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Four Series

Let $a_n = \left(-\frac{2}{3}\right)^n$. Do the following series converge or diverge?

$$\sum_{n=0}^{\infty} a_n \qquad\qquad \sum_{n=0}^{\infty} |a_n|$$

Let $b_n = \frac{(-1)^n}{n}$. Do the following series converge or diverge?

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$$\sum_{n=1}^{\infty} b_n \qquad\qquad \sum_{n=1}^{\infty} |b_n|$$

converge diverge

The series

$$\sum_{n=0}^{\infty}\left(-\frac{2}{3}\right)^n$$

is called absolutely convergent, because the series converges and if we replace the terms being added by their absolute values, that series *still* converges.

The series

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{n}$$

is called conditionally convergent, because the series converges, but if we replace the terms being added by their absolute values, that series *diverges*.

Absolute and conditional convergence

(a) A series $\sum_{n=1}^{\infty} a_n$ is said to **converge absolutely** if the series $\sum_{n=1}^{\infty} |a_n|$ converges.

(b) If $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ diverges we say that $\sum_{n=1}^{\infty} a_n$ is **conditionally convergent**.

Theorem

If the series $\sum_{n=1}^{\infty} |a_n|$ converges then the series $\sum_{n=1}^{\infty} a_n$ also converges.
That is, absolute convergence implies convergence.

If $\sum a_n$ ...	and $\sum \lvert a_n \rvert$ ...	then we say $\sum a_n$ is ...

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diverges	diverges	divergent
diverges	converges	not possible!

Does the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

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Q

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Alternating series test:

Let $a_n = \frac{1}{n^2}$. Note a_n has positive, decreasing terms, approaching 0 as n grows. Then $\sum\limits_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges by the alternating series test.

Absolute convergence implies convergence:

The series $\sum\limits_{n=1}^{\infty} \left| \frac{(-1)^n}{n^2} \right|$ is the same as the p-series $\sum\limits_{n=1}^{\infty} \frac{1}{n^2}$, which converges by the p-test. Then $\sum\limits_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges absolutely, therefore it converges.

Does the series

$$\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$$

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Q

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$$\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$$

converge or diverge?

The terms of this series are sometimes positive and sometimes negative, but they do not strictly alternate, so the alternating series test does not apply.

Note that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a convergent series, and $\frac{|\sin n|}{n^2} \leq \frac{1}{n^2}$ for all n. Then by the comparison test, $\sum_{n=1}^{\infty} \frac{|\sin n|}{n^2}$ converges.

Then $\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$ converges absolutely, hence it converges.