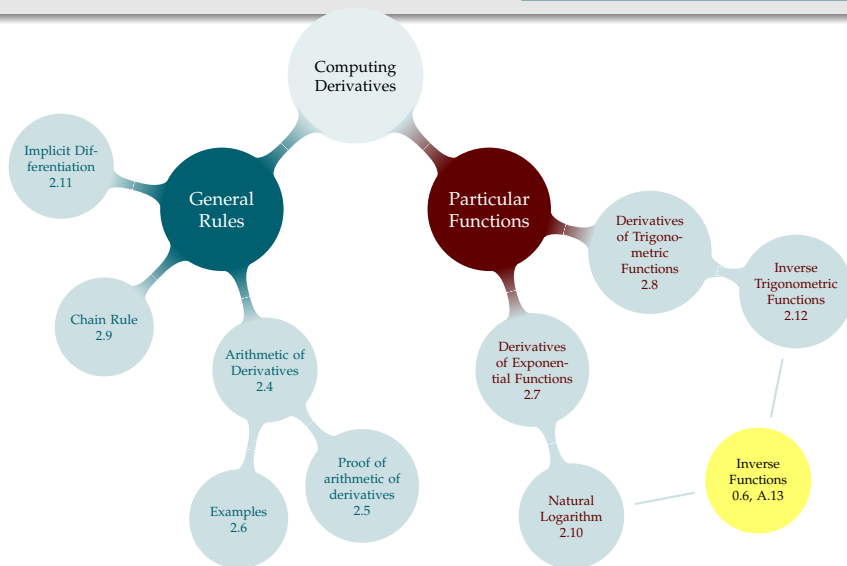


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INVERTIBILITY GAME

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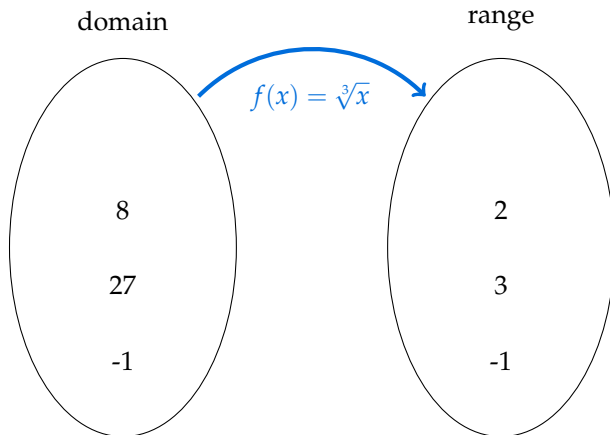
Round 1: $f(x) = 2x$

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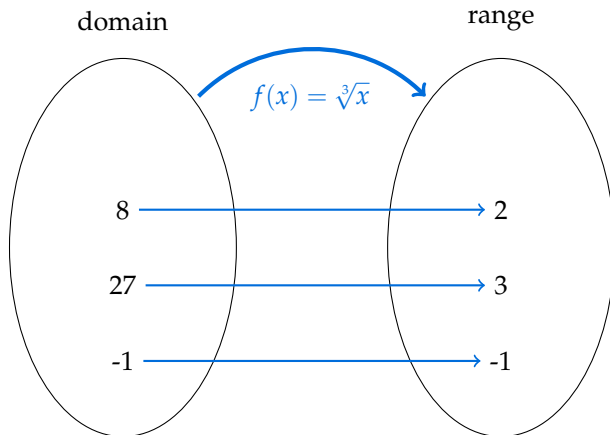
Round 3: $f(x) = |x|$

