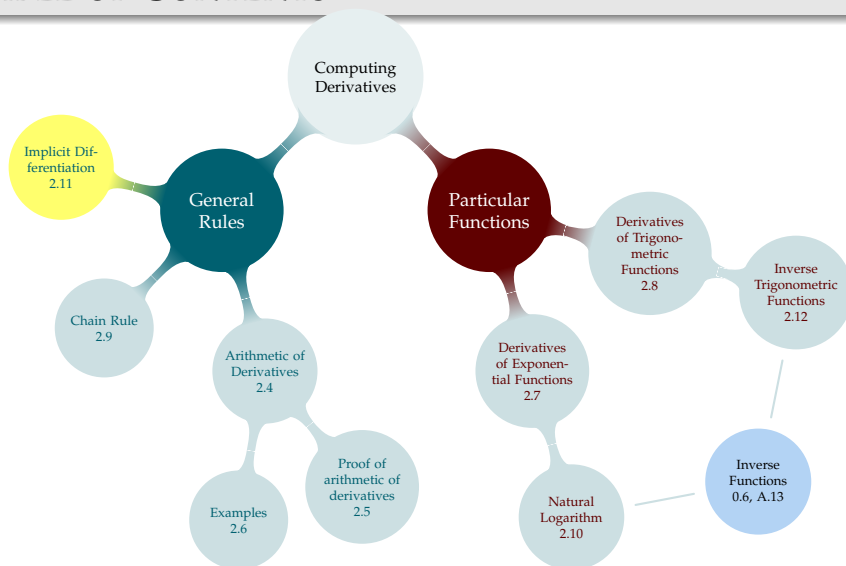


# TABLE OF CONTENTS



# IMPLICITLY DEFINED FUNCTIONS

$$y^2 + x^2 + xy + x^2y = 1$$

## IMPLICITLY DEFINED FUNCTIONS

$$y^2 + x^2 + xy + x^2y = 1$$

Which of the following points are on the curve?

$(0, 1)$ ,  $(0, -1)$ ,  $(0, 0)$ ,  $(1, 1)$

# IMPLICITLY DEFINED FUNCTIONS

$$y^2 + x^2 + xy + x^2y = 1$$

Which of the following points are on the curve?

$(0, 1)$ ,  $(0, -1)$ ,  $(0, 0)$ ,  $(1, 1)$

$(0, 1)$  and  $(0, -1)$

## IMPLICITLY DEFINED FUNCTIONS

$$y^2 + x^2 + xy + x^2y = 1$$

Which of the following points are on the curve?

$(0, 1), (0, -1), (0, 0), (1, 1)$

$(0, 1)$  and  $(0, -1)$

If  $x = -3$ , what is  $y$ ?

# IMPLICITLY DEFINED FUNCTIONS

$$y^2 + x^2 + xy + x^2y = 1$$

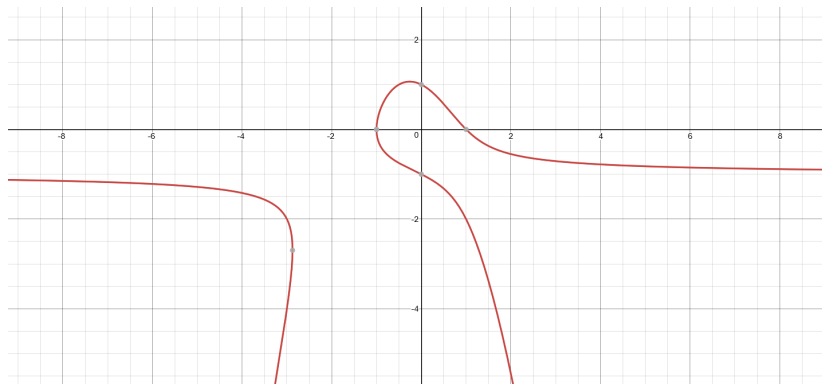
Which of the following points are on the curve?

