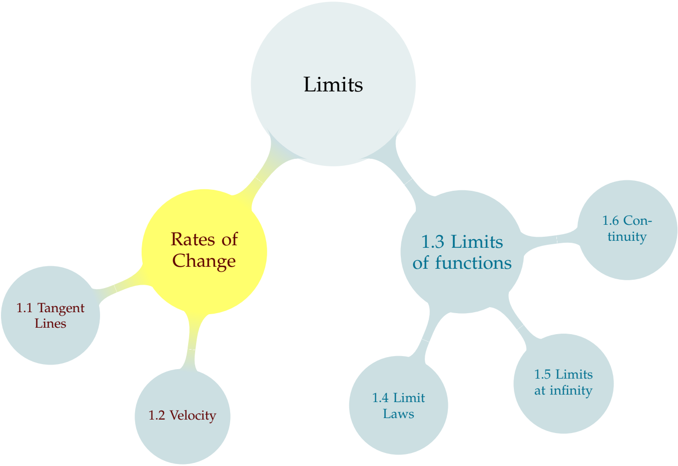
Table of Contents

SKIP CHAPTER INTRO

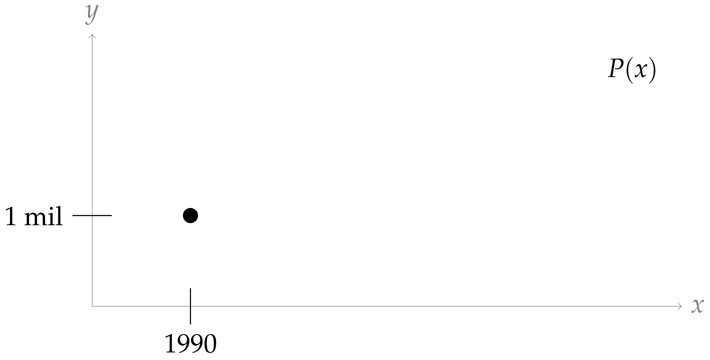


Rates of Change

Suppose the population of a small country was 1 million individuals in 1990, and is growing at a steady rate of 20,000 individuals per year.

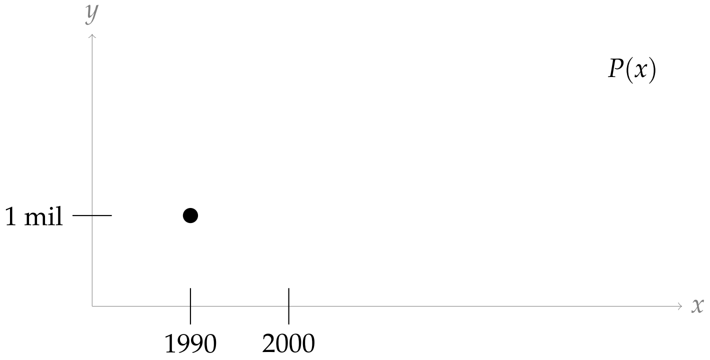
Rates of Change

Suppose the population of a small country was 1 million individuals in 1990, and is growing at a steady rate of 20,000 individuals per year.



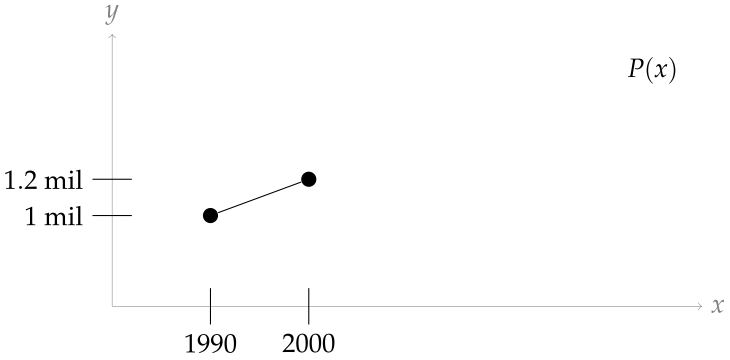
Rates of Change

Suppose the population of a small country was 1 million individuals in 1990, and is growing at a steady rate of 20,000 individuals per year.



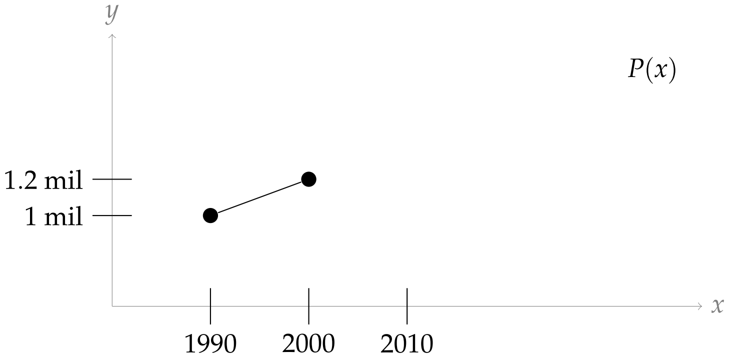
Rates of Change

Suppose the population of a small country was 1 million individuals in 1990, and is growing at a steady rate of 20,000 individuals per year.



