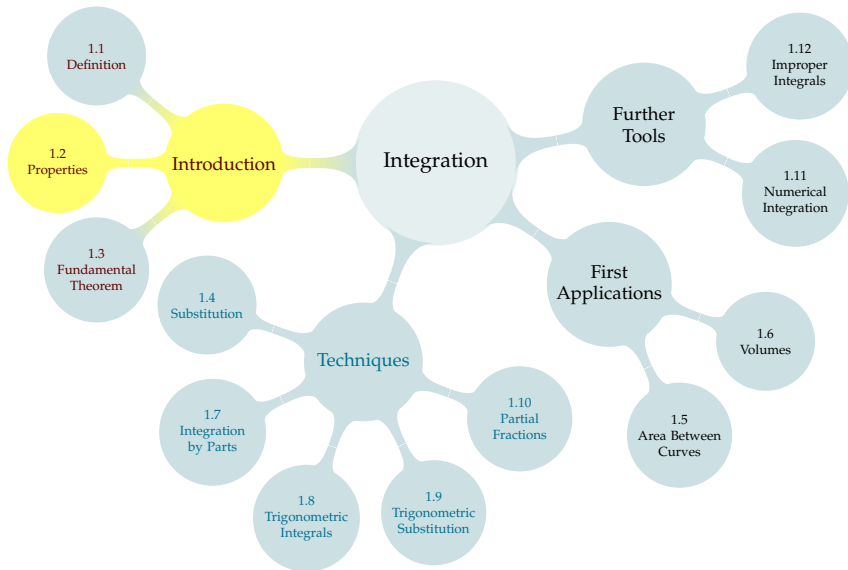


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We defined the definite integral using a limit and a sum.

Definition

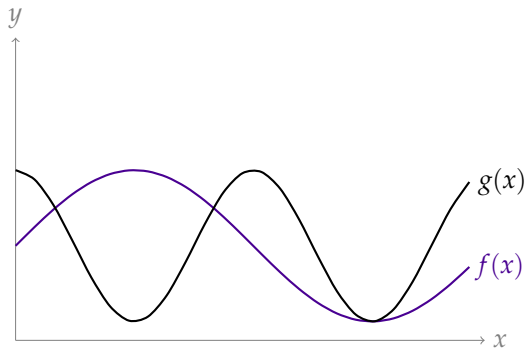
Let a and b be two real numbers and let $f(x)$ be a function that is defined for all x between a and b . Then we define $\Delta x = \frac{b-a}{N}$ and

$$\int_a^b f(x) \, dx = \lim_{N \rightarrow \infty} \sum_{i=1}^N f(x_{i,N}^*) \cdot \Delta x$$

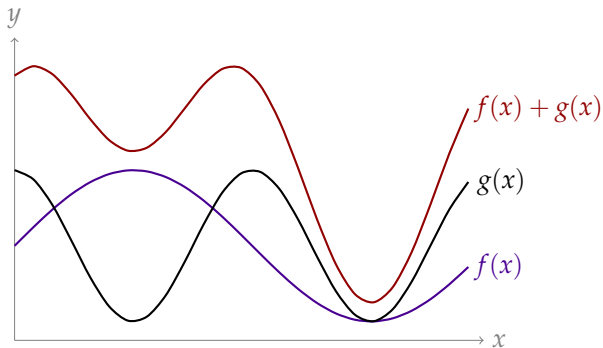
when the limit exists and when the choice of $x_{i,N}^*$ in the i^{th} interval doesn't matter.

Many of the operations that work nicely with sums and limits will also work nicely with integrals.

