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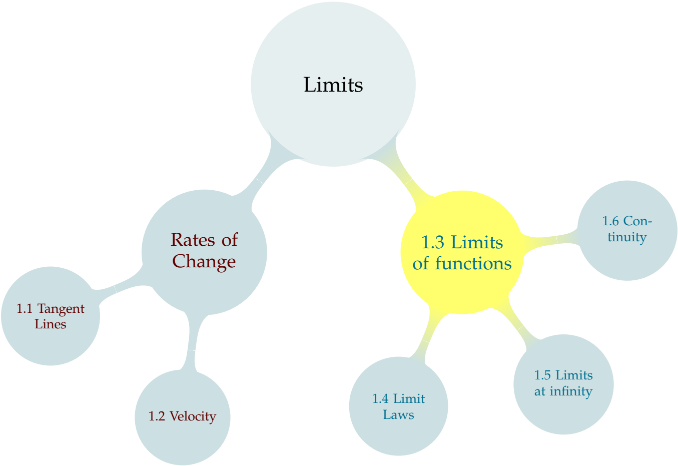


Notation 1.3.1 and Definition 1.3.3

$$\lim_{x \to a} f(x) = L$$

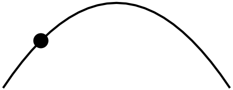
where a and L are real numbers

We read the above as "the limit as x goes to a of $f(x)$ is L."

Its meaning is: as x gets very close to (but not equal to) a, $f(x)$ gets very close to L.

Finding Slopes of Tangent Lines

We NEED limits to find slopes of tangent lines.



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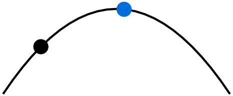

Slope of secant line: $\dfrac{\Delta y}{\Delta x}$, $\Delta x \neq 0$.

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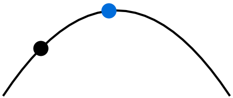

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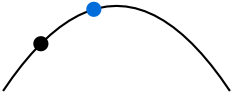

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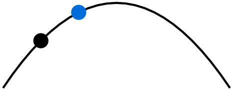

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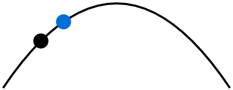

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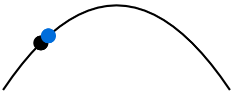

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Finding Slopes of Tangent Lines

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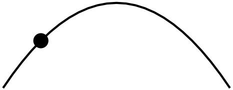

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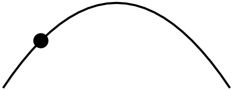

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If the position of an object at time t is given by $s(t)$, then its instantaneous velocity is given by

$$\lim_{h \to 0} \frac{s(t+h) - s(t)}{h}$$

Evaluating Limits

$$\text{Let } f(x) = \frac{x^3 + x^2 - x - 1}{x - 1}.$$

We want to evaluate $\lim\limits_{x \to 1} f(x)$.

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What is $f(1)$?

Q

ans

Evaluating Limits

$$\text{Let } f(x) = \frac{x^3 + x^2 - x - 1}{x - 1}.$$

We want to evaluate $\lim_{x \to 1} f(x)$.

What is $f(1)$? DNE (can't divide by zero)

Evaluating Limits

$$\text{Let } f(x) = \frac{x^3 + x^2 - x - 1}{x - 1}.$$

We want to evaluate $\lim_{x\to 1} f(x)$.

Use the tables below to guess $\lim_{x\to 1} f(x)$

x	$f(x)$
0.9	3.61
0.99	3.9601
0.999	3.99600
0.9999	3.99960

x	$f(x)$
1.1	4.41
1.01	4.0401
1.001	4.00400
1.0001	4.00040

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1.1	4.41
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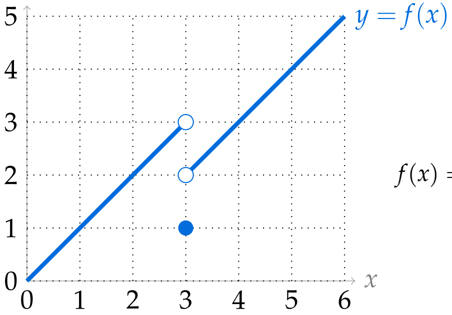
1.0001	4.00040

$$\boxed{\lim_{x \to 1} f(x) = 4}$$

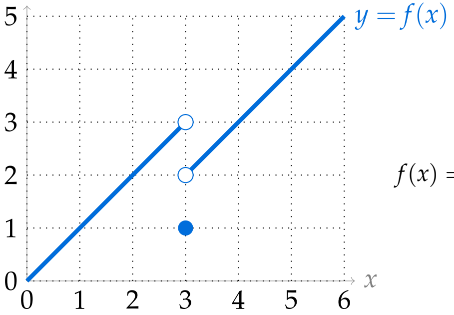
One-Sided Limits



$$f(x) = \begin{cases} x & \text{if } x < 3 \\ 1 & \text{if } x = 3 \\ x - 1 & \text{if } x > 3 \end{cases}$$

What do you think $\lim_{x \to 3} f(x)$ should be?

