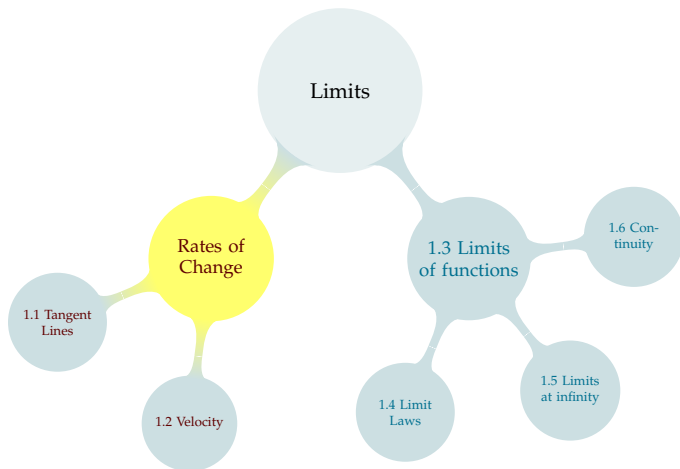


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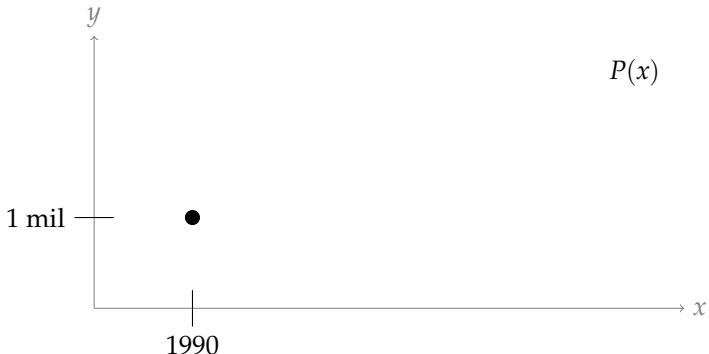


# RATES OF CHANGE

Suppose the population of a small country was 1 million individuals in 1990, and is growing at a steady rate of 20,000 individuals per year.

