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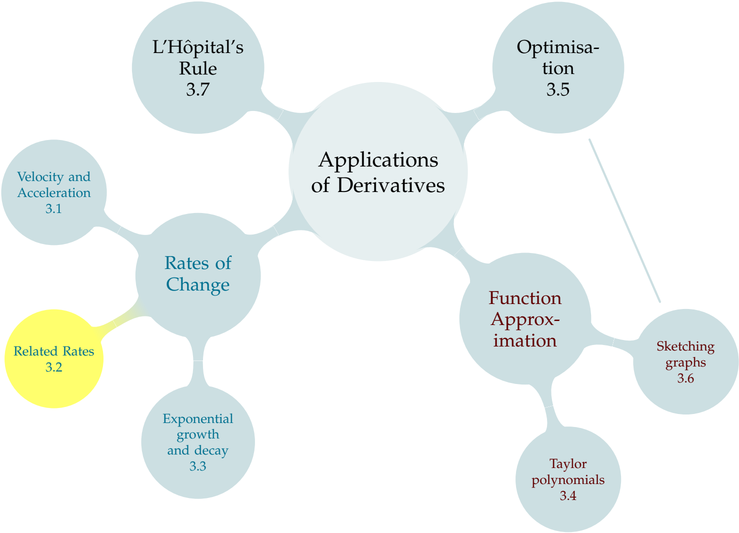

Related Rates - Introduction

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As usual, there are more examples here than you’ll probably get to in class.

Suppose P and Q are quantities that are changing over time, t. Suppose they are related by the equation

$$3P^2 = 2Q^2 + Q + 3.$$

If $\frac{dP}{dt}(t) = 5$ when $P(t) = 1$ and $Q(t) = 0$, then what is $\frac{dQ}{dt}$ at that time?

Related rates problems often involve some kind of geometric or trigonometric modeling

A garden hose can pump out a cubic meter of water in about 20 minutes. Suppose you're filling up a rectangular backyard pool, 3 meters wide and 6 meters long, with a garden hose. How fast is the water rising?

Solving Related Rates

1. Draw a Picture
2. Write what you know, and what you want to know. Note units.
3. Relate all your relevant variables in one equation.
4. Differentiate both sides (with respect to the appropriate variable!)
5. Solve for what you want.

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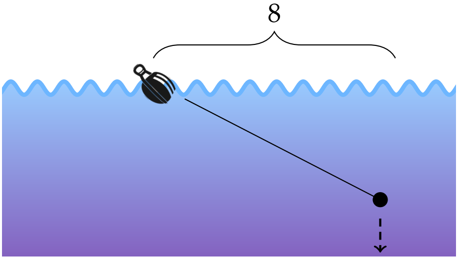
Repeat in class: “If we know how to things are related, we can differentiate to find how their derivatives are related.”

For the first few examples, the answers are given with this structure. After that, it’s more condensed. Students will likely appreciate if you (at least verbally) continue mentioning these steps, but at some point it’s good to leave them and let students develop a little more flexibility and confidence.

A weight is attached to a rope, which is attached to a pulley on a boat, at water level. The weight is taken 8 (horizontal) metres from its attachment point on the boat, then dropped in the water.
The weight sinks straight down. The rope stays taught as it is let out at a constant rate of one metre per second, and two seconds have passed. How fast is the weight descending?

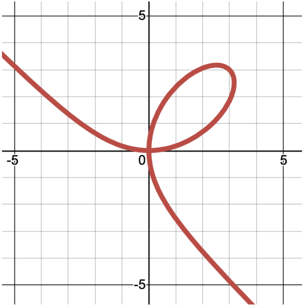


You are pouring water through a funnel with an extremely small hole. The funnel lets water out at 100mL per second, and you are pouring water into the funnel at 300mL per second. The funnel is shaped like a cone with height 20 cm and with the diameter at the top also 20 cm. (Ignore the hole in the bottom.) How fast is the height of the water in the funnel rising when it is 10 cm high?

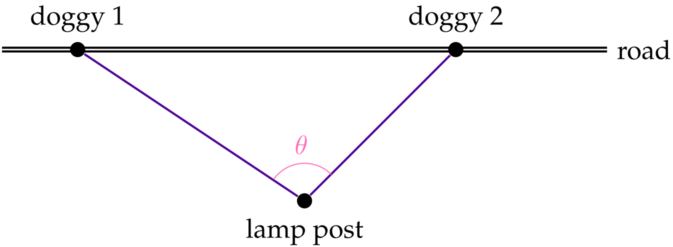
A cone with radius r and height h has volume $\frac{\pi}{3}r^2h$.

A sprinkler is 3m from a long, straight wall. The sprinkler sprays water in a circle, making three revolutions per minute. Let P be the point on the wall closest to the sprinkler. The water hits the wall at some spot, and that spot moves as the sprinkler rotates. When the spot where the water hits the wall is 1m away from P, how fast is the spot moving horizontally?
(You may assume the water travels from the sprinkler to the wall instantaneously.)

A roller coaster has a track shaped in part like the folium of Descartes: $x^3 + y^3 = 6xy$. When it is at the position $(3, 3)$, its horizontal position is changing at 2 units per second in the negative direction. How fast is its vertical position changing?



Two dogs are tied with elastic leashes to a lamp post that is 2 metres from a straight road. At first, both dogs are on the road, at the closest part of the road to the lamp post. Then, they start running in opposite directions: one dog runs 3 metres per second, and the other runs 2 metres per second. After one second of running, how fast is the angle made by the two leashes increasing?



A crow is one kilometre due east of the math building, heading east at 5 kph. An eagle is two kilometres due north of the math building, heading north at 7kph. How fast is the distance between the two birds increasing at this instant?

A triangle has one side that is 1cm long, and another side that is 2cm, and the third side is formed by an elastic band that can shrink and stretch. The two fixed sides are rotated so that the angle they form, θ, grows by 1.5 radians each second. Find the rate of change of the area inside the triangle when $\theta = \pi/4$.

Included Work

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 ‘Switch’ by K Staelin is in the public domain (accessed 7 July 2021), 8

 screenshot of graph using Desmos Graphing Calculator, `https://www.desmos.com/calculator` (accessed 7 July 2021), 11