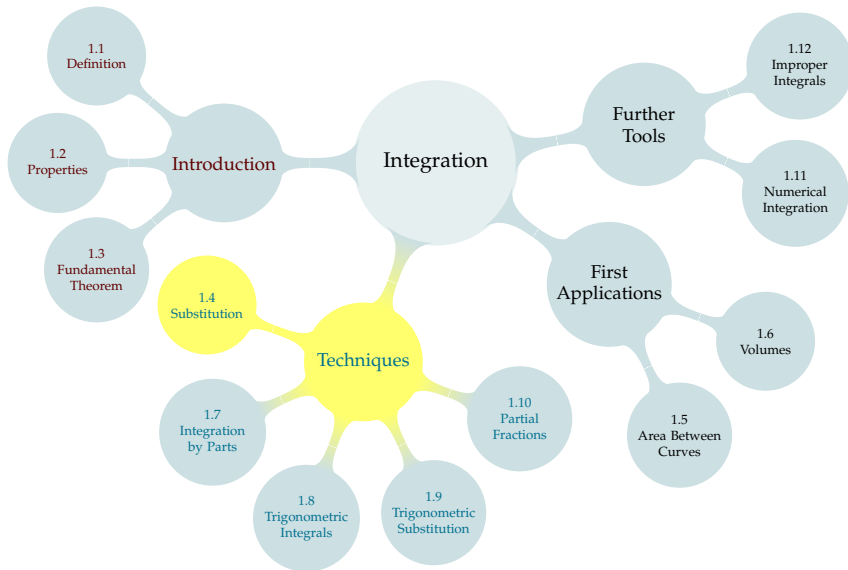


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ANTIDERIVATIVES

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ANTIDERIVATIVES

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ANTIDERIVATIVES

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ANTIDERIVATIVES

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Hard to guess the antiderivative without seeing the derivative first!

ANTIDERIVATIVES

Chain Rule:

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Hallmark of the chain rule: an “inside” function, with that function’s derivative multiplied.

SOLVE BY INSPECTION

$$\int 2xe^{x^2+1} dx$$

$$\int \frac{1}{x} \cos(\log x) dx$$

$$\int 3(\sin x + 1)^2 \cos x dx$$

(Look for an “inside” function, with its derivative multiplied.)

SOLVE BY INSPECTION

$$\int 2xe^{x^2+1} dx = e^{x^2+1} + C$$

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(Look for an “inside” function, with its derivative multiplied.)

UNDOING THE CHAIN RULE

Chain Rule:

$$\frac{d}{dx} \{f(u(x))\} = f'(u(x)) \cdot u'(x)$$

(Here, $u(x)$ is our “inside function”)

Antiderivative Fact:

$$\int f'(u(x)) \cdot u'(x) \, dx = f(u(x)) + C$$

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Shorthand: call $u(x)$ simply u ;
since $\frac{du}{dx} = u'(x)$, call $u'(x) \, dx$ simply du .

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$$\int f'(u(x)) \cdot u'(x) \, dx = \int f'(u) \, du \Big|_{u=u(x)} = f(u(x)) + C$$

This is the **substitution rule**.

We saw these integrals before, and solved them by inspection. Now try using the language of substitution.

$$\int 2xe^{x^2+1} dx$$

$$\int \frac{1}{x} \cos(\log x) dx$$

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We saw these integrals before, and solved them by inspection. Now try using the language of substitution.

$$\int 2xe^{x^2+1} dx$$

Using u as shorthand for $x^2 + 1$, and du as shorthand for $2x dx$:

$$\int 2xe^{x^2+1} dx = \int e^u du = e^u + C = e^{x^2+1} + C$$

$$\int \frac{1}{x} \cos(\log x) dx$$

Using u as shorthand for $\log x$, and du as shorthand for $\frac{1}{x} dx$:

$$\int \frac{1}{x} \cos(\log x) dx = \int \cos(u) du = \sin(u) + C = \sin(\log x) + C$$

$$\int 3(\sin x + 1)^2 \cos x dx$$

Using u as shorthand for $\sin x + 1$, and du as shorthand for $\cos x dx$:

$$\int 3(\sin x + 1)^2 \cos x dx = \int 3u^2 du = u^3 + C = (\sin x + 1)^3 + C$$

$$\int (3x^2) \sin(x^3 + 1) \, dx =$$

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“Inside” function: $x^3 + 1$.

