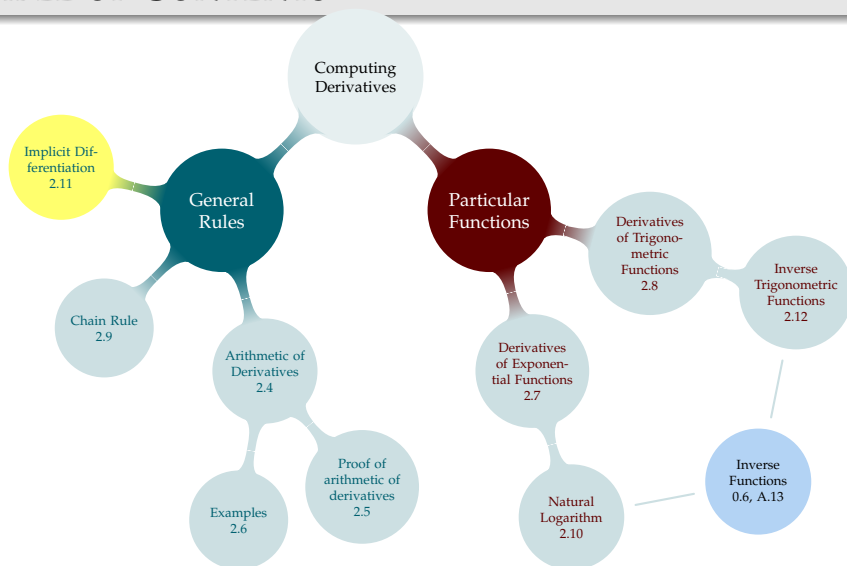# TABLE OF CONTENTS

IMPLICITLY DEFINED FUNCTIONS

$$y^2 + x^2 + xy + x^2y = 1$$

Which of the following points are on the curve?
$(0, 1)$, $(0, -1)$, $(0, 0)$, $(1, 1)$

If $x = -3$, what is y?

Implicitly Defined Functions

2022-03-10

Things to emphasize:

- One x might have multiple y

- You won't be asked to graph these

- We can solve questions without *needing* to see their graphs

- Locally looks like a function (explain "locally") so all the stuff that worked before still works now if we restrict where we're looking

$$y^2 + x^2 + xy + x^2y = 1$$
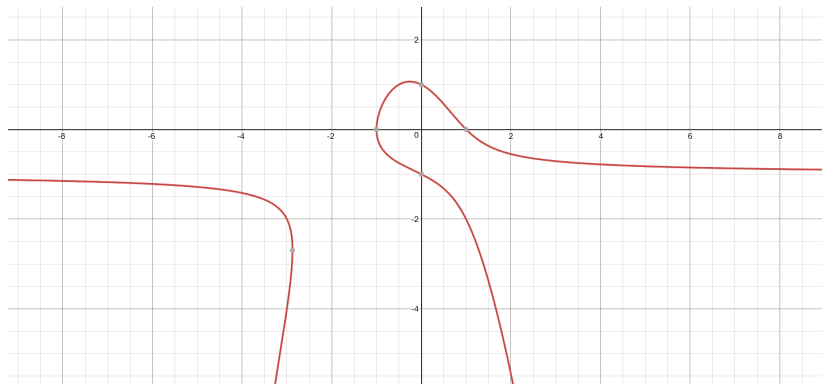
Still has a slope: $\frac{\Delta y}{\Delta x}$
Locally, y is still a function of x.

$$y^2 + x^2 + xy + x^2y = 1$$

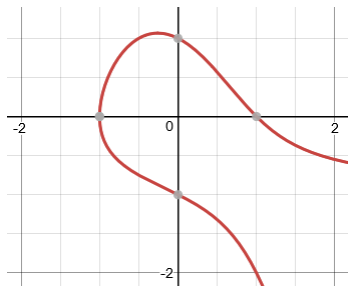Consider y as a function of x. Can we find $\frac{\mathrm{d}y}{\mathrm{d}x}$?

$\frac{\mathrm{d}}{\mathrm{d}x}[y] =$ $\qquad$ $\frac{\mathrm{d}}{\mathrm{d}x}[x] =$ $\qquad$ $\frac{\mathrm{d}}{\mathrm{d}x}[1] =$

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

Necessarily, $\frac{\mathrm{d}y}{\mathrm{d}x}$ depends on **both** y and x. Why?

NOW
YOU
Suppose $x^4 y + y^4 x = 2$. Find $\frac{dy}{dx}$ at the point $(1, 1)$.

Now you: Suppose $x^2y + y^3x = 2$. Find $\frac{dy}{dx}$ at the point $(1,1)$.

Students often struggle knowing when to replace variables with constants.

NOW
YOU Suppose $\dfrac{3y^2 + 2y + y^3}{x^2 + 1} = x$. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 0$, and the equations of the associated tangent line(s).

Use implicit differentiation to differentiate $\log(x)$, $x > 0$.

$$\log x = y(x)$$
$$x = e^{y(x)}$$

Use implicit differentiation to differentiate $\log |x|$, $x < 0$.

Use implicit differentiation to differentiate $\log_a(x)$, where $a > 0$ is a constant and $x > 0$.

Use implicit differentiation to differentiate $\log_a |x|$, $a > 0$.

Included Work