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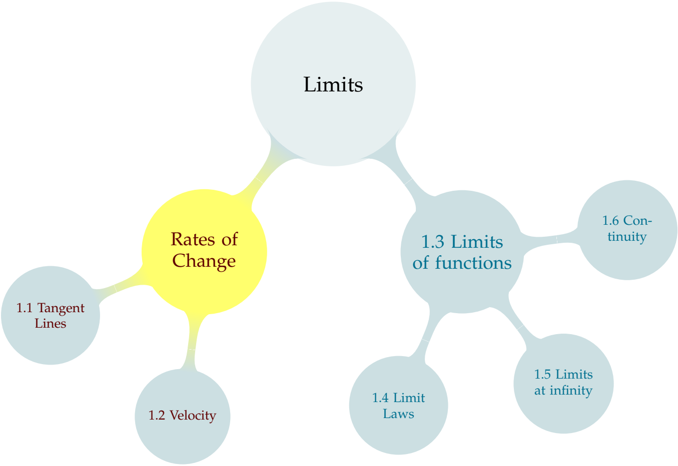

Rates of Change

Suppose the population of a small country was 1 million individuals in 1990, and is growing at a steady rate of 20,000 individuals per year.

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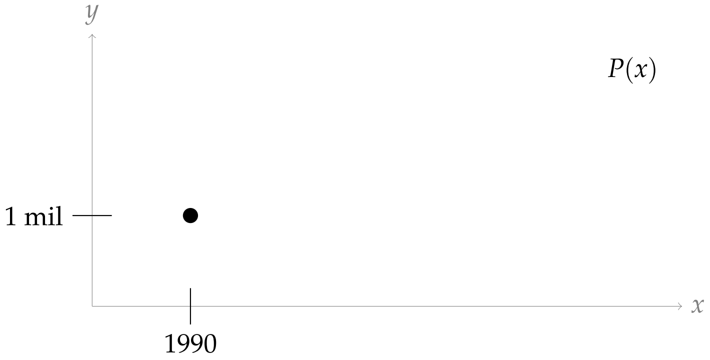

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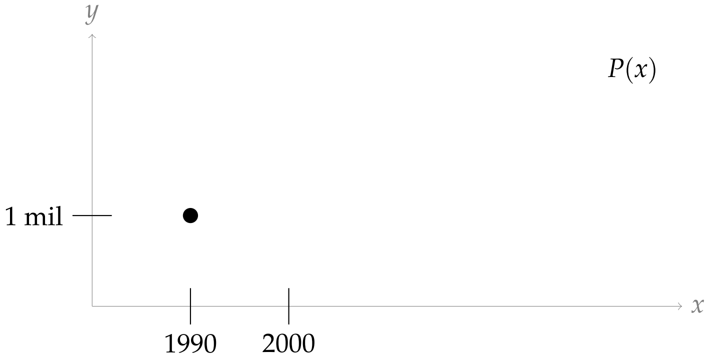

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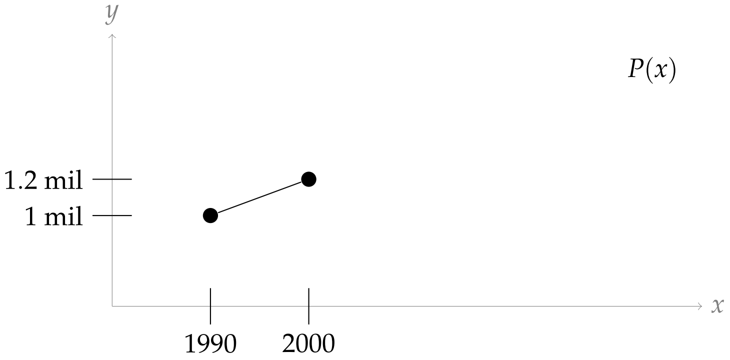

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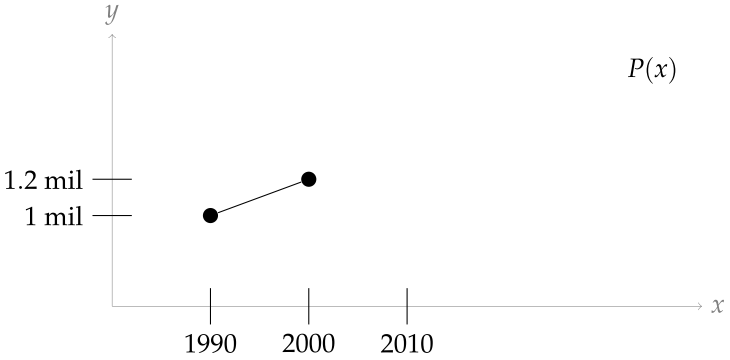

