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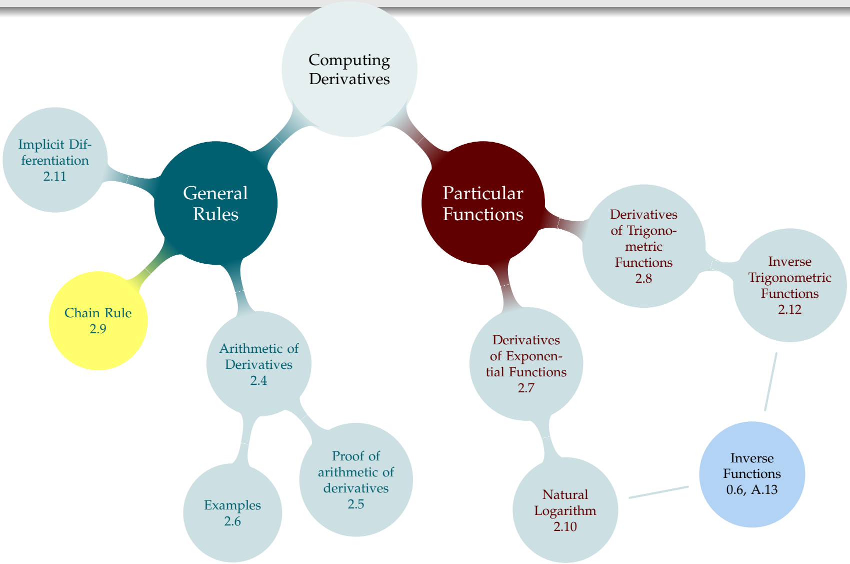
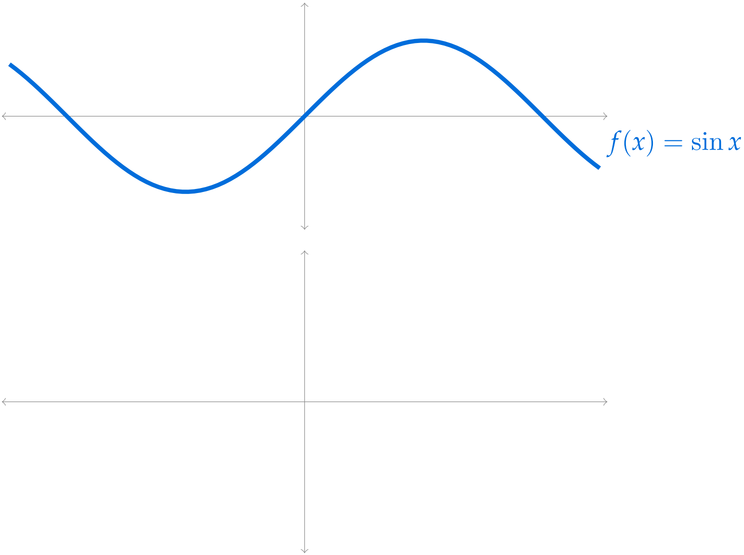


Intuition: $\sin x$ versus $\sin(2x)$



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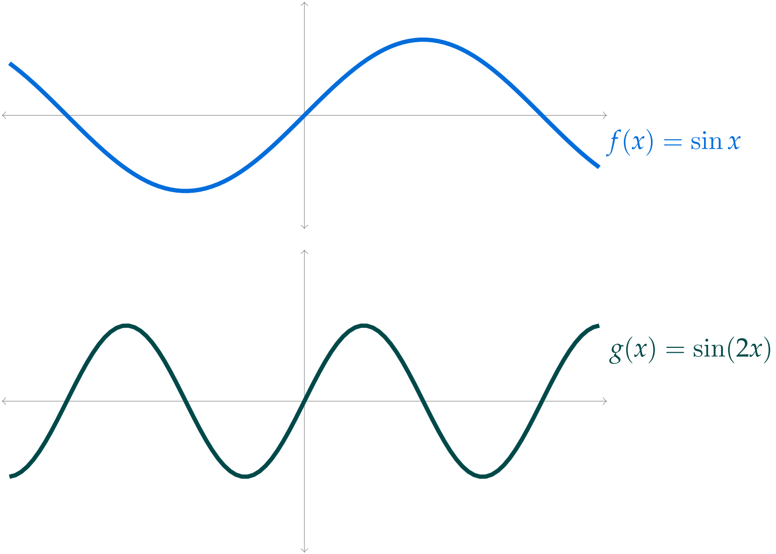

