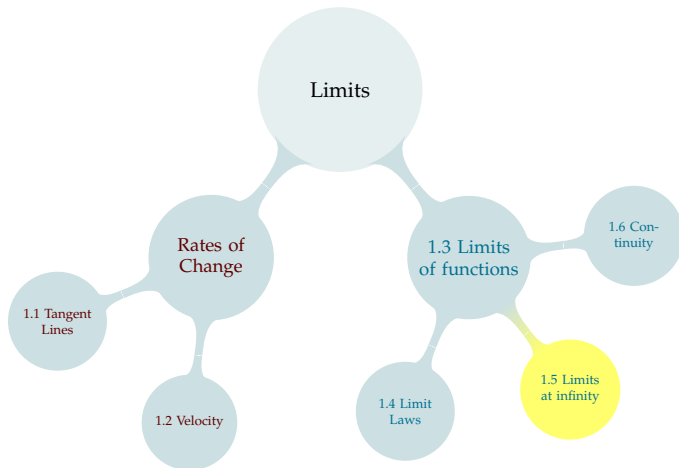


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# END BEHAVIOR

We write:

$$\lim_{x \rightarrow \infty} f(x) = L$$

to express that, as  $x$  grows larger and larger,  $f(x)$  approaches  $L$ .

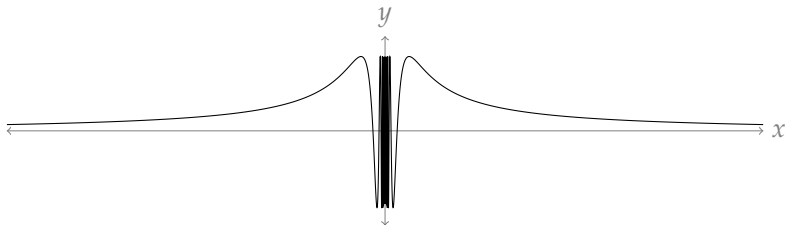
Similarly, we write:

$$\lim_{x \rightarrow -\infty} f(x) = L$$

to express that, as  $x$  grows more and more strongly negative,  $f(x)$  approaches  $L$ .

If  $L$  is a number, we call  $y = L$  a **horizontal asymptote** of  $f(x)$ .

# HORIZONTAL ASYMPTOTES



$y = 0$  is a horizontal asymptote for  $y = \sin\left(\frac{1}{x}\right)$

## COMMON LIMITS AT INFINITY

$$\lim_{x \rightarrow \infty} 13 =$$

$$\lim_{x \rightarrow -\infty} 13 =$$

$$\lim_{x \rightarrow \infty} x^3 =$$

$$\lim_{x \rightarrow -\infty} x^3 =$$

$$\lim_{x \rightarrow \infty} \frac{1}{x} =$$

$$\lim_{x \rightarrow -\infty} \frac{1}{x} =$$

$$\lim_{x \rightarrow -\infty} x^{5/3} =$$

$$\lim_{x \rightarrow -\infty} x^{2/3} =$$

$$\lim_{x \rightarrow \infty} x^2 =$$

$$\lim_{x \rightarrow -\infty} x^2 =$$

## ARITHMETIC WITH LIMITS AT INFINITY

$$\lim_{x \rightarrow \infty} \left( x + \frac{x^2}{10} \right) =$$



$$\lim_{x \rightarrow \infty} \left( x - \frac{x^2}{10} \right) =$$

$$\lim_{x \rightarrow -\infty} (x^2 + x^3 + x^4) =$$

$$\lim_{x \rightarrow -\infty} (x + 13) (x^2 + 13)^{1/3} =$$

## CALCULATING LIMITS AT INFINITY

$$\lim_{x \rightarrow \infty} \frac{x^2 + 2x + 1}{x^3}$$

## CALCULATING LIMITS AT INFINITY

$$\lim_{x \rightarrow -\infty} (x^{7/3} - x^{5/3})$$

Again: factor out largest power of  $x$ .

# CALCULATING LIMITS AT INFINITY

Suppose the height of a bouncing ball is given by  $h(t) = \frac{\sin(t)+1}{t}$ , for  $t \geq 1$ . What happens to the height over a long period of time?



# CALCULATING LIMITS AT INFINITY



$$\lim_{x \rightarrow \infty} \sqrt{x^4 + x^2 + 1} - \sqrt{x^4 + 3x^2}$$

NOW  
YOU



Evaluate  $\lim_{x \rightarrow -\infty} \frac{\sqrt{3+x^2}}{3x}$

## Included Work



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