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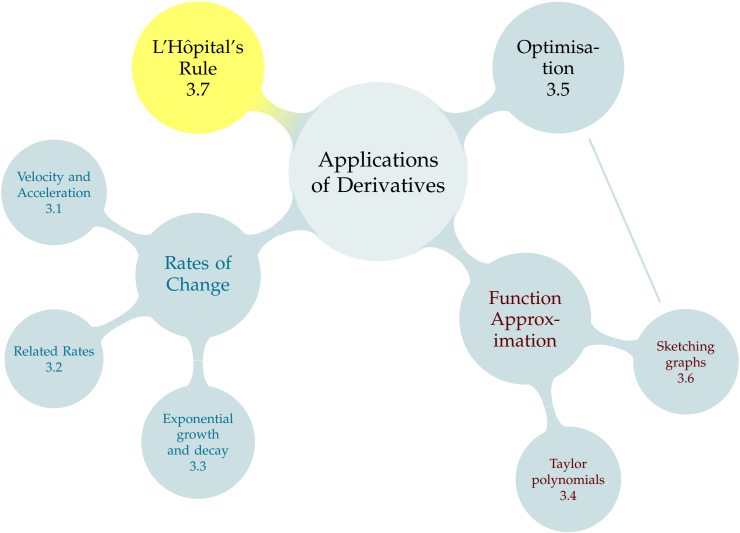

Back to Limits!

$$\lim_{x\to\infty}\frac{x^2}{5} \qquad \lim_{x\to\infty}\frac{5}{x^2} \qquad \lim_{x\to 0}\frac{x^2}{5} \qquad \lim_{x\to 0}\frac{5}{x^2}$$

Back to Limits!

$$\lim_{x\to\infty} \frac{x^2}{5} = \infty \qquad \lim_{x\to\infty} \frac{5}{x^2} \qquad \lim_{x\to 0} \frac{x^2}{5} \qquad \lim_{x\to 0} \frac{5}{x^2}$$

ans

Back to Limits!

$$\lim_{x\to\infty} \frac{x^2}{5} = \infty \qquad \lim_{x\to\infty} \frac{5}{x^2} = 0 \qquad \lim_{x\to 0} \frac{x^2}{5} \qquad \lim_{x\to 0} \frac{5}{x^2}$$

ans

Back to Limits!

$$\lim_{x\to\infty} \frac{x^2}{5} = \infty \qquad \lim_{x\to\infty} \frac{5}{x^2} = 0 \qquad \lim_{x\to 0} \frac{x^2}{5} = 0 \qquad \lim_{x\to 0} \frac{5}{x^2}$$

ans

BACK TO LIMITS!

$$\lim_{x\to\infty} \frac{x^2}{5} = \infty \qquad \lim_{x\to\infty} \frac{5}{x^2} = 0 \qquad \lim_{x\to 0} \frac{x^2}{5} = 0 \qquad \lim_{x\to 0} \frac{5}{x^2} = \infty$$

Back to Limits!

$\lim\limits_{x\to\infty} \dfrac{x^2}{5} = \infty$ $\lim\limits_{x\to\infty} \dfrac{5}{x^2} = 0$ $\lim\limits_{x\to 0} \dfrac{x^2}{5} = 0$ $\lim\limits_{x\to 0} \dfrac{5}{x^2} = \infty$

Indeterminate Forms – Definition 3.7.1

Suppose $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} g(x) = 0$. Then the limit

$$\lim_{x\to a} \frac{f(x)}{g(x)}$$

is an indeterminate form of the type $\frac{0}{0}$.
Suppose $\lim\limits_{x\to a} F(x) = \lim\limits_{x\to a} G(x) = \infty$ (or $-\infty$). Then the limit

$$\lim_{x\to a} \frac{F(x)}{G(x)}$$

is an indeterminate form of the type $\frac{\infty}{\infty}$.