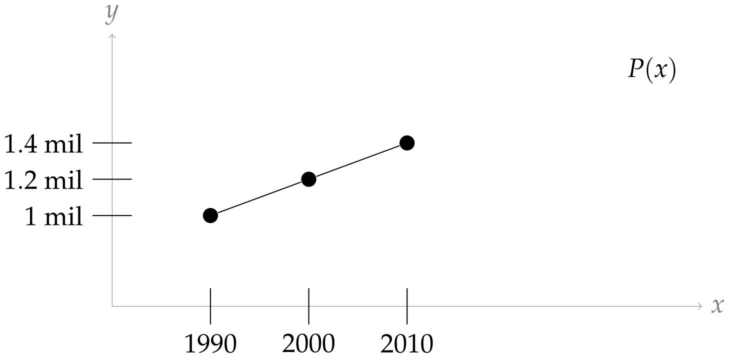
Rates of Change

Suppose the population of a small country was 1 million individuals in 1990, and is growing at a steady rate of 20,000 individuals per year.



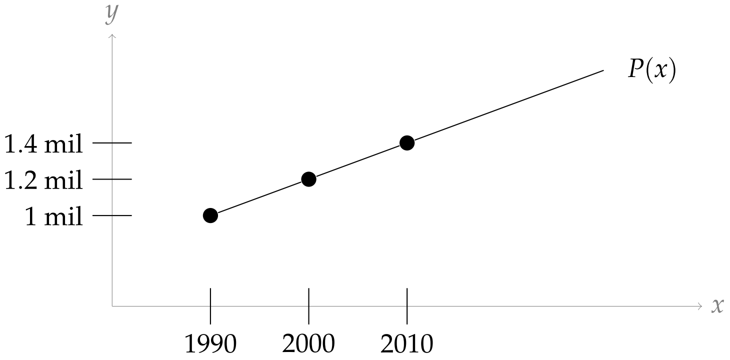
Rates of Change

Suppose the population of a small country was 1 million individuals in 1990, and is growing at a steady rate of 20,000 individuals per year.



Rates of Change

Suppose the population of a small country was 1 million individuals in 1990, and is growing at a steady rate of 20,000 individuals per year.

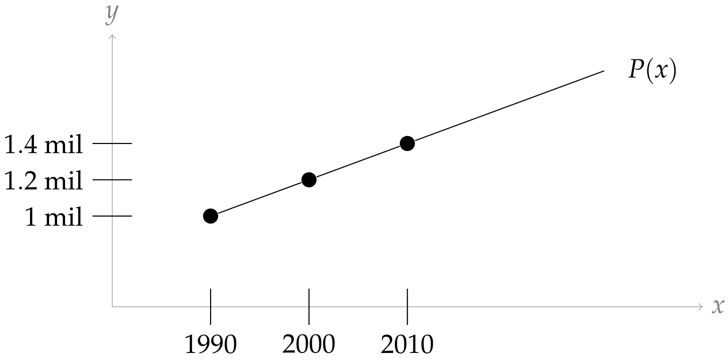


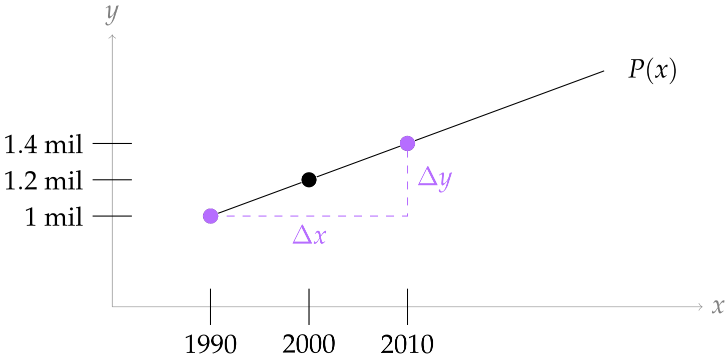
Definition

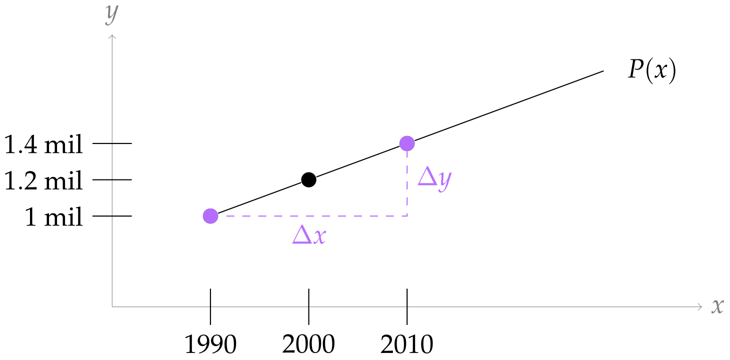
The **slope** of a line that passes through the points (x_1, y_1) and (x_2, y_2) is "rise over run"

