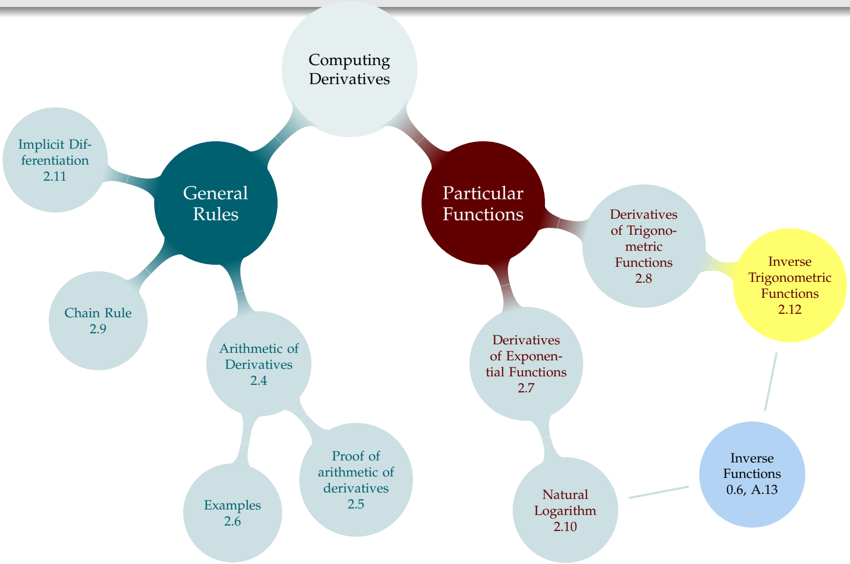
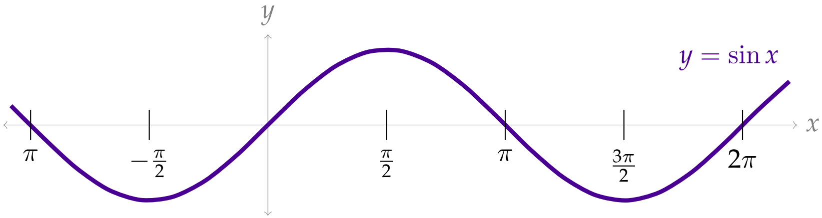
Table of Contents



INVERTIBILITY GAME



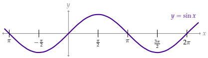
I'm thinking of a number x. Your hint: $\sin(x) = 0$. What number am I thinking of?

I'm thinking of a number x, and x is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Your hint: $\sin(x) = 0$. What number am I thinking of?

2022-03-10

2.12: Inverse Trig

Invertibility Game

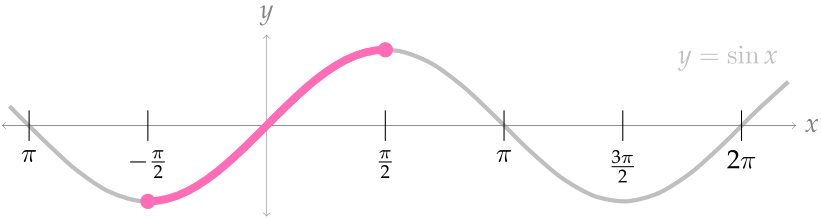


I'm thinking of a number x. Your hint: $\sin(x) = 0$. What number am I thinking of?

I'm thinking of a number x, and x is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Your hint: $\sin(x) = 0$. What number am I thinking of?

We want to invert sine, but it isn't invertible. Start with invertibility game to refresh memories.

Arcsine



$\arcsin(x)$ is the inverse of $\sin x$ restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\arcsin x$ is the (unique) number θ such that:

- $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, and
- $\sin \theta = x$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0)$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right)$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right)$
- $\arcsin\left(\frac{\pi}{2}\right)$
- $\arcsin\left(\frac{\pi}{4}\right)$

Arcsine

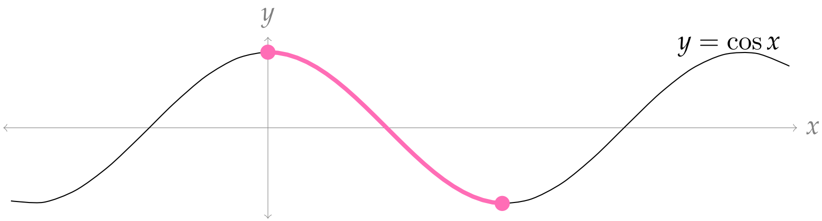
Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0)$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right)$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right)$
- $\arcsin\left(\frac{\pi}{2}\right)$
- $\arcsin\left(\frac{\pi}{4}\right)$

"These are the sines you should have memorized, or be able to figure out from reference triangles." "Remember sine is an odd function" Make clear they won't have to figure out $\arcsin(\pi/4)$. (In second-semester calculus, they will learn how to approximate it numerically.)

ARCCOSINE

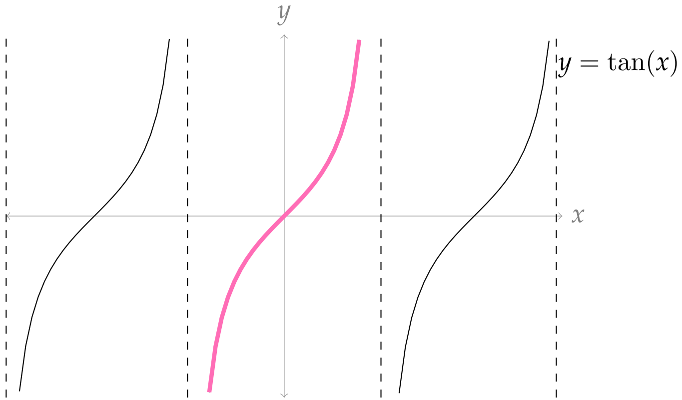


$\arccos(x)$ is the inverse of $\cos x$ restricted to $[0, \pi]$.