$(0, 1)$ ,  $(0, -1)$ ,  $(0, 0)$ ,  $(1, 1)$

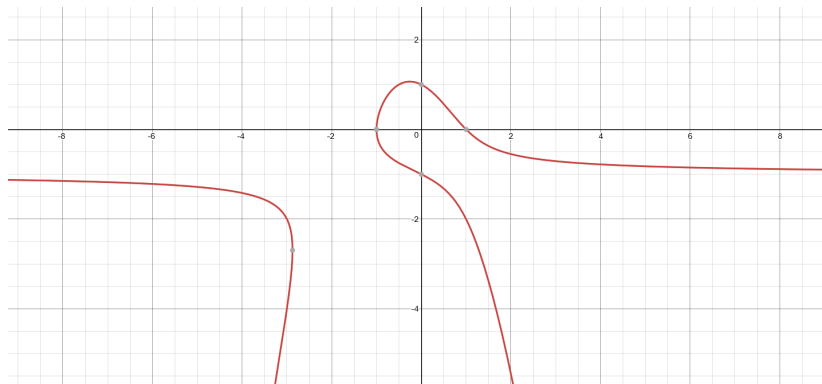
$(0, 1)$  and  $(0, -1)$

If  $x = -3$ , what is  $y$ ?  $y = -2$  and  $y = -4$

$$y^2 + x^2 + xy + x^2y = 1$$



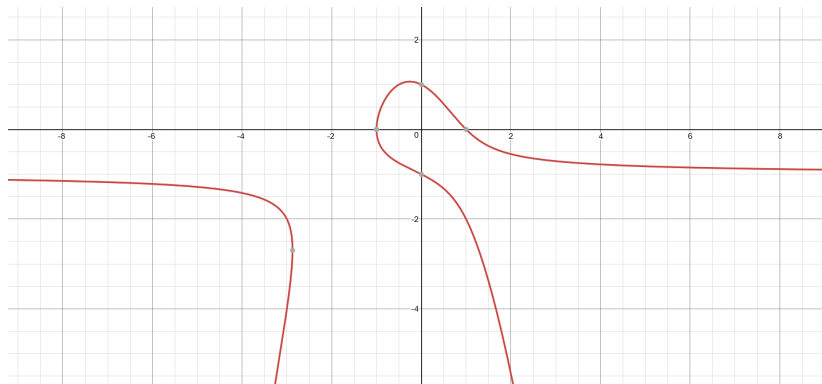
$$y^2 + x^2 + xy + x^2y = 1$$



Still has a slope:  $\frac{\Delta y}{\Delta x}$



$$y^2 + x^2 + xy + x^2y = 1$$



Still has a slope:  $\frac{\Delta y}{\Delta x}$

**Locally**,  $y$  is still a function of  $x$ .

$$y^2 + x^2 + xy + x^2y = 1$$

Consider  $y$  as a function of  $x$ . Can we find  $\frac{dy}{dx}$ ?

$$\frac{d}{dx}[y] =$$

$$y^2 + x^2 + xy + x^2y = 1$$

Consider  $y$  as a function of  $x$ . Can we find  $\frac{dy}{dx}$ ?

$$\frac{d}{dx}[y] = \frac{dy}{dx} = y'$$

$$y^2 + x^2 + xy + x^2y = 1$$

Consider  $y$  as a function of  $x$ . Can we find  $\frac{dy}{dx}$ ?

$$\frac{d}{dx}[y] = \frac{dy}{dx} = y' \qquad \frac{d}{dx}[x] =$$

$$y^2 + x^2 + xy + x^2y = 1$$

Consider  $y$  as a function of  $x$ . Can we find  $\frac{dy}{dx}$ ?

$$\frac{d}{dx}[y] = \frac{dy}{dx} = y' \qquad \frac{d}{dx}[x] = 1$$

$$y^2 + x^2 + xy + x^2y = 1$$

Consider  $y$  as a function of  $x$ . Can we find  $\frac{dy}{dx}$ ?

