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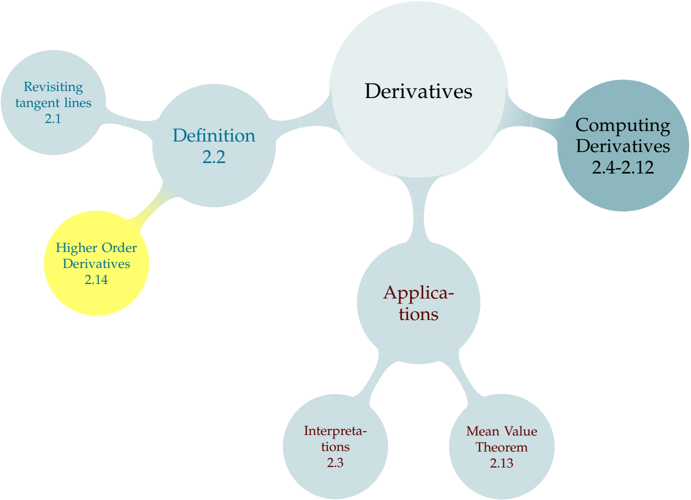

Higher Order Derivatives

Evaluate $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\mathrm{d}}{\mathrm{d}x}\left[x^5 - 2x^2 + 3\right]\right]$

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The derivative of a derivative is called the **second derivative**, written

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Similarly, the derivative of a second derivative is a third derivative, etc.

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- $f''(x)$ and $f^{(2)}(x)$ and $\frac{\mathrm{d}^2 f}{\mathrm{d}x^2}(x)$ all mean $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}}{\mathrm{d}x}f(x)\right)$
- $f'''(x)$ and $f^{(3)}(x)$ and $\frac{\mathrm{d}^3 f}{\mathrm{d}x^3}(x)$ all mean $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}}{\mathrm{d}x}f(x)\right)\right)$
- $f^{(4)}(x)$ and $\frac{\mathrm{d}^4 f}{\mathrm{d}x^4}(x)$ both mean $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}}{\mathrm{d}x}f(x)\right)\right)\right)$
- and so on.

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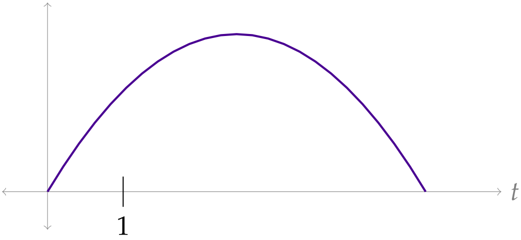

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$$\begin{aligned} s(t) &= t(5-t) = 5t - t^2 \\ s'(t) &= 5 - 2t \\ s'(1) &= 5 - 2(1) = 3 = \text{vel} \end{aligned}$$

Units of velocity: $\frac{\Delta s}{\Delta t} = \frac{m}{s}$

Units of acceleration: $\frac{\Delta s'}{\Delta t} = \frac{m/s}{s} = \frac{m}{s^2}$

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$$s(t) = t(5-t) = 5t - t^2$$

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Concept Check

True or False: If $f'(1) = 18$, then $f''(1) = 0$, since the $\frac{\mathrm{d}}{\mathrm{d}x}\{18\} = 0$.

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B. $f'(0) = 0$
C. $f''(0) = 0$
D. $f'''(0) = 0$
E. $f^{(4)}(0) = 0$

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A. $f(0) = 0$ $\quad f(x) = ax^2 + bx + c$
B. $f'(0) = 0$ $\quad f'(x) = 2ax + b$
C. $f''(0) = 0$ $\quad f''(x) = 2a$
D. $f'''(0) = 0$ $\quad f'''(x) = 0$ ✓
E. $f^{(4)}(0) = 0$ $\quad f^{(4)}(x) = 0$ ✓

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Q

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Which of the following is always true of a CUBIC polynomial $f(x)$?

A. $f(0) = 0$ $\quad f(x) = ax^3 + bx^2 + cx + d$
B. $f'(0) = 0$ $\quad f'(x) = 3ax^2 + 2bx + c$
C. $f''(0) = 0$ $\quad f''(x) = 6ax + 2b$
D. $f'''(0) = 0$ $\quad f'''(x) = 6a$
E. $f^{(4)}(0) = 0$ $\quad f^{(4)}(x) = 0$ ✓

A

Implicit Differentiation

Suppose $y(x)$ is a function such that

$$y(x) = y^3x + x^2 - 1$$

Find $y''(x)$ at the point $(-2, 1)$.

Implicit Differentiation

Suppose $y(x)$ is a function such that

$$y(x) = y^3x + x^2 - 1$$

Find $y''(x)$ at the point $(-2, 1)$. We start by differentiating both sides of the function. Remember that y is a function, not a variable.

$$\begin{aligned} y(x) &= y(x)^3x + x^2 + 1 \\ \frac{\mathrm{d}y}{\mathrm{d}x}(x) &\overset{\text{prod}}{=} y(x)^3 + 3xy(x)^2\frac{\mathrm{d}y}{\mathrm{d}x}(x) + 2x \end{aligned} \qquad (*)$$

Let's differentiate both sides again. Remember we have a rule for the product of three functions.

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 3y^2\frac{\mathrm{d}y}{\mathrm{d}x} + 3\left(y^2\frac{\mathrm{d}y}{\mathrm{d}x} + x \cdot 2y\frac{\mathrm{d}y}{\mathrm{d}x} \cdot \frac{\mathrm{d}y}{\mathrm{d}x} + xy^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right) + 2 \qquad (**)$$

When $x = -2$ and $y = 1$, using $(*)$, we find

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(-2,1)} = 1^3 + 3(-2)(1^2)\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(-2,1)} + 2(-2) = -3 - 6\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(-2,1)}$$

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(-2,1)} = -\frac{3}{7}$$

We set $x = -2$, $y = 1$, and $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3}{7}$ in equation $(**)$. Now by $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$, we actually mean $\left.\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right|_{(-2,1)}$, but to avoid clutter we don't write it that way until the end.

$$\begin{aligned}
\frac{\mathrm{d}^2y}{\mathrm{d}x^2} &= 3(1)\left(-\frac{3}{7}\right) + 3\left((1)\left(-\frac{3}{7}\right) + (-2)\cdot 2(1)\left(-\frac{3}{7}\right)\cdot\left(-\frac{3}{7}\right) + (-2)(1)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right) + 2 \\
&= -\frac{9}{7} + 3\left(-\frac{3}{7} - \frac{36}{49} - 2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right) + 2 \\
&= \left(2 - \frac{18}{7} - \frac{108}{49}\right) - 6\frac{\mathrm{d}^2y}{\mathrm{d}x^2} \\
7\frac{\mathrm{d}^2y}{\mathrm{d}x^2} &= -\frac{136}{7^2} \\
\left.\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right|_{(-2,1)} &= -\frac{136}{7^3}
\end{aligned}$$

Included Work