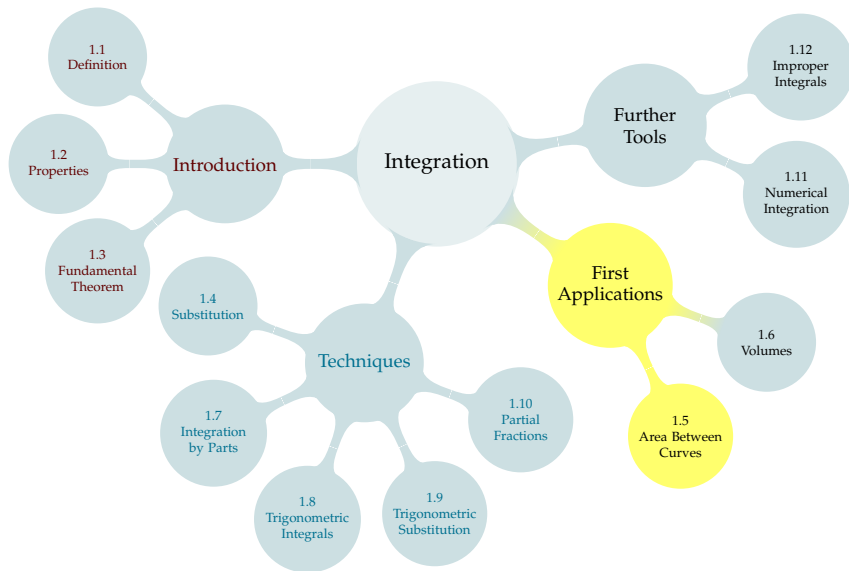
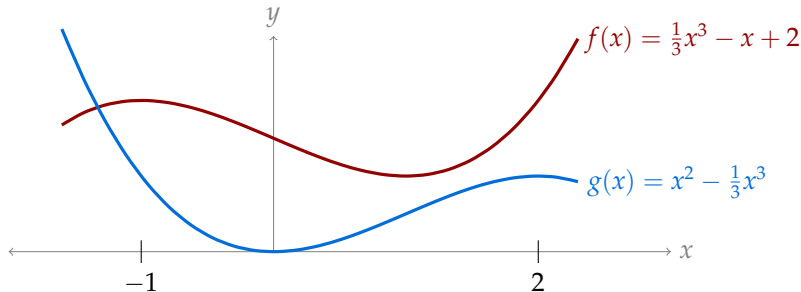


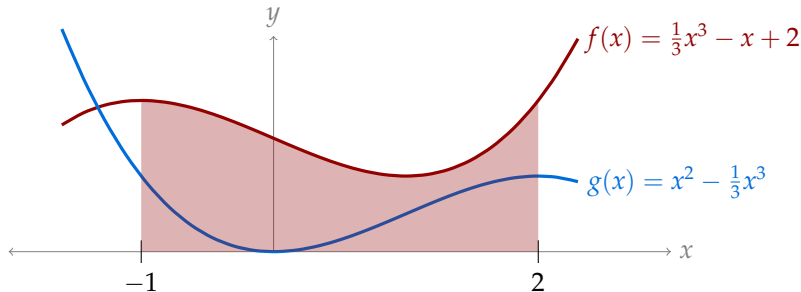
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Find the area between  $f(x)$  and  $g(x)$  from  $x = -1$  to  $x = 2$ .

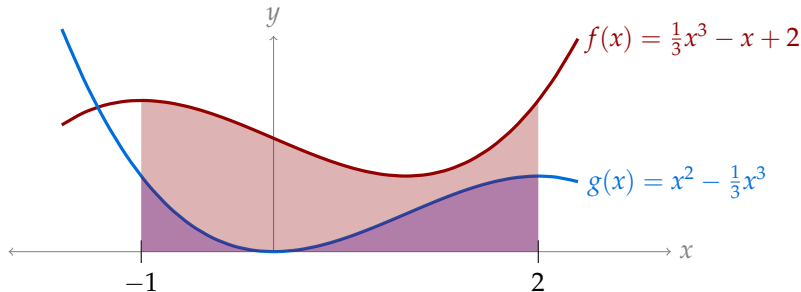


Find the area between  $f(x)$  and  $g(x)$  from  $x = -1$  to  $x = 2$ .



