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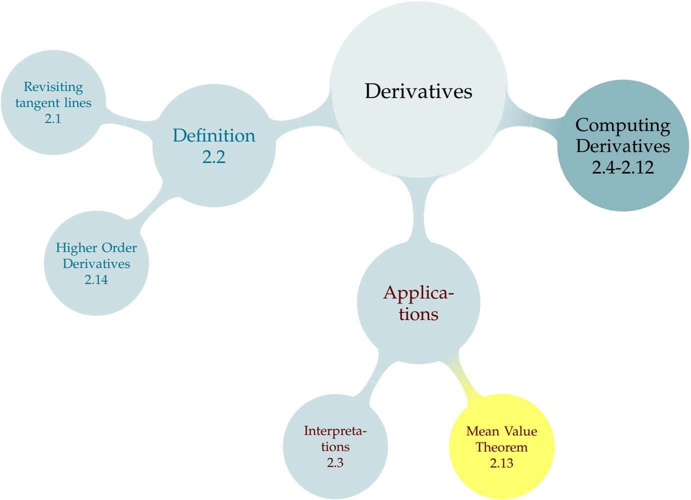
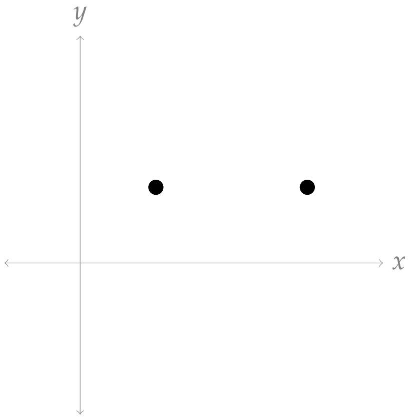


Rolle's Theorem



Rolle's Theorem – Theorem 2.13.1

Let a and b be real numbers, with $a < b$. And let f be a function with the properties:

-
-
- and $f(a) = f(b)$.

Then there exists a number c with $a < c < b$ such that

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Let a and b be real numbers, with $a < b$. And let f be a function with the properties:

- $f(x)$ is continuous for every x with $a \leq x \leq b$;
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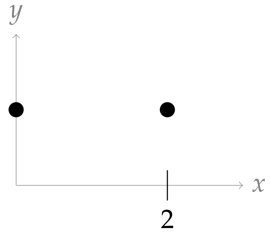
Rolle's Theorem – Theorem 2.13.1

Let $f(x)$ be continuous on the interval $[a, b]$, differentiable on (a, b), and let $f(a) = f(b)$. Then there is a number c strictly between a and b such that $f'(c) = 0$.

Example: Let $f(x) = x^3 - 2x^2 + 1$, and observe $f(2) = f(0) = 1$. Since $f(x)$ is a polynomial, it is continuous and differentiable everywhere.



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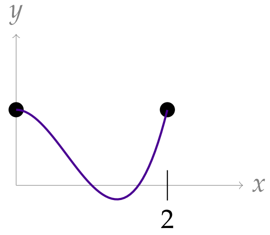

$$\begin{aligned} 0 = f'(x) &= 3x^2 - 4x \\ &= x(3x - 4) \end{aligned}$$

$$x = 0 \text{ and } \boxed{x = \frac{4}{3}}$$

$$f'\left(\frac{4}{3}\right) = 0$$

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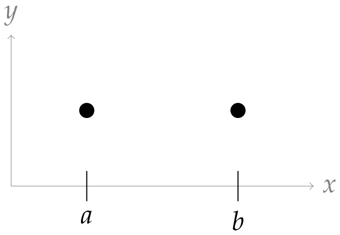

Suppose $a < b$ and $f(a) = f(b)$, $f(x)$ is continuous over $[a, b]$, and $f(x)$ is differentiable over (a, b).

How many different values of x between a and b have $f'(x) = 0$?

