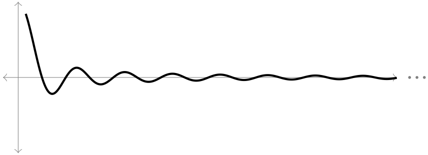
Table of Contents

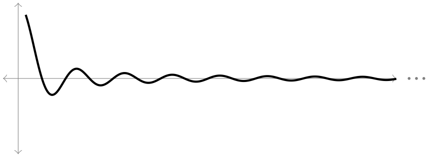
An integral is improper if one or both of the following happen:

- The region of integration is unbounded, e.g. $\int_1^\infty \frac{\sin x}{x}\,\mathrm{d}x$



An integral is improper if one or both of the following happen:

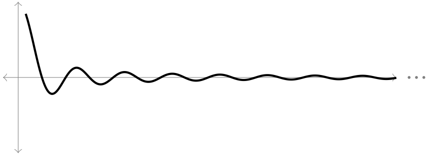
- The region of integration is unbounded, e.g. $\int_1^\infty \frac{\sin x}{x}\,\mathrm{d}x$



$$\Delta x = \frac{b-a}{n} = \frac{\infty}{n}\ ???$$

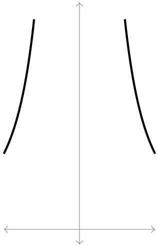
An integral is improper if one or both of the following happen:

- The region of integration is unbounded, e.g. $\int_1^\infty \frac{\sin x}{x}\,\mathrm{d}x$



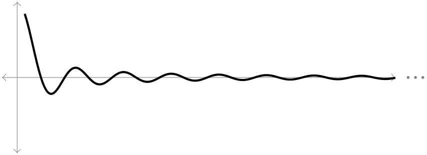
$$\Delta x = \frac{b-a}{n} = \frac{\infty}{n} \text{ ???}$$

- The integrand is unbounded over the interval, e.g. $\int_{-1}^{1} \frac{1}{x^2}\,\mathrm{d}x$



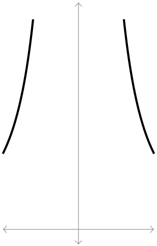
An integral is improper if one or both of the following happen:

- The region of integration is unbounded, e.g. $\int_1^\infty \frac{\sin x}{x}\,\mathrm{d}x$



$\Delta x = \frac{b-a}{n} = \frac{\infty}{n}$???

- The integrand is unbounded over the interval, e.g. $\int_{-1}^{1} \frac{1}{x^2}\,\mathrm{d}x$



$f(0)\Delta x =$???

Strategy

In both cases, we eliminate the offending parts of the integral using limits.

$$\int_1^{\infty} \frac{\sin x}{x}\, \mathrm{d}x =$$

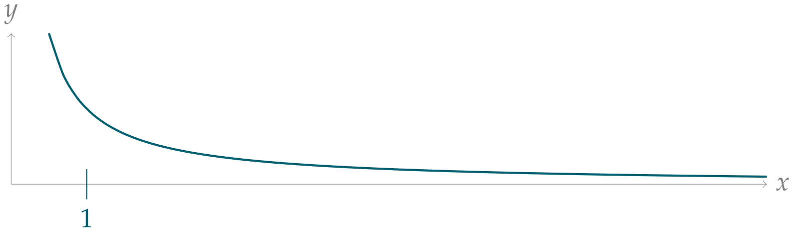
$$\int_0^3 \frac{1}{x}\, \mathrm{d}x =$$

If the limit doesn't exist, we say the integral diverges. Otherwise it converges.

$$\int_1^\infty \frac{1}{x}\,\mathrm{d}x =$$

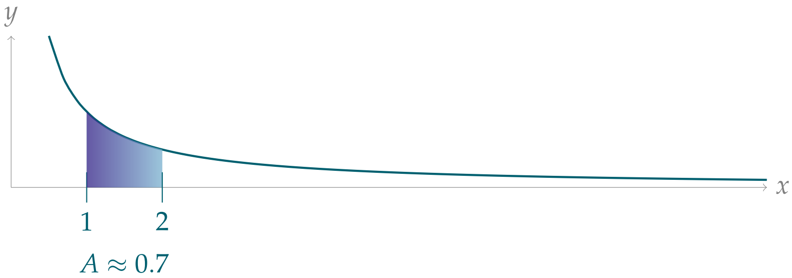
$$\int_1^\infty \frac{1}{x}\,\mathrm{d}x = \lim_{a\to\infty}\left[\int_1^a \frac{1}{x}\,\mathrm{d}x\right]$$
$$= \lim_{a\to\infty}[\log a] = \infty$$

We say this integral diverges because the limit is not a number.



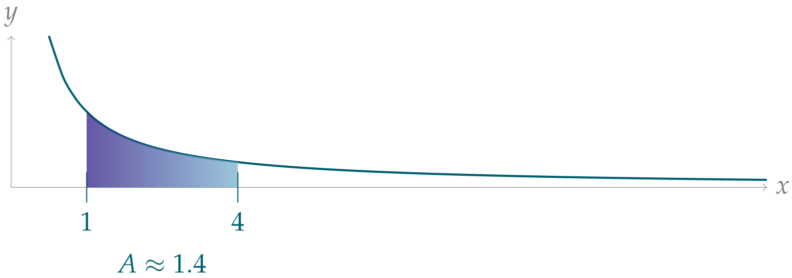
$$\int_1^\infty \frac{1}{x}\,\mathrm{d}x = \lim_{a\to\infty}\left[\int_1^a \frac{1}{x}\,\mathrm{d}x\right]$$
$$= \lim_{a\to\infty}[\log a] = \infty$$

We say this integral diverges because the limit is not a number.



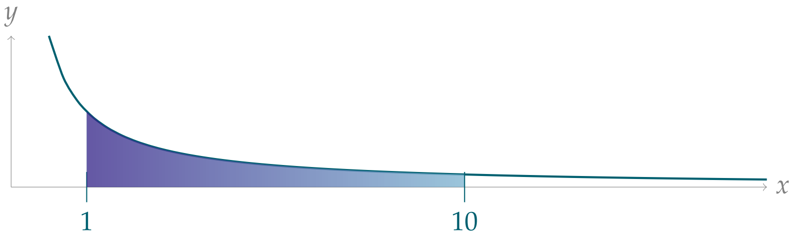
$$\int_1^\infty \frac{1}{x}\,\mathrm{d}x = \lim_{a\to\infty}\left[\int_1^a \frac{1}{x}\,\mathrm{d}x\right] = \lim_{a\to\infty}[\log a] = \infty$$

We say this integral diverges because the limit is not a number.



$$
\begin{aligned}
\int_1^\infty \frac{1}{x}\,\mathrm{d}x &= \lim_{a\to\infty}\left[\int_1^a \frac{1}{x}\,\mathrm{d}x\right] \\
&= \lim_{a\to\infty}\left[\log a\right] = \infty
\end{aligned}
$$

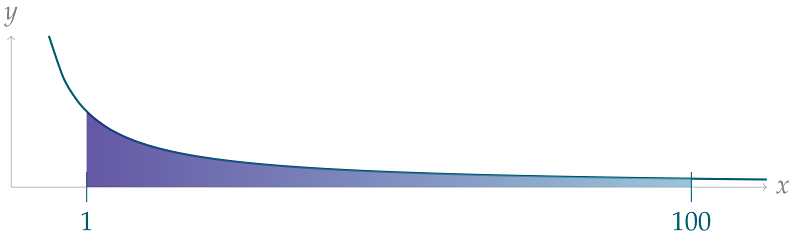
We say this integral diverges because the limit is not a number.



$A \approx 2.3$

$$\int_1^\infty \frac{1}{x}\,\mathrm{d}x = \lim_{a\to\infty}\left[\int_1^a \frac{1}{x}\,\mathrm{d}x\right]$$
$$= \lim_{a\to\infty}[\log a] = \infty$$

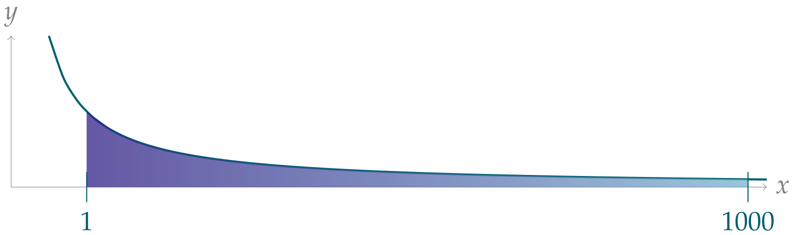
We say this integral diverges because the limit is not a number.



$A \approx 4.6$

$$\int_1^\infty \frac{1}{x}\,\mathrm{d}x = \lim_{a\to\infty}\left[\int_1^a \frac{1}{x}\,\mathrm{d}x\right] = \lim_{a\to\infty}\left[\log a\right] = \infty$$

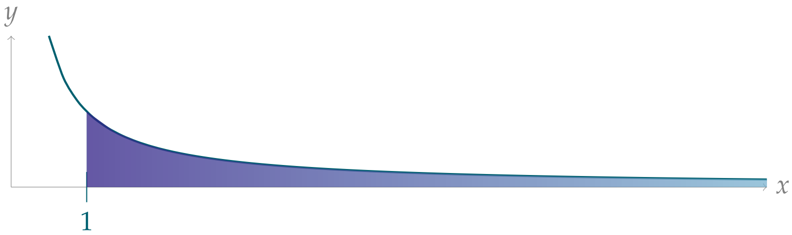
We say this integral diverges because the limit is not a number.



$A \approx 6.9$

$$\int_1^\infty \frac{1}{x}\,\mathrm{d}x = \lim_{a\to\infty}\left[\int_1^a \frac{1}{x}\,\mathrm{d}x\right]$$
$$= \lim_{a\to\infty}\,[\log a] = \infty$$

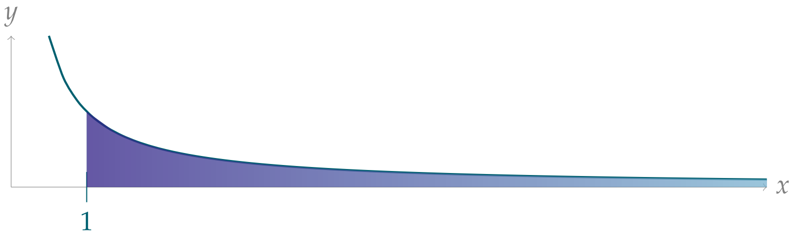
We say this integral diverges because the limit is not a number.