Round 4: $f(x) = \sin x$

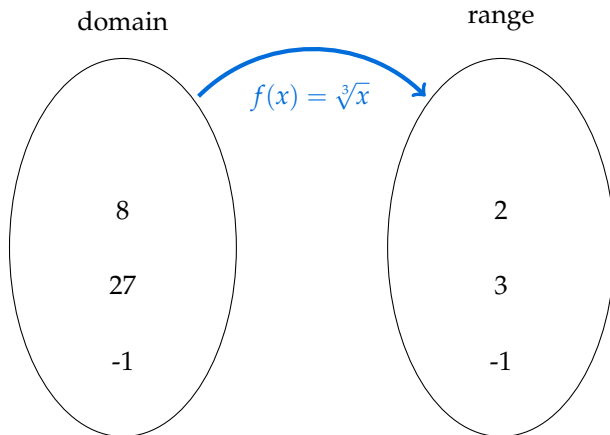
FUNCTIONS ARE MAPS



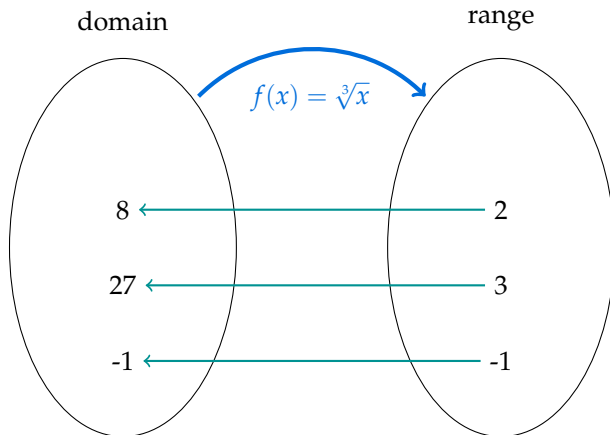
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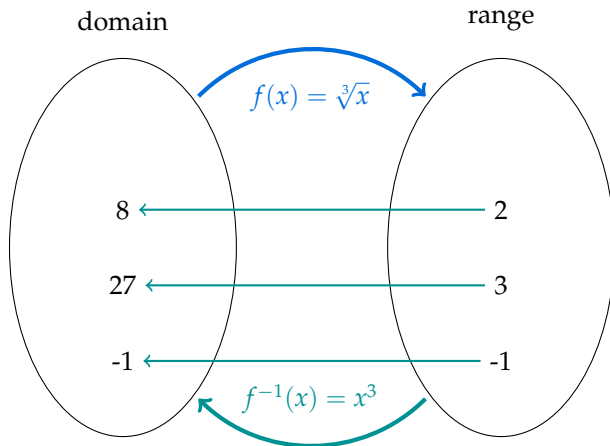
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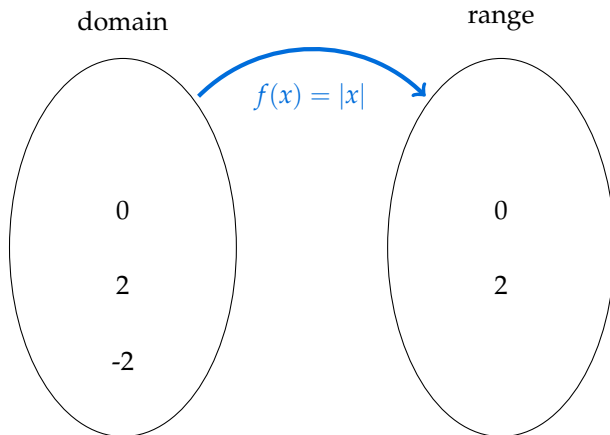
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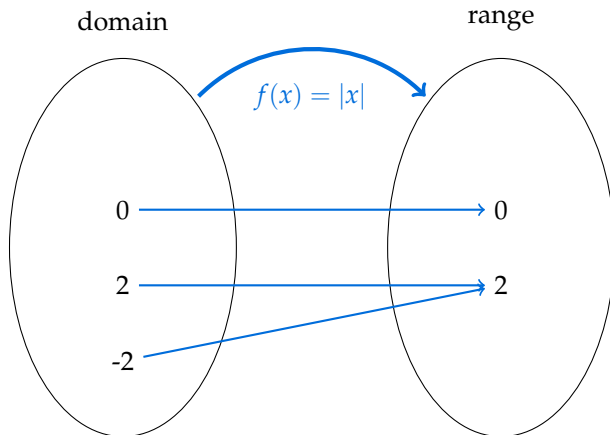
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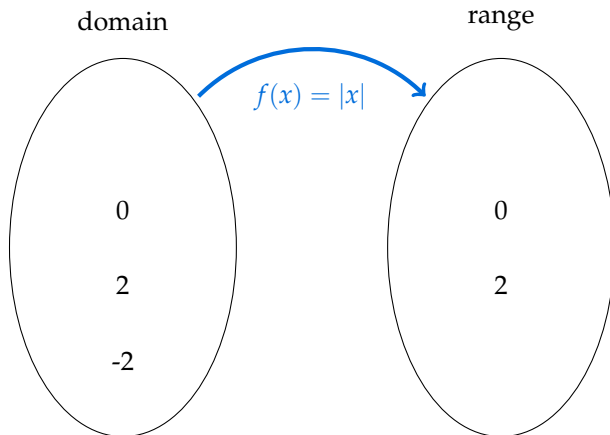
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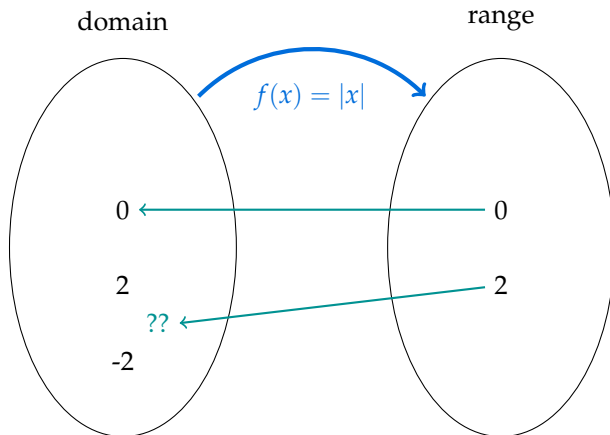
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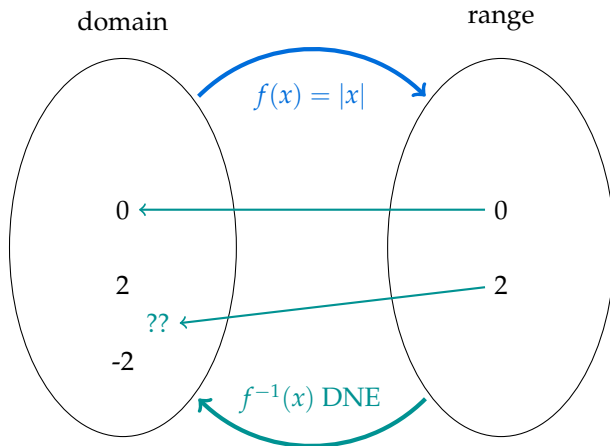
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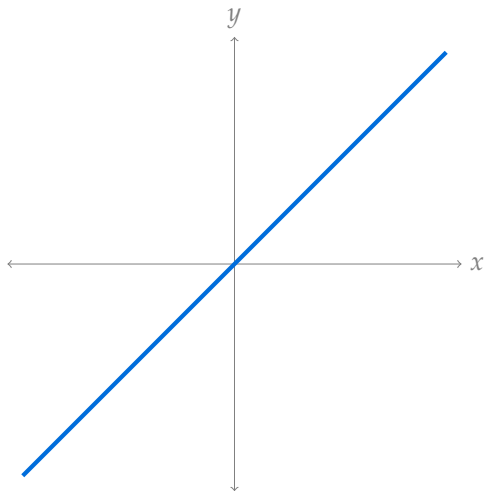


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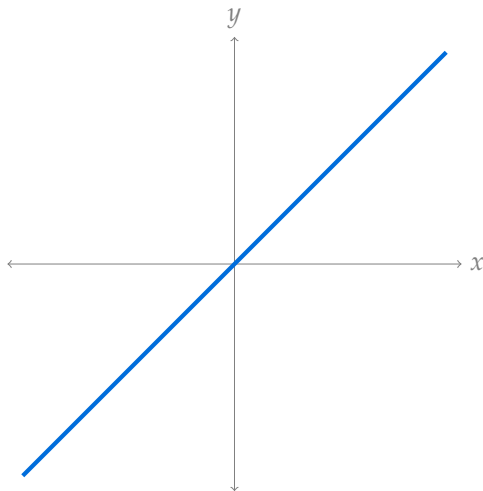
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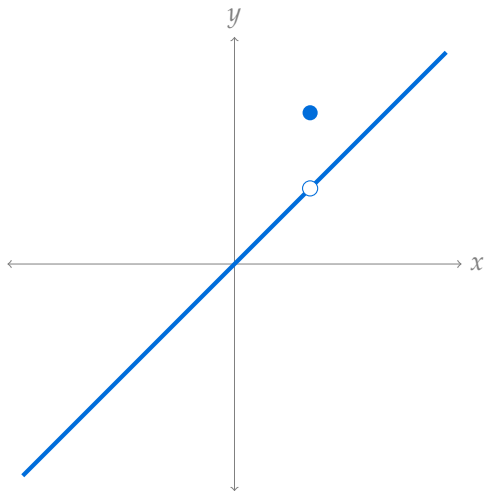
A. invertible

