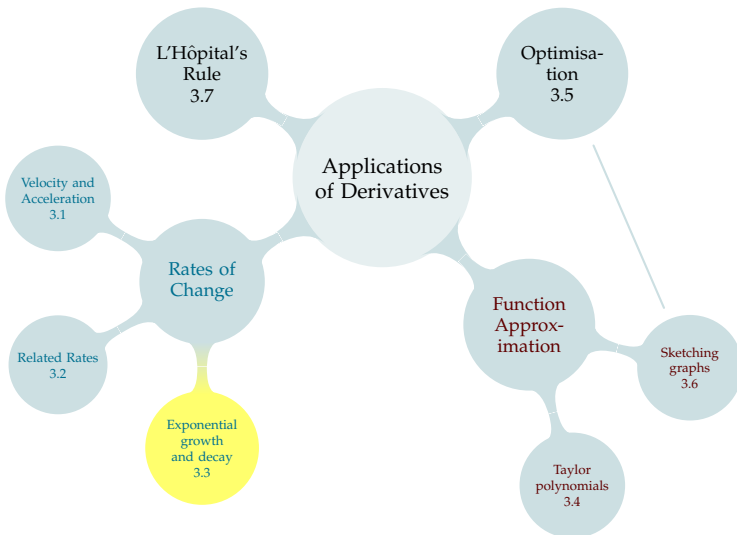


TABLE OF CONTENTS



RADIOACTIVE DECAY

The number of atoms in a sample that decay in a given time interval is proportional to the number of atoms in the sample.

RADIOACTIVE DECAY

The number of atoms in a sample that decay in a given time interval is proportional to the number of atoms in the sample.

Differential Equation

Let $Q = Q(t)$ be the amount of a radioactive substance at time t . Then for some positive constant k :

$$\frac{dQ}{dt} = -kQ$$

RADIOACTIVE DECAY

The number of atoms in a sample that decay in a given time interval is proportional to the number of atoms in the sample.

Differential Equation

Let $Q = Q(t)$ be the amount of a radioactive substance at time t . Then for some positive constant k :

$$\frac{dQ}{dt} = -kQ$$

Solution – Theorem 3.3.2

Let $Q(t) = Ce^{-kt}$, where k and C are constants. Then:

RADIOACTIVE DECAY

The number of atoms in a sample that decay in a given time interval is proportional to the number of atoms in the sample.

Differential Equation

Let $Q = Q(t)$ be the amount of a radioactive substance at time t . Then for some positive constant k :

$$\frac{dQ}{dt} = -kQ$$

Solution – Theorem 3.3.2

Let $\boxed{Q(t) = Ce^{-kt}}$, where k and C are constants. Then:

$$\frac{dQ}{dt}(t) = C \cdot e^{-kt} \cdot (-k) = -kCe^{-kt} = -kQ(t)$$

RADIOACTIVE DECAY

Quantity of a Radioactive Isotope

$$Q(t) = Ce^{-kt}$$

$Q(t)$: quantity at time t

RADIOACTIVE DECAY

Quantity of a Radioactive Isotope

$$Q(t) = Ce^{-kt}$$

$Q(t)$: quantity at time t

What is the sign of $Q(t)$?

- A. positive or zero
- B. negative or zero
- C. could be either
- D. I don't know

RADIOACTIVE DECAY

Quantity of a Radioactive Isotope

$$Q(t) = Ce^{-kt}$$

$Q(t)$: quantity at time t

What is the sign of $Q(t)$?

- A. positive or zero
- B. negative or zero
- C. could be either
- D. I don't know

RADIOACTIVE DECAY

Quantity of a Radioactive Isotope

$$Q(t) = Ce^{-kt}$$

$Q(t)$: quantity at time t

What is the sign of $Q(t)$?

- A. positive or zero
- B. negative or zero
- C. could be either
- D. I don't know

What is the sign of C ?

