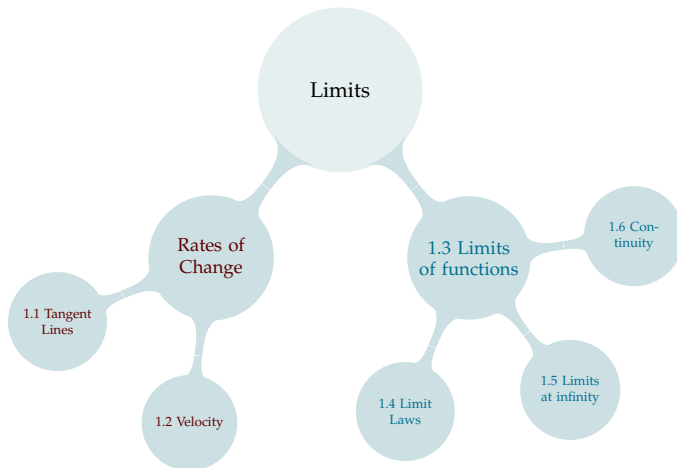


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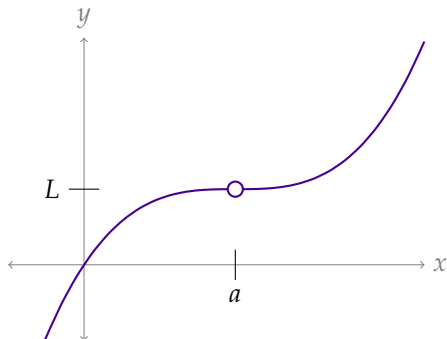
1.7 (Optional) Making the Informal a Little More Formal



Now that we've seen the limits of functions as x goes to positive and negative infinity, let's look at limits as x approaches a real number.

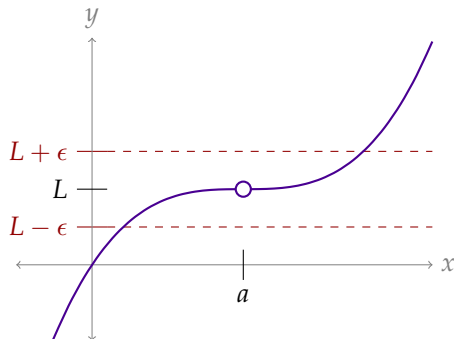
$$\lim_{x \rightarrow a} f(x) = L$$

Informally: If x is close enough (but not equal to) a , then y is close enough to L .



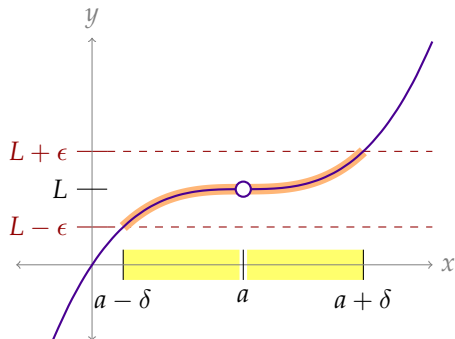
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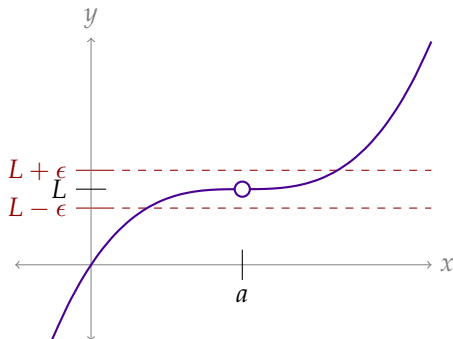
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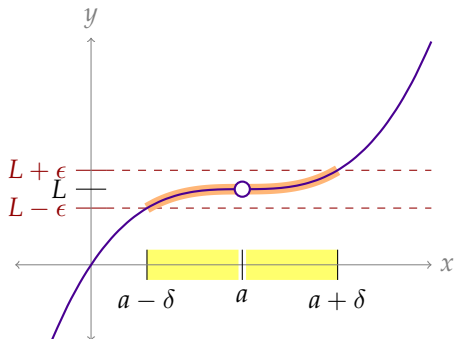
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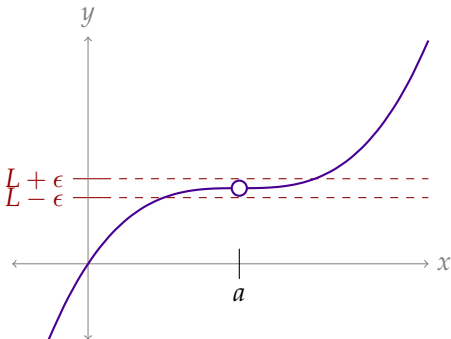
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Informally: If x is close enough (but not equal to) a , then y is close enough to L .



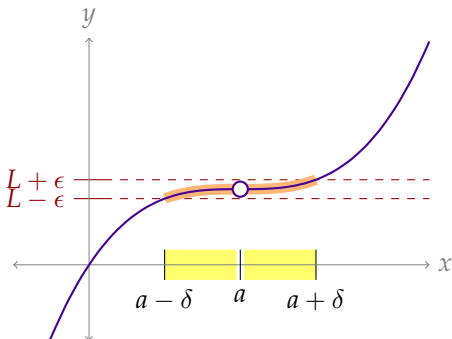
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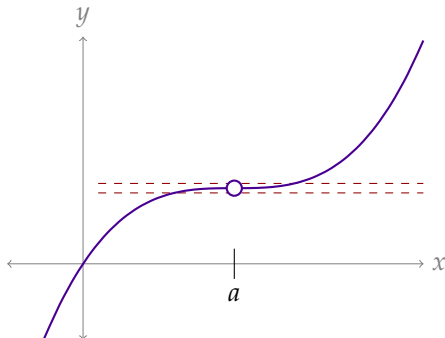
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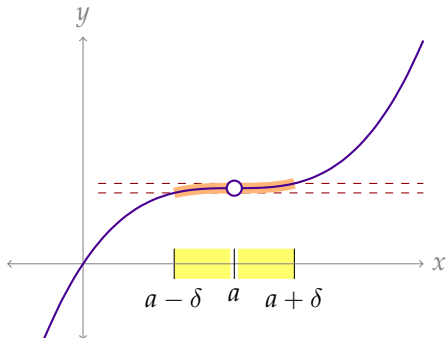
$$\lim_{x \rightarrow a} f(x) = L$$

Informally: If x is close enough (but not equal to) a , then y is close enough to L .

