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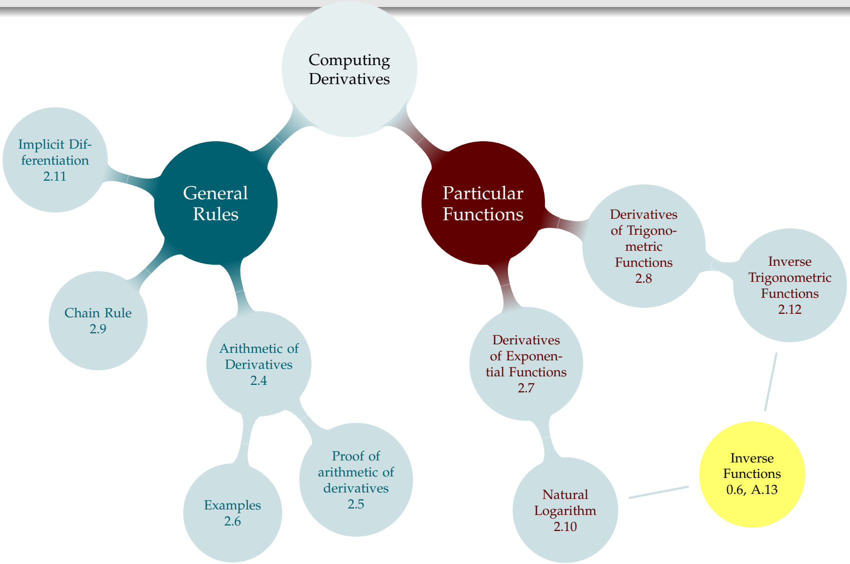
Skip review of inverse functions and logarithms



Invertibility Game

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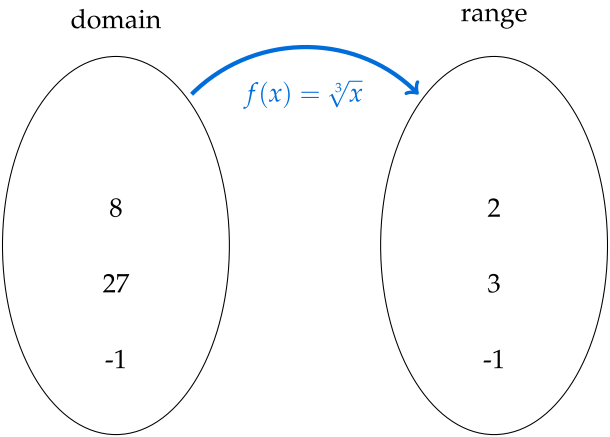
Round 3: $f(x) = |x|$

Round 4: $f(x) = \sin x$

Functions are Maps



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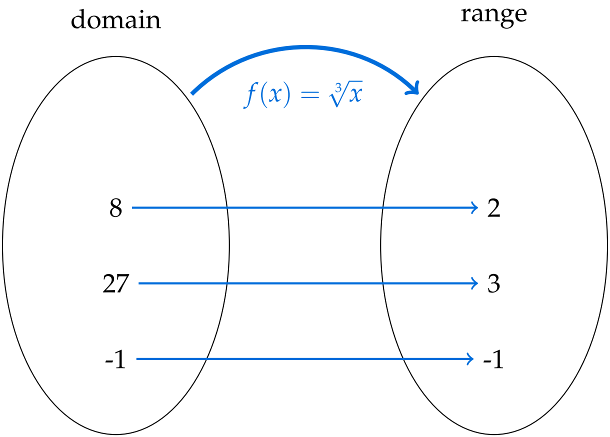

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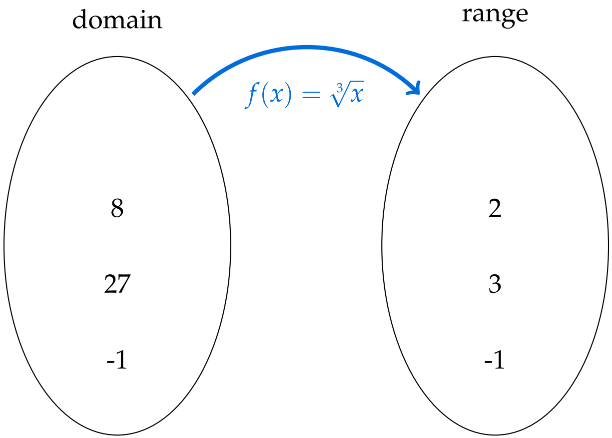

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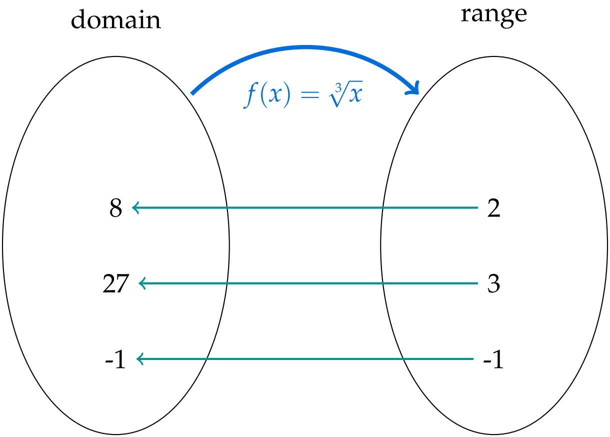

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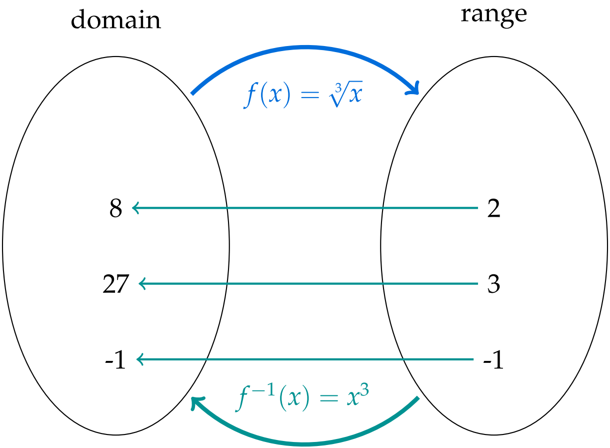

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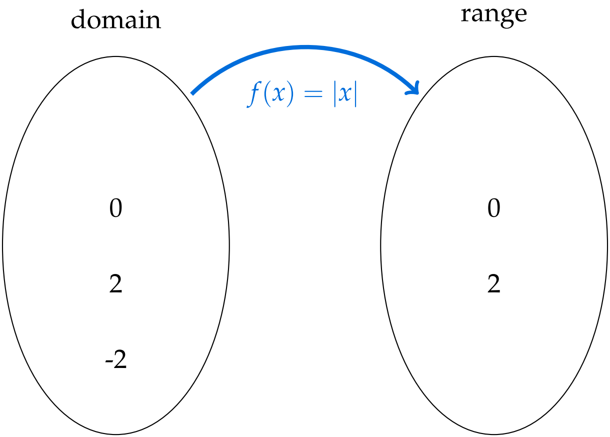

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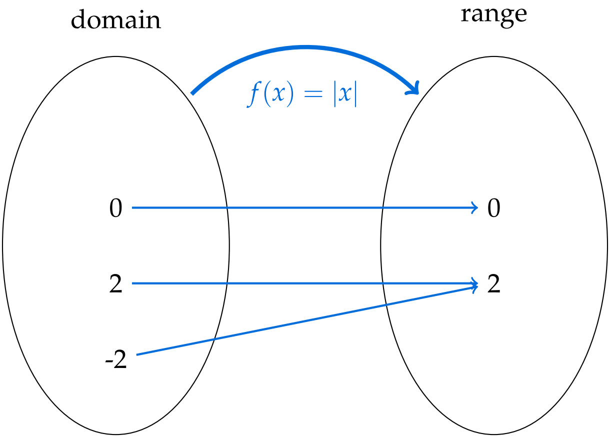

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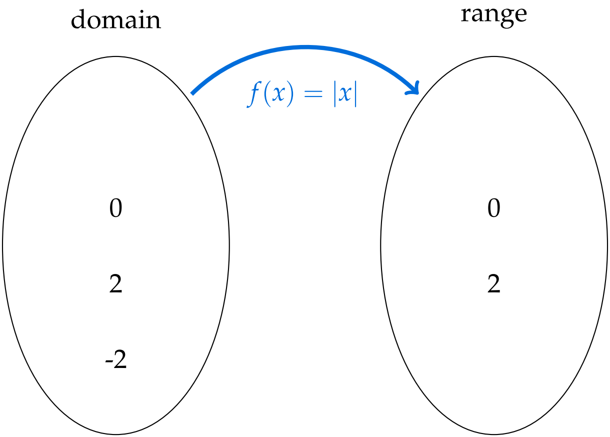

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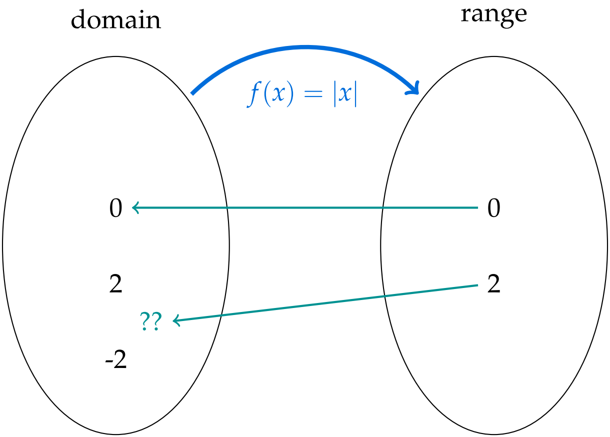

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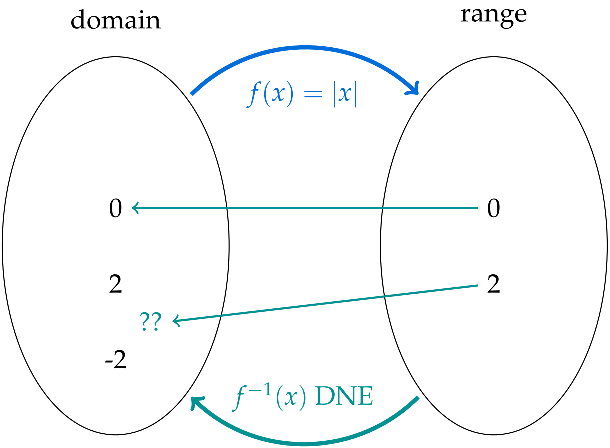
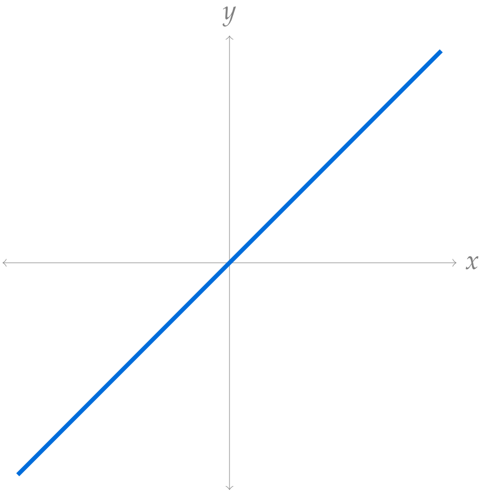
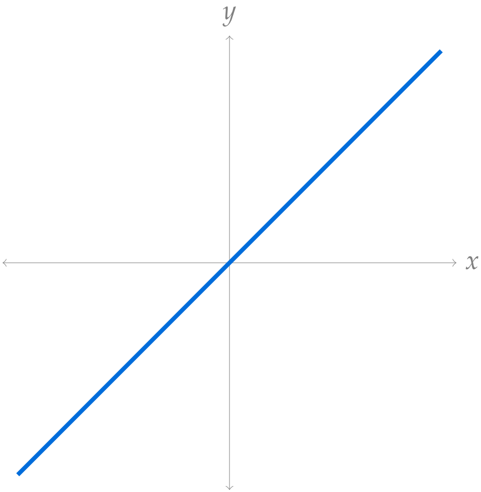




A. invertible B. not invertible



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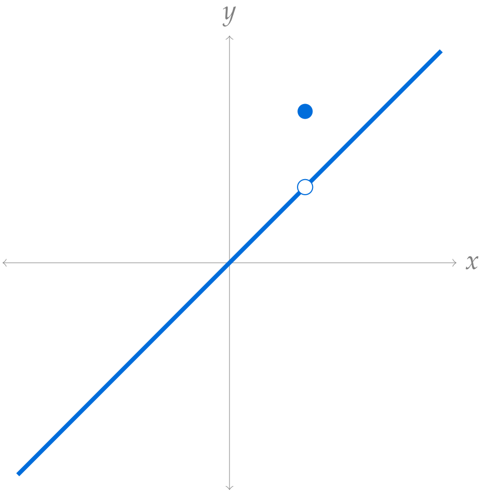

