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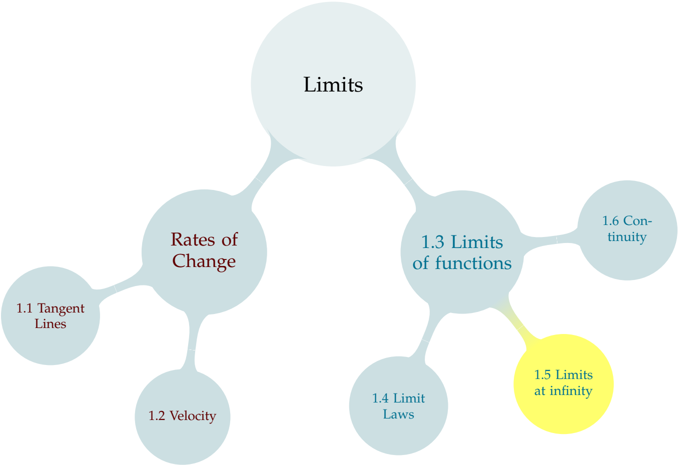


End Behavior

We write:

$$\lim_{x \to \infty} f(x) = L$$

to express that, as x grows larger and larger, $f(x)$ approaches L.

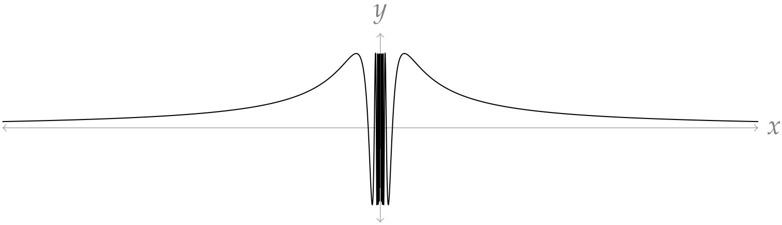
Similarly, we write:

$$\lim_{x \to -\infty} f(x) = L$$

to express that, as x grows more and more strongly negative, $f(x)$ approaches L.

If L is a number, we call $y = L$ a horizontal asymptote of $f(x)$.

Horizontal Asymptotes



$y = 0$ is a horizontal asymptote for $y = \sin\left(\frac{1}{x}\right)$

Common Limits at Infinity

$$\lim_{x\to\infty} 13 =$$

$$\lim_{x\to-\infty} 13 =$$

$$\lim_{x\to\infty} x^3 =$$

$$\lim_{x\to-\infty} x^3 =$$

$$\lim_{x\to\infty} \frac{1}{x} =$$

$$\lim_{x\to-\infty} \frac{1}{x} =$$

$$\lim_{x\to-\infty} x^{5/3} =$$

$$\lim_{x\to-\infty} x^{2/3} =$$

$$\lim_{x\to\infty} x^2 =$$

$$\lim_{x\to-\infty} x^2 =$$

Arithmetic with Limits at Infinity

Now You

$$\lim_{x\to\infty}\left(x+\frac{x^2}{10}\right)=$$

$$\lim_{x\to\infty}\left(x-\frac{x^2}{10}\right)=$$

$$\lim_{x\to-\infty}\left(x^2+x^3+x^4\right)=$$

$$\lim_{x\to-\infty}(x+13)\left(x^2+13\right)^{1/3}=$$

Calculating Limits at Infinity

$$\lim_{x\to\infty} \frac{x^2+2x+1}{x^3}$$

Calculating Limits at Infinity

$$\lim_{x \to -\infty} \left(x^{7/3} - x^{5/3}\right)$$

Again: factor out largest power of x.

Calculating Limits at Infinity

Suppose the height of a bouncing ball is given by $h(t) = \frac{\sin(t)+1}{t}$, for $t \geq 1$. What happens to the height over a long period of time?

Calculating Limits at Infinity

Now You

$$\lim_{x\to\infty} \sqrt{x^4+x^2+1}-\sqrt{x^4+3x^2}$$

Now You

Evaluate $\displaystyle\lim_{x\to-\infty} \frac{\sqrt{3+x^2}}{3x}$

Included Work

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