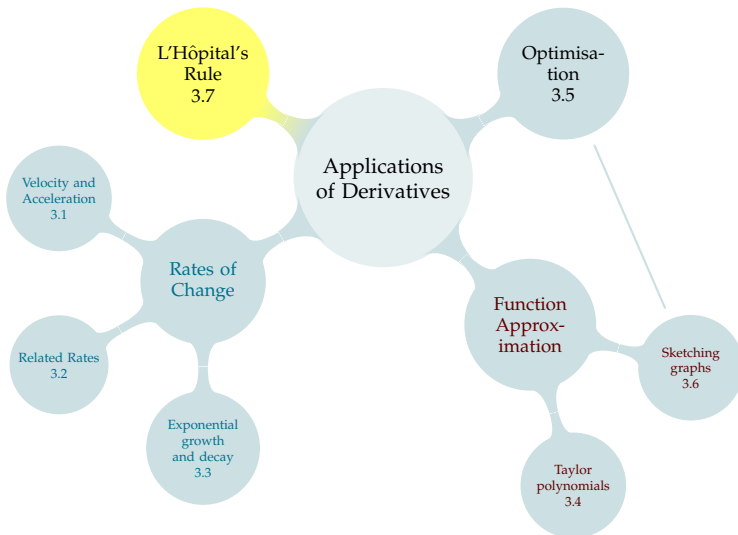


TABLE OF CONTENTS



BACK TO LIMITS!

$$\lim_{x \rightarrow \infty} \frac{x^2}{5}$$

$$\lim_{x \rightarrow \infty} \frac{5}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{x^2}{5}$$

$$\lim_{x \rightarrow 0} \frac{5}{x^2}$$

Indeterminate Forms – Definition 3.7.1

Suppose $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$. Then the limit

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$$

is an **indeterminate form** of the type $\frac{0}{0}$.

Suppose $\lim_{x \rightarrow a} F(x) = \lim_{x \rightarrow a} G(x) = \infty$ (or $-\infty$). Then the limit

$$\lim_{x \rightarrow a} \frac{F(x)}{G(x)}$$

is an **indeterminate form** of the type $\frac{\infty}{\infty}$.

When you see an indeterminate form, you need to do more work.

3.7: L'Hôpital's Rule and Indeterminate Forms

Back to Limits!

BACK TO LIMITS!

$$\lim_{x \rightarrow 0} \frac{x^2}{e^x} \quad \lim_{x \rightarrow \infty} \frac{5}{x^2} \quad \lim_{x \rightarrow 0} \frac{x^2}{e^x} \quad \lim_{x \rightarrow \infty} \frac{5}{x^2}$$

Indeterminate Forms - Definition 3.7.1

Suppose $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$. Then the limit

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$$

is an *indeterminate form* of the type $\frac{0}{0}$.

Suppose $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = \infty$ (or $-\infty$). Then the limit

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$$

is an *indeterminate form* of the type $\frac{\infty}{\infty}$.

When you see an indeterminate form, you need to do more work.

Often students misinterpret the phrase “indeterminate” as “unknowable,” so I like to emphasize the “more work needed” part. This is one of two common mistakes, the other being the assumption that $n/n = 1$ even when $n = 0$ or n is not a number.

INDETERMINATE FORMS

$$\lim_{x \rightarrow 5} \frac{x^2 - 3x - 10}{x - 5}$$

indeterminate form of the type $\frac{0}{0}$

$$\lim_{x \rightarrow \infty} \frac{3x^2 - 4x + 2}{8x^2 - 5}$$

indeterminate form of the type $\frac{\infty}{\infty}$

INDETERMINATE FORMS AND THE DERIVATIVE

$$\lim_{x \rightarrow 0} \frac{3 \sin x - x^4}{x^2 + \cos x - e^x}$$

indeterminate form of the type $\frac{0}{0}$

L'Hôpital's Rule: First Part – Theorem 3.7.2

Let f and g be functions such that $\lim_{x \rightarrow a} f(x) = 0 = \lim_{x \rightarrow a} g(x)$.

If $f'(a)$ and $g'(a)$ exist and $g'(a) \neq 0$, then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$.

If f and g are differentiable on an open interval containing a , and if $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$ exists, then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$.

This works even for $a = \pm\infty$.

Extremely Important Note:
L'Hôpital's Rule only works on indeterminate forms.

L'Hôpital's Rule: Second Part – Theorem 3.7.2

Let f and g be functions such that $\lim_{x \rightarrow a} f(x) = \infty = \lim_{x \rightarrow a} g(x)$.

If $f'(a)$ and $g'(a)$ exist and $g'(a) \neq 0$, then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$.

If f and g are differentiable on an open interval containing a , and if $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$ exists, then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$.

This works even for $a = \pm\infty$.