Back to Limits!

$$\lim_{x\to\infty} \frac{x^2}{5} = \infty \qquad \lim_{x\to\infty} \frac{5}{x^2} = 0 \qquad \lim_{x\to 0} \frac{x^2}{5} = 0 \qquad \lim_{x\to 0} \frac{5}{x^2} = \infty$$

Indeterminate Forms – Definition 3.7.1

Suppose $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} g(x) = 0$. Then the limit

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Suppose $\lim\limits_{x\to a} F(x) = \lim\limits_{x\to a} G(x) = \infty$ (or $-\infty$). Then the limit

$$\lim_{x\to a} \frac{F(x)}{G(x)}$$

is an indeterminate form of the type $\frac{\infty}{\infty}$.

When you see an indeterminate form, you need to do more work.

Indeterminate Forms

$$\lim_{x\to 5} \frac{x^2 - 3x - 10}{x - 5}$$

indeterminate form of the type $\frac{0}{0}$

Indeterminate Forms

$$\lim_{x\to 5} \frac{x^2-3x-10}{x-5}$$

indeterminate form of the type $\frac{0}{0}$

To evaluate, factor the top:

$$\lim_{x\to 5} \frac{x^2-3x-10}{x-5} = \lim_{x\to 5} \frac{(x-5)(x+2)}{x-5} = \lim_{x\to 5} x+2 = \boxed{7}$$

Indeterminate Forms

$$\lim_{x\to 5} \frac{x^2-3x-10}{x-5}$$

indeterminate form of the type $\frac{0}{0}$

To evaluate, factor the top:

$$\lim_{x\to 5} \frac{x^2-3x-10}{x-5} = \lim_{x\to 5} \frac{(x-5)(x+2)}{x-5} = \lim_{x\to 5} x+2 = \boxed{7}$$

$$\lim_{x\to \infty} \frac{3x^2-4x+2}{8x^2-5}$$

indeterminate form of the type $\frac{\infty}{\infty}$

Q

ans

Indeterminate Forms

$$\lim_{x\to 5} \frac{x^2-3x-10}{x-5}$$

indeterminate form of the type $\frac{0}{0}$

To evaluate, factor the top:

$$\lim_{x\to 5} \frac{x^2-3x-10}{x-5} = \lim_{x\to 5} \frac{(x-5)(x+2)}{x-5} = \lim_{x\to 5} x+2 = \boxed{7}$$

$$\lim_{x\to\infty} \frac{3x^2-4x+2}{8x^2-5}$$

indeterminate form of the type $\frac{\infty}{\infty}$

To evaluate, pull out x^2:

$$\lim_{x\to\infty} \frac{3x^2-4x+2}{8x^2-5} = \lim_{x\to\infty} \frac{x^2(3-\frac{4}{x}+\frac{2}{x^2})}{x^2(8-\frac{5}{x^2})} = \lim_{x\to\infty} \frac{3-\frac{4}{x}+\frac{2}{x^2}}{8-\frac{5}{x^2}} =$$

$$\lim_{x\to\infty} \frac{3-0+0}{8-0} = \boxed{\frac{3}{8}}$$

Indeterminate Forms and the Derivative

$$\lim_{x\to 0} \frac{3\sin x - x^4}{x^2+\cos x - e^x}$$

indeterminate form of the type $\frac{0}{0}$

Q

ans

Indeterminate Forms and the Derivative

$$\lim_{x\to 0} \frac{3\sin x - x^4}{x^2+\cos x - e^x}$$

indeterminate form of the type $\frac{0}{0}$

Suppose $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} g(x) = 0$. Suppose also that f and g are continuous and differentiable at a, and $g'(a) \neq 0$. Then:

$$\begin{aligned}
\lim_{x\to a}\frac{f(x)}{g(x)} &= \lim_{x\to a}\frac{f(x)-0}{g(x)-0} = \lim_{x\to a}\frac{f(x)-f(a)}{g(x)-g(a)} \\
&= \lim_{x\to a}\frac{f(x)-f(a)}{g(x)-g(a)}\,\frac{(x-a)^{-1}}{(x-a)^{-1}} \\
&= \lim_{x\to a}\left(\frac{\frac{f(x)-f(a)}{x-a}}{\frac{g(x)-g(a)}{x-a}}\right) = \frac{f'(a)}{g'(a)}
\end{aligned}$$

