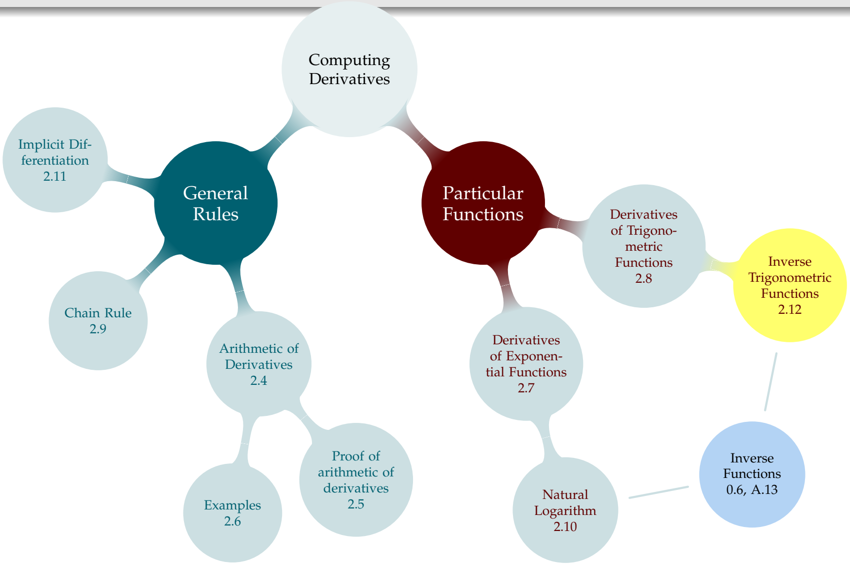
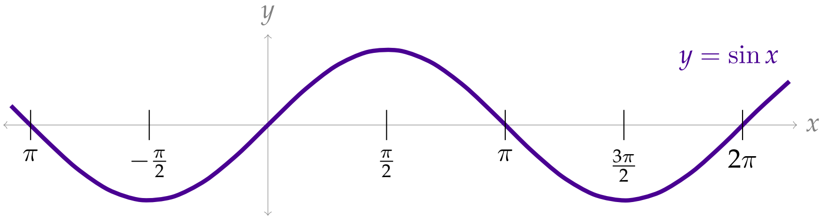
Table of Contents

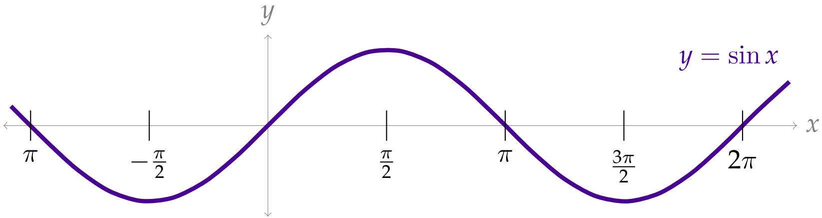
SKIP DEFINITIONS OF INVERSE TRIG FUNCTIONS



Invertibility Game

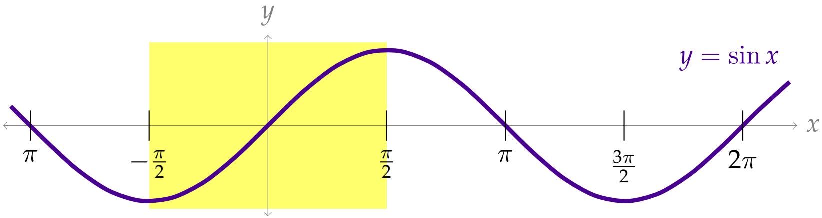


Invertibility Game



I'm thinking of a number x. Your hint: $\sin(x) = 0$. What number am I thinking of?

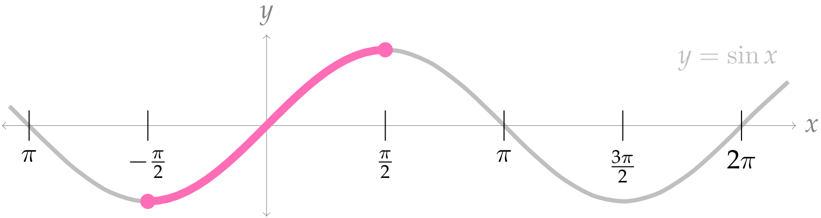
Invertibility Game



I'm thinking of a number x. Your hint: $\sin(x) = 0$. What number am I thinking of?