One-Sided Limits



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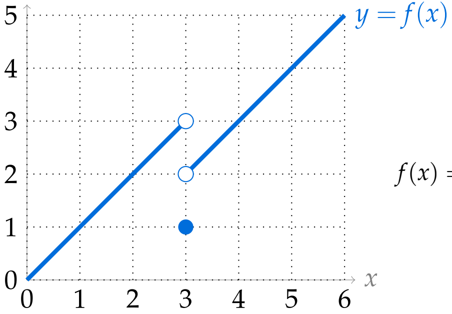


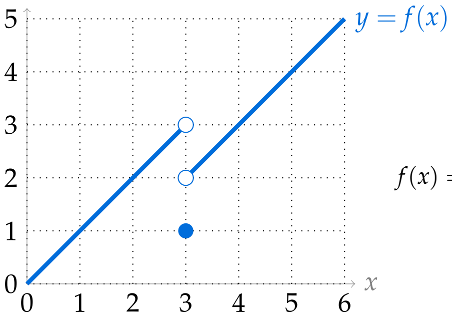
$$f(x) = \begin{cases} x & \text{if } x < 3 \\ 1 & \text{if } x = 3 \\ x-1 & \text{if } x > 3 \end{cases}$$

Evaluate: $\underbrace{\lim_{x\to 3^-} f(x)}_{\text{from the left}}$ $\underbrace{\lim_{x\to 3^+} f(x)}_{\text{from the right}}$

Q

ans

One-Sided Limits

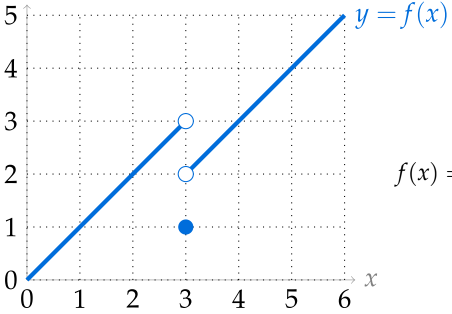


$$f(x) = \begin{cases} x & \text{if } x < 3 \\ 1 & \text{if } x = 3 \\ x - 1 & \text{if } x > 3 \end{cases}$$

Evaluate: $\underbrace{\lim_{x \to 3^-} f(x)}_{\text{from the left}} = 3 \qquad \underbrace{\lim_{x \to 3^+} f(x)}_{\text{from the right}}$

ans

ONE-SIDED LIMITS



$$f(x) = \begin{cases} x & \text{if } x < 3 \\ 1 & \text{if } x = 3 \\ x - 1 & \text{if } x > 3 \end{cases}$$

Evaluate: $\underbrace{\lim_{x \to 3^-} f(x)}_{\text{from the left}} = 3 \qquad \underbrace{\lim_{x \to 3^+} f(x)}_{\text{from the right}} = 2$

Definition 1.3.7

The limit as x goes to a **from the left** of $f(x)$ is written

$$\lim_{x \to a^-} f(x)$$

We only consider values of x that are less than a.

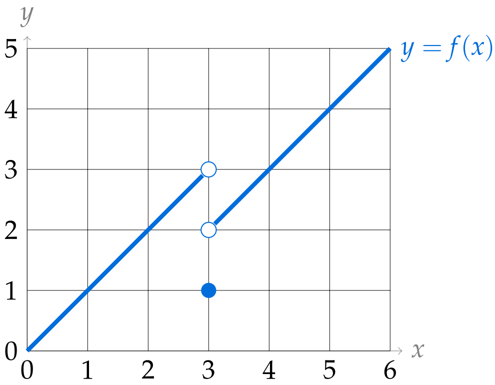
The limit as x goes to a **from the right** of $f(x)$ is written

$$\lim_{x \to a^+} f(x)$$

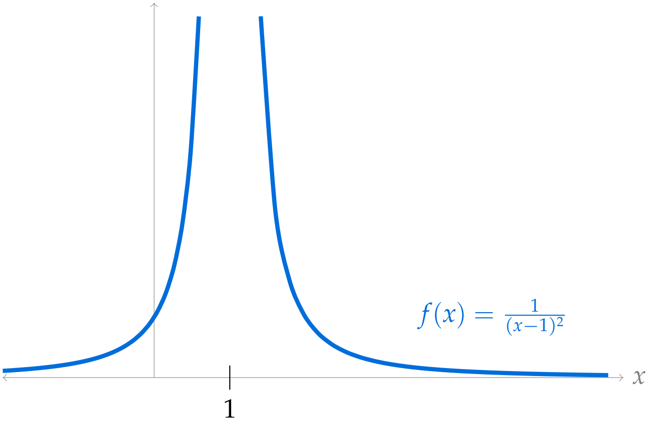
We only consider values of x greater than a.

Theorem 1.3.8

In order for $\lim_{x \to a} f(x)$ to exist, both one-sided limits must exist and be equal.



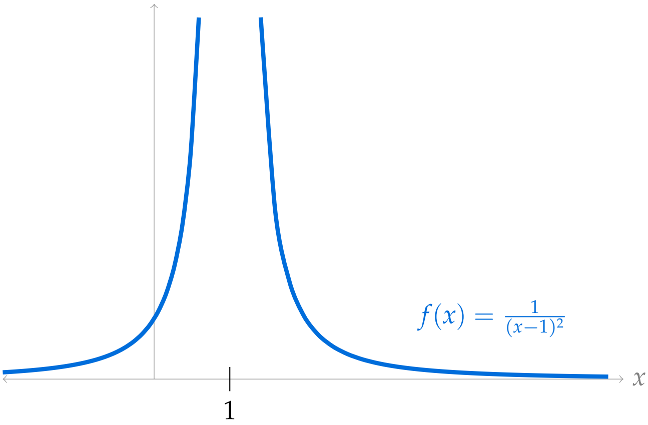
Consider the function $f(x) = \dfrac{1}{(x-1)^2}$. For what value(s) of x is $f(x)$ not defined?



ans

Based on the graph below, what would you like to write for:

