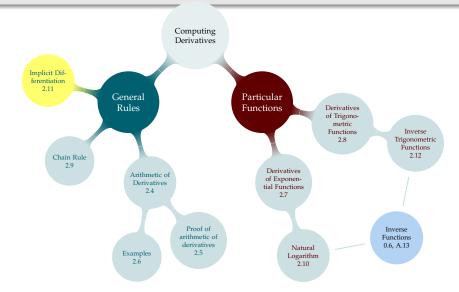
TABLE OF CONTENTS



IMPLICITLY DEFINED FUNCTIONS

$$y^2 + x^2 + xy + x^2y = 1$$

Which of the following points are on the curve? (0,1), (0,-1), (0,0), (1,1)

If
$$x = -3$$
, what is y ?

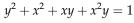
2.11: Implicit Diff

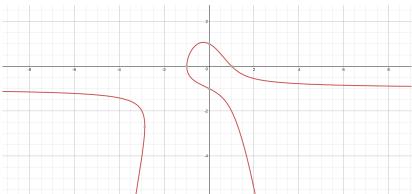
-Implicitly Defined Functions

$$\label{eq:matter} \begin{split} \overline{\text{IMPLICITLY DEFINED FUNCTIONS}} \\ y^2 + x^2 + yy + x^2y = 1 \\ \\ \text{Which of the following points are on the curve?} \\ &(0,1),(0,-1),(0,0),(1,1) \\ \\ \text{If } x = -\lambda \text{, what is } y^2 \end{split}$$

Things to emphasize:

- One x might have multiple y
- You won't be asked to graph these
- We can solve questions without needing to see their graphs
- Locally looks like a function (explain "locally") so all the stuff that worked before still works now if we restrict where we're looking





Still has a slope: $\frac{\Delta y}{\Delta x}$ **Locally**, *y* is still a function of *x*.

$$y^2 + x^2 + xy + x^2y = 1$$

Consider *y* as a function of *x*. Can we find $\frac{dy}{dx}$?

$$\frac{\mathrm{d}}{\mathrm{d}x}[y] =$$

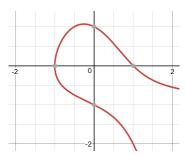
$$\frac{\mathrm{d}}{\mathrm{d}x}[y] = \frac{\mathrm{d}}{\mathrm{d}x}[x] =$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[1] =$$

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

Necessarily, $\frac{dy}{dx}$ depends on **both** y and x. Why?





Suppose $x^4y + y^4x = 2$. Find $\frac{dy}{dx}$ at the point (1,1).

Students often struggle knowing when to replace variables with constants.

Suppose
$$\frac{3y^2 + 2y + y^3}{x^2 + 1} = x$$
. Find $\frac{dy}{dx}$ when $x = 0$, and

the equations of the associated tangent line(s).

Use implicit differentiation to differentiate $\log(x)$, x > 0.

$$\log x = y(x)$$
$$x = e^{y(x)}$$

Use implicit differentiation to differentiate $\log |x|$, x < 0.

Use implicit differentiation to differentiate $\log_a(x)$, where a > 0 is a constant and x > 0.

Use implicit differentiation to differentiate $\log_a |x|$, a > 0.

Included Work

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screenshot of graph using Desmos Graphing Calculator, https://www.desmos.com/calculator (accessed 19 October 2017), 4