93-S3-Q12
Solution + Discussion

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Abstract

In this document we will go through the solution to the 93-S3-Q12 question and provide a discussion of the question at the end. There are also hints on the first page to aid you in finding a solution. There is no single method that results in an answer to a STEP question, there are a multitude of different paths that end up at the same solution. However, some methods are more straight forward and you are encouraged to take the path of least resistance.

Hints

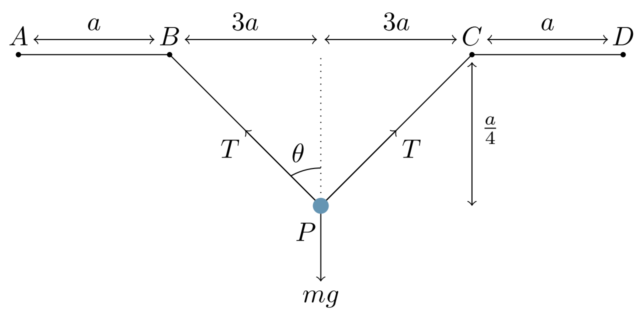
First part: Draw a diagram and use the fact that the system is in equilibrium.

Second part: Draw a diagram and use conservation of energy.

Third part: Show that $T_Q - T_R > 0$.

Solution

Drawing a diagram,



we can calculate the tension in each string as

$$T = \frac{\lambda x}{l} = \frac{kmg}{2a}\left(\frac{a\sqrt{145}}{2}\right) = \frac{1}{4}kmg\sqrt{145}$$

and as the system is in equilibrium,

$$2T\cos\theta = mg$$

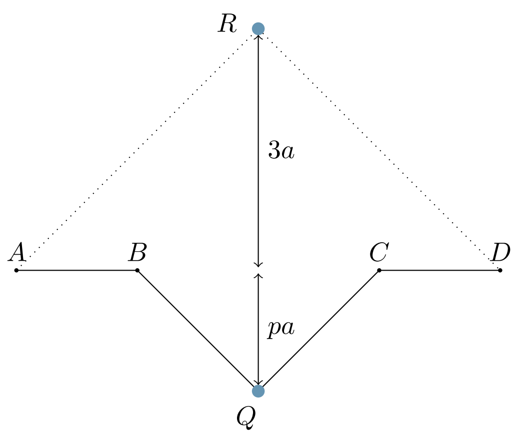
where θ is the angle of the tension of the string with the vertical.

Hence,

$$2 \times \frac{1}{4}kmg\sqrt{145} \times \frac{1}{\sqrt{145}} = mg \Rightarrow k = 2$$

using some trigonometry.

Beginning with a diagram,



we will consider the energies at point Q,

$$\text{At } Q: \quad x_{\text{total}} = 2a\sqrt{9+p^2}, \quad E_e = \frac{\lambda x_{\text{total}^2}}{2l} = 2mga(9+p^2) \text{ J}$$
$$\Rightarrow E_{\text{total}} = 2mga(9+p^2) \text{ J}$$

and at point R,

$$\text{At } R: \quad x_{\text{total}} = 8a, \quad E_e = \frac{\lambda x_{\text{total}^2}}{2l} = 32mga \text{ J}, \quad E_p = mga(p+3) \text{ J}$$
$$\Rightarrow E_{\text{total}} = mga(35+p) \text{ J}.$$

Using conservation of energy,

$$2mga(9+p^2) = mga(35+p)$$

which gives

$$2p^2 - p - 17 = 0.$$

Solving the quadratic gives $p = (1 \pm \sqrt{137})/4$ and as p is positive we have $p = (1+\sqrt{137})/4$.To show the tension at Q is greater than the tension at R we will show that the extension at Q is greater than the extension at R.

Looking at the difference between the extensions,

$$x_Q - x_R = 2a\sqrt{p^2+9} - 8a = 2a\{\sqrt{p^2+9} - 4\}$$

and we see that

$$p^2 = \frac{69}{8} + \frac{\sqrt{137}}{8} > \frac{64}{8} = 8 > 7 \Rightarrow p^2 + 9 > 16.$$

Thus,

$$x_Q - x_R = 2a\{\sqrt{p^2+9} - 4\} > 2a\{\sqrt{16} - 4\} = 0 \Rightarrow x_Q > x_R.$$

Discussion

If you are attempting a mechanics question without a diagram of the system then you are tying the carriage in front of the horses. First and foremost, draw a diagram. Once you have a diagram, the first part is straight forward once you resolve the tension in each string with the weight.

The second part requires a new diagram showing the middle of the string now at point R. Calculating the total extension at Q and R then using conservation of energy yields the required quadratic.

Finally, the third part requires showing that the tension at Q is greater than the tension at R. If our goal is to show that

$$T_Q > T_R$$

it is often easier to equivalently show that

$$T_Q - T_R > 0$$

and this subtlety often leads to a more simple show that answer. Once set up, all that is left is some trivial inequality work to show that $T_Q - T_R > 0$.

Overall, this question relies on a clear, accurate diagram with well laid out workings.