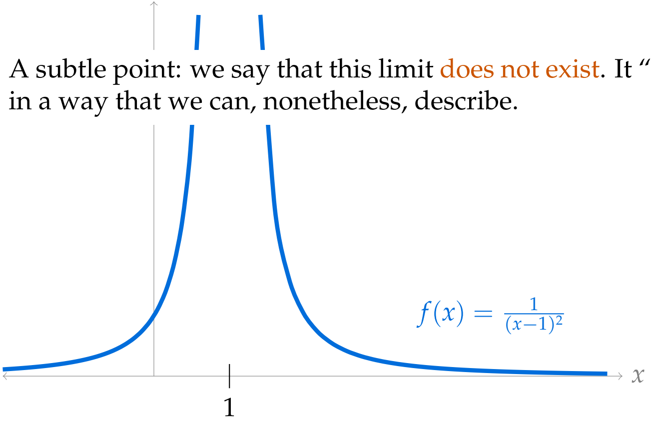
$$\lim_{x \to 1} f(x) =$$



ans

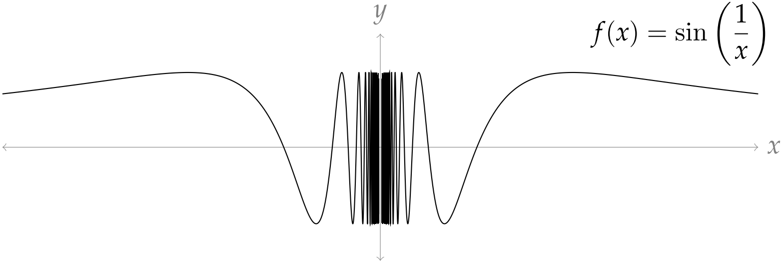
Based on the graph below, what would you like to write for:

$$\lim_{x \to 1} f(x) = \infty$$



A subtle point: we say that this limit does not exist. It "does not exist" in a way that we can, nonetheless, describe.

A Stranger Limit Example



What is $\lim\limits_{x\to\infty} f(x)$?

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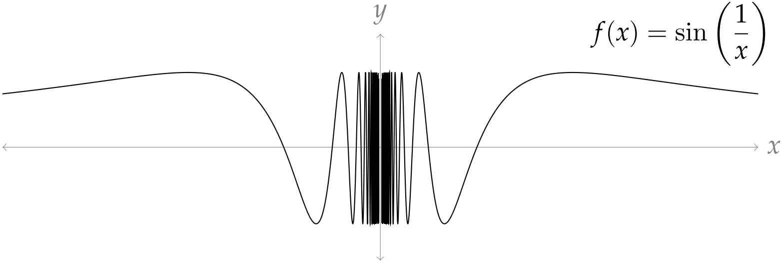
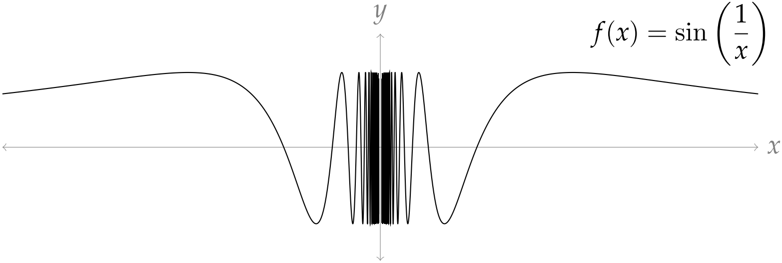


What is $\lim_{x\to\infty} f(x)$?

$\lim_{x\to\infty} f(x) = 0$

A Stranger Limit Example

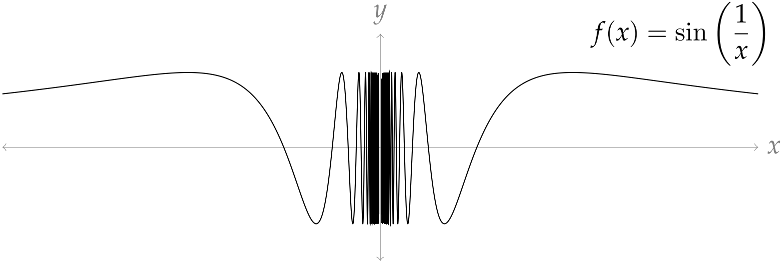


What is $\lim_{x \to 0} f(x)$?

Q

ans

A Stranger Limit Example



What is $\lim_{x \to 0} f(x)$?

$\lim_{x \to 0} f(x)$ does not exist.

We can call this behaviour "infinite wiggling."

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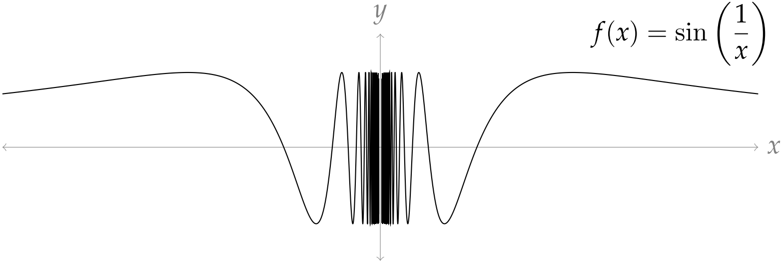
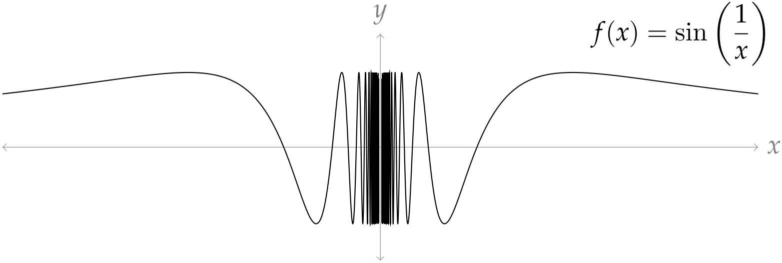


What is $\lim_{x \to \pi} f(x)$?

