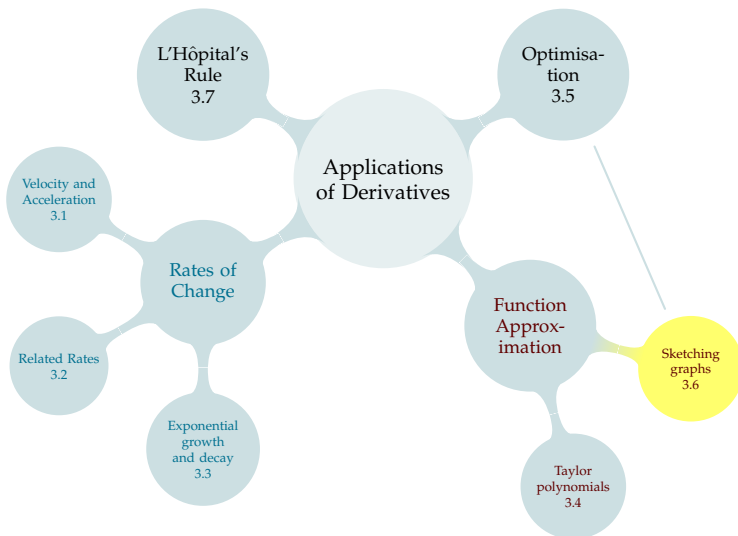


# TABLE OF CONTENTS



# CURVE SKETCHING

Review: find the domain of the following function.

$$f(x) = \frac{\sqrt{3-x^2}}{\log(x+1)}$$

Where might you expect  $f(x)$  to have a vertical asymptote? What does the function look like nearby?

(Recall: a vertical asymptote occurs at  $x = a$  if the function has an infinite discontinuity at  $a$ . That is,  $\lim_{x \rightarrow a^\pm} f(x) = \pm\infty$ .)

Where is  $f(x) = 0$ ?

What happens to  $f(x)$  near its other endpoint,  $x = -1$ ?

# CURVE SKETCHING

Good things to check:

- Domain
- Vertical asymptotes:  $\lim_{x \rightarrow a} f(x) = \pm\infty$
- Intercepts:  $x = 0, f(x) = 0$
- Horizontal asymptotes and end behavior:  $\lim_{x \rightarrow \pm\infty} f(x)$

# CURVE SKETCHING

Identify: domain, vertical asymptotes, intercepts, and horizontal asymptotes

$$f(x) = \frac{x - 2}{(x + 3)^2}$$

# CURVE SKETCHING

Identify: domain, vertical asymptotes, intercepts, and horizontal asymptotes

$$f(x) = \frac{(x+2)(x-3)^2}{x(x-5)}$$

# FIRST DERIVATIVE

Add complexity: Increasing/decreasing, critical and singular points.

$$f(x) = \frac{1}{2}x^4 - \frac{4}{3}x^3 - 15x^2$$

What does the graph of the following function look like?

$$f(x) = \frac{1}{3}x^3 + 2x^2 + 4x + 24$$

What does the graph of the following function look like?

$$f(x) = e^{\frac{x+1}{x-1}}$$



# SIGNS OF FACTORED FUNCTIONS

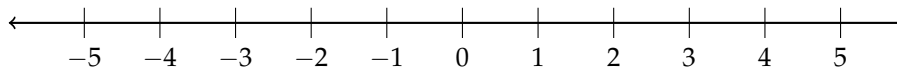
$$f(x) = (x - 1) (x - 2)^2 (x - 3)$$



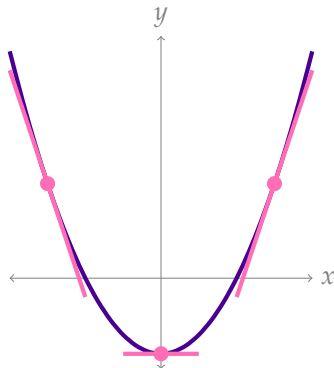
# SIGNS OF FACTORED FUNCTIONS

$$f(x) = (x - 3)(x - 1)^2x(x + 2)^3(x + 5)^4$$

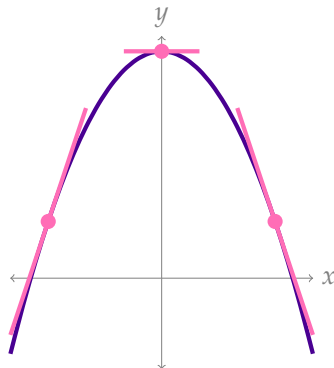
Where is  $f(x)$  positive? Where is it negative?



