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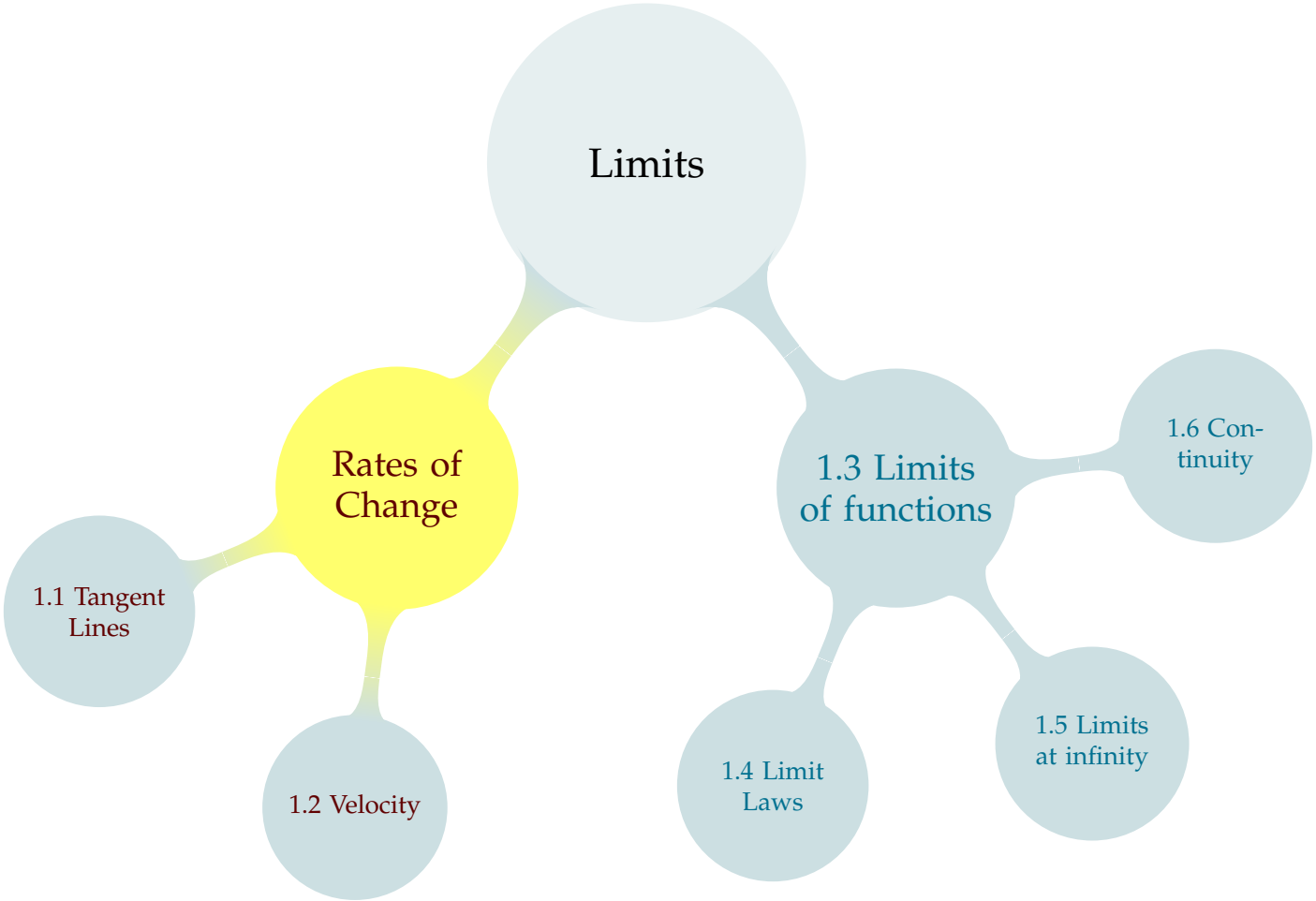
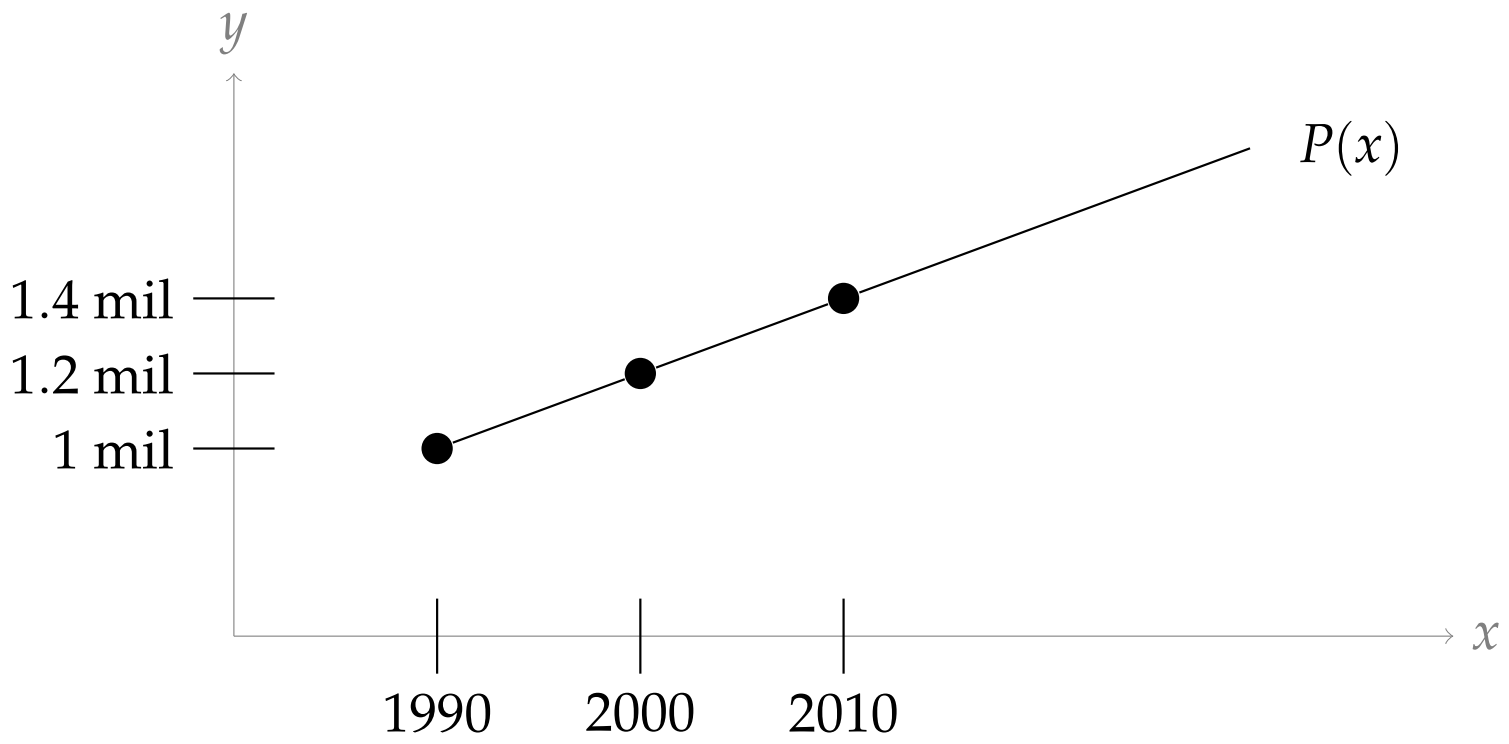