- A. positive or zero
- B. negative or zero
- C. could be either
- D. I don't know

RADIOACTIVE DECAY

Quantity of a Radioactive Isotope

$$Q(t) = Ce^{-kt}$$

$Q(t)$: quantity at time t

What is the sign of $Q(t)$?

- A. positive or zero
- B. negative or zero
- C. could be either
- D. I don't know

What is the sign of C ?

- A. positive or zero
- B. negative or zero
- C. could be either
- D. I don't know

Seaborgium Decay

The amount of ^{266}Sg (Seaborgium-266) in a sample at time t (measured in seconds) is given by

$$Q(t) = Ce^{-kt}$$

Let's approximate the half life of ^{266}Sg as 30 seconds. That is, every 30 seconds, the size of the sample halves.

What are C and k ?

$$Q(t) = Ce^{-kt}$$

Every 30 seconds, the size of the sample halves. What are C and k ?



$$Q(t) = Ce^{-kt}$$

Every 30 seconds, the size of the sample halves. What are C and k ?

- (1) $Q(0)$ is the amount of ^{266}Sg at time 0: usually, the initial sample size.

C is the quantity at time 0

- (2) The half-life of ^{266}Sg is 30 seconds. So, if we're measuring t in seconds, $Q(30) = \frac{1}{2}Q(0)$.

$$\frac{1}{2}C = \frac{1}{2}Q(0) = Q(30) = Ce^{-30k}$$

$$k = \frac{\log 2}{30}$$



A sample of radioactive matter is stored in a lab in 2000. In the year 2002, it is tested and found to contain 10 units of a particular radioactive isotope. In the year 2005, it is tested and found to contain only 2 units of that same isotope. How many units of the isotope were present in the year 2000?



The quantity of the isotope t years after 2000 is given by

$$Q(t) = Ce^{-kt}$$

where $C = Q(0)$ is the amount in the initial sample. Then the question asks us to solve for C , given

$$10 = Q(2) = Ce^{-2k} \quad \text{and} \quad 2 = Q(5) = Ce^{-5k}$$

Then

$$C = 10e^{2k} = 2e^{5k}$$

$$e^{3k} = 5 \quad \implies \quad e^k = \sqrt[3]{5}$$

$$C = 10e^{2k} = 10(\sqrt[3]{5})^2 \approx 29$$

$$Q'(t) = kQ(t)$$

The number of atoms in a sample that decay in a given time interval is proportional to the number of atoms in the sample.

$$Q'(t) = kQ(t)$$

The number of atoms in a sample that decay in a given time interval is proportional to the number of atoms in the sample.

The rate of growth of a population in a given time interval is proportional to the number of individuals in the population, when the population has ample resources.

$$Q'(t) = kQ(t)$$

The number of atoms in a sample that decay in a given time interval is proportional to the number of atoms in the sample.

The rate of growth of a population in a given time interval is proportional to the number of individuals in the population, when the population has ample resources.

The amount of interest a bank account accrues in a given time interval is proportional to the balance in that bank account.

Exponential Growth – Theorem 3.3.2

Let $Q = Q(t)$ satisfy:

$$\frac{dQ}{dt} = kQ$$

for some constant k . Then for some constant $C = Q(0)$,

$$Q(t) = Ce^{kt}$$

Exponential Growth – Theorem 3.3.2

Let $Q = Q(t)$ satisfy:

$$\frac{dQ}{dt} = kQ$$

for some constant k . Then for some constant $C = Q(0)$,

$$Q(t) = Ce^{kt}$$

Suppose $y(t)$ is a function with the properties that

$$\frac{dy}{dt} + 3y = 0 \quad \text{and} \quad y(1) = 2.$$

What is $y(t)$?

Suppose $y(t)$ is a function with the properties that

$$\frac{dy}{dt} + 3y = 0 \quad \text{and} \quad y(1) = 2.$$

What is $y(t)$?

