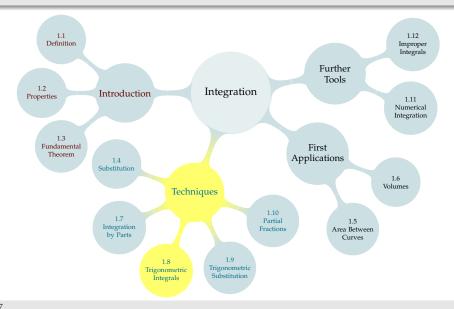
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Evaluating $\int \tan^m x \sec^n dx$

To evaluate $\int \tan^m x \sec^n dx$, we can use:

- ▶ $u = \sec x$ if m is odd and $n \ge 1$
- ▶ $u = \tan x$ if n is even and $n \ge 2$
- $ightharpoonup u = \cos x \text{ if } m \text{ is odd}$
- ► $u = \tan x$ if m is even and n = 0 (after using $\tan^2 x = \sec^2 x 1$, maybe several times)

Remaining case: n odd and m is even.

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Remaining case: n odd and m is even.

The general remaining case is known, but complicated. Instead of treating it exhaustively, we'll show examples of two methods.

$$\int \sec x \, dx$$

We saw a way of integrating secant with the following trick:

$$\int \sec x \, dx = \int \sec x \left(\frac{\sec x + \tan x}{\sec x + \tan x} \right) dx = \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$$
$$= \int \frac{1}{u} du \quad \text{with } u = \sec x + \tan x$$

Another trick: this time let $u = \sin x$, $du = \cos x dx$:

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Another trick: this time let $u = \sin x$, $du = \cos x dx$:

$$\int \sec x \, dx = \int \frac{1}{\cos x} dx = \int \frac{\cos x}{\cos^2 x} dx$$
$$= \int \frac{1}{1 - \sin^2 x} \cos x \, dx = \int \frac{1}{1 - u^2} du$$

The integrand $\frac{1}{1-u^2}$ is a rational function of u (i.e. a ratio of two polynomials). There is a procedure, called Partial Fractions, that can be used to evaluate all integrals of rational functions. We will learn it in Section 1.10.

 $\int \sec^3 x \, dx$

We can integrate around in a circle (with integration by parts) to evaluate $\int \sec^3 x \, dx$. Let $u = \sec x$, $dv = \sec^2 x \, dx$. Then $du = \sec x \tan x \, dx$ and $v = \tan x$.

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$$\int \sec^3 x \, dx = \sec x \tan x - \int \sec x \tan^2 x \, dx$$

$$= \sec x \tan x - \int \sec x \left(\sec^2 x - 1\right) dx$$

$$= \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$$

$$= \sec x \tan x - \int \sec^3 x \, dx + \log|\sec x + \tan x| + C'$$

$$2 \int \sec^3 x \, dx = \sec x \tan x + \log|\sec x + \tan x| + C'$$

$$\int \sec^3 x \, dx = \frac{1}{2} \left(\sec x \tan x + \log|\sec x + \tan x|\right) + C$$
with $C = C'/2$.