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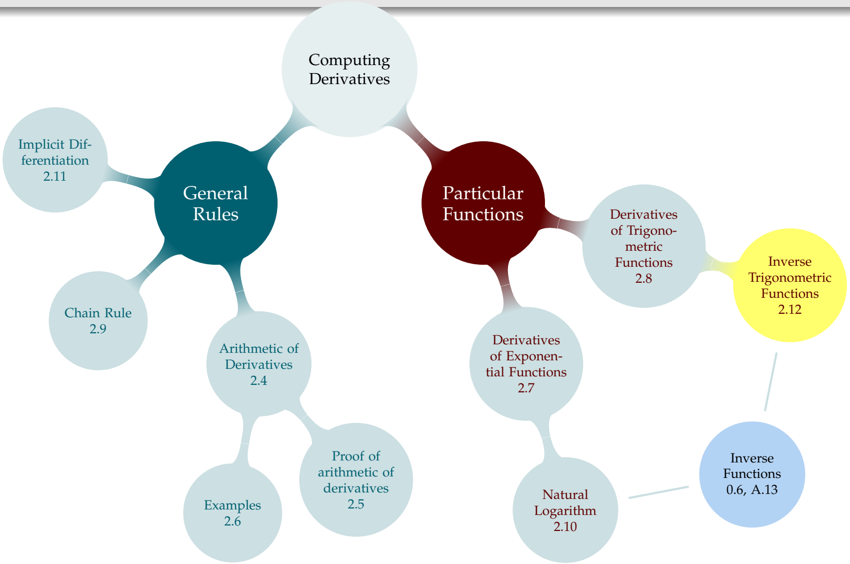
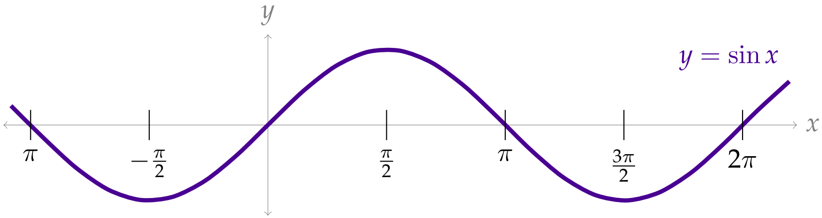


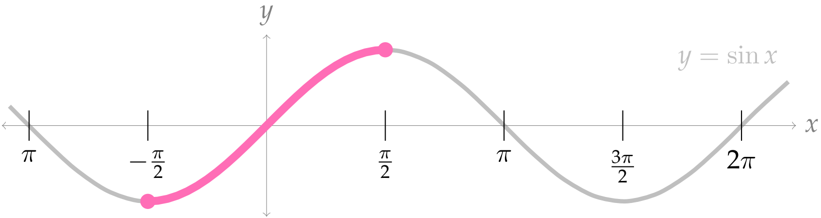
INVERTIBILITY GAME



I'm thinking of a number x. Your hint: $\sin(x) = 0$. What number am I thinking of?

I'm thinking of a number x, and x is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Your hint: $\sin(x) = 0$. What number am I thinking of?

Arcsine



$\arcsin(x)$ is the inverse of $\sin x$ restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\arcsin x$ is the (unique) number θ such that:

- $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, and
- $\sin \theta = x$

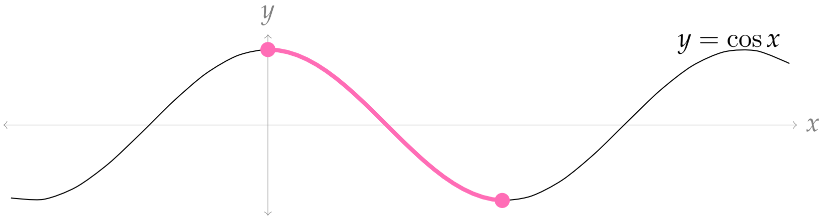
Arcsine

Reference Angles:

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- $\arcsin(0)$
- $\arcsin\left(\frac{1}{\sqrt{2}}\right)$
- $\arcsin\left(-\frac{1}{\sqrt{2}}\right)$
- $\arcsin\left(\frac{\pi}{2}\right)$
- $\arcsin\left(\frac{\pi}{4}\right)$

Arccosine

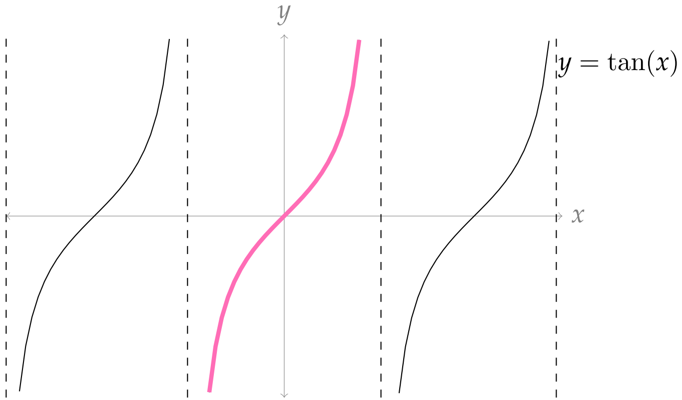


$\arccos(x)$ is the inverse of $\cos x$ restricted to $[0, \pi]$.

