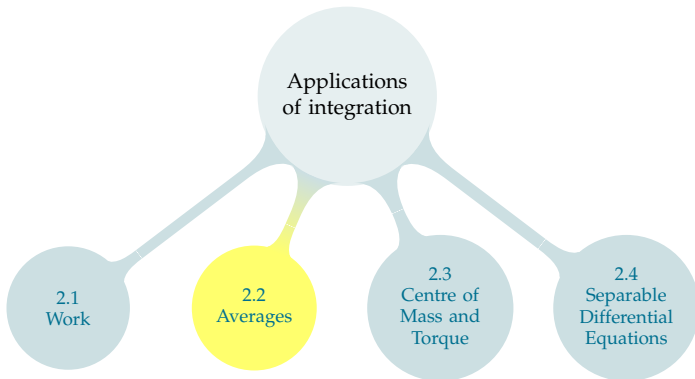
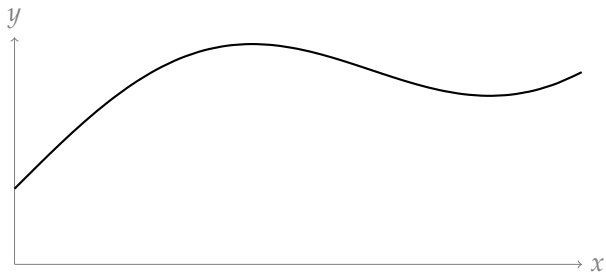
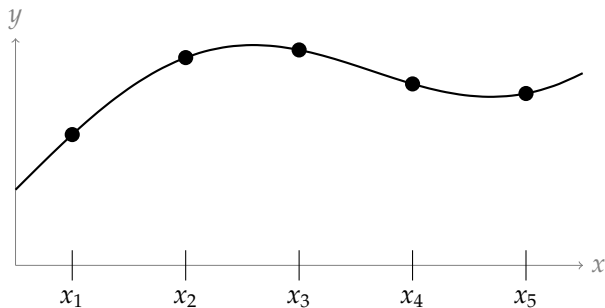


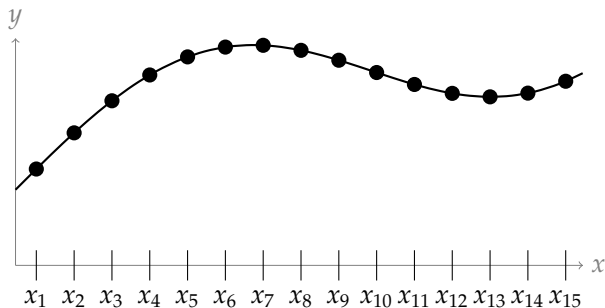
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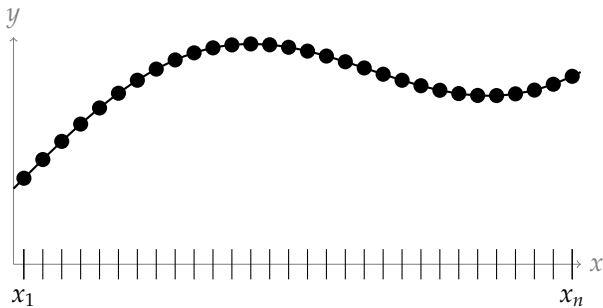




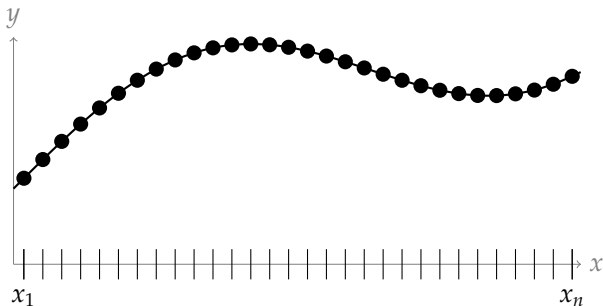
$$\text{Average} \approx \frac{f(x_1) + \cdots + f(x_5)}{5}$$