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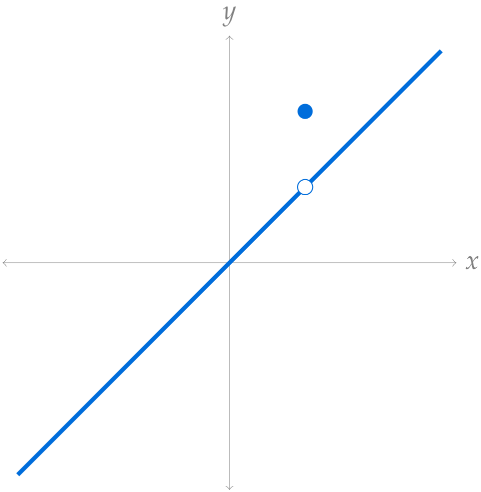

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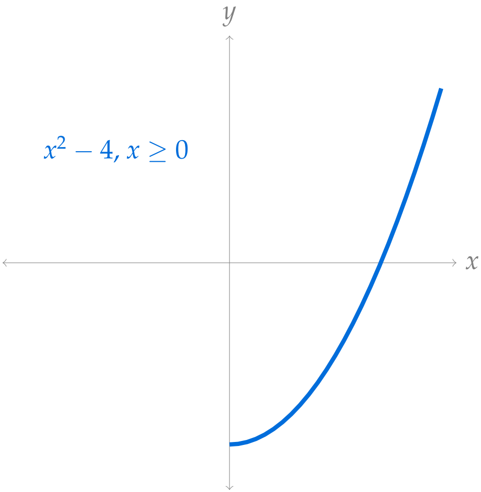

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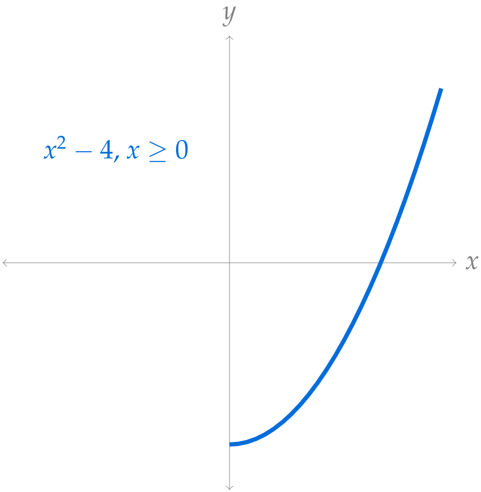

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Relationship between $f(x)$ and $f^{-1}(x)$

Let f be an invertible function.
What is $f^{-1}(f(x))$?

A. x

B. 1

C. 0

D. not sure

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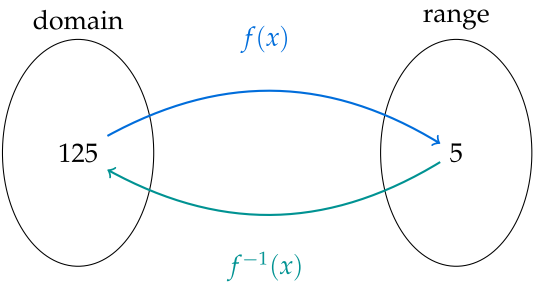

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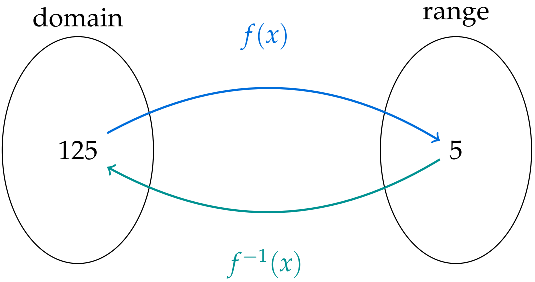

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Suppose $f(x) = \sqrt[3]{19 + x^3}$. What is $f^{-1}(3)$? (simplify your answer)

Q

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What is $f^{-1}(10)$? (do not simplify)

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What is $f^{-1}(10)$? (do not simplify)

What is $f^{-1}(x)$?

Q

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In order for a function to be invertible , different x values cannot map to the same y value.
We call such a function **one-to-one**, or **injective**.

Suppose $f(x) = \sqrt[3]{19+x^3}$. What is $f^{-1}(3)$? (simplify your answer)
$f(2) = 3$, so $f^{-1}(3) = 2$

What is $f^{-1}(10)$? (do not simplify)
$\sqrt[3]{19+y^3} = 10$ tells us $f^{-1}(10) = \sqrt[3]{10^3-19}$

What is $f^{-1}(x)$?
$\sqrt[3]{19+y^3} = x$ tells us $f^{-1}(x) = \sqrt[3]{x^3-19}$

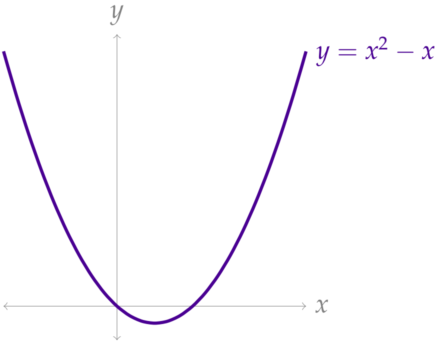
Let $f(x) = x^2 - x$.

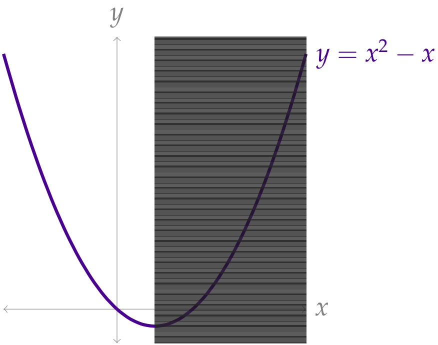
1. Sketch a graph of $f(x)$, and choose a (large) domain over which it is invertible.

2. For the domain you chose, evaluate $f^{-1}(20)$.

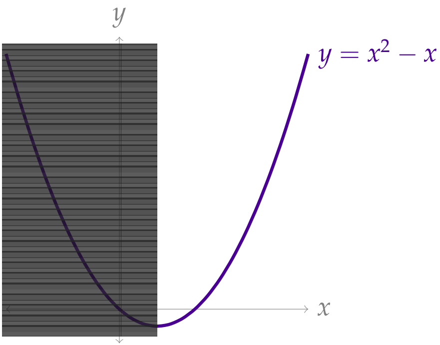
3. For the domain you chose, evaluate $f^{-1}(x)$.

4. What are the domain and range of $f^{-1}(x)$? What are the (restricted) domain and range of $f(x)$?

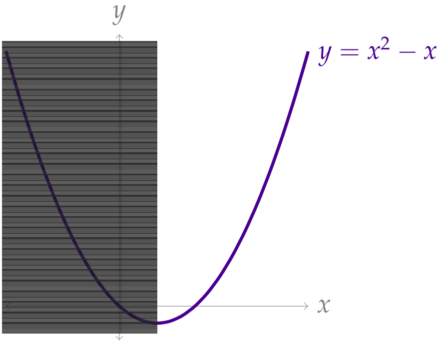

y
$y = x^2 - x$
x



Domain: $\left(-\infty, \frac{1}{2}\right]$



Domain: $\left[\frac{1}{2}, \infty\right)$



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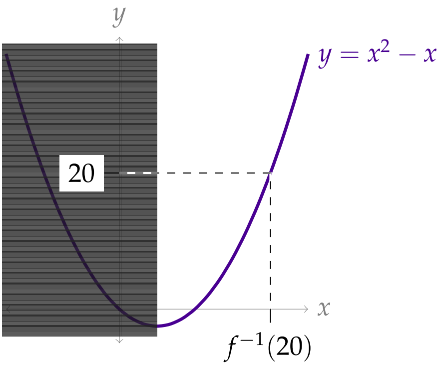
$f^{-1}(20) =$



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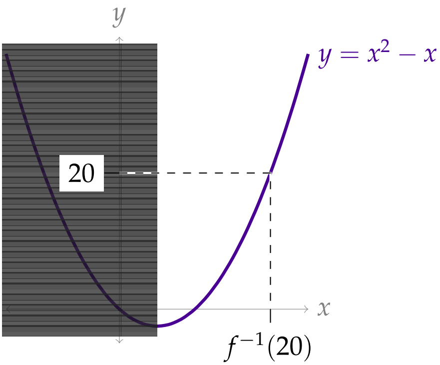
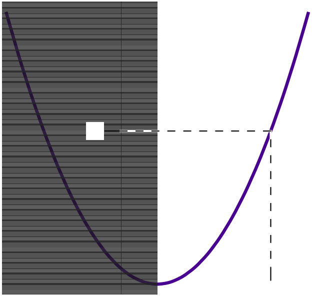


$f^{-1}(20) = 5$

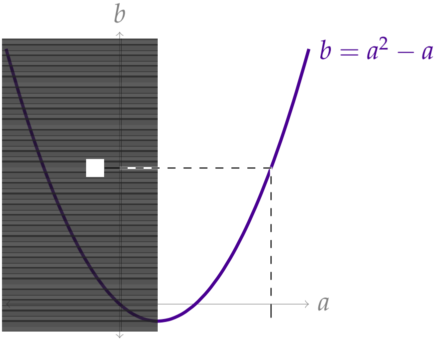
$$20 = x^2 - x$$

$$0 = x^2 - x - 20 = (x-5)(x+4)$$

$$x = 5$$



$$f^{-1}(x) =$$



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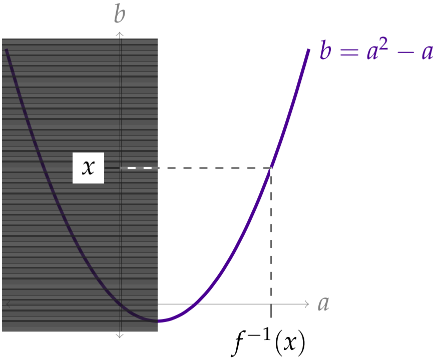

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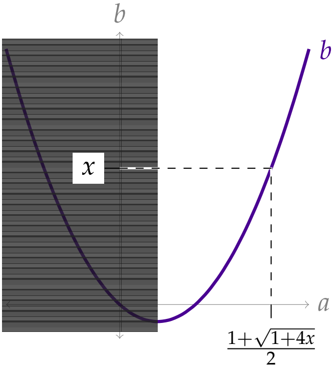


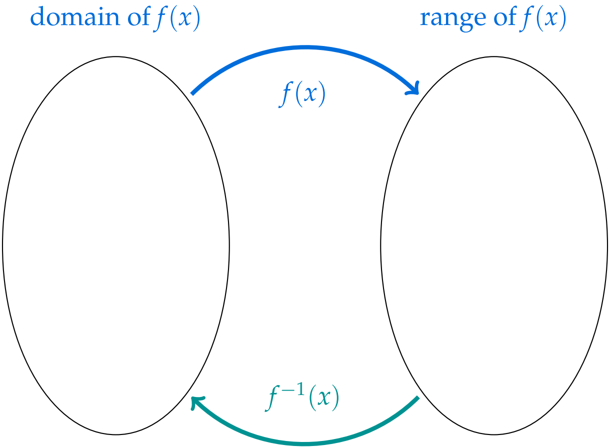
$$f^{-1}(x) = \frac{1+\sqrt{1+4x}}{2}$$

$$a^2 - a = x, \text{ find } a$$

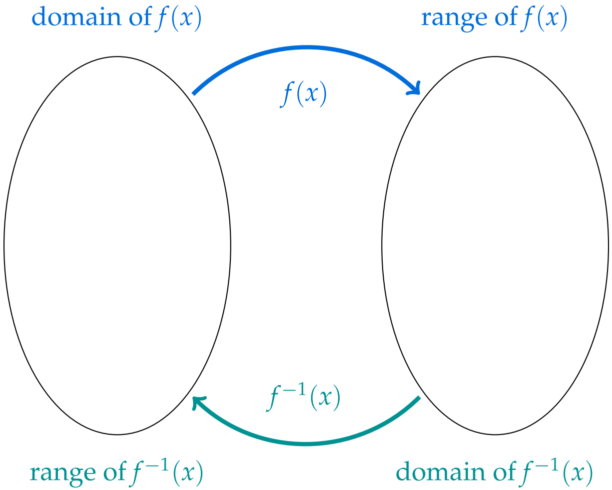
$$a^2 - a - x = 0$$

$$a = \frac{1 \pm \sqrt{1+4x}}{2}$$

$$f^{-1}(x) = \frac{1+\sqrt{1+4x}}{2}$$



domain of $f(x)$
range of $f(x)$
$f(x)$
$f^{-1}(x)$


domain of $f(x)$
range of $f(x)$
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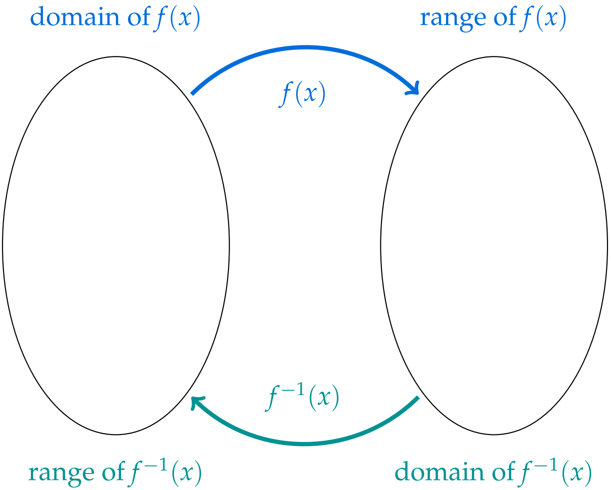
range of $f^{-1}(x)$
domain of $f^{-1}(x)$

