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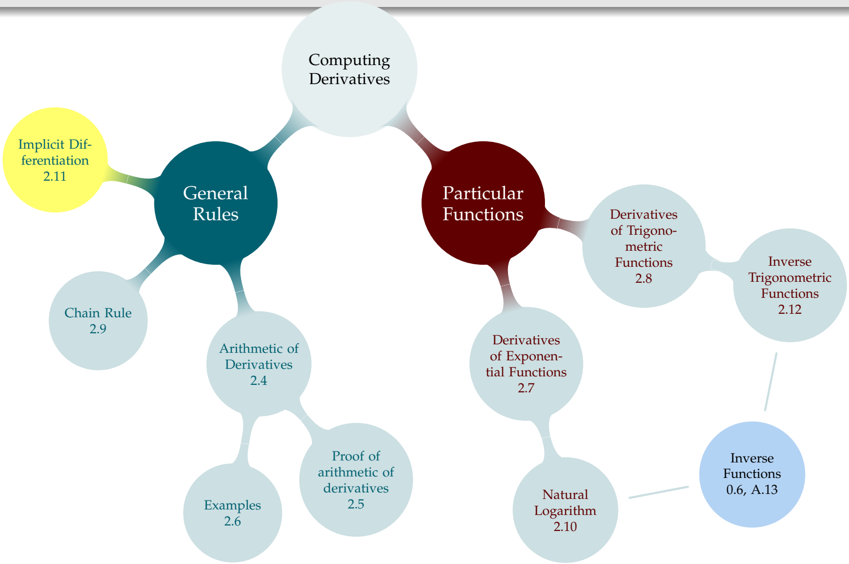

Implicitly Defined Functions

$$y^2 + x^2 + xy + x^2y = 1$$

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If $x = -3$, what is y?

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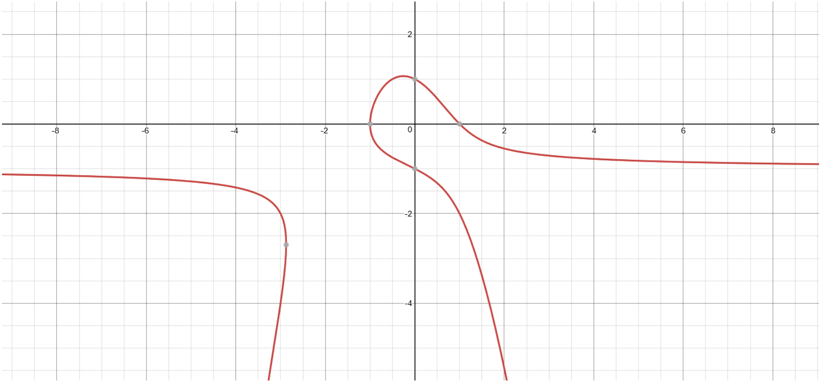
$(0,1)$ and $(0,-1)$

If $x = -3$, what is y? $y = -2$ and $y = -4$

A

$$y^2 + x^2 + xy + x^2y = 1$$



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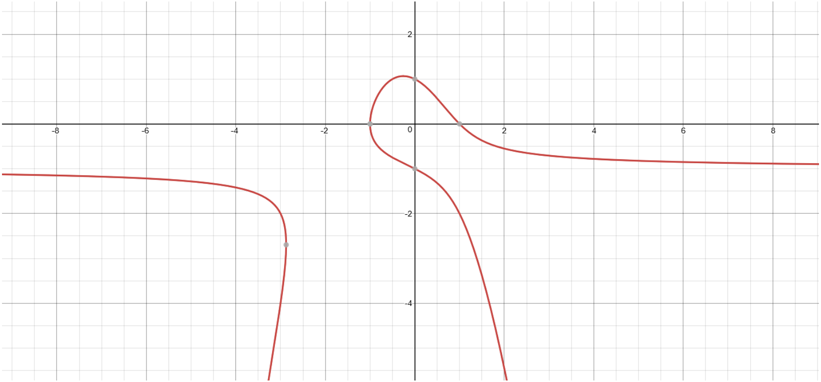
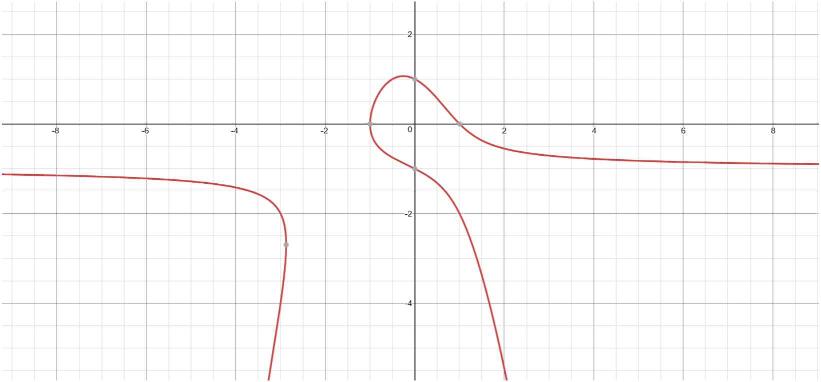