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

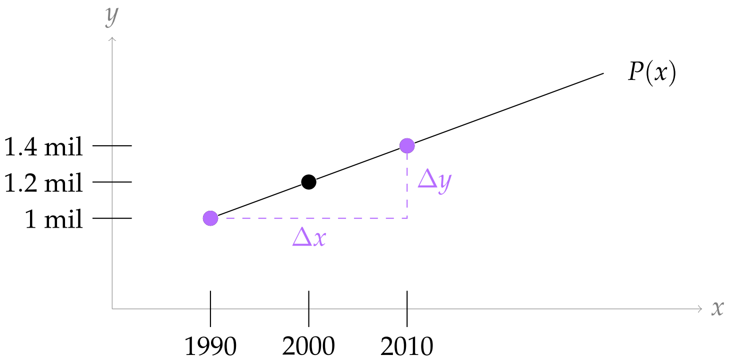
This is also called the **rate of change** of the function.
If a line has equation $y = mx + b$, its slope is m.


y
x
P(x)
1.4 mil
1.2 mil
1 mil
1990
2000
2010


y
$P(x)$
1.4 mil
1.2 mil
1 mil
Δy
Δx
x
1990
2000
2010



$$\text{Rate of change: } \frac{400,000 \text{ people}}{20 \text{ years}} = 20,000 \ \frac{\text{people}}{\text{year}}$$

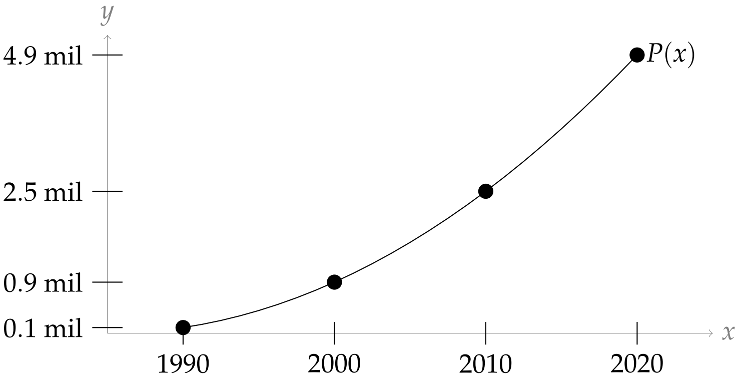


Rate of change: $\dfrac{400,000 \text{ people}}{20 \text{ years}} = 20,000 \dfrac{\text{people}}{\text{year}}$

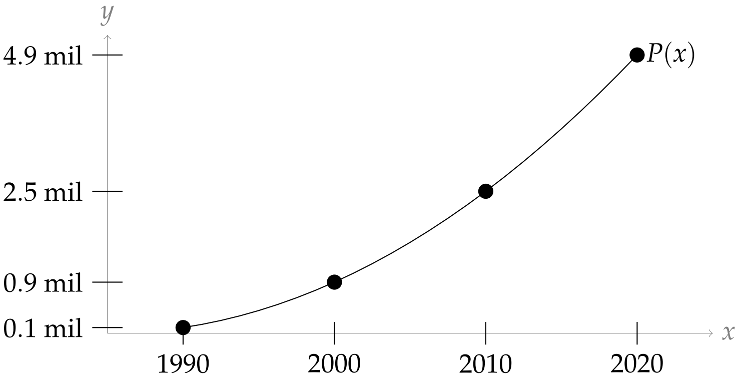
(doesn't depend on the year)

Suppose the population of a small country is given in the chart below.

Suppose the population of a small country is given in the chart below.

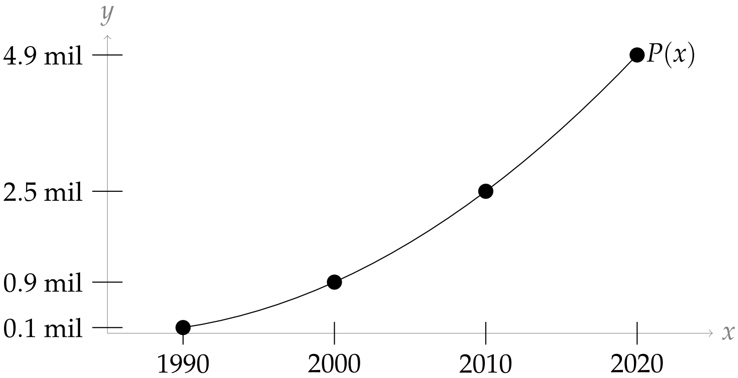


Suppose the population of a small country is given in the chart below.