Suppose $y(t)$ is a function with the properties that

$$\frac{dy}{dt} + 3y = 0 \quad \text{and} \quad y(1) = 2.$$

What is $y(t)$?

$$\frac{dy}{dt} = -3y, \text{ so } y(t) = Ce^{-3t} \text{ by the result above.}$$

To solve for C , we set $t = 1$:

$$2 = y(1) = Ce^{-3} \implies C = 2e^3$$

So,

$$y(t) = 2e^3 \cdot e^{-3t} = 2e^{3(1-t)}$$

POPULATION GROWTH

Suppose a petri dish starts with a culture of 100 bacteria cells and a limited amount of food and space. The population of the culture at different times is given in the table below. At approximately what time did the culture start to show signs of limited resources?

time	population
0	100
1	1000
3	100000
5	1000000

POPULATION GROWTH

Suppose a petri dish starts with a culture of 100 bacteria cells and a limited amount of food and space. The population of the culture at different times is given in the table below. At approximately what time did the culture start to show signs of limited resources?

time	population
0	100
1	1000
3	100000
5	1000000

All the populations before $t = 5$ follow $B(t) = 100 \cdot 10^t = 100e^{t \log 10}$.
At $t = 5$ they do not; so some time between $t = 3$ and $t = 5$, the bacteria started reproducing at a slower rate.

FLU SEASON

The CDC keeps records ([link](#)) on the number of flu cases in the US by week. At the start of the flu season, the 40th week of 2014, there are 100 cases of a particular strain. Five weeks later (at week 45), there are 506 cases. What do you think was the first week to have 5,000 cases? What about 10,000 cases?



FLU SEASON

Stacked Column Chart WHO//NREVSS

Influenza Positive Tests Reported to CDC, National Summary,
2014-15 Season, week ending Oct 02, 2015

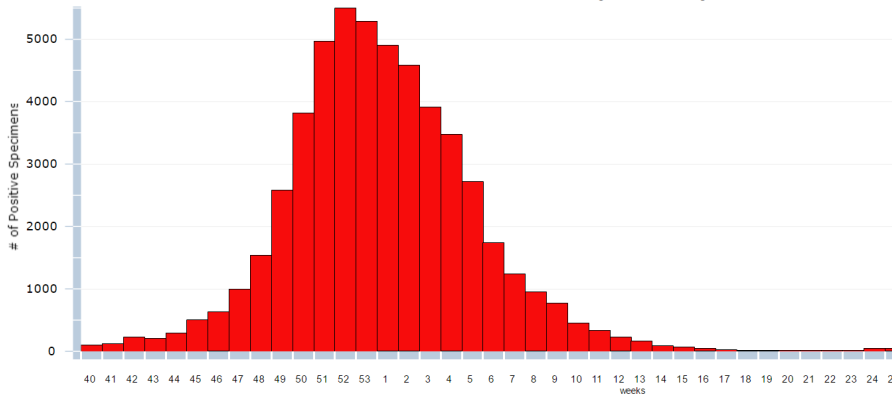
Reported by: U.S. WHO//NREVSS Collaborating Laboratories and ILINet

[Download Image](#)

[Download Data](#)



Click and drag to create rectangle to zoom



☒ Check All ☐ Percent Positive

☐ A (H1)

☒ A (H3)

☐ A (Subtyping not Performed)

☐ H3N2v

☐ A (Unable to Subtype)

☐ A (H1N1)pdm09

☐ B

FLU SEASON

Let $t = 0$ be the 40th week of 2014. Then we can model the spread of the virus like so:

$$P(t) = 100e^{kt}$$

We have one other data point: $506 = P(5) = 100e^{5k}$, so we get $e^k = 5.06^{1/5}$.
Now our equation is:

$$P(t) = 100(5.06)^{t/5}$$

We set it equal to 5000 and solve: $5000 = 100(5.06)^{t/5}$ implies

$$(5.06)^{t/5} = 50 \implies \frac{t}{5} \log(5.06) = \log 50 \implies t = \frac{5 \log 50}{\log(5.06)} \approx 12.06$$

Data from the CDC says Week 51 ($t = 11$) had 4972 cases, and Week 52 ($t = 12$) had 5498 cases.