Rates of Change

Suppose the population of a small country was 1 million individuals in 1990, and is growing at a steady rate of 20,000 individuals per year.

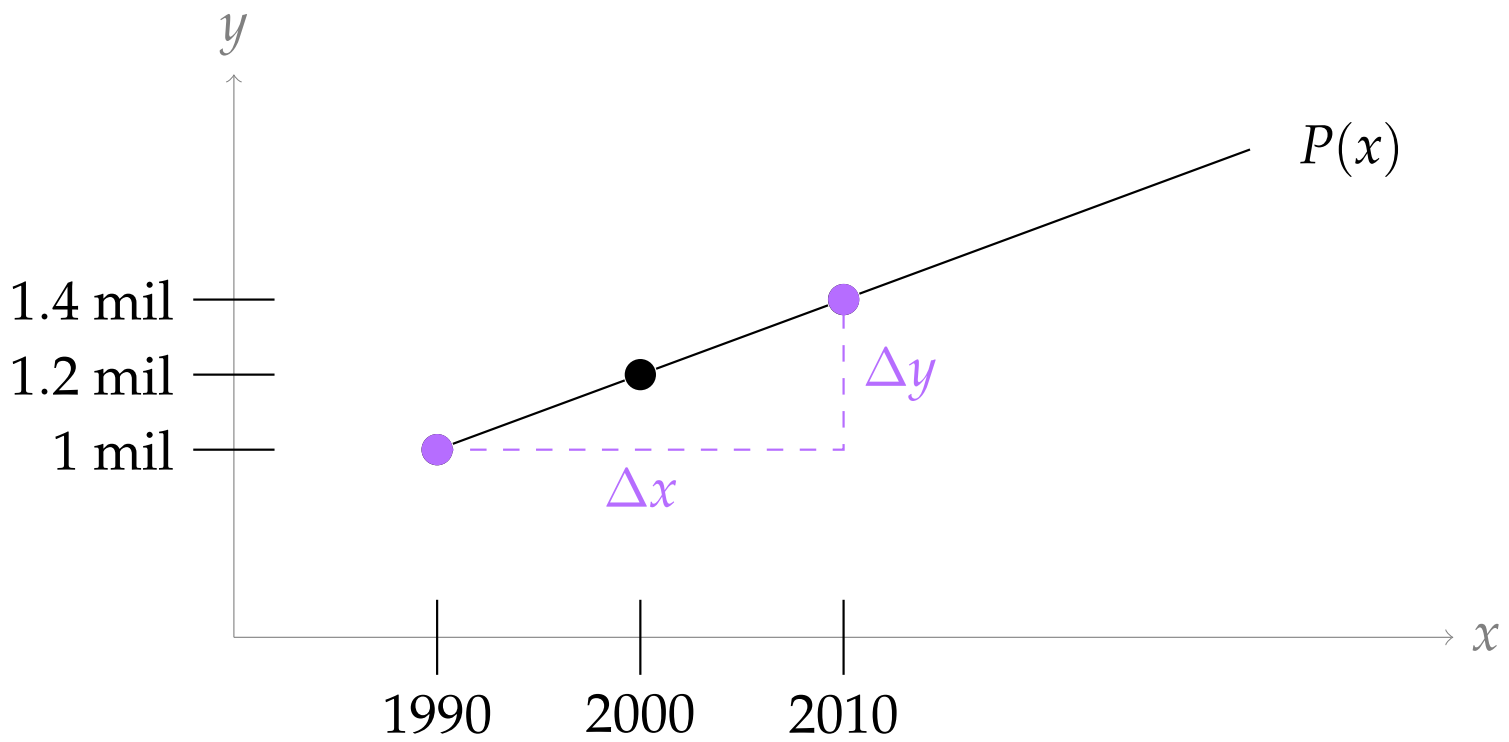


Definition

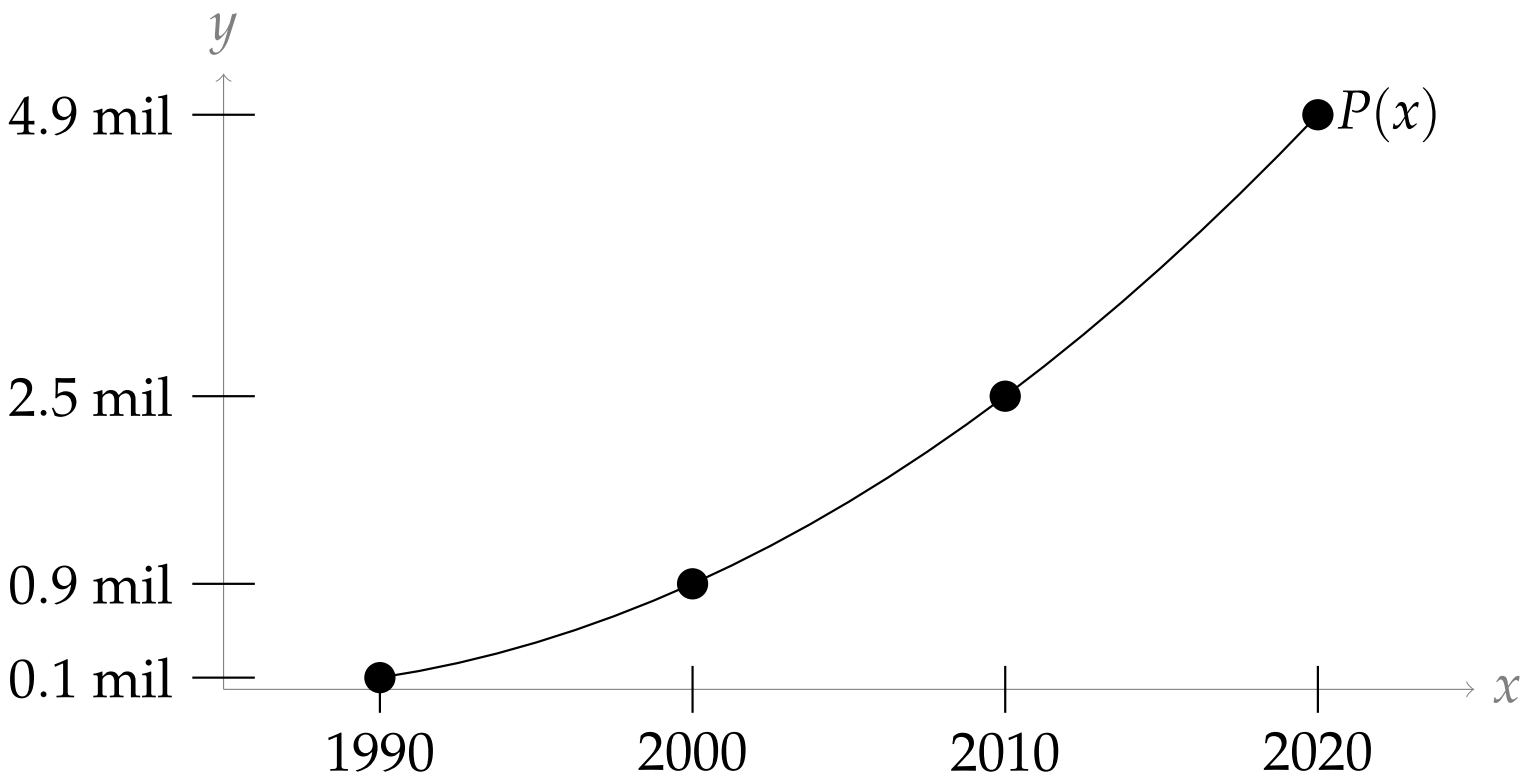
The **slope** of a line that passes through the points (x_1, y_1) and (x_2, y_2) is "rise over run"

