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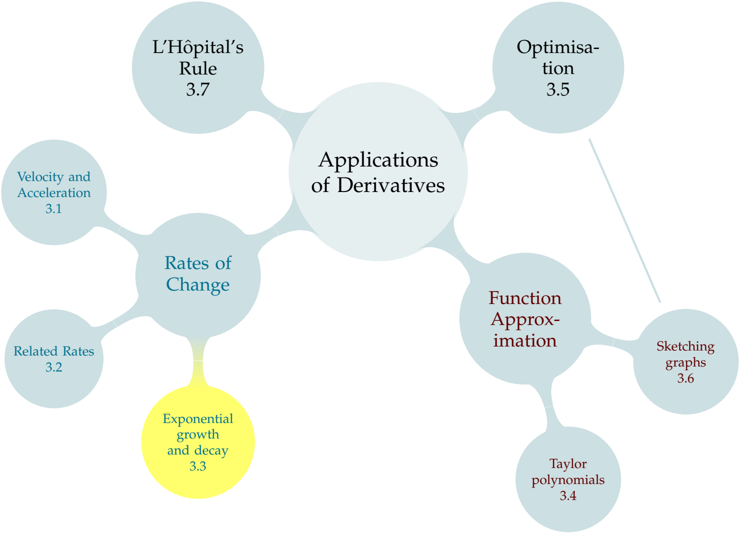

Radioactive Decay

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Let $\boxed{Q(t) = Ce^{-kt}}$, where k and C are constants. Then:

$$\frac{dQ}{dt}(t) = C \cdot e^{-kt} \cdot (-k) = -kCe^{-kt} = -kQ(t)$$

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Quantity of a Radioactive Isotope

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B. negative or zero
C. could be either
D. I don't know

Q

ans

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What is the sign of C?

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What is the sign of C?

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B. negative or zero
C. could be either
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Seaborgium Decay

The amount of ${}^{266}Sg$ (Seaborgium-266) in a sample at time t (measured in seconds) is given by

$$Q(t) = Ce^{-kt}$$

Let's approximate the half life of ${}^{266}Sg$ as 30 seconds. That is, every 30 seconds, the size of the sample halves.

What are C and k?

$$Q(t) = Ce^{-kt}$$

Every 30 seconds, the size of the sample halves. What are C and k?

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Every 30 seconds, the size of the sample halves. What are C and k?

(1) $Q(0)$ is the amount of ^{266}Sg at time 0: usually, the initial sample size.
C is the quantity at time 0

(2) The half-life of ^{266}Sg is 30 seconds. So, if we're measuring t in seconds, $Q(30) = \frac{1}{2}Q(0)$.

$$\frac{1}{2}C = \frac{1}{2}Q(0) = Q(30) = Ce^{-30k}$$

$$k = \frac{\log 2}{30}$$

A sample of radioactive matter is stored in a lab in 2000. In the year 2002, it is tested and found to contain 10 units of a particular radioactive isotope. In the year 2005, it is tested and found to contain only 2 units of that same isotope. How many units of the isotope were present in the year 2000?

The quantity of the isotope t years after 2000 is given by

$$Q(t) = Ce^{-kt}$$

where $C = Q(0)$ is the amount in the initial sample. Then the question asks us to solve for C, given

$$10 = Q(2) = Ce^{-2k} \quad \text{and} \quad 2 = Q(5) = Ce^{-5k}$$

Then

$$C = 10e^{2k} = 2e^{5k}$$

$$e^{3k} = 5 \quad \implies \quad e^{k} = \sqrt[3]{5}$$

$$C = 10e^{2k} = 10\left(\sqrt[3]{5}\right)^{2} \approx 29$$

$Q'(t) = kQ(t)$

The number of atoms in a sample that decay in a given time interval is proportional to the number of atoms in the sample.

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The amount of interest a bank account accrues in a given time interval is proportional to the balance in that bank account.