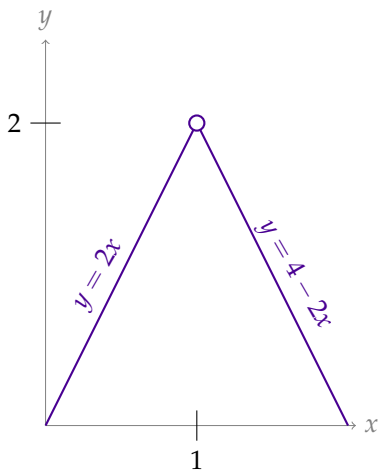


$$\lim_{x \rightarrow a} f(x) = L$$

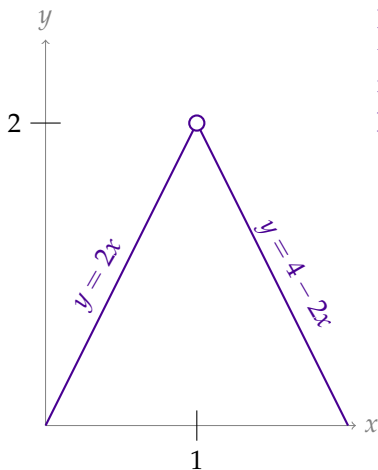
Informally: If x is close enough (but not equal to) a , then y is close enough to L .



Let $f(x) = \begin{cases} 2x & \text{if } x < 1 \\ 4 - 2x & \text{if } x > 1 \end{cases}$. Then $\lim_{x \rightarrow 1} |x| = 2$.

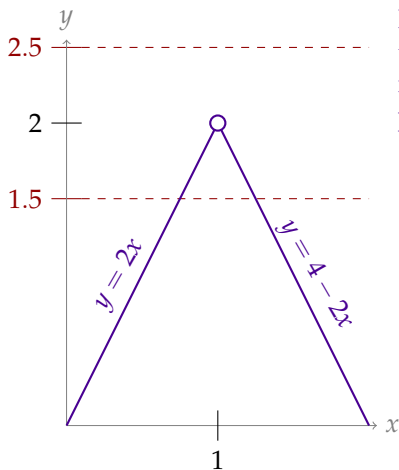


Let $f(x) = \begin{cases} 2x & \text{if } x < 1 \\ 4 - 2x & \text{if } x > 1 \end{cases}$. Then $\lim_{x \rightarrow 1} |f(x)| = 2$.



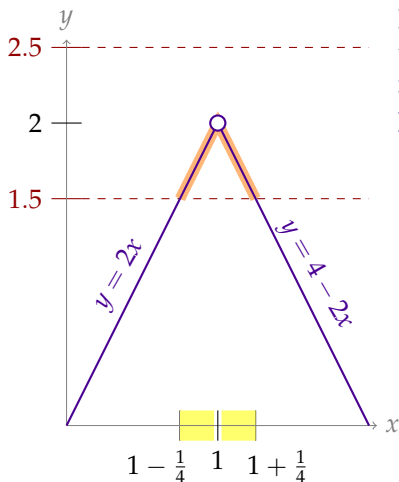
Find a positive number δ such that $|f(x) - 2| < \frac{1}{2}$ for all x in the interval $(1 - \delta, 1 + \delta)$, except possibly $x = 1$.

Let $f(x) = \begin{cases} 2x & \text{if } x < 1 \\ 4 - 2x & \text{if } x > 1 \end{cases}$. Then $\lim_{x \rightarrow 1} |f(x)| = 2$.



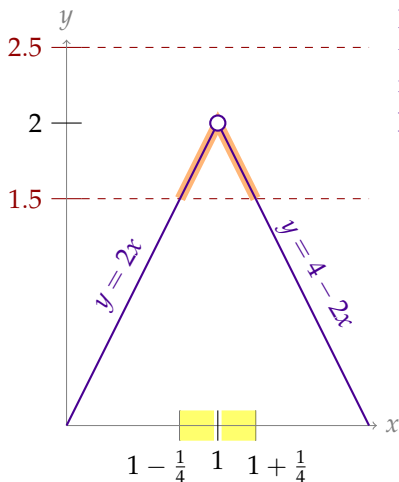
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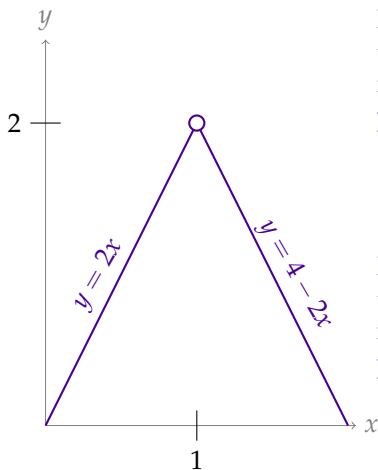
Let $f(x) = \begin{cases} 2x & \text{if } x < 1 \\ 4 - 2x & \text{if } x > 1 \end{cases}$. Then $\lim_{x \rightarrow 1} |x| = 2$.



Find a positive number δ such that $|f(x) - 2| < \frac{1}{2}$ for all x in the interval $(1 - \delta, 1 + \delta)$, except possibly $x = 1$.

$$\delta = \frac{1}{4}$$

Let $f(x) = \begin{cases} 2x & \text{if } x < 1 \\ 4 - 2x & \text{if } x > 1 \end{cases}$. Then $\lim_{x \rightarrow 1} |x| = 2$.

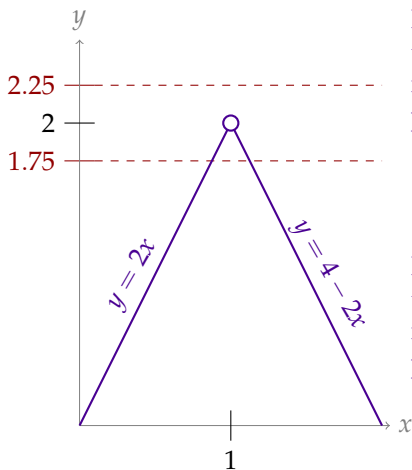


Find a positive number δ such that $|f(x) - 2| < \frac{1}{2}$ for all x in the interval $(1 - \delta, 1 + \delta)$, except possibly $x = 1$.

