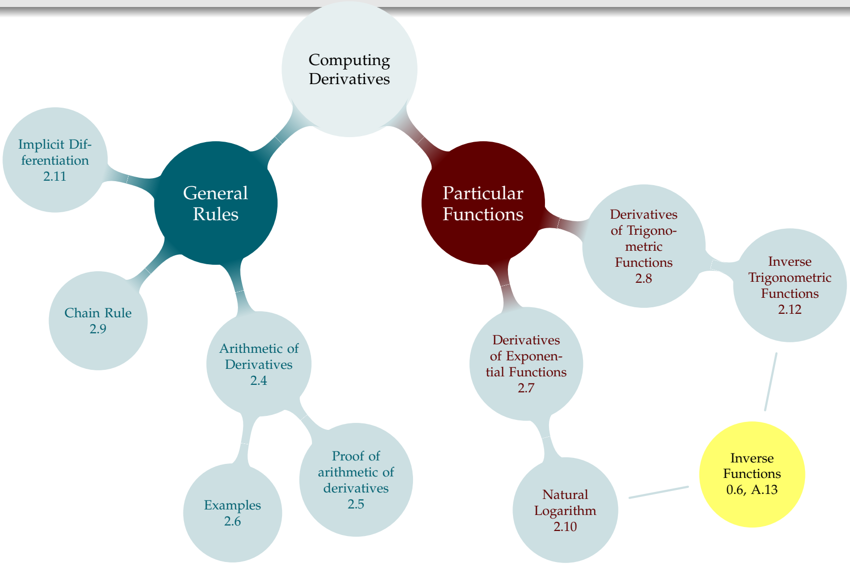
TABLE OF CONTENTS

SKIP REVIEW OF INVERSE FUNCTIONS AND LOGARITHMS



Invertibility Game

- A function $y = f(x)$ is known to both players

Invertibility Game

- A function $y = f(x)$ is known to both players
- Player A chooses a secret value x in the domain of $f(x)$

Invertibility Game

- A function $y = f(x)$ is known to both players
- Player A chooses a secret value x in the domain of $f(x)$
- Player A tells Player B what $f(x)$ is

Invertibility Game

- A function $y = f(x)$ is known to both players
- Player A chooses a secret value x in the domain of $f(x)$
- Player A tells Player B what $f(x)$ is
- Player B tries to guess Player A's x-value.

Invertibility Game

- A function $y = f(x)$ is known to both players
- Player A chooses a secret value x in the domain of $f(x)$
- Player A tells Player B what $f(x)$ is
- Player B tries to guess Player A's x-value.

Invertibility Game

- A function $y = f(x)$ is known to both players
- Player A chooses a secret value x in the domain of $f(x)$
- Player A tells Player B what $f(x)$ is
- Player B tries to guess Player A's x-value.

Round 1: $f(x) = 2x$

Invertibility Game

- A function $y = f(x)$ is known to both players
- Player A chooses a secret value x in the domain of $f(x)$
- Player A tells Player B what $f(x)$ is
- Player B tries to guess Player A's x-value.

Round 1: $f(x) = 2x$

Round 2: $f(x) = \sqrt[3]{x}$

INVERTIBILITY GAME

- A function $y = f(x)$ is known to both players
- Player A chooses a secret value x in the domain of $f(x)$
- Player A tells Player B what $f(x)$ is
- Player B tries to guess Player A's x-value.

Round 1: $f(x) = 2x$

Round 2: $f(x) = \sqrt[3]{x}$

Round 3: $f(x) = |x|$

Invertibility Game

- A function $y = f(x)$ is known to both players
- Player A chooses a secret value x in the domain of $f(x)$
- Player A tells Player B what $f(x)$ is
- Player B tries to guess Player A's x-value.

