# Notes for CLP–2 Slides

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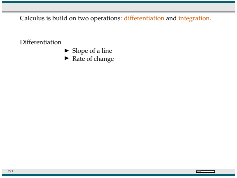
This document contains notes about the mathematical and pedagogical content of selected slides. For more general notes about compiling and editing slides, or interpreting the symbols on slides, please see ReadMe.pdf.

The page numbers in this document refer to the ‘full’ version of each section.

If you use the included python scripts for compiling the slides, page numbers in this document should synchronize with any changes you’ve made. This document depends on the .aux files from the full versions of the individual sections, so if you compile it without using the included python scripts, you must make sure these aux files are present.

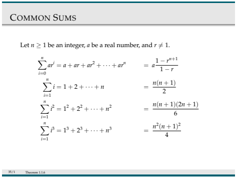



1_1Definition
p 2

Introductory remarks to motivate the upcoming chapter. We start out in a very pure-math way, so it's nice to remind students that applications will come.

Differentiation:
when we first learned it, we used it very literally: slopes of lines, rates of change. Later, it was a tool for more sophisticated operations, like numerical approximations and optimization.

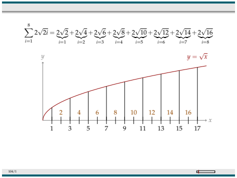
Integration will progress similarly: we'll start by interpreting it quite literally as the area under a curve, then we'll find other uses.




1_1Definition p 35

The formula for a geometric sum will come up again when we do series. The Gaussian sum formula $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ has a proof that is quick and satisfying, but it gives a false hope that the other sum formulas have similarly straightforward justifications.

In both cases, a common anxiety among students is that they might have to come up with such lovely and clever proofs by themselves on the next homework.




1_1Definition
p 104

Students
who are rusty with sigma notation often
really struggle to find the connection
between the Riemann sum and the area,
so I like to go rectangle-by-rectangle.
I think the repetition
helps the interpretation sink in.

We defined the definite integral as

$$\int_a^b f(x)\,\mathrm{d}x = \lim_{N\to\infty}\sum_{i=1}^{N} \Delta x \cdot f\left(x^*_{i,N}\right)$$

where $\Delta x = \frac{b-a}{N}$ and $x^*_{i,N}$ is a point in the interval $[a+(i-1)\Delta x\,,\ a+i\Delta x]$.

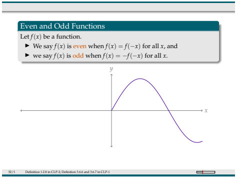
We have seen in previous classes that limits don't always exist. We will verify that this limit does indeed exist, and is equal to the desired area (at least in the most common cases).



`1_1_7_OptionalDefinition`
`p 2`

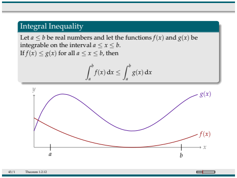
We show in the notes that the limit exists, but we're a little sneaky in assuming that it's equal to the area under the curve. You can get this inequality without assuming integrals give areas:

$$\begin{aligned}
f(x) &\leq g(x)\\
\implies f(x^*_{i,n}) &\leq g(x^*_{i,n})\\
\implies f(x^*_{i,n})\frac{b-a}{n} &\leq g(x^*_{i,n})\frac{b-a}{n}\\
\implies \sum_{i=1}^{n} f(x^*_{i,n})\frac{b-a}{n} &\leq \sum_{i=1}^{n} g(x^*_{i,n})\frac{b-a}{n}\\
\implies \lim_{n\to\infty}\sum_{i=1}^{n} f(x^*_{i,n})\frac{b-a}{n} &\leq \lim_{n\to\infty}\sum_{i=1}^{n} g(x^*_{i,n})\frac{b-a}{n}\\
\implies \int_a^b f(x)\,\mathrm{d}x &\leq \int_a^b g(x)\,\mathrm{d}x
\end{aligned}$$




1_2Properties
p 32

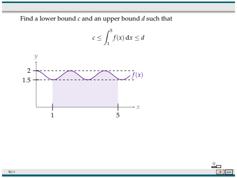
It can be hard, at first,
to visualize reflecting a curve across both axes. Flipping back and forth between slides 2 and 3 can be an easier way of demonstrating odd symmetry.




