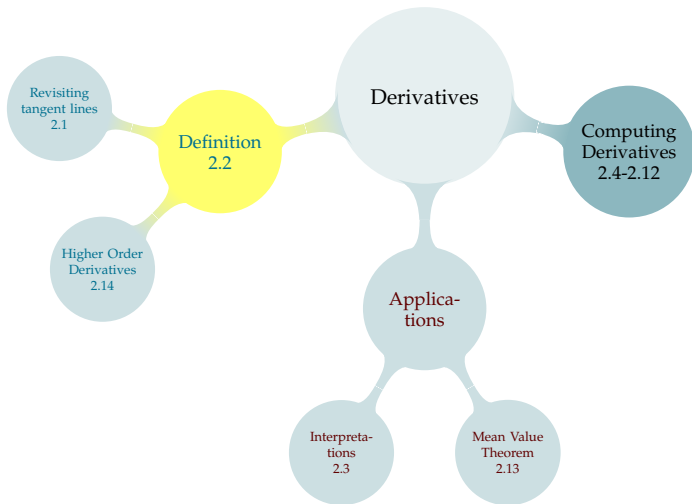


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### Definition 2.2.1

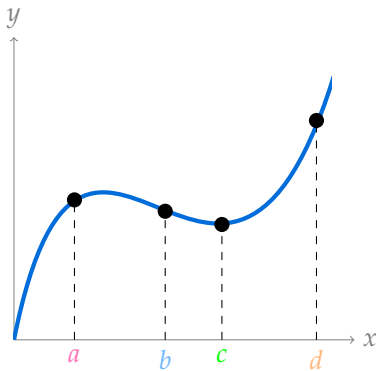
$$\text{So, } f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}.$$

$f'(a)$  is also the **instantaneous rate of change** of  $f$  at  $a$ .

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

If  $f'(a) < 0$ , then  $f$  is **decreasing** at  $a$ . Its graph “points down.”

## PRACTICE: INCREASING AND DECREASING



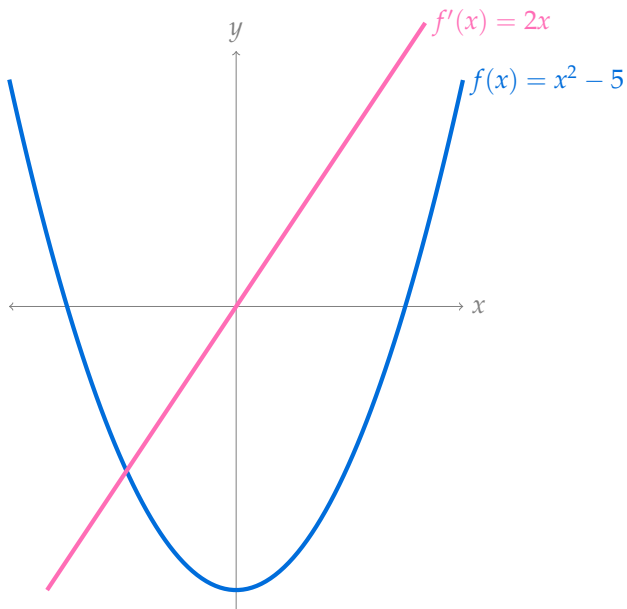
Where is  $f'(x) < 0$ ?

Where is  $f'(x) > 0$ ?

Where is  $f'(x) \approx 0$ ?

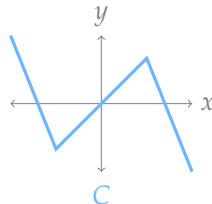
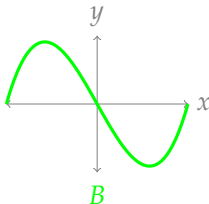
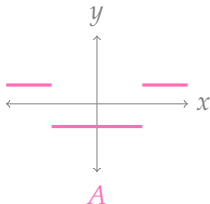
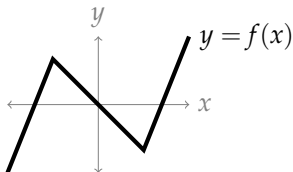
Use the definition of the derivative to find the slope of the tangent line to  $f(x) = x^2 - 5$  at the point  $x = 3$ .