$f(x) = x^2 - x$, domain: $\left[\frac{1}{2}, \infty\right)$



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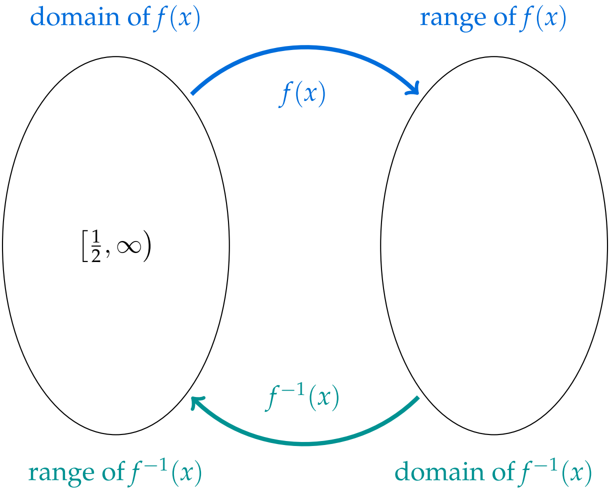

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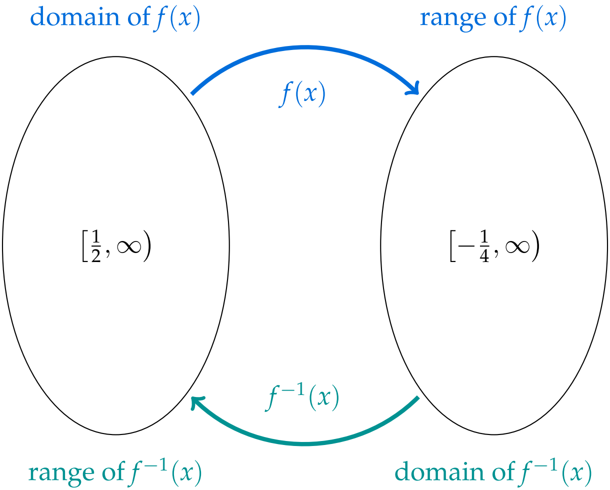
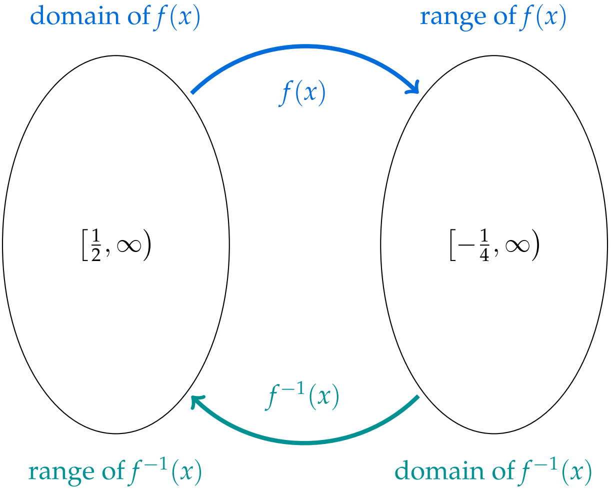


$f(x) = x^2 - x$, domain: $\left[\frac{1}{2}, \infty\right)$

$f^{-1}(x) = \frac{1+\sqrt{1+4x}}{2}$



INVERTIBILITY GAME: $f(x) = e^x$

- I'm thinking of an x. Your clue: $f(x) = e$. What is my x?

ans

INVERTIBILITY GAME: $f(x) = e^x$

- I'm thinking of an x. Your clue: $f(x) = e$. What is my x?
- I'm thinking of an x. Your clue: $f(x) = 1$. What is my x?

Invertibility game: $f(x) = e^x$

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Invertibility Game: $f(x) = e^x$

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- I'm thinking of an x. Your clue: $f(x) = e^3$. What is my x?

INVERTIBILITY GAME: $f(x) = e^x$

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- I'm thinking of an x. Your clue: $f(x) = e^3$. What is my x?
- I'm thinking of an x. Your clue: $f(x) = 0$. What is my x?

INVERTIBILITY GAME: $f(x) = e^x$

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- I'm thinking of an x. Your clue: $f(x) = 0$. What is my x? Trick question: no x gives $f(x) = 0$.

INVERTIBILITY GAME: $f(x) = e^x$ $f^{-1}(x) = \log_e x$

- I'm thinking of an x. Your clue: $f(x) = e$. What is my x? $x = 1$
- I'm thinking of an x. Your clue: $f(x) = 1$. What is my x? $x = 0$
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INVERTIBILITY GAME: $f(x) = e^x$

$f^{-1}(x) = \log_e x$

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  $\log_e(e) = 1$

- I'm thinking of an x. Your clue: $f(x) = 1$. What is my x? $x = 0$

  $\log_e(1) = 0$

- I'm thinking of an x. Your clue: $f(x) = \frac{1}{e}$. What is my x? $x = -1$

  $\log_e\left(\frac{1}{e}\right) = -1$

- I'm thinking of an x. Your clue: $f(x) = e^3$. What is my x? $x = 3$

  $\log_e(e^3) = 3$

- I'm thinking of an x. Your clue: $f(x) = 0$. What is my x? Trick question: no x gives $f(x) = 0$.

  $\log_e(x)$ is undefined at $x = 0$

1. Suppose $0 < x < 1$. Then $\log_e(x)$ is...

2. Suppose $-1 < x < 0$. Then $\log_e(x)$ is...

3. Suppose $e < x$. Then $\log_e(x)$ is...

A. positive
B. negative
C. greater than one
D. less than one
E. undefined

~~ans~~

Exponents and Logarithms

$f(x) = e^x$ $\qquad$ $f^{-1}(x) = \log_e(x) = \ln(x) = \log(x)$

EXPONENTS AND LOGARITHMS

$$f(x) = e^x \qquad f^{-1}(x) = \log_e(x) = \ln(x) = \log(x)$$

x	e^x	
0	1	
1	e	
-1	$\frac{1}{e}$	
n	e^n	

Exponents and Logarithms

$$f(x) = e^x \qquad\qquad f^{-1}(x) = \log_e(x) = \ln(x) = \log(x)$$

x	e^x	e fact $\leftrightarrow$ $\log_e$ fact	x	$\log_e(x)$
0	1			
1	e			
-1	$\frac{1}{e}$			
n	e^n			

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x	e^x	e fact $\leftrightarrow$ $\log_e$ fact	x	$\log_e(x)$
0	1	$e^0 = 1 \leftrightarrow \log_e(1) = 0$	1	0
1	e			
-1	$\frac{1}{e}$			
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$f(x) = e^x$ $\qquad$ $f^{-1}(x) = \log_e(x) = \ln(x) = \log(x)$

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1	e	$e^1 = e \leftrightarrow \log_e(e) = 1$	e	1
-1	$\frac{1}{e}$			
n	e^n			

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1	e	$e^1 = e \leftrightarrow \log_e(e) = 1$	e	1
-1	$\frac{1}{e}$	$e^{-1} = \frac{1}{e} \leftrightarrow \log_e(\frac{1}{e}) = -1$	$\frac{1}{e}$	-1
n	e^n			

EXPONENTS AND LOGARITHMS

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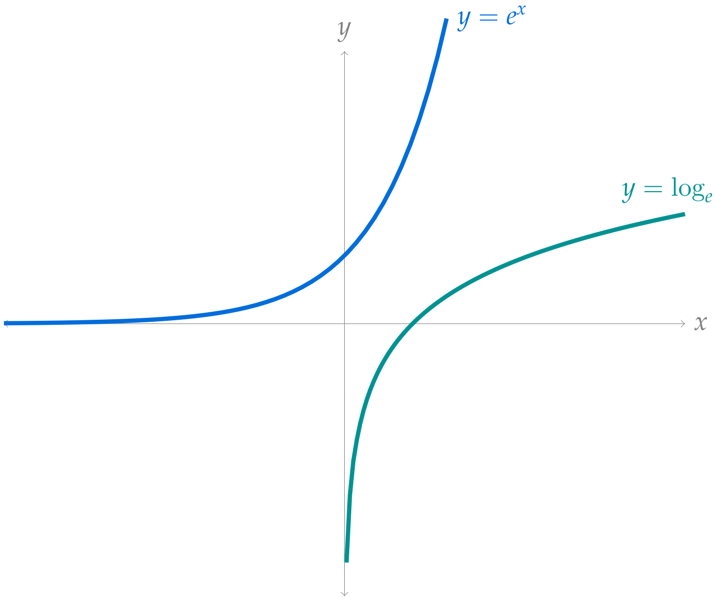
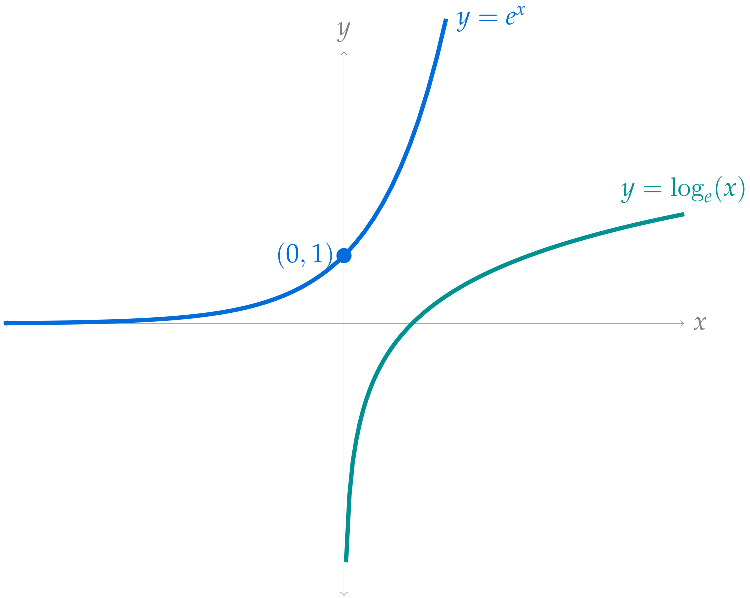
x	e^x	e fact $\leftrightarrow$ $\log_e$ fact	x	$\log_e(x)$
0	1	$e^0 = 1 \leftrightarrow \log_e(1) = 0$	1	0
1	e	$e^1 = e \leftrightarrow \log_e(e) = 1$	e	1
-1	$\frac{1}{e}$	$e^{-1} = \frac{1}{e} \leftrightarrow \log_e\left(\frac{1}{e}\right) = -1$	$\frac{1}{e}$	-1
n	e^n	$e^n = e^n \leftrightarrow \log_e(e^n) = n$	e^n	n


$y = e^x$
y
$y = \log_e(x)$
x


$y = e^x$
y
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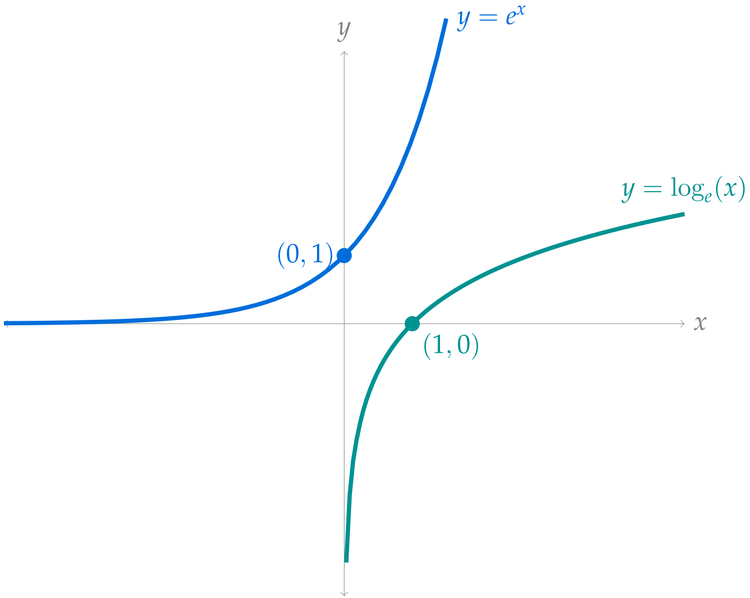
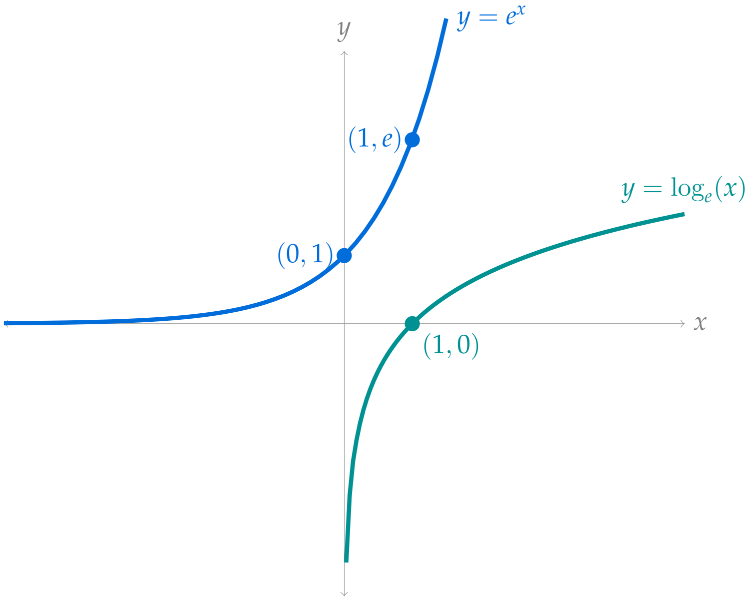
$(0, 1)$
x



