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FOUR SERIES

Let $a_n = \left(-\frac{2}{3}\right)^n$. Do the following series converge or diverge?

$$\sum_{n=0}^{\infty} a_n$$

$$\sum_{n=0}^{\infty} |a_n|$$

Let $b_n = \frac{(-1)^n}{n}$. Do the following series converge or diverge?

$$\sum_{n=1}^{\infty} b_n$$

$$\sum_{n=1}^{\infty} |b_n|$$

The series

$$\sum_{n=0}^{\infty} \left(-\frac{2}{3}\right)^n$$

is called **absolutely convergent**, because the series converges and if we replace the terms being added by their absolute values, that series *still* converges.

The series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n}$$

is called **conditionally convergent**, because the series converges, but if we replace the terms being added by their absolute values, that series *diverges*.

Absolute and conditional convergence

(a) A series $\sum_{n=1}^{\infty} a_n$ is said to **converge absolutely** if the series

$$\sum_{n=1}^{\infty} |a_n| \text{ converges.}$$

(b) If $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ diverges we say that

$$\sum_{n=1}^{\infty} a_n \text{ is **conditionally convergent**.}$$

Theorem

If the series $\sum_{n=1}^{\infty} |a_n|$ converges then the series $\sum_{n=1}^{\infty} a_n$ also converges.

That is, absolute convergence implies convergence.

If $\sum a_n \dots$	and $\sum a_n \dots$	then we say $\sum a_n$ is ...
converges	converges	
converges	diverges	
diverges	diverges	
diverges	converges	

Does the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

converge or diverge?

Does the series

$$\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$$

converge or diverge?