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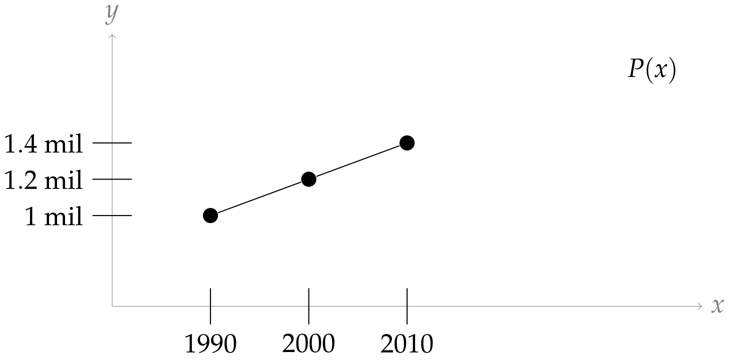

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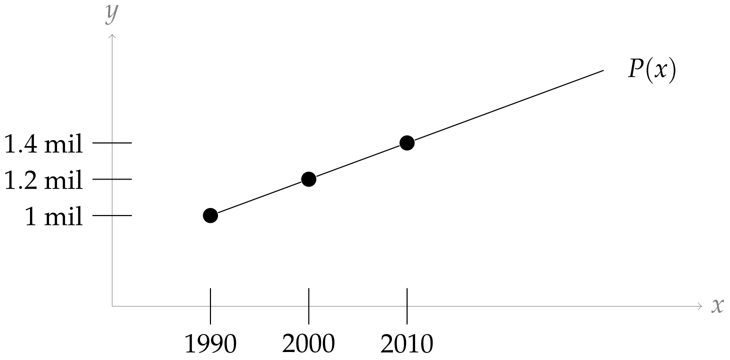
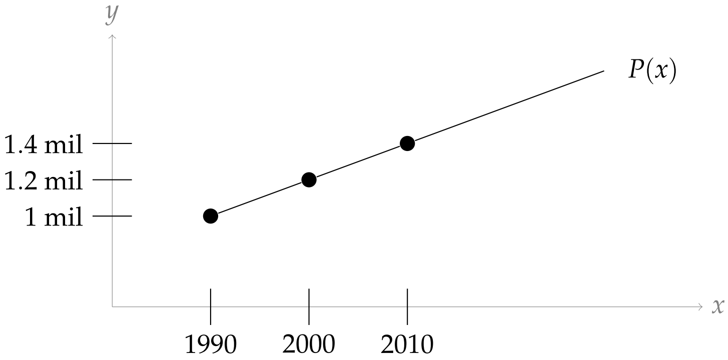


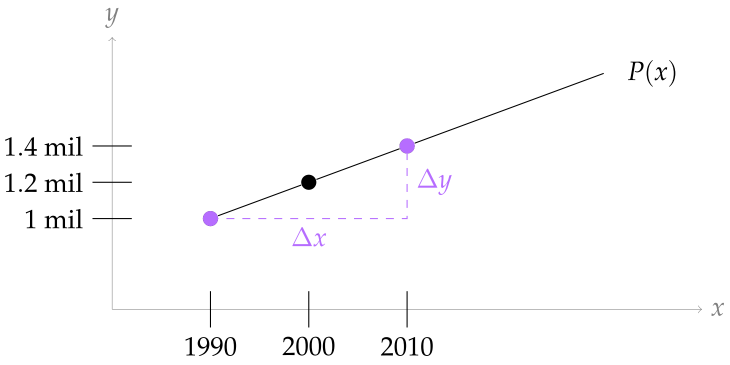
Definition

The **slope** of a line that passes through the points (x_1, y_1) and (x_2, y_2) is "rise over run"

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

This is also called the **rate of change** of the function.
If a line has equation $y = mx + b$, its slope is m.