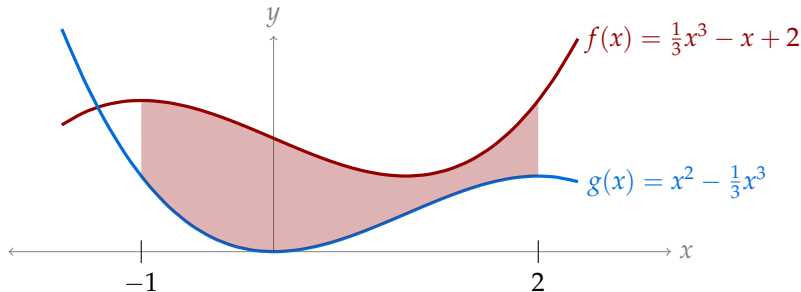
$$\int_{-1}^2 f(x) \, dx$$

Find the area between  $f(x)$  and  $g(x)$  from  $x = -1$  to  $x = 2$ .



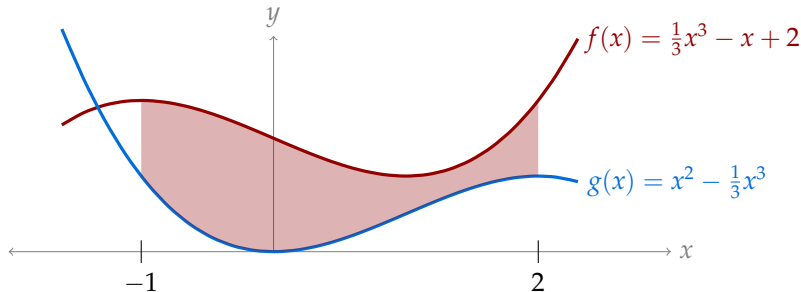
$$\int_{-1}^2 f(x) \, dx \quad \int_{-1}^2 g(x) \, dx$$

Find the area between  $f(x)$  and  $g(x)$  from  $x = -1$  to  $x = 2$ .



$$\int_{-1}^2 f(x) \, dx - \int_{-1}^2 g(x) \, dx$$

Find the area between  $f(x)$  and  $g(x)$  from  $x = -1$  to  $x = 2$ .



$$\int_{-1}^2 f(x) \, dx - \int_{-1}^2 g(x) \, dx = \int_{-1}^2 [f(x) - g(x)] \, dx$$

Find the area between  $f(x)$  and  $g(x)$  from  $x = -1$  to  $x = 2$ .

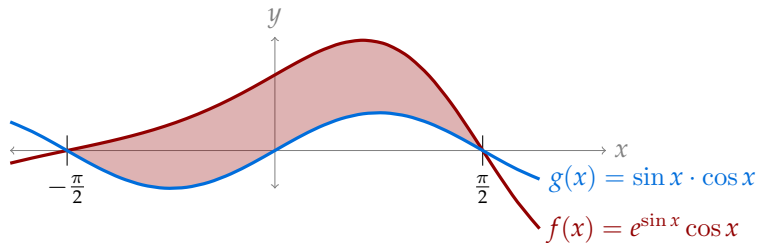
$$\begin{aligned} \int_{-1}^2 f(x) \, dx - \int_{-1}^2 g(x) \, dx &= \int_{-1}^2 [f(x) - g(x)] \, dx \\ &= \int_{-1}^2 \left[ \frac{1}{3}x^3 - x + 2 - x^2 + \frac{1}{3}x^3 \right] \, dx \end{aligned}$$

Find the area between  $f(x)$  and  $g(x)$  from  $x = -1$  to  $x = 2$ .