A

Indeterminate Forms and the Derivative

$$\lim_{x\to 0} \frac{3\sin x - x^4}{x^2+\cos x - e^x}$$

indeterminate form of the type $\frac{0}{0}$

$$\begin{aligned}\lim_{x\to 0} \frac{3\sin x - x^4}{x^2+\cos x - e^x} &= \frac{\frac{\mathrm{d}}{\mathrm{d}x}[3\sin x - x^4]\big|_{x=0}}{\frac{\mathrm{d}}{\mathrm{d}x}[x^2+\cos x - e^x]\big|_{x=0}} \\ &= \frac{[3\cos x - 4x^3]\big|_{x=0}}{[2x-\sin x - e^x]\big|_{x=0}} \\ &= \frac{3-0}{0-0-1} = \boxed{-3}\end{aligned}$$

L'Hôpital's Rule: First Part – Theorem 3.7.2

Let f and g be functions such that $\lim_{x \to a} f(x) = 0 = \lim_{x \to a} g(x)$.

If $f'(a)$ and $g'(a)$ exist and $g'(a) \neq 0$, then $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$.

If f and g are differentiable on an open interval containing a, and if $\lim_{x \to a} \frac{f'(x)}{g'(x)}$ exists, then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$.

This works even for $a = \pm\infty$.

Extremely Important Note:
L'Hôpital's Rule only works on indeterminate forms.

L'Hôpital's Rule: Second Part – Theorem 3.7.2

Let f and g be functions such that $\lim_{x\to a} f(x) = \infty = \lim_{x\to a} g(x)$.

If $f'(a)$ and $g'(a)$ exist and $g'(a) \neq 0$, then $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$.

If f and g are differentiable on an open interval containing a, and if $\lim_{x\to a} \frac{f'(x)}{g'(x)}$ exists, then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$.

This works even for $a = \pm\infty$.

Extremely Important Note:
L'Hôpital's Rule only works on indeterminate forms.

Evaluate:

$$\lim_{x\to 2} \frac{3x\tan(x-2)}{x-2}$$

Evaluate:

$$\lim_{x\to 2} \frac{3x\tan(x-2)}{x-2}$$

$$\lim_{x\to 2} \frac{3x\tan(x-2)}{x-2} \qquad \text{form } \frac{0}{0}$$

$$\overset{\text{l'H}}{=} \frac{3\left[x\sec^2(x-2)+\tan(x-2)\right]_{x=2}}{1}$$

$$=3\left[2\sec^2 0+\tan 0\right] = \boxed{6}$$

Little Harder

$$\lim_{x\to 0} \frac{x^4}{e^x - \cos x - x}$$

indeterminate form of the type $\frac{0}{0}$

Q

ans

Little Harder

$$\lim_{x \to 0} \frac{x^4}{e^x - \cos x - x}$$

indeterminate form of the type $\frac{0}{0}$

$$\lim_{x \to 0} \frac{x^4}{e^x - \cos x - x} \overset{?}{\underset{?}{}} \overset{?}{\underset{?}{=}} \left. \frac{4x^3}{e^x + \sin x - 1} \right|_{x=0} = \frac{0}{0}$$

oops

Little Harder

$$\lim_{x\to 0} \frac{x^4}{e^x - \cos x - x}$$

indeterminate form of the type $\frac{0}{0}$

$$\lim_{x\to 0} \frac{x^4}{e^x - \cos x - x} \stackrel{?}{?} \stackrel{?}{=}? \left.\frac{4x^3}{e^x + \sin x - 1}\right|_{x=0} = \frac{0}{0}$$

oops

Iterate!

$$\begin{aligned} \lim_{x\to 0} \frac{x^4}{e^x - \cos x - x} &\stackrel{\text{l'H}}{=} \lim_{x\to 0} \frac{4x^3}{e^x + \sin x - 1} \\ &\stackrel{\text{l'H}}{=} \lim_{x\to 0} \frac{12x^2}{e^x + \cos x} = \frac{0}{2} = \boxed{0} \end{aligned}$$

