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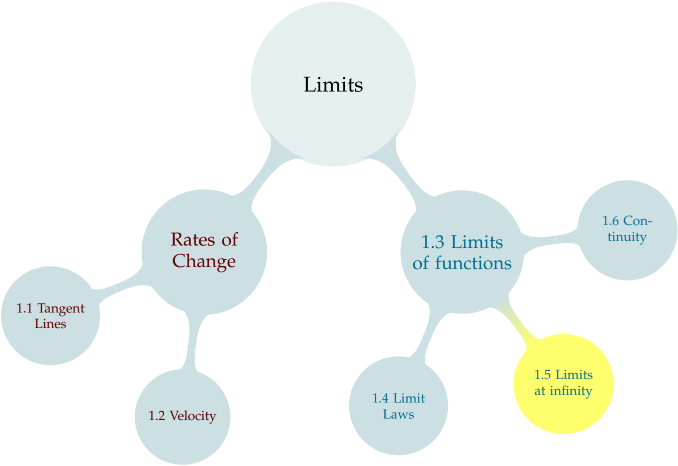


END BEHAVIOR

We write:

$$\lim_{x\to\infty} f(x) = L$$

to express that, as x grows larger and larger, $f(x)$ approaches L.

Similarly, we write:

$$\lim_{x\to-\infty} f(x) = L$$

to express that, as x grows more and more strongly negative, $f(x)$ approaches L.

End Behavior

We write:

$$\lim_{x\to\infty} f(x) = L$$

to express that, as x grows larger and larger, $f(x)$ approaches L.

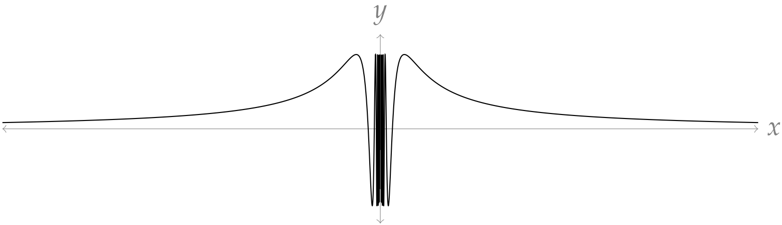
Similarly, we write:

$$\lim_{x\to-\infty} f(x) = L$$

to express that, as x grows more and more strongly negative, $f(x)$ approaches L.

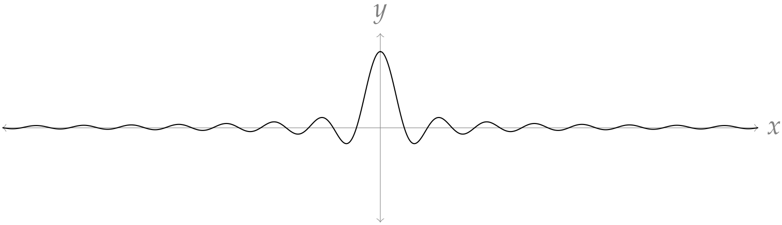
If L is a number, we call $y = L$ a horizontal asymptote of $f(x)$.

Horizontal Asymptotes



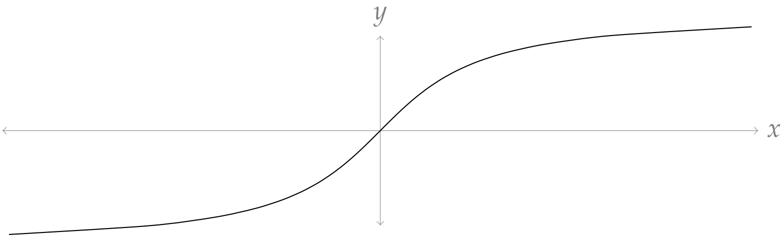
$y = 0$ is a horizontal asymptote for $y = \sin\left(\frac{1}{x}\right)$

Horizontal Asymptotes



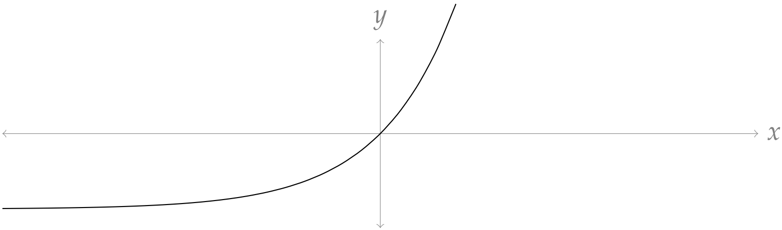
$y = 0$ is a horizontal asymptote for $y = \frac{\sin x}{x}$

Horizontal Asymptotes



$y = \frac{\pi}{2}$ and $y = -\frac{\pi}{2}$ are horizontal asymptotes for $y = \arctan(x)$

