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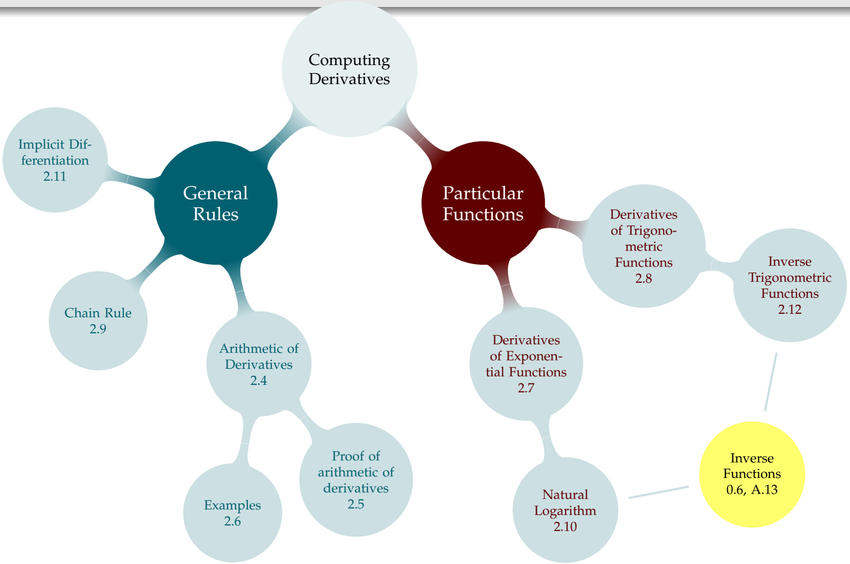
Skip review of inverse functions and logarithms



Invertibility Game

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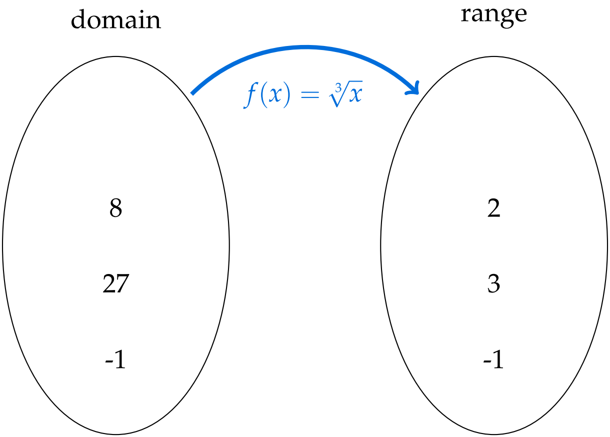
Round 3: $f(x) = |x|$

Round 4: $f(x) = \sin x$

Functions are Maps



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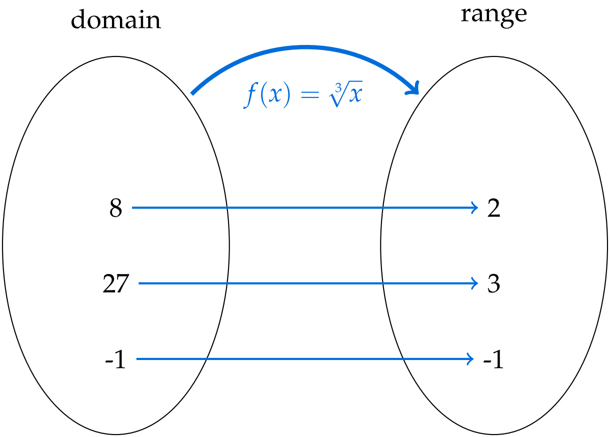

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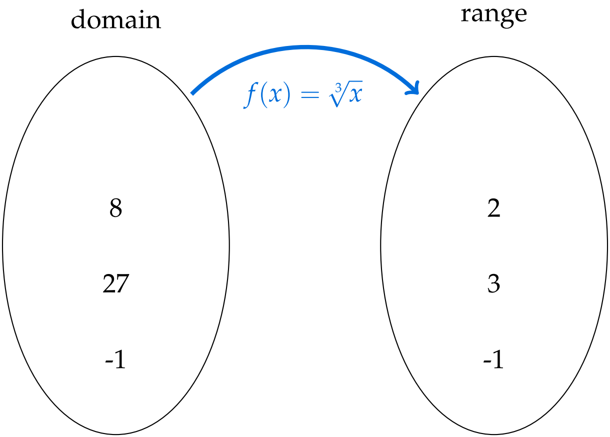

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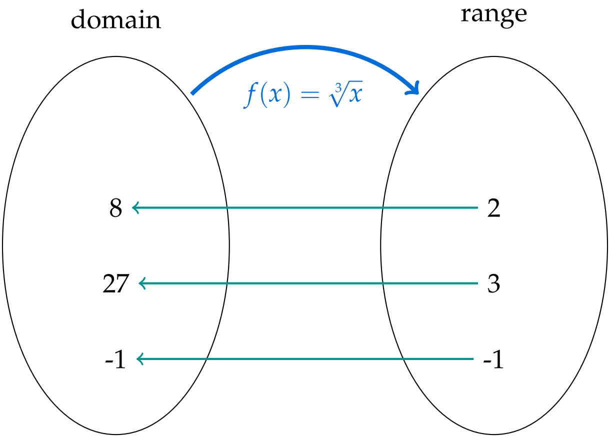

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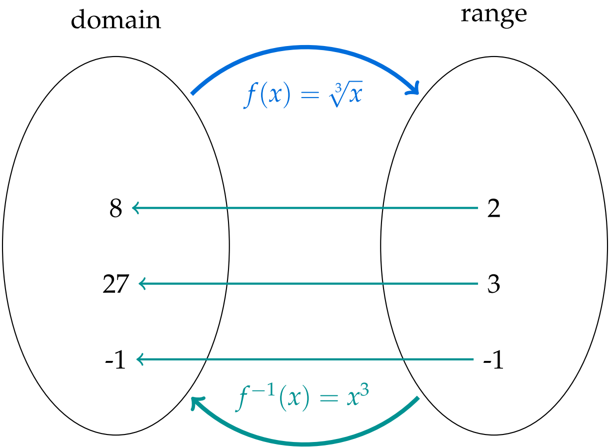

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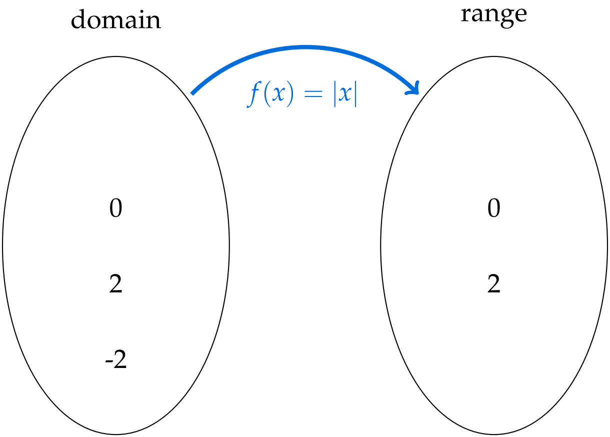

