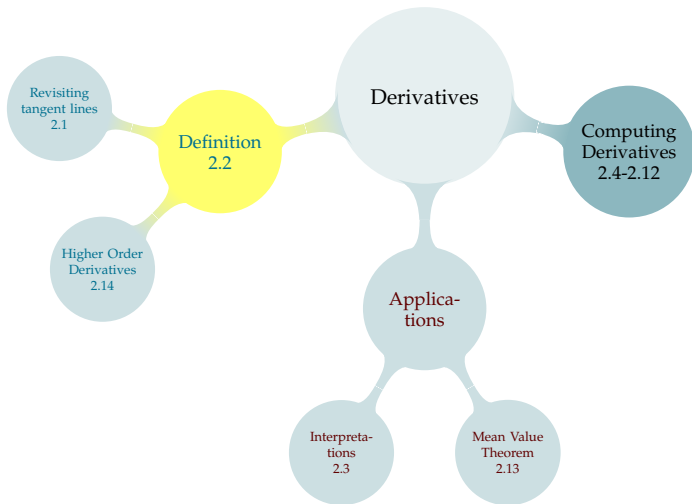


1/25

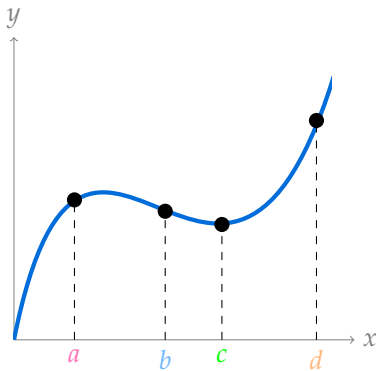


$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

If $f'(a) < 0$, then f is **decreasing** at a . Its graph “points down.”

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PRACTICE: INCREASING AND DECREASING



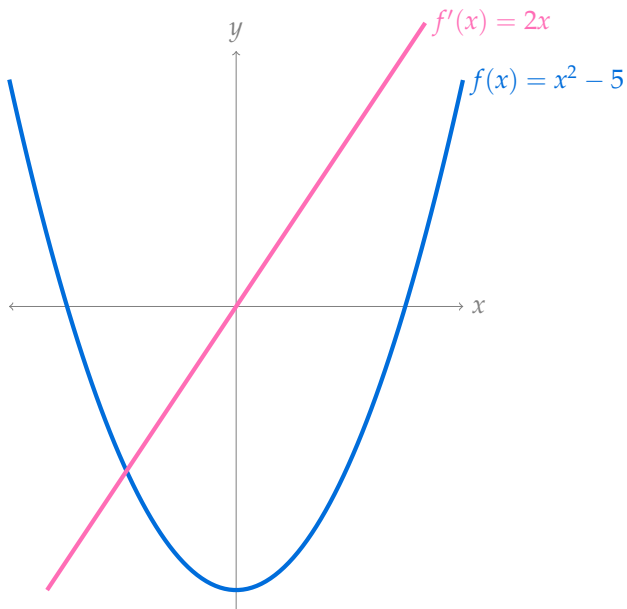
Where is $f'(x) < 0$?

Where is $f'(x) > 0$?

Where is $f'(x) \approx 0$?

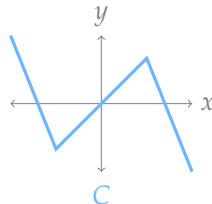
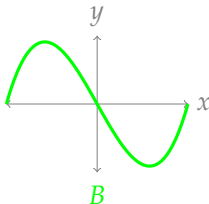
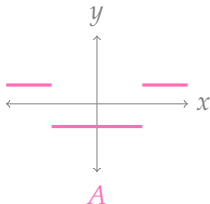
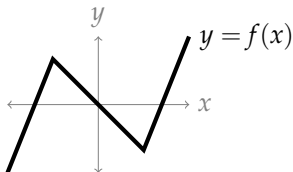
Use the definition of the derivative to find the slope of the tangent line to $f(x) = x^2 - 5$ at the point $x = 3$.

