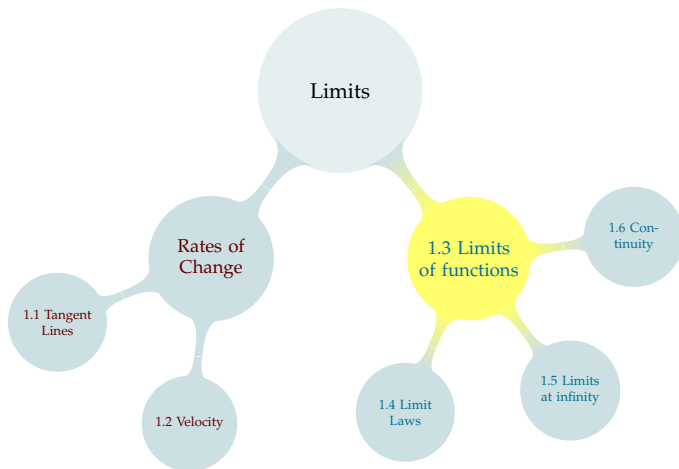


# TABLE OF CONTENTS



## Notation 1.3.1 and Definition 1.3.3

$$\lim_{x \rightarrow a} f(x) = L$$

where  $a$  and  $L$  are real numbers

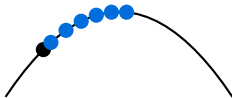
We read the above as “the limit as  $x$  goes to  $a$  of  $f(x)$  is  $L$ .”

Its meaning is: as  $x$  gets very close to (but not equal to)  $a$ ,  $f(x)$  gets very close to  $L$ .

# FINDING SLOPES OF TANGENT LINES



We NEED limits to find slopes of tangent lines.



Slope of secant line:  $\frac{\Delta y}{\Delta x}$ ,  $\Delta x \neq 0$ .

Slope of tangent line: can't do the same way.

If the position of an object at time  $t$  is given by  $s(t)$ , then its instantaneous velocity is given by

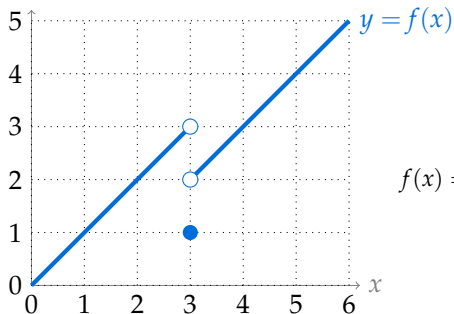
$$\lim_{h \rightarrow 0} \frac{s(t+h) - s(t)}{h}$$

# EVALUATING LIMITS

$$\text{Let } f(x) = \frac{x^3 + x^2 - x - 1}{x - 1}.$$

We want to evaluate  $\lim_{x \rightarrow 1} f(x)$ .

## ONE-SIDED LIMITS



$$f(x) = \begin{cases} x & \text{if } x < 3 \\ 1 & \text{if } x = 3 \\ x - 1 & \text{if } x > 3 \end{cases}$$

What do you think  $\lim_{x \rightarrow 3} f(x)$  should be?

### Definition 1.3.7

The limit as  $x$  goes to  $a$  **from the left** of  $f(x)$  is written

$$\lim_{x \rightarrow a^-} f(x)$$

We only consider values of  $x$  that are **less than**  $a$ .

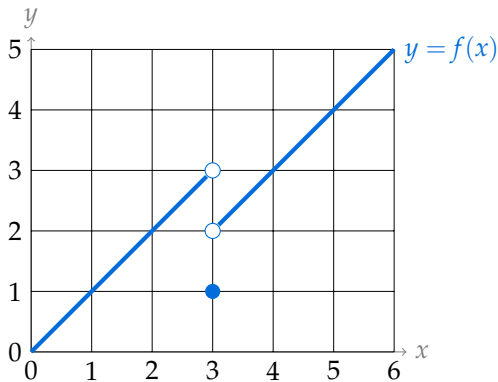
The limit as  $x$  goes to  $a$  **from the right** of  $f(x)$  is written

$$\lim_{x \rightarrow a^+} f(x)$$

We only consider values of  $x$  **greater than**  $a$ .

