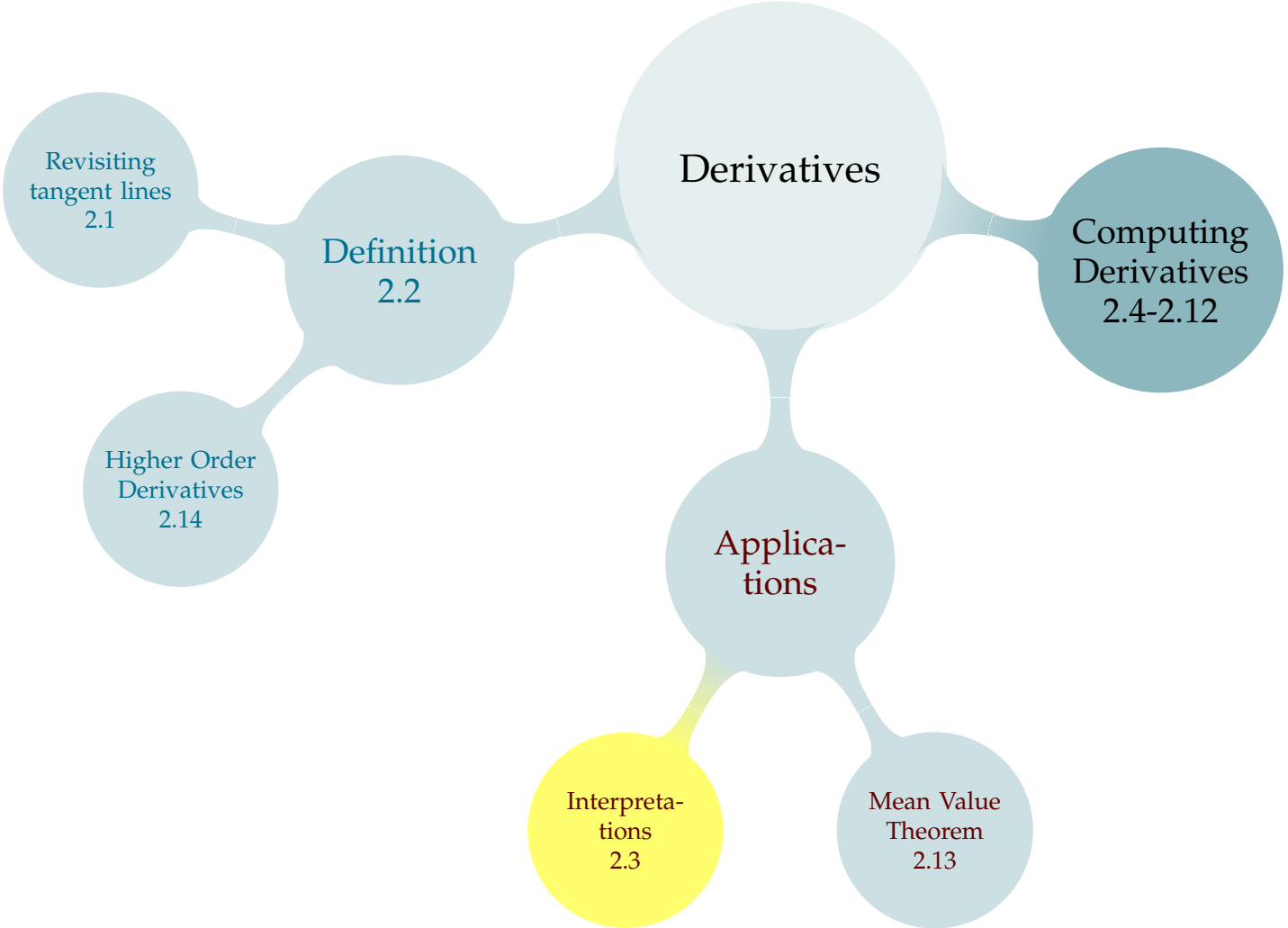
Table of Contents



Interpreting the Derivative

The derivative of $f(x)$ at a, written $f'(a)$, is the instantaneous rate of change of $f(x)$ when $x = a$.