B. not invertible



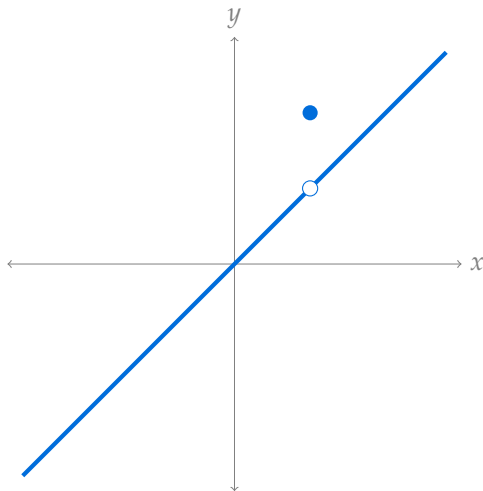
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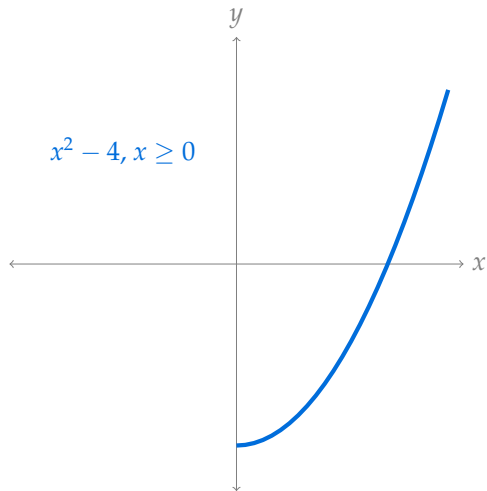
A. invertible

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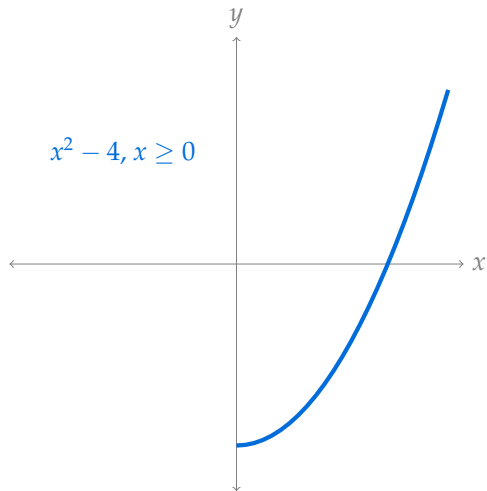
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RELATIONSHIP BETWEEN $f(x)$ AND $f^{-1}(x)$

Let f be an invertible function.

What is $f^{-1}(f(x))$?

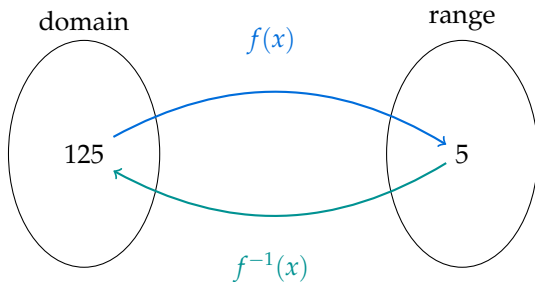
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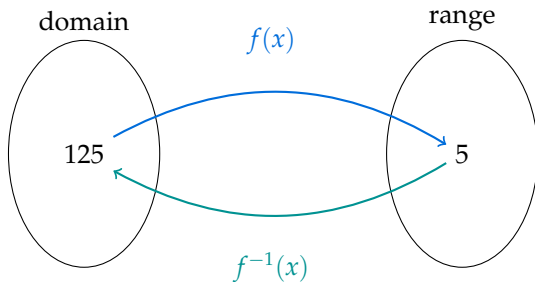


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What is $f^{-1}(10)$? (do not simplify)

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What is $f^{-1}(x)$?



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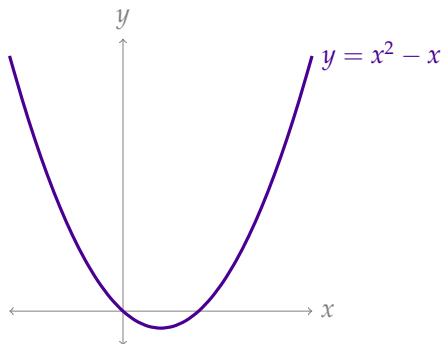
Suppose $f(x) = \sqrt[3]{19 + x^3}$. What is $f^{-1}(3)$? (simplify your answer)
 $f(2) = 3$, so $f^{-1}(3) = 2$

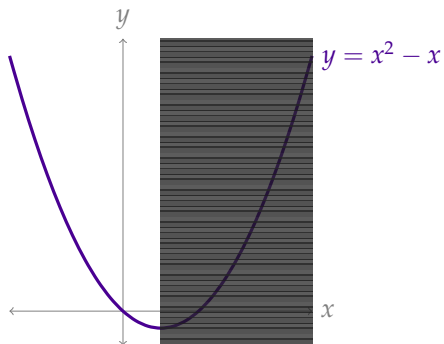
What is $f^{-1}(10)$? (do not simplify)
 $\sqrt[3]{19 + y^3} = 10$ tells us $f^{-1}(10) = \sqrt[3]{10^3 - 19}$

What is $f^{-1}(x)$?
 $\sqrt[3]{19 + y^3} = x$ tells us $f^{-1}(x) = \sqrt[3]{x^3 - 19}$

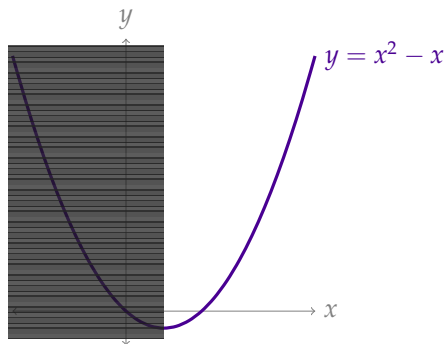
$$\text{Let } f(x) = x^2 - x.$$

1. Sketch a graph of $f(x)$, and choose a (large) domain over which it is invertible.
2. For the domain you chose, evaluate $f^{-1}(20)$.
3. For the domain you chose, evaluate $f^{-1}(x)$.
4. What are the domain and range of $f^{-1}(x)$? What are the (restricted) domain and range of $f(x)$?