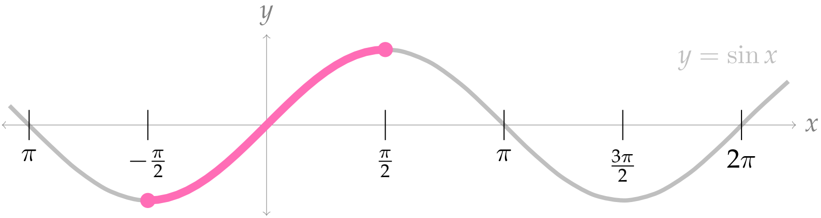
I'm thinking of a number x, and x is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Your hint: $\sin(x) = 0$. What number am I thinking of?

Arcsine



$\arcsin(x)$ is the inverse of $\sin x$ restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Arcsine



$\arcsin(x)$ is the inverse of $\sin x$ restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\arcsin x$ is the (unique) number θ such that:

- $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, and
- $\sin \theta = x$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$-\frac{\pi}{6}$	$-\frac{1}{2}$
$-\frac{\pi}{4}$	$-\frac{1}{\sqrt{2}}$
$-\frac{\pi}{3}$	$-\frac{\sqrt{3}}{2}$
$-\frac{\pi}{2}$	-1

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0)$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right)$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right)$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$
- $\arcsin\left(\frac{\pi}{2}\right)$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$
- $\arcsin\left(\frac{\pi}{2}\right)$ undefined

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$
- $\arcsin\left(\frac{\pi}{2}\right)$ undefined
- $\arcsin\left(\frac{\pi}{4}\right)$

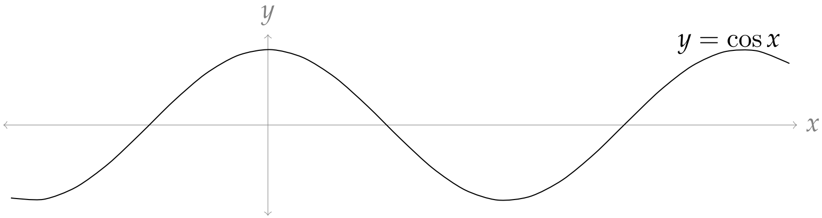
Arcsine

Reference Angles:

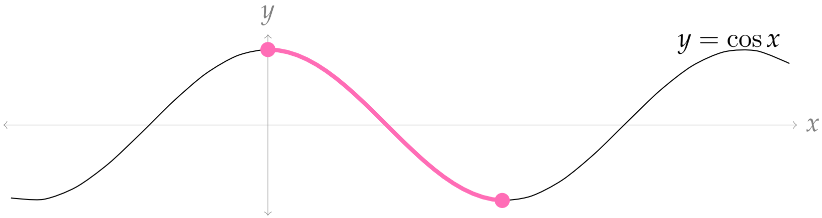
θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$
- $\arcsin\left(\frac{\pi}{2}\right)$ undefined
- $\arcsin\left(\frac{\pi}{4}\right)$ defined, but we haven't covered tools (yet) to figure it out

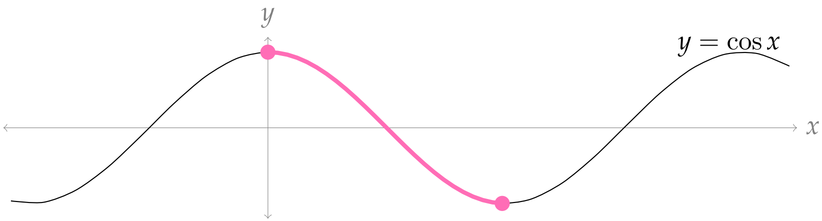
Arccosine



Arccosine



ARCCOSINE

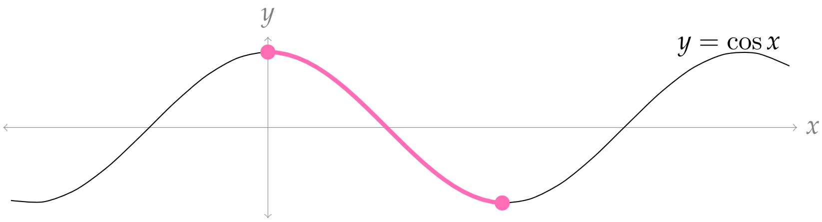


$\arccos(x)$ is the inverse of $\cos x$ restricted to $[0, \pi]$.

