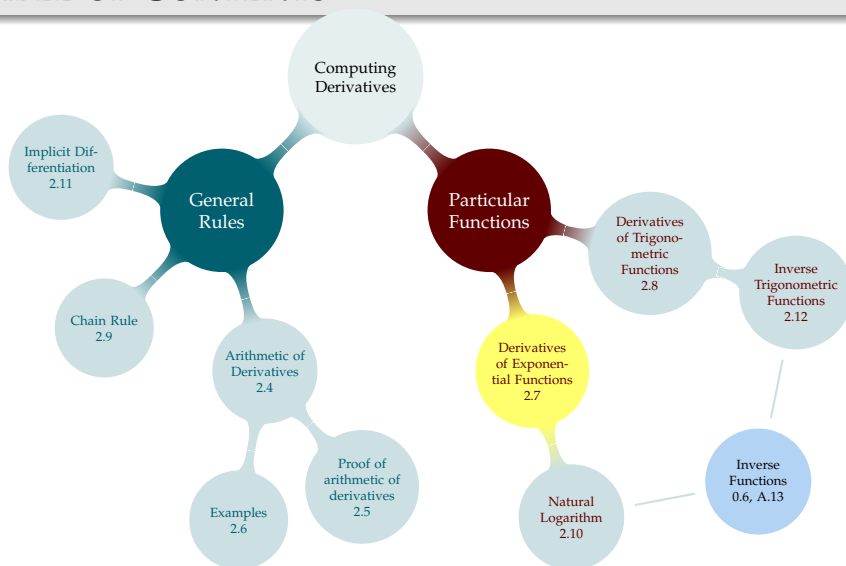
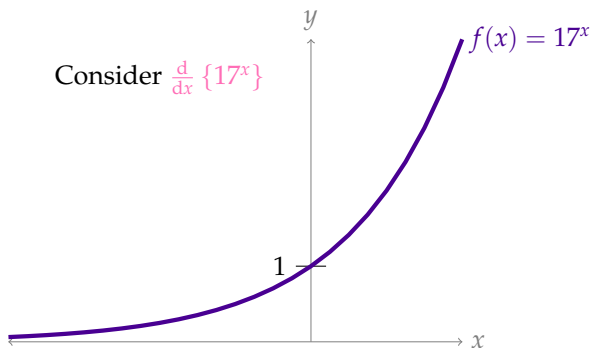


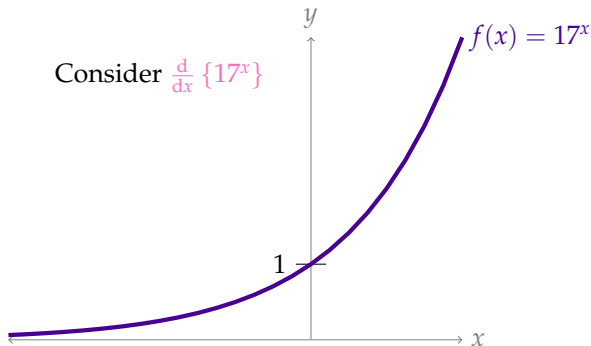
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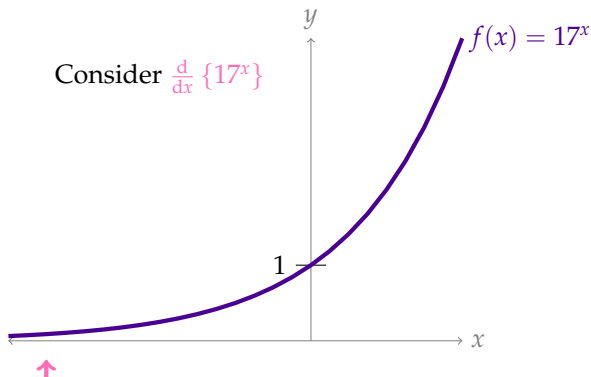


EXPONENTIAL FUNCTIONS



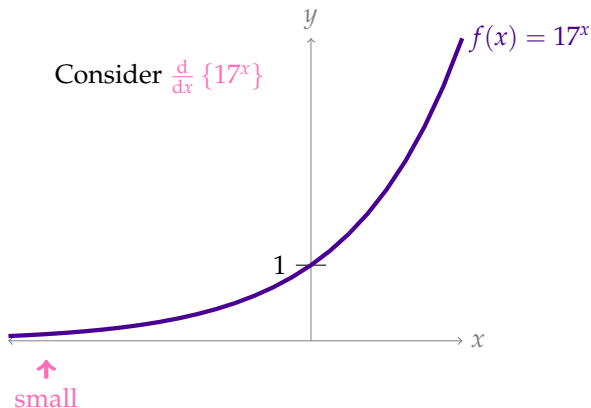
$f(x)$ is always increasing, so $f'(x)$ is always positive.

