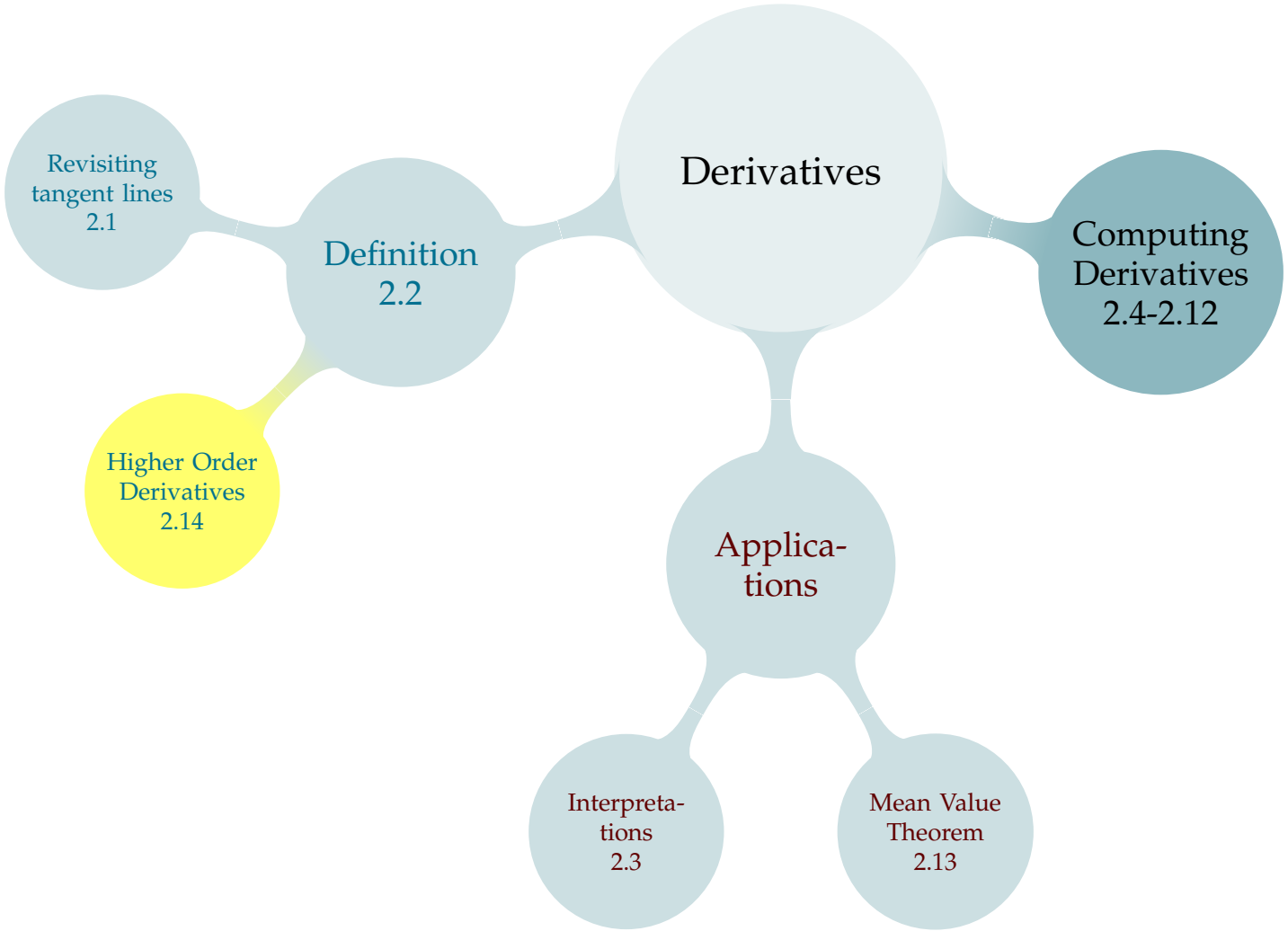
Table of Contents



Higher Order Derivatives

Evaluate $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\mathrm{d}}{\mathrm{d}x}[x^5 - 2x^2 + 3]\right]$

$$\frac{\mathrm{d}}{\mathrm{d}x}[x^5 - 2x^2 + 3] =$$

Notation 2.14.1

The derivative of a derivative is called the **second derivative**, written

$$f''(x) \qquad \text{or} \qquad \frac{\mathrm{d}^2 y}{\mathrm{d}x^2}(x)$$

Similarly, the derivative of a second derivative is a third derivative, etc.

Notation 2.14.1

- $f''(x)$ and $f^{(2)}(x)$ and $\frac{\mathrm{d}^2 f}{\mathrm{d}x^2}(x)$ all mean $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}}{\mathrm{d}x}f(x)\right)$
- $f'''(x)$ and $f^{(3)}(x)$ and $\frac{\mathrm{d}^3 f}{\mathrm{d}x^3}(x)$ all mean $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}}{\mathrm{d}x}f(x)\right)\right)$
- $f^{(4)}(x)$ and $\frac{\mathrm{d}^4 f}{\mathrm{d}x^4}(x)$ both mean $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}}{\mathrm{d}x}f(x)\right)\right)\right)$
- and so on.

Typical Example: Acceleration

- Velocity: rate of change of position
- Acceleration: rate of change of velocity.

The position of an object at time t is given by $s(t) = t(5 - t)$. *Time is measured in seconds, and position is measured in metres.*

1. Sketch the graph giving the position of the object.
2. What is the velocity of the object when $t = 1$? Include units.
3. What is the acceleration of the object when $t = 1$? Include units.

Concept Check

True or False: If $f'(1) = 18$, then $f''(1) = 0$, since the $\frac{d}{dx}\{18\} = 0$.

Which of the following is always true of a QUADRATIC polynomial $f(x)$?

A. $f(0) = 0$
B. $f'(0) = 0$
C. $f''(0) = 0$
D. $f'''(0) = 0$
E. $f^{(4)}(0) = 0$

Which of the following is always true of a CUBIC polynomial $f(x)$?

A. $f(0) = 0$
B. $f'(0) = 0$
C. $f''(0) = 0$
D. $f'''(0) = 0$
E. $f^{(4)}(0) = 0$

Implicit Differentiation

Suppose $y(x)$ is a function such that

$$y(x) = y^3x + x^2 - 1$$

Find $y''(x)$ at the point $(-2, 1)$.

Included Work