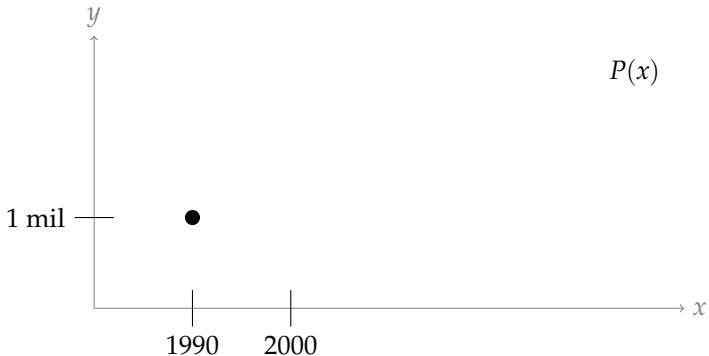
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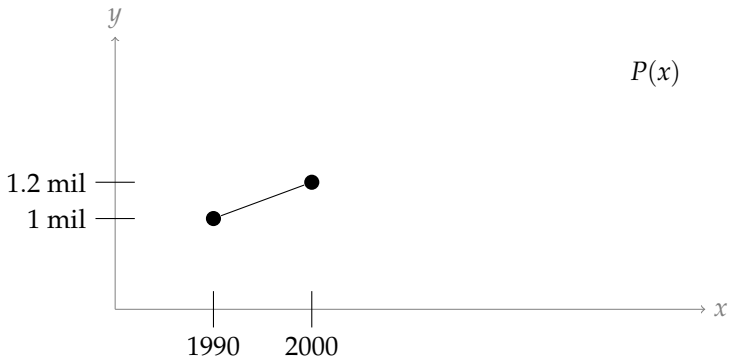
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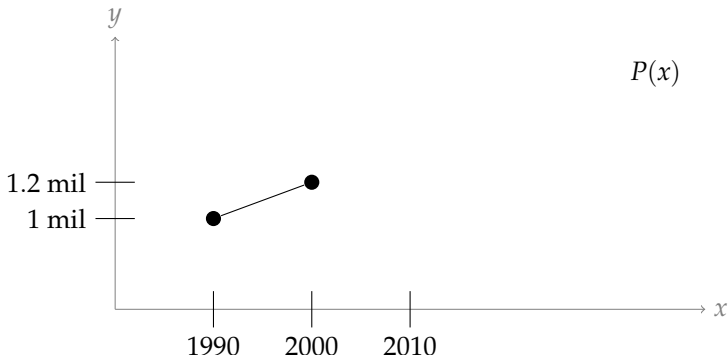
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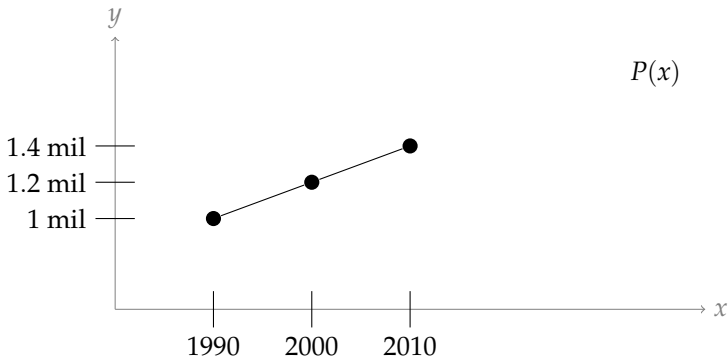
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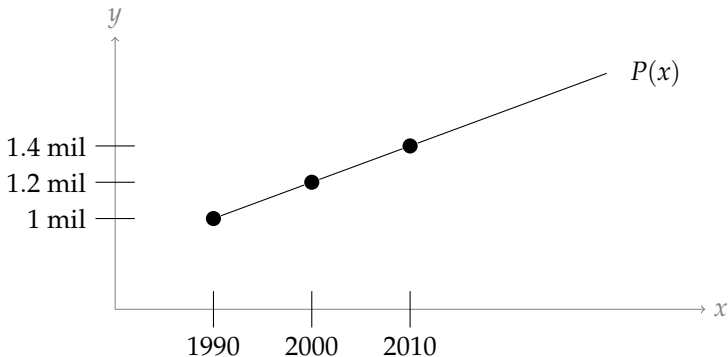
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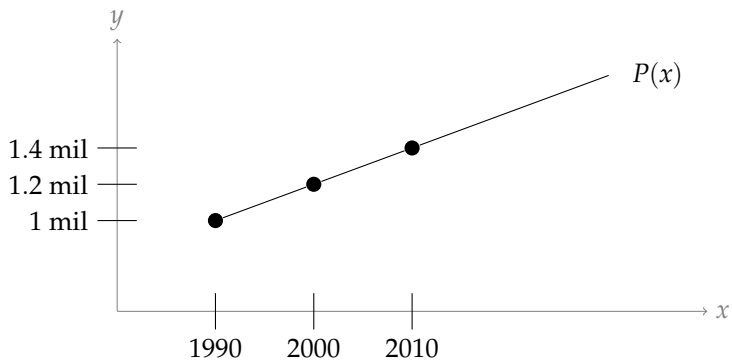
## Definition

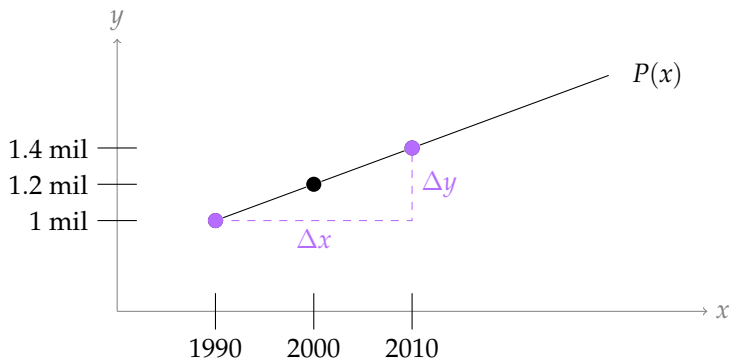
The **slope** of a line that passes through the points  $(x_1, y_1)$  and  $(x_2, y_2)$  is “rise over run”