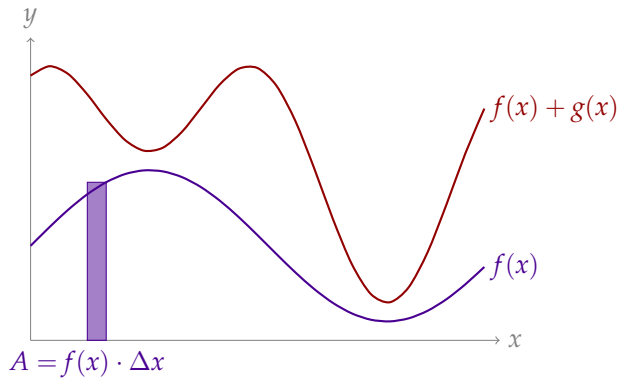
ADDING (AND SUBTRACTING) FUNCTIONS



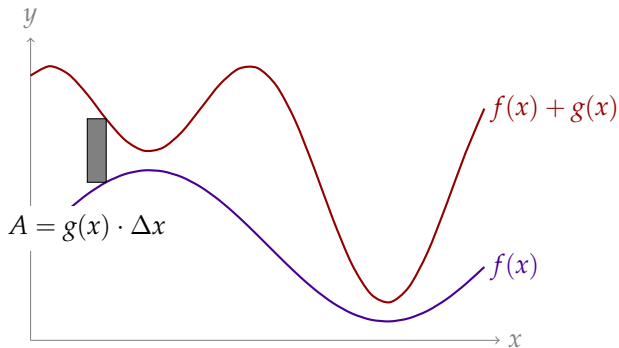
ADDING (AND SUBTRACTING) FUNCTIONS



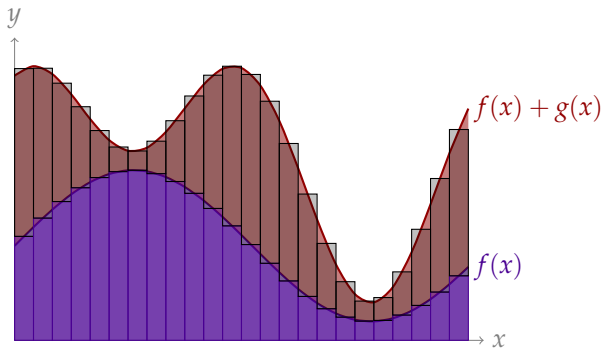
ADDING (AND SUBTRACTING) FUNCTIONS



ADDING (AND SUBTRACTING) FUNCTIONS

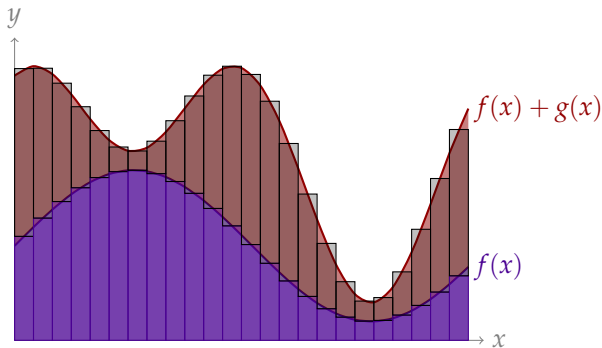


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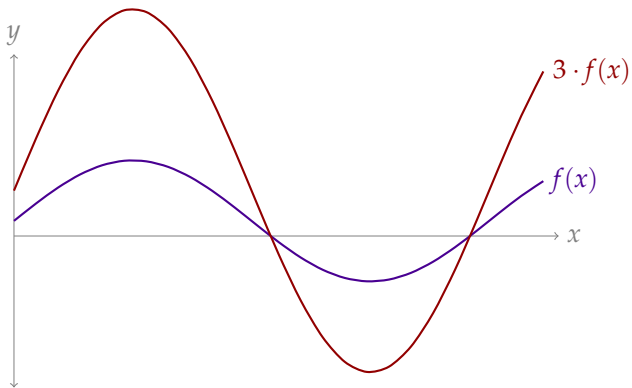
$$\int_a^b (f(x) + g(x)) dx = \int_a^b f(x) dx + \int_a^b g(x) dx$$

ADDING (AND SUBTRACTING) FUNCTIONS

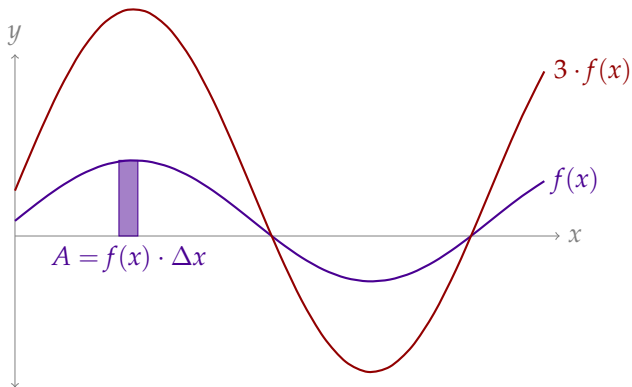


$$\int_a^b (f(x) - g(x)) dx = \int_a^b f(x) dx - \int_a^b g(x) dx$$

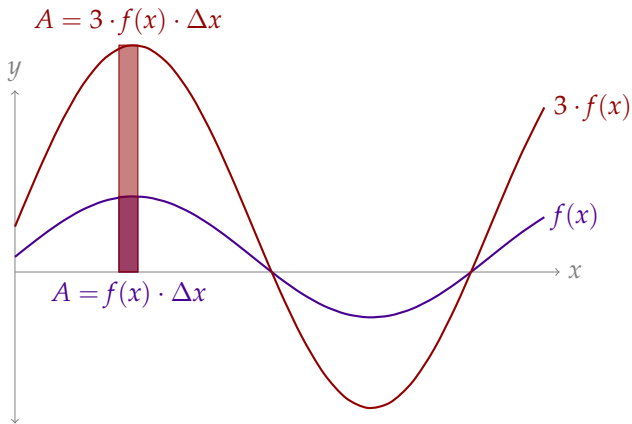
MULTIPLYING A FUNCTION BY A CONSTANT



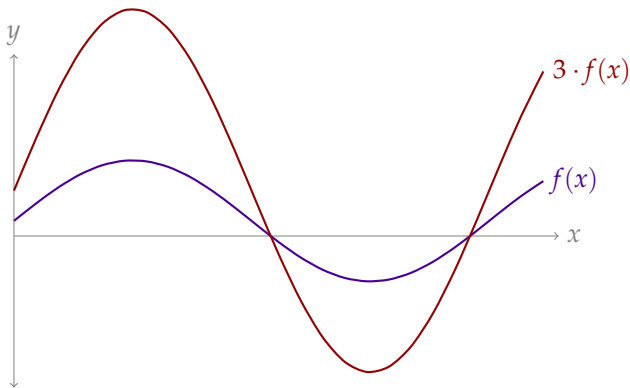
MULTIPLYING A FUNCTION BY A CONSTANT



MULTIPLYING A FUNCTION BY A CONSTANT



MULTIPLYING A FUNCTION BY A CONSTANT



$$\int_a^b c \cdot f(x) \, dx = c \int_a^b f(x) \, dx$$

ARITHMETIC OF INTEGRATION

When a , b , and c are real numbers, and the functions $f(x)$ and $g(x)$ are integrable on an interval containing a and b :