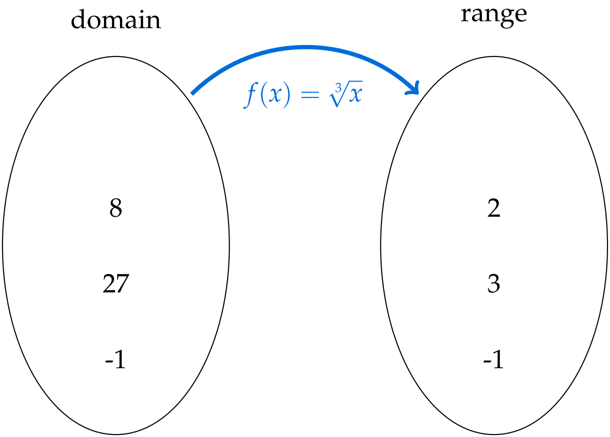
Round 1: $f(x) = 2x$

Round 2: $f(x) = \sqrt[3]{x}$

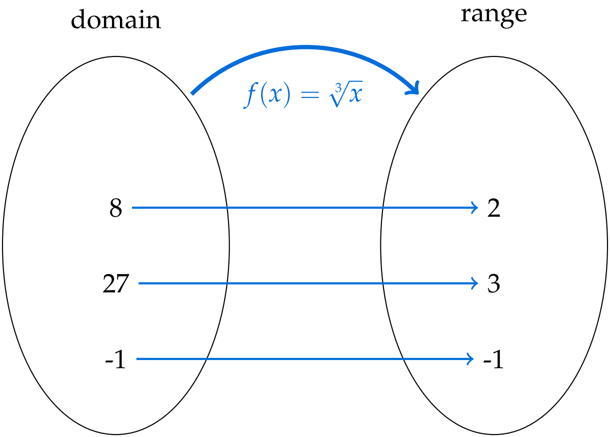
Round 3: $f(x) = |x|$

Round 4: $f(x) = \sin x$

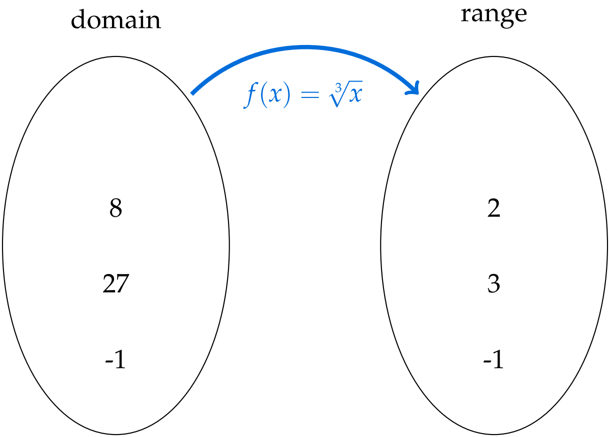
FUNCTIONS ARE MAPS



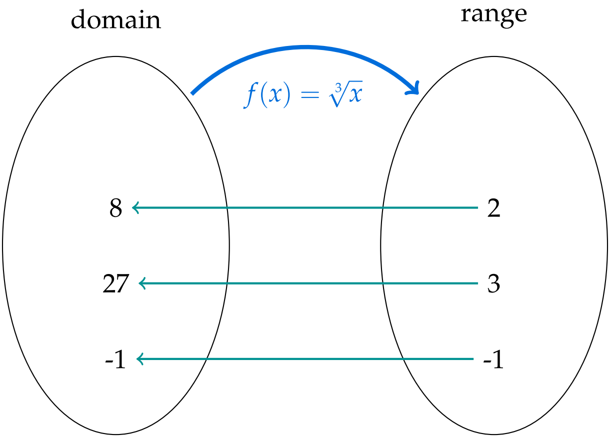
Functions are Maps



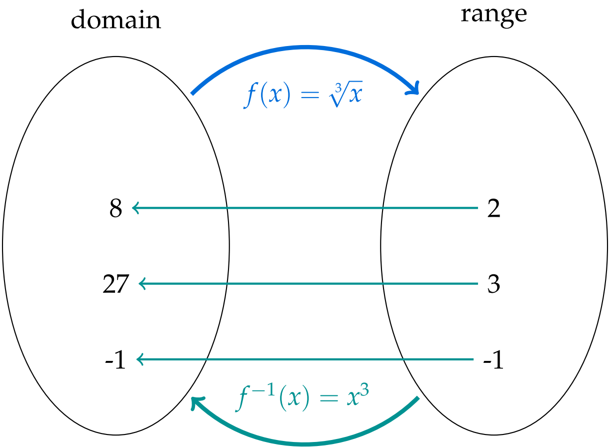
Functions are Maps



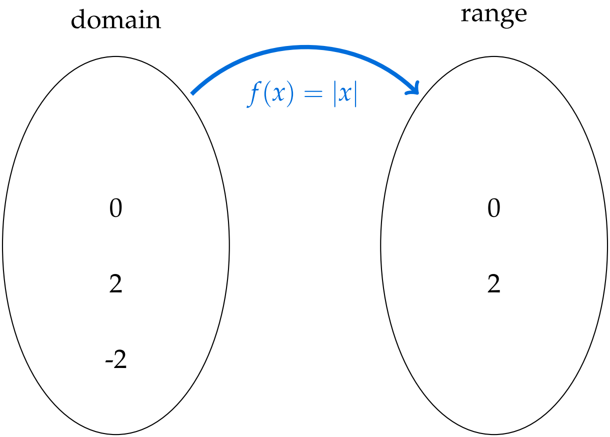
Functions are Maps



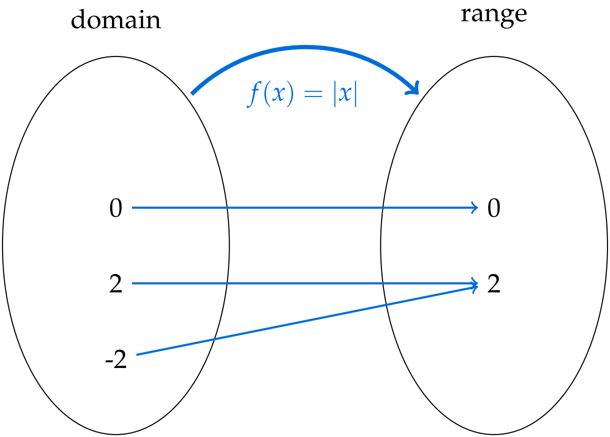
Functions are Maps



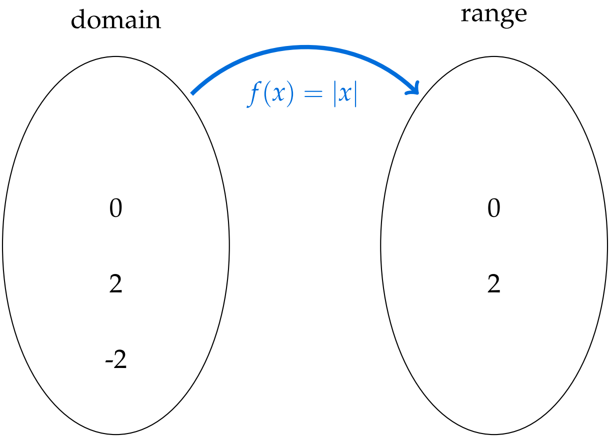
Functions are Maps



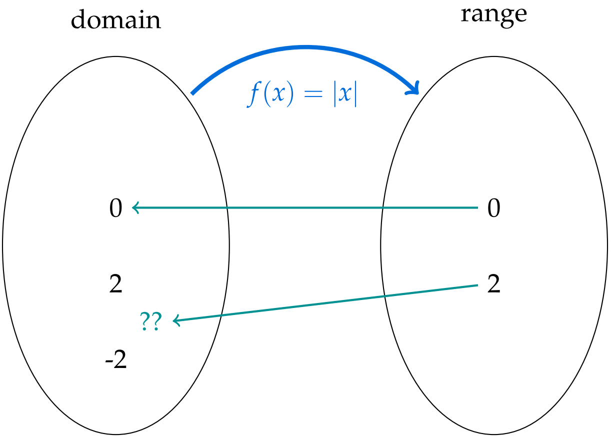
Functions are Maps



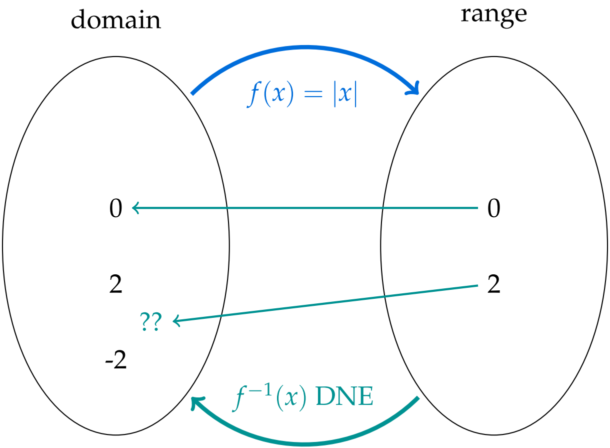
Functions are Maps

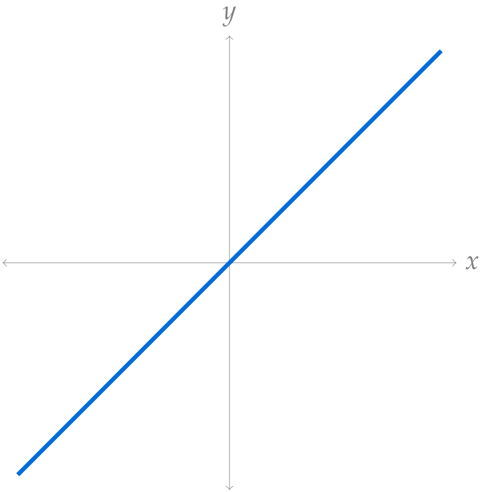


Functions are Maps

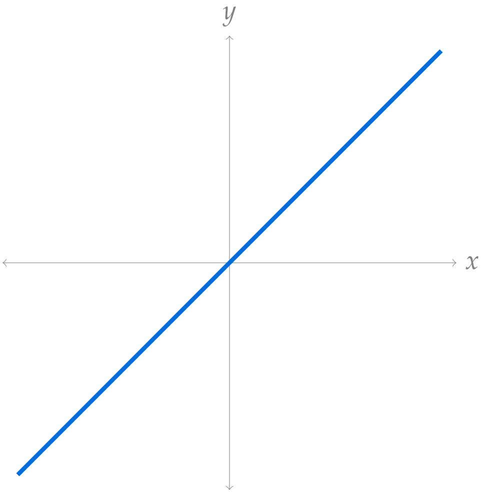


Functions are Maps



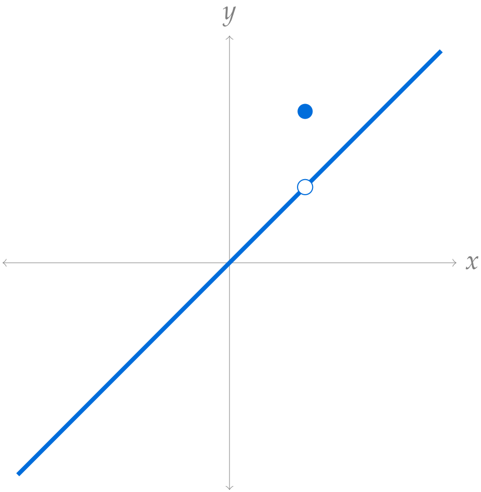


A. invertible B. not invertible



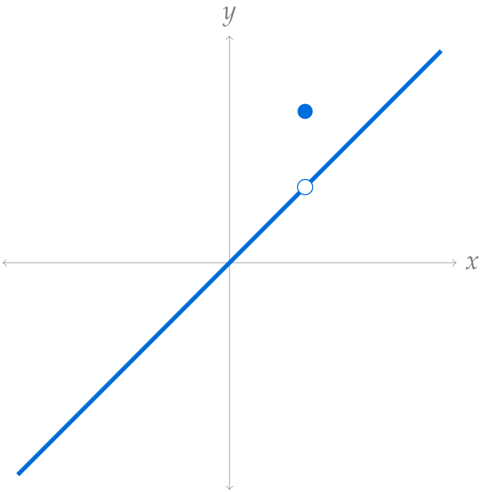
A. invertible

B. not invertible



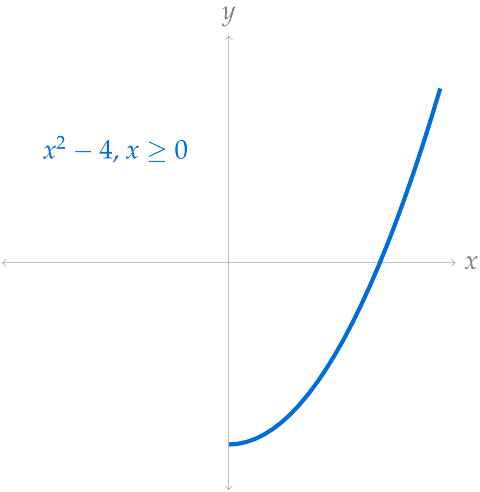
A. invertible

B. not invertible

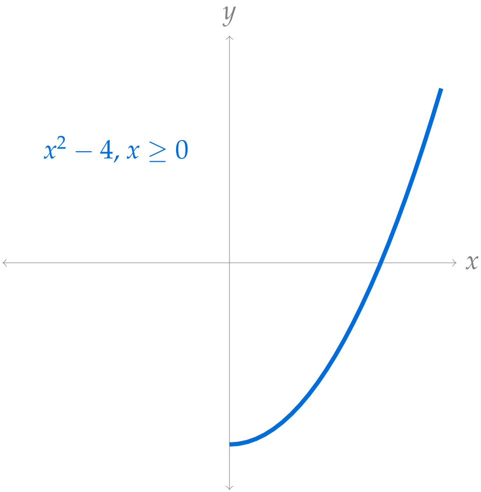


A. invertible

B. not invertible



A. invertible B. not invertible



A. invertible B. not invertible

Relationship between $f(x)$ and $f^{-1}(x)$

Let f be an invertible function.
What is $f^{-1}(f(x))$?

A. x

B. 1

C. 0

D. not sure

Relationship between $f(x)$ and $f^{-1}(x)$

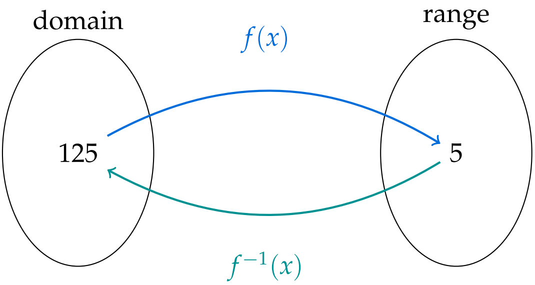
Let f be an invertible function.
What is $f^{-1}(f(x))$?

A. x

B. 1

C. 0

D. not sure



Relationship between $f(x)$ and $f^{-1}(x)$

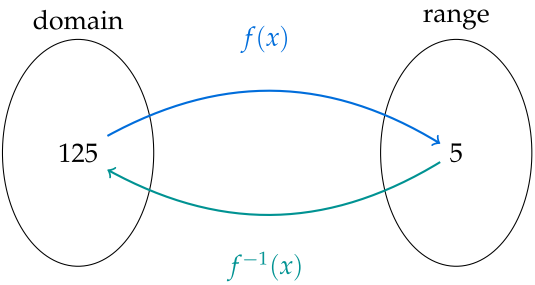
Let f be an invertible function.
What is $f^{-1}(f(x))$?

A. x

B. 1

C. 0

D. not sure



Invertibility

In order for a function to be invertible

Invertibility

In order for a function to be invertible , different x values cannot map to the same y value.

Invertibility

In order for a function to be invertible , different x values cannot map to the same y value.
We call such a function **one-to-one**, or **injective**.

Invertibility

In order for a function to be invertible , different x values cannot map to the same y value.
We call such a function **one-to-one**, or **injective**.

Suppose $f(x) = \sqrt[3]{19 + x^3}$. What is $f^{-1}(3)$? (simplify your answer)

Q

ans

Invertibility

In order for a function to be invertible , different x values cannot map to the same y value.
We call such a function **one-to-one**, or **injective**.

Suppose $f(x) = \sqrt[3]{19 + x^3}$. What is $f^{-1}(3)$? (simplify your answer)

What is $f^{-1}(10)$? (do not simplify)

Q

ans

Invertibility

In order for a function to be invertible , different x values cannot map to the same y value.
We call such a function **one-to-one**, or **injective**.

Suppose $f(x) = \sqrt[3]{19 + x^3}$. What is $f^{-1}(3)$? (simplify your answer)

What is $f^{-1}(10)$? (do not simplify)

What is $f^{-1}(x)$?

Invertibility

In order for a function to be invertible , different x values cannot map to the same y value.
We call such a function **one-to-one**, or **injective**.

Suppose $f(x) = \sqrt[3]{19 + x^3}$. What is $f^{-1}(3)$? (simplify your answer)
$f(2) = 3$, so $f^{-1}(3) = 2$

What is $f^{-1}(10)$? (do not simplify)
$\sqrt[3]{19 + y^3} = 10$ tells us $f^{-1}(10) = \sqrt[3]{10^3 - 19}$

What is $f^{-1}(x)$?
$\sqrt[3]{19 + y^3} = x$ tells us $f^{-1}(x) = \sqrt[3]{x^3 - 19}$

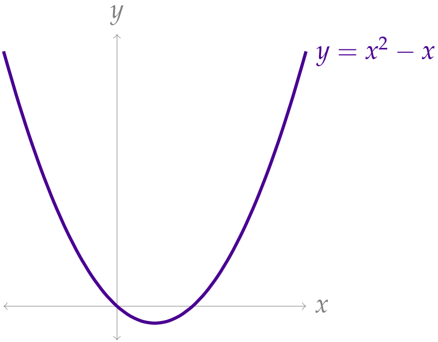
Let $f(x) = x^2 - x$.

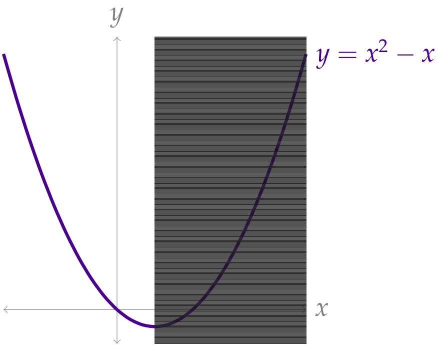
1. Sketch a graph of $f(x)$, and choose a (large) domain over which it is invertible.

2. For the domain you chose, evaluate $f^{-1}(20)$.

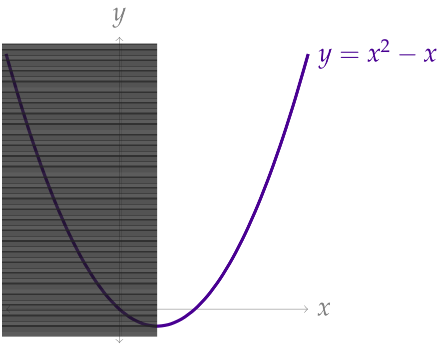
3. For the domain you chose, evaluate $f^{-1}(x)$.

4. What are the domain and range of $f^{-1}(x)$? What are the (restricted) domain and range of $f(x)$?

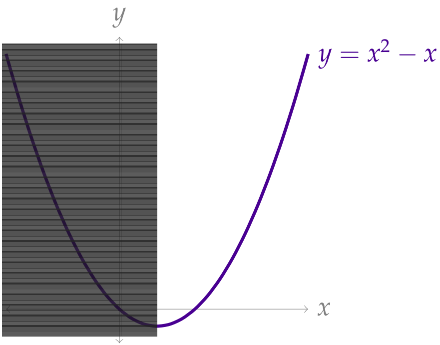

y
$y = x^2 - x$
x



Domain: $\left(-\infty, \frac{1}{2}\right]$

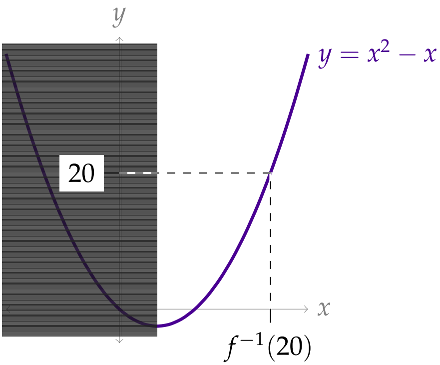


Domain: $\left[\frac{1}{2}, \infty\right)$

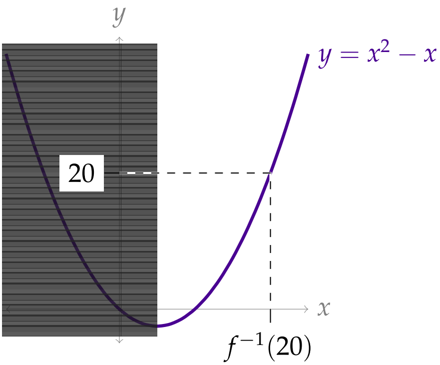


Domain: $\left[\frac{1}{2}, \infty\right)$

$f^{-1}(20) =$



$$f^{-1}(20) =$$

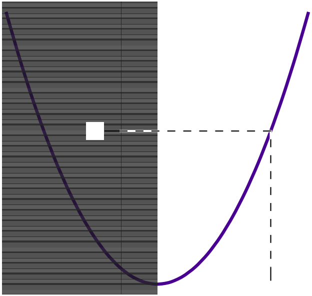


$f^{-1}(20) = 5$

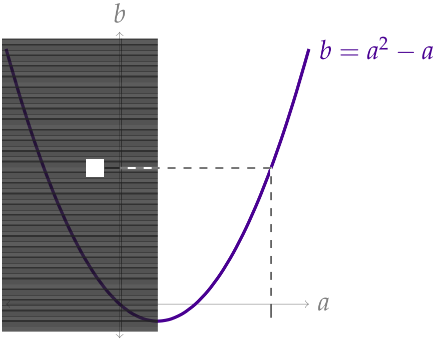
$$20 = x^2 - x$$

$$0 = x^2 - x - 20 = (x-5)(x+4)$$

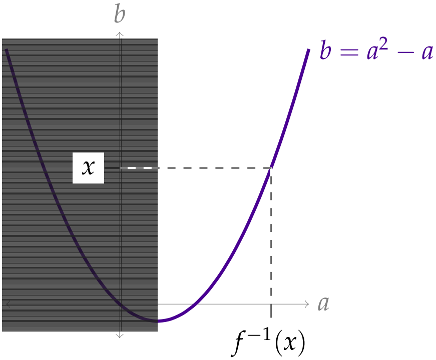
$$x = 5$$



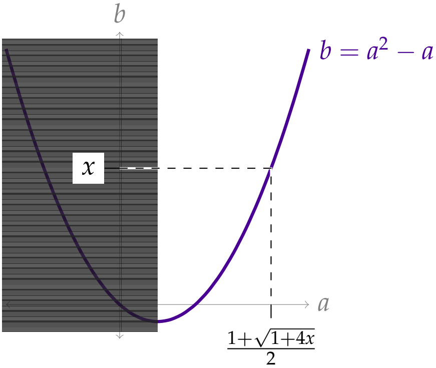
$$f^{-1}(x) =$$



$$f^{-1}(x) =$$



$$f^{-1}(x) =$$



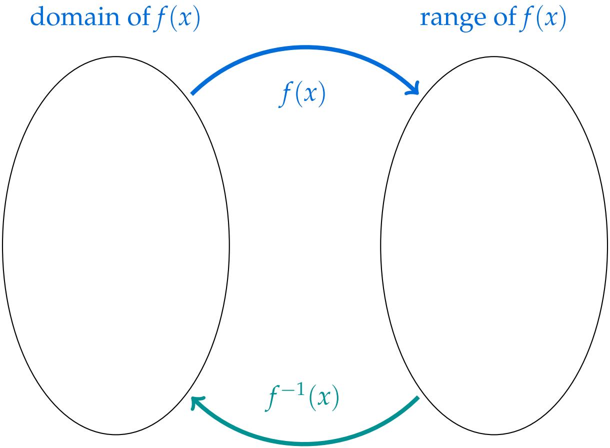
$$f^{-1}(x) = \frac{1+\sqrt{1+4x}}{2}$$

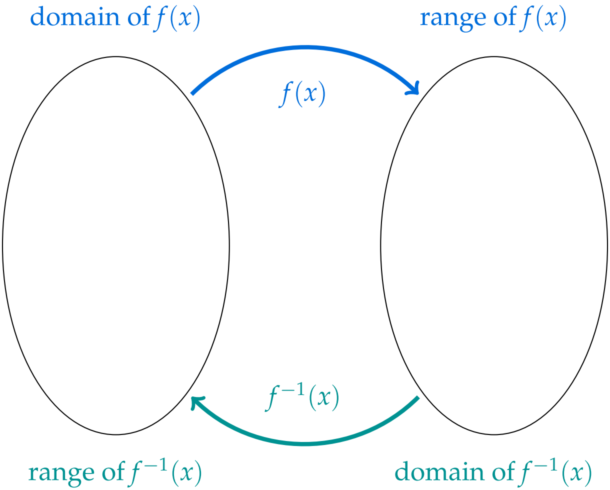
$$a^2 - a = x, \text{ find } a$$

$$a^2 - a - x = 0$$

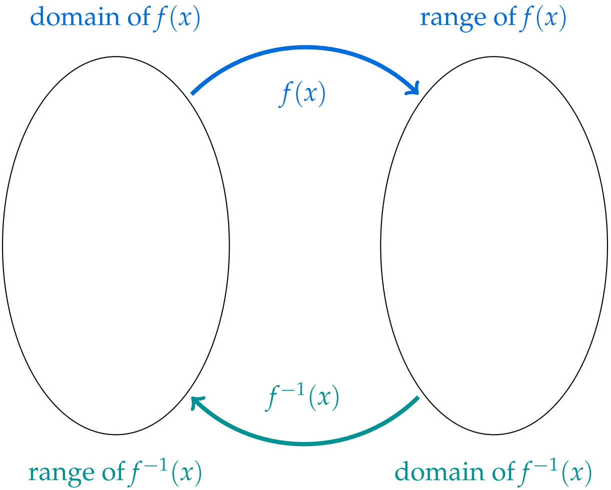
$$a = \frac{1 \pm \sqrt{1+4x}}{2}$$

$$f^{-1}(x) = \frac{1+\sqrt{1+4x}}{2}$$

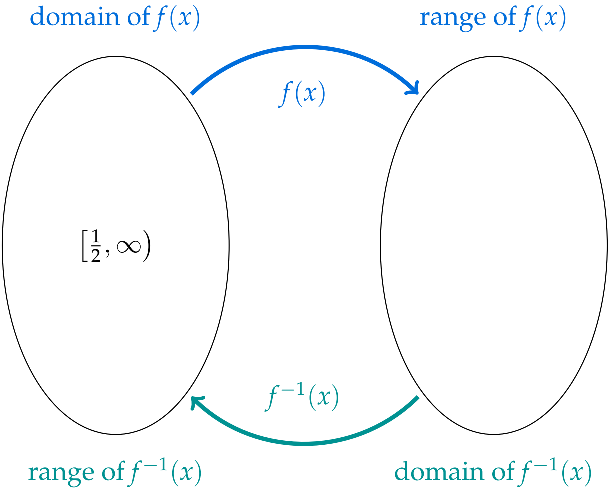

domain of $f(x)$
range of $f(x)$
$f(x)$
$f^{-1}(x)$


domain of $f(x)$
range of $f(x)$
$f(x)$
$f^{-1}(x)$
range of $f^{-1}(x)$
domain of $f^{-1}(x)$

