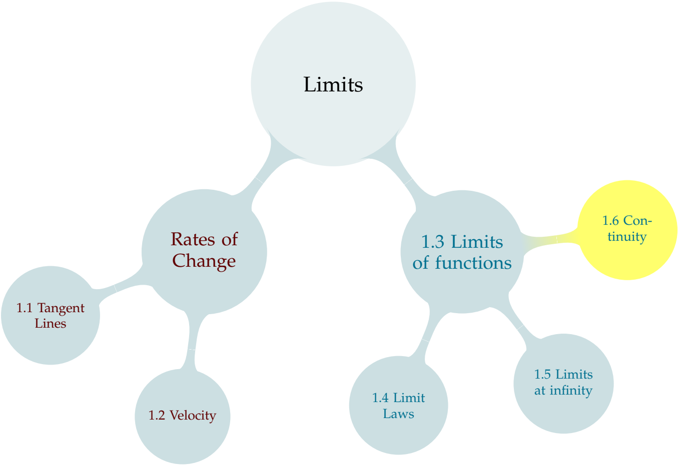
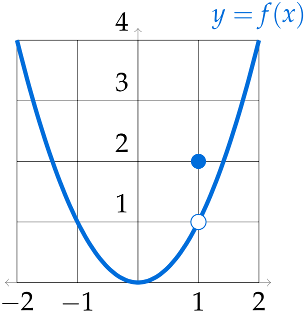
Table of Contents



Continuity

Definition 1.6.1

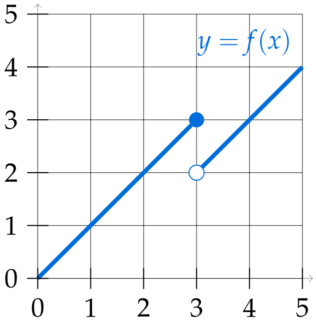
A function $f(x)$ is continuous at a point a if $\lim_{x \to a} f(x)$ exists AND is equal to $f(a)$.



Does $f(x)$ exist at $x = 1$?
Is $f(x)$ continuous at $x = 1$?

Definitions 1.6.1 and 1.6.2

A function $f(x)$ is continuous from the left at a point a if $\lim_{x \to a^-} f(x)$ exists AND is equal to $f(a)$.



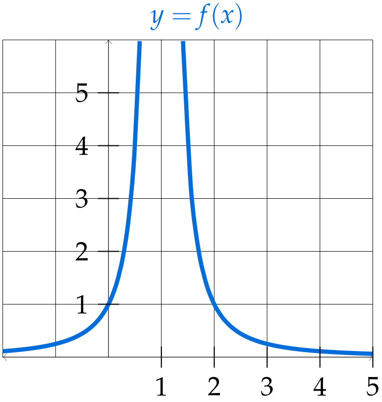
Is $f(x)$ continuous at $x = 3$?

Is $f(x)$ continuous from the left at $x = 3$?

Is $f(x)$ continuous from the right at $x = 3$?

Definition

A function $f(x)$ is continuous at a point a if $\lim_{x \to a} f(x)$ exists AND is equal to $f(a)$.



Definition

A function $f(x)$ is continuous at a point a if $\lim_{x \to a} f(x)$ exists AND is equal to $f(a)$.

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$$

Is $f(x)$ continuous at 0?

Continuous Functions

Functions made by adding, subtracting, multiplying, dividing, and taking appropriate powers of polynomials are continuous for every point in their domain.

$$f(x) = \frac{x^2}{2x-10} - \left(\frac{x^2+2x-1}{x-1} + \frac{\sqrt[5]{25-x} - \frac{1}{x}}{x+2} \right)^{1/3}$$

A continuous function is continuous for every point in $\mathbb{R}$.

We say $f(x)$ is continuous over (a, b) if it is continuous at every point in (a, b).

Common Functions – Theorem 1.6.8

Functions of the following types are continuous over their domains:

- polynomials and rationals
- roots and powers
- trig functions and their inverses
- exponential and logarithm
- The products, sums, differences, quotients, powers, and compositions of continuous functions

Where is the following function continuous?

$$f(x) = \left(\frac{\sin x}{(x-2)(x+3)} + e^{\sqrt{x}} \right)^3$$

A Technical Point

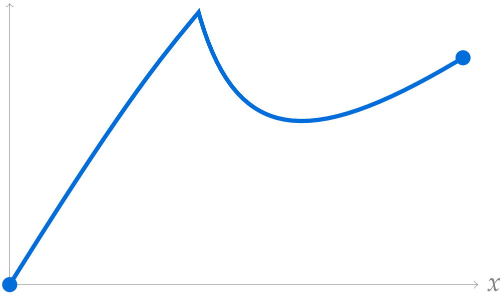
Definition 1.6.3

A function $f(x)$ is continuous on the closed interval $[a, b]$ if:

- $f(x)$ is continuous over (a, b), and
- $f(x)$ is continuous from the left at b, and
- $f(x)$ is continuous from the right at a

Intermediate Value Theorem (IVT) – Theorem 1.6.12

Let $a < b$ and let $f(x)$ be continuous over $[a, b]$. If y is any number between $f(a)$ and $f(b)$, then there exists c in (a, b) such that $f(c) = y$.