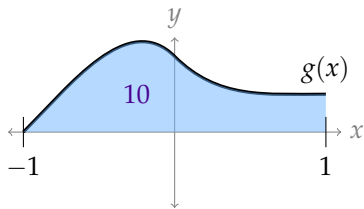
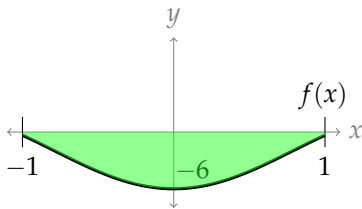
$$(a) \int_a^b [f(x) + g(x)] \, dx = \int_a^b f(x) \, dx + \int_a^b g(x) \, dx$$

$$(b) \int_a^b [f(x) - g(x)] \, dx = \int_a^b f(x) \, dx - \int_a^b g(x) \, dx$$

$$(c) \int_a^b c \cdot f(x) \, dx = c \int_a^b f(x) \, dx \quad \text{when } c \text{ is constant}$$

ARITHMETIC OF INTEGRATION

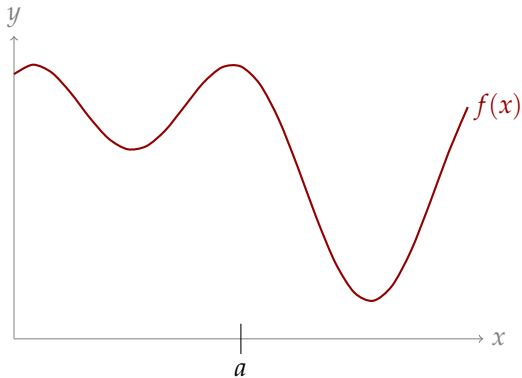
Suppose $\int_{-1}^1 f(x) \, dx = -6$ and $\int_{-1}^1 g(x) \, dx = 10$.



$$\int_{-1}^1 (2f(x) + g(x)) \, dx =$$

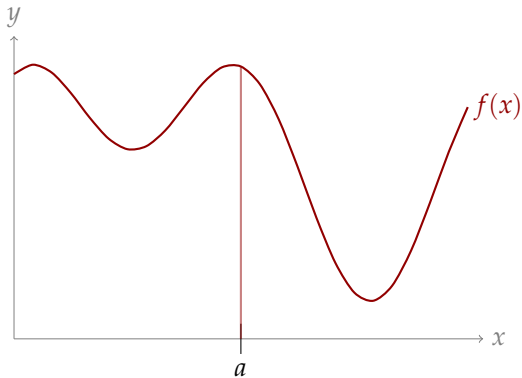


INTERVAL OF INTEGRATION



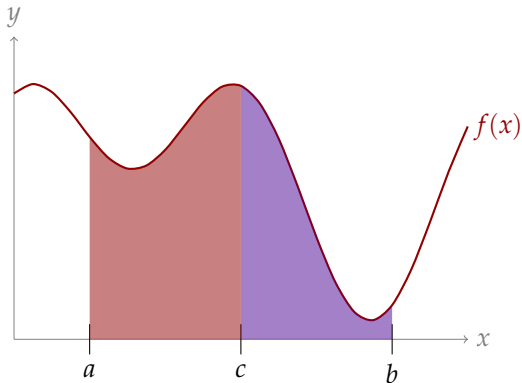
$$\int_a^a f(x) \, dx =$$

INTERVAL OF INTEGRATION



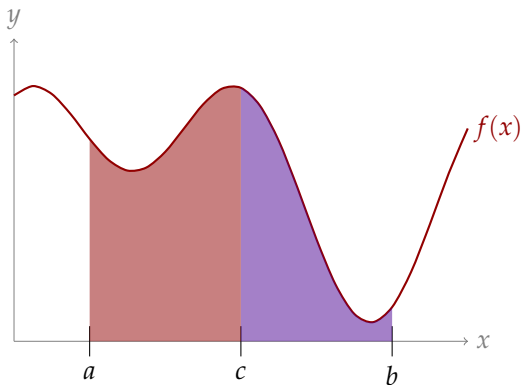
$$\int_a^a f(x) \, dx = 0$$

INTERVAL OF INTEGRATION



What rule do you think is being illustrated?