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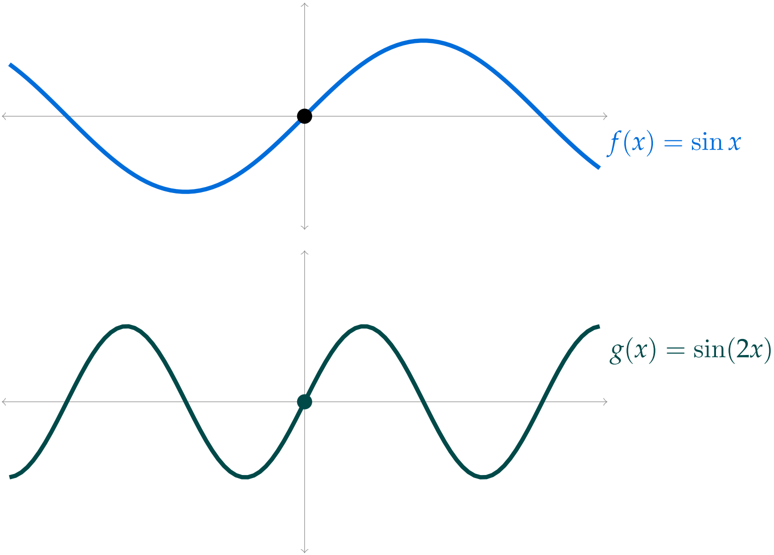

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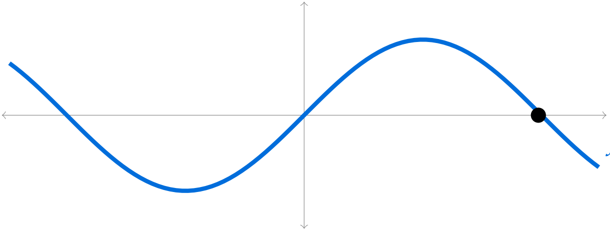

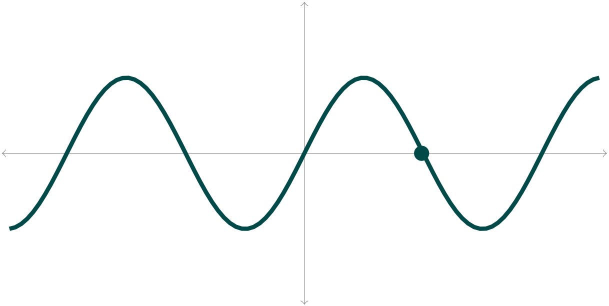

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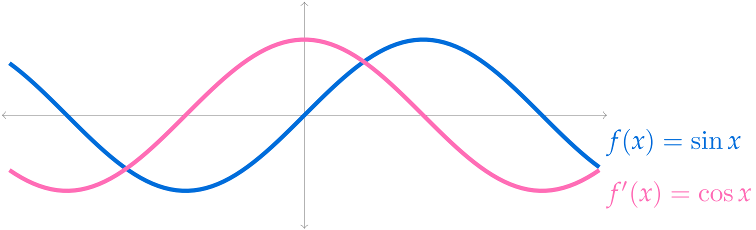