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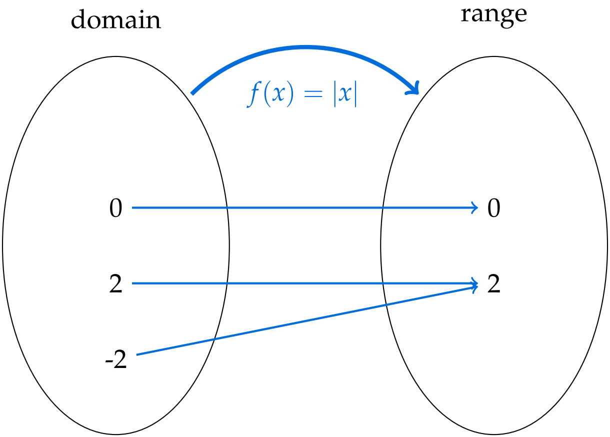

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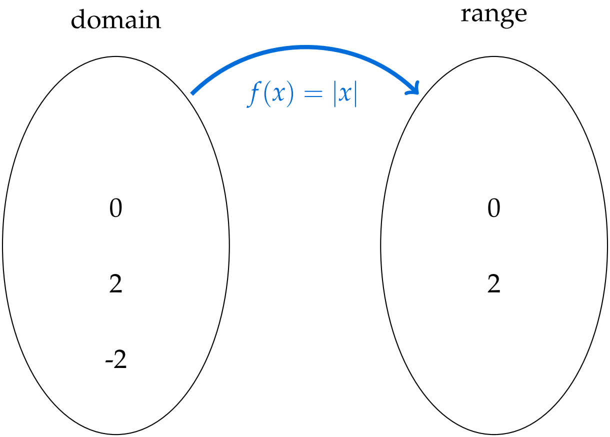
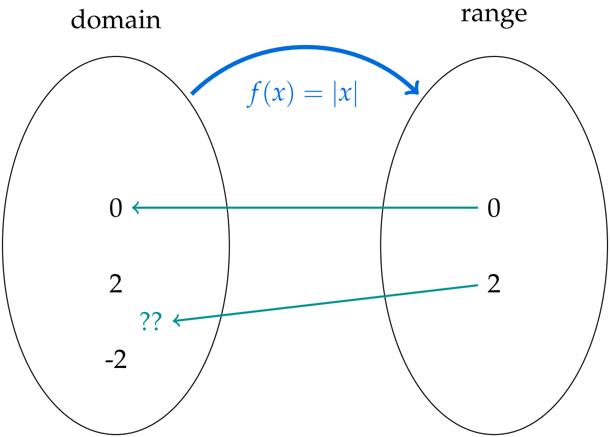


FUNCTIONS ARE MAPS



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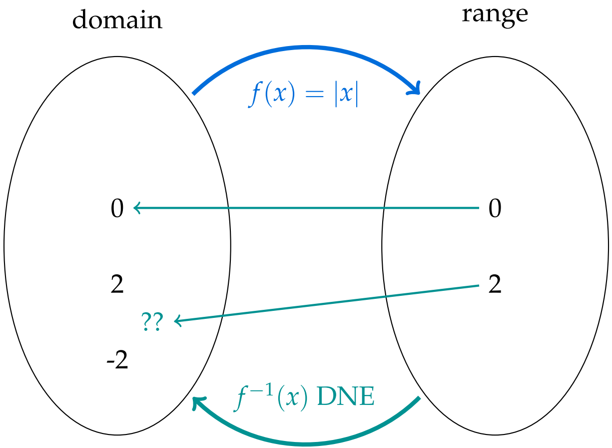
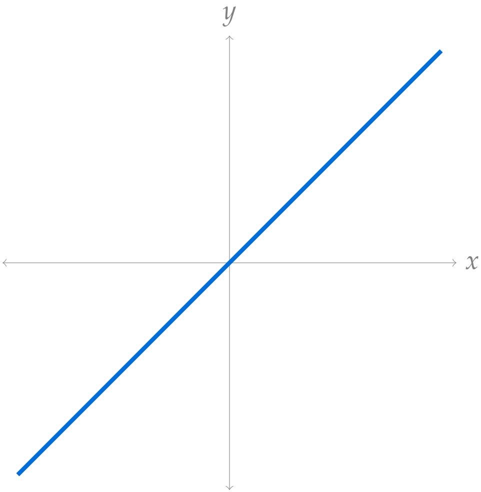
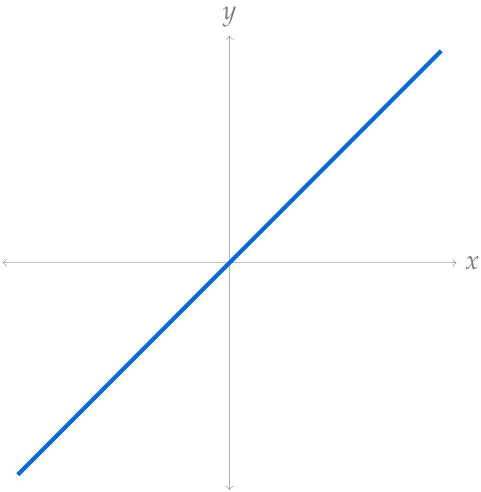




A. invertible

B. not invertible



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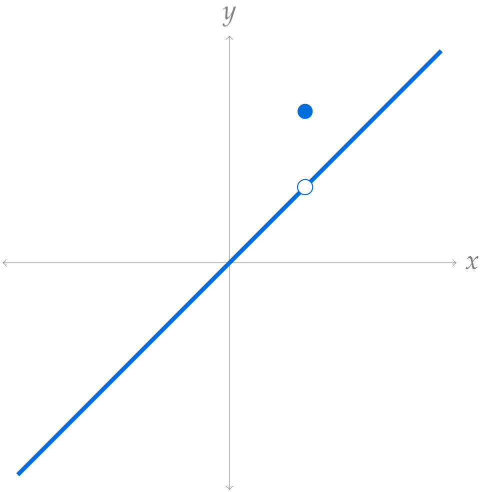

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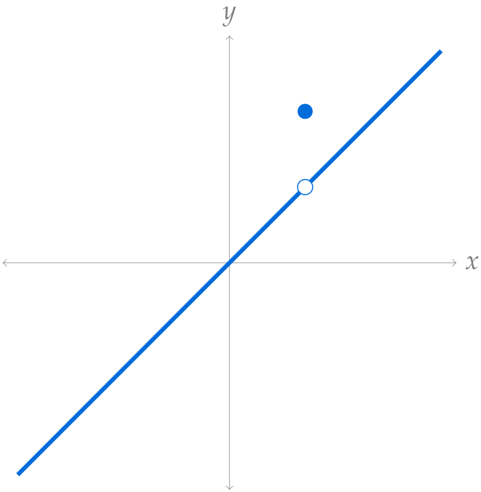

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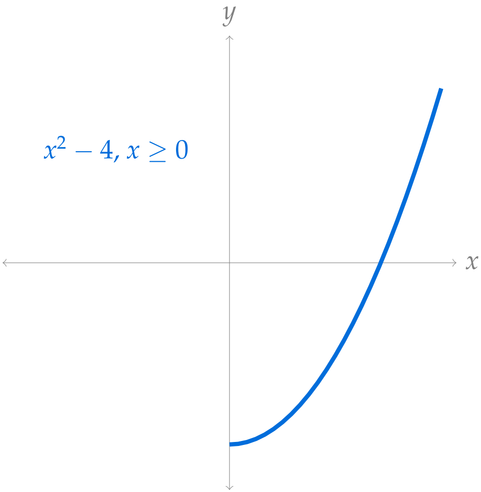

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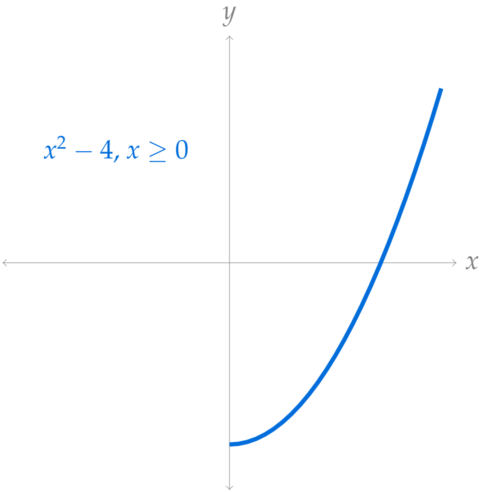

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Relationship between $f(x)$ and $f^{-1}(x)$

Let f be an invertible function.
What is $f^{-1}(f(x))$?