### Theorem 1.3.8

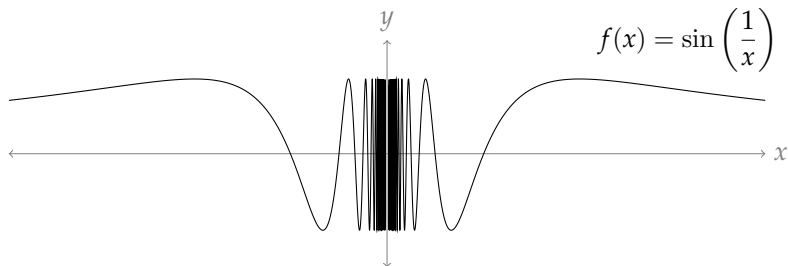
In order for  $\lim_{x \rightarrow a} f(x)$  to exist, both one-sided limits must exist and be equal.



Consider the function  $f(x) = \frac{1}{(x-1)^2}$ . For what value(s) of  $x$  is  $f(x)$  **not** defined?

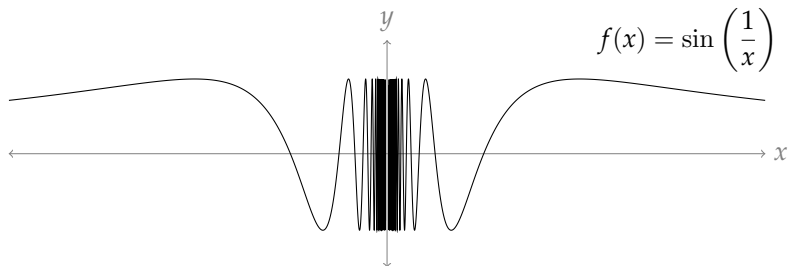


# A STRANGER LIMIT EXAMPLE



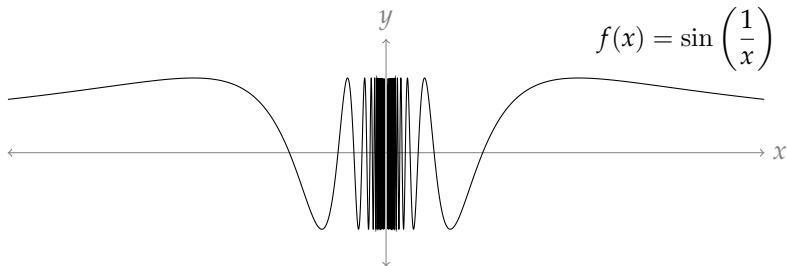
What is  $\lim_{x \rightarrow \infty} f(x)$ ?

# A STRANGER LIMIT EXAMPLE



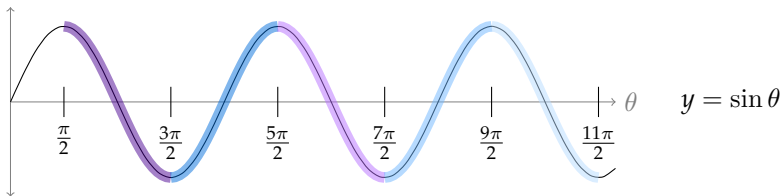
What is  $\lim_{x \rightarrow 0} f(x)$  ?

# A STRANGER LIMIT EXAMPLE



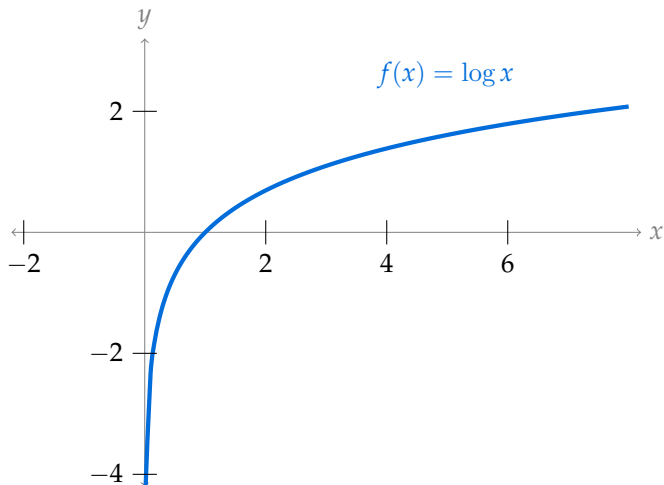
What is  $\lim_{x \rightarrow \pi} f(x)$  ?