$y = e^x$
y
$y = \log_e(x)$
$(0, 1)$
x
$(1, 0)$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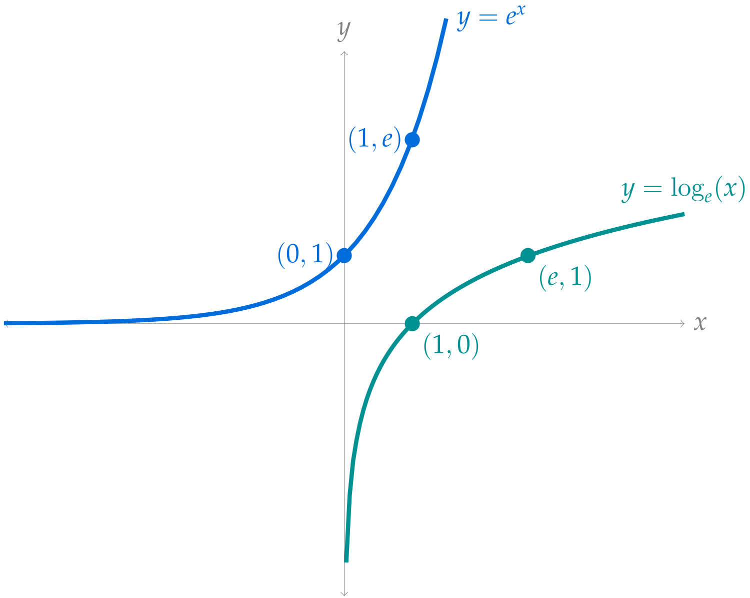

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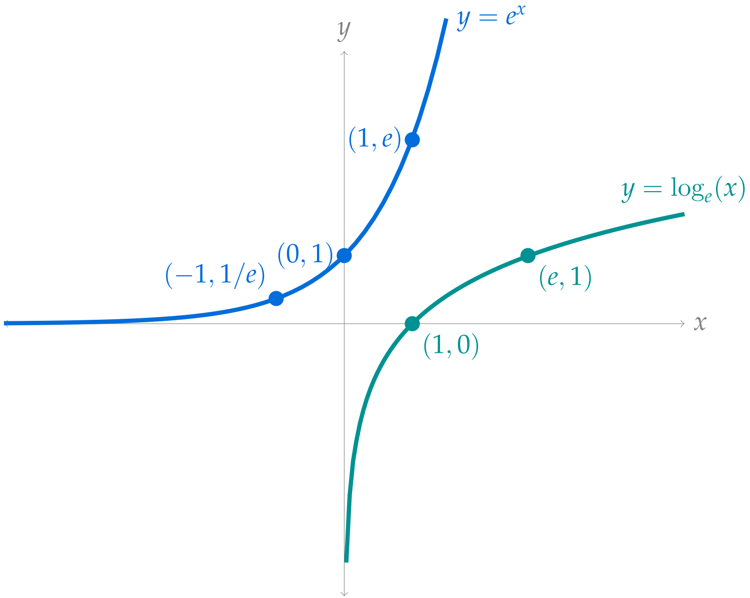
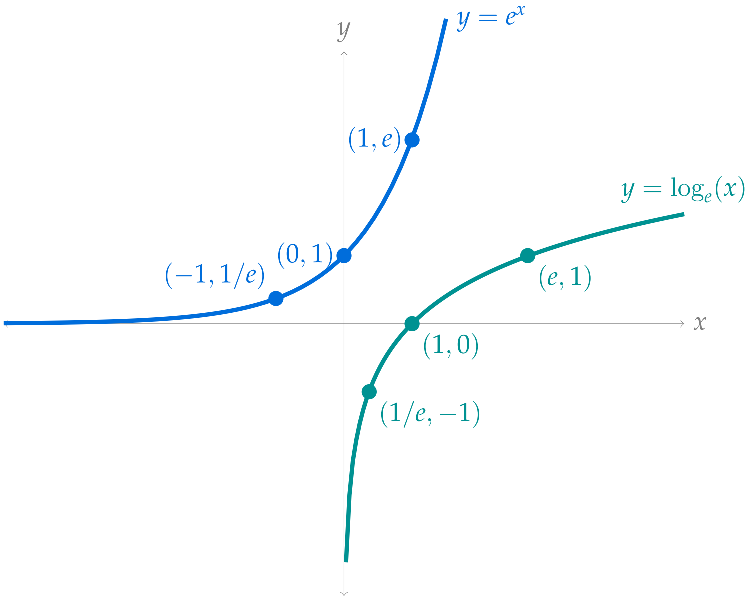
y
$(1, e)$
$y = \log_e(x)$
$(0, 1)$
$(e, 1)$
x
$(1, 0)$



$y = e^x$
y
$(1, e)$
$y = \log_e(x)$
$(0, 1)$
$(-1, 1/e)$
$(e, 1)$
x
$(1, 0)$



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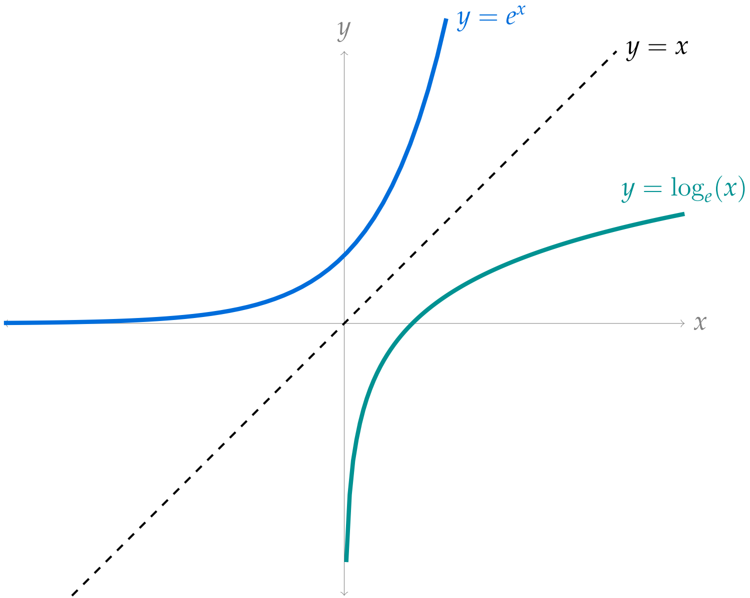
$(e, 1)$
x
$(1, 0)$
$(1/e, -1)$


$y = e^x$
y
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$y = \log_e(x)$
x

Logs of Other Bases: $\log_n(x)$ is the inverse of n^x

$\log_{10} 10^8 =$

A. 0

B. 8

C. 10

D. other

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$\log_2 16 =$

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B. 2
C. 3
D. other

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A. 0
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$\log_2 16 =$

A. 1
B. 2
C. 3
D. other ✓ $2^4 = 16$ so $\log_2 16 = 4$

Logarithm Rules

Let A and B be positive, and let n be any real number.

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Proof: $\log(A/B) = \log\left(\frac{e^{\log A}}{e^{\log B}}\right) = \log(e^{\log A - \log B}) = \log A - \log B$

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Write as a single logarithm:

$$f(x) = \log\left(\frac{10}{x^2}\right) + 2\log x + \log(10+x)$$

ans

Logarithm Rules

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$\log(A \cdot B) = \log(A) + \log(B)$
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$$\begin{aligned}
f(x) &= \log\left(\frac{10}{x^2}\right) + 2\log x + \log(10+x) \\
&= \log 10 - \log(x^2) + 2\log x + \log(10+x) \\
&= \log 10 - 2\log x + 2\log x + \log(10+x) \\
&= \log 10 + \log(10+x) = \log(10(10+x)) \\
&= \log(100+10x)
\end{aligned}$$

Base Change

Fact: $b^{\log_b(a)} = a$

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$$\Rightarrow \log\left(b^{\log_b(a)}\right) = \log(a)$$

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$$\Rightarrow \log_b(a) = \frac{\log(a)}{\log(b)}$$

Base Change

$$\begin{aligned} \text{Fact:} \qquad b^{\log_b(a)} &= a \\ \Rightarrow \log\left(b^{\log_b(a)}\right) &= \log(a) \\ \Rightarrow \log_b(a)\log(b) &= \log(a) \\ \Rightarrow \log_b(a) &= \frac{\log(a)}{\log(b)} \end{aligned}$$

In general, for positive a, b, and c:

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Suppose your calculator can only compute logarithms base 10. What would you enter to calculate $\log(17)$?

Suppose your calculator can only compute natural logarithms. What would you enter to calculate $\log_2(57)$?

Suppose your calculator can only compute logarithms base 2. What would you enter to calculate $\log(2)$?

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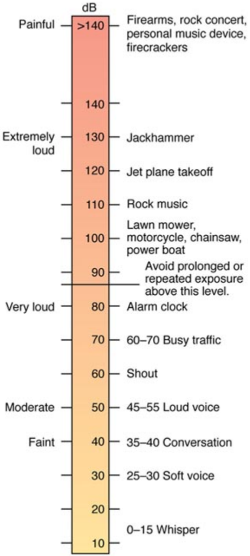
Suppose your calculator can only compute natural logarithms. What would you enter to calculate $\log_2(57)$? $\frac{\log(57)}{\log(2)}$

Suppose your calculator can only compute logarithms base 2. What would you enter to calculate $\log(2)$? $\frac{\log_2 2}{\log_2 e} = \frac{1}{\log_2 e}$



Decibels: For a particular measure of the power P of a sound wave, the decibels of that sound is:

$$10\log_{10}(P)$$

So, every ten decibels corresponds to a sound being ten **times** louder.

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A lawnmower emits a 100dB sound. How much sound will two lawnmowers make?

A. 100 dB
B. 110 dB
C. 200 dB
D. other

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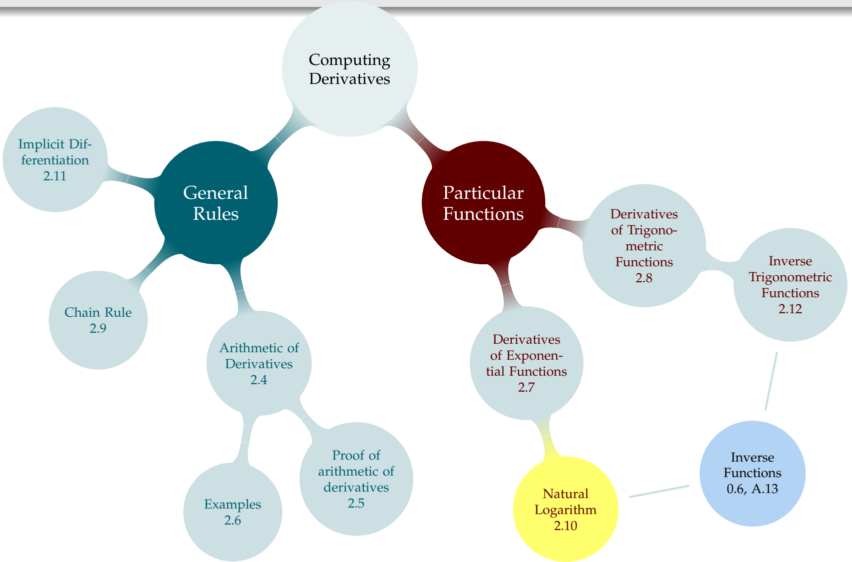

Differentiating the Natural Logarithm

Calculate $\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\}$.

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One Weird Trick:

$$x = e^{\log_e x}$$

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One Weird Trick:

$$\begin{aligned} x &= e^{\log_e x} \\ \frac{\mathrm{d}}{\mathrm{d}x}\{x\} &= \frac{\mathrm{d}}{\mathrm{d}x}\left\{e^{\log_e x}\right\} \\ 1 &= e^{\log_e x} \cdot \frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\} = x \cdot \frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\} \\ \frac{1}{x} &= \frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\} \end{aligned}$$

Differentiating the Natural Logarithm

Calculate $\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\}$.

