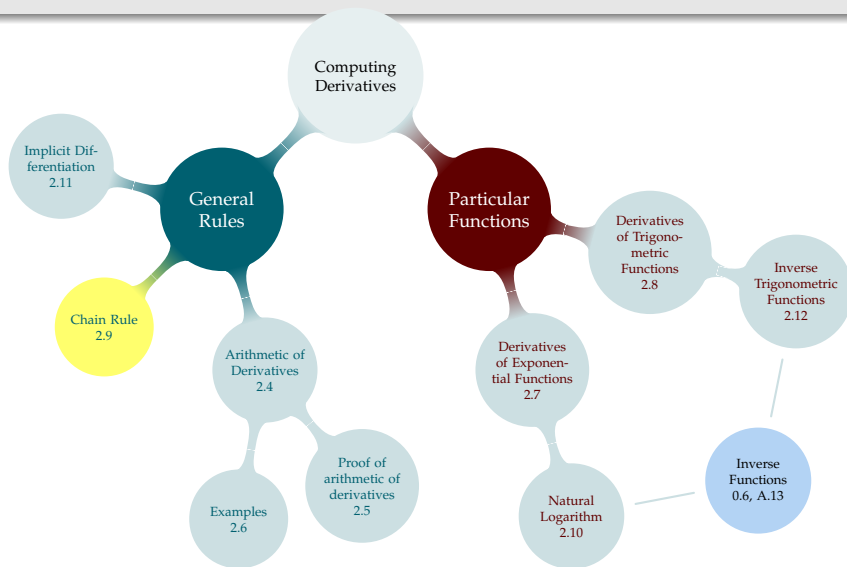
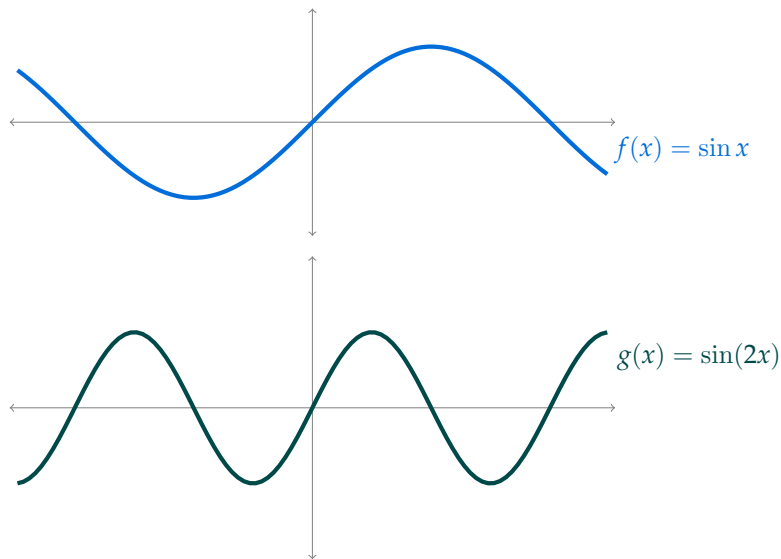


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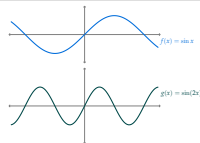


INTUITION: $\sin x$ VERSUS $\sin(2x)$ 

2.9: Chain Rule

Intuition: $\sin x$ versus $\sin(2x)$

INTUITION: $\sin x$ VERSUS $\sin(2x)$



Intuition: $\sin 2x$ changes its y -values “twice as fast” as $\sin x$, making it “twice as steep.” So it’s not enough to differentiate the outside function – something else has to happen.

COMPOUND FUNCTIONS

Video: 2:27-3:50

Morton, Jennifer. (2014). *Balancing Act: Otters, Urchins and Kelp*.
Available from [https://www.kqed.org/quest/67124/
balancing-act-otters-urchins-and-kelp](https://www.kqed.org/quest/67124/balancing-act-otters-urchins-and-kelp)

└ 2.9: Chain Rule

└ Compound Functions

Chain rule works on functions-of-functions; otters/urchins/kelp are a nice example

KELP POPULATION

 k kelp population u urchin population o otter population p public policy

$k(u)$

$k(u(o))$

$k(u(o(p)))$

These are examples of compound functions.

