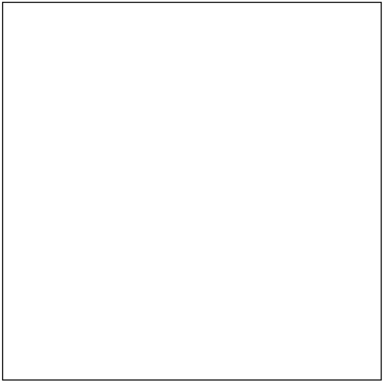
Table of Contents

Sequences and Series

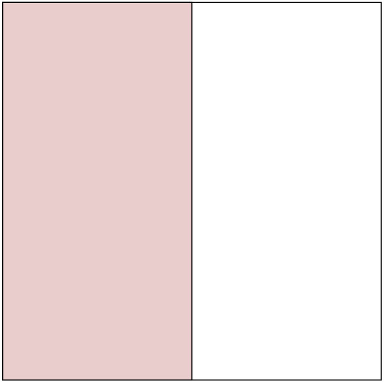
A sequence is a list of numbers
A series is the sum of such a list.

Sequences and Series



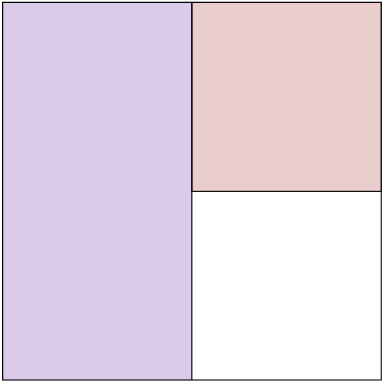
Square of Area 1

Size of Tiles: $\frac{1}{2}$



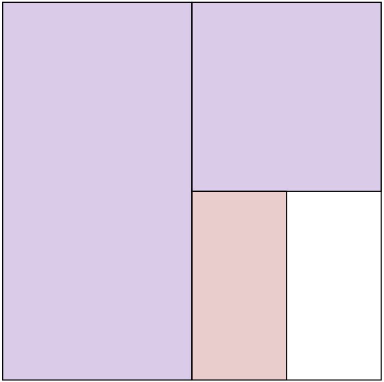
Covered Area: $\frac{1}{2}$

Size of Tiles: $\frac{1}{2}$, $\frac{1}{2^2}$



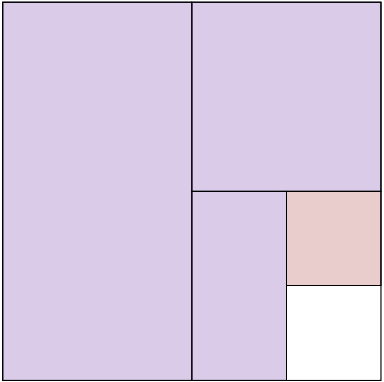
Covered Area: $\frac{1}{2}+\frac{1}{2^2}$

Size of Tiles: $\frac{1}{2}$, $\frac{1}{2^2}$, $\frac{1}{2^3}$



Covered Area: $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3}$

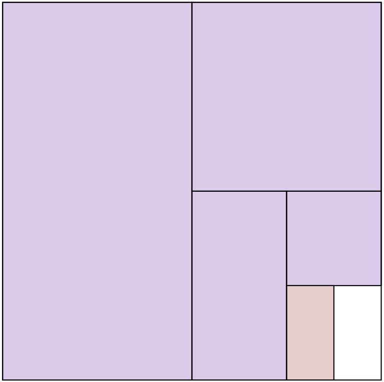
Size of Tiles: $\frac{1}{2}\,,\ \frac{1}{2^2}\,,\ \frac{1}{2^3}\,,\ \frac{1}{2^4}$



Covered Area: $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}$

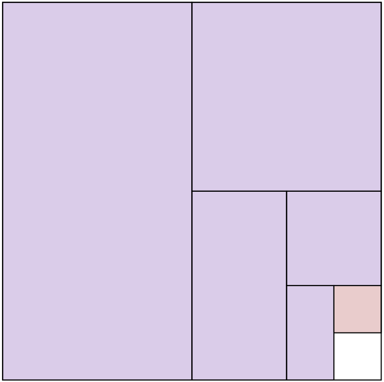
Sequences and Series

Size of Tiles: $\frac{1}{2}\ ,\ \frac{1}{2^2}\ ,\ \frac{1}{2^3}\ ,\ \frac{1}{2^4}\ ,\ \frac{1}{2^5}$



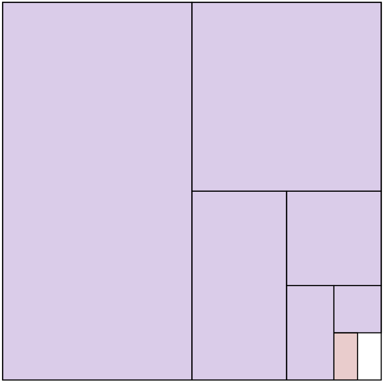
Covered Area: $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\frac{1}{2^5}$

Size of Tiles: $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \frac{1}{2^5}, \frac{1}{2^6}$



Covered Area: $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\frac{1}{2^5}+\frac{1}{2^6}$

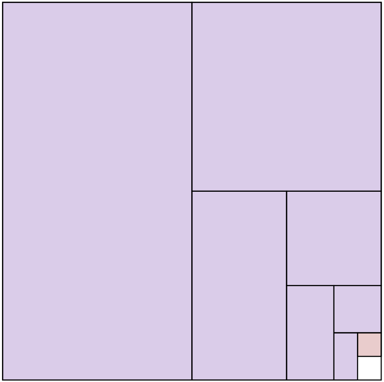
Size of Tiles: $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \frac{1}{2^5}, \frac{1}{2^6}, \frac{1}{2^7}$



Covered Area: $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7}$

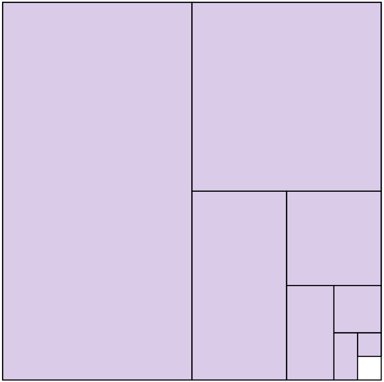
Sequences and Series

Size of Tiles: $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \frac{1}{2^5}, \frac{1}{2^6}, \frac{1}{2^7}, \frac{1}{2^8}$



Covered Area: $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\frac{1}{2^5}+\frac{1}{2^6}+\frac{1}{2^7}+\frac{1}{2^8}$

Size of Tiles: $\frac{1}{2}\,,\ \frac{1}{2^2}\,,\ \frac{1}{2^3}\,,\ \frac{1}{2^4}\,,\ \frac{1}{2^5}\,,\ \frac{1}{2^6}\,,\ \frac{1}{2^7}\,,\ \frac{1}{2^8}\,,\ \frac{1}{2^9}$

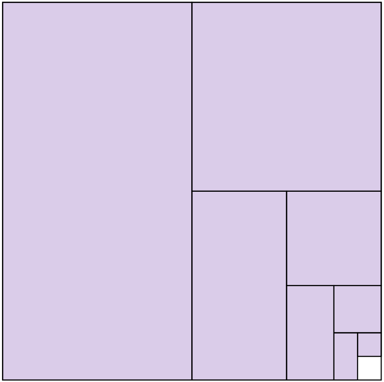


Covered Area: $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\frac{1}{2^5}+\frac{1}{2^6}+\frac{1}{2^7}+\frac{1}{2^8}+\frac{1}{2^9}$

Sequences and Series

Size of Tiles: $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \frac{1}{2^5}, \frac{1}{2^6}, \frac{1}{2^7}, \frac{1}{2^8}, \frac{1}{2^9}, \ldots$

Sequence



Covered Area: $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\frac{1}{2^5}+\frac{1}{2^6}+\frac{1}{2^7}+\frac{1}{2^8}+\frac{1}{2^9}+\cdots$

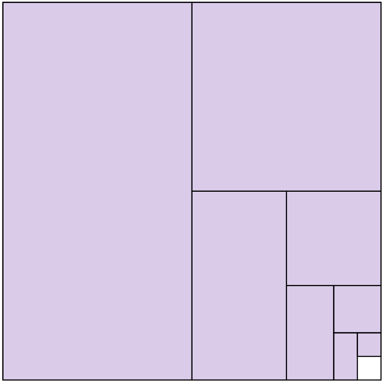
Sequences and Series

Size of Tiles: $\frac{1}{2}\,,\ \frac{1}{2^2}\,,\ \frac{1}{2^3}\,,\ \frac{1}{2^4}\,,\ \frac{1}{2^5}\,,\ \frac{1}{2^6}\,,\ \frac{1}{2^7}\,,\ \frac{1}{2^8}\,,\ \frac{1}{2^9}\,,\ \cdots$

Sequence

List of numbers,

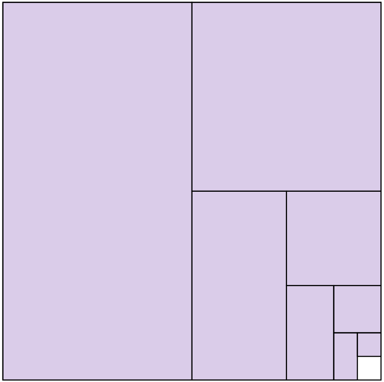
approaching



Covered Area: $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\frac{1}{2^5}+\frac{1}{2^6}+\frac{1}{2^7}+\frac{1}{2^8}+\frac{1}{2^9}+\cdots$

Sequences and Series

Size of Tiles: $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \frac{1}{2^5}, \frac{1}{2^6}, \frac{1}{2^7}, \frac{1}{2^8}, \frac{1}{2^9}, \cdots$



$\boxed{\text{Sequence}}$

List of numbers,

approaching **zero**.

Covered Area: $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\frac{1}{2^5}+\frac{1}{2^6}+\frac{1}{2^7}+\frac{1}{2^8}+\frac{1}{2^9}+\cdots$

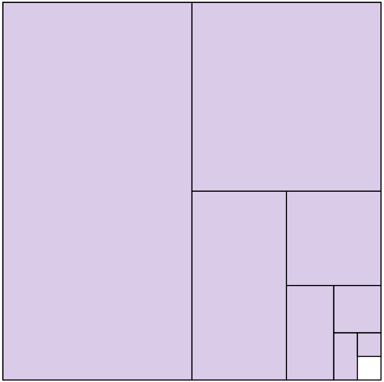
Sequences and Series

Size of Tiles: $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \frac{1}{2^5}, \frac{1}{2^6}, \frac{1}{2^7}, \frac{1}{2^8}, \frac{1}{2^9}, \dots$

Sequence

List of numbers,

approaching **zero**.

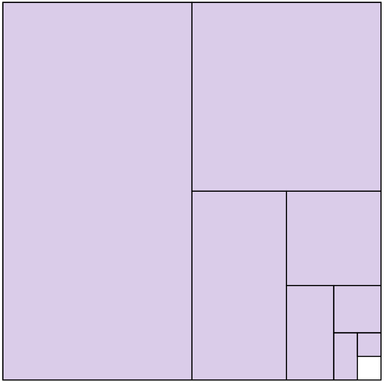


Series

Covered Area: $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} + \frac{1}{2^8} + \frac{1}{2^9} + \cdots$

Sequences and Series

Size of Tiles: $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \frac{1}{2^5}, \frac{1}{2^6}, \frac{1}{2^7}, \frac{1}{2^8}, \frac{1}{2^9}, \ldots$



Sequence

List of numbers,

approaching **zero**.

Series

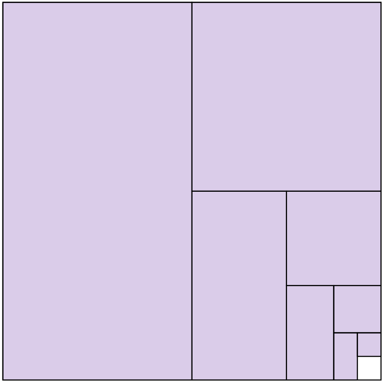
Sum of numbers,

approaching

Covered Area: $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} + \frac{1}{2^8} + \frac{1}{2^9} + \cdots$

Sequences and Series

Size of Tiles: $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \frac{1}{2^5}, \frac{1}{2^6}, \frac{1}{2^7}, \frac{1}{2^8}, \frac{1}{2^9}, \ldots$



Sequence

List of numbers,

approaching **zero**.

Series

Sum of numbers,

approaching **one**.

