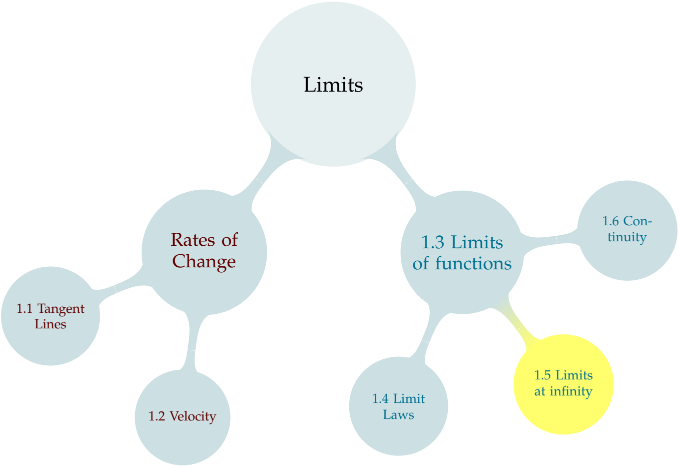
Table of Contents



End Behavior

We write:

$$\lim_{x \to \infty} f(x) = L$$

to express that, as x grows larger and larger, $f(x)$ approaches L.

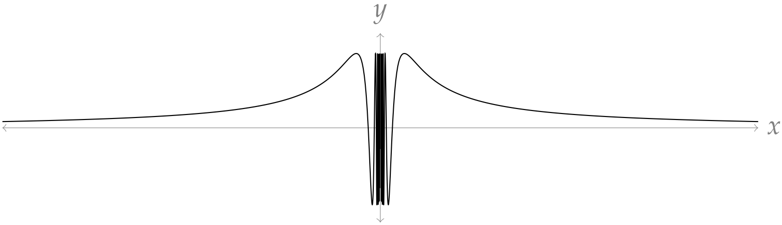
Similarly, we write:

$$\lim_{x \to -\infty} f(x) = L$$

to express that, as x grows more and more strongly negative, $f(x)$ approaches L.

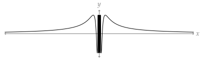
If L is a number, we call $y = L$ a horizontal asymptote of $f(x)$.

Horizontal Asymptotes



$y = 0$ is a horizontal asymptote for $y = \sin\left(\frac{1}{x}\right)$

HORIZONTAL ASYMPTOTES



$y = 0$ is a horizontal asymptote for $y = \sin\left(\frac{1}{x}\right)$

Don't have to spend a long time on these – can click through pretty quickly. Students often think that a HA only occurs when a function gets infinitely close to a value without actually reaching that valueHA doesn't have to be 0, doesn't have to be the same on both sides, can also have one side with HA and one without

Common Limits at Infinity

$$\lim_{x\to\infty} 13 =$$

$$\lim_{x\to-\infty} 13 =$$

$$\lim_{x\to\infty} \frac{1}{x} =$$

$$\lim_{x\to-\infty} \frac{1}{x} =$$

$$\lim_{x\to\infty} x^2 =$$

$$\lim_{x\to-\infty} x^2 =$$

$$\lim_{x\to\infty} x^3 =$$

$$\lim_{x\to-\infty} x^3 =$$

$$\lim_{x\to-\infty} x^{5/3} =$$

$$\lim_{x\to-\infty} x^{2/3} =$$

Arithmetic with Limits at Infinity

$$\lim_{x\to\infty}\left(x+\frac{x^2}{10}\right)=$$

Now You

$$\lim_{x\to\infty}\left(x-\frac{x^2}{10}\right)=$$

$$\lim_{x\to-\infty}\left(x^2+x^3+x^4\right)=$$

$$\lim_{x\to-\infty}(x+13)\left(x^2+13\right)^{1/3}=$$

Arithmetic with Limits at Infinity

Now You

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$$\lim_{x\to-\infty}(x+13)\left(x^2+13\right)^{1/3}=$$

Students often have a hard time treating infinity not exactly like a number. Good to point out infinity - infinity isn't necessarily 0, etc. This is a good one to encourage students to chat with their neighbours about. Could also raise hands: who thinks it's 0/inf/-inf, etc.

Calculating Limits at Infinity

$$\lim_{x\to\infty} \frac{x^2+2x+1}{x^3}$$

$$\lim_{x\to\infty} \frac{x^2+2x+1}{x^3}$$

After revealing the trick, can give students some time to start on their own. It should be review.Emphasize that we can only do “arithmetic” like this when the limits individually exist.

Calculating Limits at Infinity

$$\lim_{x \to -\infty} \left(x^{7/3} - x^{5/3}\right)$$

Again: factor out largest power of x.

Calculating Limits at Infinity

Suppose the height of a bouncing ball is given by $h(t) = \frac{\sin(t)+1}{t}$, for $t \geq 1$. What happens to the height over a long period of time?

Calculating Limits at Infinity

Now You

$$\lim_{x\to\infty} \sqrt{x^4+x^2+1}-\sqrt{x^4+3x^2}$$

Calculating Limits at Infinity

Now You

$$\lim_{x\to\infty} \sqrt{x^4+x^2+1}-\sqrt{x^4+3x^2}$$

It's nice to go over this a few times. Students try; then it goes on the projector; then click through the answer as a review. At each step, talk about how you recognize a situation from before, which helps you decide what to do.

Now You

Evaluate $\lim_{x \to -\infty} \frac{\sqrt{3 + x^2}}{3x}$

Now You

Evaluate $\lim_{x \to -\infty} \frac{\sqrt{3+x^2}}{3x}$

This is a good one to do in groups ("with your neighbour"). The negative is quite tricky. It helps understanding if students have already tried it on their own. It also brings together several techniques that are probably rusty if not actually brand new.

Included Work

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