Still has a slope: $\frac{\Delta y}{\Delta x}$

$$y^2 + x^2 + xy + x^2y = 1$$



Still has a slope: $\frac{\Delta y}{\Delta x}$
Locally, y is still a function of x.

$$y^2 + x^2 + xy + x^2y = 1$$

Consider y as a function of x. Can we find $\frac{\mathrm{d}y}{\mathrm{d}x}$?

$\frac{\mathrm{d}}{\mathrm{d}x}[y] =$

$$y^2 + x^2 + xy + x^2y = 1$$

Consider y as a function of x. Can we find $\frac{\mathrm{d}y}{\mathrm{d}x}$?

$\frac{\mathrm{d}}{\mathrm{d}x}[y] = \frac{\mathrm{d}y}{\mathrm{d}x} = y'$

$$y^2 + x^2 + xy + x^2y = 1$$

Consider y as a function of x. Can we find $\frac{dy}{dx}$?

$$\frac{d}{dx}[y] = \frac{dy}{dx} = y' \qquad \frac{d}{dx}[x] =$$

ans

$$y^2 + x^2 + xy + x^2y = 1$$

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$$\frac{d}{dx}[y] = \frac{dy}{dx} = y' \qquad \frac{d}{dx}[x] = 1 \qquad \frac{d}{dx}[1] = 0$$

Differentiate both sides with respect to x.

$$\begin{aligned}
0 &= 2y\frac{dy}{dx} + 2x + \left(x\frac{dy}{dx} + (1)y\right) + \left(x^2\frac{dy}{dx} + 2xy\right)\\
0 &= \frac{dy}{dx}\left(2y + x + x^2\right) + (2x + y + 2xy)\\
-(2x + y + 2xy) &= \frac{dy}{dx}\left(2y + x + x^2\right)\\
-\frac{2x + y + 2xy}{2y + x + x^2} &= \frac{dy}{dx}
\end{aligned}$$

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

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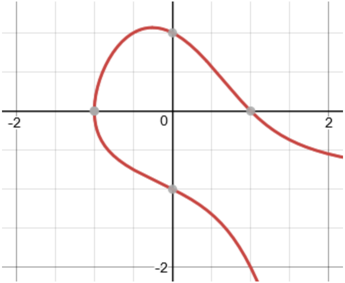
Necessarily, $\frac{\mathrm{d}y}{\mathrm{d}x}$ depends on **both** y and x. Why?



ans

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$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(1,0)} =$$

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(1,-2)} =$$

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

Necessarily, $\frac{\mathrm{d}y}{\mathrm{d}x}$ depends on **both** y and x. Why?

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(1,0)} = -\frac{2(1) + 0 + 2(1)(0)}{2(0) + 1 + 1} = -\frac{2}{2} = -1$$

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(1,-2)} = -\frac{2(1) - 2 + 2(1)(-2)}{2(-2) + 1 + 1} = -2$$

$$y^2 + x^2 + xy + x^2y = 1$$

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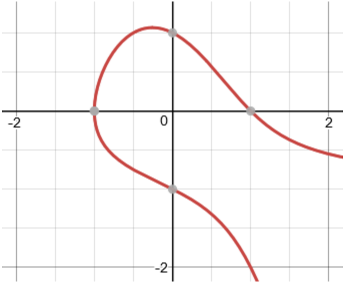
Points with the same x-value may have different slopes. We need both the x-value and the y-value to figure out which point we're talking about.

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x + y + 2xy}{2y + x + x^2}$$



Points with the same x-value may have different slopes. We need both the x-value and the y-value to figure out which point we're talking about.

NOW YOU

Suppose $x^4y + y^4x = 2$. Find $\frac{dy}{dx}$ at the point $(1, 1)$.

