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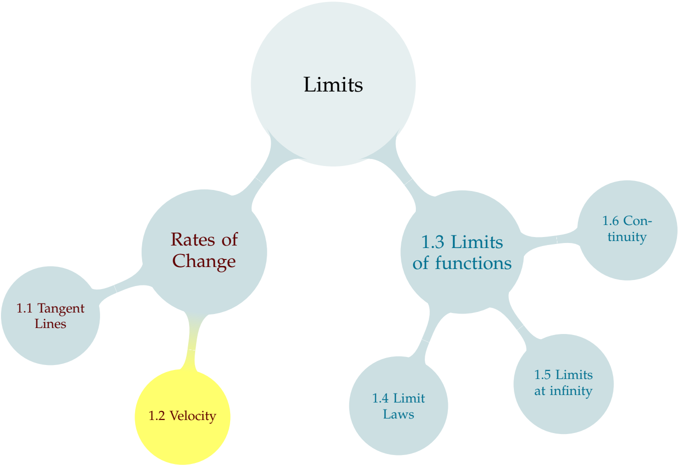
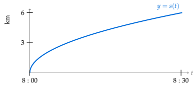



$y = s(t)$
km
6
3
t
8 : 00
8 : 30



8:07 is highlighted to emphasize that I was not ravelling at a constant speed: half the distance was covered in only 7 minutes, the other half took 23.

I've found that the bike example is a good concept check. When students answer the multiple choice question there's usually a lot who get it wrong, so it's an opportunity to fix misconceptions.

It took $\frac{1}{2}$ hour to bike 6 km. 12 kph represents the:

A. secant line to $y = s(t)$ from $t = 8:00$ to $t = 8:30$
B. slope of the secant line to $y = s(t)$ from $t = 8:00$ to $t = 8:30$
C. tangent line to $y = s(t)$ at $t = 8:30$
D. slope of the tangent line to $y = s(t)$ at $t = 8:30$

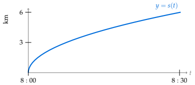
At 8:25, the speedometer on my bike reads 5 kph. 5 kph represents the:

A. secant line to $y = s(t)$ from $t = 8:00$ to $t = 8:25$

B. slope of the secant line to $y = s(t)$ from $t = 8:00$ to $t = 8:25$

C. tangent line to $y = s(t)$ at $t = 8:25$

D. slope of the tangent line to $y = s(t)$ at $t = 8:25$



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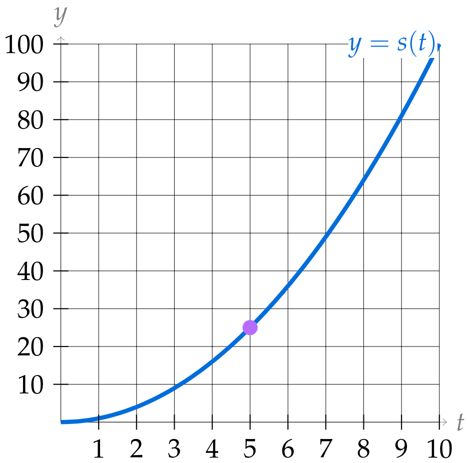
C. tangent line to $y = s(t)$ at $t = 8:25$
D. slope of the tangent line to $y = s(t)$ at $t = 8:25$

Now we move into using limits for instantaneous rates of change. Students are usually tired of the bike by now so we change the example.
For next slide: in "one way," remind verbally that instantaneous rate of change is slope of tangent line. Use a straight edge to draw the tangent and make use of the graph paper to get a decent approximation.
We use y for the vertical axis instead of h because h is used later for something else

Suppose the distance from the ground s (in meters) of a helium-filled balloon at time t over a 10-second interval is given by $s(t) = t^2$. Try to estimate how fast the balloon is rising when $t = 5$.



Let's look for an algebraic way of determining the velocity of the balloon when $t = 5$.

Our First Limit

Average Velocity, $t = 5$ to $t = 5 + h$:

$$\begin{aligned}\frac{\Delta s}{\Delta t} &= \frac{s(5+h) - s(5)}{h} \\ &= \frac{(5+h)^2 - 5^2}{h} \\ &= 10 + h \qquad \text{when } h \neq 0\end{aligned}$$

When h is very small,

$$\text{Vel} \approx 10$$

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Now we recap what we did: the informal calculation first, then using it to introduce limit notation

Limit Notation

We write:

$$\lim_{h \to 0} (10 + h) = 10$$

We say: "The limit as h goes to 0 of $(10 + h)$ is 10."

It means: As h gets extremely close to 0, $(10 + h)$ gets extremely close to 10.