$A \approx 1000$

$$\int_1^\infty \frac{1}{x}\,\mathrm{d}x = \lim_{a\to\infty}\left[\int_1^a \frac{1}{x}\,\mathrm{d}x\right]$$
$$= \lim_{a\to\infty}[\log a] = \infty$$

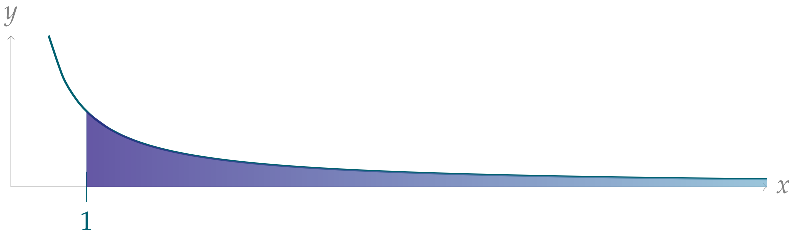
We say this integral diverges because the limit is not a number.



$A \approx 1000000$

$$\int_1^\infty \frac{1}{x}\,\mathrm{d}x = \lim_{a\to\infty}\left[\int_1^a \frac{1}{x}\,\mathrm{d}x\right]$$
$$= \lim_{a\to\infty}[\log a] = \infty$$

We say this integral diverges because the limit is not a number.



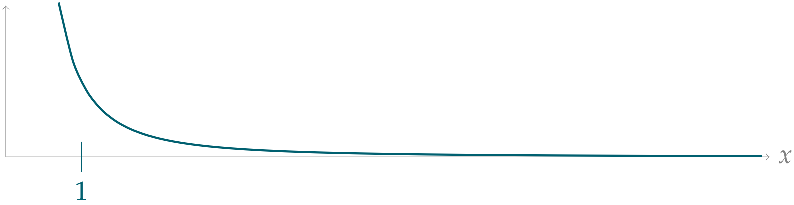
$A \approx 1000000000000$ etc

$$\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x =$$

ans

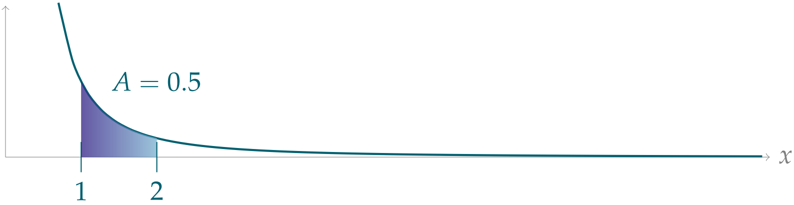
$$
\begin{aligned}
\int_1^{\infty} \frac{1}{x^2}\,\mathrm{d}x &= \lim_{a\to\infty}\left[\int_1^{a} \frac{1}{x^2}\,\mathrm{d}x\right] \\
&= \lim_{a\to\infty}\left[-\frac{1}{a}+1\right] = 1
\end{aligned}
$$

We say this integral converges because the limit is a number.



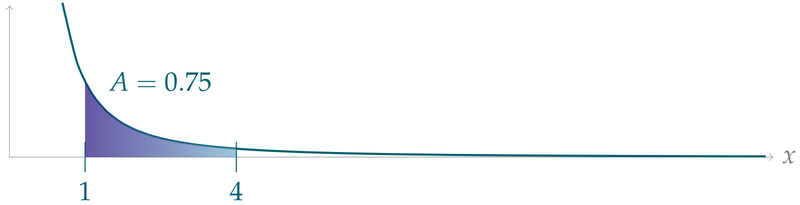
$$\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x = \lim_{a\to\infty}\left[\int_1^a \frac{1}{x^2}\,\mathrm{d}x\right] = \lim_{a\to\infty}\left[-\frac{1}{a}+1\right] = 1$$

We say this integral converges because the limit is a number.



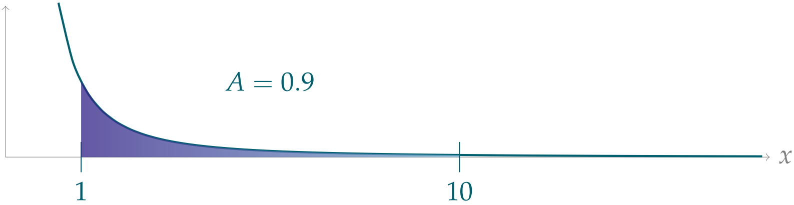
$$\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x = \lim_{a\to\infty}\left[\int_1^a \frac{1}{x^2}\,\mathrm{d}x\right]$$
$$= \lim_{a\to\infty}\left[-\frac{1}{a}+1\right] = 1$$

We say this integral converges because the limit is a number.



$$
\begin{aligned}
\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x &= \lim_{a\to\infty}\left[\int_1^a \frac{1}{x^2}\,\mathrm{d}x\right] \\
&= \lim_{a\to\infty}\left[-\frac{1}{a}+1\right] = 1
\end{aligned}
$$

We say this integral converges because the limit is a number.

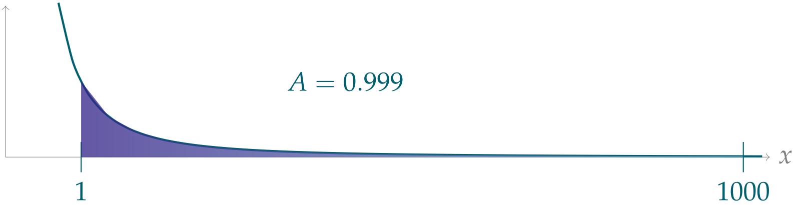


$$\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x = \lim_{a\to\infty}\left[\int_1^a \frac{1}{x^2}\,\mathrm{d}x\right]$$
$$= \lim_{a\to\infty}\left[-\frac{1}{a}+1\right] = 1$$

We say this integral converges because the limit is a number.

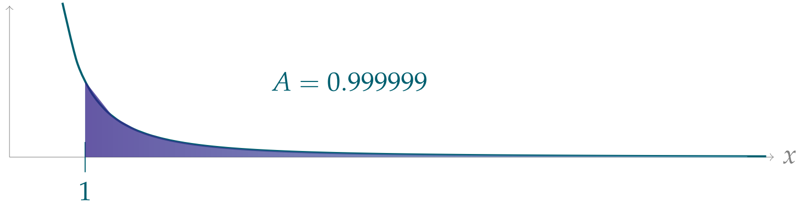
$$\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x = \lim_{a\to\infty}\left[\int_1^a \frac{1}{x^2}\,\mathrm{d}x\right]$$
$$= \lim_{a\to\infty}\left[-\frac{1}{a}+1\right] = 1$$

We say this integral converges because the limit is a number.



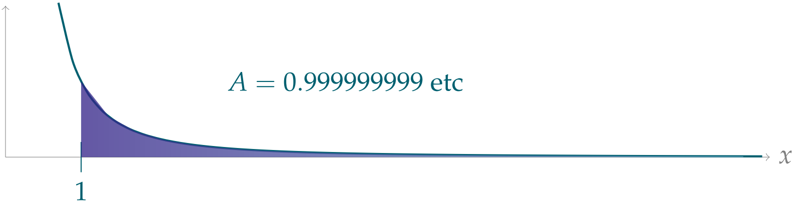
$$\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x = \lim_{a\to\infty}\left[\int_1^a \frac{1}{x^2}\,\mathrm{d}x\right]$$
$$= \lim_{a\to\infty}\left[-\frac{1}{a}+1\right] = 1$$

We say this integral converges because the limit is a number.



$$\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x = \lim_{a\to\infty}\left[\int_1^a \frac{1}{x^2}\,\mathrm{d}x\right]$$
$$= \lim_{a\to\infty}\left[-\frac{1}{a}+1\right] = 1$$

We say this integral converges because the limit is a number.

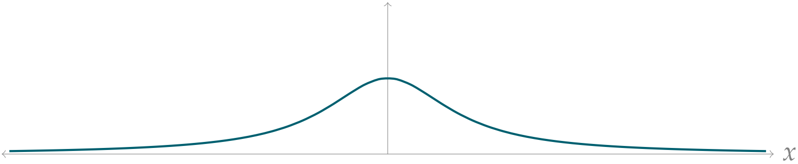


Evaluate $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\, dx$

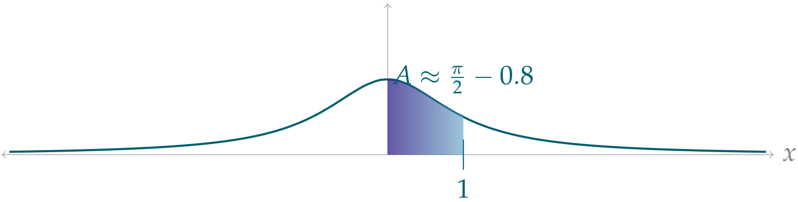
When an integral has multiple sources of impropriety, we break it up into integrals that have only one source each. If all of them converge, the original integral converges. If any of them diverges, the original integral diverges as well.

Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,\mathrm{d}x$

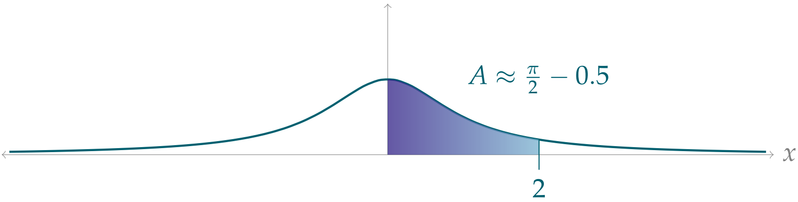
Evaluate $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,\mathrm{d}x$



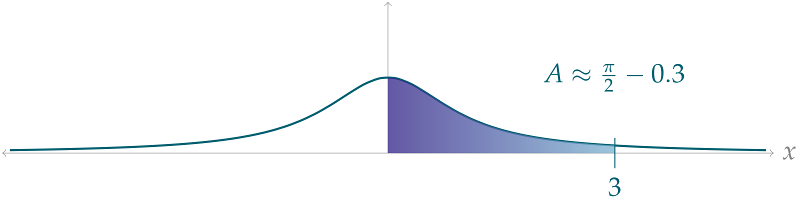
Evaluate $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,\mathrm{d}x$



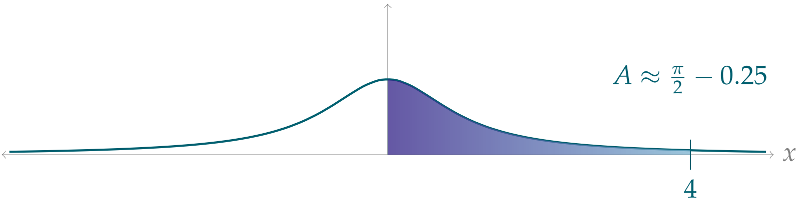
Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,\mathrm{d}x$



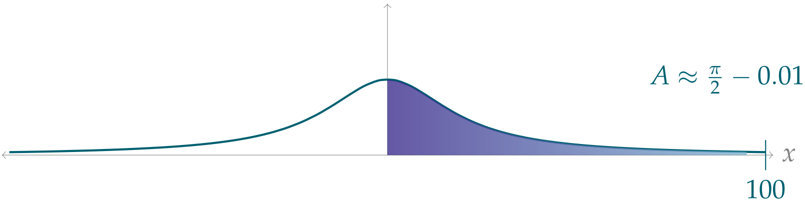
Evaluate $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,\mathrm{d}x$



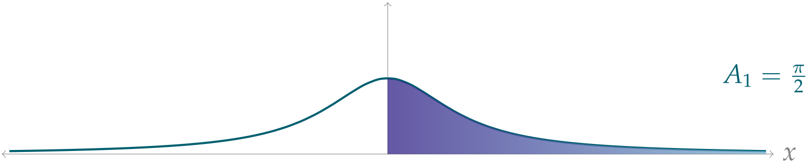
Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,\mathrm{d}x$



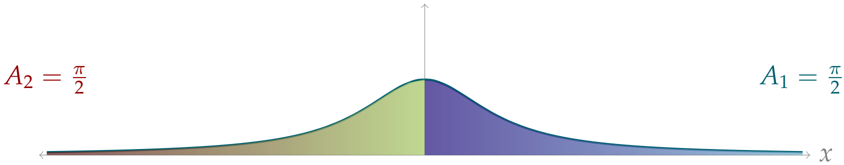
Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,\mathrm{d}x$



Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,\mathrm{d}x$

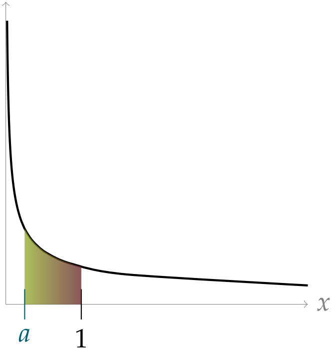


Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,\mathrm{d}x$

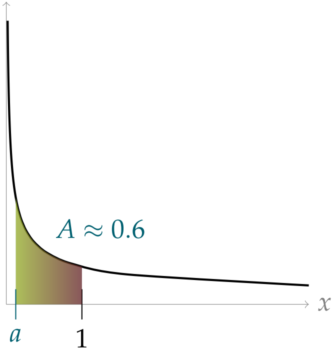


Evaluate $\displaystyle\int_0^1 \frac{1}{2\sqrt{x}}\,\mathrm{d}x$

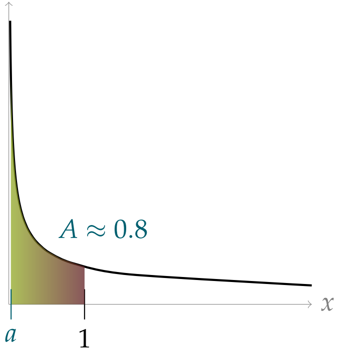
Evaluate $\displaystyle\int_0^1 \frac{1}{2\sqrt{x}}\,\mathrm{d}x$



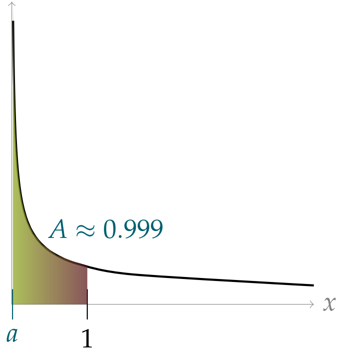
Evaluate $\displaystyle\int_0^1 \frac{1}{2\sqrt{x}}\,\mathrm{d}x$



Evaluate $\displaystyle\int_0^1 \frac{1}{2\sqrt{x}}\,\mathrm{d}x$

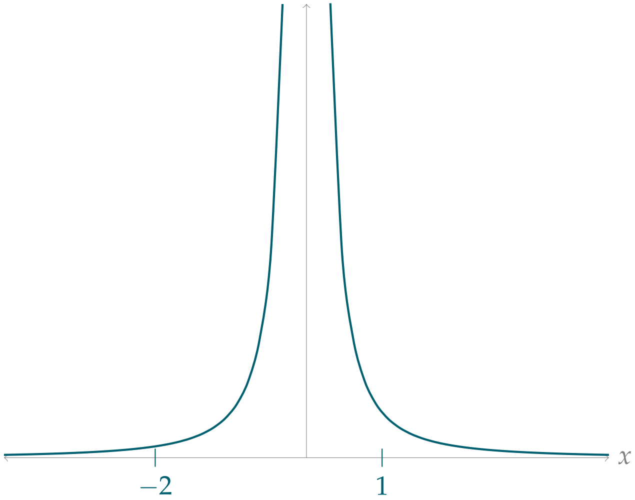


Evaluate $\int_0^1 \frac{1}{2\sqrt{x}}\,\mathrm{d}x$

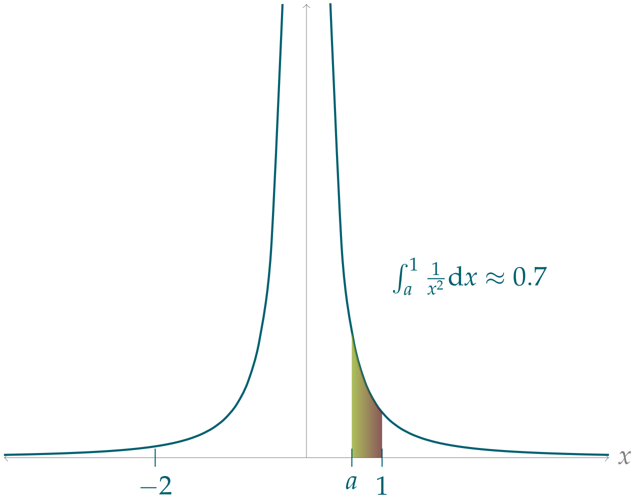


Evaluate $\int_{-2}^{1} \frac{1}{x^2}\, dx$

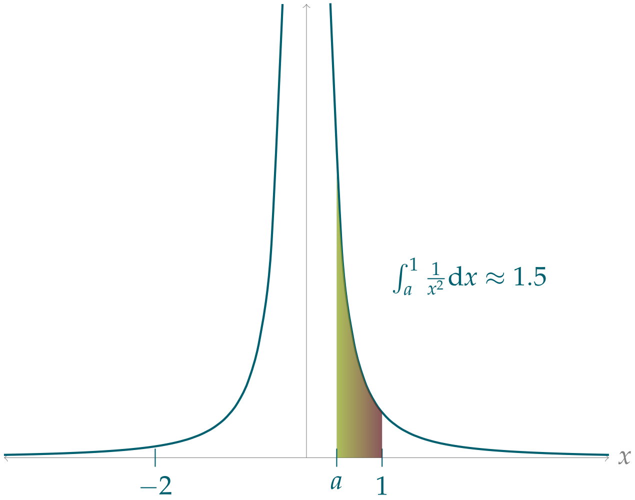
Evaluate $\int_{-2}^{1} \frac{1}{x^2}\, \mathrm{d}x$



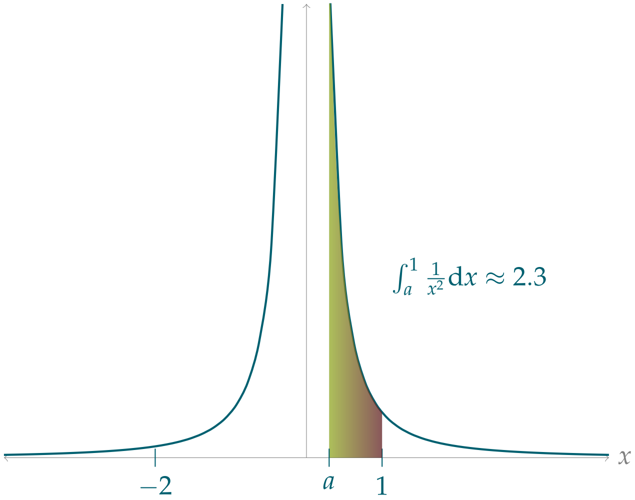
Evaluate $\int_{-2}^{1} \frac{1}{x^2}\,\mathrm{d}x$



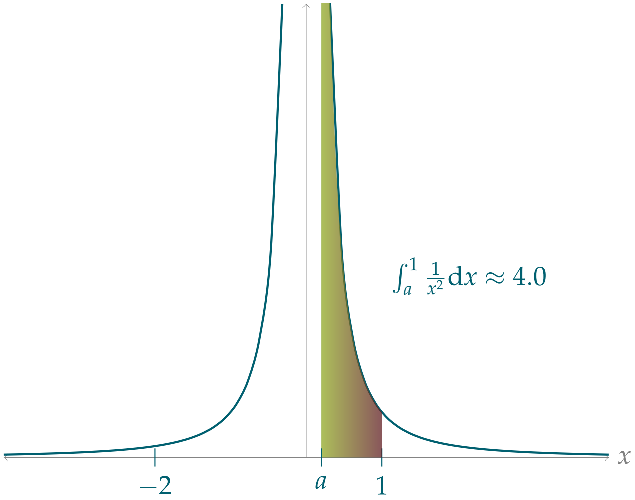
Evaluate $\int_{-2}^{1} \frac{1}{x^2}\,\mathrm{d}x$



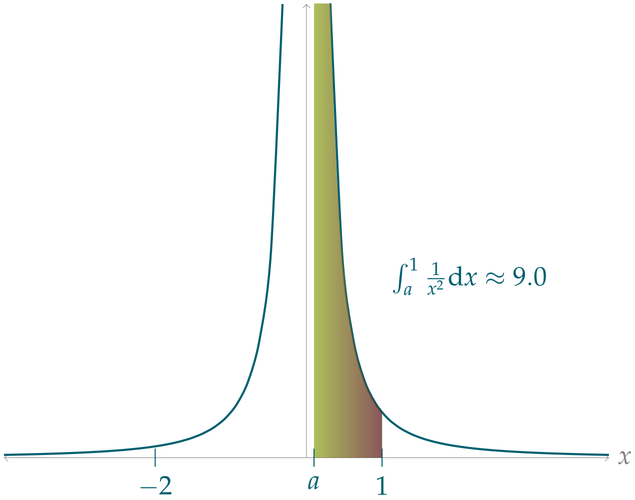
Evaluate $\displaystyle\int_{-2}^{1} \frac{1}{x^2}\,\mathrm{d}x$