Let's keep the function  $f(x) = x^2 - 5$ . We just showed  $f'(3) = 6$ .  
We can also find its derivative at an arbitrary point  $x$ :



# INCREASING AND DECREASING

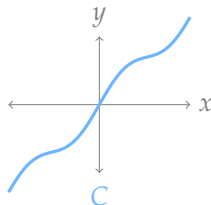
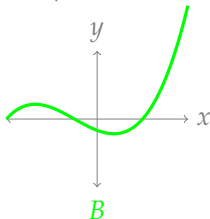
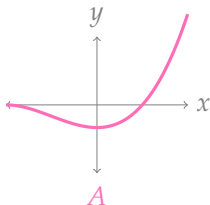
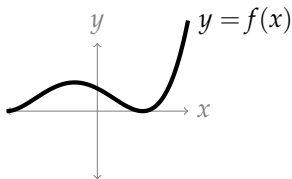
In black is the curve  $y = f(x)$ . Which of the coloured curves corresponds to  $y = f'(x)$ ?





# INCREASING AND DECREASING

In black is the curve  $y = f(x)$ . Which of the coloured curves corresponds to  $y = f'(x)$ ?



## Derivative as a Function – Definition 2.2.6

Let  $f(x)$  be a function.

The derivative of  $f(x)$  with respect to  $x$  is given by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists. Notice that  $x$  will be a part of your final expression: this is a **function**.

If  $f'(x)$  exists for all  $x$  in an interval  $(a, b)$ , we say that  $f$  is **differentiable on  $(a, b)$** .

## Notation 2.2.8

The “prime” notation  $f'(x)$  and  $f'(a)$  is sometimes called Newtonian notation. We will also use Leibnitz notation:

$$\frac{df}{dx}$$

function

$$\frac{df}{dx}(a)$$

number

$$\frac{d}{dx}f(x)$$

function

$$\frac{d}{dx}f(x)\Big|_{x=a}$$

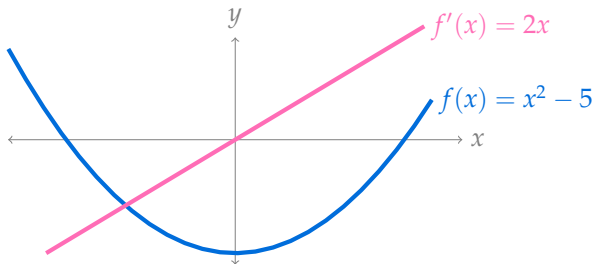
number

Newtonian Notation:

$$f(x) = x^2 + 5 \qquad f'(x) = 2x \qquad f'(3) = 6$$

Leibnitz Notation:

$$\frac{df}{dx} = \qquad \frac{df}{dx}(3) = \qquad \frac{d}{dx}f(x) = \qquad \frac{d}{dx}f(x)\Big|_{x=3} =$$



## Alternate Definition – Definition 2.2.1

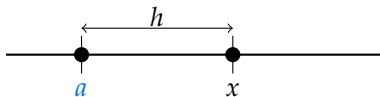
Calculating

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

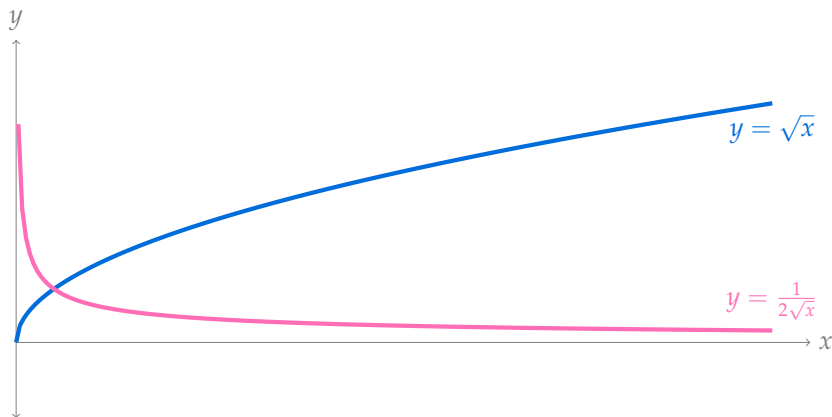
is the same as calculating

$$f'(x) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}.$$

Notice in these scenarios,  $h = x - a$ .



