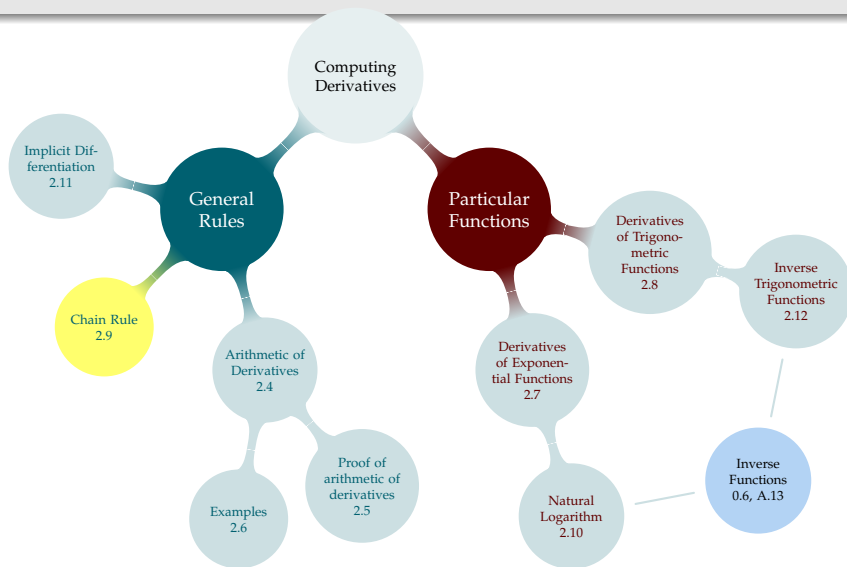
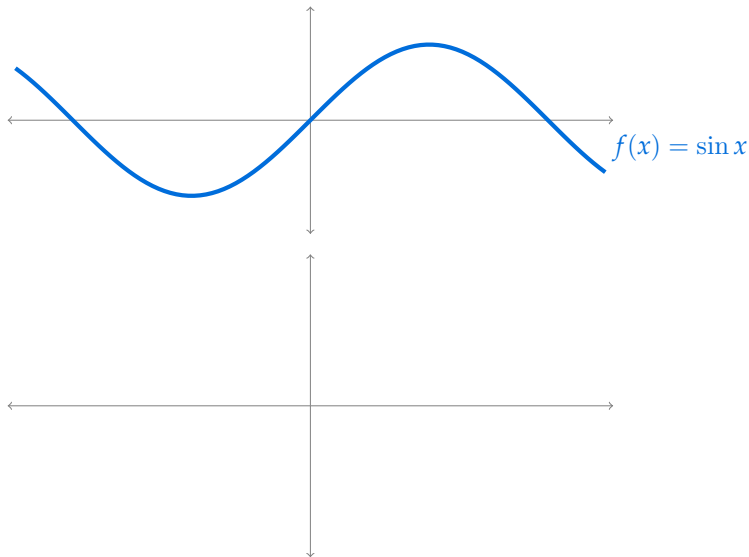
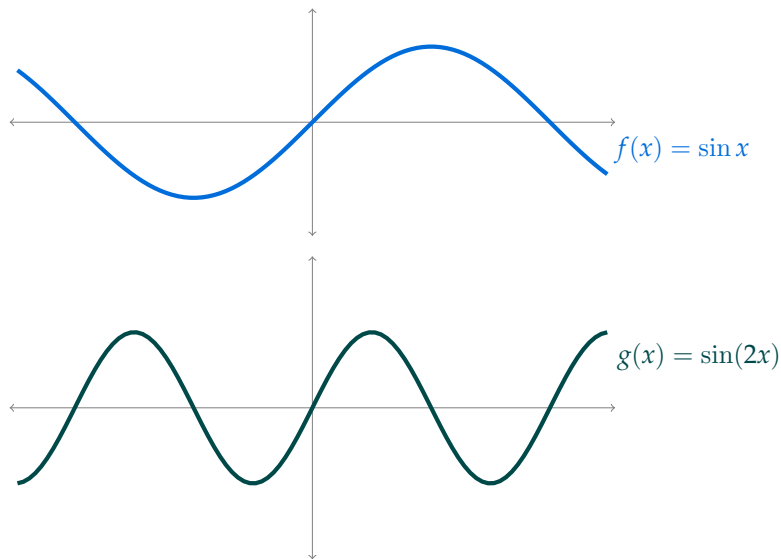
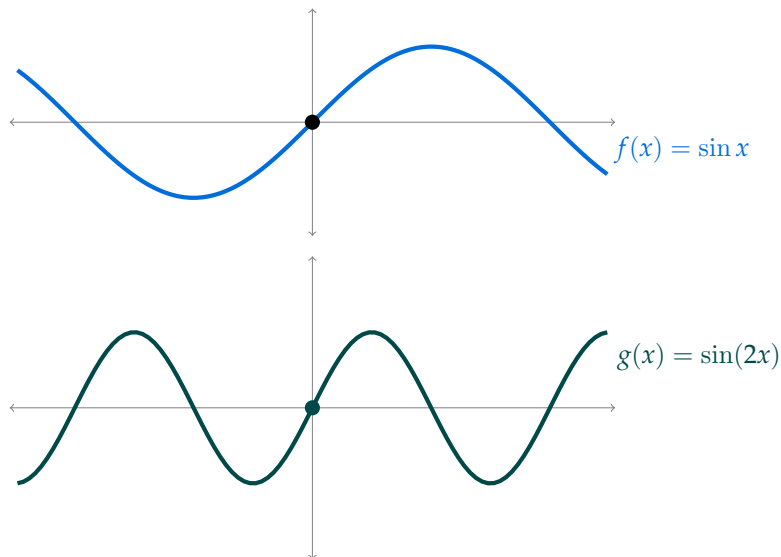


