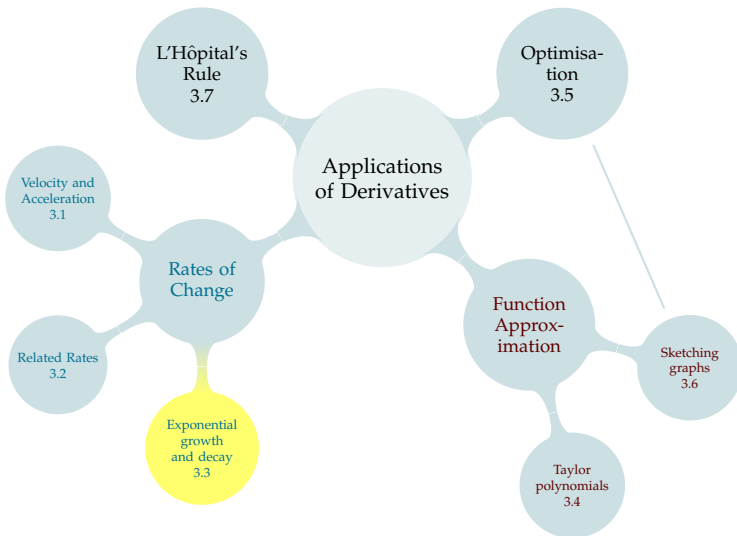


# TABLE OF CONTENTS



# RADIOACTIVE DECAY

The number of atoms in a sample that decay in a given time interval is proportional to the number of atoms in the sample.

## Differential Equation

Let  $Q = Q(t)$  be the amount of a radioactive substance at time  $t$ . Then for some positive constant  $k$ :

$$\frac{dQ}{dt} = -kQ$$

## Solution – Theorem 3.3.2

Let  $\boxed{Q(t) = Ce^{-kt}}$ , where  $k$  and  $C$  are constants. Then:

## Quantity of a Radioactive Isotope

$Q(t)$ : quantity at time  $t$

A. positive or zero  
B. negative or zero  
C. could be either  
D. I don't know

A. positive or zero  
B. negative or zero  
C. could be either  
D. I don't know

## Seaborgium Decay

The amount of  $^{266}\text{Sg}$  (Seaborgium-266) in a sample at time  $t$  (measured in seconds) is given by

$$Q(t) = Ce^{-kt}$$

Let's approximate the half life of  $^{266}\text{Sg}$  as 30 seconds. That is, every 30 seconds, the size of the sample halves.

What are  $C$  and  $k$ ?

A sample of radioactive matter is stored in a lab in 2000. In the year 2002, it is tested and found to contain 10 units of a particular radioactive isotope. In the year 2005, it is tested and found to contain only 2 units of that same isotope. How many units of the isotope were present in the year 2000?

$$Q'(t) = kQ(t)$$

The number of atoms in a sample that decay in a given time interval is proportional to the number of atoms in the sample.

The rate of growth of a population in a given time interval is proportional to the number of individuals in the population, when the population has ample resources.

The amount of interest a bank account accrues in a given time interval is proportional to the balance in that bank account.

## Exponential Growth – Theorem 3.3.2

Let  $Q = Q(t)$  satisfy:

$$\frac{dQ}{dt} = kQ$$

for some constant  $k$ . Then for some constant  $C = Q(0)$ ,

$$Q(t) = Ce^{kt}$$

Suppose  $y(t)$  is a function with the properties that

$$\frac{dy}{dt} + 3y = 0 \quad \text{and} \quad y(1) = 2.$$

What is  $y(t)$ ?

# POPULATION GROWTH

Suppose a petri dish starts with a culture of 100 bacteria cells and a limited amount of food and space. The population of the culture at different times is given in the table below. At approximately what time did the culture start to show signs of limited resources?

time	population
0	100
1	1000
3	100000
5	1000000



## FLU SEASON

The CDC keeps records ([link](#)) on the number of flu cases in the US by week. At the start of the flu season, the 40th week of 2014, there are 100 cases of a particular strain. Five weeks later (at week 45), there are 506 cases. What do you think was the first week to have 5,000 cases? What about 10,000 cases?