$$\frac{d}{dx}[y] = \frac{dy}{dx} = y'$$

$$\frac{d}{dx}[x] = 1$$

$$\frac{d}{dx}[1] =$$

$$y^2 + x^2 + xy + x^2y = 1$$

Consider  $y$  as a function of  $x$ . Can we find  $\frac{dy}{dx}$ ?

$$\frac{d}{dx}[y] = \frac{dy}{dx} = y'$$

$$\frac{d}{dx}[x] = 1$$

$$\frac{d}{dx}[1] = 0$$

$$y^2 + x^2 + xy + x^2y = 1$$

Consider  $y$  as a function of  $x$ . Can we find  $\frac{dy}{dx}$ ?

$$\frac{d}{dx}[y] = \frac{dy}{dx} = y'$$

$$\frac{d}{dx}[x] = 1$$

$$\frac{d}{dx}[1] = 0$$

Differentiate both sides with respect to  $x$ .

$$0 = 2y \frac{dy}{dx} + 2x + \left( x \frac{dy}{dx} + (1)y \right) + \left( x^2 \frac{dy}{dx} + 2xy \right)$$

$$0 = \frac{dy}{dx} (2y + x + x^2) + (2x + y + 2xy)$$

$$- (2x + y + 2xy) = \frac{dy}{dx} (2y + x + x^2)$$

$$-\frac{2x + y + 2xy}{2y + x + x^2} = \frac{dy}{dx}$$



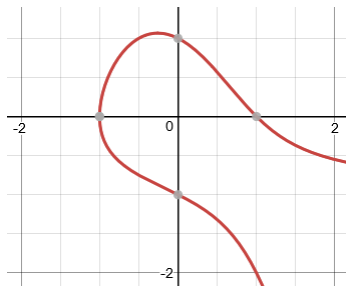
$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{dy}{dx} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{dy}{dx} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

Necessarily,  $\frac{dy}{dx}$  depends on **both**  $y$  and  $x$ . Why?



$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{dy}{dx} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

Necessarily,  $\frac{dy}{dx}$  depends on **both**  $y$  and  $x$ . Why?

$$\left. \frac{dy}{dx} \right|_{(1,0)} =$$

$$\left. \frac{dy}{dx} \right|_{(1,-2)} =$$

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{dy}{dx} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

Necessarily,  $\frac{dy}{dx}$  depends on **both**  $y$  and  $x$ . Why?

$$\begin{aligned}\left.\frac{dy}{dx}\right|_{(1,0)} &= -\frac{2(1) + 0 + 2(1)(0)}{2(0) + 1 + 1} \\ &= -\frac{2}{2} = -1\end{aligned}$$

$$\begin{aligned}\left.\frac{dy}{dx}\right|_{(1,-2)} &= -\frac{2(1) - 2 + 2(1)(-2)}{2(-2) + 1 + 1} \\ &= -2\end{aligned}$$

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{dy}{dx} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

Necessarily,  $\frac{dy}{dx}$  depends on **both**  $y$  and  $x$ . Why?

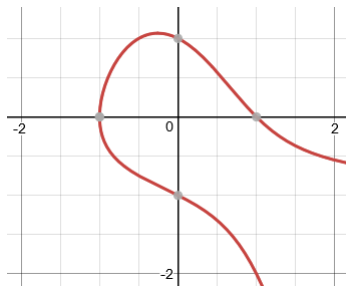
$$\begin{aligned}\left.\frac{dy}{dx}\right|_{(1,0)} &= -\frac{2(1) + 0 + 2(1)(0)}{2(0) + 1 + 1} \\ &= -\frac{2}{2} = -1\end{aligned}$$

$$\begin{aligned}\left.\frac{dy}{dx}\right|_{(1,-2)} &= -\frac{2(1) - 2 + 2(1)(-2)}{2(-2) + 1 + 1} \\ &= -2\end{aligned}$$

Points with the same  $x$ -value may have different slopes. We need both the  $x$ -value and the  $y$ -value to figure out which point we're talking about.