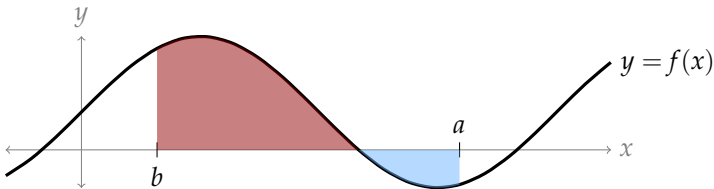
INTERVAL OF INTEGRATION



What rule do you think is being illustrated?

$$\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx$$

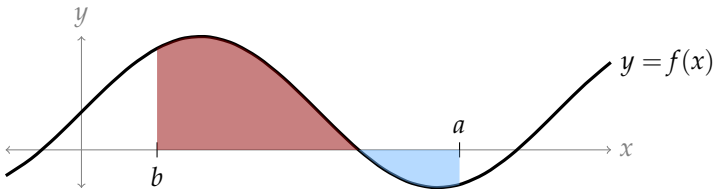
WHAT HAPPENS IN $\int_a^b f(x) \, dx$ WHEN $b < a$?



$$\int_a^b f(x) \, dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_{i,n}^*) \cdot \frac{b-a}{n}$$

This is the definition of a definite integral *whether or not* $a < b$.

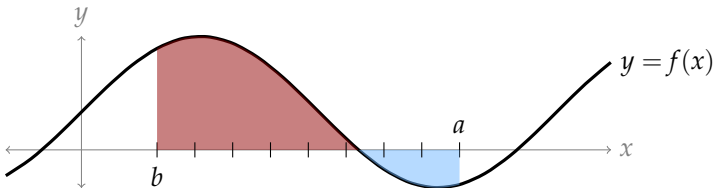
WHAT HAPPENS IN $\int_a^b f(x) \, dx$ WHEN $b < a$?



Choose a number of intervals, n .

$$\int_a^b f(x) \, dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_{i,n}^*) \cdot \frac{b-a}{n}$$

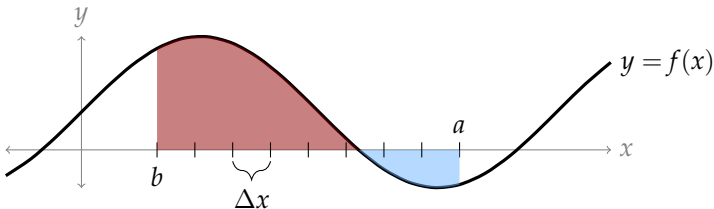
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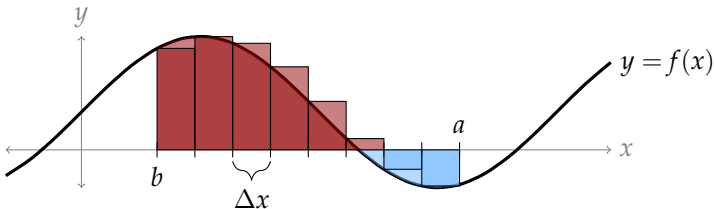


Choose a number of intervals, n .

The (signed) width of each interval is $\Delta x = \frac{b-a}{n}$, which is negative

$$\int_a^b f(x) \, dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_{i,n}^*) \cdot \frac{b-a}{n}$$

WHAT HAPPENS IN $\int_a^b f(x) \, dx$ WHEN $b < a$?



Choose a number of intervals, n .

The (signed) width of each interval is $\Delta x = \frac{b-a}{n}$, which is negative

$$\begin{aligned}\int_a^b f(x) \, dx &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_{i,n}^*) \cdot \frac{b-a}{n} \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_{i,n}^*) \left(-\frac{a-b}{n} \right) = - \int_b^a f(x) \, dx\end{aligned}$$

PROPERTY OF DEFINITE INTEGRALS

$$\int_a^b f(x) \, dx = - \int_b^a f(x) \, dx$$

As strictly a measure of area, not usually a super useful fact – but helps later when we do arithmetic with integrals.

PROPERTY OF DEFINITE INTEGRALS

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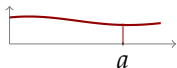
As strictly a measure of area, not usually a super useful fact – but helps later when we do arithmetic with integrals.

It's also useful that the definition works without having to worry about which limit of integration (a or b) is larger.

ARITHMETIC FOR DOMAIN OF INTEGRATION

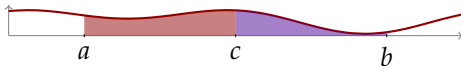
When a , b , and c are constants, and $f(x)$ is integrable over a domain containing all three:

$$(a) \int_a^a f(x) \, dx = 0$$

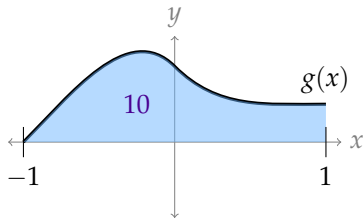
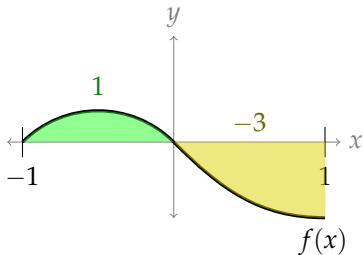


$$(b) \int_a^b f(x) \, dx = - \int_b^a f(x) \, dx \qquad \Delta x = \frac{b-a}{n} = -\frac{a-b}{n}$$

$$(c) \int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx \text{ for constant } c$$

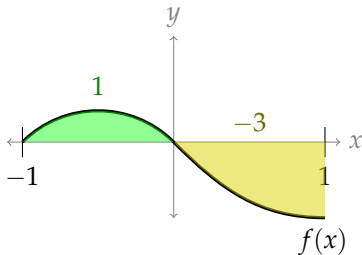


Suppose $\int_{-1}^0 f(x) \, dx = 1$, $\int_0^1 f(x) \, dx = -3$, and $\int_{-1}^1 g(x) \, dx = 10$.



$$\int_{-1}^1 (2f(x) + g(x)) \, dx =$$

Suppose $\int_{-1}^0 f(x) \, dx = 1$ and $\int_0^1 f(x) \, dx = -3$.