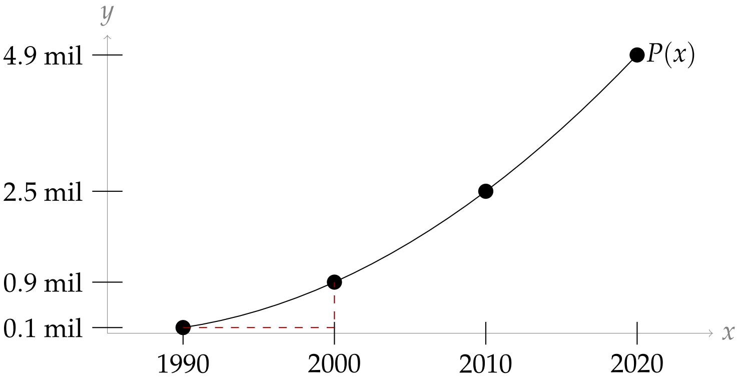
Rate of change $\dfrac{\Delta \text{ pop}}{\Delta \text{ time}}$

Suppose the population of a small country is given in the chart below.



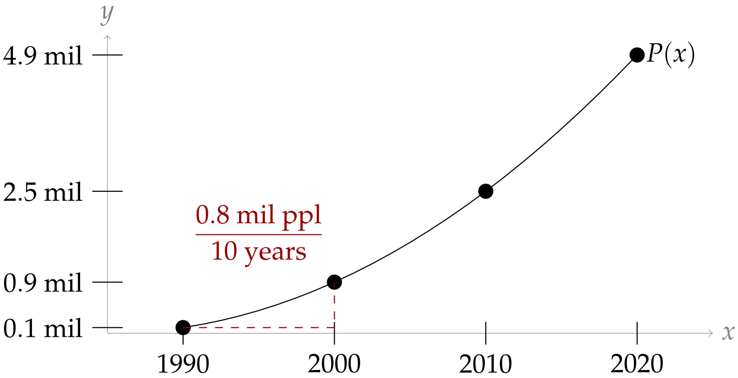
Rate of change $\dfrac{\Delta \text{ pop}}{\Delta \text{ time}}$ depends on time interval

Suppose the population of a small country is given in the chart below.



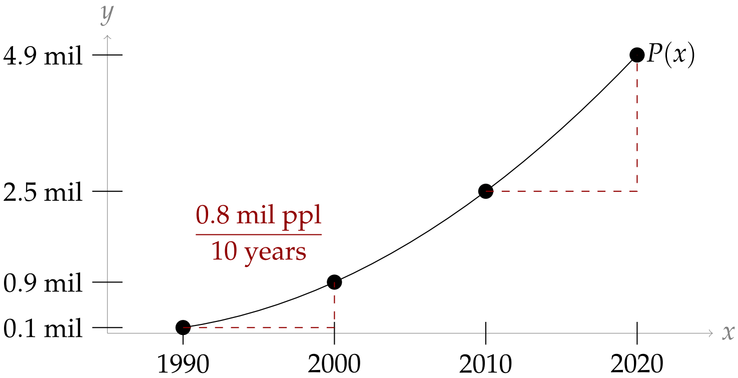
Rate of change $\dfrac{\Delta \text{ pop}}{\Delta \text{ time}}$ depends on time interval

Suppose the population of a small country is given in the chart below.



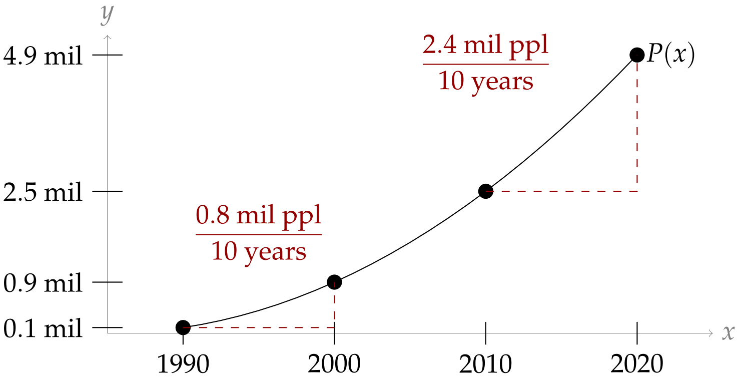
Rate of change $\dfrac{\Delta\ \text{pop}}{\Delta\ \text{time}}$ depends on time interval

Suppose the population of a small country is given in the chart below.



