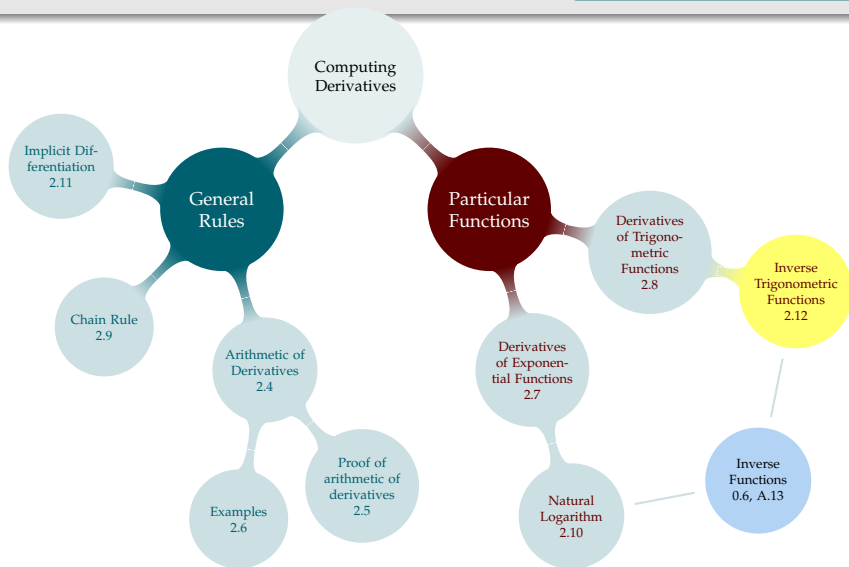
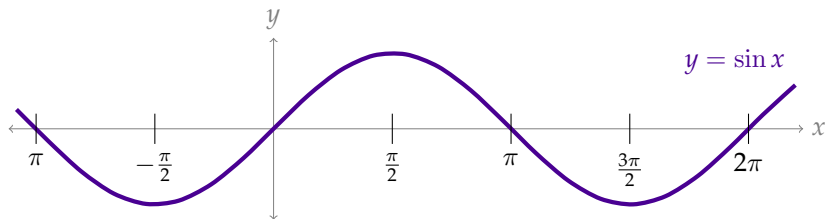


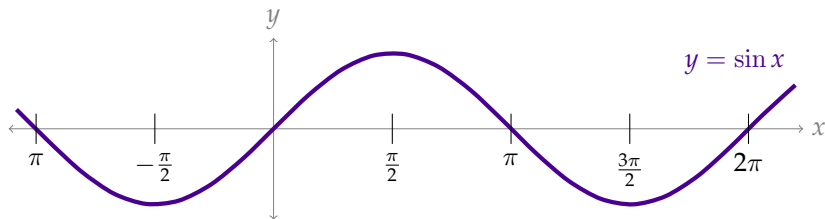
TABLE OF CONTENTS

[▶ SKIP DEFINITIONS OF INVERSE TRIG FUNCTIONS](#)

INVERTIBILITY GAME

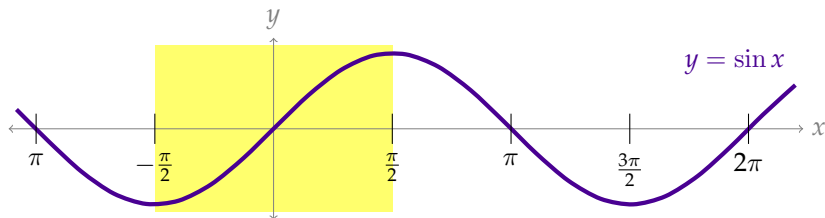


INVERTIBILITY GAME



I'm thinking of a number x . Your hint: $\sin(x) = 0$. What number am I thinking of?