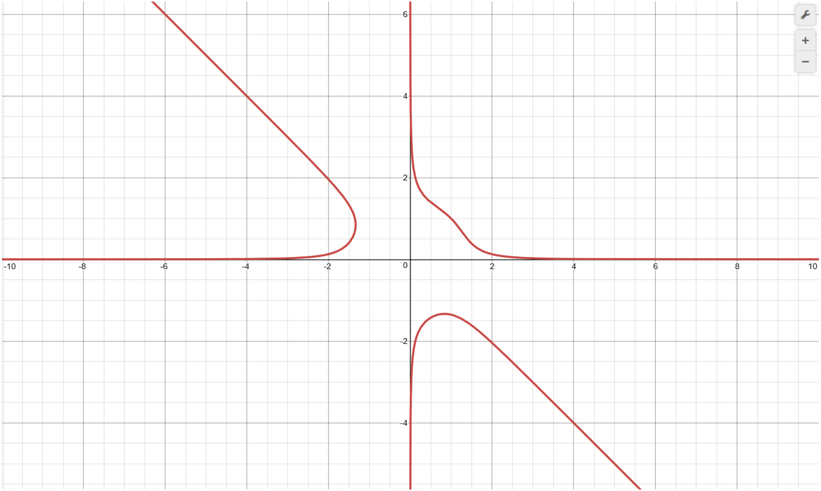
Now You

Suppose $x^4y + y^4x = 2$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point $(1, 1)$.

$$\begin{aligned} x^4 y(x) + y(x)^4 x &= 2 \\ 4x^3 y(x) + x^4 \frac{\mathrm{d}y}{\mathrm{d}x}(x) + y(x)^4 + 4y(x)^3 \frac{\mathrm{d}y}{\mathrm{d}x}(x)\, x &= 0 \end{aligned}$$

We may only replace variables with constants *after* differentiating. When $x = 1$ and $y(1) = 1$,

$$\begin{aligned} 4(1)^3 y(1) + (1)^4 \frac{\mathrm{d}y}{\mathrm{d}x}(1) + y(1)^4 + 4y(1)^3 \frac{\mathrm{d}y}{\mathrm{d}x}(1) &= 0 \\ 4 + \frac{\mathrm{d}y}{\mathrm{d}x}(1) + 1 + 4\frac{\mathrm{d}y}{\mathrm{d}x}(1) &= 0 \\ 5\frac{\mathrm{d}y}{\mathrm{d}x}(1) &= -5 \\ \frac{\mathrm{d}y}{\mathrm{d}x}(1) &= -1 \end{aligned}$$



$$x^4y + y^4x = 2$$

Now You Suppose $\dfrac{3y^2+2y+y^3}{x^2+1}=x$. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $x=0$, and the equations of the associated tangent line(s).

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To avoid the quotient rule, we start by simplifying our expression.

$$\frac{3y(x)^2+2y(x)+y(x)^3}{x^2+1}=x$$
$$3y(x)^2+2y(x)+y(x)^3=x^3+x$$
$$6y(x)\frac{\mathrm{d}y}{\mathrm{d}x}(x)+2\frac{\mathrm{d}y}{\mathrm{d}x}(x)+3y(x)^2\frac{\mathrm{d}y}{\mathrm{d}x}(x)=3x^2+1$$

When $x=0$:

$$\frac{\mathrm{d}y}{\mathrm{d}x}(0)=\frac{1}{6y(0)+2+3y(0)^2}$$

We need to know y to find $\frac{\mathrm{d}y}{\mathrm{d}x}$. We want all points where $x = 0$.

$$\begin{aligned} 3y(0)^2 + 2y(0) + y(0)^3 &= 0 \\ y(0)\left(y(0)^2 + 3y(0) + 2\right) &= 0 \\ y(0)\left(y(0)+1\right)\left(y(0)+2\right) &= 0 \\ y(0) = 0,\ y(0) = -1,\ y(0) &= -2 \end{aligned}$$