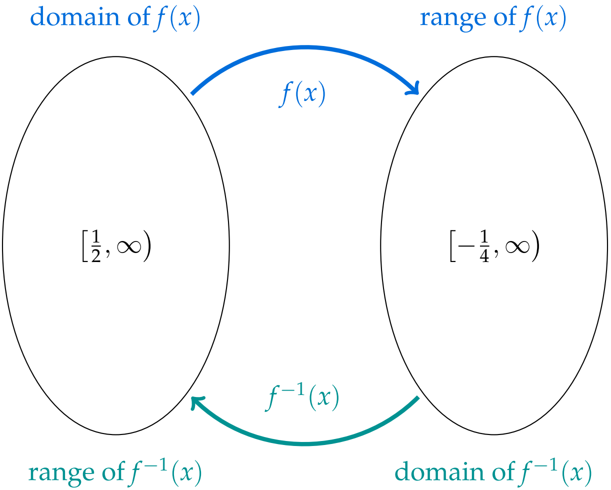
$f(x) = x^2 - x$, domain: $\left[\frac{1}{2}, \infty\right)$



$f(x) = x^2 - x$, domain: $\left[\frac{1}{2}, \infty\right)$

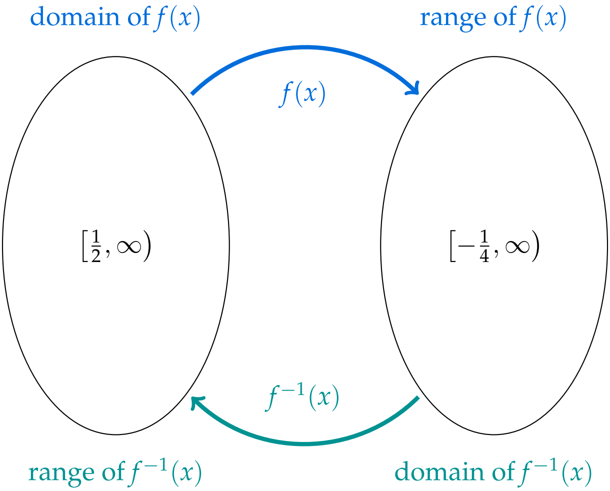


$f(x) = x^2 - x$, domain: $\left[\frac{1}{2}, \infty\right)$



$f(x) = x^2 - x$, domain: $\left[\frac{1}{2}, \infty\right)$

$f^{-1}(x) = \frac{1+\sqrt{1+4x}}{2}$



Invertibility game: $f(x) = e^x$

- I'm thinking of an x. Your clue: $f(x) = e$. What is my x?

ans

INVERTIBILITY GAME: $f(x) = e^x$

- I'm thinking of an x. Your clue: $f(x) = e$. What is my x?
- I'm thinking of an x. Your clue: $f(x) = 1$. What is my x?

ans

INVERTIBILITY GAME: $f(x) = e^x$

- I'm thinking of an x. Your clue: $f(x) = e$. What is my x?
- I'm thinking of an x. Your clue: $f(x) = 1$. What is my x?
- I'm thinking of an x. Your clue: $f(x) = \frac{1}{e}$. What is my x?

INVERTIBILITY GAME: $f(x) = e^x$

- I'm thinking of an x. Your clue: $f(x) = e$. What is my x?
- I'm thinking of an x. Your clue: $f(x) = 1$. What is my x?
- I'm thinking of an x. Your clue: $f(x) = \frac{1}{e}$. What is my x?
- I'm thinking of an x. Your clue: $f(x) = e^3$. What is my x?

INVERTIBILITY GAME: $f(x) = e^x$

- I'm thinking of an x. Your clue: $f(x) = e$. What is my x?
- I'm thinking of an x. Your clue: $f(x) = 1$. What is my x?
- I'm thinking of an x. Your clue: $f(x) = \frac{1}{e}$. What is my x?
- I'm thinking of an x. Your clue: $f(x) = e^3$. What is my x?
- I'm thinking of an x. Your clue: $f(x) = 0$. What is my x?

INVERTIBILITY GAME: $f(x) = e^x$

- I'm thinking of an x. Your clue: $f(x) = e$. What is my x? $x = 1$
- I'm thinking of an x. Your clue: $f(x) = 1$. What is my x? $x = 0$
- I'm thinking of an x. Your clue: $f(x) = \frac{1}{e}$. What is my x? $x = -1$
- I'm thinking of an x. Your clue: $f(x) = e^3$. What is my x? $x = 3$
- I'm thinking of an x. Your clue: $f(x) = 0$. What is my x? Trick question: no x gives $f(x) = 0$.

Invertibility game: $f(x) = e^x$ $\quad f^{-1}(x) = \log_e x$

- I'm thinking of an x. Your clue: $f(x) = e$. What is my x? $x = 1$
- I'm thinking of an x. Your clue: $f(x) = 1$. What is my x? $x = 0$
- I'm thinking of an x. Your clue: $f(x) = \frac{1}{e}$. What is my x? $x = -1$
- I'm thinking of an x. Your clue: $f(x) = e^3$. What is my x? $x = 3$
- I'm thinking of an x. Your clue: $f(x) = 0$. What is my x? Trick question: no x gives $f(x) = 0$.

Invertibility game: $f(x) = e^x$ $\quad f^{-1}(x) = \log_e x$

- I'm thinking of an x. Your clue: $f(x) = e$. What is my x? $x = 1$

  $\log_e(e) = 1$

- I'm thinking of an x. Your clue: $f(x) = 1$. What is my x? $x = 0$

  $\log_e(1) = 0$

- I'm thinking of an x. Your clue: $f(x) = \frac{1}{e}$. What is my x? $x = -1$

  $\log_e\left(\frac{1}{e}\right) = -1$

- I'm thinking of an x. Your clue: $f(x) = e^3$. What is my x? $x = 3$

  $\log_e(e^3) = 3$

- I'm thinking of an x. Your clue: $f(x) = 0$. What is my x? Trick question: no x gives $f(x) = 0$.

  $\log_e(x)$ is undefined at $x = 0$

1. Suppose $0 < x < 1$. Then $\log_e(x)$ is...

2. Suppose $-1 < x < 0$. Then $\log_e(x)$ is...

3. Suppose $e < x$. Then $\log_e(x)$ is...

A. positive
B. negative
C. greater than one
D. less than one
E. undefined

~~ans~~

Exponents and Logarithms

$$f(x) = e^x \qquad\qquad f^{-1}(x) = \log_e(x) = \ln(x) = \log(x)$$

Exponents and Logarithms

$$f(x) = e^x \qquad\qquad f^{-1}(x) = \log_e(x) = \ln(x) = \log(x)$$

x	e^x	
0	1	
1	e	
-1	$\frac{1}{e}$	
n	e^n	

EXPONENTS AND LOGARITHMS

$$f(x) = e^x \qquad f^{-1}(x) = \log_e(x) = \ln(x) = \log(x)$$

x	e^x	e fact $\leftrightarrow$ $\log_e$ fact	x	$\log_e(x)$
0	1			
1	e			
-1	$\frac{1}{e}$			
n	e^n			

Exponents and Logarithms

$$f(x) = e^x \qquad\qquad f^{-1}(x) = \log_e(x) = \ln(x) = \log(x)$$

x	e^x	e fact $\leftrightarrow$ $\log_e$ fact	x	$\log_e(x)$
0	1	$e^0 = 1 \leftrightarrow \log_e(1) = 0$	1	0
1	e			
-1	$\frac{1}{e}$			
n	e^n			

Exponents and Logarithms

$$f(x) = e^x \qquad\qquad f^{-1}(x) = \log_e(x) = \ln(x) = \log(x)$$

x	e^x	e fact $\leftrightarrow$ $\log_e$ fact	x	$\log_e(x)$
0	1	$e^0 = 1 \leftrightarrow \log_e(1) = 0$	1	0
1	e	$e^1 = e \leftrightarrow \log_e(e) = 1$	e	1
-1	$\frac{1}{e}$			
n	e^n			

EXPONENTS AND LOGARITHMS

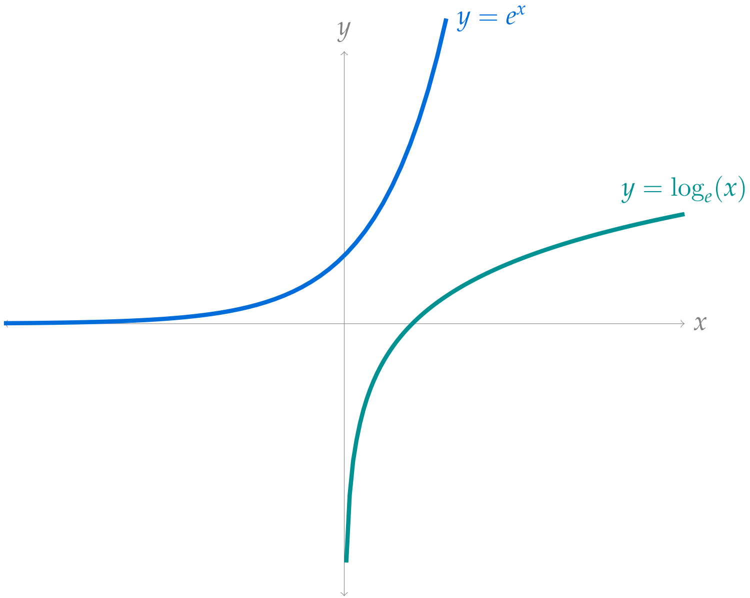
$$f(x) = e^x \qquad f^{-1}(x) = \log_e(x) = \ln(x) = \log(x)$$

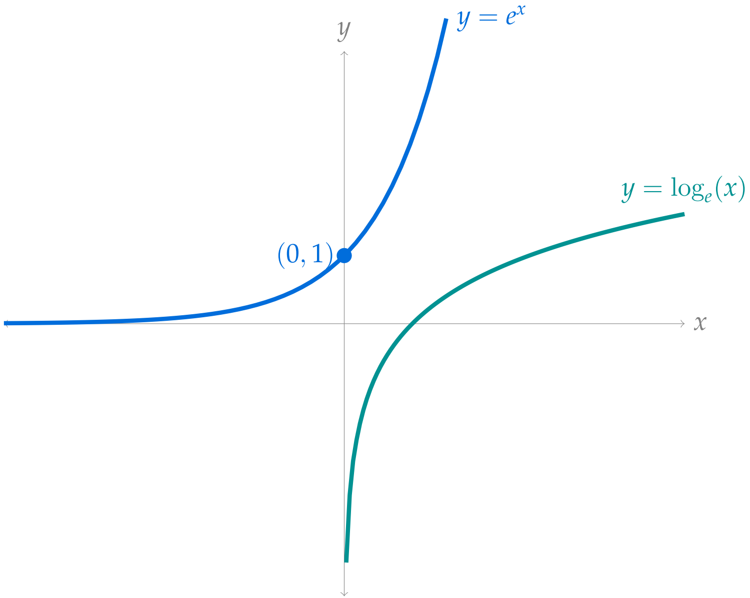
x	e^x	e fact $\leftrightarrow$ $\log_e$ fact	x	$\log_e(x)$
0	1	$e^0 = 1 \leftrightarrow \log_e(1) = 0$	1	0
1	e	$e^1 = e \leftrightarrow \log_e(e) = 1$	e	1
-1	$\frac{1}{e}$	$e^{-1} = \frac{1}{e} \leftrightarrow \log_e\left(\frac{1}{e}\right) = -1$	$\frac{1}{e}$	-1
n	e^n			

Exponents and Logarithms

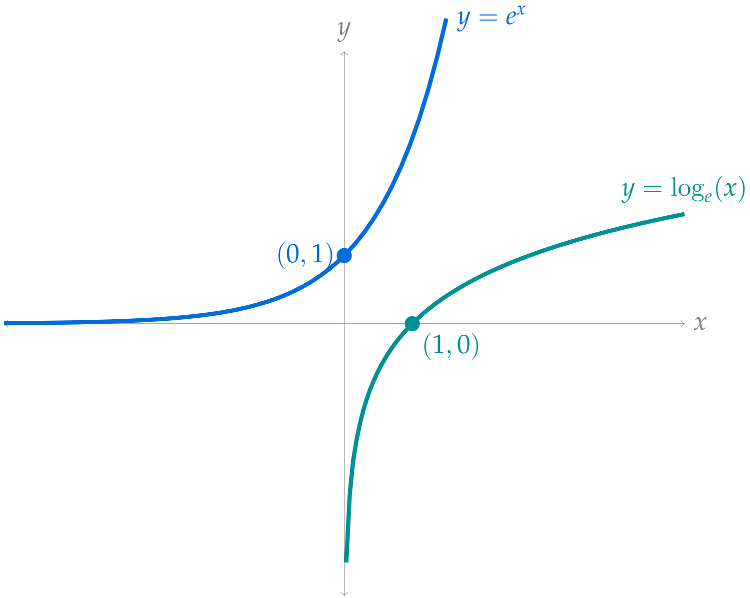
$$f(x) = e^x \qquad\qquad f^{-1}(x) = \log_e(x) = \ln(x) = \log(x)$$

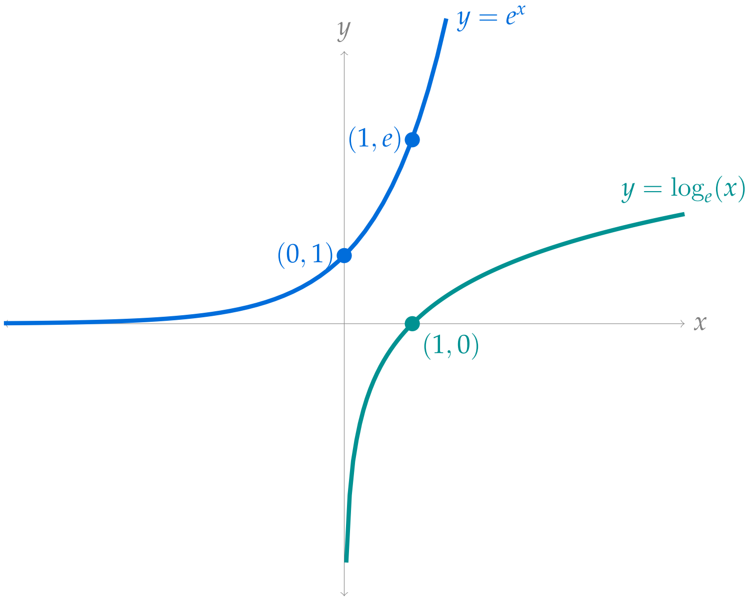
x	e^x	e fact $\leftrightarrow$ $\log_e$ fact	x	$\log_e(x)$
0	1	$e^0 = 1 \leftrightarrow \log_e(1) = 0$	1	0
1	e	$e^1 = e \leftrightarrow \log_e(e) = 1$	e	1
-1	$\frac{1}{e}$	$e^{-1} = \frac{1}{e} \leftrightarrow \log_e\left(\frac{1}{e}\right) = -1$	$\frac{1}{e}$	-1
n	e^n	$e^n = e^n \leftrightarrow \log_e(e^n) = n$	e^n	n