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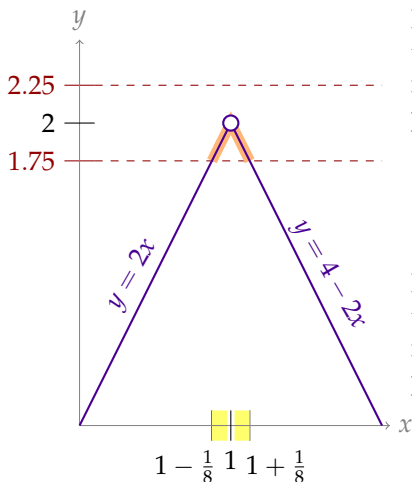


Find a positive number δ such that $|f(x) - 2| < \frac{1}{2}$ for all x in the interval $(1 - \delta, 1 + \delta)$, except possibly $x = 1$.

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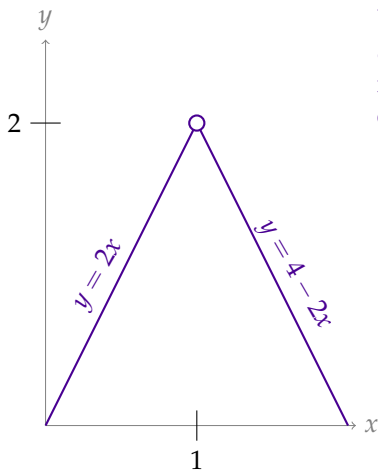
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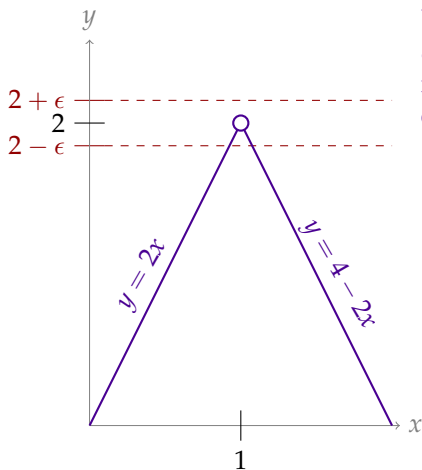
$$\delta = \frac{1}{8}$$

Let $f(x) = \begin{cases} 2x & \text{if } x < 1 \\ 4 - 2x & \text{if } x > 1 \end{cases}$. Then $\lim_{x \rightarrow 1} |f(x)| = 2$.



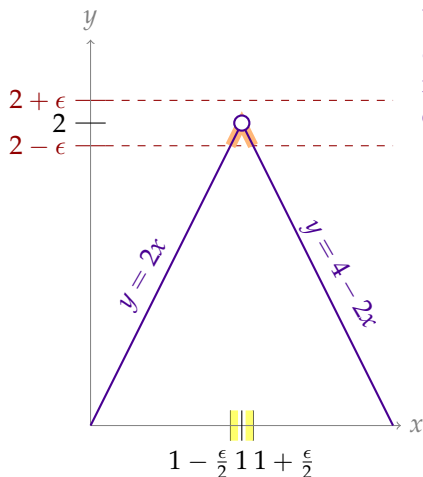
Let $\epsilon > 0$. Find a positive number δ such that $|f(x) - 2| < \epsilon$ for all x in the interval $(1 - \delta, 1 + \delta)$, except possibly $x = 1$.

Let $f(x) = \begin{cases} 2x & \text{if } x < 1 \\ 4 - 2x & \text{if } x > 1 \end{cases}$. Then $\lim_{x \rightarrow 1} |x| = 2$.



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Let $\epsilon > 0$. Find a positive number δ such that $|f(x) - 2| < \epsilon$ for all x in the interval $(1 - \delta, 1 + \delta)$, except possibly $x = 1$.

$$\delta = \frac{\epsilon}{2}$$

Definition 1.7.1

Let $a \in \mathbb{R}$ and let $f(x)$ be a function defined everywhere in a neighbourhood of a , except possibly at a . We say that

the limit as x approaches a of $f(x)$ is L

and write

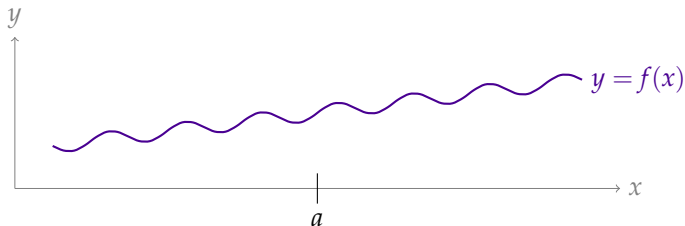
$$\lim_{x \rightarrow a} f(x) = L$$

if and only if for every $\epsilon > 0$ there exists $\delta > 0$ so that

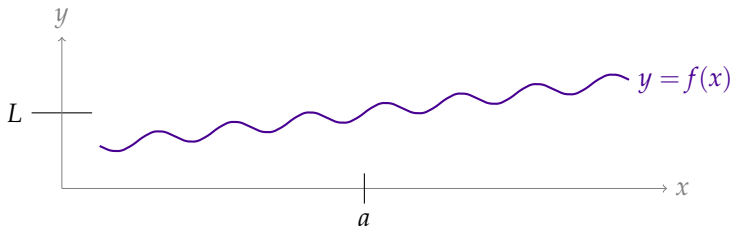
$$|f(x) - L| < \epsilon \text{ whenever } 0 < |x - a| < \delta$$

Note that an equivalent way of writing this very last statement is

$$\text{if } 0 < |x - a| < \delta \text{ then } |f(x) - L| < \epsilon.$$



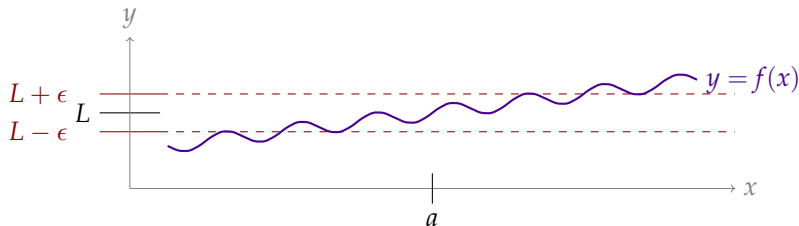
Let $a \in \mathbb{R}$ and let $f(x)$ be a function defined everywhere in a neighbourhood of a , except possibly at a .



Let $a \in \mathbb{R}$ and let $f(x)$ be a function defined everywhere in a neighbourhood of a , except possibly at a .