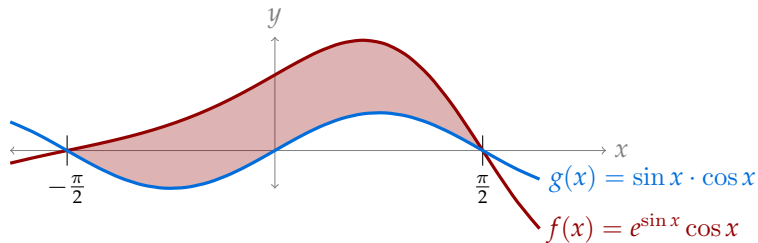
$$\begin{aligned} \int_{-1}^2 f(x) \, dx - \int_{-1}^2 g(x) \, dx &= \int_{-1}^2 [f(x) - g(x)] \, dx \\ &= \int_{-1}^2 \left[ \frac{1}{3}x^3 - x + 2 - x^2 + \frac{1}{3}x^3 \right] \, dx \\ &= \int_{-1}^2 \left[ \frac{2}{3}x^3 - x^2 - x + 2 \right] \, dx \\ &= \left[ \frac{1}{6}x^4 - \frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x \right]_{-1}^2 \\ &= \frac{16}{6} - \frac{8}{3} - \frac{4}{2} + 4 - \left( \frac{1}{6} + \frac{1}{3} - \frac{1}{2} - 2 \right) \\ &= 2 - (-2) = 4 \end{aligned}$$



Find the (unsigned) area between  $f(x)$  and  $g(x)$  from  $x = -\frac{\pi}{2}$  to  $x = \frac{\pi}{2}$ .

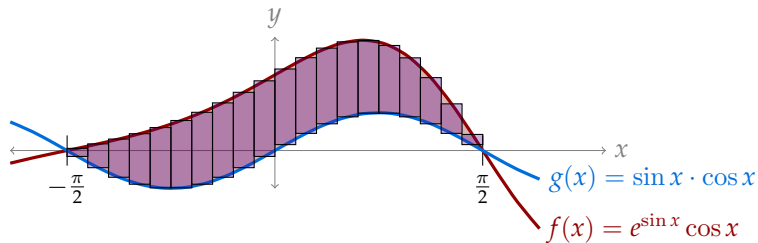


Find the (unsigned) area between  $f(x)$  and  $g(x)$  from  $x = -\frac{\pi}{2}$  to  $x = \frac{\pi}{2}$ .



$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [f(x) - g(x)] dx$$

Find the (unsigned) area between  $f(x)$  and  $g(x)$  from  $x = -\frac{\pi}{2}$  to  $x = \frac{\pi}{2}$ .



$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [f(x) - g(x)] dx$$

Find the (unsigned) area between  $f(x)$  and  $g(x)$  from  $x = -\frac{\pi}{2}$  to  $x = \frac{\pi}{2}$ .

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [f(x) - g(x)] dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (e^{\sin x} \cos x - \sin x \cos x) dx$$

Find the (unsigned) area between  $f(x)$  and  $g(x)$  from  $x = -\frac{\pi}{2}$  to  $x = \frac{\pi}{2}$ .

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [f(x) - g(x)] dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (e^{\sin x} \cos x - \sin x \cos x) dx$$

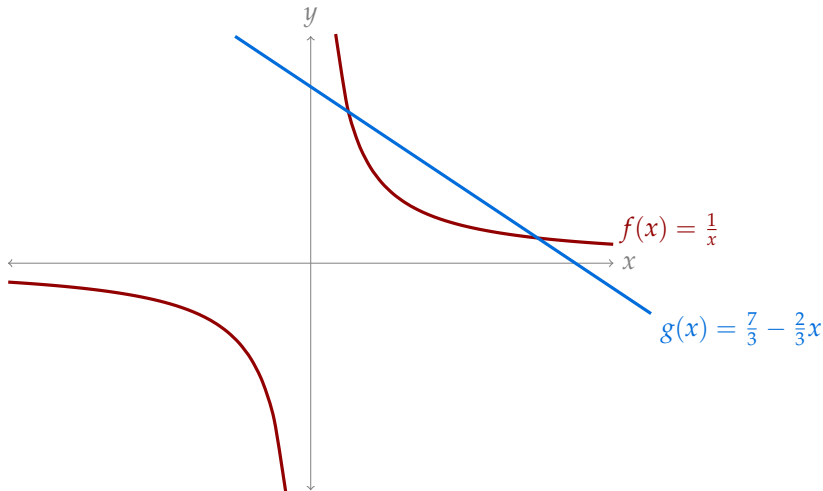
Let  $u = \sin x$ .

