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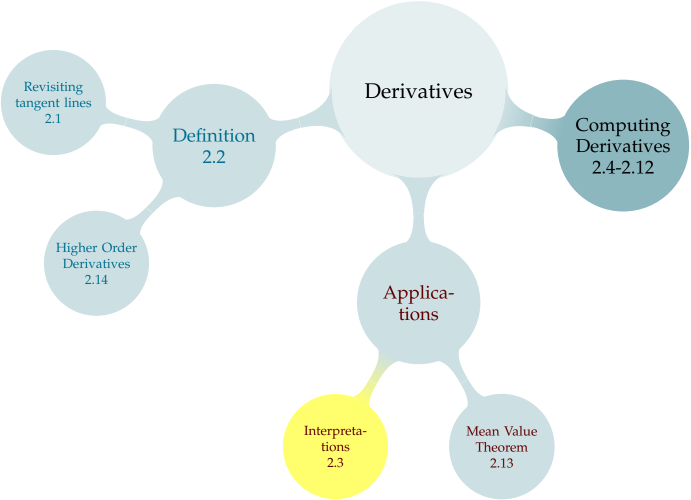

Interpreting the Derivative

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At midnight of January 1, 2012, the world population was increasing at a rate of 156 people each minute

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How fast your profit is increasing as you sell more widgets, measured in dollars per widget, at the time you sell Widget #100. So, roughly the profit earned from the sale of the 101st widget.

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Suppose $h(t)$ gives the height of a rocket t seconds after liftoff. What is the interpretation of $h'(t)$?

The speed at which the rocket is rising at time t.

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The rate (measured in molecules per second) at which the number of molecules of a certain type is changing. Roughly, how many molecules of that type are being added (or taken away, if negative) per second at time t.

A

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Suppose $G(w)$ gives the diameter in millimetres of steel wire needed to safely support a load of w kg. Suppose further that $G'(100) = 0.01$. How do you interpret $G'(100) = 0.01$?

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When your load is about 100 kg, you need to increase the diameter of your wire by about 0.01 mm for each kg increase in your load.

A paper[1] on the impacts of various factors in average life expectancy contains the following:

> *The only statistically significant variable in the model is physician density. The coefficient for this variable 20.67 indicating that a one unit increase in physician density leads to a 20.67 unit increase in life expectancy. This variable is also statistically significant at the 1% level demonstrating that this variable is very strongly and positively correlated with quality of healthcare received. This denotes that access to healthcare is very impactful in terms of increasing the quality of health in the country.*

[1] Natasha Deshpande, Anoosha Kumar, Rohini Ramaswami, *The Effect of National Healthcare Expenditure on Life Expectancy*, page 12.
Remark: physician density is measured as number of doctors per 1000 members of the population.

If $L(p)$ is the average life expectancy in an area with a density p of physicians, write the statement as a derivative: "a one unit increase in physician density leads to a 20.67 unit increase in life expectancy."

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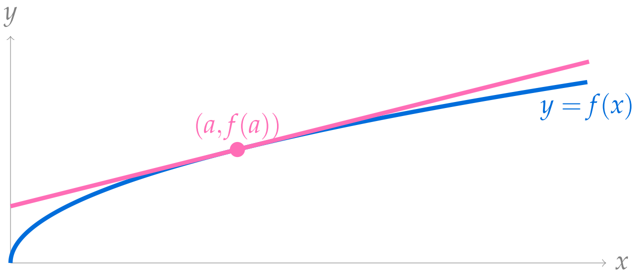
$L'(p) = 20.67$

Equation of the Tangent Line

The tangent line to $f(x)$ at a has slope $f'(a)$ and passes through the point $(a, f(a))$.



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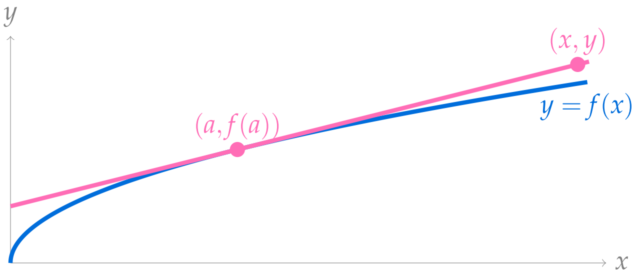

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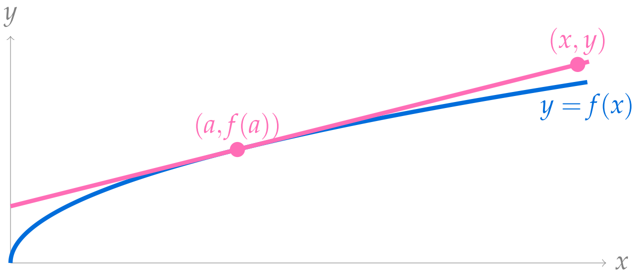

Slope of the tangent line:

$$f'(a) = \frac{\text{rise}}{\text{run}} = \frac{y - f(a)}{x - a}$$

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Rearranging: $y - f(a) = f'(a)(x - a)$ (equation of tangent line)

Tangent Line Equation – Theorem 2.3.2

The tangent line to the function $f(x)$ at point a is:

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Find the equation of the tangent line to the curve $f(x) = \sqrt{x}$ at $x = 9$. (Recall $\frac{\mathrm{d}}{\mathrm{d}x}\left[\sqrt{x}\right] = \frac{1}{2\sqrt{x}}$).

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$$a = 9, \quad f(a) = 3, \quad f'(a) = \frac{1}{2\sqrt{9}} = \frac{1}{6}$$

$$y - 3 = \frac{1}{6}(x - 9)$$

Memorize

The tangent line to the function $f(x)$ at point a is:

$$(y - f(a)) = f'(a)(x - a)$$

Now You

Let $s(t) = 3 - 0.8t^2$. Then $s'(t) = -1.6t$. Find the equation for the tangent line to the function $s(t)$ when $t = 1$.

ans

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$$a = 1, \quad s(a) = 2.2, \quad s'(a) = -1.6$$

$$y - 2.2 = -1.6(x - 1)$$

Included Work

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Natasha Deshpande, Anoosha Kumar, Rohini Ramaswami. (2014). The Effect of National Healthcare Expenditure on Life Expectancy, page 12. *College of Liberal Arts - Ivan Allen College (IAC), School of Economics: Econometric Analysis Undergraduate Research Papers.* https://smartech.gatech.edu/handle/1853/51648 (accessed July 2021), 13