A

Evaluate:

$$\lim_{x\to\infty} \frac{\log x}{\sqrt{x}}$$

Q

ans

Evaluate:

$$\lim_{x\to\infty} \frac{\log x}{\sqrt{x}}$$

$$\lim_{x\to\infty} \frac{\log x}{x} \overset{\text{l'H}}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}}$$

$$= \lim_{x\to\infty} \frac{2\sqrt{x}}{x}$$

$$= \lim_{x\to\infty} \frac{2}{\sqrt{x}} = \boxed{0}$$

A

Other Indeterminate Forms

$$\lim_{x\to\infty} e^{-x}\log x$$

form $0 \cdot \infty$

Q

ans

Other Indeterminate Forms

$$\lim_{x\to\infty} e^{-x}\log x \qquad \text{form } 0\cdot\infty$$

$$\begin{aligned}\lim_{x\to\infty} e^{-x}\log x &= \lim_{x\to\infty}\frac{\log x}{e^x} \qquad \text{form } \frac{\infty}{\infty}\\ &\stackrel{\text{l'H}}{=} \lim_{x\to\infty}\frac{1/x}{e^x}\\ &= \lim_{x\to\infty}\frac{1}{xe^x} = \boxed{0}\end{aligned}$$

A

Vote Vote Vote

Which of the following can you immediately apply L'Hôpital's rule to?

A. $\dfrac{e^x}{2e^x+1}$

B. $\displaystyle\lim_{x\to 0} \frac{e^x}{2e^x+1}$

C. $\displaystyle\lim_{x\to \infty} \frac{e^x}{2e^x+1}$

D. $\displaystyle\lim_{x\to \infty} e^{-x}(2e^x+1)$

E. $\displaystyle\lim_{x\to 0} \frac{e^x}{x^2}$

Q

ans

Vote Vote Vote

Which of the following can you <u>immediately</u> apply L’Hôpital’s rule to?

A. $\dfrac{e^x}{2e^x+1}$

B. $\displaystyle\lim_{x\to 0} \frac{e^x}{2e^x+1}$

C. $\displaystyle\lim_{x\to \infty} \frac{e^x}{2e^x+1}$

D. $\displaystyle\lim_{x\to \infty} e^{-x}(2e^x+1)$

E. $\displaystyle\lim_{x\to 0} \frac{e^x}{x^2}$

Votey McVoteface

Suppose you want to use L'Hôpital's rule to evaluate $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)}$, which has the form $\frac{0}{0}$. How does the quotient rule fit into this problem?

A. You should use the quotient rule because the function you are differentiating is a quotient.
B. You will not use the quotient rule because you differentiate the numerator and the denominator separately
C. You may use the quotient rule because perhaps $f(x)$ or $g(x)$ is itself in the form of a quotient
D. You will not use L'Hôpital's rule because $\frac{0}{0}$ is not an appropriate indeterminate form
E. You will not use L'Hôpital's rule because, since the top has limit zero, the whole function has limit 0

Q

ans

VOTEY MCVOTEFACE

Suppose you want to use L'Hôpital's rule to evaluate $\lim_{x \to a} \frac{f(x)}{g(x)}$, which has the form $\frac{0}{0}$. How does the quotient rule fit into this problem?

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E. You will not use L'Hôpital's rule because, since the top has limit zero, the whole function has limit 0

A

MORE QUESTIONS

Which of the following is NOT an indeterminate form?

A. $\frac{\infty}{\infty}$ for example, $\displaystyle\lim_{x\to\infty} \frac{e^x}{x^2}$

B. $\frac{0}{0}$ for example, $\displaystyle\lim_{x\to 0} \frac{e^x - 1}{x}$

C. $\frac{0}{\infty}$ for example, $\displaystyle\lim_{x\to 0^+} \frac{x}{\log x}$

D. $0 \cdot \infty$ for example, $\displaystyle\lim_{x\to\infty} x(\arctan(x) - \pi/2)$

E. all of the above are indeterminate forms

Q

MORE QUESTIONS

Which of the following is NOT an indeterminate form?

