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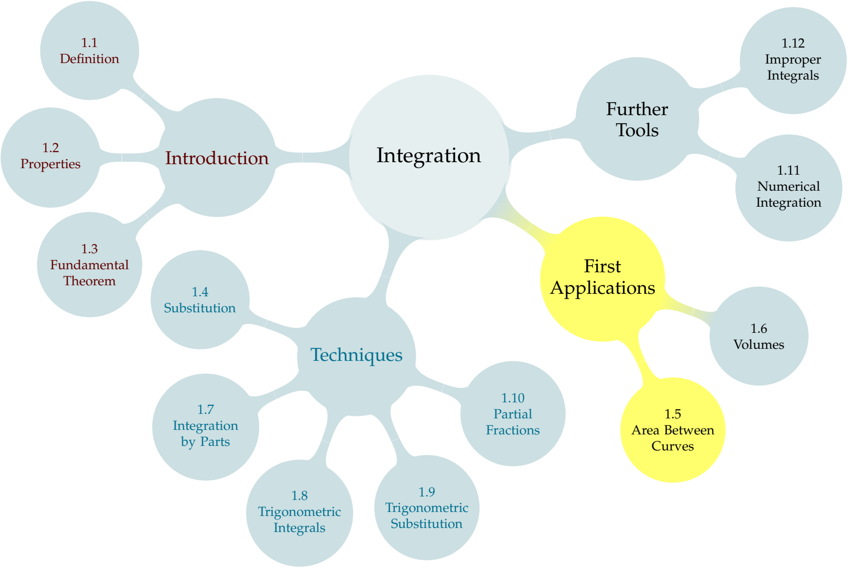
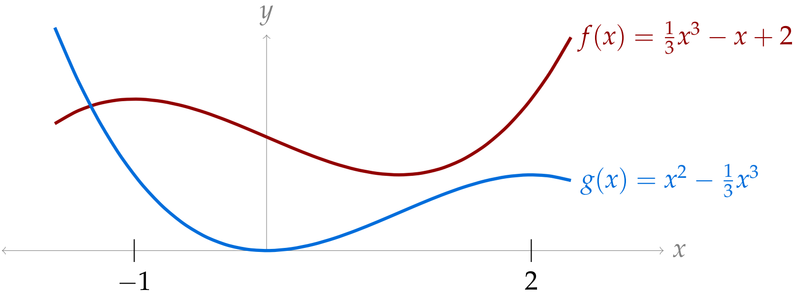
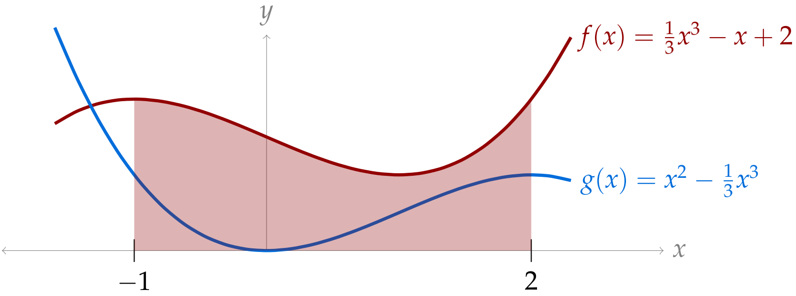


Find the area between $f(x)$ and $g(x)$ from $x = -1$ to $x = 2$.

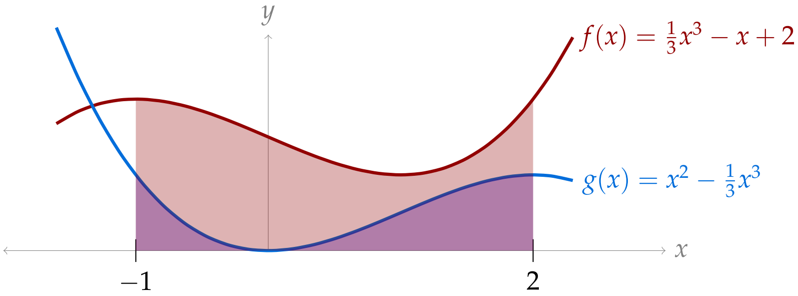


Find the area between $f(x)$ and $g(x)$ from $x = -1$ to $x = 2$.