1_2Properties
p 43

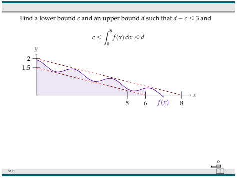
Integral inequalities
are used later in convergence results.

See Theorem 1.12.17, Convergence Tests for Improper Integrals,
and Theorem 3.3.5, The Integral Test.




1_2Properties
p 50

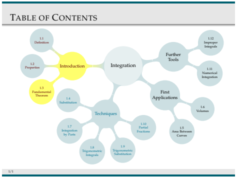
Some students
are uncomfortable with questions
that include subjective judgements. It
can be helpful to explain that we want a
bound that is reasonable, without doing
"too much" work. (It can also be helpful
to explain how such a question could
be asked, and marked, on an exam.)




1_2Properties
p 52

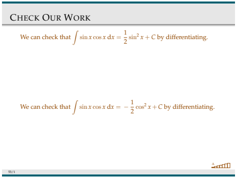
The previous question had a fairly straightforward choice of bounds. For this one, we can do less work for the upper bound by computing the area of a triangle, or slightly more by computing the area of a trapezoid.

The added requirement that $|[c, d]|$ be smaller than some tolerance decreases the ambiguity. (Such phrasing might be useful on an exam question.)



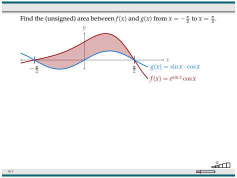

1_3Fundamental Theorem
p 1

When students see the Fundamental Theorem, they often ask what the point of Riemann sums was. It's a nice opportunity to emphasize conceptual understanding. We need a good foundation to understand *why* the difference of antiderivatives (say) leads to the area under the curve.



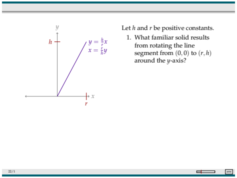

`1_4Substitution`
`p 53`

The brown ’check our work’ frames aren’t *all* meant to be worked through in class. I might do one or two, but then I click through them and verbally remind students how easy it is to check their work on these types of problems.



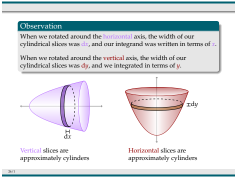

1_5AreaBetweenCurves p 9

In the first example, there was no difference between "integral" and "area," so we could "take away" area in an intuitive way. In this example, we reckon with negative area. By looking at the subdividing rectangles, we can see that the height of the rectangle is given by f-g, regardless of the sign of f and the sign of g.



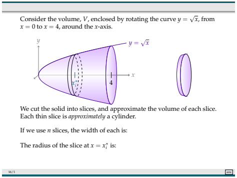

1_6Volumes
p 22

Until the last step, there's not much motivation for giving the radius as $\frac{r}{h}y$, instead of simply $x$. It's OK for students to do it both ways, but once it comes time to evaluate the integral, point out that we don't have a good method for dealing with mixed variables.



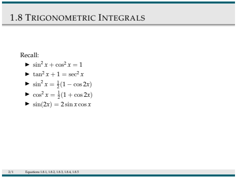

1_6Volumes
p 26

Students usually
ask here what happens if we take the slices in the other way. The slices are still shapes we could probably approximate, it's just more complicated. Later on there are some examples of solids not formed by rotation, so you can let them know we'll see more general examples later.



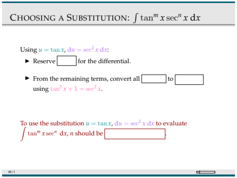

1_6Volumes
p 16

First,
we set up a Riemann sum semi-formally.
Then we show a more informal way.



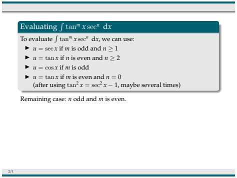

1_8TrigIntegrals
p 2

Nearly every antiderivative has a corresponding "check our work" page. I only do a few in class, but flipping by them is a good opportunity to remind students that they *could* easily check such an answer, if they wanted.



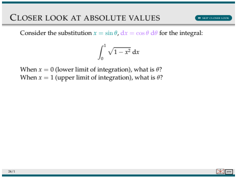

1_8TrigIntegrals
p 88

Since this is so similar to
the others we've done, can leave up the
slide with blanks for students to fill in.