Derivative of Natural Logarithm – Theorem 2.10.1

$$\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\} = \frac{1}{x} \qquad (x > 0)$$

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$$\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e |x|\} =$$

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Derivative of Natural Logarithm

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Differentiate: $f(x) = \log_e |x^2 + 1|$

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Differentiate: $f(x) = \log_e |x^2 + 1|$

We use the chain rule:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left\{\log_e \left|\boxed{x^2+1}\right|\right\} &= \frac{1}{x^2+1} \cdot (2x) \\ &= \frac{2x}{x^2+1}\end{aligned}$$

Derivatives of Logarithms – Corollary 2.10.6

For $a > 0$:

$$\frac{\mathrm{d}}{\mathrm{d}x}[\log_a |x|] = \frac{1}{x \log a}$$

In particular:

$$\frac{\mathrm{d}}{\mathrm{d}x}[\log |x|] = \frac{1}{x}$$

Derivatives of Logarithms – Corollary 2.10.6

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We use the chain rule:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\log_e \left|\boxed{\cot x}\right|\right] = \frac{1}{\cot x} \cdot \left(-\csc^2 x\right) = \frac{-\csc^2 x}{\cot x}$$

Logarithmic Differentiation - A Fancy Trick

- $\log(f \cdot g) = \log f + \log g$
- $\log\left(\frac{f}{g}\right) = \log f - \log g$
- $\log\left(f^g\right) = g \log f$

Logarithmic Differentiation - A Fancy Trick

- $\log(f \cdot g) = \log f + \log g$

  multiplication turns into addition

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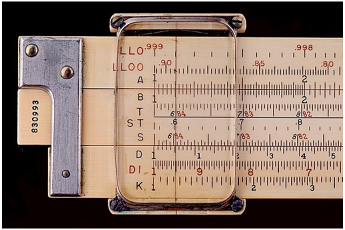
  We can exploit these properties to differentiate!

Logarithmic Differentiation

In general, if $f(x) \neq 0$, $\frac{\mathrm{d}}{\mathrm{d}x}\left[\log|f(x)|\right] = \frac{f'(x)}{f(x)}$.

$$f(x) = \left(\frac{(2x+5)^4(x^2+1)}{x+3}\right)^5$$

Find $f'(x)$.



Logarithmic Differentiation - A Fancy Trick

$$f(x) = \left(\frac{(2x+5)^4(x^2+1)}{x+3}\right)^5$$

Q

ans

Logarithmic Differentiation - A Fancy Trick

$$f(x) = \left(\frac{(2x+5)^4(x^2+1)}{x+3}\right)^5$$

$$\begin{aligned}
\log(f(x)) &= \log\left[\left(\frac{(2x+5)^4(x^2+1)}{x+3}\right)^5\right] \\
&= 5\log\left[\frac{(2x+5)^4(x^2+1)}{x+3}\right] \\
&= 5\Big[4\log(2x+5) + \log(x^2+1) - \log(x+3)\Big] \\
\frac{f'(x)}{f(x)} &= 5\left[4\frac{2}{2x+5} + \frac{2x}{x^2+1} - \frac{1}{x+3}\right] \\
f'(x) &= \left(\frac{(2x+5)^4(x^2+1)}{x+3}\right)^5 \cdot 5\left[4\frac{2}{2x+5} + \frac{2x}{x^2+1} - \frac{1}{x+3}\right]
\end{aligned}$$

Logarithmic Differentiation - A Fancy Trick

Differentiate:

$$f(x) = x^x$$

Q

ans

Logarithmic Differentiation - A Fancy Trick

Differentiate:

$$f(x) = x^x$$

$$\begin{aligned}
\log(f(x)) &= \log\left[x^x\right] \\
&= x \log x \\
\frac{f'(x)}{f(x)} &= x \cdot \frac{1}{x} + \log x \cdot 1 \\
&= 1 + \log x \\
f'(x) &= x^x\left[1 + \log x\right]
\end{aligned}$$

Logarithmic Differentiation - A Fancy Trick

Differentiate:

$$f(x) = \left(\frac{(x^{15} - 9x^2)^{10}(x + x^2 + 1)}{(x^7 + 7)(x+1)(x+2)(x+3)} \right)^5$$

Q

ans

Logarithmic Differentiation - A Fancy Trick

Differentiate:

$$f(x) = \left(\frac{(x^{15}-9x^2)^{10}(x+x^2+1)}{(x^7+7)(x+1)(x+2)(x+3)}\right)^5$$

$$\begin{aligned}
\log(f(x)) &= \log\left[\left(\frac{(x^{15}-9x^2)^{10}(x+x^2+1)}{(x^7+7)(x+1)(x+2)(x+3)}\right)^5\right]\\
&= 5\log\left[\frac{(x^{15}-9x^2)^{10}(x+x^2+1)}{(x^7+7)(x+1)(x+2)(x+3)}\right]\\
&= 5\left[10\log\left(x^{15}-9x^2\right)+\log(x+x^2+1)-\log(x^7+7)-\log(x+1)-\log(x+2)-\log(x+3)\right]\\
\frac{f'(x)}{f(x)} &= 5\left[10\frac{15x^{14}-18x}{x^{15}-9x^2}+\frac{1+2x}{x+x^2+1}-\frac{7x^6}{x^7+7}-\frac{1}{x+1}-\frac{1}{x+2}-\frac{1}{x+3}\right]\\
f'(x) &= \left(\frac{(x^{15}-9x^2)^{10}(x+x^2+1)}{(x^7+7)(x+1)(x+2)(x+3)}\right)^5\cdot 5\left[10\frac{15x^{14}-18x}{x^{15}-9x^2}+\frac{1+2x}{x+x^2+1}-\frac{7x^6}{x^7+7}-\frac{1}{x+1}-\frac{1}{x+2}-\frac{1}{x+3}\right]
\end{aligned}$$

$$f(x) = \frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x}$$

Q

ans

$$f(x) = \frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x}$$

$$\begin{aligned}
\frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x} &= f(x) \\
\log\left|\frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x}\right| &= \log|f(x)| \\
\left[\log|x^8 - e^x| + \log|x^{1/2} + 5| - 5\log|\csc x|\right] &= \log|f(x)| \\
\frac{\mathrm{d}}{\mathrm{d}x}\left[\log|x^8 - e^x| + \log|x^{1/2} + 5| - 5\log|\csc x|\right] &= \frac{\mathrm{d}}{\mathrm{d}x}\log|f(x)| \\
\frac{8x^7 - e^x}{x^8 - e^x} + \frac{\frac{1}{2}x^{-1/2}}{x^{1/2} + 5} - 5\frac{-\csc x\cot x}{\csc x} &= \frac{f'(x)}{f(x)} \\
\frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x}\left(\frac{8x^7 - e^x}{x^8 - e^x} + \frac{\frac{1}{2}x^{-1/2}}{x^{1/2} + 5} + 5\frac{\csc x\cot x}{\csc x}\right) &= f'(x)
\end{aligned}$$

$$f(x) = (x^2 + 17)(32x^5 - 8)(x^{98} - x^{57} + 32x^2)^4(32x^{10} - 10x^{32})$$

Find $f'(x)$.

Q

ans

$$f(x) = (x^2+17)(32x^5-8)(x^{98}-x^{57}+32x^2)^4(32x^{10}-10x^{32})$$

Find $f'(x)$.

$$\begin{aligned}
(x^2+17)(32x^5-8)(x^{98}-x^{57}+32x^2)^4(32x^{10}-10x^{32}) &= f(x) \\
\log\left|(x^2+17)(32x^5-8)(x^{98}-x^{57}+32x^2)^4(32x^{10}-10x^{32})\right| &= \log|f(x)| \\
\log|x^2+17| + \log|32x^5-8| + 4\log|x^{98}-x^{57}+32x^2| + \log|32x^{10}-10x^{32}| &= \log|f(x)| \\
\frac{\mathrm{d}}{\mathrm{d}x}\left[\log|x^2+17| + \log|32x^5-8| + 4\log|x^{98}-x^{57}+32x^2| + \log|32x^{10}-10x^{32}|\right] &= \frac{\mathrm{d}}{\mathrm{d}x}[\log|f(x)|] \\
\frac{2x}{x^2+17} + \frac{160x^4}{32x^5-8} + 4\frac{98x^{97}-57x^{56}+64x}{x^{98}-x^{57}+32x^2} + \frac{320x^9-320x^{31}}{32x^{10}-10x^{32}} &= \frac{f'(x)}{f(x)} \\
\left((x^2+17)(32x^5-8)(x^{98}-x^{57}+32x^2)^4(32x^{10}-10x^{32})\right)\cdot & \\
\left(\frac{2x}{x^2+17} + \frac{160x^4}{32x^5-8} + 4\frac{98x^{97}-57x^{56}+64x}{x^{98}-x^{57}+32x^2} + \frac{320x^9-320x^{31}}{32x^{10}-10x^{32}}\right) &= f'(x)
\end{aligned}$$

A

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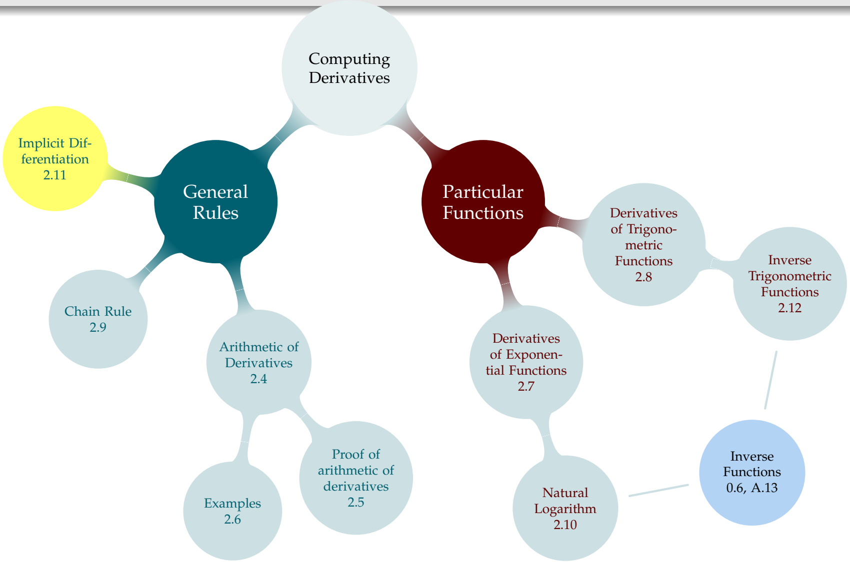

Implicitly Defined Functions

$$y^2 + x^2 + xy + x^2y = 1$$

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Which of the following points are on the curve?

$(0, 1)$, $(0, -1)$, $(0, 0)$, $(1, 1)$

Q

ans

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$(0,1)$ and $(0,-1)$

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Which of the following points are on the curve?

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$(0, 1)$ and $(0, -1)$

If $x = -3$, what is y?

Q

ans

Implicitly Defined Functions

$$y^2 + x^2 + xy + x^2y = 1$$

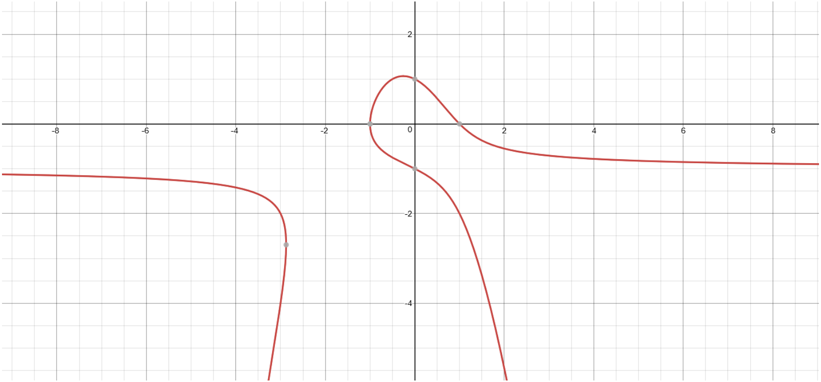
Which of the following points are on the curve?

$(0, 1)$, $(0, -1)$, $(0, 0)$, $(1, 1)$

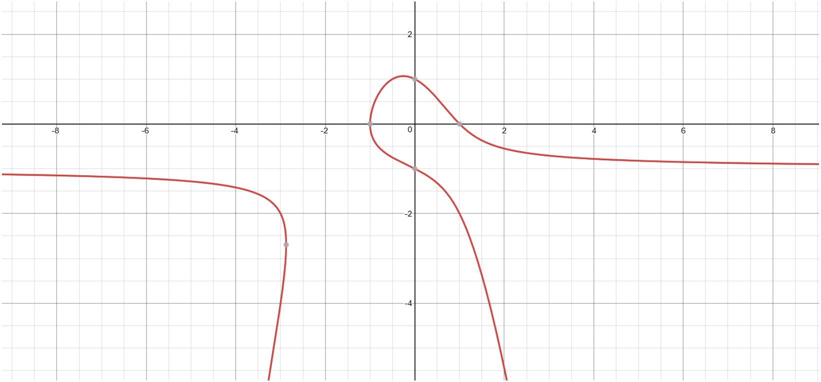
$(0, 1)$ and $(0, -1)$

If $x = -3$, what is y? $y = -2$ and $y = -4$

$$y^2 + x^2 + xy + x^2y = 1$$

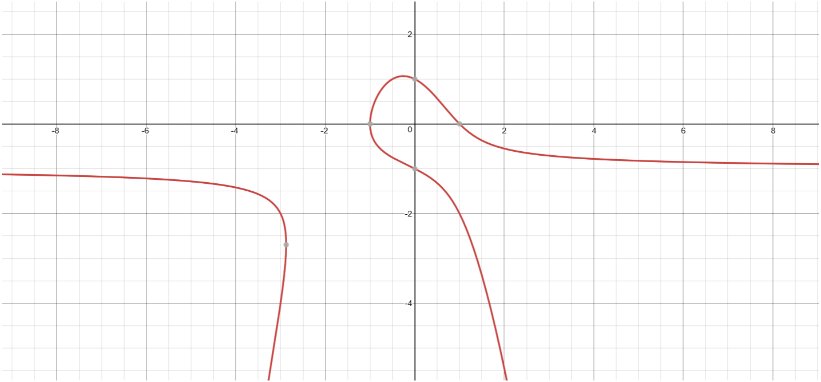


$$y^2 + x^2 + xy + x^2y = 1$$



Still has a slope: $\frac{\Delta y}{\Delta x}$

$$y^2 + x^2 + xy + x^2y = 1$$



Still has a slope: $\frac{\Delta y}{\Delta x}$
Locally, y is still a function of x.

$$y^2 + x^2 + xy + x^2y = 1$$

Consider y as a function of x. Can we find $\frac{dy}{dx}$?

$$\frac{d}{dx}[y] =$$

ans

$$y^2 + x^2 + xy + x^2y = 1$$

Consider y as a function of x. Can we find $\frac{\mathrm{d}y}{\mathrm{d}x}$?

$$\frac{\mathrm{d}}{\mathrm{d}x}[y] = \frac{\mathrm{d}y}{\mathrm{d}x} = y'$$

$$y^2 + x^2 + xy + x^2y = 1$$

Consider y as a function of x. Can we find $\frac{dy}{dx}$?

$$\frac{d}{dx}[y] = \frac{dy}{dx} = y' \qquad \frac{d}{dx}[x] =$$

$$y^2 + x^2 + xy + x^2y = 1$$

Consider y as a function of x. Can we find $\frac{dy}{dx}$?

$$\frac{d}{dx}[y] = \frac{dy}{dx} = y' \qquad \frac{d}{dx}[x] = 1$$

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Consider y as a function of x. Can we find $\frac{dy}{dx}$?