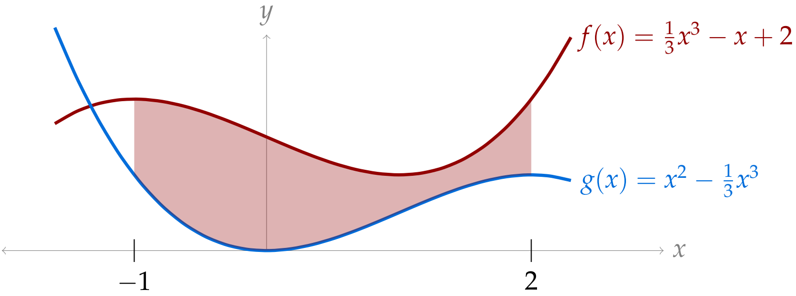
$$\int_{-1}^{2} f(x)\,\mathrm{d}x$$

Find the area between $f(x)$ and $g(x)$ from $x = -1$ to $x = 2$.



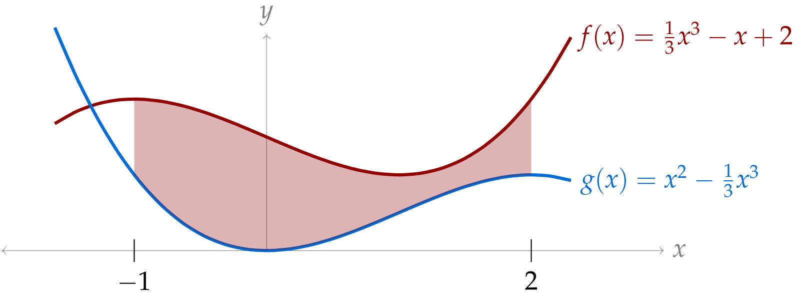
$$\int_{-1}^{2} f(x)\,\mathrm{d}x \qquad \int_{-1}^{2} g(x)\,\mathrm{d}x$$

Find the area between $f(x)$ and $g(x)$ from $x = -1$ to $x = 2$.



$$\int_{-1}^{2} f(x)\,\mathrm{d}x - \int_{-1}^{2} g(x)\,\mathrm{d}x$$

Find the area between $f(x)$ and $g(x)$ from $x = -1$ to $x = 2$.



$$\int_{-1}^{2} f(x)\,\mathrm{d}x - \int_{-1}^{2} g(x)\,\mathrm{d}x = \int_{-1}^{2} \left[f(x) - g(x)\right]\mathrm{d}x$$

Find the area between $f(x)$ and $g(x)$ from $x = -1$ to $x = 2$.

$$\int_{-1}^{2} f(x)\,\mathrm{d}x - \int_{-1}^{2} g(x)\,\mathrm{d}x = \int_{-1}^{2} \Big[f(x) - g(x)\Big]\mathrm{d}x$$

$$= \int_{-1}^{2} \left[\frac{1}{3}x^3 - x + 2 - x^2 + \frac{1}{3}x^3\right] \mathrm{d}x$$

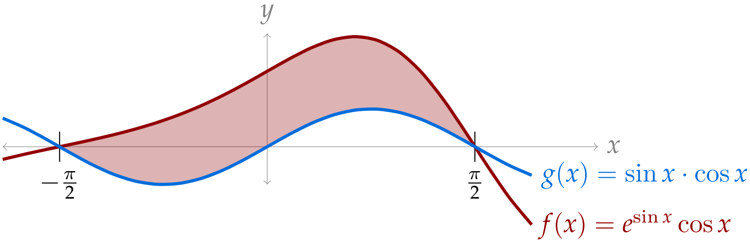
Q

ans

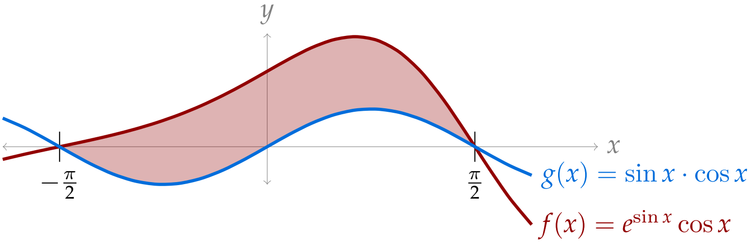
Find the area between $f(x)$ and $g(x)$ from $x = -1$ to $x = 2$.