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

This is also called the **rate of change** of the function.
If a line has equation $y = mx + b$, its slope is m.


y
P(x)
1.4 mil
1.2 mil
1 mil
Δy
Δx
x
1990
2000
2010

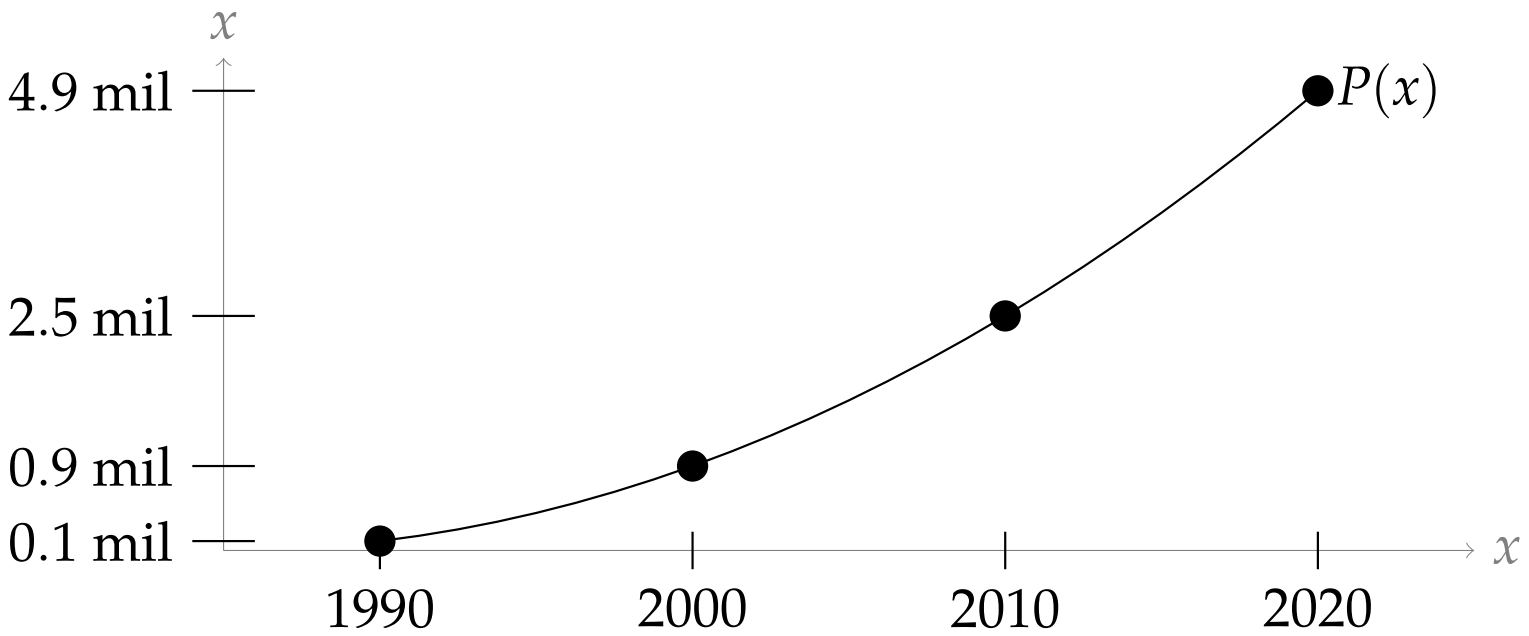
Suppose the population of a small country is given in the chart below.



