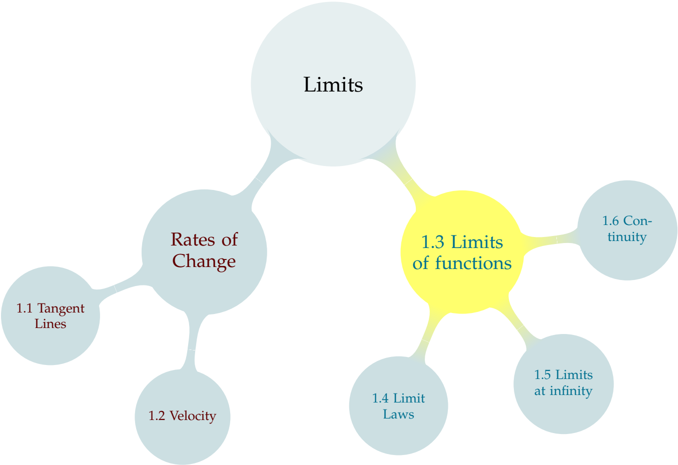
Table of Contents



Notation 1.3.1 and Definition 1.3.3

$$\lim_{x \to a} f(x) = L$$

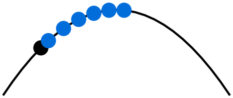
where a and L are real numbers

We read the above as "the limit as x goes to a of $f(x)$ is L."

Its meaning is: as x gets very close to (but not equal to) a, $f(x)$ gets very close to L.

Finding Slopes of Tangent Lines

We NEED limits to find slopes of tangent lines.



Slope of secant line: $\frac{\Delta y}{\Delta x}$, $\Delta x \neq 0$.
Slope of tangent line: can't do the same way.

If the position of an object at time t is given by $s(t)$, then its instantaneous velocity is given by

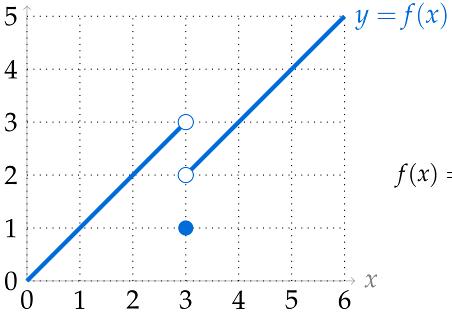
$$\lim_{h \to 0} \frac{s(t+h) - s(t)}{h}$$

Evaluating Limits

$$\text{Let } f(x) = \frac{x^3 + x^2 - x - 1}{x - 1}.$$

We want to evaluate $\lim_{x \to 1} f(x)$.

One-Sided Limits



$$f(x) = \begin{cases} x & \text{if } x < 3 \\ 1 & \text{if } x = 3 \\ x - 1 & \text{if } x > 3 \end{cases}$$

What do you think $\lim\limits_{x \to 3} f(x)$ should be?

Definition 1.3.7

The limit as x goes to a **from the left** of $f(x)$ is written

$$\lim_{x \to a^-} f(x)$$

We only consider values of x that are less than a.

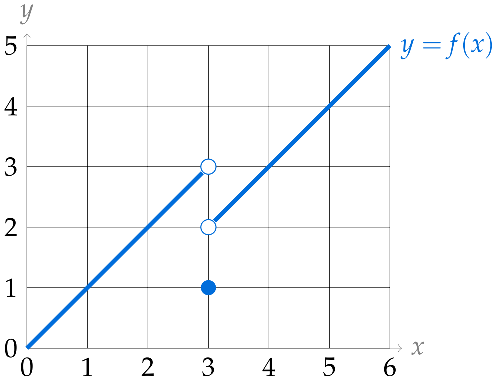
The limit as x goes to a **from the right** of $f(x)$ is written

$$\lim_{x \to a^+} f(x)$$

We only consider values of x greater than a.

