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## Examples

$\frac{d}{dx}[x^2] = 2x$ , so  $x^2$  is an antiderivative of  $2x$ .

$\frac{d}{dx}[x^2 + 5] = 2x$ , so  $x^2 + 5$  is (also) an antiderivative of  $2x$ .

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What is the most general antiderivative of  $2x$ ?

$x^2 + c$ , where we understand  $c$  as some constant (a number not depending on  $x$ ).

# ANTIDERIVATIVES

Find the most general antiderivative for the following equations.

$$f(x) = 17$$

$$f(x) = m$$

where  $m$  is a constant.

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$$17x + c$$

$$f(x) = m$$

where  $m$  is a constant.

$$mx + c$$



differentiation fact

$$\frac{d}{dx}[x^2] = 2x$$

 $\implies$ 

antidifferentiation fact

antideriv of  $2x$  :

$$x^2 + c$$

differentiation fact

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$$\text{antideriv of } 2x : \quad x^2 + c$$

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$$\text{antideriv of } 2x : \quad x^2 + c$$

$$\text{antideriv of } x : \quad \frac{1}{2}x^2 + c$$

$$\text{Check: } \frac{d}{dx} \left[ \frac{1}{2}x^2 + c \right] =$$

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$$\frac{d}{dx}[x^2] = 2x$$

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$$\text{Check: } \frac{d}{dx} \left[ \frac{1}{2}x^2 + c \right] = x$$

$$\frac{d}{dx}[x^3] = 3x^2$$

 $\implies$ 

$$\frac{d}{dx}[x^4] = 4x^3$$

 $\implies$ 

$$\frac{d}{dx}[x^5] = 5x^4$$

 $\implies$ antideriv of  $x^n$ :

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$$\text{antideriv of } x^3: \quad \frac{1}{4}x^4 + c$$

$$\frac{d}{dx}[x^5] = 5x^4$$

 $\implies$ 

$$\text{antideriv of } 5x^4: \quad x^5 + c$$

$$\text{antideriv of } x^4: \quad \frac{1}{5}x^5 + c$$

$$\text{antideriv of } x^n: \quad \frac{1}{n+1}x^{n+1} + c$$

$$\text{Check: } \frac{d}{dx} \left[ \frac{1}{n+1}x^{n+1} + c \right] = x^n$$

## Power Rule for Antidifferentiation

The most general antiderivative of  $x^n$  is  $\frac{1}{n+1}x^{n+1} + c$  if  $n \neq -1$

$$\blacktriangleright \frac{d}{dx} [ \quad ] = x^5$$

$$\blacktriangleright \frac{d}{dx} [ \quad ] = x^3$$

$$\blacktriangleright \frac{d}{dx} [ \quad ] = \frac{1}{2}x^3$$



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$$\blacktriangleright \frac{d}{dx} [ \quad ] = 5x^2 - 15x + 3$$

$$\blacktriangleright \frac{d}{dx} [ \quad ] = 13 \left( 5x^{14} - 3x^{3/7} + 52e^x \right)$$

Find the most general antiderivatives.

$$f(x) = \cos x$$

$$f(x) = \sin x$$

$$f(x) = \sec^2 x$$

$$f(x) = \frac{1}{1+x^2}$$

$$f(x) = \frac{1}{1+x^2+2x}$$

Find the most general antiderivatives.

$$f(x) = \cos x$$

$$\sin x + c$$

$$f(x) = \sin x$$

$$-\cos x + c$$

$$f(x) = \sec^2 x$$

$$\tan x + c$$

$$f(x) = \frac{1}{1+x^2}$$

$$\arctan x + c$$

$$f(x) = \frac{1}{1+x^2+2x}$$

$$\frac{-1}{x+1}$$

Find the most general antiderivatives.

$$f(x) = 17 \cos x + x^5$$

$$f(x) = \frac{23}{5 + 5x^2}$$

$$f(x) = \frac{23}{5 + 125x^2}$$

Find the most general antiderivatives.

