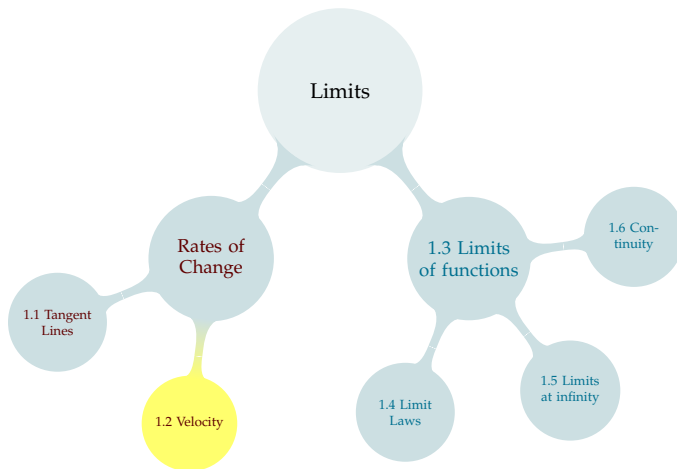
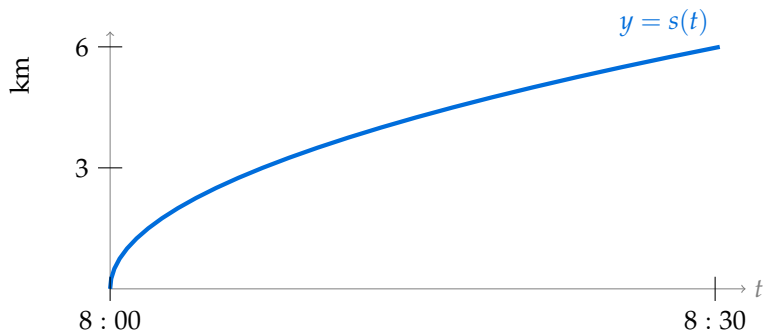
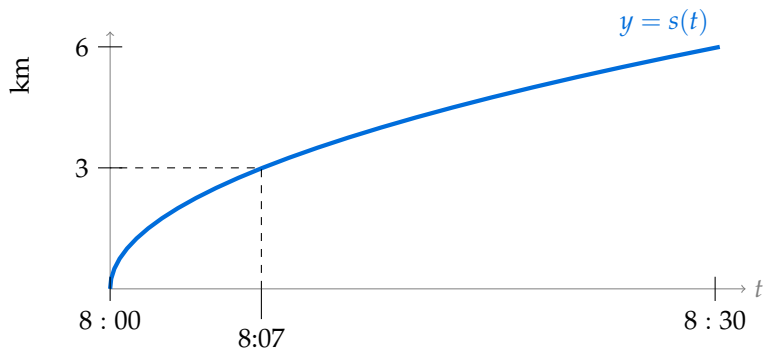
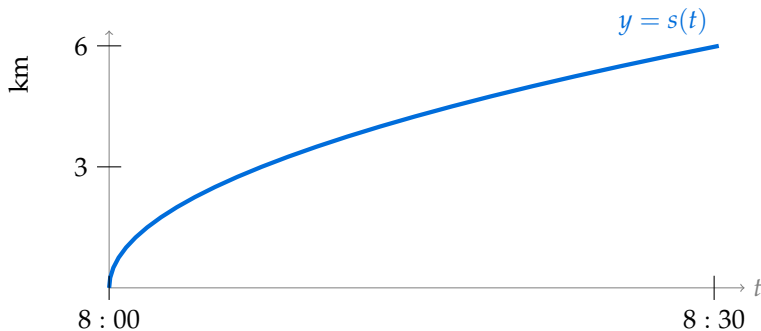


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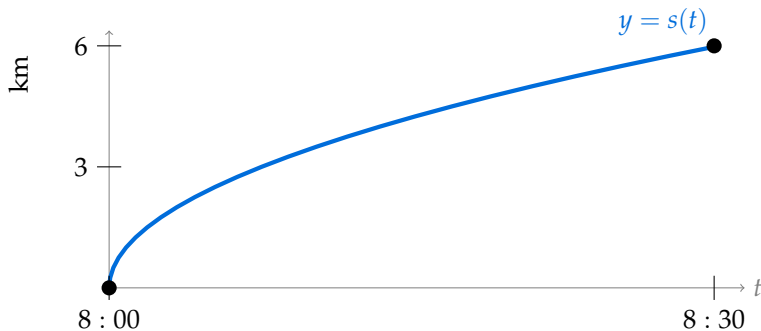