A Stranger Limit Example

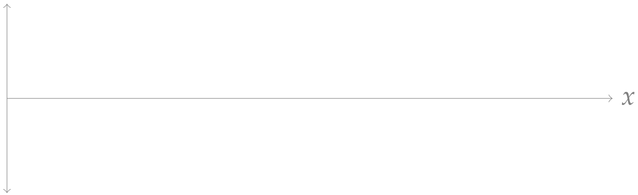


What is $\lim_{x \to \pi} f(x)$?

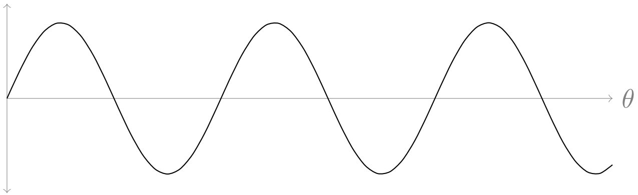
$\lim_{x \to \pi} f(x) = \sin\left(\frac{1}{\pi}\right)$

Optional: Sketching $f(x) = \sin\left(\frac{1}{x}\right)$

SKIP SKETCHING



$y = \sin\frac{1}{x}$



$y = \sin\theta$

Optional: Sketching $f(x) = \sin\left(\frac{1}{x}\right)$

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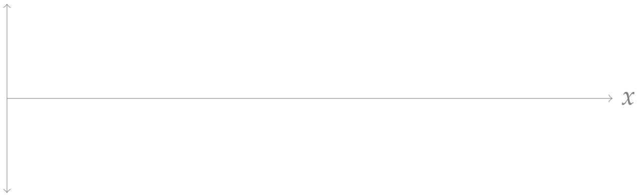

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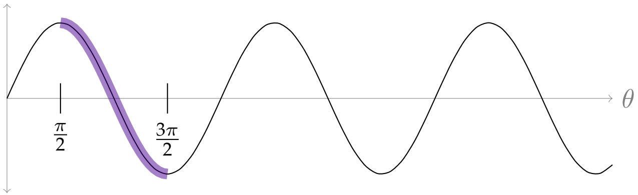

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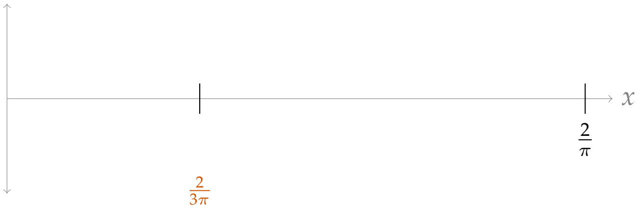

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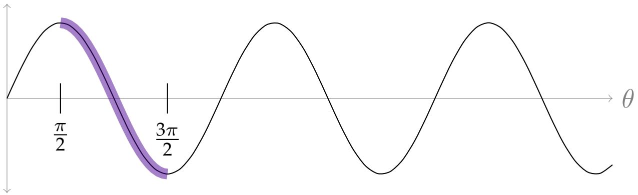

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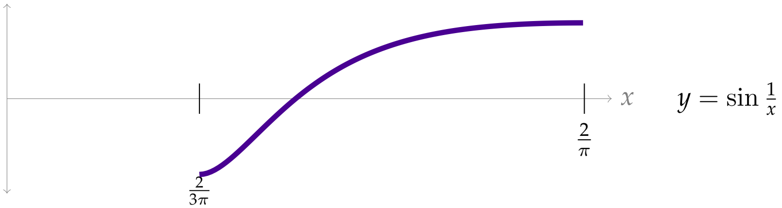

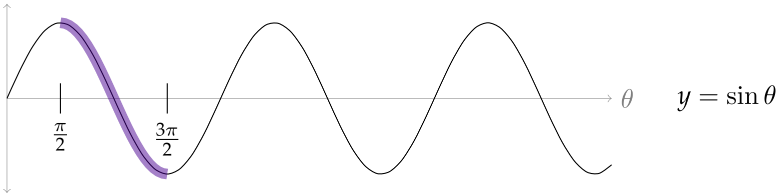

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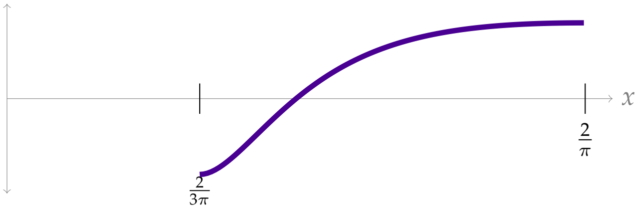

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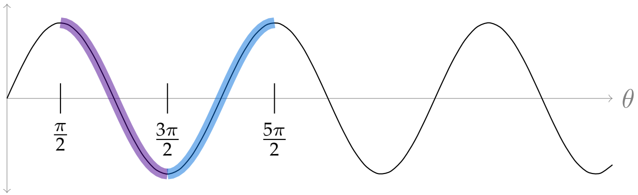

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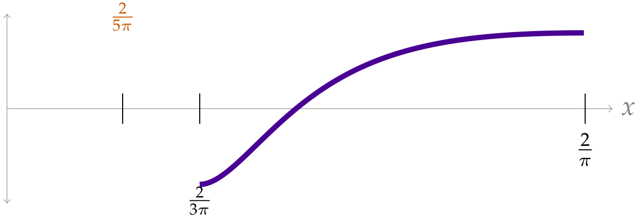