## Newton's Law of Cooling – Equation 3.3.7

The rate of change of temperature of an object is proportional to the difference in temperature between that object and its surroundings.

$$\frac{dT}{dt}(t) = K[T(t) - A]$$

where  $T(t)$  is the temperature of the object at time  $t$ ,  $A$  is the (constant) ambient temperature of the surroundings, and  $K$  is some constant depending on the object.

$$\frac{dT}{dt}(t) = K[T(t) - A]$$

$T(t)$  is the temperature of the object,  $A$  is the ambient temperature,  $K$  is some constant.

What is true of  $K$ ?

- A.  $K \geq 0$
- B.  $K \leq 0$
- C.  $K = 0$
- D.  $K$  could be positive, negative, or zero, depending on the object
- E. I don't know

## Newton's Law of Cooling – Equation 3.3.7

$$\frac{dT}{dt}(t) = K[T(t) - A]$$

$T(t)$  is the temperature of the object,  $A$  is the ambient temperature, and  $K$  is some constant.

$$T(t) = [T(0) - A]e^{Kt} + A$$

is the only function satisfying Newton's Law of Cooling

If  $T(10) < A$ , then:

- A.  $K > 0$
- B.  $T(0) > 0$
- C.  $T(0) > A$
- D.  $T(0) < A$

Evaluate  $\lim_{t \rightarrow \infty} T(t)$ .

- A.  $A$
- B.  $0$
- C.  $\infty$
- D.  $T(0)$

What assumptions are we making that might not square with the real world?

### Newton's Law of Cooling – Equation 3.3.7

$$\frac{dT}{dt} = K[T(t) - A]$$

$T(t)$  is the temperature of the object,  $A$  is the ambient temperature, and  $K$  is some constant.

### Temperature of a Cooling Body – Corollary 3.3.8

$$T(t) = [T(0) - A]e^{Kt} + A$$

A farrier forms a horseshoe heated to  $400^{\circ}\text{C}$ , then dunks it in a river at room-temperature ( $25^{\circ}\text{C}$ ). The water boils for 30 seconds. The horseshoe is safe for the horse when it's  $40^{\circ}\text{C}$ . When can the farrier put on the horseshoe?



$$T(t) = [T(0) - A]e^{Kt} + A$$

A glass of just-boiled tea is put on a porch outside. After ten minutes, the tea is  $40^\circ$ , and after 20 minutes, the tea is  $25^\circ$ . What is the temperature outside?

In 1963, the US Fish and Wildlife Service recorded a bald eagle population of 487 breeding pairs. In 1993, that number was 4015. How many breeding pairs would you expect there were in 2006? What about 2015?



## [link: Wood Bison Restoration in Alaska, Alaska Department of Fish and Game](#)

Excerpt:

*Based on experience with reintroduced populations elsewhere, wood bison would be expected to increase at a rate of 15%-25% annually after becoming established.... With an average annual growth rate of 20%, an initial precalving population of 50 bison would increase to 500 in approximately 13 years.*

NOW  
YOU



Are they using our same model?

# COMPOUND INTEREST

Suppose you invest \$10,000 in an account that accrues interest each month. After one month, your balance (with interest) is \$10,100. How much money will be in your account after a year?

Compound interest is calculated according to the formula  $Pe^{rt}$ , where  $r$  is the interest rate and  $t$  is time.

# CARRYING CAPACITY

For a population of size  $P$  with unrestricted access to resources, let  $\beta$  be the average number of offspring each breeding pair produces per generation, where a generation has length  $t_g$ . Then  $b = \frac{\beta-2}{2t_g}$  is the net birthrate (births minus deaths) per member per unit time. This yields  $\frac{dP}{dt}(t) = bP(t)$ , hence:

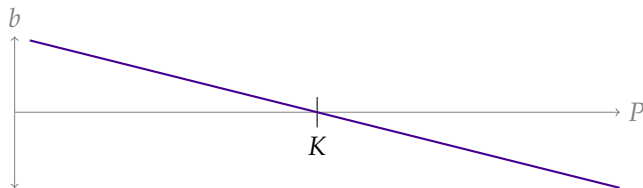
But as resources grow scarce,  $b$  might change.