Should $\frac{d}{do}k(u(o))$ be positive or negative?

A. positive

B. negative

C. I'm not sure

Should $k'(u)$ be positive or negative?

A. positive

B. negative

C. I'm not sure

2.9: Chain Rule

Kelp Population

KELP POPULATION

k kelp population
 u undine population
 σ otter population
 p public policy

$k(u)$ $k(u(\sigma))$ $k(u(\sigma(p)))$

These are examples of compound functions.

Should $\frac{dk}{du}(u(\sigma))$ be positive or negative?

A. positive B. negative C. I'm not sure

Should $k'(u)$ be positive or negative?

A. positive B. negative C. I'm not sure

It's nice to show an example of a function whose “derivative” can be two different things (depending on the variable). Now that our heads are in “function of a function” territory, chain rule. I usually just flash this slide to emphasize that all these rules are shorthand for a calculation using the definition of a derivative.

DIFFERENTIATING COMPOUND FUNCTIONS

$$\begin{aligned}
 \frac{d}{dx}\{f(g(x))\} &= \lim_{h \rightarrow 0} \frac{f(g(x+h)) - f(g(x))}{h} \\
 &= \lim_{h \rightarrow 0} \frac{f(g(x+h)) - f(g(x))}{h} \left(\frac{g(x+h) - g(x)}{g(x+h) - g(x)} \right) \\
 &= \lim_{h \rightarrow 0} \frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)} \cdot \frac{g(x+h) - g(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)} \cdot \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{f\left(\boxed{g(x+h)}\right) - f\left(\boxed{g(x)}\right)}{\boxed{g(x+h)} - \boxed{g(x)}} \cdot g'(x)
 \end{aligned}$$

Set $H = g(x+h) - g(x)$. As $h \rightarrow 0$, we also have $H \rightarrow 0$. So

$$\begin{aligned}
 &= \lim_{H \rightarrow 0} \frac{f(g(x) + H) - f(g(x))}{H} \cdot g'(x) \\
 &= f'(g(x)) \cdot g'(x)
 \end{aligned}$$

CHAIN RULE

Chain Rule – Theorem 2.9.3

Suppose f and g are differentiable functions. Then

$$\frac{d}{dx}\{f(g(x))\} = f'(g(x))g'(x) = \frac{df}{dg}(g(x))\frac{dg}{dx}(x)$$

In the case of kelp, $\frac{d}{d\text{o}}k(u(o)) = \frac{dk}{d\text{u}}(u(o))\frac{d\text{u}}{d\text{o}}(o)$

Chain Rule

Suppose f and g are differentiable functions. Then

$$\frac{d}{dx}\{f(g(x))\} = f'(g(x)) g'(x) = \frac{df}{dg}(g(x)) \frac{dg}{dx}(x)$$

Example: suppose $F(x) = \sin(e^x + x^2)$.

2.9: Chain Rule

Chain Rule

Suppose f and g are differentiable functions. Then

$$\frac{d}{dx}\{f(g(x))\} = f'(g(x))g'(x) = \frac{df}{dg}(g(x))\frac{dg}{dx}(x)$$

Example: suppose $F(x) = \sin(e^x + x^2)$.

I generally put the “inside” function in a box, to emphasize we’re treating the whole thing as one variable

$$F(v) = \left(\frac{v}{v^3 + 1} \right)^6$$

NOW
YOU



Let $f(x) = (10^x + \csc x)^{1/2}$. Find $f'(x)$.

NOW
YOU



Suppose $o(t) = e^t$, $u(o) = \frac{1}{o + \sin(o)}$, and $t \geq 10$ (so all

these functions are defined). Using the chain rule, find $\frac{d}{dt} u(o(t))$.

Note: your answer should depend only on t : not o .

Evaluate $\frac{d}{dx} \left\{ x^2 + \sec \left(x^2 + \frac{1}{x} \right) \right\}$

Evaluate $\frac{d}{dx} \left\{ \frac{1}{x + \frac{1}{x + \frac{1}{x}}} \right\}$

Included Work



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