$\arccos(x)$ is the (unique) number θ such that:

- $\cos(\theta) = x$ and
- $0 \leq \theta \leq \pi$

Arccosine

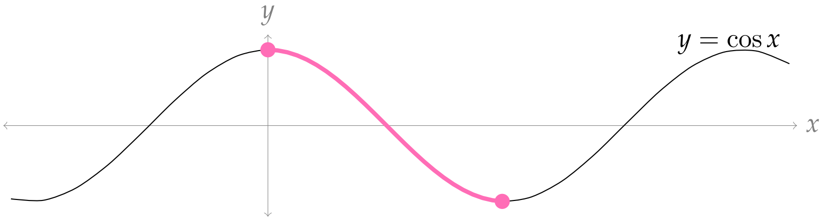


$\arccos(x)$ is the inverse of $\cos x$ restricted to $[0, \pi]$.

$\arccos(x)$ is the (unique) number θ such that:

- $\cos(\theta) = x$ and $\leftarrow\leftarrow\leftarrow$ inverse
- $0 \leq \theta \leq \pi$

Arccosine

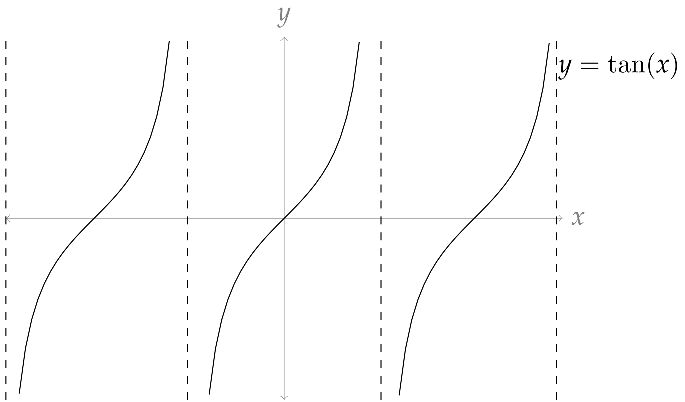


$\arccos(x)$ is the inverse of $\cos x$ restricted to $[0, \pi]$.

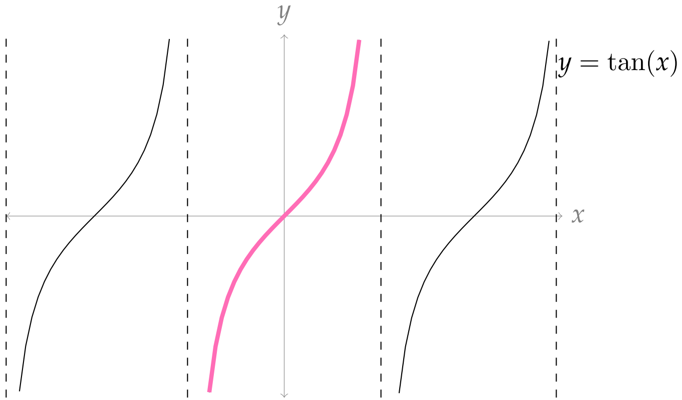
$\arccos(x)$ is the (unique) number θ such that:

- $\cos(\theta) = x$ and $\longleftarrow\!\longleftarrow\!\longleftarrow$ inverse
- $0 \leq \theta \leq \pi$ $\longleftarrow\!\longleftarrow\!\longleftarrow$ inverse exists

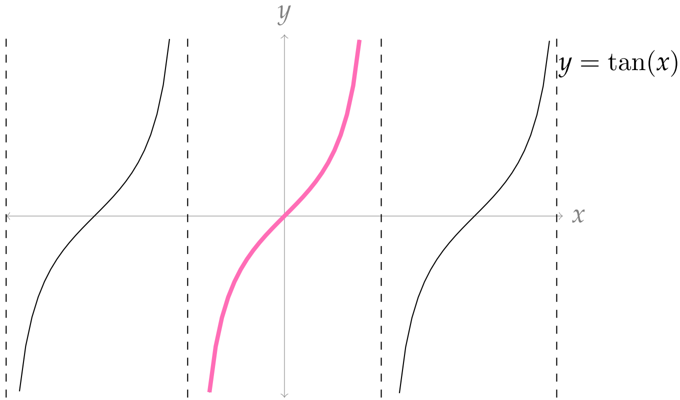
Arctangent



Arctangent



Arctangent



$\arctan(x) = \theta$ means:
(1) $\tan(\theta) = x$ and
(2) $-\pi/2 < \theta < \pi/2$

