Tandon CS Bridge: HW #3

Due on July 30, 2023

Ratan Dey Extended 24-week

Kunhua Huang

kh4092

Problem 7

Solve the following questions from the Discrete Math zyBook:

1. Excercise 3.1.1, sections a-g

   Use the definitions for the sets given below to determine whether each statement is true or false:

   $A = \{x \in Z : x\ is\ an\ integer\ multiple\ of\ 3\}$
   $B = \{x \in Z : x\ is\ a\ perfect\ square\ \}$
   $C = \{4, 5, 9, 10\}$
   $D = \{2, 4, 11, 14\}$
   $E = \{3, 6, 9\}$
   $F = \{4, 6, 16\}$

   An integer x is a perfect square if there is an integer y such that $x = y^2$.

   a. $27 \in A$
   b. $27 \in B$
   c. $100 \in B$
   d. $E \subseteq C\ or\ C \subseteq E$
   e. $E \subseteq A$
   f. $A \subset E$
   g. $E \in A$

   Solution

   a. true
   b. false
   c. true
   d. false
   e. true
   f. false
   g. false

2. Excercise 3.1.2, sections a-e

   Use the definitions for the sets given below to determine whether each statement is true or false:

   $A = \{x \in \mathbb{Z} : x\ is\ an\ integer\ multiple\ of\ 3\}$
   $B = \{x \in \mathbb{Z} : x\ is\ a\ perfect\ square\ \}$
   $C = \{4, 5, 9, 10\}$
   $D = \{2, 4, 11, 14\}$
   $E = \{3, 6, 9\}$
   $F = \{4, 6, 16\}$

   An integer x is a perfect square if there is an integer y such that $x = y^2$.

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a. $15 \subset A$

b. $\{15\} \subset A$

c. $\emptyset \subset C$

d. $D \subseteq D$

e. $\emptyset \in B$

f. $A\ is\ an\ infinite\ set$

g. $B\ is\ a\ finite\ set$

h. $|E| = 3$

i. $|E| = |F|$

Solution

a. true

b. false

c. true

d. true

e. true

f. true

g. false

h. true

i. true

3. Excercise 3.1.5, section b, d

Express each set using set builder notation. Then if the set is finite, give its cardinality. Otherwise, indicate that the set is infinite.

(a) $\{3, 6, 9, 12, \ldots\}$

(b) $\{0, 10, 20, 30, \ldots, 1000\}$

Solution

(a) $A = \{x \in \mathbb{Z}^+ : \ x\ is\ an\ integer\ multiple\ of\ 3\}$

(b) $A = \{x \in \mathbb{Z} : \ x\ is\ an\ integer\ multiple\ of\ 10\}$

4. Excercise 3.2.1, sections a-k

Let $X = \{1, \{1\}, \{1, 2\}, 2, \{3\}, 4\}$. Which statements are true?

(a) $2 \in X$

(b) $\{2\} \subseteq X$

(c) $\{2\} \in X$

(d) $3 \in X$

(e) $\{1, 2\} \in X$

(f) $\{1, 2\} \subseteq X$

(g) $\{2, 4\} \subseteq X$

(h) $\{2, 4\} \in X$

(i) $\{2, 3\} \subseteq X$

(j) $\{2, 3\} \in X$

(k) $|X| = 7$

Solution

a, b, e, f, and g are true.

Problem 8

Solve Exercise 3.2.4, section b from the Discrete Math zyBook.

Let $A = \{1, 2, 3\}$. What is $\{X \in P(A) : \ 2 \in X\}$?

Solution

$\{\emptyset, \{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$

Problem 9

Solve the following questions from the Discrete Math zyBook:

1. Excercise 3.3.1, sections c-e.

   Define the sets A, B, C, and D as follows:

   $A = \{-3, 0, 1, 4, 17\}$
   $B = \{-12, -5, 1, 4, 6\}$
   $C = \{x \in \mathbb{Z} : \ x \ is \ odd\}$
   $D = \{x \in \mathbb{Z} : \ x \ is \ positive\}$
   For each of the following set expressions, if the corresponding set is finite, express the set using roster natation. Otherwise, indicate that the set is infinite.