$$\text{Average} \approx \frac{f(x_1) + \cdots + f(x_{15})}{15}$$

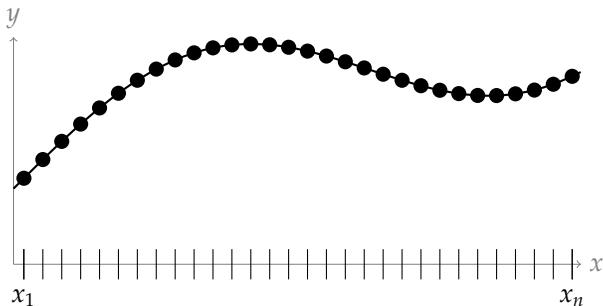


$$\text{Average} \approx \frac{f(x_1) + \cdots + f(x_n)}{n}$$



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$$\text{Average} = \lim_{n \rightarrow \infty} \left[ \frac{1}{n} \sum_{i=1}^n f(x_i) \right]$$



$$\text{Average} \approx \frac{f(x_1) + \cdots + f(x_n)}{n}$$

$$\begin{aligned} \text{Average} &= \lim_{n \rightarrow \infty} \left[ \frac{1}{n} \sum_{i=1}^n f(x_i) \right] = \lim_{n \rightarrow \infty} \left[ \frac{(b-a)}{(b-a)n} \sum_{i=1}^n f(x_i) \right] \\ &= \lim_{n \rightarrow \infty} \left[ \frac{1}{b-a} \sum_{i=1}^n f(x_i) \Delta x \right] = \frac{1}{b-a} \int_a^b f(x) dx \end{aligned}$$

## Average

Let  $f(x)$  be an integrable function defined on the interval  $a \leq x \leq b$ . The average value of  $f$  on that interval is

$$f_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x) \, dx$$



## Average

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The temperature in a certain city at time  $t$  (measured in hours past midnight) is given by

$$T(t) = t - \frac{t^2}{30}$$

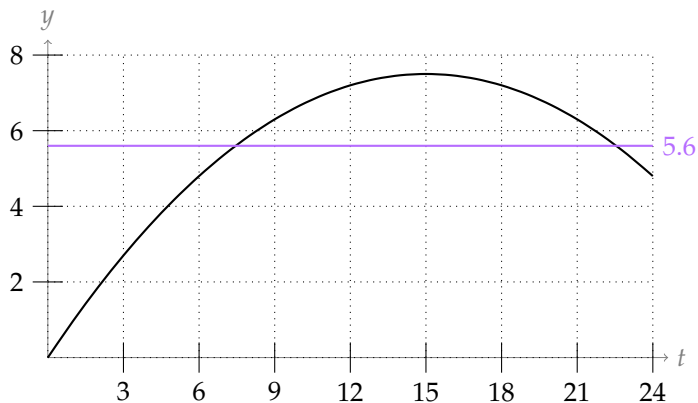
What was the average temperature of one day (from  $t = 0$  to  $t = 24$ )?

The temperature in a certain city at time  $t$  (measured in hours past midnight) is given by

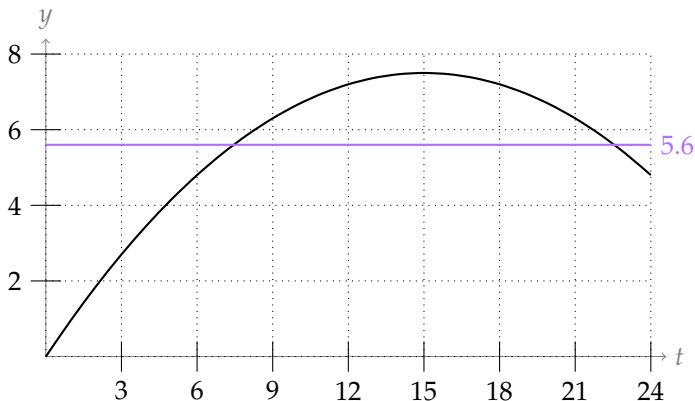
$$T(t) = t - \frac{t^2}{30}$$

What was the average temperature of one day (from  $t = 0$  to  $t = 24$ )?