Arcsecant, arcsine, and arccotangent

$$\mathrm{arcsec}(x) =$$

Arcsecant, arcsine, and arccotangent

$$\mathrm{arcsec}(x) =$$

$$\begin{aligned}
\mathrm{arcsec}(x) &= y \\
\sec y &= x \\
\frac{1}{\cos y} &= x \\
\cos y &= \tfrac{1}{x} \\
y &= \arccos\left(\tfrac{1}{x}\right) \\
\mathrm{arcsec}(x) &= \arccos\left(\tfrac{1}{x}\right)
\end{aligned}$$

Arcsecant, arcsine, and arccotangent

$$\operatorname{arcsec}(x) = \arccos\left(\tfrac{1}{x}\right)$$

$$\begin{aligned}
\operatorname{arcsec}(x) &= y \\
\sec y &= x \\
\frac{1}{\cos y} &= x \\
\cos y &= \tfrac{1}{x} \\
y &= \arccos\left(\tfrac{1}{x}\right) \\
\operatorname{arcsec}(x) &= \arccos\left(\tfrac{1}{x}\right)
\end{aligned}$$

Arcsecant, arcsine, and arccotangent

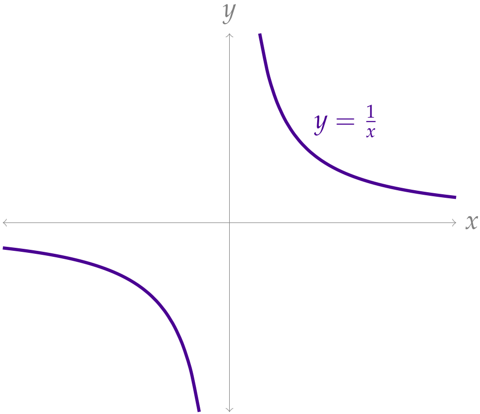
$$\operatorname{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$

$$\begin{aligned}
\operatorname{arccsc}(x) &= y \\
\csc y &= x \\
\frac{1}{\sin y} &= x \\
\sin y &= \frac{1}{x} \\
y &= \arcsin\left(\frac{1}{x}\right) \\
\operatorname{arccsc}(x) &= \arcsin\left(\frac{1}{x}\right)
\end{aligned}$$

$$\operatorname{arccot}(\mathrm{x}) = \arctan\left(\frac{1}{x}\right)$$

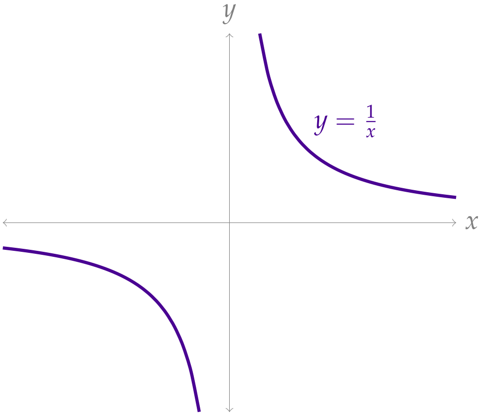
$$\begin{aligned}
\operatorname{arccot}(\mathrm{x}) &= y \\
\cot y &= x \\
\frac{1}{\tan y} &= x \\
\tan y &= \frac{1}{x} \\
y &= \arctan\left(\frac{1}{x}\right) \\
\operatorname{arccot}(\mathrm{x}) &= \arctan\left(\frac{1}{x}\right)
\end{aligned}$$

$$\operatorname{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$$



$$\operatorname{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$$

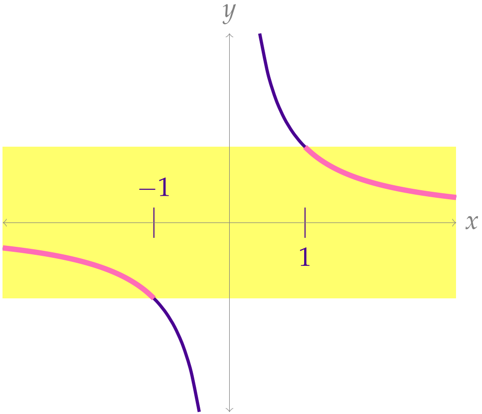
The domain of $\arccos(y)$ is $-1 \leq y \leq 1$, so the domain of $\operatorname{arcsec}(y)$ is