A. 0 or 1

B. 1

C. 0, 1, or more

D. 1 or more

E. I'm not sure

Q

ans

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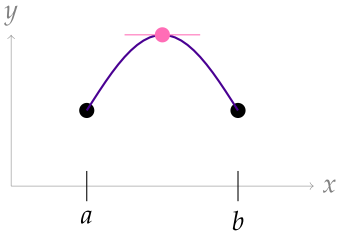

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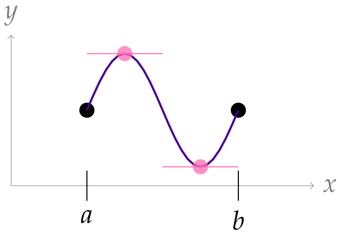

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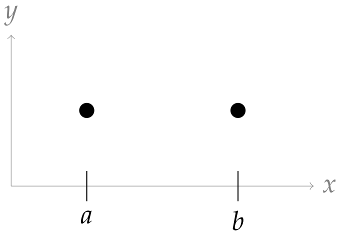

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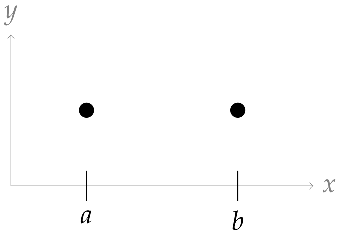

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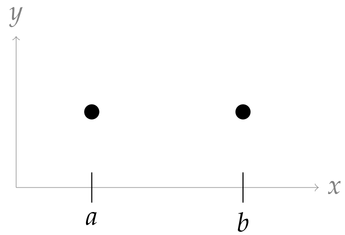

Suppose $a < b$ and $f(a) = f(b)$, $f(x)$ is continuous over $[a, b]$, and $f(x)$ is differentiable over (a, b).

Can f have an infinite number of points where $f'(x) = 0$ between a and b?

A. Sure! 

B. No way! 

C. Only if a and b are infinitely far apart 

D. I'm not sure

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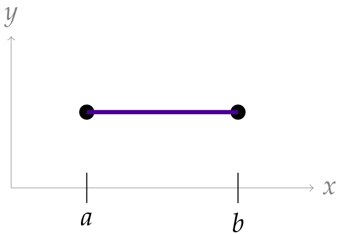

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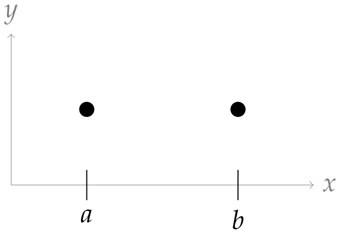

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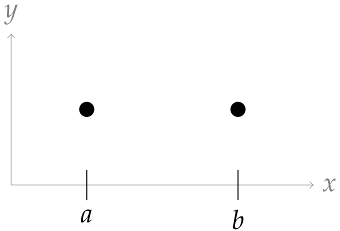

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Suppose $f(x)$ is continuous and differentiable for all real numbers, and $f(x)$ has precisely seven roots, all different. How many roots does $f'(x)$ have?

A. precisely six
B. precisely seven
C. at most seven
D. at least six

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Suppose $f(x)$ is continuous and differentiable for all real numbers, and $f'(x)$ is also continuous and differentiable for all real numbers, and $f(x)$ has precisely seven roots, all different. How many roots does $f''(x)$ have?

A. precisely six
B. precisely five
C. at most five
D. at least five

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Suppose $f(x)$ is continuous and differentiable for all real numbers, and there are precisely three places where $f'(x) = 0$. How many distinct roots does $f(x)$ have?

A. at most three

B. at most four

C. at least three

D. at least four

Q

ans

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Suppose $f(x)$ is continuous and differentiable for all real numbers, and $f'(x) = 0$ for precisely three values of x. How many distinct values x exist with $f(x) = 17$?