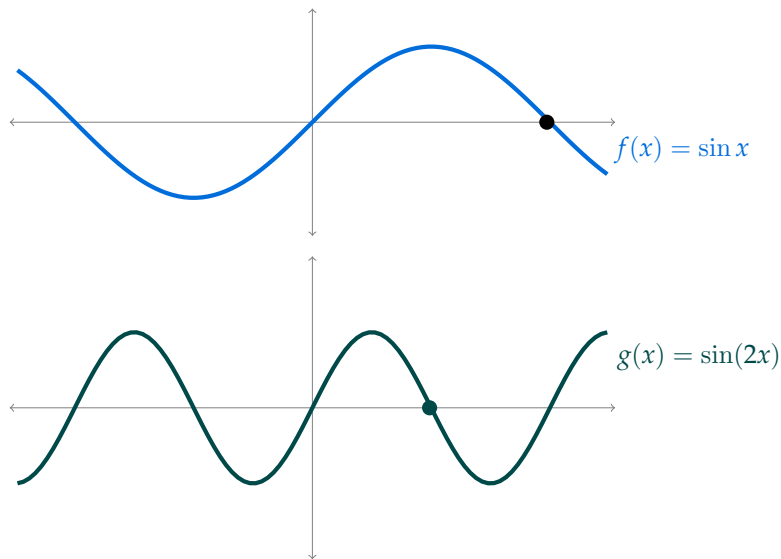
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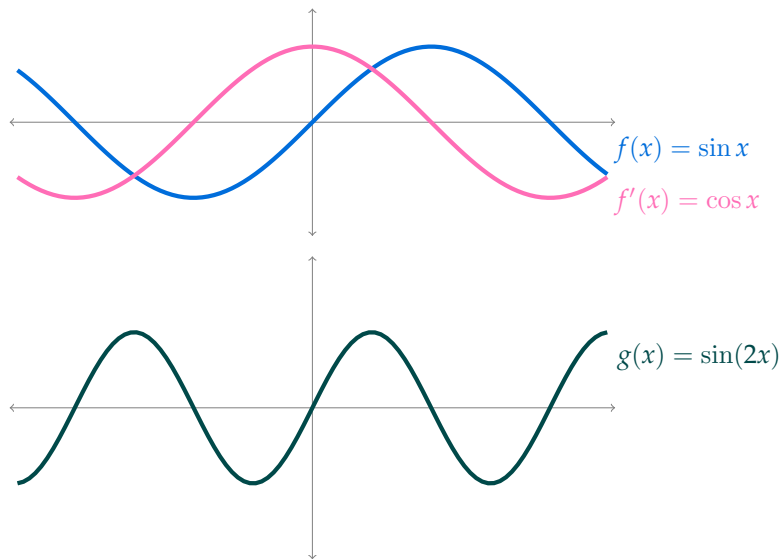