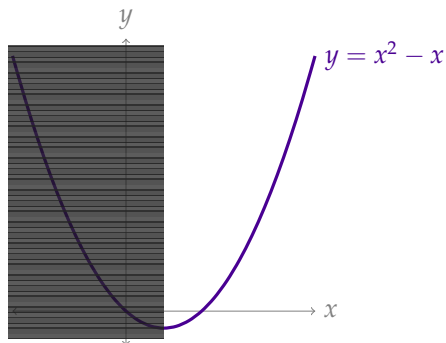




Domain: $(-\infty, \frac{1}{2}]$

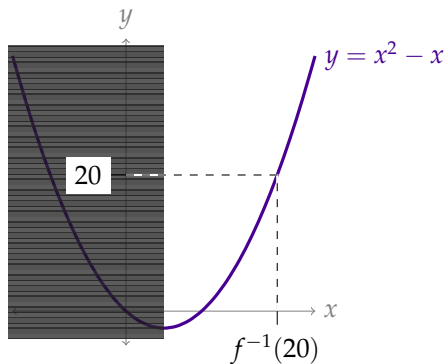


Domain: $[\frac{1}{2}, \infty)$

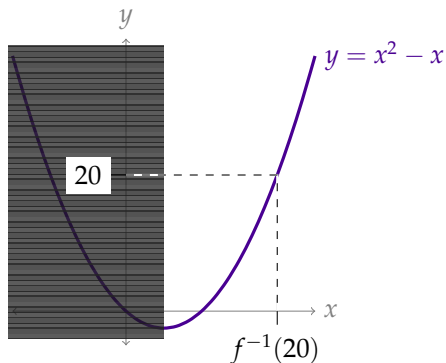


$$f^{-1}(20) =$$

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$$f^{-1}(20) =$$

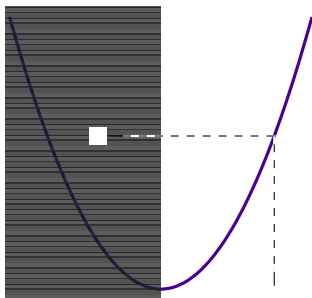


$$f^{-1}(20) = 5$$

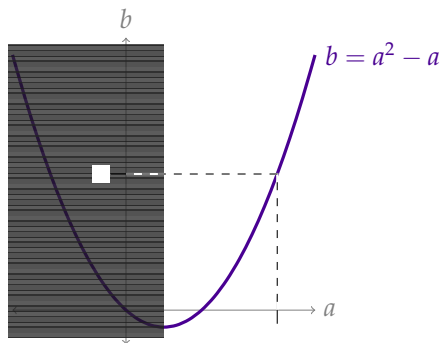
$$20 = x^2 - x$$

$$0 = x^2 - x - 20 = (x - 5)(x + 4)$$

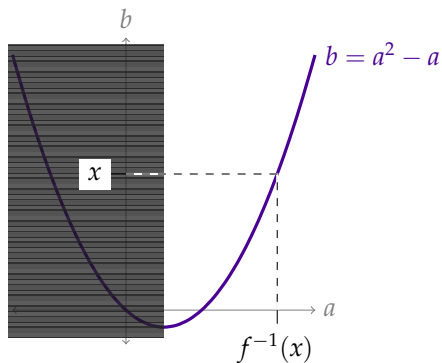
$$x = 5$$



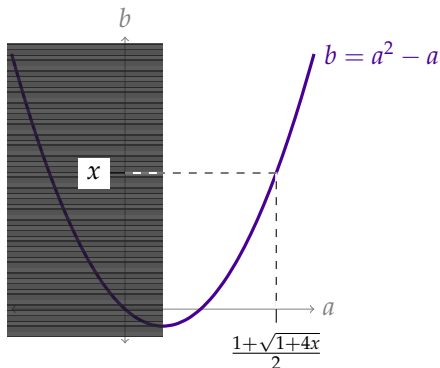
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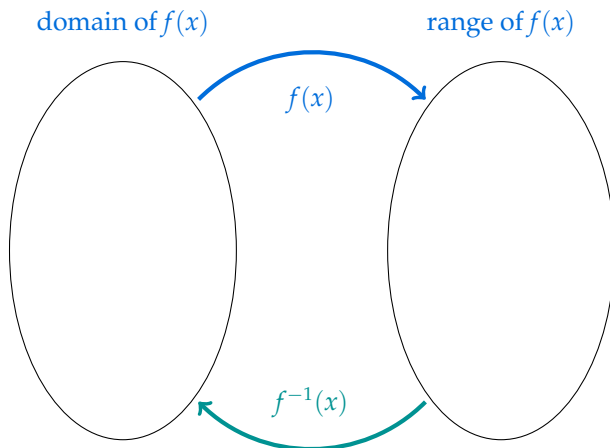
$$f^{-1}(x) = \frac{1 + \sqrt{1 + 4x}}{2}$$

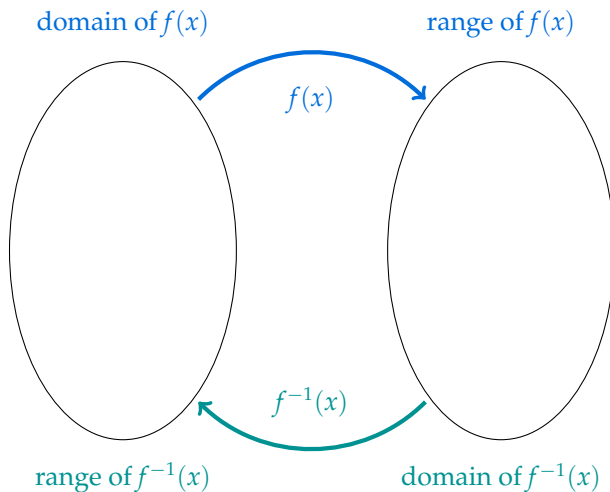
$$a^2 - a = x, \text{ find } a$$

$$a^2 - a - x = 0$$

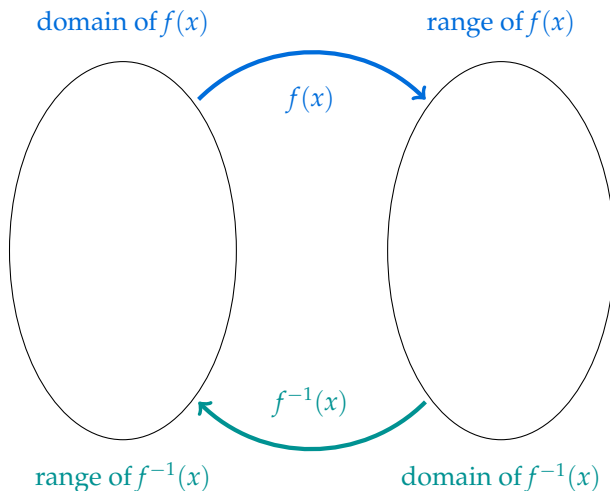
$$a = \frac{1 \pm \sqrt{1 + 4x}}{2}$$

$$f^{-1}(x) = \frac{1 + \sqrt{1 + 4x}}{2}$$

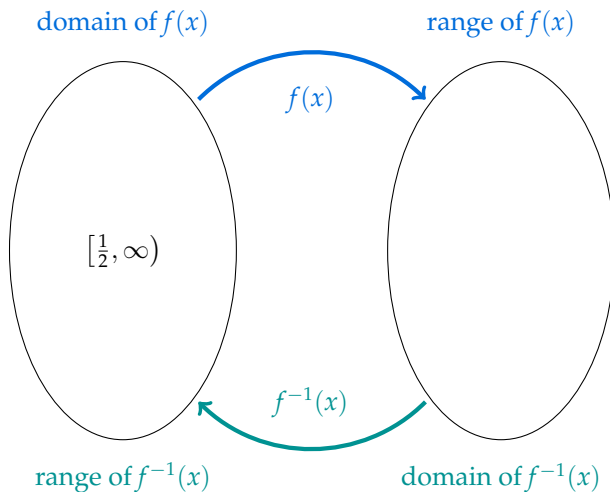




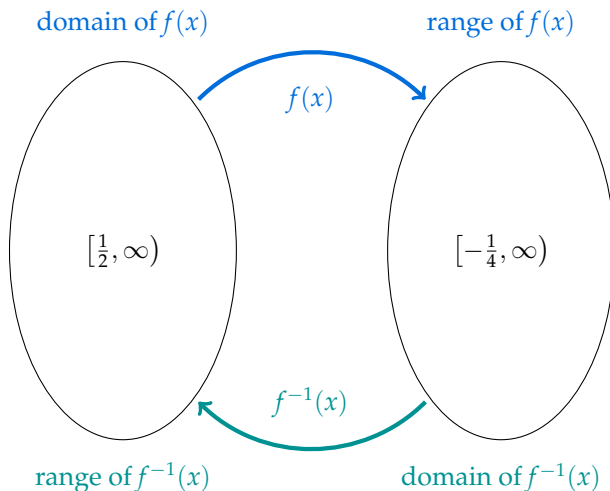
$$f(x) = x^2 - x, \text{ domain: } \left[\frac{1}{2}, \infty\right)$$



