4.1 Antiderivatives

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$\frac{\mathrm{d}}{\mathrm{d}x}[x^2 + 5] = 2x$, so $x^2 + 5$ is (also) an antiderivative of $2x$.

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What is the most general antiderivative of $2x$?

$\boxed{x^2 + c}$, where we understand c as some constant (a number not depending on x).

ANTIDERIVATIVES

Find the most general antiderivative for the following equations.

$$f(x) = 17$$

$$f(x) = m$$

where m is a constant.

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$17x + c$

$$f(x) = m$$

where m is a constant.
$mx + c$

differentiation fact		antidifferentiation fact	
$\frac{\mathrm{d}}{\mathrm{d}x}[x^2] = 2x$	$\Longrightarrow$	antideriv of $2x$:	$x^2 + c$

ans

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$\frac{\mathrm{d}}{\mathrm{d}x}[x^2] = 2x$	$\Longrightarrow$	antideriv of $2x$:	$x^2 + c$
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$\frac{\mathrm{d}}{\mathrm{d}x}[x^3] = 3x^2$	$\Longrightarrow$		
$\frac{\mathrm{d}}{\mathrm{d}x}[x^4] = 4x^3$	$\Longrightarrow$		
$\frac{\mathrm{d}}{\mathrm{d}x}[x^5] = 5x^4$	$\Longrightarrow$		

antideriv of x^n:

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		antideriv of x^3	$\frac{1}{4}x^4 + c$
$\frac{d}{dx}[x^5] = 5x^4$	$\Longrightarrow$	antideriv of $5x^4$:	$x^5 + c$
		antideriv of x^4:	$\frac{1}{5}x^5 + c$
		antideriv of x^n:	$\frac{1}{n+1}x^{n+1} + c$
		Check: $\frac{d}{dx}\left[\frac{1}{n+1}x^{n+1} + c\right] = x^n$	

Power Rule for Antidifferentiation

The most general antiderivative of x^n is $\frac{1}{n+1}x^{n+1} + c$ if $n \neq -1$

- $\frac{\mathrm{d}}{\mathrm{d}x}\Big[\qquad\qquad \Big] = x^5$
- $\frac{\mathrm{d}}{\mathrm{d}x}\Big[\qquad\qquad \Big] = x^3$
- $\frac{\mathrm{d}}{\mathrm{d}x}\Big[\qquad\qquad \Big] = \frac{1}{2}x^3$

Q Q Q

ans

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- $\dfrac{\mathrm{d}}{\mathrm{d}x}\Big[\qquad \Big] = 5x^2 - 15x + 3$
- $\dfrac{\mathrm{d}}{\mathrm{d}x}\Big[\qquad \Big] = 13\left(5x^{14} - 3x^{3/7} + 52e^x\right)$

Find the most general antiderivatives.

$$f(x) = \cos x$$

$$f(x) = \sin x$$

$$f(x) = \sec^2 x$$

$$f(x) = \frac{1}{1+x^2}$$

$$f(x) = \frac{1}{1+x^2+2x}$$

Find the most general antiderivatives.

$$f(x) = \cos x$$

$\sin x + c$

$$f(x) = \sin x$$

$-\cos x + c$

$$f(x) = \sec^2 x$$

$\tan x + c$

$$f(x) = \frac{1}{1+x^2}$$

$\arctan x + c$

$$f(x) = \frac{1}{1+x^2+2x}$$

$\frac{-1}{x+1}$

Find the most general antiderivatives.

$$f(x) = 17\cos x + x^5$$

$$f(x) = \frac{23}{5+5x^2}$$

$$f(x) = \frac{23}{5+125x^2}$$

Q

ans

Find the most general antiderivatives.

$$f(x) = 17\cos x + x^5$$

$17\sin x + \frac{1}{6}x^6 + c$

$$f(x) = \frac{23}{5 + 5x^2}$$

$\frac{23}{5}\arctan x + c$

$$f(x) = \frac{23}{5 + 125x^2}$$

$\frac{23}{25}\arctan(5x) + c$

Find the most general antiderivatives.

$$f(x) = \frac{1}{x},\ x > 0$$

$$f(x) = 5x^2 - 32x^5 - 17$$

$$f(x) = \csc x \cot x$$

$$f(x) = \frac{5}{\sqrt{1-x^2}} + 17$$

Q

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$\ln x + c$

$$f(x) = 5x^2 - 32x^5 - 17$$

$\frac{5}{3}x^3 - \frac{16}{3}x^6 - 17x + c$

$$f(x) = \csc x \cot x$$

$-\csc x + c$

$$f(x) = \frac{5}{\sqrt{1-x^2}} + 17$$

$5 \arcsin x + 17x + c$

A

Chose Your Own Adventure

Antiderivative of $\sin x \cos x$:

A. $\cos x \sin x + c$

B. $-\cos x \sin x + c$

C. $\sin^2 x + c$

D. $\frac{1}{2} \sin^2 x + c$

E. $\frac{1}{2} \cos^2 x \sin^2 x + c$

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In general, antiderivatives of x^n have the form $\frac{1}{n+1}x^{n+1}$. What is the single exception?

A. $n = -1$

B. $n = 0$

C. $n = 1$

D. $n = e$

E. $n = 1/2$

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All the Adventures are Calculus, Though

Suppose the velocity of a particle at time t is given by $v(t) = t^2 + \cos t + 3$. What function gives its position?

A. $s(t) = 2t - \sin t$

B. $s(t) = 2t - \sin t + c$

C. $s(t) = t^3 + \sin t + 3t + c$

D. $s(t) = \frac{1}{3}t^3 + \sin t + 3t + c$

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Suppose the velocity of a particle at time t is given by $v(t) = t^2 + \cos t + 3$, and its position at time 0 is given by $s(0) = 5$. What function gives its position?

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B. $s(t) = \frac{1}{3}t^3 + \sin t + 3t + 5$
C. $s(t) = \frac{1}{3}t^3 + \sin t + 3t + c$
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Find all functions $f(x)$ with $f(1) = 5$ and $f'(x) = e^{3x+5}$.

Find all functions $f(x)$ with $f(1) = 5$ and $f'(x) = e^{3x+5}$.

Antiderivative of e^{3x+5} is $\frac{1}{3}e^{3x+5} + c$. So we only need to solve for c.

$$5 = f(1) = \frac{1}{3}e^{3+5} + c$$

implies

$$c = 5 - \frac{e^8}{3}$$

So

$$f(x) = \frac{1}{3}e^{3x+5} + 5 - \frac{e^8}{3}$$

Let $Q(t)$ be the amount of a radioactive isotope in a sample. Suppose the sample is losing $50e^{-5t}$ mg per second to decay. If $Q(1) = 10e^{-5}$mg, find the equation for the amount of the isotope at time t.

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We have

$$\frac{dQ}{dt} = -50e^{-5t}$$

Note the negative, since our sample is getting smaller.

Then antidifferentiating, we find $Q(t) = 10e^{-5t} + c$ for some constant c.

Then since $Q(1) = 10e^{-5}$, we see $c = 0$.

$$\boxed{Q(t) = 10e^{-5t}}$$

Suppose $f'(t) = 2t + 7$. What is $f(10) - f(3)$?

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From antidifferentiation, we have $f(t) = t^2 + 7t + c$. Then

$$f(10) - f(3) = [100 + 70 + c] - [9 + 21 + c] = 170 - 30 = 140$$