EXPONENTIAL FUNCTIONS



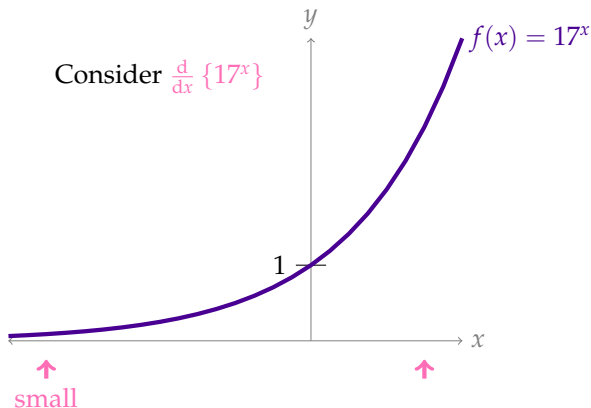
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EXPONENTIAL FUNCTIONS



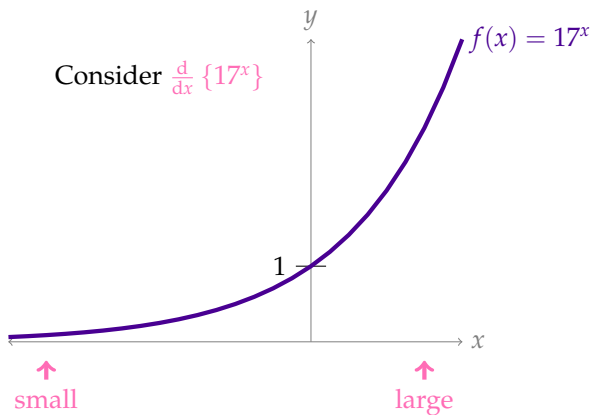
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EXPONENTIAL FUNCTIONS



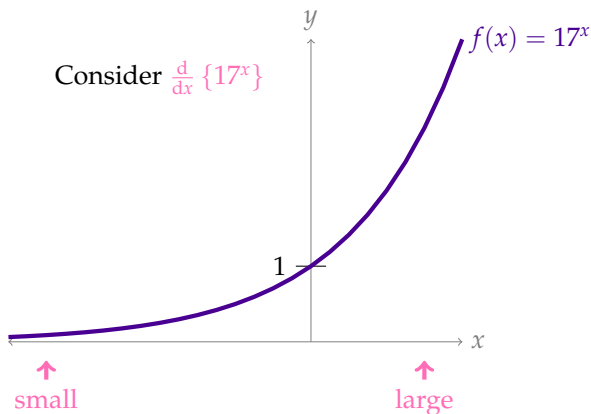
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EXPONENTIAL FUNCTIONS



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EXPONENTIAL FUNCTIONS



$f(x)$ is always increasing, so $f'(x)$ is always positive.
 $f'(x)$ might look similar to $f(x)$.

EXPONENTIAL FUNCTIONS

$$\frac{d}{dx}\{17^x\} =$$

EXPONENTIAL FUNCTIONS

$$\begin{aligned}\frac{d}{dx}\{17^x\} &= \lim_{h \rightarrow 0} \frac{17^{x+h} - 17^x}{h} \\ &= \lim_{h \rightarrow 0} \frac{17^x 17^h - 17^x}{h} \\ &= \lim_{h \rightarrow 0} \frac{17^x(17^h - 1)}{h} \\ &= 17^x \lim_{h \rightarrow 0} \frac{(17^h - 1)}{h} \\ &= 17^x (\text{ times a constant })\end{aligned}$$

$$\frac{d}{dx}\{17^x\} = 17^x \cdot \underbrace{\lim_{h \rightarrow 0} \frac{(17^h - 1)}{h}}_{\text{constant}}$$

Given what you know about $\frac{d}{dx}\{17^x\}$, **is it possible** that

$$\lim_{h \rightarrow 0} \frac{17^h - 1}{h} = 0?$$

- A. Sure, there's no reason we've seen that would make it impossible.
- B. No, it couldn't be 0, that wouldn't make sense.
- C. I do not feel equipped to answer this question.

$$\frac{d}{dx}\{17^x\} = 17^x \cdot \underbrace{\lim_{h \rightarrow 0} \frac{(17^h - 1)}{h}}_{\text{constant}}$$

Given what you know about $\frac{d}{dx}\{17^x\}$, **is it possible** that

$$\lim_{h \rightarrow 0} \frac{17^h - 1}{h} = \infty?$$

- A. Sure, there's no reason we've seen that would make it impossible.
- B. No, it couldn't be ∞ , that wouldn't make sense.
- C. I do not feel equipped to answer this question.

$$\frac{d}{dx}\{17^x\} = 17^x \cdot \underbrace{\lim_{h \rightarrow 0} \frac{(17^h - 1)}{h}}_{\text{constant}}$$

