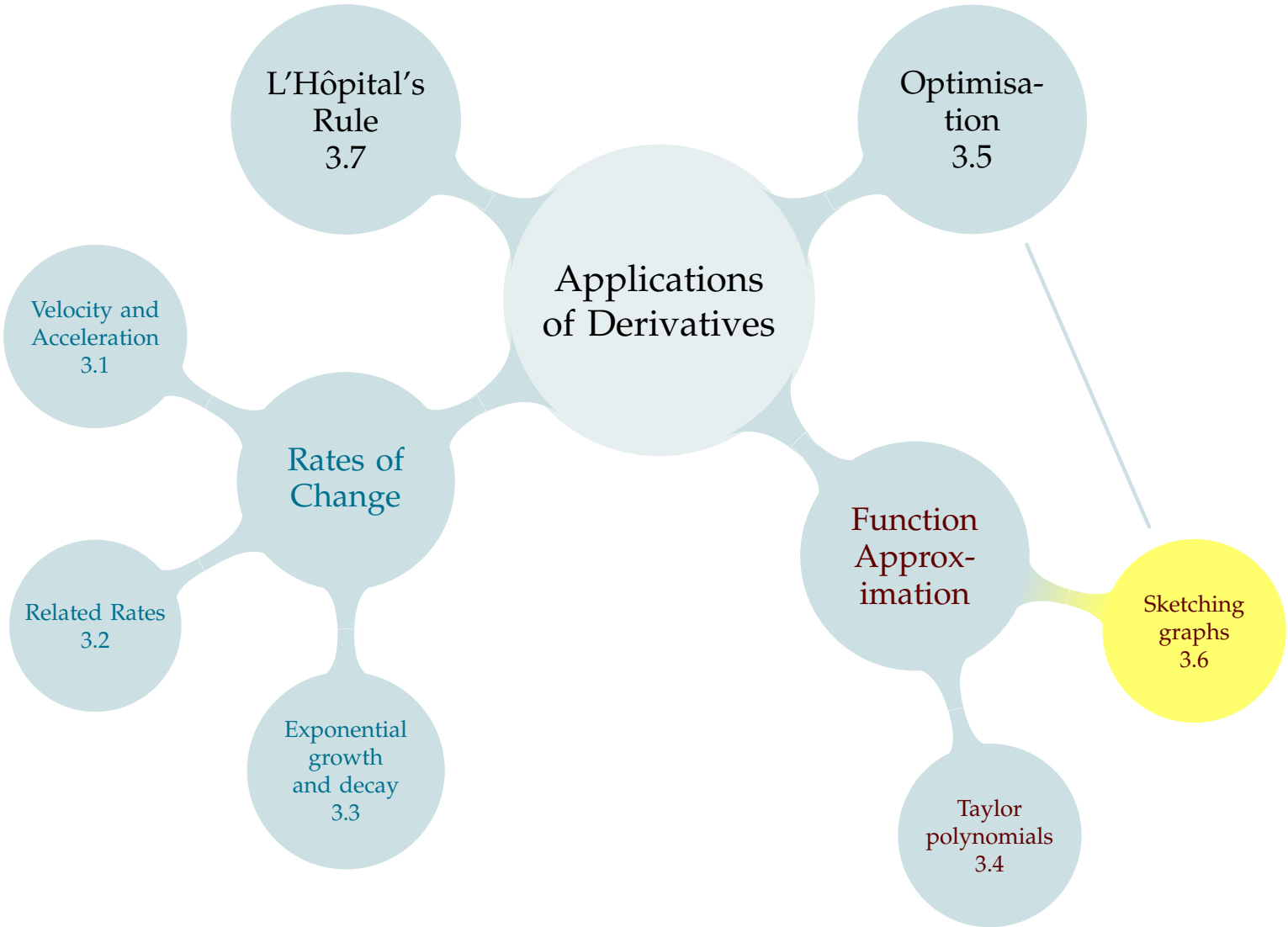
Table of Contents



CURVE SKETCHING

Review: find the domain of the following function.

$$f(x) = \frac{\sqrt{3 - x^2}}{\log(x + 1)}$$

Where might you expect $f(x)$ to have a vertical asymptote? What does the function look like nearby?
(Recall: a vertical asymptote occurs at $x = a$ if the function has an infinite discontinuity at a. That is, $\lim_{x \to a^{\pm}} f(x) = \pm\infty$.)

Where is $f(x) = 0$?

What happens to $f(x)$ near its other endpoint, $x = -1$?

Curve Sketching

Good things to check:

- Domain
- Vertical asymptotes: $\lim_{x \to a} f(x) = \pm\infty$
- Intercepts: $x = 0, f(x) = 0$
- Horizontal asymptotes and end behavior: $\lim_{x \to \pm\infty} f(x)$

CURVE SKETCHING

Identify: domain, vertical asymptotes, intercepts, and horizontal asymptotes

$$f(x) = \frac{x-2}{(x+3)^2}$$

Curve Sketching

Identify: domain, vertical asymptotes, intercepts, and horizontal asymptotes

$$f(x) = \frac{(x+2)(x-3)^2}{x(x-5)}$$

First Derivative

Add complexity: Increasing/decreasing, critical and singular points.

$$f(x) = \frac{1}{2}x^4 - \frac{4}{3}x^3 - 15x^2$$

What does the graph of the following function look like?

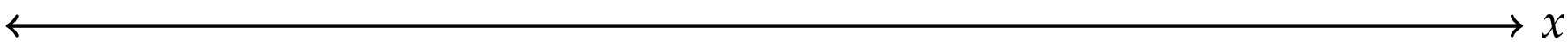
$$f(x) = \frac{1}{3}x^3 + 2x^2 + 4x + 24$$

What does the graph of the following function look like?

$$f(x) = e^{\frac{x+1}{x-1}}$$

Signs of Factored Functions

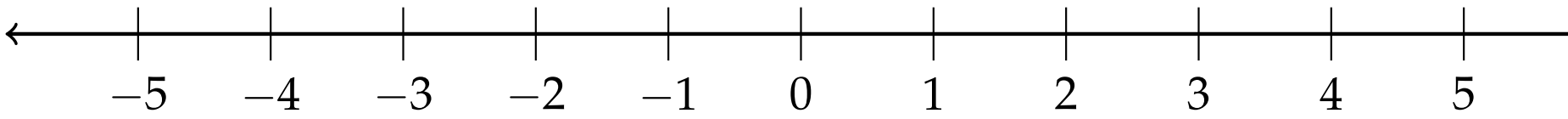
$$f(x) = \quad (x-1) \quad (x-2)^2 \quad (x-3)$$