## CARRYING CAPACITY

$b$  is the net birthrate (births minus deaths) per member per unit time.

If  $K$  is the carrying capacity of an ecosystem, we can model

$$b = b_0\left(1 - \frac{P}{K}\right).$$



NOW  
YOU



Describe to your neighbour what the following mean in

terms of the model:

- ▶  $b > 0, b = 0, b < 0$
- ▶  $P = 0, P > 0, P < 0$

## CARRYING CAPACITY

Then:

$$\frac{dP}{dt}(t) = b_0 \underbrace{\left(1 - \frac{P(t)}{K}\right)}_{\text{per capita birthrate}} P(t)$$

This is an example of a differential equation that we don't have the tools to solve. (If you take more calculus, though, you'll learn how!) It's also an example of a way you might tweak a model so its assumptions better fit what you observe.

## RADIOCARBON DATING

Researchers at Charlie Lake in BC have found evidence<sup>1</sup> of habitation dating back to around 8500 BCE. For instance, a butchered bison bone was radiocarbon dated to about 10,500 years ago.

Suppose a comparable bone of a bison alive today contains  $1\mu\text{g}$  of  $^{14}\text{C}$ . If the half-life of  $^{14}\text{C}$  is about 5730 years, roughly how much  $^{14}\text{C}$  do you think the researchers found in the sample?

- A. About  $\frac{1}{10,500} \mu\text{g}$
- B. About  $\frac{1}{4} \mu\text{g}$
- C. About  $\frac{1}{2} \mu\text{g}$

- D. About  $1 \mu\text{g}$
- E. I'm not sure how to estimate this

---

<sup>1</sup><http://pubs.aina.ucalgary.ca/arctic/Arctic49-3-265.pdf>

Suppose a body is discovered at 3:45 pm, in a room held at  $20^{\circ}$ , and the body's temperature is  $27^{\circ}$ , not the normal  $37^{\circ}$ . At 5:45 pm, the temperature of the body has dropped to  $25.3^{\circ}$ . When did the inhabitant of the body die?

## Included Work



'Brain' by Eucalyp is licensed under [CC BY 3.0](#) (accessed 8 June 2021), 17, 20



U.S. WHO/NREVSS Collaborating Laboratories and ILNet. 'Stacked Column Chart WHO//NREVSS' Centers for Disease Control and Prevention. No longer available from <http://gis.cdc.gov/grasp/fluview/fluportaldashboard.html> (accessed 20 October 2015), 9

Alaska Department of Fish and Game, Division of Wildlife Conservation. (April 2007). Wood Bison Restoration in Alaska: A Review of Environmental and Regulatory Issues and Proposed Decisions for Project Implementation, p. 11.  
[http://www.adfg.alaska.gov/static/species/speciesinfo/woodbison/pdfs/er\\_no\\_appendices.pdf](http://www.adfg.alaska.gov/static/species/speciesinfo/woodbison/pdfs/er_no_appendices.pdf) (accessed 2015 or 2016), 17

Driver et.al. Stratigraphy, Radiocarbon Dating, and Culture History of Charlie Lake Cave, British Columbia. *ARCTICVOL.* 49, no. 3 (September 1996) pp. 265 – 277.  
<http://pubs.aina.ucalgary.ca/arctic/Arctic49-3-265.pdf> (accessed 2015 or 2016), 22



Public Domain by Man vyi via [https://commons.wikimedia.org/wiki/File:West\\_Show\\_Jersey\\_2010\\_farrier\\_f.jpg](https://commons.wikimedia.org/wiki/File:West_Show_Jersey_2010_farrier_f.jpg), accessed October 2015, 14