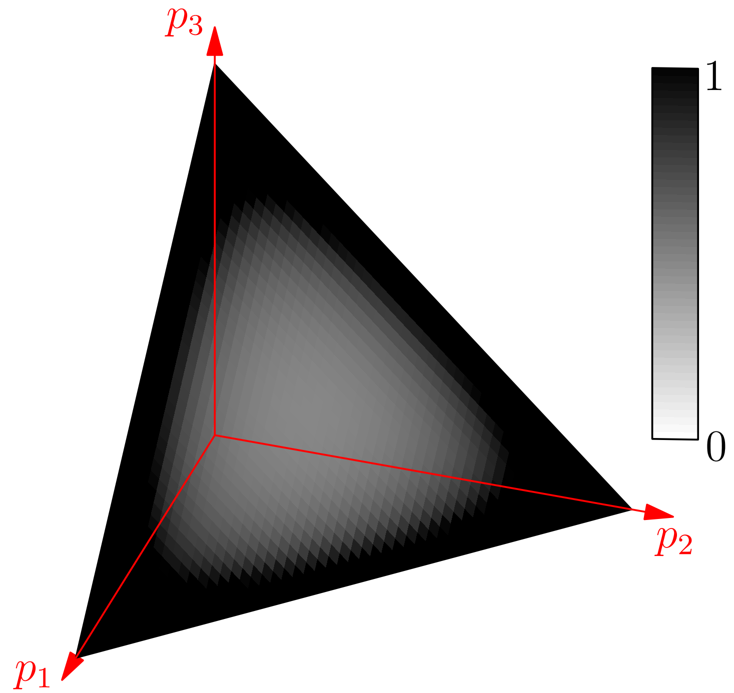
Dirichlet Allocation

- Most documents are predominantly about a few topics and most topic have a small number of words associated to them

- We can generate sparse vectors $\boldsymbol{\theta}^{(d)}$ and $\boldsymbol{\phi}^{(t)}$ from a Dirichlet distribution with small parameters $\boldsymbol{\alpha}$

$$\mathrm{Dir}(\boldsymbol{p}|\boldsymbol{\alpha}) = \Gamma\left(\sum_i \alpha_i\right) \prod_{i=1}^{n} \frac{p_i^{\alpha_i - 1}}{\Gamma(\alpha_i)}$$

- $\sum_i p_i = 1$



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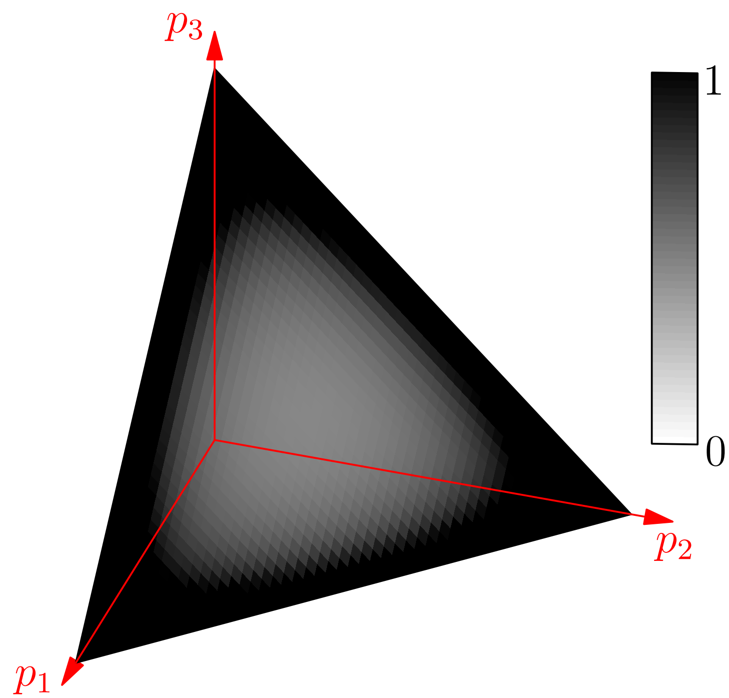