1_8_3Optional TrigIntegrals p 2

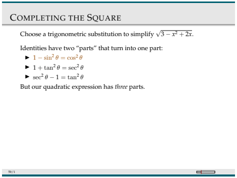
The handout version has the solutions printed on it. I imagine the computations of this section being shown in full and explained at a high-level, rather than expecting students to work through them on their own or really getting into the line-by-line details.




1_9TrigSubstitution p 24

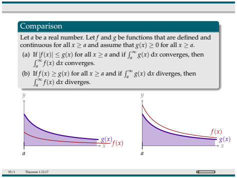
The slides for Section 1.9 are written so that, if you choose, you can tell your students to ignore the absolute values, and always let $\sqrt{\cos^2\theta} = \cos\theta$ (and so on) when doing trigonometric substitutions. Explanations about the absolute value bits are on the upcoming slides, and later on in the various solutions when we use the secant substitution. If you skip the upcoming slides, and work through solutions on your own in class, then students won't see a discussion of the absolute value issues.

The CLP problem book does not include any examples where the two cases of a secant substitution give different answers, and we don't discuss such examples here.



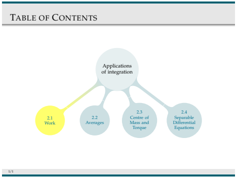

1_9TrigSubstitution
p 59

Completing the square is a technique we'll use again during partial fractions.




1_12Improper Integrals p 93

This is a good time to remind students of the way we deal with improper integrals. We can emphasize the limiting process, as opposed to trying to visualise "infinite area" or "finite area with infinite length." That's what the animations are for. "The area under $f$ from here to here is less than $g$, which is less than some finite number; the area under $f$ from here to *here* is still less than that same finite number," etc.

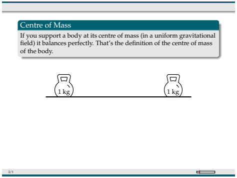



2_1Work
p 1

This section involves some physics. Students will need the following:

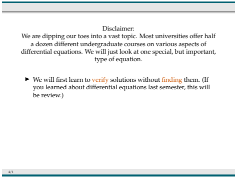
- Force is mass times acceleration.
- If you're lifting something against gravity, acceleration due to gravity is a constant $g$.
- If you want to include units, the units used are newtons and joules (in addition to units that would be familiar to students without a physics background, such as metres).

We've omitted the examples from the text that involve potential and kinetic energy, to minimize assumed prerequisite knowledge.




2_3CentreOfMass
p 2

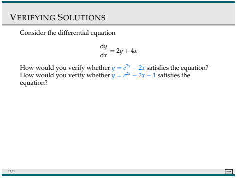
The justification for the formulas we're using for Centre of Mass can be found in the CLP–2 text, optional section 2.3.2




2_4SeparableDiffEq
p 4

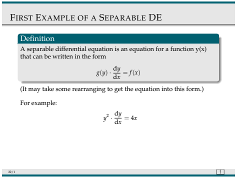
The book dives right in to solving differential equations, but in my experience, students benefit from some orientation first. Many take time to understand how to check that a function actually solves a differential equation. So, in these slides, we take some time to become familiar with DEs *before* we talk about solving them.

A common protest from students with this introduction is that we're checking answers without knowing where those answers came from. With appropriate reassurances that we are going to "get there," and that this is just to get us used to things, I still find this introduction to be well worth the time.



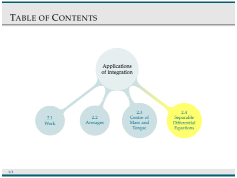

2_4SeparableDiffEq p 12

I find it helpful to emphasize the mechanics of the substitution. Students are often bewildered at first by distinguishing what gets replaced from what doesn't, so I go through each term: "Here's a $y$, so that gets replaced; this is a constant, so it stays; this is just $x$, not $y$, so it stays too." I draw boxes around each instance of $y$ and $\frac{dy}{dx}$ as I go, to make them stand out for the next step, where they're replaced.




`2_4SeparableDiffEq p 22`

Rather than start with the general derivation of the "rule" for solving separable differential equations, we go through a concrete example first. This helps with two things: First, some students have a hard time thinking of $u$-substitution going in this direction. (They did it once before with trigonometric substitution.) Second, it demonstrates that the "rule" (which seems to do some pretty dodgy things) is justified.




Carbon dating, Newon's law of cooling, and population growth are also covered in CLP–1, sections 3.3.1, 3.3.2, and 3.3.3, respectively.

2_4_2-2_4_6OptionalDEApplications
p 1