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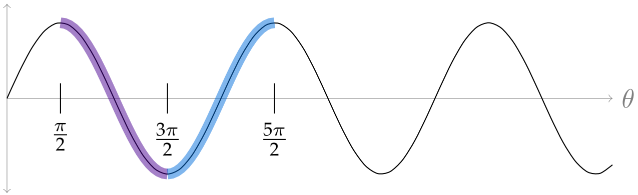

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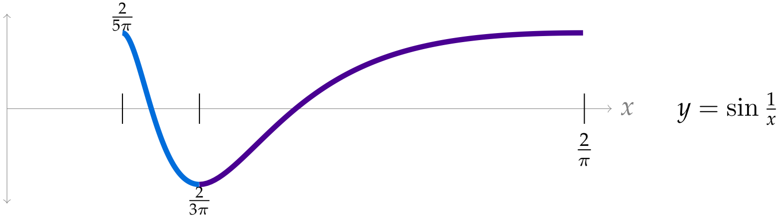

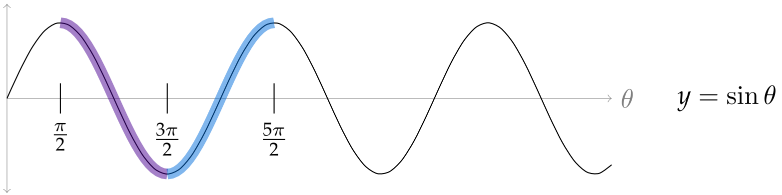

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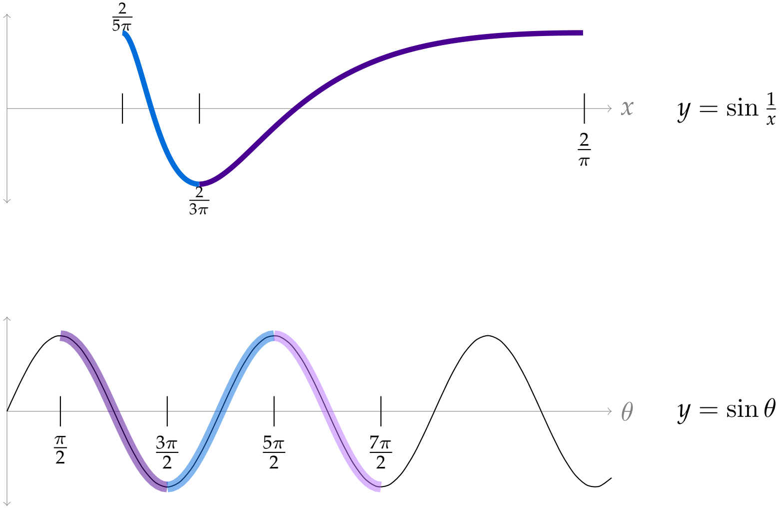

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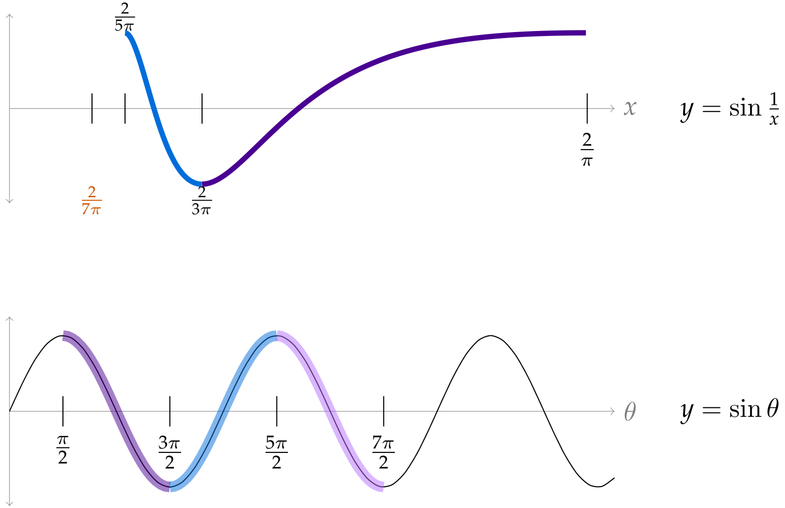
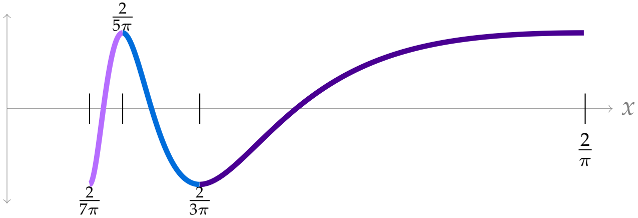
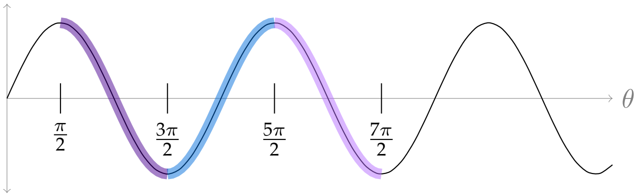


Optional: Sketching $f(x) = \sin\left(\frac{1}{x}\right)$

Skip sketching



$y = \sin\frac{1}{x}$



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OPTIONAL: SKETCHING $f(x) = \sin\left(\frac{1}{x}\right)$