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“Inside” function: $x^3 + 1$. Its derivative: $3x^2$

Shorthand: $x^3 + 1 \rightarrow u$, $3x^2 \, dx \rightarrow du$

$$\int (3x^2) \sin(x^3 + 1) \, dx = \int \sin(u) \, du \Big|_{u=x^3+1}$$

“Inside” function: $x^3 + 1$. Its derivative: $3x^2$

Shorthand: $x^3 + 1 \rightarrow u$, $3x^2 \, dx \rightarrow du$

$$\begin{aligned}\int (3x^2) \sin(x^3 + 1) \, dx &= \int \sin(u) \, du \Big|_{u=x^3+1} \\ &= -\cos(u) + C \Big|_{u=x^3+1}\end{aligned}$$

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“Inside” function: $x^3 + 1$. Its derivative: $3x^2$

Shorthand: $x^3 + 1 \rightarrow u$, $3x^2 dx \rightarrow du$

Warning 1: We don’t just change dx to du . We need to couple dx with the derivative of our inside function.

After all, we’re undoing the chain rule! We need to have an “inside derivative.”

$$\begin{aligned}\int (3x^2) \sin(x^3 + 1) \, dx &= \int \sin(u) \, du \Big|_{u=x^3+1} \\ &= -\cos(u) + C \Big|_{u=x^3+1} \\ &= \cos(x^3 + 1) + C\end{aligned}$$

“Inside” function: $x^3 + 1$. Its derivative: $3x^2$

Shorthand: $x^3 + 1 \rightarrow u$, $3x^2 \, dx \rightarrow du$

Warning 2: The final answer is a function of x .

We used the substitution rule to conclude

$$\int (3x^2) \sin(x^3 + 1) \, dx = -\cos(x^3 + 1) + C$$

We can check that our antiderivative is correct by differentiating.

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$$\int (3x^2) \sin(x^3 + 1) \, dx = -\cos(x^3 + 1) + C$$

We can check that our antiderivative is correct by differentiating.

$$\frac{d}{dx} \{-\cos(x^3 + 1) + C\} = \sin(x^3 + 1)(3x^2)$$

We saw:

$$\int 3x^2 \sin(x^3 + 1) \, dx = -\cos(x^3 + 1) + C$$

So, we can evaluate:

$$\int_0^1 3x^2 \sin(x^3 + 1) \, dx$$

We saw:

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So, we can evaluate:

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Alternately, we can put in the limits of integration as we substitute. The bounds are originally given as values of x ; we simply change them to values of u .

If $u(x) = x^3 + 1$, then $u(0) = 1$ and $u(1) = 2$.

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If $u(x) = x^3 + 1$, then $u(0) = 1$ and $u(1) = 2$.

$$\underbrace{\int_0^1}_{x\text{-values}} 3x^2 \sin(x^3 + 1) \, dx = \underbrace{\int_1^2}_{u\text{-values}} \sin(u) \, du = -\cos(2) + \cos(1)$$

NOTATION: LIMITS OF INTEGRATION

$$\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin^3 x} dx$$

NOTATION: LIMITS OF INTEGRATION

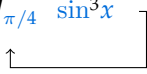
$$\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin^3 x} dx$$

Let $u = \sin x$, $du = \cos x dx$. Note the limits (or bounds) of integration $\pi/4$ and $\pi/2$ are values of x , not u : they follow the differential, unless otherwise specified.