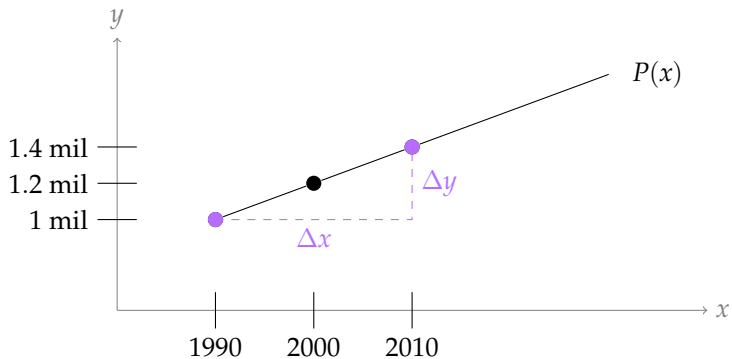
$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

This is also called the **rate of change** of the function.

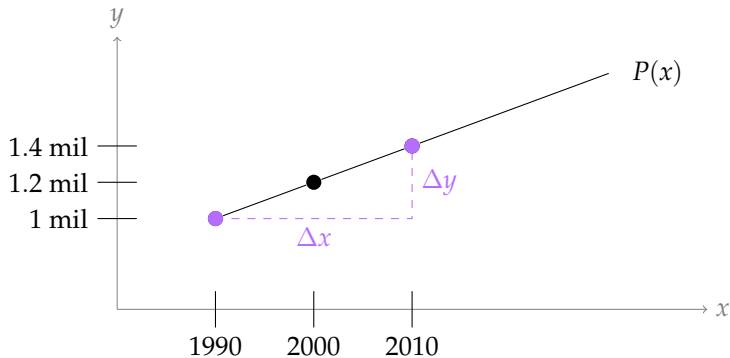
If a line has equation  $y = mx + b$ , its slope is  $m$ .







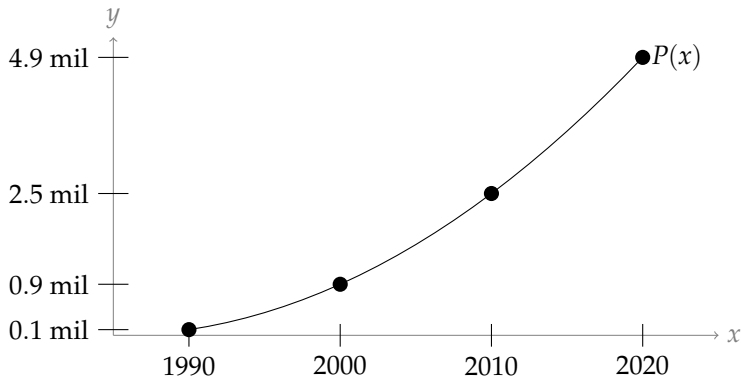
$$\text{Rate of change: } \frac{400,000 \text{ people}}{20 \text{ years}} = 20,000 \frac{\text{people}}{\text{year}}$$



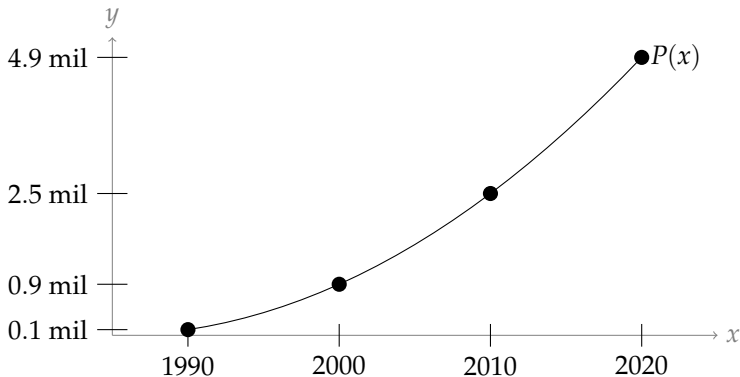
Rate of change:  $\frac{400,000 \text{ people}}{20 \text{ years}} = 20,000 \frac{\text{people}}{\text{year}}$   
(doesn't depend on the year)

Suppose the population of a small country is given in the chart below.