# OPTIONAL: SKETCHING $f(x) = \sin\left(\frac{1}{x}\right)$



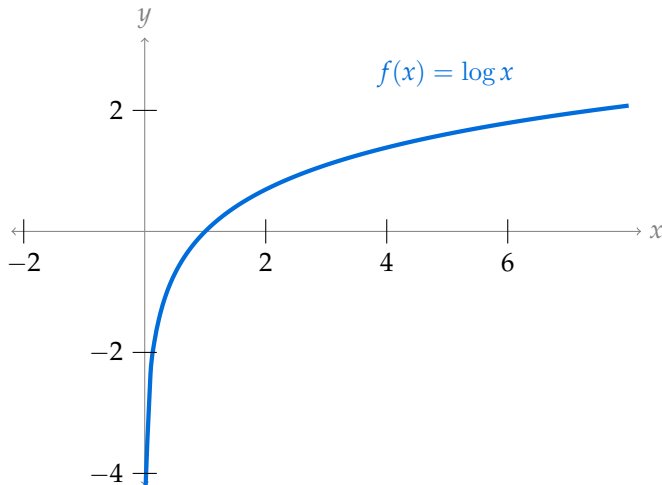
# LIMITS AND THE NATURAL LOGARITHM

Where is  $f(x)$  defined, and where is it not defined?

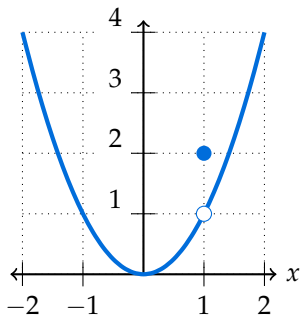


# LIMITS AND THE NATURAL LOGARITHM

What can you say about the limit of  $f(x)$  near 0?



$$f(x) = \begin{cases} x^2 & x \neq 1 \\ 2 & x = 1 \end{cases}$$



What is  $\lim_{x \rightarrow 1} f(x)$ ?

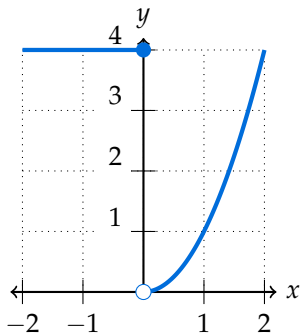
A.  $\lim_{x \rightarrow 1} f(x) = 2$

B.  $\lim_{x \rightarrow 1} f(x) = 1$

C.  $\lim_{x \rightarrow 1} f(x)$  DNE

D. none of the above

$$f(x) = \begin{cases} 4 & x \leq 0 \\ x^2 & x > 0 \end{cases}$$



What is  $\lim_{x \rightarrow 0} f(x)$ ? What is  $\lim_{x \rightarrow 0^+} f(x)$ ? What is  $f(0)$ ?

A.  $\lim_{x \rightarrow 0^+} f(x) = 4$

B.  $\lim_{x \rightarrow 0^+} f(x) = 0$

C.  $\lim_{x \rightarrow 0^+} f(x) = \begin{cases} 4 & x \leq 0 \\ 0 & x > 0 \end{cases}$

D. none of the above



Suppose  $\lim_{x \rightarrow 3^-} f(x) = 1$  and  $\lim_{x \rightarrow 3^+} f(x) = 1.5$ .

Does  $\lim_{x \rightarrow 3} f(x)$  exist?

- A. Yes, certainly, because the limits from both sides exist.
- B. No, never, because the limit from the left is not the same as the limit from the right.
- C. Can't tell. For some functions it might exist, for others not.

Suppose  $\lim_{x \rightarrow 3^-} f(x) = 22 = \lim_{x \rightarrow 3^+} f(x)$ .

Does  $\lim_{x \rightarrow 3} f(x)$  exist?

- A. Yes, certainly, because the limits from both sides exist and are equal to each other.
- B. No, never, because we only talk about one-sided limits when the actual limit doesn't exist.
- C. Can't tell. We need to know the value of the function at  $x = 3$ .

## Included Work



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