Definition

Let $y = f(x)$ be a curve that passes through (x_1, y_1) and (x_2, y_2). Then the **average rate of change** of $f(x)$ when $x_1 \leq x \leq x_2$ is

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$


x
4.9 mil
2.5 mil
0.9 mil
0.1 mil
P(x)
x
1990
2000
2010
2020

Average Rate of Change and Slope

The average rate of change of a function $f(x)$ on the interval $[a, b]$ (where $a \neq b$) is "change in output" divided by "change in input:"

$$\frac{f(b) - f(a)}{b - a}$$

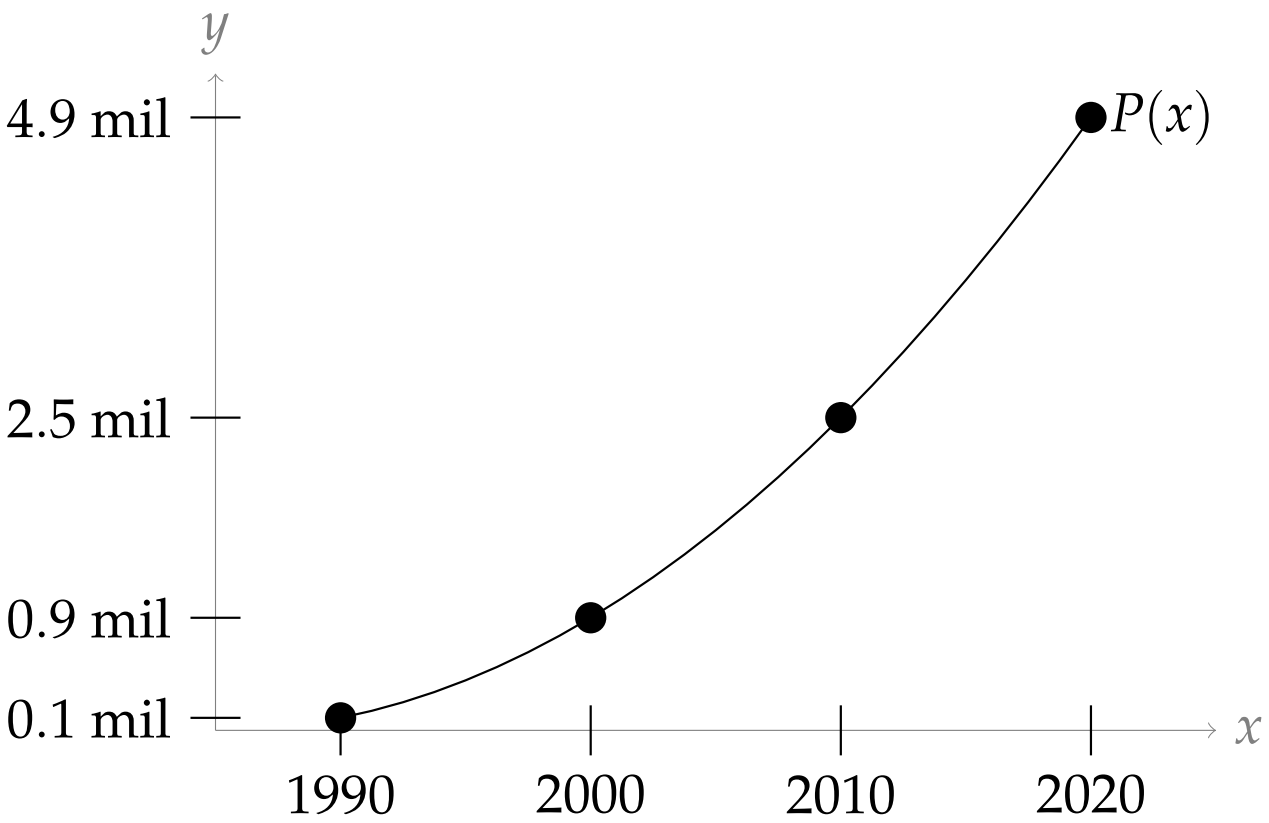
If the function $f(x)$ is a line, then the slope of the line is "rise over run,"