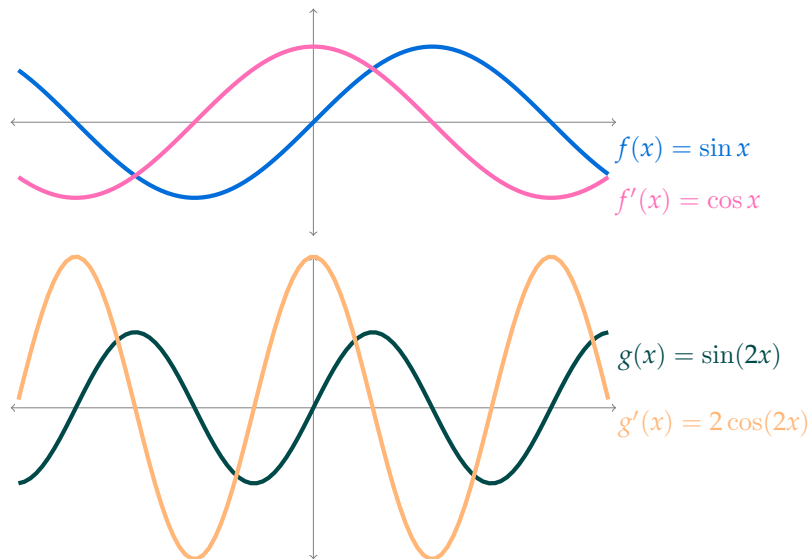
INTUITION: $\sin x$ VERSUS $\sin(2x)$ 

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COMPOUND FUNCTIONS

Video: 2:27-3:50

Morton, Jennifer. (2014). *Balancing Act: Otters, Urchins and Kelp*.
Available from [https://www.kqed.org/quest/67124/
balancing-act-otters-urchins-and-kelp](https://www.kqed.org/quest/67124/balancing-act-otters-urchins-and-kelp)

KELP POPULATION

k kelp population
 u urchin population
 o otter population

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$k(u)$

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$$k(u(o))$$

KELP POPULATION

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 p public policy

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These are examples of compound functions.

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A. positive

B. negative

C. I'm not sure

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DIFFERENTIATING COMPOUND FUNCTIONS

$$\frac{d}{dx}\{f(g(x))\} =$$

DIFFERENTIATING COMPOUND FUNCTIONS

$$\begin{aligned}
 \frac{d}{dx}\{f(g(x))\} &= \lim_{h \rightarrow 0} \frac{f(g(x+h)) - f(g(x))}{h} \\
 &= \lim_{h \rightarrow 0} \frac{f(g(x+h)) - f(g(x))}{h} \left(\frac{g(x+h) - g(x)}{g(x+h) - g(x)} \right) \\
 &= \lim_{h \rightarrow 0} \frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)} \cdot \frac{g(x+h) - g(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)} \cdot \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{f\left(\boxed{g(x+h)}\right) - f\left(\boxed{g(x)}\right)}{\boxed{g(x+h)} - \boxed{g(x)}} \cdot g'(x)
 \end{aligned}$$

Set $H = g(x+h) - g(x)$. As $h \rightarrow 0$, we also have $H \rightarrow 0$. So

$$\begin{aligned}
 &= \lim_{H \rightarrow 0} \frac{f(g(x) + H) - f(g(x))}{H} \cdot g'(x) \\
 &= f'(g(x)) \cdot g'(x)
 \end{aligned}$$

CHAIN RULE

Chain Rule – Theorem 2.9.3

Suppose f and g are differentiable functions. Then

$$\frac{d}{dx}\{f(g(x))\} = f'(g(x))g'(x) = \frac{df}{dg}(g(x))\frac{dg}{dx}(x)$$

In the case of kelp, $\frac{d}{d\text{o}}k(u(o)) = \frac{dk}{d\text{u}}(u(o))\frac{d\text{u}}{d\text{o}}(o)$

Chain Rule

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Example: suppose $F(x) = \sin(e^x + x^2)$.

Chain Rule

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Example: suppose $F(x) = \sin(e^x + x^2)$.

We can differentiate $\sin(x)$, so let's set $g(x) = e^x + x^2$ and $f(g) = \sin(g)$. Then $F(x) = f(g(x))$.

$$g'(x) = e^x + 2x \text{ and } \frac{df}{dg}(g) = \cos(g) \text{ and}$$

$$\frac{df}{dg}(g(x)) = \frac{df}{dg}\left(\boxed{e^x + x^2}\right) = \cos\left(\boxed{e^x + x^2}\right)$$

$$\text{So, } F'(x) = \frac{df}{dg}(g(x)) \frac{dg}{dx}(x) = \cos(e^x + x^2) (e^x + 2x)$$

$$F(v) = \left(\frac{v}{v^3 + 1} \right)^6$$

$$F(v) = \left(\frac{v}{v^3 + 1} \right)^6$$

$$\begin{aligned} F'(v) &= 6 \left(\boxed{\frac{v}{v^3 + 1}} \right)^5 \cdot \frac{(v^3 + 1)(1) - (v)(3v^2)}{(v^3 + 1)^2} \\ &= 6 \left(\boxed{\frac{v}{v^3 + 1}} \right)^5 \cdot \frac{-2v^3 + 1}{(v^3 + 1)^2} \end{aligned}$$

NOW
YOU



Let $f(x) = (10^x + \csc x)^{1/2}$. Find $f'(x)$.