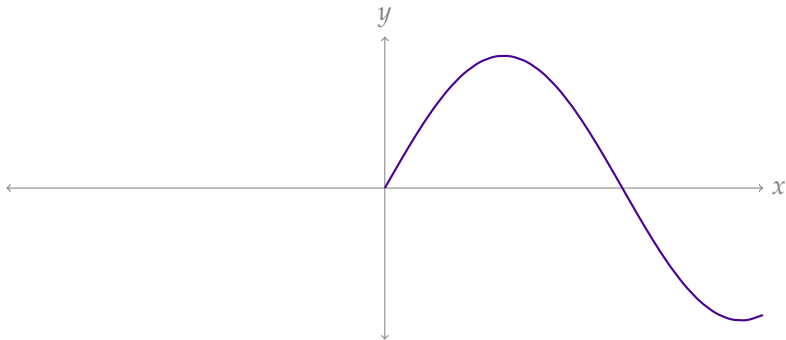


$$\int_{-1}^3 f(x) \, dx + \int_3^0 f(x) \, dx =$$

Even and Odd Functions

Let $f(x)$ be a function.

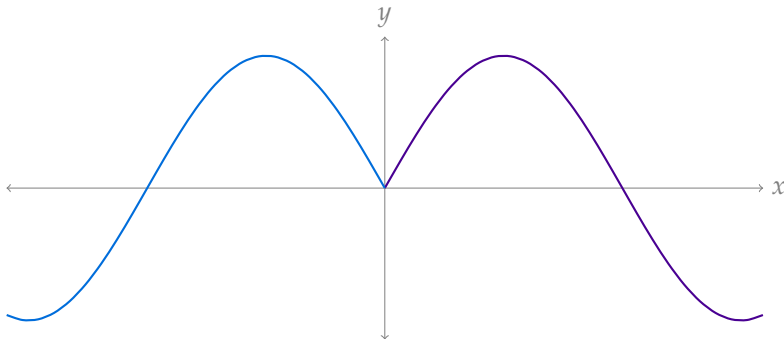
- ▶ We say $f(x)$ is **even** when $f(x) = f(-x)$ for all x , and
- ▶ we say $f(x)$ is **odd** when $f(x) = -f(-x)$ for all x .



Even and Odd Functions

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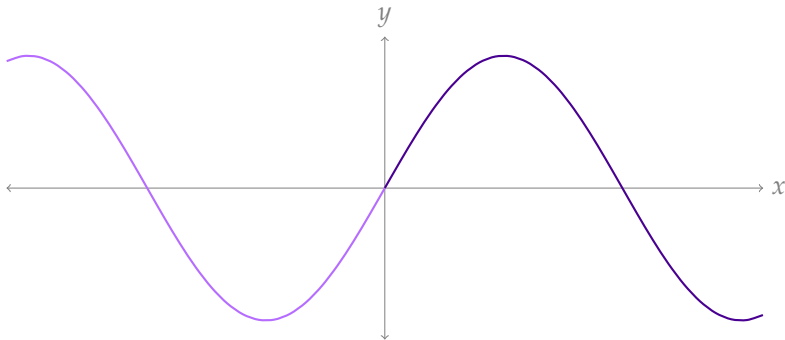
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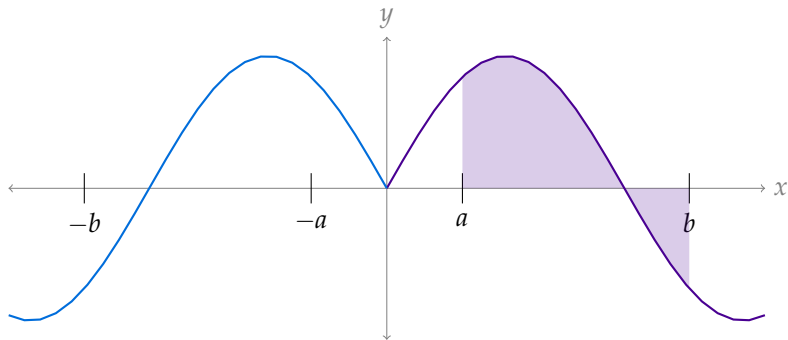
Even and Odd Functions

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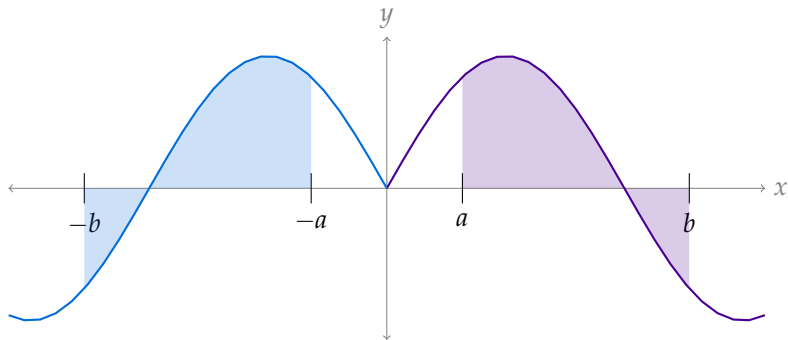
INTEGRALS OF EVEN FUNCTIONS