y
$P(x)$
1.4 mil
1.2 mil
1 mil
x
1990
2000
2010


y
$P(x)$
1.4 mil
1.2 mil
1 mil
Δy
Δx
x
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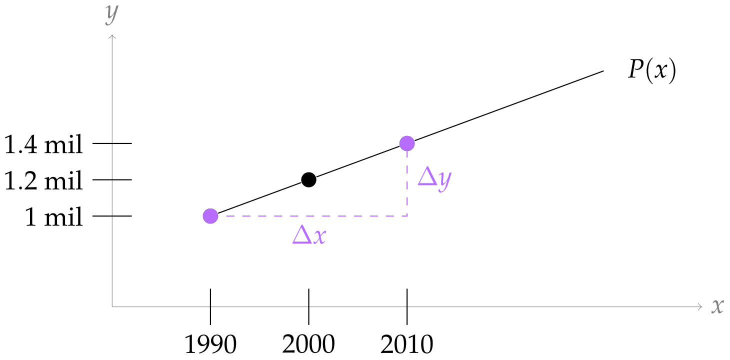
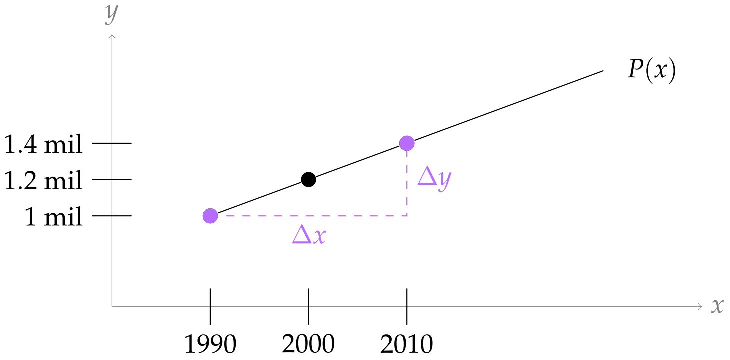


Rate of change: $\dfrac{400,000 \text{ people}}{20 \text{ years}} = 20,000 \dfrac{\text{people}}{\text{year}}$



Rate of change: $\dfrac{400,000 \text{ people}}{20 \text{ years}} = 20,000 \dfrac{\text{people}}{\text{year}}$

(doesn't depend on the year)

Suppose the population of a small country is given in the chart below.

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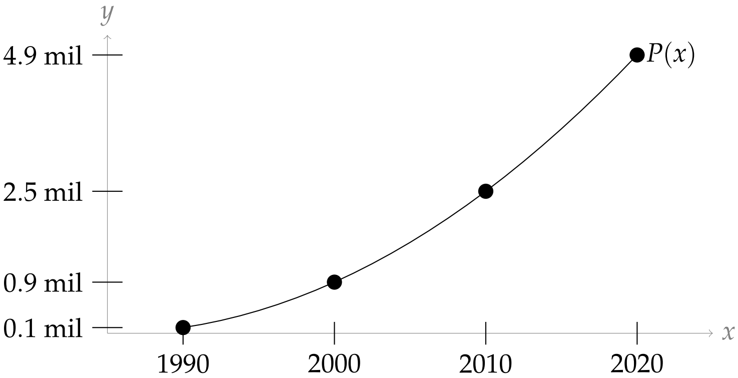

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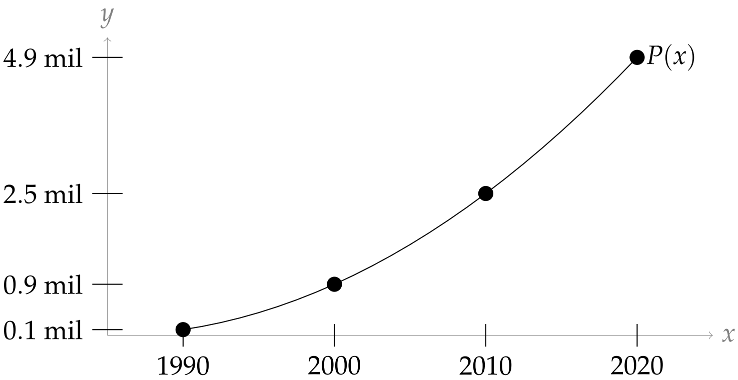