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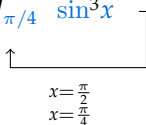
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$x = \frac{\pi}{2}$
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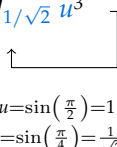
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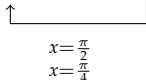
$$\int_{1/\sqrt{2}}^1 \frac{1}{u^3} du$$


$u = \sin\left(\frac{\pi}{2}\right) = 1$
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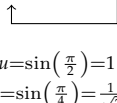
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different

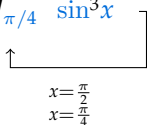
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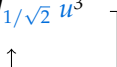
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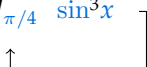
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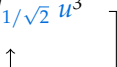
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not standard, but OK

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Let $u = \sin x$, $du = \cos x dx$.

$$\begin{aligned} \int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin^3 x} dx &= \int_{1/\sqrt{2}}^1 \frac{1}{u^3} du \\ &= \int_{1/\sqrt{2}}^1 u^{-3} du \\ &= \left[\frac{1}{-2u^2} \right]_{1/\sqrt{2}}^1 \\ &= -\frac{1}{2} - (-1) = \frac{1}{2} \end{aligned}$$

TRUE OR FALSE?

1. Using $u = x^2$,

$$\int e^{x^2} dx = \int e^u du$$

2. Using $u = x^2 + 1$,

$$\int_0^1 x \sin(x^2 + 1) dx = \int_0^1 \frac{1}{2} \sin u du$$

TRUE OR FALSE?

1. Using $u = x^2$,

$$\int e^{x^2} dx = \int e^u du$$

False: missing $u'(x)$.

$$du = (2x dx) \neq dx$$

2. Using $u = x^2 + 1$,

$$\int_0^1 x \sin(x^2 + 1) dx = \int_0^1 \frac{1}{2} \sin u du$$

False: limits of integration didn't translate.

When $x = 0$, $u = 0^2 + 1 = 1$, and when $x = 1$, $u = 1^2 + 1 = 2$.

Evaluate $\int_0^1 x^7 (x^4 + 1)^5 dx$.

Evaluate $\int_0^1 x^7 (x^4 + 1)^5 dx$.

$$u = x^4 + 1, \quad du = 4x^3 dx$$

$$u(0) = 1, \quad u(1) = 2$$

$$x^4 = u - 1, \quad x^3 dx = \frac{1}{4} du$$

$$\begin{aligned} \int_0^1 x^7 (x^4 + 1)^5 dx &= \int_0^1 (x^4) \cdot (x^4 + 1)^5 \cdot x^3 dx \\ &= \int_1^2 (u - 1) \cdot u^5 \cdot \frac{1}{4} du \\ &= \frac{1}{4} \int_1^2 (u^6 - u^5) du \\ &= \frac{1}{4} \left[\frac{1}{7} u^7 - \frac{1}{6} u^6 \right]_1^2 \\ &= \frac{1}{4} \left[\frac{2^7}{7} - \frac{2^6}{6} - \frac{1}{7} + \frac{1}{6} \right] \end{aligned}$$

Time permitting, more examples using the substitution rule

Evaluate $\int \sin x \cos x \, dx$.

Evaluate $\int \sin x \cos x \, dx$.

Let $u = \sin x$, $du = \cos x \, dx$:

$$\int \sin x \cos x \, dx = \int u \, du = \frac{1}{2}u^2 + C = \frac{1}{2}\sin^2 x + C$$

Or, let $u = \cos x$, $du = -\sin x \, dx$:

$$\int \cos x \sin x \, dx = -\int u \, du = -\frac{1}{2}u^2 + C = -\frac{1}{2}\cos^2 x + C$$

Recall $\sin^2 x + \cos^2 x = 1$ for all x , so $\frac{1}{2}\sin^2 x = -\frac{1}{2}\cos^2 x + \frac{1}{2}$. The two answers look different, but they only differ by a constant, which can be absorbed in the arbitrary constant C . If we rename the second C to C' so that the second answer is $-\frac{1}{2}\cos^2 x + C'$, then $C' = C + \frac{1}{2}$.

CHECK OUR WORK

We can check that $\int \sin x \cos x \, dx = \frac{1}{2} \sin^2 x + C$ by differentiating.

We can check that $\int \sin x \cos x \, dx = -\frac{1}{2} \cos^2 x + C$ by differentiating.