A. at most three

B. at most four

C. at least three

D. at least four

Q

ans

Rolle's Theorem – Theorem 2.13.1

Let $f(x)$ be continuous on the interval $[a, b]$, differentiable on (a, b), and let $f(a) = f(b)$. Then there is a number c strictly between a and b such that $f'(c) = 0$.

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Applications of Rolle's Theorem

Prove that the function $f(x) = x^3 + x - 1$ has at most one real root.

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Prove that the function $f(x) = x^3 + x - 1$ has at most one real root.

How many roots does $f(x)$ have?

2 or more

$f'(c) = 0$ somewhere

0 or 1

anything can happen

Applications of Rolle's Theorem

Prove that the function $f(x) = x^3 + x - 1$ has at most one real root.

Note that $f(x)$ is continuous and differentiable over all real numbers. So, by Rolle's Theorem, if it has two roots, then $f'(x) = 0$ for some x.

$f'(x) = 3x^2 + 1$, and this is always at least one, so it's never zero. Therefore, by Rolle's Theorem, $f(x)$ does not have two roots; so it has at most one.

$$0 \quad 1 \quad \cancel{2} \quad \cancel{3} \quad \cancel{4} \quad \cancel{5} \quad \cdots$$

Applications of Rolle's Theorem

Prove that the function $f(x) = x^3 + x - 1$ has at most one real root.

Logical Structure:

- If A is true, then B is true.
- B is false.
- Therefore, A is false.

- If $f(x)$ has two (or more) roots, then $f'(x)$ has a root.
- $f'(x)$ does not have a root.
- Therefore, $f(x)$ does not have two (or more) roots.

Applications of Rolle's Theorem

Prove that the function $f(x) = x^3 + x - 1$ has at most one real root.

How would you show that $f(x)$ has precisely one real root?

Applications of Rolle's Theorem

Prove that the function $f(x) = x^3 + x - 1$ has at most one real root.

How would you show that $f(x)$ has precisely one real root?

We know it has 0 or 1 root.

$$\boxed{0 \qquad 1} \qquad \cancel{2} \qquad \cancel{3} \qquad \cancel{4} \qquad \cancel{5} \qquad \cdots$$

We need to show it has “not zero” roots. So, we would find a root.

Applications of Rolle's Theorem

Prove that the function $f(x) = x^3 + x - 1$ has at most one real root.

How would you show that $f(x)$ has precisely one real root?

Inconveniently, it's hard to actually solve $0 = x^3 + x - 1$. So, we use the IVT (Section 1.6).

Intermediate Value Theorem (IVT) – Theorem 1.6.12

Let $a < b$ and let $f(x)$ be continuous over $[a, b]$. If y is any number between $f(a)$ and $f(b)$, then there exists c in (a, b) such that $f(c) = y$.

Applications of Rolle's Theorem

Prove that the function $f(x) = x^3 + x - 1$ has at most one real root.

How would you show that $f(x)$ has precisely one real root?

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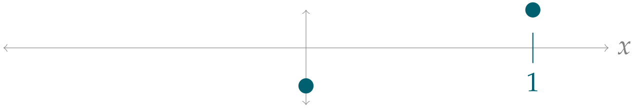
Intermediate Value Theorem (IVT) – Theorem 1.6.12

Let $a < b$ and let $f(x)$ be continuous over $[a, b]$. If y is any number between $f(a)$ and $f(b)$, then there exists c in (a, b) such that $f(c) = y$.

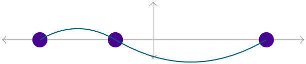
Since $f(1) > 0$ and $f(0) < 0$, and $f(x)$ is a continuous function, by the IVT $f(x)$ has a root (between 0 and 1).



Use Rolle's Theorem to show that the function $f(x) = \frac{1}{3}x^3 + 3x^2 + 9x - 3$ has at most two distinct real roots.

Use Rolle's Theorem to show that the function $f(x) = \frac{1}{3}x^3 + 3x^2 + 9x - 3$ has at most two distinct real roots.