Rate of change $\frac{\Delta \text{ pop}}{\Delta \text{ time}}$ depends on time interval

Suppose the population of a small country is given in the chart below.

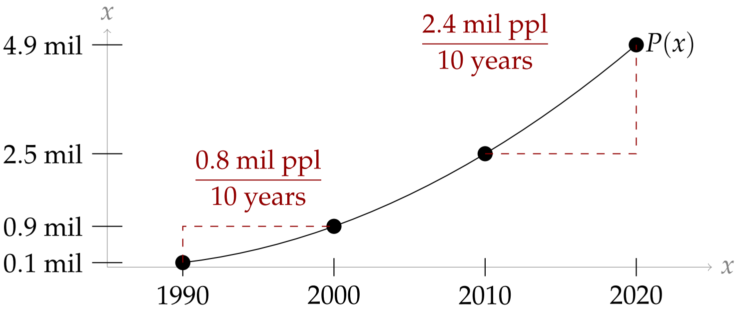


Rate of change $\dfrac{\Delta \text{ pop}}{\Delta \text{ time}}$ depends on time interval

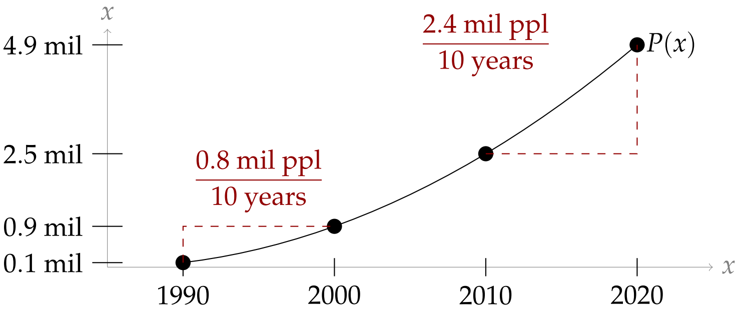
Definition

Let $y = f(x)$ be a curve that passes through (x_1, y_1) and (x_2, y_2). Then the **average rate of change** of $f(x)$ when $x_1 \leq x \leq x_2$ is

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

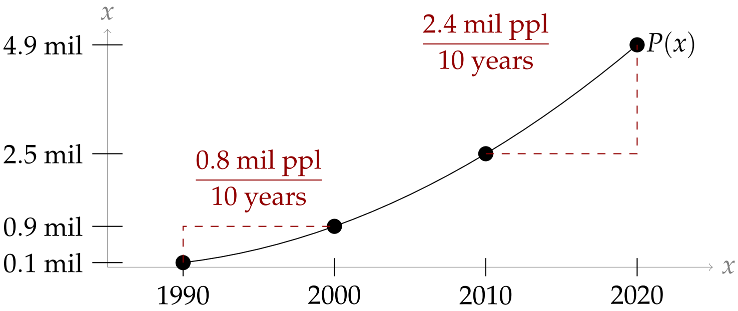


Average rate of change from 1990 to 2000:



Average rate of change from 1990 to 2000:
$80,000$ people per year.

Average rate of change from 2010 to 2020:



Average rate of change from 1990 to 2000:
$80,000$ people per year.

Average rate of change from 2010 to 2020:
$240,000$ people per year.

Average Rate of Change and Slope

The average rate of change of a function $f(x)$ on the interval $[a, b]$ (where $a \neq b$) is "change in output" divided by "change in input:"

$$\frac{f(b) - f(a)}{b - a}$$

Average Rate of Change and Slope

The average rate of change of a function $f(x)$ on the interval $[a, b]$ (where $a \neq b$) is "change in output" divided by "change in input:"

$$\frac{f(b) - f(a)}{b - a}$$

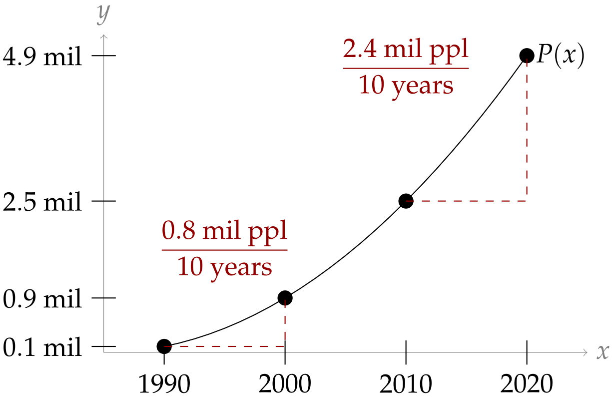
If the function $f(x)$ is a line, then the slope of the line is "rise over run,"