Rate of change $\dfrac{\Delta \text{ pop}}{\Delta \text{ time}}$

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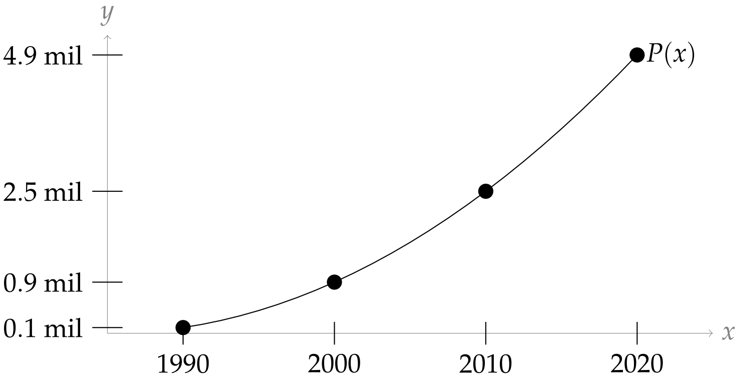
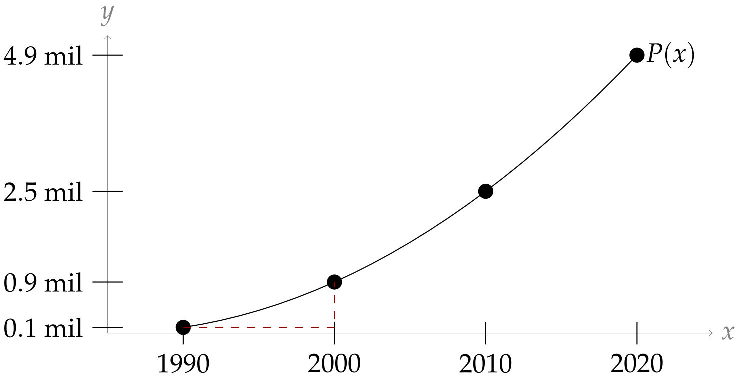


Rate of change $\dfrac{\Delta \text{ pop}}{\Delta \text{ time}}$ depends on time interval

Suppose the population of a small country is given in the chart below.



Rate of change $\dfrac{\Delta\ \text{pop}}{\Delta\ \text{time}}$ depends on time interval

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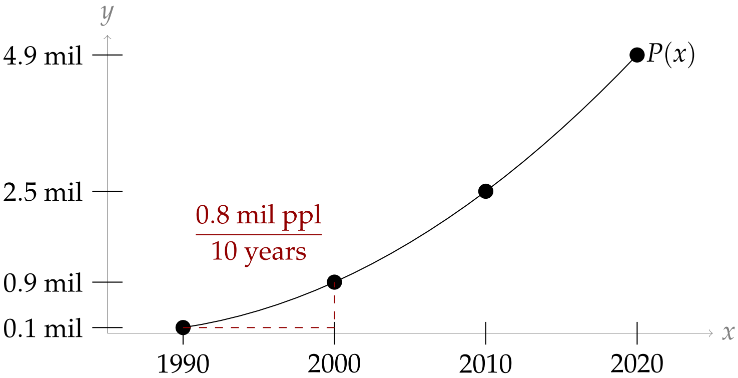

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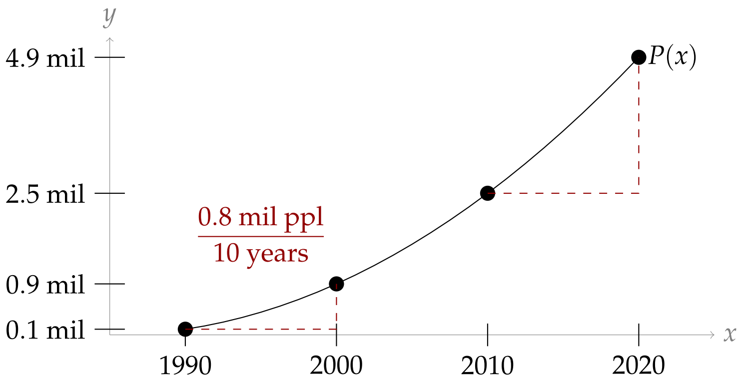

Rate of change $\frac{\Delta \text{ pop}}{\Delta \text{ time}}$ depends on time interval