A. $\frac{\infty}{\infty}$ for example, $\displaystyle\lim_{x\to\infty} \frac{e^x}{x^2}$

B. $\frac{0}{0}$ for example, $\displaystyle\lim_{x\to 0} \frac{e^x - 1}{x}$

C. $\frac{0}{\infty}$ for example, $\displaystyle\lim_{x\to 0^+} \frac{x}{\log x}=0$

D. $0 \cdot \infty$ for example, $\displaystyle\lim_{x\to\infty} x(\arctan(x) - \pi/2)$

E. all of the above are indeterminate forms

A

I HAVE SO MANY QUESTIONS

Which of the following is NOT an indeterminate form?

A. 1^∞ for example, $\lim_{x\to\infty}\left(\frac{x+1}{x}\right)^x$

B. 0^∞ for example, $\lim_{x\to\infty}\left(\frac{1}{x}\right)^x$

C. ∞^0 for example, $\lim_{x\to\infty} x^{\frac{1}{x}}$

D. 0^0 for example, $\lim_{x\to 0^+} x^x$

E. all of the above are indeterminate forms

F. none of the above are indeterminate forms

Q

ans

I have so many questions

Which of the following is NOT an indeterminate form?

A. 1^∞ for example, $\displaystyle\lim_{x\to\infty}\left(\frac{x+1}{x}\right)^x$

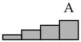
B. 0^∞ for example, $\displaystyle\lim_{x\to\infty}\left(\frac{1}{x}\right)^x = 0$

C. ∞^0 for example, $\displaystyle\lim_{x\to\infty} x^{\frac{1}{x}}$

D. 0^0 for example, $\displaystyle\lim_{x\to 0^+} x^x$

E. all of the above are indeterminate forms

F. none of the above are indeterminate forms



Exponential Indeterminate Forms

$$\lim_{x\to\infty} x^{1/x}$$

Q

ans

Exponential Indeterminate Forms

$$\lim_{x\to\infty} x^{1/x}$$

$$\begin{aligned}
\lim_{x\to\infty} x^{1/x} &= \lim_{x\to\infty} e^{\left(\log\left(x^{1/x}\right)\right)} \\
&= \lim_{x\to\infty} e^{\left(\frac{\log x}{x}\right)} \\
&= e^{\left(\lim_{x\to\infty} \frac{\log x}{x}\right)} \\
&\overset{\text{l'H}}{=} e^{\left(\lim_{x\to\infty} \frac{1}{x}\right)} \\
&= e^0 = \boxed{1}
\end{aligned}$$

Exponential Indeterminate Forms

$$\lim_{x\to\infty} \left(1+\frac{2}{x}\right)^{3x}$$

Q

ans

Exponential Indeterminate Forms

$$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x}$$

First we calculate:

$$\begin{aligned}
\lim_{x\to\infty}\log\left(\left(1+\frac{2}{x}\right)^{3x}\right) &= \lim_{x\to\infty} 3x\log\left(1+\frac{2}{x}\right)\\
&= \lim_{x\to\infty}\frac{3\log\left(1+\frac{2}{x}\right)}{x^{-1}}\\
&\stackrel{\text{l'H}}{=} \lim_{x\to\infty}\frac{3\left(\frac{-2x^{-2}}{1+2/x}\right)}{-x^{-2}}\\
&= \lim_{x\to\infty}\frac{6}{1+2/x} = 6
\end{aligned}$$

So, now:

$$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x} = \boxed{e^6}$$

Evaluate:

$$\lim_{x\to\infty} \frac{\log x}{\log \sqrt{x}}$$

$$\lim_{x\to\infty} (\log x)^{\sqrt{x}}$$

$$\lim_{x\to 0} \frac{\arcsin x}{x}$$

ans

Evaluate:

$$\lim_{x\to\infty} \frac{\log x}{\log\sqrt{x}}$$

Easier to simplify first.