A. x

B. 1

C. 0

D. not sure

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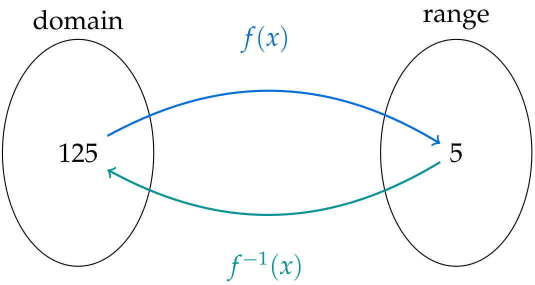

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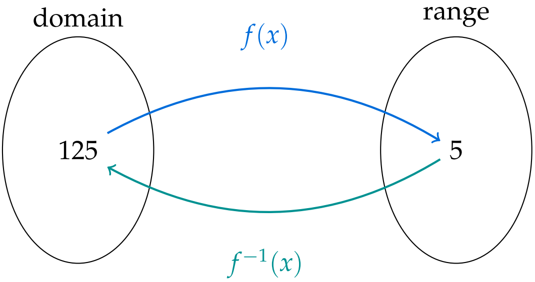

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Suppose $f(x) = \sqrt[3]{19 + x^3}$. What is $f^{-1}(3)$? (simplify your answer)

Q

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What is $f^{-1}(10)$? (do not simplify)

What is $f^{-1}(x)$?

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ans

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In order for a function to be invertible , different x values cannot map to the same y value.
We call such a function **one-to-one**, or **injective**.

Suppose $f(x) = \sqrt[3]{19 + x^3}$. What is $f^{-1}(3)$? (simplify your answer)
$f(2) = 3$, so $f^{-1}(3) = 2$

What is $f^{-1}(10)$? (do not simplify)
$\sqrt[3]{19 + y^3} = 10$ tells us $f^{-1}(10) = \sqrt[3]{10^3 - 19}$

What is $f^{-1}(x)$?
$\sqrt[3]{19 + y^3} = x$ tells us $f^{-1}(x) = \sqrt[3]{x^3 - 19}$

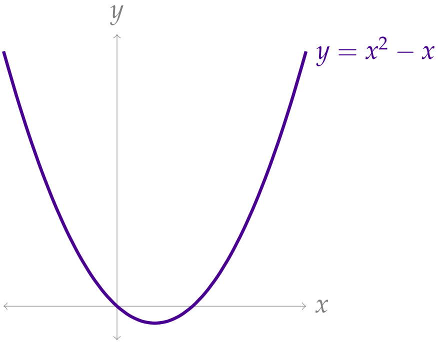
Let $f(x) = x^2 - x$.

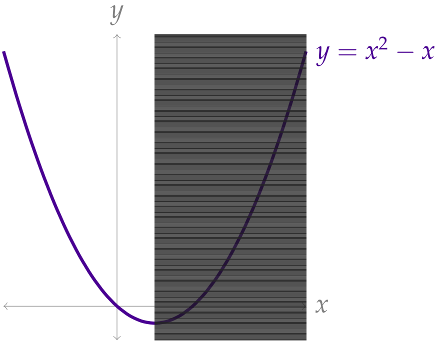
1. Sketch a graph of $f(x)$, and choose a (large) domain over which it is invertible.

2. For the domain you chose, evaluate $f^{-1}(20)$.

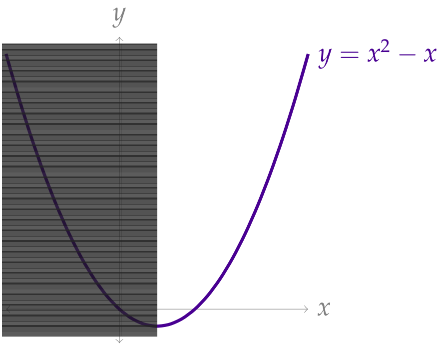
3. For the domain you chose, evaluate $f^{-1}(x)$.

4. What are the domain and range of $f^{-1}(x)$? What are the (restricted) domain and range of $f(x)$?

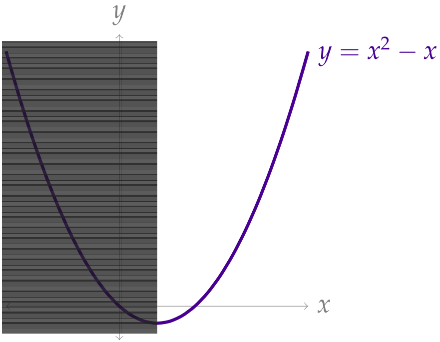

y
$y = x^2 - x$
x



Domain: $\left(-\infty, \frac{1}{2}\right]$



Domain: $\left[\frac{1}{2}, \infty\right)$



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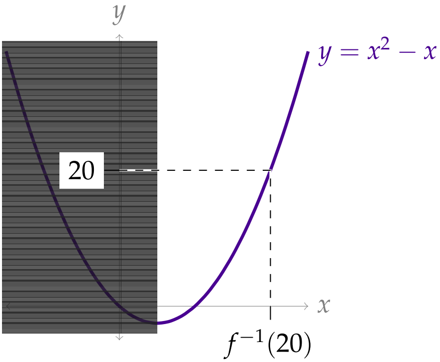
$f^{-1}(20) =$



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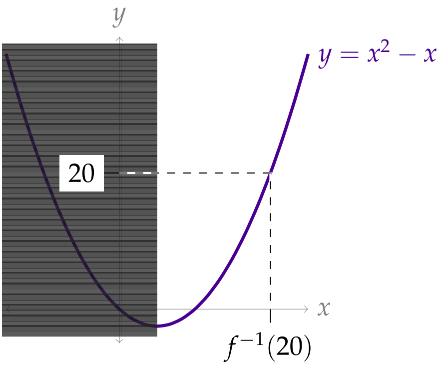
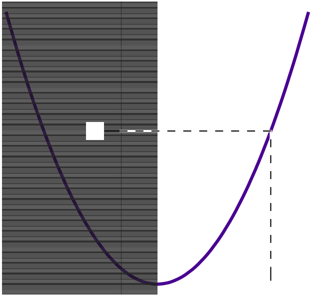
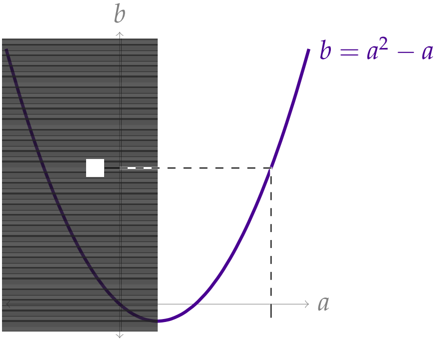