$$\frac{f(b) - f(a)}{b - a}$$

If a function is a line, its slope is the same as its average rate of change, which is the same for every interval.

If a function is not a line, its average rate of change might be different for different intervals, and we don't have a definition (yet) for its "slope."

How fast was this population growing in the year 2010? (What was its instantaneous rate of change?)



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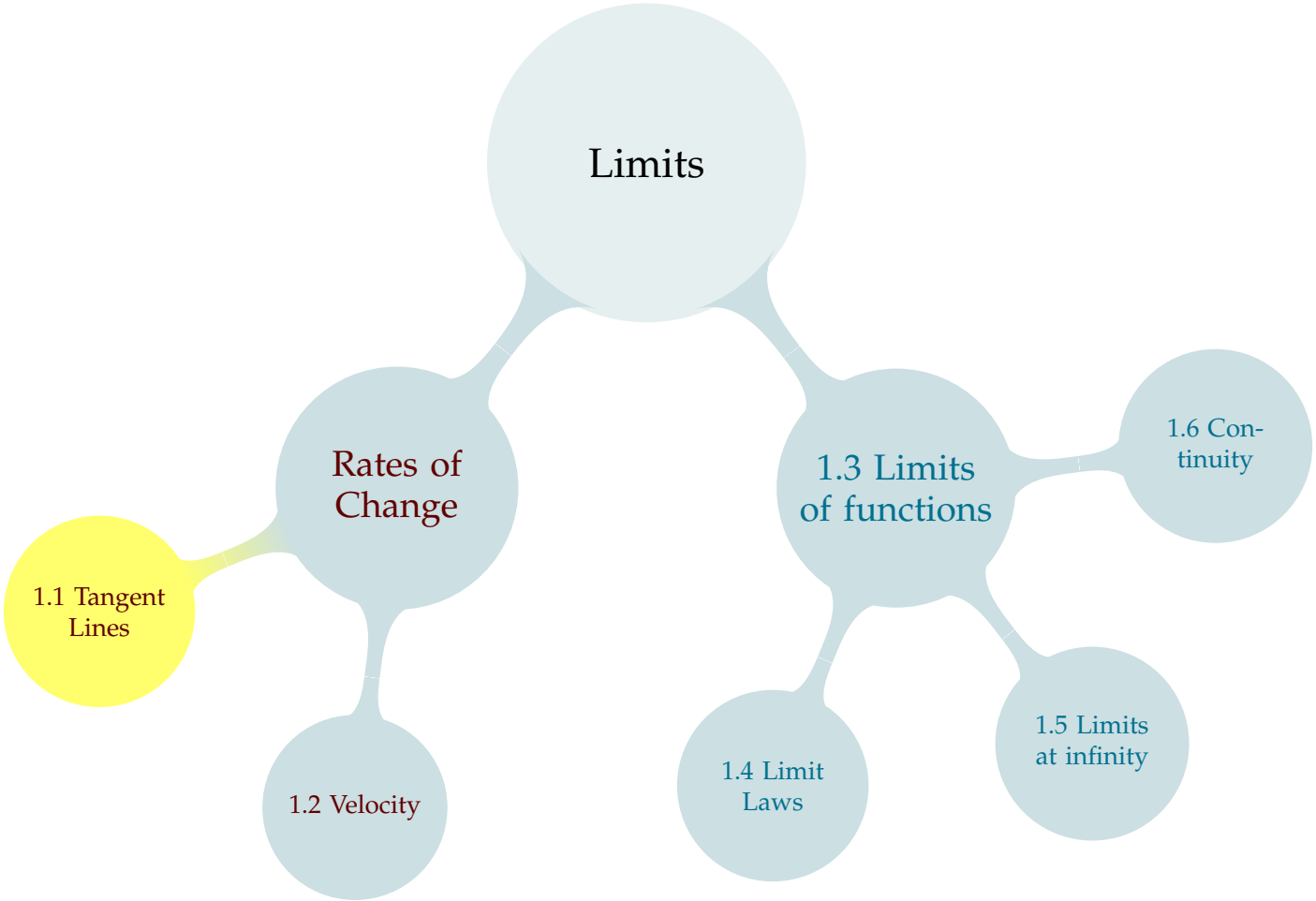