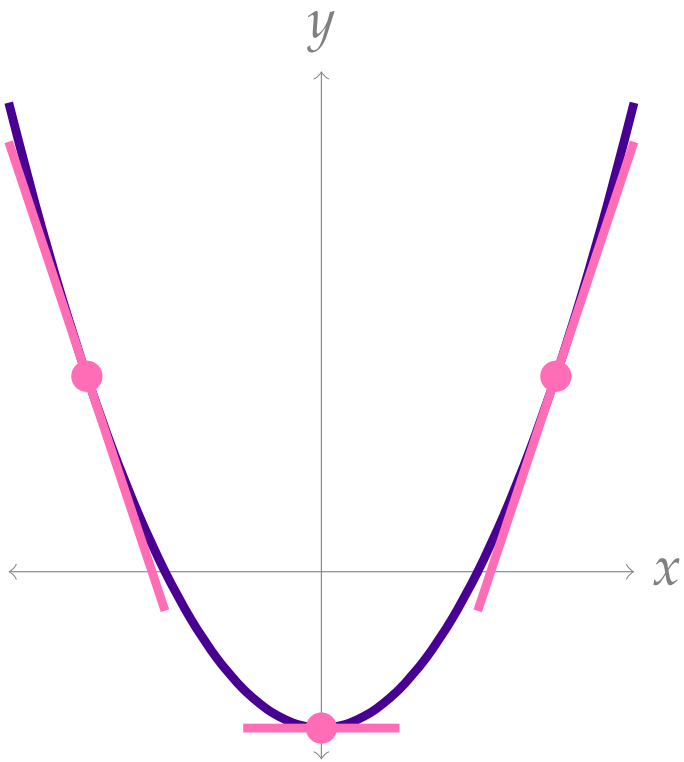
Signs of Factored Functions

$$f(x) = (x-3)(x-1)^2x(x+2)^3(x+5)^4$$

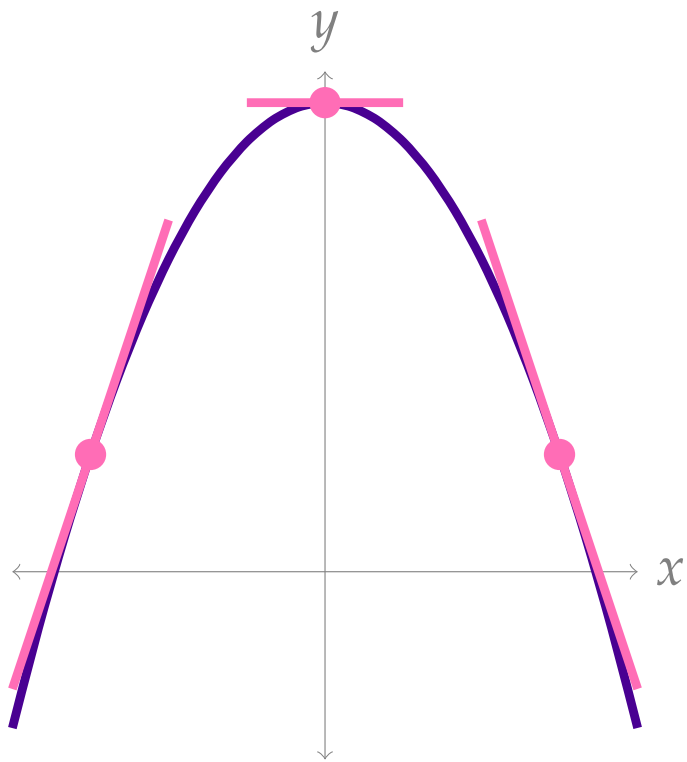
Where is $f(x)$ positive? Where is it negative?



Concavity



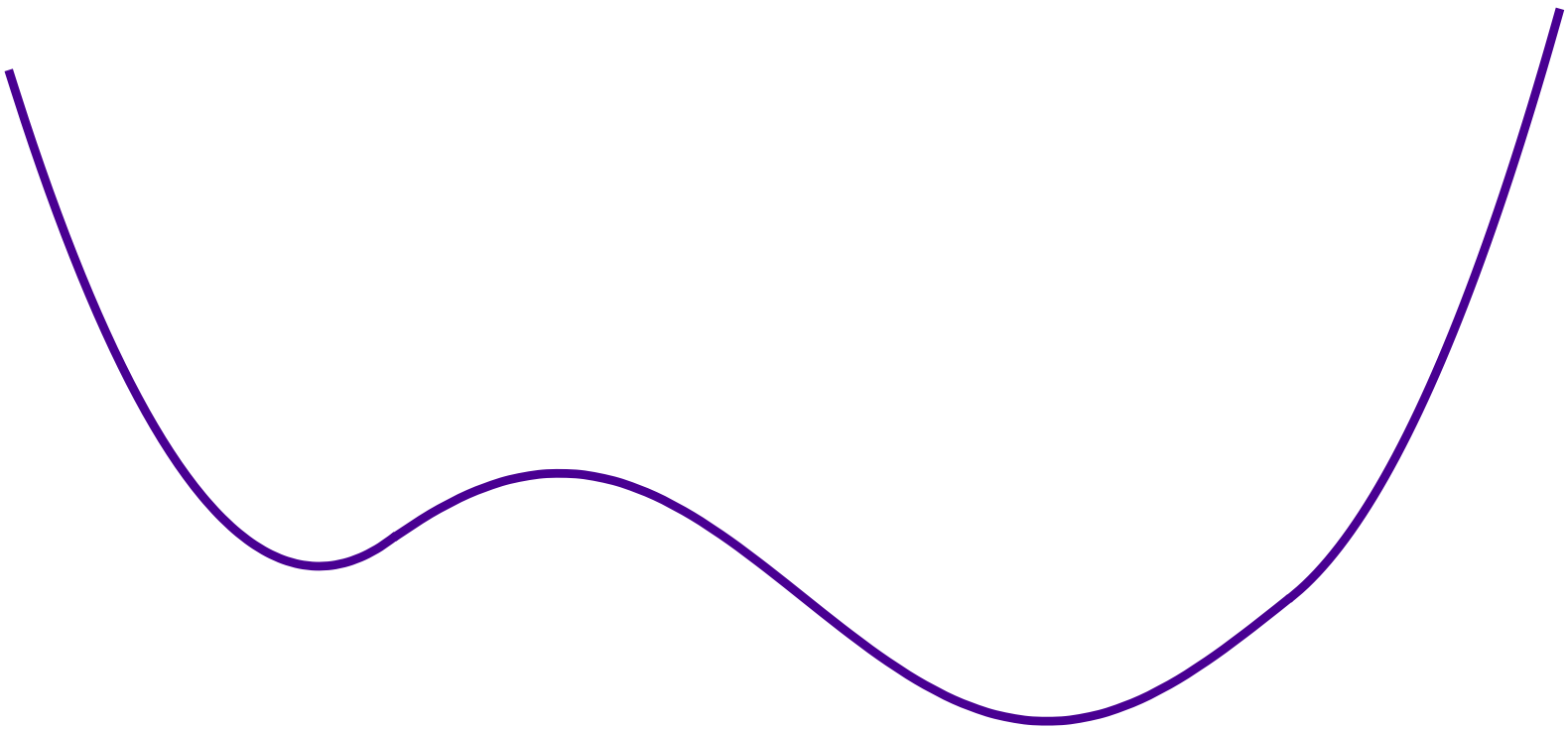
- Slopes are increasing
- $f''(x) > 0$
- "concave up"
- tangent line below curve



