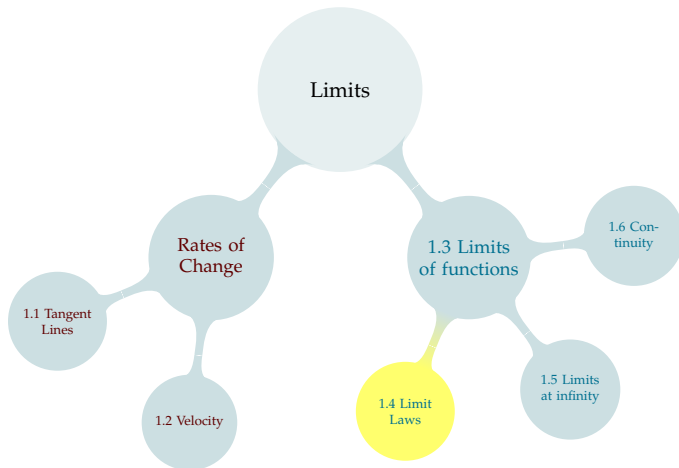


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Can't find in the same way: 3 not in domain

Algebra with Limits: Theorem 1.4.2

Suppose $\lim_{x \rightarrow a} f(x) = F$ and $\lim_{x \rightarrow a} g(x) = G$, where F and G are both real numbers. Then:

- $\lim_{x \rightarrow a} (f(x) + g(x)) = F + G$
- $\lim_{x \rightarrow a} (f(x) - g(x)) = F - G$
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Calculate: $\lim_{x \rightarrow 1} \left[\frac{2x + 4}{x + 2} + 13 \left(\frac{x + 5}{3x} \right) \left(\frac{x^2}{2x - 1} \right) \right]$

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$$\begin{aligned}\text{Calculate: } & \lim_{x \rightarrow 1} \left[\frac{2x+4}{x+2} + 13 \left(\frac{x+5}{3x} \right) \left(\frac{x^2}{2x-1} \right) \right] \\ &= \lim_{x \rightarrow 1} \left(\frac{2x+4}{x+2} \right) + \left(\lim_{x \rightarrow 1} 13 \right) \left(\lim_{x \rightarrow 1} \frac{x+5}{3x} \right) \left(\lim_{x \rightarrow 1} \frac{x^2}{2x-1} \right) \\ &= \left(\frac{2(1)+4}{1+2} \right) + (13) \left(\frac{(1)+5}{3(1)} \right) \left(\frac{1^2}{2(1)-1} \right) \\ &= (2) + 13(2)(1) \\ &= 28\end{aligned}$$

LIMITS INVOLVING POWERS AND ROOTS

Which of the following gives a real number?

A. $4^{\frac{1}{2}}$

B. $(-4)^{\frac{1}{2}}$

C. $4^{-\frac{1}{2}}$

D. $(-4)^{-\frac{1}{2}}$

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Powers of Limits – Theorem 1.4.8

If n is a positive integer, and $\lim_{x \rightarrow a} f(x) = F$ (where F is a real number), then:

$$\lim_{x \rightarrow a} (f(x))^n = F^n.$$

Furthermore, **unless** n is even and F is negative,

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$$\lim_{x \rightarrow 4} (x + 5)^{1/2} = \left[\lim_{x \rightarrow 4} (x + 5) \right]^{1/2} = 9^{1/2} = 3$$

CAUTIONARY TALES

$$\blacktriangleright \lim_{x \rightarrow 0} \frac{(5+x)^2 - 25}{x}$$

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Suppose you want to evaluate $\lim_{x \rightarrow 1} f(x)$, but $f(1)$ doesn't exist. What does that tell you?

- A $\lim_{x \rightarrow 1} f(x)$ may exist, and it may not exist.
- B We can find $\lim_{x \rightarrow 1} f(x)$ by plugging in 1 to $f(x)$.
- C Since $f(1)$ doesn't exist, it is not meaningful to talk about $\lim_{x \rightarrow 1} f(x)$.
- D Since $f(1)$ doesn't exist, automatically we know $\lim_{x \rightarrow 1} f(x)$ does not exist.
- E $\lim_{x \rightarrow 1} f(x)$ does not exist if we are "dividing by zero," but may exist otherwise.

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Which of the following statements is true about $\lim_{x \rightarrow 0} \frac{\sin x}{x^3 - x^2 + x}$?

A $\lim_{x \rightarrow 0} \frac{\sin x}{x^3 - x^2 + x} = \frac{\sin 0}{0^3 - 0^2 + 0} = \frac{0}{0}$

B Since the function $\frac{\sin x}{x^3 - x^2 + x}$ is not rational, its limit at 0 does not exist.