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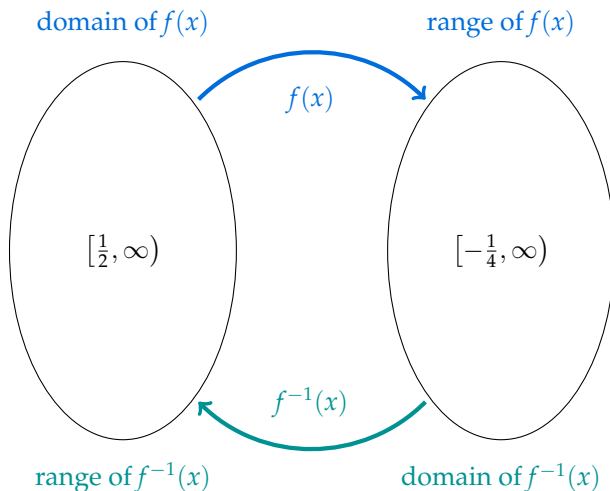


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INVERTIBILITY GAME: $f(x) = e^x$

$f^{-1}(x) = \log_e x$

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 $\log_e(e) = 1$
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 $\log_e\left(\frac{1}{e}\right) = -1$
- ▶ I'm thinking of an x . Your clue: $f(x) = e^3$. What is my x ? $x = 3$
 $\log_e(e^3) = 3$
- ▶ I'm thinking of an x . Your clue: $f(x) = 0$. What is my x ? **Trick question: no x gives $f(x) = 0$.**
 $\log_e(x)$ is undefined at $x = 0$

1. Suppose $0 < x < 1$. Then $\log_e(x)$ is...
2. Suppose $-1 < x < 0$. Then $\log_e(x)$ is...
3. Suppose $e < x$. Then $\log_e(x)$ is...
 - A. positive
 - B. negative
 - C. greater than one
 - D. less than one
 - E. undefined

EXPONENTS AND LOGARITHMS

$$f(x) = e^x$$

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x	e^x	
0	1	
1	e	
-1	$\frac{1}{e}$	
n	e^n	

EXPONENTS AND LOGARITHMS

$$f(x) = e^x$$

$$f^{-1}(x) = \log_e(x) = \ln(x) = \text{log}(x)$$

x	e^x	$e \text{ fact} \leftrightarrow \log_e \text{ fact}$	x	$\log_e(x)$
0	1			
1	e			
-1	$\frac{1}{e}$			
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1	e			
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1	e	$e^1 = e \leftrightarrow \log_e(e) = 1$	e	1
-1	$\frac{1}{e}$			
n	e^n			

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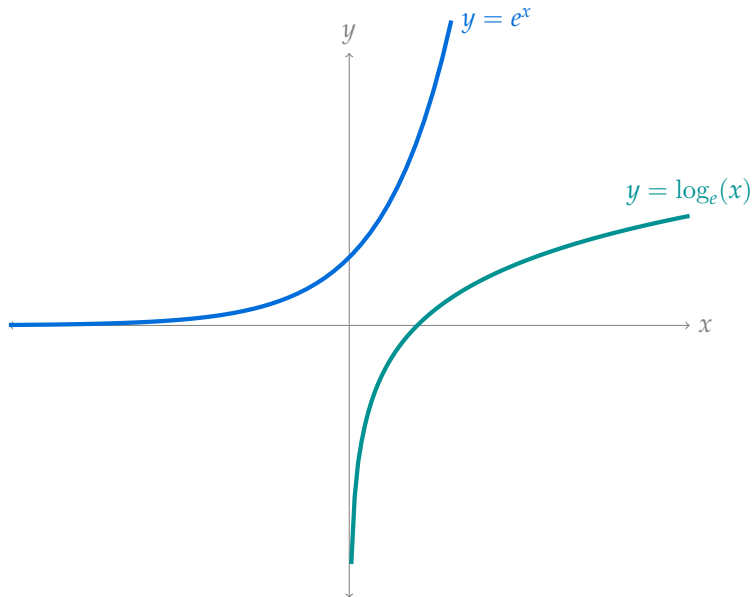
x	e^x	$e \text{ fact} \leftrightarrow \log_e \text{ fact}$	x	$\log_e(x)$
0	1	$e^0 = 1 \leftrightarrow \log_e(1) = 0$	1	0
1	e	$e^1 = e \leftrightarrow \log_e(e) = 1$	e	1
-1	$\frac{1}{e}$	$e^{-1} = \frac{1}{e} \leftrightarrow \log_e(\frac{1}{e}) = -1$	$\frac{1}{e}$	-1
n	e^n			