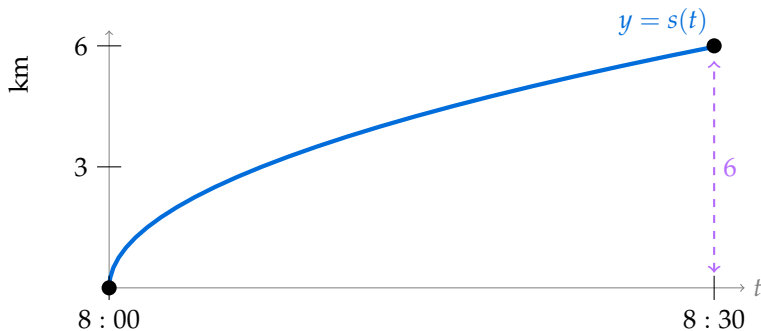
It took  $\frac{1}{2}$  hour to bike 6 km. 12 kph represents the:

- A. secant line to  $y = s(t)$  from  $t = 8:00$  to  $t = 8:30$
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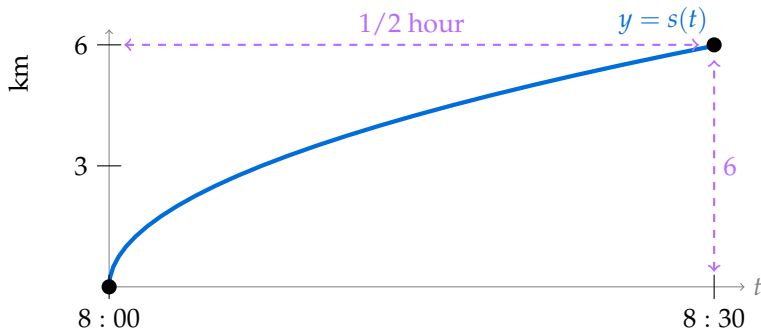
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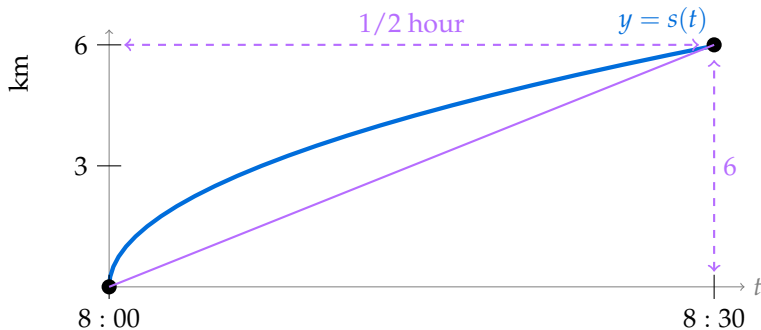
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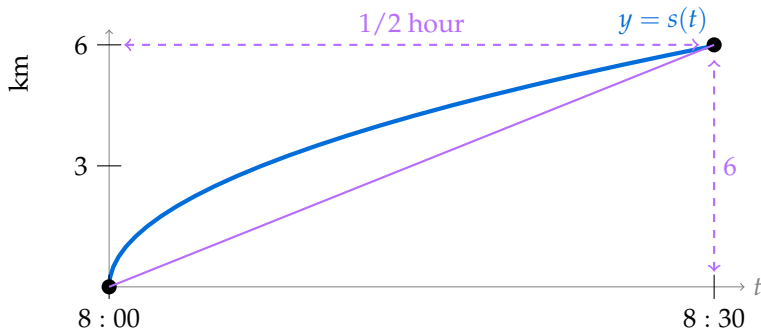