How could we find out what $\lim_{h \rightarrow 0} \frac{(17^h - 1)}{h}$ is?

$$\frac{d}{dx}\{17^x\} = 17^x \cdot \underbrace{\lim_{h \rightarrow 0} \frac{(17^h - 1)}{h}}_{\text{constant}}$$

How could we find out what $\lim_{h \rightarrow 0} \frac{(17^h - 1)}{h}$ is?

h	$\frac{17^h - 1}{h}$
0.001	2.83723068608
0.00001	2.83325347992
0.0000001	2.83321374583
0.000000001	2.83321344163

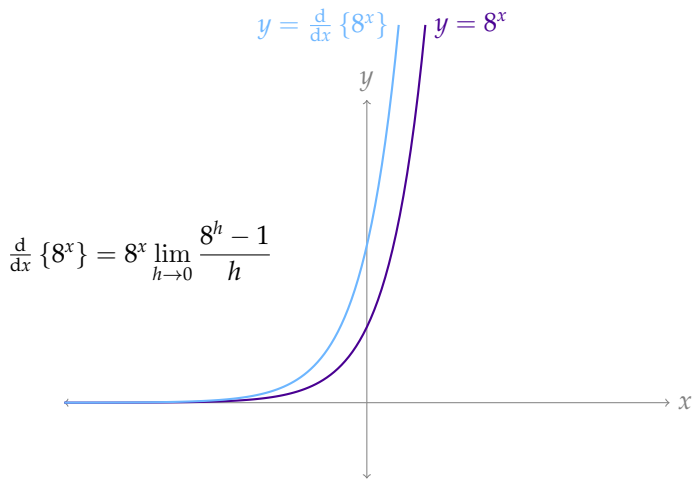
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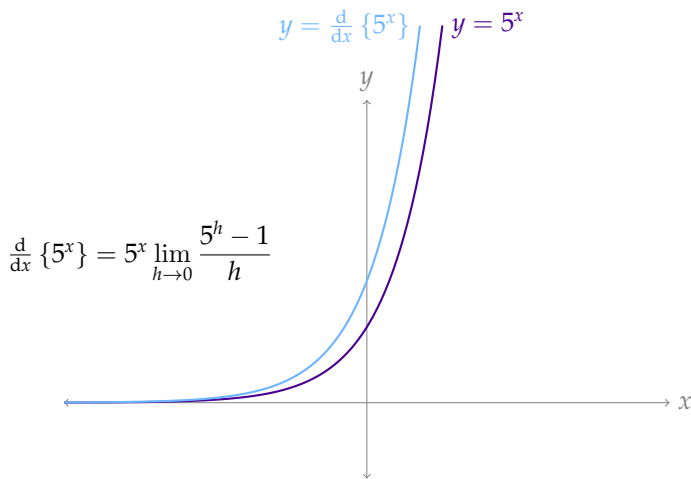
In general, for any positive number a ,