Again we use the structure:

- If $f(x)$ has three distinct roots, then $f'(x)$ has two (or more) distinct roots. 
- $f'(x)$ does not have two (or more) distinct roots.
- Therefore, $f(x)$ does not have three distinct roots.

So, all we need to do is make sure the conditions of Rolle's Theorem are satisfied, and show that $f'(x)$ does not have two (or more) distinct roots.

Use Rolle's Theorem to show that the function $f(x) = \frac{1}{3}x^3 + 3x^2 + 9x - 3$ has at most two distinct real roots.

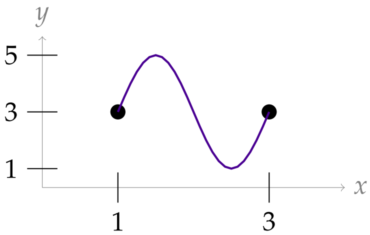
Since $f(x)$ is continuous and differentiable over all real numbers, the conditions of Rolle's Theorem are satisfied.

$f'(x) = x^2 + 6x + 9 = (x+3)^2$, which only has ONE root, namely $x = -3$.

Therefore, $f'(x)$ does not have two distinct roots, so $f(x)$ does not have three distinct roots.

So, $f(x)$ has at most two distinct roots.

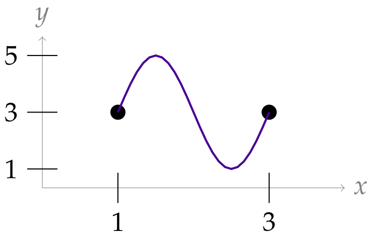
Average Rate of Change



What is the average rate of change of $f(x)$ from $x = 1$ to $x = 3$?

A. 0
B. 1
C. 2
D. 4
E. I'm not sure

Average Rate of Change



$$\frac{\Delta y}{\Delta x} = \frac{3-3}{3-1} = \frac{0}{2} = 0$$

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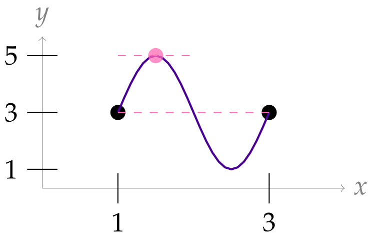

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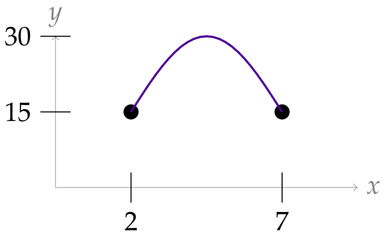
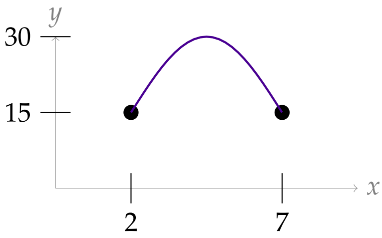


What is the average rate of change of $f(x)$ from $x = 2$ to $x = 7$?

A. 0
B. 3
C. 5
D. 15
E. I'm not sure

ans

Average Rate of Change



$$\frac{\Delta y}{\Delta x} = \frac{15-15}{7-2} = \frac{0}{5} = 0$$

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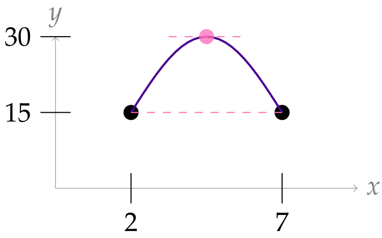

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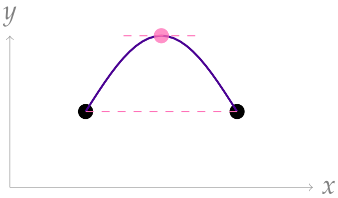
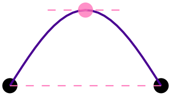
Rolle's Theorem and Average Rate of Change

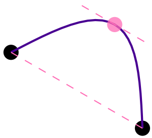
Suppose $f(x)$ is continuous on the interval $[a, b]$, differentiable on the interval (a, b), and $f(a) = f(b)$. Then there exists a number c strictly between a and b such that

$$f'(c) = 0 = \frac{f(b) - f(a)}{b - a}.$$

So there exists a point where the derivative is the same as the average rate of change.