$\arccos(x)$ is the (unique) number θ such that:

- $\cos(\theta) = x$ and
- $0 \leq \theta \leq \pi$

Arctangent



$\arctan(x) = \theta$ means:
(1) $\tan(\theta) = x$ and
(2) $-\pi/2 < \theta < \pi/2$

ARCSECANT, ARCSINE, AND ARCCOTANGENT

$$\mathrm{arcsec}(x) =$$

Arcsecant, arcsine, and arccotangent

$$\operatorname{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$

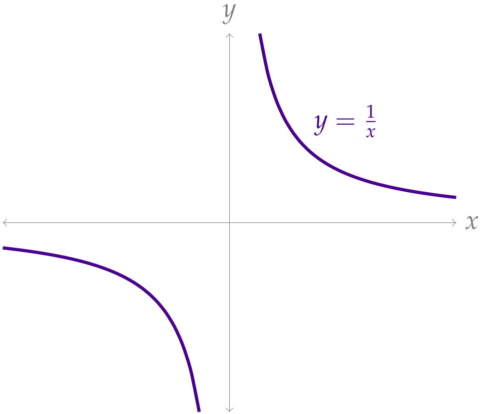
$$\begin{aligned}
\operatorname{arccsc}(x) &= y \\
\csc y &= x \\
\frac{1}{\sin y} &= x \\
\sin y &= \frac{1}{x} \\
y &= \arcsin\left(\tfrac{1}{x}\right) \\
\operatorname{arccsc}(x) &= \arcsin\left(\tfrac{1}{x}\right)
\end{aligned}$$

$$\operatorname{arccot}(\mathrm{x}) = \arctan\left(\frac{1}{x}\right)$$

$$\begin{aligned}
\operatorname{arccot}(\mathrm{x}) &= y \\
\cot y &= x \\
\frac{1}{\tan y} &= x \\
\tan y &= \tfrac{1}{x} \\
y &= \arctan\left(\tfrac{1}{x}\right) \\
\operatorname{arccot}(\mathrm{x}) &= \arctan\left(\tfrac{1}{x}\right)
\end{aligned}$$

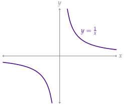
$$\text{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$$

The domain of $\arccos(y)$ is $-1 \le y \le 1$, so the domain of arcsec(y) is



$\text{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$

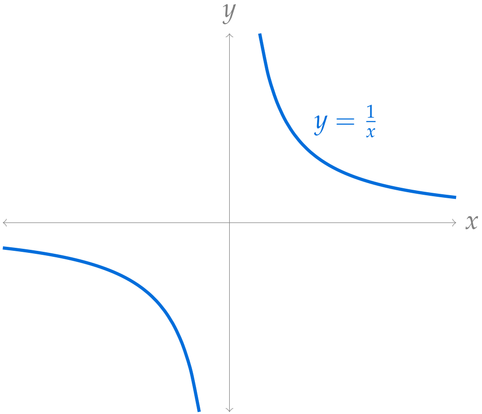
The domain of $\arccos(y)$ is $-1 \le y \le 1$, so the domain of $\text{arcsec}(y)$ is



This can be really hard for students to get the first time around. Concrete examples may help: "$\text{arcsec}(2) = \arccos(1/2)$, that's fine; $\text{arcsec}(1/2) = \arccos(2)$, that's not fine."

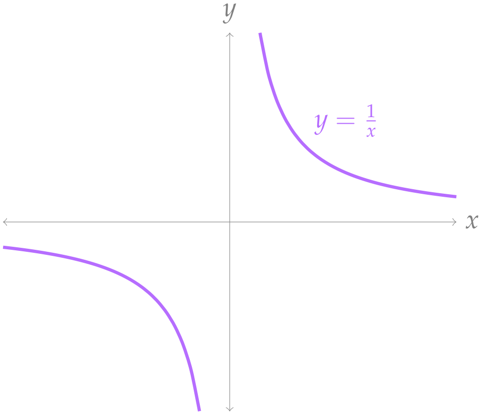
$$\operatorname{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$

Domain of $\arcsin(y)$ is $-1 \le y \le 1$, so the domain of $\operatorname{arccsc}(x)$ is



$$\text{arccot}(x) = \arctan\left(\frac{1}{x}\right)$$

Domain of $\arctan(x)$ is all real numbers, so the domain of arccot(x) is



$$y = \arcsin x$$

Find $\frac{dy}{dx}$.

$$y = \arctan x$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

$$y = \arccos x$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

To differentiate arcsecant, arccosecant, and arccotangent, you can use the chain rule!

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\operatorname{arccsc}(x)\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[\arcsin\left(\frac{1}{x}\right)\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[\arcsin\left(x^{-1}\right)\right]$$

Derivatives of Inverse Trigonometric Functions – Theorem 2.12.7

Memorize:

$$\frac{\mathrm{d}}{\mathrm{d}x}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\arccos x] = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\arcsin x] = \frac{1}{1+x^2}$$

Be able to derive:

$$\frac{\mathrm{d}}{\mathrm{d}x}[\operatorname{arccsc} x] = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\operatorname{arcsec} x] = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\operatorname{arccot} x] = -\frac{1}{1+x^2}$$