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$$\lim_{x \rightarrow a} f(x) = L$$

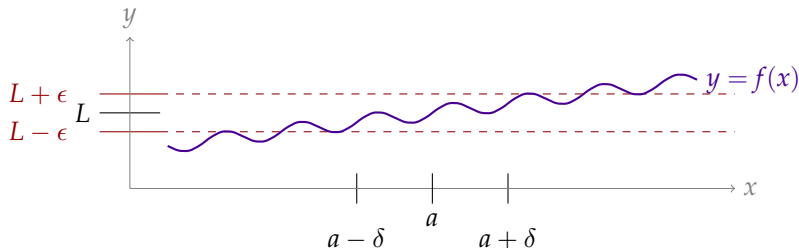


Let $a \in \mathbb{R}$ and let $f(x)$ be a function defined everywhere in a neighbourhood of a , except possibly at a .

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if and only if for every $\epsilon > 0$

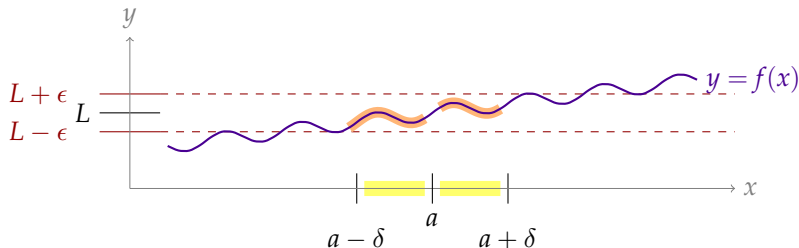


Let $a \in \mathbb{R}$ and let $f(x)$ be a function defined everywhere in a neighbourhood of a , except possibly at a .

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$$\lim_{x \rightarrow a} f(x) = L$$

if and only if for every $\epsilon > 0$ there exists $\delta > 0$



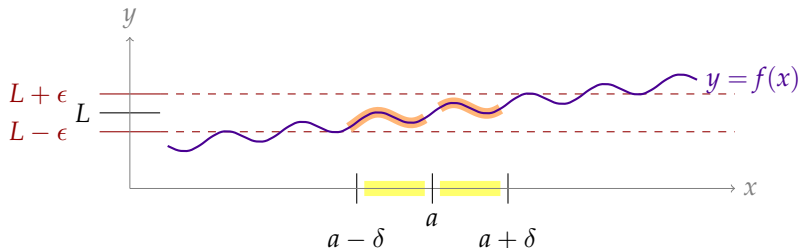
Let $a \in \mathbb{R}$ and let $f(x)$ be a function defined everywhere in a neighbourhood of a , except possibly at a .

We write

$$\lim_{x \rightarrow a} f(x) = L$$

if and only if for every $\epsilon > 0$ there exists $\delta > 0$ so that

$$|f(x) - L| < \epsilon \text{ whenever } 0 < |x - a| < \delta$$



Let $a \in \mathbb{R}$ and let $f(x)$ be a function defined everywhere in a neighbourhood of a , except possibly at a .

We write

$$\lim_{x \rightarrow a} f(x) = L$$

if and only if for every $\epsilon > 0$ there exists $\delta > 0$ so that

$$|f(x) - L| < \epsilon \text{ whenever } 0 < |x - a| < \delta$$

Definition 1.7.1

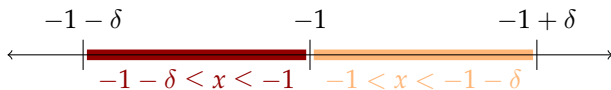
Let $a \in \mathbb{R}$ and let $f(x)$ be a function defined everywhere in a neighbourhood of a , except possibly at a . We say that $\lim_{x \rightarrow a} f(x) = L$ if and only if for every $\epsilon > 0$ there exists $\delta > 0$ so that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

Using Definition 1.7.1, prove that $\lim_{x \rightarrow -1} |x + 1| = 0$.

Using Definition 1.7.1, prove that $\lim_{x \rightarrow -1} |x + 1| = 0$.

By inspection (look at the graph of $y = |x + 1|$), we should use $\delta = \epsilon$.

Proof: Let $f(x) = |x + 1|$ and for any positive ϵ , let $\delta = \epsilon$.



If $-1 < x < -1 + \delta$:

$$|f(x) - 0| = ||x + 1| - 0| = |x + 1| = x + 1 < (-1 + \delta) + 1 = \delta = \epsilon$$

If $-1 - \delta < x < -1$:

$$|f(x) - 0| = ||x + 1| - 0| = |x + 1| = -x - 1 < -(-1 - \delta) - 1 = \delta = \epsilon$$

So if $0 < |x - (-1)| < \delta$, then $|f(x) - 0| < \epsilon$. That is, $\lim_{x \rightarrow -1} f(x) = 0$. \square

Definition 1.7.1

Let $a \in \mathbb{R}$ and let $f(x)$ be a function defined everywhere in a neighbourhood of a , except possibly at a . We say that $\lim_{x \rightarrow a} f(x) = L$ if and only if for every $\epsilon > 0$ there exists $\delta > 0$ so that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

$$\text{Let } f(x) = \begin{cases} x + 1 & x < 0 \\ 1 - x^2 & x > 0 \end{cases}.$$

Using Definition 1.7.1, prove that $\lim_{x \rightarrow 0} f(x) = 1$.

First, we need to find δ for any given ϵ . Suppose $x > 0$ and $|f(x) - 1| < \epsilon$:

$$|f(x) - 1| < \epsilon$$

$$|1 - x^2 - 1| < \epsilon$$

$$x^2 < \epsilon$$

$$0 < x < \sqrt{\epsilon}$$

Now, suppose $x < 0$ and $|f(x) - 1| < \epsilon$:

$$|f(x) - 1| < \epsilon$$

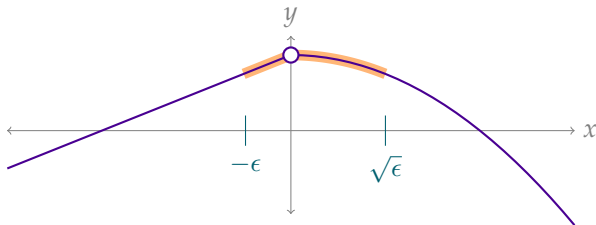
$$|x + 1 - 1| < \epsilon$$

$$|x| < \epsilon$$

$$-x < \epsilon$$

$$0 > x > -\epsilon$$

Now we know the interval over which $|f(x) - 1| < \epsilon$, but it's actually more information than we need. We don't need the exact interval; we just need some value of δ such that $0 < |x - 1| < \delta$ guarantees $|f(x) - 1| < \epsilon$.



