4.1 Antiderivatives

Basic Question

What function has derivative f(x)?

If F'(x) = f(x), we call F(x) an antiderivative of f(x).

Examples

 $\frac{d}{dx}[x^2] = 2x$, so x^2 is an <u>antiderivative</u> of 2x.

 $\frac{d}{dx}[x^2 + 5] = 2x$, so $x^2 + 5$ is (also) an antiderivative of 2x.

What is the most general antiderivative of 2x?

ANTIDERIVATIVES

Find the most general antiderivative for the following equations.

$$f(x) = 17$$

$$f(x) = m$$

where m is a constant.

This is a good time to remind students about lines.

differentiation fact

antidifferentiation fact

$$\frac{\mathrm{d}}{\mathrm{d}x}[x^2] = 2x$$

antideriv of
$$2x$$
:

$$\frac{\mathrm{d}}{\mathrm{d}x}[x^3] = 3x^2$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[x^5] = 5x^4$$

 $\frac{\mathrm{d}}{\mathrm{d}x}[x^4] = 4x^3$

$$\Longrightarrow$$

antideriv of x^n :

Power Rule for Antidifferentiation

The most general antiderivative of x^n is $\frac{1}{n+1}x^{n+1} + c$ if $n \neq -1$

$$ightharpoonup \frac{\mathrm{d}}{\mathrm{d}x}$$

$$=x^5$$

$$ightharpoonup \frac{\mathrm{d}}{\mathrm{d}x}$$

$$= x^3$$

$$ightharpoonup \frac{\mathrm{d}}{\mathrm{d}x}$$

$$\bigg] = \frac{1}{2}x^3$$

Power Rule for Antidifferentiation

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$$ightharpoonup \frac{\mathrm{d}}{\mathrm{d}x}$$

$$\bigg] = 5x^2 - 15x + 3$$

$$ightharpoonup \frac{\mathrm{d}}{\mathrm{d}x}$$

$$] = 13 \left(5x^{14} - 3x^{3/7} + 52e^x \right)$$

Now is a good time to remind students that power functions and exponential functions aren't the same

Find the most general antiderivatives.

$$f(x) = \cos x$$

$$f(x) = \sin x$$

$$f(x) = \sec^2 x$$

$$f(x) = \frac{1}{1 + x^2}$$

$$f(x) = \frac{1}{1 + x^2 + 2x}$$

Find the most general antiderivatives.

$$f(x) = 17\cos x + x^5$$

$$f(x) = \frac{23}{5 + 5x^2}$$

$$f(x) = \frac{23}{5 + 125x^2}$$

Not all of these are to actually do – more to show that there can be trickiness, and that trickiness will be a big part of next semester. Let students work on them in order, only the fastest students will get to the last examples on each slide.

Find the most general antiderivatives.

$$f(x) = \frac{1}{x}, \ x > 0$$

$$f(x) = 5x^2 - 32x^5 - 17$$

$$f(x) = \csc x \cot x$$

$$f(x) = \frac{5}{\sqrt{1 - x^2}} + 17$$

CHOSE YOUR OWN ADVENTURE

Antiderivative of $\sin x \cos x$:

A.
$$\cos x \sin x + c$$

B.
$$-\cos x \sin x + c$$

C.
$$\sin^2 x + c$$

D.
$$\frac{1}{2}\sin^2 x + c$$

E.
$$\frac{1}{2}\cos^2 x \sin^2 x + c$$

In general, antiderivatives of x^n have the form $\frac{1}{n+1}x^{n+1}$. What is the single exception?

A.
$$n = -1$$

B.
$$n = 0$$

C.
$$n = 1$$

D.
$$n = e$$

E.
$$n = 1/2$$

ALL THE ADVENTURES ARE CALCULUS, THOUGH

Suppose the velocity of a particle at time t is given by $v(t) = t^2 + \cos t + 3$. What function gives its position?

A.
$$s(t) = 2t - \sin t$$

B.
$$s(t) = 2t - \sin t + c$$

C.
$$s(t) = t^3 + \sin t + 3t + c$$

D.
$$s(t) = \frac{1}{3}t^3 + \sin t + 3t + c$$

E.
$$s(t) = \frac{1}{3}t^2 - \sin t + 3t + c$$

Suppose the velocity of a particle at time t is given by $v(t) = t^2 + \cos t + 3$, and its position at time 0 is given by s(0) = 5. What function gives its position?

A.
$$s(t) = \frac{1}{2}t^3 + \sin t + 3t$$

B.
$$s(t) = \frac{3}{3}t^3 + \sin t + 3t + 5$$

C.
$$s(t) = \frac{1}{3}t^3 + \sin t + 3t + c$$

D.
$$s(t) = 5t + c$$

E.
$$s(t) = 5t + 5$$

Find all functions f(x) with f(1) = 5 and $f'(x) = e^{3x+5}$.

Let Q(t) be the amount of a radioactive isotope in a sample. Suppose the sample is losing $50e^{-5t}$ mg per second to decay. If $Q(1) = 10e^{-5}$ mg, find the equation for the amount of the isotope at time t.

Suppose f'(t) = 2t + 7. What is f(10) - f(3)?