$$
\begin{aligned}
\int_{-1}^{2} f(x)\,\mathrm{d}x - \int_{-1}^{2} g(x)\,\mathrm{d}x &= \int_{-1}^{2} \Big[f(x) - g(x)\Big]\mathrm{d}x \\
&= \int_{-1}^{2} \left[\frac{1}{3}x^3 - x + 2 - x^2 + \frac{1}{3}x^3\right]\mathrm{d}x \\
&= \int_{-1}^{2} \left[\frac{2}{3}x^3 - x^2 - x + 2\right]\mathrm{d}x \\
&= \left[\frac{1}{6}x^4 - \frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x\right]_{-1}^{2} \\
&= \frac{16}{6} - \frac{8}{3} - \frac{4}{2} + 4 - \left(\frac{1}{6} + \frac{1}{3} - \frac{1}{2} - 2\right) \\
&= 2 - (-2) = 4
\end{aligned}
$$

Find the (unsigned) area between $f(x)$ and $g(x)$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$.

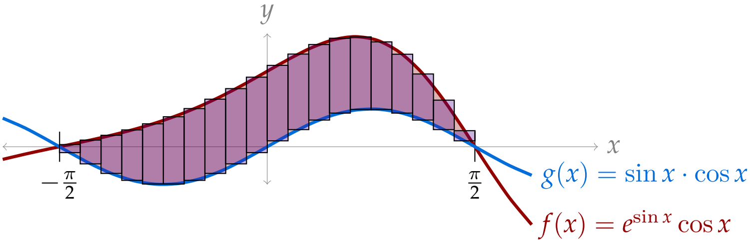


Find the (unsigned) area between $f(x)$ and $g(x)$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$.



$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[f(x) - g(x)\right] \mathrm{d}x$$

Find the (unsigned) area between $f(x)$ and $g(x)$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$.



$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \Big[f(x) - g(x)\Big] \mathrm{d}x$$

Find the (unsigned) area between $f(x)$ and $g(x)$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$.

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[f(x) - g(x)\right] \mathrm{d}x = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(e^{\sin x} \cos x - \sin x \cos x\right) \mathrm{d}x$$

Find the (unsigned) area between $f(x)$ and $g(x)$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$.

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[f(x) - g(x)\right] \mathrm{d}x = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(e^{\sin x} \cos x - \sin x \cos x\right) \mathrm{d}x$$

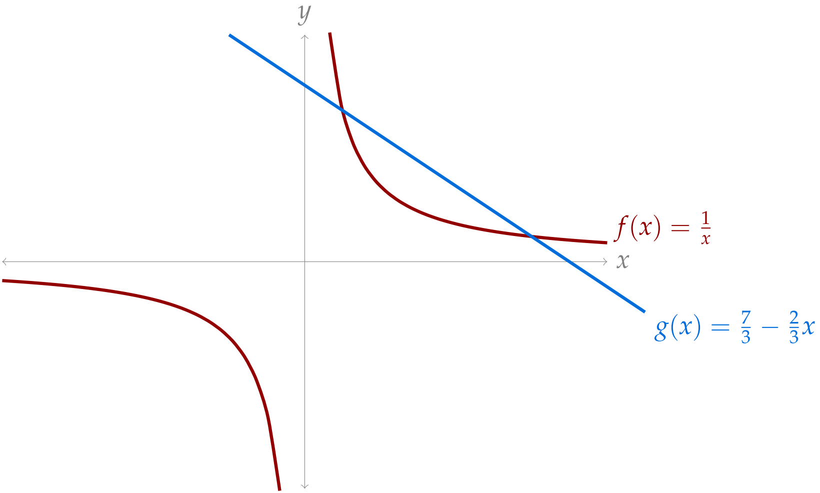
Let $u = \sin x$.
Then: $\mathrm{d}u = \cos x \, \mathrm{d}x$, $u\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$, $u\left(\frac{-\pi}{2}\right) = \sin\left(\frac{-\pi}{2}\right) = -1$.