$$\lim_{x\to\infty} (\log x)^{\sqrt{x}}$$

Not an indeterminate form: huge number to a huge power. Limit is infinity.

$$\lim_{x\to 0} \frac{\arcsin x}{x}$$

L'Hôpital: $\lim\limits_{x\to 0} \dfrac{\frac{1}{\sqrt{1-x^2}}}{1} = 1$

More Examples

$$\lim_{x\to\infty} \sqrt{2x^2+1} - \sqrt{x^2+x}$$

$$\lim_{x\to 0} \sqrt[x^2]{\sin^2 x}$$

$$\lim_{x\to 0} \sqrt[x^2]{\cos x}$$

$$\lim_{x\to\infty} \sqrt{2x^2+1} - \sqrt{x^2+x}$$

A

$\lim\limits_{x\to\infty} \sqrt{2x^2+1} - \sqrt{x^2+x}$ has the indeterminate form $\infty - \infty$. To get a better idea of what's going on, let's rationalize.

$$\begin{aligned}
&\lim_{x\to\infty} \sqrt{2x^2+1} - \sqrt{x^2+x} \\
&= \lim_{x\to\infty} \left(\sqrt{2x^2+1} - \sqrt{x^2+x}\right)\left(\frac{\sqrt{2x^2+1}+\sqrt{x^2+x}}{\sqrt{2x^2+1}+\sqrt{x^2+x}}\right) \\
&= \lim_{x\to\infty} \frac{(2x^2+1)-(x^2+x)}{\sqrt{2x^2+1}+\sqrt{x^2+x}} = \lim_{x\to\infty} \frac{x^2-x+1}{\sqrt{2x^2+1}+\sqrt{x^2+x}}
\end{aligned}$$

Here, we have the indeterminate form $\frac{\infty}{\infty}$, so l'Hôpital's Rule applies. However, if we try to use it here, we quickly get a huge mess. Instead, remember how we dealt with these kinds of limits in the past: factor out the highest power of the denominator, which is x.

$$\lim_{x\to\infty} \frac{x^2 - x + 1}{\sqrt{2x^2+1} + \sqrt{x^2+x}} = \lim_{x\to\infty} \frac{x\left(x - 1 + \frac{1}{x}\right)}{\sqrt{x^2(2+\frac{1}{x^2})} + \sqrt{x^2(1+\frac{1}{x})}}$$

$$= \lim_{x\to\infty} \frac{x\left(x - 1 + \frac{1}{x}\right)}{x\left(\sqrt{2+\frac{1}{x^2}} + \sqrt{1+\frac{1}{x}}\right)}$$

$$= \lim_{x\to\infty} \underbrace{\frac{x - 1 + \frac{1}{x}}{\sqrt{2+\frac{1}{x^2}} + \sqrt{1+\frac{1}{x}}}}_{\substack{\text{num}\to\infty \\ \text{den}\to\sqrt{2}+1}}$$

$$= \infty$$

$$\lim_{x\to 0} \sqrt[x^2]{\sin^2 x}$$

$$\lim_{x\to 0} \sqrt[x^2]{\sin^2 x}$$

$\displaystyle\lim_{x\to 0} \sin^2 x = 0$, and $\displaystyle\lim_{x\to 0} \frac{1}{x^2} = \infty$, so we have the form 0^∞. (Note that $\sin^2 x$ is positive, so our root is defined.) This is not an indeterminate form: $\displaystyle\lim_{x\to 0} \sqrt[x^2]{\sin^2 x} = 0$.

$$\lim_{x \to 0} \sqrt[x^2]{\cos x}$$

$$\lim_{x\to 0} \sqrt[x^2]{\cos x}$$

$\lim\limits_{x\to 0}\cos x = 1$ and $\lim\limits_{x\to 0}\dfrac{1}{x^2} = \infty$, so $\lim\limits_{x\to 0}(\cos x)^{\frac{1}{x^2}}$ has the indeterminate form 1^∞. We want to use l'Hôpital, but we need to get our function into a fractional indeterminate form. So, we'll use a logarithm.