Exponential Growth – Theorem 3.3.2

Let $Q = Q(t)$ satisfy:

$$\frac{dQ}{dt} = kQ$$

for some constant k. Then for some constant $C = Q(0)$,

$$Q(t) = Ce^{kt}$$

Exponential Growth – Theorem 3.3.2

Let $Q = Q(t)$ satisfy:

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Suppose $y(t)$ is a function with the properties that

$$\frac{dy}{dt} + 3y = 0 \quad \text{and} \quad y(1) = 2.$$

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$\frac{\mathrm{d}y}{\mathrm{d}t} = -3y$, so $y(t) = Ce^{-3t}$ by the result above.

To solve for C, we set $t = 1$:

$$2 = y(1) = Ce^{-3} \implies C = 2e^3$$

So,

$$y(t) = 2e^3 \cdot e^{-3t} = 2e^{3(1-t)}$$

Population Growth

Suppose a petri dish starts with a culture of 100 bacteria cells and a limited amount of food and space. The population of the culture at different times is given in the table below. At approximately what time did the culture start to show signs of limited resources?

time	population
0	100
1	1000
3	100000
5	1000000

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time	population
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5	1000000

All the populations before $t = 5$ follow $B(t) = 100 \cdot 10^t = 100e^{t \log 10}$. At $t = 5$ they do not; so some time between $t = 3$ and $t = 5$, the bacteria started reproducing at a slower rate.

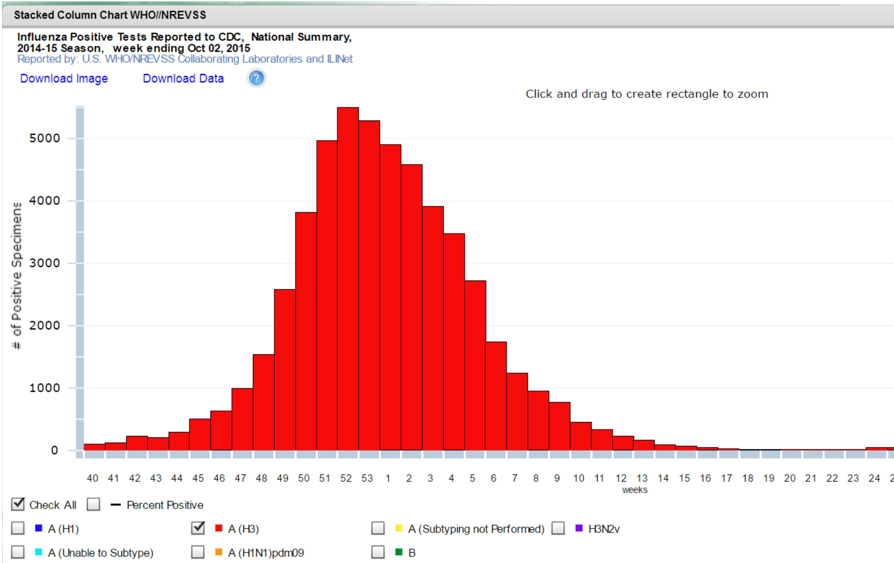
FLU SEASON

The CDC keeps records (link) on the number of flu cases in the US by week. At the start of the flu season, the 40th week of 2014, there are 100 cases of a particular strain. Five weeks later (at week 45), there are 506 cases. What do you think was the first week to have 5,000 cases? What about 10,000 cases?