Horizontal Asymptotes



$y = -1$ is a horizontal asymptote for $y = e^x - 1$

Common Limits at Infinity

$$\lim_{x\to\infty} 13 =$$

$$\lim_{x\to-\infty} 13 =$$

$$\lim_{x\to\infty} x^3 =$$

$$\lim_{x\to-\infty} x^3 =$$

$$\lim_{x\to\infty} \frac{1}{x} =$$

$$\lim_{x\to-\infty} \frac{1}{x} =$$

$$\lim_{x\to-\infty} x^{5/3} =$$

$$\lim_{x\to-\infty} x^{2/3} =$$

$$\lim_{x\to\infty} x^2 =$$

$$\lim_{x\to-\infty} x^2 =$$

Q

ans

Common Limits at Infinity

$$\lim_{x\to\infty} 13 = 13$$
$$\lim_{x\to-\infty} 13 = 13$$

$$\lim_{x\to\infty} \frac{1}{x} = 0$$
$$\lim_{x\to-\infty} \frac{1}{x} = 0$$

$$\lim_{x\to\infty} x^2 = \infty$$
$$\lim_{x\to-\infty} x^2 = \infty$$

$$\lim_{x\to\infty} x^3 = \infty$$
$$\lim_{x\to-\infty} x^3 = -\infty$$

$$\lim_{x\to-\infty} x^{5/3} = -\infty$$
$$\lim_{x\to-\infty} x^{2/3} = \infty$$

Arithmetic with Limits at Infinity

Now You

$$\lim_{x\to\infty}\left(x+\frac{x^2}{10}\right)=$$

$$\lim_{x\to\infty}\left(x-\frac{x^2}{10}\right)=$$

$$\lim_{x\to-\infty}\left(x^2+x^3+x^4\right)=$$

$$\lim_{x\to-\infty}(x+13)\left(x^2+13\right)^{1/3}=$$

Arithmetic with Limits at Infinity

Now You

$$\lim_{x\to\infty}\left(x+\frac{x^2}{10}\right)=\infty$$

$$\lim_{x\to\infty}\left(x-\frac{x^2}{10}\right)=\lim_{x\to\infty} x\left(1-\frac{x}{10}\right)=-\infty$$

$$\lim_{x\to-\infty}\left(x^2+x^3+x^4\right)=\lim_{x\to-\infty} x^4\left(\frac{1}{x^2}+\frac{1}{x}+1\right)=\infty$$

$$\lim_{x\to-\infty}(x+13)\left(x^2+13\right)^{1/3}=-\infty$$

Calculating Limits at Infinity

$$\lim_{x\to\infty} \frac{x^2+2x+1}{x^3}$$

Q

ans

Calculating Limits at Infinity

$$\lim_{x\to\infty} \frac{x^2 + 2x + 1}{x^3}$$

Trick: factor out largest power of denominator.

Q

Calculating Limits at Infinity

$$\lim_{x\to\infty} \frac{x^2+2x+1}{x^3}$$

$$\lim_{x\to\infty} \frac{x^2+2x+1}{x^3} = \lim_{x\to\infty} \frac{x^2+2x+1}{x^3}\left(\frac{\frac{1}{x^3}}{\frac{1}{x^3}}\right)$$

$$= \lim_{x\to\infty} \frac{\frac{1}{x}+\frac{2}{x^2}+\frac{1}{x^3}}{1} = \frac{\lim\limits_{x\to\infty}\frac{1}{x}+\lim\limits_{x\to\infty}\frac{2}{x^2}+\lim\limits_{x\to\infty}\frac{1}{x^3}}{\lim\limits_{x\to\infty} 1}$$

$$= \frac{0+0+0}{1} = 0$$

Calculating Limits at Infinity

$$\lim_{x \to -\infty} \left(x^{7/3} - x^{5/3}\right)$$

Q

Calculating Limits at Infinity

$$\lim_{x \to -\infty} \left(x^{7/3} - x^{5/3}\right)$$

Again: factor out largest power of x.

Q

ans

Calculating Limits at Infinity

$$\lim_{x \to -\infty} (x^{7/3} - x^{5/3})$$

Again: factor out largest power of x.

$$(x^{7/3} - x^{5/3}) = x^{7/3}\left(1 - \frac{1}{x^{2/3}}\right)$$

$$\left(\text{note: } \lim_{x \to -\infty} x^{7/3} = -\infty\right)$$

$$\left(\text{note also: } \lim_{x \to -\infty} \left(1 - \frac{1}{x^{2/3}}\right) = 1\right)$$

So, $\lim_{x \to -\infty} (x^{7/3} - x^{5/3}) = -\infty$

Calculating Limits at Infinity

Suppose the height of a bouncing ball is given by $h(t) = \frac{\sin(t)+1}{t}$, for $t \geq 1$. What happens to the height over a long period of time?