Using the same formula, $10000 = 100(5.06)^{t/5}$ yields $t = \frac{5 \log 100}{\log 5.06} \approx 14.2$ weeks; but the data shows that the flu season peaked with around 5,000 cases a week, and never got much higher.



Newton's Law of Cooling – Equation 3.3.7

The rate of change of temperature of an object is proportional to the difference in temperature between that object and its surroundings.

Newton's Law of Cooling – Equation 3.3.7

The rate of change of temperature of an object is proportional to the difference in temperature between that object and its surroundings.

$$\frac{dT}{dt}(t) = K[T(t) - A]$$

where $T(t)$ is the temperature of the object at time t , A is the (constant) ambient temperature of the surroundings, and K is some constant depending on the object.

$$\frac{dT}{dt}(t) = K[T(t) - A]$$

$T(t)$ is the temperature of the object, A is the ambient temperature, K is some constant.

What is true of K ?

- A. $K \geq 0$
- B. $K \leq 0$
- C. $K = 0$
- D. K could be positive, negative, or zero, depending on the object
- E. I don't know

$$\frac{dT}{dt}(t) = K[T(t) - A]$$

$T(t)$ is the temperature of the object, A is the ambient temperature, K is some constant.

What is true of K ?

- A. $K \geq 0$
- B. $K \leq 0$
- C. $K = 0$
- D. K could be positive, negative, or zero, depending on the object
- E. I don't know

Newton's Law of Cooling – Equation 3.3.7

$$\frac{dT}{dt}(t) = K[T(t) - A]$$

$T(t)$ is the temperature of the object, A is the ambient temperature, and K is some constant.

$$T(t) = [T(0) - A]e^{Kt} + A$$

is the only function satisfying Newton's Law of Cooling

Newton's Law of Cooling – Equation 3.3.7

$$\frac{dT}{dt}(t) = K[T(t) - A]$$

$T(t)$ is the temperature of the object, A is the ambient temperature, and K is some constant.

$$T(t) = [T(0) - A]e^{Kt} + A$$

is the only function satisfying Newton's Law of Cooling

If $T(10) < A$, then:

- A. $K > 0$
- B. $T(0) > 0$
- C. $T(0) > A$
- D. $T(0) < A$

Newton's Law of Cooling – Equation 3.3.7

$$\frac{dT}{dt}(t) = K[T(t) - A]$$

$T(t)$ is the temperature of the object, A is the ambient temperature, and K is some constant.

$$T(t) = [T(0) - A]e^{Kt} + A$$

is the only function satisfying Newton's Law of Cooling

If $T(10) < A$, then:

- A. $K > 0$
- B. $T(0) > 0$
- C. $T(0) > A$
- D. $T(0) < A$

Evaluate $\lim_{t \rightarrow \infty} T(t)$.

- A. A
- B. 0
- C. ∞
- D. $T(0)$

Newton's Law of Cooling – Equation 3.3.7

$$\frac{dT}{dt}(t) = K[T(t) - A]$$

$T(t)$ is the temperature of the object, A is the ambient temperature, and K is some constant.

$$T(t) = [T(0) - A]e^{Kt} + A$$

is the only function satisfying Newton's Law of Cooling

If $T(10) < A$, then:

- A. $K > 0$
- B. $T(0) > 0$
- C. $T(0) > A$
- D. $T(0) < A$

Evaluate $\lim_{t \rightarrow \infty} T(t)$.

- A. A
- B. 0
- C. ∞
- D. $T(0)$



What assumptions are we making that might not square with the real world?

