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## FOUR SERIES

Let  $a_n = \left(-\frac{2}{3}\right)^n$ . Do the following series converge or diverge?

$$\sum_{n=0}^{\infty} a_n$$

$$\sum_{n=0}^{\infty} |a_n|$$

Let  $b_n = \frac{(-1)^n}{n}$ . Do the following series converge or diverge?

$$\sum_{n=1}^{\infty} b_n$$

$$\sum_{n=1}^{\infty} |b_n|$$

The series

$$\sum_{n=0}^{\infty} \left(-\frac{2}{3}\right)^n$$

is called **absolutely convergent**, because the series converges and if we replace the terms being added by their absolute values, that series *still* converges.

The series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n}$$

is called **conditionally convergent**, because the series converges, but if we replace the terms being added by their absolute values, that series *diverges*.

## Absolute and conditional convergence

(a) A series  $\sum_{n=1}^{\infty} a_n$  is said to **converge absolutely** if the series

$$\sum_{n=1}^{\infty} |a_n| \text{ converges.}$$

(b) If  $\sum_{n=1}^{\infty} a_n$  converges but  $\sum_{n=1}^{\infty} |a_n|$  diverges we say that

$$\sum_{n=1}^{\infty} a_n \text{ is **conditionally convergent**.}$$

## Theorem

If the series  $\sum_{n=1}^{\infty} |a_n|$  converges then the series  $\sum_{n=1}^{\infty} a_n$  also converges.

That is, absolute convergence implies convergence.

If $\sum a_n \dots$	and $\sum  a_n  \dots$	then we say $\sum a_n$ is ...

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converges	diverges	conditionally convergent
diverges	diverges	divergent
diverges	converges	not possible!

Does the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

converge or diverge?

Does the series

$$\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$$

converge or diverge?