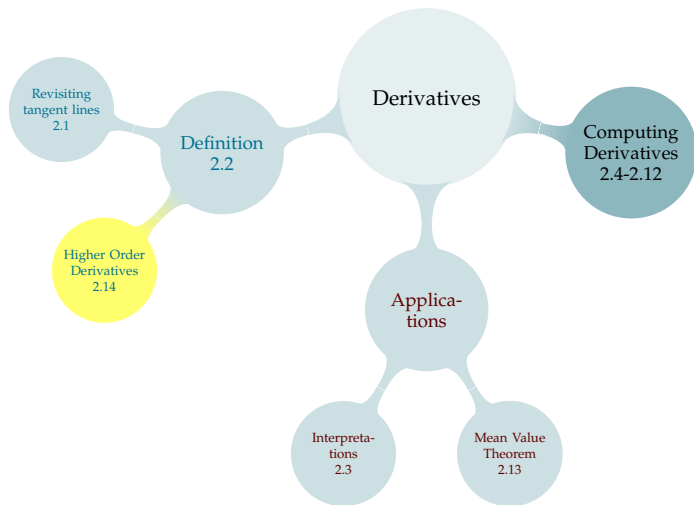


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HIGHER ORDER DERIVATIVES

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Similarly, the derivative of a second derivative is a third derivative, etc.

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- ▶ $f''(x)$ and $f^{(2)}(x)$ and $\frac{d^2f}{dx^2}(x)$ all mean $\frac{d}{dx}\left(\frac{d}{dx}f(x)\right)$
- ▶ $f'''(x)$ and $f^{(3)}(x)$ and $\frac{d^3f}{dx^3}(x)$ all mean $\frac{d}{dx}\left(\frac{d}{dx}\left(\frac{d}{dx}f(x)\right)\right)$
- ▶ $f^{(4)}(x)$ and $\frac{d^4f}{dx^4}(x)$ both mean $\frac{d}{dx}\left(\frac{d}{dx}\left(\frac{d}{dx}\left(\frac{d}{dx}f(x)\right)\right)\right)$
- ▶ and so on.

TYPICAL EXAMPLE: ACCELERATION

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The position of an object at time t is given by $s(t) = t(5 - t)$. *Time is measured in seconds, and position is measured in metres.*

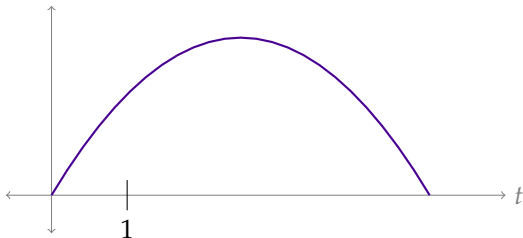
1. Sketch the graph giving the position of the object.
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$$s'(1) = 5 - 2(1) = 3 = \text{vel}$$

$$\text{Units of velocity: } \frac{\Delta s}{\Delta t} = \frac{m}{s}$$

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$$-2 = \text{acc}$$

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CONCEPT CHECK

True or False: If $f'(1) = 18$, then $f''(1) = 0$,
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Which of the following is always true of a QUADRATIC polynomial $f(x)$?

- A. $f(0) = 0$
- B. $f'(0) = 0$
- C. $f''(0) = 0$
- D. $f'''(0) = 0$
- E. $f^{(4)}(0) = 0$

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polynomial $f(x)$?**

- A. $f(0) = 0$ $f(x) = ax^2 + bx + c$
- B. $f'(0) = 0$ $f'(x) = 2ax + b$
- C. $f''(0) = 0$ $f''(x) = 2a$
- D. $f'''(0) = 0$ $f'''(x) = 0$ ✓
- E. $f^{(4)}(0) = 0$ $f^{(4)}(x) = 0$ ✓

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**Which of the following is
always true of a CUBIC
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- A. $f(0) = 0$ $f(x) = ax^3 + bx^2 + cx + d$
- B. $f'(0) = 0$ $f'(x) = 3ax^2 + 2bx + c$
- C. $f''(0) = 0$ $f''(x) = 6ax + 2b$
- D. $f'''(0) = 0$ $f'''(x) = 6a$
- E. $f^{(4)}(0) = 0$ $f^{(4)}(x) = 0$ ✓



IMPLICIT DIFFERENTIATION

Suppose $y(x)$ is a function such that

$$y(x) = y^3x + x^2 - 1$$

Find $y''(x)$ at the point $(-2, 1)$.

IMPLICIT DIFFERENTIATION

Suppose $y(x)$ is a function such that

$$y(x) = y^3x + x^2 - 1$$

Find $y''(x)$ at the point $(-2, 1)$. We start by differentiating both sides of the function. Remember that y is a function, not a variable.

$$y(x) = y(x)^3x + x^2 + 1$$

$$\frac{dy}{dx}(x) \stackrel{\text{prod}}{=} y(x)^3 + 3xy(x)^2 \frac{dy}{dx}(x) + 2x \quad (*)$$

Let's differentiate both sides again. Remember we have a rule for the product of three functions.

$$\frac{d^2y}{dx^2} = 3y^2 \frac{dy}{dx} + 3 \left(y^2 \frac{dy}{dx} + x \cdot 2y \frac{dy}{dx} \cdot \frac{dy}{dx} + xy^2 \frac{d^2y}{dx^2} \right) + 2 \quad (**)$$

When $x = -2$ and $y = 1$, using (*), we find

$$\left. \frac{dy}{dx} \right|_{(-2,1)} = 1^3 + 3(-2)(1^2) \left. \frac{dy}{dx} \right|_{(-2,1)} + 2(-2) = -3 - 6 \left. \frac{dy}{dx} \right|_{(-2,1)}$$

$$\left. \frac{dy}{dx} \right|_{(-2,1)} = -\frac{3}{7}$$

We set $x = -2$, $y = 1$, and $\frac{dy}{dx} = -\frac{3}{7}$ in equation (**). Now by $\frac{d^2y}{dx^2}$, we actually mean

$\left. \frac{d^2y}{dx^2} \right|_{(-2,1)}$, but to avoid clutter we don't write it that way until the end.

$$\frac{d^2y}{dx^2} = 3(1) \left(-\frac{3}{7} \right) + 3 \left((1) \left(-\frac{3}{7} \right) + (-2) \cdot 2(1) \left(-\frac{3}{7} \right) \cdot \left(-\frac{3}{7} \right) + (-2)(1) \frac{d^2y}{dx^2} \right) + 2$$

$$= -\frac{9}{7} + 3 \left(-\frac{3}{7} - \frac{36}{49} - 2 \frac{d^2y}{dx^2} \right) + 2$$

$$= \left(2 - \frac{18}{7} - \frac{108}{49} \right) - 6 \frac{d^2y}{dx^2}$$

$$7 \frac{d^2y}{dx^2} = -\frac{136}{7^2}$$

$$\left. \frac{d^2y}{dx^2} \right|_{(-2,1)} = -\frac{136}{7^3}$$

Included Work



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