$f^{-1}(20) = 5$

$$20 = x^2 - x$$
$$0 = x^2 - x - 20 = (x-5)(x+4)$$
$$x = 5$$



$$f^{-1}(x) =$$



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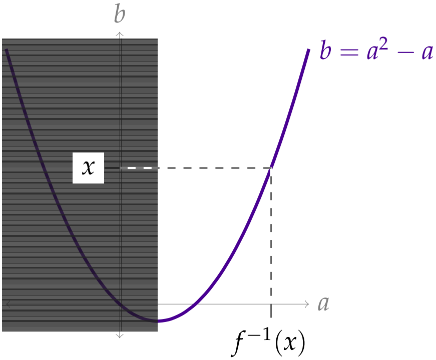

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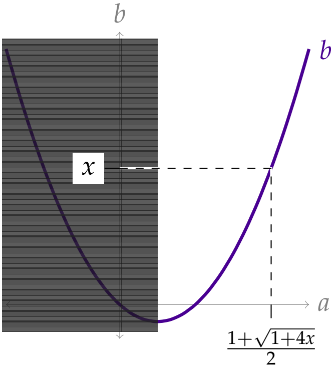


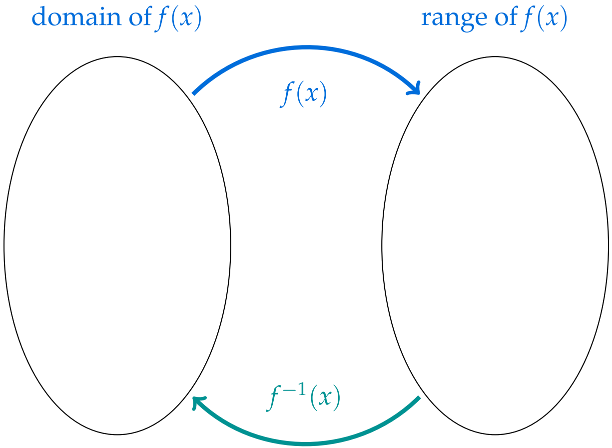
$$f^{-1}(x) = \frac{1+\sqrt{1+4x}}{2}$$

$$a^2 - a = x, \text{ find } a$$

$$a^2 - a - x = 0$$

$$a = \frac{1 \pm \sqrt{1+4x}}{2}$$

$$f^{-1}(x) = \frac{1+\sqrt{1+4x}}{2}$$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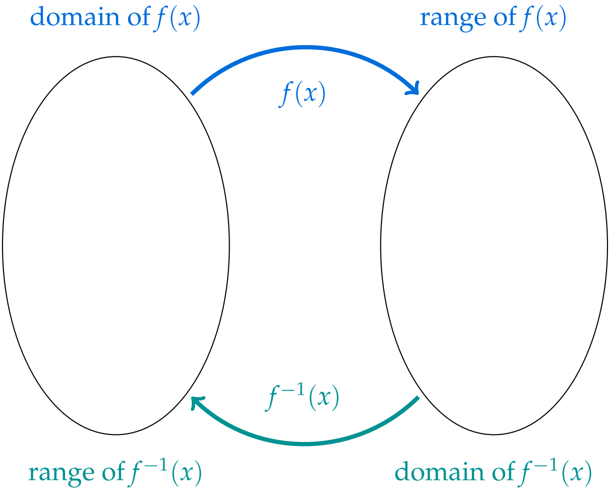
range of $f(x)$
$f(x)$
$f^{-1}(x)$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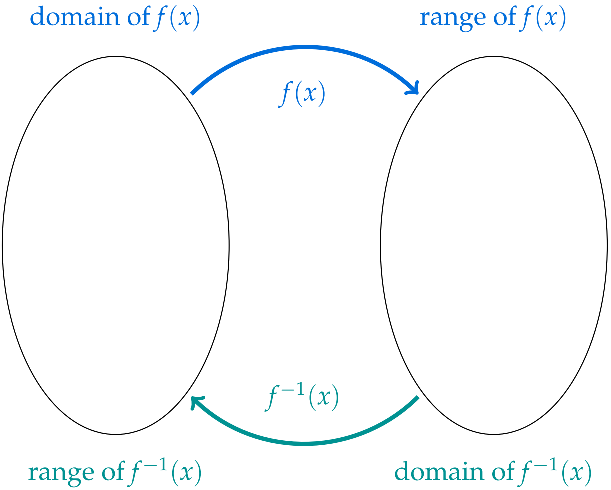
range of $f^{-1}(x)$
domain of $f^{-1}(x)$

$f(x) = x^2 - x$, domain: $\left[\frac{1}{2}, \infty\right)$



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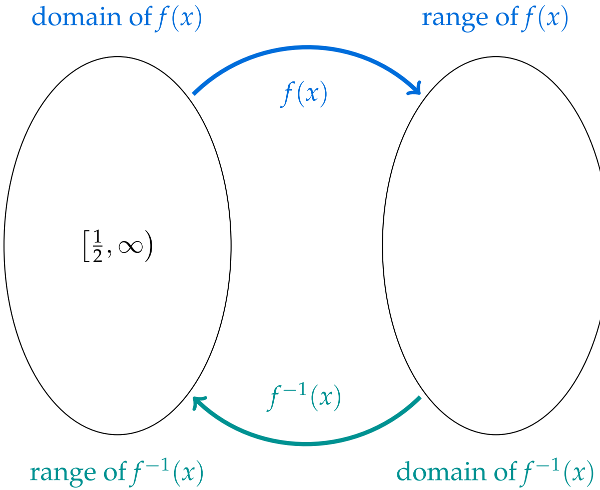

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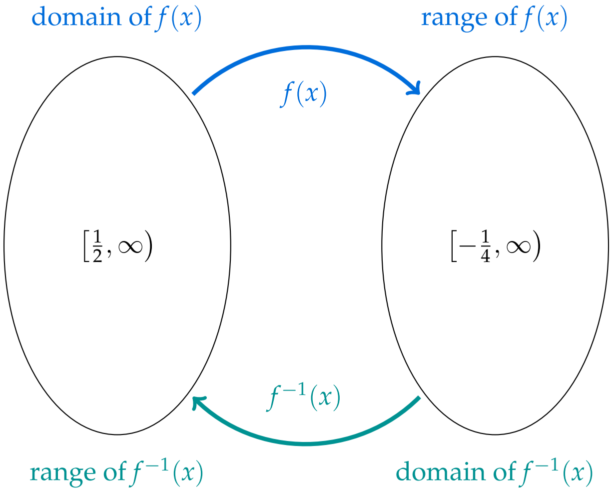

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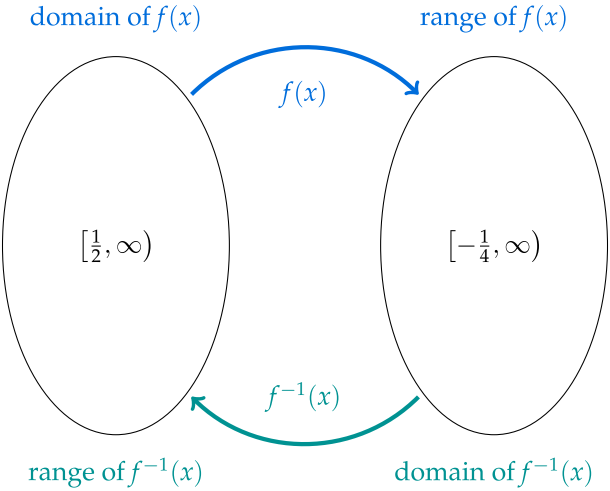
$f^{-1}(x) = \frac{1+\sqrt{1+4x}}{2}$



INVERTIBILITY GAME: $f(x) = e^x$

- I'm thinking of an x. Your clue: $f(x) = e$. What is my x?