$$\frac{dy}{dx} = \frac{1}{6y+2+3y^2}$$

$(0, 0)$

$$\left.\frac{dy}{dx}\right|_{(0,0)} = \frac{1}{2}$$

$$y - 0 = \frac{1}{2}(x-0)$$

$$y = \frac{1}{2}x$$

$(0, -1)$

$$\left.\frac{dy}{dx}\right|_{(0,-1)} = \frac{1}{-6+2+3} = -1$$

$$y-(-1) = -1(x-0)$$

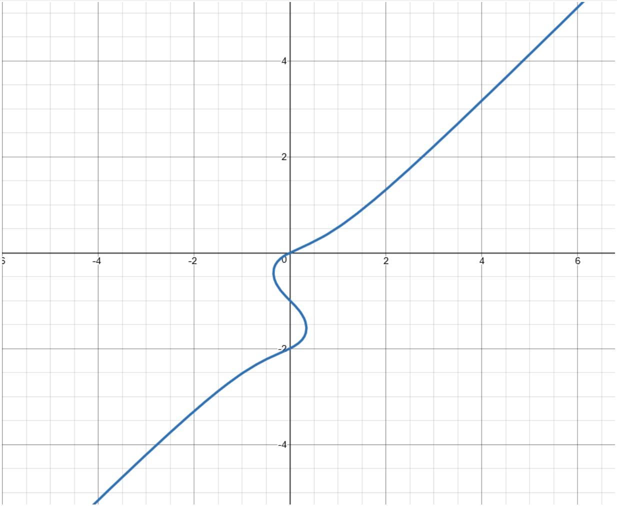
$$y = -x-1$$

$(0, -2)$

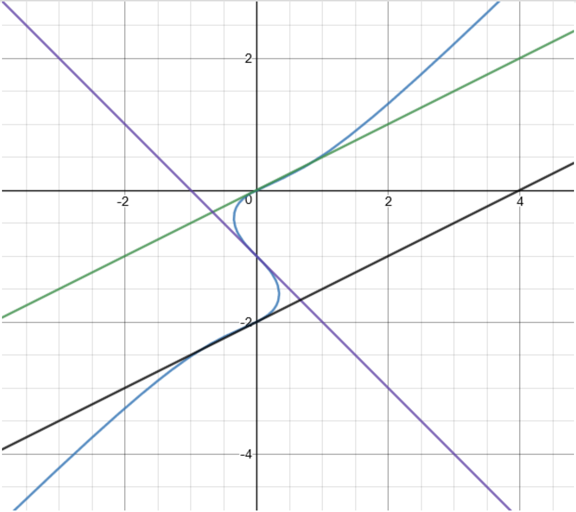
$$\left.\frac{dy}{dx}\right|_{(0,-2)} = \frac{1}{-12+2+12} = \frac{1}{2}$$

$$y-(-2) = \frac{1}{2}(x-0)$$

$$y = \frac{1}{2}x - 2$$



$$\frac{3y^2 + 2y + y^3}{x^2 + 1} = x$$



$$\frac{3y^2 + 2y + y^3}{x^2 + 1} = x$$

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

ans

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

$$\log x = y(x)$$
$$x = e^{y(x)}$$

ans

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

$$\begin{aligned} \log x &= y(x) \\ x &= e^{y(x)} \\ 1 &= e^{y(x)} \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\ \frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{1}{e^{y(x)}} = \frac{1}{x} \end{aligned}$$

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

$$\begin{aligned} \log x &= y(x) \\ x &= e^{y(x)} \\ 1 &= e^{y(x)} \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\ \frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{1}{e^{y(x)}} = \frac{1}{x} \end{aligned}$$

Use implicit differentiation to differentiate $\log|x|$, $x < 0$.

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

Use implicit differentiation to differentiate $\log|x|$, $x < 0$.

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

Use implicit differentiation to differentiate $\log|x|$, $x < 0$.

$$\begin{aligned}
\log|x| &= y(x) \\
\log(-x) &= y(x) \\
-x &= e^{y(x)} \\
-1 = e^{y(x)} \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) & \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{-1}{e^{y(x)}} = \frac{-1}{-x} = \frac{1}{x}
\end{aligned}$$

Use implicit differentiation to differentiate $\log_a(x)$, where $a > 0$ is a constant and $x > 0$.

Use implicit differentiation to differentiate $\log_a |x|$, $a > 0$.

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$$\begin{aligned}
\log_a x &= y(x) \\
x &= a^{y(x)} \\
1 &= a^{y(x)} \cdot \log_e a \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{1}{a^{y(x)} \cdot \log_e a} = \frac{1}{x \log_e a}
\end{aligned}$$

Use implicit differentiation to differentiate $\log_a |x|$, $a > 0$.

Use implicit differentiation to differentiate $\log_a |x|$, $a > 0$.

If $x > 0$, it's what we just computed. So assume $x < 0$.

$$\begin{aligned}
\log_a |x| &= y(x) \\
\log_a(-x) &= y(x) \\
-x &= a^{y(x)} \\
-1 &= a^{y(x)} \cdot \log_e a \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{-1}{a^{y(x)} \cdot \log_e a} = \frac{-1}{x \log_e a}
\end{aligned}$$

Included Work

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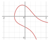
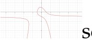
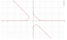
 screenshot of graph using Desmos Graphing Calculator, https://www.desmos.com/calculator (accessed 19 October 2017), 18, 22

 screenshot of graph using Desmos Graphing Calculator, https://www.desmos.com/calculator (accessed 19 October 2017), 7–9

 screenshot of graph using Desmos Graphing Calculator, https://www.desmos.com/calculator (accessed 24 October 2018), 25

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