$$= \int_{-1}^{1} (e^u - u) \, \mathrm{d}u$$

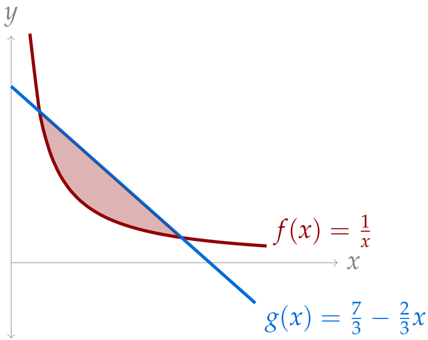
$$= \left[e^u - \frac{1}{2}u^2\right]_{-1}^{1}$$

$$= e - \frac{1}{e}$$

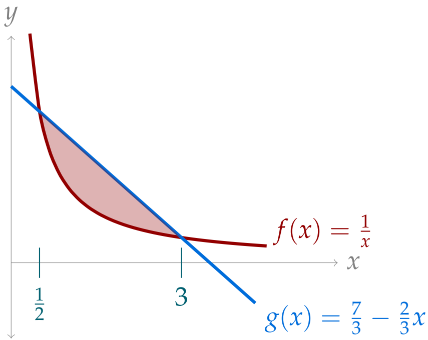
Find the (unsigned) area of the finite region bounded by $f(x)$ and $g(x)$.



Find the (unsigned) area of the finite region bounded by $f(x)$ and $g(x)$.



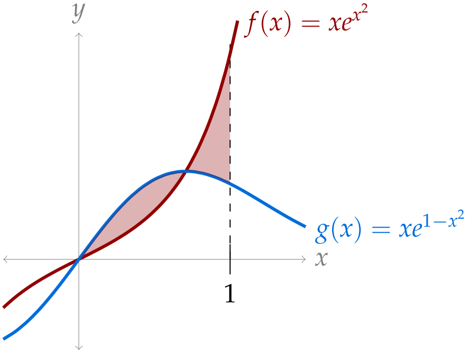
Find the (unsigned) area of the finite region bounded by $f(x)$ and $g(x)$.



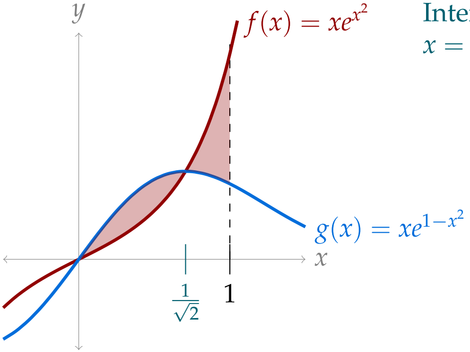
$$\frac{1}{x} = \frac{7-2x}{3}$$
$$3 = 7x - 2x^2$$
$$0 = 2x^2 - 7x + 3$$
$$x = \frac{7 \pm \sqrt{7^2 - 4(2)(3)}}{4}$$
$$= \frac{7 \pm \sqrt{25}}{4}$$
$$x = \frac{1}{2} \text{ and } x = 3$$

$$\int_{\frac{1}{2}}^{3} [g(x) - f(x)]\,dx = \int_{\frac{1}{2}}^{3} \left[\frac{7}{3} - \frac{2}{3}x - \frac{1}{x}\right] dx = \left[\frac{7}{3}x - \frac{1}{3}x^2 - \log x\right]_{\frac{1}{2}}^{3}$$
$$= (7 - 3 - \log 3) - \left(\frac{7}{6} - \frac{1}{12} - \log\frac{1}{2}\right) = \frac{35}{12} - \log 6$$

Find the (unsigned) area in the figure below between the curves $f(x)$ and $g(x)$ from $x = 0$ to $x = 1$.



Find the (unsigned) area in the figure below between the curves $f(x)$ and $g(x)$ from $x = 0$ to $x = 1$.