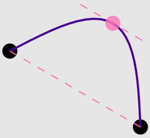

y
x





Mean Value Theorem – Theorem 2.13.4

Let $f(x)$ be continuous on the interval $[a, b]$ and differentiable on (a, b). Then there is a number c strictly between a and b such that:



$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

That is: there is some point c in (a, b) where the instantaneous rate of change of the function is equal to the average rate of change of the function on the interval $[a, b]$.

Suppose you are driving along a long, straight highway with no shortcuts. The speed limit is 100 kph. A police officer notices your car going 90 kph, and uploads your plate and the time they saw you to their database. 150 km down this same straight road, 75 minutes later, another police officer notices your car going 85kph, and uploads your plates to the database. Then they pull you over, and give you a speeding ticket. Why were they justified?

You travelled 150 km in 75 minutes. Since a moving car has a position that is continuous and differentiable, the MVT tells us that at some point, your instantaneous velocity was $\frac{150}{75}$ kilometers per minute, which works out to $\frac{150 \cdot 60}{75} = 120$ kph. So even though you weren't speeding when the officers saw you, you were definitely speeding some time in between.

Alternately, if you were going at most 100kph, then you would need at least 90 minutes to travel 150 kilometers.

According to this website, Canada geese may fly 1500 miles in a single day under favorable conditions. It also says their top speed is around 70mph. Does this seem like a typo? (If it contradicts the Mean Value Theorem, it's probably a typo.)

Q

ans

We can assume that the position of a goose is continuous and differentiable. Then the MVT tells us that a goose that travels 1500 miles in a day (24 hours) achieves, at some instant, a speed of $\frac{1500}{24}$ mph. Since $\frac{1500}{24} = 62.5$, these two facts seem compatible (and amazing!).

The record for fastest wheel-driven land speed is around 700 kph. [1] However, non-wheel driven cars (such as those powered by jet engines) have achieved higher speeds. [2]
Suppose a driver of a jet-powered car starts a 10km race at 12:00, and finishes at 12:01. Did they beat 700kph?

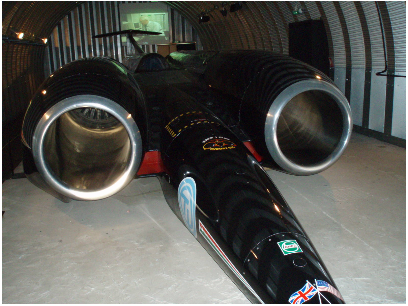
[1] (at time of writing) George Poteet, https://en.wikipedia.org/wiki/Wheel-driven_land_speed_record

[2] https://en.wikipedia.org/wiki/Land_speed_record

Q

ans

Maybe, but not necessarily. We are only guaranteed by the MVT that at some point they reached the following speed: $\frac{10}{(1/60)} = 600$ kph.



Suppose you want to download a file that is 3000 MB (slightly under 3GB). Your internet provider guarantees you that your download speeds will always be between 1 MBPS (MB per second) and 5 MBPS (because you bought the cheap plan). Using the Mean Value Theorem, give an upper and lower bound for how long the download can take (assuming your providers aren't lying, and your device is performing adequately).

We assume the download is continuous and differentiable, so we can use the MVT.
Let T be the time (in seconds) the download takes. The MVT tells us that at some point, our speed was exactly $\frac{3000}{T}$, so it must be true that

$$1 \leq \frac{3000}{T} \leq 5$$

So, $\frac{3000}{5} \leq T \leq 3000$. That is, T is between 600 and 3000 seconds, or between 10 and 50 minutes.