If $0 < |x - 1| < \min\{\epsilon, \sqrt{\epsilon}\}$, then $0 < |x - 1| < \epsilon$ and $0 < |x - 1| < \sqrt{\epsilon}$ are both true. So we set $\delta = \min\{\epsilon, \sqrt{\epsilon}\}$. (For $\epsilon < 1$, that is $\delta = \epsilon$.)

Proof: $f(x) = \begin{cases} x + 1 & x < 0 \\ 1 - x^2 & x > 0 \end{cases}.$

For any $\epsilon > 0$, let $\delta = \min\{\epsilon, \sqrt{\epsilon}\}$. Suppose $0 < |x - 0| < \delta$.

► If $x > 0$, then

$$\begin{aligned} |f(x) - 1| &= |(1 - x^2) - 1| = |-x^2| = x^2 \\ &< \delta^2 \leq \sqrt{\epsilon}^2 = \epsilon \end{aligned}$$

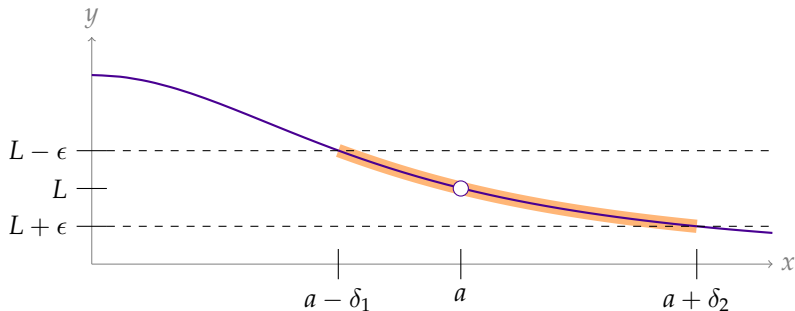
► If $x < 0$, then

$$\begin{aligned} |f(x) - 1| &= |(x + 1) - 1| = |x| \\ &< \delta \leq \epsilon \end{aligned}$$

So whenever $0 < |x - 0| < \delta$, then $|f(x) - 1| < \epsilon$. So $\lim_{x \rightarrow 0} f(x) = 1$. □

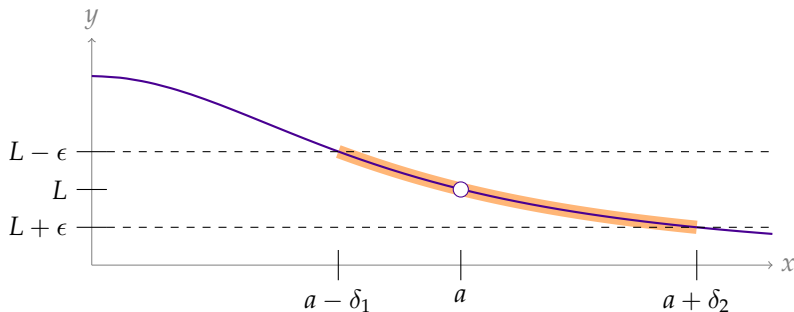
GENERAL PRINCIPLES

Suppose $|f(x) - L| < \epsilon$ whenever $a - \delta_1 < x < a$ and whenever $a < x < a + \delta_2$.



GENERAL PRINCIPLES

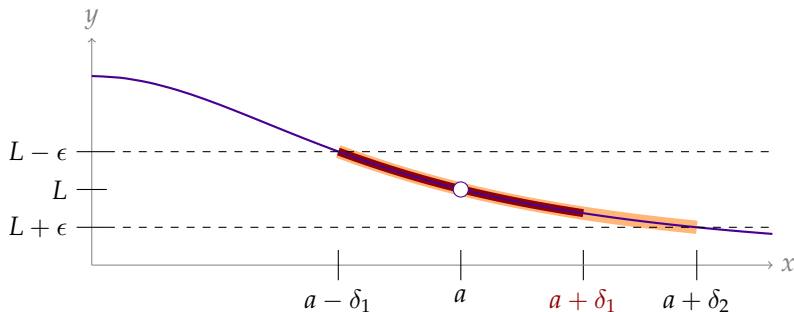
Suppose $|f(x) - L| < \epsilon$ whenever $a - \delta_1 < x < a$ and whenever $a < x < a + \delta_2$.



Consider values of x such that $0 < |x - a| < \min\{\delta_1, \delta_2\}$.

GENERAL PRINCIPLES

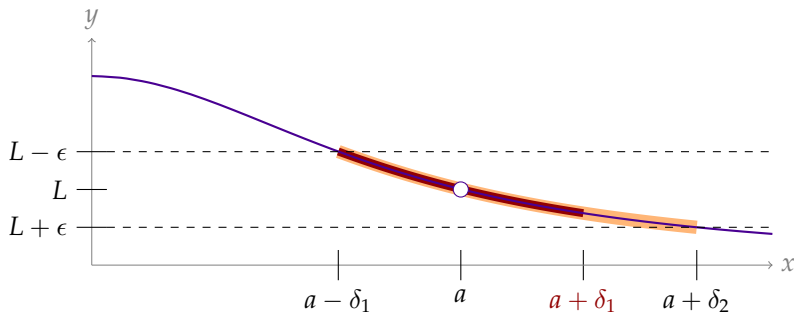
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Consider values of x such that $0 < |x - a| < \min\{\delta_1, \delta_2\}$.

GENERAL PRINCIPLES

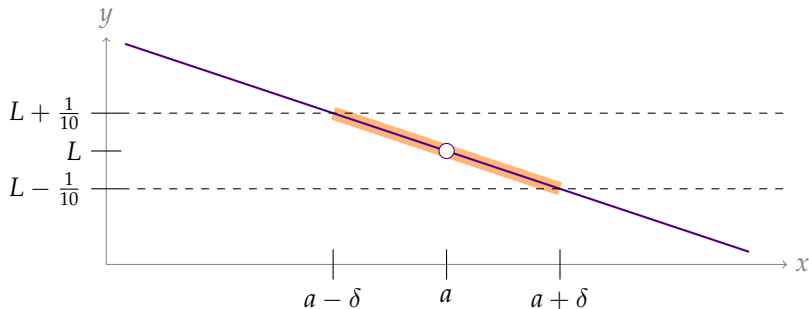
Suppose $|f(x) - L| < \epsilon$ whenever $a - \delta_1 < x < a$ and whenever $a < x < a + \delta_2$.