# CONCAVITY



- ▶ Slopes are increasing
- ▶  $f''(x) > 0$
- ▶ “concave up”
- ▶ tangent line below curve

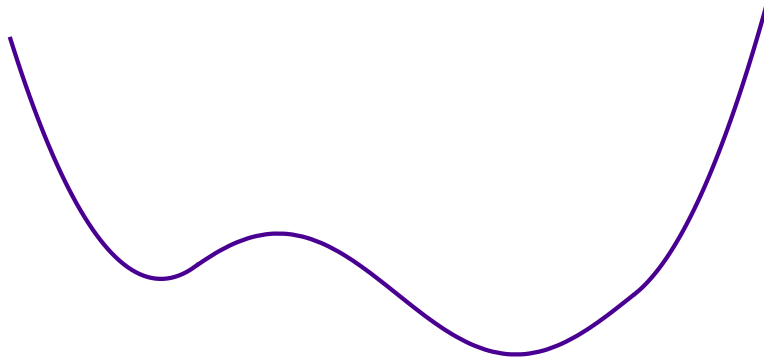


- ▶ Slopes are decreasing
- ▶  $f''(x) < 0$
- ▶ “concave down”
- ▶ tangent line above curve

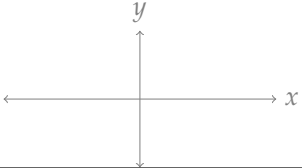
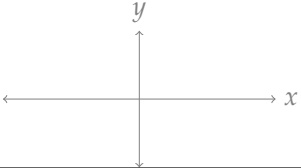
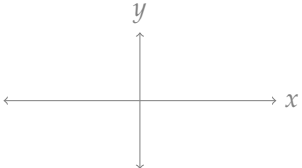
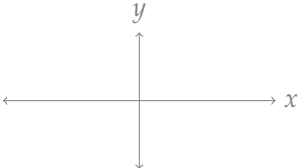
# MNEMONIC



# CONCAVITY



Sketch graphs with the following properties, or explain that none exist.

	concave up	concave down
increasing		
decreasing		

# POLL QUESTIONS

Describe the concavity of the function  $f(x) = e^x$ .

- A. concave up
- B. concave down
- C. concave up for  $x < 0$ ; concave down for  $x > 0$
- D. concave down for  $x < 0$ ; concave up for  $x > 0$
- E. I'm not sure

Is it possible to be concave up and decreasing?

- A. Yes
- B. No
- C. I'm not sure

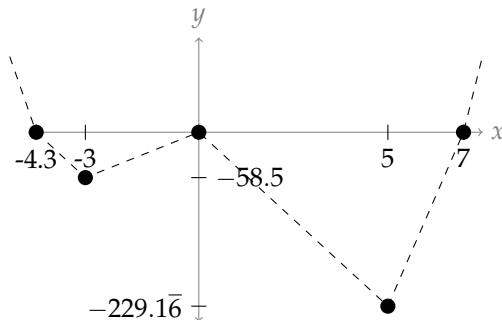
Suppose a function  $f(x)$  is defined for all real numbers, and is concave up on the interval  $[0, 1]$ . Which of the following must be true?

- A.  $f'(0) < f'(1)$
- B.  $f'(0) > f'(1)$
- C.  $f'(0)$  is positive
- D.  $f'(0)$  is negative
- E. I'm not sure

# REVISITING A PREVIOUS EXAMPLE

[◀ original example](#)

$$f(x) = \frac{1}{2}x^4 - \frac{4}{3}x^3 - 15x^2$$



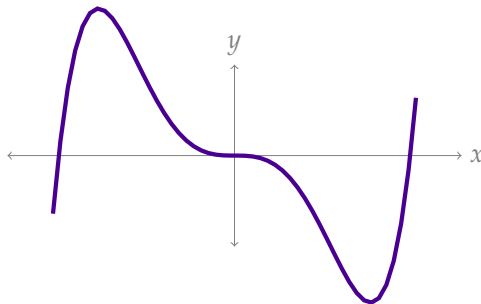
$$f''(x) = 6x^2 - 8x - 30 = 2(x - 3)(3x + 5)$$