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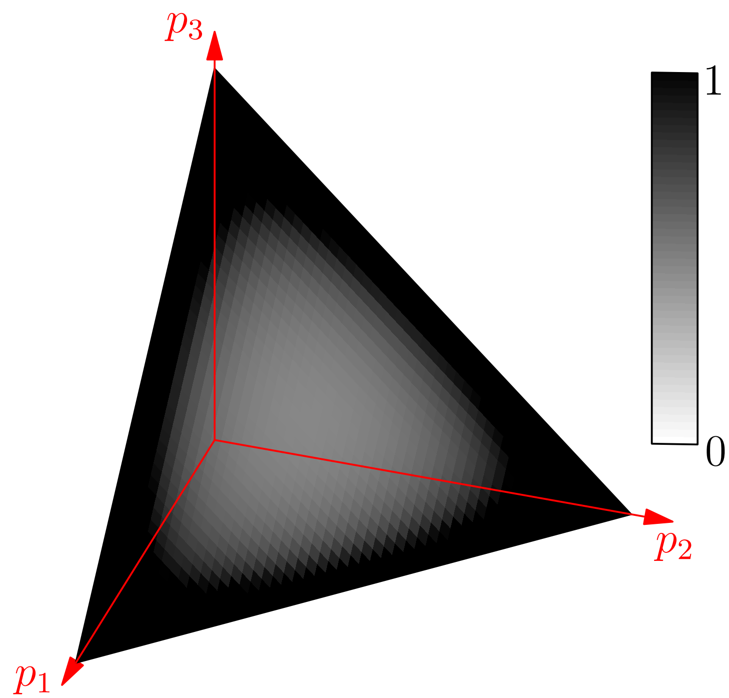

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- Where $\mathrm{Cat}(i|\boldsymbol{p}) = p_i$ is the categorical distribution (we choose one of a number of options)
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$$\forall d \in \mathcal{C} \ \wedge \ \forall i \in \{1, 2, \ldots, N_d\} \quad \tau_i^{(d)} \sim \mathrm{Cat}(\boldsymbol{\theta}^{(d)}), \ w_i^{(d)} \sim \mathrm{Cat}(\boldsymbol{\phi}^{(\tau_i^{(d)})})$$

- Where $\mathrm{Cat}(i|\boldsymbol{p}) = p_i$ is the categorical distribution (we choose one of a number of options)

- This model is known as **Latent Dirichlet Allocation**

Generating Document

- To generate a document we choose a topic for each word and a word for each topic

$$\begin{aligned} \forall d \in \mathcal{C} \quad & \boldsymbol{\theta}^{(d)} \sim \mathrm{Dir}(\alpha \mathbf{1}) \\ \forall t \in \mathcal{T} \quad & \boldsymbol{\phi}^{(t)} \sim \mathrm{Dir}(\beta \mathbf{1}) \\ \forall d \in \mathcal{C} \ \wedge \ \forall i \in \{1, 2, \ldots, N_d\} \quad & \tau_i^{(d)} \sim \mathrm{Cat}(\boldsymbol{\theta}^{(d)}), \ w_i^{(d)} \sim \mathrm{Cat}(\boldsymbol{\phi}^{(\tau_i^{(d)})}) \end{aligned}$$