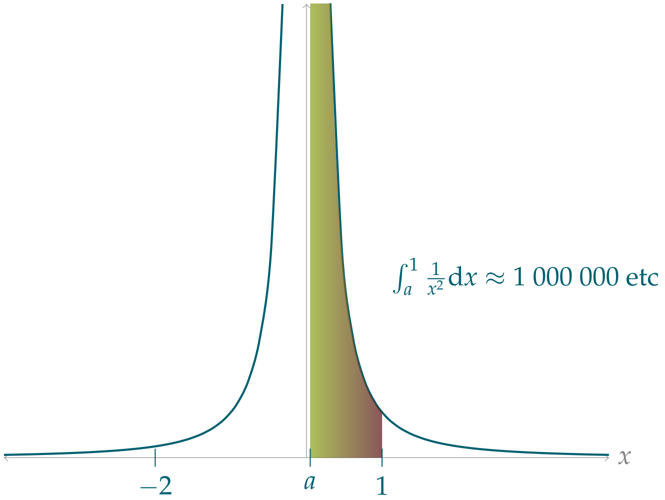
Evaluate $\int_{-2}^{1} \frac{1}{x^2}\,\mathrm{d}x$



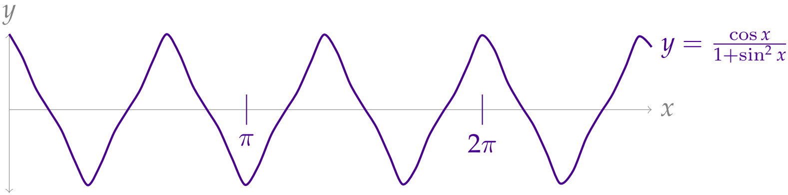
Evaluate $\int_{-2}^{1} \frac{1}{x^2}\, \mathrm{d}x$

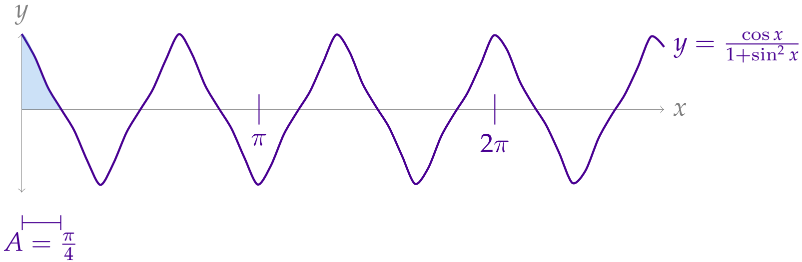


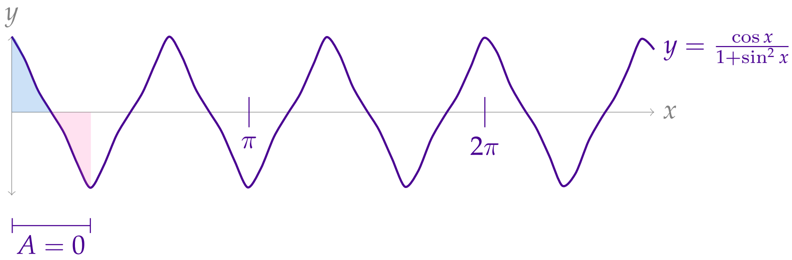
Evaluate $\int_{-2}^{1} \frac{1}{x^2}\,\mathrm{d}x$

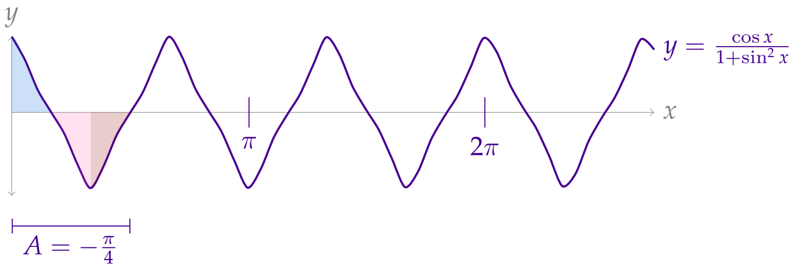


Evaluate $\displaystyle\int_0^\infty \frac{\cos x}{1+\sin^2 x}\,\mathrm{d}x$, or show that it diverges.

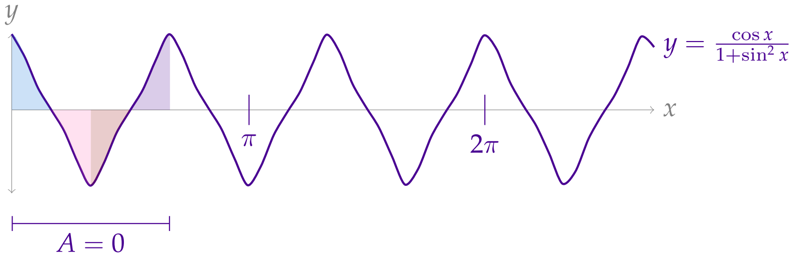

y
x
π
2π
$y = \frac{\cos x}{1+\sin^2 x}$


y
$y = \frac{\cos x}{1+\sin^2 x}$
x
π
2π
$A = \frac{\pi}{4}$

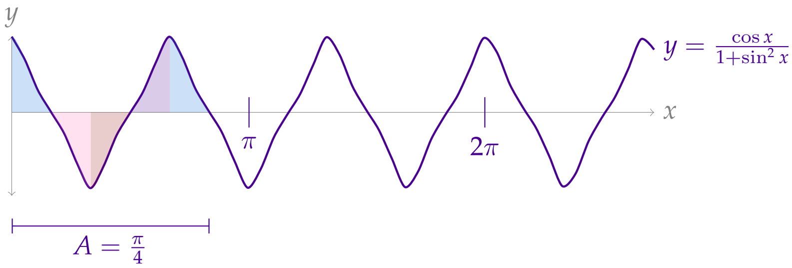

y
$y = \frac{\cos x}{1+\sin^2 x}$
x
π
2π
$A = 0$



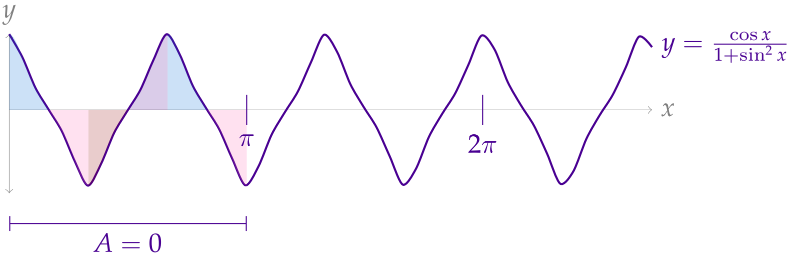
y
$y = \frac{\cos x}{1+\sin^2 x}$
x
π
2π
$A = -\frac{\pi}{4}$



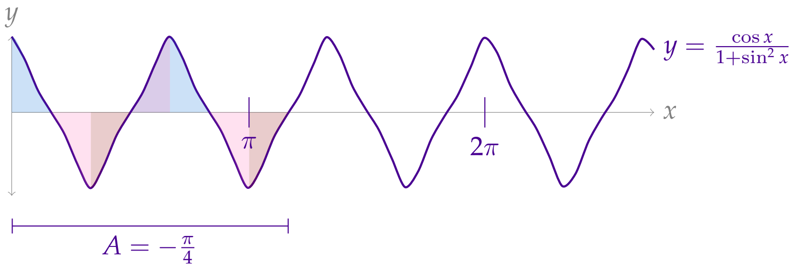
y
$y = \frac{\cos x}{1+\sin^2 x}$
x
π
2π
$A = 0$



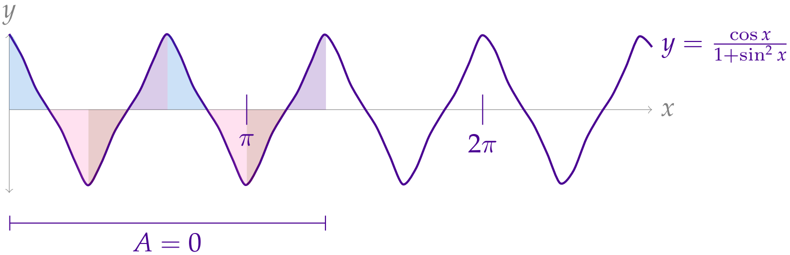
y
x
π
2π
$y = \frac{\cos x}{1+\sin^2 x}$
$A = \frac{\pi}{4}$

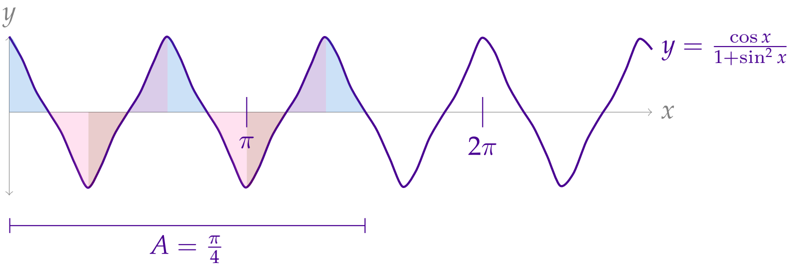


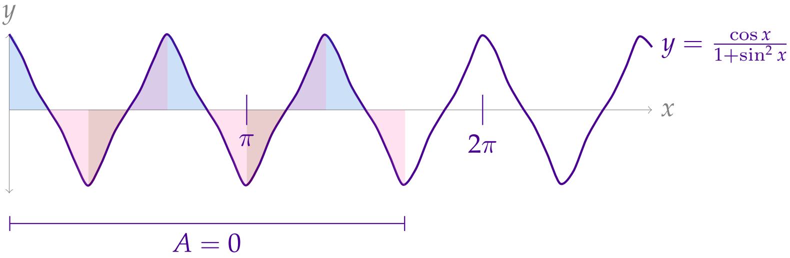
y
$y = \frac{\cos x}{1+\sin^2 x}$
x
π
2π
$A = 0$