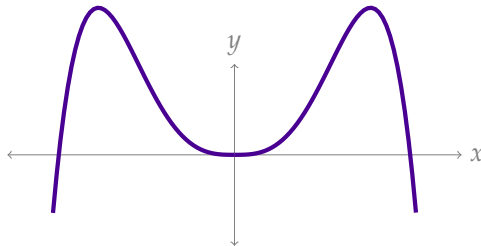
Sketch:

$$f(x) = x^5 - 15x^3$$

# EVEN AND ODD FUNCTIONS



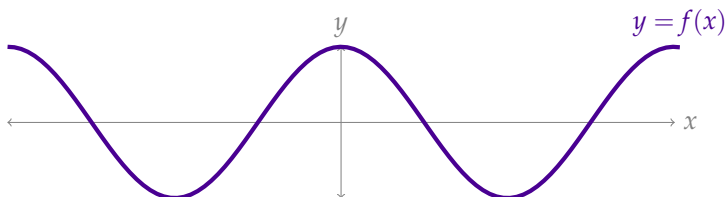
# EVEN AND ODD FUNCTIONS



## Even Function – Definition 3.6.6

A function  $f(x)$  is **even** if, for all  $x$  in its domain,

$$f(-x) = f(x)$$



# EVEN FUNCTIONS

## Even Function – Definition 3.6.6

A function  $f(x)$  is **even** if, for all  $x$  in its domain,

$$f(-x) = f(x)$$

Examples:

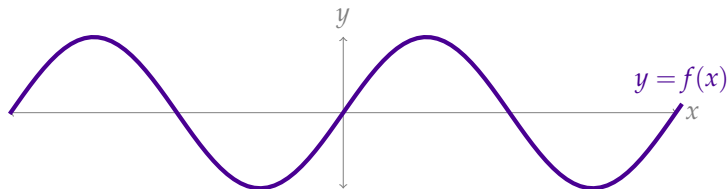
$$f(x) = x^2$$

$$f(x) = x^4$$

$$f(x) = \cos(x)$$

$$f(x) = \frac{x^4 + \cos(x)}{x^{16} + 7}$$

# ODD FUNCTIONS



Suppose  $f(1) = 2$ . Then  $f(-1) =$

Suppose  $f(3) = -2$ . Then  $f(-3) =$

## Odd Function – Definition 3.6.7

A function  $f(x)$  is **odd** if, for all  $x$  in its domain,

$$f(-x) = -f(x)$$

# ODD FUNCTIONS

## Odd Function – Definition 3.6.7

A function  $f(x)$  is **odd** if, for all  $x$  in its domain,

$$f(-x) = -f(x)$$

Examples:

$$f(x) = x$$

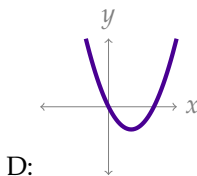
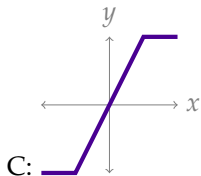
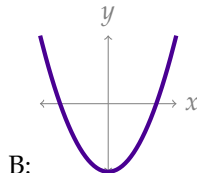
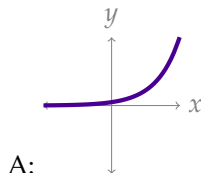
$$f(x) = x^3$$

$$f(x) = \sin(x)$$

$$f(x) = \frac{x(1 + x^2)}{x^2 + 5}$$

# POLL TIME

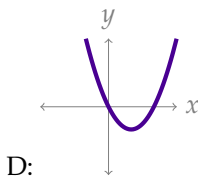
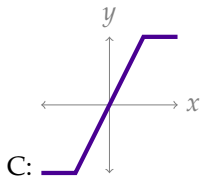
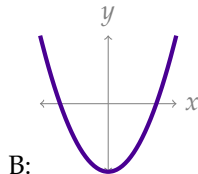
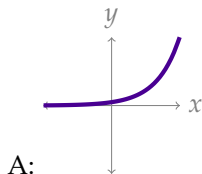
Pick out the **odd** function.





# POLL TIME

Pick out the **even** function.