Covered Area: $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} + \frac{1}{2^8} + \frac{1}{2^9} + \cdots$

Quick Review: Sigma Notation

Recall:

$$\sum_{n=1}^{5} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2}$$

Quick Review: Sigma Notation

Recall:

$$\sum_{n=1}^{5} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2}$$

We informally interpret:

$$\sum_{n=1}^{\infty} \frac{1}{n^2} =$$

Quick Review: Sigma Notation

Recall:

$$\sum_{n=1}^{5} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2}$$

We informally interpret:

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{1}{7^2} + \frac{1}{8^2} + \frac{1}{9^2} + \frac{1}{10^2} + \cdots$$

(a more rigorous definition will be discussed soon)

Let a_n and b_n be sequences, and let C be a constant.

$$\sum_{n=1}^{\infty}(C \cdot a_n) =$$

A. $\displaystyle\sum_{n=1}^{\infty} C \cdot \sum_{n=1}^{\infty} a_n$

B. $\displaystyle\sum_{n=1}^{\infty} C + \sum_{n=1}^{\infty} a_n$

C. $\displaystyle C \sum_{n=1}^{\infty} a_n$

D. $\displaystyle a_n \sum_{n=1}^{\infty} C$

E. none of the above

Q

ans

Let a_n and b_n be sequences, and let C be a constant.

$$\sum_{n=1}^{\infty}(C \cdot a_n) =$$

A. $\sum_{n=1}^{\infty} C \cdot \sum_{n=1}^{\infty} a_n$

B. $\sum_{n=1}^{\infty} C + \sum_{n=1}^{\infty} a_n$

C. $C \sum_{n=1}^{\infty} a_n$

D. $a_n \sum_{n=1}^{\infty} C$

E. none of the above

A

Let a_n and b_n be sequences, and let C be a constant.

$$\sum_{n=1}^{\infty}(a_n + b_n) =$$

A. $\sum_{n=1}^{\infty} a_n \cdot \sum_{n=1}^{\infty} b_n$

B. $\sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$

C. $a_n + \sum_{n=1}^{\infty} b_n$

D. $a_n \sum_{n=1}^{\infty} b_n$

E. none of the above

Q

ans

Let a_n and b_n be sequences, and let C be a constant.

$$\sum_{n=1}^{\infty}(a_n + b_n) =$$

A. $\sum_{n=1}^{\infty} a_n \cdot \sum_{n=1}^{\infty} b_n$

B. $\sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$

C. $a_n + \sum_{n=1}^{\infty} b_n$

D. $a_n \sum_{n=1}^{\infty} b_n$

E. none of the above

A

Let a_n and b_n be sequences, and let C be a constant.

$$\sum_{n=1}^{\infty} (a_n)^C =$$

A. $\displaystyle\sum_{n=1}^{\infty} C \cdot \sum_{n=1}^{\infty} a_n$

B. $\displaystyle\left(\sum_{n=1}^{\infty} a_n\right)^C$

C. $\displaystyle C^n \sum_{n=1}^{\infty} a_n$

D. $\displaystyle\sum_{n=1}^{\infty} C(a_n)^{C-1}$

E. none of the above

Q

Let a_n and b_n be sequences, and let C be a constant.

$$\sum_{n=1}^{\infty} (a_n)^C =$$

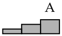
A. $\sum_{n=1}^{\infty} C \cdot \sum_{n=1}^{\infty} a_n$

B. $\left(\sum_{n=1}^{\infty} a_n \right)^C$

C. $C^n \sum_{n=1}^{\infty} a_n$

D. $\sum_{n=1}^{\infty} C (a_n)^{C-1}$

E. none of the above



SERIES PHILOSOPHY

What does it really mean to add up infinitely many things?

$$1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + \cdots$$

Series Philosophy

What does it really mean to add up infinitely many things?

$$\underbrace{1-1}_{0}+\underbrace{1-1}_{0}+\underbrace{1-1}_{0}+\underbrace{1-1}_{0}+\underbrace{1-1}_{0}+\underbrace{1-1}_{0}+\cdots$$

Series Philosophy

What does it really mean to add up infinitely many things?

$$\underbrace{1}_{1} \underbrace{-1+1}_{0} \underbrace{-1+1}_{0} \underbrace{-1+1}_{0} \underbrace{-1+1}_{0} \underbrace{-1+1}_{0} \underbrace{-1+1}_{0} \cdots$$

Series Philosophy

What does it really mean to add up infinitely many things?

$$1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + \cdots$$

$$1 \qquad 1 \qquad 1$$

Series Philosophy

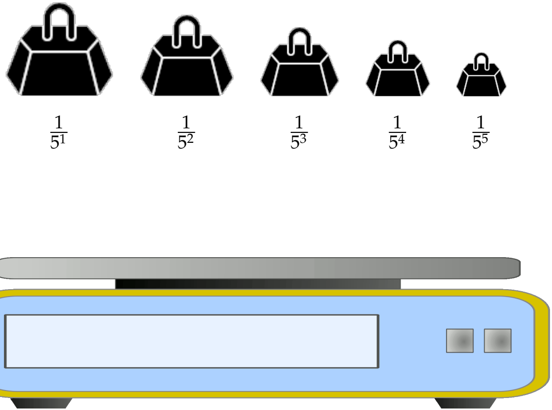
What does it really mean to add up infinitely many things?

$$1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + \cdots$$

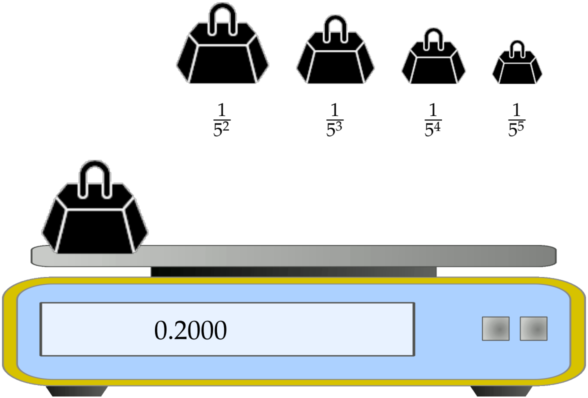
We need an unambiguous definition.

How can we add up infinitely many things?

Sequence of Partial Sums



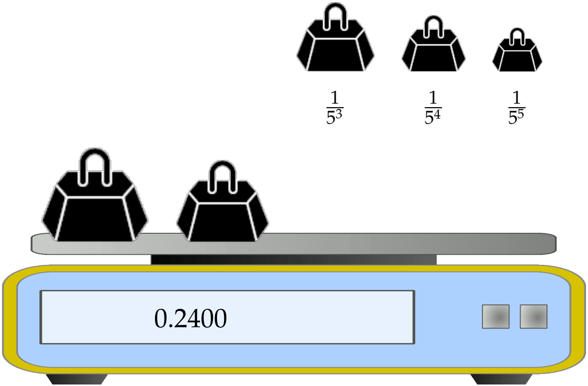
$S_1 = 0.2000$



How can we add up infinitely many things?
Sequence of Partial Sums

$S_1 = 0.2000$

$S_2 = 0.2400$

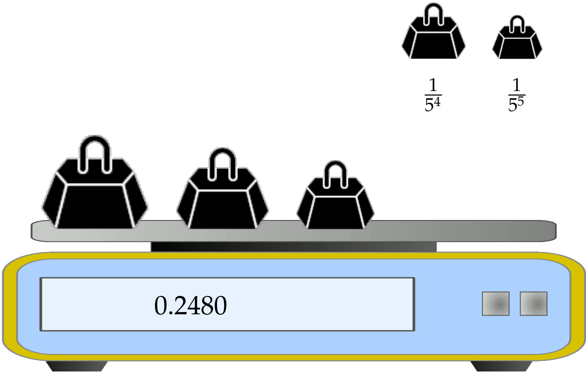


How can we add up infinitely many things?
Sequence of Partial Sums

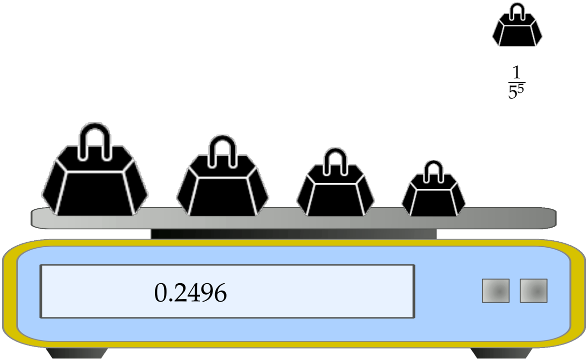
$$S_1 = 0.2000$$

$$S_2 = 0.2400$$

$$S_3 = 0.2480$$



HOW CAN WE ADD UP INFINITELY MANY THINGS?
SEQUENCE OF PARTIAL SUMS



$S_1 = 0.2000$

$S_2 = 0.2400$

$S_3 = 0.2480$

$S_4 = 0.2496$

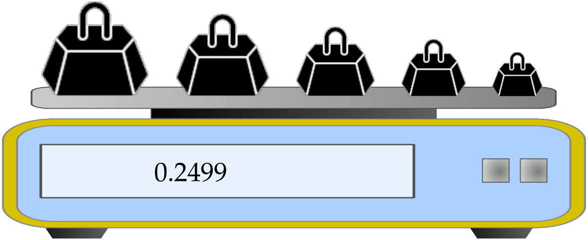
$S_1 = 0.2000$

$S_2 = 0.2400$

$S_3 = 0.2480$

$S_4 = 0.2496$

$S_5 = 0.2499$



Partial sums let us think about series (sums) using the tools we've developed for sequences (lists).

$$
\begin{aligned}
a_1 &= \frac{1}{5} = 0.2 \\
a_2 &= \frac{1}{5^2} = 0.04 \\
a_3 &= \frac{1}{5^3} = 0.008 \\
a_4 &= \frac{1}{5^4} = 0.0016 \\
a_5 &= \frac{1}{5^5} = 0.00032
\end{aligned}
$$

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$$a_1 = \frac{1}{5} = 0.2 \qquad\qquad S_1 = 0.2$$

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$$a_3 = \frac{1}{5^3} = 0.008 \qquad S_3 = 0.248$$

$$a_4 = \frac{1}{5^4} = 0.0016$$

$$a_5 = \frac{1}{5^5} = 0.00032$$

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$$a_2 = \frac{1}{5^2} = 0.04 \qquad S_2 = 0.24$$

$$a_3 = \frac{1}{5^3} = 0.008 \qquad S_3 = 0.248$$

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$$a_3 = \frac{1}{5^3} = 0.008 \qquad\qquad S_3 = 0.248$$

$$a_4 = \frac{1}{5^4} = 0.0016 \qquad\qquad S_4 = 0.2496$$

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$$a_3 = \frac{1}{5^3} = 0.008 \qquad S_3 = 0.248$$

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$$a_4 = \frac{1}{5^4} = 0.0016 \qquad S_4 = 0.2496$$

$$a_5 = \frac{1}{5^5} = 0.00032 \qquad S_5 = 0.24992$$

We define $\sum_{n=1}^{\infty} a_n = \lim_{N\to\infty} \sum_{n=1}^{N} a_n = \lim_{N\to\infty} S_N$.

$$a_1 = \frac{1}{5} = 0.2 \qquad S_1 = 0.2$$
$$a_2 = \frac{1}{5^2} = 0.04 \qquad S_2 = 0.24$$
$$a_3 = \frac{1}{5^3} = 0.008 \qquad S_3 = 0.248$$
$$a_4 = \frac{1}{5^4} = 0.0016 \qquad S_4 = 0.2496$$
$$a_5 = \frac{1}{5^5} = 0.00032 \qquad S_5 = 0.24992$$
$$a_6 = \frac{1}{5^6} = 0.000064 \qquad S_6 = 0.249984$$
$$a_7 = \frac{1}{5^7} = 0.0000128 \qquad S_7 = 0.2499968$$
$$a_8 = \frac{1}{5^8} = 0.00000256 \qquad S_8 = 0.24999936$$

From the sequence of partial sums, we guess

$$\sum_{n=1}^{\infty} \qquad = \lim_{N\to\infty} S_N \quad =$$

ans

We define $\displaystyle\sum_{n=1}^{\infty} a_n = \lim_{N\to\infty} \sum_{n=1}^{N} a_n = \lim_{N\to\infty} S_N$.

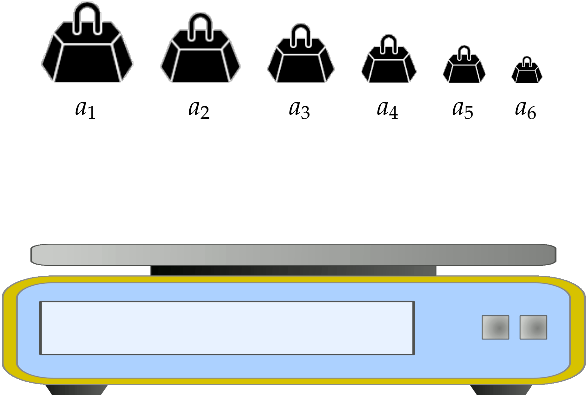
$$
\begin{aligned}
a_1 &= \frac{1}{5} = 0.2 & S_1 &= 0.2 \\
a_2 &= \frac{1}{5^2} = 0.04 & S_2 &= 0.24 \\
a_3 &= \frac{1}{5^3} = 0.008 & S_3 &= 0.248 \\
a_4 &= \frac{1}{5^4} = 0.0016 & S_4 &= 0.2496
\end{aligned}
$$

$$
\begin{aligned}
a_5 &= \frac{1}{5^5} = 0.00032 & S_5 &= 0.24992 \\
a_6 &= \frac{1}{5^6} = 0.000064 & S_6 &= 0.249984 \\
a_7 &= \frac{1}{5^7} = 0.0000128 & S_7 &= 0.2499968 \\
a_8 &= \frac{1}{5^8} = 0.00000256 & S_8 &= 0.24999936
\end{aligned}
$$

From the sequence of partial sums, we guess

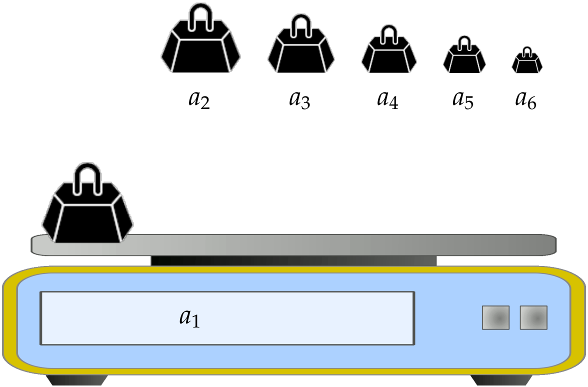
$$\sum_{n=1}^{\infty} \frac{1}{5^n} = \lim_{N\to\infty} S_N = \frac{1}{4}$$