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{dy}{dx} = -\frac{2x + y + 2xy}{2y + x + x^2}$$



Points with the same  $x$ -value may have different slopes. We need both the  $x$ -value and the  $y$ -value to figure out which point we're talking about.

NOW  
YOU



Suppose  $x^4y + y^4x = 2$ . Find  $\frac{dy}{dx}$  at the point  $(1, 1)$ .

NOW  
YOU



Suppose  $x^4y + y^4x = 2$ . Find  $\frac{dy}{dx}$  at the point  $(1, 1)$ .

$$x^4y(x) + y(x)^4x = 2$$

$$4x^3y(x) + x^4\frac{dy}{dx}(x) + y(x)^4 + 4y(x)^3\frac{dy}{dx}(x)x = 0$$

We may only replace variables with constants *after* differentiating.  
When  $x = 1$  and  $y(1) = 1$ ,

$$4(1)^3y(1) + (1)^4\frac{dy}{dx}(1) + y(1)^4 + 4y(1)^3\frac{dy}{dx}(1) = 0$$

$$4 + \frac{dy}{dx}(1) + 1 + 4\frac{dy}{dx}(1) = 0$$

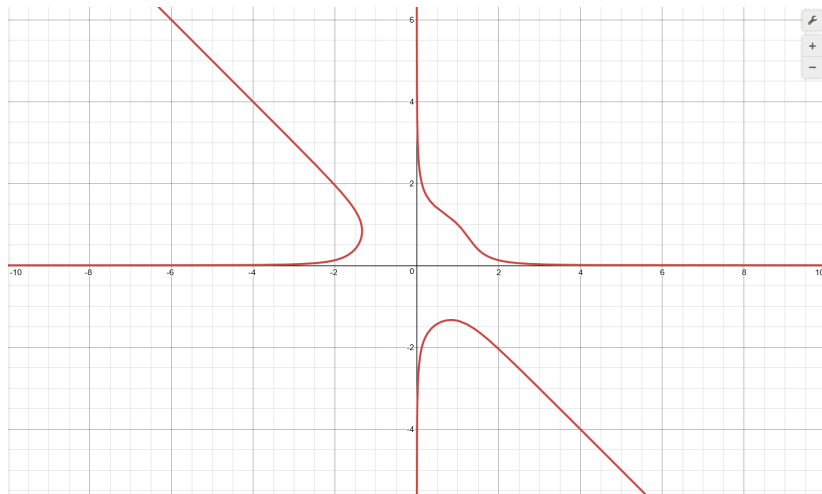
$$5\frac{dy}{dx}(1) = -5$$

$$\frac{dy}{dx}(1) = -1$$



## 2.11: Implicit Diff

○○○○○○●○○○○○○○○



$$x^4y + y^4x = 2$$

Now  
You

Suppose  $\frac{3y^2 + 2y + y^3}{x^2 + 1} = x$ . Find  $\frac{dy}{dx}$  when  $x = 0$ , and the equations of the associated tangent line(s).

NOW  
YOU



Suppose  $\frac{3y^2 + 2y + y^3}{x^2 + 1} = x$ . Find  $\frac{dy}{dx}$  when  $x = 0$ , and the equations of the associated tangent line(s).