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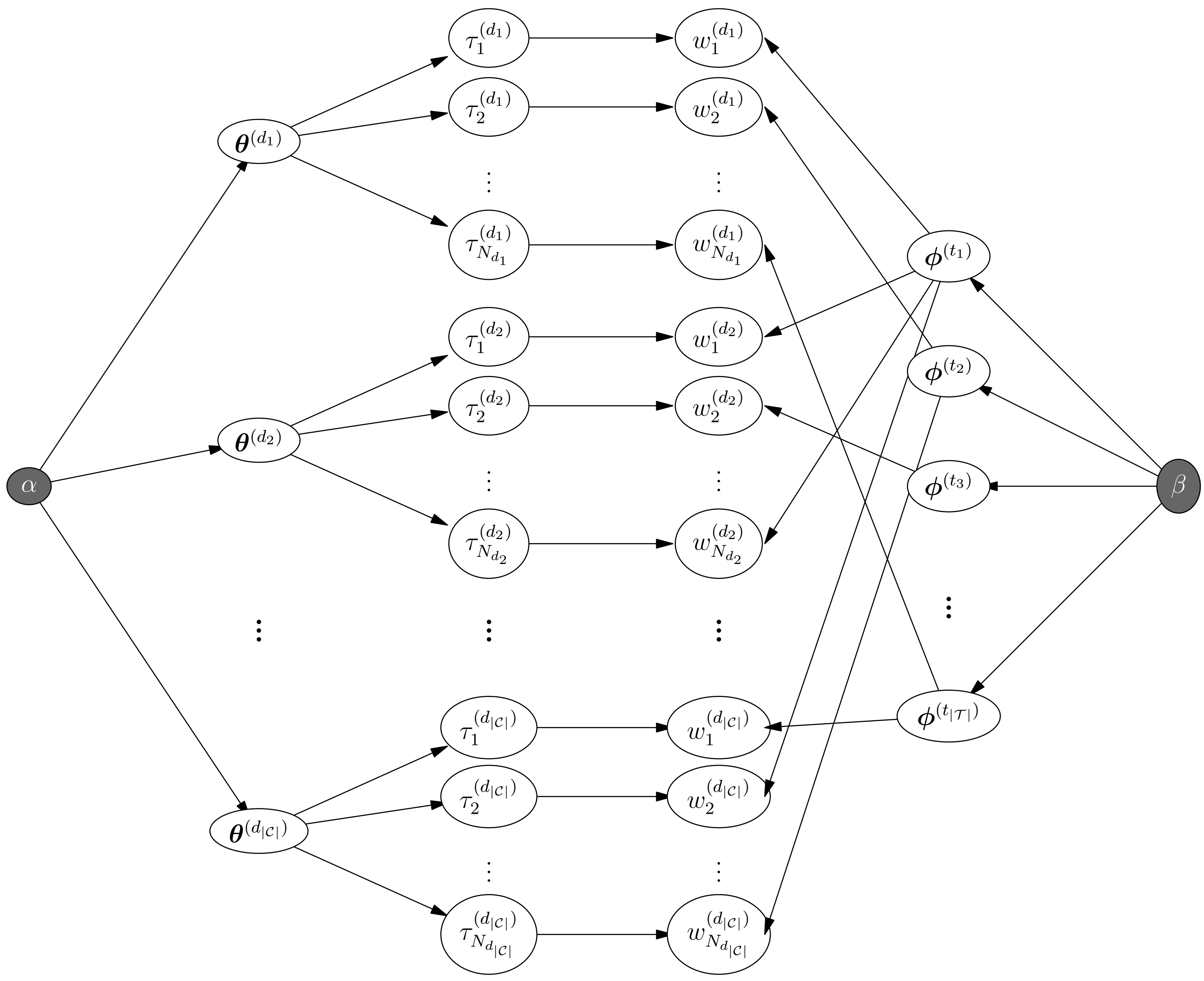
LDA Graphical Model (version 1)