# EVEN MORE POLL TIIIIIME

Suppose  $f(x)$  is an **odd** function, continuous, defined for all real numbers. What is  $f(0)$ ? Pick the best answer.

- A.  $f(0) = f(-0)$
- B.  $f(0) = -f(0)$
- C.  $f(0) = 0$
- D. all of the above are true
- E. none of the above are necessarily true

# EVEN MORE AND MORE POLL TIIIIIME

Suppose  $f(x)$  is an **even** function, continuous, defined for all real numbers. What is  $f(0)$ ? Pick the best answer.

- A.  $f(0) = f(-0)$
- B.  $f(0) = -f(0)$
- C.  $f(0) = 0$
- D. all of the above are true
- E. none of the above are necessarily true

# OK OK... LAST ONE

Suppose  $f(x)$  is an **even** function, differentiable for all real numbers.  
What can we say about  $f'(x)$ ?

- A.  $f'(x)$  is also even
- B.  $f'(x)$  is odd
- C.  $f'(x)$  is constant
- D. all of the above are true
- E. none of the above are necessarily true

# PERIODICITY

## Periodic – Definition 3.6.10

A function is **periodic** with period  $P > 0$  if

$$f(x) = f(x + P)$$

whenever  $x$  and  $x + P$  are in the domain of  $f$ , and  $P$  is the smallest such (positive) number

Examples:  $\sin(x)$ ,  $\cos(x)$  both have period  $2\pi$ ;  $\tan(x)$  has period  $\pi$ .

Ignoring concavity, sketch  $f(x) = \sin(\sin x)$ .

Challenge: ignoring exact locations of extrema, sketch  
 $g(x) = \sin(2\pi \sin x)$ .

# LET'S GRAPH

$$f(x) = (x^2 - 64)^{1/3}$$

$$f'(x) = \frac{2x}{3(x^2 - 64)^{2/3}};$$

$$f''(x) = \frac{-2(\frac{1}{3}x^2 + 64)}{3(x^2 - 64)^{5/3}}$$

# LET'S GRAPH

$$f(x) = \frac{x^2 + x}{(x + 1)(x^2 + 1)^2}$$

Note: for  $x \neq -1$ ,  $f(x) = \frac{x(x + 1)}{(x + 1)(x^2 + 1)^2} = \frac{x}{(x^2 + 1)^2}$

$$g(x) := \frac{x}{(x^2 + 1)^2}$$

$$g'(x) = \frac{1 - 3x^2}{(x^2 + 1)^3}$$

$$g''(x) = \frac{12x(x^2 - 1)}{(x^2 + 1)^4}$$



# LET'S GRAPH

$$f(x) = x(x-1)^{2/3}$$

- $f'(x) = \frac{5x-3}{3\sqrt[3]{x-1}}$
- $f''(x) = \frac{2(5x-6)}{9(\sqrt[3]{x-1})^4}$

►  $f(3/5) \approx 0.3$

►  $f(6/5) \approx 0.4$

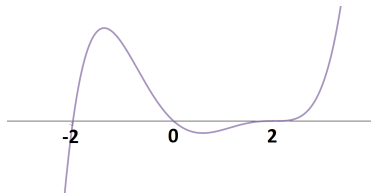
## Ch 3.6 Review: matching

# MATCH THE FUNCTION TO ITS GRAPH

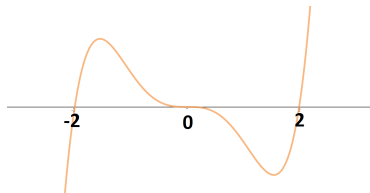
A.  $f(x) = x^3(x+2)(x-2) = x^5 - 4x^3$

B.  $f(x) = x(x+2)^3(x-2) = x^5 + 4x^4 - 16x^2 - 16x$

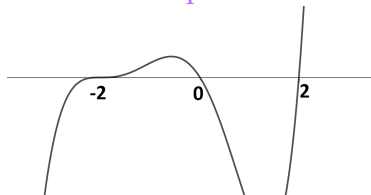
C.  $f(x) = x(x+2)(x-2)^3 = x^5 - 4x^4 + 16x^2 - 16x$