Intermediate Value Theorem (IVT) – Theorem 1.6.12

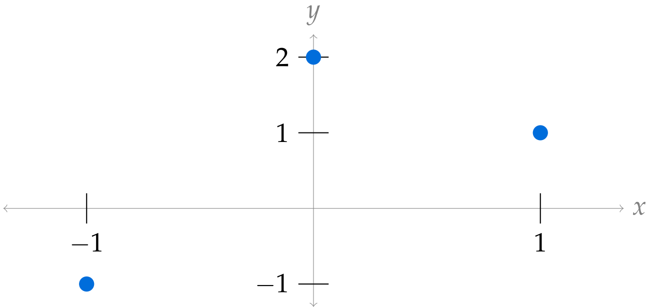
Let $a < b$ and let $f(x)$ be continuous over $[a, b]$. If y is any number between $f(a)$ and $f(b)$, then there exists c in (a, b) such that $f(c) = y$.

Suppose your favourite number is 45.54. At noon, your car is parked, and at 1pm you're driving 100kph.

Using IVT to Find Roots: "Bisection Method"

Let $f(x) = x^5 - 2x^4 + 2$. Find any value x for which $f(x) = 0$. Let's find some points:

$f(0) = 2$ $\qquad f(1) = 1$ $\qquad f(-1) = -1$



Using IVT to Find Roots: "Bisection Method"

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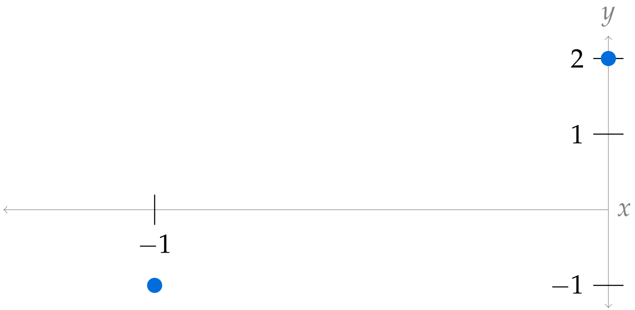


Use the Intermediate Value Theorem to show that there exists some solution to the equation $\ln x \cdot e^x = 4$, and give a reasonable interval where that solution might occur.

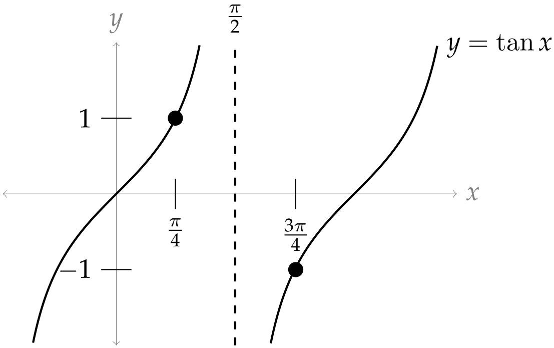
Now You Use the Intermediate Value Theorem to give a reasonable interval where the following is true: $e^x = \sin(x)$. (Don't use a calculator – use numbers you can easily evaluate.)

Now You

Is there any value of x so that $\sin x = \cos(2x) + \frac{1}{4}$?

NOW YOU Is the following reasoning correct?

- $f(x) = \tan x$ is continuous over its domain, because it is a trigonometric function.
- In particular, $f(x)$ is continuous over the interval $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$.
- $f\left(\frac{\pi}{4}\right) = 1$, and $f\left(\frac{3\pi}{4}\right) = -1$.
- Since $f\left(\frac{3\pi}{4}\right) < 0 < f\left(\frac{\pi}{4}\right)$, by the Intermediate Value Theorem, there exists some number c in the interval $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ such that $f(c) = 0$.


y
$\frac{\pi}{2}$
$y = \tan x$
1
−1
x
$\frac{\pi}{4}$
$\frac{3\pi}{4}$

Continuity

Section 1.6 Review

Suppose $f(x)$ is continuous at $x = 1$. Does $f(x)$ have to be defined at $x = 1$?

Suppose $f(x)$ is continuous at $x = 1$ and $\lim_{x \to 1^-} f(x) = 30$.

True or false: $\lim_{x \to 1^+} f(x) = 30$.

Suppose $f(x)$ is continuous at $x = 1$ and $f(1) = 22$. What is $\lim_{x \to 1} f(x)$?

Suppose $\lim_{x \to 1} f(x) = 2$. Must it be true that $f(1) = 2$?

$$f(x) = \begin{cases} ax^2 & x \geq 1 \\ 3x & x < 1 \end{cases}$$

For which value(s) of a is $f(x)$ continuous?

$$f(x) = \begin{cases} \frac{\sqrt{3}x+3}{x^2-3} & x \neq \pm\sqrt{3} \\ a & x = \pm\sqrt{3} \end{cases}$$

For which value(s) of a is $f(x)$ continuous at $x = -\sqrt{3}$?

$$f(x) = \begin{cases} \frac{\sqrt{3}x+3}{x^2-3} & x \neq \pm\sqrt{3} \\ a & x = \pm\sqrt{3} \end{cases}$$

For which value(s) of a is $f(x)$ continuous at $x = \sqrt{3}$?

Included Work