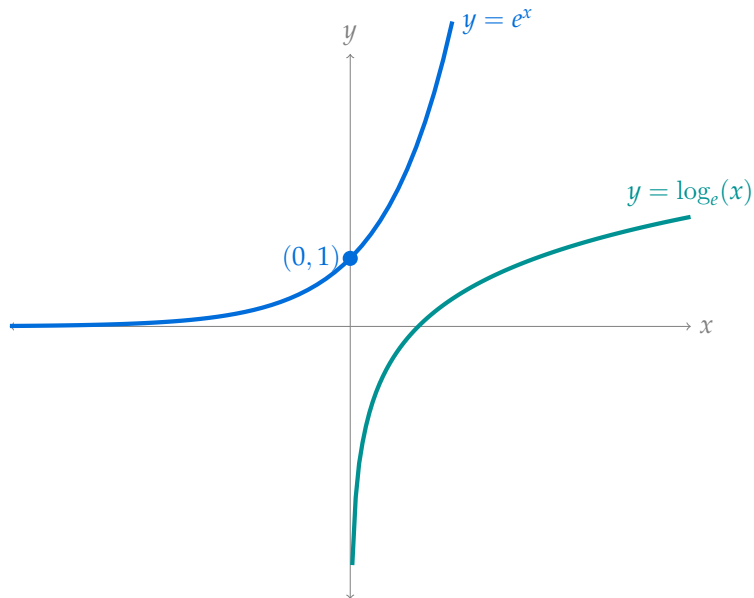
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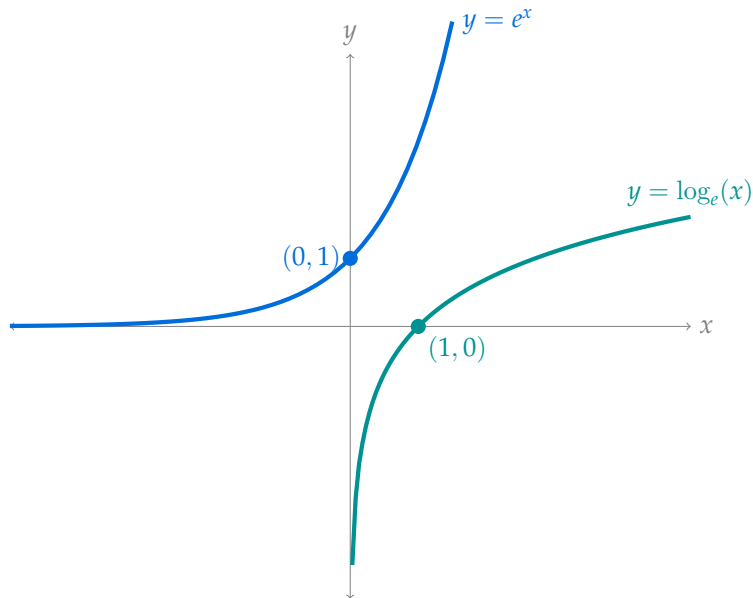
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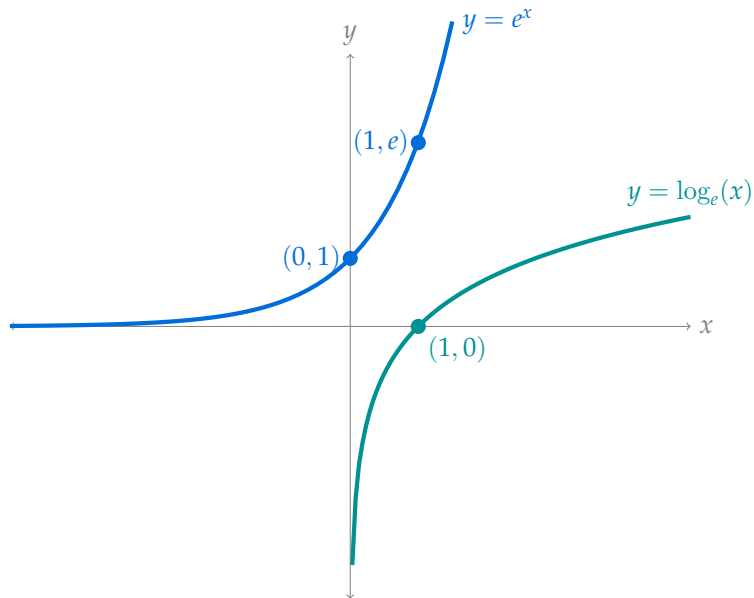
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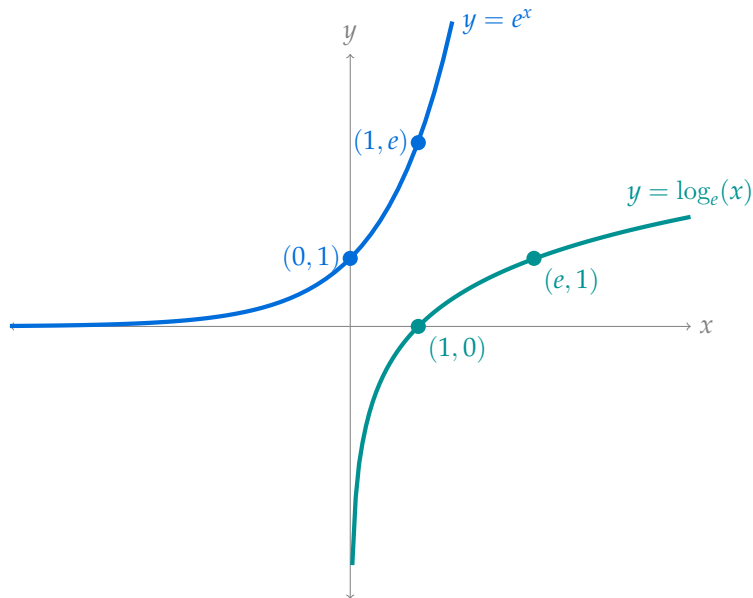
x	e^x	$e \text{ fact} \leftrightarrow \log_e \text{ fact}$	x	$\log_e(x)$
0	1	$e^0 = 1 \leftrightarrow \log_e(1) = 0$	1	0
1	e	$e^1 = e \leftrightarrow \log_e(e) = 1$	e	1
-1	$\frac{1}{e}$	$e^{-1} = \frac{1}{e} \leftrightarrow \log_e(\frac{1}{e}) = -1$	$\frac{1}{e}$	-1
n	e^n	$e^n = e^n \leftrightarrow \log_e(e^n) = n$	e^n	n