Suppose $1 \leq f'(t) \leq 5$ for all values of t, and $f(0) = 0$. What are the possible solutions to $f(t) = 3000$?

Since f is continuous and differentiable, we can use the MVT.

$$\frac{f(t)-f(0)}{t-0}=\frac{3000}{t}=f'(c)$$

for some value c between 0 and t.
So,

$$1 \leq \frac{3000}{t} \leq 5$$

hence

$$600 \leq t \leq 3000$$

Corollary to the MVT

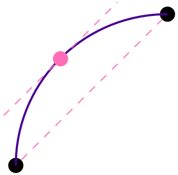
Let $a < b$ be numbers in the domain of $f(x)$ and $g(x)$, which are continuous over $[a, b]$ and differentiable over (a, b).

If $f'(x) = 0$ for all x in (a, b), then

Corollary to the MVT

Let $a < b$ be numbers in the domain of $f(x)$ and $g(x)$, which are continuous over $[a, b]$ and differentiable over (a, b).

If $f'(x) = 0$ for all x in (a, b), then $f(x)$ is constant in that interval. That is, $f(c) = f(d)$ for all c, d in $[a, b]$.



If $f(c) \neq f(d)$, then $\frac{f(d)-f(c)}{d-c} \neq 0$, so $f'(e) \neq 0$ for some e.

Corollary to the MVT

Let $a < b$ be numbers in the domain of $f(x)$ and $g(x)$, which are continuous over $[a, b]$ and differentiable over (a, b).

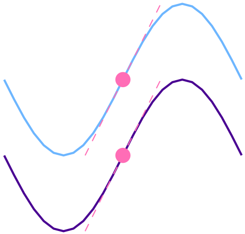
If $f'(x) = g'(x)$ for all x in (a, b), then

Q

Corollary to the MVT

Let $a < b$ be numbers in the domain of $f(x)$ and $g(x)$, which are continuous over $[a, b]$ and differentiable over (a, b).

If $f'(x) = g'(x)$ for all x in (a, b), then $f(x) = g(x) + A$ for some constant value A.



Define a new function $k(x) = f(x) - g(x)$. Then $k'(x) = 0$ everywhere, so (by the last corollary) $k(x) = A$ for some constant A.

A

Corollary to the MVT

Let $a < b$ be numbers in the domain of $f(x)$ and $g(x)$, which are continuous over $[a, b]$ and differentiable over (a, b).

If $f'(x) > 0$ for all x in (a, b), then

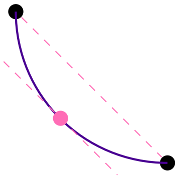
Q

ans

Corollary to the MVT

Let $a < b$ be numbers in the domain of $f(x)$ and $g(x)$, which are continuous over $[a, b]$ and differentiable over (a, b).

If $f'(x) > 0$ for all x in (a, b), then $f(x)$ is increasing. That is, $f(c) < f(d)$ for all $c < d$ in $[a, b]$.



If $f(c) > f(d)$ and $c < d$, then $\frac{f(d)-f(c)}{d-c} = \frac{\text{(negative)}}{\text{(positive)}} < 0$. Then $f'(e) < 0$ for some e between c and d.

Corollary to the MVT

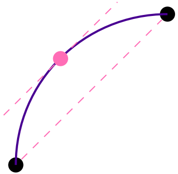
Let $a < b$ be numbers in the domain of $f(x)$ and $g(x)$, which are continuous over $[a, b]$ and differentiable over (a, b).

If $f'(x) < 0$ for all x in (a, b), then

Corollary to the MVT

Let $a < b$ be numbers in the domain of $f(x)$ and $g(x)$, which are continuous over $[a, b]$ and differentiable over (a, b).

If $f'(x) < 0$ for all x in (a, b), then $f(x)$ is decreasing. That is, $f(d) < f(c)$ for all $c < d$ in $[a, b]$.