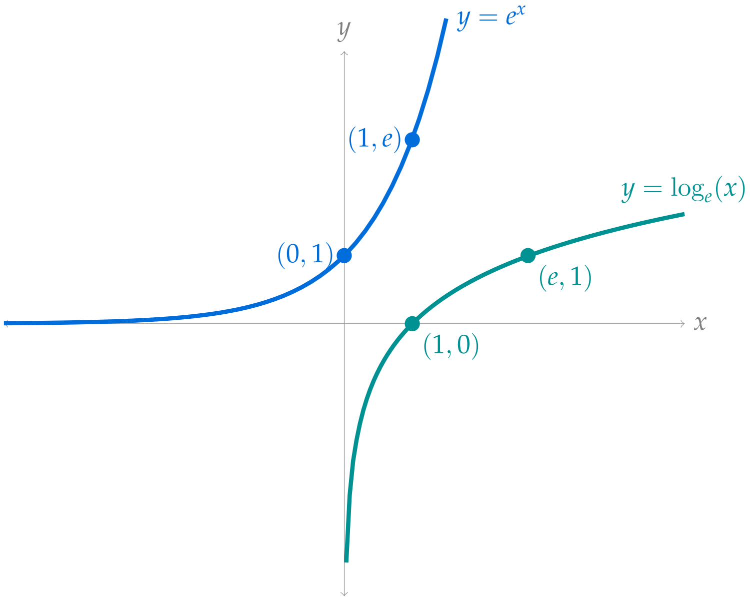

$y = e^x$
y
$y = \log_e(x)$
x



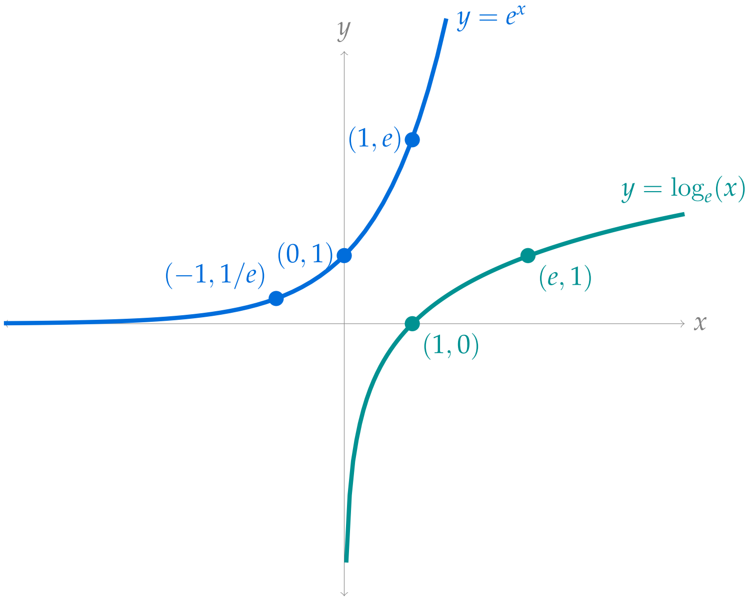
$y = e^x$
y
$y = \log_e(x)$
$(0,1)$
x


$y = e^x$
y
$y = \log_e(x)$
$(0, 1)$
x
$(1, 0)$

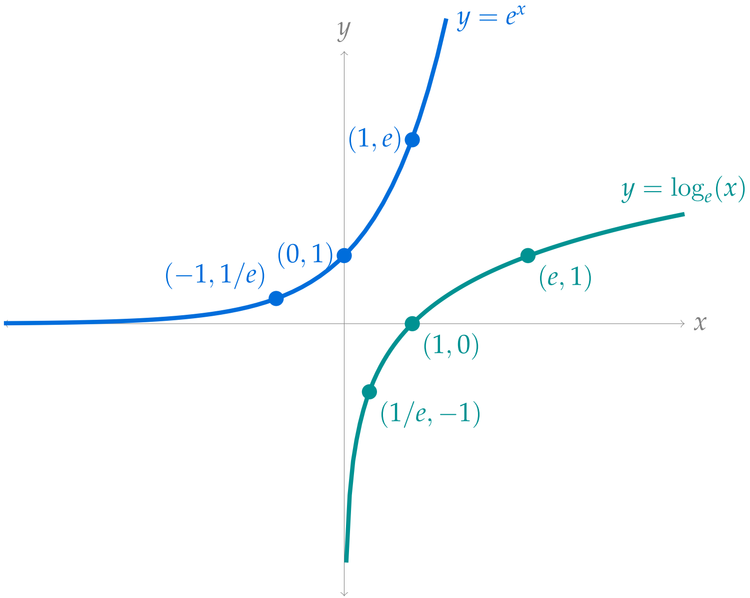

$y = e^x$
y
$(1, e)$
$y = \log_e(x)$
$(0, 1)$
x
$(1, 0)$



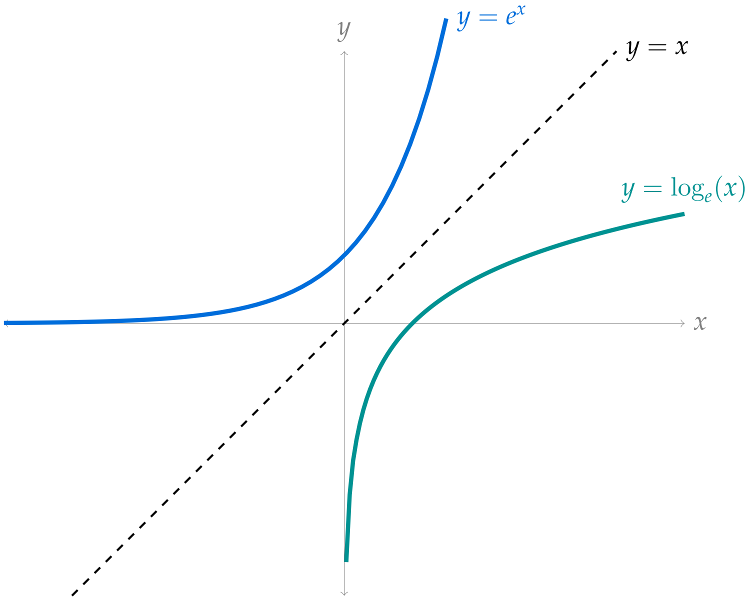
$y = e^x$
y
$(1, e)$
$y = \log_e(x)$
$(0, 1)$
$(e, 1)$
x
$(1, 0)$



$y = e^x$
y
$(1, e)$
$y = \log_e(x)$
$(0, 1)$
$(-1, 1/e)$
$(e, 1)$
x
$(1, 0)$



$y = e^x$
y
$(1, e)$
$y = \log_e(x)$
$(0, 1)$
$(-1, 1/e)$
$(e, 1)$
x
$(1, 0)$
$(1/e, -1)$


$y = e^x$
y
$y = x$
$y = \log_e(x)$
x

Logs of Other Bases: $\log_n(x)$ is the inverse of n^x

$\log_{10} 10^8 =$

A. 0
B. 8
C. 10
D. other

Logs of Other Bases: $\log_n(x)$ is the inverse of n^x

$\log_{10} 10^8 =$

A. 0

B. 8 ✓

C. 10

D. other

Logs of Other Bases: $\log_n(x)$ is the inverse of n^x

$\log_{10} 10^8 =$

A. 0

B. 8 ✓

C. 10

D. other

$\log_2 16 =$

A. 1

B. 2

C. 3

D. other

LOGS OF OTHER BASES: $\log_n(x)$ IS THE INVERSE OF n^x

$\log_{10} 10^8 =$

A. 0

B. 8 ✓

C. 10

D. other

$\log_2 16 =$

A. 1

B. 2

C. 3

D. other ✓ $2^4 = 16$ so $\log_2 16 = 4$

Logarithm Rules

Let A and B be positive, and let n be any real number.

Logarithm Rules

Let A and B be positive, and let n be any real number.

$\log(A \cdot B) =$

Logarithm Rules

Let A and B be positive, and let n be any real number.

$\log(A \cdot B) = \log(A) + \log(B)$

Proof: $\log(A \cdot B) = \log(e^{\log A} e^{\log B}) = \log(e^{\log A + \log B}) = \log(A) + \log(B)$

Logarithm Rules

Let A and B be positive, and let n be any real number.

$\log(A \cdot B) = \log(A) + \log(B)$

Proof: $\log(A \cdot B) = \log(e^{\log A} e^{\log B}) = \log(e^{\log A + \log B}) = \log(A) + \log(B)$

$\log(A/B) =$

Logarithm Rules

Let A and B be positive, and let n be any real number.

$\log(A \cdot B) = \log(A) + \log(B)$

Proof: $\log(A \cdot B) = \log(e^{\log A} e^{\log B}) = \log(e^{\log A + \log B}) = \log(A) + \log(B)$

$\log(A/B) = \log(A) - \log(B)$

Proof: $\log(A/B) = \log\left(\frac{e^{\log A}}{e^{\log B}}\right) = \log(e^{\log A - \log B}) = \log A - \log B$

Logarithm Rules

Let A and B be positive, and let n be any real number.

$\log(A \cdot B) = \log(A) + \log(B)$

Proof: $\log(A \cdot B) = \log(e^{\log A} e^{\log B}) = \log(e^{\log A + \log B}) = \log(A) + \log(B)$

$\log(A/B) = \log(A) - \log(B)$

Proof: $\log(A/B) = \log\left(\frac{e^{\log A}}{e^{\log B}}\right) = \log(e^{\log A - \log B}) = \log A - \log B$

$\log(A^n)$

Logarithm Rules

Let A and B be positive, and let n be any real number.

$\log(A \cdot B) = \log(A) + \log(B)$

Proof: $\log(A \cdot B) = \log(e^{\log A} e^{\log B}) = \log(e^{\log A + \log B}) = \log(A) + \log(B)$

$\log(A/B) = \log(A) - \log(B)$

Proof: $\log(A/B) = \log\left(\frac{e^{\log A}}{e^{\log B}}\right) = \log(e^{\log A - \log B}) = \log A - \log B$

$\log(A^n) = n \log(A)$

Proof: $\log(A^n) = \log\left(\left(e^{\log A}\right)^n\right) = \log\left(e^{n \log A}\right) = n \log A$

Logarithm Rules

Let A and B be positive, and let n be any real number.

$\log(A \cdot B) = \log(A) + \log(B)$

$\log(A/B) = \log(A) - \log(B)$

$\log(A^n) = n \log(A)$

Logarithm Rules

Let A and B be positive, and let n be any real number.

$\log(A \cdot B) = \log(A) + \log(B)$

$\log(A/B) = \log(A) - \log(B)$

$\log(A^n) = n\log(A)$

Write as a single logarithm:

$$f(x) = \log\left(\frac{10}{x^2}\right) + 2\log x + \log(10+x)$$

ans

Logarithm Rules

Let A and B be positive, and let n be any real number.
$\log(A \cdot B) = \log(A) + \log(B)$
$\log(A/B) = \log(A) - \log(B)$
$\log(A^n) = n \log(A)$

$$\begin{aligned} f(x) &= \log\left(\frac{10}{x^2}\right) + 2\log x + \log(10+x) \\ &= \log 10 - \log(x^2) + 2\log x + \log(10+x) \\ &= \log 10 - 2\log x + 2\log x + \log(10+x) \\ &= \log 10 + \log(10+x) = \log(10(10+x)) \\ &= \log(100+10x) \end{aligned}$$

Base Change

Fact: $b^{\log_b(a)} = a$

Base Change

$$
\begin{aligned}
\text{Fact:} \qquad b^{\log_b(a)} &= a \\
\Rightarrow \log\left(b^{\log_b(a)}\right) &= \log(a) \\
\Rightarrow \log_b(a)\log(b) &= \log(a) \\
\Rightarrow \log_b(a) &= \frac{\log(a)}{\log(b)}
\end{aligned}
$$

Base Change

$$
\begin{aligned}
\text{Fact:} \qquad b^{\log_b(a)} &= a \\
\Rightarrow \log\left(b^{\log_b(a)}\right) &= \log(a) \\
\Rightarrow \log_b(a)\log(b) &= \log(a) \\
\Rightarrow \log_b(a) &= \frac{\log(a)}{\log(b)}
\end{aligned}
$$

In general, for positive a, b, and c:

$$\boxed{\log_b(a) = \frac{\log_c(a)}{\log_c(b)}}$$

In general, for positive a, b, and c:

$$\boxed{\log_b(a) = \frac{\log_c(a)}{\log_c(b)}}$$

Suppose your calculator can only compute logarithms base 10. What would you enter to calculate $\log(17)$?

Suppose your calculator can only compute natural logarithms. What would you enter to calculate $\log_2(57)$?

Suppose your calculator can only compute logarithms base 2. What would you enter to calculate $\log(2)$?

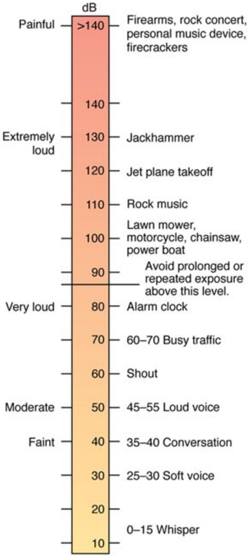
In general, for positive a, b, and c:

$$\boxed{\log_b(a) = \frac{\log_c(a)}{\log_c(b)}}$$

Suppose your calculator can only compute logarithms base 10. What would you enter to calculate $\log(17)$? $\frac{\log_{10}(17)}{\log_{10}(e)}$

Suppose your calculator can only compute natural logarithms. What would you enter to calculate $\log_2(57)$? $\frac{\log(57)}{\log(2)}$

Suppose your calculator can only compute logarithms base 2. What would you enter to calculate $\log(2)$? $\frac{\log_2 2}{\log_2 e} = \frac{1}{\log_2 e}$



Decibels: For a particular measure of the power P of a sound wave, the decibels of that sound is:

$$10 \log_{10}(P)$$

So, every ten decibels corresponds to a sound being ten **times** louder.

Decibels: For a particular measure of the power P of a sound wave, the decibels of that sound is:

$$10\log_{10}(P)$$

So, every ten decibels corresponds to a sound being ten **times** louder.

A lawnmower emits a 100dB sound. How much sound will two lawnmowers make?