$$\frac{f(b) - f(a)}{b - a}$$

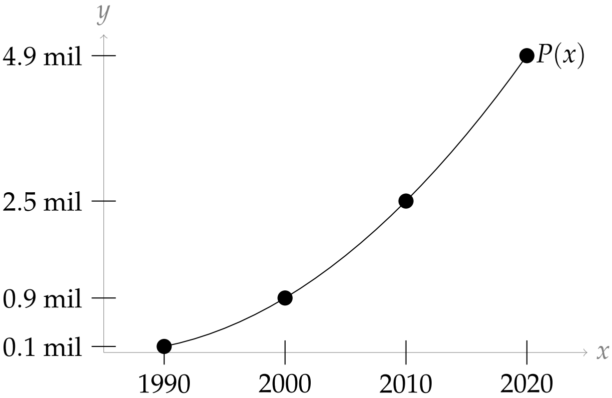
If a function is a line, its slope is the same as its average rate of change, which is the same for every interval.

If a function is not a line, its average rate of change might be different for different intervals, and we don't have a definition (yet) for its "slope."

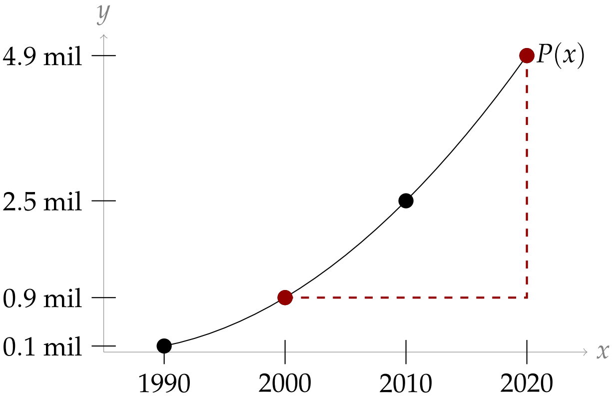
How fast was this population growing in the year 2010? (What was its instantaneous rate of change?)



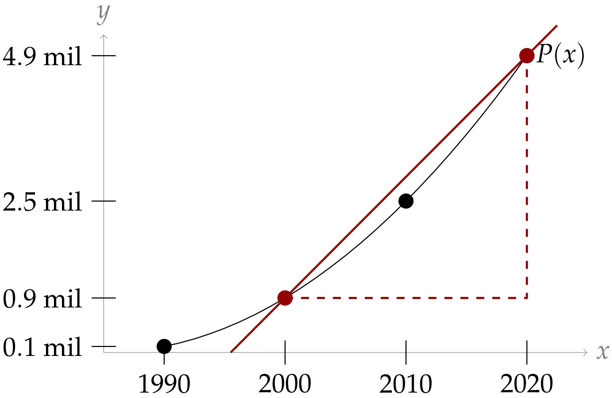
How fast was this population growing in the year 2010? (What was its instantaneous rate of change?)



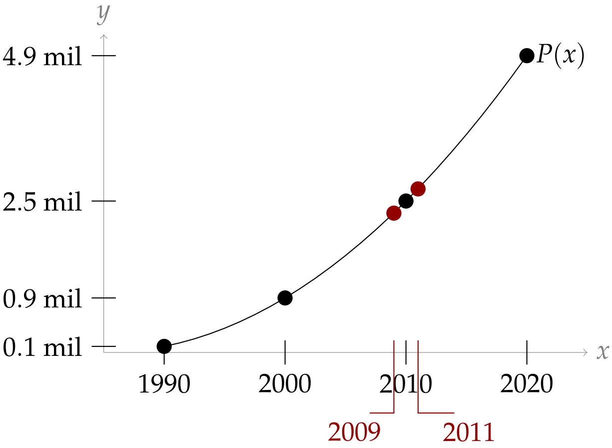
How fast was this population growing in the year 2010? (What was its instantaneous rate of change?)



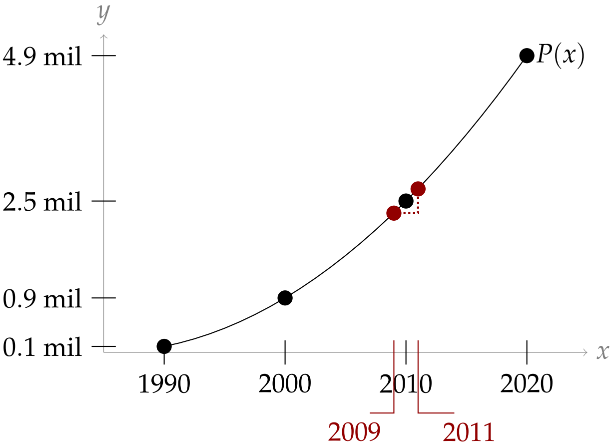
How fast was this population growing in the year 2010? (What was its instantaneous rate of change?)