Let  $f(x) = \sqrt{x}$ . Using the definition of a derivative, calculate  $f'(x)$ .



Review:

$$\lim_{x \rightarrow 0^+} \sqrt{x} = \qquad \lim_{x \rightarrow \infty} \sqrt{x} = \qquad \lim_{x \rightarrow \infty} \frac{1}{2\sqrt{x}} =$$

$$\lim_{x \rightarrow 0^+} \frac{1}{2\sqrt{x}} =$$



NOW  
YOU

Using the definition of the derivative, calculate

$$\frac{d}{dx} \left\{ \frac{1}{x} \right\}.$$



Using the definition of the derivative, calculate  $\frac{d}{dx} \left\{ \frac{2x}{x+1} \right\}$ .

Using the definition of the derivative, calculate  $\frac{d}{dx} \left\{ \frac{1}{\sqrt{x^2 + x}} \right\}$ .

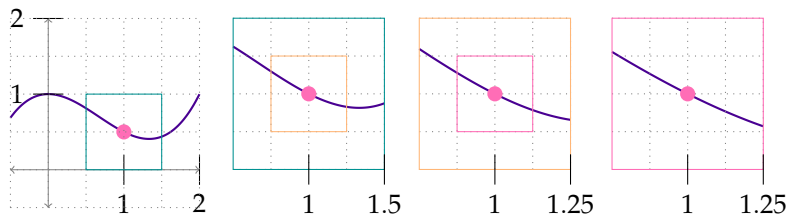
## Memorize

The derivative of a function  $f$  at a point  $a$  is given by the following limit, if it exists:

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

# ZOOMING IN

For a smooth function, if we zoom in at a point, we see a line:



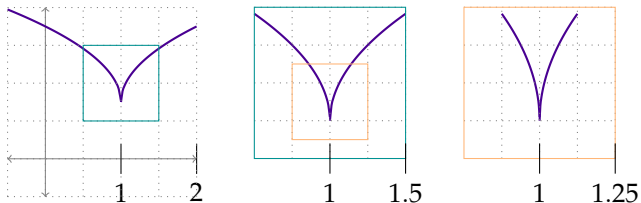
In this example, the slope of our zoomed-in line looks to be about:

$$\frac{\Delta y}{\Delta x} \approx -\frac{1}{2}$$

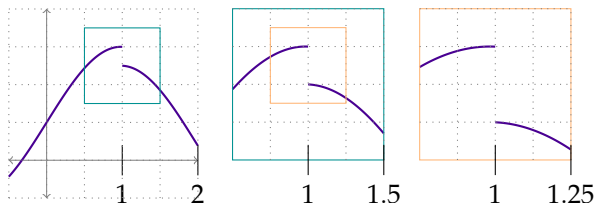
# ZOOMING IN ON FUNCTIONS THAT AREN'T SMOOTH

For a function with a cusp or a discontinuity, even though we zoom in very closely, we don't see simply a single straight line.

Cusp:



Discontinuity:



## Alternate Definition – Definition 2.2.1

Calculating

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

is the same as calculating

$$f'(x) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}.$$

Notice in these scenarios,  $h = x - a$ .

The derivative of  $f(x)$  **does not exist** at  $x = a$  if

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

does not exist.

Note this is the slope of the tangent line to  $y = f(x)$  at  $x = a$ ,  $\frac{\Delta y}{\Delta x}$ .

# WHEN DERIVATIVES DON'T EXIST

What happens if we try to calculate a derivative where none exists?

Find the derivative of  $f(x) = x^{1/3}$  at  $x = 0$ .

## Theorem 2.2.14

If the function  $f(x)$  is differentiable at  $x = a$ , then  $f(x)$  is also continuous at  $x = a$ .

Proof:



Let  $f(x)$  be a function and let  $a$  be a constant in its domain. Draw a picture of each scenario, or say that it is impossible.

$f(x)$ continuous at $x = a$ $f(x)$ differentiable at $x = a$	$f(x)$ continuous at $x = a$ $f(x)$ differentiable at $x = a$
$f(x)$ continuous at $x = a$ $f(x)$ differentiable at $x = a$	$f(x)$ continuous at $x = a$ $f(x)$ differentiable at $x = a$

## Included Work



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