A. 100 dB
B. 110 dB
C. 200 dB
D. other

Decibels: For a particular measure of the power P of a sound wave, the decibels of that sound is:

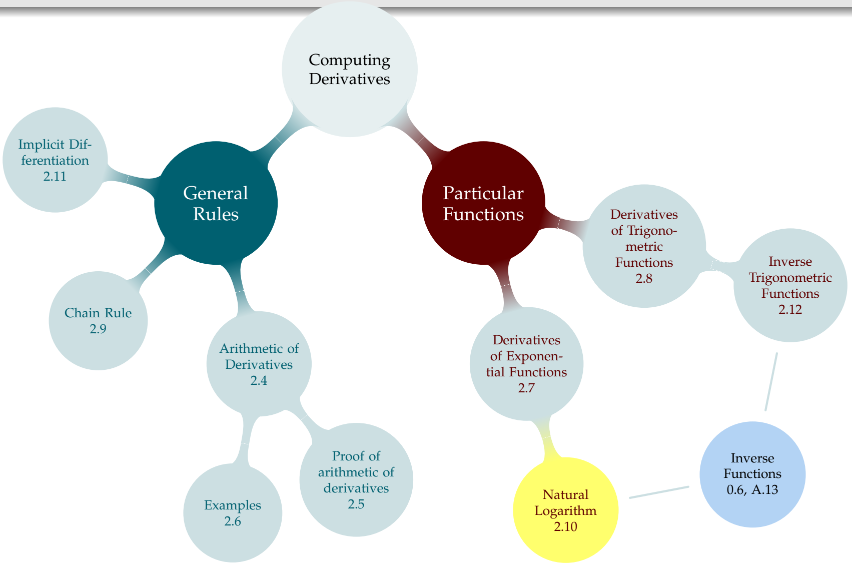
$$10\log_{10}(P)$$

So, every ten decibels corresponds to a sound being ten **times** louder.

A lawnmower emits a 100dB sound. How much sound will two lawnmowers make?

A. 100 dB
B. 110 dB
C. 200 dB
D. other ✓ more than 100, less than 110

Table of Contents



Differentiating the Natural Logarithm

Calculate $\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\}$.

Differentiating the Natural Logarithm

Calculate $\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\}$.
One Weird Trick:

$$x = e^{\log_e x}$$

Differentiating the Natural Logarithm

Calculate $\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\}$.
One Weird Trick:

$$\begin{aligned} x &= e^{\log_e x} \\ \frac{\mathrm{d}}{\mathrm{d}x}\{x\} &= \frac{\mathrm{d}}{\mathrm{d}x}\left\{e^{\log_e x}\right\} \\ 1 &= e^{\log_e x} \cdot \frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\} = x \cdot \frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\} \\ \frac{1}{x} &= \frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\} \end{aligned}$$

Differentiating the Natural Logarithm

Calculate $\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\}$.

Derivative of Natural Logarithm – Theorem 2.10.1

$$\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\} = \frac{1}{x} \qquad (x > 0)$$

Differentiating the Natural Logarithm

Calculate $\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\}$.

Derivative of Natural Logarithm – Theorem 2.10.1

$$\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\} = \frac{1}{x} \qquad (x > 0)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e |x|\} =$$

Differentiating the Natural Logarithm

Calculate $\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\}$.

Derivative of Natural Logarithm – Theorem 2.10.1

$$\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\} = \frac{1}{x} \qquad (x > 0)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e |x|\} = \frac{1}{x} \qquad (x \neq 0)$$

Derivative of Natural Logarithm

$$\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e |x|\} = \frac{1}{x} \qquad (x \neq 0)$$

Differentiate: $f(x) = \log_e |x^2 + 1|$

Derivative of Natural Logarithm

$$\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e|x|\} = \frac{1}{x} \qquad (x \neq 0)$$

Differentiate: $f(x) = \log_e|x^2+1|$

We use the chain rule:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left\{\log_e\left|\boxed{x^2+1}\right|\right\} &= \frac{1}{x^2+1}\cdot(2x)\\ &= \frac{2x}{x^2+1}\end{aligned}$$

Derivatives of Logarithms – Corollary 2.10.6

For $a > 0$:

$$\frac{\mathrm{d}}{\mathrm{d}x}[\log_a |x|] = \frac{1}{x \log a}$$

In particular:

$$\frac{\mathrm{d}}{\mathrm{d}x}[\log |x|] = \frac{1}{x}$$

Derivatives of Logarithms – Corollary 2.10.6

For $a > 0$:

$$\frac{\mathrm{d}}{\mathrm{d}x}[\log_a |x|] = \frac{1}{x \log a}$$

In particular:

$$\frac{\mathrm{d}}{\mathrm{d}x}[\log |x|] = \frac{1}{x}$$

Differentiate: $f(x) = \log_e |\cot x|$

Derivatives of Logarithms – Corollary 2.10.6

For $a > 0$:

$$\frac{\mathrm{d}}{\mathrm{d}x}[\log_a |x|] = \frac{1}{x \log a}$$

In particular:

$$\frac{\mathrm{d}}{\mathrm{d}x}[\log |x|] = \frac{1}{x}$$

Differentiate: $f(x) = \log_e |\cot x|$

We use the chain rule:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\log_e \left|\boxed{\cot x}\right|\right] = \frac{1}{\cot x} \cdot \left(-\csc^2 x\right) = \frac{-\csc^2 x}{\cot x}$$

Logarithmic Differentiation - A Fancy Trick

- $\log(f \cdot g) = \log f + \log g$

- $\log\left(\frac{f}{g}\right) = \log f - \log g$

- $\log\left(f^g\right) = g \log f$

Logarithmic Differentiation - A Fancy Trick

- $\log(f \cdot g) = \log f + \log g$

  multiplication turns into addition

- $\log\left(\frac{f}{g}\right) = \log f - \log g$

  division turns into subtraction

- $\log\left(f^g\right) = g \log f$

  exponentiation turns into multiplication

Logarithmic Differentiation - A Fancy Trick

- $\log(f \cdot g) = \log f + \log g$

  multiplication turns into addition

- $\log\left(\frac{f}{g}\right) = \log f - \log g$

  division turns into subtraction

- $\log\left(f^g\right) = g \log f$

  exponentiation turns into multiplication

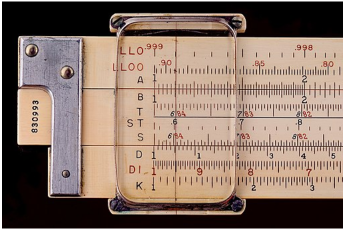
  We can exploit these properties to differentiate!

Logarithmic Differentiation

In general, if $f(x) \neq 0$, $\frac{\mathrm{d}}{\mathrm{d}x}\left[\log|f(x)|\right] = \frac{f'(x)}{f(x)}$.

$$f(x) = \left(\frac{(2x+5)^4(x^2+1)}{x+3}\right)^5$$

Find $f'(x)$.



Logarithmic Differentiation - A Fancy Trick

$$f(x) = \left(\frac{(2x+5)^4(x^2+1)}{x+3} \right)^5$$

Q

ans

Logarithmic Differentiation - A Fancy Trick

$$f(x) = \left(\frac{(2x+5)^4(x^2+1)}{x+3}\right)^5$$

$$\begin{aligned}
\log(f(x)) &= \log\left[\left(\frac{(2x+5)^4(x^2+1)}{x+3}\right)^5\right] \\
&= 5\log\left[\frac{(2x+5)^4(x^2+1)}{x+3}\right] \\
&= 5\Big[4\log(2x+5) + \log(x^2+1) - \log(x+3)\Big] \\
\frac{f'(x)}{f(x)} &= 5\left[4\frac{2}{2x+5} + \frac{2x}{x^2+1} - \frac{1}{x+3}\right] \\
f'(x) &= \left(\frac{(2x+5)^4(x^2+1)}{x+3}\right)^5 \cdot 5\left[4\frac{2}{2x+5} + \frac{2x}{x^2+1} - \frac{1}{x+3}\right]
\end{aligned}$$

Logarithmic Differentiation - A Fancy Trick

Differentiate:

$$f(x) = x^x$$

Q

ans

Logarithmic Differentiation - A Fancy Trick

Differentiate:

$$f(x) = x^x$$

$$\begin{aligned}
\log(f(x)) &= \log\left[x^x\right] \\
&= x \log x \\
\frac{f'(x)}{f(x)} &= x \cdot \frac{1}{x} + \log x \cdot 1 \\
&= 1 + \log x \\
f'(x) &= x^x\left[1 + \log x\right]
\end{aligned}$$

Logarithmic Differentiation - A Fancy Trick

Differentiate:

$$f(x) = \left(\frac{(x^{15}-9x^2)^{10}(x+x^2+1)}{(x^7+7)(x+1)(x+2)(x+3)}\right)^5$$

Q

ans

Logarithmic Differentiation - A Fancy Trick

Differentiate:

$$f(x) = \left(\frac{(x^{15}-9x^2)^{10}(x+x^2+1)}{(x^7+7)(x+1)(x+2)(x+3)}\right)^5$$

$$\begin{aligned}
\log(f(x)) &= \log\left[\left(\frac{(x^{15}-9x^2)^{10}(x+x^2+1)}{(x^7+7)(x+1)(x+2)(x+3)}\right)^5\right] \\
&= 5\log\left[\frac{(x^{15}-9x^2)^{10}(x+x^2+1)}{(x^7+7)(x+1)(x+2)(x+3)}\right] \\
&= 5\Big[10\log\left(x^{15}-9x^2\right)+\log(x+x^2+1)-\log(x^7+7)-\log(x+1)-\log(x+2)-\log(x+3)\Big] \\
\frac{f'(x)}{f(x)} &= 5\left[10\frac{15x^{14}-18x}{x^{15}-9x^2}+\frac{1+2x}{x+x^2+1}-\frac{7x^6}{x^7+7}-\frac{1}{x+1}-\frac{1}{x+2}-\frac{1}{x+3}\right]
\end{aligned}$$

$$f'(x) = \left(\frac{(x^{15}-9x^2)^{10}(x+x^2+1)}{(x^7+7)(x+1)(x+2)(x+3)}\right)^5\cdot 5\left[10\frac{15x^{14}-18x}{x^{15}-9x^2}+\frac{1+2x}{x+x^2+1}-\frac{7x^6}{x^7+7}-\frac{1}{x+1}-\frac{1}{x+2}-\frac{1}{x+3}\right]$$

A

$$f(x) = \frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x}$$

Q

ans

$$f(x) = \frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x}$$

$$\begin{aligned}
\frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x} &= f(x) \\
\log\left|\frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x}\right| &= \log|f(x)| \\
\left[\log|x^8 - e^x| + \log|x^{1/2} + 5| - 5\log|\csc x|\right] &= \log|f(x)| \\
\frac{\mathrm{d}}{\mathrm{d}x}\left[\log|x^8 - e^x| + \log|x^{1/2} + 5| - 5\log|\csc x|\right] &= \frac{\mathrm{d}}{\mathrm{d}x}\log|f(x)| \\
\frac{8x^7 - e^x}{x^8 - e^x} + \frac{\frac{1}{2}x^{-1/2}}{x^{1/2} + 5} - 5\frac{-\csc x \cot x}{\csc x} &= \frac{f'(x)}{f(x)} \\
\frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x}\left(\frac{8x^7 - e^x}{x^8 - e^x} + \frac{\frac{1}{2}x^{-1/2}}{x^{1/2} + 5} + 5\frac{\csc x \cot x}{\csc x}\right) &= f'(x)
\end{aligned}$$

A

$$f(x) = (x^2 + 17)(32x^5 - 8)(x^{98} - x^{57} + 32x^2)^4(32x^{10} - 10x^{32})$$

Find $f'(x)$.

Q

ans

$$f(x) = (x^2+17)(32x^5-8)(x^{98}-x^{57}+32x^2)^4(32x^{10}-10x^{32})$$

Find $f'(x)$.

$$(x^2+17)(32x^5-8)(x^{98}-x^{57}+32x^2)^4(32x^{10}-10x^{32}) = f(x)$$

$$\log\left|(x^2+17)(32x^5-8)(x^{98}-x^{57}+32x^2)^4(32x^{10}-10x^{32})\right| = \log|f(x)|$$

$$\log|x^2+17| + \log|32x^5-8| + 4\log|x^{98}-x^{57}+32x^2| + \log|32x^{10}-10x^{32}| = \log|f(x)|$$

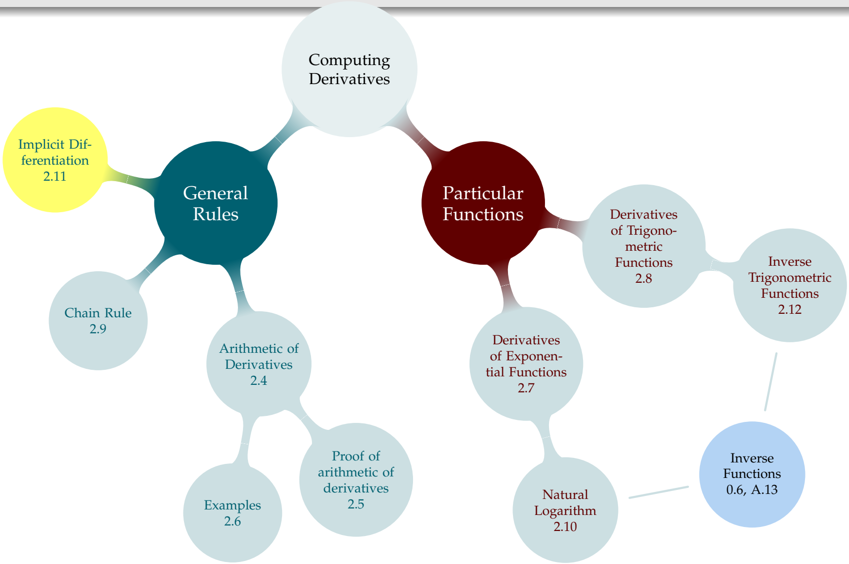
$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\log|x^2+17| + \log|32x^5-8| + 4\log|x^{98}-x^{57}+32x^2| + \log|32x^{10}-10x^{32}|\right] = \frac{\mathrm{d}}{\mathrm{d}x}[\log|f(x)|]$$

$$\frac{2x}{x^2+17} + \frac{160x^4}{32x^5-8} + 4\frac{98x^{97}-57x^{56}+64x}{x^{98}-x^{57}+32x^2} + \frac{320x^9-320x^{31}}{32x^{10}-10x^{32}} = \frac{f'(x)}{f(x)}$$

$$\left((x^2+17)(32x^5-8)(x^{98}-x^{57}+32x^2)^4(32x^{10}-10x^{32})\right)\cdot$$
$$\left(\frac{2x}{x^2+17} + \frac{160x^4}{32x^5-8} + 4\frac{98x^{97}-57x^{56}+64x}{x^{98}-x^{57}+32x^2} + \frac{320x^9-320x^{31}}{32x^{10}-10x^{32}}\right) = f'(x)$$

A

Table of Contents



Implicitly Defined Functions

$$y^2 + x^2 + xy + x^2y = 1$$

Implicitly Defined Functions

$$y^2 + x^2 + xy + x^2y = 1$$

Which of the following points are on the curve?