NOTATION: $S_N = \sum_{n=1}^{N} a_n$



NOTATION: $S_N = \sum_{n=1}^{N} a_n$

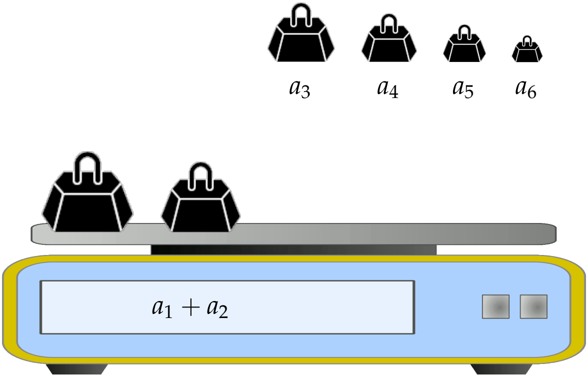
$S_1 = a_1$



NOTATION: $S_N = \sum_{n=1}^{N} a_n$

$$S_1 = a_1$$

$$S_2 = a_1 + a_2$$



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$$S_1 = a_1$$

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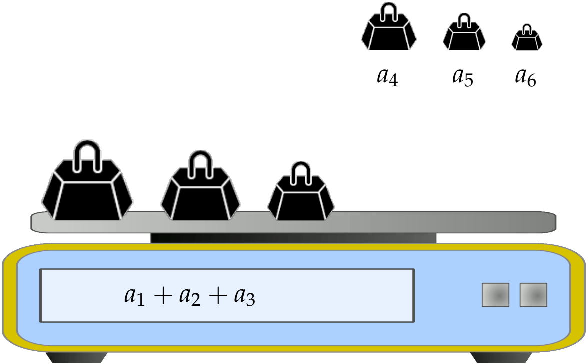
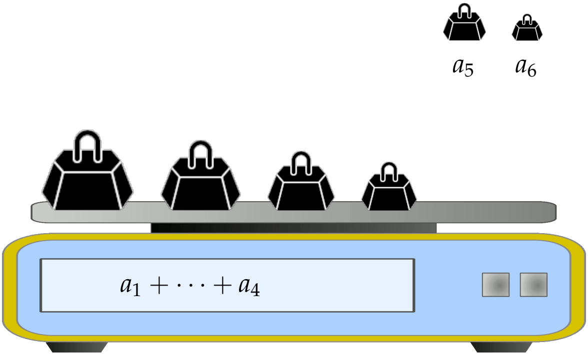


NOTATION: $S_N = \sum_{n=1}^{N} a_n$



$$S_1 = a_1$$

$$S_2 = a_1 + a_2$$

$$S_3 = a_1 + a_2 + a_3$$

$$S_4 = a_1 + \cdots + a_4$$

Notation: $S_N = \sum_{n=1}^{N} a_n$

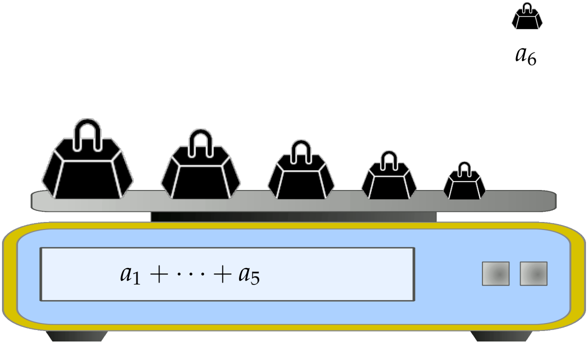
$$S_1 = a_1$$

$$S_2 = a_1 + a_2$$

$$S_3 = a_1 + a_2 + a_3$$

$$S_4 = a_1 + \cdots + a_4$$

$$S_5 = a_1 + \cdots + a_5$$



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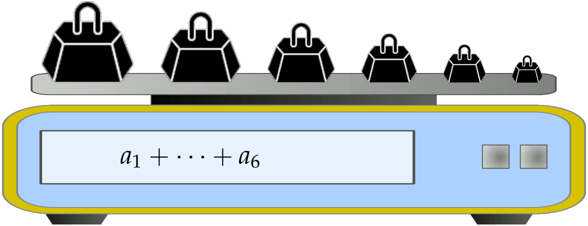


$$S_1 = a_1$$

$$S_2 = a_1 + a_2$$

$$S_3 = a_1 + a_2 + a_3$$

$$S_4 = a_1 + \cdots + a_4$$

$$S_5 = a_1 + \cdots + a_5$$

$$S_6 = a_1 + \cdots + a_6$$

Notation Practice

Suppose $\sum_{n=1}^{\infty} a_n$ has partial sums $S_N = \sum_{n=1}^{N} a_n = \dfrac{N}{N+1}$.

- Evaluate $\sum_{n=1}^{100} a_n$.

- Evaluate $\sum_{n=1}^{\infty} a_n$.

Q

ans

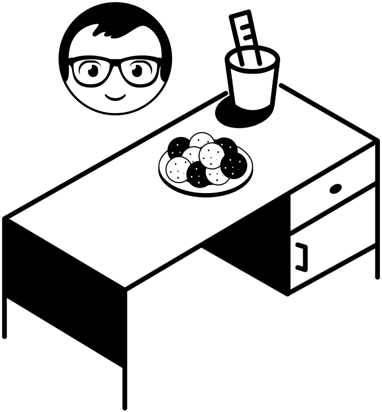
Notation Practice

Suppose $\sum_{n=1}^{\infty} a_n$ has partial sums $S_N = \sum_{n=1}^{N} a_n = \frac{N}{N+1}$.

- Evaluate $\sum_{n=1}^{100} a_n$. $\quad \sum_{n=1}^{100} a_n = S_{100} = \frac{100}{101}$

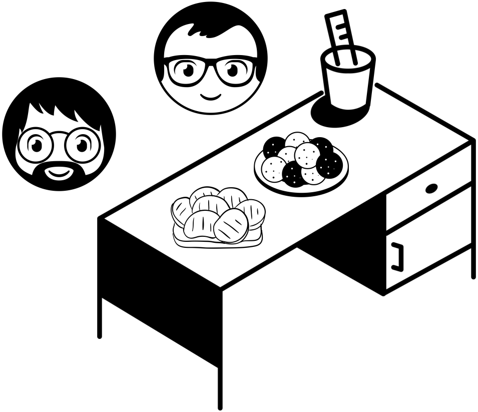
- Evaluate $\sum_{n=1}^{\infty} a_n$. $\quad \sum_{n=1}^{\infty} a_n = \lim_{N\to\infty} S_N = \lim_{N\to\infty} \frac{N}{N+1} = 1$

Notation Practice



Andrew brings a plate of cookies to the professor's desk. When he puts them down, there are 10 cookies on the desk.

Notation Practice

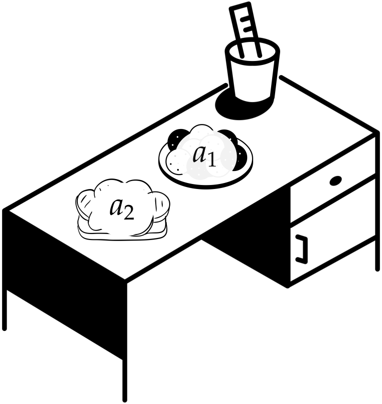


Andrew brings a plate of cookies to the professor's desk. When he puts them down, there are 10 cookies on the desk.

Then, Joel brings a plate of cookies. When he puts them down, there are 19 cookies on the desk.

How many cookies did each person bring?

Notation Practice



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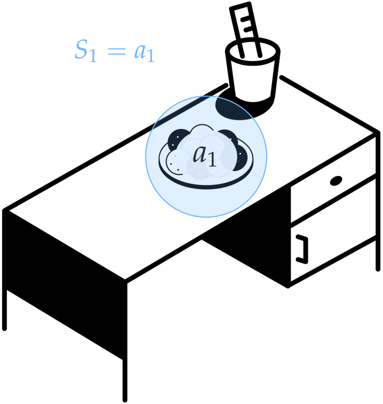
Then, Joel brings a plate of cookies. When he puts them down, there are 19 cookies on the desk.

How many cookies did each person bring?

Andrew brought 10, and Joel brought $19 - 10 = 9$.

Notation Practice



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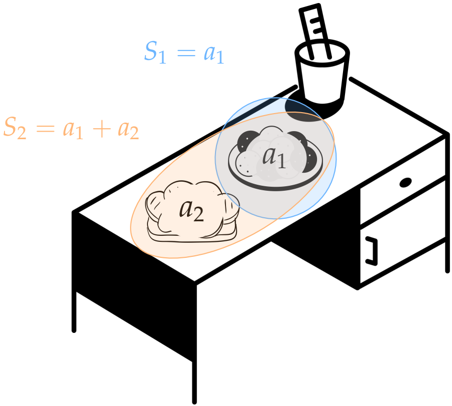
NOTATION PRACTICE



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Notation Practice

Suppose $\sum_{n=1}^{\infty} a_n$ has partial sums $S_N = \sum_{n=1}^{N} a_n = \frac{N}{N+1}$.

- Find a_1.

- Give an explicit expression for a_n, when $n > 1$.

Q

ans

Notation Practice

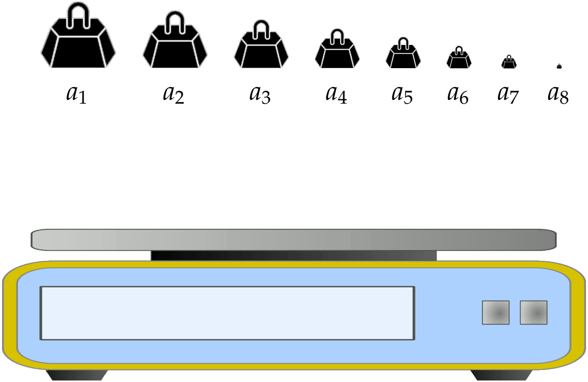
Suppose $\sum_{n=1}^{\infty} a_n$ has partial sums $S_N = \sum_{n=1}^{N} a_n = \dfrac{N}{N+1}$.

- Find a_1. $\quad a_1 = \sum_{n=1}^{1} a_n = S_1 = \dfrac{1}{2}$
- Give an explicit expression for a_n, when $n > 1$.

$$\begin{aligned} a_n &= \left(\sum_{k=1}^{n} a_k\right) - \left(\sum_{k=1}^{n-1} a_k\right) = S_n - S_{n-1} \\ &= \frac{n}{n+1} - \frac{n-1}{n} = \frac{1}{n(n+1)} \end{aligned}$$

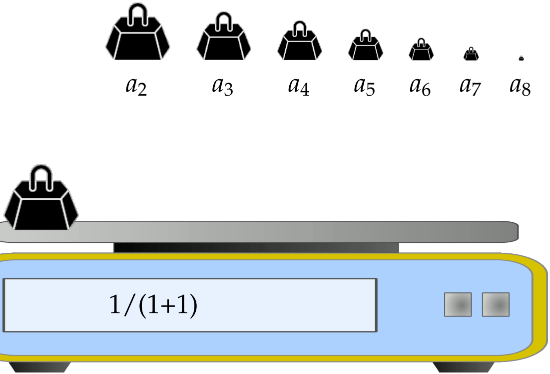
A

$$S_N = \sum_{n=1}^{N} a_n = \frac{N}{N+1}$$



$$S_N = \sum_{n=1}^{N} a_n = \frac{N}{N+1}$$

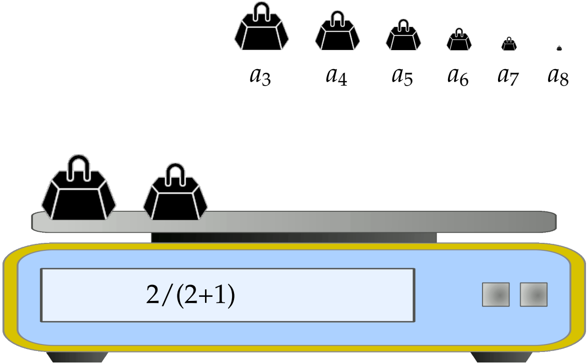
$S_1 = 1/(1+1)$



$$S_N = \sum_{n=1}^{N} a_n = \frac{N}{N+1}$$

$S_1 = 1/(1+1)$

$S_2 = 2/(2+1)$

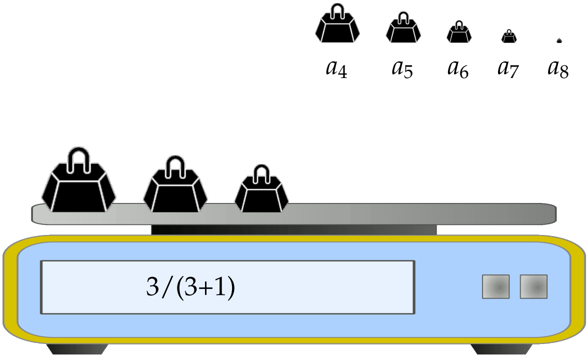


$$S_N = \sum_{n=1}^{N} a_n = \frac{N}{N+1}$$

$S_1 = 1/(1+1)$

$S_2 = 2/(2+1)$

$S_3 = 3/(3+1)$



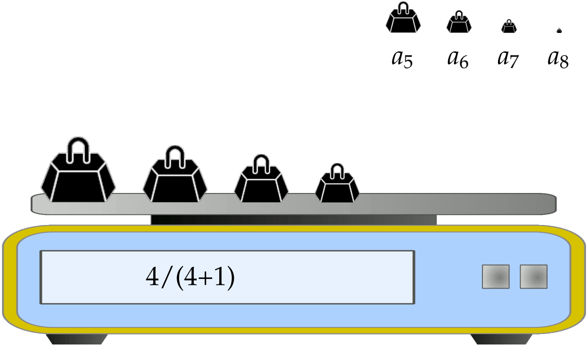
$$S_N = \sum_{n=1}^{N} a_n = \frac{N}{N+1}$$

$S_1 = 1/(1+1)$

$S_2 = 2/(2+1)$

$S_3 = 3/(3+1)$

$S_4 = 4/(4+1)$



Definition

The N^{th} partial sum of the series $\sum_{n=1}^{\infty} a_n$ is the sum of its first N terms

$$S_N = \sum_{n=1}^{N} a_n.$$

The partial sums form a sequence $\{S_N\}_{N=1}^{\infty}$. If this sequence of partial sums converges $S_N \to S$ as $N \to \infty$ then we say that the series $\sum_{n=1}^{\infty} a_n$ converges to S and we write

$$\sum_{n=1}^{\infty} a_n = S$$

If the sequence of partial sums diverges, we say that the series diverges.