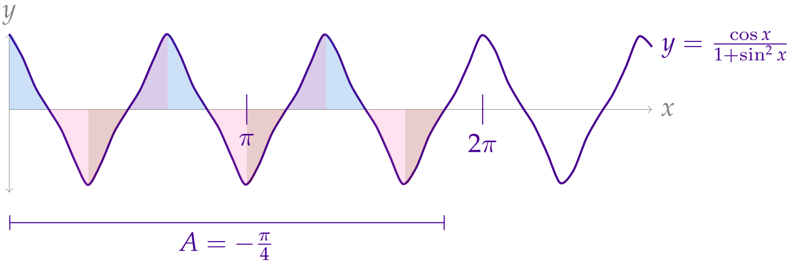
y
$y = \frac{\cos x}{1+\sin^2 x}$
x
π
2π
$A = -\frac{\pi}{4}$

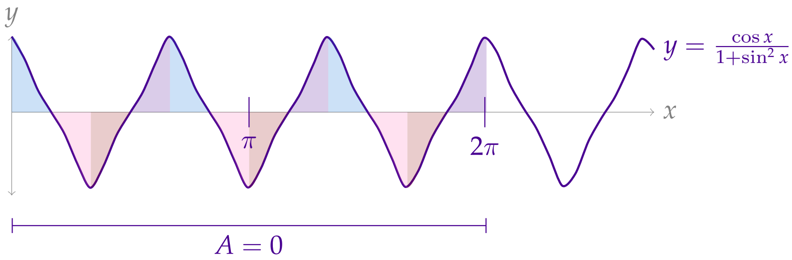

y
x
π
2π
$y = \frac{\cos x}{1+\sin^2 x}$
$A = 0$

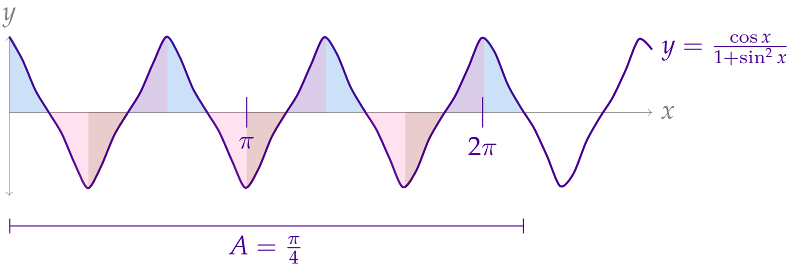

y
$y = \frac{\cos x}{1+\sin^2 x}$
x
π
2π
$A = \frac{\pi}{4}$



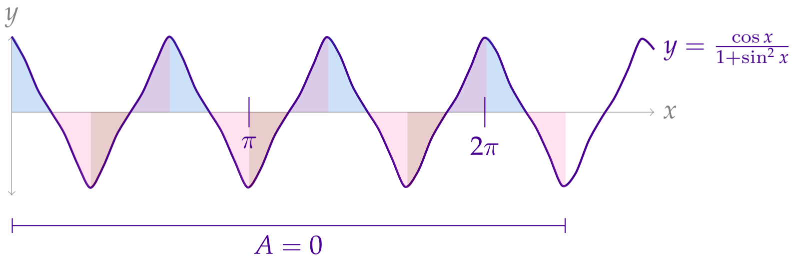
y
x
π
2π
$y = \frac{\cos x}{1+\sin^2 x}$
$A = 0$


y
x
$y = \frac{\cos x}{1+\sin^2 x}$
π
2π
$A = -\frac{\pi}{4}$

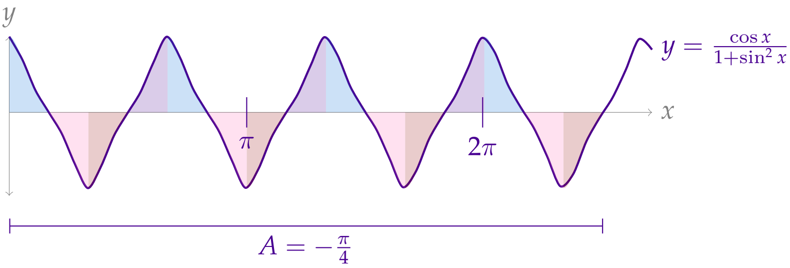

y
$y = \frac{\cos x}{1+\sin^2 x}$
x
π
2π
$A = 0$

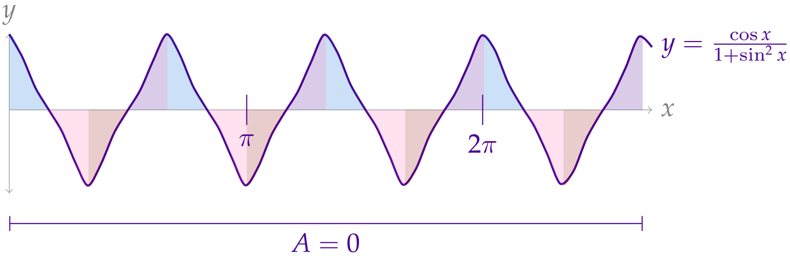


y
$y = \frac{\cos x}{1+\sin^2 x}$
x
π
2π
$A = \frac{\pi}{4}$



y
x
π
2π
$y = \frac{\cos x}{1+\sin^2 x}$
$A = 0$


y
$y = \frac{\cos x}{1+\sin^2 x}$
x
π
2π
$A = -\frac{\pi}{4}$

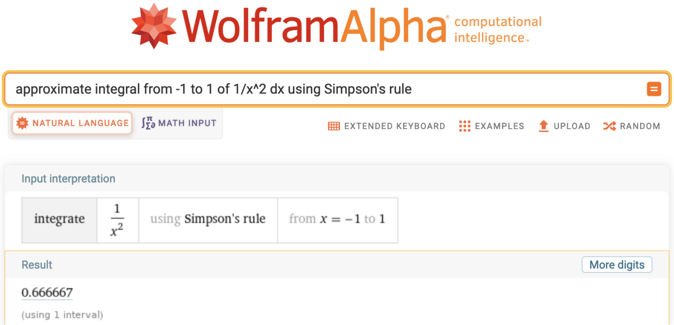

y
x
π
2π
$y = \frac{\cos x}{1+\sin^2 x}$
$A = 0$

Warning: Sneaky Divergence

If you don't realize that an integral diverges, you can generate answers that look plausible but are secretly nonsense.

For example, attempting to use the Fundamental Theorem of Calculus in the example $\displaystyle\int_{-2}^{1} \frac{1}{x^2}\mathrm{d}x$ gives $\displaystyle\left[-\frac{1}{x}\right]_{-2}^{1} = -\frac{3}{2}$: a poor approximation for positive infinity!

WARNING: SNEAKY DIVERGENCE



This mistake can be especially dangerous using computer algebra systems, where you spend less time thinking about the integral and so have fewer chances to notice that something is awry. As of this writing, WolframAlpha gives no warnings when you ask it to approximate $\int_{-1}^{1} \frac{1}{x^2}\,dx$ using Simpson's Rule: it tells you the approximation with one parabola is $\frac{2}{3}$.

Evaluate $\int_0^1 \frac{1}{x^p} \mathrm{d}x$ and $\int_1^\infty \frac{1}{x^p} \mathrm{d}x$ when p is constant.

p-test

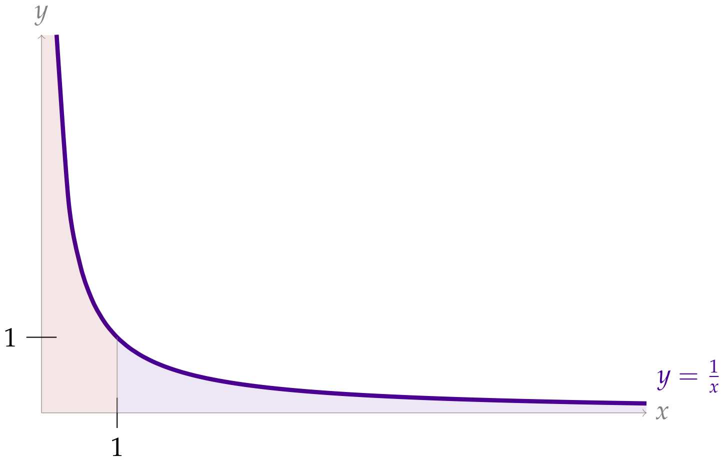
Let p be a constant.

If $p < 1$, then $\displaystyle\int_0^1 \frac{1}{x^p}\,\mathrm{d}x$ converges

If $p \geq 1$, then $\displaystyle\int_0^1 \frac{1}{x^p}\,\mathrm{d}x$ diverges

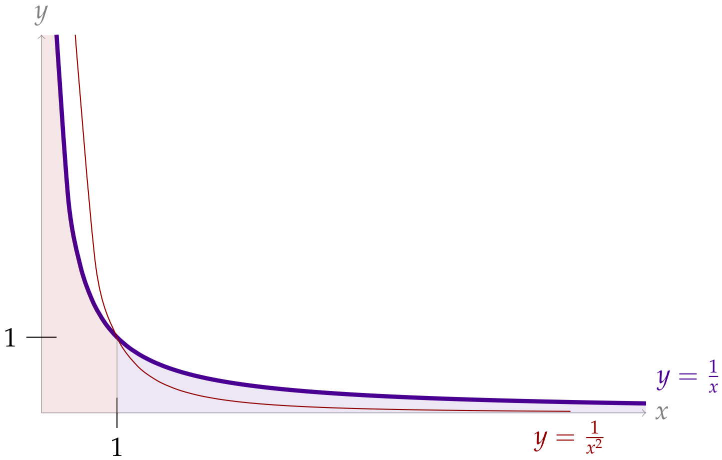
If $p > 1$, then $\displaystyle\int_1^\infty \frac{1}{x^p}\,\mathrm{d}x$ converges

If $p \leq 1$, then $\displaystyle\int_1^\infty \frac{1}{x^p}\,\mathrm{d}x$ diverges