$(0, 1)$, $(0, -1)$, $(0, 0)$, $(1, 1)$

Q

ans

Implicitly Defined Functions

$$y^2 + x^2 + xy + x^2y = 1$$

Which of the following points are on the curve?

$(0, 1)$, $(0, -1)$, $(0, 0)$, $(1, 1)$

$(0, 1)$ and $(0, -1)$

IMPLICITLY DEFINED FUNCTIONS

$$y^2 + x^2 + xy + x^2y = 1$$

Which of the following points are on the curve?

$(0, 1)$, $(0, -1)$, $(0, 0)$, $(1, 1)$

$(0, 1)$ and $(0, -1)$

If $x = -3$, what is y?

Q

ans

Implicitly Defined Functions

$$y^2 + x^2 + xy + x^2y = 1$$

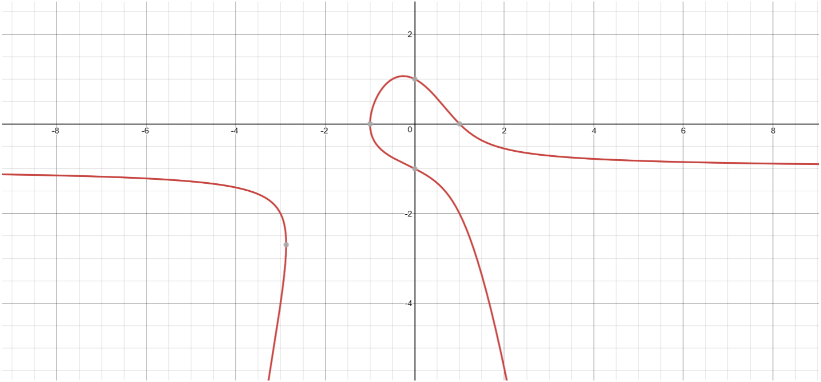
Which of the following points are on the curve?

$(0, 1)$, $(0, -1)$, $(0, 0)$, $(1, 1)$

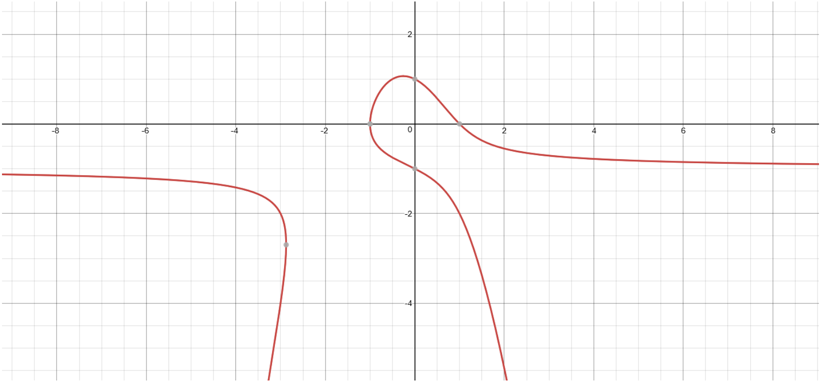
$(0, 1)$ and $(0, -1)$

If $x = -3$, what is y? $y = -2$ and $y = -4$

$$y^2 + x^2 + xy + x^2y = 1$$

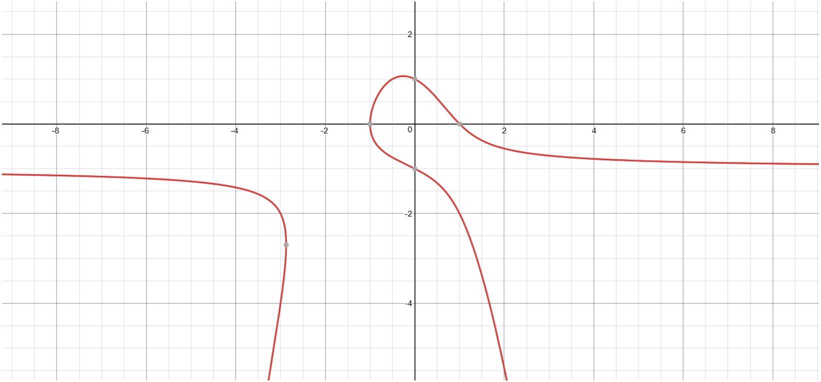


$$y^2 + x^2 + xy + x^2y = 1$$



Still has a slope: $\frac{\Delta y}{\Delta x}$

$$y^2 + x^2 + xy + x^2y = 1$$



Still has a slope: $\frac{\Delta y}{\Delta x}$
Locally, y is still a function of x.

$$y^2 + x^2 + xy + x^2y = 1$$

Consider y as a function of x. Can we find $\frac{dy}{dx}$?

$\frac{d}{dx}[y] =$

ans

$$y^2 + x^2 + xy + x^2y = 1$$

Consider y as a function of x. Can we find $\frac{dy}{dx}$?

$$\frac{d}{dx}[y] = \frac{dy}{dx} = y'$$

ans

$$y^2 + x^2 + xy + x^2y = 1$$

Consider y as a function of x. Can we find $\frac{dy}{dx}$?

$$\frac{d}{dx}[y] = \frac{dy}{dx} = y' \qquad\qquad \frac{d}{dx}[x] =$$

ans

$$y^2 + x^2 + xy + x^2y = 1$$

Consider y as a function of x. Can we find $\frac{dy}{dx}$?

$$\frac{d}{dx}[y] = \frac{dy}{dx} = y' \qquad\qquad \frac{d}{dx}[x] = 1$$

$$y^2 + x^2 + xy + x^2y = 1$$

Consider y as a function of x. Can we find $\frac{dy}{dx}$?

$\frac{d}{dx}[y] = \frac{dy}{dx} = y'$ $\qquad$ $\frac{d}{dx}[x] = 1$ $\qquad$ $\frac{d}{dx}[1] =$

ans

$$y^2 + x^2 + xy + x^2y = 1$$

Consider y as a function of x. Can we find $\frac{dy}{dx}$?

$$\frac{d}{dx}[y] = \frac{dy}{dx} = y' \qquad \frac{d}{dx}[x] = 1 \qquad \frac{d}{dx}[1] = 0$$

ans

$$y^2 + x^2 + xy + x^2y = 1$$

Consider y as a function of x. Can we find $\frac{\mathrm{d}y}{\mathrm{d}x}$?

$$\frac{\mathrm{d}}{\mathrm{d}x}[y] = \frac{\mathrm{d}y}{\mathrm{d}x} = y' \qquad \frac{\mathrm{d}}{\mathrm{d}x}[x] = 1 \qquad \frac{\mathrm{d}}{\mathrm{d}x}[1] = 0$$

Differentiate both sides with respect to x.

$$\begin{aligned}
0 &= 2y\frac{\mathrm{d}y}{\mathrm{d}x} + 2x + \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + (1)y\right) + \left(x^2\frac{\mathrm{d}y}{\mathrm{d}x} + 2xy\right)\\
0 &= \frac{\mathrm{d}y}{\mathrm{d}x}\left(2y + x + x^2\right) + (2x + y + 2xy)\\
-(2x + y + 2xy) &= \frac{\mathrm{d}y}{\mathrm{d}x}\left(2y + x + x^2\right)\\
-\frac{2x + y + 2xy}{2y + x + x^2} &= \frac{\mathrm{d}y}{\mathrm{d}x}
\end{aligned}$$

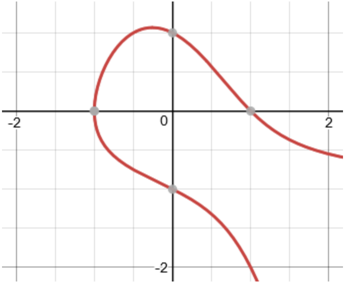
$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

Necessarily, $\frac{\mathrm{d}y}{\mathrm{d}x}$ depends on **both** y and x. Why?



$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

Necessarily, $\frac{\mathrm{d}y}{\mathrm{d}x}$ depends on **both** y and x. Why?

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(1,0)} =$$

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(1,-2)} =$$

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

Necessarily, $\frac{\mathrm{d}y}{\mathrm{d}x}$ depends on **both** y and x. Why?

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(1,0)} = -\frac{2(1) + 0 + 2(1)(0)}{2(0) + 1 + 1} = -\frac{2}{2} = -1$$

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(1,-2)} = -\frac{2(1) - 2 + 2(1)(-2)}{2(-2) + 1 + 1} = -2$$

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{dy}{dx} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

Necessarily, $\frac{dy}{dx}$ depends on **both** y and x. Why?

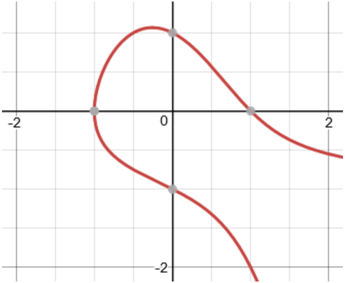
$$\left.\frac{dy}{dx}\right|_{(1,0)} = -\frac{2(1) + 0 + 2(1)(0)}{2(0) + 1 + 1} = -\frac{2}{2} = -1$$

$$\left.\frac{dy}{dx}\right|_{(1,-2)} = -\frac{2(1) - 2 + 2(1)(-2)}{2(-2) + 1 + 1} = -2$$

Points with the same x-value may have different slopes. We need both the x-value and the y-value to figure out which point we're talking about.

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x + y + 2xy}{2y + x + x^2}$$



Points with the same x-value may have different slopes. We need both the x-value and the y-value to figure out which point we're talking about.

Now You

Suppose $x^4y + y^4x = 2$. Find $\frac{dy}{dx}$ at the point $(1, 1)$.

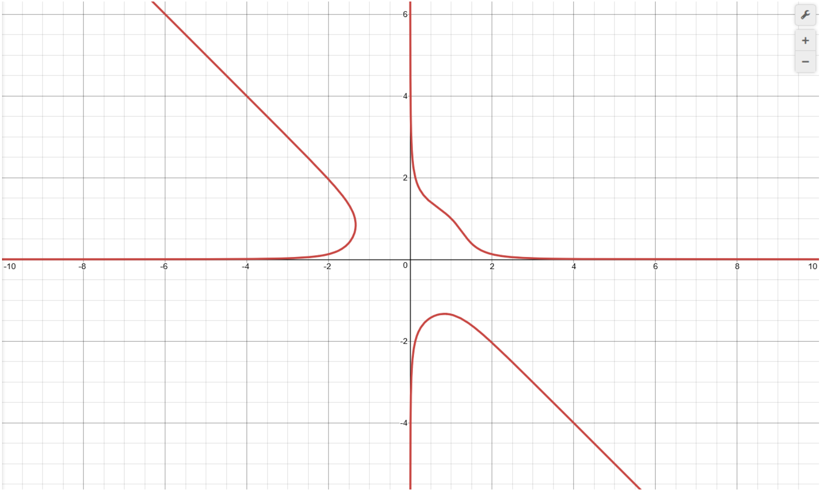
Q

Now You Suppose $x^4y + y^4x = 2$. Find $\frac{dy}{dx}$ at the point $(1,1)$.

$$\begin{aligned} x^4y(x) + y(x)^4x &= 2 \\ 4x^3y(x) + x^4\frac{dy}{dx}(x) + y(x)^4 + 4y(x)^3\frac{dy}{dx}(x)\,x &= 0 \end{aligned}$$

We may only replace variables with constants *after* differentiating. When $x = 1$ and $y(1) = 1$,

$$\begin{aligned} 4(1)^3y(1) + (1)^4\frac{dy}{dx}(1) + y(1)^4 + 4y(1)^3\frac{dy}{dx}(1) &= 0 \\ 4 + \frac{dy}{dx}(1) + 1 + 4\frac{dy}{dx}(1) &= 0 \\ 5\frac{dy}{dx}(1) &= -5 \\ \frac{dy}{dx}(1) &= -1 \end{aligned}$$



$$x^4y + y^4x = 2$$

NOW YOU Suppose $\dfrac{3y^2+2y+y^3}{x^2+1}=x$. Find $\dfrac{dy}{dx}$ when $x=0$, and the equations of the associated tangent line(s).

Q

ans

Now You Suppose $\dfrac{3y^2+2y+y^3}{x^2+1}=x$. Find $\dfrac{dy}{dx}$ when $x=0$, and the equations of the associated tangent line(s).

To avoid the quotient rule, we start by simplifying our expression.

$$\frac{3y(x)^2+2y(x)+y(x)^3}{x^2+1}=x$$

$$3y(x)^2+2y(x)+y(x)^3=x^3+x$$

$$6y(x)\frac{dy}{dx}(x)+2\frac{dy}{dx}(x)+3y(x)^2\frac{dy}{dx}(x)=3x^2+1$$

When $x=0$:

$$\frac{dy}{dx}(0)=\frac{1}{6y(0)+2+3y(0)^2}$$

We need to know y to find $\frac{dy}{dx}$. We want all points where $x = 0$.

$$\begin{aligned} 3y(0)^2 + 2y(0) + y(0)^3 &= 0 \\ y(0)\left(y(0)^2 + 3y(0) + 2\right) &= 0 \\ y(0)\left(y(0) + 1\right)\left(y(0) + 2\right) &= 0 \\ y(0) = 0,\ y(0) = -1,\ y(0) &= -2 \end{aligned}$$

$$\frac{dy}{dx} = \frac{1}{6y + 2 + 3y^2}$$

$(0, 0)$

$$\left.\frac{dy}{dx}\right|_{(0,0)} = \frac{1}{2}$$

$$y - 0 = \frac{1}{2}(x - 0)$$

$$y = \frac{1}{2}x$$

$(0, -1)$

$$\left.\frac{dy}{dx}\right|_{(0,-1)} = \frac{1}{-6+2+3} = -1$$

$$y - (-1) = -1(x - 0)$$

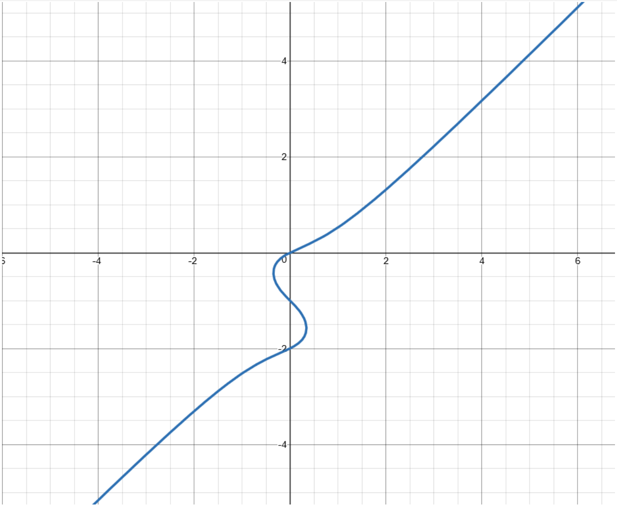
$$y = -x - 1$$

$(0, -2)$

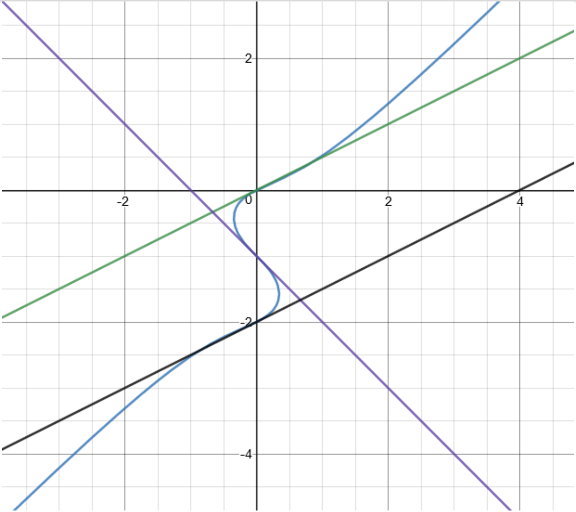
$$\left.\frac{dy}{dx}\right|_{(0,-2)} = \frac{1}{-12+2+12} = \frac{1}{2}$$

$$y - (-2) = \frac{1}{2}(x - 0)$$

$$y = \frac{1}{2}x - 2$$



$$\frac{3y^2 + 2y + y^3}{x^2 + 1} = x$$



$$\frac{3y^2+2y+y^3}{x^2+1}=x$$

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

$$\begin{aligned} \log x &= y(x) \\ x &= e^{y(x)} \end{aligned}$$

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

$$\begin{aligned}
\log x &= y(x) \\
x &= e^{y(x)} \\
1 &= e^{y(x)} \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{1}{e^{y(x)}} = \frac{1}{x}
\end{aligned}$$

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

$$\begin{aligned} \log x &= y(x) \\ x &= e^{y(x)} \\ 1 &= e^{y(x)} \cdot \frac{dy}{dx}(x) \\ \frac{dy}{dx}(x) &= \frac{1}{e^{y(x)}} = \frac{1}{x} \end{aligned}$$

Use implicit differentiation to differentiate $\log|x|$, $x < 0$.