Let's keep the function $f(x) = x^2 - 5$. We just showed $f'(3) = 6$.
We can also find its derivative at an arbitrary point x :



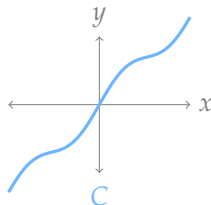
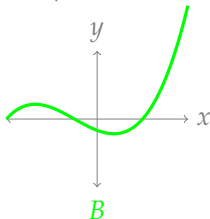
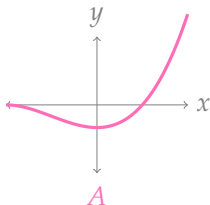
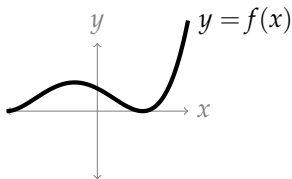
INCREASING AND DECREASING

In black is the curve $y = f(x)$. Which of the coloured curves corresponds to $y = f'(x)$?



INCREASING AND DECREASING

In black is the curve $y = f(x)$. Which of the coloured curves corresponds to $y = f'(x)$?



Derivative as a Function – Definition 2.2.6

Let $f(x)$ be a function.

The derivative of $f(x)$ with respect to x is given by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists. Notice that x will be a part of your final expression: this is a **function**.

If $f'(x)$ exists for all x in an interval (a, b) , we say that f is **differentiable on (a, b)** .

Notation 2.2.8

The “prime” notation $f'(x)$ and $f'(a)$ is sometimes called Newtonian notation. We will also use Leibnitz notation:

$$\frac{df}{dx}$$

function

$$\frac{df}{dx}(a)$$

number

$$\frac{d}{dx}f(x)$$

function

$$\frac{d}{dx}f(x)\Big|_{x=a}$$

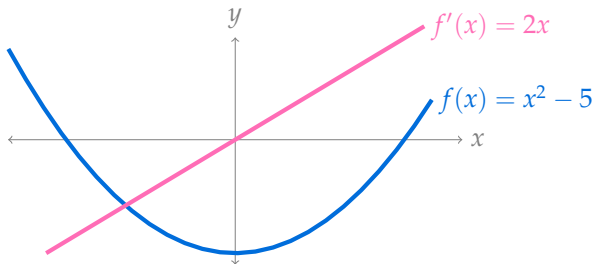
number

Newtonian Notation:

$$f(x) = x^2 + 5 \qquad f'(x) = 2x \qquad f'(3) = 6$$

Leibnitz Notation:

$$\frac{df}{dx} = \qquad \frac{df}{dx}(3) = \qquad \frac{d}{dx}f(x) = \qquad \frac{d}{dx}f(x)\bigg|_{x=3} =$$



Alternate Definition – Definition 2.2.1

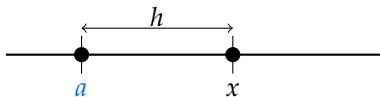
Calculating

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

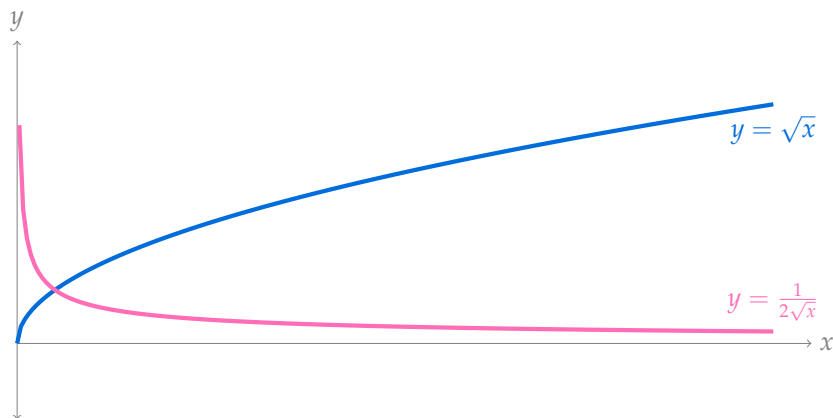
is the same as calculating

$$f'(x) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}.$$

Notice in these scenarios, $h = x - a$.



