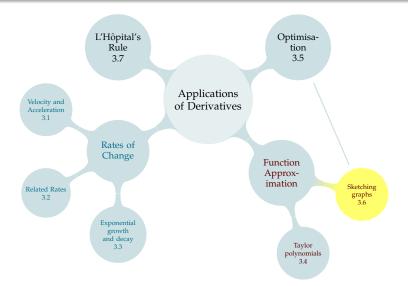
TABLE OF CONTENTS



—Table of Contents



It can be very frustrating to learn how to solve a problem only partially, but this is what we do in this section. There are many pieces to curve sketching, and by focusing on one at a time, we teach students how to start sketching before we teach them how to finish sketching. It can be helpful in class to remind students that the remaining pieces are going to be taught soon.

CURVE SKETCHING

3.6.1: Domain, Intercepts, Asymptotes

•000

Review: find the domain of the following function.

$$f(x) = \frac{\sqrt{3 - x^2}}{\log(x + 1)}$$

Where might you expect f(x) to have a vertical asymptote? What does the function look like nearby? (Recall: a vertical asymptote occurs at x = a if the function has an infinite discontinuity at a. That is, $\lim_{x\to a^{\pm}} f(x) = \pm \infty$.)

Where is
$$f(x) = 0$$
?

What happens to f(x) near its other endpoint, x = -1?

Good things to check:

- Domain
- Vertical asymptotes: $\lim_{x \to a} f(x) = \pm \infty$
- Intercepts: x = 0, f(x) = 0
- Horizontal asymptotes and end behavior: $\lim_{x \to \pm \infty} f(x)$

0000

Identify: domain, vertical asymptotes, intercepts, and horizontal asymptotes

$$f(x) = \frac{x-2}{(x+3)^2}$$

-Curve Sketching

Graphing takes many steps, and it's often frustrating to see a final result without knowing how to get there. So when I show graphs that we haven't generated in class, I like to add information like: we will learn how to do this part later; look at how the intercepts are where we expect; look at the horizontal asymptotes that we predicted; etc.

0000

Identify: domain, vertical asymptotes, intercepts, and horizontal asymptotes

$$f(x) = \frac{(x+2)(x-3)^2}{x(x-5)}$$

Add complexity: Increasing/decreasing, critical and singular points.

$$f(x) = \frac{1}{2}x^4 - \frac{4}{3}x^3 - 15x^2$$

What does the graph of the following function look like?

$$f(x) = \frac{1}{3}x^3 + 2x^2 + 4x + 24$$

What does the graph of the following function look like?

$$f(x) = e^{\frac{x+1}{x-1}}$$

SIGNS OF FACTORED FUNCTIONS

3.6.1: Domain, Intercepts, Asymptotes

$$f(x) = (x-1) (x-2)^2 (x-3)$$



Signs of Factored Functions

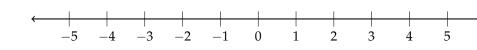
-3.6.2: First Derivative

Signs of Factored Functions $f(x) = -(x-1) - (x-2)^2 - (x-3)$

Students often learned in high school that they should test points between consecutive roots, which is much slower.

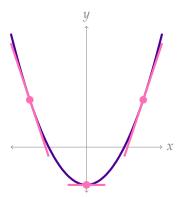
$$f(x) = (x-3)(x-1)^2x(x+2)^3(x+5)^4$$

Where is f(x) positive? Where is it negative?

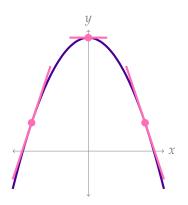


CONCAVITY

3.6.1: Domain, Intercepts, Asymptotes



- ► Slopes are increasing
- ► f''(x) > 0
- ► "concave up"
- ► tangent line below curve



- ► Slopes are decreasing
- ► f''(x) < 0
- ► "concave down"
- ► tangent line above curve

MNEMONIC





CONCAVITY

3.6.1: Domain, Intercepts, Asymptotes





__Concavity

-3.6.3: Concavity



"Just by staring at it, decide where this function is concave up, and where it is concave down."

	concave up	concave down
increasing	$ \begin{array}{c} y \\ \uparrow \\ \downarrow \\ \end{array} $	$ \begin{array}{c} y \\ \uparrow \\ \downarrow \\ \end{array} $
decreasing	$\longleftrightarrow x$	$\begin{matrix} y \\ \uparrow \\ \downarrow \end{matrix} \rightarrow x$

It is a very common misconception that e.g.	concave up functions are also		
increasing. Be wary of conflating first and second derivatives.			

Describe the concavity of the function $f(x) = e^x$.

- A. concave up
- B. concave down
- C. concave up for x < 0; concave down for x > 0
- D. concave down for x < 0; concave up for x > 0
- E. I'm not sure

Is it possible to be concave up and decreasing?

A. Yes

B. No

C. I'm not sure

Suppose a function f(x) is defined for all real numbers, and is concave up on the interval [0,1]. Which of the following must be true?