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- I'm thinking of an x. Your clue: $f(x) = e$. What is my x?
- I'm thinking of an x. Your clue: $f(x) = 1$. What is my x?

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- I'm thinking of an x. Your clue: $f(x) = \frac{1}{e}$. What is my x?

INVERTIBILITY GAME: $f(x) = e^x$

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- I'm thinking of an x. Your clue: $f(x) = e^3$. What is my x?

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- I'm thinking of an x. Your clue: $f(x) = e^3$. What is my x?
- I'm thinking of an x. Your clue: $f(x) = 0$. What is my x?

INVERTIBILITY GAME: $f(x) = e^x$

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- I'm thinking of an x. Your clue: $f(x) = 0$. What is my x? Trick question: no x gives $f(x) = 0$.

INVERTIBILITY GAME: $f(x) = e^x$ $f^{-1}(x) = \log_e x$

- I'm thinking of an x. Your clue: $f(x) = e$. What is my x? $x = 1$
- I'm thinking of an x. Your clue: $f(x) = 1$. What is my x? $x = 0$
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INVERTIBILITY GAME: $f(x) = e^x$ $\quad f^{-1}(x) = \log_e x$

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  $\log_e(e) = 1$
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  $\log_e\left(\frac{1}{e}\right) = -1$
- I'm thinking of an x. Your clue: $f(x) = e^3$. What is my x? $x = 3$
  $\log_e(e^3) = 3$
- I'm thinking of an x. Your clue: $f(x) = 0$. What is my x? Trick question: no x gives $f(x) = 0$.
  $\log_e(x)$ is undefined at $x = 0$

1. Suppose $0 < x < 1$. Then $\log_e(x)$ is...

2. Suppose $-1 < x < 0$. Then $\log_e(x)$ is...

3. Suppose $e < x$. Then $\log_e(x)$ is...

A. positive
B. negative
C. greater than one
D. less than one
E. undefined

Exponents and Logarithms

$$f(x) = e^x \qquad f^{-1}(x) = \log_e(x) = \ln(x) = \log(x)$$

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$$f(x) = e^x \qquad\qquad f^{-1}(x) = \log_e(x) = \ln(x) = \log(x)$$

x	e^x	
0	1	
1	e	
-1	$\frac{1}{e}$	
n	e^n	

Exponents and Logarithms

$$f(x) = e^x \qquad\qquad f^{-1}(x) = \log_e(x) = \ln(x) = \log(x)$$

x	e^x	e fact $\leftrightarrow$ $\log_e$ fact	x	$\log_e(x)$
0	1			
1	e			
-1	$\frac{1}{e}$			
n	e^n			

Exponents and Logarithms

$f(x) = e^x$

$f^{-1}(x) = \log_e(x) = \ln(x) = \log(x)$

x	e^x	e fact $\leftrightarrow$ $\log_e$ fact	x	$\log_e(x)$
0	1	$e^0 = 1 \leftrightarrow \log_e(1) = 0$	1	0
1	e			
-1	$\frac{1}{e}$			
n	e^n			

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1	e	$e^1 = e \leftrightarrow \log_e(e) = 1$	e	1
-1	$\frac{1}{e}$			
n	e^n			

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1	e	$e^{1} = e \leftrightarrow \log_e(e) = 1$	e	1
-1	$\frac{1}{e}$	$e^{-1} = \frac{1}{e} \leftrightarrow \log_e\left(\frac{1}{e}\right) = -1$	$\frac{1}{e}$	-1
n	e^n			

Exponents and Logarithms

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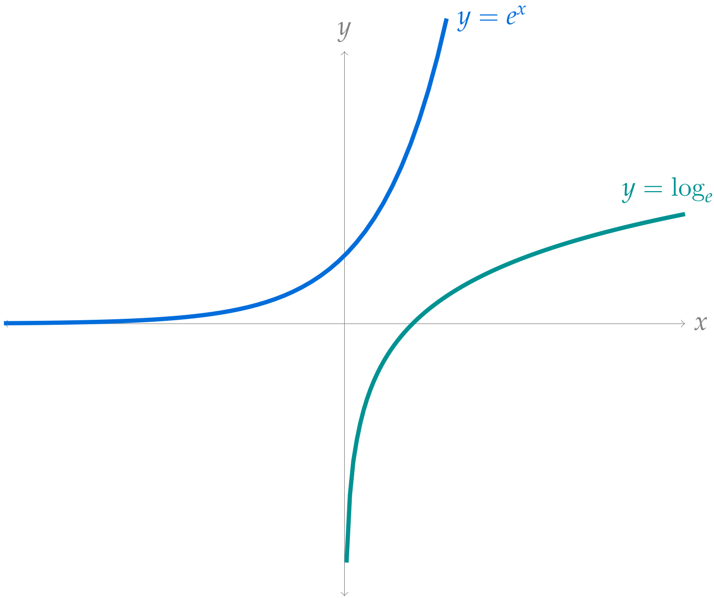
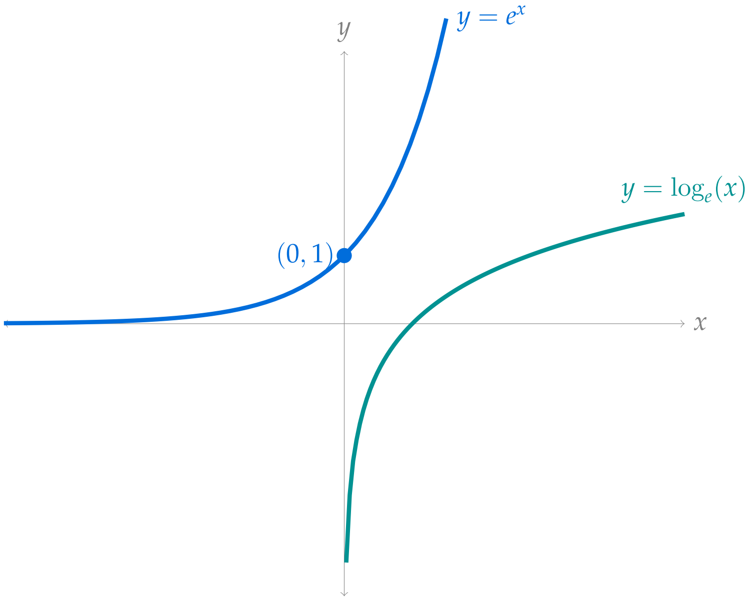
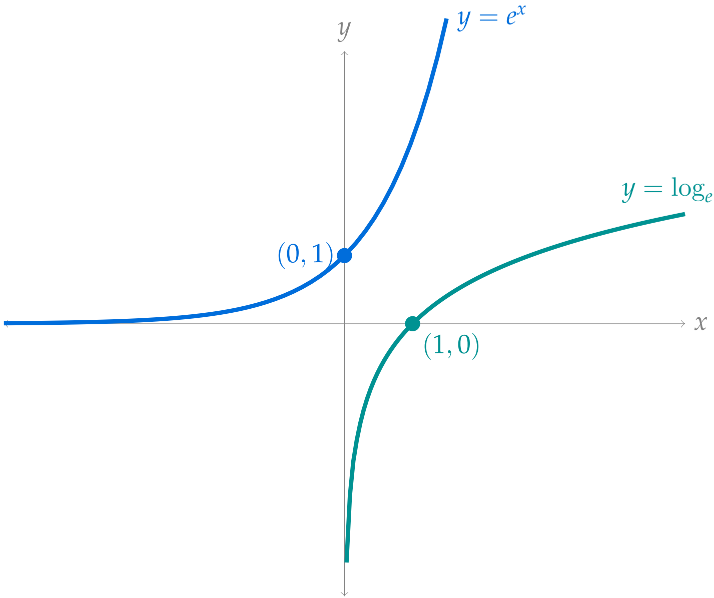
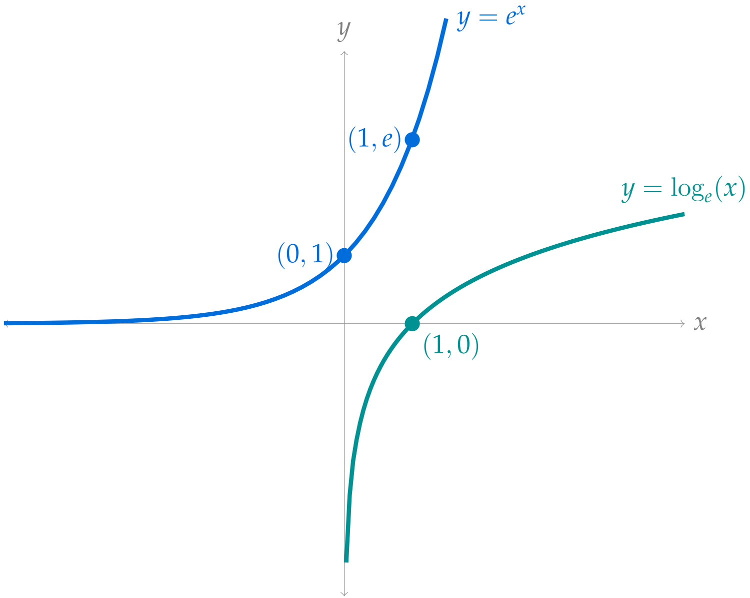
1	e	$e^1 = e \leftrightarrow \log_e(e) = 1$	e	1
-1	$\frac{1}{e}$	$e^{-1} = \frac{1}{e} \leftrightarrow \log_e(\frac{1}{e}) = -1$	$\frac{1}{e}$	-1
n	e^n	$e^n = e^n \leftrightarrow \log_e(e^n) = n$	e^n	n


