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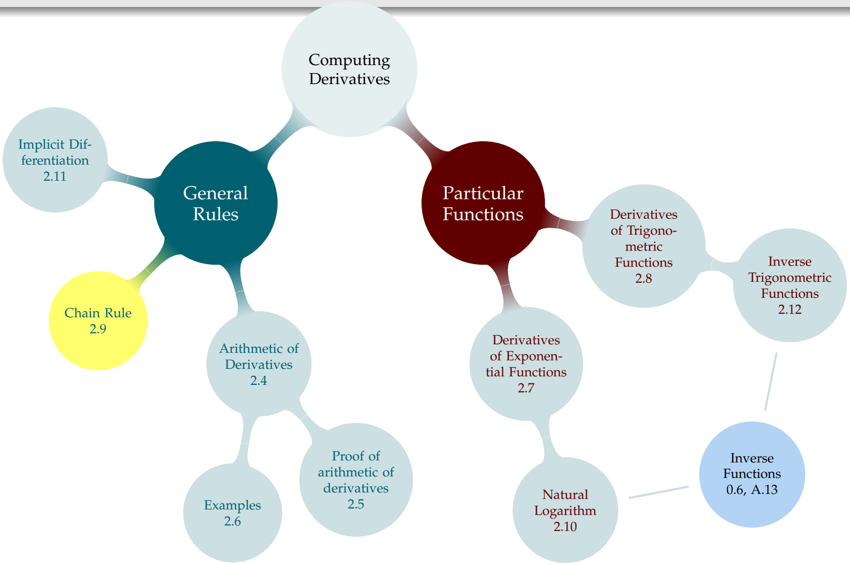
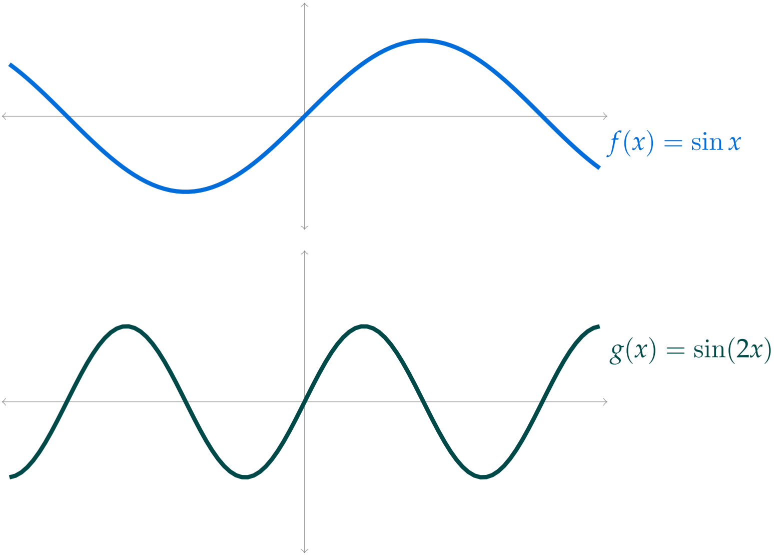


INTUITION: $\sin x$ VERSUS $\sin(2x)$



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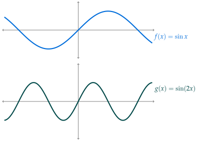

Intuition: sin 2x changes its y-values "twice as fast" as sin x, making it "twice as steep." So it's not enough to differentiate the outside function – something else has to happen.

Compound Functions

Video: 2:27-3:50

Morton, Jennifer. (2014). *Balancing Act: Otters, Urchins and Kelp*. Available from https://www.kqed.org/quest/67124/balancing-act-otters-urchins-and-kelp

Chain rule works on functions-of-functions; otters/urchins/kelp are a nice example

Compound Functions

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Kelp Population

k	kelp population
u	urchin population
o	otter population
p	public policy

$k(u)$ $\qquad$ $k(u(o))$ $\qquad$ $k(u(o(p)))$

These are examples of compound functions.

Should $\frac{\mathrm{d}}{\mathrm{d}o}k\big(u(o)\big)$ be positive or negative?
A. positive $\qquad$ B. negative $\qquad$ C. I'm not sure

Should $k'(u)$ be positive or negative?
A. positive $\qquad$ B. negative $\qquad$ C. I'm not sure

Kelp Population

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These are examples of compound functions.