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

Use implicit differentiation to differentiate $\log|x|$, $x < 0$.

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

Use implicit differentiation to differentiate $\log|x|$, $x < 0$.

$$\begin{aligned}
\log|x| &= y(x) \\
\log(-x) &= y(x) \\
-x &= e^{y(x)} \\
-1 = e^{y(x)} \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) & \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{-1}{e^{y(x)}} = \frac{-1}{-x} = \frac{1}{x}
\end{aligned}$$

Use implicit differentiation to differentiate $\log_a(x)$, where $a > 0$ is a constant and $x > 0$.

Use implicit differentiation to differentiate $\log_a |x|$, $a > 0$.

Use implicit differentiation to differentiate $\log_a(x)$, where $a > 0$ is a constant and $x > 0$.

Use implicit differentiation to differentiate $\log_a(x)$, where $a > 0$ is a constant and $x > 0$.

$$\begin{aligned}
\log_a x &= y(x) \\
x &= a^{y(x)} \\
1 &= a^{y(x)} \cdot \log_e a \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{1}{a^{y(x)} \cdot \log_e a} = \frac{1}{x \log_e a}
\end{aligned}$$

Use implicit differentiation to differentiate $\log_a |x|$, $a > 0$.

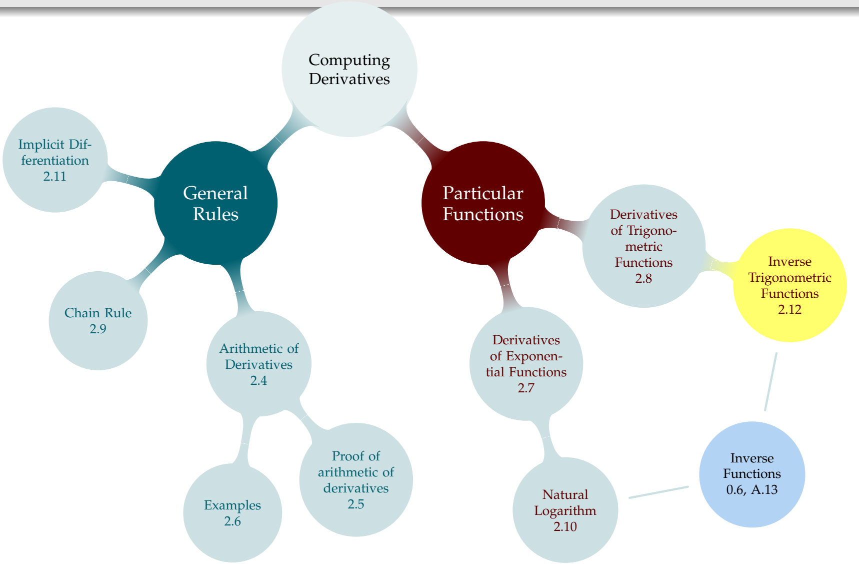
Use implicit differentiation to differentiate $\log_a |x|$, $a > 0$.

If $x > 0$, it's what we just computed. So assume $x < 0$.

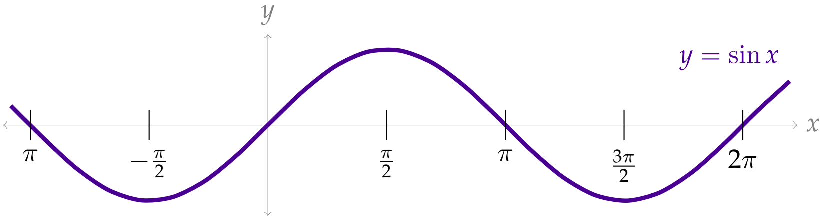
$$\begin{aligned}
\log_a |x| &= y(x) \\
\log_a(-x) &= y(x) \\
-x &= a^{y(x)} \\
-1 &= a^{y(x)} \cdot \log_e a \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{-1}{a^{y(x)} \cdot \log_e a} = \frac{-1}{x \log_e a}
\end{aligned}$$

Table of Contents

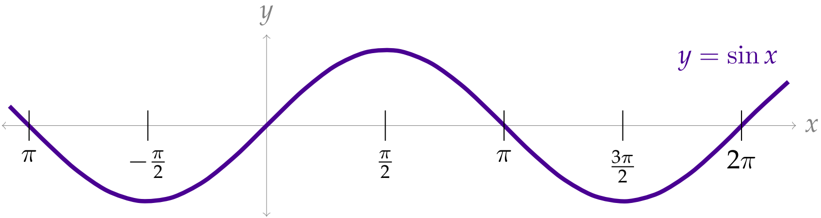
SKIP DEFINITIONS OF INVERSE TRIG FUNCTIONS



Invertibility Game

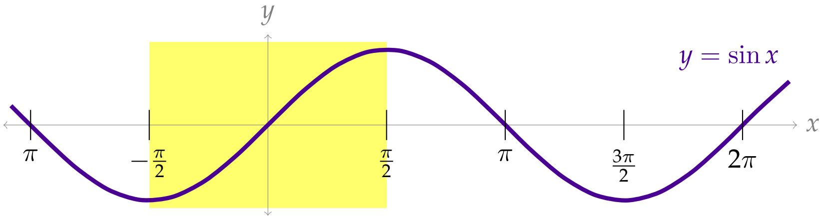


INVERTIBILITY GAME



I'm thinking of a number x. Your hint: $\sin(x) = 0$. What number am I thinking of?

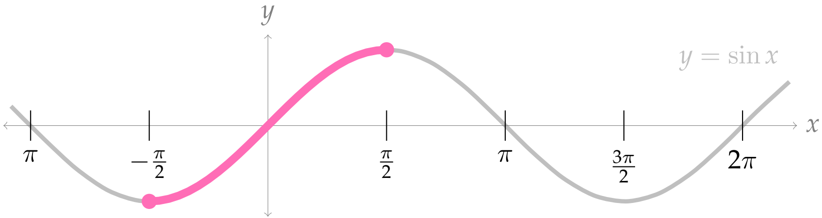
Invertibility Game



I'm thinking of a number x. Your hint: $\sin(x) = 0$. What number am I thinking of?

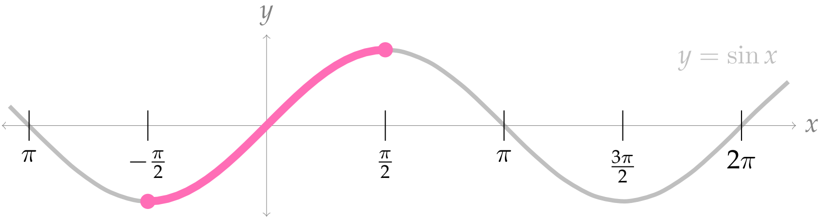
I'm thinking of a number x, and x is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Your hint: $\sin(x) = 0$. What number am I thinking of?

ARCSINE



$\arcsin(x)$ is the inverse of $\sin x$ restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Arcsine



$\arcsin(x)$ is the inverse of $\sin x$ restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\arcsin x$ is the (unique) number θ such that:

- $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, and
- $\sin \theta = x$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$—\ \frac{\pi}{6}$	$—\ \frac{1}{2}$
$—\ \frac{\pi}{4}$	$—\ \frac{1}{\sqrt{2}}$
$—\ \frac{\pi}{3}$	$—\ \frac{\sqrt{3}}{2}$
$—\ \frac{\pi}{2}$	$—\ 1$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0)$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right)$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right)$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$
- $\arcsin\left(\frac{\pi}{2}\right)$

ARCSINE

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$
- $\arcsin\left(\frac{\pi}{2}\right)$ undefined

Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$
- $\arcsin\left(\frac{\pi}{2}\right)$ undefined
- $\arcsin\left(\frac{\pi}{4}\right)$

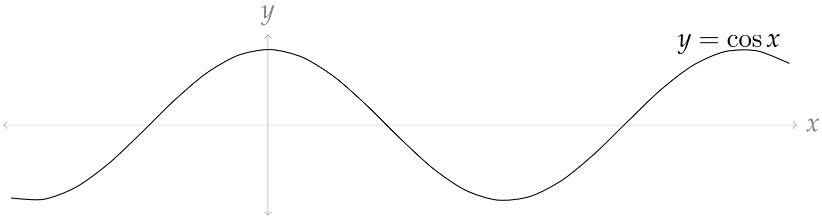
Arcsine

Reference Angles:

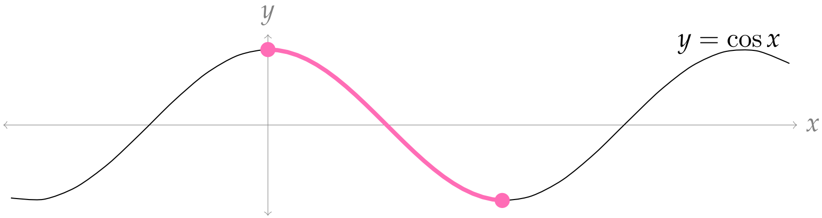
θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0) = 0$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$
- $\arcsin\left(\frac{\pi}{2}\right)$ undefined
- $\arcsin\left(\frac{\pi}{4}\right)$ defined, but we haven't covered tools (yet) to figure it out

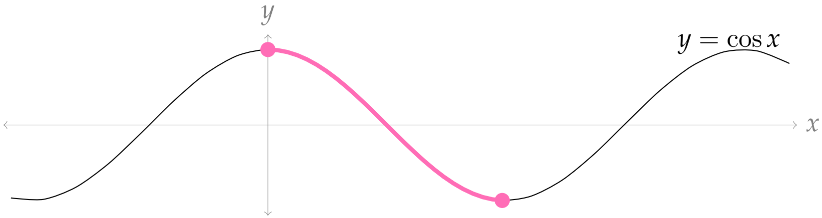
Arccosine



Arccosine



ARCCOSINE

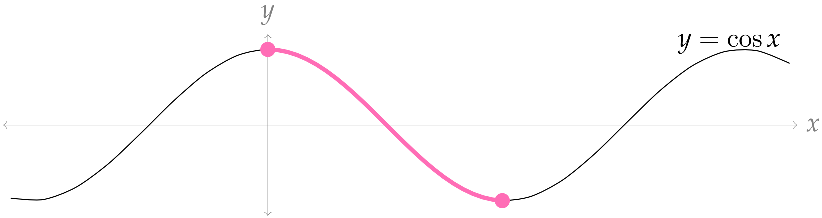


$\arccos(x)$ is the inverse of $\cos x$ restricted to $[0, \pi]$.

$\arccos(x)$ is the (unique) number θ such that:

- $\cos(\theta) = x$ and
- $0 \leq \theta \leq \pi$

Arccosine

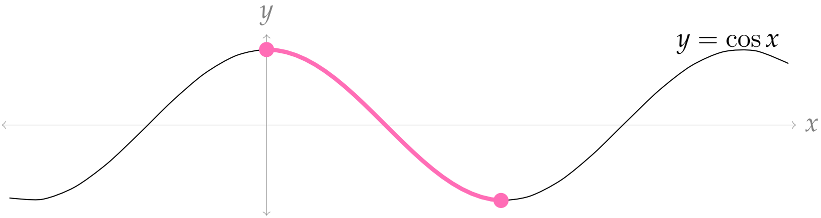


$\arccos(x)$ is the inverse of $\cos x$ restricted to $[0, \pi]$.

$\arccos(x)$ is the (unique) number θ such that:

- $\cos(\theta) = x$ and $\leftarrow\leftarrow\leftarrow$ inverse
- $0 \le \theta \le \pi$

Arccosine

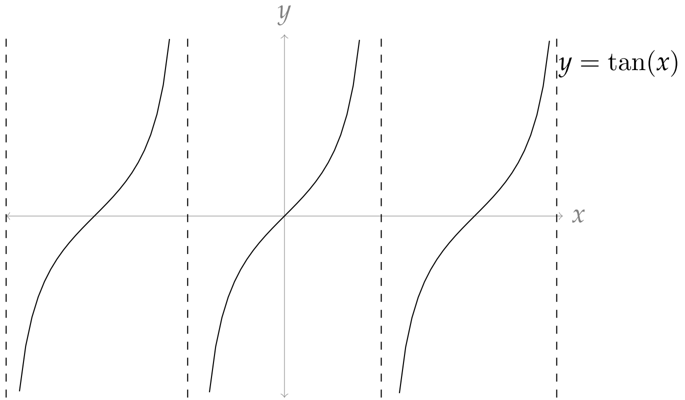


$\arccos(x)$ is the inverse of $\cos x$ restricted to $[0, \pi]$.