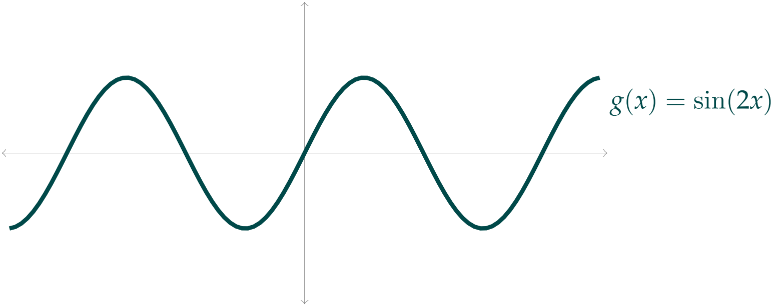

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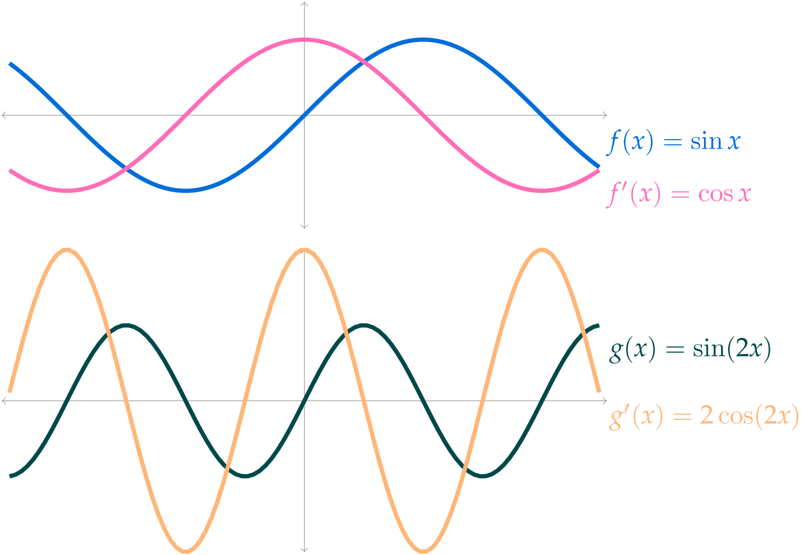

Compound Functions

Video: 2:27-3:50

Morton, Jennifer. (2014). *Balancing Act: Otters, Urchins and Kelp*. Available from https://www.kqed.org/quest/67124/balancing-act-otters-urchins-and-kelp

Kelp Population

k	kelp population
u	urchin population
o	otter population

Kelp Population

k	kelp population
u	urchin population
o	otter population

$k(u)$

Kelp Population

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u	urchin population
o	otter population

$k(u)$ $\qquad$ $k(u(o))$

Kelp Population

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u	urchin population
o	otter population
p	public policy

$k(u)$ $\qquad$ $k(u(o))$ $\qquad$ $k(u(o(p)))$

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These are examples of compound functions.

Kelp Population

k kelp population
u urchin population
o otter population
p public policy

$k(u)$ $\qquad$ $k(u(o))$ $\qquad$ $k(u(o(p)))$

These are examples of compound functions.

Should $\frac{\mathrm{d}}{\mathrm{d}o}k\big(u(o)\big)$ be positive or negative?
A. positive $\qquad$ B. negative $\qquad$ C. I'm not sure

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Should $k'(u)$ be positive or negative?
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Differentiating Compound Functions

$$\frac{\mathrm{d}}{\mathrm{d}x}\{f(g(x))\} =$$

ans

Differentiating Compound Functions

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\{f(g(x))\} &= \lim_{h\to 0}\frac{f(g(x+h))-f(g(x))}{h}\\
&= \lim_{h\to 0}\frac{f(g(x+h))-f(g(x))}{h}\left(\frac{g(x+h)-g(x)}{g(x+h)-g(x)}\right)\\
&= \lim_{h\to 0}\frac{f(g(x+h))-f(g(x))}{g(x+h)-g(x)}\cdot\frac{g(x+h)-g(x)}{h}\\
&= \lim_{h\to 0}\frac{f(g(x+h))-f(g(x))}{g(x+h)-g(x)}\cdot\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}\\
&= \lim_{h\to 0}\frac{f\left(\boxed{g(x+h)}\right)-f\left(\boxed{g(x)}\right)}{\boxed{g(x+h)}-\boxed{g(x)}}\cdot g'(x)
\end{aligned}
$$

Set $H = g(x+h)-g(x)$. As $h\to 0$, we also have $H\to 0$. So

$$
\begin{aligned}
&= \lim_{H\to 0}\frac{f(g(x)+H)-f(g(x))}{H}\cdot g'(x)\\
&= f'(g(x))\cdot g'(x)
\end{aligned}
$$