Then:  $du = \cos x dx$ ,  $u\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$ ,  $u\left(-\frac{\pi}{2}\right) = \sin\left(-\frac{\pi}{2}\right) = -1$ .

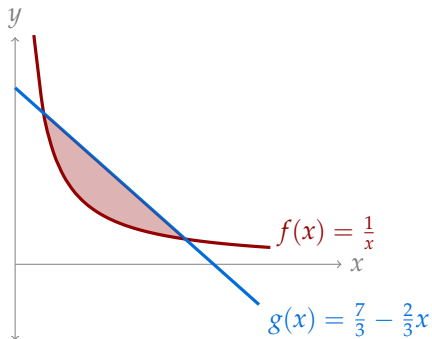
$$\begin{aligned} &= \int_{-1}^1 (e^u - u) du \\ &= \left[ e^u - \frac{1}{2}u^2 \right]_{-1}^1 \\ &= e - \frac{1}{e} \end{aligned}$$



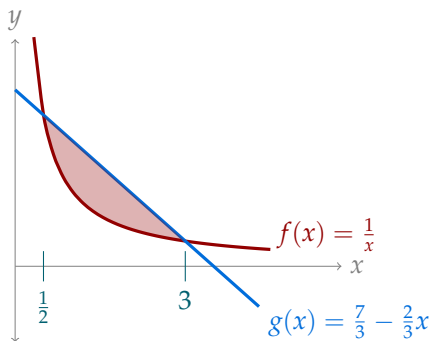
Find the (unsigned) area of the finite region bounded by  $f(x)$  and  $g(x)$ .



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Find the (unsigned) area of the finite region bounded by  $f(x)$  and  $g(x)$ .



$$\frac{1}{x} = \frac{7 - 2x}{3}$$

$$3 = 7x - 2x^2$$

$$0 = 2x^2 - 7x + 3$$

$$x = \frac{7 \pm \sqrt{7^2 - 4(2)(3)}}{4}$$

$$= \frac{7 \pm \sqrt{25}}{4}$$

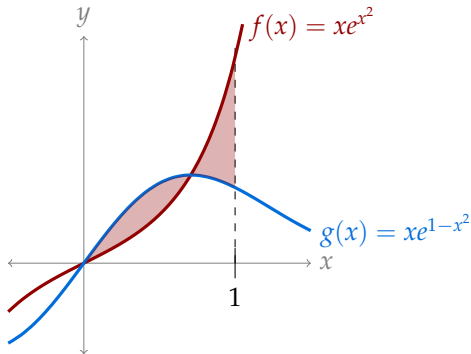
$$x = \frac{1}{2} \text{ and } x = 3$$

$$\begin{aligned} \int_{\frac{1}{2}}^3 [g(x) - f(x)] dx &= \int_{\frac{1}{2}}^3 \left[ \frac{7}{3} - \frac{2}{3}x - \frac{1}{x} \right] dx = \left[ \frac{7}{3}x - \frac{1}{3}x^2 - \log x \right]_{\frac{1}{2}}^3 \\ &= (7 - 3 - \log 3) - \left( \frac{7}{6} - \frac{1}{12} - \log \frac{1}{2} \right) = \frac{35}{12} - \log 6 \end{aligned}$$

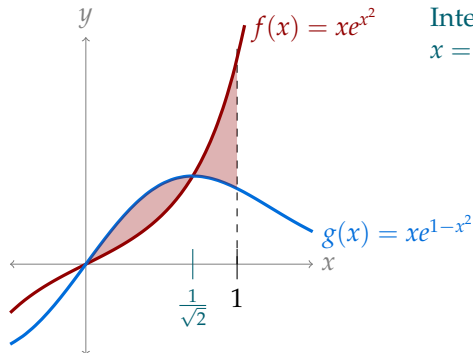




Find the (unsigned) area in the figure below between the curves  $f(x)$  and  $g(x)$  from  $x = 0$  to  $x = 1$ .



