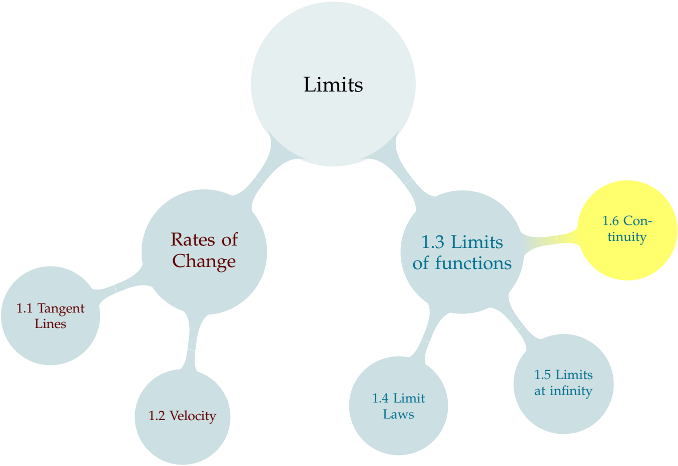
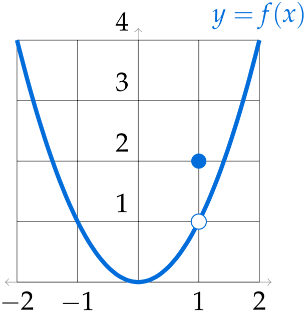
Table of Contents



Continuity

Definition 1.6.1

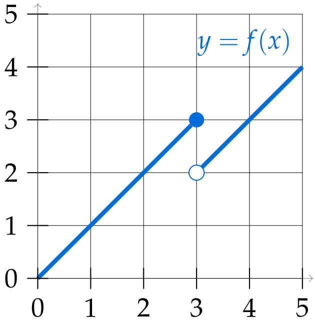
A function $f(x)$ is continuous at a point a if $\lim_{x\to a} f(x)$ exists AND is equal to $f(a)$.



Does $f(x)$ exist at $x = 1$?
Is $f(x)$ continuous at $x = 1$?

Definitions 1.6.1 and 1.6.2

A function $f(x)$ is continuous from the left at a point a if $\lim\limits_{x\to a^-} f(x)$ exists AND is equal to $f(a)$.



Is $f(x)$ continuous at $x = 3$?

Is $f(x)$ continuous from the left at $x = 3$?

Is $f(x)$ continuous from the right at $x = 3$?

Definitions 1.6.1 and 1.6.2

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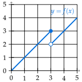


Is $f(x)$ continuous at $x = 3$?

Is $f(x)$ continuous from the left at $x = 3$?

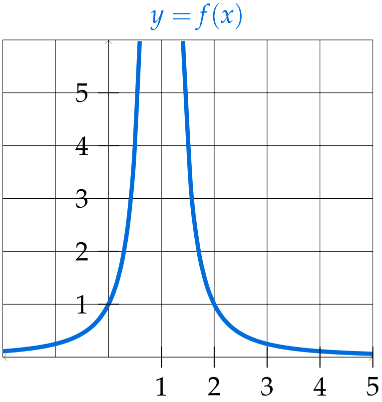
Is $f(x)$ continuous from the right at $x = 3$?

Writing down strict def of jump discontinuity doesn't seem super helpful. Can still say it's when the limit from the left and right both exist but don't match. Also, rather than writing down def of "cont from right," can use pen to change the def of "from the left".

Good time to emphasize limits as expectations. From the right, you *expect* to hit $y = 2$, but you don't, so it's discontinuous. From the right, you *expect* to hit $y = 3$, *and you do*, so it's continuous.

Definition

A function $f(x)$ is continuous at a point a if $\lim_{x \to a} f(x)$ exists AND is equal to $f(a)$.



Definition

A function $f(x)$ is continuous at a point a if $\lim_{x\to a} f(x)$ exists AND is equal to $f(a)$.