Let $f(x) = \sqrt{x}$. Using the definition of a derivative, calculate $f'(x)$.



Review:

$$\lim_{x \rightarrow 0^+} \sqrt{x} = \quad \lim_{x \rightarrow \infty} \frac{1}{2\sqrt{x}} =$$

$$\lim_{x \rightarrow 0^+} \frac{1}{2\sqrt{x}} =$$



NOW
YOU

Using the definition of the derivative, calculate

$$\frac{d}{dx} \left\{ \frac{1}{x} \right\}.$$

Using the definition of the derivative, calculate $\frac{d}{dx} \left\{ \frac{2x}{x+1} \right\}$.

Using the definition of the derivative, calculate $\frac{d}{dx} \left\{ \frac{1}{\sqrt{x^2 + x}} \right\}$.

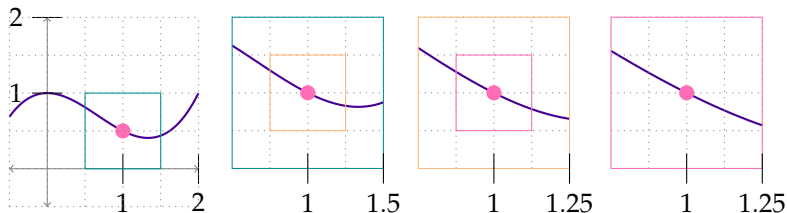
Memorize

The derivative of a function f at a point a is given by the following limit, if it exists:

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

ZOOMING IN

For a smooth function, if we zoom in at a point, we see a line:



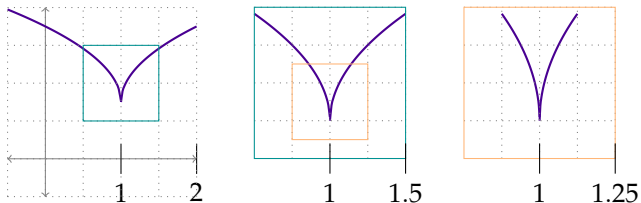
In this example, the slope of our zoomed-in line looks to be about:

$$\frac{\Delta y}{\Delta x} \approx -\frac{1}{2}$$

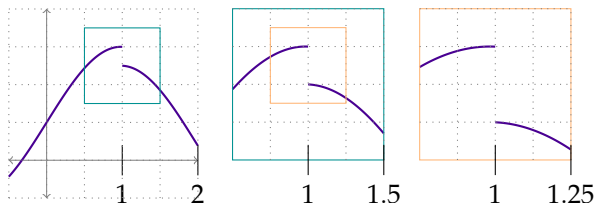
ZOOMING IN ON FUNCTIONS THAT AREN'T SMOOTH

For a function with a cusp or a discontinuity, even though we zoom in very closely, we don't see simply a single straight line.

Cusp:



Discontinuity:



Alternate Definition – Definition 2.2.1

Calculating

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

is the same as calculating

$$f'(x) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}.$$

Notice in these scenarios, $h = x - a$.

The derivative of $f(x)$ **does not exist** at $x = a$ if

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

does not exist.

Note this is the slope of the tangent line to $y = f(x)$ at $x = a$, $\frac{\Delta y}{\Delta x}$.

WHEN DERIVATIVES DON'T EXIST

What happens if we try to calculate a derivative where none exists?

Find the derivative of $f(x) = x^{1/3}$ at $x = 0$.

Theorem 2.2.14

If the function $f(x)$ is differentiable at $x = a$, then $f(x)$ is also continuous at $x = a$.

Proof:

Let $f(x)$ be a function and let a be a constant in its domain. Draw a picture of each scenario, or say that it is impossible.

$f(x)$ continuous at $x = a$ $f(x)$ differentiable at $x = a$	$f(x)$ continuous at $x = a$ $f(x)$ differentiable at $x = a$
$f(x)$ continuous at $x = a$ $f(x)$ differentiable at $x = a$	$f(x)$ continuous at $x = a$ $f(x)$ differentiable at $x = a$

Included Work



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