Find the (unsigned) area in the figure below between the curves  $f(x)$  and  $g(x)$  from  $x = 0$  to  $x = 1$ .



Intersections at  $x = 0$  and  $x = \pm \frac{1}{\sqrt{2}}$ :

$$xe^{x^2} = xe^{1-x^2}$$

$$e^{x^2} = e^{1-x^2} \text{ or } x = 0$$

$$x^2 = 1 - x^2$$

$$x = \pm \frac{1}{\sqrt{2}}$$

$$\text{Area} = \int_0^{\frac{1}{\sqrt{2}}} [g(x) - f(x)] dx + \int_{\frac{1}{\sqrt{2}}}^1 [f(x) - g(x)] dx$$

Find the (unsigned) area in the figure below between the curves  $f(x)$  and  $g(x)$  from  $x = 0$  to  $x = 1$ .

$$\begin{aligned}\text{Area} &= \int_0^{\frac{1}{\sqrt{2}}} [g(x) - f(x)] dx + \int_{\frac{1}{\sqrt{2}}}^1 [f(x) - g(x)] dx \\ &= \int_0^{\frac{1}{\sqrt{2}}} [xe^{1-x^2} - xe^{x^2}] dx + \int_{\frac{1}{\sqrt{2}}}^1 [xe^{x^2} - xe^{1-x^2}] dx\end{aligned}$$

Find the (unsigned) area in the figure below between the curves  $f(x)$  and  $g(x)$  from  $x = 0$  to  $x = 1$ .

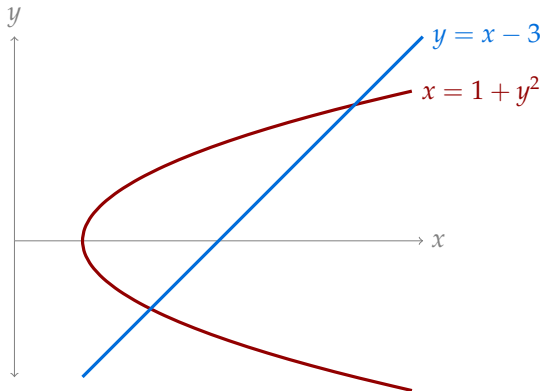
$$\begin{aligned}\text{Area} &= \int_0^{\frac{1}{\sqrt{2}}} [g(x) - f(x)] dx + \int_{\frac{1}{\sqrt{2}}}^1 [f(x) - g(x)] dx \\ &= \int_0^{\frac{1}{\sqrt{2}}} [xe^{1-x^2} - xe^{x^2}] dx + \int_{\frac{1}{\sqrt{2}}}^1 [xe^{x^2} - xe^{1-x^2}] dx\end{aligned}$$

$$\text{Aside: } \underbrace{\int xe^{1-x^2} dx}_{u=1-x^2, du=-2x dx} = -\frac{1}{2}e^{1-x^2} + C \quad \underbrace{\int xe^{x^2} dx}_{u=x^2, du=2x dx} = \frac{1}{2}e^{x^2} + C$$

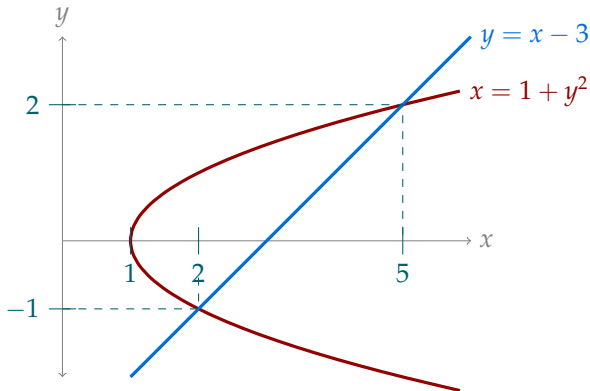
$$\begin{aligned}\text{Area} &= \left[ -\frac{1}{2}e^{1-x^2} - \frac{1}{2}e^{x^2} \right]_0^{\frac{1}{\sqrt{2}}} + \left[ \frac{1}{2}e^{x^2} - \left( -\frac{1}{2}e^{1-x^2} \right) \right]_{\frac{1}{\sqrt{2}}}^1 \\ &= -\frac{1}{2} \left[ \left( e^{\frac{1}{2}} + e^{\frac{1}{2}} \right) - (e^1 + e^0) \right] + \frac{1}{2} \left[ (e^1 + e^0) - \left( e^{\frac{1}{2}} + e^{\frac{1}{2}} \right) \right] \\ &= e - 2\sqrt{e} + 1\end{aligned}$$



