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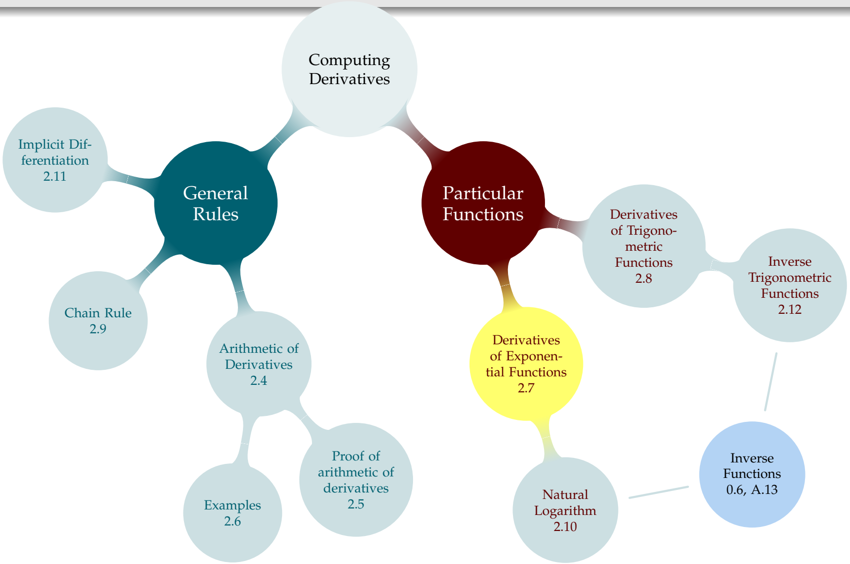
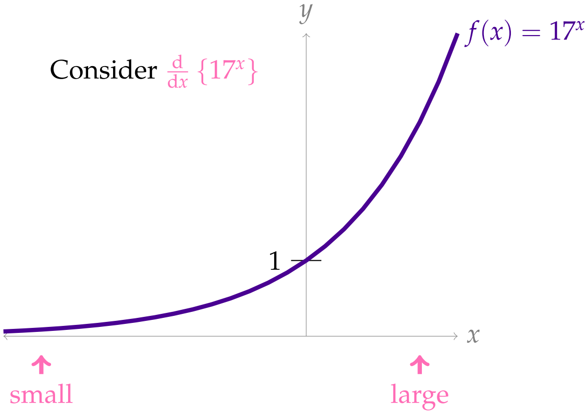


Exponential Functions



$f(x)$ is always increasing, so $f'(x)$ is always positive.
$f'(x)$ might look similar to $f(x)$.

Exponential Functions

$$\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\} =$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\} = 17^x \cdot \underbrace{\lim_{h\to 0}\frac{(17^h - 1)}{h}}_{\text{constant}}$$

Given what you know about $\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\}$, **is it possible** that
$\displaystyle\lim_{h\to 0}\frac{17^h - 1}{h} = 0$?

A. Sure, there's no reason we've seen that would make it impossible.
B. No, it couldn't be 0, that wouldn't make sense.
C. I do not feel equipped to answer this question.

2.7: Derivs of Exponential Functions

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The second question is there as a second chance for students who missed the first one

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Given what you know about $\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\}$, **is it possible** that $\lim_{h\to 0}\frac{17^h - 1}{h} = \infty$?

A. Sure, there's no reason we've seen that would make it impossible.

B. No, it couldn't be ∞, that wouldn't make sense.

C. I do not feel equipped to answer this question.

2.7: Derivs of Exponential Functions

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h	$\dfrac{17^h - 1}{h}$
0.001	2.83723068608
0.00001	2.83325347992
0.0000001	2.83321374583
0.000000001	2.83321344163

2.7: Derivs of Exponential Functions

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"It's not clear what constant this is, but it is a constant."

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\} &= \lim_{h\to 0}\frac{17^{x+h}-17^x}{h}\\
&= \lim_{h\to 0}\frac{17^x 17^h-17^x}{h}\\
&= \lim_{h\to 0}\frac{17^x(17^h-1)}{h}\\
&= 17^x\lim_{h\to 0}\frac{(17^h-1)}{h}
\end{aligned}$$

