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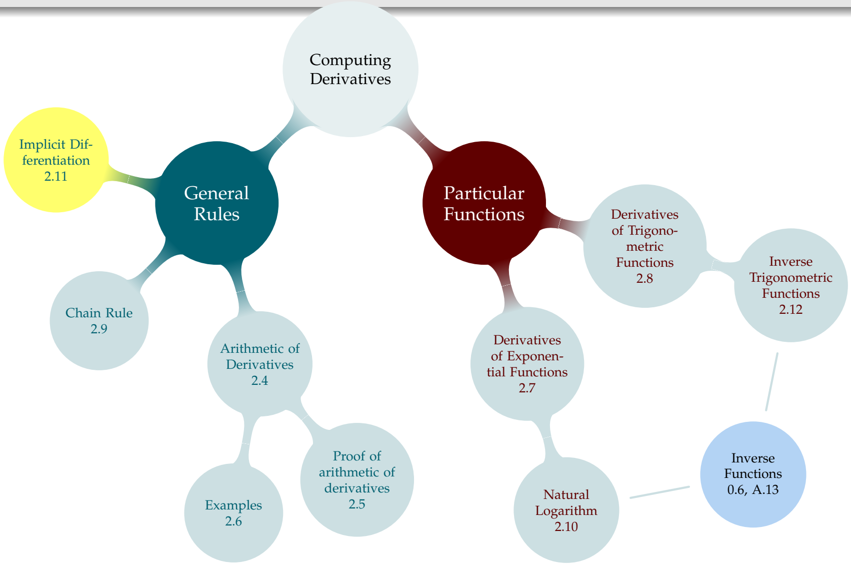

Implicitly Defined Functions

$$y^2 + x^2 + xy + x^2y = 1$$

Which of the following points are on the curve?

$(0, 1)$, $(0, -1)$, $(0, 0)$, $(1, 1)$

If $x = -3$, what is y?

IMPLICITLY DEFINED FUNCTIONS

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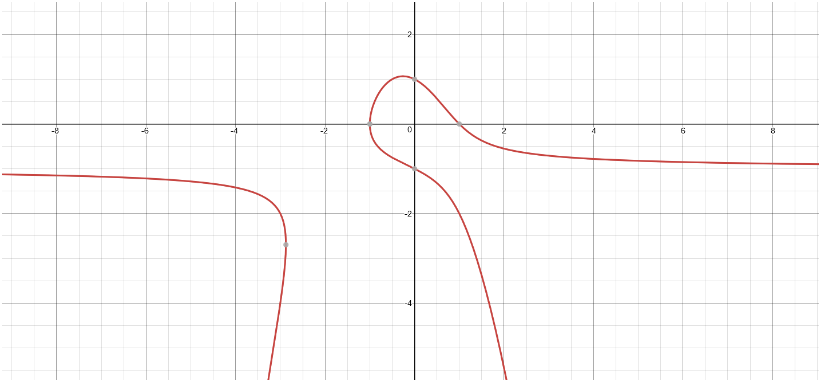
Things to emphasize:

- One x might have multiple y
- You won't be asked to graph these
- We can solve questions without *needing* to see their graphs
- Locally looks like a function (explain "locally") so all the stuff that worked before still works now if we restrict where we're looking

$$y^2 + x^2 + xy + x^2y = 1$$



Still has a slope: $\frac{\Delta y}{\Delta x}$
Locally, y is still a function of x.

$$y^2 + x^2 + xy + x^2y = 1$$

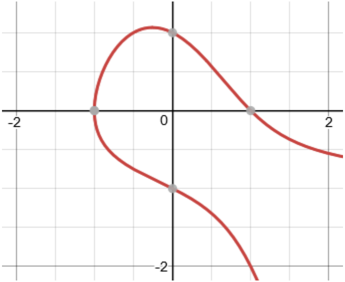
Consider y as a function of x. Can we find $\frac{dy}{dx}$?

$\frac{d}{dx}[y] =$ $\frac{d}{dx}[x] =$ $\frac{d}{dx}[1] =$

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{dy}{dx} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

Necessarily, $\frac{dy}{dx}$ depends on **both** y and x. Why?



Now You

Suppose $x^4y + y^4x = 2$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point $(1, 1)$.

Now You Suppose $x^4y + y^4x = 2$. Find $\frac{dy}{dx}$ at the point $(1,1)$.

Students often struggle knowing when to replace variables with constants.

Now You Suppose $\dfrac{3y^2+2y+y^3}{x^2+1}=x$. Find $\dfrac{dy}{dx}$ when $x=0$, and the equations of the associated tangent line(s).

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

$$\begin{aligned} \log x &= y(x) \\ x &= e^{y(x)} \end{aligned}$$

Use implicit differentiation to differentiate $\log|x|$, $x < 0$.

Use implicit differentiation to differentiate $\log_a(x)$, where $a > 0$ is a constant and $x > 0$.

Use implicit differentiation to differentiate $\log_a |x|$, $a > 0$.

Included Work

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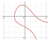
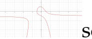
 screenshot of graph using Desmos Graphing Calculator, https://www.desmos.com/calculator (accessed 19 October 2017), 6

 screenshot of graph using Desmos Graphing Calculator, https://www.desmos.com/calculator (accessed 19 October 2017), 4