Intersections at $x = 0$ and $x = \pm\frac{1}{\sqrt{2}}$:

$$\begin{aligned} xe^{x^2} &= xe^{1-x^2} \\ e^{x^2} &= e^{1-x^2} \text{ or } x = 0 \\ x^2 &= 1 - x^2 \\ x &= \pm\frac{1}{\sqrt{2}} \end{aligned}$$

$$\text{Area} = \int_0^{\frac{1}{\sqrt{2}}} \left[g(x) - f(x)\right] \mathrm{d}x + \int_{\frac{1}{\sqrt{2}}}^{1} \left[f(x) - g(x)\right] \mathrm{d}x$$

Find the (unsigned) area in the figure below between the curves $f(x)$ and $g(x)$ from $x = 0$ to $x = 1$.

$$\begin{aligned}\text{Area} &= \int_0^{\frac{1}{\sqrt{2}}} \left[g(x) - f(x)\right] \mathrm{d}x + \int_{\frac{1}{\sqrt{2}}}^1 \left[f(x) - g(x)\right] \mathrm{d}x \\ &= \int_0^{\frac{1}{\sqrt{2}}} \left[xe^{1-x^2} - xe^{x^2}\right] \mathrm{d}x + \int_{\frac{1}{\sqrt{2}}}^1 \left[xe^{x^2} - xe^{1-x^2}\right] \mathrm{d}x\end{aligned}$$

A

Find the (unsigned) area in the figure below between the curves $f(x)$ and $g(x)$ from $x=0$ to $x=1$.

$$\text{Area} = \int_0^{\frac{1}{\sqrt{2}}} \Big[g(x) - f(x)\Big]\,\mathrm{d}x + \int_{\frac{1}{\sqrt{2}}}^{1} \Big[f(x) - g(x)\Big]\,\mathrm{d}x$$

$$= \int_0^{\frac{1}{\sqrt{2}}} \Big[xe^{1-x^2} - xe^{x^2}\Big]\,\mathrm{d}x + \int_{\frac{1}{\sqrt{2}}}^{1} \Big[xe^{x^2} - xe^{1-x^2}\Big]\,\mathrm{d}x$$

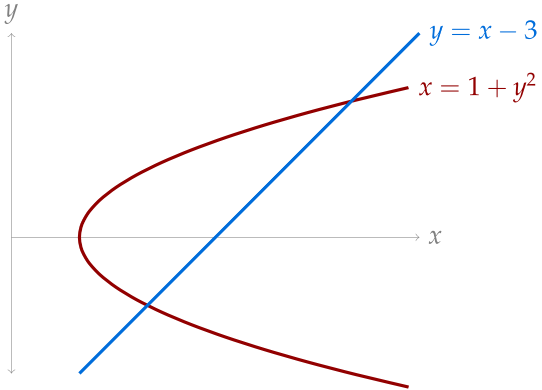
Aside: $\underbrace{\int xe^{1-x^2}\,\mathrm{d}x}_{u=1-x^2,\ \mathrm{d}u=-2x\,\mathrm{d}x} = -\frac{1}{2}e^{1-x^2} + C \qquad \underbrace{\int xe^{x^2}\,\mathrm{d}x}_{u=x^2,\ \mathrm{d}u=2x\,\mathrm{d}x} = \frac{1}{2}e^{x^2} + C$

$$\text{Area} = \left[-\frac{1}{2}e^{1-x^2} - \frac{1}{2}e^{x^2}\right]_0^{\frac{1}{\sqrt{2}}} + \left[\frac{1}{2}e^{x^2} - \left(-\frac{1}{2}e^{1-x^2}\right)\right]_{\frac{1}{\sqrt{2}}}^{1}$$

$$= -\frac{1}{2}\left[\left(e^{\frac{1}{2}} + e^{\frac{1}{2}}\right) - \left(e^{1} + e^{0}\right)\right] + \frac{1}{2}\left[\left(e^{1} + e^{0}\right) - \left(e^{\frac{1}{2}} + e^{\frac{1}{2}}\right)\right]$$

$$= e - 2\sqrt{e} + 1$$

Set up, but do not evaluate, integral(s) to find the (unsigned) area of the finite region bounded by $x = 1 + y^2$ and $y = x - 3$.