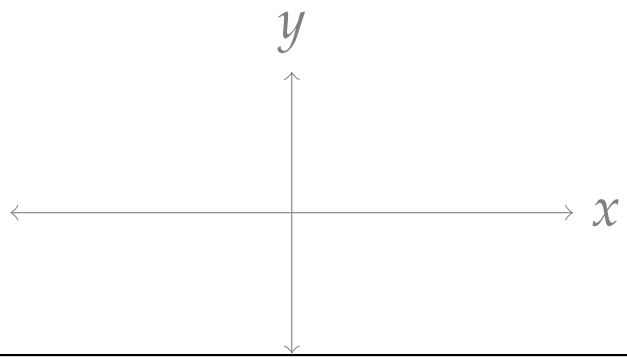
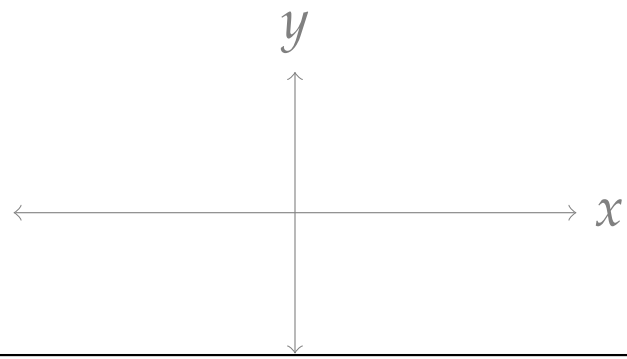
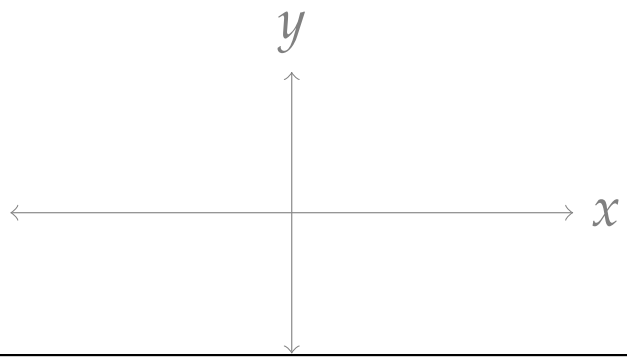
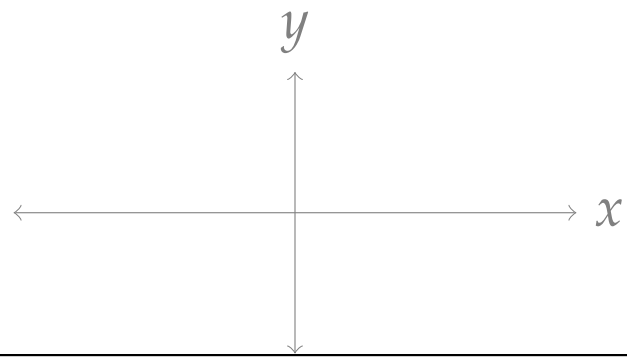
- Slopes are decreasing
- $f''(x) < 0$
- "concave down"
- tangent line above curve

Mnemonic

Concavity



Sketch graphs with the following properties, or explain that none exist.

	concave up	concave down
increasing		
decreasing		

Poll Questions

Describe the concavity of the function $f(x) = e^x$.

A. concave up
B. concave down
C. concave up for $x < 0$; concave down for $x > 0$
D. concave down for $x < 0$; concave up for $x > 0$
E. I'm not sure

Is it possible to be concave up and decreasing?

A. Yes B. No C. I'm not sure

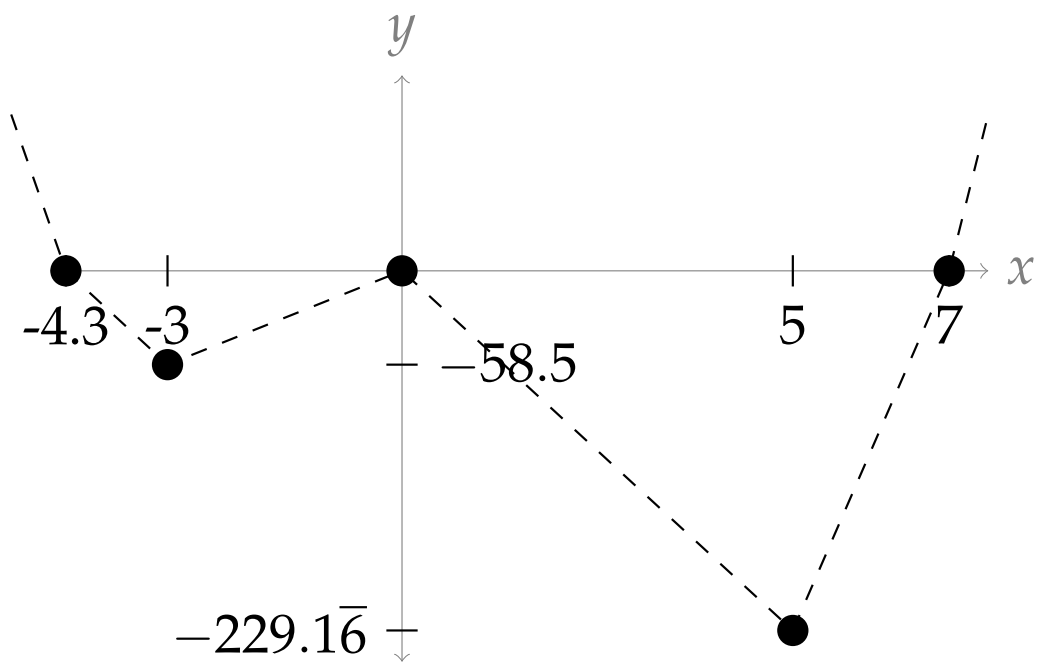
Suppose a function $f(x)$ is defined for all real numbers, and is concave up on the interval $[0, 1]$. Which of the following must be true?