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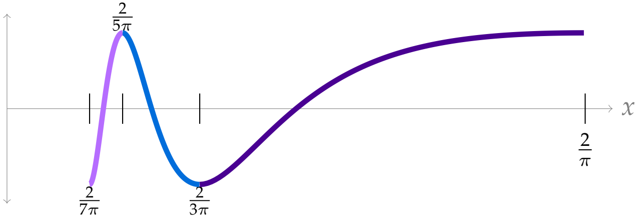

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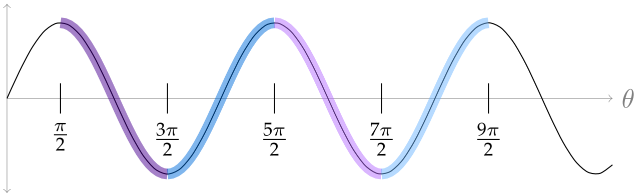

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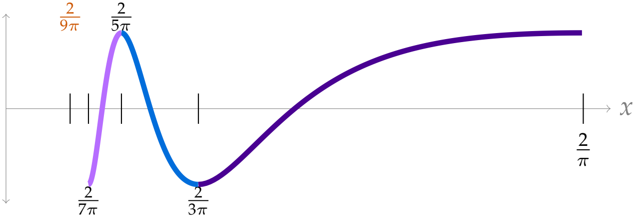

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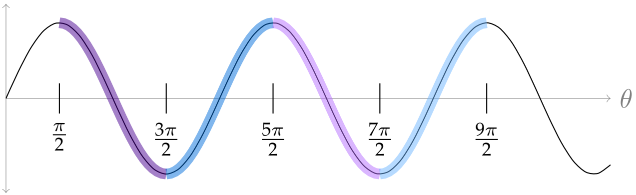

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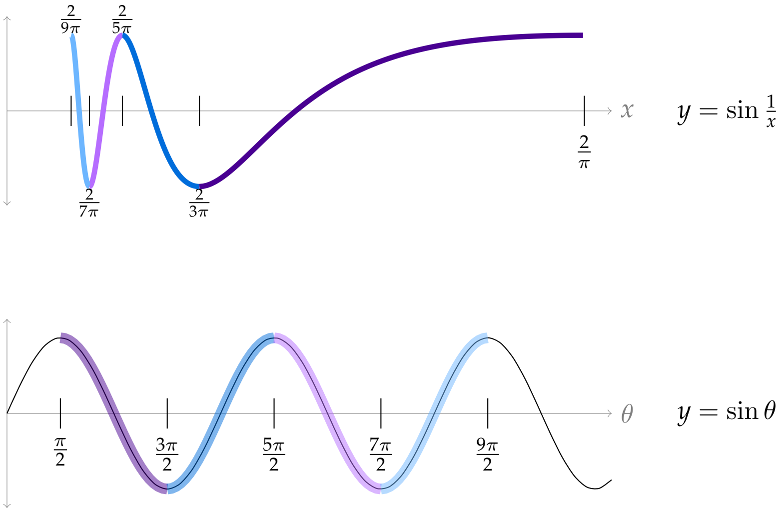

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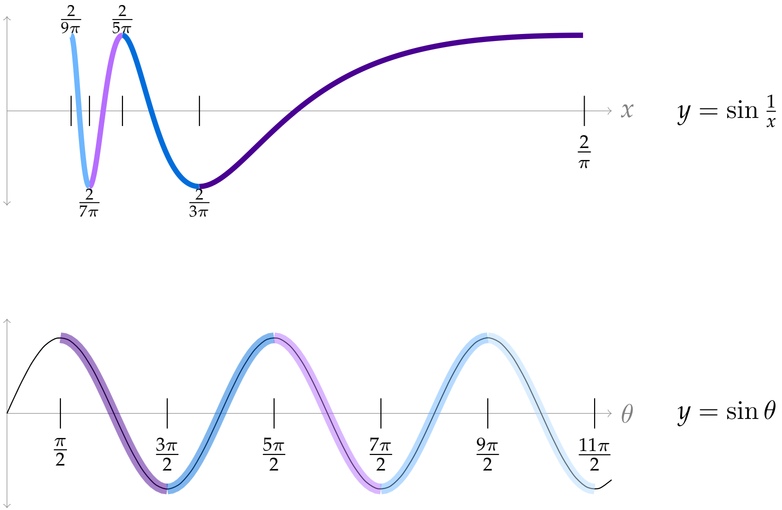
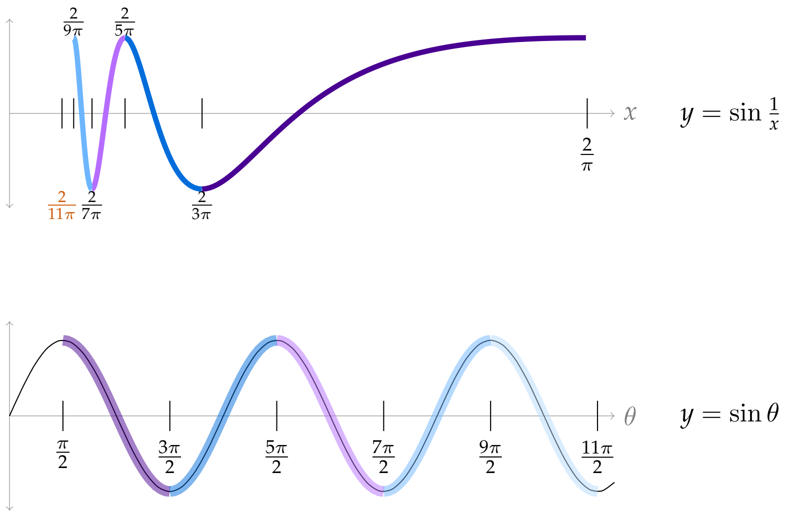