$$\frac{d}{dx}\{a^x\} = a^x \lim_{h \rightarrow 0} \frac{a^h - 1}{h}$$

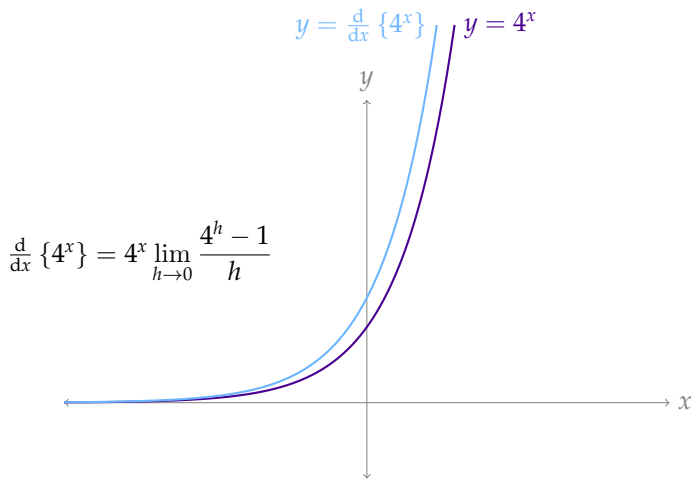
EXPONENTIAL FUNCTIONS



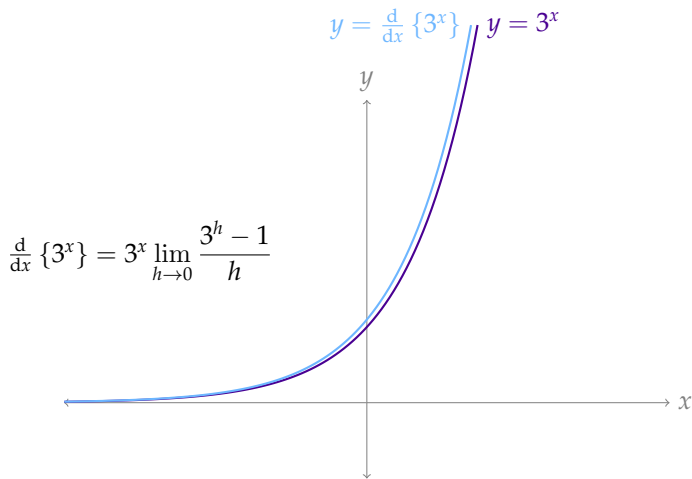
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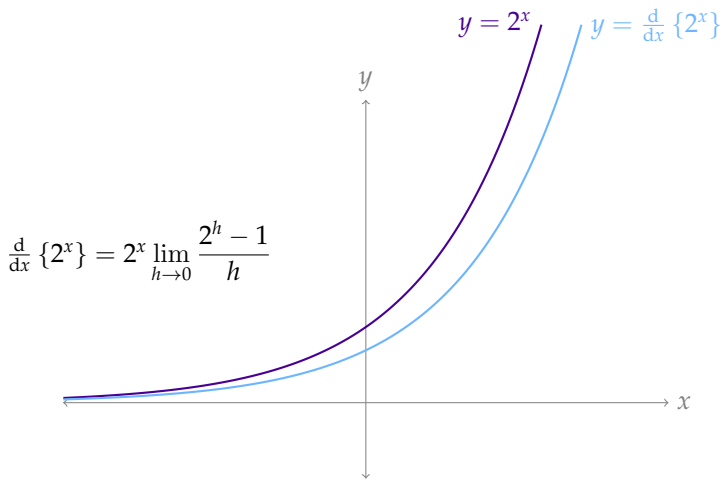
EXPONENTIAL FUNCTIONS



EXPONENTIAL FUNCTIONS



EXPONENTIAL FUNCTIONS



In general, for any positive number a , $\frac{d}{dx}\{a^x\} = a^x \lim_{h \rightarrow 0} \frac{a^h - 1}{h}$

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Euler's Number – Theorem 2.7.4

We define e to be the unique number satisfying

$$\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$$

In general, for any positive number a , $\frac{d}{dx}\{a^x\} = a^x \lim_{h \rightarrow 0} \frac{a^h - 1}{h}$

Euler's Number – Theorem 2.7.4

We define e to be the unique number satisfying

$$\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$$

$e \approx 2.7182818284590452353602874713526624\dots$ (Wikipedia)

Theorem 2.7.4 and Corollary 2.10.6

Using this definition of e ,

$$\frac{d}{dx}\{e^x\} = e^x \underbrace{\lim_{h \rightarrow 0} \frac{e^h - 1}{h}}_1 = e^x$$

Theorem 2.7.4 and Corollary 2.10.6

Using this definition of e ,

$$\frac{d}{dx}\{e^x\} = e^x \underbrace{\lim_{h \rightarrow 0} \frac{e^h - 1}{h}}_1 = e^x$$

In general, $\lim_{h \rightarrow 0} \frac{a^h - 1}{h} = \log_e(a)$, so $\frac{d}{dx}\{a^x\} = a^x \log_e(a)$

That $\lim_{h \rightarrow 0} \frac{a^h - 1}{h} = \log_e(a)$ and $\frac{d}{dx}\{a^x\} = a^x \log_e(a)$ are consequences of

$$a^x = (e^{\log_e(a)})^x = e^{x \log_e(a)}$$

For the details, see the end of Section 2.7.

Things to Have Memorized

$$\frac{d}{dx} \{e^x\} = e^x$$

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Let $f(x) = \frac{e^x}{3x^5}$. When is the tangent line to $f(x)$ horizontal?

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Horizontal tangent line \Leftrightarrow slope of tangent line is zero $\Leftrightarrow f'(x) = 0$

$$0 = f'(x) = \frac{3x^5 e^x - e^x (15x^4)}{(3x^5)^2} = \left(\frac{e^x}{9x^{10}} \right) (3x^4) (x - 5)$$

$$x = 0 \text{ or } x = 5$$

But, since $f(x)$ is not defined at zero, the tangent line is only horizontal at

$$x = 5$$

Evaluate $\frac{d}{dx} \{e^{3x}\}$

Suppose the deficit, in millions, of a fictitious country is given by

$$f(x) = e^x(4x^3 - 12x^2 + 14x - 4)$$

where x is the number of years since the current leader took office. Suppose the leader has been in power for exactly two years.

1. Is the deficit increasing or decreasing?

Suppose the deficit, in millions, of a fictitious country is given by

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1. Is the deficit increasing or decreasing?
2. Is the rate at which the deficit is growing increasing or decreasing?

Included Work



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