C Since the numerator and denominator of $\frac{\sin x}{x^3 - x^2 + x}$ are both 0 when $x = 0$, the limit exists.

D Since the function $\frac{\sin x}{x^3 - x^2 + x}$ is not defined at 0, plugging in $x = 0$ will not tell us the limit.

E Since the function $\frac{\sin x}{x^3 - x^2 + x}$ consists of the quotient of polynomials and trigonometric functions, its limit exists everywhere.

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Which of the following statements is true about $\lim_{x \rightarrow 1} \frac{\sin x}{x^3 - x^2 + x}$?

A $\lim_{x \rightarrow 1} \frac{\sin x}{x^3 - x^2 + x} = \frac{\sin 1}{1^3 - 1^2 + 1} = \sin 1$

B Since the function $\frac{\sin x}{x^3 - x^2 + x}$ is not rational, its limit at 1 does not exist.

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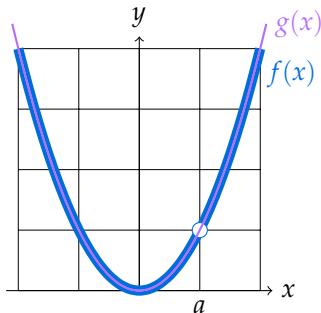
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Functions that Differ at a Single Point – Theorem 1.4.12

Suppose $\lim_{x \rightarrow a} g(x)$ exists, and $f(x) = g(x)$
when x is close to a (but not necessarily equal to a).

Then $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x)$.



Evaluate $\lim_{x \rightarrow 1} \frac{x^3 + x^2 - x - 1}{x - 1}$.

Evaluate $\lim_{x \rightarrow 1} \frac{x^3 + x^2 - x - 1}{x - 1}$.

$$\begin{aligned}\frac{x^3 + x^2 - x - 1}{x - 1} &= \frac{(x + 1)^2(x - 1)}{x - 1} \\ &= (x + 1)^2 \text{ whenever } x \neq 1\end{aligned}$$

$$\text{So, } \lim_{x \rightarrow 1} \frac{x^3 + x^2 - x - 1}{x - 1} = \lim_{x \rightarrow 1} (x + 1)^2 = 4$$

Evaluate $\lim_{x \rightarrow 5} \frac{\sqrt{x+20} - \sqrt{4x+5}}{x-5}$

Evaluate $\lim_{x \rightarrow 5} \frac{\sqrt{x+20} - \sqrt{4x+5}}{x-5}$

$$\begin{aligned}
 \frac{\sqrt{x+20} - \sqrt{4x+5}}{x-5} &= \frac{\sqrt{x+20} - \sqrt{4x+5}}{x-5} \left(\frac{\sqrt{x+20} + \sqrt{4x+5}}{\sqrt{x+20} + \sqrt{4x+5}} \right) \\
 &= \frac{(x+20) - (4x+5)}{(x-5)(\sqrt{x+20} + \sqrt{4x+5})} \\
 &= \frac{-3x+15}{(x-5)(\sqrt{x+20} + \sqrt{4x+5})} \\
 &= \frac{-3}{\sqrt{x+20} + \sqrt{4x+5}}
 \end{aligned}$$

So,

$$\begin{aligned}
 \lim_{x \rightarrow 5} \frac{\sqrt{x+20} - \sqrt{4x+5}}{x-5} &= \lim_{x \rightarrow 5} \frac{-3}{\sqrt{x+20} + \sqrt{4x+5}} \\
 &= \frac{-3}{\sqrt{5+20} + \sqrt{4(5)+5}} = \frac{-3}{10}
 \end{aligned}$$

A FEW STRATEGIES FOR CALCULATING LIMITS

First, hope that you can **directly substitute** (plug in). If your function is made up of the **sum, difference, product, quotient, or power of polynomials**, you can do this **provided** the function exists where you're taking the limit.

$$\lim_{x \rightarrow 1} \left(\sqrt{35 + x^5} + \frac{x - 3}{x^2} \right)^3 =$$

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$$\begin{aligned}\lim_{x \rightarrow 1} \left(\sqrt{35 + x^5} + \frac{x - 3}{x^2} \right)^3 &= \\ \left(\sqrt{35 + 1^5} + \frac{1 - 3}{1^2} \right)^3 &= 64\end{aligned}$$

To take a limit outside the domain of a function (that is made up of the sum, difference, product, quotient, or power of polynomials) try to **simplify and cancel**.

$$\lim_{x \rightarrow 0} \frac{x + 7}{\frac{1}{x} - \frac{1}{2x}}$$

To take a limit outside the domain of a function (that is made up of the sum, difference, product, quotient, or power of polynomials) try to **simplify and cancel**.