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$$

Is $f(x)$ continuous at 0?

Definition

A function $f(x)$ is continuous at a point a if $\lim_{x \to a} f(x)$ exists AND is equal to $f(a)$.

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$$

Is $f(x)$ continuous at 0?

"Sometimes, especially without a graph, it can be very difficult to tell. Remember that we already sketched this example. If we hadn't, answering would be tough.

Continuous Functions

Functions made by adding, subtracting, multiplying, dividing, and taking appropriate powers of polynomials are continuous for every point in their domain.

$$f(x) = \frac{x^2}{2x-10} - \left(\frac{x^2+2x-1}{x-1} + \frac{\sqrt[5]{25-x} - \frac{1}{x}}{x+2} \right)^{1/3}$$

A continuous function is continuous for every point in $\mathbb{R}$.

We say $f(x)$ is continuous over (a, b) if it is continuous at every point in (a, b).

Common Functions – Theorem 1.6.8

Functions of the following types are continuous over their domains:

- polynomials and rationals
- roots and powers
- trig functions and their inverses
- exponential and logarithm
- The products, sums, differences, quotients, powers, and compositions of continuous functions

Where is the following function continuous?

$$f(x) = \left(\frac{\sin x}{(x-2)(x+3)} + e^{\sqrt{x}} \right)^3$$

A Technical Point

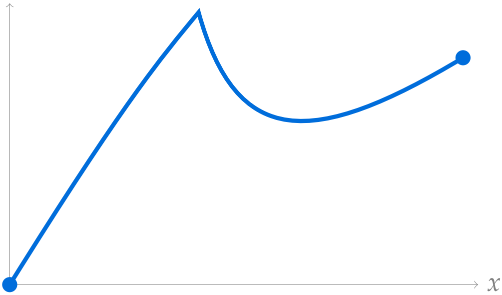
Definition 1.6.3

A function $f(x)$ is continuous on the closed interval $[a, b]$ if:

- $f(x)$ is continuous over (a, b), and
- $f(x)$ is continuous from the left at b, and
- $f(x)$ is continuous from the right at a

Intermediate Value Theorem (IVT) – Theorem 1.6.12

Let $a < b$ and let $f(x)$ be continuous over $[a, b]$. If y is any number between $f(a)$ and $f(b)$, then there exists c in (a, b) such that $f(c) = y$.



Intermediate Value Theorem (IVT) – Theorem 1.6.12

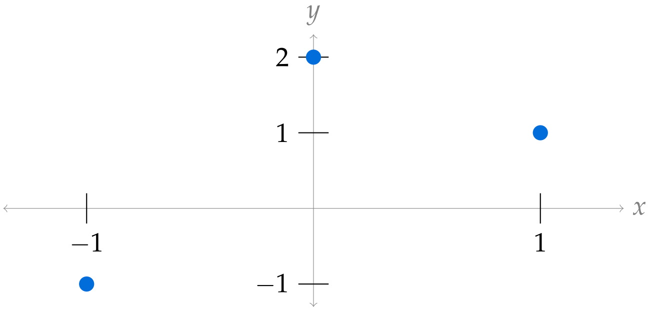
Let $a < b$ and let $f(x)$ be continuous over $[a, b]$. If y is any number between $f(a)$ and $f(b)$, then there exists c in (a, b) such that $f(c) = y$.

Suppose your favourite number is 45.54. At noon, your car is parked, and at 1pm you're driving 100kph.

Using IVT to Find Roots: "Bisection Method"

Let $f(x) = x^5 - 2x^4 + 2$. Find any value x for which $f(x) = 0$. Let's find some points:

$f(0) = 2$ $f(1) = 1$ $f(-1) = -1$



Using IVT to Find Roots: "Bisection Method"

Let $f(x) = x^5 - 2x^4 + 2$. Find any value x for which $f(x) = 0$.