Consider values of x such that $0 < |x - a| < \min\{\delta_1, \delta_2\}$. For these values, it is (still) the case that $|f(x) - L| < \epsilon$.

GENERAL PRINCIPLES

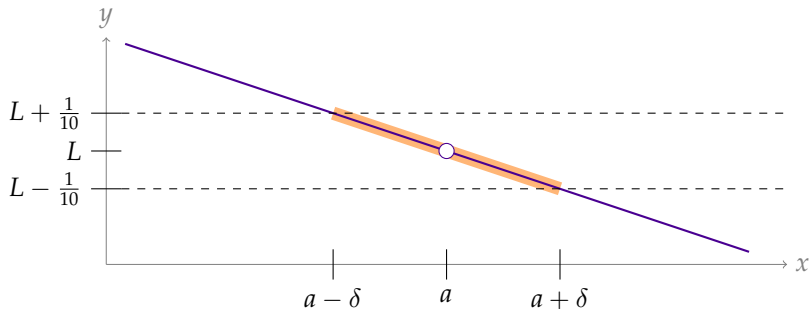
Suppose $|f(x) - L| < \frac{1}{10}$ for all x such that $0 < |x - a| < \delta$.



► skip ϵ small

GENERAL PRINCIPLES

Suppose $|f(x) - L| < \frac{1}{10}$ for all x such that $0 < |x - a| < \delta$.

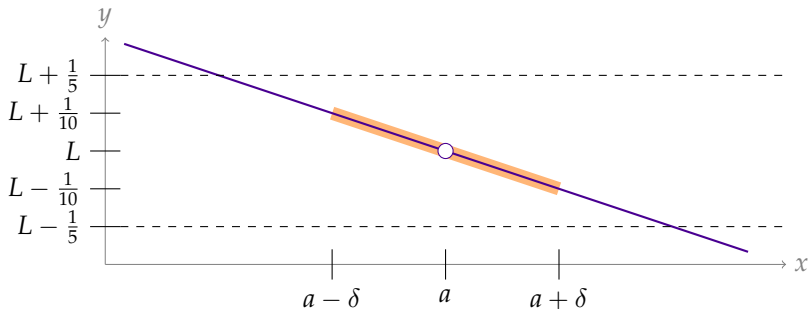


Can you give values of x where $|f(x) - L| < \frac{1}{5}$?

► skip ϵ small

GENERAL PRINCIPLES

Suppose $|f(x) - L| < \frac{1}{10}$ for all x such that $0 < |x - a| < \delta$.



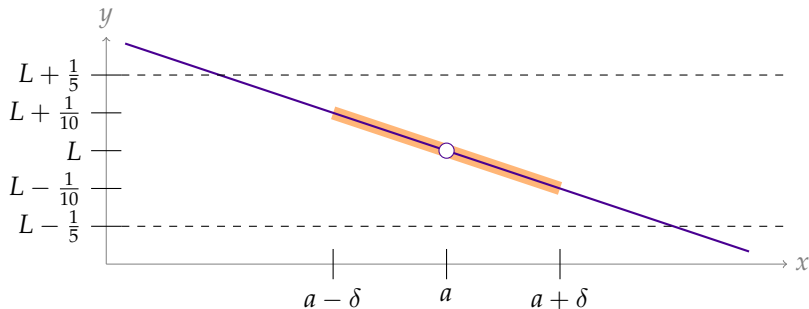
Can you give values of x where $|f(x) - L| < \frac{1}{5}$?

► skip ϵ small

GENERAL PRINCIPLES

Suppose $|f(x) - L| < \frac{1}{10}$ for all x such that $0 < |x - a| < \delta$.

Then also $|f(x) - L| < \frac{1}{5}$ for all x such that $0 < |x - a| < \delta$.



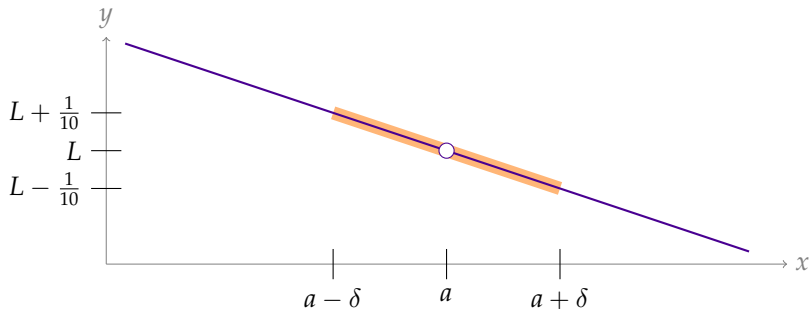
Can you give values of x where $|f(x) - L| < \frac{1}{5}$?

► skip ϵ small

GENERAL PRINCIPLES

Suppose $|f(x) - L| < \frac{1}{10}$ for all x such that $0 < |x - a| < \delta$.

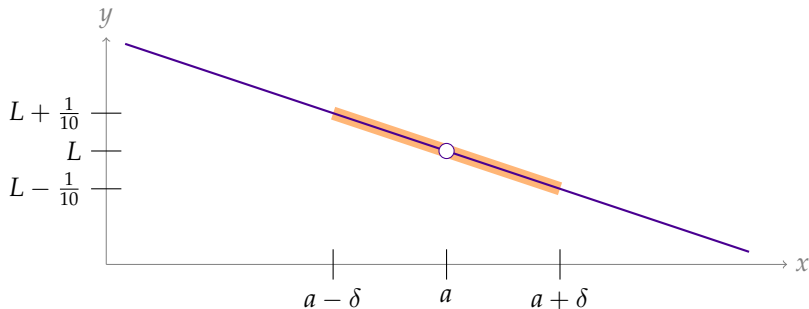
Then also $|f(x) - L| < \frac{1}{2}$ for all x such that $0 < |x - a| < \delta$.



GENERAL PRINCIPLES

Suppose $|f(x) - L| < \frac{1}{10}$ for all x such that $0 < |x - a| < \delta$.