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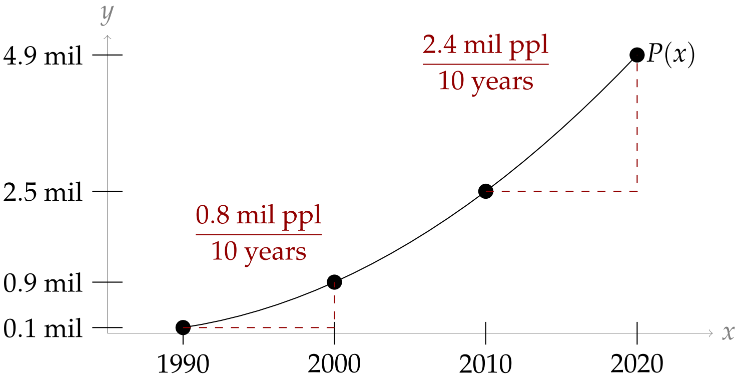
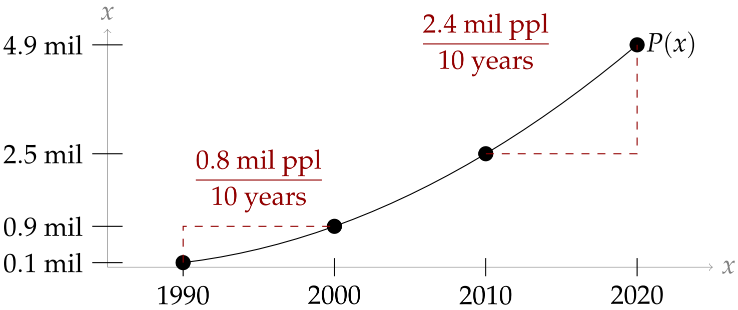


Rate of change $\dfrac{\Delta\text{ pop}}{\Delta\text{ time}}$ depends on time interval

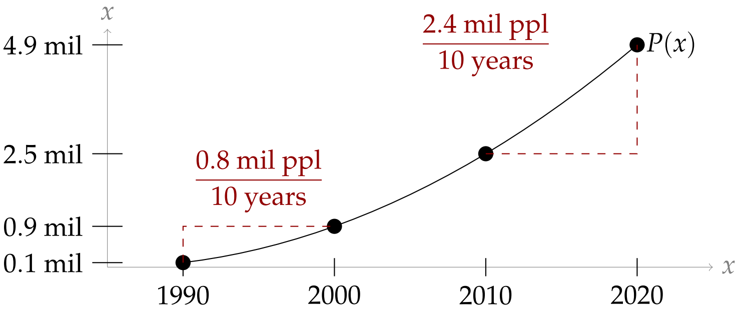
Definition

Let $y = f(x)$ be a curve that passes through (x_1, y_1) and (x_2, y_2). Then the **average rate of change** of $f(x)$ when $x_1 \leq x \leq x_2$ is

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

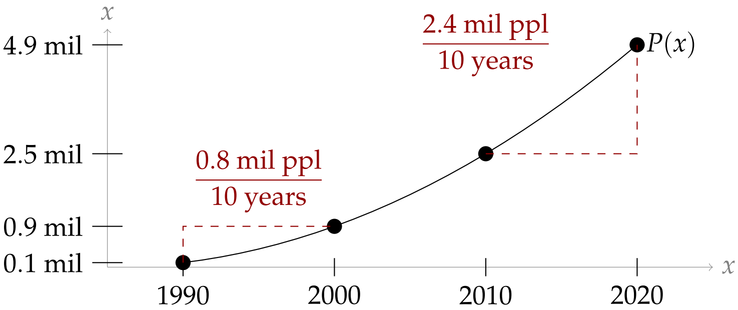


Average rate of change from 1990 to 2000:



Average rate of change from 1990 to 2000:
$80,000$ people per year.

Average rate of change from 2010 to 2020:



Average rate of change from 1990 to 2000:
$80,000$ people per year.

Average rate of change from 2010 to 2020:
$240,000$ people per year.

Average Rate of Change and Slope

The average rate of change of a function $f(x)$ on the interval $[a, b]$ (where $a \neq b$) is "change in output" divided by "change in input:"

$$\frac{f(b) - f(a)}{b - a}$$

Average Rate of Change and Slope

The average rate of change of a function $f(x)$ on the interval $[a, b]$ (where $a \neq b$) is "change in output" divided by "change in input:"

$$\frac{f(b) - f(a)}{b - a}$$

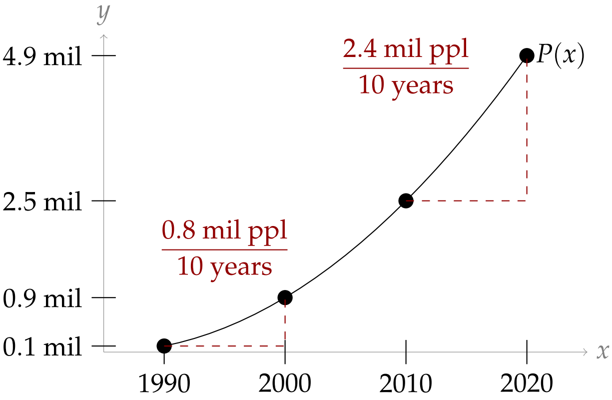
If the function $f(x)$ is a line, then the slope of the line is "rise over run,"

$$\frac{f(b) - f(a)}{b - a}$$

If a function is a line, its slope is the same as its average rate of change, which is the same for every interval.