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{x+7}{\frac{1}{x} - \frac{1}{2x}} &= \lim_{x \rightarrow 0} \frac{x+7}{\frac{2}{2x} - \frac{1}{2x}} \\ &= \lim_{x \rightarrow 0} \frac{x+7}{\frac{1}{2x}} = \lim_{x \rightarrow 0} 2x(x+7) = 0\end{aligned}$$

To take a limit outside the domain of a function (that is made up of the sum, difference, product, quotient, or power of polynomials) try to **simplify and cancel**.

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{x+7}{\frac{1}{x} - \frac{1}{2x}} &= \lim_{x \rightarrow 0} \frac{x+7}{\frac{2}{2x} - \frac{1}{2x}} \\ &= \lim_{x \rightarrow 0} \frac{x+7}{\frac{1}{2x}} = \lim_{x \rightarrow 0} 2x(x+7) = 0\end{aligned}$$

Otherwise, you can try graphing the function, or making a table of values, to get a better picture of what is going on.

DENOMINATORS APPROACHING ZERO

$$\lim_{x \rightarrow 1} \frac{1}{(x-1)^2}$$

$$\lim_{x \rightarrow 1} \frac{-1}{(x-1)^2}$$

$$\lim_{x \rightarrow 1^-} \frac{1}{x-1}$$

$$\lim_{x \rightarrow 1^+} \frac{1}{x-1}$$

DENOMINATORS APPROACHING ZERO

$$\lim_{x \rightarrow 1} \frac{1}{(x-1)^2} = \infty$$

$$\lim_{x \rightarrow 1} \frac{-1}{(x-1)^2} = -\infty$$

$$\lim_{x \rightarrow 1^-} \frac{1}{x-1} = -\infty$$

$$\lim_{x \rightarrow 1^+} \frac{1}{x-1} = \infty$$

DENOMINATORS APPROACHING ZERO



$$\lim_{x \rightarrow 2^+} \frac{x}{x^2 - 4}$$

$$\lim_{x \rightarrow 2^-} \frac{x}{4 - x^2}$$

$$\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4}$$

DENOMINATORS APPROACHING ZERO



$$\lim_{x \rightarrow 2^+} \frac{x}{x^2 - 4} = \infty$$

$$\lim_{x \rightarrow 2^-} \frac{x}{4 - x^2} = \infty$$

$$\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4} = \frac{1}{4}$$

Squeeze Theorem – Theorem 1.4.17

Suppose, when x is near (but not necessarily equal to) a , we have functions $f(x)$, $g(x)$, and $h(x)$ so that

$$f(x) \leq g(x) \leq h(x)$$

and $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x)$. Then $\lim_{x \rightarrow a} g(x) = \lim_{x \rightarrow a} f(x)$.

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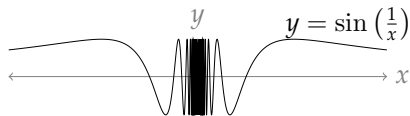
$$\lim_{x \rightarrow 0} x^2 \sin \left(\frac{1}{x} \right)$$

Evaluate:

$$\lim_{x \rightarrow 0} x^2 \sin \left(\frac{1}{x} \right)$$

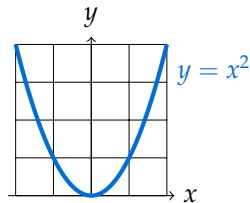
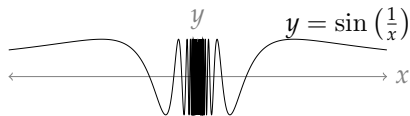
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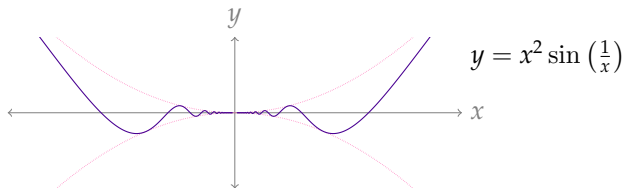
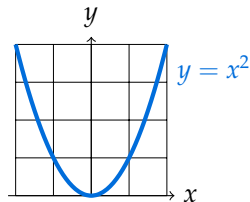
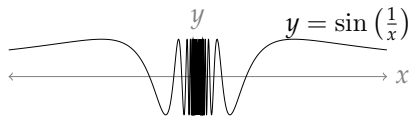
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$$\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right)$$

$$\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right)$$

$$-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$$

$$\text{so } -x^2 \leq x^2 \sin\left(\frac{1}{x}\right) \leq x^2$$

$$\text{and also } \lim_{x \rightarrow 0} -x^2 = 0 = \lim_{x \rightarrow 0} x^2$$

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Therefore, by the Squeeze Theorem, $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right) = 0$.

Included Work



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