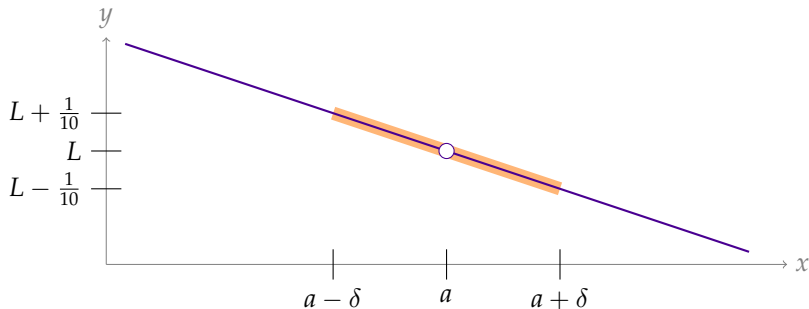
Then also $|f(x) - L| < 1$ for all x such that $0 < |x - a| < \delta$.



GENERAL PRINCIPLES

Suppose $|f(x) - L| < \frac{1}{10}$ for all x such that $0 < |x - a| < \delta$.

Then also $|f(x) - L| < 100$ for all x such that $0 < |x - a| < \delta$.



► skip ϵ small

GENERAL PRINCIPLES

Definition 1.7.1

Let $a \in \mathbb{R}$ and let $f(x)$ be a function defined everywhere in a neighbourhood of a , except possibly at a . We say that $\lim_{x \rightarrow a} f(x) = L$ if and only if **for every** $\epsilon > 0$ there exists $\delta > 0$ so that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

GENERAL PRINCIPLES

Definition 1.7.1

Let $a \in \mathbb{R}$ and let $f(x)$ be a function defined everywhere in a neighbourhood of a , except possibly at a . We say that $\lim_{x \rightarrow a} f(x) = L$ if and only if **for every $\epsilon > 0$** there exists $\delta > 0$ so that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

It is enough to show that **for every ϵ such that $0 < \epsilon < c$** (where c is some constant) there exists $\delta > 0$ so that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

That means it doesn't hurt your proof if you say something like "we assume $\epsilon < 1$ ".

GENERAL PRINCIPLES

Definition 1.7.1

Let $a \in \mathbb{R}$ and let $f(x)$ be a function defined everywhere in a neighbourhood of a , except possibly at a . We say that $\lim_{x \rightarrow a} f(x) = L$ if and only if **for every** $\epsilon > 0$ there exists $\delta > 0$ so that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

It is enough to show that **for every** ϵ such that $0 < \epsilon < c$ (where c is some constant) there exists $\delta > 0$ so that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

That means it doesn't hurt your proof if you say something like "we assume $\epsilon < 1$ ".

In a previous example, we chose

$$\delta = \min\{\epsilon, \sqrt{\epsilon}\}$$

It would be OK to say "we can assume $\epsilon < 1$; set $\delta = \epsilon$."

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Let $a \in \mathbb{R}$ and let $f(x)$ be a function defined everywhere in a neighbourhood of a , except possibly at a . We say that $\lim_{x \rightarrow a} f(x) = L$ if and only if for every $\epsilon > 0$ there exists $\delta > 0$ so that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

Using Definition 1.7.1, prove that $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4} = \frac{1}{4}$.

We start, as usual, by finding δ . Note

$$\frac{x-2}{x^2-4} = \frac{x-2}{(x+2)(x-2)} = \frac{1}{x+2}$$

whenever $x \neq 2$.

$$\begin{aligned} \left| \frac{1}{x+2} - \frac{1}{4} \right| &< \epsilon \\ -\epsilon &< \frac{1}{x+2} - \frac{1}{4} < \epsilon \\ \frac{1}{4} - \epsilon &< \frac{1}{x+2} < \frac{1}{4} + \epsilon \\ \frac{1-4\epsilon}{4} &< \frac{1}{x+2} < \frac{1+4\epsilon}{4} \\ \frac{4}{1-4\epsilon} &> x+2 > \frac{4}{1+4\epsilon} \\ \frac{4}{1-4\epsilon} - 2 &> x > \frac{4}{1+4\epsilon} - 2 \end{aligned}$$

$$\frac{2+8\epsilon}{1-4\epsilon} > x > \frac{2-8\epsilon}{1+4\epsilon}$$

We want our bounds to look like $2 - \delta_1$ and $2 + \delta_2$.

$$\begin{aligned} \frac{2-8\epsilon+16\epsilon}{1-4\epsilon} &> x > \frac{2+8\epsilon-16\epsilon}{1+4\epsilon} \\ 2 + \frac{16\epsilon}{1-4\epsilon} &> x > 2 - \frac{16\epsilon}{1+4\epsilon} \end{aligned}$$

For x in the interval found, $|f(x) - \frac{1}{4}| < \epsilon$. The interval is not exactly in the form $2 - \delta < x < 2 + \delta$, but it's close. Remember a smaller interval will also have the property $|f(x) - \frac{1}{4}| < \epsilon$. So, set

$$\delta = \min \left\{ \frac{16\epsilon}{1-4\epsilon}, \frac{16\epsilon}{1+4\epsilon} \right\} = \frac{16\epsilon}{1+4\epsilon}.$$



Proof: For any $\epsilon > 0$, let $\delta = \frac{16\epsilon}{1+4\epsilon}$. Suppose $0 < |x - 2| < \delta$. Note that since $x \neq 2$, we have $f(x) = \frac{x-2}{(x+2)(x-2)} = \frac{1}{x+2}$.

► If $x > 2$, then $2 < x < 2 + \delta$:

$$\begin{aligned} \left| f(x) - \frac{1}{4} \right| &= \left| \frac{1}{(x+2)} - \frac{1}{4} \right| = \frac{1}{4} - \frac{1}{(x+2)} \\ &< \frac{1}{4} - \frac{1}{(2+\delta)+2} = \frac{1}{4} - \frac{1}{4+\delta} \\ &= \frac{1}{4} - \frac{1}{4 + \frac{16\epsilon}{1+4\epsilon}} < \frac{1}{4} - \frac{1}{4 + \frac{16\epsilon}{1-4\epsilon}} \end{aligned}$$

(using the fact that $\delta = \frac{16\epsilon}{1+4\epsilon} < \frac{16\epsilon}{1-4\epsilon}$)

$$= \frac{1}{4} - \frac{1}{\frac{4-16\epsilon+16\epsilon}{1-4\epsilon}} = \frac{1}{4} - \frac{1-4\epsilon}{4} = \frac{4\epsilon}{4} = \epsilon$$