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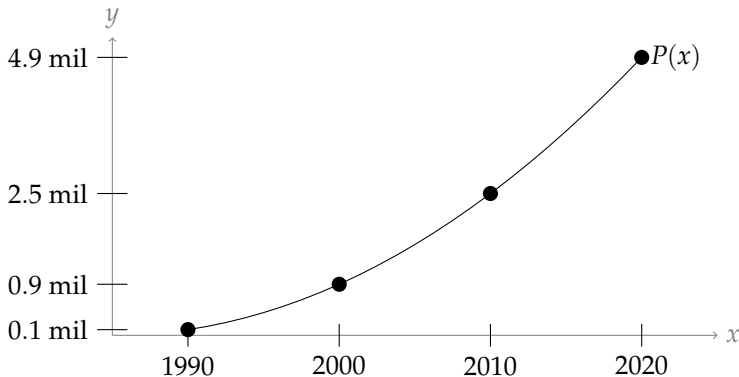
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Rate of change  $\frac{\Delta \text{pop}}{\Delta \text{time}}$

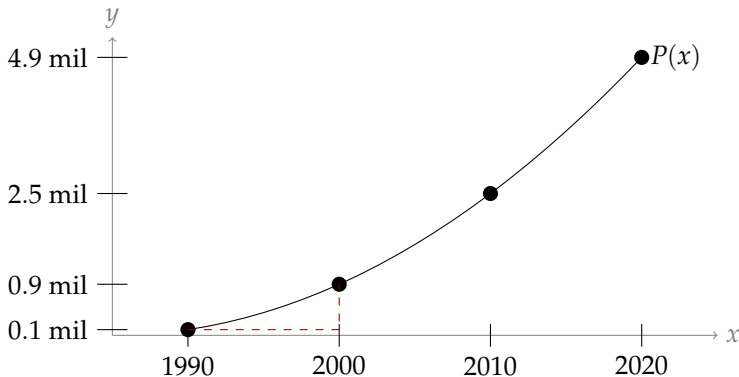


Suppose the population of a small country is given in the chart below.



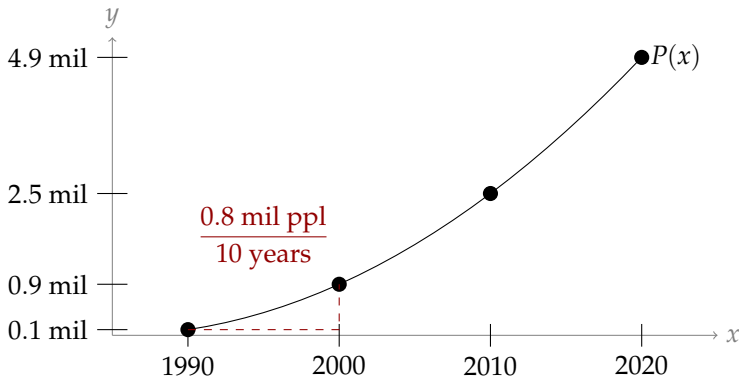
Rate of change  $\frac{\Delta \text{pop}}{\Delta \text{time}}$  depends on time interval

Suppose the population of a small country is given in the chart below.



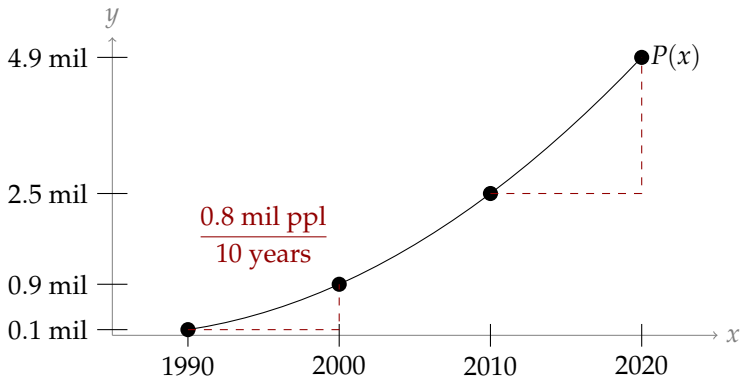
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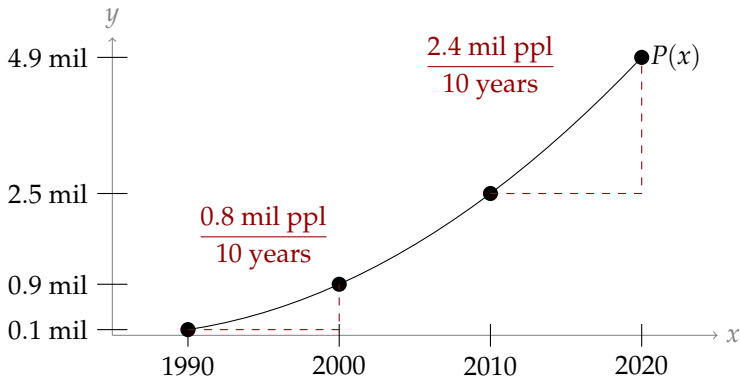
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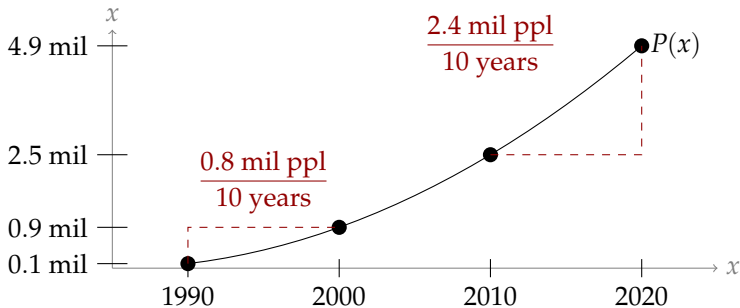