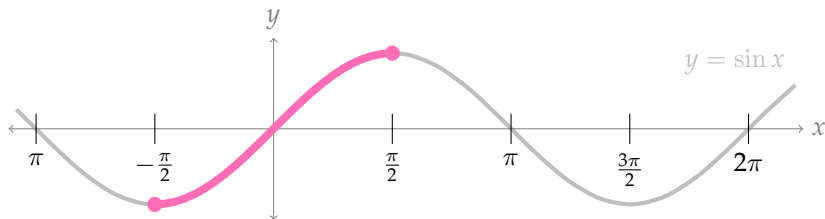
INVERTIBILITY GAME



I'm thinking of a number x . Your hint: $\sin(x) = 0$. What number am I thinking of?

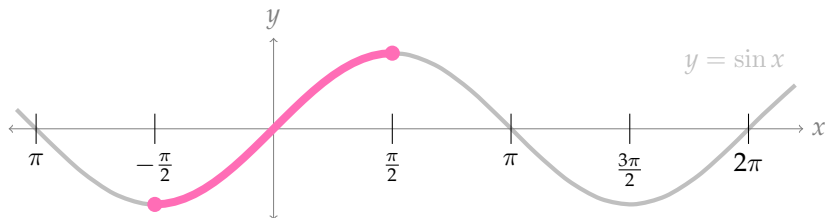
I'm thinking of a number x , and x is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Your hint: $\sin(x) = 0$. What number am I thinking of?

ARCSINE



$\arcsin(x)$ is the inverse of $\sin x$ restricted to $[-\frac{\pi}{2}, \frac{\pi}{2}]$

ARCSINE



$\arcsin(x)$ is the inverse of $\sin x$ restricted to $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\arcsin x$ is the (unique) number θ such that:

- ▶ $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, and
- ▶ $\sin \theta = x$

ARCSINE

Reference Angles:

θ	$\sin \theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

ARCSINE

Reference Angles:

θ	$\sin \theta$
0	0
$-\frac{\pi}{6}$	$-\frac{1}{2}$
$-\frac{\pi}{4}$	$-\frac{1}{\sqrt{2}}$
$-\frac{\pi}{3}$	$-\frac{\sqrt{3}}{2}$
$-\frac{\pi}{2}$	-1

ARCSINE

Reference Angles:

► $\arcsin(0)$

θ	$\sin \theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

ARCSINE

Reference Angles:

► $\arcsin(0) = 0$

θ	$\sin \theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

ARCSINE

Reference Angles:

θ	$\sin \theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

$$\blacktriangleright \arcsin(0) = 0$$

$$\blacktriangleright \arcsin\left(\frac{1}{\sqrt{2}}\right)$$

ARCSINE

Reference Angles:

θ	$\sin \theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

$$\blacktriangleright \arcsin(0) = 0$$

$$\blacktriangleright \arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

ARCSINE

Reference Angles:

θ	$\sin \theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

$$\blacktriangleright \arcsin(0) = 0$$

$$\blacktriangleright \arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$\blacktriangleright \arcsin\left(-\frac{1}{\sqrt{2}}\right)$$

ARCSINE

Reference Angles:

θ	$\sin \theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

$$\blacktriangleright \arcsin(0) = 0$$

$$\blacktriangleright \arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$\blacktriangleright \arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$$

ARCSINE

Reference Angles:

θ	$\sin \theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

$$\blacktriangleright \arcsin(0) = 0$$

$$\blacktriangleright \arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$\blacktriangleright \arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$$

$$\blacktriangleright \arcsin\left(\frac{\pi}{2}\right)$$

ARCSINE

Reference Angles:

θ	$\sin \theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

$$\blacktriangleright \arcsin(0) = 0$$

$$\blacktriangleright \arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$\blacktriangleright \arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$$

$$\blacktriangleright \arcsin\left(\frac{\pi}{2}\right) \text{undefined}$$

ARCSINE

Reference Angles:

θ	$\sin \theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

$$\blacktriangleright \arcsin(0) = 0$$

$$\blacktriangleright \arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$\blacktriangleright \arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$$

$$\blacktriangleright \arcsin\left(\frac{\pi}{2}\right) \text{undefined}$$

$$\blacktriangleright \arcsin\left(\frac{\pi}{4}\right)$$

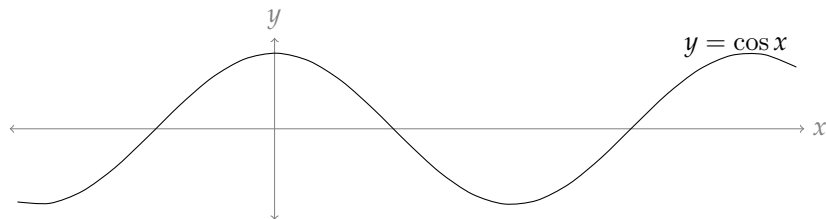
ARCSINE

Reference Angles:

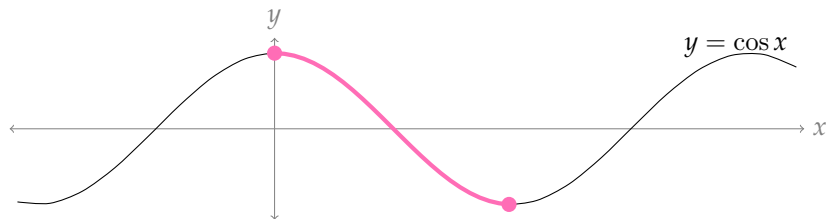
θ	$\sin \theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

- ▶ $\arcsin(0) = 0$
- ▶ $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- ▶ $\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$
- ▶ $\arcsin\left(\frac{\pi}{2}\right)$ undefined
- ▶ $\arcsin\left(\frac{\pi}{4}\right)$ defined, but we haven't covered tools (yet) to figure it out

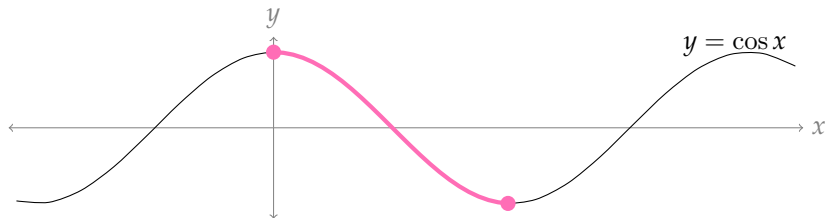
ARCCOSINE



ARCCOSINE



ARCCOSINE

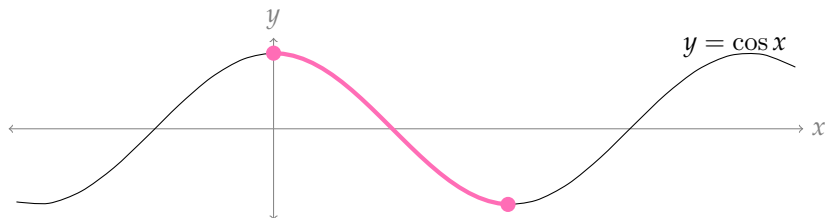


$\arccos(x)$ is the inverse of $\cos x$ restricted to $[0, \pi]$.

$\arccos(x)$ is the (unique) number θ such that:

- ▶ $\cos(\theta) = x$ and
- ▶ $0 \leq \theta \leq \pi$

ARCCOSINE

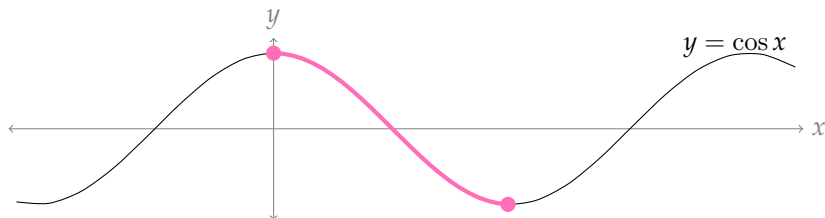


$\arccos(x)$ is the inverse of $\cos x$ restricted to $[0, \pi]$.