Geometric Series

Let a and r be two fixed real numbers with $a \neq 0$. The series

$$a + ar + ar^2 + ar^3 + \cdots$$

is called the **geometric series** with first term a and ratio r.

We call r the *ratio* because it is the quotient of consecutive terms:

$$\frac{ar^{n+1}}{ar^n} = r$$

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We call r the *ratio* because it is the quotient of consecutive terms:

$$\frac{ar^{n+1}}{ar^n} = r$$

Another useful way of identifying geometric series is to determine whether all pairs of consecutive terms have the same ratio.

- Geometric: $1 + \frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \frac{1}{5^4} + \cdots$
- Geometric: $\sum_{n=0}^{\infty} \frac{1}{2^n}$
- Not geometric: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots$

Consider the partial sum S_N of a geometric series:

$$S_N = a + ar + ar^2 + ar^3 + \cdots + ar^N$$

Consider the partial sum S_N of a geometric series:

$$S_N = a + ar + ar^2 + ar^3 + \cdots + ar^N$$

$$rS_N =$$

Consider the partial sum S_N of a geometric series:

$$\begin{aligned} S_N &= a + ar + ar^2 + ar^3 + \cdots + ar^N \\ rS_N &= \quad\quad ar + ar^2 + ar^3 + \cdots + ar^N + ar^{N+1} \end{aligned}$$

Consider the partial sum S_N of a geometric series:

$$
\begin{aligned}
S_N &= a + ar + ar^2 + ar^3 + \cdots + ar^N \\
rS_N &= \qquad ar + ar^2 + ar^3 + \cdots + ar^N + ar^{N+1} \\
rS_N - S_N &=
\end{aligned}
$$

Consider the partial sum S_N of a geometric series:

$$\begin{aligned} S_N &= a + ar + ar^2 + ar^3 + \cdots + ar^N \\ rS_N &= \phantom{a + {}} ar + ar^2 + ar^3 + \cdots + ar^N + ar^{N+1} \\ rS_N - S_N &= -a \qquad\qquad + \qquad\qquad ar^{N+1} \end{aligned}$$

Consider the partial sum S_N of a geometric series:

$$\begin{aligned} S_N &= a + ar + ar^2 + ar^3 + \cdots + ar^N \\ rS_N &= \qquad ar + ar^2 + ar^3 + \cdots + ar^N + ar^{N+1} \\ rS_N - S_N &= -a \qquad\qquad\qquad + \qquad\qquad\qquad ar^{N+1} \\ S_N(r-1) &= ar^{N+1} - a \end{aligned}$$

If $r \neq 1$, then

$$S_N = \frac{ar^{N+1} - a}{r-1} = a\frac{r^{N+1} - 1}{r-1}$$

Geometric Series and Partial Sums

Let a and r be constants with $a \neq 0$, and let N be a natural number.

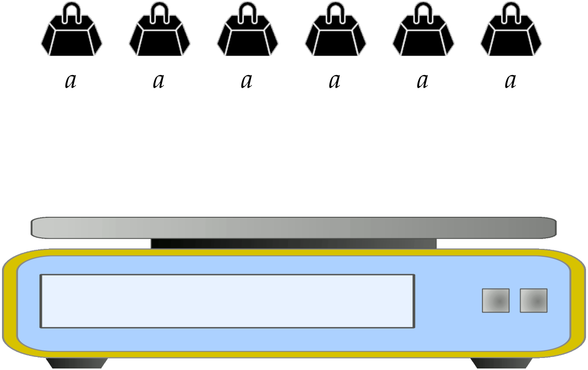
- If $r \neq 1$, then $a + ar + ar^2 + ar^3 + \cdots + ar^N = a\dfrac{r^{N+1} - 1}{r - 1}$.
- If $r = 1$, then $a + ar + ar^2 + ar^3 + \cdots + ar^N =$
- If $|r| < 1$, then $\displaystyle\sum_{n=0}^{\infty} ar^n =$
- If $r = 1$, then $\displaystyle\sum_{n=0}^{\infty} ar^n$
- If $r = -1$, then $\displaystyle\sum_{n=0}^{\infty} ar^n$
- If $|r| > 1$, then $\displaystyle\sum_{n=0}^{\infty} ar^n$

Geometric Series and Partial Sums

Let a and r be constants with $a \neq 0$, and let N be a natural number.

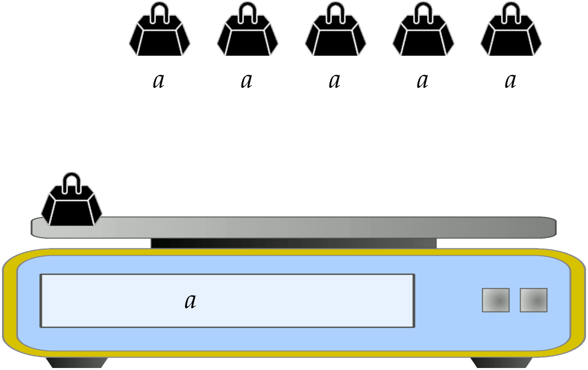
- If $r \neq 1$, then $a + ar + ar^2 + ar^3 + \cdots + ar^N = a\dfrac{r^{N+1} - 1}{r - 1}$.
- If $r = 1$, then $a + ar + ar^2 + ar^3 + \cdots + ar^N = (N+1)a$.
- If $|r| < 1$, then $\displaystyle\sum_{n=0}^{\infty} ar^n = \lim_{N\to\infty} a\frac{r^{N+1} - 1}{r - 1} = a\frac{1}{1 - r}$
- If $r = 1$, then $\displaystyle\sum_{n=0}^{\infty} ar^n$ diverges
- If $r = -1$, then $\displaystyle\sum_{n=0}^{\infty} ar^n$ diverges
- If $|r| > 1$, then $\displaystyle\sum_{n=0}^{\infty} ar^n$ diverges

$$\sum_{n=0}^{\infty} ar^n, r = 1, a \neq 0$$



$$\sum_{n=0}^{\infty} ar^n, r = 1, a \neq 0$$

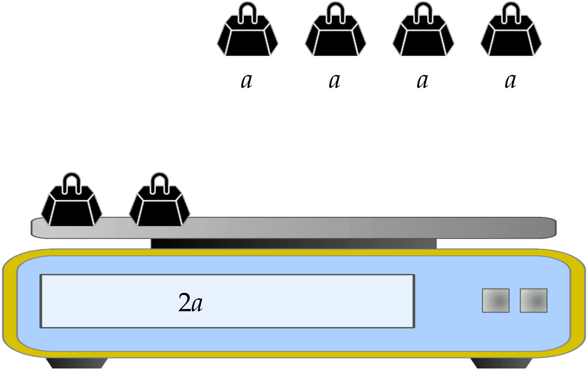
$S_0 = a$



$$\sum_{n=0}^{\infty} ar^n, r = 1, a \neq 0$$

$S_0 = a$

$S_1 = 2a$

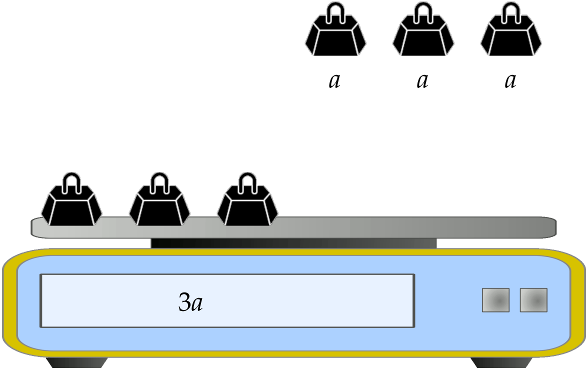


$$\sum_{n=0}^{\infty} ar^n, r = 1, a \neq 0$$

$$S_0 = a$$

$$S_1 = 2a$$

$$S_2 = 3a$$



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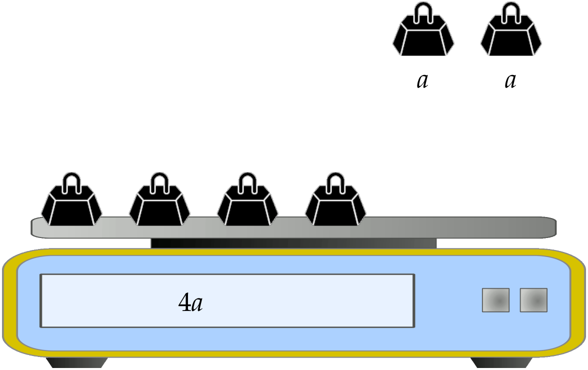


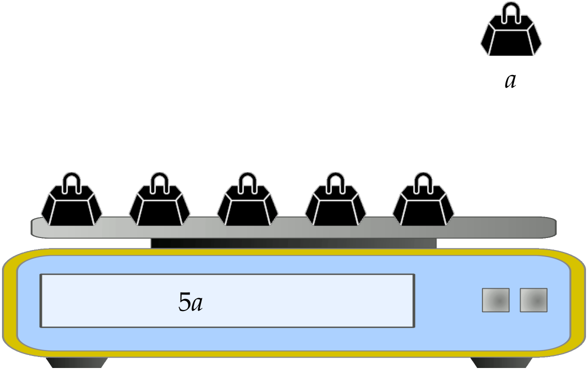
$S_0 = a$

$S_1 = 2a$

$S_2 = 3a$

$S_3 = 4a$

$$\sum_{n=0}^{\infty} ar^n, r = 1, a \neq 0$$



$S_0 = a$

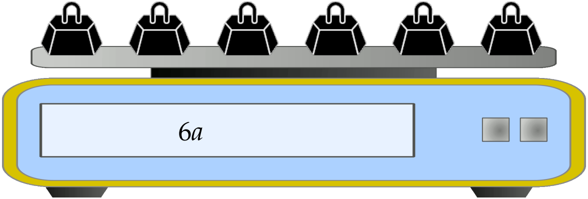
$S_1 = 2a$

$S_2 = 3a$

$S_3 = 4a$

$S_4 = 5a$

$$\sum_{n=0}^{\infty} ar^n, r = 1, a \neq 0$$



$$S_0 = a$$

$$S_1 = 2a$$

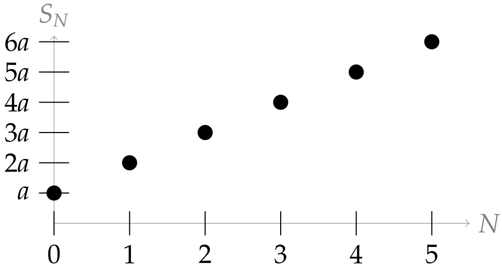
$$S_2 = 3a$$

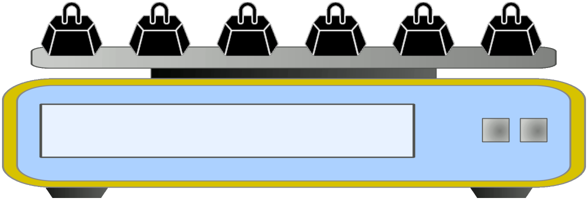
$$S_3 = 4a$$

$$S_4 = 5a$$

$$S_5 = 6a$$

$$\sum_{n=0}^{\infty} ar^n, r = 1, a \neq 0$$





$S_0 = a$

$S_1 = 2a$

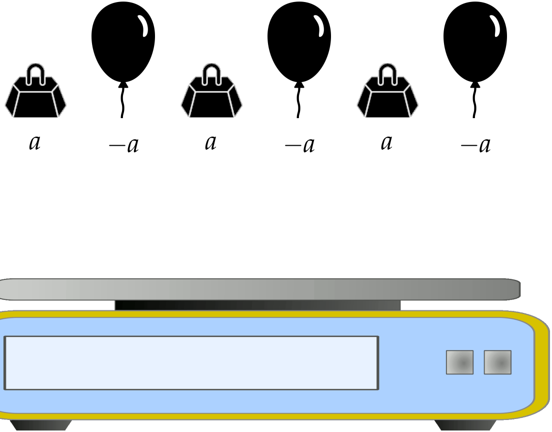
$S_2 = 3a$

$S_3 = 4a$

$S_4 = 5a$

$S_5 = 6a$

$$\sum_{n=0}^{\infty} ar^n, r = -1, a \neq 0$$



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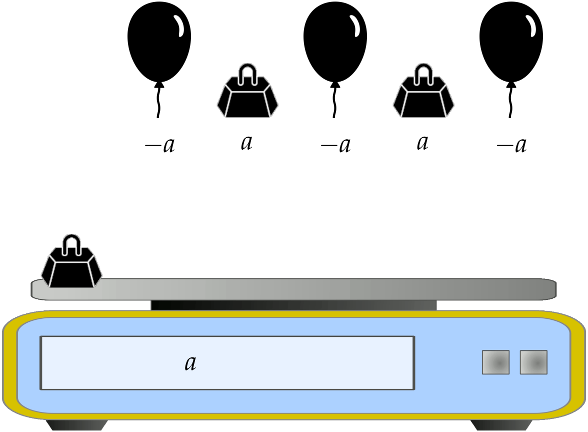
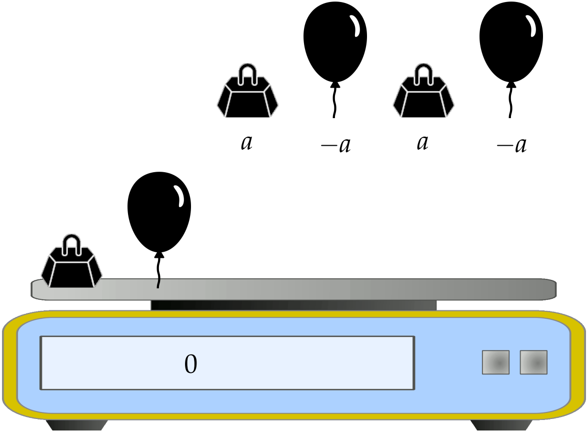
$S_0 = a$