Rate of change  $\frac{\Delta \text{pop}}{\Delta \text{time}}$  depends on time interval

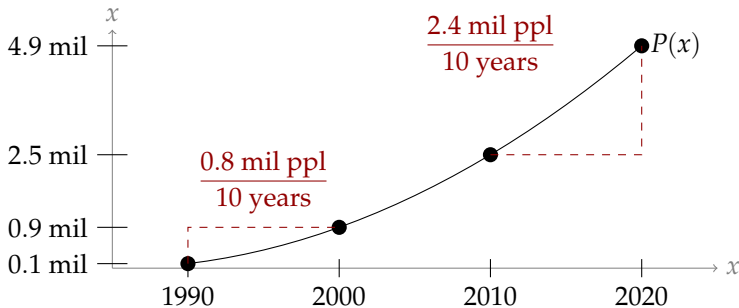
## Definition

Let  $y = f(x)$  be a curve that passes through  $(x_1, y_1)$  and  $(x_2, y_2)$ . Then the **average rate of change** of  $f(x)$  when  $x_1 \leq x \leq x_2$  is

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



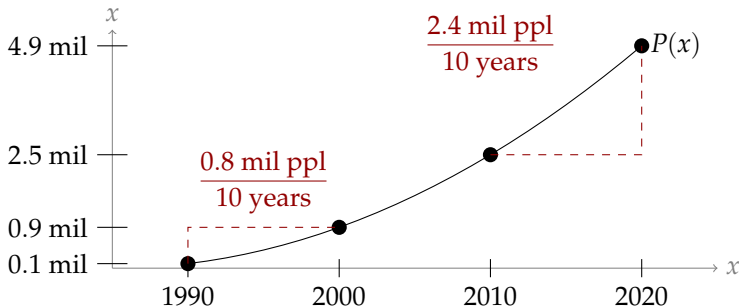
Average rate of change from 1990 to 2000:



Average rate of change from 1990 to 2000:  
80,000 people per year.

Average rate of change from 2010 to 2020:





Average rate of change from 1990 to 2000:  
80,000 people per year.

Average rate of change from 2010 to 2020:  
240,000 people per year.

## Average Rate of Change and Slope

The **average rate of change** of a function  $f(x)$  on the interval  $[a, b]$  (where  $a \neq b$ ) is “change in output” divided by “change in input:”

$$\frac{f(b) - f(a)}{b - a}$$

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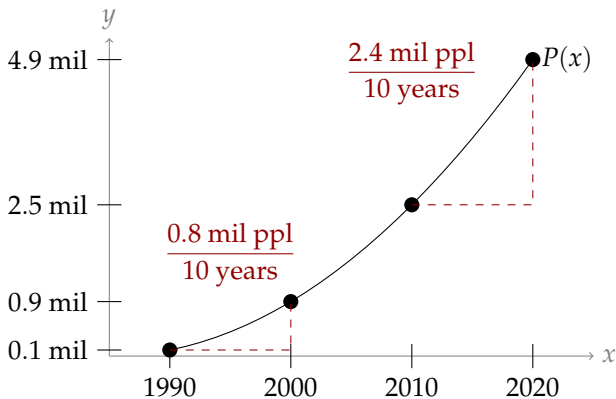
If the function  $f(x)$  is a **line**, then the slope of the line is “rise over run,”