   (a) $A \cap C$
   (b) $A \cup (B \cap C)$
   (c) $A \cap B \cap C$
   (d) $A \cup C$
   (e) $(A \cup B) \cap C$
   (f) $A \cup (C \cap D)$

   Solution

   (a) $\{-3, 1, 17\}$
   (b) $\{-5. - 3, 0, 1, 4, 17\}$
   (c) $\{1\}$

2. Excercise 3.3.3, sections a, b, e, f

   Use the following definitions to express each union or intersection given. You can use roster or set builder notation in your responses, but no set operations, For each definition, $i \in \mathbb{Z}^+$

   $A_i = \{i^0, i^1, i^2\}$
   $B_i = \{x \in \mathbb{R} : \ -i <= x <= 1/i\}$
   $C_i = \{x \in \mathbb{R}; \ -1/i <= x <= 1/i\}$

   (a) $\bigcap_{i=2}^{5} A_i$
   (b) $\bigcup_{i=2}^{5} A_i$
   (c) $\bigcap_{i=1}^{100} B_i$
   (d) $\bigcup_{i=1}^{100} B_i$
   (e) $\bigcap_{i=1}^{100} C_i$
   (f) $\bigcup_{i=1}^{100} C_i$

   Solution

(a) $\{1\}$

(b) $\{1, 2, 3, 4, 5, 9, 16, 25\}$

(c) $\{x \in \mathbb{R} : -1 <= x <= 1/100\}$

(d) $\{x \in \mathbb{R} : -100 <= x <= 1\}$

(e) $\{x \in \mathbb{R} : -1/100 <= x <= 1/100\}$

(f) $\{x \in \mathbb{R} : -1 <= x <= 1\}$

3. Excercise 3.3.4, sections b, d

Use the set definitions $A = \{a, b\}$ and $B = \{b, c\}$ to express each set below. Use roster notation in your solutions.

(a) $P(A \cup B)$

(b) $P(A) \cup P(B)$

Solution

(a) $A \cup B = \{a, b, c\}$
$P(A \cup B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

(b) $P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
$P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$
$P(A) \cup P(B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$

Problem 10

Solve the following questions from the Descrete Math zyBook:

1. Excercise 3.5.1, sections b, c

   The sets A, B, and C are defined as follows:

   $A = \{tall, grande, venti\}$
   $B = \{foam, non - foam\}$
   $C = \{non - fat, whole\}$

   Use the definition for A, B, and C to answer the questions. Express the elements using n-tuple notation, not string notation.

   (a) Write an element from the set $B \times A \times C$.
   (b) Write the set $B \times C$ using roster notation.

   Solution

   (a) $(foam, tall, non - fat)$
   (b) $\{(foam, non - fat), (foam, whole), (non - foam, non - fat), (non - foam, whole)\}$

2. Excercise 3.5.3, sections b, c, e

   Indicate which of the following statements are true.

   (a) $\mathbb{Z}^2 \subseteq \mathbb{R}^2$
   (b) $\mathbb{Z}^2 \cap \mathbb{Z}^3 = \emptyset$
   (c) For any three sets, A, B, and C, if $A \subseteq B$, then $A \times C \subseteq B \times C$

   Solution

   (a) true
   (b) true
   (c) true

3. Excercise 3.5.6, sections d, e

   Express the following sets using the roster method. Express the elements as strings, not n-tuples.

   (a) $\{xy : \text{ where } x \in \{0\} \cup \{0\}^2 \text{ and } y \in \{1\} \cup \{1\}^2\}$
   (b) $\{xy : x \in \{aa, ab\} \text{ and } y \in \{a\} \cup \{a\}^2\}$

   Solution

   (a) $\{01, 011, 001, 0011\}$
   (b) $\{aaa, aaaa, aba, abaa\}$

Problem 10 continued on next page. . .