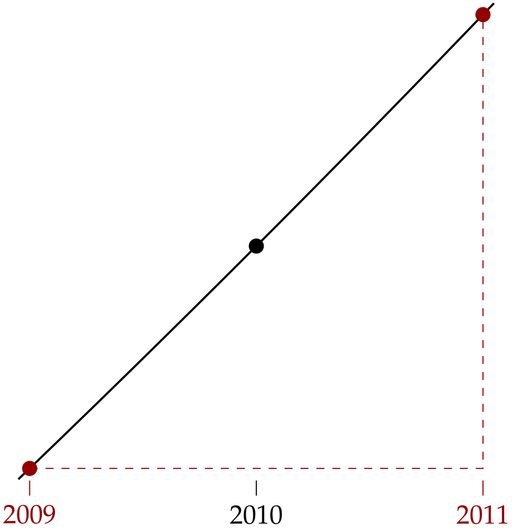
How fast was this population growing in the year 2010? (What was its instantaneous rate of change?)



How fast was this population growing in the year 2010? (What was its instantaneous rate of change?)



How fast was this population growing in the year 2010?



How fast was this population growing in the year 2010?

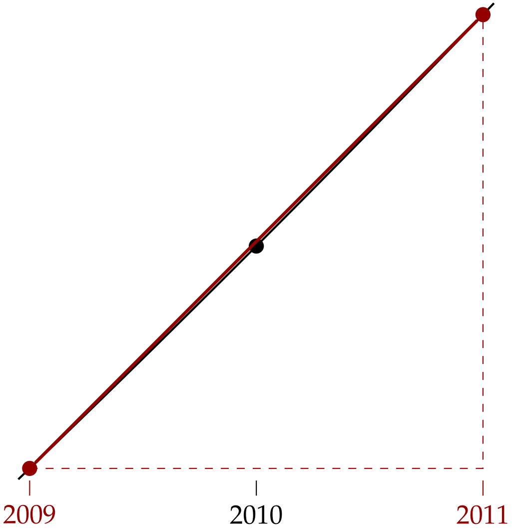
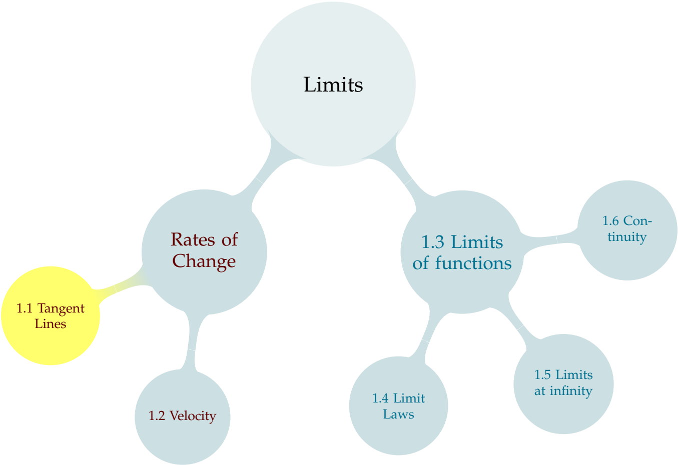
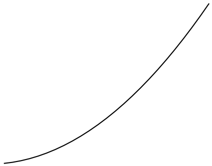


Table of Contents



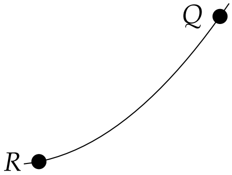
Definition

The **secant line** to the curve $y = f(x)$ through points R and Q is a line that passes through R and Q.



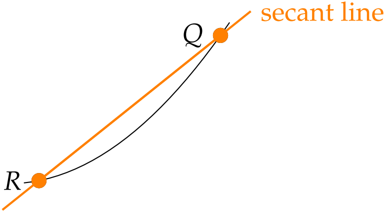
Definition

The **secant line** to the curve $y = f(x)$ through points R and Q is a line that passes through R and Q.



Definition

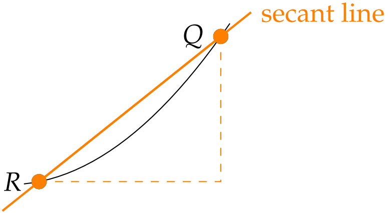
The **secant line** to the curve $y = f(x)$ through points R and Q is a line that passes through R and Q.



Definition

The **secant line** to the curve $y = f(x)$ through points R and Q is a line that passes through R and Q.