Q

ans

Flu Season


Stacked Column Chart WHO//NREVSS
Influenza Positive Tests Reported to CDC, National Summary,
2014-15 Season, week ending Oct 02, 2015
Reported by: U.S. WHO/NREVSS Collaborating Laboratories and ILINet
Download Image
Download Data
Click and drag to create rectangle to zoom
# of Positive Specimens
5000
4000
3000
2000
1000
0
40 41 42 43 44 45 46 47 48 49 50 51 52 53 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
weeks
Check All
Percent Positive
A (H1)
A (H3)
A (Subtyping not Performed)
H3N2v
A (Unable to Subtype)
A (H1N1)pdm09
B

Flu Season

Let $t = 0$ be the 40th week of 2014. Then we can model the spread of the virus like so:

$$P(t) = 100e^{kt}$$

We have one other data point: $506 = P(5) = 100e^{5k}$, so we get $e^k = 5.06^{1/5}$. Now our equation is:

$$P(t) = 100(5.06)^{t/5}$$

We set it equal to 5000 and solve: $5000 = 100(5.06)^{t/5}$ implies

$$(5.06)^{t/5} = 50 \implies \frac{t}{5}\log(5.06) = \log 50 \implies t = \frac{5\log 50}{\log(5.06)} \approx 12.06$$

Data from the CDC says Week 51 ($t = 11$) had 4972 cases, and Week 52 ($t = 12$) had 5498 cases.

Using the same formula, $10000 = 100(5.06)^{t/5}$ yields $t = \frac{5\log 100}{\log 5.06} \approx 14.2$ weeks; but the data shows that the flu season peaked with around 5,000 cases a week, and never got much higher.

Newton's Law of Cooling – Equation 3.3.7

The rate of change of temperature of an object is proportional to the difference in temperature between that object and its surroundings.

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$$\frac{dT}{dt}(t) = K[T(t) - A]$$

where $T(t)$ is the temperature of the object at time t, A is the (constant) ambient temperature of the surroundings, and K is some constant depending on the object.

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$T(t)$ is the temperature of the object, A is the ambient temperature, K is some constant.

What is true of K?

A. $K \geq 0$

B. $K \leq 0$

C. $K = 0$

D. K could be positive, negative, or zero, depending on the object

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is the only function satisfying Newton's Law of Cooling

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If $T(10) < A$, then:

A. $K > 0$

B. $T(0) > 0$

C. $T(0) > A$

D. $T(0) < A$

Newton's Law of Cooling – Equation 3.3.7

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D. $T(0) < A$

Evaluate $\lim_{t \to \infty} T(t)$.

A. A
B. 0
C. ∞
D. $T(0)$

Newton's Law of Cooling – Equation 3.3.7

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Evaluate $\lim_{t \to \infty} T(t)$.

A. A

B. 0

C. ∞

D. $T(0)$

What assumptions are we making that might not square with the real world?

Newton's Law of Cooling – Equation 3.3.7

$$\frac{dT}{dt} = K[T(t) - A]$$

$T(t)$ is the temperature of the object, A is the ambient temperature, and K is some constant.

Temperature of a Cooling Body – Corollary 3.3.8

$$T(t) = [T(0) - A]e^{Kt} + A$$

Q

ans

A farrier forms a horseshoe heated to 400° C, then dunks it in a river at room-temperature (25° C). The water boils for 30 seconds. The horseshoe is safe for the horse when it's 40° C. When can the farrier put on the horseshoe?

Q

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$$T(t) = [T(0) - A]e^{Kt} + A$$

Q

ans

We know: $T(0) = 400$, $T(30) = 100$, and $A = 25$. We want to find K.

$$100 = T(30) = [T(0) - A]e^{30K} + A = 375e^{30K} + 25$$
$$\Rightarrow 75 = 375e^{30K} \Rightarrow \frac{1}{5} = e^{30K} \Rightarrow K = \frac{-\log 5}{30}$$

Now, we set $T(t) = 40$ and solve for t:

$$\begin{aligned}
40 &= T(t) = 375e^{\frac{-\log 5}{30}t} + 25 \\
15 &= 375e^{\frac{-log5}{30}t} = 375 \cdot 5^{-t/30} \\
\frac{1}{25} &= 5^{-t/30} \\
25 &= 5^{t/30} \\
2 &= t/30
\end{aligned}$$

So the farrier can put the shoe on after 60 seconds in the water.