A. $f'(0) < f'(1)$
B. $f'(0) > f'(1)$
C. $f'(0)$ is positive
D. $f'(0)$ is negative
E. I'm not sure

Revisiting a previous example

original example

$$f(x) = \frac{1}{2}x^4 - \frac{4}{3}x^3 - 15x^2$$

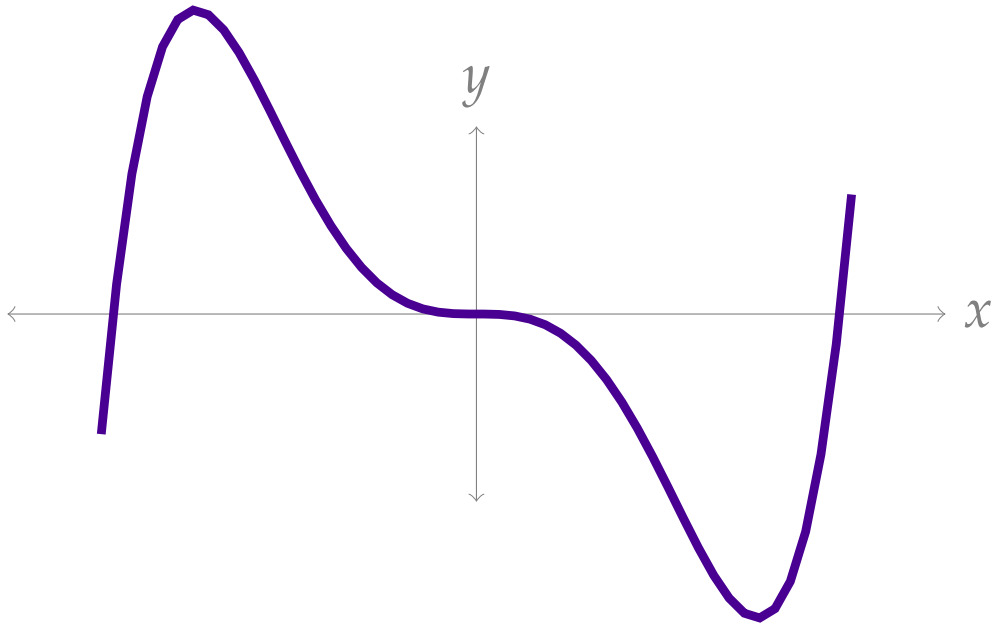


$$f''(x) = 6x^2 - 8x - 30 = 2(x-3)(3x+5)$$

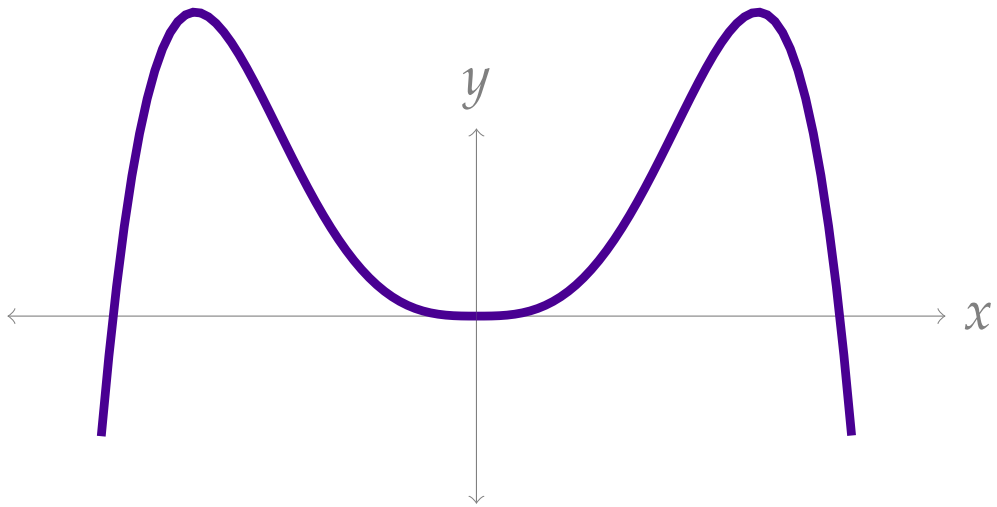
Sketch:

$$f(x) = x^5 - 15x^3$$

Even and Odd Functions



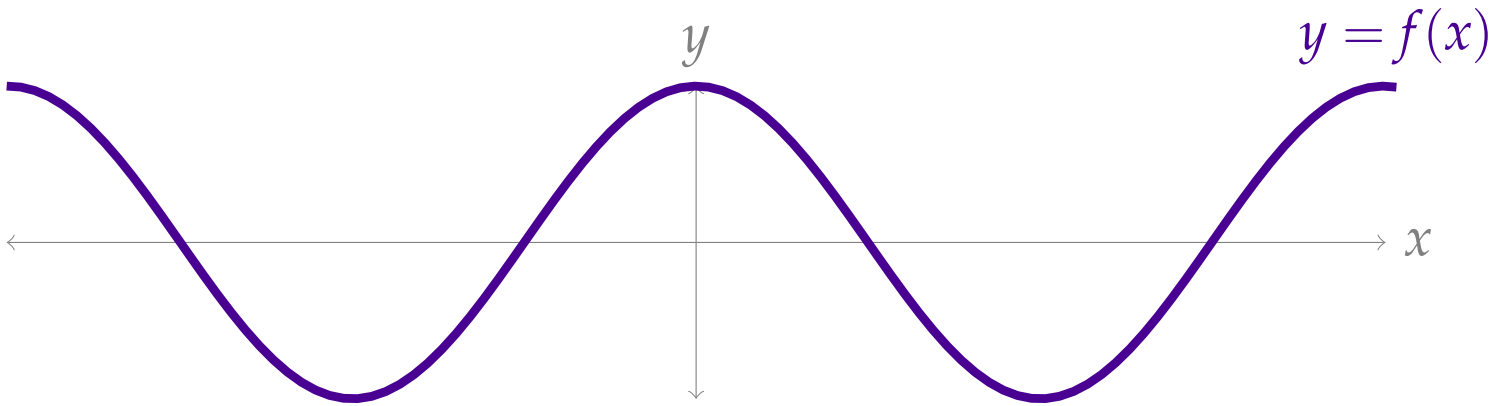
Even and Odd Functions



Even Function – Definition 3.6.5

A function $f(x)$ is even if, for all x in its domain,

$$f(-x) = f(x)$$



Even Functions

Even Function – Definition 3.6.5

A function $f(x)$ is even if, for all x in its domain,

$$f(-x) = f(x)$$

Examples:

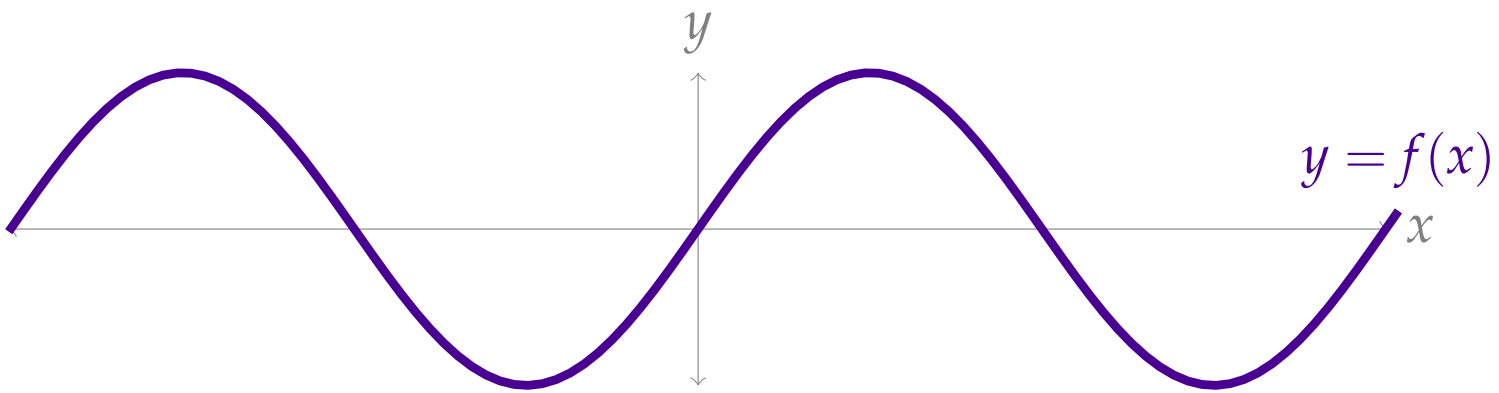
$f(x) = x^2$

$f(x) = x^4$

$f(x) = \cos(x)$

$f(x) = \dfrac{x^4 + \cos(x)}{x^{16} + 7}$

Odd Functions



Suppose $f(1) = 2$. Then $f(-1) =$
Suppose $f(3) = -2$. Then $f(-3) =$

Odd Function – Definition 3.6.6

A function $f(x)$ is odd if, for all x in its domain,

$$f(-x) = -f(x)$$

Odd Functions

Odd Function – Definition 3.6.6

A function $f(x)$ is odd if, for all x in its domain,

$$f(-x) = -f(x)$$

Examples:

$f(x) = x$

$f(x) = x^3$

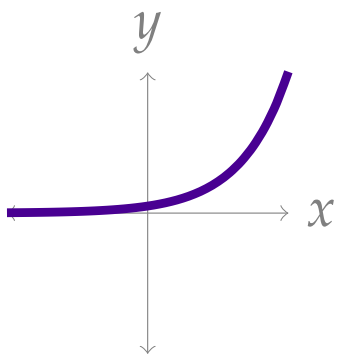
$f(x) = \sin(x)$

$f(x) = \dfrac{x(1+x^2)}{x^2+5}$

Poll Tiiime

Pick out the odd function.

A:



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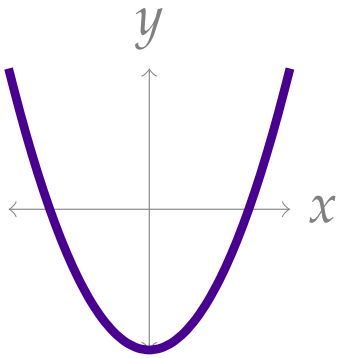
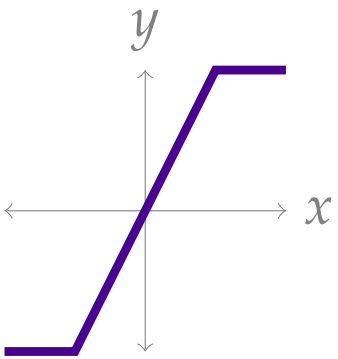
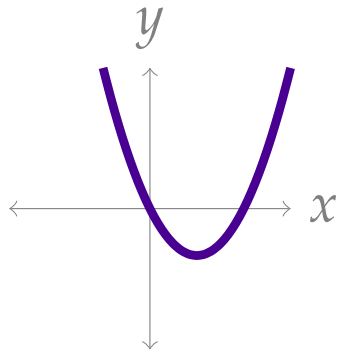


C:



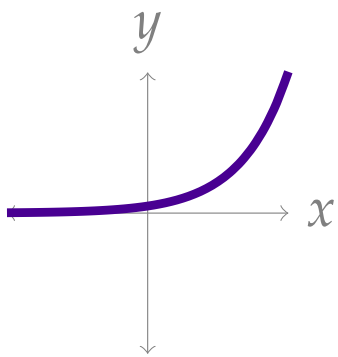
D:



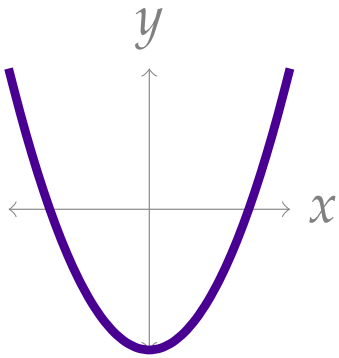
Poll Tiiime

Pick out the even function.