If a function is not a line, its average rate of change might be different for different intervals, and we don't have a definition (yet) for its "slope."

How fast was this population growing in the year 2010? (What was its instantaneous rate of change?)



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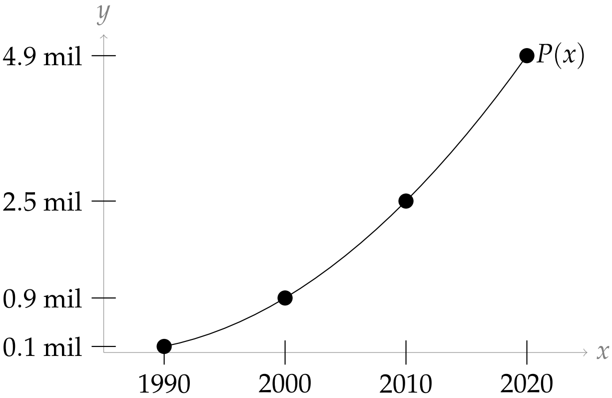

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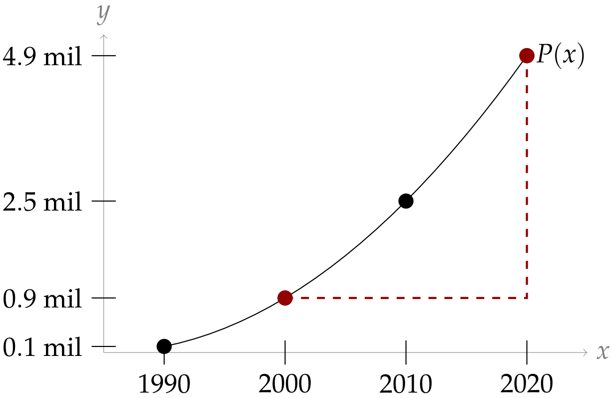

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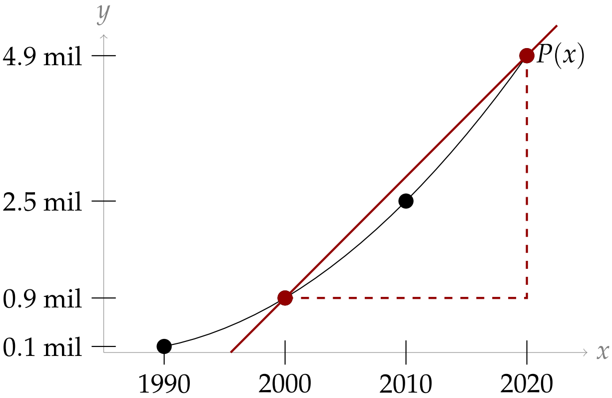

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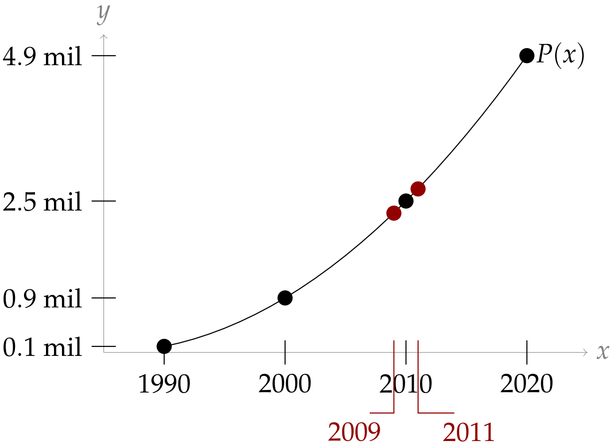