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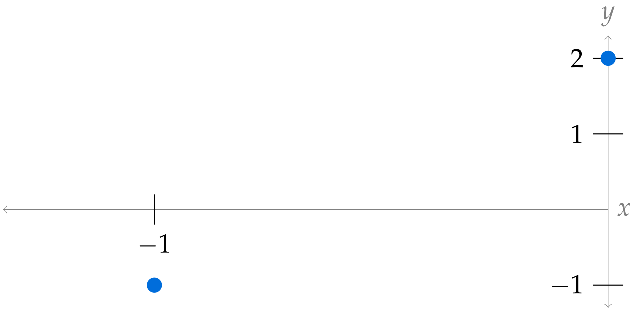

Use the Intermediate Value Theorem to show that there exists some solution to the equation $\ln x \cdot e^x = 4$, and give a reasonable interval where that solution might occur.

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Often doing this once isn't enough for it to stick, hence the second student-work problem

NOW YOU Use the Intermediate Value Theorem to give a reasonable interval where the following is true: $e^x = \sin(x)$. (Don't use a calculator – use numbers you can easily evaluate.)

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Now You Is there any value of x so that $\sin x = \cos(2x) + \frac{1}{4}$?

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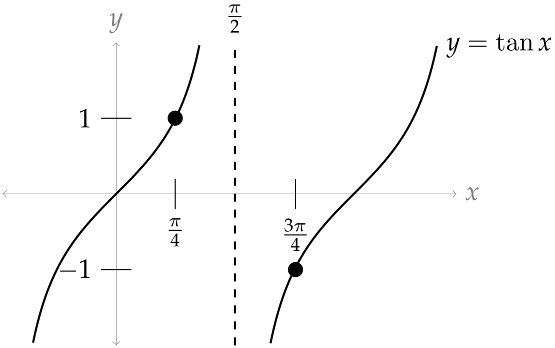
Now You

Is the following reasoning correct?

- $f(x) = \tan x$ is continuous over its domain, because it is a trigonometric function.
- In particular, $f(x)$ is continuous over the interval $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$.
- $f\left(\frac{\pi}{4}\right) = 1$, and $f\left(\frac{3\pi}{4}\right) = -1$.
- Since $f\left(\frac{3\pi}{4}\right) < 0 < f\left(\frac{\pi}{4}\right)$, by the Intermediate Value Theorem, there exists some number c in the interval $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ such that $f(c) = 0$.


y
$\frac{\pi}{2}$
$y = \tan x$
1
-1
$\frac{\pi}{4}$
$\frac{3\pi}{4}$
x

Continuity

Section 1.6 Review

Suppose $f(x)$ is continuous at $x = 1$. Does $f(x)$ have to be defined at $x = 1$?

Suppose $f(x)$ is continuous at $x = 1$ and $\lim_{x \to 1^-} f(x) = 30$.

True or false: $\lim_{x \to 1^+} f(x) = 30$.

Suppose $f(x)$ is continuous at $x = 1$ and $f(1) = 22$. What is $\lim_{x \to 1} f(x)$?

Suppose $\lim_{x\to 1} f(x) = 2$. Must it be true that $f(1) = 2$?

$$f(x) = \begin{cases} ax^2 & x \geq 1 \\ 3x & x < 1 \end{cases}$$

For which value(s) of a is $f(x)$ continuous?

$$f(x) = \begin{cases} \frac{\sqrt{3}x+3}{x^2-3} & x \neq \pm\sqrt{3} \\ a & x = \pm\sqrt{3} \end{cases}$$

For which value(s) of a is $f(x)$ continuous at $x = -\sqrt{3}$?

$$f(x) = \begin{cases} \frac{\sqrt{3}x+3}{x^2-3} & x \neq \pm\sqrt{3} \\ a & x = \pm\sqrt{3} \end{cases}$$

For which value(s) of a is $f(x)$ continuous at $x = \sqrt{3}$?

Included Work

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