Suppose $f(x)$ is **even**. Then

$$\int_a^b f(x) \, dx =$$

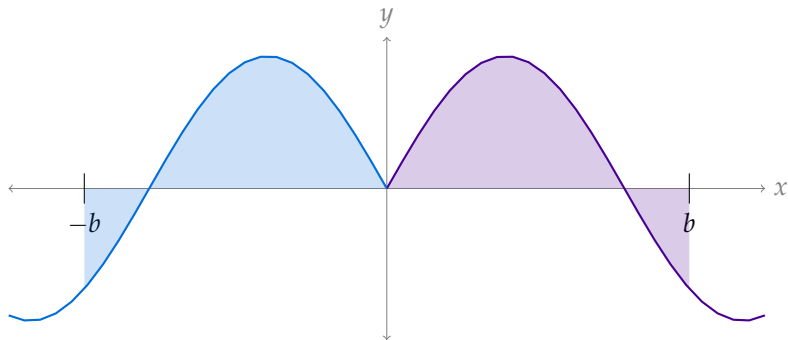
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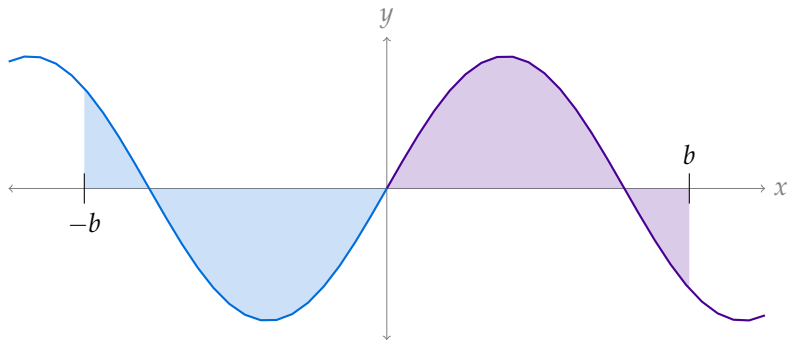
INTEGRALS OF EVEN FUNCTIONS



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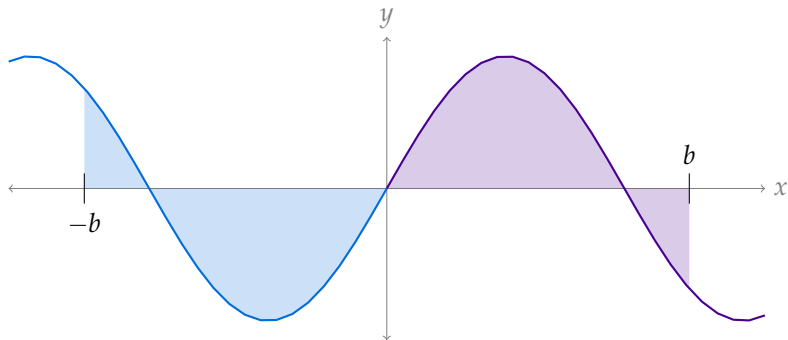
INTEGRALS OF ODD FUNCTIONS



Suppose $f(x)$ is **odd**. Then

$$\int_{-b}^b f(x) \, dx =$$

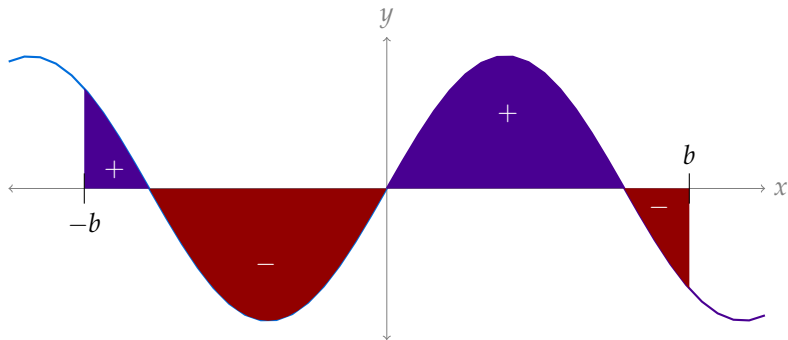
INTEGRALS OF ODD FUNCTIONS



Suppose $f(x)$ is **odd**. Then

$$\int_{-b}^b f(x) \, dx =$$

INTEGRALS OF ODD FUNCTIONS



Suppose $f(x)$ is odd. Then

$$\int_{-b}^b f(x) \, dx =$$

Theorem 1.2.11 (Even and Odd)

Let $a > 0$.