Q
ans

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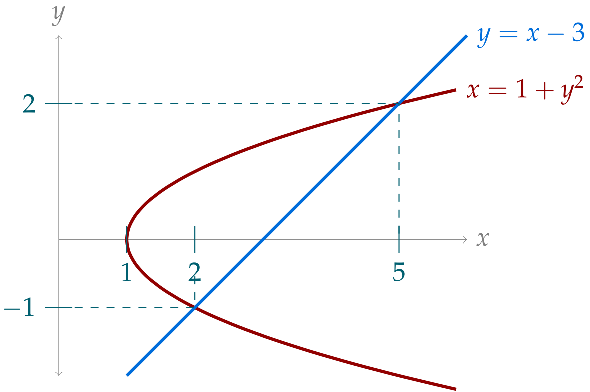

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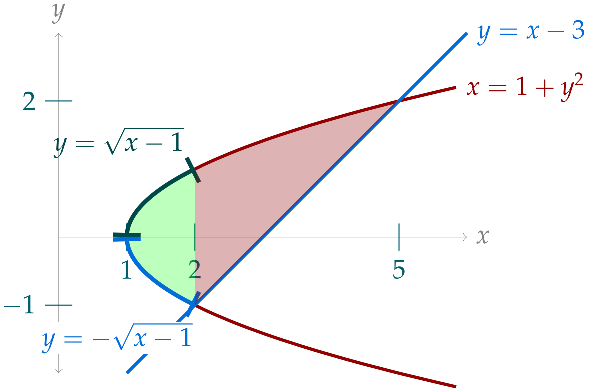
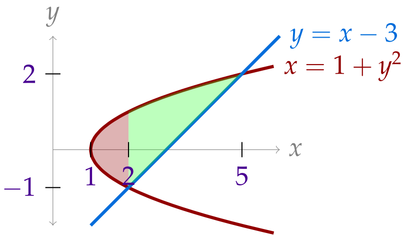
Set up, but do not evaluate, integral(s) to find the (unsigned) area of the finite region bounded by $x = 1 + y^2$ and $y = x - 3$.

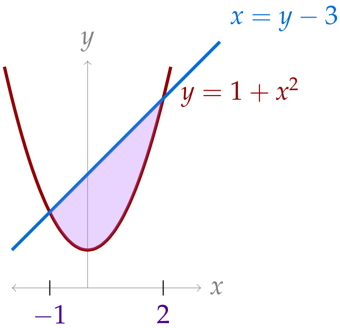


Option 1: $$\int_1^2 \left[\sqrt{x-1} - \left(-\sqrt{x-1}\right)\right] \mathrm{d}x + \int_2^5 \left[\sqrt{x-1} - (x-3)\right] \mathrm{d}x$$

Set up, but do not evaluate, integral(s) to find the (unsigned) area of the finite region bounded by $x = 1 + y^2$ and $y = x - 3$.

Option 2: Swapping x and y results in a figure with the same area.





Set up, but do not evaluate, integral(s) to find the (unsigned) area of the finite region bounded by $x = 1 + y^2$ and $y = x - 3$.
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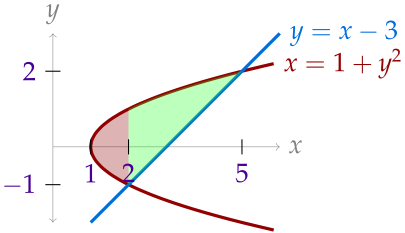

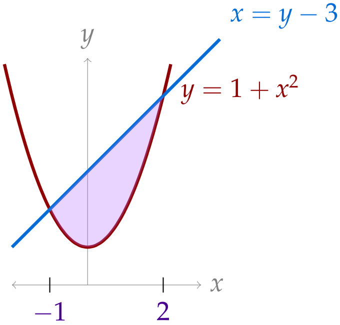

$$\int_{-1}^{2} \left[(x+3) - (1+x^2)\right] \, dx$$

Included Work

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