$\frac{d}{dx}[y] = \frac{dy}{dx} = y'$ $\qquad$ $\frac{d}{dx}[x] = 1$ $\qquad$ $\frac{d}{dx}[1] =$

$$y^2 + x^2 + xy + x^2y = 1$$

Consider y as a function of x. Can we find $\frac{dy}{dx}$?

$$\frac{d}{dx}[y] = \frac{dy}{dx} = y' \qquad \frac{d}{dx}[x] = 1 \qquad \frac{d}{dx}[1] = 0$$

ans

$$y^2 + x^2 + xy + x^2y = 1$$

Consider y as a function of x. Can we find $\frac{dy}{dx}$?

$$\frac{d}{dx}[y] = \frac{dy}{dx} = y' \qquad \frac{d}{dx}[x] = 1 \qquad \frac{d}{dx}[1] = 0$$

Differentiate both sides with respect to x.

$$\begin{aligned}
0 &= 2y\frac{dy}{dx} + 2x + \left(x\frac{dy}{dx} + (1)y\right) + \left(x^2\frac{dy}{dx} + 2xy\right) \\
0 &= \frac{dy}{dx}\left(2y + x + x^2\right) + (2x + y + 2xy) \\
-(2x + y + 2xy) &= \frac{dy}{dx}\left(2y + x + x^2\right) \\
-\frac{2x + y + 2xy}{2y + x + x^2} &= \frac{dy}{dx}
\end{aligned}$$

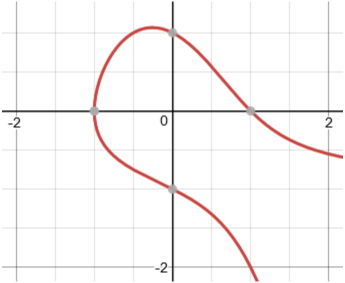
$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{dy}{dx} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

Necessarily, $\frac{dy}{dx}$ depends on **both** y and x. Why?



ans

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

Necessarily, $\frac{\mathrm{d}y}{\mathrm{d}x}$ depends on **both** y and x. Why?

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(1,0)} = \qquad \qquad \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(1,-2)} =$$

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{dy}{dx} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

Necessarily, $\frac{dy}{dx}$ depends on **both** y and x. Why?

$$\left.\frac{dy}{dx}\right|_{(1,0)} = -\frac{2(1) + 0 + 2(1)(0)}{2(0) + 1 + 1} = -\frac{2}{2} = -1$$

$$\left.\frac{dy}{dx}\right|_{(1,-2)} = -\frac{2(1) - 2 + 2(1)(-2)}{2(-2) + 1 + 1} = -2$$

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

Necessarily, $\frac{\mathrm{d}y}{\mathrm{d}x}$ depends on **both** y and x. Why?

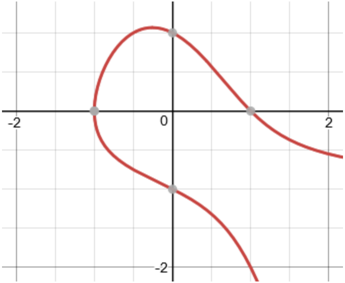
$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(1,0)} = -\frac{2(1) + 0 + 2(1)(0)}{2(0) + 1 + 1} = -\frac{2}{2} = -1$$

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(1,-2)} = -\frac{2(1) - 2 + 2(1)(-2)}{2(-2) + 1 + 1} = -2$$

Points with the same x-value may have different slopes. We need both the x-value and the y-value to figure out which point we're talking about.

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x + y + 2xy}{2y + x + x^2}$$



Points with the same x-value may have different slopes. We need both the x-value and the y-value to figure out which point we're talking about.

Now You

Suppose $x^4y + y^4x = 2$. Find $\frac{dy}{dx}$ at the point $(1, 1)$.

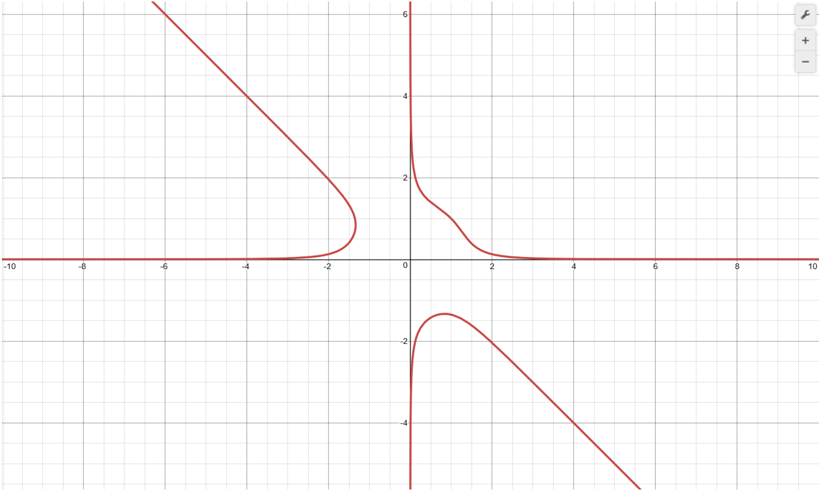
Now You

Suppose $x^4y + y^4x = 2$. Find $\frac{dy}{dx}$ at the point $(1, 1)$.

$$
\begin{aligned}
x^4y(x) + y(x)^4x &= 2 \\
4x^3y(x) + x^4\frac{dy}{dx}(x) + y(x)^4 + 4y(x)^3\frac{dy}{dx}(x)\,x &= 0
\end{aligned}
$$

We may only replace variables with constants *after* differentiating.
When $x = 1$ and $y(1) = 1$,

$$
\begin{aligned}
4(1)^3y(1) + (1)^4\frac{dy}{dx}(1) + y(1)^4 + 4y(1)^3\frac{dy}{dx}(1) &= 0 \\
4 + \frac{dy}{dx}(1) + 1 + 4\frac{dy}{dx}(1) &= 0 \\
5\frac{dy}{dx}(1) &= -5 \\
\frac{dy}{dx}(1) &= -1
\end{aligned}
$$



$$x^4y + y^4x = 2$$

Now You Suppose $\dfrac{3y^2+2y+y^3}{x^2+1}=x$. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $x=0$, and the equations of the associated tangent line(s).

Now You Suppose $\dfrac{3y^2+2y+y^3}{x^2+1}=x$. Find $\dfrac{dy}{dx}$ when $x=0$, and the equations of the associated tangent line(s).

To avoid the quotient rule, we start by simplifying our expression.

$$\frac{3y(x)^2+2y(x)+y(x)^3}{x^2+1}=x$$

$$3y(x)^2+2y(x)+y(x)^3=x^3+x$$

$$6y(x)\frac{dy}{dx}(x)+2\frac{dy}{dx}(x)+3y(x)^2\frac{dy}{dx}(x)=3x^2+1$$

When $x=0$:

$$\frac{dy}{dx}(0)=\frac{1}{6y(0)+2+3y(0)^2}$$

We need to know y to find $\frac{dy}{dx}$. We want all points where $x = 0$.

$$\begin{aligned} 3y(0)^2 + 2y(0) + y(0)^3 &= 0 \\ y(0)\left(y(0)^2 + 3y(0) + 2\right) &= 0 \\ y(0)\left(y(0)+1\right)\left(y(0)+2\right) &= 0 \\ y(0) = 0,\ y(0) = -1,\ y(0) &= -2 \end{aligned}$$

A

$$\frac{dy}{dx} = \frac{1}{6y + 2 + 3y^2}$$

$(0, 0)$

$$\left.\frac{dy}{dx}\right|_{(0,0)} = \frac{1}{2}$$

$$y - 0 = \frac{1}{2}(x - 0)$$

$$y = \frac{1}{2}x$$

$(0, -1)$

$$\left.\frac{dy}{dx}\right|_{(0,-1)} = \frac{1}{-6+2+3} = -1$$

$$y - (-1) = -1(x - 0)$$

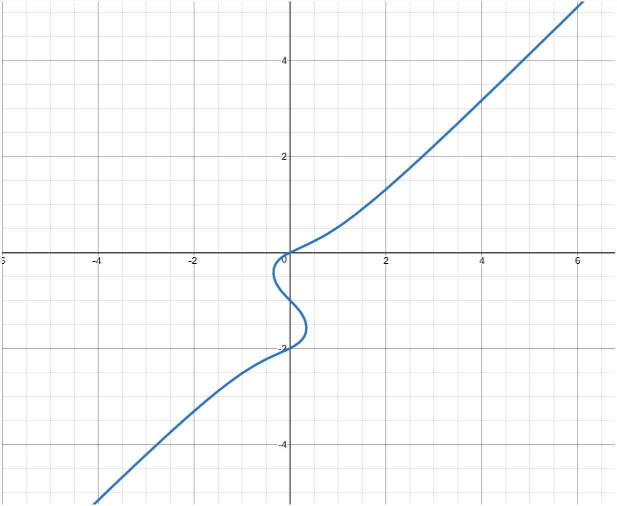
$$y = -x - 1$$

$(0, -2)$

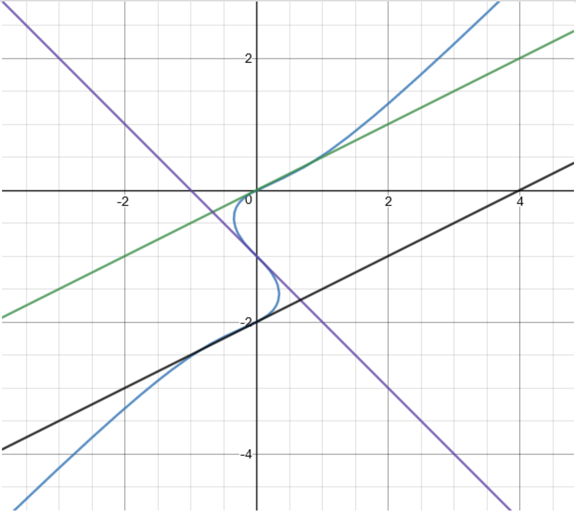
$$\left.\frac{dy}{dx}\right|_{(0,-2)} = \frac{1}{-12+2+12} = \frac{1}{2}$$

$$y - (-2) = \frac{1}{2}(x - 0)$$

$$y = \frac{1}{2}x - 2$$



$$\frac{3y^2 + 2y + y^3}{x^2 + 1} = x$$



$$\frac{3y^2 + 2y + y^3}{x^2 + 1} = x$$

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

$$\begin{aligned} \log x &= y(x) \\ x &= e^{y(x)} \end{aligned}$$

ans

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

$$\begin{aligned}
\log x &= y(x) \\
x &= e^{y(x)} \\
1 &= e^{y(x)} \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{1}{e^{y(x)}} = \frac{1}{x}
\end{aligned}$$

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

$$\begin{aligned} \log x &= y(x) \\ x &= e^{y(x)} \\ 1 &= e^{y(x)} \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\ \frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{1}{e^{y(x)}} = \frac{1}{x} \end{aligned}$$

Use implicit differentiation to differentiate $\log|x|$, $x < 0$.

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

Use implicit differentiation to differentiate $\log|x|$, $x < 0$.

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

Use implicit differentiation to differentiate $\log|x|$, $x < 0$.

$$\begin{aligned}
\log|x| &= y(x)\\
\log(-x) &= y(x)\\
-x &= e^{y(x)}\\
-1 = e^{y(x)} \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x)\\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{-1}{e^{y(x)}} = \frac{-1}{-x} = \frac{1}{x}
\end{aligned}$$

Use implicit differentiation to differentiate $\log_a(x)$, where $a > 0$ is a constant and $x > 0$.