CHECK OUR WORK

We can check that $\int \sin x \cos x \, dx = \frac{1}{2} \sin^2 x + C$ by differentiating.

$$\frac{d}{dx} \left\{ \frac{1}{2} \sin^2 x + C \right\} = \frac{2}{2} \sin x \cdot \cos x = \sin x \cos x$$

Our answer works.

We can check that $\int \sin x \cos x \, dx = -\frac{1}{2} \cos^2 x + C$ by differentiating.

$$\frac{d}{dx} \left\{ -\frac{1}{2} \cos^2 x + C \right\} = -\frac{2}{2} \cos x \cdot (-\sin x) = \sin x \cos x$$

This answer works too.

Evaluate $\int \frac{\log x}{3x} dx$.

Evaluate $\int \frac{\log x}{3x} dx$.

Let $u = \log x$, $du = \frac{1}{x} dx$:

$$\begin{aligned}\int \frac{\log x}{3} \cdot \frac{1}{x} dx &= \frac{1}{3} \int u \, du \\ &= \frac{1}{6} u^2 + C \\ &= \frac{1}{6} \log^2 x + C\end{aligned}$$

CHECK OUR WORK

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We can check that $\int \frac{\log x}{3x} dx = \frac{1}{6} \log^2 x + C$ by differentiating.

$$\frac{d}{dx} \left\{ \frac{1}{6} \log^2 x + C \right\} = \frac{2}{6} \log x \cdot \frac{1}{x} = \frac{\log x}{3x}$$

Our answer works.

Evaluate $\int \frac{e^x}{e^x + 15} dx$.

Evaluate $\int x^4(x^5 + 1)^8 dx$.

Evaluate $\int \frac{e^x}{e^x + 15} dx$.

Let $u = e^x + 15$, $du = e^x dx$

$$\int \frac{e^x}{e^x + 15} dx = \int \frac{1}{u} du = \log |u| + C = \log |e^x + 15| + C$$

In this case, since $e^x + 15 > 0$, the absolute values on $|e^x + 15|$ are optional.

Evaluate $\int x^4(x^5 + 1)^8 dx$.

Evaluate $\int \frac{e^x}{e^x + 15} dx$.

Evaluate $\int x^4(x^5 + 1)^8 dx$.

Let $u = x^5 + 1$, $du = 5x^4 dx$. Then, $x^4 dx = \frac{1}{5} du$.

$$\begin{aligned}\int x^4(x^5 + 1)^8 dx &= \int \frac{1}{5}(u)^8 du \\ &= \frac{1}{5} \cdot \frac{1}{9} u^9 + C = \frac{1}{45}(x^5 + 1)^9 + C\end{aligned}$$



CHECK OUR WORK

We can check that $\int \frac{e^x}{e^x + 15} dx = \log |e^x + 15| + C$ by differentiating.

We can check that $\int x^4(x^5 + 1)^8 dx = \frac{1}{45}(x^5 + 1)^9 + C$ by differentiating.



CHECK OUR WORK

We can check that $\int \frac{e^x}{e^x + 15} dx = \log |e^x + 15| + C$ by differentiating.

$$\frac{d}{dx} \{ \log |e^x + 15| + C \} = \frac{1}{e^x + 15} \cdot e^x = \frac{e^x}{e^x + 15}$$

Our answer works.

We can check that $\int x^4(x^5 + 1)^8 dx = \frac{1}{45}(x^5 + 1)^9 + C$ by differentiating.

$$\frac{d}{dx} \left\{ \frac{1}{45}(x^5 + 1)^9 + C \right\} = \frac{9}{45}(x^5 + 1)^8 \cdot 5x^4 = (x^5 + 1)^8 x^4$$

Our answer works.



Evaluate $\int_4^8 \frac{s}{s-3} ds$. Be careful to use correct notation.

Evaluate $\int_4^8 \frac{s}{s-3} ds$. Be careful to use correct notation.

Let $u = s - 3$, $du = ds$.