Extremely Important Note:
L'Hôpital's Rule only works on indeterminate forms.

Evaluate:

$$\lim_{x \rightarrow 2} \frac{3x \tan(x - 2)}{x - 2}$$

LITTLE HARDER

$$\lim_{x \rightarrow 0} \frac{x^4}{e^x - \cos x - x}$$

indeterminate form of the type $\frac{0}{0}$

Evaluate:

$$\lim_{x \rightarrow \infty} \frac{\log x}{\sqrt{x}}$$

OTHER INDETERMINATE FORMS

$$\lim_{x \rightarrow \infty} e^{-x} \log x$$

form $0 \cdot \infty$

VOTE VOTE VOTE

Which of the following can you immediately apply L'Hôpital's rule to?

A. $\frac{e^x}{2e^x + 1}$

B. $\lim_{x \rightarrow 0} \frac{e^x}{2e^x + 1}$

C. $\lim_{x \rightarrow \infty} \frac{e^x}{2e^x + 1}$

D. $\lim_{x \rightarrow \infty} e^{-x}(2e^x + 1)$

E. $\lim_{x \rightarrow 0} \frac{e^x}{x^2}$

VOTEY McVOTEFACE

Suppose you want to use L'Hôpital's rule to evaluate $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$, which has the form $\frac{0}{0}$. How does the quotient rule fit into this problem?

- A. You should use the quotient rule because the function you are differentiating is a quotient.
- B. You will not use the quotient rule because you differentiate the numerator and the denominator separately
- C. You may use the quotient rule because perhaps $f(x)$ or $g(x)$ is itself in the form of a quotient
- D. You will not use L'Hôpital's rule because $\frac{0}{0}$ is not an appropriate indeterminate form
- E. You will not use L'Hôpital's rule because, since the top has limit zero, the whole function has limit 0

MORE QUESTIONS

Which of the following is NOT an indeterminate form?

- A. $\frac{\infty}{\infty}$ for example, $\lim_{x \rightarrow \infty} \frac{e^x}{x^2}$
- B. $\frac{0}{0}$ for example, $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$
- C. $\frac{0}{\infty}$ for example, $\lim_{x \rightarrow 0^+} \frac{x}{\log x}$
- D. $0 \cdot \infty$ for example, $\lim_{x \rightarrow \infty} x(\arctan(x) - \pi/2)$
- E. all of the above are indeterminate forms

I HAVE SO MANY QUESTIONS

Which of the following is NOT an indeterminate form?

A. 1^∞ for example, $\lim_{x \rightarrow \infty} \left(\frac{x+1}{x} \right)^x$

B. 0^∞ for example, $\lim_{x \rightarrow \infty} \left(\frac{1}{x} \right)^x$

C. ∞^0 for example, $\lim_{x \rightarrow \infty} x^{\frac{1}{x}}$

D. 0^0 for example, $\lim_{x \rightarrow 0^+} x^x$

E. all of the above are indeterminate forms

F. none of the above are indeterminate forms

3.7: L'Hôpital's Rule and Indeterminate Forms

I have so many questions

1^∞ definitely takes some explaining

I HAVE SO MANY QUESTIONS

Which of the following is NOT an indeterminate form?

- A. 1^∞ for example, $\lim_{x \rightarrow \infty} \left(\frac{x+1}{x}\right)^x$
- B. 0^∞ for example, $\lim_{x \rightarrow \infty} \left(\frac{1}{x}\right)^x$
- C. ∞^0 for example, $\lim_{x \rightarrow \infty} x^{\frac{1}{x}}$
- D. 0^0 for example, $\lim_{x \rightarrow 0^+} x^x$
- E. all of the above are indeterminate forms
- F. none of the above are indeterminate forms

EXPONENTIAL INDETERMINATE FORMS

$$\lim_{x \rightarrow \infty} x^{1/x}$$

EXPONENTIAL INDETERMINATE FORMS

$$\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^{3x}$$

Evaluate:

$$\lim_{x \rightarrow \infty} \frac{\log x}{\log \sqrt{x}}$$

$$\lim_{x \rightarrow \infty} (\log x)^{\sqrt{x}}$$

$$\lim_{x \rightarrow 0} \frac{\arcsin x}{x}$$

MORE EXAMPLES

$$\lim_{x \rightarrow \infty} \sqrt{2x^2 + 1} - \sqrt{x^2 + x}$$

$$\lim_{x \rightarrow 0} \sqrt[x^2]{\sin^2 x}$$

$$\lim_{x \rightarrow 0} \sqrt[x^2]{\cos x}$$

Sketch the graph of $f(x) = x \log x$.

Note: when you want to know $\lim_{x \rightarrow 0} f(x)$, you'll need to use L'Hôpital.

Evaluate $\lim_{x \rightarrow 0^+} (\csc x)^x$