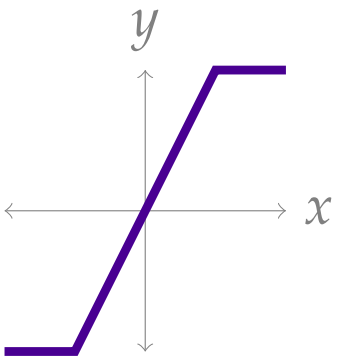
A:



B:



C:



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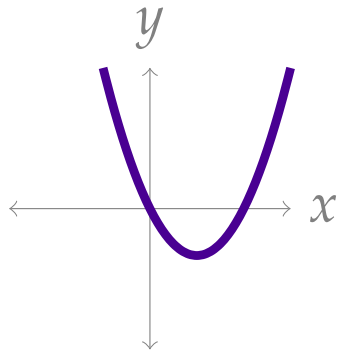


Even more Poll tiiiiime

Suppose $f(x)$ is an odd function, continuous, defined for all real numbers. What is $f(0)$? Pick the best answer.

A. $f(0) = f(-0)$

B. $f(0) = -f(0)$

C. $f(0) = 0$

D. all of the above are true

E. none of the above are necessarily true

Even more and more Poll tiiiiime

Suppose $f(x)$ is an even function, continuous, defined for all real numbers. What is $f(0)$? Pick the best answer.

A. $f(0) = f(-0)$

B. $f(0) = -f(0)$

C. $f(0) = 0$

D. all of the above are true

E. none of the above are necessarily true

OK OK... LAST ONE

Suppose $f(x)$ is an even function, differentiable for all real numbers. What can we say about $f'(x)$?

A. $f'(x)$ is also even

B. $f'(x)$ is odd

C. $f'(x)$ is constant

D. all of the above are true

E. none of the above are necessarily true

Periodicity

Periodic – Definition 3.6.9

A function is periodic with period $P > 0$ if

$$f(x) = f(x + P)$$

whenever x and $x + P$ are in the domain of f, and P is the smallest such (positive) number

Examples: $\sin(x)$, $\cos(x)$ both have period 2π; $\tan(x)$ has period π.

Ignoring concavity, sketch $f(x) = \sin(\sin x)$.

Challenge: ignoring exact locations of extrema, sketch $g(x) = \sin(2\pi \sin x)$.

Let's Graph

$$f(x) = (x^2 - 64)^{1/3}$$

$$f'(x) = \frac{2x}{3(x^2 - 64)^{2/3}};$$

$$f''(x) = \frac{-2(\frac{1}{3}x^2 + 64)}{3(x^2 - 64)^{5/3}}$$

Let's Graph

$$f(x) = \frac{x^2 + x}{(x+1)(x^2+1)^2}$$

Note: for $x \neq -1, f(x) = \frac{x(x+1)}{(x+1)(x^2+1)^2} = \frac{x}{(x^2+1)^2}$

$$g(x) := \frac{x}{(x^2+1)^2}$$

$$g'(x) = \frac{1 - 3x^2}{(x^2+1)^3}$$

$$g''(x) = \frac{12x(x^2-1)}{(x^2+1)^4}$$

Let's Graph

$$f(x) = x(x-1)^{2/3}$$

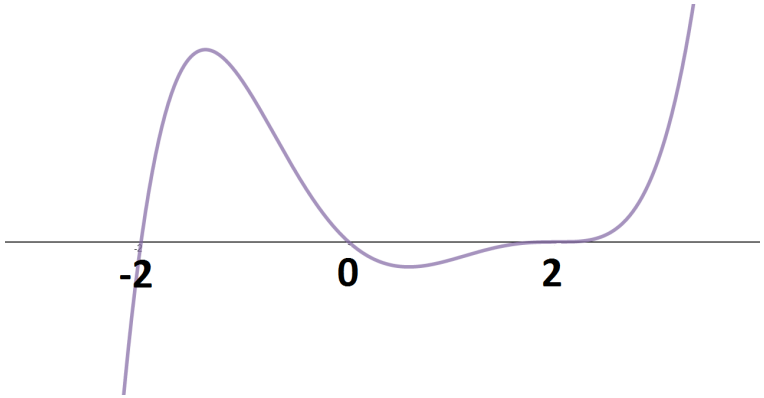
- $f'(x) = \dfrac{5x-3}{3\sqrt[3]{x-1}}$
- $f''(x) = \dfrac{2(5x-6)}{9(\sqrt[3]{x-1})^4}$