- A. f'(0) < f'(1)
- B. f'(0) > f'(1)
- C. f'(0) is positive
- D. f'(0) is negative
- E. I'm not sure

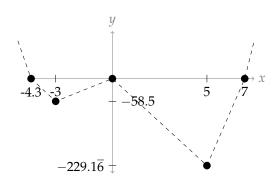
REVISITING A PREVIOUS EXAMPLE

◆ original example

$$f(x) = \frac{1}{2}x^4 - \frac{4}{3}x^3 - 15x^2$$

3.6.3: Concavity

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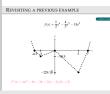


$$f''(x) = 6x^2 - 8x - 30 = 2(x - 3)(3x + 5)$$

3.6.1: Domain, Intercepts, Asymptotes

-3.6.3: Concavity

Revisiting a previous example



Rather than start from scratch, here's an example we already have a skeleton for. The second derivative factors nicely, so we can quickly see how to add concavity to our sketches.

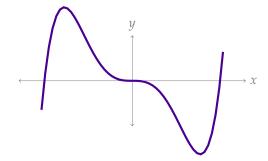
Sketch:

$$f(x) = x^5 - 15x^3$$

Mention symmetry, to motivate next subsection.

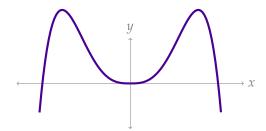
EVEN AND ODD FUNCTIONS

3.6.1: Domain, Intercepts, Asymptotes



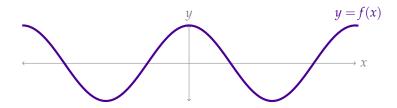
EVEN AND ODD FUNCTIONS

3.6.1: Domain, Intercepts, Asymptotes



A function f(x) is even if, for all x in its domain,

$$f(-x) = f(x)$$



Even Function – Definition 3.6.6

A function f(x) is even if, for all x in its domain,

$$f(-x) = f(x)$$

Examples:

$$f(x) = x^2$$

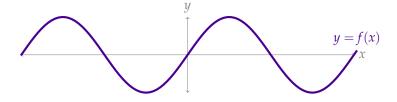
$$f(x) = x^4$$

$$f(x) = \cos(x)$$

$$f(x) = \cos(x) f(x) = \frac{x^4 + \cos(x)}{x^{16} + 7}$$

ODD FUNCTIONS

3.6.1: Domain, Intercepts, Asymptotes



Suppose
$$f(1) = 2$$
. Then $f(-1) =$
Suppose $f(3) = -2$. Then $f(-3) =$

Odd Function – Definition 3.6.7

A function f(x) is odd if, for all x in its domain,

$$f(-x) = -f(x)$$

Odd Function – Definition 3.6.7

A function f(x) is odd if, for all x in its domain,

$$f(-x) = -f(x)$$

Examples:

$$f(x) = x$$

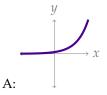
$$f(x) = x^3$$

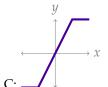
$$f(x) = \sin(x)$$

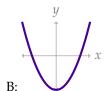
$$f(x) = \sin(x)$$
$$f(x) = \frac{x(1+x^2)}{x^2+5}$$

3.6.1: Domain, Intercepts, Asymptotes POLL TIIIME

Pick out the odd function.





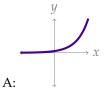


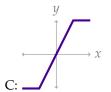


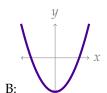
POLL TIIIME

3.6.1: Domain, Intercepts, Asymptotes

Pick out the even function.









EVEN MORE POLL TIIIIME

Suppose f(x) is an odd function, continuous, defined for all real numbers. What is f(0)? Pick the best answer.

A.
$$f(0) = f(-0)$$

B.
$$f(0) = -f(0)$$

C.
$$f(0) = 0$$

D. all of the above are true

E. none of the above are necessarily true

EVEN MORE AND MORE POLL TIIIIIME

Suppose f(x) is an even function, continuous, defined for all real numbers. What is f(0)? Pick the best answer.

A.
$$f(0) = f(-0)$$

B.
$$f(0) = -f(0)$$

C.
$$f(0) = 0$$

D. all of the above are true

E. none of the above are necessarily true

OK OK... LAST ONE

3.6.1: Domain, Intercepts, Asymptotes

Suppose f(x) is an even function, differentiable for all real numbers. What can we say about f'(x)?

- A. f'(x) is also even
- B. f'(x) is odd
- C. f'(x) is constant
- D. all of the above are true
- E. none of the above are necessarily true

PERIODICITY

Periodic – Definition 3.6.10

A function is periodic with period P > 0 if

$$f(x) = f(x + P)$$

whenever x and x + P are in the domain of f, and P is the smallest such (positive) number

Examples: $\sin(x)$, $\cos(x)$ both have period 2π ; $\tan(x)$ has period π .