$y = e^x$
y
$y = \log_e(x)$
x


$y = e^x$
y
$y = \log_e(x)$
$(0, 1)$
x


y
$y = e^x$
$y = \log_e(x)$
$(0, 1)$
x
$(1, 0)$


$y = e^x$
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x
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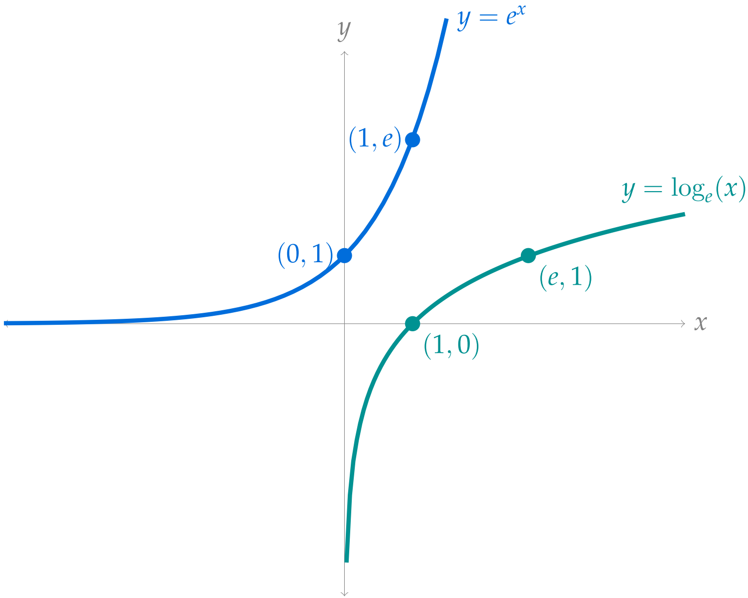
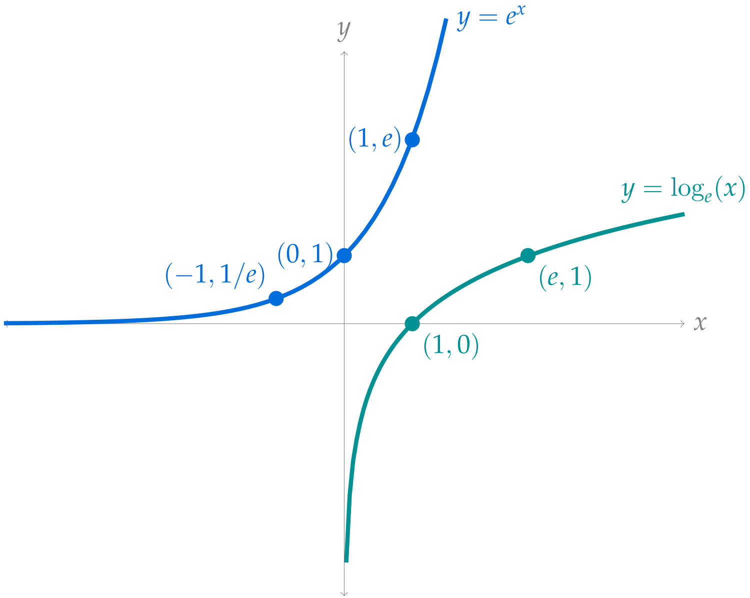
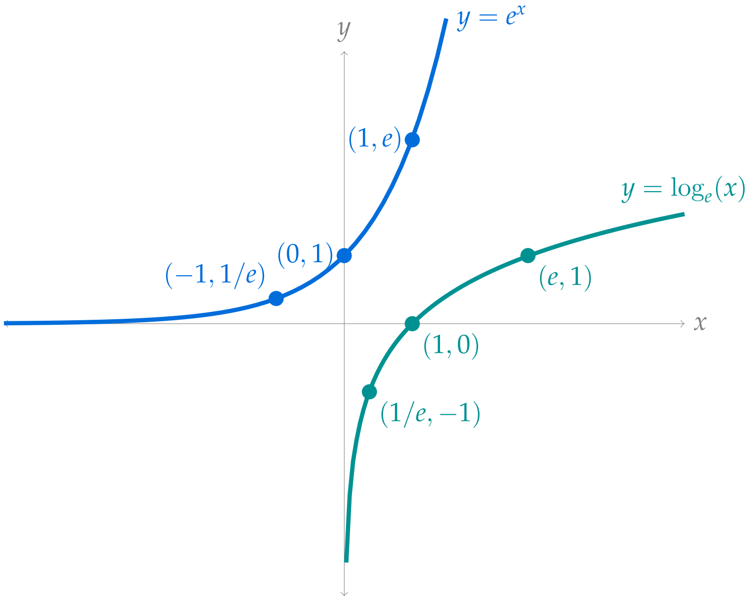