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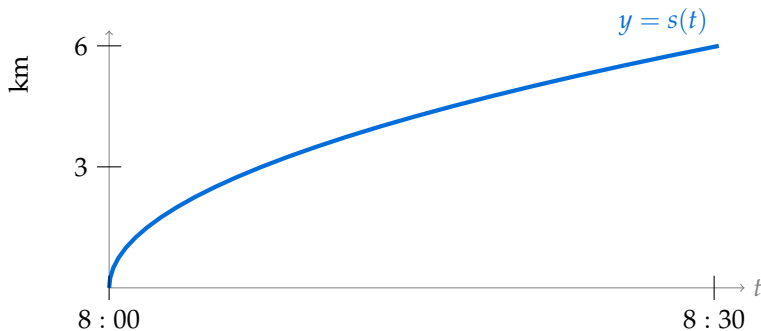
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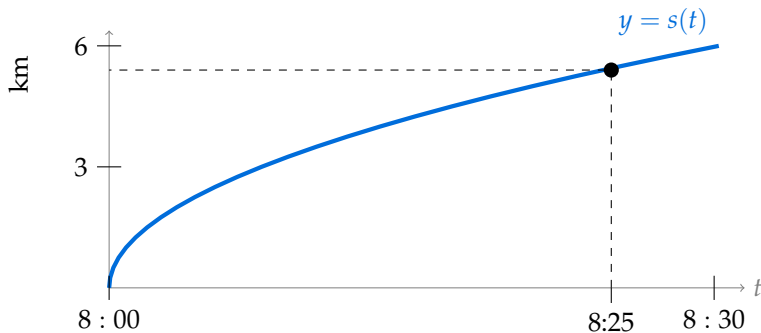
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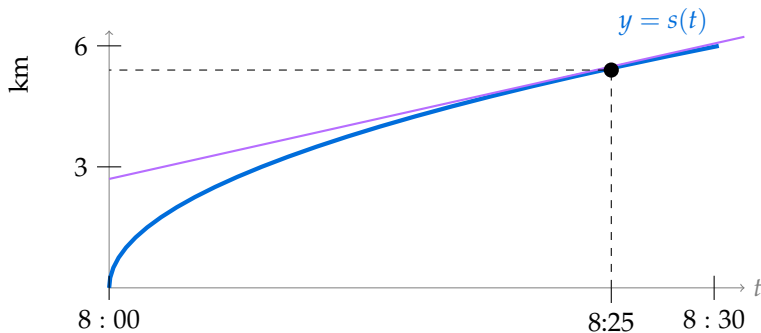
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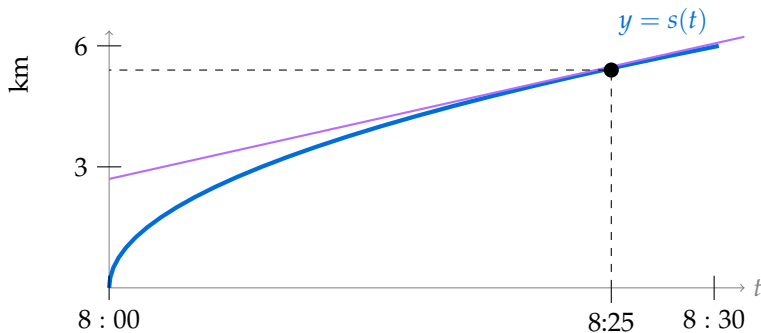
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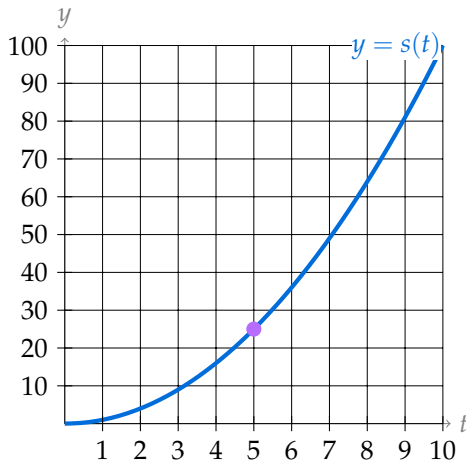