$$\frac{f(b) - f(a)}{b - a}$$

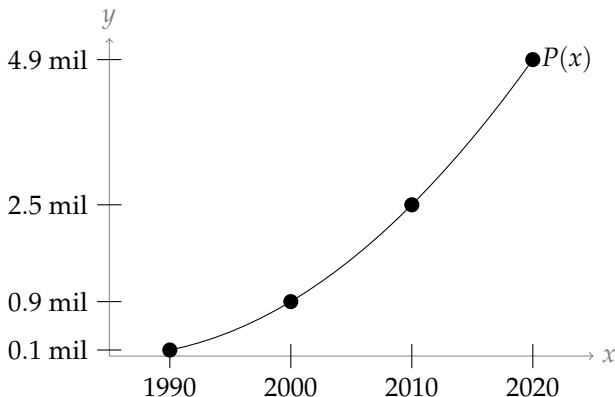
If a function is a line, its slope is the same as its average rate of change, which is the same for every interval.

If a function is not a line, its average rate of change might be different for different intervals, and we don't have a definition (yet) for its "slope."

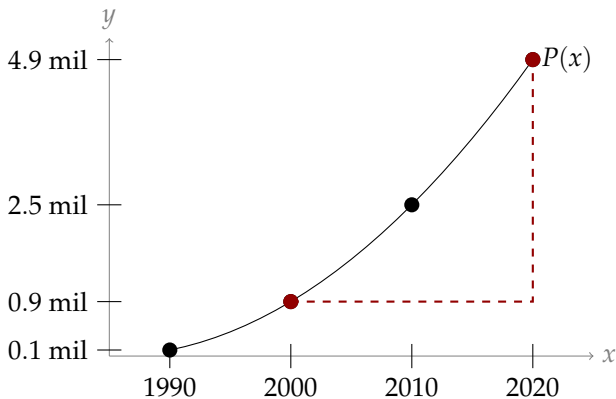
How fast was this population growing in the year 2010? (What was its **instantaneous** rate of change?)



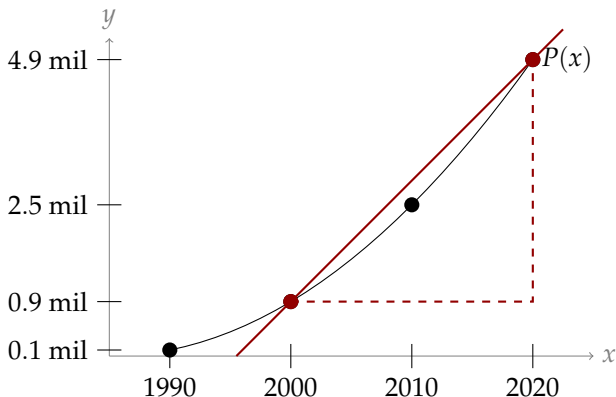
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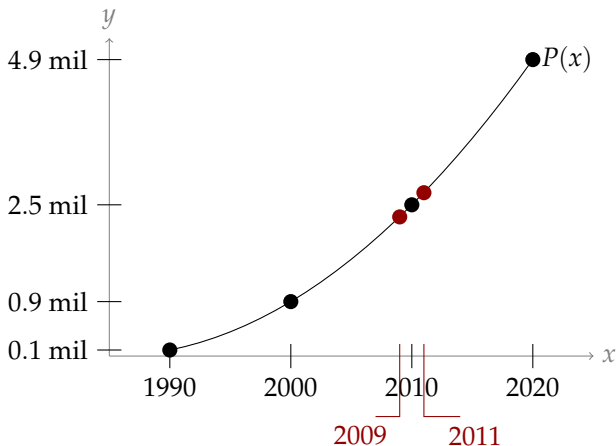