Let's check that our answer makes some intuitive sense.

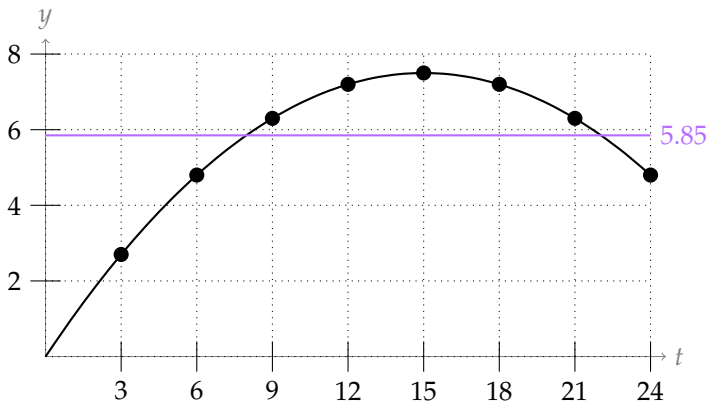


Let's check that our answer makes some intuitive sense.



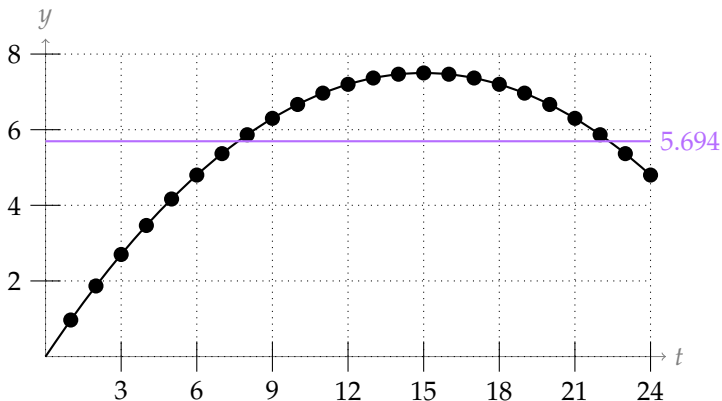
Since the temperature is always between 0 and 8, we expect the average to be between 0 and 8

Let's also recall the motivation for our definition



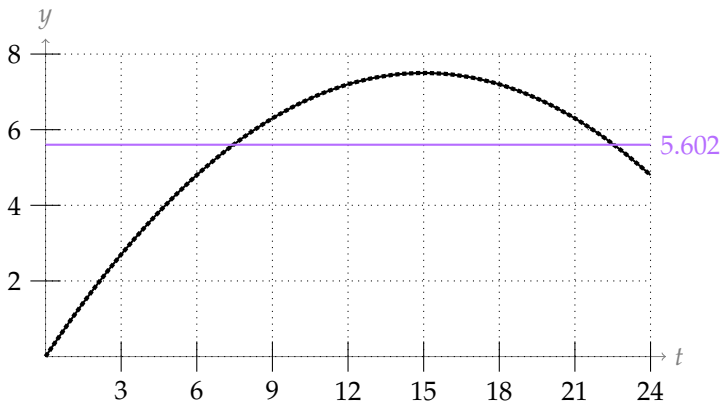
Average temperature, measured every three hours

Let's also recall the motivation for our definition



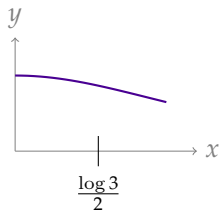
Average temperature, measured every hour

Let's also recall the motivation for our definition



Average temperature, measured every minute

Find the average value of the function  $f(x) = \frac{e^x}{e^{2x} + 1}$  over the interval  $\left[0, \frac{\log 3}{2}\right]$ .





# AVERAGE VELOCITY

Let  $x(t)$  be the position at time  $t$  of a car moving along the  $x$ -axis. The velocity of the car at time  $t$  is the derivative  $v(t) = x'(t)$ . The average velocity of the car over the time interval  $a \leq t \leq b$  is:

## Included Work



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