Q

ans

Calculating Limits at Infinity

Suppose the height of a bouncing ball is given by $h(t) = \frac{\sin(t)+1}{t}$, for $t \geq 1$. What happens to the height over a long period of time?

$$\begin{array}{ccccc} 0 & \leq & \dfrac{\sin(t)+1}{t} & \leq & \dfrac{2}{t} \\ \lim\limits_{t\to\infty} 0 & = & 0 & = & \lim\limits_{t\to\infty} \dfrac{2}{t} \end{array}$$

So, by the Squeeze Theorem,

$$\lim_{t\to\infty} \frac{\sin(t)+1}{t} = 0$$

Calculating Limits at Infinity

Now You

$$\lim_{x\to\infty} \sqrt{x^4+x^2+1}-\sqrt{x^4+3x^2}$$

Calculating Limits at Infinity

Now You

$$\lim_{x\to\infty} \sqrt{x^4+x^2+1}-\sqrt{x^4+3x^2}$$

Multiply function by conjugate:

$$\left(\sqrt{x^4+x^2+1}-\sqrt{x^4+3x^2}\right)\left(\frac{\sqrt{x^4+x^2+1}+\sqrt{x^4+3x^2}}{\sqrt{x^4+x^2+1}+\sqrt{x^4+3x^2}}\right)$$

$$=\frac{-2x^2+1}{\sqrt{x^4+x^2+1}+\sqrt{x^4+3x^2}}$$

Calculating Limits at Infinity

Now You

$$\lim_{x\to\infty} \sqrt{x^4+x^2+1} - \sqrt{x^4+3x^2}$$

Multiply function by conjugate:

$$\left(\sqrt{x^4+x^2+1} - \sqrt{x^4+3x^2}\right)\left(\frac{\sqrt{x^4+x^2+1} + \sqrt{x^4+3x^2}}{\sqrt{x^4+x^2+1} + \sqrt{x^4+3x^2}}\right)$$

$$= \frac{-2x^2+1}{\sqrt{x^4+x^2+1} + \sqrt{x^4+3x^2}}$$

Factor out highest power: x^2 (same as $\sqrt{x^4}$)

$$\frac{-2x^2+1}{\sqrt{x^4+x^2+1} + \sqrt{x^4+3x^2}}\left(\frac{1/x^2}{1/\sqrt{x^4}}\right)$$

$$= \frac{-2+\frac{1}{x^2}}{\sqrt{1+\frac{1}{x^2}+\frac{1}{x^4}} + \sqrt{1+\frac{3}{x^2}}}$$

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Calculating Limits at Infinity

Multiply function by conjugate:

$$\left(\sqrt{x^4+x^2+1}-\sqrt{x^4+3x^2}\right)\left(\frac{\sqrt{x^4+x^2+1}+\sqrt{x^4+3x^2}}{\sqrt{x^4+x^2+1}+\sqrt{x^4+3x^2}}\right)$$
$$=\frac{-2x^2+1}{\sqrt{x^4+x^2+1}+\sqrt{x^4+3x^2}}$$

Factor out highest power: x^2 (same as $\sqrt{x^4}$)

$$\frac{-2x^2+1}{\sqrt{x^4+x^2+1}+\sqrt{x^4+3x^2}}\left(\frac{1/x^2}{1/\sqrt{x^4}}\right)$$
$$=\frac{-2+\frac{1}{x^2}}{\sqrt{1+\frac{1}{x^2}+\frac{1}{x^4}}+\sqrt{1+\frac{3}{x^2}}}$$

$$\lim_{x\to\infty}\frac{-2+\frac{1}{x^2}}{\sqrt{1+\frac{1}{x^2}+\frac{1}{x^4}}+\sqrt{1+\frac{3}{x^2}}}=\frac{-2+0}{\sqrt{1+0+0}+\sqrt{1+0}}=\frac{-2}{2}=-1$$

Now You

Evaluate $\displaystyle\lim_{x\to-\infty}\frac{\sqrt{3+x^2}}{3x}$

Q

ans

Now You

Evaluate $\displaystyle\lim_{x\to-\infty}\frac{\sqrt{3+x^2}}{3x}$

We factor out the largest power of the denominator, which is is x.

$$\lim_{x\to-\infty}\frac{\sqrt{3+x^2}}{3x}\left(\frac{1/x}{1/x}\right)=\lim_{x\to-\infty}\frac{\frac{\sqrt{3+x^2}}{x}}{3}$$

When $x<0$, $\sqrt{x^2}=|x|=-x$

$$\begin{aligned}
&=\lim_{x\to-\infty}\frac{1}{3}\frac{\sqrt{3+x^2}}{-\sqrt{x^2}}\\
&=\lim_{x\to-\infty}-\frac{1}{3}\sqrt{\frac{3+x^2}{x^2}}\\
&=\lim_{x\to-\infty}-\frac{1}{3}\sqrt{\frac{3}{x^2}+1}\\
&=-\frac{1}{3}
\end{aligned}$$

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Included Work

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