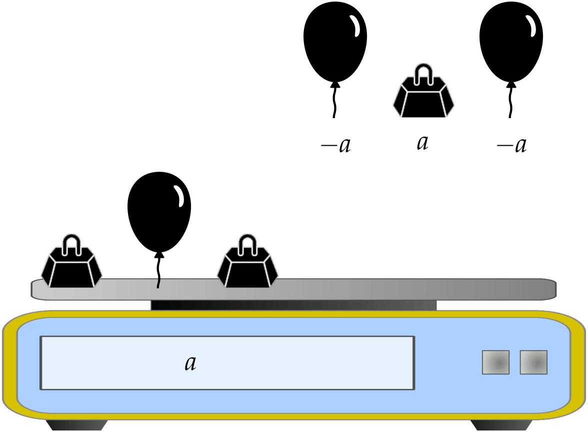
$\sum_{n=0}^{\infty} ar^n, r = -1, a \neq 0$



$S_0 = a$

$S_1 = 0$

$\sum_{n=0}^{\infty} ar^n, r = -1, a \neq 0$

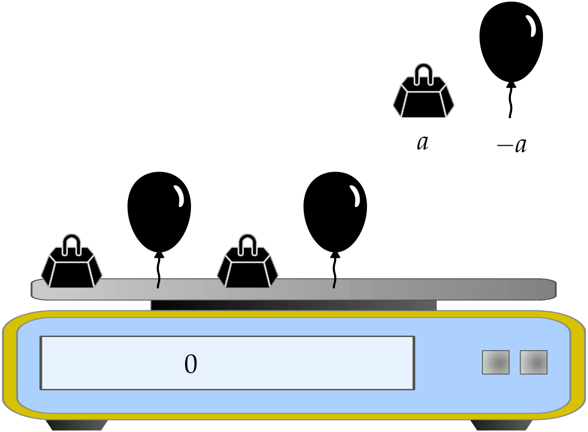


$S_0 = a$

$S_1 = 0$

$S_2 = a$

$\sum_{n=0}^{\infty} ar^n, r = -1, a \neq 0$



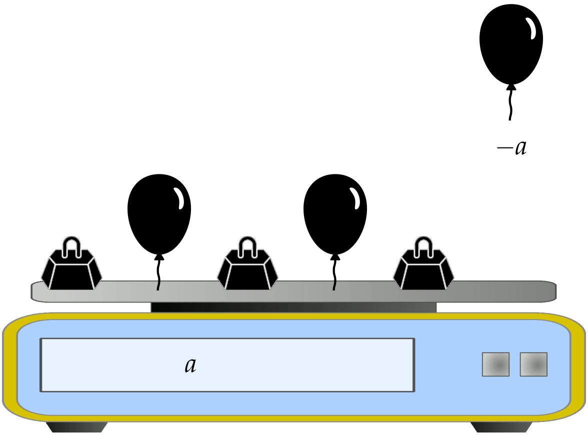
$S_0 = a$

$S_1 = 0$

$S_2 = a$

$S_3 = 0$

$$\sum_{n=0}^{\infty} ar^n, r = -1, a \neq 0$$



$S_0 = a$

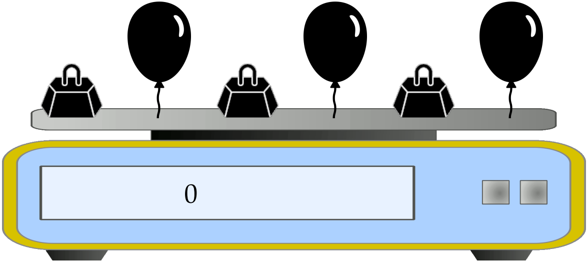
$S_1 = 0$

$S_2 = a$

$S_3 = 0$

$S_4 = a$

$$\sum_{n=0}^{\infty} ar^n, r = -1, a \neq 0$$



$S_0 = a$

$S_1 = 0$

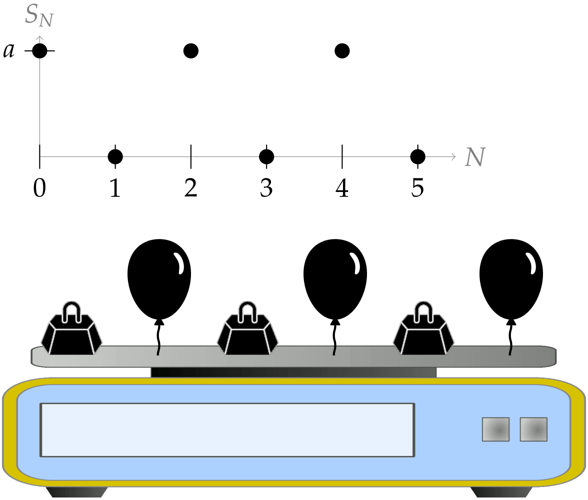
$S_2 = a$

$S_3 = 0$

$S_4 = a$

$S_5 = 0$

$\sum_{n=0}^{\infty} ar^n, r = -1, a \neq 0$



$S_0 = a$

$S_1 = 0$

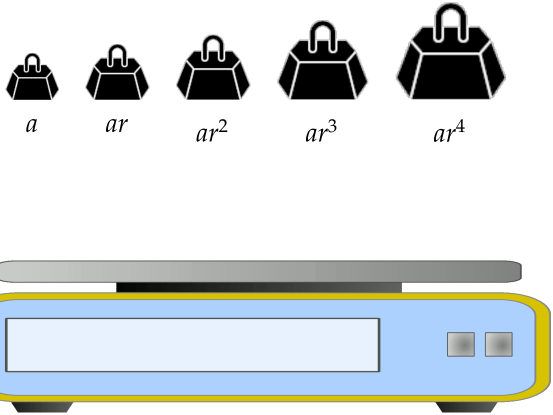
$S_2 = a$

$S_3 = 0$

$S_4 = a$

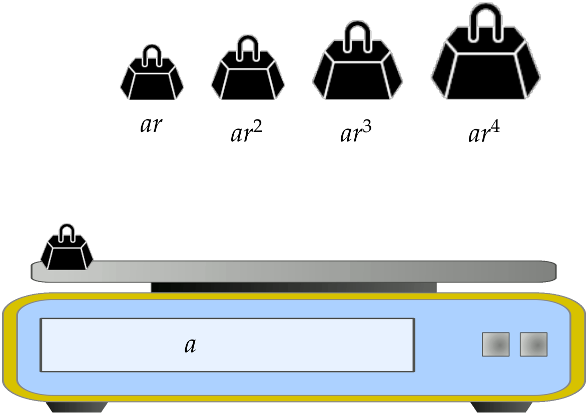
$S_5 = 0$

$$\sum_{n=0}^{\infty} ar^n, r > 1, a \neq 0$$



$$\sum_{n=0}^{\infty} ar^n, r > 1, a \neq 0$$

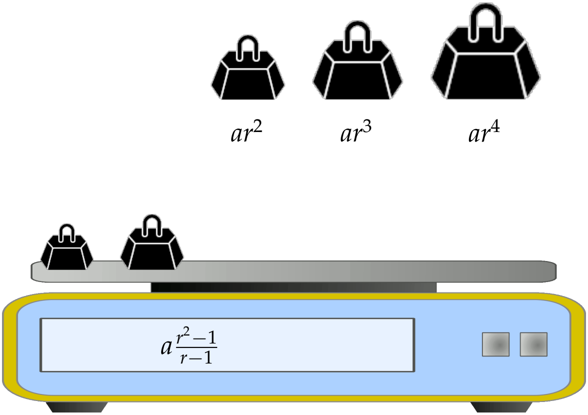
$S_0 = a$



$\sum_{n=0}^{\infty} ar^n, r > 1, a \neq 0$

$$S_0 = a$$

$$S_1 = a\frac{r^2-1}{r-1}$$

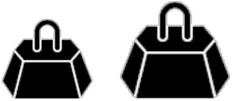


$$\sum_{n=0}^{\infty} ar^n, r > 1, a \neq 0$$

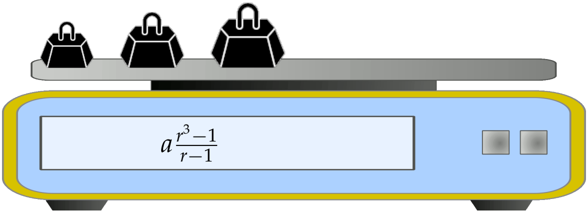
$$S_0 = a$$

$$S_1 = a\frac{r^2-1}{r-1}$$

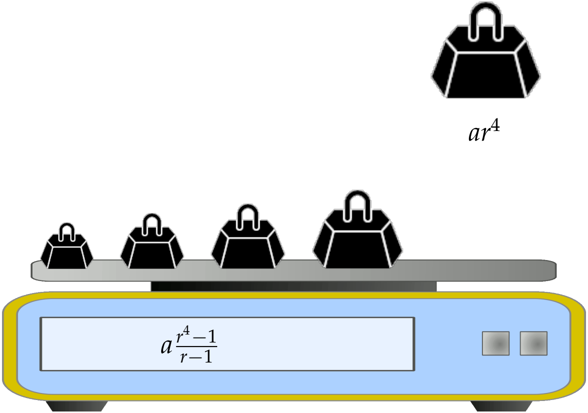
$$S_2 = a\frac{r^3-1}{r-1}$$



ar^3 ar^4



$$\sum_{n=0}^{\infty} ar^n, r > 1, a \neq 0$$



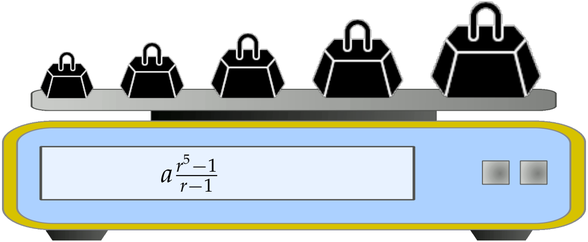
$$S_0 = a$$

$$S_1 = a\frac{r^2-1}{r-1}$$

$$S_2 = a\frac{r^3-1}{r-1}$$

$$S_3 = a\frac{r^4-1}{r-1}$$

$$\sum_{n=0}^{\infty} ar^n, r > 1, a \neq 0$$



$$S_0 = a$$

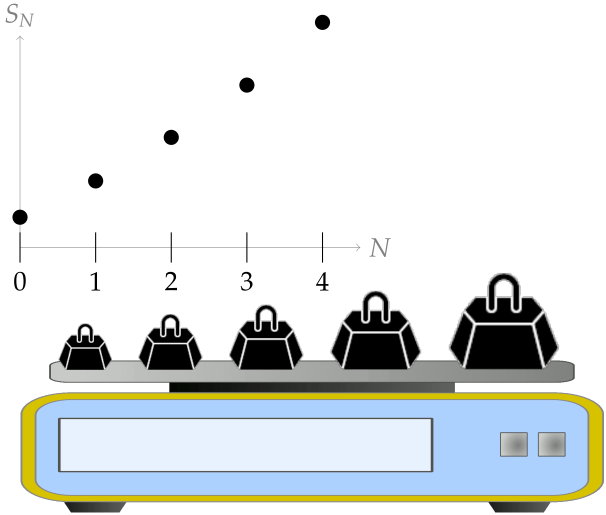
$$S_1 = a\frac{r^2-1}{r-1}$$

$$S_2 = a\frac{r^3-1}{r-1}$$

$$S_3 = a\frac{r^4-1}{r-1}$$

$$S_4 = a\frac{r^5-1}{r-1}$$

$\sum_{n=0}^{\infty} ar^n, r > 1, a \neq 0$



$$S_0 = a$$

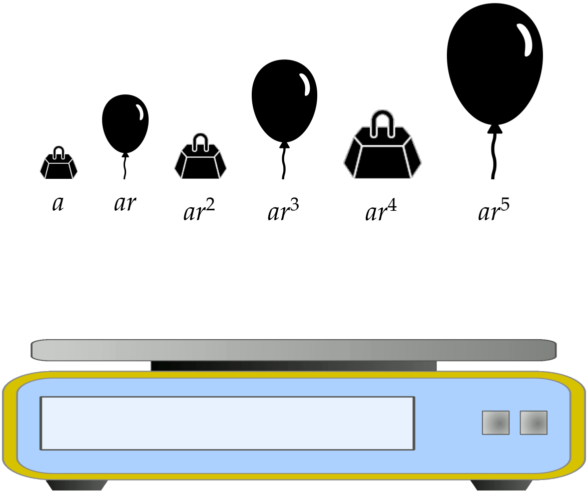
$$S_1 = a\frac{r^2-1}{r-1}$$

$$S_2 = a\frac{r^3-1}{r-1}$$

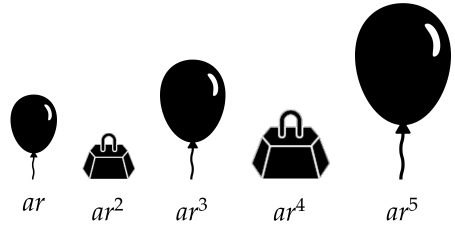
$$S_3 = a\frac{r^4-1}{r-1}$$

$$S_4 = a\frac{r^5-1}{r-1}$$

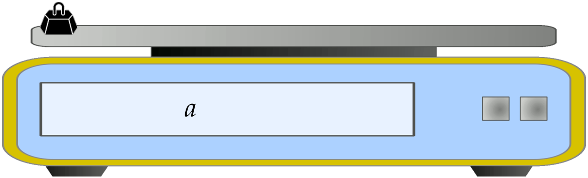
$$\sum_{n=0}^{\infty} ar^n, r < -1, a \neq 0$$