(a) If $f(x)$ is an **even** function, then

$$\int_{-a}^a f(x) \, dx = 2 \int_0^a f(x) \, dx$$

(b) If $f(x)$ is an **odd** function, then

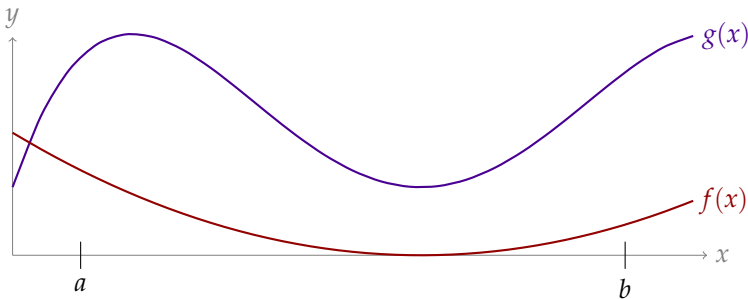
$$\int_{-a}^a f(x) \, dx = 0$$

Integral Inequality

Let $a \leq b$ be real numbers and let the functions $f(x)$ and $g(x)$ be integrable on the interval $a \leq x \leq b$.

If $f(x) \leq g(x)$ for all $a \leq x \leq b$, then

$$\int_a^b f(x) \, dx \leq \int_a^b g(x) \, dx$$

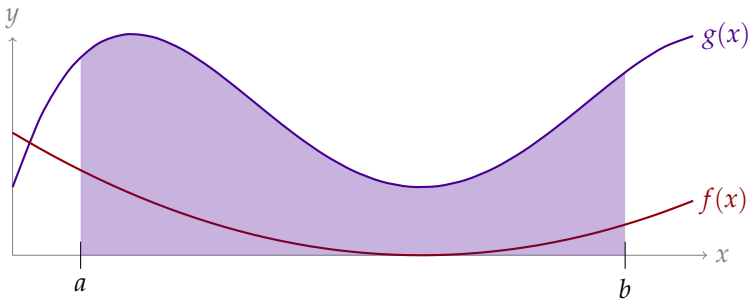


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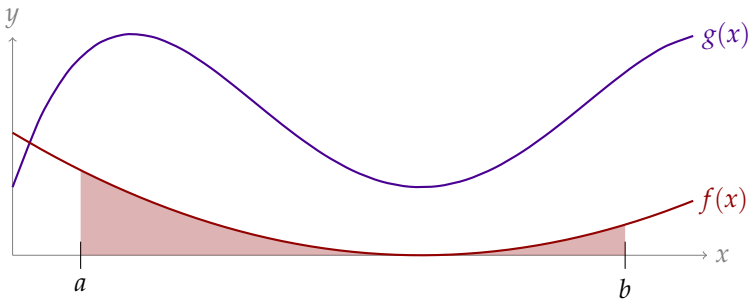


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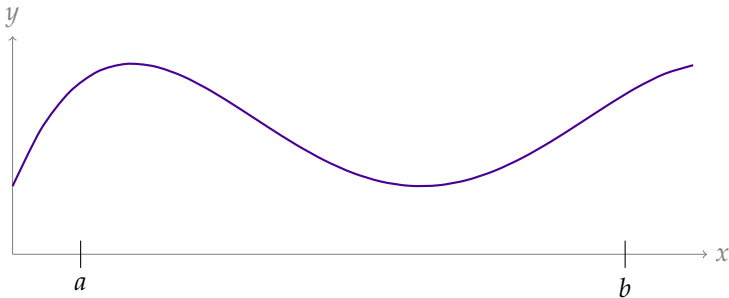


Integral Inequality

Let $a \leq b$ and $m \leq M$ be real numbers and let the function $f(x)$ be integrable on the interval $a \leq x \leq b$.

If $m \leq f(x) \leq M$ for all $a \leq x \leq b$, then

$$m(b-a) \leq \int_a^b f(x) dx \leq M(b-a)$$

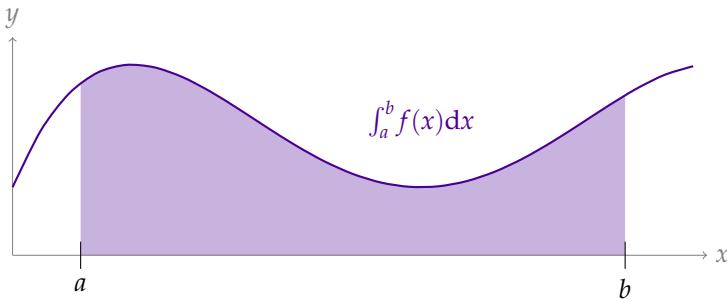


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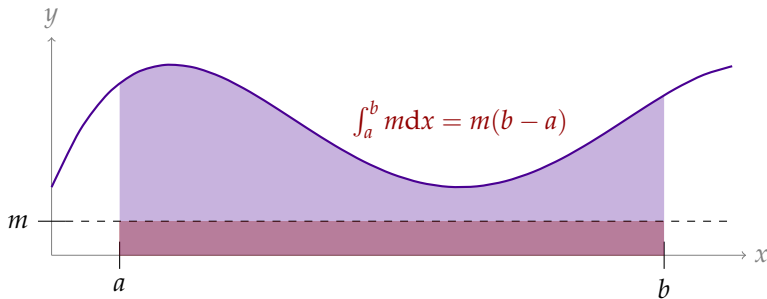