$\int_0^1 \frac{1}{x}\,\mathrm{d}x$ diverges

$\int_1^\infty \frac{1}{x}\,\mathrm{d}x$ diverges

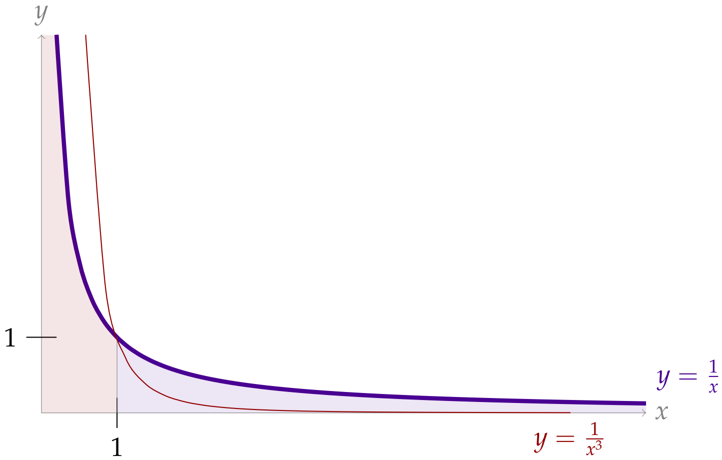


$\int_0^1 \frac{1}{x}\,\mathrm{d}x$ diverges

$\int_1^\infty \frac{1}{x}\,\mathrm{d}x$ diverges

$\int_0^1 \frac{1}{x^2}\,\mathrm{d}x$ diverges

$\int_0^1 \frac{1}{x^2}\,\mathrm{d}x$ converges

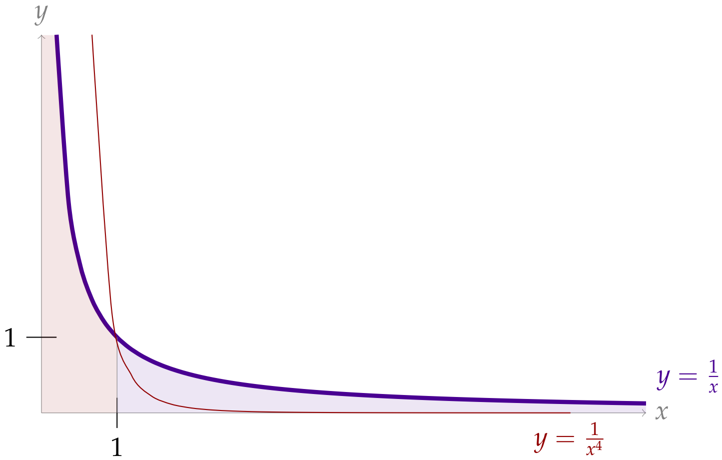


$\int_0^1 \frac{1}{x}\,\mathrm{d}x$ diverges

$\int_1^\infty \frac{1}{x}\,\mathrm{d}x$ diverges

$\int_0^1 \frac{1}{x^3}\,\mathrm{d}x$ diverges

$\int_0^1 \frac{1}{x^3}\,\mathrm{d}x$ converges

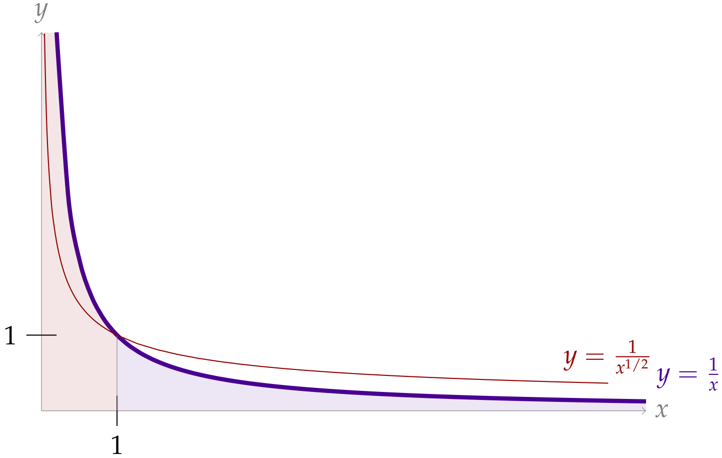


$\int_0^1 \frac{1}{x}\, \mathrm{d}x$ diverges

$\int_1^\infty \frac{1}{x}\, \mathrm{d}x$ diverges

$\int_0^1 \frac{1}{x^4}\, \mathrm{d}x$ diverges

$\int_0^1 \frac{1}{x^4}\, \mathrm{d}x$ converges

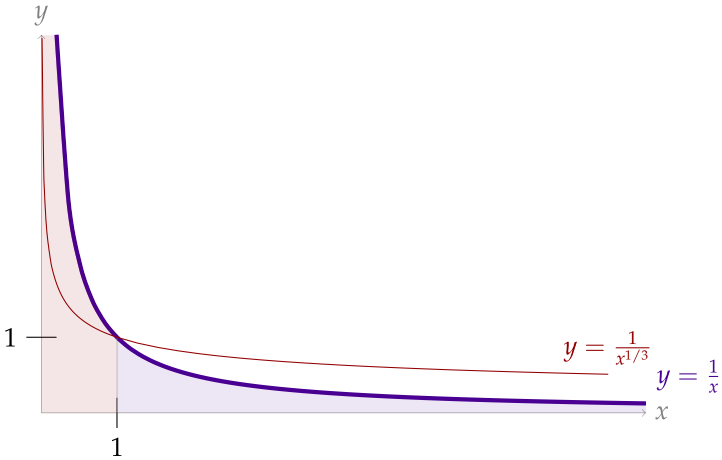


$\int_0^1 \frac{1}{x}\, \mathrm{d}x$ diverges

$\int_1^\infty \frac{1}{x}\, \mathrm{d}x$ diverges

$\int_0^1 \frac{1}{x^{1/2}}\, \mathrm{d}x$ converges

$\int_0^1 \frac{1}{x^{1/2}}\, \mathrm{d}x$ diverges

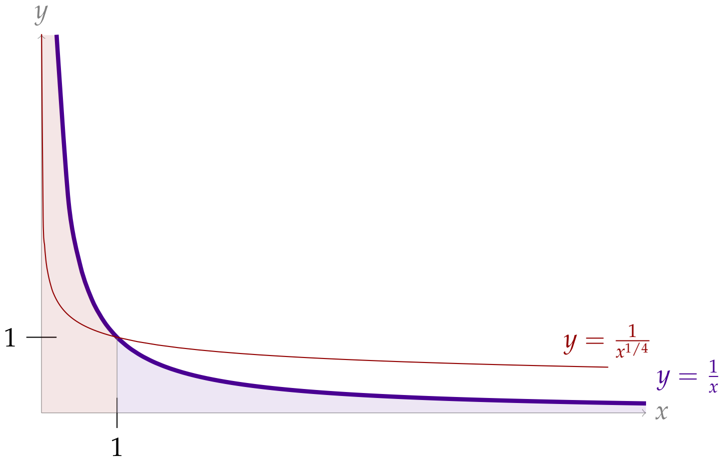


$\int_0^1 \frac{1}{x}\,\mathrm{d}x$ diverges

$\int_1^\infty \frac{1}{x}\,\mathrm{d}x$ diverges

$\int_0^1 \frac{1}{x^{1/3}}\,\mathrm{d}x$ converges

$\int_0^1 \frac{1}{x^{1/3}}\,\mathrm{d}x$ diverges



$\int_0^1 \frac{1}{x}\,\mathrm{d}x$ diverges

$\int_1^\infty \frac{1}{x}\,\mathrm{d}x$ diverges

$\int_0^1 \frac{1}{x^{1/4}}\,\mathrm{d}x$ converges

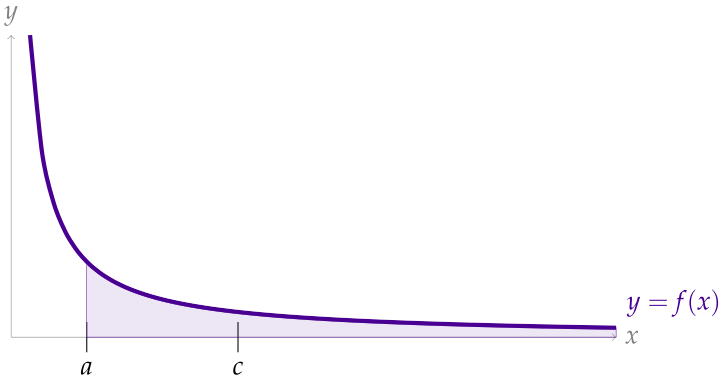
$\int_0^1 \frac{1}{x^{1/4}}\,\mathrm{d}x$ diverges

Decide whether each integral converges or diverges.

- $\displaystyle\int_0^1 \frac{1}{x^{1/3}}\,\mathrm{d}x$
- $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\,\mathrm{d}x$
- $\displaystyle\int_0^1 \frac{1}{x}\,\mathrm{d}x$
- $\displaystyle\int_0^1 \frac{1}{x^{1.5}}\,\mathrm{d}x$
- $\displaystyle\int_1^\infty \frac{1}{x^{1/3}}\,\mathrm{d}x$
- $\displaystyle\int_1^\infty \frac{1}{\sqrt{x}}\,\mathrm{d}x$
- $\displaystyle\int_1^\infty \frac{1}{x}\,\mathrm{d}x$
- $\displaystyle\int_1^\infty \frac{1}{x^{1.5}}\,\mathrm{d}x$

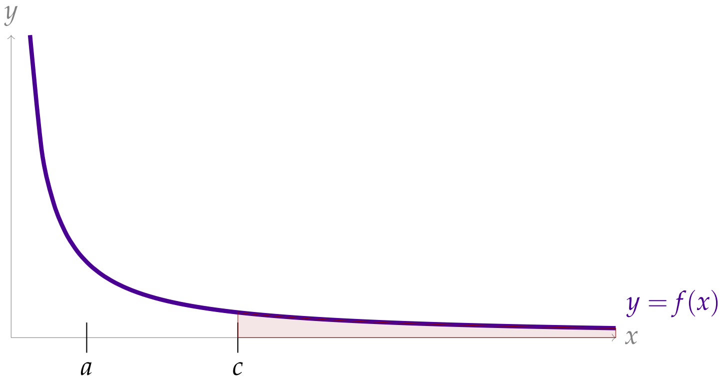
Theorem 1.12.20

Let a and c be real numbers with $a < c$ and let the function $f(x)$ be continuous for all $x \geq a$. Then the improper integral $\int_a^\infty f(x)\,\mathrm{d}x$ converges if and only if the improper integral $\int_c^\infty f(x)\,\mathrm{d}x$ converges.



Theorem 1.12.20

Let a and c be real numbers with $a < c$ and let the function $f(x)$ be continuous for all $x \geq a$. Then the improper integral $\int_a^\infty f(x)\,\mathrm{d}x$ converges if and only if the improper integral $\int_c^\infty f(x)\,\mathrm{d}x$ converges.



Decide whether each integral converges or diverges.