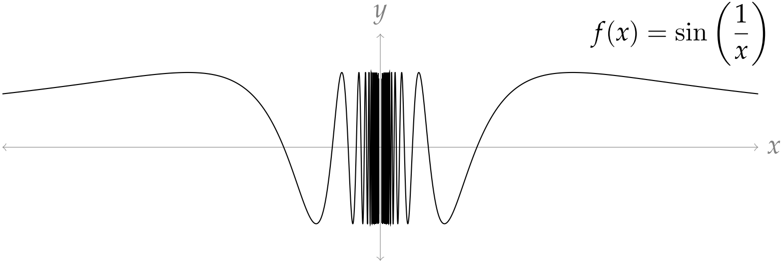
Theorem 1.3.8

In order for $\lim\limits_{x \to a} f(x)$ to exist, both one-sided limits must exist and be equal.



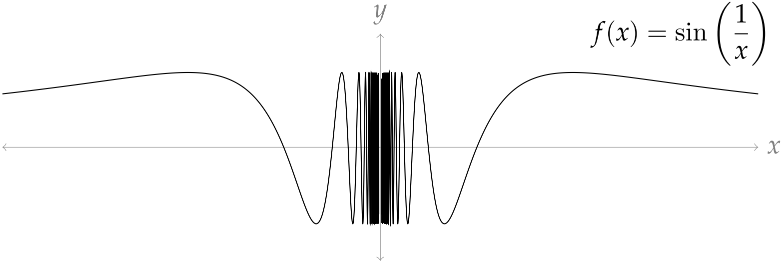
Consider the function $f(x) = \dfrac{1}{(x-1)^2}$. For what value(s) of x is $f(x)$ not defined?

A Stranger Limit Example



What is $\lim_{x\to\infty} f(x)$?

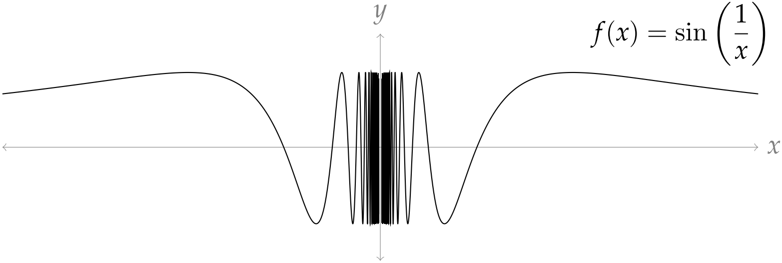
A Stranger Limit Example



$$f(x) = \sin\left(\frac{1}{x}\right)$$

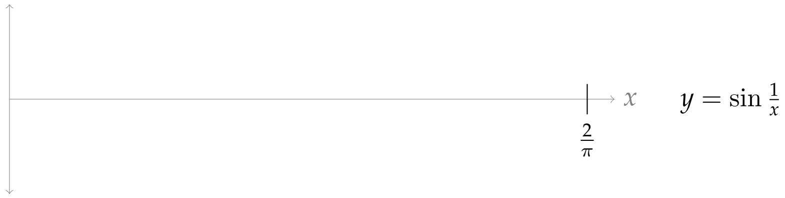
What is $\lim_{x \to 0} f(x)$?

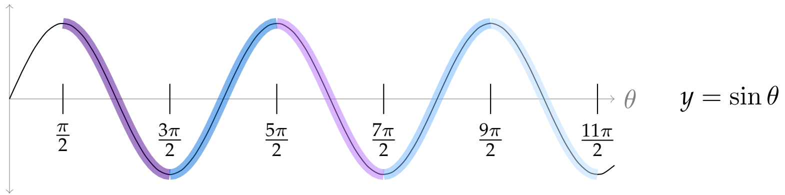
A Stranger Limit Example



What is $\lim_{x \to \pi} f(x)$?

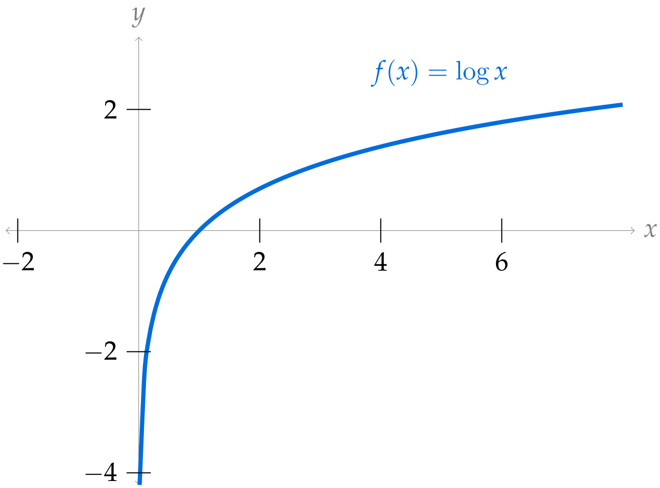
Optional: Sketching $f(x) = \sin\left(\frac{1}{x}\right)$