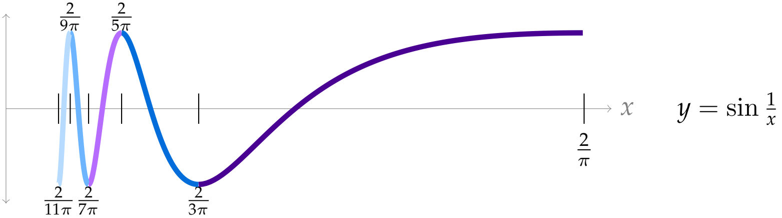
Optional: Sketching $f(x) = \sin\left(\frac{1}{x}\right)$

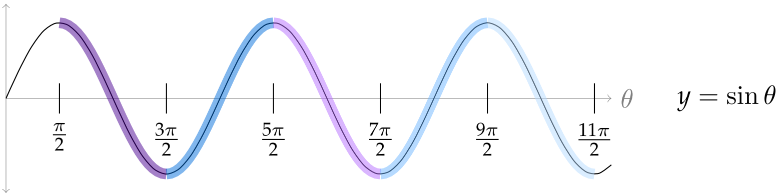
SKIP SKETCHING



Optional: Sketching $f(x) = \sin\left(\frac{1}{x}\right)$

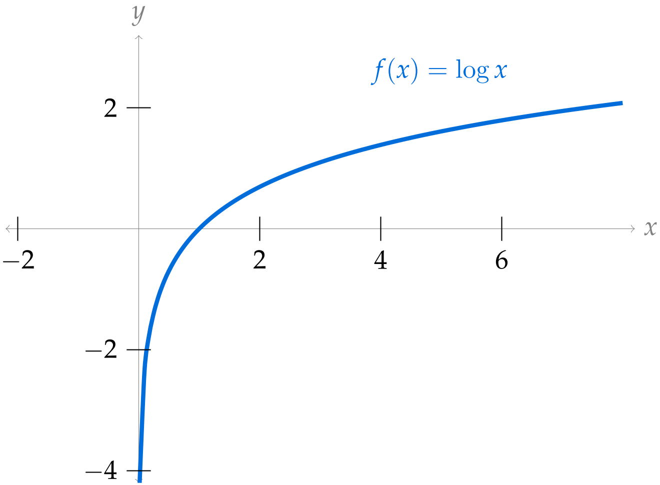
SKIP SKETCHING





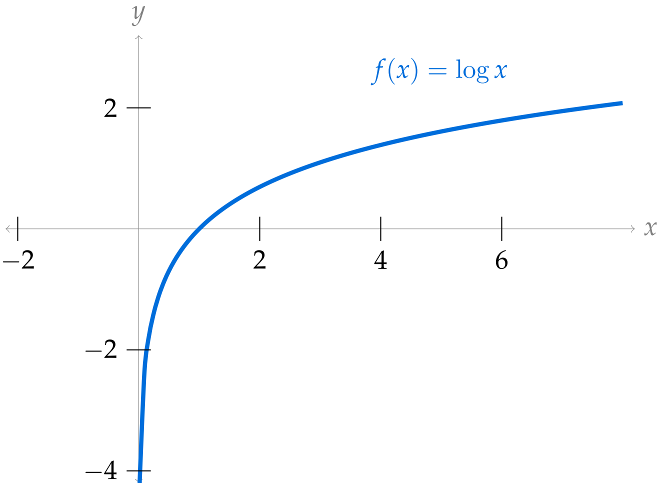
LIMITS AND THE NATURAL LOGARITHM

Where is $f(x)$ defined, and where is it not defined?



Limits and the Natural Logarithm

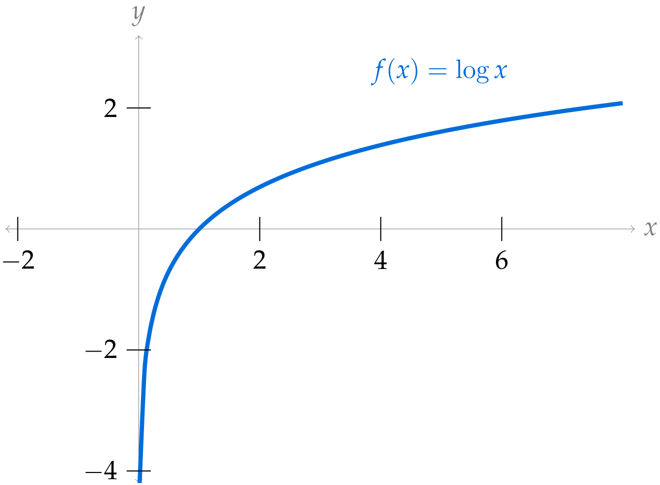
What can you say about the limit of $f(x)$ near 0?



ans

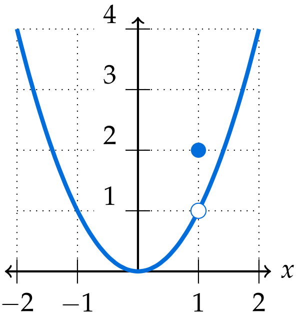
Limits and the Natural Logarithm

What can you say about the limit of $f(x)$ near 0? $\lim\limits_{x \to 0^+} \log(x) = -\infty$



Section 1.3 Review

$$f(x) = \begin{cases} x^2 & x \neq 1 \\ 2 & x = 1 \end{cases}$$



What is $\lim_{x \to 1} f(x)$?