- $\int_0^9 \frac{1}{x^{0.3}}\,\mathrm{d}x$
- $\int_0^{81} \frac{1}{x^2}\,\mathrm{d}x$
- $\int_0^{\frac{1}{2}} \frac{1}{x^3}\,\mathrm{d}x$
- $\int_{15}^{\infty} \frac{1}{x^{0.3}}\,\mathrm{d}x$
- $\int_{0.4}^{\infty} \frac{1}{x^2}\,\mathrm{d}x$
- $\int_{\frac{1}{2}}^{\infty} \frac{1}{x^3}\,\mathrm{d}x$

It is very common to encounter integrals that are too complicated to evaluate explicitly. Numerical approximation schemes, evaluated by computer, are often used instead. You want to be sure that at least the integral converges before feeding it into a computer.

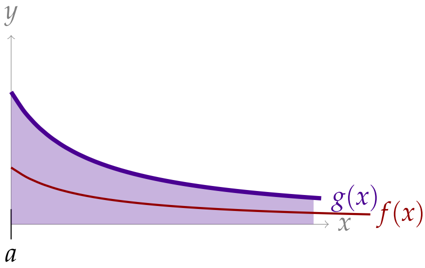
Fortunately it is usually possible to determine whether or not an improper integral converges even when you cannot evaluate it explicitly.

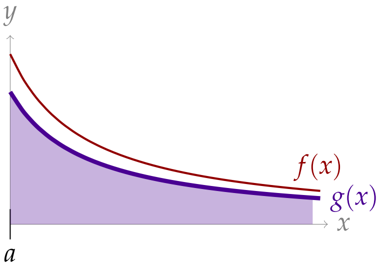
Comparison

Let a be a real number. Let f and g be functions that are defined and continuous for all $x \geq a$ and assume that $g(x) \geq 0$ for all $x \geq a$.

(a) If $|f(x)| \leq g(x)$ for all $x \geq a$ and if $\int_a^\infty g(x)\,\mathrm{d}x$ converges, then $\int_a^\infty f(x)\,\mathrm{d}x$ converges.

(b) If $f(x) \geq g(x)$ for all $x \geq a$ and if $\int_a^\infty g(x)\,\mathrm{d}x$ diverges, then $\int_a^\infty f(x)\,\mathrm{d}x$ diverges.



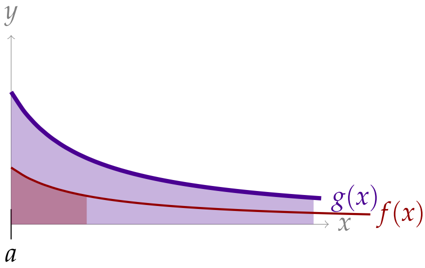


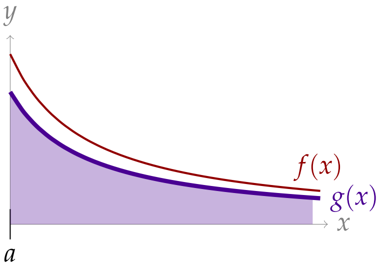
Comparison

Let a be a real number. Let f and g be functions that are defined and continuous for all $x \geq a$ and assume that $g(x) \geq 0$ for all $x \geq a$.

(a) If $|f(x)| \leq g(x)$ for all $x \geq a$ and if $\int_a^\infty g(x)\,\mathrm{d}x$ converges, then $\int_a^\infty f(x)\,\mathrm{d}x$ converges.

(b) If $f(x) \geq g(x)$ for all $x \geq a$ and if $\int_a^\infty g(x)\,\mathrm{d}x$ diverges, then $\int_a^\infty f(x)\,\mathrm{d}x$ diverges.



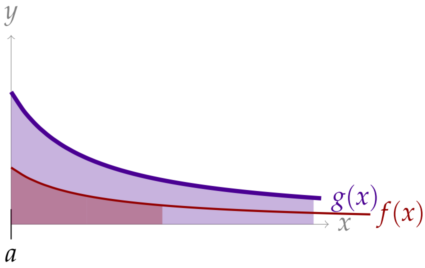


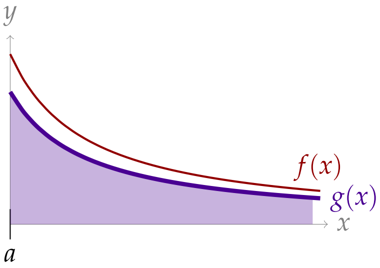
Comparison

Let a be a real number. Let f and g be functions that are defined and continuous for all $x \geq a$ and assume that $g(x) \geq 0$ for all $x \geq a$.

(a) If $|f(x)| \leq g(x)$ for all $x \geq a$ and if $\int_a^\infty g(x)\,\mathrm{d}x$ converges, then $\int_a^\infty f(x)\,\mathrm{d}x$ converges.

(b) If $f(x) \geq g(x)$ for all $x \geq a$ and if $\int_a^\infty g(x)\,\mathrm{d}x$ diverges, then $\int_a^\infty f(x)\,\mathrm{d}x$ diverges.



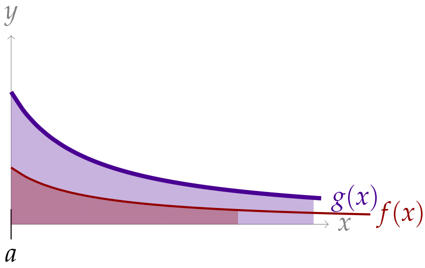


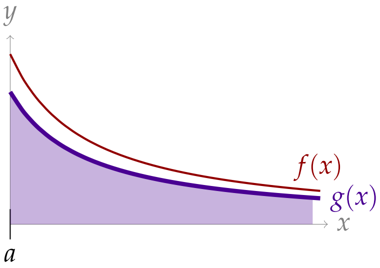
Comparison

Let a be a real number. Let f and g be functions that are defined and continuous for all $x \geq a$ and assume that $g(x) \geq 0$ for all $x \geq a$.

(a) If $|f(x)| \leq g(x)$ for all $x \geq a$ and if $\int_a^\infty g(x)\,\mathrm{d}x$ converges, then $\int_a^\infty f(x)\,\mathrm{d}x$ converges.

(b) If $f(x) \geq g(x)$ for all $x \geq a$ and if $\int_a^\infty g(x)\,\mathrm{d}x$ diverges, then $\int_a^\infty f(x)\,\mathrm{d}x$ diverges.



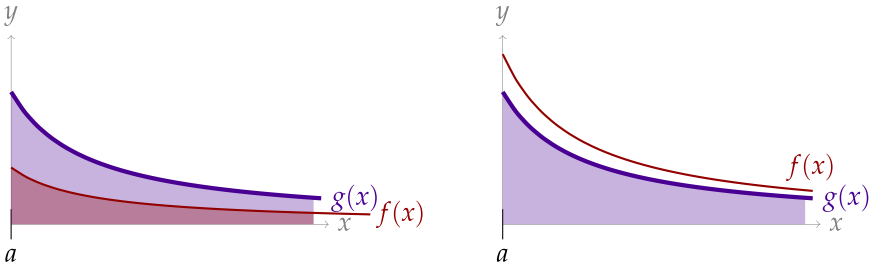


Comparison

Let a be a real number. Let f and g be functions that are defined and continuous for all $x \geq a$ and assume that $g(x) \geq 0$ for all $x \geq a$.

(a) If $|f(x)| \leq g(x)$ for all $x \geq a$ and if $\int_a^\infty g(x)\,\mathrm{d}x$ converges, then $\int_a^\infty f(x)\,\mathrm{d}x$ converges.

(b) If $f(x) \geq g(x)$ for all $x \geq a$ and if $\int_a^\infty g(x)\,\mathrm{d}x$ diverges, then $\int_a^\infty f(x)\,\mathrm{d}x$ diverges.

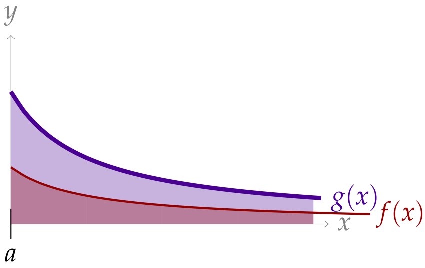


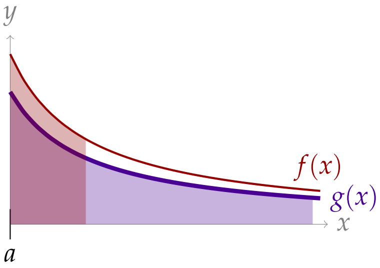
Comparison

Let a be a real number. Let f and g be functions that are defined and continuous for all $x \geq a$ and assume that $g(x) \geq 0$ for all $x \geq a$.

(a) If $|f(x)| \leq g(x)$ for all $x \geq a$ and if $\int_a^\infty g(x)\,\mathrm{d}x$ converges, then $\int_a^\infty f(x)\,\mathrm{d}x$ converges.

(b) If $f(x) \geq g(x)$ for all $x \geq a$ and if $\int_a^\infty g(x)\,\mathrm{d}x$ diverges, then $\int_a^\infty f(x)\,\mathrm{d}x$ diverges.



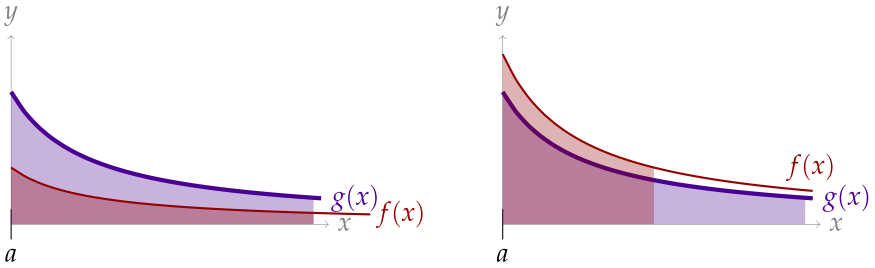


Comparison

Let a be a real number. Let f and g be functions that are defined and continuous for all $x \geq a$ and assume that $g(x) \geq 0$ for all $x \geq a$.

(a) If $|f(x)| \leq g(x)$ for all $x \geq a$ and if $\int_a^\infty g(x)\,\mathrm{d}x$ converges, then $\int_a^\infty f(x)\,\mathrm{d}x$ converges.

(b) If $f(x) \geq g(x)$ for all $x \geq a$ and if $\int_a^\infty g(x)\,\mathrm{d}x$ diverges, then $\int_a^\infty f(x)\,\mathrm{d}x$ diverges.