A glass of just-boiled tea is put on a porch outside. After ten minutes, the tea is 40°, and after 20 minutes, the tea is 25°. What is the temperature outside?

$T(0) = 100$, so

$$T(10) = [100 - A]e^{10K} + A = 100e^{10K} + A\left(1 - e^{10K}\right) = 40$$
$$T(20) = [100 - A]e^{20K} + A = 100e^{20K} + A\left(1 - e^{20K}\right) = 25$$

Solving both for A, we get $A = \dfrac{40 - 100e^{10K}}{1 - e^{10K}} = \dfrac{25 - 100e^{20K}}{1 - e^{20K}}$

Although this looks complicated, if we set $x = e^{10k}$, it simplifies to something we can easily solve.

$$A = \frac{40 - 100e^{10K}}{1 - e^{10K}} = \frac{25 - 100e^{20K}}{1 - e^{20K}}$$

$$A = \frac{40 - 100x}{1 - x} = \frac{25 - 100x^2}{1 - x^2}$$

$$(40 - 100x)(1 - x^2) = (25 - 100x^2)(1 - x)$$

$$(40 - 100x)(1 + x)(1 - x) = (25 - 100x^2)(1 - x)$$

$$(40 - 100x)(1 + x) = 25 - 100x^2$$

$$40 - 60x - 100x^2 = 25 - 100x^2$$

$$40 - 60x = 25$$

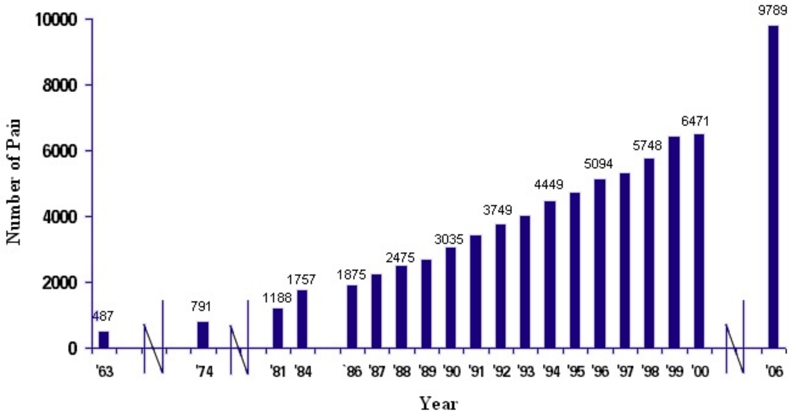
$$x = \frac{1}{4}$$

$$A = \frac{40 - 100x}{1 - x} = \frac{40 - \frac{100}{4}}{1 - \frac{1}{4}} = 20$$

It is 20 degrees outside.

A

In 1963, the US Fish and Wildlife Service recorded a bald eagle population of 487 breeding pairs. In 1993, that number was 4015. How many breeding pairs would you expect there were in 2006? What about 2015?



9789
10000
8000
6000
4000
2000
0
Number of Pair
6471
5748
5094
4449
3749
3035
2475
1875
1757
1188
791
487
'63
'74
'81
'84
'86
'87
'88
'89
'90
'91
'92
'93
'94
'95
'96
'97
'98
'99
'00
'06
Year

Since we don't have a better model, let's assume the population P of nesting pairs follows:

$$P(t) = P(0)e^{Kt}$$

for some constant K.

To fit the data we have, let $t = 0$ represent 1963, so $P(0) = 487$. Then

$$4015 = P(30) = 487e^{30K}$$

so $e^K = \left(\frac{4015}{487}\right)^{1/30}$.