A. $\lim_{x\to 1} f(x) = 2$

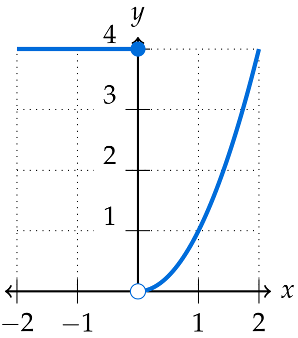
B. $\lim_{x\to 1} f(x) = 1$

C. $\lim_{x\to 1} f(x)$ DNE

D. none of the above

~~ans~~

$$f(x) = \begin{cases} 4 & x \leq 0 \\ x^2 & x > 0 \end{cases}$$



What is $\lim_{x \to 0} f(x)$?

A. $\lim_{x \to 0} f(x) = 4$

B. $\lim_{x \to 0} f(x) = 0$

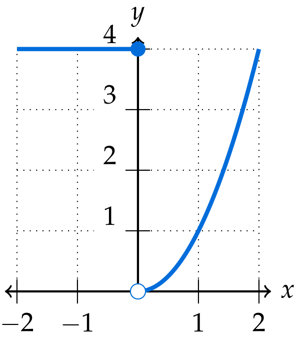
C. $\lim_{x \to 0} f(x) = \begin{cases} 4 & x \leq 0 \\ 0 & x > 0 \end{cases}$

D. none of the above

Q

ans

$$f(x) = \begin{cases} 4 & x \leq 0 \\ x^2 & x > 0 \end{cases}$$



What is $\lim_{x \to 0} f(x)$?

A. $\lim_{x \to 0} f(x) = 4$

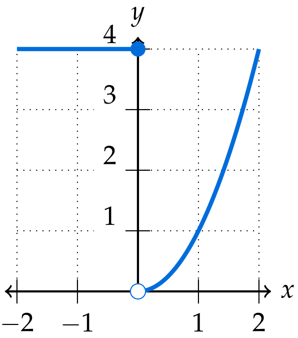
B. $\lim_{x \to 0} f(x) = 0$

C. $\lim_{x \to 0} f(x) = \begin{cases} 4 & x \leq 0 \\ 0 & x > 0 \end{cases}$

D. none of the above

$\lim_{x \to 0} f(x)$ DNE

$$f(x) = \begin{cases} 4 & x \leq 0 \\ x^2 & x > 0 \end{cases}$$



What is $\lim\limits_{x \to 0^+} f(x)$?

A. $\lim\limits_{x \to 0} f(x) = 4$

B. $\lim\limits_{x \to 0} f(x) = 0$

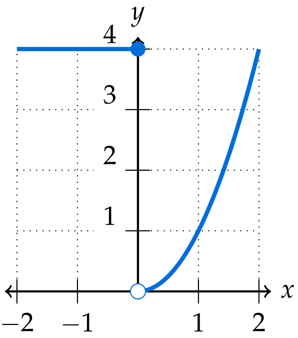
C. $\lim\limits_{x \to 0} f(x) = \begin{cases} 4 & x \leq 0 \\ 0 & x > 0 \end{cases}$

D. none of the above

Q

ans

$$f(x) = \begin{cases} 4 & x \leq 0 \\ x^2 & x > 0 \end{cases}$$



What is $\lim\limits_{x \to 0^+} f(x)$?

A. $\lim\limits_{x \to 0^+} f(x) = 4$

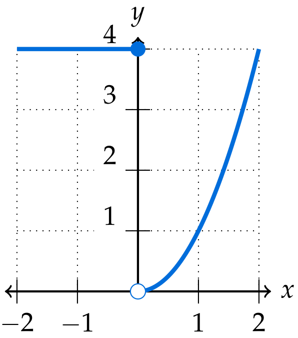
B. $\lim\limits_{x \to 0^+} f(x) = 0$

C. $\lim\limits_{x \to 0^+} f(x) = \begin{cases} 4 & x \leq 0 \\ 0 & x > 0 \end{cases}$

D. none of the above

$$f(x) = \begin{cases} 4 & x \leq 0 \\ x^2 & x > 0 \end{cases}$$

What is $f(0)$?



Q

ans

Suppose $\lim_{x\to 3^-} f(x) = 1$ and $\lim_{x\to 3^+} f(x) = 1.5$.

Does $\lim_{x\to 3} f(x)$ exist?

A. Yes, certainly, because the limits from both sides exist.

B. No, never, because the limit from the left is not the same as the limit from the right.

C. Can't tell. For some functions is might exist, for others not.

Q

ans

Suppose $\lim_{x \to 3^-} f(x) = 1$ and $\lim_{x \to 3^+} f(x) = 1.5$.

Does $\lim_{x \to 3} f(x)$ exist?

A. Yes, certainly, because the limits from both sides exist.

B. No, never, because the limit from the left is not the same as the limit from the right.

C. Can't tell. For some functions is might exist, for others not.

Suppose $\lim_{x \to 3^-} f(x) = 22 = \lim_{x \to 3^+} f(x)$.

Does $\lim_{x \to 3} f(x)$ exist?

A. Yes, certainly, because the limits from both sides exist and are equal to each other.

B. No, never, because we only talk about one-sided limits when the actual limit doesn't exist.

C. Can't tell. We need to know the value of the function at $x = 3$.

Q

ans

Suppose $\lim_{x \to 3^-} f(x) = 22 = \lim_{x \to 3^+} f(x)$.

Does $\lim_{x \to 3} f(x)$ exist?

A. Yes, certainly, because the limits from both sides exist and are equal to each other.

B. No, never, because we only talk about one-sided limits when the actual limit doesn't exist.

C. Can't tell. We need to know the value of the function at $x = 3$.

A

Included Work