If $f(c) < f(d)$ and $c < d$, then $\frac{f(d)-f(c)}{d-c} = \frac{\text{(positive)}}{\text{(positive)}} > 0$. Then $f'(e) > 0$ for some e between c and d.

Mean Value Theorem – Theorem 2.13.4

Let $f(x)$ be continuous on the interval $[a, b]$ and differentiable on (a, b). Then there is a number c strictly between a and b such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

WARNING: The MVT has two hypotheses.

- $f(x)$ has to be continuous on $[a, b]$.
- $f(x)$ has to be differentiable on (a, b).

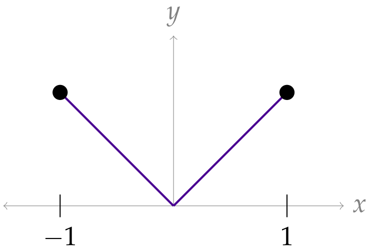
If either of these hypotheses are violated, the conclusion of the MVT can fail. Here are two examples.

Mean Value Theorem – Theorem 2.13.4

Let $f(x)$ be continuous on the interval $[a, b]$ and differentiable on (a, b). Then there is a number c strictly between a and b such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Example: Let $a = -1$, $b = 1$ and $f(x) = |x|$.



Mean Value Theorem – Theorem 2.13.4

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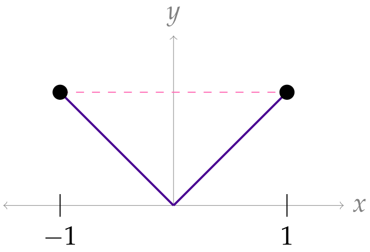

Mean Value Theorem – Theorem 2.13.4

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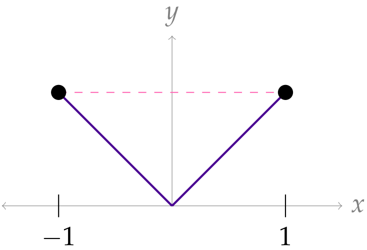


$$f'(x) = \begin{cases} 1 & \text{if } x>0 \\ -1 & \text{if } x<0 \\ \text{undefined} & \text{if } x=0 \end{cases}$$

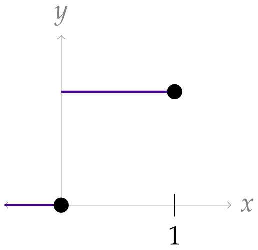
$f'(x)$ is never $0 = \frac{f(b)-f(a)}{b-a}$.

Mean Value Theorem – Theorem 2.13.4

Let $f(x)$ be continuous on the interval $[a, b]$ and differentiable on (a, b). Then there is a number c strictly between a and b such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Example: Let $a = 0$, $b = 1$ and $f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 & \text{if } x > 0 \end{cases}$.



Mean Value Theorem – Theorem 2.13.4

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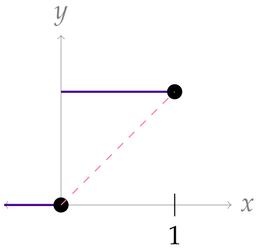

Mean Value Theorem – Theorem 2.13.4

Let $f(x)$ be continuous on the interval $[a, b]$ and differentiable on (a, b). Then there is a number c strictly between a and b such that

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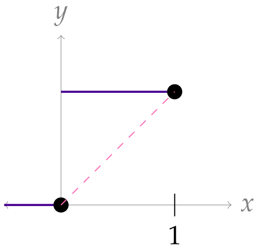

$$f'(x) = \begin{cases} 0 & \text{if } x > 0 \\ 0 & \text{if } x < 0 \\ \text{undefined} & \text{if } x = 0 \end{cases}$$

$f'(x)$ is never $1 = \frac{f(b)-f(a)}{b-a}$.

Included Work

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