Chain Rule

Chain Rule – Theorem 2.9.3

Suppose f and g are differentiable functions. Then

$$\frac{\mathrm{d}}{\mathrm{d}x}\{f(g(x))\} = f'(g(x))\,g'(x) = \frac{\mathrm{d}f}{\mathrm{d}g}(g(x))\,\frac{\mathrm{d}g}{\mathrm{d}x}(x)$$

In the case of kelp, $\frac{\mathrm{d}}{\mathrm{d}o}k(u(o)) = \frac{\mathrm{d}k}{\mathrm{d}u}(u(o))\,\frac{\mathrm{d}u}{\mathrm{d}o}(o)$

Chain Rule

Suppose f and g are differentiable functions. Then

$$\frac{\mathrm{d}}{\mathrm{d}x}\{f\left(g(x)\right)\} = f'\left(g(x)\right)g'(x) = \frac{\mathrm{d}f}{\mathrm{d}g}\left(g(x)\right)\frac{\mathrm{d}g}{\mathrm{d}x}(x)$$

Example: suppose $F(x) = \sin(e^x + x^2)$.

ans

Chain Rule

Suppose f and g are differentiable functions. Then

$$\frac{\mathrm{d}}{\mathrm{d}x}\{f(g(x))\} = f'(g(x))\,g'(x) = \frac{\mathrm{d}f}{\mathrm{d}g}(g(x))\,\frac{\mathrm{d}g}{\mathrm{d}x}(x)$$

Example: suppose $F(x) = \sin(e^x + x^2)$.

We can differentiate $\sin(x)$, so let's set $g(x) = e^x + x^2$ and $f(g) = \sin(g)$. Then $F(x) = f(g(x))$.

$g'(x) = e^x + 2x$ and $\frac{\mathrm{d}f}{\mathrm{d}g}(g) = \cos(g)$ and
$\frac{\mathrm{d}f}{\mathrm{d}g}(g(x)) = \frac{\mathrm{d}f}{\mathrm{d}g}\left(\boxed{e^x + x^2}\right) = \cos\left(\boxed{e^x + x^2}\right)$

So, $F'(x) = \frac{\mathrm{d}f}{\mathrm{d}g}(g(x))\,\frac{\mathrm{d}g}{\mathrm{d}x}(x) = \cos(e^x + x^2)\,(e^x + 2x)$

$$F(v) = \left(\frac{v}{v^3 + 1} \right)^6$$

Q

ans

$$F(v) = \left(\frac{v}{v^3+1}\right)^6$$

$$\begin{aligned} F'(v) &= 6\left(\boxed{\frac{v}{v^3+1}}\right)^5 \cdot \frac{(v^3+1)(1)-(v)(3v^2)}{(v^3+1)^2} \\ &= 6\left(\boxed{\frac{v}{v^3+1}}\right)^5 \cdot \frac{-2v^3+1}{(v^3+1)^2} \end{aligned}$$

Now You Let $f(x) = (10^x + \csc x)^{1/2}$. Find $f'(x)$.

Now You Suppose $o(t) = e^t$, $u(o) = \frac{1}{o+\sin(o)}$, and $t \geq 10$ (so all these functions are defined). Using the chain rule, find $\frac{\mathrm{d}}{\mathrm{d}t} u(o(t))$.
Note: your answer should depend only on t: not o.

Now You

Let $f(x) = (10^x + \csc x)^{1/2}$. Find $f'(x)$.

Q

ans

Now You

Let $f(x) = (10^x + \csc x)^{1/2}$. Find $f'(x)$.

$$f(x) = (\boxed{10^x + \csc x})^{1/2}$$

Using the chain rule,

$$\begin{aligned} f'(x) &= \frac{1}{2}(\boxed{10^x + \csc x})^{-1/2}(10^x \log_e 10 - \csc x \cot x) \\ &= \frac{10^x \log_e 10 - \csc x \cot x}{2\sqrt{10^x + \csc x}} \end{aligned}$$