$\arccos(x)$ is the (unique) number θ such that:

- ▶ $\cos(\theta) = x$ and ←←← inverse
- ▶ $0 \leq \theta \leq \pi$

ARCCOSINE

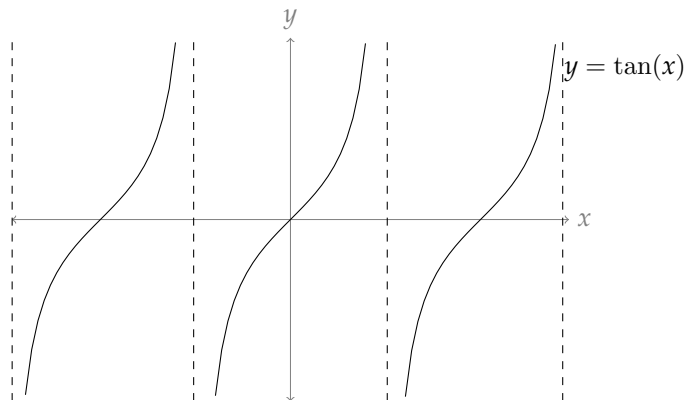


$\arccos(x)$ is the inverse of $\cos x$ restricted to $[0, \pi]$.

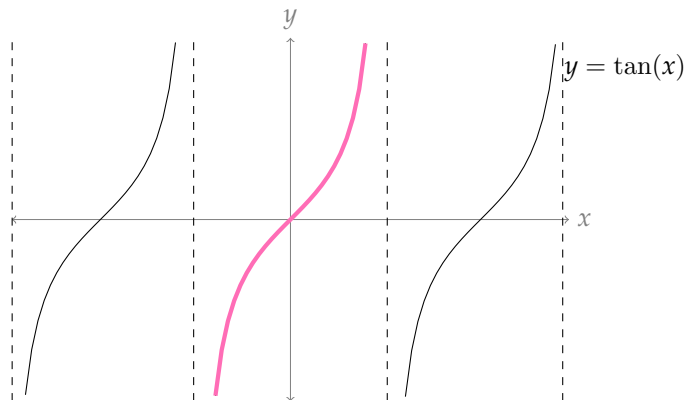
$\arccos(x)$ is the (unique) number θ such that:

- ▶ $\cos(\theta) = x$ and ←←← inverse
- ▶ $0 \leq \theta \leq \pi$ ←←← inverse exists

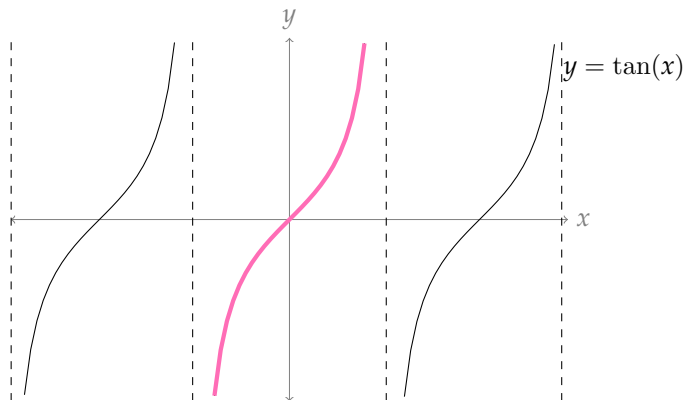
ARCTANGENT



ARCTANGENT



ARCTANGENT



$\arctan(x) = \theta$ means:

- (1) $\tan(\theta) = x$ and
- (2) $-\pi/2 < \theta < \pi/2$

ARCSECANT, ARCSINE, AND ARCCOTANGENT

$$\operatorname{arcsec}(x) =$$

ARCSECANT, ARCSINE, AND ARCCOTANGENT

$$\operatorname{arcsec}(x) =$$

$$\operatorname{arcsec}(x) = y$$

$$\sec y = x$$

$$\frac{1}{\cos y} = x$$

$$\cos y = \frac{1}{x}$$

$$y = \arccos\left(\frac{1}{x}\right)$$

$$\operatorname{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$$

ARCSECANT, ARCSINE, AND ARCCOTANGENT

$$\operatorname{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$$

$$\operatorname{arcsec}(x) = y$$

$$\sec y = x$$

$$\frac{1}{\cos y} = x$$

$$\cos y = \frac{1}{x}$$

$$y = \arccos\left(\frac{1}{x}\right)$$

$$\operatorname{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$$

ARCSECANT, ARCSINE, AND ARCCOTANGENT

$$\operatorname{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$

$$\operatorname{arccsc}(x) = y$$

$$\csc y = x$$

$$\frac{1}{\sin y} = x$$

$$\sin y = \frac{1}{x}$$

$$y = \arcsin\left(\frac{1}{x}\right)$$

$$\operatorname{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$

$$\operatorname{arccot}(x) = \arctan\left(\frac{1}{x}\right)$$

$$\operatorname{arccot}(x) = y$$

$$\cot y = x$$

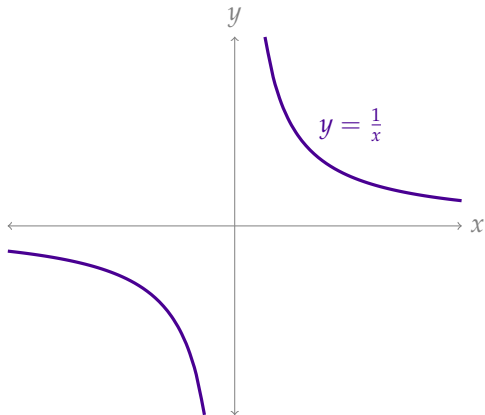
$$\frac{1}{\tan y} = x$$

$$\tan y = \frac{1}{x}$$

$$y = \arctan\left(\frac{1}{x}\right)$$

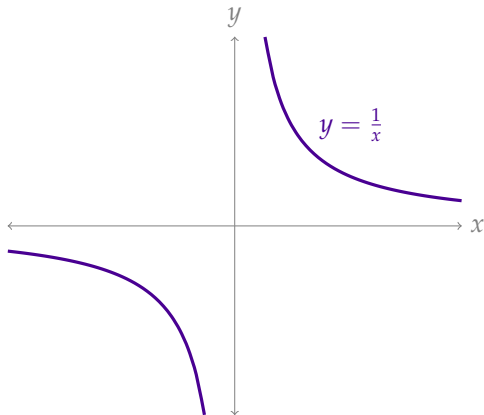
$$\operatorname{arccot}(x) = \arctan\left(\frac{1}{x}\right)$$

$$\operatorname{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$$



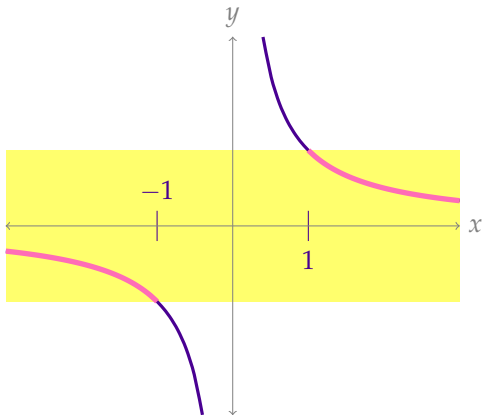
$$\operatorname{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$$

The domain of $\arccos(y)$ is $-1 \leq y \leq 1$, so the domain of $\operatorname{arcsec}(y)$ is



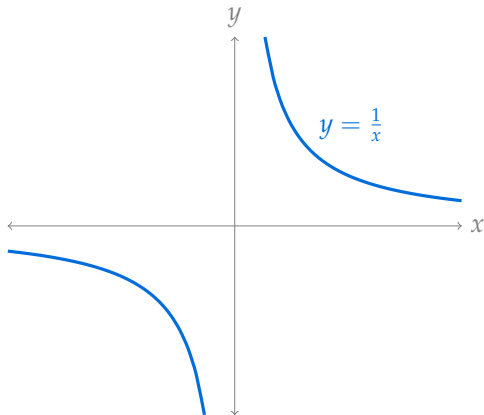
$$\operatorname{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$$

The domain of $\arccos(y)$ is $-1 \leq y \leq 1$, so the domain of $\operatorname{arcsec}(y)$ is