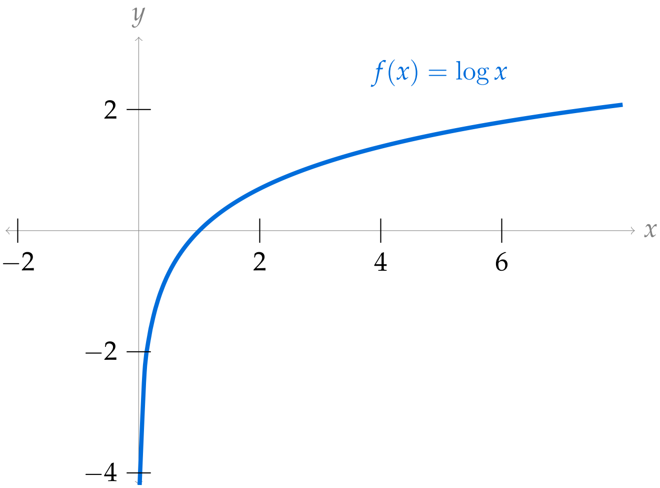
Limits and the Natural Logarithm

Where is $f(x)$ defined, and where is it not defined?

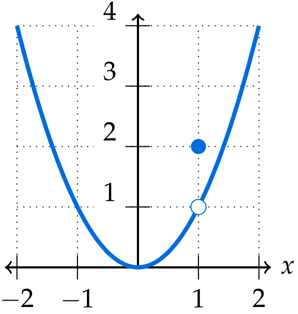


Limits and the Natural Logarithm

What can you say about the limit of $f(x)$ near 0?



$$f(x) = \begin{cases} x^2 & x \neq 1 \\ 2 & x = 1 \end{cases}$$



What is $\lim\limits_{x \to 1} f(x)$?

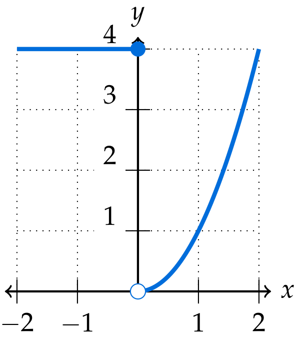
A. $\lim\limits_{x \to 1} f(x) = 2$

B. $\lim\limits_{x \to 1} f(x) = 1$

C. $\lim\limits_{x \to 1} f(x)$ DNE

D. none of the above

$$f(x) = \begin{cases} 4 & x \leq 0 \\ x^2 & x > 0 \end{cases}$$



What is $\lim\limits_{x\to 0} f(x)$? What is $\lim\limits_{x\to 0^+} f(x)$? What is $f(0)$?

A. $\lim\limits_{x\to 0^+} f(x) = 4$

B. $\lim\limits_{x\to 0^+} f(x) = 0$

C. $\lim\limits_{x\to 0^+} f(x) = \begin{cases} 4 & x \leq 0 \\ 0 & x > 0 \end{cases}$

D. none of the above

Suppose $\lim_{x \to 3^-} f(x) = 1$ and $\lim_{x \to 3^+} f(x) = 1.5$.

Does $\lim_{x \to 3} f(x)$ exist?

A. Yes, certainly, because the limits from both sides exist.

B. No, never, because the limit from the left is not the same as the limit from the right.

C. Can't tell. For some functions is might exist, for others not.

Suppose $\lim_{x \to 3^-} f(x) = 22 = \lim_{x \to 3^+} f(x)$.

Does $\lim_{x \to 3} f(x)$ exist?

A. Yes, certainly, because the limits from both sides exist and are equal to each other.

B. No, never, because we only talk about one-sided limits when the actual limit doesn't exist.

C. Can't tell. We need to know the value of the function at $x = 3$.

Included Work

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