► If $x < 2$, then $2 - \delta < x < 2$:

$$\begin{aligned}
 \left| f(x) - \frac{1}{4} \right| &= \left| \frac{1}{(x+2)} - \frac{1}{4} \right| = \frac{1}{(x+2)} - \frac{1}{4} \\
 &< \frac{1}{(2-\delta)+2} - \frac{1}{4} = \frac{1}{4-\delta} - \frac{1}{4} \\
 &= \frac{1}{4 - \frac{16\epsilon}{1+4\epsilon}} - \frac{1}{4} = \frac{1}{\frac{4+16\epsilon-16\epsilon}{1+4\epsilon}} - \frac{1}{4} \\
 &= \frac{1+4\epsilon}{4} - \frac{1}{4} = \epsilon
 \end{aligned}$$

We have shown that whenever $0 < |x - 2| < \delta$, then

$$|f(x) - \frac{1}{4}| < \epsilon. \text{ So, } \lim_{x \rightarrow 2} \frac{x-2}{x^2-4} = \frac{1}{4}.$$



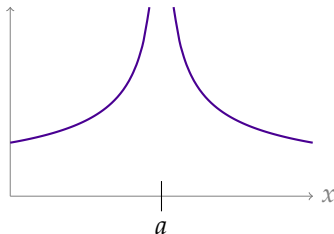
INFINITE LIMITS

Definition 1.8.1 (b)

Let a be a real number and $f(x)$ be a function defined for all $x \neq a$. We write

$$\lim_{x \rightarrow a} f(x) = \infty$$

if and only if for every $P > 0$ there exists $\delta > 0$ so that $f(x) > P$ whenever $0 < |x - a| < \delta$.



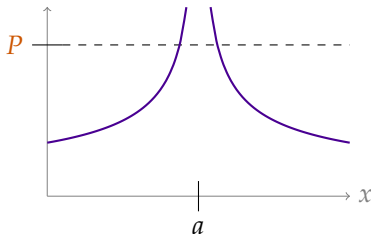
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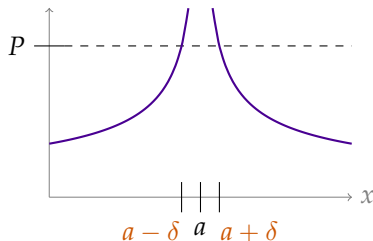
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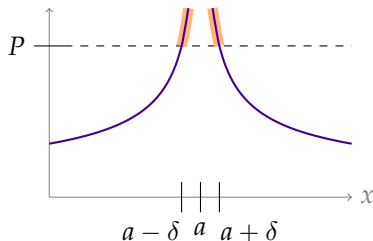
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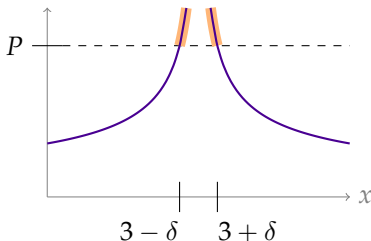


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Let $f(x) = \frac{1}{(x-3)^2}$. Using Definition 1.8.1, prove or disprove that

$$\lim_{x \rightarrow 3} f(x) = \infty$$

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First, we find δ . Let $P > 0$.

So we choose $\delta = \frac{1}{\sqrt{P}}$.

$$f(x) > P$$

$$\frac{1}{(x-3)^2} > P$$

$$(x-3)^2 < \frac{1}{P}$$

$$|x-3| < \frac{1}{\sqrt{P}}$$

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Let a be a real number and $f(x)$ be a function defined for all $x \neq a$. We write

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if and only if for every $P > 0$ there exists $\delta > 0$ so that $f(x) > P$ whenever $0 < |x - a| < \delta$.

Proof: For any $P > 0$, set $\delta = \frac{1}{\sqrt{P}}$. So, $\lim_{x \rightarrow 3} \frac{1}{(x-3)^2} = \infty$.
 If $0 < |x - 3|$, then $x \neq 3$, so $f(x)$ exists. If, furthermore, $|x - 3| < \delta$, then:

$$\begin{aligned} f(x) &= \frac{1}{(x-3)^2} \\ &> \frac{1}{\delta^2} = \frac{1}{\left(\frac{1}{\sqrt{P}}\right)^2} = P \end{aligned}$$

Definition 1.8.1 (b)

Let a be a real number and $f(x)$ be a function defined for all $x \neq a$. We write

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if and only if for every $P > 0$ there exists $\delta > 0$ so that $f(x) > P$ whenever $0 < |x - a| < \delta$.

Let $f(x) = \frac{1}{x-2}$. Using Definition 1.8.1, prove or disprove that

$$\lim_{x \rightarrow 2} f(x) = \infty$$

Definition 1.8.1 (b)

Let a be a real number and $f(x)$ be a function defined for all $x \neq a$. We write

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if and only if for every $P > 0$ there exists $\delta > 0$ so that $f(x) > P$ whenever $0 < |x - a| < \delta$.

Definition 1.8.1 (b)

Let a be a real number and $f(x)$ be a function defined for all $x \neq a$. We write

$$\lim_{x \rightarrow a} f(x) = \infty$$

if and only if for every $P > 0$ there exists $\delta > 0$ so that $f(x) > P$ whenever $0 < |x - a| < \delta$.

Note that $x - 2 < 0$ when $x < 2$. So for any $P > 0$, whenever $x < 2$, we have $f(x) < P$. That tells us that $\lim_{x \rightarrow 2} f(x) \neq \infty$.

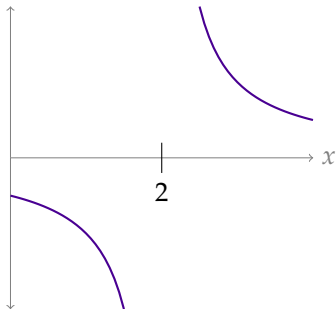
Proof: Let $P = 1$. For any $\delta > 0$, set $x_0 = 2 - \frac{\delta}{2}$. Then $0 < |x_0 - 2| < \delta$, but since $x < 2$, we have $\frac{1}{x-2} < 0 < P$. That is, there does not exist any $\delta > 0$ such that $f(x) > P$ whenever $0 < |x - 2| < \delta$. Therefore $\lim_{x \rightarrow 2} f(x) \neq \infty$.

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Let a be a real number and $f(x)$ be a function defined for all $x \neq a$. We write

$$\lim_{x \rightarrow a} f(x) = \infty$$

if and only if for every $P > 0$ there exists $\delta > 0$ so that $f(x) > P$ whenever $0 < |x - a| < \delta$.



Included Work



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