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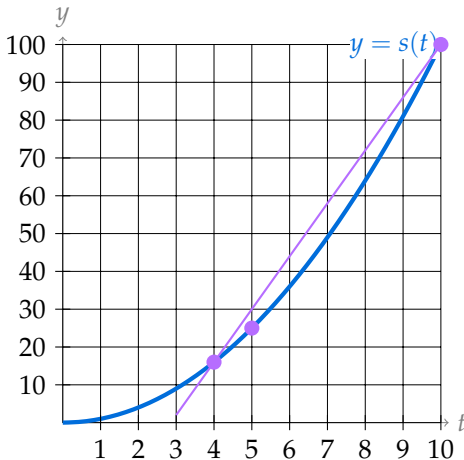
Suppose the distance from the ground  $s$  (in meters) of a helium-filled balloon at time  $t$  over a 10-second interval is given by  $s(t) = t^2$ . Try to estimate how fast the balloon is rising when  $t = 5$ .

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Estimate the  
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tangent line to  
the curve

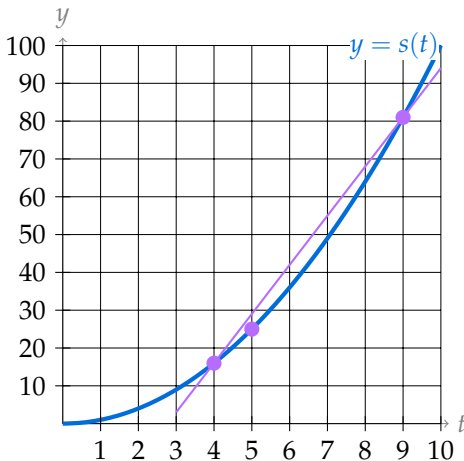
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Another way:  
Calculate  
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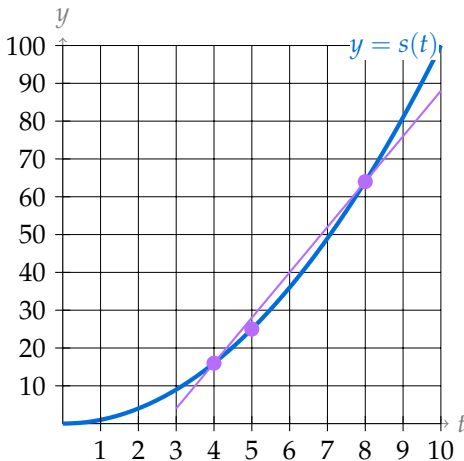


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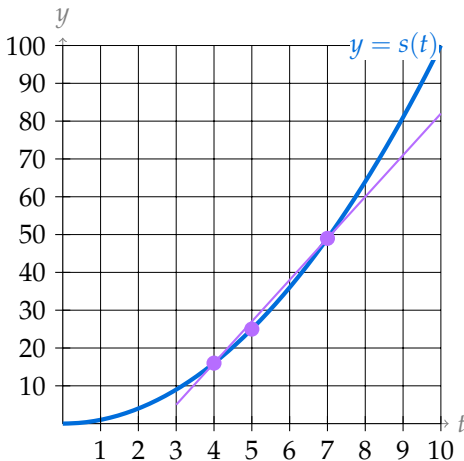
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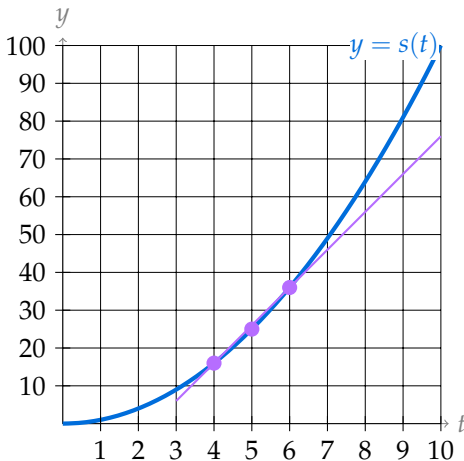
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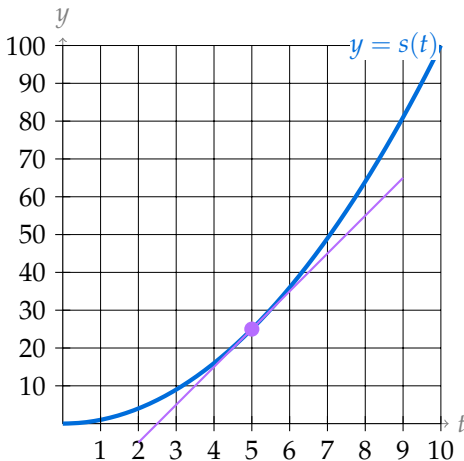
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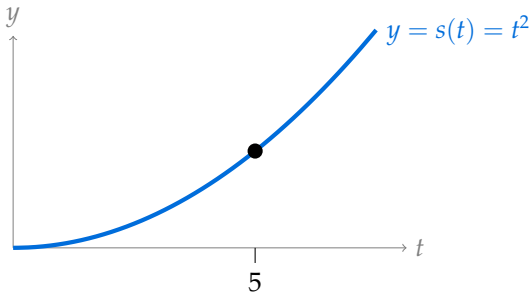
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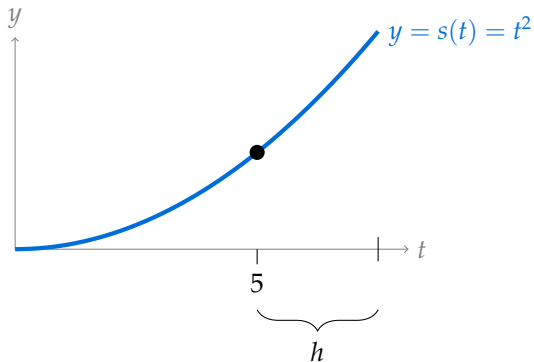


