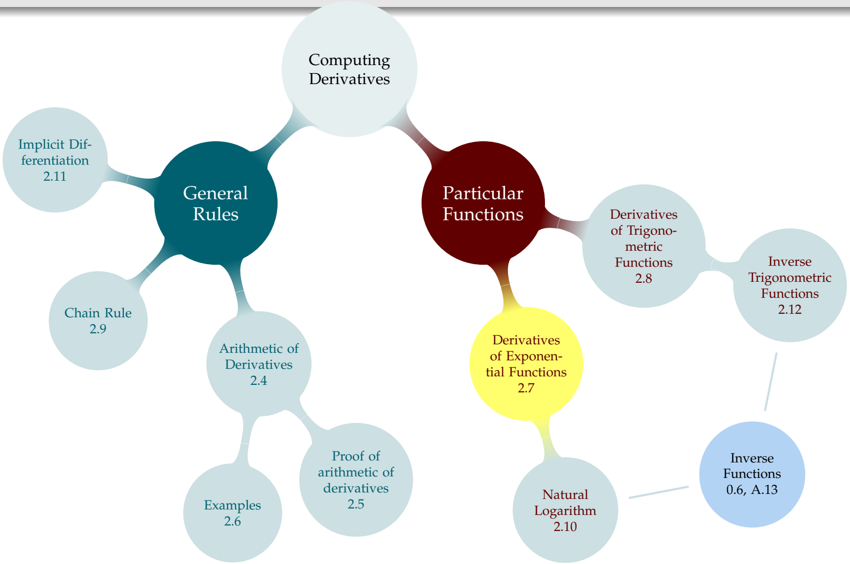
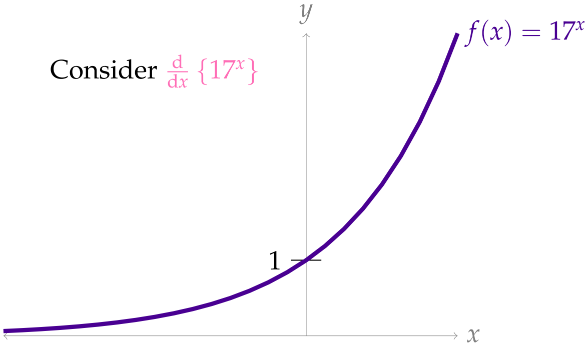
Table of Contents



Exponential Functions

Consider $\frac{\mathrm{d}}{\mathrm{d}x}\left\{17^x\right\}$



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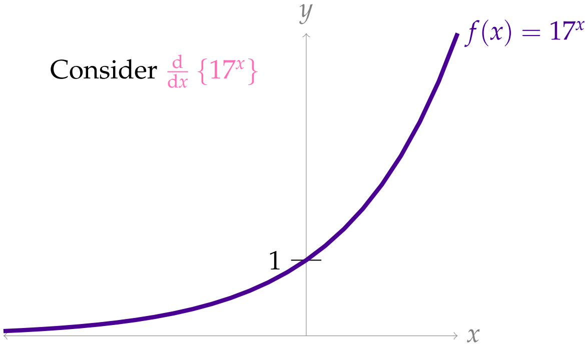

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$f(x)$ is always increasing, so $f'(x)$ is always positive.

EXPONENTIAL FUNCTIONS

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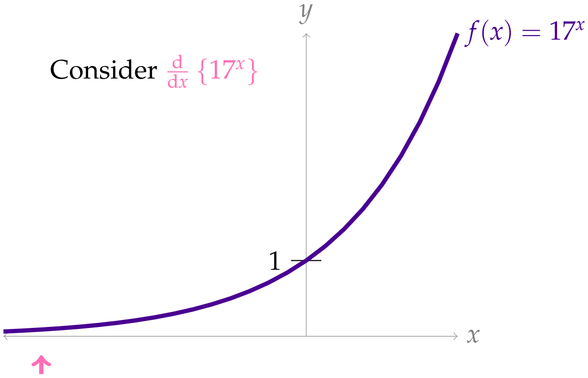

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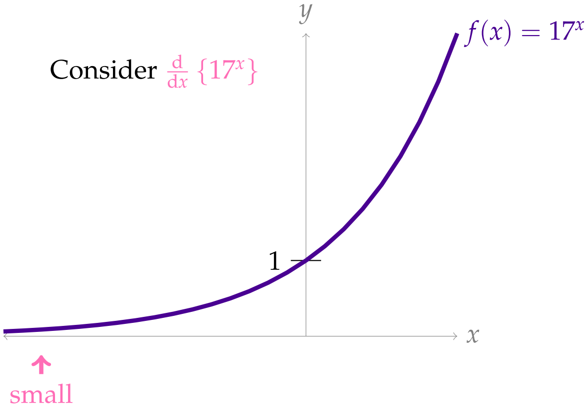