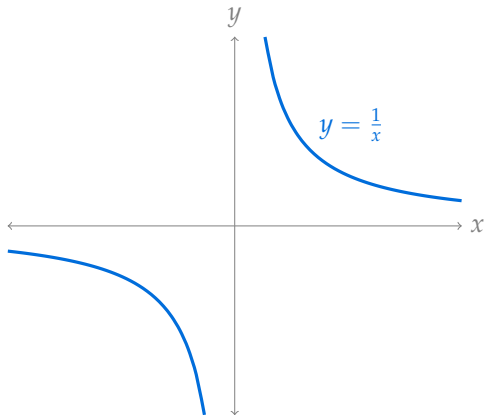
$$(-\infty, -1] \cup [1, \infty).$$

$$\operatorname{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$



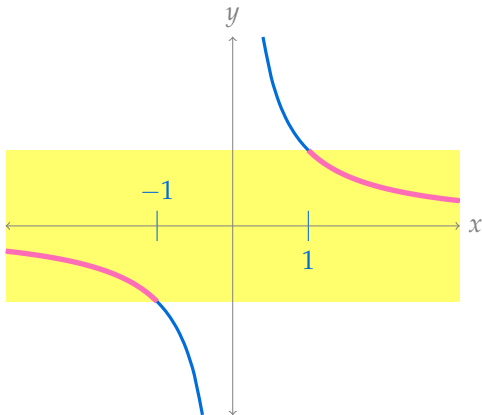
$$\operatorname{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$

Domain of $\arcsin(y)$ is $-1 \leq y \leq 1$, so the domain of $\operatorname{arccsc}(x)$ is



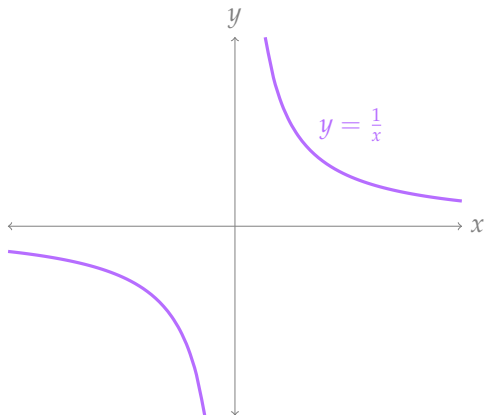
$$\operatorname{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$

Domain of $\arcsin(y)$ is $-1 \leq y \leq 1$, so the domain of $\operatorname{arccsc}(x)$ is



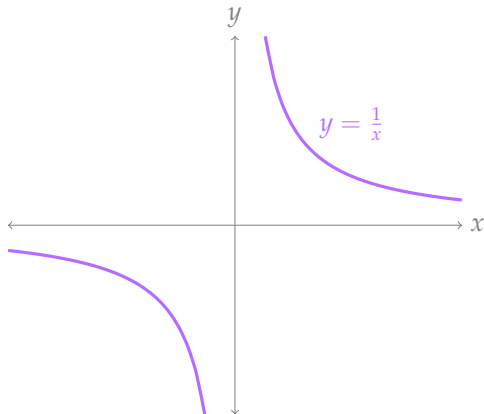
$$(-\infty, -1] \cup [1, \infty).$$

$$\operatorname{arccot}(x) = \arctan\left(\frac{1}{x}\right)$$



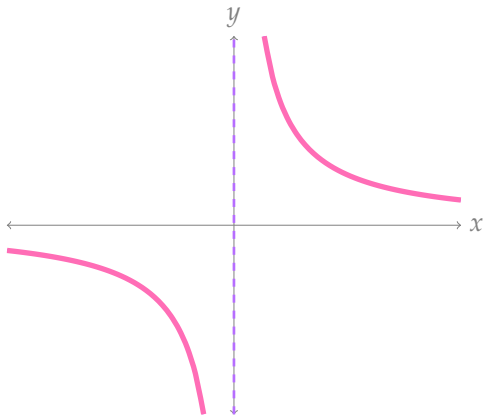
$$\operatorname{arccot}(x) = \arctan\left(\frac{1}{x}\right)$$

Domain of $\arctan(x)$ is all real numbers, so the domain of $\operatorname{arccot}(x)$ is



$$\operatorname{arccot}(x) = \arctan\left(\frac{1}{x}\right)$$

Domain of $\arctan(x)$ is all real numbers, so the domain of $\operatorname{arccot}(x)$ is



$$(-\infty, 0) \cup (0, \infty).$$

$$y = \arcsin x$$

Find $\frac{dy}{dx}$.