Now we use this to predict $P(43)$ (since 2006 is 43 years after 1963) and $P(52)$ (since 2015 is 52 years after 1963).

$$P(43) = 487(e^K)^{43} = 487\left(\frac{4015}{487}\right)^{43/30} \approx 10016$$

So we guess in 2016 there were about 10,016 breeding pairs in the lower 48.

$$P(52) = 487(e^K)^{52} = 487\left(\frac{4015}{487}\right)^{52/30} \approx 18860$$

link: Wood Bison Restoration in Alaska, Alaska Department of Fish and Game

Excerpt:

> *Based on experience with reintroduced populations elsewhere, wood bison would be expected to increase at a rate of 15%-25% annually after becoming established.... With an average annual growth rate of 20%, an initial precalving population of 50 bison would increase to 500 in approximately 13 years.*

Now You

Are they using our same model?

Q

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NOW YOU Are they using our same model?

Our model gives the same result.

Compound Interest

Suppose you invest \$10,000 in an account that accrues interest each month. After one month, your balance (with interest) is \$10,100. How much money will be in your account after a year?

Q

ans

Compound Interest

Suppose you invest \$10,000 in an account that accrues interest each month. After one month, your balance (with interest) is \$10,100. How much money will be in your account after a year?

Compound interest is calculated according to the formula Pe^{rt}, where r is the interest rate and t is time.

Q

ans

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Compound interest is calculated according to the formula Pe^{rt}, where r is the interest rate and t is time.

Measuring time in months,

$$\begin{aligned} 10000e^{r\cdot 1} &= 10100 \\ e^r &= \frac{10100}{10000} = 1.01 \\ 10000e^{12r} &= 10000\cdot(e^r)^{12} = 10000\cdot 1.01^{12} \approx 11268.25 \end{aligned}$$

Carrying Capacity

For a population of size P with unrestricted access to resources, let β be the average number of offspring each breeding pair produces per generation, where a generation has length t_g. Then $b = \frac{\beta-2}{2t_g}$ is the net birthrate (births minus deaths) per member per unit time. This yields $\frac{dP}{dt}(t) = bP(t)$, hence:

Q

ans

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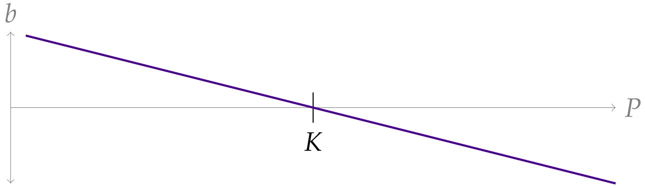
$$P(t) = P(0)e^{bt}$$

But as resources grow scarce, b might change.

Carrying Capacity

b is the net birthrate (births minus deaths) per member per unit time. If K is the carrying capacity of an ecosystem, we can model $b = b_0(1 - \frac{P}{K})$.



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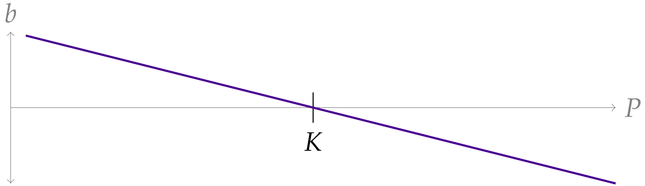

Now You  Describe to your neighbour what the following mean in terms of the model:

- $b > 0, b = 0, b < 0$
- $P = 0, P > 0, P < 0$

Q

ans

Carrying Capacity

Then:

$$\frac{dP}{dt}(t) = \underbrace{b_0 \left(1 - \frac{P(t)}{K}\right)}_{\text{per capita birthrate}} P(t)$$

Carrying Capacity

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This is an example of a differential equation that we don't have the tools to solve. (If you take more calculus, though, you'll learn how!) It's also an example of a way you might tweak a model so its assumptions better fit what you observe.

Radiocarbon Dating

Researchers at Charlie Lake in BC have found evidence[1] of habitation dating back to around 8500 BCE. For instance, a butchered bison bone was radiocarbon dated to about 10,500 years ago.