Ignoring concavity, sketch $f(x) = \sin(\sin x)$.

3.6.1: Domain, Intercepts, Asymptotes

Challenge: ignoring exact locations of extrema, sketch $g(x) = \sin(2\pi \sin x)$.

The second one really is quite tough; you can let the faster students work on it while the rest of the class is working on f(x). Then the solution is more-or-less in the slides if they want to look, with no need to do it in class.

LET'S GRAPH

3.6.1: Domain, Intercepts, Asymptotes

$$f(x) = (x^2 - 64)^{1/3}$$

3.6.3: Concavity

$$f'(x) = \frac{2x}{3(x^2 - 64)^{2/3}};$$

$$f''(x) = \frac{-2(\frac{1}{3}x^2 + 64)}{3(x^2 - 64)^{5/3}}$$

Takes too much time in class to differentiate, so derivatives given

$$f(x) = \frac{x^2 + x}{(x+1)(x^2+1)^2}$$

Note: for
$$x \neq -1$$
, $f(x) = \frac{x(x+1)}{(x+1)(x^2+1)^2} = \frac{x}{(x^2+1)^2}$
$$g(x) := \frac{x}{(x^2+1)^2}$$

$$g'(x) = \frac{1 - 3x^2}{(x^2 + 1)^3}$$
$$g''(x) = \frac{12x(x^2 - 1)}{(x^2 + 1)^4}$$

LET'S GRAPH

3.6.1: Domain, Intercepts, Asymptotes

$$f(x) = x(x-1)^{2/3}$$

$$f'(x) = \frac{5x - 3}{3\sqrt[3]{x - 1}}$$

•
$$f'(x) = \frac{5x - 3}{3\sqrt[3]{x - 1}}$$

• $f''(x) = \frac{2(5x - 6)}{9(\sqrt[3]{x - 1})^4}$

- ► $f(3/5) \approx 0.3$
- ► $f(6/5) \approx 0.4$

Ch 3.6 Review: matching

3.6.1: Domain, Intercepts, Asymptotes

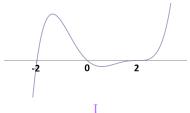
-3.6.5: A checklist for sketching

Matching (as opposed to sketching) is a nice way to review specific ideas, like when a function changes signs.

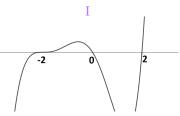
MATCH THE FUNCTION TO ITS GRAPH

A.
$$f(x) = x^3(x+2)(x-2) = x^5 - 4x^3$$

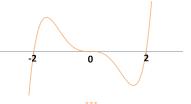
B. $f(x) = x(x+2)^3(x-2) = x^5 + 4x^4 - 16x^2 - 16x$
C. $f(x) = x(x+2)(x-2)^3 = x^5 - 4x^4 + 16x^2 - 16x$



3.6.1: Domain, Intercepts, Asymptotes



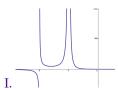
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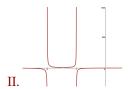


A.
$$f(x) = \frac{x-1}{(x+1)(x+2)}$$

B. $f(x) = \frac{(x-1)^2}{(x+1)(x+2)}$

B.
$$f(x) = \frac{(x-1)^2}{(x+1)(x+2)}$$



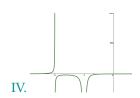


C.
$$f(x) = \frac{x-1}{(x+1)^2(x+2)}$$

C.
$$f(x) = \frac{x-1}{(x+1)^2(x+2)}$$

D. $f(x) = \frac{(x-1)^2}{(x+1)^2(x+2)}$





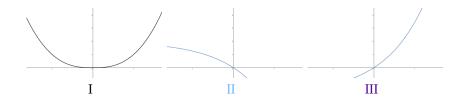
MATCH THE FUNCTION TO ITS GRAPH

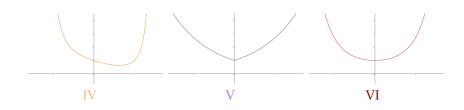
A.
$$f(x) = |x|^e$$

$$B. f(x) = e^{|x|}$$

$$C. f(x) = e^{x^2}$$

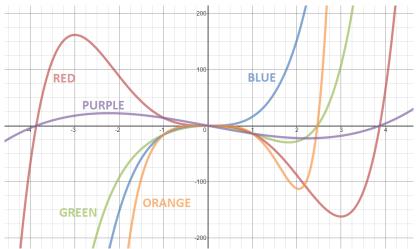
$$D. f(x) = e^{x^4 - x}$$





3.6.1: Domain, Intercepts, Asymptotes

A.
$$f(x) = x^5 + 15x^3$$
 B. $f(x) = x^5 - 15x^3$ C. $f(x) = x^5 - 15x^2$ D. $f(x) = x^3 - 15x$ E. $f(x) = x^7 - 15x^4$



Included Work