NOW YOU Suppose $o(t) = e^t$, $u(o) = \frac{1}{o+\sin(o)}$, and $t \geq 10$ (so all these functions are defined). Using the chain rule, find $\frac{\mathrm{d}}{\mathrm{d}t} u\big(o(t)\big)$.
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Note: your answer should depend only on t: not o.

$$\begin{aligned}
o'(t) &= e^t \\
u'(o) &= \frac{(o+\sin o)(0) - (1)(1+\cos o)}{(o+\sin o)^2} \\
&= \frac{-(1+\cos o)}{(o+\sin o)^2} \\
\frac{\mathrm{d}}{\mathrm{d}t} u\big(o(t)\big) &= u'\big(o(t)\big)\, o'(t) \\
&= -e^t \left(\frac{1+\cos\big(o(t)\big)}{\big[o(t)+\sin(o(t))\big]^2} \right) \\
&= -e^t \left(\frac{1+\cos(e^t)}{\big[e^t+\sin(e^t)\big]^2} \right)
\end{aligned}$$

A

More Examples

Now You Evaluate $\dfrac{\mathrm{d}}{\mathrm{d}x}\left\{x^2+\sec\left(x^2+\dfrac{1}{x}\right)\right\}$

Now You Evaluate $\dfrac{\mathrm{d}}{\mathrm{d}x}\left\{\dfrac{1}{x+\dfrac{1}{x+\frac{1}{x}}}\right\}$

Evaluate $\dfrac{\mathrm{d}}{\mathrm{d}x}\left\{x^2+\sec\left(x^2+\dfrac{1}{x}\right)\right\}$

Evaluate $\frac{d}{dx}\left\{x^2+\sec\left(x^2+\frac{1}{x}\right)\right\}$

$$\frac{d}{dx}\left\{x^2+\sec\left(\boxed{x^2+\frac{1}{x}}\right)\right\}$$
$$=2x+\sec\left(\boxed{x^2+\frac{1}{x}}\right)\cdot\tan\left(\boxed{x^2+\frac{1}{x}}\right)\cdot\frac{d}{dx}\left\{\boxed{x^2+\frac{1}{x}}\right\}$$
$$=2x+\sec\left(\boxed{x^2+\frac{1}{x}}\right)\cdot\tan\left(\boxed{x^2+\frac{1}{x}}\right)\cdot\frac{d}{dx}\left\{\boxed{x^2+x^{-1}}\right\}$$
$$=2x+\sec\left(\boxed{x^2+\frac{1}{x}}\right)\cdot\tan\left(\boxed{x^2+\frac{1}{x}}\right)\cdot\left(2x-x^{-2}\right)$$

Notice: That first term, $2x$, is not multiplied by anything else.

Evaluate $\dfrac{\mathrm{d}}{\mathrm{d}x}\left\{\dfrac{1}{x+\dfrac{1}{x+\frac{1}{x}}}\right\}$

Q

Evaluate $\dfrac{\mathrm{d}}{\mathrm{d}x}\left\{\dfrac{1}{x+\dfrac{1}{x+\frac{1}{x}}}\right\}$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{\frac{1}{x+\dfrac{1}{x+\frac{1}{x}}}\right\} = \frac{\mathrm{d}}{\mathrm{d}x}\left\{\left(x+\left(x+x^{-1}\right)^{-1}\right)^{-1}\right\}$$

$$= -\left(\boxed{x+\left(x+x^{-1}\right)^{-1}}\right)^{-2}\cdot\frac{\mathrm{d}}{\mathrm{d}x}\left\{\boxed{x+\left(x+x^{-1}\right)^{-1}}\right\}$$

$$= -\left(\boxed{x+\left(x+x^{-1}\right)^{-1}}\right)^{-2}\cdot\left[1+(-1)\left(\boxed{x+x^{-1}}\right)^{-2}\cdot\frac{\mathrm{d}}{\mathrm{d}x}\left\{\boxed{x+x^{-1}}\right\}\right]$$

$$= -\left(\boxed{x+\left(x+x^{-1}\right)^{-1}}\right)^{-2}\cdot\left[1+(-1)\left(\boxed{x+x^{-1}}\right)^{-2}\cdot\left(1-x^{-2}\right)\right]$$

A

Included Work

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