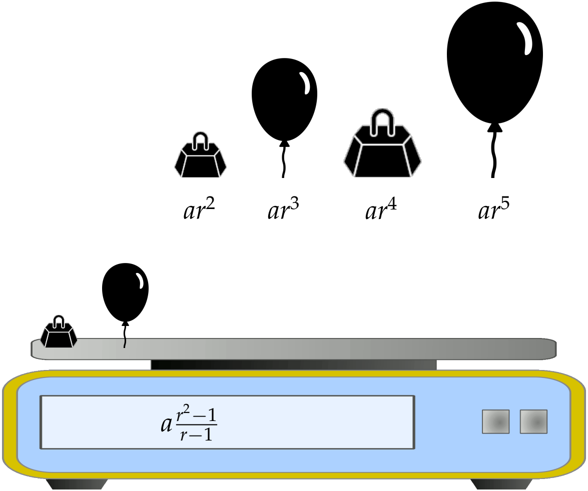
$\sum_{n=0}^{\infty} ar^n, r < -1, a \neq 0$



$S_0 = a$



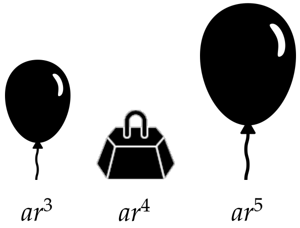
$\sum_{n=0}^{\infty} ar^n, r < -1, a \neq 0$



$S_0 = a$

$S_1 = a\frac{r^2-1}{r-1}$

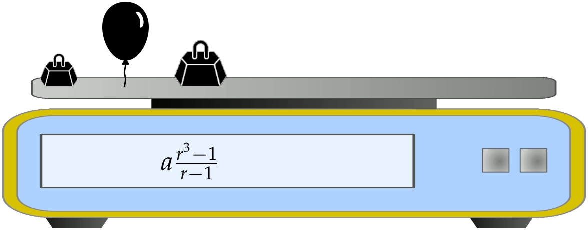
$$\sum_{n=0}^{\infty} ar^n, r < -1, a \neq 0$$



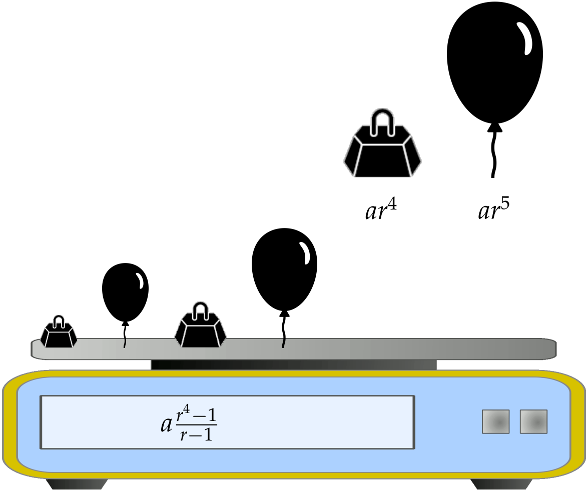
$$S_0 = a$$

$$S_1 = a\frac{r^2-1}{r-1}$$

$$S_2 = a\frac{r^3-1}{r-1}$$



$\sum_{n=0}^{\infty} ar^n, r < -1, a \neq 0$



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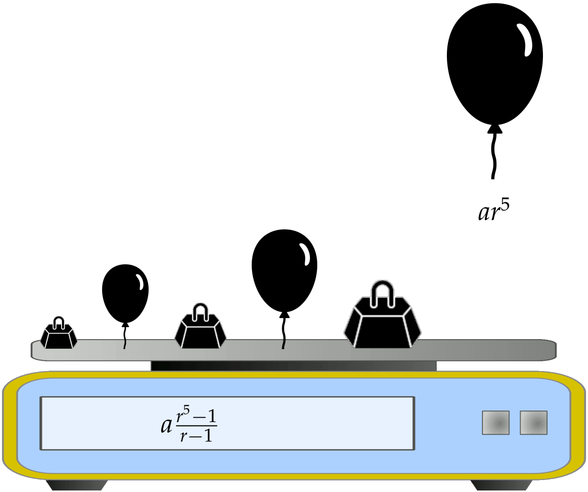
$$\sum_{n=0}^{\infty} ar^n, r < -1, a \neq 0$$



$$S_0 = a$$

$$S_1 = a\frac{r^2-1}{r-1}$$

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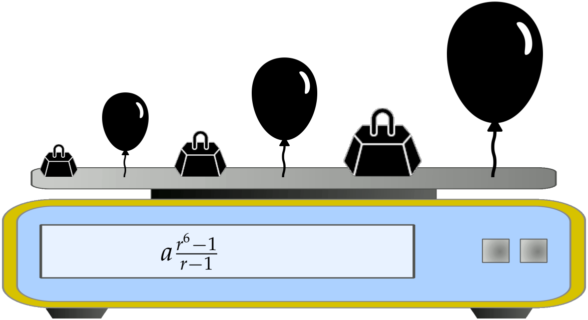
$$S_4 = a\frac{r^5-1}{r-1}$$

$\sum_{n=0}^{\infty} ar^n, r < -1, a \neq 0$



$$S_0 = a$$

$$S_1 = a\frac{r^2-1}{r-1}$$

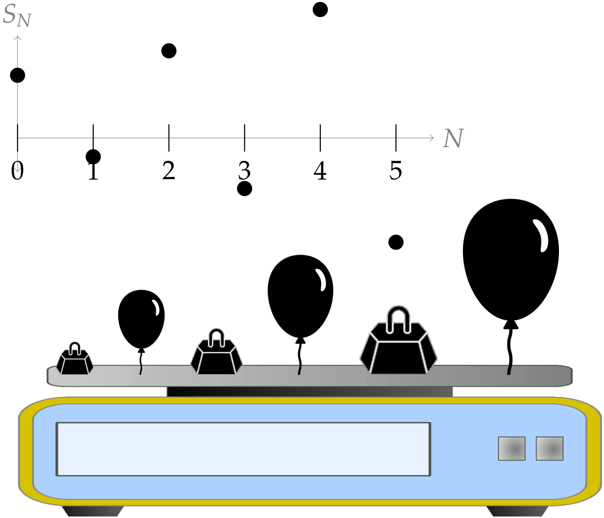
$$S_2 = a\frac{r^3-1}{r-1}$$

$$S_3 = a\frac{r^4-1}{r-1}$$

$$S_4 = a\frac{r^5-1}{r-1}$$

$$S_5 = a\frac{r^6-1}{r-1}$$

$\sum_{n=0}^{\infty} ar^n, r < -1, a \neq 0$



$$S_0 = a$$

$$S_1 = a\frac{r^2-1}{r-1}$$

$$S_2 = a\frac{r^3-1}{r-1}$$

$$S_3 = a\frac{r^4-1}{r-1}$$

$$S_4 = a\frac{r^5-1}{r-1}$$

$$S_5 = a\frac{r^6-1}{r-1}$$

Geometric Series

New bitcoins are produced when a particular type of computational problem is solved. Every time 210,000 solutions are found, the number of bitcoins each solution can produce is cut in half.

- Each of the first 210,000 solutions can produce 50 bitcoins.
- Each of the next 210,000 solutions can produce $\frac{50}{2}$ bitcoins.
- Each of the next 210,000 solutions can produce $\frac{50}{2^2}$ bitcoins.
- Each of the next 210,000 solutions can produce $\frac{50}{2^3}$ bitcoins.

Assume that this continues forever, and that bitcoins are infinitely divisible.[1]How many bitcoins can possibly be produced?

[1]Actually the smallest allowed division of a bitcoin is 10^{-8}.

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Assume that this continues forever, and that bitcoins are infinitely divisible.[1]How many bitcoins can possibly be produced?

We start by writing the total number of bitcoin produced as a series. Since we want to know an upper bound, we'll assume that infinite solutions can be found and used to make bitcoin.

$$210\,000(50)+210\,000\left(\frac{50}{2}\right)+210\,000\left(\frac{50}{2^2}\right)+\cdots=\sum_{n=0}^{\infty}(210\,000)\left(\frac{50}{2^n}\right)$$

[1]Actually the smallest allowed division of a bitcoin is 10^{-8}.

Geometric Series

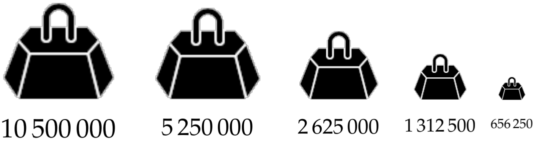
$$\sum_{n=0}^{\infty} (210\,000)\left(\frac{50}{2^n}\right) =$$

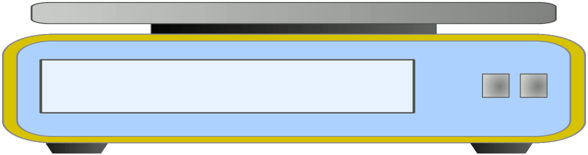
Geometric Series

$$\begin{aligned}\sum_{n=0}^{\infty}(210\,000)\left(\frac{50}{2^n}\right) &= \sum_{n=0}^{\infty}(210\,000\cdot 50)\left(\frac{1}{2}\right)^n \\ &= (210\,000\cdot 50)\frac{1}{1-\frac{1}{2}} \\ &= (210\,000\cdot 50)(2) \\ &= 21\,000\,000\end{aligned}$$

So there will never be more than 21,000,000 bitcoins produced this way.

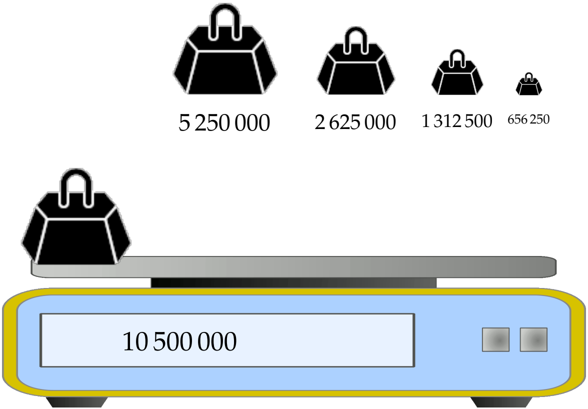
$$\sum_{n=0}^{\infty} 210\,000 \left(\frac{50}{2^n} \right) = 21\,000\,000$$





$$\sum_{n=0}^{\infty} 210\,000 \left(\frac{50}{2^n}\right) = 21\,000\,000$$

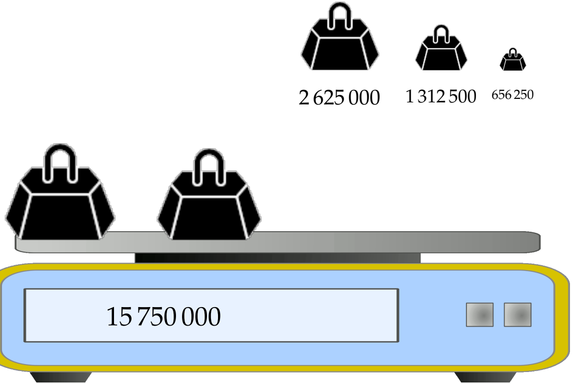
$$S_0 = 10\,500\,000$$



$$\sum_{n=0}^{\infty} 210\,000 \left(\frac{50}{2^n}\right) = 21\,000\,000$$

$$S_0 = 10\,500\,000$$

$$S_1 = 15\,750\,000$$

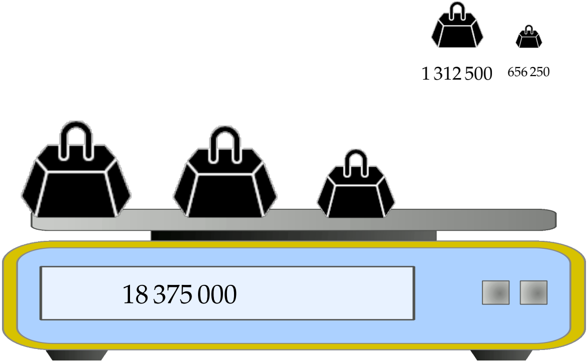


$$\sum_{n=0}^{\infty} 210\,000 \left(\frac{50}{2^n}\right) = 21\,000\,000$$

$$S_0 = 10\,500\,000$$

$$S_1 = 15\,750\,000$$

$$S_2 = 18\,375\,000$$



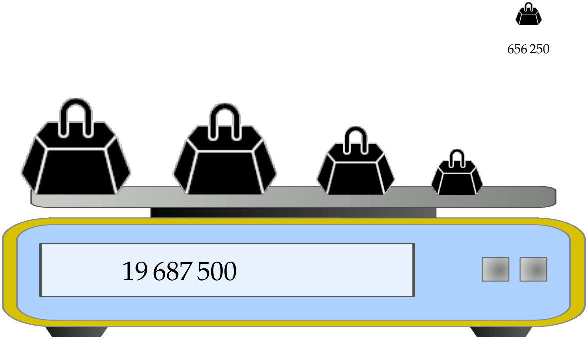
$$\sum_{n=0}^{\infty} 210\,000 \left(\frac{50}{2^n}\right) = 21\,000\,000$$

$$S_0 = 10\,500\,000$$

$$S_1 = 15\,750\,000$$

$$S_2 = 18\,375\,000$$

$$S_3 = 19\,687\,500$$



$$\sum_{n=0}^{\infty} 210\,000 \left(\frac{50}{2^n}\right) = 21\,000\,000$$

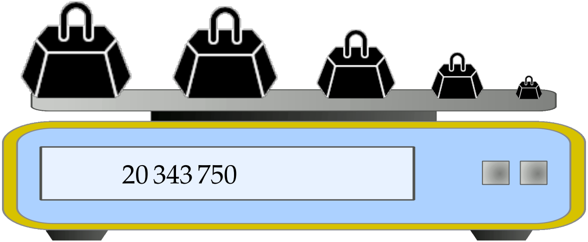
$$S_0 = 10\,500\,000$$

$$S_1 = 15\,750\,000$$

$$S_2 = 18\,375\,000$$

$$S_3 = 19\,687\,500$$

$$S_4 = 20\,343\,750$$



Arithmetic of Series

Let S, T, and C be real numbers. Let the two series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge to S and T respectively. Then

$$\sum_{n=1}^{\infty} [a_n + b_n] = S + T$$

$$\sum_{n=1}^{\infty} [a_n - b_n] = S - T$$

$$\sum_{n=1}^{\infty} [Ca_n] = CS$$

Geometric Series and Partial Sums

Let a and r be fixed numbers, and let N be a positive integer. Then

$$\sum_{n=0}^{N} ar^n = \begin{cases} a \cdot \frac{1-r^{N+1}}{1-r} & \text{if } r \neq 1 \\ a(N+1) & \text{if } r = 1 \end{cases}$$

so

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r} \text{ provided } |r| < 1$$