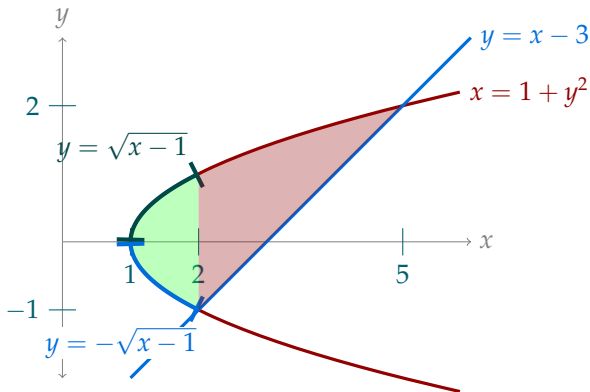
Set up, but do not evaluate, integral(s) to find the (unsigned) area of the finite region bounded by  $x = 1 + y^2$  and  $y = x - 3$ .



Set up, but do not evaluate, integral(s) to find the (unsigned) area of the finite region bounded by  $x = 1 + y^2$  and  $y = x - 3$ .



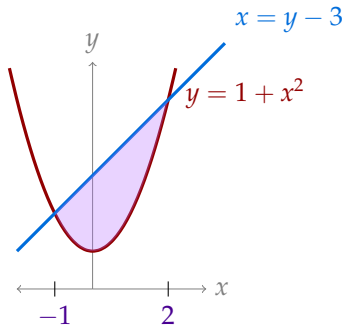
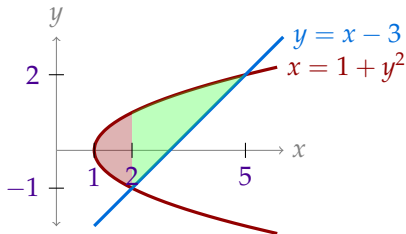
Set up, but do not evaluate, integral(s) to find the (unsigned) area of the finite region bounded by  $x = 1 + y^2$  and  $y = x - 3$ .



Option 1: 
$$\int_1^2 \left[ \sqrt{x-1} - \left( -\sqrt{x-1} \right) \right] dx + \int_2^5 \left[ \sqrt{x-1} - (x-3) \right] dx$$

Set up, but do not evaluate, integral(s) to find the (unsigned) area of the finite region bounded by  $x = 1 + y^2$  and  $y = x - 3$ .

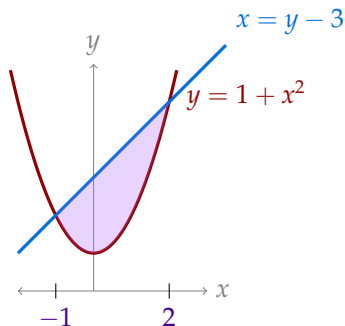
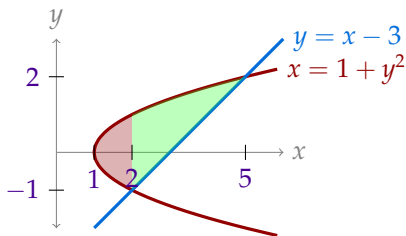
Option 2: Swapping  $x$  and  $y$  results in a figure with the same area.





Set up, but do not evaluate, integral(s) to find the (unsigned) area of the finite region bounded by  $x = 1 + y^2$  and  $y = x - 3$ .

Option 2: Swapping  $x$  and  $y$  results in a figure with the same area.



$$\int_{-1}^2 [(x + 3) - (1 + x^2)] \, dx$$

## Included Work



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