$y = e^x$
y
$(1, e)$
$y = \log_e(x)$
$(0, 1)$
$(e, 1)$
x
$(1, 0)$


$y = e^x$
y
$(1, e)$
$y = \log_e(x)$
$(0, 1)$
$(-1, 1/e)$
$(e, 1)$
x
$(1, 0)$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$(1, e)$
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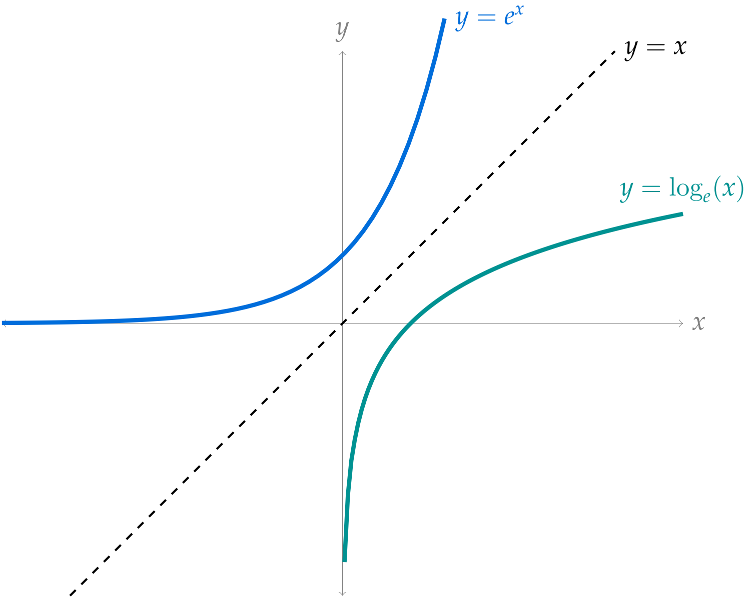
$(e, 1)$
x
$(1, 0)$
$(1/e, -1)$


$y = e^x$
y
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x

LOGS OF OTHER BASES: $\log_n(x)$ IS THE INVERSE OF n^x

$\log_{10} 10^8 =$

A. 0
B. 8
C. 10
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D. other ✓ $2^4 = 16$ so $\log_2 16 = 4$

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Let A and B be positive, and let n be any real number.

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Write as a single logarithm:

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$$
\begin{aligned}
f(x) &= \log\left(\frac{10}{x^2}\right) + 2\log x + \log(10+x) \\
&= \log 10 - \log(x^2) + 2\log x + \log(10+x) \\
&= \log 10 - 2\log x + 2\log x + \log(10+x) \\
&= \log 10 + \log(10+x) = \log(10(10+x))) \\
&= \log(100+10x)
\end{aligned}
$$

Base Change

Fact: $\quad b^{\log_b(a)} = a$

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$$\Rightarrow \log\left(b^{\log_b(a)}\right) = \log(a)$$

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In general, for positive a, b, and c:

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Suppose your calculator can only compute logarithms base 10. What would you enter to calculate $\log(17)$?

Suppose your calculator can only compute natural logarithms. What would you enter to calculate $\log_2(57)$?

Suppose your calculator can only compute logarithms base 2. What would you enter to calculate $\log(2)$?

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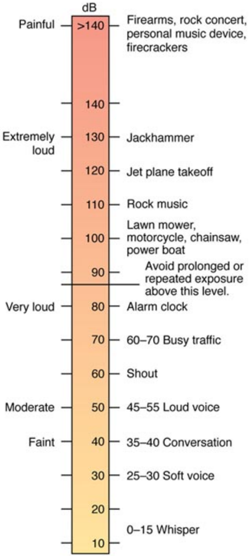
Suppose your calculator can only compute natural logarithms. What would you enter to calculate $\log_2(57)$? $\frac{\log(57)}{\log(2)}$

Suppose your calculator can only compute logarithms base 2. What would you enter to calculate $\log(2)$? $\frac{\log_2 2}{\log_2 e} = \frac{1}{\log_2 e}$



Decibels: For a particular measure of the power P of a sound wave, the decibels of that sound is:

$$10\log_{10}(P)$$

So, every ten decibels corresponds to a sound being ten **times** louder.

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A. 100 dB
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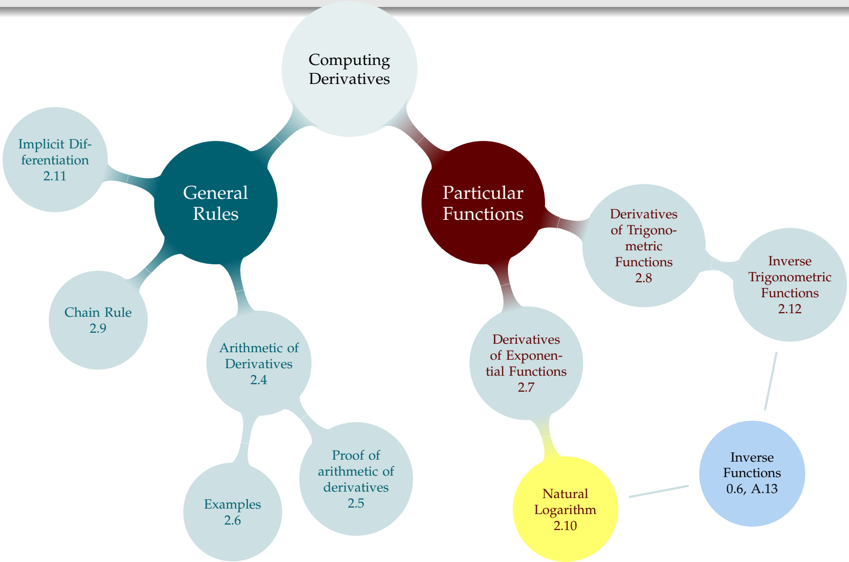
Table of Contents



Differentiating the Natural Logarithm

Calculate $\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\}$.

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One Weird Trick:

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One Weird Trick:

$$\begin{aligned} x &= e^{\log_e x} \\ \frac{\mathrm{d}}{\mathrm{d}x}\{x\} &= \frac{\mathrm{d}}{\mathrm{d}x}\left\{e^{\log_e x}\right\} \\ 1 &= e^{\log_e x} \cdot \frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\} = x \cdot \frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\} \\ \frac{1}{x} &= \frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\} \end{aligned}$$

Differentiating the Natural Logarithm

Calculate $\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e x\}$.

Derivative of Natural Logarithm – Theorem 2.10.1

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$$\frac{\mathrm{d}}{\mathrm{d}x}\{\log_e |x|\} =$$

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Derivative of Natural Logarithm

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We use the chain rule:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left\{\log_e \left|\boxed{x^2+1}\right|\right\} &= \frac{1}{x^2+1} \cdot (2x)\\ &= \frac{2x}{x^2+1}\end{aligned}$$

Derivatives of Logarithms – Corollary 2.10.6

For $a > 0$:

$$\frac{\mathrm{d}}{\mathrm{d}x}[\log_a |x|] = \frac{1}{x \log a}$$

In particular:

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We use the chain rule:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\log_e \left|\boxed{\cot x}\right|\right] = \frac{1}{\cot x} \cdot \left(- \csc^2 x\right) = \frac{-\csc^2 x}{\cot x}$$

Logarithmic Differentiation - A Fancy Trick

- $\log(f \cdot g) = \log f + \log g$

- $\log\left(\frac{f}{g}\right) = \log f - \log g$

- $\log\left(f^{g}\right) = g \log f$

Logarithmic Differentiation - A Fancy Trick

- $\log(f \cdot g) = \log f + \log g$

  multiplication turns into addition

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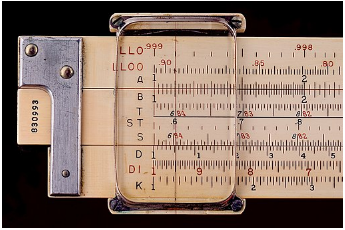
  We can exploit these properties to differentiate!