Evaluate $\displaystyle\sum_{n=0}^{\infty} \left(\frac{2}{3^n} + \frac{4}{5^n} \right)$

$$\sum_{n=0}^{\infty}\left(\frac{2}{3^n}+\frac{4}{5^n}\right) =$$

ans

$$
\begin{aligned}
\sum_{n=0}^{\infty}\left(\frac{2}{3^n}+\frac{4}{5^n}\right) &= \sum_{n=0}^{\infty}\frac{2}{3^n}+\sum_{n=0}^{\infty}\frac{4}{5^n} \\
&= \sum_{n=0}^{\infty}2\left(\frac{1}{3}\right)^n+\sum_{n=0}^{\infty}4\left(\frac{1}{5}\right)^n \\
&= \frac{2}{1-\frac{1}{3}}+\frac{4}{1-\frac{1}{5}} \\
&= \frac{2}{2/3}+\frac{4}{4/5} \\
&= 3+5=8
\end{aligned}
$$

Geometric Series and Partial Sums

Let a and r be fixed numbers, and let N be a positive integer. Then

$$\sum_{n=0}^{N} ar^n = \begin{cases} a \cdot \frac{1-r^{N+1}}{1-r} & \text{if } r \neq 1 \\ a(N+1) & \text{if } r = 1 \end{cases}$$

so

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r} \text{ provided } |r| < 1$$

Evaluate $\displaystyle\sum_{n=6}^{\infty} \left(\frac{3^{n-1}}{5^{2n}} \right)$

$$\sum_{n=6}^{\infty} \left(\frac{3^{n-1}}{5^{2n}} \right) =$$

ans

$$\sum_{n=6}^{\infty}\left(\frac{3^{n-1}}{5^{2n}}\right)=\sum_{n=6}^{\infty}\frac{1}{3}\left(\frac{3^n}{5^{2n}}\right)=\sum_{n=6}^{\infty}\frac{1}{3}\left(\frac{3^n}{25^n}\right)=\sum_{n=6}^{\infty}\frac{1}{3}\left(\frac{3}{25}\right)^n$$

Set $k=n-6$. Then $n=k+6$ and when $n=6, k=0$.

$$=\sum_{k=0}^{\infty}\frac{1}{3}\left(\frac{3}{25}\right)^{k+6}=\sum_{k=1}^{\infty}\frac{1}{3}\left(\frac{3}{25}\right)^{6}\left(\frac{3}{25}\right)^{k}$$

$$=\frac{1}{3}\left(\frac{3}{25}\right)^{6}\cdot\frac{1}{1-3/25}=\frac{3^5}{25^6}\cdot\frac{25}{22}=\frac{3^5}{25^5\cdot 22}$$

Geometric Series and Partial Sums

Let a and r be fixed numbers, and let N be a positive integer. Then

$$\sum_{n=0}^{N} ar^n = \begin{cases} a \cdot \frac{1-r^{N+1}}{1-r} & \text{if } r \neq 1 \\ a(N+1) & \text{if } r = 1 \end{cases}$$

so

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r} \text{ provided } |r| < 1$$

Evaluate $\displaystyle\sum_{n=0}^{\infty} \left(\frac{2^{2n}}{3^n}\right)$

$$\sum_{n=0}^{\infty} \left(\frac{2^{2n}}{3^n} \right) =$$

A

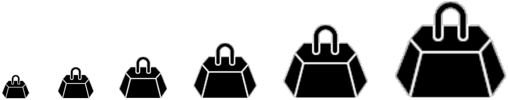
ans

$$\sum_{n=0}^{\infty}\left(\frac{2^{2n}}{3^n}\right) = \sum_{n=0}^{\infty}\left(\frac{4^n}{3^n}\right)$$
$$= \sum_{n=1}^{\infty}\left(\frac{4}{3}\right)^n$$

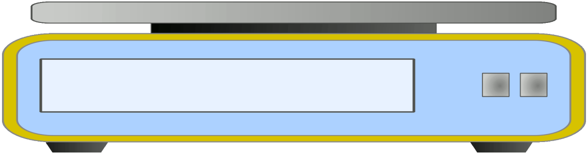
Since $\left|\frac{4}{3}\right| > 1$, the series diverges.

A

$$\sum_{n=0}^{\infty} \left(\frac{2^{2n}}{3^n} \right) \text{ diverges}$$

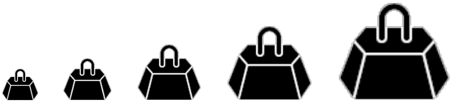


$1 \quad \frac{4}{3} \quad \frac{16}{9} \quad \frac{64}{27} \quad \frac{256}{81} \quad \frac{1024}{243}$

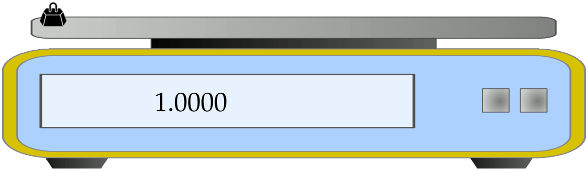


$$\sum_{n=0}^{\infty} \left(\frac{2^{2n}}{3^n} \right) \text{ diverges}$$

$$S_0 = 1.0000$$



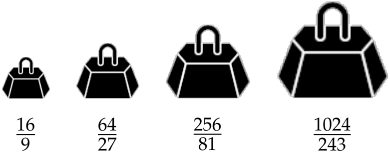
$\frac{4}{3}$ $\frac{16}{9}$ $\frac{64}{27}$ $\frac{256}{81}$ $\frac{1024}{243}$

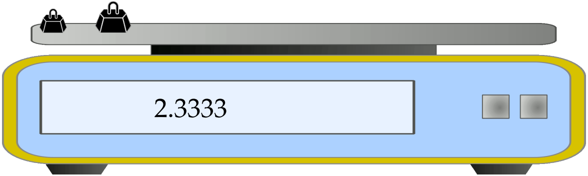


$$\sum_{n=0}^{\infty}\left(\frac{2^{2n}}{3^n}\right) \text{ diverges}$$

$$S_0 = 1.0000$$

$$S_1 = 2.3333$$



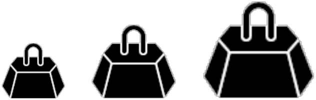


$$\sum_{n=0}^{\infty} \left(\frac{2^{2n}}{3^n} \right) \text{ diverges}$$

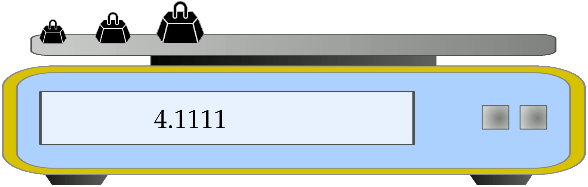
$S_0 = 1.0000$

$S_1 = 2.3333$

$S_2 = 4.1111$



$\frac{64}{27}$ $\frac{256}{81}$ $\frac{1024}{243}$



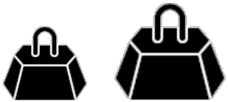
$$\sum_{n=0}^{\infty} \left(\frac{2^{2n}}{3^n} \right) \text{ diverges}$$

$S_0 = 1.0000$

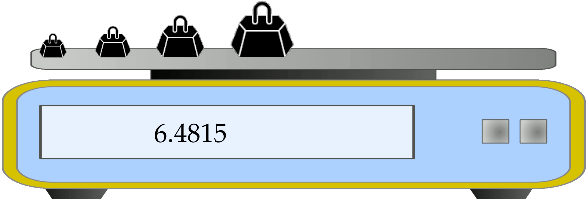
$S_1 = 2.3333$

$S_2 = 4.1111$

$S_3 = 6.4815$



$\frac{256}{81}$ $\frac{1024}{243}$



$$\sum_{n=0}^{\infty}\left(\frac{2^{2n}}{3^n}\right) \text{ diverges}$$

$\frac{1024}{243}$

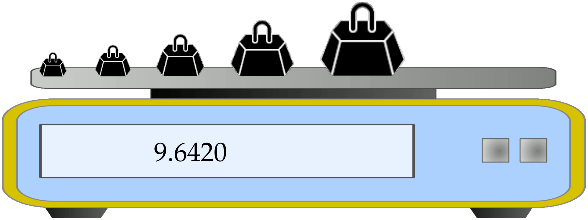
$S_0 = 1.0000$

$S_1 = 2.3333$

$S_2 = 4.1111$

$S_3 = 6.4815$

$S_4 = 9.6420$



$$\sum_{n=0}^{\infty} \left(\frac{2^{2n}}{3^n} \right) \text{ diverges}$$

$S_0 = 1.0000$

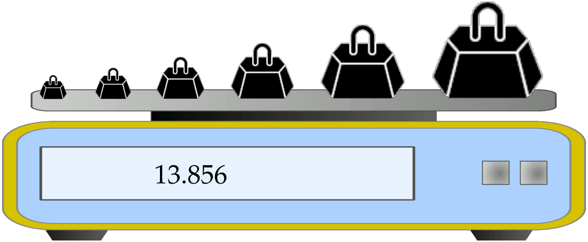
$S_1 = 2.3333$

$S_2 = 4.1111$

$S_3 = 6.4815$

$S_4 = 9.6420$

$S_5 = 13.856$



Telescoping Sums

Evaluate $\displaystyle\sum_{n=1}^{800}\left(\frac{1}{n}-\frac{1}{n+1}\right)$.

Evaluate $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$.

Q

ans

Telescoping Sums

Evaluate $\displaystyle\sum_{n=1}^{800}\left(\frac{1}{n}-\frac{1}{n+1}\right)$.

Evaluate $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$.

$$\begin{array}{lccc}
a_1: & \frac{1}{1} & - & \frac{1}{2}\\
a_2: & \frac{1}{2} & - & \frac{1}{3}\\
a_3: & \frac{1}{3} & - & \frac{1}{4}\\
a_4: & \frac{1}{4} & - & \frac{1}{5}\\
\vdots & & & \\
a_{N-1}: & \frac{1}{N-1} & - & \frac{1}{N}\\
a_N: & \frac{1}{N} & - & \frac{1}{N+1}
\end{array}$$

$$\begin{array}{lccc}
S_1 = & \frac{1}{1} & - & \frac{1}{2}\\
S_2 = & & & \\
S_3 = & & & \\
S_4 = & & & \\
\vdots & & & \\
S_N = & & &
\end{array}$$

Q

ans

Telescoping Sums

Evaluate $\displaystyle\sum_{n=1}^{800}\left(\frac{1}{n}-\frac{1}{n+1}\right)$.

Evaluate $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$.

$$\begin{array}{llll}
a_1: & \frac{1}{1} & - & \frac{1}{2} \\
a_2: & \frac{1}{2} & - & \frac{1}{3} \\
a_3: & \frac{1}{3} & - & \frac{1}{4} \\
a_4: & \frac{1}{4} & - & \frac{1}{5} \\
\vdots & & & \\
a_{N-1}: & \frac{1}{N-1} & - & \frac{1}{N} \\
a_N: & \frac{1}{N} & - & \frac{1}{N+1}
\end{array}$$

$$\begin{array}{llll}
S_1 = & \frac{1}{1} & - & \frac{1}{2} \\
S_2 = & \frac{1}{1} & - & \frac{1}{3} \\
S_3 = & & & \\
S_4 = & & & \\
\vdots & & & \\
S_N = & & &
\end{array}$$

Q

ans

Telescoping Sums

Evaluate $\displaystyle\sum_{n=1}^{800}\left(\frac{1}{n}-\frac{1}{n+1}\right)$.

Evaluate $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$.

$$\begin{array}{rccc}
a_1: & \frac{1}{1} & - & \frac{1}{2} \\
a_2: & \frac{1}{2} & - & \frac{1}{3} \\
a_3: & \frac{1}{3} & - & \frac{1}{4} \\
a_4: & \frac{1}{4} & - & \frac{1}{5} \\
\vdots & & & \\
a_{N-1}: & \frac{1}{N-1} & - & \frac{1}{N} \\
a_N: & \frac{1}{N} & - & \frac{1}{N+1}
\end{array}$$

$$\begin{array}{rccc}
S_1 = & \frac{1}{1} & - & \frac{1}{2} \\
S_2 = & \frac{1}{1} & - & \frac{1}{3} \\
S_3 = & \frac{1}{1} & - & \frac{1}{4} \\
S_4 = & & & \\
\vdots & & & \\
S_N = & & &
\end{array}$$

Q

ans

Telescoping Sums

Evaluate $\displaystyle\sum_{n=1}^{800}\left(\frac{1}{n}-\frac{1}{n+1}\right)$.