- $f(3/5) \approx 0.3$
- $f(6/5) \approx 0.4$

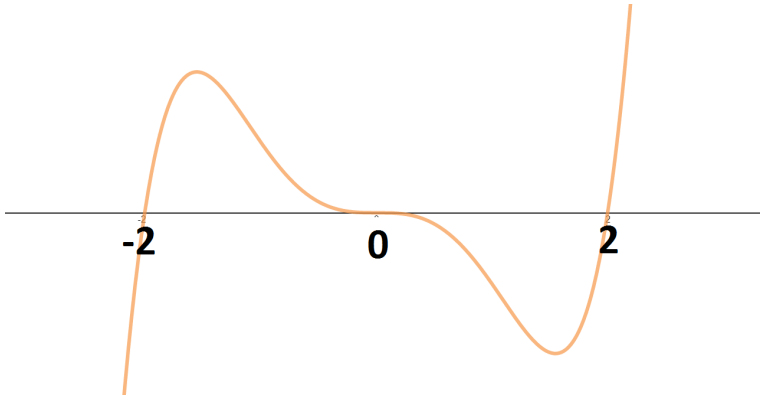
Ch 3.6 Review: matching

Match the Function to its Graph

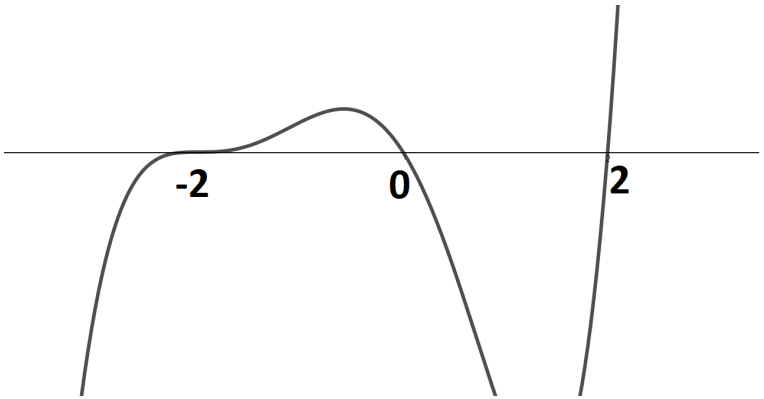
A. $f(x) = x^3(x+2)(x-2) = x^5 - 4x^3$
B. $f(x) = x(x+2)^3(x-2) = x^5 + 4x^4 - 16x^2 - 16x$
C. $f(x) = x(x+2)(x-2)^3 = x^5 - 4x^4 + 16x^2 - 16x$



I



III



II

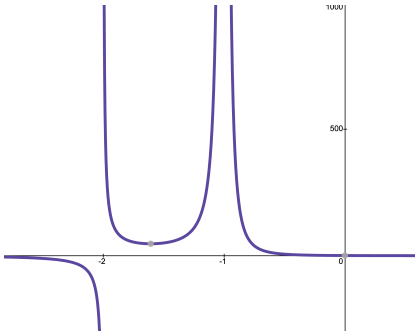
A. $f(x) = \dfrac{x-1}{(x+1)(x+2)}$

B. $f(x) = \dfrac{(x-1)^2}{(x+1)(x+2)}$

C. $f(x) = \dfrac{x-1}{(x+1)^2(x+2)}$

D. $f(x) = \dfrac{(x-1)^2}{(x+1)^2(x+2)}$

I.



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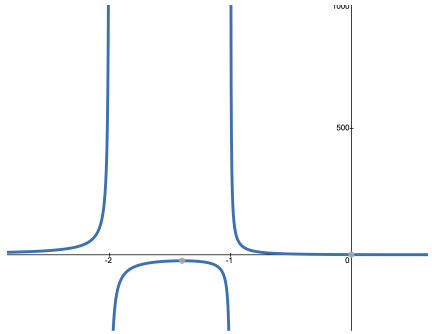

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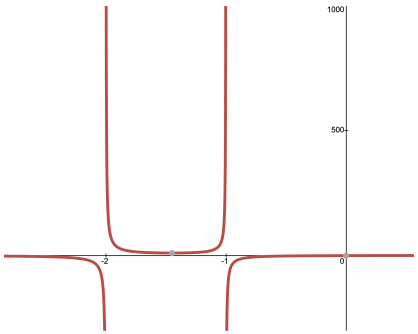
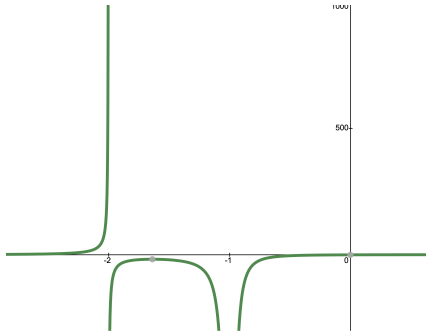
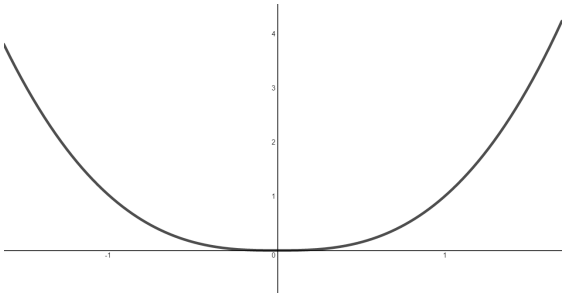


IV.



Match the Function to its Graph

A. $f(x) = |x|^e$ B. $f(x) = e^{|x|}$ C. $f(x) = e^{x^2}$ D. $f(x) = e^{x^4 - x}$



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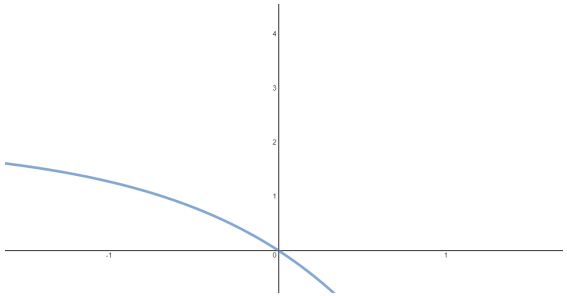
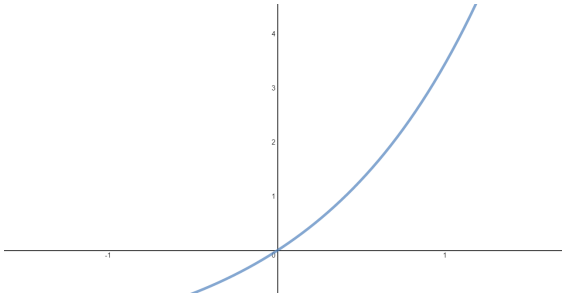


II



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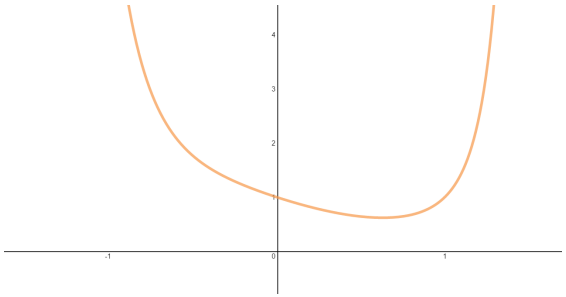
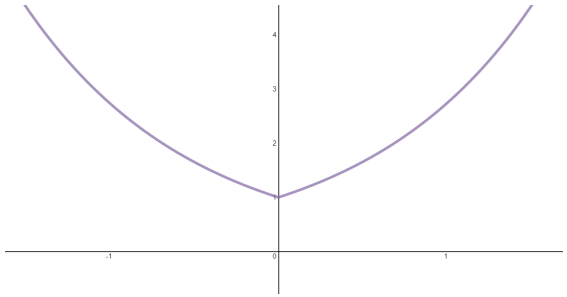
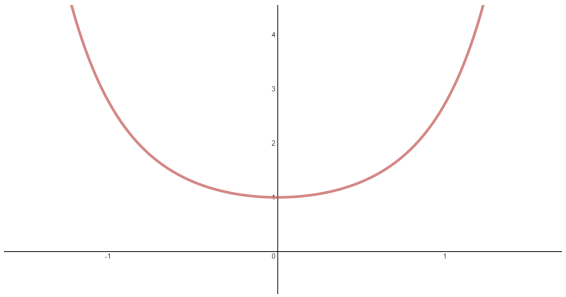