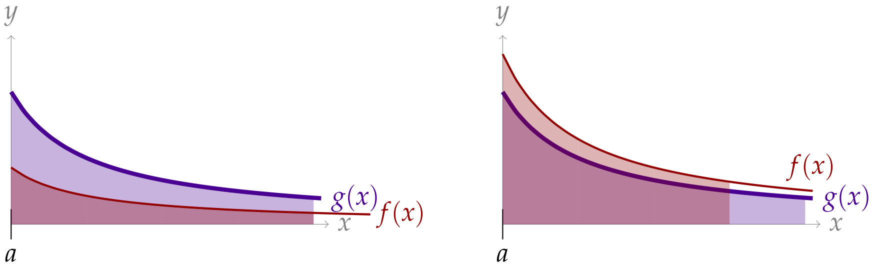


Comparison

Let a be a real number. Let f and g be functions that are defined and continuous for all $x \geq a$ and assume that $g(x) \geq 0$ for all $x \geq a$.

(a) If $|f(x)| \leq g(x)$ for all $x \geq a$ and if $\int_a^\infty g(x)\, \mathrm{d}x$ converges, then $\int_a^\infty f(x)\, \mathrm{d}x$ converges.

(b) If $f(x) \geq g(x)$ for all $x \geq a$ and if $\int_a^\infty g(x)\, \mathrm{d}x$ diverges, then $\int_a^\infty f(x)\, \mathrm{d}x$ diverges.

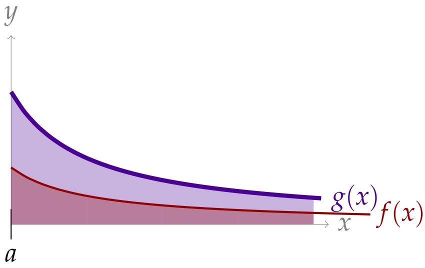


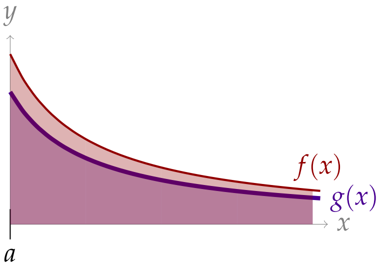
Comparison

Let a be a real number. Let f and g be functions that are defined and continuous for all $x \geq a$ and assume that $g(x) \geq 0$ for all $x \geq a$.

(a) If $|f(x)| \leq g(x)$ for all $x \geq a$ and if $\int_a^\infty g(x)\,\mathrm{d}x$ converges, then $\int_a^\infty f(x)\,\mathrm{d}x$ converges.

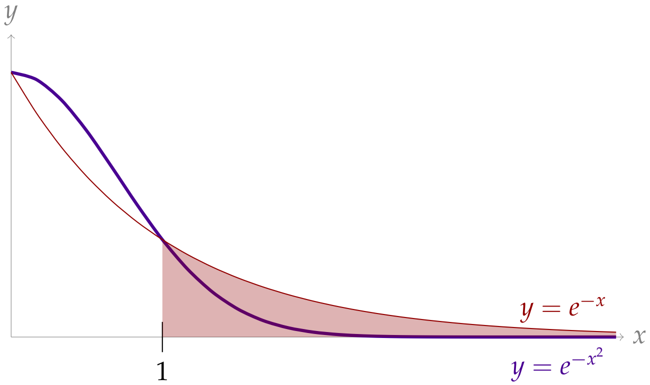
(b) If $f(x) \geq g(x)$ for all $x \geq a$ and if $\int_a^\infty g(x)\,\mathrm{d}x$ diverges, then $\int_a^\infty f(x)\,\mathrm{d}x$ diverges.



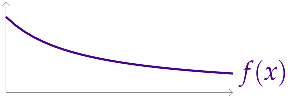
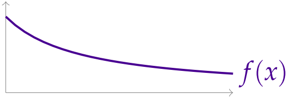
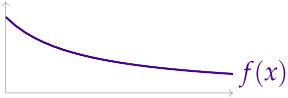
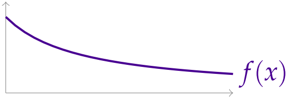


Does the integral $\int_1^\infty e^{-x^2}$ converge or diverge?

Does the integral $\int_1^{\infty} e^{-x^2}$ converge or diverge?



Let functions $f(x)$ and $g(x)$ be positive and continuous for all $x \geq a$.

	$\int_a^\infty g(x)\,\mathrm{d}x$ converges	$\int_a^\infty g(x)\,\mathrm{d}x$ diverges
$f(x) \leq g(x)$ for all $x \geq a$		
$f(x) \geq g(x)$ for all $x \geq a$		

For each example below, decide whether the statement is a valid use of the comparison theorem.

- $\displaystyle\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x$ converges and $0 \le \frac{1}{x^2} \le \frac{2+\sin x}{x^2}$ for $x \ge 1$. So by the comparison test, $\displaystyle\int_1^\infty \frac{2+\sin x}{x^2}\,\mathrm{d}x$ converges as well.

- $\displaystyle\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x$ converges and $0 \le \frac{e^{-x}}{x^2} \le \frac{1}{x^2}$ for $x \ge 1$. So by the comparison test, $\displaystyle\int_1^\infty \frac{e^{-x}}{x^2}\,\mathrm{d}x$ converges as well.

- $\displaystyle\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x$ converges and $-\frac{1}{x} \le \frac{1}{x^2}$ for $x \ge 1$. So by the comparison test, $\displaystyle\int_1^\infty \frac{-1}{x}\,\mathrm{d}x$ converges as well.

For each example below, decide whether the statement is a valid use of the comparison theorem.

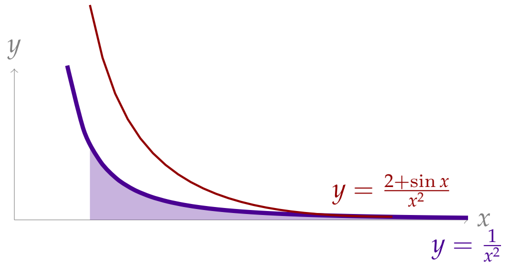
- $\displaystyle\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x$ converges and $0 \leq \frac{1}{x^2} \leq \frac{2+\sin x}{x^2}$ for $x \geq 1$. So by the comparison test, $\displaystyle\int_1^\infty \frac{2+\sin x}{x^2}\,\mathrm{d}x$ converges as well.

Q

ans

For each example below, decide whether the statement is a valid use of the comparison theorem.

- $\displaystyle\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x$ converges and $0 \le \frac{1}{x^2} \le \frac{2+\sin x}{x^2}$ for $x \ge 1$. So by the comparison test, $\displaystyle\int_1^\infty \frac{2+\sin x}{x^2}\,\mathrm{d}x$ converges as well.



invalid

For each example below, decide whether the statement is a valid use of the comparison theorem.

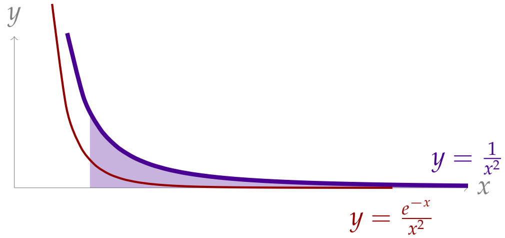
- $\displaystyle\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x$ converges and $0 \le \frac{e^{-x}}{x^2} \le \frac{1}{x^2}$ for $x \ge 1$. So by the comparison test, $\displaystyle\int_1^\infty \frac{e^{-x}}{x^2}\,\mathrm{d}x$ converges as well.

Q

ans

For each example below, decide whether the statement is a valid use of the comparison theorem.

- $\displaystyle\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x$ converges and $0 \le \frac{e^{-x}}{x^2} \le \frac{1}{x^2}$ for $x \ge 1$. So by the comparison test, $\displaystyle\int_1^\infty \frac{e^{-x}}{x^2}\,\mathrm{d}x$ converges as well.



valid

For each example below, decide whether the statement is a valid use of the comparison theorem.

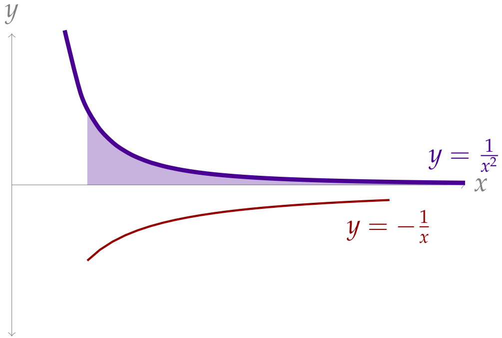
- $\displaystyle\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x$ converges and $-\frac{1}{x} \leq \frac{1}{x^2}$ for $x \geq 1$. So by the comparison test, $\displaystyle\int_1^\infty \frac{-1}{x}\,\mathrm{d}x$ converges as well.

Q

ans

For each example below, decide whether the statement is a valid use of the comparison theorem.

- $\displaystyle\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x$ converges and $-\frac{1}{x} \leq \frac{1}{x^2}$ for $x \geq 1$. So by the comparison test, $\displaystyle\int_1^\infty \frac{-1}{x}\,\mathrm{d}x$ converges as well.



invalid

Limiting comparison

Let $-\infty < a < \infty$. Let f and g be functions that are defined and continuous for all $x \geq a$ and assume that $g(x) \geq 0$ for all $x \geq a$. If the limit

$$\lim_{x \to \infty} \frac{f(x)}{g(x)}$$

exists and is nonzero, then either $\int_a^\infty f(x)\,\mathrm{d}x$ and $\int_a^\infty g(x)\,\mathrm{d}x$ both converge, or they both diverge.

Limiting comparison

Let $-\infty < a < \infty$. Let f and g be functions that are defined and continuous for all $x \geq a$ and assume that $g(x) \geq 0$ for all $x \geq a$. If the limit

$$\lim_{x\to\infty} \frac{f(x)}{g(x)}$$

exists and is nonzero, then either $\int_a^\infty f(x)\,\mathrm{d}x$ and $\int_a^\infty g(x)\,\mathrm{d}x$ both converge, or they both diverge.

Use limiting comparison to determine whether $\displaystyle\int_1^\infty \frac{1}{x+10}\,\mathrm{d}x$ converges or diverges.

Use limiting comparison to determine whether $\displaystyle\int_1^\infty \frac{1}{x+10}\,\mathrm{d}x$ converges or diverges.

Included Work

 ‘Notebook’ by Iconic is licensed under CC BY 3.0 (accessed 9 June 2021), 26, 95, 103

 WolframAlpha (accessed 25 August 2021) , 68