Newton's Law of Cooling – Equation 3.3.7

$$\frac{dT}{dt} = K[T(t) - A]$$

$T(t)$ is the temperature of the object, A is the ambient temperature, and K is some constant.

Temperature of a Cooling Body – Corollary 3.3.8

$$T(t) = [T(0) - A]e^{Kt} + A$$

A farrier forms a horseshoe heated to 400°C , then dunks it in a river at room-temperature (25°C). The water boils for 30 seconds. The horseshoe is safe for the horse when it's 40°C . When can the farrier put on the horseshoe?



A farrier forms a horseshoe heated to 400°C , then dunks it in a river at room-temperature (25°C). The water boils for 30 seconds. The horseshoe is safe for the horse when it's 40°C . When can the farrier put on the horseshoe?

$$T(t) = [T(0) - A]e^{Kt} + A$$



We know: $T(0) = 400$, $T(30) = 100$, and $A = 25$. We want to find K .

$$100 = T(30) = [T(0) - A]e^{30K} + A = 375e^{30K} + 25$$

$$\Rightarrow 75 = 375e^{30K} \Rightarrow \frac{1}{5} = e^{30K} \Rightarrow K = \frac{-\log 5}{30}$$

Now, we set $T(t) = 40$ and solve for t :

$$40 = T(t) = 375e^{\frac{-\log 5}{30}t} + 25$$

$$15 = 375e^{\frac{-\log 5}{30}t} = 375 \cdot 5^{-t/30}$$

$$\frac{1}{25} = 5^{-t/30}$$

$$25 = 5^{t/30}$$

$$2 = t/30$$

So the farrier can put the shoe on after 60 seconds in the water.



A glass of just-boiled tea is put on a porch outside. After ten minutes, the tea is 40° , and after 20 minutes, the tea is 25° . What is the temperature outside?



$$T(0) = 100, \text{ so}$$

$$T(10) = [100 - A]e^{10K} + A = 100e^{10K} + A(1 - e^{10K}) = 40$$

$$T(20) = [100 - A]e^{20K} + A = 100e^{20K} + A(1 - e^{20K}) = 25$$

$$\text{Solving both for } A, \text{ we get } A = \frac{40 - 100e^{10K}}{1 - e^{10K}} = \frac{25 - 100e^{20K}}{1 - e^{20K}}$$

Although this looks complicated, if we set $x = e^{10k}$, it simplifies to something we can easily solve.



$$A = \frac{40 - 100e^{10K}}{1 - e^{10K}} = \frac{25 - 100e^{20K}}{1 - e^{20K}}$$

$$A = \frac{40 - 100x}{1 - x} = \frac{25 - 100x^2}{1 - x^2}$$

$$(40 - 100x)(1 - x^2) = (25 - 100x^2)(1 - x)$$

$$(40 - 100x)(1 + x)(1 - x) = (25 - 100x^2)(1 - x)$$

$$(40 - 100x)(1 + x) = 25 - 100x^2$$

$$40 - 60x - 100x^2 = 25 - 100x^2$$

$$40 - 60x = 25$$

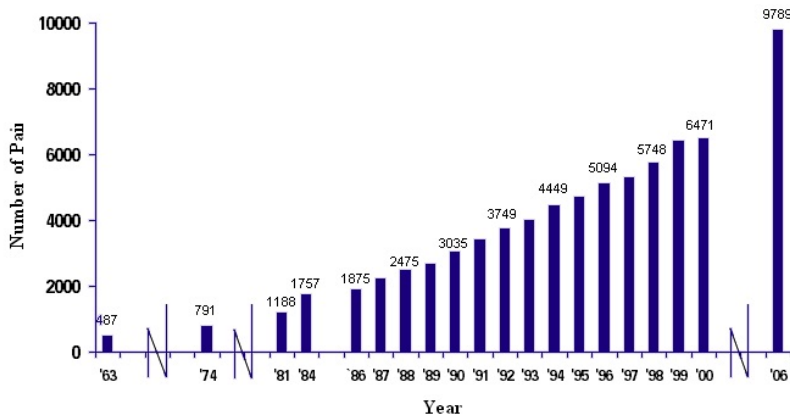
$$x = \frac{1}{4}$$

$$A = \frac{40 - 100x}{1 - x} = \frac{40 - \frac{100}{4}}{1 - \frac{1}{4}} = 20$$

It is 20 degrees outside.