ans

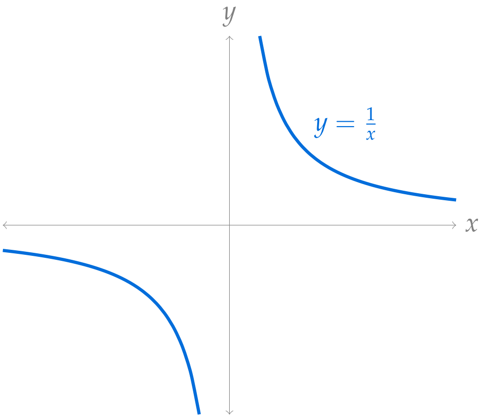
$$\text{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$$

The domain of $\arccos(y)$ is $-1 \leq y \leq 1$, so the domain of $\text{arcsec}(y)$ is



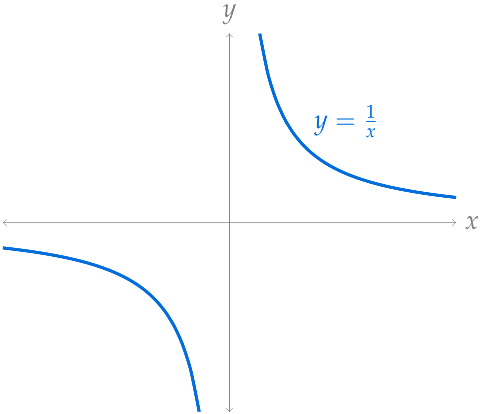
$(-\infty, -1] \cup [1, \infty)$.

$$\operatorname{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$



$$\operatorname{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$

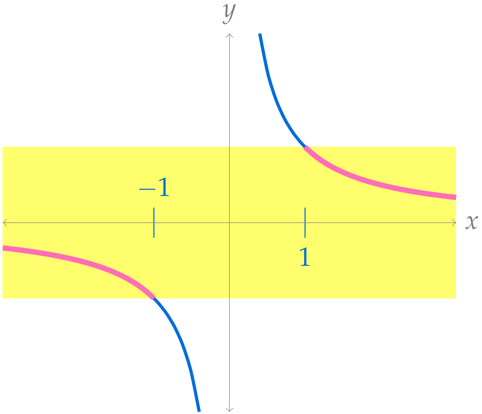
Domain of $\arcsin(y)$ is $-1 \leq y \leq 1$, so the domain of $\operatorname{arccsc}(x)$ is



ans

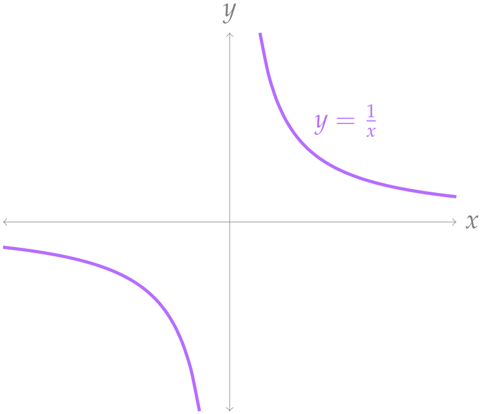
$$\text{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$

Domain of $\arcsin(y)$ is $-1 \leq y \leq 1$, so the domain of $\text{arccsc}(x)$ is



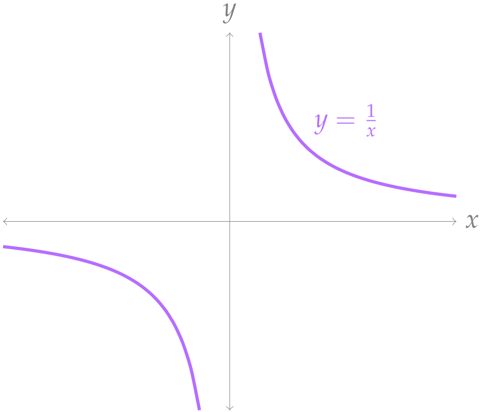
$(-\infty, -1] \cup [1, \infty)$.