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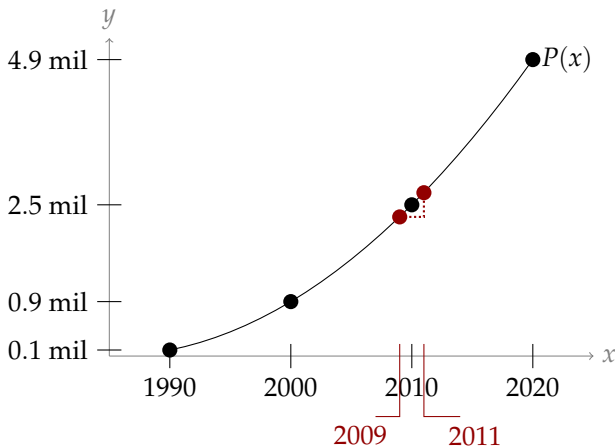




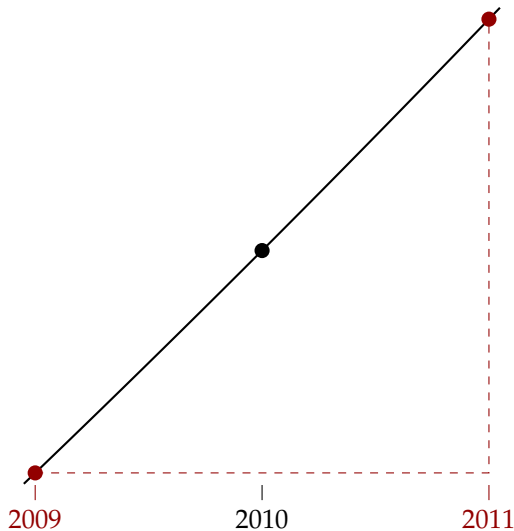
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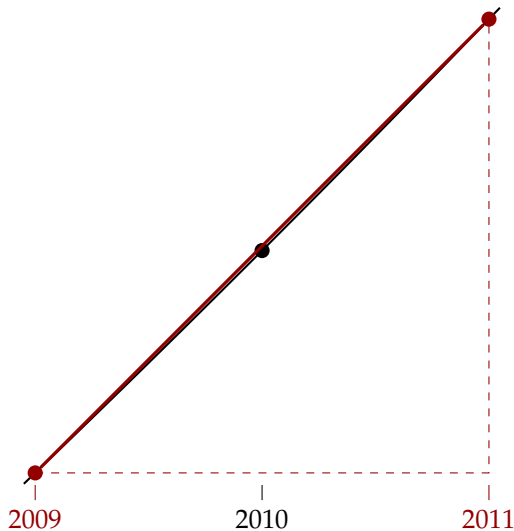
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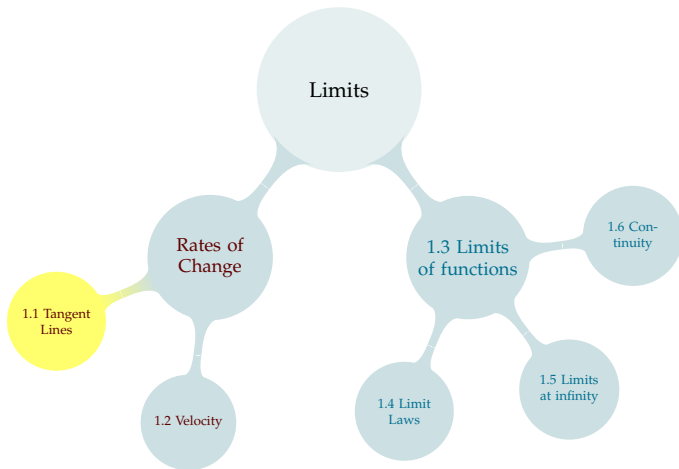
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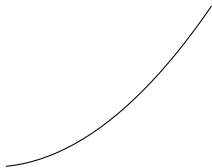


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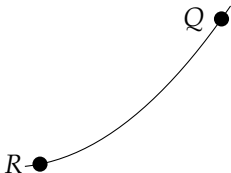
## Definition

The **secant line** to the curve  $y = f(x)$  through points  $R$  and  $Q$  is a line that passes through  $R$  and  $Q$ .



