Time-series linear regression review

- $y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_K x_{K,t} + \varepsilon_t$ for $t = 1, \ldots, T$
  - $E(\varepsilon_t) = 0$
  - $\text{Var}(\varepsilon_t) = \sigma_\varepsilon^2$
- Matrix notation $\Rightarrow \underset{T\times 1}{\mathbf{y}} = \underset{[T\times(K+1)]}{\mathbf{X}} \underset{[(K+1)\times 1]}{\boldsymbol{\beta}} + \underset{T\times 1}{\boldsymbol{\varepsilon}}$
  - $\mathbf{y} = \begin{bmatrix} y_1 & y_2 & \cdots & y_T \end{bmatrix}'$
  - $\mathbf{X} = \begin{bmatrix} \boldsymbol{\iota}_T & \mathbf{x}_1 & \cdots & \mathbf{x}_K \end{bmatrix}$
    - $\mathbf{x}_k = \begin{bmatrix} x_{k,1} & x_{k,2} & \cdots & x_{k,T} \end{bmatrix}'$ for $k = 1, \ldots, K$
  - $\boldsymbol{\beta} = \begin{bmatrix} \beta_0 & \beta_1 & \cdots & \beta_K \end{bmatrix}'$
  - $\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_T \end{bmatrix}'$

OLS estimation

- Objective function $\Rightarrow \underset{\hat{\boldsymbol{\beta}}}{\arg\min}\ (\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}})'(\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}})$
- $\boldsymbol{\beta}$ estimate $\Rightarrow \hat{\boldsymbol{\beta}} = \begin{bmatrix} \hat{\beta}_0 & \hat{\beta}_1 & \cdots & \hat{\beta}_K \end{bmatrix}' = (\mathbf{X}'\mathbf{X})^{-1}(\mathbf{X}'\mathbf{y})$
- Residual vector $\Rightarrow \hat{\boldsymbol{\varepsilon}} = \begin{bmatrix} \hat{\varepsilon}_1 & \hat{\varepsilon}_2 & \cdots & \hat{\varepsilon}_T \end{bmatrix}' = \mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}$
- σ_ε^2 estimate $\Rightarrow \hat{\sigma}_\varepsilon^2 = \frac{\hat{\boldsymbol{\varepsilon}}'\hat{\boldsymbol{\varepsilon}}}{T-(K+1)}$
- Covariance matrix $\Rightarrow \hat{\boldsymbol{\Sigma}}_{\hat{\boldsymbol{\beta}}} = \hat{\sigma}_\varepsilon^2\,(\mathbf{X}'\mathbf{X})^{-1}$
  - Standard errors $\Rightarrow \hat{\mathbf{s}} = \left[\mathrm{diag}\left(\hat{\boldsymbol{\Sigma}}_{\hat{\boldsymbol{\beta}}}\right)\right]^{0.5}$
  - t-statistics $\Rightarrow t_i = \frac{\hat{\beta}_i}{\hat{s}_i}$ for $i = 0, 1, \ldots, K$

Goodness of fit

- Coefficient of determination $\Rightarrow R^2 = 1 - \frac{\text{SSE}}{\text{TSS}}$
  - $\text{SSE} = \hat{\varepsilon}'\hat{\varepsilon}$
  - $\text{TSS} = \sum_{t=1}^{T} (y_t - \hat{\mu}_y)^2 = (\mathbf{y} - \hat{\mu}_y \boldsymbol{\iota}_T)'(\mathbf{y} - \hat{\mu}_y \boldsymbol{\iota}_T)$
    - $\hat{\mu}_y = \frac{1}{T} \sum_{t=1}^{T} y_t = \frac{\boldsymbol{\iota}_T' \mathbf{y}}{T}$
- Adjusted $R^2 \Rightarrow \bar{R}^2 = R^2 - (1 - R^2)\left[\frac{T}{T-(K+1)}\right]$