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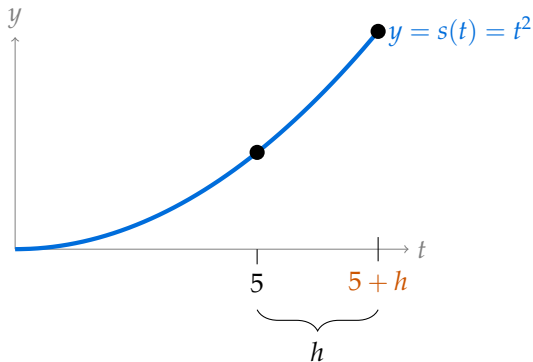
Let's look for an algebraic way of determining the velocity of the balloon when  $t = 5$ .



Suppose the interval  $[5, \quad ]$  has length  $h$ . What is the right endpoint of the interval?

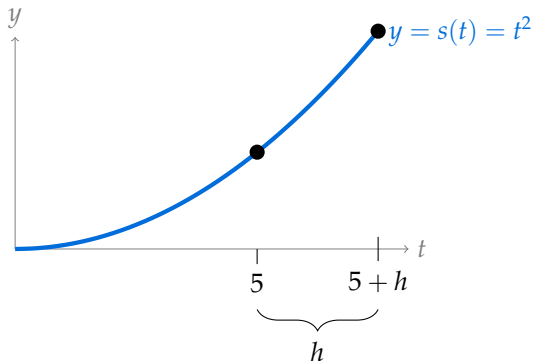


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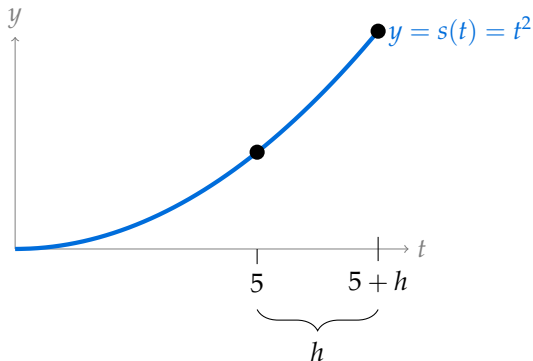




Write the equation for the average (vertical) velocity from  $t = 5$  to  $t = 5 + h$ .

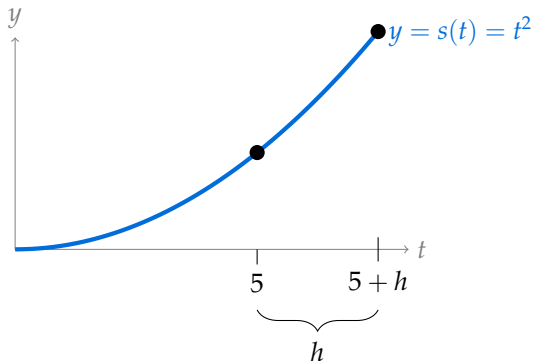


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$$\text{vel} = \frac{\Delta \text{ height}}{\Delta \text{ time}} = \frac{s(5+h) - s(5)}{(5+h) - 5} = \frac{(5+h)^2 - 5^2}{h}$$

What happens to the velocity when  $h$  is very, very small?



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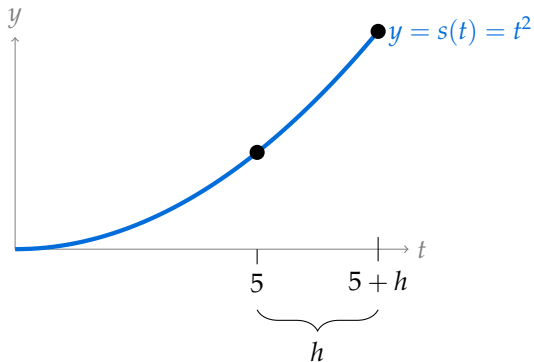
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When  $h$  is very small,

$$\approx 10$$



What do you think is the slope of the tangent line to the graph when  $t = 5$ ?



# OUR FIRST LIMIT

Average Velocity,  $t = 5$  to  $t = 5 + h$ :

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It means: As  $h$  gets extremely close to 0,  $(10 + h)$  gets extremely close to 10.

## Included Work



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