In general, for any positive number a,

$$\frac{\mathrm{d}}{\mathrm{d}x}\{a^x\} = a^x\lim_{h\to 0}\frac{a^h-1}{h}$$

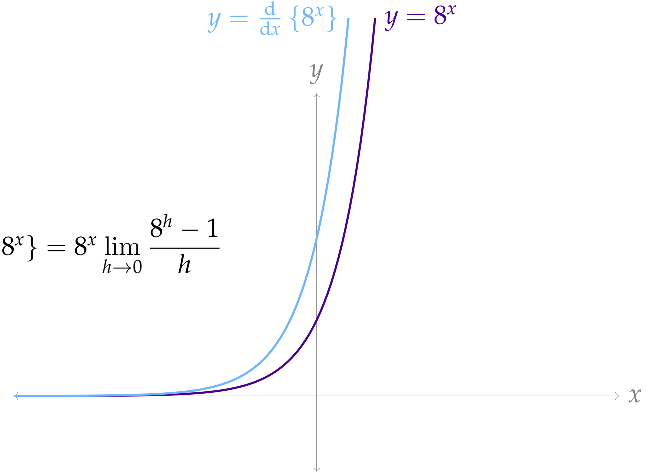
$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\} &= \lim_{h\to 0}\frac{17^{x+h}-17^x}{h}\\ &= \lim_{h\to 0}\frac{17^x 17^h-17^x}{h}\\ &= \lim_{h\to 0}\frac{17^x(17^h-1)}{h}\\ &= 17^x\lim_{h\to 0}\frac{(17^h-1)}{h}\end{aligned}$$

In general, for any positive number a,

$$\frac{\mathrm{d}}{\mathrm{d}x}\{a^x\} = a^x\lim_{h\to 0}\frac{a^h-1}{h}$$

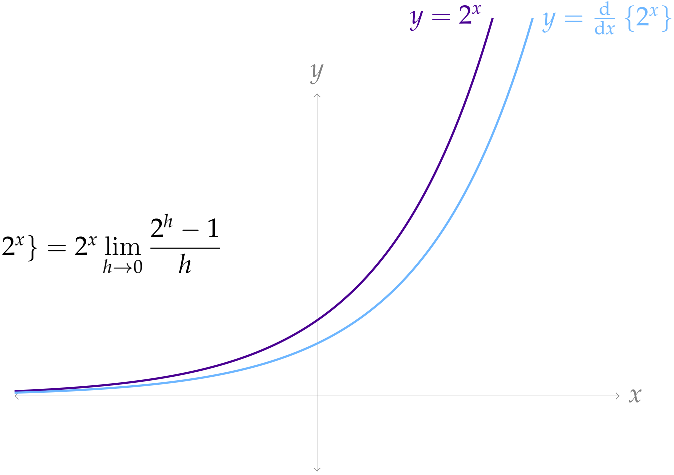
Point out that 17 could have been replaced with any other number. Go through and cross out 17, write in a.

Exponential Functions



$$\frac{d}{dx}\{8^x\} = 8^x \lim_{h\to 0} \frac{8^h - 1}{h}$$

Exponential Functions



$$\frac{d}{dx}\{2^x\} = 2^x \lim_{h\to 0} \frac{2^h - 1}{h}$$

In general, for any positive number a, $\frac{\mathrm{d}}{\mathrm{d}x}\{a^x\} = a^x \lim_{h\to 0} \dfrac{a^h - 1}{h}$

Euler's Number – Theorem 2.7.4

We define e to be the unique number satisfying

$$\lim_{h\to 0} \frac{e^h - 1}{h} = 1$$

$e \approx 2.71828182845904523536028747135266 24\ldots$ (Wikipedia)

Theorem 2.7.4 and Corollary 2.10.6

Using this definition of e,

$$\frac{\mathrm{d}}{\mathrm{d}x}\{e^x\} = e^x \underbrace{\lim_{h\to 0}\frac{e^h-1}{h}}_{1} = e^x$$

In general, $\lim_{h\to 0}\frac{a^h-1}{h} = \log_e(a)$, so $\frac{\mathrm{d}}{\mathrm{d}x}\{a^x\} = a^x\log_e(a)$

That $\lim_{h\to 0}\frac{a^h-1}{h} = \log_e(a)$ and $\frac{\mathrm{d}}{\mathrm{d}x}\{a^x\} = a^x\log_e(a)$ are consequences of

$$a^x = \left(e^{\log_e(a)}\right)^x = e^{x\log_e(a)}$$

For the details, see the end of Section 2.7.

Things to Have Memorized

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{e^x\right\} = e^x$$

When a is any constant,

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{a^x\right\} = a^x \log_e(a)$$

Let $f(x) = \frac{e^x}{3x^5}$. When is the tangent line to $f(x)$ horizontal?

Evaluate $\frac{\mathrm{d}}{\mathrm{d}x}\left\{e^{3x}\right\}$

Evaluate $\frac{d}{dx}\left\{e^{3x}\right\}$

2 ways: product rule with $e^x \cdot e^x \cdot e^x$, or previous rule with $\left(e^3\right)^x$

Suppose the deficit, in millions, of a fictitious country is given by

$$f(x) = e^x(4x^3 - 12x^2 + 14x - 4)$$

where x is the number of years since the current leader took office. Suppose the leader has been in power for exactly two years.

1. Is the deficit increasing or decreasing?

2. Is the rate at which the deficit is growing increasing or decreasing?