Interpreting the Derivative

The derivative of $f(x)$ at a, written $f'(a)$, is the instantaneous rate of change of $f(x)$ when $x = a$.

Suppose $P(t)$ gives the number of people in the world at t minutes past midnight, January 1, 2012. Suppose further that $P'(0) = 156$. How do you interpret $P'(0) = 156$?

Interpreting the Derivative

The derivative of $f(x)$ at a, written $f'(a)$, is the instantaneous rate of change of $f(x)$ when $x = a$.

Suppose $P(n)$ gives the total profit, in dollars, earned by selling n widgets. How do you interpret $P'(100)$?

Interpreting the Derivative

The derivative of $f(x)$ at a, written $f'(a)$, is the instantaneous rate of change of $f(x)$ when $x = a$.

Suppose $h(t)$ gives the height of a rocket t seconds after liftoff. What is the interpretation of $h'(t)$?

Interpreting the Derivative

The derivative of $f(x)$ at a, written $f'(a)$, is the instantaneous rate of change of $f(x)$ when $x = a$.

Suppose $M(t)$ is the number of molecules of a chemical in a test tube t seconds after a reaction starts. Interpret $M'(t)$.

Interpreting the Derivative

The derivative of $f(x)$ at a, written $f'(a)$, is the instantaneous rate of change of $f(x)$ when $x = a$.

Suppose $G(w)$ gives the diameter in millimetres of steel wire needed to safely support a load of w kg. Suppose further that $G'(100) = 0.01$. How do you interpret $G'(100) = 0.01$?

A paper[1] on the impacts of various factors in average life expectancy contains the following:

> *The only statistically significant variable in the model is physician density. The coefficient for this variable 20.67 indicating that a one unit increase in physician density leads to a 20.67 unit increase in life expectancy. This variable is also statistically significant at the 1% level demonstrating that this variable is very strongly and positively correlated with quality of healthcare received. This denotes that access to healthcare is very impactful in terms of increasing the quality of health in the country.*

[1] Natasha Deshpande, Anoosha Kumar, Rohini Ramaswami, *The Effect of National Healthcare Expenditure on Life Expectancy*, page 12.
Remark: physician density is measured as number of doctors per 1000 members of the population.

If $L(p)$ is the average life expectancy in an area with a density p of physicians, write the statement as a derivative: "a one unit increase in physician density leads to a 20.67 unit increase in life expectancy."

Equation of the Tangent Line

The tangent line to $f(x)$ at a has slope $f'(a)$ and passes through the point $(a, f(a))$.

Tangent Line Equation – Theorem 2.3.2

The tangent line to the function $f(x)$ at point a is:

$$(y - f(a)) = f'(a)(x - a)$$

Point-Slope Formula

In general, a line with slope m passing through point (x_1, y_1) has the equation:

$$(y - y_1) = m(x - x_1)$$

Find the equation of the tangent line to the curve $f(x) = \sqrt{x}$ at $x = 9$. (Recall $\frac{\mathrm{d}}{\mathrm{d}x}\left[\sqrt{x}\right] = \frac{1}{2\sqrt{x}}$).

Memorize

The tangent line to the function $f(x)$ at point a is:

$$(y - f(a)) = f'(a)(x - a)$$

NOW YOU

Let $s(t) = 3 - 0.8t^2$. Then $s'(t) = -1.6t$. Find the equation for the tangent line to the function $s(t)$ when $t = 1$.

Included Work

'Brain' by Eucalyp is licensed under CC BY 3.0 (accessed 8 June 2021), 13

Natasha Deshpande, Anoosha Kumar, Rohini Ramaswami. (2014). The Effect of National Healthcare Expenditure on Life Expectancy, page 12. *College of Liberal Arts - Ivan Allen College (IAC), School of Economics: Econometric Analysis Undergraduate Research Papers*. https://smartech.gatech.edu/handle/1853/51648 (accessed July 2021), 8