To avoid the quotient rule, we start by simplifying our expression.

$$\frac{3y(x)^2 + 2y(x) + y(x)^3}{x^2 + 1} = x$$

$$3y(x)^2 + 2y(x) + y(x)^3 = x^3 + x$$

$$6y(x) \frac{dy}{dx}(x) + 2 \frac{dy}{dx}(x) + 3y(x)^2 \frac{dy}{dx}(x) = 3x^2 + 1$$

When  $x = 0$ :

$$\frac{dy}{dx}(0) = \frac{1}{6y(0) + 2 + 3y(0)^2}$$

We need to know  $y$  to find  $\frac{dy}{dx}$ . We want all points where  $x = 0$ .

$$3y(0)^2 + 2y(0) + y(0)^3 = 0$$

$$y(0)(y(0)^2 + 3y(0) + 2) = 0$$

$$y(0)(y(0) + 1)(y(0) + 2) = 0$$

$$y(0) = 0, y(0) = -1, y(0) = -2$$

$$\frac{dy}{dx} = \frac{1}{6y + 2 + 3y^2}$$

$$(0, 0)$$

$$(0, -1)$$

$$(0, -2)$$

$$\left. \frac{dy}{dx} \right|_{(0,0)} = \frac{1}{2}$$

$$y - 0 = \frac{1}{2}(x - 0)$$

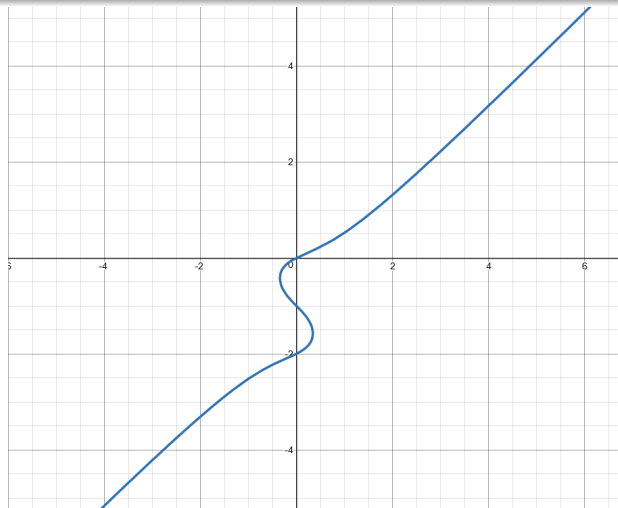
$$y = \frac{1}{2}x$$

$$\begin{aligned} \left. \frac{dy}{dx} \right|_{(0,-1)} &= \frac{1}{-6+2+3} \\ &= -1 \end{aligned}$$

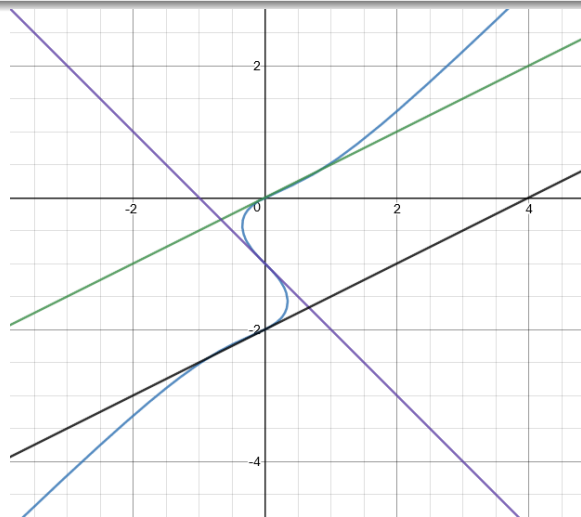
$$\begin{aligned} \left. \frac{dy}{dx} \right|_{(0,-2)} &= \frac{1}{-12+2+12} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} y - (-1) &= -1(x - 0) \\ y &= -x - 1 \end{aligned}$$

$$\begin{aligned} y - (-2) &= \frac{1}{2}(x - 0) \\ y &= \frac{1}{2}x - 2 \end{aligned}$$



$$\frac{3y^2 + 2y + y^3}{x^2 + 1} = x$$



$$\frac{3y^2 + 2y + y^3}{x^2 + 1} = x$$

Use implicit differentiation to differentiate  $\log(x)$ ,  $x > 0$ .