Use implicit differentiation to differentiate $\log_a |x|$, $a > 0$.

Use implicit differentiation to differentiate $\log_a(x)$, where $a > 0$ is a constant and $x > 0$.

Use implicit differentiation to differentiate $\log_a(x)$, where $a > 0$ is a constant and $x > 0$.

$$\begin{aligned}
\log_a x &= y(x) \\
x &= a^{y(x)} \\
1 &= a^{y(x)} \cdot \log_e a \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{1}{a^{y(x)} \cdot \log_e a} = \frac{1}{x \log_e a}
\end{aligned}$$

Use implicit differentiation to differentiate $\log_a |x|$, $a > 0$.

Use implicit differentiation to differentiate $\log_a |x|$, $a > 0$.

If $x > 0$, it's what we just computed. So assume $x < 0$.

$$\begin{aligned}
\log_a |x| &= y(x) \\
\log_a(-x) &= y(x) \\
-x &= a^{y(x)} \\
-1 &= a^{y(x)} \cdot \log_e a \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{-1}{a^{y(x)} \cdot \log_e a} = \frac{-1}{x \log_e a}
\end{aligned}$$

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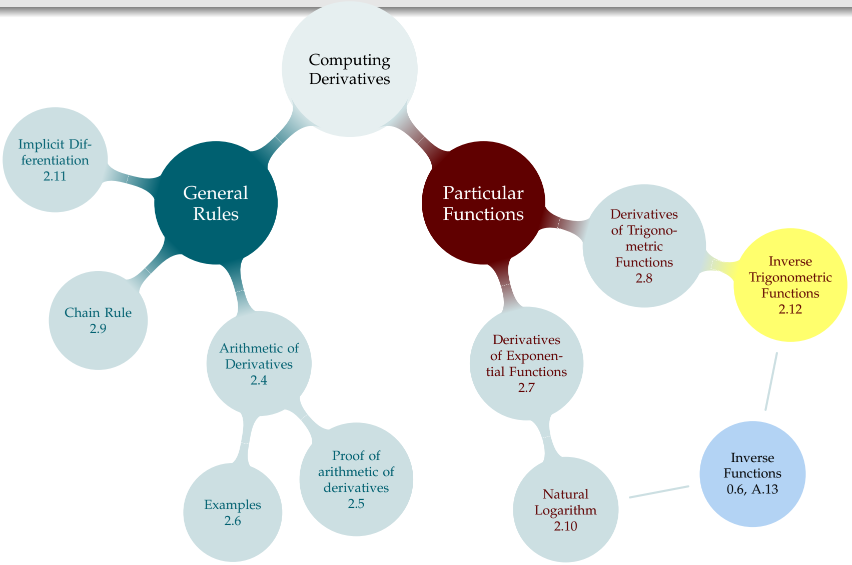
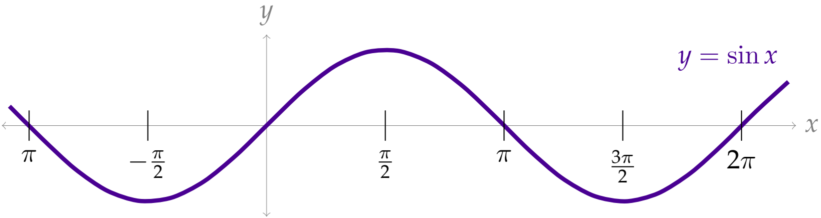
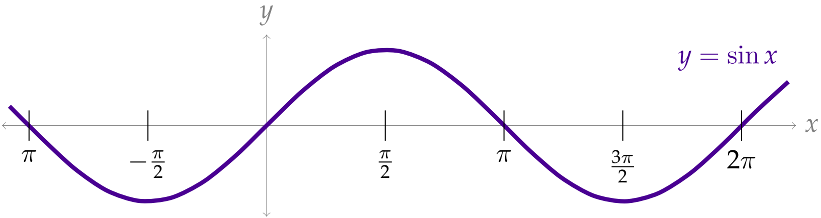
SKIP DEFINITIONS OF INVERSE TRIG FUNCTIONS



Invertibility Game

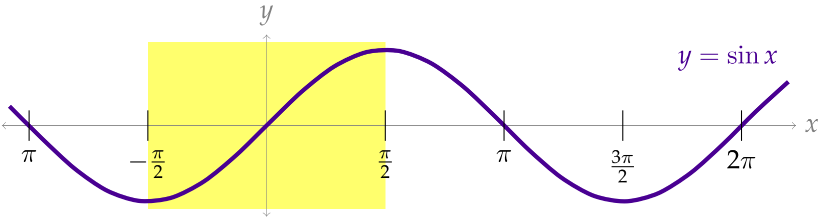


INVERTIBILITY GAME



I'm thinking of a number x. Your hint: $\sin(x) = 0$. What number am I thinking of?

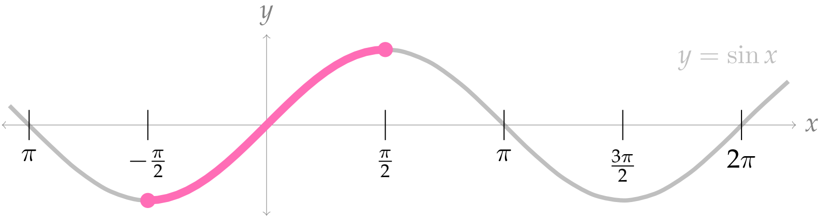
Invertibility Game



I'm thinking of a number x. Your hint: $\sin(x) = 0$. What number am I thinking of?

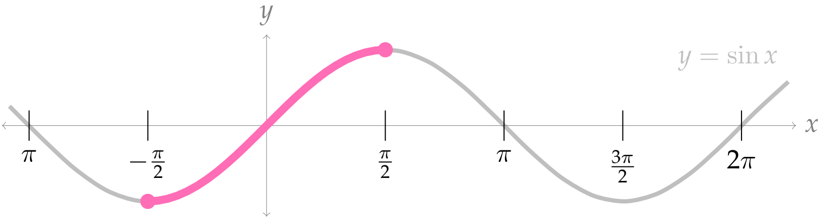
I'm thinking of a number x, and x is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Your hint: $\sin(x) = 0$. What number am I thinking of?

Arcsine



$\arcsin(x)$ is the inverse of $\sin x$ restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Arcsine



$\arcsin(x)$ is the inverse of $\sin x$ restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\arcsin x$ is the (unique) number θ such that:

- $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, and
- $\sin \theta = x$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
— $\frac{\pi}{6}$	— $\frac{1}{2}$
— $\frac{\pi}{4}$	— $\frac{1}{\sqrt{2}}$
— $\frac{\pi}{3}$	— $\frac{\sqrt{3}}{2}$
— $\frac{\pi}{2}$	— 1

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0)$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right)$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right)$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$
- $\arcsin\left(\frac{\pi}{2}\right)$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$
- $\arcsin\left(\frac{\pi}{2}\right)$ undefined

ARCSINE

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$
- $\arcsin\left(\frac{\pi}{2}\right)$ undefined
- $\arcsin\left(\frac{\pi}{4}\right)$

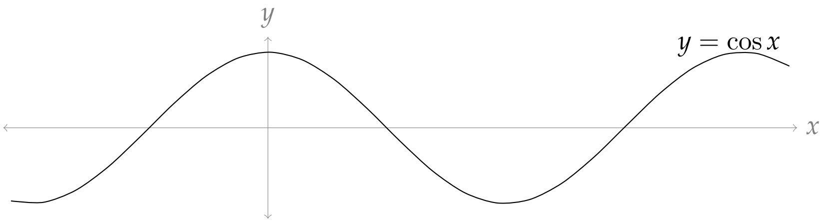
Arcsine

Reference Angles:

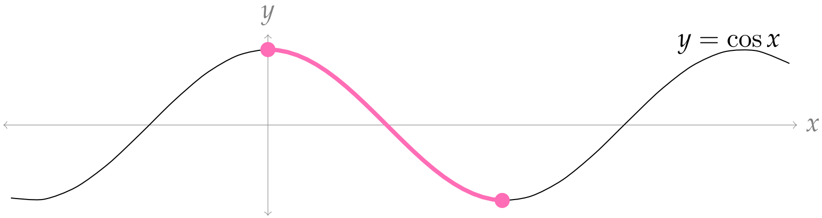
θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$
- $\arcsin\left(\frac{\pi}{2}\right)$ undefined
- $\arcsin\left(\frac{\pi}{4}\right)$ defined, but we haven't covered tools (yet) to figure it out

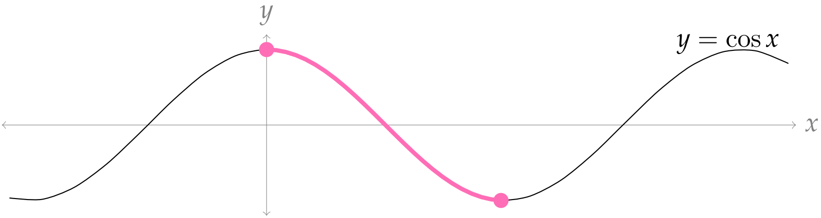
Arccosine



Arccosine



Arccosine

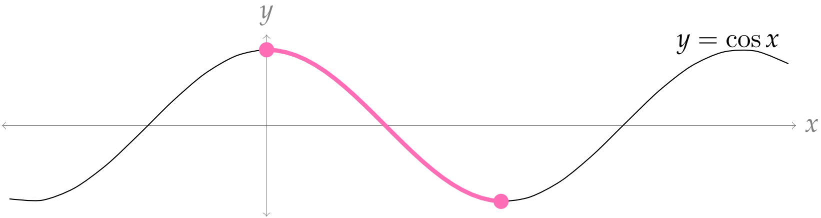


$\arccos(x)$ is the inverse of $\cos x$ restricted to $[0, \pi]$.

$\arccos(x)$ is the (unique) number θ such that:

- $\cos(\theta) = x$ and
- $0 \leq \theta \leq \pi$

Arccosine

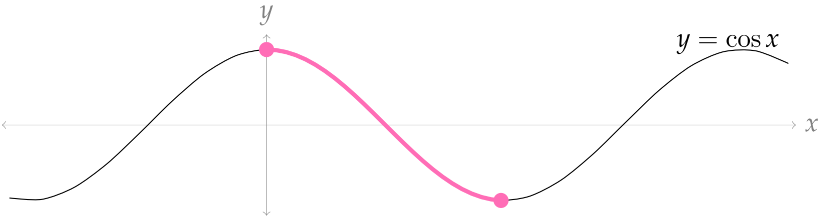


$\arccos(x)$ is the inverse of $\cos x$ restricted to $[0, \pi]$.

$\arccos(x)$ is the (unique) number θ such that:

- $\cos(\theta) = x$ and $\leftarrow\leftarrow\leftarrow$ inverse
- $0 \leq \theta \leq \pi$

Arccosine

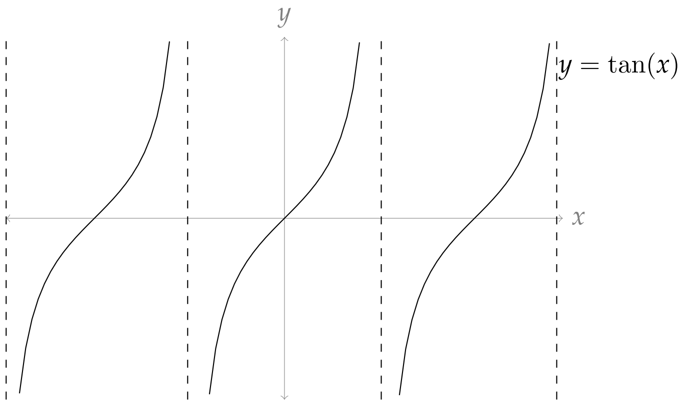


$\arccos(x)$ is the inverse of $\cos x$ restricted to $[0, \pi]$.