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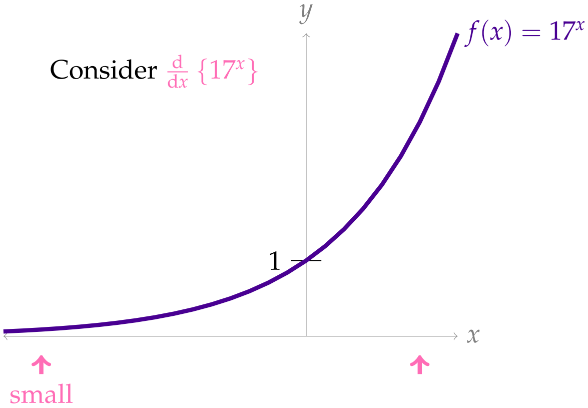

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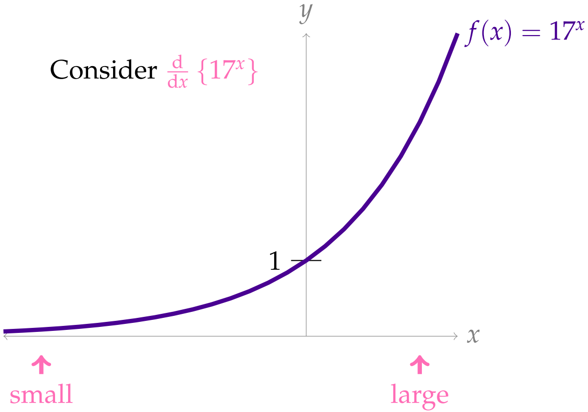
Exponential Functions



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EXPONENTIAL FUNCTIONS

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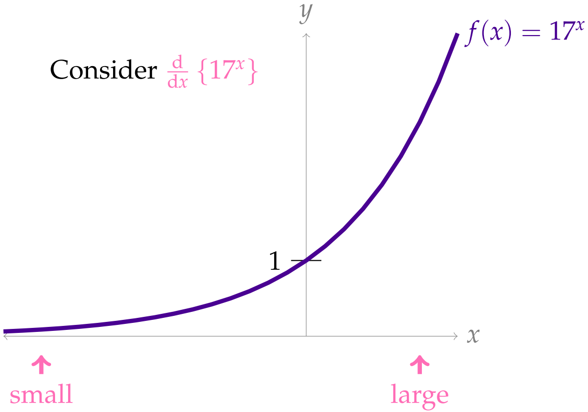

$f(x)$ is always increasing, so $f'(x)$ is always positive.
$f'(x)$ might look similar to $f(x)$.

Exponential Functions

$$\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\} =$$

Exponential Functions

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\} &= \lim_{h\to 0}\frac{17^{x+h}-17^x}{h} \\
&= \lim_{h\to 0}\frac{17^x 17^h-17^x}{h} \\
&= \lim_{h\to 0}\frac{17^x(17^h-1)}{h} \\
&= 17^x \lim_{h\to 0}\frac{(17^h-1)}{h} \\
&= 17^x(\text{ times a constant })
\end{aligned}
$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\} = 17^x \cdot \underbrace{\lim_{h\to 0}\frac{(17^h - 1)}{h}}_{\text{constant}}$$

Given what you know about $\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\}$, **is it possible** that $\displaystyle\lim_{h\to 0}\frac{17^h - 1}{h} = 0$?

A. Sure, there's no reason we've seen that would make it impossible.
B. No, it couldn't be 0, that wouldn't make sense.
C. I do not feel equipped to answer this question.

$$\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\} = 17^x \cdot \underbrace{\lim_{h\to 0}\frac{(17^h-1)}{h}}_{\text{constant}}$$

Given what you know about $\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\}$, **is it possible** that $\lim\limits_{h\to 0}\dfrac{17^h-1}{h} = \infty$?

A. Sure, there's no reason we've seen that would make it impossible.
B. No, it couldn't be ∞, that wouldn't make sense.
C. I do not feel equipped to answer this question.

$$\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\} = 17^x \cdot \underbrace{\lim_{h\to 0}\frac{(17^h - 1)}{h}}_{\text{constant}}$$

How could we find out what $\lim_{h\to 0}\frac{(17^h - 1)}{h}$ is?