$$\operatorname{arccot}(x) = \arctan\left(\frac{1}{x}\right)$$



$$\text{arccot}(x) = \arctan\left(\frac{1}{x}\right)$$

Domain of $\arctan(x)$ is all real numbers, so the domain of $\text{arccot}(x)$ is



ans

$$\text{arccot(x)} = \arctan\left(\frac{1}{x}\right)$$

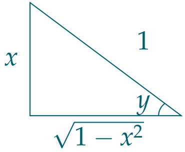
Domain of $\arctan(x)$ is all real numbers, so the domain of $\text{arccot}(x)$ is

$(-\infty, 0) \cup (0, \infty)$.

$$y = \arcsin x$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

$$
\begin{aligned}
y(x) &= \arcsin x \\
x &= \sin y(x) \\
\frac{\mathrm{d}}{\mathrm{d}x}[x] &= \frac{\mathrm{d}}{\mathrm{d}x}[\sin y(x)] \\
1 &= \cos y(x) \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{1}{\cos y(x)} \\
&= \frac{\text{hyp}}{\text{adj}} \\
&= \frac{1}{\sqrt{1-x^2}}
\end{aligned}
$$

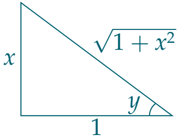


ans

$$y = \arctan x$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

$$\begin{aligned}
y(x) &= \arctan x \\
x &= \tan y(x) \\
\frac{\mathrm{d}}{\mathrm{d}x}[x] &= \frac{\mathrm{d}}{\mathrm{d}x}[\tan y(x)] \\
1 &= \sec^2 y(x) \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \cos^2 y(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \left(\frac{\text{adj}}{\text{hyp}}\right)^2 = \left(\frac{1}{\sqrt{1+x^2}}\right)^2 \\
&= \frac{1}{1+x^2}
\end{aligned}$$

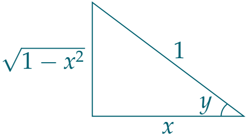


ans

$$y = \arccos x$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

$$\begin{aligned}
y(x) &= \arccos x \\
x &= \cos y(x) \\
\frac{\mathrm{d}}{\mathrm{d}x}[x] &= \frac{\mathrm{d}}{\mathrm{d}x}[\cos y(x)] \\
1 &= -\sin y(x) \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{-1}{\sin y(x)} \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{-\text{hyp}}{\text{opp}} = \frac{-1}{\sqrt{1-x^2}}
\end{aligned}$$



ans

To differentiate arcsecant, arccosecant, and arccotangent, you can use the chain rule!

To differentiate arcsecant, arccosecant, and arccotangent, you can use the chain rule!

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\operatorname{arccsc}(x)\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[\arcsin\left(\frac{1}{x}\right)\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[\arcsin\left(x^{-1}\right)\right]$$

To differentiate arcsecant, arccosecant, and arccotangent, you can use the chain rule!

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\operatorname{arccsc}(x)\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[\arcsin\left(\frac{1}{x}\right)\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[\arcsin\left(x^{-1}\right)\right]$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\arcsin\left(\boxed{x^{-1}}\right)\right] = \frac{1}{\sqrt{1-\left(\boxed{x^{-1}}\right)^2}} \cdot \boxed{\left(-x^{-2}\right)} = \frac{-1}{x^2\sqrt{1-x^{-2}}}$$

$$= \frac{-1}{\sqrt{x^4}\sqrt{1-x^{-2}}} = \frac{-1}{\sqrt{x^2}\sqrt{x^2}\sqrt{1-x^{-2}}} = \frac{-1}{\sqrt{x^2}\sqrt{x^2-1}} = \frac{-1}{|x|\sqrt{1-x^2}}$$

Derivatives of Inverse Trigonometric Functions – Theorem 2.12.7

Memorize:

$$\frac{\mathrm{d}}{\mathrm{d}x}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\arccos x] = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\arcsin x] = \frac{1}{1+x^2}$$

Be able to derive:

$$\frac{\mathrm{d}}{\mathrm{d}x}[\operatorname{arccsc} x] = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\operatorname{arcsec} x] = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\operatorname{arccot} x] = -\frac{1}{1+x^2}$$