I



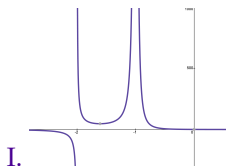
III



II

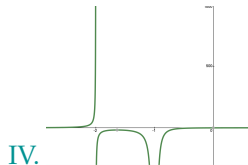
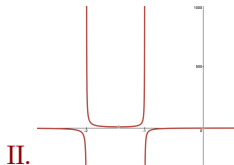
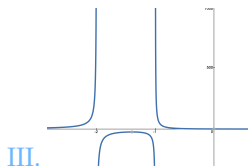
A.  $f(x) = \frac{x-1}{(x+1)(x+2)}$

B.  $f(x) = \frac{(x-1)^2}{(x+1)(x+2)}$



C.  $f(x) = \frac{x-1}{(x+1)^2(x+2)}$

D.  $f(x) = \frac{(x-1)^2}{(x+1)^2(x+2)}$



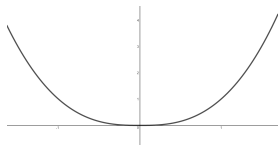
# MATCH THE FUNCTION TO ITS GRAPH

A.  $f(x) = |x|^e$

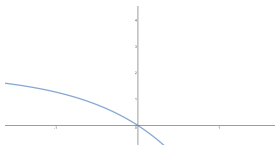
B.  $f(x) = e^{|x|}$

C.  $f(x) = e^{x^2}$

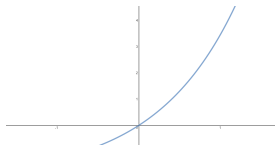
D.  $f(x) = e^{x^4-x}$



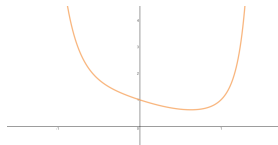
I



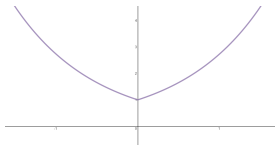
II



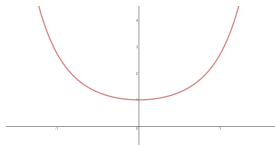
III



IV



V



VI

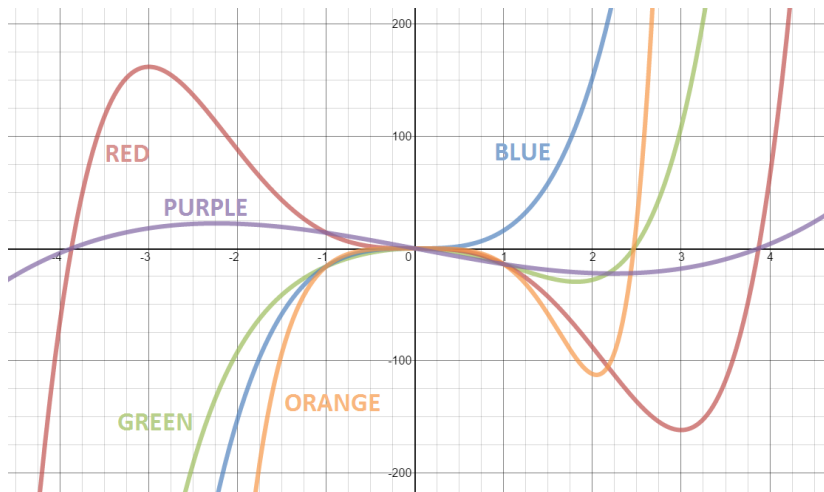
A.  $f(x) = x^5 + 15x^3$

B.  $f(x) = x^5 - 15x^3$

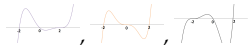
C.  $f(x) = x^5 - 15x^2$

D.  $f(x) = x^3 - 15x$

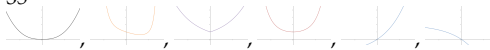
E.  $f(x) = x^7 - 15x^4$



## Included Work



screenshots of graphs generated using Desmos Graphing Calculator <https://www.desmos.com/calculator> (accessed 13 November 2015), 35



screenshots of graphs generated using Desmos Graphing Calculator <https://www.desmos.com/calculator>, (accessed 13 November 2015), 37



screenshots from graphs generated using Desmos Graphing Calculator <https://www.desmos.com/calculator> (accessed 16 July 2021), 36



screenshot from graphs generated using Desmos Graphing Calculator <https://www.desmos.com/calculator>, with text added (accessed 13 November 2015), 38