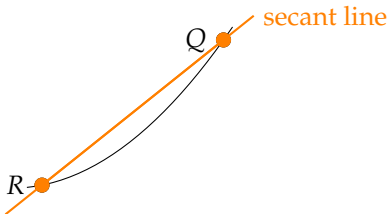
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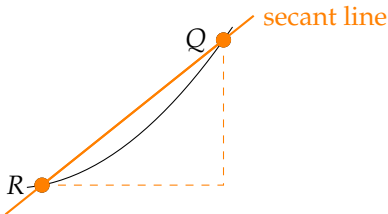




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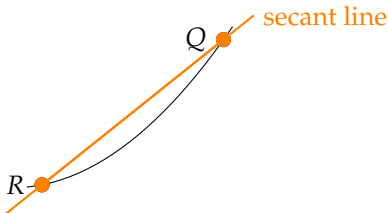
We call the slope of the secant line the **average rate of change of  $f(x)$  from  $R$  to  $Q$** .



## Definition

The **tangent line** to the curve  $y = f(x)$  at point  $P$  is a line that

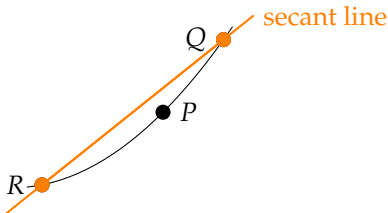
- passes through  $P$  and
- has the same slope as  $f(x)$  at  $P$ .



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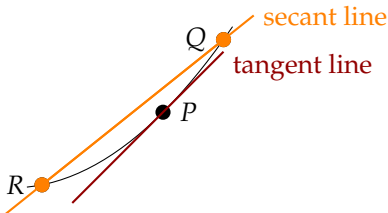
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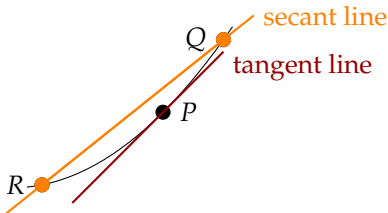


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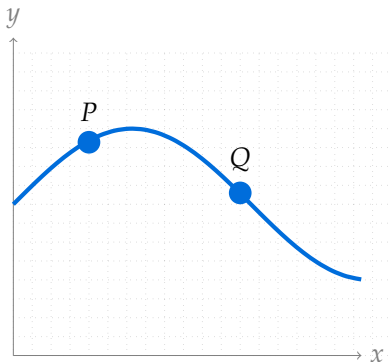
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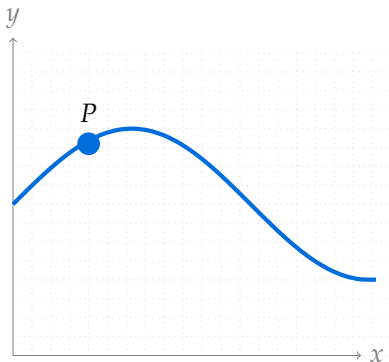
We call the slope of the tangent line the **instantaneous rate of change of  $f(x)$  at  $P$** .



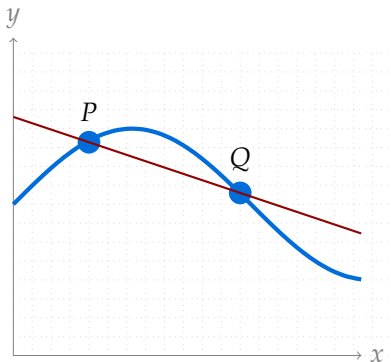
On the graph below, draw the secant line to the curve through points  $P$  and  $Q$ .



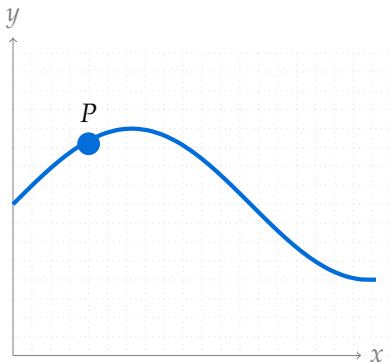
On the graph below, draw the tangent line to the curve at point  $P$ .



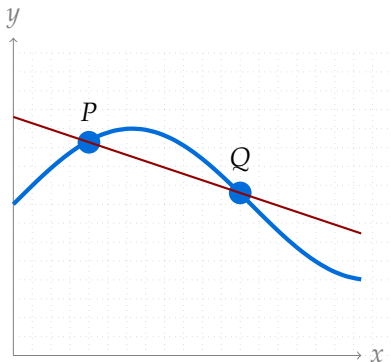
On the graph below, draw the secant line to the curve through points  $P$  and  $Q$ .



On the graph below, draw the tangent line to the curve at point  $P$ .



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