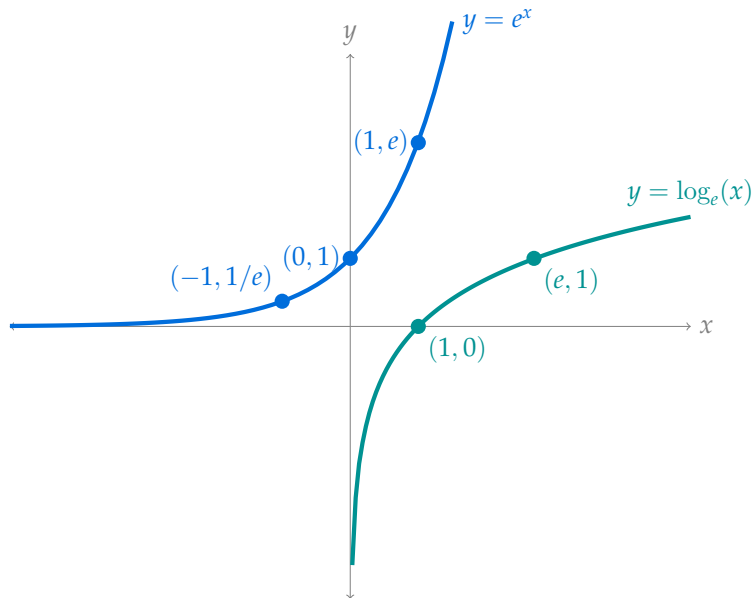


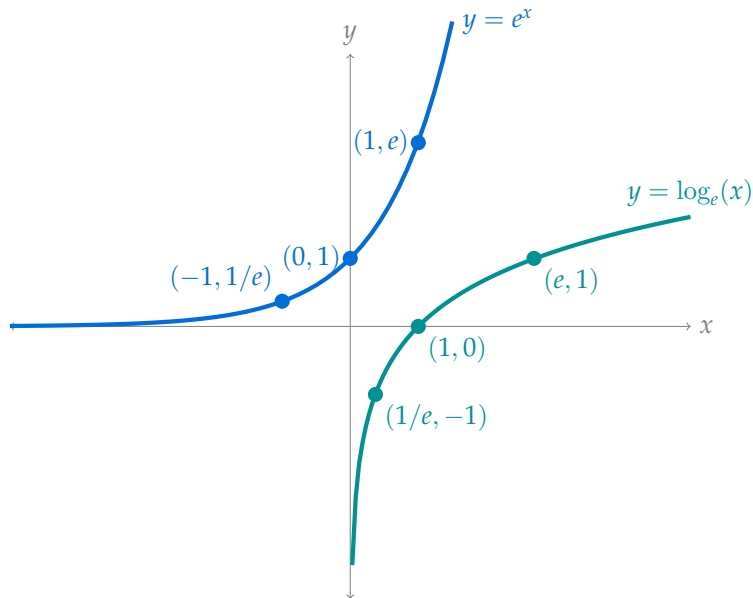


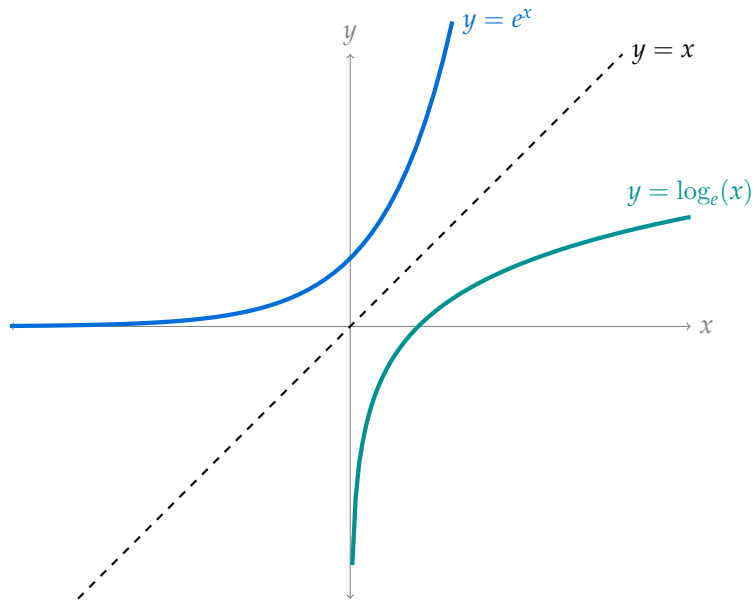












LOGS OF OTHER BASES: $\log_n(x)$ IS THE INVERSE OF n^x

$$\log_{10} 10^8 =$$

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Logarithm Rules

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Write as a single logarithm:

$$f(x) = \log\left(\frac{10}{x^2}\right) + 2 \log x + \log(10 + x)$$

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$$\begin{aligned} f(x) &= \log\left(\frac{10}{x^2}\right) + 2 \log x + \log(10 + x) \\ &= \log 10 - \log(x^2) + 2 \log x + \log(10 + x) \\ &= \log 10 - 2 \log x + 2 \log x + \log(10 + x) \\ &= \log 10 + \log(10 + x) = \log(10(10 + x)) \\ &= \log(100 + 10x) \end{aligned}$$

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Suppose your calculator can only compute logarithms base 10. What would you enter to calculate $\log(17)$?

Suppose your calculator can only compute natural logarithms. What would you enter to calculate $\log_2(57)$?

Suppose your calculator can only compute logarithms base 2. What would you enter to calculate $\log(2)$?

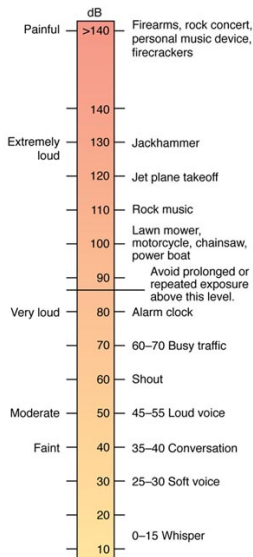
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Suppose your calculator can only compute logarithms base 2. What would you enter to calculate $\log(2)$? $\frac{\log_2 2}{\log_2 e} = \frac{1}{\log_2 e}$



Decibels: For a particular measure of the power P of a sound wave, the decibels of that sound is:

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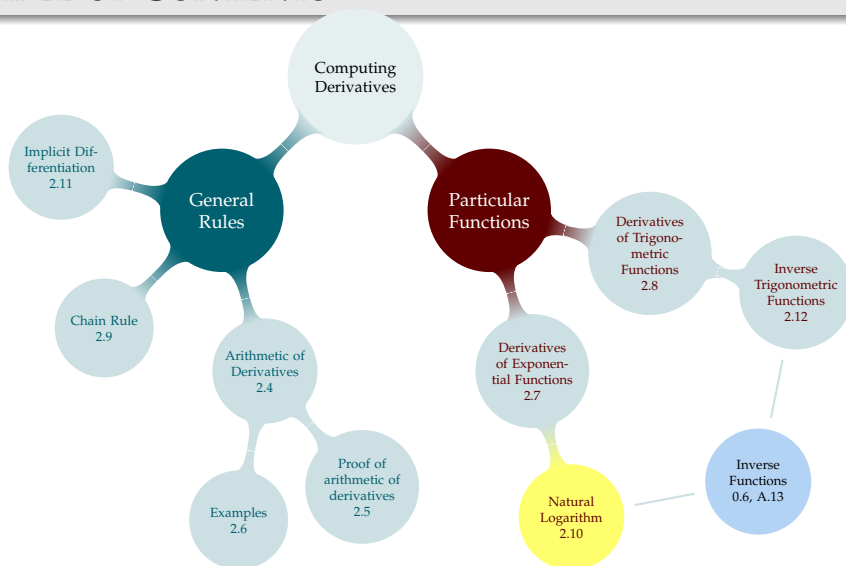
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Differentiate: $f(x) = \log_e |x^2 + 1|$

We use the chain rule:

$$\begin{aligned} \frac{d}{dx} \left\{ \log_e \left| \boxed{x^2 + 1} \right| \right\} &= \frac{1}{x^2 + 1} \cdot (2x) \\ &= \frac{2x}{x^2 + 1} \end{aligned}$$

Derivatives of Logarithms – Corollary 2.10.6

For $a > 0$:

$$\frac{d}{dx} [\log_a |x|] = \frac{1}{x \log a}$$

In particular:

$$\frac{d}{dx} [\log |x|] = \frac{1}{x}$$

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We use the chain rule:

$$\frac{d}{dx} [\log_e |\boxed{\cot x}|] = \frac{1}{\cot x} \cdot (-\csc^2 x) = \frac{-\csc^2 x}{\cot x}$$

LOGARITHMIC DIFFERENTIATION - A FANCY TRICK

$$\blacktriangleright \log(f \cdot g) = \log f + \log g$$

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► $\log(f \cdot g) = \log f + \log g$

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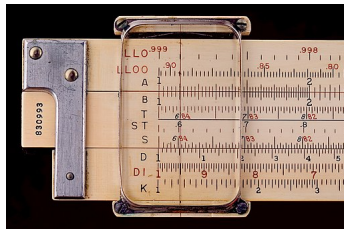
We can exploit these properties to differentiate!