4. Excercise 3.5.7, sections c, f, g

   Use the following set definitions to specify each set in roster notation. Except where noted, express elements of Cartesian products as strings.

   $A = \{a\}$
   $B = \{b, c\}$
   $C = \{a, b, d\}$

   (a) $(A \times B) \cup (A \times C)$
   (b) $P(A \times B)$
   (c) $P(A) \times P(B)$. Use ordered pair notation for elements of the Cartesian product.

   Solution

   (a) $\{ab, ac\} \cup \{aa, ab, ad\}$
   $\{abaa, abab, abad, acaa, acab, acad\}$
   (b) $\{\emptyset, ab, ac, abac\}$
   (c) $\{\emptyset, \{a\}\} \times \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$
   $\{\emptyset, (\{a\}, \{b\}), (\{a\}, \{c\}), (\{a\}, \{b, c\})\}$

Problem 11

Solve the following questions from the Discrete Math zyBook:

1. Excercise 3.6.2, sections b, c

   Use the set identities given in the table to prove the following new identities. Label each step in your proof with the set identity used to establish that step.

   (a) $(B \cup A) \cap (\bar{B} \cup A) = A$

   (b) $\overline{(A \cap \overline{B})} = \bar{A} \cup B$

   Solution

   (a) *Proof.*

1.	$\leftrightarrow A \cup (\bar{B} \cap B)$	*Associative Laws*
2.	$\leftrightarrow A \cup \emptyset$	*Complement Laws*
3.	$\leftrightarrow A$	*Domination Law*

   □

   (b) *Proof.*

1.	$\leftrightarrow \bar{A} \cup \bar{\bar{B}}$	*De Morgan's Law*
2.	$\leftrightarrow \bar{A} \cup B$	*Double Complement Law*

   □

2. Excercise 3.6.3, sections b, d

   A set equation is not an identity if there are examples for the variables denoting the sets that cause the equation to be false. Show that each set equation given below is not a set identity.

   (a) $A - (B \cap A) = A$

   (b) $(B - A) \cup A = A$

   Solution

   (a) *Proof.* Let $A = \{1, 2\}$, $B = \{1, 2, 3\}$,
   $A - (B \cap A) = \emptyset \neq A$
   It is not a set identity. □

   (b) *Proof.* Let $A = \{1, 2, \}$, $B = \{1, 2, 3\}$,
   $(B - A) \cup A = \{1, 2, 3\} \neq A$
   It is not a set identity. □

3. Excercise 3.6.4, sections b, c

   The set subtraction law states that $A - B = A \cap \bar{B}$. Use the set subtraction law as well as the other set identities given in the table to prove each of the following new identities. Label each step in your proof with the set identity used to establish that step.

   (a) $A \cap (B - A) = \emptyset$

Problem 11 continued on next page. . .

(b) $A \cup (B - A) = A \cup B$

Solution

(a) *Proof.*

1.	$\leftrightarrow A \cap (B \cap \bar{A})$	*Subtraction Law*
2.	$\leftrightarrow (A \cap \bar{A}) \cap B$	*Associative Law & Commutative Law*
3.	$\leftrightarrow \emptyset \cap B$	*Complement Law*
4.	$\leftrightarrow \emptyset$	*Domination Law*

$\square$

(b) *Proof.*

1.	$\leftrightarrow A \cup (B \cap \bar{A})$	*Subtraction Law*
2.	$\leftrightarrow (A \cup B) \cap (A \cup \bar{A})$	*Distributive Law*
3.	$\leftrightarrow (A \cup B) \cap U$	*Complement Laws*
4.	$\leftrightarrow A \cup B$	*Identity Laws*

$\square$