Then $s = u + 3$, $u(4) = 1$ and $u(8) = 5$.

$$\begin{aligned}\int_4^8 \frac{s}{s-3} ds &= \int_1^5 \frac{u+3}{u} du \\&= \int_1^5 \left(1 + \frac{3}{u}\right) du \\&= [u + 3 \log |u|]_1^5 \\&= [5 + 3 \log 5] - [1 + 3 \log 1] \\&= 4 + 3 \log 5\end{aligned}$$



Evaluate $\int x^9(x^5 + 1)^8 dx$.

Evaluate $\int x^9(x^5 + 1)^8 dx$.

Let $u = x^5 + 1$, $du = 5x^4 dx$.

Then $x^4 dx = \frac{1}{5} du$, $x^5 = u - 1$.

$$\begin{aligned}\int x^9(x^5 + 1)^8 dx &= \int (x^4) \cdot (x^5) \cdot (x^5 + 1)^8 dx \\&= \int \left(\frac{1}{5}\right) \cdot (u - 1) \cdot u^8 du = \frac{1}{5} \int (u^9 - u^8) du \\&= \frac{1}{5} \left[\frac{1}{10} u^{10} - \frac{1}{9} u^9 \right] + C \\&= \frac{1}{5} \left[\frac{(x^5 + 1)^{10}}{10} - \frac{(x^5 + 1)^9}{9} \right] + C\end{aligned}$$



CHECK OUR WORK

We can check that $\int x^9(x^5 + 1)^8 \, dx = \frac{1}{5} \left[\frac{(x^5 + 1)^{10}}{10} - \frac{(x^5 + 1)^9}{9} \right] + C$
by differentiating.



CHECK OUR WORK

We can check that $\int x^9(x^5 + 1)^8 dx = \frac{1}{5} \left[\frac{(x^5 + 1)^{10}}{10} - \frac{(x^5 + 1)^9}{9} \right] + C$
by differentiating.

$$\begin{aligned} & \frac{d}{dx} \left\{ \frac{1}{5} \left[\frac{(x^5 + 1)^{10}}{10} - \frac{(x^5 + 1)^9}{9} \right] + C \right\} \\ &= \frac{1}{5} [(x^5 + 1)^9 \cdot 5x^4 - (x^5 + 1)^8 \cdot 5x^4] \\ &= x^4(x^5 + 1)^9 - x^4(x^5 + 1)^8 \\ &= x^4(x^5 + 1)^8[(x^5 + 1) - 1] \\ &= x^4(x^5 + 1)^8[x^5] \\ &= x^9(x^5 + 1)^8 \end{aligned}$$

Our answer works.



PARTICULARLY TRICKY SUBSTITUTION

Evaluate $\int \frac{1}{e^x + e^{-x}} dx$.



PARTICULARLY TRICKY SUBSTITUTION

Evaluate $\int \frac{1}{e^x + e^{-x}} dx$.

Let $u = e^x$, $du = e^x dx$. Then $dx = \frac{du}{e^x} = \frac{du}{u}$.

$$\begin{aligned}\int \frac{1}{e^x + e^{-x}} dx &= \int \frac{1}{u + \frac{1}{u}} \frac{du}{u} \\ &= \int \frac{1}{u^2 + 1} du \\ &= \arctan(u) + C \\ &= \arctan(e^x) + C\end{aligned}$$



CHECK OUR WORK

We can check that $\int \frac{1}{e^x + e^{-x}} dx = \arctan(e^x) + C$ by differentiating.



CHECK OUR WORK

We can check that $\int \frac{1}{e^x + e^{-x}} dx = \arctan(e^x) + C$ by differentiating.

$$\begin{aligned}\frac{d}{dx} \{\arctan(e^x) + C\} &= \frac{1}{(e^x)^2 + 1} \cdot e^x \\ &= \frac{e^x}{(e^x)^2 + 1} \\ &= \frac{e^x}{(e^x)^2 + 1} \cdot \frac{e^{-x}}{e^{-x}} \\ &= \frac{1}{e^x + e^{-x}}\end{aligned}$$

Our answer works.



Included Work



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