$$y := (\cos x)^{\frac{1}{x^2}}$$

$$\log y = \log\left((\cos x)^{\frac{1}{x^2}}\right) = \frac{1}{x^2}\log(\cos x) = \frac{\log\cos x}{x^2}$$

$$\lim_{x\to 0}\log y = \lim_{x\to 0}\underbrace{\frac{\log\cos x}{x^2}}_{\substack{\text{num}\to 0\\ \text{den}\to 0}} = \lim_{x\to 0}\frac{\frac{-\sin x}{\cos x}}{2x} = \lim_{x\to 0}\underbrace{\frac{-\tan x}{2x}}_{\substack{\text{num}\to 0\\ \text{den}\to 0}} = \lim_{x\to 0}\frac{-\sec^2 x}{2}$$

$$= \lim_{x\to 0}\frac{-1}{2\cos^2 x} = -\frac{1}{2}$$

Therefore, $\lim\limits_{x\to 0} y = \lim\limits_{x\to 0} e^{\log y} = e^{-1/2} = \dfrac{1}{\sqrt{e}}$

Sketch the graph of $f(x) = x \log x$.
Note: when you want to know $\lim_{x \to 0} f(x)$, you'll need to use L'Hôpital.

Evaluate $\lim_{x \to 0^+} (\csc x)^x$

$$f(x) = x \log x$$

A

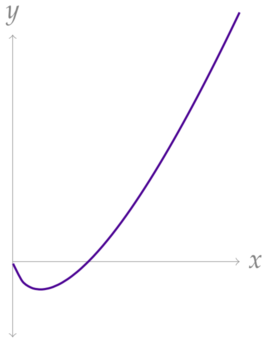
$$f(x) = x \log x$$

- Domain: $x > 0$
- HA: none
- Intercepts: $(1, 0)$
- $y = \log x$ has a VA at $x = 0$. For our function, let's see what its behaviour is near 0:

$$\lim_{x \to 0^+} x \log x = \lim_{x \to 0^+} \frac{\log x}{1/x} \qquad \text{form } \frac{\infty}{\infty}$$

$$\stackrel{\text{l'H}}{=} \lim_{x \to 0^+} \frac{1/x}{-1/x^2} = \lim_{x \to 0^+} -x = 0$$

- $f'(x) = 1 + \log x$;
  CP at $x = \frac{1}{e}$ (and $y = -\frac{1}{e}$);
  As x gets close to 0, $f'(x)$ goes to negative infinity, so near $x = 0$ our line looks vertical.
  Decreasing on $(0, \frac{1}{e})$ and increasing on $(1/e, \infty)$
- $f''(x) = \frac{1}{x}$; concave up on entire domain


y
x

Evaluate $\lim\limits_{x \to 0^+} (\csc x)^x$

Evaluate $\lim_{x\to 0^+} (\csc x)^x$

Indeterminate form: ∞^0

$$\begin{aligned} y &= (\csc x)^x \\ \log y &= x \log(\csc x) \\ \lim_{x\to 0^+} \log y &= \lim_{x\to 0^+} x\log(\csc x) \\ &= \lim_{x\to 0^+} x\log\left(\frac{1}{\sin x}\right) \\ &= \lim_{x\to 0^+} -x\log(\sin x) \end{aligned}$$

indeterminate form $0\cdot\infty$

$$\begin{aligned} &= \lim_{x\to 0^+} \frac{\log(\sin x)}{-1/x} \\ &\overset{\text{l'H}}{=} \lim_{x\to 0^+} \frac{\frac{\cos x}{\sin x}}{-1/x^2} \\ &= \lim_{x\to 0^+} \frac{-x^2}{\tan x} \end{aligned}$$

indeterminate form $\frac{0}{0}$

$$\begin{aligned} &\overset{\text{l'H}}{=} \lim_{x\to 0^+} \frac{-2x}{\sec^2 x} \\ &= \frac{0}{1} = 0 \end{aligned}$$

conclusion

$$\lim_{x\to 0^+} \log y = 0$$

$$\lim_{x\to 0^+} y = e^0 = \boxed{1}$$

Included Work

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