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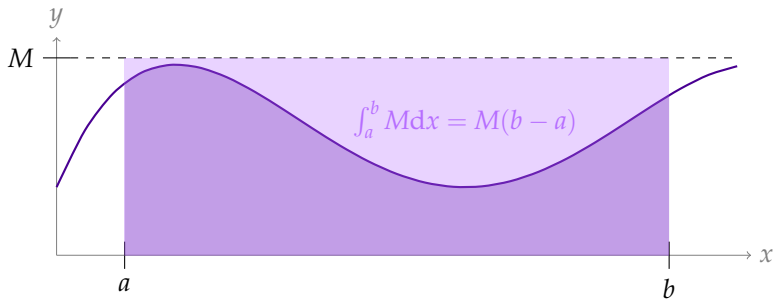


Integral Inequality

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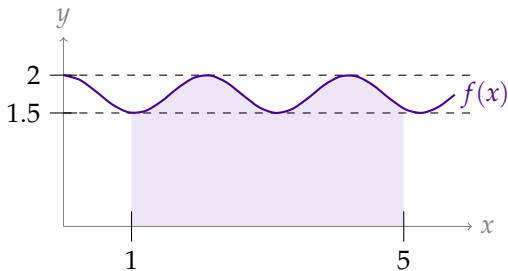
If $m \leq f(x) \leq M$ for all $a \leq x \leq b$, then

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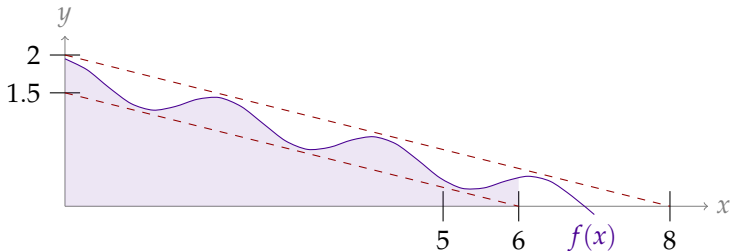
Find a lower bound c and an upper bound d such that

$$c \leq \int_1^5 f(x) \, dx \leq d$$



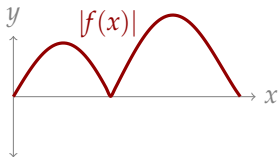
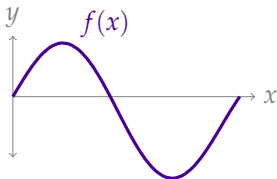
Find a lower bound c and an upper bound d such that $d - c \leq 3$ and

$$c \leq \int_0^6 f(x) \, dx \leq d$$



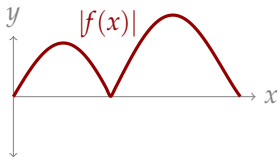
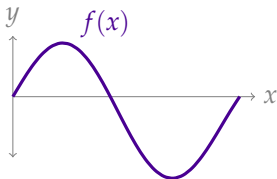
ABSOLUTE VALUES

$$f(x) \leq |f(x)| \text{ for any } f(x)$$



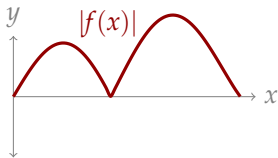
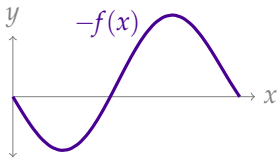
ABSOLUTE VALUES

$$f(x) \leq |f(x)| \text{ for any } f(x)$$
$$-f(x) \leq |f(x)| \text{ for any } f(x)$$



ABSOLUTE VALUES

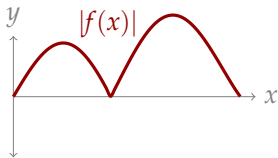
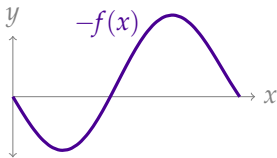
$$f(x) \leq |f(x)| \text{ for any } f(x)$$
$$-f(x) \leq |f(x)| \text{ for any } f(x)$$



ABSOLUTE VALUES

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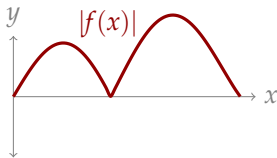
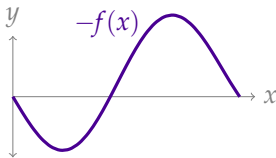
$$-f(x) \leq |f(x)| \text{ for any } f(x)$$



$$\int_a^b f(x) \, dx \leq \int_a^b |f(x)| \, dx \quad \text{and} \quad \int_a^b -f(x) \, dx \leq \int_a^b |f(x)| \, dx$$

ABSOLUTE VALUES

$$\begin{aligned} f(x) &\leq |f(x)| \text{ for any } f(x) \\ -f(x) &\leq |f(x)| \text{ for any } f(x) \end{aligned}$$



$$\int_a^b f(x) \, dx \leq \int_a^b |f(x)| \, dx \quad \text{and} \quad \int_a^b -f(x) \, dx \leq \int_a^b |f(x)| \, dx$$

$$\left| \int_a^b f(x) \, dx \right| \leq \int_a^b |f(x)| \, dx$$

because $\left| \int_a^b f(x) \, dx \right|$ is either $\int_a^b f(x) \, dx$ or $-\int_a^b f(x) \, dx$.

Included Work

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