NOW
YOU



Suppose $o(t) = e^t$, $u(o) = \frac{1}{o + \sin(o)}$, and $t \geq 10$ (so all

these functions are defined). Using the chain rule, find $\frac{d}{dt} u(o(t))$.

Note: your answer should depend only on t : not o .

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Let $f(x) = (10^x + \csc x)^{1/2}$. Find $f'(x)$.

$$f(x) = (\boxed{10^x + \csc x})^{1/2}$$

Using the chain rule,

$$\begin{aligned} f'(x) &= \frac{1}{2} (\boxed{10^x + \csc x})^{-1/2} (10^x \log_e 10 - \csc x \cot x) \\ &= \frac{10^x \log_e 10 - \csc x \cot x}{2\sqrt{10^x + \csc x}} \end{aligned}$$

Now
You

Suppose $o(t) = e^t$, $u(o) = \frac{1}{o + \sin(o)}$, and $t \geq 10$ (so all

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these functions are defined). Using the chain rule, find $\frac{d}{dt}u(o(t))$.

Note: your answer should depend only on t : not o .

$$o'(t) = e^t$$

$$\begin{aligned} u'(o) &= \frac{(o + \sin o)(0) - (1)(1 + \cos o)}{(o + \sin o)^2} \\ &= \frac{-(1 + \cos o)}{(o + \sin o)^2} \end{aligned}$$

$$\begin{aligned} \frac{d}{dt}u(o(t)) &= u'(o(t)) o'(t) \\ &= -e^t \left(\frac{1 + \cos(o(t))}{[o(t) + \sin(o(t))]^2} \right) \\ &= -e^t \left(\frac{1 + \cos(e^t)}{[e^t + \sin(e^t)]^2} \right) \end{aligned}$$



MORE EXAMPLES



Evaluate $\frac{d}{dx} \left\{ x^2 + \sec \left(x^2 + \frac{1}{x} \right) \right\}$



Evaluate $\frac{d}{dx} \left\{ \frac{1}{x + \frac{1}{x + \frac{1}{x}}} \right\}$

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$$\frac{d}{dx} \left\{ x^2 + \sec \left(\boxed{x^2 + \frac{1}{x}} \right) \right\}$$

$$= 2x + \sec \left(\boxed{x^2 + \frac{1}{x}} \right) \cdot \tan \left(\boxed{x^2 + \frac{1}{x}} \right) \cdot \frac{d}{dx} \left\{ \boxed{x^2 + \frac{1}{x}} \right\}$$

$$= 2x + \sec \left(\boxed{x^2 + \frac{1}{x}} \right) \cdot \tan \left(\boxed{x^2 + \frac{1}{x}} \right) \cdot \frac{d}{dx} \left\{ \boxed{x^2 + x^{-1}} \right\}$$

$$= 2x + \sec \left(\boxed{x^2 + \frac{1}{x}} \right) \cdot \tan \left(\boxed{x^2 + \frac{1}{x}} \right) \cdot (2x - x^{-2})$$

Notice: That first term, $2x$, is not multiplied by anything else.

Evaluate $\frac{d}{dx} \left\{ \frac{1}{x + \frac{1}{x + \frac{1}{x}}} \right\}$

Evaluate $\frac{d}{dx} \left\{ \frac{1}{x + \frac{1}{x}} \right\}$

$$\frac{d}{dx} \left\{ \frac{1}{x + \frac{1}{x}} \right\} = \frac{d}{dx} \left\{ \left(x + (x + x^{-1})^{-1} \right)^{-1} \right\}$$

$$= - \left(x + (x + x^{-1})^{-1} \right)^{-2} \cdot \frac{d}{dx} \left\{ x + (x + x^{-1})^{-1} \right\}$$

$$= - \left(x + (x + x^{-1})^{-1} \right)^{-2} \cdot \left[1 + (-1) \left(x + x^{-1} \right)^{-2} \cdot \frac{d}{dx} \left\{ x + x^{-1} \right\} \right]$$

$$= - \left(x + (x + x^{-1})^{-1} \right)^{-2} \cdot \left[1 + (-1) \left(x + x^{-1} \right)^{-2} \cdot (1 - x^{-2}) \right]$$

Included Work



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