Logarithmic Differentiation

In general, if $f(x) \neq 0$, $\frac{d}{dx} [\log |f(x)|] = \frac{f'(x)}{f(x)}$.

$$f(x) = \left(\frac{(2x+5)^4(x^2+1)}{x+3} \right)^5$$

Find $f'(x)$.



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$$\log(f(x)) = \log \left[\left(\frac{(2x+5)^4(x^2+1)}{x+3} \right)^5 \right]$$

$$= 5 \log \left[\frac{(2x+5)^4(x^2+1)}{x+3} \right]$$

$$= 5 \left[4 \log(2x+5) + \log(x^2+1) - \log(x+3) \right]$$

$$\frac{f'(x)}{f(x)} = 5 \left[4 \frac{2}{2x+5} + \frac{2x}{x^2+1} - \frac{1}{x+3} \right]$$

$$f'(x) = \left(\frac{(2x+5)^4(x^2+1)}{x+3} \right)^5 \cdot 5 \left[4 \frac{2}{2x+5} + \frac{2x}{x^2+1} - \frac{1}{x+3} \right]$$

LOGARITHMIC DIFFERENTIATION - A FANCY TRICK

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LOGARITHMIC DIFFERENTIATION - A FANCY TRICK

Differentiate:

$$f(x) = x^x$$

$$\log(f(x)) = \log[x^x]$$

$$= x \log x$$

$$\frac{f'(x)}{f(x)} = x \cdot \frac{1}{x} + \log x \cdot 1$$

$$= 1 + \log x$$

$$f'(x) = x^x [1 + \log x]$$

LOGARITHMIC DIFFERENTIATION - A FANCY TRICK

Differentiate:

$$f(x) = \left(\frac{(x^{15} - 9x^2)^{10}(x + x^2 + 1)}{(x^7 + 7)(x + 1)(x + 2)(x + 3)} \right)^5$$

LOGARITHMIC DIFFERENTIATION - A FANCY TRICK

Differentiate:

$$f(x) = \left(\frac{(x^{15} - 9x^2)^{10}(x + x^2 + 1)}{(x^7 + 7)(x + 1)(x + 2)(x + 3)} \right)^5$$

$$\log(f(x)) = \log \left[\left(\frac{(x^{15} - 9x^2)^{10}(x + x^2 + 1)}{(x^7 + 7)(x + 1)(x + 2)(x + 3)} \right)^5 \right]$$

$$= 5 \log \left[\frac{(x^{15} - 9x^2)^{10}(x + x^2 + 1)}{(x^7 + 7)(x + 1)(x + 2)(x + 3)} \right]$$

$$= 5 \left[10 \log(x^{15} - 9x^2) + \log(x + x^2 + 1) - \log(x^7 + 7) - \log(x + 1) - \log(x + 2) - \log(x + 3) \right]$$

$$\frac{f'(x)}{f(x)} = 5 \left[10 \frac{15x^{14} - 18x}{x^{15} - 9x^2} + \frac{1 + 2x}{x + x^2 + 1} - \frac{7x^6}{x^7 + 7} - \frac{1}{x + 1} - \frac{1}{x + 2} - \frac{1}{x + 3} \right]$$

$$f'(x) = \left(\frac{(x^{15} - 9x^2)^{10}(x + x^2 + 1)}{(x^7 + 7)(x + 1)(x + 2)(x + 3)} \right)^5 \cdot 5 \left[10 \frac{15x^{14} - 18x}{x^{15} - 9x^2} + \frac{1 + 2x}{x + x^2 + 1} - \frac{7x^6}{x^7 + 7} - \frac{1}{x + 1} - \frac{1}{x + 2} - \frac{1}{x + 3} \right]$$



$$f(x) = \frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x}$$

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$$\log \left| \frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x} \right| = \log |f(x)|$$

$$\left[\log |x^8 - e^x| + \log |x^{1/2} + 5| - 5 \log |\csc x| \right] = \log |f(x)|$$

$$\frac{d}{dx} \left[\log |x^8 - e^x| + \log |x^{1/2} + 5| - 5 \log |\csc x| \right] = \frac{d}{dx} \log |f(x)|$$

$$\frac{8x^7 - e^x}{x^8 - e^x} + \frac{\frac{1}{2}x^{-1/2}}{x^{1/2} + 5} - 5 \frac{-\csc x \cot x}{\csc x} = \frac{f'(x)}{f(x)}$$

$$\frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x} \left(\frac{8x^7 - e^x}{x^8 - e^x} + \frac{\frac{1}{2}x^{-1/2}}{x^{1/2} + 5} + 5 \frac{\csc x \cot x}{\csc x} \right) = f'(x)$$

$$f(x) = (x^2 + 17)(32x^5 - 8)(x^{98} - x^{57} + 32x^2)^4(32x^{10} - 10x^{32})$$

Find $f'(x)$.



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$$\log \left| (x^2 + 17)(32x^5 - 8)(x^{98} - x^{57} + 32x^2)^4(32x^{10} - 10x^{32}) \right| = \log |f(x)|$$

$$\log |x^2 + 17| + \log |32x^5 - 8| + 4 \log |x^{98} - x^{57} + 32x^2| + \log |32x^{10} - 10x^{32}| = \log |f(x)|$$

$$\frac{d}{dx} \left[\log |x^2 + 17| + \log |32x^5 - 8| + 4 \log |x^{98} - x^{57} + 32x^2| + \log |32x^{10} - 10x^{32}| \right] = \frac{d}{dx} [\log |f(x)|]$$

$$\frac{2x}{x^2 + 17} + \frac{160x^4}{32x^5 - 8} + 4 \frac{98x^{97} - 57x^{56} + 64x}{x^{98} - x^{57} + 32x^2} + \frac{320x^9 - 320x^{31}}{32x^{10} - 10x^{32}} = \frac{f'(x)}{f(x)}$$

$$\left((x^2 + 17)(32x^5 - 8)(x^{98} - x^{57} + 32x^2)^4(32x^{10} - 10x^{32}) \right) \cdot$$

$$\left(\frac{2x}{x^2 + 17} + \frac{160x^4}{32x^5 - 8} + 4 \frac{98x^{97} - 57x^{56} + 64x}{x^{98} - x^{57} + 32x^2} + \frac{320x^9 - 320x^{31}}{32x^{10} - 10x^{32}} \right) = f'(x)$$



Included Work



Anonymous. (2012) [Decibel Scale of Frequently Heard Sounds](#).
Biology Forums. <http://biology-forums.com/index.php?action=gallery;sa=view;id=6156> (accessed 7 October 2015) , 97



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