In 1963, the US Fish and Wildlife Service recorded a bald eagle population of 487 breeding pairs. In 1993, that number was 4015. How many breeding pairs would you expect there were in 2006? What about 2015?



Since we don't have a better model, let's assume the population P of nesting pairs follows:

$$P(t) = P(0)e^{Kt}$$

for some constant K .

To fit the data we have, let $t = 0$ represent 1963, so $P(0) = 487$. Then

$$4015 = P(30) = 487e^{30K}$$

$$\text{so } e^K = \left(\frac{4015}{487}\right)^{1/30}.$$

Now we use this to predict $P(43)$ (since 2006 is 43 years after 1963) and $P(52)$ (since 2015 is 52 years after 1963).

$$P(43) = 487(e^K)^{43} = 487\left(\frac{4015}{487}\right)^{43/30} \approx 10016$$

So we guess in 2016 there were about 10,016 breeding pairs in the lower 48.

$$P(52) = 487(e^K)^{52} = 487\left(\frac{4015}{487}\right)^{52/30} \approx 18860$$

link: Wood Bison Restoration in Alaska, Alaska Department of Fish and Game

Excerpt:

Based on experience with reintroduced populations elsewhere, wood bison would be expected to increase at a rate of 15%-25% annually after becoming established.... With an average annual growth rate of 20%, an initial precalving population of 50 bison would increase to 500 in approximately 13 years.

NOW
YOU



Are they using our same model?

link: Wood Bison Restoration in Alaska, Alaska Department of Fish and Game

Excerpt:

Based on experience with reintroduced populations elsewhere, wood bison would be expected to increase at a rate of 15%-25% annually after becoming established.... With an average annual growth rate of 20%, an initial precalving population of 50 bison would increase to 500 in approximately 13 years.

NOW
YOU



Are they using our same model?

Our model gives the same result.



COMPOUND INTEREST

Suppose you invest \$10,000 in an account that accrues interest each month. After one month, your balance (with interest) is \$10,100. How much money will be in your account after a year?

COMPOUND INTEREST

Suppose you invest \$10,000 in an account that accrues interest each month. After one month, your balance (with interest) is \$10,100. How much money will be in your account after a year?

Compound interest is calculated according to the formula Pe^{rt} , where r is the interest rate and t is time.

COMPOUND INTEREST

Suppose you invest \$10,000 in an account that accrues interest each month. After one month, your balance (with interest) is \$10,100. How much money will be in your account after a year?

Compound interest is calculated according to the formula Pe^{rt} , where r is the interest rate and t is time.

Measuring time in months,

$$10000e^{r \cdot 1} = 10100$$

$$e^r = \frac{10100}{10000} = 1.01$$

$$10000e^{12r} = 10000 \cdot (e^r)^{12} = 10000 \cdot 1.01^{12} \approx 11268.25$$



CARRYING CAPACITY

For a population of size P with unrestricted access to resources, let β be the average number of offspring each breeding pair produces per generation, where a generation has length t_g . Then $b = \frac{\beta-2}{2t_g}$ is the net birthrate (births minus deaths) per member per unit time. This yields $\frac{dP}{dt}(t) = bP(t)$, hence:

CARRYING CAPACITY

For a population of size P with unrestricted access to resources, let β be the average number of offspring each breeding pair produces per generation, where a generation has length t_g . Then $b = \frac{\beta-2}{2t_g}$ is the net birthrate (births minus deaths) per member per unit time. This yields $\frac{dP}{dt}(t) = bP(t)$, hence:

$$P(t) = P(0)e^{bt}$$



CARRYING CAPACITY

For a population of size P with unrestricted access to resources, let β be the average number of offspring each breeding pair produces per generation, where a generation has length t_g . Then $b = \frac{\beta-2}{2t_g}$ is the net birthrate (births minus deaths) per member per unit time. This yields $\frac{dP}{dt}(t) = bP(t)$, hence:

$$P(t) = P(0)e^{bt}$$

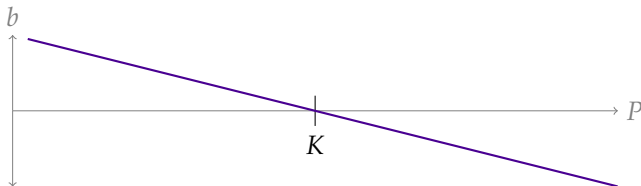
But as resources grow scarce, b might change.

CARRYING CAPACITY

b is the net birthrate (births minus deaths) per member per unit time.

If K is the carrying capacity of an ecosystem, we can model

$$b = b_0\left(1 - \frac{P}{K}\right).$$

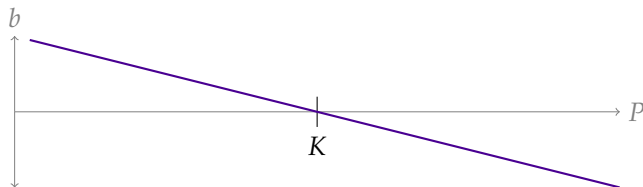


CARRYING CAPACITY

b is the net birthrate (births minus deaths) per member per unit time.

If K is the carrying capacity of an ecosystem, we can model

$$b = b_0\left(1 - \frac{P}{K}\right).$$



Describe to your neighbour what the following mean in

terms of the model:

- ▶ $b > 0, b = 0, b < 0$
- ▶ $P = 0, P > 0, P < 0$

CARRYING CAPACITY

Then:

$$\frac{dP}{dt}(t) = b_0 \underbrace{\left(1 - \frac{P(t)}{K}\right)}_{\text{per capita birthrate}} P(t)$$

CARRYING CAPACITY

Then:

$$\frac{dP}{dt}(t) = b_0 \underbrace{\left(1 - \frac{P(t)}{K}\right)}_{\text{per capita birthrate}} P(t)$$

This is an example of a differential equation that we don't have the tools to solve. (If you take more calculus, though, you'll learn how!) It's also an example of a way you might tweak a model so its assumptions better fit what you observe.

RADIOCARBON DATING

Researchers at Charlie Lake in BC have found evidence¹ of habitation dating back to around 8500 BCE. For instance, a butchered bison bone was radiocarbon dated to about 10,500 years ago.

Suppose a comparable bone of a bison alive today contains $1\mu\text{g}$ of ^{14}C . If the half-life of ^{14}C is about 5730 years, roughly how much ^{14}C do you think the researchers found in the sample?

¹<http://pubs.aina.ucalgary.ca/arctic/Arctic49-3-265.pdf>

RADIOCARBON DATING

Researchers at Charlie Lake in BC have found evidence of habitation dating back to around 8500 BCE. For instance, a butchered bison bone was radiocarbon dated to about 10,500 years ago.

Suppose a comparable bone of a bison alive today contains $1\mu\text{g}$ of ^{14}C . If the half-life of ^{14}C is about 5730 years, roughly how much ^{14}C do you think the researchers found in the sample?

- A. About $\frac{1}{10,500} \mu\text{g}$
- B. About $\frac{1}{4} \mu\text{g}$
- C. About $\frac{1}{2} \mu\text{g}$

- D. About $1 \mu\text{g}$
- E. I'm not sure how to estimate this



RADIOCARBON DATING

First, an estimate; 10500 is not so far off from $2(5730)$, i.e. two half-lives, so we might guess that there is roughly a $(\frac{1}{2})^2 = \frac{1}{4}$ of a microgram left.