$\arccos(x)$ is the (unique) number θ such that:

- $\cos(\theta) = x$ and
- $0 \leq \theta \leq \pi$

ARCTANGENT



$\arctan(x) = \theta$ means:
(1) $\tan(\theta) = x$ and
(2) $-\pi/2 < \theta < \pi/2$

Arcsecant, arcsine, and arccotangent

$$\operatorname{arcsec}(x) =$$

Arcsecant, arcsine, and arccotangent

$$\text{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$

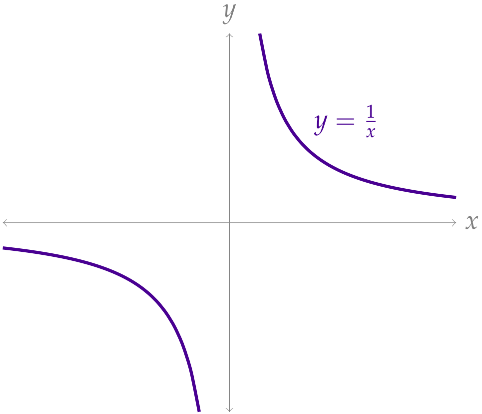
$$\begin{aligned}
\text{arccsc}(x) &= y \\
\csc y &= x \\
\frac{1}{\sin y} &= x \\
\sin y &= \frac{1}{x} \\
y &= \arcsin\left(\tfrac{1}{x}\right) \\
\text{arccsc}(x) &= \arcsin\left(\tfrac{1}{x}\right)
\end{aligned}$$

$$\text{arccot(x)} = \arctan\left(\frac{1}{x}\right)$$

$$\begin{aligned}
\text{arccot(x)} &= y \\
\cot y &= x \\
\frac{1}{\tan y} &= x \\
\tan y &= \tfrac{1}{x} \\
y &= \arctan\left(\tfrac{1}{x}\right) \\
\text{arccot(x)} &= \arctan\left(\tfrac{1}{x}\right)
\end{aligned}$$

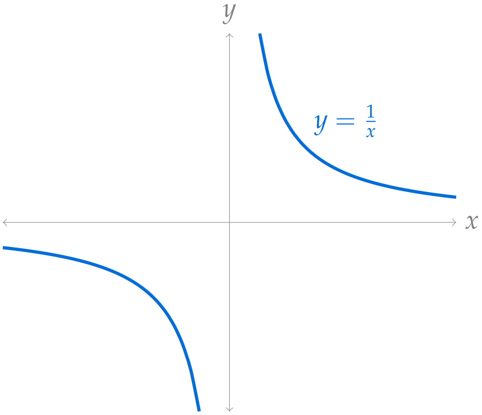
$$\operatorname{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$$

The domain of $\arccos(y)$ is $-1 \le y \le 1$, so the domain of $\operatorname{arcsec}(y)$ is



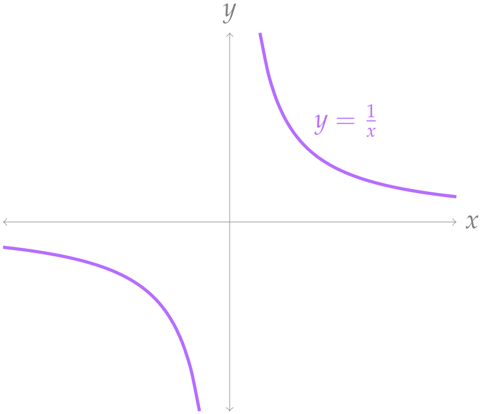
$$\text{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$

Domain of $\arcsin(y)$ is $-1 \leq y \leq 1$, so the domain of $\text{arccsc}(x)$ is



$$\text{arccot(x)} = \arctan\left(\frac{1}{x}\right)$$

Domain of $\arctan(x)$ is all real numbers, so the domain of $\text{arccot}(x)$ is



$$y = \arcsin x$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

$$y = \arctan x$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

$$y = \arccos x$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

To differentiate arcsecant, arccosecant, and arccotangent, you can use the chain rule!

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\operatorname{arccsc}(x)\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[\arcsin\left(\frac{1}{x}\right)\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[\arcsin\left(x^{-1}\right)\right]$$

Derivatives of Inverse Trigonometric Functions – Theorem 2.12.7

Memorize:

$$\frac{\mathrm{d}}{\mathrm{d}x}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\arccos x] = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\arcsin x] = \frac{1}{1+x^2}$$

Be able to derive:

$$\frac{\mathrm{d}}{\mathrm{d}x}[\operatorname{arccsc} x] = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\operatorname{arcsec} x] = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\operatorname{arccot} x] = -\frac{1}{1+x^2}$$