Logarithmic Differentiation

In general, if $f(x) \neq 0$, $\frac{\mathrm{d}}{\mathrm{d}x}\left[\log|f(x)|\right] = \frac{f'(x)}{f(x)}$.

$$f(x) = \left(\frac{(2x+5)^4(x^2+1)}{x+3}\right)^5$$

Find $f'(x)$.



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Logarithmic Differentiation - A Fancy Trick

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$$\begin{aligned}
\log(f(x)) &= \log\left[\left(\frac{(2x+5)^4(x^2+1)}{x+3}\right)^5\right] \\
&= 5\log\left[\frac{(2x+5)^4(x^2+1)}{x+3}\right] \\
&= 5\left[4\log(2x+5) + \log(x^2+1) - \log(x+3)\right] \\
\frac{f'(x)}{f(x)} &= 5\left[4\frac{2}{2x+5} + \frac{2x}{x^2+1} - \frac{1}{x+3}\right] \\
f'(x) &= \left(\frac{(2x+5)^4(x^2+1)}{x+3}\right)^5 \cdot 5\left[4\frac{2}{2x+5} + \frac{2x}{x^2+1} - \frac{1}{x+3}\right]
\end{aligned}$$

Logarithmic Differentiation - A Fancy Trick

Differentiate:

$$f(x) = x^x$$

Q

Logarithmic Differentiation - A Fancy Trick

Differentiate:

$$f(x) = x^x$$

$$\begin{aligned}
\log(f(x)) &= \log\left[x^x\right] \\
&= x \log x \\
\frac{f'(x)}{f(x)} &= x \cdot \frac{1}{x} + \log x \cdot 1 \\
&= 1 + \log x \\
f'(x) &= x^x\left[1 + \log x\right]
\end{aligned}$$

Logarithmic Differentiation - A Fancy Trick

Differentiate:

$$f(x) = \left(\frac{(x^{15}-9x^2)^{10}(x+x^2+1)}{(x^7+7)(x+1)(x+2)(x+3)}\right)^5$$

Q

ans

Logarithmic Differentiation - A Fancy Trick

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$$f(x) = \left(\frac{(x^{15}-9x^2)^{10}(x+x^2+1)}{(x^7+7)(x+1)(x+2)(x+3)}\right)^5$$

$$\begin{aligned}
\log(f(x)) &= \log\left[\left(\frac{(x^{15}-9x^2)^{10}(x+x^2+1)}{(x^7+7)(x+1)(x+2)(x+3)}\right)^5\right] \\
&= 5\log\left[\frac{(x^{15}-9x^2)^{10}(x+x^2+1)}{(x^7+7)(x+1)(x+2)(x+3)}\right] \\
&= 5\left[10\log\left(x^{15}-9x^2\right)+\log(x+x^2+1)-\log(x^7+7)-\log(x+1)-\log(x+2)-\log(x+3)\right] \\
\frac{f'(x)}{f(x)} &= 5\left[10\frac{15x^{14}-18x}{x^{15}-9x^2}+\frac{1+2x}{x+x^2+1}-\frac{7x^6}{x^7+7}-\frac{1}{x+1}-\frac{1}{x+2}-\frac{1}{x+3}\right]
\end{aligned}$$

$$f'(x) = \left(\frac{(x^{15}-9x^2)^{10}(x+x^2+1)}{(x^7+7)(x+1)(x+2)(x+3)}\right)^5 \cdot 5\left[10\frac{15x^{14}-18x}{x^{15}-9x^2}+\frac{1+2x}{x+x^2+1}-\frac{7x^6}{x^7+7}-\frac{1}{x+1}-\frac{1}{x+2}-\frac{1}{x+3}\right]$$

A

$$f(x) = \frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x}$$

Q

ans

$$f(x) = \frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x}$$

$$\begin{aligned}
\frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x} &= f(x) \\
\log\left|\frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x}\right| &= \log|f(x)| \\
\left[\log|x^8 - e^x| + \log|x^{1/2} + 5| - 5\log|\csc x|\right] &= \log|f(x)| \\
\frac{\mathrm{d}}{\mathrm{d}x}\left[\log|x^8 - e^x| + \log|x^{1/2} + 5| - 5\log|\csc x|\right] &= \frac{\mathrm{d}}{\mathrm{d}x}\log|f(x)| \\
\frac{8x^7 - e^x}{x^8 - e^x} + \frac{\frac{1}{2}x^{-1/2}}{x^{1/2} + 5} - 5\frac{-\csc x \cot x}{\csc x} &= \frac{f'(x)}{f(x)} \\
\frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x}\left(\frac{8x^7 - e^x}{x^8 - e^x} + \frac{\frac{1}{2}x^{-1/2}}{x^{1/2} + 5} + 5\frac{\csc x \cot x}{\csc x}\right) &= f'(x)
\end{aligned}$$

$$f(x) = (x^2+17)(32x^5-8)(x^{98}-x^{57}+32x^2)^4(32x^{10}-10x^{32})$$

Find $f'(x)$.

Q

ans

$$f(x) = (x^2+17)(32x^5-8)(x^{98}-x^{57}+32x^2)^4(32x^{10}-10x^{32})$$

Find $f'(x)$.

$$\begin{aligned}
(x^2+17)(32x^5-8)(x^{98}-x^{57}+32x^2)^4(32x^{10}-10x^{32}) &= f(x)\\
\log\left|(x^2+17)(32x^5-8)(x^{98}-x^{57}+32x^2)^4(32x^{10}-10x^{32})\right| &= \log|f(x)|\\
\log|x^2+17|+\log|32x^5-8|+4\log|x^{98}-x^{57}+32x^2|+\log|32x^{10}-10x^{32}| &= \log|f(x)|\\
\frac{\mathrm{d}}{\mathrm{d}x}\left[\log|x^2+17|+\log|32x^5-8|+4\log|x^{98}-x^{57}+32x^2|+\log|32x^{10}-10x^{32}|\right] &= \frac{\mathrm{d}}{\mathrm{d}x}[\log|f(x)|]\\
\frac{2x}{x^2+17}+\frac{160x^4}{32x^5-8}+4\frac{98x^{97}-57x^{56}+64x}{x^{98}-x^{57}+32x^2}+\frac{320x^9-320x^{31}}{32x^{10}-10x^{32}} &= \frac{f'(x)}{f(x)}\\
\left((x^2+17)(32x^5-8)(x^{98}-x^{57}+32x^2)^4(32x^{10}-10x^{32})\right)\cdot\qquad&\\
\left(\frac{2x}{x^2+17}+\frac{160x^4}{32x^5-8}+4\frac{98x^{97}-57x^{56}+64x}{x^{98}-x^{57}+32x^2}+\frac{320x^9-320x^{31}}{32x^{10}-10x^{32}}\right) &= f'(x)
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Included Work