We know $Q(t) = Ce^{-kt} = e^{-kt} \mu\text{g}$. We want to find $Q(10500)$, so we need to solve for k . Since we know the half-life: to do this, solve

$$\frac{1}{2} = e^{-k \cdot 5730} \quad \text{to get} \quad k = \frac{\log 2}{5730}$$

Now:

$$Q(10500) = e^{-\frac{\log 2}{5730} \cdot 10500} = 2^{-\frac{10500}{5730}} \approx 0.28 \mu\text{g}$$



Suppose a body is discovered at 3:45 pm, in a room held at 20° , and the body's temperature is 27° , not the normal 37° . At 5:45 pm, the temperature of the body has dropped to 25.3° . When did the inhabitant of the body die?

Set our time so that $t = 0$ is 3:45pm and $t = 2$ is 5:45pm. Then $T(0) = 27$, $T(2) = 25.4$, and $A = 20$. Now:

$$T(t) = [27 - 20]e^{Kt} + 20 = 7e^{Kt} + 20$$

Using what we know about 5:45pm:

$$7e^{2K} + 20 = T(2) = 25.3$$

so

$$7e^{2K} = 5.3 \implies e^{2K} = \frac{5.3}{7} \implies e^K = \left(\frac{5.3}{7}\right)^{1/2}$$

Now:

$$T(t) = 7e^{Kt} + 20 = 7\left(\frac{5.3}{7}\right)^{t/2} + 20$$

So we set $T(t) = 37$ and solve for t .



$$7 \left(\frac{5.3}{7} \right)^{t/2} + 20 = 37$$

$$7 \left(\frac{5.3}{7} \right)^{t/2} = 17$$

$$\left(\frac{5.3}{7} \right)^{t/2} = \frac{17}{7}$$

$$\frac{t}{2} = \frac{\log(17/7)}{\log(5.3/7)}$$

$$t = 2 \frac{\log(17/7)}{\log(5.3/7)} \approx -6.4$$

So the person died about 6.4 hours before 3:45pm. Now 0.4 hours is 24 minutes. So 6 hours and 24 minutes before 3:45 pm is 6 hours before 3:21pm, which is 9:21 am.



Included Work



'Brain' by Eucalyp is licensed under CC BY 3.0 (accessed 8 June 2021), 46, 47, 54, 55



'Chart of Bald Eagle Breeding Pairs in Lower 48 States'. Eagle nesting data, US Fish and Wildlife Service Midwest Region (accessed 19 October 2016), 44



'Notebook' by Iconic is licensed under CC BY 3.0 (accessed 9 June 2021), 11, 20, 37



U.S. WHO/NREVSS Collaborating Laboratories and ILNet. 'Stacked Column Chart WHO//NREVSS' Centers for Disease Control and Prevention. No longer available from

<http://gis.cdc.gov/grasp/fluview/fluportaldashboard.html> (accessed 20 October 2015), 26

Alaska Department of Fish and Game, Division of Wildlife Conservation. (April 2007). Wood Bison Restoration in Alaska: A Review of Environmental and Regulatory Issues and Proposed Decisions for Project Implementation, p. 11.

http://www.adfg.alaska.gov/static/species/speciesinfo/woodbison/pdfs/er_no_appendices.pdf (accessed 2015 or 2016), 46, 47

Driver et.al. Stratigraphy, Radiocarbon Dating, and Culture History of Charlie Lake Cave, British Columbia. *ARCTICVOL.* 49, no. 3 (September 1996) pp. 265 – 277.

<http://pubs.aina.ucalgary.ca/arctic/Arctic49-3-265.pdf> (accessed 2015 or 2016), 58, 59

Public Domain by Man vyi via https://commons.wikimedia.org/wiki/File:West_Show_Jersey_2010_farrier_f.jpg, accessed October 2015, 37, 38