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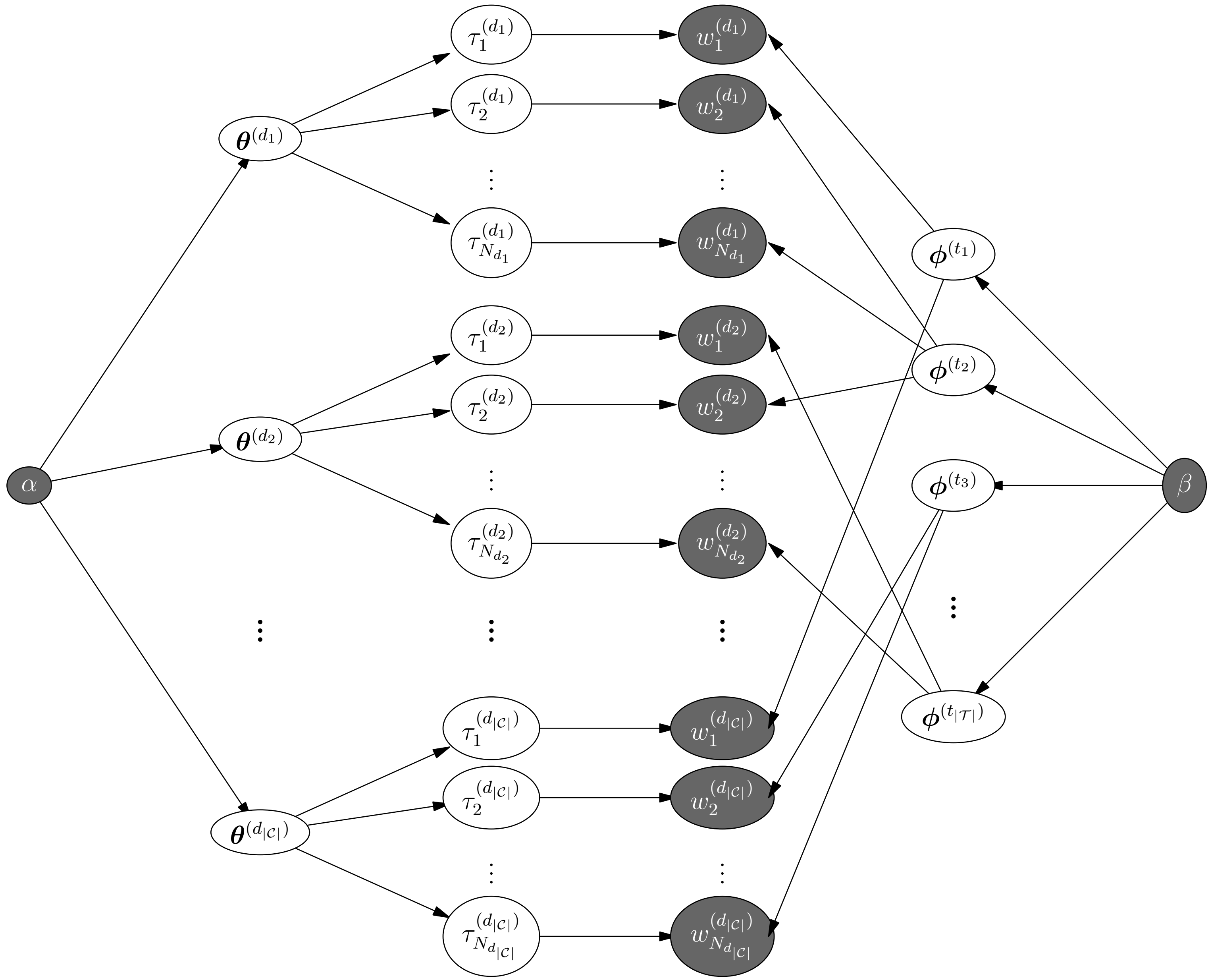
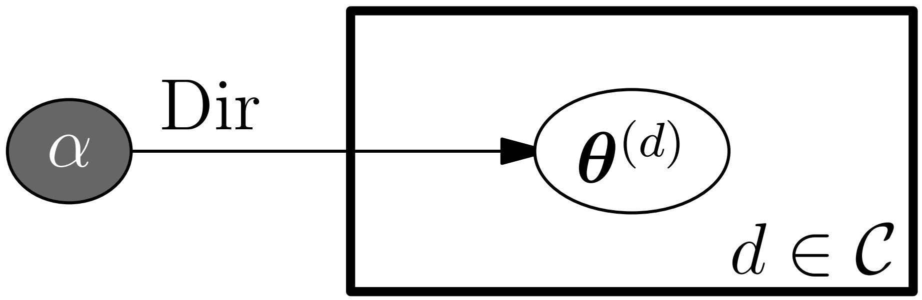


Plate Diagrams

- Drawing every random variable is tedious (and not really possible)
- A short-hand is to draw a box (plate) meaning repeat



- That is we generate vectors $\boldsymbol{\theta}^d$ from a Dirchelet distribution $\mathrm{Dir}\left(\boldsymbol{\theta}|\alpha\mathbf{1}\right)$ for all documents in corpus $\mathcal{C}$

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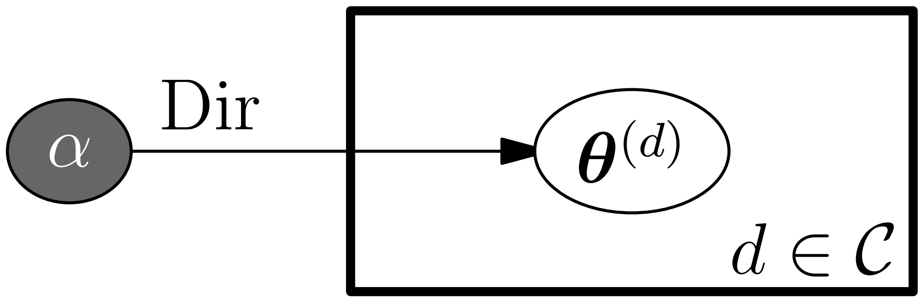