Evaluate $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$.

$$\begin{array}{lccc}
a_1: & \frac{1}{1} & - & \frac{1}{2} \\
a_2: & \frac{1}{2} & - & \frac{1}{3} \\
a_3: & \frac{1}{3} & - & \frac{1}{4} \\
a_4: & \frac{1}{4} & - & \frac{1}{5} \\
\vdots & & & \\
a_{N-1}: & \frac{1}{N-1} & - & \frac{1}{N} \\
a_N: & \frac{1}{N} & - & \frac{1}{N+1}
\end{array}$$

$$\begin{array}{lccc}
S_1 = & \frac{1}{1} & - & \frac{1}{2} \\
S_2 = & \frac{1}{1} & - & \frac{1}{3} \\
S_3 = & \frac{1}{1} & - & \frac{1}{4} \\
S_4 = & \frac{1}{1} & - & \frac{1}{5} \\
\vdots & & & \\
S_N = & & &
\end{array}$$

Q

ans

Telescoping Sums

Evaluate $\displaystyle\sum_{n=1}^{800}\left(\frac{1}{n}-\frac{1}{n+1}\right)$.

Evaluate $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$.

$$\begin{array}{llcl}
a_1: & \frac{1}{1} & - & \frac{1}{2} \\
a_2: & \frac{1}{2} & - & \frac{1}{3} \\
a_3: & \frac{1}{3} & - & \frac{1}{4} \\
a_4: & \frac{1}{4} & - & \frac{1}{5} \\
\vdots & & & \\
a_{N-1}: & \frac{1}{N-1} & - & \frac{1}{N} \\
a_N: & \frac{1}{N} & - & \frac{1}{N+1}
\end{array}$$

$$\begin{array}{lccc}
S_1 = & \frac{1}{1} & - & \frac{1}{2} \\
S_2 = & \frac{1}{1} & - & \frac{1}{3} \\
S_3 = & \frac{1}{1} & - & \frac{1}{4} \\
S_4 = & \frac{1}{1} & - & \frac{1}{5} \\
\vdots & & & \\
S_N = & & &
\end{array}$$

Q

ans

Telescoping Sums

Evaluate $\displaystyle\sum_{n=1}^{800}\left(\frac{1}{n}-\frac{1}{n+1}\right)$.

Evaluate $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$.

$$\begin{array}{lccc}
a_1: & \frac{1}{1} & - & \frac{1}{2} \\
a_2: & \frac{1}{2} & - & \frac{1}{3} \\
a_3: & \frac{1}{3} & - & \frac{1}{4} \\
a_4: & \frac{1}{4} & - & \frac{1}{5} \\
\vdots & & & \\
a_{N-1}: & \frac{1}{N-1} & - & \frac{1}{N} \\
a_N: & \frac{1}{N} & - & \frac{1}{N+1}
\end{array}$$

$$\begin{array}{lcccc}
S_1 = & \frac{1}{1} & - & \frac{1}{2} & \\
S_2 = & \frac{1}{1} & - & \frac{1}{3} & \\
S_3 = & \frac{1}{1} & - & \frac{1}{4} & \\
S_4 = & \frac{1}{1} & - & \frac{1}{5} & \\
\vdots & & & & \\
S_N = & \frac{1}{1} & - & \frac{1}{N+1} & = \frac{N}{N+1}
\end{array}$$

Q

ans

Telescoping Sums

Evaluate $\displaystyle\sum_{n=1}^{800}\left(\frac{1}{n}-\frac{1}{n+1}\right)$.

Evaluate $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$.

$$\begin{array}{lccc}
a_1: & \frac{1}{1} & - & \frac{1}{2} \\
a_2: & \frac{1}{2} & - & \frac{1}{3} \\
a_3: & \frac{1}{3} & - & \frac{1}{4} \\
a_4: & \frac{1}{4} & - & \frac{1}{5} \\
\vdots & & & \\
a_{N-1}: & \frac{1}{N-1} & - & \frac{1}{N} \\
a_N: & \frac{1}{N} & - & \frac{1}{N+1}
\end{array}$$

$$\begin{array}{lcccl}
S_1 = & \frac{1}{1} & - & \frac{1}{2} & \\
S_2 = & \frac{1}{1} & - & \frac{1}{3} & \\
S_3 = & \frac{1}{1} & - & \frac{1}{4} & \\
S_4 = & \frac{1}{1} & - & \frac{1}{5} & \\
\vdots & & & & \\
S_N = & \frac{1}{1} & - & \frac{1}{N+1} & = \frac{N}{N+1}
\end{array}$$

$$\sum_{n=1}^{800}\left(\frac{1}{n}-\frac{1}{n+1}\right)=S_{800}=\frac{800}{801}$$

$$\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)=\lim_{N\to\infty}S_N=1$$

Q

ans

Evaluate $\displaystyle\sum_{n=1}^{1000} \log\left(\frac{n+1}{n}\right)$.

Evaluate $\displaystyle\sum_{n=1}^{\infty} \log\left(\frac{n+1}{n}\right)$.

Q

ans

Evaluate $\sum_{n=1}^{1000} \log\left(\frac{n+1}{n}\right)$.

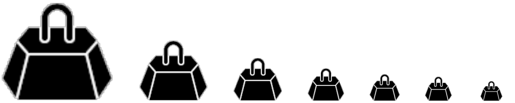
Evaluate $\sum_{n=1}^{\infty} \log\left(\frac{n+1}{n}\right)$.

$$\begin{array}{lcl}
a_1 & : & \log(2) - \cancel{\log(1)} \\
a_2 & : & \log(3) - \log(2) \\
a_3 & : & \log(4) - \log(3) \\
\vdots & & \\
a_{n-1} & : & \log(n) - \log(n-1) \\
a_n & : & \log(n+1) - \log(n)
\end{array}
\qquad
\begin{array}{lcl}
S_1 & = & \log(2) \\
S_2 & = & \log(3) \\
S_3 & = & \log(4) \\
 & & \\
 & & \\
S_n & = & \log(n+1)
\end{array}$$

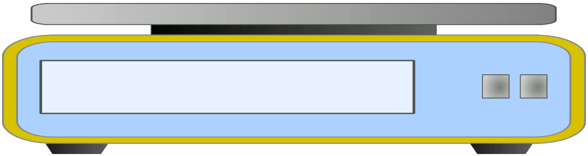
So, $\sum_{n=1}^{1000} \log\left(\frac{n+1}{n}\right) = S_{1000} = \log(1001)$ and

$$\sum_{n=1}^{\infty} \log\left(\frac{n+1}{n}\right) = \lim_{n\to\infty} \log(n+1) = \infty$$

$$\sum_{n=1}^{\infty} \log\left(\frac{n+1}{n}\right) \text{ diverges}$$

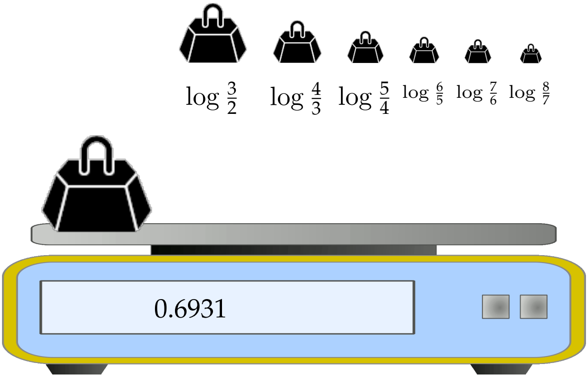


$\log 2$ $\log \frac{3}{2}$ $\log \frac{4}{3}$ $\log \frac{5}{4}$ $\log \frac{6}{5}$ $\log \frac{7}{6}$ $\log \frac{8}{7}$



$$\sum_{n=1}^{\infty} \log\left(\frac{n+1}{n}\right) \text{ diverges}$$

$$S_1 = 0.6931$$

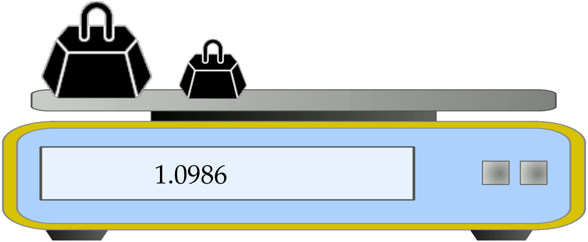


$$\sum_{n=1}^{\infty} \log\left(\frac{n+1}{n}\right) \text{ diverges}$$

$S_1 = 0.6931$

$S_2 = 1.0986$

$\log \frac{4}{3}$ $\log \frac{5}{4}$ $\log \frac{6}{5}$ $\log \frac{7}{6}$ $\log \frac{8}{7}$



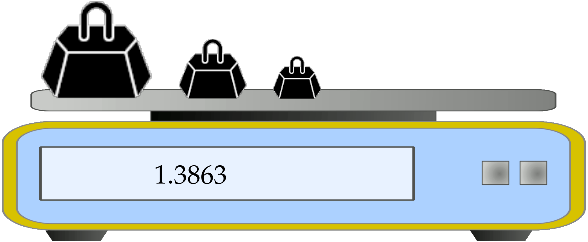
$$\sum_{n=1}^{\infty} \log\left(\frac{n+1}{n}\right) \text{ diverges}$$

$S_1 = 0.6931$

$S_2 = 1.0986$

$S_3 = 1.3863$

$\log \frac{5}{4}$ $\log \frac{6}{5}$ $\log \frac{7}{6}$ $\log \frac{8}{7}$



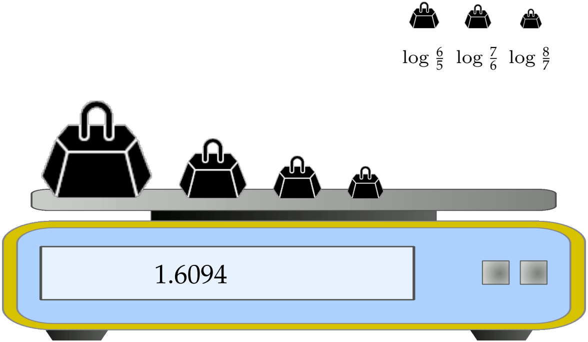
$$\sum_{n=1}^{\infty} \log\left(\frac{n+1}{n}\right) \text{ diverges}$$

$S_1 = 0.6931$

$S_2 = 1.0986$

$S_3 = 1.3863$

$S_4 = 1.6094$



$$\sum_{n=1}^{\infty} \log\left(\frac{n+1}{n}\right) \text{ diverges}$$

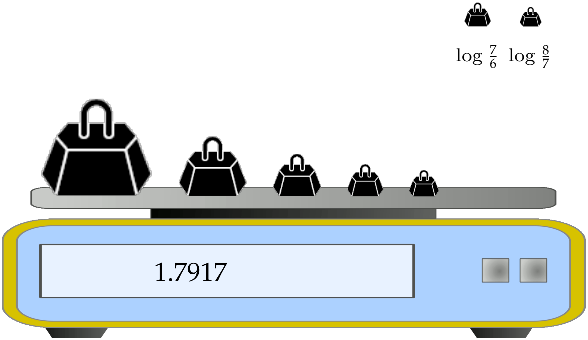
$S_1 = 0.6931$

$S_2 = 1.0986$

$S_3 = 1.3863$

$S_4 = 1.6094$

$S_5 = 1.7917$



$$\sum_{n=1}^{\infty} \log\left(\frac{n+1}{n}\right) \text{ diverges}$$

$S_1 = 0.6931$

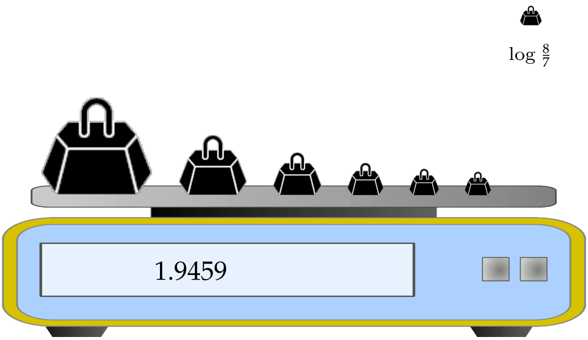
$S_2 = 1.0986$

$S_3 = 1.3863$

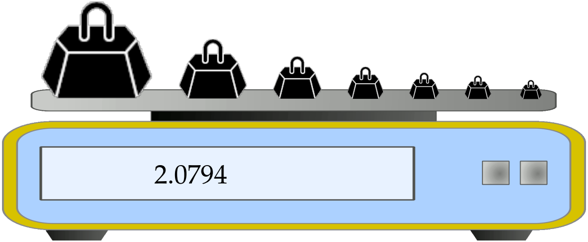
$S_4 = 1.6094$

$S_5 = 1.7917$

$S_6 = 1.9459$



$$\sum_{n=1}^{\infty} \log\left(\frac{n+1}{n}\right) \text{ diverges}$$



$S_1 = 0.6931$

$S_2 = 1.0986$

$S_3 = 1.3863$

$S_4 = 1.6094$

$S_5 = 1.7917$

$S_6 = 1.9459$

$S_7 = 2.0794$

Included Work

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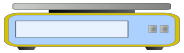 'Waage/Libra' by B. Lachner is in the public domain (accessed April 2021, edited), 33–38, 52–58, 68–72, 84–114, 119–124, 135–141, 152–159

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