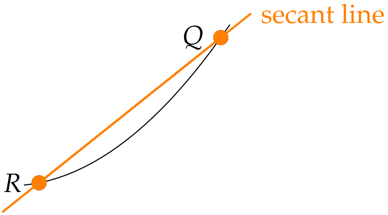
We call the slope of the secant line the **average rate of change of $f(x)$ from R to Q.**



Definition

The **tangent line** to the curve $y = f(x)$ at point P is a line that

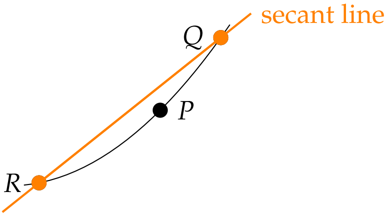
- passes through P and
- has the same slope as $f(x)$ at P.



Definition

The **tangent line** to the curve $y = f(x)$ at point P is a line that

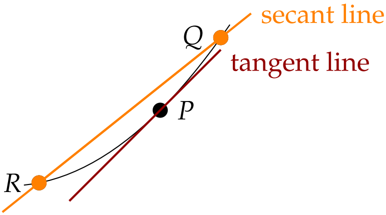
- passes through P and
- has the same slope as $f(x)$ at P.



Definition

The **tangent line** to the curve $y = f(x)$ at point P is a line that

- passes through P and
- has the same slope as $f(x)$ at P.

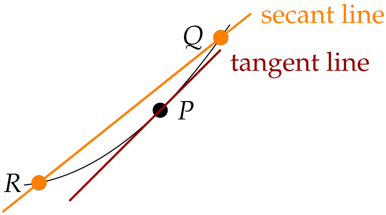


Definition

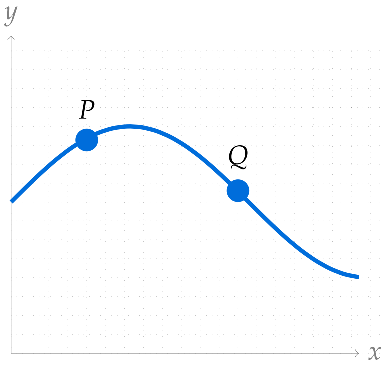
The **tangent line** to the curve $y = f(x)$ at point P is a line that

- passes through P and
- has the same slope as $f(x)$ at P.

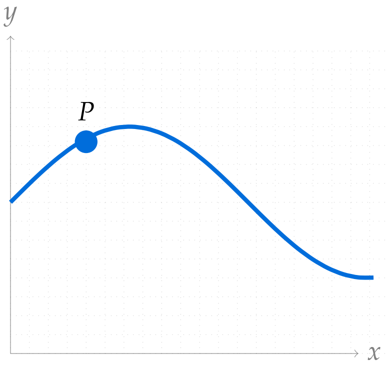
We call the slope of the tangent line the **instantaneous rate of change of $f(x)$ at P.**



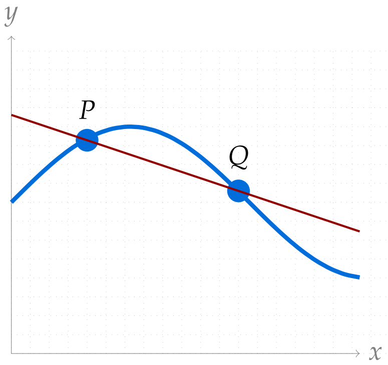
On the graph below, draw the secant line to the curve through points P and Q.



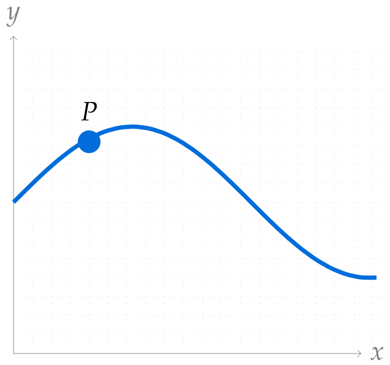
On the graph below, draw the tangent line to the curve at point P.



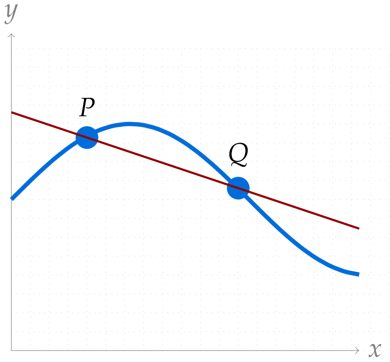
On the graph below, draw the secant line to the curve through points P and Q.



On the graph below, draw the tangent line to the curve at point P.



On the graph below, draw the secant line to the curve through points P and Q.



On the graph below, draw the tangent line to the curve at point P.