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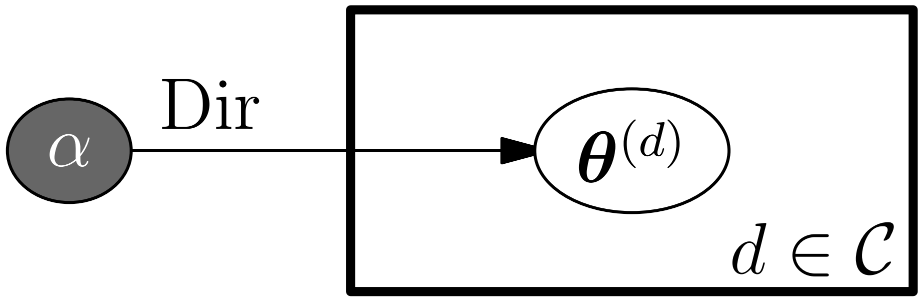

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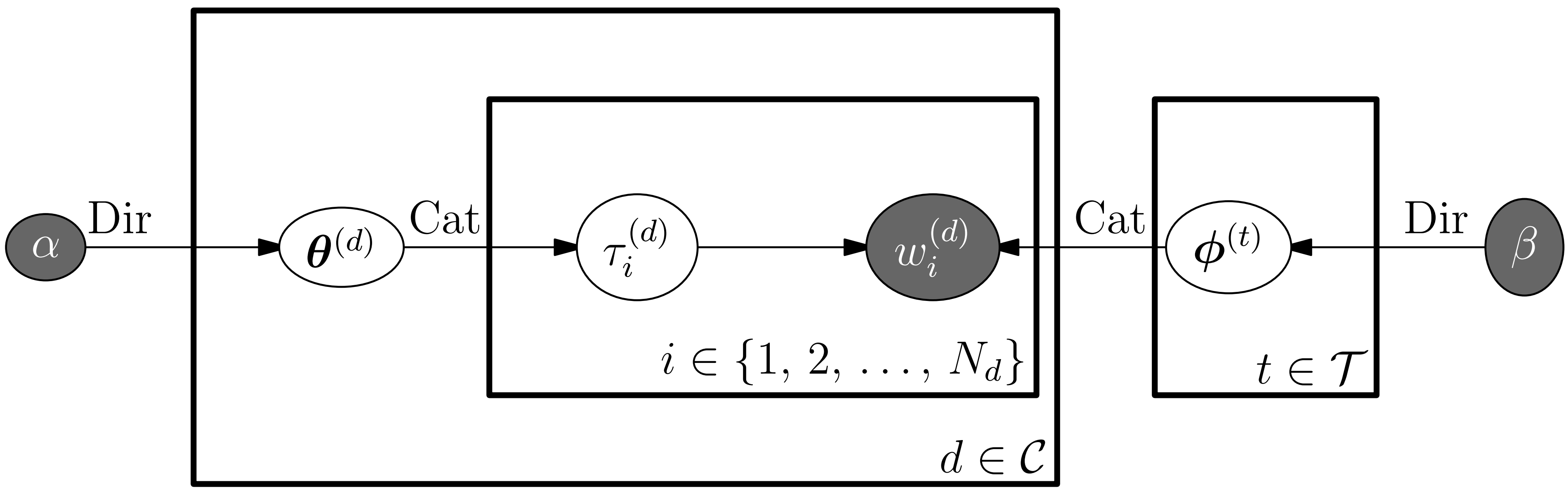

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- Personally, I find it hard to read, but you get used to it