$$y(x) = \arcsin x$$

$$x = \sin y(x)$$

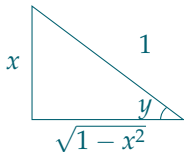
$$\frac{d}{dx}[x] = \frac{d}{dx}[\sin y(x)]$$

$$1 = \cos y(x) \cdot \frac{dy}{dx}(x)$$

$$\frac{dy}{dx}(x) = \frac{1}{\cos y(x)}$$

$$= \frac{\text{hyp}}{\text{adj}}$$

$$= \frac{1}{\sqrt{1-x^2}}$$



Find $\frac{dy}{dx}$.

$$y = \arctan x$$

$$y(x) = \arctan x$$

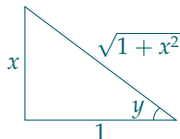
$$x = \tan y(x)$$

$$\frac{d}{dx}[x] = \frac{d}{dx}[\tan y(x)]$$

$$1 = \sec^2 y(x) \cdot \frac{dy}{dx}(x)$$

$$\frac{dy}{dx}(x) = \cos^2 y(x)$$

$$\begin{aligned} \frac{dy}{dx}(x) &= \left(\frac{\text{adj}}{\text{hyp}} \right)^2 = \left(\frac{1}{\sqrt{1+x^2}} \right)^2 \\ &= \frac{1}{1+x^2} \end{aligned}$$



Find $\frac{dy}{dx}$.

$$y = \arccos x$$

$$y(x) = \arccos x$$

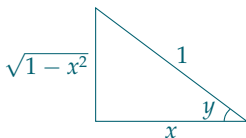
$$x = \cos y(x)$$

$$\frac{d}{dx}[x] = \frac{d}{dx}[\cos y(x)]$$

$$1 = -\sin y(x) \cdot \frac{dy}{dx}(x)$$

$$\frac{dy}{dx}(x) = \frac{-1}{\sin y(x)}$$

$$\frac{dy}{dx}(x) = \frac{-\text{hyp}}{\text{opp}} = \frac{-1}{\sqrt{1-x^2}}$$



To differentiate arcsecant, arccosecant, and arccotangent, you can use the chain rule!

To differentiate arcsecant, arccosecant, and arccotangent, you can use the chain rule!

$$\frac{d}{dx} [\operatorname{arccsc}(x)] = \frac{d}{dx} \left[\arcsin \left(\frac{1}{x} \right) \right] = \frac{d}{dx} [\arcsin (x^{-1})]$$

To differentiate arcsecant, arccosecant, and arccotangent, you can use the chain rule!

$$\frac{d}{dx} [\operatorname{arccsc}(x)] = \frac{d}{dx} \left[\arcsin \left(\frac{1}{x} \right) \right] = \frac{d}{dx} [\arcsin (x^{-1})]$$

$$\begin{aligned} \frac{d}{dx} \left[\arcsin \left(\boxed{x^{-1}} \right) \right] &= \frac{1}{\sqrt{1 - \left(\boxed{x^{-1}} \right)^2}} \cdot \boxed{\left(-x^{-2} \right)} = \frac{-1}{x^2 \sqrt{1 - x^{-2}}} \\ &= \frac{-1}{\sqrt{x^4} \sqrt{1 - x^{-2}}} = \frac{-1}{\sqrt{x^2} \sqrt{x^2} \sqrt{1 - x^{-2}}} = \frac{-1}{\sqrt{x^2} \sqrt{x^2 - 1}} = \frac{-1}{|x| \sqrt{1 - x^2}} \end{aligned}$$

Derivatives of Inverse Trigonometric Functions – Theorem 2.12.7

Memorize:

$$\frac{d}{dx} [\arcsin x] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} [\arccos x] = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} [\arcsin x] = \frac{1}{1+x^2}$$

Be able to derive:

$$\frac{d}{dx} [\operatorname{arccsc} x] = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx} [\operatorname{arcsec} x] = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx} [\operatorname{arccot} x] = -\frac{1}{1+x^2}$$