$$f(x) = 17 \cos x + x^5$$

$$17 \sin x + \frac{1}{6}x^6 + c$$

$$f(x) = \frac{23}{5 + 5x^2}$$

$$\frac{23}{5} \arctan x + c$$

$$f(x) = \frac{23}{5 + 125x^2}$$

$$\frac{23}{25} \arctan(5x) + c$$

Find the most general antiderivatives.

$$f(x) = \frac{1}{x}, x > 0$$

$$f(x) = 5x^2 - 32x^5 - 17$$

$$f(x) = \csc x \cot x$$

$$f(x) = \frac{5}{\sqrt{1-x^2}} + 17$$

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$$f(x) = \frac{1}{x}, x > 0$$

$$\ln x + c$$

$$f(x) = 5x^2 - 32x^5 - 17$$

$$\frac{5}{3}x^3 - \frac{16}{3}x^6 - 17x + c$$

$$f(x) = \csc x \cot x$$

$$- \csc x + c$$

$$f(x) = \frac{5}{\sqrt{1-x^2}} + 17$$

$$5 \arcsin x + 17x + c$$



## CHOOSE YOUR OWN ADVENTURE

Antiderivative of  $\sin x \cos x$ :

- A.  $\cos x \sin x + c$
- B.  $-\cos x \sin x + c$
- C.  $\sin^2 x + c$
- D.  $\frac{1}{2} \sin^2 x + c$
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In general, antiderivatives of  $x^n$  have the form  $\frac{1}{n+1}x^{n+1}$ . What is the single exception?

- A.  $n = -1$
- B.  $n = 0$
- C.  $n = 1$
- D.  $n = e$
- E.  $n = 1/2$

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# ALL THE ADVENTURES ARE CALCULUS, THOUGH

Suppose the velocity of a particle at time  $t$  is given by  $v(t) = t^2 + \cos t + 3$ . What function gives its position?

A.  $s(t) = 2t - \sin t$

B.  $s(t) = 2t - \sin t + c$

C.  $s(t) = t^3 + \sin t + 3t + c$

D.  $s(t) = \frac{1}{3}t^3 + \sin t + 3t + c$

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Suppose the velocity of a particle at time  $t$  is given by  $v(t) = t^2 + \cos t + 3$ , and its position at time 0 is given by  $s(0) = 5$ . What function gives its position?

- A.  $s(t) = \frac{1}{3}t^3 + \sin t + 3t$
- B.  $s(t) = \frac{1}{3}t^3 + \sin t + 3t + 5$
- C.  $s(t) = \frac{1}{3}t^3 + \sin t + 3t + c$
- D.  $s(t) = 5t + c$
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Find all functions  $f(x)$  with  $f(1) = 5$  and  $f'(x) = e^{3x+5}$ .



Find all functions  $f(x)$  with  $f(1) = 5$  and  $f'(x) = e^{3x+5}$ .

Antiderivative of  $e^{3x+5}$  is  $\frac{1}{3}e^{3x+5} + c$ . So we only need to solve for  $c$ .

$$5 = f(1) = \frac{1}{3}e^{3+5} + c$$

implies

$$c = 5 - \frac{e^8}{3}$$

So

$$f(x) = \frac{1}{3}e^{3x+5} + 5 - \frac{e^8}{3}$$



Let  $Q(t)$  be the amount of a radioactive isotope in a sample. Suppose the sample is losing  $50e^{-5t}$  mg per second to decay. If  $Q(1) = 10e^{-5}$  mg, find the equation for the amount of the isotope at time  $t$ .



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We have

$$\frac{dQ}{dt} = -50e^{-5t}$$

Note the negative, since our sample is getting smaller.

Then antidifferentiating, we find  $Q(t) = 10e^{-5t} + c$  for some constant  $c$ .

Then since  $Q(1) = 10e^{-5}$ , we see  $c = 0$ .

$$Q(t) = 10e^{-5t}$$



Suppose  $f'(t) = 2t + 7$ . What is  $f(10) - f(3)$ ?

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From antidifferentiation, we have  $f(t) = t^2 + 7t + c$ . Then

$$f(10) - f(3) = [100 + 70 + c] - [9 + 21 + c] = 170 - 30 = 140$$