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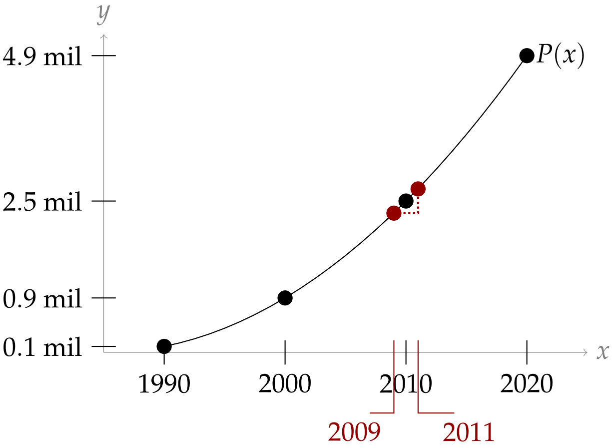

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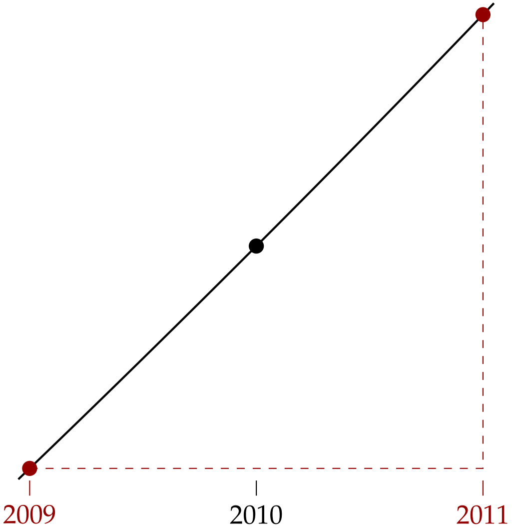

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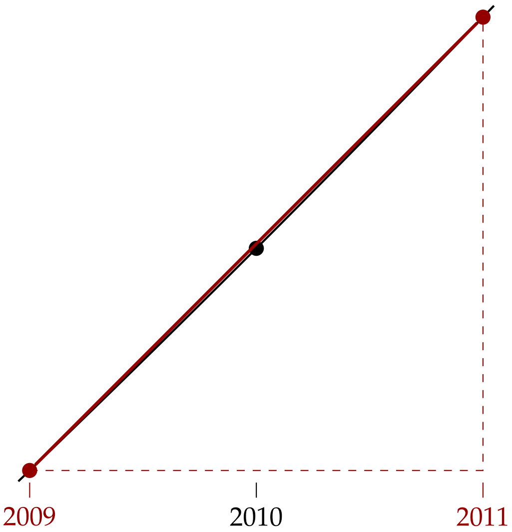

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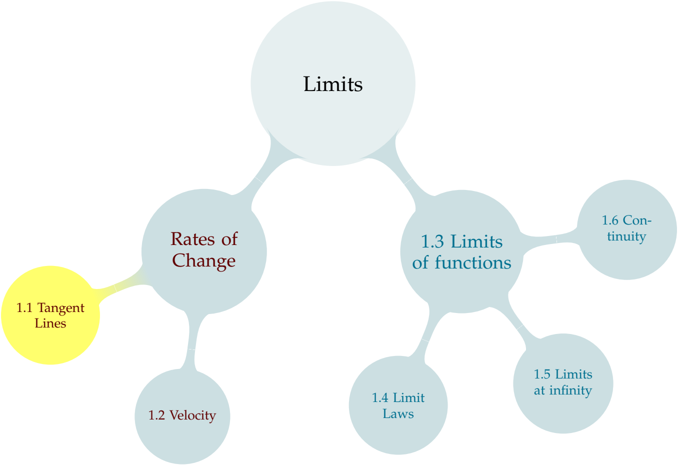
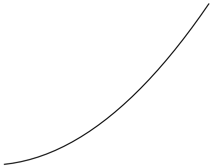


Definition

The **secant line** to the curve $y = f(x)$ through points R and Q is a line that passes through R and Q.



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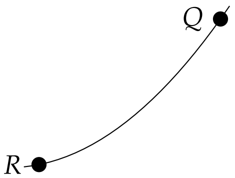

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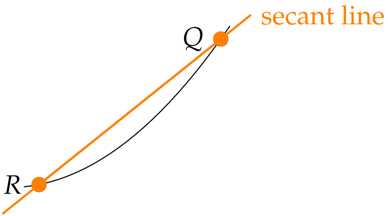
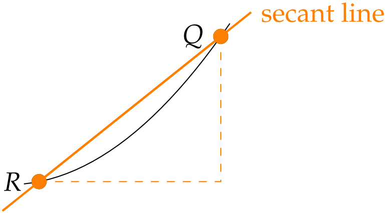


Definition

The **secant line** to the curve $y = f(x)$ through points R and Q is a line that passes through R and Q.