Suppose a comparable bone of a bison alive today contains 1μg of ^{14}C. If the half-life of ^{14}C is about 5730 years, roughly how much ^{14}C do you think the researchers found in the sample?

[1] http://pubs.aina.ucalgary.ca/arctic/Arctic49-3-265.pdf

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A. About $\frac{1}{10{,}500}$ μg

B. About $\frac{1}{4}$ μg

C. About $\frac{1}{2}$ μg

D. About 1 μg

E. I'm not sure how to estimate this

Radiocarbon Dating

First, an estimate; 10500 is not so far off from 2(5730), i.e. two half-lives, so we might guess that there is roughly a $(\frac{1}{2})^2 = \frac{1}{4}$ of a microgram left.

We know $Q(t) = Ce^{-kt} = e^{-kt}$ μg. We want to find $Q(10500)$, so we need to solve for k. Since we know the half-life: to do this, solve

$$\frac{1}{2} = e^{-k \cdot 5730} \quad \text{to get} \quad k = \frac{\log 2}{5730}$$

Now:

$$Q(10500) = e^{-\frac{\log 2}{5730} \cdot 10500} = 2^{-\frac{10500}{5730}} \approx 0.28\ \mu\text{g}$$

Suppose a body is discovered at 3:45 pm, in a room held at 20°, and the body's temperature is 27°, not the normal 37°. At 5:45 pm, the temperature of the body has dropped to 25.3°. When did the inhabitant of the body die?

Q

Set our time so that $t = 0$ is 3:45pm and $t = 2$ is 5:45pm. Then $T(0) = 27$, $T(2) = 25.4$, and $A = 20$. Now:

$$T(t) = [27 - 20]e^{Kt} + 20 = 7e^{Kt} + 20$$

Using what we know about 5:45pm:

$$7e^{2K} + 20 = T(2) = 25.3$$

so

$$7e^{2K} = 5.3 \implies e^{2K} = \frac{5.3}{7} \implies e^{K} = \left(\frac{5.3}{7}\right)^{1/2}$$

Now:

$$T(t) = 7e^{Kt} + 20 = 7\left(\frac{5.3}{7}\right)^{t/2} + 20$$

So we set $T(t) = 37$ and solve for t.

$$
\begin{aligned}
7\left(\frac{5.3}{7}\right)^{t/2} + 20 &= 37 \\
7\left(\frac{5.3}{7}\right)^{t/2} &= 17 \\
\left(\frac{5.3}{7}\right)^{t/2} &= \frac{17}{7} \\
\frac{t}{2} &= \frac{\log(17/7)}{\log(5.3/7)} \\
t &= 2\frac{\log(17/7)}{\log(5.3/7)} \approx -6.4
\end{aligned}
$$

So the person died about 6.4 hours before 3:45pm. Now 0.4 hours is 24 minutes. So 6 hours and 24 minutes before 3:45 pm is 6 hours before 3:21pm, which is 9:21 am.

Included Work

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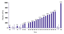 'Chart of Bald Eagle Breeding Pairs in Lower 48 States'. Eagle nesting data, US Fish and Wildlife Service Midwest Region (accessed 19 October 2016), 44

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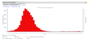 U.S. WHO/NREVSS Collaborating Laboratories and ILNet. 'Stacked Column Chart WHO//NREVSS' Centers for Disease Control and Prevention. No longer available from http://gis.cdc.gov/grasp/fluview/fluportaldashboard.html (accessed 20 October 2015), 26

Alaska Department of Fish and Game, Division of Wildlife Conservation. (April 2007). Wood Bison Restoration in Alaska: A Review of Environmental and Regulatory Issues and Proposed Decisions for Project Implementation, p. 11. http://www.adfg.alaska.gov/static/species/speciesinfo/woodbison/pdfs/er_no_appendices.pdf (accessed 2015 or 2016), 46, 47