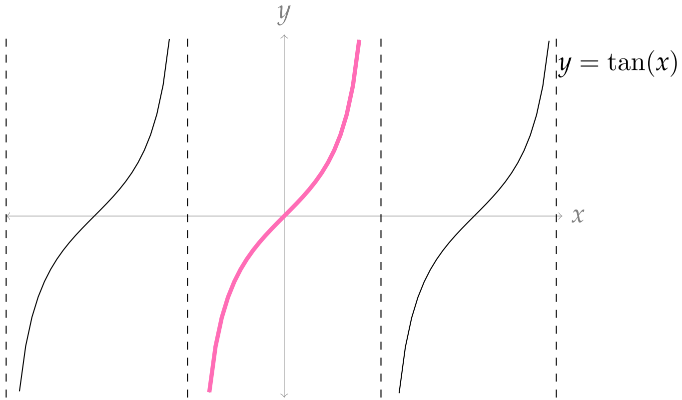
$\arccos(x)$ is the (unique) number θ such that:

- $\cos(\theta) = x$ and $\quad\leftarrow\leftarrow\leftarrow$ inverse
- $0 \leq \theta \leq \pi$ $\quad\leftarrow\leftarrow\leftarrow$ inverse exists

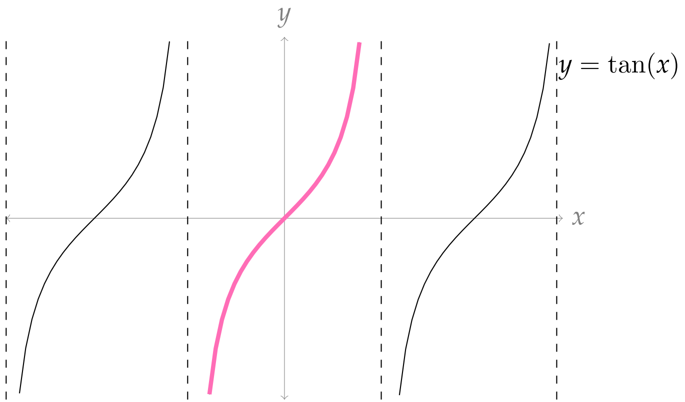
Arctangent



Arctangent



ARCTANGENT



$\arctan(x) = \theta$ means:

(1) $\tan(\theta) = x$ and

(2) $-\pi/2 < \theta < \pi/2$

Arcsecant, arcsine, and arccotangent

$$\text{arcsec}(x) =$$

Arcsecant, arcsine, and arccotangent

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$$\begin{aligned}
\text{arcsec}(x) &= y \\
\sec y &= x \\
\frac{1}{\cos y} &= x \\
\cos y &= \tfrac{1}{x} \\
y &= \arccos\left(\tfrac{1}{x}\right) \\
\text{arcsec}(x) &= \arccos\left(\tfrac{1}{x}\right)
\end{aligned}$$

Arcsecant, arcsine, and arccotangent

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Arcsecant, arcsine, and arccotangent

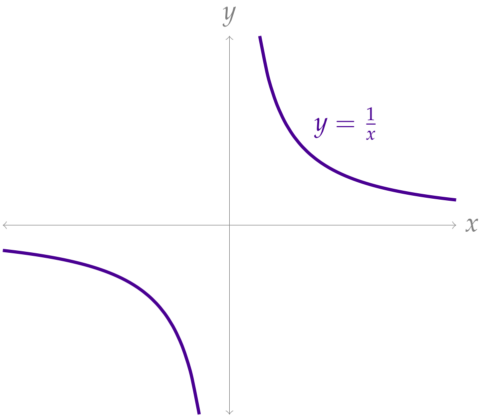
$$\operatorname{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$

$$\begin{aligned}
\operatorname{arccsc}(x) &= y \\
\csc y &= x \\
\frac{1}{\sin y} &= x \\
\sin y &= \frac{1}{x} \\
y &= \arcsin\left(\frac{1}{x}\right) \\
\operatorname{arccsc}(x) &= \arcsin\left(\frac{1}{x}\right)
\end{aligned}$$

$$\operatorname{arccot}(\mathrm{x}) = \arctan\left(\frac{1}{x}\right)$$

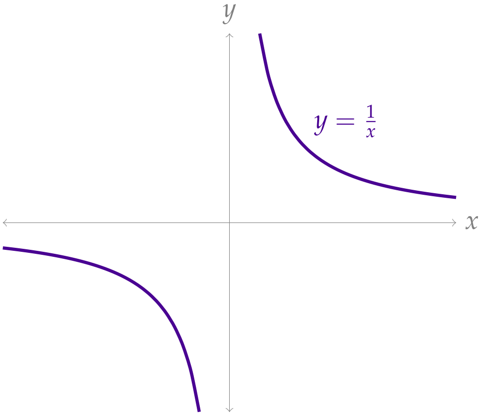
$$\begin{aligned}
\operatorname{arccot}(\mathrm{x}) &= y \\
\cot y &= x \\
\frac{1}{\tan y} &= x \\
\tan y &= \frac{1}{x} \\
y &= \arctan\left(\frac{1}{x}\right) \\
\operatorname{arccot}(\mathrm{x}) &= \arctan\left(\frac{1}{x}\right)
\end{aligned}$$

$$\text{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$$



$$\text{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$$

The domain of $\arccos(y)$ is $-1 \leq y \leq 1$, so the domain of $\text{arcsec}(y)$ is



ans

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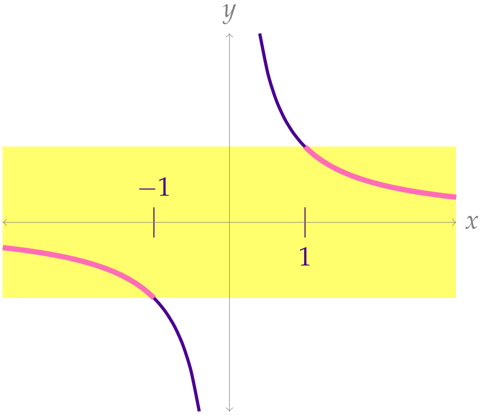
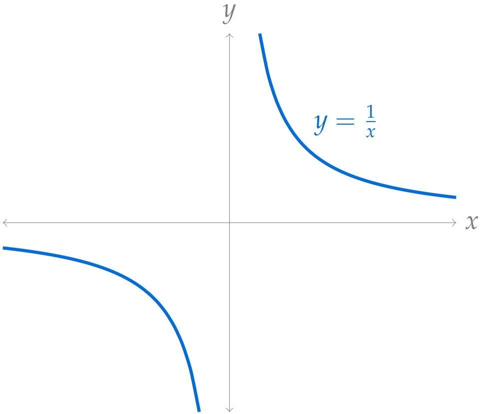


$(-\infty, -1] \cup [1, \infty)$.

$$\operatorname{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$



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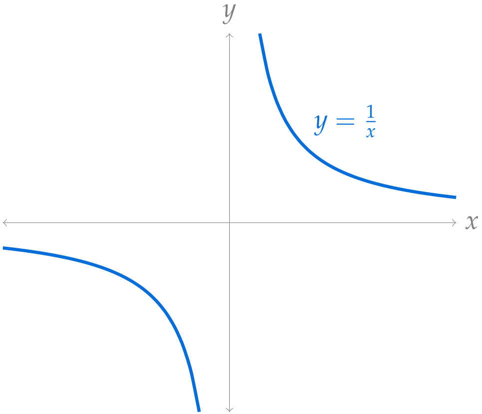

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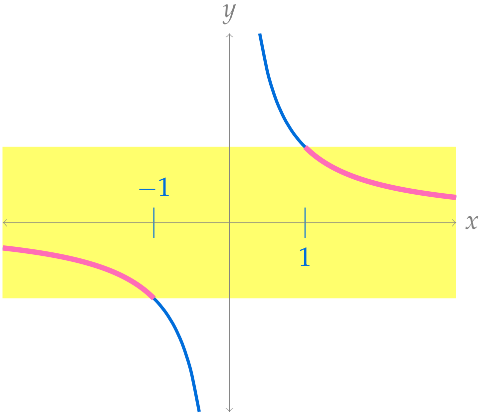
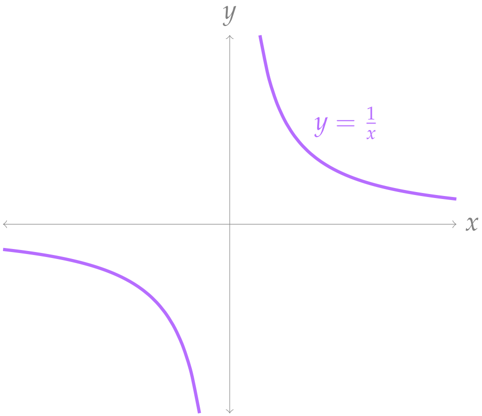


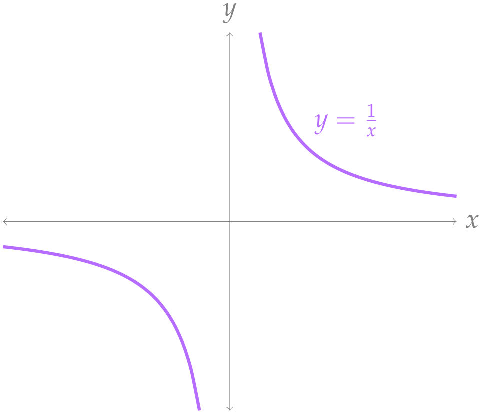
$(-\infty, -1] \cup [1, \infty)$.

$$\operatorname{arccot}(x) = \arctan\left(\frac{1}{x}\right)$$



$$\text{arccot}(x) = \arctan\left(\frac{1}{x}\right)$$

Domain of $\arctan(x)$ is all real numbers, so the domain of $\text{arccot}(x)$ is



ans

$$\text{arccot}(x) = \arctan\left(\frac{1}{x}\right)$$

Domain of $\arctan(x)$ is all real numbers, so the domain of $\text{arccot}(x)$ is

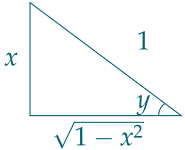
$(-\infty, 0) \cup (0, \infty)$.

$$y = \arcsin x$$

Find $\frac{dy}{dx}$.

ans

$$\begin{aligned}
y(x) &= \arcsin x \\
x &= \sin y(x) \\
\frac{\mathrm{d}}{\mathrm{d}x}[x] &= \frac{\mathrm{d}}{\mathrm{d}x}[\sin y(x)] \\
1 &= \cos y(x) \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{1}{\cos y(x)} \\
&= \frac{\text{hyp}}{\text{adj}} \\
&= \frac{1}{\sqrt{1-x^2}}
\end{aligned}$$

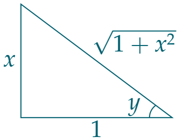


$$y = \arctan x$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

Q

$$
\begin{aligned}
y(x) &= \arctan x \\
x &= \tan y(x) \\
\frac{\mathrm{d}}{\mathrm{d}x}[x] &= \frac{\mathrm{d}}{\mathrm{d}x}[\tan y(x)] \\
1 &= \sec^2 y(x) \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \cos^2 y(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \left(\frac{\text{adj}}{\text{hyp}}\right)^2 = \left(\frac{1}{\sqrt{1+x^2}}\right)^2 \\
&= \frac{1}{1+x^2}
\end{aligned}
$$

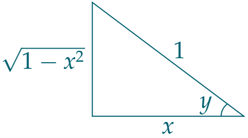


$$y = \arccos x$$

Find $\frac{dy}{dx}$.

Q

$$
\begin{aligned}
y(x) &= \arccos x \\
x &= \cos y(x) \\
\frac{\mathrm{d}}{\mathrm{d}x}[x] &= \frac{\mathrm{d}}{\mathrm{d}x}[\cos y(x)] \\
1 &= -\sin y(x) \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{-1}{\sin y(x)} \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{-\mathrm{hyp}}{\mathrm{opp}} = \frac{-1}{\sqrt{1-x^2}}
\end{aligned}
$$



ans

To differentiate arcsecant, arccosecant, and arccotangent, you can use the chain rule!

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$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\operatorname{arccsc}(x)\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[\arcsin\left(\frac{1}{x}\right)\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[\arcsin\left(x^{-1}\right)\right]$$

To differentiate arcsecant, arccosecant, and arccotangent, you can use the chain rule!

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$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\arcsin\left(\boxed{x^{-1}}\right)\right] = \frac{1}{\sqrt{1-\left(\boxed{x^{-1}}\right)^2}} \cdot \boxed{\left(-x^{-2}\right)} = \frac{-1}{x^2\sqrt{1-x^{-2}}}$$

$$= \frac{-1}{\sqrt{x^4}\sqrt{1-x^{-2}}} = \frac{-1}{\sqrt{x^2}\sqrt{x^2}\sqrt{1-x^{-2}}} = \frac{-1}{\sqrt{x^2}\sqrt{x^2-1}} = \frac{-1}{|x|\sqrt{1-x^2}}$$

Derivatives of Inverse Trigonometric Functions – Theorem 2.12.7

Memorize:

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arccos x] = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arcsin x] = \frac{1}{1+x^2}$$

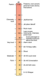
Be able to derive:

$$\frac{d}{dx}[\operatorname{arccsc} x] = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\operatorname{arcsec} x] = \frac{1}{|x|\sqrt{x^2-1}}$$

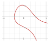
$$\frac{d}{dx}[\operatorname{arccot} x] = -\frac{1}{1+x^2}$$

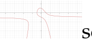
Included Work

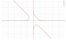
 Anonymous. (2012) Decibel Scale of Frequently Heard Sounds. Biology Forums. http://biology-forums.com/index.php?action=gallery;sa=view;id=6156 (accessed 7 October 2015) , 97

 'Brain' by Eucalyp is licensed under CC BY 3.0 (accessed 8 June 2021), 148, 149, 151, 152

 'Slide rule – KEUFFEL & ESSER CO. N.Y. ' by s58y is licensed under CC BY 2.0 (accessed 20 July 2021) , 115

 screenshot of graph using Desmos Graphing Calculator, https://www.desmos.com/calculator (accessed 19 October 2017), 143, 147

 screenshot of graph using Desmos Graphing Calculator, https://www.desmos.com/calculator (accessed 19 October 2017), 132–134

 screenshot of graph using Desmos Graphing Calculator, https://www.desmos.com/calculator (accessed 24 October 2018), 150

 screenshot of graph using Desmos Graphing Calculator, https://www.desmos.com/calculator, (accessed 24 October 2018), 155