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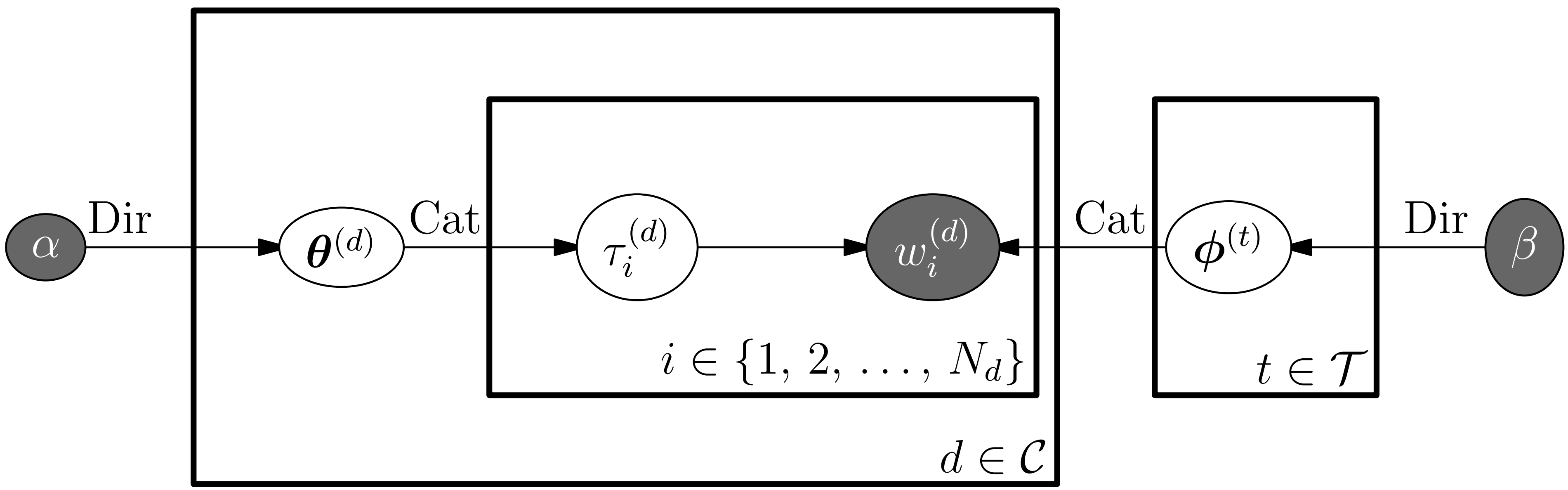

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Probabilistic Model

- The graphical Model is shorthand for the variables

$$\boldsymbol{W} = (\boldsymbol{w}^{(d)} | d \in \mathcal{C}) \quad \text{with} \quad \boldsymbol{w}^{(d)} = (w_1^{(d)}, w_2^{(d)}, \dots, w_{N_d}^{(d)}), \quad \text{and} \quad w_i^{(d)} \in \mathcal{V}$$

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$$\boldsymbol{\Phi} = (\boldsymbol{\phi}^{(t)} | t \in \mathcal{T}) \quad \text{with} \quad \boldsymbol{\phi}^{(t)} = (\phi_w^{(t)} | w \in \mathcal{V}) \in \Lambda^{|\mathcal{V}|}$$

- Distributed according to

$$\mathbb{P}\left(\boldsymbol{W}, \boldsymbol{T}, \boldsymbol{\Theta}, \boldsymbol{\Phi} \middle| \alpha, \beta\right) = \left(\prod_{t \in \mathcal{T}} \mathrm{Dir}\left(\boldsymbol{\phi}^{(t)} \middle| \beta \mathbf{1}\right) \right)$$
$$\left(\prod_{d \in \mathcal{C}} \mathrm{Dir}\left(\boldsymbol{\theta}^{(d)} \middle| \alpha \mathbf{1}\right) \prod_{i=1}^{N_d} \mathrm{Cat}\left(\tau_i^{(d)} \middle| \boldsymbol{\theta}^{(d)}\right) \mathrm{Cat}\left(w_i^{(d)} \middle| \boldsymbol{\phi}^{(\tau_i^{(d)})}\right) \right)$$

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Finding Topics

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- But we are interested in the words associated with each topic $\boldsymbol{\phi}^{(t_i)}$
- And the topics associated with each document $\boldsymbol{\theta}^{(d)}$
- To compute them we need to sample the probability distribution
- One way to do this is using Monte Carlo methods (see next lecture)

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Summary

- Building probabilistic models is an intricate process
- Graphical models provide a representation showing the causal relationship between random variables
- This allows us to break down the joint probability of all the variables into conditional probabilities
- This is useful for building the model, but also can speed up evaluating expectations
- Making observations changes the probabilities of random variables
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