Tandon CS Bridge: HW #6

Due on August 18, 2023

Ratan Dey Extended 24-week

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Problem 5

Use the definition of θ in order to show the following:

1. $5n^3 + 2n^2 + 3n = \theta(n^3)$
2. $\sqrt[2]{7n^2 + 2n - 8} = \theta(n)$

Solution

1. *Proof.* For the lower bound, when $C_1 = 4$, for $n \geq 0$,
   $5n^3 \leq 5n^3 + 2n^2 + 3n$
   For the upper bound,
   $5n^3 + 2n^2 + 3n \leq 6n^3$, when $C_2 = 6$, for $n \geq 3$.
   $C_1 \cdot g(n) \leq 5n^3 + 2n^2 + 3n \leq C_2 \cdot g(n)$
   $\leftrightarrow 4n^3 \leq 5n^3 + 2n^2 + 3n \leq 6n^3$
   $5n^3 + 2n^2 + 3n = \theta(n^3)$ □

2. *Proof.* For $n \geq 4$, $\sqrt[2]{n^2} = \sqrt[2]{7}n \leq \sqrt[2]{7n^2 + 2n - 8}$
   $C_1 = \sqrt[2]{7}$
   For the upper bound, when $C_2 = \sqrt[2]{7} + 1$, for $n \geq 0$,
   $\sqrt[2]{7n^2 + 2n - 8} \leq (\sqrt[2]{7} + 1)n$
   $C_1 \cdot g(n) \leq \sqrt[2]{7n^2 + 2n - 8} \leq C_2 \cdot g(n)$
   $\leftrightarrow \sqrt[2]{7}n \leq \sqrt[2]{7n^2 + 2n - 8} \leq (\sqrt[2]{7} + 1)n$
   $\sqrt[2]{7n^2 + 2n - 8} = \theta(n)$ □