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How could we find out what $\lim_{h\to 0}\frac{(17^h - 1)}{h}$ is?

h	$\frac{17^h - 1}{h}$
0.001	2.83723068608
0.00001	2.83325347992
0.0000001	2.83321374583
0.000000001	2.83321344163

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\} &= \lim_{h\to 0} \frac{17^{x+h} - 17^x}{h} \\
&= \lim_{h\to 0} \frac{17^x 17^h - 17^x}{h} \\
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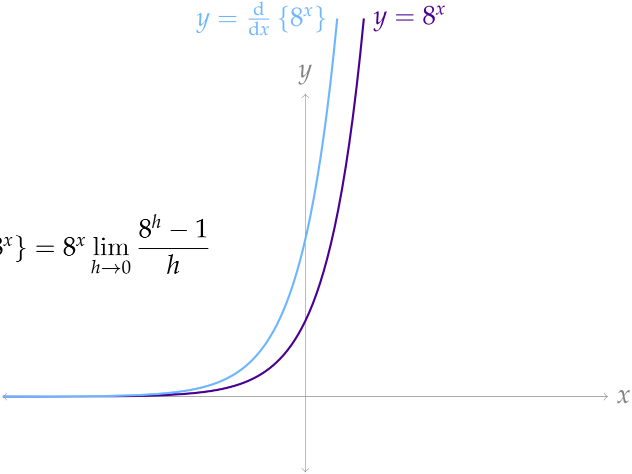
\end{aligned}$$

In general, for any positive number a,

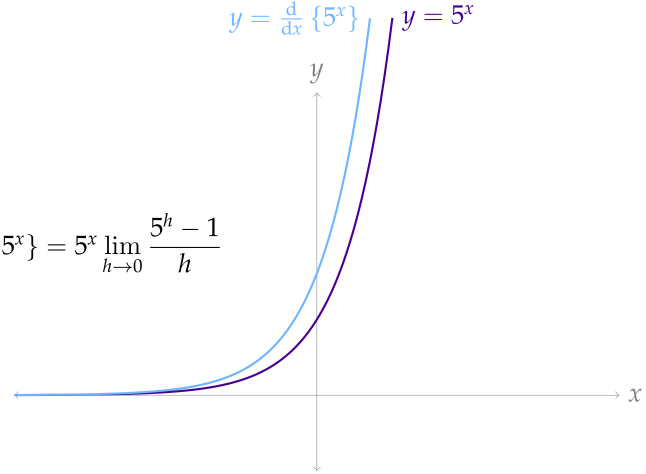
$$\frac{\mathrm{d}}{\mathrm{d}x}\{a^x\} = a^x \lim_{h\to 0}\frac{a^h-1}{h}$$

Exponential Functions



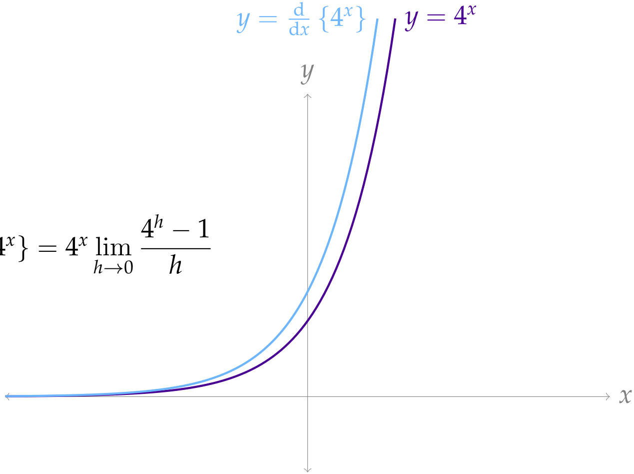
$$\frac{d}{dx}\{8^x\} = 8^x \lim_{h\to 0} \frac{8^h - 1}{h}$$

Exponential Functions



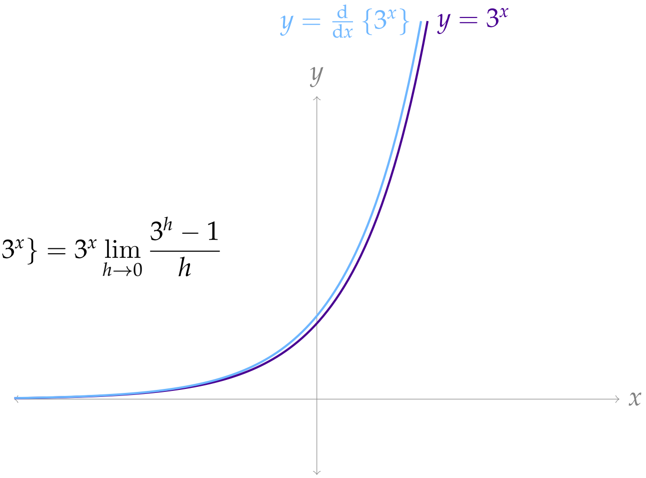
$$\frac{d}{dx}\{5^x\} = 5^x \lim_{h\to 0} \frac{5^h - 1}{h}$$