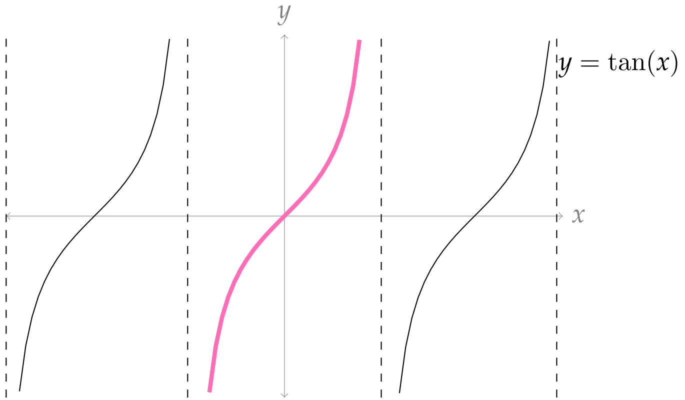
$\arccos(x)$ is the (unique) number θ such that:

- $\cos(\theta) = x$ and $\leftarrow\leftarrow\leftarrow$ inverse
- $0 \leq \theta \leq \pi$ $\leftarrow\leftarrow\leftarrow$ inverse exists

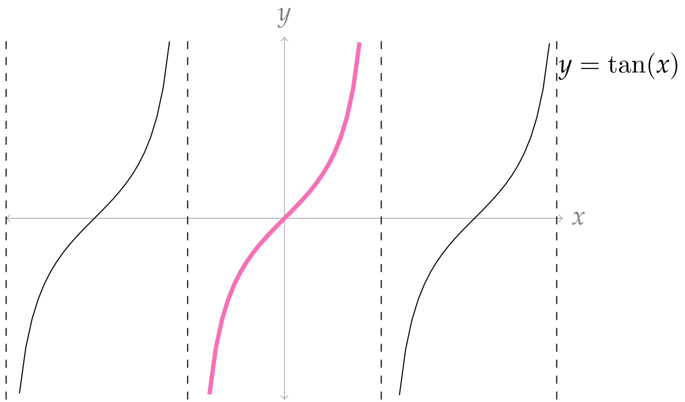
Arctangent



Arctangent



Arctangent



$\arctan(x) = \theta$ means:
(1) $\tan(\theta) = x$ and
(2) $-\pi/2 < \theta < \pi/2$

Arcsecant, arcsine, and arccotangent

$$\operatorname{arcsec}(x) =$$

Arcsecant, arcsine, and arccotangent

$$\operatorname{arcsec}(x) =$$

$$\begin{aligned}
\operatorname{arcsec}(x) &= y \\
\sec y &= x \\
\frac{1}{\cos y} &= x \\
\cos y &= \tfrac{1}{x} \\
y &= \arccos\left(\tfrac{1}{x}\right) \\
\operatorname{arcsec}(x) &= \arccos\left(\tfrac{1}{x}\right)
\end{aligned}$$

Arcsecant, arcsine, and arccotangent

$$\text{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$$

$$\begin{aligned}
\text{arcsec}(x) &= y \\
\sec y &= x \\
\frac{1}{\cos y} &= x \\
\cos y &= \frac{1}{x} \\
y &= \arccos\left(\frac{1}{x}\right) \\
\text{arcsec}(x) &= \arccos\left(\frac{1}{x}\right)
\end{aligned}$$

Arcsecant, arcsine, and arccotangent

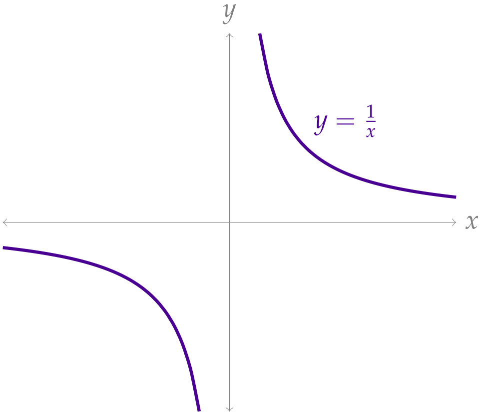
$$\text{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$

$$\begin{aligned}
\text{arccsc}(x) &= y \\
\csc y &= x \\
\frac{1}{\sin y} &= x \\
\sin y &= \frac{1}{x} \\
y &= \arcsin\left(\frac{1}{x}\right) \\
\text{arccsc}(x) &= \arcsin\left(\frac{1}{x}\right)
\end{aligned}$$

$$\text{arccot(x)} = \arctan\left(\frac{1}{x}\right)$$

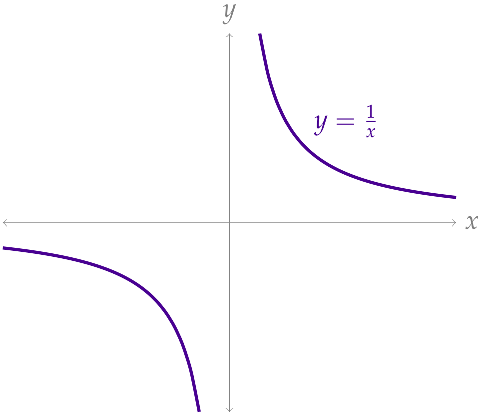
$$\begin{aligned}
\text{arccot(x)} &= y \\
\cot y &= x \\
\frac{1}{\tan y} &= x \\
\tan y &= \frac{1}{x} \\
y &= \arctan\left(\frac{1}{x}\right) \\
\text{arccot(x)} &= \arctan\left(\frac{1}{x}\right)
\end{aligned}$$

$$\mathrm{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$$



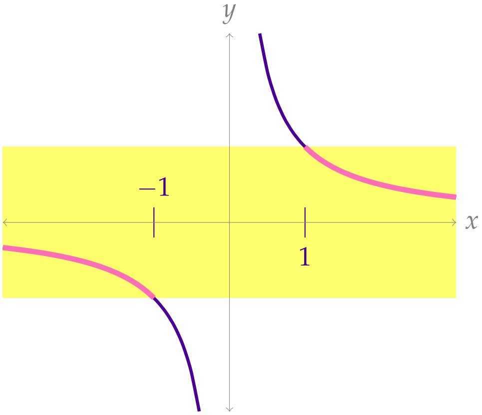
$$\operatorname{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$$

The domain of $\arccos(y)$ is $-1 \le y \le 1$, so the domain of $\operatorname{arcsec}(y)$ is



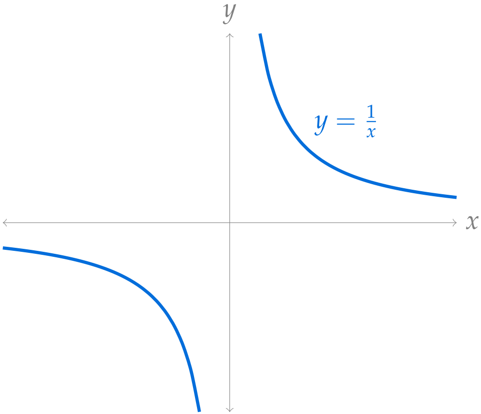
$$\text{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$$

The domain of $\arccos(y)$ is $-1 \le y \le 1$, so the domain of $\text{arcsec}(y)$ is



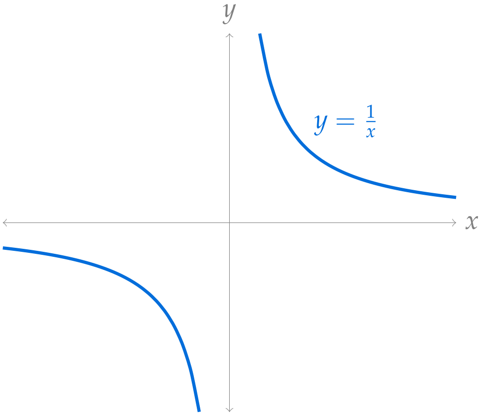
$(-\infty, -1] \cup [1, \infty)$.

$$\text{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$



$$\text{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$

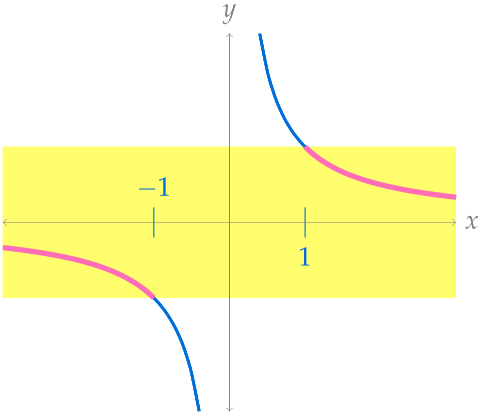
Domain of $\arcsin(y)$ is $-1 \leq y \leq 1$, so the domain of $\text{arccsc}(x)$ is



ans

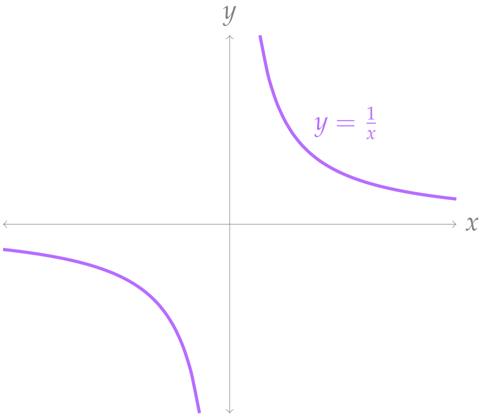
$$\operatorname{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$

Domain of $\arcsin(y)$ is $-1 \leq y \leq 1$, so the domain of $\operatorname{arccsc}(x)$ is



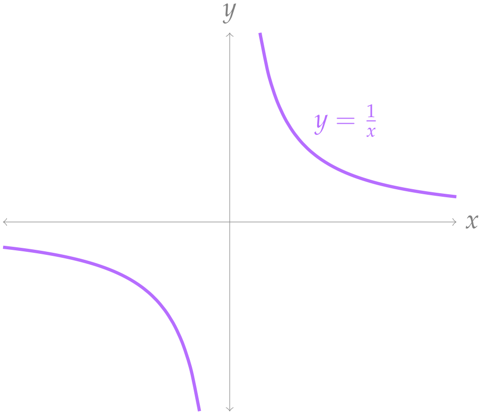
$(-\infty, -1] \cup [1, \infty)$.

$$\text{arccot}(x) = \arctan\left(\frac{1}{x}\right)$$



$$\text{arccot}(x) = \arctan\left(\frac{1}{x}\right)$$

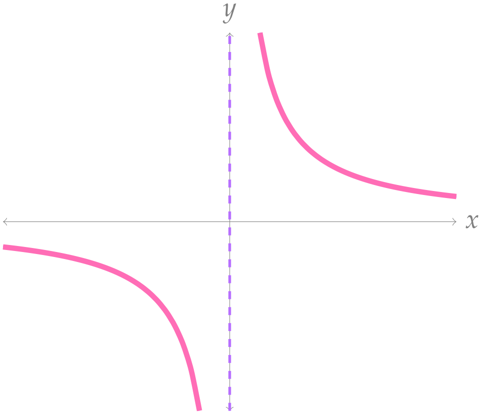
Domain of $\arctan(x)$ is all real numbers, so the domain of $\text{arccot}(x)$ is



ans

$$\text{arccot(x)} = \arctan\left(\frac{1}{x}\right)$$

Domain of $\arctan(x)$ is all real numbers, so the domain of $\text{arccot}(x)$ is



$(-\infty, 0) \cup (0, \infty)$.

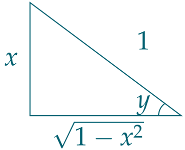
$$y = \arcsin x$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

Q

ans

$$\begin{aligned}
y(x) &= \arcsin x \\
x &= \sin y(x) \\
\frac{\mathrm{d}}{\mathrm{d}x}[x] &= \frac{\mathrm{d}}{\mathrm{d}x}[\sin y(x)] \\
1 &= \cos y(x) \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{1}{\cos y(x)} \\
&= \frac{\text{hyp}}{\text{adj}} \\
&= \frac{1}{\sqrt{1-x^2}}
\end{aligned}$$

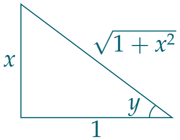


ans

$$y = \arctan x$$

Find $\frac{dy}{dx}$.

$$
\begin{aligned}
y(x) &= \arctan x \\
x &= \tan y(x) \\
\frac{\mathrm{d}}{\mathrm{d}x}[x] &= \frac{\mathrm{d}}{\mathrm{d}x}[\tan y(x)] \\
1 &= \sec^2 y(x) \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \cos^2 y(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \left(\frac{\text{adj}}{\text{hyp}}\right)^2 = \left(\frac{1}{\sqrt{1+x^2}}\right)^2 \\
&= \frac{1}{1+x^2}
\end{aligned}
$$



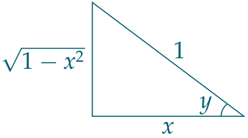
$$y = \arccos x$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

Q

ans

$$\begin{aligned}
y(x) &= \arccos x \\
x &= \cos y(x) \\
\frac{\mathrm{d}}{\mathrm{d}x}[x] &= \frac{\mathrm{d}}{\mathrm{d}x}[\cos y(x)] \\
1 &= -\sin y(x) \cdot \frac{\mathrm{d}y}{\mathrm{d}x}(x) \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{-1}{\sin y(x)} \\
\frac{\mathrm{d}y}{\mathrm{d}x}(x) &= \frac{-\text{hyp}}{\text{opp}} = \frac{-1}{\sqrt{1-x^2}}
\end{aligned}$$



To differentiate arcsecant, arccosecant, and arccotangent, you can use the chain rule!

To differentiate arcsecant, arccosecant, and arccotangent, you can use the chain rule!

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\operatorname{arccsc}(x)\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[\arcsin\left(\frac{1}{x}\right)\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[\arcsin\left(x^{-1}\right)\right]$$

To differentiate arcsecant, arccosecant, and arccotangent, you can use the chain rule!

$$\frac{d}{dx}[\operatorname{arccsc}(x)] = \frac{d}{dx}\left[\arcsin\left(\frac{1}{x}\right)\right] = \frac{d}{dx}\left[\arcsin\left(x^{-1}\right)\right]$$

$$\frac{d}{dx}\left[\arcsin\left(\boxed{x^{-1}}\right)\right] = \frac{1}{\sqrt{1-\left(\boxed{x^{-1}}\right)^2}} \cdot \boxed{\left(-x^{-2}\right)} = \frac{-1}{x^2\sqrt{1-x^{-2}}}$$

$$= \frac{-1}{\sqrt{x^4}\sqrt{1-x^{-2}}} = \frac{-1}{\sqrt{x^2}\sqrt{x^2}\sqrt{1-x^{-2}}} = \frac{-1}{\sqrt{x^2}\sqrt{x^2-1}} = \frac{-1}{|x|\sqrt{1-x^2}}$$

Derivatives of Inverse Trigonometric Functions – Theorem 2.12.7

Memorize:

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arccos x] = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arcsin x] = \frac{1}{1+x^2}$$

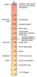
Be able to derive:

$$\frac{d}{dx}[\operatorname{arccsc} x] = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\operatorname{arcsec} x] = \frac{1}{|x|\sqrt{x^2-1}}$$

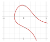
$$\frac{d}{dx}[\operatorname{arccot} x] = -\frac{1}{1+x^2}$$

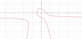
Included Work

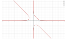

Anonymous. (2012) Decibel Scale of Frequently Heard Sounds. Biology Forums. http://biology-forums.com/index.php?action=gallery;sa=view;id=6156 (accessed 7 October 2015) , 97


'Brain' by Eucalyp is licensed under CC BY 3.0 (accessed 8 June 2021), 148, 149, 151, 152


'Slide rule – KEUFFEL & ESSER CO. N.Y. ' by s58y is licensed under CC BY 2.0 (accessed 20 July 2021) , 115


screenshot of graph using Desmos Graphing Calculator, https://www.desmos.com/calculator (accessed 19 October 2017), 143, 147


screenshot of graph using Desmos Graphing Calculator, https://www.desmos.com/calculator (accessed 19 October 2017), 132–134


screenshot of graph using Desmos Graphing Calculator, https://www.desmos.com/calculator (accessed 24 October 2018), 150


screenshot of graph using Desmos Graphing Calculator, https://www.desmos.com/calculator, (accessed 24 October 2018), 155