IV



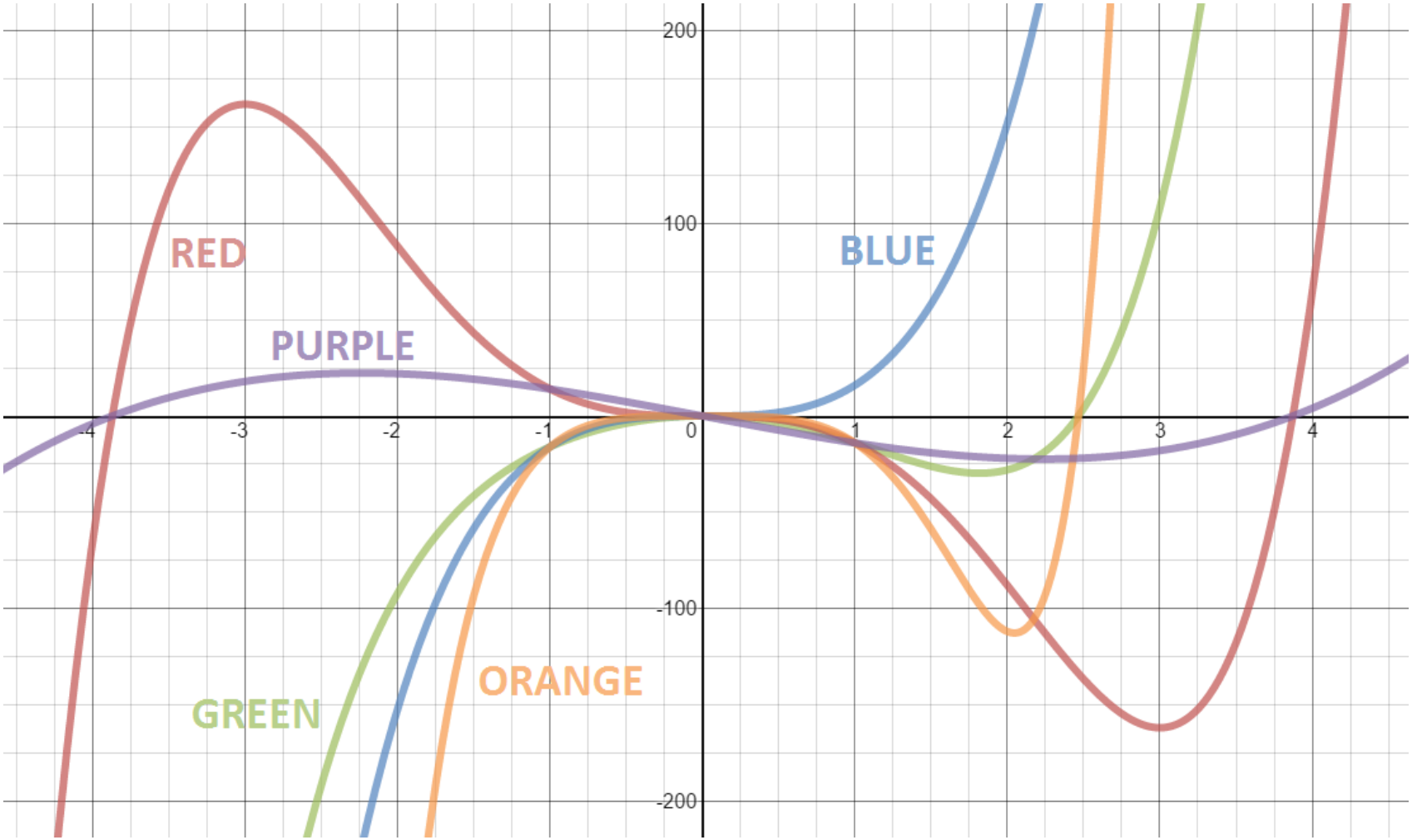
V



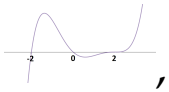
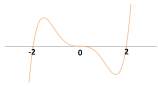
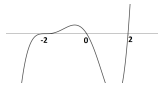
VI

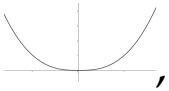
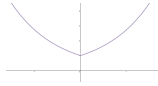
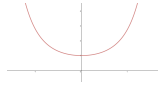
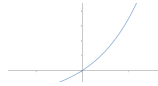
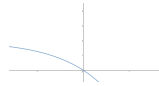
A. $f(x) = x^5 + 15x^3$ B. $f(x) = x^5 - 15x^3$ C. $f(x) = x^5 - 15x^2$

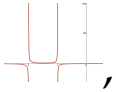
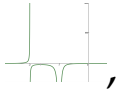
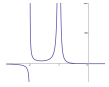
D. $f(x) = x^3 - 15x$ E. $f(x) = x^7 - 15x^4$

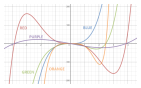


Included Work

 ,  ,  screenshots of graphs generated using Desmos Graphing Calculator https://www.desmos.com/calculator (accessed 13 November 2015), 35

 ,  ,  ,  ,  ,  screenshots of graphs generated using Desmos Graphing Calculator https://www.desmos.com/calculator, (accessed 13 November 2015), 37

 ,  ,  ,  screenshots from graphs generated using Desmos Graphing Calculator https://www.desmos.com/calculator (accessed 16 July 2021), 36

 screenshot from graphs generated using Desmos Graphing Calculator https://www.desmos.com/calculator, with text added (accessed 13 November 2015), 38