Exponential Functions



$$\frac{d}{dx}\{4^x\} = 4^x \lim_{h\to 0} \frac{4^h - 1}{h}$$

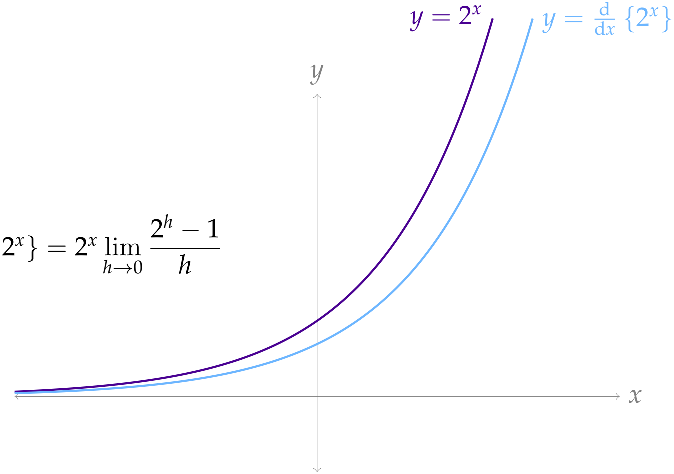
Exponential Functions



$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{3^x\right\} = 3^x \lim_{h\to 0} \frac{3^h - 1}{h}$$

Exponential Functions

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{2^x\right\} = 2^x \lim_{h\to 0}\frac{2^h-1}{h}$$



In general, for any positive number a, $\frac{\mathrm{d}}{\mathrm{d}x}\{a^x\} = a^x \lim_{h\to 0} \dfrac{a^h - 1}{h}$

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Euler's Number – Theorem 2.7.4

We define e to be the unique number satisfying

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

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Euler's Number – Theorem 2.7.4

We define e to be the unique number satisfying

$$\lim_{h\to 0} \frac{e^h - 1}{h} = 1$$

$e \approx 2.71828182845904523536028747135266 24\ldots$ (Wikipedia)

Theorem 2.7.4 and Corollary 2.10.6

Using this definition of e,

$$\frac{\mathrm{d}}{\mathrm{d}x}\{e^x\} = e^x \underbrace{\lim_{h\to 0} \frac{e^h - 1}{h}}_{1} = e^x$$

Theorem 2.7.4 and Corollary 2.10.6

Using this definition of e,

$$\frac{\mathrm{d}}{\mathrm{d}x}\{e^x\} = e^x \underbrace{\lim_{h\to 0} \frac{e^h - 1}{h}}_{1} = e^x$$

In general, $\lim_{h\to 0} \frac{a^h - 1}{h} = \log_e(a)$, so $\frac{\mathrm{d}}{\mathrm{d}x}\{a^x\} = a^x \log_e(a)$

That $\lim_{h\to 0} \frac{a^h - 1}{h} = \log_e(a)$ and $\frac{\mathrm{d}}{\mathrm{d}x}\{a^x\} = a^x \log_e(a)$ are consequences of

$$a^x = \left(e^{\log_e(a)}\right)^x = e^{x \log_e(a)}$$

For the details, see the end of Section 2.7.

Things to Have Memorized

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{e^x\right\} = e^x$$

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Let $f(x) = \frac{e^x}{3x^5}$. When is the tangent line to $f(x)$ horizontal?

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Horizontal tangent line $\Leftrightarrow$ slope of tangent line is zero $\Leftrightarrow f'(x) = 0$

$$0 = f'(x) = \frac{3x^5e^x - e^x(15x^4)}{(3x^5)^2} = \left(\frac{e^x}{9x^{10}}\right)(3x^4)(x-5)$$

$$x = 0 \text{ or } x = 5$$

But, since $f(x)$ is not defined at zero, the tangent line is only horizontal at

$$x = 5$$

Evaluate $\frac{\mathrm{d}}{\mathrm{d}x}\left\{e^{3x}\right\}$

Suppose the deficit, in millions, of a fictitious country is given by

$$f(x) = e^x(4x^3 - 12x^2 + 14x - 4)$$

where x is the number of years since the current leader took office. Suppose the leader has been in power for exactly two years.

1. Is the deficit increasing or decreasing?

~~ans~~

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1. Is the deficit increasing or decreasing?

2. Is the rate at which the deficit is growing increasing or decreasing?

Included Work

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