Definition

The **secant line** to the curve $y = f(x)$ through points R and Q is a line that passes through R and Q.

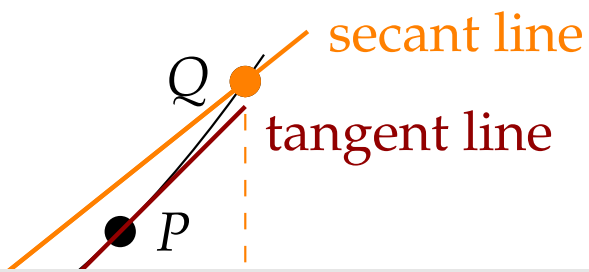
We call the slope of the secant line the **average rate of change of** $f(x)$ **from** R **to** Q**.**

Definition

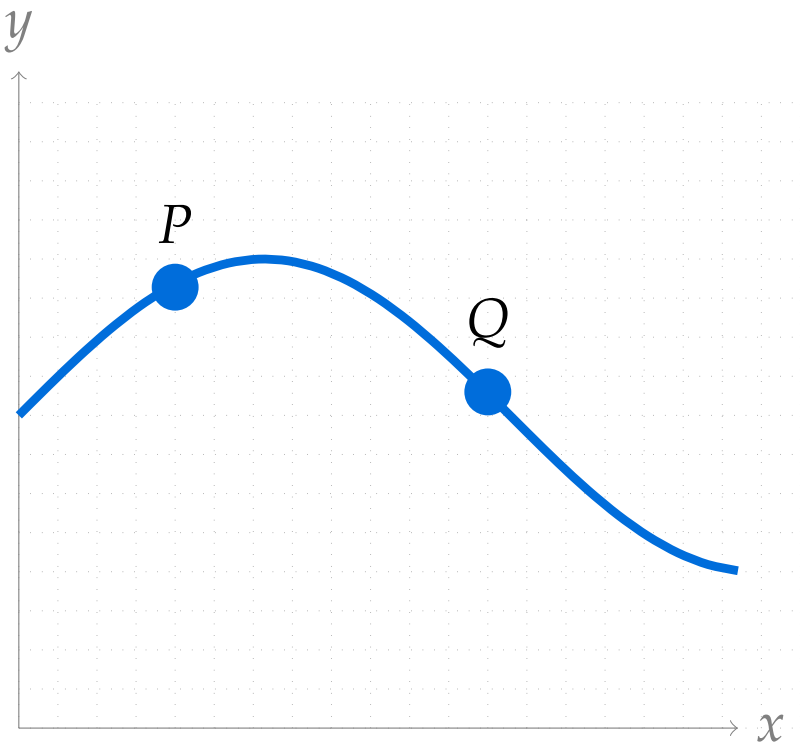
The **tangent line** to the curve $y = f(x)$ at point P is a line that

- passes through P and
- has the same slope as $f(x)$ at P.

We call the slope of the tangent line the **instantaneous rate of change of** $f(x)$ **at** P**.**



On the graph below, draw the secant line to the curve through points P and Q.



On the graph below, draw the tangent line to the curve at point P.