Should $\frac{d}{do}k(u(o))$ be positive or negative?
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Should $k'(u)$ be positive or negative?
A. positive B. negative C. I'm not sure

It's nice to show an example of a function whose "derivative" can be two different things (depending on the variable). Now that our heads are in "function of a function" territory, chain rule. I usually just flash this slide to emphasize that all these rules are shorthand for a calculation using the definition of a derivative.

DIFFERENTIATING COMPOUND FUNCTIONS

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\{f(g(x))\} &= \lim_{h\to 0}\frac{f(g(x+h))-f(g(x))}{h} \\
&= \lim_{h\to 0}\frac{f(g(x+h))-f(g(x))}{h}\left(\frac{g(x+h)-g(x)}{g(x+h)-g(x)}\right) \\
&= \lim_{h\to 0}\frac{f(g(x+h))-f(g(x))}{g(x+h)-g(x)}\cdot\frac{g(x+h)-g(x)}{h} \\
&= \lim_{h\to 0}\frac{f(g(x+h))-f(g(x))}{g(x+h)-g(x)}\cdot\lim_{h\to 0}\frac{g(x+h)-g(x)}{h} \\
&= \lim_{h\to 0}\frac{f\left(\boxed{g(x+h)}\right)-f\left(\boxed{g(x)}\right)}{\boxed{g(x+h)}-\boxed{g(x)}}\cdot g'(x)
\end{aligned}
$$

Set $H = g(x+h) - g(x)$. As $h \to 0$, we also have $H \to 0$. So

$$
\begin{aligned}
&= \lim_{H\to 0}\frac{f(g(x)+H)-f(g(x))}{H}\cdot g'(x) \\
&= f'(g(x))\cdot g'(x)
\end{aligned}
$$

Chain Rule

Chain Rule – Theorem 2.9.3

Suppose f and g are differentiable functions. Then

$$\frac{\mathrm{d}}{\mathrm{d}x}\{f(g(x))\} = f'(g(x))\, g'(x) = \frac{\mathrm{d}f}{\mathrm{d}g}(g(x))\frac{\mathrm{d}g}{\mathrm{d}x}(x)$$

In the case of kelp, $\frac{\mathrm{d}}{\mathrm{d}o}k(u(o)) = \frac{\mathrm{d}k}{\mathrm{d}u}(u(o))\frac{\mathrm{d}u}{\mathrm{d}o}(o)$

Chain Rule

Suppose f and g are differentiable functions. Then

$$\frac{\mathrm{d}}{\mathrm{d}x}\{f\left(g(x)\right)\} = f'\left(g(x)\right) g'(x) = \frac{\mathrm{d}f}{\mathrm{d}g}\left(g(x)\right)\frac{\mathrm{d}g}{\mathrm{d}x}(x)$$

Example: suppose $F(x) = \sin(e^x + x^2)$.

Chain Rule

Suppose f and g are differentiable functions. Then

$$\frac{\mathrm{d}}{\mathrm{d}x}\{f(g(x))\} = f'(g(x))\,g'(x) = \frac{\mathrm{d}f}{\mathrm{d}g}(g(x))\,\frac{\mathrm{d}g}{\mathrm{d}x}(x)$$

Example: suppose $F(x) = \sin(e^x + x^2)$.

I generally put the “inside” function in a box, to emphasize we’re treating the whole thing as one variable

$$F(v) = \left(\frac{v}{v^3+1}\right)^6$$

Now You Let $f(x) = (10^x + \csc x)^{1/2}$. Find $f'(x)$.

Now You Suppose $o(t) = e^t$, $u(o) = \frac{1}{o+\sin(o)}$, and $t \geq 10$ (so all these functions are defined). Using the chain rule, find $\frac{\mathrm{d}}{\mathrm{d}t} u\big(o(t)\big)$.
Note: your answer should depend only on t: not o.

Evaluate $\dfrac{\mathrm{d}}{\mathrm{d}x}\left\{x^2+\sec\left(x^2+\dfrac{1}{x}\right)\right\}$

Evaluate $\dfrac{\mathrm{d}}{\mathrm{d}x}\left\{\dfrac{1}{x+\dfrac{1}{x+\frac{1}{x}}}\right\}$

Included Work

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