Use implicit differentiation to differentiate  $\log(x)$ ,  $x > 0$ .

$$\log x = y(x)$$

$$x = e^{y(x)}$$

Use implicit differentiation to differentiate  $\log(x)$ ,  $x > 0$ .

$$\log x = y(x)$$

$$x = e^{y(x)}$$

$$1 = e^{y(x)} \cdot \frac{dy}{dx}(x)$$

$$\frac{dy}{dx}(x) = \frac{1}{e^{y(x)}} = \frac{1}{x}$$

Use implicit differentiation to differentiate  $\log(x)$ ,  $x > 0$ .

$$\log x = y(x)$$

$$x = e^{y(x)}$$

$$1 = e^{y(x)} \cdot \frac{dy}{dx}(x)$$

$$\frac{dy}{dx}(x) = \frac{1}{e^{y(x)}} = \frac{1}{x}$$

Use implicit differentiation to differentiate  $\log |x|$ ,  $x < 0$ .

Use implicit differentiation to differentiate  $\log(x)$ ,  $x > 0$ .

Use implicit differentiation to differentiate  $\log|x|$ ,  $x < 0$ .

Use implicit differentiation to differentiate  $\log(x)$ ,  $x > 0$ .

Use implicit differentiation to differentiate  $\log|x|$ ,  $x < 0$ .

$$\log|x| = y(x)$$

$$\log(-x) = y(x)$$

$$-x = e^{y(x)}$$

$$-1 = e^{y(x)} \cdot \frac{dy}{dx}(x)$$

$$\frac{dy}{dx}(x) = \frac{-1}{e^{y(x)}} = \frac{-1}{-x} = \frac{1}{x}$$

Use implicit differentiation to differentiate  $\log_a(x)$ , where  $a > 0$  is a constant and  $x > 0$ .

Use implicit differentiation to differentiate  $\log_a |x|$ ,  $a > 0$ .

Use implicit differentiation to differentiate  $\log_a(x)$ , where  $a > 0$  is a constant and  $x > 0$ .

Use implicit differentiation to differentiate  $\log_a(x)$ , where  $a > 0$  is a constant and  $x > 0$ .

$$\log_a x = y(x)$$

$$x = a^{y(x)}$$

$$1 = a^{y(x)} \cdot \log_e a \cdot \frac{dy}{dx}(x)$$

$$\frac{dy}{dx}(x) = \frac{1}{a^{y(x)} \cdot \log_e a} = \frac{1}{x \log_e a}$$



Use implicit differentiation to differentiate  $\log_a |x|$ ,  $a > 0$ .

Use implicit differentiation to differentiate  $\log_a |x|$ ,  $a > 0$ .

If  $x > 0$ , it's what we just computed. So assume  $x < 0$ .

$$\log_a |x| = y(x)$$

$$\log_a (-x) = y(x)$$

$$-x = a^{y(x)}$$

$$-1 = a^{y(x)} \cdot \log_e a \cdot \frac{dy}{dx}(x)$$

$$\frac{dy}{dx}(x) = \frac{-1}{a^{y(x)} \cdot \log_e a} = \frac{-1}{x \log_e a}$$

## Included Work



'Brain' by Eucalyp is licensed under CC BY 3.0 (accessed 8 June 2021), 23, 24, 26, 27



screenshot of graph using Desmos Graphing Calculator,  
<https://www.desmos.com/calculator> (accessed 19 October 2017), 18, 22



screenshot of graph using Desmos Graphing Calculator,  
<https://www.desmos.com/calculator> (accessed 19 October 2017), 7–9



screenshot of graph using Desmos Graphing Calculator,  
<https://www.desmos.com/calculator> (accessed 24 October 2018), 25



screenshot of graph using Desmos Graphing Calculator,  
<https://www.desmos.com/calculator>, (accessed 24 October 2018), 30



screenshot of graph using Desmos Graphing Calculator,  
<https://www.desmos.com/calculator> (accessed 24 October 2018), 31



'Notebook' by Iconic is licensed under CC BY 3.0 (accessed 9 June 2021), 35, 38