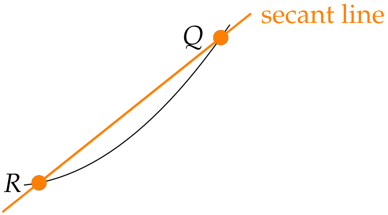
We call the slope of the secant line the **average rate of change of** $f(x)$ **from** R **to** Q**.**



Definition

The **tangent line** to the curve $y = f(x)$ at point P is a line that

- passes through P and
- has the same slope as $f(x)$ at P.



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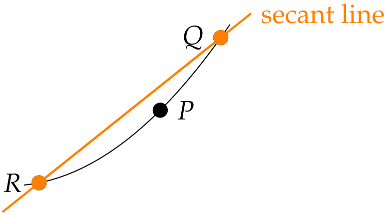

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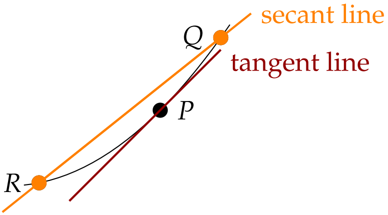
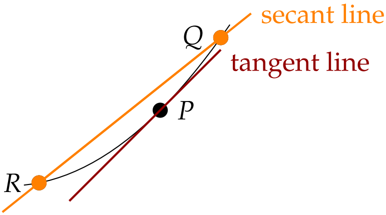


Definition

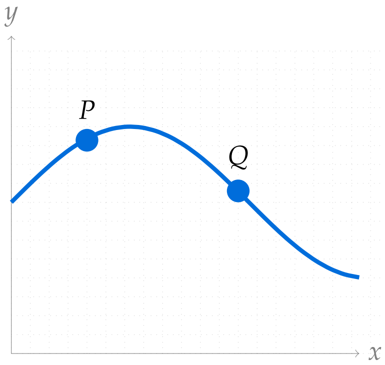
The **tangent line** to the curve $y = f(x)$ at point P is a line that

- passes through P and
- has the same slope as $f(x)$ at P.

We call the slope of the tangent line the **instantaneous rate of change of $f(x)$ at P.**



On the graph below, draw the secant line to the curve through points P and Q.



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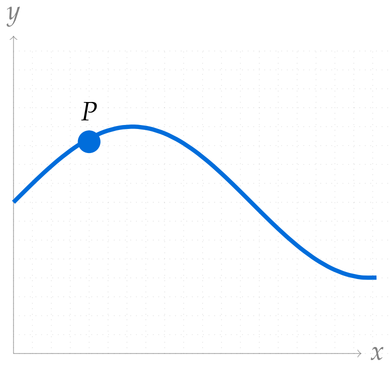

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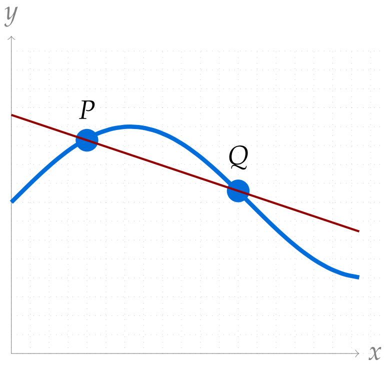

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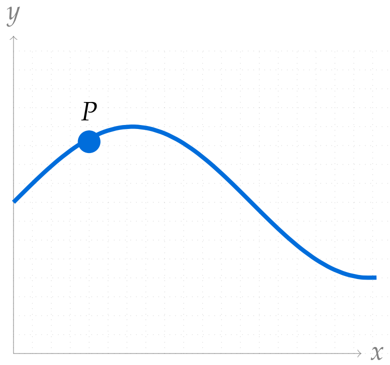

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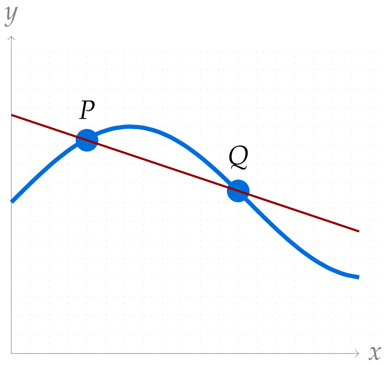

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